Linear Buckling Analysis of square plate with stiffener profiles

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Abstract

A thin-walled plate, subjected to compressive loading is vulnerable to failure due to buckling, beyond the critical buckling load. The value of the critical buckling load in a plate is improved by adding stiffeners. This paper presents an Eigenvalue buckling analysis for a thin-walled plate using Finite Element Method. A shell structure of the thin plate was designed and loaded with CCFF boundary conditions to perform the eigenvalue buckling analysis in Abaqus. Four design patterns, two standards (ortho-grid and 45-degree waffle), and two custom-designed patterns were taken as an example to demonstrate that adding stiffeners on a thin plate results in an increased value of the critical buckling load.

1 Introduction

A thin plate is a structural design governed by a relation of the thickness being comparatively small in value than the planar dimension. Mathematical expression, if 't' is the thickness and 'L' for the width or length:

$$t/L << 1 \tag{1}$$

The thin plate structures are employed in numerous applications; aerospace vehicles, truss, beam structures, raft foundations, etc. The cross-section of a thin plate, subjected to heavy compressive stress is vulnerable to buckling. Buckling is a measure of the instability of any structure (in this case, the thin plates). The effect of buckling is observed in the plates via bowing, warping, wrinkling, or twisting deformation, later accompanied by the development of excessive compressive stresses throughout the plate or within some portion of the plate [4]. The excessive stress in the structure gets released by changing the shape i.e., distortion in the planar region of the plates through permanent deformation from the initial design.

Figure 1: Example of Deformation due to Buckling.

Grid stiffened plates are a combination of thin late and a pattern of stiffener designed over it. The stiffeners added over a base plate improves the overall stiffness, hence increases the critical buckling load on the plate. Critical buckling load is a specific value beyond which permanent deformation in the structure is observed due to buckling. Li and Bettess [5] suggested that adding stiffeners to a plate design has a twofold effect on the structure by enhancing the stiffness and increasing the threshold of critical buckling load for the design. They mentioned the example of an “I” beam being a better choice than a rectangular beam in carrying lateral loads even if both shaped beams have the same cross-section area. Ebadi [1] investigated the control and decrease of cracks using stiffeners over a base plate and summarized that the addition of stiffeners, limited and decreased the secondary cracks and prevented crushing by decreasing concentrated cracking of the structure. This investigation verifies that the addition of stiffeners on a plate increases its overall stiffness and stretches the critical buckling load range for deformation under buckling.

The next task in designing a stiffened plate is the pattern of stiffeners over the plate and its effect on the buckling performance. The most general pattern for stiffeners includes the ortho-grid (consisting of straight lines horizontally and vertically), 45-degree waffle, and custom-designed curvilinear patterns [2]. Lie and Hao [3] performed a numerical analysis of various stiffener patterns; the standard ortho-grid, 45-degree waffle with constant thickness, variable thickness, and curvilinear stiffener patterns comparing the critical buckling load for each design. Their approach was to change different variables, the thickness of stiffeners, the height of stiffeners, layout pattern and analyzed each design by performing an eigenvalue buckling analysis. They compared the critical buckling load obtained from the analysis and presented a curvilinear pattern of stiffeners that improved the buckling performance by 73% from the initial ortho-grid design with constant thickness and height of stiffeners.

Following the work presented by Lie and Hao, this paper presents a comparison of three stiffener patterns: ortho-grid, 45-degree waffle, and custom-designed pattern by performing eigenvalue buckling analysis on each plate and obtaining the critical buckling load for each structure. Throughout the analysis in this paper, the height and thickness of stiffeners were kept constant. The boundary condition and dimension of the plate were referred from Lie and Hao, to compare the obtained results with a standard-presented work.

