$$\int_{-a}^{a} f(x)dx = \begin{cases} 2\int_0^a f(x)dx & if\ f(x) is\ even \\ 0 & if\ f(x) is\ odd \end{cases}$$

$$\int_{-\pi}^{\pi} cosmx.cosnx\ dx = \begin{cases} 0, & m \neq n \\ \pi, & m = n > 0 \\ 2\pi, & m = n = 0 \end{cases}$$

$$\int_{-\pi}^{\pi} sinmx.sinnx\ dx = \begin{cases} 0, & m \neq n, m = n = 0 \\ \pi, & m = n > 0 \end{cases}$$

$$sinn\pi = 0, cosn\pi = (-1)^n, n\epsilon z, cos2n\pi = 1$$

$$\sin\left(n + \frac{1}{2}\right)\pi = (-1)^n, n\epsilon z, cos\left(n + \frac{1}{2}\right)\pi = 0$$

$$\sin\frac{n\pi}{2} = \begin{cases} (-1)^{\frac{n-1}{2}}, & n\ is\ odd \\ 0, & n\ is\ even \end{cases}$$

Problem: find the fourier series to represent $f(x) = x^2$ in the interval $[0, 2\pi]$

Solution: we know that the fourier series of $f(x)$ in $[0,2\pi]$is given by

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n \cos nx + b_n\ sinnx)$$

Where $a_0 = \frac{1}{\pi}\int_0^{2\pi} f(x)dx$

$$a_n = \frac{1}{\pi}\int_0^{2\pi} f(x)\cos nx\, dx$$

$$b_n = \frac{1}{\pi}\int_0^{2\pi} f(x)\sin nx\, dx$$

Now $a_0 = \frac{1}{\pi}\int_0^{2\pi} x^2 dx$

$$= \frac{1}{\pi}\left[\frac{x^3}{3}\right]_0^{2\pi}$$

$$= \frac{1}{3\pi}[(2\pi)^3 - 0^3]$$

$$a_0 = \frac{8\pi^2}{3}$$

$$a_n = \frac{1}{\pi}\int_0^{2\pi} f(x)\cos nx\, dx$$

$$= \frac{1}{\pi}\left[\int_0^{2\pi} x^2 \cos nx\, dx\right]$$

$$= \frac{1}{\pi}\left[x^2 \int \cos nx\, dx - \int \frac{d}{dx}(x^2)\int \cos nx\, dx\ \ dx\right]_0^{2\pi}$$

$$= \frac{1}{\pi}\left[\frac{x^2 sinnx}{n} - \int 2x\frac{sinnx}{n}\ dx\right]_0^{2\pi}$$

$$= \frac{1}{\pi}\left[\frac{x^2 sinnx}{n} - \frac{2}{n}\int xsin\, nx\, dx\right]_0^{2\pi}$$

$$= \frac{1}{\pi}\left[\frac{x^2 sinnx}{n} - \frac{2}{n}\left(x\left(-\frac{cosnx}{n}\right) - \int (1)\left(-\frac{cosnx}{n}\right)dx\right)\right]_0^{2\pi}$$

$$= \frac{1}{\pi}\left[\frac{x^2 sinnx}{n} + \frac{2xcos\ nx}{n^2} - \frac{2}{n^3}\sin nx\right]_0^{2\pi}$$

$$= \frac{1}{\pi}\left[\frac{(2\pi)^2 sin\ n.2\pi}{n} + \frac{2.2\pi.cos\ n.2\pi}{n^2} - \frac{2}{n^3}\sin n.2\pi - \frac{0^2.sinn.0}{n} - \frac{2.0.cos\ n.0}{n^2} + \frac{2}{n^3}\sin n.0\right]$$

