In [1]:

```
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
import seaborn as sns
from statsmodels.graphics.regressionplots import influence_plot
import statsmodels.formula.api as smf
import pandas.util.testing as tm
```

```
<ipython-input-1-19fc16e3f5f4>:7: FutureWarning: pandas.util.testing is deprecated. Use the functions in
the public API at pandas.testing instead.
  import pandas.util.testing as tm
```

Importing the dataset

In [2]:

```
df= pd.read_csv('/Users/acer/Sandesh Pal/Data Science Assgn/Multiple REgression/50_Startups.csv')
```

In [3]:

```
df.columns
```

Out[3]:

```
Index(['R&D Spend', 'Administration', 'Marketing Spend', 'State', 'Profit'], dtype='object')
```

In [4]:

```
df.info() #no-null values
```

```
<class 'pandas.core.frame.DataFrame'>
RangeIndex: 50 entries, 0 to 49
Data columns (total 5 columns):
 #   Column           Non-Null Count  Dtype  
---  ------           --------------  -----  
 0   R&D Spend        50 non-null     float64
 1   Administration   50 non-null     float64
 2   Marketing Spend  50 non-null     float64
 3   State            50 non-null     object 
 4   Profit           50 non-null     float64
dtypes: float64(4), object(1)
memory usage: 2.1+ KB
```

In [5]:

```
df['State'].value_counts() #Checking the categories of State variable
```

Out[5]:

```
New York      17
California    17
Florida       16
Name: State, dtype: int64
```

In [6]:

```
#Getting dummy variables
df1= pd.get_dummies(df,columns=['State'])
```

In [7]:

```
df1.head()
```

Out[7]:

	R&D Spend	Administration	Marketing Spend	Profit	State_California	State_Florida	State_New York
0	165349.20	136897.80	471784.10	192261.83	0	0	1
1	162597.70	151377.59	443898.53	191792.06	1	0	0
2	153441.51	101145.55	407934.54	191050.39	0	1	0
3	144372.41	118671.85	383199.62	182901.99	0	0	1
4	142107.34	91391.77	366168.42	166187.94	0	1	0

In [8]:

```
df1.columns #renaming the columns
```

Out[8]:

```
Index(['R&D Spend', 'Administration', 'Marketing Spend', 'Profit',
       'State_California', 'State_Florida', 'State_New York'],
      dtype='object')
```

In [14]:

```
df1 = df1.rename({'R&D Spend':'rnd', 'Administration':'admin', 'Marketing Spend':'ms', 'Profit':'p',
       'State_California':'ca', 'State_Florida':'fl', 'State_New York':'ny'},axis=1)
```

In [15]:

```
df1.columns
```

Out[15]:

```
Index(['rnd', 'admin', 'ms', 'p', 'ca', 'fl', 'ny'], dtype='object')
```

In [16]:

```
df1[df1.duplicated()].shape #checking for duplicates# Building the regression model
```

Out[16]:

```
(0, 7)
```

In [17]:

```
df1.corr() #Correlation of profit with admin is very less
```

Out[17]:

	rnd	admin	ms	p	ca	fl	ny
rnd	1.000000	0.241955	0.724248	0.972900	-0.143165	0.105711	0.039068
admin	0.241955	1.000000	-0.032154	0.200717	-0.015478	0.010493	0.005145
ms	0.724248	-0.032154	1.000000	0.747766	-0.168875	0.205685	-0.033670
p	0.972900	0.200717	0.747766	1.000000	-0.145837	0.116244	0.031368
ca	-0.143165	-0.015478	-0.168875	-0.145837	1.000000	-0.492366	-0.515152
fl	0.105711	0.010493	0.205685	0.116244	-0.492366	1.000000	-0.492366
ny	0.039068	0.005145	-0.033670	0.031368	-0.515152	-0.492366	1.000000

Building the regression model

In [18]:

```
model1 = smf.ols('p~rnd+admin+ms+ca+fl+ny',data=df1).fit()
```

In [19]:

```
model1.summary()
```

Out[19]:

OLS Regression Results

Dep. Variable:	p	**R-squared:**	0.951
Model:	OLS	**Adj. R-squared:**	0.945
Method:	Least Squares	**F-statistic:**	169.9
Date:	Wed, 23 Jun 2021	**Prob (F-statistic):**	1.34e-27
Time:	13:01:38	**Log-Likelihood:**	-525.38
No. Observations:	50	**AIC:**	1063.
Df Residuals:	44	**BIC:**	1074.
Df Model:	5		
Covariance Type:	nonrobust		

	coef	std err	t	P>|t|	[0.025	0.975]
Intercept	3.763e+04	5073.636	7.417	0.000	2.74e+04	4.79e+04
rnd	0.8060	0.046	17.369	0.000	0.712	0.900
admin	-0.0270	0.052	-0.517	0.608	-0.132	0.078
ms	0.0270	0.017	1.574	0.123	-0.008	0.062
ca	1.249e+04	2449.797	5.099	0.000	7554.868	1.74e+04
fl	1.269e+04	2726.700	4.654	0.000	7195.596	1.82e+04
ny	1.245e+04	2486.364	5.007	0.000	7439.285	1.75e+04

Omnibus:	14.782	**Durbin-Watson:**	1.283
Prob(Omnibus):	0.001	**Jarque-Bera (JB):**	21.266
Skew:	-0.948	**Prob(JB):**	2.41e-05
Kurtosis:	5.572	**Cond. No.**	2.63e+21

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
[2] The smallest eigenvalue is 5.63e-31. This might indicate that there are
strong multicollinearity problems or that the design matrix is singular.

