

# **Autonomous Vehicle: Dynamic Model, Control & Planning**

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# 1 Introduction

This document outlines the dynamic model, control and planning for car like autonomous robot. The this document will focus on motion control and local planning problems.



Figure 1: System architecture for autonomous vehicle

## 2 Modeling for Planning and Control

### 2.1 Kinematic model of a car like robot

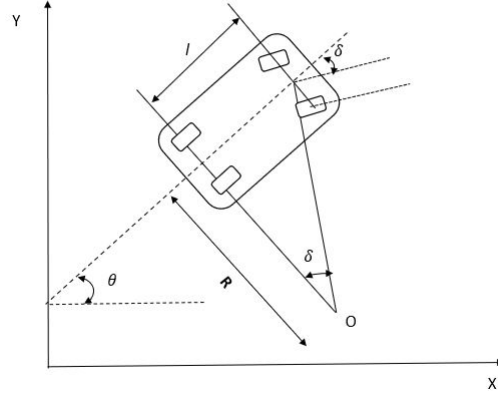


Figure 2: Kinematics of a car like mobile robot

The differential constraint for rear wheel is given by:

$$R = \frac{l}{\tan \delta} \quad (1)$$

$$\dot{x}_r = v_r \cos(\theta) \quad (2)$$

$$\dot{y}_r = v_r \sin(\theta) \quad (3)$$

$$\dot{\theta} = \frac{v_r}{l} \tan \delta \quad (4)$$

$$\dot{v}_r = a \quad (5)$$

The differential constrain can also be written in terms of motion of forward wheel,

$$\dot{x}_f = v_f \cos(\theta + \delta) \quad (6)$$

$$\dot{y}_f = v_f \sin(\theta + \delta) \quad (7)$$

$$\dot{\theta} = \frac{v_f}{l} \tan \delta \quad (8)$$

The front wheel speed  $v_f$ , is related to rear wheel speed  $v_r$  by:

$$\frac{v_r}{v_f} = \cos(\delta) \quad (9)$$

The planning and control problems for this model involve selecting the steering angle  $\delta$  within the mechanical limits of the vehicle  $\delta \in [\delta_{min}, \delta_{max}]$ , and forward speed  $v_r$  within an acceptable range,  $v_r \in [v_{min}, v_{max}]$  [1].

A simplification that is sometimes utilized is to select the heading rate  $\omega$  instead of the steering angle  $\delta$ . These quantities are related by

$$\delta = \arctan\left(\frac{l\omega}{v_r}\right) \quad (10)$$

simplifying the heading dynamics to

$$\dot{\theta} = \omega, \quad \omega \in \left[\frac{v_r}{l} \tan(\delta_{min}), \frac{v_r}{l} \tan(\delta_{max})\right] \quad (11)$$

### 3 Vehicle Control

### 4 Trajectory Planning

### References

- [1] Brian Paden, Michal Čáp, Sze Zheng Yong, Dmitry Yershov, and Emilio Frazzoli. A survey of motion planning and control techniques for self-driving urban vehicles. *IEEE Transactions on intelligent vehicles*, 1(1):33–55, 2016.