Is the normal density properly normalised?

Is the normal density properly normalised?

$$\int_{-\infty}^{\infty} \frac{e^{-x^2/2}}{\sqrt{2\pi}}\, dx$$

Is the normal density properly normalised?

$$\left(\int_{-\infty}^{\infty} \frac{e^{-x^2/2}}{\sqrt{2\pi}}\,dx\right)^2 = \int_{-\infty}^{\infty} \frac{e^{-x^2/2}}{\sqrt{2\pi}}\,dx \cdot \int_{-\infty}^{\infty} \frac{e^{-y^2/2}}{\sqrt{2\pi}}\,dy$$

Is the normal density properly normalised?

$$\left(\int_{-\infty}^{\infty} \frac{e^{-x^2/2}}{\sqrt{2\pi}}\, dx\right)^2 = \int_{-\infty}^{\infty} \frac{e^{-x^2/2}}{\sqrt{2\pi}}\, dx \cdot \int_{-\infty}^{\infty} \frac{e^{-y^2/2}}{\sqrt{2\pi}}\, dy$$

$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-(x^2+y^2)/2}\, dy\, dx$$

Is the normal density properly normalised?

$$\left(\int_{-\infty}^{\infty} \frac{e^{-x^2/2}}{\sqrt{2\pi}}\,dx\right)^2 = \int_{-\infty}^{\infty} \frac{e^{-x^2/2}}{\sqrt{2\pi}}\,dx \cdot \int_{-\infty}^{\infty} \frac{e^{-y^2/2}}{\sqrt{2\pi}}\,dy$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-(x^2+y^2)/2}\,dy\,dx$$

rectangular-to-polar transformation

$x = r\cos\theta, \quad y = r\sin\theta, \quad dy\,dx = r\,d\theta\,dr$

Is the normal density properly normalised?

$$\left(\int_{-\infty}^{\infty} \frac{e^{-x^2/2}}{\sqrt{2\pi}}\,dx\right)^2 = \int_{-\infty}^{\infty} \frac{e^{-x^2/2}}{\sqrt{2\pi}}\,dx \cdot \int_{-\infty}^{\infty} \frac{e^{-y^2/2}}{\sqrt{2\pi}}\,dy$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-(x^2+y^2)/2}\,dy\,dx$$

$$= \frac{1}{2\pi}\int_{0}^{\infty}\int_{0}^{2\pi} e^{-r^2/2}\,r\,d\theta\,dr$$

rectangular-to-polar transformation

$x = r\cos\theta, \quad y = r\sin\theta, \quad dy\,dx = r\,d\theta\,dr$

Is the normal density properly normalised?

$$\left(\int_{-\infty}^{\infty} \frac{e^{-x^2/2}}{\sqrt{2\pi}}\, dx\right)^2 = \int_{-\infty}^{\infty} \frac{e^{-x^2/2}}{\sqrt{2\pi}}\, dx \cdot \int_{-\infty}^{\infty} \frac{e^{-y^2/2}}{\sqrt{2\pi}}\, dy$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-(x^2+y^2)/2}\, dy\, dx$$

rectangular-to-polar transformation

$x = r\cos\theta, \quad y = r\sin\theta, \quad dy\, dx = r\, d\theta\, dr$

$$= \frac{1}{2\pi}\int_{0}^{\infty}\int_{0}^{2\pi} e^{-r^2/2}\, r\, d\theta\, dr$$

$$= \frac{1}{2\pi}\int_{0}^{\infty} e^{-r^2/2}\, r \int_{0}^{2\pi} d\theta\, dr$$

Is the normal density properly normalised?

$$
\begin{aligned}
\left(\int_{-\infty}^{\infty} \frac{e^{-x^2/2}}{\sqrt{2\pi}}\,dx\right)^2 &= \int_{-\infty}^{\infty} \frac{e^{-x^2/2}}{\sqrt{2\pi}}\,dx \cdot \int_{-\infty}^{\infty} \frac{e^{-y^2/2}}{\sqrt{2\pi}}\,dy \\
&= \frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-(x^2+y^2)/2}\,dy\,dx \\
&= \frac{1}{2\pi}\int_{0}^{\infty}\int_{0}^{2\pi} e^{-r^2/2}\,r\,d\theta\,dr \\
&= \frac{1}{2\pi}\int_{0}^{\infty} e^{-r^2/2}\,r\int_{0}^{2\pi} d\theta\,dr \\
&= \frac{1}{2\pi}\int_{0}^{\infty} e^{-r^2/2}\,r\,dr \cdot 2\pi
\end{aligned}
$$

rectangular-to-polar transformation

$x = r\cos\theta, \quad y = r\sin\theta, \quad dy\,dx = r\,d\theta\,dr$

Is the normal density properly normalised?

