MESQUITE Mesh Quality Improvement Toolkit

User's Guide

Patrick Knupp
The Sandia National Laboratories
Albuquerque NM USA

Lori Freitag-Diachin Livermore National Laboratory Livermore CA USA

Boyd Tidwell Elemental Technologies Inc. American Fork, UT USA

Last Updated: 17 May, 2013

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Chapter 1

Introduction to Mesquite

1.1 Overview of Mesh Quality

Mesh quality refers to geometric properties of a mesh such as local volume, smoothness, shape, and orientation that, if not properly controlled, can adversely affect solution accuracy or computational efficiency of numerical simulations. In this section we give an overview of the role of mesh quality in the context of computer simulations of physical phenomena.

Simulation of many phenomena in the physical world involves computing numerical solutions to partial differential equations (PDE's). Commonly used approaches to computing numerical solutions such as finite volume and finite element methods require the use of approximations to the continuum operators in the PDE and a mesh or grid to subdivide the physical domain into small subregions. Together, the approximations and the mesh define a discretization. The difference between the exact solution to the PDE and the numerical solution is known as the discretization error. A convergent discretization means that the discretization error will asymptotically approach zero as the characteristic mesh size "h" approaches zero. Decreasing mesh size to reduce discretization error to nearly zero is often impractical in realistic simulations due to limited computing resources. One way to increase the accuracy of simulations with the same computer resources is to adapt the mesh to the domain and to the numerical solution. In adaptive refinement, the local mesh volume (or size) is made smaller in locations where the local discretization error is large and is made larger in locations where the error is small. In local h-refinement, mesh volume is made smaller by locally subdividing the mesh. In r-refinement, mesh volume is made smaller by moving mesh nodes closer together. Geometric adaptation can also be important in improving simulation accuracy. In regions of high domain curvature one adapts the mesh to the domain geometry by creating locally smaller mesh sizes. We see, then, that local mesh size (or volume) is a critical parameter in determining the accuracy of a simulation.

Aside from local mesh size, several other geometric mesh properties can affect solution accuracy. These include mesh smoothness, local mesh angles, aspect ratio, and orientation. For example, in some discretization methods there will be a loss of accuracy if the mesh is not smooth. In other cases, aspect ratios and orientation must be carefully adapted to the solution in order to maintain a certain level of accuracy. Simulations using meshes or domains that evolve in time (such as in ALE simulations) usually require that initially good geometric mesh properties be retained throughout the simulation time period. It is thus often important to control other geometric mesh properties in addition to local mesh size within an adaptive simulation.

In addition to solution accuracy, geometric mesh properties can also affect the amount of computer time required to obtain the numerical solution. Simulation codes usually employ iterative solvers to solve systems of equations and thus obtain numerical solutions to PDE's. The rate at which these solvers converge is determined by the spectral radius of a certain matrix. The spectral radius of the matrix is affected by, among other things, geometric properties of the mesh. Poor mesh quality can thus adversely impact solution efficiency.

Adaptive meshing techniques require an initial mesh to begin the adaptation procedure. Poor quality of the initial mesh (relative to the adapted mesh) can be difficult to overcome or, at least, reduce the efficiency of the adaptive procedure. For example, if the initial mesh contains locally inverted elements, these can often be fixed before the adaptive procedure begins. As another example, if it is known à priori that small angles will be needed on the boundary of the domain to obtain reasonable simulation accuracy, one should try to first create the small angles in the initial mesh to improve the efficiency of the subsequent adaptive meshing procedure.

Many simulations, particularly those in industry, are performed in a non-adaptive setting. That is to say, an initial mesh is generated and used throughout the calculation. The mesh is not changed as the solution is computed. Mesh quality remains important for such calculations. First, for complicated geometric domains it is often difficult to obtain good initial mesh quality. This is particularly true for non-simplicial meshes but can be true for simplicial meshes as well. A common requirement is that the mesh be smooth. Many simulation codes will not run to completion if the initial mesh contains a local volume which is negative. These must be eliminated before a simulation can begin. Analysts performing non-adaptive calculations often have considerable experience in using a variety of meshes on their problem and have a good à priori idea of what constitutes good mesh quality for a given problem. They thus desire to control the usual geometric mesh properties of the non-adapted mesh carefully.

1.2 How Mesh Quality Is Improved

Mesh quality can and should be considered during many stages of the mesh generation process from defeaturing CAD models to creation and adaptation of the mesh. Thus, for example, certain non-essential features of a CAD model, if eliminated, would go a long way to improving the quality of the mesh, depending upon the meshing scheme. Other critical meshing parameters which can affect mesh quality include geometric domain partitions, interval size and count, interaction of meshes within large assemblies of parts, biasing requirements, corner picking, etc. Choices made during the mesh generation phase of an analysis may have a large impact on initial mesh quality. Mesh quality can thus be improved by changing the way in which the domain is meshed.

Once the meshing stage is completed, one can improve mesh quality by techniques such as vertex movement and local topology modification. In vertex movement schemes, one seeks to reposition existing mesh vertices to achieve better quality. If vertex movement is undertaken within an adaptive setting, it is commonly referred to as r-refinement. Classic examples of vertex movement methods include Laplace smoothing [10] and Winslow smoothing [23]. It is helpful, in vertex movement schemes, to first be able to measure mesh quality so that one can explain in what sense one has improved it. Given a metric to measure mesh quality, one can formulate a numerical optimization problem which guides vertex movement to find the optimal mesh and thus improve its quality. Numerical optimization methods recently developed for unstructured meshes include [6, 13, 9, 8, 11, 12, 4].

A large number of mesh quality metrics have been devised to measure mesh quality. Many of these metrics are independent of any solution properties and are thus not useful in adaptive meshing. However, there are a number of weighted quality metrics which can be tied to the numerical solution via error indicators or other information for adaptive meshing.

Another way to improve mesh quality is to use local topological modification methods in which mesh vertices or elements are locally created and/or destroyed. These methods are very successful when applied to simplicial meshes, often within an adaptive context. Local topology modification is less effective on non-simplicial meshes.

Mesh quality improvement remains an important on-going research area. There remain, for example, open questions with regard to metrics which can be used in adaptive settings, theoretical questions on problem formulation, and how to obtain improved meshes quickly. An important subset of Mesquite capabilities is based on a mathematical theory that we are developing which we call the Target-matrix

paradigm (TMP). The basic idea is similar to that from Harmonic mappings, as applied to mesh generation: use only a few very soundly formulated quality metrics and adapt the mesh to a wide variety of specialized purposes via specification of the mapping on the target manifold. However, TMP is formulated as a discrete optimization problem, which allows direct control over important properties such as invertibility which must hold even if the asymptotic limit is not reached. The mathematics behind the Target-matrix paradigm can be found in [14, 21, 5, 15, 17, 16, 18].

Although mesh quality improvement algorithms have been widely implemented in both meshing and applications codes, it has always been difficult to improve the quality of a mesh created in one software package using an improvement algorithm which has been implemented in another. This difficulty and others have inspired the creation of the Mesquite software library. This library is described in the next section.

1.3 Mesquite Goals

Mesquite (Mesh Quality Improvement Toolkit) is designed to provide a stand-alone, portable, comprehensive suite of mesh quality improvement algorithms and components that can be used to construct custom quality improvement algorithms. The design is flexible so that the algorithms can be applied to many different mesh element types and orders and referenced to both isotropic and anisotropic ideal elements. Mesquite provides a robust and effective mesh improvement toolkit that allows both meshing researchers application scientists to benefit from the latest developments in mesh quality control and improvement.

Mesquite design goals are derived from a mathematical framework and are focused on providing a versatile, comprehensive, inter-operable, robust, and efficient library of mesh quality improvement algorithms that can be used by the non-expert and extended and customized by experts. In this section we highlight the current status of Mesquite in several of our design goal areas.

Versatile. Mesquite works on structured, unstructured, and hybrid meshes in both two and three dimensions. The design permits improvements to meshes composed of triangular, tetrahedral, quadrilateral, hexahedral, prismatic (wedge) and pyramidal elements. Support for general polyhedral elements may be added at a future time. It currently incorporates only methods for node movement; plans for topology modification and hybrid improvement strategies lie in the future. Node movement strategies include both local patch-based iteration schemes for one or a few free vertices and global objective functions which improve all vertices simultaneously. Mesquite will be applicable to both adaptive and nonadaptive meshing and to both low- and high-order discretization schemes, but currently works with non-adaptive meshes containing linear elements.

Comprehensive. Mesquite will address a large variety of mesh quality improvement goals including mesh volume control (sizing, invertibility), mesh angles, aspect ratios, and orientation. Specific goals include mesh untangling, mesh smoothing, shape improvement, anisotropic smoothing, mesh rezoning for ALE, mesh alignment, and deforming mesh algorithms. These goals can be pursued in both adaptive and non-adaptive settings. The software is customizable, enabling users to insert their own quality metrics, objective functions, and algorithms and also provides mechanisms for creating combined approaches that use one or more improvement algorithms.

Inter-operable. To ensure that Mesquite is inter-operable with a large number of mesh generation packages, Mesquite defines a generic interface for accessing application mesh and domain data. Additionally, Mesquite provides an adapter to interface with the common interfaces for mesh and geometry query currently under developed by the ITAPS center. These interfaces provide uniform access to mesh geometry and topology and will be implemented by all ITAPS center software including several DOE-supported mesh generation packages. We are working with the ITAPS interface design team to ensure that Mesquite has efficient access to mesh and geometry information through strategies such as information caching and agglomeration. We are also participating in the design of interfaces needed to support

topological changes generated by mesh swapping and flipping algorithms and to constrain vertices to the surface of a geometrical model.

Efficient. The outer layers of Mesquite use object-oriented design in C++ while the inner kernels use optimizable coding constructs such as arrays and inlined functions. To ensure efficient use of computationally intensive optimization algorithms, we employ inexpensive smoothers, such as Laplacian smoothing, as "preconditioners" for the more expensive optimization techniques. In addition, mesh culling algorithms can be used to smooth only those areas of the mesh that require improvement. Considerable attention has been devoted to understanding and implementing a variety of termination criteria that can be used to control the computational cost of the optimization algorithms.

Robust. Sound software engineering principles and robust numerical algorithms are employed in Mesquite. A comprehensive suite of test problems and a unit testing framework have been developed to verify the correct execution of the code.

Mesquite is not intended to be a mesh generation tool. It can serve as a post-processor to a mesh generation procedure, a mesh pre-processor to a non-adaptive simulation code, or as an algorithm for in-core adaptive mesh quality improvement. As a software library, Mesquite is intended to be linked to either a meshing code or to a simulation code.

1.4 Mesquite Concepts

Mesquite software design is based on a mathematical framework that improves mesh quality by solving an optimization problem to guide the movement of mesh vertices. The user inputs a mesh or submesh consisting of vertices, elements, and the relationships between them. The quality of each vertex or element in the mesh is described by a local quality metric that is a function of a subset of the mesh vertices. The global quality of the mesh is formed by taking the global norm or the average of the local mesh qualities. The global quality is thus a function of the positions of all the mesh vertices. If this function can be used in a well-posed minimization problem (e.g., it is bounded below and has one or more local minimums), mesh vertices are moved by Mesquite toward the vertex positions of the optimal mesh, thus improving the quality according to the criterion defined by the local quality metric. By changing the local quality metric one can achieve a variety of mesh quality improvement goals such as mesh untangling, shape improvement, and size adaptation.

Users of Mesquite should have in mind a goal or set of goals which define the quality of the mesh which is to be improved. The goal determines which quality metric or metrics one will use in the optimization problem. Other user inputs will include an objective function template which describes the norm or average they wish to use in defining the global mesh quality. For example, an L-infinity norm will tend to improve the worst-case local quality while an L-2 norm will improve the RMS quality of the global mesh. Once the global quality (objective function) is defined, the user can select a numerical optimization scheme (solver) within Mesquite such as a steepest descent, conjugate gradient, or feasible Newton method. A variety of termination criteria can be selected singly or in combination to tell the solver when to halt. These are useful in controlling the trade-off between the accuracy of the minimization procedure vs. how much CPU is consumed. There is also an important flag that determines whether the optimization problem will be solved via a succession of optimizations on local patches followed by a complete pass over the global mesh or if it will be solved using a global patch in which all mesh vertices are moved simultaneously. Advantages and disadvantages of each of these approaches is currently under study.

Sometimes hybrid mesh optimization schemes are useful, for example, in first untangling a mesh and then improving the shape of its elements. For sequences of optimization problems Mesquite uses the concept of an instruction queue. The queue determines the order in which the optimization problems are solved, using the output from the previous optimization step as the input to the next optimization step. The queue defines a master quality improver that defines the ultimate mesh quality improvement goal.

The queue can also be used to include steps to assess mesh quality say before and after each optimization step within the queue. The quality assessor measures various aspects of quality in the mesh and may include other quality metrics besides the one used to define the optimization problem.

Optimization problems can be solved directly by minimizing the objective function or indirectly by positioning mesh vertices at a stationary point of the global objective function. Stationary points are defined by setting the gradient of the objective function to zero. The indirect method is akin to iteratively solving a system of linear (or nonlinear) equations. Currently, such systems are solved in Mesquite and other mesh quality software by using the local patch method that is akin to a Gauss-Seidel iteration. The prime example of this in Mesquite is Laplace smoothing. In the future we may include methods for solving global systems of equations in Mesquite to obtain solutions more quickly. In the past, some mesh smoothing algorithms have been formulated as a local iterative method that cannot be derived by setting the gradient of an objective function to zero. Such methods are frowned upon in Mesquite since one cannot state what mesh quality metric is improved. However, if such methods are included in future versions of Mesquite, they will be done in a manner similar to the local Laplace smoothing algorithm in Mesquite.

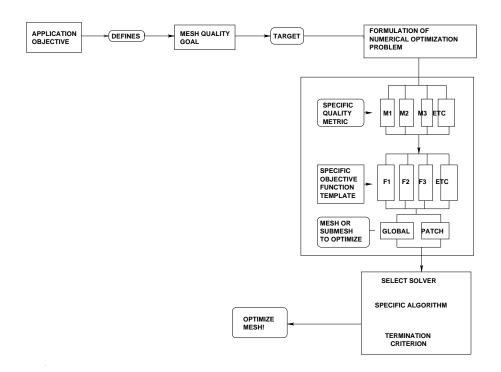


Figure 1.1: The Mesquite Paradigm

1.5 How to use this User's Manual

This user's manual

- provides an introduction to mesh quality and basic Mesquite concepts (Chapter 1),
- instructs novice users on how to download and install Mesquite (Chapter 2),
- provides a tutorial on Mesquite's simplified user's interface and Mesquite's detailed API (Chapter 3).
- describes how to load a mesh in Mesquite via files (Chapter 4), and

- describes Mesquite interactions with domain geometry (Chapter 6), and
- describes Mesquite Wrappers (Chapter 7),

Consult the doxygen documentation for the API reference as well as details on the software. There are two sets of doxygen documentations available:

- The developer doxygen doc is located in mesquite/doc/developer/. From that directory, you must run 'doxygen Mesquite.dox'.
- The user doxygen doc (API doc) is located in mesquite/doc/user/doxygen. From that directory, you must run 'doxygen Mesquite-user.dox'.

The doxygen command will generate two directories: an html directory containing the file index.html that you can open with your web browser, and a latex directory containing a Makefile that will generate a dvi file.

Chapter 2

Installing Mesquite

2.1 Requirements

2.1.1 Downloading Mesquite

The Mesquite distribution (in source form) may be obtained at the following URL:

http://www.cs.sandia.gov/~web1400/1400_download.html

2.1.2 Supported Platforms and Build Requirements

The Mesquite source code will compile in any environment comforming to the ISO/IEC 9899-1999 (C99), ISO/IEC 14882-1998 (C++98) and ISO/IEC 9945:2003 (POSIX) standards. It may also compile under many other environments.

Mesquite requires a reasonably standards-conforming C++ compiler and corresponding libraries. No additional libraries are required to build the core Mesquite library. Several optional features have additional requirements. These are listed in the next section.

Mesquite uses the GNU autotools build system. The Makefiles generated by the configure script should work on any Unix-like platform using the build tools (e.g. make) provided with that platform. The minimal requires beyond a C++ compiler are a Bourne shell (typically /bin/sh) implementation and a minimal corresponding command environment and an implementation of the make utility.

Support for building Mesquite with Microsoft Visual Studio is no longer available as of Mesquite version 2.0. Mesquite can still be built with Visual Studio but features added since the 2.0 release, such as building the iMeshP (Parallel) version, is not supported via CMake.

2.1.3 Optional Libraries and Utilities

• Unit tests: Mesquite provides a series of unit tests that may be used to verify the correct behavior of a build of the Mesquite library. These tests are implemented using CppUnit framework. The CppUnit framework must be installed to compile and run these tests. It is available at this URL:

http://cppunit.sourceforge.net

To run the unit tests on Unix-like platforms:

- build and install CppUnit.
- If using CMake:
 - o Set Mesquite_ENABLE_TEST to ON
 - o search for 'cppunit' in CMake.
 - o set CPPUNIT_INCLUDES to '<cppunit_install_dir>/include'
 - o set CPPUNIT_LIBRARY to '<cppunit_install_dir>/lib/libcppunit.so'
 - o Congigure and then Generate to produce CMake files

```
o After generation, exit CMake and cd to the build directory specified
in CMake.
o Enter:
    $ make
    $ ctest

- If using GNU Autotools:
    o From the top-level mesquite directory, Enter:
        $ ./configure --with-cppunit=<cppunit_install_dir>
        $ make
        $ make check
```

• ExodusII support: To enable support for reading or writing ExodusII files in Mesquite, the header files for the ExodusII library must be available and the library supporting ExodusII, the ExodusII library and possibly the NetCDF library must be available. If you need to build the ExodusII libraries from source, you will need to build the NetCDF library first. ExodusII files can be read via the MeshImpl::read_exodus(...) function and written via the MeshImpl::write_exodus(...) function. The ExodusII library can be obtained at the following URL:

```
http://sourceforge.net/projects/exodusii
```

The NetCDF library can be obtained at the following URL:

http://www.unidata.ucar.edu/downloads/netcdf/index.jsp

2.2 Release and Debug Builds

Mesquite can be built in either a Release or Debug mode. On Unix-like systems the type of build is specified via an option on the configure command as detailed in Section 2.3.2. On Windows systems the type of build is selected through the CMake utility as explained in Section 2.3.3. For Mac systems, the build type is specified in the do-configure script as shown in Section 2.4.2. The default mode on all systems is Release.

2.3 Building Mesquite

After downloading and unpacking the Mesquite source, the next step is to configure and build and install the Mesquite library.

2.3.1 Compiling on Unix-like systems

This section presents the steps required to compile Mesquite with the default options. It is typically required that Mesquite be "installed" before it is used in an application. The default installation location is the system-wide /usr/local directory. It is more common to specify an alternate directory in which to install the Mesquite library and headers. This can be done using the --prefix option to the configure script. Additional options are available for fine-grained control of installation locations.

- 1. Change your working directory to the top-level Mesquite source directory (typically mesquite-<version>/).
- 2. Run the configure script with the command:

```
./configure --prefix=<installdir>,
```

replacing <installedir> with the location in which the finished Mesquite library is to be placed.

3. Compile Mesquite with the command: make

- 4. Optionally verify that Mesquite compiled correctly with the command: make check
- 5. Move resulting files into the destination (install) directory with the command: make install

If the configure step failed, please consult the following section describing some of the optional arguments to the ./configure script.

CMake can also be used to build Mesquite on Unix-like systems. With a version of CMake in your path, the command ccmake will invoke a text-based version of CMake, the command cmake-gui runs a graphical version. Instructions for configuring with CMake are found in Section 2.3.3. After generating the make files, exit CMake and cd to the specified build directory and enter the command: make.

2.3.2 Options for Unix-like systems

This section describes the options available for customizing the build system and the resulting Mesquite library. An brief description of these and other options is available with the command: ./configure --help.

The following values may be specified as environmental variables, as arguments to the configure script using the NAME=VALUE syntax, or as arguments to make using the NAME=VALUE syntax. The value of these variables (if set) during the configure step will become the default for the compile step. The value of any of these variables will override the default if specified during the compile step.

CXX The C++ compiler command

CXXFLAGS Arguments to the C++ compiler, such as those specifying debug symbols or the optimization level.

CC The C compiler command

CFLAGS Command line arguments to be used for the C compiler.

DOXYGEN The doxygen API documentation generation tool.

Most options to the configure script are either of the form

--with-FEATURE[=ARG] or --enable-FEATURE[=ARG].

Some options may accept an additional argument following an '=' character. For each –with-FEATURE option, there is also a corresponding –without-FEATURE option. Similarly, there is a –disable-FEATURE option corresponding to each –enable-FEATURE option. The negative forms fo the options (–without-FEATURE and –disable-FEATURE) do not accept an additional argument. Only the positive form of each option is stated in the description below.

The following general build and debug options may be specified during the configure step:

- --enable-debug Select a subset of the following options that make the most sense for developers of Mesquite, implies: --enable-debug-symbols, --enable-debug-output=1,2, --enable-trap-fpe, --enable-silent-rules
- -enable-release This is the default behavior unless –enable-debug is specified. It selects a subset of the following options that typically work best for using Mesquite in a production application.
- --enable-compile-optimized Compile with the available optimizations that improve performance without any significant drawbacks (the -O2 compiler flag.)
- --enable-debug-symbols Include debugging information in the compiled Mesquite objects (the -g compiler flag).
- --enable-debug-assertions Include internal consistancy checks that abort when an error is detected.
- --enable-debug-output=n,m,... Enable the output of debug and status messages to file descriptor 1 (stdout). An list of integer debug flags for which to enable output may be specified as a commaseparated list of values. The default is to enable debug flags 1 and 2 if this option is specified without any explicit debug flag values.

