Lecture 13. Use and Interpretation of Dummy Variables

Stop worrying for 1 lecture and learn to appreciate the uses that "dummy variables" can be put to

Using dummy variables to measure average differences

Using dummy variables when more than 2 discrete categories

Using dummy variables for policy analysis

Using dummy variables to net out seasonality

Dummy variables – where the variable takes only one of two values – are useful tools in econometrics, since often interested in variables that are *qualitative* rather than *quantitative*

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In practice this means interested in variables that split the sample into two distinct groups in the following way

D = 1 if the criterion is satisfied

D = 0 if not

Eg. Male/Female so that the dummy variable "Male" would be coded 1 if male and 0 if female

(though could equally create another variable "Female" coded 1 if female and 0 if male)

Model is $LnW = b_0 + b_1Age + b_2Male$

where Male = 1 or 0

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where Male = 1 or 0

For men therefore the predicted wage

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For women

$$LnW_{women} = b_0 + b_1 Age + b_2*(0)$$

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$$LnW = b_0 + b_1Age + b_2Male$$

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For men therefore the predicted wage

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$$= b_0 + b_1 Age + b_2$$

For women

$$LnW_{women} = b_0 + b_1 Age + b_2*(0)$$

$$b_0 + b_1 Age$$

$$\overline{\hat{Y}} = \overline{Y}$$

(see lecture 2)

So in the case of a regression model with log wages as the dependent variable, $LnW = b_0 + b_1Age + b_2Male$

the average of the fitted values equals the average of log wages

$$Ln(W) = L\overline{n}W$$

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$$\overline{\wedge}$$
 $\overline{\wedge}$ $\overline{\wedge}$ $LnW^{men} - LnW^{women} = LnW^{men} - LnW^{women}$

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$$= LnW \underset{holdsymbol}{men} - LnW \underset{women}{women}$$

$$= b_0 + b_1 Age + b_2 - b_0 + b_1 Age$$

$$= b_2$$

which is just the coefficient on the male dummy variable

It also follows that the constant, b_0 , measures the intercept of default group (women) with age set to zero and $b_0 + b_2$ is the intercept for men

The (average) difference in pay between men and women is then

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which is just the coefficient on the male dummy variable

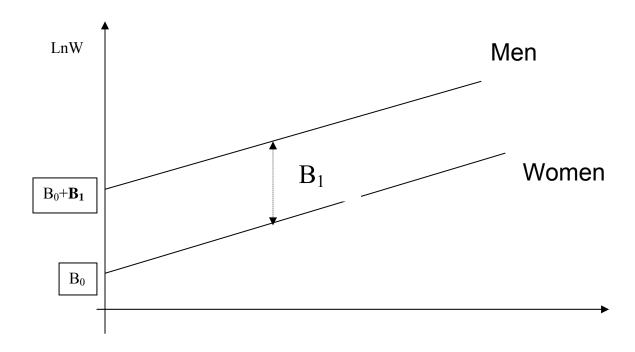
So the coefficients on dummy variables measure the average difference between the group coded with the value "1" and the group coded with the value "0" (the "default" or "base group")

It also follows that the constant, b_0 , now measures the notional value of the dependent variable (in this case log wages) of the default group (in this case women) with age set to zero

and b₀ + b₂ is the intercept and notional value of log wages at age zero for men

So to measure *average* difference between two groups

 $LnW = \beta_0 + \beta_1 Group Dummy$



A simple regression of the log of hourly wages on age using the data set ps4data.dta gives

. reg lhwa	age age							
Source	SS	df	MS			Number of obs	=	12098
	+			_		F(1, 12096)	=	235.55
Model	75.4334757	1	75.433475	7		Prob > F	=	0.0000
Residual	3873.61564	12096	.32023938	8		R-squared	=	0.0191
	+			_		Adj R-squared	=	0.0190
Total	3949.04911	12097	.32644863	3		Root MSE	=	.5659
lhwage	Coef.	Std. I	 Err.	t	P> t	[95% Conf.	Int	erval]
age	.0070548	.00045	597 15	.348	0.000	.0061538	. (0079558
_cons	1.693719	.01869	945 90	.600	0.000	1.657075	1	.730364

