Research Project on Quantum Entanglement

CMSC 643

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Executive Summary

We are given the following set of entangled states:

$$\begin{cases} |EPR\rangle_{ij} & = & \frac{|0_i 1_j\rangle - |1_i 0_j\rangle}{\sqrt{2}} \\ |GHZ\rangle_{ijk} & = & \frac{|0_i 0_j 0_k\rangle + |1_i 1_j 1_k\rangle}{\sqrt{2}} \\ |Werner\rangle_{ijk} & = & \frac{|0_i 0_j 1_k\rangle + |0_i 1_j 0_k\rangle + |1_i 0_j 0_k\rangle}{\sqrt{3}} \end{cases}$$

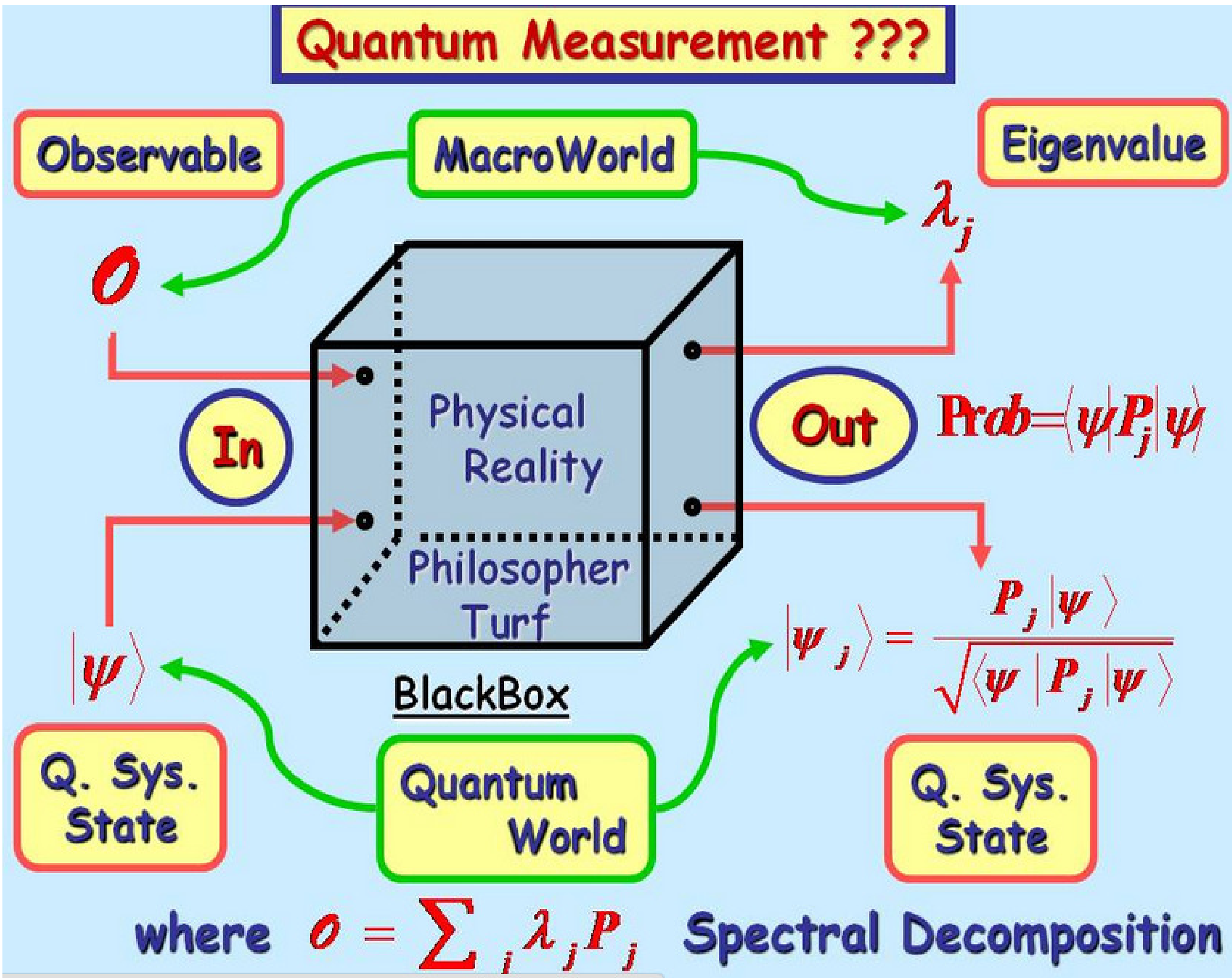
We have to find the results of Bell Measurements, where the Bell Basis vectors (for the Hilbert space for 2-qubit systems) are as follows:

$$\begin{cases} |\Psi^-\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle) \\ |\Psi^+\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle) \\ |\Phi^-\rangle = \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle) \\ |\Phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle) \end{cases}$$

The results of (selective) Bell Measurements on the Entangled pairs will be interesting in the sense that they will result in Entanglement Swapping that can have applications in quantum repeaters.

Learnt from the Project

The following represents the crux of Quantum Measurement, as taken from Dr. Lomonaco's slides:



Projector: Matrices with properties $P.P = P$

Ortho-normal Projectors: Satisfies Kronecker Delta**:** $\delta_{ij} = P_i.P_j, \ \delta_{ij} = \begin{cases} 0, & i \neq j \\ 1, & i = j \end{cases}$

Projectors Span the Eigen Space: $\sum_i P_i = I$

Measurement:

1) Normalize the quantum state $|\varphi\rangle$ to be measured.

2) Compute the orthonormal basis vectors $\{V_i\}_{i=1}^{n}$ spanning the eigenspace of the observable Ω. (find the eignvalues and orhto-normal eigenvectors of Hermitian Ω).

