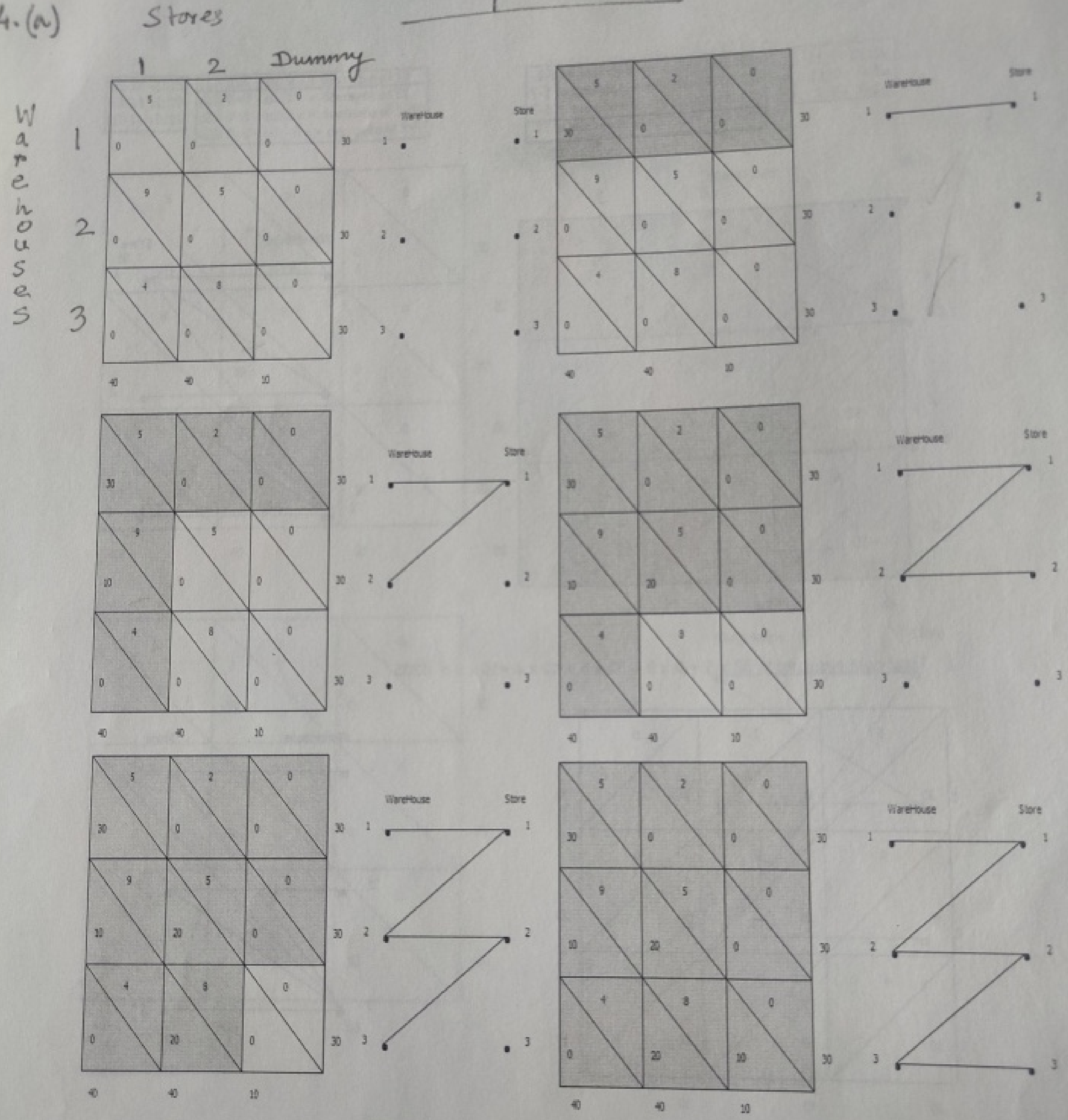
10/10

Sandipan Dey

Math 685 HW 4'5

4.(a)

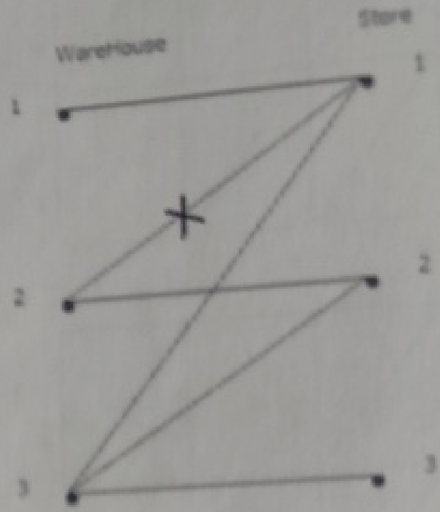


The total cost (by North-West Corner method) = $30\times5+10\times9+20\times5+20\times8+10\times0=500\$$

u1: 10	v1: 13
u2: 6	v2: 11
u3: 1	v3: 3

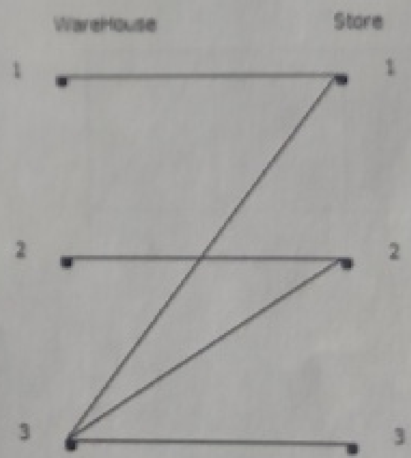
c12: 2 v2 - u1: 1 => decrease of $-1
c13: 0 v3 - u1: -7 => decrease of $-7
c23: 0 v3 - u2: -3 => decrease of $-3
$\boxed{\text{c31: 4 v1 - u3: 12 => decrease of \$8}}$

	1	2	3	
1	5 / 30	2 / 0	0 / 0	30
2	9 / 10 -10	5 / 20 +10	0 / 0	30
3	4 / 0 +10	8 / 20 -10	0 / 10	30
	40	40	10	



Total cost reducing to 30 x 5 + 0 x 9 + 30 x 5 + 10 x 4 +10 x 8 = 420$

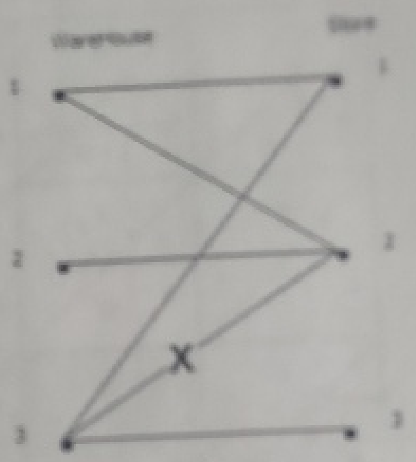
5 / 30	2 / 0	0 / 0	30
9 / 0	5 / 30	0 / 0	30
4 / 10	8 / 10	0 / 10	30
40	40	10	



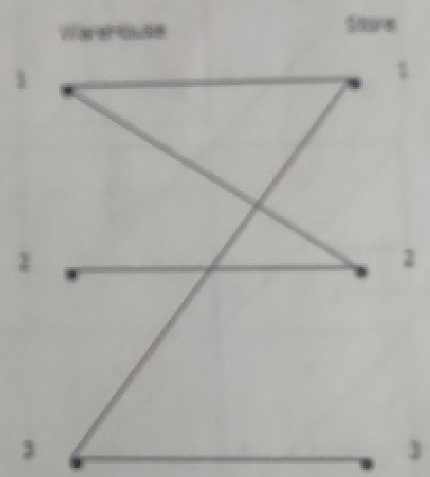
u1: 10	v1: 15
u2: 14	v2: 19
u3: 11	v3: 11

c12: 2 v2 - u1: 9 => decrease of 57
c13: 0 v3 - u1: 1 => decrease of 51
c21: 9 v1 - u2: 1 => decrease of 5-8
c23: 0 v3 - u2: -3 => decrease of 5-3

5 / 20 / -10	2 / 0 / +10	0 / 0	30
9 / 0	5 / 20	0 / 0	30
4 / 10 / +10	8 / 10 / -10	0 / 10	30
40	40	10	



5 / 20	2 / 10	0 / 0	30
9 / 0	5 / 30	0 / 0	30
4 / 20	8 / 0	0 / 10	30
40	40	10	



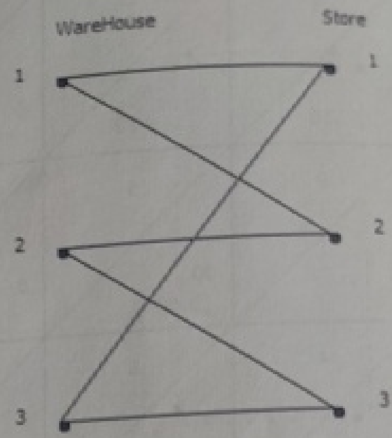
Total cost = 390

.(b)

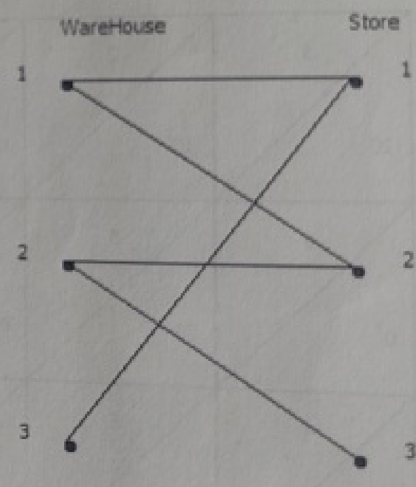
u1: 10 v1: 15
u2: 7 v2: 12
u3: 11 v3: 11

c13: 0 v3 - u1: 1 => decrease of $1
c21: 9 v1 - u2: 8 => decrease of $-1
c23: 0 v3 - u2: 4 => decrease of $4
c32: 8 v2 - u3: 1 => decrease of $-7

