A Setting up a Covariance Matrix

You measured 3D coordinates with a GPS device. The standard deviation of a coordinate is given as $\sigma_x = \sigma_y =$ 1cm and $\sigma_z = 3$cm. Further, the correlation between an x, y, and z coordinate is always zero. The correlation between x_i and x_j for $(i \neq j)$ is given as

$$\rho_{x_i,x_j} = \frac{1}{|i-j|} \cdot 0.5$$

similarly, for y_i and $y_j$for $(i \neq j)$, we have

$$\rho_{y_i,y_j} = \frac{1}{|i-j|} \cdot 0.5$$

and for z_i and z_j for $(i \neq j)$, we have

$$\rho_{z_i,z_j} = \frac{1}{|i-j|} \cdot 0.5.$$

1. **(10)** Compute the covariance matrix of the measurements, assuming the observation vector is

$$\begin{bmatrix} x_1 & \dots & x_n & y_1 & \dots & y_n & z_1 & \dots & z_n \end{bmatrix}.$$

   Please report the matrix using the visualization with imagesc and colorbar in Matlab.

B Variance Propagation I

You have measured n points $p_i = (x_i, y_i)$ that lie on a straight line. We model the straight line as follows

$$y = mx + b.$$

The measured points with their covariance information are given as an additional Matlab file `ex02_data.mat`.

2. **(15)** Compute the parameters of the straight line for every pair of points. Please report the parameters and pairs of points for which m is lowest and highest, and for which b is highest and lowest.
3. **(25)** Perform the variance propagation for every straight line. Please report the parameters and pairs of points for which σ_m is lowest and highest, and for which σ_b is highest and lowest.

Please give the formulas and describe your solution.

C Variance Propagation II

You have measured n points $p_i = (x_i, y_i)$. You want to compute the distances between every point i and $i+1$.

$$d_{i,i+1} = \sqrt{(x_i - x_{i+1})^2 + (y_i - y_{i-1})^2}.$$

The measured points with their covariance information are given as an additional Matlab file `ex02_data.mat`.

4. **(20)** Compute the distances between every point i and $i+1$. Please report only the distances for the following pairs: pair $1, 2$, pair $11, 12$, pair $21, 22$, and pair $31, 32$.
5. **(30)** Perform the variance propagation such that you obtain the covariance matrix of all distances. Please report the matrix using the visualization with imagesc and colorbar in Matlab. Further, please report the distance with the lowest and highest standard deviation.

Please give the formulas and describe your solution.
Total: 100

D Variance Propagation III

You have measured n points $p_i = (x_i, y_i)$. Further, you want to compute the angle at point i from the x-axis to point $i+1$.

$$r_{i,i+1} = \text{atan2}\left(\frac{y_{i+1} - y_i}{x_{i+1} - x_i}\right).$$

The measured points with their covariance information are given as an additional Matlab file `ex02_data.mat`.

6. **(20)** Compute the angles at every point i to point $i+1$.
7. **(30)** Perform the variance propagation such that you obtain the covariance matrix of all angles.