Institute of Geodesy and Geoinformation für Geodäsie und Geoinformation

University of Bonn

Course: Statistics and Adjustment Theory
Homework Assignment 1 - Random Variables
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1 Rolling Two Dices

Task 1 The total number of elementary events is : 6 * 6 = 36 The elementary events are given below;

$$\Omega = \begin{Bmatrix} (1,1) & (1,2) & (1,3) & (1,4) & (1,5) & (1,6) \\ (2,1) & (2,2) & (2,3) & (2,4) & (2,5) & (2,6) \\ (3,1) & (3,2) & (3,3) & (3,4) & (3,5) & (3,6) \\ (4,1) & (4,2) & (4,3) & (4,4) & (4,5) & (4,6) \\ (5,1) & (5,2) & (5,3) & (5,4) & (5,5) & (5,6) \\ (6,1) & (6,2) & (6,3) & (6,4) & (6,5) & (6,6) \end{Bmatrix}$$

Where, Sample space "Ω" is is the set of all elementary events.

Task 2 Simulation of MATLAB

With the use of the randi function in MATLAB, plotting of absolute and relative frequencies was determined by applying both 10 times and 1000 times.
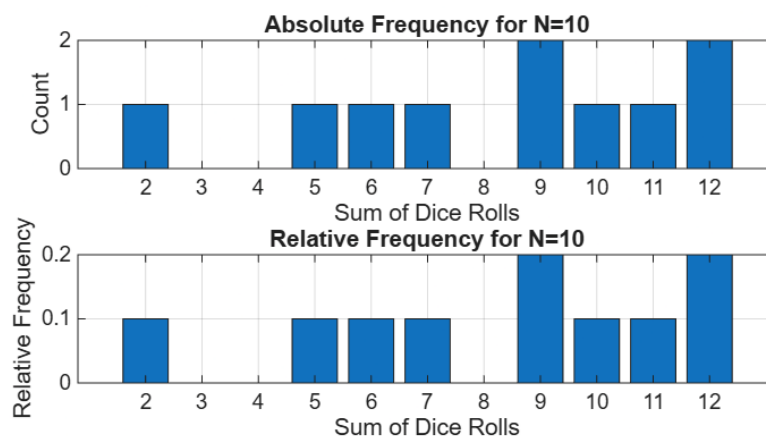
Figure 2 visualizes the observations.

Figure 1: Absolute and Relative Frequency for N=10

Explanation: With the application of the *randi* function, the absolute and relative frequencies were determined on the MATLAB platform. The figure demonstrates that the sum of dice rolls with possibility from 2-12 have the respective frequencies from 0 to 2 in a random experiment.
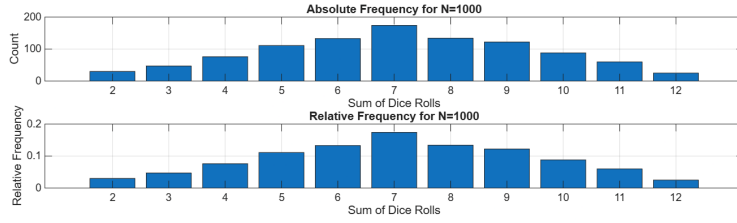
Figure 2: Absolute and Relative Frequency for N=1000

Explanation: With the application of the *randi* function, the absolute and relative frequencies were determined on the MATLAB platform.

Task 3 Probability of the Event

The probability of the elementary event is :

$$Probability = \frac{1}{Total\,Number\,of\,Events} = \frac{1}{36}$$

Task 4 The probability for any event which is a subset of the set of the elementary events is given by:

$$P(A) = \frac{Number\,of\,Elementary\,events\,in\,A}{Total\,Number\,of\,elementary\,events\,in\,\Omega} = \frac{|A|}{|\Omega|} = \frac{|A|}{36}$$

Where event (A) is any subset of the sample space Ω.

