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Towards an axiomatic theory of probability
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Axioms of probability measure

- *Positivity*: $\mathbf{P}(A) \geq 0$.
- *Normalisation*: $\mathbf{P}(\Omega) = 1$.
- *Additivity*: If $\{A_j, j \geq 1\}$ are pairwise disjoint events, $\mathbf{P}\left(\bigcup_j A_j\right) = \sum_j \mathbf{P}(A_j)$.

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Basic properties of probability measure

- *The impossible event*: $\mathbf{P}(\emptyset) = 0$.
- *Boundedness*: $0 \leq \mathbf{P}(A) \leq 1$.
- *Monotonicity*: If $A \subseteq B$, then $\mathbf{P}(A) \leq \mathbf{P}(B)$.
- *Boole's inequality*: $\mathbf{P}(A \cup B) \leq \mathbf{P}(A) + \mathbf{P}(B)$,

$$\mathbf{P}(A_1 \cup A_2 \cup \cdots \cup A_n) \leq \mathbf{P}(A_1) + \mathbf{P}(A_2) + \cdots + \mathbf{P}(A_n).$$

- *Additivity*: $\mathbf{P}(A) + \mathbf{P}\,(A^c) = 1$.
- *Inclusion–exclusion*: $\mathbf{P}(A \cup B) = \mathbf{P}(A) + \mathbf{P}(B) - \mathbf{P}(A \cap B)$.

The probabilist's trinity $(\Omega, \mathcal{F}, \mathbf{P})$

- What is the collection of conceptual outcomes ω of the chance experiment?

  Sample space: Ω

- What are the events $A, B, C, \ldots$ of interest?

  Algebra of events: $\mathcal{F}$

- What are the chances associated with the events?

  Probability measure: $\mathbf{P}(\cdot)$