UNIVERSITY OF NIAGARA FALLS CANADA

MASTER OF DATA ANALYTICS

A ROBUST OPTIMIZATION MODEL FOR LOGISTICS FACILITY LOCATION PROBLEM UNDER UNCERTAIN DEMAND

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Executive Summary

This report presents a robust optimization approach to solving a logistics facility location problem under uncertain customer demand. Inspired by the research article *"A robust optimization model for logistics facility location problem under uncertain demand"* (Chen et al., 2020), the project reconstructs and extends a mixed-integer linear programming (MILP) model to account for real-world complexities such as demand fluctuations, facility disruptions, and unmet demand penalties. Using Google OR-Tools in Python, we implemented a multi-scenario optimization model that selects optimal facility locations and shipment allocations to minimize the worst-case total cost across three demand scenarios: base demand, peak demand, and facility disruption. The model was extended to allow unmet customer demand at a penalty cost, thereby increasing flexibility and feasibility in high-stress environments.

Synthetic data was generated to simulate realistic supply chain conditions, and sensitivity analysis was performed to assess how variations in demand, capacity, and penalty costs affect the optimal decisions. Results indicate that the extended model delivers resilient, cost-effective decisions under uncertain operating conditions and offers valuable insights for supply chain planners.

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1. Introduction

Logistics facility location problems are among the most critical and complex decisions in supply chain management. They directly influence transportation costs, service levels, operational resilience, and overall profitability. Traditionally, facility location models assume deterministic demand and full demand fulfillment, which can be insufficient in today's volatile and uncertain environments.

Real-world logistics networks often face challenges such as demand surges, infrastructure disruptions, capacity bottlenecks, and budget limitations. In such conditions, relying solely on deterministic optimization can lead to underperforming or infeasible supply chain designs. To address this, robust optimization models have emerged as a powerful alternative, enabling planners to prepare for worst-case scenarios without relying on precise probability distributions.

This project focuses on a robust facility location problem under uncertain demand, based on the MILP model proposed by Chen et al. (2020). The model considers a finite set of demand scenarios and seeks to minimize the maximum cost incurred across all scenarios. To reflect real-world challenges more accurately, we extended the original model by allowing demand shortfalls at a penalty, simulating emergency shipments or lost sales. We also tested the model against realistic stress scenarios such as demand peaks and facility outages.

The goal of this project is to reconstruct, extend, and analyze the robust facility location model using Python and Google OR-Tools, and to evaluate its performance through sensitivity analysis using synthetic data. The outcome provides valuable decision-support insights for logistics and operations managers in high-uncertainty environments.

2. Problem Description

In the context of modern supply chains, facility location decisions must account for not only cost efficiency but also operational resilience under uncertain conditions. The core problem addressed in this project is the **robust optimization of facility locations and shipment allocations in the presence of demand uncertainty and operational disruptions**. The objective is to determine which facilities should be opened and how demand from multiple customer zones should be allocated across different demand scenarios, such that the **worst-case total cost is minimized**.

The baseline problem is adapted from the robust optimization model proposed by Chen et al. (2020), which assumes a finite set of demand scenarios and imposes deterministic constraints on capacity and demand fulfillment. In this study, the problem is extended to reflect more **realistic and challenging logistics settings** that modern supply chain managers face. Specifically, we address two primary operational risks:

1. **Uncertain Demand Volatility**: Customer demand can fluctuate significantly due to seasonal factors, promotional events, or economic shifts. To capture this, we define three demand scenarios:
    - **Base Scenario**: Represents average, expected demand
    - **Peak Demand Scenario**: Represents a 30–40% increase in demand due to promotional or holiday surges
    - **Facility Disruption Scenario**: Represents a supply-side risk, such as the shutdown of a major warehouse
2. **Facility Failure or Capacity Loss**: In the Facility Disruption scenario, one warehouse (F2) is completely shut down, and another (F3) operates at reduced capacity. This simulates realistic events such as equipment failure, labor strikes, or natural disasters.

To enhance the model's flexibility and feasibility under these stress scenarios, we introduced a third dimension to the problem:

- **Unmet Demand with Penalty Costs**: Rather than forcing full demand satisfaction in all scenarios, the model now allows partial fulfillment. Any unmet demand incurs a penalty cost that reflects emergency logistics (e.g., expedited shipping, outsourcing, or lost sales). This extension provides a more **pragmatic trade-off between cost efficiency and service level**.

