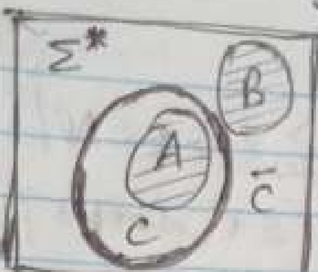
Automata

1.

Sandipan Dey 24/30 HW-3

(4.18.)



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$A \cap B = \varphi$

$\Rightarrow (A \cap B)' = \bar{A} \cup \bar{B} = \Sigma^*$

$A \subseteq \bar{B}$, $B \subseteq \bar{A}$ (separation)

$A \subseteq C$, $B \subseteq \bar{C}$

$x \in A \Rightarrow x \in C$

$x \in B \Rightarrow x \in \bar{C}$

1) $\begin{cases} A \in \text{co-TR} \Rightarrow \exists \text{ TM}, M_{\bar{A}} \mid M_{\bar{A}} \text{ accepts } \bar{A} \\ B \in \text{co-TR} \Rightarrow \exists \text{TM}, M_{\bar{B}} \mid M_{\bar{B}} \text{ accepts } \bar{B} \end{cases}$

Now, $\bar{C} \subseteq \bar{A} \Leftrightarrow x \in \bar{C} \Rightarrow x \in \bar{A}$
also $C \subseteq \bar{B} \Leftrightarrow x \in C \Rightarrow x \in \bar{B}$ $\Rightarrow$ $\begin{cases} x \in \bar{A} & \text{if } x \in \bar{C} \\ \in \bar{B} & \text{ow} \end{cases}$

(separation by C)

Let's construct TM M_C to ~~decide~~ language C: decide

M (x)

1. Run $M_{\bar{A}}$ and $M_{\bar{B}}$ on x simultaneously.
2. If $M_{\bar{A}}$ accepts first /* $x \in \bar{A}$ */
   M_c rejects. /* $x \in \bar{C}$ */
3. If $M_{\bar{B}}$ accepts first /* $x \in \bar{B}$ */
   M_c accepts. /* $x \in C$ */

tie breaker?

2) It's not possible that both $M_{\bar{A}}$ and $M_{\bar{B}}$ run forever since it implies $x \notin \bar{A} \cup \bar{B}$, but $\bar{A} \cup \bar{B} = \Sigma^*$, hence $\nexists x$.
Hence M_c always halts. If it halts at step-3, M_c accepts $\Rightarrow x \notin B$ and if it halts at step-2, M_c ~~accept~~ rejects $\Rightarrow x \notin A$.

2. $\text{COFINITE} = \{\langle M \rangle \mid \text{the complement of } L(M) \text{ is finite}\}$

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$= \{\langle M \rangle \mid \bar{L}(M) \text{ is finite}\}$

$= \{\langle M \rangle \mid \Sigma^* - L(M) \text{ is finite}\}$ No.

$= ALL_{TM} - FINITE = ALL_{TM} \cap \overline{FINITE}$

$ALL_{TM} - FINITE = ALL_{TM}$ since none of the M that accepts Σ^* have finite language

$= \{\langle M \rangle \mid \forall x \in \Sigma^*, \exists t \in \mathbb{N} [M(x) \text{ accepts in } \leq t \text{ steps}]\}$

do this without set operations

$- \{\langle M \rangle \mid \exists x \in \Sigma^*, \forall y \in \Sigma^*, y >_{lex} x, t \in \mathbb{N}$
$[M(y) \text{ does not accept in } \leq t \text{ steps}]\}$

$= \{ \langle M \rangle \mid \forall x \in \Sigma^*, \exists y \in \Sigma^*, x >_{lex} y, t \in \mathbb{N} \mid$

$M[x]$ accepts in $\leq t$ steps $\}$

different t's

$SUBSET = \{ \langle M_1, M_2 \rangle \mid L(M_1) \subseteq L(M_2) \}$

$= \{ \langle M_1, M_2 \rangle \mid \forall x \in \Sigma^*$

$\exists t \in \mathbb{N}$ [$M_1(x)$ accepts in $\leq t$ steps

$\Rightarrow M_2(x)$ accepts in $\leq t$ steps] $\}$

not negate quantifier too.

$= \{ \langle M_1, M_2 \rangle \mid \forall x \in \Sigma^*,$

$\exists t \in \mathbb{N}$ [$\neg$($M_1(x)$ accepts in $\leq t$ steps)

$\vee$ ($M_2(x)$ accepts in $\leq t$ steps)] $\}$

$= \{ \langle M_1, M_2 \rangle \mid \forall x \in \Sigma^*,$

$\exists t \in \mathbb{N}$ [$M_1(x)$ does not accept in $\leq t$ steps

or $M_2(x)$ accepts in $\leq t$ steps] $\}$

1 3. Let's construct ~~an an enumerator~~ a TM M that ~~enumerat~~

simulates the LBAs $\mathcal{L}_1, \mathcal{L}_2, \ldots$

$M(x)$

1. find $|Q|, |T|, n$ for all the LBAs.
   minLBA $\leftarrow \infty$

   but this is undecidable!

2. for $i = 1$ to ∞ do
   for $j = 1$ to ∞ do
   construct Enumerator E_i to enumerate $\mathcal{L}_i, \mathcal{L}_j$?
   run till $|Q| n |T|^n$ steps for each string
   ~~dec... if minLBA~~
   ~~if minLBA $> \langle \mathcal{L}_j \rangle$ then and $\mathcal{L}_i$... $L(\mathcal{L}_i) = L(\mathcal{L}_j)$~~
   ~~minLBA $\leftarrow \langle \mathcal{L}_i \rangle$.~~
   if $\langle \mathcal{L}_i \rangle < \langle \mathcal{L}_j \rangle$ and $L(\mathcal{L}_i) = L(\mathcal{L}_j)$ and
   minLBA $> \langle \mathcal{L}_i \rangle$
   then minLBA $\leftarrow \langle \mathcal{L}_i \rangle$
   endif

   where do you output?