$$\left(\int_{-\infty}^{\infty} \frac{e^{-x^2/2}}{\sqrt{2\pi}}\,dx\right)^2 = \int_{-\infty}^{\infty} \frac{e^{-x^2/2}}{\sqrt{2\pi}}\,dx \cdot \int_{-\infty}^{\infty} \frac{e^{-y^2/2}}{\sqrt{2\pi}}\,dy$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-(x^2+y^2)/2}\,dy\,dx$$

rectangular-to-polar transformation
$x = r\cos\theta, \quad y = r\sin\theta, \quad dy\,dx = r\,d\theta\,dr$

$$= \frac{1}{2\pi}\int_{0}^{\infty}\int_{0}^{2\pi} e^{-r^2/2}\,r\,d\theta\,dr$$

$$= \frac{1}{2\pi}\int_{0}^{\infty} e^{-r^2/2}\,r\int_{0}^{2\pi} d\theta\,dr$$

$$= \frac{1}{2\pi}\int_{0}^{\infty} e^{-r^2/2}\,r\,dr \cdot 2\pi$$

$$= \int_{0}^{\infty} e^{-r^2/2}\,r\,dr$$

Is the normal density properly normalised?

$$
\begin{aligned}
\left(\int_{-\infty}^{\infty} \frac{e^{-x^2/2}}{\sqrt{2\pi}}\,dx\right)^2 &= \int_{-\infty}^{\infty} \frac{e^{-x^2/2}}{\sqrt{2\pi}}\,dx \cdot \int_{-\infty}^{\infty} \frac{e^{-y^2/2}}{\sqrt{2\pi}}\,dy \\
&= \frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-(x^2+y^2)/2}\,dy\,dx \\
&= \frac{1}{2\pi}\int_{0}^{\infty}\int_{0}^{2\pi} e^{-r^2/2}\,r\,d\theta\,dr \\
&= \frac{1}{2\pi}\int_{0}^{\infty} e^{-r^2/2}\,r\int_{0}^{2\pi} d\theta\,dr \\
&= \frac{1}{2\pi}\int_{0}^{\infty} e^{-r^2/2}\,r\,dr \cdot 2\pi \\
&= \int_{0}^{\infty} e^{-r^2/2}\,r\,dr
\end{aligned}
$$

rectangular-to-polar transformation

$x = r\cos\theta, \quad y = r\sin\theta, \quad dy\,dx = r\,d\theta\,dr$

change of variable

$t = r^2/2, \quad dt = r\,dr$

Is the normal density properly normalised?

$$
\begin{aligned}
\left(\int_{-\infty}^{\infty} \frac{e^{-x^2/2}}{\sqrt{2\pi}}\,dx\right)^2 &= \int_{-\infty}^{\infty} \frac{e^{-x^2/2}}{\sqrt{2\pi}}\,dx \cdot \int_{-\infty}^{\infty} \frac{e^{-y^2/2}}{\sqrt{2\pi}}\,dy \\
&= \frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-(x^2+y^2)/2}\,dy\,dx \\
&= \frac{1}{2\pi}\int_{0}^{\infty}\int_{0}^{2\pi} e^{-r^2/2}\,r\,d\theta\,dr \\
&= \frac{1}{2\pi}\int_{0}^{\infty} e^{-r^2/2}\,r\int_{0}^{2\pi} d\theta\,dr \\
&= \frac{1}{2\pi}\int_{0}^{\infty} e^{-r^2/2}\,r\,dr\cdot 2\pi \\
&= \int_{0}^{\infty} e^{-r^2/2}\,r\,dr \\
&= \int_{0}^{\infty} e^{-t}\,dt
\end{aligned}
$$

rectangular-to-polar transformation

$x = r\cos\theta, \quad y = r\sin\theta, \quad dy\,dx = r\,d\theta\,dr$

change of variable

$t = r^2/2, \quad dt = r\,dr$

Is the normal density properly normalised?

$$\left(\int_{-\infty}^{\infty} \frac{e^{-x^2/2}}{\sqrt{2\pi}}\,dx\right)^2 = \int_{-\infty}^{\infty} \frac{e^{-x^2/2}}{\sqrt{2\pi}}\,dx \cdot \int_{-\infty}^{\infty} \frac{e^{-y^2/2}}{\sqrt{2\pi}}\,dy$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-(x^2+y^2)/2}\,dy\,dx$$

rectangular-to-polar transformation

$x = r\cos\theta, \quad y = r\sin\theta, \quad dy\,dx = r\,d\theta\,dr$

$$= \frac{1}{2\pi}\int_{0}^{\infty}\int_{0}^{2\pi} e^{-r^2/2}\,r\,d\theta\,dr$$

$$= \frac{1}{2\pi}\int_{0}^{\infty} e^{-r^2/2}\,r \int_{0}^{2\pi} d\theta\,dr$$

$$= \frac{1}{2\pi}\int_{0}^{\infty} e^{-r^2/2}\,r\,dr \cdot 2\pi$$

$$= \int_{0}^{\infty} e^{-r^2/2}\,r\,dr$$

change of variable

$t = r^2/2, \quad dt = r\,dr$

$$= \int_{0}^{\infty} e^{-t}\,dt$$

$$= -e^{-t}\Big|_{0}^{\infty} = -0 + 1 = 1$$