

# Assignment-4

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<https://github.com/satyasm45/Summer-Internship/tree/main/Assignment-4/Codes>

and latex-tikz codes from

<https://github.com/satyasm45/Summer-Internship/tree/main/Assignment-4>

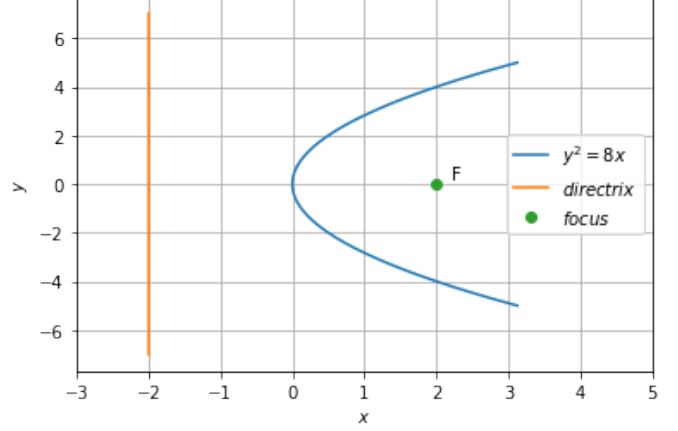


Fig. 2.1: Parabola  $y^2 = 8x$

## 1 QUESTION NO. 2.30

Find the equation of the parabola with focus  $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$  and directrix  $\begin{pmatrix} 1 & 0 \end{pmatrix} \mathbf{x} = -2$ .

## 2 EXPLANATION

**Definition 1.** A parabola is a curve where any point is at an equal distance from: a fixed point (the focus  $\mathbf{F}$ ), and, a fixed straight line (the directrix  $\mathbf{n}^T \mathbf{x} = c$ ).

**Lemma 2.1.** The distance of a point  $\mathbf{P}$  from a line  $\mathbf{n}^T \mathbf{x} = c$  is given by:

$$\frac{|c - \mathbf{P}^T \mathbf{n}|}{\|\mathbf{n}\|} \quad (2.0.1)$$

Using Definition 1 and Lemma 2.1 for any point  $\mathbf{x}$  on parabola we have:

$$\|\mathbf{x} - \mathbf{F}\|^2 = \frac{(c - \mathbf{x}^T \mathbf{n})^2}{\|\mathbf{n}\|^2} \quad (2.0.2)$$

**Theorem 2.1.** The equation of a parabola with focus  $\mathbf{F}$ , directrix  $\mathbf{n}^T \mathbf{x} = c$  and  $\lambda = \|\mathbf{n}\|^2$  is given by:

$$\mathbf{x}^T (\lambda \mathbf{I} - \mathbf{n} \mathbf{n}^T) \mathbf{x} + 2(c \mathbf{n} - \lambda \mathbf{F})^T \mathbf{x} + \lambda \|\mathbf{F}\|^2 - c^2 = 0 \quad (2.0.3)$$

*Proof.* Using (2.0.2)

$$\lambda (\mathbf{x} - \mathbf{F})^T (\mathbf{x} - \mathbf{F}) = (c - \mathbf{x}^T \mathbf{n})^2 \quad (2.0.4)$$

$$\lambda (\mathbf{x}^T \mathbf{x} - 2\mathbf{F}^T \mathbf{x} + \|\mathbf{F}\|^2) = c^2 + (\mathbf{x}^T \mathbf{n})^2 - 2c\mathbf{x}^T \mathbf{n} \quad (2.0.5)$$

$$\lambda \mathbf{x}^T \mathbf{x} - (\mathbf{x}^T \mathbf{n})^2 - 2\lambda \mathbf{F}^T \mathbf{x} + 2c\mathbf{n}^T \mathbf{x} = c^2 - \lambda \|\mathbf{F}\|^2 \quad (2.0.6)$$

$$\lambda \mathbf{x}^T \mathbf{I} \mathbf{x} - \mathbf{x}^T \mathbf{n} \mathbf{n}^T \mathbf{x} + 2(c\mathbf{n} - \lambda \mathbf{F})^T \mathbf{x} = c^2 - \lambda \|\mathbf{F}\|^2 \quad (2.0.7)$$

$$\mathbf{x}^T (\lambda \mathbf{I} - \mathbf{n} \mathbf{n}^T) \mathbf{x} + 2(c\mathbf{n} - \lambda \mathbf{F})^T \mathbf{x} + \lambda \|\mathbf{F}\|^2 - c^2 = 0 \quad (2.0.8)$$

□

Given information:

$$\mathbf{F} = \begin{pmatrix} 2 \\ 0 \end{pmatrix}, \mathbf{n} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, c = -2, \lambda = 1 \quad (2.0.9)$$

Substituting values of  $\mathbf{F}, \mathbf{n}, c, \lambda$  from (2.0.9):

$$\mathbf{x}^T \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} + 2 \begin{pmatrix} -4 & 0 \end{pmatrix} \mathbf{x} + 0 = 0 \quad (2.0.10)$$

Replacing  $\mathbf{x}$  by  $\begin{pmatrix} x \\ y \end{pmatrix}$  in (2.0.10) gives:

$$y^2 = 8x \quad (2.0.11)$$