- --enable-function-timers Enable time-profiling of some portions of Mesquite.
- -enable-trap-fpe Enable generation of a floating-point exception signal for arithmetic errors (e.g. division by zero.) This is an option intended for Mesquite developers. Enabling this will typically cause the application using Mesquite to abort when such an error is encountered.
- --enable-namespace=NAME Specify an alternate namespace so as to avoid symbol conflicts between multiple versions of Mesquite. See Section 2.3.4.
- --disable-option-checking Ignore unrecognized --enable--with options.
- --enable-shared[=PKGS] Build shared libraries [default=no]
- --enable-static[=PKGS] Build static libraries [default=yes]
- --enable-silent-rules Less verbose build output (undo: 'make V=1')
- --disable-silent-rules Verbose build output (undo: 'make V=0')
- --enable-fast-native Compile with fast math options, target build CPU, etc.
- --enable-quiet-make Much less verbose output during build. Default is no unless --enable-debug was specified.
- --enable-32bit Force 32-bit object code.
- --enable-64bit Force 64-bit object code.
- --disable-dependency-tracking Speeds up one-time build.
- --enable-dependency-tracking Do not reject slow dependency extractors.
- --disable-libtool-lock Avoid locking (might break parallel builds)
- --enable-api-doc=FORMATLIST Generate Doxygen docs for the API. Available formats are HTML, PDF, PS, ALL. Default is no docs. Multiple values must be separated by commas with no spaces. When the 'make' command is entered after configuring for api docs, the requested documents will be created in the mesquite/doc/user/doxygen directory.
- --enable-user-guide=FORMATLIST Generate Mesquite Users Guide. Available formats are PDF ,PS, ALL. Default is no docs. Multiple values must be separated by commas with no spaces. When the 'make' command is entered after configuring for the users guide, the requested documents will be created in the mesquite/doc/user directory.

The following options specify optional Mesquite components and the location of the corresponding dependencies.

- --with-cppunit=DIR The CppUnit library is required to compile and run the tests to verify that a particular build of the Mesquite library is working correctly. If the CppUnit library is not installed in a default location where the ./configure script can find it, this option may be used to specify the location.
- --with-exodus=DIR Enable support for reading and writing ExodusII files, and optionally specify the location where the ExodusII library and headers required for this option are installed.
- --without-cppunit Disable CppUnit tests.
- --without-exodus Disable exodusII support (default).
- --with-netcdf=DIR Specify the location of the NetCDF library required by the ExodusII library. The default is to look in the ExodusII directory.
- --without-netcdf Skip NetCDF check.

- --with-mpi=DIR Enable parallel support.
- --with-pic Try to use only PIC/non-PIC objects [default=use both].
- --with-gnu-ld Assume the C compiler uses GNU ld [default=no].
- --with-imesh=PATH Path to iMesh-Defs.inc (enable iMesh).
- --with-imeshp=PATH Path to iMeshP-Defs.inc (enable iMeshP).

2.3.3 Compiling on Microsoft Windows (CMake build)

The Mesquite source includes the necessary input files to generate Microsoft Visual Studio project files using the CMake utility. You will need to download and install the CMake utility for Windows if you have not already done so. It is available at:

http://www.cmake.org/cmake/resources/software.html

Using the graphical version of the CMake utility, select the folder containing the Mesquite source and enter a folder in which you would like the CMake output and compiled code to be stored. Select the Configure button. You will be presented with a group of configuration options. Modify any desired options and click the Configure button again. Each time you change one or more configuration options, you must click the Configure button to update the list of available options.

The 'CMAKE_BUILD_TYPE' opton controls the build mode that will be used by Visual Studio. Set the option to either 'RELEASE' or 'DEBUG. The build mode can also be changed from within Visual Studio itself after the generated project file has be loaded.

When you have finished changing build options, click the Generate button to generate Visual Studio input files and exit the CMake utility.

The build folder you specified in the CMake utility should now contain the necessary input files to build Mesquite using Microsoft Visual C++.

2.3.4 Linking Multiple Versions of Mesquite

Sometimes it is necessary to have multiple different versions of a library such as Mesquite linked into the same application. This situation typically arises when an application needs both Mesquite and some other library that depends on an older version of Mesquite. Without taking steps to avoid symbol name conflicts such a situation will often result in surprising, strange, and difficult to diagnose runtime errors.

Mesquite provides the ability to specify an alternate namespace and a standard namespace alias to assist with addressing such situations. The "namespace" Mesquite is typically an alias to the true internal C++ namespace containing all Mesquite code. Applications can and should use that alias rather than the internal namespace to avoid the need to modify application code whenever the internal namespace changes.

The internal namespace can be changed with the configure option --enable-namespace=MyNS or the cmake option Mesqutie_NAMESPACE, where the value "MyNS" can be replaced with any string that is an acceptable C++ namespace label. The default namespace is MesquiteN, with the Mesquite major version substituted for N. Specifying an alternate internal namespace results in different mangled symbol names in the compiled library, thus avoiding symbol name conflicts.

If the requested namespace is anything other than "Mesquite", then Mesquite will always provide the alias namespace Mesquite = MESQUITE_NS; so that application code may always use the Mesquite namespace.

2.4 Building Trilinos Mesquite from Scratch on a Mac

2.4.1 Installation

Trilinos depends on git and cmake. macports is a tool for downloading and installing software on macs. At Sandia, downloads to macs are done through portal 80. in /opt/local/etc/macports/macports.conf set

However leave proxy_rsync undefined.

First install macports. Type macports into google, and follow links to download and install. macports installs to /opt/local/bin and /opt/local/lib. To verify that macports is correctly installed, first type which port, then go on to step 2. Also if an install fails for some indiscernible reason, first try simply repeating the install command.

```
sudo port selfupdate
sudo port search git-core
sudo port install git-core +bash_completion
sudo port search cmake
sudo port install cmake
```

The build of Mesquite with automake and autotools depends on libtoolize, which can be downloaded similarly.

Check Trilinos (and test that git has been ported successfully) by entering:

```
git clone software.sandia.gov:/space/git/Trilinos.
```

Download clang from "http://llvm.org/releases/download.html". Select "Clang Binaries for MacOS X/x86-64" and install it (unpack the .tar.gz file).

This usually puts the downloaded tarball into HOME/Downloads, though you can specify a different download directory if you've configured your web browser that way. Choose a place to put the directory of binaries, such as HOME/pkg,

```
mkdir ~/pkg
mv clang+llvm-3.0-x86_64-apple-darwin11.tar.gz ~/pkg
tar xvzf clang+llvm-3.0-x86_64-apple-darwin11.tar.gz
```

This creates a clang+llvm-3.0-x86_64-apple-darwin11 directory. The bin subdirectory has the C and C++ compiler front ends (clang resp. clang++).

2.4.2 Building

Create a build directory for Trilinos.

```
cd Trilinos
mkdir Serial
cd Serial
```

EXTRA_ARGS=\$@

#rm -f CMakeCache.txt

Write a do-configure script for building Trilinos that uses the C and C++ compiler front ends, such as:

```
CC=$HOME/pkg/clang+llvm-3.0-x86_64-apple-darwin11/bin/clang CXX=$HOME/pkg/clang+llvm-3.0-x86_64-apple-darwin11/bin/clang++
```

```
cmake \
-D CMAKE_BUILD_TYPE:STRING=DEBUG \
-D HAVE_GCC_ABI_DEMANGLE:BOOL=ON \
-D DART_TESTING_TIMEOUT:STRING=600 \
-D Trilinos_ENABLE_Fortran:BOOL=OFF \
-D CMAKE_CXX_COMPILER:FILEPATH="$CXX" \
```

```
-D CMAKE_C_COMPILER:FILEPATH="$CC" \
-D Trilinos_ENABLE_DEBUG:BOOL=ON \
-D Trilinos_ENABLE_TESTS:BOOL=ON \
-D Trilinos_ENABLE_ALL_PACKAGES:BOOL=OFF \
-D Trilinos_ENABLE_Mesquite:BOOL=ON \
-D Tpetra_ENABLE_TESTS:BOOL=ON \
$EXTRA_ARGS \
```

Make the script executable, chmod a+x do-configure, and then run it to configure Trilinos.

It is also possible to build Mesquite on a Mac using the graphical version of CMake. Just follow the instructions in Section 2.3.3 (downloading the Mac version instead of the Windows version of course).

Next build Trilinos with make.

It is possible to build OpenMPI (e.g. 1.4.5) with clang and clang++. It is necessary to to turn off generation of the Fortran MPI bindings. OpenMPI is built in a directory completely separate from the OpenMPI source tree. Here is a configure line for building OpenMPI with Clang:

```
$ CC=$HOME/pkg/clang+llvm-3.0-x86_64-apple-darwin11/bin/clang CXX=$HOME/pkg/clang+llvm-3.0-x86_64-apple-darwin11/bin/clang++../openmpi-1.4.5/configure --prefix=$HOME/pkg/openmpi-1.4.5 --disable-mpi-f77 --disable-mpi-f90
```

Chapter 3

Examples

3.1 Short Tutorial

In this section, we write a driver code which calls the Mesquite library to improve the quality of a test mesh. This tutorial section is aimed at giving the user a feel for Mesquite: this section is not where to look for detailed information. In particular, information pertaining to loading a particular mesh format (see Chapter 4), interacting through a particular mesh interface (section 4.1), and details of defining geometric domains (see Chapter 6) are not given in this section.

First, we write a small program using Mesquite's simplified API, or wrappers, to show the fastest way to deploy Mesquite functionality to improve a mesh. The wrapper concept, as well as details about the different wrappers available, are described in section 3.1.3. Following this first example, we set up customized mesh improvement tool using Mesquite's low-level API, the details of which are described in section 3.1.4.

3.1.1 Tutorial File Template

To create and link a driver code, the Mesquite library must be installed per the instructions of section 2.3. The commands and file names specified in this section are relative to the installed testsuite/tutorial directory. It is assumed that that is the working directory. This tutorial begins with the file tutorial.cpp, which contains the following template:

The lines labeled 1-3 highlight three basic aspects of using Mesquite;

1. For convenience, Mesquite provides the header file

include/Mesquite_all_headers.hpp

which includes all Mesquite headers. Although this is the easiest way to handle the include directives, it may slow down compilation of the application.

- 2. All Mesquite classes are part of the Mesquite namespace.
- 3. The MsqError class defines an object type used to communicate Mesquite errors to the application. The calling application must pass an instance of the MsqError class or an instance of a subclass of MsqError to many Mesquite functions. The state of the error object may be checked by casting the instance of a Boolean or using it in a Boolean context. The state is cleared by calling the clear method.
- 4. In the sections that follow, we guide the user through the steps necessary to smooth a mesh using Mesquite. All new lines of code to be added to the template file start in this position and are added in the order in which they are discussed.

The code above takes a mesh file name as a command line argument and performs no action. We can compile it in the (examples/) directory with the command:

```
make -f tutorial.make
```

3.1.2 Loading a Test Mesh

Our next step is to load one of the test meshes distributed with Mesquite. These meshes are distributed in the VTK unstructured mesh format, the details of which are given in [22, 3]. This format was chosen because of its readability and ease of use. In this tutorial we use the simplest mechanism for loadling a mesh into Mesquite; different options are described in Chapter 4. In particular, to load a VTK test mesh in Mesquite, instantiate the Mesquite mesh database object, MeshImpl, and use the read_vtk member function by adding the following lines to the file template described in 3.1.1.

```
Mesquite::MeshImpl my_mesh;
my_mesh.read_vtk(argv[1], err);
if (err)
{
   std::cout << err << std::endl;
   return 1;
}</pre>
```

If the mesh read in contains more than one type of element, Mesquite will automatically handle the mixed elements with no additional effort required.

Mesquite also provides a function to write a mesh file in VTK format, given a MeshImpl object:

```
my_mesh.write_vtk("original_mesh.vtk",err);
```

Mesquite deals automatically with all types of supported elements (triangles, quadrilaterals, tetrahedra, hexahedra, wedges, and pyramids), and also hybrid meshes consisting of mixed element types. Some meshes require geometry information as well. When improving a surface mesh, Mesquite must be provided information about surface(s) the mesh is constrained to lie on and the association between mesh entities and entities of the geometric domain (surfaces, curves, etc.) Because Mesquite is inherently a 3D code, all 2D meshes must specify some geometry constraints. The details for general geometric surfaces are explained in Chapter 6. In this section, we show how to define the geometry of a 2D planar mesh, specified by a point (x, y, z) and a normal. For example, the following defines an xy-plane shifted five units in the z-direction:

```
Vector3D normal(0,0,1);
Vector3D point(0,0,5);
PlanarDomain my_mesh_plane(normal, point);
```

3.1.3 Improving the Mesh with a Wrapper Class

The simplest way to use a Mesquite mesh quality improvement procedure is to instantiate one of the wrapper classes described in Chapter 7. Here, we will instantiate the LaplacianSmoother wrapper and use it to improve the Mesh we created earlier. Mesquite can optimize the mesh without further input from the user by utilizing preset, default values. If some customization is desired, the wrapper classes also allow users to set the most important parameters of the underlying algorithms and metrics (see Chapter 7 for details).

```
Mesquite::LaplaceWrapper mesh_quality_algorithm;

MeshDomainAssoc mesh_and_domain = MeshDomainAssoc(&my_mesh, &my_mesh_plane);
mesh_quality_algorithm.run_instructions(&mesh_and_domain, err);

//Should check the error object after the instruction is ran
// to see whether the instructions were all successful.
if (err)
{
    std::cout << err << std::endl;
    return 1;
}</pre>
```

Once the algorithm has been executed using the run_instructions member function of the wrapper class, the improved mesh can be written to a new file:

```
my_mesh.write_vtk("smoothed_mesh.vtk",err);
```

This completes the code necessary for the simple wrapper example. Once the code has successfully compiled by typing the make command given in section 3.1.1, run it from the tutorial directory mesquite/testSuite/tutorial with a mesh file name as a command line argument by typing:

```
./tutorial ../../meshFiles/2D/quads/untangled/square_quad_10_rand.vtk
```

The code creates the files original_mesh.vtk and improved_mesh.vtk in the current directory. These two meshes, the original and the optimized, are shown in figure 3.1. The text output of the code, shown below, reports the inverse mean ratio quality metric statistics for the original mesh before optimization and the final mesh after optimization. The optimized mesh consists of square quadrilaterals which have an inverse mean ratio value of 1.0.

```
Evaluating quality for 100 elements.
 This mesh had 100 quadrilateral elements.
 There were no inverted elements detected.
 No entities had undefined values for any computed metric.
    Element Quality Statistics
    minimum
              average
                            rms
                                   maximum
                                             std.dev.
                         1.1738
    1.01013
              1.16655
                                   1.79134
                                             0.130322
    Number of statistics = 100
    Metric = Inverse Mean Ratio
    Element Quality not based on sample points.
```

```
Evaluating quality for 100 elements.

This mesh had 100 quadrilateral elements.

There were no inverted elements detected.

No entities had undefined values for any computed metric.
```

Element Quality Statistics

```
minimum average rms maximum std.dev.
1 1 1 1.00001 2.20663e-006
```

Number of statistics = 100 Metric = Inverse Mean Ratio Element Quality not based on sample points.

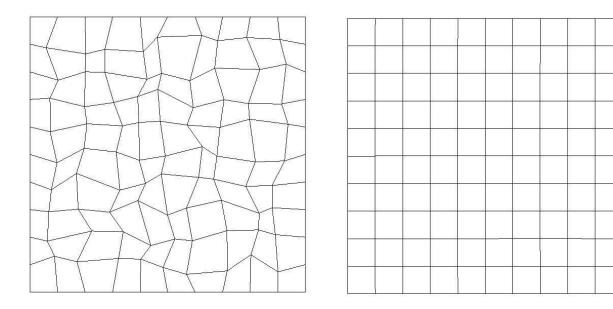


Figure 3.1: square_quad_10_rand.vtk mesh. The original mesh is on the left, the mesh smoothed with the LaplacianSmoother is shown on the right.

3.1.4 Improving the Mesh with the Low Level API

If the user requires in-depth control over the mesh quality improvement process, the use of lower-level Mesquite classes provides an extensive amount of flexibility. In particular, the user can specify the quality metric, objective function template, and optimization algorithm by instantiating particular instances of each. For each, various options such as numerical or analytical gradient and Hessian evaluations or the patch size can be selected. Furthermore, the user can fine tune the optimization algorithm performance by creating and setting the parameters of the termination criteria.

Once these core objects have been created and customized, the user creates an instruction queue and adds one or more quality improvers and quality assessors to it. The mesh optimization process is initiated with the run_instructions method on the instruction queue class.

In this section, we provide a simple example to highlight the main steps needed for this approach. The code segment given below performs the same functionality as the wrapper class highlighted in the previous section. The comment lines provide high level documentation; the details of each class and the low-level API are not described here.

```
// creates a mean ratio quality metric ...
IdealWeightInverseMeanRatio inverse_mean_ratio(err);
  // sets the objective function template
LPtoPTemplate obj_func(&inverse_mean_ratio, 2, err);
  // creates the optimization procedures
TrustRegion t_region(&obj_func);
  //performs optimization globally
t_region.use_global_patch();
  // creates a termination criterion and
 // add it to the optimization procedure
 // outer loop: default behavior: 1 iteration
  // inner loop: stop if gradient norm < eps</pre>
TerminationCriterion tc_inner;
tc_inner.add_absolute_gradient_L2_norm( 1e-4 );
t_region.set_inner_termination_criterion(&tc_inner);
  // creates a quality assessor
QualityAssessor m_ratio_qa(&inverse_mean_ratio);
  // creates an instruction queue
InstructionQueue queue;
queue.add_quality_assessor(&m_ratio_qa, err);
queue.set_master_quality_improver(&t_region, err);
queue.add_quality_assessor(&m_ratio_qa, err);
  // do optimization of the mesh_set
MeshDomainAssoc mesh_and_domain = MeshDomainAssoc(&my_mesh, &my_mesh_plane);
queue.run_instructions(&mesh_and_domain, err);
if (err) {
  std::cout << err << std::endl;</pre>
 return 2;
```

3.1.5 Mesh Improvement Examples

The left image in figure 3.2 shows a mesh that has been degraded by moving the disk from the right side of the square to the left while keeping the mesh topology fixed. The mesh file

mesquite/meshFiles/2D/vtk/quads/tangled/hole_in_square.vtk contains the information for this mesh. If you plan to run this example, note that the normal direction that defines the geometry is now (0,0,-1). This change must be made in the tutorial example code as was done in section 3.1.2, or an error message will be thrown.

```
Vector3D normal(0,0,-1);
Vector3D point(0,0,-5);
PlanarDomain my_mesh_plane(normal, point);
```

We can now improve the mesh with the wrapper mentioned in 3.1.3 or the detailed API mentioned in 3.1.4. Because we changed the normal, the driver code must be recompiled; otherwise the code and executable are as before. Once the code is recompiled, type

```
./tutorial ../../meshFiles/2D/vtk/quads/tangled/hole_in_square.vtk
```

to improve this mesh. The smoothed mesh is shown in the right image of figure 3.2. The vertex locations have been repositioned and significantly improve the quality of the mesh, as shown by the onscreen quality assessor output:

Element Quality Statistics

```
minimum average rms maximum std.dev.
1.07588 85.8391 463.357 5037.46 455.336
```

Number of statistics = 140 Metric = Inverse Mean Ratio Element Quality not based on sample points.

```
******* QualityAssessor(free only) Summary *********
```

```
Evaluating quality for 140 elements.

This mesh had 140 quadrilateral elements.

There were no inverted elements detected.

No entities had undefined values for any computed metric.
```

Element Quality Statistics

```
minimum average rms maximum std.dev.
1.01896 1.83479 1.91775 3.36336 0.557969
```

```
Number of statistics = 140
Metric = Inverse Mean Ratio
Element Quality not based on sample points
```

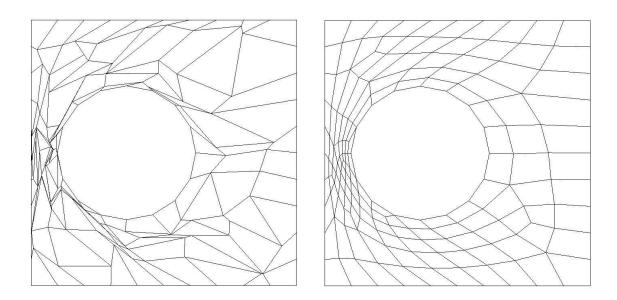


Figure 3.2: hole_in_square.vtk mesh. The original mesh is on the left, the mesh smoothed with Mesquite is shown on the right.

3.1.6 Regression Testing

Regression testing encompasses running unit tests as well as comparing results data against "blessed" or "gold" data. An example of comparing results of a smoothed mesh against a gold version is in mesquite/testSuite/parallel_smooth_laplace/par_hex_smooth_laplace.cpp. This utilizes a function in MeshUtil, meshes_are_different, to compare two MeshImpl objects (within a specified numerical tolerance). It is recommended that both unit testing and gold-comparison testing be included in your test code development.

Chapter 4

Getting Mesh Into Mesquite

The application must provide Mesquite with data on which to operate. The two fundamental classes of information Mesquite requires are:

- Mesh vertex coordinates and element connectivity, and
- Constraints on vertex movement.

In this chapter we will assume that the only constraint available for vertex movement is to flag the vertices as fixed. More advanced constraints such as vertices following geometric curves or surfaces are discussed in the following chapter.

The mesh data expected as input to Mesquite is a set of vertices and elements. Each vertex has associated with it a fixed flag, a "byte", and x, y, and z coordinate values. The fixed flag is used as input only. It indicates whether or not the corresponding vertex position should be fixed (i.e., coordinates not allowed to change) during the optimization. The "byte" is one byte of Mesquite-specific working data associated with each vertex (currently only used for vertex culling.) The coordinate values for each vertex serve as both input and output: as input they are the initial positions of the vertices and as output they are the optimized positions.

Each element of the input mesh has associated with it a type and a list of vertices. The type is one of the values defined in Mesquite::EntityTopology (Mesquite.hpp). It species the topology (type of shape) of the element. Currently supported element types are triangles, quadrilaterals, tetrahedra, hexahedra, triangular wedges, and pyramids. The list of vertices (commonly referred to as the "connectivity") define the geometry (location and variation of shape) for the element. The vertices are expected to be in a pre-defined order specific to the element topology. Mesquite uses the canonical ordering defined in the ExodusII specification[19].

For some more advanced capabilities, Mesquite may require the ability to attach arbitrary pieces of data (called "tags") to mesh elements or vertices. For more on tags see Section 4.5.

