

Assignment-4

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Download all python codes from

<https://github.com/satyasm45/Summer-Internship/tree/main/Assignment-4/Codes>

and latex-tikz codes from

<https://github.com/satyasm45/Summer-Internship/tree/main/Assignment-4>

So, (2.0.10) is the equation of the parabola. Replacing \mathbf{x} by $\begin{pmatrix} x \\ y \end{pmatrix}$ will give us the general second degree equation as:

$$y^2 = 8x \quad (2.0.11)$$

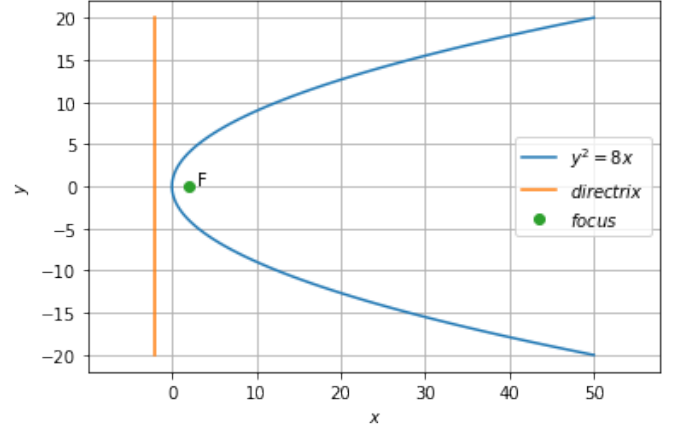


Fig. 2.1: Parabola $y^2 = 8x$

1 QUESTION No. 2.30

Find the equation of the parabola with focus $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$ and directrix $\begin{pmatrix} 1 & 0 \end{pmatrix} \mathbf{x} = -2$.

2 EXPLANATION

A parabola is a curve where any point is at an equal distance from: a fixed point (the focus \mathbf{F}), and, a fixed straight line (the directrix). Let \mathbf{x} lie on the parabola and \mathbf{P} lie on the line. Therefore:

$$\|\mathbf{x} - \mathbf{F}\|^2 = \frac{((\mathbf{P} - \mathbf{x})^T \mathbf{n})^2}{\|\mathbf{n}\|^2} \quad (2.0.1)$$

Here \mathbf{n} is normal vector of the directrix given by $\mathbf{n} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$. Now,

$$\|\mathbf{x} - \mathbf{F}\|^2 = (\mathbf{P}^T \mathbf{n} - \mathbf{x}^T \mathbf{n})^2 \quad (2.0.2)$$

$$\|\mathbf{x} - \mathbf{F}\|^2 = (-2 - \mathbf{x}^T \mathbf{n})^2 \quad (2.0.3)$$

$$(\mathbf{x} - \mathbf{F})^T (\mathbf{x} - \mathbf{F}) = (-2 - \mathbf{x}^T \mathbf{n})^2 \quad (2.0.4)$$

$$\mathbf{x}^T \mathbf{x} - 2\mathbf{F}^T \mathbf{x} + \|\mathbf{F}\|^2 = 4 + (\mathbf{x}^T \mathbf{n})^2 + 4\mathbf{x}^T \mathbf{n} \quad (2.0.5)$$

$$\mathbf{x}^T \mathbf{x} - (\mathbf{x}^T \mathbf{n})^2 - 2\mathbf{F}^T \mathbf{x} - 4\mathbf{n}^T \mathbf{x} = \|\mathbf{F}\|^2 - 4 \quad (2.0.6)$$

$$\mathbf{x}^T \mathbf{x} - (\mathbf{x}^T \mathbf{n})^2 - 2 \begin{pmatrix} 2 & 0 \end{pmatrix} \mathbf{x} - 4 \begin{pmatrix} 1 & 0 \end{pmatrix} \mathbf{x} = 0 \quad (2.0.7)$$

$$\mathbf{x}^T \mathbf{I} \mathbf{x} - \mathbf{x}^T \mathbf{n} \mathbf{n}^T \mathbf{x} + 2 \begin{pmatrix} -4 & 0 \end{pmatrix} \mathbf{x} = 0 \quad (2.0.8)$$

$$\mathbf{x}^T \left(\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \begin{pmatrix} 1 \\ 0 \end{pmatrix} \begin{pmatrix} 1 & 0 \end{pmatrix} \right) \mathbf{x} + 2 \begin{pmatrix} -4 & 0 \end{pmatrix} \mathbf{x} = 0 \quad (2.0.9)$$

$$\mathbf{x}^T \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} + 2 \begin{pmatrix} -4 & 0 \end{pmatrix} \mathbf{x} + 0 = 0 \quad (2.0.10)$$