Data Mining

Session 6 – Main Theme
Mining Frequent Patterns, Association, and Correlations

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Adapted from course textbook resources
Data Mining Concepts and Techniques (2nd Edition)
Jiawei Han and Micheline Kamber

Agenda

1. **Session Overview**
2. **Mining Frequent Patterns, Association, and Correlations**
3. **Summary and Conclusion**

What is the class about?

- Course description and syllabus:
  - » http://www.nyu.edu/classes/jcf/g22.3033-002/
  - » http://www.cs.nyu.edu/courses/spring10/G22.3033-002/index.html

- Textbooks:
  - » ***Data Mining: Concepts and Techniques (2nd Edition)***

Jiawei Han, Micheline Kamber
Morgan Kaufmann
ISBN-10: 1-55860-901-6, ISBN-13: 978-1-55860-901-3, (2006)

  - » ***Microsoft SQL Server 2008 Analysis Services Step by Step***

Scott Cameron
Microsoft Press
ISBN-10: 0-73562-620-0, ISBN-13: 978-0-73562-620-31 1st Edition (04/15/09)

Session Agenda

- Basic concepts and a roadmap
- Scalable frequent itemset mining methods
- Mining various kinds of association rules
- From association to correlation analysis
- Constraint-based association mining
- Mining colossal patterns
- Summary

Icons / Metaphors

 Information

 Common Realization

 Knowledge/Competency Pattern

 Governance

 Alignment

 Solution Approach

Agenda

1 Session Overview

2 Mining Frequent Patterns, Association, and Correlations

3 Summary and Conclusion

Mining Frequent Patterns, Association and Correlations – Sub-Topics

- Basic concepts and a road map
- Scalable frequent itemset mining methods
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What Is Frequent Pattern Analysis?

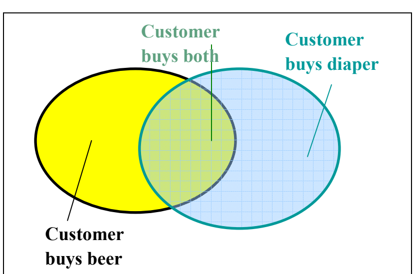
- Frequent pattern: a pattern (a set of items, subsequences, substructures, etc.) that occurs frequently in a data set
- First proposed by Agrawal, Imielinski, and Swami [AIS93] in the context of frequent itemsets and association rule mining
- Motivation: Finding inherent regularities in data
  - What products were often purchased together?— Beer and diapers?!
  - What are the subsequent purchases after buying a PC?
  - What kinds of DNA are sensitive to this new drug?
  - Can we automatically classify web documents?
- Applications
  - Basket data analysis, cross-marketing, catalog design, sale campaign analysis, Web log (click stream) analysis, and DNA sequence analysis.

Why Is Freq. Pattern Mining Important?

- Freq. pattern: An intrinsic and important property of datasets
- Foundation for many essential data mining tasks
  - Association, correlation, and causality analysis
  - Sequential, structural (e.g., sub-graph) patterns
  - Pattern analysis in spatiotemporal, multimedia, time-series, and stream data
  - Classification: discriminative, frequent pattern analysis
  - Cluster analysis: frequent pattern-based clustering
  - Data warehousing: iceberg cube and cube-gradient
  - Semantic data compression: fascicles
  - Broad applications

Basic Concepts: Frequent Patterns

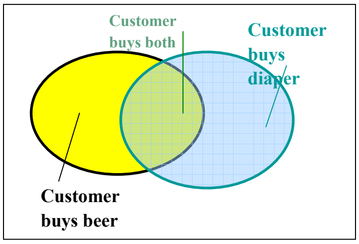
Tid	Items bought
10	Beer, Nuts, Diaper
20	Beer, Coffee, Diaper
30	Beer, Diaper, Eggs
40	Nuts, Eggs, Milk
50	Nuts, Coffee, Diaper, Eggs, Milk



- itemset: A set of one or more items
- k-itemset $X = \{x_1, \ldots, x_k\}$
- *(absolute) support*, or, *support count* of X: Frequency or occurrence of an itemset X
- *(relative) support*, *s*, is the fraction of transactions that contains X (i.e., the probability that a transaction contains X)
- An itemset X is *frequent* if X's support is no less than a *minsup* threshold

Basic Concepts: Association Rules

Tid	Items bought
10	Beer, Nuts, Diaper
20	Beer, Coffee, Diaper
30	Beer, Diaper, Eggs
40	Nuts, Eggs, Milk
50	Nuts, Coffee, Diaper, Eggs, Milk



- Find all the rules $X \rightarrow Y$ with minimum support and confidence
  - support, *s*, probability that a transaction contains $X \cup Y$
  - confidence, *c,* conditional probability that a transaction having X also contains *Y*
- *Let minsup = 50%, minconf = 50%*
- *Freq. Pat.:* Beer:3, Nuts:3, Diaper:4, Eggs:3, {Beer, Diaper}:3
- Association rules: (many more!)
  - *Beer* → *Diaper* (60%, 100%)
  - *Diaper* → *Beer* (60%, 75%)

Closed Patterns and Max-Patterns

- A long pattern contains a combinatorial number of sub-patterns, e.g., $\{a_1, \ldots, a_{100}\}$ contains $\binom{100}{1} + \binom{100}{2} + \ldots + \binom{100}{100} = 2^{100} - 1 = 1.27 \times 10^{30}$ sub-patterns!
- Solution: *Mine closed patterns and max-patterns instead*
- An itemset X is closed if X is *frequent* and there exists *no super-pattern* Y ɔ X, *with the same support* as X (proposed by Pasquier, et al. @ ICDT'99)
- An itemset X is a max-pattern if X is frequent and there exists no frequent super-pattern Y ɔ X (proposed by Bayardo @ SIGMOD'98)
- Closed pattern is a lossless compression of freq. patterns
  - Reducing the # of patterns and rules

Closed Patterns and Max-Patterns

- Exercise. DB = {<a_1, …, a_{100}>, < a_1, …, a_{50}>}
  - Min_sup = 1.
- What is the set of closed itemset?
  - <a_1, …, a_{100}>: 1
  - < a_1, …, a_{50}>: 2
- What is the set of max-pattern?
  - <a_1, …, a_{100}>: 1
- What is the set of all patterns?
  - !!

Computational Complexity of Frequent Itemset Mining

- How many itemsets are potentially to be generated in the worst case?
  - The number of frequent itemsets to be generated is senstive to the minsup threshold
  - When minsup is low, there exist potentially an exponential number of frequent itemsets
  - The worst case: M^N where M: # distinct items, and N: max length of transactions
- The worst case complexty vs. the expected probability
  - Ex. Suppose Walmart has 10^4 kinds of products
    - The chance to pick up one product 10^{-4}
    - The chance to pick up a particular set of 10 products: ~10^{-40}
    - What is the chance this particular set of 10 products to be frequent 10^3 times in 10^9 transactions?

Mining Frequent Patterns, Association and Correlations – Sub-Topics

- Basic concepts and a road map
- Scalable frequent itemset mining methods
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The Downward Closure Property and Scalable Mining Methods

- The downward closure property of frequent patterns
  - Any subset of a frequent itemset must be frequent
  - If **{beer, diaper, nuts}** is frequent, so is **{beer, diaper}**
  - i.e., every transaction having {beer, diaper, nuts} also contains {beer, diaper}
- Scalable mining methods: Three major approaches
  - Apriori (Agrawal & Srikant@VLDB'94)
  - Freq. pattern growth (FPgrowth—Han, Pei & Yin @SIGMOD'00)
  - Vertical data format approach (Charm—Zaki & Hsiao @SDM'02)

Apriori: A Candidate Generation & Test Approach

- Apriori pruning principle: If there is any itemset which is infrequent, its superset should not be generated/tested! (Agrawal & Srikant @VLDB'94, Mannila, et al. @ KDD' 94)
- Method:
  - Initially, scan DB once to get frequent 1-itemset
  - Generate length (k+1) candidate itemsets from length k frequent itemsets
  - Test the candidates against DB
  - Terminate when no frequent or candidate set can be generated

The Apriori Algorithm—An Example

$Sup_{min} = 2$

Database TDB

Tid	Items
10	A, C, D
20	B, C, E
30	A, B, C, E
40	B, E

1st scan → C_1

Itemset	sup
{A}	2
{B}	3
{C}	3
{D}	1
{E}	3

→ L_1

Itemset	sup
{A}	2
{B}	3
{C}	3
{E}	3

C_2

Itemset
{A, B}
{A, C}
{A, E}
{B, C}
{B, E}
{C, E}

2nd scan → C_2

Itemset	sup
{A, B}	1
{A, C}	2
{A, E}	1
{B, C}	2
{B, E}	3
{C, E}	2

→ L_2

Itemset	sup
{A, C}	2
{B, C}	2
{B, E}	3
{C, E}	2

C_3

Itemset
{B, C, E}

3rd scan → L_3

Itemset	sup
{B, C, E}	2

The Apriori Algorithm (Pseudo-Code)

C_k: Candidate itemset of size k
L_k : frequent itemset of size k

L_1 = {frequent items};
for (k = 1; L_k !=∅; k++) **do begin**
  C_{k+1} = candidates generated from L_k;
  for each transaction t in database do
    increment the count of all candidates in C_{k+1} that are contained in t
  L_{k+1} = candidates in C_{k+1} with min_support
  end
return $\cup_k L_k$;

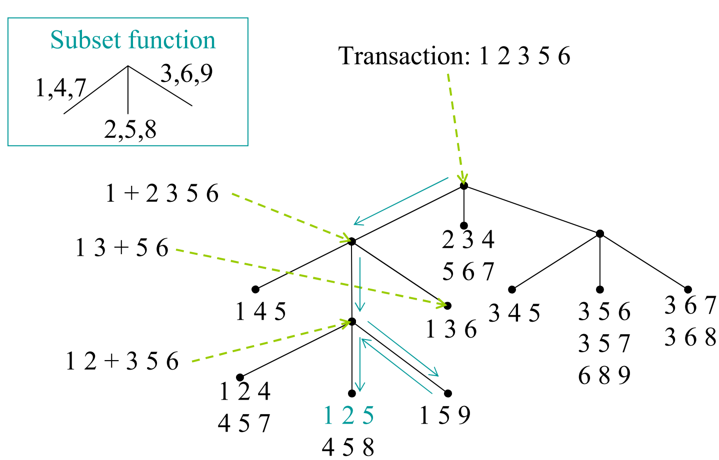
Implementation of Apriori

- How to generate candidates?
  - Step 1: self-joining L_k
  - Step 2: pruning
- Example of Candidate-generation
  - L_3={*abc, abd, acd, ace, bcd*}
  - Self-joining: L_3*L_3
    - *abcd* from *abc* and *abd*
    - *acde* from *acd* and *ace*
  - Pruning:
    - *acde* is removed because *ade* is not in L_3
  - C_4 = {*abcd*}

How to Count Supports of Candidates?

