Project 1

Course: FYS-STK4155
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Exercise 1.

a) The goal for this part of the exercise is trying to fit a linear model (linear in terms of regression coefficients) to the Franke function given as

$$\begin{aligned} f(x,y) = &\frac{3}{4}\exp\left(-\frac{(9x-2)^2}{4} - \frac{(9y-2)^2}{4}\right) + \frac{3}{4}\exp\left(-\frac{(9x+1)^2}{49} - \frac{(9y+1)}{10}\right) \\ &+ \frac{1}{2}\exp\left(-\frac{(9x-7)^2}{4} - \frac{(9y-3)^2}{4}\right) - \frac{1}{5}\exp\left(-(9x-4)^2 - (9y-7)^2\right) \end{aligned} \tag{1}$$

where polynomial combinations of x and y in the span $x, y \in [0, 1]$ will be the explanatory variables. The linear regression equation then takes the form

$$\widehat{f(x,y)} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\epsilon} \tag{2}$$

where $\widehat{f(x,y)}$ is the least squares estimate of the Franke function, $\mathbf{X}$ is the $n \times p$ design matrix consisting of the aforementioned polynomial combinations of x and y, $\boldsymbol{\beta}$ are the $p \times 1$ regression coefficients and $\boldsymbol{\epsilon}$ is just random noise/unobserved random variables. In order to get the best estimate for the Franke function, we want to choose $\boldsymbol{\beta}$-values so that we minimize the residual sum of squares (thereby the name "least squares"). Solving equation 2 with the intent to minimize the residual sum of squares yields the estimates for the $\boldsymbol{\beta}$-values

$$\hat{\boldsymbol{\beta}} = (\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T y \tag{3}$$

where T marks the transpose of the matrix. We can now get the estimate $\hat{\boldsymbol{\beta}}$ with equation 3 and then compute the estimate of the Franke function as seen in equation 2. Doing so

References

- Reference