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Coordinate Systems MGE-01 WS2526

Exercise 3

December 9th, 2025
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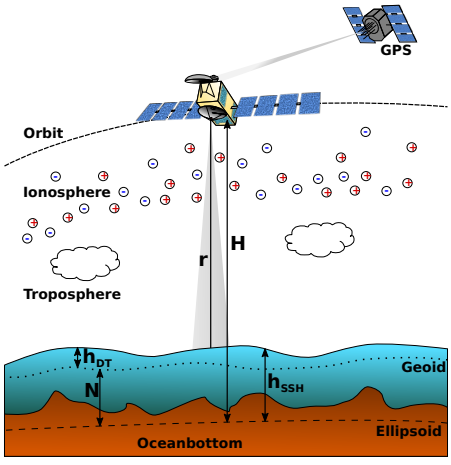
Introduction

Radar altimeter satellites such as Jason-2, -3, Saral/Altika, Cryosat-2, Sentinel-3, etc., measure the range from the spacecraft to the sea surface. The ellipsoidal sea surface height (SSH) is then obtained as the satellite's height above the ellipsoid minus the measured range. Typical measurement accuracy is 2-3 cm, but by averaging many measurements one achieves mm-accuracy and better. The satellites' 3D positions are obtained from precise GNSS tracking. Altimetric SSH can be compared to SSH determined at tide gauges provided they are equipped with GNSS. In radar altimetry it is common to use the Topex/Poseidon (T/P) ellipsoid.



Radar altimetry principle (image credit: Bernd Uebbing)

Task 3.1 (10 Points)

Given are the Cartesian coordinates of a satellite at time t,

$$\boldsymbol{x}_{\text{sat}} = \begin{bmatrix} x_{\text{sat}} \\ y_{\text{sat}} \\ z_{\text{sat}} \end{bmatrix} = \begin{bmatrix} 4\,831\,342.4634\,\text{m} \\ 2\,833\,965.0779\,\text{m} \\ 5\,289\,590.6351\,\text{m} \end{bmatrix}$$

and the ellipsoidal longitude, latitude and height of a nearby (few km) tide gauge station w.r.t. to the Topex/Poseidon ellipsoid

$$\begin{aligned} \lambda_{\text{tg}}^{\text{T/P}} &= 30.329\,000\,100^\circ \\ \varphi_{\text{tg}}^{\text{T/P}} &= 43.592\,000\,088^\circ \\ h_{\text{tg}}^{\text{T/P}} &= 30.888\,\text{m}. \end{aligned}$$

1. Compute ellipsoidal coordinates (longitude $\lambda_{\text{sat}}^{\text{T/P}}$, ellipsoidal latitude $\varphi_{\text{sat}}^{\text{T/P}}$, ellipsoidal height $h_{\text{sat}}^{\text{T/P}}$) of the satellite w.r.t. the T/P ellipsoid. [50%]
2. What would be the difference in height if we were to use the GRS80 ellipsoid instead? [10%]
3. How large is the difference between geodetic/ellipsoidal (T/P ellipsoid) and geocentric/spherical latitude of the satellite ϕ_{sat}? How big is the error expressed in meters, if we were accidentally to compute the geocentric latitude instead of the ellipsoidal one? [20%]
4. How far is the euclidean distance between the altimeter footprint (sub-satellite point on the ellipsoid) and the tide gauge? [20%]

Task 3.2 (10 Points)

The satellite orbit has, at time t, an inclination of $i = 66.036\,006\,500^\circ$, right ascension of ascending node (RAAN) of $\Omega = 335.188\,990\,200^\circ$ and argument of perigee $\omega = 289.450\,123\,600^\circ$. We assume here the orbit is perfectly circular, and the revolution time is $T = 110.0000\,\text{min}$. Range measurements are provided every $\Delta t = 1\,\text{s}$.

1. Compute the ellipsodial coordinates of the satellite footprint for one revolution, i.e. for times $[t, t + \Delta t, t + 2\Delta t, \ldots, t + n\Delta t]$. Hint: you will need to compute the true anomaly for all points in time. [50%]
2. For the whole orbital revolution, generate a global map of the satellites's groundtrack. The plot should contain coastlines and axis labels. [50%]

Formal Regulations

Your solution must include

- step-by-step explanation of the way of solving (what equations are used)
- all intermediate results
- all results must be provided with (the correct) units
- all results must be provided with the relevant number of digits
- all the above must be in a machine-readable format (i.e. not as scanned hand-written text)
- the codes that you used