- Why counting supports of candidates a problem?
  - The total number of candidates can be very huge
  - One transaction may contain many candidates
- Method:
  - Candidate itemsets are stored in a *hash-tree*
  - *Leaf* node of hash-tree contains a list of itemsets and counts
  - *Interior* node contains a hash table
  - *Subset function*: finds all the candidates contained in a transaction

Example: Counting Supports of Candidates



Candidate Generation: An SQL Implementation

- SQL Implementation of candidate generation
  - Suppose the items in L_{k-1} are listed in an order
  - Step 1: self-joining L_{k-1}
    - insert into $\boldsymbol{C_k}$
    - select $\boldsymbol{p.item_1, p.item_2, \ldots, p.item_{k-1}, q.item_{k-1}}$
    - from $\boldsymbol{L_{k-1}\ p, L_{k-1}\ q}$
    - where $\boldsymbol{p.item_1=q.item_1, \ldots, p.item_{k-2}=q.item_{k-2}, p.item_{k-1} < q.item_{k-1}}$
  - Step 2: pruning
    - forall ***itemsets c in*** $\boldsymbol{C_k}$ do
      - forall ***(k-1)-subsets s of c*** do
        - **if** *(s is not in* L_{k-1}*)* **then delete** *c* **from** C_k
- Use object-relational extensions like UDFs, BLOBs, and Table functions for efficient implementation [S. Sarawagi, S. Thomas, and R. Agrawal. Integrating association rule mining with relational database systems: Alternatives and implications. SIGMOD'98]

Further Improvements of Mining Methods

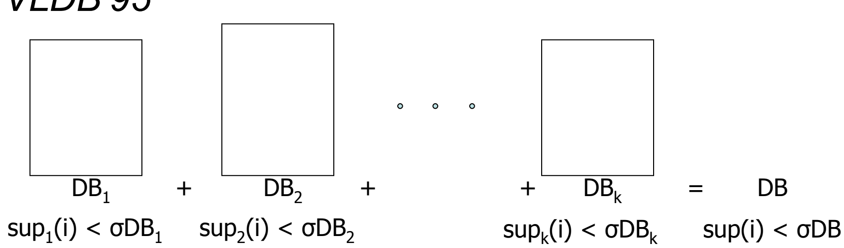
- AFOPT (Liu, et al. @ KDD'03)
  - A "push-right" method for mining condensed frequent pattern (CFP) tree
- Carpenter (Pan, et al. @ KDD'03)
  - Mine data sets with small rows but numerous columns
  - Construct a row-enumeration tree for efficient mining
- FPgrowth+ (Grahne and Zhu, FIMI'03)
  - Efficiently Using Prefix-Trees in Mining Frequent Itemsets, Proc. ICDM'03 Int. Workshop on Frequent Itemset Mining Implementations (FIMI'03), Melbourne, FL, Nov. 2003
- TD-Close (Liu, et al, SDM'06)

Further Improvement of the Apriori Method

- Major computational challenges
  - Multiple scans of transaction database
  - Huge number of candidates
  - Tedious workload of support counting for candidates
- Improving Apriori: general ideas
  - Reduce passes of transaction database scans
  - Shrink number of candidates
  - Facilitate support counting of candidates

Partition: Scan Database Only Twice

- Any itemset that is potentially frequent in DB must be frequent in at least one of the partitions of DB
  - Scan 1: partition database and find local frequent patterns
  - Scan 2: consolidate global frequent patterns
- A. Savasere, E. Omiecinski and S. Navathe, *VLDB'95*



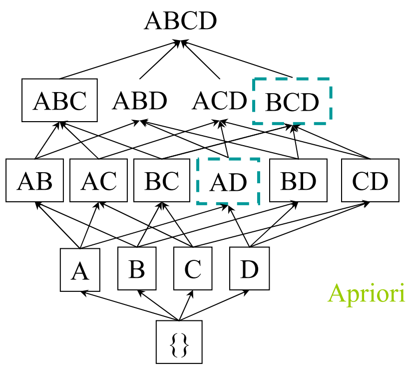
DHP: Reduce the Number of Candidates

- A *k*-itemset whose corresponding hashing bucket count is below the threshold cannot be frequent
  - Candidates: a, b, c, d, e
  - Hash entries: {ab, ad, ae} {bd, be, de} …
  - Frequent 1-itemset: a, b, d, e
  - ab is not a candidate 2-itemset if the sum of count of {ab, ad, ae} is below support threshold
- J. Park, M. Chen, and P. Yu. An effective hash-based algorithm for mining association rules. In *SIGMOD'95*

Sampling for Frequent Patterns

- Select a sample of original database, mine frequent patterns within sample using Apriori
- Scan database once to verify frequent itemsets found in sample, only *borders* of closure of frequent patterns are checked
  - Example: check *abcd* instead of *ab, ac, …, etc.*
- Scan database again to find missed frequent patterns
- H. Toivonen. Sampling large databases for association rules. In *VLDB'96*

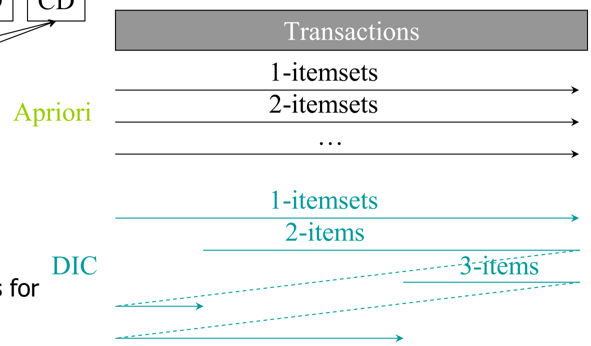
DIC: Reduce Number of Scans



Itemset lattice

S. Brin R. Motwani, J. Ullman, and S. Tsur. Dynamic itemset counting and implication rules for market basket data. In *SIGMOD'97*

- Once both A and D are determined frequent, the counting of AD begins
- Once all length-2 subsets of BCD are determined frequent, the counting of BCD begins



Pattern-Growth Approach: Mining Frequent Patterns Without Candidate Generation

- Bottlenecks of the Apriori approach
  - Breadth-first (i.e., level-wise) search
  - Candidate generation and test
    - Often generates a huge number of candidates
- The FPGrowth Approach (J. Han, J. Pei, and Y. Yin, SIGMOD' 00)
  - Depth-first search
  - Avoid explicit candidate generation
- Major philosophy: Grow long patterns from short ones using local frequent items only
  - "abc" is a frequent pattern
  - Get all transactions having "abc", i.e., project DB on abc: DB|abc
  - "d" is a local frequent item in DB|abc → abcd is a frequent pattern

Construct FP-tree from a Transaction Database

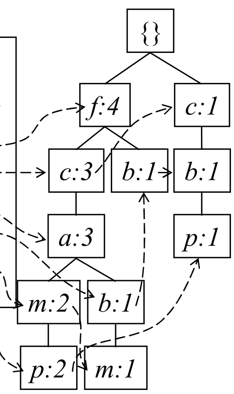
TID	*Items bought*	*(ordered) frequent items*
100	***{f, a, c, d, g, i, m, p}***	***{f, c, a, m, p}***
200	***{a, b, c, f, l, m, o}***	***{f, c, a, b, m}***
300	***{b, f, h, j, o, w}***	***{f, b}***
400	***{b, c, k, s, p}***	***{c, b, p}***
500	***{a, f, c, e, l, p, m, n}***	***{f, c, a, m, p}***

min_support = 3

1. Scan DB once, find frequent 1-itemset (single item pattern)
2. Sort frequent items in frequency descending order, f-list
3. Scan DB again, construct FP-tree

Header Table

Item	*frequency*	*head*
f	*4*	
c	*4*	
a	*3*	
b	*3*	
m	*3*	
p	*3*	



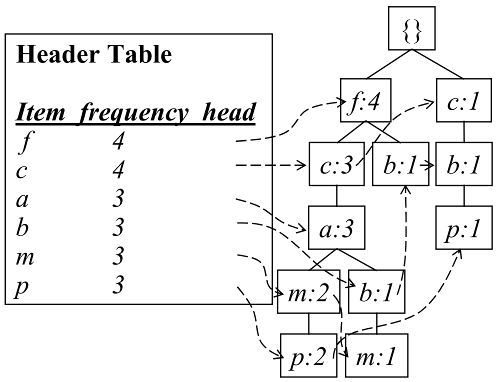
F-list = f-c-a-b-m-p

Partition Patterns and Databases

- Frequent patterns can be partitioned into subsets according to f-list
  - F-list = f-c-a-b-m-p
  - Patterns containing p
  - Patterns having m but no p
  - …
  - Patterns having c but no a nor b, m, p
  - Pattern f
- Completeness and non-redundancy

Find Patterns Having P From P-conditional Database

- Starting at the frequent item header table in the FP-tree
- Traverse the FP-tree by following the link of each frequent item *p*
- Accumulate all of *transformed prefix paths* of item *p* to form *p*'s conditional pattern base

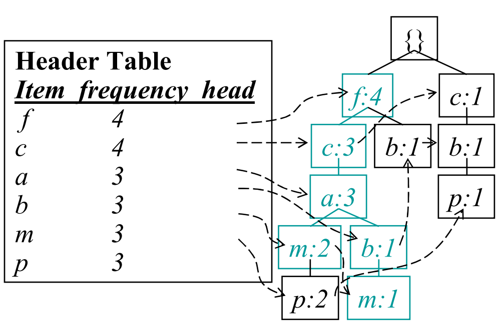


Conditional **pattern bases**

item	*cond. pattern base*
c	***f:3***
a	***fc:3***
b	***fca:1, f:1, c:1***
m	***fca:2, fcab:1***
p	***fcam:2, cb:1***

From Conditional Pattern-bases to Conditional FP-trees

- For each pattern-base
  - Accumulate the count for each item in the base
  - Construct the FP-tree for the frequent items of the pattern base



m-conditional **pattern base:**
fca:2, fcab:1

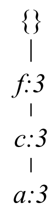
→

{}
|
f:3 →
|
c:3
|
a:3

m-conditional **FP-tree**

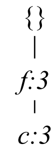
All frequent patterns relate to *m*
m,
fm, cm, am,
fcm, fam, cam,
fcam

Recursion: Mining Each Conditional FP-tree



m-conditional **FP-tree**

Cond. pattern base of "am": (fc:3)

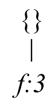


am-conditional **FP-tree**

Cond. pattern base of "cm": (f:3)

cm-conditional **FP-tree**

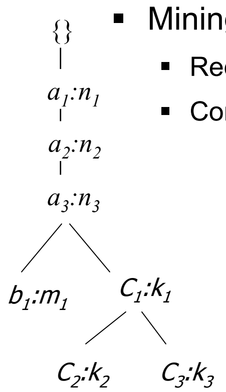
Cond. pattern base of "cam": (f:3)

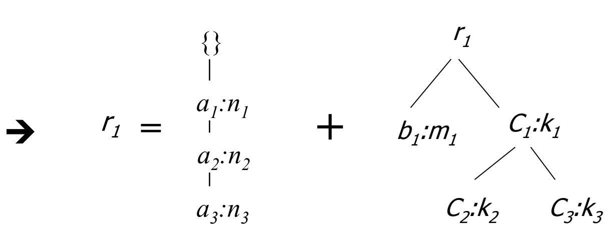


cam-conditional **FP-tree**

A Special Case: Single Prefix Path in FP-tree

- Suppose a (conditional) FP-tree T has a shared single prefix-path P
- Mining can be decomposed into two parts
  - Reduction of the single prefix path into one node
  - Concatenation of the mining results of the two parts





Benefits of the FP-tree Structure

- Completeness
  - Preserve complete information for frequent pattern mining
  - Never break a long pattern of any transaction
- Compactness
  - Reduce irrelevant info—infrequent items are gone
  - Items in frequency descending order: the more frequently occurring, the more likely to be shared
  - Never be larger than the original database (not count node-links and the *count* field)