4.1 The Mesquite:: Mesh Interface

The Mesquite::Mesh class (MeshInterface.hpp) defines the interface Mesquite uses to interact with mesh data. In C++ this means that the class defines a variety of pure virtual (or abstract) functions for accessing mesh data. An application may implement a subclass of Mesquite::Mesh, providing implementations of the virtual methods that allow Mesquite direct access to the applications in-memory mesh representation.

The Mesquite:: Mesh interface defines functions for operations such as:

- Get a list of all mesh vertices.
- Get a list of all mesh elements.
- Get a property of a vertex (coordinates, fixed flag, etc.)
- Set a property of a vertex (coordinates, "byte", etc.)

- Get the type of an element
- Get the vertices in an element

It also defines other operations that are only used for certain optimization algorithms:

- Get the list of elements for which a specific vertex occurs in the connectivity list.
- Define a "tag" and use it to associate data with vertices or elements.

Mesh entities (vertices and elements) are referenced in the Mesquite::Mesh interface using 'handles'. There must be a unique handle space for all vertices, and a separate unique handle space for all elements. That is, there must be a one-to-one mapping between handle values and vertices, and a one-to-one mapping between handle values and elements. The storage type of the handles is specified by

```
Mesquite::Mesh::VertexHandle and Mesquite::Mesh::ElementHandle.
```

The handle types are of sufficient size to hold either a pointer or an index, allowing the underlying implementation of the Mesquite::Mesh interface to be either pointer-based or index-based. All mesh entities are referenced using handles. For example, the Mesquite::Mesh interface declares a method to retrieve the list of all vertices as an array of handles and a method to update the coordinates of a vertex where the vertex is specified as a handle.

It is recommended that the application invoke the Mesquite optimizer for subsets of the mesh rather than the entire mesh whenever it makes sense to do so. For example, if a mesh of two geometric volumes is to be optimized and all mesh vertices lying on geometric surfaces are constrained to be fixed (such vertices will not be moved during the optimization) then optimizing each volume separately will produce the same result as optimizing both together.

4.2 Accessing Mesh In Arrays

One common representation of mesh in applications is composed of simple coordinate and index arrays. Mesquite provides the ArrayMesh implementation of the Mesquite::Mesh interface to allow Mesquite to access such array-based mesh definitions directly. The mesh must be defined as follows:

- Vertex coordinates must be stored in an array of double-precision floating-point values. The coordinate values must be interleaved, meaning that the x, y, and z coordinate values for a single vertex are contiguous in memory.
- The mesh must be composed of a single element type.
- The element connectivity (vertices in each element) must be stored in an interleaved format as an array of long integers. The vertices in each element are specified by an integer i, where the location of the coordinates of the corresponding vertex is located at position 3*i in the vertex coordinates array.
- The fixed/boundary state of the vertices must be stored in an array of integer values, where a value of 1 indicates a fixed vertex and a value of 0 indicates a free vertex. The values in this array must be in the same order as the corresponding vertex coordinates in the coordinate array.

The following is a simple example of using the ArrayMesh object to pass Mesquite a mesh containing four quadrilateral elements.

```
0, 2, 0,
                       1, 2, 0,
                       2, 2, 0 ;
    /* quadrilateral element connectivity (vertices) */
 long quads [] = \{ 0, 1, 4, 3, \}
                   1, 2, 5, 4,
                   3, 4, 7, 6,
                   4, 5, 8, 7 };
    /* all vertices except the center one are fixed */
  int fixed[] = \{ 1, 1, 1, 1, \dots \}
                   1, 0, 1,
                  1, 1, 1 };
/** create an ArrayMesh to pass the above mesh into Mesquite **/
  ArrayMesh mesh (
                    /* 3D mesh (three coord values per vertex) */
      3,
                    /* nine vertices */
                    /* the vertex coordinates */
      coords,
                    /* the vertex fixed flags */
      fixed,
                    /* four elements */
      QUADRILATERAL, /* elements are quadrilaterals */
                    /* element connectivity */
      quads);
/** smooth the mesh **/
    /* Need surface to constrain 2D elements to */
  PlanarDomain domain ( PlanarDomain::XY );
  MsqError err;
  ShapeImprover shape_wrapper;
  if (err) {
    std::cout << err << std::endl;
    exit (2);
  }
 MeshDomainAssoc mesh_and_domain = MeshDomainAssoc(&mesh, &domain);
  shape_wrapper.run_instructions( &mesh_and_domain, err );
  if (err) {
    std::cout << "Error_smoothing_mesh:" << std::endl
              << err << std::endl;
  }
/** Output the new location of the center vertex **/
  std::cout << "New_vertex_location:_(_"
            << coords[12] << ","
            << coords[13] << ","
            << coords[14] << "") << std::endl;</pre>
```

NOTE: When using the ArrayMesh interface, the application is responsible for managing the storage of the mesh data. The ArrayMesh does NOT copy the input mesh.

4.3 Reading Mesh From Files

4.3.1 VTK and ExodusII Files

Mesquite provides a concrete implementation of the Mesquite::Mesh named Mesquite::MeshImpl. This implementation is capable of reading mesh from VTK[22, 3] and optionally ExodusII files. See Sections 2.1.3 and 2.3 for more information regarding the optional support for ExodusII files.

4.3.2 Reading and Writing VTK Files

VTK files can be read into Mesquite via the MeshImpl::read_vtk(...) method. VTK files can be written using the MeshImpl::write_vtk(...) method. The current implementation writes version 3.0 of the VTK file format and is capable of reading any version of the file format up to 3.0. Mesquite only supports the legacy VTK file format; it does not support XML format. The capibilities and limitations of Mesquite VTK file processing is as follows:

- cannot read binary files
- can read the following Dataset types:

```
"STRUCTURED_POINTS",
"STRUCTURED_GRID",
"UNSTRUCTURED_GRID",
"POLYDATA",
"RECTILINEAR_GRID",
"FIELD"
```

- can only write "UNSTRUCTURED_GRID" Dataset type
- cannot read triangle strips
- can read/write following VTK attribute data:

```
"SCALARS",
"COLOR_SCALARS",
"VECTORS",
"NORMALS",
"TEXTURE_COORDINATES",
"TENSORS",
"FIELD"
```

- will accept either "FIELD" or "DATASET FIELD" for block name value
- polygonal meshes can be read and written by Mesquite using VTK format. However, only the LaplaceSmoother and the SmartLaplaceSmoother are currently able to work with this mesh type.
- supports fixed and slaved POINT Dataset Attributes with dataNames of "fixed" and "slaved". These are used to specify which nodes are to be considered fixed and slaved.

The 'fixed' flag for vertices can be specified in VTK files by defining a SCALAR POINT_DATA attribute with values of 0 or 1, where 1 indicates that the corresponding vertex is fixed. The Mesquite::MeshImpl class is capable of reading and storing tag data (see Section 4.5) using VTK attributes and field data. A example of a Mesquite compatible VTK file can be found in Section 4.6.

4.3.3 VTK Cell Types

VTK files can contain the following types of cells:

- 1. VTK_VERTEX
- 2. VTK_POLY_VERTEX
- 3. VTK_LINE
- 4. VTK_POLY_LINE
- 5. VTK_TRIANGLE
- 6. VTK_TRIANGLE_STRIP
- 7. VTK_POLYGON
- 8. VTK_PIXEL
- 9. VTK_QUAD
- 10. VTK_TETRA
- 11. VTK_VOXEL
- 12. VTK_HEXAHEDRON
- 13. VTK_WEDGE
- 14. VTK_PYRAMID
- 15. VTK_PENTAGONAL_PRISM
- 16. VTK_HEXAGONAL_PRISM
- 21. VTK_QUADRATIC_EDGE
- 22. VTK_QUADRATIC_TRIANGLE
- 23. VTK_QUADRATIC_QUAD
- 24. VTK_QUADRATIC_TETRA
- 25. VTK_QUADRATIC_HEXAHEDRON
- 26. VTK_QUADRATIC_WEDGE
- 27. VTK_QUADRATIC_PYRAMID
- 28. VTK_BI_QUADRATIC_QUAD
- 29. VTK_TRI_QUADRATIC_HEXAHEDRON
- 30. VTK_QUADRATIC_LINEAR_QUAD
- 31. VTK_QUADRATIC_LINEAR_WEDGE
- 32. VTK_QUADRATIC_WEDGE
- 33. VTK_QUADRATIC_WEDGE
- 34. VTK_BI_QUADRATIC_HEXAHEDRON

Mesquite supports the following VTK cell types (followed by the VTK Cell Types number):

- TRIANGLE (5, 22)
- QUADRILATERAL (9, 23, 28)
- POLYGON (7)
- TETRAHEDRON (10, 24)
- HEXAHEDRON (12, 25, 29)
- PRISM (13, 26)
- PYRAMID (14, 27)

4.4 ITAPS iMesh Interface

4.4.1 Introduction

The ITAPS Working Group has defined a standard API for exchange of mesh data between applications. The iMesh interface[20] defines a superset of the functionality required for the Mesquite::Mesh interface. Mesquite can access mesh through an iMesh interface using the Mesquite::MsqIMesh class declared

in MsqIMesh.hpp. This class is an "adaptor": it presents the iMesh interface as the Mesquite::Mesh interface.

The primary advantage of this method of providing mesh data to mesquite is that it is designed for interoperability. The same API can be used to provide other tools and services access to the mesh data. And there are stand-alone mesh data base libraries that already implement this API such as MOAB[2] and FMDB[1]. It is also possible to implement the iMesh interface in Fortran.

4.4.2 Overview

A Mesquite::MsqIMesh instance must be provided with at least two pieces of information: The iMesh_Instance handle and an iBase_EntitySetHandle. The optional iBase_TagHandle for the "fixed tag" must frequently be provided as well. The iMesh_Instance specifies the instance of the database containing the mesh. The iBase_EntitySetHandle handle specifies the subset of that mesh that is to be optimized by Mesquite. If the entire mesh is to be optimized then the "root set" should be specified for this argument. The quality of all elements in this set will be used to drive the mesh optimization. All vertices adjacent to any elements in the set will be moved as a part of the optimization unless they are explicitly designated as fixed. The "fixed tag" is used to indicate such vertices. Every vertex adjacent to the input elements should be tagged with a single integer value of either zero or one for the "fixed tag". A value of one indicates that the vertex is fixed while a value of zero indicates that the vertex location is to be optimized by Mesquite.

The boundary of the mesh must always be constrained in some way for the mesh optimization to produce valid results. For a volume mesh this can be accomplished by either designating the vertices on the mesh boundary as fixed or by specifying a geometric domain (e.g. surfaces, curves, etc) that the boundary vertices are constrained to lie on. For a surface mesh some geometric domain must always be specified (e.g. a surface) and it is still necessary to specify which vertices are fixed unless the geometric domain also includes the bounding geometric curves constraining the movement of the boundary mesh vertices¹. Geometric domains are the topic of Chapter 6. Further discussion and examples in this section will be limited to volume meshes and true 2D meshes, both with the boundary vertices designated as fixed via the "fixed tag".

Designating vertices as fixed is the responsibility of the application using Mesquite. This responsibility is left to the application (as opposed to providing some utility in Mesquite to find the "skin" of a mesh) for several reasons. An application can often obtain the set of vertices bounding a region of mesh directly through data not available to Mesquite. For example if the application has a B-rep solid model for which the mesh is a discretization then it typically can obtain the bounding vertices as the set of vertices associated with the bounding geometric entities. Further, there exist cases where the fixed vertices are more than just those on the topological boundary of the mesh. For example, consider the mesh of a conic surface that includes a vertex at the apex of the cone. Such a vertex must be designated as fixed because the lack of a valid surface normal at that point will interfere with the correct functioning of Mesquite. Such a vertex cannot be reliably identified given only the mesh. However, identifying such vertices typically happens naturally when obtaining the set of fixed vertices from the association with bounding geometric entities. Finally, the optimal implementation of a "skinning" operation depends greatly on details of the mesh representation that Mesquite is not aware of and is not otherwise concerned with.

4.4.3 Practical Details

The Mesquite::MsqIMesh class caches data related to the input iBase_EntitySetHandle upon construction. If the contents of the referenced entity set change, or the vertices associated with elements contained in that set change, then the application should either re-create the Mesquite::MsqIMesh instance or notify an existing instance of the change by calling the set_active_set member function. Similarly, while the implementation does not at the time of this writing cache data related to the "fixed tag", it may do so in the future. For forward compatibility the application should consider calling the set_fixed_tag method of Mesquite::MsqIMesh to notify the instance that the value of the tag may have changed for some mesh vertices.

¹A surface mesh that forms a topological sphere has no boundary and therefore need not have vertices designated as fixed or otherwise constrained as long as the entire geometric domain is continuous.

The current version of Mesquite uses the following functions from the iMesh interface:

- iMesh_getRootSet
- iMesh_getGeometricDimension
- iMesh_getEntities
- iMesh_getNumOfType
- iMesh_isEntContained
- iMesh_getEntArrTopo
- iMesh_getEntArrAdj
- iMesh_getVtxArrCoords
- iMesh_setVtxCoord
- iMesh_createTag
- iMesh_destroyTag
- iMesh_getTagName
- iMesh_getTagSizeBytes
- iMesh_getTagType
- iMesh_getTagHandle
- iMesh_getIntArrData
- iMesh_getIntData
- iMesh_getArrData
- iMesh_setArrData
- iMesh_setIntData
- iMesh_setIntArrData

An implementation should provide complete implementations of all of these methods to guarantee compatibility with all possible Mesquite algorithms.

4.4.4 Volume Example

The following example demonstrates the use of the ShapeImprover wrapper with an implementation of the iMesh interface. It is assumed that the application has implemented the iMesh interface to provide access to its own data or is using an existing implementation of the iMesh interface to store its mesh data. The example illustrates the setup necessary to correctly pass a subset of a mesh to mesquite and how to designate boundary vertices as fixed using the "fixed tag". The input to the example function is the iMesh_Instance handle and an iBase_EntitySetHandle specifying both the elements for which to improve the quality and the free vertices. The example code uses this application-supplied designation of which vertices are fixed to initialize the "fixed tag".

```
#include <MsqError.hpp>
#include <ShapeImprover.hpp>
#include <MsqIMesh.hpp>
#include <vector>
#include <iostream>
#include <iMesh.h>
using namespace Mesquite;
/** \ brief \ Call \ Mesquite \ Shape Improver \ wrapper \ for \ volume \ mesh
 * Smooth mesh accessed through ITAPS APIs using Mesquite
 * ShapeImprover.
 *\param mesh_instance iMesh API instance
 *\backslash param\ mesh\ A\ set\ defined\ in\ 'mesh\_instance'\ that\ contains
              *both* the set of elements to smooth *and* the
               set of interior vertices that are to be moved
               to improve the quality of the mesh. This set
               must *not* contain vertices on the boundary of
               the volume mesh.
 *\rvert return\ mesquite\ error\ code\ or\ imesh\ error\ code
           (0 for success in all cases.)
 */
int shape_improve_volume( iMesh_Instance mesh_instance,
                           iBase_EntitySetHandle mesh )
{
  MsqPrintError err(std::cerr);
  int ierr;
  iBase_EntityHandle *ptr1, *ptr2;
  int *ptr3, *ptr4;
  int i5, i6, i7, i8, i9, i10, i11;
  const int elem_dim = 3;
  const int max_vtx_per_elem = 8;
    // create adapter (should also create fixed tag)
  MsqIMesh mesh_adapter( mesh_instance, mesh, elem_dim, err );
  if (err) return err.error_code();
    // get tag for marking vertices as fixed
    // Note: we assume here that the tag has already been created.
  iBase\_TagHandle\ fixed\_tag = 0;
  iMesh_getTagHandle ( mesh_instance,
                       "fixed",
                       &fixed_tag,
                       &ierr,
                       strlen("fixed"));
  if (iBase_SUCCESS != ierr) return ierr;
    // get all vertices in mesh
  int count, num_vtx;
  iMesh_getNumOfType( mesh_instance, mesh, elem_dim, &count, &ierr );
  if (iBase_SUCCESS != ierr) return ierr;
  std::vector<iBase_EntityHandle> elems(count), verts(max_vtx_per_elem*count);
```

```
std::vector<int> indices(max_vtx_per_elem*count), offsets(count+1);
ptr1 = \&elems[0];
ptr2 = &verts[0];
ptr3 = \&indices[0];
ptr4 = \&offsets[0];
i5 = elems.size();
i7 = verts.size();
i8 = indices.size();
i10 = offsets.size();
iMesh_getAdjEntIndices( mesh_instance, mesh,
                        elem_dim, iMesh_ALL_TOPOLOGIES, iBase_VERTEX,
                        &ptr1, &i5, &i6,
                        &ptr2, &i7, &num_vtx,
                        &ptr3, &i8, &i9,
                        &ptr4, &i10, &i11, &ierr);
if (iBase_SUCCESS != ierr) return ierr;
verts.resize( num_vtx );
  // set fixed flag on all vertices
std::vector<int> tag_data(num_vtx, 1);
iMesh_setIntArrData ( mesh_instance, &verts[0], verts.size(),
                     fixed_tag, &tag_data[0], tag_data.size(), &ierr);
if (iBase_SUCCESS != ierr) return ierr;
  // clear fixed flag for vertices contained directly in set
iMesh_getNumOfType( mesh_instance, mesh, iBase_VERTEX, &count, &ierr );
if (iBase_SUCCESS != ierr) return ierr;
verts.resize(count);
ptr1 = &verts[0];
i5 = verts.size();
iMesh_getEntities( mesh_instance, mesh, iBase_VERTEX, iMesh_ALL_TOPOLOGIES,
                   &ptr1, &i5, &i6, &ierr);
if (iBase_SUCCESS != ierr) return ierr;
tag_data.clear();
tag_data.resize( verts.size(), 0 );
iMesh_setIntArrData ( mesh_instance, &verts[0], verts.size(),
                     fixed_tag, &tag_data[0], tag_data.size(), &ierr);
if (iBase_SUCCESS != ierr) return ierr;
  // Finally, smooth the mesh
ShapeImprover smoother;
smoother.run_instructions(&mesh_adapter, err);
if (err) return err.error_code();
return 0;
```

4.4.5 Two-dimensional Example

This section presents an example of how to use Mesquite to optimize a 2D mesh. It is a modification of the example from the previous section with changes shown in blue. As Mesquite operates only on 3D meshes (either volume or surface), a 2D mesh is optimized by treating it as a surface mesh constrained to the XY plane.

```
#include <MsqError.hpp>
#include <ShapeImprover.hpp>
#include <MsqIMesh.hpp>
#include <PlanarDomain.hpp>
#include <vector>
#include <iostream>
#include <iMesh.h>
using namespace Mesquite;
/**\brief Call Mesquite ShapeImprover wrapper for 2D mesh
 * Smooth mesh accessed through ITAPS APIs using Mesquite
 * ShapeImprover.
 *\param mesh_instance iMesh API instance
 *\backslash param\ mesh\ A\ set\ defined\ in\ 'mesh\_instance'\ that\ contains
              *both* the set of elements to smooth *and* the
               set of interior vertices that are to be moved
               to improve the quality of the mesh. This set
              must *not* contain vertices on the boundary of
               the mesh.
 *\rvert return\ mesquite\ error\ code\ or\ imesh\ error\ code
          (0 for success in all cases.)
 */
int shape_improve_2D ( iMesh_Instance mesh_instance,
                       iBase_EntitySetHandle mesh )
{
  MsqPrintError err(std::cerr);
  int ierr;
  iBase_EntityHandle *ptr1, *ptr2;
  int *ptr3, *ptr4;
  int i5, i6, i7, i8, i9, i10, i11;
  const int elem_dim = 2;
  const int max_vertex_per_elem = 4;
    // create adapter (should also create fixed tag)
  MsqIMesh mesh_adapter( mesh_instance, mesh, elem_dim, err );
  if (err) return err.error_code();
    // get tag for marking vertices as fixed
    // Note: we assume here that the tag has already been created.
  iBase\_TagHandle\ fixed\_tag = 0;
  iMesh_getTagHandle ( mesh_instance,
                       "fixed",
                       &fixed_tag,
                      &ierr,
                       strlen("fixed"));
  if (iBase_SUCCESS != ierr) return ierr;
    // get all vertices in mesh
  int count, num_vtx;
  iMesh_getNumOfType( mesh_instance, mesh, elem_dim, &count, &ierr );
  if (iBase_SUCCESS != ierr) return ierr;
  std::vector<iBase_EntityHandle> elems(count), verts(max_vtx_per_elem*count);
```

```
std::vector<int> indices(max_vtx_per_elem*count), offsets(count+1);
ptr1 = \&elems[0];
ptr2 = &verts[0];
ptr3 = \&indices[0];
ptr4 = \&offsets[0];
i5 = elems.size();
i7 = verts.size();
i8 = indices.size();
i10 = offsets.size();
iMesh_getAdjEntIndices( mesh_instance, mesh,
                        elem_dim, iMesh_ALL_TOPOLOGIES, iBase_VERTEX,
                        &ptr1, &i5, &i6,
                        &ptr2, &i7, &num_vtx,
                        &ptr3, &i8, &i9,
                        &ptr4, &i10, &i11, &ierr);
if (iBase_SUCCESS != ierr) return ierr;
verts.resize( num_vtx );
  // set fixed flag on all vertices
std::vector<int> tag_data(num_vtx, 1);
iMesh_setIntArrData ( mesh_instance, &verts[0], verts.size(),
                     fixed_tag, &tag_data[0], tag_data.size(), &ierr);
if (iBase_SUCCESS != ierr) return ierr;
  // clear fixed flag for vertices contained directly in set
iMesh_getNumOfType( mesh_instance, mesh, iBase_VERTEX, &count, &ierr );
if (iBase_SUCCESS != ierr) return ierr;
verts.resize(count);
ptr1 = &verts[0];
i5 = verts.size();
iMesh_getEntities( mesh_instance, mesh, iBase_VERTEX, iMesh_ALL_TOPOLOGIES,
                   &ptr1, &i5, &i6, &ierr);
if (iBase_SUCCESS != ierr) return ierr;
tag_data.clear();
tag_data.resize( verts.size(), 0 );
iMesh_setIntArrData ( mesh_instance, &verts[0], verts.size(),
                     fixed_tag, &tag_data[0], tag_data.size(), &ierr);
if (iBase_SUCCESS != ierr) return ierr;
  // Finally, smooth the mesh
ShapeImprover smoother;
PlanarDomain xyplane (PlanarDomain::XY);
MeshDomainAssoc mesh_and_domain = MeshDomainAssoc(&mesh_adapter, &xyplane);
smoother.run_instructions(&mesh_and_domain, err);
if (err) return err.error_code();
return 0;
```

}

4.5 Tags

4.5.1 Using Tags

Mesquite has the ability to attach arbitrary pieces of data to mesh elements or vertices via the use of tags. Assigning tag data to a vertex or element is a two step process. First, the tag itself is created to describe the tag name, type, and size. Second, the actual data value is associated with the vertex or element using the tag as a descriptor.

