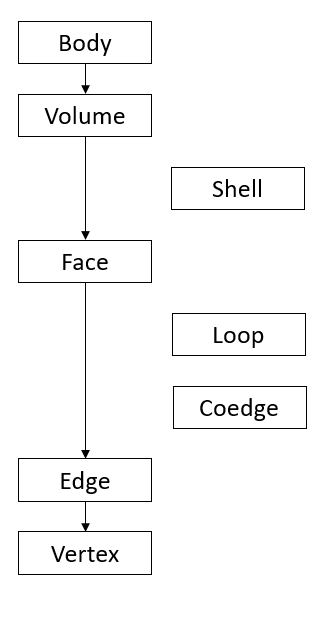
SGM User Manual

**Basic Data Types**

**Entities**

**Topology**

SGM Topology is summarized by the following diagram;

SGM has entity types for three-dimensional Volumes, two-dimensional Faces, one-dimensional Edges, and zero-dimensional Vertices. In addition, SGM has a Body entity type that contains zero or more Volumes and an optional cloud of points. Other topology entities types such as Shells, Loops and Coedges are intentionally left out of SGM since all their information may be derived from the other types on demand.

In addition to a reduced topology set, SGM also has a reduced set of flipping flags. However, one flipping flag remains on the face, which indicates if the surface normal and face normal flipped. To replace Coedges, SGM Faces maintain a map of which side the faces is on for each edge on the face.

Hence, SGM Topology is sufficient to define any bound subset of three-dimensional Euclidian space such that the boundary may be defined by a piecewise differentiable set of surfaces, curves, and / or points. This means that Bodies in SGM may be zero, one, two, or three dimensional, and may be parts of each dimension from zero to three. In addition, SGM supports non-manifold non-connected and non-orientable Bodies.

Body

A Body is made up of a set of Volumes and / or a set of points. The points and volumes must be pairwise disjoint. In other words, a body is made up of the connected components of a subset of space. The zero-dimensional components are just points and the other components are Volumes. A Volume may contain only one edge with a point-curve, which is equivalent to a single point. However, defining a large set of points by using a Volume for each point is more expensive in terms of memory. Hence, the need to have a set of points in addition to a set of Volumes.

A Body that only contains points is called a “Point Cloud”. A Body that contains only Volumes with edges is called a “Wire-Body”. A Body that contains only double-sided faces is called a “Sheet Body”. A Body that contains only single sided faces is called a “Solid Body”. However, SGM Bodies may contain elements of any or all types of bodies and Bodies that do are called “Mixed Bodies”.

Volume

A Volume is a connected component of a Body that consists of a set of boundaries Faces of the two and three-dimensional parts of the Volume, and a set of Edges for the one-dimensional parts of the Volume. The Edges and Faces of a Volume should have pairwise disjoint interiors. In addition, to a set of Faces and Edges the Volume also knows it Body.

The single sided Faces of a Volume can be further divided into connected components of single sided Faces that are called “Shells”. Once found there are two types of shells, the outer shell and any number of inner shells, which are also called voids.

Volumes may also contain interior Faces, and Edges, which are called “Membrane” Faces or Edges that are not used in finding shells.

Face

A Face is a connected subset of a Surface with a piecewise differentiable boundary and a connected interior. Faces may have interior Edges. However, the Edges may not divide the Face, which is to say that face minus its edges must be connected. Hence a Face consists of a Surface along with a set of Edges. In addition, a Face also knows if it is on the left, right, or both side(s) of each of its edge(s), and if it normal vectors match or are opposite to its Surface’s normal vectors. Moreover, a Face knows it Volume.

Faces also maintain a number to indicate how many sides of the face are exposed to the outside. A Face may be zero, single, or double sided. Zero sided faces are called “Membrane” faces and are inside their Volume, single sided faces make up the boundary of the three-dimensional part of a Volume, and double-sided faces make up the “Sheet Body” parts of Volume.

The edges of a Face may be divide up into connected sets, which are called “Loops”. Moreover, the Edges of a “Loop” may be ordered, along with a corresponding set of flags that indicate how one may walk around a boundary component of the face so that the face is always on the left or right, depending on its corresponding flag. In the case of interior edges and / or seams an edge may appear twice in the “Loop” with different flags for each appearance. Finding points on the boundary of Face in three-dimensional space just requires evaluating the Curve(s) of the Edge(s). However, finding the points in the Face’s Surface’s parameter space is more difficult, given that one point may have up to four different values in parameter space. To aid in finding such points a special evaluator exists that take the three-dimensional point, the Edge, and the “Loop” flag and figures out the correct values.

Edge

An Edge is a connected subset of a Curve. The subset is defined by a one dimensional internal that is a subset of the domain of its Curves. In addition, to containing a Curve an Edge also contains zero, one or two Vertices(s) and it knows its Faces or it Volume if it is a “Wire Edge”.

Vertex

A Vertex is part of the boundary of a set of Edges, it knows it point and its Edges.

**Geometry**

SGM geometry consists of Curves and Surfaces. SGM has a large set of different types of Curves and Surfaces for three reasons. First, even though in theory one can represent all curves and surfaces with approximating NURBs, NURB surfaces are significantly heavier than and other curve and surface types in terms of memory, and they are very significantly slower to find points on they and closest points to them. The second reason why SGM has a large set of curves and surface types is to support the clean translation to and from of STEP files. The third reason why SGM includes different types of curves and surfaces are to support specialty uses of the modeler or to test parts of the modeler.

A SGM Curve is a function from a closed and bounded interval in the real line to three-dimensional Euclidian space that is differentiable and one-to-one, on the interior of its domain. A SGM Surface is a function from a coordinate aligned closed and bounded rectangle in the XY-plane to three-dimensional Euclidian space that is differentiable and one-to-one, on the interior of its domain.

The point that a Curve maps the lower bound of its domain to is called its start point and the upper bound of its domain to is called it end point. If the start and end point of a curve match, then the curve is called closed. If in addition the start and end derivatives match then the curve is called periodic.

As a matter of tradition, the coordinates used to describe the domain of a surface are U for X, and V for Y. Hence, by the lower U domain we mean the part of the domain at is at the lower X bound, and upper U domain and lower and upper V domains are similarly defined. SGM Surfaces do not need to be one-to-one on the boundary of their domain. However, they have the added restriction that if they are not one-to-one, then they must map a whole side of their domain to the opposite side, or to a single point. In the case that one side is mapped to the opposite side the surface is called closed in U or V, and in the case that a side is mapped to a single point the surface is said to be singular at lower or upper U or V. Moreover, in the case of a closed surface the mapping from one side to the other must be one-to-one and may not flip the other coordinate value. In other words, a mobius strip may not be made out of one surface. However, it may be made from two. As with curves a surface that is closed in U or V where the partial derivatives with respect to U or V also match on the whole boundary side is called periodic in U or V.

When it comes to geometry, three-dimensional geometry is the primary definition over the use of two-dimensional geometry of the parameter space of a surface. Hence, two surfaces are considered the same if they match geometrically even if they do not match parametrically. However, when geometry is transformed, specifically when it is scaled, the parametrization of curves and surfaces also scales so that cached parameter values are unchanged.

**Curves**

SGM supports Lines, Circles, Ellipses, Parabolas, Hyperbolas, Point-Curves, NUBs, NURBs, Hermite, Helices, and Torus-Knots.

**Conic Sections**

Conic Sections are a special set of planar curves that make up intersections between a Plane and Cone. One point determines a Point-Curve, two points determine a Line, three points determine a Circle, and five planar points are sufficient to determine any Conic Section.

Lines

Lines in SGM are “essentially” infinite, which means that their domain may be set to be from -SGM\_MAX to SGM\_MAX, or [-1E+12,1E+12]. However, the domain of a Line may be set to be smaller. In SGM Lines are defined by an Origin Point, a Axis Unit Vector, and a scale value, given by the following equation;

f(t) = Origin + Axis\*(scale\*t).

Circles

Circles in SGM may have any domain of length less than or equal to two pi. However, the domain of a circle is most commonly [0,2pi]. A Circle is defined by a Center Point, a radius, and an orthogonal set of X Y and Z Axis Unit Vectors. The Z Axis is called the circles Normal Vector. Given these values a circle is defined by;

f(t) = Center + (cos(t)\*XAxis+sin(t)\*YAxis)\*radius.

Ellipses

An Ellipse in SGM may have any domain of length less than or equal to two pi. However, the domain of an ellipse is most commonly [0,2pi]. An Ellipse is defined by a Center Point, an xRadius, a yRadius and an orthogonal set of X Y and Z Axis Unit Vectors. The Z Axis is called the ellipses Normal Vector. Given these values an ellipse is defined by

f(t) = Center + cos(t)\*yRadius\*XAxis + sin(t)\*xRadius\*YAxis.

Parabolas

Parabolas in SGM are “essentially” infinite, which means that their domain may be set to be from -SGM\_MAX to SGM\_MAX, or [-1E+12,1E+12]. However, the domain of a Parabola may be set to be smaller. In SGM Parabolas are defined by an Origin Point, an orthogonal set of X Y and Z Axis Unit Vectors, and a constant “a” as follows;

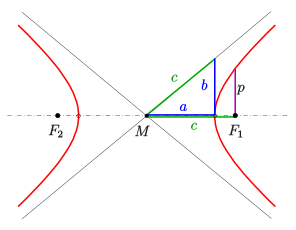
f(t) = Origin + XAxis\*t + YAxis\*a\*t^2.

Hyperbolas

Parabolas in SGM are “essentially” infinite, which means that their domain may be set to be from -SGM\_MAX to SGM\_MAX, or [-1E+12,1E+12]. However, the domain of a Parabola may be set to be smaller. In SGM Parabolas are defined by an Origin Point, an orthogonal set of X Y and Z Axis Unit Vectors, and two constants “a” and “b” as follows;

f(t) = Origin + XAxis\*t + YAxis\*(a\*sqrt(1+t^2/b^2)).

The constant “a” is the distance from f(0) to the Origin, and the constant “b” is the distance from f(0) to the asymptote and shown below. XAxis is in direction of “b” and the YAxis is in the direction of “a” shown below. It should be noted that sometimes in the literature X and Y are swapped. In SGM the XAxis goes in the direction that it does so that f(t) is a well-defined function, which is to say that f(t) is unique. Also, it should be noted that only the half on the right is used. To create the other half a different curve needs to be made with the negative value of YAxis.



Point-Curves

Point-Curves are the only curves that are permitted to have zero derivatives in the interior of their domains. By default, the domain of a Point-Curve is set to [0,0]. However, it may be set to any non-empty value, with the intent to match parameter space that it spans at a singularity.

f(t) = Origin.

**Piecewise Defined Curves**

A piecewise defined curve is a curve that is made from concatenating a finite number of either polynomial or rational expressions together. A polynomial curve is a function where each coordinate is defined by a polynomial. By a rational expression is a function where each coordinate is defined by a polynomial divided by a polynomial.

NURB Curves

A NURB stands for a Non-Uniform-Rational-B-spline. The “NU” indicating that the lengths of the domains of the part of the curve do not have to be uniformly spaced. The “R” indicating that the curves are rational expressions, and the “B” standing for the word “Basis”, which refers to a basis in a vector space defined, in this case by the set of all polynomial or rational expressions.

The significant of the “B” is in how NURBs are defined. Instead of defining the three sets of coefficients for each dimension and each defining segment. A NURB may be defined by a vector of points and weights along with a possibly different length monotonically increasing vector of real numbers called a “knot” vector. The vector of points are called control points and in SGM the weights are combined with the control points to form a four-dimensional point for the form (x,y,x,w). The number of knots is related to the number of control points, given the degree of the defining polynomials by the following formula;

Degree = Knots – Control Points -1

Given a degree, n, and a knot vector, then for each control point, , one can define a polynomial function called a bases function, . Given a parameter value t, vector of weights,, and control points a NURB is defined as the following sum;

f(t)

How to construct the bases functions for a NURB and a detailed discussion of properties of NURBs is a subject that is beyond the scope of this user manual. For more information the reader is directed to “The NURB Book” by Les A, Piegl and Wayne Tiller.

However, there are three points that are worth commenting on in this manual. First with the exceptions of the end points the control points DO NOT lie on the curve. Moreover, the knot vector does not contain parameters that directly correspond the control points. However, SGM has functions that will create a NURB that passes through a set of points, by finding the control points that achieve this goal. Second, all conic sections can be fit perfectly with a NURB, but in doing so the parameterization of the curve may be compromised. Specifically, circles may be perfectly fit with NURBs but not with an arc length or angle based parameterization. Moreover, there are some curves, such as helices that can only be approximated with NURBs and as the desired fit tolerance approaches zero, the number of required control points grows without bounds. However, not all is bad in that degree three NURBs are defined in such a way that even across the boundaries of their defining segments the curves have C2 continuity, and infinite continuity interior to each span.

NUB Curves

Since f(x)=1 is a polynomial, all polynomials are rational expressions. However, not all rational expressions are polynomials. In the event that all the rational expressions are polynomials we drop the “R” and use the expression NUB for a Non-Uniform-B-spline. Hence, a NUB can be defined using the same bases functions as a NURB but with the simpler equation as follows;

f(t)

It should be noted that most NURBs used in CAD are NUBs in that all of their weights are set to one. Moreover, most NUBs are of degree three. However, degree two NURBs are used when defining a curve from three points and degree one is used for two points or for piecewise linear curves. In some cases, degree five is used when C2 continuity is desired. Likewise, higher degree NURBs can be used when high levels of continuity are desired, and / or a single span is desired to design a shape that is not fit well by a lower degree polynomial.

Hermite Curves

Hermite curves are a simpler form of piecewise polynomial curves that are defined by a vector of parameters, points and tangent vectors all of the same size. In the case of a Hermite curve the defining points do lie on the curve at the given parameters, and the tangent vectors are the tangent vectors of the curve at the given parameters. However, Hermite curves are only C1 at the defining points and are of degree three.

