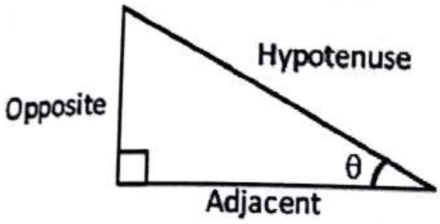
18 TRIGONOMETRY

DEFINITION OF THE TRIG FUNCTIONS

Right Triangle Definition

For this definition we assume that

$0 < \theta < \frac{\pi}{2}$ or $0° < \theta < 90°$



$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$ $\operatorname{cosec}\theta = \frac{\text{hypotenuse}}{\text{opposite}}$

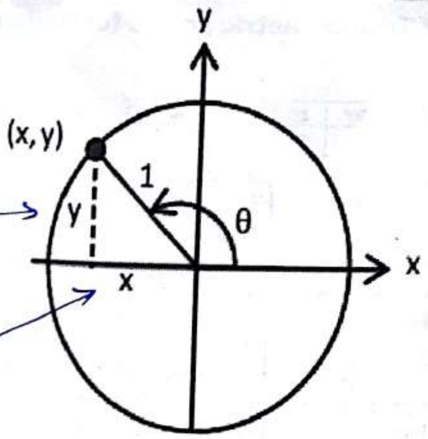
$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$ $\sec\theta = \frac{\text{hypotenuse}}{\text{adjacent}}$

$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$ $\cot\theta = \frac{\text{adjacent}}{\text{opposite}}$

Unit Circle Definition

For this definition θ is any angle

here, 'y' has a +ve value



here 'x' has a negative value

$\sin\theta = \frac{y}{1} = y$ $\operatorname{cosec}\theta = \frac{1}{y}$

$\cos\theta = \frac{x}{1} = x$ $\sec\theta = \frac{1}{x}$

$\tan\theta = \frac{y}{x}$ $\cot\theta = \frac{x}{y}$

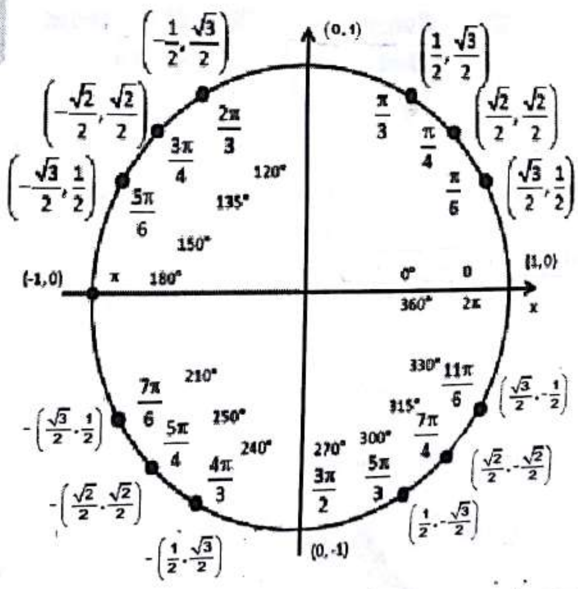
Trigonometry Relations

1. $\sin\theta = \frac{1}{\operatorname{cosec}\theta}$
2. $\cos\theta = \frac{1}{\sec\theta}$
3. $\tan\theta = \frac{1}{\cot\theta}$
4. $\tan\theta = \frac{\sin\theta}{\cos\theta}$
5. $\cot\theta = \frac{\cos\theta}{\sin\theta}$
6. $\sin^2\theta + \cos^2\theta = 1$
7. $\sec^2\theta - \tan^2\theta = 1$
8. $\sec\theta - \tan\theta = \frac{1}{\sec\theta + \tan\theta}$
9. $\operatorname{cosec}^2\theta - \cot^2\theta = 1$

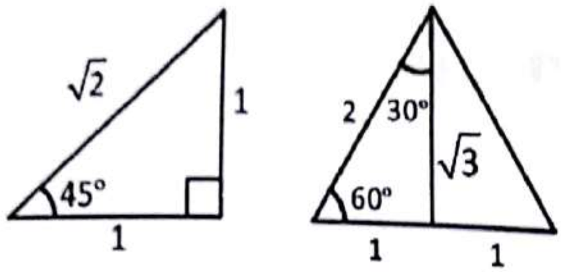
$\Rightarrow \operatorname{cosec}\theta - \cot\theta = \frac{1}{\operatorname{cosec}\theta + \cot\theta}$

Trigonometric identities

Unit Circle



For any ordered pair on the unit circle (x, y) : $\cos\theta = x$ and $\sin\theta = y$



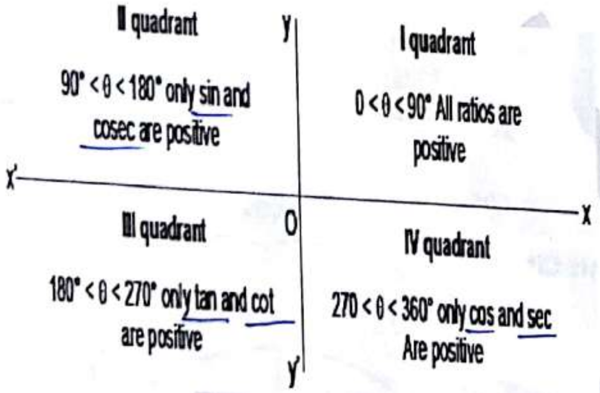
$\cos 45° = \frac{1}{\sqrt{2}}$ $\quad \cos 60 = \frac{1}{2}$ $\quad \cos 30 = \frac{\sqrt{3}}{2}$

$\sin 45° = \frac{1}{\sqrt{2}}$ $\quad \sin 60° = \frac{\sqrt{3}}{2}$ $\quad \sin 30° = \frac{1}{2}$

$\tan 45° = 1$ $\quad \tan 60° = \sqrt{3}$ $\quad \tan 30° = \frac{1}{\sqrt{3}}$

Details about Quadrats :

1. In I quadrant, all the trigonometric ratios are positve.
2. In II quadrant sin and cosec are positive, other trigonometric ratios are negative.
3. In III quadrant tan and cot are positive, other trigonometric ratios are negative.
4. In IV quadrant cos and sec are positve, other tirgonometric ratios are negative.



Allied Angles :

$90° \pm \theta$, $180° \pm \theta$, $270° \pm \theta$, $360° \pm \theta$ are called "allied angles".

Cofunction Formulae

$$\sin\left(\frac{\pi}{2} - \theta\right) = \cos\theta$$

$$\cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta$$

$$\tan\left(\frac{\pi}{2} - \theta\right) = \cot\theta$$

$$\text{cosec}\left(\frac{\pi}{2} - \theta\right) = \sec\theta$$

$$\sec\left(\frac{\pi}{2} - \theta\right) = \text{cosec}\,\theta$$

$$\cot\left(\frac{\pi}{2} - \theta\right) = \tan\theta$$

Formulas for Negatives

$\sin(-\theta) = -\sin\theta$

$\cos(-\theta) = \cos\theta$

$\tan(-\theta) = -\tan\theta$

$\text{cosec}(-\theta) = -\text{cosec}\,\theta$

$\sec(-\theta) = \sec\theta$

$\cot(-\theta) = -\cot\theta$

$\pi/2$ Phase Shift

$$\sin\left(\theta + \frac{\pi}{2}\right) = \cos\theta$$

$$\cos\left(\theta + \frac{\pi}{2}\right) = -\sin\theta$$

$$\tan\left(\theta + \frac{\pi}{2}\right) = -\cot\theta$$

$$\text{cosec}\left(\theta + \frac{\pi}{2}\right) = \sec\theta$$

$$\sec\left(\theta + \frac{\pi}{2}\right) = -\text{cosec}\,\theta$$

$$\cot\left(\theta + \frac{\pi}{2}\right) = -\tan\theta$$

π Phase Shift

$\sin(\theta + \pi) = -\sin\theta$

$\cos(\theta + \pi) = -\cos\theta$

$\tan(\theta + \pi) = \tan\theta$

$\text{cosec}(\theta + \pi) = -\text{cosec}\,\theta$

$\sec(\theta + \pi) = -\sec\theta$

$\cot(\theta + \pi) = \cot\theta$

Values of Trigonometric Ratios for Certain Angles

	0°	30°	45°	60°	90°	180°	270°
sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	0	−1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	−1	0
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	not defined	0	not defined
cot	not defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0	not defined	0
sec	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	not defined	−1	not defined
cosec	not defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1	not defined	−1

Compound Angles

Formulae :

1st set

1. $\sin(A + B) = \sin A \cos B + \cos A \sin B$
2. $\sin(A - B) = \sin A \cos B - \cos A \sin B$
3. $\sin(A + B)\sin(A - B) = \sin^2 A - \sin^2 B = \cos^2 B - \cos^2 A$

2nd set

4. $\cos(A + B) = \cos A \cos B - \sin A \sin B$
5. $\cos(A - B) = \cos A \cos B + \sin A \sin B$
6. $\cos(A + B)\cos(A - B) = \cos^2 A - \sin^2 B = \cos^2 B - \sin^2 A$

3rd set
7. $\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$

8. $\tan(A-B) = \dfrac{\tan A - \tan B}{1 + \tan A \tan B}$

9. $\tan(A+B)\tan(A-B) = \dfrac{\tan^2 A - \tan^2 B}{1 - \tan^2 A \tan^2 B}$

4th set
10. $\cot(A+B) = \dfrac{\cot A \cot B - 1}{\cot B + \cot A}$

11. $\cot(A-B) = \dfrac{\cot A \cot B + 1}{\cot B - \cot A}$

12. $\cot(A+B)\cot(A-B) = \dfrac{\cot^2 A \cot^2 B - 1}{\cot^2 B - \cot^2 A}$

5th set
13. $\tan\left(\dfrac{\pi}{4}+\theta\right) = \dfrac{1+\tan\theta}{1-\tan\theta} = \dfrac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta}$

14. $\tan\left(\dfrac{\pi}{4}-\theta\right) = \dfrac{1-\tan\theta}{1+\tan\theta} = \dfrac{\cos\theta-\sin\theta}{\cos\theta+\sin\theta}$

6th set
15. $\sin 15^\circ = \dfrac{\sqrt{3}-1}{2\sqrt{2}} = \cos 75^\circ$

16. $\cos 15^\circ = \dfrac{\sqrt{3}+1}{2\sqrt{2}} = \sin 75^\circ$

17. $\tan 15^\circ = 2 - \sqrt{3} = \cot 75^\circ$

18. $\cot 15^\circ = 2 + \sqrt{3} = \tan 75^\circ$

Multiple Angles

1st set → 19. $\sin 2A = 2\sin A\cos A = \dfrac{2\tan A}{1+\tan^2 A}$

2nd set → 21. $\cos 2A = \cos^2 A - \sin^2 A = 1 - 2\sin^2 A$

$= 2\cos^2 A - 1 = \dfrac{1-\tan^2 A}{1+\tan^2 A}$

3rd set → 22. $\tan 2A = \dfrac{2\tan A}{1-\tan^2 A}$

4th set → 23. $\cot 2A = \dfrac{\cot^2 A - 1}{2\cot A}$

2nd → 24. $1 + \cos 2A = 2\cos^2 A$

7th set
25. $\sin 3A = 3\sin A - 4\sin^3 A$

26. $\cos 3A = 4\cos^3 A - 3\cos A$

27. $\tan 3A = \dfrac{3\tan A - \tan^3 A}{1 - 3\tan^2 A}$

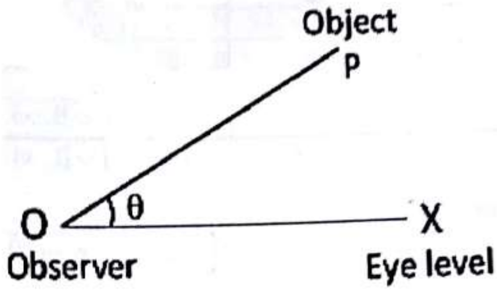
28. $\cot 3A = \dfrac{3\cot A - \cot^3 A}{1 - 3\cot^2 A}$

8th set

	18°	36°
sin	$\dfrac{\sqrt{5}-1}{4}$	$\dfrac{\sqrt{10-2\sqrt{5}}}{4}$
cos	$\dfrac{\sqrt{10+2\sqrt{5}}}{4}$	$\dfrac{\sqrt{5}+1}{4}$

HEIGHTS AND DISTANCES

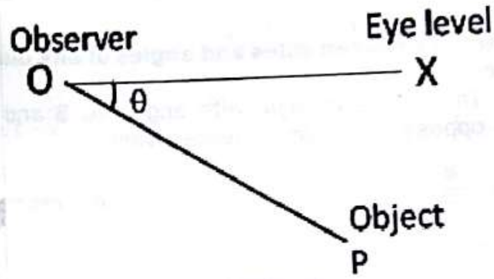
Angle of Elevation



Let 'O' be the observation Point and OX eye level.

If object 'P' is above eye level then the angle made by OP with OX is called angle of elevation

Angle of Depression



If object is below eye level then the angle made by OP with OX is called angle of depression.

Product to Sum

$\cos\alpha\cos\beta = \dfrac{\cos(\alpha-\beta)+\cos(\alpha+\beta)}{2}$

$\sin\alpha\sin\beta = \dfrac{\cos(\alpha-\beta)-\cos(\alpha+\beta)}{2}$

$\sin\alpha\cos\beta = \dfrac{\sin(\alpha+\beta)+\sin(\alpha-\beta)}{2}$

$\cos\alpha\sin\beta = \dfrac{\sin(\alpha+\beta)-\sin(\alpha-\beta)}{2}$

part of 1st & 2nd set

Sum to Product

$\sin\alpha + \sin\beta = 2\sin\left(\dfrac{\alpha+\beta}{2}\right)\cos\left(\dfrac{\alpha-\beta}{2}\right)$

$\sin\alpha - \sin\beta = 2\cos\left(\dfrac{\alpha+\beta}{2}\right)\sin\left(\dfrac{\alpha-\beta}{2}\right)$

$\cos\alpha + \cos\beta = 2\cos\left(\dfrac{\alpha+\beta}{2}\right)\cos\left(\dfrac{\alpha-\beta}{2}\right)$

$\cos\alpha - \cos\beta = (-2)\sin\left(\dfrac{\alpha+\beta}{2}\right)\sin\left(\dfrac{\alpha-\beta}{2}\right)$

obtained from the formulae of sin 2A & cos 2A

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Ranges of Trigonometric Ratios

S.No.	Function	Range
1.	sinx	$[-1, 1]$
2.	cosx	$[-1, 1]$
3.	tanx	R
4.	cotx	R
5.	secx	$(-\infty, -1] \cup [1, \infty)$
6.	cosec x	$(-\infty, -1] \cup [1, \infty)$

Range

1. The maximum value of $\boxed{a \cos\theta + b \sin\theta + c}$ is $\boxed{c + \sqrt{a^2 + b^2}}$
2. The minimum value of $\boxed{a \cos\theta + b \sin\theta + c}$ is $\boxed{c - \sqrt{a^2 + b^2}}$
3. If a and b are positive real numbers such that a > b, then the minimum value of $\boxed{a \sec\theta - b \tan\theta}$ is $\boxed{\sqrt{a^2 - b^2}}$

Relations between sides and angles of any plane triangle

In a plane triangle with angles A, B and C and sides opposite a, b and c respectively,

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = \text{ diameter of circumscribed circle}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$a = b \cos C + c \cos B$$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\tan\frac{A-B}{2} = \frac{a-b}{a+b}\cot\frac{C}{2}$$

$$\text{area} = \frac{1}{2}ab\sin C = \frac{1}{2}bc\sin A$$

$$= \frac{1}{2}ca\sin B = \sqrt{s(s-a)(s-b)(s-c)}$$

where

$$s = \frac{1}{2}(a+b+c)$$

TYPE – I

1. In circular measure, the value of the angle 11° 15' is

A) $\frac{\pi^c}{16}$ B) $\frac{\pi^c}{8}$

C) $\frac{\pi^c}{4}$ D) $\frac{\pi^c}{12}$

2. In a triangle ABC, $\angle ABC = 75°$ and $\angle ACB = \frac{\pi^c}{4}$. The circular measure of $\angle BAC$ is

A) $\frac{5\pi}{12}$ radian B) $\frac{\pi}{3}$ radian

C) $\frac{\pi}{6}$ radian D) $\frac{\pi}{2}$ radian

3. The circular measure of an angle of an isosceles triangle is $\frac{5\pi}{9}$. Circular measure of one of the other angles must be

A) $\frac{5\pi}{18}$ B) $\frac{5\pi}{9}$

C) $\frac{2\pi}{9}$ D) $\frac{4\pi}{9}$

4. The degree measure of 1 radian (taking $\pi = \frac{22}{7}$) is

A) 57°61' 22" (approx.)

B) 57° 16' 22" (approx.)

C) 57°22' 16" (approx.)

D) 57°32' 16" (approx.)

5. $\left(\frac{3\pi}{5}\right)$ radians is equal to

A) 100° B) 120°

C) 108° D) 180°

6. If the sum of two angles is 135° and their difference is $\frac{\pi}{12}$, then the circular measure of the greater angle is

A) $\frac{2\pi}{3}$ B) $\frac{3\pi}{5}$

C) $\frac{5\pi}{12}$ D) $\frac{\pi}{3}$

7. If $0 \le \theta \le \frac{\pi}{2}$ and $\sec^2\theta + \tan^2\theta = 7$, then θ is

A) $\frac{5\pi}{12}$ radian B) $\frac{\pi}{3}$ radian

C) $\frac{\pi}{5}$ radian D) $\frac{\pi}{6}$ radian

8. If the sum and difference of two angles are $\frac{22}{9}$ radian and 36° respectively, then the value of smaller angle in degree taking the value of π as $\frac{22}{7}$ is :

A) 52° B) 60°

C) 56° D) 48°

TYPE – II

1. The minimum value of $2\sin^2\theta + 3\cos^2\theta$ is

A) 0 B) 3

C) 2 D) 1

2. If cosec 39° = x, the value of $\frac{1}{\operatorname{cosec}^2 51°} + \sin^2 39° + \tan^2 51° - \frac{1}{\sin^2 51° \sec^2 39°}$ is

A) $\sqrt{x^2-1}$ B) $\sqrt{1-x^2}$

C) x^2-1 D) $1-x^2$

3. The value of tan 4°.tan 43°.tan 47°.tan 86° is

A) 2 B) 3

C) 1 D) 4

4. If $\frac{\tan\theta + \cot\theta}{\tan\theta - \cot\theta} = 2, (0 \le \theta \le 90°)$, then the value of $\sin\theta$ is

A) $\frac{2}{\sqrt{3}}$ B) $\frac{\sqrt{3}}{2}$

C) $\frac{1}{2}$ D) 1

5. If cos x + cos y = 2, the value of sin x + sin y is

A) 0 B) 1

C) 2 D) –1

6. The value of tan 1° tan2° tan3°....... tan89° is :

A) 1 B) 0

C) $\sqrt{3}$ D) $\frac{1}{\sqrt{3}}$

7. The measure of the angles of a Triangle are in the ratio 2:7 : 11. Measures of angles are

A) 16°, 56°, 88°

B) 18°, 63°, 99°

C) 20°, 70°, 90°

D) 25°, 175°, 105°

8. The angles of a triangle are (x + 5)°, (2x - 3)° and (3x + 4)°. The value of x is

A) 30 B) 31

C) 29 D) 28

9. The value of cot 10° . cot 20° cot 60° . cot 70° . cot 80° is

A) 1 B) –1

C) $\sqrt{3}$ D) $\frac{1}{\sqrt{3}}$

10. If θ be an acute angle and $7\sin^2\theta + 3\cos^2\theta = 4$, then the value of tan θ is

A) $\sqrt{3}$ B) $\frac{1}{\sqrt{3}}$

C) 1 D) 0

11. The value of $\sin^2 1° + \sin^2 5° + \sin^2 9° + ... + \sin^2 89°$ is

A) $11\frac{1}{2}$ B) $11\sqrt{2}$

C) 11 D) $\frac{11}{\sqrt{2}}$

12. The numerical value of cot 18° $\left(\cot 72° \cos^2 22° + \frac{1}{\tan 72° \sec^2 68°}\right)$ is

A) 1 B) $\sqrt{2}$

C) 3 D) $\frac{1}{\sqrt{3}}$

13. If $\tan 15° = 2-\sqrt{3}$, the value of tan 15° cot 75° + tan 75° cot 15° is

A) 14 B) 12

C) 10 D) 8

14. If x, y are acute angles, 0 < x + y < 90° and sin (2x - 20°) = cos (2y + 20°), then the value of tan (x + y) is :

A) $\frac{1}{\sqrt{3}}$ B) $\frac{\sqrt{3}}{2}$

C) $\sqrt{3}$ D) 1

15. If $\angle A$ and $\angle B$ are complementary to each other, then the value of $\sec^2 A + \sec^2 B - \sec^2 A . \sec^2 B$ is

A) 1 B) –1

C) 2 D) 0

16. $\sin^2 5° + \sin^2 6° + ... + \sin^2 84° + \sin^2 85° = ?$

A) $39\frac{1}{2}$ B) $40\frac{1}{2}$

C) 40 D) $39\frac{1}{\sqrt{2}}$

17. $\sin^2 5° + \sin^2 10° + \sin^2 15° + ... + \sin^2 85° + \sin^2 90°$

A) $7\frac{1}{2}$ B) $8\frac{1}{2}$

C) 9 D) $9\frac{1}{2}$

18. The value of

$\frac{\sin 39^\circ}{\cos 51^\circ} + 2 \tan 11^\circ \tan 31^\circ$

$\tan 45^\circ \tan 59^\circ$ tan. $79^\circ - 3 (\sin^2 21^\circ + \sin^2 69^\circ)$ is :

A) 2 B) −1
C) 1 D) 0

19. If $\frac{\cos^2\theta}{\cot^2\theta - \cos^2\theta} = 3$ and $0^\circ < \theta < 90^\circ$, then the value of θ is :

A) 30° B) 45°
C) 60° D) None of these

20. IfA = tan 11° tan 29°, B = 2 cot 61° cot 79°, then :

A) A = 2B B) A = −2B
C) 2A = B D) 2A = − B

21. If $\sin 17^\circ = \frac{x}{y}$, then the value of (sec 17°− sin 73°) is

A) $\frac{y^2}{x\sqrt{y^2-x^2}}$

B) $\frac{x^2}{y\sqrt{y^2-x^2}}$

C) $\frac{x^2}{y\sqrt{x^2-y^2}}$

D) $\frac{y^2}{x\sqrt{x^2-y^2}}$

22. If $0 < \theta < 90$, the value of $\sin\theta + \cos\theta$ is

A) equal to 1
B) greater than 1
C) less than 1
D) equal to 2

23. The expression $\frac{\tan 57^\circ + \cot 37^\circ}{\tan 33^\circ + \cot 53^\circ}$ is equal to

A) tan 33° cot 57°
B) tan 57° cot 37°
C) tan 33° cot 53°
D) tan 53° cot 37°

24. The value of $\frac{\cot 30^\circ - \cot 75^\circ}{\tan 15^\circ - \tan 60^\circ}$ is

A) 0 B) 1
C) $\sqrt{3}-1$ D) −1

25. The value of cot θ .tan(90° − θ) − sec(90° − θ) cosecθ + (sin² 25° + sin² 65°) +

$\sqrt{3}$ (tan 5° tan 15°. tan 30°. tan75°. tan 85°) is :

A) 1 B) −1
C) 2 D) 0

26. If sin (3x − 20°) = cos (3y + 20°), then the value of (x + y) is

A) 20° B) 30°
C) 40° D) 45°

27. If cos θ cosec 23° = 1, the value of θ is

A) 23° B) 37°
C) 63° D) 67°

28. If 2 (cos² θ − sin² θ) = 1, θis a positive acute angle, then the value of θ is

A) 60° B) 30°
C) 45° D) $22\frac{1}{2}^\circ$

29. The value of (tan35° tan45° tan55°) is

A) $\frac{1}{2}$ B) 2
C) 0 D) 1

30. If sec (7θ + 28°) = cosec (30° − 3θ) then the value of θ is

A) 8° B) 5°
C) 60° D) 9°

31. If $\tan\left(\frac{\pi}{2} - \frac{\theta}{2}\right) = \sqrt{3}$, the value of $\cos\theta$ is

A) 0 B) $\frac{1}{\sqrt{2}}$
C) $\frac{1}{2}$ D) 1

32. If 7 sin² θ + 3 cos²θ = 4 (0° ≤ θ ≤ 90°), then value of θ is

A) $\frac{\pi}{2}$ B) $\frac{\pi}{3}$
C) $\frac{\pi}{6}$ D) $\frac{\pi}{4}$

33. If $\sec\theta = x + \frac{1}{4x} (0^\circ < \theta < 90^\circ)$, then $\sec\theta + \tan\theta$ is equal to

A) $\frac{x}{2}$ B) 2x
C) x D) $\frac{1}{2x}$

34. The value of cos 1° cos 2° cos 3°....... cos 177° cos 178° cos 179° is :

A) 0 B) $\frac{1}{2}$
C) 1 D) $\frac{1}{\sqrt{2}}$

35. The value of (sin² 25° + sin² 65°) is :

A) $\frac{\sqrt{3}}{2}$ B) 1
C) 0 D) $\frac{2}{\sqrt{3}}$

36. If $\sec\theta + \tan\theta = \sqrt{3}$ $(0^\circ \le \theta \le 90^\circ)$, then tan3θ is

A) undefined B) $\frac{1}{\sqrt{3}}$
C) $\frac{1}{\sqrt{2}}$ D) $\sqrt{3}$

37. If sin (60° − θ) = cos (ψ − 30°), then the value of tan (ψ − θ) is (assume that θ and ψ are both positive acute angles with θ < 60° and ψ > 30°).

A) $\frac{1}{\sqrt{3}}$ B) 0
C) $\sqrt{3}$ D) 1

38. If a sin θ + b cos θ = c then the value of a cos θ − b sin θis :

A) $\pm\sqrt{-a^2+b^2+c^2}$
B) $\pm\sqrt{a^2+b^2-c^2}$
C) $\pm\sqrt{a^2-b^2-c^2}$
D) $\pm\sqrt{a^2-b^2+c^2}$

39. If $\sin(A-B) = \frac{1}{2}$ and $\cos(A+B) = \frac{1}{2}$ where A > B > 0 and A + B is an acute angle, then the value B is

A) $\frac{\pi}{6}$ B) $\frac{\pi}{12}$
C) $\frac{\pi}{4}$ D) $\frac{\pi}{2}$

40. Maximum value of (2 sin θ + 3 cos θ) is

A) 2 B) $\sqrt{13}$
D) $\sqrt{15}$ D) 1

41. The value of 152 (sin 30° + 2 cos² 45° + 3 sin 30° + 4 cos² 45° + ... + 17 sin 30°+ 18 cos² 45°) is

A) an integer but not a perfect square
B) a rational number but not an integer
C) a perfect square of an integer
D) irrational

42. Evaluate : 3 cos 80° cosec 10° + 2 cos 59° cosec 31°

A) 1 B) 3
C) 2 D) 5

43. sin² θ − 3 sin θ + 2 = 0 will be true if

A) 0 ≤ θ < 90 B) 0 < θ < 90
C) θ = 0° D) θ = 90°

44. If $\tan\alpha = n\tan\beta$ and $\sin\alpha = m\sin\beta$, then $\cos^2\alpha$ is

A) $\frac{m^2}{n^2+1}$ B) $\frac{m^2}{n^2}$

C) $\frac{m^2-1}{n^2-1}$ D) $\frac{m^2+1}{n^2+1}$

45. If $\tan\theta = \frac{3}{4}$ and θ is acute, then cosec θ

A) $\frac{4}{5}$ B) $\frac{5}{3}$

C) 2 D) $\frac{1}{2}$

46. If $\operatorname{cosec}\theta - \cot\theta = \frac{7}{2}$, the value of cosec θ is :

A) $\frac{47}{28}$ B) $\frac{51}{28}$

C) $\frac{53}{28}$ D) $\frac{49}{28}$

47. If $x\sin 45° = y\operatorname{cosec} 30°$, then $\frac{x^4}{y^4}$ is equal to

A) 4^3 B) 6^3

C) 2^3 D) 8^3

48. If $5\tan\theta = 4$, then $\frac{5\sin\theta - 3\cos\theta}{5\sin\theta + 2\cos\theta}$ is equal to

A) $\frac{2}{3}$ B) $\frac{1}{4}$

C) $\frac{1}{6}$ D) $\frac{1}{3}$

49. $2\operatorname{cosec}^2 23° \cot^2 67° - \sin^2 23° - \sin^2 67° - \cot^2 67°$ is equal to

A) 1 B) $\sec^2 23°$

C) $\tan^2 23°$ D) 0

50. The equation $\cos^2\theta = \frac{(x+y)^2}{4xy}$ is only possible when

A) $x = -y$ B) $x > y$

C) $x = y$ D) $x < y$

51. The value of $\operatorname{cosec}^2 18° - \frac{1}{\cot^2 72°}$ is

A) $\frac{1}{\sqrt{3}}$ B) $\frac{\sqrt{2}}{3}$

C) $\frac{1}{2}$ D) 1

52. If $\alpha + \beta = 90°$, then the value of $(1 - \sin^2\alpha)\,(1-\cos^2\alpha)\times(1+\cot^2\beta)\,(1+\tan^2\beta)$ is

A) 1 B) −1

C) 0 D) 2

53. $\frac{2\sin 68°}{\cos 22°} - \frac{2\cot 15°}{5\tan 75°} - \frac{3\tan 45°.\tan 20°.\tan 40°.\tan 50°.\tan 70°}{5}$ is equal to

A) −1 B) 0

C) 1 D) 2

54. The value of tan 10° tan 15° tan 75° tan 80° is

A) 0 B) 1

C) −1 D) 2

55. The minimum value of $4\tan^2\theta + 9\cot^2\theta$ is equal to

A) 0 B) 5

C) 12 D) 13

56. If sin 7x = cos 11x , then the value of tan 9x + cot 9x is

A) 1 B) 2

C) 3 D) 4

57. If $\tan^2\alpha = 1 + 2\tan^2\beta$ (α, β are positive acute angles), then $\sqrt{2}\cos\alpha - \cos\beta$ is equal to

A) 0 B) $\sqrt{2}$

C) 1 D) −1

58. The product cos1° cos2° cos3° cos4° .... cos 100° is equal to

A) −1 B) $\frac{1}{4}$

C) 1 D) 0

59. If $2(\cos^2\theta - \sin^2\theta) = 1$ (θ is a positive acute angle), then cot θ is equal to

A) $-\sqrt{3}$ B) $\frac{1}{\sqrt{3}}$

C) 1 D) $\sqrt{3}$

60. If tan (2θ + 45°) = cot 3θ where (2θ + 45°) and 3θ are acute angles, then the value of θ is

A) 5° B) 9°

C) 12° D) 15°

61. If θ be acute angle and $\cos\theta = \frac{15}{17}$, then the value of $\cot(90° - \theta)$ is

A) $\frac{2\sqrt{8}}{15}$ B) $\frac{8}{15}$

C) $\frac{\sqrt{2}}{17}$ D) $\frac{8\sqrt{2}}{17}$

62. If $\sec^2\theta + \tan^2\theta = \frac{7}{12}$, then $\sec^4\theta - \tan^4\theta =$

A) $\frac{7}{12}$ B) $\frac{1}{2}$

C) $\frac{5}{12}$ D) 1

63. If $0 < x < \frac{\pi}{2}$ and secx = cosecy, then the value of sin (x + y) is:

A) 0 B) 1

C) $\frac{1}{2}$ D) $\frac{1}{\sqrt{3}}$

64. If A, B and C be the angles of a triangle, then put of the following, the incorrect relation is:

A) $\sin\frac{A+B}{2} = \cos\frac{C}{2}$

B) $\cos\left(\frac{A+B}{2}\right) = \sin\frac{C}{2}$

C) $\tan\left(\frac{A+B}{2}\right) = \sec\frac{C}{2}$

D) $\cot\left(\frac{A+B}{2}\right) = \tan\frac{C}{2}$

65. If $\sin\alpha + \cos\beta = 2$ $(0° \le \beta < \alpha \le 90°)$, then $\sin\left(\frac{2\alpha+\beta}{3}\right) =$

A) $\sin\frac{\alpha}{2}$ B) $\cos\frac{\alpha}{3}$

C) $\sin\frac{\alpha}{3}$ D) $\cos\frac{2\alpha}{3}$

66. If $\cos^4\theta - \sin^4\theta = \frac{2}{3}$, then the value of $2\cos^2\theta - 1$ is

A) 0 B) 1

C) $\frac{2}{3}$ D) $\frac{3}{2}$

67. If $\sin\alpha\sec(30° + \alpha) = 1$ $(0 < \alpha < 60°)$, then the value of $\sin\alpha + \cos 2\alpha$ is

A) 1 B) $\frac{2+\sqrt{3}}{2\sqrt{3}}$

C) 0 D) $\sqrt{2}$

68. If tan θ = 1, then the value of $\frac{8\sin\theta + 5\cos\theta}{\sin^3\theta - 2\cos^3\theta + 7\cos\theta}$ is

A) 2 B) $2\frac{1}{2}$

C) 3 D) $\frac{4}{5}$

69. If θ be a positive acute angle satisfying $\cos^2\theta + \cos^4\theta = 1$, then the value of $\tan^2\theta + \tan^4\theta$ is

A) $\frac{3}{2}$ B) 1

C) $\frac{1}{2}$ D)0

70. If $\tan\theta = \frac{4}{3}$, then the value of $\frac{3\sin\theta + 2\cos\theta}{3\sin\theta - 2\cos\theta}$ is
A) 0.5 B) -0.5
C) 3.0 D) -3.0

71. The simplified value of $(\sec A - \cos A)^2 + (\mathrm{cosec}\, A - \sin A)^2 - (\cot A - \tan A)^2$ is
A) 0 B) $\frac{1}{2}$
C) 1 D) 2

72. If θ be acute and $\tan\theta + \cot\theta = 2$, then the value of $\tan^5\theta + \cot^{10}\theta$ is
A) 1 B) 2
C) 3 D) 4

73. If $\sin\theta - \cos\theta = \frac{7}{13}$ and $0 < \theta < 90°$, then the value of $\sin\theta + \cos\theta$ is
A) $\frac{17}{13}$ B) $\frac{13}{17}$
C) $\frac{1}{13}$ D) $\frac{1}{17}$

74. If $2\cos\theta - \sin\theta = \frac{1}{\sqrt{2}}$, $(0° < \theta < 90°)$ the value of $2\sin\theta + \cos\theta$ is
A) $\frac{1}{\sqrt{2}}$ B) $\sqrt{2}$
C) $\frac{3}{\sqrt{2}}$ D) $\frac{\sqrt{2}}{3}$

75. If $\frac{\sin\theta + \cos\theta}{\sin\theta - \cos\theta} = 3$, then the value of $\sin^4\theta - \cos^4\theta$ is
A) $\frac{1}{5}$ B) $\frac{2}{5}$
C) $\frac{3}{5}$ D) $\frac{4}{5}$

76. If $\sec^2\theta + \tan^2\theta = 7$, then the value of θ when $0° \le \theta \le 90°$, is
A) 60° B) 30°
C) 0° D) 90°

77. The simplified value of $(\sec x \sec y + \tan x \tan y)^2 - (\sec x \tan y + \tan x \sec y)^2$ is :
A) -1 B) 0
C) $\sec^2 x$ D) 1

78. If $\sin\theta + \mathrm{cosec}\theta = 2$, then value of $\sin^{100}\theta + \mathrm{cosec}^{100}\theta$ is equal to:
A) 1 B) 2
C) 3 D) 100

79. If $A = \sin^2\theta + \cos^4\theta$, for any value of θ, then the value of A is
A) $1 \le A \le 2$
B) $\frac{3}{4} \le A \le 1$
C) $\frac{13}{16} \le A \le 1$
D) $\frac{3}{4} \le A \le \frac{13}{16}$

80. If $\sin\theta + \mathrm{cosec}\theta = 2$, then the value of $\sin^5\theta + \mathrm{cosec}^5\theta$ when $0° \le \theta\ \theta\ 90°$, is
A) 0 B) 1
C) 10 D) 2

81. If $\tan 2\theta . \tan 4\theta = 1$, then the value of $\tan 3\theta$ is
A) $\sqrt{3}$ B) 0
C) 1 D) $\frac{1}{\sqrt{3}}$

82. If $\cos^2\alpha + \cos^2\beta = 2$, then the value of $\tan^3\alpha + \sin^5\beta$ is :
A) -1 B) 0
C) 1 D) $\frac{1}{\sqrt{3}}$

83. If θ is a positive acute angle and $\tan 2\theta \tan 3\theta = 1$, then the value of $\left(2\cos^2\frac{5\theta}{2} - 1\right)$ is
A) $-\frac{1}{2}$ B) 1
C) 0 D) $\frac{1}{2}$

84. In a right-angled triangle XYZ, right-angled at Y, if $XY = 2\sqrt{6}$ and $XZ - YZ = 2$, then $\sec X + \tan X$ is
A) $\frac{1}{\sqrt{6}}$ B) $\sqrt{6}$
C) $2\sqrt{6}$ D) $\frac{\sqrt{6}}{2}$

85. The minimum value of $\sin^2\theta + \cos^2\theta + \sec^2\theta + \mathrm{cosec}^2\theta + \tan^2\theta + \cot^2\theta$ is
A)1 B) 3
C) 5 D) 7

86. If $\cos 20° = m$ and $\cos 70° = n$, then the value of $m^2 + n^2$ is
A) 1 B) $\frac{3}{2}$
C) $\frac{1}{\sqrt{2}}$ D) $\frac{1}{2}$

87. If $\cos\theta + \sec\theta = 2$, the value of $\cos^6\theta + \sec^6\theta$ is
A) 4 B) 8
C) 1 D) 2

88. The numerical value of $\frac{5}{\sec^2\theta} + \frac{2}{1+\cot^2\theta} + 3\sin^2\theta$ is
A) 5 B) 2
C) 3 D) 4

89. The numerical value of $\left(\frac{1}{\cos\theta} + \frac{1}{\cot\theta}\right)\left(\frac{1}{\cos\theta} - \frac{1}{\cot\theta}\right)$ is
A) 0 B) -1
C) +1 D) 2

90. If $\frac{\sin\theta + \cos\theta}{\sin\theta - \cos\theta} = \frac{5}{4}$, the value of $\frac{\tan^2\theta + 1}{\tan^2\theta - 1}$ is
A) $\frac{25}{16}$ B) $\frac{41}{9}$
C) $\frac{41}{40}$ D) $\frac{40}{41}$

91. If $\tan 7\theta \tan 2\theta = 1$, then the value of $\tan 3\theta$ is
A) $\sqrt{3}$ B) $-\frac{1}{\sqrt{3}}$
C) $\frac{1}{\sqrt{3}}$ D) $-\sqrt{3}$

92. The value of $(2\cos^2\theta - 1)\left(\frac{1+\tan\theta}{1-\tan\theta} + \frac{1-\tan\theta}{1+\tan\theta}\right)$ is
A) 4 B) 1
C) 3 D) 2

93. If $\sec\theta + \tan\theta = 2$, then the value of $\sec\theta$ is
A) $\frac{4}{5}$ B) 5
C) $\frac{5}{4}$ D) $\sqrt{2}$

94. If $\mathrm{cosec}\theta - \sin\theta = l$ and $\sec\theta - \cos\theta = m$, then the value of $l^2m^2(l^2 + m^2 + 3)$ is
A) -1 B) 0
C) 1 D) 2

95. If $\frac{2\sin\theta - \cos\theta}{\cos\theta + \sin\theta} = 1$, then value of $\cot\theta$ is
A) $\frac{1}{2}$ B) $\frac{1}{3}$
C) 3 D) 2

96. If $\tan\theta = 2$, then the value of $\frac{8\sin\theta + 5\cos\theta}{\sin^3\theta + 2\cos^3\theta + 3\cos\theta}$ is
A) $\frac{21}{5}$ B) $\frac{8}{5}$
C) $\frac{7}{5}$ D) $\frac{16}{5}$

97. If $\tan\theta + \cot\theta = 2$, then the value of $\tan^{100}\theta + \cot^{100}\theta$ is
A) 2 B) 0
C) 1 D) $\sqrt{3}$

98. $\frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta}$ is equal to
A) $1 - \tan\theta - \cot\theta$
B) $1 + \tan\theta - \cot\theta$
C) $1 - \tan\theta + \cot\theta$
D) $1 + \tan\theta + \cot\theta$

99. If $\sin\theta + \operatorname{cosec}\theta = 2$, then the value of $\sin^9\theta + \operatorname{cosec}^9\theta$ is :
A) 3 B) 2
C) 4 D) 1

100. If $\sec\theta + \tan\theta = 2 + \sqrt{5}$, then the value of $\sin\theta + \cos\theta$ is :
A) $\frac{3}{\sqrt{5}}$ B) $\sqrt{5}$
C) $\frac{7}{\sqrt{5}}$ D) $\frac{1}{\sqrt{5}}$

101. The value of $(1 + \cot\theta - \operatorname{cosec}\theta)(1 + \tan\theta + \sec\theta)$ is equal to
A) 1 B) 2
C) 0 D) −1

102. If $\tan\theta + \cot\theta = 2$, then the value of $\tan^n\theta + \cot^n\theta$ ($0° < \theta < 90°$, n is an integer) is
A) 2 B) 2^n
A) 2n D) 2^{n+1}

103. If $\frac{\sin\theta}{x} = \frac{\cos\theta}{y}$, then $\sin\theta - \cos\theta$ is equal to
A) $x - y$ B) $x + y$
C) $\frac{x-y}{\sqrt{x^2+y^2}}$ D) $\frac{y-x}{\sqrt{x^2+y^2}}$

104. If $x = a\sec\theta\cos\phi$, $y = b\sec\theta\sin\phi$, $z = c\tan\theta$, then the value of $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2}$ is
A) 1 B) 4
C) 9 D) 0

105. If $\frac{\sec\theta + \tan\theta}{\sec\theta - \tan\theta} = \frac{5}{3}$, then $\sin\theta$ is equal to
A) $\frac{1}{4}$ B) $\frac{1}{3}$
C) $\frac{2}{3}$ D) $\frac{3}{4}$

106. If $\cos x + \cos^2 x = 1$, the numerical value of $(\sin^{12}x + 3\sin^{10}x + 3\sin^8 x + \sin^6 x - 1)$ is :
A) −1 B) 2
C) 0 D) 1

107. If $(1 + \sin\alpha)(1 + \sin\beta)(1 + \sin\gamma) = (1 - \sin\alpha)(1 - \sin\beta)(1 - \sin\gamma)$, then each side is equal to
A) $\pm\cos\alpha\cos\beta\cos\gamma$
B) $\pm\sin\alpha\sin\beta\sin\gamma$
C) $\pm\sin\alpha\cos\beta\cos\gamma$
D) $\pm\sin\alpha\sin\beta\cos\gamma$

108. The numerical value of $\frac{1}{1+\cot^2\theta} + \frac{3}{1+\tan^2\theta} + 2\sin^2\theta$ will be
A) 2 B) 5
C) 6 D) 3

109. The value of $\frac{4}{1+\tan^2\alpha} + \frac{1}{1+\cot^2\alpha} + 3\sin^2\alpha$ is
A) 4 B) −1
C) 2 D) 3

110. The value of $3(\sin x - \cos x)^4 + 6(\sin x + \cos x)^2 + 4(\sin^6 x + \cos^6 x)$ is
A) 14 B) 11
C) 12 D) 13

111. The value of $\sec\theta\left(\frac{1+\sin\theta}{\cos\theta} + \frac{\cos\theta}{1+\sin\theta}\right) - 2\tan^2\theta$ is
A) 4 B) 1
C) 2 D) 0

112. If $\tan\theta + \cot\theta = 2$, then the value of $\tan^2\theta + \cot^2\theta$ is
A) 2 B) 1
C) $\sqrt{2}$ D) 0

113. The eliminant of θ from $x\cos\theta - y\sin\theta = 2$ and $x\sin\theta + y\cos\theta = 4$ will give
A) $x^2 + y^2 = 20$
B) $3x^2 + y^2 = 20$
C) $x^2 - y^2 = 20$
D) $3x^2 - y^2 = 10$

114. The value of $\left[\frac{\cos^2 A(\sin A + \cos A)}{\operatorname{cosec}^2 A(\sin A - \cos A)} + \frac{\sin^2 A(\sin A - \cos A)}{\sec^2 A(\sin A + \cos A)}\right]$ $(\sec^2 A - \operatorname{cosec}^2 A)$
A) 1 B) 3
C) 2 D) 4

115. The value of $\frac{1}{\operatorname{cosec}\theta - \cot\theta} - \frac{1}{\sin\theta}$ is
A) 1 B) $\cot\theta$
C) $\operatorname{cosec}\theta$ D) $\tan\theta$

116. If $\cos\theta + \sin\theta = \sqrt{2}\cos\theta$, then $\cos\theta - \sin\theta$ is
A) $\sqrt{2}\tan\theta$
B) $-\sqrt{2}\cos\theta$
C) $-\sqrt{2}\sin\theta$
D) $\sqrt{2}\sin\theta$

117. If $\cos^4\theta - \sin^4\theta = \frac{2}{3}$, then the value of $1 - 2\sin^2\theta$ is
A) $\frac{4}{3}$ B) 0
C) $\frac{2}{3}$ D) $\frac{1}{3}$

118. The value of $\frac{1}{(1+\tan^2\theta)} + \frac{1}{(1+\cot^2\theta)}$ is
A) $\frac{1}{4}$ B) 1
C) $\frac{5}{4}$ D) $\frac{4}{3}$

119. If $\sin\theta - \cos\theta = \frac{1}{2}$ then value of $\sin\theta + \cos\theta$ is :
A) −2 B) ±2
C) $\frac{\sqrt{7}}{2}$ D) 2

120. The value of $\frac{\sin A}{1+\cos A} + \frac{\sin A}{1-\cos A}$ is $(0° < A < 90°)$
A) 2 cosec A B) 2 sec A
C) 2 sin A D) 2 cos A

121. If $r\sin\theta = 1$, $r\cos\theta = \sqrt{3}$ then the value of $(\sqrt{3}\tan\theta + 1)$ is
A) $\sqrt{3}$ B) $\frac{1}{\sqrt{3}}$
C) 1 D) 2

122. If $x\cos\theta - y\sin\theta = \sqrt{x^2+y^2}$ and $\frac{\cos^2\theta}{a^2} + \frac{\sin^2\theta}{b^2} = \frac{1}{x^2+y^2}$ then the correct relation is
A) $\frac{x^2}{b^2} - \frac{y^2}{a^2} = 1$
B) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$
C) $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$
D) $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

123. If $\tan\theta - \cot\theta = 0$, find the value of $\sin\theta + \cos\theta$.
A) 0 B) 1
C) $\sqrt{2}$ D) 2

124. The greatest value of $\sin^4\theta + \cos^4\theta$ is
A) 2 B) 3
C) $\frac{1}{2}$ D) 1

125. If $3\sin\theta + 5\cos\theta = 5$, then $5\sin\theta - 3\cos\theta$ is equal to
A) ±3 B) ± 5
C) 1 D) ± 2

126. If $\sin\theta + \sin^2\theta = 1$, then the value of $\cos^2\theta + \cos^4\theta$ is
A) 2 B) 4
C) 0 D) 1

127. If $\tan\theta + \cot\theta = 2$ then the value of θ is
A) 45° B) 60°
C) 90° D) 30°

128. If $\cos \pi x = x^2 - x + \frac{5}{4}$, the value of x will be
A) 0
B) 1
C) −1
D) None of the above

129. The numerical value of
$$1 + \frac{1}{\cot^2 63^o} - \sec^2 27^o + \frac{1}{\sin^2 63^o} - \operatorname{cosec}^2 27^o \text{ is}$$
A) 1 B) 2
C) −1 D) 0

130. If $x = \frac{\cos\theta}{1-\sin\theta}$, then $\frac{\cos\theta}{1+\sin\theta}$ is equal to
A) $x - 1$ B) $\frac{1}{x}$
C) $\frac{1}{x+1}$ D) $\frac{1}{1-x}$

131. In Δ ABC, $\angle B = 90^o$ and AB : BC = 2 : 1. The value of sin A + cot C is
A) $3+\sqrt{5}$ B) $\frac{2+\sqrt{5}}{2\sqrt{5}}$
C) $2+\sqrt{5}$ D) $3\sqrt{5}$

132. If $\sin\frac{\pi x}{2} = x^2 - 2x + 2$, then the value of x is
A) 0
B) 1
C) −1
D) None of these

133. The value of
$$\frac{\sin 43^o}{\cos 47^o} + \frac{\cos 19^o}{\sin 71^o} - 8\cos^2 60^o \text{ is}$$
A) 0 B) 1
C) 2 D) −1

134. The value of
$$\left(\sin^2 7\frac{1^0}{2} + \sin^2 82\frac{1^0}{2} + \tan^2 2^o.\tan^2 88^o\right) \text{ is}$$
A) 1 B) 2
C) 0 D) 4

135. Find the value of $1 - 2\sin^2\theta + \sin^4\theta$.
A) $\sin^4\theta$ B) $\cos^4\theta$
C) $\operatorname{cosec}^4\theta$ D) $\sec^4\theta$

136. The simplest value of cot 9° cot 27°cot63° cot 81° is
A) 0 B) 1
C) −1 D) $\sqrt{3}$

137. If $(1 + \sin A)(1 + \sin B)(1 + \sin C) = (1 - \sin A)(1 - \sin B)(1 - \sin C)$, $0 < A, B, C < \frac{\pi}{2}$ then each side is equal to
A) sin A sin B sin C
B) cos A cos B cos C
C) tan A tan B tan C
D) 1

138. The value of θ, which satisfies the equation $\tan^2\theta + 3 = 3\sec\theta$, $0^o \le \theta < 90^o$ is
A) 15° or 0° B) 30° or 0°
C) 45° or 0° D) 60° or 0°

139. If $\sin\theta = 0.7$, then $\cos\theta$, $0 \le \theta < 90^o$, is
A) 0.3 B) $\sqrt{0.49}$
C) $\sqrt{0.51}$ D) $\sqrt{0.9}$

