Sandia National Laboratories

Exceptional service in the national interest

Measurement Decision Risk

Measurement uncertainty and conformity assessment

Primary Standards Lab

SAND2022-5860 TR

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Class Agenda

1. Decision making and conformity assessment
2. Specific and global risk
3. Determining prior information
4. Simplified risk metrics
5. Risk management and guardbanding
6. When TUR doesn’t work

Introductions

Introduce yourself

- Name
- Organization
- Background (EE, ME, Quality, etc.)
- Type of measurement (calibration, product inspection, etc.)

Learning Objectives

1. Define “measurement decision” and how uncertainty in a measurement leads to risk in the decision
2. Calculate test uncertainty ratios given tolerance limits and measurement uncertainty
3. Understand the limitations and assumptions of the test uncertainty ratio and 4:1 metric
4. Understand the difference between specific and global risk and the formulas used to compute them
5. Use Sandia’s uncertainty calculator to calculate specific and global risk for various measurement cases
6. Calculate guardband acceptance limits based on TUR or targeted risk levels
7. Understand risk in one-sided limits
8. Select appropriate acceptance limits and decision rules based on the available information for a measurement

Prerequisites

- Basic understanding of Measurement Uncertainty Evaluation
- Familiarity with Probability Density Functions and Calculus (i.e. double integrals)

Q1. What quantity are you measuring?			Q2: How accurate do the measurements need to be?			Q3: How will you ensure your equipment can make this measurement?			
Quantity Measured	MC	Requirement Number	Value or Range of Values Measured	Tolerance Limits	Guardbanded Acceptance Limit	Equipment Used (M&TE)	Measurement Uncertainty	TUR (>4 desired)	Calibrate (Yes/No)

This class: what does TUR and guardband really mean about a measurement?

Reference Materials

- JCGM 106:2012. Evaluation of measurement data – The role of measurement uncertainty in conformity assessment. https://www.bipm.org/en/publications/guides

- S. Crowder, C. Delker, E. Forrest, N. Martin. *Introduction to Statistics in Metrology*. Springer, 2020. (esp. Chapter 5)

This class follows notation in JCGM 106. Literature on risk evaluation is very inconsistent and often leads to confusion. See Appendix slides for translation of conventions to other authors' work.

Section 1
Measurement Decisions and Conformity Assessment

Section 1 – Measurement Decisions and Conformity Assessment

Objective

- Define “measurement decision” and “conformity assessment” and describe how uncertainty in measurement leads to risk in the decision outcome

Content

- Risk assessment
- Types of measurement decisions
- Conformity assessment
- Risk in decision outcome
- Standards and Requirements

Examples of decisions based on measurement

- **Calibration**: Is the power supply operating within tolerance?
- **Manufacturing**: Does the widget meet its engineering specifications?
- **R&D**: Should the grant proposal be funded based on initial data?
- **Application**: Should the aircraft take off based on measured air speed?
- **Commerce**: How much should you pay for that tank of gasoline?
- **Entertainment**: Can we use this football in the Super Bowl?

All measurements have uncertainty, so all decisions have risk of being incorrect!

What is risk? (Quality engineer's definition)

- Risk describes how uncertain events may cause loss to a particular individual or group
- Complete identification of risk requires definition of:
  - Undesirable Events
  - Likelihood or uncertainty of undesirable events
  - Consequences of undesirable events, quantification of loss

What is risk? (Quality engineer's definition)

Risk describes how uncertain events may cause loss to a particular individual or group.

Identification of: Undesirable events, probability of those events, and quantification of cost of those events

Overall Risk = (Probability of an undesirable outcome) × (Consequences of the undesirable outcome)

Likelihood Tier	Consequence Tier				
	Catastrophic	Severe	Moderate	Low	Negligible
Very High	VH	VH	H	M	L
High	VH	H	M	M	L
Moderate	H	M	M	L	L
Low	M	M	L	L	N
Negligible	L	L	L	N	N

See SNL Policy QA001.2 for general guidance on evaluating consequence and likelihood levels.

Definition: Conformity Assessment

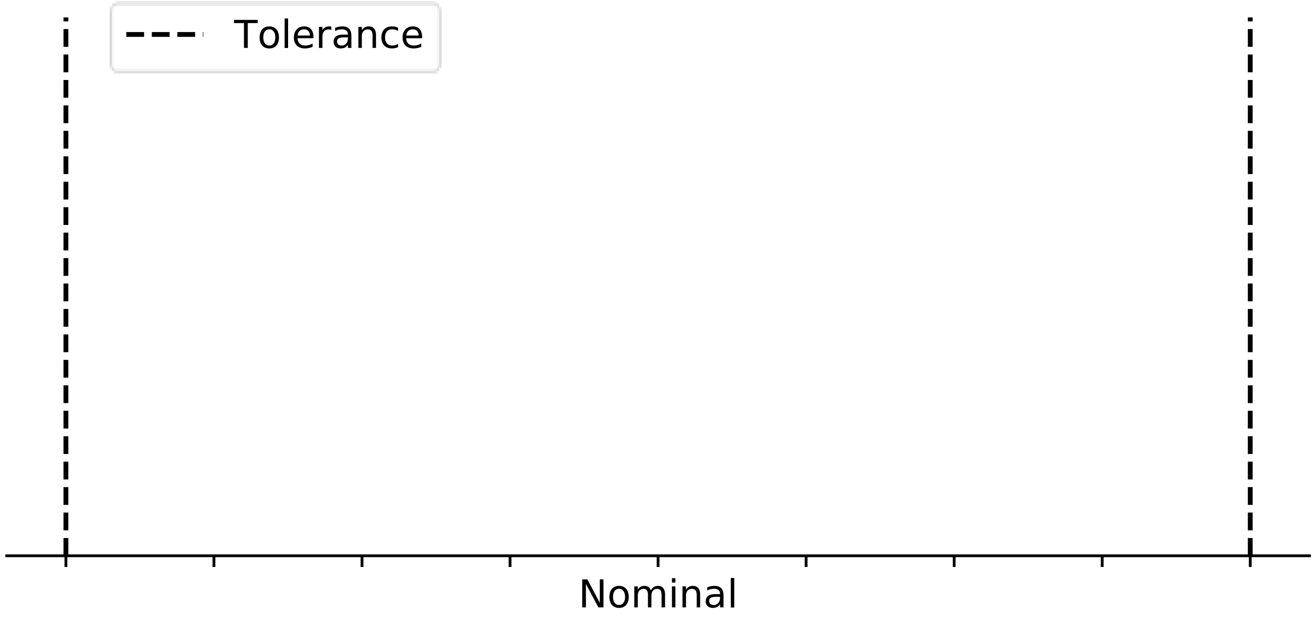
Conformity Assessment is any activity undertaken to determine, directly or indirectly, whether a product, process, system, person, or body meets relevant standards and fulfills specified requirements.

– JCGM 106

Measurement Decision

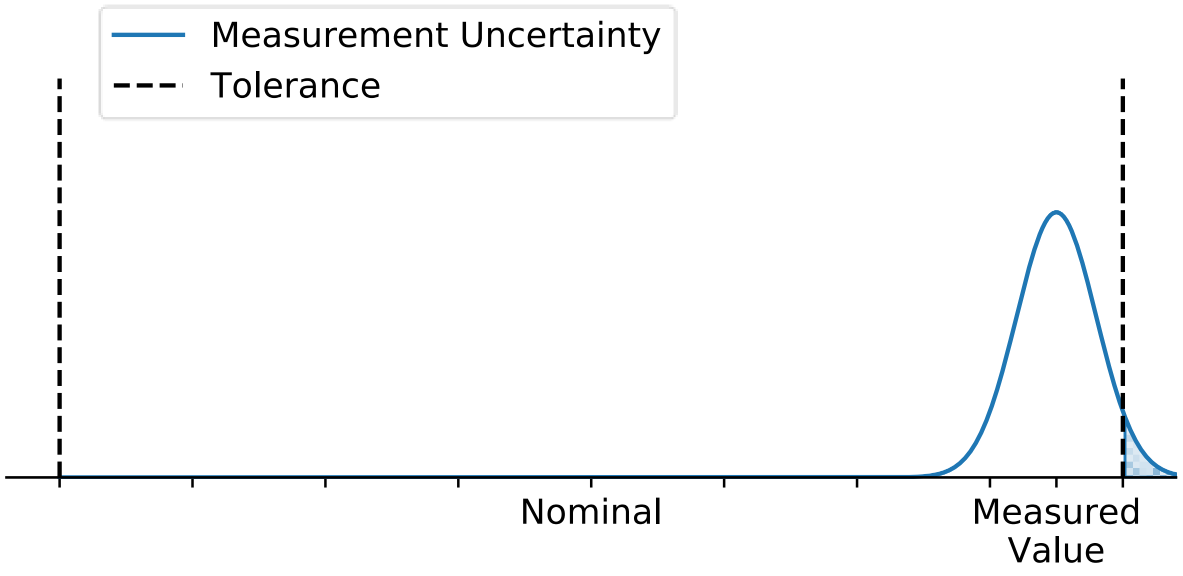
To meet requirements, a measurement must be between the dashed lines

- Calibration limits
- Engineering specification on manufactured components
- Airspeed required for safe takeoff
- Etc.



Measurement Decision

The True Value is never known because all measurements have uncertainty!



What is risk? (Metrologist's definition)

Measurement Risk = Probability of accepting a bad DUT or rejecting a good DUT

Likelihood Tier	Consequence Tier				
	Catastrophic	Severe	Moderate	Low	Negligible
Very High	VH	VH	H	M	L
High	VH	H	M	M	L
Moderate	H	M	M	L	L
Low	M	M	L	L	N
Negligible	L	L	L	N	N

This course is concerned with quantifying and mitigating the probability of an undesirable outcome based on a measurement.

Measurement Risk is mitigated by 1) calibrating equipment and 2) Measurement Assurance Plans that determine and document this likelihood

Binary Decision Rule – Four possible outcomes

	True Value In Tolerance	True Value Out of Tolerance
Measured Value PASS	True Accept	False Accept (Consumer Risk)
Measured Value FAIL	False Reject (Producer Risk)	True Reject

Question

Which of the following does not apply to the complete definition of risk?:

A. Undesirable Events
B. Likelihood or uncertainty of undesirable events
C. Guardbanding
D. Quantification of loss and affected parties due to those events

Consequences of Bad Measurements – Deflategate

Jeffrey Beall/CC-BY-SA 4.0

Exponent Engineering: The Effect of Various Environmental and Physical Factors on the Measured Internal Pressure of NFL Footballs, 2015.

Use a Measurement Assurance Plan to document measurement adequacy

Step 1 – List product requirements

- NFL Rule Book

Q1. What quantity are you measuring?			Q2: How accurate do the measurements need to be?			Q3: How will you ensure your equipment can make this measurement?			
Quantity Measured	MC	Requirement Number	Value or Range of Values Measured	Tolerance Limits	Guardbanded Acceptance Limit	Equipment Used (M&TE)	Measurement Uncertainty	TUR (>4 desired)	Calibrate (Yes/No)
Football Air Pressure	*****	**Rule 2, Section 1**	**13 "pounds" (psig)**	**± 0.5 "pounds" (psig)**					

Use a Measurement Assurance Plan to document measurement adequacy

Step 2 – Evaluate the equipment and measurement uncertainty

- Pressure gage uncertainty from analysis in Crowder, et. al.

Q1. What quantity are you measuring?			Q2: How accurate do the measurements need to be?			Q3: How will you ensure your equipment can make this measurement?			
Quantity Measured	MC	Requirement Number	Value or Range of Values Measured	Tolerance Limits	Guardbanded Acceptance Limit	Equipment Used (M&TE)	Measurement Uncertainty	TUR (>4 desired)	Calibrate (Yes/No)
Football Air Pressure	*	**Rule 2, Section 1**	**13 "pounds" (psig)**	**± 0.5 "pounds" (psig)**		**Wilson (?) pressure gauge**	**± 1 psig**		**Y**

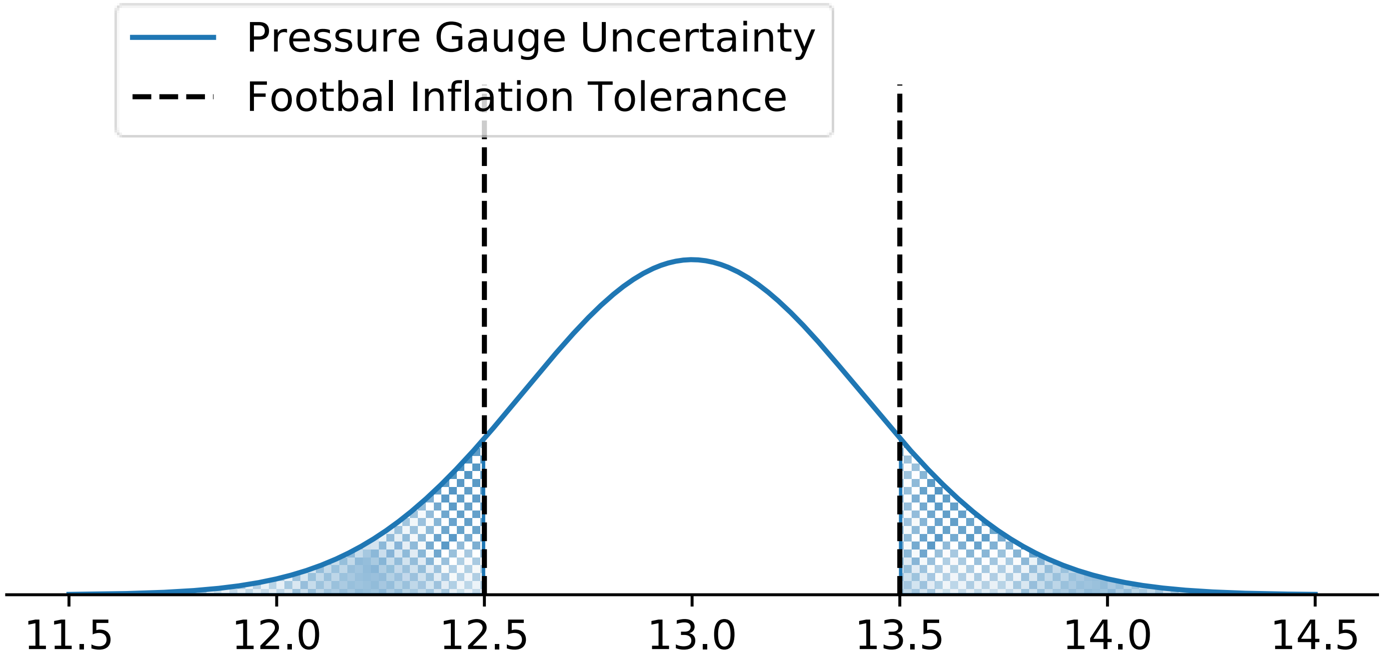
Use a Measurement Assurance Plan to document measurement adequacy

Step 3 – Calculate the Test Uncertainty Ratio (TUR).

- TUR = (± Tolerance Limits)/(± Measurement Uncertainty @ 95%)
- General rule: If TUR ≥ 4, the measurement is OK.

Q1. What quantity are you measuring?			Q2: How accurate do the measurements need to be?			Q3: How will you ensure your equipment can make this measurement?			
Quantity Measured	MC	Requirement Number	Value or Range of Values Measured	Tolerance Limits	Guardbanded Acceptance Limit	Equipment Used (M&TE)	Measurement Uncertainty	TUR (>4 desired)	Calibrate (Yes/No)
Football Air Pressure	*****	**Rule 2, Section 1**	**13 "pounds" (psig)**	**± 0.5 "pounds" (psig)**		**Wilson (?) pressure gauge**	**± 1 psig**	**0.5**	**Y**

Deflategate Measurement – What happens when your TUR is too low



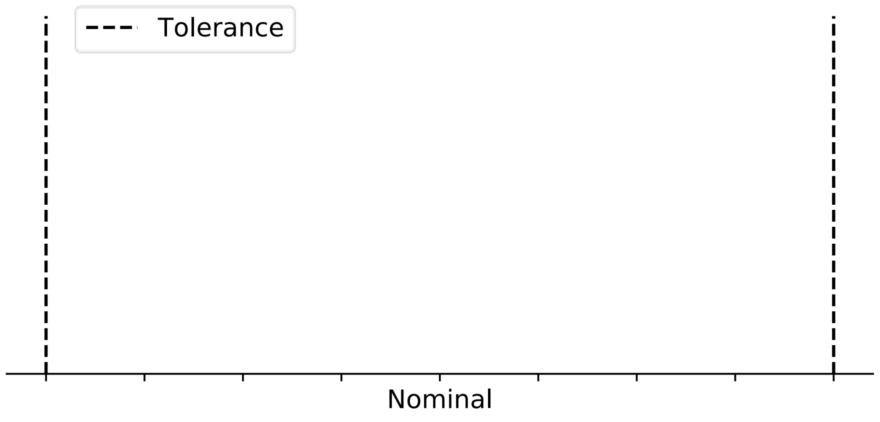
With a measured value of 13 psig, **~32% probability of a false accept!**

Another questionable football measurement: what's the uncertainty/risk when they "bring out the chains"?

What is this TUR thing, anyway?

The remainder of this class:

- What’s so special about TUR > 4:1?
- How do we calculate actual risk probabilities?
- What if I’m very risk averse? Or very risk tolerant?
- What if my manufacturing process is very tightly controlled? Or is very out of control? Or I have no idea what it is?
- What about one-sided limits where TUR is not defined?



Requirements and Policy

ISO17025 7.8.6.1 – When a statement of conformity to a specification is provided, the laboratory shall document the decision rule employed, taking into account the level of risk (such as false accept and false reject).

ANSI/NCSL Z540.3 – the probability that incorrect acceptance decisions will result from calibration tests shall not exceed 2%. Where it is not practicable to estimate this probability, the TUR shall be equal to or greater than 4:1.

D&P Manual, 13.2 – ensure that the collective uncertainty of the measurement process (at a 95% level of confidence) shall not exceed 25% of the acceptable tolerance (e.g. manufacturer's specification) for each characteristic of the M&TE being certified, unless an appropriate guard banding technique is used.

General Specification 9900000 – When test accuracy ratios (TAR) of 4:1 or greater are maintained, the resulting values can be directly compared to the specified limits.

Question

M.C.

How is the TUR defined?

A. (± Mean)/(± Measurement Uncertainty)
B. (± Measurement Uncertainty)/(± Tolerance Limits)
C. (± Tolerance Limits)/(± Guardband)
D. (± Tolerance Limits)/(± Measurement Uncertainty)

Section 2
Specific and Global Risk

Calculating risk probabilities

Section 2– Specific and Global Risk

Objective

- Define and give examples of “specific risk”, “global risk”, and calculate probability of conformance, probability of false accept, and probability of false reject.

Content

- Review of measurement uncertainty
- Probability of Conformance
- Incorrect Decision Making
- Specific Risk
- Global Risk
- Software for Risk Evaluation

Type A and Type B Uncertainty Evaluation

Type A: Uncertainty derived from statistical analysis of current test data

- Estimate the bell curve using measurements
- Std Dev: $\sqrt{\frac{1}{N-1}\sum(x_i - \bar{x})^2}$
- Std Uncertainty: $u_A(x) = s(x)/\sqrt{N}$

Type B: Uncertainty derived from other sources

- Assign a probability distribution and normalize it:
  - Normal: Given ±U with confidence and/or k value… $u = U/k$
  - Uniform: Given ±U with no other information… $u = a/\sqrt{3}$
  - Triangular: Given ±U, but more likely to be near center… $u = a/\sqrt{6}$

A full uncertainty evaluation for direct measurements requires both Type A and Type B:

$$u_c = \sqrt{u_{A_1}^2 + u_{A_2}^2 \ldots + u_{B_1}^2 + u_{B_2}^2 \ldots} = \sqrt{\sum_{i=1}^{N_A} u_{A_i}^2 + \sum_{i=1}^{N_B} u_{B_i}^2}$$

Question

T/F

Manufacturer's specifications for vertical accuracy would fall under Type B measurement uncertainty

- T
- F

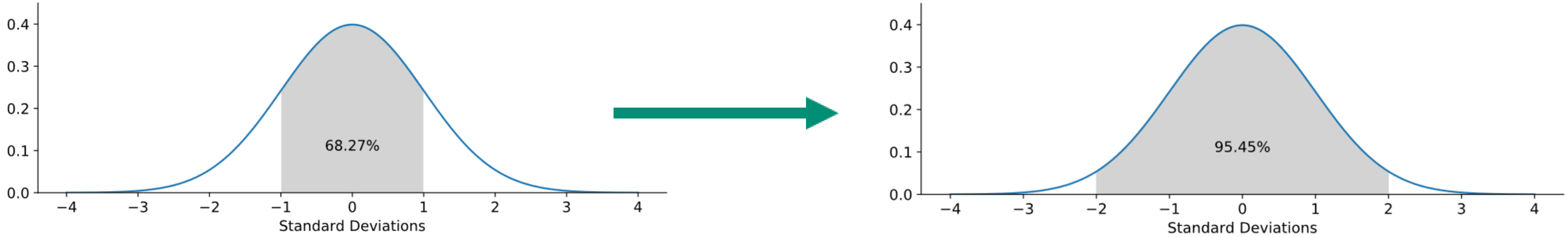
Expanded Uncertainty

One standard deviation, which covers 68% of the probability, isn't very reassuring. A higher level of confidence is desired.

Multiply u_c by a **coverage factor** k (usually 2) to get **expanded uncertainty** U.

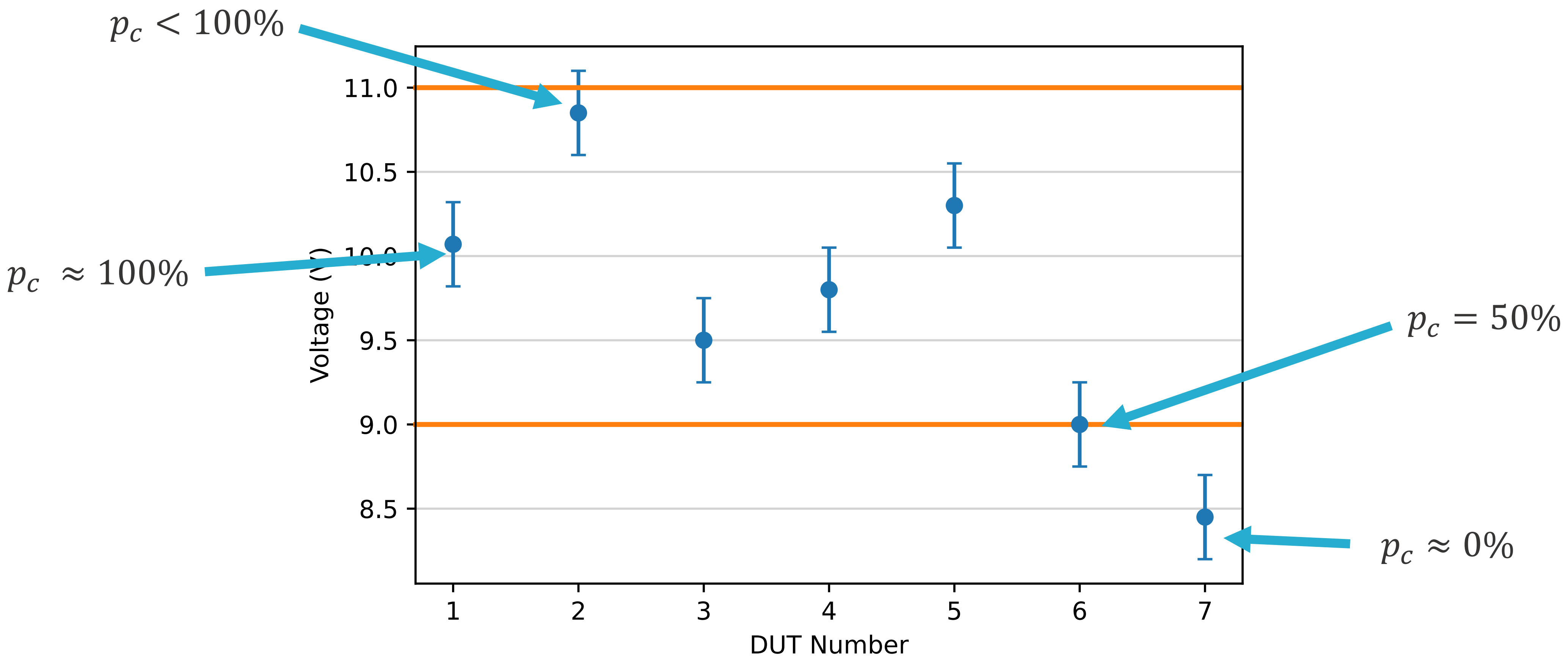
U_{95} is sometimes used to denote 95% expanded uncertainty.

k depends on **degrees of freedom** – the number of independent pieces of information used to determine the standard deviation.



Probability of Conformance (p_c)

Given a measurement result (with uncertainty), what is the probability the device under test (DUT) meets requirements?

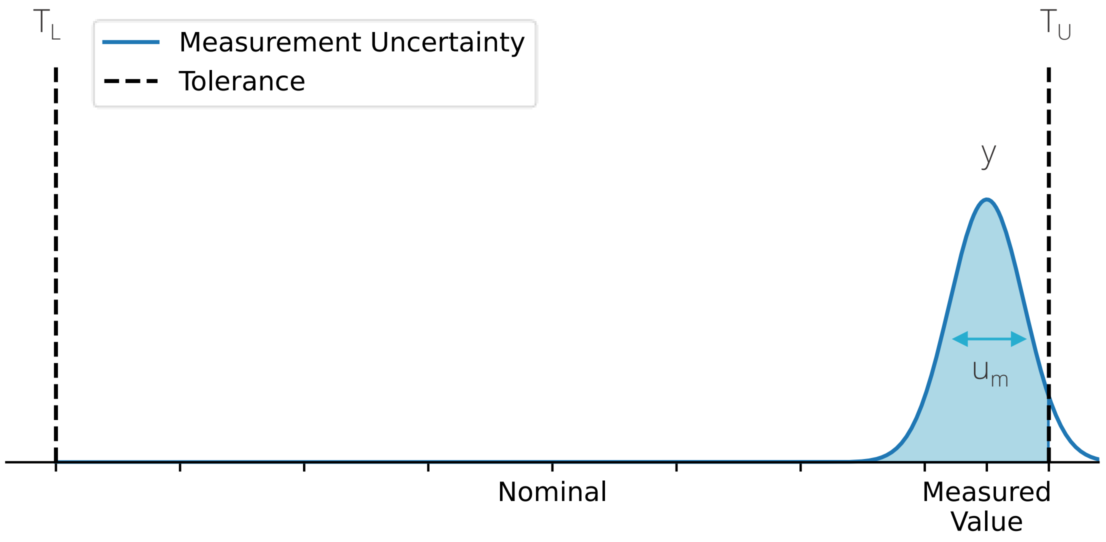


Probability of Conformance – Quantified

Probability that the true value falls within the limits (integrate the PDF inside the limits)

$$p_c = \int_{T_L}^{T_U} p_m(t - y)\, dt$$

- p_m = PDF of measurement uncertainty
- y = Measured Value (known)
- t = True Value (unknown; integrate)
- T_L, T_U = Lower and Upper Tolerance limits



With normal distribution of standard deviation u_m:

$$p_c = \frac{1}{u_m\sqrt{2\pi}} \int_{T_L}^{T_U} e^{-\frac{(t-y)^2}{2u_m^2}}\, dt$$

In Excel (one side): “NORM.DIST(TU, y, um, TRUE)”

Example – Speed Limit Enforcement (Example 1 in JCGM106)

Highway Patrol used radar to measure your speed. Given these values, what is the probability you were speeding?

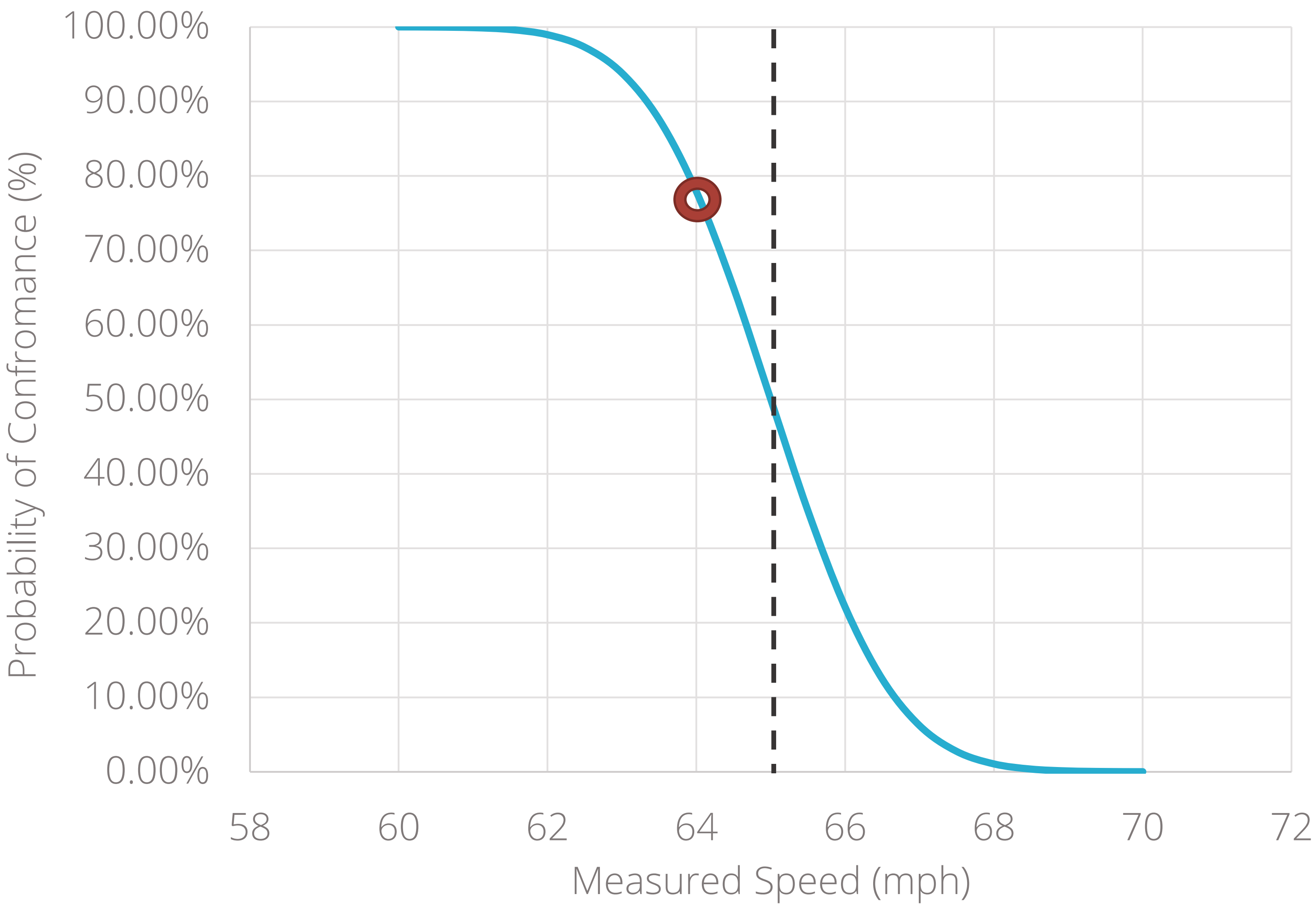
- Upper speed limit = 65 mph
- Measured value = 64 mph
- Standard uncertainty = 1.3 mph

- Probability of conformance can be calculated:
  - T_U = 65
  - T_L = $-\infty$
  - y = 64
  - u_m = 1.3
  - t = integration variable

$$p_c = \frac{1}{u_m\sqrt{2\pi}} \int_{T_L}^{T_U} e^{-\frac{(t-y)^2}{2u_m^2}}\, dt$$

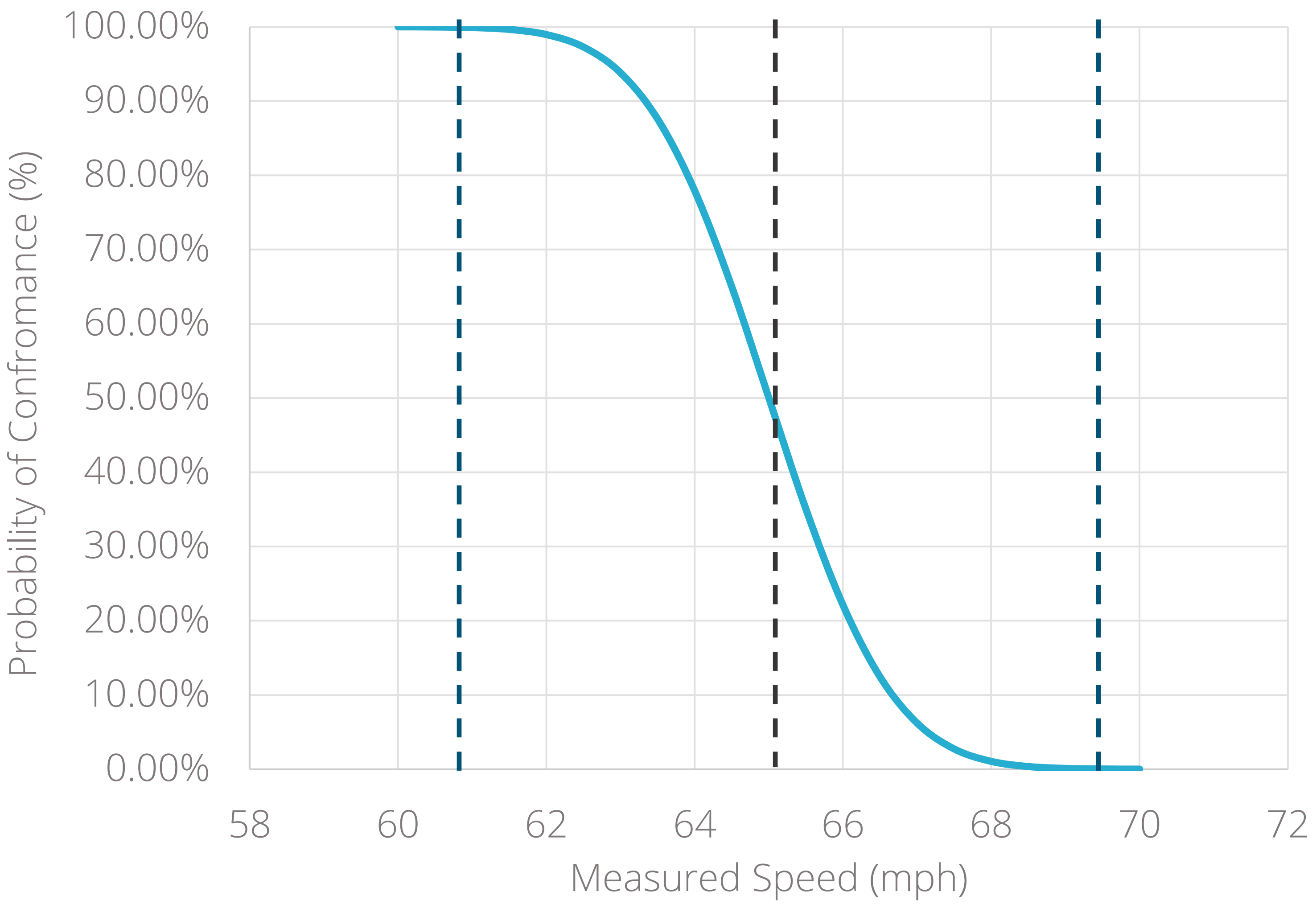
Can use Excel to plot p_c over range of measured values: NORM.DIST(TU, y, um, TRUE)

Probability of Conformance Plot – Speed Limit Enforcement



High confidence you’re not speeding Uncertain region High confidence you are speeding

Probability of Conformance Plot – Speed Limit Enforcement



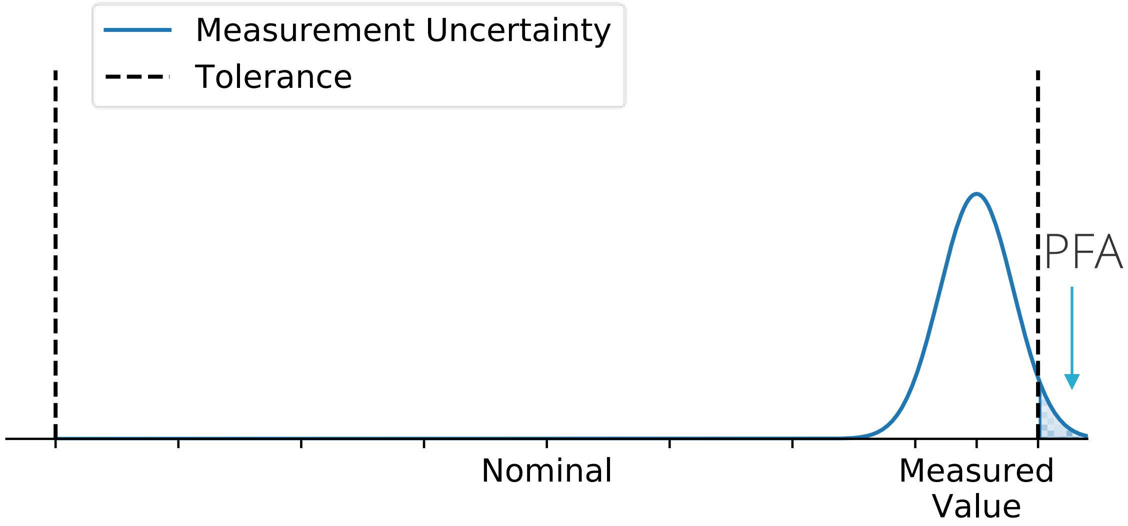
Can use this plot to find thresholds that give confidence

Specific Risk – adds a decision to the measurement result

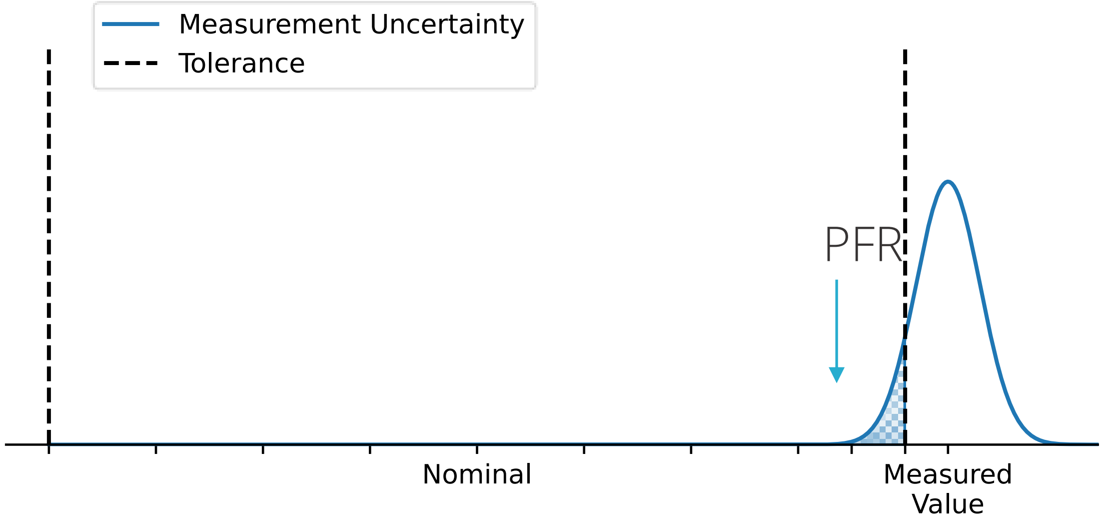
Specific Risk: probability that a decision based on a specific measurement result is incorrect

- Specific false accept: decision = pass; true value = fail
- Specific false reject: decision = fail; true value = pass

Specific risk is found by integrating the shaded area – outside the limits (False Accept) or inside the limits (False Reject).



Mostly pass, but some fail. I.e., the decision was pass, but it could have failed



Mostly fail, but some pass. I.e., the decision was fail, but it could have passed

Specific Risk – Quantified

When the decision is pass ($T_L \le y \le T_U$):

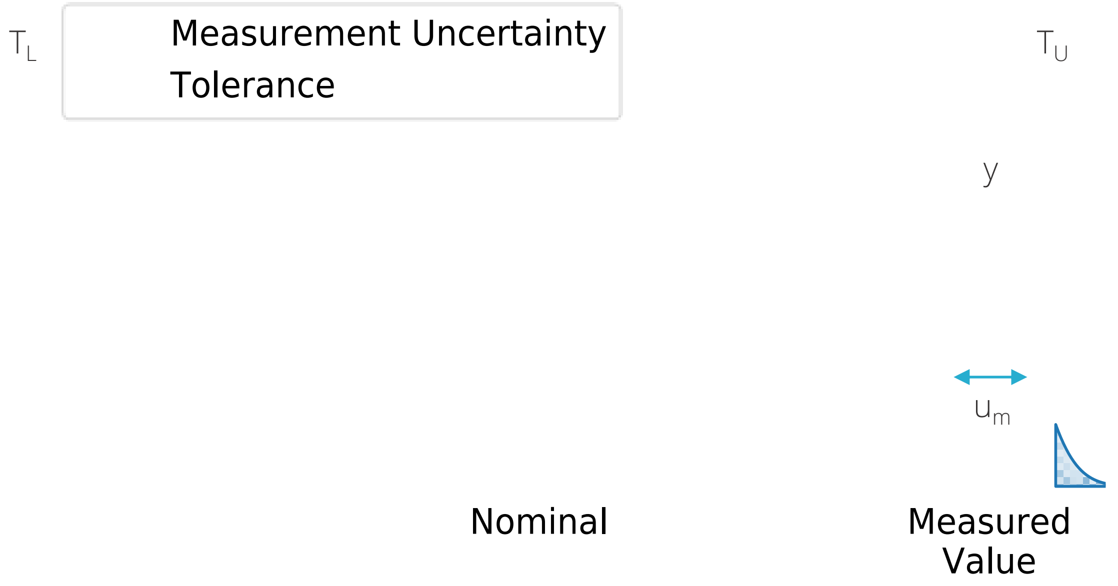
- $p_{FA}|_y = \int_{-\infty}^{T_L} p_m(t-y)\,dt + \int_{T_U}^{\infty} p_m(t-y)\,dt = 1 - p_c$

When the decision is fail ($y < T_L \; or \; y > T_U$):

- $p_{FR}|_y = \int_{T_L}^{T_U} p_m(t-y)\,dt = p_c$

With normal distribution of standard deviation u_m:

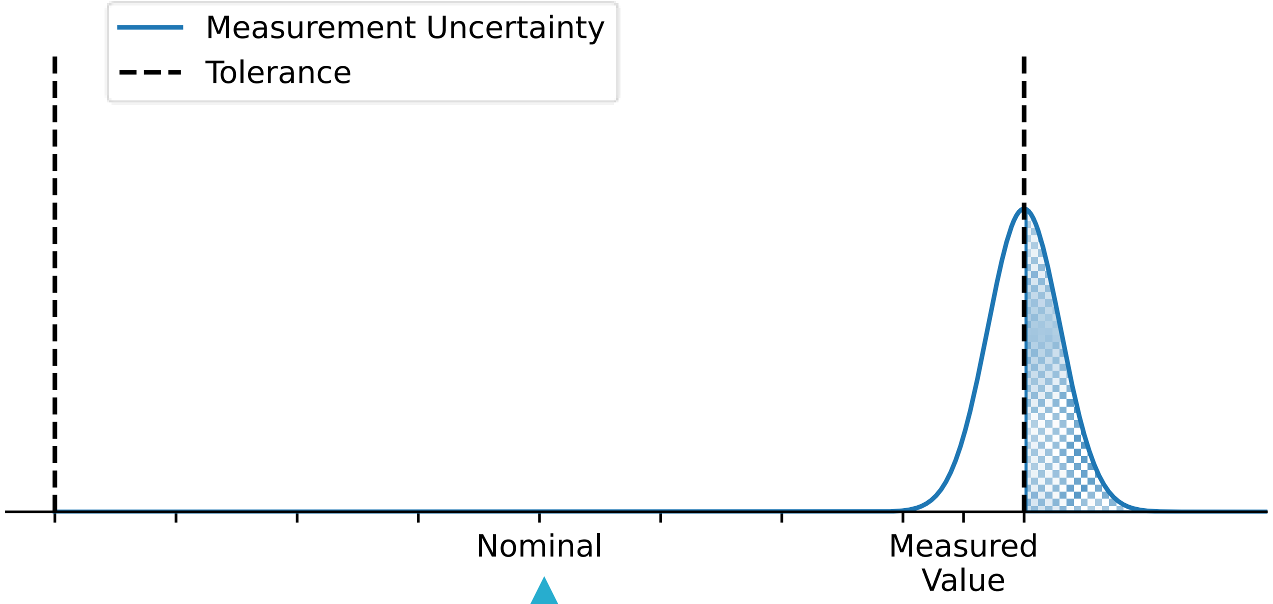
- $p_{FA}|_y = \int_{-\infty}^{T_L} \frac{1}{u_m\sqrt{2\pi}} e^{-\frac{(t-y)^2}{2u_m^2}}\,dt + \int_{T_U}^{\infty} \frac{1}{u_m\sqrt{2\pi}} e^{-\frac{(t-y)^2}{2u_m^2}}\,dt$
- $p_{FR}|_y = \int_{T_L}^{T_U} \frac{1}{u_m\sqrt{2\pi}} e^{-\frac{(t-y)^2}{2u_m^2}}\,dt$



In Excel (one side): “1 - NORM.DIST(TU, y, um, TRUE)”

Risk Probability

There can be a 50% risk my measurement decision is wrong?!



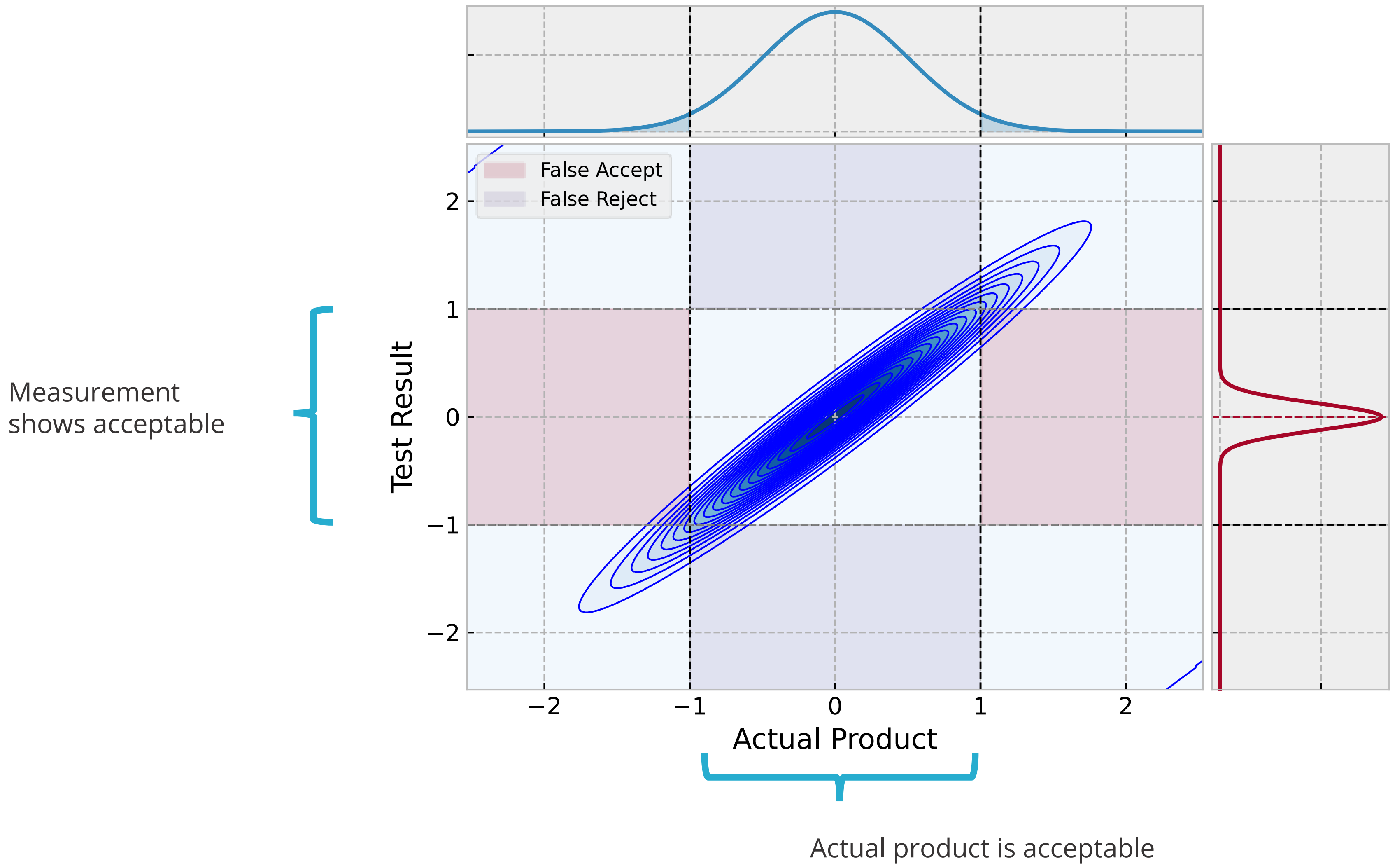
Yes, but…

Most DUTs should be near here

Low probability of finding a DUT here

Global Risk

Global Risk combines these two probabilities – the PDF of the measurement uncertainty and the PDF of all the DUTs being measured

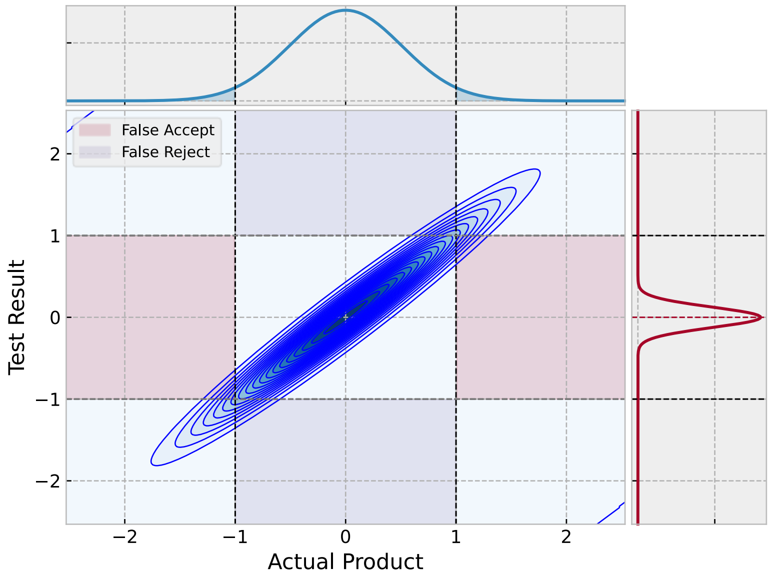


Global Risk

Measurement uncertainty PDF: get from uncertainty analysis (i.e. GUM methods)

Product PDF: Need prior information about the DUTs!

- Historical data on DUTs of same or similar design
- Observed Out of Tolerance (OOT) rate of same or similar calibration assets
- Information from manufacturing process
- Details in next section…



Global Risk – Quantified (Equation 19 and 20 in JCGM106)

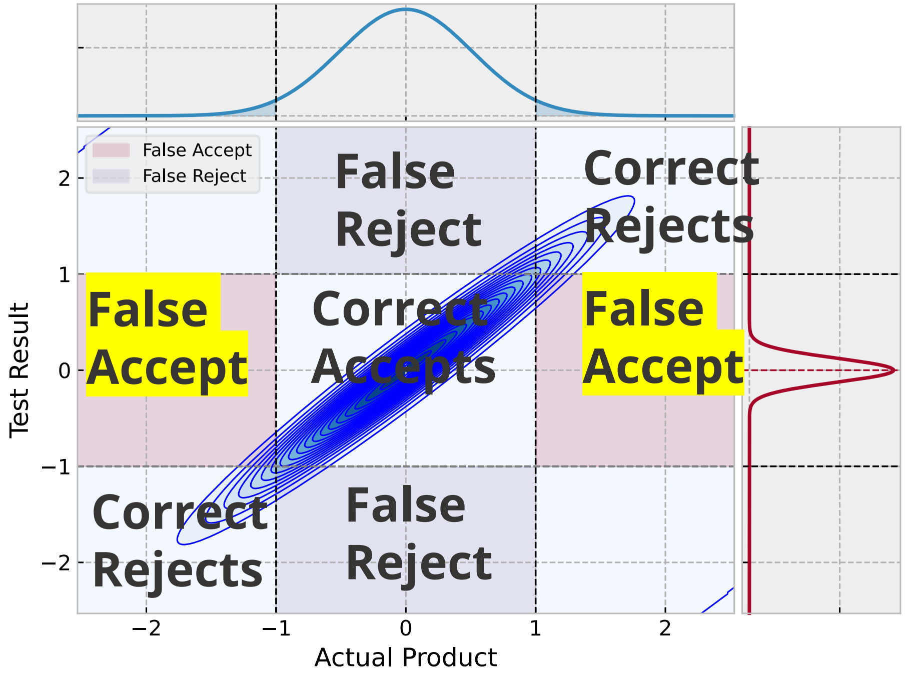
Global Probability of False Accept (PFA):

$$PFA = \int_{-\infty}^{T_L} \int_{T_L}^{T_U} p_m(t-y)\, p_p(t)\, dy\, dt + \int_{T_U}^{+\infty} \int_{T_L}^{T_U} p_m(t-y)\, p_p(t)\, dy\, dt$$

The inside integral (TL to TU) is integrating the "correct accept" regions.
The outside integral (-∞ to TL) is integrating the "fail" regions
On the LHS.
(TU to +∞) is integrating the "fail" regions RHS

Variables:

y = measured value (integrate over all measured values within limits)
t = true value (integrate over all true values outside limits)
T_U = Upper tolerance limit
T_L = Lower tolerance limit
p_m = PDF of measurement uncertainty
p_p = PDF of products/DUTs



In Excel: Not easy. Can be done numerically or by Monte-Carlo.

Global Risk – Quantified (Equation 19 and 20 in JCGM106)

Global Probability of False Reject (PFR):

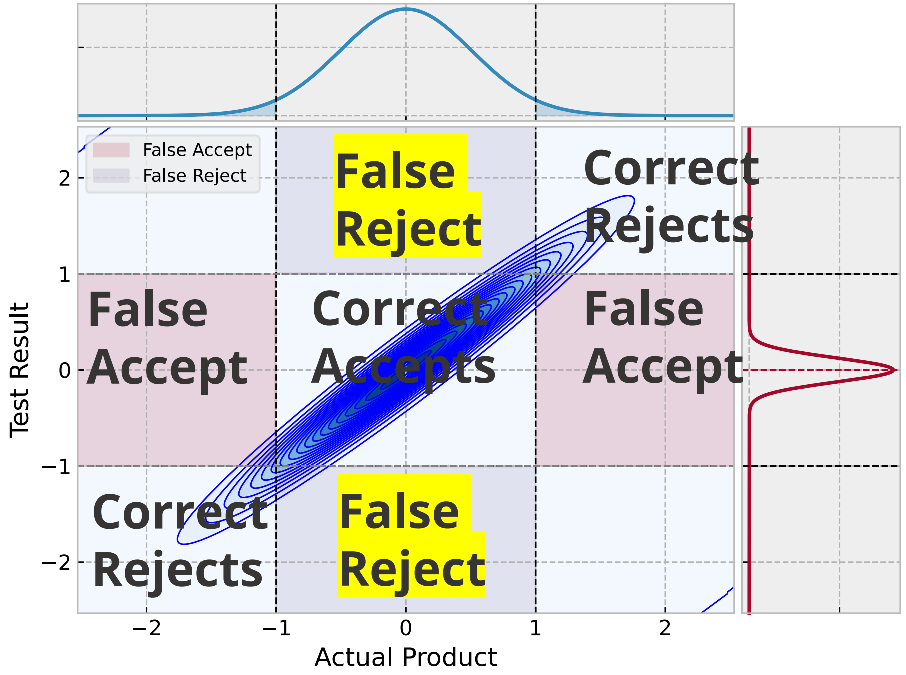
$$PFR = \int_{T_L}^{T_U} \int_{-\infty}^{T_L} p_m(t-y)\ p_p(t)\ dy\ dt + \int_{T_L}^{T_U} \int_{T_U}^{+\infty} p_m(t-y)\ p_p(t)\ dy\ dt$$

The inside integral (TL to TU) is integrating the "correct accept" regions.

The outside integral (-∞ to TL) is integrating the "fail" regions

On the LHS.

(TU to +∞) is integrating the "fail" regions RHS



Variables:

- y = measured value (integrate over all measured values within limits)
- t = true value (integrate over all true values outside limits)
- T_U = Upper tolerance limit
- T_L = Lower tolerance limit
- p_m = PDF of measurement uncertainty
- p_p = PDF of products/DUTs

In Excel: Not easy. Can be done numerically or by Monte-Carlo.

Some common variations on PFA/PFR equations

Three different equations for PFA/PFR that produce identical results:

- JCGM 106, Equation 19 and 20 (generalized for any PDF)
- JCGM 106, Equation A.15 and A.16 (simplified for normal PDFs)
- Deaver's Equation 6 and 7 in "How to Maintain Confidence 1993" (Normal PDFs, but uses different notation for TUR)
- Crowder's "Statistics in Metrology" equivalent to JCGM 19 and 20, but rearranged.

See appendix slides for details and comparison.

Our notation attempts to follow JCGM:

- $T_U, \; T_L$ = upper and lower tolerance limits
- $A_U, \; A_L$ = upper and lower acceptance limits
- u_m = Measurement uncertainty
- u_0 = Std. Deviation of products
- y_0 = Nominal value of products
- p_p = Probability density function of products
- p_m = Probability density function of measurement
- $\varphi(\eta;\; \mu, \sigma^2)$ = normal PDF with mean μ and std.dev. σ at true value η

Question

What does global risk consider that specific risk does not?

A. Distribution of products
B. Measurement uncertainty
C. Tolerance limits
D. One measured value

Exercise – Voltmeter calibration

Manufacturer's specification on a voltmeter at reading 10.00 V is ± 0.05 V

Historical calibration data on this model of meter shows a 10 % out-of-tolerance rate. In other words, the DUTs follow a distribution of $\mu = 10.00$ V and $\sigma = 0.03$ V.

The $k = 1$ uncertainty (standard deviation) in the calibration measurement is 0.010 V.

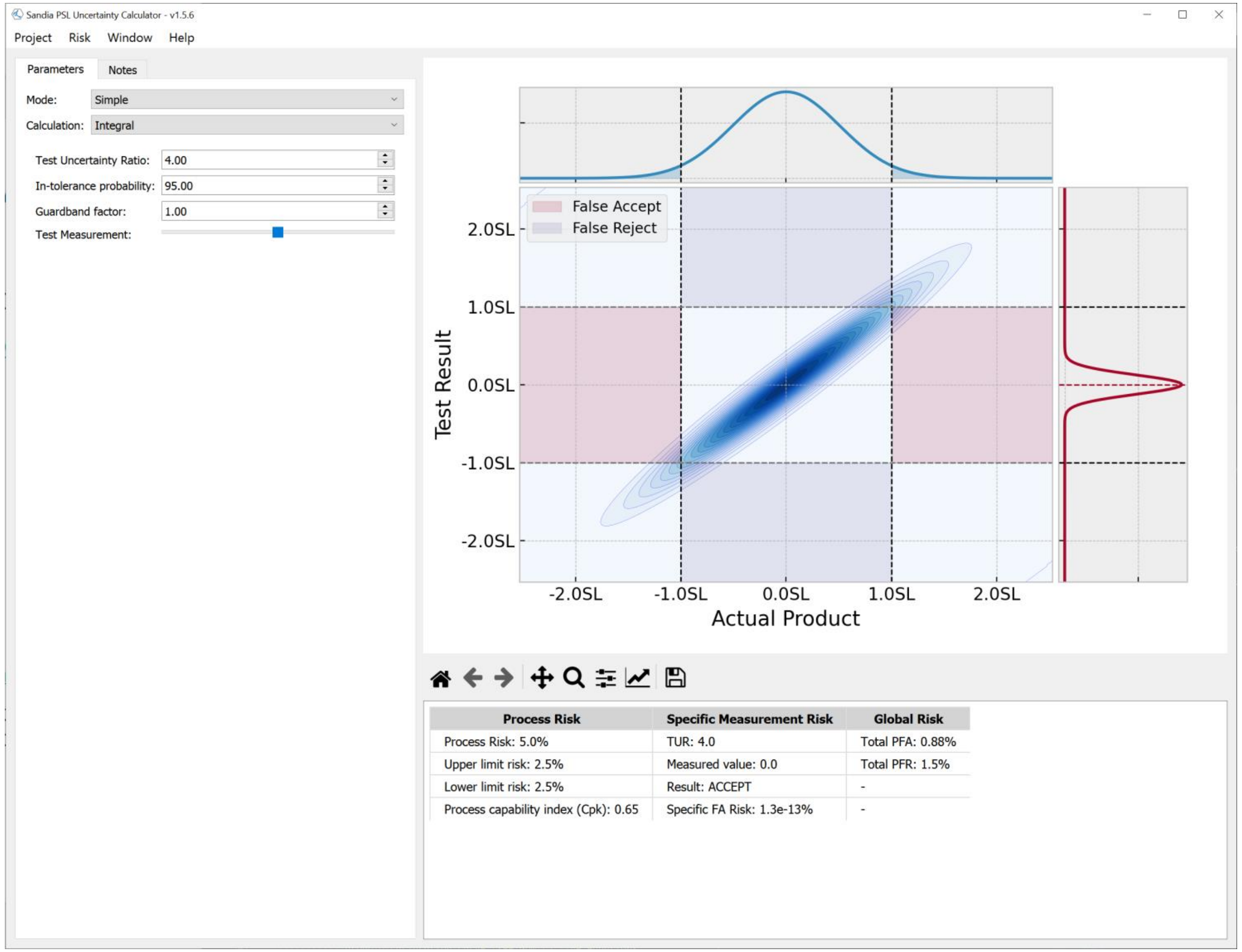
Calculate the probability of false accept and probability of false reject for any voltmeter of this model.

Software to the rescue!

Sandia Uncertainty Calculator (Suncal)

- https://sandiapsl.github.io

See Appendix slides for installation and usage notes.



Exercise – Use Suncal to calculate PFA and PFR of voltmeter problem

- Tolerance Limits: $T_U,\ T_L$ = 9.95, 10.05
- Measurement standard deviation: u_m = 0.01
- Product standard deivation: u_0 = 0.03
- Product mean: y_0 = 10.00

$$PFA = \int_{-\infty}^{T_L} \left(\int_{T_L}^{T_U} \frac{1}{u_m\sqrt{2\pi}} e^{-\frac{1}{2u_m^2}(y-t)^2} dy \right) \frac{1}{u_0\sqrt{2\pi}} e^{-\frac{1}{2u_0^2}(t-y_0)^2} dt + \int_{T_U}^{+\infty} \left(\int_{T_L}^{T_U} \frac{1}{u_m\sqrt{2\pi}} e^{-\frac{1}{2u_m^2}(y-t)^2} dy \right) \frac{1}{u_0\sqrt{2\pi}} e^{-\frac{1}{2u_0^2}(t-y_0)^2} dt = 1.9\%$$

$$PFR = \int_{T_L}^{T_U} \left(\int_{-\infty}^{T_L} \frac{1}{u_m\sqrt{2\pi}} e^{-\frac{1}{2u_m^2}(y-t)^2} dy \right) \frac{1}{u_0\sqrt{2\pi}} e^{-\frac{1}{2u_0^2}(t-y_0)^2} dt + \int_{T_L}^{T_U} \left(\int_{T_U}^{+\infty} \frac{1}{u_m\sqrt{2\pi}} e^{-\frac{1}{2u_m^2}(y-t)^2} dy \right) \frac{1}{u_0\sqrt{2\pi}} e^{-\frac{1}{2u_0^2}(t-y_0)^2} dt = 3.6\%$$

Section 3 Historical Product Data

Determining the product distribution

Section 3 – Historical Product Data

Objective

- Determine appropriate probability distributions and parameters based on prior knowledge about a measurement and process

Content

- Estimating product distribution in calibration labs
- Estimating product distribution in manufacturing

Information needed to calculate risk (“prior knowledge”)

1. Tolerance Limits (engineering/design)
2. Measurement distribution (measurement uncertainty analysis – GUM)
3. **Product distribution** (knowledge of manufacturing or calibration process)

Where do we get the product distribution in Calibration Labs?

- Calibration labs usually set uncertainties and intervals to achieve a desired OOT rate
- Estimate product distribution using In-Tolerance Probability (itp = 1 - OOT)
  - ITP is sometimes called End-of-Period Reliability (EOPR).

- Assumes a normal distribution centered between tolerance limits:

$$\sigma_p \approx \frac{(T_U - T_L)/2}{F^{-1}\left(\frac{1 + itp}{2}\right)}$$

(F^{-1} is inverse standard normal)

Example – Estimating Product Distribution from ITP

Calibration of 100 Ω ± 1 Ω Resistors

Historical data shows 6% out-of-tolerance rate

- $itp = 1 - 6\% = 0.94$

$$\sigma_p \approx \frac{(T_U - T_L)/2}{F^{-1}\left(\frac{1+itp}{2}\right)} = \frac{(101-99)/2}{F^{-1}\left(\frac{1+.94}{2}\right)} = 0.532$$

In Excel, NORM.S.INV is F^{-1}:   "=(101-99)/2/NORM.S.INV((1+.94)/2)"

Where do we get the product distribution in Manufacturing?

Measure a sample of the manufactured parts, preferably with highly accurate measurement

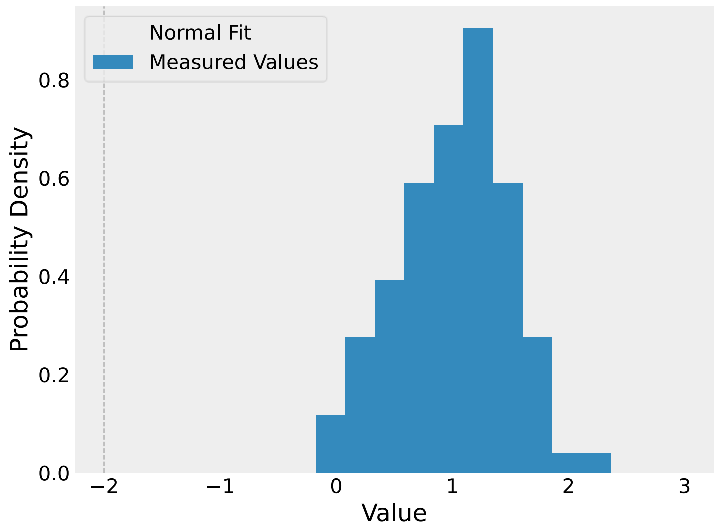
- Characterize the sample data (mean μ, standard deviation s)
- Make a histogram of product sample
- Fit a probability distribution to the histogram

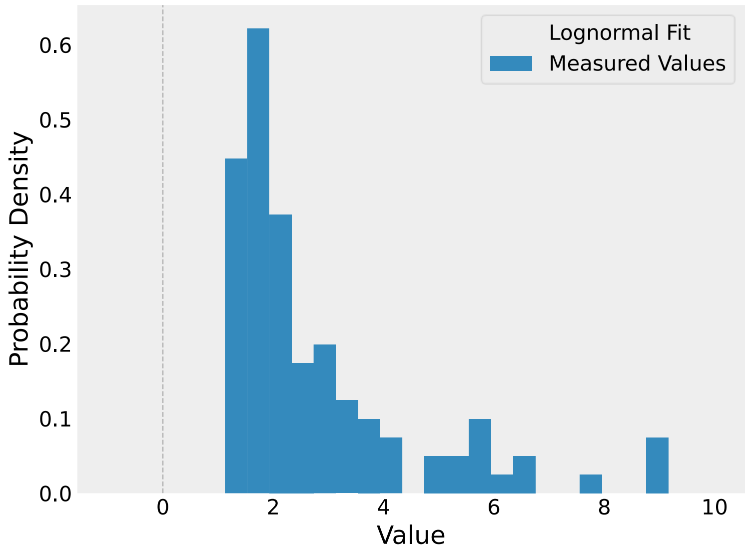
Common choices:

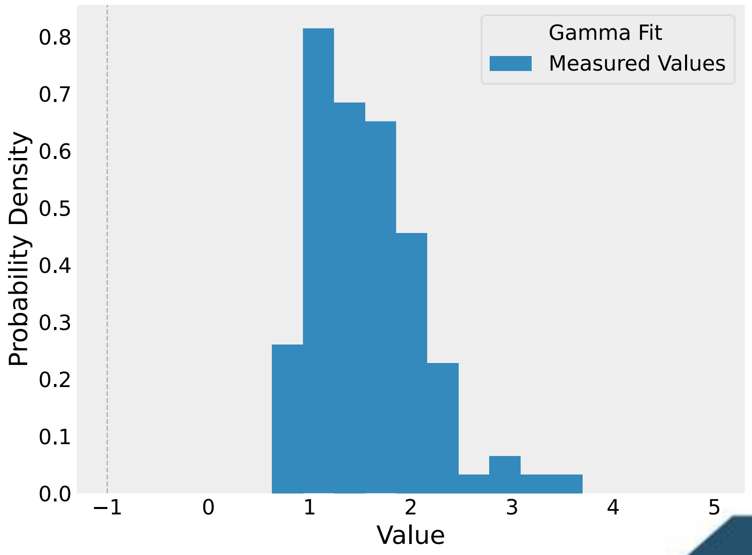
- Normal distribution – the default choice
- Lognormal distribution – many physical growth processes; reliability data
- Gamma distribution – useful for values that must be positive, but are near zero (see JCGM106 B.3)

Distribution Fitting

- Normal distribution: $N(\mu, s)$
- Lognormal distribution: $Lognorm(\mu, s)$
- Gamma distribution: $\text{gamma}\left(\alpha = \frac{\mu^2}{s^2}; \beta = \frac{\mu}{s^2}\right)$
  - μ is sampled mean; s is sampled standard deviation







Example – Distribution Fit From Manufacturing Data

Manufacturing 0.1 Ω resistors

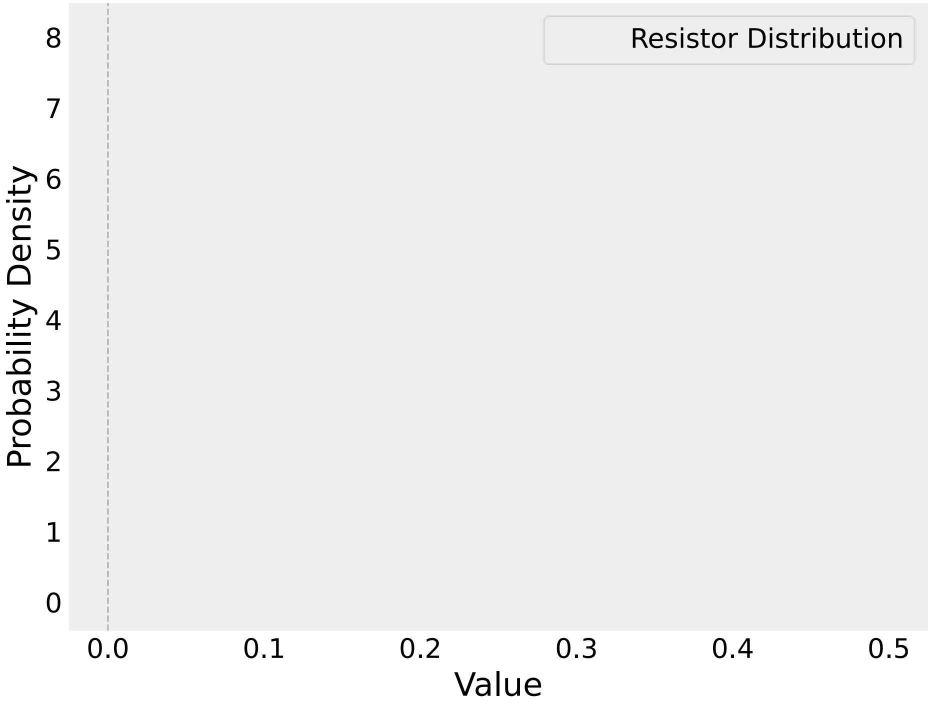
Sampled from a lot of 100:

- Mean = 0.1196 Ω
- Standard Deviation = 0.0542 Ω

Resistors can never be negative – but the nominal value is close to zero → Use gamma distribution

- $\alpha = \frac{\mu^2}{s^2} = \frac{0.1196^2}{0.0542^2} = 4.86$
- $\beta = \frac{\mu}{s^2} = \frac{0.1196}{0.0542^2} = 40.67$

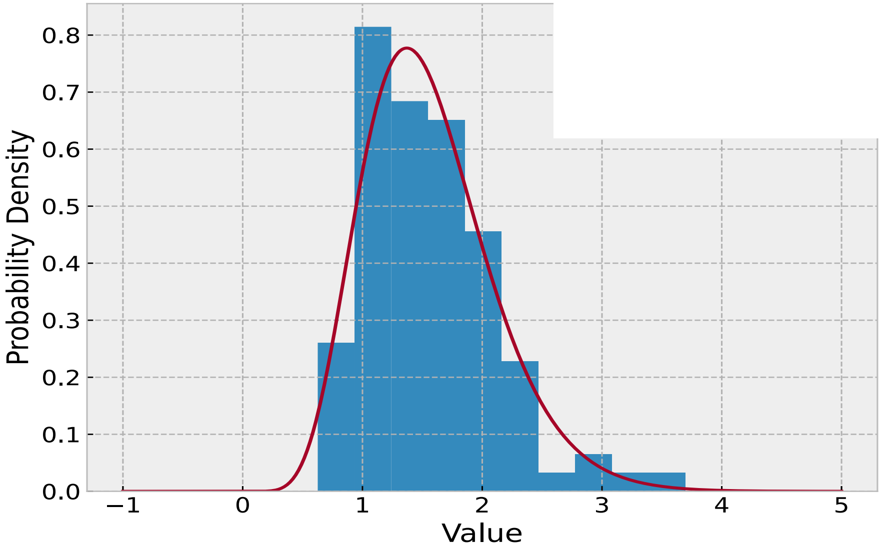
* See Appendix for fitting distributions using Suncal



Question

T/F

The following distribution should be treated as a normal distribution if it demonstrates a distribution of resistance measurements?



Section 4
Simplified Risk Metrics

Test Uncertainty Ratios

Section 4 – Simplified Risk Metrics

Objective

- Define test uncertainty ratio and show how it relates to false accept and reject risk.

Content

- Information Requirements
- Test Uncertainty Ratio
- TUR Assumptions
- Risk Curves

Information Requirements

- Tolerance Limits (engineering/design)
- Measurement distribution (measurement uncertainty analysis – GUM)
- Product distribution (knowledge of manufacturing or calibration process)

Between the need for detailed information, combined with the messy PFA and PFR integrals (historically difficult to solve), some simplified metrics for quantifying risk have been developed.

Test Uncertainty Ratio

- TUR = [± Tolerance Limit] / [± Measurement Uncertainty @ 95% confidence]
- Or in terms of absolute tolerance limits: $TUR = \frac{\pm T}{\pm U} = \frac{T_U - T_L}{2U_{95}}$

- Note: U_{95} = expanded uncertainty expressed at 95% confidence $\cong 2\, u_m$

TAR or TUR?

Most common definitions found in literature:

- TUR = [± Tolerance Limit] / [± Measurement Uncertainty @ 95% confidence]
- TAR = [± Tolerance Limit] / [± Equipment Accuracy Only @ 95%]

9900000 and PSLM:

- TAR = product characteristic tolerance divided by the collective measurement uncertainty of the equipment based on a confidence level of 95% or better [99M]
- TUR = certification tolerance of device being calibrated divided by process uncertainty of the process used to calibrate [PSLM]

JCGM 106:

- “Care has to be taken when such [TAR or TUR] rules are encountered because they are sometimes ambiguously or incompletely defined.”
- Measurement Capability Index = [± Tolerance Limit] / [± Measurement Uncertainty @ 95%]

What does TUR have to do with risk?

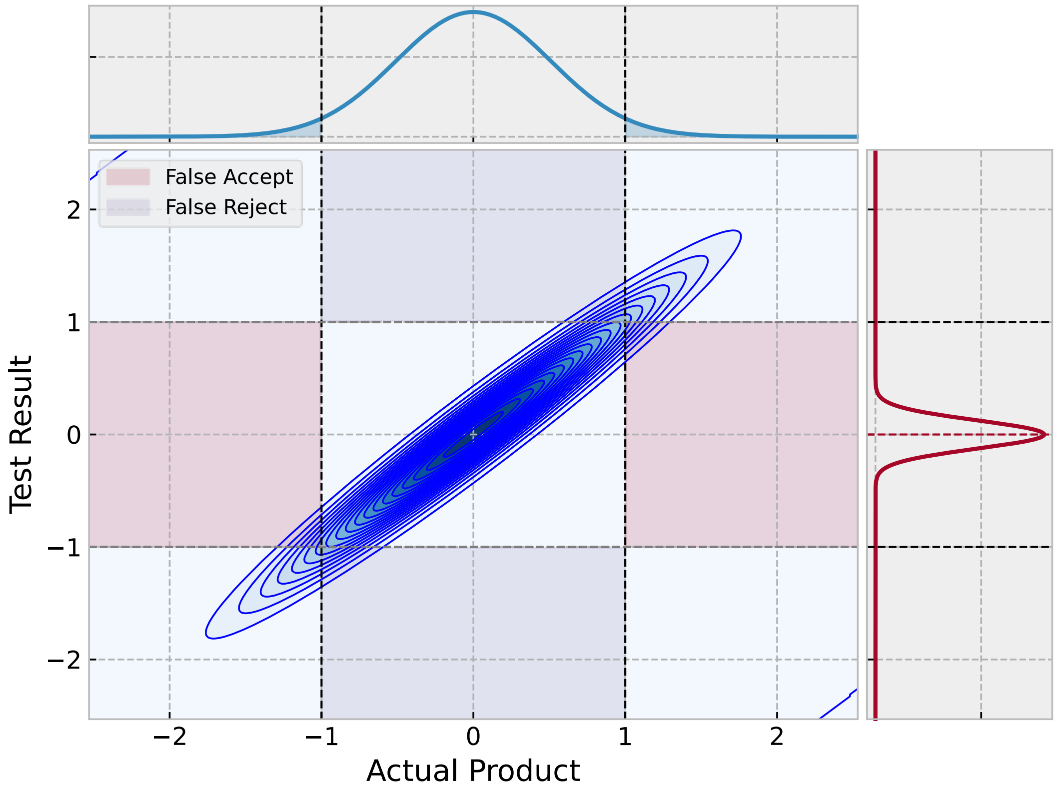
$TUR = \frac{T_U - T_L}{2U_{95}}$ → Measurement Distribution

$itp = 2\,\Phi\left(\frac{T_U - T_L}{2u_0}\right) - 1$ → Product Distribution

Φ = standard normal PDF

(Assuming normal distributions with $y_0 = 0$)

TUR

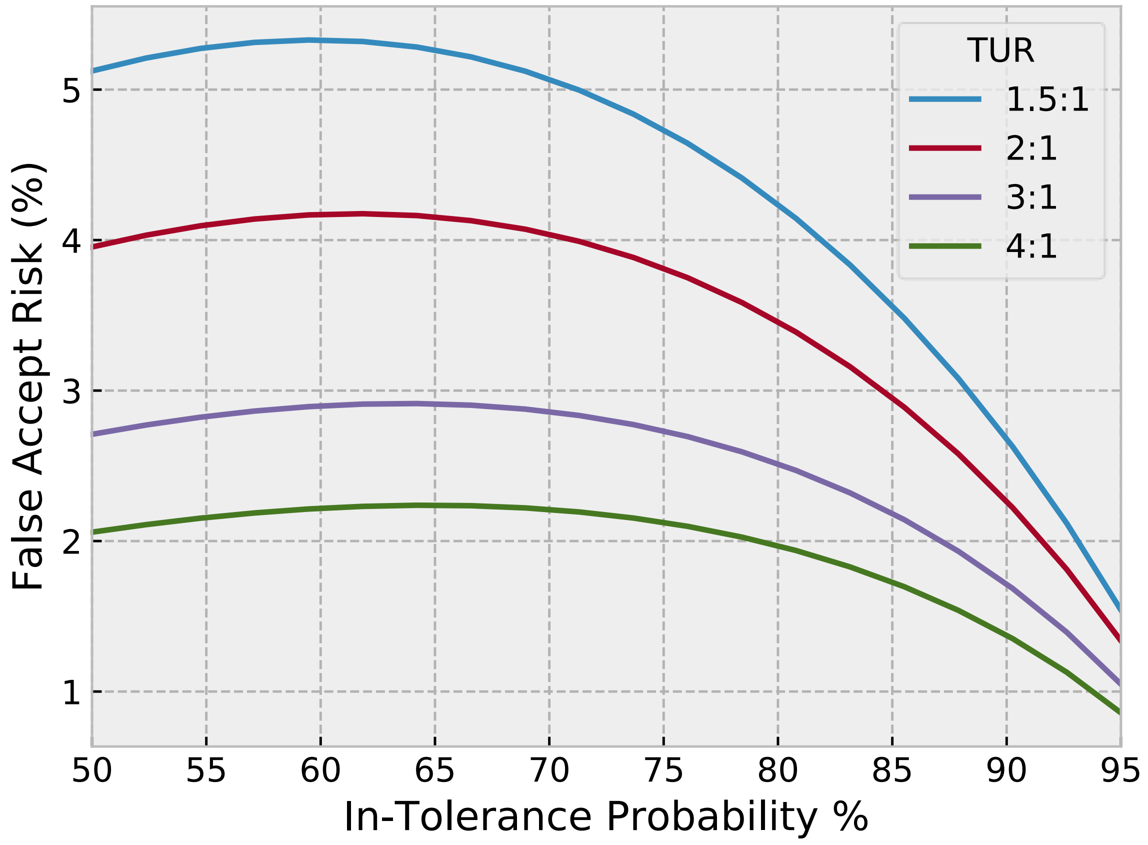


ITP distribution

PFA versus TUR and itp – why we use 4:1 “rule”

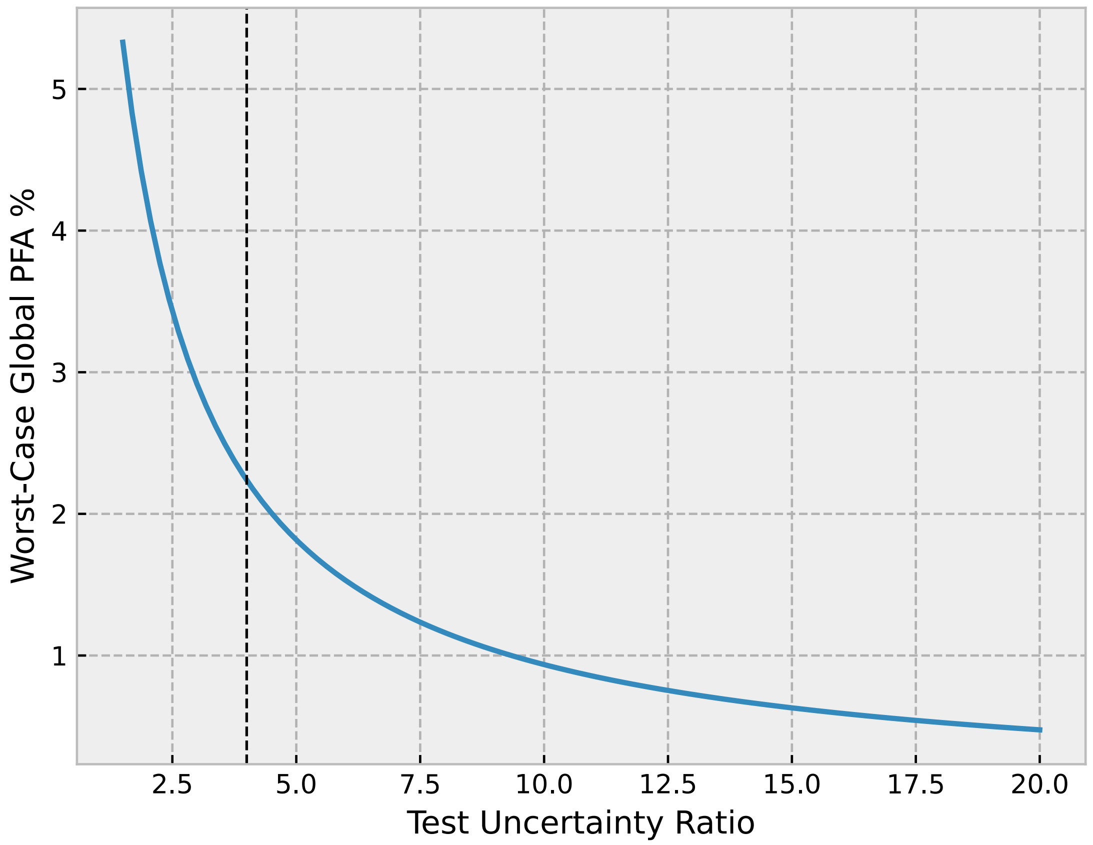
- TUR and itp condense all the required knowledge into two ratios

- PFA is always less than 2% for all TUR bigger than 4, given a typical itp above 80%.



TUR and worst-case PFA

Extract the peaks along each TUR from the previous graph…



Historical Note

From Scott Mimbs (NASA), Measurement Decision Risk: The Importance of Definitions. NCSLI Workshop & Symposium, 2007:

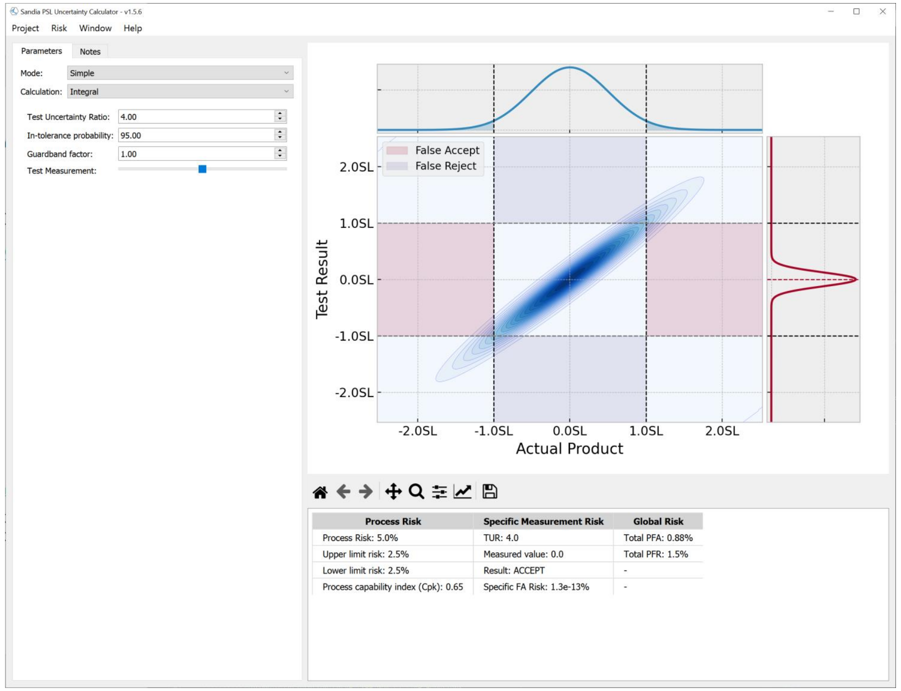
- “In the mid-50's, computing consumer risk was a very arduous task (requiring use of a slide rule), which [Jerry] Hayes decided not to require U.S. Navy contractors to perform.”
- “The [Navy’s] practice at the time was to use a 10:1 ratio, but that value was considered unsupportable by the nation’s calibration support and measurement traceability infrastructure…”
- “A consumer risk of 1% was selected, which calculated to be about a 3:1 accuracy ratio … decided to pad the ratio … thus the 4:1 ratio requirement was developed and established as Navy policy.”
- “NASA used the 10:1 for all calibration requirements through the first moon landing in 1969. After that, calibration requirements were changed to 4:1 while test measurement requirements remained at 10:1.”
- **“When Hayes allowed the use [of TUR]… the idea was supposed to be temporary until better computing power became available…”**

Now we have two risk calculation tools

TUR as a quick metric for simple risk evaluation

Jan1959/CC BY-SA 4.0

Software for full false accept false reject evaluation



Sandia Uncertainty Calculator

It’s 2024 – Computing power is just a bit better than it was in 1955.

Exercise – Resistor Example JCGM106 9.5.3 (page 27)

Manufacturing precision resistors:

- Nominal Resistance: 1500 Ω
- Tolerance interval: 1499.8 Ω to 1500.2 Ω
- Sample of resistors measured using high-precision meter has mean 1500.0 Ω, standard deviation 0.12 Ω (product distribution)

Resistors are inspected

- Measurement process has uncertainty $u_m = 0.04$ Ω (k = 1)

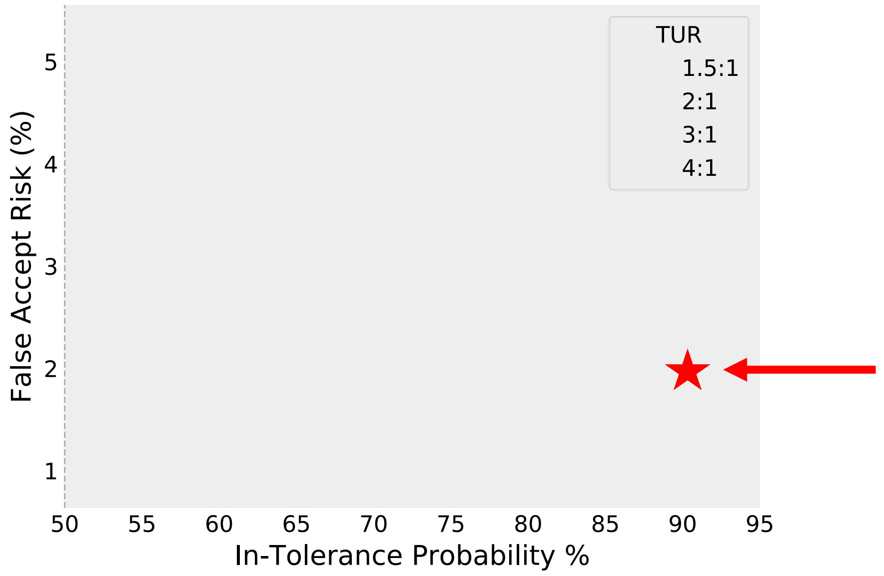
- **What is the TUR?**
- **What is the global PFA?**
- **What is the ITP? (If the manufacturer accepted all resistors without any measurement, what percent would be conforming?)**
- **Does TUR make sense as a PFA metric?**

Resistor Example – Add a screening process

What is the TUR? → $\frac{\pm 0.2}{\pm(2\times 0.04)}$ **= 2.5**

What is the global false accept risk? → **1.9 %**

What percent are conforming if there were no inspection made? → **100% - 9.6% = 90.4%**



Process Risk	Specific Measurement Risk	Global Risk
Process Risk: 9.6%	TUR: 2.5	Total PFA: 1.9%
Upper limit risk: 4.8%	Measured value: 1500	Total PFR: 3.6%
Lower limit risk: 4.8%	Result: ACCEPT	-
Process capability index (Cpk): 0.56	Specific FA Risk: 0.000057%	-

TUR makes some assumptions. Are they justified?

- > 80% itp in product distribution
- Unbiased distributions – product distribution centered between limits
- Normal distributions
- 2% PFA is acceptable (using 4:1 rule)

But don't forget, to perform a full PFA analysis requires prior knowledge of product distribution, often not known.

TUR assumptions

TUR > 4 “rule” says 2% likelihood is ok…

Likelihood Tier	Consequence Tier				
	Catastrophic	Severe	Moderate	Low	Negligible
Very High	VH	VH	H	M	L
High	VH	H	M	M	L
Moderate	H	M	M	L	L
Low	M	M	L	L	N
Negligible	L	L	L	N	N

But TUR doesn’t know about consequence tier…

Section 5
Risk Management

Guardbanding to mitigate risk

Section 5 – Risk Management

Objective

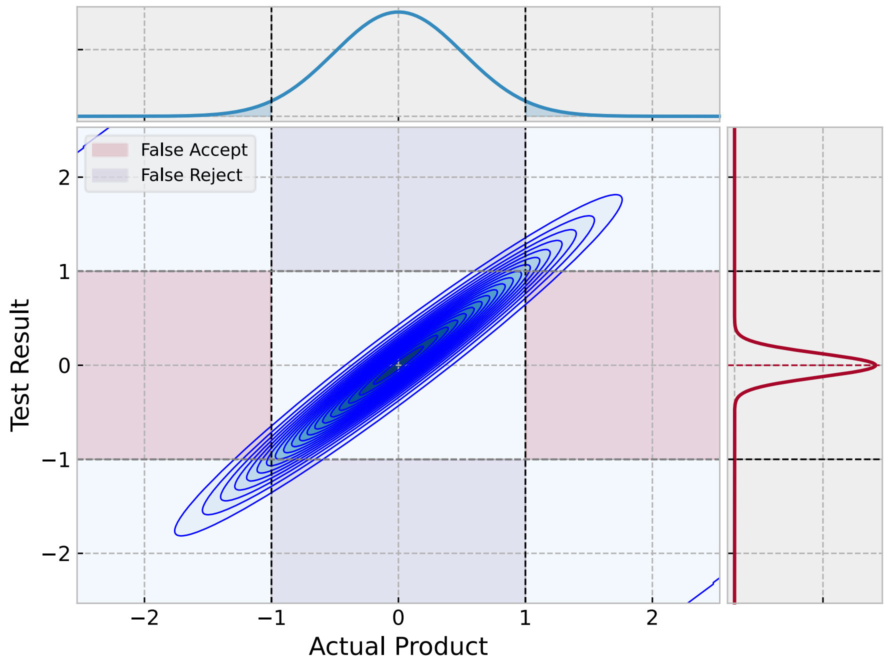
- Discuss and analyze guardbanding methods used for reducing false accept risk.

Content

- Guardbanding
- Risk evaluation with guardband
- Guardband methods

Four ways of reducing false accept risk

1. Reduce measurement uncertainty ($)
2. Improve process control ($$)
3. Convince design engineers to relax the tolerance (??)
4. **Apply guardbanded acceptance limits**



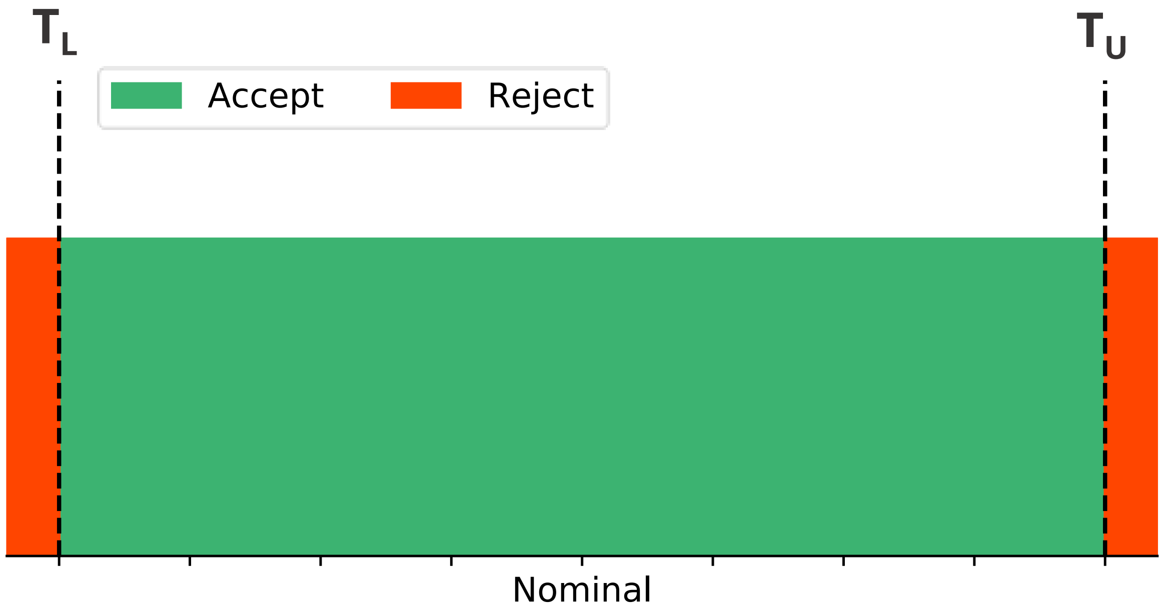
Guardbanding defines acceptance limits lower than tolerance limits

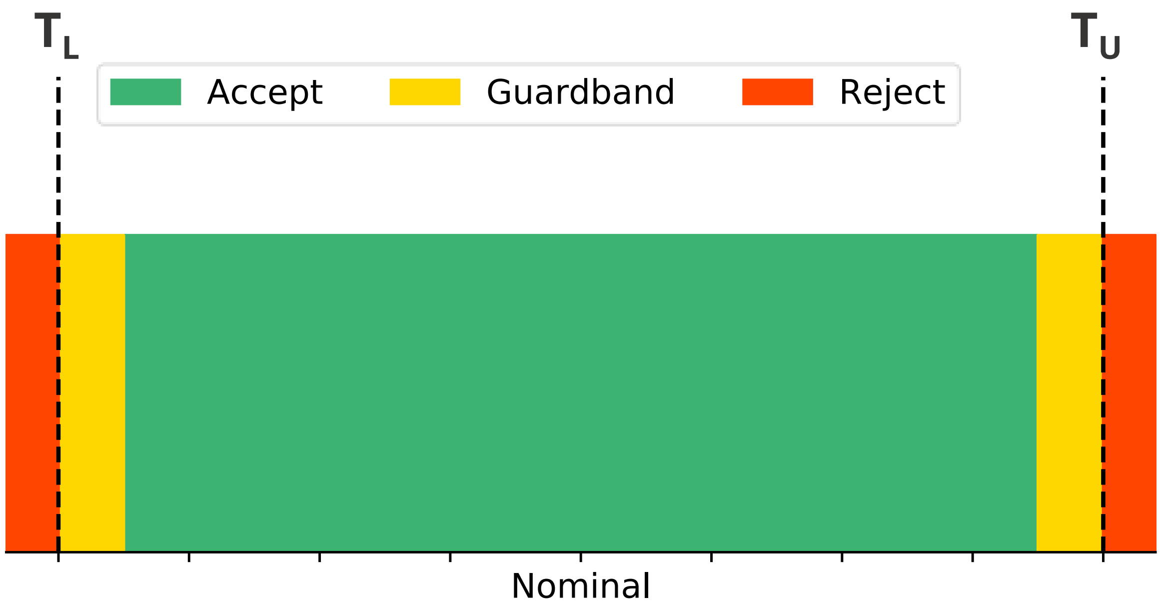
Typical rule: Apply guardband if TUR < 4

Determine Acceptance Limits A_L and A_U

Parts measuring in the green are accepted

Parts measuring in yellow are rejected, even though the measured value indicates it is within tolerance





Guardband factor is the multiplier used to reduce tolerance limits

- New acceptance limit = Original limit × guardband factor

- With Symmetric Limits
  - $\pm A = \pm T \times GBF$

- Or in terms of non-symmetric limits:
  - $A_L = T_L + (1 - GBF)\left(\frac{T_U - T_L}{2}\right)$
  - $A_U = T_U - (1 - GBF)\left(\frac{T_U - T_L}{2}\right)$

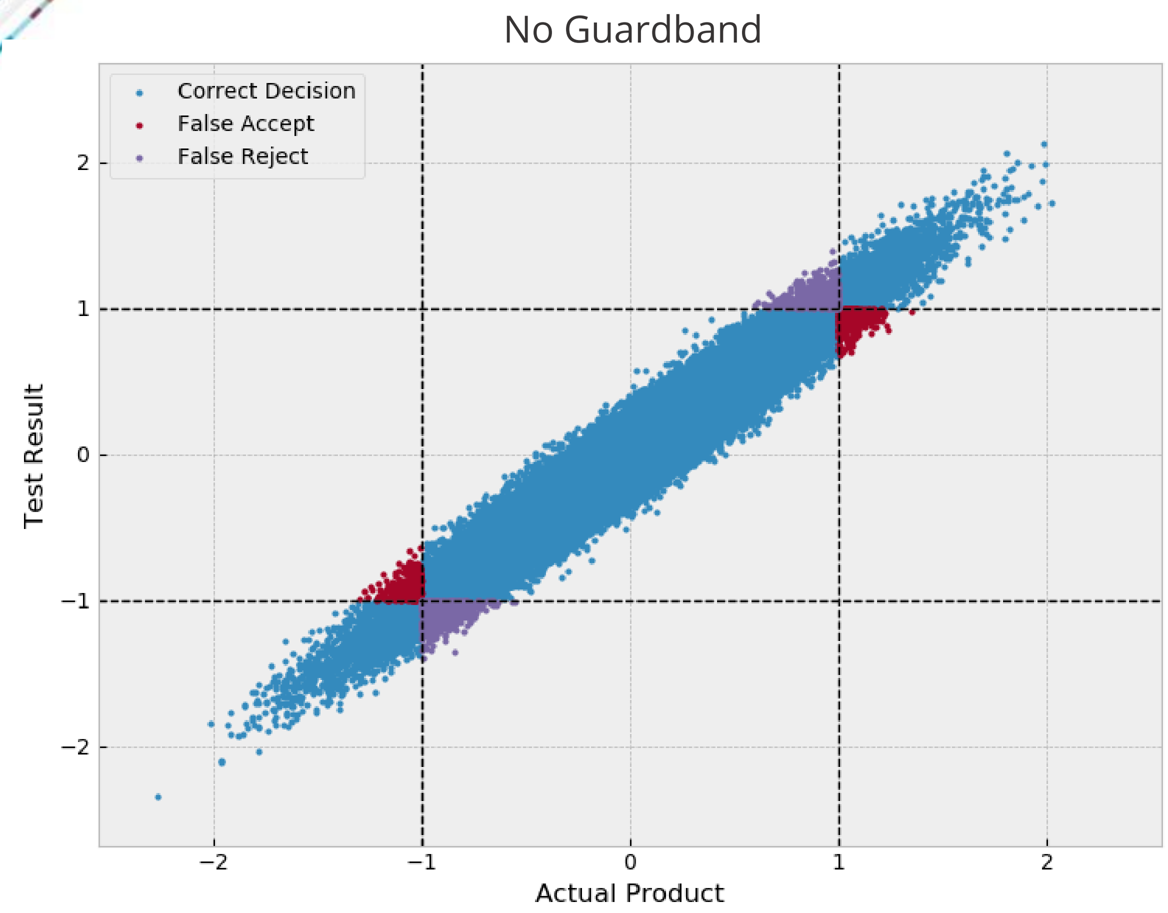
Guardbanded Risk – calculated from same integrals with modified limits

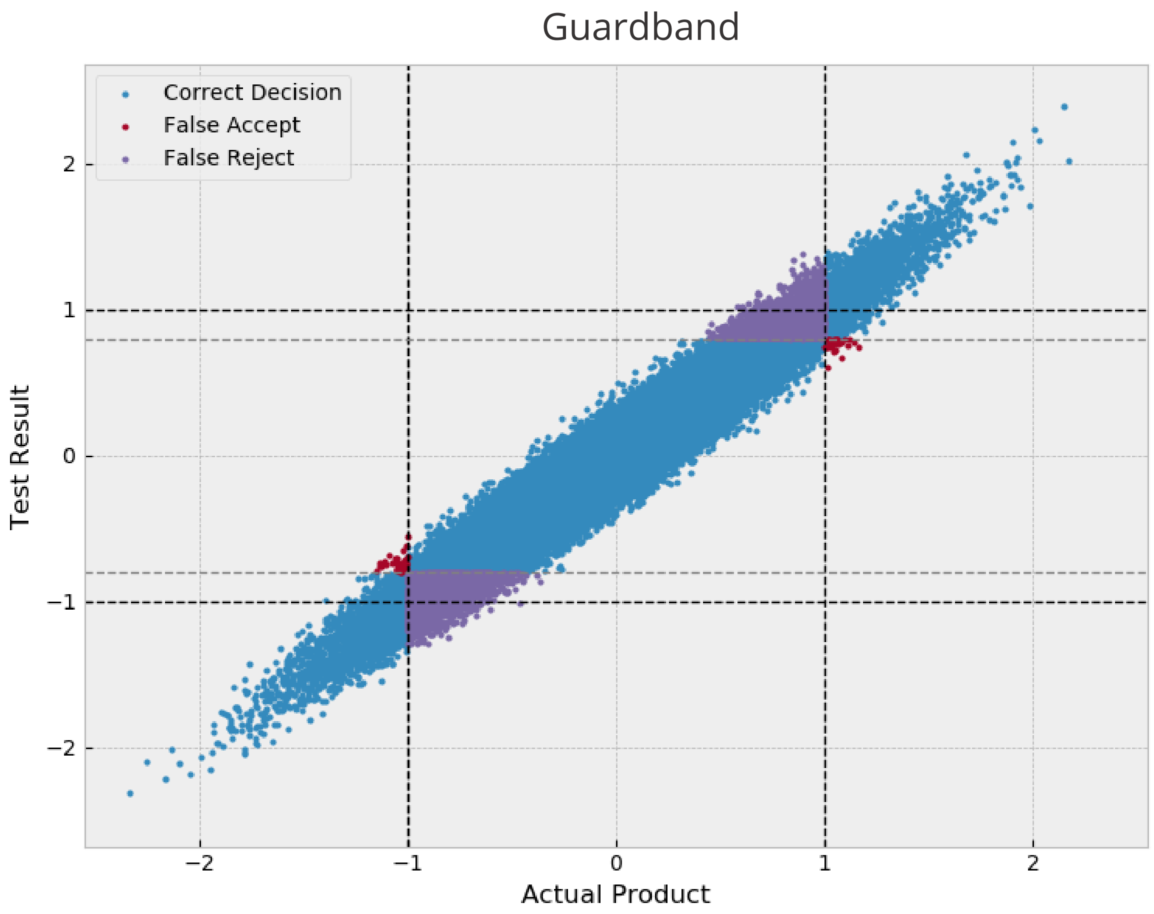
$$PFA = \int_{-\infty}^{T_L} \int_{A_L}^{A_U} p_m(t-y)\, p_p(t)\, dy\, dt + \int_{T_U}^{+\infty} \int_{A_L}^{A_U} p_m(t-y)\, p_p(t)\, dy\, dt$$

$$PFR = \int_{T_L}^{T_U} \int_{-\infty}^{A_L} p_m(t-y)\ p_p(t)\, dy\, dt + \int_{T_L}^{T_U} \int_{A_U}^{+\infty} p_m(t-y)\ p_p(t)\, dy\, dt$$

p_m= Measurement PDF
p_p= Product PDF
y = Measured Value
t = Possible True Values
T_L, T_U = Lower and Upper Tolerance Limits
A_L, A_U = Lower and Upper Acceptance Limits

Guardbanded Risk – visualized using Monte Carlo





Notice the tradeoff between PFA and PFR!

Common guardbanding methods

Most methods for calculating an appropriate guardband are based on TUR

1. RSS Method
   - $GBF = \sqrt{1 - 1/TUR^2}$

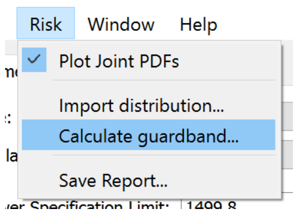
2. U_{95} Method
   - $GBF = 1 - 1/TUR$
   - Equivalent to $A = T - U_{95}$; subtract the 95% measurement uncertainty
   - "Method 5" in NCSLI's Handbook to Z540.3

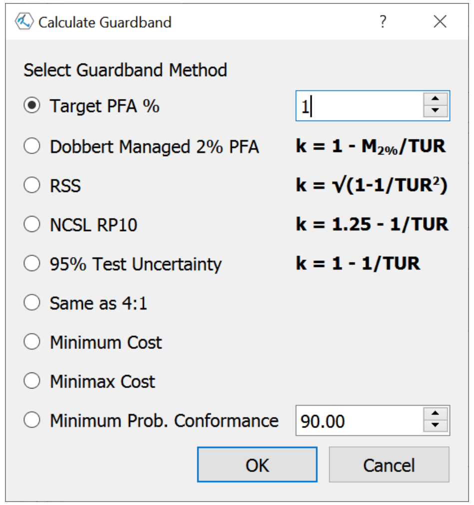
3. "Method 6" :
   - GBF = $1 - M_2/TUR$
   - $M_2 = 1.04 - \exp(0.38\ln(TUR) - 0.54)$
   - Guaranteed PFA < 2% for any ITP and TUR combination
   - "Method 6" in NCSLI's Handbook to Z540.3

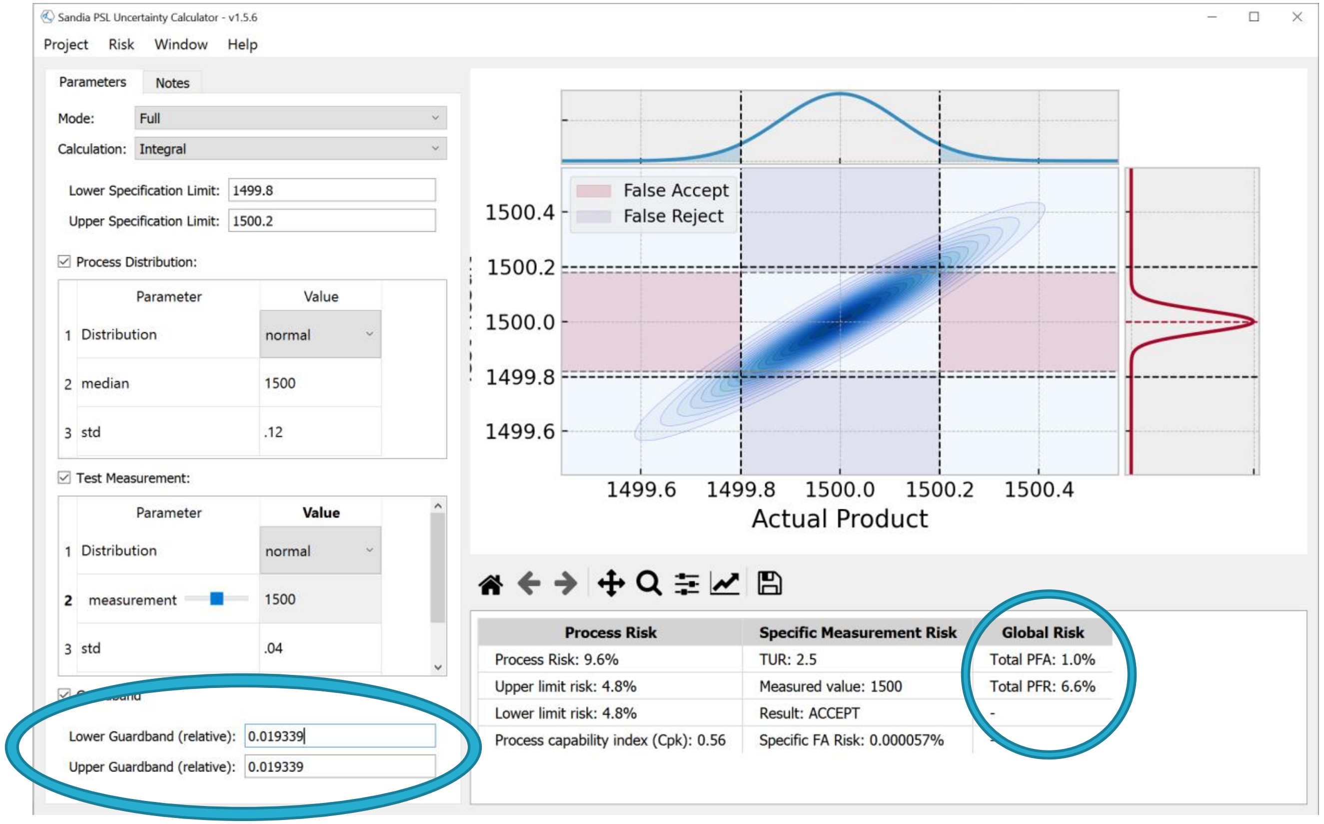
4. Target a desired PFA
   - Reverse the PFA integrals and solve for A_L, A_U that result in the desired PFA.
   - "Method 1" and "Method 2" in NCSLI's Handbook to Z540.3

Resistor Example continued with guardbanding

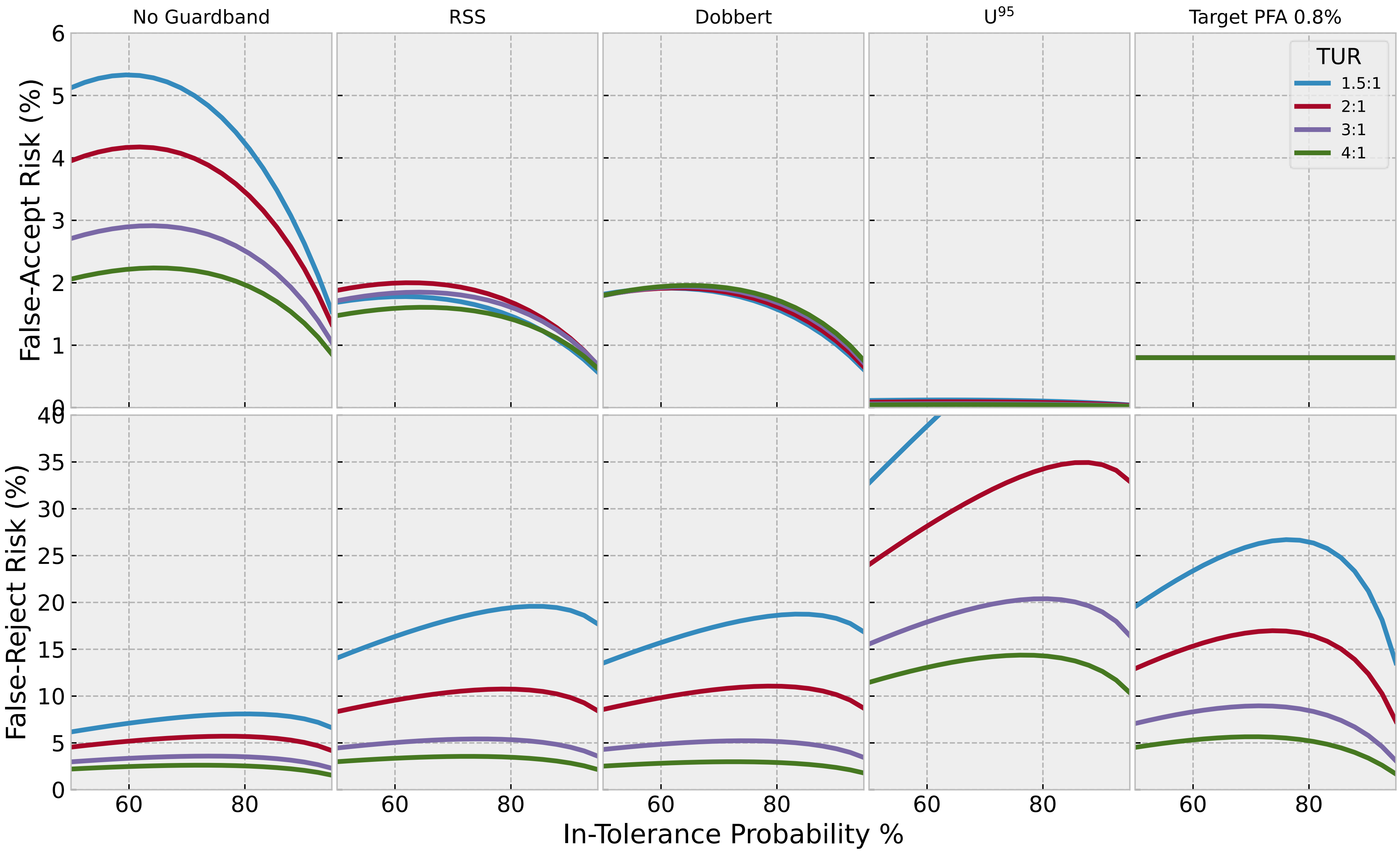
Use Suncal's guardband calculator to try different methods







How guardband methods reduce risk differently



Guardbanding comments and recommendations

Use the **RSS Method** when:

- Need an easy to remember equation
- Calculations are done by hand or in Excel

Use **Method 6** (Dobbert's Equation) when:

- Calculations are all performed in automated software
- OR if a PFA other than 2% is desired (rederive the coefficients)
- NOTE: this method will set A > T (accepting products outside the limits) if TUR > ~4.5!

Use $\boldsymbol{U_{95}}$ (Method 5) when:

- You don't like math and are ok with huge PFR
- Need to be extra-conservative
- You have a true one-sided limit and unknown product mean, where TUR is not defined

Use **Target PFA** method when:

- An exact PFA is desired, and fairly high PFR is acceptable.
- Computing power and software is available

Another option:

- Minimize total expected cost of false accept and false reject (see Easterling's papers)

Which is more important - Global Risk vs. Specific Risk?

The above guardbanding methods all work to reduce global risk. A guardband could also be computed to allow a maximum specific risk.

Consider specific risk as the “important number” when:

- No history or prior knowledge of unit being measured
- Cost of false accept far exceeds cost of false reject (need to be conservative)

Consider global risk when:

- Prior knowledge can be assumed on unit being measured
- Process is in tight control (very high ITP)

Unconditional Risk vs. Conditional Risk?

	Accepted	Rejected	Total
In Tolerance	83	7	90
Out of Tolerance	1	9	10
Total	84	16	100

Of **ALL** the parts that were made:

- PFA = 1 / 100 = 1.0% → P(OOT, Accept), unconditional false accept risk
- PFR = 7 / 100 = 7.0% → P(IT, Reject)

Of the parts **DELIVERED** to the customer:

- Conditional PFA (CPFA) = 1 / 84 = 1.2% → P(OOT | Accept)
- Conditional PFR (CPFR) = 7 / 16 = 44% → P(IT | Reject)

Note: Typical guardband methods are designed to control unconditional PFA.
Customers may be more concerned with CPFA. CPFA is always greater than PFA!

Question

T/F

The traditional RSS guardbanding equation is:

1-1/TUR

Section 6
When TUR doesn't work

One-sided limits, non-normal or biased distributions, etc.

Section 6 – When TUR doesn't work

Objective

- Discuss options for guardbanding and risk mitigation when TURs assumptions break down

Content

- TUR Assumptions
- One-sided tolerances
- Non-normal or biased distributions
- Exercise

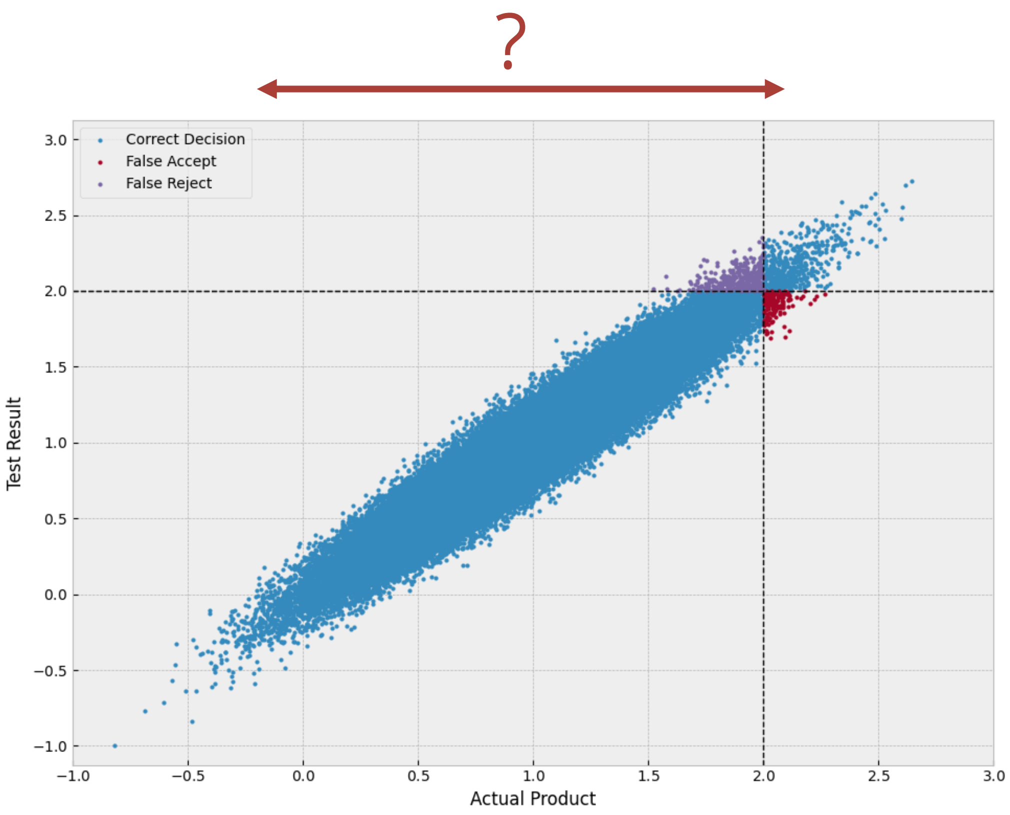
Recall the assumptions made by TUR and 4:1 rule

- > 80% itp in product distribution
- Unbiased distributions – product distribution centered between two limits
- Normal distributions
- 2% PFA is acceptable (using 4:1 rule)

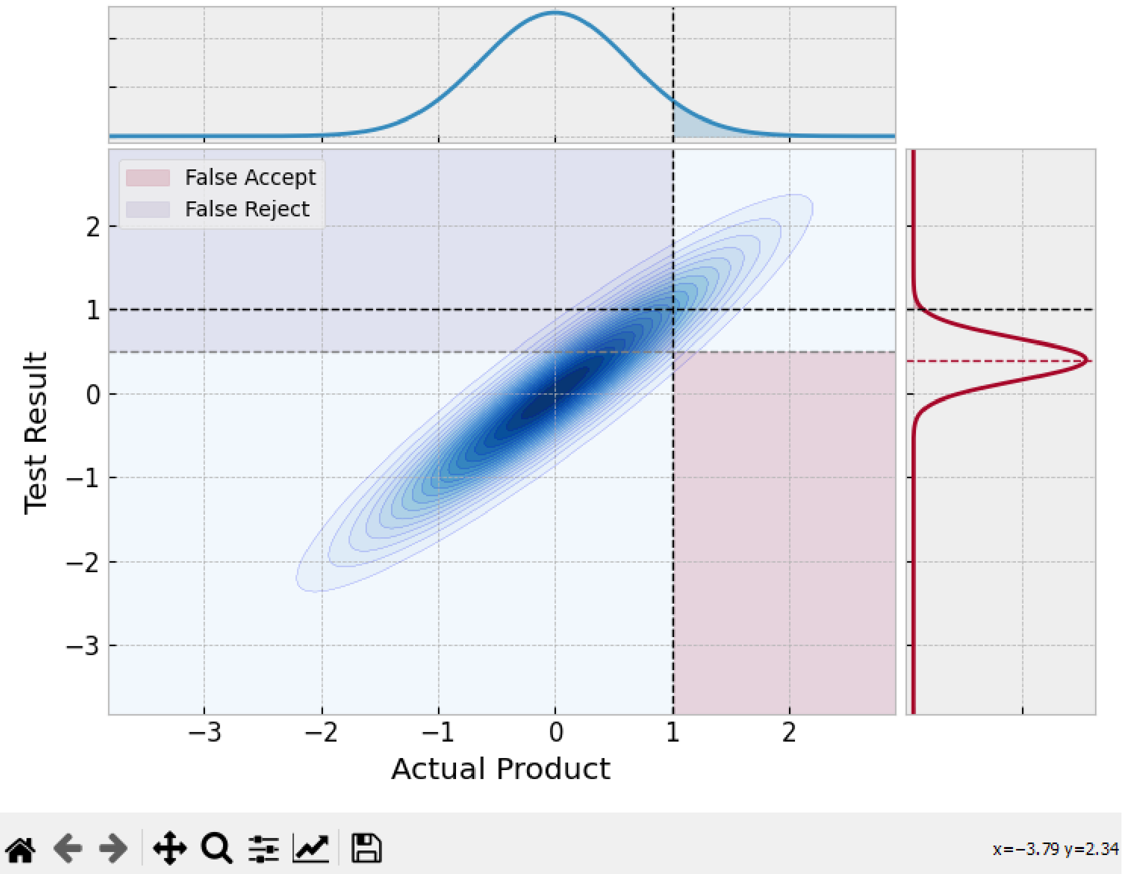
One-sided tolerances

Cannot calculate TUR = $(T_U - T_L)/2U_{95}$.

- No simple way to assume product distribution y_0 and u_0
- Typical solution when product distribution is unknown: use U_{95} guardbanding on single-sided limit: $A = T \; - U_{95}$

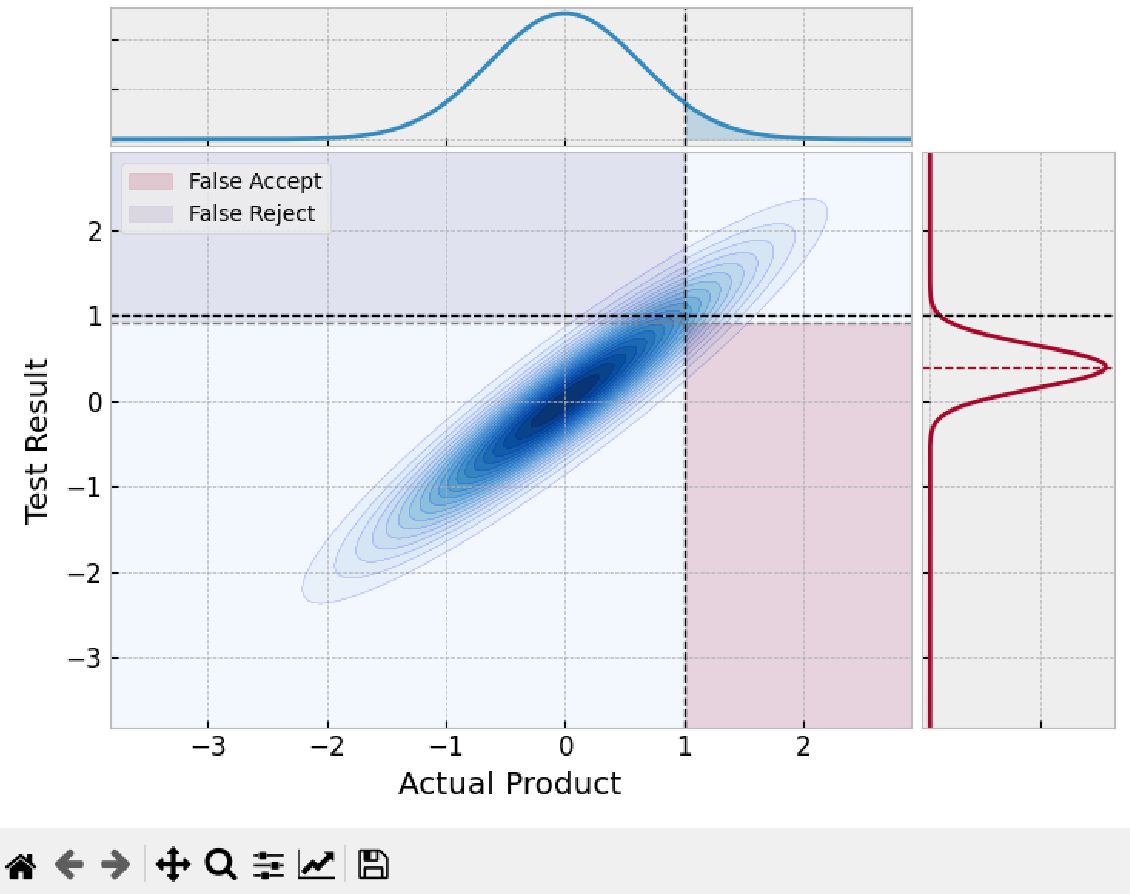


One-sided tolerances – Traditional U_{95} can be overly conservative



Process Risk	Specific Measurement Risk	Global Risk
Process Risk: 5.8%	TUR: inf	Total PFA: 0.031%
Upper limit risk: 5.8%	Measured value: 0.40	Total PFR: 17%
Lower limit risk: 0.0%	Result: ACCEPT	
Process capability index (Cpk): 0.52	Specific FA Risk: 0.82%	-

U_{95} Guardband



Process Risk	Specific Measurement Risk	Global Risk
Process Risk: 5.8%	TUR: inf	Total PFA: 0.80%
Upper limit risk: 5.8%	Measured value: 0.40	Total PFR: 4.1%
Lower limit risk: 0.0%	Result: ACCEPT	
Process capability index (Cpk): 0.52	Specific FA Risk: 0.82%	-

Target 0.8% Guardband

Assuming T_L = 0 is dangerous!

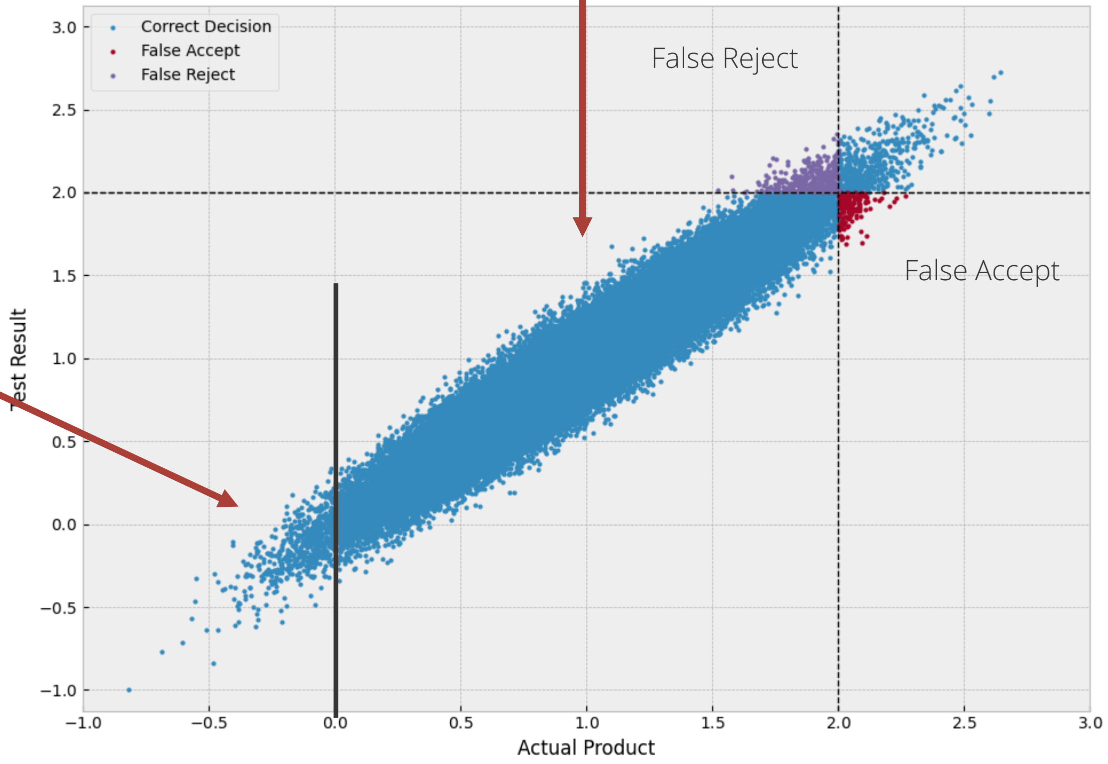
$$TUR = \frac{T_U - T_L}{2U_{95}}$$

$$ITP = 2\,\Phi\left(\frac{T_U - T_L}{2u_0}\right) - 1$$

Maximum limit; set $T_L = 0$?

Assumes average product is exactly half the limit

Assumes product will fall below zero with the same probability they fall above the maximum limit



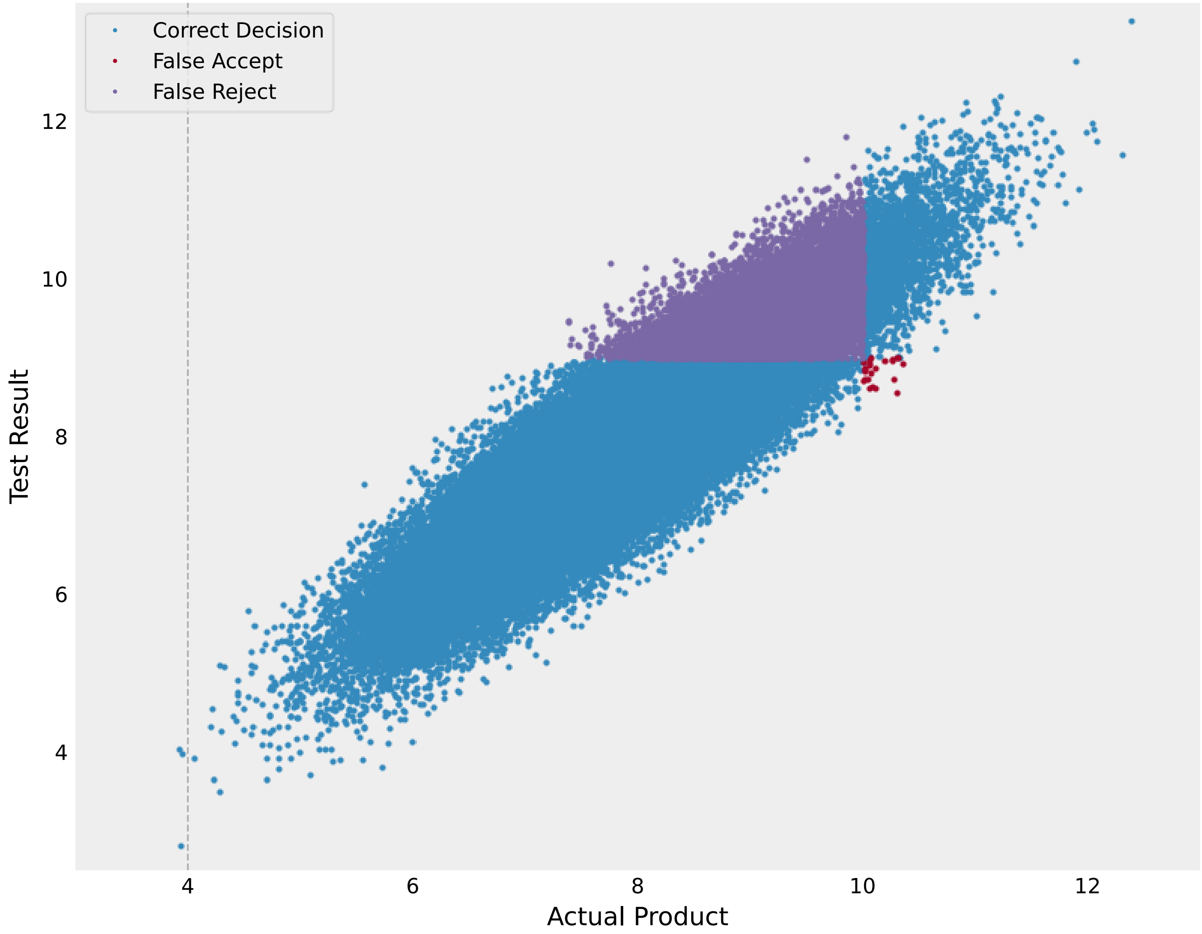
The implied zero limit does not result in a valid representation of products, so this TUR with T_L=0 is not a valid representation of risk!

One-sided Option 1: Always apply U_{95} Guardbanding

Acceptance Limit $A_U = T_U - U_{95}$

or $A_L = T_L - U_{95}$

(Subtract off the uncertainty)



Does not consider DUT population, so can be overly conservative (high false rejects)

Does not make any assumptions about DUTs.

Current NSE recommendation as defined in 9900000

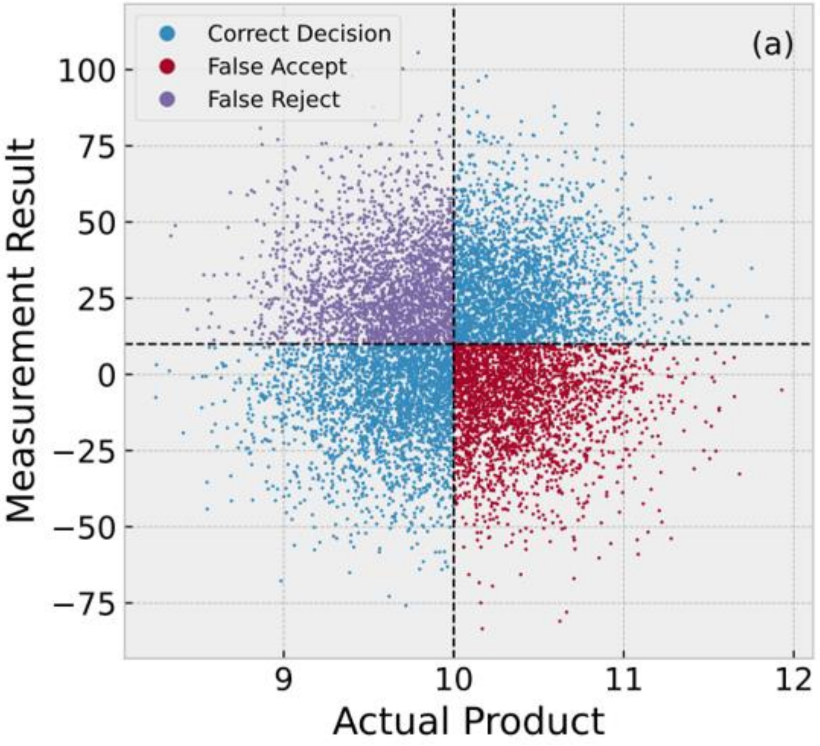
One-sided Option 2: Reduced U95 Guardband

Acceptance Limit $A_U = T_U - 0.871\ U_{95}$

or $A_L = T_L + 0.871\ U_{95}$

Not as overly-conservative as U95 method.

Results in < 2% PFA when process mean is within the tolerance.



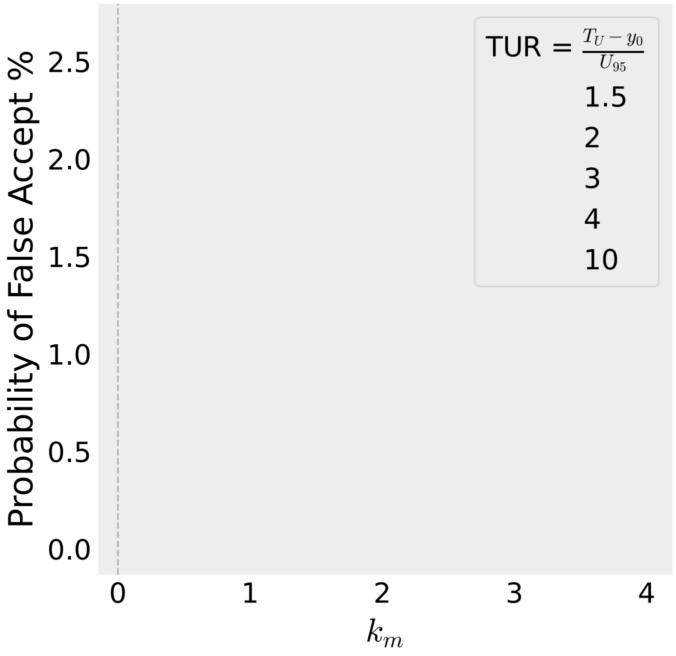
Considers this as worst-case

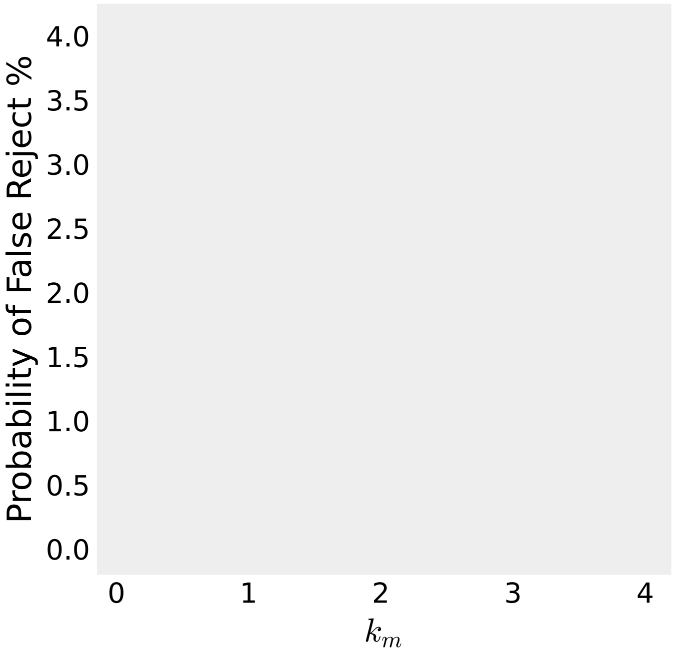
9900000 AU: Not explicitly included, but satisfies the new requirements

One-sided Option 3: Use a modified TUR > 4 rule

Calculate TUR by using a **known** DUT population mean:

$$TUR = \frac{T_U - y_0}{U_{95}}$$





$k_m = \frac{T_U - y_0}{u_0}$,

Number of standard deviations between the tolerance and DUT population mean (because one-sided ITP can't be calculated either)

9900000 AU: Added as a new option

One-sided Option 4: Just do the calculus

If y_0 is known, u_0 is probably known too (via sampling the population).

You have everything you need to just solve* for the ideal guardband directly…

$$PFA = 2\% = \int_{T_U}^{+\infty} \left(\int_{-\infty}^{A_U} \frac{1}{u_m\sqrt{2\pi}} e^{-\frac{1}{2u_m^2}(y-t)^2} dy \right) \frac{1}{u_0\sqrt{2\pi}} e^{-\frac{1}{2u_0^2}(t-y_0)^2} dt$$

*Suncal software can do the math for you.

**This is really the best option for all guardband calculations, not just with one-sided limits. It's not the 1950s anymore – we can use computers to do math!

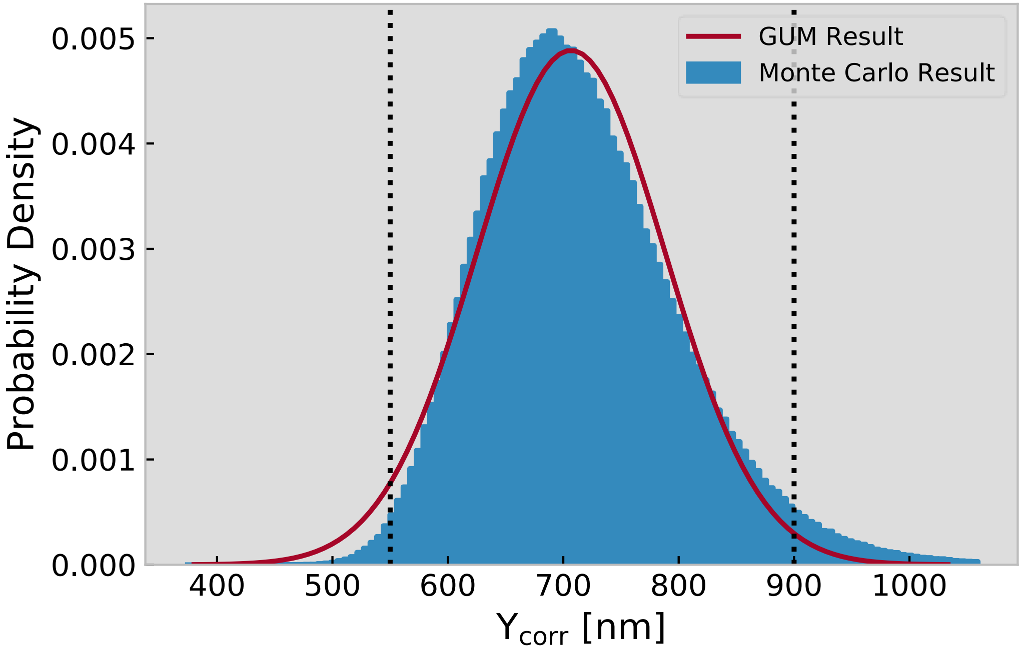
9900000 AU: Added as a new option

Non-normal distributions

When either distribution is not normal/gaussian, use the proper PDF in the PFA/PFR equations:

$$PFA = \int_{-\infty}^{T_L}\int_{T_L}^{T_U} p_m(t-y)\, p_p(t)\, dy\, dt + \int_{T_U}^{+\infty}\int_{T_L}^{T_U} p_m(t-y)\, p_p(t)\, dy\, dt$$

See "Evaluating Risk in an Abnormal World", NCSLI Measure, 13.2 (2020).



	Specific Risk Calculated Using:	
Limit	Normal PDF (GUM)	Histogram PDF (MC)
< 900 nm	0.90%	2.97%

Exercise – Ball Bearings (JCGM106 9.5.4)

Manufacturing precision ball bearings. Radial error motion must be less than 2 μm

- A perfect bearing has 0 radial error motion
- Process characterized by sampling: mean 1 μm; standard deviation 0.5 μm.
- Each bearing measured with standard uncertainty 0.25 μm (normal distribution)
- Maximum allowed PFA is 0.1%.

Questions to get started:

- **What are the tolerance limits?**
- **What is the TUR? Does TUR make sense?**
- **What PDF should be used for product distribution?**

Exercise – Ball Bearings (JCGM106 9.5.4)

Because radial error motion can never be negative, but process is close to zero, a gamma distribution can be used. (See JCGM106 B.3)

Lower limit is implied at 0.

TUR does not apply! (Distribution is gamma, customer requires 0.1% PFA max)

Gamma Distribution:

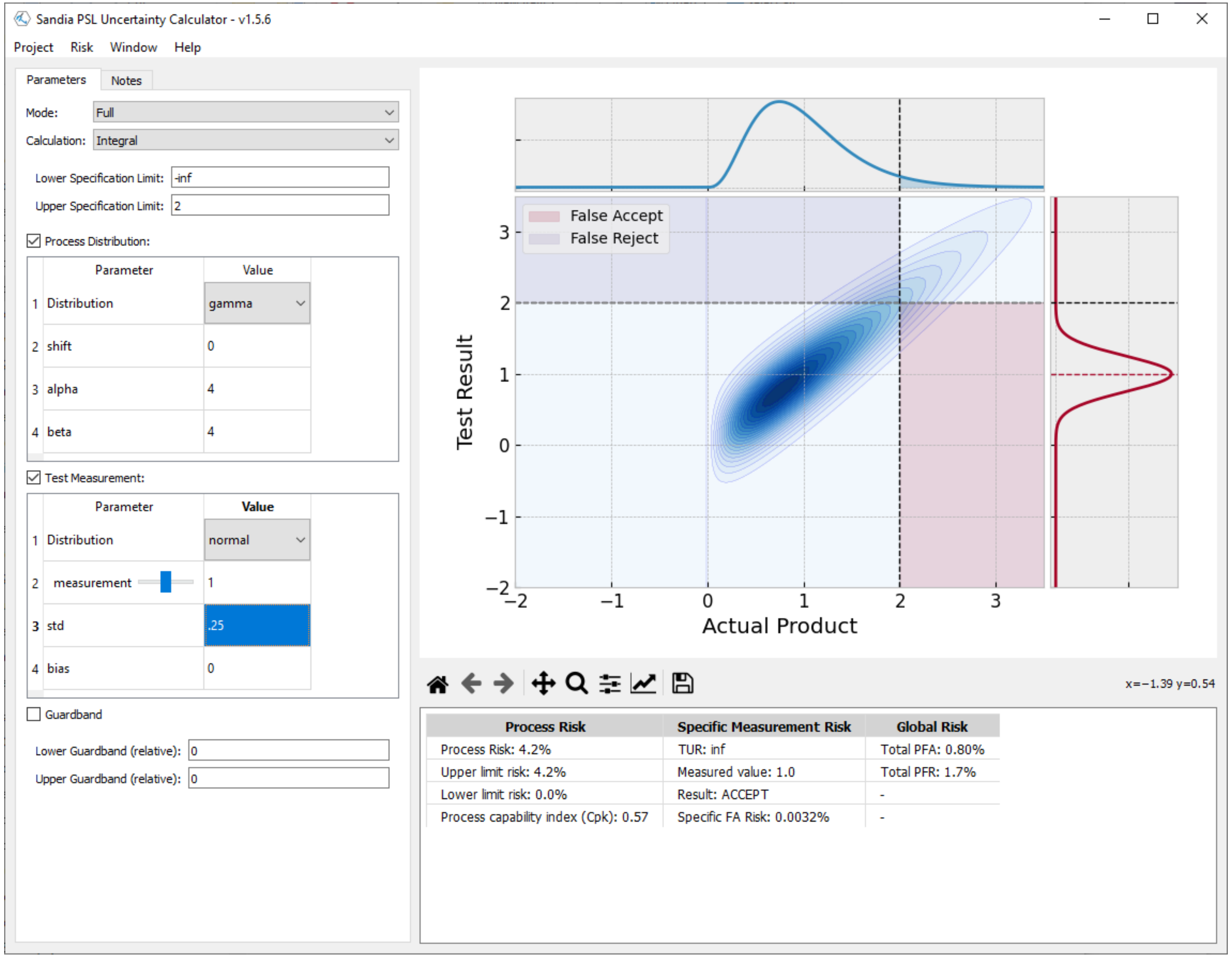
$$\alpha = \frac{\mu^2}{s^2} = \frac{1\ \mu m^2}{0.5\ \mu m^2} = 4$$

$$\beta = \frac{\mu}{s^2} = \frac{1\ \mu m}{0.5\ \mu m^2} = 4$$

Exercise – Ball Bearings (JCGM106 9.5.4)

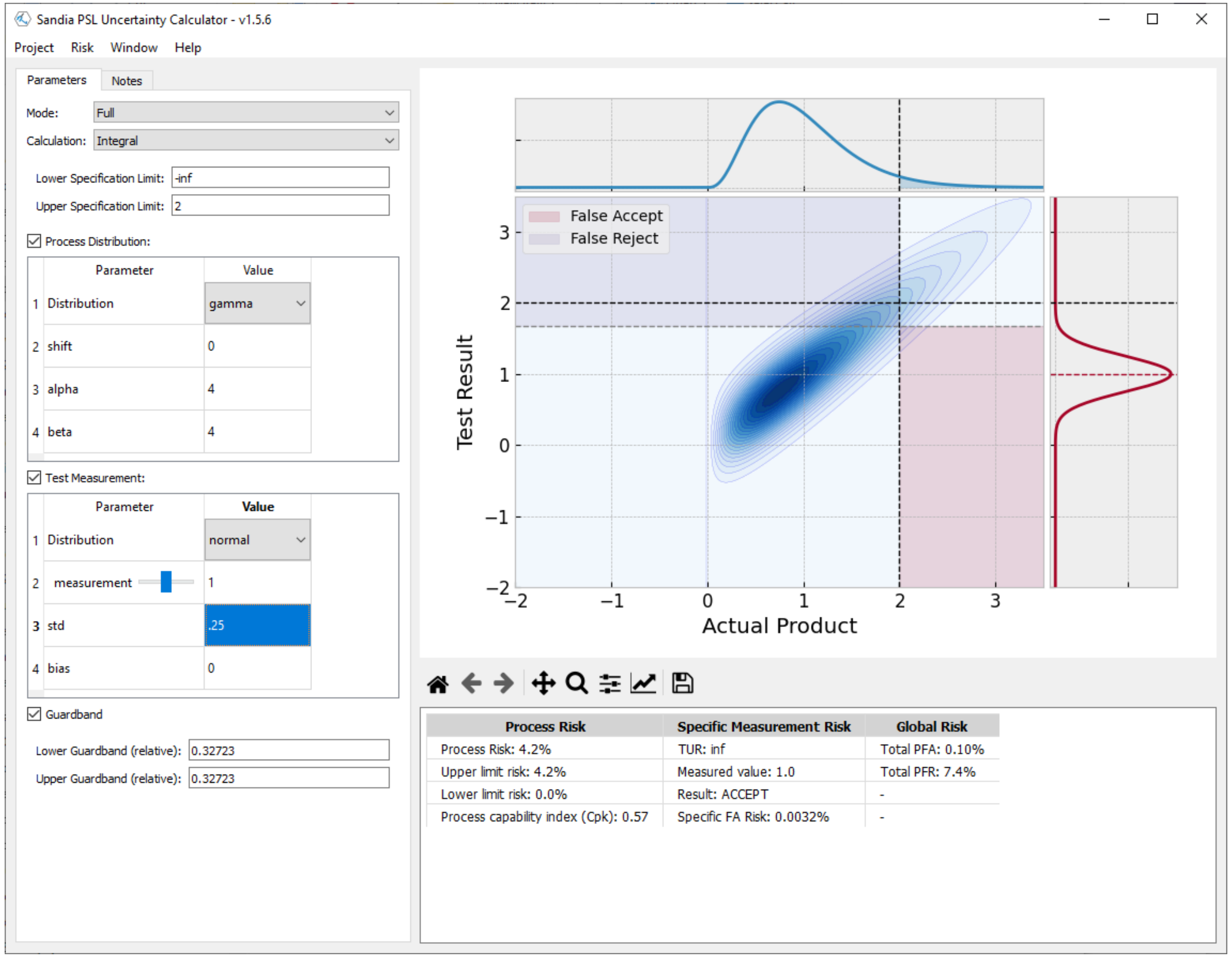
- Requirement: < 2 μm
- Process sampled: mean 1 μm; standard deviation 0.5 μm.
  - Gamma distribution, $\alpha = 4$, $\beta = 4$
- Measurement uncertainty: 0.25 μm (normal distribution, $k = 1$)
- Maximum allowed PFA is 0.1%.

Exercise – Ball Bearings (JCGM106 9.5.4)



Exercise – Ball Bearings (JCGM106 9.5.4)

Use the guardband finder (Risk Menu) to target the required PFA of 0.1%.



Wrap-Up

Decision Rules

A “decision rule” is a formal statement of how a decision is made, taking into account measurement uncertainty and risk. Examples:

- Simple Acceptance: Accept any product measuring within the limits
- Guarded Acceptance: Accept products within reduced limits applied when TUR < 4.

Other options:

- Conditional Acceptance: Products within the guardband are “undetermined”
- Nonbinary Rules: Products that are too big can be sent back for rework, products too small must be scrapped

All decision rules must be agreed upon between the metrology organization and customer

Changes to General Requirements 9900000 Issue AU (Jan 2024)

1. Test Accuracy Ratio (TAR) based evaluation — **Clarified calculation, +examples**
   - Two-sided Tolerances — No Change
   - One-sided Tolerances — **Defined TAR-based options**
2. Specification of acceptance equipment in drawing — **Clarifications**
3. Direct evaluation of false accept risk — **New option**

Aligns with new DPBPS R028 Metrology Program requirements (effective August 2025)

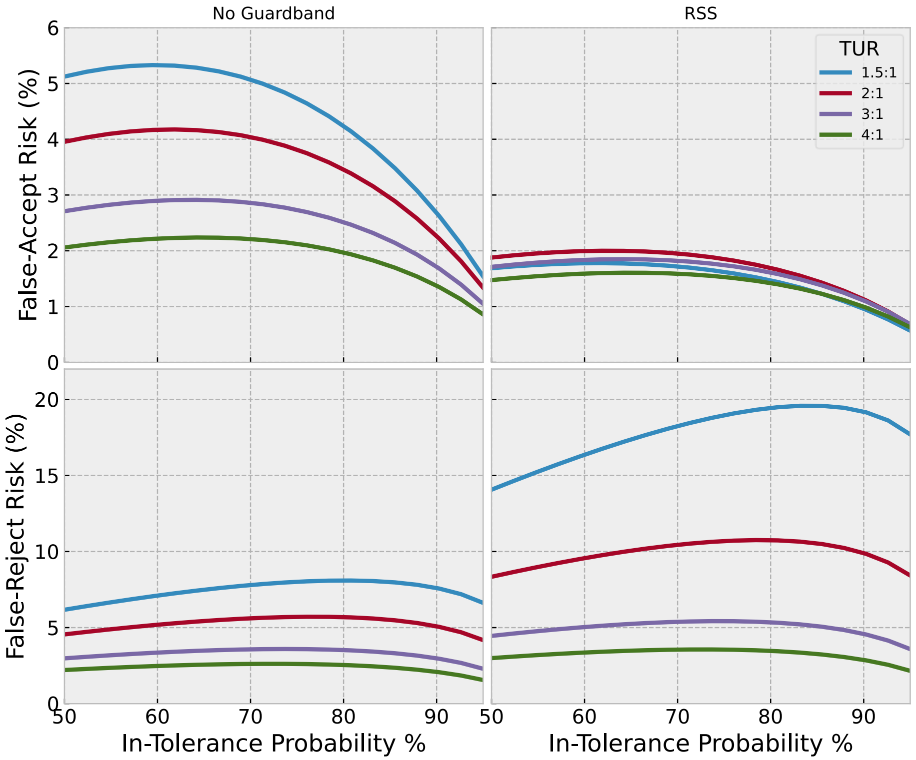
Back to Measurement Assurance Plans

- Use TUR if its assumptions are justified
- If not, evaluate false accept/reject and/or worst-case probability of conformance to determine if risk is acceptable

Q1. What quantity are you measuring?			Q2: How accurate do the measurements need to be?			Q3: How will you ensure your equipment can make this measurement?			
Quantity Measured	MC	Requirement Number	Value or Range of Values Measured	Tolerance Limits	Guardbanded Acceptance Limit	Equipment Used (M&TE)	Measurement Uncertainty	TUR (>4 desired)	Calibrate (Yes/No)

Bottom line

How you evaluate risk and assign acceptance limits all depends on what data is available and how much risk, either global or specific, is acceptable for the given application.



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Variables Glossary

T_U: Upper tolerance limit
T_L: Lower tolerance limit
A_U: Upper acceptance limit
A_L: Lower acceptance limit
p_m: Probability density function of measurement
p_p: Probability density function of products (prior knowledge)
y_0: Mean of product distribution
u_0: Standard deviation of product distribution
u_m: Standard deviation of measurement distribution
y: Measured Value
TUR: Test Uncertainty Ratio = $\frac{T_U - T_L}{2U_{95}} = \frac{T_U - T_L}{4u_m} = \frac{\pm T}{\pm U_{95}}$
itp: In-tolerance probability
U_{95}: Expanded (95% coverage) measurement uncertainty

Appendix A
Equation Variations

Differences in notation used in various publications

JCGM106 Eq. 19 and 20 with normal distributions substituted

$$PFA = \int_{-\infty}^{T_L} \left(\int_{A_L}^{A_U} \frac{1}{u_m\sqrt{2\pi}} e^{-\frac{1}{2u_m^2}(y-t)^2} dy \right) \frac{1}{u_0\sqrt{2\pi}} e^{-\frac{1}{2u_0^2}(t-y_0)^2} dt$$
$$+ \int_{T_U}^{+\infty} \left(\int_{A_L}^{A_U} \frac{1}{u_m\sqrt{2\pi}} e^{-\frac{1}{2u_m^2}(y-t)^2} dy \right) \frac{1}{u_0\sqrt{2\pi}} e^{-\frac{1}{2u_0^2}(t-y_0)^2} dt$$

u_m = Standard Deviation of Measurement Unc.
u_0 = Standard Deviation of Product Dist.
y_0= Mean of Product Distribution.
y = Measured Value
T_L, T_U = Lower and Upper tolerance limits
A_L, A_U = Lower and Upper acceptance limits

$$PFR = \int_{T_L}^{T_U} \left(\int_{-\infty}^{A_L} \frac{1}{u_m\sqrt{2\pi}} e^{-\frac{1}{2u_m^2}(y-t)^2} dy \right) \frac{1}{u_0\sqrt{2\pi}} e^{-\frac{1}{2u_0^2}(t-y_0)^2} dt$$
$$+ \int_{T_L}^{T_U} \left(\int_{A_U}^{+\infty} \frac{1}{u_m\sqrt{2\pi}} e^{-\frac{1}{2u_m^2}(y-t)^2} dy \right) \frac{1}{u_0\sqrt{2\pi}} e^{-\frac{1}{2u_0^2}(t-y_0)^2} dt$$

JCGM106 Normal Probability Density and Distribution Functions

The normal probability density function for the measurand *Y*:

$$g(\eta|\eta_m) = \frac{1}{u\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{\xi-\mu}{u}\right)\right] \coloneqq \varphi(\eta; y, u^2)$$

The probability of a normally distributed measurand *Y* is denoted by;

$$\Pr(a \le Y \le b|\eta_m)$$
$$= \Phi\left(\frac{b-y}{u}\right) - \Phi\left(\frac{a-y}{u}\right)$$

The normal distribution function is denoted by:

$$\Phi(z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z} \exp(-t^2/2)dt$$

JCGM106 Equations A.15 and A.16 (for normal distributions)

Equation A.15 in JCGM106 for false accept probability:

$$PFA = \int_{-\infty}^{(T_L-y_0)/u_0} F(z)\varphi(z)dz + \int_{(T_U-y_0)/u_0}^{\infty} F(z)\varphi(z)dz$$

Where $\varphi(z) = \frac{1}{\sqrt{2\pi}}\exp\left(-\frac{z^2}{2}\right)$,

$$F(z) = \Phi\left(\frac{A_U-y_0-u_0z}{u_m}\right) - \Phi\left(\frac{A_L-y_0-u_0z}{u_m}\right)$$

$\Phi(x)$ = standard normal distribution function = $\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x} e^{-t^2/2}dt$

This equation is equivalent to equation 9. By changing variables, one integration was replaced with the Φ function. May be slightly faster to execute in software that has efficient implementations of $\Phi(x)$.

Deaver's Equations for PFA and PFR (normal distributions)

Deaver's "How to Maintain Confidence" (1993) paper gives alternate equations for PFA and PFR (Eq 6 and 7) that also assumes normal distributions, but his paper uses different notation:

- Deaver uses a different definition of TUR, and gives it the symbol R:
  - $R = u_0/u_m$.
- Specification limit is defined in terms of number of standard deviations of the process distribution:
  - $SL = (TU - y_0)/u_0$
- Our $TUR = \frac{T_U - T_L}{2U_{95}} = \frac{SL}{R}$.
- Guardband Factor K:
  - $K = (AU - y_0)/(TU - y_0)$
  - $K = 1$ means no guardband is applied.
- SL and K assume T_U and T_L are symmetric about y_0.

Notation translator

	JCGM 106	Crowder	Dobbert	Deaver
Tolerance Limits	T_L, T_U	SL_L, SL_U	$\pm T$ (Relative to 0)	$\pm SL \cdot \sigma$ (Tolerance defined as # stdevs from nominal)
Acceptance Limits	A_L, A_U	AL_L, AL_U	$\pm A$ (Relative to 0)	$\pm K \cdot SL \cdot \sigma$ (Guradband factor K)
Product Mean	y_0	μ_p	0	0
Product Std. dev.	u_0	σ_p	σ_0	σ
Measurement Std. dev.	u_m	σ_m	σ_m	σ/R (R = σ/σ_m is what Deaver calls TUR)
TUR	$\frac{T_U - T_L}{4u_m}$	$\frac{SL_U - SL_L}{4\sigma_m}$	$T/(2\sigma_m)$	SL/R (Gives equivalent JCGM106 TUR in terms of Deaver's notation)

Appendix B
Additional Considerations

Potentially useful stuff we don’t have time for in a 4-hour class

Probability of Conformance using Microsoft Excel

Excel formula NORM.DIST computes the normal PDF.

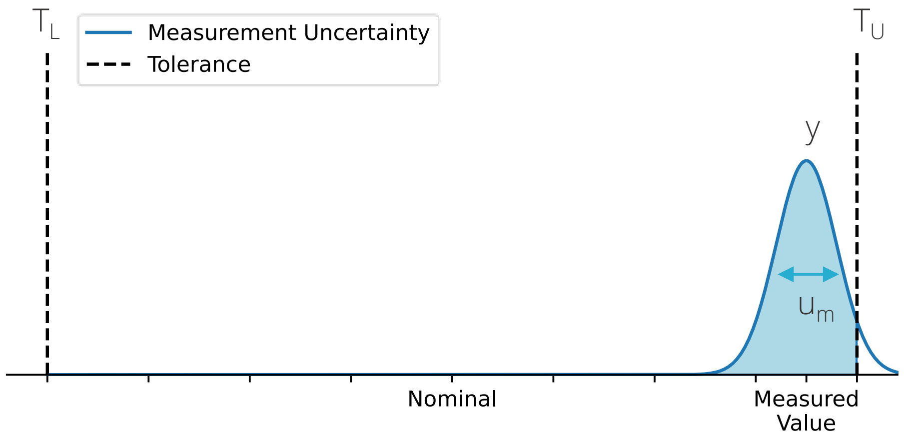
One-sided (maximum) limit:

- p_c = NORM.DIST(TU, y, um, TRUE) → probability of conformance (integrate below TU)

Two-sided limit:

- p_c = NORM.DIST(TU, y, um, TRUE) – NORM.DIST(TL, y, um, TRUE)

The TRUE parameter tells it to compute the cumulative PDF (which does the integral)



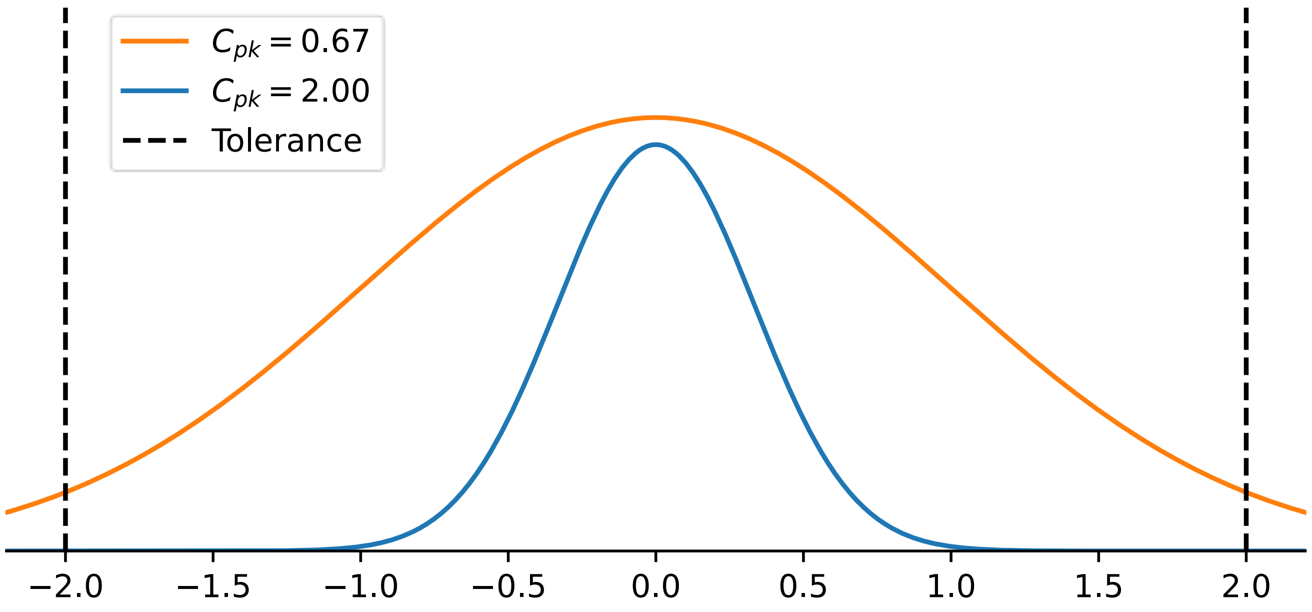
Process Capability Index C_{pk}

Metric used to describe quality of process distribution (observed mean y_0 and std.dev. u_0). Related to itp. Measurement uncertainty is not considered.

If C_{pk} is high enough, risk is minimal regardless of measurement uncertainty

- $C_{pk} = \min\left[\frac{T_U - y_0}{3u_0}, \frac{y_0 - T_L}{3u_0}\right]$

Cpk	Sigma Level	Process Yield
0.33	1	68.27%
0.67	2	95.45%
1.00	3	99.73%
1.33	4	99.99%
1.67	5	99.9999%
2.0	6	99.9999998%



Guardbanding methods as described in NCSL's Guide to Z540

Require numerical minimization techniques:

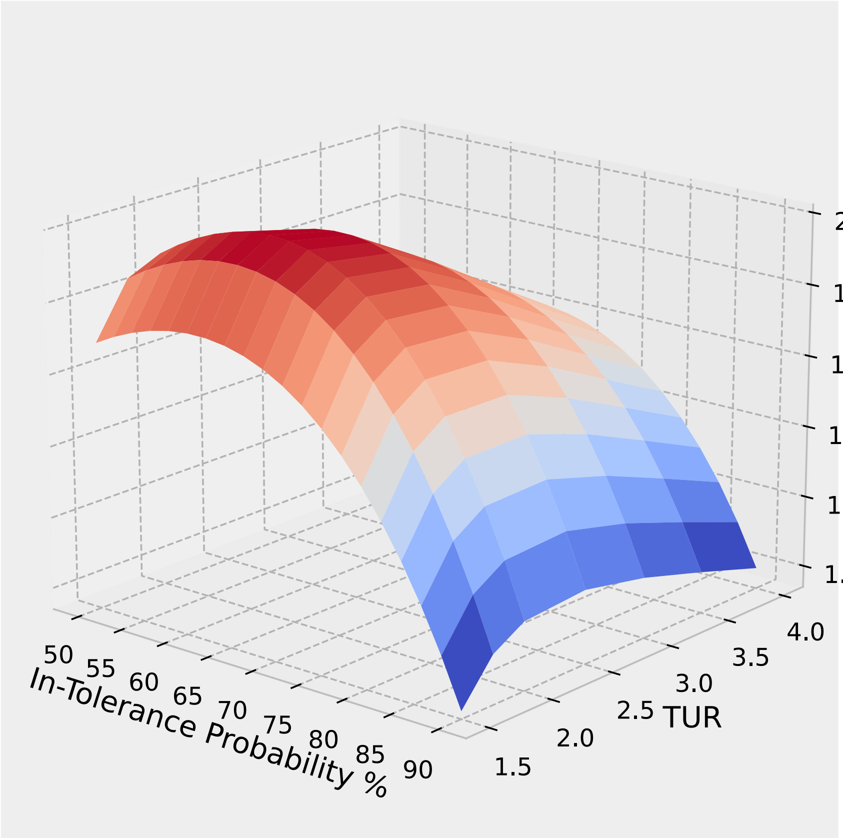
- Method 1: Solve for PFA = 2%, at each test point
- Method 2: Solve for PFA = 2%, at equipment level
- Method 3: Solve for probability of false accept GIVEN acceptance result
- Method 4: Bayesian statistics approach

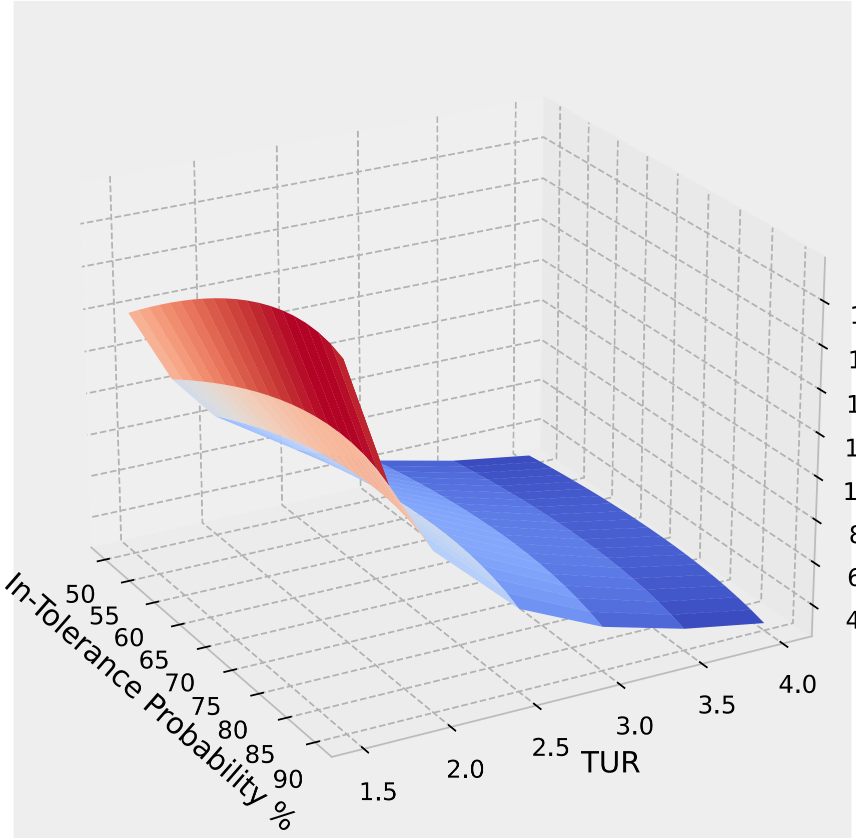
Calculate by hand, Excel, etc. (analytical solution):

- Method 5: Subtract 95% Uncertainty → $A = T - U_{95}$
- Method 6: TUR-based, managed guardband (Dobbert's Method)

Risk in 3D!

With RSS Guardbanding



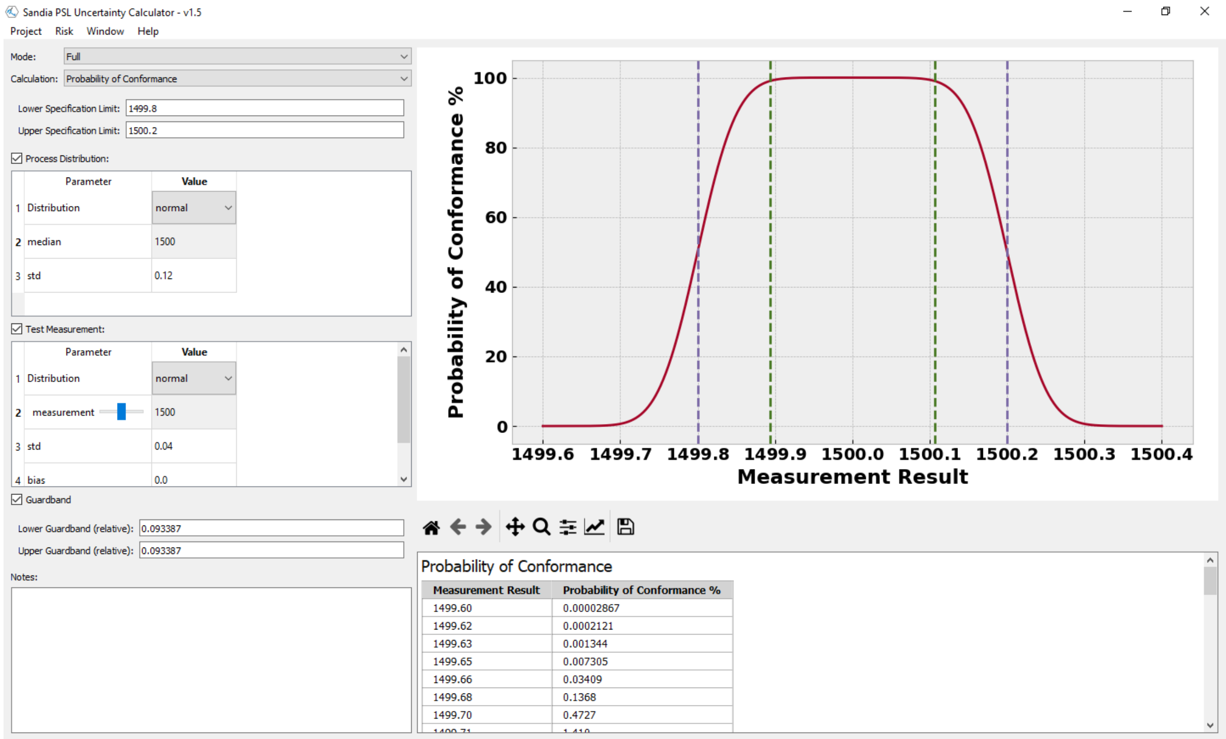


Specific-Risk Guardbanding

Remember you can always have up to 50% specific risk?

If specific risk is a concern, or process distribution is unknown, can set guardband based solely on it.

Resistor Problem from JCGM 106 with Minimum Probability of Conformance set to 99%:



Appendix C
Suncal Risk Calculations

Calculator Download (Desktop Version)

Download the latest version of the calculator from
https://github.com/sandialabs/suncal/releases/latest

(v1.5.2+ required for this course, to use gamma distribution)

Files:

- SandiaUncertCalc.exe – standalone Windows executable, no installation required
- SandiaUncertCalc.zip – Same as above, but in zip format for download through restrictive firewalls
- SandiaUncertCalcInstall.exe – installer for Windows. Runs a bit faster than above
- SandiaUncertCalc-OSX-x.x.zip – Package for Mac OS
- Suncalmanual.pdf – User's manual
- Examples.zip – Example uncertainty problems to load in to the calculator

Features

Given: Measurement model, input values and uncertainties

Use: Uncertainty Propagation

Given: Measurement model, output requirement, need input uncertainty

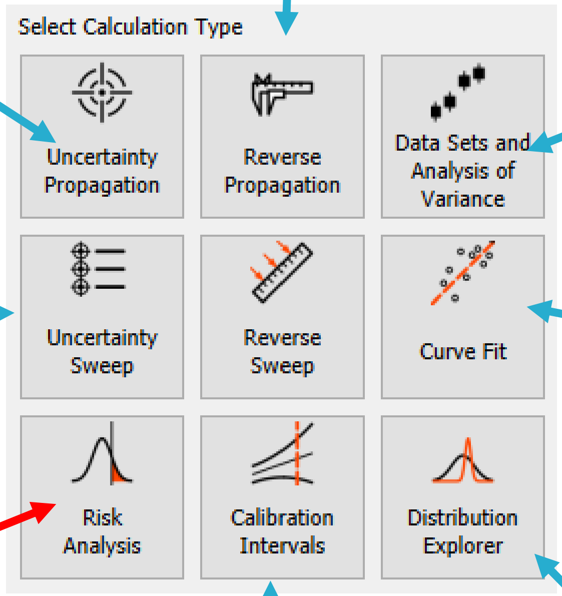
Use: Reverse Propagation

Given: 2-Dimensional data (e.g. N measurements × M days)

Use: Data Sets and ANOVA

Given: Measurement model, inputs could take several values

Use: Sweeps



Given: (x, y) data, need relationship between x and y

Use: Curve Fit

Given: Probability Distributions and tolerance limits

Use: Risk Analysis

Given: Historical calibration pass/fail data

Use: Calibration Intervals

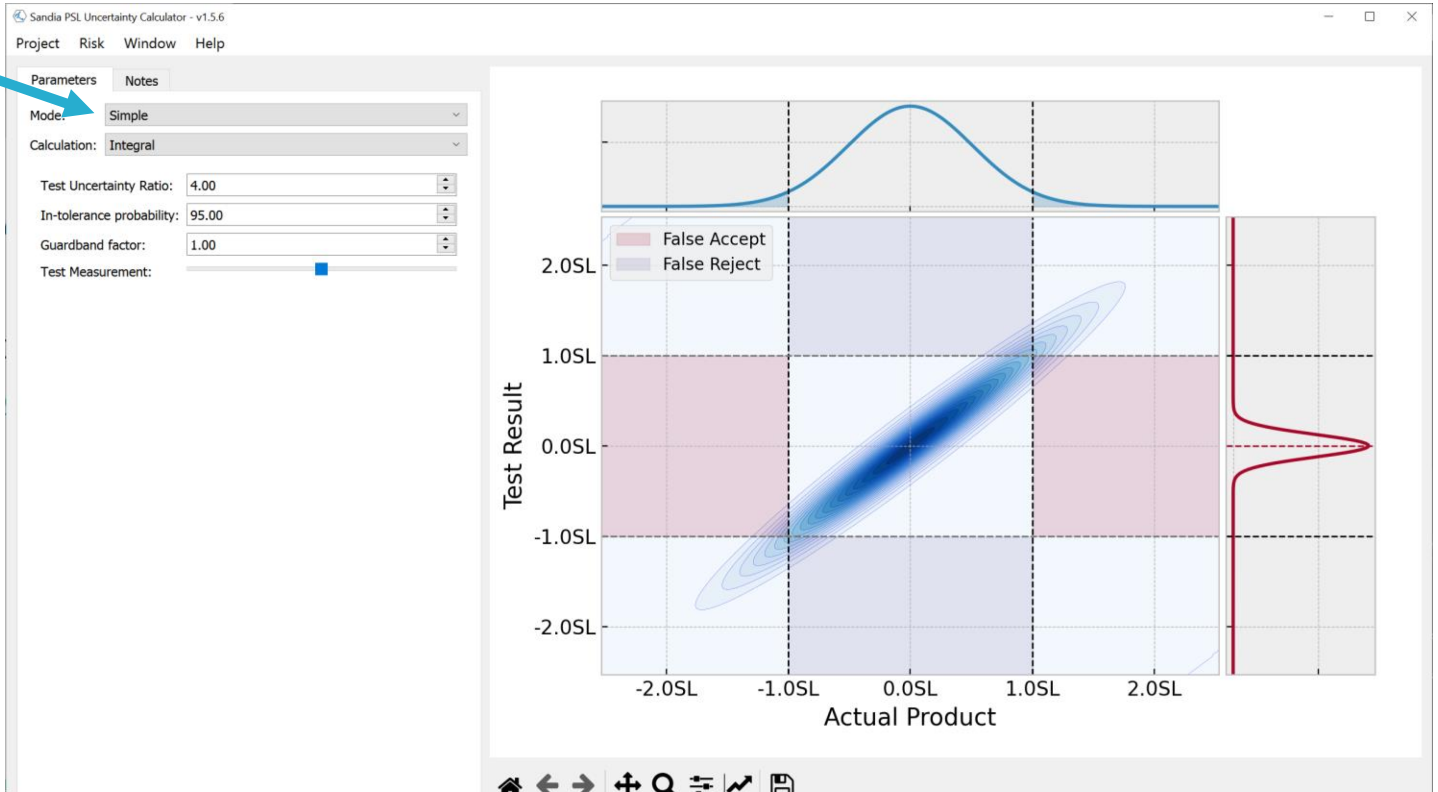
Given: Probability Distributions to visualize

Use: Distribution Explorer

Simple Mode

Calculate PFA/PFR by entering TUR, ITP, and GBF.

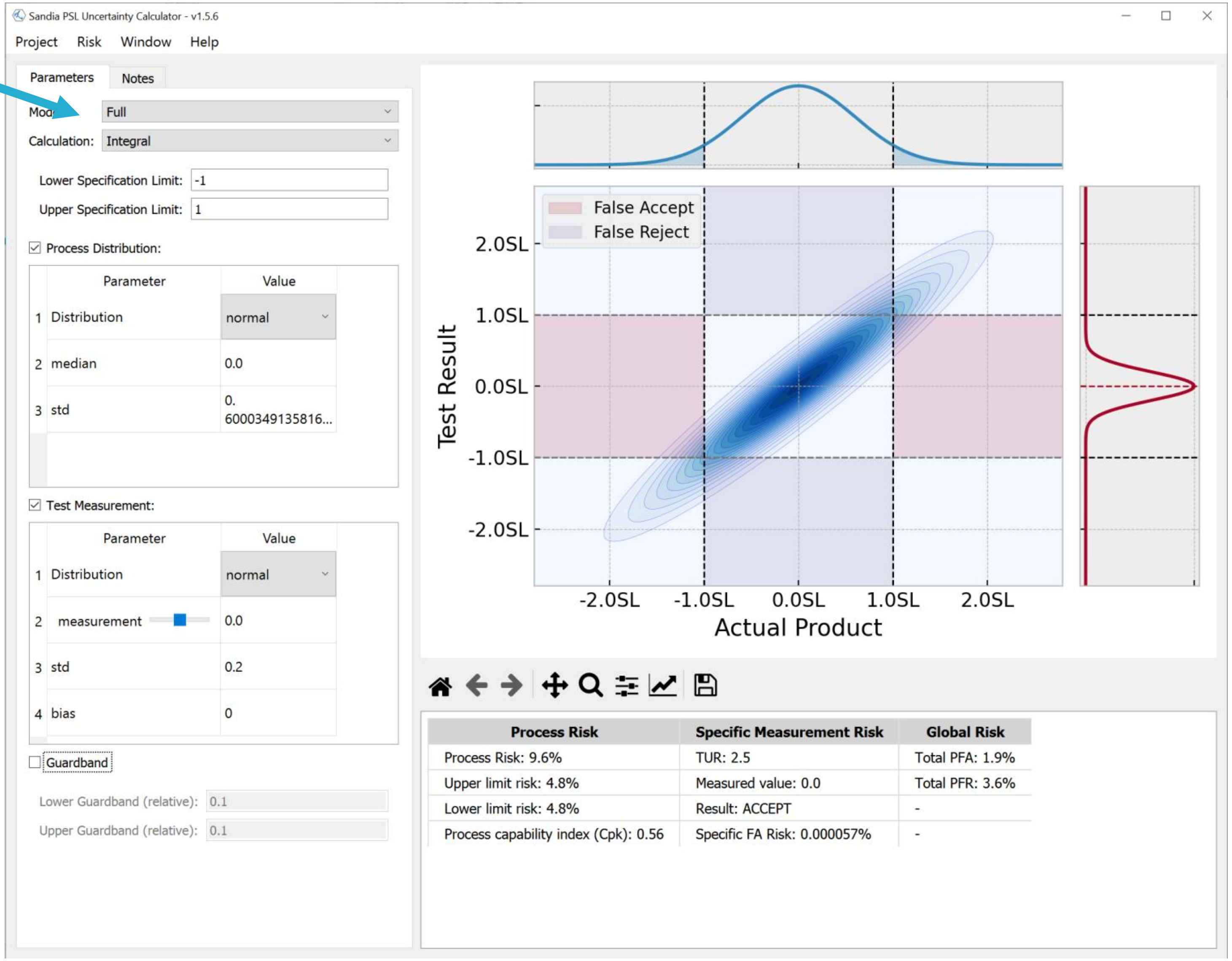
Assumes normal distributions.



Process Risk	Specific Measurement Risk	Global Risk
Process Risk: 5.0%	TUR: 4.0	Total PFA: 0.88%
Upper limit risk: 2.5%	Measured value: 0.0	Total PFR: 1.5%
Lower limit risk: 2.5%	Result: ACCEPT	-
Process capability index (Cpk): 0.65	Specific FA Risk: 1.3e-13%	-

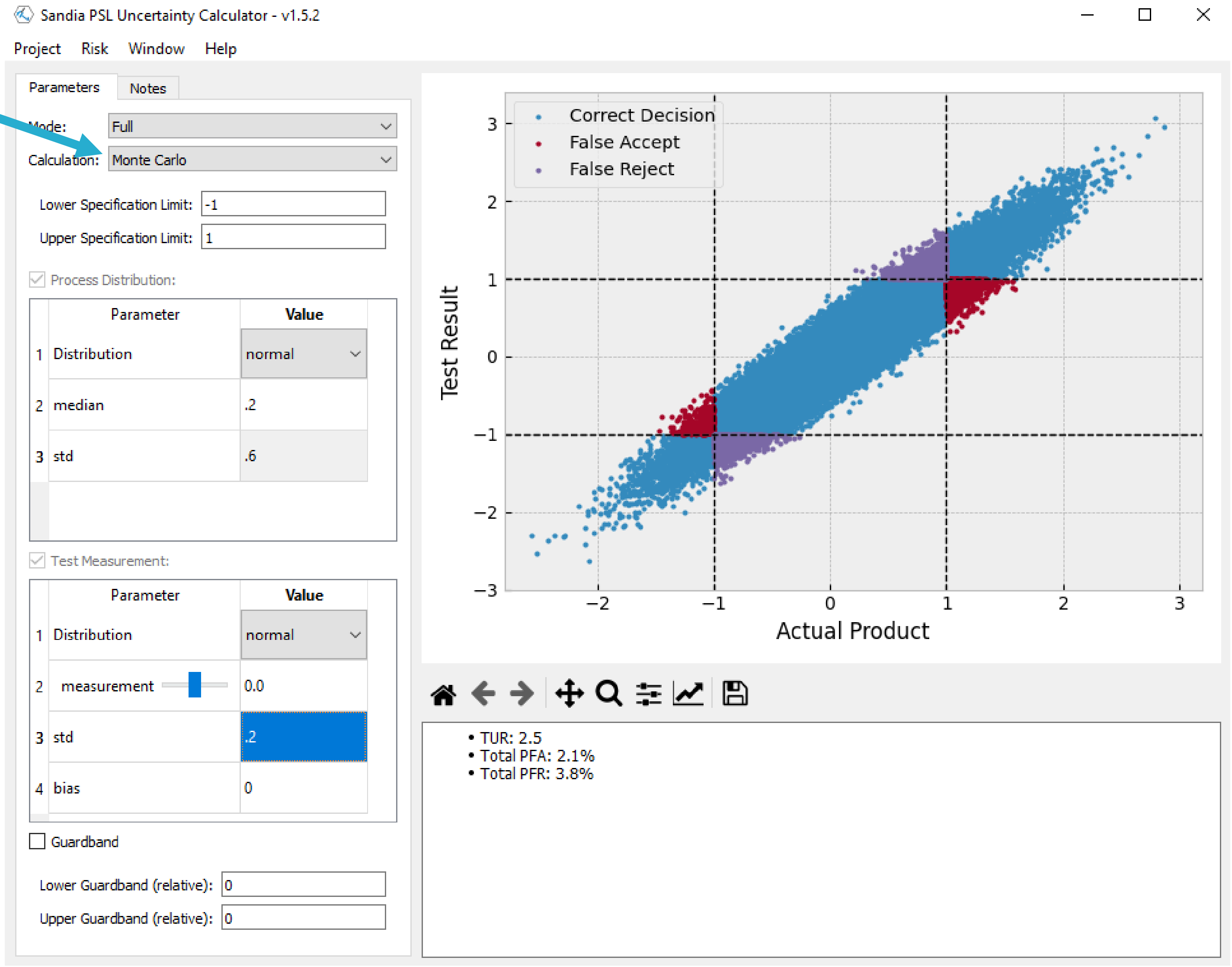
Full Mode

Enter exact process and test distribution values to calculate PFA/PFR

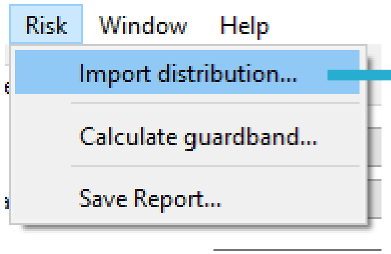


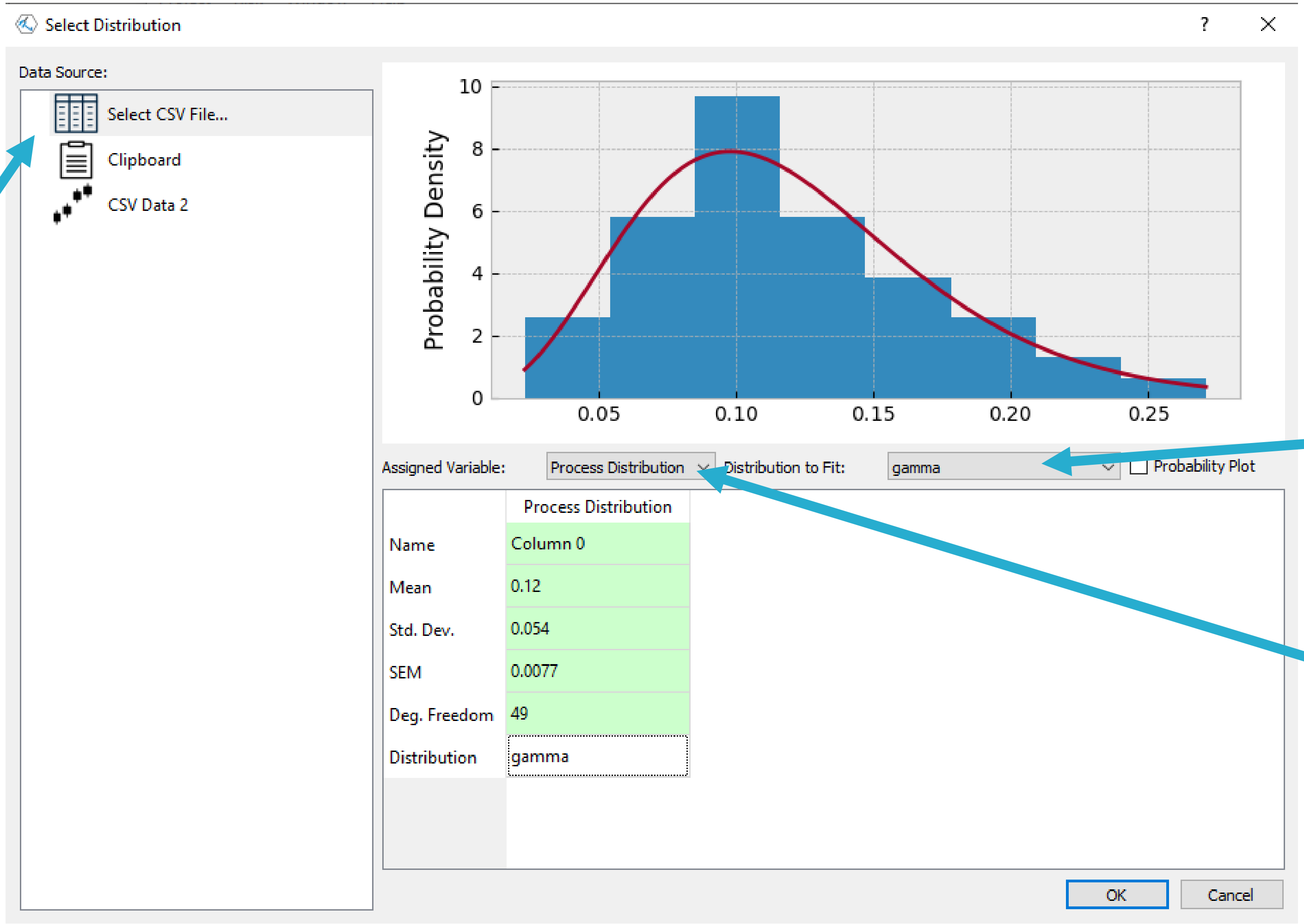
Monte Carlo

Visualize the probability and PFA/PFR integrals by changing the calculation type to Monte Carlo.



Fitting distributions to sampled data



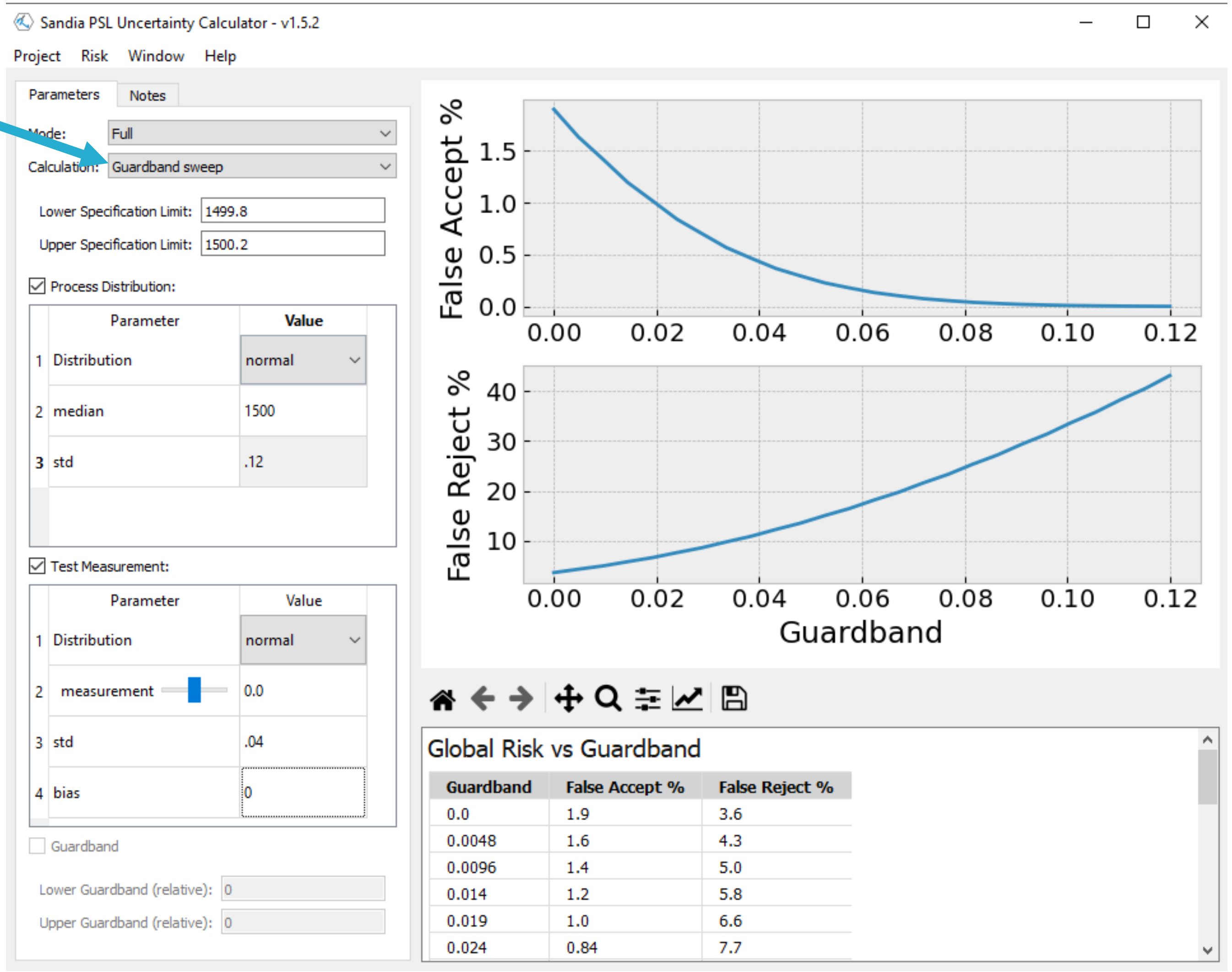


Select source of data here (CSV file or paste from clipboard/Excel)

Select distribution to fit

Specify which distribution in risk calculation (process or measurement)

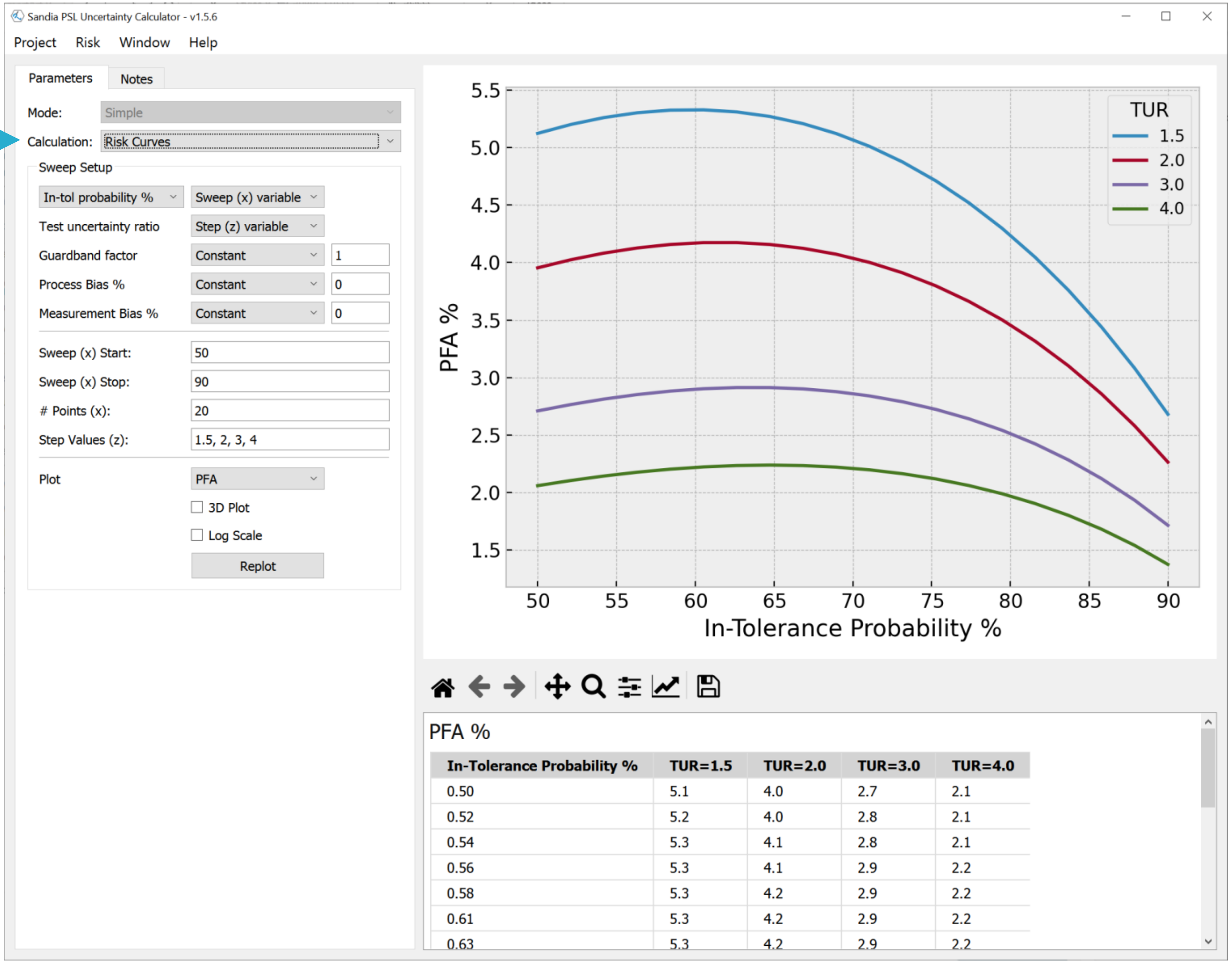
Sweeping guardband to find optimal value



Sweep can aid in finding optimal tradeoff between PFA and PFR

Risk Curves Mode

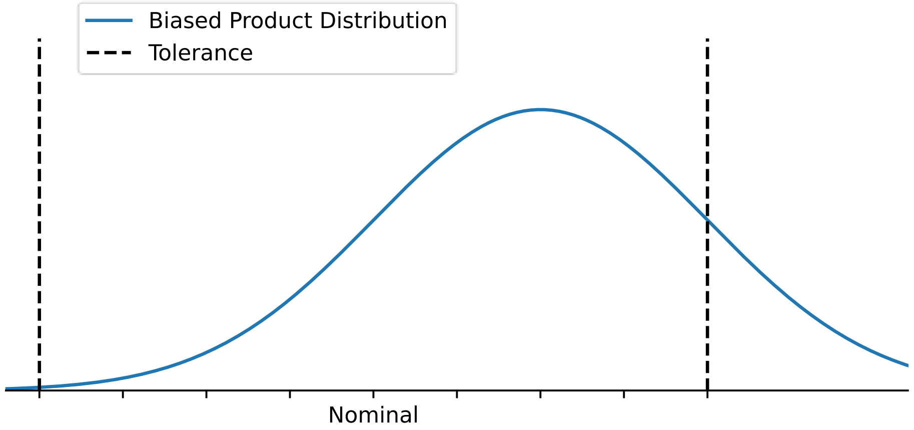
Highly configurable plotting of relationships between ITP, TUR, GBF, and Bias.

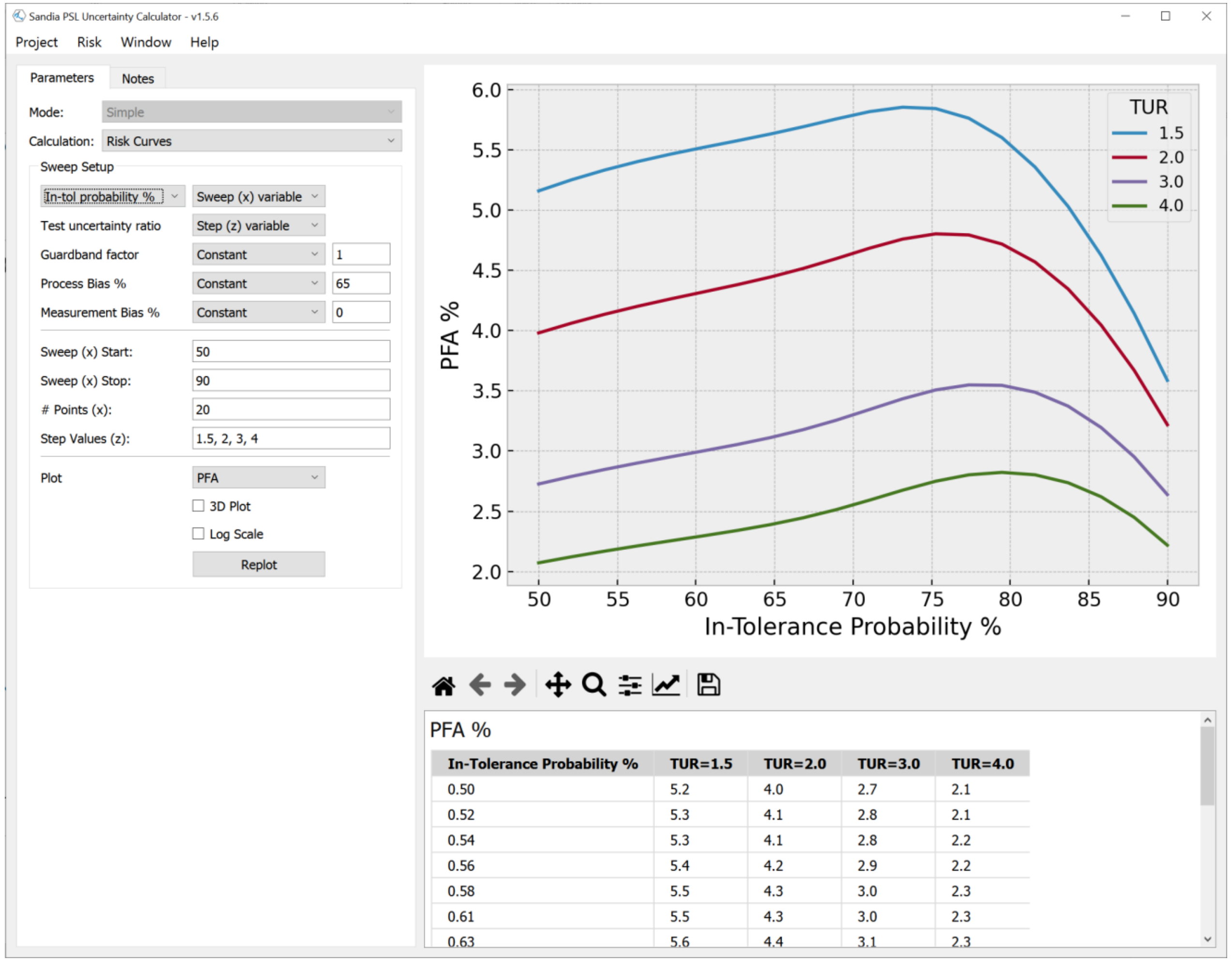


Risk with Biased Distributions

Use Suncal's risk curves tool to plot PFA vs ITP **with a bias**

- itp: 50% to 95%
- TUR: 1.5, 2, 3, and 4
- Bias: constant 65%





Suncal – Web (beta version)

https://sandialabs.github.io/suncal/suncal/index.html

- Same backend code with web-based interface
- No download/install necessary
- Calculations run locally in your browser (use Chrome or Edge)

Suncal Web – Risk Basic

Integral

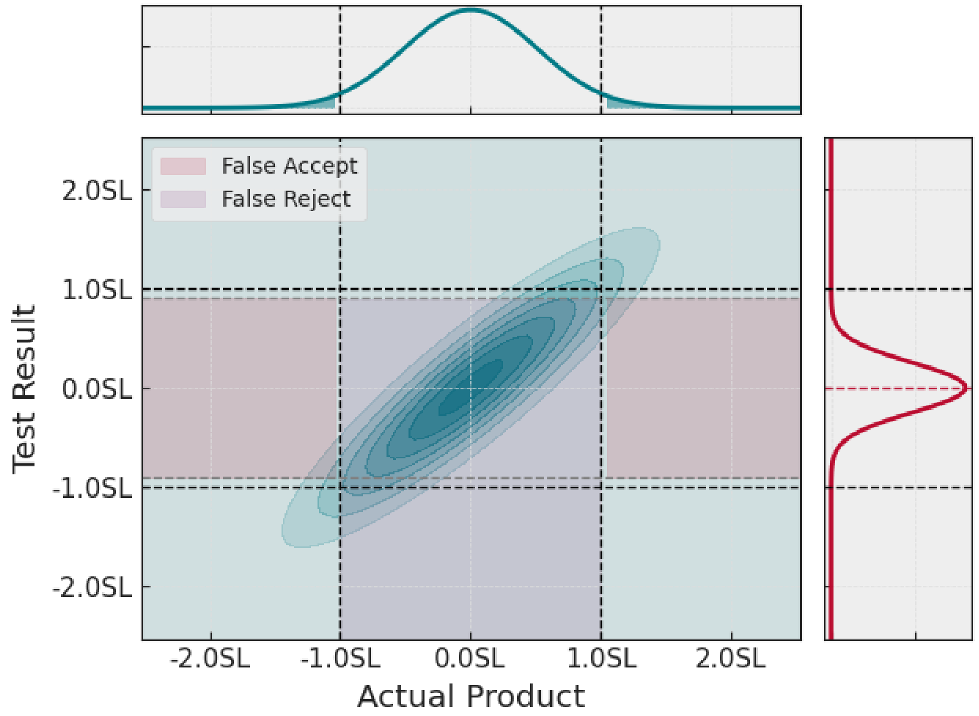
Simple

Total Uncertainty Ratio: 2.0

In-tolerance Probability: 95.0

Guardband Factor: 0.9

Test Measurement:



Process Risk	Specific Measurement Risk	Global Risk
Process Risk: 5.0%	TUR: 2.0	Total PFA: 0.82%
Upper limit risk: 2.5%	Measured value: 0.0	Total PFR: 7.1%
Lower limit risk: 2.5%	Result: ACCEPT	-
Process capability index (Cpk): 0.65	Specific FA Risk: 0.0063%	-

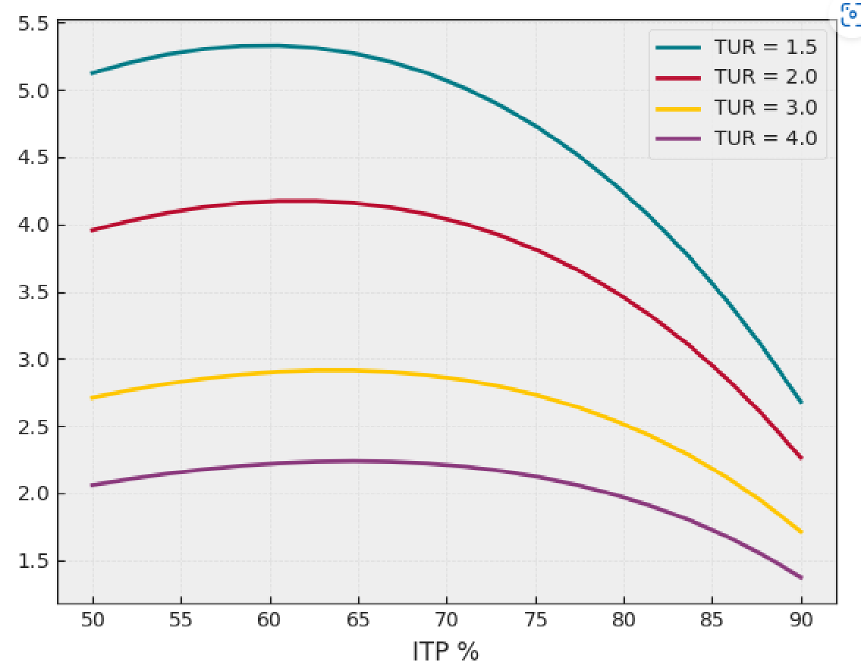
Suncal Web – Risk Curves

Risk Curves

In-tol probability %	Sweep (x) variable	90
Test Uncertainty Ratio	Step (z) variable	4
Guardband Factor	Constant	1
Process Bias %	Constant	0
Measurement Bias %	Constant	0

Sweep (x) Start:	50
Sweep (x) Stop:	90
# Points (x):	20
Step Values (z):	1.5, 2, 3, 4

Plot	PFA
	☐ 3DPlot
	☐ Log Scale
	Plot



ITP %	TUR = 1.5	TUR = 2.0	TUR = 3.0	TUR = 4.0
0.5	5.1	4.0	2.7	2.1
0.52	5.2	4.0	2.8	2.1
0.54	5.3	4.1	2.8	2.1