ASSIGNMENT 2

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Download all python codes from

https://github.com/balumurisandhyarani550/ASSIGNMENT2/tree/main/ASSIGNMENT4/CODES

and latex-tikz codes from

https://github.com/balumurisandhyarani550/ASSIGNMENT2/tree/main/ASSIGNMENT4

1 Question No 2.8

Which of the following pairs of linear equations are consistent/inconsistent, obtain the solution:

1)

$$\begin{pmatrix}1 & 1\end{pmatrix}\mathbf{x} = 5$$
$$\begin{pmatrix}2 & 2\end{pmatrix}\mathbf{x} = 10 \tag{1.0.1}$$

2)

$$\begin{pmatrix}1 & -1\end{pmatrix}\mathbf{x} = 8$$
$$\begin{pmatrix}3 & -3\end{pmatrix}\mathbf{x} = 16 \tag{1.0.2}$$

2 SOLUTION

1)

$$\begin{pmatrix}1 & 1\end{pmatrix}\mathbf{x} = 5$$
$$\begin{pmatrix}2 & 2\end{pmatrix}\mathbf{x} = 10 \tag{2.0.1}$$

The above equations can be expressed as the matrix equation

$$\begin{pmatrix}1 & 1\\2 & 2\end{pmatrix}\mathbf{x} = \begin{pmatrix}5\\10\end{pmatrix} \tag{2.0.2}$$

The augmented matrix for the above equation is row reduced as follows

$$\begin{pmatrix}1 & 1 & 5\\2 & 2 & 10\end{pmatrix} \xleftrightarrow{R_2 \leftarrow R_2 - 2R_1} \begin{pmatrix}1 & 1 & 5\\0 & 0 & 0\end{pmatrix} \tag{2.0.3}$$

$\because$ row reduction of the 2×3 matrix

$$\begin{pmatrix}1 & 1 & 5\\2 & 2 & 10\end{pmatrix} \tag{2.0.4}$$

results in a matrix with 1 nonzero row, its rank is 1. Similarly, the rank of the matrix

$$\begin{pmatrix}1 & 1\\2 & 2\end{pmatrix} \tag{2.0.4}$$

is also 1.

$$\begin{aligned}\because Rank\begin{pmatrix}1 & 1\\2 & 2\end{pmatrix} &= Rank\begin{pmatrix}1 & 1 & 5\\2 & 2 & 10\end{pmatrix}\\ &= dim\begin{pmatrix}1 & 1\\2 & 2\end{pmatrix}\\ &= 1\end{aligned} \tag{2.0.5}$$

$\therefore$ Given lines (1.0.1) have infinitely many solutions so we can say they concide. The given lines are consistent.

2)

$$\begin{pmatrix}1 & -1\end{pmatrix}\mathbf{x} = 8$$
$$\begin{pmatrix}3 & -3\end{pmatrix}\mathbf{x} = 16 \tag{2.0.6}$$

The above equations can be expressed as the matrix equation

$$\begin{pmatrix}1 & -1\\3 & -3\end{pmatrix}//\mathbf{x} = \begin{pmatrix}8\\16\end{pmatrix} \tag{2.0.7}$$

The augmented matrix for the above equation is row reduced as follows

$$\begin{pmatrix}1 & -1 & 8\\3 & -3 & 16\end{pmatrix} \xleftrightarrow{R_2 \leftarrow R_2 - 3R_1} \begin{pmatrix}1 & -1 & 8\\0 & 0 & -8\end{pmatrix} \tag{2.0.8}$$

$$\tag{2.0.9}$$

$\because$ row reduction of the 2×3 matrix

$$\begin{pmatrix}1 & -1 & 8\\3 & -3 & 16\end{pmatrix} \tag{2.0.10}$$

results in a matrix with 2 nonzero rows, its rank is 2. Similarly, the rank of the matrix

$$\begin{pmatrix}1 & -1\\3 & -3\end{pmatrix} \tag{2.0.11}$$

is also 1.

$$\because Rank\begin{pmatrix}1 & -1\\ 3 & -3\end{pmatrix} \neq Rank\begin{pmatrix}1 & -1 & 8\\ 3 & -3 & 16\end{pmatrix} \tag{2.0.12}$$

$\therefore$ Given lines (1.0.2) have no solution so we say they are parallel. The givens lines are inconsistent. PLOT OF GIVEN LINES -

Plot of (1.0.1) -

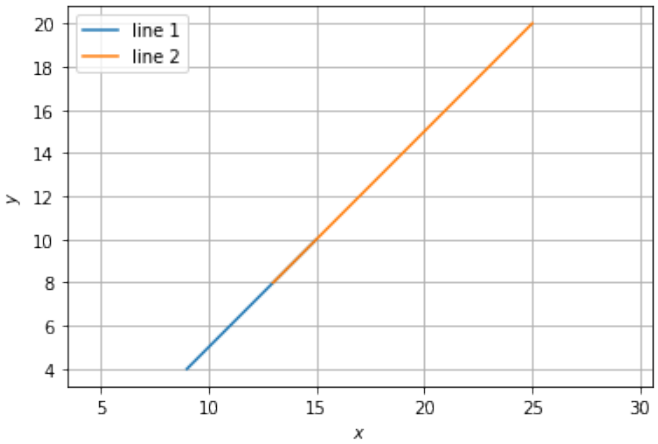

Fig. 2.1: SAME LINES

Plot of (1.0.2)

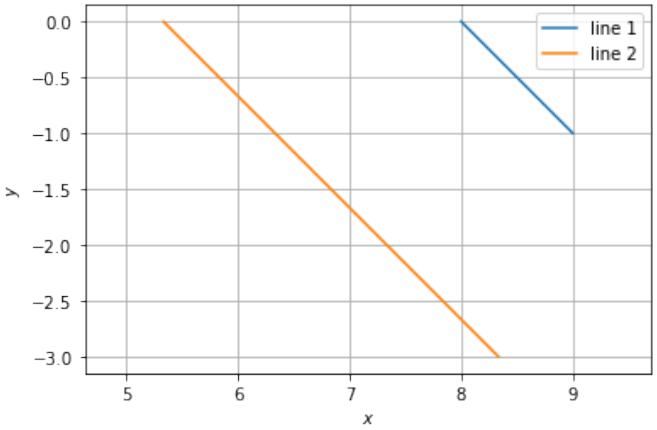

Fig. 2.2: Parallel lines