Universiteit van Amsterdam

Introduction Computational Science

Cellular Automata 4

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1. *ECA rule 184 is known as the 'traffic rule'. Implement it; use periodic boundary conditions. Explain in what way it models bottleneck-free congestion by looking at its state transition table.*

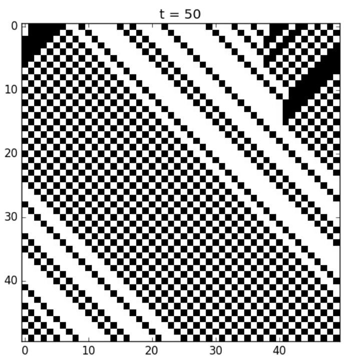
   Rule 184 uses the following transition table;

   Table 1: Transition table of rule 184

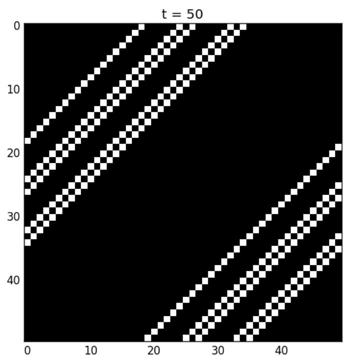
Current	111	110	101	100	011	010	001	000
New	1	0	1	1	1	0	0	0

   If we look at this table we can see that *cars* only move one way. But only if there is no *car* on the right next to him. If the current state is 100 or 101 the *car* from the left wil go one place to the right. The car will stay if the state is *111* or *011* and it will not move in other states.

2. *Show the evolution of a CA of size N = 50 cells for 50 time steps for the 'car' densities 0.4 and 0.9. Describe briefly what you see.*



(a) *Evolution of CA with density of 0.4*



(b) *Evolution of CA with density of 0.9*

As you can see in the figure, when there is any congestion, rule 184 will create room around the *car*. When all the congestion has been solved every car will move one cell at a time. With a density of 0.4 the congestion is solved at t = 15 from then on every car can 'flow' freely. With a density of 0.9 the cars will not reach a state without congestion. Every time step more cars are standing still then are moving.

3. *These are two simulated experiments, or simulations in short, namely the experiment of letting a given density of cars drive on a stretch of road. Name as many advantages as you can think of for simulating these experiments as opposed to using real cars, drivers, and roads*

   (a) No costs for gasoline
   (b) No people needed
   (c) No cars needed
   (d) Variables can be changed very easily
   (e) It is much quicker
   (f) You don't have to block the roads
   (g) There are no measurement errors

4. *Write a function which calculates a 'car flow' value for a given initial state for the CA. We will define it as the number of 1s ('cars') that cross your system boundary on the right-hand side, per unit time. This represents a measurement that we can compute on a (simulated) experiment. Use a sufficiently large number of time steps, denoted T, to measure this reliably, e.g., $T \geq 1000$. Pick any $N \geq 50$.*

   Plot this car flow as function of the initial density of cars, using at least 20 density values in the range [0.0, 1.0]. For each density value you should generate multiple initial states and plot average the car flow for each initial state. Let us denote the number of initial states that you sample and average over by R. Is there a clear phase transition, and if so, at which density value?

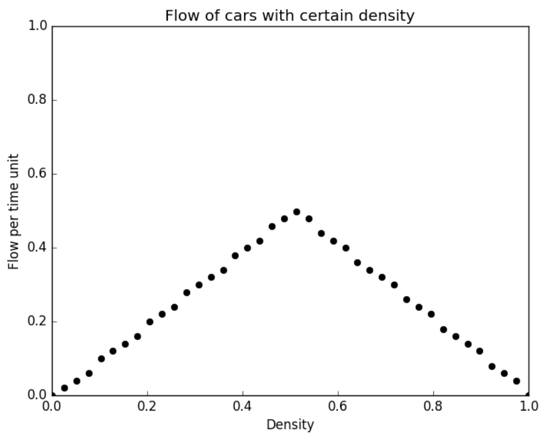


Figure 2: Car flow

As you can see in the figure there is a clear phase transition when the density is 0.5 Before that the flow will rise after it passed a density of 0.5 it will decrease again. This was tested $N = 50$, $T = 1000$, $R = 10$ and 40 different density values. We can clarify this with logic, the ideal situation is when half of the cells have cars in it, they can move without congestion at a maximal rate. If fewer cars they will still drive without congestion but not every cell is occupied. When adding more cars then half of the cells then cars can't move all the time and congestion happens.