Since $\sin n\pi = 0, \cos n\pi = (-1)^n , \cos 2n\pi = 1$

$$= \frac{1}{\pi}\left[\frac{4\pi}{n^2}\right]$$

$$a_n = \frac{4}{n^2}$$

$$b_n = \frac{1}{\pi}\left[\int_0^{2\pi} f(x)\sin nx\ dx\right]$$

$$= \frac{1}{\pi}\left[\int_0^{2\pi} x^2\sin nx\ dx\right]$$

$$= \frac{1}{\pi}\left[x^2\int \sin nx\ dx - \int \frac{d}{dx}(x^2)\left(\int \sin nx\ dx\right)dx\right]_0^{2\pi}$$

$$= \frac{1}{\pi}\left[\frac{x^2(-\cos nx)}{n} - \int 2x\frac{(-\cos nx)}{n}dx\right]_0^{2\pi}$$

$$= \frac{1}{\pi}\left[\frac{x^2(-\cos nx)}{n} + \frac{2}{n}\int xcos\ nx\ dx\right]_0^{2\pi}$$

$$= \frac{1}{\pi}\left[\frac{x^2(-\cos nx)}{n} + \frac{2}{n}\left[\frac{x(sin\ nx)}{n} - \frac{(1)(-\cos nx)}{n^2}\right]\right]_0^{2\pi}$$

$$= \frac{1}{\pi}\left[-\frac{x^2\cos nx}{n} + \frac{2}{n^2}x\sin nx + \frac{2}{n^3}cos\ nx\right]_0^{2\pi}$$

$$= \frac{1}{\pi}[-\frac{(2\pi)^2 \cos 2n\pi}{n} + \frac{2}{n^2}(2\pi)\sin 2n\pi + \frac{2}{n^3} cos\ 2n\pi - (-0 + 0 + \frac{2}{n^3} cos\ n(0))]$$

$$= \frac{1}{\pi}\left[-\frac{4\pi^2}{n} + 0 + \frac{2}{n^3} - \frac{2}{n^3}\right]$$

since $\sin n\pi = 0, \cos 2n\pi = 1$

$$b_n = -\frac{4\pi}{n}$$

The fourier series expansion for f(x) is

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n \cos nx + b_n\ sinnx)$$

$$x^2 = \frac{\frac{8}{3}\pi^2}{2} + \sum_{n=1}^{\infty}(\frac{4}{n^2}\cos nx - \frac{4\pi}{n}\ sinnx)$$

$$x^2 = \frac{4}{3}\pi^2 + \sum_{n=1}^{\infty}(\frac{4}{n^2}\cos nx - \frac{4\pi}{n}\ sinnx)$$

Problem: find the fourier series of periodic function defined as $f(x) = \begin{cases} -\pi, & -\pi < x < 0 \\ x, & 0 < x < \pi \end{cases}$ hence deduce that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots \ldots \ldots = \frac{\pi^2}{8}$

Solution: given function

$$f(x) = \begin{cases} -\pi, & -\pi < x < 0 \\ x, & 0 < x < \pi \end{cases}$$

We know that the fourier series of $f(x)$ in the interval $[-\pi, \pi]$ is given by

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n \cos nx + b_n\, sinnx)$$

Where $a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)dx$

$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\, dx$

$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\, dx$

$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)dx$

$= \frac{1}{\pi}[\int_{-\pi}^{0} f(x)dx + \int_{0}^{\pi} f(x)dx\,]$

$= \frac{1}{\pi}[\int_{-\pi}^{0} -\pi\, dx + \int_{0}^{\pi} xdx\,]$

$= \frac{1}{\pi}[(-\pi x)\begin{matrix}0\\-\pi\end{matrix} + (\frac{x^2}{2})\begin{matrix}\pi\\0\end{matrix}]$

$= \frac{1}{\pi}[(-\pi.0 + \pi.-\pi) + (\frac{\pi^2}{2} - \frac{0^2}{2})]$

$= \frac{1}{\pi}[-\pi^2 + \frac{\pi^2}{2}]$

$a_0 = -\frac{\pi}{2}$

$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\, dx$

$= \frac{1}{\pi}[\int_{-\pi}^{0} f(x)\cos nx\, dx + \int_{0}^{\pi} f(x)\cos nx\, dx\,]$

