20.10.05

Rectangle Stabbing Problem:

$$\min \sum_{h \in H} x_h + \sum_{v \in V} x_v$$

$$\sum_{h \in H(r)} x_h + \sum_{v \in V(r)} x_v \geq 1 \quad \forall r$$

$$x_v, x_h \in \{0, 1\}$$

based on LP optimal soln, $x^* = (x_h^*, x_v^*)$

$$\min (\quad\quad)$$

$$A[x_h \; 0] \geq 1 \text{ if } A[r] \cdot \langle x_h^* \; 0 \rangle \geq 1/2$$

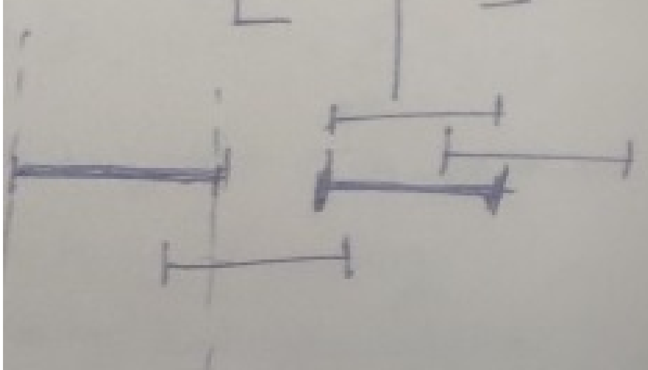
$$A[0 \; x_v] \geq 1 \text{ if } A[r] \langle 0 \; x_v^* \rangle \geq 1/2$$

$$x_h, x_v \in \{0, 1\}.$$

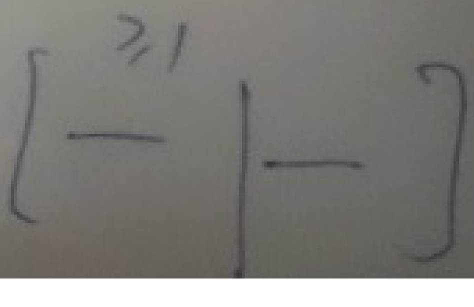
$$A' \; \begin{array}{cc} x_h & x_v \\ \hline \multicolumn{2}{c}{\left[\begin{array}{c|c} & 0 \\ \hline 0 & \end{array}\right]} \end{array} \geq 1$$

min x

$A'x \geq 1$



Number of independent intervals are same as number of lines



Integer optimal to P' can be constructed ~~~~ in $O(n \lg n)$ [If not sorted, we have to sort]

Fact : $2x^*$ ~~~~ it is feasible in P'.

OPT' be the optimal soln to P'.

$OPT' \leq 2x^*$.

Performance Ratio is $\frac{OPT'}{x^*} \leq 2$

value associated with optimal integral soln

value associated with optimal soln

provided the soln that is constructed over there is indeed optimal soln.

~~Prop~~ Problem above the line & below the line is independent

If Matrix A' is TUM (totally Unimodular)

P : min cx

$A'x \geq 1$

$x \in \{0,1\}^n$

then ~~there only~~ all the optimal solutions to P are integral (0/1).

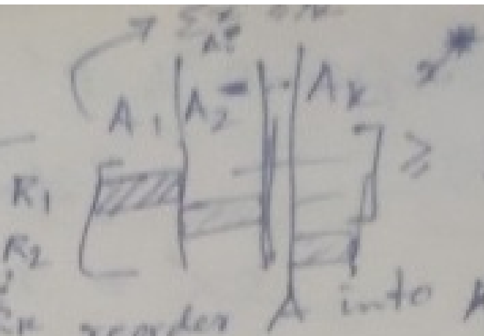
$A''x = 1$

$x = A''^{-1} 1$.

Reading Assignment

Proof of the statement.

Question (Research)



reorder A into K parts

min cx
$Ax \geq 1$
$x \in \{0,1\}$

x^* is fractional

~~Each~~: If each submatrix $(A_{:, R_i})$ $\forall i$ is TUM then

$\rho r = K$.

~~what~~ Given such matrix A what is the smallest K?

Generalize ~~to~~ to K balanced matrices.

we can compute in polynomial time

doesnot have any odd cycles $2K+1$ for any k

3-odd cycle $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$

5-odd cycle
$$\begin{matrix} 1 & 1 & & & \\ & 1 & 1 & & \\ & & 1 & 1 & \\ & & & 1 & 1 \\ 1 & & & & 1 \end{matrix}$$

single element.
performance ratio
n

$x^* = \left[x^*_{A_1} \cdots x^*_{A_n} \right]$.
$\geq \frac{1}{K}$

Q: How to solve point seperation problem using rectangle stabbing?

Consider rectangles ~~defining~~ defined by every pair of points

Grecia, Calinescu, Karloff

CCCG 04

~~[illegible]~~ IJCGA?

LP is expensive n^5 for coefficient matrix 0-1 matrix

E. Tardos, 1985

$A[0,1]$ ϵ strongly Polynomial.