Now introduce a male dummy variable (1= male, 0 otherwise) as an **intercept dummy.** This specification says the slope effect (of age) is the same for men and women, but that the intercept (or the **average difference** in pay between men and women) is different

. reg lhw age male

Source	SS	df	MS		Number of obs	
Model Residual 	264.053053 3684.99606 3949.04911	12095 .304	.026526 4671026 5448633		F(2, 12095) Prob > F R-squared Adj R-squared Root MSE	= 433.34 = 0.0000 = 0.0669 = 0.0667 = .55197
lhw	Coef.	Std. Err.	t	P> t	[95% Conf.	Interval]
age male _cons	.0066816 .2498691 1.583852	.0004486 .0100423 .0187615	14.89 24.88 84.42	0.000 0.000 0.000	.0058022 .2301846 1.547077	.0075609 .2695537 1.620628

Hence

average wage difference between men and women $=(b_0 - (b_0 + b_2)) = b_2 = 25\%$ more on average

Note that if we define a dummy variables as female (1= female, 0 otherwise) then

. reg lhwage age female									
Source	SS	df		MS		Number of obs	=	12098	
	+					F(2, 12095)	=	433.34	
Model	264.053053	2	132.	026526		Prob > F	=	0.0000	
Residual	3684.99606	12095	.304	671026		R-squared	=	0.0669	
	+					Adj R-squared	=	0.0667	
Total	3949.04911	12097	.326	448633		Root MSE	=	.55197	
lhwage	Coef.	Std.	Err.	t	P> t	[95% Conf.	In	terval]	
	+								
age	.0066816	.0004	1486	14.89	4 0.000	.0058022		0075609	
female	2498691	.0100)423	-24.882	2 0.000	2695537		2301846	
_cons	1.833721	.0190	0829	96.093	3 0.000	1.796316	1	.871127	

The coefficient estimate on the dummy variable is the same but the sign of the effect is reversed (now negative). This is because the reference (default) category in this regression is now men

Model is now
$$LnW = b_0 + b_1Age + b_2female$$

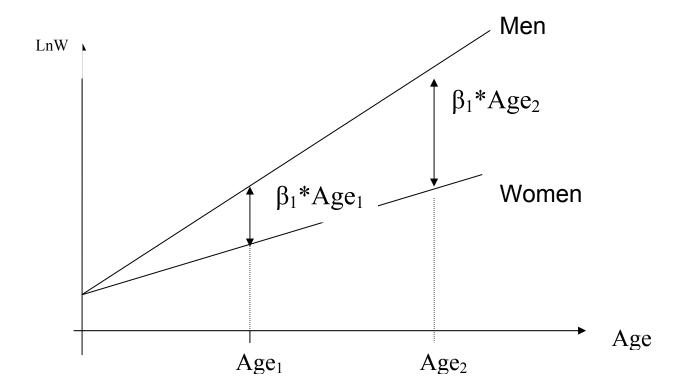
so constant, b_0 , measures average earnings of default group (men) and $b_0 + b_2$ is average earnings of women

So now

average wage difference between men and women
$$=(b_0 - (b_0 + b_2)) = b_2 = -25\%$$
 less on average

Hence it does not matter which way the dummy variable is defined as long as you are clear as to the appropriate reference category.

2) To measure *Difference in Slope Effects* between two groups $LnW = \beta_0 + \beta_1 Group Dummy*Slope Variable$



(Dummy Variable Interaction Term)

Model is now
$$LnW = b_0 + b_1Age + b_2Female*Age$$

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 if female = 0

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$$dLnW/dAge = b_1$$
 if female = 0
= $b_1 + b_2$ if female = 1

So effect of 1 extra year of age on earnings

= .0097 if male

= (.0097 - .0065) if female

Can include both an intercept and a slope dummy variable in the same regression to decide whether differences were caused by differences in intercepts or the slope variables