140. The value of $\sin^2 65^o + \sin^2 25^o + \cos^2 35^o + \cos^2 55^o$ is
A) 0 B) 1
C) 2 D) $\frac{1}{2}$

141. If x sin 60° . tan 30° = sec 60° . cot 45°, then the value of x is
A) 2 B) $2\sqrt{3}$
C) 4 D) $4\sqrt{3}$

142. If $\theta = 60^o$, then
$$\frac{1}{2}\sqrt{1+\sin\theta} + \frac{1}{2}\sqrt{1-\sin\theta} \text{ is}$$
equal to
A) $\cot\frac{\theta}{2}$ B) $\sec\frac{\theta}{2}$
C) $\sin\frac{\theta}{2}$ D) $\cos\frac{\theta}{2}$

143. If $\frac{2\tan^2 30^o}{1-\tan^2 30^o} + \sec^2 45^o - \sec^2 0^o = x\sec 60^o$, then the value of x is
A) 2 B) 1
C) 0 D) −1

144. If $\tan\theta = \frac{\sin\alpha - \cos\alpha}{\sin\alpha + \cos\alpha}$, then $\sin\alpha + \cos\alpha$ is
A) $\pm\sqrt{2}\sin\theta$
B) $\pm\sqrt{2}\cos\theta$
C) $\pm\frac{1}{\sqrt{2}}\sin\theta$
D) $\pm\frac{1}{\sqrt{2}}\cos\theta$

145. If $7\sin^2\theta + 3\cos^2\theta = 4$, $(0^o < \theta < 90^o)$, then the value of $\tan\theta$ is
A) $\frac{1}{\sqrt{3}}$ B) $\frac{1}{2}$
C) 1 D) $\sqrt{3}$

146. If $\tan 9^o = \frac{p}{q}$, then the value of $\frac{\sec^2 81^o}{1+\cot^2 81^o}$ is
A) $\frac{q}{p}$ B) 1
C) $\frac{p^2}{q^2}$ D) $\frac{q^2}{p^2}$

147. If $\sec\theta + \tan\theta = 5$, then the value of $\frac{\tan\theta + 1}{\tan\theta - 1}$ is
A) $\frac{11}{7}$ B) $\frac{13}{7}$
C) $\frac{15}{7}$ D) $\frac{17}{7}$

148. If $\tan^2\theta = 1 - e^2$, then the value of $\sec\theta + \tan^3\theta \operatorname{cosec}\theta$ is
A) $(2+e^2)^{\frac{3}{2}}$ B) $(2-e^2)^{\frac{1}{2}}$
C) $(2+e^2)^{\frac{1}{2}}$ D) $(2-e^2)^{\frac{3}{2}}$

149. Which one of the following is true for $0^o < \theta < 90^o$?
A) $\cos\theta \le \cos^2\theta$
B) $\cos\theta > \cos^2\theta$
C) $\cos\theta < \cos^2\theta$
D) $\cos\theta \ge \cos^2\theta$

150. If x sin 60° tan 30° - tan²45° = cosec 60° cot 30° - sec²45°, then x =
A) 2 B) −2
C) 6 D) −4

151. If $x = a\sec\alpha\cos\beta$, $y = b\sec\alpha\sin\beta$, $z = c\tan\alpha$, then the value of $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2}$ is
A) 2 B) 0
C) 1 D) −1

152. If $\frac{\cos\alpha}{\cos\beta} = a$ and $\frac{\sin\alpha}{\sin\beta} = b$, the value of $\sin^2\beta$ in terms of a and b is
A) $\frac{a^2+1}{a^2-b^2}$ B) $\frac{a^2-b^2}{a^2+b^2}$
C) $\frac{a^2-1}{a^2-b^2}$ D) $\frac{a^2-1}{a^2+b^2}$

153. The value of
$$\frac{\cos^2 60^o + 4\sec^2 30^o - \tan^2 45^o}{\sin^2 30^o + \cos^2 30^o} \text{ is}$$
A) $\frac{64}{\sqrt{3}}$ B) $\frac{55}{12}$
C) $\frac{67}{12}$ D) $\frac{67}{10}$

154. The value of $\sin^2 30^\circ \cos^2 45^\circ + 5\tan^2 30^\circ + \frac{3}{2}\sin^2 90^\circ - 3\cos^2 90^\circ$ is

A) $3\frac{7}{24}$ B) $3\frac{3}{24}$

C) $3\frac{1}{24}$ D) $3\frac{5}{24}$

155. If $\cos^2\theta - \sin^2\theta = \frac{1}{3}$, where $0 \le \theta \le \frac{\pi}{2}$, then the value of $\cos^4\theta - \sin^4\theta$ is

A) $\frac{1}{3}$ B) $\frac{2}{3}$

C) $\frac{1}{9}$ D) $\frac{2}{9}$

156. If $\tan\theta = \frac{1}{\sqrt{11}}$ and $0 < \theta < \frac{\pi}{2}$, then the value of $\frac{\operatorname{cosec}^2\theta - \sec^2\theta}{\operatorname{cosec}^2\theta + \sec^2\theta}$ is

A) $\frac{3}{4}$ B) $\frac{4}{5}$

C) $\frac{5}{6}$ D) $\frac{6}{7}$

157. The value of $\frac{1}{\sqrt{2}}\sin\frac{\pi}{6}\cos\frac{\pi}{4} - \cot\frac{\pi}{3}\sec\frac{\pi}{6} + \frac{5\tan\frac{\pi}{4}}{12\sin\frac{\pi}{2}}$ is equal to

A) 0 B) 1

C) 2 D) $\frac{3}{2}$

158. If $\sin\theta = \frac{3}{5}$, then the value of $\frac{\tan\theta + \cos\theta}{\cot\theta + \operatorname{cosec}\theta}$ is equal to

A) $\frac{29}{60}$ B) $\frac{31}{60}$

C) $\frac{34}{60}$ D) $\frac{37}{60}$

159. If a cos θ + b sin θ = p and a sin θ − b cos θ = q, then the relation between a, b, p and q is

A) $a^2 - b^2 = p^2 - q^2$

B) $a^2 + b^2 = p^2 + q^2$

C) $a + b = p + q$

D) $a - b = p - q$

160. If $(\sin\alpha + \operatorname{cosec}\alpha)^2 + (\cos\alpha + \sec\alpha)^2 = k + \tan^2\alpha + \cot^2\alpha$, then the value of k is

A) 1 B) 7

C) 3 D) 5

161. If $\sin 21^\circ = \frac{x}{y}$, then $\sec 21^\circ - \sin 69^\circ$ is equal to

A) $\frac{x^2}{y\sqrt{y^2 - x^2}}$

B) $\frac{y^2}{x\sqrt{y^2 - x^2}}$

C) $\frac{y^2}{y\sqrt{x^2 - y^2}}$

D) $\frac{y^2}{x\sqrt{x^2 - y^2}}$

162. If sec α + tan α = 2, then the value of sin α is (assume that 0 < α < 90°)

A) 0.4 B) 0.5

C) 0.6 D) 0.8

163. If 3 sin θ + 5 cos θ = 5, then the value of 5 sin θ − 3 cos θ will be

A) ±3 B) ±5

C) ±2 D) ±1

164. If θ is an acute angle and tan θ + cot θ = 2, then the value of $\tan^5\theta + \cot^5\theta$ is

A) 1 B) 2

C) 3 D) 4

165. The simple value of tan 1° . tan 2° . tan 3° .... tan 89° is

A) $\frac{1}{2}$ B) 0

C) 1 D) $\frac{2}{3}$

166. If $x\sin^2 60^\circ - \frac{3}{2}\sec 60^\circ \tan^2 30^\circ + \frac{4}{5}\sin^2 45^\circ \tan^2 60^\circ = 0$ then x is

A) $-\frac{1}{15}$ B) −4

C) $-\frac{4}{15}$ D) − 2

167. If $7\sin\alpha = 24\cos\alpha$ $0 < \alpha < \frac{\pi}{2}$, then the value of 14 tan α − 75 cos α − 7 sec α is equal to

A) 3 B) 4

C) 1 D) 2

168. The value of x which satisfies the equation $2\operatorname{cosec}^2 30^\circ + x\sin^2 60^\circ - \frac{3}{4}\tan^2 30^\circ = 10$ is

A) 2 B) 3

C) 0 D) 1

169. If $2\sin\theta + \cos\theta = \frac{7}{3}$ then the value of $(\tan^2\theta - \sec^2\theta)$ is

A) 0 B) −1

C) $\frac{3}{7}$ D) $\frac{7}{3}$

170. If $29\tan\theta = 31$, then the value of $\frac{1 + 2\sin\theta\cos\theta}{1 - 2\sin\theta\cos\theta}$ is equal to

A) 810 B) 900

C) 540 D) 490

171. ABCD is a rectangle of which AC is a diagonal. The value of $(\tan^2\angle CAD + 1)\sin^2\angle BAC$ is

A) 2 B) $\frac{1}{4}$

C) 1 D) 0

172. If tan x = sin 45°. cos 45° + sin 30° then the value of x is

A) 30° B) 45°

C) 60° D) 90°

173. For any real values of θ, $\sqrt{\frac{\sec\theta - 1}{\sec\theta + 1}} = ?$

A) cotθ − cosecθ

B) secθ − tanθ

C) cosecθ − cotθ

D) tanθ − secθ

174. If the sum and difference of two angles are 135° and $\frac{\pi}{12}$ respectively, then the value of the angles in degree measure are

A) 70°, 65° B) 75°, 60°

C) 45°, 90° D) 80°, 55°

175. In a ΔABC, $\angle B = \frac{\pi}{3}$, $\angle C = \frac{\pi}{4}$ and D divides BC internally in the ratio 1 : 3 then $\frac{\sin\angle BAD}{\sin\angle CAD}$ is equal to

A) $\frac{1}{\sqrt{2}}$ B) $\frac{1}{\sqrt{3}}$

C) $\frac{1}{\sqrt{6}}$ D) $\sqrt{6}$

176. If sin 3A = cos (A - 26°), where 3A is an acute angle then the value of A is

A) 29° B) 26°

C) 23° D) 28°

177. Value of $\sec^2\theta - \frac{\sin^2\theta - 2\sin^4\theta}{2\cos^4\theta - \cos^2\theta}$ is

A) 1 B) 2

C) −1 D) 0

178. If x = a (sinθ + cosθ), y = b (sinθ − cosθ) then the value of $\frac{x^2}{a^2} + \frac{y^2}{b^2}$ is

A) 0 B) 1
C) 2 D) −2

179. If sin 5 θ = cos 20° (0° < θ < 90°) then the value of θ is
A) 4° B) 22°
C) 10° D) 14°

180. If 0° < θ < 90° and 2 secθ = 3 cosec²θ, then θ is
A) $\frac{\pi}{6}$ B) $\frac{\pi}{4}$
C) $\frac{\pi}{3}$ D) $\frac{\pi}{5}$

181. $\sqrt{\frac{1+\sin\theta}{1-\sin\theta}}+\sqrt{\frac{1-\sin\theta}{1+\sin\theta}}$ is equal to
A) 2 cos θ B) 2 sinθ
C) 2 cot θ D) 2 sec θ

182. If $\cos\theta=\frac{3}{5}$, then the value of sinθ . secθ . tanθ is
A) $\frac{9}{16}$ B) $\frac{16}{9}$
C) $\frac{3}{4}$ D) $\frac{4}{3}$

183. If 0° < A < 90°, then the value of tan²A + cot² A − sec² A cosec² A is
A) 0 B) 1
C) 2 D) −2

184. If α and β are positive acute angles, sin (4α− β) = 1 and $\cos(2\alpha+\beta)=\frac{1}{2}$, then the value of sin (α +2β) is
A) 0 B) 1
C) $\frac{\sqrt{3}}{2}$ D) $\frac{1}{\sqrt{2}}$

185. If θ is a positive acute angle and cosec $\theta=\sqrt{3}$, then the value of cot θ − cosec θ is
A) $\frac{3(\sqrt{2}-\sqrt{3})}{3}$
B) $\frac{\sqrt{2}(3+\sqrt{3})}{3}$
C) $\frac{\sqrt{2}(3-\sqrt{3})}{3}$
D) $\frac{3\sqrt{2}+\sqrt{3}}{3}$

186. If θ is a positive acute angle and 4 cos² θ− 4 cosθ + 1 = 0, then the value of tan (θ − 15°) is equal to
A) 0 B) 1
C) $\sqrt{3}$ D) $\frac{1}{\sqrt{3}}$

187. If (r cosθ − $\sqrt{3}$)² + (r sinθ− 1)² = 0 then the value of $\frac{r\tan\theta+\sec\theta}{r\sec\theta+\tan\theta}$ is equal to
A) $\frac{4}{5}$ B) $\frac{5}{4}$
C) $\frac{\sqrt{3}}{4}$ D) $\frac{\sqrt{5}}{4}$

188. The value of $\frac{\sin 25^\circ\cos 65^\circ+\cos 25^\circ\sin 65^\circ}{\tan^2 70^\circ-\operatorname{cosec}^2 20^\circ}$
A) −1 B) 0
C) 1 D) 2

189. If sin (θ + 18°) = cos 60° (0 < θ < 90°), then the value of cos 5θ is
A) $\frac{1}{2}$ B) 0
C) $\frac{1}{\sqrt{2}}$ D) 1

190. If $\tan\theta=\frac{3}{4}$, then the value of $\frac{4\sin^2\theta-2\cos^2\theta}{4\sin^2\theta+3\cos^2\theta}$ is equal to
A) $\frac{1}{21}$ B) $\frac{2}{21}$
C) $\frac{4}{21}$ D) $\frac{8}{21}$

191. If $\frac{\cos\alpha}{\cos\beta}=a$, $\frac{\sin\alpha}{\sin\beta}=b$, then $\sin^2\beta$ is equal to
A) $\frac{a^2-1}{a^2+b^2}$ B) $\frac{a^2+1}{a^2-b^2}$
C) $\frac{a^2-1}{a^2-b^2}$ D) $\frac{a^2+1}{a^2+b^2}$

192. Let A, B, C, D be the angles of a quadrilateral. If they are concyclic. then the value of cos A + cos B + cos C + cos D is
A) 0 B) 1
C) −1 D) 2

193. If $\sqrt{3}$ tan θ = 3 sin θ, then the value of (sin²θ − cos²θ) is
A) 1 B) 3
C) $\frac{1}{3}$ D) None

194. If sin(A + B) = sin A cos B + cos A sin B, then the value of sin75° is
A) $\frac{\sqrt{3}+1}{\sqrt{2}}$ B) $\frac{\sqrt{2}+1}{2\sqrt{2}}$
C) $\frac{\sqrt{3}+1}{2\sqrt{2}}$ D) $\frac{\sqrt{3}+1}{2}$

195. ABC is a right angled triangle, right angled at B and ∠A = 60° and AB = 20 cm, then the ratio of sides BC and CA is
A) $\sqrt{3}:1$ B) $1:\sqrt{3}$
C) $\sqrt{3}:\sqrt{2}$ D) $\sqrt{3}:2$

196. If tan 2θ . tan 3θ = 1, where 0° < θ < 90° then the value of θ is
A) $22\frac{1}{2}^\circ$ B) 18°
C) 24° D) 30°

197. If cos²α − sin²α = tan²β, then the value of cos²β − sin²β is
A) cot²α B) cot²β
C) tan²α D) tan²β

198. If tan (A + B) = $\sqrt{3}$ and $\tan(A-B)=\frac{1}{\sqrt{3}}$, $(\angle A+\angle B)<90^\circ$, A ≥ B, then ∠A is
A) 90° B) 30°
C) 45° D) 60°

199. The value of $\frac{\sin\theta-2\sin^3\theta}{2\cos^3\theta-\cos\theta}$ is equal to
A) sinθ B) cosθ
C) tanθ D) cotθ

200. If r sinθ = $\frac{7}{2}$ and r cosθ = $\frac{7\sqrt{3}}{2}$, then value of r is
A) 4 B) 3
C) 5 D) 7

201. If $\theta+\phi=\frac{\pi}{2}$ and $\sin\theta=\frac{1}{2}$, then the value of sin ϕ is
A) 1 B) $\frac{1}{\sqrt{2}}$
C) $\frac{1}{2}$ D) $\frac{\sqrt{3}}{2}$

202. If 0° < θ < 90° and 2 sin²θ + 3 cosθ = 3, then the value of θ is
A) 30° B) 0°
C) 45° D) 75°

203. The value of θ (0 ≤ θ ≤ 90°) satisfying 2 sin²θ = 3 cos θ is
A) 60° B) 30°
C) 90° D) 45°

204. If a (tanθ + cotθ) = 1, sinθ + cos θ = b with 0° < θ < 90°, then a relation between a and b is
A) b² = 2 (a + 1)
B) b² = 2(a − 1)
C) 2a = b² − 1
D) 2a = b² + 1

205. If A is an acute angle and cot A + cosec A = 3, then the value of sin A is

A) 1 B) $\frac{3}{5}$

C) $\frac{4}{5}$ D) 0

206. The simplest value of $\sin^2x + 2\tan^2x - 2\sec^2x + \cos^2x$ is
A) 1 B) 0
C) −1 D) 2

207. If $x = a\sec\theta$ and $y = b\tan\theta$ then $\frac{x^2}{a^2} - \frac{y^2}{b^2} = ?$
A) 1 B) 2
C) 3 D) 4

208. The value of $\sin^2 1^\circ + \sin^2 2^\circ + \sin^2 3^\circ + ...+ \sin^2 89^\circ$ is
A) 22 B) 44
C) $22\frac{1}{2}$ D) $44\frac{1}{2}$

209. The value of $\frac{\cos^3\theta + \sin^3\theta}{\cos\theta + \sin\theta} + \frac{\cos^3\theta - \sin^3\theta}{\cos\theta - \sin\theta}$ is equal to
A) −1 B) 1
C) 2 D) 0

210. If $\sin 17^\circ = \frac{x}{y}$, then $\sec 17^\circ - \sin 73^\circ$ is equal to
A) $\frac{y}{\sqrt{y^2 - x^2}}$
B) $\frac{y^2}{(x\sqrt{y^2 - x^2})}$
C) $\frac{x}{(y\sqrt{y^2 - x^2})}$
D) $\frac{x^2}{(y\sqrt{y^2 - x^2})}$

211. If θ is a positive acute angle and $\operatorname{cosec}\theta + \cot\theta = \sqrt{3}$, then the value of $\operatorname{cosec}\theta$ is
A) $\frac{1}{\sqrt{3}}$ B) $\sqrt{3}$
C) $\frac{2}{\sqrt{3}}$ D) 1

212. If $\cos\alpha + \sec\alpha = \sqrt{3}$ then the value of $\cos^3\alpha + \sec^3\alpha$ is
A) 2 B) 1
C) 0 D) 4

213. If $\sin\theta + \cos\theta = \sqrt{2}\cos\theta$, then the value of $\cot\theta$ is
A) $\sqrt{2}+1$ B) $\sqrt{2}-1$
C) $\sqrt{3}-1$ D) $\sqrt{3}+1$

214. If $\cos^4\theta - \sin^4\theta = \frac{2}{3}$, then the value of $1 - 2\sin^2\theta$ is
A) $\frac{2}{3}$ B) $\frac{3}{2}$
C) 1 D) 0

215. The value of $\frac{\cot 30^\circ - \cot 75^\circ}{\tan 15^\circ - \tan 60^\circ}$ is equal to
A) − 1 B) 0
C) 1 D) 2

216. If $\sin\theta + \cos\theta = p$ and $\sec\theta + \operatorname{cosec}\theta = q$, then the value of $q(p^2 - 1)$ is
A) 1 B) p
C) 2p D) 2

217. If $\sin(3\alpha - \beta) = 1$ and $\cos(2\alpha + \beta) = \frac{1}{2}$, then the value of $\tan\alpha$ is
A) 0 B) $\frac{1}{\sqrt{3}}$
C) 1 D) $\sqrt{3}$

218. If $\sin(60^\circ - x) = \cos(y + 60^\circ)$, then the value of $\sin(x - y)$ is
A) $\frac{1}{\sqrt{2}}$ B) $\frac{1}{2}$
C) $\frac{\sqrt{3}}{2}$ D) 1

219. If $x = a\sec\theta$, $y = b\tan\theta$, then $\frac{x^2}{a^2} - \frac{y^2}{b^2}$ is
A) −1 B) 0
C) 1 D) 2

220. a, b, c are the lengths of three sides of a triangle ABC. If a, b, c are related by the relation $a^2 + b^2 + c^2 = ab + bc + ca$, then the value of $\sin^2A + \sin^2B + \sin^2C$ is
A) $\frac{3}{4}$ B) $\frac{3\sqrt{3}}{2}$
C) $\frac{3}{2}$ D) $\frac{9}{4}$

221. If $a\sin\theta + b\cos\theta = c$, then $a\cos\theta - b\sin\theta$ is equal to
A) $\pm\sqrt{a + b - c}$
B) $\pm\sqrt{a^2 + b^2 + c^2}$
C) $\pm\sqrt{a^2 + b^2 - c^2}$
D) $\pm\sqrt{c^2 + a^2 - b^2}$

222. If $\sin\theta + \cos\theta = \sqrt{2}\sin(90^\circ - \theta)$, then the value of $\cot\theta$ is
A) $-\sqrt{2}-1$ B) $\sqrt{2}-1$
C) $\sqrt{2}+1$ D) $-\sqrt{2}+1$

223. If θ is a positive acute angle and $3(\sec^2\theta + \tan^2\theta) = 5$, then the value of $\cos 2\theta$ is
A) $\frac{1}{2}$ B) $\frac{1}{\sqrt{2}}$
C) $\frac{\sqrt{3}}{2}$ D) 1

224. If $x\cos^2 30^\circ . \sin 60^\circ = \frac{\tan^2 45^\circ . \sec 60^\circ}{\operatorname{cosec} 60^\circ}$ then the value of x is
A) $\frac{1}{\sqrt{3}}$ B) $\frac{1}{\sqrt{2}}$
C) $2\frac{2}{3}$ D) $\frac{1}{2}$

225. If $\tan\alpha = 2$, then the value of $\frac{\operatorname{cosec}^2\alpha - \sec^2\alpha}{\operatorname{cosec}^2\alpha + \sec^2\alpha}$ is
A) $-\frac{15}{9}$ B) $-\frac{3}{5}$
C) $\frac{3}{5}$ D) $\frac{17}{5}$

226. If $\sin(\theta + 30^\circ) = \frac{3}{\sqrt{12}}$, then the value of $\cos^2\theta$ is
A) $\frac{1}{4}$ B) $\frac{\sqrt{3}}{2}$
C) $\frac{3}{4}$ D) $\frac{1}{2}$

227. If $0 \le \theta \le 90^\circ$ and $4\cos^2\theta - 4\sqrt{3}\cos\theta + 3 = 0$ then the value of θ is
A) 30° B) 45°
C) 90° D) 60°

228. If $\sec\theta - \cos\theta = \frac{3}{2}$ where θ is a positive acute angle, then the value of $\sec\theta$ is
A) $-\frac{1}{2}$ B) 1
C) 2 D) 0

229. If $\tan(5x - 10^\circ) = \cot(5y + 20^\circ)$, the value of $(x + y)$ is
A) 15° B) 16°
C) 24° D) 20°

230. If $\sin\theta + \sin^2\theta = 1$, then the value of $\cos^{12}\theta + 3\cos^{10}\theta + 3\cos^8\theta + \cos^6\theta - 1$ is
A) 1 B) 2
C) 3 D) 0

231. The value of $\tan 11^\circ \tan 17^\circ \tan 79^\circ \tan 73^\circ$ is
A) $\frac{1}{2}$ B) 0
C) 1 D) $\frac{1}{\sqrt{2}}$

232. If for any acute angle A, $\sin A + \sin^2 A = 1$, then the value of $\cos^2 A + \cos^4 A$ is
A) −1 B) 1
C) 2 D) 0

233. The value of $(1 + \sec 20^\circ + \cot 70^\circ)(1 - \operatorname{cosec} 20^\circ + \tan 70^\circ)$ is equal to

A) 0 B) 1
C) 2 D) −1

234. If 0° < A < 90°, the value of $\frac{\tan A - \sec A - 1}{\tan A + \sec A + 1}$ is
A) $\frac{\sin A - 1}{\cos A}$ B) $\frac{1 - \sin A}{\cos A}$
C) $\frac{1 - \cos A}{\sin A}$ D) $\frac{\sin A + 1}{\cos A}$

235. If α is an cute angle and 2 sin α + 15 cos² α = 7 then the value of cot α is
A) $\frac{4}{3}$ B) $\frac{4}{5}$
C) $\frac{5}{4}$ D) $\frac{3}{4}$

236. If sin (A − B) = sin A cos B − cosA sinB, then sin 15° will be
A) $\frac{\sqrt{3}+1}{2\sqrt{2}}$ B) $\frac{\sqrt{3}}{2\sqrt{2}}$
C) $\frac{\sqrt{3}-1}{-\sqrt{2}}$ D) $\frac{\sqrt{3}-1}{2\sqrt{2}}$

237. If sec x + cos x = 2, then the value of $\sec^{16} x + \cos^{16} x$ will be
A) $\sqrt{3}$ B) 2
C) 1 D) 0

238. If $\sin^4\theta + \cos^4\theta = 2\sin^2\theta\cos^2\theta$, θ is an acute angle, then the value of tan θ is
A) 1 B) 2
C) $\sqrt{2}$ D) 0

239. The maximum value of $\sin^4\theta + \cos^4\theta$ is
A) $\frac{1}{3}$ B) 1
C) 2 D) 3

240. Find the value of tan 4° tan 43° tan 47° tan 86°
A) $\frac{2}{3}$ B) 1
C) $\frac{1}{2}$ D) 2

241. If x cosθ − sinθ = 1, then x² + (1 +x²) sinθ equals
A) 2 B) 1
C) −1 D) 0

242. If sin θ + sin² θ = 1 then cos² θ + cos⁴ θ is equal to
A) None B) 1
C) $\frac{\sin\theta}{\cos^2\theta}$ D) $\frac{\cos^2\theta}{\sin\theta}$

243. The numerical value of $\frac{\cos^2 45^\circ}{\sin^2 60^\circ} + \frac{\cos^2 60^\circ}{\sin^2 45^\circ} - \frac{\tan^2 30^\circ}{\cot^2 45^\circ} - \frac{\sin^2 30^\circ}{\cot^2 30^\circ}$ is
A) $1\frac{1}{4}$ B) $\frac{3}{4}$
C) $\frac{1}{4}$ D) $\frac{1}{2}$

244. The value of tan1°tan2°tan3° ....tan89° is
A) 1
B) −1
C) 0
D) None of the options

245. If $\frac{\cos\alpha}{\sin\beta} = n$ and $\frac{\cos\alpha}{\cos\beta} = m$, then the value of $\cos^2\beta$ is
A) $\frac{m^2}{m^2+n^2}$ B) $\frac{1}{m^2+n^2}$
C) $\frac{n^2}{m^2+n^2}$ D) 0

246. If 0° ≤ A ≤ 90°, the simplified form of the given expression sin A cos A (tan A − cot A) is
A) 1 B) 1 − 2 sin² A
C) 2 sin² A − 1 D) 1 − cos² A

247. If θ is an acute angle and $\tan^2\theta + \frac{1}{\tan^2\theta} = 2$, then the value of θ is :
A) 60° B) 45°
C) 15° D) 30°

248. If tan θ + cotθ = 5, then tan²θ + cot² θ is
A) 23 B) 25
C) 26 D) 24

249. The value of sin²22° + sin²68° + cot²30° is
A) 4 B) 3
C) $\frac{3}{4}$ D) $\frac{5}{4}$

250. The minimum value of 2sin²θ + 3cos²θ is
A) 3 B) 4
C) 2 D) 1

251. If θ be acute angle and tan (4θ− 50°) = cot(50° − θ), then the value of θ in degrees is :
A) 20 B) 50
C) 40 D) 30

252. If 5 sinθ = 3, the numerical value of $\frac{\sec\theta - \tan\theta}{\sec\theta + \tan\theta}$ is
A) $\frac{1}{2}$ B) $\frac{1}{5}$
C) $\frac{1}{3}$ D) $\frac{1}{4}$

253. If secθ + tanθ = p,(p ≠ 0) then secθ is equal to
A) $\left(p - \frac{1}{p}\right)$, p ≠ 0
B) $2\left(p - \frac{1}{p}\right)$, p ≠ 0
C) $\left(p + \frac{1}{p}\right)$, p ≠ 0
D) $\frac{1}{2}\left(p + \frac{1}{p}\right)$, p ≠ 0

254. If 1 + cos²θ = 3 sinθ cosθ, then the integral value of $\cot\theta\left(0 < \theta < \frac{\pi}{2}\right)$ is
A) 1 B) 2
C) 0 D) 3

255. The value of the following is : 3(sin⁴θ + cos⁴θ) + 2(sin⁶θ + cos⁶θ) + 12sin²θ cos²θ
A) 0 B) 3
C) 2 D) 5

256. If secθ + tanθ = 2 + $\sqrt{5}$, then the value of sinθ is (0° ≤ θ ≤ 90°)
A) $\frac{\sqrt{3}}{2}$ B) $\frac{2}{\sqrt{5}}$
C) $\frac{1}{\sqrt{5}}$ D) $\frac{4}{5}$

257. If $\frac{\sec\theta + \tan\theta}{\sec\theta - \tan\theta} = 2\frac{51}{79}$ then the value of sin θ is
A) $\frac{39}{72}$ B) $\frac{65}{144}$
C) $\frac{35}{72}$ D) $\frac{91}{144}$

258. If tan A + cot A = 2, then the value of tan¹⁰A + cot¹⁰A is
A) 4 B) 2
C) 2^{10} D) 1

259. The value of cos²30° + sin²60° + tan²45° + sec²60° + cos0° is
A) $4\frac{1}{2}$ B) $5\frac{1}{2}$
C) $6\frac{1}{2}$ D) $7\frac{1}{2}$

260. If cos x + cos² x = 1, then sin⁸x + 2 sin⁶x + sin⁴x is equal to
A) 0 B) 3
C) 2 D) 1

261. In ΔABC, ∠C = 90° and AB = c, BC = a, CA = b; then the value of (cosec B − cos A) is
A) $\frac{c^2}{ab}$ B) $\frac{b^2}{ca}$
C) $\frac{a^2}{bc}$ D) $\frac{bc}{a^2}$

262. If $\tan\theta - \cot\theta = 0$ and θ is positive acute angle, then the value of $\frac{\tan(\theta+15^\circ)}{\tan(\theta-15^\circ)}$ is

A) 3 B) $\frac{1}{\sqrt{3}}$

C) $\frac{1}{3}$ D) $\sqrt{3}$

263. The value of cot 41°. cot 42° . cot 43°. cot 44°. cot 45° . cot 46°. cot 47°. cot 48° . cot 49°

A) 1 B) 0

C) $\frac{\sqrt{3}}{2}$ D) $\frac{1}{\sqrt{2}}$

264. If $x = a\sin\theta - b\cos\theta$, $y = a\cos\theta + b\sin\theta$, then which of the following is true?

A) $\frac{x^2}{y^2}+\frac{a^2}{b^2}=1$

B) $x^2+y^2=a^2-b^2$

C) $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

D) $x^2+y^2=a^2+b^2$

265. If $\sec\theta - \tan\theta = \frac{1}{\sqrt{3}}$, the value of $\sec\theta \cdot \tan\theta$ is

A) $\frac{2}{3}$ B) $\frac{2}{\sqrt{3}}$

C) $\frac{4}{\sqrt{3}}$ D) $\frac{1}{\sqrt{3}}$

266. If $5\cos\theta + 12\sin\theta = 13$, $0^\circ < \theta < 90^\circ$, then the value of $\sin\theta$ is

A) $\frac{5}{13}$ B) $-\frac{12}{13}$

C) $\frac{6}{13}$ D) $\frac{12}{13}$

267. If $7\sin^2\theta + 3\cos^2\theta = 4$, then the value of $\tan\theta$ is (θ is acute)

A) $\frac{1}{\sqrt{3}}$ B) $\frac{1}{\sqrt{2}}$

C) $\sqrt{3}$ D) 1

268. The value of (cosec a − sin a) (sec a − cos a) (tan a + cot a) is

A) 1 B) 6

C) 2 D) 4

269. If $\sin A + \sin^2 A = 1$, then the value of $\cos^2 A + \cos^4 A$ is

A) 2 B) $1\frac{2}{3}$

C) $1\frac{1}{2}$ D) 1

270. If $\tan A = n\tan B$ and $\sin A = m\sin B$, then the value of $\cos^2 A$ is

A) $\frac{m^2-1}{n^2+1}$ B) $\frac{m^2+1}{n^2-1}$

C) $\frac{m^2+1}{n^2+1}$ D) $\frac{m^2-1}{n^2-1}$

271. If $\sin\theta + \cos\theta = \sqrt{2}\sin(90^\circ - \theta)$ then $\cot\theta$ is equal to

A) $\sqrt{2}$ B) 0

C) $\sqrt{2}-1$ D) $\sqrt{2}+1$

272. The value of the following is $\frac{(\tan 20^\circ)^2}{(\operatorname{cosec}70^\circ)^2}+\frac{(\cot 20^\circ)^2}{(\sec 70^\circ)^2}+$ 2tan 15° . tan 45° . tan 75°

A) 1 B) 4

C) 3 D) 2

273. The value of the following is $\left(\frac{\sin 47^\circ}{\cos 43^\circ}\right)^2+\left(\frac{\cos 43^\circ}{\sin 47^\circ}\right)^2-4\cos^2 45^\circ$

A) −1 B) 0

C) 1 D) $\frac{1}{2}$

274. If $0^\circ < \theta < 90^\circ$ and $\operatorname{cosec}\theta = \cot^2\theta$ then the value of the expression $\operatorname{cosec}^4\theta - 2\operatorname{cosec}^3\theta + \cot^2\theta$ is equal to:

A) 2 B) 0

C) 1 D) 3

275. If $4\sin^2\theta - 1 = 0$ and angle θ is less than 90° the value of $\cos^2\theta + \tan^2\theta$ is : (Take $0^\circ < \theta < 90^\circ$)

A) $\frac{17}{15}$ B) $\frac{13}{12}$

C) $\frac{11}{9}$ D) $\frac{12}{11}$

276. Find numerical value of $\frac{9}{\operatorname{cosec}^2\theta}+4\cos^2\theta+\frac{5}{1+\tan^2\theta}$

A) 5 B) 7

C) 9 D) 4

277. If $\tan\theta + \sec\theta = 3$, θ being acute, the value of $5\sin\theta$ is :

A) $\frac{5}{2}$ B) $\frac{\sqrt{3}}{5}$

C) $\frac{5}{\sqrt{3}}$ D) 4

278. If $\cos\theta = \frac{p}{\sqrt{p^2+q^2}}$ then the value of $\tan\theta$ is

A) $\frac{q}{\sqrt{p^2-q^2}}$ B) $\frac{q}{p}$

C) $\frac{p}{p^2+q^2}$ D) $\frac{q}{\sqrt{p^2+q^2}}$

279. If A, B. and C be the angles of a triangle, then out of the following, the incorrect relation is:

A) $\cos\left(\frac{A+B}{2}\right)=\sin\frac{C}{2}$

B) $\tan\left(\frac{A+B}{2}\right)=\sec\frac{C}{2}$

C) $\cot\left(\frac{A+B}{2}\right)=\tan\frac{C}{2}$

D) $\sin\frac{A+B}{2}=\cos\frac{C}{2}$

280. The value of the expression $\sin^2 1^\circ + \sin^2 11^\circ + \sin^2 21^\circ + \sin^2 31^\circ + \sin^2 41^\circ + \sin^2 45^\circ + \sin^2 49^\circ + \sin^2 59^\circ + \sin^2 69^\circ + \sin^2 79^\circ + \sin^2 89^\circ$ is :

A) 0 B) $5\frac{1}{2}$

C) 5 D) $4\frac{1}{2}$

281. If $x = a(\sin\theta + \cos\theta)$ and $y = b(\sin\theta - \cos\theta)$, then the value of $\frac{x^2}{a^2}+\frac{y^2}{b^2}$ is

A) 4 B) 3

C) 1 D) 2

282. If $\cos\theta + \sin\theta = m$ and $\sec\theta + \operatorname{cosec}\theta = n$ then the value of $n(m^2-1)$ is equal to :

A) 2m B) mn

C) 4mn D) 2n

283. If $\frac{x - x\tan^2 30^\circ}{1+\tan^2 30^\circ} = \sin^2 30^\circ + 4\cot^2 45^\circ - \sec^2 60^\circ$, then the value of x is :

A) $\frac{1}{4}$ B) $\frac{1}{5}$

C) $\frac{1}{2}$ D) $\frac{1}{\sqrt{3}}$

284. If $\cos A + \sin A = \sqrt{2}\cos A$ then $\cos A - \sin A$ is equal to : (where $0^\circ < A < 90^\circ$)

A) $\sqrt{2}\sin A$ B) 2 sin A

C) $2\sqrt{\sin A}$ D) $\sqrt{2\sin A}$

285. If $\frac{\sin\theta+\cos\theta}{\sin\theta-\cos\theta}=3$ then the value of $\sin^4\theta$ is

A) $\frac{16}{25}$ B) $\frac{1}{5}$

C) $\frac{4}{5}$ D) $\frac{3}{5}$

286. If $\sin 2\theta = \frac{\sqrt{3}}{2}$ then the value of $\sin 3\theta$ is equal to (Take $0^\circ \le \theta \le 90^\circ$)

A) $\frac{1}{2}$ B) 1

C) 0 D) $\frac{\sqrt{3}}{2}$

287. Value of the expression :

$$\frac{1+2\sin 60^\circ\cos 60^\circ}{\sin 60^\circ+\cos 60^\circ}+\frac{1-2\sin 60^\circ\cos 60^\circ}{\sin 60^\circ-\cos 60^\circ}$$ is

A) $2\sqrt{3}$ B) 0

C) $\sqrt{3}$ D) 2

288. If $\alpha+\beta=90^\circ$, then the expression $\frac{\tan\alpha}{\tan\beta}+\sin^2\alpha+\sin^2\beta$ is equal to :

A) $\sec^2\beta$ B) $\tan^2\alpha$

C) $\tan^2\beta$ D) $\sec^2\alpha$

289. The value of x in the equation

$$\tan^2\frac{\pi}{4}-\cos^2\frac{\pi}{3}=x\sin\frac{\pi}{4}\cos\frac{\pi}{4}\tan\frac{\pi}{3}$$ is

A) $\frac{2}{\sqrt{3}}$ B) $\frac{3\sqrt{3}}{4}$

C) $\frac{1}{\sqrt{3}}$ D) $\frac{\sqrt{3}}{2}$

290. If $\sin A - \cos A = \frac{\sqrt{3}-1}{2}$, then the value of sin A . cos A is

A) $\frac{\sqrt{3}}{2}$ B) $\frac{3}{2}$

C) $\frac{\sqrt{3}}{4}$ D) $\frac{1}{\sqrt{3}}$

291. If $\sin(90^\circ-\theta)+\cos\theta=\sqrt{2}\cos(90^\circ-\theta)$, then the value of $\operatorname{cosec}\theta$ is

A) $\frac{2}{3}$ B) $\sqrt{\frac{3}{2}}$

C) $\frac{1}{\sqrt{2}}$ D) $\frac{1}{\sqrt{3}}$

292. If $\tan\left(\frac{\pi}{2}-\frac{\alpha}{2}\right)=\sqrt{3}$, then the value of $\cos\alpha$ is

A) $\frac{1}{\sqrt{2}}$ B) $\frac{1}{2}$

C) 0 D) $\frac{\sqrt{3}}{2}$

293. The value of cos 1° cos 2° cos 3° ... cos 180° is

A) 0 B) 1

C) $\frac{\sqrt{3}}{2}$ D) $\frac{1}{2}$

TYPE – III

1. If the angle of elevation of the Sun changes from 30° to 45°, the length of the shadow of a pillar decreases by 20 metres. The height of the pillar is

A) $20(\sqrt{3}-1)$m

B) $20(\sqrt{3}+1)$m

C) $10(\sqrt{3}-1)$m

D) $10(\sqrt{3}+1)$m

2. One flies a kite with a thread 150 metre long. If the thread of the kite makes an angle of 60° with the horizontal line, then the height of the kite from the ground (assuming the thread to be in a straight line) is

A) 50 metre

B) $75\sqrt{3}$ metre

C) $25\sqrt{3}$ metre

D) 80 metre

3. The angle of elevation of the top of a tower from two points A and B lying on the horizontal through the foot of the tower are respectively 15° and 30°. If A and B are on the same side of the tower and AB = 48 metre, then the height of the tower is :

A) $24\sqrt{3}$ metre B) 24 metre

C) $24\sqrt{2}$ metre D) 96 metre

4. At a point on a horizontal line through the base of a monument, the angle of elevation of the top of the monument is found to be such that its tangent is $\frac{1}{5}$. On walking 138 metres towards the monument the secant of the angle of elevation is found to be $\frac{\sqrt{193}}{12}$. The height of the monument (in metre) is

A) 35 B) 49

C) 42 D) 56

5. The distance between two pillars of length 16 metres and 9 metres is x metres. If two angles of elevation of their respective top from the bottom of the other are complementary to each other, then the value of x (in metres) is

A) 15 B) 16

C) 12 D) 9

6. The angle of elevation of the top of a building from the top and bottom of a tree are x and y respectively. If the height of the tree is h metre, then (in metre) the height of the building is

A) $\frac{h\cot x}{\cot x+\cot y}$

B) $\frac{h\cot y}{\cot x+\cot y}$

C) $\frac{h\cot x}{\cot x-\cot y}$

D) $\frac{h\cot y}{\cot x-\cot y}$

7. The angle of elevation of the top of a tower from a point A on the ground is 30°. On moving a distance of 20 metres towards the foot of the tower to a point B, the angle of elevation increases to 60°. The height of the tower is

A) $\sqrt{3}$ m B) $5\sqrt{3}$ m

C) $10\sqrt{3}$ m D) $20\sqrt{3}$ m

8. Two poles of equal height are standing opposite to each other on either side of a road which is 100m wide. From a point between them on road, angle of elevation of their tops are 30° and 60°. The height of each pole (in metre) is

A) $25\sqrt{3}$ B) $20\sqrt{3}$

C) $28\sqrt{3}$ D) $30\sqrt{3}$

9. A telegraph post is bent at a point above the ground due to storm. Its top just meets the ground at a distance of $8\sqrt{3}$ metres from its foot and makes an angle of 30° then the height of the post is :

A) 16 metres B) 23 metres

C) 24 metres D) 10 metres

10. The angle of elevation of the top of a building and the top of the chimney on the roof of the building from a point on the ground are x and 45° respectively. The height of building is h metre. Then the height of the chimney, (in metre) is :

A) h cot x + h B) h cot x – h

C) h tan x – h D) h tan x + h

11. Two posts are x metres apart and the height of one is double that of the other. If from the mid-point of the line joining their feet, an observer finds

the angular elevations of their tops to be complementary, then the height (in metres) of the shorter post is

A) $\frac{x}{2\sqrt{2}}$ B) $\frac{x}{4}$

C) $x\sqrt{2}$ D) $\frac{x}{\sqrt{2}}$

12. An aeroplane when flying at a height of 5000m from the ground passes vertically above another aeroplane at an instant, when the angles of elevation of the two aeroplanes from the same point on the ground are 60° and 45° respectively. The vertical distance between the aeroplanes at that instant is

A) $5000(\sqrt{3}-1)$ m

B) $5000(3-\sqrt{3})$ m

C) $5000\left(1-\frac{1}{\sqrt{3}}\right)$m

D) 4500 m

13. A man standing at a point P is watching the top of a tower, which makes an angle of elevation of 30°. The man walks some distance towards the tower and then his angle of elevation of the top of the tower is 60°. If the height of the tower is 30 m, then the distance he moves is

A) 22 m B) $22\sqrt{3}$ m

C) 20 m D) $20\sqrt{3}$ m

14. The distance between two vertical poles is 60 m. The height of one of the poles is double the height of the other. The angle of elevation of the top of the poles from the middle point of the line segment joining their feet are complementary to each other. The height of the poles are :

A) 10 m and 20 m

B) 20 m and 40 m

C) 20.9 m and 41.8m

D) $15\sqrt{2}$ m and $30\sqrt{2}$ m

15. An aeroplane when flying at a height of 3125m from the ground passes vertically below another plane at an instant when the angle of elevation of the two planes from the same point on the ground are 30° and 60° respectively. The distance between the two planes at that instant is

A) 6520 m B) 6000 m

C) 5000 m D) 6250 m

16. The shadow of the tower becomes 60 metres longer when the altitude of the sun changes from 45° to 30°. Then the height of the tower is

A) $20(\sqrt{3}+1)$ m

B) $24(\sqrt{3}+1)$ m

C) $30(\sqrt{3}+1)$ m

D) $30(\sqrt{3}-1)$ m

17. A vertical post 15 ft high is broken at a certain height and its upper part, not completely separated, meets the ground at an angle of 30°. Find the height at which the post is broken.