When a tag itself is created, it is given a name and associated data type. Valid Mesquite Tag Types are: BYTE, BOOL, INT, DOUBLE, and HANDLE. Tags are created using the method MeshImpl::tag_create() or one of it derived forms.

Mesquite can also handle VTK tags. Valid VTK Tag Types are: SCALAR, COLOR, VECTOR, NORMAL, TEXTURE, TENSOR, and FIELD. VTK tags and data are assigned as part of the reading a VTK file operation and can be created and manipulated from within Mesquite. The VTK tags and data are persistent for the VTK entitles within Mesquite including when they are written back to a file.

Once the tag is created, it is used to associate data to a vertex or element mesh entity using the methods MeshImpl::tag_set_vertex_data() or MeshImpl::tag_set_element_data(). Tagged data is recovered via MeshImpl::tag_get_vertex_data() or MeshImpl::tag_get_element_data().

4.5.2 Vector Example

The following is a simple example of using tags to associate vectors with elements of a mesh. It creates a simple mesh of two quads then creates a single tag consisting of 3 doubles to represent the vector. After creating the values for the two vectors, it associates the vectors with the elements of the mesh via a call to tag_set_element_data().

```
#include "Mesquite.hpp"
#include "MeshImpl.hpp"
#include "MsqError.hpp"
int main(int argc, char* argv[])
  const double vertices [] = \{ 0, 0, 0, \dots \}
                               0, 1, 0,
                               1, 0, 0,
                               1, 1, 0,
                               2, 1, 0,
                               2, 0, 0;
  const int conn[] = \{0, 1, 3, 2, 1, 5, 4, 3\};
  bool fixed [] = {true, true, true, true, true};
  Mesquite::MsqError err;
  std::vector < Mesquite::Mesh::ElementHandle > elements;
    // Create the mesh
  Mesquite::MeshImpl mesh(6, 2, Mesquite::QUADRILATERAL, fixed, vertices, conn);
  mesh.get_all_elements(elements, err);
  std::vector < double > v_tags (elements.size() * 3);
    // Create tag
  Mesquite:: TagHandle v_tag =
                 mesh.tag_create("VECTOR", Mesquite::Mesh::DOUBLE, 3, NULL, err);
  std::vector<double> v_coords(6);
```

4.5.3 2x2 Matrix on vertices using Tags Example

}

The follow code example shows another way tags can be used in Mesquite. It attaches a 2x2 matrix to each vertex in a mesh by creating four tags and four arrays of doubles. The value for the matrices are stored in the arrays with each array representing one value of the matrix. Then the four tags are associated with the four arrays to create tagged data for each vertex representing the 2x2 matrix for it.

```
int main(int argc, char* argv[])
double tri_verts[] = \{ 0.0, 0.0, 0.0, 
                       1.0, 0.0, 0.0,
                       0.0, 1.0, 0.0,
                       1.0, 1.0, 0.0 };
 const int tri_elems [] = \{ 0, 1, 2, 
 bool fixed [] = {true, true, true};
 Mesquite:: MsqError err;
 std::vector<Mesquite::Mesh::ElementHandle>elements;
   // Create the mesh
 Mesquite::MeshImpl mesh(4, 2, Mesquite::TRIANGLE, fixed, tri_verts, tri_elems);
 mesh.get_all_elements(elements, err);
 std::vector < double > lid1 (elements.size());
 std::vector < double > lid2 (elements.size ());
 std::vector < double > lid3 (elements.size());
 std::vector < double > lid4 (elements.size ());
 double adTestOri [4] = \{0.0, 0.0, 0.0, 0.0\};
 for (unsigned int i=0; i < elements.size(); i++)
   Compute eigenvectors and eigenvalues to be used to set the Target
```

```
(actual steps left out for simplicity of example)
  **********************
  if (adTestOri[1]*adTestOri[1]+adTestOri[3]*adTestOri[3] >
      adTestOri[0] * adTestOri[0] + adTestOri[2] * adTestOri[2])
    lid1[i] = -adTestOri[1];
    lid2 [i] = adTestOri [0];
    lid3[i] = -adTestOri[3];
    lid4 [i] = adTestOri[2];
  }
  else
    lid1[i] = adTestOri[0];
    lid2[i] = adTestOri[1];
    lid3 [i] = adTestOri[2];
    lid4 [i] = adTestOri[3];
  }
}
const char LOCAL_ID_NAME1[] = "LOCAL_ID1";
const char LOCAL_ID_NAME2[] = "LOCAL_ID2";
const char LOCAL_ID_NAME3[] = "LOCAL_ID3";
const char LOCAL_ID_NAME4[] = "LOCAL_ID4";
  // Create tags
Mesquite::TagHandle lid_tag1 =
      mesh.tag_create(LOCAL_ID_NAME1, Mesquite::Mesh::DOUBLE, 1, NULL, err);
Mesquite::TagHandle lid_tag2 =
      mesh.tag_create(LOCAL_ID_NAME2, Mesquite::Mesh::DOUBLE, 1, NULL, err);
Mesquite::TagHandle lid_tag3 =
      mesh.tag_create(LOCAL_ID_NAME3, Mesquite::Mesh::DOUBLE, 1, NULL, err);
Mesquite::TagHandle lid_tag4 =
      mesh.tag_create(LOCAL_ID_NAME4, Mesquite::Mesh::DOUBLE, 1, NULL, err);
size_t num_cells=(elements.size());
  // Associate arrays representing 2x2 matricies to verticies
mesh.tag_set_vertex_data(
    lid_tag1 , num_cells , Mesquite :: arrptr(elements) , Mesquite :: arrptr(lid1) , err);
mesh.tag_set_vertex_data(
    lid_tag2 , num_cells , Mesquite :: arrptr(elements) , Mesquite :: arrptr(lid2), err);
mesh.tag_set_vertex_data(
    lid_tag3 , num_cells , Mesquite :: arrptr(elements) , Mesquite :: arrptr(lid3), err);
mesh.tag_set_vertex_data(
    lid_tag4 , num_cells , Mesquite:: arrptr(elements), Mesquite:: arrptr(lid4), err);
return 0;
```

4.6 Slaved Verticies

Mesquite supports three types of vertices: fixed, free, and slaved. Fixed vertices cannot be moved by optimization routines, free vertices are eligible to be moved, and slaved vertices are handled in a specific manner as described in this section.

Slaved vertices are used primarily with a mesh that contains higher order elements. They are typically

vertices at edge or face centers as well as element centers. In the cases where these higher order nodes cannot or should not be used in an optimization or quality metric evaluation, they are marked as slaved. This allows the operations to proceed as if those nodes were not present in the mesh but the slaved nodes can still be moved to retain the proper spatial relationship with any free nodes that were moved by the optimization.

How vertices can be moved by optimization routines is controlled by two flags in the MeshImplData class: fixed and slaved. When the fixed flag is false the vertex is considered to be free, true signifies a fixed vertex. True for the slaved flag marks the vertex as slaved. The combination of fixed=true and slaved=true is not valid and will result in failures during processing of the mesh containing the vertices.

There are three ways in Mesquite to mark vertices as slaved:

1. When reading in a mesh from a VTK file, a dataset attribute of 'slaved' can be used to define which of the points in the mesh are slaved. The following VTK input file describes a very contrived mesh of a single quad with four fixed vertices at the corners and a slaved vertex at the mid-point of each edge. In the input file, the first four points defined are the corners, the next for are the mid-point nodes.

```
# vtk DataFile Version 3.0
Mesquite Mesh
ASCII
DATASET UNSTRUCTURED_GRID
POINTS 8 double
0 0 0
1 0 0
1 1 0
0 1 0
0.5 0 0
1 0.5 0
0.5 1 0
0 0.5 0
CELLS 1 9
8 0 1 2 3 4 5 6 7
CELL_TYPES 1
23
POINT_DATA 8
SCALARS fixed int
LOOKUP_TABLE default
1
1
1
0
0
0
SCALARS GLOBAL_ID int 1
LOOKUP_TABLE default
1
2
3
4
5
6
7
SCALARS slaved int 1
LOOKUP_TABLE default
```

```
0
0
1
1
1
1
1
CELL_DATA 1
SCALARS GLOBAL_ID int 1
LOOKUP_TABLE default
```

2. A vertex can be marked as slaved by calling the method

MeshImplData::slave_vertex(size_t index, bool flag, MsqError& err) with the parameter 'flag' being "true" to signify slaved, "false" for not slaved.

3. The VertexBoundarySlaver class can be inserted in the instruction queue before any optimization to determine which higher-order nodes are slaved as a function of their distance from the boundary of the mesh. It will attempt to automatically mark vertices as slaved according to the input parameters 'depth' and 'max_boundary_domain_dimension'. 'depth' is the number of elements inwards from the boundary for which all contained higher-order nodes will be free variables in the optimization. Any vertex further from the boundary will be slaved. A depth of zero will result in all higher-order nodes being slaved except free nodes on the boundary. 'max_boundary_domain_dimension' specifies the definition of "boundary". If greater than or equal to 4, then the set of all fixed vertices is assumed to be the boundary. If less than four, then all vertices constrained to a domain with the specified number of fewer degrees of freedom (constrained to a geometric entity with an equal or smaller topological dimension) will be considered to be the boundary. If not specified, the boundary will be assumed to be indicated by fixed vertices.

Assuming a mesh has already been created, the following code snippet calls the VertexBoundarySlaver class to determine which vertices are to be slaved:

```
unsigned depth = 1;
unsigned boundary = 2;
Settings settings;
settings.set_slaved_ho_node_mode( Settings::SLAVE_CALCULATED );
SlaveBoundaryVertices determine_slaved_verts( depth, boundary );
MeshDomainAssoc mesh_and_domain = MeshDomainAssoc(&mesh, &domain);
determine_slaved_verts.loop_over_mesh( &mesh_and_domain, &settings, err );
```

Chapter 5

Mesquite Features

This chapter is summarizes various aspects of Mesquite that will be useful in understanding the topics that follow.

5.1 Solvers

Many of the quality improvers in Mesquite work by minimizing the value of an objective function (see Section 5.2), where the objective function is a function of the mesh vertex coordinates. The term *solver* is often used to refer to the portion of the code in the concrete subclass of VertexMover that implements the inner loop of the optimization for quality improvers that optimize an explicit objective-function based (the code that implements the function-minimization algorithm.)

The remainder of this section lists the solvers available in Mesquite along with a small code example for each showing how to invoke the solver.

5.1.1 Relaxation Smoothers

Note that the relaxation smoothers do NOT require an objective function.

5.1.1.1 LaplacianSmoother

Implements the Laplacian smoothing for a patch with one free vertex. It moves the free center vertex to the average of the neighboring vertices.

```
int main(int argc, char* argv[])
{
  const char input_file [] = MESH_FILES_DIR "2D/vtk/quads/untangled/square_quad_2.vtk";

  /* Read a VTK Mesh file */
  MsqPrintError err(cout);
  Mesquite::MeshImpl mesh;
  mesh.read_vtk( input_file , err);
  if (err) return 1;

  // creates an intruction queue
  InstructionQueue queue1;

  // creates a mean ratio quality metric ...
  ConditionNumberQualityMetric shape_metric;
  EdgeLengthQualityMetric lapl_met;
  lapl_met.set_averaging_method(QualityMetric::RMS);
```

```
// creates the laplacian smoother procedures
LaplacianSmoother lapl1;
QualityAssessor stop_qa=QualityAssessor(&shape_metric);
stop_qa.add_quality_assessment(&lapl_met);
  TerminationCriterion sc2;
sc2.add_iteration_limit(10);
if (err) return 1;
lapl1.set_outer_termination_criterion(&sc2);
queue1.add_quality_assessor(&stop_qa, err);
if (err) return 1;
queue1.set_master_quality_improver(&lapl1, err);
if (err) return 1;
queue1.add_quality_assessor(&stop_qa, err);
if (err) return 1;
Planar Domain plane (Vector 3D (0,0,1), Vector 3D (0,0,5));
 // launches optimization on mesh
MeshDomainAssoc mesh_and_domain = MeshDomainAssoc(&mesh, &plane);
queue1.run_instructions(&mesh_and_domain, err);
if (err) return 1;
return 0;
```

5.1.1.2 SmartLaplacianSmoother

Does same as the Laplacian smoother, but doesn't invert elements. Invoked same way as the LaplacianSmoother. If initial mesh in non-inverted, the SmartLaplacianSmoother performs Laplace smoothing while trying not to invert the mesh.

5.1.1.3 Randomize

The randomize smoother moves a free vertex to a random location within a local patch. This smoother is provided as a convenience to those who wish to generate a poor quality mesh from an initial mesh in order to test another mesh improvement algorithm.

```
int main()
{
    Mesquite::MeshImpl mesh;
    MsqPrintError err(cout);
    mesh.read_vtk(VTK_2D_DIR "quads/untangled/tangled_quad.vtk", err);
    if (err) return 1;

// Set Domain Constraint
    Vector3D pnt(0,0,0);
    Vector3D s_norm(0,0,1);
    PlanarDomain msq_geom(s_norm, pnt);

// creates an intruction queue
InstructionQueue queue1;
```

```
Randomize rand(.05);

TerminationCriterion sc_rand;
sc_rand.add_iteration_limit( 10 );

rand.set_outer_termination_criterion(&sc_rand);

queuel.set_master_quality_improver(&rand, err);
if (err) return 1;

MeshDomainAssoc mesh_and_domain = MeshDomainAssoc(&mesh, &msq_geom);
queuel.run_instructions(&mesh_and_domain, err);
if (err) return 1;

return 0;
}
```

5.1.2 OptSolvers

5.1.2.1 ConjugateGradient

Optimizes the objective function using the Polack-Ribiere scheme.

```
int main()
  Mesquite::MeshImpl mesh;
 MsqPrintError err(cout);
 mesh.read_vtk(VTK_2D_DIR "quads/untangled/tangled_quad.vtk", err);
  if (err) return 1;
   // Set Domain Constraint
  Vector3D pnt(0,0,0);
  Vector3D s_norm(0,0,1);
 PlanarDomain msq_geom(s_norm, pnt);
    // creates an intruction queue
  InstructionQueue queue1;
   // creates a mean ratio quality metric ...
  ConditionNumberQualityMetric shape-metric;
  UntangleBetaQualityMetric untangle(2);
 LInfTemplate obj_func(&untangle);
  if (err) return 1;
   // creates the steepest descent optimization procedures
  ConjugateGradient cg(&obj_func, err);
  if (err) return 1;
  QualityAssessor stop_qa=QualityAssessor(&shape_metric);
  QualityAssessor stop_qa2=QualityAssessor(&shape_metric);
 stop_qa.add_quality_assessment(&untangle);
```

```
queue1.add_quality_assessor(&stop_qa,err);
if (err) return 1;

queue1.set_master_quality_improver(&cg, err);
if (err) return 1;
queue1.add_quality_assessor(&stop_qa2,err);
if (err) return 1;

MeshDomainAssoc mesh_and_domain = MeshDomainAssoc(&mesh, &msq_geom);
queue1.run_instructions(&mesh_and_domain, err);
if (err) return 1;

return 0;
}
```

5.1.2.2 FeasibleNewton

Implements the newton non-linear programming algorithm in order to move a free vertex to an optimal position given an ObjectiveFunction object and a QualityMetric object. This implementation of Feasible Newton works well on volume meshes and on surfaces meshes using PlanarDomain that lie in the X-Y coordinate plane. It should not be used on non-planar surface meshes.

```
int main()
  MsqPrintError err(cout);
  Mesquite:: MeshImpl mesh;
 mesh.read_vtk(MESH_FILES_DIR "3D/vtk/tets/untangled/tire.vtk", err);
  if (err) return 1;
   // creates an intruction queue
 InstructionQueue queue1;
    // creates a mean ratio quality metric ...
  IdealWeightInverseMeanRatio mean(err);
  if (err) return 1;
 LPtoPTemplate obj_func(&mean, 1, err);
  if (err) return 1;
    // creates the optimization procedures
  FeasibleNewton fn(&obj_func);
   //perform optimization globally
  fn.use_global_patch();
  if (err) return 1;
  queue1.set_master_quality_improver(&fn, err);
  if (err) return 1;
  queue1.run_instructions(&mesh, err);
  if (err) return 1;
 return 0;
```

5.1.2.3 SteepestDescent

A very basic implementation of the steepest descent optimization algorithm. It works on patches of any size but the step size is automatically computed and is not accessible through the interface. It is for testing purposes only.

```
int main()
 MsqPrintError err(std::cout);
 Mesquite::MeshImpl mesh;
 mesh.read_vtk(MESH_FILES_DIR
           "3D/vtk/hexes/untangled/hexes_4by2by2.vtk", err);
   // creates an instruction queue
 InstructionQueue queue1;
   // creates a mean ratio quality metric ...
 IdealWeightInverseMeanRatio mean_ratio(err);
  if (err) return 1;
 ConditionNumberQualityMetric cond_num;
  mean_ratio.set_averaging_method(QualityMetric::LINEAR);
   // ... and builds an objective function with it
 LPtoPTemplate obj_func(&mean_ratio, 2, err);
  if (err) return 1;
  // creates the steepest descent optimization procedures
 SteepestDescent sd( &obj_func );
 sd.use_global_patch();
  QualityAssessor stop_qa=QualityAssessor(&mean_ratio);
  if (err) return 1;
 stop_qa.add_quality_assessment(&cond_num);
  if (err) return 1;
  TerminationCriterion tc2;
  tc2.add_iteration_limit(1);
 sd.set_inner_termination_criterion(&tc2);
 queue1.add_quality_assessor(&stop_qa, err);
  queue1.set_master_quality_improver(&sd, err);
  if (err) return 1;
 queue1.add_quality_assessor(&stop_qa, err);
  if (err) return 1;
 mesh.write_vtk("original_mesh.vtk", err);
 if (err) return 1;
 queue1.run_instructions(&mesh, err);
  if (err) return 1;
 return 0;
```

5.1.2.4 NonGradient

The NonGradient class is a concrete vertex mover which performs derivative free minimization based on the Amoeba Method, as implemented in Numerical Recipes in C. It can be used with any Objective Function template and metric, but is particularly helpful when the Objective Function is non-differentiable, as in the MAX template. In fact, it is strongly recommended that the MaxTemplate class only be used with the NonGradient solver. Using the MaxTemplate class with any other solver class can produce less than optimal results. Mesquite will issue a warning if the MaxTemplate class is used with any solver other than NonGradient.

NonGradient can only handle patches containing exactly one free vertex. To create a PatchSet with one free vertex per patch, call the 'use_element_on_vertex_patch()' member function of the NonGradient class as shown in the example below.

The NonGradient class can be used with both Non-barrier and Barrier metrics. The code sample below shows how it is used with a Barrier metric. An example of using it with a Non-barrier metric can be found in the nongradient_test in the Mesquite testSuite.

```
int main()
  MsqPrintError err(std::cout);
  Planar Domain xy Plane (Planar Domain:: XY, -5);
 #define FILE_NAME "bad_circle_tri.vtk"
 const char file_name[] = MESH_FILES_DIR "2D/vtk/tris/untangled/" FILE_NAME
    // Barrier / Max Objective Function Test
  Mesquite::MeshImpl mesh_max;
 mesh_max.read_vtk(file_name, err);
  if (err)
    std::cerr << "Failed_to_read_file." << std::endl;
    return 1;
  IdealShapeTarget target_max;
  TShapeB1 mu;
  TQualityMetric tqMetric_max( &target_max, &mu );
 ElementQM* metricPtr_max;
  ObjectiveFunctionTemplate* objFunctionPtr_max;
 ElementMaxQM maxMetric( &tqMetric_max );
 MaxTemplate maxObjFunction(&maxMetric); // max(max)
 LPtoPTemplate PtoPObjMaxfunction(&maxMetric, 1.0, err); // max(max)
 // Processing for Max Objective Function
 NonGradient max_opt(&maxObjFunction); // optimization procedure
 max_opt.setSimplexDiameterScale(0);
 max_opt.use_element_on_vertex_patch(); // local patch
 max_opt.set_debugging_level(0);
 //\ Construct\ and\ register\ the\ Quality\ Assessor\ instances
  QualityAssessor max_initial_qa=QualityAssessor(&maxMetric, 10);
```

```
QualityAssessor maxObj_max_optimal_qa=QualityAssessor(&maxMetric, 10);

//*************************

TerminationCriterion innerTC, outerTC;

outerTC.add_iteration_limit(40);
innerTC.add_iteration_limit(20);
max_opt.set_outer_termination_criterion(&outerTC);
max_opt.set_inner_termination_criterion(&innerTC);
Mesquite::MeshDomainAssoc mesh_and_domain

= MeshDomainAssoc(&mesh_max, &xyPlane);
queue1.run_instructions(&mesh_and_domain, err);
if (err) return 1;

return 0;
}
```

5.1.2.5 QuasiNewton

Port of Todd Munson's quasi-Newton solver to Mesquite.

```
int main()
 MsqPrintError err(std::cout);
 Mesquite:: MeshImpl mesh;
 mesh.read_vtk(MESH_FILES_DIR "3D/vtk/hexes/untangled/hexes_4by2by2.vtk", err);
   // creates an intruction queue
 InstructionQueue queue1;
   // creates a mean ratio quality metric ...
 IdealWeightInverseMeanRatio mean_ratio(err);
  if (err) return 1;
 EdgeLengthQualityMetric cond_num;
 mean_ratio.set_averaging_method(QualityMetric::LINEAR);
   // build an objective function
 LPtoPTemplate obj_func(&mean_ratio, 2, err);
  if (err) return 1;
 QuasiNewton qn( &obj_func );
 qn.use_global_patch();
  Quality Assessor stop_qa=Quality Assessor(&mean_ratio);
  if (err) return 1;
 stop_qa.add_quality_assessment(&cond_num);
  if (err) return 1;
  TerminationCriterion tc2;
 tc2.add_iteration_limit(1);
 qn.set_inner_termination_criterion(&tc2);
 queue1.add_quality_assessor(&stop_qa, err);
```

```
queue1.set_master_quality_improver(&qn, err);
if (err) return 1;
queue1.add_quality_assessor(&stop_qa,err);
if (err) return 1;

queue1.run_instructions(&mesh, err);
if (err) return 1;

return 0;
}
```

5.1.2.6 TrustRegion

Port of Todd Munson's trust region solver to Mesquite. The TrustRegion optimizer is invoked in the same way as in the previous QuasiNewton example.