2 METHODOLOGY

2.1 Plate Design and Boundary Conditions

Abaqus was used to model and perform eigenvalue buckling analysis on the thin plate structure. A square plate of 1050mm X 1050mm was designed with a skin thickness of 2.6mm for the thin plate on the base. The stiffener patterns were 13.40mm in height with a thickness of 1.4mm. The distance between two stiffeners in the ortho-grid plate and 45-degree waffle grid plate was 100mm. Clamped, Clamped, Free, Free (CCFF) boundary conditions were used for the plate in this analysis. The left and right edges of the plate were restricted by out-of-the-plane displacement and rotation along the y-axis, whereas the top and bottom edges of the plate were set free.

$$P_{left} = sin(\frac{3\pi y}{1000}) + 1 \tag{2}$$

$$P_{right} = -2sin(\frac{\pi(y+125)}{500}) + 1 \tag{3}$$

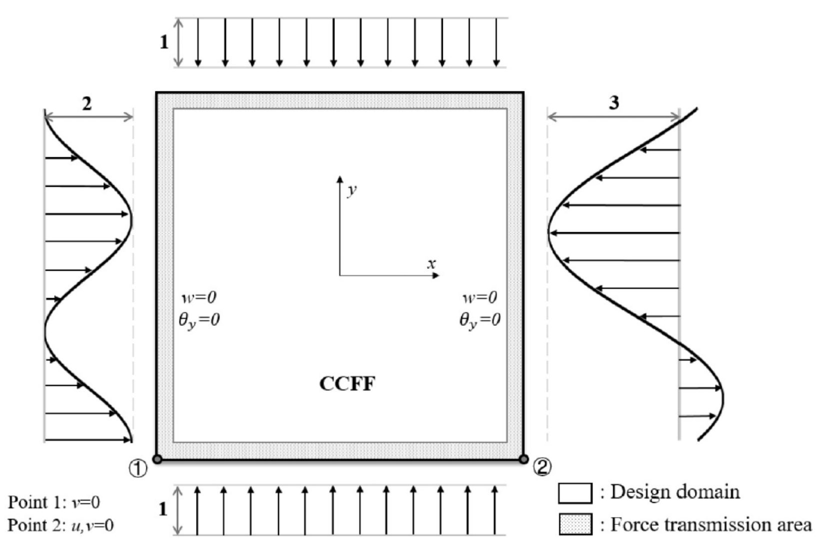


Figure 2: Boundary Conditions for the Design [3].

The boundary conditions applied on the plate:
BC1: Compressive force of 1N on the top and bottom edge of the plate
BC2: Sinusoidal compressive load on the side edges (Equation 2 and Equation 3)
BC3: Constraining the deformation in y-axis on the bottom-left edge
BC4: Constraining the deformation in the x and y axes on the bottom-right edge
BC5: Constraining the deformation in z-axis and rotation in y-axis on both side edges.

Table 1: Properties of Aluminum Alloy 2139.

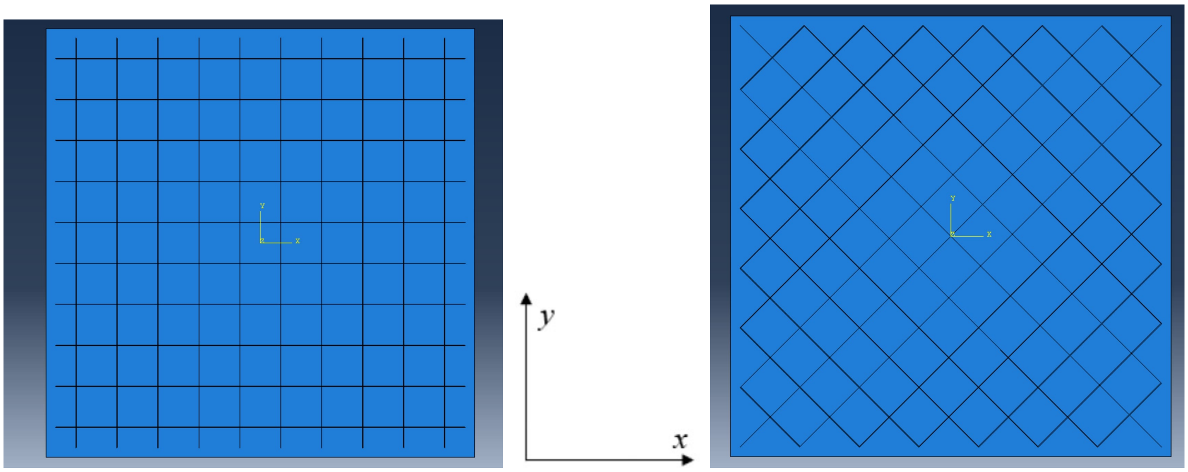
Type	**Value**
Young's Modulus	72.50 GPa
Poisson's ratio	0.33
Density	2.80E-6 kg/mm^3