$= \frac{1}{\pi}[\int_{-\pi}^{0} -\pi\cos nx\, dx + \int_{0}^{\pi} x\cos nx\, dx\,]$

$$= \frac{1}{\pi}[\left(-\frac{\pi sinnx}{n}\right)\begin{matrix}0\\-\pi\end{matrix} + (x\frac{sinnx}{n} - (1)\left(-\frac{cosnx}{n^2}\right))_0^\pi$$

$$= \frac{1}{\pi}[-\frac{\pi}{n}(\sin(0) - \sin n(-\pi))$$

$$+ \pi\frac{\sin n\pi}{n} + \frac{1}{n^2}cos\, n\pi - 0 - \frac{1}{n^2}cos\ 0\ ]$$

$= \frac{1}{\pi}[\frac{1}{n^2}(-1)^n - \frac{1}{n^2}]$

since $\sin n\pi = 0, cosn\pi = (-1)^n$

$a_n = \frac{1}{\pi n^2}[(-1)^n - 1]$

$b_n = \frac{1}{\pi}[\int_{-\pi}^{\pi} f(x)\sin nx\ dx]$

$= \frac{1}{\pi}[\int_{-\pi}^{0} f(x)\sin nx\ dx + \int_0^\pi f(x)\sin nx\ dx]$

`$= \frac{1}{\pi}\left[\int_{-\pi}^{0} -\pi\sin nx\ dx + \int_0^\pi x\sin nx\ dx\right]$

$= \frac{1}{\pi}[-\pi\left(-\frac{\cos nx}{n}\right)\begin{matrix}0\\-\pi\end{matrix} + (x\left(-\frac{\cos nx}{n}\right) -$
$(1)\left(-\frac{\sin nx}{n^2}\right))_0^\pi]$

$$= \frac{1}{\pi}[-\pi\left(-\frac{\cos n0}{n} + \frac{\cos n(-\pi)}{n}\right)$$

$$+ -\pi\frac{\cos n\pi}{n}$$

$$+ (\sin n\pi)\frac{}{n^2} + 0 - \sin 0/n^2\ ]$$

$$= \frac{1}{\pi}[\frac{\pi}{n} - \frac{\pi(-1)^n}{n} - \frac{\pi(-1)^n}{n}]$$

$$b_n = [\frac{1}{n} - \frac{2(-1)^n}{n}]$$

Hence the fourier series of $f(x)$ is

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n \cos nx + b_n \, sinnx)$$

$f(x) = \frac{-\frac{\pi}{2}}{2} + \sum_{n=1}^{\infty}(\frac{1}{\pi n^2}[(-1)^n - 1]\cos nx + [\frac{1}{n} - \frac{2(-1)^n}{n}] \, sinnx)$

$= -\frac{\pi}{4} - \frac{2}{\pi}(\cos x + \frac{cos3x}{3^2} + \frac{cos5x}{5^2} + \cdots \ldots) + (3sinx - \frac{sin2x}{2} + \frac{sin3x}{3} - \frac{sin4x}{4} + \cdots \ldots))$

Put $x = 0$ in above equation then

$f(0) = -\frac{\pi}{4} - \frac{2}{\pi}(\cos 0 + \frac{cos3.0}{3^2} + \frac{cos5.0}{5^2} + \cdots \ldots) + (3sin0 - \frac{sin2.0}{2} + \frac{sin3.0}{3} - \frac{sin4.0}{4} + \cdots \ldots)$

$$f(0) = -\frac{\pi}{4} - \frac{2}{\pi}(\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots \ldots \ldots)$$

Since $f(x)$ is discontinuous at x=0, $f(0-0) = -\pi$

$$f(0+0) = 0$$

$$f(0) = \frac{1}{2}(f(0-0) + f(0+0))$$

$$= -\frac{\pi}{2}$$

Hence

$$f(0) = -\frac{\pi}{4} - \frac{2}{\pi}\left(\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots \ldots \ldots\right)$$

$$-\frac{\pi}{2} + \frac{\pi}{4} = -\frac{2}{\pi}\left(\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots \ldots \ldots\right)$$

$$\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots \ldots \ldots = \frac{\pi^2}{8}$$

