Question 3

True.

Proof:

Assume n is even. Let n = 2m, where m is an integer.

$n^2 + n + 1 = (2m)^2 + 2m + 1$

$= 4m^2 + 2m + 1$

$= 2(2m^2 + m) + 1$

$= $ odd.

Assume n is odd. Let n = 2m - 1, where m is an integer.

$n^2 + n + 1 = (2m - 1)^2 + (2m - 1) + 1$

$= 4m^2 - 4m + 1 + 2m - 1 + 1$

$= 4m^2 - 2m + 1$

$= 2(2m^2 - m) + 1$

$= $ odd.

Therefore $n^2 + n + 1$ is always odd.