The Frequent Pattern Growth Mining Method

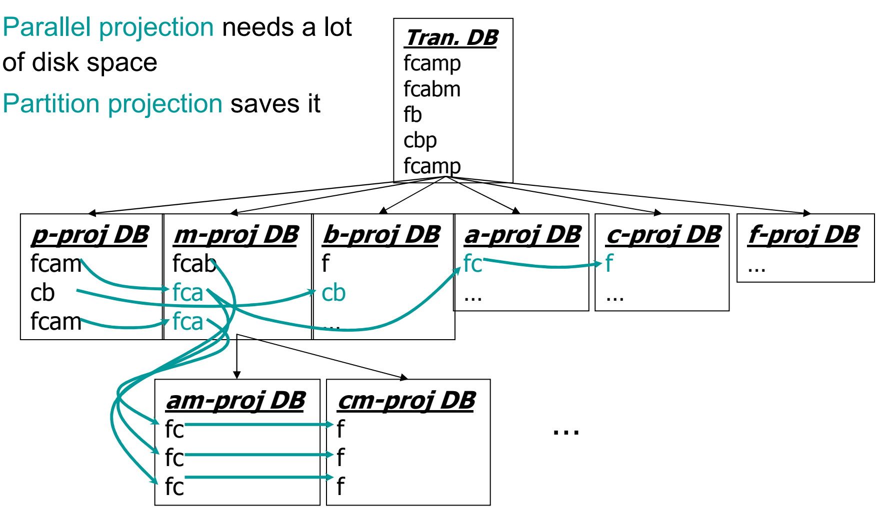
- Idea: Frequent pattern growth
  - Recursively grow frequent patterns by pattern and database partition
- Method
  - For each frequent item, construct its conditional pattern-base, and then its conditional FP-tree
  - Repeat the process on each newly created conditional FP-tree
  - Until the resulting FP-tree is empty, or it contains only one path—single path will generate all the combinations of its sub-paths, each of which is a frequent pattern

Scaling FP-growth by Database Projection

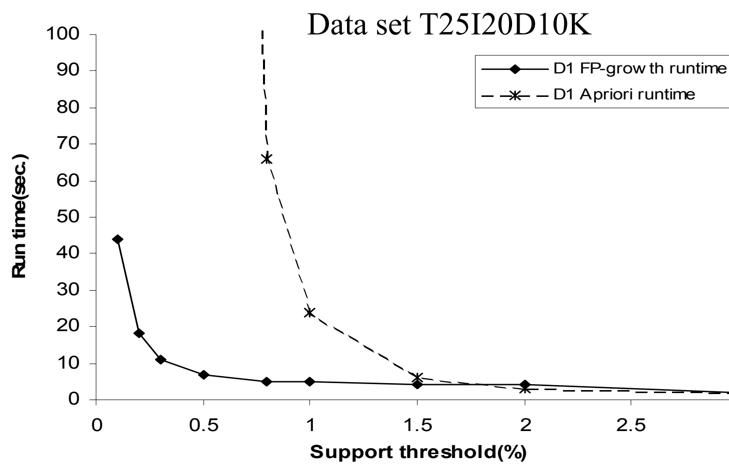
- What about if FP-tree cannot fit in memory?
  - DB projection
- First partition a database into a set of projected DBs
- Then construct and mine FP-tree for each projected DB
- Parallel projection vs. partition projection techniques
  - Parallel projection
    - Project the DB in parallel for each frequent item
    - Parallel projection is space costly
    - All the partitions can be processed in parallel
  - Partition projection
    - Partition the DB based on the ordered frequent items
    - Passing the unprocessed parts to the subsequent partitions

Partition-Based Projection

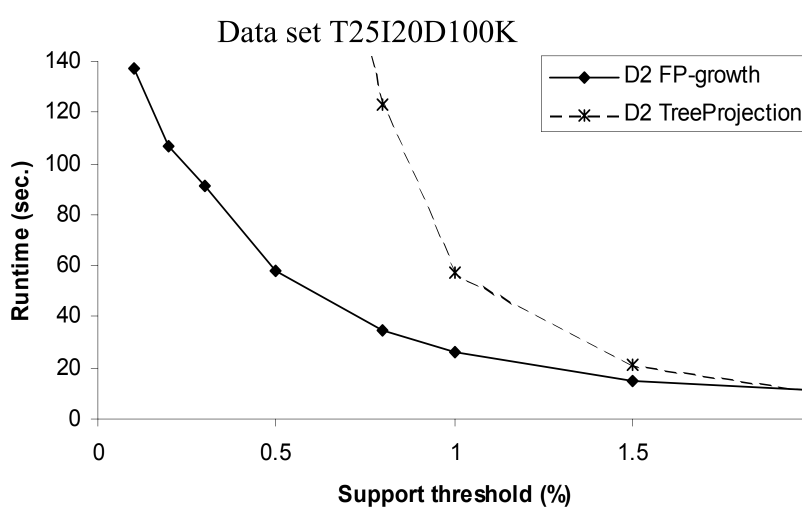
- Parallel projection needs a lot of disk space
- Partition projection saves it



FP-Growth vs. Apriori: Scalability With the Support Threshold



FP-Growth vs. Tree-Projection: Scalability with the Support Threshold



Advantages of the Pattern Growth Approach

- Divide-and-conquer:
  - Decompose both the mining task and DB according to the frequent patterns obtained so far
  - Lead to focused search of smaller databases
- Other factors
  - No candidate generation, no candidate test
  - Compressed database: FP-tree structure
  - No repeated scan of entire database
  - Basic ops: counting local freq items and building sub FP-tree, no pattern search and matching
- A good open-source implementation and refinement of FPGrowth
  - FPGrowth+ (Grahne and J. Zhu, FIMI'03)

Extension of Pattern Growth Mining Methodology

- Mining closed frequent itemsets and max-patterns
  - CLOSET (DMKD'00), FPclose, and FPMax (Grahne & Zhu, Fimi'03)
- Mining sequential patterns
  - PrefixSpan (ICDE'01), CloSpan (SDM'03), BIDE (ICDE'04)
- Mining graph patterns
  - gSpan (ICDM'02), CloseGraph (KDD'03)
- Constraint-based mining of frequent patterns
  - Convertible constraints (ICDE'01), gPrune (PAKDD'03)
- Computing iceberg data cubes with complex measures
  - H-tree, H-cubing, and Star-cubing (SIGMOD'01, VLDB'03)
- Pattern-growth-based Clustering
  - MaPle (Pei, et al., ICDM'03)
- Pattern-Growth-Based Classification
  - Mining frequent and discriminative patterns (Cheng, et al, ICDE'07)

MaxMiner: Mining Max-patterns

Tid	Items
10	A,B,C,D,E
20	B,C,D,E,
30	A,C,D,F

- 1st scan: find frequent items
  - A, B, C, D, E
- 2nd scan: find support for
  - AB, AC, AD, AE, ABCDE
  - BC, BD, BE, BCDE
  - CD, CE, CDE, DE,

Potential max-patterns

- Since BCDE is a max-pattern, no need to check BCD, BDE, CDE in later scan
- R. Bayardo. Efficiently mining long patterns from databases. *SIGMOD'98*

Mining Frequent Closed Patterns: CLOSET

- Flist: list of all frequent items in support ascending order
  - Flist: d-a-f-e-c
- Divide search space
  - Patterns having d
  - Patterns having d but no a, etc.

Min_sup=2

TID	Items
10	a, c, d, e, f
20	a, b, e
30	c, e, f
40	a, c, d, f
50	c, e, f

- Find frequent closed pattern recursively
  - Every transaction having d also has cfa → cfad is a frequent closed pattern
- J. Pei, J. Han & R. Mao. CLOSET: An Efficient Algorithm for Mining Frequent Closed Itemsets", DMKD'00.

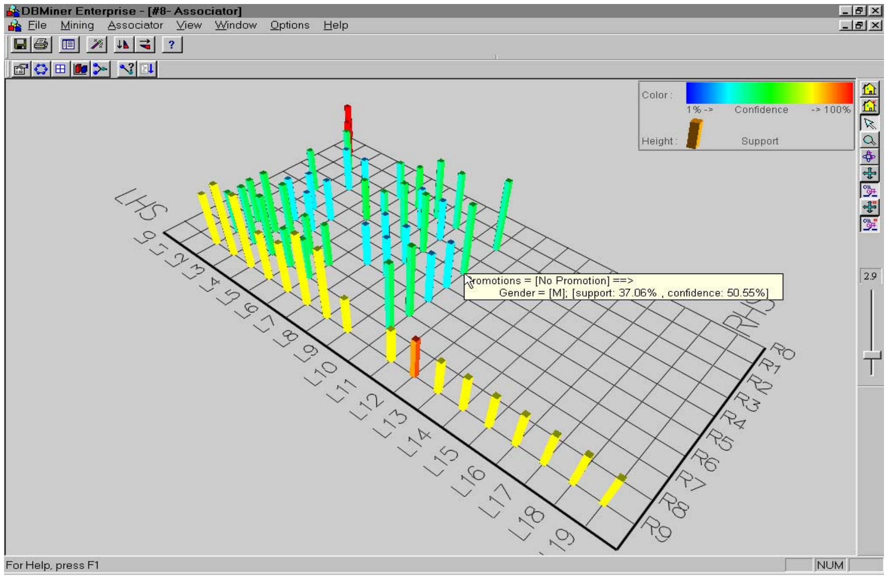
CLOSET+: Mining Closed Itemsets by Pattern-Growth

- Itemset merging: if Y appears in every occurrence of X, then Y is merged with X
- Sub-itemset pruning: if Y ɔ X, and sup(X) = sup(Y), X and all of X's descendants in the set enumeration tree can be pruned
- Hybrid tree projection
  - Bottom-up physical tree-projection
  - Top-down pseudo tree-projection
- Item skipping: if a local frequent item has the same support in several header tables at different levels, one can prune it from the header table at higher levels
- Efficient subset checking

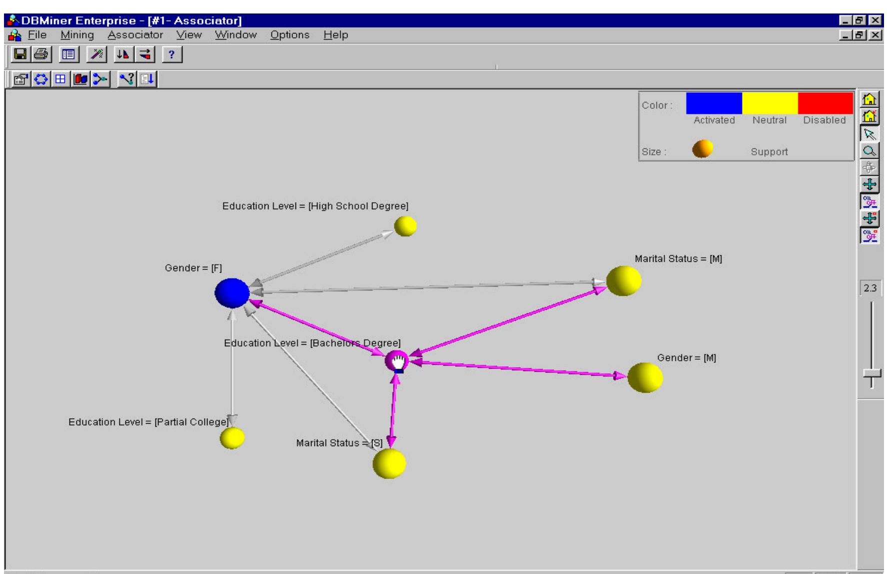
CHARM / ECLAT: Mining by Exploring Vertical Data Format

