Maxcut

Property 1

LP(G) at least

$\#\{\text{edges}\} \cdot 0.99$

Random graphs

$$G \sim G(n, C/n)$$

$$\Pr(\#\{\text{cycles of length at most } g-1\} < n^{1/2}) < 0.5$$

Markov's Inequality

X: non-negative random variable
a: positive real number

$$\mathbf{Pr}(\mathbf{X} \geq \mathbf{a}) \leq \frac{\mathbf{E}(\mathbf{X})}{\mathbf{a}}$$

Proof

Step function $$\mathbf{H_a(x)} = \begin{cases} \mathbf{a} & \mathbf{if\ x \geq a,} \\ \mathbf{0} & \mathbf{if\ x < a.} \end{cases}$$

$$\mathbf{X \geq H_a(X),}$$

$$\mathbf{E(X) \geq E(H_a(X)) = a \cdot Pr(X \geq a).}$$

Goal: Pr(#{cycles of length at most g-1}>$n^{1/2}$) < 0.5

Use Markov's inequality!

G~ G(n,C/n) such that n=C^{3g}

X = #{cycles of length at most g-1}, a=$C^{1.5g}$

$$\Pr(X \geq C^{1.5g}) \leq \frac{E(X)}{C^{1.5g}}$$

$$\mathbf{E(X)} \le \sum_{\ell=3}^{g-1} \frac{n^\ell}{\ell}\left(\frac{C}{n}\right)^\ell = \sum_{\ell=3}^{g-1} \frac{C^\ell}{\ell} \le C^g,$$

Hence, $\Pr(X \ge C^{1.5g}) \le \frac{E(X)}{C^{1.5g}} \le \frac{C^g}{C^{1.5g}} = \frac{1}{C^{g/2}} \le \frac{1}{n^{1/6}},$

Choosing n>(0.5)^6=64, $\Pr(X < n^{1/2}) > 0.5.$

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