Linearized Analysis

of

Power System Dynamics

with PST

Contents

1 Introduction

An interconnected alternating current power system operates, in the steady state, with a constant frequency throughout the system. The frequency is maintained by the system's synchronous generators, and their speed governing systems. In the absence of speed governors, an AC interconnected power system operates with the same frequency throughout the system, but that frequency will vary as the system load varies. Following a change in system load, the generators' speeds will be reduced and the relative angles between the generators will automatically adjust until, in the steady state the speeds of all generators will be such that they all produce the same frequency, and the additional load is supplied. Speed governors act to control the generators' speeds close to the nominal value. Final frequency control may be performed manually by system operators or by slow automatic controls.

Large and small disturbances are constantly applied to power systems. Small disturbances occur due the varying nature of power system loads. Large disturbances are caused by faults which may occur on the power system. The most common faults are those due to natural causes, for example lightening striking power system transmission lines. Power system dynamic analysis is necessary to ensure that the power system remains operative under different system conditions and following the most common types of fault.

For small system changes, the power system dynamics is essentially linear. A linearized power system dynamic model will indicate whether or not a power system's steady state is viable (stable) or not viable (unstable). In addition, a linearized power system dynamic model can be used to design the controls which may be necessary to maintain the system's stability to small changes.

Power systems are not linear. This means that as a system changes, e.g., as the load on the system increases, its dynamic characteristics also change, and an initially stable system may be come unstable. Ideally, the limit of stability should be higher that the limits on the system posed by the thermal capacity of the system's components. In practice, some naturally occurring faults may cause a system to be limited to less than the thermal limit. In other systems, the system's operation may be limited to less than the thermal limit by oscillations between system elements, caused by small changes in load, increasing in amplitude (becoming unstable) rather than dying away. This is not at all desirable, and in a well designed power system this type of oscillatory stability should not occur until the thermal limit is exceeded.

2 The Mathematics of Dynamic Oscillations

Dynamic oscillations are caused by the interchange of potential and kinetic energy following changes in a systems variables from a steady state equilibrium condition. The simplest physical oscillatory system is that of a mass hanging from a spring. The equations of motion of the mass and spring system are

$$M\frac{d^2x}{dt^2} = -Kx + Mg \tag{2.1}$$

M is the mass, K is the spring constant and g is the force of gravity. In the steady state

$$x = x_o = \frac{Mg}{K} \tag{2.2}$$

If the mass is pulled down a small distance Δx and then released, the spring will pull it back towards the equilibrium position. However, in the ideal case of equation 2.1, when x reaches x_o the mass will still be moving, and the speed will not reach zero until x is equal to $x_o + \Delta x$. In this system the energy supplied to the system to make the initial change in the mass position is not dissipated. At $x = x_o \pm \Delta x$ the systems kinetic energy will be zero, and its potential energy will be $\frac{1}{2}K\Delta x^2 + Mg\Delta x$. At $x = x_o$, the systems potential energy will be zero and the kinetic energy will be at its maximum. The mass will oscillate sinusoid ally.

$$x = x_o - \Delta x \cos\left(\sqrt{\frac{K}{M}}t\right) \tag{2.3}$$

In practice, the change in the potential energy of the mass will eventually be dissipated. If there is a force proportional to the speed of the mass, the mass will eventually settle down to its original equilibrium condition. In this case the equation of motion of 2.1 is changed to

$$M\frac{d^2x}{dt^2} = -Kx - D\frac{dx}{dt} + Mg \tag{2.4}$$

The oscillations in x will now have the form

$$x = x_o + \Delta x_m \exp(-\alpha t)\cos(\omega t - \phi) \tag{2.5}$$

the values of α and ω are determined to satisfy the differential equation 2.4. The values of Δx_m and ϕ are determined to satisfy the initial conditions, $\frac{dx}{dt} = 0$, and $x = x_o - \Delta x$.

$$\begin{aligned} \frac{dx}{dt} &= -\Delta x_m e^{-\alpha t}\left[\alpha\cos(\omega t - \phi) + \omega\sin(\omega t - \phi)\right] \\ \frac{d^2x}{dt^2} &= \Delta x_m e^{-\alpha t}\left[\left(\alpha^2 - \omega^2\right)\cos(\omega t - \phi) + 2\alpha\omega\sin(\omega t - \phi)\right] \end{aligned} \tag{2.6}$$

For x to be equal to $x_o - \Delta x$ at t = 0,

$$\Delta x_m \cos\phi = -\Delta x \tag{2.7}$$

for $\frac{dx}{dt}$ to be equal to 0 at t = 0,

$$\alpha\cos\phi + \omega\sin\phi = 0 \tag{2.8}$$

For the differential equation of 2.4 to be satisfied

$$M\left[\left(\alpha^2 - \omega^2\right)\cos(\omega t - \phi) + 2\alpha\omega\sin(\omega t - \phi)\right] - D\left[\alpha\cos(\omega t - \phi) + \omega\sin(\omega t - \phi)\right] + K\cos(\omega t - \phi) = 0 \tag{2.9}$$

equating the coefficients of $\cos(\omega t - \phi)$ and $\sin(\omega t - \phi)$ to zero gives

$$\begin{aligned} M\left(\alpha^2 - \omega^2\right) - D\alpha + K &= 0 \\ 2M\alpha\omega + D\omega &= 0 \end{aligned} \tag{2.10}$$

Thus

$$\begin{aligned} \alpha &= -\frac{D}{2K} \\ \omega^2 &= \left(\frac{K}{M} - \frac{D^2}{4M^2}\right) \end{aligned} \tag{2.11}$$

Equation 2.11 is valid provided that $\frac{D}{4M} < K$, which is the condition that the mass-spring-damper system is oscillatory.

From 2.8

$$\begin{aligned} \tan\phi &= -\frac{\alpha}{\omega} \\ \cos\phi &= \frac{\omega}{\sqrt{\alpha^2 + \omega^2}} \\ \Delta x_m &= -\frac{\sqrt{\alpha^2 + \omega^2}}{\omega}\Delta x \end{aligned} \tag{2.12}$$

More complicated linear systems can be broken down into combinations of first order differential equations of the form

$$\frac{dx}{dt} = \beta x + y \tag{2.13}$$

By hand, the algebra can be tedious for larger systems. Even a single synchronous generator and its controls may require 20 coupled first order differential equations to describe its dynamic performance. Thus, methods other than those shown above are used to determine a systems oscillatory dynamics.

The equations of complex dynamic systems may be expressed as a number of interconnected first order differential equations ,i.e.,

$$\frac{d\mathbf{x}}{dt} = \mathbf{f}(\mathbf{x}, \mathbf{y}, t) \tag{2.14}$$

where **x** is a vector of states, that is variables which completely describe the state of the system, **y** is a vector of system inputs, and t is time.

2.1 Modal Analysis

For a linear system, equation 2.14 can be written as

$$\frac{d\mathbf{x}}{dt} = \mathbf{Ax} + \mathbf{Bu} \tag{2.15}$$

where **x** is a vector of system states
A is a square matrix of dimension equal to the number of states
B is a matrix which defines the proportion of each input that is applied to each state equation
u is a vector of system inputs

The system's output is generally expressed as

$$\mathbf{y} = \mathbf{Cx} + \mathbf{Du} \tag{2.16}$$

y is a vector of system outputs
C is the output matrix
D is the feed forward matrix

The solution to 2.15 is of the form

$$\mathbf{x} = \sum_{i=1}^{n} u_i z_i \tag{2.17}$$

where

u_i is the i^{th} right eigenvector of **A**
z_i is the i^{th} mode

From 2.17 it can be seen that a right eigenvector defines the relative proportion of its associated mode in the system's states. The right eigenvector is often called the mode shape.

Each mode satisfies a linear differential equation of the form

$$\frac{dz_i}{dt} = \lambda_i z_i + v_i \mathbf{Bu} \tag{2.18}$$

where

λ_i is the i^{th} eigenvalue of **A**
v_i is the i^{th} left eigenvector of **A**

The i^{th} mode is a scalar function of time which depends on the i^{th} eigenvalue, the i^{th} left eigenvector and the input to the system.

The most general form of the solution is obtained by multiplying both sides of equation 2.18 by $e^{-\lambda_i t}$ to give

$$\begin{aligned}
&e^{-\lambda_i t}\frac{dz_i}{dt} - \lambda_i e^{-\lambda_i t} z_i = e^{-\lambda_i t} v_i \mathbf{Bu} \\
&\frac{d}{dt}\left(e^{-\lambda_i t} z_i\right) = e^{-\lambda_i t} v_i \mathbf{Bu} \\
&e^{-\lambda_i t} z_i(t) - z_i(0) = \int_0^t e^{-\lambda_i \tau} v_i \mathbf{Bu}(\tau)d\tau \\
&z_i(t) = e^{\lambda_i t} z_i(0) + \int_0^t e^{-\lambda_i(\tau - t)} v_i \mathbf{Bu}(\tau)d\tau
\end{aligned} \tag{2.19}$$

An eigenvalue (λ) of a matrix **A**, is defined as satisfying the following equation

$$\det\left[\mathbf{A} - \lambda \mathbf{I}\right] = 0 \tag{2.20}$$

The i^{th} right eigenvector satisfies

$$\mathbf{A}u_i = \lambda_i u_i \tag{2.21}$$

The i^{th} left eigenvector satisfies

$$v_i \mathbf{A} = \lambda_i v_i \tag{2.22}$$

The eigenvectors are not unique: they may be multiplied by any number and still satisfy the defining equations. The left and right eigenvectors for a particular eigenvalue may be scaled so that the product $v_i u_i = 1$. Then, in most systems, pre multiplication of the collection of right eigenvectors by the collection of left eigenvectors gives a unit diagonal matrix, and premultiplication of the state matrix A by the collection of left eigenvectors and post multiplication by the collection of right eigenvectors gives a diagonal matrix of eigenvalues, i.e.,

$$\mathbf{VAU} = \mathbf{\Lambda} \tag{2.23}$$

where

- $\mathbf{\Lambda}$ is a diagonal matrix of the eigenvalues
- $\mathbf{V}$ is a matrix of left eigenvalues
- $\mathbf{U}$ is a matrix of right eigenvalues

The matrices are ordered to be consistent, i.e., the r^{th} row of $\mathbf{V}$, and the r^{th} column of $\mathbf{U}$ are the left and right eigenvectors associated with the eigenvalue that is the r^{th} diagonal of $\mathbf{\Lambda}$.

To form a state space model for the mass/spring system of equation 2.4, the two states may be taken as the change in position of the mass from equilibrium and the speed of the mass. The state equations are

$$\frac{d}{dt}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{K}{M} & -\frac{D}{M} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \tag{2.24}$$

the eigenvalues satisfy

$$\begin{aligned}
&\det\begin{bmatrix} -\lambda & 1 \\ -\frac{K}{M} & -\frac{D}{M} - \lambda \end{bmatrix} = 0 \\
&\lambda^2 + \frac{D}{M}\lambda + \frac{K}{M} = 0 \\
&\lambda = -\frac{D}{2M} \pm \sqrt{\left(\left(\frac{D}{2M}\right)^2 - \frac{K}{M}\right)}
\end{aligned} \tag{2.25}$$

If $\frac{K}{M} > \frac{D^2}{4M^2}$, the eigenvalues are complex. The oscillation has the form $Ae^{\lambda t} + A^* e^{\lambda^* t}$.

Second order oscillatory systems, such as this are often put into the standard form

$$\lambda^2 + 2\zeta\omega_o\lambda + \omega_o^2 = 0 \tag{2.26}$$

where

ζ is called the damping ratio, and ω_o is called the undamped natural angular frequency.

The solution of equation 2.26 is

$$\lambda = -\zeta\omega_o \pm i\omega_o\sqrt{(1-\zeta^2)} \tag{2.27}$$

if $\zeta < 1$.

For the mass/spring system

$$\begin{aligned} \omega_o^2 &= \frac{K}{M} \\ \zeta &= \frac{D}{2M\omega_o} \end{aligned} \tag{2.28}$$

Oscillations in power systems do not need to be heavily damped, and a damping ratio of greater than 0.05 is normally satisfactory. The step response of a second order system with damping ratio 0.05 and undamped natural angular frequency of 1 is shown in Figure 1.

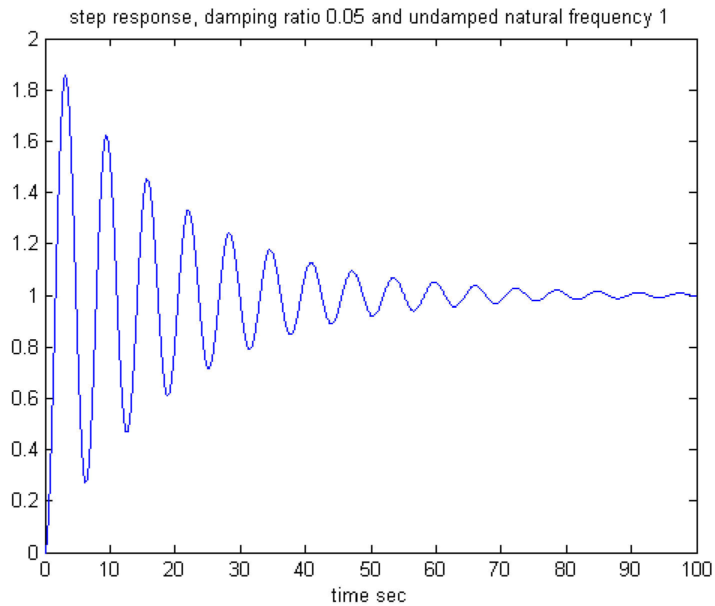

Figure 1 Step response of a second order system

Some systems have a number of equal eigenvalues. When the equal eigenvalues correspond physically to a series combination, the system is said to have nonlinear divisors. Using the rules above would give two equal eigenvectors and the inverse of the eigenvector matrix would be singular. A simple example of such a system is shown in Figure 2.

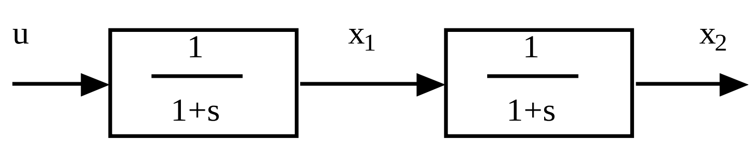

Figure 2 Simple system with equal eigenvalues and non-linear divisors

The state equations for this system are

$$\frac{d}{dt}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u \tag{2.29}$$

The two eigenvalues are equal to -1. Both eigenvectors are equal, i.e.,

$$u = \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix} \tag{2.30}$$

A system with non-linear divisors cannot be diagonalized. However, it may be reduced to Jordan canonical form, in which the eigenvalues are on the diagonal, but eigenvalues that are non-linear divisors have unity in the first upper diagonal.

To put equation 2.29 into Jordan canonical form the states are just reordered to give

$$\frac{d}{dt}\begin{bmatrix} x_2 \\ x_1 \end{bmatrix} = \begin{bmatrix} -1 & 1 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} x_2 \\ x_1 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u \tag{2.31}$$

The system of Figure 3 also has two equal eigenvalues.

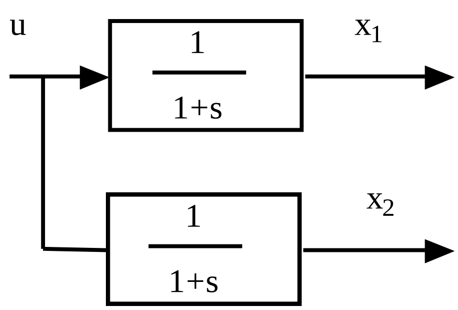

Figure 3 Simple system with equal eigenvalues but linear divisors

The state equations for this system are

$$\frac{d}{dt}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u \tag{2.32}$$

The eigenvalues are equal, but the state matrix is diagonal and the eigenvectors are distinct.

In both cases the two modes have the same fundamental decay (e^{-t}). With a unit step input to the system of Figure 2

$$x_1 = (1 - e^{-t}) \tag{2.33}$$

This is the input into the second block, and from 2.19

$$\begin{aligned} x_2 &= \int_0^t e^{(\tau - t)}(1 - e^{-\tau})d\tau \\ &= 1 - e^{-t} - te^{-t} \end{aligned} \tag{2.34}$$

In the system of Figure 3

$$x_1 = x_2 = 1 - e^{-t} \tag{2.35}$$

In system measurements, equal modes cannot be separately identified. In power system analysis, this can sometimes lead to identification problems, since there are many oscillatory modes having close if not exactly equal frequencies. In analysis, the eigenvectors associated with equal eigenvalues with linear divisors may be combined to give an alternative valid eigenvector.

Figure 4 shows a single line diagram of a simple power system.

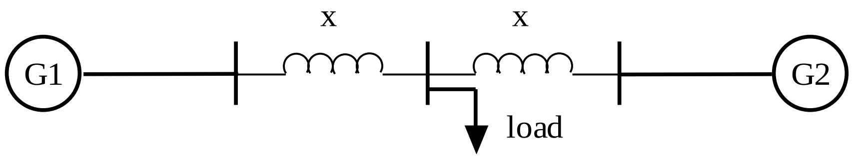

Figure 4 Single line diagram of two generator power system.

The generators have identical loading and parameters. The state space model of the system with classical generators is

$$\frac{d}{dt}\begin{bmatrix}\Delta\delta_1\\ \Delta\omega_1\\ \Delta\delta_2\\ \Delta\omega_2\end{bmatrix}=\begin{bmatrix}0 & 1 & 0 & 0\\ -22.168 & 0 & 22.168 & 0\\ 0 & 0 & 0 & 1\\ 22.168 & 0 & -22.168 & 0\end{bmatrix}\begin{bmatrix}\Delta\delta_1\\ \Delta\omega_1\\ \Delta\delta_2\\ \Delta\omega_2\end{bmatrix} \tag{2.36}$$

The state matrix is singular. There are two zero eigenvalues and a complimentary pair of imaginary eigenvalues. The MATLAB **eig** function gives the following result

```
[u,l]=eig(a); l = diag(l)

l =

   0.0000 + 6.7258i
   0.0000 - 6.7258i
  -0.0000
        0
```

The one zero eigenvalue is associated with the sum of the generator angle columns in the state matrix being zero. If this sum is not zero, it implies that the initializing load flow has not converged with sufficient accuracy. The other effectively zero eigenvalue is associated with the fact that in this system the rate of change of generator speed is proportional to the difference between the rotor angles.

The eigenvector matrix produced by the MATLAB **eig** function is

```
u =

   0.0000 - 0.1040i   0.0000 + 0.1040i   -0.7071              0.7071
   0.6994             0.6994             -0.0000              0.0000
  -0.0000 + 0.1040i  -0.0000 - 0.1040i   -0.7071              0.7071
  -0.6994 + 0.0000i  -0.6994 - 0.0000i   -0.0000              0.0000
```

The eigenvector matrix is singular since the last two columns are equal to within a constant multiplier.

Using the stsp object’s overloaded version of **eig**, the state matrix is reduced to Jordan canonical form.

```
ld =

        0        1.0000             0                  0
        0        0                  0                  0
        0        0                  0 - 6.7258i        0
        0        0                  0                  0 + 6.7258i
```

The associated eigenvector matrix is

```
u =

   0.7071        0              0.1051i         - 0.1051i
        0        0.7071         0.7071            0.7071
   0.7071        0              0.1051i         + 0.1051i
        0        0.7071        -0.7071          - 0.7071
```

The second zero eigenvalue may be eliminated by adding damping to the generators' shafts. This is done mathematically by adding a negative quantity to the speed diagonals of the state matrix

$$\frac{d}{dt}\begin{bmatrix}\Delta\delta_1\\ \Delta\omega_1\\ \Delta\delta_2\\ \Delta\omega_2\end{bmatrix}=\begin{bmatrix}0 & 1 & 0 & 0\\ -22.168 & -1 & 22.168 & 0\\ 0 & 0 & 0 & 1\\ 22.168 & 0 & -22.168 & -1\end{bmatrix}\begin{bmatrix}\Delta\delta_1\\ \Delta\omega_1\\ \Delta\delta_2\\ \Delta\omega_2\end{bmatrix}$$
$$\frac{dx}{dt}=ax \tag{2.37}$$

```
[u,l]=eig(a)

u =

  -0.0077 - 0.1037i  -0.0077 + 0.1037i  -0.7071             -0.5000
   0.6994             0.6994             0.0000              0.5000
   0.0077 + 0.1037i   0.0077 - 0.1037i  -0.7071             -0.5000
  -0.6994 - 0.0000i  -0.6994 + 0.0000i   0.0000              0.5000

l =

  -0.5000 + 6.7072i        0                  0                   0
        0            -0.5000 - 6.7072i        0                   0
        0                  0            -0.0000                   0
        0                  0                  0             -1.0000
```

It can be seen that the eigenvalues are now distinct and the eigenvector matrix is not singular. The oscillatory modes are damped.

To obtain the above system model, a steady state power flow model of the system was obtained which was accurate to a tolerance of 10^{-15}. In models of larger power systems, it is not usual to use such a small tolerance in the power flow solution; 10^{-4} is a typical tolerance used in power flow analysis of large power system models. For this system, with the initial conditions set by a power flow solved to 10^{-4} tolerance and with no added damping, the MATLAB **eig** function gives

```
[u,l]=eig(a)

u =

0 - 0.1040i         0 + 0.1040i   0.7071             0.7071
0.6994              0.6994        0 + 0.0030i        0 - 0.0030i
0 + 0.1040i         0 - 0.1040i   0.7071             0.7071
-0.6994             -0.6994       0 + 0.0030i        0 - 0.0030i
```

```
l =

        0 + 6.7258i         0                   0                 0
        0                   0 - 6.7258i         0                 0
        0                   0                   0 + 0.0042i       0
        0                   0                   0                 0 - 0.0042i
```

For this system the eigenvector matrix is not singular and the two theoretically zero eigenvalues have been calculated as a pair of imaginary eigenvalues with small magnitude. The other eigenvalues and eigenvectors are identical to those found using the more accurately converged power flow solution.

2.2 Interpreting Modal Analysis

Transients in a linear dynamic system die away following a change provided that the real parts of all eigenvalues are negative. A system in which **all** transients decay is termed a **stable**, while a system in which **any** transient grows is termed **unstable**.

If a power system is unstable, it is effectively inoperative. However, because power systems are nonlinear, a linearized model with an complex eigenvalue having a positive real part may result in system oscillations of constant amplitude limited by the system's nonlinearity. Alternatively, a growing oscillation may cause the system to move to a nonviable operating condition and collapse. In either case the operation of the system is compromised.

Power systems are large dynamic structures, and it is often quite difficult to understand their dynamics fully. The interchange of power between synchronous generators has essentially oscillatory dynamic properties. To operate successfully, power systems have a number of controls. Those associated with the systems generators have a considerable effect on the stability of the natural oscillations between the generators. Modal analysis of a system can be used to understand a particular system's limitations, so that oscillatory instability can be avoided.

In any linear system, a mode's right eigenvector gives the relative amplitude of the mode which would be observed between the system's dynamic states. If an eigenvector coefficient is zero for a particular state, the mode cannot be seen in measurements of that state. The state with the largest eigenvector magnitude will have the largest amplitude of oscillation for that mode.

Observability is used to describe how each system mode appears in the system's outputs. If a mode can be detected in a measurement of a system output it is said to be observable in that output. Observability can be easily determined from a knowledge of the right eigenvector associated with a mode (u_i) and the system's output matrix **C**. If the product of the j^{th} row of **C** and u_i is non-zero, then the i^{th} mode is observable in the j^{th} output.

Controllability is used to describe how a mode responds when the system's inputs are disturbed. If a mode responds to a change a system input, that system is said to be controllable by that input. Controllability can be determined from a knowledge of the system's input matrix **B**. If the product of the i^{th} row of the left eigenvector (v_i) with the j^{th} column of the input matrix is non-zero, then the i^{th} mode is controllable by the j^{th} input.

A mode which is neither observable in an output, nor controllable by an input will remain unchanged when that output is fed back to that input.

2.2.1 Sensitivity

It is often useful to know the sensitivity of eigenvalues to a change in an elements in the state matrix. This sensitivity can be determined using the left and right eigenvectors. The sensitivity of the j^{th} eigenvalue to a change in the element in the state matrix in the r^{th} row and the s^{th} column is

$$\frac{\partial \lambda_j}{\partial A_{rs}} = v_j(r)u_j(s) \tag{2.38}$$

Note: Equation 2.38 is not valid for an eigenvalue which is a non-linear divisor.

A particularly useful sensitivity is that due to changes in the diagonal elements of the state matrix. This is called the participation factor in power system analysis. The sensitivity of j^{th} eigenvalue to a change in the i^{th} diagonal element is

$$p_{ji} = v_j(i)u_j(i) \tag{2.39}$$

In a power system, it may be desirable to add damping to a generator angle oscillation mode. The participation factor associated with the generator speed states will indicate those generators at which the addition of a damping coefficient will modify the mode's damping. If the real part of the speed participation factor for the mode is positive, the addition of damping at that generator will add damping to the mode. If the real part of the participation factor is negative, adding damping at that generator decreases the damping of the mode.

Participation factors are non-dimensional and easy to compute. They are a fast way to determine possible candidate machines for damping control of unstable or lightly stable rotor angle oscillatory modes. In practice damping cannot be added to a generator's shaft directly, however, the speed participation factor remains a good indicator of generators at which damping controls, such as power system stabilizers, may be effective.

The sensitivity of eigenvalues to feedback between a particular input and output may be calculated from the systems residues. Residues are the coefficients in the numerator of the partial fraction expansion of the transfer function between a system's outputs and inputs. The residues may be determined most easily from the Laplace transform of equations 2.16, 2.17 and 2.18.

$$\begin{aligned}
\bar{\mathbf{y}} &= \mathbf{C}\bar{\mathbf{x}} + \mathbf{D}\bar{\mathbf{u}} \\
\bar{\mathbf{x}} &= \sum_{i=1}^{n} u_i \bar{z}_i \\
s\bar{z}_i &= \lambda_i \bar{z}_i + v_i \mathbf{B}\bar{\mathbf{u}} \\
\bar{\mathbf{x}} &= \sum_{i=1}^{n} \frac{u_i v_i \mathbf{B}\bar{\mathbf{u}}}{s-\lambda_i} \\
\bar{\mathbf{y}} &= \left[\sum_{i=1}^{n} \frac{\mathbf{C}u_i v_i \mathbf{B}}{s-\lambda_i} + \mathbf{D}\right]\bar{\mathbf{u}} = \left[\sum_{i=1}^{n} \frac{\mathbf{r_i}}{s-\lambda_i} + \mathbf{D}\right]\bar{\mathbf{u}}
\end{aligned} \tag{2.40}$$

$\mathbf{r_i}$ is the i^{th} residue. Each residue is a matrix with the same dimensions as **D**.

With a single input and output the residues are complex scalars. In such a case consider that the system input is

$$\bar{u} = \Delta k\ \bar{y} \tag{2.41}$$

The eigenvalues are the solution of

$$1 - \left(\sum_{1}^{n_s} \frac{r_i}{s-\lambda_i} + D\right)\Delta k = 0 \tag{2.42}$$

Assume that a solution is

$$s = \lambda_r + \Delta\lambda_r \tag{2.43}$$

Thus

$$1 - \left(\sum_{1}^{n_s} \frac{r_i}{\lambda_r - \lambda_i + \Delta\lambda_r} + D \right) \Delta k = 0 \tag{2.44}$$

As Δk tends to zero, the only nonzero term in the summation is $r_k \dfrac{\Delta k}{\Delta\lambda_r}$. Thus, from 2.44, the sensitivity of the r^{th} eigenvalue to positive feedback between output and input is

$$\frac{\partial\lambda_i}{\partial k} = r_i \tag{2.45}$$

A zero residue indicates that the corresponding eigenvalue is either uncontrollable by that input or unobservable in that output.

3 Modelling Interconnected Power Systems

The single line diagram of the two-area system is shown in Figure 5.

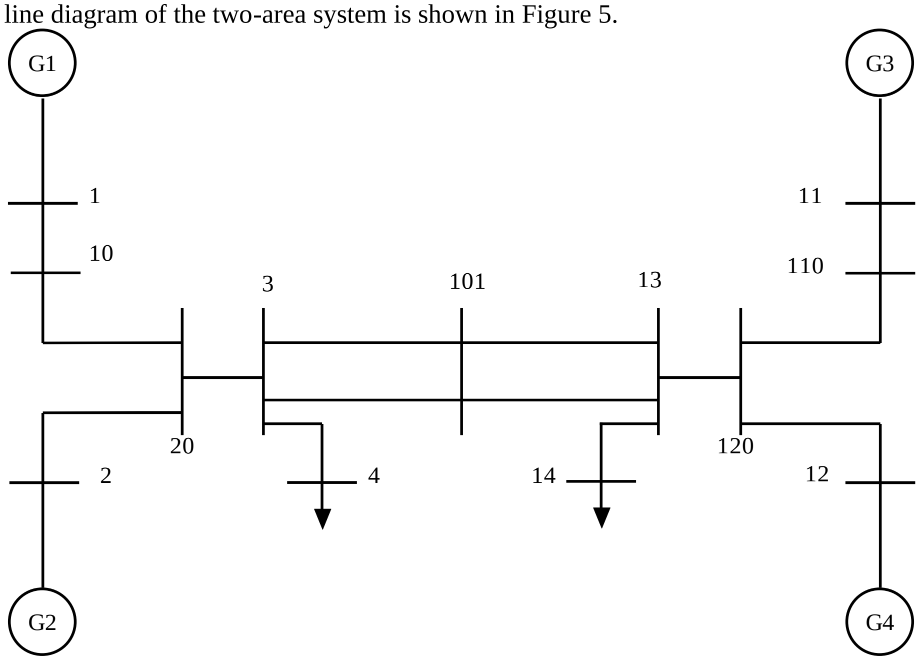

Figure 5 Two area system

The process of developing a system model is quite complex, even for a system as small as the two-area system. Small signal stability programs exists which form the required models of generators and their controls. Here, the MATLAB based toolbox PSTV2 is used. PSTV2 inputs data files, solves the power flow. It initializes the systems dynamic states, and then, by perturbing each state in turn, calculating the corresponding rate of change of state, and dividing by the perturbation, it forms the columns of the state matrix. Using this the full linearized model is formed: the data files are given in Appendix 1.

3.1 Classical Generator Models

The following MATLAB instruction builds the required linear model for the case where the generators are represented by classical models.

```
Two-area test case with em generator models
50% constant current load

inner load flow iterations
     3
 tap iterations
     1

Performing linearization
non-linear loads
disturb generator
disturb generator
disturb generator
disturb generator
calculating eigenvalues and eigenvectors
Current plot held
```

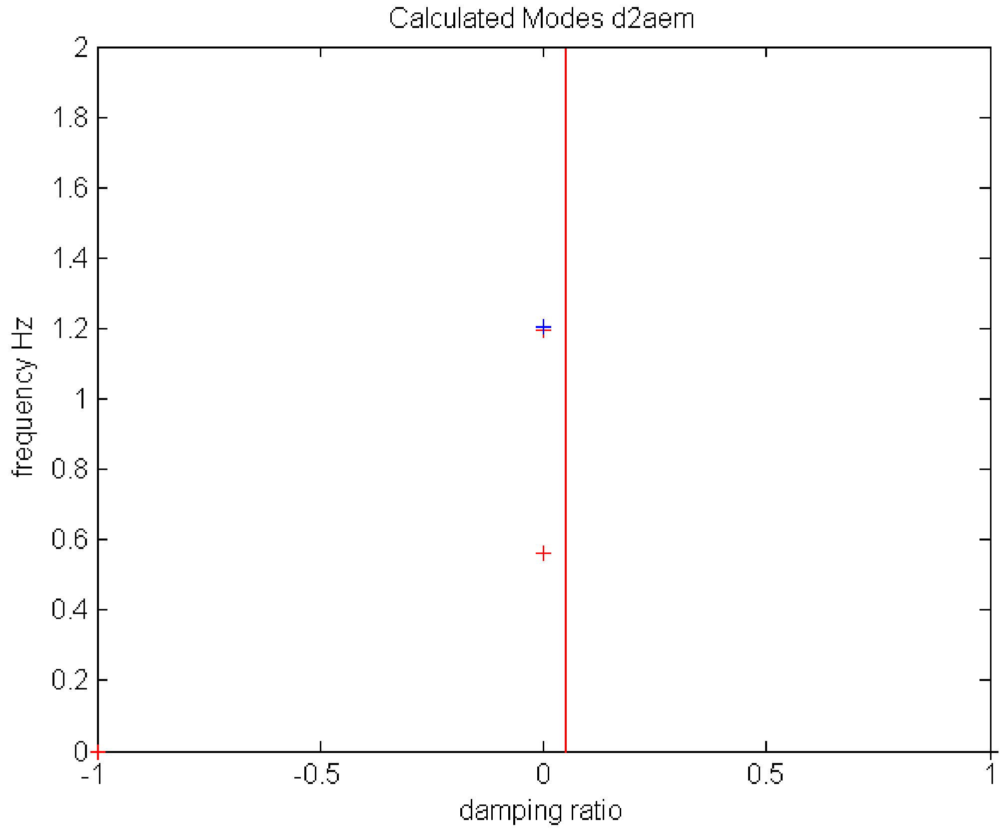

Figure 6 Modes for the two area system with classical generator models

```
[l damp freq]
ans =
    -0.011059                            1                        0
     0.011059                           -1                        0
  2.3498e-012 -     3.5319i -6.6532e-013                    0.56211
  2.3498e-012 +     3.5319i -6.6532e-013                    0.56211
  2.2673e-013 -     7.5092i -3.0194e-014                     1.1951
  2.2673e-013 +     7.5092i -3.0194e-014                     1.1951
 -1.2872e-014 -     7.5746i  1.6993e-015                     1.2055
 -1.2872e-014 +     7.5746i  1.6993e-015                     1.2055
```

Each of the four generators has two dynamic states; the change in rotor angle and speed.

The first two eigenvalues in this list should be zero. That they are nonzero, is due to the accuracy of the power flow solution: the power flow determines the network's initial conditions. In this example it was solved to the normal tolerance of 1e-4 PU, which represents 0.01 MW in this system model. The low frequency oscillatory modes indicate an inter-area oscillation with generators 1 and 2 oscillating against generators 3 and 4. The two pairs of higher frequency oscillatory modes are almost equal. These modes describe the oscillations between the generators in each area,. i.e., generator 1 oscillating against generator 2, and generator 3 oscillating against generator 4.

Eigenvectors are used to deduce the pattern of oscillation.
The eigenvector for the inter area mode is

```
u(ang_idx,4)
ans =
     -0.48004 -4.9069e-013i
     -0.38871 -1.4883e-013i
            1
      0.87625 +2.8258e-013i
```

The two local modes are almost equal. This leads to some problems in finding the source of the mode using the eigenvectors. For exactly equal eigenvalues, which are linear divisors, any combination of eigenvectors for the two equal eigenvalues is also an eigenvector. If we examine the eigenvectors for modes 6 and 8, it seems that the two modes are also inter-area modes with all generators taking part in the oscillations.

```
u(ang_idx,[6 8])
ans =
     -0.76428 -3.7448e-013i      0.44353 +3.2691e-012i
      0.85168 +5.3336e-013i     -0.51412 - 3.615e-012i
      -0.8882 -3.4303e-013i     -0.77895 +6.5218e-014i
            1                          1
```

However, this is misleading. If, the inertia constants of generators 4 and 5 are changed from 6.5s to 5.5 s, the eigenvealues are

```
[(1:NumStates)' l]
ans =
            1                -0.011339
            2                 0.011339
            3                3.201e-012 -     3.7445i
            4                3.201e-012 +     3.7445i
            5               2.5181e-014 -     7.5339i
            6               2.5181e-014 +     7.5339i
            7               2.0427e-013 -     8.2085i
            8               2.0427e-013 +     8.2085i
```

The angle components of the eigenvectors for modes 6 and 8 are now

```
u(ang_idx,[6 8])
ans =
     -0.88323 + 1.228e-014i      0.02117 +2.8936e-013i
            1                   -0.034477 -2.4343e-013i
       -0.123 -5.9565e-014i      -0.81704 -2.4961e-013i
     0.030575 +1.8057e-014i            1
```

The two eigenvectors are now quite ditinct and show mode 6 to be predominantly an oscillation between generators 1 and 2, and mode 8 to be an oscillation between generators 3 and 4. The responses to a positive step change in the mechanical torque at generator 1 and a similar negative change at generator 2 for the case with all inertia constants equal are shown in Figures 7 and 8.

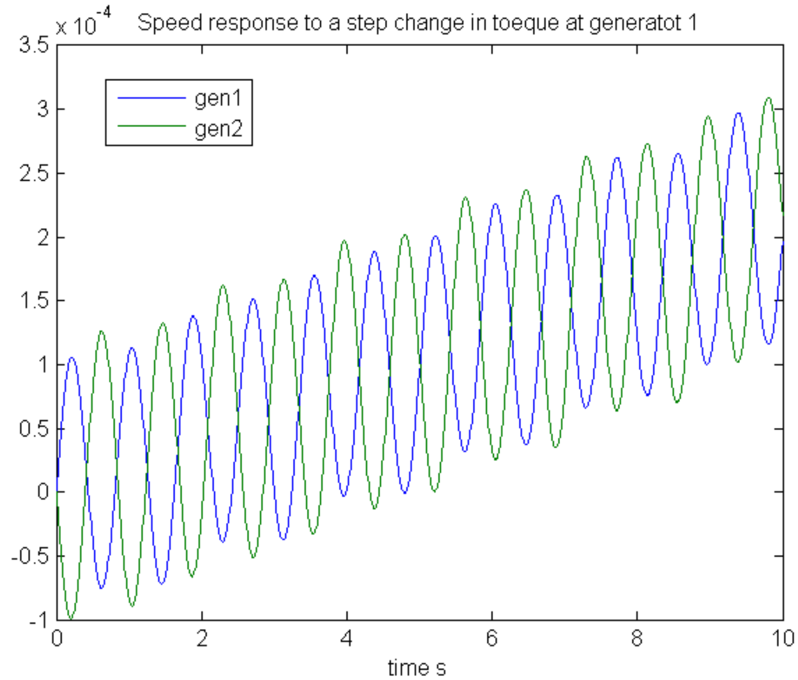

Figure 7 Speed response of generators 1 and 2 to a step change in torque at generator 1

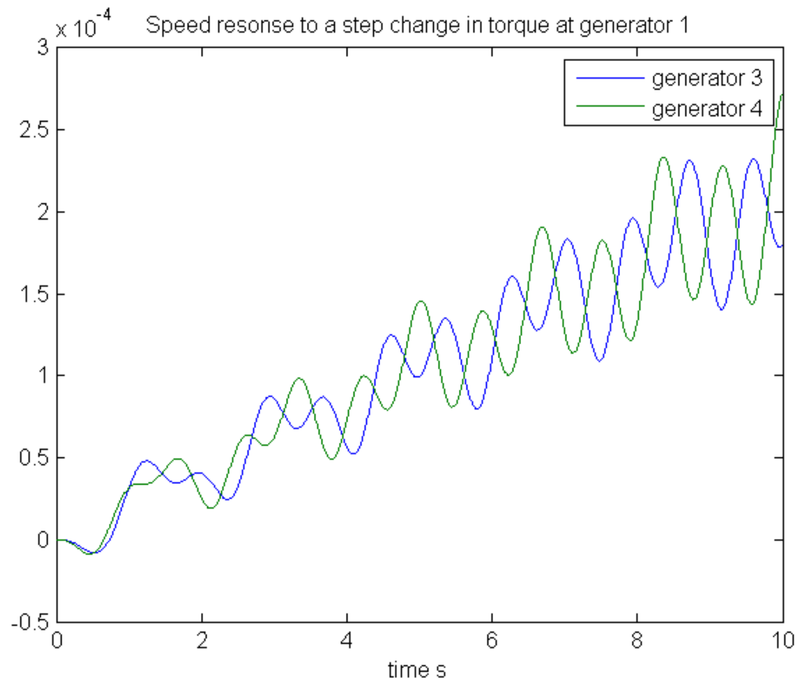

Figure 8 Speeed response of generators 3 and 4 to a step change in torque at generator 1

From Figure 7, it can be seen that the first mode is excited strongly immediately, with generator 1 oscillating against generator 2. The oscillations between generators 3 and 4, shown in Figure 8, take some time to build up. The process is called parametric resonance, and is a function of the closeness of the local modes.

The plots were obtained using

```
b_pm1 = zeros(8,1);
b_pm1(2)=1/2/6.5;
b_pm1(4)=-1/2/6.5;
spmspd12 = stsp(a_mat,b_pm1,c_spd([1 2],:),zeros(2,1));
spmspd34 = stsp(a_mat,b_pm1,c_spd([3 4],:),zeros(2,1));
[r12,t]=stepres(spmspd12,0.01,10,.01);
[r34,t]=stepres(spmspd34,0.01,10,.01);
```

If the inertia constants of generators 3 and 4 are reduced to 4, the responses of the generator speeds are changed drastically, to those shown in Figures 9 and 10.

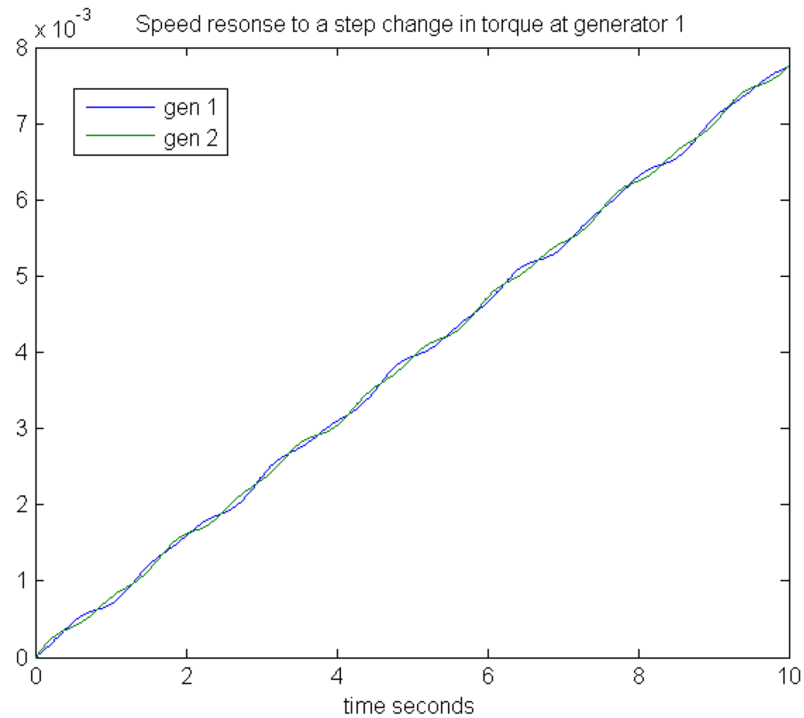

Figure 9 Speeed response of generators 3 and 4 to a step change in torque at generator 1

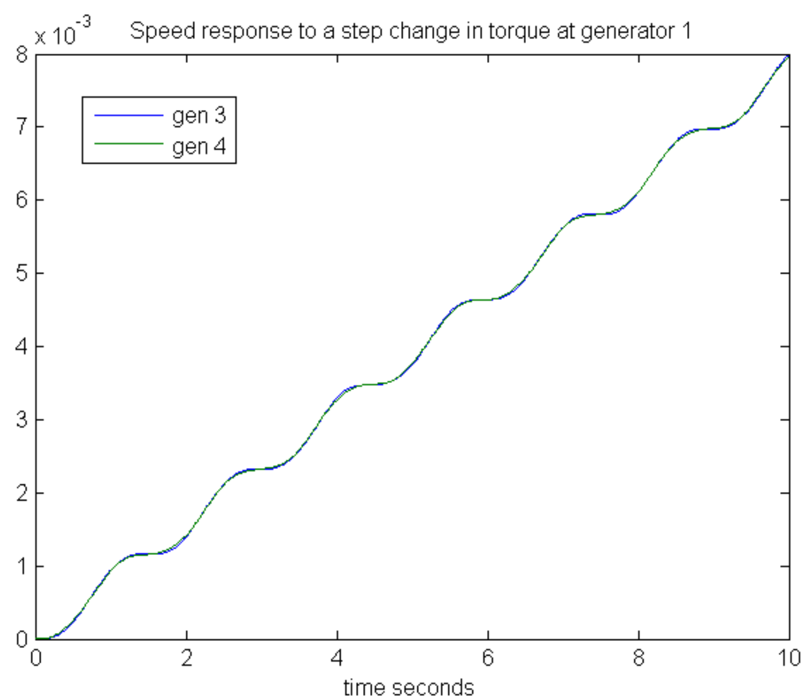

Figure 10 Speeed response of generators 3 and 4 to a step change in torque at generator 1

3.2 Subtransient generator models

With generators modelled including the rotor windings, the main picture is similar, but the response is altered. The eigenvalues are

```
l =
    -0.020016
     0.021786
     -0.05457
     -0.06015
      0.11151
     -0.28072
      -2.2252
      -3.2273
     -0.11522 -      3.4177i
     -0.11522 +      3.4177i
      -4.7467
      -4.7647
     -0.53819 -      6.7387i
     -0.53819 +      6.7387i
     -0.55253 -      6.7925i
     -0.55253 +      6.7925i
      -28.977
      -30.315
      -33.692
      -34.839
      -36.018
      -36.211
      -37.127
      -37.211
```

The eigenvectors associated with the first two modes are

```
u(:,1:2)

          0.5          0.5
 -2.6547e-005  2.8895e-005
  6.0615e-006   5.256e-006
  6.0296e-006  5.0546e-006
  6.2908e-006  7.4248e-006
  8.1913e-006  9.7027e-006
      0.50001      0.50001
 -2.6548e-005  2.8896e-005
 -4.3035e-006  -7.133e-006
 -4.2589e-006 -7.2135e-006
  2.9867e-006   3.478e-006
  3.8889e-006  4.5451e-006
      0.49999      0.49999
 -2.6547e-005  2.8895e-005
  8.2052e-006  4.3556e-006
    8.12e-006  4.4048e-006
  6.9556e-006  6.0702e-006
  9.0569e-006  7.9325e-006
      0.50001          0.5
 -2.6548e-005  2.8895e-005
 -7.2778e-006 -1.7389e-006
 -7.2023e-006 -1.7585e-006
  2.1139e-006  4.5802e-006
  2.7525e-006  5.9854e-006
```

Participation factors give the sensitivity an eigenvalue to a change in a diagonal element of the state matrix.

The angle participation vectors for the first three modes are

```
a abs(p(ang_idx,1:3))
ans =
      0.29491    0.0063559    0.0038176
       0.2913    0.0028171    0.0053215
      0.21108    0.0065771   0.00065912
      0.20275     0.006269  2.8359e-005
```

The effect of this positive real mode may be seen in a transient simulation: a three phase fault at bus 3, normally cleared. The voltage response at bus 3 is shown in Figure 11.

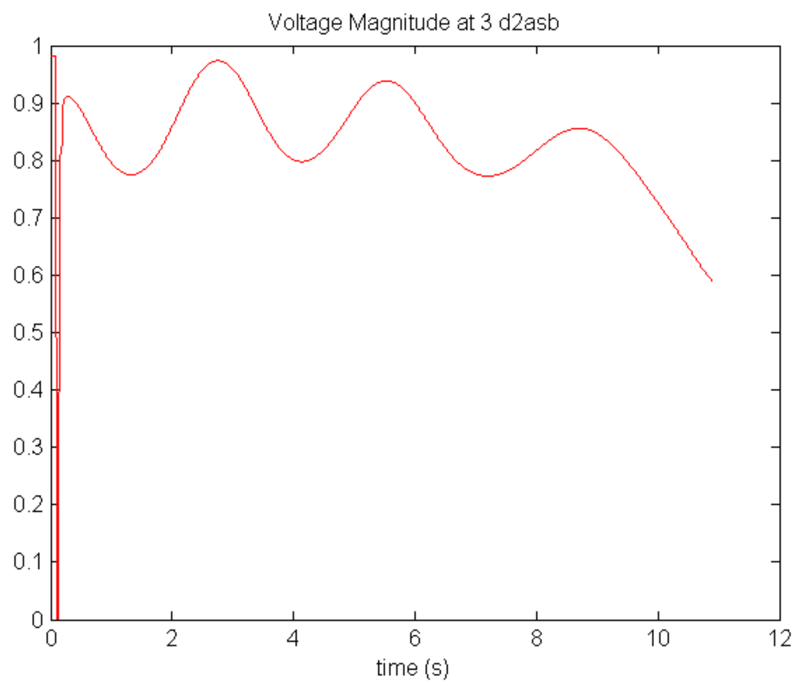

Figure 11 Response of bus 3 voltage magnitude to a normally clear three phase fault

3.3 Exciters and governors on all generators

With static exciters and thermal turbines and governors on all units, the inter-area mode is unstable.

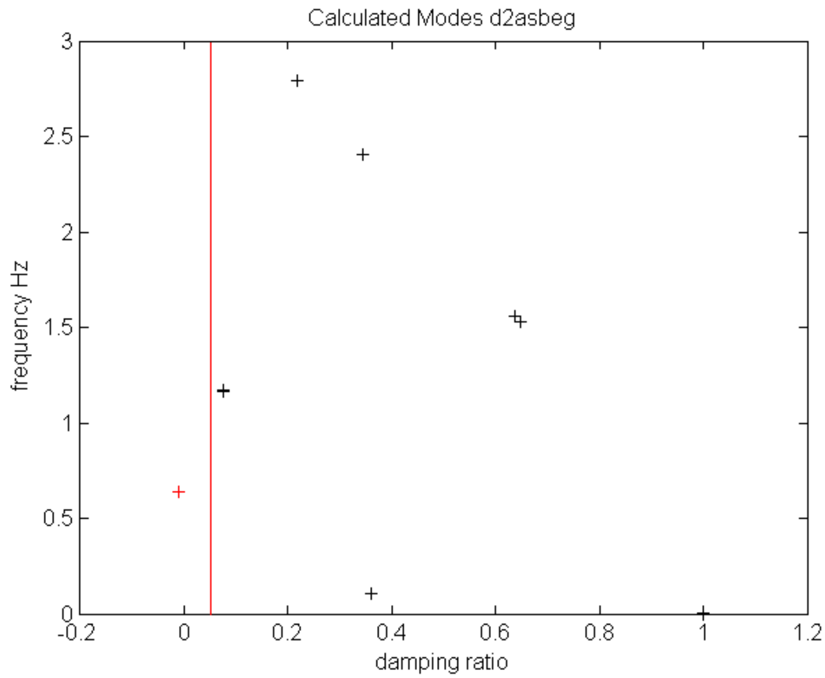

Figure 12 Modes for two-area system with exciters and governors

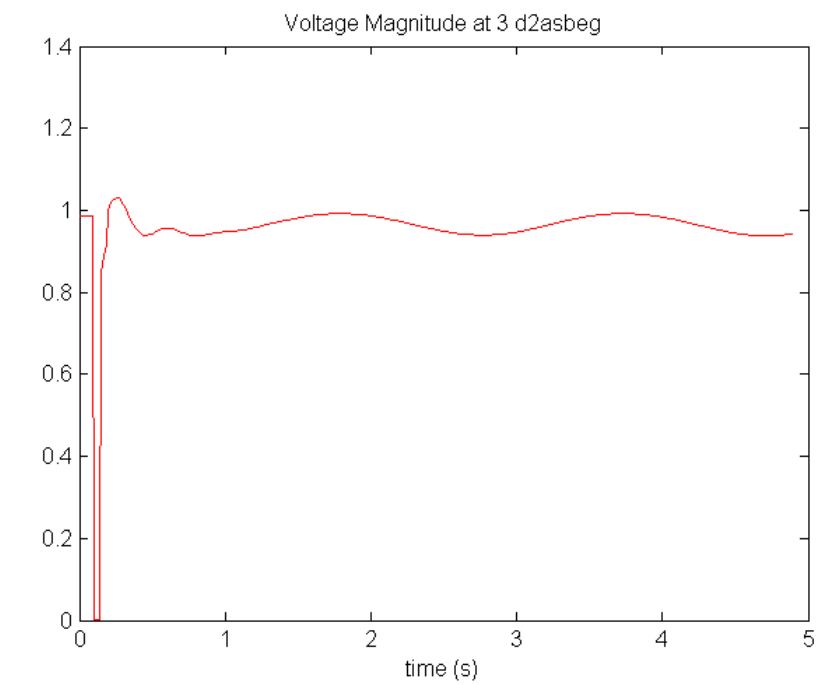

Figure 13 Response to a thrre phase fault

The response to a three phase fault shows the unstable interarea oscillation.

4 Feedback Control

Feedback control of a power system's synchronous generators is essential to the robustness of the system. There are two main control loops on each generator; voltage control through the generator's exciter, and speed control thorough the generator's prime mover. Fast acting voltage control helps a generator to remain in synchronism with the rest of the system following faults. Speed control maintains the system's frequency as the load on the system changes. The speed control is normally quite slow to act, and most oscillatory instabilities in power systems are associated with excitation voltage control.

4.1 Voltage Control

To see the effect of a fast exciter at generator 1, a state space model of the d2asb system with input equal to the field voltage of generator 1 and output equal to the magnitude of bus 1 is constructed. A root locus with the gain of the negative feedback of the output to the input, through a transfer function $\frac{1}{1+s0.05}$ shows how a fast exciter will affect the modes.

The state space model for investigating the effect of voltage feedback is obtained as follows:

```
b_efd = zeros(NumStates,1);
b_efd(3) = 0.125;
sefdvm1 = stsp(a_mat,b_efd,c_v(1,:),0);
sexc = lag_stsp(1,.05);
rlexc1 = rtlocus(sefdvm1,-sexc,0,1,500);
plot(l,'k+')
hold
Current plot held
lz = zeros(sefdvm1);
plot(lz,'ko')
axis([-10 1 0 10])
plot(rlexc1,'k.')
labxyarg
dr_plot(0,20,0.05,'k');
grid
plot(rlexc1(:,101),'r*')
```

The poles of the root locus are just the eigenvalues of the system without controls.
The zeros are obtained using the stsp function zeros.

```
lz    =  zeros(sefdvm1)
lz =
     0.015195
    -0.016743
     -0.04921
     -0.06389
     0.072315
      -2.7696
      -3.3616
  -0.00096585 -     3.7346i
  -0.00096585 +     3.7346i
      -4.3979
       -4.756
     -0.53979 -     6.7815i
     -0.53979 +     6.7815i
     -0.60326 -     7.0932i
     -0.60326 +     7.0932i
      -29.444
```

```
-30.515
-34.081
-36.108
-36.222 -    0.20931i
-36.222 +    0.20931i
-37.192
-66.667
    Inf
    Inf
```

There are two zeros at infinity, and two theoretically at zero. The two zeros at zero imply that the open loop zero eigenvalues are either not observable or not controllable with this input/output pair.

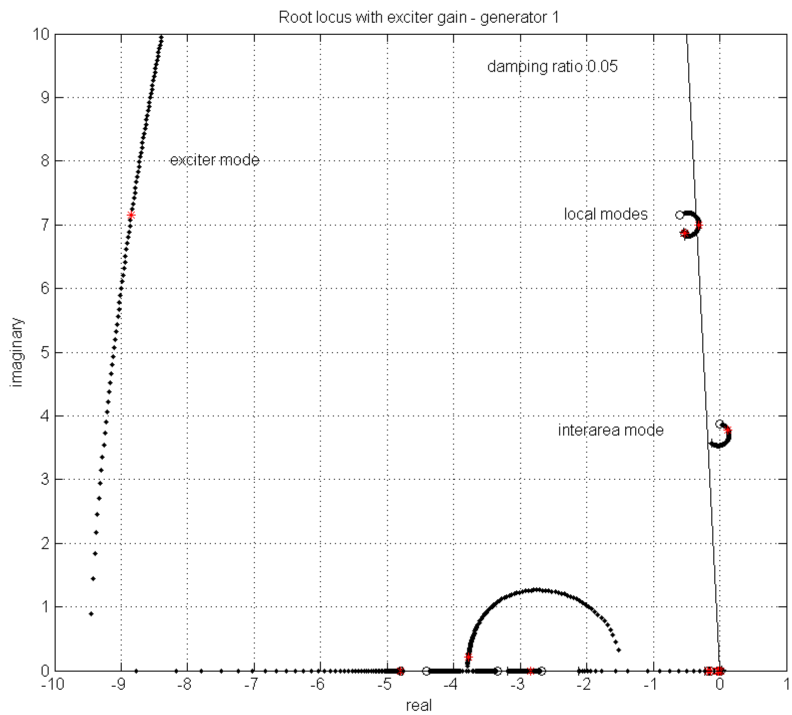

Figure 14 Root locus with exciter gain generator 1

The root locus is shown in Figure 14. The damping of the electromechanical modes associated with generator 1 are reduced by the negative feedback. At a gain of 100, shown by * the eigenvalues are:

```
rlexc1(:,101)
   0.019115
    -0.0225
   -0.03611
   -0.13887
   -0.17772
    -2.8451
    0.12564 -      3.777i
    0.12564 +      3.777i
    -3.7801 -    0.20878i
    -3.7801 +    0.20878i
    -4.8068
   -0.52287 -     6.8736i
   -0.52287 +     6.8736i
   -0.31439 -     6.9953i
```

```
-0.31439 +     6.9953i
 -8.8446 -     7.1607i
 -8.8446 +     7.1607i
 -29.064
  -30.36
 -34.027
 -35.897 -   0.090064i
 -35.897 +   0.090064i
 -36.187
 -37.191
 -38.639
```

The effect of the negative feedback is to destabilize the inter-area mode , to reduce the damping of the local mode associated with oscillations between generator 1 and 2 , and introduce an oscillatory exciter mode. The local mode associated with oscillations between generators 3 and 4 is unchanged. Figure 16 shows that only one local mode is modified by the exciter on generator 1.

There is an oscillatory mode introduced by the voltage feedback, which is normally termed an exciter mode. The damping of the exciter mode reduces as the exciter gain increases, but, in this system, it remains adequately stable even for very high feedback gain.

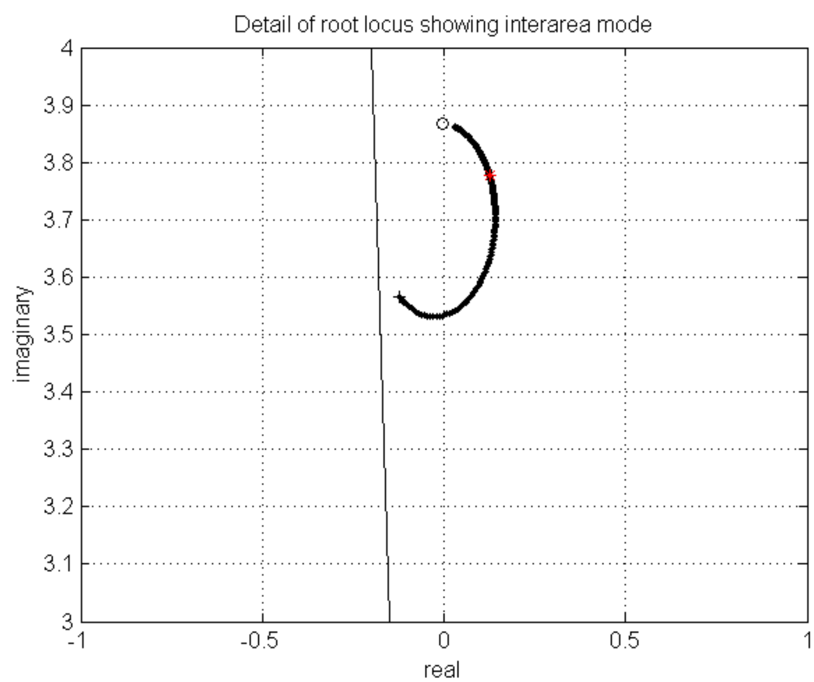

Figure 15 Detail of root locus showing inter-area mode

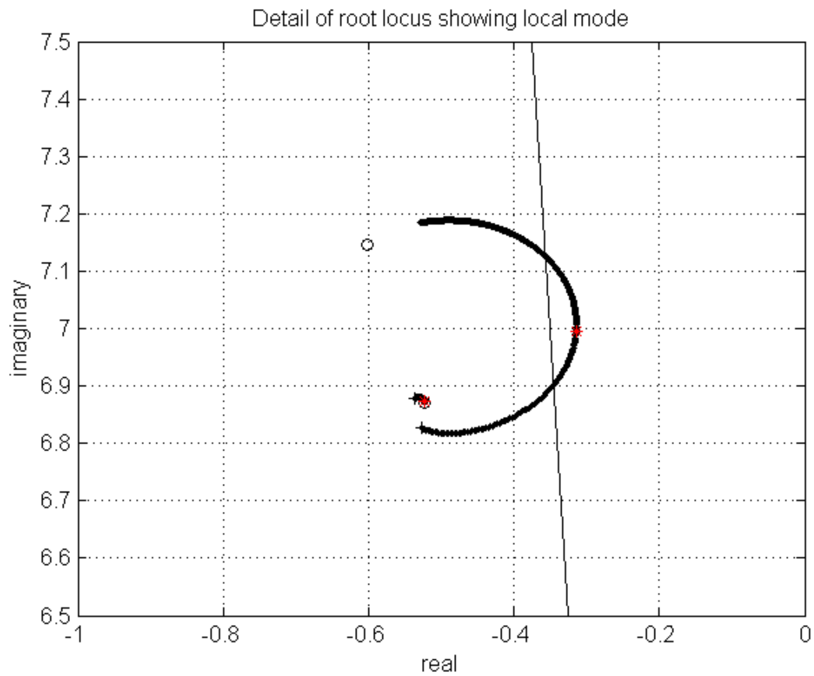

Figure 16 Detail of root locus showing local modes

The residues for the voltage feedback are obtained using

```
rsc = residue(sefdvm1);
rs = zeros(24,1);
for k=1:24;rs(k)=rsc{k};end
[(1:24)' l rs]
1                  -0.024178                 0.00020019
2                   0.026483 -  0.0026733i     0.015113 +    0.03778i
3                   0.026483 +  0.0026733i     0.015113 -    0.03778i
4                   -0.13836                 0.00063465
5                   -0.16298                   0.017579
6                   -0.21895                   0.055966
7                    -2.1279                  -0.023031
8                    -3.1955                 -0.0014622
9                   -0.12149 -      3.565i   -0.0064657 -  0.0034932i
10                  -0.12149 +      3.565i   -0.0064657 +  0.0034932i
11                    -4.769                  0.0042536
12                   -4.8094                 0.00031356
13                  -0.52723 -     6.8265i   -0.0025241 - 0.00044147i
14                  -0.52723 +     6.8265i   -0.0025241 + 0.00044147i
15                  -0.53609 -     6.8787i  -0.00041313 + 0.00056961i
16                  -0.53609 +     6.8787i  -0.00041313 - 0.00056961i
17                   -28.725                 -0.0041899
18                   -30.217                 -0.0014789
19                   -33.757                 -0.0060624
20                   -34.973                  -0.011342
21                   -35.959                 0.00027568
22                   -36.157                -0.00021369
23                   -37.159                 -0.0038982
24                   -37.232                 -0.0049012
```

A detail of the root locus of the inter-area modes with exciter gain, and the sensitivity to negative feedback obtained from the residue are shown in Figure 17.

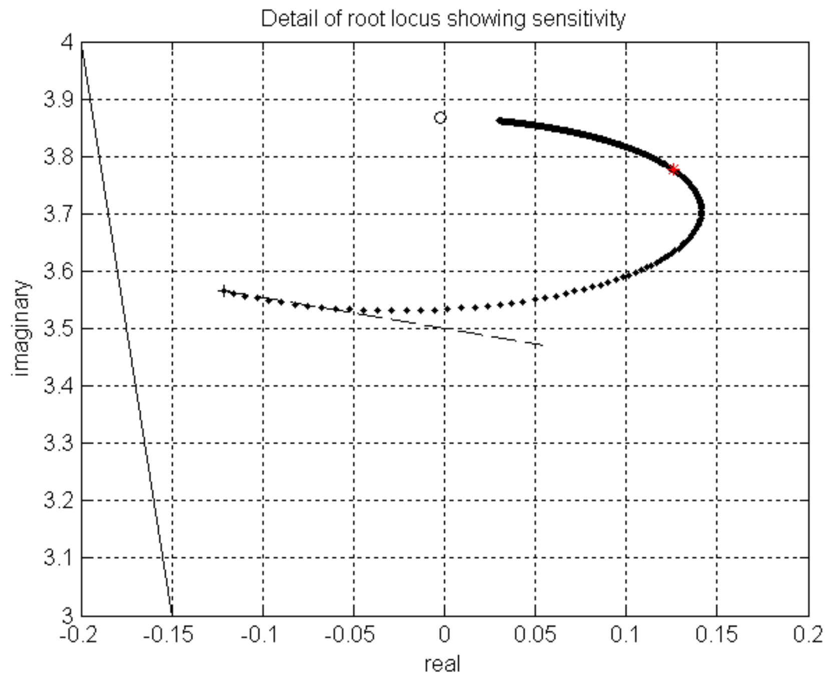

Figure 17 Root locus of inter-area mode and sensitivity calculated from residue

The dashed line was plotted using

```
Sl10 = l(10)-0.2*cos(angle(rs(10)))-i*0.2*sin(angle(rs(10)));
plot([l(10);Sl10],'k--')
```

Each dot represents an increment of gain equal to 1. Thus the sensitivity is valid only for a change in gain of 10.

The inter-area mode tends to a zero as the feedback gain increases.

4.2 Speed Control

There are no governors modelled in this system. As a second example, the effect of adding negative feedback between the generator speed output and the generator mechanical power input is analyzed.

The state space object for this feedback is obtained using

```
b_pm = zeros(NumStates,1);b_pm(2,:)=1/2/6.5;
spmspd1 = stsp(a_mat,b_pm(:,1),c_spd(1,:),0);
```

The zeros for this input and output are obtained using

```
lz = zeros(spmspd1)
           0
      -0.0696 -   0.048778i
      -0.0696 +   0.048778i
     -0.15676
     -0.22213
      -2.5695
      0.14481 -     2.9629i
      0.14481 +     2.9629i
      -3.2089
      -4.8081
      -4.9865
     -0.39187 -     5.3429i
     -0.39187 +     5.3429i
     -0.53207 -     6.8607i
     -0.53207 +     6.8607i
```

```
-28.734
-30.217
-33.757
-34.973
-35.994
-36.196
-37.171
-37.252
    Inf
    Inf
```

The root locus is shown in Figure 18. The symbol * indicates a gain of 25, which corresponds to a 4% droop.

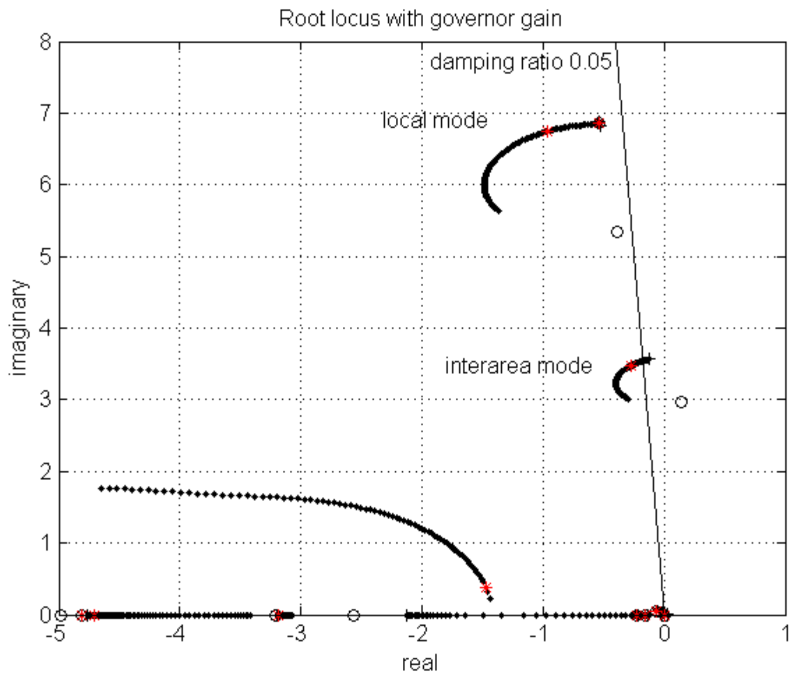

Figure 18 Root locus with negative feedback of generator speed to mechanical power input, * gain of 25

The feedback has made the electromechanical mode associated with generators 1 and 2, and the inter-area mode more stable. It has also modified one of the zero eigenvalues, as can be seen in the detailed locus shown in Figure 19.

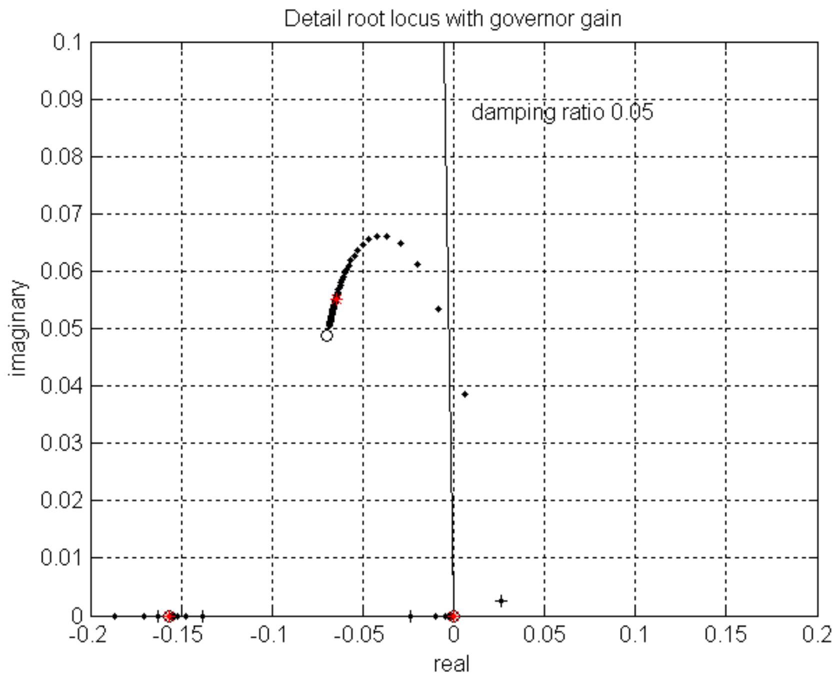

Figure 19 Detail of root locus

One zero eigenvalue still remains. The speed feedback produces a very low frequency, stable oscillatory mode at a gain of 25.

4.3 Combined Voltage and Speed Control

With a static exciter model with a gain of 100, an avr time constant of 0.05 s, and a thermal turbine governor with a droop of 4%, the system's eigenvalues are

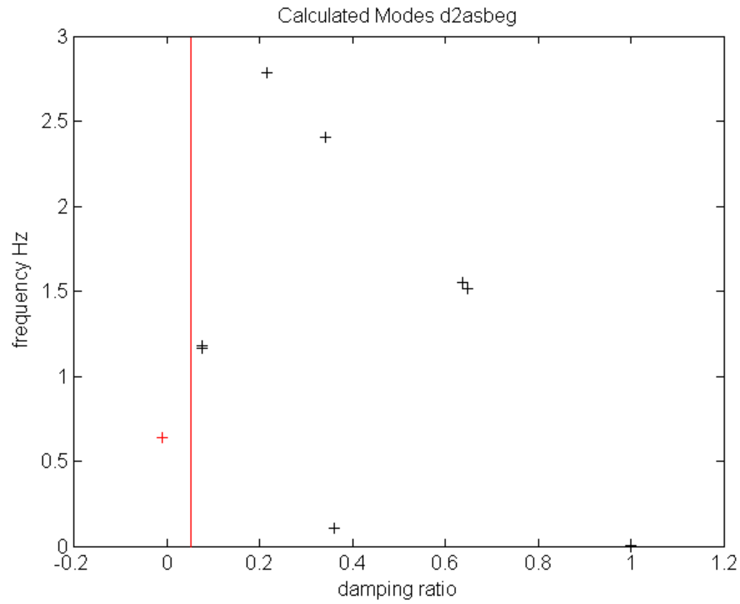

Figure 20 Eigenvalues with controls on generator 1

There is one eigenvalue close to zero; the inter-area mode is unstable; and the local electromechanical modes are stable, but now quite distinct. There are other low frequency and high frequency oscillatory modes. The response of the system to a three phase fault at bus 3, shows the unstable inter-area mode, but does not indicate voltage collapse.

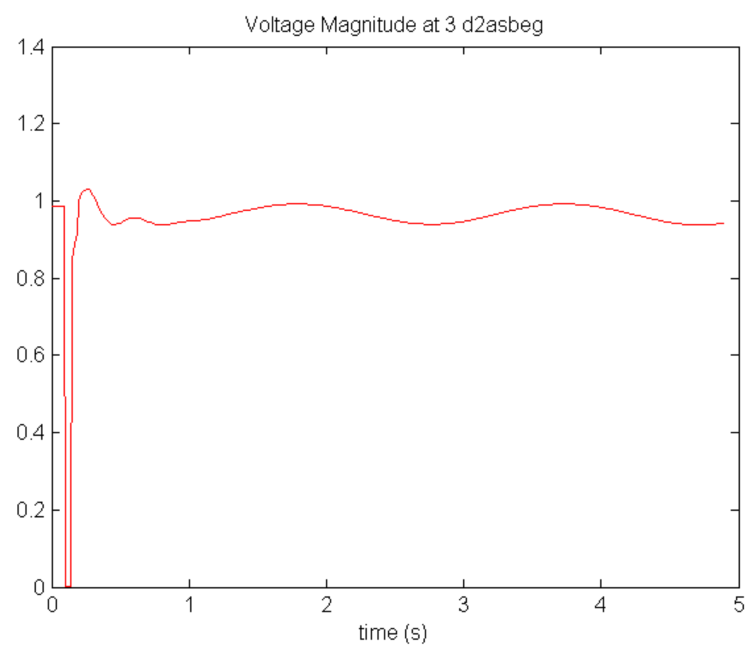

Figure 21 Response of bus 3 voltage magnitude to a three phase fault

4.4 Damping Control

With an unstable inter-area mode, the two-area system with exciter and governor at generator 1 is clearly unsatisfactory. One may stabilize the inter-area mode by reducing the flow between buses 3 and 13, the power transferred between the two areas. However, this is a restriction on the economic operation of the system. The alternative is to use feedback control to stabilize the inter-area oscillation. Power System Stabilizers are commonly used for this purpose. In general, a power system stabilizer takes a generator's speed as input, and its output is fed to the voltage reference input of the generator's excitation system.
In the following, the system model has static exciters and thermal turbine governors on all generators. As shown in Figure 22, the inter-area mode (15,16 +) remains unstable.

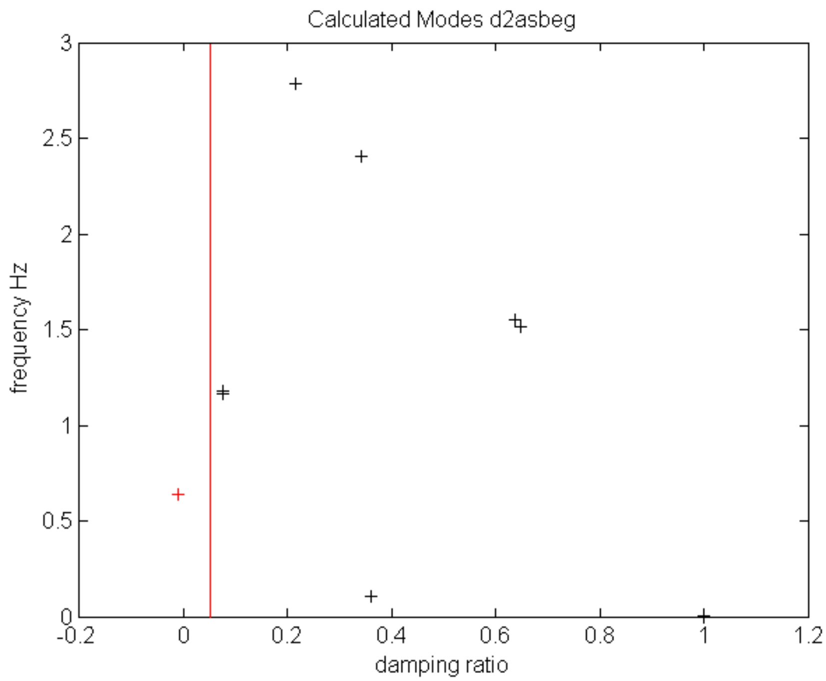

Figure 22 System modes with exciters and governors on all generators

The speed components of the participation factor associated with the mode to be controlled indicate which generators are candidates for power system stabilizer installation.

```
spd_idx = ang_idx+1
     2
    13
    24
    35
p(spd_idx,16)=
      0.12932 -   0.025568i
     0.079142 -  0.0026664i
      0.16841 +    0.01043i
      0.13482 -  0.0099147i
```

This shows that a torque proportional to the negative of speed added to the shafts of generators 3 or 4, would add more damping to the inter-area mode than if it was added to generators 1 or 2. However, to see the general effect of a power system stabilizer on generator 1, its speed is fed back through a high pass filter (washout) to the avr reference input. The washout filter has the form

$$\frac{s10}{1+s10}$$

The state space object for with exciter reference voltage as input and the generator speed as output is

```
svrspd1 = stsp(a_mat,b_vr(:,1),c_spd(1,:),0);
```

The state space object for the 10 second washout is

```
swo = wo_stsp(10);
```

The zeros for the speed to voltage reference feedback are obtained using

```
lz = zeros(svrspd1);
```

The root locus is obtained using

```
rlvrspd1 = rtlocus(svrspd1,swo,0,1,100);
```

The root locus is shown in Figure 23.

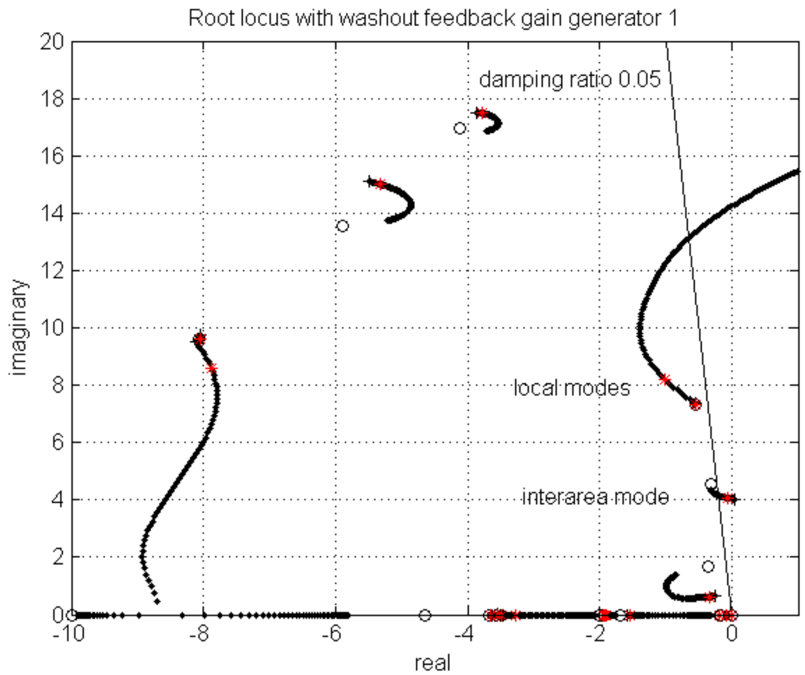

Figure 23 Root locus with feed back gain speed generator 1 to exciter voltage reference

It can be seen that the inter-area and the local modes between generator 1 and generator 2 are modified by the feedback. The frequency of the local mode associated with generator 1 is increased, and its damping is increased at first and then reduced. The inter-area mode's damping is increased, and its frequency is increased slightly. The amount of inter-area mode damping is restricted by a zero. The governor mode's damping is increased. The exciter mode associated the generator 1 is reduced in frequency. A power system stabilizer at generator 1 is clearly feasible to damp the inter-area mode, but it requires to be more carefully designed.

The most practical method to determine the required power system stabilizer phase lead is to try to match the phase lag between the exciter voltage reference and the generator electric power output, when the generator angles and speeds do not change[1]. If the stabilizer exactly compensates this phase, it produces a torque on the generator shaft in antiphase to the generator speed, i.e., it acts as an ideal damper. In MATLAB this model can be obtained by deleting the speed and angle rows of the state matrix and the b matrix associated with the exciter reference, and the speed and angle columns of the state matrix and the c matrix associated with the generator power.

```
a=a_mat;b= b_vr(:,1);c=c_p(1,:);
rot_idx = sort([ang_idx; ang_idx+1]);
a(rot_idx,:)=[];b(rot_idx)=[];
```

---

[1] E.V. Larsen and D.A. Swann, "Applying Power System Stabilizers, Part I: General Concepts, Part II: Performance Objectives and Tuning Concepts, Part II: Practical Considerations", IEEE Trans. on Power Apparatus and Systems, PAS-100, 1981, pp. 3017-3046.

```
a(:,rot_idx)=[];c(rot_idx)=[];
spssi = stsp(a,b,c,0);
[f,ympssi,yapssi]=fr_stsp(spssi,'lin',0.01,.01,2);
plot(f,-yapssi)
```

The negative of the phase lead between the voltage reference input and the electrical power output gives the ideal power system stabilizer phase lead shown in Figure 24.

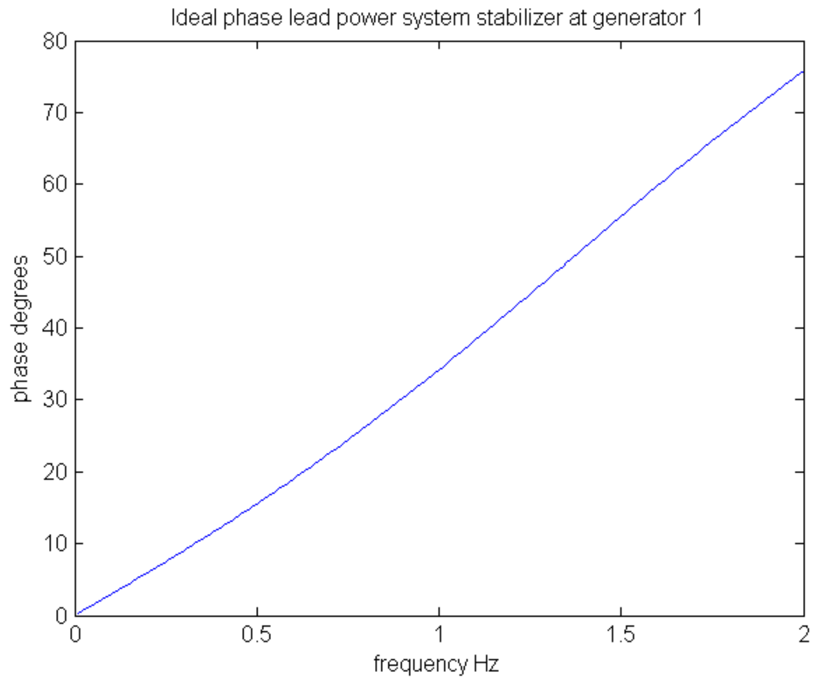

Figure 24 Ideal Power System Stabilizer Phase Lead

Obviously, this cannot be measured. However, Larsen and Swann suggest that the ideal phase lead is close to the negative of the phase lead between the exciter voltage reference and the phase of the generator voltage magnitude. This can be measured on-site quite easily, provided that the system conditions are such that all modes are stable.

For the two area system, if the power flow between area 1 and area 2 is reduced to 0 MW, the inter-area modes are stable.

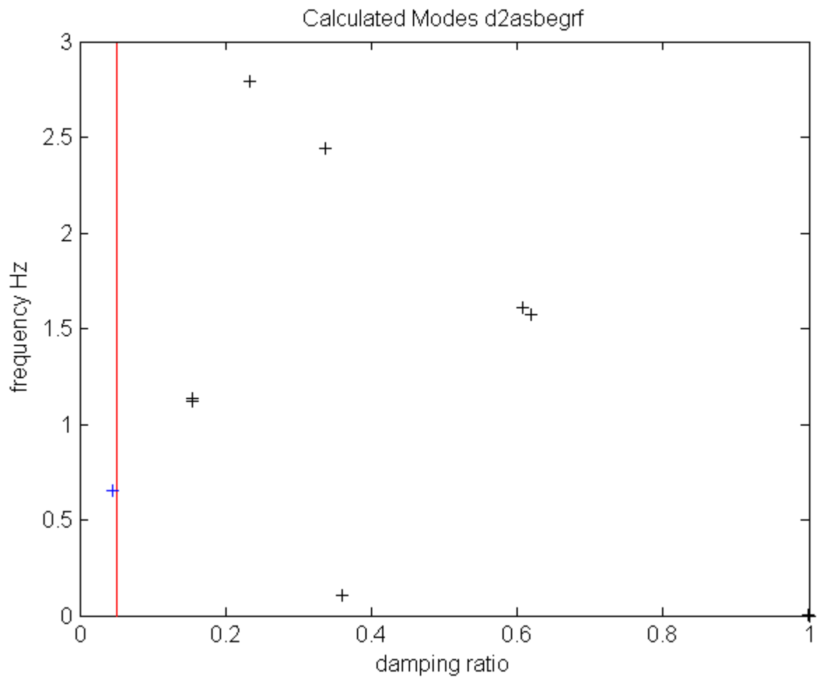

Figure 25 System modes with reduced flow

In this case the negative of the phase between the exciter voltage reference and the terminal voltage magnitude at generator 1 is shown in Figure 26, and compared with the ideal power system stabilizer phase lead for the same operating condition.

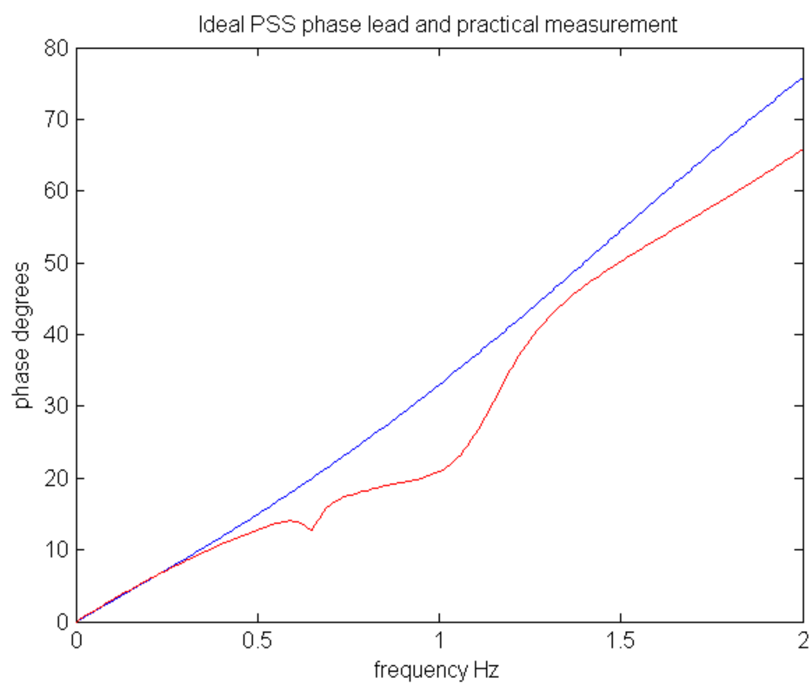

Figure 26 Ideal PSS phase lead (blue) and negative of phase of voltage magnitude (red), generator 1, with inter-area flow reduced to zero

By comparison with Figure 25, it can be seen that the ideal phase lead is not sensitive to operating conditions.

Trial power system stabilizers have the following parameters

Trial Number	T_{wo}	T1	T2	T3	T4
1	10	.08	.02	.08	.02
2	10	.1	.02	.1	.02
3	10	.05	.02	.05	.02
4	10	.05	.01	.05	.01
5	10	.05	.01	.08	.01
6	10	.05	.015	.08	.015
7	10	.05	.02	.08	.015

The trial power system stabilizer phase leads are compared with the ideal phase lead in Figure 27.

The root locus with the gain of a PSS6 is shown in Figure 28. The eigenvalues with a power system stabilizer gains of 10 , 20, and 30 are shown by *. It can be seen, that the stabilizer acts to damp the local mode associated with generators 1 and 2, and the inter-area mode. A line representing a damping ratio of 0.05 is shown in the figure. The exciter mode frequency is increased and its damping is reduced by this power system stabilizer.

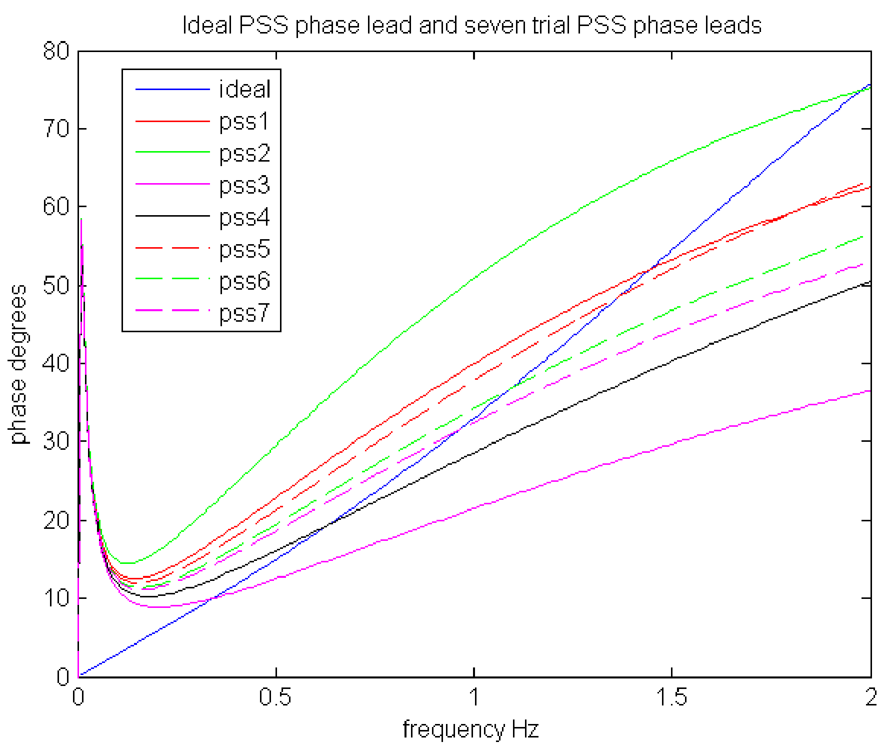

Figure 27 Ideal PSS phase lead and seven trial PSS designs

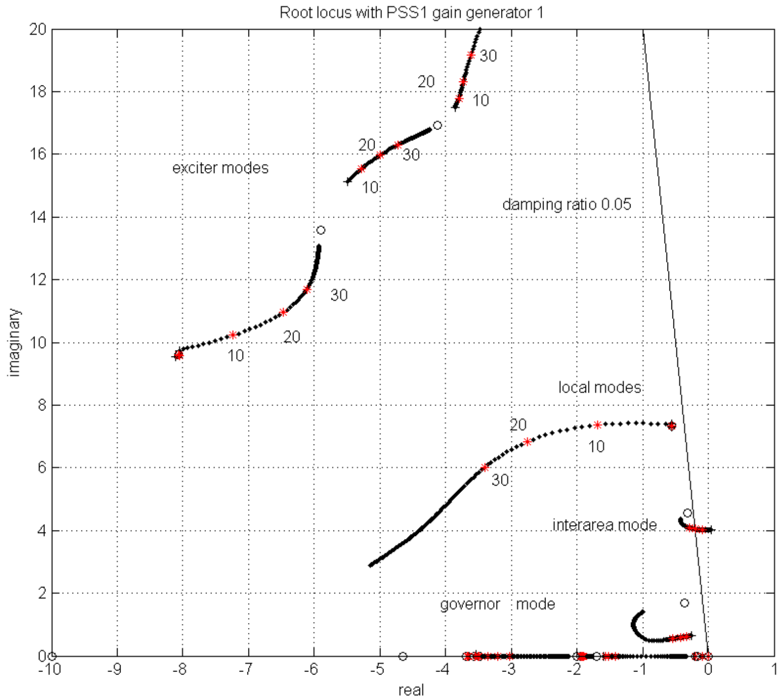

Figure 28 Root locus with gain of power system stabilizer 1

4.5 Power System Stabilizers on all Generators

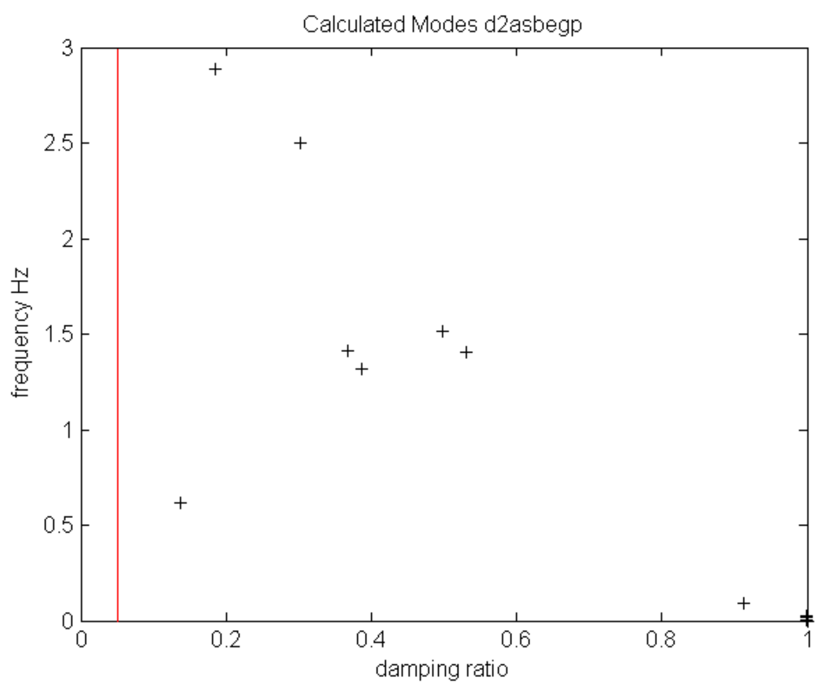

Figure 29 System modes with power system stabilizers on all generators

The system is satisfactorily damped , i.e., all modes have damping ratios greater than 0.05. The voltage magnitude at bus three following a normally cleared three phase fault is shown in Figure 30 .

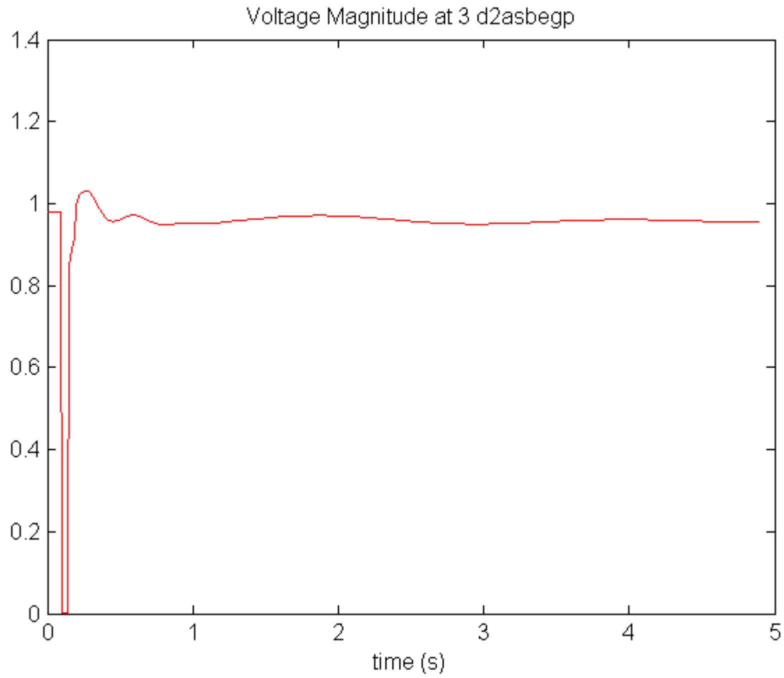

Figure 30 Response of bus 3 terminal voltage to a normally cleared three phase fault

5 System response to small changes

With a linearized system model, response calculations are valid only for small changes which do not violate the linearization of the system's nonlinear model. Responses to small step changes in exciter voltage reference and governor power reference inputs are meaningful, since the results can be compared with measurements made at a generator. The effect of ambient changes in a system's loads on the system can also be simulated correctly using a linearized model of the system.

In PST, the response is obtained using state space models with the input and output chosen appropriately.

The available outputs are the bus voltage magnitudes and angles (radians); the generator speeds, electrical power, field voltage, field current and prime mover torque (PU); SVC suceptance; active and reactive load conductance and suceptance. The available inputs are the exciter reference voltages; the torques applied to the generator shafts (for generators no governors modelled); the governor power reference; the SVC reference; active load modulation reference; reactive load modulation reference.

The effect of a step change of the exciter voltage reference at generator 1 on the voltage at its terminal bus is shown in Figure 31. The calculation is performed as follows.

```
svrvm1 = stsp(a_mat,b_vr(:,1),c_v(1,:),0);
[rvrvm1,t]=stepres(svrvm1,0.05,20,.05);
```

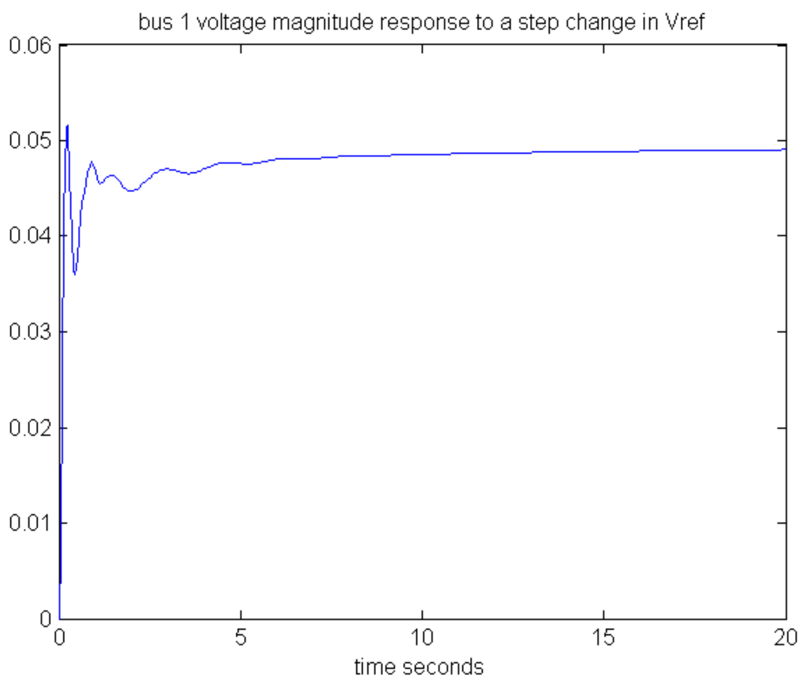

Figure 31 Response of generator 1 terminal voltage to a step input in V_{ref}

The response of the turbine output torque to a change in the governor power reference at generator 1 is shown in Figure 32. The calculation is performed as follows:

```
sprpm1 = stsp(a_mat,b_pr(:,1),c_pm,zeros(4,1));
[rpr1pm,t]=stepres(sprpm1,0.1,100,0.01);
```

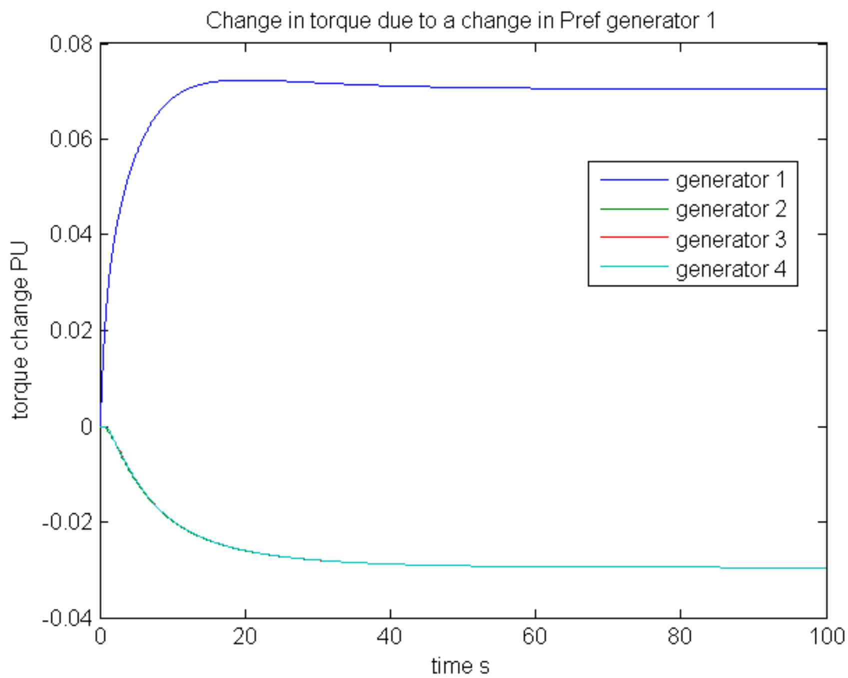

Figure 32 Response of turbine output torque to a change in P_{ref} at generator 1

The response of the voltage magnitude at buses 3 and 13 to changes in the active load at buses 4 and 14 can be examined using the maximum singular value of the voltages as the frequency of the change varies. The load variation capability is enabled by supplying the required data in the input file

```
lmod_con = [...
1 4  100 1 -1 1 0.05;
2 14 100 1 -1 1 0.05;
];
```

This allows modulation of the active load at buses 4 and 14. A 0.02s time constant is specified and the maximum load modulation is ±100 MW.

```
slmmv313=stsp(a_mat,b_lmod,c_v([3 8],:),zeros(2));
[f,ylmv,smnlmv,smxlmv,cnlmv]=fr_mstsp(slmmv313,'log',0.01,1.1,100);
semilogx(f,smxlmv)
```

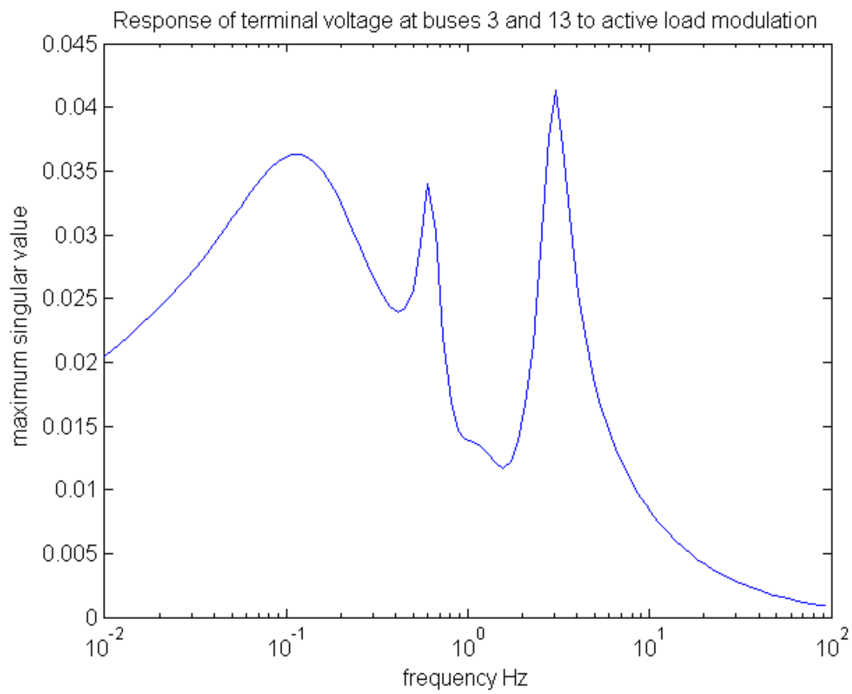

Figure 33 Frequency response of bus voltage magnitude to active load change

The maximum singular value gives the greatest magnitude response in any direction. The modes associated with the governor, inter-area mode and an exciter mode are clearly seen.

The response of the change in frequency at buses 3 and 13 to active load modulation is shown in Figure 34.

The state space object for the change in voltage angle is formed initially and then post multiplied by a differentiator **sd**, and premultiplied by the conversion factor from radians/s to Hz.
The each differentiator has the following transfer function

$$\frac{s}{1+0.01s}$$

```
slmav313=stsp(a_mat,b_lmod,c_ang([3 8],:),zeros(2));
sd = [dif_stsp;dif_stsp];
slmf313 = (1/2/pi).*slmav313.*sd;
[f,ylf,smnlf,smxlf,cnlf]=fr_mstsp(slmf313,'log',0.01,1.1,100);
semilogx(f,smxlf)
```

It can be seen that the governor mode dominates the response. This implies that the system may experience frequency problems if either of the loads had a large pulsating load component.

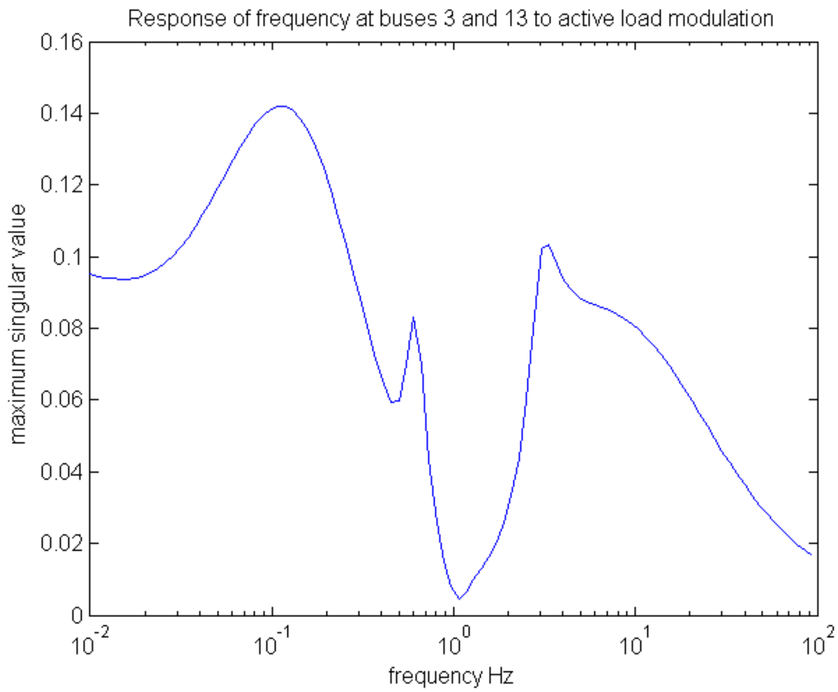

Figure 34 Maximum singular value of the response of frequency at buses 3 and 13 to active load modulation

6 Power System Stabilizer Placement

In the two area system used for the previous examples, it was shown that the speed participation factors associated with the electromechanical modes indicate where power system stabilizers may be placed. In more complex systems, power system stabilizer placement is not so straightforward. Such a system is shown in Figure 35. The system has some interesting properties. It consists of 5 areas. Areas 1, 2 and 3 each contain a single large generator: these areas represent aggregated equivalents. Area 4 is modelled in more detail, but has one equivalent generator in addition to more normally sized generators. Area 5 has the most detailed modelling..

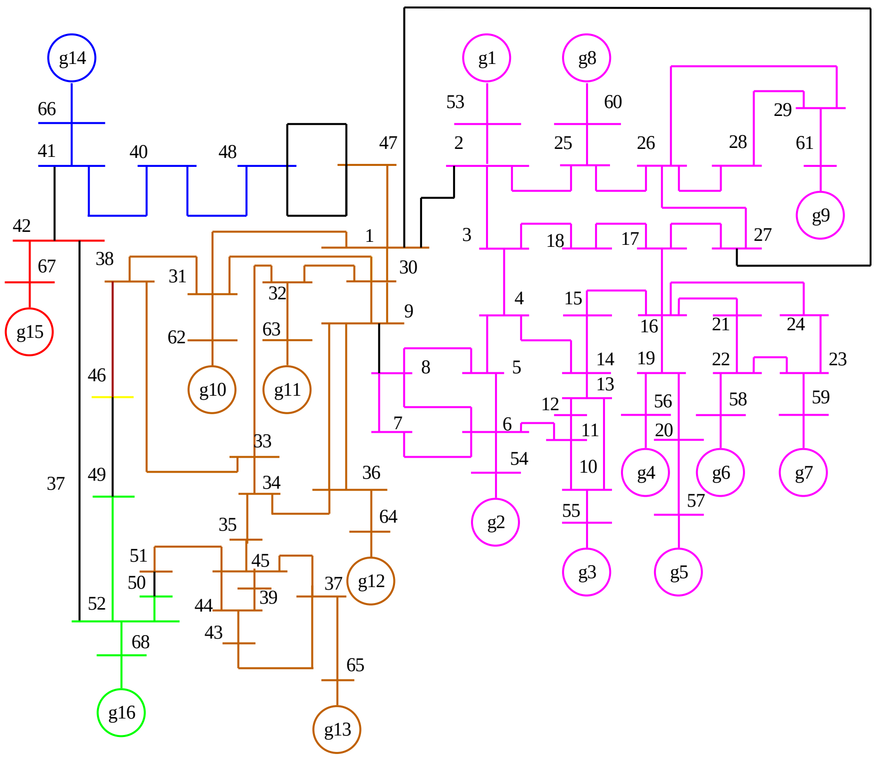

Figure 35 16 Generator System

With static exciters, and turbines and governors modelled on all generators, and no power system stabilizers, the system has unstable electromechanical modes

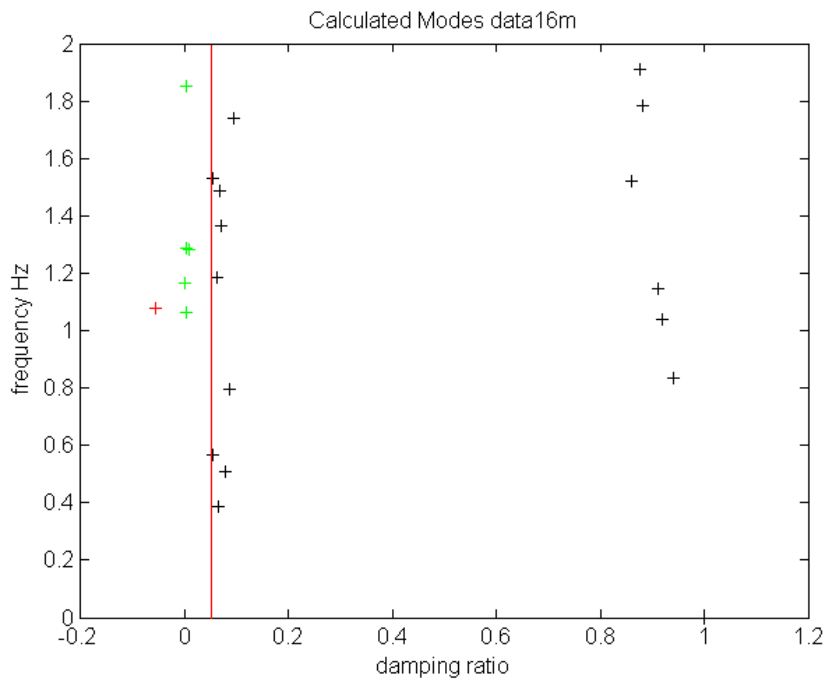

Figure 36 16 generator system with no power system stabilizers

Power system stabilizers may be used to stabilize the system. The real parts of the speed participation factors indicate the modes which may be stabilized by power system stabilizers.
The rotor angle modes are

mode	Eigenvalue	Damping ratio	Frequency
18	-0.15509 + 2.4164i	0.064052	0.38458
20	-0.24816 + 3.1861i	0.077652	0.50709
23	-0.19358 + 3.5519i	0.05442	0.5653
28	-0.42588 + 4.9846i	0.08513	0.79332
31	-0.019345 + 6.6877i	0.0028926	1.0644
33	0.38621 + 6.7784i	-0.056884	1.0788
36	-0.0092545 + 7.3256i	0.0012633	1.1659
38	-0.46374 + 7.4513i	0.062115	1.1859
41	-0.057186 + 8.0716i	0.0070847	1.2846
43	-0.032292 + 8.1019i	0.0039857	1.2895
45	-0.60866 + 8.5831i	0.070736	1.366
48	-0.63688 + 9.3315i	0.068092	1.4852
51	-0.52961 + 9.6156i	0.054995	1.5304
54	-1.0422 + 10.939i	0.094846	1.7409
56	-0.032582 + 11.627i	0.0028024	1.8504

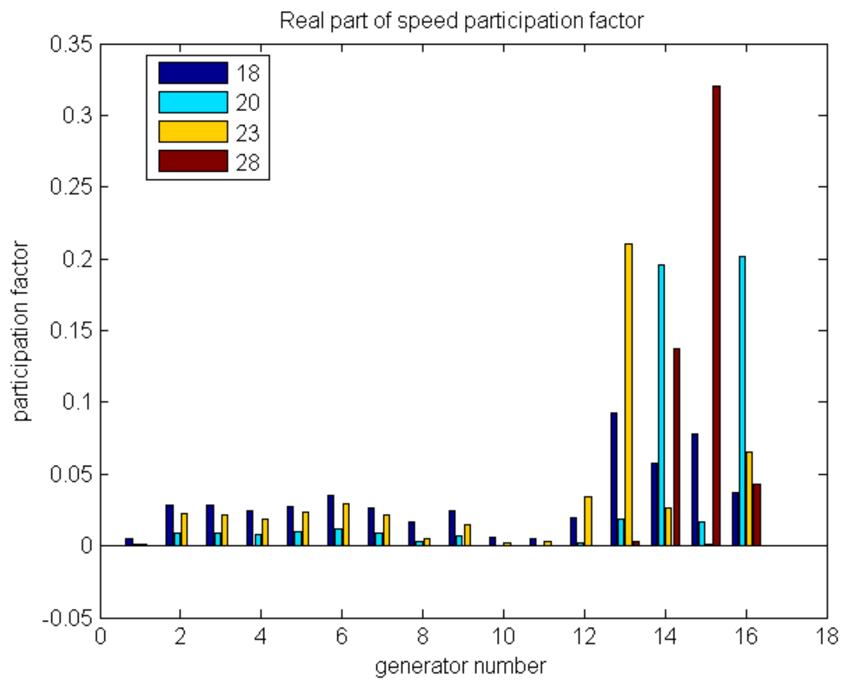

Figure 37 Real part of speed participation factor modes 18, 20, 23 and 28

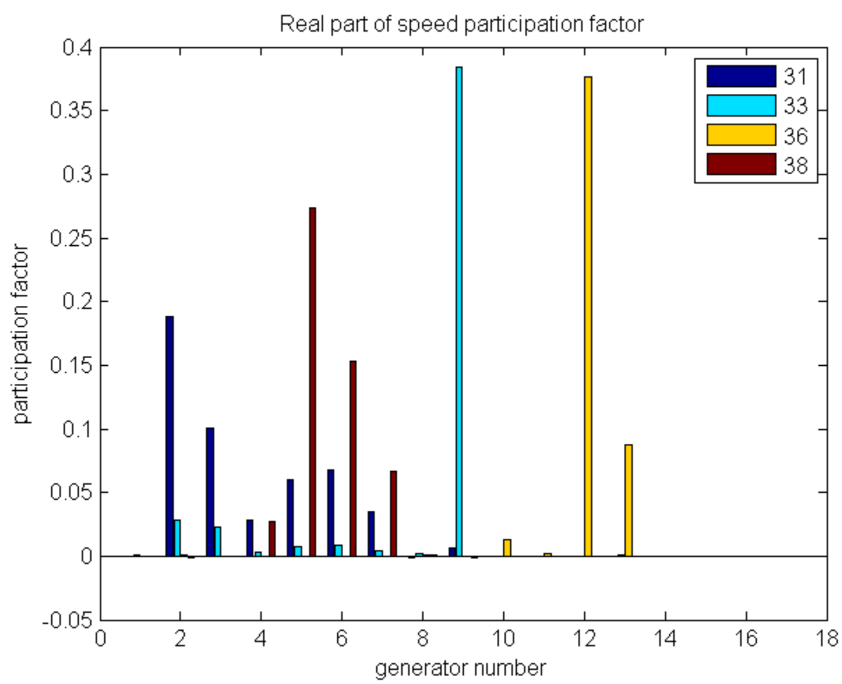

Figure 38 Real part of speed participation factor, modes 31, 33, 36 and 38

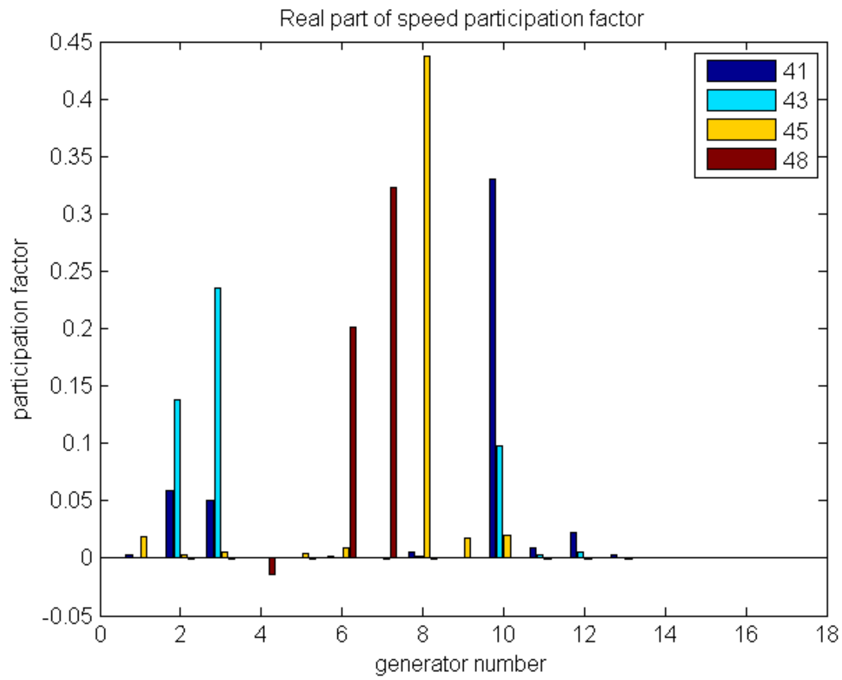

Figure 39 Real part of speed participation factor, modes 40, 43, 45, 48

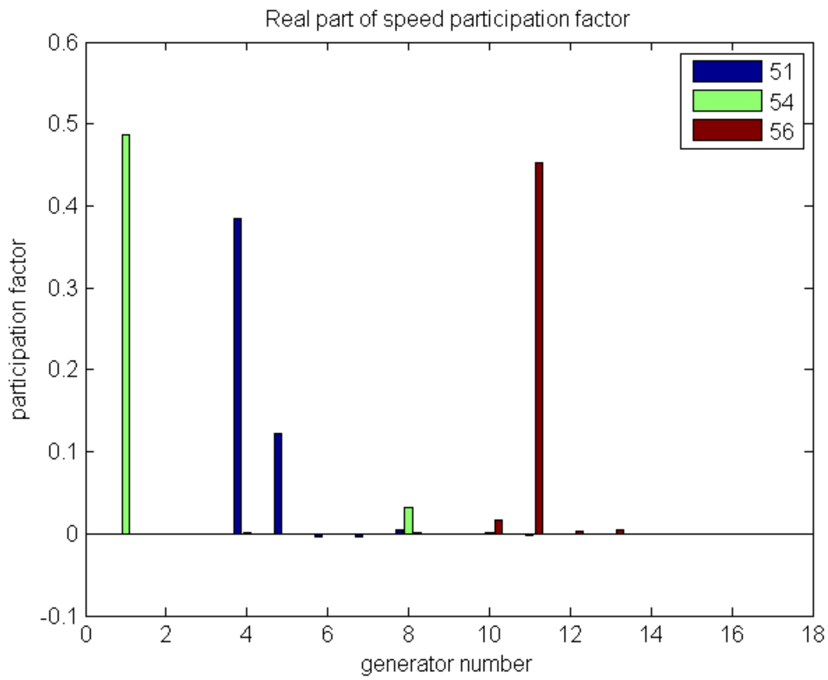

Figure 40 Real part of speed participation factor, modes 51, 54 and 56

It can be seen, from Figures 37 to 40, that the number of generators which participate in an electromechanical mode reduces as the mode's frequency increases. The very low frequency modes are inter-area modes, while the high frequency modes are local modes. The highest frequency mode (mode 56) has the largest real part of speed participation at generator 11. The mode is just stable.,with a damping ratio of 0.0028024. A power system stabilizer at generator 11 will be necessary.

By examination of the bar charts, the generators having the highest participation factor in electromechanical modes are as follows

18	-0.15509 + 2.4164i	13
20	-0.24816 + 3.1861i	14
23	-0.19358 + 3.5519i	13
28	-0.42588 + 4.9846i	15
31	-0.019345 + 6.6877i	2
33	0.38621 + 6.7784i	9
36	-0.0092545 + 7.3256i	12
38	-0.46374 + 7.4513i	5
41	-0.057186 + 8.0716i	10
43	-0.032292 + 8.1019i	3
45	-0.60866 + 8.5831i	8
48	-0.63688 + 9.3315i	7
51	-0.52961 + 9.6156i	4
54	-1.0422 + 10.939i	1
56	-0.032582 + 11.627i	11

6.1 Generator 11

Mode 56 is essentially generator 11 oscillating against the rest of the system. Consequently, this mode may be stabilized by a power system stabilizer at generator 11. The ideal power system stabilizer phase lead for generator 11 together with the phase of three trial power system stabilizers, is shown in Figure 41. The stabilizers' state space objects were formed using

```
spss1 = wo_stsp(10).*ldlg_stsp(1,.02,.1).*ldlg_stsp(1,.02,.1);
spss2 = wo_stsp(10).*ldlg_stsp(1,.02,.08).*ldlg_stsp(1,.01,.05);
spss3 = wo_stsp(10).*ldlg_stsp(1,.03,.08).*ldlg_stsp(1,.01,.05);
spss4 = wo_stsp(10).*ldlg_stsp(1,.02,.05).*ldlg_stsp(1,.02,.05);
```

The root locus with the gain of pss4 is shown in Figure 42. The mode is stabilized satisfactorily with a power system stabilizer gain of 5 (shown by *). No other electromechanical modes are altered. Note that the under compensation of pss4 causes the frequency of the mode to increase, as well as the damping, with inceasing gain.

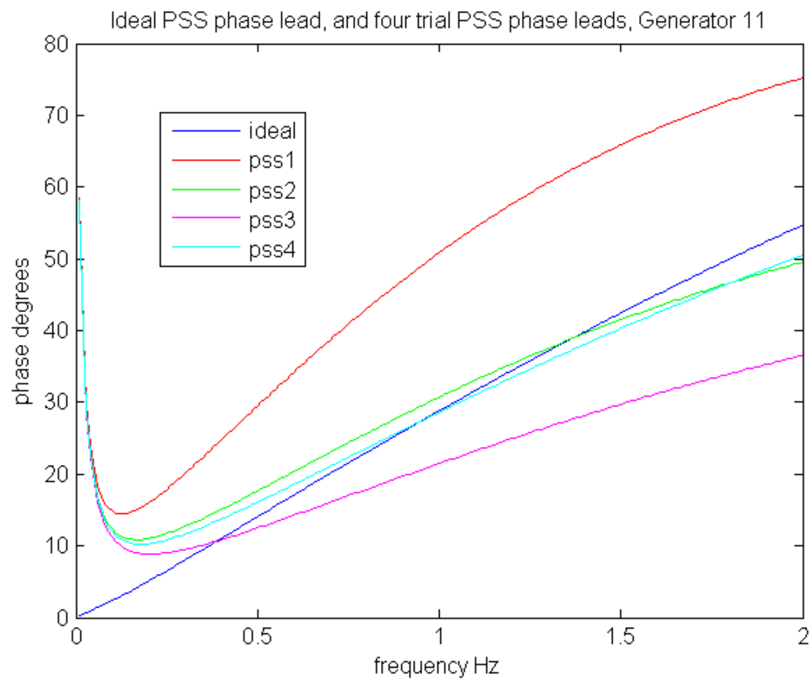

Figure 41 Ideal and trial power system stabilizer phase leads generator 11

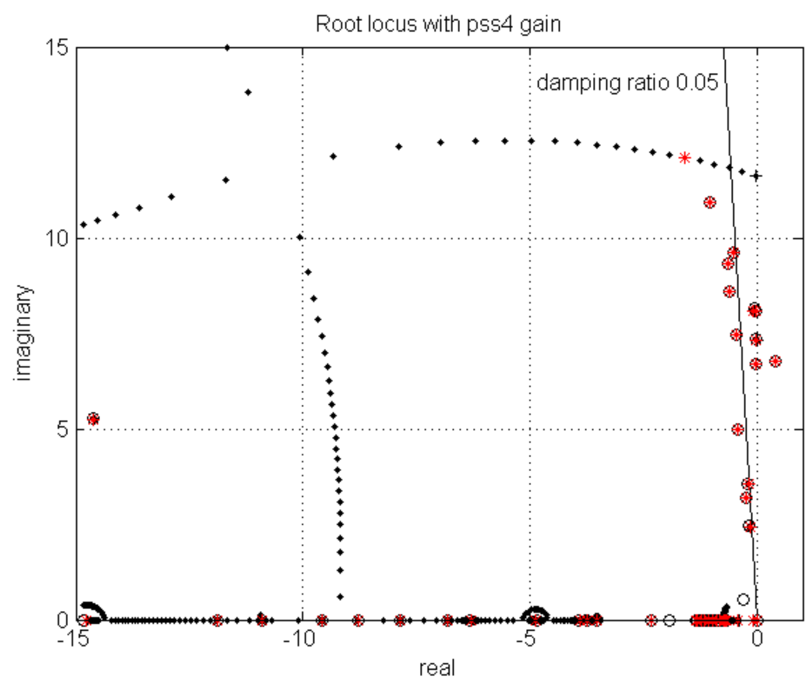

Figure 42 Root locus with PSS3 gain at generator 11

6.2 Generator 3

A power system stabilizer at generator 3 should add damping to modes 31 and 43. The stabilizer's ideal phase lead and three trials are shown in Figure 43. A power system stabilizer at generator 3 requires more phase lead than the stabilizer at generator 11. The characteristics of PSS1 gives an almost exact match . A root locus with power system stabilizer gain is shown in Figure 44. A gain of 10 is shown by *. Figure 45 shows the effect of PSS 2, It can be seen that with this power system stabilizer, the frequency of the stabilized mode is increased slightly as the gain is increased, whereas with PSS1, the modes frequency is reduced.
Other electromechanical modes are modified by the power system stabilizer on generator 3. A detail of the root locus with PSS2 is shown in Figure 46.

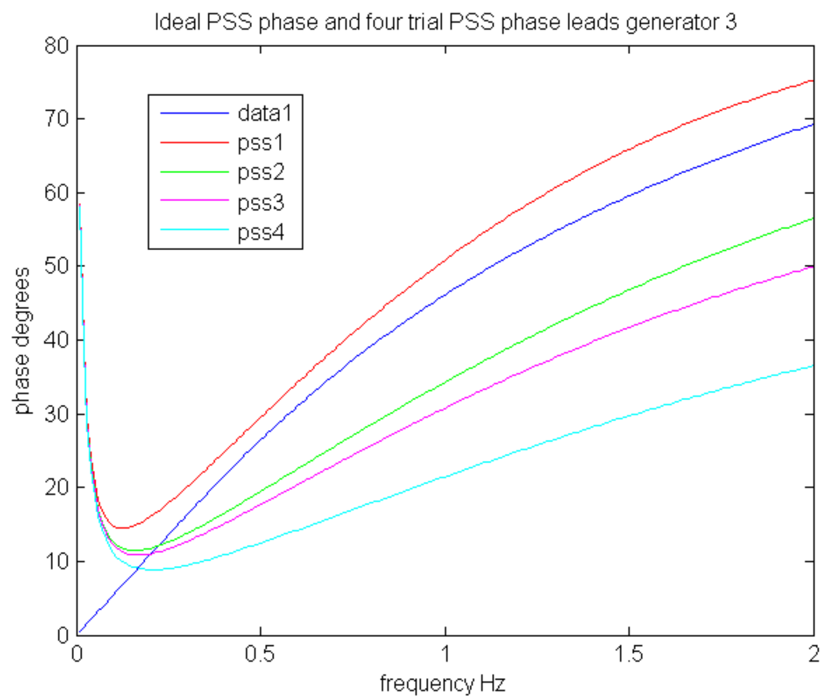

Figure 43 Ideal power system stabilizer phase lead and fours trials generator 3

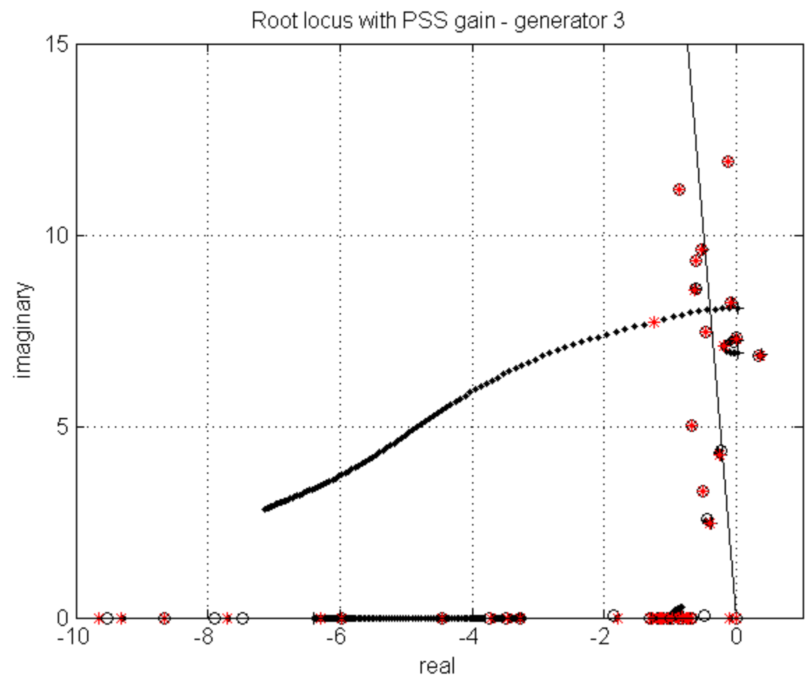

Figure 44 Root locus with pss1 gain generator 3, * PSS gain 10

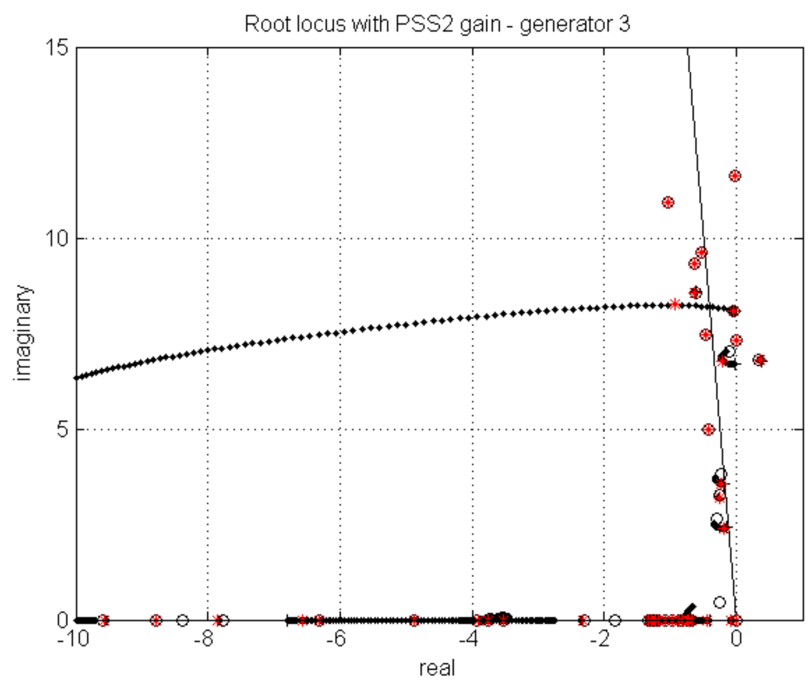

Figure 45 Root locus with pss2 gain generator 3, * PSS gain 10

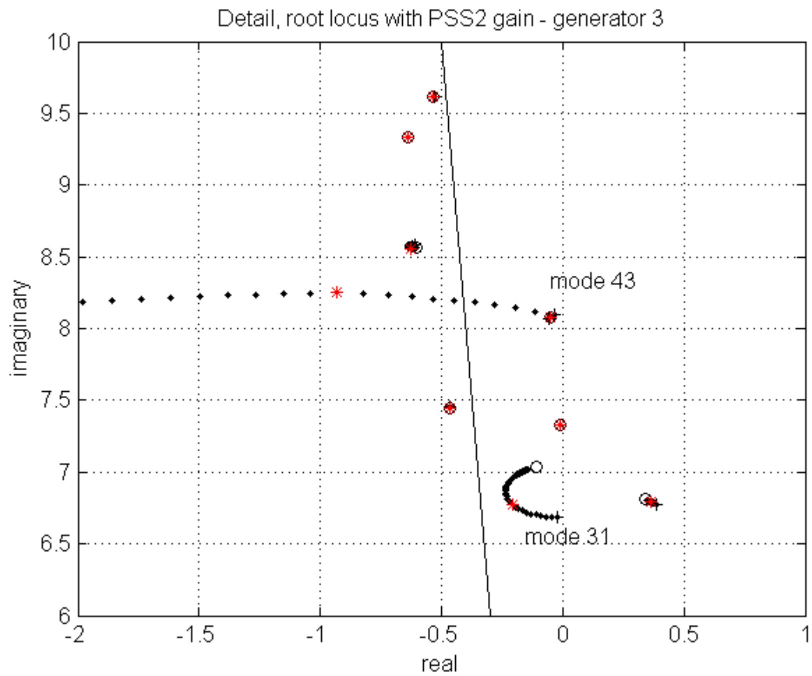

Figure 46 Detail of root locus with pss2 gain generator 3, * PSS gain 10

6.3 Generator 9

According to the generator speed participation factors, a power system stabilizer at generator 9 will add damping to mode 33. The ideal and four trial power system stabilizer phase leads are shown in Figure 47.

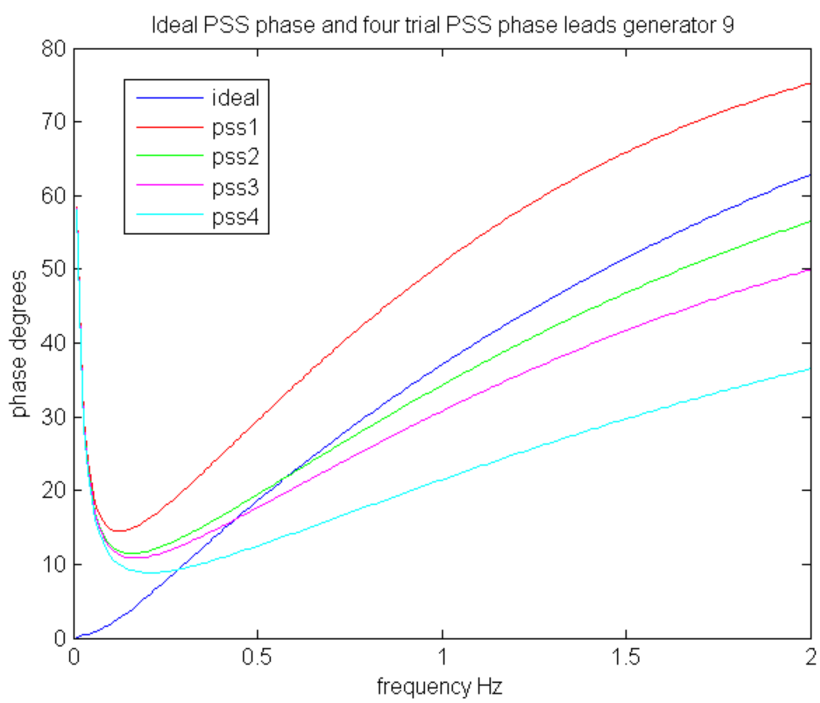

Figure 47 Ideal and four trial power system stabilizer phase leads for generator 9

The root locus with the gain of PSS2 is shown in Figure 48. One mode is stabilized, and others are modified. A detail of the root locus is shown in Figure 49. The * indicates a power system stabilizer gain of 10. Damping is added to modes 31 and 33 by a power system stabilizer on generator 9. However, this power system stabilizer effectively damps mode 31. The incease in damping of mode 33 is restricted by a zero, as shown in Figure 49.

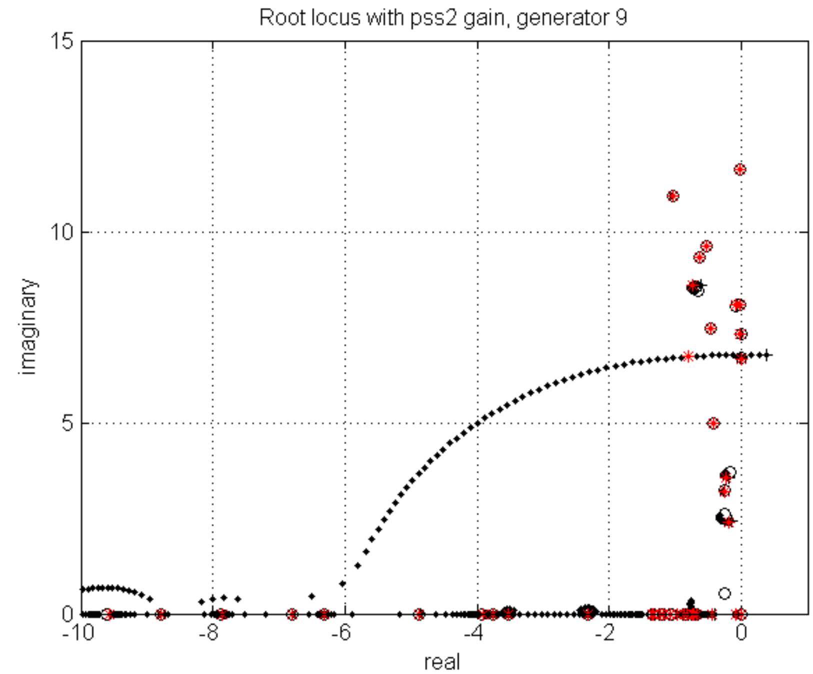

Figure 48 Root locus with power system stabilizer gain, generator 9

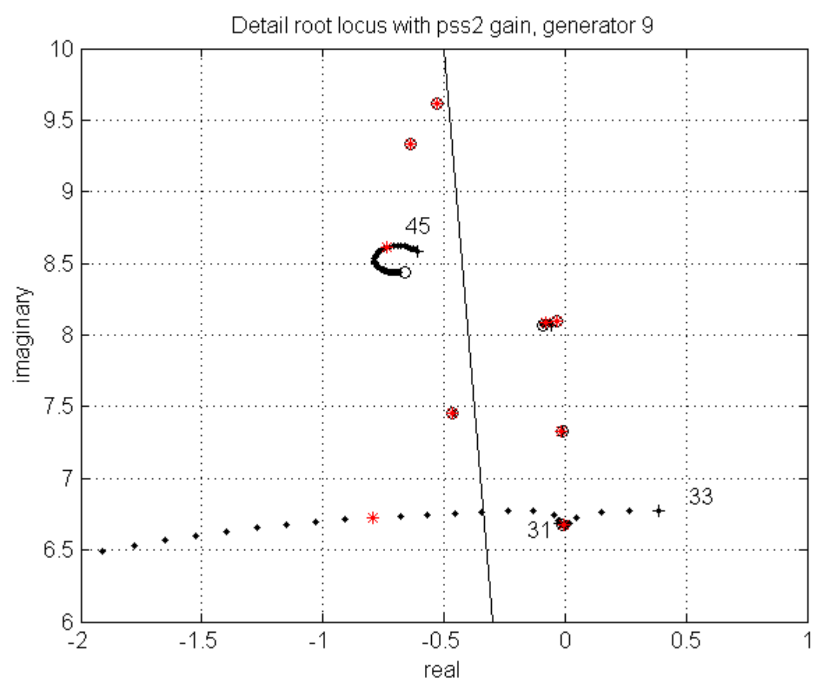

Figure 49 Detail of root locus with power system stabilizer gain at generator 9

6.4 Generator 2

A power system stabilizer at generator 2 should add damping to modes 31 and 41 and 43. The ideal phase lead for generator 2 and four trial PSS phase leads are shown in Figure 50. The root locus with power system stabilizer gain is shown in Figure 51. A detail of the locus is shown in Figure 52..

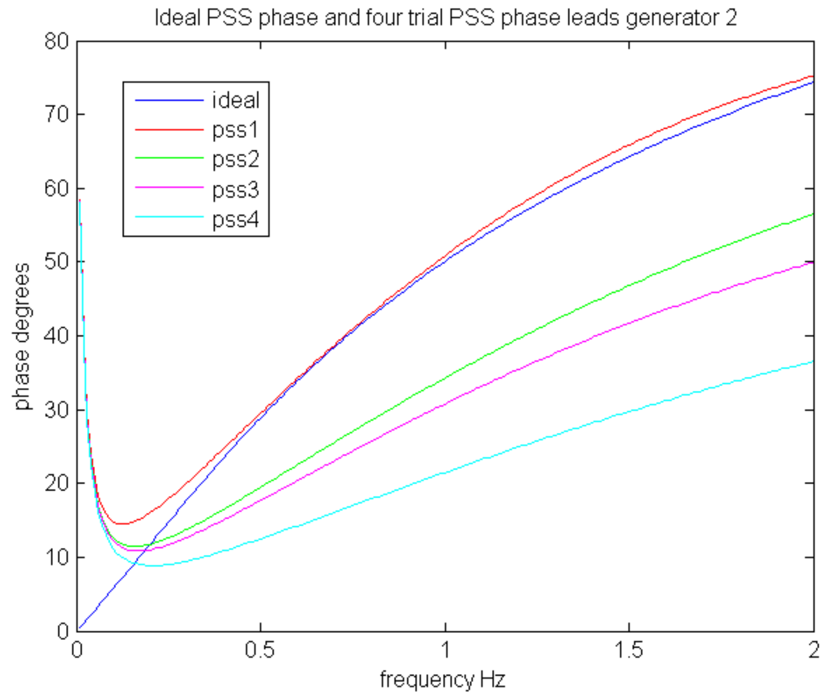

Figure 50 Ideal power system stabilizer phase lead and four trials generator 2

The under compensation of PSS2 adds synchronizing torque to generator 2..

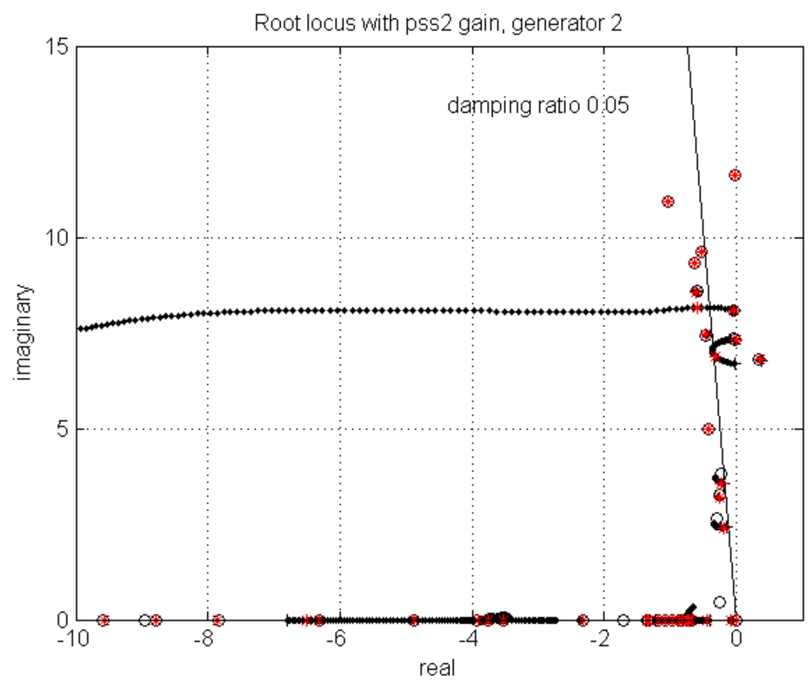

Figure 51 Root locus with power system stabilizer gain - generator 2

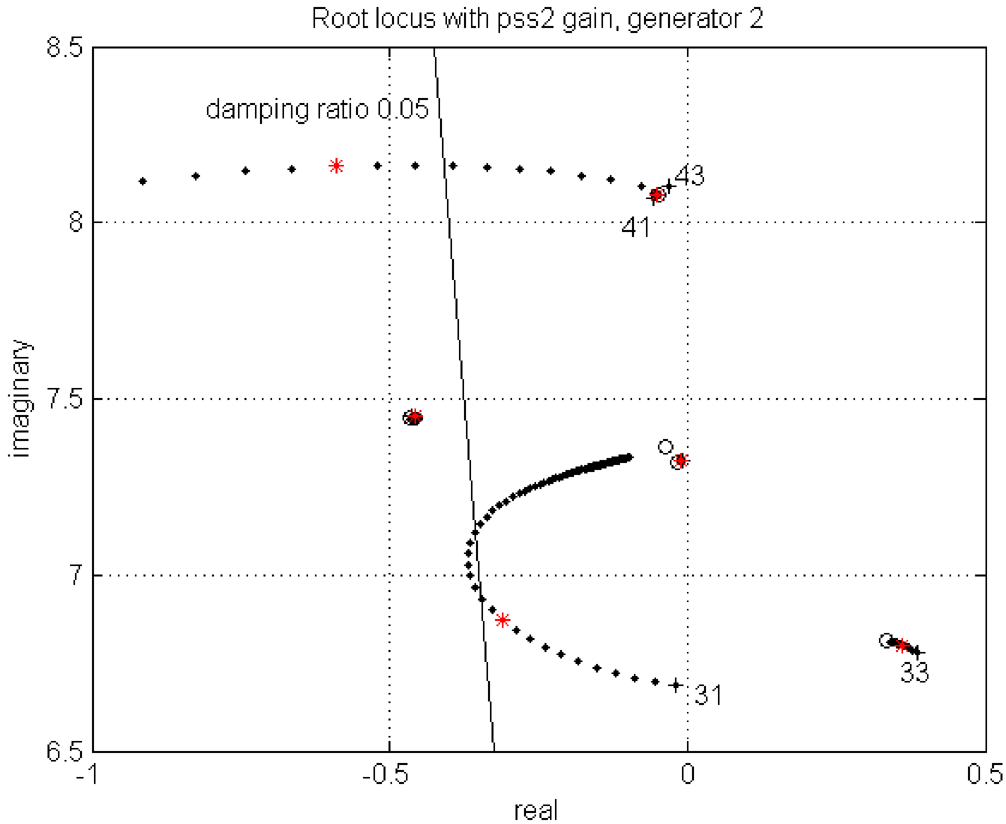

Figure 52 Detail showing modes modified by a power system stabilizer at generator 2

Again, the participation factor gives an accurate indication of the sensitivity of the damping to changes in power system stabilizer gain.

6.5 Effect of in-service Power System Stabilizers

Each of the root loci above are for a single power system stabilizer. While the ideal power system stabilizer phase lead for a particular generator does not depend on other generators, the effect of a stabilizer will be changed by other power system stabilizers.

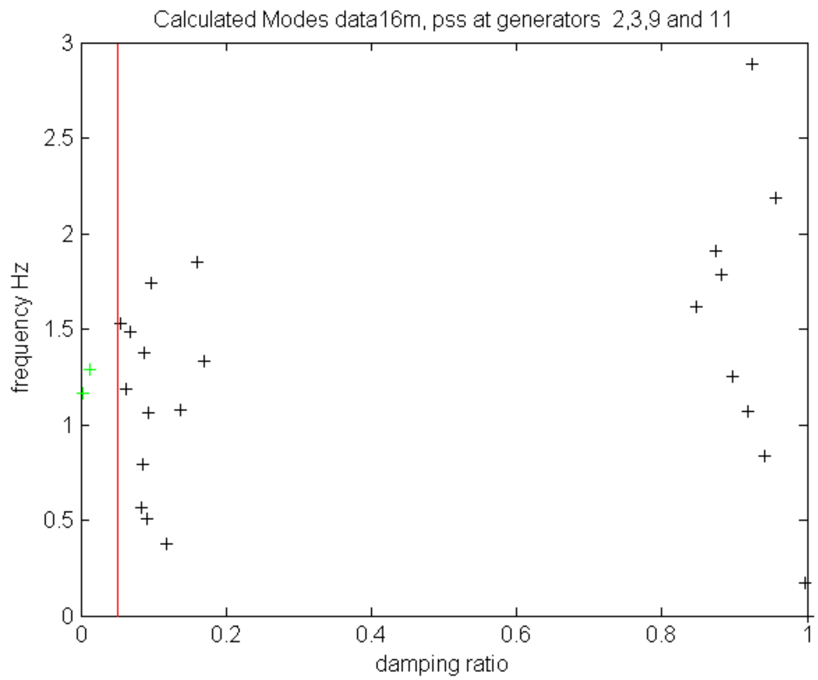

Figure 53 System Modes with PSS at generators 2,3,9 and 11

6.6 Fully Stabilized System

The ideal phase lead at any generator is independent of power system stabilizers elsewhere in the system. This means that all power system stabilizer phase leads may be determined from a single base system. On the other hand, the power system stabilizer gain is best set with those power system stabilizers already designed added to the model. The power system stabilizer parameters for generators 2 to 12, obtained using this procedure, are shown in Table 1.

Table 1. Power System Stabilizer Parameters Generators 2 to 12

Generator	Gain	T_w	T_1	T_2	T_3	T_4
1	10	10	0.1	0.02	0.08	0.02
2	10	10	0.08	0.02	0.08	0.02
3	10	10	0.08	0.02	0.05	0.01
4	10	10	0.08	0.02	0.08	0.02
5	10	10	0.08	0.02	0.08	0.02
6	10	10	0.1	0.02	0.1	0.02
7	10	10	0.08	0.02	0.08	0.02
8	4	10	0.08	0.02	0.08	0.02
9	10	10	0.08	0.02	0.05	0.01
10	10	10	0.1	0.02	0.1	0.02
11	5	10	0.08	0.03	0.05	0.01
12	10	10	0.1	0.02	0.1	0.02

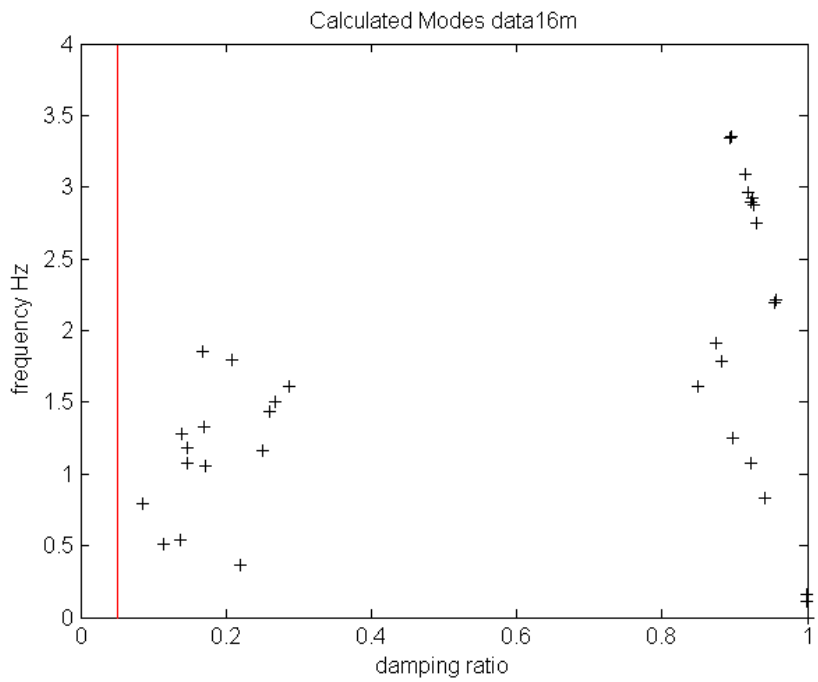

Figure 54 Dominant eigenvalues, power system stabilizers on generators 1 to 12

Even for this relatively small system, power system stabilizer design takes considerable care. In the North American interconnected systems, many generators have power system stabilizers that were put into service to stabilize oscillations local to the generators. If they were designed correctly initially, there should be little need to adjust the tuning. However, for generators put into service more than 30 years ago, it may be cost effective to review the power system stabilizer settings, since at that time local oscillations were generally more troublesome than the lower frequency inter-area oscillations, and power system stabilizers were often tuned to add damping at only the local mode.

Power system stabilizers should always be put into service when systems are upgraded from slow to fast exciters. If oscillations are found to be limiting on a system which has power system stabilizers, their settings should be checked and reset if necessary.

7 Electronic Controls

In some systems, power system stabilizers are not feasible for oscillation damping control. For example, in systems where the generators have slow exciters, the required phase shift for adequate stability may be excessive. Very high power system stabilizer phase lead makes the stabilizer gain large at high frequencies, and this may cause operating problems. In such systems, electronic damping control may be cost effective for under damped or unstable inter-area modes. Electronic system controls, or Flexible AC Transmission Systems (FACTS) are thyristor based system elements. They include Static VAr Compensators (SVCs), and Thyristor Controlled Series Capacitors (TCSCs). An SVC may be used to maintain the voltage at a network bus, and a TCSC may be used to control the flow in a transmission line. Both may be used to control inter-area oscillations, provided that they are placed such that the modes are controllable by the device and observable in its available inputs.

In this section, the two-area system will be used to illustrate the capabilities of these controls for damping the inter-area mode. In the system model, the exciters are slow acting dc exciters. Each generator has a thermal turbine and governor.

7.1 Static VAr Compensator

A Static VAr Compensator controls voltage by modulating the bus shunt capacitance or reactance. Under dynamic conditions, modulation of the SVC reference may be used to damp those electromechanical inter-area modes that are controllable and observable at the SVC. The two area system has three likely locations for SVCs for damping the inter-area mode, buses 3, 13 and 101. Each location will be examined in detail. Each SVC has a rating of 200 MVA, and a range of ±200MVA. The SVC voltage regulation control has a gain of 10 and a time constant of 0.05 s
With no SVCs in-service, the system's modes are shown in Figure 55.

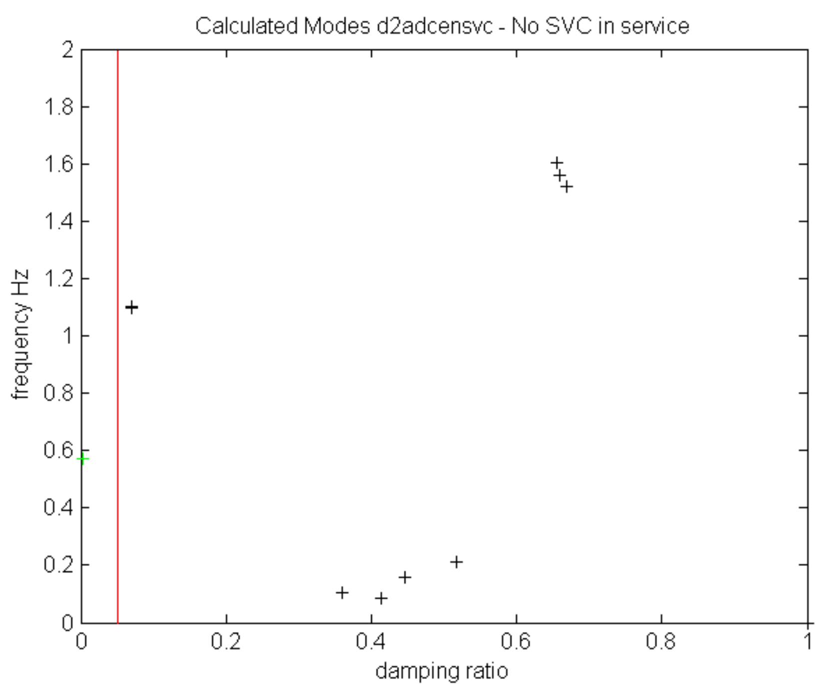

Figure 55 Modes with no SVC in-service

The generator local modes have frequencies of 1.0958 Hz and 1.1032 Hz, and damping ratios 0.06926 and 0.070156. The interarea mode is marginally stable having with frequency 0.57066 Hz and damping ratio 0.0024145. The governor modes are those with frequencies less than 0.3Hz and the exciter modes are those with frequencies greater than 1.5Hz.

7.1.1 SVC Location

The effect of an SVC at three possible locations may most easily be determined by examing the relationship between the system's poles and the zeros associated with reactive load modulation at each bus. The state space models for active load modulation at buses 3, 13 and 101 are

```
srlmodcimt3 = stsp(a_mat,b_rlmod(:,2),c_ilmf(5,:),0);
srlmodcimt13 = stsp(a_mat,b_rlmod(:,3),-c_ilmf(11,:),0);
srlmodcimt101 = stsp(a_mat,b_rlmod(:,1),c_ilmt(5,:),0);
```

The zeros associated with these models will indicate whether an SVC at the buses will have any effect on the inter-area mode.

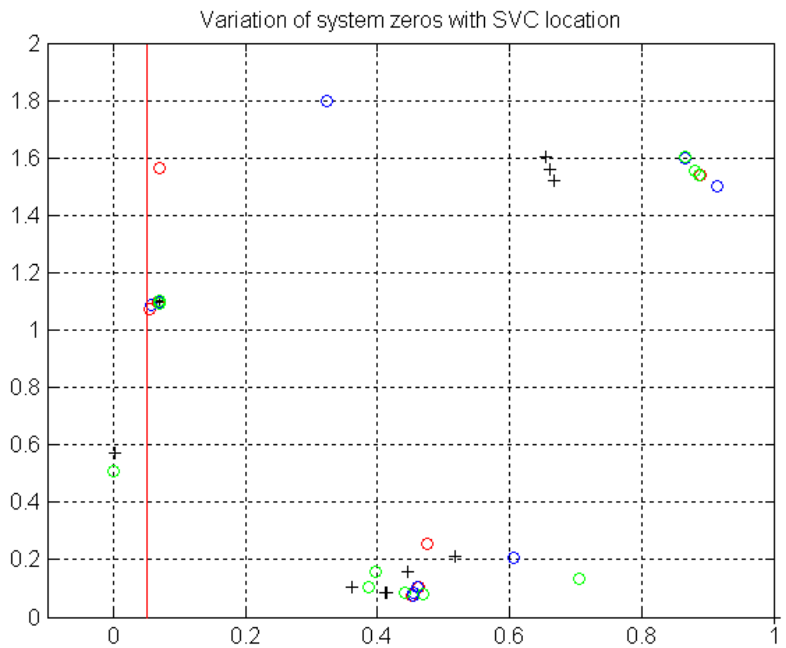

Figure 56 Poles and zeros with reactive load modulation, poles +, zeros: o bus 3;o bus 13; o bus 101

The interarea mode has a frequency of 0.57066 Hz, and there is a zero close to this pole with reactive load modulation at bus 101..

7.1.2 Damping Control

Positive feedback of line current magnitude is used to stabilize the system with SVC.

7.1.2.1 SVC at Bus 3

The system with input svc reference and output line current magnitude at bus 3 is formed using

```
ssvc3 = stsp(a_mat,b_svc,c_ilmf(5,:),0);
ssvcd3 = dif_stsp.*ssvc3;
lz = zeros(ssvc3);
```

The rootlocus with positive feedback gain is formed using

```
rlcon3 = rtlocus(ssvcd3,1,0,.01,10);
```

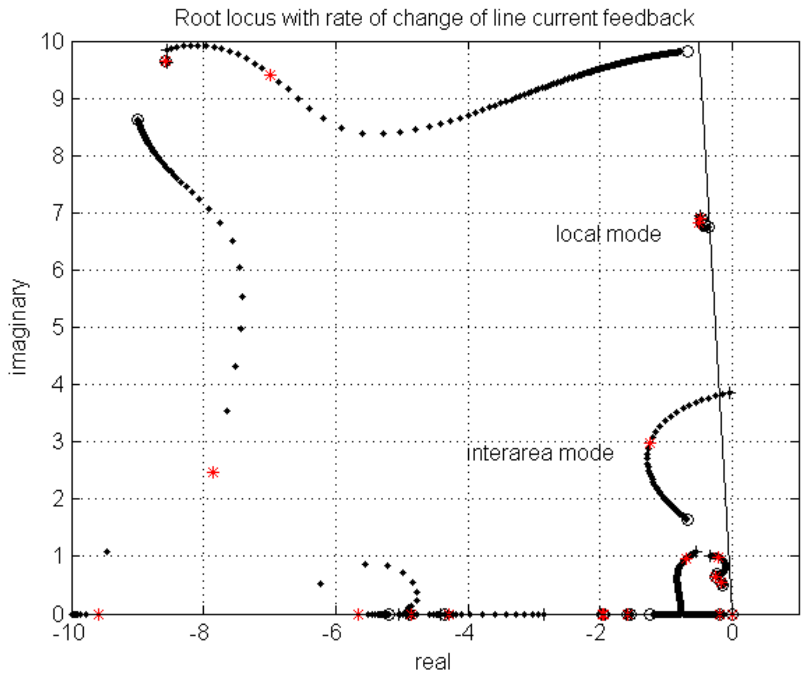

Figure 57 Root locus with damping control gain SVC at bus 3, * gain 0.15

With feedback shaped by the transfer function $K_d(s) = \dfrac{s+1}{5s+1}$, the locus is modified to that shown in Figure 58. A gain of 2 inceases the damping sufficiently without significantly reducing the frequency of the interarea mode.
ks = ldlg_stsp(1,5,1);
rlcon3 = rtlocus(ssvcd3,ks,0,.1,100);

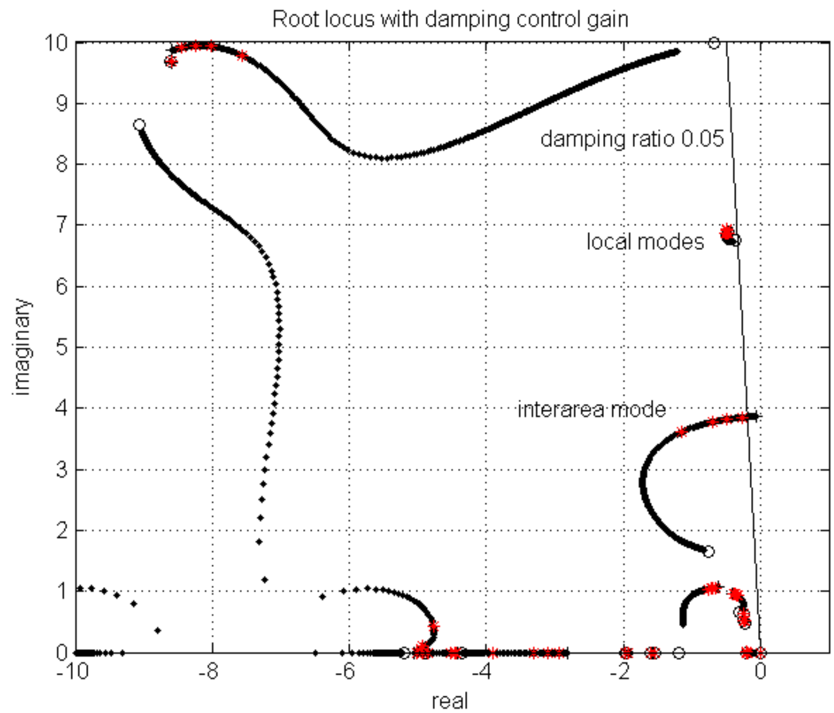

Figure 58 Root locus with modified damping control gain, * gains 0.1,0.2,0.3,0.5

7.1.3 SVC at Bus 13

The system with input svc reference and output line current magnitude at bus 13 is formed using

```
ssvc13 = stsp(a_mat,b_svc,c_ilmf(11,:),0);
ssvcd13 = dif_stsp.*ssvc13;
sc1 = ldlg_stsp(1,5,1);
rlcon13 = rtlocus(ssvcd13,-sc1,0,.01,10);
```

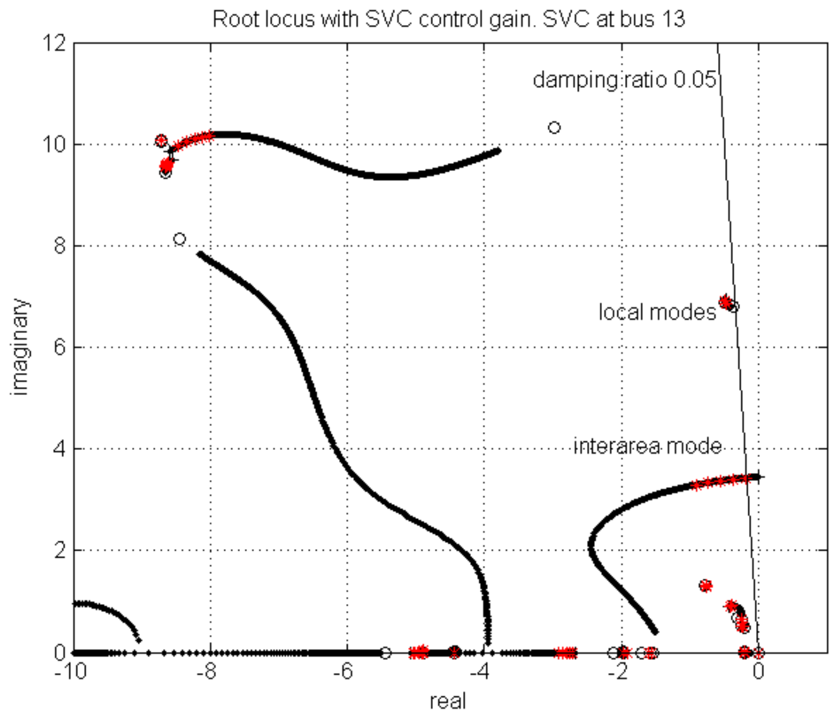

Figure 59 Root locus with SVC damping control gain, SVC at bus 13, * gains 1, 2, 3, 4

The damping characteristics are similar to those with the SVC placed at bus 3. Satisfactory damping of the inter-area mode and the control modes is obtained with a gain of 0.2.

7.1.4 SVC at Bus 101

The system with input svc reference and output line current magnitude at bus 101 is formed using

```
ssvc101 = stsp(a_mat,b_svc,c_ilmt(11,:),0);
ssvcd101 = dif_stsp.*ssvc101;
sc1 = ldlg_stsp(1,5,1);
rlcon101 = rtlocus(ssvcd101,sc1,0,.1,10);
```

The root locus shown in Figure 61, shows that an SVC at bus 101 is able to stabilize the interarea mode, but that it requires consideraly higher feedback gain to acheive a damping ratio of 0.05.

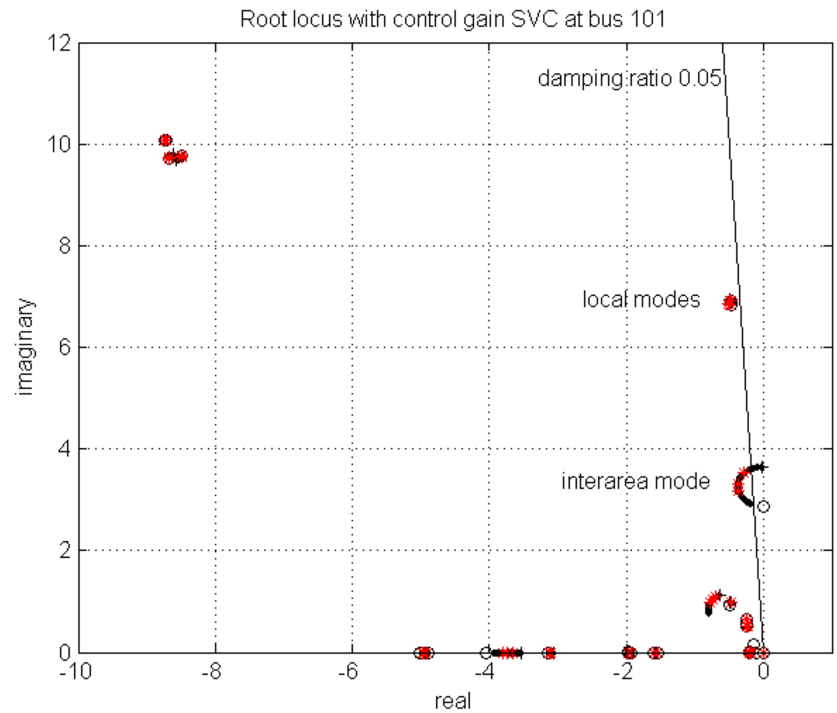

Figure 60 Root locus with SVC damping control gain, SVC at bus 101, * gains 1 to 3

7.2 Thyristor Controlled Series Capacitor

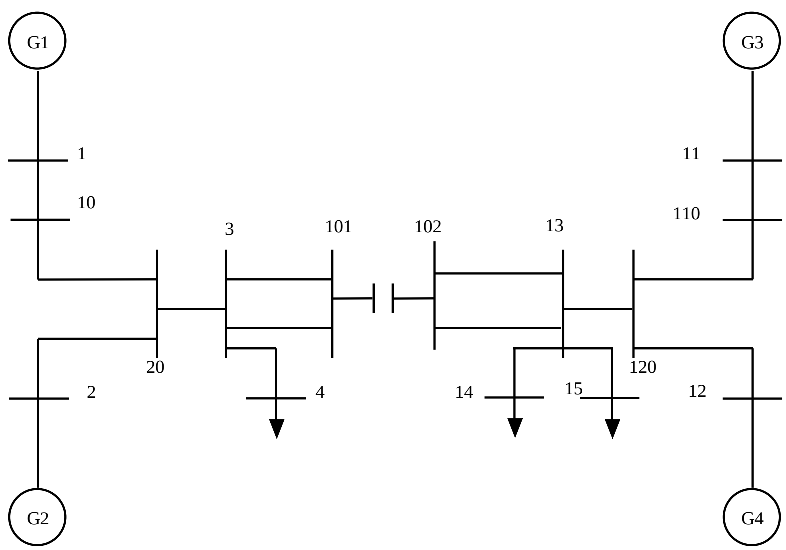

Figure 61 Single line diagram of two -area system with TCSC

In this system, there is a single tie line between the two areas, The line reactance is 40% compensated with a series capacitance between buses 101 and 102. The generators and their controls are identical to those used in the SVC study. The systems modes are shown in Figure 62.

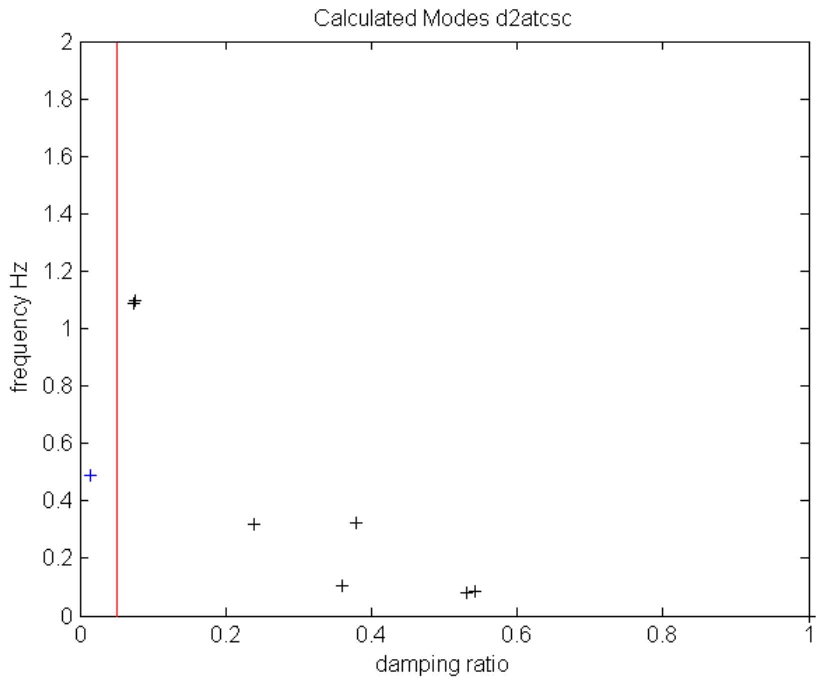

Figure 62 System modes two area system with TCSC

The modes with the series capacitor are very similar to the modes with two tie lines.

For damping control, the change in the voltage magnitude at bus 101 will be used to modulate the series capacitance. The dominant poles and zeros are shown in Figure 63.

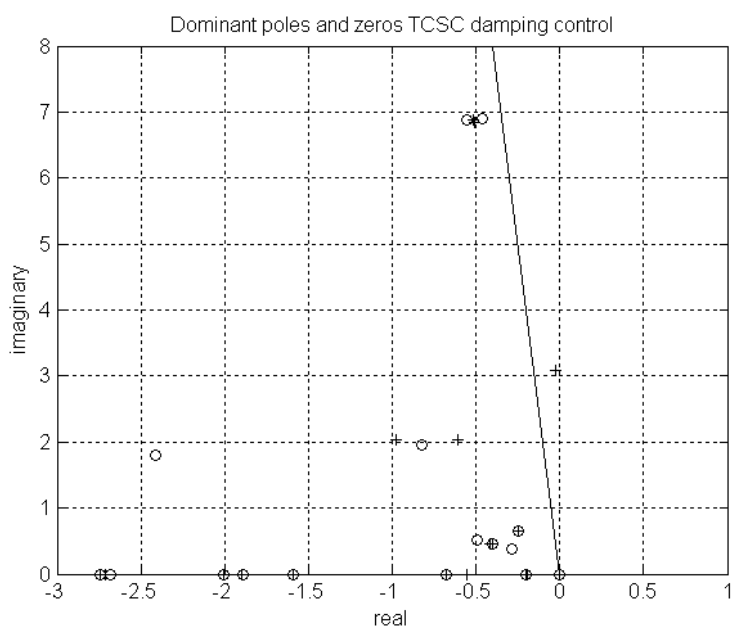

Figure 63 Dominant poles and zeros TCSC damping control

The local, governor and exciter modes are not controllable by the TCSC modulation.

Figure 64 shows the root locus with TCSC damping control gain. The control input is the voltage magnitude at bus 101. The control has the transfer function

$$C(s) = \frac{100s(s+10)^2}{(s+1)(s+2)^2(s+20)}$$

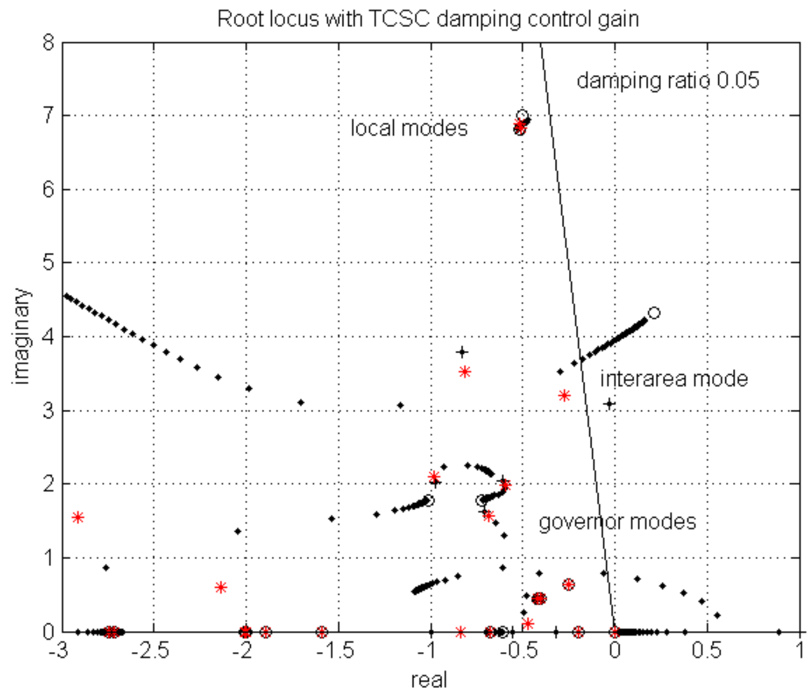

Figure 64 Root locus with TCSC damping control gain, * gain 1

The frequency response of the control is shown in Figure 65.

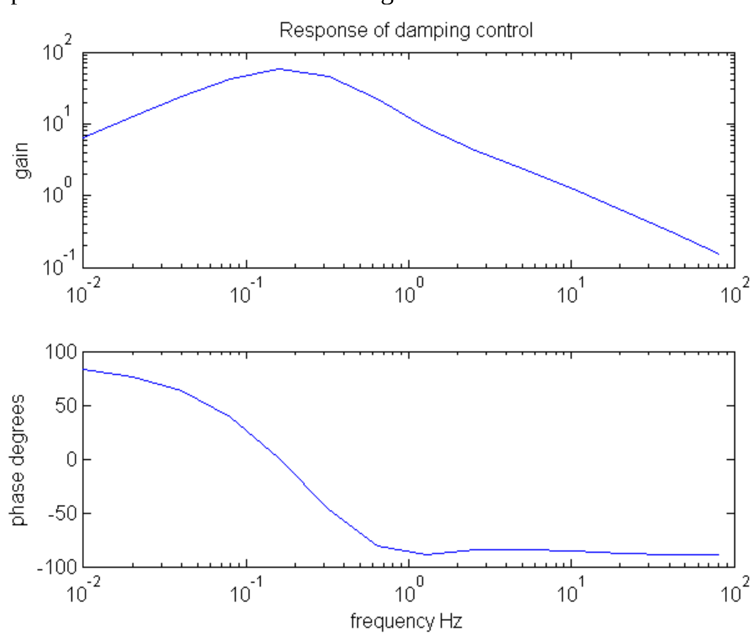

Figure 65 Frequency response of damping control

The damping control, with a gain of 15, stabilizes the inter-area mode satisfactorily. At higher gains the mode reduces in frequency and becomes unstable. With a stabilizing control gain of 10, the system's response to a normally cleared three phase fault at bus 102, is shown in Figure 66.

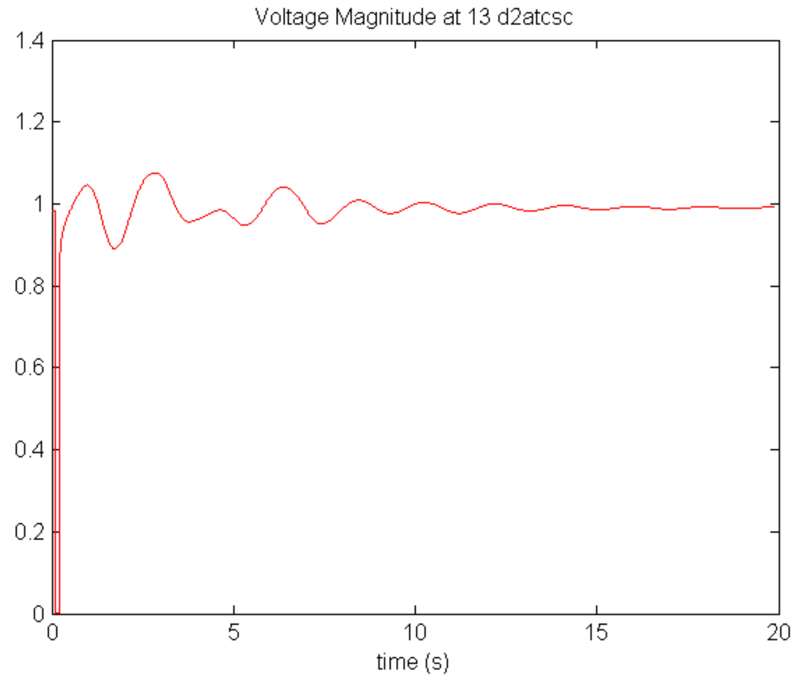

Figure 66 Response to a three phase fault at bus 13

8 Robust Performance

Most large interconnected power systems are robust to changes in system configuration and load. Generally, the systems have been designed, or have evolved, so that only extreme faults cause the system to become unstable. If the systems are well designed and operated, even such faults should not lead to system collapse. However, with deregulation of the power system industry, the source of power is more difficult to determine, and this may lead to operation of the system under conditions which have not been considered. Ideally, properly designed stabilizing controls should be provided on all large synchronous generators which supply power to the system. However, the possibility remains that some generators may lack stabilizing controls, such as power system stabilizers, or that their governors may be set to control power rather than speed. Wind farms that use induction generators require reactive power support from the network, and additionally provide no system frequency control. In the future, the constant frequency and voltage that we have come to expect may not be so naturally attainable.

8.1 Effects of Load Type

The 16 machine system will be used to illustrate the effects of different types of loads on the systems performance. The active loads will be considered to be

1. Constant impedance active and reactive loads
2. 50% constant impedance and 50% constant current active loads, constant impedance reactive loads
3. Constant current active loads and constant impedance reactive loads
4. Constant power active loads and constant impedance reactive loads
5. A mixture of induction motors picking up 50% of the active power load, the remaining active power loads are 50% constant impedance and 50% constant current, and the remaining reactive power loads are constant impedance

The frequencies and damping ratios of the modes are shown in Figure 67. It can be seen that these modes do vary with the type of load, but the systems modes are adequately damped for all load types.

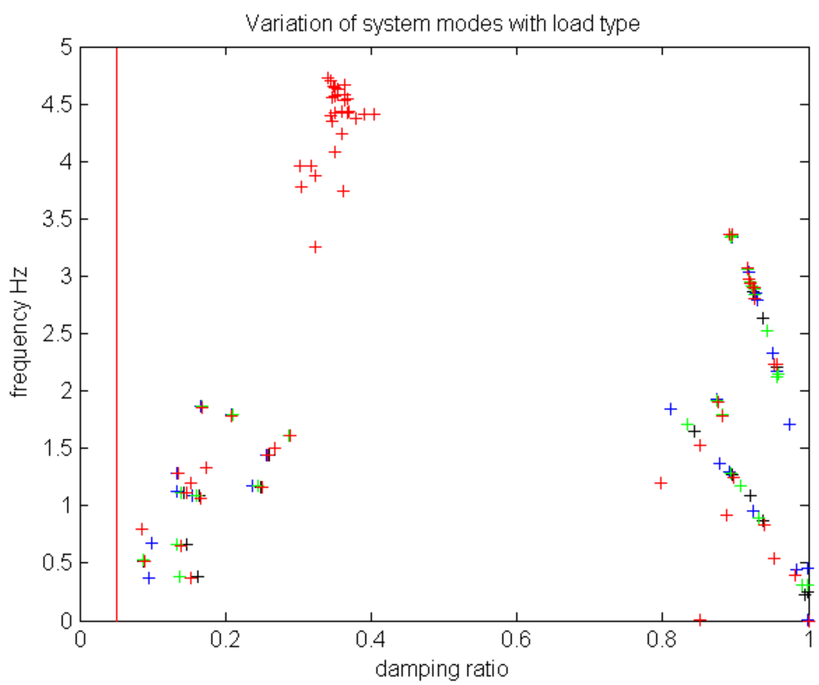

Figure 67 Variation of Modes with Load Type: + 50/50 consti constz, + const power, + constant current, + const impedance and induction motors

The systems bus voltage magnitude response to load variation for these load models is shown in Figure 68, and the per unit bus frequency response is shown in Figure 69. The lowest magnitude changes occur when the active load is modelled as constant impedance, and the highest changes when the active load is modelled with induction motors. At low frequencies the response with induction motor load is very close to that of the response with a 50% constant current, 50% constant impedance load. At higher frequencies, over 3 Hz, the variations in bus voltage magnitude and frequency are less with motor loads than with static loads.

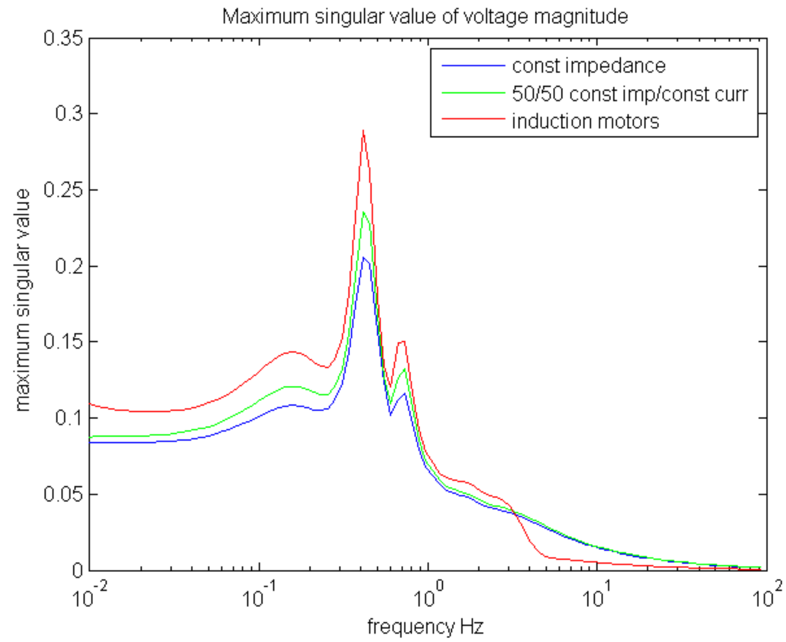

Figure 68 The maximum singular value of the bus voltage magnitude to load modulation

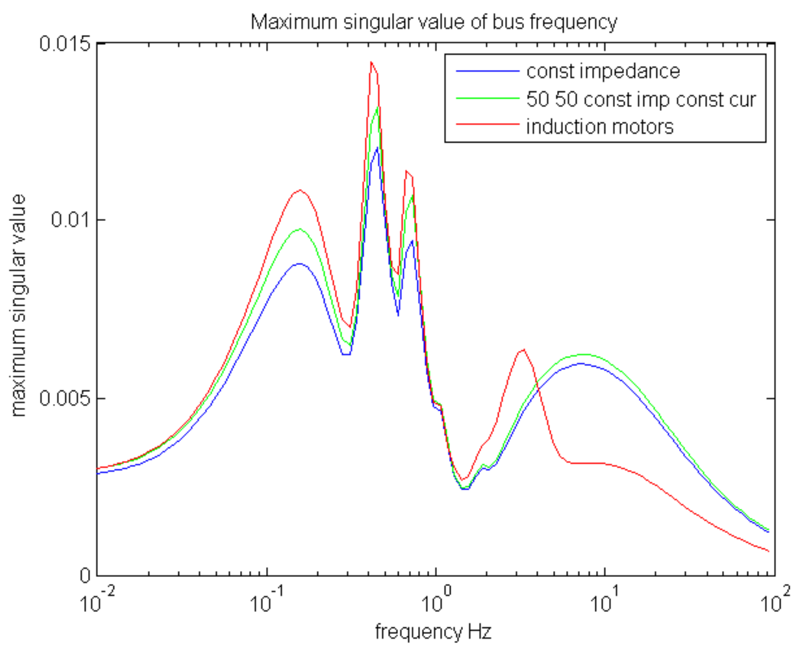

Figure 69 Variation of the maximum singular value bus frequency (PU) to load modulation

8.2 Effects of System Structure

A system's oscillations should remain stable for a double contingency. This matches the transient stability requirement that a system remains stable when a fault occurs with a critical unit out-of-service. Normally, a system's oscillatory stability limit should be higher than the transient stability limit.

For the 16 generator system, the double contingency is taken as the loss of two of the lines connecting area 4 to the rest of the system. The disconnected lines are between bus 1 and bus 2 and between bus 8 and bus 9.

The calculated modes are shown in the Figure 70.

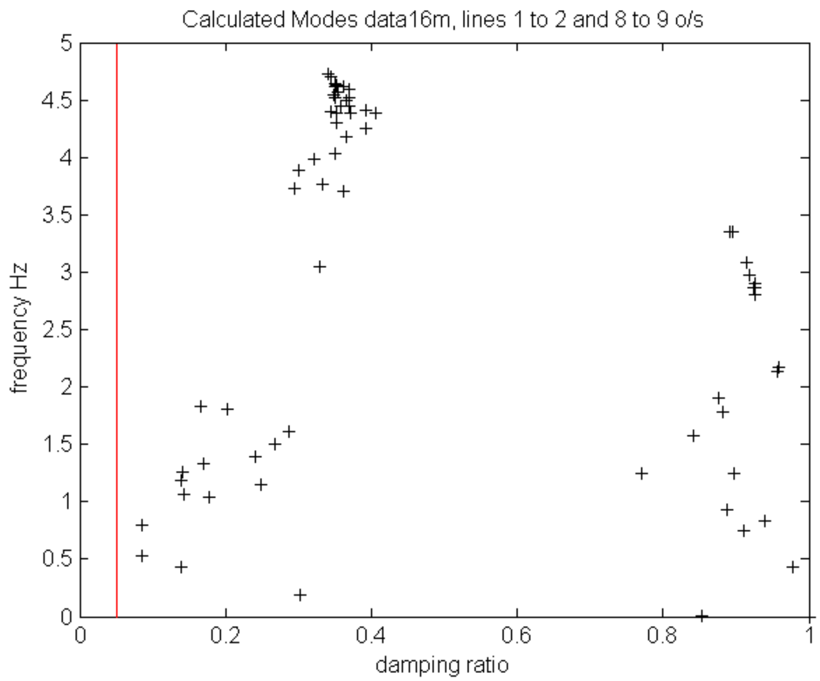

Figure 70 System modes with two lines out-of-service

Figure 71 shows the response of the bus voltage magnitude to active load variation, and Figure 72 shows the response of the bus frequency, with two line out-of-service. It can be seen, that the loss of the two lines has caused the effect of load modulation to be doubled from the case with all lines in-service.
The transient response to a fault at bus 2 on line 1-2, with line 8 to 9 out-of-service is shown in Figure 73.

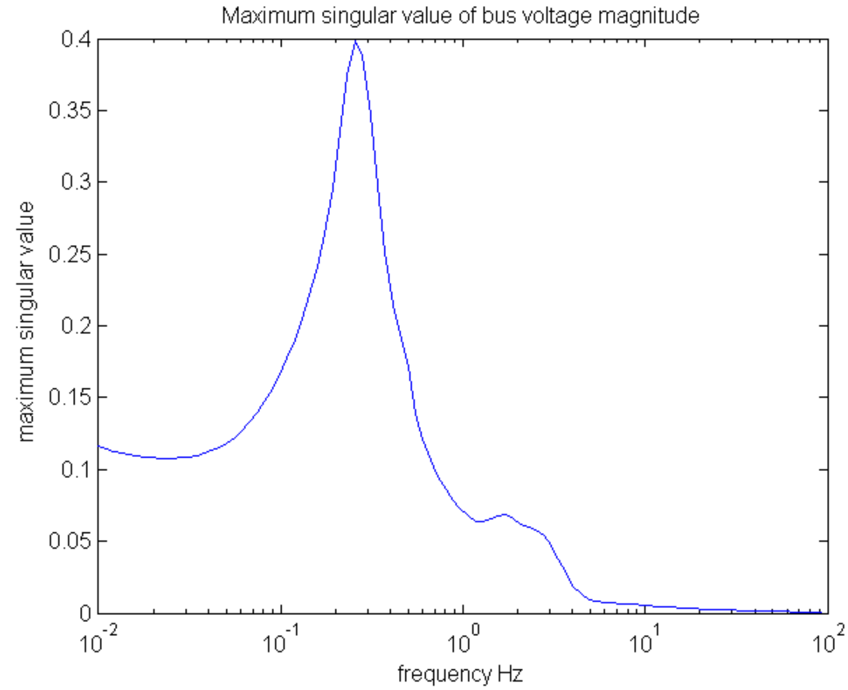

Figure 71 Response of bus voltage magnitude to active load modulation

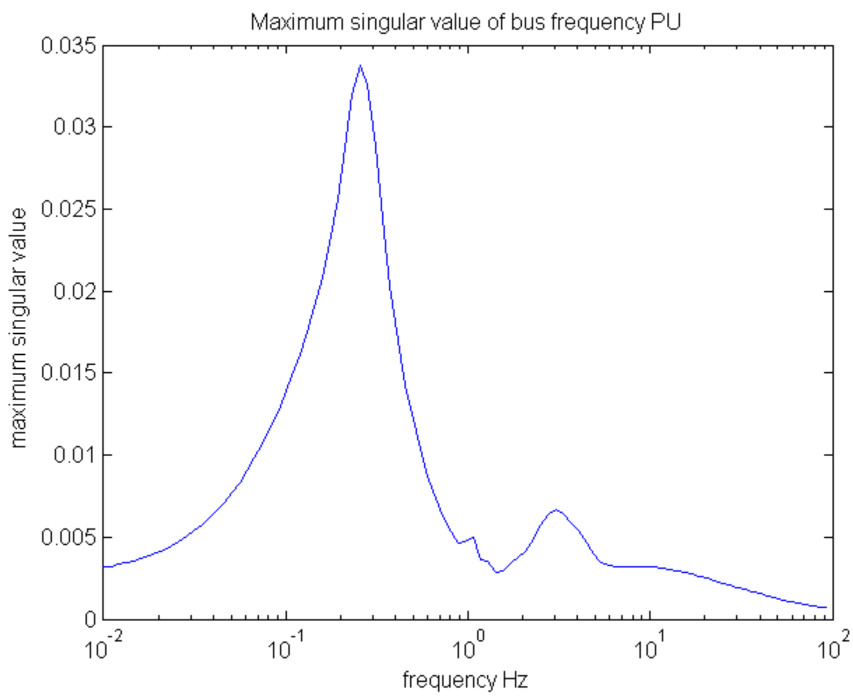

Figure 72 Response of bus frequency (PU) to active load modulation

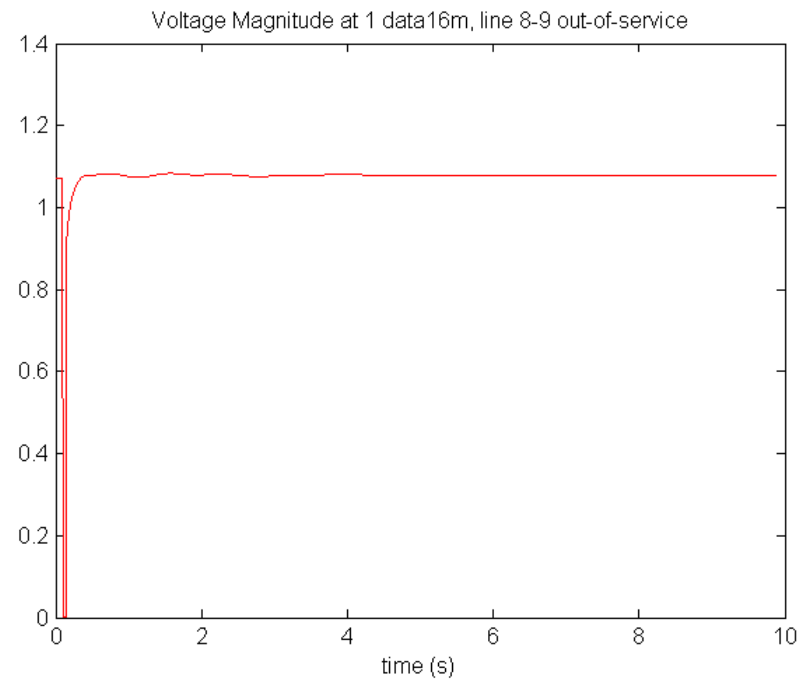

Figure 73 Response to a three phase fault at bus 1 on line 1-2 with line 8 to 9 out-of-servive

9 Appendix 1 Example System Data

9.1 Two Area System

```
% Two Area Test Case
% sub transient generators with static exciters, turbine/governors
% 50% constant current active loads
% load modulation
% with power system stabilizers

disp('Two-area test case with subtransient generator models')
disp('Static exciters')
disp('turbine/governors')
disp('power system stabilizers')
% bus data format
% bus:
% col1 number
% col2 voltage magnitude(pu)
% col3 voltage angle(degree)
% col4 p_gen(pu)
% col5 q_gen(pu),
% col6 p_load(pu)
% col7 q_load(pu)
% col8 G shunt(pu)
% col9 B shunt(pu)
% col10 bus_type
%       bus_type - 1, swing bus
%               - 2, generator bus (PV bus)
%               - 3, load bus (PQ bus)
% col11 q_gen_max(pu)
% col12 q_gen_min(pu)
% col13 v_rated (kV)
% col14 v_max  pu
% col15 v_min  pu


bus = [...
1  1.03    18.5   7.00   1.61  0.00  0.00  0.00  0.00 2  5.0  -1.0  22.0   1.1  .9;
2  1.01    8.80   7.00   1.76  0.00  0.00  0.00  0.00 2  5.0  -1.0  22.0   1.1  .9;
3  0.9781  -6.1   0.00   0.00  0.00  0.00  0.00  3.00 3  0.0   0.0  230.0  1.5  .5;
4  0.95   -10     0.00   0.00  9.76  1.00  0.00  0.00 3  0.0   0.0  115.0  1.05 .95;
10 1.0103  12.1   0.00   0.00  0.00  0.00  0.00  0.00 3  0.0   0.0  230.0  1.5  .5;
11 1.03    -6.8   7.16   1.49  0.00  0.00  0.00  0.00 1  5.0  -1.0  22.0   1.1  .9;
12 1.01    -16.9  7.00   1.39  0.00  0.00  0.00  0.00 2  5.0  -1.0  22.0   1.1  .9;
13 0.9899  -31.8  0.00   0.00  0.00  0.00  0.00  5.00 3  0.0   0.0  230.0  1.5  .5;
14 0.95    -35    0.00   0.00  17.65 1.00  0.00  0.00 3  0.0   0.0  115.0  1.05  .95;
20 0.9876    2.1  0.00   0.00  0.00  0.00  0.00  0.00 3  0.0   0.0  230.0  1.5  .5;
101 1.00    -19.3  0.00   0.00  0.00  0.00  0.00  2.00 3  2.0   0.0  500.0  1.5  .5;
110 1.0125  -13.4  0.00   0.00  0.00  0.00  0.00  0.00 3  0.0   0.0  230.0  1.5  .5;
120 0.9938  -23.6  0.00   0.00  0.00  0.00  0.00  0.00 3  0.0   0.0  230.0  1.5  .5;
   ];
```

```
% line data format
% line:
%       col1    from bus
%       col2    to bus
%       col3    resistance(pu)
%       col4    reactance(pu)
%       col5    line charging(pu)
%       col6    tap ratio
%       col7    tap phase
%       col8    tapmax
%       col9    tapmin
%       col10   tapsize

line = [...
1   10  0.0     0.0167   0.00    1.0  0. 0.  0.  0.;
2   20  0.0     0.0167   0.00    1.0  0. 0.  0.  0.;
3    4  0.0     0.005     0.00   1.0  0. 1.2 0.8 0.02;
3   20  0.001   0.0100   0.0175  1.0  0. 0.  0.  0.;
3   101 0.011   0.110    0.1925  1.0  0. 0.  0.  0.;
3   101 0.011   0.110    0.1925  1.0  0. 0.  0.  0.;
10  20  0.0025  0.025    0.0437  1.0  0. 0.  0.  0.;
11  110 0.0     0.0167   0.0     1.0  0. 0.  0.  0.;
12  120 0.0     0.0167   0.0     1.0  0. 0.  0.  0.;
13  101 0.011   0.11     0.1925  1.0  0. 0.  0.  0.;
13  101 0.011   0.11     0.1925  1.0  0. 0.  0.  0.;
13   14  0.0    0.005    0.00    1.0  0. 1.2 0.8 0.02;
13  120 0.001   0.01     0.0175  1.0  0. 0.  0.  0.;
110 120 0.0025  0.025    0.0437  1.0  0. 0.  0.  0.;
];

% Machine data format
% Machine data format
%        1. machine number,
%        2. bus number,
%        3. base mva,
%        4. leakage reactance x_l(pu),
%        5. resistance r_a(pu),
%        6. d-axis sychronous reactance x_d(pu),
%        7. d-axis transient reactance x'_d(pu),
%        8. d-axis subtransient reactance x"_d(pu),
%        9. d-axis open-circuit time constant T'_do(sec),
%       10. d-axis open-circuit subtransient time constant
%                 T"_do(sec),
%       11. q-axis sychronous reactance x_q(pu),
%       12. q-axis transient reactance x'_q(pu),
%       13. q-axis subtransient reactance x"_q(pu),
%       14. q-axis open-circuit time constant T'_qo(sec),
%       15. q-axis open circuit subtransient time constant
%                 T"_qo(sec),
%       16. inertia constant H(sec),
%       17. damping coefficient d_o(pu),
%       18. dampling coefficient d_1(pu),
%       19. bus number
%
% note: all the following machines use sub-transient model

mac_con = [ ...

1 1 900 0.200  0.00    1.8  0.30  0.25 8.00  0.03...
                       1.7  0.55  0.24 0.4   0.05...
                       6.5  0  0  1  0.0654  0.5743;
2 2 900 0.200  0.00    1.8  0.30  0.25 8.00  0.03...
                       1.7  0.55  0.25 0.4   0.05...
                       6.5  0  0  2  0.0654  0.5743;
3 11 900 0.200  0.00   1.8  0.30  0.25 8.00  0.03...
                       1.7  0.55  0.24 0.4   0.05...
                       6.5  0  0  3  0.0654  0.5743;
4 12 900 0.200  0.00   1.8  0.30  0.25 8.00  0.03...
                       1.7  0.55  0.25 0.4   0.05...
                       6.5  0  0  4  0.0654  0.5743];
```

```
% simple exciter model, type 0; there are three exciter models
exc_con = [...
0 1 0.01 200.0  0.05     0     0    5.0  -5.0...
    0    0      0     0     0    0    0      0      0    0   0;
0 2 0.01 200.0  0.05     0     0    5.0  -5.0...
    0    0      0     0     0    0    0      0      0    0   0;
0 3 0.01 200.0  0.05     0     0    5.0  -5.0...
    0    0      0     0     0    0    0      0      0    0   0;
0 4 0.01 200.0  0.05     0     0    5.0  -5.0...
    0    0      0     0     0    0    0      0      0    0   0];
% power system stabilizer model
%       col1    type 1 speed input; 2 power input
%       col2    generator number
%       col3    pssgain*washout time constant
%       col4    washout time constant
%       col5    first lead time constant
%       col6    first lag time constant
%       col7    second lead time constant
%       col8    second lag time constant
%       col9    maximum output limit
%       col10   minimum output limit
pss_con = [...
     1  1  100  10 0.05 0.015 0.08 0.01 0.2 -0.05;
     1  2  100  10 0.05 0.015 0.08 0.01 0.2 -0.05;
     1  3  100  10 0.05 0.015 0.08 0.01 0.2 -0.05;
     1  4  100  10 0.05 0.015 0.08 0.01 0.2 -0.05
 ];


% governor model
% tg_con matrix format
%column       data                   unit
%  1    turbine model number (=1)
%  2    machine number
%  3    speed set point   wf          pu
%  4    steady state gain 1/R         pu
%  5    maximum power order  Tmax     pu on generator base
%  6    servo time constant   Ts      sec
%  7    governor time constant  Tc    sec
%  8    transient gain time constant T3       sec
%  9    HP section time constant   T4 sec
% 10    reheater time constant    T5  sec

tg_con = [...
1  1  1  25.0  1.0  0.1  0.5 0.0 1.25 5.0;
1  2  1  25.0  1.0  0.1  0.5 0.0 1.25 5.0;
1  3  1  25.0  1.0  0.1  0.5 0.0 1.25 5.0;
1  4  1  25.0  1.0  0.1  0.5 0.0 1.25 5.0];

% passive load type definition
%       col1    bus number
%       col2    proportion of constant active power load
%       col3    proportion of constant reactive power load
%       col4    proportion of constant active current load
%       col5    proportion of constant reactive current load
load_con = [4  0  0  .5  0;
            14 0  0  .5  0];% 50/50 constant inpedance constant current
disp('50% constant current load')
%disp('load modulation')
%active load modulation enabled
%       col1    load number
%       col2    bus number
%       col3    MVA rating
%       col4    maximum output limit pu
%       col4    minimum output limit pu
%       col6    Time constant
lmod_con = [...
1 4  100 1 -1 1 0.02;
2 14 100 1 -1 1 0.02;
];
```

```
rlmod_con = [...
%1 4   100 1  -1  1  0.05;
%2 14  100 1  -1  1  0.05;
];


%Switching file defines the simulation control
% row 1 col1  simulation start time (s) (cols 2 to 6 zeros)
%       col7  initial time step (s)
% row 2 col1  fault application time (s)
%       col2  bus number at which fault is applied
%       col3  bus number defining far end of faulted line
%       col4  zero sequence impedance in pu on system base
%       col5  negative sequence impedance in pu on system base
%       col6  type of fault  - 0 three phase
%                            - 1 line to ground
%                            - 2 line-to-line to ground
%                            - 3 line-to-line
%                            - 4 loss of line with no fault
%                            - 5 loss of load at bus
%                            - 6 no action
%       col7  time step for fault period (s)
% row 3 col1  near end fault clearing time (s) (cols 2 to 6 zeros)
%       col7  time step for second part of fault (s)
% row 4 col1  far end fault clearing time (s) (cols 2 to 6 zeros)
%       col7  time step for fault cleared simulation (s)
% row 5 col1  time to change step length (s)
%       col7  time step (s)
%
%
%
% row n col1 finishing time (s)  (n indicates that intermediate rows may be inserted)

sw_con = [...
0    0    0    0    0    0    0.01;%sets intitial time step
0.1  3    101  0    0    0    0.01; %three phase fault at bus 3
0.15 0    0    0    0    0    0.01; %clear near end
0.20 0    0    0    0    0    0.01; %clear remote end
%0.50 0    0    0    0    0    0.01; % increase time step
%1.0  0    0    0    0    0    0.01; % increase time step
10.0  0    0    0    0    0    0]; % end simulation

%ibus_con = [0 1 1 1];% sets generators 2, 3 and 4 to be infinite buses
%behind source impedance in small signal stability model
```

9.2 Two Area System with TCSC

```
% Two Area Test Case
% subtransient generator models
% dc exciters
% turbine/governor
% 50% constant current/50% constant impedance loads
% TCSC
% single tie
% pre-fault

% bus data format
% bus:
% col1 number
% col2 voltage magnitude(pu)
% col3 voltage angle(degree)
% col4 p_gen(pu)
% col5 q_gen(pu),
% col6 p_load(pu)
% col7 q_load(pu)
% col8 G shunt(pu)
% col9 B shunt(pu)
% col10 bus_type
%       bus_type - 1, swing bus
%                - 2, generator bus (PV bus)
%                - 3, load bus (PQ bus)
% col11 q_gen_max(pu)
```

```
% col12 q_gen_min(pu)
% col13 v_rated (kV)
% col14 v_max  pu
% col15 v_min  pu
% post fault - open VAR limits freeze taps
bus = [...
   1  1.03     18.5    7.00   1.61  0.00  0.00  0.00  0.00 1   10.0  -4.0  22.0   1.1  .9;
   2  1.01      8.80   7.00   1.76  0.00  0.00  0.00  0.00 2   10.0  -4.0  22.0   1.1  .9;
   3  1.0      -6.1    0.00   0.00  0.00  0.00  0.00  3.00 3    0.0   0.0  230.0  1.5  .5;
   4  0.97    -10      0.00   0.00  9.76  1.00  0.00  0.00 3    0.0   0.0  115.0  1.5  .9;
   10 1.0103   12.1    0.00   0.00  0.00  0.00  0.00  0.00 3    0.0   0.0  230.0  1.5  .5;
   11 1.03     -6.8    7.16   1.49  0.00  0.00  0.00  0.00 2   10.0  -4.0  22.0   1.1  .9;
   12 1.01    -16.9    7.00   1.39  0.00  0.00  0.00  0.00 2   10.0  -4.0  22.0   1.1  .9;
   13 1.0     -31.8    0.00   0.00  0.00  0.00  0.00  5.00 3    0.0   0.0  230.0  1.5  .5;
   14 0.97    -38      0.00   0.00  17.17 1.00  0.00  0.00 3    0.0   0.0  115.0  1.5  .9;
   15 0.97    -38      0.00   0.00  0.50  0.00  0.00  0.00 3    0.0   0.0  115.0  1.5  .9;
   20 0.9876    2.1    0.00   0.00  0.00  0.00  0.00  0.00 3    0.0   0.0  230.0  1.5  .5;
  101 1.05    -19.3    0.00   0.50  0.00  0.00  0.00  1.00 2    1.0   0.0  230.0  1.5  .5;
  102 1.05    -19.3    0.00   0.50  0.00  0.00  0.00  1.0  2    1.0   0.0  230.0  1.5  .5;
  110 1.0125  -13.4    0.00   0.00  0.00  0.00  0.00  0.00 3    0.0   0.0  230.0  1.5  .5;
  120 0.9938  -23.6    0.00   0.00  0.00  0.00  0.00  0.00 3    0.0   0.0  230.0  1.5  .5
];


% line data format
% line: from bus, to bus, resistance(pu), reactance(pu),
%       line charging(pu), tap ratio, tap phase, tapmax, tapmin, tapsize

line = [...
1   10  0.0     0.0167   0.00    1.0  0. 0.  0.  0.;
2   20  0.0     0.0167   0.00    1.0  0. 0.  0.  0.;
3    4  0.0     0.005    0.00    1.0  0. 0.  0.  0.;
3   20  0.001   0.01     0.0175  1.0  0. 0.  0.  0.;
3   101 0.011   0.11     0.1925  1.0  0. 0.  0.  0.;
10  20  0.0025  0.025    0.0437  1.0  0. 0.  0.  0.;
11  110 0.0     0.0167   0.0     1.0  0. 0.  0.  0.;
12  120 0.0     0.0167   0.0     1.0  0. 0.  0.  0.;
13   14 0.0     0.005    0.00    1.0  0. 0.  0.  0.;
13   15 0.0     0.01     0.00    1.0  0. 0.  0.  0.;
13  102 0.011   0.11     0.1925  1.0  0. 0.  0.  0.;
101 102 0.00    -0.088   0.00    1.0  0. 0.  0.  0.;% 40% series compensation in line
13  120 0.001   0.01     0.0175  1.0  0. 0.  0.  0.;
110 120 0.0025  0.025    0.0437  1.0  0. 0.  0.  0.];


% Machine data format
% Machine data format
%       1. machine number,
%       2. bus number,
%       3. base mva,
%       4. leakage reactance x_l(pu),
%       5. resistance r_a(pu),
%       6. d-axis sychronous reactance x_d(pu),
%       7. d-axis transient reactance x'_d(pu),
%       8. d-axis subtransient reactance x"_d(pu),
%       9. d-axis open-circuit time constant T'_do(sec),
%      10. d-axis open-circuit subtransient time constant
%                T"_do(sec),
%      11. q-axis sychronous reactance x_q(pu),
%      12. q-axis transient reactance x'_q(pu),
%      13. q-axis subtransient reactance x"_q(pu),
%      14. q-axis open-circuit time constant T'_qo(sec),
%      15. q-axis open circuit subtransient time constant
%                T"_qo(sec),
%      16. inertia constant H(sec),
%      17. damping coefficient d_o(pu),
%      18. dampling coefficient d_1(pu),
%      19. bus number
%
% note: all the following machines use sub-transient model
```

```
mac_con = [ ...

1 1 900 0.200  0.0025  1.8  0.30  0.25 8.00  0.03...
                       1.7  0.55  0.25 0.4   0.05...
                       6.5  0  0  1;
2 2 900 0.200  0.0025  1.8  0.30  0.25 8.00  0.03...
                       1.7  0.55  0.25 0.4   0.05...
                       6.5  0  0  2;
3 11 900 0.200  0.0025  1.8  0.30  0.25 8.00  0.03...
                       1.7  0.55  0.25 0.4   0.05...
                       6.5  0  0  11;
4 12 900 0.200  0.0025  1.8  0.30  0.25 8.00  0.03...
                       1.7  0.55  0.25 0.4   0.05...
                       6.5  0  0  12];

% all dc exciters, no pss

exc_con = [...
1 1 0.01 20.0   0.06  0     0    1.0   -0.9...
0.0  0.46   3.1   0.33  2.3  0.1   0.1   2.0    0    0   0;
1 2 0.01 20.0   0.06  0     0    1.0   -0.9...
    0.0  0.46   3.1   0.33  2.3  0.1   0.1   2.0    0    0   0;
1 3 0.01 20.0   0.06  0     0    1.0   -0.9...
    0.0  0.46   3.1   0.33  2.3  0.1   0.1   2.0    0    0   0;
1 4 0.01 20.0   0.06  0     0    1.0   -0.9...
   0.0  0.46   3.1   0.33  2.3  0.1   0.1   2.0    0    0   0];


% governor model
% tg_con matrix format
%column        data         unit
%  1    turbine model number (=1)
%  2    machine number
%  3    speed set point   wf        pu
%  4    steady state gain 1/R       pu
%  5    maximum power order  Tmax   pu on generator base
%  6    servo time constant   Ts    sec
%  7    governor time constant  Tc  sec
%  8    transient gain time constant T3 sec
%  9    HP section time constant   T4   sec
% 10    reheater time constant    T5    sec

tg_con = [...
1  1  1  25.0  1.0  0.1  0.5 0.0 1.25 5.0;
1  2  1  25.0  1.0  0.1  0.5 0.0 1.25 5.0;
1  3  1  25.0  1.0  0.1  0.5 0.0 1.25 5.0;
1  4  1  25.0  1.0  0.1  0.5 0.0 1.25 5.0];


% non-conforming load
% col 1           bus number
% col 2           fraction const active power load
% col 3           fraction const reactive power load
% col 4           fraction const active current load
% col 5           fraction const reactive current load
load_con = [...
3   0  0   0   0;
4   0  0   .5  0;
13  0  0   0   0;
14  0  0   .5  0;
101 0  0   0  0;
102 0  0   0  0];
disp('0.5 constant current load, reactive load modulation at buses 3,4,13,14 and 101')
disp('active load moduation at bus 4 and 14, tcsc between bus 101 and bus 102')

% real load modulation
lmod_con = [...
   1 4  100 1 -1 1 0.05;
   2 14 100 1 -1 1 0.05];
%reactive load modulation
rlmod_con = [...
```

```
2 4   100 1  -1  1  0.05;
4 14  100 1  -1  1  0.05];
% modulation at buses 4,14
%tcsc
%col1 - tcsc number
%col2 - from bus number
%col3 - to bus number
%col4 - Control Gain
%col5 - Control Time Costant
%col6 - Maximum B - On system base
%col7 - Minimum B - On system base
tcsc_con = [ 1 101 102 1 .05 3 -3];
svc_dc = [];
tcsc_dc=[];
cd('C:\Program Files\MATLAB\R2007a\PSTV2\chapter10\results')
load control6
%tcsc_dc{1}=100.*sc3.*scr;%control state space object
tcsc_dc{1}= 10.*sc3;
tcsc_dc{2}=1;%tcsc number
tcsc_dc{3}=101;%bus number
tcsc_dc{4}=3;% control max output limit
tcsc_dc{5}=-3;% control min output limit
%tcsc_dc{6}=14;% number of control states
tcsc_dc{6}=4;% number of control states
%Switching file defines the simulation control
% row 1 col1  simulation start time (s) (cols 2 to 6 zeros)
%       col7  initial time step (s)
% row 2 col1  fault application time (s)
%       col2  bus number at which fault is applied
%       col3  bus number defining far end of faulted line
%       col4  zero sequence impedance in pu on system base
%       col5  negative sequence impedance in pu on system base
%       col6  type of fault  - 0 three phase
%                            - 1 line to ground
%                            - 2 line-to-line to ground
%                            - 3 line-to-line
%                            - 4 loss of line with no fault
%                            - 5 loss of load at bus
%                            - 6 no fault
%       col7  time step for fault period (s)
% row 3 col1  near end fault clearing time (s) (cols 2 to 6 zeros)
%       col7  time step for second part of fault (s)
% row 4 col1  far end fault clearing time (s) (cols 2 to 6 zeros)
%       col7  time step for fault cleared simulation (s)
% row 5 col1  time to change step length (s)
%       col7  time step (s)
%
%
%
% row n col1 finishing time (s)  (n indicates that intermediate rows may be inserted)

sw_con = [...
0    0    0    0    0    0    0.005;%sets intitial time step
0.1  13   15   0    0    0    0.005; %three phase fault at bus 13
0.2 0   0     0    0    0    0.005
0.3  0   0     0    0    0    0.005
0.5  0    0    0    0    0    0.01;
20.0  0    0    0    0    0    0]; % end simulation
% monitor all line flows
lmon_con = [1:length(line(:,1))]';
```

9.3 16 machine system

```
% data16m.m (16 Machine System Data)
% This is a 16-machine system with 86 transmission lines and
% 68 buses. Data are extracted from the GE final report
% entitled "Singular Perturbations, Coherency and
% Aggregation of Dynamic Systems," pp.6-42, July 1981.
% generator data modified - base changed to make xd = 1.8
% static exciters on all generators
% thermal governors on all generators
%

% Bus data format
% bus:
% col1 number
% col2 voltage magnitude(pu)
% col3 voltage angle(degree)
% col4 p_gen(pu)
% col5 q_gen(pu),
% col6 p_load(pu)
% col7 q_load(pu)
% col8 G shunt(pu)
% col9 B shunt(pu)
% col10 bus_type
%       bus_type - 1, swing bus
%                - 2, generator bus (PV bus)
%                - 3, load bus (PQ bus)
% col11 q_gen_max(pu)
% col12 q_gen_min(pu)


bus = [ 1 1.00    0.00   0.00   0.00   2.527  1.1856  0.00 0.00  3  0  0;
       2 1.00    0.00   0.00   0.00   0.00   0.00    0.00 0.00  3  0  0;
       3 1.00    0.00   0.00   0.00   3.22   0.02    0.00 0.00  3  0  0;
       4 1.00    0.00   0.00   0.00   5.00   1.840   0.00 0.00  3  0  0;
       5 1.00    0.00   0.00   0.00   0.00   0.00    0.00 0.00  3  0  0;
       6 1.00    0.00   0.00   0.00   0.00   0.00    0.00 0.00  3  0  0;
       7 1.00    0.00   0.00   0.00   2.34   0.84    0.00 0.00  3  0  0;
       8 1.00    0.00   0.00   0.00   5.22   1.77    0.00 0.00  3  0  0;
       9 1.00    0.00   0.00   0.00   1.04   1.25    0.00 0.00  3  0  0;
      10 1.00    0.00   0.00   0.00   0.00   0.00    0.00 0.00  3  0  0;
      11 1.00    0.00   0.00   0.00   0.00   0.00    0.00 0.00  3  0  0;
      12 1.00    0.00   0.00   0.00   0.09   0.88    0.00 0.00  3  0  0;
      13 1.00    0.00   0.00   0.00   0.00   0.00    0.00 0.00  3  0  0;
      14 1.00    0.00   0.00   0.00   0.00   0.00    0.00 0.00  3  0  0;
      15 1.00    0.00   0.00   0.00   3.200  1.5300  0.00 0.00  3  0  0;
      16 1.00    0.00   0.00   0.00   3.290  0.32    0.00 0.00  3  0  0;
      17 1.00    0.00   0.00   0.00   0.00   0.00    0.00 0.00  3  0  0;
      18 1.00    0.00   0.00   0.00   1.58   0.30    0.00 0.00  3  0  0;
      19 1.00    0.00   0.00   0.00   0.00   0.00    0.00 0.00  3  0  0;
      20 1.00    0.00   0.00   0.00   6.800  1.03    0.00 0.00  3  0  0;
      21 1.00    0.00   0.00   0.00   1.740  1.15    0.00 0.00  3  0  0;
      22 1.00    0.00   0.00   0.00   0.00   0.00    0.00 0.00  3  0  0;
      23 1.00    0.00   0.00   0.00   1.480  0.85    0.00 0.00  3  0  0;
      24 1.00    0.00   0.00   0.00   3.09  -0.92    0.00 0.00  3  0  0;
      25 1.00    0.00   0.00   0.00   2.24   0.47    0.00 0.00  3  0  0;
      26 1.00    0.00   0.00   0.00   1.39   0.17    0.00 0.00  3  0  0;
      27 1.00    0.00   0.00   0.00   2.810  0.76    0.00 0.00  3  0  0;
      28 1.00    0.00   0.00   0.00   2.060  0.28    0.00 0.00  3  0  0;
      29 1.00    0.00   0.00   0.00   2.840  0.27    0.00 0.00  3  0  0;
      30 1.00    0.00   0.00   0.00   0.00   0.00    0.00 0.00  3  0  0;
      31 1.00    0.00   0.00   0.00   0.00   0.00    0.00 0.00  3  0  0;
      32 1.00    0.00   0.00   0.00   0.00   0.00    0.00 0.00  3  0  0;
      33 1.00    0.00   0.00   0.00   1.12   0.00    0.00 0.00  3  0  0;
      34 1.00    0.00   0.00   0.00   0.00   0.00    0.00 0.00  3  0  0;
      35 1.00    0.00   0.00   0.00   0.00   0.00    0.00 0.00  3  0  0;
      36 1.00    0.00   0.00   0.00   1.02  -0.1946  0.00 0.00  3  0  0;
      37 1.00    0.00   0.00   0.00  60.00   3.00    0.00 0.00  3  0  0;
      38 1.00    0.00   0.00   0.00   0.00   0.00    0.00 0.00  3  0  0;
      39 1.00    0.00   0.00   0.00   2.67   0.126   0.00 0.00  3  0  0;
      40 1.00    0.00   0.00   0.00   0.6563 0.2353  0.00 0.00  3  0  0;
```

```
        41 1.00    0.00   0.00   0.00  10.00   2.50    0.00 0.00  3  0  0;
        42 1.00    0.00   0.00   0.00  11.50   2.50    0.00 0.00  3  0  0;
        43 1.00    0.00   0.00   0.00   0.00   0.00    0.00 0.00  3  0  0;
        44 1.00    0.00   0.00   0.00   2.6755 0.0484  0.00 0.00  3  0  0;
        45 1.00    0.00   0.00   0.00   2.08   0.21    0.00 0.00  3  0  0;
        46 1.00    0.00   0.00   0.00   1.507  0.285   0.00 0.00  3  0  0;
        47 1.00    0.00   0.00   0.00   2.0312 0.3259  0.00 0.00  3  0  0;
        48 1.00    0.00   0.00   0.00   2.4120 0.022   0.00 0.00  3  0  0;
        49 1.00    0.00   0.00   0.00   1.6400 0.29    0.00 0.00  3  0  0;
        50 1.00    0.00   0.00   0.00   2.00  -1.47    0.00 0.00  3  0  0;
        51 1.00    0.00   0.00   0.00   4.37  -1.22    0.00 0.00  3  0  0;
        52 1.00    0.00   0.00   0.00  24.70   1.23    0.00 0.00  3  0  0;
        53 1.045   0.00   2.50   0.00   0.00   0.00    0.00 0.00  2  999  -999;
        54 0.98    0.00   5.45   0.00   0.00   0.00    0.00 0.00  2  999  -999;
        55 0.983   0.00   6.50   0.00   0.00   0.00    0.00 0.00  2  999  -999;
        56 0.997   0.00   6.32   0.00   0.00   0.00    0.00 0.00  2  999  -999;
        57 1.011   0.00   5.052  0.00   0.00   0.00    0.00 0.00  2  999  -999;
        58 1.050   0.00   7.00   0.00   0.00   0.00    0.00 0.00  2  999  -999;
        59 1.063   0.00   5.60   0.00   0.00   0.00    0.00 0.00  2  999  -999;
        60 1.03    0.00   5.40   0.00   0.00   0.00    0.00 0.00  2  999  -999;
        61 1.025   0.00   8.00   0.00   0.00   0.00    0.00 0.00  2  999  -999;
        62 1.010   0.00   5.00   0.00   0.00   0.00    0.00 0.00  2  999  -999;
        63 1.000   0.00  10.000  0.00   0.00   0.00    0.00 0.00  2  999  -999;
        64 1.0156  0.00  13.50   0.00   0.00   0.00    0.00 0.00  2  999  -999;
        65 1.011   0.00  35.91   0.00   0.00   0.00    0.00 0.00  1  0  0;
        66 1.00    0.00  17.85   0.00   0.00   0.00    0.00 0.00  2  999  -999;
        67 1.000   0.00  10.00   0.00   0.00   0.00    0.00 0.00  2  999  -999;
        68 1.000   0.00  40.00   0.00   0.00   0.00    0.00 0.00  2  999  -999;
];


% Line data format
% line: from bus, to bus, resistance(pu), reactance(pu),
%       line charging(pu), tap ratio, phase shift(deg)
disp('all lines in service')
line = [ ...
         1   2  0.0070  0.0822  0.3493  0      0.;
         1  30  0.0008  0.0074  0.48    0      0.;
         2   3  0.0013  0.0151  0.2572  0      0.;
         2  25  0.007   0.0086  0.146   0      0.;
         2  53  0.      0.0181  0.      1.025  0.;
         3   4  0.0013  0.0213  0.2214  0.     0.;
         3  18  0.0011  0.0133  0.2138  0.     0.;
         4   5  0.0008  0.0128  0.1342  0.     0.;
         4  14  0.0008  0.0129  0.1382  0.     0.;
         5   6  0.0002  0.0026  0.0434  0.     0.;
         5   8  0.0008  0.0112  0.1476  0.     0.;
         6   7  0.0006  0.0092  0.1130  0.     0.;
         6  11  0.0007  0.0082  0.1389  0.     0.;
         6  54  0.      0.0250  0.      1.07   0.;
         7   8  0.0004  0.0046  0.078   0.     0.;
         8   9  0.0023  0.0363  0.3804  0.     0.;
         9  30  0.0019  0.0183  0.29    0.     0.;
        10  11  0.0004  0.0043  0.0729  0.     0.;
        10  13  0.0004  0.0043  0.0729  0.     0.;
        10  55  0.      0.02    0.      1.07   0.;
        12  11  0.0016  0.0435  0.      1.06   0.;
        12  13  0.0016  0.0435  0.      1.06   0.;
        13  14  0.0009  0.0101  0.1723  0.     0.;
        14  15  0.0018  0.0217  0.366   0.     0.;
        15  16  0.0009  0.0094  0.171   0.     0.;
        16  17  0.0007  0.0089  0.1342  0.     0.;
        16  19  0.0016  0.0195  0.3040  0.     0.;
        16  21  0.0008  0.0135  0.2548  0.     0.;
        16  24  0.0003  0.0059  0.0680  0.     0.;
        17  18  0.0007  0.0082  0.1319  0.     0.;
        17  27  0.0013  0.0173  0.3216  0.     0.;
        19  20  0.0007  0.0138  0.      1.06   0.;
        19  56  0.0007  0.0142  0.      1.07   0.;
        20  57  0.0009  0.0180  0.      1.009  0.;
```

```
        21  22  0.0008  0.0140  0.2565  0.      0.;
        22  23  0.0006  0.0096  0.1846  0.      0.;
        22  58  0.      0.0143  0.      1.025   0.;
        23  24  0.0022  0.0350  0.3610  0.      0.;
        23  59  0.0005  0.0272  0.      0.      0.;
        25  26  0.0032  0.0323  0.5310  0.      0.;
        25  60  0.0006  0.0232  0.      1.025   0.;
        26  27  0.0014  0.0147  0.2396  0.      0.;
        26  28  0.0043  0.0474  0.7802  0.      0.;
        26  29  0.0057  0.0625  1.0290  0.      0.;
        28  29  0.0014  0.0151  0.2490  0.      0.;
        29  61  0.0008  0.0156  0.      1.025   0.;
         9  30  0.0019  0.0183  0.29    0.      0.;
         9  36  0.0022  0.0196  0.34    0.      0.;
         9  36  0.0022  0.0196  0.34    0.      0.;
        36  37  0.0005  0.0045  0.32    0.      0.;
        34  36  0.0033  0.0111  1.45    0.      0.;
        35  34  0.0001  0.0074  0.      0.946   0.;
        33  34  0.0011  0.0157  0.202   0.      0.;
        32  33  0.0008  0.0099  0.168   0.      0.;
        30  31  0.0013  0.0187  0.333   0.      0.;
        30  32  0.0024  0.0288  0.488   0.      0.;
         1  31  0.0016  0.0163  0.25    0.      0.;
        31  38  0.0011  0.0147  0.247   0.      0.;
        33  38  0.0036  0.0444  0.693   0.      0.;
        38  46  0.0022  0.0284  0.43    0.      0.;
        46  49  0.0018  0.0274  0.27    0.      0.;
         1  47  0.0013  0.0188  1.31    0.      0.;
        47  48  0.0025  0.0268  0.40    0.      0.;
        47  48  0.0025  0.0268  0.40    0.      0.;
        48  40  0.0020  0.022   1.28    0.      0.;
        35  45  0.0007  0.0175  1.39    0.      0.;
        37  43  0.0005  0.0276  0.      0.      0.;
        43  44  0.0001  0.0011  0.      0.      0.;
        44  45  0.0025  0.073   0.      0.      0.;
        39  44  0.      0.0411  0.      0.      0.;
        39  45  0.      0.0839  0.      0.      0.;
        45  51  0.0004  0.0105  0.72    0.      0.;
        50  52  0.0012  0.0288  2.06    0.      0.;
        50  51  0.0009  0.0221  1.62    0.      0.;
        49  52  0.0076  0.1141  1.16    0.      0.;
        52  42  0.0040  0.0600  2.25    0.      0.;
        42  41  0.0040  0.0600  2.25    0.      0.;
        41  40  0.0060  0.0840  3.15    0.      0.;
        31  62  0.      0.026   0.      1.04    0.;
        32  63  0.      0.013   0.      1.04    0.;
        36  64  0.      0.0075  0.      1.04    0.;
        37  65  0.      0.0033  0.      1.04    0.;
        41  66  0.      0.0015  0.      1.      0.;
        42  67  0.      0.0015  0.      1.      0.;
        52  68  0.      0.0030  0.      1.      0.;
         1  27  0.032   0.32    0.41    1.      0.];


% Machine data format
%       1. machine number,
%       2. bus number,
%       3. base mva,
%       4. leakage reactance x_l(pu),
%       5. resistance r_a(pu),
%       6. d-axis sychronous reactance x_d(pu),
%       7. d-axis transient reactance x'_d(pu),
%       8. d-axis subtransient reactance x"_d(pu),
%       9. d-axis open-circuit time constant T'_do(sec),
%      10. d-axis open-circuit subtransient time constant
%                T"_do(sec),
%      11. q-axis sychronous reactance x_q(pu),
%      12. q-axis transient reactance x'_q(pu),
%      13. q-axis subtransient reactance x"_q(pu),
%      14. q-axis open-circuit time constant T'_qo(sec),
%      15. q-axis open circuit subtransient time constant
```

```
%                T"_qo(sec),
%      16. inertia constant H(sec),
%      17. damping coefficient d_o(pu),
%      18. dampling coefficient d_1(pu),
%      19. bus number
%      20. saturation factor S(1.0)
%      21. saturation factor S(1.2)
% note: all the following machines use subtransient reactance model

mac_con = [...
  1  53    1800  0.0125  0  1.8    0.558     0.45  10.2  0.05 ...
                            1.242  0.504     0.45   1.5  0.035 ...
                            2.3333  0   0  53  0   0;
  2  54  610.17   0.035  0  1.8      0.42529  0.30508  6.56  0.05 ...
                            1.7207   0.3661   0.30508  1.5   0.035 ...
                            4.9494  0   0  54  0   0;
  3  55  721.44  0.0304  0  1.8    0.38309  0.32465   5.7  0.05 ...
                            1.7098 0.36072  0.32465   1.5  0.035 ...
                            4.9623  0   0  55  0   0;
  4  56  687.02  0.0295  0  1.8     0.29954  0.24046  5.69  0.05 ...
                            1.7725  0.27481  0.24046   1.5  0.035 ...
                            4.1629  0   0  56  0   0;
  5  57  545.45   0.027  0  1.8     0.36  0.27273   5.4  0.05 ...
                            1.6909  0.32727  0.27273  0.44  0.035 ...
                            4.7667  0   0  57  0   0;
  6  58  708.66  0.0224  0  1.8  0.35433  0.28346   7.3  0.05 ...
                            1.7079   0.3189  0.28346   0.4  0.035 ...
                            4.9107  0   0  58  0   0;
  7  59  610.17  0.0322  0  1.8  0.29898  0.24407  5.66  0.05 ...
                            1.7817  0.27458  0.24407   1.5  0.035 ...
                            4.3267  0   0  59  0   0;
  8  60  620.69   0.028  0  1.8  0.35379  0.27931   6.7  0.05 ...
                            1.7379  0.31034  0.27931  0.41  0.035 ...
                            3.915  0   0  60  0   0;
  9  61   854.7  0.0298  0  1.8  0.48718  0.38462  4.79  0.05 ...
                            1.7521  0.42735  0.38462  1.96  0.035 ...
                            4.0365  0   0  61  0   0;
 10  62  1065.1  0.0199  0  1.8  0.48675  0.42604  9.37  0.05 ...
                            1.2249  0.47929  0.42604   1.5  0.035 ...
                            2.9106  0   0  62  0   0;
 11  63  1406.3  0.0103  0  1.8  0.25312  0.16875   4.1  0.05 ...
                            1.7297  0.21094  0.16875   1.5  0.035 ...
                            2.0053  0   0  63  0   0;
 12  64  1782.2   0.022  0  1.8  0.55248  0.44554   7.4  0.05 ...
                            1.6931  0.49901  0.44554   1.5  0.035 ...
                            5.1791  0   0  64  0   0;
 13  65   12162   0.003  0  1.8  0.33446  0.24324   5.9  0.05 ...
                            1.7392  0.30405  0.24324   1.5  0.035 ...
                            4.0782  4   0  65  0   0;
 14  66   10000  0.0017  0  1.8    0.285     0.23   4.1  0.05 ...
                            1.73     0.25     0.23   1.5  0.035 ...
                            3  3   0  66  0   0;
 15  67   10000  0.0017  0  1.8    0.285     0.23   4.1  0.05 ...
                            1.73     0.25     0.23   1.5  0.035  ...
                            3  3   0  67  0   0;
 16  68   10112  0.0041  0  1.8  0.35899  0.27809   7.8  0.05 ...
                            1.6888  0.30337  0.27809   1.5  0.035 ...
                            4.45  4   0  68  0   0;
];

% identical simple exciters on all generators
exc_con = [...
   0 1 0 200.  0.05 0 0 5.  -5.  0    0 0 0 0 0 0 0 0 0 0;
   0 2 0 200.  0.05 0 0 5.  -5.  0    0 0 0 0 0 0 0 0 0 0;
   0 3 0 200.  0.05 0 0 5.  -5.  0    0 0 0 0 0 0 0 0 0 0;
   0 4 0 200.  0.05 0 0 5.  -5.  0    0 0 0 0 0 0 0 0 0 0;
   0 5 0 200.  0.05 0 0 5.  -5.  0    0 0 0 0 0 0 0 0 0 0;
   0 6 0 200.  0.05 0 0 5.  -5.  0    0 0 0 0 0 0 0 0 0 0;
   0 7 0 200.  0.05 0 0 5.  -5.  0    0 0 0 0 0 0 0 0 0 0;
   0 8 0 200.  0.05 0 0 5.  -5.  0    0 0 0 0 0 0 0 0 0 0;
   0 9 0 200.  0.05 0 0 5.  -5.  0    0 0 0 0 0 0 0 0 0 0;
```

```
   0 10 0 200. 0.05 0 0 5.  -5.  0    0 0 0 0 0 0 0 0 0 0;
   0 11 0 200. 0.05 0 0 5.  -5.  0    0 0 0 0 0 0 0 0 0 0;
   0 12 0 200. 0.05 0 0 5.  -5.  0    0 0 0 0 0 0 0 0 0 0;
   0 13 0 200. 0.05 0 0 5.  -5.  0    0 0 0 0 0 0 0 0 0 0;
   0 14 0 200. 0.05 0 0 5.  -5.  0    0 0 0 0 0 0 0 0 0 0;
   0 15 0 200. 0.05 0 0 5.  -5.  0    0 0 0 0 0 0 0 0 0 0;
   0 16 0 200  0.05 0 0 5.  -5.  0    0 0 0 0 0 0 0 0 0 0];
% power system stabilizerS
%       col1    type 1 speed input; 2 power input
%       col2    generator number
%       col3    pssgain*washout time constant
%       col4    washout time constant
%       col5    first lead time constant
%       col6    first lag time constant
%       col7    second lead time constant
%       col8    second lag time constant
%       col9    maximum output limit
%       col10   minimum output limit
pss_con = [...
     1  2  100  10 0.08 0.02 0.08 0.02 0.2 -0.05;
     1  3  100  10 0.08 0.02 0.05 0.01 0.2 -0.05;
     1  4  100  10 0.08 0.02 0.08 0.02 0.2 -0.05;
     1  5  100  10 0.08 0.02 0.08 0.02 0.2 -0.05;
     1  6  100  10 0.08 0.02 0.08 0.02 0.2 -0.05;
     1  7  100  10 0.08 0.02 0.08 0.02 0.2 -0.05;
     1  8   40  10 0.08 0.02 0.08 0.02 0.2 -0.05;
     1  9  100  10 0.05 0.02 0.05 0.02 0.2 -0.05;
     1  10 100  10 0.08 0.02 0.08 0.02 0.2 -0.05;
     1  11  60  10 0.05 0.02 0.05 0.02 0.2 -0.05;
     1  12 100  10 0.10 0.02 0.10 0.02 0.2 -0.05;

 ];


%column       data                  unit
%  1    turbine model number (=1)
%  2    machine number
%  3    speed set point   wf          pu
%  4    steady state gain 1/R         pu
%  5    maximum power order  Tmax     pu on generator base
%  6    servo time constant   Ts      sec
%  7    governor time constant  Tc    sec
%  8    transient gain time constant T3       sec
%  9    HP section time constant   T4 sec
% 10    reheater time constant    T5  sec
disp('thermal turbine/governors on all units')
tg_con = [...
1  1  1  25.0  1.0  0.1  0.5 0.0 1.25 5.0;
1  2  1  25.0  1.0  0.1  0.5 0.0 1.25 5.0;
1  3  1  25.0  1.0  0.1  0.5 0.0 1.25 5.0;
1  4  1  25.0  1.0  0.1  0.5 0.0 1.25 5.0;
1  5  1  25.0  1.0  0.1  0.5 0.0 1.25 5.0;
1  6  1  25.0  1.0  0.1  0.5 0.0 1.25 5.0;
1  7  1  25.0  1.0  0.1  0.5 0.0 1.25 5.0;
1  8  1  25.0  1.0  0.1  0.5 0.0 1.25 5.0;
1  9  1  25.0  1.0  0.1  0.5 0.0 1.25 5.0;
1 10  1  25.0  1.0  0.1  0.5 0.0 1.25 5.0;
1 11  1  25.0  1.0  0.1  0.5 0.0 1.25 5.0;
1 12  1  25.0  1.0  0.1  0.5 0.0 1.25 5.0;
1 13  1  25.0  1.0  0.1  0.5 0.0 1.25 5.0;
1 14  1  25.0  1.0  0.1  0.5 0.0 1.25 5.0;
1 15  1  25.0  1.0  0.1  0.5 0.0 1.25 5.0;
1 16  1  25.0  1.0  0.1  0.5 0.0 1.25 5.0;
];



%ibus_con=ones(16,1); ibus_con(16)=0;
%Switching file defines the simulation control
% row 1 col1  simulation start time (s) (cols 2 to 6 zeros)
```

```
%       col7  initial time step (s)
% row 2 col1  fault application time (s)
%       col2  bus number at which fault is applied
%       col3  bus number defining far end of faulted line
%       col4  zero sequence impedance in pu on system base
%       col5  negative sequence impedance in pu on system base
%       col6  type of fault  - 0 three phase
%                            - 1 line to ground
%                            - 2 line-to-line to ground
%                            - 3 line-to-line
%                            - 4 loss of line with no fault
%                            - 5 loss of load at bus
%       col7  time step for fault period (s)
% row 3 col1  near end fault clearing time (s) (cols 2 to 6 zeros)
%       col7  time step for second part of fault (s)
% row 4 col1  far end fault clearing time (s) (cols 2 to 6 zeros)
%       col7  time step for fault cleared simulation (s)
% row 5 col1  time to change step length (s)
%       col7  time step (s)
%
%
%
% row n col1 finishing time (s)  (n indicates that intermediate rows may be inserted)

sw_con = [...
0    0    0    0    0    0    0.01;%sets intitial time step
0.1  1    27    0    0    0    0.005; %apply three phase fault at bus 1, on line 1-2
0.15 0    0    0    0    0    0.005; %clear fault at bus
0.20 0    0    0    0    0    0.005; %clear remote end
0.50 0    0    0    0    0    0.01; % increase time step
1.0  0    0    0    0    0   0.01; % increase time step
5.0  0    0    0    0    0    0]; % end simulation
%
%
% non-conforming load
% col 1           bus number
% col 2           fraction const active power load
% col 3           fraction const reactive power load
% col 4           fraction const active current load
% col 5           fraction const reactive current load

% non-conforming load on all load buses
disp('active loads 50/50 constant current/constant impedance; reactive loads constant
impedance')
load_con = [...
1  0 0 .5 0;
3  0 0 .5 0;
4  0 0 .5 0;
7  0 0 .5 0;
8  0 0 .5 0;
9  0 0 .5 0;
15 0 0 .5 0;
16 0 0 .5 0;
18 0 0 .5 0;
20 0 0 .5 0;
21 0 0 .5 0;
23 0 0 .5 0;
24 0 0 .5 0;
25 0 0 .5 0;
26 0 0 .5 0;
27 0 0 .5 0;
28 0 0 .5 0;
29 0 0 .5 0;
33 0 0 .5 0;
37 0 0 .5 0;
39 0 0 .5 0;
40 0 0 .5 0;
41 0 0 .5 0;
42 0 0 .5 0;
44 0 0 .5 0;
45 0 0 .5 0;
```

```
46 0 0 .5 0;
47 0 0 .5 0;
48 0 0 .5 0;
49 0 0 .5 0;
50 0 0 .5 0;
51 0 0 .5 0;
52 0 0 .5 0;
];
%load_con(:,4)=0;% constant impedance loads
lmod_con = [...
1  1  100 1 -1 1 0.05 ;
2  3  100 1 -1 1 0.05 ;
3  4  100 1 -1 1 0.05 ;
4  7  100 1 -1 1 0.05 ;
5  8  100 1 -1 1 0.05 ;
6  9  100 1 -1 1 0.05 ;
7  15 100 1 -1 1 0.05 ;
8  16 100 1 -1 1 0.05 ;
9  18 100 1 -1 1 0.05 ;
10 20 100 1 -1 1 0.05 ;
11 21 100 1 -1 1 0.05 ;
12 23 100 1 -1 1 0.05 ;
13 24 100 1 -1 1 0.05 ;
14 25 100 1 -1 1 0.05 ;
15 26 100 1 -1 1 0.05 ;
16 27 100 1 -1 1 0.05 ;
17 28 100 1 -1 1 0.05 ;
18 29 100 1 -1 1 0.05 ;
19 33 100 1 -1 1 0.05 ;
20 37 100 1 -1 1 0.05 ;
21 39 100 1 -1 1 0.05 ;
22 40 100 1 -1 1 0.05 ;
23 41 100 1 -1 1 0.05 ;
24 42 100 1 -1 1 0.05 ;
25 44 100 1 -1 1 0.05 ;
26 45 100 1 -1 1 0.05 ;
27 46 100 1 -1 1 0.05 ;
28 47 100 1 -1 1 0.05 ;
29 48 100 1 -1 1 0.05 ;
30 49 100 1 -1 1 0.05 ;
31 50 100 1 -1 1 0.05 ;
32 51 100 1 -1 1 0.05 ;
33 52 100 1 -1 1 0.05 ;
];
lmod_con = [];
% induction motor data
% 1. Motor Number
% 2. Bus Number
% 3. Motor MVA Base
% 4. rs pu
% 5. xs pu - stator leakage reactance
% 6. Xm pu - magnetizing reactance
% 7. rr pu
% 8. xr pu - rotor leakage reactance
% 9. H  s  - motor plus load inertia constant
% 10. rr1 pu - second cage resistance
% 11. xr1 pu - intercage reactance
% 12. dbf    - deepbar factor
% 13. isat pu - saturation current
% 15. fraction of bus power drawn by motor ( if zero motor starts at t=0)

% Motor Load Data
% format for motor load data - mld_con
% 1 motor number
% 2 bus number
% 3 stiction load pu on motor base (f1)
% 4 stiction load coefficient (i1)
% 5 external load  pu on motor base(f2)
% 6 external load coefficient (i2)
%
% load has the form
```

```
% tload = f1*slip^i1 + f2*(1-slip)^i2
ind_con = [ ...
  1   1  150  .03274 .08516 3.7788  .06164 .06005  1.0 0.01354 0.07517 0     3 0 .5;
  2   3  200  .03274 .08516 3.7788  .06164 .06005  1.0 0.01354 0.07517 0     3 0 .5;
  3   4  350  .03274 .08516 3.7788  .06164 .06005  1.0 0.01354 0.07517 0     3 0 .5;
  4   7  150  .03274 .08516 3.7788  .06164 .06005  1.0 0.01354 0.07517 0     3 0 .5;
  5   8  325  .03274 .08516 3.7788  .06164 .06005  1.0 0.01354 0.07517 0     3 0 .5;
  6   9   60  .03274 .08516 3.7788  .06164 .06005  1.0 0.01354 0.07517 0     3 0 .5;
  7   12   5  .03274 .08516 3.7788  .06164 .06005  1.0 0.01354 0.07517 0     3 0 .5;
  8   15 200  .03274 .08516 3.7788  .06164 .06005  1.0 0.01354 0.07517 0     3 0 .5;
  9   16 200  .03274 .08516 3.7788  .06164 .06005  1.0 0.01354 0.07517 0     3 0 .5;
  10  18 100  .03274 .08516 3.7788  .06164 .06005  1.0 0.01354 0.07517 0     3 0 .5;
  11  20 400  .03274 .08516 3.7788  .06164 .06005  1.0 0.01354 0.07517 0     3 0 .5;
  12  21 100  .03274 .08516 3.7788  .06164 .06005  1.0 0.01354 0.07517 0     3 0 .5;
  13  23 100  .03274 .08516 3.7788  .06164 .06005  1.0 0.01354 0.07517 0     3 0 .5;
  14  24 175  .03274 .08516 3.7788  .06164 .06005  1.0 0.01354 0.07517 0     3 0 .5;
  15  25 150  .03274 .08516 3.7788  .06164 .06005  1.0 0.01354 0.07517 0     3 0 .5;
  16  26 100  .03274 .08516 3.7788  .06164 .06005  1.0 0.01354 0.07517 0     3 0 .5;
  17  27 200  .03274 .08516 3.7788  .06164 .06005  1.0 0.01354 0.07517 0     3 0 .5;
  18  28 120  .03274 .08516 3.7788  .06164 .06005  1.0 0.01354 0.07517 0     3 0 .5;
  19  29 200  .03274 .08516 3.7788  .06164 .06005  1.0 0.01354 0.07517 0     3 0 .5;
  20  33  75  .03274 .08516 3.7788  .06164 .06005  1.0 0.01354 0.07517 0     3 0 .5;
  21  36  75  .03274 .08516 3.7788  .06164 .06005  1.0 0.01354 0.07517 0     3 0 .5;
  22  40  50  .03274 .08516 3.7788  .06164 .06005  1.0 0.01354 0.07517 0     3 0 .5;
  23  45 120  .03274 .08516 3.7788  .06164 .06005  1.0 0.01354 0.07517 0     3 0 .5;
  24  46 100  .03274 .08516 3.7788  .06164 .06005  1.0 0.01354 0.07517 0     3 0 .5;
  25  47 120  .03274 .08516 3.7788  .06164 .06005  1.0 0.01354 0.07517 0     3 0 .5;
  26  48 200  .03274 .08516 3.7788  .06164 .06005  1.0 0.01354 0.07517 0     3 0 .5;
  27  49 110  .03274 .08516 3.7788  .06164 .06005  1.0 0.01354 0.07517 0     3 0 .5;
  28  50 130  .03274 .08516 3.7788  .06164 .06005  1.0 0.01354 0.07517 0     3 0 .5;
  29  51 300  .03274 .08516 3.7788  .06164 .06005  1.0 0.01354 0.07517 0     3 0 .5;
];
ind_con = [];

mld_con = [ ...
1  1  .1  1  .7  2;
2  3  .1  1  .7  2;
3  4  .1  1  .7  2;
4  7  .1  1  .7  2;
5  8  .1  1  .7  2;
6  9  .1  1  .7  2;
7  12 .1  1  .7  2;
8  15 .1  1  .7  2;
9  16 .1  1  .7  2;
10 18 .1  1  .7  2;
11 20 .1  1  .7  2;
12 21 .1  1  .7  2;
13 23 .1  1  .7  2;
14 24 .1  1  .7  2;
15 25 .1  1  .7  2;
16 26 .1  1  .7  2;
17 27 .1  1  .7  2;
18 28 .1  1  .7  2;
19 29 .1  1  .7  2;
20 33 .1  1  .7  2;
21 36 .1  1  .7  2;
22 40 .1  1  .7  2;
23 45 .1  1  .7  2;
24 46 .1  1  .7  2;
25 47 .1  1  .7  2;
26 48 .1  1  .7  2;
27 49 .1  1  .7  2;
28 50 .1  1  .7  2;
29 51 .1  1  .7  2;
];
```