Home Work 1
Com S 435/535

Due Sep 12, 9:30AM

When asked to calculate probabilities of events, your solutions must be mathematically rigorous and you must write all the steps of your derivation. You will not receive full credit for merely stating the probability and/or providing an intuitive or high level argument as why your solution is correct.

1. For each of the following experiments, write the sample space and the probability function. (50 Points)

   (a) Toss a biased coin (probability of head 1/3) n times.

   (b) Toss a fair coin. If the outcome is head, then throw a balanced dice; otherwise toss a fair coin.

   (c) Let $S = \{1, 2, \cdots, n\}$. Let $T = \emptyset$.
   For every $i \in S$ {
   Toss a fair coin.
   If the outcome is Head, $T = T\{i\}$
   }
   Output T.

   (d) Toss a balance dice twice. Output the following 3-tuple $\langle a_1, a_2, a_1 * a_2 \rangle$: Where a_1 is the outcome of the first dice, and a_2 is the outcome of the second dice.

   (e) Let $S = \{1, 2, \cdots, n\}$. Uniformly at random pick x from $\{1, 2, \cdots, n\}$. Uniformly at random pick y from $\{1, 2, \cdots, n\} - \{x\}$. Output $\langle x, y \rangle$.

2. Consider the experiment from 1e. Let m be a number between 1 and n. Calculate the probability that one of the numbers picked is m. (20 Points)

3. Let $S = \{1, 2, \cdots, n\}$. Consider the following experiment uniformly at random pick x from S; uniformly at random pick y from S. Show that the probability that x equals y is $1/n$. (20 Points)

4. We say that a binary sequence $a = a_1 a_2 \cdots a_n$ dominates a binary sequence $b = b_1 b_2 \cdots b_n$ if the following holds.

   $$\forall i \quad b_i = 1 \Rightarrow a_i = 1$$

   Consider the following experiment: Toss a fair coin n times to obtain a binary sequence a (view head as 1 and tail as 0). Again Toss a fair coin n times to obtain a binary sequence b. Calculate the exact probability that a dominates b. (50 Points)

5. Suppose you threw two balanced dice. Consider following events.

   (a) E_1: Outcome of the first dice is 3.

   (b) E_2: Sum of the outcomes is 7

   (c) E_3: Sum of the outcomes is 9.

   Answer the following questions. (30 Points)

   (a) What are probabilities of E_1, E_2, and E_3.

   (b) Are E_1 and E_2 independent? Prove your answer.

   (c) Are E_1 and E_3 independent? Prove your answer.

Guidelines:

- You are allowed to discuss with your classmates, while doing your homework. However, I strongly suggest that you think about the problems on your own before discussing.
- Definition of *classmates*: Students who are taking CS 435/535 in Fall 17.
- However, You should write the final solutions alone, without consulting your classmates. Your writing should demonstrate that you understand the proofs completely. If we suspect that you wrote the proofs without understanding, we may ask you to explain the proofs in person. In such scenarios, failure to explain proofs will be taken as evidence of *academic dishonesty*.
- For each problem, you should acknowledge the students with whom you discussed. This will not affect your grade. Failure to acknowledge is considered *academic dishonesty*, and it will affect your grade.
- Any student found guilty of academic dishonesty will receive "F" in the. course.
- When proofs are required, make them both clear and rigorous. Do not hand wave. Even when proofs are not required, you should justify your answers and explain your work.
- Late homeworks are not accepted.