As per model 1, admin and ms are not significant, hence checking individually

In [20]:

```
model1a = smf.ols('p~admin',data=df1).fit()
```

In [21]:

```
model1a.summary()
```

Out[21]:

OLS Regression Results

Dep. Variable:	p	**R-squared:**	0.040
Model:	OLS	**Adj. R-squared:**	0.020
Method:	Least Squares	**F-statistic:**	2.015
Date:	Wed, 23 Jun 2021	**Prob (F-statistic):**	0.162
Time:	13:02:59	**Log-Likelihood:**	-599.63
No. Observations:	50	**AIC:**	1203.
Df Residuals:	48	**BIC:**	1207.
Df Model:	1		
Covariance Type:	nonrobust		

	coef	**std err**	**t**	**P>|t|**	**[0.025**	**0.975]**
Intercept	7.697e+04	2.53e+04	3.040	0.004	2.61e+04	1.28e+05
admin	0.2887	0.203	1.419	0.162	-0.120	0.698

Omnibus:	0.126	**Durbin-Watson:**	0.099
Prob(Omnibus):	0.939	**Jarque-Bera (JB):**	0.110
Skew:	0.093	**Prob(JB):**	0.947
Kurtosis:	2.866	**Cond. No.**	5.59e+05

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
[2] The condition number is large, 5.59e+05. This might indicate that there are
strong multicollinearity or other numerical problems.

As per model 1a, admin is not significant.

In [22]:

```
model1b = smf.ols('p~ms',data=df1).fit()
```

In [23]:

```
model1b.summary()
```

Out[23]:

OLS Regression Results

Dep. Variable:	p	**R-squared:**	0.559
Model:	OLS	**Adj. R-squared:**	0.550
Method:	Least Squares	**F-statistic:**	60.88
Date:	Wed, 23 Jun 2021	**Prob (F-statistic):**	4.38e-10
Time:	13:03:48	**Log-Likelihood:**	-580.18
No. Observations:	50	**AIC:**	1164.
Df Residuals:	48	**BIC:**	1168.
Df Model:	1		
Covariance Type:	nonrobust		

	coef	**std err**	**t**	**P>|t|**	**[0.025**	**0.975]**
Intercept	6e+04	7684.530	7.808	0.000	4.46e+04	7.55e+04
ms	0.2465	0.032	7.803	0.000	0.183	0.310

Omnibus:	4.420	**Durbin-Watson:**	1.178
Prob(Omnibus):	0.110	**Jarque-Bera (JB):**	3.882
Skew:	-0.336	**Prob(JB):**	0.144
Kurtosis:	4.188	**Cond. No.**	4.89e+05

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
[2] The condition number is large, 4.89e+05. This might indicate that there are
strong multicollinearity or other numerical problems.

As per model 1b, ms is signoficant

Proceeding for model deletion dignostics now

Cook's distance

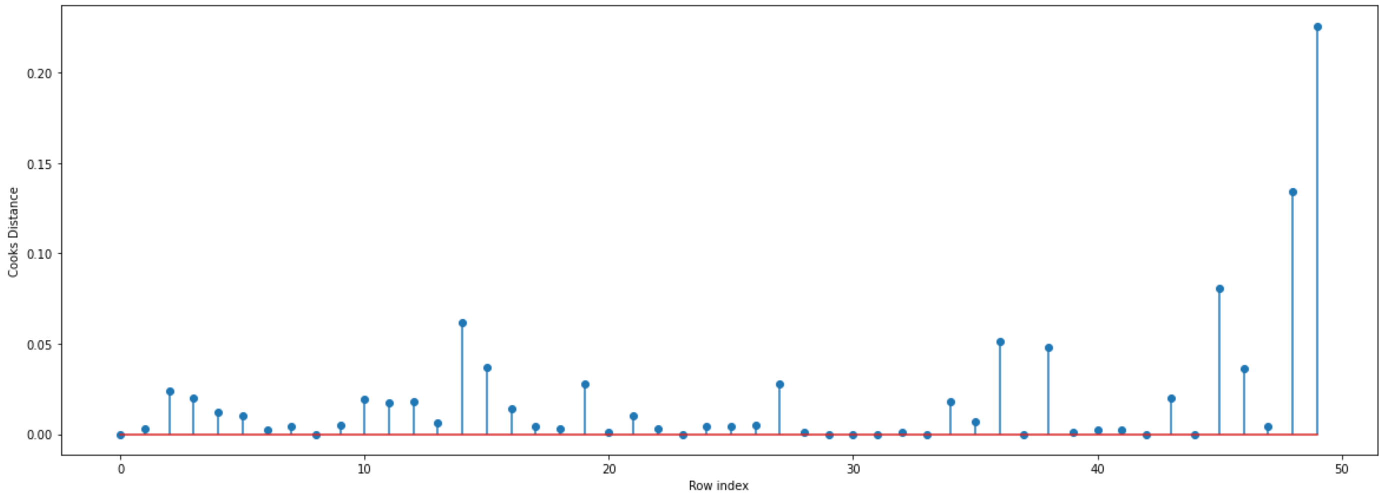
Iteration -1

In [24]:

```
model_influence = model1.get_influence()
(c, _) = model_influence.cooks_distance
```

