Time Series Data Analysis and Forecasting

MSDS

Module 6,7

Topic Covered

- Time-series analysis in financial markets
- Volatility modeling
- Structure of a model
- ARCH model
- Properties of ARCH
- GARCH model
- Intergrated GARCH model
- Event studies
- abnormal returns

Time-series analysis in financial markets

- Financial markets refer broadly to any marketplace that entitles the trading of securities, commodities, and other fungibles
- financial security market includes stock market, cryptocurrency market, etc. Bahadur et al. (2019).
- Among the three markets, which are stock, cryptocurrency, and commodity, the stock markets are well known to people while the other two are not.

Time-series analysis in financial markets

- Returns over one day are typically small, and their average is close to zero. At the same time, their variances and standard deviations can be relatively large.
- Over the course of a few years, several very large returns (in magnitude) are typically observed. These relative outliers happen on only a handful of days, but they account for substantial movements in asset prices.
- Because of these extreme returns, distribution of daily asset returns is not normal, but heavy-tailed, and sometimes skewed.
- individual stock returns have even greater variability and more extreme observations than index returns.

What is Volatility in financial markets?

- Volatility is a statistical measure of the dispersion of returns for a given security or market index.
- In most cases, **higher the volatility**, the **riskier the security**.
- Volatility is often measured from either the standard deviation or variance between returns from that same security or market index

What is Volatility in financial markets?

- It represents how large an **asset's prices** swing around the mean price—it is a statistical measure of its dispersion of returns.
- We can measure volatility, including beta coefficients, option pricing models, and standard deviations of returns.
- Volatile assets are often considered riskier than less volatile assets because the price is expected to be less predictable.
- **Implied volatility** measures how volatile the market will be while **historical volatility** measures price changes over predetermined periods of time
- Volatility is an important variable for calculating options prices.

Implied volatility(IV)

- The term implied volatility refers to a metric that captures the market's view of likelihood of future changes in a given security's price.
- Investors can use implied volatility to **project future moves** and supply and demand, and often employ it to price options contracts.
- Supply and demand and time value are major determining factors for calculating implied volatility.
- Implied volatility usually increases in **bearish markets** and decreases when the market is **bullish**.

High vs low Volatility

Range from -4% to 6%

Range from -20% to 15%

How to calculate volatility?

- Volatility is calculated using variance and standard deviation (the standard deviation is the square root of the variance).
- Since volatility describes changes over a specific period of time, take the standard deviation and multiply that by the square root of number of periods in question:

$$Volatility = \sigma\sqrt{T}$$

where:

- σ =standard deviation of returns
- T = number of periods in the time horizon

Example- How to calculate volatility?

1. Find the mean of the data set.
2. Calculate the difference between each data value and the mean.
3. Square the deviations. This will eliminate negative values.
4. Add the squared deviations together.
5. Divide the sum of the squared deviations by the number of data values.

Example-

Price	Mean	Deviation	Deviation Squared	Variance	Standard Deviation
1	5.5	-4.5	20.25		
2	5.5	-3.5	12.25		
3	5.5	-2.5	6.5		
4	5.5	-1.5	2.5		
5	5.5	-0.5	0.25		
6	5.5	0.5	0.25		
7	5.5	1.5	2.25		
8	5.5	2.5	6.25		
9	5.5	3.5	12.25		
10	5.5	4.5	20.25		
55			82.5	8.25	2.872281323

Volatility Modeling

- It provides simple approach to calculating value at risk of a financial position in risk management.
- It also plays an important role in asset allocation under the mean-variance framework.
- modeling volatility of a time series can improve the efficiency in parameter estimation and the accuracy in interval forecast
- univariate volatility models include the autoregressive conditional heteroscedastic (**ARCH**) model, generalized ARCH (**GARCH**) model, exponential GARCH (**EGARCH**) model, the conditional heteroscedastic autoregressive moving average (**CHARMA**) model, the random coefficient autoregressive (**RCA**) model.