2.2 Numerical Approach

A shell model of dimension [1050mmx1050mm] was designed as a base plate in ABAQUS with a shell thickness of 2.6mm. The corresponding stiffener pattern: ortho-grid and 45-degree waffle were designed on one of the faces and extruded to 13.4mm with a shell thickness of 1.4mm. The model was applied with appropriate Boundary Conditions as mentioned in the problem statement. Throughout the design pattern the boundary conditions, the thickness of base and stiffeners, height of stiffeners were constant. Each plate design went through eigenvalue buckling analysis to obtain the eigenvalue buckling load using Abaqus software, which was used to calculate the critical buckling factor i.e. eigenvalue divided by the length of the plate (1000mm).

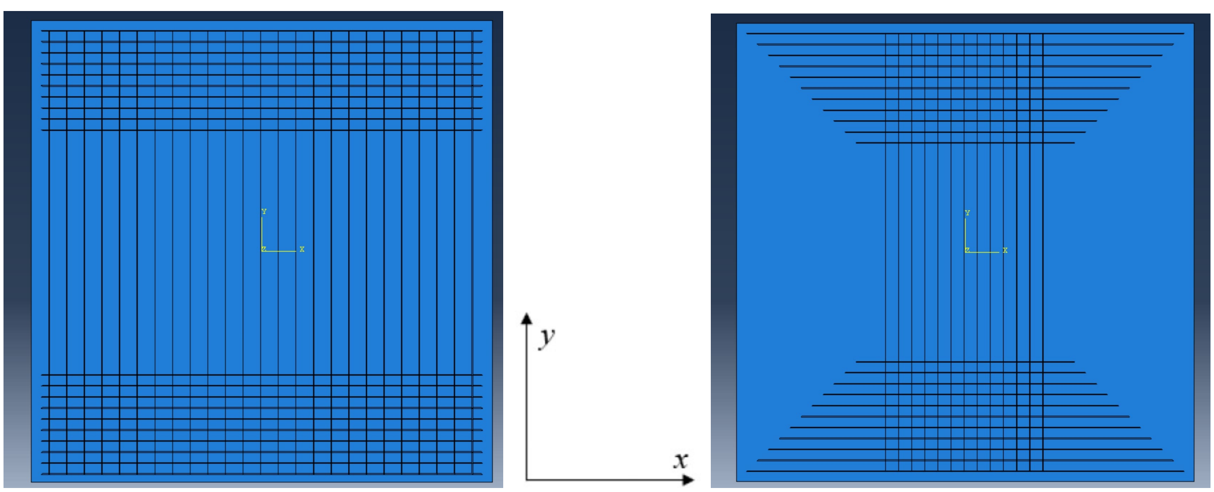
Table 2: Mesh Properties- Element Number.

Ortho-grid	2885
45-degree waffle	12018
Design Pattern 1	9290
Design Pattern 2	5886



(a) Orthogonal Design (b) 45-degree waffle

Figure 3: Design Pattern- Orthogonal and 45 deg Waffle.



(a) Design Pattern 1 (b) Design Pattern 2

Figure 4: Custom Design Patterns.

3 Results and Analysis

The ortho-grid stiffener plate was kept as a standard plate to compare the remaining three stiffener designs (45-degree waffle and two custom design patterns). The critical buckling load for all three eigenmode shapes obtained from Abaqus for each plate, shown in "Table 3". The ortho-grid pattern has a lower critical buckling factor when compared to the 45-degree waffle and two custom design patterns for each eigenmode. The critical buckling factor values demonstrate that the pattern of stiffeners on a plate affects the buckling performance as the values are different for each design pattern.