In [25]:

```
fig = plt.subplots(figsize=(20, 7))
plt.stem(np.arange(len(df1)), np.round(c, 3))
plt.xlabel('Row index')
plt.ylabel('Cooks Distance')
plt.show()
```



In [26]:

```
np.argmax(c),np.max(c)
```

Out[26]:

```
(49, 0.22625094501509324)
```

In [27]:

```
df2 = df1.drop([49],axis=0).reset_index()
```

In [28]:

```
df2 = df2.drop(['index'],axis=1)
```

In [29]:

```
model2 = smf.ols('p~rnd+admin+ms+ca+fl+ny',data=df2).fit()
```

In [30]:

```
model2.summary()
```

Out[30]:

OLS Regression Results

Dep. Variable:	p	**R-squared:**	0.962
Model:	OLS	**Adj. R-squared:**	0.957
Method:	Least Squares	**F-statistic:**	216.6
Date:	Wed, 23 Jun 2021	**Prob (F-statistic):**	2.51e-29
Time:	13:05:49	**Log-Likelihood:**	-505.97
No. Observations:	49	**AIC:**	1024.
Df Residuals:	43	**BIC:**	1035.
Df Model:	5		
Covariance Type:	nonrobust		

	coef	**std err**	**t**	**P>|t|**	**[0.025**	**0.975]**
Intercept	3.915e+04	4249.909	9.213	0.000	3.06e+04	4.77e+04
rnd	0.7836	0.039	20.056	0.000	0.705	0.862
admin	-0.0220	0.044	-0.505	0.616	-0.110	0.066
ms	0.0258	0.014	1.804	0.078	-0.003	0.055
ca	1.422e+04	2081.663	6.833	0.000	1e+04	1.84e+04
fl	1.266e+04	2276.728	5.561	0.000	8068.684	1.73e+04
ny	1.227e+04	2076.431	5.909	0.000	8082.828	1.65e+04

Omnibus:	0.051	**Durbin-Watson:**	1.667
Prob(Omnibus):	0.975	**Jarque-Bera (JB):**	0.207
Skew:	0.061	**Prob(JB):**	0.902
Kurtosis:	2.705	**Cond. No.**	3.47e+21

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
[2] The smallest eigenvalue is 3.21e-31. This might indicate that there are
strong multicollinearity problems or that the design matrix is singular.

Rsquared value has increased, ms and admin are still insignificant

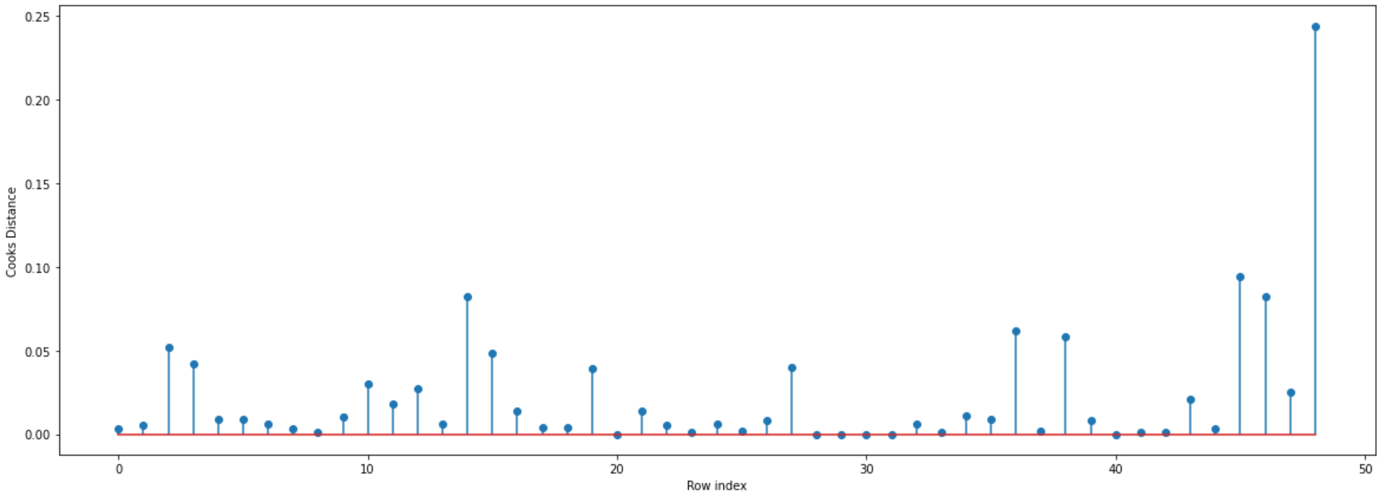
Iteration 2

In [31]:

```
model_influence = model2.get_influence()
(c1, _) = model_influence.cooks_distance
```