Model Building

1. Specify a **mean equation** by testing for serial dependence in the data and, if necessary, building an econometric model (e.g., an ARMA model) for the return series to remove any linear dependence.
2. Use the **residuals of the mean** equation to test for ARCH effects.
3. Specify a volatility model if **ARCH effects** are statistically significant and perform a joint estimation of the mean and volatility equations.
4. Check the **fitted model** carefully and refine it if necessary.

ARCH model

- Autoregressive conditional heteroskedasticity (ARCH) is a statistical model used to analyze **volatility** in time series in order to forecast future volatility.
- ARCH modeling is used to estimate risk by providing a model of volatility that more closely resembles real markets.
- Basic idea is that

  (a) shock a_t of an asset return is serially uncorrelated but dependent

  (b) dependence of a_t can be described by a simple quadratic function of its lagged values.
- ARCH(m) model assumes…

$$a_t = \sigma_t \epsilon_t, \quad \sigma_t^2 = \alpha_0 + \alpha_1 a_{t-1}^2 + \cdots + \alpha_m a_{t-m}^2,$$

ARCH(1) Model

ARCH(1) model for the variance of model y_t is that conditional on y_{t-1} , variance at time t is

$$\mathrm{Var}(y_t|y_{t-1}) = \sigma_t^2 = \alpha_0 + \alpha_1 y_{t-1}^2$$

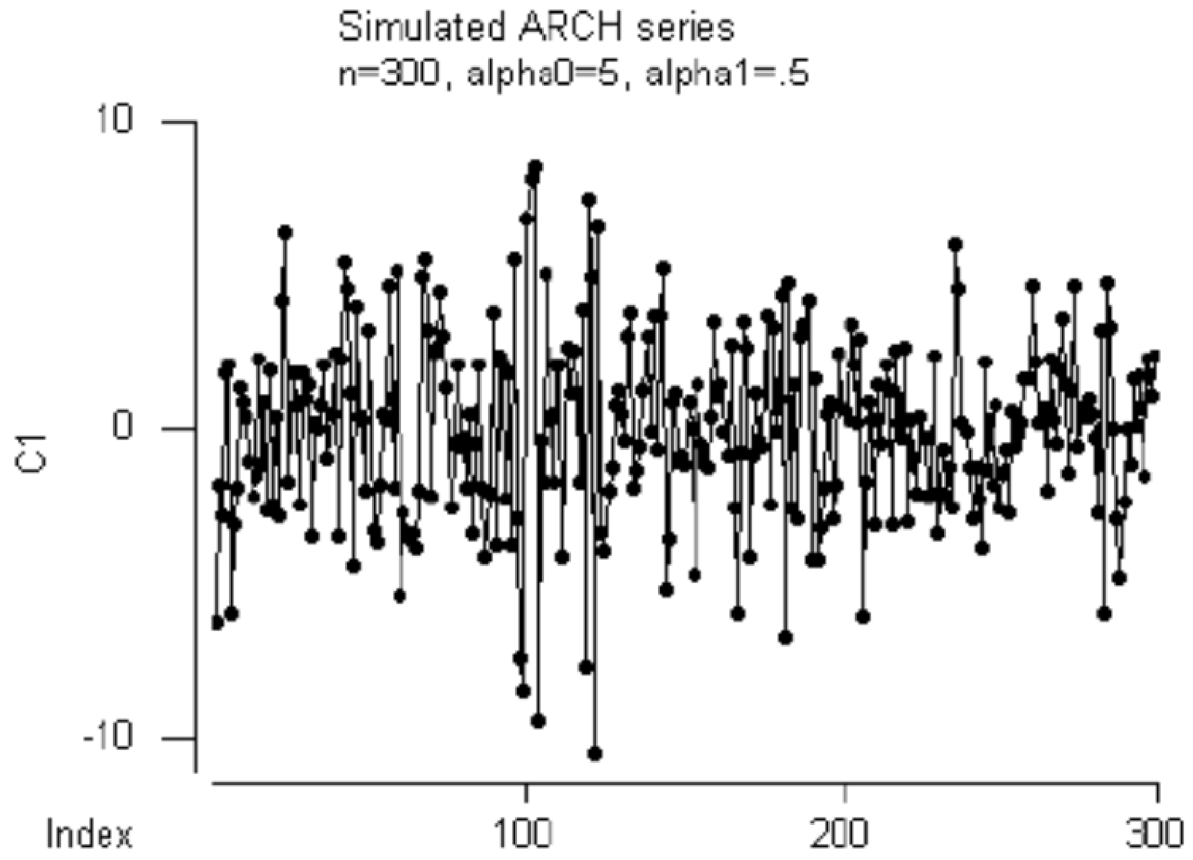
impose the constraints $\alpha_0 \geq 0$ and $\alpha_1 \geq 0$ to avoid negative variance.