5. *Now plot the same graph but for a very low T (e.g., T = 5) and a very low number of initial conditions R per density value (e.g., R = 3). What is the effect of such 'undersampling'? Show a plot with undersampled results.*

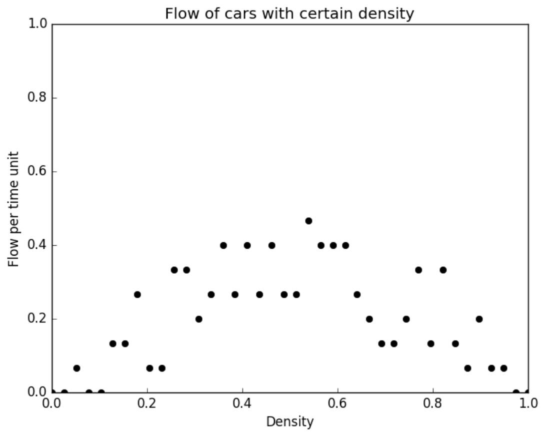


Figure 3: Car flow undersampled

As we can see in the plot we get variation in our results when undersampling (T = 5, R = 3). Some are way to high and some are too low. When taking a low T not all the congestion will be solved so we would get wrong results anyway, besides that we only test it 3 times so we don't get a good average result.

6. *Implement a function which takes the "car flow versus density" data points1 of exercise 4 as input and returns an automatically estimated 'position' (density value) of the phase transition as output (termed 'critical density', a scalar). (Note: the returned value does not have to be equal to one of the input densities, i.e., it may be interpolated.) Mention briefly how you implemented this.*

   After we exported the csv from our previous function. We will import the data and swap the axis. Then we can interpolate this to get a good approximation of the right density at a certain flow.

At which value for T = Tmin do we have at least 90 percent probability of inferring the correct critical density? Estimate this probability by repeating your automatic detection many (at least 10) times for each T. The fraction of 'correct' values is then the probability of inferring the correct critical density. We'll say that a returned critical density value is 'correct' if it is within 0.05 of the real value. For each density value use R = 10 and keep it fixed. Show a plot of "probability correct" as function of T from which it is easy to estimate Tmin by visual inspection.

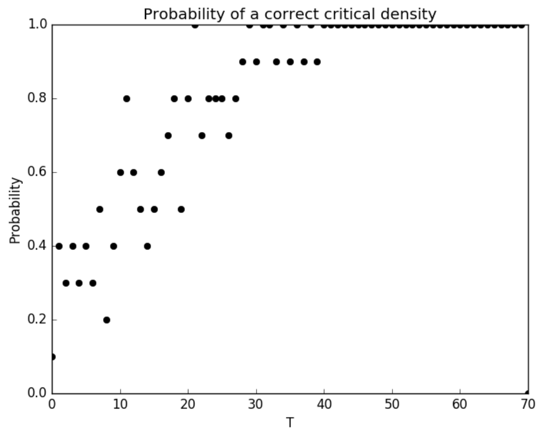


Figure 4: Probability of correct critical density

We can see that T minus is at T = 28.

7. *Now it is time to analyze the simulation results, regarding the phenomenon that we started with. Let us say that our minimal model captures the basic phenomenon very well (namely, the existence of a phase transition), using only minimal ingredients (collision avoidance). What can we conclude about the importance of other possible ingredients, such as the gender of drivers or the sizes of their cars, for explaining the existence of the phase transition? Explain why.*

   In models it is common to keep them simple. Don't make them more difficult then necessary. This principle is called Occam's razor. Of course we can not conclude anything about the other ingredients. We can see that the model is working with this minimalistic implementation and a phase transition exists.