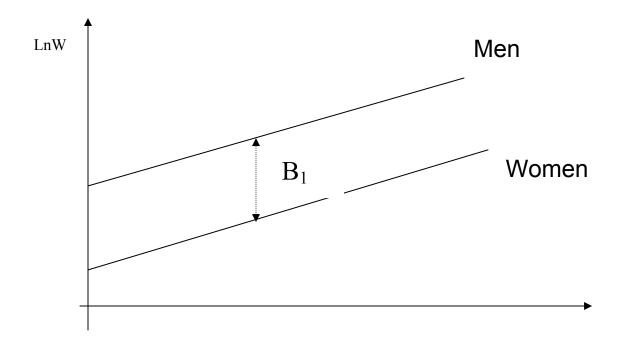
. reg lhwa	age age female SS	e femage df	MS	_	Number of obs		12098 311.80
Model Residual	283.506857 3665.54226		4.5022855		Prob > F R-squared Adj R-squared	=	0.0000 0.0718 0.0716
Total	3949.04911 	12097 .	326448633	3	Root MSE	=	.55053
lhwage	Coef.	Std. Er	r.	t P> t	[95% Conf.	In	terval]
age female femage _cons	.0100393 .0308822 0071846 1.701176	.000613 .036446 .000896	5 0. 8 -8.	.376 0.000 .847 0.397 .012 0.000 .457 0.000	0405588 0089425		.011241 1023233 0054268 .750608

In this example the average differences in pay between men and women appear to be driven by factors which cause the slopes to differ (ie the rewards to extra years of experience are much lower for women than men)- Note that this model is equivalent to running separate regressions for men and women – since allowing both intercept and slope to vary

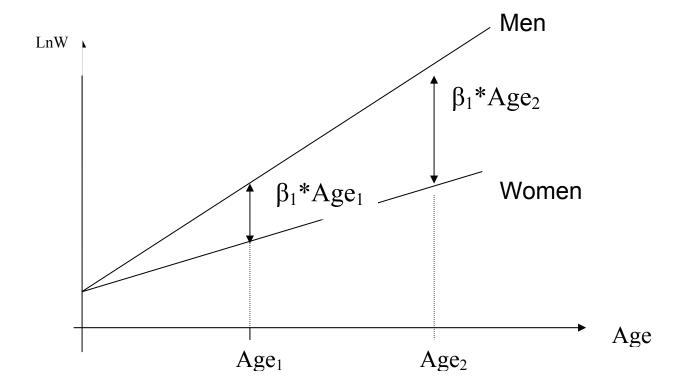
Using & Understanding Dummy Variables

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D_{South} = 1 if live in the South, 0 otherwise

However

As a rule should always include **one less** dummy variable in the model than there are categories, otherwise will introduce multicolinearity into the model

Example of Dummy Variable Trap

Suppose interested in estimating the effect of (5) different qualifications on pay

A regression of the log of hourly earnings on dummy variables for each of 5 education categories gives the following output

. reg lhwa	age age postgi	ad grad hig	hint low no	one		
Source	SS	df	MS		Number of obs	= 12098
	+				F(5, 12092)	= 747.70
Model	932.600688	5 186.	520138		Prob > F	= 0.0000
Residual	3016.44842	12092 .249	458189		R-squared	= 0.2362
	+				Adj R-squared	= 0.2358
Total	3949.04911	12097 .326	448633		Root MSE	= .49946
lhwage	Coef.	Std. Err.	t	P> t	[95% Conf.	<pre>Interval]</pre>
	+					
age	.010341	.0004148	24.931	0.000	.009528	.0111541
postgrad	(dropped)					
grad	0924185	.0237212	-3.896	0.000	1389159	045921
highint	4011569	.0225955	-17.754	0.000	4454478	356866
low	6723372	.0209313	-32.121	0.000	7133659	6313086
none	9497773	.0242098	-39.231	0.000	9972324	9023222
_cons	2.110261	.0259174	81.422	0.000	2.059459	2.161064

5 dummy variables exhaust the set of possible categories, so the sum of these 5 dummy variables is always one for each observation in the data set.

