22.09.05.

Review

Set cover

Greedy has pr. H_n.

Maximum Coverage Problem

Greedy has pr $1-\left(1-\frac{1}{k}\right)^k$

IPL, Khuller (Weighted MAX-Coverage)

Input: A set of points in 2D.

Problem: To seperate all the pairs of points using minimum number of Horizontal vertical lines.

$1 \equiv (a,c)$

$2 \equiv (b,c)$

$3 \equiv (a,b)$

$4 \equiv (a,b)$

Set cover problem

set is a line.

Every segment is intersected by a line

For every line the pairs intersected by the line are in the set

~~a, b, c are~~

each line corresponds to a bunch of segments, that becomes a member of set.

logn

2 approximation

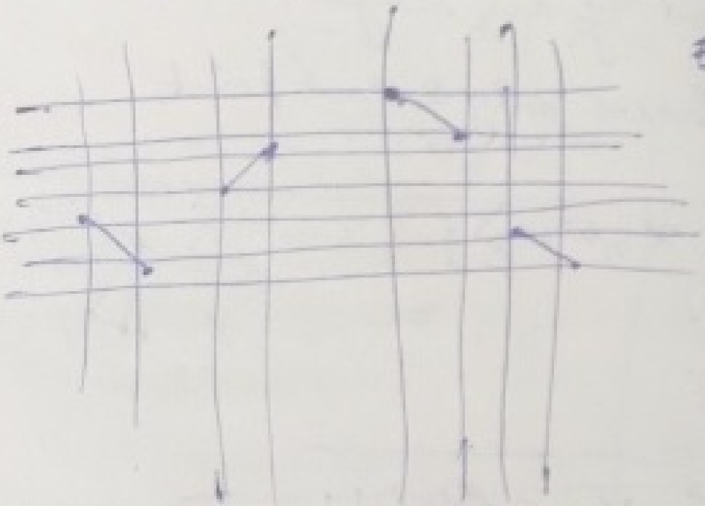
Linear Programming

line

? Segment a, b are independent if they do not intersect a common line. segment ?

inclusive

S as a maximal independent set of segments. [every other segment will intersec



- ~~If S is~~ S defines a partition & each cell contains atmost 1 point

We have a 4-approx algorithm.

~~Vector~~

Vertex Cover

<u>SC</u> and frequency (f) of an element $u \in U$ as the number of S_is that contain u.

VC ≡ SC in which every element has frequency 2.

<u>$G = (V, E)$</u> and $S_u = \{ e \in E \mid e$ is incident on $u \}$, $u \in V$

every edge is <u>covered</u> (we should have picked an endpoint of an edge).

each edge will have only 2 end points

$\boxed{\text{IP:}}$ $\min \sum_{u \in V} x_n w_n \geq 1$

$x_u + x_v \geq 1, \quad (u, v) \in E$

$x_u \in \{0, 1\} \quad \forall u \in U$

Extreme point solution is a solution ~~is an optimal solution~~ which can't be expressed as a convex combination of two feasible solutions.

Th [Nemhauser & Trotter].

Every extreme point solution is half-integral

all values are $\{0, 1, 1/2\}$

Proof (EPS)

x^* is extreme point solution but not half integral.

$V_+ = \{u \mid x^*_u > 1/2\}$

$V_- = \{u \mid x^*_u < 1/2\}$.

$V \setminus (V_+ \cup V_-)$

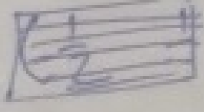
y $u \in V_+,\ X_u = X^*_u - \epsilon$ $u \in V_-,\ X_u = X^*_u + \epsilon$	Z $V_+ \to X^*_u + \epsilon$ $V_- \to X^*_u - \epsilon$

$x^* = \frac{1}{2}(y+z)$.

$(>1/2 > 1/2)$

$(<1/2 > 1/2)$

Claim: Find a half-integral solution ~~this~~ gives a 2-approximation soln.



$$\begin{pmatrix} 1 & \frac{1}{2} & 0 \\ \downarrow & \downarrow & \downarrow \\ P & Q & R \end{pmatrix}$$

$OPT \geq w(P) + \frac{1}{2} w(Q)$

$ALG \leq w(P) + w(Q)$

$\therefore \frac{ALG}{OPT}$, 2-approx algo

G is k-colorable & the coloring is given

Claim:

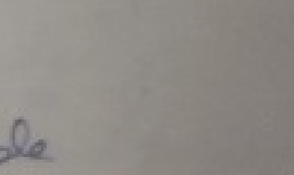
Vertex Cover can be approximated within

$2 - 2/k$.

Every planar Graph is 4-colourable

$\boxed{\text{Every vertex}}$ $r \in R$. [all the neighbours of R are in P, because if it has some neighbours in Q, then that edge is not covered]

- For any vertex in R all its neighbours ~~again~~ are in P
- $S \subseteq Q$ then $R \cup S$ is independent if S is independent



[R can't have an edge, otherwise R is NOT OPTIMAL]

S pick ~~least~~ most heavy color class from $\mathcal{G}$ call it S

$$w(S) \geq \frac{W(\mathcal{G})}{k}.$$

$\overline{(R \cup S)}$ is vertex cover &

$$wt(\overline{R \cup S}) \leq w(P) + \frac{k-1}{k} W(\mathcal{G})$$

$R \cup S$ does not intersect with k.

$$\leq \frac{2(k-1)}{k}\left(w(P) + \frac{1}{2} W(\mathcal{G})\right).$$

Pr = Performance ration $\leq \frac{2(k-1)}{k} = 2 - \frac{2}{k}$.

Strongest possible result known about vertex cover.

Gonzalez, Teo, IPL

$$\left(2 - \frac{\log\log v}{\log v}\right).$$

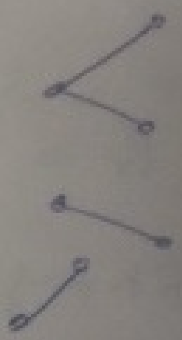
Dual of Vertex Cover problem

(D) $\sum_{(u,v)\in E} y(u,v)$ (Matching Problem)

$$\sum_{v \in V} y(u,v) \le 1 \quad \forall u$$

$$y(u,v) \in \{0,1\}$$

$G = (V,E)$, $M \subseteq E$ is a matching if no two edges in M share a vertex.



FMP if $y(u,v) \in \{0,1\}$ is removed ~~NP-hard~~.
↓
fractional → Polynomially solvable.

original MP is ~~not~~ NP hard

- v is M-saturated if some edge in M is incident on V, else v is M-unsaturated.
- M-alternating path: is a path whose edges alternate in ~~$E \setminus M$ & M~~ M & $E \setminus M$
- M-augmenting path: is an odd length M-alternating path that starts & ends in a M-unsaturated vertex

Th (Berge 1957)

M is maximum in $G \Leftrightarrow \nexists$ any augmenting path

Hat Why matching can be solved in polynomial time.