

ASSIGNMENT 5

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Download all python codes from

[https://github.com/balumurisandhyarani550/
Assignment5/tree/main/Assignment5](https://github.com/balumurisandhyarani550/Assignment5/tree/main/Assignment5)

and latex-tikz codes from

[https://github.com/balumurisandhyarani550/
Assignment5/tree/main/Assignment5](https://github.com/balumurisandhyarani550/Assignment5/tree/main/Assignment5)

Therefore , the corresponding roots are 0 , 1.

Eigen vectors corresponding to $\lambda = 0, 1$ respectively.

$$p_1 = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad (2.0.9)$$

$$p_2 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad (2.0.10)$$

Using eigenvalue decomposition,

$$\mathbf{D} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, \mathbf{P} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad (2.0.11)$$

Now,

$$\begin{pmatrix} \mathbf{u}^T + \eta \mathbf{p}_1^T \\ \mathbf{V} \end{pmatrix} \mathbf{c} = \begin{pmatrix} -f \\ \eta \mathbf{p}_1 - \mathbf{u} \end{pmatrix} \quad (2.0.12)$$

\therefore Vertex \mathbf{c} is given by

$$\begin{pmatrix} \frac{7}{2} & -1 \\ 1 & 0 \\ 0 & 0 \end{pmatrix} \mathbf{c} = \begin{pmatrix} -10 \\ \frac{-7}{2} \\ 0 \end{pmatrix} \quad (2.0.13)$$

$$\Rightarrow \begin{pmatrix} \frac{7}{2} & -1 \\ 1 & 0 \end{pmatrix} \mathbf{c} = \begin{pmatrix} -10 \\ \frac{-7}{2} \end{pmatrix} \quad (2.0.14)$$

$$\Rightarrow \mathbf{c} = \begin{pmatrix} \frac{-7}{4} \\ \frac{-9}{4} \end{pmatrix} \quad (2.0.15)$$

Now,

1 QUESTION No 2.18(QUAD FORMS)

Find the zero's of the quadratic polynomial $x^2 + 7x + 10$ and verify the relationship between the zero's and the coefficients.

2 SOLUTION

Given

$$y = x^2 + 7x + 10 \quad (2.0.1)$$

$$x^2 - y + 7x + 10 = 0 \quad (2.0.2)$$

compare with standard form of equation

$$ax^2 + bxy + cy^2 + 2dx + 2ey + f = 0 \quad (2.0.3)$$

$$a = 1, b = 0, c = 0, d = \frac{7}{2}, e = \frac{-1}{2}, f = 10$$

Here,

$$\mathbf{V} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \mathbf{u} = \begin{pmatrix} \frac{7}{2} \\ \frac{-1}{2} \end{pmatrix}, f = 10 \quad (2.0.4)$$

$$|V| = \begin{vmatrix} 1 & 0 \\ 0 & 0 \end{vmatrix} = 0 \quad (2.0.5)$$

Find the eigen values of corresponding \mathbf{V}

$$|V - \lambda I| = 0 \quad (2.0.6)$$

$$\begin{pmatrix} 1 - \lambda & 0 \\ 0 & -\lambda \end{pmatrix} = 0 \quad (2.0.7)$$

$$\lambda = 0, 1 \quad (2.0.8)$$

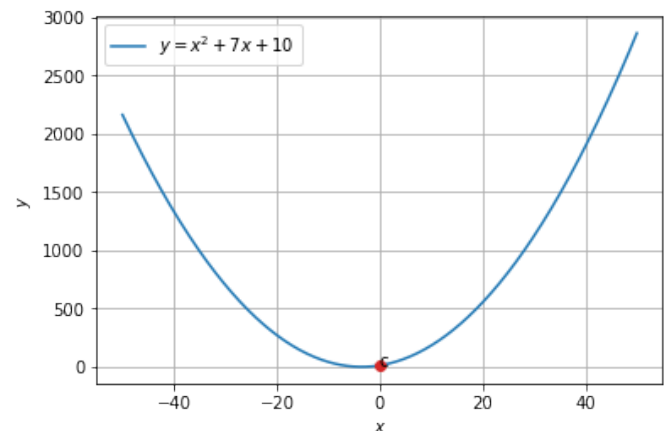


Fig. 2.1: $y = x^2 + 7x + 10$

$$\mathbf{p}_1^T \mathbf{c} = \begin{pmatrix} 0 & 1 \end{pmatrix} \begin{pmatrix} \frac{-7}{2} \\ \frac{-9}{4} \end{pmatrix} \quad (2.0.16)$$

$$= \frac{-9}{4} \quad (2.0.17)$$

and,

$$\mathbf{p}_2^T \mathbf{V} \mathbf{p}_2 = \begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad (2.0.18)$$

$$= 1 \quad (2.0.19)$$

\therefore

$$(\mathbf{p}_1^T \mathbf{c})(\mathbf{p}_2^T \mathbf{V} \mathbf{p}_2) = \frac{-9}{4} < 0 \quad (2.0.20)$$

Hence, the given equation has real roots.