

# Assignment 3

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Download all python codes from

<https://github.com/satyasm45/Summer-Internship/tree/main/Assignment-3/Codes>

and latex-tikz codes from

<https://github.com/satyasm45/Summer-Internship/tree/main/Assignment-3>

## 1 QUESTION No. 2.55

Let **A** and **B** be the centres of two circles of equal radii 3 such that each one of them passes through the centre of the other. Let them intersect at **C** and **D**. Is  $AB \perp CD$ ?

## 2 SOLUTION

To perform the given construction let us assume

$$\mathbf{A} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (2.0.1)$$

Based on the constraints given in the question, **B** will lie on the circle with center as **A** and radius 3. Without loss of generality, let us assume:

$$\mathbf{B} = \begin{pmatrix} 3 \\ 0 \end{pmatrix} \quad (2.0.2)$$

Then,

$$\|\mathbf{B} - \mathbf{A}\| = \|\mathbf{A} - \mathbf{B}\| = \|\mathbf{B}\| = 3 \quad (\because \mathbf{A} = 0) \quad (2.0.3)$$

The centers and radii of the two circles are now given in table 2.1

	Circle 1	Circle 2
Centre	$\mathbf{A} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$	$\mathbf{B} = \begin{pmatrix} 3 \\ 0 \end{pmatrix}$
Radius	$r_1 = 3$	$r_2 = 3$

TABLE 2.1: Input values

Let us define:

$$\mathbf{u} = \begin{pmatrix} \cos \theta \\ \sin \theta \end{pmatrix}, \theta \in [0, 2\pi]. \quad (2.0.4)$$

Then any point  $\begin{pmatrix} x_1 \\ y_1 \end{pmatrix}$  on circle 1 is given by :

$$\begin{pmatrix} x_1 \\ y_1 \end{pmatrix} = \mathbf{A} + r_1 \mathbf{u} = 3 \begin{pmatrix} \cos \theta \\ \sin \theta \end{pmatrix} \quad (\because \mathbf{A} = 0) \quad (2.0.5)$$

This is the locus of Circle 1.

Similarly, locus of circle 2 is given by:

$$\begin{pmatrix} x_2 \\ y_2 \end{pmatrix} = \mathbf{B} + r_2 \mathbf{u} = \begin{pmatrix} 3 \\ 0 \end{pmatrix} + 3 \begin{pmatrix} \cos \theta \\ \sin \theta \end{pmatrix} \quad (2.0.6)$$

Using the locus of the circles Fig. 2.1 was plotted.

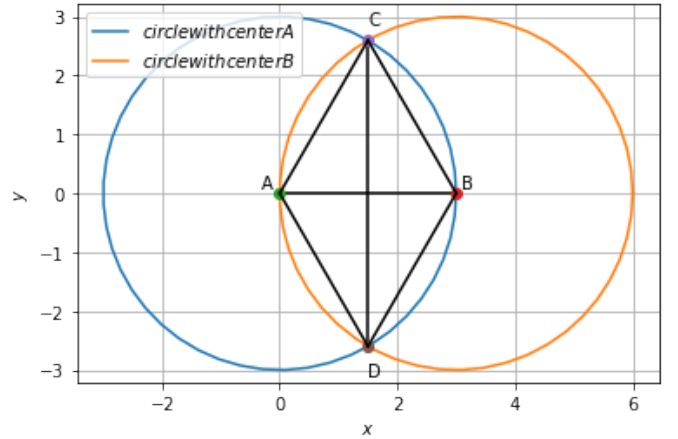


Fig. 2.1: Circles with their points of intersection

We have **C** and **D** as points of intersection. So,

$$\|\mathbf{D} - \mathbf{A}\| = \|\mathbf{C} - \mathbf{A}\| = r_1 = 3 \quad (2.0.7)$$

$$\|\mathbf{D} - \mathbf{B}\| = \|\mathbf{C} - \mathbf{B}\| = r_2 = 3 \quad (2.0.8)$$

Therefore, in quadrilateral ACBD we have

$$\|\mathbf{D} - \mathbf{B}\| = \|\mathbf{C} - \mathbf{B}\| = \|\mathbf{D} - \mathbf{A}\| = \|\mathbf{C} - \mathbf{A}\| = 3 \quad (2.0.9)$$

So, ACBD is a Rhombus. For a Rhombus we have diagonals bisect each other at right angles.

Therefore it can be concluded that  $AB \perp CD$ .