1a) James Neuhaus

(1)

$$z_j^\ell = \sum_k a_j^{\ell-1} w_{kj} + b_j^\ell$$

$$a_j^\ell = f(z_j^\ell) \qquad \frac{\partial a_j^\ell}{\partial z_j^\ell} = f'(z_j^\ell)$$

$$\frac{\partial z_j^\ell}{\partial a_j^{\ell-1}} = \sum_k w_{kj}^\ell \Rightarrow \frac{\partial z_j^{\ell+1}}{\partial a_j^\ell} = \sum_k w_{kj}^{\ell+1}$$

$$\Delta_j^L = \frac{\partial C}{\partial z_j^L} = \frac{\partial C}{\partial a_j^L}\frac{\partial a_j^L}{\partial z_j^L} = \frac{\partial C}{\partial a_j^L} f'(z_j^L)$$

$\ell < L$

$$\Delta_j^\ell = \frac{\partial C}{\partial z_j^L} = \sum_k \underbrace{\frac{\partial C}{\partial z_k^{\ell+1}}}_{\to \Delta_k^{\ell+1}} \frac{\partial z_k^{\ell+1}}{\partial a_j^\ell} \underbrace{\frac{\partial a_j^\ell}{\partial z_j^\ell}}_{\to f'(z_j^\ell)} = \sum_k \Delta_k^{\ell+1} \left[w^{(\ell+1)T}\right]_{kj} f'(z_j^\ell)$$

1b

$$\frac{\partial C}{\partial w_{ij}^\ell} = \underbrace{\frac{\partial C}{\partial z_j^\ell}}_{\Delta_j^\ell} \frac{\partial z_j^\ell}{\partial w_{ij}^\ell} = \Delta_j^\ell \, a_i^{\ell-1} \cancel{\frac{\partial w_{ij}}{\partial w_{ij}}} = a_i^{\ell-1} \Delta_j^\ell$$

$$\frac{\partial C}{\partial b_j^\ell} = \frac{\partial C}{\partial z_j^\ell} \cancel{\frac{\partial z_j^\ell}{\partial b_j^\ell}} = \Delta_j^\ell$$

d) $a_j^l = z_j^l$

$$\Delta_j^L = \frac{\partial C}{\partial z_j^L} = \frac{\partial C}{\partial a_j^L}\frac{\partial a_j^L}{\partial z_j^L} = \frac{\partial C}{\partial a_j^L}$$

$l<L$

$$\Delta_j^l = \frac{\partial C}{\partial z_j^l} = \sum_k \frac{\partial C}{\partial z_k^{l+1}}\frac{\partial z_k^{l+1}}{\partial a_j}\cancelto{1}{\frac{\partial a_j^l}{\partial z_j^l}} = \sum_k \Delta_k^{l+1}\left[W^{(l+1)T}\right]_{kj}$$

$$\frac{\partial C}{\partial w_{ij}^l} = a_j^{l-1}\Delta_j^l \qquad \frac{\partial C}{\partial b_j^l} = \Delta_j^l$$