Problem: Expand the function $f(x) = x^2$ as a Fourier series in $[-\pi, \pi]$ and hence deduce $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} + \cdots \ldots . = \frac{\pi^2}{12}$

Solution: given $f(x) = x^2$

Since x^2 is even , if $f(x)$ is even then the fourier series of $f(x)$ is

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx)$$

Where

$$a_n = \frac{2}{\pi} \int_0^{\pi} f(x) \cos nx \, dx, n = 0,1,2,3 \ldots \ldots .$$

$$a_0 = \frac{2}{\pi}\int_0^{\pi} f(x)dx$$

$$= \frac{2}{\pi}\int_0^{\pi} x^2 dx$$

$$= \frac{2}{\pi}[\frac{x^3}{3}]_0^{\pi}$$

$$= \frac{2}{\pi}[\frac{\pi^3}{3} - \frac{0^3}{3}]$$

$$a_0 = \frac{2}{3}\pi^2$$

$a_n = \frac{2}{\pi}\int_0^{\pi} f(x)\cos nx\, dx$

$$= \frac{2}{\pi}\int_0^{\pi} x^2 \cos nx\, dx$$

$$= \frac{2}{\pi}[x^2 \int \cos nx\, dx - \int \frac{d}{dx}(x^2) \int \cos nx\, dx\ \ dx]_0^{\pi}$$

$$= \frac{2}{\pi}[\frac{x^2 sinnx}{n} - \int 2x \frac{sinnx}{n}\ dx]_0^{\pi}$$

$$= \frac{2}{\pi}[\frac{x^2 sinnx}{n} - \frac{2}{n}\int xsin\, nx\, dx]_0^{\pi}$$

$$= \frac{2}{\pi}[\frac{x^2 sinnx}{n} - \frac{2}{n}\left(x\left(-\frac{cosnx}{n}\right) - \int(1)\left(-\frac{cosnx}{n}\right)dx\right)]_0^\pi$$

$$= \frac{2}{\pi}[\frac{x^2 sinnx}{n} + \frac{2xcos\, nx}{n^2} - \frac{2}{n^3}\sin nx]\begin{matrix}\pi\\0\end{matrix}$$

$$= \frac{2}{\pi}[\frac{\pi^2 sinn\pi}{n} + \frac{2\pi cos\, n\pi}{n^2} - \frac{2}{n^3}\sin n\pi - 0 - 0 + \frac{2}{n^3}sin0]$$

$$= \frac{2}{\pi}[\frac{2\pi}{n^2}(-1)^n]$$

$$= \frac{4}{n^2}(-1)^n$$

Therefore $f(x) = \frac{\frac{2}{3}\pi^2}{2} + \sum_{n=1}^{\infty}\frac{4}{n^2}(-1)^n \cos nx$

$$x^2 = \frac{\pi^2}{3} + 4(-\frac{cosx}{1^2} + \frac{cos2x}{2^2} - \frac{cos3x}{3^2} + \cdots \ldots.)$$

Put $x = 0$, in above equation we get

$0 = \frac{\pi^2}{3} - 4(\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} + \cdots \ldots.)$

$\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} + \cdots \ldots. = \frac{\pi^2}{12}$

Problem: obtain the half range sine and cosine series for $f(x) = \frac{\pi x(\pi - x)}{8}$ in the range $0 \le x \le \pi$.

Solution : given $f(x) = \frac{\pi x(\pi - x)}{8}$

$f(x) = \frac{\pi(\pi x - x^2)}{8}$

Half Range sine series is $b_n = \frac{2}{\pi}\int_0^\pi f(x)\sin nx\, dx$

$$= \frac{2}{\pi}\int_0^\pi \frac{\pi(\pi x - x^2)}{8}\sin nx\, dx$$

$$= \frac{1}{4}[\int_0^\pi \pi x \sin nx\, dx - \int_0^\pi x^2 \sin nx\, dx]$$

$= \frac{1}{4}[\pi\left(x\left(-\frac{cosnx}{n}\right) - \left(-\frac{sinnx}{n^2}\right)\right) - (x^2\left(-\frac{cosnx}{n}\right) - (2x)\left(-\frac{sinnx}{n^2}\right) + 2(\frac{cosnx}{n^3}))]_0^\pi$

