UMBC™

AN HONORS UNIVERSITY IN MARYLAND

Blue Book

Student's Name Sandipan Dey

Date ____________ Subject CMSC 651

Professor's name: Dr Richard Chang

Comments:

18 + 12 + 16 = 46/60

Integrity:
A Value That Endures

"By enrolling in this course, each student assumes the responsibilities of an active participant in UMBC's scholarly community in which everyone's academic work and behavior are held to the highest standards of honesty. Cheating, fabrication, plagiarism, and helping others to commit these acts are all forms of academic dishonesty, and they are wrong. Academic misconduct could result in disciplinary action that may include, but is not limited to, suspension or dismissal. To read the full Student Academic Conduct Policy, consult the UMBC Student Handbook, the Faculty Handbook, or the UMBC Policies section of the UMBC Directory."

UMBC Faculty Senate
February 13, 2001

18/20

1. (a) $f(n) = \begin{cases} n, & n \equiv 1 \pmod 2 \\ n^3, & n \equiv 0 \pmod 2 \end{cases}$

Clearly, $f(n) = O(n^3)$, ~~Hence~~ and $f(n) = \Omega(n)$

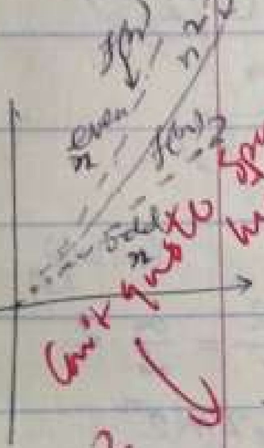
~~Hence~~ $DSPACE[n^2]$

$= \{ L \mid L$ is a language ~~accepted~~ decided by a deterministic TM in space $O(n^2) \}$

$DSPACE[f(n)]$

$= \{ L \mid L$ is a language ~~accepted~~ decided by a DTM in space $O(f(n)) \}$

$= \{ L \mid L$ is a language ~~accepted~~ decided by a DTM in space $O(n^3) \}$



Can't quote the space hierarchy theorem in this direction.

-2

Since $f(n) \neq O(n^2)$ and ~~by sp~~ $n^2 = o(n^3)$, by space hierarchy theorem $\exists$ a language A that can be decided in $n^3 = O(n^3)$ space but not in $n^2 = o(n^3)$ space, hence $DSPACE[n^2] \subsetneq DSPACE[f(n)]$ since $\exists A \in DSPACE[f(n)] - DSPACE[n^2]$ by space hierarchy theorem.

not the case that $DSPACE[n^2] \subseteq DSPACE[f(n)]$

b. $DTIME[2^n]$

$= \{ L \mid L$ is a language ~~accepted~~ decided by a DTM in $O(2^n)$ time $\}$

$DTIME[3^n]$

$= \{ L \mid L$ is a language decided by a DTM in $O(3^n)$ time $\}$

but $2^n = o(3^n)$, since $\lim_{n\to\infty} \frac{2^n}{3^n} = 0$

and by time hierarchy theorem,

$\exists$ a language A that can be ~~accepted~~ decided in $O(3^n)$

but can't be ~~accepted~~ in $o\left(\frac{3^n}{\lg 3^n}\right) = o\left(\frac{3^n}{n \lg 3}\right)$

$= o\left(\frac{3^n}{n}\right)$ space.

But again, $2^n = o\left(\frac{3^n}{n}\right)$ since,

$L = \lim_{n\to\infty} \frac{n.2^n}{3^n} = \frac{\infty}{\infty}$ (L'Hospital)

$= \lim_{n\to\infty} \frac{n.2^n \lg_e 2 + 2^n}{3^n \log_e 3}$

$= \frac{\ln 2}{\ln 3} . \lim_{n\to\infty} \frac{2^n}{3^n}$

$= \frac{\ln 2}{\ln 3} . L$

$\Rightarrow L\left(1 - \frac{\ln 2}{\ln 3}\right) = 0$

$\Rightarrow L = 0$

hence $\exists$ language $A \in DTIME(3^n) - DTIME(2^n)$.