In [32]:

```
fig = plt.subplots(figsize=(20, 7))
plt.stem(np.arange(len(df2)), np.round(c1, 3))
plt.xlabel('Row index')
plt.ylabel('Cooks Distance')
plt.show()
```



In [33]:

```
np.argmax(c1),np.max(c1)
```

Out[33]:

```
(48, 0.24352817571403915)
```

In [34]:

```
df3 = df2.drop([48],axis=0).reset_index()
```

In [35]:

```
df3 = df3.drop(['index'],axis=1)
```

In [36]:

```
model3= smf.ols('p~rnd+admin+ms+ca+fl+ny',data=df3).fit()
```

In [37]:

```
model3.summary()
```

Out[37]:

OLS Regression Results

Dep. Variable:	p	**R-squared:**	0.963
Model:	OLS	**Adj. R-squared:**	0.958
Method:	Least Squares	**F-statistic:**	217.6
Date:	Wed, 23 Jun 2021	**Prob (F-statistic):**	7.02e-29
Time:	13:07:39	**Log-Likelihood:**	-493.23
No. Observations:	48	**AIC:**	998.5
Df Residuals:	42	**BIC:**	1010.
Df Model:	5		
Covariance Type:	nonrobust		

	coef	**std err**	**t**	**P>|t|**	**[0.025**	**0.975]**
Intercept	4.409e+04	4569.825	9.648	0.000	3.49e+04	5.33e+04
rnd	0.7888	0.037	21.160	0.000	0.714	0.864
admin	-0.0621	0.045	-1.381	0.174	-0.153	0.029
ms	0.0179	0.014	1.275	0.209	-0.010	0.046
ca	1.53e+04	2035.915	7.516	0.000	1.12e+04	1.94e+04
fl	1.415e+04	2261.128	6.259	0.000	9588.163	1.87e+04
ny	1.464e+04	2223.793	6.581	0.000	1.01e+04	1.91e+04

Omnibus:	0.267	**Durbin-Watson:**	1.835
Prob(Omnibus):	0.875	**Jarque-Bera (JB):**	0.453
Skew:	0.108	**Prob(JB):**	0.797
Kurtosis:	2.576	**Cond. No.**	1.39e+21

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
[2] The smallest eigenvalue is 2.02e-30. This might indicate that there are
strong multicollinearity problems or that the design matrix is singular.

Rsquared value has increased, ms and admin are still insignificant

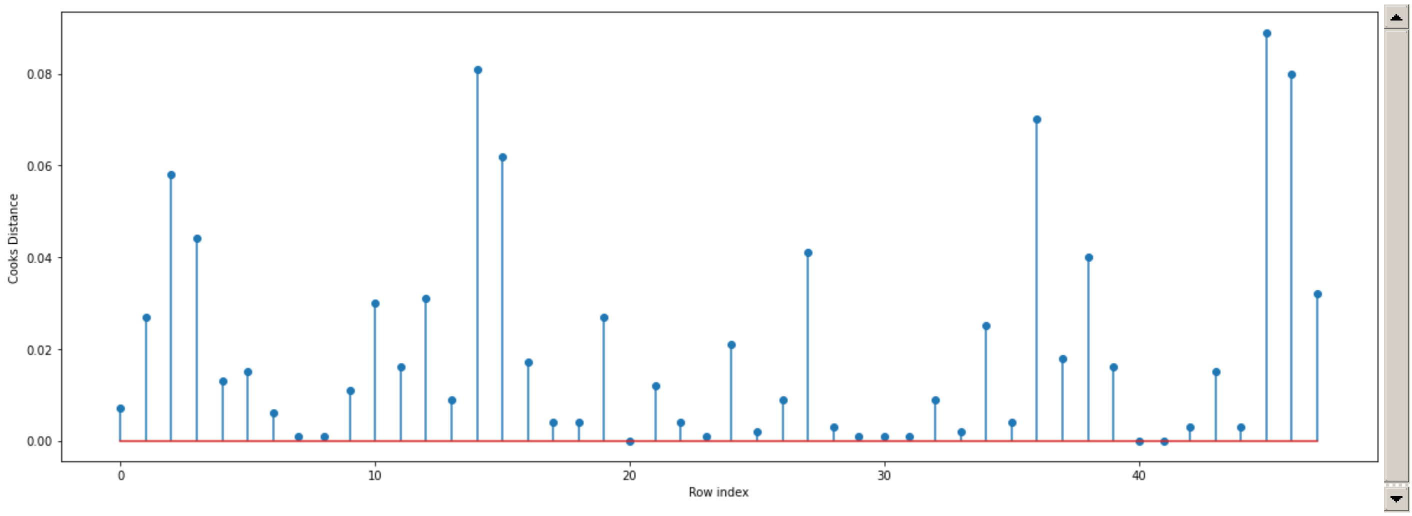
Iteration 3

In [38]:

```
model_influence = model3.get_influence()
(c2, _) = model_influence.cooks_distance
```

In [39]:

```
fig = plt.subplots(figsize=(20, 7))
plt.stem(np.arange(len(df3)), np.round(c2, 3))
plt.xlabel('Row index')
plt.ylabel('Cooks Distance')
plt.show()
```



Since cook's distance of most of the points seems to lie in the same range, hence we can conculde that there are no more influence points.