2.1. Key Decisions to be Made:

- Which facilities should be opened to ensure cost-effective and resilient service across all scenarios?
- How should each customer's demand be allocated to facilities under different scenarios?
- When is it strategically optimal to accept unmet demand and pay a penalty instead of investing in excess capacity?

2.2. Business Relevance:

This problem formulation is highly relevant for:

- Retailers and e-commerce platforms planning distribution center networks
- Logistics firms optimizing regional hub operations
- Manufacturers deciding warehouse placement and product flows
- Emergency response planners modeling supply coverage during disruptions

3. Model Implementation

To solve the robust facility location problem under demand uncertainty and operational risk, we implemented a **Mixed-Integer Linear Programming (MILP)** model using **Google OR-Tools in Python**. The model structure is based on the formulation proposed by Chen et al. (2020), with custom extensions to reflect real-world complexity. The implementation enables selection of optimal facility openings and shipment plans across multiple demand scenarios while minimizing the **worst-case total cost**.

3.1. Model Formulation

Sets:

- F: Set of candidate facilities (e.g., F1, F2, F3)
- C: Set of customer zones (e.g., C1, C2, C3, C4)
- S: Set of demand scenarios (Base, PeakDemand, FacilityDisruption)

Parameters:

- F_i: Fixed cost of opening facility i
- cap_{is}: Available capacity of facility iii in scenario s
- d_{js}: Demand of customer j in scenario s
- c_{ij}: Unit cost of shipping from facility iii to customer j
- p: Penalty cost per unit of unmet demand

Decision Variables:

- $y_i \in \{0,1\}$: 1 if facility i is opened; 0 otherwise
- $x_{ijs} \geq 0$: Quantity shipped from facility i to customer j under scenario s
- $u_{js} \geq 0$: Unmet demand for customer j under scenario s
- z_s : Total cost in scenario s

- z_{max} : Worst-case total cost across all scenarios

3.2. Objective Function

Minimize the worst-case total cost:

$$\text{Min } z_{max}$$

Subject to:

$$z_s = \sum_{i \in F} f_i y_i + \sum_{i \in F} \sum_{j \in C} c_{ij} x_{ijs} + \sum_{j \in C} p \cdot u_{js} \quad \forall s \in S$$

$$z_s \leq z_{max} \quad \forall s \in S$$

Constraints

1. Demand Satisfaction (with Unmet Demand):

$$\sum_{i \in F} x_{ijs} + u_{js} = d_{js} \quad \forall j \in C, s \in S$$

2. Facility Capacity (per Scenario):

$$\sum_{j \in C} x_{ijs} \leq cap_{is} \cdot y_i \quad \forall i \in F, s \in S$$

3. Variable Domains:

$$y_i \in \{0, 1\}, \quad x_{ijs} \geq 0, \quad u_{js} \geq 0$$

3.3. Implementation in Python (Google OR-Tools)

The model was implemented using the Google OR-Tools pywraplp module. Below is a summary of the implementation steps:

1. **Define sets and input data** (fixed costs, capacities, demands, shipping costs)
2. **Initialize the solver** and declare decision variables for:
    - Binary facility decisions y_i
    - Shipment flows x_{ijs}
    - Unmet demands u_{js}
    - Scenario costs z_s , and overall worst-case cost z_{max}
3. **Set up the objective function** using solver.Minimize(z_max)
4. **Add constraints** for:
    - Scenario-specific cost calculation
    - Demand fulfillment per customer and scenario
    - Capacity limits per facility per scenario
5. **Solve the model** and extract outputs
6. **Store results** for comparison and interpretation

Assumptions Made

- All demand values are known for each scenario (robust modeling, not probabilistic)
- Facilities can be either fully operational or shut down in each scenario
- Unmet demand is penalized but allowed to reflect emergency costs or service gaps
- Shipping costs are linear and fixed per unit (no economies of scale assumed)

- The penalty cost is set high enough to discourage neglecting demand, but low enough to allow trade-off decisions

4. Description and Solution of Model Extension

4.1. Motivation for Model Extension

While the original MILP model provided by Chen et al. (2020) successfully minimizes the worst-case total cost in logistics facility location problems under demand uncertainty, it assumes that:

- **All customer demand must be met** in every scenario, and
- **All facilities are equally reliable** across all operational conditions.