Obs. Constant postgrad grad higher low noquals Sum

Obs.	Constant	postgrad	d grad	higher	low	noquals	Sum
1	1	1	0	0	0	0	

Obs.	Constant	postgrad	d grad	higher	low	noquals	Sum
1	1	1	0	0	0	0	1

Obs.	Constant	postg	rad grad	higher	low	noquals	Sum
1	1	1	0	0	0	0	1
2	1	0	1	0	0	0	

Obs.	Constant	postg	rad grad	higher	low	noquals	Sum
1	1	1	0	0	0	0	1
2	1	0	1	0	0	0	1

Obs.	Constant	t postgi	rad grad	higher	low	noquals	Sum
1	1	1	0	0	0	0	1
2	1	0	1	0	0	0	1
3	1	0	0	0	0	1	1

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1	1	1	0	0	0	0	1
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Given the presence of a constant using 5 dummy variables leads to pure multicolinearity, becase the sum=1 which is the same as the value of the constant)

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3	1	0	0	0	0	1	1

Given the presence of a constant using 5 dummy variables leads to pure multicolinearity, (the sum=1 = value of the constant)

So can't include all 5 dummies and the constant in the same model

Obs.	Constant	postgrad	grad	higher	low
1	1	1	0	0	0
2	1	0	1	0	0
3	1	0	0	0	0

Obs.	Constant	postgrad	grad	higher	low	Sum of dummies
1	1	1	0	0	0	
2	1	0	1	0	0	
3	1	0	0	0	0	

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1	1	1	0	0	0	1
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1	1	1	0	0	0	1
2	1	0	1	0	0	1
3	1	0	0	0	0	0

Suppose drop the no quals dummy in the example above, the model is then

Obs.	Constar	nt postgrad	grad	higher	low	Sum of dummies
1	<mark>1</mark>	1	0	0	0	<mark>1</mark>
2	<mark>1</mark>	0	1	0	0	<mark>1</mark>
3	<mark>1</mark>	0	0	0	0	<mark>O</mark>

and so the sum is no longer collinear with the constant

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Doesn't matter which one you drop, though convention says drop the dummy variable corresponding to the most common category.

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3	<mark>1</mark>	0	0	0	0	<mark>O</mark>

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Doesn't matter which one you drop, though convention says drop the dummy variable corresponding to the most common category.

However changing the "default" category does change the coefficients, since all dummy variables are measured relative to this default reference category

Example: Dropping the postgraduate dummy (which Stata did automatically before when faced with the dummy variable trap) just replicates the above results. All the education dummy variables pay effects are measured relative to the missing postgraduate dummy variable (which effectively is now picked up by the constant term)

	reg lhw age	grad highint	low no	ne					
	Source	SS	df		MS		Number of obs	=	12098
-		+					F(5, 12092)	=	747.70
	Model	932.600688	5	186	.520138		Prob > F	=	0.0000
	Residual	3016.44842	12092	.249	9458189		R-squared	=	0.2362
_		+					Adj R-squared	=	0.2358
	Total	3949.04911	12097	.326	5448633		Root MSE	=	.49946
-									
	lhw	Coef.	Std.	Err.	t	P> t	[95% Conf.	In	terval]
-		+							
	age	.010341	.0004	148	24.93	0.000	.009528		0111541
	grad	0924185	.0237	212	-3.90	0.000	1389159	_	.045921
	highint	4011569	.0225	955	-17.75	0.000	4454478	_	.356866
	low	6723372	.0209	313	-32.12	0.000	7133659		6313086
	none	9497773	.0242	098	-39.23	0.000	9972324		9023222
	_cons	2.110261	.0259	174	81.42	0.000	2.059459	2	.161064

coefficients on education dummies are all negative since all categories earn less than the default group of postgraduates