- Vertical format: $t(AB) = \{T_{11}, T_{25}, \ldots\}$
  - tid-list: list of trans.-ids containing an itemset
- Deriving closed patterns based on vertical intersections
  - $t(X) = t(Y)$: X and Y always happen together
  - $t(X) \subset t(Y)$: transaction having X always has Y
- Using diffset to accelerate mining
  - Only keep track of differences of tids
  - $t(X) = \{T_1, T_2, T_3\}$, $t(XY) = \{T_1, T_3\}$
  - Diffset $(XY, X) = \{T_2\}$
- Eclat/MaxEclat (Zaki et al. @KDD'97), VIPER(P. Shenoy et al.@SIGMOD'00), CHARM (Zaki & Hsiao@SDM'02)

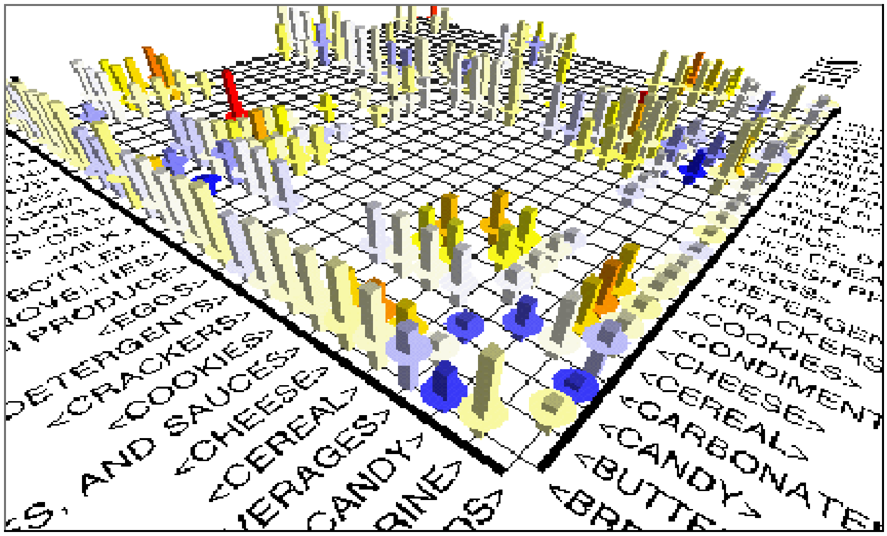
Visualization of Association Rules: Plane Graph



Visualization of Association Rules: Rule Graph



Visualization of Association Rules (SGI/MineSet 3.0)



Mining Frequent Patterns, Association and Correlations – Sub-Topics

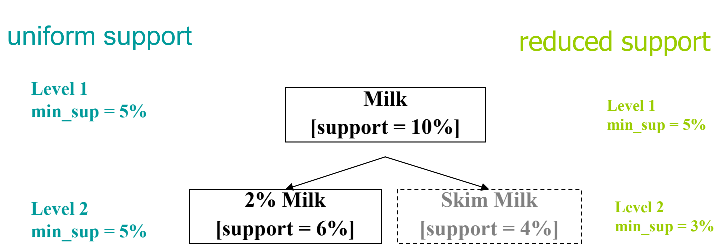
- Basic concepts and a road map
- Scalable frequent itemset mining methods
- Mining various kinds of association rules
- From association to correlation analysis
- Constraint-based association mining
- From association to correlation analysis
- Mining colossal patterns
- Summary

Mining Various Kinds of Association Rules

- Mining multilevel association
- Mining multidimensional association
- Mining quantitative association
- Mining interesting correlation patterns

Mining Multiple-Level Association Rules

- Items often form hierarchies
- Flexible support settings
  - Items at the lower level are expected to have lower support
- Exploration of *shared* multi-level mining (Agrawal & Srikant@VLB'95, Han & Fu@VLDB'95)



Multi-level Association: Redundancy Filtering

- Some rules may be redundant due to “ancestor” relationships between items
- Example
  - milk $\Rightarrow$ wheat bread [support = 8%, confidence = 70%]
  - 2% milk $\Rightarrow$ wheat bread [support = 2%, confidence = 72%]
- We say the first rule is an ancestor of the second rule
- A rule is redundant if its support is close to the “expected” value, based on the rule’s ancestor

Mining Multi-Dimensional Association

- Single-dimensional rules:
  - buys(X, “milk”) $\Rightarrow$ buys(X, “bread”)
- Multi-dimensional rules: $\geq$ 2 dimensions or predicates
  - Inter-dimension assoc. rules (*no repeated predicates*)
    - age(X,”19-25”) $\wedge$ occupation(X,“student”) $\Rightarrow$ buys(X, “coke”)
  - hybrid-dimension assoc. rules (*repeated predicates*)
    - age(X,”19-25”) $\wedge$ buys(X, “popcorn”) $\Rightarrow$ buys(X, “coke”)
- Categorical Attributes: finite number of possible values, no ordering among values—data cube approach
- Quantitative Attributes: Numeric, implicit ordering among values—discretization, clustering, and gradient approaches

Mining Quantitative Associations

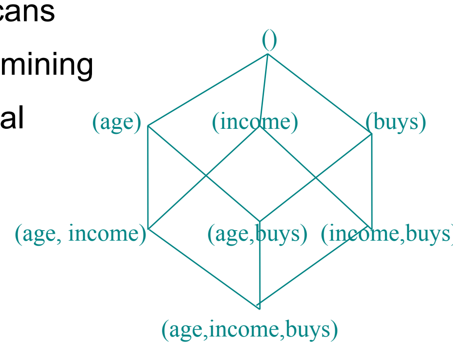
- Techniques can be categorized by how numerical attributes, such as age or salary are treated:

1. Static discretization based on predefined concept hierarchies (data cube methods)
2. Dynamic discretization based on data distribution (quantitative rules, e.g., Agrawal & Srikant@SIGMOD96)
3. Clustering: Distance-based association (e.g., Yang & Miller@SIGMOD97)
    - One dimensional clustering then association
4. Deviation: (such as Aumann and Lindell@KDD99)

   Sex = female => Wage: mean=$7/hr (overall mean = $9)

Static Discretization of Quantitative Attributes

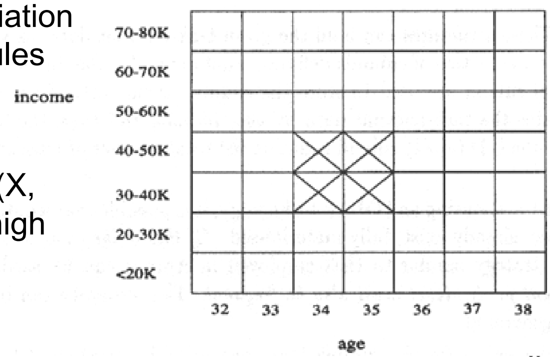
- Discretized prior to mining using concept hierarchy.
- Numeric values are replaced by ranges
- In relational database, finding all frequent k-predicate sets will require k or k+1 table scans
- Data cube is well suited for mining
- The cells of an n-dimensional cuboid correspond to the predicate sets
- Mining from data cubes can be much faster



Quantitative Association Rules

- Proposed by Lent, Swami and Widom ICDE'97
- Numeric attributes are *dynamically* discretized
  - Such that the confidence or compactness of the rules mined is maximized
- 2-D quantitative association rules: $A_{quan1} \wedge A_{quan2} \Rightarrow A_{cat}$
- Cluster *adjacent* association rules to form general rules using a 2-D grid
- Example

age(X, "34-35") $\wedge$ income(X, "30-50K") $\Rightarrow$ buys(X, "high resolution TV")



Mining Other Interesting Patterns

- Flexible support constraints (Wang, et al. @ VLDB'02)
  - Some items (e.g., diamond) may occur rarely but are valuable
  - Customized sup_{min} specification and application
- Top-K closed frequent patterns (Han, et al. @ ICDM'02)
  - Hard to specify sup_{min}, but top-k with $length_{min}$ is more desirable
  - Dynamically raise sup_{min} in FP-tree construction and mining, and select most promising path to mine

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Interestingness Measure: Correlations (Lift)

- *play basketball* $\Rightarrow$ *eat cereal* [40%, 66.7%] is misleading
  - The overall % of students eating cereal is 75% > 66.7%.
- *play basketball* $\Rightarrow$ *not eat cereal* [20%, 33.3%] is more accurate, although with lower support and confidence
- Measure of dependent/correlated events: lift

$$lift = \frac{P(A \cup B)}{P(A)P(B)}$$

$$lift(B,C) = \frac{2000/5000}{3000/5000 * 3750/5000} = 0.89$$

$$lift(B,\neg C) = \frac{1000/5000}{3000/5000 * 1250/5000} = 1.33$$

	Basketball	Not basketball	Sum (row)
Cereal	2000	1750	3750
Not cereal	1000	250	1250
Sum(col.)	3000	2000	5000

Are *lift* and χ^2 Good Measures of Correlation?

- *"Buy walnuts $\Rightarrow$ buy milk* [1%, 80%]" is misleading if 85% of customers buy milk
- Support and confidence are not good to indicate correlations
- Over 20 interestingness measures have been proposed (see Tan, Kumar, Sritastava @KDD'02)
- Which are good ones?

