ASSIGNMENT 6

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Download all python codes from

https://github.com/balumurisandhyarani550/Assignment6/tree/main/Assignment6

Latex-tikz codes from

https://github.com/balumurisandhyarani550/Assignment6/tree/main/Assignment6

1 Question No 2.72(b)

In each of the following find the equation for the ellipse that satisfies the given conditions:

1) Vertices$\begin{pmatrix} 0 \\ \pm 13 \end{pmatrix}$, foci $\begin{pmatrix} 0 \\ \pm 5 \end{pmatrix}$

2 Solution

Let

$$a = \sqrt{\frac{\mathbf{u}^\top \mathbf{V}^{-1}\mathbf{u} - f}{\lambda_1}}, b = \sqrt{\frac{\mathbf{u}^\top \mathbf{V}^{-1}\mathbf{u} - f}{\lambda_2}} \tag{2.0.1}$$

$$Now, c^2 = a^2 - b^2 \tag{2.0.2}$$

$$\implies 25 = \frac{\mathbf{u}^\top \mathbf{V}^{-1}\mathbf{u} - f}{\lambda_1} - \frac{\mathbf{u}^\top \mathbf{V}^{-1}\mathbf{u} - f}{\lambda_2} \tag{2.0.3}$$

given vertices $\begin{pmatrix} 0 \\ \pm a \end{pmatrix}$=$\begin{pmatrix} 0 \\ \pm 13 \end{pmatrix}$

$$\implies a = 13 \tag{2.0.4}$$

$$\sqrt{\frac{\mathbf{u}^\top \mathbf{V}^{-1}\mathbf{u} - f}{\lambda_1}} = 13 \tag{2.0.5}$$

$$\lambda_1 = \frac{\mathbf{u}^\top \mathbf{V}^{-1}\mathbf{u} - f}{169} \tag{2.0.6}$$

$$\implies \sqrt{\frac{\mathbf{u}^\top \mathbf{V}^{-1}\mathbf{u} - f}{\lambda_2}} = 12 \tag{2.0.7}$$

$$\lambda_2 = \frac{\mathbf{u}^\top \mathbf{V}^{-1}\mathbf{u} - f}{144} \tag{2.0.8}$$

Final equation is :

$$\frac{\mathbf{y}^\top D y}{\mathbf{u}^\top \mathbf{V}^{-1}\mathbf{u} - f} = 1 \tag{2.0.9}$$

$$\tag{2.0.10}$$

putting (2.0.6) and(2.0.8) in the above equation.

$$\implies \frac{\mathbf{y}^\top \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix}}{\mathbf{u}^\top \mathbf{V}^{-1}\mathbf{u} - f} = 1$$

$$\implies \mathbf{y}^\top \begin{pmatrix} \frac{1}{169} & 0 \\ 0 & \frac{1}{144} \end{pmatrix} = 1$$

Plot of ellipse:

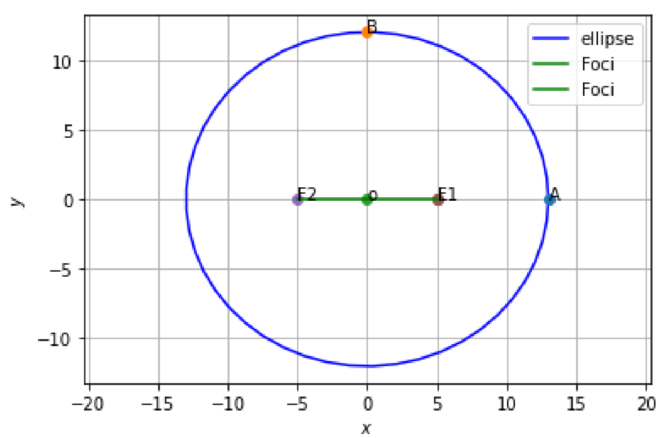

Fig. 2.1: Ellipse $\frac{x^2}{169} + \frac{y^2}{144} = 1$