X Lessons

The Lucas numbers

The Lucas numbers are closely related to the Fibonacci numbers and satisfy the same recursion relation $L_{n+1} = L_n + L_{n-1}$, but with starting values $L_1 = 1$ and $L_2 = 3$. The Lucas numbers are an example of a generalized Fibonacci sequence that satisfies $f_{n+1} = f_n + f_{n-1}$ with starting values $f_1 = p$ and $f_2 = q$.

(a) Using mathematical induction, prove that $f_{n+2} = F_n p + F_{n+1} q$.

(b) Form a table containing the first 12 Fibonacci and Lucas numbers that can be referred to in the next two problems.

(c) Prove that $L_n = F_{n-1} + F_{n+1}$.

(d) Prove that $F_n = \frac{1}{5}\left(L_{n-1} + L_{n+1}\right)$.

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There are two ways you can submit your answer to this prompt:

*1) **Easier option:** You can write up the proof on a piece of paper, take a picture or scan it, and then post it as your reply by clicking on the Camera or Picture icon. Make sure the proof is legible so others can comment on it.*

2) Advanced option: *You can type in your answer using Mathjax input. Read more here - Mathjax Basic Tutorial.*
Important Notes: *Coursera requires you to use double $$ at the start and end of each expression (single $ does not work). HKUST cannot support any technical problems encountered with Mathjax input in Coursera.*

Participation is optional

Type your response here...

WK   +4

8 learners have submitted a response.