3) Compute the projectors $P_i = V_i^{+}.\ V_i$.

4) After measurement, the state of the quantum system will “collapse” to any of the n states (corresponding to its n ortho-normal eigenspace) with certain probabilities. Probability that the quantum system with initial state $|\varphi\rangle$ will be in state $|\varphi_i\rangle$ after the Measurement is, $p_i = \langle \varphi | P_i | \varphi \rangle$,

with $\sum_i p_i = \sum_i \langle \varphi | P_i | \varphi \rangle = 1$ (must be in one of the states).

5) The i^{th} probable state after measurement can be computed as, $\varphi_i = \frac{P_i . |\varphi\rangle}{\sqrt{\langle \varphi | P_i | \varphi \rangle}}$.

For this project, we already had basis vectors as 4 Orthonormal Bell Basis vectors, from which we computed 4 Orthonormal Bell Projectors (as shown in the output).

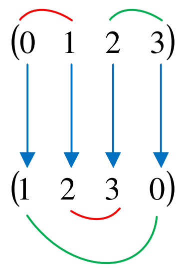
Normalization:

The quantum states must be normalized (since probability amplitudes).

Permutation Matrices:

We can easily go from one permutation of the qubits to another by multiplying a suitable permutation matrix to the first one.

The example shown below shows how to obtain such a permutation matrix for Entanglement Swapping:



									After
0	0	0	0	0	0	0	0	0	0
1	0	0	0	1	0	0	1	0	2
2	0	0	1	0	0	1	0	0	4
3	0	0	1	1	0	1	1	0	6
4	0	1	0	0	1	0	0	0	8
5	0	1	0	1	1	0	1	0	10
6	0	1	1	0	1	1	0	0	12
7	0	1	1	1	1	1	1	0	14
8	1	0	0	0	0	0	0	1	1
9	1	0	0	1	0	0	1	1	3
10	1	0	1	0	0	1	0	1	5
11	1	0	1	1	0	1	1	1	7
12	1	1	0	0	1	0	0	1	9
13	1	1	0	1	1	0	1	1	11
14	1	1	1	0	1	1	0	1	13
15	1	1	1	1	1	1	1	1	15

Hence, Permutation operator: (0) (15) (1 2 4 8) (3 6 12 9) (5 10) (7 14 13 11) with the following permutation matrix (performs just a left circular shift):

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
8	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
9	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
12	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
13	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
14	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1

Entanglement Swapping

Before measurement: EPR pairs (0 1) & (2 3) were entangled.
After measurement: EPR pairs (0 3) & (1 2) will be entangled, as will be shown in the output, since the state will be same as the one obtained applying the permutation matrix for swapping (entangled) bits.

Selective Measurement

1) In order to obtain a Bell Measurement of the qubits 1 and 2 of the (4-qubit) state $\varphi = |EPR\rangle_{01} \otimes |EPR\rangle_{23}$, we compute $I_2 \otimes P_{Bell} \otimes I_2$, where P_{Bell} is one of the 4 projectors for the Bell Measurement.

2) In order to obtain a Bell Measurement of the qubits 0 and 5 of the (6-qubit) state $\varphi = |GHZ\rangle_{012} \otimes |GHZ\rangle_{345}$, we need a trick. First compute the unitary permutation matrix Π that performs the permutation $(012345) \rightarrow (345012)$, which can be done by a 3-bit left circular shift.

   Next compute $I_2 \otimes P_{Bell} \otimes I_2$, where P_{Bell} is one of the 4 projectors for the Bell Measurement. Again apply the inverse permutation $\Pi^{-1} = \Pi^T$ (since unitary) to obtain the final states. Hence the entire operation can be described as the composition $\Pi^T..I_2 \otimes P_{Bell} \otimes I_2.\Pi$.

   This can be shown from the output of the question 3.

