1. S is the set $\{1, 2, 3, \ldots, 1000000\}$. Show that for any subset A of S with 101 elements we can find 100 distinct elements x_i of S, such that the sets $\{a + x_i a \in A\}$ are all pairwise disjoint.

2. Find all pairs (m, n) of positive integers such that $\frac{m^2}{2mn^2 - n^3 + 1}$ is a positive integer.

3. A convex hexagon has the property that for any pair of opposite sides the distance between their midpoints is $\frac{\sqrt{3}}{2}$ times the sum of their lengths Show that all the hexagon's angles are equal.

4. $ABCD$ is cyclic. The feet of the perpendicular from D to the lines AB, BC, CA are P, Q, R respectively. Show that the angle bisectors of ABC and CDA meet on the line AC iff $RP = RQ$.

5. Given n>2 and reals $x_1 \leq x_2 \leq \ldots \leq x_n$, show that $\left(\sum_{i,j} \left|x_i - x_j\right|^2\right) \leq \frac{2}{3}\left(n^2 - 1\right) \sum_{i,j} \left(x_i - x_j\right)^2$Show that we have equality iff the sequence is an arithmetic progression.

6. Show that for each prime p, there exists a prime q such that $n^p - p$ is not divisible by q for any positive integer n.