Assignment-8

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Download all python codes from

https://github.com/balumurisandhyarani/Assignment8/tree/main/Assignment8

and latex-tikz codes from

https://github.com/balumurisandhyarani550/Assignment8/tree/main/Assignment8

1 QUESTION No-2.42

Show that the matrix

$$B^\top \mathbf{AB} \tag{1.0.1}$$

is symmetric or skew symmetric according as A is symmetric or skew symmetric.

2 Solution

If A be symmetric i.e.,

$$\mathbf{A}^\top = \mathbf{A} \tag{2.0.1}$$

then

$$\begin{aligned}(\mathbf{B}^\top\mathbf{AB})^\top &= [\mathbf{B}^\top(\mathbf{AB})]^\top && (2.0.2)\\ &= \left(AB\right)^\top\left(B^\top\right)^\top && (2.0.3)\\ &= (\mathbf{B}^\top\mathbf{A}^\top)\mathbf{B} && (2.0.4)\\ &= \mathbf{B}^\top\mathbf{A}^\top\mathbf{B} && (2.0.5)\\ &= \mathbf{B}^\top\mathbf{AB} && (2.0.6)\\ & && (2.0.7)\end{aligned}$$

Hence

$$\mathbf{B}^\top\mathbf{AB} \tag{2.0.8}$$

is symmetric.
If A is skew symmetric i. e.,

$$\mathbf{A}^\top = -\mathbf{A} \tag{2.0.9}$$

then

$$\begin{aligned}(\mathbf{B}^\top\mathbf{AB})^\top &= [\mathbf{B}^\top(\mathbf{AB})]^\top && (2.0.10)\\ &= (\mathbf{AB})^\top\left(B^\top\right)^\top && (2.0.11)\\ &= (\mathbf{B}^\top\mathbf{A}^\top)\mathbf{B} && (2.0.12)\\ &= \mathbf{B}^\top\left(-A\right)\mathbf{B} && (2.0.13)\\ &= -(\mathbf{B}^\top\mathbf{AB}) && (2.0.14)\end{aligned}$$

therefore

$$\mathbf{B}^\top\mathbf{AB} \tag{2.0.15}$$

is skew symmetric.