

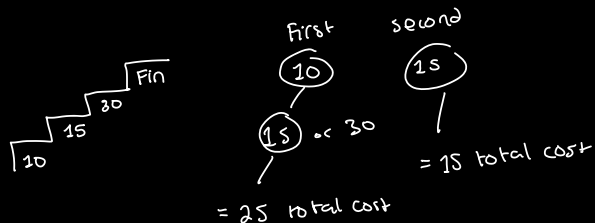
Problem: For staircase, the i -th step is assigned a non-negative cost indicated by a cost array.

When pay cost for step \rightarrow climb ONE or TWO steps

Find minimum cost to reach top of staircase

first step can be FIRST or SECOND step

$$\text{cost} = [10, 15, 30]$$



How to recognize dp problem?
optimization \rightarrow min
 \rightarrow max

① Based in Recursion \rightarrow base case

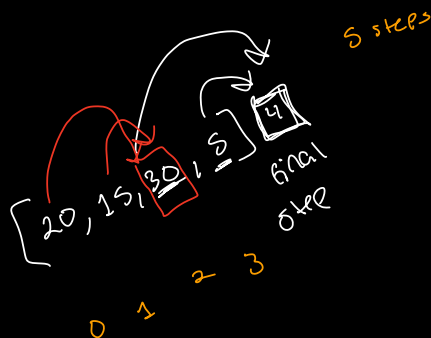
② Recurrence relation

formula for basis of recursive sol'n

func() :

base case

func()



$n=4$

$$\text{cost}(n) = \min(\text{minCost}(n-1), \text{minCost}(n-2)) + \text{cost}(n)$$

5 30 3 2

Top Down

$$\text{cost}(n-1) + \min(\text{minCost}(n-3), \text{minCost}(n-4))$$

recurrence relation

$$\text{minCost}(i) = \text{cost}[i] + \min(\text{minCost}(i-1), \text{minCost}(i-2))$$

base case

$$\begin{cases} i < 0: \text{return } 0 \\ i == 0: \text{return cost}[0] \\ i == 1: \text{return cost}[1] \end{cases}$$

const minCostClimbingStairs = function(cost) {
 const n = cost.length;
 return Math.min(minCost(n-1, cost),
 minCost(n-2, cost));

}

const minCost = (i, cost) =>
 if (i < 0) return 0
 if (i == 0 or i == 1) return cost[i]
 (minCost(i-1, cost),

return $\text{cost}[i] + \min(\dots, \dots)$
 $\min \text{cost}(i-2, \text{cost})$

recurrence relation

Time: 2^n

Space: (n)

call stack only contains of single branch down to bottom of binary tree AT WORST

— stack contains max height of tree

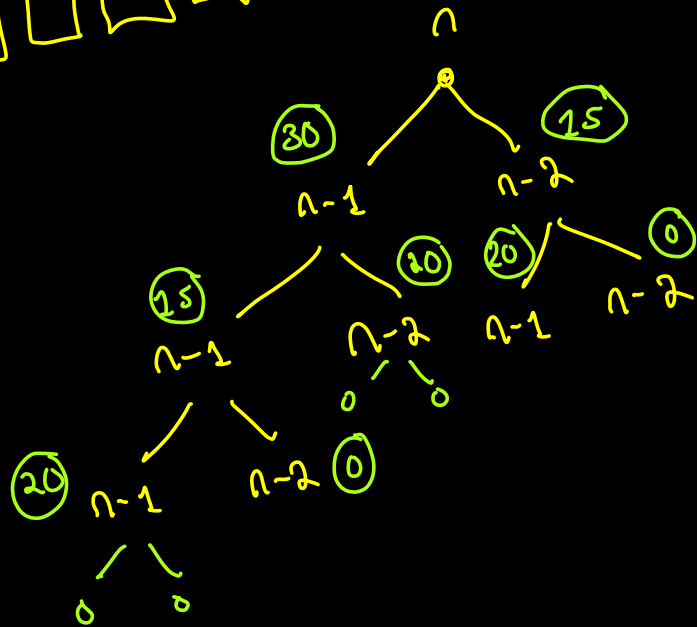
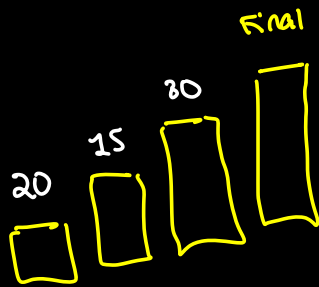
$$2^0 = 1$$