The variance at time t is connected to the value of the series at time t – 1. A relatively large value of y_{t-1}^2 gives a relatively large value of the variance at time t. This means that the value of y_t is less predictable at time t −1 than at times after a relatively small value of y_{t-1}^2.

Example ARCH(1) Model

The following plot is a time series plot of a simulated series (n = 300) for the ARCH model

$$\mathrm{Var}(y_t|y_{t-1}) = \sigma_t^2 = 5 + 0.5y_{t-1}^2.$$



GARCH Model

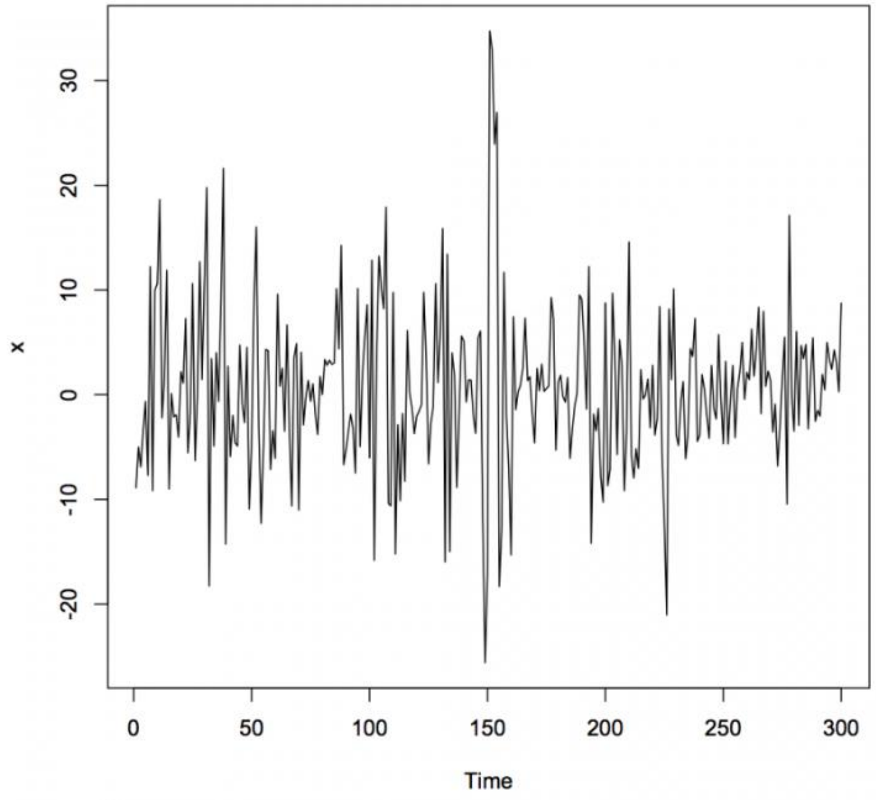
- A GARCH (generalized autoregressive conditionally heteroskedasticity) model uses values of the **past squared observations** and **past variances** to model the variance at time t.
- As an example, a GARCH(1,1) is