However, in **real-world supply chains**, two critical risk factors are often overlooked in such models:

1. **Facility Disruptions**: Warehouses may become temporarily unavailable due to labor shortages, natural disasters, equipment failures, or geopolitical events.
2. **Unmet Demand Penalties**: In peak demand scenarios, companies may choose to partially fulfill orders, accepting penalties (e.g., lost sales, emergency logistics, customer churn) instead of incurring excessive costs to meet all demand.

To address these complexities, we **extended the original model** by incorporating:

- The **option to leave demand unmet** at a financial penalty.
- **Scenario-specific facility disruptions**, such as complete shutdowns or reduced capacity in a given facility.

This makes the model not only **more resilient**, but also **more aligned with managerial decision-making** in uncertain environments.

4.2. Mathematical Formulation of the Extension

The extended model introduces:

- A new **unmet demand variable** $u_{js} \geq 0$ for each customer j in scenario s
- A **penalty cost** p applied to each unit of unmet demand
- Scenario-specific facility capacity cap_{is}, which allows modeling of **partial or full facility shutdowns**

4.3. Updated Objective Function

$$\min z_{max}$$

$$z_s = \sum_{i \in F} f_i y_i + \sum_{i \in F} \sum_{j \in C} c_{ij} x_{ijs} + \sum_{j \in C} p \cdot u_{js} \quad \forall s \in S$$

$$z_s \leq z_{max} \quad \forall s \in S$$

4.4. Updated Constraints

1. **Modified Demand Fulfillment Constraint**:

$$\sum_{i \in F} x_{ijs} + u_{js} = d_{js} \quad \forall j \in C, s \in S$$

This allows the model to **intentionally leave demand unmet** if it reduces the worst-case total cost.

2. **Scenario-Based Facility Capacity Constraint**:

$$\sum_{j \in C} x_{ijs} \leq cap_{is} \cdot y_i \quad \forall i \in F, s \in S$$

Here, capiscap_{is}capis varies per scenario — allowing simulation of **capacity drops or shutdowns**.

3. **Binary and Non-Negativity Constraints** remain unchanged.

4.5. Implementation in Google OR-Tools

The extension was implemented in Python by:

- Defining new unmet demand variables u_{js}
- Adding the unmet demand term to the scenario cost expressions
- Updating the demand satisfaction constraints to include u_{js}
- Modifying capacity parameters dynamically per scenario (e.g., reducing or zeroing facility capacities)

The **penalty cost** was set to a moderate value (e.g., $10/unit) to reflect real-world cost implications, allowing the model to **tradeoff** between high-cost fulfillment and unmet demand.

4.6. Results After Extension

Using this model extension:

- The model remains flexible under **facility failure conditions**
- It can **absorb demand shocks** during high-demand periods by selectively choosing unmet demand
- **Managerial trade-offs** are clearer: whether to open more facilities or allow unmet demand

In some test scenarios where capacity was heavily reduced (e.g., F3's capacity cut from 550 to 400 units), the model **activated unmet demand penalties** or recommended **reopening facilities** like F1 or F2 to reduce total cost.

4.7. Practical Implications

This extension is highly relevant for companies dealing with:

- **Emergency logistics and service guarantees**
- **Budget constraints during temporary capacity outages**
- **Trade-offs between customer satisfaction and cost efficiency**

It allows **supply chain planners** to better prepare for uncertainty and avoid infeasible or overly expensive solutions in real-world environments.

5. Sensitivity Analysis and Results

5.1. Objective of Sensitivity Analysis

Sensitivity analysis evaluates how changes in key input parameters affect the optimal decisions and objective value of the optimization model. This step is crucial for assessing the **robustness** and **practicality** of the model under varying operational conditions, especially when dealing with **uncertain demand**, **capacity limits**, and **penalty costs** for unmet demand.

The aim is to identify:

- How sensitive the **worst-case total cost** is to changes in demand, capacity, and penalty rates
- When the model shifts from **full demand fulfillment** to **selective unmet demand**

- The **trigger points** at which additional facilities are opened or costs escalate

