W-6

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Let $f(n) = n^2$ and the language $A' = \text{pad}(A, f(n))$.

$\therefore A' = \{ x\#^{n^2-n} : x \in A \wedge n = |x| \}$.

Now, $A \in \text{TIME}(n^6)$

$\Longleftrightarrow \exists$ TM M_A that decides A in $O(n^6)$ time.

A' is f-padded version of A.

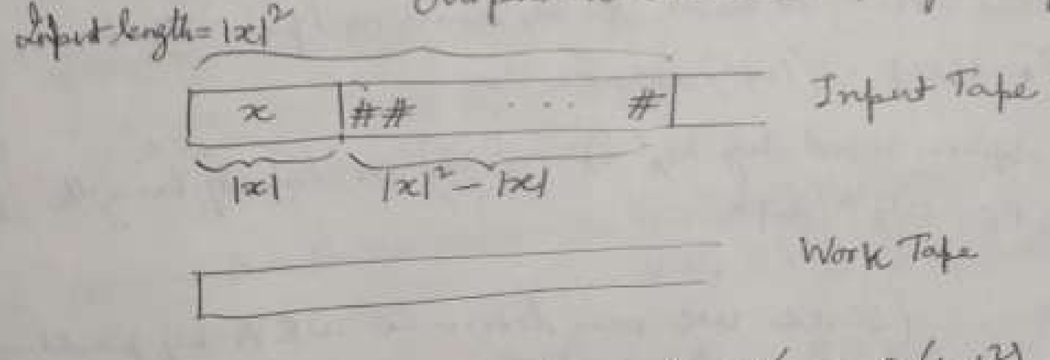
From M_A, let's construct a TM M_A' that decides A'.

M_A' = "on input y:

1. check if $\exists x \in \Sigma^* \mid y = x\#^{|x|^2-|x|}$ in $O(|x|^2)$ time,
   /* count backwards the number of #-s and check if there are $|x|^2-|x|$ such #s, in $O(|x|^2-|x|) = O(|x|^2)$ time */

1.1. If Not, output REJECT

1.2. Else, run M_A on x for $O(|x|^6)$ time and output its answer." /* ignoring the #s */ "



Clearly the running time for $M_A' = O(|x|^2) + O(|x|^6) = O(|x|^6)$.

but input length of $M_A' = |x|^2 = n$ (let)

$\Rightarrow$ runtime $= O((|x|^2)^3) = O(n^3)$ w.r.t. the length of input

$\Rightarrow A' \in \text{TIME}(n^3)$

(Also, M_A' always halts since M_A " ")

2.

10

To prove: $P \neq SPACE[n^3]$

By contradiction

Let's assume $P = SPACE[n^3]$, to the contrary.

By Space Hierarchy Theorem $\exists$ a language $A \mid A \in SPACE[n^6]$ but $A \notin SPACE[n^3]$, since $n^3 = o(n^6)$ ... ①

Now, the language $A' = pad(A, n^2) \in SPACE[n^3] = P$, we can show using the similar technique as in 9.13 as follows:

Construct TM, $M_{A'}$ from M_A (deciding A)

$M_{A'}(y)$

Check if y is of the form $\underbrace{x}_{|x|}\underbrace{\#^{|x|^2-|x|}}_{|x|^2-|x|}$

/* Count number of # using $\log(|x|^2-|x|) = O(\log|x|^2)$ space */

if no then REJECT

else run M on x and output whatever it does

/* uses $O(|x|^6)$ space */

Hence total space used by $M_{A'}$ is $O(|x|^6)$ on input with length $|x|^2$.

$\Rightarrow A' \in SPACE[n^3] = P$.

$\Rightarrow A \in P$ (since we can determine $w \in A$ by padding w with $|w|^2 - |w|$ #'s and then testing whether $w' = w\#^{|w|^2-|w|} \in A'$ in polynomial time)

But $P = SPACE[n^3]$, by assumption

$\Rightarrow A \in SPACE[n^3]$ ... ②

① & ② $\Rightarrow\Leftarrow$

(Proved)