Changing the default category to the no qualifications group gives

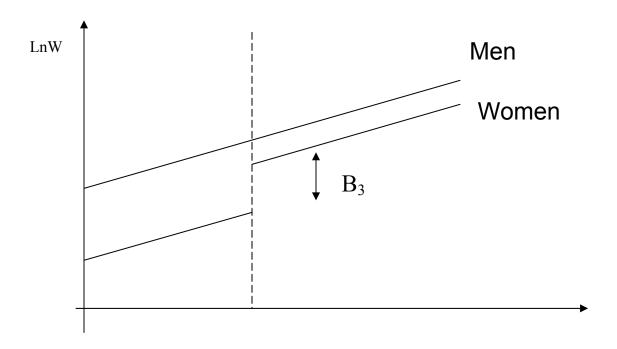
reg lhw age	postgrad grad	d highint	Tow				
Source	SS	df	MS		Number of obs	=	12098
 	+				F(5, 12092)	=	747.70
Model	932.600688	5 18	6.520138		Prob > F	=	0.0000
Residual	3016.44842	12092 .2	49458189		R-squared	=	0.2362
 	+				Adj R-squared	=	0.2358
Total	3949.04911	12097 .3	26448633		Root MSE	=	.49946
lhw	Coef.	Std. Err	·. t	P> t	[95% Conf.	Int	terval]
 	+						
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postgrad	.9497773	.0242098	39.23	0.000	.9023222		9972324
grad	.8573589	.0189204	45.31	0.000	.8202718		.894446
highint	.5486204	.0174109	31.51	0.000	.5144922	. !	5827486
low	.2774401	.0151439	18.32	0.000	.2477555	•	3071246
_cons	1.160484	.0231247	50.18	0.000	1.115156	1	.205812

and now the coefficients are all positive (relative to those with no quals.)

Dummy Variables and Policy Analysis

One important practical use of a regression is to try and evaluate the "treatment effect" of a policy intervention.

3) To Measure Effects of *Change in the Average Behaviour* of two groups, one subject to a policy the other not (the Difference-in-Difference Estimator)



Treatment/Policy affects only **a sub-section** of the population Eg A drug, EMA, Change in Tuition Fees, Minimum Wage

and may lead to a change in behaviour for the treated group - as captured by a change in the intercept (or slope) **after** the intervention (treatment) takes place

Dummy Variables and Policy Analysis

One important practical use of a regression is to try and evaluate the "treatment effect" of a policy intervention.

Usually this means comparing outcomes for those affected by a policy that is of concern to economists

Eg a law on taxing cars in central London – creates a "treatment" group, (eg those who drive in London) and those not, (the "control" group).

Other examples targeted tax cuts, minimum wages, area variation in schooling practices, policing

In principle one could set up a dummy variable to denote membership of the treatment group (or not) and run the following regression

$$LnW = a + b*Treatment Dummy + u$$
 (1)

In principle one could set up a dummy variable to denote membership of the treatment group (or not) and run the following regression

$$LnW = a + b*Treatment Dummy + u$$
 (1)

where Treatment = 1 if exposed to a treatment = 0 if not

reg price newham if time>3 & (newham==1 | croydon==1) reg price newham if time<=3 & (newham==1 | croydon==1) reg price newham after afternew if time>3 & (newham==1 | croydon==1)

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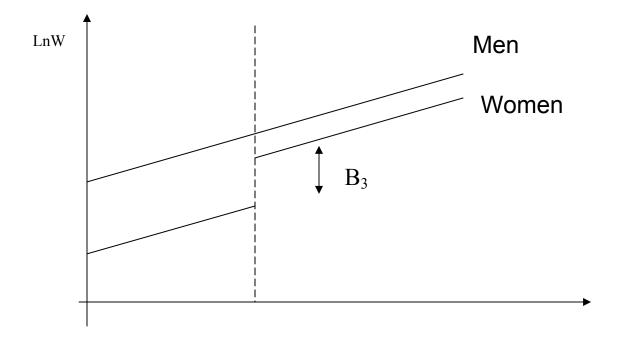
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- the "difference in difference estimator"



 $[Y_t^2 - Y_t^1]$ = Effect of Policy + other influences

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(assuming the effect of other influences is the same for both treatment and control groups)

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(assuming the effect of other influences is the same for both treatment and control groups)

Hence the need to try and choose a control group that is similar to the treatment group (apart from the experience of the treatment)

In practice this estimator can be obtained by combining (pooling) the data over the periods before and after and running the following regression

LnW = a + a₂After + b₁Treatment Dummy + b₂After*Treatment Dummy

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a is the average wage of the control group in the base year, a_2 , is the average wage of the control group in the second year, b_1 gives the difference on wages between treatment and control group in the base year

b₂ is the "difference in difference" estimator – the change in wages for the treatment group relative to the control in the second period.

Why?