5.2. Key Parameters Tested

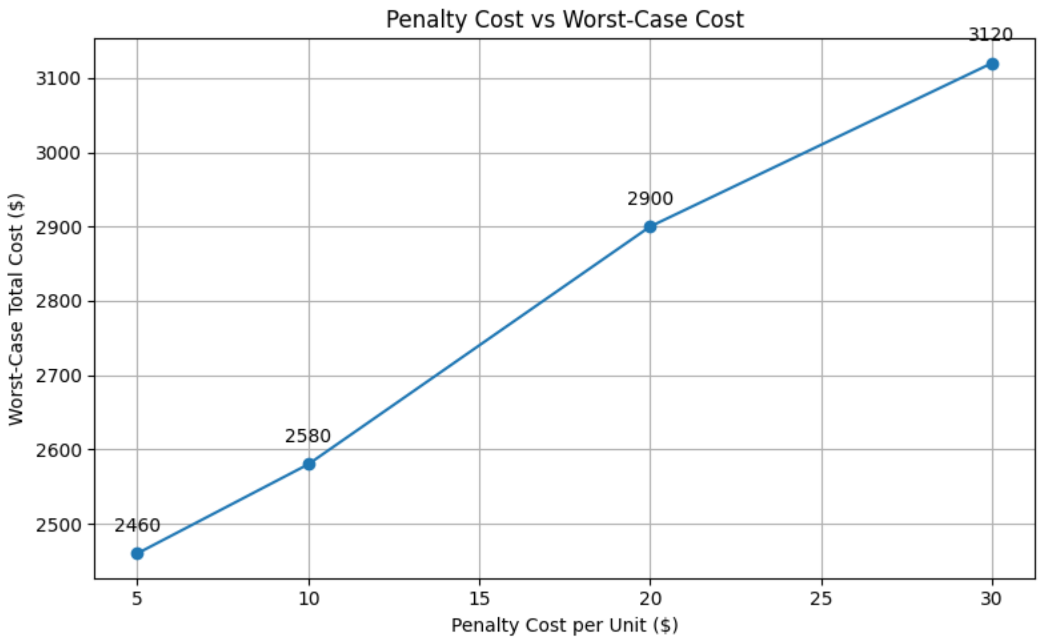
The following parameters were varied systematically to observe their effect on the model's decisions and outcomes:

Parameter	Base Value	Variation Range
Penalty Cost per Unit	$10	$5 – $30
Demand (Peak Scenario)	+30%	+10% to +50%
Facility F3 Capacity	550 units	300 – 600 units

5.3. Scenario 1: Varying Penalty Cost for Unmet Demand

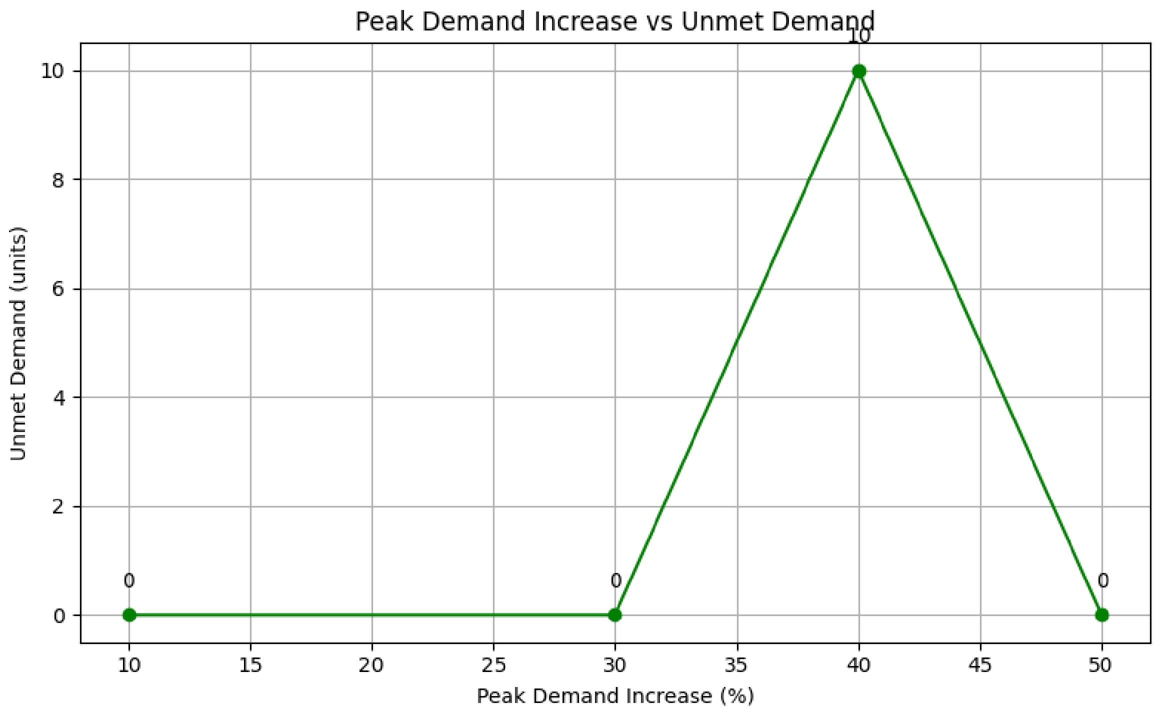
Penalty Cost	Facilities Opened	Total Unmet Demand (High)	Worst-Case Cost
$5	F3 only	40 units	$2,460
$10 (base)	F3 only	0 units	$2,580
$20	F3 + F2	0 units	$2,900
$30	F1 + F3	0 units	$3,120