5 / 20 (-10)	2 / 10 (+10)	0 / 0	30
9 / 0	5 / 30 (-10)	0 / 0 (+10)	30
4 / 20 (+10)	8 / 0	0 / 10 (-10)	30
40	40	10	



5 / 10	2 / 20	0 / 0	30
9 / 0	5 / 20	0 / 10	30
4 / 30	8 / 0	0 / 0	30
40	40	10	



u1: 10 v1: 15
u2: 7 v2: 12
u3: 11 v3: 7

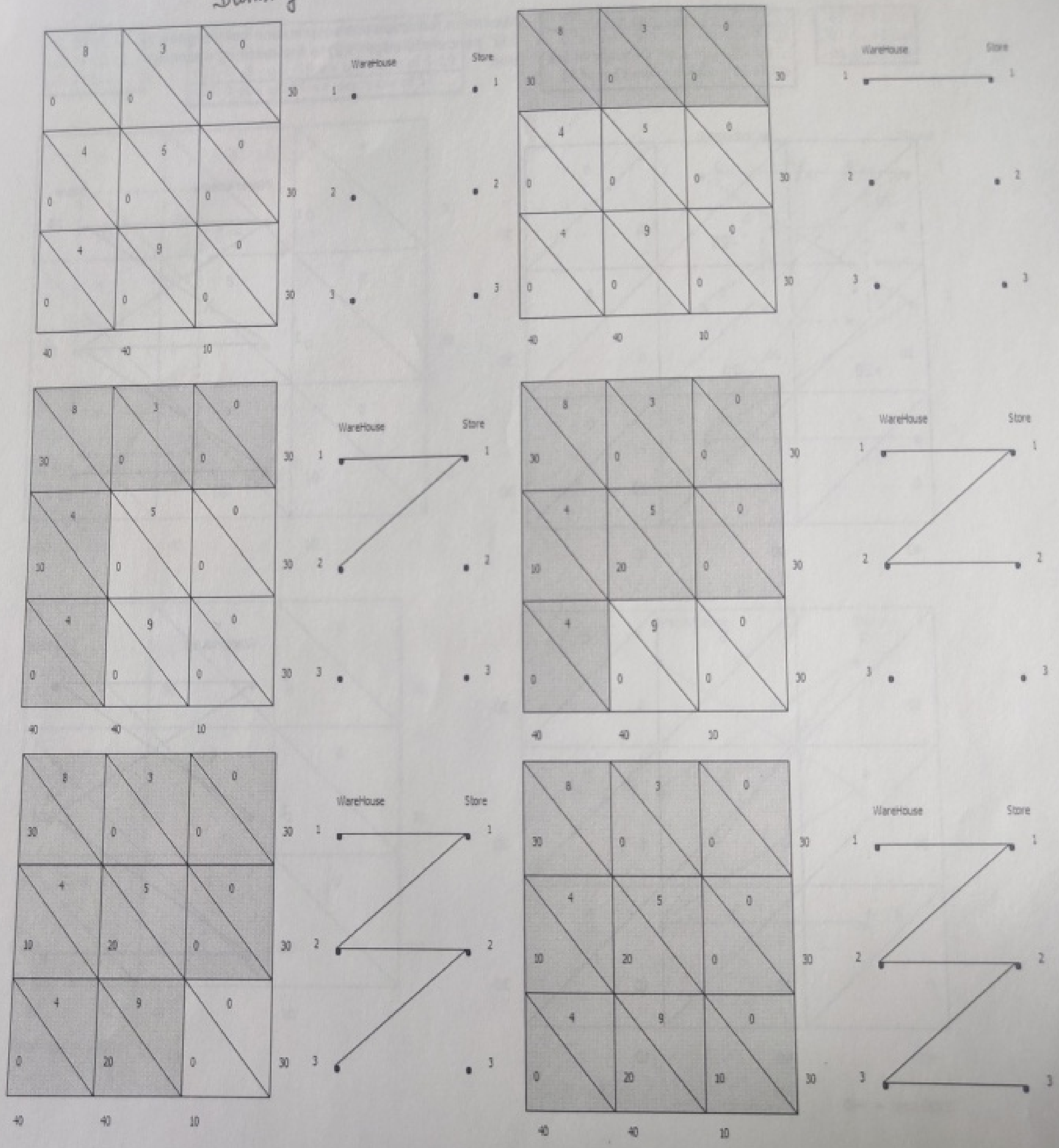
c13: 0 v3 - u1: -3 => decrease of $-3
c21: 9 v1 - u2: 8 => decrease of $-1
c32: 8 v2 - u3: 1 => decrease of $-7
c33: 0 v3 - u3: - 4 => decrease of $- 4

No positive reduction in the transportation cost => reached Optimal Solution!

Optimal cost = 310

4.(b)

Dummy



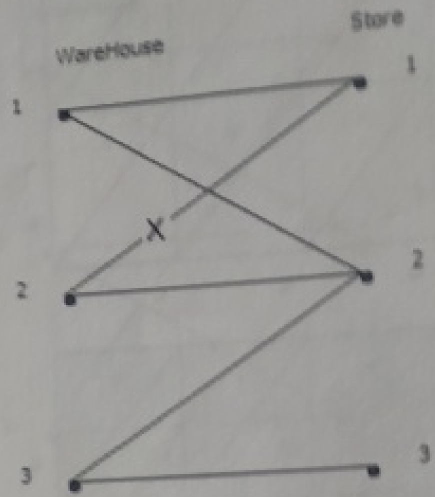
The total cost (by NWCST method) $= 30\times8+10\times4+20\times5+20\times9+10\times0=560\$$

u1: 10 v1: 18
u2: 14 v2: 19
u3: 10 v3: 10

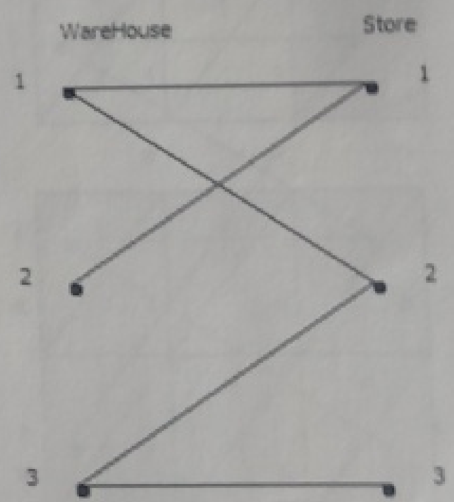
c12: 3 v2 - u1: 9 => decrease of $6
c13: 0 v3 - u1: 0 => decrease of $0
c23: 0 v3 - u2: -4 => decrease of $-4
c31: 4 v1 - u3: 8 => decrease of $4

Maximum reduction in transportation cost is given by c12, hence add edge (1,2) to the exisiting spanning tree

8 -20 30	3 0 +20	0 0	30
4 10 +20	5 20 -20	0 0	30
4 0	9 20	0 10	30
40	40	10	



8 10	3 20	0 0	30
4 30	5 0	0 0	30
4 0	9 20	0 10	30
40	40	10	

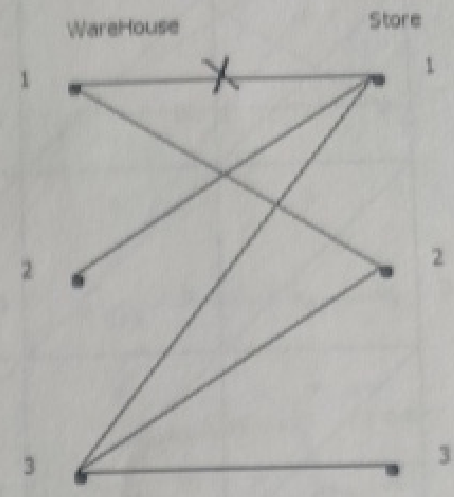


Total cost = 440

u1: 10 v1: 18
u2: 14 v2: 13
u3: 4 v3: 4

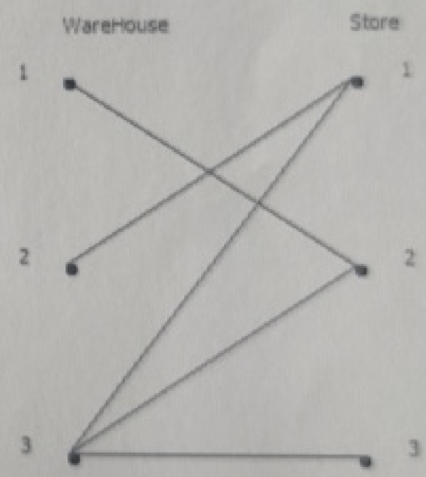
c13: 0 v3 - u1: -6 => decrease of $-6
c22: 5 v2 - u2: -1 => decrease of $-6
c23: 0 v3 - u2: -10 => decrease of $-10
c31: 4 v1 - u3: 14 => decrease of $10