$$2^1 = 2$$

$$2^2 = 4$$

$$2^3 = 8$$

for every call, call 2 recursive functions



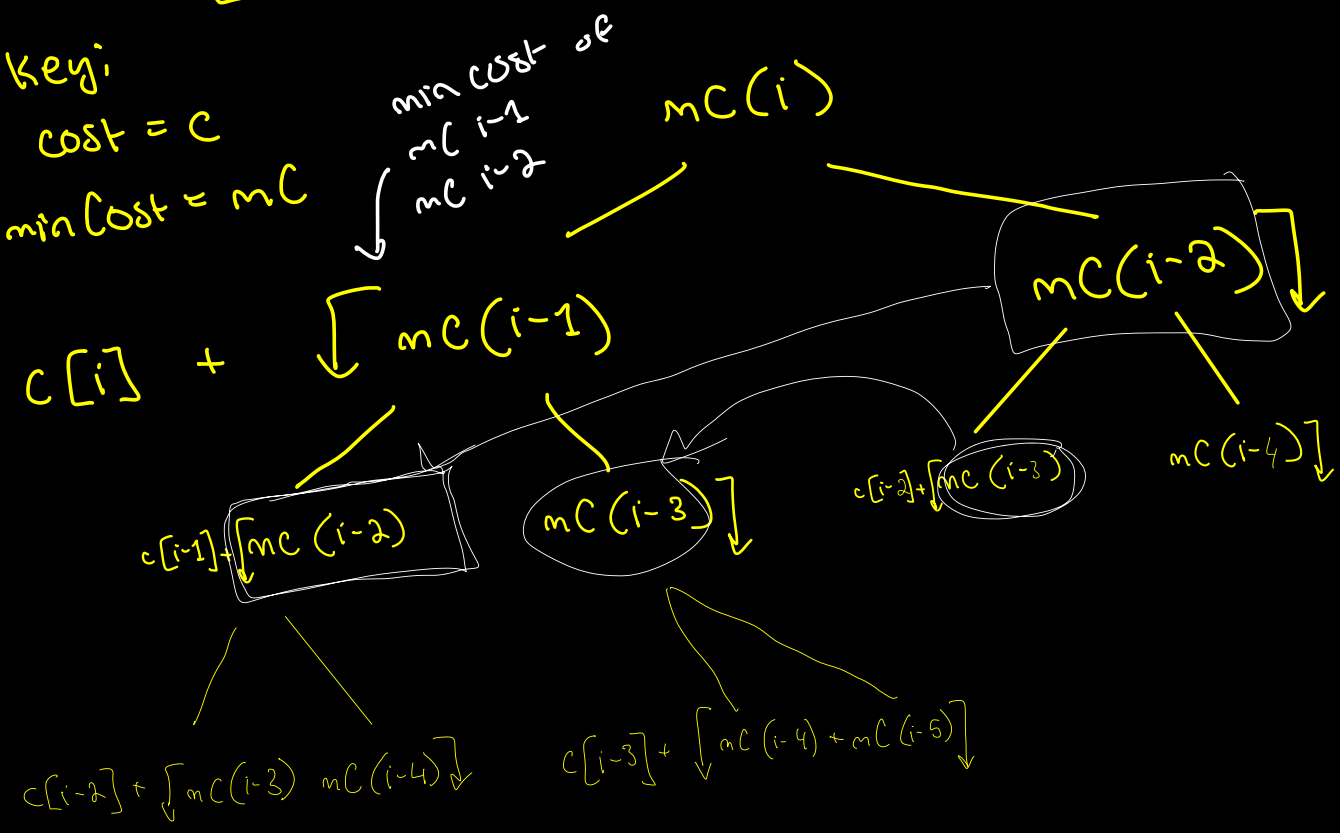
double the calls for

every value
of n

- ① define recurrence relation
- ② create (brute force) recursive function

③ memoize — state based tree

key:
cost = c
min cost = mC



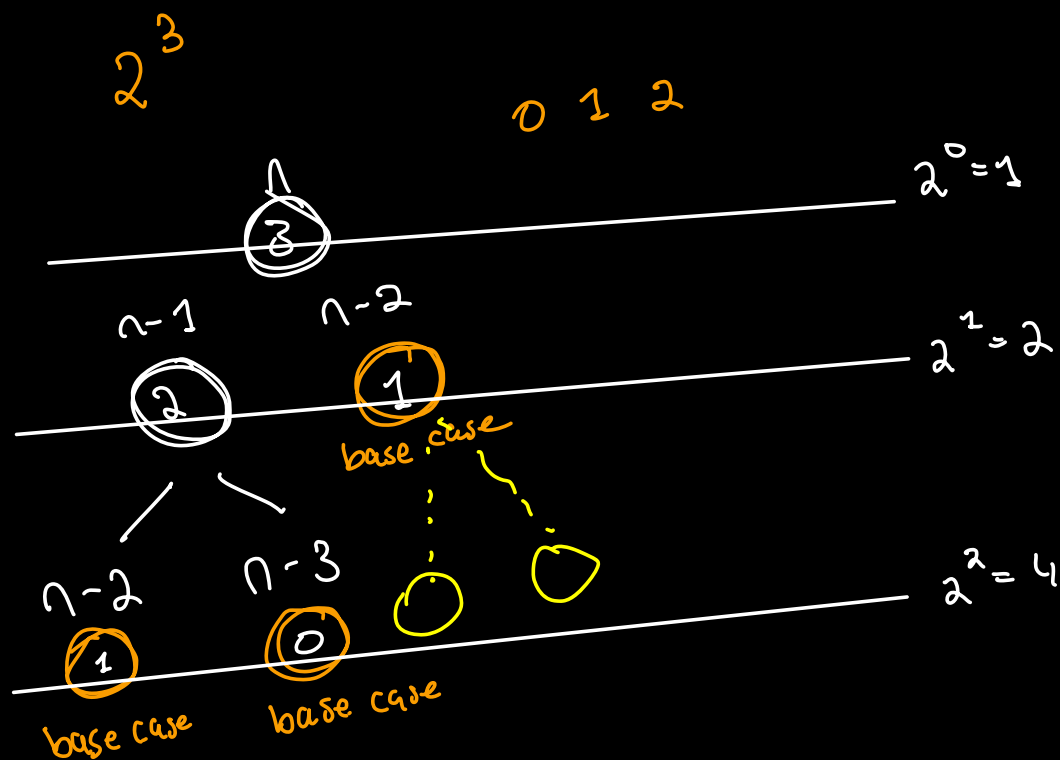