Source Code

```
(*-----------------------------------------------------------------------------*)
(*---------------- Sandipan Dey, UMBC CSEE ---------------------------------------*)
(*---------------- The Source Code for Quantum Bell Package ----------------------*)
(*---------------- Major Functions ------------------------------------------------
                1. ComputeBellBasis
                2. ComputeEPRPairs
                3. ComputeBellProjectors
                4. ComputeBellProbStates
                5. ShowEntangledOutputTables
-------------------------------------------------------------------------------*)

BeginPackage["QuantumBell`"]

Begin["`Private`"]

       Bra[k_, b_] := {b[[k + 1]]};
       Ket[k_, b_] := Bra[k, b]†;
       BraVec[coeff_, b_] := KetVec[coeff, b]†;
       KetVec[coeff_, b_] := Module[{n, v, i, j},
           n = Dimensions[b][[1]]; v = Table[0, {i, n}, {j, 1}]; Do [v = v + coeff[[i]] * Ket[i - 1, b], {i, n}]; v
       ];
       InnerProduct[Ψ1_, Ψ2_] := Ψ1†.Ψ2;
       InnerProductReal[Ψ1_, Ψ2_] := Ψ1ᵀ.Ψ2;
       TensorProduct[Ψ1_, Ψ2_] := KroneckerProduct[Ψ1, Ψ2];
       TensorProduct[Ψ1_, Ψ2_, Ψ3_] := KroneckerProduct[KroneckerProduct[Ψ1, Ψ2], Ψ3];
       Basis[n_] := IdentityMatrix[n];

(* Computes the Bell Basis *)
ComputeBellBasis[] := Module[{b, p00, p01, p10, p11, b1, b2, b3, b4},
        b = Basis[2];
        p00 = TensorProduct[Ket[0, b], Ket[0, b]] ;
        p11 = TensorProduct[Ket[1, b], Ket[1, b]] ;
        p01 = TensorProduct[Ket[0, b], Ket[1, b]] ;
        p10 = TensorProduct[Ket[1, b], Ket[0, b]] ;
        b1 = (p00 + p11) / Sqrt[2];
        b2 = (p01 + p10) / Sqrt[2];
        b3 = (p01 - p10) / Sqrt[2];
        b4 = (p00 - p11) / Sqrt[2];
        {b1ᵀ, b2ᵀ, b3ᵀ, b4ᵀ}
];
```

```
(* Computes EPR Pairs *)
ComputeEPRPairs[] := Module[{b},
   b = Basis[2]; (TensorProduct[Ket[0, b], Ket[1, b]] - TensorProduct[Ket[1, b], Ket[0, b]]) / Sqrt[2]];
(* Computes GHZ Pairs *)
ComputeGHZPairs[] := Module[{b},
   b = Basis[2]; (TensorProduct[Ket[0, b], Ket[0, b], Ket[0, b]] + TensorProduct[Ket[1, b], Ket[1, b], Ket[1, b]]) / Sqrt[2]];
(* Computes Werner Pairs *)
ComputeWernerPairs[] :=  Module[{b},
    b = Basis[2];
   (TensorProduct[Ket[0, b], Ket[0, b], Ket[1, b]] + TensorProduct[Ket[0, b], Ket[1, b],
                                                  + TensorProduct[Ket[1, b], Ket[0, b], Ket[0, b]]) /  Sqrt[3]];
(* Computes EPR Pairs (0,1) and (2,3) entangled*)
ComputeEPRPairs0123Entangled[] := Module[{BIT01, BIT23, BIT0123},
    BIT01 = BIT23 = ComputeEPRPairs[];
    BIT0123 = TensorProduct[BIT01, BIT23]; (* (0, 1), (2, 3) Entangled *)
    {BIT01, BIT23, BIT0123}
];

(* Computes GHZ Pairs (0,1,2) and (3,4,5) entangled*)
ComputeGHZPairs012345Entangled[] := Module[{BIT012, BIT345, BIT012345},
    BIT012 =  BIT345 = ComputeGHZPairs[];
    BIT012345 = TensorProduct[BIT012, BIT345]; (* (0, 1, 2), (3, 4, 5) Entangled *)
    {BIT012, BIT345, BIT012345}
];

(* Computes Werner Pair (0,1,2) and GHZ Pair (3,4,5) entangled*)
ComputeWernerGHZPairs012345Entangled[] := Module[{BIT012, BIT345, BIT012345},
    BIT012 = ComputeWernerPairs[];
    BIT345 = ComputeGHZPairs[];
    BIT012345 = TensorProduct[BIT012, BIT345]; (* (0, 1, 2), (3, 4, 5) Entangled *)
    {BIT012, BIT345, BIT012345}
];

(* Computes Werner Pair (0,1,2) and EPR Pair (3,4) entangled*)
ComputeWernerEPRPairs01234Entangled[] := Module[{BIT012, BIT34, BIT01234},
    BIT012 = ComputeWernerPairs[];
    BIT34 = ComputeEPRPairs[];
    BIT01234 = TensorProduct[BIT012, BIT34]; (* (0, 1, 2), (3, 4) Entangled *)
    {BIT012, BIT34, BIT01234}
];
```

```
(* ComputeBellProjectors: Computes the Projectors *)
(* Inputs ⇒ V: Bell Basis containing orthonormal eignevectors, n: Dimension of the EigenSpace *)
(* Output ⇒ n Projectors *)
ComputeBellProjectors[V_, n_] := Module[{P, pVerify, oVerify, i, j, ZeroMatrix, ket, braw},
   pVerify = oVerify = True;
   P = Table[0, {i, n}, {j, 1}];
   Do[ket = V[[i]]ᵀ; braw = ket†; P[[i]] = ket.braw, {i, n}];
   ZeroMatrix = Table[0, {i, n}, {j, n}];
   Do[Do[If[P[[i]].P[[j]] != ZeroMatrix and i != j, oVerify = False,], {i, n}], {j, n}]; (*Verify Kronecker*)
   {P, pVerify}
];

(* ComputeBellProbStates: Computes the Probabilities and the States *)
(* Inputs ⇒ P: n Projectors, Ψ: Quantum System, n: Dimension of the EigenSpace *)
(* Output ⇒ n Probabilities and the States *)
ComputeBellProbStates[P_, Ψ_, n_, I1_, I2_] := Module[{prob, state, Proj, i},
    Proj = Table[TensorProduct[I1, P[[i]], I2], {i, n}];
    prob = Table[Expand[Ψ†.Proj[[i]].Ψ][[1]][[1]], {i, n}]; (*Probabilities*)
    state = Table[Map[Simplify, Transpose[{Normalize[Transpose[Proj[[i]].Ψ][[1]]]}]], {i, n}]; (*States*)
    {prob, state}
];

(* ComputeBellProbStatesWithUnitaryTransform: Computes the Probabilities and the States after Unitary Transformation *)
(* Inputs ⇒ P: n Projectors, Ψ: Quantum System, n: Dimension of the EigenSpace, U: the Unitary Matrix *)
(* Output ⇒ n Probabilities and the States *)
ComputeBellProbStatesWithUnitaryTransform[P_, Ψ_, n_, I1_, I2_, U_] := Module[{prob, state, Proj, i},
   Proj = Table[TensorProduct[I1, P[[i]], I2], {i, n}];
   prob = Table[Expand[Ψ†.Proj[[i]].Ψ][[1]][[1]], {i, n}]; (*Probabilities*)
   state = Table[Map[Simplify, Transpose[{Normalize[Uᵀ.Transpose[Proj[[i]].U.Ψ][[1]]]}]], {i, n}]; (*States*)
   {prob, state}
];

(* GetPermutationMatrix: Computes the Permutation Matrix given the permutations *)
(* Inputs ⇒ p: the permutation Π, n: number of elements in the permutation *)
(* e.g., for the Π = (0) (1 2 4 8) (5 10) (6 12 9 3) (7 14 13 11) (15), input {0 2 4 6 8 10 12 14 1 3 5 7 9 11 13 15} *)
(* Output ⇒ the Permutation Matrix *)
GetPermutationMatrix[p_, n_] := Module[{permMat, i, j},
   permMat = Table[0, {i, n}, {j, n}];
   Do[permMat[[i]][[p[[i]]]] = 1, {i, n}];
   permMat
];

(* GetPermutationMatrix: Computes the Permutation Matrix given the permutations *)
(* Inputs ⇒ n: number of elements in the permutation, r: number of bits to shit *)
(* Output ⇒ the Permutation Matrix *)
GetPermutationMatrixByLeftRotation[n_, r_] := Module[{permMat, i, j, f, s, max},
   max = 2^n - 1;