8 10 -10	3 20 +10	0 0	30
4 30	5 0	0 0	30
4 0 +10	9 20 -10	0 10	30
40	40	10	



8 0	3 30	0 0	30
4 30	5 0	0 0	30
4 10	9 10	0 10	30
40	40	10	

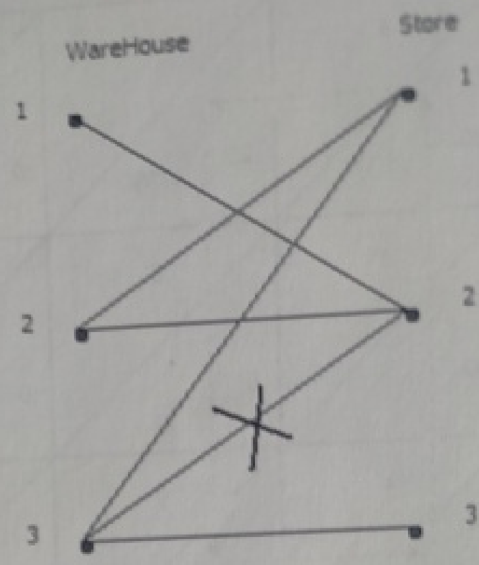
Total cost = 340



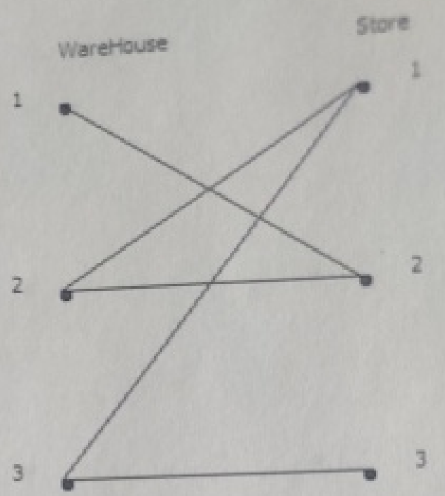
u1: 10 v1: 8
u2: 4 v2: 13
u3: 4 v3: 4

c11: 8 v1 - u1: -2 => decrease of $-10
c13: 0 v3 - u1: -6 => decrease of $-6
c22: 5 v2 - u2: 9 => decrease of $4
c23: 0 v3 - u2: 0 => decrease of $0

8 0	3 30	0 0	30
4 30 -10	5 0 +10	0 0	30
4 10 +10	9 10 -10	0 10	30
40	40	10	



8 0	3 30	0 0	30
4 20	5 10	0 0	30
4 20	9 0	0 10	30
40	40	10	



Spanning Tree

u1: 10	v1: 12
u2: 8	v2: 13
u3: 8	v3: 8

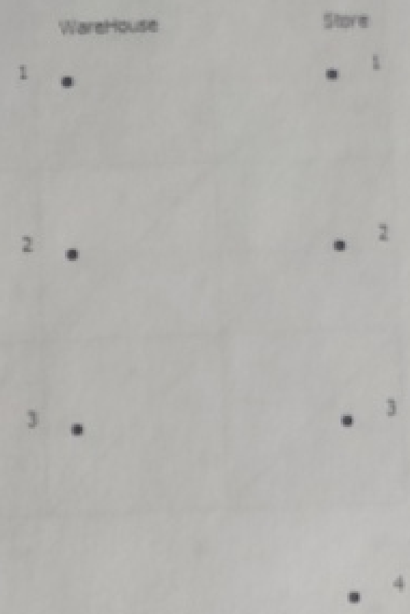
c11: 8 v1 - u1: 2 = > decrease of $-6
c13: 0 v3 - u1: - 4 = > decrease of $-4
c23: 0 v3 - u2: 0 = > decrease of $0
c32: 9 v2 - u3: 4 = > decrease of $-5

Reached the Optimal Solution.

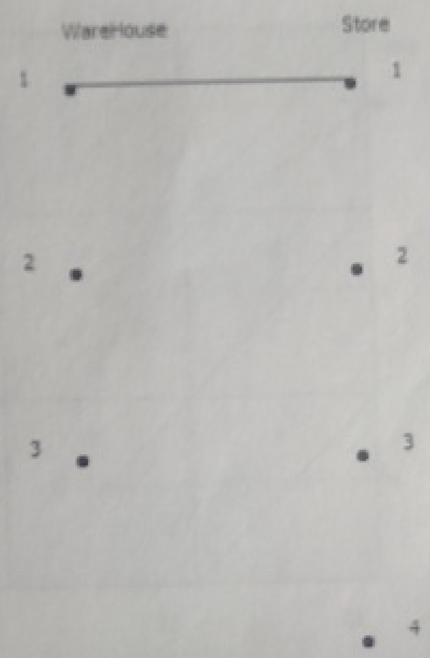
Optimal cost = 300$

6. (a)

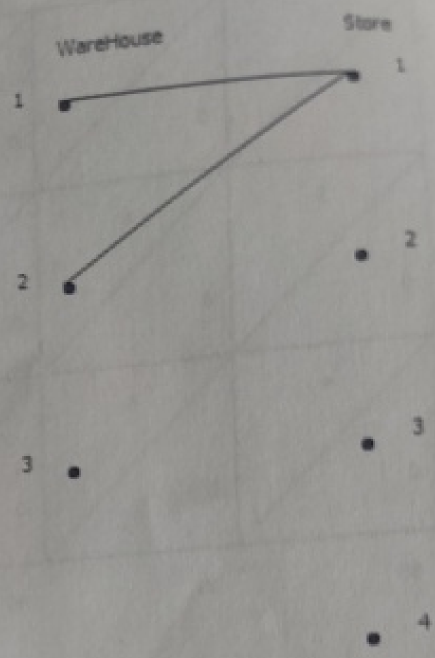
7 / 0	2 / 0	5 / 0	0 / 0	40
3 / 0	5 / 0	4 / 0	0 / 0	30
4 / 0	6 / 0	3 / 0	0 / 0	50
50	10	40	20	



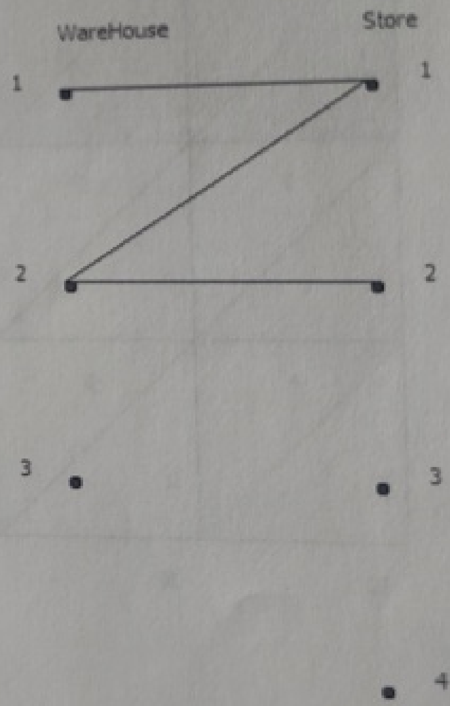
7 / 40	2 / 0	5 / 0	0 / 0	40
3 / 0	5 / 0	4 / 0	0 / 0	30
4 / 0	6 / 0	3 / 0	0 / 0	50
50	10	40	20	



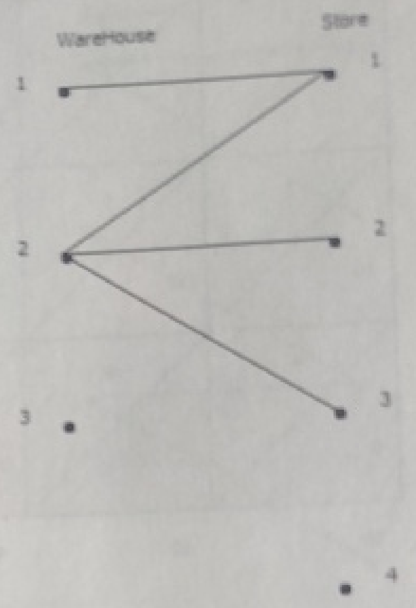
7 / 40	2 / 0	5 / 0	0 / 0	40
3 / 10	5 / 0	4 / 0	0 / 0	30
4 / 0	6 / 0	3 / 0	0 / 0	50
50	10	40	20	



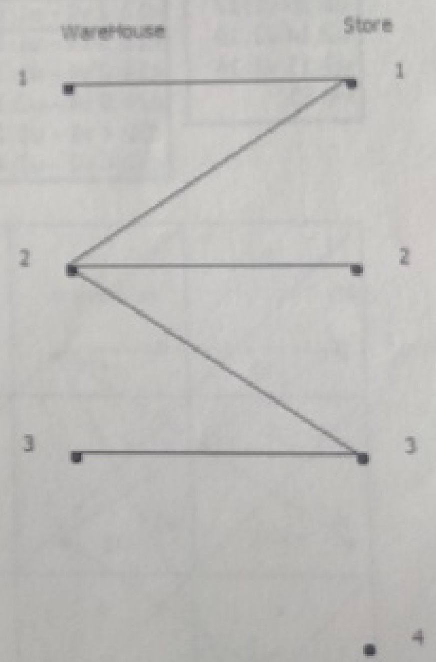
7 / 40	2 / 0	5 / 0	0 / 0	40
3 / 10	5 / 10	4 / 0	0 / 0	30
4 / 0	6 / 0	3 / 0	0 / 0	50
50	10	40	20	



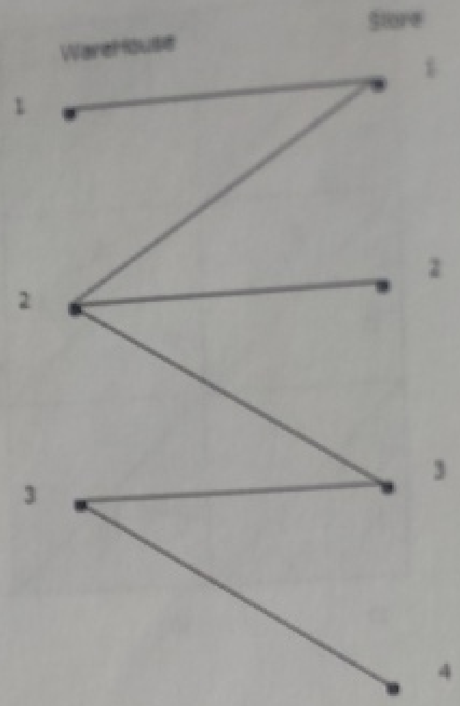
7 40	2 0	5 0	0 0	40
3 10	5 10	4 10	0 0	30
4 0	6 0	3 0	0 0	50
50	10	40	20	