In [40]:

```
#Checking VIF value to see if there's some collinearity in the variables

rsq_rnd = smf.ols("rnd~admin+ms", data= df3).fit().rsquared
ViF_rnd = 1/(1-rsq_rnd)

rsq_adm = smf.ols("admin~rnd+ms", data=df3).fit().rsquared
ViF_adm = 1/(1-rsq_adm)

rsq_mar = smf.ols("ms~admin+rnd", data= df3).fit().rsquared
ViF_mar = 1/(1-rsq_mar)

d1 = {'Variables':['rnd','admin','ms'],'VIF':[ViF_rnd,ViF_adm,ViF_mar]}
Vif_frame = pd.DataFrame(d1)
Vif_frame
```

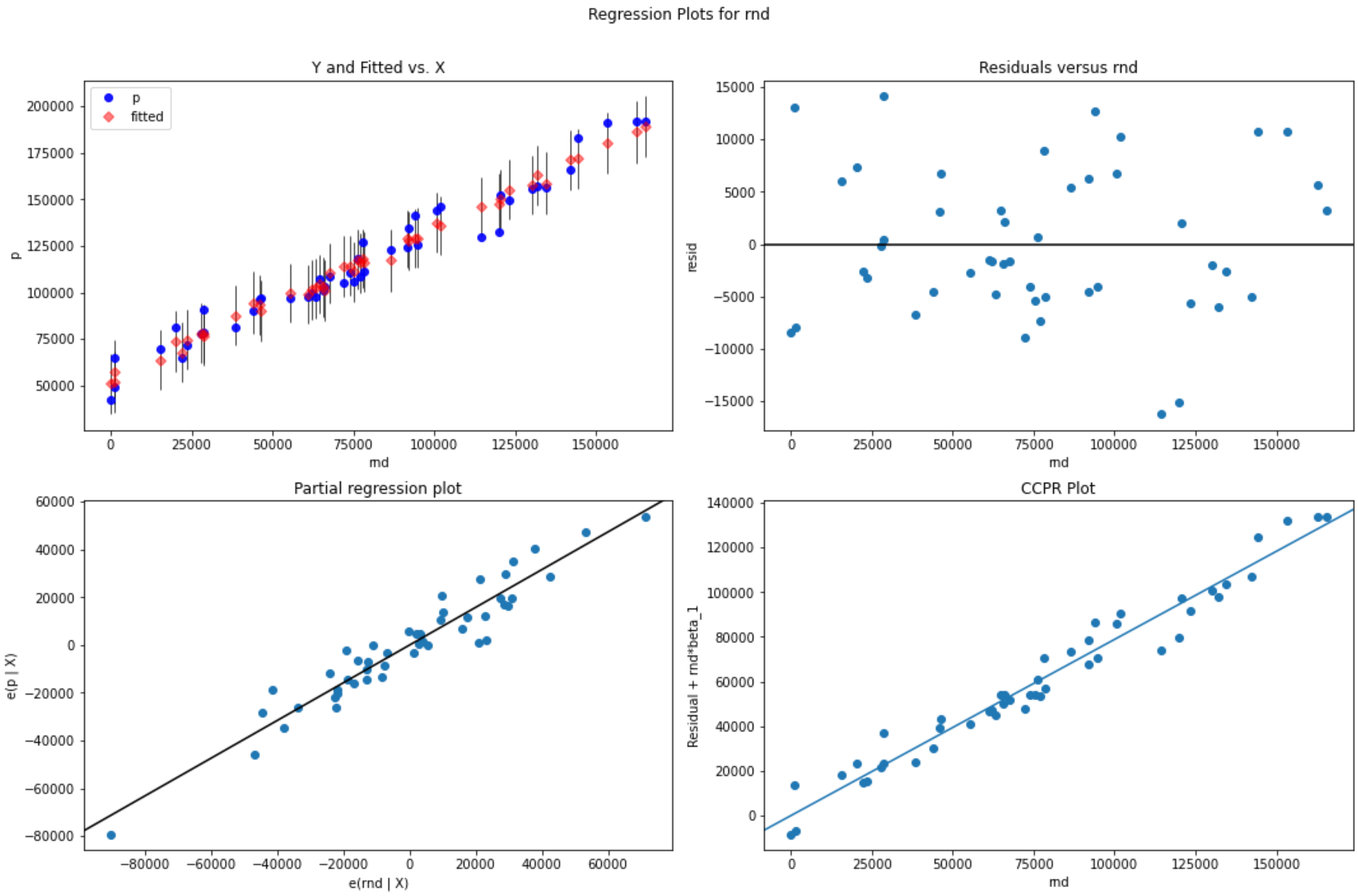
Out[40]:

	Variables	VIF
0	rnd	2.250972
1	admin	1.196016
2	ms	2.229867

Since all VIFs are below 10, hence there's no interdependency among the variables

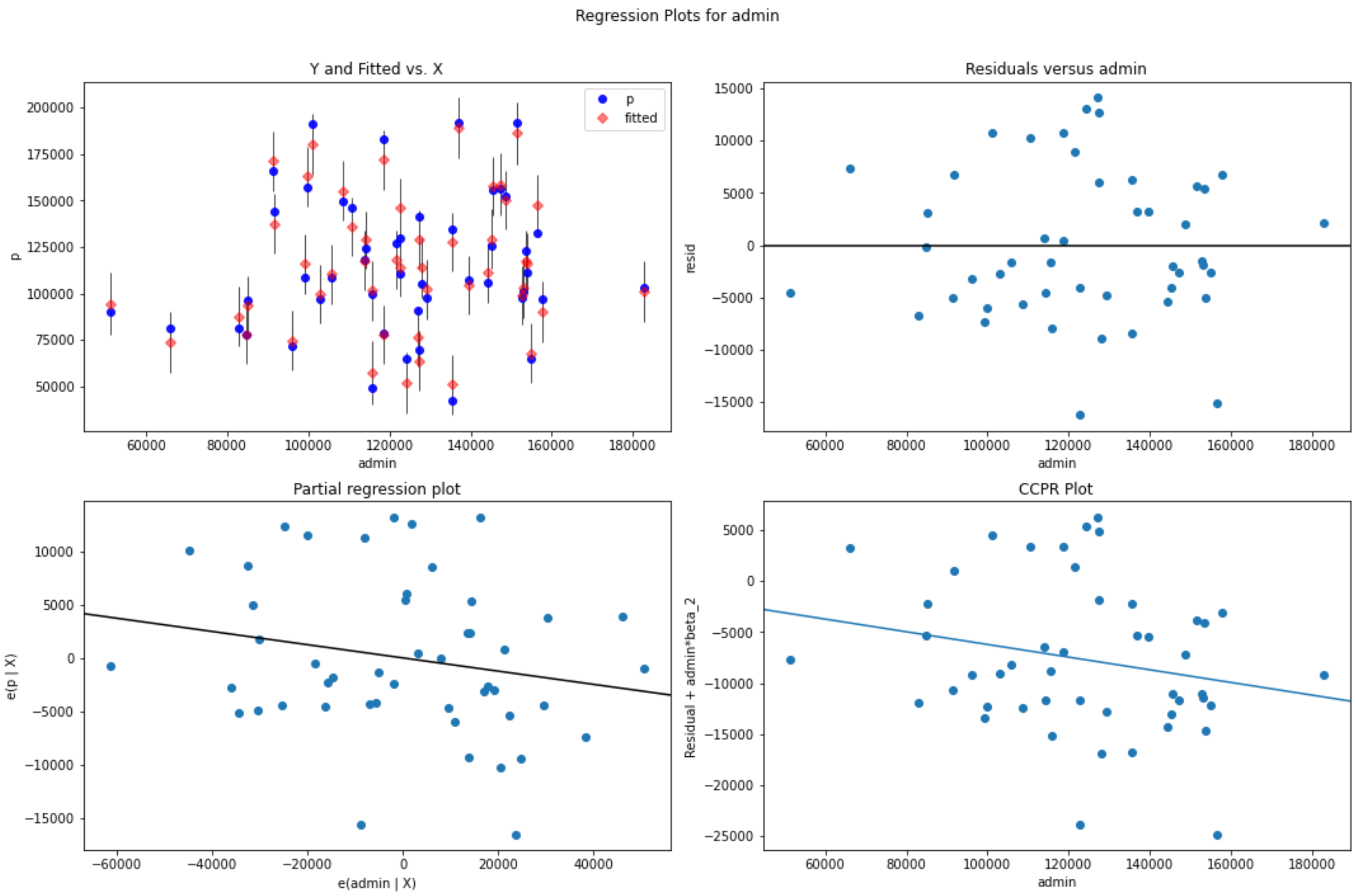
In [41]:

```
import statsmodels.api as sm
fig = plt.figure(figsize=(15,10))
fig = sm.graphics.plot_regress_exog(model3, "rnd", fig=fig)
plt.show()
```



