The usual culprits

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integer parameter n

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- Natural model of randomness in finite settings; urn models.
- Probability calculations are combinatorial: if $A \subseteq \{1, 2, \ldots, n\}$ then

$$\mathbf{P}(A) = \frac{1}{n} \cdot \text{card}(A) = \frac{\text{\# of outcomes favorable for } A}{\text{total \# of outcomes}}$$

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integer parameter n, real parameter $0 < p < 1$; $q = 1 - p$

With p co-opted to represent a parameter of the distribution, introduce new notation $b(k)$ instead of $p(k)$ for the mass function.

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- Models polls, accumulated successes in repeated trials. Arises in statistical tests.

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real parameter $\lambda > 0$

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- Models rare events, point processes, arrivals and departures in queues.

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- Models waiting times, time to failure, run lengths.