A) 10 ft

B) 5ft

C) $15\sqrt{3}(2-\sqrt{3})$ ft

D) $5\sqrt{3}$ ft

18. The shadow of a tower is $\sqrt{3}$ times its height. Then the angle of elevation of the top of the tower is

A) 45° B) 30°

C) 60° D) 90°

19. A man 6 ft tall casts a shadow 4 ft long, at the same time when a flag pole casts a shadow 50 ft long. The height of the flag pole is

A) 80 ft B) 75 ft

C) 60 ft D) 70 ft

20. The angle of elevation of an aeroplane from a point on the ground is 60°. After 15 seconds flight, the elevation changes to 30°. If the aeroplane is flying at a height of $1500\sqrt{3}$ m, find the speed of the plane

A) 300 m/sec B) 200 m/sec

C) 100 m/sec D) 150 m/sec

21. The angle of elevation of the top of a tower from the point P and Q at distance of 'a' and 'b' respectively from the base of the tower and in the same straight line with it are complementary. The height of the tower is

A) $\sqrt{ab}$ B) $\frac{a}{b}$

C) ab D) a^2b^2

22. The angle of elevation of a tower from a distance 100 m from its foot is 30°. Height of the tower is :

A) $\frac{100}{\sqrt{3}}$ m B) $50\sqrt{3}$ m

C) $\frac{200}{\sqrt{3}}$ m D) $100\sqrt{3}$ m

23. A kite is flying at a height of 50 metre. If the length of string is 100 metre then the inclination of string to the horizontal ground in degree measure is

A) 90 B) 60

C) 45 D) 30

24. If the angle of elevation of a balloon from two consecutive kilometre-stones along a road are 30° and 60° respectively, then the height of the balloon above the ground will be

A) $\frac{\sqrt{3}}{2}$ km B) $\frac{1}{2}$ km

C) $\frac{2}{\sqrt{3}}$ km D) $3\sqrt{3}$ km

25. A vertical stick 12 cm long casts a shadow 8 cm long on the ground. At the same time, a tower casts a shadow 40 m long on the ground. The height of the tower is

A) 72 m B) 60 m

C) 65 m D) 70 m

26. A tower standing on a horizontal plane subtends a certain angle at a point 160 m apart from the foot of the tower. On advancing 100 m towards it, the tower is found to subtend an angle twice as before. The height of the tower is

A) 80 m B) 100 m

C) 160 m D) 200 m

27. The angle of elevation of a tower from a distance 50 m from its foot is 30°. The height of the tower is

A) $50\sqrt{3}$ m B) $\frac{50}{\sqrt{3}}$ m

C) $75\sqrt{3}$ m D) $\frac{75}{\sqrt{3}}$ m

28. The length of the shadow of a vertical tower on level ground increases by 10 metres when the altitude of the sun changes from 45° to 30°. Then the height of the tower is

A) $5\sqrt{3}$ metre
B) $10(\sqrt{3}+1)$ metre
C) $5(\sqrt{3}+1)$ metre
D) $10\sqrt{3}$ metre

29. The elevation of the top of a tower from a point on the ground is 45°. On travelling 60m from the point towards the tower, the elevation of the top becomes 60°. The height of the tower (in metres) is
A) 30
B) $30(3-\sqrt{3})$
C) $30(3+\sqrt{3})$
D) $30\sqrt{3}$

30. From two points on the ground lying on a straight line through the foot of a pillar, the two angles of elevation of the top of the pillar are complementary to each other. If the distance of the two points from the foot of the pillar are 9 metres and 16 metres and the two points lie on the same side of the pillar, then the height of the pillar is
A) 5 m B) 10 m
C) 7 m D) 12 m

31. From a point P on the ground the angle of elevation of the top of a 10 m tall building is 30°. A flag is hoisted at the top of the building and the angle of elevation of the top of the flagstaff from P is 45°. Find the length of the flagstaff.
(Take $\sqrt{3} = 1.732$)
A) $10(\sqrt{3}+2)$ m
B) $10(\sqrt{3}+1)$ m
C) $10\sqrt{3}$ m
D) 7.32 m

32. The angle of elevation of the top of a vertical tower situated perpendicularly on a plane is observed as 60° from a point P on the same plane. From another point Q, 10m vertically above the point P, the angle of depression of the foot of the tower is 30°. The height of the tower is
A) 15 m B) 30 m
C) 20 m D) 25 m

33. From a point 20 m away from the foot of a tower, the angle of elevation of the top of the tower is 30°. The height of the tower is
A) $10\sqrt{3}$ m B) $20\sqrt{3}$ m
C) $\frac{10}{\sqrt{3}}$ m D) $\frac{20}{\sqrt{3}}$ m

34. The angle of elevation of a ladder leaning against a house is 60° and the foot of the ladder is 6.5 metres from the house. The length of the ladder is
A) $\frac{13}{\sqrt{3}}$ metres
B) 13 metres
C) 15 metres
D) 3.25 metres

35. The angle of elevation of sun changes from 30° to 45°, the length of the shadow of a pole decreases by 4 metres, the height of the pole is
(Assume $\sqrt{3} = 1.732$)
A) 1.464 m B) 9.464 m
C) 3.648 cm D) 5.464 m

36. A vertical pole and a vertical tower are standing on the same level ground. Height of the pole is 10 metres. From the top of the pole the angle of elevation of the top of the tower and angle of depression of the foot of the tower are 60° and 30° respectively. The height of the tower is
A) 20 m B) 30 m
C) 40 m D) 50 m

37. The length of the shadow of a vertical tower on level ground increases by 10 metres when the altitude of the sun changes from 45° to 30°. Then the height of the tower is
A) $5(\sqrt{3}+1)$ metres
B) $5(\sqrt{3}-1)$ metres
C) $5\sqrt{3}$ metres
D) $\frac{5}{\sqrt{3}}$ metres

38. If a pole of 12 m height casts a shadow of $4\sqrt{3}$ m long on the ground, then the sun's angle of elevation at that instant is
A) 30° B) 60°
C) 45° D) 90°

39. The angle of elevation of the top of a tower from a point on the ground is 30° and moving 70 metres towards the tower it becomes 60°. The height of the tower is
A) 10 metre
B) $\frac{10}{\sqrt{3}}$ metre
C) $10\sqrt{3}$ metre
D) $35\sqrt{3}$ metre

40. The shadow of a tower standing on a level plane is found to be 30 metre longer when the Sun's altitude changes from 60° to 45°. The height of the tower is
A) $15(3+\sqrt{3})$ metre
B) $15(\sqrt{3}+1)$ metre
C) $15(\sqrt{3}-1)$ metre
D) $15(3-\sqrt{3})$ metre

41. The angle of elevation of the top of a tower of height $100\sqrt{3}$ metre from a point at a distance of 100 metre from the foot of the tower on a horizontal plane is
A) 45° B) 60°
C) 30° D) $22\frac{1}{2}°$

42. The shadow of a tower standing on a level plane is found to be 40m longer when the sun's altitude is 45°, than when it is 60° The height of the tower is
A) $30(3+\sqrt{3})$ metre
B) $40(3+\sqrt{3})$ metre
C) $20(3+\sqrt{3})$ metre
D) $10(3+\sqrt{3})$ metre

43. From two points on the ground and lying on a straight line through the foot of a pillar, the two angles of elevation of the top of the pillar are complementary to each other. If the distances of the two points from the foot of the pillar are 12 metres and 27 metres and the two points lie on the same side of the pillar, then the height (in metres) of the pillar is
A) 12 B) 18
C) 15 D) 16

44. If the height of a pole is $2\sqrt{3}$ metre and the length of its shadow is 2 metre, then the angle of elevation of the sun is
A) 90° B) 45°
C) 30° D) 60°

45. A 10 metre long ladder is placed against a wall. It is inclined at an angle of 30° to the ground. The distance (in m) of the foot of the ladder from the wall is (Given $\sqrt{3} = 1.732$)
A) 8.16 B) 7.32
C) 8.26 D) 8.66

46. The angle of elevation of a tower from a distance of 100 metre from its foot is 30°. Then the height of the tower is
A) $50\sqrt{3}$ metre
B) $100\sqrt{3}$ metre
C) $\frac{50}{\sqrt{3}}$ metre
D) $\frac{100}{\sqrt{3}}$ metre

47. A kite is flying at the height of 75 m – from the ground. The string makes an angle θ(where $\cot\theta = \frac{8}{15}$) with the level ground. Assuming that there is no slack in the string the length of the string
A) 85 metre B) 65 metre
C) 75 metre D) 40 metre

48. Two towers AB and CD have lengths 45m and 15m respectively. The angle of elevation from the bottom of the tower B to the top of the tower A is 60°. If the angle of elevation from the bottom of tower A to the top of the tower B is θ then value of sin θ is:
A) $\frac{1}{\sqrt{2}}$ B) $\frac{1}{2}$
C) $\frac{\sqrt{3}}{2}$ D) $\frac{2}{\sqrt{3}}$

49. If a 48 m tall building has a shadow of $48\sqrt{3}$ m., then the angle of elevation of the sun is
A) 15° B) 60°
C) 45° D)30°

50. A telegraph post is bent at a point above the ground due to storm. Its top just touches the ground at a distance of $10\sqrt{3}$ metre from its foot and makes an angle of 30° with the horizontal. Then height (in metres) of the telegraph post is
A) 30 B) 24
C) 20 D) 25

51. TF is a tower with F on the ground. The angle of elevation of T from A is x° such that tan $x^o = \frac{2}{5}$ andAF = 200m. The angle of elevation of T from a nearer point B is y° with BF = 80m. The value of y° is
A) 60° B) 30°
C) 75° D) 45°

52. If the angle of elevation of the sun changes from 45° to 60°, then the length of the shadow of a pillar decreases by 10 m. The height of the pillar is :
A) $5(3-\sqrt{5})$ metre
B) $5(\sqrt{3}+1)$ metre
C) $15(\sqrt{3}+1)$ metre
D) $5(3+\sqrt{3})$ metre

53. The ratio of the length of a rod and its shadow is $1:\sqrt{3}$. The angle of elevation of the sun is :
A) 90° B) 30°
C) 45° D) 60°

TYPE – IV

1. There are two vertical posts, one on each side of a road, just opposite to each other. One post is 108 metre high. From the top of this post, the angle of depression of the top and foot of the other post are 30° and 60° respectively. The height of the other post (in metre) is
A) 36 B) 72
C) 108 D) 110

2. There are two temples, one on each bank of a river, just opposite to each other. One temple is 54m high. From the top of this temple, the angles of depression of the top and the foot of the other temple are 30° and 60° respectively. The length of the temple is :
A) 18 m B) 36 m
C) $36\sqrt{3}$ m D) $18\sqrt{3}$ m

3. The top of two poles of height 24 m and 36 m are connected by a wire. If the wire makes an angle of 60° with the horizontal, then the length of the wire is
A) 6 m B) $8\sqrt{3}$ m
C) 8 m D) $6\sqrt{3}$ m

4. From the top of a hill 200 m high, the angle of depression of the top and the bottom of a tower are observed to be 30° and 60°. The height of the tower is (in m) :
A) $\frac{400\sqrt{3}}{3}$ B) $166\frac{2}{3}$
C) $133\frac{1}{3}$ D) $200\sqrt{3}$

5. From a tower 125 metres high, the angle of depression of two objects, which are in horizontal line through the base of the tower, are 45° and 30° and they are on the same side of the tower. The distance (in metres) between the objects is
A) $125\sqrt{3}$
B) $125(\sqrt{3}-1)$
C) $125/(\sqrt{3}-1)$
D) $125(\sqrt{3}+1)$

6. From the top of a tower of height 180 m the angles of depression of two objects on either sides of the tower are 30° and 45°. Then the distance between the objects are
A) $180(3+\sqrt{3})$ m
B) $180(3-\sqrt{3})$ m
C) $180(\sqrt{3}-1)$ m
D) $180(\sqrt{3}+1)$ m

7. From the peak of a hill which is 300 m high, the angle of depression of two sides of a bridge lying on a ground are 45° and 30° (both ends of the bridge are on the same side of the hill). Then the length of the bridge is
A) $300(\sqrt{3}-1)$ m
B) $300(\sqrt{3}+1)$ m
C) $300\sqrt{3}$ m
D) $\frac{300}{\sqrt{3}}$ m

8. From an aeroplane just over a river, the angle of depression of two palm trees on the opposite bank of the river are found to be 60° and 30° respectively. If the breadth of the river is 400 metres, then the

height of the aeroplane above the river at that instant is (Assume $\sqrt{3}$ = 1.732)

A) 173.2 metres
B) 346.4 metres
C) 519.6 metres
D) 692.8 metres

9. From the top of a light-house at a height 20 metres above sea-level, the angle of depression of a ship is 30°. The distance of the ship from the foot of the light house is

A) 20 m B) $20\sqrt{3}$ m
C) 30 m D) $30\sqrt{3}$ m

10. From an aeroplane just over a straight road, the angles of depression of two consecutive kilo-metre stones situated at opposite sides of the aeroplane were found to be 60° and 30° respectively. The height (in km) of the aeroplane from the road at that instant, is

A) $\frac{\sqrt{3}}{2}$ B) $\frac{\sqrt{3}}{3}$
C) $\frac{\sqrt{3}}{4}$ D) $\sqrt{3}$

11. The angle of depression of a point situated at a distance of 70 m from the base of a tower is 60°. The height of the tower is :

A) $35\sqrt{3}$ m B) $70\sqrt{3}$ m
C) $\frac{70\sqrt{3}}{3}$ m D) 70 m

TYPE – V

1. A pole stands vertically, inside a scalene triangular park ABC. If the angle of elevation of the top of the pole from each corner of the park is same, then in ΔABC, the foot of the pole is at the

A) centroid
B)circumcentre
C) incentre
D) orthocentre

2. The base of a triangle is $120\sqrt{3}$ cm and two angles at the base are 30° and 60° respectively. The altitude of the triangle is

A) 120 cm B) 60 cm
C) $100\sqrt{3}$ cm D) 90 cm

3. The two banks of a canal are straight and parallel. A, B, C are three persons of whom A stands on one bank and B and C on the opposite banks. B finds the angle ABC is 30°, while C finds that the angle ACB 60°. If B and C are 100 metres apart, the breadth of the canal is

A) $\frac{25}{\sqrt{3}}$ metres
B) $20\sqrt{3}$ metres
C) $25\sqrt{3}$ metres
D) $\frac{20}{\sqrt{3}}$ metres

4. A person of height 6ft. wants to pluck a fruit which is on a $\frac{26}{3}$ ft. hight tree. If the person is standing $\frac{8}{\sqrt{3}}$ ft. away from the base of the tree, then at what angle should he throw a stone so that it hits the fruit?

A) 75° B) 30°
C) 45° D) 60°

2016 – 2017 QUESTIONS

1. The circular measure of the included angle formed by the hour hand and minute hand of a clock at 3 p.m. will be

A) $\frac{\pi}{4}$ B) $\frac{\pi}{3}$
C) $\frac{5\pi}{12}$ D) $\frac{\pi}{2}$

2. Which of the following relations is correct for $0 < \theta < 90°$?

A) $\sin\theta = \sin^2\theta$
B) $\sin\theta < \sin^2\theta$
C) $\sin\theta > \sin^2\theta$
D) $\sin\theta = \operatorname{cosec}\theta$

3. If θ is an acute angle and $\sin(\theta + 18°) = \frac{1}{2}$, then the value of θ in circular measure is :

A) $\frac{\pi}{12}$ radians
B) $\frac{\pi}{15}$ radians
C) $\frac{2\pi}{5}$ radians
D) $\frac{3\pi}{13}$ radians

4. What is the measure of central angle of the arc whose length is 11 cm and radius of the circle is 14 cm?

A) 45° B) 60°
C) 75° D) 90°

5. If $(a^2 - b^2)\sin\theta + 2ab\cos\theta = a^2 + b^2$, then $\tan\theta$ = ?

A) $\frac{2ab}{a^2 - b^2}$ B) $\frac{a^2 - b^2}{2ab}$
C) $\frac{ab}{a^2 - b^2}$ D) $\frac{a^2 - b^2}{ab}$

6. If $2y\cos\theta = x\sin\theta$ and $2x\sec\theta - y\operatorname{cosec}\theta = 3$, then the value of $(x^2 + 4y^2)$ is

A) 1 B) 2
C) 3 D) 4

7. If $\tan\theta_1 = 1$, $\sin\theta_2 \frac{1}{\sqrt{2}}$, then the value of $\sin(\theta_1 + \theta_2)$ equal to A) −1 B) 0
C) 1 D) $\frac{1}{2}$

8. Find the value of $\tan\theta(1 + \sec 2\theta)(1 + \sec 4\theta)(1 + \sec 8\theta)$.

A) $\tan 10\theta$ B) $\tan 8\theta$
C) $\tan 12\theta$ D) 1

9. The value of $\frac{\sin 65°}{\cos 25°}$ is

A) 0 B) 1
C) 2 D)Not defined

10. The value of $(\sec^2 45° - \cot^2 45°) - (\sin^2 30° + \sin^2 60°)$ is

A) 1 B) $2\sqrt{3}$
C) 0 D) $\frac{1}{\sqrt{2}}$

11. The value of the following Is :

$$\frac{\sin\theta\ \operatorname{cosec}\theta\ \tan\theta\ \cot\theta}{\sin^2\theta + \cos^2\theta}$$

A) 1 B) $\tan\theta$
C) 0 D) 2

12. $\angle Y$ is the right angle of the trianlge XYZ. If $XY = 2\sqrt{6}$ cm and XZ-YZ = 2cm, then the value of (secX + tanX) is :

A) $\frac{1}{\sqrt{6}}$ B) $\frac{1}{2\sqrt{3}}$
C) $2\sqrt{6}$ D) $\sqrt{6}$

13. Find the value of 8 cos 10° cos 20° cos 40°.

A) tan80°
B) cot10°
C) tan80° or cot 10°
D) None of these

14. If $\pi\sin\theta = 1$, $\pi\cos\theta = 1$, then the value of $\left\{\sqrt{3}\tan\left(\frac{2}{3}\theta\right) + 1\right\}$ is

A) 1 B) $\sqrt{3}$

C) 2 D) $\frac{1}{\sqrt{3}}$

15. If in a triangle ABC, sin A = cosB, then the value of cos C is

A) $\frac{\sqrt{3}}{2}$ B) 0

C) 1 D) $\frac{1}{\sqrt{2}}$

16. If $x^2 = \sin^2 30° + \cot^2 45° - \sec^2 60°$, then the value of $x (x > 0)$ is

A) $-\frac{1}{2}$ B) 1

C) 0 D) $\frac{1}{2}$

17. $7\sin^2\theta + 3\cos^2\theta = 4$ then the value of $\sec\theta + \operatorname{cosec}\theta$ is

A) $\frac{2}{\sqrt{3}} - 2$

B) $\frac{2}{\sqrt{3}} + 2$

C) $\frac{2}{\sqrt{3}}$

D) None of these

18. The value of tan80° tan 10° + $\sin^2 70° + \sin^2 20°$ is

A) 0 B) 1

C) 2 D) $\frac{\sqrt{3}}{2}$

19. Find the value of $\left(\frac{\sin 27°}{\cos 63°}\right)^2 + \left(\frac{\cos 63°}{\sin 27°}\right)^2$

A) 0 B) 2

C) 3 D) 1

20. If $\sin\theta + \cos\theta = 1$. then $\sin\theta.\cos\theta$, is equal to

A) 0 B) 1

C) $\frac{1}{2}$ D) $-\frac{1}{2}$

21. If sinC - sinD = x, then the value of xis

A) $2\sin\left[\frac{C+D}{2}\right]\cos\left[\frac{(C-D)}{2}\right]$

B) $2\cos\left[\frac{C+D}{2}\right]\cos\left[\frac{(C-D)}{2}\right]$

C) $2\cos\left[\frac{C+D}{2}\right]\sin\left[\frac{(C-D)}{2}\right]$

D) $2\sin\left[\frac{C+D}{2}\right]\sin\left[\frac{(D-C)}{2}\right]$

22. If $x = a\cos\theta\cos\theta$, $y = a\cos\theta\sin\theta$ and $z = a\sin\theta$, then the value of $(x^2 + y^2 + z^2)$ is

A) $2a^2$ B) $4a^2$

C) $9a^2$ D) a^2

23. If tan θ = tan 30°. tan 60° and θ is an acute angle, then 2θ is equal to

A) 30° B) 45°

C) 90° D) 0°

24. The value of $(1 + \tan^2\theta)(1 - \sin^2\theta)$ is

A) 2 B) 1

C) −1 D) −2

25. If $\sin\theta = \frac{\sqrt{3}}{2}$ and $0° < \theta < 90°$, then the value of tan (θ − 15°) is

A) 1 B) $\sqrt{3}$

C) $\frac{1}{\sqrt{3}}$ D) $\sqrt{2}$

26. If $r\sin\theta = \sqrt{3}$ and $r\cos\theta = 1$, when values of r and θ are : $(0° \le 9 \le 90°)$

A) r= 1, θ = 30°

B) $r = \frac{1}{2}$, θ = 30°

C) $r = \sqrt{3}$, θ = 30°

D) r = 2, θ = 60°

27. x, y be two acute angles, x + y < 90° and sin(2x − 20°) = cos (2y + 20°), the value of tan (x + y) is

A) $\sqrt{3}$ B) $\frac{1}{\sqrt{3}}$

C) 1 D) $2 + \sqrt{2}$

28. (1 + sec 20° + cot 70°)(1 - cosec 20° + tan70°) is equal to

A) 0 B) 1

C) 2 D) 3

29. The value of $\frac{\sin\theta}{1+\cos\theta} + \frac{\sin\theta}{1-\cos\theta}$ is :

A) 2sinθ B) 2cosθ

C) 2secθ D) 2cosecθ

30. The value of $\cos^2 20° + \cos^2 70°$ is:

A) $\sqrt{2}$ B) 2

C) $\frac{1}{\sqrt{2}}$ D) 1

31. If sinθ+ cosecθ = 2, then the value of $(\sin^{-7}\theta + \operatorname{cosec}^7\theta)$ is

A) 2^7 B) 2^{-7}

C) 2 D) 2^{-1}

32. The value of $\frac{\sin^2 63° + \sin^2 27°}{\cos^2 17° + \cos^2 73°}$ is :

A) 0 B) 1

C) 2 D) −1

33. The value of $\cos^2 20° + \cos^2 70°$ is:

A) 0 B) 1

C) $\frac{1}{2}$ D) $\frac{1}{\sqrt{3}}$

34. If a.sin 45°. cos 45°. tan60° = $\tan^2 45°$ − cos 60°, then find the value of a.

A) $\frac{1}{\sqrt{3}}$ B) $\sqrt{3}$

C) 1 D) $\frac{\sqrt{3}}{2}$

35. The value of tan 315° cot (−405°) is equal to

A) −1 B) 1

C) 0 D) 2

36. If sinθ + cosecθ = 2, the value of $\sin^n\theta + \operatorname{cosec}^n\theta$ is :

A) 2^n B) $2^{\frac{1}{n}}$

C) 2 D) 0

37. If sin A + $\sin^2 A$ =1 then what is the value of $\cos^2 A + \cos^4 A$?

A) $\frac{1}{2}$ B) 1

C) 2 D) 3

38. If θ is positive acute angle and $4\sin^2\theta = 3$, then the value of $\left(\tan\theta - \cot\frac{\theta}{2}\right)$ is :

A) 1 B) 0

C) $\sqrt{3}$ D) $\frac{1}{\sqrt{3}}$

39. θ is a positive acute angle and sinθ − cosθ = 0, then the value of (secθ + cosecθ) is :

A) 2 B) $\sqrt{2}$

C) $2\sqrt{2}$ D) $3\sqrt{2}$

40. If $\alpha + \beta = 90°$ and $\alpha : \beta = 2 : 1$, then the ratio of cos α to cosβ is :

A) $1 : \sqrt{3}$ B) 1 : 3

C) $1 : \sqrt{2}$ D) 1 : 2

41. If sec (4x − 50°) = cosec (50° − x), then the value of x is

A) 45° B) 90°

C) 30° D) 60°

42. The value of (cos 53° − sin 37°) is

A) 0 B) 1

C) 2 sin 37° D) 2 cos 53°

43. If $7\sin^2\theta + 3\cos^2\theta = 4$, and $0° < \theta < 90°$, then the value of tan θ is :

A) $\frac{1}{\sqrt{2}}$ B) $\frac{1}{\sqrt{3}}$

C) $\sqrt{\frac{3}{2}}$ D) 1

44. If tan (A – B) = x, then the value of x is

A) $\frac{(\tan A + \tan B)}{(1 - \tan A \tan B)}$

B) $\frac{(\tan A + \tan B)}{(1 + \tan A \tan B)}$

C) $\frac{(\tan A - \tan B)}{(1 - \tan A \tan B)}$

D) $\frac{(\tan A - \tan B)}{(1 + \tan A \tan B)}$

45. What is the value of $\sin\left(\frac{11\pi}{6}\right)$?

A) $\frac{2}{\sqrt{3}}$ B) $\frac{-2}{\sqrt{3}}$

C) $\frac{-1}{2}$ D) $\frac{1}{2}$

46. If sin P + cosec P = 2, then the value of $\sin^7P + \text{cosec}^7P$ is

A) 1 B) 2

C) 3 D) 0

47. The value of the expression $2(\sin^6\theta + \cos^6\theta) - 3(\sin^4\theta + \cos^4\theta) + 1$ is

A) –1 B) 0

C) 1 D) 2

48. If x = cosecθ – sinθ and y = secθ – cosθ, then the relation between x and y is

A) $x^2 + y^2 + 3 = 1$

B) $x^2y^2(x^2 + y^2 + 3) = 1$

C) $x^2(x^2 + y^2 - 5) = 1$

D) $y^2(x^2 + y^2 - 5) = 1$

49. A tower is 50 metre high. Its shadow is x metres shorter when the sun's altitude is 45° than when it is 30°. The value of x in metre is :

A) $50\sqrt{3}$ B) $50(\sqrt{3}-1)$

C) $50(\sqrt{3}+1)$ D) 50

50. A vertical tower stands on a horizontal plane and is surmounted by a vertical flag staff of height h. At a point on the plane, the angle of elevation of the bottom of the flag staff is α and that of the top of the flag staff is β Then the height of the tower is

A) h tanθ

B) $\frac{h\tan\alpha}{\tan\beta - \tan\alpha}$

C) $\frac{h\tan\alpha}{\tan\alpha - \tan\beta}$

C) None of these

51. The angle of elevation of a ladder leaning against a wall is 60° and the foot of the ladder is 4.6 metre away from the wall. The length of the ladder is

A) 2.3 metre B) 4.6 metre

C) 9.2 metre D) 7.8 metre

52. The angle of elevation of the sun when the length of the shadow of a pole is equal to its height is :

A) 30° B) 45°

C) 60° D) 90°

53. Two ships are sailing in the sea on the two sides of a light house. The angles of elevation of the top of the light house as observed from the two ships are 30° and 45° respectively. If the light house is 100m high, the distance be-tween the two ships is :

(take $\sqrt{3} = 1.73$)

A) 173 metre B) 200 metre

C) 273 metre D) 300 metre

54. The respective ratio between the height of tower and the point at some distance from its foot is $5\sqrt{3}:5$. What will be the angle of elevation of the top of the tower?

A) 30° B) 60°

C) 90° D) 45°

55. The thread of a kite makes angle 60° with the horizontal plane. If the length of the thread be 80 m, then the vertical height of the kite will be

A) $50\sqrt{3}$ metre

B) $45\sqrt{3}$ metre

C) $43\sqrt{3}$ metre

D) $40\sqrt{3}$ metre

56. A 1.6 m tall observer is 45 metres away from a tower. The angle of elevation from his eye to the top of the tower is 30°, then the height of the tower in metres is (Take $\sqrt{3} = 1.73$)

A) 25.98 B) 26.58

C) 27.58 D) 27.98

57. If the length of the shadow of a vertical pole be $\sqrt{3}$ times the height of the pole, the angle of elevation of the sun is :

A) 60° B) 45°

C) 30° D) 90°

58. If the angle of elevation of a cloud from a point 200m above a lake is 30° and the angle of depression of its reflection in the lake is 60°. Then the height of the cloud above the lake is:

A) 100 m B) 200 m

C) 300 m D) 400 m

59. If the angle of elevation of the top of a pillar from the ground level is raised from 30° to 60°, the length of the shadow of a pillar of height $50\sqrt{3}$ will be decreased by

A) 60 metre B) 75 metre

C) 100 metre D) 50 metre

60. Two persons are on either side of a temple, 75 m high, observe the angle of elevation of the top of the temple to be 30° and 60° respectively. The distance between the persons is:

A) 173.2 metre

B) 100 metre

C) 157.7 metre

D) 273.2 metre

61. The length of shadow of a tower is $\sqrt{3}$ times that of its length. The angle of elevation of the sun is :

A) 45° B) 30°

C) 60° D) None

62. The upper part of a tree broken at a certain height makes an angle of 60° with the ground at a distance of 10 metre from its foot. The original height of the tree was

A) $20\sqrt{3}$ metre

B) $10\sqrt{3}$ metre

C) $10(2+\sqrt{3})$ metre

D) $10(2-\sqrt{3})$ metre

63. The angles of elevation of the top of a tower from two points at a distance of 4 m and 9 m from the base of the tower and in the same straight line with it are complementary. The height of the tower is :
A) 4 metre B) 7 metre
C) 9 metre D) 6 metre

64. Two men are on opposite sides of a tower. They measure the angles of elevation of the top of the tower as 30° and 45° respectively. If the height of the tower is 50 metre, the distance be-tween the two men is (Take $\sqrt{3} = 1.73$)
A) 136.5 metre
B) $50\sqrt{3}$ metre
C) $100\sqrt{3}$ metre
D) 135.5 metre

65. A hydrogen filled balloon ascending at the rate of 18 kmph was drifted by wind. Its angle of elevation at 10th and 15th minutes were found to be 60° and 45° respectively. The wind speed (in whole numbers) during the last five minutes, approximately, is equal to
A) 7km./hr. B) 11 km./hr.
C) 26km./hr. D) 33 km./hr.

66. A pilot in an aeroplane at an altitude of 200 metre observes two points lying on either side of a river. If the angles of depression of the two points be 45° and 60°, then the width of the river is
A) $\left(200+\frac{200}{\sqrt{3}}\right)$ metre
B) $\left(200-\frac{200}{\sqrt{3}}\right)$ metre
C) $400\sqrt{3}$ metre
D) $\left(\frac{400}{\sqrt{3}}\right)$ metre

67. A man on the top of a tower, standing on the sea shore, finds that a boat coming towards him takes 10 minutes for the angle of depression to change from 30° to 60°. How soon the boat reach the sea-shore ?
A) 5 minutes B) 7 minutes
C) 10 minutes D) 15 minutes

68. The cliff of a mountain is 180 m high and the angles of depression of two ships on the either side of cliff are 30° and 60°. What is the distance between the two ships?
A) 400 metre
B) $400\sqrt{3}$ metre
C) 415.68 metre
D) 398.6 metre

69. A helicopter, at an altitude of 1500 metre, finds that two ships are sailing towards it, in the same direction. The angles of depression of the ships as observed from the helicopter are 60° and 30° respectively. Distance between the two ships, in metre is
A) $1000\sqrt{3}$ B) $\frac{1000}{\sqrt{3}}$
C) $500\sqrt{3}$ D) $\frac{500}{\sqrt{3}}$

70. From the top of a building 60 metre high, the angles of depression of the top and bottom of a tower are observed to be 30° and 60° respectively. The height of the tower in metre is :
A) 40 B) 45
C) 50 D) 55

71. A boat is moving away from an observation tower. It makes an angle of depression of 60° with an observer's eye when at a distance of 50 metre from the tower. After 8 seconds, the angle of depression becomes 30°. By assuming that it is running in still water, the approximate speed of the boat is :
A) 33km/hr B) 42km/hr
C) 45km/hr D) 50km/hr

72. The angles of depression of two ships from the top of a light house are 60° and 45° towards east. If the ships are 300 metre apart, the height of the light house is
A) $200\left(3+\sqrt{3}\right)$ metre
B) $250\left(3+\sqrt{3}\right)$ metre
C) $150\left(3+\sqrt{3}\right)$ metre
D) $160\left(3+\sqrt{3}\right)$ metre

SHORT ANSWERS

Type – I

1.(A)	2.(B)	3.(C)	4.(B)
5.(C)	6.(C)	7.(B)	8.(A)

Type – II

1.(C)	2.(C)	3.(C)	4.(B)
5.(A)	6.(A)	7.(B)	8.(C)
9.(D)	10.(B)	11.(A)	12.(A)
13.(A)	14.(D)	15.(D)	16.(B)
17.(D)	18.(D)	19.(C)	20.(C)
21.(B)	22.(B)	23.(B)	24.(D)
25.(A)	26.(B)	27.(D)	28.(B)
29.(D)	30.(A)	31.(C)	32.(C)
33.(B)	34.(A)	35.(B)	36.(A)
37.(C)	38.(B)	39.(B)	40.(B)
41.(C)	42.(D)	43.(D)	44.(C)
45.(B)	46.(C)	47.(A)	48.(C)
49.(B)	50.(C)	51.(D)	52.(A)
53.(C)	54.(B)	55.(C)	56.(B)
57.(A)	58.(D)	59.(D)	60.(B)
61.(B)	62.(A)	63.(B)	64.(C)
65.(B)	66.(C)	67.(A)	68.(A)
69.(B)	70.(C)	71.(C)	72.(B)
73.(A)	74.(C)	75.(C)	76.(A)
77.(D)	78.(B)	79.(B)	80.(D)
81.(C)	82.(B)	83.(C)	84.(B)
85.(D)	86.(A)	87.(D)	88.(A)
89.(C)	90.(C)	91.(C)	92.(D)
93.(C)	94.(C)	95.(A)	96.(A)
97.(A)	98.(D)	99.(B)	100.(A)
101.(B)	102.(A)	103.(C)	104.(A)
105.(A)	106.(C)	107.(A)	108.(D)
109.(A)	110.(D)	111.(C)	112.(A)
113.(A)	114.(C)	115.(B)	116.(D)
117.(C)	118.(B)	119.(C)	120.(A)
121.(D)	122.(B)	123.(C)	124.(D)
125.(A)	126.(D)	127.(A)	128.(D)
129.(D)	130.(B)	131.(B)	132.(B)
133.(A)	134.(B)	135.(B)	136.(B)
137.(B)	138.(D)	139.(C)	140.(C)
141.(C)	142.(D)	143.(B)	144.(B)
145.(A)	146.(D)	147.(D)	148.(D)
149.(B)	150.(A)	151.(C)	152.(C)

153.(C)	154.(A)	155.(A)	156.(C)
157.(A)	158.(B)	159.(B)	160.(B)
161.(A)	162.(C)	163.(A)	164.(B)
165.(C)	166.(C)	167.(D)	168.(B)
169.(B)	170.(B)	171.(C)	172.(B)
173.(C)	174.(B)	175.(C)	176.(A)
177.(A)	178.(C)	179.(D)	180.(C)
181.(D)	182.(B)	183.(D)	184.(D)
185.(A)	186.(B)	187.(A)	188.(A)
189.(A)	190.(A)	191.(C)	192.(A)
193.(C)	194.(C)	195.(D)	196.(B)
197.(C)	198.(C)	199.(C)	200.(D)
201.(D)	202.(B)	203.(A)	204.(C)
205.(B)	206.(C)	207.(A)	208.(D)
209.(C)	210.(D)	211.(C)	212.(C)
213.(A)	214.(A)	215.(A)	216.(C)
217.(B)	218.(B)	219.(C)	220.(D)
221.(C)	222.(C)	223.(A)	224.(C)
225.(B)	226.(C)	227.(A)	228.(C)
229.(B)	230.(D)	231.(C)	232.(B)
233.(C)	234.(A)	235.(D)	236.(D)
237.(B)	238.(A)	239.(B)	240.(B)
241.(B)	242.(B)	243.(B)	244.(A)
245.(C)	246.(C)	247.(B)	248.(A)
249.(A)	250.(C)	251.(D)	252.(D)
253.(D)	254.(A)	255.(D)	256.(B)
257.(B)	258.(B)	259.(D)	260.(D)
261.(C)	262.(A)	263(A)	264.(D)
265.(A)	266.(D)	267.(A)	268.(A)
269.(D)	270.(D)	271.(D)	272.(C)
273.(B)	274.(B)	275.(B)	276.(C)
277.(D)	278.(B)	279.(B)	280.(B)
281.(D)	282.(A)	283.(C)	284.(A)
285.(A)	286.(B)	287.(C)	288.(D)
289.(D)	290.(C)	291.(B)	292.(B)
293.(A)			

Type – III

1.(D)	2.(B)	3.(B)	4.(C)
5.(C)	6.(C)	7.(C)	8.(A)
9.(C)	10.(B)	11.(A)	12.(C)
13.(D)	14.(D)	15.(D)	16.(C)
17.(B)	18.(B)	19.(B)	20.(B)
21.(A)	22.(A)	23.(D)	24.(A)
25.(B)	26.(A)	27.(B)	28.(C)
29.(C)	30.(D)	31.(D)	32.(B)
33.(D)	34.(B)	35.(D)	36.(C)
37.(A)	38.(B)	39.(D)	40.(A)
41.(B)	42.(C)	43.(B)	44.(D)
45.(D)	46.(D)	47.(A)	48.(B)
49.(D)	50.(A)	51.(D)	52.(D)
53.(B)			

Type – IV

1.(B)	2.(B)	3.(B)	4.(C)
5.(B)	6.(D)	7.(A)	8.(A)
9.(B)	10.(C)	11.(B)	

Type – V

1.(B)	2.(D)	3.(C)	4.(B)

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1.(D)	2.(C)	3.(B)	4.(A)
5.(B)	6.(D)	7.(C)	8.(B)
9.(B)	10.(C)	11.(A)	12.(D)
13.(C)	14.(C)	15.(B)	16.(D)
17.(B)	18.(C)	19.(B)	20.(A)
21.(C)	22.(D)	23.(C)	24.(B)
25.(A)	26.(D)	27.(C)	28.(C)
29.(D)	30.(D)	31.(*)	32.(B)
33.(B)	34.(A)	35.(B)	36.(C)
37.(B)	38.(B)	39.(C)	40.(A)
41.(C)	42.(A)	43.(B)	44.(D)
45.(C)	46.(B)	47.(B)	48.(B)
49.(B)	50.(B)	51.(C)	52.(B)
53.(C)	54.(B)	55.(D)	56.(C)
57.(C)	58.(D)	59.(C)	60.(A)
61.(B)	62.(C)	63.(D)	64.(A)
65.(D)	66.(A)	67.(A)	68.(C)
69.(A)	70.(A)	71.(C)	72.(C)

Solutions

TYPE – I

1. (A) $11^\circ 15'$. In circular measure we know

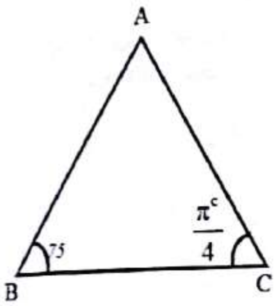
$180^\circ = \pi^c$

$11^\circ 15' = 11^\circ + \frac{15^\circ}{60}$

$11 + \frac{1}{4} = \frac{45^\circ}{4}$

$\frac{\pi \times \frac{45}{4}}{180} = \frac{\pi}{180} \times \frac{45}{4} = \frac{\pi^c}{16}$

2. (B)



Given

$\angle ABC = 75$

$\angle ACB = \frac{\pi^c}{4}$

$\angle BAC = ?$ (In circular measure)

$\angle ABC = 75^\circ$

$= \frac{\pi \times 75}{180} = \frac{5\pi^c}{12}$

$A + B + C = \pi^c$

$= \pi - \frac{5\pi}{12} - \frac{\pi}{4} = \frac{4\pi}{12}$

$A = \frac{\pi}{3}$

3. (C) Sum of remaining two angles

$= \pi - \frac{5\pi}{9} = \frac{4\pi}{9}$

Each angle $= \frac{1}{2} \times \frac{4\pi}{9} = \frac{2\pi}{9}$

4. (B) $\because \pi^c = 180^\circ$

1 radian $= \frac{180^\circ}{\pi}$

$= \frac{180 \times 7}{22} = \frac{630}{11} = 57\frac{3^\circ}{11}$

$= 57^\circ \frac{3}{11} \times 60' = 57^\circ \frac{180'}{11}$

$= 57^\circ 16' \frac{4}{11} \times 60'' = 57^\circ 16' 22''$

5. (C) $\frac{3\pi}{5}$ radian $= \frac{180 \times \frac{3\pi}{5}}{\pi}$

$= 180 \times \frac{3}{5} = 108^\circ$

6. (C) Given sum of two angle A + B = 135°

Difference $A - B = \frac{\pi^c}{12}$

Find greater angle in (circular measure)

$A + B = 135^\circ = \frac{135 \times \pi}{180}$ radian

$A + B = \frac{3\pi}{4}$

$A - B = \frac{\pi}{12}$

Adding these two equations

$2A = \frac{5\pi}{6} \Rightarrow A = \frac{5\pi}{12}$

7. (B) $\sec^2\theta + \tan^2\theta = 7$

$1 + \tan^2\theta + \tan^2\theta = 7$

$2\tan^2\theta = 7 - 1 = 6$

$\tan^2\theta = 3$

$\tan\theta = \sqrt{3}$

$\theta = 60^\circ$

$= 60 \times \frac{\pi}{180} = \frac{\pi^c}{3}$

8. (A) $A + B = \frac{22^c}{9} = \frac{22}{9} \times \frac{180}{\pi}$

$= \frac{22}{9} \times \frac{180 \times 7}{22} = 140$

A + B = 140

A − B = 36 on adding 2A = 176

A = 88 B = 52

TYPE – II

1. (C) Min value of

$2\sin^2\theta + 3\cos^2\theta$

$= 2\sin^2\theta + 2\cos^2\theta + \cos^2\theta$

$= 2(\sin^2\theta + \cos^2\theta) + \cos^2\theta$

$= 2 + \cos^2\theta \left[\because \sin^2\theta + \cos^2\theta = 1\right]$

$\because$ Minimum value of $\cos\theta = -1$

But $\cos^2\theta \geq 0$, when $\theta = 90$

$\left[\because \cos 0^\circ = 1, \cos 90^\circ = 0\right]$

$\therefore$ Required min value = 2 + 0 = 2

Shortcut : $a\sin^2\theta + b\cos^2\theta$

if a > b	if a < b
max = a	max = b
min = b	min = a

2. (C) Given cosec 39° = x value of

$\frac{1}{\text{cosec}^2 51} + \sin^2 39 + \tan^2 51$

$- \frac{1}{\sin^2 51 . \sec^2 39}$

$= \sin^2 51 + \sin^2 39 + \cot^2 39$

$- \frac{1}{\cos^2 39 . \sec^2 39}$

$= 1 + \cot^2 39 - 1 = \cot^2 39$

$= \text{cosec}^2 39 - 1 = x^2 - 1$

(If A + B = 90

$\sin^2 A + \sin^2 B = 1$

$\cos^2 A + \cos^2 B = 1$

$\sin^2 A = \cos^2 B$

$\tan^2 A = \cot^2 B$)

3. (C) tan 4° . tan43° . tan47° . tan86°

$\boxed{\text{If } A + B = 90 \Rightarrow \tan A . \tan B = 1}$

tan4 . tan86 . tan43 . tan47

1 × 1 = 1

4. (B) If $\frac{\tan\theta + \cot\theta}{\tan\theta - \cot\theta} = 2$ find $\sin\theta$

By componendo & dividendo

$\frac{2\tan\theta}{2\cot\theta} = \frac{3}{1}$

$\tan^2\theta = 3$

$\tan\theta = \sqrt{3}$

$\sin\theta = \frac{\sqrt{3}}{2}$

5. (A) cosx + cosy = 2

sinx + siny =

max . (cosx) = 1 When x = 0

cosx + cosy = 2 is possible only when both cosx = cosy = 1

i.e., x, y must be '0'

x = y = 0

sinx + siny = 0

6. (A) tan 1 tan 2 tan3......... tan 89

If A + B = 90, tanA . tanB = 1

tan1 . tan2 .... tan45 .... tan88 . tan89

1 × 1 × 1 .... × 1 = 1

7. (B) Let the measure of three angles of triangle are 2x, 7x and 11x respectively.

$\therefore$ 2x + 7x + 11x = 180°

$\Rightarrow$ 20x = 180°

$\Rightarrow x = \frac{180}{20} = 9^\circ$

$\therefore$ First angle = 2x = 2 × 9 =

180°

Second angle = 7x = 7 × 9 = 63°

Third angle = 11x = 11 × 9 = 99°

8. (C) Sum of angles of a triangle = 180°

$\therefore$ x + 5 + 2x − 3 + 3x + 4 = 180°

$\Rightarrow 6x + 6 = 180°$

$\Rightarrow 6x = 180 - 6 = 174°$

$\Rightarrow x = \frac{174}{6} = 29$

9. (D) cot 10° . cot 20° cot60°cot 70. cot 80°

If A + B = 90, cotA . cotB = 1

(cot 10 . cot 80) (cot 20 . cot70) cot 60

$1 \times 1 \times \frac{1}{\sqrt{3}} = \frac{1}{\sqrt{3}}$

10. (B) $7\sin^2\theta + 3\cos^2\theta = 4$

$4\sin^2\theta + 3(\sin^2\theta + \cos^2\theta)$

$4\sin^2\theta = 1$

$\sin\theta = \frac{1}{2}$

$\Rightarrow \tan\theta = \frac{1}{\sqrt{3}}$

11. (A) $\sin^2 1° + \sin^2 5° + \sin^2 9° + \ldots + \sin^2 89°$

No.of terms

$= \frac{\text{last term - first term}}{\text{diff}} + 1$

$= \frac{89-1}{4} + 1 = 23$

Sum of terms $= \frac{23}{2} = 11\frac{1}{2}$

$\because \sin^2 A + \sin^2 B = 1$

If A + B = 90

12. (A)

$\cot 18°\left(\cot 72°.\cos^2 22° + \frac{1}{\tan 72°.\sec^2 68°}\right)$

$= \cot 18° . \cot 72° . \cos^2 22° + \frac{\cot 18°}{\tan 72°.\sec^2 68°}$

$= \cos^2 22° + \cos^2 68°$

$= \cos^2 22° + \sin^2 22° = 1$

$\left[\begin{array}{l}\because \tan(90°-\theta) = \cot\theta \\ \sin(90° - \theta) = \cos\theta \\ \sin^2\theta + \cos^2\theta = 1\end{array}\right]$

13. (A)

tan 15° . cot75° + tan75° . cot15°

= tan15° . cot(90° − 15°) + tan(90° − 15°). cot15°

= $\tan^2 15° + \cot^2 15°$ ... (i)

$\left[\begin{array}{l}\because \tan(90°-\theta) = \cot\theta \\ \cot(90° - \theta) = \tan\theta\end{array}\right]$

$\tan 15° = 2 - \sqrt{3}$

$\therefore \cot 15°$

$= \frac{1}{2-\sqrt{3}} = \frac{2+\sqrt{3}}{(2-\sqrt{3})(2+\sqrt{3})}$

$= 2 + \sqrt{3}$

$\therefore \tan^2 15° + \cot^2 15°$

$= (2-\sqrt{3})^2 + (2+\sqrt{3})^2$

$= 2(4 + 3) = 14$

14. (D) sin(2x − 20) = cos(2y + 20)

sinA = cosB if A + B = 90

2x − 20 + 2y + 20 = 90

2x + 2y = 90

x + y = 45

tan(x + y) = 1

15. (D) If ∠A & ∠B are complimentary to each other then

$\sec^2 A + \sec^2 B = \sec^2 A.\sec^2 B$

$\sec^2 A.\sec^2 B - \sec^2 A.\sec^2 B = 0$

16. (B) $\sin^2 5 + \sin^2 6 + \ldots + \sin^2 85$

no. of terms $= \frac{\text{last - first}}{\text{diff}} + 1$

$= \frac{85-5}{1} + 1 = 81$

sum of terms $= \frac{81}{2} = 40\frac{1}{2}$

17. (D) $\sin^2 5 + \sin^2 10 + \ldots + \sin^2 85 + \sin^2 90$

no.of terms $= \frac{\text{last - first}}{\text{diff}} + 1$

$= \frac{85-5}{5} + 1 = 17$

Sum of terms

$= \frac{17}{2} = 8\frac{1}{2} + \sin^2 90$

$= 8\frac{1}{2} + 1 = 9\frac{1}{2}$

18. (D) $\frac{\sin 39°}{\cos 51°} + 2\tan 11°.\tan 79°.$

tan31°.tan59°.tan45° −3(sin²21° + sin²69°)

$= \frac{\sin 39°}{\cos(90°-39°)} + 2\tan 11°.\cot 11°$

tan31°.cot31° − 3(sin²21° + cos²21°)

= 1 + 2 − 3 = 0

$[\tan\theta.\cot\theta = 1, \sin^2\theta + \cos^2\theta = 1]$

19. (C) $\frac{\cos^2\theta}{\cot^2\theta - \cos^2\theta} = 3$

$\Rightarrow \cos^2\theta = 3\cot^2\theta - 3\cos^2\theta$

$\Rightarrow 4\cos^2\theta = 3\cot^2\theta$

$\Rightarrow 4\cos^2\theta = 3\frac{\cos^2\theta}{\sin^2\theta}$

$\Rightarrow 4\cos^2\theta - \frac{3\cos^2\theta}{\sin^2\theta} = 0$

$\Rightarrow \cos^2\theta\left(4 - \frac{3}{\sin^2\theta}\right) = 0$

$\therefore 4 - \frac{3}{\sin^2\theta} = 0 \ \& \ \cos^2\theta = 0$

$\Rightarrow 4\sin^2\theta = 3$

$\Rightarrow \sin^2\theta = \frac{\sqrt{3}}{2} = \sin 60°$

$\Rightarrow \theta = 60°$

20. (C) A = tan11° . tan29°

B = 2 cot61° . cot79°

= 2 cot(90° − 29°) cot(90° − 11°)

= 2 tan29° . tan11° = 2A

$[\because \cot(90°-\theta) = \tan\theta]$

21. (B) $\sin 17° = \frac{x}{y}$

$\cos 17° = \sqrt{1 - \sin^2 17°}$

$= \sqrt{1 - \frac{x^2}{y^2}} = \sqrt{\frac{y^2 - x^2}{y^2}}$

$= \frac{\sqrt{y^2 - x^2}}{y}$

$\therefore \sec 17° = \frac{y}{\sqrt{y^2 - x^2}}$

sin 73° = sin(90° − 17°)

= cos17°

$\therefore$ sec 17° − sin73°

$= \frac{y}{\sqrt{y^2 - x^2}} - \frac{\sqrt{y^2 - x^2}}{y}$

$= \frac{y^2 - y^2 + x^2}{y\sqrt{y^2 - x^2}} = \frac{x^2}{y\sqrt{y^2 - x^2}}$

22. (B) The value of sinθ + cosθ minimum is 1

The value of sinθ + cosθ maximum is $\sqrt{2}$

23. (B) $\frac{\tan 57° + \cot 37°}{\tan 33° + \cot 53°}$

$= \frac{\cot 33° + \tan 53°}{\tan 33° + \cot 53°}$

$\left[\begin{array}{l}\because \tan(90° - \theta = \cot\theta, \\ \cot(90°-\theta) = \tan\theta\end{array}\right]$

$= \frac{\frac{1}{\tan 33°} + \tan 53°}{\tan 33° + \frac{1}{\tan 53°}}$

$= \frac{1 + \tan 53°.\tan 33°}{\tan 33°.\tan 53° + 1} \times \frac{\tan 53°}{\tan 33°}$

= tan53° . cot33°

= cot37° . tan57°

24. (D) $\cot 30^\circ = \cot(90^\circ - 60^\circ) = \tan 60^\circ$
$\cot 75^\circ = \cot(90^\circ - 15^\circ) = \tan 15^\circ$
$\therefore \dfrac{\cot 30^\circ - \cot 75^\circ}{\tan 15^\circ - \tan 60^\circ}$
$= \dfrac{\tan 60^\circ - \tan 15^\circ}{\tan 15^\circ - \tan 60^\circ} = -1$

25. (A)
$\cot\theta.\tan(90^\circ-\theta) - \sec(90^\circ-\theta).\operatorname{cosec}\theta + (\sin^2 25^\circ + \sin^2 65^\circ) + \sqrt{3}(\tan 5^\circ.\tan 15^\circ.\tan 30^\circ.\tan 75^\circ.\tan 85^\circ)$
$(\sin^2 25^\circ + \cos^2 25^\circ) + \sqrt{3}(\tan 5^\circ.\cot 5^\circ.\tan 15^\circ.\cot 15^\circ.\tan 30^\circ)$
$= (\cot^2\theta - \operatorname{cosec}^2\theta) + (\sin^2 25^\circ + \cos^2 25^\circ) + \sqrt{3} \times \dfrac{1}{\sqrt{3}}$
$= -1 + 1 + 1 = 1$
$[\sin(90^\circ-\theta) = \cos\theta; \operatorname{cosec}^2\theta - \cot^2\theta = 1; \tan(90^\circ-\theta) = \cot\theta; \sec(90^\circ-\theta) = \operatorname{cosec}\theta]$

26. (B) sin A = cosB If A + B = 90
$3x - 20 + 3y + 20 = 90^\circ$
$x + y = 30^\circ$

27. (D) $\cos\theta.\operatorname{cosec}23^\circ = 1$
$\Rightarrow \operatorname{cosec}23^\circ = \dfrac{1}{\cos\theta} = \sec\theta$
$\Rightarrow \operatorname{cosec}23^\circ = \operatorname{cosec}(90^\circ-\theta)$
$\Rightarrow 23^\circ = 90^\circ - \theta$
$\Rightarrow \theta = 90^\circ - 23^\circ = 67^\circ$

28. (B) $2(\cos^2\theta - \sin^2\theta) = 1$
$\Rightarrow \cos^2\theta - \sin^2\theta = \dfrac{1}{2}$
$\Rightarrow 1 - 2\sin^2\theta = \dfrac{1}{2}$
$\Rightarrow 2\sin^2\theta = 1 - \dfrac{1}{2}$
$\Rightarrow 2\sin^2\theta = \dfrac{1}{2} \Rightarrow \sin^2\theta = \dfrac{1}{4}$
$\Rightarrow \sin\theta = \pm\dfrac{1}{2} = \sin 30^\circ$
$\left[\begin{array}{l} \because \theta \text{ is + ve angle} \\ \therefore \theta \neq \dfrac{-1}{2} \end{array}\right]$
$\Rightarrow \theta = 30^\circ$

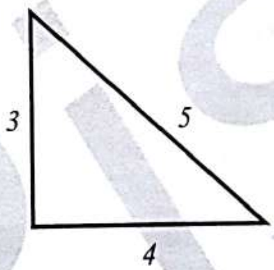
29. (D) tanA . tanB = 1 if A + B = 90

30. (A) sinA = cosB if A + B = 90

31. (C) $\tan\left(\dfrac{\pi}{2} - \dfrac{\theta}{2}\right) = \sqrt{3}$
$\Rightarrow \cot\dfrac{\theta}{2} = \sqrt{3} = \cot 30^\circ$
$\Rightarrow \dfrac{\theta}{2} = 30^\circ \Rightarrow \theta = 60^\circ$
$\therefore \cos\theta = \cos 60^\circ = \dfrac{1}{2}$

32. (C) $7\sin^2\theta + 3\cos^2\theta = 4$
$\Rightarrow 7\sin^2\theta + 3(1 - \sin^2\theta) = 4$
$\Rightarrow 7\sin^2\theta + 3 - 3\sin^2\theta = 4$
$\Rightarrow 4\sin^2\theta = 4 - 3 = 1$
$\Rightarrow \sin^2\theta = \dfrac{1}{4}$
$\Rightarrow \sin\theta = \dfrac{1}{2} = \sin\dfrac{\pi}{6}$
$\left[\begin{array}{l} \text{Note} : \sin\theta \neq \dfrac{-1}{2} \\ \therefore 0 < \theta < 90 \end{array}\right]$
$\Rightarrow \theta = \dfrac{\pi}{6}$

33. (B) $\sec\theta = x + \dfrac{1}{4x}$
Let x = 1
$\sec\theta = 1 + \dfrac{1}{4} = \dfrac{5}{4}$