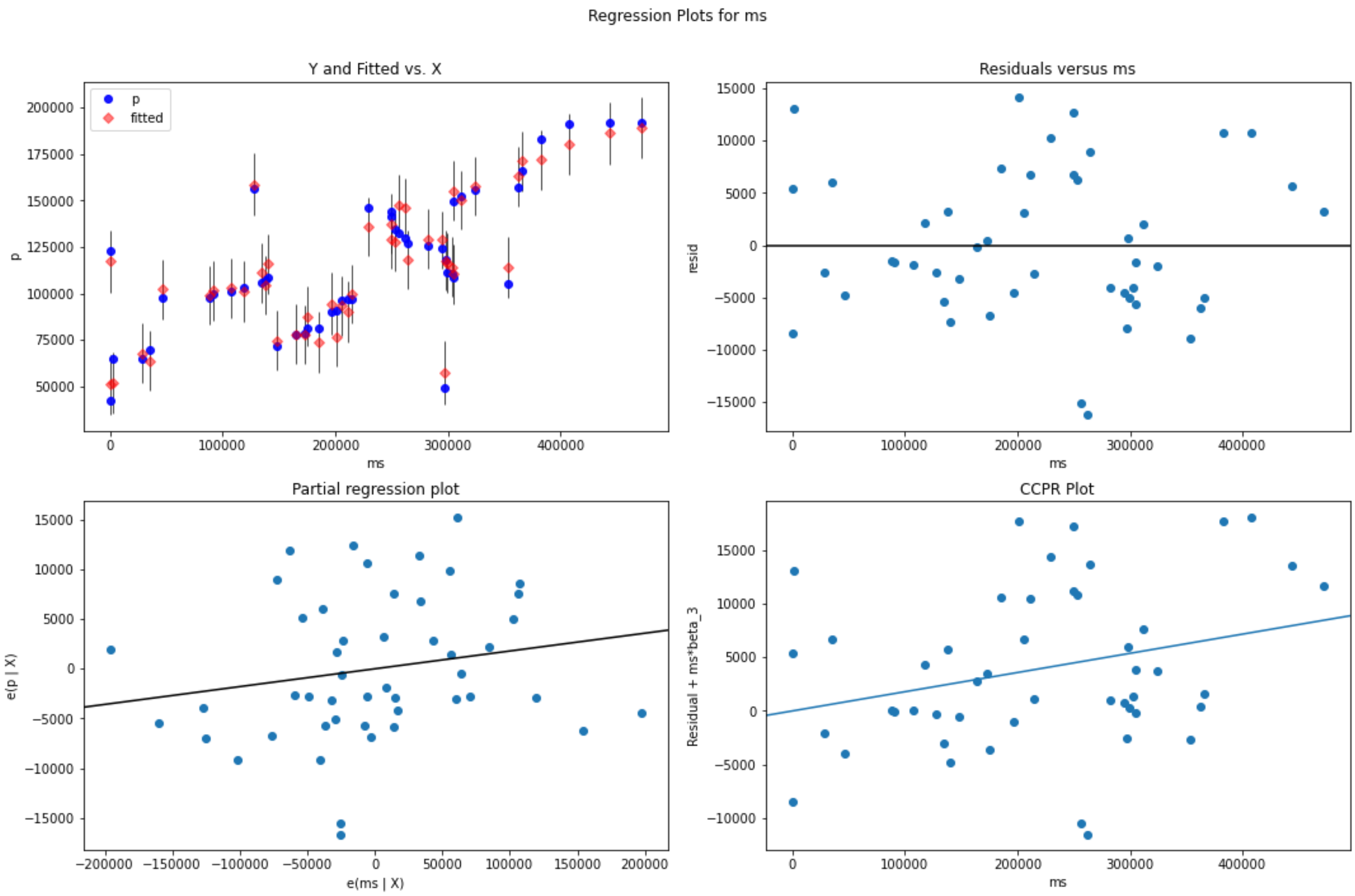
In [42]:

```
fig = plt.figure(figsize=(15,10))
fig = sm.graphics.plot_regress_exog(model3, "admin", fig=fig)
plt.show()
```



In [43]:

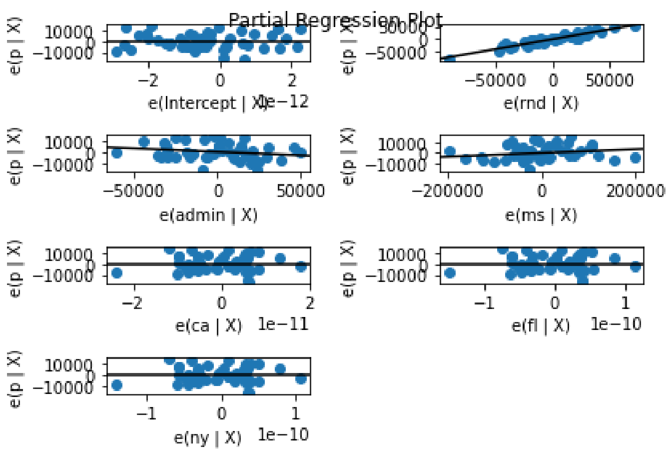
```
fig = plt.figure(figsize=(15,10))
fig = sm.graphics.plot_regress_exog(model3, "ms", fig=fig)
plt.show()
```