5.2 Objective Function

5.2.1 Definition

An objective function is a scalar function of all the vertex coordinates in the active mesh. The mesh vertex locations are optimized so as to minimize the objective function value.

The objective functions implemented in Mesquite can be divided into two general categories: template objective functions and composite objective functions. Template objective functions have an associated QualityMetric instance and typically implement some kind of averaging scheme. Composite objective functions modify the values of one or two other objective functions, such as scaling the value or summing two values.

Most solvers in Mesquite also require the gradient of the objective function (the partial derivatives of the objective function with respect to the coordinate values of the free vertices in the patch.)

Some quality improvers (currently FeasibleNewton) need to know the Hessian (second partial derivatives) of the objective function. Only an ObjectiveFunction implementation capable of providing Hessian data may be used with such a solver. Mesquite provides numerical approximation of objective function gradient values, and of quality metric gradient and Hessian values, but not objective function Hessian values. Further, Mesquite is capable of working only with Hessians of objective functions that have sparse Hessian matrices. The Hessian matrix may only contain non-zero terms for vertex pairs that share at least one element. This limitation is explicit in the implementation of the MsqHessian class, and is implicit in other areas of the code (such as portions of the ObjectiveFunction interface relating to use in a BCD algorithm.) Some ObjectiveFunction implementations such as CompositeOFMultiply (the product of two other objective function values) and StdDevTemplate have a dense Hessian and therefore cannot be used with solvers requiring a Hessian.

Objective Functions can be used with all QualityMetric classes and with the target metric classes (TMetric and AWMetric). When using a Barrier Metric derived from TMetricBarrier or AWMetricBarrier, the mesh to be optimized cannot contain inverted elements. Any attempt to use an objective function on an inverted mesh with a barrier target metric will cause Mesquite to terminate execution with a BARRIER_VIOLATED error.

5.2.2 Objective Function Implementations

The ObjectiveFunctionTemplate is a base class for those objective function implementations that are some kind of average of quality metric values. The ObjectiveFunctionTemplate class provides the API for associating a QualityMetric with an objective function and it provides a common implementation for initializing coordinate descent optimizations.

The composite objective functions modify one or two existing objective functions (e.g. adding two together). All composite ObjectiveFunctions support analytical gradients and coordinate descent optimization if the underlying ObjectiveFunctions do. Similarly, all except CompositeOFMultiply support

analytical Hessians as long as their underlying ObjectiveFunctions do. CompositeOFMultiply does not have a suitably sparse Hessian.

Two of the most commonly used objective functions are:

LPtoPTemplate - Calculates the L_p objective function raised to the pth power. That is, it sums the p-th powers of (the absolute value of) the quality metric values.

PMeanPTemplate - This class implements an objective function that is the power-mean of the quality metric evaluations raised to the power-mean power. That is, the sum of each quality metric value raised to a power, divided by the total number of quality metric values.

5.2.3 Example

Below is a simplified version of the ShapeImprover Wrapper that has been annotated to illustrate the use of an objective function.

```
void ShapeImprover::run_wrapper( MeshDomainAssoc* mesh_and_domain,
                                 ParallelMesh* pmesh,
                                 Settings * settings ,
                                 QualityAssessor* qa,
                                 MsqError& err )
   // Quality Metrics
   // Used to obtain a target matrix for the quality assessment. Creates a
   // target matrix based on the element type and its ideal shape
  IdealShapeTarget target;
   // Local sample point quality metric is TShapeB1, a shape metric
   // with barrier
 TShapeB1 mu;
   // A quality metric defined using 2D and 3D target metrics, where
   // the active matrix compared to the target by the underlying
   // metrics is the Jacobian matrix of the mapping function at a
   // given sample point. Evaluates the quality metric at the sample
    // point by forming T in terms of the active and target matrices.
  TQualityMetric metric_0 ( &target , &mu );
   // A composite metric that converts the sample-based metric
   // (metric_0) to an element-based metric by averaging the metric
    // values at each sample point within the element
 ElementPMeanP metric ( 1.0, &metric_0 );
   // QualityAssessor
 qa->add_quality_assessment(&metric);
   // an objective function that is the power-mean of the
   // quality metric evaluations raised to the power-mean power.
 PMeanPTemplate obj_func( 1.0, &metric );
   // Optimizes the objective function using the Polack-Ribiere scheme.
  ConjugateGradient improver( &obj_func );
    // Treat the mesh as a single patch
```

```
improver.use_global_patch();
TerminationCriterion inner;
inner.add_absolute_vertex_movement_edge_length(0.005);
improver.set_inner_termination_criterion(&inner);
InstructionQueue q;
q.set_master_quality_improver(&improver, err); MSQERRRTN(err);
q.add_quality_assessor(qa, err); MSQERRRTN(err);
q.run_common(mesh_and_domain, pmesh, settings, err); MSQERRRTN(err);
}
```

In the above example, the quality metric TShapB1 can be replaced by any of the many quality metrics that are derived from the TMetric class.

5.3 MsqError

Almost every function in the Mesquite API accepts an instance of the MsqError class and uses that instance to flag the occurance of any errors. For brevity, this argument is not shown or discussed for any function in the API. The reader may assume an implicit final argument of type MsqError& for almost every function shown or discussed in this document, the exceptions being those functions that cannot fail

The MsqError class can be treated as a Boolean, where a true state indicates an error. It can also be sent to a C++ output stream to print the error code, error message, and call stack (trace of nested function calls beginning with the topmost API call down to the function at which the error occured.) An application will typically use an MsqError as follows:

```
if (err) {
  std::cout << err << std::endl;
  return FAILURE;
}</pre>
```

The MsqError class also provides functions to progamatically extract data from such as the error message, error code, and call stack lines.

Mesquite also provides several macros to assist developers in using the MsqError class within Mesquite. The MSQ_SETERR macro is used to flag an initial error condition. The following examples show typical uses of this macro:

```
// literal error message and error code
MSQ_SETERR(err)( "My_error_message", MsqError::UNKNOWNERROR );
// error code and printf-style formatted error message
MSQ_SETERR(err)( MsqError::INVALID_ARG, "Argument_'foo'_=_%d", foo );
// error code and default message for that error code
MSQ_SETERR(err)( MsqError::OUT_OF_MEMORY );
```

The MSQ_CHKERR macro evalutes to true if an error has been flagged, and false otherwise. Further, if an error has been flagged, it appends the location of the MSQ_CHKERR invokation to the call stack maintained in the MsqError instance. This is the mechanism by which Mesquite generates the call-stack data. The following is an example of how this macro is typically used:

```
if (MSQ_CHKERR(err))
return FAILURE;
```

The macro may also seen used similar to the following example:

```
return !MSQ_CHKERR(err) && result_bool;
```

This statement will result in a return value of false if either an error has been flaged or result_bool is false. The order of the tests is important in this example. The MSQ_CHKERR macro must occur first so that the call stack is updated, regardless of the value of result_bool.

MSQ_ERRRTN and MSQ_ERRZERO are convenience macros for developers. They are defined as follows:

```
#define MSQ_ERRRTN(ERR) if (MSQ_CHKERR(ERR)) return #define MSQ_ERRZERO(ERR) if (MSQ_CHKERR(ERR)) return 0
```

Chapter 6

Constraining Mesh to a Geometric Domain

Vertex positions may be constrained to a geometric domain by providing Mesquite with an optional instance of the Mesquite::MeshDomain interface. This interface provides two fundamental capabilities: mesh-geometry classification, and interrogation of local geometric properties. The methods defined in the Mesquite::MeshDomain interface combine both queries into single operation. Queries are passed a mesh entity handle (see Section 4.1), and are expected to interrogate the geometric domain that the specified mesh entity is classified to.

If Mesquite is used to optimize the mesh of a B-Rep solid model (the data model used by all modern CAD systems), then the domain is composed of geometric vertices, curves, surfaces, and volumes. Curves are bounded by end vertices, surfaces are bounded by loops (closed chains) of curves, and volumes are bounded by groups of surfaces. Mesquite expects each surface element (triangle, quadrilateral, etc.) to be associated with a 2D domain (surface). Vertices may be associated with a geometric entity that either contains adjacent mesh elements or bounds the geometric entity containing the adjacent elements. Mesquite does not use geometric volumes. A query for the closest location on the domain for a vertex or element whose classification is a geometric volume should simply return the input position.

It is possible to define an optimization problem such that mesh classification data need not be provided in a Mesquite::MeshDomain implementation. This is done by optimizing the mesh associated with each simple geometric component of the domain separately, with the boundary vertices flagged as fixed. The following pseudo-code illustrates such an approach for a B-Rep type geometric domain:

```
for each geometric vertex
mark associated vertex as fixed
end-for
for each curve
do any application-specific optimization of curve node placement
mark associated mesh vertices as fixed
end-for
for each surface
define Mesquite::MeshDomain for surface geometry
invoke Mesquite to optimize surface mesh
mark all associated mesh vertices as fixed
end-for
for each volume
invoke Mesquite to optimize volume mesh w/o Mesquite::MeshDomain
end-for
```

6.1 The ITAPS iGeom and iRel Interfaces

Mesquite can access mesh domain data through the *iGeom* and *iRel* interface defined by the ITAPS Work Group. These interfaces provide APIs for accessing B-Rep geometric data and associating mesh with geometry (classification), respectively. Mesquite provides the Mesquite::MsqIGeomclass (MsqIGeom.hpp) as an adaptor for interfacing with applications that present the *iGeom* and *iRel* interfaces. The use of the *iRel* interface is optional. If all the mesh vertices are constrained to a single geometric surface, it is sufficient to provide only an iGeom instance to Mesquite::MsqIGeom. If vertices are constrained to different geometric entities, then the *iRel* interface must be provided to Mesquite::MsqIGeom so Mesquite can determine which iMesh entity a given vertex is constrained to lie in.

6.2 Simple Geometric Domains

Mesquite provides several implementations of the Mesquite::MeshDomain interface for simple geometric primitives. All MeshDomains in Mesquite are geometric surfaces upon which meshes consisting of triangles and/or quadrilaterals can exist. Mesquite does not have any implementations of 3D geometric regions. The domains available in Mesquite include:

- PlanarDomain: An unbounded planar surface.
- XYPlanarDomain: An unbounded planar surface that exists in the XY-plane.
- SphericalDomain: A closed spherical surface.
- CylinderDomain: An unbounded cylindrical surface.
- BoundedCylinderDomain: A bounded cylindrical surface.
- ConicDomain: An unbounded cone with a circular cross-section.
- XYRectangle: An bounded rectangular domain in the XY-Plane.

The PlanarDomain is often used to map \mathbb{R}^2 optimization problems to \mathbb{R}^3 . The others are used primarily for testing purposes.

Notes about Domains:

- The BoundedCylinderDomain provides some simplistic mesh-geometry classification capabilities. The others do not provide any classification functionality. Creating a bounded Cylinder is a two-step process. First, a cylinder is created via the constructor by specifying a radius, a vector defining the direction of the axis, and a point through which the axis passes. Second, the bounding part is specified by calling one of the two overloaded methods "create_curve()". Both versions accept a distance from the axis where the circular curve to act at the bounding box will be placed along with vertices to be considered bound to the curve. The vertices are specified by either a list or a mesh depending on which version of the method is used.
- The ConicDomain is not bounded at the apex. It extends infinitely in both directions.
- The XYPlanarDomain is the only MeshDomain type that can be used with FeasibleNewton optimization. FeasibleNewton also operates on volume meshes.
- The XYRectangle domain is a simple 2D domain used for free-smooth testing. The specified rectangle can be in the XY, YZ, or ZX plane. The constructor takes as input a point (x,y,z), a height and width, and a plane. A corresponding bounding box is then created in the specified plane. The method "setup(iMesquite::Mesh* mesh, Mesquite::MsqError& err)" can then be used to determine if a particular mesh lies completely in the defined bounded rectangle. If any of the vertices of the mesh lie outside the rectangle a non-zero err value will be returned.

6.3 Associating a Mesh with a Domain

The MeshDomainAssoc class is used to associate a Mesh class instance with a MeshDomain class instance. When the constructor for the MeshDomainAssoc class is called, a check is performed to determine if the characteristics of the mesh are compatible with the associated domain.

The constructor for the MeshDomainAssoc class:

where:

- 'mesh' is the mesh instance being associated.
- 'domain' is the domain being associated.
- 'full_compatibility_check' controls how many vertices will be checked for compatibility with the associated domain. When false (the default), only the first vertex of the mesh is checked for compatibility. When true, all vertices of the mesh are checked.
- 'proceed' controls what Mesquite will do if the compatibility check fails. When false (the default), mesquite terminates execution. When true, execution continues.
- 'skip_compatibility_check' when true, does not perform the compatibility check. When false (the default), the check is performed.

Only the first two parameters are required when calling the constructor. All other parameters only need to be specified when something other than the default behavior is desired.

The MeshDomainAssoc class is designed to work with planar surface (2D) meshes only. When using volume (3D) meshes that require a MeshDomainAssoc instance to be created in order to call a function or method, the domain parameter in the constructor should be specified as a NULL.

In a given program, many different instances of the MeshDomainAssoc class can be created; one for each combination of a specific mesh, domain, and different values for the optional parameters.

An example of how MeshDomainAssoc is used can be found in the paraboloid_domain_test located in the testSuite directory.

Chapter 7

Mesquite Wrapper Descriptions

Applications which desire to access Mesquite capabilities without delving into the low-level API can invoke wrappers to perform basic mesh quality improvement tasks that, except for a few user-defined inputs, are fully automatic. The wrappers target classic mesh optimization problems that occur repeatedly across many applications. See section 3.1.3 for an example of how to invoke a wrapper. This chapter provides a summary of the current Mesquite wrappers.

Note that the wrappers do not, by themselves, completely define the optimization problem. The user still has to set the fixed/free flags, and the values of the termination criteria.

7.1 Laplace-smoothing

Name: LaplaceWrapper

Example:

Purpose: Produce a smooth mesh.

Notes: This is a local patch relaxation-solver. A 'smart' Laplacian solver is also available in Mesquite,

but it is not used in this wrapper.

Limitations/assumptions: No invertibility guarantee.

Input Termination Criterion: Stop after 10 global iterations.

Under the Cover:

Hardwired Parameters: None

Mesh/Element Type: Any supported type.

Global/Local: Local Patch with Culling

Evaluating quality for 64 elements.

This mesh had 64 quadrilateral elements.

THERE ARE 28 INVERTED ELEMENTS.

(Elements invalid at 108 of 256 sample locations.)

28 OF 64 ENTITIES EVALUATED TO AN UNDEFINED VALUE FOR Inverse Mean Ratio

Element Quality Statistics

minimum average rms maximum std.dev.

****** QualityAssessor(free only) Summary *********

0 1.48854 2.00094 3.15625 1.33716

Number of statistics = 64 Metric = Inverse Mean Ratio Element Quality not based on sample points.

****** QualityAssessor(free only) Summary *********

Evaluating quality for 64 elements.

This mesh had 64 quadrilateral elements.

There were no inverted elements detected.

No entities had undefined values for any computed metric.

Element Quality Statistics

Number of statistics = 64 Metric = Inverse Mean Ratio Element Quality not based on sample points.

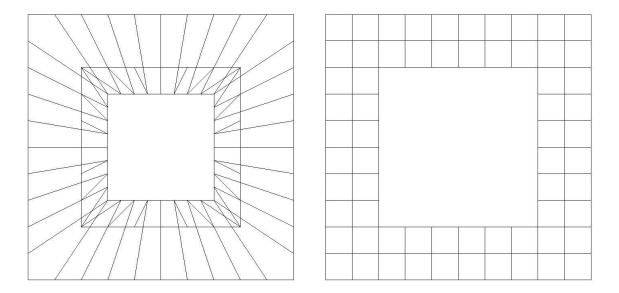


Figure 7.1: LaplaceWrapper, file: inverted-hole-1.vtk mesh. The original mesh is on the left, the mesh smoothed with the LaplacianSmoother is shown on the right.

7.2 Shape-Improvement

Name: ShapeImprover

Purpose: Make the shape of an element as close as possible to that of the ideal/regular element shape. For example, make triangular and tetrahedral elements equilateral. The wrapper will use a non-barrier metric on meshes that contain inverted elements and will use a barrier metric if the mesh does not contain inverted elements. The default CPU time limit is 300 seconds.

Notes: There is no guarantee that the wrapper will be able to successfully untangle a mesh that contains inverted elements.

Limitations/assumptions:

Input Termination Criterion: CPU time limit of 300 seconds or maximum absolute vertex movement of 10 percent of the minimum edge length.

Under the Cover:

Metric: TMPQualityMetric(Shape/ShapeBarrier)

Objective Function: Algebraic mean of quality metric values

Mesh/Element Type: Any supported type.

Solver: Conjugate Gradient

Global/Local: Global

Example:

```
****** QualityAssessor(free only) Summary *********
```

Evaluating quality for 40 elements.

This mesh had 40 quadrilateral elements.

There were no inverted elements detected.

No entities had undefined values for any computed metric.

Element Quality Statistics

```
minimum average rms maximum std.dev. 0.0767863 0.232261 0.262717 0.404594 0.122781
```

Number of statistics = 40

Metric = ElementPMeanP(TShapeB1)

Element Quality not based on sample points.

```
****** QualityAssessor(free only) Summary *********
```

Evaluating quality for 40 elements.

This mesh had 40 quadrilateral elements.

There were no inverted elements detected.

No entities had undefined values for any computed metric.

Element Quality Statistics

```
minimum average rms maximum std.dev. 0.124926 0.204118 0.207417 0.328688 0.0368497
```

Number of statistics = 40

Metric = ElementPMeanP(TShapeB1)

Element Quality not based on sample points.

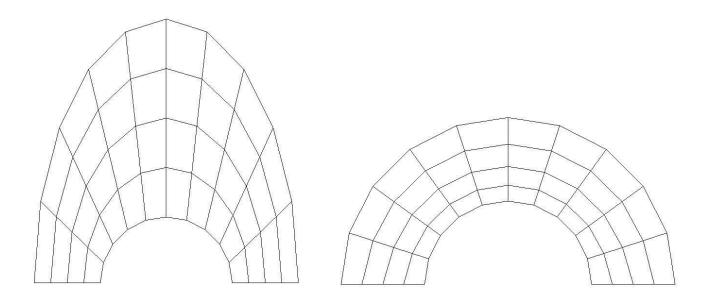


Figure 7.2: ShapeImproverWrapper, file: tfi_horse10x4-12.vtk mesh. The original mesh is on the left, the improved mesh is shown on the right.

7.3 Untangler

Name: UntangleWrapper

Purpose: Untangle elements. Prioritizes untangling over element shape or other mesh quality measures.

Notes: A second optimization to improve element quality after untangling is often necessary.

Limitations/assumptions: There is no guarantee that the optimal mesh computed using this wrapper will, in fact, be untangled.

Input Termination Criterion: CPU time limit (not used if input value is non-positive) or fraction of mean edge length (default is 0.005). It also terminates if all elements are untangled, such that it should not modify an input mesh with no inverted elements.

Under the Cover:

Metric: TUntangleBeta or TUntangleMu(TSizeNB1) or TUntangleMu(TShapeSizeNB1)

Objective Function: Algebraic mean of quality metric values

Mesh/Element Type: Any supported type.

Solver: Steepest Descent

Global/Local: Local with culling, optionally Jacobi

Example:

```
****** QualityAssessor(free only) Summary *********
```

Evaluating quality for 1024 elements.

This mesh had 1024 quadrilateral elements.

THERE ARE 9 INVERTED ELEMENTS.

(Elements invalid at 9 of 4096 sample locations.)

No entities had undefined values for any computed metric.

Element Quality Statistics

```
minimum average rms maximum std.dev.
0 48.5379 210.965 2915.69 205.305
```

Number of statistics = 1024

Metric = ElementPMeanP(untangle(2D:TShapeSize2DNB1; 3D:TShapeSize3DNB1))

Element Quality not based on sample points.

```
****** QualityAssessor(free only) Summary *********
```

Evaluating quality for 1024 elements.

This mesh had 1024 quadrilateral elements.

There were no inverted elements detected.

No entities had undefined values for any computed metric.

Element Quality Statistics

```
minimum average rms maximum std.dev.
0 1.46636 23.8476 462.591 23.8025
```

Number of statistics = 1024

Metric = ElementPMeanP(untangle(2D:TShapeSize2DNB1; 3D:TShapeSize3DNB1))

Element Quality not based on sample points.



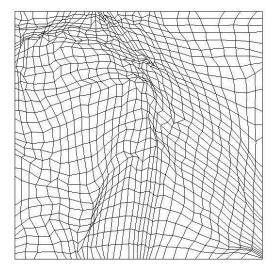


Figure 7.3: UntangleWrapper, TShapeSizeNB1 option, file: shest_grid32.vtk mesh. The original mesh is on the left, the untangled mesh is shown on the right.

7.4 Minimum Edge-Length Improvement

Name: PaverMinEdgeLengthWrapper

Purpose: Make all the edges in the mesh of equal length while moving toward the ideal shape. Intended for explicit PDE codes whose time-step limination is governed by the minimum edge-length in the mesh.

