ASSIGNMENT 5

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Download all python codes from

https://github.com/balumurisandhyarani550/Assignment5/tree/main/Assignment5

and latex-tikz codes from

https://github.com/balumurisandhyarani550/Assignment5/tree/main/Assignment5

1 Question No 2.18(Quad forms)

Find the zero's of the quadratic polynomial $x^2 + 7x+10$ and verify the relationship between the zero's and the coefficients.

2 Solution

Given

$$y = x^2 + 7x + 10 \tag{2.0.1}$$

$$x^2 - y + 7x + 10 = 0 \tag{2.0.2}$$

compare with standard form of equation

$$ax^2 + bxy + cy^2 + 2dx + 2ey + f = 0 \tag{2.0.3}$$

a = 1 , b = 0 , c = 0 , d = $\frac{7}{2}$, e = $\frac{-1}{2}$, f = 10
Here,

$$\mathbf{V} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \mathbf{u} = \begin{pmatrix} \frac{7}{2} \\ \frac{-1}{2} \end{pmatrix}, f = 10 \tag{2.0.4}$$

Using eigenvalue decomposition,

$$\mathbf{D} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, \mathbf{P} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \tag{2.0.5}$$

Now,

$$\begin{pmatrix} \mathbf{u}^T + \eta \mathbf{p_1}^T \\ \mathbf{V} \end{pmatrix} \mathbf{c} = \begin{pmatrix} -f \\ \eta \mathbf{p_1} - \mathbf{u} \end{pmatrix} \tag{2.0.6}$$

$\therefore$Vertex $\mathbf{c}$ is given by

$$\begin{pmatrix} \frac{7}{2} & -1 \\ 1 & 0 \\ 0 & 0 \end{pmatrix} \mathbf{c} = \begin{pmatrix} -10 \\ \frac{-7}{2} \\ 0 \end{pmatrix} \tag{2.0.7}$$

$$\implies \begin{pmatrix} \frac{7}{2} & -1 \\ 1 & 0 \end{pmatrix} \mathbf{c} = \begin{pmatrix} -10 \\ \frac{-7}{2} \end{pmatrix} \tag{2.0.8}$$

$$\implies \mathbf{c} = \begin{pmatrix} \frac{-7}{2} \\ \frac{-9}{4} \end{pmatrix} \tag{2.0.9}$$

$$\begin{pmatrix} x1 \\ x2 \end{pmatrix} = \begin{pmatrix} \frac{-7}{2} \\ \frac{-9}{4} \end{pmatrix} \tag{2.0.10}$$

Now,

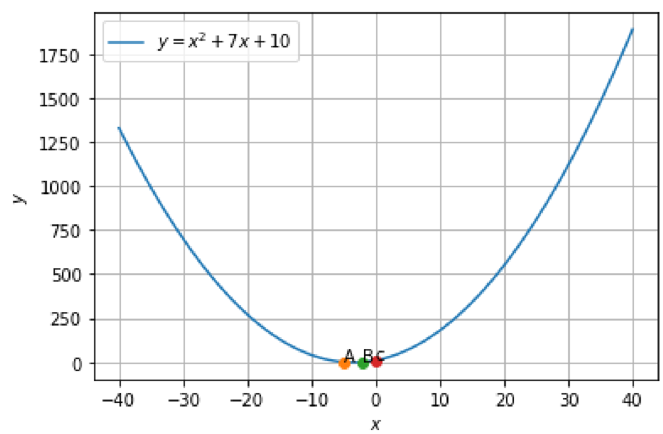

Fig. 2.1: $y = x^2 + 7x + 10$

$$\mathbf{p_1}^T \mathbf{c} = \begin{pmatrix} 0 & 1 \end{pmatrix} \begin{pmatrix} \frac{-7}{2} \\ \frac{-9}{4} \end{pmatrix} \tag{2.0.11}$$

$$= \frac{-9}{4} \tag{2.0.12}$$

and,

$$\mathbf{p_2}^T \mathbf{V} \mathbf{p_2} = \begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{2.0.13}$$

$$= 1 \tag{2.0.14}$$

$\because$

$$(\mathbf{p_1}^T \mathbf{c})(\mathbf{p_2}^T \mathbf{V} \mathbf{p_2}) = \frac{-9}{4} < 0 \tag{2.0.15}$$

Hence,the given equation has real roots.
Now,

$$\mathbf{a} = \mathbf{e_1}^T \mathbf{V} \mathbf{e_1} \tag{2.0.16}$$

$$\implies \begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \mathbf{1} \tag{2.0.17}$$

$$\mathbf{b} = \mathbf{2u}^T \mathbf{e_1} \tag{2.0.18}$$

$$\implies \mathbf{2} \begin{pmatrix} \frac{7}{2} & \frac{-1}{2} \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \mathbf{7} \tag{2.0.19}$$

$$\mathbf{c} = \mathbf{f} = \mathbf{10} \tag{2.0.20}$$

$$\tag{2.0.21}$$

therefore the roots are

$$\mathbf{x} = \left(\frac{\left(-b\right) + \sqrt{\mathbf{b}^2 - \mathbf{4ac}}}{\mathbf{2a}} \right) \tag{2.0.22}$$

$$= \frac{\left(-7\right) + \sqrt{9}}{\mathbf{2}} = -\mathbf{2} \tag{2.0.23}$$

$$\implies \mathbf{x} = \left(\frac{-\mathbf{b} - \sqrt{\mathbf{b}^2 - \mathbf{4ac}}}{\mathbf{2a}} \right) \tag{2.0.24}$$

$$= \frac{\left(-7\right) - \sqrt{9}}{\mathbf{2}} = -\mathbf{5} \tag{2.0.25}$$

$$\tag{2.0.26}$$