LnW = a + a₂After + b₁Treatment Dummy + b₂After*Treatment Dummy

 $LnW = a + a_2After + b_1Treatment Dummy + b_2After*Treatment Dummy$ If After=0 and Treatment Dummy = 0

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$$(a + a2 + b_1 + b_2) - (a + b_1)$$

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So the change in wages for the treatment group is

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and the change in wages for the control group is

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so the "difference in difference" estimator

Change in wages for treatment – change in wages for control

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= Change in wages for treatment - change in wages for control

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Example In July 2005, London won the rights to host the 2012 Olympic games. Shortly afterward there were media reports that house prices were rising "fast" in areas close to the Olympic site. Can evaluate whether this was true by using Newham as the Treatment area (the borough in which the Olympic site is located) and a similar London borough further away from the site as a "control".

The data set *olympics.dta* has monthly data on house prices over time in Newham & Hounslow. The dummy variable "newham" takes the value 1 if the house price observation is from Newham and 0 if not. The dummy variable "after" takes the value 1 if the month was after the Olympic announcement and 0 otherwise. The interaction term "afternew"

g afternew=after*newham

takes the value 1 only if the month is after the event and the observation is in Newham. The coefficient on this term will be the difference-in-difference estimator (the differential effect of the Olympic bid on house prices in newham relative to the control area of croydon.

. reg price af	fter if newham	==1						
Source	SS	df	MS	3		Number of obs	=	81
	+					F(1, 79)	=	38.09
Model	3.8272e+10	1	3.8272∈	10+ ف		Prob > F	=	0.0000
Residual	7.9385e+10	79	1.0049∈	90+د		R-squared	=	0.3253
	+					Adj R-squared	=	0.3167
Total	1.1766e+11	80	1.4707∈	90+د		Root MSE	=	31700
price	Coef.	Std.	Err.	t	P> t	[95% Conf.	Int	cerval]
	+							
after	53378.93	8649.	343	6.17	0.000	36162.84	70	0595.02
_cons	165035	3962.	462 4	11.65	0.000	157147.9		172922

House prices were indeed higher in Newham after the Olympic announcement, but...

	reg price af	ter if hounsl	ow==1						
	Source	SS	df		MS		Number of obs	=	80
-	+						F(1, 78)	=	36.75
	Model	2.9394e+10	1	2.939	4e+10		Prob > F	=	0.0000
	Residual	6.2388e+10	78	7998	46080		R-squared	=	0.3203
-	+						Adj R-squared	=	0.3115
	Total	9.1782e+10	79	1.161	8e+09		Root MSE	=	28282
-									
	price	Coef.	Std.	Err.	t	P> t	[95% Conf.	Int	cerval]

	+					
after	46857.27	7729.537	6.06	0.000	31468.94	62245.6
_cons	205399.7	3563.14	57.65	0.000	198306.1	212493.4