Note: Data structure used to memoize == Data structure iterated thru
(in this case: array)

?

which side will hit
the base case first?

→ the right most branch

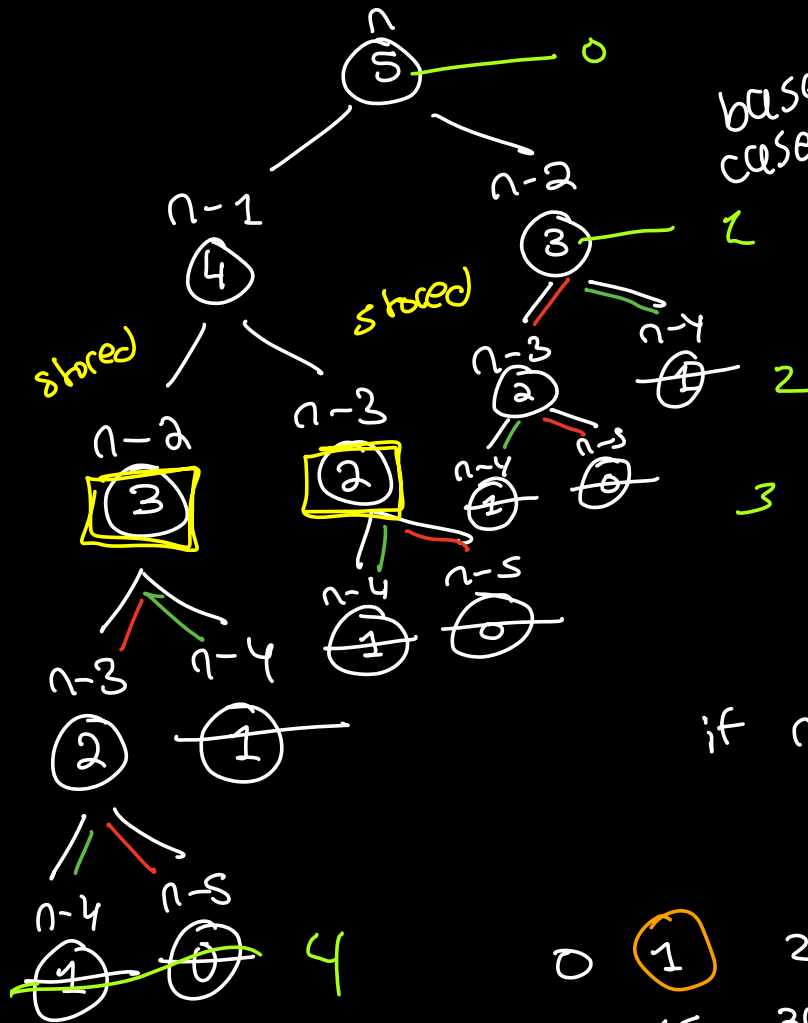
memoize the right most branch,
then if traversing thru - if
the value hasn't been seen before -
include it in the array!



