

PHYS-UA 210 Computational Physics

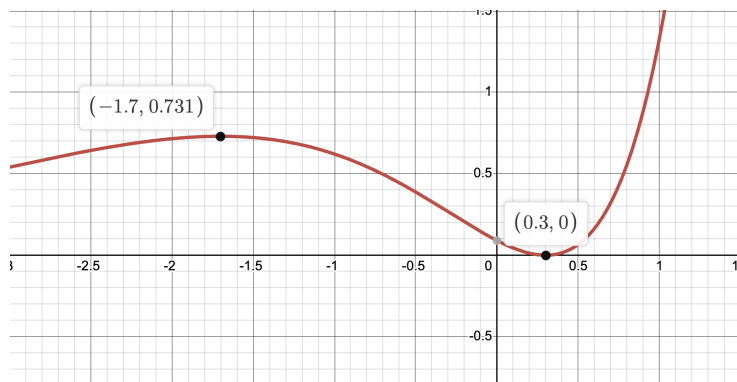
Problem Set 07

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Question 1: Brent's Method

Function to be minimized: $f(x) = (x - 0.3)^2 e^x$



Calculating the minimum of the function using Brent's method and SciPy's implementation:

```
Using Brent's method:  
0.30000001192092896
```

```
Using SciPy's method:  
-1917.273245625146
```

Over the interval $[-1, 5]$ the code was used on, Brent's method yields about 0.3 which is very close to the true minimum value. However, SciPy's implementation significantly strays from the true value. Though the lower bound does not go below -1, given that the function tends to zero as x goes to negative infinity,

perhaps the minimization method is yielding that large negative number where the function tends

Question 2: Logistic Regression

Logistic Regression helps in modeling the probability of a discrete outcome given an input variable. In this question, the outcome of a binary question with the answer 'yes' as 1 and 'no' as 0 is modelled with respect to the age of the population.

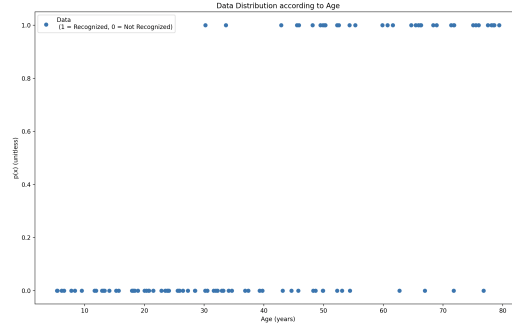
The probability of the logistic function is given as:

$$p(x) = \frac{1}{1 + \exp(-(\beta_0 + \beta_1 x))}$$

Here, β_0 and β_1 are the parameters which the model depends on, x is the age of the person the question is being asked and $p(x)$ is the probability of the answer being yes.

The aim of logistic regression is: finding the most fitting coefficients β_0 and β_1 according to the data provided meaning, finding the coefficient by the negative log-likelihood method and using optimization to minimize it.

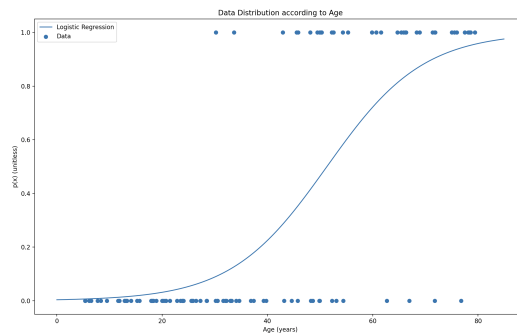
Initially, the data is distributed as follows:



By computing the negative log-likelihood and minimizing it, the optimal values for β_0 and β_1 along with their errors and the covariance matrix are:

```
Optimal parameters:  
b0 = -5.620231143131335  
b1 = 0.10956336528478754  
  
Errors:  
Error in b0 = 0.705756054226042  
Error in b1 = 0.014100511140330244  
  
Covariance Matrix:  
C = [[ 4.98091608e-01 -9.62404926e-03]  
      [-9.62404926e-03 1.98824414e-04]]
```

Finally, the linear regression model is given as follows:



The answers do make sense: the older people are more likely to answer yes as the phrase is old school and the model fits this behavior in general. The positive value of β_1 ensures that as x increases $\exp(-\beta_1)$ decreases which increases $p(x)$.

Please find my GitHub repository through this: [link](#).