1

Assignment-8

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Download all python codes from

therefore

is skew symmetric.

https://github.com/balumurisandhyarani/
Assignment8/tree/main/Assignment8

 $\mathbf{B}^{\mathsf{T}}\mathbf{A}\mathbf{B} \tag{2.0.15}$

and latex-tikz codes from

https://github.com/balumurisandhyarani550/ Assignment8/tree/main/Assignment8

1 QUESTION No-2.42

Show that the matrix $\mathbf{B}^{\mathsf{T}}\mathbf{A}\mathbf{B}$ is symmetric or skew symmetric according as \mathbf{A} is symmetric or skew symmetric.

2 Solution

If A be symmetric i.e.,

$$\mathbf{A}^{\mathsf{T}} = \mathbf{A} \tag{2.0.1}$$

then

$$(\mathbf{B}^{\mathsf{T}}\mathbf{A}\mathbf{B})^{\mathsf{T}} = [\mathbf{B}^{\mathsf{T}}(\mathbf{A}\mathbf{B})]^{\mathsf{T}}$$
(2.0.2)

$$(\mathbf{A}\mathbf{B})^{\mathsf{T}}(\mathbf{B}^{\mathsf{T}})^{\mathsf{T}}$$
 (2.0.3)

$$= (\mathbf{B}^{\mathsf{T}} \mathbf{A}^{\mathsf{T}}) \mathbf{B} \tag{2.0.4}$$

$$= \mathbf{B}^{\mathsf{T}} \mathbf{A}^{\mathsf{T}} \mathbf{B} \tag{2.0.5}$$

$$= \mathbf{B}^{\mathsf{T}} \mathbf{A} \mathbf{B} \tag{2.0.6}$$

(2.0.7)

Hence

$$\mathbf{B}^{\mathsf{T}}\mathbf{A}\mathbf{B} \tag{2.0.8}$$

is symmetric.

If A is skew symmetric i. e.,

$$\mathbf{A}^{\mathsf{T}} = -\mathbf{A} \tag{2.0.9}$$

then

$$(\mathbf{B}^{\mathsf{T}}\mathbf{A}\mathbf{B})^{\mathsf{T}} = [\mathbf{B}^{\mathsf{T}}(\mathbf{A}\mathbf{B})]^{\mathsf{T}}$$
 (2.0.10)

$$= (\mathbf{A}\mathbf{B})^{\mathsf{T}} (\mathbf{B}^{\mathsf{T}})^{\mathsf{T}} \tag{2.0.11}$$

$$= (\mathbf{B}^{\mathsf{T}} \mathbf{A}^{\mathsf{T}}) \mathbf{B} \tag{2.0.12}$$

$$= \mathbf{B}^{\mathsf{T}}(-\mathbf{A})\mathbf{B} \tag{2.0.13}$$

$$= -(\mathbf{B}^{\mathsf{T}} \mathbf{A} \mathbf{B}) \tag{2.0.14}$$