   permMat = Table[0, {i, max + 1}, {j, max + 1}];
   Do[f = BitAnd[BitShiftLeft[i, r], max]; s = BitShiftRight[i, n - r]; permMat[[i + 1]][[f + s + 1]] = 1, {i, 0, max}];
   permMat
];
```

```
(* ShowEntangledOutputTables: Shows the Output Tables *)
(* Inputs ⇒ V: Bell basis orthonormal eigmevectors, P: Projectors, Ψ: The EPR pairs,
           prob: Probabilities, state: States, n: Dimension of the EigenSpace *)
(* Output ⇒ None *)
ShowEntangledOutputTables[question_, Ψ1_, Ψ2_, Ψ_, V_, P_, prob_, state_, n_, I1_, I2_, pVerify_, col1_, col2_, col3_, col4_, col5_, si_, st_, sv_, so_, sp_] :=
 Module[{inputTable, verifyTable, outputTable, projTable, permTable, permMat, i, j, k, statestring1, statestring2, statestring},
  {statestring1, statestring2, statestring} = Switch[question, 1, {"|EPR⟩01", "|EPR⟩23", "|EPR⟩01⊗|EPR⟩23"},
                                               2, {"|GHZ⟩012", "|GHZ⟩345", "|GHZ⟩012⊗|GHZ⟩345"},
                                               3, {"|GHZ⟩012", "|GHZ⟩345", "|GHZ⟩012⊗|GHZ⟩345"},
                                               4, {"|Werner⟩012", "|GHZ⟩345", "|Werner⟩01⊗|GHZ⟩345"},
                                               5, {"|Werner⟩012", "|EPR⟩34", "|Werner⟩012⊗|EPR⟩34"}
                                        ];


  inputTable = Table[If[i == 0, Switch[j, 1, statestring1 <> ToString[Dimensions[Ψ1]],
                          2, statestring2 <> ToString[Dimensions[Ψ2]],
                          3, statestring <> ToString[Dimensions[Ψ]]],
                         Switch[j, 1, MatrixForm[Ψ1], 2, MatrixForm[Ψ2], 3, MatrixForm[Ψ]]],
         {i, 0, 1}, {j, 3}];
  projTable = Table[If[i == 0, Switch[j, 1, "", 2, "Projectors"],
                      Switch[j, 1, i, 2, MatrixForm[TensorProduct[I1, P[[i]], I2]]]],
         {i, 0, n}, {j, 2}];
  verifyTable = Table[If[k == 1, Switch[j, 1, "∑P = I", 2, "Pi.Pi = Pi", 3, "∑p = 1", 4, "Pi.Pj=0, i≠j"],
                           Switch[j, 1, Sum[P[[i]], {i, n}] == IdentityMatrix[Dimensions[P][[2]]],
                                  2, pVerify,
                                  3, If[Sum[prob[[i]], {i, n}] == 1, True, False],
                                  4, True]],
         {k, 2}, {j, 4}];

  outputTable = Table[If[i == 0, Switch[j, 1, "Bell Basis", 2, "Bell Projectors", 3, "I(x)Projector(x)I", 4, "Probability", 5, "State"],
                           Switch[j, 1, MatrixForm[V[[i]]ᵀ],
                             2, MatrixForm[P[[i]]],
                             3, MatrixForm[TensorProduct[I1, P[[i]], I2]],
                             4, prob[[i]],
                             5, MatrixForm[state[[i]]]]],
         {i, 0, n}, {j, 5}];
```

```
(* Check by swapping the entangled bits by permutation matrix *)
(* Entanglement swapping permutation: i.e., (0) (1 2 4 8) (5 10) (6 12 9 3) (7 14 13 11) (15) *)
permMat = Switch[question,
            1, GetPermutationMatrixByLeftRotation[4, 1],
            (* permMat = GetPermutationMatrix[{1,3, 5, 7, 9, 11, 13, 15, 2, 4, 6, 8, 10, 12, 14, 16 },16]; *)
            3, GetPermutationMatrixByLeftRotation[6, 3]
           ];
permTable = Switch[question,
                1, Table[If[i == 0,
                   Switch[j, 1, "|EPR⟩01|EPR⟩23",
                             2, "(0) (1 2 4 8) (5 10) (6 12 9 3) (7 14 13 11) (15)",
                             3, "|EPR⟩12|EPR⟩30"],
                   Switch[j, 1, MatrixForm[Ψ], 2, MatrixForm[permMat], 3, MatrixForm[permMat.Ψ]]],
                   {i, 0, 1}, {j, 3}],
                3, Table[If[i == 0,
                   Switch[j, 1, "|GHZ⟩012⊗|GHZ⟩345",
                             2, "(0 1 2 3 4 5) -> (3 4 5 0 1 2)",
                             3, "P(012345) -> (345012) . |GHZ⟩012⊗|GHZ⟩345"],
                  Switch[j, 1, MatrixForm[Ψ], 2, MatrixForm[permMat], 3, MatrixForm[permMat.Ψ]]],
                  {i, 0, 1}, {j, 3}]
            ];
(* Show Outputs *)
Grid[inputTable, Alignment -> Center, Spacings -> {si, 1}, Frame -> All, ItemStyle -> "Text", Background -> {{None, None}, {col1, None}}]
Grid[projTable, Alignment -> Center, Spacings -> {st, 1}, Frame -> All, ItemStyle -> "Text", Background -> {{None, None}, {col2, None}}]
Grid[verifyTable, Alignment -> Center, Spacings -> {sv, 1}, Frame -> All, ItemStyle -> "Text", Background -> {{None, None}, {col3, None}}]
Grid[outputTable, Alignment -> Center, Spacings -> {so, 1}, Frame -> All, ItemStyle -> "Text", Background -> {{None, None}, {col4, None}}]
If[question == 1 || question == 3, Grid[permTable, Alignment -> Center, Spacings -> {sp, 1}, Frame -> All, ItemStyle -> "Text", Background -> {{None, None}, {col5, None}}] ,]
];
```

```
(* Measure the Entangled System *)
MeasureEntangledSystemWithBellBasis[question_, si_:12, st_:10, sv_:10, so_:6, sp_:6] :=
      Module[{pair1, pair2, pairs, I1dim, I2dim, V, P, prob, state, pVerify},
   (* Compute Pairs *)
   {pair1, pair2, pairs} = Switch[question, 1, ComputeEPRPairs0123Entangled[],
                                  2, ComputeGHZPairs012345Entangled[],
                                  3, ComputeGHZPairs012345Entangled[],
                                  4, ComputeWernerGHZPairs012345Entangled[],
                                  5, ComputeWernerEPRPairs01234Entangled[]];
   {I1dim, I2dim} = Switch[question, 1, {2, 2}, 2, {4, 4}, 3, {4, 4}, 4, {4, 4}, 5, {4, 2}];

   (* Compute Bell Basis *)
   V = ComputeBellBasis[];
   (* Compute Projectors *)
   {P, pVerify} = ComputeBellProjectors[V, 4];
   (* Compute Probabilities and States *)
   {prob, state} = ComputeBellProbStates[P, pairs, 4, IdentityMatrix[I1dim], IdentityMatrix[I2dim]];

   (* Shows Output Tables *)
   ShowEntangledOutputTables[question, pair1, pair2, pairs, V, P, prob, state, 4,
    IdentityMatrix[I1dim], IdentityMatrix[I2dim], pVerify, Red, Blue, Green, Yellow, Pink, si, st, sv, so, sp]
];

(* Question 1 *)
MeasureEntangledSystemWithBellBasis[1]

(* Question 2 *)
MeasureEntangledSystemWithBellBasis[2]

(* Question 3 *)
MeasureEntangledSystemWithBellBasis[3]

(* Question 4 *)
MeasureEntangledSystemWithBellBasis[4]

(* Question 5 *)
MeasureEntangledSystemWithBellBasis[5]
(**)

End[]

EndPackage[]
```