7 40	2 0	5 0	0 0	40
3 10	5 10	4 10	0 0	30
4 0	6 0	3 30	0 0	50
50	10	40	20	



7 40	2 0	5 0	0 0	40
3 10	5 10	4 10	0 0	30
4 0	6 0	3 30	0 20	50
50	10	40	20	

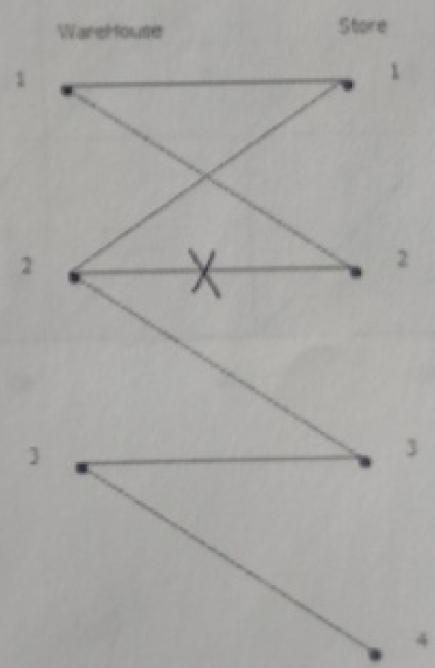


Total cost = 490$

u1: 10	v1: 17
u2: 14	v2: 19
u3: 15	v3: 18
	v4: 15

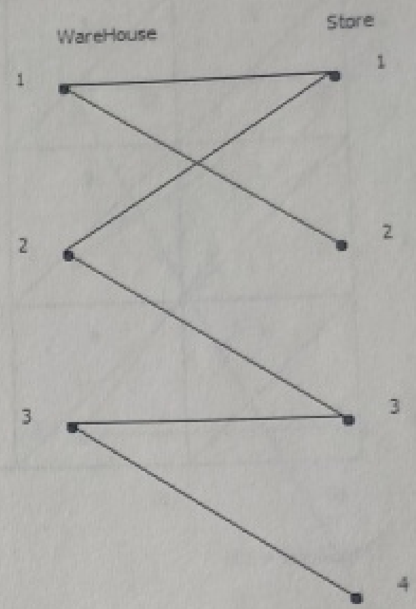
c12: 2 v2 - u1: 9 => decrease of $7
c13: 5 v3 - u1: 8 => decrease of $3
c14: 0 v4 - u1: 5 => decrease of $5
c24: 0 v4 - u2: 1 => decrease of $1
c31: 4 v1 - u3: 2 => decrease of $-2
c32: 6 v2 - u3: 4 => decrease of $-2

7 40 -10	2 0 +10	5 0	0 0	40
3 10 +10	5 10 -10	4 10	0 0	30
4 0	6 0	3 30	0 20	50
50	10	40	20	



7 30	2 10	5 0	0 0	40
3 20	5 0	4 10	0 0	30
4 0	6 0	3 30	0 20	50
50	10	40	20	

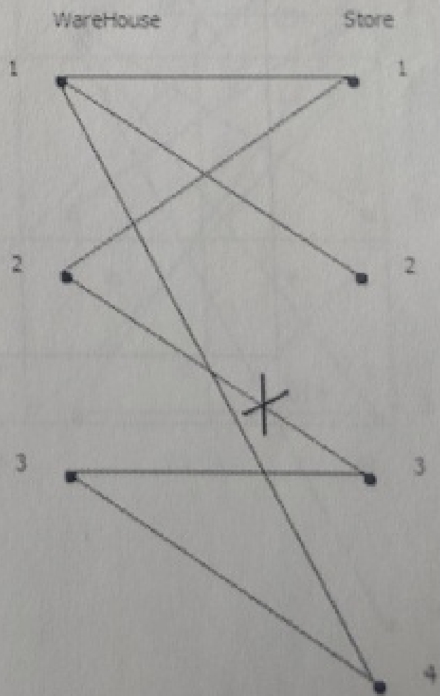
Total cost = 420

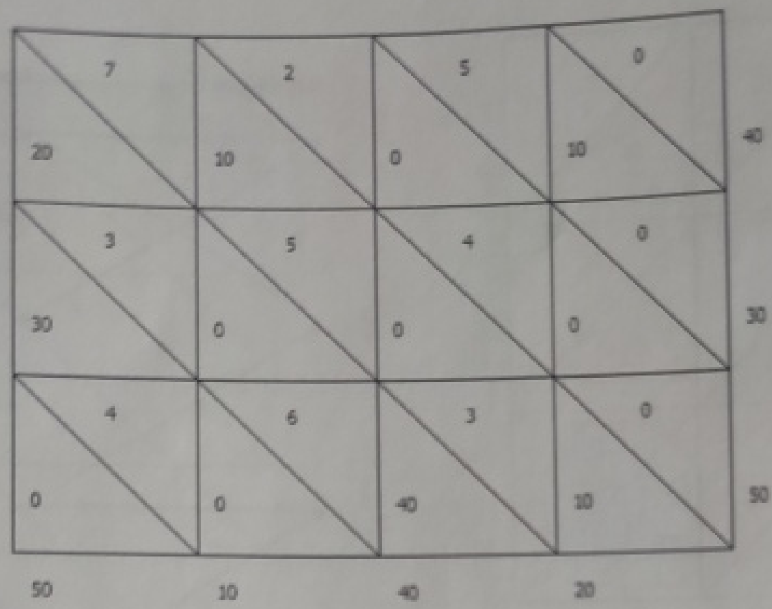


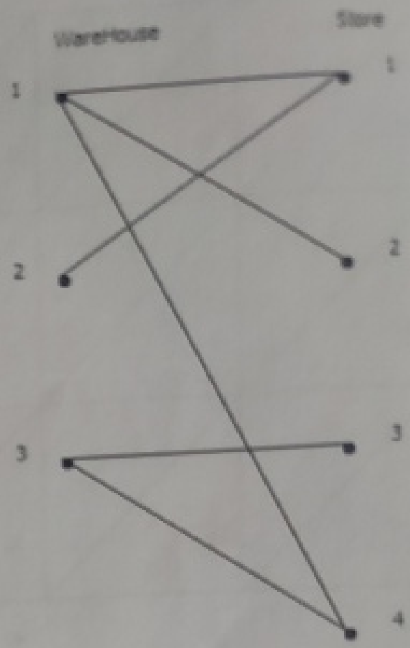
u1: 10 v1: 17
u2: 14 v2: 12
u3: 15 v3: 18
v4: 15

c13: 5 v3 - u1: 8 => decrease of $3
c14: 0 v4 - u1: 5 => decrease of $5
c22: 5 v2 - u2: -2 => decrease of $-7
c24: 0 v4 - u2: 1 => decrease of $1
c31: 4 v1 - u3: 2 => decrease of $-2
c32: 6 v2 - u3: -3 => decrease of $-9

7 30 -10	2 10	5 0	0 0 +10	40
3 20 +10	5 0	4 10 -10	0 0	30
4 0	6 0	3 30 +10	0 20 -10	50
50	10	40	20	



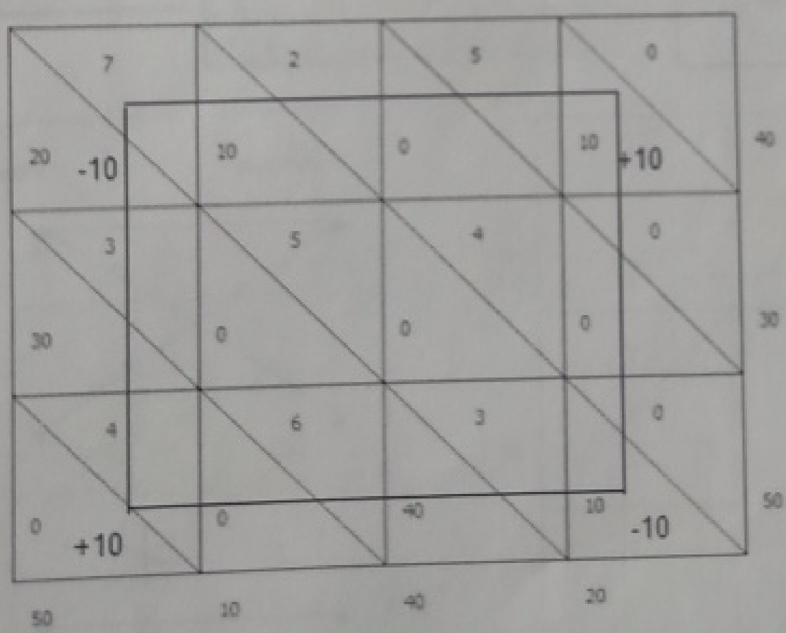


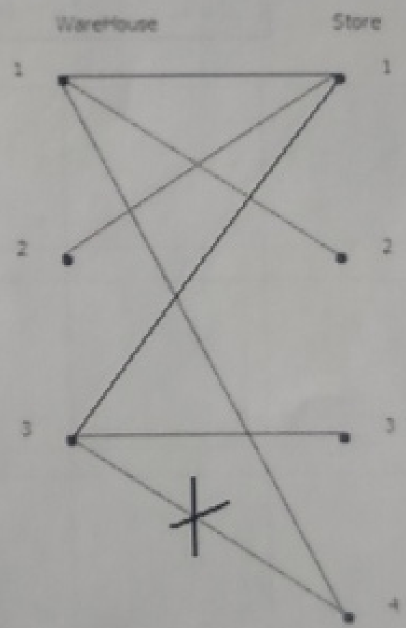


Total cost = 370

u1: 10	v1: 17
u2: 14	v2: 12
u3: 10	v3: 13
v4: 10	

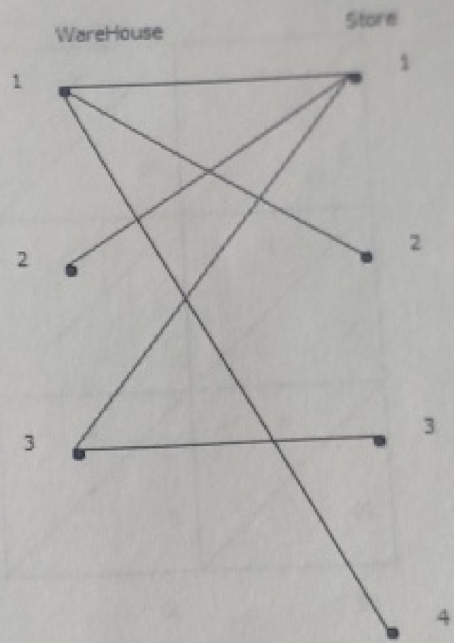
c13: 5, v3 - u1: 3 => decrease of $-2
c22: 5, v2 - u2: -2 => decrease of $-7
c23: 4, v3 - u2: -1 => decrease of $-5
c24: 0, v4 - u2: -4 => decrease of $-4
c31: 4, v1 - u3: 7 => decrease of $3
c32: 6, v2 - u3: 2 => decrease of $-4