$\sec\theta + \tan\theta$
$\dfrac{5}{4} + \dfrac{3}{4} = \dfrac{8}{4} = 2$
Substitute x = 1 from options

34. (A) cos 90° = 0
$\therefore$ cos 1° . cos2° ... cos 179° = 0

35. (B) If A + B = 90
sinA = cosB
tanA = cotB
tanA . tanB = 1
cotA . cotB = 1
$\sin^2 A + \sin^2 B = 1$
$\cos^2 A + \cos^2 B = 1$

36. (A) $\sec\theta + \tan\theta = \sqrt{3}$ ... (i)
$\because \sec^2\theta - \tan^2\theta = 1$
$\Rightarrow (\sec\theta - \tan\theta)(\sec\theta + \tan\theta) = 1$
$\Rightarrow \sec\theta - \tan\theta = \dfrac{1}{\sqrt{3}}$ ... (ii)
By subtracting (ii) from (i)
$\sec\theta + \tan\theta - \sec\theta + \tan\theta$
$= \sqrt{3} - \dfrac{1}{\sqrt{3}}$
$\Rightarrow 2\tan\theta = \dfrac{3-1}{\sqrt{3}}$
$\Rightarrow \tan\theta = \dfrac{1}{\sqrt{3}} = \tan 30^\circ$
$\Rightarrow \theta = 30^\circ$
$\therefore \tan 3\theta = \tan 90^\circ$
= undefined

37. (C) $\sin(60^\circ - \theta) = \cos(\Psi - 30^\circ)$
$= \sin(90^\circ - \Psi + 30^\circ)$
$= \sin(120^\circ - \Psi)$
$\Rightarrow 60^\circ - \theta = 120^\circ - \Psi$
$\Rightarrow \Psi - \theta = 60^\circ$
$\therefore \tan(\Psi - \theta) = \tan 60^\circ = \sqrt{3}$

38. (B) Given a sinθ + bcosθ = c ... (1)
Let acosθ − bsinθ = x ... (2)
Eq. (1) & (2) squaring & adding
$a^2 + b^2 = c^2 + x^2$
$x = \pm\sqrt{a^2 + b^2 - c^2}$

39. (B) $\sin(A - B) = \dfrac{1}{2} = \sin 30^\circ$
$\Rightarrow A - B = 30^\circ$
Again,
$\cos(A + B) = \dfrac{1}{2} = \cos 60^\circ$
$\Rightarrow A + B = 60^\circ$
$\therefore A + B + A - B$
$= 30^\circ + 60^\circ = 90^\circ$
$\Rightarrow 2A = 90^\circ$
$\Rightarrow A = 45^\circ$
$\therefore A - B = 30^\circ$
$\Rightarrow B = A - 30^\circ$
$= 45^\circ - 30^\circ = 15^\circ$
$= \dfrac{15 \times \pi}{180} = \dfrac{\pi}{12}$ radian

40. (B) Max value of
$2\sin\theta + 3\cos\theta$
$a\sin\theta \pm b\cos\theta$
max.value $= \sqrt{a^2 + b^2}$
min.value $= -\sqrt{a^2 + b^2}$
$\sqrt{4 + 9} = \sqrt{13}$

41. (C) 152 (sin30° + 2cos²45° + 3sin 30° + 4 cos²45° + ... + 17 sin30° + 18 cos²45°)
$\dfrac{152}{2}(1 + 2 + 3 + ...18)$
$76\left(\dfrac{18 \times 19}{2}\right) = 12996$
$\sqrt{12996} = 114$

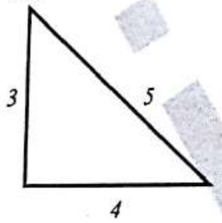
42. (D) 3 cos 80° . cosec10° + 2cos59° . cosec31°
$= 3\cos(90° - 10°) . \text{cosec } 10° + 2\cos(90° - 31°) . \text{cosec}31°$
$= 3 \sin 10° . \text{cosec}10° + 2 \sin31° . \text{cosec}31°$
$= 3 + 2 = 5$

$[\because \cos(90°-\theta) = \sin\theta; \sin\theta.\text{cosec}\theta = 1]$

43. (D) $\sin^2\theta - 3\sin\theta + 2 = 0$
$\Rightarrow \sin^2\theta - 2\sin\theta - \sin\theta + 2 = 0$
$\Rightarrow \sin\theta(\sin\theta - 2) - 1(\sin\theta - 2) = 0$
$\Rightarrow (\sin\theta - 1)(\sin\theta - 2) = 0$
$\Rightarrow \sin\theta = 1 = \sin 90°$
$\Rightarrow \theta = 90°$ and $\sin\theta \neq 2$

44. (C) $\tan\alpha = n \tan\beta$
$\sin\alpha = m \sin\beta$
$\cos^2\alpha = \tan\alpha = n\tan\beta$
$\frac{\sin\alpha}{\cos\alpha} = \frac{n\sin\beta}{\cos\beta}$
$\frac{m\sin\beta}{\cos\alpha} = \frac{n\sin\beta}{\cos\beta}$
$m\cos\beta = n\cos\alpha$ ...(1)
$m\sin\beta = \sin\alpha$ ...(2)
Adding after squaring
$m^2\cos^2\beta + m^2\sin^2\beta$
$= n^2\cos^2\alpha + \sin^2\alpha$
$m^2 = n^2\cos^2\alpha + 1 - \cos^2\alpha$
$m^2 - 1 = (n^2 - 1)\cos^2\alpha$
$\cos^2\alpha = \frac{m^2 - 1}{n^2 - 1}$

45. (B)



$\tan\theta = \frac{3}{4}$
$\text{cosec}\theta = \frac{H}{P} = \frac{5}{3}$

46. (C) $\text{cosec}\theta - \cot\theta = \frac{7}{2}$
$\text{cosec}\theta + \cot\theta = \frac{2}{7}$
$(\because \text{cosec}^2\theta - \cot^2\theta = 1)$
Adding
$2\text{cosec}\theta = \frac{2}{7} + \frac{7}{2} = \frac{53}{14}$
$\text{cosec}\theta = \frac{53}{28}$

47. (A) x sin 45° = y cosec 30°
$\frac{x}{y} = \frac{\text{cosec}30}{\sin45} = \frac{2}{\frac{1}{\sqrt{2}}} = 2\sqrt{2}$
$\frac{x^4}{y^4} = (2\sqrt{2})^4 = 2^4\ 2^2 = 2^6 = 4^3$

48. (C)
$5\tan\theta = 4$ then $\frac{5\sin\theta - 3\cos\theta}{5\sin\theta + 2\cos\theta}$
= divide with $\cos\theta$
$\frac{5\tan\theta - 3}{5\tan\theta + 2} = \frac{4-3}{4+2} = \frac{1}{6}$

49. (B) $2\text{cosec}^2 23\ \cot^2 67 - \sin^2 23$
$-\sin^2 67 - \cot^2 67$
$= 2\text{cosec}^2 23\ \cot^2(90-23)$
$-\frac{(\sin^2 23 + \sin^2 67)}{1} - \cot^2 67$
$= 2\ \text{cosec}^2 23 . \tan^2 23 - 1 - \cot^2 67$
$= \frac{2}{\sin^2 23}\ \frac{\sin^2 23}{\cos^2 23} - 1 - \tan^2 23$
$= 2\sec^2 23 - (1 + \tan^2 23)$
$= 2\sec^2 23 - \sec^2 23 = \sec^2 23$

50. (C) $\cos^2\theta = \frac{(x+y)^2}{4xy}$
$\frac{(x+y)^2}{4xy} = 1$ $(\because \max . \cos^2\theta = 1)$
$4xy = (x+y)^2$
$4xy = x^2 + y^2 + 2xy$
$(x-y)^2 = 0$
$x - y = 0$
$x = y$

51. (D) $\text{cosec}^2 18 - \frac{1}{\cot^2 72}$
$= \text{cosec}^2 18 - \tan^2 72$
$= \text{cosec}^2 18 - \cot^2 18$
$[\because \tan^2 A = \cot^2 B \text{ if } A + B = 90]$
$= 1\ [\because \text{cosec}^2\theta - \cot^2\theta = 1]$

52. (A) $\alpha + \beta = 90$
$= (1 - \sin^2\alpha)(1 - \cos^2\alpha)$
$(1 + \cot^2\beta)(1 + \tan^2\beta)$
$= \cos^2\alpha . \sin^2\alpha . \text{cosec}^2\beta . \sec^2\beta$
Put $\alpha = \beta = 45$
$= \frac{1}{2} . \frac{1}{2} . 2 . 2 = 1$
(or)
$= \sin^2\beta . \sin^2\alpha . \text{cosec}^2\beta$
$. \text{cosec}^2\alpha$
$[\because \alpha + \beta = 90] = 1$

53. (C) $\frac{2\sin68}{\cos22} - \frac{2\cot15}{5\tan75}$
$-\frac{3\tan45.\tan20.\tan40.\tan50.\tan70}{5}$
$= \frac{2\sin68}{\sin68} - \frac{2\cot15}{5\cot15}$
$[\because \sin A = \cos B \text{ atn}A = \cot B \text{ if } A + B = 90]$
$-\frac{3 \times 1.(\tan20.\tan70)(\tan40.\tan50)}{5}$
$= 2 - \frac{2}{5} - \frac{3}{5}$
$[\because \tan A.\tan B = 1 \text{ if } A + B = 90] = 1$

54. (B) tan 10 . tan 15 . tan 75 . tan 80
tan 10 . tan 80 . tan 15 . tan 75
$1 \times 1 = 1$
$[\because \tan A . \tan B = 1$ if $A + B = 90]$

55. (C) min value of
$4\tan^2\theta + 9\cot^2\theta$
$ax^2 + \frac{b}{x^2}$
min.value $= 2\sqrt{ab}$
max.value $= \infty$
$4\tan^2\theta + \frac{9}{\tan^2\theta}$
min.val $= 2\sqrt{4.9}$
$= 2\sqrt{36} = 2 \times 6 = 12$

56. (B) sin 7x = cos 11x
tan 9x + cot9x =
$\because \sin A = \cos B$ if $A + B = 90$
$7x + 11x = 90$
$18x = 90$
$x = 5$
$\tan 45 + \cot 45 = 1 + 1 = 2$

57. (A)
$\tan^2\alpha = 1 + 2\tan^2\beta$, α, β + ve acute angles
$\tan^2\alpha = 1 + 2\tan^2\beta$
$\sec^2\alpha - 1 = 1 + 2(\sec^2\beta - 1)$
$[\because \text{sce}^2\theta - \tan^2\theta = 1]$
$\sec^2\alpha - 1 = 2\sec^2\beta - 1$
$\frac{1}{\cos^2\alpha} = \frac{2}{\cos^2\beta}$
$\cos^2\beta = 2\cos^2\alpha$
$\cos\beta = \sqrt{2}\cos\alpha$
$\therefore \sqrt{2}\cos\alpha - \cos\beta = 0$
(or)
$\tan^2\alpha = 1 + 2\tan^2\beta$
Put $\beta = 45°$
$\tan^2\alpha = 1 + 2 = 3$
$\tan\alpha = \sqrt{3}$

$\alpha = 60$

$\sqrt{2}\cos 60 - \cos 45$

$= \sqrt{2} \times \frac{1}{2} - \frac{1}{\sqrt{2}}$

$= \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} = 0$

58. (D)

$(\cos 1^\circ \cos 2^\circ \ldots \cos 90^\circ \cos 100) = 0$

59. (D) $2(\cos^2\theta - \sin^2\theta) = 1$

$\cos 2\theta = \frac{1}{2}$

$2\theta = 60$

$\theta = 30$

$\cot 30 = \sqrt{3}$

60. (B) If $\tan(2\theta + 45) = \cot 3\theta$

then $\theta = 2\theta + 45 + 3\theta = 90$

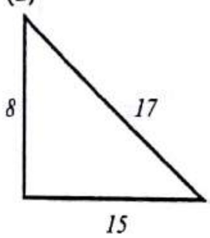
$[\therefore \tan A = \cot B \text{ if } A + B = 90^\circ]$

$5\theta + 45 = 90$

$5\theta = 45$

$\theta = 9$

61. (B)



$\cos\theta = \frac{15}{17}$

$\cot(90 - \theta)$

$\tan\theta = \frac{8}{15}$

62. (A) $\sec^2\theta + \tan^2\theta = \frac{7}{12}$

$\sec^4\theta - \tan^4\theta = (\sec^2\theta)^2 - (\tan^2\theta)^2$

$(\sec^2\theta - \tan^2\theta)(\sec^2\theta + \tan^2\theta)$

$1 \times \frac{7}{12} = \frac{7}{12}$

63. (B) $\sec x = \operatorname{cosec} y$

$\frac{1}{\cos x} = \frac{1}{\sin y}$

$\sin y = \cos x$

$x + y = 90$

$\sin(x + y) = \sin 90 = 1$

64. (C) $A + B + C = \pi = 180$

$\frac{A+B}{2} = \frac{180}{2} - \frac{C}{2}$

$\sin\left(\frac{A+B}{2}\right) = \sin\left(\frac{\pi}{2} - \frac{C}{2}\right) = \cos\frac{C}{2}$

$\cos\left(\frac{A+B}{2}\right) = \sin\frac{C}{2}$

$\cot\left(\frac{A+B}{2}\right) = \tan\frac{C}{2}$

$\tan\left(\frac{A+B}{2}\right) = \cot\frac{C}{2}$

65. (B) $\frac{\sin\alpha}{1} + \frac{\cos\beta}{1} = 2$

It is possible only when

$\alpha = 90, \beta = 0$

$\sin\frac{180}{3} = \sin 60 = \frac{\sqrt{3}}{2}$

From options takes (b)

$\cos\frac{\alpha}{3} = \cos 30 = \frac{\sqrt{3}}{2}$

66. (C) $\cos^4\theta - \sin^4\theta = \frac{2}{3}$

$(\cos^2\theta)^2 - (\sin^2\theta)^2 = \frac{2}{3}$

$(\cos^2\theta + \sin^2\theta)(\cos^2\theta - \sin^2\theta) = \frac{2}{3}$

$\cos^2\theta - \sin^2\theta = \frac{2}{3}$

$2\cos^2\theta - 1 = \frac{2}{3}$

67. (A) $\sin\alpha \sec(30 + \alpha) = 1$

$\frac{\sin\alpha}{\cos(30+\alpha)} = 1$

$\sin\alpha = \cos(30 + \alpha)$

$\alpha + 30 + \alpha = 90$

$[\because \sin A = \cos B \text{ if } A + B = 90]$

$2\alpha = 60$

$\alpha = 30$

$\sin\alpha + \cos 2\alpha = \frac{1}{2} + \frac{1}{2} = 1$

68. (A) $\tan\theta = 1$

$\frac{8\sin\theta + 5\cos\theta}{\sin^3\theta - 2\cos^3\theta + 7\cos\theta} =$

given $\tan\theta = 1 \Rightarrow \theta = 45$

$\frac{8\sin 45 + 5\cos 45}{\sin^3 45 - 2\cos^3 45 + 7\cos 45}$

$= \frac{8 \times \frac{1}{\sqrt{2}} + 5 \times \frac{1}{\sqrt{2}}}{\left(\frac{1}{\sqrt{2}}\right)^3 - 2\left(\frac{1}{\sqrt{2}}\right)^3 + 7\left(\frac{1}{\sqrt{2}}\right)} = 2$

69. (B) $\cos^2\theta + \cos^4\theta = 1$

$\cos^4\theta = 1 - \cos^2\theta = \sin^2\theta$

$\cos^2\theta . \cos^2\theta = \sin^2\theta$

$\cos^2\theta = \frac{\sin^2\theta}{\cos^2\theta} = \tan^2\theta$

$\cos^2\theta + \cos^4\theta = \tan^2\theta + \tan^4\theta = 1$

70. (C) $\tan\theta = \frac{4}{3}$

$\frac{3\sin\theta + 2\cos\theta}{3\sin\theta - 2\cos\theta}$ divide with $\cos\theta$

$\frac{3\tan\theta + 2}{3\tan\theta - 2} = \frac{3.\frac{4}{3} + 2}{3.\frac{4}{3} - 2} = \frac{6}{2} = 3$

71. (C)

$(\sec A - \cos A)^2 + (\operatorname{cosec} A - \sin A)^2 - (\cot A - \tan A)^2$

$= \sec^2 A + \cos^2 A - 2\sec A.\cos A + \operatorname{cosec}^2 A + \sin^2 A - 2\operatorname{cosec} A.\sin A - \cot^2 A - \tan^2 A + 2\cot A \tan A$

$= \sec^2 A - \tan^2 A + \cos^2 A + \sin^2 A + \operatorname{cosec}^2 A - \cot^2 A - 2$

$= 3 - 2 = 1$

(or)

put $\theta = 45$

$= (\sec 45 - \cos 45)^2 + (\operatorname{cosec} 45 - \sin 45)^2 - (\cot 45 - \tan 45)^2$

$= \left(\sqrt{2} - \frac{1}{\sqrt{2}}\right)^2 + \left(\sqrt{2} - \frac{1}{\sqrt{2}}\right)^2 - (1-1)^2$

$= \frac{1}{2} + \frac{1}{2} - 0 = 1$

72. (B) $\frac{\tan\theta}{1} + \frac{\cot\theta}{1} = 2$

Possible only when $\theta = 45$

$\tan^5\theta + \cot^{10}\theta = 1 + 1 = 2$

$x + \frac{1}{x} = 2$ is possible only when $x = 1$

73. (A) $\sin\theta - \cos\theta = \frac{7}{13}$

$\sin\theta - \cos\theta = x$

$\Rightarrow \sin\theta + \cos\theta = \sqrt{2 - x^2}$

$\sin\theta + \cos\theta = \sqrt{2 - \frac{49}{169}}$

$= \sqrt{\frac{289}{169}} = \frac{17}{13}$

74. (C) $2\cos\theta - \sin\theta = \frac{1}{\sqrt{2}}$

Put $\theta = 45$

$2\cos 45 - \sin 45 = \frac{1}{\sqrt{2}}$

$2 \times \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}}$

$\frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}}$

It satisfies so θ = 45

$2\sin\theta + \cos\theta = 2.\sin 45 + \cos 45$

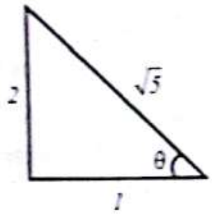
$= \frac{2}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \frac{3}{\sqrt{2}}$

75. (C) $\frac{\sin\theta + \cos\theta}{\sin\theta - \cos\theta} = 3$

$\sin\theta + \cos\theta = 3\sin\theta - 3\cos\theta$

$2\sin\theta = 4\cos\theta$

$\frac{\sin\theta}{\cos\theta} = \frac{2}{1}$



$\therefore \sin^4\theta - \cos^4\theta$

$= (\sin^2\theta - \cos^2\theta)(\sin^2\theta - \cos^2\theta)$

$= 1(\sin^2\theta - \cos^2\theta)$

$= \left(\frac{2}{\sqrt{5}}\right)^2 - \left(\frac{1}{\sqrt{5}}\right)^2$

$= \frac{4}{5} - \frac{1}{5} = \frac{3}{5}$

76. (A) $\sec^2\theta + \tan^2\theta = 7$

$1 + \tan^2\theta + \tan^2\theta = 7$

$2\tan^2\theta = 6$

$\tan\theta = \sqrt{3} = \tan 60$

$\theta = 60$

77. (D) $(\sec x \sec y + \tan x \tan y)^2$

$- (\sec x \tan y + \tan x \sec y)^2$

put $x = y = 45$

$(\sec 45 \sec 45 + \tan 45 . \tan 45)^2$

$-(\sec 45 \tan 45 + \tan 45 \sec 45)^2$

$= (\sqrt{2}.\sqrt{2} + 1.1)^2$

$- (\sqrt{2} \times 1 + 1 \times \sqrt{2})^2$

$= (2 + 1)^2 - (2\sqrt{2})^2$

$= 9 - 8 = 1$

78. (B) $\sin\theta + \operatorname{cosec}\theta = 2$

$x + \frac{1}{x} = 2 \Rightarrow x = 1$

$\sin^{100}\theta + \operatorname{cosec}^{100}\theta$

$= 1 + 1 = 2$

79. (B) $A = \sin^2\theta + \cos^4\theta$

Put $\theta = 45$ for min.value

$A = \sin^2 45 + \cos^4 45$

$= \frac{1}{2} + \frac{1}{4} = \frac{3}{4}$

Put $\theta = 90$ for max.value

$A = \sin^2 90 + \cos^4 90 = 1 + 0 = 1$

A lies in $\frac{3}{4} \le A \le 1$

80. (D) $\sin\theta + \operatorname{cosec}\theta = 2$

$x + \frac{1}{x} = 2 \Rightarrow x = 1$

$\sin^5\theta + \operatorname{cosec}^5\theta = 1 + 1 = 2$

81. (C) $\tan 2\theta . \tan 4\theta = 1$

$2\theta + 4\theta = 90$

$[\because \tan A.\tan B = 1 \text{ if } A + B = 90]$

$6\theta = 90$

$\theta = 15$

$\tan 3\theta = \tan 45 = 1$

82. (B) $\frac{\cos^2\alpha}{1} + \frac{\cos^2\beta}{1} = 2$

Possible when $\alpha = \beta = 0$

$\tan^3\alpha + \sin^5\beta = 0 + 0 = 0$

83. (C) $\tan 2\theta . \tan 3\theta = 1$

$2\theta + 3\theta = 90$

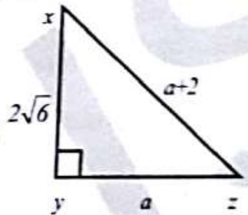
$5\theta = 90$

$\theta = 18$

$2\cos^2\frac{5\theta}{2} - 1 = 2\cos^2 45 - 1$

$= 2.\frac{1}{2} - 1 = 0$

84. (B)



$xz - yz = 2$

$(2\sqrt{6})^2 + a^2 = (a + 2)^2$

$24 + a^2 = (a + 2)^2$

Put values of 'a' to satisfy

a = 5

$\sec x + \tan x = \frac{7}{2\sqrt{6}} + \frac{5}{2\sqrt{6}}$

$= \frac{12}{2\sqrt{6}} = \frac{6}{\sqrt{6}} = \sqrt{6}$

85. (D) min . value

$\sin^2\theta + \cos^2\theta + \sec^2\theta$

$+ \operatorname{cosec}^2\theta + \tan^2\theta + \cot^2\theta$

$= \sin^2\theta + \operatorname{cosec}^2\theta + \cos^2\theta$

$+ \sec^2\theta + \tan^2\theta + \cot^2\theta$

$= \sin^2\theta + 1 + \cot^2\theta + \cos^2\theta$

$+ 1 + \tan^2\theta + \tan^2\theta + \cot^2\theta$

$= 2 + 1 + 2(\tan^2\theta + \cot^2\theta)$

$= 3 + 2(\tan^2\theta + \cot^2\theta)$

min.value $ax^2 + \frac{b}{x^2} = 2\sqrt{ab}$

$2\sqrt{1.1} = 2$

$= 3 + 2.2 = 7$

86. (A) $\cos 20° = m$ and $\cos 70° = n$

$\therefore m^2 + n^2 = \cos^2 20° + \cos^2 70°$

$= \cos^2(90° - 70°) + \cos^2 70°$

$\Rightarrow \sin^2 70° + \cos^2 70° = 1$

87. (D) $\cos\theta + \sec\theta = 2$

$x + \frac{1}{x} = 2 \Rightarrow x = 1$

$\cos^6\theta + \sec^6\theta = 1 + 1 = 2$

88. (A)

$\frac{5}{\sec^2\theta} + \frac{2}{1 + \cot^2\theta} + 3\sin^2\theta$

$= 5\cos^2\theta + \frac{2}{\operatorname{cosec}^2\theta} + 3\sin^2\theta$

$= 5\cos^2\theta + 2\sin^2\theta + 3\sin^2\theta$

$= 5(\cos^2\theta + \sin^2\theta) = 5$

89. (C)

$\left(\frac{1}{\cos\theta} + \frac{1}{\cot\theta}\right)\left(\frac{1}{\cos\theta} - \frac{1}{\cot\theta}\right)$

$(\sec\theta + \tan\theta)(\sec\theta - \tan\theta)$

$\sec^2\theta - \tan^2\theta = 1$

90. (C) $\frac{\sin\theta + \cos\theta}{\sin\theta - \cos\theta} = \frac{5}{4}$

$4\sin\theta + 4\cos\theta$

$= 5\sin\theta - 5\cos\theta$

$9\cos\theta = \sin\theta \Rightarrow \tan\theta = 9$

$\frac{\tan^2\theta + 1}{\tan^2\theta - 1} = \frac{81 + 1}{81 - 1} = \frac{82}{80} = \frac{41}{40}$

91. (C) $\tan 7\theta \tan 2\theta = 1$

$\tan A . \tan B = 1$ if $A + B = 90$

$7\theta + 2\theta = 90$

$9\theta = 90$

$\theta = 10$

$\tan 3\theta = \tan 3\theta = \frac{1}{\sqrt{3}}$

92. (D)

$(2\cos^2\theta - 1)\left(\frac{1 + \tan\theta}{1 - \tan\theta} + \frac{1 - \tan\theta}{1 + \tan\theta}\right)$

$= (2\cos^2\theta - (\sin^2\theta + \cos^2\theta))$

$\left(\frac{(1 + \tan\theta)^2 + (1 - \tan\theta)^2}{1 - \tan^2\theta}\right)$

$= (\cos^2\theta - \sin^2\theta)$

$\left(\frac{1 + \tan^2\theta + 2\tan\theta + 1 + \tan^2\theta - 2\tan\theta}{1 - \frac{\sin^2\theta}{\cos^2\theta}}\right)$

$= (\cos^2\theta - \sin^2\theta)$

$\frac{2 + 2\tan^2\theta}{\cos^2\theta - \sin^2\theta} \times \cos^2\theta$

$= 2(1 + \tan^2\theta) \times \cos^2\theta$

$= 2\sec^2\theta \cdot \cos^2\theta = 2$

(or)

Put $\theta = 0°$

$= (2\cos^2\theta - 1)\times\left(\frac{1+\tan\theta}{1-\tan\theta} + \frac{1-\tan\theta}{1+\tan\theta}\right)$

$= (2\times1-1)\left(\frac{1+0}{1-0} + \frac{1-0}{1+0}\right)$

$= (2-1)(1+1) = 2$

93. (C) $\sec\theta + \tan\theta = 2$

$\sec\theta - \tan\theta = \frac{1}{2}$

$2\sec\theta = 2 + \frac{1}{2} = \frac{5}{2}$

$\sec\theta = \frac{5}{4}$

94. (C) $\operatorname{cosec}\theta - \sin\theta = l$

$\sec\theta - \cos\theta = m$

$l^2 m^2 (l^2 + m^2 + 3) =$ put

$\theta = 45$

$l = \operatorname{cosec}45 - \sin 45$

$= \sqrt{2} - \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}}$

$m = \sec 45 - \cos 45$

$= \sqrt{2} - \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}}$

$l^2 m^2 (l^2 + m^2 + 3)$

$= \frac{1}{2}\times\frac{1}{2}\left(\frac{1}{2}+\frac{1}{2}+3\right)$

$= \frac{1}{4}(4) = 1$

95. (A) $\frac{2\sin\theta - \cos\theta}{\cos\theta + \sin\theta} = 1$

divide with $\sin\theta$

$\frac{2-\cot\theta}{1+\cot\theta} = 1$

$2 - \cot\theta = 1 + \cot\theta$

$1 = 2\cot\theta$

$\cot\theta = \frac{1}{2}$

96. (A) $\frac{8\sin\theta + 5\cos\theta}{\sin^3\theta + 2\cos^3\theta + 3\cos\theta}$

Divide with '$\cos\theta$'

(given $\tan\theta = 2$)

$\frac{8\tan\theta + 5}{\tan\theta.\sin^2\theta + 2\cos^2\theta + 3}$

$= \frac{8.2+5}{2\sin^2\theta + 2\cos^2\theta + 3}$

$= \frac{21}{2(\sin^2\theta + \cos^2\theta) + 3}$

$= \frac{21}{2+3} = \frac{21}{5}$

97. (A) $\tan\theta + \cot\theta = 2$

$x + \frac{1}{x} = 2 \Rightarrow x = 1$

$\tan^{100}\theta + \cot^{100}\theta = 1 + 1 = 2$

98. (D) $\frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta}$

$= \frac{\tan\theta}{1-\frac{1}{\tan\theta}} + \frac{\frac{1}{\tan\theta}}{1-\tan\theta}$

$= \frac{\tan^2\theta}{\tan\theta-1} + \frac{1}{\tan\theta(1-\tan\theta)}$

$= \frac{\tan^2\theta}{\tan\theta-1} - \frac{1}{\tan\theta(\tan\theta-1)}$

$= \frac{\tan^3\theta-1}{\tan\theta(\tan\theta-1)}$

$= \frac{(\tan\theta-1)(\tan^2\theta+\tan\theta+1)}{\tan\theta(\tan\theta-1)}$

$= \frac{\tan^2\theta+\tan\theta+1}{\tan\theta}$

$= \tan\theta + \cot\theta + 1$

99. (B) $\sin\theta + \operatorname{cosec}\theta = 2$

$x + \frac{1}{x} = 2 \Rightarrow x = 1$

$\sin^9\theta + \operatorname{cosec}^9\theta = 1 + 1 = 2$

100. (A) $\sec\theta + \tan\theta = 2 + \sqrt{5}$

$\sec\theta - \tan\theta = \frac{1}{2+\sqrt{5}} = \sqrt{5} - 2$

On adding

$2\sec\theta = 2 + \sqrt{5} + \sqrt{5} - 2 = 2\sqrt{5}$

$\sec\theta = \sqrt{5}$

$\cos\theta = \frac{1}{\sqrt{5}}$

$\sin\theta = \sqrt{1-\cos^2\theta} = \sqrt{1-\frac{1}{5}}$

$\sin\theta = \frac{2}{\sqrt{5}}$

$\sin\theta + \cos\theta = \frac{2}{\sqrt{5}} + \frac{1}{\sqrt{5}} = \frac{3}{\sqrt{5}}$

101. (B) $(1 + \cot\theta - \operatorname{cosec}\theta)$
$(1 + \tan\theta + \sec\theta)$

$= \left(1 + \frac{\cos\theta}{\sin\theta} - \frac{1}{\sin\theta}\right)$
$\left(1 + \frac{\sin\theta}{\cos\theta} + \frac{1}{\cos\theta}\right)$

$= \left(\frac{\sin\theta + \cos\theta - 1}{\sin\theta}\right)$
$\left(\frac{\cos\theta + \sin\theta + 1}{\cos\theta}\right)$

$= \frac{(\sin\theta + \cos\theta)^2 - 1}{\sin\theta.\cos\theta}$

$= \frac{\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta - 1}{\sin\theta.\cos\theta}$

$= \frac{2\sin\theta\cos\theta}{\sin\theta\cos\theta} = 2$

(or)

Put $\theta = 45$

$(1 + \cot 45 - \operatorname{cosec}45)$
$(1 + \tan 45 + \sec 45)$

$(1 + 1 - \sqrt{2})(1 + 1 + \sqrt{2})$

$(2 - \sqrt{2})(2 + \sqrt{2})$

$4 - 2 = 2$

102. (A) $\tan\theta + \cot\theta = 2$

$x + \frac{1}{x} = 2 \Rightarrow x = 1$

$\tan^n\theta + \cot^n\theta = 1 + 1 = 2$

103. (C) $\frac{\sin\theta}{x} = \frac{\cos\theta}{y} = \frac{1}{k}$

$x = k\sin\theta,\ y = k\cos\theta$

$x^2 + y^2 = k^2(\sin^2\theta + \cos^2\theta) = k^2$

$k = \sqrt{x^2+y^2}$

$\therefore \sin\theta - \cos\theta = \frac{x}{k} - \frac{y}{k} = \frac{x-y}{k}$

$= \frac{x-y}{\sqrt{x^2+y^2}}$

104. (A) $x = a\sec\theta\cos\phi$

$y = b\sec\theta\sin\phi$

$z = c\tan\theta$

$\frac{x}{a} = \sec\theta \cdot \cos\phi$

$\frac{y}{b} = \sec\theta \cdot \sin\phi$

$\frac{z}{c} = \tan\theta$

$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2}$

$= \sec^2\theta\cos^2\phi + \sec^2\theta$
$\sin^2\phi - \tan^2\theta$

$\sec^2\theta(\cos^2\phi + \sin^2\phi) - \tan^2\theta$

$\sec^2\theta - \tan^2\theta = 1$

105. (A) $\frac{\sec\theta + \tan\theta}{\sec\theta - \tan\theta} = \frac{5}{3}$

$3\sec\theta + 3\tan\theta = 5\sec\theta - 5\tan\theta$

$8\tan\theta = 2\sec\theta$

$4\tan\theta = \sec\theta$

$\frac{4\sin\theta}{\cos\theta} = \frac{1}{\cos\theta}$

$\sin\theta = \frac{1}{4}$

106. (C) $\cos x + \cos^2 x = 1$

$\cos x = 1 - \cos^2 x = \sin^2 x$

$\therefore \sin^2 x + 3\sin^{10} x + 3\sin^8$
$x + \sin^6 x - 1$

$= (\sin^4 x + \sin^2 x)^3 - 1$

$= (\cos^2 x + \sin^2 x)^3 - 1$

$= 1 - 1 = 0$

107. (A) $(1+\sin\alpha)(1+\sin\beta)(1+\sin\gamma)$

$= (1-\sin\alpha)(1-\sin\beta)(1-\sin\gamma)$

Let

$x = (1+\sin\alpha)(1+\sin\beta)(1+\sin\gamma)$

$= (1-\sin\alpha)(1-\sin\beta)(1-\sin\gamma)$

$x.x = (1-\sin^2\alpha)$

$(1-\sin^2\beta)(1-\sin^2\gamma)$

$x^2 = \cos^2\alpha \cdot \cos^2\beta \cdot \cos^2\gamma$

$x = \pm\cos\alpha.\cos\beta.\cos\gamma$

108. (D)

$$\frac{1}{1+\cot^2\theta} + \frac{3}{1+\tan^2\theta} + 2\sin^2\theta$$

$$\frac{1}{\operatorname{cosec}^2\theta} + \frac{3}{\sec^2\theta} + 2\sin^2\theta$$

$\sin^2\theta + 3\cos^2\theta + 2\sin^2\theta$

$3(\sin^2\theta + \cos^2\theta) = 3$

(or) put $\theta = 45$

$$= \frac{1}{1+\cot^2 45} + \frac{3}{1+\tan^2 45} + 2\sin^2 45$$

$$= \frac{1}{1+1} + \frac{3}{1+1} + \frac{2}{2}$$

$$= \frac{1}{2} + \frac{3}{2} + 1 = \frac{6}{2} = 3$$

109. (A)

$$\frac{4}{1+\tan^2\alpha} + \frac{1}{1+\cot^2\alpha} + 3\sin^2\alpha$$

$$\frac{4}{\sec^2\alpha} + \frac{1}{\operatorname{cosec}^2\alpha} + 3\sin^2\alpha$$

$4\cos^2\alpha + 1\sin^2\alpha + 3\sin^2\alpha$

$4(\cos^2\alpha + \sin^2\alpha) = 4$

(or)

Put $\alpha = 45$

$$= \frac{4}{1+1} + \frac{1}{1+1} + \frac{3}{2}$$

$$= 2 + \frac{1}{2} + \frac{3}{2} = 2 + \frac{1}{2} + \frac{3}{2}$$

$$= \frac{8}{2} = 4$$

110. (D) $3(\sin x - \cos x)^4 + 6(\sin x + \cos x)^2$

$+4(\sin^6 x + \cos^6 x)$

Put $x = 90$

$3(\sin 90 - \cos 90)^4$

$+6(\sin 90 + \cos 90)^2$

$+4(\sin^6 90 + \cos^6 90)$

$= 3(1-0)^4 + 6(1+0)^2 + 4(1^6 + 0)$

$= 3 + 6 + 4 = 13$

111. (C)

$$\sec\theta\left(\frac{1+\sin\theta}{\cos\theta} + \frac{\cos\theta}{1+\sin\theta}\right) - 2\tan^2\theta$$

Take $\theta = 0$

$$= \sec 0\left(\frac{1+\sin 0}{\cos 0} + \frac{\cos 0}{1+\sin 0}\right) - 2\tan^2 0$$

$$= 1 \times \left(\frac{1+0}{1} + \frac{1}{1+0}\right)$$

$= 1(1+1) = 2$

112. (A) $\tan\theta + \cot\theta = 2$

$$x + \frac{1}{x} = 2 \Rightarrow x = 1$$

$\tan^2\theta + \cot^2\theta = 1 + 1 = 2$

113. (A) $x\cos\theta - y\sin\theta = 2$

$x\sin\theta + y\cos\theta = 4$

squaring

$x^2\cos^2\theta + y^2\sin^2\theta$

$-2xy\sin\theta\cos\theta = 4$

$x^2\sin^2\theta + y^2\cos^2\theta$

$+2xy\sin\theta\cos\theta = 16$

Adding

$\cos^2\theta(x^2+y^2) +$

$\sin^2\theta(x^2+y^2) = 20$

$(x^2+y^2)\times(\cos^2\theta + \sin^2\theta) = 20$

$x^2 + y^2 = 20$

114. (C) $$\left[\frac{\cos^2 A(\sin A + \cos A)}{\operatorname{cosec}^2 A(\sin A - \cos A)} + \frac{\sin^2 A(\sin A - \cos A)}{\sec^2 A(\sin A + \cos A)}\right]$$

$$\times\left[\frac{1}{\cos^2 A} - \frac{1}{\sin^2 A}\right]$$

$$= \frac{(\sin A + \cos A)^2 + (\sin A - \cos A)^2}{\operatorname{cosec}^2 A.(\sin^2 A - \cos^2 A).\sec^2 A}$$

$$\times\frac{\sin^2 A - \cos^2 A}{\cos^2 A.\sin^2 A} = 2$$

115. (B) $$\frac{1}{\operatorname{cosec}\theta - \cot\theta} - \frac{1}{\sin\theta}$$

$$\frac{\operatorname{cosec}^2\theta - \cot^2\theta}{\operatorname{cosec}\theta - \cot\theta} - \frac{1}{\sin\theta}$$

$\operatorname{cosec}\theta + \cot\theta - \operatorname{cosec}\theta = \cot\theta$

116. (D) $\cos\theta + \sin\theta = \sqrt{2}\cos\theta$

Let $\cos\theta - \sin\theta = x$

Squaring & Adding

$= 2\times(\cos^2\theta + \sin^2\theta)$

$= (\sqrt{2}\cos\theta)^2 + x^2$

$2 - 2\cos^2\theta = x^2$

$2\sin^2\theta = x^2$

$x = \sqrt{2}\sin\theta$

117. (C) $\cos^4\theta - \sin^4\theta = \frac{2}{3}$

$(\cos^2\theta - \sin^2\theta)(\cos^2\theta + \sin^2\theta) = \frac{2}{3}$

$\cos^2\theta - \sin^2\theta = \frac{2}{3}$

$1 - 2\sin^2\theta = \frac{2}{3}$

118. (B) $$\frac{1}{(1+\tan^2\theta)} + \frac{1}{1+\cot^2\theta)}$$

$$= \frac{1}{\sec^2\theta} + \frac{1}{\operatorname{cosec}^2\theta}$$

$\cos^2\theta + \sin^2\theta = 1$

119. (C) $\sin\theta - \cos\theta = \frac{1}{2}$

$\sin\theta + \cos\theta = x$

Squaring & Adding

$$2(\sin^2\theta + \cos^2\theta) = \frac{1}{4} + x^2$$

$$x^2 = 2 - \frac{1}{4} = \frac{7}{4}$$

$$x = \frac{\sqrt{7}}{2}$$

120. (A) $$\frac{\sin A}{1+\cos A} + \frac{\sin A}{1-\cos A}$$

$$= \frac{\sin A(1-\cos A) + \sin A(1+\cos A)}{1-\cos^2 A}$$

$$= \frac{\sin A - \sin A\cos A + \sin A + \sin A\cos A}{\sin^2 A}$$

$$\frac{2\sin A}{\sin^2 A} = \frac{2}{\sin A} = 2\operatorname{cosec} A$$

121. (D) If $r\sin\theta = 1$

$r\cos\theta = \sqrt{3}$

$$\frac{r\sin\theta}{r\cos\theta} = \frac{1}{\sqrt{3}}$$

$$\tan\theta = \frac{1}{\sqrt{3}}$$

$$\sqrt{3}.\tan\theta + 1 = \sqrt{3}\times\frac{1}{\sqrt{3}} + 1 = 2$$

122. (B)

$x\cos\theta - y\sin\theta = \sqrt{x^2+y^2}$

$$\frac{\cos^2\theta}{a^2} + \frac{\sin^2\theta}{b^2} = \frac{1}{x^2+y^2}$$

$$\frac{x.\cos\theta}{\sqrt{x^2+y^2}} + \frac{(-y\sin\theta)}{\sqrt{x^2+y^2}} = 1$$

$\cos^2\theta + \sin^2\theta = 1$

$$\therefore \cos\theta = \frac{x}{\sqrt{x^2+y^2}}$$

$$\sin\theta = \frac{-y}{\sqrt{x^2+y^2}}$$

Put these values in given

equation

$$\frac{x^2}{(\sqrt{x^2+y^2})^2}\times\frac{1}{a^2}+\frac{y^2}{(\sqrt{x^2+y^2})^2}$$

$$\frac{1}{b^2}=\frac{1}{x^2+y^2}$$

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

123. (C) $\tan\theta-\cot\theta=0$

$\tan\theta=\cot\theta$

$\theta=45$

$\sin\theta+\cos\theta=\sin 45+\cos 45$

$=\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}=\sqrt{2}$

124. (D) Mod. Value $\sin^4\theta+\cos^4\theta$

In expressions like

$\sin^{2n}\theta+\cos^{2n}\theta$

max.value = 1

For min.value put $\theta = 45$

(or)

$\sin^4\theta+\cos^4\theta=(\sin^2\theta+\cos^2\theta)^2$

$-2\sin^2\theta\cos^2\theta$

$=1-2\sin^2\theta\cos^2\theta$

$=1-\frac{1}{2}(2\sin\theta\cos\theta)^2$

$=1-\frac{1}{2}\sin^2 2\theta$

$\text{max.value}=1-\frac{1}{2}\times(0)=1$

$\text{min.value}=1-\frac{1}{2}\times(1)=\frac{1}{2}$

125. (A) $3\sin\theta+5\cos\theta=5$

$5\sin\theta-3\cos\theta=x$

On squaring & Adding

$9\sin^2\theta+25\cos^2\theta+25\sin^2\theta$

$+9\cos^2\theta=25+x^2$

$34(\sin^2\theta+\cos^2\theta)=25+x^2$

$x^2=34-25=9$

$x=\pm 3$

126. (D) $\sin\theta+\sin^2\theta=1$

$\sin\theta=1-\sin^2\theta=\cos^2\theta$

$\sin^2\theta=\cos^4\theta$

$\cos^2\theta+\cos^4\theta=\cos^2\theta$

$+\sin^2\theta=1$

127. (A) $\tan\theta+\cot\theta=2$

$\Rightarrow\theta=45$

128. (D) $\cos\pi x=x^2-x+\frac{5}{4}$

$=x^2-2.x.\frac{1}{2}+\frac{1}{4}-\frac{1}{4}+\frac{5}{4}$

$=\left(x-\frac{1}{2}\right)^2+1>1$

$=-1\le\cos x\le 1$

'x' value none of the given options

129. (D) $1+\frac{1}{\cot^2 63}=\sec^2 27$

$+\frac{1}{\sin^2 63}=\cos ec^2 27$

$1+\tan^2 63-\sec^2 27$

$+\cos ec^2 63-\cos ec^2 27$

$1+\cot^2 27-\sec^2 27+\sec^2 27$

$-\cos ec^2 27$

$1+\cot^2 27-\cos ec^2 27$

$1-1=0$

130. (B) $x=\frac{\cos\theta}{1-\sin\theta}\times\frac{1+\sin\theta}{1+\sin\theta}$

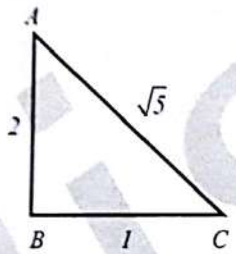
$=\frac{\cos\theta(1+\sin\theta)}{1-\sin^2\theta}$

$=\frac{\cos\theta(1+\sin\theta)}{\cos^2\theta}$

$x=\frac{1+\sin\theta}{\cos\theta}$

$\frac{\cos\theta}{1+\sin\theta}=\frac{1}{x}$

131. (B)



$\sin A+\cot c=\frac{1}{\sqrt{5}}+\frac{1}{2}$

$=\frac{2+\sqrt{5}}{2\sqrt{5}}$

132. (B) $\sin\frac{\pi x}{2}=x^2-2x+2$

put x value from options

Let x = 1

$\sin\frac{\pi}{2}=1-2+2=1$

sin 90 = 1

satisfies so x = 1

133. (A) $\frac{\sin 43}{\cos 47}+\frac{\cos 19}{\sin 71}-8\cos^2 60$

$\sin A=\cos B$ if $A+B=90$

$1+1-8\times\frac{1}{4}$

$2-2=0$

134. (B) $\frac{\sin^2 7\frac{1}{2}^o+\sin^2 82\frac{1}{2}^o}{1}$

$+\frac{\tan^2 2.\tan^2 88^o}{1}$

$1+1=2$

If $A+B=90$

$\sin^2 A+\sin^2 B=1$

$\tan^2 A.\tan^2 B=1$

135. (B) $1-2\sin^2\theta+\sin^4\theta$

$(1-\sin^2\theta)^2=(\cos^2\theta)^2=\cos^4\theta$

136. (B) $\cot 9^o.\cot 27^o\cot 63^o\cot 81^o$

$(\cot 9^o.\cot 81^o).(\cot 27.\cot 68)$

$1\times 1=1$

$\cot A.\cot B=1$ if $A+B=90$

137. (B) (1+sinA) (1+sinB) (1+sinC)

= (1−sinA)(1−sinB)(1−sinC)

Let

x = (1+sinA) (1+sinB)(1+sinC)

=(1−sinA)(1−sinB)(1−sinC)

$x.x=(1-\sin^2 A)(1-\sin^2 B)(1-\sin^2 1)$

$x^2=\cos^2 A\cos^2 B\cos^2 C$

$x=\pm\cos A.\cos B.\cos C$

Given $0<A, B, C<\frac{\pi}{2}$

$x=\cos A.\cos B.\cos C$

138. (D) $\tan^2\theta+3=3\sec\theta$

$\sec^2\theta-1+3=3\sec\theta$

$\sec^2\theta-3\sec\theta+2=0$

$\sec^2\theta-2\sec\theta-\sec\theta+2=0$

$\sec\theta(\sec\theta-2)-1(\sec\theta-2)=0$

$(\sec\theta-1)(\sec\theta-2)=0$

$\sec\theta=1$ or 2

$\theta=0^o$ or 60^o

139. (C) $\sin\theta=0.7$

$\cos\theta=\sqrt{1-\sin^2\theta}$

$=\sqrt{1-(0.7)^2}=\sqrt{1-0.49}$

$=\sqrt{0.51}$

140. (C)

$\frac{\sin^2 65+\sin^2 25}{1}+\frac{\cos^2 35+\cos^2 55}{1}$

$1+1=2$

$\sin^2 A+\sin^2 B=1$

If A + B = 90

$\cos^2 A+\cos^2 B=1$

141. (C) $x.\sin 60^o.\tan 30^o$

$=\sec 60^o.\cot 45^o$

$\Rightarrow x\times\frac{\sqrt{3}}{2}\times\frac{1}{\sqrt{3}}=2\times 1$

$\Rightarrow x=2\times 2=4$

142. (D) $\frac{1}{2}\sqrt{1+\sin\theta}+\frac{1}{2}\sqrt{1-\sin\theta}$

$=\frac{1}{2}\left(\sqrt{1+\sin 60^o}+\sqrt{1-\sin 60^o}\right)$

$=\frac{1}{2}\left(\sqrt{1+\frac{\sqrt{3}}{2}}+\sqrt{1-\frac{\sqrt{3}}{2}}\right)$

$=\frac{1}{2\sqrt{2}}\left(\sqrt{2+\sqrt{3}}+\sqrt{2-\sqrt{3}}\right)$

$= \frac{1}{2\sqrt{2}} \times \frac{1}{\sqrt{2}}\left(\sqrt{4+2\sqrt{3}} + \sqrt{4-2\sqrt{3}}\right)$

$= \frac{1}{4}\left(\sqrt{(\sqrt{3}+1)^2} + \sqrt{(\sqrt{3}-1)^2}\right)$

$= \frac{1}{4}\left(\sqrt{3}+1+\sqrt{3}-1\right)$

$= \frac{2\sqrt{3}}{4} = \frac{\sqrt{3}}{2} = \cos 30^o$

$= \cos\frac{\theta}{2}$

143. (B) $\frac{2\tan^2 30^o}{1-\tan^2 30^o} + \sec^2 45^o - \sec^2 0^o$

$= x \sec 60^o$

$\Rightarrow \frac{2\times\left(\frac{1}{\sqrt{3}}\right)^2}{1-\left(\frac{1}{\sqrt{3}}\right)^2} + (\sqrt{2})^2 - 1 = x \times 2$

$\Rightarrow \frac{\frac{2}{3}}{1-\frac{1}{3}} + 2 - 1 = x \times 2$

$\Rightarrow \frac{2}{3} \times \frac{3}{2} + 1 = x \times 2$

$\Rightarrow 2 = x \times 2 \Rightarrow x = \frac{2}{2} = 1$

144. (B) $\tan\theta = \frac{\sin\alpha - \cos\alpha}{\sin\alpha + \cos\alpha}$

$\therefore 1 + \tan^2\theta$

$= 1 + \frac{(\sin\alpha - \cos\alpha)^2}{(\sin\alpha + \cos\alpha)^2}$

$\Rightarrow \sec^2\theta =$

$= \frac{(\sin\alpha + \cos\alpha)^2 + (\sin\alpha - \cos\alpha)^2}{(\sin\alpha + \cos\alpha)^2}$

$\Rightarrow \sec^2\theta = \frac{2(\sin^2\alpha + \cos^2\alpha)}{(\sin\alpha + \cos\alpha)^2}$