symbol	measure	range	formula
ϕ	ϕ-coefficient	-1 ... 1	$\frac{P(A,B)-P(A)P(B)}{\sqrt{P(A)P(B)(1-P(A))(1-P(B))}}$
Q	Yule's Q	-1 ... 1	$\frac{P(A,B)P(\overline{A},\overline{B})-P(A,\overline{B})P(\overline{A},B)}{P(A,B)P(\overline{A},\overline{B})+P(A,\overline{B})P(\overline{A},B)}$
Y	Yule's Y	-1 ... 1	$\frac{\sqrt{P(A,B)P(\overline{A},\overline{B})}-\sqrt{P(A,\overline{B})P(\overline{A},B)}}{\sqrt{P(A,B)P(\overline{A},\overline{B})}+\sqrt{P(A,\overline{B})P(\overline{A},B)}}$
k	Cohen's	-1 ... 1	$\frac{P(A,B)+P(\overline{A},\overline{B})-P(A)P(B)-P(\overline{A})P(\overline{B})}{1-P(A)P(B)-P(\overline{A})P(\overline{B})}$
PS	Piatetsky-Shapiro's	-0.25 ... 0.25	$P(A,B)-P(A)P(B)$
F	Certainty factor	-1 ... 1	$\max(\frac{P(B\mid A)-P(B)}{1-P(B)}, \frac{P(A\mid B)-P(A)}{1-P(A)})$
AV	added value	-0.5 ... 1	$\max(P(B\mid A)-P(B), P(A\mid B)-P(A))$
K	Klosgen's Q	-0.33 ... 0.38	$\sqrt{P(A,B)}\max(P(B\mid A)-P(B), P(A\mid B)-P(A))$
g	Goodman-kruskal's	0 ... 1	$\frac{\Sigma_j \max_k P(A_j,B_k)+\Sigma_k \max_j P(A_j,B_k)-\max_j P(A_j)-\max_k P(B_k)}{2-\max_j P(A_j)-max_k P(B_k)}$
M	Mutual Information	0 ... 1	$\frac{\Sigma_i\Sigma_j P(A_i,B_j)\log\frac{P(A_i,B_j)}{P(A_i)P(B_j)}}{\min(-\Sigma_i P(A_i)\log P(A_i)\log P(A_i), -\Sigma_i P(B_i)\log P(B_i)\log P(B_i))}$
J	J-Measure	0 ... 1	$\max(P(A,B)\log(\frac{P(B\mid A)}{P(B)})+P(A\overline{B})\log(\frac{P(\overline{B}\mid A)}{P(\overline{B})}), P(A,B)\log(\frac{P(A\mid B)}{P(A)})+P(\overline{A}B)\log(\frac{P(\overline{A}\mid B)}{P(\overline{A})}))$
G	Gini index	0 ... 1	$\max(P(A)[P(B\mid A)^2+P(\overline{B}\mid A)^2]+P(\overline{A})[P(B\mid\overline{A})^2+P(\overline{B}\mid\overline{A})^2]-P(B)^2-P(\overline{B})^2, P(B)[P(A\mid B)^2+P(\overline{A}\mid B)^2]+P(\overline{B})[P(A\mid\overline{B})^2+P(\overline{A}\mid\overline{B})^2]-P(A)^2-P(\overline{A})^2)$
s	support	0 ... 1	$P(A,B)$
c	confidence	0 ... 1	$max(P(B\mid A), P(A\mid B))$
L	Laplace	0 ... 1	$\max(\frac{NP(A,B)+1}{NP(A)+2}, \frac{NP(A,B)+1}{NP(B)+2})$
IS	Cosine	0 ... 1	$\frac{P(A,B)}{\sqrt{P(A)P(B)}}$
γ	coherence(Jaccard)	0 ... 1	$\frac{P(A,B)}{P(A)+P(B)-P(A,B)}$
α	all_confidence	0 ... 1	$\frac{P(A,B)}{\max(P(A),P(B))}$
o	odds ratio	0 ... ∞	$\frac{P(A,B)P(\overline{A},\overline{B})}{P(\overline{A},B)P(A,\overline{B})}$
V	Conviction	0.5 ... ∞	$\max(\frac{P(A)P(\overline{B})}{P(A\overline{B})}, \frac{P(B)P(\overline{A})}{P(B\overline{A})})$
λ	lift	0 ... ∞	$\frac{P(A,B)}{P(A)P(B)}$
S	Collective strength	0 ... ∞	$\frac{P(A,B)+P(\overline{AB})}{P(A)P(B)+P(\overline{A})P(\overline{B})}\times\frac{1-P(A)P(B)-P(\overline{A})P(\overline{B})}{1-P(A,B)-P(\overline{AB})}$
χ^2	χ^2	0 ... ∞	$\Sigma_i\frac{(P(A_i)-E_i)^2}{E_i}$

Null-Invariant Measures

Table 6: Properties of interestingness measures. Note that none of the measures satisfies all the properties.

Symbol	Measure	Range	P1	P2	P3	O1	O2	O3	O3'	O4
ϕ	ϕ-coefficient	$-1\cdots0\cdots1$	Yes	Yes	Yes	Yes	No	Yes	Yes	No
λ	Goodman-Kruskal's	$0\cdots1$	Yes	No	No	Yes	No	No*	Yes	No
α	odds ratio	$0\cdots1\cdots\infty$	Yes*	Yes	Yes	Yes	Yes	Yes*	Yes	No
Q	Yule's Q	$-1\cdots0\cdots1$	Yes	Yes	Yes	Yes	Yes	Yes	Yes	No
Y	Yule's Y	$-1\cdots0\cdots1$	Yes	Yes	Yes	Yes	Yes	Yes	Yes	No
κ	Cohen's	$-1\cdots0\cdots1$	Yes	Yes	Yes	Yes	No	No	Yes	No
M	Mutual Information	$0\cdots1$	Yes	Yes	Yes	No**	No	No*	Yes	No
J	J-Measure	$0\cdots1$	Yes	No	No	No**	No	No	No	No
G	Gini index	$0\cdots1$	Yes	No	No	No**	No	No*	Yes	No
s	Support	$0\cdots1$	No	Yes	No	Yes	No	No	No	No
c	Confidence	$0\cdots1$	No	Yes	No	No**	No	No	No	Yes
L	Laplace	$0\cdots1$	No	Yes	No	No**	No	No	No	No
V	Conviction	$0.5\cdots1\cdots\infty$	No	Yes	No	No**	No	No	Yes	No
I	Interest	$0\cdots1\cdots\infty$	Yes*	Yes	Yes	Yes	No	No	No	No
IS	Cosine	$0\cdots\sqrt{P(A,B)}\cdots1$	No	Yes	Yes	Yes	No	No	No	Yes
PS	Piatetsky-Shapiro's	$-0.25\cdots0\cdots0.25$	Yes	Yes	Yes	Yes	No	Yes	Yes	No
F	Certainty factor	$-1\cdots0\cdots1$	Yes	Yes	Yes	No**	No	No	Yes	No
AV	Added value	$-0.5\cdots0\cdots1$	Yes	Yes	Yes	No**	No	No	No	No
S	Collective strength	$0\cdots1\cdots\infty$	No	Yes	Yes	Yes	No	Yes*	Yes	No
ζ	Jaccard	$0\cdots1$	No	Yes	Yes	Yes	No	No	No	Yes
K	Klosgen's	$(\frac{2}{\sqrt{3}}-1)^{1/2}[2-\sqrt{3}-\frac{1}{\sqrt{3}}]\cdots0\cdots\frac{2}{3\sqrt{3}}$	Yes	Yes	Yes	No**	No	No	No	No

where:
- P1: $O(\mathbf{M})=0$ if $det(\mathbf{M})=0$, *i.e.*, whenever A and B are statistically independent.
- P2: $O(\mathbf{M_2})>O(\mathbf{M_1})$ if $\mathbf{M_2}=\mathbf{M_1}+[k\ \ -k;\ -k\ \ k]$.
- P3: $O(\mathbf{M_2})<O(\mathbf{M_1})$ if $\mathbf{M_2}=\mathbf{M_1}+[0\ k;\ 0\ \ -k]$ or $\mathbf{M_2}=\mathbf{M_1}+[0\ 0;\ k\ \ -k]$.
- O1: Property 1: Symmetry under variable permutation.
- O2: Property 2: Row and Column scaling invariance.
- O3: Property 3: Antisymmetry under row or column permutation.
- O3': Property 4: Inversion invariance.
- O4: Property 5: Null invariance.
- Yes*: Yes if measure is normalized.
- No*: Symmetry under row or column permutation.
- No**: No unless the measure is symmetrized by taking $\max(M(A,B),M(B,A))$.

Comparison of Interestingness Measures

- Null-(transaction) invariance is crucial for correlation analysis
- Lift and χ^2 are not null-invariant
- 5 null-invariant measures

	Milk	No Milk	Sum (row)
Coffee	m, c	~m, c	c
No Coffee	m, ~c	~m, ~c	~c
Sum(col.)	m	~m	Σ

Measure	Definition	Range	Null-Invariant
$\chi^2(a,b)$	$\sum_{i,j=0,1} \frac{(e(a_i,b_j)-o(a_i,b_j))^2}{e(a_i,b_j)}$	$[0,\infty]$	No
$Lift(a,b)$	$\frac{P(ab)}{P(a)P(b)}$	$[0,\infty]$	No
$AllConf(a,b)$	$\frac{sup(ab)}{max\{sup(a),sup(b)\}}$	$[0,1]$	Yes
$Coherence(a,b)$	$\frac{sup(ab)}{sup(a)+sup(b)-sup(ab)}$	$[0,1]$	Yes
$Cosine(a,b)$	$\frac{sup(ab)}{\sqrt{sup(a)sup(b)}}$	$[0,1]$	Yes
$Kulc(a,b)$	$\frac{sup(ab)}{2}(\frac{1}{sup(a)}+\frac{1}{sup(b)})$	$[0,1]$	Yes
$MaxConf(a,b)$	$max\{\frac{sup(ab)}{sup(a)},\frac{sup(ab)}{sup(b)}\}$	$[0,1]$	Yes

Table 3. Interestingness measure definitions.

Null-transactions w.r.t. m and c

Kulczynski measure (1927)

Null-invariant

Data set	mc	$\overline{m}c$	$m\overline{c}$	$\overline{mc}$	χ^2	$Lift$	$AllConf$	$Coherence$	$Cosine$	$Kulc$	$MaxConf$
D_1	10,000	1,000	1,000	100,000	90557	9.26	0.91	0.83	0.91	0.91	0.91
D_2	10,000	1,000	1,000	100	0	1	0.91	0.83	0.91	0.91	0.91
D_3	100	1,000	1,000	100,000	670	8.44	0.09	0.05	0.09	0.09	0.09
D_4	1,000	1,000	1,000	100,000	24740	25.75	0.5	0.33	0.5	0.5	0.5
D_5	1,000	100	10,000	100,000	8173	9.18	0.09	0.09	0.29	0.5	0.91
D_6	1,000	10	100,000	100,000	965	1.97	0.01	0.01	0.10	0.5	0.99

Table 2. Example data sets.

Subtle: They disagree

Analysis of DBLP Coauthor Relationships

Recent DB conferences, removing balanced associations, low sup, etc.

ID	Author a	Author b	$sup(ab)$	$sup(a)$	$sup(b)$	$Coherence$	$Cosine$	$Kulc$
1	Hans-Peter Kriegel	Martin Ester	28	146	54	0.163 (2)	0.315 (7)	0.355 (9)
2	Michael Carey	Miron Livny	26	104	58	0.191 (1)	0.335 (4)	0.349 (10)
3	Hans-Peter Kriegel	Joerg Sander	24	146	36	0.152 (3)	0.331 (5)	0.416 (8)
4	Christos Faloutsos	Spiros Papadimitriou	20	162	26	0.119 (7)	0.308 (10)	0.446 (7)
5	Hans-Peter Kriegel	Martin Pfeifle	18	146	18	0.123 (6)	0.351 (2)	0.562 (2)
6	Hector Garcia-Molina	Wilburt Labio	16	144	18	0.110 (9)	0.314 (8)	0.500 (4)
7	Divyakant Agrawal	Wang Hsiung	16	120	16	0.133 (5)	0.365 (1)	0.567 (1)
8	Elke Rundensteiner	Murali Mani	16	104	20	0.148 (4)	0.351 (3)	0.477 (6)
9	Divyakant Agrawal	Oliver Po	12	120	12	0.100 (10)	0.316 (6)	0.550 (3)
10	Gerhard Weikum	Martin Theobald	12	106	14	0.111 (8)	0.312 (9)	0.485 (5)

Table 5. Experiment on DBLP data set.

Advisor-advisee relation: Kulc: high, coherence: low, cosine: middle

- Tianyi Wu, Yuguo Chen and Jiawei Han, "Association Mining in Large Databases: A Re-Examination of Its Measures", Proc. 2007 Int. Conf. Principles and Practice of Knowledge Discovery in Databases (PKDD'07), Sept. 2007

Which Null-Invariant Measure Is Better?