7 / 10	2 / 10	5 / 0	0 / 20	40
3 / 30	5 / 0	4 / 0	0 / 0	30
4 / 10	6 / 0	3 / 40	0 / 0	50
50	10	40	20	

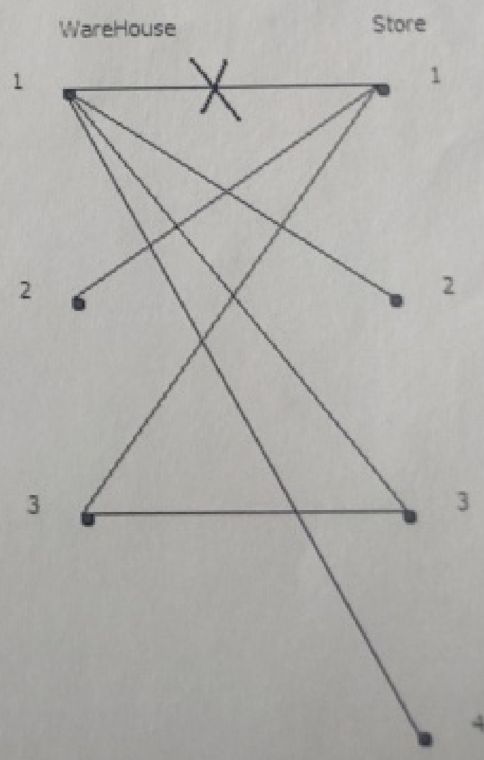
Total cost = 340



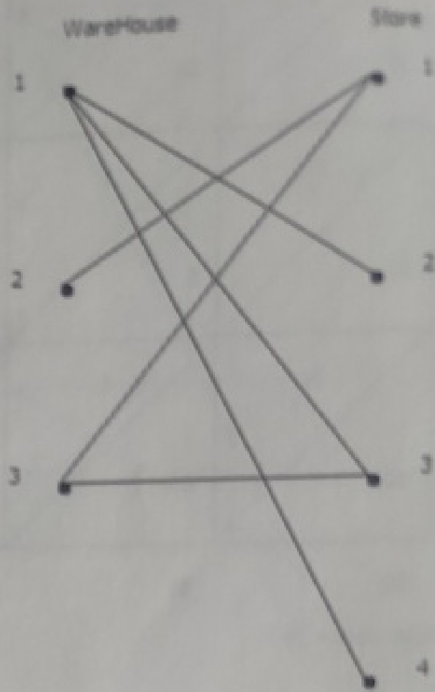
u1: 10	v1: 17
u2: 14	v2: 12
u3: 13	v3: 16
v4: 10	

c13: 5, v3- u1: 6 => decrease of $1
c22: 5, v2 - u2: -2 => decrease of $-7
c23: 4, v3- u2: 2 => decrease of $-2
c24: 5, v4- u2: -4 => decrease of $-9
c32: 6, v2- u3: -1 => decrease of $-7
c34: 0, v4- u3: -3 => decrease of $-3

7 / -10 / 10	2 / 10	5 / 0 / +10	0 / 20	40
3 / 30	5 / 0	4 / 0	0 / 0	30
4 / 10 / +10	6 / 0	3 / 40 / -10	0 / 0	50
50	10	40	20	



7 0	2 10	5 10	0 20	40
3 30	5 0	4 0	0 0	30
4 20	6 0	3 30	0 0	50
50	10	40	20	



u1: 10 v1: 16
u2: 13 v2: 12
u3: 12 v3: 15
v4: 10

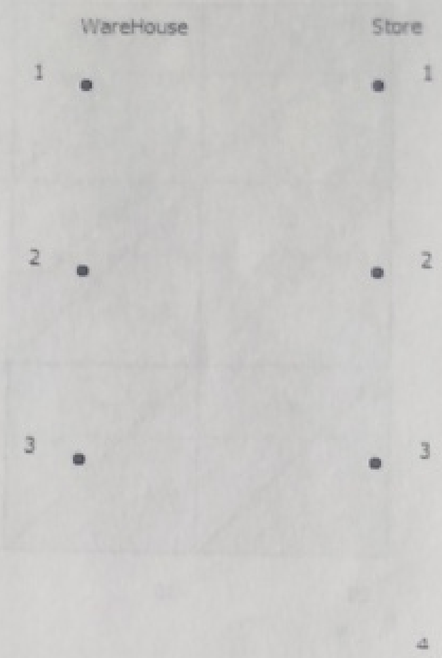
c11: 7, v1 - u1: 6 => decrease of $-1
c22: 5, v2 - u2: -1 => decrease of $-6
c23: 4, v3 - u2: 2 => decrease of $-2
c24: 0, v4 - u2: -3 => decrease of $-3
c32: 6, v2 - u3: 0 => decrease of $-6
c34: 0, v4 - u3: -2 => decrease of $-2

At last the Optimal solution!!!

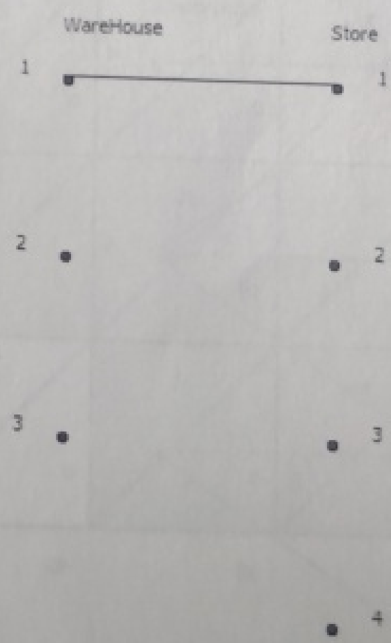
Optimal cost = 330$

6. (b)

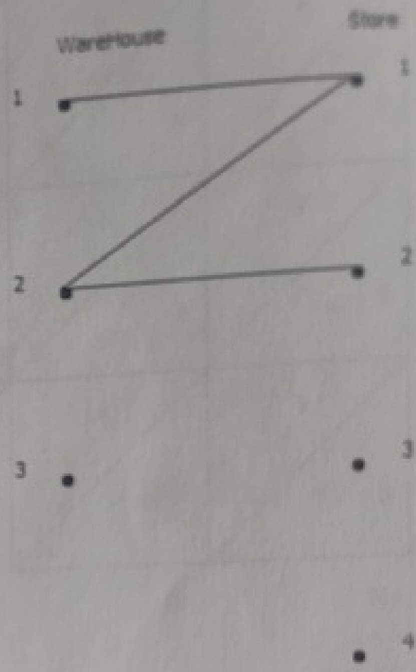
			Dummy	
8 / 0	3 / 0	2 / 0	0 / 0	40
3 / 0	5 / 0	7 / 0	0 / 0	30
6 / 0	4 / 0	5 / 0	0 / 0	50
50	10	40	20	



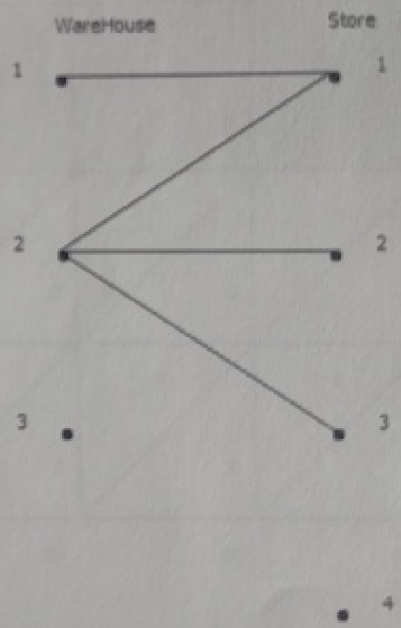
8 / 40	3 / 0	2 / 0	0 / 0	40
3 / 0	5 / 0	7 / 0	0 / 0	30
6 / 0	4 / 0	5 / 0	0 / 0	50
50	10	40	20	



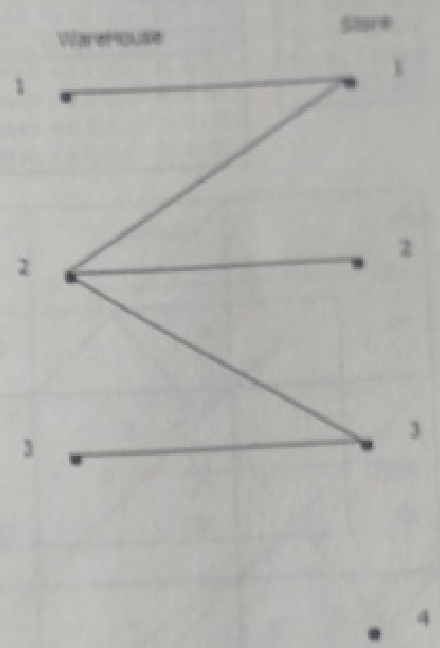
8 / 40	3 / 0	2 / 0	0 / 0	40
3 / 10	5 / 10	7 / 0	0 / 0	30
6 / 0	4 / 0	5 / 0	0 / 0	50
50	10	40	20	



