SECTION - A

1. For what values of k, the system of linear equation

   (a) $x + y + z+ = 2$
   (b) $2x + y - z = 3$
   (c) $3x + 2y + k = 4$
      has a unique solution ?

2. If $\vec{a} = 4\hat{i} - \hat{j} + \hat{k}$ and $\vec{b} = 2\hat{i} - 2\hat{j} + \hat{k}$, them find a unit vector parallel to the vector $\vec{a} + \vec{b}$.

3. Find λ and μ if
   $(\hat{i} + 3\hat{j} + 9\hat{k}) \times (3\hat{i}-\lambda\hat{j}+\mu\hat{k}) = \vec{0}$.

4. Write the sum of intercepts cut off by the plane $\vec{r}. (2\hat{i} + \hat{j} - \hat{k}) - 5 = 0$ on the three axis.

5. If $A = \begin{bmatrix} cos\alpha & sin\alpha \\ -sin\alpha & cos\alpha \end{bmatrix}$, $find \ \alpha \ satisfying \ 0 < \alpha < \frac{\pi}{2}$ when $A+A^T = \sqrt{2}I_2$; where A^T is traspose of A.

6. If A is a 3×3 matrix and $|3A| = k|A|$, then write the value of k.

7. Find: $\int(x+3) \sqrt{3 - 4x - x^2} \, dx$.

8. Evaluate: $\int_{-2}^{2} \frac{x^2}{1+5x} \, dx$.

9. Find the equation of tangents to the curve $y = x^3 + 2x - 4$, which are perpendicular to line $x + 14y + 3 = 0$

10.
$$If f(x) = \begin{cases} \frac{\sin(a+1)x+2 \sin x}{x}, & x < 0 \\ 2, & x = 0 \\ \frac{\sqrt{1+bx}-1}{x}, & x > 0 \end{cases}$$

   is continuous at x=0, then find the values of a and b.

11. Solve for $x : tan^{-1} (x - 1) + tan^{-1}(x + 1) = tan^{-1} 3x$.

1

12. prove that $tan^{-1}[\frac{6x-8x^3}{1-12x^2}] - tan^{-1}[\frac{4x}{1-4x^2}] = tan^{-1}2x; |2x| < \frac{1}{\sqrt{3}}$.

13. If x $cos(a + y) = cosy$ then prove that $\frac{dy}{dx} = \frac{cos^2(a+y)}{sina}$.

    Hence show that sina $\frac{d^2y}{dx^2} + sin2(a+y)\frac{dy}{dx} = 0$.

14. Find $\frac{dy}{dx}$ if $y = sin^{-1}[\frac{6x-4\sqrt{1-4x^2}}{5}]$

15. A bag X contains 4 white balls and 2 black balls, while another bag Y contains 3 white balls and 3 black balls. Two balls are drawn (without replacement) at random from one of the bags and were found to be one white and one black. Find the probability that the balls were drawn from bag Y.

16. A and B throw a pair of dice alternately, till one of them gets a total of 10 and wins the game. Find their respective probabilities of winning, if A starts first.

17. Find the coordinates of the foot of perpendicular drawn from the point $A(-1, 8, 4)$ to the line joining the points $B(0, -1, 3)$ and $C(2, -3, -1)$. Hence find the image of the A in the line BC.

18. Show that the four points $A(4, 5, 1)$, $B(0, -1, 1)$, $C(3, 9, 4)$ and $D(-4, 4, 4)$ are coplanar.

19. A typist charges ₹145 for typing 10 English and 3 Hind pages,while charges for typing 3 english and 10 Hindi pages are ₹180. Using matrices, find the charges of typing one English and one Hindi page separately. However typist charged only ₹2 per page from a poor student Shyam for 5 Hindi pages. How much less was charged from this poor boy ? Which values are reflected in this problem ?

20. Find the particular solution of the differential equation
    $2y\ e^{\frac{x}{y}}$ dx+(y-2x $e^{\frac{x}{y}}$ dy = 0)
    given that $x = 0$ when $y = 1$.

21. Find the particular solution of differential equation : $\frac{dy}{dx} = -\frac{x+ycosx}{1+sinx}$
    given that y=1 when x=0.

22. Find : $\int \frac{(2x-5)e^{2x}}{(2x-3)^3} dx$

23. Find : $\int \frac{x^2+x+1}{(x^2+1)(x+2)} dx$

24. Prove that $y = \frac{4sin\theta}{2+cos\theta} - \theta$ is an increasing function of θ on $[0, \frac{\pi}{2}]$.

25. Show that semi-vertical angle of a cone of maximum volume and given slant height is $\cos^{-1}(\frac{1}{\sqrt{3}})$

26. Using properties of determinants, prove that
$$\begin{vmatrix} (x+y)^2 & zx & zy \\ zx & (z+y)^2 & xy \\ zy & xy & (z+x)^2 \end{vmatrix} = 2xyz(x+y+z)^3$$

27. If $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$ and $A^3 - 6A^2 + 7A + kI_3 = O \, find \, k$

28. Using the method of integration, find the area of the triangular region whose vertices are $(2, -2), (4, 3)$ and $(1, 2)$.

29. Let $A = R \times R$ and * be a binary operation on A defined by
$(a, b) * (c, d) = (a + c, b + d)$
Show that * is commutative and associative.
Find the identity element for * on A Also find the inverse of every element $(a, b)\epsilon A$.

30. Three numbers are selected at random (without replacement) from first six positive integers. Let X donete the largest of the three numbers obtained. Find the probability di.stribution of X. Also, find the mean and variane of the distribution.

31. A retired person wants to invest an amount of ₹50, 000. His broker recommends investing in two type of bonds $'A'$ and $'B'$ yielding 10% and 90% return respectively on the invested amount. He decides to invest at least ₹20, 000 in bond $'A'$ and at least ₹10, 000 in bond $'B'$. He also wants to invest at leasst as much in bond $'A'$ as in bond $'B'$. Solve this linear programming problem graphically to maximise his returns.

32. Find the equation of the plane which contains the line of intersection of the planes.
$\overrightarrow{r}.(\hat{i} - 2\hat{j} - 3\hat{k}) - 4 = 0$ and

$\vec{r}.(-2\hat{i} + \hat{j} + \hat{k}) + 5 = 0$

and whose interrcept on x-axis is equal to that of on y-axis.