Output

(1)

$\vert EPR\rangle_{01}$ {4, 1}	$\vert EPR\rangle_{23}$ {4, 1}	$\vert EPR\rangle_{01} \vert EPR\rangle_{23}$ {16, 1}
$\begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \\ 0 \end{pmatrix}$	$\begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \\ 0 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{2} \\ -\frac{1}{2} \\ 0 \\ 0 \\ -\frac{1}{2} \\ \frac{1}{2} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$

$\vert EPR\rangle_{01} \vert EPR\rangle_{23}$	(0) (1 2 4 8) (5 10) (6 12 9 3) (7 14 13 11) (15)	$\vert EPR\rangle_{03} \vert EPR\rangle_{12}$
$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{2} \\ -\frac{1}{2} \\ 0 \\ 0 \\ -\frac{1}{2} \\ \frac{1}{2} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0 \\ 0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ 0 \\ -\frac{1}{2} \\ 0 \\ \frac{1}{2} \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{2} \\ 0 \\ -\frac{1}{2} \\ 0 \\ 0 \\ 0 \end{pmatrix}$

$\sum P = I$	$P_i . P_i = P_i$	$\sum p = 1$	$P_i . P_j = 0,\ i \neq j$
True	True	True	True

	Projectors
1	$\begin{pmatrix} \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} \end{pmatrix}$
2	$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$
3	$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & -\frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & -\frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$
4	$\begin{pmatrix} \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} \end{pmatrix}$