Notes: Based on Target-matrix paradigm.

Limitations/assumptions: Initial mesh must be non-inverted. User does not want to preserve or create anisotropic elements.

Input Termination Criterion: maximum iterations (default=50), maximum absolute vertex movement

Under the Cover:

Hardwired Parameters: None

Metric: Target2DShapeSizeBarrier or Target3DShapeSizeBarrier

Tradeoff Coefficient: 1.0

Objective Function: Linear Average over the Sample Points

Mesh/Element Type: Any supported type.

Solver: Trust Region Global/Local: Global

Example:

******* QualityAssessor(free only) Summary *********

Evaluating quality for 8 elements. This mesh had 8 quadrilateral elements.

There were no inverted elements detected.

No entities had undefined values for any computed metric.

Element Quality Statistics

minimum average rms maximum std.dev. 0.357275 0.983461 1.33806 3.27555 0.907303

Number of statistics = 8

Metric = ElementPMeanP(TShapeSizeB1)

Element Quality not based on sample points.

Element Quality Statistics

 minimum
 average
 rms
 maximum
 std.dev.

 0.538516
 1.11317
 1.15398
 1.51327
 0.304184

Number of statistics = 12

Metric = EdgeLength

Element Quality not based on sample points.

******* QualityAssessor(free only) Summary *********

Evaluating quality for 8 elements.

This mesh had 8 quadrilateral elements.

There were no inverted elements detected.

No entities had undefined values for any computed metric.

Element Quality Statistics

minimum average rms maximum std.dev. 0.00135009 0.0017804 0.00179488 0.00221721 0.000227498

Number of statistics = 8

Metric = ElementPMeanP(TShapeSizeB1)

Element Quality not based on sample points.

Element Quality Statistics

minimum average rms maximum std.dev. 0.994086 1.0004 1.00041 1.00293 0.00253389

Number of statistics = 12

Metric = EdgeLength

Element Quality not based on sample points.

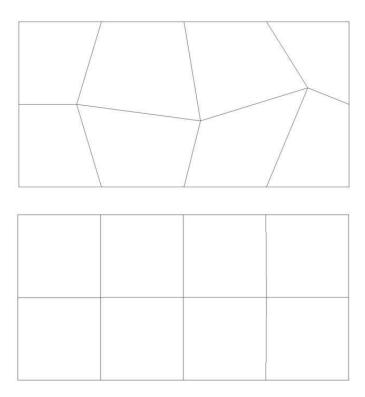


Figure 7.4: PaverMinEdgeLengthWrapper, file: quads_4by2_bad.vtk mesh. The original mesh is on the top, the improved mesh is shown on the bottom.

7.5 Improve the Shapes in a Size-adapted Mesh

Name: SizeAdaptShapeWrapper

Purpose: Make the shape of an element as close as possible to that of the ideal element shape, while preserving, as much as possible, the size of each element in the mesh. To be used on isotropic initial meshes that are already size-adapted.

Notes: Based on Target-matrix Paradigm.

Limitations/assumptions: Initial mesh must be non-inverted. User wants to preserve sizes of elements in initial mesh and does not want to preserve or create anisotropic elements.

Input Termination Criterion: maximum iterations (default=50), maximum absolute vertex movement

Under the Cover:

Hardwired Parameters: None

Metric: Target2DShapeSizeBarrier or Target3DShapeSizeBarrier

Tradeoff Coefficient: 1.0

Objective Function: Linear Average over the Sample Points

Mesh/Element Type: Any supported type.

Solver: Trust Region Global/Local: Global

Example:

******** QualityAssessor(free only) Summary *********

This mesh had 3456 quadrilateral elements.

There were no inverted elements detected.

No entities had undefined values for any computed metric.

Element Quality Statistics

minimum average rms maximum std.dev. 0.00447499 0.23505 0.288066 0.710249 0.166533

Number of statistics = 3456

Metric = ElementPMeanP(TShapeSizeB1)

Element Quality not based on sample points.

Element Quality Statistics

minimum average rms maximum std.dev. 0.163072 0.574857 0.62062 1.01908 0.2339

Number of statistics = 13824

Metric = EdgeLength

Element Quality not based on sample points.

****** QualityAssessor(free only) Summary *********

Evaluating quality for 3456 elements.

This mesh had 3456 quadrilateral elements.

There were no inverted elements detected.

No entities had undefined values for any computed metric.

Element Quality Statistics

minimum average rms maximum std.dev. 0.00558463 0.0591772 0.0767815 0.303106 0.048923

Number of statistics = 3456

Metric = ElementPMeanP(TShapeSizeB1)

Element Quality not based on sample points.

Element Quality Statistics

 minimum
 average
 rms
 maximum
 std.dev.

 0.161144
 0.562074
 0.607611
 1.11686
 0.230789

Number of statistics = 13824

Metric = EdgeLength

Element Quality not based on sample points.

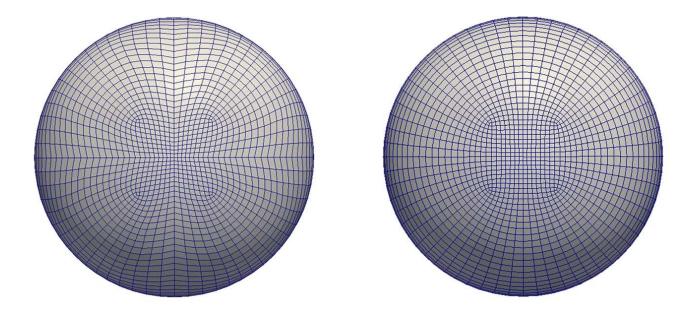


Figure 7.5: SizeAdaptShapeWrapper, file: bias-sphere-quads.vtk mesh. The original mesh is on the left, the improved mesh is shown on the right.

7.6 Improve Sliver Tets in a Viscous CFD Mesh

Name: ViscousCFDTetShapeWrapper

Purpose: Improve the shape of sliver elements in the far-field of a CFD mesh while preserving an existing

layer of sliver elements in the boundary layer. Notes: Based on Target-matrix paradigm.

Limitations/assumptions: Tetrahedral meshes only.

Input Termination Criterion: Iteration Count (default=50) or Maximum Absolution Vertex Movement

Under the Cover:

Hardwired Parameters: In tradeoff coefficient model.

Metric: Target2DShape+Target2DShapeSizeOrient (or 3D) (or Barrier)

Tradeoff Coefficient: Based on element dihedral angle Objective Function: Linear average over Sample Points

Mesh/Element Type: Tetrahedra

Solver: Trust Region Global/Local: Global

Additional Information:

Article: Introducing the Target-matrix Paradigm for mesh optimization via node-movement", Engineering with Computers, Sept. 2011.

7.7 Deforming Domain

Name: DeformingDomainWrapper

Purpose: Use initial mesh on undeformed geometric domain to guide optimization of mesh moved to deformed geometric domain.

Notes: Uses a non-barrier metric which means that the wrapper could potentially invert/tangle elements. Limitations/assumptions: Application responsible for explicit handling of mesh on geometric curves and points. Initial mesh before moving to deformed domain must be available.

Input Termination Criterion: CPU time limit (not used if input value is non-positive) or fraction of mean edge length (default is 0.005).

Under the Cover:

Metric: TMPQualityMetric(TShapeNB1 or TShapeSizeNB1 or TShapeSizeOrientNB1)

Objective Function: Algebraic mean of quality metric values

Mesh/Element Type: Any supported type.

Solver: Steepest Descent

Global/Local: Local with culling

Additional Information:

Article: "Updating meshes on deforming domains", Communications in Numerical Methods in Engineering, 24:467-476, 2008...

Example:

```
************ QualityAssessor(free only) Summary **********

Evaluating quality for 216 elements.

This mesh had 216 quadrilateral elements.
```

THERE ARE 56 INVERTED ELEMENTS.

(Elements invalid at 220 of 864 sample locations.)

No entities had undefined values for any computed metric.

Element Quality Statistics

```
minimum average rms maximum std.dev.
4.57621e-005 7.63416 20.8252 67.0217 19.3754
```

```
Number of statistics = 216
```

Metric = ElementPMeanP(TShapeNB1)

Element Quality not based on sample points.

```
****** QualityAssessor(free only) Summary *********
```

Evaluating quality for 216 elements.

This mesh had 216 quadrilateral elements.

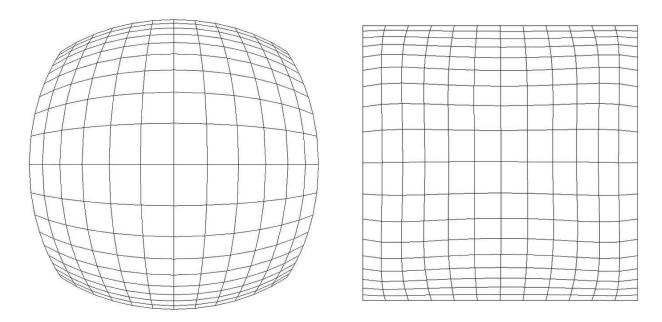
There were no inverted elements detected.

No entities had undefined values for any computed metric.

Element Quality Statistics

```
minimum average rms maximum std.dev. 0.000763758 0.0202682 0.0252805 0.0947676 0.0151097
```

Number of statistics = 216
Metric = ElementPMeanP(TShapeNB1)
Element Quality not based on sample points.



 $\label{eq:continuous} Figure~7.6:~Deforming Domain Wrapper,~file:~sph-10-zsquare.vtk~mesh.~The~original~mesh~is~on~the~left,~the~improved~mesh~is~shown~on~the~right.$

Chapter 8

Optimization Strategies

8.1 The Generalized Optimization Loop

In Mesquite a generalization of the optimization strategy is used to implement a wide variety of optimization strategies. Before discussing the different types of optimization strategies that can be implemented with Mesquite we will first need to discuss the generalized strategy.

The mesh can be decomposed into subsets called *patches*. The specifics of this mesh decomposition are discussed in Section 8.2. The optimization is done by repeatedly iterating over the set of patches, optimizing each separately.

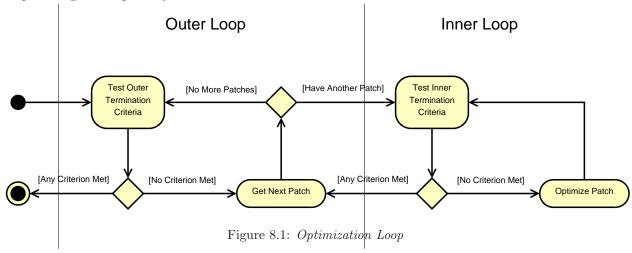


Figure 8.1 depicts the generalization of optimization strategies in Mesquite. The generalized optimization is composed of three loops shown as non-overlapping square cycles in the diagram. The test to exit each loop is performed at the decision points (diamonds) in the diagram. The loops are logically nested from left to right, such that the right most loop is performed within a single iteration of the loop to the left of it. The inner- and outer-most loops terminate based on user-definable termination criterion. The center loop is the iteration over the set of patches composing the mesh.

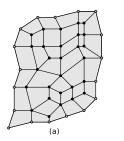
The inner-most loop (the right-most cycle in the diagram) represents the iterative optimization of the mesh contained in a single patch. This optimization is done until the *inner termination criterion* is met. Once the inner criterion is met the optimizer advances to the next patch and the inner loop is entered again to optimize that patch. Once each patch has been optimized the *outer termination criterion* is tested. If the criterion has not been met then the loop over the set of patches is repeated.

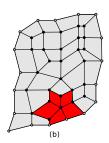
The set of outer termination criteria determine when the optimization of the entire mesh is complete. The set of inner termination criteria determine when the optimization of a single patch is complete. Both sets of criteria are tested before entering their respective loops. If a criterion is met before the loop starts then no iterations of the corresponding loop will be performed.

The outer loop(s) are implemented in the VertexMover class. The inner loop is implemented in subclasses. The LaplacianSmoother class in Mesquite provides a traditional Laplace smoother. For this class the mesh is decomposed into patches that each contain a single free vertex and the adjacent elements, one patch for each free vertex in the mesh. For Laplace smoothing the inner (per-vertex) optimization is not iterative. The inner loop always has an implicit termination criterion of a single iteration. Any other inner termination criterion will still be tested before performing the relaxation of the free vertex in the patch such that if any such criterion is met no optimization of the vertex will be performed. However, culling (Section 8.8 can have a similar effect while typically producing better results. Passes are made over the entire mesh until one of the specified outer termination criterion is met.

8.2 Patches

Mesquite can operate on a decomposition of the mesh into subsets called *patches*. Each patch is optimized individually. The overall mesh optimization is performed by repeatedly iterating over the set of patches. Mesquite provides two build-in mesh decompositions¹: element-on-vertex patches and a global patch.







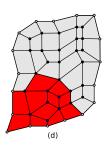


Figure 8.2: Miscellaneous patch configurations.

The global patch is a "decomposition" where the entire mesh is contained in a single patch. This is used in the global optimization strategy discussed in Section 8.4. Figure 8.2c illustrates the global patch.

The element-on-vertex decomposition subdivides the mesh into a single patch for each free vertex. Each patch includes the layer of elements adjacent to the free vertex. A element-on-vertex patch is illustrated in Figure 8.2b. This decomposition is typically used for all optimization strategies discussed in this chapter except for global optimization. Any other decomposition except global may be used for any of the optimization strategies. All of the discussed strategies other than global do not make sense for a global patch.

Any patch decomposition can be used with Mesquite. While no other decomposition strategy is provided with Mesquite, the any implementation of the PatchSet interface can be associated with any quality improver that supports it (any subclass of PatchSetUser, currently all except LaplacianSmoother). An implementation of that interface is expected to provide three things:

- 1. An enumeration of all the patches in the decomposition of the mesh
- 2. For each patch, the set of vertices to optimize
- 3. For each patch, the set of elements for which the quality is to be optimized (typically all elements containing the vertices to be optimized.)

A normal decomposition will be done such that each free vertex in the mesh is optimized in exactly one patch, but Mesquite does not enforce this. Having a free vertex be optimized in no patch will result in that vertex effectively being fixed for the optimization. A decomposition that optimized the same vertices in multiple patches is allowable, and should have no adverse side effects unless doing a Jacobi optimization (Section 8.9).

The listing below shows how a custom implementation of the PatchSet interface can be used with the SteepestDescent solver.

¹Mesquite includes an additional decomposition of the mesh into single-element patches which is not suitable for use in optimization. It is used internally for quality assessment and other purposes

```
MyMeshDecomposition my_patch_set;
SteepestDescent quality_improver( &objective_function );
quality_improver.use_patch_set(&my_patch_set);
```

In any of the examples later in this chapter that call use_element_on_vertex_patch(), that call may be substituted with a call to use_patch_set to use some decomposition other than single-vertex patches.

8.3 PatchSetUser and PatchSet

A PatchSetUser is an optimizer for which the application may specify how the mesh is decomposed into patches. PatchSetUser provides two pre-defined patch schemes. The use_global_patch() method will result in a single patch containing the entire mesh. The use_element_on_vertex_patch() method will result in a patch for every free vertex in the mesh, containing only the free vertex and its adjacent elements. An alternate scheme for subdividing the mesh into patches may be specified by providing a custom implementation of the PatchSet interface.

The PatchSet interface defines two methods: get_patch_handles and get_patch. The get_patch_handles method returns a list of handles (or identifiers), one for each potential patch in a mesh. The get_patch method returns the free vertices and elements in a patch, given one of the handles passed back from the get_patch_handles method.

The GlobalPatch class is the implementation of the PatchSet interface that provides a single patch for the entire mesh. The VertexPatches class provides the Laplacian-like decomposition of the mesh into a patch for every free vertex. The ElementPatches class is used internally in places other than the main optimization loop, such as initializing BCD data and in the QualityAssessor class. It decomposes the mesh into single-element patches with no free vertices.

8.4 Global

For a global optimization an objective function that measures the quality of the mesh is minimized using a numerical solver. The coordinates of all of the free vertices in the mesh are the free variables in the optimization. This is the default mode of operation for most solver-based implementations of the QualityImprover interface.

A global optimization is simplest form of the generalized optimization loop. In this mode the mesh is "decomposed" into a single patch containing the entire mesh. The outer loops in Figure 8.1 are executed only once. The entire optimization process happens in the inner loop. For global optimization the outer termination criterion is the default of a single iteration. The inner termination criterion should be used to terminate the optimization process. Setting some other outer termination criterion is not prohibited, but will result in a much less efficient optimization process. There is no logical difference between inner and outer termination criterion, but each iteration of the outer loop begins with a clean solver state which will result in less efficient operation of the solver. Even steepest descent, the simplest solver, calculates a initial step size based on the previous iteration of the inner loop.

The listing below shows how global optimization can be selected.

```
// Create global optimizer instance
SteepestDescent improver( &objective_function );
improver.use_global_patch();

// Set only inner termination criterion for
// global optimization
TerminationCriterion inner;
inner.add_absolute_vertex_movement( 1e-3 );
improver.set_inner_termination_criterion( &inner );

// Run optimization
InstructionQueue queue;
```

```
queue.set_master_quality_improver( &improver, err );
queue.run_instructions( &mesh, err );
```

8.5 Nash Game vs. Block Coordinate Descent

Quality improvers that have an explicit ObjectiveFunction may be used with the block coordinate descent algorithm rather than the default Nash game algorithm if the ObjectiveFunction implementation supports this mode of operation. In a Nash game optimization (the default), the objective function that is optimized by the inner loop is evaluated over only the patch being optimized. In a block coordinate descent algorithm, the objective function to be optimized in the inner loop is evaluated over the entire mesh. Only the influence of the current patch vertices on the global objective function is considered during the optimization of each patch.

8.6 Nash Game

```
// Create Nash optimizer instance
SteepestDescent improver( &objective_function );
improver.use_element_on_vertex_patch();

// Set inner and outer termination criterion for
// non-global patch
TerminationCriterion inner, outer;
outer.add_absolute_vertex_movement( 1e-3 );
inner.add_iteration_limit( 2 );
improver.set_outer_termination_criterion( &outer );
improver.set_inner_termination_criterion( &inner );

// Run optimization
InstructionQueue queue;
queue.set_master_quality_improver( &improver, err );
queue.run_instructions( &mesh, err );
```

8.7 Block Coordinate Descent

```
// Create BCD optimizer instance
SteepestDescent improver( & objective_function );
improver.use_element_on_vertex_patch();
improver.do_block_coordinate_descent_optimization();

// Set inner and outer termnation criterion for
// non-global patch
TerminationCriterion inner, outer;
outer.add_relative_quality_improvement( 1e-2 );
inner.add_iteration_limit( 2 );
improver.set_outer_termination_criterion( & outer );
improver.set_inner_termination_criterion( & inner );

// Run optimization
InstructionQueue queue;
queue.set_master_quality_improver( & improver, err );
queue.run_instructions( & mesh, err );
```

8.8 Culling

```
// Create Nash optimizer with culling
SteepestDescent improver(&objective_function);
improver.use_element_on_vertex_patch();
// The culling criterion is effectively an outer
// termination criterion because optimization will
// always stop when all patches are culled. We
// must explicitly pass an empty outer termination
/\!/ criterion to replace the default of one iteration.
// Additional outer termination criteria may also be
// specifed.
TerminationCriterion inner, outer;
inner.cull_on_absolute_vertex_movement( 1e-3 );
inner.add_iteration_limit( 2 );
improver.set_outer_termination_criterion( &outer );
improver.set_inner_termination_criterion( &inner );
// Run optimization
InstructionQueue queue;
queue.set_master_quality_improver(&improver, err);
queue.run_instructions(&mesh, err);
```

8.9 Jacobi

```
// Create Jacobi optimizer instance
SteepestDescent improver( & objective_function );
improver.use_element_on_vertex_patch();
improver.do_jacobi_optimization();

// Set inner and outer termination criterion for
// non-global patch
TerminationCriterion inner, outer;
outer.add_absolute_vertex_movement( 1e-3 );
inner.add_iteration_limit( 2 );
improver.set_outer_termination_criterion( & outer );
improver.set_inner_termination_criterion( & inner );

// Run optimization
InstructionQueue queue;
queue.set_master_quality_improver( & improver, err );
queue.run_instructions( & mesh, err );
```

Chapter 9

Analyzing Optimizer Behavior

This chapter provides a brief overview of some of the tools provided in Mesquite for assisting with the analysis and visualization of the Mesquite optimization process. The tools discussed in this section can be used to provide additional output. External tools such as Paraview, VisIt, or GNU Plot must be used to visualize the data.

9.1 Assessing Quality

The QualityAssessor class provides a summary of the mesh quality. It can be used with Non Target-paradigm metrics (QualityMetric classes) as well as Target-paradigm metrics (TMetric classes). For simplicity, the following discussion refers to the QualityMetrics classes but the concepts apply to the TMetric classes as well. The QualityAssessor class can be used in a direct fashion as shown in the example below or via the InstructionQueue class as described in Section 3.1.4. An instance of the QualityMetric class can be specified for the QualityAssessor instance at creation to be used to assess the mesh quality. Additional QualityMetric instances can be created using the Assessor class and by adding them to the QualityAssessor instance via the "add_quality_assessment" method. If no QualityMetrics are specified, the only assessment that will be performed is a simple count of inverted elements. One or more instances of the QualityAssessor class may be inserted in the InstructionQueue at any point to print a summary of the mesh quality at that time during the optimization.

9.1.1 Stopping Assessment

A stopping assessment can be specified for each QualityAssessor instance. The "stopping assessment" directs the assessment code calculate a value using the power mean data to use that value as the return value for the loop_over_mesh call. If no power mean is specified for a QualityAssessor instance, a simple average of all metric values calculated during the assessment is returned from loop_over_mesh. Only one stopping assessment with its associated power mean can be specified for a particular QualityAssessor instance. There are three different ways to specify a stopping assessment: when the QualityAssessor instance is created using a constructor, when a quality assessment is added via the add_quality_assessment () method, and directly with the set_stopping_assessment() method. Since only one stopping assessment can be defined for a each instance of QualityAssessor, the last action that causes the stopping assessment to be set will be the one used for the assessment no matter how many metrics have been included.