$\Rightarrow \frac{1}{\cos^2\theta} = \frac{2}{(\sin\alpha + \cos\alpha)^2}$

$\Rightarrow \frac{1}{\cos\theta} = \frac{\pm\sqrt{2}}{\sin\alpha + \cos\alpha}$

$\Rightarrow \sin\alpha + \cos\alpha = \pm\sqrt{2}\cos\theta$

145. (A) $7\sin^2\theta + 3\cos^2\theta = 4$

On dividing both sides by $\cos^2\theta$

$7\tan^2\theta + 3 = 4\sec^2\theta$

$\Rightarrow 7\tan^2\theta + 3 = 4(1+\tan^2\theta)$

$\Rightarrow 7\tan^2\theta + 3 = 4 + 4\tan^2\theta$

$\Rightarrow 7\tan^2\theta - 4\tan^2\theta = 4 - 3$

$\Rightarrow 3\tan^2\theta = 1$

$\Rightarrow \tan^2\theta = \frac{1}{3}$

$\Rightarrow \tan\theta = \frac{1}{\sqrt{3}}$

146. (D) $\tan 9^o = \frac{p}{q}$

$\therefore \frac{\sec^2 81^o}{1+\cot^2 81^o} = \frac{\sec^2 81^o}{\operatorname{cosec}^2 81^o}$

$= \frac{1}{\cos^2 81^o} \times \sin^2 81^o$

$= \tan^2 81^o = \tan^2(90^o - 9^o)$

$= \cot^2 9^o = \frac{q^2}{p^2}$

147. (D) $\sec\theta + \tan\theta = 5$

$\therefore \sec^2\theta - \tan^2\theta = 1$

$\Rightarrow (\sec\theta - \tan\theta)(\sec\theta + \tan\theta) = 1$

$\Rightarrow \sec\theta - \tan\theta = \frac{1}{5}$

$\therefore (\sec\theta + \tan\theta) - (\sec\theta - \tan\theta)$

$= 5 - \frac{1}{5} = \frac{25-1}{5}$

$\Rightarrow 2\tan\theta = \frac{24}{5} \Rightarrow \tan\theta = \frac{12}{5}$

$\therefore \frac{\tan\theta + 1}{\tan\theta - 1} = \frac{\frac{12}{5}+1}{\frac{12}{5}-1}$

$= \frac{12+5}{12-5} = \frac{17}{7}$

148. (D) $\tan^2\theta = 1 - e^2$

$\therefore \sec\theta + \tan^3\theta.\operatorname{cosec}\theta$

$= \sec\theta + \tan^2\theta.\tan\theta.\operatorname{cosec}\theta$

$= \sec\theta + \tan^2\theta.\frac{\sin\theta}{\cos\theta}.\frac{1}{\sin\theta}$

$= \sec\theta + \tan^2\theta.\sec\theta$

$= \sec\theta.(1+\tan^2\theta)$

$= (1+\tan^2\theta)^{\frac{1}{2}}.(1+\tan^2\theta)$

$= (1+\tan^2\theta)^{\frac{3}{2}} = (1+1-e^2)^{\frac{3}{2}}$

$= (2-e^2)^{\frac{3}{2}}$

149. (B) When $\theta = 60^o$

$\cos\theta = \frac{1}{2}, \cos^2\theta = \frac{1}{4}$

$\therefore \cos\theta > \cos^2\theta$

150. (A) $x\sin 60^o \tan 30^o - \tan^2 45^o$

$= \operatorname{cosec}60^o . \cot 30^o - \sec^2 45^o$

$\Rightarrow x . \frac{\sqrt{3}}{2} . \frac{1}{\sqrt{3}} - 1$

$= \frac{2}{\sqrt{3}} \times \sqrt{3} - (\sqrt{2})^2$

$\Rightarrow \frac{x}{2} - 1 = 2 - 2 = 0$

$\Rightarrow \frac{x}{2} = 1 \Rightarrow x = 2$

151. (C) $x = a \sec\alpha . \cos\beta$

$\Rightarrow \frac{x}{a} = \sec\alpha . \cos\beta$

Similarly,

$\frac{y}{b} = \sec\alpha . \sin\beta$, $\frac{z}{c} = \tan\alpha$

$\therefore \frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2}$

$= \sec^2\alpha.\cos^2\beta + \sec^2\alpha.\sin^2\beta - \tan^2\alpha$

$= \sec^2\alpha(\cos^2\beta + \sin^2\beta) - \tan^2\alpha$

$= \sec^2 - \tan^2\alpha = 1$

152. (C) $\frac{\cos\alpha}{\cos\beta} = a \Rightarrow \cos\alpha = a\cos\beta$

On squaring both sides,

$\cos^2\alpha = a^2\cos^2\beta$

$\Rightarrow 1 - \sin^2\alpha = a^2(1-\sin^2\beta)$ ... (i)

Again, $\sin\alpha = b\sin\beta$

$\Rightarrow \sin^2\alpha = b^2\sin^2\beta$

$\therefore$ From equation (i),

$1 - b^2\sin^2\beta = a^2 - a^2\sin^2\beta$

$\Rightarrow a^2\sin^2\beta - b^2\sin^2\beta = a^2 - 1$

$\Rightarrow \sin^2\beta(a^2 - b^2) = a^2 - 1$

$\Rightarrow \sin^2\beta = \frac{a^2-1}{a^2-b^2}$

153. (B) Expression

$= \frac{\cos^2 60^o + 4\sec^2 30^o - \tan^2 45^o}{\sin^2 30^o + \cos^2 30^o}$

$= \left(\frac{1}{2}\right)^2 + 4\left(\frac{2}{\sqrt{3}}\right)^2 - 1$

$[\because \sin^2\theta + \cos^2\theta = 1]$

$= \frac{1}{4} + \frac{16}{3} - 1$

$= \frac{3+64-12}{12} = \frac{55}{12}$

154. (A) $\sin^2 30^o \cos^2 45^o + 5\tan^2 30^o + \frac{3}{2}\sin^2 90^o - 3\cos^2 90^o$

$= \left(\frac{1}{2}\right)^2 \times \left(\frac{1}{\sqrt{2}}\right)^2 + 5\times\left(\frac{1}{\sqrt{3}}\right)^2 + \frac{3}{2}\times 1 - 3\times 0$

$= \frac{1}{4} \times \frac{1}{2} + 5 \times \frac{1}{3} + \frac{3}{2}$

$= \frac{1}{8} + \frac{5}{3} + \frac{3}{2} = \frac{3+40+36}{24}$

$= \frac{79}{24} = 3\frac{7}{24}$

155. (A) $\cos^2\theta - \sin^2\theta = \frac{1}{3}$

$\cos^4\theta - \sin^4\theta =$

$= (\cos^2\theta + \sin^2\theta)(\cos^2\theta - \sin^2\theta)$

$= 1 \times \frac{1}{3} = \frac{1}{3}$

156. (C) $\tan\theta = \frac{1}{\sqrt{11}}$; $\cot\theta = \sqrt{11}$

$\therefore \frac{\operatorname{cosec}^2\theta - \sec^2\theta}{\operatorname{cosec}^2\theta + \sec^2\theta}$

$= \frac{1+\cot^2\theta - (1+\tan^2\theta)}{1+\cot^2\theta + 1 + \tan^2\theta}$

$= \frac{\cot^2\theta - \tan^2\theta}{\cot^2\theta + \tan^2\theta + 2}$

$= \frac{(\sqrt{11})^2 - \left(\frac{1}{\sqrt{11}}\right)^2}{(\sqrt{11})^2 + \left(\frac{1}{\sqrt{11}}\right)^2 + 2}$

$= \frac{11 - \frac{1}{11}}{11 + \frac{1}{11} + 2} = \frac{\frac{121-1}{11}}{\frac{121+1+22}{11}}$

$= \frac{120}{144} = \frac{5}{6}$

157. (A) Expression

$= \frac{1}{\sqrt{2}}\sin\frac{\pi}{6} . \cos\frac{\pi}{4}$

$- \cot\frac{\pi}{3} . \sec\frac{\pi}{6} + \frac{5\tan\frac{\pi}{4}}{12\sin\frac{\pi}{2}}$

$= \frac{1}{\sqrt{2}} \times \frac{1}{2} \times \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{3}} \times \frac{2}{\sqrt{3}} + \frac{5\times1}{12\times1}$

$= \frac{1}{4} - \frac{2}{3} + \frac{5}{12}$

$= \frac{3-8+5}{12} = 0$

158. (B) $\sin\theta = \frac{3}{5}$

$\therefore \cos\theta = \sqrt{1-\sin^2\theta}$

$= \sqrt{1-\left(\frac{3}{5}\right)^2} = \sqrt{1-\frac{9}{25}}$

$= \sqrt{\frac{16}{25}} = \frac{4}{5}$

$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{3}{5}}{\frac{4}{5}} = \frac{3}{4}$

$\cot\theta = \frac{1}{\tan\theta} = \frac{4}{3}$

$\operatorname{cosec}\theta = \frac{1}{\sin\theta} = \frac{5}{3}$

$\therefore \frac{\tan\theta + \cos\theta}{\cot\theta + \operatorname{cosec}\theta} = \frac{\frac{3}{4} + \frac{4}{5}}{\frac{4}{3} + \frac{5}{3}}$

$= \frac{\frac{15+16}{20}}{\frac{4+5}{3}} = \frac{31}{20} \times \frac{3}{9} = \frac{31}{60}$

159. (B) $a\cos\theta + b\sin\theta = p$

$a\sin\theta - b\cos\theta = q$

On squaring and adding,

$a^2\cos^2\theta + b^2\sin^2\theta + 2ab$

$\sin\theta.\cos\theta + a^2\sin^2\theta + b^2$

$\cos^2\theta. - 2ab\sin\theta.\cos\theta = p^2 + q^2$

$\Rightarrow a^2\cos^2\theta + a^2\sin^2\theta + b^2$

$\sin^2\theta + b^2\cos^2\theta = p^2 + q^2$

$\Rightarrow a^2(\cos^2\theta + \sin^2\theta) + b^2(\sin^2\theta + \cos^2\theta)$

$= p^2 + q^2$

$\Rightarrow a^2 + b^2 = p^2 + q^2$

160. (B)

$(\sin\alpha + \operatorname{cosec}\alpha)^2 + (\cos\alpha + \sec\alpha)^2$

$= k + \tan^2\alpha + \cot^2\alpha$

$\Rightarrow \sin^2\alpha + \operatorname{cosec}^2\alpha + 2\sin\alpha.$

$\operatorname{cosec}\alpha + \cos^2\alpha + \sec^2\alpha + 2\cos\alpha.$

$\sec\alpha = k + \tan^2\alpha + \cot^2\alpha$

$\Rightarrow 5 + \operatorname{cosec}^2\alpha + \sec^2\alpha$

$= k + \tan^2\alpha + \cot^2\alpha$

$\Rightarrow 7 + \cot^2\alpha + \tan^2\alpha$

$= k + \tan^2\alpha + \cot^2\alpha$

$\Rightarrow k = 7$

161. (A) $\sin 21^\circ = \frac{x}{y}$

$\cos 21^\circ = \sqrt{1-\sin^2 21^\circ}$

$= \sqrt{1-\frac{x^2}{y^2}} = \frac{\sqrt{y^2-x^2}}{y}$

$\therefore \sec 21^\circ = \frac{y}{\sqrt{y^2-x^2}}$

$\therefore \sec 21^\circ - \sin 69^\circ$

$= \sec 21^\circ - \sin(90^\circ - 21^\circ)$

$= \sec 21^\circ - \cos 21^\circ$

$= \frac{y}{\sqrt{y^2-x^2}} - \frac{\sqrt{y^2-x^2}}{y}$

$= \frac{y^2-(y^2-x^2)}{y\sqrt{y^2-x^2}} = \frac{x^2}{y\sqrt{y^2-x^2}}$

162. (C) $\sec\alpha + \tan\alpha = 2$

$\Rightarrow \frac{1}{\cos\alpha} + \frac{\sin\alpha}{\cos\alpha} = 2$

$\Rightarrow \frac{1+\sin\alpha}{\cos\alpha} = 2$

$\Rightarrow 1 + \sin\alpha = 2\cos\alpha$

$\Rightarrow (1+\sin\alpha)^2 = 4\cos^2\alpha$

$\Rightarrow 1 + \sin^2\alpha + 2\sin\alpha = 4(1-\sin^2\alpha)$

$\Rightarrow 1 + \sin^2\alpha + 2\sin\alpha = 4 - 4\sin^2\alpha$

$\Rightarrow 5\sin^2\alpha + 2\sin\alpha + 1 - 4 = 0$

$\Rightarrow 5\sin^2\alpha + 2\sin\alpha - 3 = 0$

$\Rightarrow 5\sin^2\alpha + 5\sin\alpha - 3\sin\alpha - 3 = 0$

$\Rightarrow \sin\alpha(\sin\alpha + 1) - 3(\sin\alpha + 1) = 0$

$\Rightarrow (5\sin\alpha - 3)(\sin\alpha + 1) = 0$

$\because \alpha < 90^\circ$

$\therefore 5\sin\alpha - 3 = 0$

$\Rightarrow 5\sin\alpha = 3$

$\Rightarrow \sin\alpha = \frac{3}{5} = 0.6$

163. (A) $3\sin\theta + 5\cos\theta = 5$ ... (i)

$5\sin\theta - 3\cos\theta = x$ (let) ... (ii)

On squaring and adding both the equations,

$(3\sin\theta + 5\cos\theta)^2 +$

$(5\sin\theta - 3\cos\theta)^2$

$= 5^2 + x^2$

$\Rightarrow 9\sin^2\theta + 25\cos^2\theta + 30\sin\theta$

$.\cos\theta + 25\sin^2\theta + 9\cos^2\theta - 30$

$\sin\theta.\cos\theta = 25 + x^2$

$\Rightarrow 9\sin^2\theta + 9\cos^2\theta + 25\cos^2\theta +$

$25\sin^2\theta = 25 + x^2$

$\Rightarrow 9(\sin^2\theta + \cos^2\theta) + 25$

$(\cos^2\theta + \sin^2\theta) = 25 + x^2$

$\Rightarrow 9 + 25 = 25 + x^2$

$\Rightarrow x^2 = 9 \Rightarrow x = \pm 3$

164. (B) $\tan\theta + \cot\theta = 2$

$\Rightarrow \tan\theta + \frac{1}{\tan\theta} = 2$

$\Rightarrow \frac{\tan^2\theta + 1}{\tan\theta} = 2$

$\Rightarrow \tan^2\theta + 1 = 2\tan\theta$

$\Rightarrow \tan^2\theta - 2\tan\theta + 1 = 0$

$\Rightarrow (\tan\theta - 1)^2 = 0$

$\Rightarrow \tan\theta - 1 = 0 \Rightarrow \tan\theta = 1$

165. (C)

$\tan 1^\circ . \tan 2^\circ . \tan 3^\circ \ldots \tan 89^\circ$

$= (\tan 1^\circ.\tan 89^\circ)(\tan 2^\circ.\tan 88^\circ) \ldots. \tan 45^\circ$

$= (\tan 1^\circ.\tan(90^\circ - 1^\circ))(\tan 2^\circ \tan(90^\circ - 2^\circ)) \ldots \tan 45^\circ$

$= (\tan 1^\circ.\cot 1^\circ)(\tan 2^\circ.\cot 2^\circ) \ldots 45^\circ$

$= 1.1 \ldots 1. = 1$

$[\tan(90^\circ - \theta) = \cot\theta]$

166. (C)

$x\sin^2 60^\circ - \frac{3}{2}\sec 60^\circ.\tan^2 30^\circ + \frac{4}{5}\sin^2 45^\circ.\tan^2 60^\circ = 0$

$\Rightarrow x.\left(\frac{\sqrt{3}}{2}\right)^2 - \frac{3}{2}\times 2.\left(\frac{1}{\sqrt{3}}\right)^2 + \frac{4}{5}\times\left(\frac{1}{\sqrt{2}}\right)^2\times(\sqrt{3})^2 = 0$

$\Rightarrow \frac{3x}{4} - \frac{3}{2}\times 2\times\frac{1}{3} + \frac{4}{5}\times\frac{1}{2}\times 3 = 0$

$\Rightarrow \frac{3x}{4} - 1 + \frac{6}{5} = 0$

$\Rightarrow \frac{3x}{4} = 1 - \frac{6}{5} = \frac{5-6}{5} = \frac{-1}{5}$

$\Rightarrow x = -\frac{1}{5}\times\frac{4}{3} = \frac{-4}{15}$

167. (D) $7\sin\alpha = 24\cos\alpha$

$\Rightarrow \frac{\sin\alpha}{\cos\alpha} = \frac{24}{7} \Rightarrow \tan\alpha = \frac{24}{7}$

$\therefore \sec\alpha = \sqrt{1+\tan^2\alpha}$

$= \sqrt{1+\left(\frac{24}{7}\right)^2}$

$= \sqrt{1+\frac{576}{49}} = \sqrt{\frac{49+576}{49}}$

$= \sqrt{\frac{625}{49}} = \frac{25}{7}$

$\therefore \cos\alpha = \frac{1}{\sec\alpha} = \frac{7}{25}$

$\therefore 14\tan\alpha - 75\cos\alpha - 7\sec\alpha$

$= 14\times\frac{24}{7} - 75\times\frac{7}{25} - 7\times\frac{25}{7}$

$= 48 - 21 - 25 = 2$

168. (B)

$2\operatorname{cosec}^2 30^\circ + x\sin^2 60^\circ - \frac{3}{4}\tan^2 30^\circ = 10$

$\Rightarrow 2\times(2)^2 + x\times\left(\frac{\sqrt{3}}{2}\right)^2 - \frac{3}{4}\times\left(\frac{1}{\sqrt{3}}\right)^2 = 10$

$\Rightarrow 8 + \frac{3x}{4} - \frac{3}{4}\times\frac{1}{3} = 10$

$\Rightarrow \frac{3x}{4} = 10 + \frac{1}{4} - 8$

$\Rightarrow \frac{3x}{4} = \frac{9}{4} \Rightarrow 3x = 9$

$\Rightarrow x = \frac{9}{3} = 3$

169. (B) $\tan^2\theta - \sec^2\theta = -(\sec^2\theta - \tan^2\theta) = -1$

170. (B) $29\tan\theta = 31 \Rightarrow \tan\theta = \frac{31}{29}$

$\frac{1+2\sin\theta.\cos\theta}{1-2\sin\theta\cos\theta}$

$= \frac{\sin^2\theta+\cos^2\theta+2\sin\theta.\cos\theta}{\sin^2\theta+\cos^2\theta-2\sin.\cos\theta}$

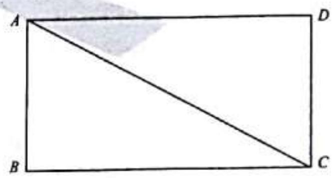
$= \frac{(\sin\theta+\cos\theta)^2}{(\sin\theta-\cos\theta)^2}$

$= \left(\frac{\frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\cos\theta}}{\frac{\sin\theta}{\cos\theta}-\frac{\cos\theta}{\cos\theta}}\right)^2 = \left(\frac{\tan\theta+1}{\tan\theta-1}\right)^2$

$= \left(\frac{\frac{31}{29}+1}{\frac{31}{29}-1}\right)^2 = \left(\frac{\frac{31+29}{29}}{\frac{31-29}{29}}\right)^2$

$= \left(\frac{60}{2}\right)^2 = (30)^2 = 900$

171. (C)



$\angle ACD = 45^\circ$

$\angle BAC = 45^\circ$

$\therefore (\tan^2\angle CAD + 1).\sin^2\angle BAC$

$= (\tan^2 45^\circ + 1).\sin^2 45^\circ$

$= (1+1)\times\left(\frac{1}{\sqrt{2}}\right)^2 = 2\times\frac{1}{2} = 1$

172. (B) $\tan x = \sin 45^\circ.\cos 45^\circ + \sin 30^\circ$

$= \frac{1}{\sqrt{2}}.\frac{1}{\sqrt{2}} + \frac{1}{2} = \frac{1}{2}+\frac{1}{2} = 1$

$\therefore \tan x = \tan 45^\circ \Rightarrow x = 45^\circ$

173. (C)

$\sqrt{\frac{\sec\theta-1}{\sec\theta+1}} = \sqrt{\frac{\frac{1}{\cos\theta}-1}{\frac{1}{\cos\theta}+1}}$

$= \sqrt{\frac{\frac{1-\cos\theta}{\cos\theta}}{\frac{1+\cos\theta}{\cos\theta}}}$

$= \sqrt{\frac{1-\cos\theta}{1+\cos\theta}} = \sqrt{\frac{(1-\cos\theta)(1-\cos\theta)}{(1+\cos\theta)(1-\cos\theta)}}$

(Rationalising the numerator and the denominator)

$= \sqrt{\frac{(1-\cos\theta)^2}{1-\cos^2\theta}} = \sqrt{\frac{(1-\cos\theta)^2}{\sin^2\theta}}$

$= \operatorname{cosec}\theta - \cot\theta$

174. (B) Let the angles be A and B where A > B

$\therefore A + B = 135^\circ$ and, $A - B = \frac{\pi}{12}$

$= \frac{\pi}{12}\times\frac{180^\circ}{\pi} = 15^\circ$

On adding

$A + B + A - B$

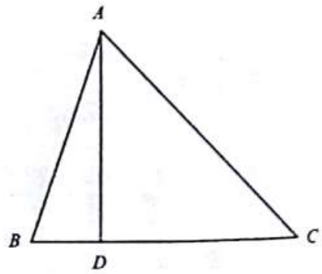
$= 135^\circ + 15^\circ = 150^\circ$

$\Rightarrow 2A = 150^\circ \Rightarrow A = \frac{150}{2} = 75^\circ$

$\therefore A + B = 135^\circ$

$\Rightarrow B = 135^\circ - 75^\circ = 60^\circ$

175. (C)



$\angle B = \frac{\pi}{3}, \angle = \frac{\pi}{4}$

and $\frac{BD}{DC} = \frac{1}{3}$

From ΔABD,

$\frac{BD}{\sin BAD} = \frac{AD}{\sin ABD}$

$\Rightarrow \frac{BD}{\sin BAD} = \frac{AD}{\sin\frac{\pi}{3}}$

$\Rightarrow \dfrac{BD}{\sin BAD} = \dfrac{AD}{\frac{\sqrt{3}}{2}}$

$\Rightarrow AD = \dfrac{\sqrt{3}}{2} \cdot \dfrac{BD}{\sin BAD}$ ... (i)

Fromn ΔADC

$\dfrac{CD}{\sin DAC} = \dfrac{AD}{\sin ACD}$

$\Rightarrow \dfrac{CD}{\sin DAC} = \dfrac{AD}{\sin\frac{\pi}{4}}$

$\Rightarrow AD = \dfrac{1}{\sqrt{2}} \cdot \dfrac{CD}{\sin DAC}$ ... (ii)

$\dfrac{\sqrt{3}}{2} \cdot \dfrac{BD}{\sin BAD} = \dfrac{1}{\sqrt{2}} \cdot \dfrac{CD}{\sin DAC}$

$\Rightarrow \dfrac{\sin BAD}{\sin DAC} = \dfrac{\sqrt{3}}{2} \times \sqrt{2} \times \dfrac{BD}{CD}$

$\Rightarrow \dfrac{\sin BAD}{\sin DAC} = \dfrac{\sqrt{3}}{2} \times \sqrt{2} \times \dfrac{1}{3}$

$= \dfrac{1}{\sqrt{2} \times \sqrt{3}} = \dfrac{1}{\sqrt{6}}$

176. (A) $\sin 3A = \cos(A - 26^\circ)$

$\Rightarrow \cos(90^\circ - 3A) = \cos(A - 26^\circ)$

$\Rightarrow 90^\circ - 3A = A - 26^\circ$

$\Rightarrow 90^\circ + 26^\circ = 3A + A$

$\Rightarrow 4A = 116$

$\Rightarrow A = \dfrac{116}{4} = 29^\circ$

177. (A) $\sec^2\theta - \dfrac{\sin^2\theta - 2\sin^4\theta}{2\cos^4\theta - \cos^2\theta}$

$= \sec^2\theta - \dfrac{\sin^2\theta(1 - 2\sin^2\theta)}{\cos^2\theta(2\cos^2\theta - 1)}$

$= \sec^2\theta - \dfrac{\sin^2\theta(1 - 2(1 - \cos^2\theta))}{\cos^2\theta(2\cos^2\theta - 1)}$

$= \sec^2\theta - \tan^2\theta\dfrac{(2\cos^2\theta - 1)}{2\cos^2\theta - 1}$

$= \sec^2\theta - \tan^2\theta = 1$

178. (C) $x = a(\sin\theta + \cos\theta)$ and $y = b(\sin\theta - \cos\theta)$

$\Rightarrow \dfrac{x}{a} = \sin\theta + \cos\theta$ and

$\dfrac{y}{b} = \sin\theta - \cos\theta$

$\therefore \dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = (\sin\theta + \cos\theta)^2 + (\sin\theta - \cos\theta)^2$

$= \sin^2\theta + \cos^2\theta + 2\sin\theta.\cos\theta + \sin^2\theta + \cos^2\theta - 2\sin\theta.\cos\theta$

$= 2(\sin^2\theta + \cos^2\theta) = 2$

179. (D) $\sin 5\theta = \cos 20^\circ$

$\Rightarrow \sin 5\theta = \sin(90 - 20)$

$= \sin 70^\circ$

$\Rightarrow 5\theta = 70^\circ$

$\Rightarrow \theta = \dfrac{70}{5} = 14^\circ$

180. (C) $2\sec\theta = 3\operatorname{cosec}^2\theta$

$\Rightarrow \dfrac{2}{\cos\theta} = \dfrac{3}{\sin^2\theta} = \dfrac{3}{1 - \cos^2\theta}$

$\Rightarrow 2 - 2\cos^2\theta = 3\cos\theta$

$\Rightarrow 2\cos^2\theta + 3\cos\theta - 2 = 0$

$\Rightarrow 2\cos^2\theta + 4\cos\theta - \cos\theta - 2 = 0$

$\Rightarrow 2\cos\theta(\cos\theta + 2) - 1(\cos\theta + 2) = 0$

$\Rightarrow (2\cos\theta - 1)(\cos\theta + 2) = 0$

$\therefore 2\cos\theta - 1 = 0$ as $\cos\theta + 2 \neq 0$

$\Rightarrow \cos\theta = \dfrac{1}{2} = \cos 60^\circ$ or $\cos\dfrac{\pi}{3}$

$\Rightarrow \theta = \dfrac{\pi}{3}$

181. (D) Expression

$= \sqrt{\dfrac{1 + \sin\theta}{1 - \sin\theta}} + \sqrt{\dfrac{1 - \sin\theta}{1 + \sin\theta}}$

$= \sqrt{\dfrac{(1 + \sin\theta)(1 + \sin\theta)}{(1 - \sin\theta)(1 + \sin\theta)}} + \sqrt{\dfrac{(1 - \sin\theta)(1 - \sin\theta)}{(1 + \sin\theta)(1 - \sin\theta)}}$

$= \sqrt{\dfrac{(1 + \sin\theta)^2}{1 - \sin^2\theta}} + \sqrt{\dfrac{(1 - \sin\theta)^2}{1 - \sin^2\theta}}$

$= \sqrt{\dfrac{(1 + \sin\theta)^2}{\cos^2\theta}} + \sqrt{\dfrac{(1 - \sin\theta)^2}{\cos^2\theta}}$

$= \dfrac{1 + \sin\theta}{\cos\theta} + \dfrac{1 - \sin\theta}{\cos\theta}$

$= \dfrac{1 + \sin\theta + 1 - \sin\theta}{\cos\theta} = \dfrac{2}{\cos\theta}$

$= 2\sec\theta$

182. (B) $\cos\theta = \dfrac{3}{5}$

$\therefore \sec\theta = \dfrac{5}{3}$

$\therefore \tan\theta = \sqrt{\sec^2\theta - 1}$

$\sqrt{\left(\dfrac{5}{3}\right)^2 - 1}$

$\sqrt{\dfrac{25}{9} - 1} = \sqrt{\dfrac{25 - 9}{9}} = \sqrt{\dfrac{16}{9}} = \dfrac{4}{3}$

$\therefore \sin\theta . \sec\theta . \tan\theta = \dfrac{\sin\theta}{\cos\theta}.\tan\theta$

$= \tan^2\theta = \left(\dfrac{4}{3}\right)^2 = \dfrac{16}{9}$

183. (D) $\tan^2 A + \cot^2 A - \sec^2 A \operatorname{cosec}^2 A$

$= \tan^2 A + \cot^2 A - (1 + \tan^2 A)(1 + \cot^2 A)$

$= \tan^2 A + \cot^2 A - (1 + \tan^2 A + \cot^2 A + \cot^2 A.\tan^2 A)$

$= \tan^2 A + \cot^2 A - 1 - \tan^2 A - \cot^2 A - \cot^2 A.\tan^2 A$

$= -1 - 1 = -2$

$[\tan A . \cot A = 1]$

184. (D) $\sin(4\alpha - \beta) = 1 = \sin 90^\circ$

$\Rightarrow 4\alpha - \beta = 90^\circ$ ... (i)

$\cos(2\alpha - \beta) = \dfrac{1}{2} = \cos 60^\circ$

$\Rightarrow 2\alpha + \beta = 60^\circ$ ... (ii)

On adding equations (i) and (ii),

$4\alpha - \beta + 2\alpha + \beta = 90^\circ + 60^\circ$

$\Rightarrow 6\alpha = 150^\circ \Rightarrow \alpha = \dfrac{150}{6} = 25^\circ$

From equation (ii),

$2 \times 25 + \beta = 60^\circ$

$\Rightarrow \beta = 60^\circ - 50^\circ = 10^\circ$

$\therefore \sin(\alpha + 2\beta)$

$= \sin(25 + 2 \times 10)$

$= \sin 45^\circ = \dfrac{1}{\sqrt{2}}$

185. (A) $\operatorname{cosec}\theta = \sqrt{3}$

$\cot\theta = \sqrt{\operatorname{cosec}^2\theta - 1}$

$= \sqrt{(\sqrt{3})^2 - 1} = \sqrt{3 - 1} = \sqrt{2}$

$\therefore \cot\theta - \operatorname{cosec}\theta = \sqrt{2} - \sqrt{3}$

$= \dfrac{3(\sqrt{2} - \sqrt{3})}{3} = (\sqrt{2} - \sqrt{3})$

186. (B) $4\cos^2\theta - 4\cos\theta + 1 = 0$

$\Rightarrow (2\cos\theta - 1)^2 = 0$

$\Rightarrow 2\cos\theta - 1 = 0$

$\Rightarrow 2\cos\theta = 1$

$\Rightarrow \cos\theta = \dfrac{1}{2} = \cos 60^\circ$

$\Rightarrow \theta = 60^\circ$

$\therefore \tan(\theta - 15^\circ) = \tan(60^\circ - 15^\circ)$

$= \tan 45^\circ = 1$

187. (A)

$(r\cos\theta - \sqrt{3})^2 + (r\sin\theta - 1)^2 = 0$

$\Rightarrow r\cos\theta - \sqrt{3} = 0$ and $r\sin\theta - 1 = 0$

$\Rightarrow r\cos\theta = \sqrt{3}$ and $r\sin\theta = 1$

$\cos\theta = \dfrac{\sqrt{3}}{r}$ and $\sin\theta = \dfrac{1}{r}$

Put $\theta = 30^\circ \Rightarrow r = 2$

$\dfrac{r\tan\theta + \sec\theta}{r\sec\theta + \tan\theta} = \dfrac{2 \times \frac{1}{\sqrt{3}} + \frac{2}{\sqrt{3}}}{2 \times \frac{2}{\sqrt{3}} + \frac{1}{\sqrt{3}}} = \dfrac{4}{5}$

188. (A)

$$\frac{\sin 25^\circ \cos 65^\circ + \cos 25^\circ \sin 65^\circ}{\tan^2 70^\circ - \operatorname{cosec}^2 20^\circ}$$

$$= \frac{\sin 25^\circ \cos(90^\circ - 25^\circ) + \cos 25^\circ \sin(90^\circ - 25)^\circ}{\tan^2 70^\circ - \operatorname{cosec}^2(90^\circ - 70^\circ)}$$

$$\begin{bmatrix} \because \sin(90^\circ - \theta) = \cos\theta \\ \cos(90^\circ - \theta) = \sin\theta \\ \operatorname{cosec}(90^\circ - \theta) = \sec\theta \end{bmatrix}$$

$$= \frac{\sin 25^\circ \sin 25^\circ + \cos 25^\circ . \cos 25^\circ}{\tan^2 70^\circ - \sec^2 70^\circ}$$

$$= \frac{\sin^2 25^\circ + \cos^2 25^\circ}{\tan^2 70^\circ - \sec^2 70^\circ}$$

$$= \frac{1}{-1} = -1$$

$$\begin{bmatrix} \because \sec^2\theta - \tan^2\theta = 1 \\ \sin^2\theta + \cos^2\theta = 1 \end{bmatrix}$$

189. (A) $\sin(\theta + 18^\circ) = \cos 60^\circ$

$= \cos(90^\circ - 30^\circ) = \sin 30^\circ$

$\Rightarrow \theta + 18^\circ = 30^\circ$

$\Rightarrow \theta = 30^\circ - 18^\circ = 12^\circ$

$\therefore \cos 5\theta = \cos 60^\circ = \frac{1}{2}$

190. (A) $\tan\theta = \frac{3}{4} \Rightarrow \tan^2\theta = \frac{9}{16}$

Expression

$$= \frac{4\sin^2\theta - 2\cos^2\theta}{4\sin^2\theta + 3\cos^2\theta}$$

$$= \frac{4\frac{\sin^2\theta}{\cos^2\theta} - 2\frac{\cos^2\theta}{\cos^2\theta}}{4\frac{\sin^2\theta}{\cos^2\theta} + 3\frac{\cos^2\theta}{\cos^2\theta}}$$

$$= \frac{4\tan^2\theta - 2}{4\tan^2\theta + 3}$$

$$= \frac{4 \times \frac{9}{16} - 2}{4 \times \frac{9}{16} + 3}$$

$$= \frac{\frac{9}{4} - 2}{\frac{9}{4} + 3} = \frac{9-8}{9+12} = \frac{1}{21}$$

191. (C) $\frac{\cos\alpha}{\cos\beta} = a$

$\Rightarrow \frac{\cos^2\alpha}{\cos^2\beta} = a^2$

$\Rightarrow \frac{1 - \sin^2\alpha}{1 - \sin^2\beta} = a^2$

$\Rightarrow 1 - \sin^2\alpha = a^2(1 - \sin^2\beta)$

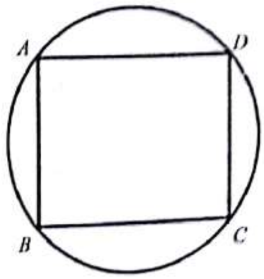
$\Rightarrow 1 - b^2\sin^2\beta = a^2 - a^2\sin^2\beta$

$\Rightarrow 1 - a^2 = b^2\sin^2\beta - a^2\sin^2\beta$

$\Rightarrow 1 - a^2 = (b^2 - a^2)\sin^2\beta$

$\Rightarrow \sin^2\beta = \frac{1 - a^2}{b^2 - a^2} = \frac{a^2 - 1}{a^2 - b^2}$

192. (A)



ABCD is a concyclic quadrilateral.

$\angle A + \angle C = \angle B + \angle D = 180^\circ$

$\therefore \angle A = 180^\circ - \angle C$

$\therefore \cos A = \cos(180^\circ - C)$

$= -\cos C$

and $\cos B = -\cos D$

$\therefore \cos A + \cos B + \cos C + \cos D$

$= \cos A + \cos B - \cos A - \cos B$

$= 0$

193. (C) $\sqrt{3}\tan\theta = 3\sin\theta$

$\Rightarrow \sqrt{3}\frac{\sin\theta}{\cos\theta} = 3\sin\theta$

$\Rightarrow \sqrt{3} = 3\cos\theta$

$\Rightarrow \cos\theta = \frac{\sqrt{3}}{3} = \frac{1}{\sqrt{3}}$

$\therefore \sin\theta = \sqrt{1 - \cos^2\theta}$

$= \sqrt{1 - \frac{1}{3}} = \sqrt{\frac{2}{3}}$

$\therefore \sin^2\theta - \cos^2\theta$

$$= \left(\sqrt{\frac{2}{3}}\right)^2 - \left(\frac{1}{\sqrt{3}}\right)^2$$

$$= \frac{2}{3} - \frac{1}{3} = \frac{1}{3}$$

194. (C) $A = 45^\circ$, $B = 30^\circ$ (let)

$\therefore \sin(A+B) = \sin A . \cos B + \cos A . \sin B$

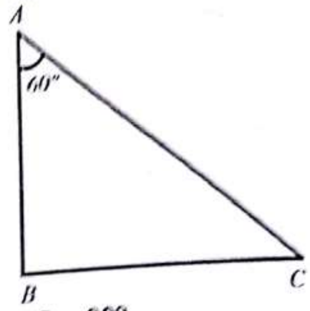
$\Rightarrow \sin(45^\circ + 30^\circ)$

$= \sin 45^\circ . \cos 30^\circ + \cos 45^\circ . \sin 30^\circ$

$$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{2}$$

$$= \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}} = \frac{\sqrt{3}+1}{2\sqrt{2}}$$

195. (D)



$\angle B = 90^\circ$

$\angle A = 60^\circ$

$\angle C = 180^\circ - 90^\circ - 60^\circ = 30^\circ$

$\cos C = \frac{BC}{CA}$

$\Rightarrow \cos 30^\circ = \frac{BC}{CA}$

$\Rightarrow \frac{\sqrt{3}}{2} = \frac{BC}{CA} = \sqrt{3} : 2$

196. (B) $\tan 2\theta . \tan 3\theta = 1$

$\Rightarrow \tan 3\theta = \frac{1}{\tan 2\theta} = \cot 2\theta$

$\Rightarrow \tan 3\theta = \tan(90^\circ - 2\theta)$

$\Rightarrow 3\theta = 90^\circ - 2\theta$

$\Rightarrow 3\theta + 2\theta = 5\theta = 90^\circ$

$\Rightarrow \theta = \frac{90^\circ}{5} = 18^\circ$

197. (C) $\cos^2\alpha - \sin^2\alpha = \tan^2\beta$

$\Rightarrow \cos^2\alpha - (1 - \cos^2\alpha) = \tan^2\beta$

$\Rightarrow 2\cos^2\alpha - 1 = \tan^2\beta$

$\Rightarrow 2\cos^2\alpha = 1 + \tan^2\beta = \sec^2\beta$

$\Rightarrow \cos^2\beta = \frac{1}{2\cos^2\alpha}$

$\sin^2\beta = 1 - \cos^2\beta$

$= 1 - \frac{1}{2\cos^2\alpha}$

$= \frac{2\cos^2\alpha - 1}{2\cos^2\alpha}$

$\therefore \cos^2\beta - \sin^2\beta$

$$= \frac{1}{2\cos^2\alpha} - \frac{2\cos^2\alpha - 1}{2\cos^2\alpha}$$

$$= \frac{1 - 2\cos^2\alpha + 1}{2\cos^2\alpha}$$

$$= \frac{2(1 - \cos^2\alpha)}{2\cos^2\alpha} = \frac{\sin^2\alpha}{\cos^2\alpha}$$

$= \tan^2\alpha$

198. (C) $\tan(A+B) = \sqrt{3} = \tan 60^\circ$

$\Rightarrow A + B = 60^\circ$ ... (i)

$\tan(A - B) = \frac{1}{\sqrt{3}} = \tan 30^\circ$

$\Rightarrow A - B = 30^\circ$ ... (ii)

$\therefore A + B + A - B = 60^\circ + 30^\circ$

$\Rightarrow 2A = 90^o$

$\Rightarrow A = \frac{90^o}{2} = 45^o$

199. (C) Expression

$= \frac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta}$

$= \frac{\sin\theta(1-2\sin^2\theta)}{\cos\theta(2\cos^2\theta - 1)}$

$= \frac{\sin\theta}{\cos\theta} \cdot \frac{(1-2(1-\cos^2\theta))}{(2\cos^2\theta-1)}$

$= \tan\theta \frac{(1+2\cos^2\theta-2)}{(2\cos^2\theta-1)}$

$= \tan\theta \cdot \frac{2\cos^2\theta-1}{2\cos^2\theta-1} = \tan\theta$

200. (D) $r\sin\theta = \frac{7}{2}$ ... (i)

$r\cos\theta = \frac{7\sqrt{3}}{2}$ ... (ii)

On squaring both equations and adding

$r^2\sin^2\theta + r^2\cos^2\theta$

$= \left(\frac{7}{2}\right)^2 + \left(\frac{7\sqrt{3}}{2}\right)^2$

$\Rightarrow r^2(\sin^2\theta + \cos^2\theta)$

$= \frac{49}{4} + \frac{147}{4}$

$\Rightarrow r^2 = \frac{49+147}{4} = \frac{196}{4} = 49$

$\therefore r = \sqrt{49} = 7$

201. (D) $\sin\theta = \frac{1}{2} = \sin 30^o = \sin\frac{\pi}{6}$

$\Rightarrow \theta = \frac{\pi}{6}$

$[\because 180^o = \pi \text{ radian}]$

$\therefore \theta + \phi = \frac{\pi}{2} \Rightarrow \frac{\pi}{6} + \phi = \frac{\pi}{2}$

$\Rightarrow \phi = \frac{\pi}{2} - \frac{\pi}{6} = \frac{3\pi - \pi}{6}$

$= \frac{2\pi}{6} = \frac{\pi}{3}$

$\therefore \sin\phi = \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$

202 (B) $2\sin^2\theta + 3\cos\theta = 3$

$\Rightarrow 2(1-\cos^2\theta) + 3\cos\theta = 3$

$\Rightarrow 2 - 2\cos^2\theta + 3\cos\theta = 3$

$\Rightarrow 2\cos^2\theta - 3\cos\theta + 1 = 0$

$\Rightarrow 2\cos^2\theta - 2\cos\theta$
$-\cos\theta + 1 = 0$

$\Rightarrow 2\cos\theta(\cos\theta - 1)$
$-1(\cos\theta - 1) = 0$

$\Rightarrow 2\cos\theta - 1 = 0 \Rightarrow 2\cos\theta = 1$

$\Rightarrow \cos\theta = \frac{1}{2} \Rightarrow \theta = 60^o$

or, $\cos\theta - 1 = 0 \Rightarrow \cos\theta = 1$

$\Rightarrow \theta = 0^o$

203. (A) $2\sin^2\theta = 3\cos\theta$

$\Rightarrow 2(1-\cos^2\theta) = 3\cos\theta$

$\Rightarrow 2 - 2\cos^2\theta = 3\cos\theta$

$\Rightarrow 2\cos^2\theta + 3\cos\theta - 2 = 0$

$\Rightarrow 2\cos^2\theta + 4\cos\theta - \cos\theta - 2 = 0$

$\Rightarrow (2\cos\theta - 1)(\cos\theta + 2) = 0$

$\Rightarrow 2\cos\theta - 1 = 0$ because $\cos\theta + 2 \neq 0$

$\Rightarrow 2\cos\theta = 1$

$\Rightarrow \cos\theta = \frac{1}{2} = \cos 60^o$

$\Rightarrow \theta = 60^o$

204. (C) $a(\tan\theta + \cot\theta) = 1$

$\Rightarrow a\left(\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}\right) = 1$

$\Rightarrow a\left(\frac{\sin^2\theta + \cos^2\theta}{\sin\theta.\cos\theta}\right) = 1$

$\Rightarrow \sin\theta \,.\, \cos\theta = a$ ... (i)

$\sin\theta + \cos\theta = b$

On squaring both sides,

$\sin^2\theta + \cos^2\theta + 2\sin\theta.\cos\theta = b^2$

$\Rightarrow 1 + 2a = b^2$

$\Rightarrow 2a = b^2 - 1$

205. (B) $\text{cosec}^2A - \cot^2A = 1$

$(\text{cosec}A + \cot A)$
$(\text{cosec}A - \cot A) = 1$

$\text{cosec}A - \cot A = \frac{1}{3}$

$\text{cosec}\,A + \cot A = 3$

On adding

$2\,\text{cosec}A = \frac{1}{3} + 3$

$= \frac{1+9}{3} = \frac{10}{3}$

$\Rightarrow \text{cosec}A = \frac{10}{3\times 2} = \frac{5}{3}$

$\therefore \sin A = \frac{3}{5}$

206. (C)

$\sin^2 x + 2\tan^2 x - 2\sec^2 x + \cos^2 x$

$= \sin^2 x + \cos^2 x - 2\sec^2 x$
$+2\tan^2 x$

$= 1 - 2(\sec^2 x - \tan^2 x)$

$= 1 - 2 = -1$

$[\sec^2 x - \tan^2 x = 1,$
$\text{sen}^2x + \cos^2 x = 1]$

207. (A) $x = a\sec\theta$

$\Rightarrow \frac{x}{a} = \sec\theta$

Again, $y = b\tan\theta$

$\Rightarrow \frac{y}{b} = \tan\theta$

$\therefore \frac{x^2}{a^2} - \frac{y^2}{b^2}$

$= \sec^2\theta - \tan^2\theta = 1$

208. (D) $\sin^2 1^o + \sin^2 2^o + \sin^2 3^o$
$+ .... + \sin^2 89^o$

$= (\sin^2 1^o + \sin^2 89^o)$
$+(\sin^2 2^o + \sin^2 88^o)$

$+ ....$ to 44 terms $+ \sin^2 45^o$

$= (\sin^2 1^o + \sin^2(90^o - 1^o))$
$+(\sin^2 2^o + \sin^2(90^o - 2^o)) + ...$ to 44

terms $+ \left(\frac{1}{\sqrt{2}}\right)^2$

$= (\sin^2 1^o + \cos^2 1^o)$
$+(\sin^2 2^o + \cos^2 2^o)$

$+ ....$ to 44 terms $+ \frac{1}{2}$,

$[\sin(90^o - \theta) = \cos\theta]$

$= 1 + 1 + ...$to44 terms $+ \frac{1}{2}$

$[\sin^2\theta + \cos^2\theta = \cos\theta]$

$= 44 + \frac{1}{2} = 44\frac{1}{2}$

209. (C) $\frac{\cos^3\theta + \sin^3\theta}{\cos\theta + \sin\theta} + \frac{\cos^3 - \sin^3\theta}{\cos\theta - \sin\theta}$

$\frac{\cos\theta + \sin\theta)(\cos^2\theta + \sin^2\theta - \cos\theta.\sin\theta)}{\cos\theta + \sin\theta}$

$\frac{(\cos\theta - \sin\theta)(\cos^2\theta + \sin^2\theta + \cos\theta.\sin\theta)}{\cos\theta - \sin\theta}$

$= \cos^2\theta + \sin^2\theta - \cos\theta.\sin\theta$
$+ \cos^2\theta + \sin^2\theta + \cos\theta.\sin\theta$

$= 1 + 1 = 2$

210. (D) $\sin 17^o = \frac{x}{y}$

$\sin 73^o = \sin(90^o - 17^o) = \cos 17^o$

$\therefore \cos 17^o = \sqrt{1 - \sin^2 17^o}$

$= \sqrt{1 - \frac{x^2}{y^2}} = \sqrt{\frac{y^2 - x^2}{y^2}}$

$= \frac{\sqrt{y^2 - x^2}}{y}$

$\therefore \sec 17^o = \frac{y}{\sqrt{y^2 - x^2}}$

$\therefore \sec 17^o - \sin 73^o$

$= \sec 17^\circ - \cos 17^\circ$

$= \dfrac{y}{\sqrt{y^2-x^2}} - \dfrac{\sqrt{y^2-x^2}}{y}$

$= \dfrac{y^2-(y^2-x^2)}{y\sqrt{y^2-x^2}}$

$= \dfrac{y^2-y^2+x^2}{y\sqrt{y^2-x^2}}$

$= \dfrac{x^2}{y\sqrt{y^2-x^2}}$

211. (C) $\operatorname{cosec}\theta + \cot\theta = \sqrt{3}$ ... (i)

$\operatorname{cosec}^2\theta - \cot^2\theta = 1$

$\Rightarrow (\operatorname{cosec}\theta + \cot\theta)$

$(\operatorname{cosec}\theta - \cot\theta) = 1$

$\Rightarrow \operatorname{cosec}\theta - \cot\theta = \dfrac{1}{\sqrt{3}}$ ... (ii)

$\therefore \operatorname{cosec}\theta + \cot\theta + \operatorname{cosec}\theta$

$-\cot\theta = \sqrt{3} + \dfrac{1}{\sqrt{3}}$

$\Rightarrow 2\operatorname{cosec}\theta = \dfrac{3+1}{\sqrt{3}}$

$\Rightarrow \operatorname{cosec}\theta = \dfrac{4}{2\sqrt{3}} = \dfrac{2}{\sqrt{3}}$

212. (C) $\cos\alpha + \sec\alpha = \sqrt{3}$

$\therefore \cos^3\alpha + \sec^3\alpha$

$= (\cos\alpha + \sec\alpha)^3$

$-3\cos\alpha.\sec\alpha(\cos\alpha + \sec\alpha)$

$= (\sqrt{3})^3 - 3 \times \sqrt{3}$

$3\sqrt{3} - 3\sqrt{3} = 0$

213. (A) $\sin\theta + \cos\theta = \sqrt{2}\cos\theta$

$\Rightarrow \sqrt{2}\cos\theta - \cos\theta = \sin\theta$

$\Rightarrow \cos\theta(\sqrt{2}-1) = \sin\theta$

$\Rightarrow \dfrac{\cos\theta}{\sin\theta} = \dfrac{1}{\sqrt{2}-1}$

$= \dfrac{\sqrt{2}+1}{(\sqrt{2}-1)(\sqrt{2}+1)} = \sqrt{2}+1$

$\cot\theta = \sqrt{2}+1$

214. (A) $\cos^4\theta - \sin^4\theta = \dfrac{2}{3}$

$\Rightarrow (\cos^2\theta + \sin^2\theta)(\cos^2\theta - \sin^2\theta) = \dfrac{2}{3}$

$[\because \cos^2\theta + \sin^2\theta = 1]$

$\Rightarrow \cos^2\theta - \sin^2\theta = \dfrac{2}{3}$

$\Rightarrow 1 - \sin^2\theta - \sin^2\theta = \dfrac{2}{3}$

$\Rightarrow 1 - 2\sin^2\theta = \dfrac{2}{3}$

215. (A) $\dfrac{\cot 30^\circ - \cot 75^\circ}{\tan 15^\circ - \tan 60^\circ}$

$= \dfrac{\cot(90^\circ-60^\circ) - \cot(90^\circ-15^\circ)}{\tan 15^\circ - \tan 60^\circ}$