Bell Basis	Bell Projectors	I(x)Projector(x)I	Probability	State
$\begin{pmatrix} \frac{1}{\sqrt{2}} \\ 0 \\ 0 \\ \frac{1}{\sqrt{2}} \end{pmatrix}$	$\begin{pmatrix} \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & \frac{1}{2} \end{pmatrix}$	$\begin{pmatrix} \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} \end{pmatrix}$	$\frac{1}{4}$	$\begin{pmatrix} -\frac{1}{2} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -\frac{1}{2} \\ 0 \\ 0 \\ -\frac{1}{2} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -\frac{1}{2} \end{pmatrix}$
$\begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$	$\frac{1}{4}$	$\begin{pmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{2} \\ 0 \\ \frac{1}{2} \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{2} \\ 0 \\ \frac{1}{2} \\ 0 \\ 0 \\ 0 \end{pmatrix}$
$\begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \\ 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & -\frac{1}{2} & 0 \\ 0 & -\frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & -\frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & -\frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$	$\frac{1}{4}$	$\begin{pmatrix} 0 \\ 0 \\ 0 \\ -\frac{1}{2} \\ 0 \\ \frac{1}{2} \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{2} \\ 0 \\ -\frac{1}{2} \\ 0 \\ 0 \\ 0 \end{pmatrix}$
$\begin{pmatrix} \frac{1}{\sqrt{2}} \\ 0 \\ 0 \\ -\frac{1}{\sqrt{2}} \end{pmatrix}$	$\begin{pmatrix} \frac{1}{2} & 0 & 0 & -\frac{1}{2} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -\frac{1}{2} & 0 & 0 & \frac{1}{2} \end{pmatrix}$	$\begin{pmatrix} \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} \end{pmatrix}$	$\frac{1}{4}$	$\begin{pmatrix} \frac{1}{2} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -\frac{1}{2} \\ 0 \\ 0 \\ -\frac{1}{2} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{2} \end{pmatrix}$

(2)

$\vert GHZ\rangle_{012}$ {8, 1}	$\vert GHZ\rangle_{345}$ {8, 1}	$\vert GHZ\rangle_{012}\vert GHZ\rangle_{345}$ {64, 1}
$\begin{pmatrix} \frac{1}{\sqrt{2}} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{\sqrt{2}} \end{pmatrix}$	$\begin{pmatrix} \frac{1}{\sqrt{2}} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{\sqrt{2}} \end{pmatrix}$	$\begin{pmatrix} \frac{1}{2} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{2} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{2} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{2} \end{pmatrix}$

$\sum P = I$	$P_i.P_i = P_i$	$\sum p = 1$	$P_i.P_j=0,\ i\neq j$
True	True	True	True

Bell Basis	Bell Projectors	I⊗Projector⊗I	Probability	State
$\begin{pmatrix} \frac{1}{\sqrt{2}} \\ 0 \\ 0 \\ \frac{1}{\sqrt{2}} \end{pmatrix}$	$\begin{pmatrix} \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & \frac{1}{2} \end{pmatrix}$	[illegible]	$\frac{1}{4}$	$\begin{pmatrix} \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} \end{pmatrix}^T$
$\begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$	[illegible]	$\frac{1}{4}$	$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}^T$

$\begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \\ 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & -\frac{1}{2} & 0 \\ 0 & -\frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$	[illegible]	$\frac{1}{4}$	$(0, 0, 0, 0, 0, 0, 0, \frac{1}{2}, 0, 0, 0, -\frac{1}{2}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -\frac{1}{2}, 0, 0, 0, \frac{1}{2}, 0, 0, 0, 0, 0, 0, 0)$
$\begin{pmatrix} \frac{1}{\sqrt{2}} \\ 0 \\ 0 \\ -\frac{1}{\sqrt{2}} \end{pmatrix}$	$\begin{pmatrix} \frac{1}{2} & 0 & 0 & -\frac{1}{2} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -\frac{1}{2} & 0 & 0 & \frac{1}{2} \end{pmatrix}$	[illegible]	$\frac{1}{4}$	$(\frac{1}{2}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -\frac{1}{2}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -\frac{1}{2}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, \frac{1}{2})$

(3)
(Applying the permutation matrix to measure bits 0 & 5)

$\vert GHZ\rangle_{012} \otimes \vert GHZ\rangle_{345}$	$\{0\ 1\ 2\ 3\ 4\ 5\} \to \{3\ 4\ 5\ 0\ 1\ 2\}$	$P_{(012345) \to (345012)} \cdot \vert GHZ\rangle_{012} \otimes \vert GHZ\rangle_{345}$
$\begin{pmatrix} \frac{1}{2} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{2} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{2} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{2} \end{pmatrix}$	$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 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0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} \frac{1}{2} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{2} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{2} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{2} \end{pmatrix}$

As we can see, the state does not change even after applying the unitary matrix.
The other outputs are similar to that of (2).

(4) (Output sampled)

$\lvert\mathrm{Werner}\rangle_{012}\{8,\ 1\}$	$\lvert\mathrm{GHZ}\rangle_{345}\{8,\ 1\}$	$\lvert\mathrm{Werner}\rangle_{012}\lvert\mathrm{GHZ}\rangle_{345}\{64,\ 1\}$
$\begin{pmatrix} 0 \\ \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \\ 0 \\ 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} \frac{1}{\sqrt{2}} \\ 0 \\ 0 \\ 0 \\ \frac{1}{\sqrt{2}} \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{\sqrt{6}} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{6}} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{\sqrt{6}} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{\sqrt{6}} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{\sqrt{6}} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$

$\sum P = I$	$P_i . P_i = P_i$	$\sum p = 1$	$P_i . P_j = 0,\ i \neq j$
True	True	True	True

Bell Basis	Bell Projectors	I (x) Projector (x) I	Probability	State
$\begin{pmatrix} \frac{1}{\sqrt{2}} \\ 0 \\ 0 \\ \frac{1}{\sqrt{2}} \end{pmatrix}$	$\begin{pmatrix} \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & \frac{1}{2} \end{pmatrix}$	$(\ll 1 \gg)$	$\frac{1}{4}$	$\begin{pmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{\sqrt{6}} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{6}} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{\sqrt{6}} \\ 0 \\ 0 \\ 0 \\ \frac{1}{\sqrt{6}} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{\sqrt{6}} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$