9.1.2 Using the Quality Assessor

The QualityAssessor class provides a number of constructors. Each allows the specification of a different set parameters to control the quality assessment. The parameters are described below including default values, if any. Note that all parameters are not used in each constructor.

Parameters used by QualityAssesor constructors:

metric: QualityMetric to register for use in assessing mesh quality. Will also be used in the setting of the stopping assessment.

- histogram_intervals: If non-zero, a histogram of quality metric values composed of the specified number of intervals will be generated. Default is zero.
- **power_mean:** If non-zero, in addition to the normal summary statistics for the quality metric, an additional general power mean with the specified power will be calculated. Is used as the value set for the stopping assessment. Default is zero.
- free_elements_only: When this option is TRUE, summary statistics are only computed over the set of elements which contain free vertices. If an element in the mesh does not contain a free vertex, its quality is not included in the summary. If an element in the mesh does not contain a free vertex, its quality cannot be improved by Mesquite. To compute the quality of all mesh elements, regardless of whether Mesquite can improve them, set this option to FALSE. Default is TRUE.
- metric_value_tag_name: If a non-null value is specified, a tag with the specified name can be associated with quality values for individual elements or vertices if metric is an element-based or vertex-based metric. If metric is not element-based or vertex-based, this argument has no effect. The specified tag can then be associated with quality values generated for a mesh. Element-based metrics can have one tagged value per element quality value while vertex-based metrics can have one tagged value per vertex quality value. Tagged quality values are created using the methods tag_set_element_data() and tag_set_vertex_data() found in the MeshImpl class. The tagged values can be retrieved using the methods tag_get_element_data() and tag_get_vertex()_data from the same class. Tags cannot be used with target metric classes. Default for metric_value_tag_name is null value.
- **inverted_element_tag_name:** If a non-null value is specified, an integer tag with the specified name will be used to store a value of 0 for normal elements and 1 for inverted elements. Default is null value.
- print_summary_to_std_out: If TRUE, summary of mesh quality will be written to std::out. If FALSE,
 quality assessment will be available via the get_results and get_all_results methods, but will not be
 printed. Default is TRUE.

output_stream IO: stream to which to write a summary of the mesh quality.

name: Name to include in output. Useful if several QualityAssessors are in use at the same time.

After the QualityAssessor instance is created, any of a number of methods can be used to set individual characteristics of the QualityAssessor object.

The add_histogram_assessment method can be used to assign values for a histogram. If the specified QualityMetric instance has not been added to the QualityAssessor instance, it will be created with the provided values. If the QualityMetric instance was previously added to the QualityAssessor instance, it is updated with the provided values. The signiture of the method is:

The paramaters min and max are used to define the lower and upper limits for the values displayed in the histogram. If zero, the value used for the histogram will be based on the actual values of the quality assessment. The intervals parameter specifies how many "steps" or "bins" will be shown on the histogram. If the specified value is negative or 0, an interval of one will be used. The metric_label parameter is a string that will be used to identify the histogram in the output. It is useful when several quality assessments are used at the same time. The remaining parameters work as described in Section 9.1.2.

Once setup for the QualityAssessor object is complete, the actual assessment is preformed by calling "loop_over_mesh". After it terminates, results can be obtained using various methods supplied by the QualityAssessor class.

For each instance of the QualityAssessor, a summary of the results will be printed after the assessments have been completed. Display of the summary can be turned off by calling disable_printing_results() before the assessment is started. The summary will include data for each of the metric assessments run by the Assessor. The printed data includes the metric name, the minimum and maximum values, the average value, the rms (root mean square), and the standard deviation. If a power mean was specified for the assessment, an additional column will display the resultant value under a header containing the power mean value used in the calculation. All values in the QualityAssessor summary table are per mesh element. Any requested histograms are then displayed. The number of values in the histogram is dependant upon the type of metric performed. For element-based metrics, the histogram contains one value per element. For vertex-based metrics, it will contain the number of target sample points per element times the number of elements.

Element quality metrics is calculated in one of four ways:

- 1. As the average of the quality values of the underlying sample points. A single quality value is calculated for each element. This functionality is provided by the ElementAvgQM class.
- 2. As the maximum of the quality values of the underlying sample points for each element. A single quality value is calculated for each element. This functionality is provide by the ElementMaxQM class.
- 3. For target-based metrics (any metric derived from the TMPQualityMetric class or TMetric class used in conjuntion with the TQualityMetric class), a quality value is produced for each element sample point. For example, a quadrangle has four sample points, one at each corner, therefore four quality values per element will be produced for a quadrangle when using a target-based metric.
- 4. All other metrics derived from the QualityMetric class that are not one of the above metrics will have one quality metric value per element. Sample points are not used.

Below is an example of a summary and histogram for an eight element mesh for two different metrics, one that included a power mean of 1.5.

```
******* QualityAssessor(free only) Summary *********
 Evaluating quality for 8 elements.
 This mesh had 8 quadrilateral elements.
 There were no inverted elements detected.
 No entities had undefined values for any computed metric.
    Element Quality Statistics
    minimum
                                                    std.dev.
                average
                                 rms
                                         maximum
    1.05817
                                         1.35948
                                                   0.0995044
                1.14257
                              1.1469
    Number of statistics = 8
    Metric = Condition Number
    Element Quality not based on sample points.
    Sample Point Quality Statistics
    minimum
                average
                             1.5-mean
                                                                 std.dev.
                                              rms
                                                      maximum
                   2.23
                              2.28262
       1.18
                                          2.33533
                                                         3.77
                                                                 0.693433
```

```
Number of statistics = 32
Metric = TSquared
```

TSquared histogram:

9.2 QualityMetrics Classes

The QualityMetric classes are divided into groups based on their general function as detailed in the following sections.

9.2.1 Shape Quality Metrics

9.2.1.1 AspectRatioGammaQualityMetric

This class computes the Aspect Ratio Gamma metric for two- and three-diminsional simplicial elements.

9.2.1.2 ConditionNumberQualityMetric

Computes the condition number of given element.

The metric does not use the sample point functionality or the compute_weighted_jacobian. It evaluates the metric at the element vertices, and uses the isotropic ideal element. It does require a feasible region, and the metric needs to be minimized

9.2.1.3 IdealWeightInverseMeanRatio

Computes the inverse mean ratio of given element.

The metric does not use the sample point functionality or the compute_weighted_jacobian. It evaluates the metric at the element vertices, and uses the isotropic ideal element. Optionally, the metric computation can be raised to the 'pow_dbl' power. This does not necessarily raise the metric value to the 'pow_dbl' power but instead raises each local metric. For example, if the corner inverse mean ratios of a quadraliteral element were m1,m2,m3, and m4 and we set pow_dbl=2 and used linear averaging, the metric value would then be m=.25(m1*m1+m2*m2+m3*m3+m4*m4). The metric does require a feasible region, and the metric needs to be minimized if pow_dbl is greater than zero and maximized if pow_dbl is less than zero. pow_dbl being equal to zero is invalid.

9.2.1.4 IdealWeightMeanRatio

Computes the mean ratio quality metric of given element.

The metric does not use the sample point functionality or the compute_weighted_jacobian. It evaluates the metric at the element vertices, and uses the isotropic ideal element. It does require a feasible region, and the metric needs to be maximized.

9.2.2 TMP Quality Metrics

9.2.2.1 AffineMapMetric

Compares targets to affine map to ideal element.

A quality metric defined using 2D and 3D target metrics, where the active (A) matrix is an affine map to the ideal, unit-side element.

9.2.2.2 AWQualityMetric

Compares targets to mapping function Jacobian matrices

A quality metric defined using 2D and 3D target metrics, where the active (A) matrix compared to the target by the underlying metrics is the Jacobian matrix of the mapping function at a given sample point. For surface elements, A is rotated to align the normal with W, such that both matrices can be reduced from 3x2 to 2x2.

9.2.2.3 TMPQualityMetric

Compare targets to mapping function Jacobian matrices

Base class for various TMP QualityMetric implementations.

9.2.2.4 TQualityMetric

Compare targets to mapping function Jacobian matrices

A quality metric defined using 2D and 3D target metrics, where the active (A) matrix compared to the target by the underlying metrics is the Jacobian matrix of the mapping function at a given sample point. For surface elements, A is rotated to align the normal with W, such that both matrices can be reduced from 3x2 to 2x2.

9.2.3 Untangle Quality Metrics

9.2.3.1 UntangleBetaQualityMetric

The untangle beta quality metric.

Given a scalar value beta and local signed element volume alpha_i, define delta_i to be alpha_i minus beta. The Untangle beta value is then defined as square root of the sum over sample points of the absolute value of delta_i minus delta_i, difference squared. That is, the root mean square of the difference, abs(delta_i) minus delta_i.

The constructor defaults to RMS AveragingMethod and ELEMENT_VERTICES evaluationMode. The default beta value is .05.

9.2.4 Volume Quality Metrics

9.2.4.1 LocalSizeQualityMetric

Computes the local size metric for a given vertex.

LocalSizeQualityMetric is a vertex based metric which computes the corner volume (or area) for the element corners attached to a given element. Then these volumes (or areas) are averaged together. The default averaging method is QualityMetric::RMS.

9.2.4.2 SizeMetric

Element size (area or volume)

This metric is intended primarily for use with QualityAssessor for obtaining a summary of the element sizes in the mesh.

9.3 Quality Assessor Code Example

A simple example using the QualityAssessor class:

```
MsqError err;
MeshImpl meshToAssess;
PlanarDomain myDomain;
Settings mySettings;
meshToAssess.clear();
  // read in mesh
const char* filename = "meshToAssess.vtk";
meshToAssess.read_vtk( filename, err);
  // create metric instance
ConditionNumberQualityMetric metric;
  // create QualityAssessor instance accepting default values
QualityAssessor qa(&metric);
  // change some of the default parameters
qa.measure_free_samples_only(false);
qa.disable_printing_results();
  // run the QualityAssessor
MeshDomainAssoc mesh_and_domain = MeshDomainAssoc(&meshToAssess, &myDomain);
qa.loop_over_mesh(&mesh_and_domain, &mySettings, err);
  // get results
const QualityAssessor::Assessor* results = qa.get_results( &metric );
int invalid_element_count = results -> get_invalid_element_count();
if ( invalid_element_count != 0 )
  std::cout << "Warning:" << invalid_element_count
                      << "_invalid_elements_found." << std::endl;</pre>
```

9.4 Common-scale Histograms

When optimizating a mesh, it can be useful to display the quality before and after optimization. This is done by adding a QualityAssessor instance to an InstructionQueue, adding a quality improver instance to the InstructionQueue, and then adding the Quality Assessor instance to the InstructionQueue a second time. This allows a comparison of the mesh quality before and after optimization. Example code for doing this:

```
#include "Mesquite_all_headers.hpp"

using namespace Mesquite;

int main()
{
    MsqPrintError err(std::cout);
    Mesquite::MeshImpl mesh;
    mesh.read_vtk("hexes_4by2by2.vtk", err);
    ConditionNumberQualityMetric cond_num;
```

```
// builds an objective function
LPtoPTemplate obj_func(&cond_num, 2, err);
if (err) return 1;
 // creates the steepest descent optimization procedures
SteepestDescent descent_method(&obj_func);
descent_method.use_global_patch();
Quality Assessor qa=Quality Assessor(&cond_num, 10);
if (err) return 1;
  TerminationCriterion tc2;
tc2.add_iteration_limit(1);
descent_method.set_inner_termination_criterion(&tc2);
 // creates an instruction queue
InstructionQueue queue1;
 // adds quality assessment to instruction queue
queue1.add_quality_assessor(&qa,err);
  // adds a improver/solver to instruction queue
queue1.set_master_quality_improver(&descent_method, err);
if (err) return 1;
 /\!/ adds another quality assessment to instruction queue
 // to show any improvement
queue1.add_quality_assessor(&qa, err);
if (err) return 1;
 // launches optimization on mesh
queue1.run_instructions(&mesh, err);
if (err) return 1;
return 0;
```

9.4.1 Creating Common-scale Histograms

Comparing before and after histograms can be difficult when there is a large difference in the resultant quality value range. In such cases, the common-scale histogram feature can be used to display two histograms with a common vertical interval scale and a common horizontal scale for the number of quality values that fall into each interval. Unlike the above example that reused the same Quality Assessor instance for the before and after histograms, the common-scale histograms require two separate Quality Assessor instances. After both the before optimization and after optimization quality assessments have been performed, the method 'scale_histograms(Quality Assessor* optimal)' can be called to create a pair of common-scale histograms. The before assessment is known as the 'initial', the after assessment is known as the 'optimal'. The histogram interval for both the initial and optimal assessments must be the same for scale_histograms() to work correctly.

Below is a portion of the previous code modified to show how to create common-scale histograms.

```
// Set up the quality assessors
QualityAssessor initial_quality_assessor=QualityAssessor(&cond_num, 10);
```

```
QualityAssessor optimal_quality_assessor=QualityAssessor(&cond_num, 10);

// assess the quality of the initial mesh
queue1.add_quality_assessor(&initial_quality_assessor, err);

queue1.set_master_quality_improver(&descent_method, err);

if (err) return 1;

// assess the quality of the final mesh
queue1.add_quality_assessor(&optimal_quality_assessor, err);

if (err) return 1;

// launches optimization on mesh
queue1.run_instructions(&mesh, err);

if (err) return 1;

// create common—scale histograms
initial_quality_assessor.scale_histograms(&optimal_quality_assessor);
```

When executed, the above code will send its output to std::cout.

9.4.2 Common-scale Histograms output example

The following is the output created by the code shown in Section 9.4.1. It consists of the initial and optimal QualityAssessor Summaries followed by the initial and optimal common scale histograms.

```
****** QualityAssessor(free only) Summary *********
 Evaluating quality for 16 elements.
 This mesh had 16 hexahedron elements.
 There were no inverted elements detected.
 No entities had undefined values for any computed metric.
    Element Quality Statistics
            average rms maximum std.dev.
1.01984 1.01999 1.04547 0.0169641
    minimum
    1.00394
    Number of statistics = 16
    Metric = Condition Number
    Element Quality not based on sample points.
  Condition Number histogram:
    (1.01-1.01) |0
  (1.01-1.02) |0
  (1.02-1.02) | 0
  (1.02-1.03) | 0
  (1.03-1.03) |0
  (1.03-1.04) | 0
  (1.04-1.04) | 0
  metric was evaluated 16 times.
******* QualityAssessor(free only) Summary *********
 Evaluating quality for 16 elements.
 This mesh had 16 hexahedron elements.
 There were no inverted elements detected.
 No entities had undefined values for any computed metric.
    Element Quality Statistics
                          maximum std.dev.
                  rms
minimum
         average
                   1.00124
                           1.00259
1.00003
         1.00124
                                    0.0010116
    Number of statistics = 16
    Metric = Condition Number
    Element Quality not based on sample points.
  Condition Number histogram:
```

```
(1.00056-1.00082) | 0
 (1.00082-1.00109) | 0
 (1.00109-1.00135) | 0
 (1.00135-1.00162) | 0
 (1.00188-1.00214) | 0
 (1.00214-1.00241) | 0
 metric was evaluated 16 times.
Condition Number histogram (initial mesh):
        ) |=======8
    1-1
    1-1.01) |0
 (1.01-1.01) | 0
 (1.01-1.02) | 0
 (1.02-1.02) |============4
 (1.02-1.03) | 0
 (1.03-1.03) | 0
 (1.03-1.04) | 0
 (1.04-1.04) | 0
 (1.04-1.05) |=======4
metric was evaluated 16 times.
 Condition Number histogram (optimal mesh):
    1-1.01) | 0
 (1.01-1.01) | 0
 (1.01-1.02) | 0
 (1.02-1.02) | 0
 (1.02-1.03) | 0
 (1.03-1.03) | 0
 (1.03-1.04) | 0
 (1.04-1.04) | 0
 (1.04-1.05) | 0
metric was evaluated 16 times.
```

9.5 Debug Output

Mesquite contains a mechanism to send status and debug messages to an output stream (e.g. stdout or std::cout). On Unix-like systems that use a configure/make autotools system debug output is enabled using the "-enable-debug" option on the configure command. This option enables Mesquite's debug capabilities but does not enable any actual debug output messages. Output messages are controlled by flags specified using the "-enable-debug-output" option on the configure command. This two step approach is used so that in release builds the debug output feature can be disabled so that turning on debug flags in a released version has no effect.

Debug messages are grouped into logical categories identified by an integer number. For example, debug flag 1 refers to warnings, debug flag 2 is used for status information about the outer optimization loop, and debug flag 3 is used for status of the inner optimization loop. The command to turn on all three flags would be: "./configure –enable-debug-output=1,2,3". When specifying debug flags using the "–enable-debug-output", the "–enable-debug" flag is implied and need not be supplied. The CMake

utility can also be used to enable debug output by setting the "Trillinos_ENABLE_DEBUG" option to "ON". As with the configure command, debug output is only enabled with no flags having been set. CMake options do not support setting of the output message flags so, when configuring Mesquite with CMake, these flags must be specified using the techniques described below.

Debug flags can be controlled through a variety of means. The --enable-debug-output configure option can be specified with a comma-separated list of integer values to specify which debug groups should be enabled by default. An application may call the MsqDebug::enable(unsigned) and MsqDebug::disable(unsigned) functions to enable or disable debug message groups. Debug message groups may also be controlled with the environmental variables MESQUITE_DEBUG and MESQUITE_NO_DEBUG. Each should have a comma-separated list of integer values as its argument. The variables enable and disable, respectively, the corresponding debug message groups.

Additional detail of the available configure command options can be found in Section 2.3.

9.6 Plotting Convergence Behavior

The Mesquite TerminationCriterion class can produce a simple table of tab-separated values for the different Mesquite termination criterion. This file can be used to plot the behavior of the optimization loop using GNU Plot, a spread sheet application, or any other suitable tool. The code listing below illustrates how this feature is activated.

```
// Create global optimizer instance
SteepestDescent improver( &objective_function );
improver.use_global_patch();

// Set only inner termination criterion for
// global optimization
TerminationCriterion inner;
inner.add_absolute_vertex_movement( 1e-3 );
inner.write_iterations( "plot.gpt" );
improver.set_inner_termination_criterion( &inner );

// Run optimization
InstructionQueue queue;
queue.set_master_quality_improver( &improver, err );
queue.run_instructions( &mesh, err );
```

For usable results the feature must be activated on the appropriate TerminationCriterion instance. For a global optimization it should be enabled for the *inner* termination criterion. For other optimization strategies (see Chapter 8) it should be enabled for the *outer* termination criterion.

The following is a sample output file:

#Iter	CPU	ObjFunc	GradL2	GradInf	Movement	Inverted
0	0	1.47419	0	0	0	0
1	0	1.147	0	0	0.657155	0
2	0	1.04779	0	0	0.402173	0
3	0	1.00572	0	0	0.357444	0
4	0	1.00006	0	0	0.150652	0
5	0	1	0	0	0.0153396	0
6	0	1	0	0	0.00015034	0
7	0	1	0	0	6.40008e-09	0

Notice that several of the columns contain only zeros. The column containing the iteration number will always contain valid values. Other values will only be included if they are calculated during the optimization loop. The objective function value will be included for any global optimization that uses an explicit objective function (currently any optimizer other than LaplacianSmoother). In the example source code above we are using the steepest descent solver with a global patch so the objective function value is also included. The other values will only be present if they are calculated for the purpose of

checking termination criteria. In the example source code we specify a termination criterion based on vertex movement, so the column labeled "movement" contains the maximum distance any vertex was moved for the corresponding iteration.

Figures 9.1 shows the result of using the above data file with the following GNU Plot commands:

```
set xlabel 'iterations'
set ylabel 'objective function value'
set y2label 'maximum vertex movement'
set y2tics 0.1
plot 'plot.gpt' using 1:3 with linespoints \
    title 'objective function', \
    'plot.gpt' using 1:6 axes x1y2 with \
    linespoints title 'vertex movement'
```

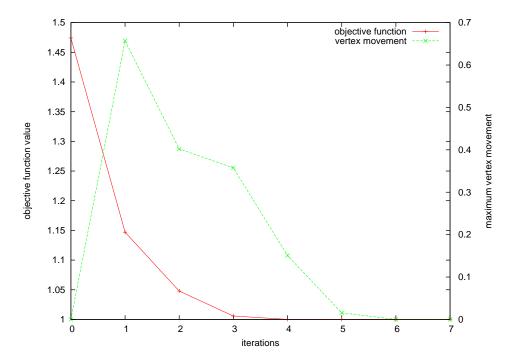


Figure 9.1: Convergence Plot

9.7 Viewing Meshes

VTK files read and written by the MeshImpl class are viewable in a plethora of visualization tools that use the VTK visualization library.

The Mesquite::MeshWriter namespace contains functions to export mesh in a variety of formats for visualization including:

- GNU Plot
- Visualization TookKit (VTK)
- Encapsulated PostScript (EPS)
- Scalable Vector Graphics (SVG)
- StereoLithography (STL)

The GNU plot format writes line data that can be used to plot a wireframe of the mesh (the mesh edges). Both 2D and 3D meshes can be exported in this format. A mesh can be plotted as a 2D projection with the GNU plot command:

plot 'filename' with linespoints

or as a rotatable 3D plot with the command:

splot 'filename' with linespoints

Figure 9.2 is the result of plotting the mesh contained in testSuite/higher_order/homogeneousPart.vtk with GNU plot.

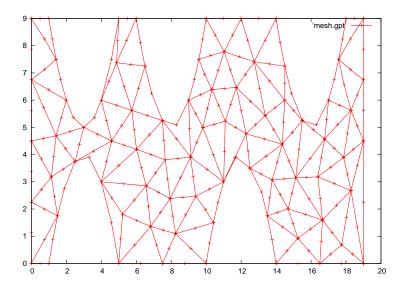


Figure 9.2: GNU Plot of 2D Quadratic Triangles

As mentioned in the previous section, the VTK file format can be used with a variety of visualization tools. Figure 9.3 shows a simple plot of the same mesh in the Paraview visualization tool.

