OKLAHOMA STATE UNIVERSITY
School of Mechanical & Aerospace Engineering

MAE 5010 - Atmospheric Flight Control

Homework #3 (Assigned: 4/17, Due: 4/26)
Output LQ Regulator/Tracker Design, Nominal Performance + Gust Rejection Analysis

1. In lecture it was mentioned that with the assumption of $C = I$ (we have available measurements of the entire state), the 3 output LQ synthesis equations reduce to the familiar algebraic Riccati equation associated with the LQR problem:

$$A^T P + PA - PBR^{-1}B^T P + Q = 0$$

Show that this is indeed the case.

2. Consider the system

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u, \quad y = \begin{bmatrix} 1 & 1 \end{bmatrix} x,$$

Implement the algorithm presented in Moerder and Calise (1985) in MATLAB (with reasonable m-file documentation). A good test case to make sure your code is working initially is the 2-state example presented at the end of Levine and Athans (1970). Once this is done, find the output feedback gain that minimizes

$$J = E\left\{ \int_0^\infty (x^T Q x + u^T R u) \; dt \right\},$$

Assume $Q = I$, and try various values for R to obtain a good response. Plot your results for several values of R and comment on the trends in overshoot and settling time as you increase R.

3. For the F-16 Lateral Regulator example presented in lecture,

   (a) Use your MATLAB implementation of the algorithm presented in Moerder and Calise (1985) to generate three sets of optimal feedback gains for $\rho = 0.1$, 0.5, and 1, assuming $q_{dr} = 50$, and $q_r = 100$.

   (b) Plot the 4 lateral dynamics states (β, ϕ, p, r) as a function of time (for all three values of ρ) using the same initial condition (i.e., 4 plots with three curves each). Is the change in control authority within the closed loop system evident from the plots?

4. In this problem you will analyze the robust stability properties of a controller designed for a two-state model of the dutch roll dynamics for a UAV with two novel control inputs. The state is $x = (\ \Delta\beta,\ \Delta r\ )$ and the inputs are $u = (\ \Delta\delta_1,\ \Delta\delta_2\ )$.

$$G_o(s) = \begin{bmatrix} \frac{6}{(0.9s+1)(0.1s+1)} & \frac{-0.05}{0.1s+1} \\ \frac{0.07}{0.3s+1} & \frac{5}{(1.8s-1)(0.06s+1)} \end{bmatrix} \qquad K(s) = \begin{bmatrix} \frac{2(s+1)}{s} & \frac{-s}{3s+1} \\ \frac{-5(s+1)}{0.8s+1} & \frac{4(0.7s+1)}{s} \end{bmatrix}$$

(a) For the following nominal plant $G_o(s)$ closed loop specifications, discuss the performance of the given controller (specifically, over which frequency interval we expect to achieve adequate nominal performance).

- *Attenuation of input disturbance signals d_I at the plant output y*
- *Avoidance of large control signals u due to reference demands r*
- *Attenuation of measurement noise signals n at the plant output y*

The MATLAB command `loopsens` may be useful here since it automatically generates a *structure* with all the closed loop transfer functions.

(b) Assume the uncertainty in the two channels can be represented using a multiplicative input uncertainty model. The first channel (weight $W_1(s)$) has a 20% error at low frequency, increases to 100% at 35 rad/s, and reaches 1000% (10X) at the high frequency range. The corresponding MATLAB command to generate the first order weight for this channel is `W1=makeweight(0.20,35,10)`. Similarly, the second channel (weight $W_2(s)$) has a 25% error at low frequency, increases to 100% at 40 rad/s, and reaches 1000% (10X) at the high frequency range. Generate magnitude plots of the two weighting functions and label them on the same graph.

(c) Assume these weights can be represented by the block diagonal weighting matrix

$$W_I(s) \quad = \quad \begin{bmatrix} W_1(s) & 0 \\ 0 & W_2(s) \end{bmatrix}$$

and check to determine whether or not the system is robust with respect to the specified multiplicative input uncertainty. Be sure to generate both the singular value plots as a function of frequency (using the MATLAB command `sigma`) and the H_∞ norm for the appropriate MIMO transfer function to verify your conclusions. Does your conclusion still hold if you raise the uncertainty level at low frequency in the second channel from 25% to 40%?

(d) Now consider a new uncertainty model, *inverse additive uncertainty*, where the family of plants G is given by $G = G_o(I - W\Delta G_o)^{-1}$. First draw the corresponding closed loop diagram and derive a H_∞ norm test for this type of uncertainty model using the small gain theorem. Assuming the same weights for the channels as in part (b), is the system robustly stable for this uncertainty model?

5. Consider the output feedback controller that you designed for the F-16 lateral regulator system in problem 3 above.

(a) What is the minimum frequency (in rad/s) above which you expect at least 10X noise rejection n at the plant output y?

(b) Assume a plant *output multiplicative uncertainty* model and derive an H_∞ norm test using the small gain theorem for this particular uncertainty model. For a weighting matrix where all four channels have 30% error at low frequency, increasing to 100% error at the crossover at 30 rad/s, and reach 1000% (10X) error at the high frequency range, is your controller robustly stable? If so, to what percentage can you increase the low frequency error bound and still verify robust stability?