- IR (Imbalance Ratio): measure the imbalance of two itemsets A and B in rule implications

$$IR(A,B) = \frac{|sup(A) - sup(B)|}{sup(A) + sup(B) - sup(A \cup B)}$$

- Kulczynski and Imbalance Ratio (IR) together present a clear picture for all the three datasets D_4 through D_6
  - D_4 is balanced & neutral
  - D_5 is imbalanced & neutral
  - D_6 is very imbalanced & neutral

Data	mc	$\overline{m}c$	$m\overline{c}$	$\overline{mc}$	*all_conf.*	*max_conf.*	*Kulc.*	*cosine*	IR
D_1	10,000	1,000	1,000	100,000	0.91	0.91	0.91	0.91	0.0
D_2	10,000	1,000	1,000	100	0.91	0.91	0.91	0.91	0.0
D_3	100	1,000	1,000	100,000	0.09	0.09	0.09	0.09	0.0
D_4	1,000	1,000	1,000	100,000	0.5	0.5	0.5	0.5	0.0
D_5	1,000	100	10,000	100,000	0.09	0.91	0.5	0.29	0.89
D_6	1,000	10	100,000	100,000	0.01	0.99	0.5	0.10	0.99

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Constraint-based (Query-Directed) Mining

- Finding all the patterns in a database autonomously? — unrealistic!
  - The patterns could be too many but not focused!
- Data mining should be an interactive process
  - User directs what to be mined using a data mining query language (or a graphical user interface)
- Constraint-based mining
  - User flexibility: provides constraints on what to be mined
  - System optimization: explores such constraints for efficient mining — constraint-based mining: constraint-pushing, similar to push selection first in DB query processing
  - Note: still find all the answers satisfying constraints, not finding some answers in "heuristic search"

Constraints in Data Mining

- Knowledge type constraint:
  - classification, association, etc.
- Data constraint — using SQL-like queries
  - find product pairs sold together in stores in Chicago in Dec.'02
- Dimension/level constraint
  - in relevance to region, price, brand, customer category
- Rule (or pattern) constraint
  - small sales (price < $10) triggers big sales (sum > $200)
- Interestingness constraint
  - strong rules: min_support ≥ 3%, min_confidence ≥ 60%

Constraint-Based Frequent Pattern Mining

- Classification of constraints based on their constraint-pushing capabilities
  - Anti-monotonic: If constraint c is violated, its further mining can be terminated
  - Monotonic: If c is satisfied, no need to check c again
  - Data anti-monotonic: If a transaction t does not satisfy c, t can be pruned from its further mining
  - Succinct: c must be satisfied, so one can start with the data sets satisfying c
  - Convertible: c is not monotonic nor anti-monotonic, but it can be converted into it if items in the transaction can be properly ordered

Anti-Monotonicity in Constraint Pushing

- A constraint C is *antimonotone* if the super pattern satisfies C, all of its sub-patterns do so too
- In other words, *anti-monotonicity:* If an itemset S **violates** the constraint, so does any of its superset
- Ex. 1. *sum(S.price)* $\leq$ *v* is anti-monotone
- Ex. 2. range(S.profit) $\leq$ 15 is anti-monotone
  - Itemset *ab* violates C
  - So does every superset of *ab*
- Ex. 3. *sum(S.Price)* $\geq$ *v* is not anti-monotone
- Ex. 4. *support count* is anti-monotone: core property used in Apriori

TDB (min_sup=2)

TID	Transaction
10	a, b, c, d, f
20	b, c, d, f, g, h
30	a, c, d, e, f
40	c, e, f, g

Item	Profit
a	40
b	0
c	-20
d	10
e	-30
f	30
g	20
h	-10

Monotonicity for Constraint Pushing

- A constraint C is *monotone* if the pattern satisfies C, we do not need to check C in subsequent mining
- Alternatively, monotonicity: *If an itemset S* ***satisfies*** *the constraint, so does any of its superset*
- Ex. 1. *sum(S.Price)* ≥ *v* is monotone
- Ex. 2. *min(S.Price)* ≤ *v* is monotone
- Ex. 3. C: range(S.profit) ≥ 15
  - Itemset *ab* satisfies C
  - So does every superset of *ab*

TDB (min_sup=2)

TID	Transaction
10	a, b, c, d, f
20	b, c, d, f, g, h
30	a, c, d, e, f
40	c, e, f, g

Item	Profit
a	40
b	0
c	-20
d	10
e	-30
f	30
g	20
h	-10

Data Antimonotonicity: Pruning Data Space

- A constraint c is *data antimonotone* if for a pattern p cannot satisfy a transaction t under c, p's superset cannot satisfy t under c either
- The key for data antimonotone is *recursive data reduction*
- Ex. 1. *sum(S.Price)* ≥ *v* is data antimonotone
- Ex. 2. *min(S.Price)* ≤ *v* is data antimonotone
- Ex. 3. C: *range(S.profit)* ≥ *25* is data antimonotone
  - Itemset {b, c}'s projected DB:
    - T10': {d, f, h}, T20': {d, f, g, h}, T30': {d, f, g}
  - since C cannot satisfy T10', T10' can be pruned

TDB (min_sup=2)

TID	Transaction
10	a, b, c, d, f, h
20	b, c, d, f, g, h
30	b, c, d, f, g
40	c, e, f, g

Item	Profit
a	40
b	0
c	-20
d	-15
e	-30
f	-10
g	20
h	-5

Succinctness

- Succinctness:
  - Given A_1, the set of items satisfying a succinctness constraint *C*, then any set *S* satisfying *C* is based on A_1 , i.e., *S* contains a subset belonging to A_1
  - Idea: Without looking at the transaction database, whether an itemset *S* satisfies constraint C can be determined based on the selection of items
  - $min(S.Price) \leq v$ is succinct
  - $sum(S.Price) \geq v$ is not succinct
- Optimization: If *C* is succinct, *C* is pre-counting pushable

The Apriori Algorithm — Example

Database D

TID	Items
100	1 3 4
200	2 3 5
300	1 2 3 5
400	2 5

Scan D

C_1

itemset	sup.
{1}	2
{2}	3
{3}	3
{4}	1
{5}	3

L_1

itemset	sup.
{1}	2
{2}	3
{3}	3
{5}	3

C_2

itemset
{1 2}
{1 3}
{1 5}
{2 3}
{2 5}
{3 5}

Scan D

C_2

itemset	sup
{1 2}	1
{1 3}	2
{1 5}	1
{2 3}	2
{2 5}	3
{3 5}	2

L_2

itemset	sup
{1 3}	2
{2 3}	2
{2 5}	3
{3 5}	2

C_3

itemset
{2 3 5}

Scan D

L_3

itemset	sup
{2 3 5}	2

Naïve Algorithm: Apriori + Constraint

Database D

TID	Items
100	1 3 4
200	2 3 5
300	1 2 3 5
400	2 5

Scan D

C_1

itemset	sup.
{1}	2
{2}	3
{3}	3
{4}	1
{5}	3

L_1

itemset	sup.
{1}	2
{2}	3
{3}	3
~~{5}~~	~~3~~

C_2

itemset
{1 2}
{1 3}
{1 5}
{2 3}
{2 5}
{3 5}

Scan D

C_2

itemset	sup
{1 2}	1
{1 3}	2
{1 5}	1
{2 3}	2
{2 5}	3
{3 5}	2

L_2

itemset	sup
{1 3}	2
~~{2 3}~~	~~2~~
~~{2 5}~~	~~3~~
~~{3 5}~~	~~2~~

C_3

itemset
{2 3 5}

Scan D

L_3

itemset	sup
~~{2 3 5}~~	~~2~~

Constraint:

Sum{S.price} < 5

The Constrained Apriori Algorithm: Push a Succinct Constraint Deep

Database D

TID	Items
100	1 3 4
200	2 3 5
300	1 2 3 5
400	2 5

Scan D

C_1

itemset	sup.
{1}	2
{2}	3
{3}	3
{4}	1
{5}	3

L_1

itemset	sup.
{1}	2
{2}	3
{3}	3
{5}	3

C_2

itemset
{1 2}
{1 3}
{1 5}
~~{2 3}~~
~~{2 5}~~
~~{3 5}~~

not immediately to be used

Scan D

C_2

itemset	sup
{1 2}	1
{1 3}	2
{1 5}	1
~~{2 3}~~	~~2~~
~~{2 5}~~	~~3~~
~~{3 5}~~	~~2~~

L_2

itemset	sup
{1 3}	2
~~{2 3}~~	~~2~~
~~{2 5}~~	~~3~~
~~{3 5}~~	~~2~~

C_3

itemset
~~{2 3 5}~~

Scan D

L_3

itemset	sup
~~{2 3 5}~~	~~2~~

Constraint:

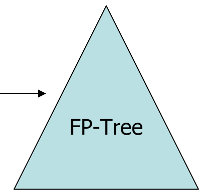
min{S.price } <= 1

The Constrained FP-Growth Algorithm: Push a Succinct Constraint Deep

TID	Items
100	1 3 4
200	2 3 5
300	1 2 3 5
400	2 5

Remove infrequent length 1

TID	Items
100	1 3
200	2 3 5
300	1 2 3 5
400	2 5



1-Projected DB

TID	Items
100	3 4
300	2 3 5

No Need to project on 2, 3, or 5

Constraint:

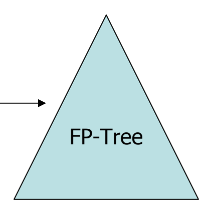
min{S.price } <= 1

The Constrained FP-Growth Algorithm:
Push a Data Antimonotonic Constraint Deep

Remove from data

TID	Items
100	1 3 4
~~200~~	~~2 3 5~~
300	1 2 3 5
~~400~~	~~2 5~~

TID	Items
100	1 3
300	1 3



Single branch, we are done

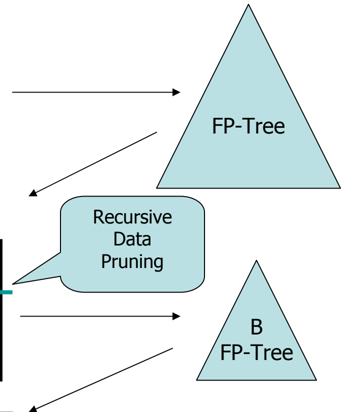
Constraint:

min{S.price } <= 1

The Constrained FP-Growth Algorithm: Push a Data Antimonotonic Constraint Deep

TID	Transaction
10	a, b, c, d, f, h
20	b, c, d, f, g, h
30	b, c, d, f, g
40	a, c, e, f, g

TID	Transaction
10	a, b, c, d, f, h
20	b, c, d, f, g, h
30	b, c, d, f, g
40	a, c, e, f, g



B-Projected DB

TID	Transaction
~~10~~	~~a, c, d, f, h~~
20	c, d, f, g, ~~h~~
30	c, d, f, g

Single branch:

bcdfg: 2

Item	Profit
a	40
b	0
c	-20
d	-15
e	-30
f	-10
g	20
h	-5

Constraint:
range{S.price } > 25
min_sup >= 2

Converting "Tough" Constraints

- Convert tough constraints into anti-monotone or monotone by properly ordering items
- Examine C: avg(S.profit) ≥ 25
  - Order items in value-descending order
    - *<a, f, g, d, b, h, c, e>*
  - If an itemset *afb* violates C
    - So does *afbh, afb**
    - It becomes anti-monotone!

