What have we learned?

Any natural and man-made phenomena exhibit a periodic behavior: after a certain amount of time, called the period, these phenomena repeat exactly. The inverse of the period, that is, the number of repetitions per unit of time, is called the frequency. Sines and cosines, simple oscillatory functions, are natural candidates as the basic building blocks to represent more complicated oscillatory signals. This is the basic goal of Fourier analysis: to decompose a signal in terms of sines and cosines.

We have two kinds of Fourier tools, Fourier analysis and Fourier synthesis. Fourier analysis allows determining the weight of each periodic basic building block in a given signal; with the analysis, we move from the time domain to the frequency domain. Fourier synthesis allows building signals with a "user-defined" frequency content: with this we move from the frequency domain to the time domain.

We have started our exploration of Fourier analysis with the simplest tool, the discrete Fourier transform (DFT) that applies to finite-length signals. If N is the length of the signals, we have seen that the set of complex exponentials

$$w_k[n] = e^{j\frac{2\pi}{N}nk}, n = 0, \ldots, N-1, k = 0, \ldots, N-1$$

forms an orthogonal basis of $\mathbb{C}^N$. The DFT is simply a change of basis in this vector space.