Hence, if we consider the event as "Sum of 7" then

$$Probability = \frac{6}{36} = \frac{1}{6}$$

Similarly, if the event is "Sum is even" then,

$$Probability = \frac{18}{36} = \frac{1}{2}$$

2   Probability Density Function

Task 5.   We know, for a function to be Probability Density Function, it must approve 2 conditions. The conditions are:

- Non-negativity: $f_X(x) >= 0$

- Integral over $\mathbb{R}$ is equal to : 1

$$\int_{-\infty}^{\infty} f_X(x)\, dx = 1.$$

Task 5. a.

$$f(x) = \begin{cases} \frac{5}{2x} - x^2 & x \in [0,1] \\ 0 & \text{otherwise} \end{cases}$$

This function is not a Probability Density Function because when $F(0) = \infty$,
It disapproves of the condition of non-negativity.

Task 5. b.

$$f(x) = \begin{cases} x - 2x^2 & x \in [0,1] \\ 0 & \text{otherwise} \end{cases}$$

This function is not a Probability Density Function because when $F(1) = -1$, which disapproves the condition of non-negativity.

Task 5. c.

$$f(x) = \begin{cases} \frac{2}{x^2} & x \in [1,2] \\ 0 & \text{otherwise} \end{cases}$$

This function is a Probability Density Function as it approves the both conditions of PDF.
i.e.

- The range of the f(x) value when $x \in [1,2]$ is always non-negative and in otherwise situation, the value is zero, which satisfies the non-negative condition.

- The integration of the given function is 1, which fulfills the second condition of having Integral over $\mathbb{R}$ is equal to 1.

Task 6. The Cumulative Distribution Function (CDF), $F(x)$ can be defined as the integral of PDF, $f(t)$, from $-\infty$ to x;

$$F(x) = P(X \leq x) = \int_{-\infty}^{x} f(t)d(t)$$

Calculation of $F(x)$ is done across three values given by $f(x)$

Case 1: $x \leq 0$

$$F(x) = \int_{-\infty}^{0} 0d(t) = 0$$

Case 2: $0 < x < 1$

$$F(x) = \int_{-\infty}^{x} f(t)d(t) = \int_{-\infty}^{0} 0d(t) + \int_{0}^{x} 1.dt = 0 + x = x$$

Case 3: $x \geq 1$

$$F(x) = \int_{-\infty}^{x} f(t)d(t) = \int_{-\infty}^{0} f(t)d(t) + \int_{0}^{1} f(t)d(t) + \int_{1}^{x} f(t)d(t)$$

Then,

$$F(x) = \int_{-\infty}^{0} 0d(t) + \int_{0}^{1} 1d(t) + \int_{1}^{x} 0d(t)$$

$$F(x) = 1 for x \geq 1$$

∴ Hence, The CDF is;

$$F(x) = \begin{cases} 0, & x \leq 0 \\ x, & 0 < x < 1 \\ 1, & x \geq 1 \end{cases}$$

Now, the Expectation Value is given by;

$$E[X] = \int_{-\infty}^{\infty} xf(x)\, dx$$

Since $f(x) = 0$ outside $(0, 1)$, we only integrate from 0 to 1:

$$E[X] = \int_0^1 x \cdot 1\, dx = \left[\frac{x^2}{2}\right]_0^1 = \frac{1}{2}$$

Task 7. The random variable x has the following given values; $E[X] = \mu = 120$
Then,

- Expectation value of y1;
  The random variable is defined as;
  $$y1 = x - 15;$$

  $$E[y1] = E[x - 15]$$
  $$E[y1] = E[x] - E[15]$$
  $$E[y1] = 120 - 15$$
  $$E[y1] = 105$$

  The expectation value of y1 is 105.

- Expectation value of y2;
  The random variable is defined as;
  $$y2 = \frac{x}{3} - 12 + y1$$

  Then,
  $$E[y2] = E[\frac{x}{3} - 12 + y1]$$
  $$E[y2] = E[\frac{x}{3}] - E[12] + E[y1]$$
  $$E[y2] = \frac{1}{3}E[x] - 12 + E[y1]$$
  $$E[y2] = \frac{1}{3}120 - 12 + 105$$
  $$E[y2] = 40 - 12 + 105$$
  $$E[y2] = 133$$

  The expectation value of y1 is 133.