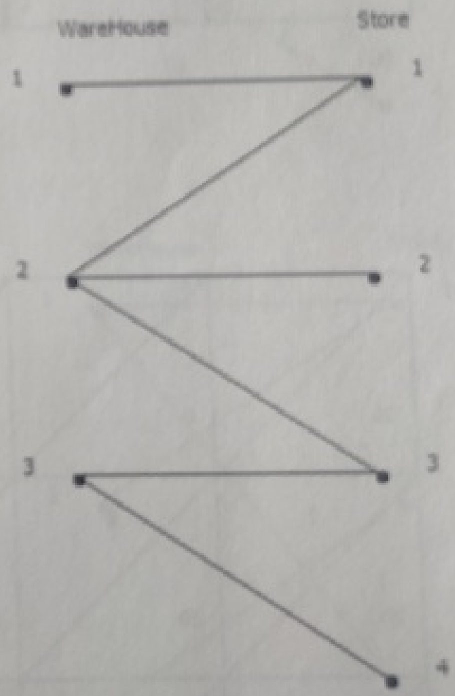
8 / 40	3 / 0	2 / 0	0 / 0	40
3 / 10	5 / 10	7 / 10	0 / 0	30
6 / 0	4 / 0	5 / 0	0 / 0	50
50	10	40	20	



8 / 40	3 / 0	2 / 0	0 / 0	40
3 / 10	5 / 10	7 / 10	0 / 0	30
6 / 0	4 / 0	5 / 30	0 / 0	50
50	10	40	20	



8 / 40	3 / 0	2 / 0	0 / 0	40
3 / 10	5 / 10	7 / 10	0 / 0	30
6 / 0	4 / 0	5 / 30	0 / 20	50
50	10	40	20	

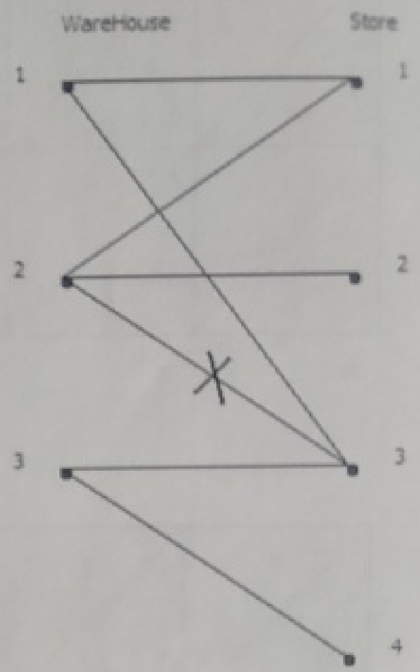


Total cost (by NWCST method) = 620$

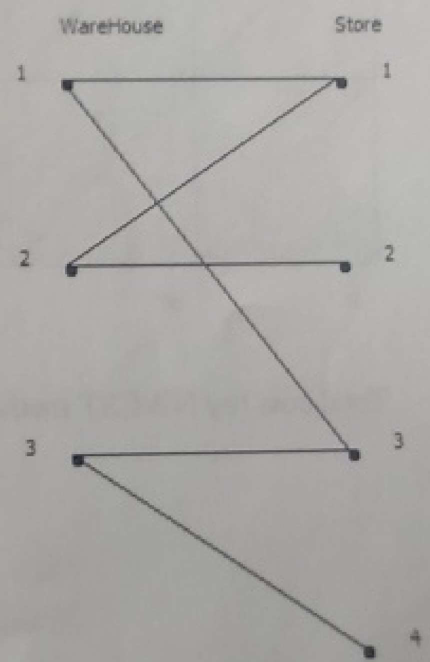
u1: 10 v1: 18
u2: 15 v2: 20
u3: 17 v3: 22
v4: 17

c12: 3 v2 - u1: 10 => decrease of $7
c13: 2 v3 - u1: 12 => decrease of $10
c14: 0 v4 - u1: 7 => decrease of $7
c24: 0 v4 - u2: 2 => decrease of $2
c31: 6 v1 - u3: 1 => decrease of $-5
c32: 4 v2 - u3: 3 => decrease of $-1

8 -10 40	3 0	2 0 +10	0 0	40
3 +10 10	5 10	7 -10 10	0 0	30
6 0	4 0	5 30	0 20	50
50	10	40	20	



8 30	3 0	2 10	0 0	40
3 20	5 10	7 0	0 0	30
6 0	4 0	5 30	0 20	50
50	10	40	20	

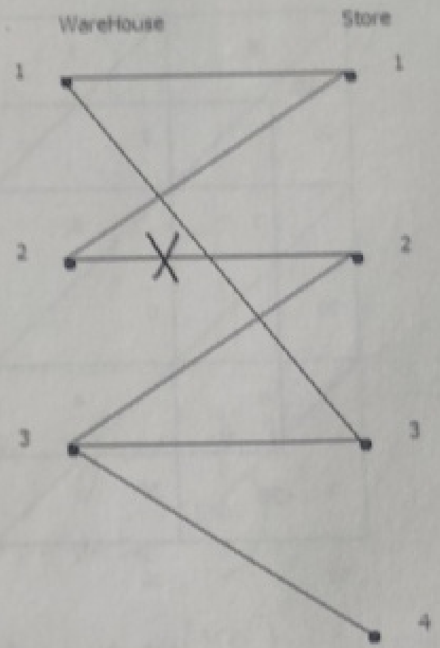


Total cost = 520

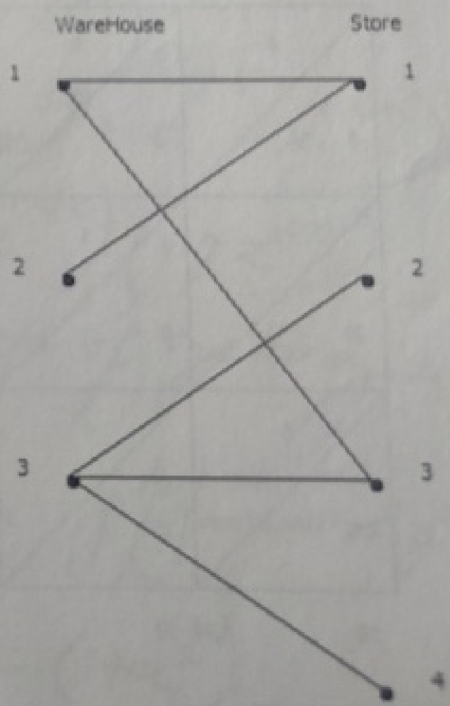
u1: 10	v1: 18			
u2: 15	v2: 20			
u3: 7	v3: 12			
v4: 7				

c12: 3 v2 - u1: 10 => decrease of $7
c14: 0 v4 - u1: -3 => decrease of $ -3
c23: 7 v3 - u2: -3 => decrease of $-10
c24: 0 v4 - u2: -8 => decrease of $ -8
c31: 6 v1 - u3: 11 => decrease of $5
c32: 4 v2 - u3: 13 => decrease of $9

8 30 -10	3 0	2 10 +10	0 0	40
3 20 +10	5 10 -10	7 0	0 0	30
6 0	4 0 +10	5 -10 30	0 20	50
50	10	40	20	



8 20	3 0	2 20	0 0	40
3 30	5 0	7 0	0 0	30
6 0	4 10	5 20	0 20	50
50	10	40	20	

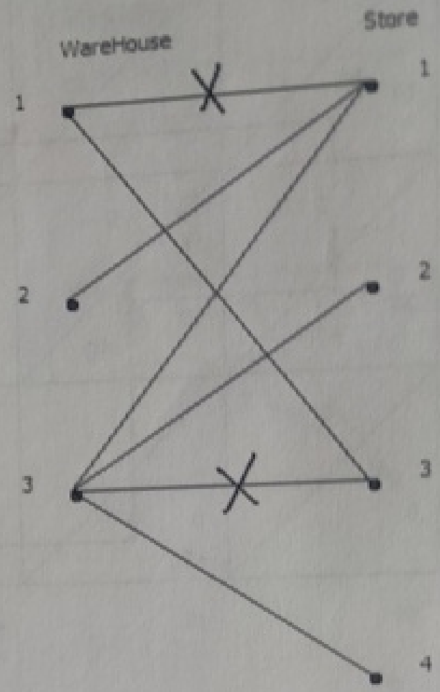


Total cost = 430

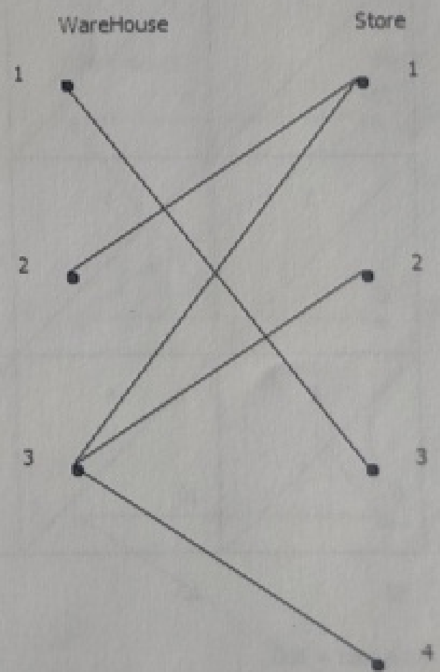
u1: 10 v1: 18
u2: 15 v2: 11
u3: 7 v3: 12
v4: 7

c12: 3 v2 - u1: 1 => decrease of $ -2
c14: 0 v4 - u1: -3. => decrease of $ -3
c22: 5 v2 - u2: -4 => decrease of $ -9
c23: 7 v3 - u2: -3 => decrease of $-10
c24: 0 v4 - u2: -8 => decrease of $ -8
c31: 6 v1 - u3: 11 => decrease of $5

8 / 20 (-20)	3 / 0	2 / 20 (+20)	0 / 0	40
3 / 30	5 / 0	7 / 0	0 / 0	30
6 / 0 (+20)	4 / 10	5 / 20 (-20)	0 / 20	50
50	10	40	20	



8 / 0	3 / 0	2 / 40	0 / 0	40
3 / 30	5 / 0	7 / 0	0 / 0	30
6 / 20	4 / 10	5 / 0	0 / 20	50
50	10	40	20	



Total cost = 330

The graph has become disconnected (forest).
start with u1 = 10 from one component and
u2 = 10 from another (tree).

u1: 10	v1: 13
u2: 10	v2: 11
u3: 7	v3: 12
v4: 7	

c11: 8, v1 - u1: 3 => decrease of $-5
c12: 3, v2 - u1: 1 => decrease of $-2
c14: 0, v4 - u1: -3 => decrease of $-3
c22: 5, v2 - u2: 1 => decrease of $-4
c23: 7, v3 - u2: 5 => decrease of $-2
c24: 0, v4 - u2: -3 => decrease of $-3
c33: 5, v3 - u3: 5 => decrease of $0

Optimal solution!!!