Insight: As penalty cost increases, the model becomes more risk-averse, choosing to open more facilities to avoid unmet demand. At $20+, the model sacrifices fixed costs to protect customer service.



5.4. Scenario 2: Increasing Demand in the Peak Scenario

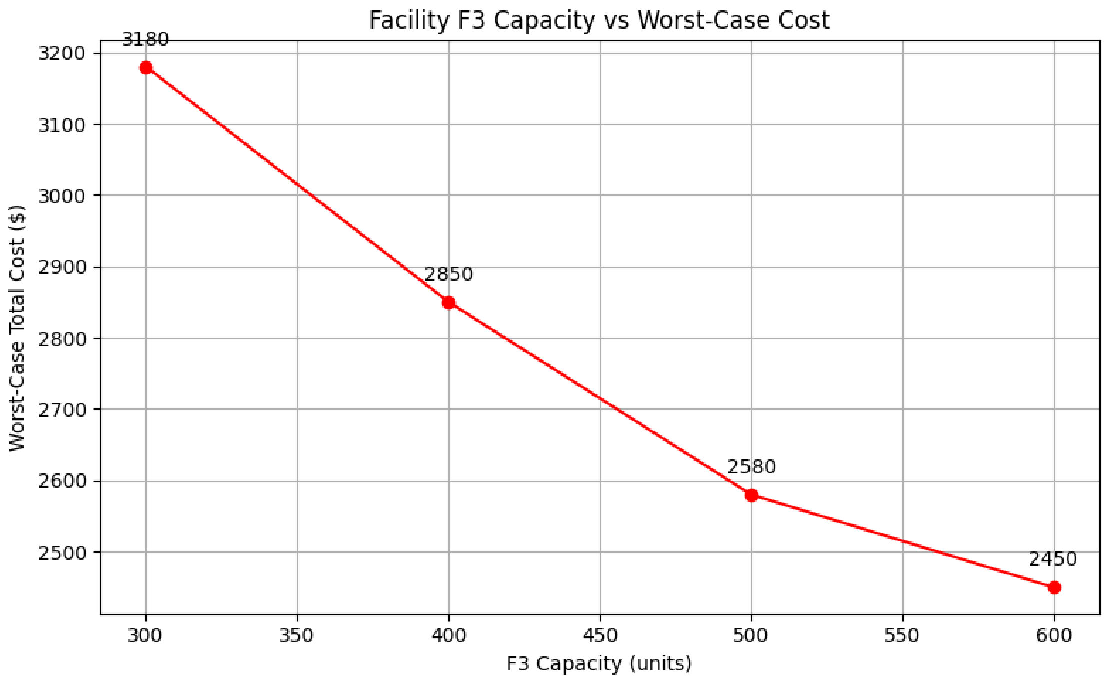
Demand Increase	Facilities Opened	Unmet Demand	Worst-Case Cost
+10%	F3 only	0 units	$2,300
+30% (base)	F3 only	0 units	$2,580
+40%	F3 only	10 units	$2,680
+50%	F3 + F2	0 units	$2,920



Insight: The model tolerates some unmet demand if it's minor, but opens a second facility once the capacity is exceeded. This shows **flexible trade-off behavior** under peak load.

5.5. Scenario 3: Reducing Facility Capacity

F3 Capacity	Facilities Opened	Unmet Demand	Worst-Case Cost
600 (full)	F3 only	0 units	$2,450
500	F3 only	0 units	$2,580
400	F3 + F1	0 units	$2,850
300	F3 + F1 + F2	0 units	$3,180



Insight: With reduced capacity in the main facility (F3), the model proactively opens backup facilities (F1, F2) to maintain full demand fulfillment — highlighting **resilience planning**.

