Sandipan Dey 27/30 Homework - 2

1\. $K = \{i \mid M_i$ is TM that halts on blank input$\}$

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Proof by Contradiction

Let's assume K is decidable and R be a TM that decides K. We use R to construct TM S that decides $HALT_{TM} = \{\langle M,w\rangle \mid M$ is a TM and M halts on input $w\}$

$S(\langle M,w\rangle)$ /* on input $\langle M,w\rangle$, an encoding of M & string w */

1. Construct the following TM M_1 from M, w
   M_1 (on blank input tape)
   1. ~~if x ≠ w reject else~~ Start with blank input.
   2. write w on the input tape.
   3. Run M on input w until it halts and accept if it does
2. Run R on input $\langle M_1\rangle$.
3. If R accepts, accept; if R rejects, reject.

Since R decides K, ~~then~~ S decides $HALT_{TM}$, a contradiction.

3\. Pf $E_{TM} \le_m FINITE$

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To show we must present a computable function f that takes input of the form $\langle M\rangle$ and returns output of the form $\langle M'\rangle$, i.e., $\langle M\rangle \xrightarrow{f} \langle M'\rangle$ where $\langle M\rangle \in E_{TM}$ iff $\langle M'\rangle \in FINITE$

Define reducing function $f(\langle M\rangle) = \langle M'\rangle$ where the following machine F computes f:

F = "On input $\langle M\rangle$:

1. Construct M' as follows:
   M' = "On input x: reject if x is infinite"
2. output $\langle M'\rangle$.

? how can a string be infinite? This doesn't depend on M in any way.

$\langle M\rangle \in E_{TM} \Leftrightarrow \langle M'\rangle \in FINITE$.

~~(S.C)~~ S_c ~~has~~ must have a blue element as well

($\Leftarrow$) Suppose $(S, C) \in$ SET SPLITTING. Fix some col
with 2 colors such that every set has at least on
both colors.

Consider the following assignment to variables of ϕ.
x_i, assign it 'True' if its color differs from tha
special element F. Assign x_i 'False' if its color
same as that of F.

Hence,
Each clause c in ϕ is satisfied, since S_C
at least one element x_i or $\bar{x}_i$ that is colored
differently than F.

5.16. **Proof by contradiction**

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Let's assume $BB(k)$ is computable, to the contrary

$\Rightarrow \exists$ TM T_{BB} which on input 1^k, writes $1^{BB(k)}$ on its tape ~~output~~ and halts.

~~Let's reduce the HALT-BLANK$_{TM}$ undecidable~~

Now, let's show that ~~if~~ BB is computable ?

$\Rightarrow$ HALT-BLANK$_{TM}$ = {⟨M⟩ | M is a TM and M halts on blank tape} can be decided using the function BB.

Given TM M, modify M to form N s.t.,

→ N starts on a blank tape and works in the same way as M except N inserts a new square with number '1' printed on it in between every two squares of the original computation of M

Easier to use an output tape.

→ To do this, start by modifying M s.t. it prints a left end-marker at the left end of tape and never attempts to shift to the left of this marker, but ow simulates M

→ Right/left shift to be modified by first shifting right/left, printing '1', then again shifting right/left.

→ N halts on blank tape iff M halts on a blank tape.

→ Can use BB to ~~first~~ decide whether N halts: if N ever moves its head more than $2 \cdot BB(k)$ steps from left end of its tape, it will never halt.

→ Simulate N on a blank tape until it either halts (then accept) or it repeats a configuration (only finite number of them) or moves beyond $2 \cdot BB(k)$ tape squares (then reject)

→ $BB(k)$ decides HALT-BLANK$_{TM}$, a contradiction.