24.10.05.

Relaxed complementary slackness Conditions

→ $\alpha_j - \beta_{\phi(i) j} = c_{\phi(j), j}$

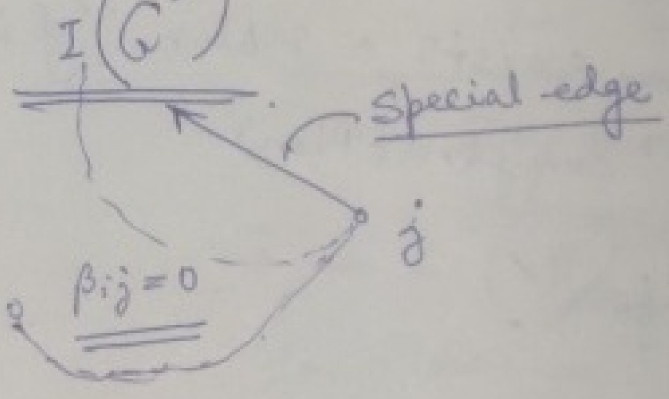
for every directly connected city j
(By construction) (A)

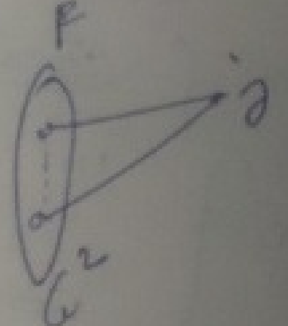
→ $\displaystyle\sum_{j:\, i=\phi(j)} \beta_{i,j} = f_i$ (B)

→ $\frac{1}{3} c_{\phi(j) j} \le \alpha_j \le c_{\phi(j), j}$.

for every indirectly connected city j.

$$\boxed{-\beta_{ij} + \alpha_j = c_{ij}}$$

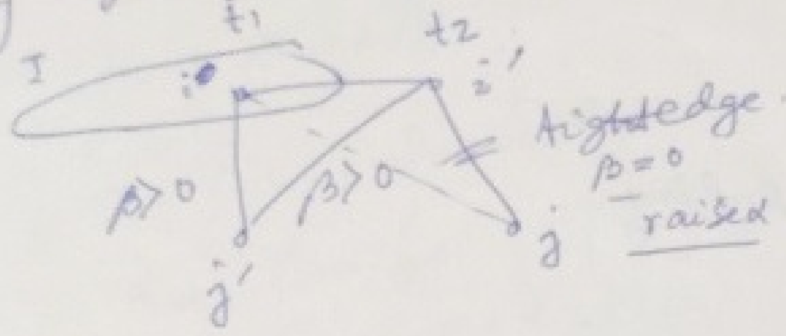




(B) Each open facility i is fully paid for by cities connected (directly) to it.

$$f_i = \sum_{j:(i,j) \text{ is special}} \beta_{ij} = \sum_{j:\phi(j)=i} \beta_{ij}$$ if (i,j) is not special $\beta_{ij} = 0$.

If ~~any~~ some city is not directly connected to i, then $\beta_{ij} = 0$



$i = \phi(j)$

j being served by i

$\alpha_j \geq c_{i'j}$

$\alpha_{j'} \geq c_{ij'}$

$\alpha_{j'} \geq c_{i'j'}$

Need $\boxed{\alpha_j \geq \alpha_{j'}}$ then

$\cancel{3}\alpha_j \geq c_{i'j} + c_{ij'} + c_{i'j'}$

$\geq \cancel{3}c_{ij}$

$\Rightarrow \alpha_j \geq c_{ij}$

— "i' is the connecting witness for j".

$$\begin{cases} \alpha_{j'} \leq \min(t_1, t_2). \\ \alpha_j \geq t_2 \end{cases}$$

$$\left[\begin{array}{l} \alpha_{j'} \leq \min(t_1, t_2). \\ \alpha_j \geq t_2. \end{array}\right]$$

uniform rate of increase α's grows by 1 unit/time

$\alpha_j \geq \alpha_{j'}$

$$\Rightarrow \sum_{\substack{i \in F \\ j \in C}} c_{ij} x_{ij} + 3 \sum_{i \in F} f_i y_i \leq 3 \sum_{j \in C} \alpha_j$$

(verify?).

The performance ratio = 3

L1: Defn

(F) PTAS

Task scheduling

Vertex Cover

Satisfiability.

Max-2-SAT (.91) (semi-definite programming)

Min-2-Sat (minimize number of clauses satisfied)

$\cong$ vertex cover

(Marathe & Ravi)

L2: Knapsack FPTAS (Ibawa & Kim)

4
(2) HW

Sorting by Reversal.

Max-SNP Complete: problems for which PTAS does not exist

APX-hard → there is enough evidence (no proof) PTAS does not exist

| α_0 | better than α_0 approx.

= 2 $P = NP$
or $NP \subseteq p^{\log n}$ DTIME

L3: Linear Programming
Weak duality
strong duality
CSC
Set Cover formulation.

L4: Randomized $O(\log n)$
Greedy method $H(n)$

MAXSAT $RR = 1/2$
max $(RR + LP) = 3/4$ ("Compound Greedy Type")

L5: Setcover via dual Fitting. (Constructing dual solution, used to analyze greedy or locally optimal methods)

Max-Coverage
$\frac{e}{e-1}$ approx.

PSP (4-approx) open?

6: Vertex Cover (??)
IP formulation.
Gavriel's
$(2 - 2/k)$ (PSR) Nemhauser Trotter partit

On dense graphs(Iwana & Sondoj)
Feige(Regular Graphs)

Every vertex has degree $\underline{d}$.

$|E| = \frac{nd}{2}$. $\boxed{O(1)}$

d.

$\underline{n/2}$

Dual of VC (Matching).

L7: Matching
fractional Matching
Maximum charge

BP
$\frac{1}{2}$ integral P(D) sol
labelling algorithm

Lovasz & Plummer
(Matching Theory).

PSP again.

L8: BP again
Primal-Dual Schema
Weighted set cover (only formulation)

L9: $\alpha = 1, \beta = f$
Relaxed CS Conditions (set cover illustrated)
and the Anaylis
Proof of "the charging Th^m"

Goemans & Williamson also in D. Hochbaum (Handbooks)

Synchronous Raising of the Duals

-10: Synchronous Raising of the Duals

HW

Facility location

3 - approx (started).

Phase I

11: Rectilinear Partitions & Rectangle Stabbing.

4

2 (Probability the right bound?).

2 approx.

Kovaleva, S. Spieksma ($\frac{e}{e-1}$ for intervals)

"Geometric Covering Problems".

B. Baker

PTAS planar graphs

D. Hochbaum (Handbook) & Maas.

12: Facility location (Phase II)

tight example.

13: Finished the analysis:

K-median using facility location.

Review.

$O(\sqrt{V}.E)$

Network Flow $O(V^3)$ 1977