Optimal cost = 330$

Ex 2 #ways of giving change for $1.

1¢ $(1+x+x^2+\dots+x^{100})$

5¢ $(1+x^5+x^{10}+\dots+x^{100})$

10¢ $(1+x^{10}+\dots+x^{100})$

25¢ $(1+x^{25}+x^{50}+x^{75}+x^{100})$

50¢ $(1+x^{50}+x^{100})$

100¢ $(1+x^{100})$

All possible ways of getting transferred

Ex 3 #ways to write an integer r as sum of distinct powers of 2

$g(x) = (1+x)(1+x^2)(1+x^4)\dots(1+x^{2^k})\dots$

$a_r = 1 \;\forall r \in \mathbb{N}$

Theorem Pf: [illegible] I show: $g(x) = 1+x+\dots+x^n+\dots = \frac{1}{1-x}$

$(1-x)(1+x)(1+x^2)(1+x^4)\dots$

$(1-x)\,g(x) = 1$

Partitions of Integers

Ferrer's Diagram

$1+1+2+3+5+8=20$

p

```
8  • • • • • • • •
5  • • • • •
3  • • •
2  • •
1  •
1  •
```

conjugate

$p^{\perp}$

```
• • • • • •
• • • •
• • •
• •
• •
•
•
•
```

> 1—1 correspondence between a partition & its conjugate

Theorem

partitions of r into k numbers (non-zero) — # ways to partition into #, max^m of which is k

—

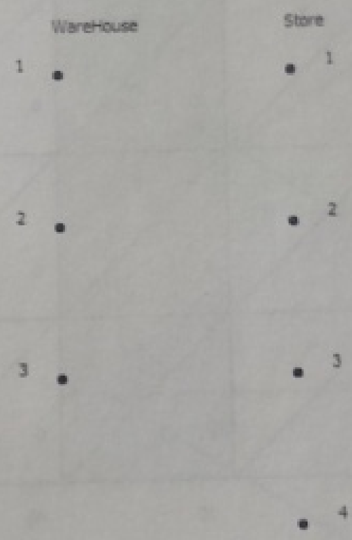
Generating function

8(c)

Dummy

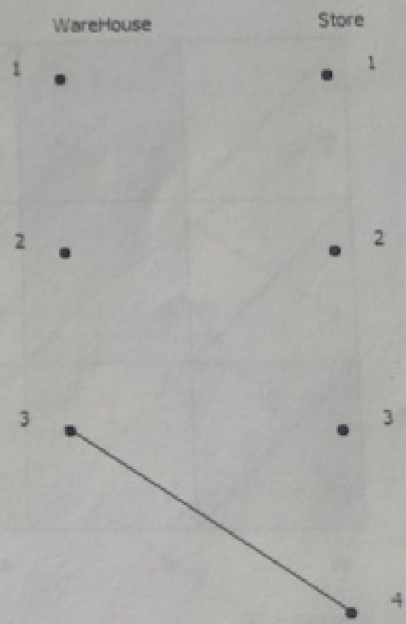
7 / 0	2 / 0	5 / 0	0 / 0	40
3 / 0	5 / 0	4 / 0	0 / 0	30
4 / 0	6 / 0	3 / 0	0 / 0	50
50	10	40	20	

Total cost = 0



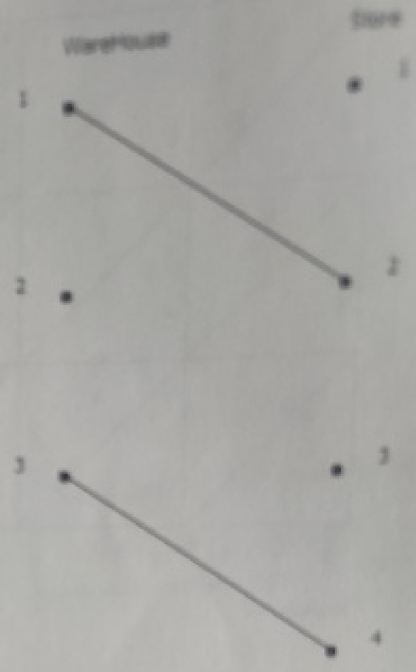
7 / 0	2 / 0	5 / 0	0 / 0	40
3 / 0	5 / 0	4 / 0	0 / 0	30
4 / 0	6 / 0	3 / 0	0 / 20	50
50	10	40	20	

Total cost = 0



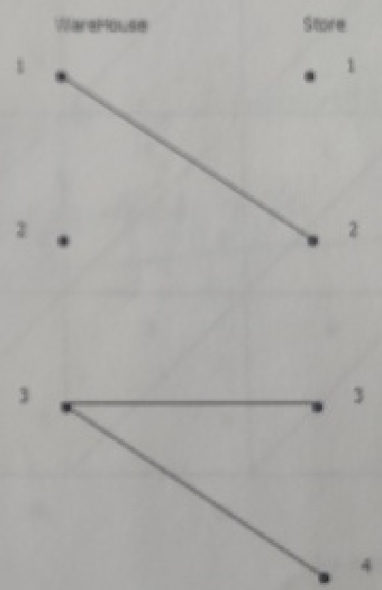
7 0	2 10	5 0	0 0	40
3 0	5 0	4 0	0 0	30
4 0	6 0	3 0	0 20	50
50	10	40	20	

Total cost = 20



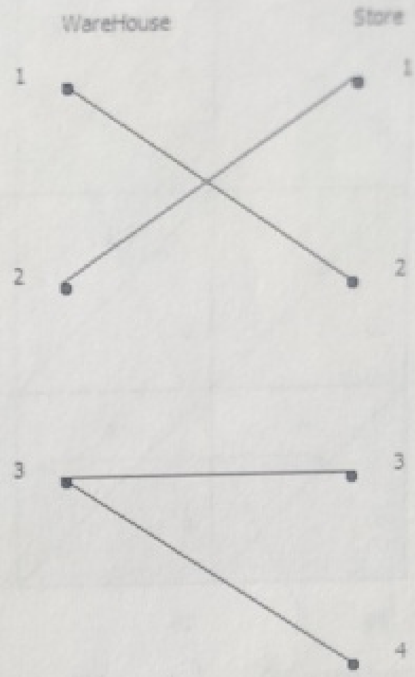
7 0	2 10	5 0	0 0	40
3 0	5 0	4 0	0 0	30
4 0	6 0	3 30	0 20	50
50	10	40	20	

Total cost = 110



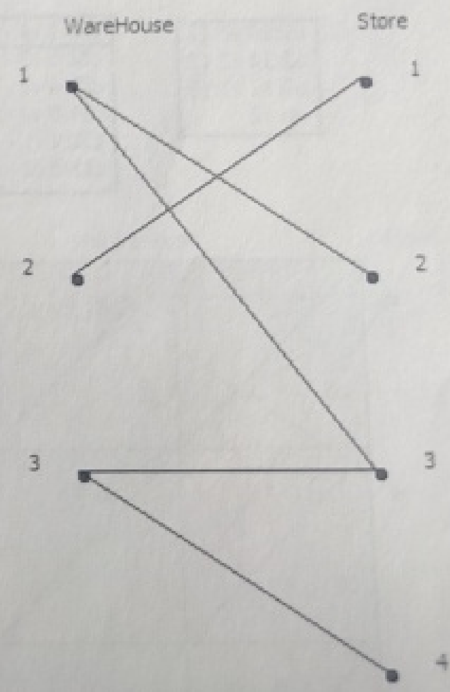
7 0	2 10	5 0	0 0	40
3 30	5 0	4 0	0 0	30
4 0	6 0	3 30	0 20	50
50	10	40	20	

Total cost = 200

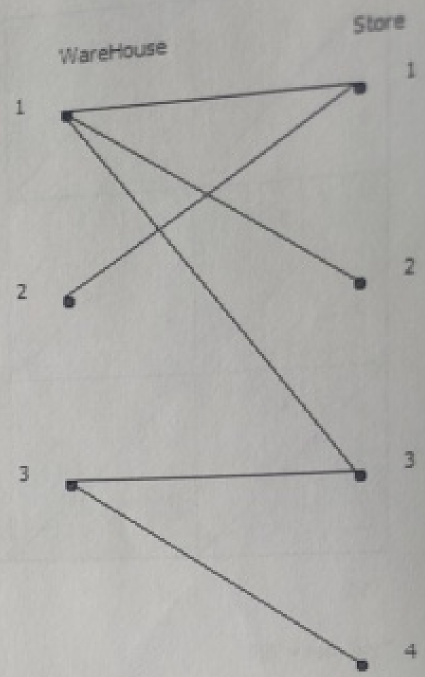


7 0	2 10	5 10	0 0	40
3 30	5 0	4 0	0 0	30
4 0	6 0	3 30	0 20	50
50	10	40	20	

Total cost = 250



7 20	2 10	5 10	0 0	40
3 30	5 0	4 0	0 0	30
4 0	6 0	3 30	0 20	50
50	10	40	20	

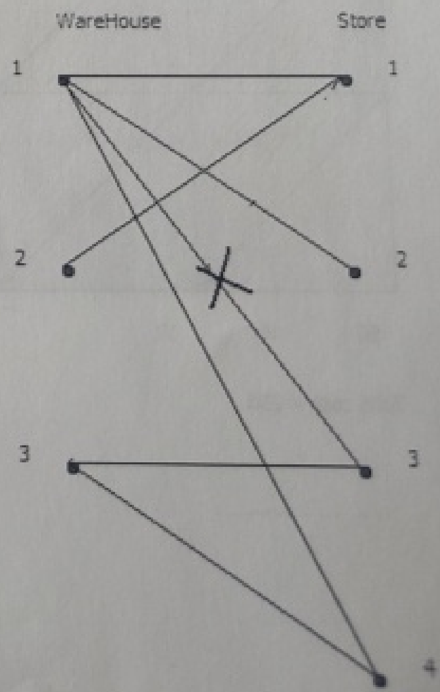


Total cost = 390 (Min^m cost method)