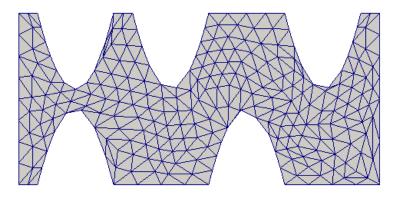


Figure 9.3: Paraview plot of 2D Quadratic Triangles

Figure 9.4 shows the output of the encapsulated PostScript writer for the mesh. The EPS writer can write only 2D projections of the mesh. The caller must specify a projection when calling MeshWriter::write_eps. The testSuite/higher_order/homogeneousPart.vtk file contains quadratic triangle elements. Compare the

mesh edges on the mesh boundary in this plot with the output in Figures 9.2 and 9.3. The EPS writer in Mesquite exports the quadratic edges as curves corresponding to the classic quadratic edge shape function:

$$E(u) = \frac{1}{2}u(u-1)V_1 + (1-u^2)V_2 + \frac{1}{2}u(u+1)V_3$$

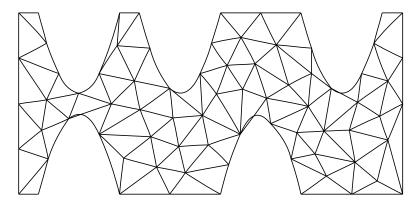


Figure 9.4: Encapsulated PostScript of 2D Quadratic Triangles

The STL file format can be used to write only linear triangles. Higher-order triangular elements will be written as linear triangles. An error will be returned if the mesh contains other element types.

9.8 Exporting Mesh Quality

The Quality Assessor class has the ability to store mesh quality values and other characteristics as tag data on mesh elements. This data can be accessed directly by applications or written to a VTK file using the MeshImpl class or the applications native mesh writer (if it is capable of writing tag data.) The example code below was used to create the VTK file from which the Paraview plot in Figure 9.5 was generated.

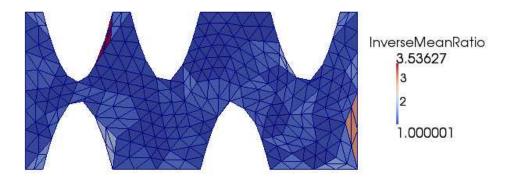


Figure 9.5: Paraview Plot Coloring Elements by Quality Metric Value

```
MsqError err;
MeshImpl mesh;
mesh.read_vtk( "homogeneousPart.vtk", err );

IdealWeightInverseMeanRatio metric;
QualityAssessor qa;
qa.add_quality_assessment(&metric,0,0,0,"InverseMeanRatio");

PlanarDomain plane(PlanarDomain::XY);
InstructionQueue queue;
queue.add_quality_assessor(&qa, err );
MeshDomainAssoc mesh_and_domain = MeshDomainAssoc(&mesh, &plane);
queue.run_instructions(&mesh_and_domain, err );

mesh.write_vtk("meshqual.vtk", err );
```

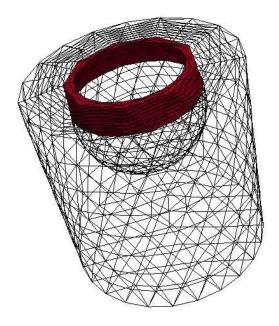


Figure 9.6: Paraview Plot Showing Inverted Elements

Figure 9.6 is a Paraview plot showing the inverted elements in a quadratic tetrahedral mesh. The mesh is plotted twice: once as a simple wireframe of the mesh boundary and a second time as solid mesh with a threshold filter on the inverted flag exported by Mesquite. The listing below shows how the QualityAssessor class can be instructed to flag inverted elements:

```
MsqError err;
MeshImpl mesh;
mesh.read_vtk( "sphereCylinder_1194_inv.vtk", err );
QualityAssessor qa;
qa.tag_inverted_elements("Inverted");
InstructionQueue queue;
queue.add_quality_assessor(&qa, err );
MeshDomainAssoc mesh_and_domain = MeshDomainAssoc(&mesh, &plane);
queue.run_instructions( &mesh_and_domain, err );
mesh.write_vtk( "meshqual.vtk", err );
```

9.9 Mesh Optimization Visualization

The Mesquite Termination Criterion class can write the complete mesh after each iteration as either VTK or GNU Plot data suitable for viewing as an animation. Similar to requesting plot data as described in Section 9.6, it is important to request this feature from the appropriate termination criterion instance. If doing a global optimization, the feature should be activated for the *inner* termination criterion. Otherwise the feature should almost always be activated for the *outer* termination criterion.

The command to request an animation of the mesh optimization in the VTK format is:

```
tc.write_mesh_steps("anim", TerminationCriterion::VTK);
```

This will produce a sequence of files named "anim.1.vtk", "anim.2.vtk", etc. The files can be opened in visualization tools such as Paraview as a single set and played back as an animation. If the optimization calculates the gradient of the objective function, that data will also be included in the file as vector data on each mesh vertex. The components of the vector on each vertex are the partial derivatives of the objective function with respect to each coordinate value of the vertex. A Paraview "glyph" filter can be used to display these vector values during the animation.

The command to request an animation of the mesh optimization in a format suitable for animating in GNU plot is:

```
tc.write_mesh_steps("anim", TerminationCriterion::GNUPLOT);
```

This will produce a sequence of files named "anim.1", "anim.2", etc. It will also export a file named "anim" that contains the necessary GNU Plot commands to display the animation.

Chapter 10

Using Mesquite in Parallel

10.1 Introduction

Large meshes are often partitioned across many parallel processors either because they are too large to fit into the memory of a single machine or in order to speed up the computation. Even if it would be possible to assemble all partitions on a single processor, smooth the mesh, and repartition the result, such an approach would be very I/O inefficient. Moreover, for larger meshes such an approach would quickly run out of memory and fail. Therefore Mesquite supports smoothing meshes in parallel.

Mesquite currently does only synchronous Nash-game or local optimizations in parallel [7]. It does not yet provide parallel solvers and therefore cannot do either block coordinate descent or truly global optimizations in parallel (minimization of an explicit, global objective function.)

For algorithms such as Laplacian smoothing that are local optimizations, optimization in parallel is essentially the same as in serial. For other optimizations that do a global minimization of an explicitly defined objective function in serial (for example ShapeImprover), the parallel optimization will be a Nash-game type optimization where the interior vertices (those not on the partition boundaries) will be optimized as a group. Each vertex on the partition boundary will then be optimized individually. While a global optimization in serial will typically have only one outer iteration, it is generally desirable to do multiple outer iterations in parallel so the Nash-game type optimization can reach convergence. Mesquite wrappers (see Chapter 7) that implement global optimizations in serial default to 10 outer iterations in parallel.

10.2 Distributed Mesh

The input mesh for use in parallel quality improvement must be partitioned based on vertices. That is, each vertex in the mesh must be assigned a single processor as its owner. For optimal performance, vertices should be evenly distributed amongst available processors and the vertices assigned to the same processor should compose a contiguously connected patch of mesh.

Each processor must also have access to all elements for which the position of its vertices influence the quality. For almost all algorithms in Mesquite, this is the set of all elements that contain one of the vertices. Further, each processor must also be able to access any additional vertices owned by other processors that are necessary to define those elements. The instances of such vertices on processors that do not own them are typically referred to as "ghosted" vertices. Elements for which copies exist on multiple processors may sometimes also be referred to as "ghosted" or "ghost" elements.

Figure 10.1 shows a mesh partitioned amongst three processors. The vertices owned by the three different processors are shown in three different colors: blue, red, and green. Elements are colored according to the processors for which copies of that element must be available. A copy of an element must be available on each processor owning at least one of the vertices of the element. Elements colored blue, red, or green need be visible only on the processor owning vertices of the corresponding color. The single grey element must have copies defined on all three processors because each of its vertices is owned by a different processor. The remaining elements must be defined on at least two processors.

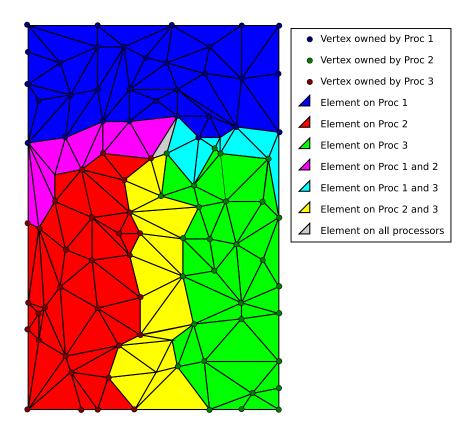


Figure 10.1: Sharing or ghosting of elements and vertices in a partitioned mesh.

For a copy of an element to be available on a processor, all of its vertices must also be available on that processor. So for all elements for which copies exist on more than one processor, the vertices contained in those elements must also exist as ghost vertices on at least one processor. That is, copies of such vertices must exist on processors other than those that are responsible for optimizing the location of that vertex. For example, copies of the yellow elements in Figure 10.1 exist on both the blue and the green processors. All blue vertices in at least one yellow element must exist as ghost vertices on the green processor and all green vertices in at least one yellow element exist as ghost copies on the blue processor. A copy of the grey element must exist on every processor. Therefore each vertex in that element exist as ghost copies on both of the other two processors that do not own it.

10.3 Input Data

Assuming the mesh exists in partitioned form the user has to provide Mesquite with three things:

- a processor ID of type int for every vertex that determines which processor owns a vertex and is in charge for smoothing this vertex,
- a global ID of type size_t for every vertex that (at least in combination with the processor ID) is globally unique,
- all necessary ghost elements and ghost nodes along the partition boundary must be provided.

The following copies of elements and vertices must exist: Elements must exist on all processors that own one or more of the vertices they reference. Vertices must exist on all processors that have some element referencing them.

The Mesquite::ParallelMesh class (ParallelMeshInterface.hpp) inherits Mesquite::Mesh and defines the interface Mesquite uses to interact with parallel mesh data. It contains the following additional pure virtual (or abstract) functions:

- get processor ids for given vertices,
- get global ids for given vertices,
- set and get a pointer to a Mesquite::ParallelHelper object.

To allow Mesquite direct access to the way you store the parallel mesh data you must inherit Mesquite::ParallelMesh and also implement your own get processor ID and get global ID functionality. The Mesquite::ParallelHelper class takes care of all the underlying communication using MPI. You will always use the Mesquite::ParallelHelperImpl implementation that we provide.

Alternatively you can turn any existing mesh of type Mesquite::Mesh into a parallel mesh of type

Mesquite::ParallelMesh by using the Mesquite::ParallelMeshImpl

implementation we provide. On creation it needs a pointer to an object of type Mesquite::Mesh and the names of two tags. It is expected that every vertex is properly tagged with the processor ID tag being of type INT and the global ID tag being of type HANDLE.

10.3.1 ParallelMesh Implementation Requirements

In addition to global and processor ID's, a tag named LOCAL_ID, with type INT, must be provided in your ParallelMesh implementation. In summary, here are the tags and their types required by Parallel Mesquite:

1	Concept name	Typical/required code string	Mesquite type
vertex processor owner id		PROCESSOR_ID (typical, implementation-dependent)	INT
ı	vertex global unique id	GLOBAL_ID (typical, implementation-dependent)	HANDLE
	vertex local id (internal use)	LOCAL_ID (required)	INT

If you obtained Mesquite from the Trilinos site, you can see a sample imlementation of ParallelMesh in the stk_percept package, at

 ${\tt Trilinos/packages/stk/stk_percept/stk_percept/mesh/mod/mesquite-interface/PerceptMesquiteMesh.*pp.}$

10.4 ITAPS iMeshP Interface

The MsqIMeshP class is an alternate implementation of the ParallelMesh interface that can be used to provide Mesquite with callbacks to access mesh and related parallel properties. The ITAPS Working Group has defined a standard API for exchange of parallel mesh data between applications. The Mesquite::MsqIMeshP class declared in MsqIMeshP.hpp is an "adaptor": it presents the iMeshP interface as the Mesquite::ParallelMesh interface.

This class will use the iMeshP API to query processor identifiers and global identifiers for mesh vertices. However, the MPI-based communication routines implemented in ParallelHelperImpl are used rather to communicate updated vertex locations between processors, rather than the mechanism provided by the iMeshP implementation.

10.5 Examples

This section contains two different examples of simple stand-alone applications that demonstrate the use of the LaplaceWrapper smoother in parallel. Both examples, in being stand-alone programs, load the mesh from one or more files. When integrating Mesquite into an existing application where it is desired that Mesquite access application mesh data in memory, the initial setup will be different. It will typically involve either providing some application-specific implementation of the Mesh and possibly ParallelMesh interfaces or instances of an application-specific iMeshP and iMesh implementation.

10.5.1 Example: Parallel Laplacian Smooth

This example uses the LaplaceWrapper wrapper in parallel using the built-in Mesh, ParallelMesh, and ParallelHelperImpl implementations. For this example to work, the mesh must be partitioned such that the mesh for each processor is saved in a separate file named part-%d.vtk, with the %d replaced

with the processor rank. Each VTK file must contain vertex attributes named GID and PID containing the global ID and owning processor rank for each vertex. Further, as this example provides no geometric domain definition, the vertices on the boundary of the mesh must be designated as "fixed" for the problem setup to be valid.

```
/* Mesquite includes */
#include <Mesquite.hpp>
#include <MeshImpl.hpp>
#include <ParallelMeshImpl.hpp>
#include <ParallelHelper.hpp>
#include <MsqError.hpp>
#include <LaplaceWrapper.hpp>
/* other includes */
#include <mpi.h>
#include <iostream>
using namespace std;
int main( int argc, char* argv[] )
  /* init MPI */
  int rank, nprocs;
  if (MPI_SUCCESS != MPI_Init(&argc, &argv)) {
   cerr << "MPI_Init failed." << endl;</pre>
   return 2;
  MPI_Comm_rank(MPI_COMM_WORLD, &rank);
  MPI_Comm_size(MPI_COMM_WORLD, &nprocs);
  /* create processor-specific file names */
  ostringstream in_name, out_name;
  in_name << "part-" << rank << ".vtk";
  out_name << "part-" << rank << "-smoothed.vtk";</pre>
  /* load different mesh files on each processor */
  Mesquite::MsqError err;
  Mesquite::MeshImpl mesh;
  mesh.read_vtk(in_name.str().c_str(), err);
  if (err) {cerr << err << endl; return 1;}</pre>
  /* create parallel mesh instance, specifying tags
   * containing parallel data */
  Mesquite::ParallelMeshImpl parallel_mesh(&mesh, "GID", "PID");
  Mesquite::ParallelHelperImpl helper;
  helper.set_communicator(MPI_COMM_WORLD);
  helper.set_parallel_mesh(&parallel_mesh);
  parallel_mesh.set_parallel_helper(&helper);
  /* do Laplacian smooth */
  LaplaceWrapper optimizer;
  optimizer.run_instructions(&parallel_mesh, err);
  if (err) {cerr << err << endl; return 1; }</pre>
  /* write mesh */
  mesh.write_vtk(out_name.str().c_str(),err);
  if (err) {cerr << err << endl; return 1;}</pre>
```

```
MPI_Finalize();
  return 0;
}
```

10.5.1.1 Implementation of Example 10.5.1

In your Mesquite distribution, there is an implementation of the example code for Lapalace smoothing in parallel, in the file mesquite/testSuite/parallel_smooth_laplace/par_hex_smooth_laplace.cpp. This code reads in a serial or parallel-split set of VTK files and smooths the mesh, then compares the result to a "gold" copy, which is useful for regression testing (see 3.1.6).

10.5.1.2 Parallel Regression Tests

In addition to the Laplace example, see

mesquite/testSuite/parallel_untangle_shape/par_hex_untangle_shape.cpp

for example use of parallel mesh untangling and shape improvement, and the associated files that begin with "par_*" under the meshFiles/2D,3D/vtk/ directories.

For example, an initial, tangled quadrilateral mesh is shown in 10.2 while the result of untangling and smoothing is shown in 10.3. A similar example with hexahedra is shown in figures 10.4 and 10.5.

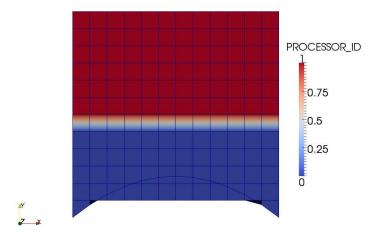


Figure 10.2: Initial, tangled quadrilateral mesh.

10.5.2 Example: Using Mesquite::Mesquite::MsqIMeshP

Similar to the example in Section 10.5.1, this example uses the LaplaceWrapper wrapper in parallel to improve element shape. However, this example assumes that either the iMeshP implementation is partitioning or that it is reading some pre-defined partitioned mesh and it relies on the iMeshP implementation to create ghost elements, assign global vertex IDs, etc.

An implementation of the iMesh and iMeshP APIs must be provided for this example to work. Mesquite can use these APIs, but does not provide them.

```
/* Mesquite includes */
#include <Mesquite.hpp>
#include <MsqIMeshP.hpp>
```

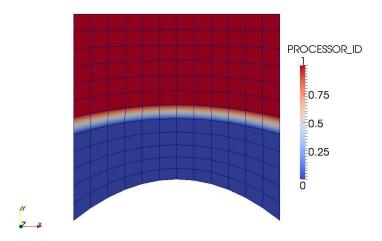


Figure 10.3: Untangled and smoothed quadrilateral mesh.

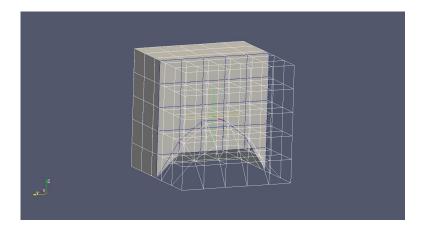


Figure 10.4: Initial, tangled hexahedra mesh.

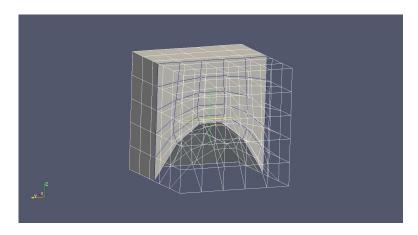


Figure 10.5: Untangled and smoothed hexahedral mesh.

```
#include <ParallelMeshImpl.hpp>
#include <ParallelHelper.hpp>
#include <MsqError.hpp>
#include <LaplaceWrapper.hpp>
/* other includes */
#include <mpi.h>
#include <iostream>
using namespace std;
int main( int argc, char* argv[] )
  const char input_file[] = "testmesh";
  const char output_file[] = "smoothmesh";
  /* init MPI */
  int rank, nprocs;
  if (MPI_SUCCESS != MPI_Init(&argc, &argv)) {
   cerr << "MPI_Init failed." << endl;</pre>
   return 2;
  MPI_Comm_rank(MPI_COMM_WORLD, &rank);
  MPI_Comm_size(MPI_COMM_WORLD, &nprocs);
  /* create a new instance of the iMesh database */
  int ierr;
  iMesh_Instance mesh;
  iMesh_newMesh(NULL, &mesh, &ierr, 0);
  if (iBase_SUCCESS != ierr) return ierr;
  iBase_EntitySetHandle root_set;
  iMesh_getRootSet(mesh, &root_set, &ierr);
  if (iBase_SUCCESS != ierr) return ierr;
  /* create a partition instance in which to read
     the partitioned mesh */
  iMeshP_PartitionHandle partition;
  iMeshP_createPartitionAll(mesh, MPI_COMM_WORLD,
                            &partition, &err);
  if (iBase_SUCCESS != ierr) return ierr;
  /* load mesh */
  iMeshP_loadAll(mesh, partition, root_set, input_file,
                 NULL, &err, strlen(input_file), 0);
  if (iBase_SUCCESS != ierr) return ierr;
  /* create 1 layer of ghost entities */
  iMeshP_createGhostEntsAll(mesh, partition, 3, 1, 1, 0, &err);
  if (iBase_SUCCESS != ierr) return ierr;
  /* create MsqIMeshP instance */
  Mesquite::MsqError err;
  Mesquite::MsqIMeshP parallel_mesh(mesh, partition, root_set,
                                     iBase_REGION, err);
  if (err) {cerr << err << endl; return 1; }</pre>
```

```
/* do Laplacian smooth */
  LaplaceWrapper optimizer;
  optimizer.run_instructions(&parallel_mesh, err);
  if (err) {cerr << err << endl; return 1; }</pre>
  /* write mesh */
  iMeshP_saveAll(mesh, partition, root_set, output_file,
                 NULL, &ierr, strlen(output_file), 0);
  if (iBase_SUCCESS != ierr) return ierr;
  /* cleanup */
  {\tt iMeshP\_destroyPartitionAll(mesh, partition, \&ierr);}
  if (iBase_SUCCESS != ierr) return ierr;
  iMesh_dtor(mesh, &ierr);
  if (iBase_SUCCESS != ierr) return ierr;
  MPI_Finalize();
  return 0;
}
```

Chapter 11

User Support

11.1 Mailing Lists

An open mailing for discussion of Mesquite usage questions is available at mesquite@software.sandia.gov. This list is open to all Mesquite users. Archived messages and subscription information are available on the list web page:

http://software.sandia.gov/mailman/listinfo/mesquite.

11.2 WWW Page

The Mesquite WWW page is located at http://www.cs.sandia.gov/optimization/knupp/Mesquite.html.

Appendix A

The Mesquite Team

The Mesquite team is composed of members from Sandia National Laboratories (SNL), Lawrence Livermore National Laboratory (LLNL), and Elemental Technologies Inc. (ETI).

The current Mesquite developement team includes:

```
Lori Freitag-Diachin (Co-PI, LLNL),
Patrick Knupp (Co-PI, SNL), and
Boyd Tidwell (ETI)
```

Past developers and other significant contributors to the development of Mesquite include:

```
Michael Brewer (SNL),

Ulrich Hetmaniuk (SNL),

Jason Kraftcheck (UW).

Thomas Leurent (ANL),

Darryl Melander (SNL), and

Todd Munson (ANL).
```

Current Mesquite developers can be contacted via e-mail at either the (private) developers' mailing list:

Mesquite-Developers@software.sandia.gov, or the (open) mailing list for all Mesquite users: Mesquite@software.sandia.gov.

Appendix B

Acknowledgments

Mesquite is supported under the DOE SciDAC Interoperable Tools and Petascale Simulation (ITAPS) project.

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