Energy-Minimizing Algebraic Multigrid Solvers for Systems of Partial Differential Equations

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The repeated implicit solution of large sparse linear systems is a key computational challenge in many parallel calculations at Sandia and often dominates the total simulation time. The solvers in the Trilinos framework are frequently used for these systems, in particular the smoothed aggregation (SA) multigrid methods in the ML library. The SA methods are well-developed but can struggle with linear systems arising from problems with multiphysics and/or severely stretched meshes, as might be found in in structural mechanics or electro-magnetics. Stretched meshes can reflect either underlying anisotropic phenomena or meshing technology limitations. In this CSRF project we have developed a family of novel energy-minimizing multigrid algorithms that address these shortcomings while providing faster solution times. These new algorithms have the flexibility to take any sparsity pattern for transfer matrices that propagate information within the multigrid hierarchy. This allows us to choose patterns tailored for problem physics. Additionally, the cost of applying these algorithms is lower than SA while still achieving accurate interpolation of key problematic error modes. This yields methods that still have very good convergence properties. In 3D linear elasticity mesh scaling studies, energy minimization is always cheaper to apply (meaning faster run times) than SA. As the mesh stretching becomes more severe, energy minimization converges faster than SA. As an example, the new method is about 2.5 times faster than SA on a 40^3 mesh where the stretching in the x direction is 100 times either the y or z directions.