$$\sigma_t^2 = \alpha_0 + \alpha_1 y_{t-1}^2 + \beta_1 \sigma_{t-1}^2$$

Example- GARCH Model

The following plot is a time series plot of a simulated series, x, (n = 300) for the GARCH(1,1) model

$$\text{Var}(x_t|x_{t-1}) = \sigma_t^2 = 5 + 0.5x_{t-1}^2 + 0.5\sigma_{t-1}^2$$



Limitation of GARCH Model

- GARCH model has limitations in predicting financial volatility.
- One limitation is **volatility persistence**, which makes it cumbersome for GARCH models to analyze stock prices.
- **presence of noise** in stock market returns, which can affect the forecasting performance of GARCH models.
- GARCH model may not be suitable for **highly volatile assets like cryptocurrencies**, as they exhibit volatility clustering and require more sophisticated models.
- To improve GARCH model, researchers have explored techniques such as **wavelet de-noising** of data, which improves accuracy of volatility forecasts.

Example Case : Stock price daily returns

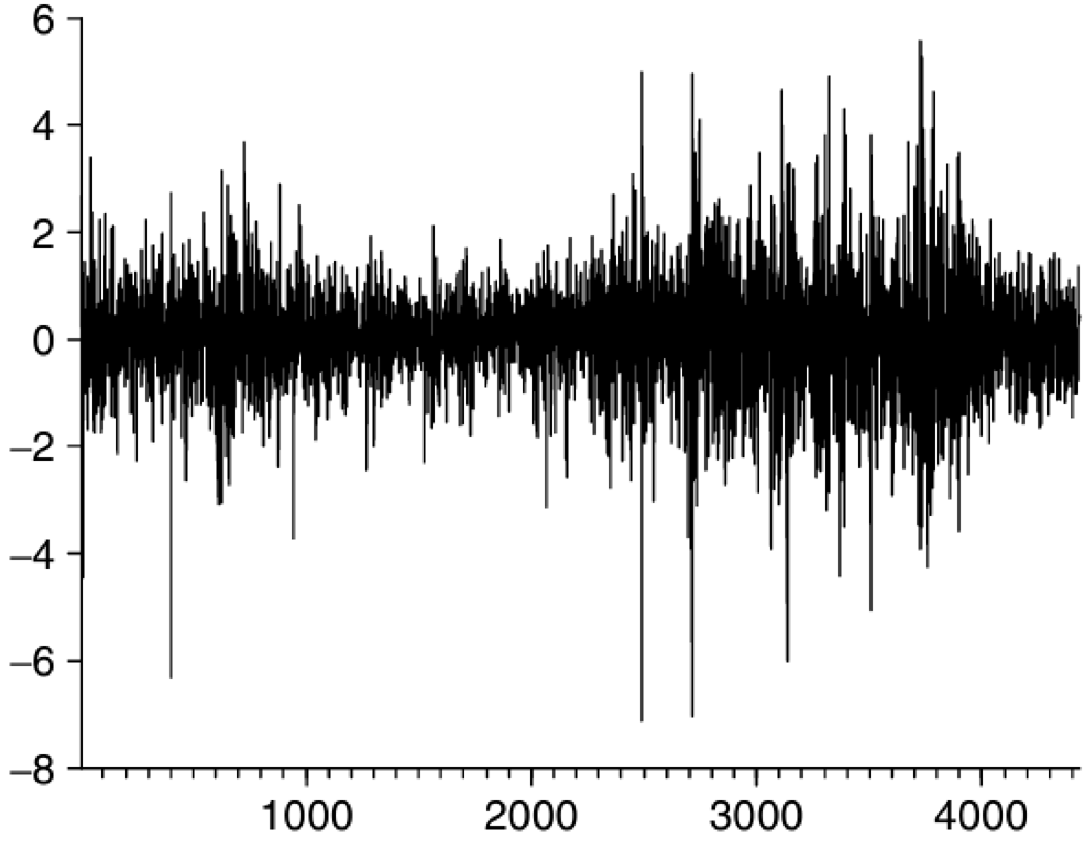


Figure 1.7 S&P500 equity index continuously compounded daily returns from 5 April 1988 to 5 April 2005.

Example Case : Stock price daily returns

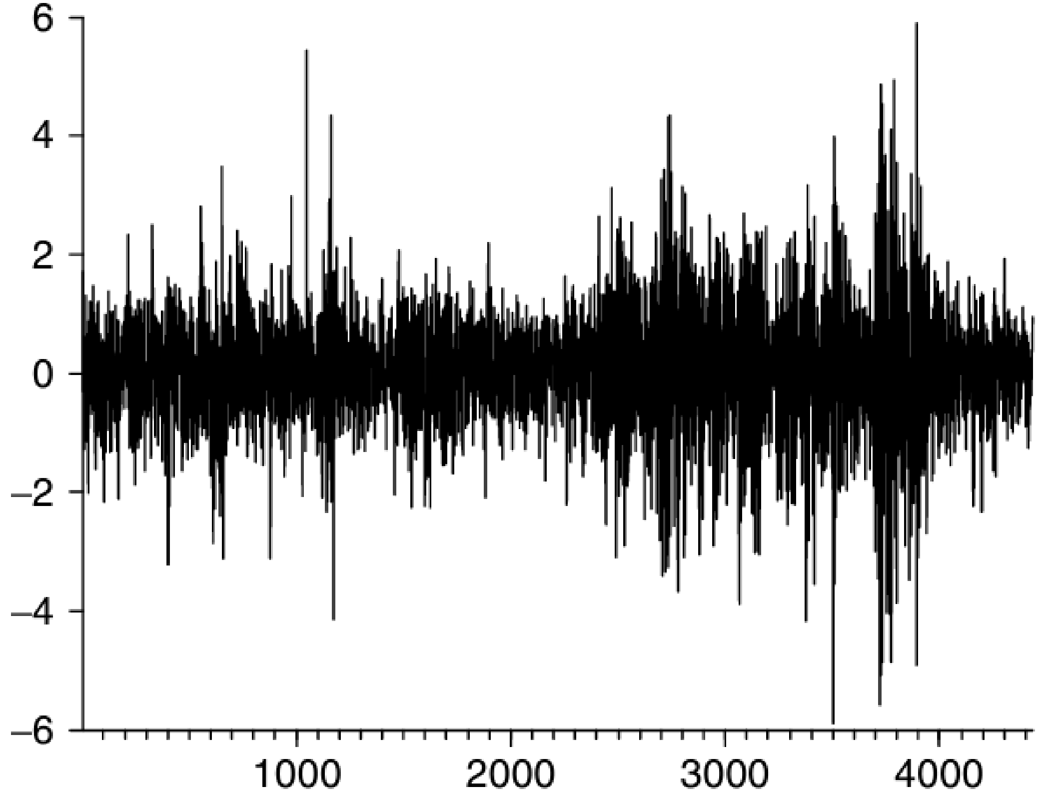


Figure 1.8 FTSE100 equity index continuously compounded daily returns from 5 April 1988 to 5 April 2005.

EGARCH Model

- Exponential Generalized Autoregressive Conditional Heteroskedastic
- developed to allow for asymmetric effects between positive and negative shocks on **conditional variance** of future observations.
- Another advantage, as pointed out by Nelson and Cao (1992), is that there are no restrictions on parameters.

EGARCH

- Advantage of the EGARCH model is that it allows to reflect how positive and negative changes in the series affect volatility.
- The conditional variance of the GARCH model is a function of the square of the past innovations, so it does not collect positive or negative changes.
- GARCH model has the disadvantage that conditional variance depends on magnitude of delayed innovations, but not of their sign.
- This problem was solved with exponential generalized autoregressive conditional heteroskedastic (EGARCH) model (Nelson 1991) by introducing a measure of the sign of innovations.