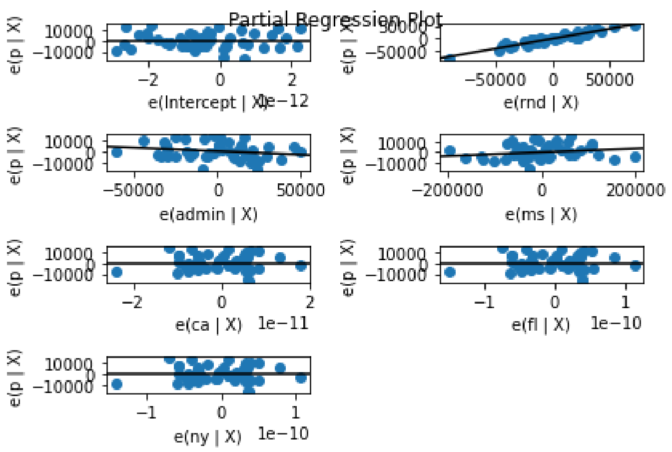


In [44]:

```
sm.graphics.plot_partregress_grid(model3)
```

Out[44]:





Since the correlation of admin with profit is less, and also as per the p-value, it's insignificant, hence we can drop the admin variable

In [45]:

```
final_model= smf.ols('p~rnd+ms+ca+fl+ny',data=df3).fit()
```

In [46]:

```
final_model.summary()
```

Out[46]:

OLS Regression Results

Dep. Variable:	p	**R-squared:**	0.961
Model:	OLS	**Adj. R-squared:**	0.958
Method:	Least Squares	**F-statistic:**	265.9
Date:	Wed, 23 Jun 2021	**Prob (F-statistic):**	1.02e-29
Time:	13:11:32	**Log-Likelihood:**	-494.30
No. Observations:	48	**AIC:**	998.6
Df Residuals:	43	**BIC:**	1008.
Df Model:	4		
Covariance Type:	nonrobust		

	coef	**std err**	**t**	**P>|t|**	**[0.025**	**0.975]**
Intercept	3.83e+04	1841.077	20.803	0.000	3.46e+04	4.2e+04
rnd	0.7692	0.035	22.072	0.000	0.699	0.840
ms	0.0251	0.013	1.908	0.063	-0.001	0.052
ca	1.353e+04	1599.496	8.461	0.000	1.03e+04	1.68e+04
fl	1.216e+04	1761.727	6.904	0.000	8609.549	1.57e+04
ny	1.26e+04	1685.592	7.478	0.000	9204.975	1.6e+04

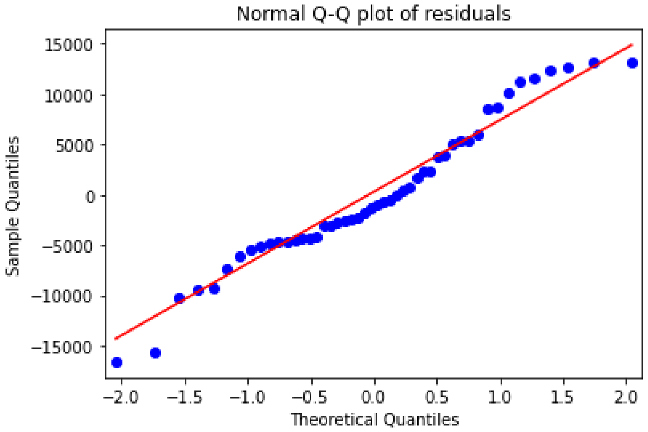
Omnibus:	0.133	**Durbin-Watson:**	1.645
Prob(Omnibus):	0.936	**Jarque-Bera (JB):**	0.304
Skew:	0.097	**Prob(JB):**	0.859
Kurtosis:	2.661	**Cond. No.**	1.41e+21

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
[2] The smallest eigenvalue is 1.66e-30. This might indicate that there are
strong multicollinearity problems or that the design matrix is singular.

Test for Normality of Residuals (Q-Q Plot)

In [47]:

```
qqplot=sm.qqplot(final_model.resid,line='q') # line = 45 to draw the diagnoal line
plt.title("Normal Q-Q plot of residuals")
plt.show()
```



Residual Plot for Homoscedasticity

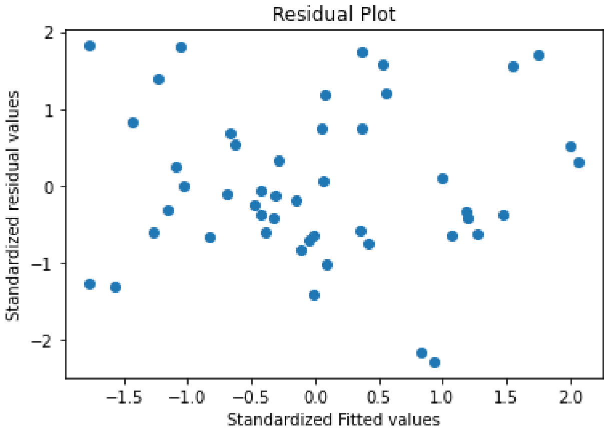
In [48]:

```
#Residual Plot for Homoscedasticity
def get_standardized_values( vals ):
    return (vals - vals.mean())/vals.std()
```

In [49]:

```
plt.scatter(get_standardized_values(final_model.fittedvalues),
            get_standardized_values(final_model.resid))

plt.title('Residual Plot')
plt.xlabel('Standardized Fitted values')
plt.ylabel('Standardized residual values')
plt.show()
```



In [ ]: