Working backwards: s_n

$$s_n = f_1 + f_2 + \cdots + f_n$$

$$f_n = u_n - f_1 u_{n-1} - f_2 u_{n-2} - \cdots - f_j u_{n-j} - \cdots - f_{n-1} u_1$$

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n	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
u_n	0	0	0	0	$\frac{1}{32}$	$\frac{1}{64}$	$\frac{1}{64}$	$\frac{1}{64}$	$\frac{1}{64}$	$\frac{17}{1024}$	$\frac{33}{2048}$	$\frac{33}{2048}$	$\frac{33}{2048}$	$\frac{33}{2048}$	$\frac{529}{32768}$	$\frac{1057}{65536}$	$\frac{1057}{65536}$	$\frac{1057}{65536}$	$\frac{1057}{65536}$	$\frac{16913}{1048576}$
f_n	0	0	0	0	$\frac{1}{32}$	$\frac{1}{64}$	$\frac{1}{64}$	$\frac{1}{64}$	$\frac{1}{64}$	$\frac{1}{64}$	$\frac{31}{2048}$	$\frac{61}{4096}$	$\frac{15}{1024}$	$\frac{59}{4096}$	$\frac{29}{2048}$	$\frac{57}{4096}$	$\frac{1793}{131072}$	$\frac{3525}{262144}$	$\frac{3465}{262144}$	$\frac{1703}{131072}$

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Slogan: If you know $\{f_1, f_2, \ldots, f_n, \ldots\}$ then you know $\{s_1, s_2, \ldots, s_n, \ldots\}$!

n	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
u_n	0	0	0	0	$\frac{1}{32}$	$\frac{1}{64}$	$\frac{1}{64}$	$\frac{1}{64}$	$\frac{1}{64}$	$\frac{17}{1024}$	$\frac{33}{2048}$	$\frac{33}{2048}$	$\frac{33}{2048}$	$\frac{33}{2048}$	$\frac{529}{32768}$	$\frac{1057}{65536}$	$\frac{1057}{65536}$	$\frac{1057}{65536}$	$\frac{1057}{65536}$	$\frac{16913}{1048576}$
f_n	0	0	0	0	$\frac{1}{32}$	$\frac{1}{64}$	$\frac{1}{64}$	$\frac{1}{64}$	$\frac{1}{64}$	$\frac{1}{64}$	$\frac{31}{2048}$	$\frac{61}{4096}$	$\frac{15}{1024}$	$\frac{59}{4096}$	$\frac{29}{2048}$	$\frac{57}{4096}$	$\frac{1793}{131072}$	$\frac{3525}{262144}$	$\frac{3465}{262144}$	$\frac{1703}{131072}$

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n	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
u_n	0	0	0	0	$\frac{1}{32}$	$\frac{1}{64}$	$\frac{1}{64}$	$\frac{1}{64}$	$\frac{1}{64}$	$\frac{17}{1024}$	$\frac{33}{2048}$	$\frac{33}{2048}$	$\frac{33}{2048}$	$\frac{33}{2048}$	$\frac{529}{32768}$	$\frac{1057}{65536}$	$\frac{1057}{65536}$	$\frac{1057}{65536}$	$\frac{1057}{65536}$	$\frac{16913}{1048576}$
f_n	0	0	0	0	$\frac{1}{32}$	$\frac{1}{64}$	$\frac{1}{64}$	$\frac{1}{64}$	$\frac{1}{64}$	$\frac{1}{64}$	$\frac{31}{2048}$	$\frac{61}{4096}$	$\frac{15}{1024}$	$\frac{59}{4096}$	$\frac{29}{2048}$	$\frac{57}{4096}$	$\frac{1793}{131072}$	$\frac{3525}{262144}$	$\frac{3465}{262144}$	$\frac{1703}{131072}$

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n	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
u_n	0	0	0	0	$\frac{1}{32}$	$\frac{1}{64}$	$\frac{1}{64}$	$\frac{1}{64}$	$\frac{1}{64}$	$\frac{17}{1024}$	$\frac{33}{2048}$	$\frac{33}{2048}$	$\frac{33}{2048}$	$\frac{33}{2048}$	$\frac{529}{32768}$	$\frac{1057}{65536}$	$\frac{1057}{65536}$	$\frac{1057}{65536}$	$\frac{1057}{65536}$	$\frac{16913}{1048576}$
f_n	0	0	0	0	$\frac{1}{32}$	$\frac{1}{64}$	$\frac{1}{64}$	$\frac{1}{64}$	$\frac{1}{64}$	$\frac{1}{64}$	$\frac{31}{2048}$	$\frac{61}{4096}$	$\frac{15}{1024}$	$\frac{59}{4096}$	$\frac{29}{2048}$	$\frac{57}{4096}$	$\frac{1793}{131072}$	$\frac{3525}{262144}$	$\frac{3465}{262144}$	$\frac{1703}{131072}$

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$$s_6 = f_1 + f_2 + f_3 + f_4 + f_5 + f_6 = 2^{-5} + 2^{-6} = 2^{-6}(2+1) = 3 \cdot 2^{-6}$$

n	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
u_n	0	0	0	0	$\frac{1}{32}$	$\frac{1}{64}$	$\frac{1}{64}$	$\frac{1}{64}$	$\frac{1}{64}$	$\frac{17}{1024}$	$\frac{33}{2048}$	$\frac{33}{2048}$	$\frac{33}{2048}$	$\frac{33}{2048}$	$\frac{529}{32768}$	$\frac{1057}{65536}$	$\frac{1057}{65536}$	$\frac{1057}{65536}$	$\frac{1057}{65536}$	$\frac{16913}{1048576}$
f_n	0	0	0	0	$\frac{1}{32}$	$\frac{1}{64}$	$\frac{1}{64}$	$\frac{1}{64}$	$\frac{1}{64}$	$\frac{1}{64}$	$\frac{31}{2048}$	$\frac{61}{4096}$	$\frac{15}{1024}$	$\frac{59}{4096}$	$\frac{29}{2048}$	$\frac{57}{4096}$	$\frac{1793}{131072}$	$\frac{3525}{262144}$	$\frac{3465}{262144}$	$\frac{1703}{131072}$

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$$s_7 = f_1 + f_2 + f_3 + f_4 + f_5 + f_6 + f_7 = 2^{-5} + 2^{-6} + 2^{-6} = 2^{-6}(2+1+1) = 2^{-4}$$

n	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
u_n	0	0	0	0	$\frac{1}{32}$	$\frac{1}{64}$	$\frac{1}{64}$	$\frac{1}{64}$	$\frac{1}{64}$	$\frac{17}{1024}$	$\frac{33}{2048}$	$\frac{33}{2048}$	$\frac{33}{2048}$	$\frac{33}{2048}$	$\frac{529}{32768}$	$\frac{1057}{65536}$	$\frac{1057}{65536}$	$\frac{1057}{65536}$	$\frac{1057}{65536}$	$\frac{16913}{1048576}$
f_n	0	0	0	0	$\frac{1}{32}$	$\frac{1}{64}$	$\frac{1}{64}$	$\frac{1}{64}$	$\frac{1}{64}$	$\frac{1}{64}$	$\frac{31}{2048}$	$\frac{61}{4096}$	$\frac{15}{1024}$	$\frac{59}{4096}$	$\frac{29}{2048}$	$\frac{57}{4096}$	$\frac{1793}{131072}$	$\frac{3525}{262144}$	$\frac{3465}{262144}$	$\frac{1703}{131072}$

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$$s_7 = f_1 + f_2 + f_3 + f_4 + f_5 + f_6 + f_7 = 2^{-5} + 2^{-6} + 2^{-6} = 2^{-6}(2+1+1) = 2^{-4}$$

n	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
u_n	0	0	0	0	$\frac{1}{32}$	$\frac{1}{64}$	$\frac{1}{64}$	$\frac{1}{64}$	$\frac{1}{64}$	$\frac{17}{1024}$	$\frac{33}{2048}$	$\frac{33}{2048}$	$\frac{33}{2048}$	$\frac{33}{2048}$	$\frac{529}{32768}$	$\frac{1057}{65536}$	$\frac{1057}{65536}$	$\frac{1057}{65536}$	$\frac{1057}{65536}$	$\frac{16913}{1048576}$
f_n	0	0	0	0	$\frac{1}{32}$	$\frac{1}{64}$	$\frac{1}{64}$	$\frac{1}{64}$	$\frac{1}{64}$	$\frac{1}{64}$	$\frac{31}{2048}$	$\frac{61}{4096}$	$\frac{15}{1024}$	$\frac{59}{4096}$	$\frac{29}{2048}$	$\frac{57}{4096}$	$\frac{1793}{131072}$	$\frac{3525}{262144}$	$\frac{3465}{262144}$	$\frac{1703}{131072}$
s_n	0	0	0	0	$\frac{1}{32}$	$\frac{3}{64}$	$\frac{1}{16}$	$\frac{5}{64}$	$\frac{3}{32}$	$\frac{7}{64}$	$\frac{255}{2048}$	$\frac{571}{4096}$	$\frac{631}{4096}$	$\frac{345}{2048}$	$\frac{187}{1024}$	$\frac{805}{4096}$	$\frac{27533}{131072}$	$\frac{58631}{262144}$	$\frac{3881}{16384}$	$\frac{32751}{131072}$

Taking stock

$$s_n = f_1 + f_2 + \cdots + f_n$$

$$f_n = u_n - f_1 u_{n-1} - f_2 u_{n-2} - \cdots - f_j u_{n-j} - \cdots - f_{n-1} u_1$$

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n	10	20	30	40	50	60	70	80	90	100
s_n	0.1094	0.2499	0.3682	0.4679	0.5519	0.6226	0.6821	0.7323	0.7745	0.8101

Taking stock

$$s_n = f_1 + f_2 + \cdots + f_n$$

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n	10	20	30	40	50	60	70	80	90	100
s_n	0.1094	0.2499	0.3682	0.4679	0.5519	0.6226	0.6821	0.7323	0.7745	0.8101

Slogan

It is not at all unlikely to see a success run of length five in a sequence of coin tosses.