**Advanced Curve Types**

Helices

A Helix is the only curve in three-dimensional Euclidian space with constant curvature other than a line and a circle. Hence, a helix can move within itself with a twisting and translating movement just like a circle can move within itself with a rotation and line can move within itself with a translation. Since these are the only curves with this property they show up quite often in mechanical parts. Moreover, a helix is one of the curve types that is not fit well by a NURB curve. Hence, either one has to fit them with a very heavy spline or one has to have a procedural representation for them. In the past these curves were so heavy that some CAD systems by marking a cylinder with data that indicated what type of threads to add to the cylinder without modeling them explicitly.

The STEP file format does not support Helices. Hence, when they are writing out they are output as a spline. Therefore, when one reads in a helix from a STEP file, they will read in a spline that will only approximate the helix. This can be a significant source of translator error that can be mitigated by measuring the curvature, and torsion of the spline and fitting an exact helix to the spline.

Torus Knots

Torus knots were added to SGM primarily for testing since they lie on a torus and cross both of the seams multiple times. Torus knots are like helices on a torus in that they uniformly wind around a torus like a helix uniformly winds around a cylinder. The curves are called “knots” since they are closed in three-dimensional Euclidian space that form a knot. The knots that they may form are characterized by how many times they curve crosses the U and V seam of their torus. To prevent self-intersecting curves the number of U and V seam crossing must be relatively prime.

**Surfaces**

SGM supports Planes, Spheres, Cylinders, Cones, Torii, NUBs, NURBs, Revolved, Extruded, and Offset Surfaces. Surfaces do not have a flipping flag which means that the normals of Sphere, Cylinders, Cones, Torii, always point out. Faces do have a flipping flag to indicate that the surface normal points in the opposite direction from the face normal.

**Simple Surfaces**

Planes

Spheres

Cylinders

A Cylinder is defined by an origin, radius and axis. The origin of the cylinder should be selected to be near the region of interest of the faces that the cylinder is owned by. In general, the origin should be a point on the axis of the cylinder near the center of mass of the surface’s faces. By placing the origin near the center of mass of the faces the parameterization is more numerically stable, than if the origin were placed far from the region of interest.

Cylinders are parameterized so that U goes around the cylinder and V goes in the linear direction. So that the parameterization will match in speed in both the U and V direction the U direction is parameterized from zero to two pi, and the linear direction is parameterized by Euclidian distance times the radius of the cylinder.

Cones

To prevent an interior self-intersection a cone in SGM is made up of only one side of the apex of the cone, and the apex is an upper V singularity of the surface. Hence, the U direction goes around the cone and the V direction goes in the linear direction of the cone, ending at the apex of the cone. Therefore, the Axis of a cone points from the origin to the apex of the cone. Like a cylinder, the origin of a cone should be a point on the axis of the cone near the center of mass of the surface’s faces. In SGM the origin must be a point on the axis, other than the apex, that is inside the half of the cone that is to be used. Also like a cylinder the U direction is parameterized from zero to two pi, and the V direction is parameterized by Euclidian distance times the radius of the cone, where the radius of the cone is the distance from the origin to the cone from the origin in a direction that is perpendicular to the axis.

Torii

**Piecewise Defined Surfaces**

NUB Surfaces

Most NURB surfaces are really just NUB surfaces, which is to say that the weights of all the control points are all equal and usually equal to one.

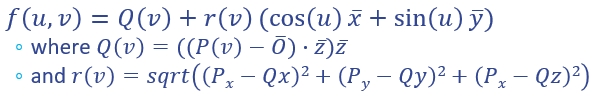
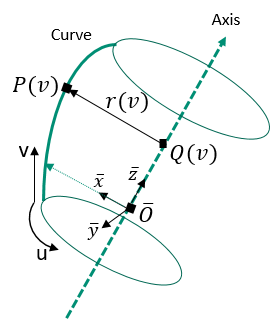
NURB Surfaces

NURB surfaces are used less often than NUB surfaces. However, since NURB surfaces can has circular parameter lines, they can be found as blend surfaces in STEP files.

**Advanced Surface Types**

Revolved

All the simple surfaces, except for a plane, can be represented as a Revolved surface.



Extruded

Both cylinders and planes can be represented as an Extruded surface.

Offset Surfaces

All of the simple surfaces can be offset by creating another simple surface of the same type but with different parameters. However, NURBs and NUBs do not have this property. Even the offset of an Extrude, Revolved, and Offset surface can be created with the same type of surface but with different parameters. However, since the offsets of NURBs, NUBs and conic sections other than lines and circles, are not the same type of curve, the defining curve in Extruded and Revolved surfaces may have to change type.

**SGM Functions**

**Checker / Testing**

**Simplicial Complex Support**

**Faceting**

**General Entity Functions**

**Interrogation Functions**

**Mathematics Functions**

**Measure Functions**

**Primitive Creation**

**Topology Traversal**

**Translators**

**STEP**

**STL**

**Facet To B-Reps**