EGARCH Formal Specification

$$x_t = \mu + a_t$$

$$\ln \sigma_t^2 = \alpha_o + \sum_{i=1}^{p} \alpha_i \left(|\epsilon_{t-i}| + \gamma_i \epsilon_{t-i}\right) + \sum_{j=1}^{q} \beta_j \ln \sigma_{t-j}^2$$

$$a_t = \sigma_t \times \epsilon_t$$

$$\epsilon_t \sim P_\nu(0, 1)$$

Where:

- x_t is the time series value at time t.
- μ is the mean of the GARCH model.
- a_t is the model's residual at time t.
- σ_t is the conditional standard deviation (i.e., volatility) at time t.
- p is the order of the ARCH component model.

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$$a_t = \sigma_t \times \epsilon_t$$

$$\epsilon_t \sim P_\nu(0, 1)$$

- $\alpha_o, \alpha_1, \alpha_2, \ldots, \alpha_p$ are the parameters of the ARCH component model.
- q is the order of the GARCH component model.
- $\beta_1, \beta_2, \ldots, \beta_q$ are the parameters of the GARCH component model.
- $[\epsilon_t]$ are the standardized residuals:

$$[\epsilon_t] \sim i.i.d$$

$$E[\epsilon_t] = 0$$

$$VAR[\epsilon_t] = 1$$

CHARMA

- Conditional Heteroskedastic ARMA model
- uses random coefficients to produce conditional heteroscedasticity
- Model is formally defined as:---

$$r_t = \mu_t + a_t, \quad a_t = \delta_{1t}a_{t-1} + \delta_{2t}a_{t-2} + \cdots + \delta_{mt}a_{t-m} + \eta_t,$$

$\boldsymbol{\eta_t}$ *is a gaussian white noise series with mean zero and variance* ${\sigma_\eta}^2$

$$\{\boldsymbol{\delta_t}\} = \{(\boldsymbol{\delta_{1t}}, \ldots., \boldsymbol{\delta_{mt}})\} \text{ is a sequence of iid random vectors with mean zero and positive co} - \text{variance matrix.}$$

If m=1, then model reduces to……….

$$\sigma_t^2 = \sigma_\eta^2 + \omega_{11}a_{t-1}^2 + 2\omega_{12}a_{t-1}a_{t-2} + \omega_{22}a_{t-2}^2,$$