$\begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$	$(\ll 1 \gg)$	$\frac{1}{4}$	$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{\sqrt{6}} \\ 0 \\ 0 \\ 0 \\ \frac{1}{\sqrt{6}} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{\sqrt{6}} \\ 0 \\ 0 \\ 0 \\ \frac{1}{\sqrt{6}} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{\sqrt{6}} \\ 0 \\ 0 \\ 0 \\ \frac{1}{\sqrt{6}} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$

$\begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \\ 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & -\frac{1}{2} & 0 \\ 0 & -\frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$	(≪1≫)	$\frac{1}{4}$	$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -\frac{1}{\sqrt{6}} \\ 0 \\ 0 \\ 0 \\ \frac{1}{\sqrt{6}} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{\sqrt{6}} \\ 0 \\ 0 \\ 0 \\ -\frac{1}{\sqrt{6}} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{\sqrt{6}} \\ 0 \\ 0 \\ 0 \\ -\frac{1}{\sqrt{6}} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$

$\begin{pmatrix} \frac{1}{\sqrt{2}} \\ 0 \\ 0 \\ -\frac{1}{\sqrt{2}} \end{pmatrix}$	$\begin{pmatrix} \frac{1}{2} & 0 & 0 & -\frac{1}{2} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -\frac{1}{2} & 0 & 0 & \frac{1}{2} \end{pmatrix}$	$(\ll 1 \gg)$	$\frac{1}{4}$	$\begin{pmatrix} 0 \\ 0 \\ 0 \\ -\frac{1}{\sqrt{6}} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{6}} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -\frac{1}{\sqrt{6}} \\ 0 \\ 0 \\ 0 \\ \frac{1}{\sqrt{6}} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -\frac{1}{\sqrt{6}} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$

(5) (Output Sampled)

$\vert Werner\rangle_{C12}\{8,\ 1\}$	$\vert EPR\rangle_{34}\{4,\ 1\}$	$\vert Werner\rangle_{C12}\vert EPR\rangle_{34}\{32,\ 1\}$
$\begin{pmatrix} 0 \\ \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \\ 0 \\ \frac{1}{\sqrt{3}} \\ 0 \\ 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \\ 0 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{\sqrt{6}} \\ -\frac{1}{\sqrt{6}} \\ 0 \\ 0 \\ \frac{1}{\sqrt{6}} \\ -\frac{1}{\sqrt{6}} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{\sqrt{6}} \\ -\frac{1}{\sqrt{6}} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$

$\sum P = I$	$P_i . P_i = P_i$	$\sum p = 1$	$P_i . P_j = 0,\ i \neq j$
True	True	True	True

Bell Basis	Bell Projectors	I(x)Projector(x)I	Probability	State
$\begin{pmatrix}\frac{1}{\sqrt{2}}\\0\\0\\\frac{1}{\sqrt{2}}\end{pmatrix}$	$\begin{pmatrix}\frac{1}{2}&0&0&\frac{1}{2}\\0&0&0&0\\0&0&0&0\\\frac{1}{2}&0&0&\frac{1}{2}\end{pmatrix}$	$\begin{pmatrix}\frac{1}{2}&0&0&0&0&0&\frac{1}{2}&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\0&\frac{1}{2}&0&0&0&0&0&\frac{1}{2}&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\\frac{1}{2}&0&0&0&0&0&\frac{1}{2}&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\0&\frac{1}{2}&0&0&0&0&0&\frac{1}{2}&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&\frac{1}{2}&0&0&0&0&0&\frac{1}{2}&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&\frac{1}{2}&0&0&0&0&0&\frac{1}{2}&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&\frac{1}{2}&0&0&0&0&0&\frac{1}{2}&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&\frac{1}{2}&0&0&0&0&0&\frac{1}{2}&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&\frac{1}{2}&0&0&0&0&0&\frac{1}{2}&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&\frac{1}{2}&0&0&0&0&0&\frac{1}{2}&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&\frac{1}{2}&0&0&0&0&0&\frac{1}{2}&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&\frac{1}{2}&0&0&0&0&0&\frac{1}{2}&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&\frac{1}{2}&0&0&0&0&0&\frac{1}{2}&0\\0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&\frac{1}{2}&0&0&0&0&0&\frac{1}{2}\\0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&\frac{1}{2}&0&0&0&0&0&\frac{1}{2}&0\\0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&\frac{1}{2}&0&0&0&0&0&\frac{1}{2}\end{pmatrix}$	$\frac{1}{4}$	$\begin{pmatrix}-\frac{1}{\sqrt{8}}\\0\\0\\0\\0\\0\\-\frac{1}{\sqrt{8}}\\0\\0\\\frac{1}{\sqrt{8}}\\0\\0\\0\\0\\0\\\frac{1}{\sqrt{8}}\\0\\\frac{1}{\sqrt{8}}\\0\\0\\0\\0\\0\\\frac{1}{\sqrt{8}}\\0\\0\\0\\0\\0\\0\\0\\0\end{pmatrix}$
$\begin{pmatrix}0\\\frac{1}{\sqrt{2}}\\\frac{1}{\sqrt{2}}\\0\end{pmatrix}$	$\begin{pmatrix}0&0&0&0\\0&\frac{1}{2}&\frac{1}{2}&0\\0&\frac{1}{2}&\frac{1}{2}&0\\0&0&0&0\end{pmatrix}$	$\begin{pmatrix}0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\0&0&\frac{1}{2}&0&\frac{1}{2}&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\0&0&0&\frac{1}{2}&0&\frac{1}{2}&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\0&0&\frac{1}{2}&0&\frac{1}{2}&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\0&0&0&\frac{1}{2}&0&\frac{1}{2}&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0&\frac{1}{2}&0&\frac{1}{2}&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0&0&\frac{1}{2}&0&\frac{1}{2}&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0&\frac{1}{2}&0&\frac{1}{2}&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0&0&\frac{1}{2}&0&\frac{1}{2}&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&\frac{1}{2}&0&\frac{1}{2}&0&0&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&\frac{1}{2}&0&\frac{1}{2}&0&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&\frac{1}{2}&0&\frac{1}{2}&0&0&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&\frac{1}{2}&0&\frac{1}{2}&0&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&\frac{1}{2}&0&\frac{1}{2}&0&0&0\\0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&\frac{1}{2}&0&\frac{1}{2}&0&0\\0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&\frac{1}{2}&0&\frac{1}{2}&0&0&0\\0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&\frac{1}{2}&0&\frac{1}{2}&0&0\\0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\end{pmatrix}$	$\frac{1}{4}$	$\begin{pmatrix}0\\0\\0\\\frac{1}{\sqrt{8}}\\0\\\frac{1}{\sqrt{8}}\\0\\0\\0\\0\\-\frac{1}{\sqrt{8}}\\0\\-\frac{1}{\sqrt{8}}\\0\\0\\0\\0\\0\\-\frac{1}{\sqrt{8}}\\0\\-\frac{1}{\sqrt{8}}\\0\\0\\0\\0\\0\\0\\0\\0\\0\\0\\0\end{pmatrix}$