u1: 10	v1: 17
u2: 14	v2: 12
u3: 12	v3: 15
v4: 12	

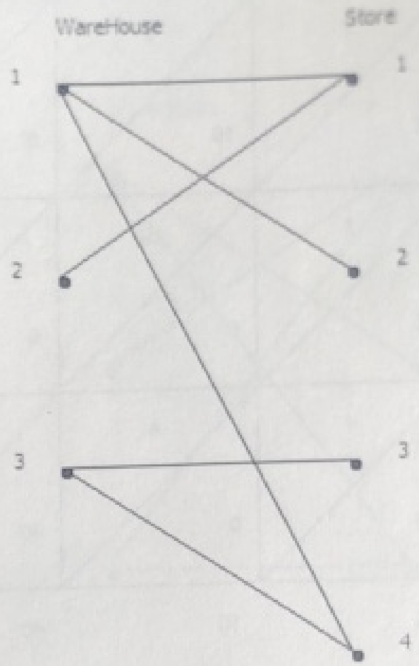
c14: 0 v4 - u1: 2 => decrease of $2
c22: 5 v2 - u2: -2 => decrease of $-7
c23: 4 v3 - u2: 1 => decrease of $-3
c24: 0 v4 - u2: -2 => decrease of $ -2
c31: 4 v1 - u3: 5 => decrease of $1
c32: 6 v2 - u3: 0 => decrease of $-6

7 20	2 10	5 -10 10	0 +10 0	40
3 30	5 0	4 0	0 0	30
4 0	6 0	3 +10 30	0 -10 20	50
50	10	40	20	



7 / 20	2 / 10	5 / 0	0 / 10	40
3 / 30	5 / 0	4 / 0	0 / 0	30
4 / 0	6 / 0	3 / 40	0 / 10	50
50	10	40	20	

Total cost = 370

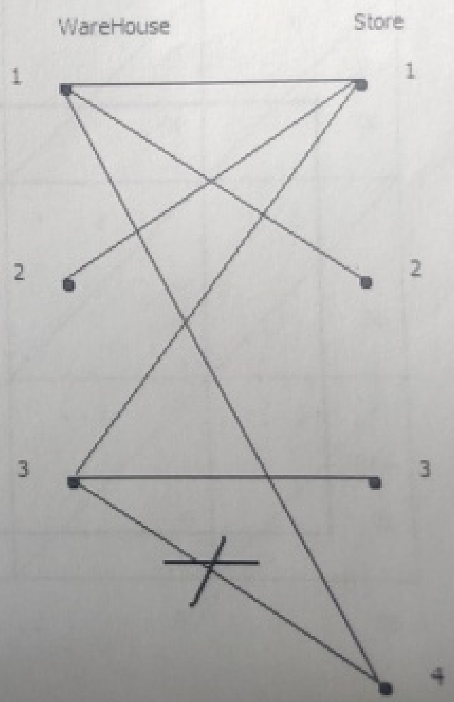


u1: 10	v1: 17
u2: 14	v2: 12
u3: 10	v3: 13
v4: 10	

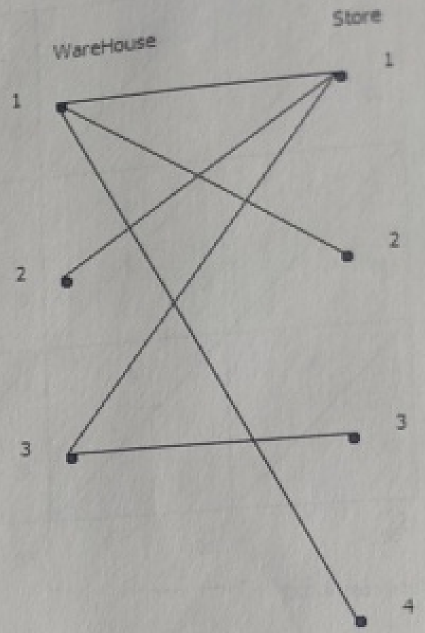
c13: 4, v3 - u1: 3 => decrease of \$-1
c22: 5, v2 - u2: -2 => decrease of \$-7
c23: 4, v3 - u2: 1 => decrease of \$-3
c24: 0, v4 - u2: -4 => decrease of \$-4
c31: 4, v1 - u3: 7 => decrease of \$3
c32: 6, v2 - u3: 2 => decrease of \$-4

7 / -10 20	2 / 10	5 / 0	0 / 10 +10	40
3 / 30	5 / 0	4 / 0	0 / 0	30
4 / 0 +10	6 / 0	3 / 40	0 / 10 -10	50
50	10	40	20	

Total cost = 370



7 / 10	2 / 10	5 / 0	0 / 20	40
3 / 30	5 / 0	4 / 0	0 / 0	30
4 / 10	6 / 0	3 / 40	0 / 0	50
50	10	40	20	

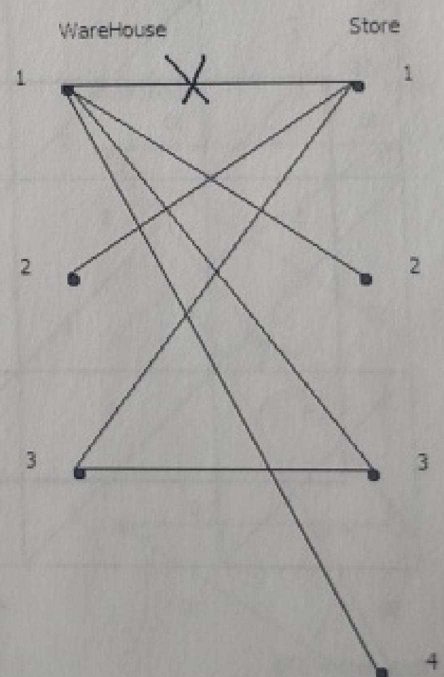


Total cost = 340

u1: 10	v1: 17
u2: 14	v2: 12
u3: 13	v3: 16
v4: 10	

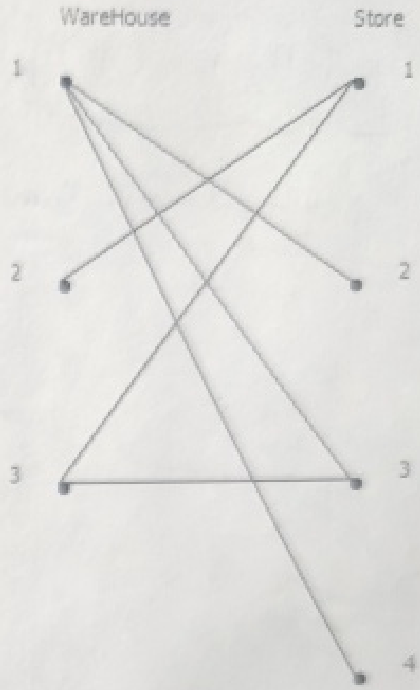
c13: 5, v3 - u1: 6 => decrease of $1
c22: 5, v2 - u2: -2 => decrease of $-7
c23: 4, v3 - u2: 2 => decrease of $-2
c24: 0, v4 - u2: -4 => decrease of $-4
c32: 6, v2 - u3: -1 => decrease of $-7
c34: 0, v4 - u3: -3 => decrease of $-3

7 / -10 / 10	2 / 10	5 / 0 +10	0 / 20	40
3 / 30	5 / 0	4 / 0	0 / 0	30
4 / +10 / 10	6 / 0	3 / 40 -10	0 / 0	50
50	10	40	20	



7 0	2 10	5 10	0 20	40
3 30	5 0	4 0	0 0	30
4 20	6 0	3 30	0 0	50
50	10	40	20	

Total cost = 330



At last we reached the optimal solution, which is same as that we got earlier.

Optimal cost = 330$

Sandipan Dey

Order

sum of integers

Partitions of $n \in N$

order does not make much sense

$n_1 + n_2 + \cdots + n_k = n$

k fixed or Variable

Unordered sum

$1 + 1 + \cdots + 1 = n$

$n = n$

$e_1 = 1s$
$e_2 = 2s$
$\vdots$
$e_k = ks$

$1+2+2+3+3+3+\cdots = n$

$1e_1 + 2e_2 + \cdots + ke_k$

a_r $\begin{Bmatrix} x^0 \\ x^1 \\ \vdots \\ x^k \end{Bmatrix} \begin{Bmatrix} (x^2)^0 \\ (x^2)^1 \\ \vdots \\ (x^2)^k \end{Bmatrix} \cdots \begin{Bmatrix} (x^l)^0 \\ (x^l)^1 \\ \vdots \\ (x^l)^k \end{Bmatrix}$

~~repitition~~ repitition

$$g(x) = (1 + x + \cdots + x^k + \cdots)(1 + x^2 + x^4 + \cdots + x^{2k})(\cdots (1 + x^l + (x^l)^2 + \cdots)$$

$$= \frac{1}{1-x} \cdot \frac{1}{1-x^2} \cdots \frac{1}{1-x^l}$$

$$g(x) = \prod_{l=1}^{\infty} \frac{1}{(1-x^l)}$$

Example a_r = # ways to express as a sum of integers.

$$(1+x)(1+x^2)\cdots(1+x^r)\cdots$$

not present

a_r coefficient of x^r

present