The last 2
levels of the tree
will contain

BASE
CASES

$n=5$



base cases

array

0	:	$\text{cost}[0] = 20$
1	:	$\text{cost}[1] = 15$
2	:	45
3	:	55
4	:	110
<hr/>		
i		

if $n == i+1$:

$\text{cost}[1] + \text{cost}[3] = 55$

0	1	2	3	4	
20	15	30	5	10	

0 1 1 → 2

$(2) \rightarrow \min(20, 15) = 15$

$15 + \text{cost}[2] = 45$

$(3) \rightarrow \min(15, 45) = 15 \quad 1 \rightarrow 3$

$15 + \text{cost}[3] = 20$

2, 3

$(0) \rightarrow 20$

$(1) \rightarrow 15$

$$\textcircled{4} \rightarrow \min(45, 20) = 20 \quad 1 \rightarrow 2 \rightarrow 4$$

$$20 + \text{cost}[4] = 30$$

$$\textcircled{5} \rightarrow \min\left(\overset{3}{20}, \overset{4}{30}\right) = \boxed{20+0}$$

let array = $-1 \cdot n$ initialize array size n w -1 values
 array[0] = cost[0]
 array[1] = cost[1]
 const minCostClimbingStairs = function(cost) {
 const n = cost.length;
 return Math.min(minCost(n-1, cost),
 minCost(n-2, cost));
 }

const minCost = (i, cost, array) =>
 if (i < 0) return 0
 if (i == 0 or i == 1) return cost[i]

right = (array[i-1] == -1) ? minCost(i-1, cost) : array[i-1]

left = (array[i-2] == -1) ? minCost(i-2, cost) : array[i-2]

array[i] = cost[i] + min(left, right)

return array[i]

})
 leetcode

Bottom-Up (Iterative)

$$\text{minCost}(0) = \text{cost}[0]$$

$$\text{minCost}(1) = \text{cost}[1]$$

$$2: \min \left(\overset{i-2}{\text{mC}(0)}, \overset{i-1}{\text{mC}(1)} \right) + \text{cost}[2]$$

$$3: \min \left(\overset{i-2}{\text{mC}(1)}, \overset{i-1}{\text{mC}(2)} \right) + \text{cost}[3]$$

$$4: \min \left(\overset{i-2}{\text{mC}(2)}, \overset{i-1}{\text{mC}(3)} \right) + \text{cost}[4]$$

Intro:

- Verify Constraints
- Create Testcases

Brute Force:

- Brainstorming & Pattern Observations
- Pseudocode
- Write code
- Run through testcases
- Analyze time and space complexity

Optimal:

- Brainstorming & Pattern Observations
- Pseudocode
- Write code
- Run through testcases
- Analyze time and space complexity

1 → recurrence relation

used to
build recursive func

2 → recursive func

Top



down

determine if
optimization using
memoization

convert

3 Top ⇒ ↑
 ↓ Bottom

4 Derive iterative
 sol'n for ↑
 Bottom

check for wasted space
S minimize
space complexity