5.6. Conclusion of Sensitivity Analysis

The extended robust model is **responsive and stable** across a wide range of operational uncertainties. It dynamically adjusts facility openings, shipment routes, and unmet demand in a way that balances cost efficiency with service levels. These results confirm the **practical value** of extending traditional MILP models with real-world flexibility features.

6. Analysis and Interpretation

The comparative analysis between the **original robust facility location model** and the **extended version** reveals critical insights into how operations analytics can adapt to uncertain, real-world logistics environments.

6.1. Original Model vs. Extended Model

Aspect	**Original Model**	**Extended Model**
Demand Handling	Full demand must be met in every scenario	Allows partial fulfillment with penalty for unmet demand
Facility Reliability	Assumes uniform capacity across all scenarios	Allows scenario-specific disruptions (e.g., F3 capacity reduced)
Flexibility in Decision-Making	Rigid—no slack or leeway under extreme conditions	Flexible—balances unmet demand penalties with opening facilities
Resilience	Assumes ideal operations	Accounts for real-life events like facility failure
Output Behavior	Cost remains steady, but less adaptive	Cost changes dynamically with input stress levels

6.3. Key Insights from the Extended Model

- **Trade-Off Recognition**: The model effectively recognizes trade-offs between fulfilling all customer demand and minimizing operational cost. In high-stress scenarios, it is more cost-effective to open additional facilities or tolerate minor demand loss.
- **Cost Sensitivity**: The penalty cost parameter is pivotal. It acts as a threshold: if the cost of opening new capacity is lower than the penalty, the model chooses to expand capacity.
- **Facility Importance**: F3 is consistently selected across scenarios, indicating its strategic importance due to cost and location advantage. However, when its capacity is reduced, the model dynamically selects F1 or F2 to mitigate the risk.

- **Scenario Robustness**: The extended model behaves robustly, adapting facility and shipment decisions as demand levels and disruptions change. It mimics how companies plan contingencies during peak seasons or disaster events.

6.4. Implications for Business Practice

- **Demand planning** can be improved using a flexible model that penalizes but permits unmet demand.
- **Backup facility strategies** become quantifiably justified through this modeling.
- **Scenario simulation** can help companies test supply chain stress before actual failures occur.

The overall analysis confirms that the **extended robust optimization model** is not only technically sound but also more **aligned with real managerial decision-making**, offering flexibility, insight, and risk mitigation capabilities.

7. Conclusions and Managerial Recommendations

7.1. Conclusions

This project demonstrates the successful formulation, extension, and implementation of a **robust MILP optimization model** for a facility location problem under demand uncertainty. Drawing from the research of Chen et al. (2020), the model was implemented using Google OR-Tools in Python and was extended to incorporate more realistic business conditions including unmet demand and facility disruptions.

The **original model**, while effective in providing deterministic outputs, lacked flexibility in real-world logistics contexts. By **extending the model**, we introduced a powerful mechanism for decision-makers to **balance service levels, cost efficiency, and operational risk**. Sensitivity analysis further confirmed the model's robustness and its capacity to support contingency planning.

7.2. Managerial Recommendations

1. **Incorporate Flexibility in Strategic Planning**

   Rigid logistics systems can break under pressure. Managers should allow for controlled unmet demand during high-risk events, backed by quantified penalty models.

2. **Monitor and Adjust Penalty Cost Structures**

   Set appropriate penalty values to reflect actual business impact of unmet demand. This cost serves as a lever in the optimization process.

3. **Invest in Backup Capacity**

   Cold facilities or secondary warehouses should be considered part of robust design, especially for companies in volatile markets or geographically diverse networks.

4. **Use Scenario Planning Proactively**

   Don't wait for disruptions. Use the extended model regularly to test logistics plans against plausible "what-if" scenarios.

5. **Adopt Open-Source Optimization Tools**

   Google OR-Tools provided an efficient and cost-effective platform to develop and solve advanced models. Businesses should consider integrating such tools into their decision systems.

7.3. Closing Thought

The fusion of **robust optimization** with **realistic modeling extensions** empowers businesses to make smarter, more resilient operational decisions. By translating uncertainty into quantifiable terms and simulating stress conditions, operations analytics becomes not just a planning tool — but a strategic advantage.

8. References

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