$\Rightarrow DTIME(2^n) \subsetneq DTIME(3^n)$

(c) By Savitch's theorem,

$$NSPACE[2^n] = DSPACE[(2^n)^2] = DSPACE[4^n].$$

Also, by Space hierarchy theorem,

$$DSPACE[4^n] \subsetneq DSPACE[5^n]$$

$$\Rightarrow NSPACE[2^n] \subsetneq DSPACE[5^n].$$

not =

(d) $DSPACE[n] = DTIME[2^n] \subsetneq DTIME[(\log n)^n]$.

by time hierarchy theorem.

3. We know PATH is NL complete.

16/20 To show that DAG-PATH is NL-Complete,

we need to show: (a) DAG-PATH $\in$ NL

(b) PATH $\leq^m_{\log}$ DAG-PATH.

(a) Construct a ~~[illegible] logspace [illegible] nondeter~~ nondeterministic logspace turing machine M that decides DAG-PATH in the following manner:

First the ~~machine~~ NTM M_P decides directed path from s to t in logspace

$M_P(\langle G, s, t\rangle)$

~~1. Start by marking s on its work tape~~ as the current vertex ($u = s$)

~~2. Repeat until no new node~~ a path starting from s, t

1.1. Non deterministically guess the next vertex on the chosen path v, s.t. $(u, v) \in E[G]$. Each time only store the current node in the worktape w/

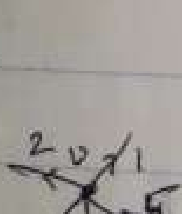
2. if t is ~~marked~~ on the path then accept, o/w reject.

Next the NTM M_A decides whether G is acylic in logspace (~~same as~~ easier than UCYCLE which is in logspace)

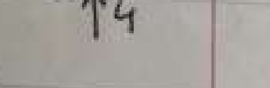
but that's undirected

$M_A(\langle G\rangle)$

1. For each vertex $v \in V[G]$ do the following:



1.1. Order the edges cyclically that are ~~incident on~~ incident on v. ~~Choose the~~ Store v ~~and~~ on its worktape and mark v. Store the current vertex c on the worktape as well (initially $c \leftarrow v$)



Worktape

1.2. ~~repeat the following~~ Non deterministically guess a path starting from v. Everytime find the next node on the path from input from E[G] and store the current node on worktape as c

1.3. if the current node $c = v$ then reject (must be a directed cycle) o/w accept

what about wrong guesses? Use Immerman

The NTM $M(\langle G, s, t\rangle)$

1. Run $M_P(\langle G,s,t\rangle)$ on $(\langle G,s,t\rangle)$
2. Run $M_A(\langle G\rangle)$ on $\langle G\rangle$.
3. If Both M_P & M_A accept, accept, o/w reject.

(b) PATH $\leq^{P}_{logm}$ DAG-PATH.

Consider the following function f computed by the turing machine F $\quad f: \langle G,s,t\rangle \rightarrow \langle G',s,t\rangle$

might mess up reachability.

$F(\langle G,s,t\rangle)$

1. Scan the edge set $E[G]$ from the input.

2. if both (u,v) and $(v,u) \in E[G]$ delete one and output $E[G']$ on the output tape ~~$\forall \vec{(u,v)} \in E[G]$, if $\vec{(v,u)} \notin E[G]$~~ add ~~to~~ $E[G']$ to get $E[G']$ [but Not (u,v)]

$G' = G$ must remove all the cycles

3. output $\langle G',s,t\rangle$

This required to store the current node on the worktape hence logspace reduction. Also, its easy to see that

$\langle G,s,t\rangle \in PATH$ iff $\langle G',s,t\rangle \in DAG\_PATH$

2. ~~INK[n] IN~~ SPACE[n] =

$INK[n] = TIME[n]$ Since can change symbols only n times not more.

12

$SPACE[n] = TIME[O(2^n)]$

not equal.