Home Work 2
Com S 435/535

Due: Sep 26, 9:30AM

There are 4 problems. Each problem is worth 40 points.

1. Suppose we have a dice with following probabilities: Probability of 1 is $1/9$, probability of 2 is $1/3$, probability of 3 is $1/9$, probability of 4 is $1/6$, probability of 5 is $1/9$ and probability of 6 is $1/6$. Consider an experiment where we tossed this biased dice n times. Compute the expectation of the following random variables.

   (a) X is the sum of all outcomes

   (b) Y is number times the out come is even

2. Let $S = \{1, \cdots, n\}$. Suppose that you uniformly at random picked a function f from S to S. Let
$$U = \{i \mid i \text{ has exactly one inverse with respect to } f\},$$
   and
$$V = \{m \mid f(m) = m\}$$
   Compute expectations of $|U|$ and $|V|$.

3. Let S be a set of size N, and $T = \{1, \cdots N^2\}$. Let h be a random function from S to T, and let
$$C_h = \{\langle x, y \rangle \mid h(x) = h(y)\}$$
   In lectures we showed that $E[|C_h|] \leq 1/2$. Compute $Var(|C_h|)$. Use Chebyshev inequality to obtain an upper bound on the following
$$\Pr[|C_h| \geq 1]$$
   You may use the following fact in your proofs. if $X_1, X_2, \cdots X_n$ be a sequence of n random variables such that for every $i \neq j$ X_i and X_j are independent. Then
$$Var(X_1 + X_2 + \cdots + X_n) = Var(X_1) + Var(X_2) + \cdots Var(X_n)$$

4. Suppose that we roll a balanced dice n times. What is the probability that the sum of all outcomes is atmost $2n$. Give an upper bound on this probability by using each of Markov, Chebyshev and Chernoff bounds.

5. Suppose that you have a biased coin with unknown bias. I.e., When you toss this coin, the outcome is head with probability p and you do not know the value of p. Your goal is estimate the value of p. You do the following experiment: Toss the coin 2000 times and let X be number of times head appears. Output $X/2000$ as an estimate for p. How good is this estimate? Show the following:

$$\Pr[p - 0.1 \leq X/2000 \leq p + 0.1] \geq 1 - \frac{1}{1000}$$

Guidelines:

- You are allowed to discuss with your classmates, while doing your homework. However, I strongly suggest that you think about the problems on your own before discussing.

- Definition of *classmates*: Students who are taking CS 435X/535X in Spring 17.

- However, You should write the final solutions alone, without consulting your classmates. Your writing should demonstrate that you understand the proofs completely. If we suspect that you wrote the proofs without understanding, we may ask you to explain the proofs in person. In such scenarios, failure to explain proofs will be taken as evidence of *academic dishonesty*.

- For each problem, you should acknowledge the students with whom you discussed. This will not affect your grade. Failure to acknowledge is considered *academic dishonesty*, and it will affect your grade.

- Any student found guilty of academic dishonesty will receive "F" in the. course.

- When proofs are required, make them both clear and rigorous. Do not hand wave. Even when proofs are not required, you should justify your answers and explain your work.

- Late homeworks are not accepted.