TDB (min_sup=2)

TID	Transaction
10	a, b, c, d, f
20	b, c, d, f, g, h
30	a, c, d, e, f
40	c, e, f, g

Item	Profit
a	40
b	0
c	-20
d	10
e	-30
f	30
g	20
h	-10

Strongly Convertible Constraints

- avg(X) ≥ 25 is convertible anti-monotone w.r.t. item value descending order R: <*a, f, g, d, b, h, c, e*>
  - If an itemset *af* violates a constraint C, so does every itemset with *af* as prefix, such as *afd*
- avg(X) ≥ 25 is convertible monotone w.r.t. item value ascending order R^{-1}: <*e, c, h, b, d, g, f, a*>
  - If an itemset *d* satisfies a constraint *C*, so does itemsets *df* and *dfa*, which having *d* as a prefix
- Thus, avg(X) ≥ 25 is strongly convertible

Item	Profit
a	40
b	0
c	-20
d	10
e	-30
f	30
g	20
h	-10

Can Apriori Handle Convertible Constraints?

- A convertible, not monotone nor anti-monotone nor succinct constraint cannot be pushed deep into the an Apriori mining algorithm
  - Within the level wise framework, no direct pruning based on the constraint can be made
  - Itemset df violates constraint C: avg(X) >= 25
  - Since adf satisfies C, Apriori needs df to assemble adf, df cannot be pruned
- But it can be pushed into frequent-pattern growth framework!

Item	Value
a	40
b	0
c	-20
d	10
e	-30
f	30
g	20
h	-10

Mining With Convertible Constraints

- C: avg(X) >= 25, min_sup=2
- List items in every transaction in value descending order R: <a, f, g, d, b, h, c, e>
  - C is convertible anti-monotone w.r.t. R
- Scan TDB once
  - remove infrequent items
    - Item h is dropped
  - Itemsets a and f are good, …
- Projection-based mining
  - Imposing an appropriate order on item projection
  - Many tough constraints can be converted into (anti)-monotone

Item	Value
a	40
f	30
g	20
d	10
b	0
h	-10
c	-20
e	-30

TDB (min_sup=2)

TID	Transaction
10	a, f, d, b, c
20	f, g, d, b, c
30	a, f, d, c, e
40	f, g, h, c, e

Handling Multiple Constraints

- Different constraints may require different or even conflicting item-ordering
- If there exists an order *R* s.t. both C_1 and C_2 are convertible w.r.t. *R,* then there is no conflict between the two convertible constraints
- If there exists conflict on order of items
  - Try to satisfy one constraint first
  - Then using the order for the other constraint to mine frequent itemsets in the corresponding projected database

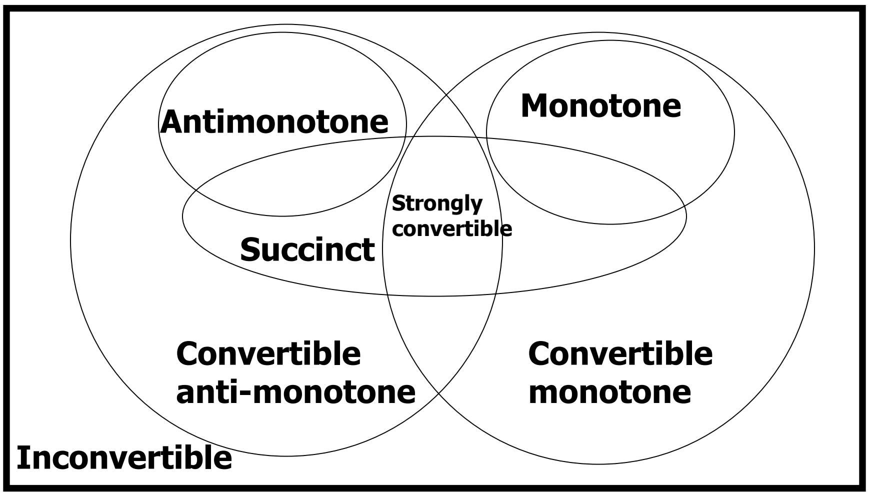
What Constraints Are Convertible?

Constraint	Convertible anti-monotone	Convertible monotone	Strongly convertible
avg(S) ≤ , ≥ v	Yes	Yes	Yes
median(S) ≤ , ≥ v	Yes	Yes	Yes
sum(S) ≤ v (items could be of any value, v ≥ 0)	Yes	No	No
sum(S) ≤ v (items could be of any value, v ≤ 0)	No	Yes	No
sum(S) ≥ v (items could be of any value, v ≥ 0)	No	Yes	No
sum(S) ≥ v (items could be of any value, v ≤ 0)	Yes	No	No
……			

Constraint-Based Mining — A General Picture

Constraint	Antimonotone	Monotone	Succinct
v ∈ S	no	yes	yes
S ⊇ V	no	yes	yes
S ⊆ V	yes	no	yes
min(S) ≤ v	no	yes	yes
min(S) ≥ v	yes	no	yes
max(S) ≤ v	yes	no	yes
max(S) ≥ v	no	yes	yes
count(S) ≤ v	yes	no	weakly
count(S) ≥ v	no	yes	weakly
sum(S) ≤ v (a ∈ S, a ≥ 0)	yes	no	no
sum(S) ≥ v (a ∈ S, a ≥ 0)	no	yes	no
range(S) ≤ v	yes	no	no
range(S) ≥ v	no	yes	no
avg(S) θ v, θ ∈ { =, ≤, ≥ }	convertible	convertible	no
support(S) ≥ ξ	yes	no	no
support(S) ≤ ξ	no	yes	no

A Classification of Constraints



Mining Frequent Patterns, Association and Correlations – Sub-Topics

- Basic concepts and a road map
- Scalable frequent itemset mining methods
- Mining various kinds of association rules
- From association to correlation analysis
- Constraint-based association mining
- Mining colossal patterns
- Summary

Why Mining Colossal Frequent Patterns?

- F. Zhu, X. Yan, J. Han, P. S. Yu, and H. Cheng, "Mining Colossal Frequent Patterns by Core Pattern Fusion", ICDE'07.
- We have many algorithms, but can we mine large (i.e., colossal) patterns? – such as just size around 50 to 100? Unfortunately, not!
- Why not? – the curse of "downward closure" of frequent patterns
  - The "downward closure" property
    - Any sub-pattern of a frequent pattern is frequent.
  - Example. If $(a_1, a_2, \ldots, a_{100})$ is frequent, then $a_1, a_2, \ldots, a_{100}$, (a_1, a_2), (a_1, a_3), …, (a_1, a_{100}), (a_1, a_2, a_3), … are all frequent! There are about 2^{100} such frequent itemsets!
  - No matter using breadth-first search (e.g., Apriori) or depth-first search (FPgrowth), we have to examine so many patterns
- Thus the downward closure property leads to explosion!

Colossal Patterns: A Motivating Example

Let's make a set of 40 transactions

T1 = 1 2 3 4 ….. 39 40

T2 = 1 2 3 4 ….. 39 40

: .

: .

: .

: .

T40=1 2 3 4 ….. 39 40

Then delete the items on the diagonal

T_1 = 2 3 4 ….. 39 40

T_2 = 1 3 4 ….. 39 40

: .

: .

: .

: .

T_{40}=1 2 3 4 …… 39

Closed/maximal patterns may partially alleviate the problem but not really solve it: We often need to mine scattered large patterns!

Let the minimum support threshold σ= 20

There are $\binom{40}{20}$ frequent patterns of size 20

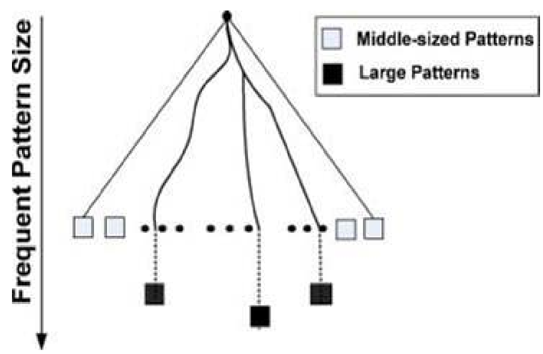
Each is closed and maximal

patterns = $\binom{n}{n/2} \approx \sqrt{2/\pi}\frac{2^n}{\sqrt{n}}$

The size of the answer set is exponential to n

Colossal Pattern Set: Small but Interesting

- It is often the case that only a small number of patterns are colossal, i.e., of large size
- Colossal patterns are usually attached with greater importance than those of small pattern sizes



Mining Colossal Patterns: Motivation and Philosophy

- Motivation: Many real-world tasks need mining colossal patterns
  - Micro-array analysis in bioinformatics (when support is low)
  - Biological sequence patterns
  - Biological/sociological/information graph pattern mining
- *No hope for completeness*
  - If the mining of mid-sized patterns is explosive in size, there is no hope to find colossal patterns efficiently by insisting "complete set" mining philosophy
- *Jumping out of the swamp of the mid-sized results*
  - What we may develop is a philosophy that may jump out of the swamp of mid-sized results that are explosive in size and jump to reach colossal patterns
- *Striving for mining almost complete colossal patterns*
  - The key is to develop a mechanism that may quickly reach colossal patterns and discover most of them

Alas, A Show of Colossal Pattern Mining!

T_1 = 2 3 4 ….. 39 40
T_2 = 1 3 4 ….. 39 40
: .
: .
: .
: .
T_{40}=1 2 3 4 …… 39
T_{41}= 41 42 43 ….. 79
T_{42}= 41 42 43 ….. 79
: .
: .
T_{60}= 41 42 43 … 79

Let the min-support threshold σ= 20

Then there are $\binom{40}{20}$ closed/maximal frequent patterns of size 20

However, there is only one with size greater than 20, (*i.e.,* colossal):

α= {41,42,…,79} of size 39

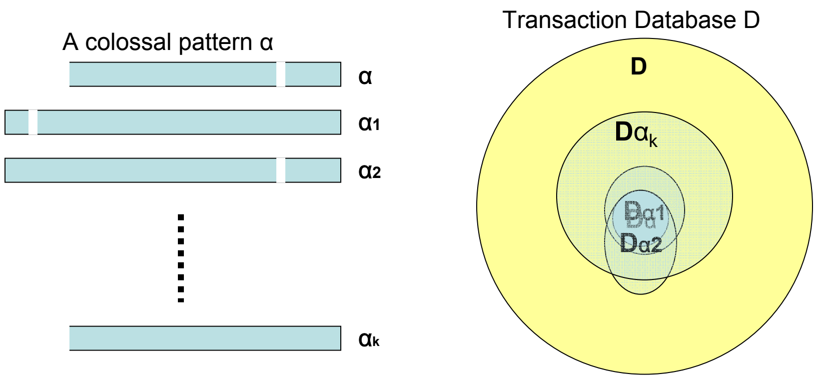
The existing fastest mining algorithms (*e.g.,* FPClose, LCM) fail to complete running

The algorithm outputs this colossal pattern in seconds

Methodology of Pattern-Fusion Strategy

- Pattern-Fusion traverses the tree in a bounded-breadth way
  - Always pushes down a frontier of a bounded-size candidate pool
  - Only a fixed number of patterns in the current candidate pool will be used as the starting nodes to go down in the pattern tree – thus avoids the exponential search space
- Pattern-Fusion identifies "shortcuts" whenever possible
  - Pattern growth is not performed by single-item addition but by leaps and bounded: agglomeration of multiple patterns in the pool
  - These shortcuts will direct the search down the tree much more rapidly towards the colossal patterns

Observation: Colossal Patterns and Core Patterns



Subpatterns α_1 to α_k cluster tightly around the colossal pattern α by sharing a similar support. We call such subpatterns *core patterns* of α