Since $sinn\pi = sin0 = 0$

$= \frac{1}{4}[\pi\left(-\frac{\pi cosn\pi}{n} + 0\right) - (-\frac{\pi^2 cosn\pi}{n} + \frac{2cosn\pi}{n^3} + 0 - \frac{2}{n^3}cos0]$

$= \frac{1}{4}[-\frac{\pi^2}{n}(-1)^n + \frac{\pi^2}{n}(-1)^n - \frac{2}{n^3}(-1)^n + \frac{2}{n^3}]$

$b_n = \frac{1}{2n^3}[1 - (-1)^n]$

The half range sine series of $f(x)$ is

$$\frac{\pi(\pi x - x^2)}{8} = \sum_1^\infty \frac{1}{2n^3}[1 - (-1)^n]\, sinnx$$

We know that half range cosine series expansion of $f(x)$ is

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx)$$

Where

$$a_n = \frac{2}{\pi} \int_0^{\pi} f(x) \cos nx \, dx, n = 0,1,2,3 \ldots \ldots .$$

$a_0 = \frac{2}{\pi} \int_0^{\pi} f(x) dx$

$= \frac{2}{\pi} \int_0^{\pi} \frac{\pi(\pi x - x^2)}{8} dx$

$= \frac{1}{4} \int_0^{\pi} (\pi x - x^2) dx$

$= \frac{1}{4} [\frac{\pi x^2}{2} - \frac{x^3}{3}]_0^{\pi}$

$= \frac{1}{4} [\frac{\pi^3}{2} - \frac{\pi^3}{3}]$

$a_0 = \frac{\pi^3}{24}$

$a_n = \frac{2}{\pi} \int_0^{\pi} f(x) \cos nx \, dx$

$= \frac{2}{\pi} \int_0^{\pi} \frac{\pi(\pi x - x^2)}{8} \cos nx \, dx$

$= \frac{1}{4} \int_0^{\pi} (\pi x - x^2) \cos nx \, dx$

$= \frac{1}{4} [\pi \int_0^{\pi} x \cos nx \, dx - \int_0^{\pi} x^2 \cos nx \, dx]$

$= \frac{1}{4}[\pi\left(x\left(\frac{sinnx}{n}\right) - \left(-\frac{cosnx}{n^2}\right)\right) - (x^2\left(\frac{sinnx}{n}\right) - 2x\left(-\frac{cosnx}{n^2}\right) + 2(-\frac{sinnx}{n^3}))]_0^\pi$

$a_n = -\frac{\pi}{4n^2}(1 + (-1)^n)$

The half range cosine series of $f(x)$ is

$$\frac{\pi(\pi x - x^2)}{8} = \frac{\frac{\pi^3}{24}}{2} + \sum_{n=1}^{\infty}(-\frac{\pi}{4n^2}(1 + (-1)^n)\cos nx)$$

$$\frac{\pi(\pi x - x^2)}{8} = \frac{\pi^3}{48} - \sum_{n=1}^{\infty}\frac{\pi}{4n^2}(1 + (-1)^n \cos nx)$$

Problem: Express $f(x) = x$ as a Fourier series in $(-l, l)$

Solution : $given\ f(x) = x$

Since $f(x) = x$ is odd function

The fourier series of odd function in $(-l, l)$ is

$f(x) = \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{l}$, where $b_n = \frac{2}{l}\int_0^l f(x)\sin\frac{n\pi x}{l}\,dx$

$b_n = \frac{2}{l}\int_0^l x\sin\frac{n\pi x}{l}\,dx$

$= \frac{2}{l}\left[x\left(-\frac{\cos\frac{n\pi x}{l}}{\frac{n\pi}{l}}\right) - (1)(-\frac{\sin\frac{n\pi x}{l}}{(\frac{n\pi}{l})^2})\right]_0^l$

$= \frac{2}{l}\left[-\frac{l}{n\pi}\left(x\cos\frac{n\pi x}{l}\right) + \left(\frac{l}{n\pi}\right)^2\sin\frac{n\pi x}{l}\right]_0^l$

