PHYS-UA 210 Computational Physics
Problem Set 04

Sandhya Sharma

October 3, 2023

Question 1

Exercise 5.6: Calculating Integral

The code calculates the integral of the function $f(x) = x^4 - 2x + 1$ from 0 to 2 using the trapezoidal rule using 20 slices.
It also estimates the error on the result using $\epsilon = \frac{1}{3}(I_2 - I_1)$ where I_2 is the integral of the function using 20 slices and I_1 is that of 10 slices.

Output:

```
f(x) = x^4 - 2x + 1
Integral of f(x) using N = 10:  4.50656
Integral of f(x) using N = 20:  4.273330000000001
Estimated error              : 0.07774333333333297
```

Here, both integrals vary from the true value of 4.4 since both are simply estimations of the real area under the curve by dividing it into trapeziums. Even the error is an estimation as we are ignoring its terms with order higher than 2.

Question 2

Exercise 5.10: Period of an Anharmonic Oscillator

Total energy of the oscillator: $E = \frac{1}{2}m(\frac{dx}{dt})^2 + V(x)$

To find the period of the oscillator:

At $t = 0$, $x = a$ so $\frac{dx}{dt} = 0$ and $E = V(a)$

$$V(a) = \frac{1}{2}m(\frac{dx}{dt})^2 + V(x)$$

$$\sqrt{(2(V(a) - V(x))} = \sqrt{m}\frac{dx}{dt}$$

$$dt = \frac{\sqrt{m}}{\sqrt{(2(V(a) - V(x))}}$$

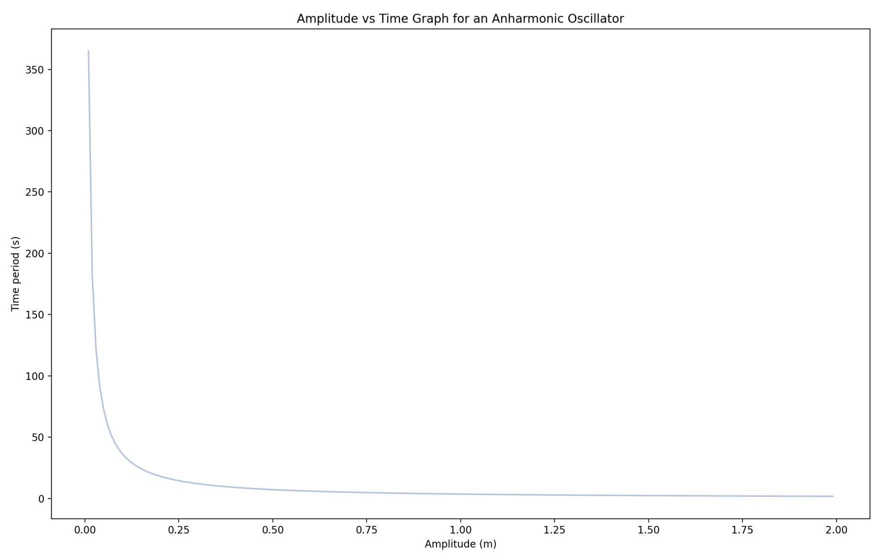
Integrating both sides with bounds: $t = [0, T/4]$ and $x = [0, a]$

$$\int_0^{T/4} dt = \int_0^a \frac{\sqrt{m}}{\sqrt{(2(V(a) - V(x))}}\, dx$$

$$T = \sqrt{8m} \int_0^a \frac{1}{\sqrt{(2(V(a) - V(x))}}\, dx$$

Calculating the period of the oscillator with $V(x) = x^4$ using Gaussian quadrature with N = 20 points for amplitudes (a) ranging from 0 to 2:

Output:

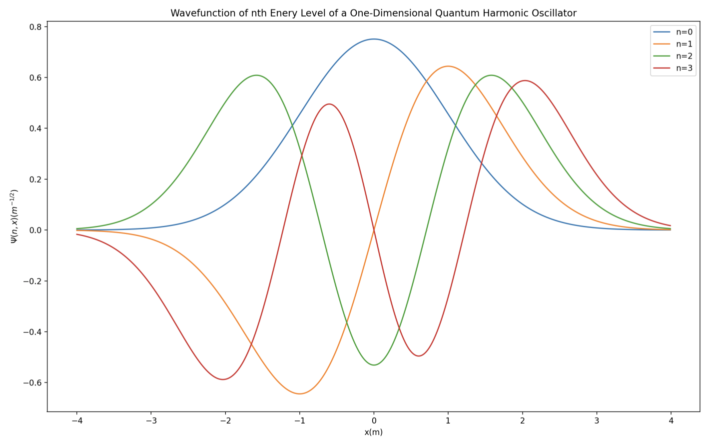


Here, as the amplitude increases, the oscillator gets faster (i.e., the time period decreases) and period diverges as the amplitude goes to zero. The system is an anharmonic oscillator and unlike SHM, the time period is not independent of amplitude. Since the restoring force is not proportional to the displacement of the system and instead depend non linearly on the displacement. This causes the time period to depend on the amplitude of oscillation.

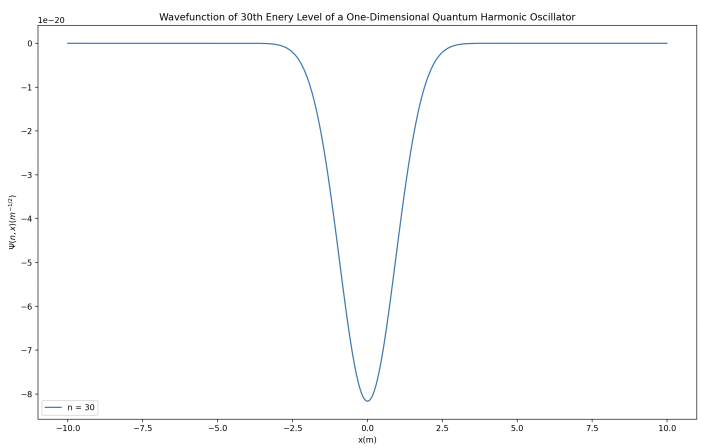
Question 3

Ex 5.13: Quantum uncertainty in the harmonic oscillator

Plot of wavefunctions of harmonic oscillator for $n = 0, 1, 2, 3$ for $x = [-4, 4]$:



Plot of wavefunction for $n = 30$ from $x = -10$ to $x = 10$:



Here, recursive functions for both the factorial and Hermite polynomials took a long time to compute so both functions were replaced with loops that do not require recursion.

Calculating the quantum uncertainty in the position of the particle using three methods of Gaussian quadrature, Gauss-Hermite quadrature and SciPy's quad function (for exact answer with zero approximation error):

```
Quantum uncertainty using Gaussian-quadrature:  2.3452078737858177
Quantum uncertainty using Gauss-Hermite quadrature:  2.32808777917093
Quantum uncertainty using SciPy's quad function: 2.345207879911715
```

Please find my GitHub repository through this link.