Robustness of Colossal Patterns

- Core Patterns

  Intuitively, for a frequent pattern α, a subpattern β is a τ-core pattern of α if β shares a similar support set with α, i.e.,

$$\frac{|D_\alpha|}{|D_\beta|} \geq \tau \qquad 0 < \tau \leq 1$$

  where τ is called the core ratio

- Robustness of Colossal Patterns

  A colossal pattern is robust in the sense that it tends to have much more core patterns than small patterns

Example: Core Patterns

- A colossal pattern has far more core patterns than a small-sized pattern
- A colossal pattern has far more core descendants of a smaller size c
- A random draw from a complete set of pattern of size c would more likely to pick a core descendant of a colossal pattern
- A colossal pattern can be generated by merging a set of core patterns

Transaction (# of Ts)	Core Patterns (τ = 0.5)
(abe) (100)	(abe), (ab), (be), (ae), (e)
(bcf) (100)	(bcf), (bc), (bf)
(acf) (100)	(acf), (ac), (af)
(abcef) (100)	(ab), (ac), (af), (ae), (bc), (bf), (be) (ce), (fe), (e), (abc), (abf), (abe), (ace), (acf), (afe), (bcf), (bce), (bfe), (cfe), (abcf), (abce), (bcfe), (acfe), (abfe), (abcef)

Robustness of Colossal Patterns

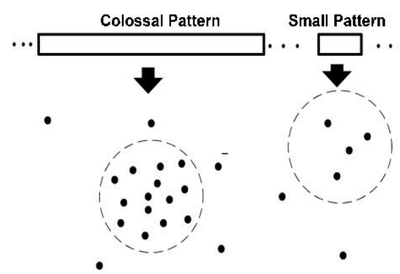
- (d,τ)-robustness: A pattern α is *(d, τ)-robust* if *d* is the maximum number of items that can be removed from α for the resulting pattern to remain a τ-core pattern of α
- For a (d,τ)-robust pattern α, it has $\Omega(2^d)$ core patterns
  - A colossal patterns tend to have a large number of core patterns
- Pattern distance: For patterns α and β, the pattern distance of α and β is defined to be $$Dist(\alpha, \beta) = 1 - \frac{|D_\alpha \cap D_\beta|}{|D_\alpha \cup D_\beta|}$$
- If two patterns α and β are both core patterns of a same pattern, they would be bounded by a "ball" of a radius specified by their core ratio τ

$$Dist(\alpha, \beta) \leq 1 - \frac{1}{2/\tau - 1} = r(\tau)$$

- Once we identify one core pattern, we will be able to find all the other core patterns by a bounding ball of radius r(τ)

Colossal Patterns Correspond to Dense Balls

- Due to their robustness, colossal patterns correspond to dense balls
  - Ω(2^d) in population
- A random draw in the pattern space will hit somewhere in the ball with high probability



Idea of Pattern-Fusion Algorithm

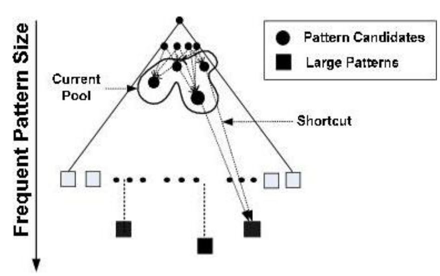
- Generate a complete set of frequent patterns up to a small size
- Randomly pick a pattern β, and β has a high probability to be a core-descendant of some colossal pattern α
- Identify all α's descendants in this complete set, and merge all of them — This would generate a much larger core-descendant of α
- In the same fashion, we select K patterns. This set of larger core-descendants will be the candidate pool for the next iteration

Pattern-Fusion: The Algorithm

- Initialization (Initial pool): Use an existing algorithm to mine all frequent patterns up to a small size, e.g., 3
- Iteration (Iterative Pattern Fusion):
  - At each iteration, k seed patterns are randomly picked from the current pattern pool
  - For each seed pattern thus picked, we find all the patterns within a bounding ball centered at the seed pattern
  - All these patterns found are fused together to generate a set of super-patterns. All the super-patterns thus generated form a new pool for the next iteration
- Termination: when the current pool contains no more than K patterns at the beginning of an iteration

Why Is Pattern-Fusion Efficient?

- A bounded-breadth pattern tree traversal
  - It avoids explosion in mining mid-sized ones
  - Randomness comes to help to stay on the right path
- Ability to identify "short-cuts" and take "leaps"
  - fuse small patterns together in one step to generate new patterns of significant sizes
  - Efficiency



Pattern-Fusion Leads to Good Approximation

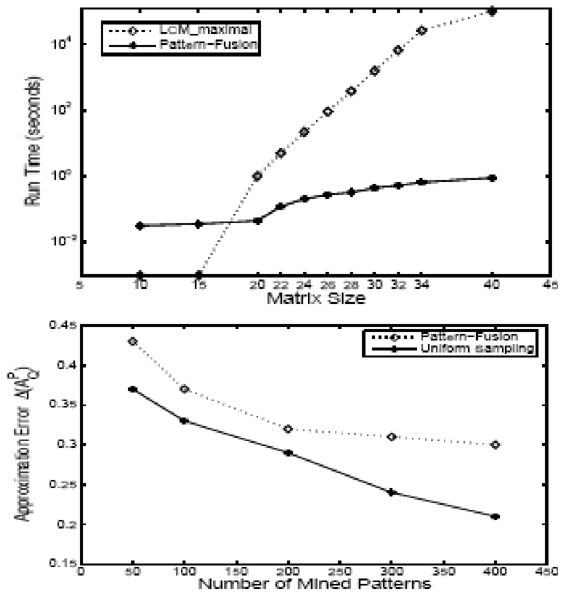
- Gearing toward colossal patterns
  - The larger the pattern, the greater the chance it will be generated
- Catching outliers
  - The more distinct the pattern, the greater the chance it will be generated

Experimental Setting

- Synthetic data set
  - $Diag_n$ an n x (n-1) table where i^{th} row has integers from 1 to n except i. Each row is taken as an itemset. min_support is n/2.
- Real data set
  - Replace: A program trace data set collected from the "replace" program, widely used in software engineering research
  - ALL: A popular gene expression data set, a clinical data on ALL-AML leukemia (www.broad.mit.edu/tools/data.html).
    - Each item is a column, representing the activitiy level of gene/protein in the same
    - Frequent pattern would reveal important correlation between gene expression patterns and disease outcomes

Experiment Results on $Diag_n$

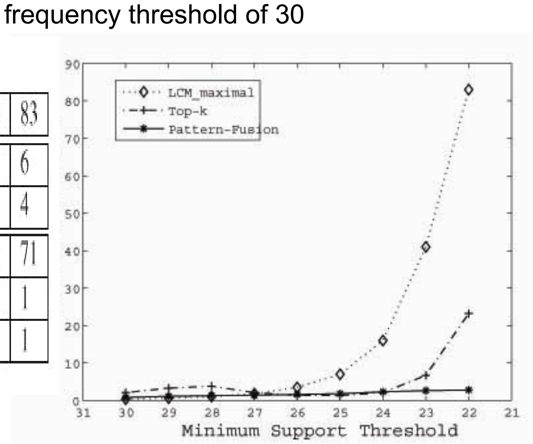
- LCM run time increases exponentially with pattern size n
- Pattern-Fusion finishes efficiently
- The approximation error of Pattern-Fusion (with min-sup 20) in comparison with the complete set) is rather close to uniform sampling (which randomly picks K patterns from the complete answer set)



Experimental Results on ALL

- ALL: A popular gene expression data set with 38 transactions, each with 866 columns
  - There are 1736 items in total
  - The table shows a high frequency threshold of 30

Pattern Size	110	107	102	91	86	84	83
The complete set	1	1	1	1	1	2	6
Pattern-Fusion	1	1	1	1	1	1	4
Pattern Size	82	77	76	75	74	73	71
The complete set	1	2	1	1	1	2	1
Pattern-Fusion	0	2	0	1	1	1	1

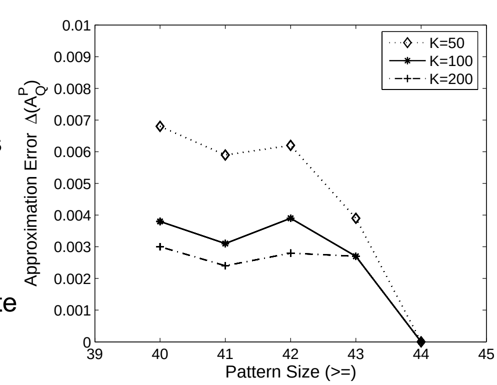


Experimental Results on REPLACE

- REPLACE
  - A program trace data set, recording 4395 calls and transitions
  - The data set contains 4395 transactions with 57 items in total
  - With support threshold of 0.03, the largest patterns are of size 44
  - They are all discovered by Pattern-Fusion with different settings of K and τ, when started with an initial pool of 20948 patterns of size <=3

Experimental Results on REPLACE

- Approximation error when compared with the complete mining result
- Example. Out of the total 98 patterns of size >=42, when K=100, Pattern-Fusion returns 80 of them
- A good approximation to the colossal patterns in the sense that any pattern in the complete set is on average at most 0.17 items away from one of these 80 patterns



Agenda

1. Session Overview
2. Mining Frequent Patterns, Association, and Correlations
3. Summary and Conclusion

Frequent-Pattern Mining: Summary

- Frequent pattern mining—an important task in data mining
- Scalable frequent pattern mining methods
  - Apriori (Candidate generation & test)
  - Projection-based (FPgrowth, CLOSET+, ...)
  - Vertical format approach (CHARM, ...)
- Mining a variety of rules and interesting patterns
- Constraint-based mining
- Mining sequential and structured patterns
- Extensions and applications

Frequent-Pattern Mining: Research Problems

- Mining fault-tolerant frequent, sequential and structured patterns
  - Patterns allows limited faults (insertion, deletion, mutation)
- Mining truly interesting patterns
  - Surprising, novel, concise, …
- Application exploration
  - E.g., DNA sequence analysis and bio-pattern classification
  - “Invisible” data mining

Ref: Basic Concepts of Frequent Pattern Mining

- (Association Rules) R. Agrawal, T. Imielinski, and A. Swami. Mining association rules between sets of items in large databases. SIGMOD'93.
- (Max-pattern) R. J. Bayardo. Efficiently mining long patterns from databases. SIGMOD'98.
- (Closed-pattern) N. Pasquier, Y. Bastide, R. Taouil, and L. Lakhal. Discovering frequent closed itemsets for association rules. ICDT'99.
- (Sequential pattern) R. Agrawal and R. Srikant. Mining sequential patterns. ICDE'95

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Further Improvements of Mining Methods

- AFOPT (Liu, et al. @ KDD'03)
  - A "push-right" method for mining condensed frequent pattern (CFP) tree
- Carpenter (Pan, et al. @ KDD'03)
  - Mine data sets with small rows but numerous columns
  - Construct a row-enumeration tree for efficient mining
- FPgrowth+ (Grahne and Zhu, FIMI'03)
  - Efficiently Using Prefix-Trees in Mining Frequent Itemsets, Proc. ICDM'03 Int. Workshop on Frequent Itemset Mining Implementations (FIMI'03), Melbourne, FL, Nov. 2003
- TD-Close (Liu, et al, SDM'06)

Assignments & Readings

- Readings

  - Chapter 5
- Individual Project #1
  - Ongoing

Next Session: Classification and Prediction