Assignment-4

Satya Sangram Mishra

Download all python codes from

and latex-tikz codes from

1 Question No. 2.30

Find the equation of the parabola with focus $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$ and directrix $\begin{pmatrix} 1 & 0 \end{pmatrix} \mathbf{x} = -2$.

2 Explanation

A parabola is a curve where any point is at an equal distance from: a fixed point (the focus $\mathbf{F}$), and. a fixed straight line (the directrix $\mathbf{n}^T \mathbf{x} = c$). Let $\mathbf{x}$ lie on the parabola and $\mathbf{P}$ lie on the line. Therefore:

$$\|\mathbf{x} - \mathbf{F}\|^2 = \frac{((\mathbf{P} - \mathbf{x})^T \mathbf{n})^2}{\|\mathbf{n}\|^2} \tag{2.0.1}$$

Here.

$$\mathbf{F} = \begin{pmatrix} 2 \\ 0 \end{pmatrix}, \mathbf{n} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, c = -2 \tag{2.0.2}$$

Let $\lambda = \|\mathbf{n}\|^2$. Now,

$$\lambda \|\mathbf{x} - \mathbf{F}\|^2 = (\mathbf{P}^T \mathbf{n} - \mathbf{x}^T \mathbf{n})^2 \tag{2.0.3}$$

$$\lambda \|\mathbf{x} - \mathbf{F}\|^2 = (c - \mathbf{x}^T \mathbf{n})^2 \quad (\because \mathbf{n}^T \mathbf{P} = c) \tag{2.0.4}$$

$$\lambda (\mathbf{x} - \mathbf{F})^T (\mathbf{x} - \mathbf{F}) = (c - \mathbf{x}^T \mathbf{n})^2 \tag{2.0.5}$$

$$\lambda (\mathbf{x}^T \mathbf{x} - 2\mathbf{F}^T \mathbf{x} + \|\mathbf{F}\|^2) = c^2 + (\mathbf{x}^T \mathbf{n})^2 - 2c\mathbf{x}^T \mathbf{n} \tag{2.0.6}$$

$$\lambda \mathbf{x}^T \mathbf{x} - (\mathbf{x}^T \mathbf{n})^2 - 2\lambda \mathbf{F}^T \mathbf{x} + 2c\mathbf{n}^T \mathbf{x} = c^2 - \lambda \|\mathbf{F}\|^2 \tag{2.0.7}$$

$$\lambda \mathbf{x}^T \mathbf{I} \mathbf{x} - \mathbf{x}^T \mathbf{n}\mathbf{n}^T \mathbf{x} + 2(c\mathbf{n} - \lambda \mathbf{F})^T \mathbf{x} = c^2 - \lambda \|\mathbf{F}\|^2 \tag{2.0.8}$$

$$\mathbf{x}^T (\lambda \mathbf{I} - \mathbf{n}\mathbf{n}^T)\mathbf{x} + 2(c\mathbf{n} - \lambda \mathbf{F})^T \mathbf{x} + \lambda \|\mathbf{F}\|^2 - c^2 = 0 \tag{2.0.9}$$

So the general equation of parabola based on the known values is given by (2.0.9). Substituting values of $\mathbf{F}, \mathbf{n}$ and c from (2.0.2) and using $\lambda = \|\mathbf{n}\|^2$:

$$\mathbf{x}^T (\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \begin{pmatrix} 1 \\ 0 \end{pmatrix} \begin{pmatrix} 1 & 0 \end{pmatrix})\mathbf{x} + 2\begin{pmatrix} -4 & 0 \end{pmatrix} \mathbf{x} = 0 \tag{2.0.10}$$

$$\mathbf{x}^T \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} + 2\begin{pmatrix} -4 & 0 \end{pmatrix} \mathbf{x} + 0 = 0 \tag{2.0.11}$$
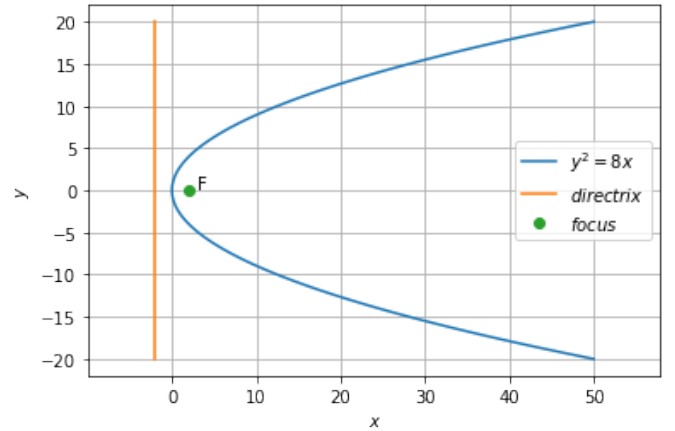
Replacing $\mathbf{x}$ by $\begin{pmatrix} x \\ y \end{pmatrix}$ gives:

$$y^2 = 8x \tag{2.0.12}$$

Fig. 2.1: Parabola $y^2 = 8x$