$= \frac{2}{l}\left[-\frac{l}{n\pi}\left(l.\cos\frac{n\pi l}{l}\right) + \left(\frac{l}{n\pi}\right)^2 \sin\frac{n\pi l}{l} - (-\frac{l}{n\pi}\left(0.\cos\frac{n\pi.0}{l}\right) + \left(\frac{l}{n\pi}\right)^2 \sin\frac{n\pi.0}{l})\right]$

$= \frac{2}{l}\left[-\frac{l}{n\pi}(l.\cos n\pi) + \left(\frac{l}{n\pi}\right)^2 \sin n\pi - (\left(\frac{l}{n\pi}\right)^2 \sin 0)\right]$

Since $\cos n\pi = (-1)^n, \sin n\pi = 0$

$= \frac{2}{l}\left[-\frac{l^2}{n\pi}(-1)^n + 0 - 0\right]$

$b_n = \frac{2l}{n\pi}(-1)^{n+1}$

Hence the fourier series expansion of $f(x)$ is

$f(x) = \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{l}$

$x = \sum_{n=1}^{\infty} \frac{2l}{n\pi}(-1)^{n+1} \sin\frac{n\pi x}{l}$

Problem : if $f(x) = |x|$ Expand $f(x)$ as fourier series in the interval $(-2,2)$.

Solution : let $f(x) = |x|$

Since f(x) is even function

Then the fourier series for even function

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{l}$$

$$a_n = \frac{2}{l}\int_0^l f(x)\cos\frac{n\pi x}{l}\ dx$$

$$a_0 = \frac{2}{2}\int_0^2 x\,dx$$

$$= \left[\frac{x^2}{2}\right]\begin{matrix}2\\0\end{matrix}$$

$$= \frac{4}{2} - \frac{0}{2}$$

$$= 2$$

$$a_n = \frac{2}{2}\int_0^2 f(x)\cos\frac{n\pi x}{2}\ dx$$

$$= \int_0^2 x\cos\frac{n\pi x}{2}\ dx$$

$$= \left[x\left(\frac{\sin\frac{n\pi x}{2}}{\frac{n\pi}{2}}\right) - (1)\left(-\frac{\cos\frac{n\pi x}{2}}{\left(\frac{n\pi}{2}\right)^2}\right)\right]\begin{matrix}2\\0\end{matrix}$$

$$= \left[2\left(\frac{\sin\frac{n\pi 2}{2}}{\frac{n\pi}{2}}\right) + \left(\frac{\cos\frac{n\pi 2}{2}}{\left(\frac{n\pi}{2}\right)^2}\right) - 2\left(\frac{\sin\frac{n\pi 0}{2}}{\frac{n\pi}{2}}\right) - \left(\frac{\cos\frac{n\pi 0}{2}}{\left(\frac{n\pi}{2}\right)^2}\right)\right]$$

$$= \left[2\frac{\sin n\pi}{\frac{n\pi}{2}} + \frac{\cos n\pi}{\left(\frac{n\pi}{2}\right)^2} - 2\frac{\sin 0}{\frac{n\pi}{2}} - \frac{\cos 0}{\left(\frac{n\pi}{2}\right)^2}\right]$$

$$= \left[\frac{(-1)^n}{\left(\frac{n\pi}{2}\right)^2} - \frac{1}{\left(\frac{n\pi}{2}\right)^2}\right]$$

$$a_n = \frac{4}{(n^2\pi^2)}[(-1)^n - 1]$$

The fourier series of $f(x)$ is

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{l}$$

$$|x| = 1 + \sum_{n=1}^{\infty} \frac{4}{(n^2\pi^2)}[(-1)^n - 1]\ \cos\frac{n\pi x}{2}$$

E-Resources and E-textbooks

A Text book of Fourier series (mathematics for Engineering) by W.Bolton.

A Text book of Fourier series (Dover books on Mathematics) by Georgi.P.Tolstov.

A Text Book of Engineering Mathematics, Vol 2 by Debashis Dutta, NIT, Warangal,A.P

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