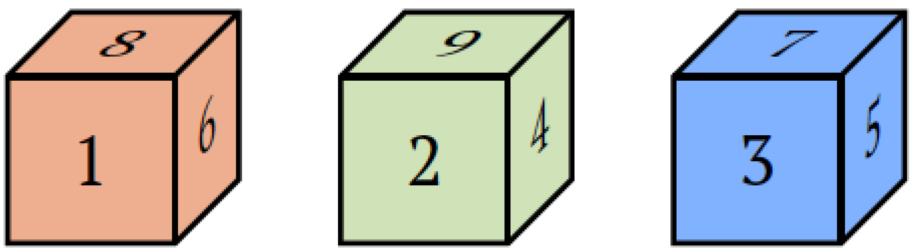
Dice Game Problem

Someone offers you to play the following game. There are several fair dice on the table with various numbers o their sides. You and your opponent pick one dice each. Both of you throw the dice and whoever has the larger number wins. You are going to play this game many times and you are, of course, interested in maximizing the probability of winning.

To give you an advantage, your opponent lets you pick your dice first: you pick a dice and then your opponent pics one of the remaining dice.

How would you select a dice?

Of course, everything depends on the specific dice that are on the table. Say, if there are several standard dice (with numbers $1, 2, 3, 4, 5, 6$ on their sides), then it does not matter which one to choose. But what if you see the dice shown in the picture below?



Here we have three dice: $[1, 1, 6, 6, 8, 8]$, $[2, 2, 4, 4, 9, 9]$, and $[3, 3, 5, 5, 7, 7]$.

Which one would you choose?

The expected value of each dice is equal to 5:

$$\frac{1+8+6}{3} = 5\,, \quad \frac{2+4+9}{3} = 5\,, \quad \frac{3+7+5}{3} = 5\,.$$

Does this mean that it does not matter which one to choose?

As we have discussed on previous week, comparing dices by their expected values is not a good idea. Instead, let us compute, for each pair of dice, which one of them beats the other one more often.

Let us start with the first two dice. The following picture shows all 36 outcomes.