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The birthday paradox

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- Another balls and urns setting: n people (balls), 365 days (urns). Outcomes: birthdays arranged in an ordered list form an ordered n-tuple $(k_1, k_2, \ldots, k_n)$ of days of the year (we ignore leap years) where *sampling is with replacement*. We assume random sampling (this at least gives a conservative estimate of the chances of coincidence).

- The number of possible outcomes: 365^n.

- The complement of the event that there is a coincidence of birthdays consists of those outcomes $(k_1, k_2, \ldots, k_n)$ where each of $k_1, k_2, \ldots, k_n$ is a distinct day of the year. This corresponds to *sampling without replacement*. The number of outcomes favourable to the event that each person has a distinct birthday is $365^{\underline{n}} = 365 \cdot (365 - 1) \cdot (365 - 2) \cdots (365 - n + 1)$.

- The probability of a birthday coincidence is $p(n) := 1 - \frac{365^{\underline{n}}}{365^n}$.

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n	$p(n)$
5	0.0271
10	0.1169
15	0.2529
20	0.4114
23	0.5073
30	0.7063
50	0.9704
70	0.9992