Introduction

This module is concerned with understanding the frequency domain representation of signals via a tool called Fourier transform. The Fourier transform is a fundamental tool in the arsenal of any engineer and it is used in virtually all scientific domains.

The ancient Greeks and, later on, Arab scientists were intrigued by the rainbow phenomenon and tried in vain to come up with an explanation for it. In 1300, a Dominican monk, Theoderich of Freiberg. held a bottle of water to the sun and recognized the colors of the rainbow, and understood the role of water drops in the phenomenon. It took another 300 years until Descartes and Newton gave a full explanation of how white light can be decomposed into colors. Newton, in particular, devised a crucial experiment proving once and for all that white light is composed of many fundamental frequencies associated with different colors. This was the birth of notion of spectrum.

The main character in the rest of the story is Joseph Fourier, a 19th-century French mathematician and physicist. Fourier provided the mathematics to really understand what Descartes and Newton described as physicists. Fourier proposed to decompose any well-behaved function into a sum of harmonic sines and cosines. How this could be done? He wrote longhand into a book called *Théorie Analytique de la Chaleur*. The reason Fourier was studying this was a physical problem. He wanted to understand the solution to the heat equation and it turns out that sines and cosines are eigenfunctions of the heat equation and therefore represent a very natural basis where to search for a solution of this partial differential equation. The original manuscript looks very complicated but today, thanks to efficient mathematical notation and electronic means to carry out the actual computations, we not only understand this quite easily but use it in everyday life (although, probably, you're not even aware of it!)

In this first lesson we will talk about Fourier analysis first from an intuitive point of view. We will start by introducing the concept of frequency domain. We will then see that we can use the concept of vector space to explain Fourier analysis as a change of basis in the space of finite-length signals. To this end, we will study the so-called Fourier basis for this space.