$= \dfrac{\tan 60^\circ - \tan 15^\circ}{\tan 15^\circ - \tan 60^\circ} = -1$

216. (C) $\sin\theta + \cos\theta = p$

$\sec\theta + \operatorname{cosec}\theta = q$

$\Rightarrow \dfrac{1}{\cos\theta} + \dfrac{1}{\sin\theta} = q$

$\Rightarrow \dfrac{\sin\theta + \cos\theta}{\sin\theta\,.\,\cos\theta} = q$

$\therefore q(p^2-1) = \left(\dfrac{\sin\theta + \cos\theta}{\sin\theta.\cos\theta}\right)$

$((\sin\theta + \cos\theta)^2 - 1)$

$= \dfrac{\sin\theta + \cos\theta}{\sin\theta.\cos\theta}.(\sin^2\theta + \cos^2\theta$

$+2\sin\theta.\cos\theta - 1)$

$= \dfrac{\sin\theta + \cos\theta}{\sin\theta.\cos\theta}.2\sin\theta.\cos\theta = 2p$

217. (B) $\sin(3\alpha - \beta) = 1 = \sin 90^\circ$

$\Rightarrow 3\alpha - \beta = 90^\circ$ ... (i)

$\cos(2\alpha + \beta) = \dfrac{1}{2} = \cos 60^\circ$

$\Rightarrow 2\alpha + \beta = 60^\circ$ ... (ii)

By adding both equations,

$3\alpha + 2\alpha = 90^\circ + 60^\circ$

$\Rightarrow 5\alpha = 150$

$\Rightarrow \alpha = \dfrac{150}{5} = 30^\circ$

$\therefore \tan\alpha = \tan 30^\circ = \dfrac{1}{\sqrt{3}}$

218. (B) $\sin(60^\circ - x) = \cos(y + 60^\circ)$

$\Rightarrow \sin(60^\circ - x) = \sin(90^\circ - y - 60^\circ)$

$[\because \sin(90^\circ - \theta) = \cos\theta]$

$\Rightarrow 60^\circ - x = 90^\circ - y - 60^\circ$

$= 30^\circ - y$

$\Rightarrow x - y = 60^\circ - 30^\circ$

$\Rightarrow x - y = 30^\circ$

$\therefore \sin(x-y) = \sin 30^\circ = \dfrac{1}{2}$

219. (C) $x = a\sec\theta \Rightarrow \dfrac{x}{a} = \sec\theta$

and $y = b\tan\theta \Rightarrow \dfrac{y}{b} = \tan\theta$

$\therefore \dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = \sec^2\theta - \tan^2\theta = 1$

220. (D) $a^2 + b^2 + c^2 = ab + bc + ca$

$\Rightarrow 2a^2 + 2b^2 + 2c^2 = 2ab$

$+2bc + 2ca$

$\Rightarrow a^2 + b^2 + b^2 + c^2 + c^2 + a^2$

$-2ab - 2bc - 2ca = 0$

$\Rightarrow a^2 + b^2 - 2ab + b^2 + c^2 - 2bc$

$+c^2 + a^2 - 2ca = 0$

$\Rightarrow (a-b)^2 + (b-c)^2 + (c-a)^2 = 0$

$\Rightarrow a - b = 0 \Rightarrow a = b$

$b - c = 0 \Rightarrow b = c$

$c - a = 0 \Rightarrow c = a$

$\therefore$ ΔABC is an equilateral triangle

$\therefore \angle A = \angle B = \angle C = 60^\circ$

$\therefore \sin^2 A + \sin^2 B + \sin^2 C$

$= 3\sin^2 A = 3 \times \sin^2 60^\circ$

$= 3 \times \left(\dfrac{\sqrt{3}}{2}\right)^2$

$= \dfrac{3 \times 3}{4} = \dfrac{9}{4}$

221. (C) $a\sin\theta + b\cos\theta = c$ ... (i)

$a\cos\theta - b\sin\theta = x$ (let) ... (ii)

On squairng equations (i) and (ii) and adding,

$a^2\sin^2\theta + b^2\cos^2\theta + 2ab\sin\theta.$

$\cos\theta$

$\cos\theta + a^2\cos^2\theta + b^2\sin^2\theta$

$-2ab\sin\theta.\cos\theta = c^2 + x^2$

$\Rightarrow a^2(\sin^2\theta + \cos^2\theta) + b^2$

$(\cos^2\theta + \sin^2\theta) = c^2 + x^2$

$\Rightarrow a^2 + b^2 = c^2 + x^2$

$\Rightarrow x^2 = a^2 + b^2 - c^2$

$\Rightarrow x = \pm\sqrt{a^2 + b^2 - c^2}$

222. (C) $\sin\theta + \cos\theta = \sqrt{2}\sin(90^\circ - \theta)$

$\Rightarrow \sin\theta + \cos\theta = \sqrt{2}\cos\theta$

$\Rightarrow \sin\theta = \sqrt{2}\cos\theta - \cos\theta$

$\Rightarrow \sin\theta = \cos\theta(\sqrt{2}-1)$

$\Rightarrow \dfrac{\cos\theta}{\sin\theta} = \dfrac{1}{\sqrt{2}-1}$

$\Rightarrow \cot\theta = \dfrac{1}{\sqrt{2}-1} \times \dfrac{(\sqrt{2}+1)}{(\sqrt{2}+1)}$

$= \dfrac{\sqrt{2}+1}{2-1} = \sqrt{2}+1$

223. (A) $3(\sec^2\theta + \tan^2\theta) = 5$

$\Rightarrow \sec^2\theta + \tan^2\theta = \dfrac{5}{3}$

$\Rightarrow \sec^2\theta + \sec^2\theta - 1 = \dfrac{5}{3}$

$\Rightarrow 2\sec^2\theta = \dfrac{5}{3} + 1 = \dfrac{8}{3}$

$\Rightarrow \sec^2\theta = \dfrac{4}{3} \Rightarrow \sec\theta = \dfrac{2}{\sqrt{3}}$

$\Rightarrow \cos\theta = \dfrac{\sqrt{3}}{2} = \cos 30^\circ$

$\Rightarrow \theta = 30^\circ$

$\therefore \cos 2\theta = \cos 60^\circ = \frac{1}{2}$

224. (C) $x.\cos^2 30^\circ.\sin 60^\circ = \frac{\tan^2 45^\circ.\sec 60^\circ}{\operatorname{cosec} 60^\circ}$

$\Rightarrow x.\left(\frac{\sqrt{3}}{2}\right)^2.\frac{\sqrt{3}}{2} = \frac{1\times 2}{\frac{2}{\sqrt{3}}}$

$\Rightarrow x \times \frac{3}{4} \times \frac{\sqrt{3}}{2} = \sqrt{3}$

$\Rightarrow x = \frac{\sqrt{3}\times 8}{3\sqrt{3}} = \frac{8}{3} = 2\frac{2}{3}$

225. (B) $\tan\alpha = 2$

$\therefore \frac{\operatorname{cosec}^2\alpha - \sec^2\alpha}{\operatorname{cosec}^2\alpha + \sec^2\alpha}$

$= \frac{1+\cot^2\alpha - 1 - \tan^2\alpha}{1+\cot^2\alpha + 1 + \tan^2\alpha}$

$= \frac{\cot^2\alpha - \tan^2\alpha}{\cot^2\alpha + \tan^2\alpha + 2}$

$= \frac{\frac{1}{4} - 4}{\frac{1}{4} + 4 + 2} = \frac{\frac{1-16}{4}}{\frac{1+16+8}{4}}$

$= \frac{-15}{25} = \frac{-3}{5}$

226. (C) $\sin(\theta + 30^\circ) = \frac{3}{\sqrt{12}}$

$= \frac{3}{2\sqrt{3}} = \frac{\sqrt{3}}{2}$

$\Rightarrow \sin(\theta + 30^\circ) = \sin 60^\circ$

$\Rightarrow \theta + 30^\circ = 60^\circ$

$\Rightarrow \theta = 60 - 30 = 30^\circ$

$\therefore \cos^2\theta = \cos^2 30^\circ$

$= \left(\frac{\sqrt{3}}{2}\right)^2 = \frac{3}{4}$

227. (A) $4\cos^2\theta - 4\sqrt{3}\cos\theta + 3 = 0$

$\Rightarrow (2\cos\theta)^2 - 2.2\cos\theta.\sqrt{3} + (\sqrt{3})^2 = 0$

$\Rightarrow (2\cos\theta - \sqrt{3})^2 = 0$

$\Rightarrow 2\cos\theta - \sqrt{3} = 0$

$\Rightarrow 2\cos\theta = \sqrt{3}$

$\Rightarrow \cos\theta = \frac{\sqrt{3}}{2} = \cos 30^\circ$

$\Rightarrow \theta = 30^\circ$

228. (C) $\sec\theta - \cos\theta = \frac{3}{2}$

$\Rightarrow \sec\theta - \frac{1}{\sec\theta} = \frac{3}{2}$

$\Rightarrow \frac{\sec^2\theta - 1}{\sec\theta} = \frac{3}{2}$

$\Rightarrow 2\sec^2\theta - 2 = 3\sec\theta$

$\Rightarrow 2\sec^2\theta - 3\sec\theta - 2 = 0$

$\Rightarrow \sec^2\theta - 4\sec\theta + \sec\theta - 2 = 0$

$\Rightarrow 2\sec\theta(\sec\theta - 2) + 1(\sec\theta - 2) = 0$

$\Rightarrow (2\sec\theta + 1)(\sec\theta - 2) = 0$

$\Rightarrow \sec\theta = 2$ because $2\sec\theta + 1 \neq 0$ θ is positive acute angle

229. (B) $\tan(5x - 10^\circ) = \cot(5y + 20^\circ)$

$\Rightarrow \tan(5x - 10^\circ) = \tan(90^\circ - 5y + 20^\circ))$

$\Rightarrow 5x - 10^\circ = 90^\circ - 5y - 20^\circ$

$\Rightarrow 5x + 5y = 70^\circ + 10^\circ$

$\Rightarrow 5(x + y) = 80^\circ$

$\Rightarrow x + y = \frac{80^\circ}{5} = 16^\circ$

230. (D) $\sin\theta + \sin^2\theta = 1$

$\Rightarrow \sin\theta = 1 - \sin^2\theta = \cos^2\theta$

$\Rightarrow \cos^2\theta + \cos^4\theta = 1$

Now,

$\cos^{12}\theta + 3\cos^{10}\theta + 3\cos^8\theta + \cos^6\theta - 1$

$= (\cos^4\theta + \cos^2\theta)^3 - 1$

$= (\sin^2\theta + \cos^2\theta)^3 - 1 = 1 - 1 = 0$

231. (C) $\tan 11^\circ.\tan 17^\circ.\tan 79^\circ.\tan 73^\circ$

$= \tan 11^\circ.\tan 17^\circ.\tan(90^\circ - 11^\circ).\tan(90^\circ - 17^\circ)$

$= \tan 11^\circ . \tan 17^\circ . \cot 11^\circ . \cot 17^\circ$

$= \tan 11^\circ . \cot 11^\circ . \tan 17^\circ . \cot 17^\circ$

$= 1 \times 1 = 1$

$[\because \tan(90^\circ - \theta) = \cot\theta; \tan\theta.\cot\theta = 1]$

232. (B) $\sin A + \sin^2 A = 1$

$\Rightarrow \sin A = 1 - \sin^2 A = \cos^2 A$

$\therefore \cos^2 A + \cos^4 A$

$= \cos^2 A + (\cos^2 A)^2$

$= \cos^2 A + \sin^2 A = 1$

233. (C) $(1 + \sec 20^\circ + \cot 70^\circ)(1 - \operatorname{cosec} 20^\circ + \tan 70^\circ)$

$= (1 + \sec 20^\circ + \tan 20^\circ)(1 - \operatorname{cosec} 20^\circ + \cot 20^\circ)$

$\left[\because \tan(90^\circ - \theta) = \cot\theta; \cot(90^\circ - \theta) = \tan\theta\right]$

$= \left(1 + \frac{1}{\cos 20^\circ} + \frac{\sin 20^\circ}{\cos 20^\circ}\right)\left(1 - \frac{1}{\sin 20^\circ} + \frac{\cos 20^\circ}{\sin 20^\circ}\right)$

$= \frac{\cos 20^\circ + 1 + \sin 20^\circ}{\cos 20^\circ} \times \frac{\sin 20^\circ - 1 + \cos 20^\circ}{\sin 20^\circ}$

$= \frac{\cos^2 20^\circ + \sin^2 20^\circ + 2\sin 20^\circ.\cos 20^\circ - 1}{\sin 20^\circ.\cos 20^\circ}$

$= \frac{1 + 2\sin 20^\circ.\cos 20^\circ - 1}{\sin 20^\circ.\cos 20^\circ} = 2$

234. (A) $\frac{\tan A - \sec A - 1}{\tan A + \sec A + 1}$

$= \frac{\tan A - \sec A - (\sec^2 A - \tan^2 A)}{\tan A + \sec A + 1}$

$= \frac{(\tan A - \sec A) - (\sec A - \tan A)(\sec A + \tan A)}{\tan A + \sec A + 1}$

$= \frac{(\tan A - \sec A) + (\tan A - \sec A)(\sec A + \tan A)}{\tan A + \sec A + 1}$

$= \frac{(\tan A - \sec A)(1 + \sec A + \tan A)}{\tan A + \sec A + 1}$

$= \tan A - \sec A = \frac{\sin A}{\cos A} - \frac{1}{\cos A}$

$= \frac{\sin A - 1}{\cos A}$

235. (D) $2\sin\alpha + 15\cos^2\alpha = 7$

$\Rightarrow 2\sin\alpha + 15(1 - \sin^2\alpha) = 7$

$\Rightarrow 2\sin\alpha + 15 - 15\sin^2\alpha = 7$

$\Rightarrow 15\sin^2\alpha - 2\sin\alpha - 15 + 7 = 0$

$\Rightarrow 15\sin^2\alpha - 2\sin\alpha - 8 = 0$

$\Rightarrow 15\sin^2\alpha - 12\sin\alpha + 10\sin\alpha - 8 = 0$

$\Rightarrow 3\sin\alpha(5\sin\alpha - 4) + 2(5\sin\alpha - 4) = 0$

$\Rightarrow (3\sin\alpha + 2)(5\sin\alpha - 4) = 0$

$\Rightarrow 5\sin\alpha - 4 = 0 \Rightarrow \sin\alpha = \frac{4}{5}$

$\sin\alpha \neq -\frac{2}{3}$ because α is acute angle.

$\therefore \operatorname{cosec}\alpha = \frac{1}{\sin\alpha} = \frac{5}{4}$

$\therefore \cot\alpha = \sqrt{\operatorname{cosec}^2\alpha - 1}$

$= \sqrt{\left(\frac{5}{4}\right)^2 - 1} = \sqrt{\frac{25}{16} - 1}$

$= \sqrt{\frac{25-16}{16}} = \sqrt{\frac{9}{16}} = \frac{3}{4}$

236. (D) Let, $A = 45^\circ$

$B = 30^\circ$

$\sin(A - B) =$

$= \sin A . \cos B - \cos A . \sin B$

$\Rightarrow \sin(45^\circ - 30^\circ)$

$= \sin 45^\circ.\cos 30^\circ - \cos 45^\circ.\sin 30^\circ$

$\Rightarrow \sin 15^\circ$

$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \times \frac{1}{2}$

$= \frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}} = \frac{\sqrt{3}-1}{2\sqrt{2}}$

237. (B) $\sec x + \cos x = 2$

$\Rightarrow \frac{1}{\cos x} + \cos x = 2$

$\Rightarrow \frac{1+\cos^2 x}{\cos x} = 2$

$\Rightarrow \cos^2 x + 1 = 2\cos x$

$\Rightarrow \cos^2 x - 2\cos x + 1 = 0$

$\Rightarrow (\cos x - 1)^2 = 0 \Rightarrow \cos - 1 = 0$

$\Rightarrow \cos x = 1$

$\therefore \sec x = 1$

$\therefore \sec^{16} x + \cos^{16} x = 1 + 1 = 2$

238. (A)

$\sin^4\theta + \cos^4\theta = 2\sin^2\theta.\cos^2\theta$

$\Rightarrow \sin^4\theta + \cos^4\theta - 2\sin^2\theta$

$\cos^2\theta = 0$

$\Rightarrow (\sin^2\theta - \cos^2\theta)^2 = 0$

$\Rightarrow \sin^2\theta - \cos^2\theta = 0$

$\Rightarrow \sin^2\theta = \cos^2\theta$

$= \tan^2\theta = 1 \Rightarrow \tan\theta = +1$

$\because \theta$ is acute angle

239. (B) Expression

$= \sin^4\theta + \cos^4\theta$

$= (\sin^2\theta)^2 + (\cos^2\theta)^2$

$= (\sin^2\theta + \cos^2\theta)^2 - 2\sin^2\theta.\cos^2\theta$

$= 1 - 2\sin^2\theta.\cos^2\theta$

$= 1 - \frac{4\sin^2\theta.\cos^2\theta}{2}$

$[\because \sin 2\theta = 2\sin\theta.\cos\theta]$

$= 1 - \frac{\sin^2 2\theta}{2}$

$= 1 - \frac{1-\cos 4\theta}{4}$

$[\because 1 - \cos 2\theta = 2\cos^2\theta]$

$= 1 - \frac{1}{4} + \frac{\cos 4\theta}{4}$

$= 1 - \frac{1}{4} + \frac{1}{4} = 1$

$(\cos 4\theta \le 1)$

(OR)

The value of $\sin^4\theta + \cos^4\theta$ will be maximum if $\theta = 0^o$

$\therefore$ Required value

$= (\sin 0)^4 + (\cos 0)^4 = 0 + 1 = 1$

240. (B)

$\tan 86^o = \cot(90^o - 86^o) = \cot 4^o$

$\tan 47^o = \cot(90^o - 47^o) = \cot 43^o$

$\therefore (\tan 4^o.\tan 86^o)(\tan 43^o.\tan 47^o)$

$= (\tan 4^o; \cot 4^o)(\tan 43^o.\cot 43^o)$

$= 1\ (\because \tan\theta.\cot\theta = 1)$

241. (B) $x\cos\theta - \sin\theta = 1$

$\Rightarrow x\cos\theta = 1 + \sin\theta$

$\Rightarrow x = \frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta}$

$\Rightarrow x = \sec\theta + \tan\theta$ ... (i)

$\because \sec^2\theta - \tan^2\theta = 1$

$\Rightarrow (\sec\theta + \tan\theta)(\sec\theta - \tan\theta) = 1$

$\Rightarrow \sec\theta - \tan\theta = \frac{1}{x}$ ... (ii)

From equation (i) + (ii),

$2\sec\theta = x + \frac{1}{x} = \frac{x^2+1}{x}$

$\Rightarrow \sec\theta = \frac{x^2+1}{2x}$

From equation (i) − (ii),

$2\tan\theta = x - \frac{1}{x} = \frac{x^2-1}{x}$

$\therefore \tan\theta = \frac{x^2-1}{2x}$

$\therefore \sin\theta = \frac{\tan\theta}{\sec\theta}$

$= \frac{x^2-1}{2x} \times \frac{2x}{x^2+1} = \frac{x^2-1}{x^2+1}$

$\therefore$ Expression

$= x^2 - (1+x^2)\sin\theta$

$= x^2 - (1+x^2) \times \frac{x^2-1}{x^2+1}$

$= x^2 - x^2 + 1 = 1$

Note : In the original equation $x^2 + (1+x^2)\sin\theta$ has been given that seems incorrect.

(or)

Put $\theta = 0^\circ$ then $x = 1$

$x^2 + (1+x^2)\sin\theta = 1 + (1+1).0$

$= 1.$

242. (B) $\sin\theta + \sin^2\theta = 1$

$\Rightarrow \sin\theta = 1 - \sin^2\theta = \cos^2\theta$

$\therefore \cos^2\theta + \cos^4\theta$

$= \cos^2\theta + (\cos^2\theta)^2$

$= \cos^2\theta + \sin^2\theta = 1$

243. (B) $\frac{\cos^2 45^o}{\sin^2 60^o} + \frac{\cos^2 60^o}{\sin^2 45^o}$

$- \frac{\tan^2 30^o}{\cot^2 45^o} - \frac{\sin^2 30^o}{\cot^2 30^o}$

$= \frac{\left(\frac{1}{\sqrt{2}}\right)^2}{\left(\frac{\sqrt{3}}{2}\right)^2} + \frac{\left(\frac{1}{2}\right)^2}{\left(\frac{1}{\sqrt{2}}\right)^2}$

$- \frac{\left(\frac{1}{\sqrt{3}}\right)^2}{(1)^2} - \frac{\left(\frac{1}{2}\right)^2}{\left(\sqrt{3}\right)^2}$

$= \frac{1}{2} \times \frac{4}{3} + \frac{1}{4} \times 2 - \frac{1}{3} \times 1 - \frac{1}{4 \times 3}$

$= \frac{2}{3} + \frac{1}{2} - \frac{1}{3} - \frac{1}{12}$

$= \frac{8+6-4-1}{12} = \frac{9}{12} = \frac{3}{4}$

244. (A) $\tan(90^o - \theta) = \cot\theta$

$\tan\theta.\cot\theta = 1$

$\tan 89^\circ = \tan(90^\circ - 1^\circ) = \cot 1^\circ$

$\tan 88^\circ = \tan(90^\circ - 2^\circ) = \cot 2^\circ$

$\therefore$ Expression

$= (\tan 1^o.\tan 89^o)$

$(\tan 2^o.\tan 88^o)$ .... $\tan 45^o$

$= (\tan 1^o.\cot 1^o).(\tan 2^o.\cot 2^o)$

..... $\tan 45^o$

$= 1.1 \ldots\ldots 1 = 1$

245. (C) $\frac{\cos\alpha}{\sin\beta} = n$ and $\frac{\cos\alpha}{\cos\beta} = m$

$\Rightarrow \cos\alpha = n\sin\beta$ and

$\cos\alpha = m\cos\beta.$

$\therefore n^2\sin^2\beta = m^2\cos^2\beta$

$\Rightarrow n^2(1-\cos^2\beta) = m^2\cos^2\beta$

$\Rightarrow n^2 - n^2\cos^2\beta = m^2\cos^2\beta$

$\Rightarrow m^2\cos^2\beta + n^2\cos^2\beta = n^2$

$\Rightarrow \cos^2\beta(m^2+n^2) = n^2$

$\Rightarrow \cos^2\beta = \frac{n^2}{m^2+n^2}$

246. (C) $\sin A.\cos A(\tan A - \cot A)$

$= \sin A.\cos A\left(\frac{\sin A}{\cos A} - \frac{\cos A}{\sin A}\right)$

$= \sin A.\cos A\left(\frac{\sin^2 A - \cos^2 A}{\sin A.\cos A}\right)$

$= \sin^2 A - \cos^2 A$

$= \sin^2 A - (1 - \sin^2 A)$

$= \sin^2 A - 1 + \sin^2 A$

$= 2\sin^2 A - 1$

247. (B) $\tan^2\theta + \frac{1}{\tan^2\theta} = 2$

$\Rightarrow \frac{\tan^4\theta + 1}{\tan^2\theta} = 2$

$\Rightarrow \tan^4\theta + 1 = 2\tan^2\theta$

$\Rightarrow \tan^4\theta - 2\tan^2\theta + 1 = 0$

$\Rightarrow (\tan^2\theta - 1)^2 = 0$

$\Rightarrow \tan^2\theta - 1 = 0$

$\Rightarrow \tan^2\theta = 1$

$\Rightarrow \tan\theta = 1 = \tan 45^o$

$\Rightarrow \theta = 45^o \because \theta$ is an acute angle

248. (A) $\tan\theta + \cot\theta = 5$
On squaring both sides,
$(\tan\theta + \cot\theta)^2 = 5^2$
$\Rightarrow \tan^2\theta + \cot^2\theta + 2\tan\theta.\cot\theta = 25$
$\Rightarrow \tan^2\theta + \cot^2\theta = 25 - 2 = 23$
$[\because \tan\theta.\cot\theta = 1]$

249. (A) $\sin^2 22^o + \sin^2 68^o + \cot^2 30^o$
$= \sin^2 22^o + \sin^2(90^o - 22^o) + (\sqrt{3})^2$
$= \sin^2 22^o + \cos^2 22^o + 3$
$[\because \sin^2\theta + \cos^2 = 1]$
$= 1 + 3 = 4$

250. (C) $2\sin^2\theta + 3\cos^2\theta$
$= 2\sin^2\theta + 2\cos^2\theta + \cos^2\theta$
$= 2(\sin^2\theta + \cos^2\theta) + \cos^2\theta$
$= 2 + \cos^2\theta$
$\therefore$ Minimum value = 2 + 0 = 2
because $\cos^2\theta \geq 0$
(or)
$a\sin^2\theta + b\cos^2\theta$
If a > b
min value = b; max value = a
If a < b
min value = a; max value = b

251. (D) $\tan(4\theta - 50^o) = \cot(50^o - \theta)$
$\Rightarrow \tan(4\theta - 50^o)$
$= \tan(90^o - (50^o - \theta))$
$\Rightarrow 4\theta - 50^o = 90^o - (50^o - \theta)$
$\Rightarrow 4\theta - 50^o = 90^o - 50^o + \theta$
$\Rightarrow 4\theta - 50^o = 40^o + \theta$
$\Rightarrow 4\theta - \theta = 40^o + 50^o$
$\Rightarrow 3\theta = 90^o \Rightarrow \theta = \frac{90^o}{3} = 30^o$

252. (D) $5\sin\theta = 3 \Rightarrow \sin\theta = \frac{3}{5}$
Expression $= \frac{\sec\theta - \tan\theta}{\sec\theta + \tan\theta}$
$= \frac{\frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta}}{\frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta}} = \frac{\frac{1-\sin\theta}{\cos\theta}}{\frac{1+\sin\theta}{\cos\theta}}$
$= \frac{1-\sin\theta}{1+\sin\theta} = \frac{1-\frac{3}{5}}{1+\frac{3}{5}} = \frac{5-3}{5+3}$
$= \frac{2}{8} = \frac{1}{4}$

253. (D) $\sec\theta + \tan\theta = p$ ... (i)
$\because \sec^2\theta - \tan^2\theta = 1$
$\Rightarrow (\sec\theta + \tan\theta)(\sec\theta - \tan\theta) = 1$
$\Rightarrow \sec\theta - \tan\theta = \frac{1}{p}$ ... (ii)
On adding both the equations,
$2\sec\theta = p + \frac{1}{p}$
$\Rightarrow \sec\theta = \frac{1}{2}\left(p + \frac{1}{p}\right)$

254. (A) $1 + \cos^2\theta = 3\sin\theta.\cos\theta$
Dividing both sides by $\sin^2\theta$,
$\frac{1}{\sin^2\theta} + \frac{\cos^2\theta}{\sin^2\theta} = \frac{3\sin\theta\cos\theta}{\sin^2\theta}$
$\Rightarrow \text{cosec}^2\theta + \cot^2\theta = 3\cot\theta$
$\Rightarrow 1 + \cot^2\theta + \cot^2\theta = 3\cot\theta$
$\Rightarrow 2\cot^2\theta - 3\cot\theta + 1 = 0$
$\Rightarrow 2\cot^2\theta - 2\cot\theta - \cot\theta + 1 = 0$
$\Rightarrow 2\cot^2\theta(\cot\theta - 1) - 1(\cot\theta - 1) = 0$
$\Rightarrow (2\cot\theta - 1)(\cot\theta - 1) = 0$
$\Rightarrow \cot\theta = \frac{1}{2}$ or 1
(or)
Put $\theta = 45°$
$1 + \frac{1}{2} = 3 \times \frac{1}{2}$
$\frac{3}{2} = \frac{3}{2}$
Satisfy the given condition then $\theta = 45°$
$\cot 45° = 1$

255. (D) Expression
$= 3(\sin^4\theta + \cos^4\theta)$
$+2(\sin^6\theta + \cos^6\theta)$
$+12\sin^2\theta.\cos^2\theta$
$= 3\{(\sin^2\theta + \cos^2\theta)^2 - 2\sin^2\theta.\cos^2\theta\}$
$+2\{(\sin^2\theta + \cos^2\theta)^3$
$-3\sin^2\theta.\cos^2\theta$
$(\sin^2\theta + \cos^2\theta) + 12\sin^2\theta.\cos^2\theta$
$[\because a^2 + b^2 = (a+b)^2 - 2ab;\ a^3 + b^3$
$(a+b)^3 - 3ab(a+b)]$
$= 3(1 - 2\sin^2\theta.\cos^2\theta)$
$+2(1 - 3\sin^2\theta$
$.\cos^2\theta) + 12\sin^2\theta.\cos^2\theta$
$= 3 - 6\sin^2\theta.\cos^2\theta + 2 - 6$
$\sin^2\theta\cos^2\theta + 12\sin^2\theta.\cos^2\theta = 5$

256. (B) $\sec\theta + \tan\theta = 2 + \sqrt{5}$
$\because \sec^2\theta - \tan^2\theta = 1$
$\Rightarrow (\sec\theta + \tan\theta)(\sec\theta - \tan\theta) = 1$
$\Rightarrow \frac{1}{\sqrt{5}+2} \times \frac{\sqrt{5}-2}{\sqrt{5}-2} = \frac{\sqrt{5}-2}{5-4}$
$= \sqrt{5} - 2$
$\therefore \sec\theta + \tan\theta + \sec\theta - \tan\theta$
$= 2 + \sqrt{5} + \sqrt{5} - 2$
$\Rightarrow 2\sec\theta = 2\sqrt{5}$
$\Rightarrow \sec\theta = \sqrt{5}$ ... (i)
Again,
$\sec\theta + \tan\theta - (\sec\theta - \tan\theta)$
$= 2 + \sqrt{5} - \sqrt{5} + 2$
$\Rightarrow 2\tan\theta = 4 \Rightarrow \tan\theta = 2$ ... (ii)
$\therefore \sin\theta = \frac{\tan\theta}{\sec\theta} = \frac{2}{\sqrt{5}}$

257. (B) $\frac{\sec\theta + \tan\theta}{\sec\theta - \tan\theta} = 2\frac{51}{79}$
$= \frac{158 + 51}{79} = \frac{209}{79}$
By componendo and dividendo,
$\frac{\sec\theta + \tan\theta + \sec\theta - \tan\theta}{\sec\theta + \tan\theta - \sec\theta + \tan\theta}$
$= \frac{209 + 79}{209 - 79}$
$\Rightarrow \frac{2\sec\theta}{2\tan\theta} = \frac{288}{130}$
$\Rightarrow \frac{\sec\theta}{\tan\theta} = \frac{144}{65}$
$\therefore \sin\theta = \frac{\tan\theta}{\sec\theta} = \frac{65}{144}$

258. (B) $\tan A + \cot A = 2$
$\Rightarrow \tan A + \frac{1}{\tan A} = 2$
$\Rightarrow \frac{\tan^2 A + 1}{\tan A} = 2$
$\Rightarrow \tan^2 A + 1 = 2\tan A$
$\Rightarrow \tan^2 A - 2\tan A + 1 = 0$
$\Rightarrow (\tan A - 1)^2 = 0$
$\Rightarrow \tan A - 1 = 0 \Rightarrow \tan A = 1$
$\cot A = 1$
$\therefore \tan^{10} A + \cot^{10} A = 1 + 1 = 2$

259. (D) $\cos^2 30^o + \sin^2 60^o + \tan^2 45^o + \sec^2 60^o + \cos 0°$
$= \left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 + (1)^2 + (2)^2 + 1$
$= \frac{3}{4} + \frac{3}{4} + 1 + 4 + 1$
$= 6 + \frac{3+3}{4}$
$= 6 + \frac{6}{4} = 6 + \frac{3}{2} = \frac{12+3}{2}$
$= \frac{15}{2} = 7\frac{1}{2}$

260. (D) $\cos x + \cos^2 x = 1$

$\Rightarrow \cos x = 1 - \cos^2 x = \sin^2 x$ ... (i)

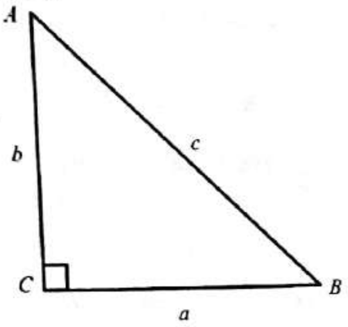
$\therefore \sin^8 x + 2\sin^6 x + \sin^4 x$

$= (\sin^4 x + \sin^2 x)^2$

$= ((\cos x)^2 + \sin^2 x)^2$

$= (\cos^2 x + \sin^2 x)^2 = 1$

261. (C)



In ΔABC,

$AB^2 = AC^2 + BC^2$

$\Rightarrow c^2 = a^2 + b^2$ ... (i)

From ΔABC,

$\operatorname{cosec} B = \frac{AB}{AC} = \frac{c}{b}$ ... (ii)

$\cos A = \frac{AC}{AB} = \frac{b}{c}$

$\therefore \operatorname{cosec} B - \cos A = \frac{c}{b} - \frac{b}{c}$

$= \frac{c^2 - b^2}{bc} = \frac{a^2}{bc}$

262. (A) If $\tan\theta = \cot\theta$

$\Rightarrow \theta = 45^\circ$

$\therefore \frac{\tan(\theta + 15^\circ)}{\tan(\theta - 15^\circ)}$

$= \frac{\tan(45^\circ + 15^\circ)}{\tan(45^\circ - 15^\circ)} = \frac{\tan 60^\circ}{\tan 30^\circ}$

$= \frac{\sqrt{3}}{\frac{1}{\sqrt{3}}} = \sqrt{3} \times \sqrt{3} = 3$

263. (A) Expression = (cot41°. cot 49°) . (cot 42° . cot 48°) (cot 43° . cot 47°) . (cot 44° . cot 46°) . cot 45° = cot 41° . tan (90° − 49°) . cot 42° . tan (90° − 48°) . cot 43° tan (90° − 47°) . cot 44° . tan (90° − 46°) . 1 =(cot 41° . tan41°) (cot42° . tan42°) . (cot 43° . tan 43°) . (cot 44° . tan 44°) . 1 = 1

$[\because \tan(90^\circ - \theta) = \cot\theta;$

$\tan\theta . \cot\theta = 1]$

264. (D) $x = a\sin\theta - b\cos\theta$ ... (i)

$y = a\cos\theta + b\sin\theta$ ... (ii)

On squaring and adding both the equations,

$x^2 + y^2 = (a\sin\theta - b\cos\theta)^2 + (a\cos\theta + b\sin\theta)^2$

$= a^2\sin^2\theta + b^2\cos^2\theta - 2ab\sin\theta.\cos\theta + a^2\cos^2\theta + b^2\sin^2\theta + 2ab\sin\theta.\cos\theta$

$= a^2(\sin^2\theta + \cos^2\theta) + b^2(\cos^2\theta + \sin^2\theta)$

$= a^2 + b^2$

$[\because \sin^2\theta + \cos^2\theta = 1]$

265. (A) $\sec\theta - \tan\theta = \frac{1}{\sqrt{3}}$ ... (i)

$\because \sec^2\theta - \tan^2\theta = 1$

$\Rightarrow (\sec\theta + \tan\theta)(\sec\theta - \tan\theta) = 1$

$\Rightarrow \sec\theta + \tan\theta = \sqrt{3}$ ... (ii)

On adding equations (i) and (ii)

$2\sec\theta = \sqrt{3} + \frac{1}{\sqrt{3}}$

$= \frac{3+1}{\sqrt{3}} = \frac{4}{\sqrt{3}}$

$\Rightarrow \sec\theta = \frac{2}{\sqrt{3}}$

Again, by equation (ii) − (i),

$2\tan\theta = \sqrt{3} - \frac{1}{\sqrt{3}}$

$= \frac{3-1}{\sqrt{3}} = \frac{2}{\sqrt{3}}$

$\therefore \sec\theta . \tan\theta$

$= \frac{2}{\sqrt{3}} \times \frac{1}{\sqrt{3}} = \frac{2}{3}$

266. (D) $5\cos\theta + 12\sin\theta = 13$

$\Rightarrow \frac{5}{13}\cos\theta + \frac{12}{13}\sin\theta = 1$

$\because \sin^2\theta + \cos^2\theta = 1$

$\therefore \sin\theta = \frac{12}{13}, \cos\theta = \frac{5}{13}$

267. (A) $7\sin^2\theta + 3\cos^2\theta = 4$

On dividing both sides by $\cos^2\theta$,

$7\frac{\sin^2\theta}{\cos^2\theta} + \frac{3\cos^2\theta}{\cos^2\theta} = \frac{4}{\cos^2\theta}$

$\Rightarrow 7\tan^2\theta + 3 = 4\sec^2\theta$

$\Rightarrow 7\tan^2\theta + 3 = 4(1 + \tan^2\theta)$

$\Rightarrow 7\tan^2\theta + 3 = 4 + 4\tan^2\theta$

$\Rightarrow 7\tan^2\theta - 4\tan^2\theta = 4 - 3$

$\Rightarrow 3\tan^2\theta = 1 \Rightarrow \tan^2\theta = \frac{1}{3}$

$\Rightarrow \tan\theta = \frac{1}{\sqrt{3}}$

268. (A) Expression

$= (\operatorname{cosec} a - \sin a)(\sec a - \cos a)$

$(\tan a + \cot a)$

$= \left(\frac{1}{\sin a} - \sin a\right)\left(\frac{1}{\cos a} - \cos a\right)$

$\left(\frac{\sin a}{\cos a} + \frac{\cos a}{\sin a}\right)$

$= \left(\frac{1 - \sin^2 a}{\sin a}\right)\left(\frac{1 - \cos^2 a}{\cos a}\right)$

$\frac{\sin^2 a + \cos^2 a}{\cos a . \sin a}$

$= \frac{\cos^2 a}{\sin a} \times \frac{\sin^2 a}{\cos a} \times \frac{1}{\cos a . \sin a} = 1$

269. (D) $\sin A + \sin^2 A = 1$

$\Rightarrow \sin A = 1 - \sin^2 A = \cos^2 A$

$\therefore \cos^2 A + \cos^4 A$

$= \cos^2 A + (\cos^2 A)^2$

$= \cos^2 A + \sin^2 A = 1$

270. (D) $\tan A = n\tan B$

$\Rightarrow \tan B = \frac{1}{n}\tan A$

$\Rightarrow \cot B = \frac{n}{\tan A}$

and $\sin A = m\sin B$

$\Rightarrow \sin B = \frac{1}{m}\sin A$

$\Rightarrow \operatorname{cosec} B = \frac{m}{\sin A}$

$\because \operatorname{cosec}^2 B - \cot^2 B = 1$

$\Rightarrow \frac{m^2}{\sin^2 A} - \frac{n^2}{\tan^2 A} = 1$

$\Rightarrow \frac{m^2}{\sin^2 A} - \frac{n^2\cos^2 A}{\sin^2 A} = 1$

$\Rightarrow \frac{m^2 - n^2\cos^2 A}{\sin^2 A} = 1$

$\Rightarrow m^2 - n^2\cos^2 A = \sin^2 A$

$\Rightarrow m^2 - n^2\cos^2 A = 1 - \cos^2 A$

$\Rightarrow m^2 - 1 = (n^2 - 1)\cos^2 A$

$\Rightarrow \cos^2 A = \frac{m^2 - 1}{n^2 - 1}$

271. (D)

$\sin\theta + \cos\theta = \sqrt{2}\sin(90^\circ - \theta)$

$\Rightarrow \sin\theta + \cos\theta = \sqrt{2}\cos\theta$

$\Rightarrow \sqrt{2}\cos\theta - \cos\theta = \sin\theta$

$\Rightarrow \cos\left(\sqrt{2} - 1\right) = \sin\theta$

$\Rightarrow \frac{\cos\theta}{\sin\theta} = \frac{1}{\sqrt{2} - 1}$

$\Rightarrow \cot\theta = \frac{1}{\sqrt{2} - 1} \times \frac{\sqrt{2} + 1}{\sqrt{2} + 1}$

$= \frac{\sqrt{2} + 1}{2 - 1} = \sqrt{2} + 1$

272. (C) $\tan 20^\circ = \tan(90^\circ - 70^\circ) = \cot 70^\circ$

$\therefore \cot 20^\circ = \tan 70^\circ$

$\tan 15^\circ = \tan(90^\circ - 75^\circ) = \cot 75^\circ$

$\therefore$ Expression $= \cot^2 70^\circ . \sin^2 70^\circ + \tan^2 70^\circ . \cos^2 70^\circ + 2 \cot 75^\circ . \tan 75^\circ . \tan 45^\circ$

$= \frac{\cos^2 70^\circ}{\sin^2 70^\circ} . \sin^2 70^\circ + \frac{\sin^2 70^\circ}{\cos^2 70^\circ} . \cos^2 70^\circ + 2 \times 1 \times 1$

$= \cos^2 70^\circ + \sin^2 70^\circ + 2$

$= 1 + 2 = 3$

$\left[\because \sin\theta . \operatorname{cosec}\theta = 1;\ \cos\theta . \sec\theta = 1;\ \tan\theta . \cot\theta = 1\right]$

273. (B)

$\sin 47^\circ = \sin(90^\circ - 43^\circ) = \cos 43^\circ$

$\therefore \left(\frac{\sin 47^\circ}{\cos 43^\circ}\right)^2 + \left(\frac{\cos 43^\circ}{\sin 47^\circ}\right)^2 - 4\cos^2 45^\circ$

$= \left(\frac{\cos 43^\circ}{\cos 43^\circ}\right) + \left(\frac{\sin 47^\circ}{\sin 47^\circ}\right)^2 - 4 \times \left(\frac{1}{\sqrt{2}}\right)^2$

$\left[\because \sin(90^\circ - \theta) = \cos\theta,\ \cos(90^\circ - \theta) = \sin\theta\right]$

$= 1 + 1 - 4 \times \frac{1}{2} = 2 - 2 = 0$

274. (B) $\operatorname{cosec}\theta = \cot^2\theta$

$\Rightarrow \operatorname{cosec}\theta = \operatorname{cosec}^2\theta - 1$

$\Rightarrow \operatorname{cosec}^2\theta - \operatorname{cosec}\theta = 1$ ... (i)

Expression

$\operatorname{cosec}^4\theta - 2\operatorname{cosec}^3\theta + \cot^2\theta$

$= \operatorname{cosec}^4\theta - \operatorname{cosec}^3\theta - \operatorname{cosec}^3\theta + \operatorname{cosec}^2\theta - 1$

$= \operatorname{cosec}^2\theta(\operatorname{cosec}^2\theta - \operatorname{cosec}\theta) - \operatorname{cosec}\theta(\operatorname{cosec}^2\theta - \operatorname{cosec}\theta) - 1$

$(\operatorname{cosec}^2\theta - \operatorname{cosec}\theta) \times (\operatorname{cosec}^2\theta - \operatorname{cosec}\theta) - 1$

$= 1 \times 1 - 1 = 0$

275. (B) $4\sin^2\theta - 1 = 0$

$\Rightarrow 4\sin^2\theta = 1$

$\Rightarrow \sin^2\theta = \frac{1}{4}$

$\Rightarrow \sin\theta = \frac{1}{2} (\because \theta < 90^\circ)$

$\therefore \sin\theta = \sin 30^\circ \Rightarrow \theta = 30^\circ$

$\therefore \cos^2\theta + \tan^2\theta$

$= \cos^2 30^\circ + \tan^2 30^\circ$

$= \left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{\sqrt{3}}\right)^2 = \frac{3}{4} + \frac{1}{3}$

$= \frac{9+4}{12} = \frac{13}{12}$

276. (C)

$\frac{9}{\operatorname{cosec}^2\theta} + 4\cos^2\theta + \frac{5}{1+\tan^2\theta}$

$= 9\sin^2\theta + 4\cos^2\theta + \frac{5}{\sec^2\theta}$

$= 9\sin^2\theta + 4\cos^2\theta + 5\cos^2\theta$

$= 9\sin^2\theta + 9\cos^2\theta$

$= 9(\sin^2\theta + \cos^2\theta) = 9 \times 1 = 9$

277. (D) $\tan\theta + \sec\theta = 3$ ... (i)

$\because \sec^2\theta - \tan^2\theta = 1$

$\Rightarrow (\sec\theta - \tan\theta)(\sec\theta + \tan\theta) = 1$

$\Rightarrow \sec\theta - \tan\theta = \frac{1}{3}$ ... (ii)

On adding equations (i) and (ii),

$2\sec\theta = 3 + \frac{1}{3} = \frac{10}{3}$

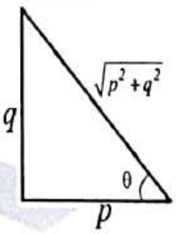
On subtracitng equation (ii) from (i),

$2\tan\theta = 3 - \frac{1}{3} = \frac{9-1}{3} = \frac{8}{3}$

$\therefore \sin\theta = \frac{\tan\theta}{\sec\theta} = \frac{8}{3} \times \frac{3}{10} = \frac{4}{5}$

$\therefore 5\sin\theta = 5 \times \frac{4}{5} = 4$

278. (B)



$\tan\theta = \frac{q}{p}$

279. (B) In a triangle ABC, $A + B + C = 180^\circ$

$\Rightarrow \frac{A}{2} + \frac{B}{2} + \frac{C}{2} = 90^\circ$

$\Rightarrow \frac{A+B}{2} = 90^\circ - \frac{C}{2}$

$\Rightarrow \tan\left(\frac{A+B}{2}\right)$

$= \tan\left(90^\circ - \frac{C}{2}\right) = \cot\frac{C}{2}$

280. (B) $\sin 89^\circ = \sin(90^\circ - 1^\circ) = \cos 1^\circ$

$\sin 79^\circ = \sin(90^\circ - 11^\circ) = \cos 11^\circ$

$\sin 69^\circ = \sin(90^\circ - 21^\circ) = \cos 21^\circ$

$\sin 59^\circ = \sin(90^\circ - 31^\circ) = \cos 31^\circ$

$\sin 49^\circ = \sin(90^\circ - 41^\circ) = \cos 41^\circ$

$\therefore$ Expression

$= (\sin^2 1^\circ + \cos^2 1^\circ) + (\sin^2 11^\circ + \cos^2 11^\circ) + (\sin^2 21^\circ + \cos^2 21^\circ) + (\sin^2 31^\circ + \cos^2 31^\circ) + (\sin^2 41^\circ + \cos^2 41^\circ) + \sin^2 45^\circ$

$= 5 + \left(\frac{1}{\sqrt{2}}\right)^2 = 5 + \frac{1}{2} = 5\frac{1}{2}$

$\left[\because \sin^2\theta + \cos^2\theta = 1\right]$

281. (D) $x = a(\sin\theta + \cos\theta)$

$\Rightarrow \frac{x}{a} = \sin\theta + \cos\theta$

and, $y = b(\sin\theta - \cos\theta)$

$\Rightarrow \frac{y}{b} = \sin\theta - \cos\theta$

$\therefore \frac{x^2}{a^2} + \frac{y^2}{b^2}$

$= (\sin\theta + \cos\theta)^2 + (\sin\theta - \cos\theta)^2$

$= 2(\sin^2\theta + \cos^2\theta) = 2$

$\left[\because (a+b)^2 + (a-b)^2 = 2(a^2+b^2)\right]$

282. (A) $\cos\theta + \sin\theta = m$ ... (i)

$\sec\theta + \operatorname{cosec}\theta = n$

$\Rightarrow \frac{1}{\cos\theta} + \frac{1}{\sin\theta} = n$

$\Rightarrow \frac{\sin\theta + \cos\theta}{\sin\theta . \cos\theta} = n$ ... (ii)

$\therefore n(m^2 - 1) = \frac{\sin\theta + \cos\theta}{\sin\theta . \cos\theta}\left[(\sin\theta + \cos\theta)^2 - 1\right]$

$= \frac{\sin\theta + \cos\theta}{\sin\theta . \cos\theta}(\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta - 1)$

$= \frac{\sin\theta + \cos\theta}{\sin\theta . \cos\theta} \times 2\sin\theta . \cos\theta$

$\left[\because \sin^2\theta + \cos^2\theta = 1\right]$

$= 2(\sin\theta + \cos\theta) = 2m$

283. (C) $\frac{x - x\tan^2 30^\circ}{1 + \tan^2 30^\circ} = \sin^3 30^\circ + 4\cot^2 45^\circ - \sec^2 60^\circ$

$\Rightarrow \frac{x - x \times \left(\frac{1}{\sqrt{3}}\right)^2}{1 + \left(\frac{1}{\sqrt{3}}\right)^2}$

$= \left(\frac{1}{2}\right)^2 + 4 \times (1)^2 - (2)^2$

$\Rightarrow \frac{x-\frac{x}{3}}{1+\frac{1}{3}} = \frac{1}{4} + 4 - 4$

$\Rightarrow \frac{3x-x}{3+1} = \frac{1}{4}$

$\Rightarrow 2x = \frac{1}{4} \times 4 = 1$

$\Rightarrow x = \frac{1}{2}$

284. (A) $\cos A + \sin A = \sqrt{2}\cos A$ ... (i)

$\cos A - \sin A = x$ (let) ... (ii)

On squaring both equation and adding

$\cos^2 A + \sin^2 A + 2\sin A.\cos A$

$+ \cos^2 A + \sin^2 A - 2\sin A \cos A$

$= 2\cos^2 A + x^2$

$\Rightarrow 2(\cos^2 A + \sin^2 A)$

$= 2\cos^2 A + x^2$

$\Rightarrow x^2 + 2\cos^2 A = 2$

$\Rightarrow x^2 = 2 - 2\cos^2 A$

$= 2(1-\cos^2 A) = 2\sin^2 A$

$\therefore x = \sqrt{2}\sin A$

285. (A) $\frac{\sin\theta+\cos\theta}{\sin\theta-\cos\theta} = \frac{3}{1}$

By componendo and dividendo,

$\frac{\sin\theta+\cos\theta+\sin\theta-\cos\theta}{\sin\theta+\cos\theta-\sin\theta+\cos\theta} = \frac{3+1}{3-1}$

$\Rightarrow \frac{2\sin\theta}{2\cos\theta} = \frac{4}{2}$

$\Rightarrow \tan\theta = 2$

$\therefore \cot\theta = \frac{1}{2}$

$\therefore \operatorname{cosec}\theta = \sqrt{1+\cot^2\theta}$

$= \sqrt{1+\frac{1}{4}} = \sqrt{\frac{5}{4}} = \frac{\sqrt{5}}{2}$

$\therefore \sin\theta = \frac{2}{\sqrt{5}}$

$\sin^4\theta = \frac{16}{25}$

286. (B) $\sin 2\theta = \frac{\sqrt{3}}{2} = \sin 60^\circ$

$\Rightarrow 2\theta = 60^\circ \Rightarrow \theta = 30^\circ$

$\therefore \sin 3\theta = \sin 90^\circ = 1$

287. (C) Expression

$= \frac{1+2\times\frac{\sqrt{3}}{2}\times\frac{1}{2}}{\frac{\sqrt{3}}{2}+\frac{1}{2}} + \frac{1-2\times\frac{\sqrt{3}}{2}\times\frac{1}{2}}{\frac{\sqrt{3}}{2}-\frac{1}{2}}$

$= \frac{1+\frac{\sqrt{3}}{2}}{\frac{\sqrt{3}+1}{2}} + \frac{1-\frac{\sqrt{3}}{2}}{\frac{\sqrt{3}-1}{2}}$

$= \frac{2+\sqrt{3}}{\sqrt{3}+1} + \frac{2-\sqrt{3}}{\sqrt{3}-1}$

$= \frac{(2+\sqrt{3})(\sqrt{3}-1)+(\sqrt{3}+1)(2-\sqrt{3})}{(\sqrt{3}+1)(\sqrt{3}-1)}$

$= \frac{2\sqrt{3}-2+3-\sqrt{3}+2\sqrt{3}-3+2-\sqrt{3}}{3-1}$

$= \frac{4\sqrt{3}-2\sqrt{3}}{2} = \frac{2\sqrt{3}}{2} = \sqrt{3}$

288. (D) $\alpha + \beta = 90^\circ$

$\Rightarrow \alpha = 90^\circ - \beta$

$\Rightarrow \tan\alpha = \tan(90^\circ-\beta) = \cot\beta$

$\sin\alpha = \sin(90^\circ-\beta) = \cos\beta$

$\therefore$ Expression

$= \frac{\cot\beta}{\tan\beta} + \cos^2\beta + \sin^2\beta$

$= \cot^2\beta + 1$

$= \operatorname{cosec}^2\beta = \operatorname{cosec}^2(90^\circ-\alpha)$

$= \sec^2\alpha$

289. (D) $\tan^2\frac{\pi}{4} - \cos^2\frac{\pi}{3}$

$= x\sin\frac{\pi}{4}.\cos\frac{\pi}{4}.\tan\frac{\pi}{3}$

$\Rightarrow 1-\left(\frac{1}{2}\right)^2 = x.\frac{1}{\sqrt{2}}\times\frac{1}{\sqrt{2}}\times\sqrt{3}$

$\Rightarrow 1-\frac{1}{4} = x\times\frac{\sqrt{3}}{2}$

$\Rightarrow \frac{4-1}{4} = x\times\frac{\sqrt{3}}{2}$

$\Rightarrow x = \frac{3}{4}\times\frac{2}{\sqrt{3}} = \frac{\sqrt{3}}{2}$

290. (C) $\sin A - \cos A = \frac{\sqrt{3}-1}{2}$

On squairng both sides,

$\sin^2 A + \cos^2 A - 2\sin A.\cos A$

$= \frac{1}{4}\left[(\sqrt{3})^2 + (1)^2 - 2\sqrt{3}\right]$

$\Rightarrow 1 - 2\sin A\cos A$

$= \frac{1}{4}(4-2\sqrt{3})$

$\Rightarrow 1 - 2\sin A\cos A = \frac{2-\sqrt{3}}{2}$

$\Rightarrow 2 - 4\sin A\cos A = 2 - \sqrt{3}$

$\Rightarrow 4\sin A.\cos A = 2 - 2 + \sqrt{3} = \sqrt{3}$

$\Rightarrow \sin A.\cos A = \frac{\sqrt{3}}{4}$

291. (B) $\sin(90^\circ-\theta) + \cos\theta$

$= \sqrt{2}\cos(90^\circ-\theta)$

$\Rightarrow \cos\theta + \cos\theta = \sqrt{2}\sin\theta$

$\Rightarrow 2\cos\theta = \sqrt{2}\sin\theta$

$\Rightarrow \cot\theta = \frac{1}{\sqrt{2}}$

$\therefore \operatorname{cosec}^2\theta = 1 + \cot^2\theta$

$= 1 + \left(\frac{1}{\sqrt{2}}\right)^2 = 1 + \frac{1}{2}$

$\Rightarrow \operatorname{cosec}^2\theta = \frac{3}{2}$

$\Rightarrow \operatorname{cosec}\theta = \sqrt{\frac{3}{2}}$

292. (B) $\tan\left(\frac{\pi}{2}-\frac{\alpha}{2}\right) = \sqrt{3}$

$\Rightarrow \cot\frac{\alpha}{2} = \sqrt{3} = \cot 30^\circ$

$\Rightarrow \frac{\alpha}{2} = 30^\circ \Rightarrow \alpha = 60^\circ$

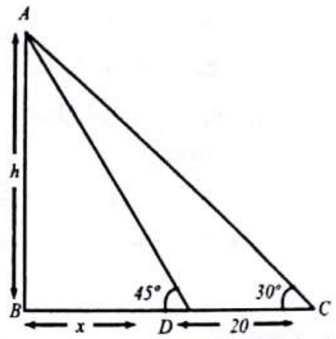
$\therefore \cos\alpha = \cos 60^\circ = \frac{1}{2}$

293. (A) $\because \cos 90^\circ = 0$

$\therefore \cos 1^\circ . \cos 2^\circ ..... \cos 180^\circ = 0$

TYPE – III

1. (D)



Let AB be a pillar of height h meter

If BD = length of shadow = x and DC = 20 m

then, BC = BD + DC

$\Rightarrow$ BC = (x + 20) metre

From Δ ABD,

$\tan 45^\circ = \frac{h}{x} \Rightarrow h = x$ ...(i)

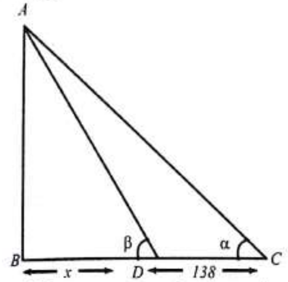
From ΔABC,

$\tan 30^\circ = \frac{AB}{BC} \Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{x+20}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{h+20}$

$\Rightarrow \sqrt{3}h = h + 20$

4. (C)



AB = monument = h metre
DC = 138 metre
BD = x metre

$\tan\alpha = \frac{1}{5}$

$\sec\beta = \frac{\sqrt{193}}{12}$

$\therefore \tan\beta = \sqrt{\sec^2\beta - 1}$

$= \sqrt{\frac{193}{144} - 1} = \sqrt{\frac{193-144}{144}}$

$= \sqrt{\frac{49}{144}} = \frac{7}{12}$

$\therefore$ From ΔABC,

$\tan\alpha = \frac{AB}{BC}$

$\Rightarrow \frac{1}{5} = \frac{h}{x+138}$

$\Rightarrow h = \frac{x+138}{5}$

$\Rightarrow 5h = x + 138$ ... (i)

From ΔABD,

$\tan\beta = \frac{h}{x} \Rightarrow \frac{7}{12} = \frac{h}{x}$

$\Rightarrow x = \frac{12h}{7}$ ... (ii)

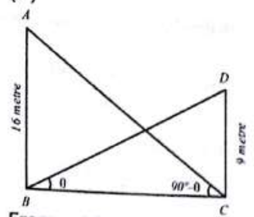
$\therefore 5h = \frac{12h}{7} + 138$ (By (i) & (ii))

$\Rightarrow 35h - 12h = 138 \times 7$

$\Rightarrow 23h = 138 \times 7$

$\Rightarrow h = \frac{138 \times 7}{23} = 42$ metre

5. (C)



From ΔABC,

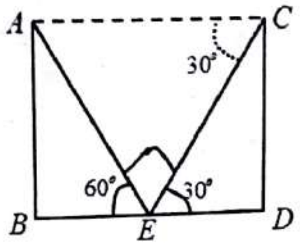
$\tan(90^o - \theta) = \frac{16}{x}$

$1 : \sqrt{3} : 2$

sides in the ratio $\downarrow \quad \downarrow$

$10\sqrt{3} \quad 20$

8. (A)



AB = CD & BD = AC = 100

In ΔAEC

Angles are in the ratio 30 : 60 : 90

$1 : \sqrt{3} : 2$

$\downarrow \qquad \downarrow$

$50 \qquad 100$

$\Rightarrow AE = 50$

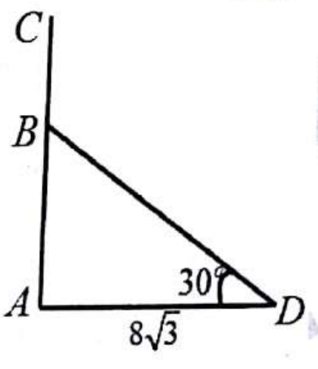
$\therefore$ In ΔABE

Angles are in the ratio 30 : 60: 90

sides are in the pole is $1 : \sqrt{3} : 2$

$\therefore$ height of the pole is $25\sqrt{3}$

9. (C)



Given that BC = BD

In ΔABD

Angles are in the ratio 30 : 60 : 90

sides are in the ratio

$1 : \sqrt{3} : 2$

$\downarrow \quad \downarrow \quad \downarrow$

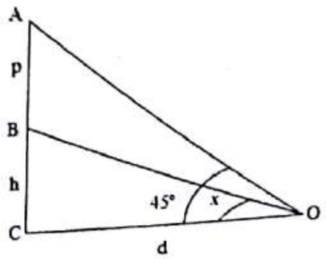
$8 \quad 8\sqrt{3} \quad 16$

$\therefore$ height of the lower AB+BC

= AB+BD = 8+16

= 24

10. (B)



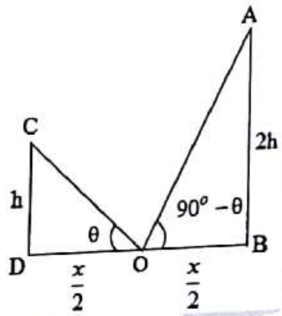
$\cot x = \frac{d}{h} \Rightarrow d = h\cot x$ ...(1)

$\cot 45 = \frac{d}{h+p} \Rightarrow h + p = d$ ...(2)

from (1) and (2)

$h + p = h\cot x \qquad p = h\cot x - h$

11. (A) CD = h metre, AB= 2 h metre



$OD = OB = \frac{x}{2}$ metre

From ΔOCD,

$\tan\theta = \frac{h}{\frac{x}{2}} = \frac{2h}{x}$

From ΔOAB,

$\tan(90° - \theta) = \frac{AB}{BO}$

$\Rightarrow \cot\theta = \frac{2h}{\frac{x}{2}} = \frac{4h}{x}$ ...(ii)

Multiplying both equations,

$\tan\theta.\cot\theta = \frac{2h}{x} \times \frac{4h}{x}$

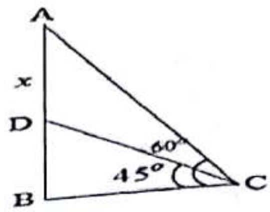
$\Rightarrow x^2 = 8h^2$

$[\because \tan\theta\cot\theta = 1]$

$\Rightarrow h^2 = \frac{x^2}{8}$

$\Rightarrow h = \frac{x}{2\sqrt{2}}$ metre

12. (C)



$\angle ACB = 60°$

$\angle DCB = 45°$

AB = 5000 metre

AD = x metre

$\therefore$ From ΔABC,

$\tan 60° = \frac{AB}{BC}$

$\Rightarrow \sqrt{3} = \frac{5000}{BC}$

$\Rightarrow BC = \frac{5000}{\sqrt{3}}$ metre

From ΔDBC,

$\tan 45° = \frac{DB}{BC}$

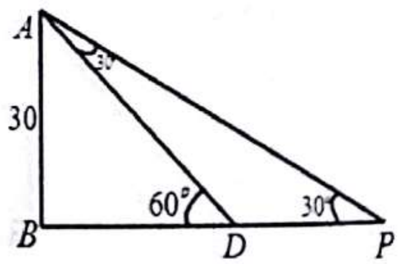
$\Rightarrow DB = BC = \frac{5000}{\sqrt{3}}$

$\therefore AD = AB - BD$

$= 5000 - \frac{5000}{\sqrt{3}}$

$= 5000\left(1 - \frac{1}{\sqrt{3}}\right)m$

13. (D)



In ΔABD

Angles are in the ratio 30 : 60 : 90

sides are in the ratio

$1 : \sqrt{3} : 2$

$\downarrow \quad \downarrow \quad \downarrow$

$10\sqrt{3} \quad 30 \quad 20\sqrt{3}$

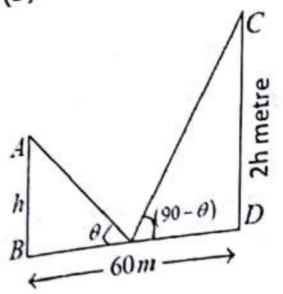
BD AB AD

We know that

ΔADP is isoceleses triangle

$AD = DP = 20\sqrt{3}$

14. (D)



BE = DE = 30 metre

$\angle AEB = \theta \therefore \angle CED = 90^o$

From ΔABE,

$\tan\ \theta = \frac{AB}{BE}$

$\Rightarrow \tan\theta = \frac{h}{30}$

$\Rightarrow h = 30\tan\theta$

From ΔCDE,

$\tan(90^\circ - \theta) = \frac{2h}{30}$

$\Rightarrow \cot\theta = \frac{h}{15} \Rightarrow h = 15\cot\theta$ ...(ii)

By multiplying both equations,

$h^2 = 30 \times 15 \times \tan\theta \times \cot\theta$

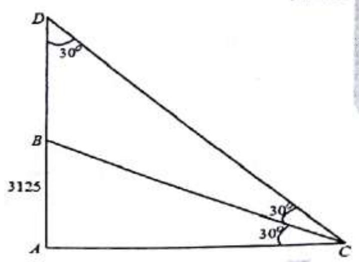
$\Rightarrow h^2 = 30 \times 15$

$[\because \tan\theta.\cot\theta = 1]$

$\Rightarrow h = 15\sqrt{2}\text{meter} = AB$

$\Rightarrow 2h = 30\sqrt{2}\text{metre} = CD$

15. (D)



ΔBCD is an isoceleses

BD = BC

in ΔABC angles are in the ratio 30 : 60 : 90

sides are in the ratio

$1 : \sqrt{3} : 2$

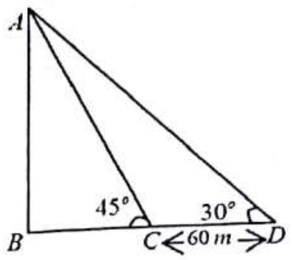
↓ ↓

3125 6250

AB BC

$\therefore$ Distance between two planes BD = BC = 6250

16. (C)



AB= Tower = h metre

$\angle ADB = 30^\circ$

$\angle ACB = 45^\circ$

CD = 60 metre

BC = x metre

From ΔABC,

$\tan 45^\circ = \frac{AB}{BC}$

$\Rightarrow 1 = \frac{h}{x} \Rightarrow h = x$

From ΔABD,

$\tan\ 30^\circ = \frac{AB}{BD}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{x+60}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{h+60}$

$\Rightarrow \sqrt{3}h = h + 60$

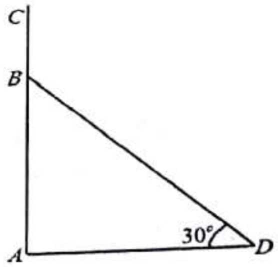
$\Rightarrow \sqrt{3}h - h = 60$

$\Rightarrow h(\sqrt{3}-1) = 60$

$\Rightarrow h = \frac{60}{\sqrt{3}-1} = \frac{60(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)}$

$= 30(\sqrt{3}+1)$ metre

17. (B)



In ΔABD angles are in the ratio 30 : 60 : 90

sides are in the ratio

$1 : \sqrt{3} : 2$

AB BD

We know that BC = BD

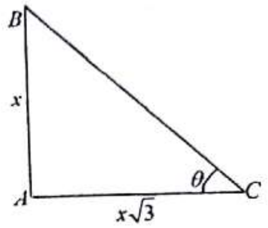
$\therefore$ AC = AB + BD

3x = 15

x = 5

$\therefore$ AB = 5

18. (B)



$\tan\theta = \frac{AB}{AC} = \frac{x}{x\sqrt{3}} = \frac{1}{\sqrt{3}}$

$\theta = 30$

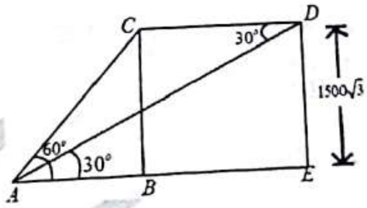
19. (B) height shadow

$6 \longrightarrow 4$

$75 \longleftarrow 50$

$\therefore$ height of the flag post is 75

20. (B)



$BC = DE = 1500\sqrt{3}$

Angles are in the ratio 30 : 60 : 90

sides are in the ratio

$1 : \sqrt{3} : 2$

↓ ↓

$1500\sqrt{3}$ 3000

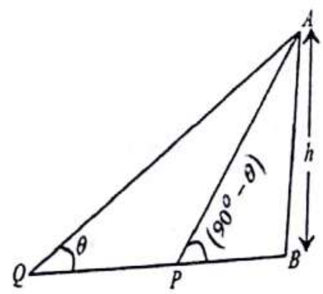
BC AC

ΔACD is isoscleses triangle.

$\therefore$ AC = CD = 3000

speed of the plane = $\frac{3000}{15}$ = 200 m/s

21. (A)



AB = Tower = h units

Let, $\angle AQB$, = θ $\therefore \angle APB = 90° - \theta$

PB = a ; BQ = b

From ΔAPB

$\tan(90^\circ - \theta) = \frac{h}{PB}$

$\Rightarrow \cot\theta = \frac{h}{a}$ ...(i)

From ΔAQB

$\tan\theta = \frac{AB}{QB} = \frac{h}{b}$ ...(ii)

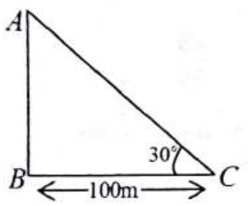
By multiplying (i) & (ii)

$\tan\theta.\cot\theta = \frac{h}{b} \times \frac{h}{a}$

$\Rightarrow h^2 = ab$

$\Rightarrow h = \sqrt{ab}$

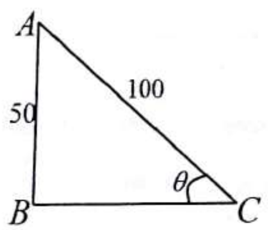
22. (A)



tan 30 = $\frac{AB}{BC}$

$\frac{1}{\sqrt{3}} = \frac{AB}{100} \Rightarrow AB = \frac{100}{\sqrt{3}}$

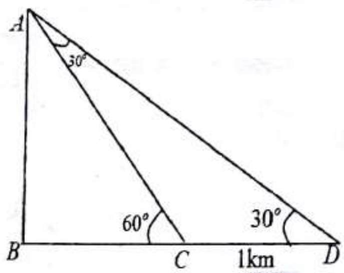
23. (D)



$\sin\theta = \frac{AB}{AC} = \frac{50}{100} = \frac{1}{2}$

$\Rightarrow \theta = 30$

24. (A)



ΔACD is isoscleses Δ

AC = CD = 1 km

in Δ ABC

Angles are in the ratio 30 : 60 : 90

sides are in the ratio

$1 : \sqrt{3} : 2$

$\downarrow \quad \downarrow$

$\frac{\sqrt{3}}{2}$ 1 km

∴ height of the ballon above the ground is $\frac{\sqrt{3}}{2}$ km.

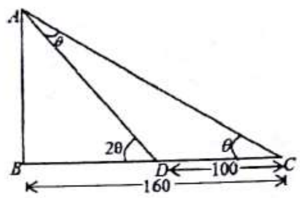
25. (B) height shadow

12 → 8

60 ← 40

∴ height of the lower is 60

26. (A)



ΔADC is an isosceles triangle

∴ AD =DC = 100

Here ABD is a right angle

$AD^2 = AB^2 + BD^2$

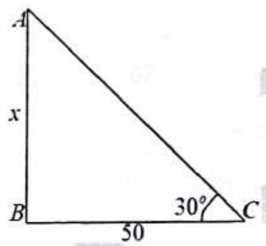
$100^2 = AB^2 + 60^2$

$AB^2 = 6400$

AB = 80

∴ height of the lower is 80

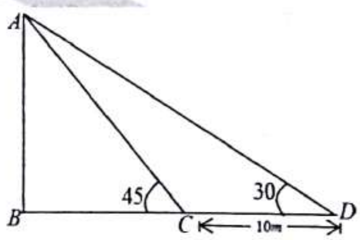
27. (B)



$\tan 30 = \frac{AB}{BC} \Rightarrow \frac{1}{\sqrt{3}} = \frac{x}{50}$

$x = \frac{50}{\sqrt{3}}$

28. (C)



$\tan 45 = \frac{AB}{BC}$

AB = BC

tan 30 = $\frac{AB}{BD}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{AB}{BC+10}$

AB +10 = $\sqrt{3}$ AB [BC =AB]

10 = $\sqrt{3}$ AB - AB

10 = AB $(\sqrt{3}-1)$

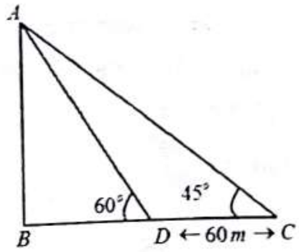
$AB = \frac{10}{\sqrt{3}-1}$

$AB = \frac{10}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$

$AB = 5(\sqrt{3}+1)$

∴ height of tower is $5(\sqrt{3}+1)$

29. (C)



AB = tower = h metre

$\angle ACB = 45°, \angle ADB = 60°$

CD = 60 metre, BD = x metre

From ΔABC,

$\tan 45° = \frac{AB}{BC}$

$\Rightarrow 1 = \frac{h}{x+60}$

$\Rightarrow h = x + 60$ ...(i)

From ΔABD,

$\tan 60° = \frac{AB}{BD}$

$\Rightarrow \sqrt{3} = \frac{h}{x} \Rightarrow h = \sqrt{3}x$

$\Rightarrow h = \sqrt{3}.(h-60)$

(from (i) x = h – 60)

$\Rightarrow \sqrt{3}h - h = 60\sqrt{3}$

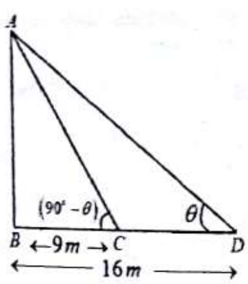
$\Rightarrow h(\sqrt{3}-1) = 60\sqrt{3}$

$\Rightarrow h = \frac{60\sqrt{3}}{\sqrt{3}-1} = \frac{60\sqrt{3}(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)}$

$= 30\sqrt{3}(\sqrt{3}+1)$

$= 30(3+\sqrt{3})$metre

30. (D)



AB = Pole = h metre

BC = 9 metre BD = 16 metre

$\angle ADB = \theta$;

$\angle ABC = 90^\circ - \theta$

From ΔABD from,

$\Rightarrow \tan\theta = \frac{h}{16}$ .... (i)

From ΔABC

$\tan(90-\theta) = \frac{h}{9}$ ... (ii)

By multiplying (i) & (ii)

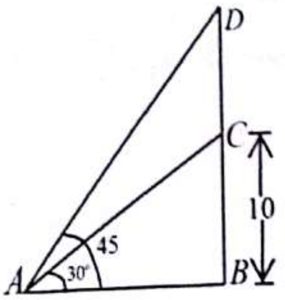
$\tan\theta.\cot\theta = \frac{h}{9} \times \frac{h}{16}$

$\Rightarrow \frac{h^2}{144} = 1$

$\Rightarrow h^2 = 144$

$\Rightarrow h = \sqrt{144} = 12$metre

31. (D)



tan 45 = $\frac{BD}{AB} \Rightarrow BD = AB$

tan 30 = $\frac{BC}{AB} \Rightarrow \frac{1}{\sqrt{3}} = \frac{10}{AB}$

$\Rightarrow AB = 10\sqrt{3}$

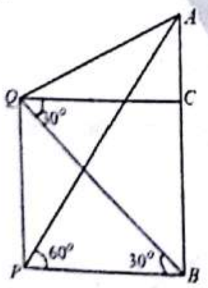
height of the flagstaff = CD = BD − BC

= AB - BC = $10\sqrt{3} - 10$

= $10(\sqrt{3}-1)$

= 7.32

32. (B)



AB = Tower = h metre

PQ=10 metre

$\angle APB = 60$,

$\angle CQB = \angle QBP = 30^\circ$

In ΔPBQ,

$\tan 30^\circ = \frac{PQ}{PB} \Rightarrow \frac{1}{\sqrt{3}} = \frac{10}{PB}$

$\Rightarrow PB = 10\sqrt{3}$metre

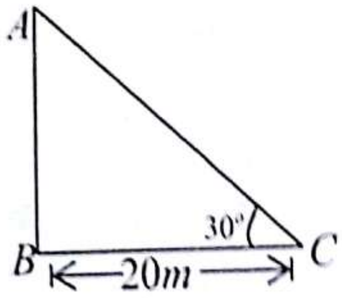
In ΔAPB,

$\tan 60^\circ = \frac{AB}{PB}$

$\Rightarrow \sqrt{3} = \frac{h}{10\sqrt{3}}$

$\Rightarrow h = \sqrt{3} \times 10\sqrt{3} = 30$metre

33. (D)



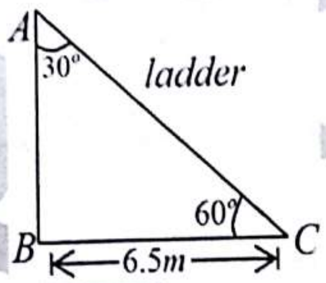
tan 30 = $\frac{AB}{BC}$

$\frac{1}{\sqrt{3}} = \frac{AB}{20}$

$AB = \frac{20}{\sqrt{3}}$

$\therefore$ height of the lower is $\frac{20}{\sqrt{3}}$

34. (B)



In Δ Angles are in the ratio 30 : 60 : 90

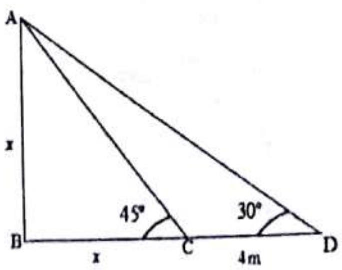
sides are in the ratio

$1 : \sqrt{3} : 2$

$\downarrow \quad \downarrow$

6.5 13

35. (D)



Let the length of pole = x mtr.

$\tan 45 = \frac{AB}{BC} \Rightarrow AB = BC = x$

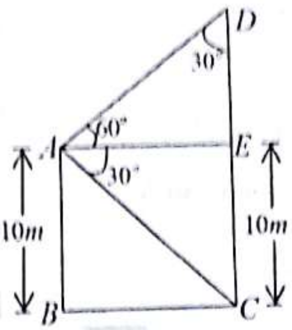
$\tan 30 = \frac{AB}{BD} \Rightarrow \frac{x}{x+4} = \frac{1}{\sqrt{3}}$

$\Rightarrow \sqrt{3}x = x + 4$

$x = \frac{4}{\sqrt{3}-1}$

$x = 2(\sqrt{3}+1) = 5.464$

36. (C)



AE || to BC

$\therefore$ AB = EC = 10m

In ΔACE angles 30 : 60 : 90

$1 : \sqrt{3} : 2$

sides $\downarrow \quad \downarrow$

10 $10\sqrt{3}$

$\therefore AE = 10\sqrt{3}$

In ΔADE angle 30, 60, 90

$1 : \sqrt{3} : 2$

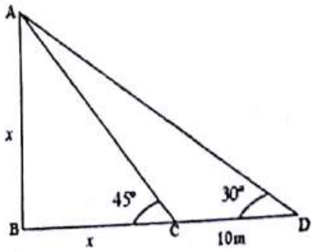
sides $\downarrow \quad \downarrow$

$10\sqrt{3}$ 30

$\therefore$ ED = 30

$\therefore$ height of tower CD = EC + ED = 10 + 30 = 40

37. (A)



$\tan 45 = \frac{AB}{BC} \Rightarrow AB = BC = x$

$\tan 30 = \frac{AB}{BD} \Rightarrow \frac{x}{x+10} = \frac{1}{\sqrt{3}}$

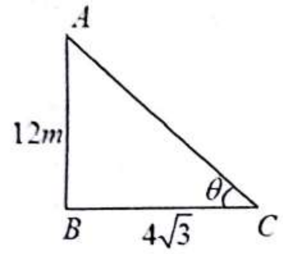
$\sqrt{3x} = x + 10$

$x = \frac{10}{\sqrt{3}-1}$

$x = 5(\sqrt{3}+1)$

$\therefore$ height of the tower is $5(\sqrt{3}+1)$

38. (B)

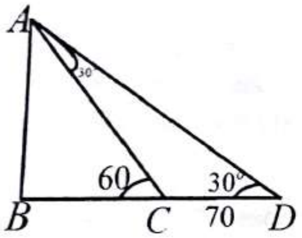


$\tan\theta = \frac{12}{4\sqrt{3}} = \sqrt{3}$

$\tan\theta = \tan 60$

$\theta = 60$

39. (D)



ΔACD is an isosceles

$\therefore$ AC = CD = 70

Angles in Δ ABC are 30 : 60 : 90

side are in the ratio

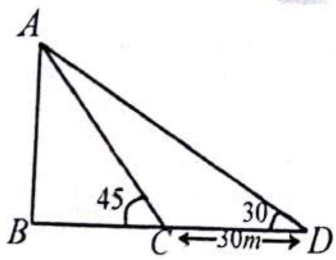
$1 : \sqrt{3} : 2$

$\downarrow \quad \downarrow$

$35\sqrt{3} \quad 70$

$\therefore$ height of the tower = $35\sqrt{3}$

40. (A)



$\tan 45 = \frac{AB}{BC}$

AB = BC

$\tan 30 = \frac{AB}{BC} \Rightarrow \frac{1}{\sqrt{3}} = \frac{AB}{BC+30}$

$AB + 30 = \sqrt{3}\,AB$ ($\because$ AB = BC)

$AB(\sqrt{3}-1) = 30$

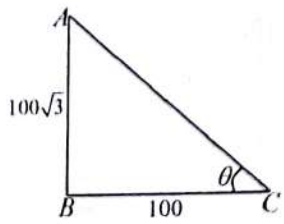
$AB = \frac{30}{\sqrt{3}-1}$

$= 15(\sqrt{3}+1)$

$\therefore$ height of the tower is

$15(\sqrt{3}+1)$

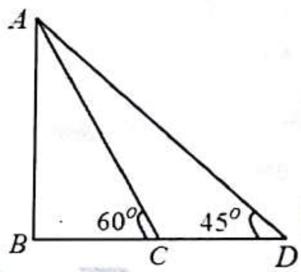
41. (B)



$\tan\theta = \frac{AB}{BC} = \frac{100\sqrt{3}}{100} = \sqrt{3}$

$\theta = 60$

42. (C)



$\angle ACB = 60°$; BC = x metre

CD = 40 metre, AB = Tower=h metre

From Δ ABC,

$\tan 60° = \frac{AB}{BC} \Rightarrow \sqrt{3} = \frac{h}{x}$

$\Rightarrow h = \sqrt{3}x$ From Δ ABD,

$\tan 45° = \frac{AB}{BD}$

$\Rightarrow 1 = \frac{h}{x+40}$

$\Rightarrow h = x + 40 = \frac{h}{\sqrt{3}} + 40$

$\Rightarrow h - \frac{h}{\sqrt{3}} = 40$

$\Rightarrow \frac{\sqrt{3}h - h}{\sqrt{3}} = 40$

$\Rightarrow (\sqrt{3}-1)h = 40\sqrt{3}$

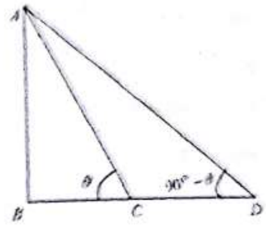
$\Rightarrow h = \frac{40\sqrt{3}}{\sqrt{3}-1}$

$= \frac{40\sqrt{3}(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)}$

$= \frac{40\sqrt{3}(\sqrt{3}+1)}{3-1}$

$= 20(3+\sqrt{3})$ metre

43. (B)



Let, $\angle ACB = \theta$

$\therefore \angle ADB = 90° - \theta$

BC = 12 metre

BD = 27 metre

AB = Pillar = h metre

From ΔABD

$\tan(90° - \theta) = \frac{AB}{BD}$

$\Rightarrow \cot\theta = \frac{h}{27}$

$\therefore \tan\theta . \cot\theta = \frac{h}{12} \times \frac{h}{27}$

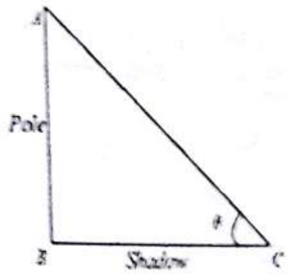
$\Rightarrow h^2 = 12 \times 27$

$\Rightarrow h = \sqrt{12 \times 27}$

$= \sqrt{2 \times 2 \times 3 \times 3 \times 3 \times 3}$

$= 2 \times 3 \times 3 = 18$ metre

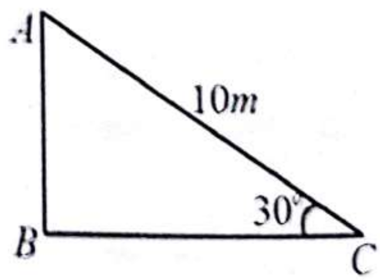
44. (D)



$\tan\theta = \frac{AB}{BC} = \frac{2\sqrt{3}}{2} = \sqrt{3}$

$\Rightarrow \tan\theta = \tan 60° \Rightarrow \theta = 60°$

45. (D)

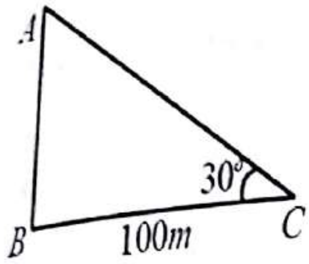


$\cos 30 = \frac{BC}{AC} \Rightarrow \frac{\sqrt{3}}{2} = \frac{BC}{10}$

$\Rightarrow BC = 5\sqrt{3}$

$BC = 8.66$

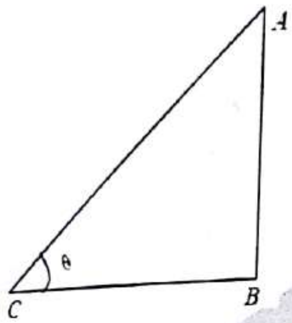
46. (D)



$\tan 30 = \dfrac{AB}{BC}$

$\dfrac{1}{\sqrt{3}} = \dfrac{AB}{100} \Rightarrow AB = \dfrac{100}{\sqrt{3}}$

47. (A)



A = Position of kite

AC = length of string

AB = 75 metre

$\cot\theta = \dfrac{8}{15}$

$\therefore \operatorname{cosec}\theta = \sqrt{1+\cot^2\theta}$

$= \sqrt{1+\left(\dfrac{8}{15}\right)^2} = \sqrt{1+\dfrac{64}{225}}$

$= \sqrt{\dfrac{225+64}{225}} = \sqrt{\dfrac{289}{225}} = \dfrac{17}{15}$

$\therefore \sin\theta = \dfrac{15}{17}$

From ΔABC

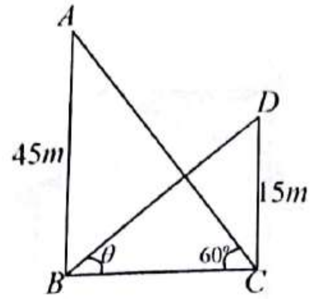
$\sin\theta = \dfrac{AB}{AC}$

$\Rightarrow \dfrac{15}{17} = \dfrac{75}{AC}$

$\Rightarrow AC \times 15 = 17 \times 75$

$\Rightarrow AC = \dfrac{17\times 75}{15} = 85$ metre

48. (B)



$\tan 60 = \dfrac{AB}{BC}$

$\sqrt{3} = \dfrac{45}{BC}$

$BC = \dfrac{45}{\sqrt{3}}$

$\tan\theta = \dfrac{DC}{BC} \Rightarrow \tan\theta = \dfrac{15}{\frac{45}{\sqrt{3}}}$

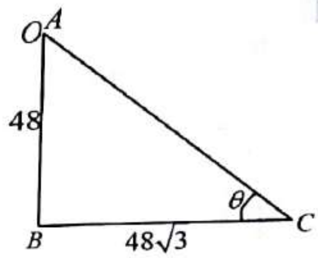
$\tan\theta = \dfrac{15\sqrt{3}}{45}$

$\tan\theta = \dfrac{1}{\sqrt{3}}$

$\therefore \theta = 30^o$

$\therefore \sin 30 = \dfrac{1}{2}$

49. (D)

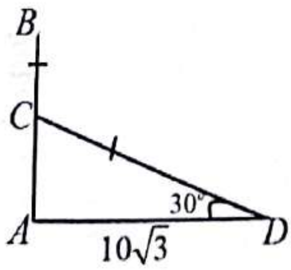


$\tan\theta = \dfrac{48}{48\sqrt{3}}$

$\tan\theta = \dfrac{1}{\sqrt{3}}$

$\therefore \theta = 30$

50. (A)



In ΔACD

Angles are in the ratio 30, 60, 90

sides are in the ratio

$1 : \sqrt{3} : 2$

$\downarrow \quad \downarrow \quad \downarrow$

$10 \quad 10\sqrt{3} \quad 20$

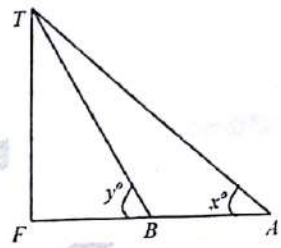
$\therefore AC = 10,\ CD = 20$

height of a telegraph pole is $AB = AC + BC = AC + CD$

$AB = 10+20$

$\therefore$ height is 30

51. (D)



TF=Tower = h metre

$\angle TAF = x°$; $\angle TBF = y°$,

BF =80 metre

in ΔAFT,

$\tan x° = \dfrac{TF}{AF}$

$\Rightarrow \dfrac{2}{5} = \dfrac{h}{200}$

$\Rightarrow h = \dfrac{2}{5} \times 200$

= 80 metre

in ΔBFT,

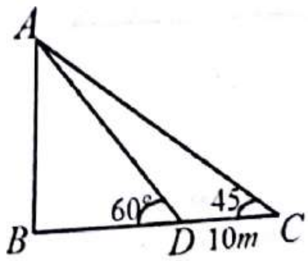
$\tan y° = \dfrac{TF}{FB}$

$\Rightarrow \tan y° = \dfrac{80}{80} = 1$

$\Rightarrow \tan y° = \tan 45°$

$\Rightarrow y = 45°$

52. (D)



From ΔABC

$\tan 45 = \frac{AB}{BC} \Rightarrow AB = BC$

From ΔABD

$\tan 60 = \frac{AB}{BD}$

$\sqrt{3} = \frac{AB}{BC - DC}$

$\sqrt{3} = \frac{AB}{AB - 10}$

$AB\sqrt{3} - 10\sqrt{3} = AB$

$10\sqrt{3} = AB(\sqrt{3} - 1)$

$AB = \frac{10\sqrt{3}}{\sqrt{3} - 1}$

$= \frac{10\sqrt{3}(\sqrt{3} + 1)}{2}$

$AB = 5(3 + \sqrt{3})$

Method-II

$\tan 60 = \frac{AB}{BD}$

$\sqrt{3} = \frac{AB}{BD}$ ...(i)

$\tan 45 = \frac{AB}{BC}$

$\frac{1}{1} = \frac{AB}{BC}$ ...(ii)

Multiply equation (2) by $\sqrt{3}$

$\frac{AB}{BC} = \frac{\sqrt{3}}{\sqrt{3}}$

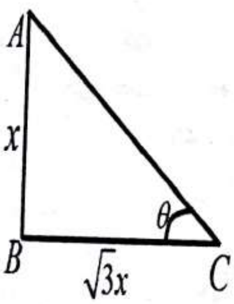
$DC = BC - BD$

$= \sqrt{3} - 1$

$\sqrt{3} - 1 \longrightarrow 10$

$\sqrt{3} \longrightarrow 5(3 + \sqrt{3})$

53. (B)



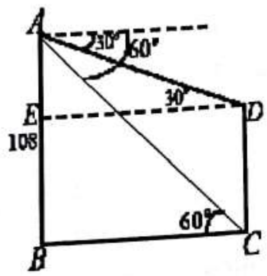
$\tan\theta = \frac{x}{\sqrt{3}x}$

$\tan\theta = \tan 30$

$\theta = 30$

TYPE-IV

1. (B)



From ΔABC

$\tan 60 = \frac{AB}{BC} \Rightarrow \sqrt{3} = \frac{108}{BC}$

$BC = \frac{108}{\sqrt{3}}$

From ΔAED

$\tan 30 = \frac{AE}{ED} \Rightarrow \frac{1}{\sqrt{3}} = \frac{AE}{\frac{108}{\sqrt{3}}}$

(Where BC = ED parallel lines)

$AE = 36$

$\therefore$ height of second pole

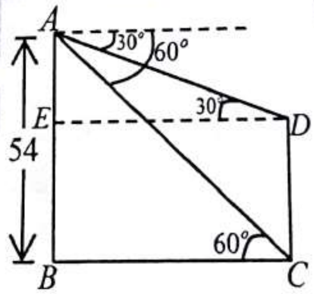
$CD = BE = AB - AE$

$= 108 - 36$

$= 72$

$\therefore$ height is 72

2. (B)



From ΔABC

$\tan 60 = \frac{AB}{BC} \Rightarrow \sqrt{3} = \frac{54}{BC}$

$\Rightarrow BC = \frac{54}{\sqrt{3}}$

From ΔAED

$\tan 30 = \frac{AE}{ED} \Rightarrow \frac{1}{\sqrt{3}} = \frac{AE}{\frac{54}{\sqrt{3}}}$

$\Rightarrow AE = 18$

$\therefore$ height of second temple

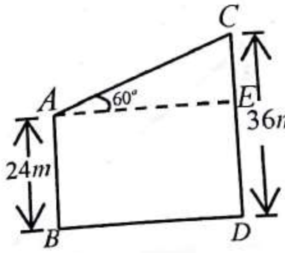
$DC = EB = AB - AE$

$= 54 - 18$

$= 36$

$\therefore$ height is 36

3. (B)



$CE = CD - AB = 36 - 24$

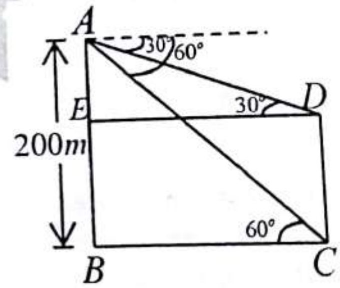
$= 12$

$\therefore \sin 60 = \frac{CE}{AC} = AC = \frac{12}{\frac{\sqrt{3}}{2}}$

$AC = 8\sqrt{3}$

$\therefore$ length of wire $= 8\sqrt{3}$

4. (C)



From ΔABC

$\tan 60 = \frac{AB}{BC} \Rightarrow \sqrt{3} = \frac{200}{BC}$

$\Rightarrow BC = \frac{200}{\sqrt{3}}$

From ΔAED

$\tan 30 = \frac{AE}{ED} = \frac{1}{\sqrt{3}} = \frac{AE}{\frac{200}{\sqrt{3}}}$

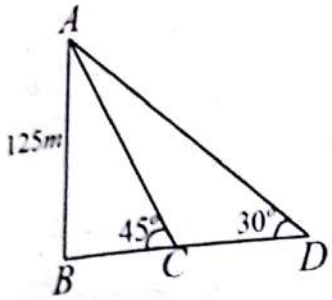
$\Rightarrow AE = \frac{200}{3}$

height of the tower.

$CD = AB - AE = 200 - \frac{200}{3}$

$= \frac{400}{3} = 133\frac{1}{3}$

5. (B)



$\tan 45 = \frac{AB}{BC} \Rightarrow AB = BC =$
125

$\tan 30 = \frac{AB}{BD} \Rightarrow \frac{1}{\sqrt{3}} = \frac{AB}{BD}$

$BD = 125\sqrt{3}$

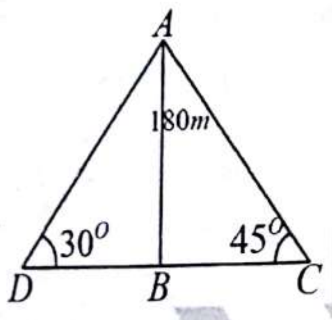
distance between two objects

CD = BD - BC

$= 125\sqrt{3} - 125$

$= 125(\sqrt{3}-1)$

6. (D)



From ΔABC

$\tan 45 = \frac{AB}{BC} \Rightarrow AB$
$= BC = 180$

From ΔABD

$\frac{AB}{BD} = \tan 30$

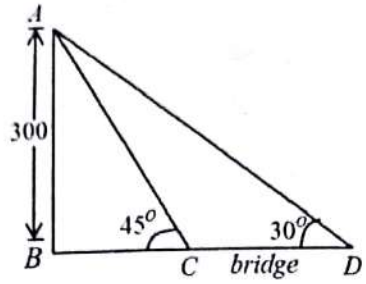
$\frac{180}{BD} = \frac{1}{\sqrt{3}} \Rightarrow BD = 180\sqrt{3}$

distance between two objects

$CD = BC + BD = 180 + 180\sqrt{3}$

$= 180(\sqrt{3}+1)$

7. (A)



From ΔABC

$\tan 45 = \frac{AB}{BC} \Rightarrow AB = BC =$
300

From ΔABD

$\tan 30 = \frac{AB}{BD} \Rightarrow \frac{1}{\sqrt{3}} = \frac{300}{BD}$

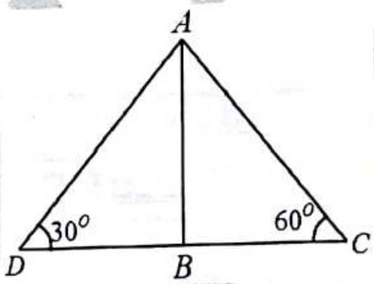
$\Rightarrow BD = 300\sqrt{3}$

length of the bridge CD = BD - BC

$= 300\sqrt{3} - 300$

$= 300(\sqrt{3}-1)$

8. (A)



From ΔABC

$\tan 60 = \frac{AB}{BC} \Rightarrow BC\sqrt{3} = AB$ ... (1)

From ΔABD

$\tan 30 = \frac{AB}{BD}$

$AB = \frac{BD}{\sqrt{3}}$ ... (2)

From (1) & (2)

$BC\sqrt{3} = \frac{BD}{\sqrt{3}}$

BD = 3BC

BC+BD = 400

BC+3BC = 400

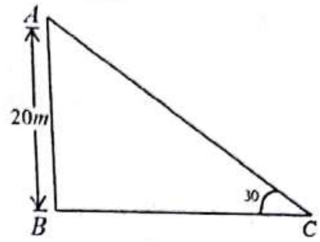
BC = 100

From (1) $AB = BC\sqrt{3}$

$= 100\sqrt{3}$

AB = 173.2

9. (B)

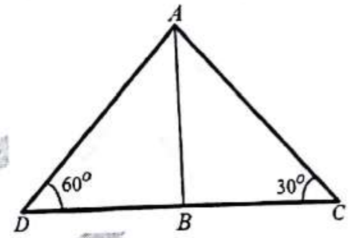


$\tan 30 = \frac{AB}{BC}$

$\frac{1}{\sqrt{3}} = \frac{20}{BC}$

$BC = 20\sqrt{3}$

10. (C)



From ΔABC

$\tan 30 = \frac{AB}{BC}$

$\frac{BC}{\sqrt{3}} = AB$ ...(1)

From ΔABD

$\tan 60 = \frac{AB}{BD}$

$\sqrt{3}\ BD = AB$ ...(2)

(1) & (2)

$\frac{BC}{\sqrt{3}} = \sqrt{3}\ BD$

BC= 3BD

BC+BD = 1km

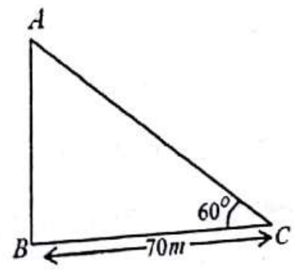
4BD = 1 km

$BD = \frac{1}{4}$km

From (2)

$AB = \frac{\sqrt{3}}{4}$ km

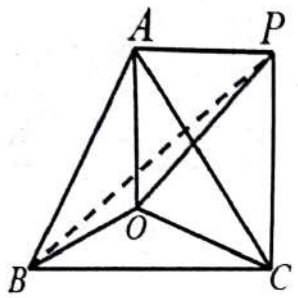
11. (B)



$\tan 60 = \frac{AB}{BC} \Rightarrow AB = 70\sqrt{3}$

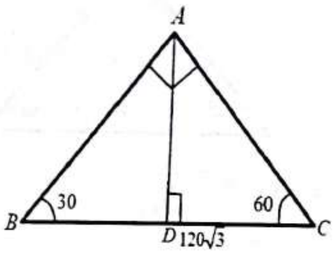
TYPE-V

1. (B)



AP=CP = BP
It is possible only when OA=OB=OC i.e. radii of circum circle. or, (circumcentre)

2. (D)



Remaining sides are the angles of $\Delta$is
sides of $\Delta$are

30,	60,	90
↓	↓	↓
$60\sqrt{3}$	180	$120\sqrt{3}$

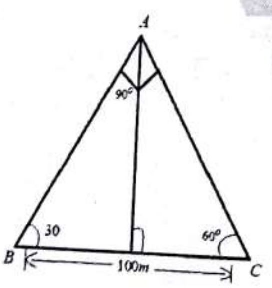
$\frac{1}{2}$ AB$\times$ AC = $\frac{1}{2}$ AD$\times$ BC

$60\sqrt{3}\times 180 = AD\times 120\sqrt{3}$

AD = 90

$\therefore$ Altitude is 90.

3. (C)



Angles are in the ratio 30 : 60 : 90
sides are in the ratio

1 :	$\sqrt{3}$:	2
↓	↓	↓
50	$50\sqrt{3}$	100

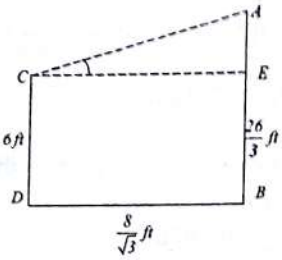
Areas $\frac{1}{2}\times AB\times AC =$

$\frac{1}{2}\times AD\times BC$

$50\times 50\sqrt{3} = 100\times AD$

AD = $25\sqrt{3}$

4. (B)



CD = EB, EC = BD
AE = AB - BE

$=\frac{26}{3}-6=\frac{8}{3}$

$\tan\theta = \frac{AE}{CE}$

$=\frac{\frac{8}{3}}{\frac{8}{\sqrt{3}}}=\frac{1}{\sqrt{3}}$

θ = 30

2016 – 2017 SOLUTIONS

1. (D) The hour hand of a watch traces an angle of 30° in an hour.
$\therefore$ Angle traced at 3 O'clock
= 3 × 30° = 90°
$\because$ 180° = π radian

$\therefore 90° = \frac{\pi}{180}\times 90°$

$=\frac{\pi}{2}$ radian

2. (C) For $0 < \theta < 90°$,
$0 < \sin\theta < 1$
$\sin\theta > \sin^2\theta$
If θ = 30°,

$\sin\theta = \sin 30° = \frac{1}{2}$

$\sin^2\theta = \sin^2 30° = \frac{1}{4}$

Clearly, $\frac{1}{2} > \frac{1}{4}$

3. (B) $\sin(\theta + 18°) = \frac{1}{2} = \sin 30°$

$\Rightarrow \theta + 18° = 30°$

$\Rightarrow \theta = 30° - 18° = 12°$

$\because 180° = \pi$ radian

$\therefore 12° = \frac{\pi}{180}\times 12$

$=\frac{\pi}{15}$ radians

4. (A) $\theta = \frac{l}{r}$ radian

$=\frac{11}{14}$ radian

$\because \pi$ radian = 180°

$\therefore$ 1 radian = $\frac{180°}{\pi}$

$\therefore \frac{11}{14}$ radian $= \frac{180}{\frac{22}{7}}\times\frac{11}{14}$

$=\frac{180\times 11\times 7}{22\times 14} = 45°$

5. (B) $(a^2 - b^2)\sin\theta + 2ab\cos\theta = (a^2 + b^2)$
On dividing by $\cos\theta$,
$(a^2 - b^2)\tan\theta + 2ab$
$= (a^2 + b^2)\sec\theta$
On squaring both sides,
$(a^2 - b^2)^2\tan^2\theta + 4a^2b^2 + 4ab(a^2 - b^2)\tan\theta$
$= (a^2 + b^2)^2\sec^2\theta$
$\Rightarrow (a^2 - b^2)^2\tan^2\theta + 4ab(a^2 - b^2)\tan\theta + 4a^2b^2$
$= (a^2 + b^2)^2(1 + \tan^2\theta)$
$\Rightarrow (a^2 + b^2)^2\tan^2\theta - (a^2 - b^2)^2\tan^2\theta\ 4ab(a^2 - b^2)^2\tan\theta + (a^2 + b^2) - 4a^2b^2 = 0$
$\Rightarrow \tan^2\theta((a^2 + b^2)^2 - (a^2 - b^2)^2) - 4ab(a^2 - b^2)\tan\theta + (a^2 - b^2)^2 = 0$
$\Rightarrow 4a^2b^2\tan^2\theta - 4ab(a^2 - b^2)\tan\theta + (a^2 - b^2)^2 = 0$
$\Rightarrow \{2ab\tan\theta - (a^2 - b^2)\}^2 = 0$
$\Rightarrow 2ab\tan\theta - (a^2 - b^2) = 0$

$\Rightarrow \tan\theta = \frac{a^2 - b^2}{2ab}$

6. (D) $2y\cos\theta = x\sin\theta$ ... (i)
$2x\sec\theta - y\,\text{cosec}\theta = 3$

$\Rightarrow 2.\frac{2y\cos\theta}{\sin\theta}.\sec\theta - y\,\text{cosec}\theta = 3$

$\Rightarrow 4y\,\text{cosec}\theta - y\,\text{cosec}\theta = 3$
$\Rightarrow 3y\,\text{cosec}\theta = 3$

$\Rightarrow y = \frac{3}{3\,\text{cosec}\theta} = \sin\theta$

From equation (i),
$2\sin\theta\,.\,\cos\theta = x\sin\theta$
$\Rightarrow x = 2\cos\theta$
$\therefore x^2 + 4y^2 = 4\cos^2\theta + 4\sin^2\theta = 4(\cos^2\theta + \sin^2\theta) = 4$

7. (C) $\tan\theta_1 = 1\ \tan 45°$
$\Rightarrow \theta_1 = 45°$

Again, $\sin\theta_2 = \frac{1}{\sqrt{2}} = \sin 45°$

$\Rightarrow \theta_2 = 45°$

$\therefore \sin(\theta_1 + \theta_2) = \sin 90° = 1$

8. (B) $\tan\theta(1 + \sec 2\theta)(1 + \sec 4\theta)(1 + \sec 8\theta)$

$= \tan\theta\left(1 + \frac{1}{\cos 2\theta}\right)\left(1 + \frac{1}{\cos 4\theta}\right)\left(1 + \frac{1}{\cos 8\theta}\right)$

$= \tan\theta\left(\frac{\cos 2\theta + 1}{\cos 2\theta}\right)\left(\frac{\cos 4\theta + 1}{\cos 4\theta}\right)\left(\frac{\cos 8\theta + 1}{\cos 8\theta}\right)$

$= \tan\theta \frac{2\cos^2\theta}{\cos 2\theta} \cdot \frac{2\cos^2 2\theta}{\cos 4\theta} \frac{2\cos^2 4\theta}{\cos 8\theta}$

$\left[\because 1 + \cos 2\theta = 2\cos^2\theta\right]$

$= 8.\frac{\tan\theta.\cos^2\theta.\cos 2\theta.\cos 4\theta}{\cos 8\theta}$

$= 4.\frac{2\sin\theta.\cos\theta.\cos 2\theta.\cos 4\theta}{\cos 8\theta}$

$= 4.\frac{\sin 2\theta.\cos 2\theta.\cos 4\theta}{\cos 8\theta}$

$= \frac{2\sin 4\theta.\cos 4\theta}{\cos 8\theta} = \frac{\sin 8\theta}{\cos 8\theta} = \tan 8\theta$

9. (B) $\frac{\sin 65°}{\cos 25°} = \frac{\sin(90° - 25°)}{\cos 25°}$

$= \frac{\cos 25°}{\cos 25°} = 1$

10. (C) $\sec^2 45° - \cot^2 45° - \sin^2 30° - \sin^2 60°$

$= \left(\sqrt{2}\right)^2 - 1 - \left(\frac{1}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2$

$= 2 - 1 - \frac{1}{4} - \frac{3}{4}$

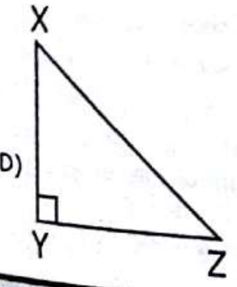
$= 1 - \frac{1}{4} - \frac{3}{4} = \frac{3}{4} - \frac{3}{4} = 0$

11. (A) Expression

$= \frac{(\sin\theta.\operatorname{cosec}\theta)(\tan\theta.\cot\theta)}{\sin^2\theta + \cos^2\theta}$

$= \frac{1 \times 1}{1} = 1$

12 (D)



$XY = 2\sqrt{6}$ cm

$XY - YZ = 2$ cm. ... (i)

$\therefore XZ^2 = XY^2 + YZ^2$

$\Rightarrow XZ^2 - YZ^2 = \left(2\sqrt{6}\right)^2$

$\Rightarrow XZ^2 - YZ^2 = 24$

$\therefore \frac{XZ^2 - YZ^2}{XZ - YZ} = \frac{24}{2}$

$\Rightarrow XZ + YZ = 12$ ... (ii)

$\Rightarrow \sec X + \tan X = \frac{XZ}{XY} + \frac{YZ}{XY}$

$= \frac{XZ + YZ}{XY} = \frac{12}{2\sqrt{6}} = \sqrt{6}$

13. (C) Expression

$= 8 \cos 10° . \cos 20° . \cos 40°$

$= 4\left(\frac{2\sin 10°.\cos 10°.\cos 20°.\cos 40°}{\sin 10°}\right)$

$= 2\left(\frac{2\sin 20°.\cos 20°.\cos 40°}{\sin 10°}\right)$

$[\because 2 \sin\theta . \cos\theta = \sin 2\theta]$

$= \left(\frac{2\sin 40°.\cos 40°}{\sin 10°}\right)$

$= \frac{\sin 80°}{\sin 10°} = \frac{\sin 80°}{\cos(90° - 10°)}$

$= \frac{\sin 80°}{\cos 80°}$ or, $\frac{\cos 10°}{\sin 10°}$

$\left[\begin{array}{l}\because \sin(90° - \theta) = \cos\theta; \\ \cos(90° - \theta) = \sin\theta\end{array}\right]$

$= \tan 80°$ or $\cot 10°$

14. (C) $\pi \sin\theta = 1$ and $\pi \cos\theta = 1$

$\therefore \frac{\pi \sin\theta}{\pi \cos\theta} = 1$

$\therefore \tan\theta = 1 = \tan 45°$

$\Rightarrow \theta = 45°$

$\therefore \sqrt{3} \tan\left(\frac{2\theta}{3}\right) + 1$

$= \sqrt{3} \tan\left(\frac{2 \times 45°}{3}\right) + 1$

$= \sqrt{3} \tan 30° + 1$

$= \sqrt{3} \times \frac{1}{\sqrt{3}} + 1 = 1 + 1 = 2$

15. (B) $\sin A = \cos B$

$\Rightarrow \sin A = \sin(90° - B)$

$\Rightarrow A = 90° - B$

$\Rightarrow A + B = 90°$

$\therefore \angle C = 90°$

$\therefore \cos C = \cos 90° = 0$

16. (D) $x^2 = \sin^2 30° + 4 \cot^2 45° - \sec^2 60°$

$= \left(\frac{1}{2}\right)^2 + 4(1)^2 - (2)^2$

$= \frac{1}{4} + 4 - 4 = \frac{1}{4}$

$\therefore x = \frac{1}{2}$

17. (B) $7 \sin^2\theta + 3\cos^2\theta = 4$

$\Rightarrow 4\sin^2\theta + 3\sin^2\theta + 3\cos^2\theta = 4$

$\Rightarrow 4\sin^2\theta + 3(\sin^2\theta + \cos^2\theta) = 4$

$\Rightarrow 4\sin^2\theta = 4 - 3$

$[\because \sin^2\theta + \cos^2\theta = 1]$

$\Rightarrow 4\sin^2\theta = 1$

$\Rightarrow \sin^2\theta = \frac{1}{4}$

$\Rightarrow \sin\theta = \frac{1}{2} = \sin 30°$

$\Rightarrow \theta = 30°$

$\therefore \sec\theta + \operatorname{cosec}\theta$

$= \sec 30° + \operatorname{cosec} 30°$

$= \frac{2}{\sqrt{3}} + 2$

18. (C) $\tan 80° . \tan 10° + \sin^2 70° + \sin^2 20°$

$= \tan(90° - 10°) . \tan 10° + \sin^2(90° - 20°) + \sin^2 20°$

$= \cot 10° . \tan 10° + \cos^2 20° + \sin^2 20°$

$= 1 + 1 = 2$

$[\sin(90° - \theta) = \cos\theta; \tan(90° - \theta) = \cot\theta; \tan\theta . \cot\theta = 1]$

19. (B) $\sin 27° = \sin(90° - 63°) = \cos 63°$

$[\because \sin(90° - \theta) = \cos\theta]$

$\therefore \left(\frac{\sin 27°}{\cos 63°}\right)^2 + \left(\frac{\cos 63°}{\sin 27°}\right)^2$

$= \left(\frac{\sin 27°}{\sin 27°}\right)^2 + \left(\frac{\sin 27°}{\sin 27°}\right)^2$

$= 1 + 1 = 2$

20. (A) $\sin\theta + \cos\theta = 1$

On squaring,

$(\sin\theta + \cos\theta)^2 = 1$

$\Rightarrow \sin^2\theta + \cos^2\theta + 2 \sin\theta . \cos\theta = 1$

$\Rightarrow 1 + 2 \sin\theta . \cos\theta = 1$

$\Rightarrow 2 \sin\theta . \cos\theta = 1 - 1 = 0$

$\Rightarrow \sin\theta . \cos\theta = 0$

21. (C) $\sin C - \sin D = 2\cos\frac{C + D}{2}$

$\sin\frac{C - D}{2} = x$

Illustration

$\sin(A + B)$

$= \sin A . \cos B + \cos A . \sin B$

$\sin(A - B)$

$= \sin A . \cos B - \cos A . \sin B$

$\therefore \sin(A + B) - \sin(A - B)$

$= 2 \cos A . \sin B$

Let, A + B = C; A − B = D

∴ On adding

$A = \frac{C+D}{2}$

On subtracting,

$B = \frac{C-D}{2}$

$\therefore \sin C - \sin D = 2\cos\frac{C+D}{2}\cdot\sin\frac{C-D}{2}$

22. (D) $x = a\cos\theta\cdot\cos\phi$

$y = a\cos\theta\cdot\sin\phi$

$z = a\sin\theta$

$\therefore x^2 + y^2 + z^2$

$= a^2\cos^2\phi\cdot\cos^2\theta + a^2\cos^2\theta\cdot\sin^2\phi + a^2\sin^2\theta$

$= a^2\cos^2\theta(\cos^2\phi + \sin^2\phi) + a^2\sin^2\theta$

$= a^2\cos^2\theta + a^2\sin^2\theta$

$= a^2(\cos^2\theta + \sin^2\theta) = a^2$

23. (C) $\tan\theta = \tan30^\circ\cdot\tan60^\circ$

$\Rightarrow \tan\theta = \frac{1}{\sqrt{3}}\times\sqrt{3} = 1$

$\Rightarrow \tan\theta = \tan45^\circ$

$\Rightarrow \theta = 45^\circ$

$\therefore 2\theta = 2\times45^\circ = 90^\circ$

24. (B) Expression

$= (1+\tan^2\theta)\cdot(1-\sin^2\theta)$

$= \sec^2\theta\cdot\cos^2\theta = 1$

$[\because \sec^2\theta - \tan^2\theta = 1 = \sin^2\theta + \cos^2\theta;\ \sec\theta\cdot\cos\theta = 1]$

25. (A) $\sin\theta = \frac{\sqrt{3}}{2} = \sin60^\circ$

$\Rightarrow \theta = 60^\circ$

$\therefore \tan(\theta - 15^\circ)$

$= \tan(60^\circ - 15^\circ) = \tan45^\circ = 1$

26. (D) $r\sin\theta = \sqrt{3}$

$r\cos\theta = 1$

On squaring and adding,

$r^2\sin^2\theta + r^2\cos^2\theta = 3 + 1$

$\Rightarrow r^2(\sin^2\theta + \cos^2\theta) = 4$

$\Rightarrow r^2 = 4 \Rightarrow r = \sqrt{4} = 2$

Again, $\frac{r\sin\theta}{r\cos\theta} = \sqrt{3}$

$\Rightarrow \tan\theta = \sqrt{3} = \tan60^\circ$

$\Rightarrow \theta = 60^\circ$

27. (C) $\sin(2x - 20^\circ) = \cos(2y + 20^\circ)$

$\Rightarrow \sin(2x - 20^\circ)$

$= \sin(90^\circ - (2y + 20^\circ))$

$\Rightarrow 2x - 20^\circ = 90^\circ - 2y - 20^\circ$

$\Rightarrow 2x + 2y = 90^\circ$

$\Rightarrow 2(x+y) = 90^\circ \Rightarrow x + y = 45^\circ$

$\therefore \tan(x+y) = \tan45^\circ = 1$

28. (C) $(1 + \sec20^\circ + \cot70^\circ)(1 - \text{cosec}20^\circ + \tan70^\circ)$

$= (1 + \sec20^\circ + \tan20^\circ)(1 - \text{cosec}20^\circ + \cot20^\circ)$

$[\because \tan(90^\circ - \theta) = \cot\theta;\ \cot(90^\circ - \theta) = \tan\theta]$

$= \left(1 + \frac{1}{\cos20^\circ} + \frac{\sin20^\circ}{\cos20^\circ}\right)\left(1 - \frac{1}{\sin20^\circ} + \frac{\cos20^\circ}{\sin20^\circ}\right)$

$= \left(\frac{\cos20^\circ + 1 + \sin20^\circ}{\cos20^\circ}\right)\left(\frac{\sin20^\circ - 1 + \cos20^\circ}{\sin20^\circ}\right)$

$= \frac{(\sin20^\circ + \cos20^\circ + 1)(\sin20^\circ + \cos20^\circ - 1)}{\sin20^\circ.\cos20^\circ}$

$= \frac{(\sin20^\circ + \cos20^\circ)^2 - 1}{\sin20^\circ.\cos20^\circ}$

$= \frac{\sin^2 20^\circ + \cos^2 20^\circ + 2\sin20^\circ.\cos20^\circ - 1}{\sin20^\circ.\cos20^\circ}$

$= \frac{1 + 2\sin20^\circ.\cos20^\circ - 1}{\sin20^\circ.\cos20^\circ} = 2$

29. (D) Expression

$= \frac{\sin\theta}{1+\cos\theta} + \frac{\sin\theta}{1-\cos\theta}$

$= \frac{\sin\theta(1-\cos\theta) + \sin\theta(1+\cos\theta)}{(1+\cos\theta)(1-\cos\theta)}$

$= \frac{\sin\theta - \sin\theta.\cos\theta + \sin\theta + \sin\theta.\cos\theta}{1-\cos^2\theta}$

$= \frac{2\sin\theta}{\sin^2\theta} = \frac{2}{\sin\theta} = 2\,\text{cosec}\theta$

30. (D) $\cos^2 20^\circ + \cos^2 70^\circ$

$= \cos^2(90^\circ - 70^\circ) + \cos^2 70^\circ$

$= \sin^2 70^\circ + \cos^2 70^\circ = 1$

$[\cos(90^\circ - \theta) = \sin\theta]$

31. $\sin\theta + \text{cosec}\theta = 2$

$\Rightarrow \sin\theta + \frac{1}{\sin\theta} = 2$

$\Rightarrow \sin^2\theta + 1 = 2\sin\theta$

$\Rightarrow \sin^2\theta - 2\sin\theta + 1 = 0$

$\Rightarrow (\sin\theta - 1)^2 = 0$

$\Rightarrow \sin\theta - 1 = 0$

$\Rightarrow \sin\theta = 1$

$\therefore \text{cosec}\theta = 1$

$\Rightarrow \sin^{-7}\theta + \text{cosec}^7\theta$

$= (1)^{-7} + (1)^7 = 2$

32. (B) $\frac{\sin^2 63^\circ + \sin^2 27^\circ}{\cos^2 17^\circ + \cos^2 73^\circ}$

$= \frac{\sin^2 63^\circ + \sin^2(90^\circ - 63^\circ)}{\cos^2 17^\circ + \cos^2(90^\circ - 17^\circ)}$

$= \frac{\sin^2 63^\circ + \cos^2 63^\circ}{\cos^2 17^\circ + \sin^2 17^\circ} = 1$

$\left[\begin{array}{l}\because \sin(90^\circ - \theta) = \cos\theta;\\ \cos(90^\circ - \theta) = \sin\theta;\\ \sin^2\theta + \cos^2\theta = 1\end{array}\right]$

33. (B) $\cos^2 20^\circ + \cos^2 70^\circ$

$= \cos^2 20^\circ + \cos^2(90^\circ - 20^\circ)$

$= \cos^2 20^\circ + \sin^2 20^\circ = 1$

$[\because \cos(90^\circ - \theta) = \sin\theta]$

34. (A) $a\sin45^\circ\cdot\cos45^\circ\cdot\tan60^\circ = \tan^2 45^\circ - \cos60^\circ$

$\Rightarrow a\times\frac{1}{\sqrt{2}}\times\frac{1}{\sqrt{2}}\times\sqrt{3} = 1 - \frac{1}{2}$

$\Rightarrow \frac{\sqrt{3}a}{2} = \frac{1}{2}$

$\Rightarrow \sqrt{3}a = 1 \qquad \Rightarrow a = \frac{1}{\sqrt{3}}$

35. (B) $\tan315^\circ\cdot\cot(-405^\circ)$

$= -\tan315^\circ\cdot\cot405^\circ$

$[\cot(-\theta) = -\cot\theta]$

$= -\tan(360^\circ - 45^\circ)\cdot\cot(360^\circ + 45^\circ)$

$= -(-\tan45^\circ)\cdot\cot45^\circ$

$= \tan45^\circ\cdot\cot45^\circ = 1$

36. (C) $\sin\theta + \text{cosec}\theta = 2$

$\Rightarrow \sin\theta + \frac{1}{\sin\theta} = 2$

$\Rightarrow \frac{\sin^2\theta + 1}{\sin\theta} = 2$

$\Rightarrow \sin2\theta + 1 = 2\sin\theta$

$\Rightarrow \sin1\theta - 2\sin\theta + 1 = 0$

$\Rightarrow (\sin\theta - 1)2 = 0$

$\Rightarrow \sin\theta - 1 = 0$

$\Rightarrow \sin\theta = 1$

$\therefore \text{cosec}\theta = 1$

$\therefore \sin^n\theta + \text{cosec}^n\theta = 1 + 1 = 2$

37. (B) $\sin A + \sin^2 A = 1$

$\Rightarrow \sin A = 1 - \sin^2 A = \cos^2 A$

$\therefore \cos^4 A = \sin^2 A$

$\therefore \cos^2 A + \cos^4 A$

$= \cos^2 A + \sin^2 A = 1$

38. (B) $4\sin^2\theta = 3$

$\Rightarrow \sin^2\theta = \frac{3}{4}$

$\Rightarrow \sin\theta = \frac{\sqrt{3}}{2} = \sin60^\circ$

$\Rightarrow \theta = 60^\circ$

$\therefore \tan\theta - \cot\frac{\theta}{2}$

$= \tan60^\circ - \cot30^\circ$

$= \sqrt{3} - \sqrt{3} = 0$

39. (C) $\sin\theta - \cos\theta = 0$

$\Rightarrow \sin\theta = \cos\theta$

$\Rightarrow \tan\theta = 1 = \tan45^\circ$

$\Rightarrow \theta = 45^\circ$

$\therefore \sec\theta + \text{cosec}\theta$

$= \sec45^\circ + \text{cosec}45^\circ$

$= \sqrt{2} + \sqrt{2} = 2\sqrt{2}$

40. (A) $\alpha : \beta = 2 : 1$

Sum of the terms of ratio

$= 2 + 1 = 3$

$\alpha + \beta = 90^\circ$

$\therefore \alpha = \frac{2}{3} \times 90^\circ = 60^\circ$

$\beta = 30^\circ$

$\therefore \frac{\cos\alpha}{\cos\beta} = \frac{\cos 60^\circ}{\cos 30^\circ} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}}$

$= 1 : \sqrt{3}$

41. (C)
$\sec(4x - 50^\circ) = \operatorname{cosec}(50^\circ - x)$
$\Rightarrow \sec(4x - 50^\circ) = \sec(90^\circ - (50^\circ - x)) = \sec(40^\circ + x)$
$\Rightarrow 4x - 50^\circ = 40^\circ + x$
$\Rightarrow 4x - x = 50^\circ + 40^\circ$
$\Rightarrow 3x = 90^\circ \Rightarrow x = \frac{90^\circ}{3} = 30^\circ$

42. (A) $\cos 53^\circ - \sin 37^\circ$
$= \cos(90^\circ - 37^\circ) - \sin 37^\circ$
$= \sin 37^\circ - \sin 37^\circ = 0$

43. (B) $7\sin^2\theta + 3\cos^2\theta = 4$
On dividing by $\cos^2\theta$,
$\frac{7\sin^2\theta}{\cos^2\theta} + \frac{3\cos^2\theta}{\cos^2\theta} = \frac{4}{\cos^2\theta}$
$\Rightarrow 7\tan^2\theta + 3 = 4\sec^2\theta = 4(1 + \tan^2\theta)$
$\Rightarrow 7\tan^2\theta + 3 = 4 + 4\tan^2\theta$
$\Rightarrow 7\tan^2\theta - 4\tan^2\theta = 4 - 3$
$\Rightarrow 3\tan^2\theta = 1$
$\Rightarrow \tan^2\theta = \frac{1}{3}$
$\Rightarrow \tan\theta = \frac{1}{\sqrt{3}}$

44. (D) $\sin(A - B) = \sin A . \cos B - \cos A . \sin B$
$\cos(A - B) = \cos A . \cos B + \sin A . \sin B$
$\therefore \frac{\sin(A-B)}{\cos(A-B)}$
$= \frac{\sin A.\cos B - \cos A.\sin B}{\cos A.\cos B + \sin A.\sin B}$
$\Rightarrow \tan(A - B)$
$= \frac{\frac{\sin A.\cos A}{\cos A.\cos B} - \frac{\cos A.\sin B}{\cos A.\cos B}}{\frac{\cos A.\cos B}{\cos A.\cos B} + \frac{\cos A.\sin B}{\cos A.\cos B}}$
(Dividing numerator and denominator by cosA cosB)
$= \frac{\tan A - \tan B}{1 + \tan A.\tan B}$

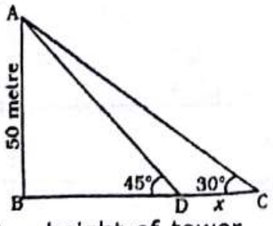
45. (C) $\sin\frac{11\pi}{6}$
$= \sin\left(2\pi - \frac{\pi}{6}\right)$
$[\because \sin(360^\circ - \theta) = \sin(2\pi - \theta) = -\sin\theta]$
$= -\sin\frac{\pi}{6} = -\frac{1}{2}$

46. (B) $\sin P + \operatorname{cosec} P = 2$
$\Rightarrow \sin P + \frac{1}{\sin P} = 2$
$\Rightarrow \frac{\sin^2 P + 1}{\sin P} = 2$
$\Rightarrow \sin^2 P + 1 = 2\sin P$
$\Rightarrow \sin^2 P - 2\sin P + 1 = 0$
$\Rightarrow (\sin P - 1)^2 = 0$
$\Rightarrow \sin P - 1 = 0 \Rightarrow \sin P = 1$
$\therefore \operatorname{cosec} P = 1$
$\therefore \sin^2 P + \operatorname{cosec}^2 P = 1 + 1 = 2$

47. (B) $(\sin^6\theta + \cos^6\theta) - 3(\sin^4\theta + \cos^4\theta) + 1$
$= 2\{(\sin^2\theta + \cos^2\theta)^3 - 3\sin^2\theta\cos^2\theta(\sin^2\theta + \cos^2\theta)\} - 3\{(\sin^2\theta + \cos^2\theta)^2 - 2\sin^2\theta . \cos^2\theta\} + 1$
$= 2(1 - 3\sin^2\theta . \cos^2\theta) - 3(1 - 2\sin^2\theta . \cos^2\theta) + 1$
$= 2 - 6\sin^2\theta . \cos^2\theta - 3 + 6\sin^2\theta = 3 - 3 = 0$

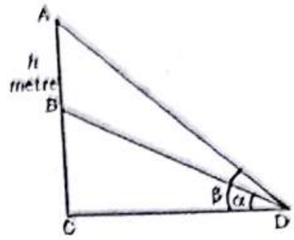
48. (B) $x = \operatorname{cosec}\theta - \sin\theta$
$= \frac{1}{\sin\theta} - \sin\theta$
$= \frac{1 - \sin^2\theta}{\sin\theta} = \frac{\cos^2\theta}{\sin\theta}$
$y = \sec\theta - \cos\theta$
$= \frac{1}{\cos\theta} - \cos\theta$
$= \frac{1 - \cos^2\theta}{\cos\theta} = \frac{\sin^2\theta}{\cos\theta}$
$\therefore x^2y^2(x^2 + y^2 + 3)$
$= \frac{\cos^4\theta}{\sin^2\theta} \times \frac{\sin^4\theta}{\cos^2\theta}\left(\frac{\cos^4\theta}{\sin^2\theta} + \frac{\sin^4\theta}{\cos^2\theta} + 3\right)$
$= \sin^2\theta . \cos^2\theta$
$\left(\frac{\cos^6\theta + \sin^6\theta + 3\sin^2\theta.\cos^2\theta}{\sin^2\theta.\cos^2\theta}\right)$
$= \cos^6\theta + \sin^6\theta + 3\sin^2\theta . \cos^2\theta(\sin^2\theta + \cos^2\theta)$
$= (\cos^2\theta + \sin^2\theta)^3 = 1$

49. (B)



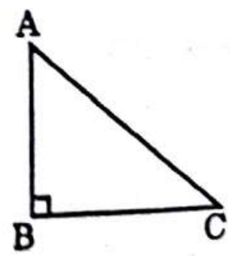
AB = height of tower
= 50 metre
$\angle ACB = 30^\circ$; $\angle ADB = 45^\circ$
In ΔABD,
$\tan 45^\circ = \frac{50}{BD}$
$\Rightarrow 1 = \frac{50}{BD} \Rightarrow BD = 50$ metre
In ΔABC,
$\tan 30^\circ = \frac{AB}{BC} \Rightarrow \frac{1}{\sqrt{3}} = \frac{50}{50 + x}$
$\Rightarrow 50 + x = 50\sqrt{3}$
$\Rightarrow x = 50\sqrt{3} - 50$
$= 50(\sqrt{3} - 1)$ metre

50. (B)



Let height of tower = BC
= y metre
AB = height of flag-staff = h metre
$\angle BDC = \alpha$; $\angle ADC = \beta$
Let, Cd = x metre
In ΔBCD,
$\tan\alpha = \frac{BC}{CD}$
$\Rightarrow \tan\alpha = \frac{y}{x}$
$\Rightarrow x = \frac{y}{\tan\alpha}$ ... (ii)
In ΔACD,
$\tan\beta = \frac{AC}{CD}$
$\Rightarrow \tan\beta = \frac{h + y}{x}$
$\Rightarrow x = \frac{h + y}{\tan\beta}$ ... (ii)
$\therefore \frac{y}{\tan\alpha} = \frac{h + y}{\tan\beta}$
$\Rightarrow y\tan\beta = h\tan\alpha + y\tan\alpha$
$\Rightarrow y\tan\beta - y\tan\alpha = h\tan\alpha$
$\Rightarrow y(\tan\beta - \tan\alpha) = h\tan\alpha$
$\Rightarrow y = \frac{h\tan\alpha}{\tan\beta - \tan\alpha}$

51. (C)



AB = Height of the wall
AC = Length of ladder = h metre
BC = b = 4.6 metre
$\angle ACB = 60^\circ$

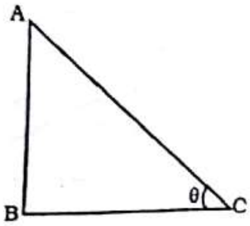
$\therefore \cos 60^\circ = \frac{BC}{AC}$

$\Rightarrow \frac{1}{2} = \frac{4.6}{h}$

$\Rightarrow h = (2 \times 4.6)$ metre

$= 9.2$ metre

52. (B)



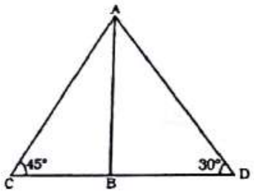
AB = BC

$\tan\theta = \frac{AB}{BC}$

$\Rightarrow \tan\theta = 1 = \tan 45^\circ$

$\Rightarrow \theta = 45^\circ$

53. (C)



AB = Light house = 100 metre

C and D are positions of ships.

Let,

BC = x metre and BD = y metre

$\angle ACB = 45^\circ$ and $\angle ADB = 30^\circ$

From ΔABC,

$\tan 45^\circ = \frac{AB}{BC}$

$\Rightarrow 1 = \frac{100}{x} \Rightarrow x = 100$ metre

From ΔABD,

$\tan 30^\circ = \frac{AB}{BD}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{100}{y}$

$\Rightarrow y = 100\sqrt{3}$ metre

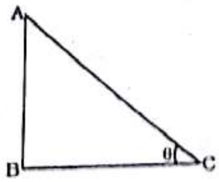
$= (100 \times 1.73)$ metre

$= 173$ metre

$\therefore$ Required distance = x + y

$= 100 + 173 = 273$ metre

54. (B)



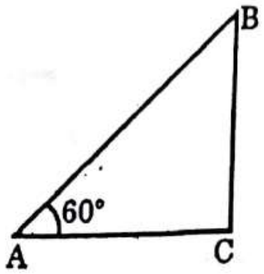
AB = Tower

BC = distance of point C

$\tan\theta = \frac{AB}{BC} = \frac{5\sqrt{3}}{5} = \sqrt{3}$

$\therefore \tan\theta = \tan 60^\circ \Rightarrow \theta = 60^\circ$

55. (D)



AB = Length of thread

= h metre

= 80 metre

$\angle BAC = 60^\circ$

BC = Vertical height of kite

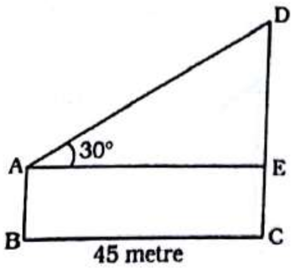
In ΔABC,

$\sin 60^\circ = \frac{BC}{AB}$

$\Rightarrow \frac{\sqrt{3}}{2} = \frac{h}{80}$

$\Rightarrow h = 80 \times \frac{\sqrt{3}}{2} = 40\sqrt{3}$ metre

56. (C)



AB = Height of observer = 1.6 metre

CD = Height of tower

= h metre

$\therefore$ DE = (h − 1.6) metre ; BC = AE = 45 metre

$\angle DAE = 30^\circ$

$\therefore \tan 30^\circ = \frac{DE}{AE}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h-1.6}{45}$

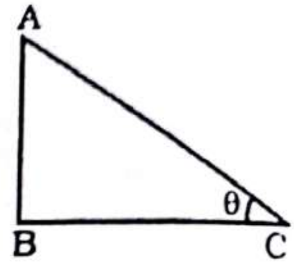
$\Rightarrow h - 1.6 = \frac{45}{\sqrt{3}} = 15\sqrt{3}$

$\Rightarrow h - 1.6 = 15 \times 1.732 = 25.98$

$\Rightarrow h = (25.98 + 1.6)$ metre

$= 27.58$ metre

57. (C)



AB = Height of pole = x units

BC = Length of shadow

$= \sqrt{3}$ x units

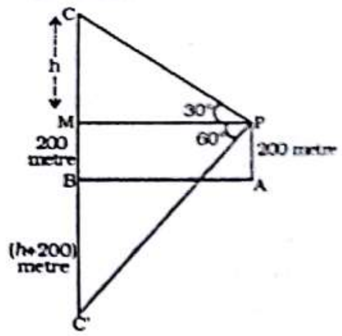
$\angle ACB = \theta$

$\therefore \tan\theta = \frac{AB}{BC}$

$\Rightarrow \tan\theta = \frac{x}{\sqrt{3}x} = \frac{1}{\sqrt{3}} = \tan 30^\circ$

$\Rightarrow \theta = 30^\circ$

58. (D)



AB is the surface of lake. C' is the reflection of cloud 'C'.

$\angle CPM = 30^\circ$ and $\angle C'PM = 60^\circ$

Let, C M = h metre

CB = (h + 200) metre

C'B = (h + 200) metre

In ΔCMP,

$\tan 30^\circ = \frac{CM}{PM}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{PM}$

$\Rightarrow PM = \sqrt{3}h$ ...(i)

In $\Delta PMC'$,

$\tan 60^\circ = \frac{C'M}{PM}$

$\Rightarrow \tan 60^\circ = \frac{C'B + BM}{PM}$

$\Rightarrow \sqrt{3} = \dfrac{h + 200 + 200}{PM}$

$\Rightarrow PM = \dfrac{h + 400}{\sqrt{3}}$ ...(ii)

From equations (i) and (ii),

$\sqrt{3}\,h = \dfrac{h + 400}{\sqrt{3}}$

$\Rightarrow 3h = h + 400$

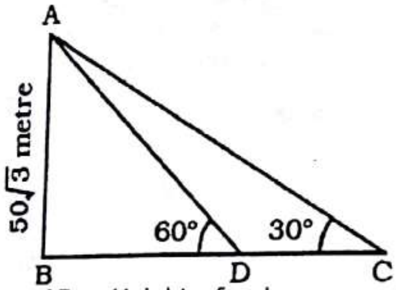
$\Rightarrow 2h = 400 \Rightarrow h = 200$

$\therefore CB = h + 200 = 400$ metre

Note : If the angle of elevation of a cloud from a point h metre above a lake is α and the angle of depression of its reflection in the lake is β, then the height of the cloud

$= \dfrac{h(\tan\beta + \tan\alpha)}{(\tan\beta - \tan\alpha)}$

59. (C)



AB = Height of pole

$= 50\sqrt{3}$ metre

BC = Length of shadow = x metre

When,

$\angle ACB = 30°$

BD = Length of shadow = y metre

When,

$\angle ADB = 60°$

In ΔABC,

$\tan 30° = \dfrac{AB}{BC}$

$\Rightarrow \dfrac{1}{\sqrt{3}} = \dfrac{50\sqrt{3}}{x}$

$\Rightarrow x = 50\sqrt{3} \times \sqrt{3} = 150$ metre

In ΔABD,

$\tan 60° = \dfrac{AB}{BD}$

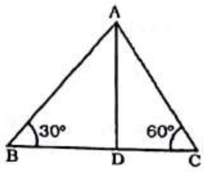
$\Rightarrow \sqrt{3} = \dfrac{50\sqrt{3}}{y}$

$\Rightarrow \sqrt{3}y = 50\sqrt{3}$

$\Rightarrow y = \dfrac{50\sqrt{3}}{\sqrt{3}} = 50$ metre

$\therefore CD = x - y = 150 - 50 = 100$

60. (A)



AD = Height of temple = 75 metre

B and C $\Rightarrow$ Positions of men

$\angle ABD = 30°$; $\angle ACD = 60°$

In ΔABD,

$\tan 30° = \dfrac{AD}{BD}$

$\Rightarrow \dfrac{1}{\sqrt{3}} = \dfrac{75}{BD}$

$\Rightarrow BD = 75\sqrt{3}$ metre

In ΔACD,

$\tan 60° = \dfrac{AD}{DC}$

$\Rightarrow \sqrt{3} = \dfrac{75}{DC}$

$\Rightarrow DC = \dfrac{75}{\sqrt{3}} = 25\sqrt{3}$ metre

$\therefore BC = BD + DC$

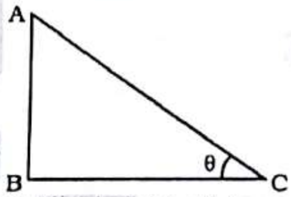
$= 75\sqrt{3} + 25\sqrt{3}$

$= 100\sqrt{3}$ metre

$= (100 \times 1.732)$ metre

$= 173.2$ metre

61. (B)



Let the height of tower be x units.

$\therefore$ Length of shadow $= \sqrt{3}x$ units

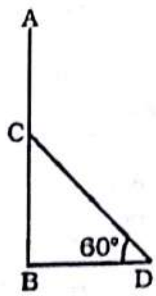
In ΔABC,

$\therefore \tan\theta = \dfrac{AB}{BC} = \dfrac{x}{\sqrt{3}x} = \dfrac{1}{\sqrt{3}}$

$\Rightarrow \tan\theta = \tan 30°$

$\Rightarrow \theta = 30°$

62. (C)



AB = Height of tree

Let the tree brak at point C.

BC = x metre

$\therefore AC = CD$

$\angle CDB = 60°$; BD = 10 metre

In ΔBCD,

$\tan 60° = \dfrac{BC}{BD} \Rightarrow \sqrt{3} = \dfrac{x}{10}$

$\Rightarrow x = 10\sqrt{3}$ metre

Again, $\sin 60° = \dfrac{BC}{CD}$

$\Rightarrow \dfrac{\sqrt{3}}{2} = \dfrac{10\sqrt{3}}{CD}$

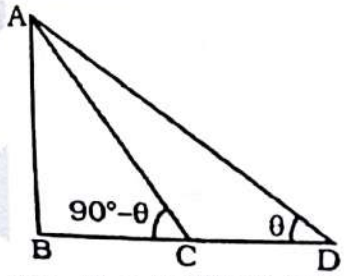
$\Rightarrow CD = \dfrac{2 \times 10\sqrt{3}}{\sqrt{3}} = 20$ metre

$\therefore$ Height of tree = AB

$= \left(20 + 10\sqrt{3}\right)$ metre

$= 10\left(2 + \sqrt{3}\right)$ metre

63. (D)



Let, AB = Height of tower = h metre

BC = 4 metre, BD = 9 metre

$\angle ACB = 90° - \theta$; $\angle ADB = \theta$

In ΔABC,

$\tan(90° - \theta) = \dfrac{AB}{BC}$

$\Rightarrow \cot\theta = \dfrac{h}{4}$ ...(i)

In ΔABD,

$\tan\theta = \dfrac{h}{9}$ ...(ii)

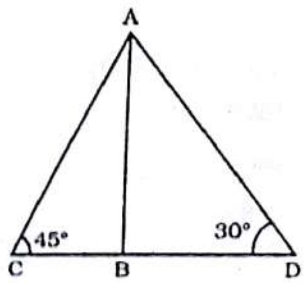
On multiplying both equations.

$\tan\theta . \cot\theta = \dfrac{h}{4} \times \dfrac{h}{9}$

$\Rightarrow \dfrac{h^2}{36} = 1 \Rightarrow h^2 = 36$

$\Rightarrow h = \sqrt{36} = 6$ metre

64. (A)



AB = Height of tower
C and D ⇒ Positions of men
BC = x metre (let)
BD = y metre
∠ACB = 45°; ∠ADB = 30°
In ΔABC,

$$\tan 45^\circ = \frac{AB}{BC}$$

$$\Rightarrow 1 = \frac{50}{x}$$

⇒ x = 50 metre
In ΔABD,

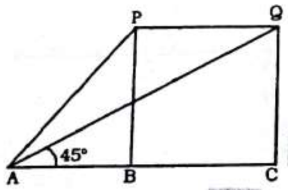
$$\tan 30^\circ = \frac{AB}{BD}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{50}{y}$$

$$\Rightarrow y = 50\sqrt{3}$$

= (50 × 1.73) metre
= 86.5 metre
∴ CD = (50 + 86.5) metre
= 136.5 metre

65. (D)



In height of balloon

$= 3800\sqrt{3}$ m, then,

$$\tan 60^\circ = \frac{BP}{AB}$$

$$\Rightarrow \sqrt{3} = \frac{3800\sqrt{3}}{AB}$$

⇒ AB = 3800 m

$$\tan 45^\circ = \frac{CQ}{AC}$$

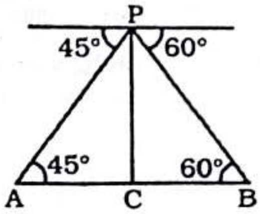
$$\Rightarrow 1 = \frac{3800\sqrt{3}}{AC}$$

$\Rightarrow AC = 3800\sqrt{3}$ m
∴ PQ = AC – AB
$= (3800\sqrt{3} - 3800)$
= 3800 × 0.732
$= 3800\ (\sqrt{3} - 1)$
= 2782 m
∴ Required speed

$$= \left(\frac{2782}{\frac{5}{60} \times 1000}\right) \text{km / hr.}$$

= 33.3 km/hr. (Approximately)

66. (A)



P = Position of pilot
PC = 200 metre
AB = width of river
AC = x metre (let)
CB = y metre (let)
∠PAC = 45° ; ∠PBC = 60°
In ΔAPC,

$$\tan 45^\circ = \frac{PC}{AC}$$

$$\Rightarrow 1 = \frac{200}{x}$$

⇒ x = 200 metre
In ΔPCB,

$$\tan 60^\circ = \frac{PC}{CB}$$

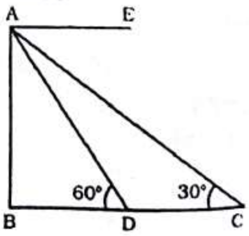
$$\Rightarrow \sqrt{3} = \frac{200}{y}$$

$$\Rightarrow y = \frac{200}{\sqrt{3}} \text{ metre}$$

∴ width of river = x + y

$$= \left(200 + \frac{200}{\sqrt{3}}\right) \text{metre}$$

67. (A)



Let speed of boat
= v metre/minute
Time taken to reach B from D
= t minutes
∠ACB = 30° ; ∠ADB = 60°
AB = tower
in ΔABC,

$$\tan 30^\circ = \frac{AB}{BC}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{AB}{\upsilon t + 10\upsilon}$$

$$\Rightarrow AB = \frac{\upsilon t + 10\upsilon}{\sqrt{3}}$$

In ΔABD,

$$\tan 60^\circ = \frac{AB}{BD}$$

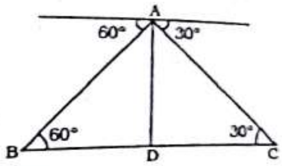
$$\Rightarrow \sqrt{3} = \frac{AB}{\upsilon t}$$

$$\Rightarrow \sqrt{3}\upsilon t = AB$$

$$\Rightarrow \sqrt{3}\upsilon t = \frac{10\upsilon + \upsilon t}{\sqrt{3}}$$

⇒ 3t = 10 + t
⇒ 2t = 10
⇒ t = 5 minutes

68. (C)



AD = Cliff = 180 metre
∠ABD = 60°, ∠ACD = 30°
From ΔABD

$$\tan 60^\circ = \frac{AD}{BD}$$

$$\Rightarrow \sqrt{3} = \frac{180}{BD}$$

$$\Rightarrow BD = \frac{180\sqrt{3}}{} = 60\sqrt{3} \text{ metre}$$

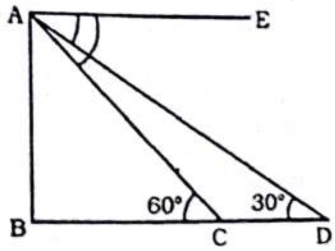
From ΔACD,

$$\tan 30^\circ = \frac{AD}{CD}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{180}{CD}$$

$\Rightarrow CD = 100\sqrt{3}$ metre
∴ BC = BD + DC
$60\sqrt{3} + 180\sqrt{3}$
$240\sqrt{3}$ metre
= (240 × 1.732) metre
= 415.68 metre

69. (A)



AD = Height of helicopter
= 1500 metre
C and D ⇒ positions of ships
∠ADB = 30° ; ∠ACB = 60°
Let, BC = x metre and BD = y metre

In ΔABD,

$\tan 30^\circ = \frac{AB}{BD}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{1500}{y}$

$\Rightarrow y = 1500\sqrt{3}$ metre ... (i)

In ΔABC,

$\tan 60^\circ = \frac{AB}{BC}$

$\Rightarrow \sqrt{3} = \frac{1500}{x}$

$\Rightarrow x = \frac{1500}{\sqrt{3}}$

$= 500\sqrt{3}$ metre ... (ii)

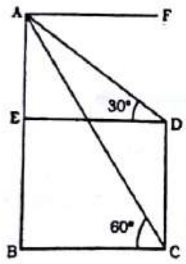
$\therefore$ Distance between ships

$= (y - x)$ metre

$= \left(1500\sqrt{3} - 500\sqrt{3}\right)$ metre

$= 1000\sqrt{3}$ metre

70. (A)



AB = Height of building = 60 metre

CD = Height of tower = h metre

$\angle FAD = \angle ADE = 30^\circ$

$\angle FAC = \angle ACB = 60^\circ$

From ΔABC,

$\tan 60^\circ = \frac{AB}{BC}$

$\Rightarrow \sqrt{3} = \frac{60}{BC}$

$\Rightarrow BC = \frac{60}{\sqrt{3}} = 20\sqrt{3}$ metre = DE

From ΔADE,

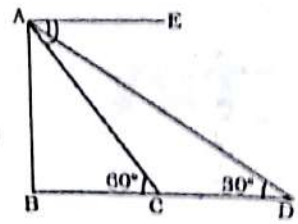
$\tan 30^\circ = \frac{AE}{ED}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{60-h}{20\sqrt{3}}$

$\Rightarrow 60 - h = \frac{20\sqrt{3}}{\sqrt{3}} = 20$

$\Rightarrow h = 60 - 20 = 40$ metre

71. (C)



AB = height of observation tower = h metre

C and D = Positions of boat

BC = 50 metre

Let, CD = x metre

$\angle ACB = 60^\circ = \angle EAC$

$\angle ADB = 30^\circ = \angle EAD$

$\tan 60^\circ = \frac{AB}{BC}$

$\Rightarrow \sqrt{3} = \frac{h}{50}$

$\Rightarrow h = 50\sqrt{3}$ metre

In ΔABD,

$\tan 30^\circ = \frac{AB}{BD}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{50\sqrt{3}}{50 + x}$

$\Rightarrow 50 + x = 50\sqrt{3} \times \sqrt{3} = 150$

$\Rightarrow x = 150 - 50 = 100$ metre

$\therefore$ Speed of boat

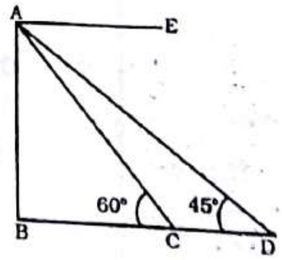
$= \frac{\text{Distance}}{\text{Time}}$

$= \left(\frac{100}{8}\right)$ m / sec.

$= \left(\frac{100}{8} \times \frac{15}{5}\right)$ kmph

= 45 kmph.

72. (C)



AB = Lamp post = h metre

C and D = Positions of ships

CD = 300 metre ; BC = x metre

$\angle ACB = 60$ metre ; $\angle ADB = 45^\circ$

In ΔABC,

$\tan 60^\circ = \frac{AB}{BC}$

$\Rightarrow \sqrt{3} = \frac{h}{x}$

$\Rightarrow h = \sqrt{3}x$ ...(i)

In ΔABD,

$\tan 45^\circ = \frac{AB}{BD}$

$\Rightarrow 1 = \frac{h}{x + 300}$

$\Rightarrow h = x + 300$

$\Rightarrow h = \frac{h}{\sqrt{3}} + 300$

$\Rightarrow h - \frac{h}{\sqrt{3}} = 300$

$\Rightarrow \frac{\sqrt{3}h - h}{\sqrt{3}} = 300$

$\Rightarrow h\left(\sqrt{3} - 1\right) = 300\sqrt{3}$

$\Rightarrow h = \frac{300\left(\sqrt{3}\right)}{\sqrt{3} - 1}$

$= \frac{300\sqrt{3}\left(\sqrt{3} + 1\right)}{\left(\sqrt{3} - 1\right)\left(\sqrt{3} + 1\right)}$

$= \frac{300\left(3 + \sqrt{3}\right)}{2}$

$= 150\left(3 + \sqrt{3}\right)$ metre