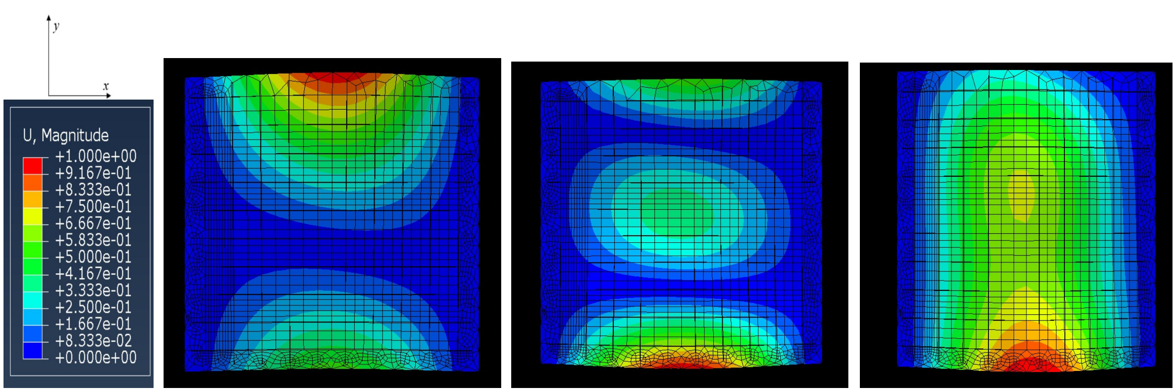
Table 3: Critical Buckling Factor.

Design	Mode 1	Mode 2	Mode 3	Mass(kg)
Ortho-grid	7.21	9.79	17.6	9.23
45-Degree Waffle	7.44	12.1	14.6	9.21
Design Pattern 1	12.6	14.9	22.4	10.74
Design Pattern 2	10.9	12.5	18.0	9.80

Table 4: Comparison of Critical Buckling Factor.

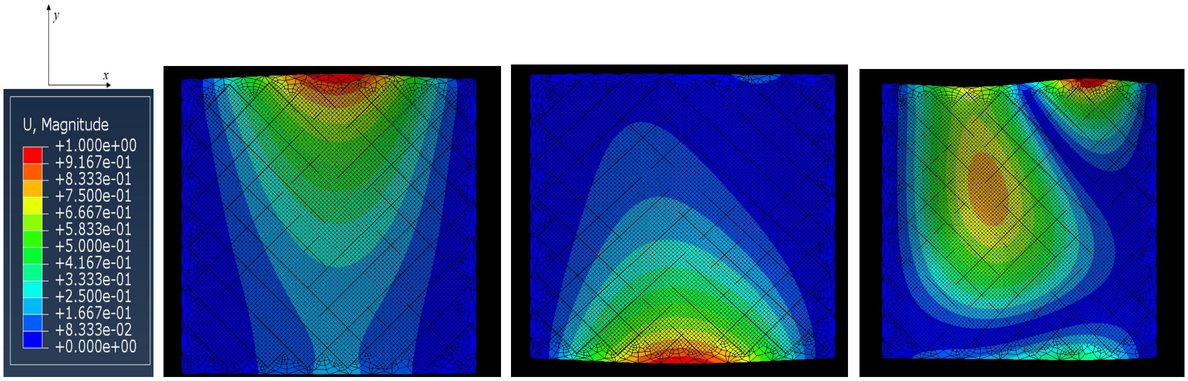
Design	Reference	Calculated
Ortho-grid	7.478	7.21
45-degree Waffle	7.749	7.44

"Table 4" compares the critical buckling factor obtained from the Abaqus analysis with the analysis performed by Li and Hao [3].