$O(n^5)$

Facility Location

~~$x_i \in F$~~ $i \in F \quad j \in C$

$$(P) \quad \min \sum_{i \in F} y_i f_i + \sum_{i \in F, j \in C} x_{ij} \cdot c_{ij}$$

$$\alpha_j \quad \sum_{i \in F} x_{ij} \geq 1 \quad \forall j \in C$$

$$\beta_{ij} \quad y_i \geq x_{ij} \quad \forall i \in F, j \in C$$

$$(D) \quad \max \sum_{j \in C} \alpha_j \quad \text{s.t.} \quad \alpha_j - \beta_{ij} \leq c_{ij} \quad \forall i \in F, j \in C$$

$$\sum_{j \in C} \beta_{ij} \leq f_i, \quad \forall i \in F$$

(1) $x_{ij} > 0 \Rightarrow \alpha_i - \beta_{ij} = c_{ij}$
(2) $y_i > 0 \Rightarrow \sum_{j \in C} \beta_{ij} = f_i$.

} Primal complementary slackness condition

(3) $\alpha_j > 0 \Rightarrow \sum_{i \in F} x_{ij} = 1$.
(4) $\beta_{ij} > 0 \Rightarrow y_i = x_{ij}$

} Dual complementary slackness condition.

Relaxed conditions are

$$\frac{1}{3} c_{\phi(j),j} \le \alpha_j - \beta_{\phi(j),j} \le c_{\phi(j),j}$$

$$\frac{1}{3} f_i \le \sum_{j : \phi(j) = i} \beta_{ij} \le f_i.$$

cities which are connected to

$\phi(j)$: ~~facility~~ facility i.

Directly connected.

$$\alpha_j - \beta_{\phi(j),j} = c_{\phi(j),j} \quad (A)$$

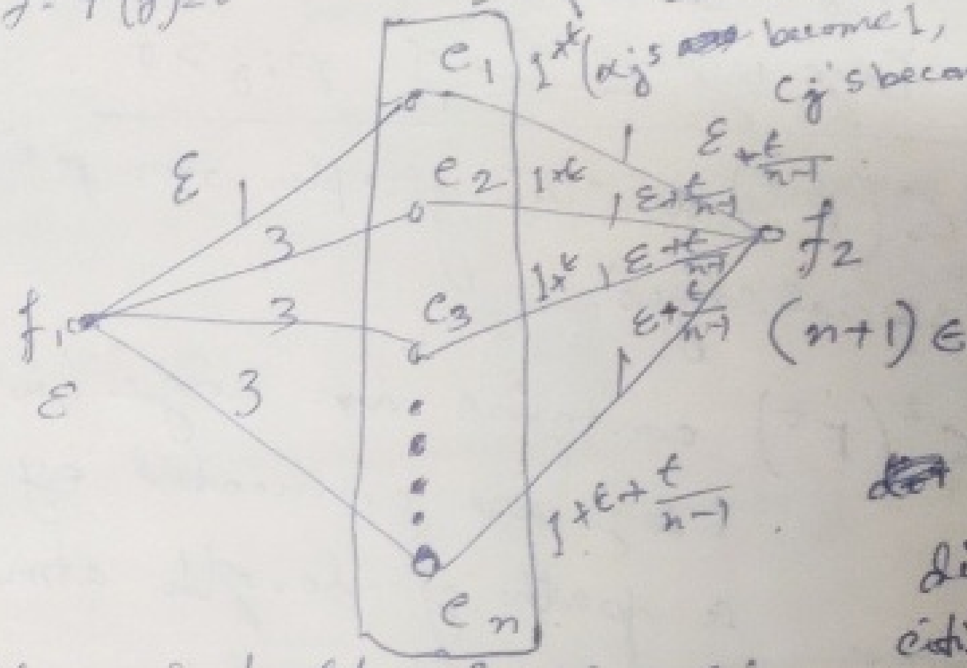
~~[illegible]~~

Indirectly connected

$\boxed{\beta_{ij} = 0 \text{ (by defination)}}$ ~~$\alpha_j - \beta_{\phi(j),j}$~~ ~~we want them in our complementary slackness~~

$\frac{1}{3} c_{\varphi(j)j} \le \alpha_j \le c_{\varphi(j)j}$. (B)

$\sum_{j:\varphi(j)=i} \beta_{ij} = f_i(c)$ [summation over all the cities that are connected to city i].



1^x (α_j's ~~...~~ become 1, c_j's become tight).

Edge is tight if $\alpha_j = c_{ij}$

~~de~~ directly connected cities correspond to tight edges.

only on the tight edges raise β until some facility is ~~paid for or some new~~ edge becomes tight

Facilities will be declared connecting witness.

se1

every city becomes connected to opened facility.

raising dual variables.

synchronized. ~~variable ra~~

Phase 2

F^t is the set of temporarily open facilities. We want to close some of ~~above~~ open facilities, because cost is too much

Edge is tight if $\alpha_i = c_{ij}$.

Edge is special if $\beta_{ij} > 0$

$G(F^t)$ as the graph over F^t & the "special edges".

$G^2(F^t)$ contains an edge (i, j) if i & j are connected by a path of length atmost 2 in $G(F^t)$.

Take any maximal independent set I in $G^2(F^t)$ and declare all facilities in I as open. Close everything in $F^t \setminus I$.

10) City j $\boxed{F_j \text{ is the set of facilities } i, \text{ s.t. } \beta_{ij} > 0 \quad c_j}$

1. If $I \cap F_j \neq \phi$ then connect j to $I \cap F_j$.

2. $I \cap F_j = \phi$

j has connecting witness i

facilities directly connected:
(if no connecting witness, algorithm would go on)
if $i \in I$ then connect j to i

else (~~by G^2 construction~~)

facilities indirectly connected:
~~$i' \in I$~~ there exists a nbhr i' of i in I (by G^2 construction).

Connect j to i'.