$\begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \\ 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & -\frac{1}{2} & 0 \\ 0 & -\frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&\frac{1}{2}&0&-\frac{1}{2}&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&\frac{1}{2}&0&-\frac{1}{2}&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&-\frac{1}{2}&0&\frac{1}{2}&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&-\frac{1}{2}&0&\frac{1}{2}&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&\frac{1}{2}&0&-\frac{1}{2}&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&\frac{1}{2}&0&-\frac{1}{2}&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&-\frac{1}{2}&0&\frac{1}{2}&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&-\frac{1}{2}&0&\frac{1}{2}&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&\frac{1}{2}&0&-\frac{1}{2}&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&\frac{1}{2}&0&-\frac{1}{2}&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&-\frac{1}{2}&0&\frac{1}{2}&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&-\frac{1}{2}&0&\frac{1}{2}&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&\frac{1}{2}&0&-\frac{1}{2}&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&\frac{1}{2}&0&-\frac{1}{2}&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&-\frac{1}{2}&0&\frac{1}{2}&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&-\frac{1}{2}&0&\frac{1}{2}&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \end{pmatrix}$	$\frac{1}{4}$	$\begin{pmatrix} 0 \\ 0 \\ 0 \\ -\frac{1}{\sqrt{8}} \\ 0 \\ \frac{1}{\sqrt{8}} \\ 0 \\ 0 \\ 0 \\ 0 \\ -\frac{1}{\sqrt{8}} \\ 0 \\ \frac{1}{\sqrt{8}} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -\frac{1}{\sqrt{8}} \\ 0 \\ \frac{1}{\sqrt{8}} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$
$\begin{pmatrix} \frac{1}{\sqrt{2}} \\ 0 \\ 0 \\ -\frac{1}{\sqrt{2}} \end{pmatrix}$	$\begin{pmatrix} \frac{1}{2} & 0 & 0 & -\frac{1}{2} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -\frac{1}{2} & 0 & 0 & \frac{1}{2} \end{pmatrix}$	$\begin{pmatrix} \frac{1}{2}&0&0&0&0&0&-\frac{1}{2}&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&\frac{1}{2}&0&0&0&0&0&-\frac{1}{2}&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ -\frac{1}{2}&0&0&0&0&0&\frac{1}{2}&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&-\frac{1}{2}&0&0&0&0&0&\frac{1}{2}&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&\frac{1}{2}&0&0&0&0&0&-\frac{1}{2}&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&\frac{1}{2}&0&0&0&0&0&-\frac{1}{2}&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&-\frac{1}{2}&0&0&0&0&0&\frac{1}{2}&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&-\frac{1}{2}&0&0&0&0&0&\frac{1}{2}&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&\frac{1}{2}&0&0&0&0&0&-\frac{1}{2}&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&\frac{1}{2}&0&0&0&0&0&-\frac{1}{2}&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&-\frac{1}{2}&0&0&0&0&0&\frac{1}{2}&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&-\frac{1}{2}&0&0&0&0&0&\frac{1}{2}&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&\frac{1}{2}&0&0&0&0&0&-\frac{1}{2}&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&\frac{1}{2}&0&0&0&0&0&-\frac{1}{2} \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&-\frac{1}{2}&0&0&0&0&0&\frac{1}{2}&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&-\frac{1}{2}&0&0&0&0&0&\frac{1}{2} \end{pmatrix}$	$\frac{1}{4}$	$\begin{pmatrix} \frac{1}{\sqrt{8}} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -\frac{1}{\sqrt{8}} \\ 0 \\ 0 \\ \frac{1}{\sqrt{8}} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -\frac{1}{\sqrt{8}} \\ 0 \\ \frac{1}{\sqrt{8}} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -\frac{1}{\sqrt{8}} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$

References

1. Noson S. Yanofsky and Micro A. Mannucci, Quantum Computing for Computer Scientists.
2. Nielsen, Chuang, Quantum Computation and Quantum Information.
3. Samuel J. Lomonaco, Jr.,Quantum Computation, A Grand Mathematical Challenge for the Twenty-First Century and the Millennium.
4. Samuel J. Lomonaco, Quantum Computation and Quantum Information, A Millennium Volume.
5. QuantWiki.