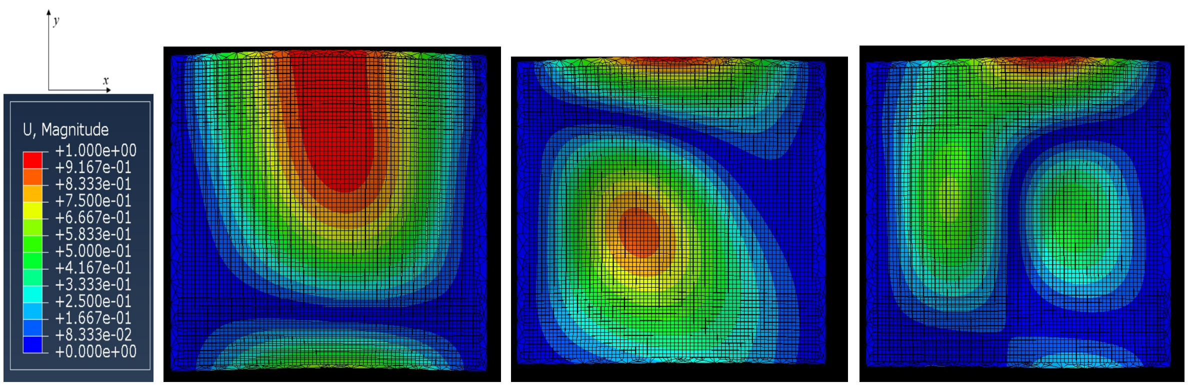
(a) Mode 1 (b) Mode 2 (c) Mode 3

Figure 5: Buckling Analysis [Deformation]- Orthogonal Design.



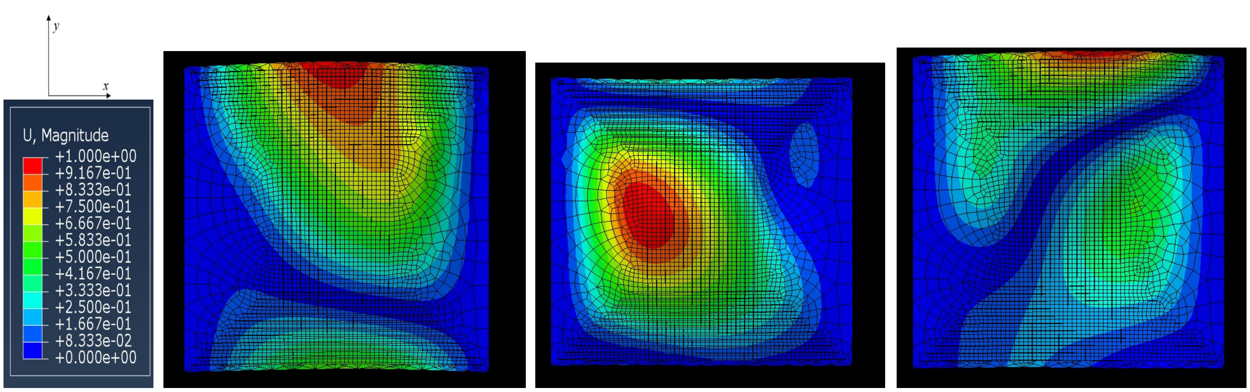
(a) Mode 1 (b) Mode 2 (c) Mode 3

Figure 6: Buckling Analysis [Deformation]- 45 degree Waffle Design.



(a) Mode 1 (b) Mode 2 (c) Mode 3

Figure 7: Buckling Analysis [Deformation]- Design Pattern 1.



(a) Mode 1 (b) Mode 2 (c) Mode 3

Figure 8: Buckling Analysis [Deformation]- Design Pattern 2.

The results obtained from the eigenvalue buckling analysis demonstrated that the placement of stiffeners on a thin plate has a significant effect on the buckling load of the structure. The placement of stiffeners in ortho-grid design and 45-degree waffle design patterns are similar where the stiffeners are covered all over the plate. The critical buckling factor in these designs is close to each other ("Table 3") [Ortho-grid: 7.21 and 45-degree waffle: 7.44]. However, moving to the custom design patterns, there is a three-unit increase in the critical buckling factor. Comparing in-between Design Pattern 1 and Design Pattern 2, Design Pattern 2 is optimum for this structure as the total mass of the structure ("Table 3") is close to the value for the orthogonal pattern and 45-degree waffle pattern.

4 Conclusion and Recommendation

This paper compared four stiffener design patterns for a base plate by performing eigenvalue buckling analysis and comparing their critical buckling factor. As predicted, the custom design patterns had higher eigenvalue loading when compared to standard ortho-grid and 45-degree waffle patterns. The ortho-grid and 45-degree waffle pattern provided a base to observe the critical locations with high deformation values. The high deformation spaces on the plate were stiffened heavily in the custom design plates.

The stiffener designed for this analysis was straight with constant thickness and height throughout the plate. This design is customizable with curvilinear stiffener patterns with variation in the height and thickness to obtain a higher critical buckling factor, hence improving the critical buckling load of the structure.

References

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