1. For what values of k, the system of linear equation

   (a) $x + y + z + = 2$

   (b) $2x + y - z = 3$

   (c) $3x + 2y + k = 4$
   has a unique solution ?

2. If $\overrightarrow{a} = 4\hat{i} - \hat{j} + \hat{k}$ and $\overrightarrow{b} = 2\hat{i} - 2\hat{j} + \hat{k}$, them find a unit vector parallel to the vector $\overrightarrow{a} + \overrightarrow{b}$.

3. Find λ and μ if
   $\left(\hat{i} + 3\hat{j} + 9\hat{k}\right) \times \left(3\hat{i} - \lambda\hat{j} + \mu\hat{k}\right) = \overrightarrow{0}$.

4. Write the sum of intercepts cut off by the plane $\overrightarrow{r}.\left(2\hat{i} + \hat{j} - \hat{k}\right) - 5 = 0$ on the three axis.

5. If $A = \begin{bmatrix} cos\alpha & sin\alpha \\ -sin\alpha & cos\alpha \end{bmatrix}, find\ \alpha\ satisfying\ 0 < \alpha < \frac{\pi}{2}$. when $A + A^T = \sqrt{2}I_2$;
   where A^T is traspose of A.

6. If A is a 3×3 matrix and $\left|3A\right| = k\left|A\right|$, then write the value of k.

7. Find: $\int (x + 3)\sqrt{3 - 4x - x^2}$ dx.

8. Evaluate: $\int_{-2}^{2} \frac{x^2}{1+5x}$ dx.

9. Find the equation of tangents to the curve $y = x^3 + 2x - 4$, which are perpendicular to line $x + 14y + 3 = 0$

10.
$$Iff(x) = \begin{cases} \frac{\sin(a+1)x + 2\sin x}{x}, & x < 0 \\ 2, & x = 0 \\ \frac{\sqrt{1+bx-1}}{x}, & x > 0 \end{cases}$$
   is continuous at x=0, then find the values of a and b.

11. Solve for x : tan^{-1} $(x - 1) + tan^{-1}$ $(x + 1) = tan^{-1}$ 3x.

12. prove that tan^{-1} $[\frac{6x-8x^3}{1-12x^2}]$ $-tan^{-1}$ $[\frac{4x}{1-4x^2}]$=tan^{-1}2x; $\left|2x\right| < \frac{1}{\sqrt{3}}$.

13. If x $cos(a+y)$=$cosy$ then prove that $\frac{dy}{dx}=\frac{cos^2(a+y)}{sina}$.
Hence show that sina $\frac{d^2y}{dx^2}$+sin2$(a+y)\frac{dy}{dx}=0$.

14. Find $\frac{dy}{dx}$ if $y=sin^{-1}[\frac{6x-4\sqrt{1-4x^2}}{5}]$

15. A bag X contains 4 white balls and 2 black balls, while another bag Y contains 3 white balls and 3 black balls. Two balls are drawn ($withoutreplacement$) at random from one of the bags and were found to be one white and one black. Find the probability that the balls were drawn from bag Y.

16. A and B throw a pair of dice alternately, till one of them gets a total of 10 and wins the game. Find their respective probabilities of winning, if A starts first.

17. Find the coordinates of the foot of perpendicular drawn from the point $A\,(-1,8,4)$ to the line joining the points $B\,(0,-1,3)$ and $C\,(2,-3,-1)$. Hence find the image of the A in the line BC.

18. Show that the four points $A\,(4,5,1)$, $B\,(0,-1,1)$, $C\,(3,9,4)$ and $D\,(-4,4,4)$ are coplanar.

19. A typist charges ₹145 for typing 10 English and 3 Hind pages,while charges for typing 3 english and 10 Hindi pages are ₹180. Using matrices, find the charges of typing one English and one Hindi page separately. However typist charged only ₹2 per page from a poor student Shyam for 5 Hindi pages. How much less was charged from this poor boy ? Which values are reflected in this problem ?

20. Find the particular solution of the differential equation
$2y\ e^{\frac{x}{y}}$ dx+$(y-2x)\ e^{\frac{x}{y}}dy=0$
given that $x=0$ when $y=1$.

21. Find the particular solution of differential equation : $\frac{dy}{dx}=-\frac{x+ycosx}{1+sinx}$
given that y=1 when x=0.

22. Find : $\int\frac{(2x-5)e^{2x}}{(2x-3)^3}dx$

23. Find : $\int\frac{x^2+x+1}{(x^2+1)(x+2)}dx$

24. Prove that $y=\frac{4sin\theta}{2+cos\theta}-\theta$ is an increasing function of θ on $[0,\frac{\pi}{2}]$.

25. Show that semi-vertical angle of a cone of maximum volume and given slant height is $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$

26. Using properties of determinants, prove that
$$\begin{bmatrix} (x+y)^2 & zx & zy \\ zx & (z+y)^2 & xy \\ zy & xy & (z+x)^2 \end{bmatrix} = 2xyz\,(x+y+z)^3$$

27. If $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$ and $A^3 - 6A^2 + 7A + kI_3 = O find k$

28. Using the method of integration, find the area of the triangular region whose vertices are $(2, -2)$,$(4, 3)$ and $(1, 2)$.

29. Let $A = R \times R$ and * be a binary operation on A defined by
$(a, b) * (c, d) = (a + c, b + d)$
Show that * is commutative and associative.
Find the identity element for * on A Also find the inverse of every element $(a, b)\, \epsilon A$.

30. Three numbers are selected at random (without replacement) from first six positive integers. Let X donete the largest of the three numbers obtained. Find the probability di.stribution of X. Also, find the mean and variane of the distribution.

31. A retired person wants to invest an amount of ₹50, 000. His broker recommends investing in two type of bonds $'A'$ and $'B'$ yielding 10% and 90% return respectively on the invested amount. He decides to invest at least ₹20, 000 in bond $'A'$ and at least ₹10, 000 in bond $'B'$. He also wants to invest at leasst as much in bond $'A'$ as in bond $'B'$. Solve this linear programming problem graphically to maximise his returns.

32. Find the equation of the plane which contains the line of intersection of the planes.
$\vec{r}.\left(\hat{i} - 2\hat{j} - 3\hat{k}\right) - 4 = 0$ and
$\vec{r}.\left(-2\hat{i} + \hat{j} + \hat{k}\right) + 5 = 0$
and whose interrcept on x-axis is equal to that of on y-axis.