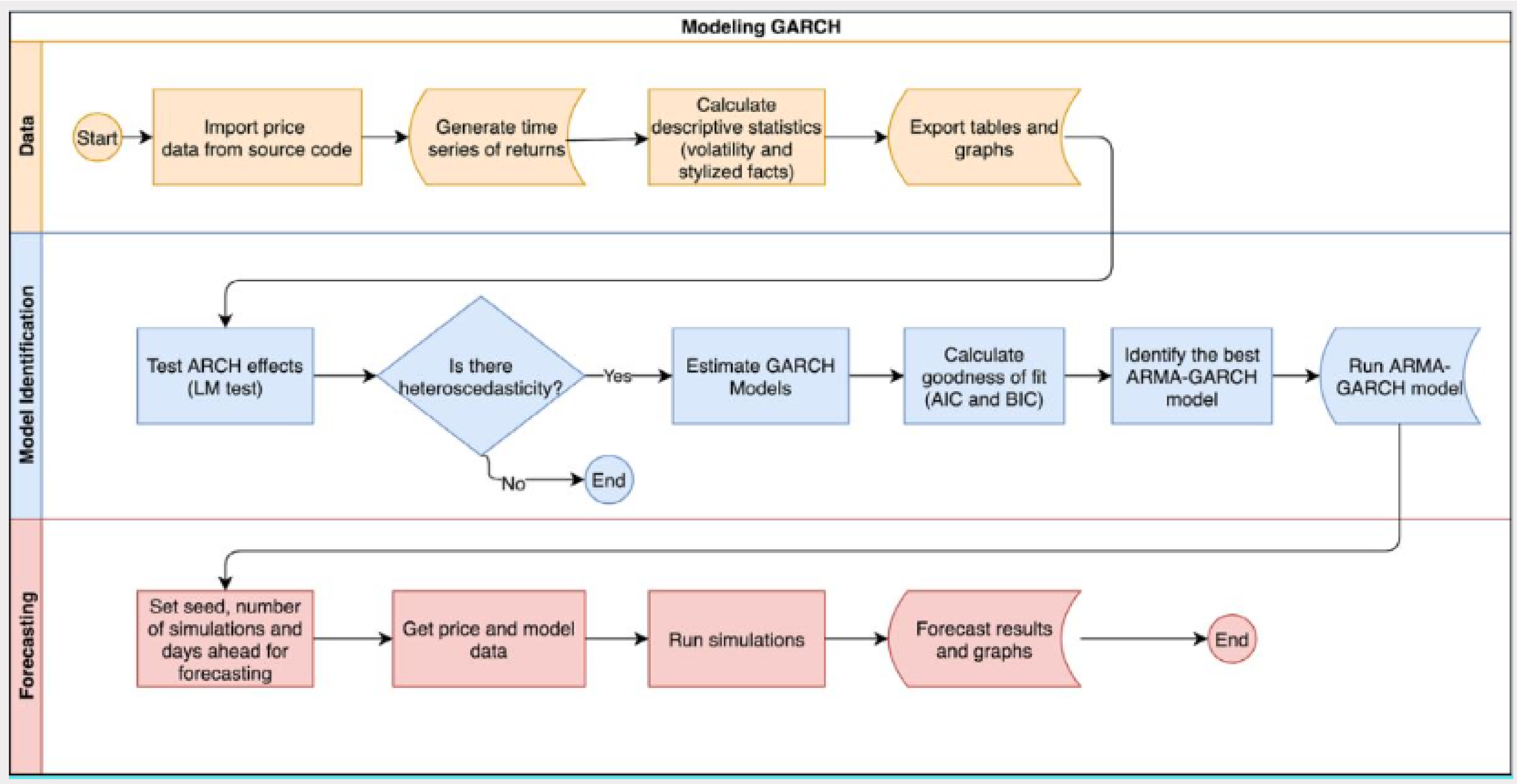
Random Coefficient Autoregressive (RCA)

- RCA model is introduced to account for variability among different subjects under study, similar to panel data analysis in econometrics and the hierarchical model in statistics.
- RCA model is a conditional heteroscedastic model
- but historically it is used to obtain a better description of conditional mean equation of the process by allowing for the parameters to evolve over time

A time series r_t is said to follow an RCA(p) model if it satisfies....

$$r_t = \phi_0 + \sum_{i=1}^{p} (\phi_i + \delta_{it}) r_{t-i} + a_t,$$

STEPS to Apply GARCH modelling for forecasting



ARCH Effect

- It is important to take into account a model that does not assume that the variance is constant because it is unlikely in the case of financial time series that the variance of the errors will remain constant throughout time.
- *If the variance of the errors is not constant, this would be known as* ***heteroscedasticity****.*
- *A time series exhibiting conditional heteroscedasticity or autocorrelation in the* ***squared series*** *is said to have autoregressive conditional heteroscedastic -**ARCH) Effects**.*
- It is important to test for ARCH Effect before applying ARCH or GARCH Models.

TEST for ARCH Effect

- It makes more sense to compute the Engle (1982) test for ARCH effects before estimating a GARCH-type model to ensure that this class of models is appropriate for the data.

Consider testing the hypotheses:

$$H_0 = No\ ARCH\ effect$$
$$H_1 = ARCH\ effect$$

The ARCH test is frequently applied to raw returns data.

Review Questions

- What is volatility in financial market?
- Describe ARCH model.
- Discuss GARCH model using example.
- Explain EGRACH model and its significance.