1. S is the set of all (h, k) with h, k non-negative integers such that $h + k<n$. Each element of S is colored red or blue, so that if (h, k) is red and $h' \leq h, k' \leq k$, then (h', k') is also red. A type 1 subset of S has n blue elements with different first member and a type 2 subset of S has n blue elements with different second member. Show that there are the same number of type 1 and type 2 subsets.

2. BC is a diameter of a circle center O. A is any point on the circle with $\angle AOC>60^o$. EF is the chord which is the perpendicular bisector of AO. D is the midpoint of the minor arc AB. The line through O parallel to AD meets AC at J. Show that J is the incenter of triangle CEF.

3. Find all pairs of integer $m>2, n>2$ such that there are infinitely many positive integers k for which $k^n + k^2 - 1$ divides $k^m + k - 1$.

4. The positive divisors of the integer $n \geq 1$ are $d_1 \leq d_2 \leq \ldots \leq d_k$, so that $d_1 = 1, d_k = n$. Let $d = d_1 d_2 + d_2 d_3 + \ldots d_k - d_k$. Show that $d \leq n^2$ and find all n for which d divides n^2.

5. Find all real-valued functions on the reals such that $\left(f(x) + f(y)\right)(f(u) + f(v)) = f(xu - yv) = f(xv - yu)$ for all x, y, u, v.

6. $n>2$ circlesof radius 1 are drawn in the plane so that no line meets more than two of the circles. Their centers are $0_1, 0_2 \ldots 0_n$. Show that $\sum_{i<} 1/0_i 0_j \leq (n-1)\frac{\pi}{4}$.