Homework 1 Solution

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Large Data ComS 535/435

September 25, 2017

1 Task 1

1.1 (a)

$\Omega = \{0, 1\}^n$ where a 0 denotes a head and a 1 denotes a tail.

The probability of a sample $s \in \Omega$ is (l denotes the number of heads):

$$P(s) = (\frac{1}{3})^l * (\frac{2}{3})^{n-l}$$

1.2 (b)

$\Omega = \{H1, H2, H3, H4, H4, H5, H6, TH, TT\}$.

$$P(xy) = \begin{cases} \frac{1}{12} & x = H \\ \frac{1}{4} & x = T \end{cases}$$

1.3 (c)

$\Omega = \{U | U \subseteq S\}$.

$$P(s) = \frac{1}{2^n}$$

1.4 (d)

$\Omega = \{< i, j, k > | 1 \leq i \leq 6, 1 \leq j \leq 6, k = i * j\}$.

$$P(s) = \frac{1}{36}$$

1.5 (e)

$\Omega = \{< i, j > | 1 \leq i \leq n, 1 \leq j \leq n, i \neq j\}$.

$$P(s) = \frac{1}{n(n-1)}$$

2 Task 2

Let E denote the event that one number picked is m. Let E_1 denote the event that the first number is m and E_2 denote the event that the second number is m. Since the first number cannot be equal to the second, E_1 and E_2 are disjoint.

$$\begin{aligned} Pr(E) &= Pr(E_1 \bigcup E_2) \\ &= Pr(E_1) + Pr(E_2) \\ &= \tfrac{1}{n} + \tfrac{n-1}{n} * \tfrac{1}{n-1} \\ &= \tfrac{2}{n} \end{aligned}$$

3 Task 3

Let E_i denote the event that $x = i$ and $y = i$. Apparently all E_i are disjoint.

$$\begin{aligned} Pr(E) &= \textstyle\sum_{i=1}^{n} Pr(E_i) \\ &= \textstyle\sum_{i=1}^{n} (Pr(x = i) * Pr(y = i)) \\ &= \textstyle\sum_{i=1}^{n} (\tfrac{1}{n} * \tfrac{1}{n}) \\ &= \tfrac{1}{n} \end{aligned}$$

4 Task 4

Let E_i denote the event that $b_i = 1 \Rightarrow a_i = 0$. Apparently, E_i is independent on other events E_j where $i \neq j$.

It is easy to see that $Pr(E_i) = \frac{3}{4}$ (because the only case fails is $b_i = 0$ and $a_i = 0$).

Based on the definition of domination, a dominates b if and only if all the events E_i hold.

$$\begin{aligned} Pr(E) &= \textstyle\prod_{i=1}^{n} Pr(E_i) \\ &= \textstyle\prod_{i=1}^{n} \tfrac{3}{4} \\ &= (\tfrac{3}{4})^n \end{aligned}$$

5 Task 5

5.1 (a)

Since E_1 contains 6 different samples ($\{< 3, i > | 1 \leq i \leq 6\}$)

$$Pr(E_1) = \frac{1}{6}$$

Since E_2 contains 6 different samples ($\{< i, 7 - i > | 1 \leq i \leq 6\}$)

$$Pr(E_2) = \frac{1}{6}$$

Since E_3 contains 4 different samples ($\{< i, 9 - i > | 3 \leq i \leq 6\}$)

$$Pr(E_2) = \frac{1}{9}$$

5.2 (b)

Yes.

Since $E_1 \bigcap E_2$ contains 1 sample which is $(< 3, 4 >)$, $Pr(E_1 \bigcap E_2) = \frac{1}{36}$.
$Pr(E_1 \bigcap E_2) = \frac{1}{36} = Pr(E_2) * Pr(E_1)$.
Hence these two event are independent.

5.3 (c)

No.

Since $E_1 \bigcap E_3$ contains 1 sample which is $(< 3, 6 >)$, $Pr(E_1 \bigcap E_2) = \frac{1}{36}$.
$Pr(E_1 \bigcap E_2) = \frac{1}{36} \neq \frac{1}{54} = Pr(E_1) * Pr(E_3)$.
Hence these two event are not independent.