they were also higher in Hounslow

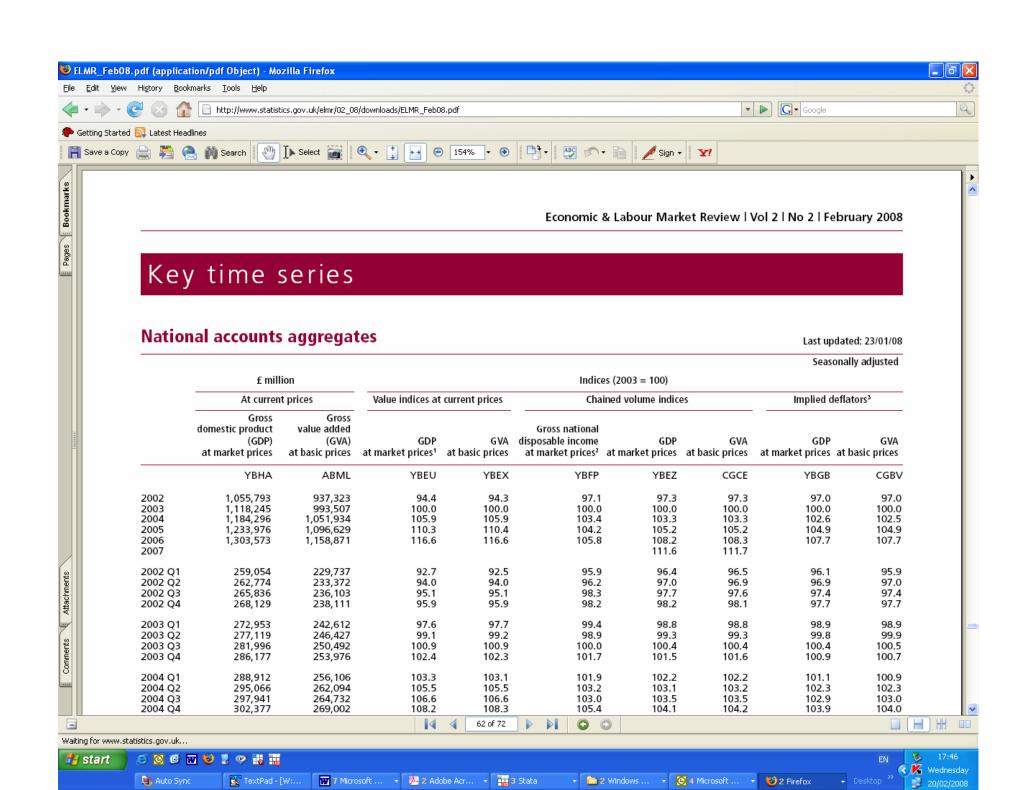
Moreover the annual rate of growth of house prices (approximated by the log of the 12 month change) was not significantly different in the after period

	. reg dlogp af	ter if newham	==1						
	Source	SS	df		MS		Number of obs	=	72
	+						F(1, 70)	=	0.74
	Model	.025162236	1	.025	162236		Prob > F	=	0.3931
	Residual	2.38491217	70	.034	070174		R-squared	=	0.0104
	+						Adj R-squared	=	-0.0037
	Total	2.4100744	71	.03	394471		Root MSE	=	.18458
	dlogp	Coef.	Std.	Err.	t	P> t	[95% Conf.	In	terval]
•	after	0440185	.0512	2209	-0.86	0.393	1461754		0581385
	_cons	.0868633	.0248	3889	3.49	0.001	.037224		1365027

and the difference-in-difference analysis confirms that there was no differential house price growth between the two areas. It seems claims of a house price effect were exaggerated.

reg	logp after	newham after	new if	newhai	m==1	hounslow:	==1		
	Source	SS	df]	MS		Number of obs	=	161
	+						F(3, 157)	=	40.90
	Model	3.70463956	3	1.234	87985		Prob > F	=	0.0000
	Residual	4.74020159	157	.0301	92367		R-squared	=	0.4387
	+						Adj R-squared	=	0.4280
	Total	8.44484115	160	.0527	80257		Root MSE	=	.17376
	logp	Coef.	Std.	Err.	t	P> t	[95% Conf.	In	terval]
	+								
	after	.2172745	.0474	896	4.58	0.000	.1234735		3110755
	newham	2308987	.0308	383	-7.49	0.000	2918101		1699872
	afternew	.0868757	.0671	047	1.29	0.197	0456688		<mark>2194202</mark>
	_cons	12.22069	.0218	916	558.24	0.000	12.17745	1	2.26393

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Series net of seasonal effects are said to be "seasonally adjusted"

$$Y_t = b_0 + b_1Q1 + b_2Q2 + b_3Q3 + b_4Trend + u_t$$

$$Y_t = b_0 + b_1Q1 + b_2Q2 + b_3Q3 + b_4Trend + u_t$$

where Trend = 1 in year 1

$$Y_t = b_0 + b_1Q1 + b_2Q2 + b_3Q3 + b_4Trend + u_t$$

$$Y_t = b_0 + b_1Q1 + b_2Q2 + b_3Q3 + b_4Trend + u_t$$

where Trend =1 in year 1

= 2 in year 2

= T in year T

$$Y_t = b_0 + b_1Q1 + b_2Q2 + b_3Q3 + b_4Trend + u_t$$

= T in year T

since $dY_t/dTrend = b_4$

given that the coefficient measures the unit change in y for a unit change in the trend variable

and the units of measurement in this case are years

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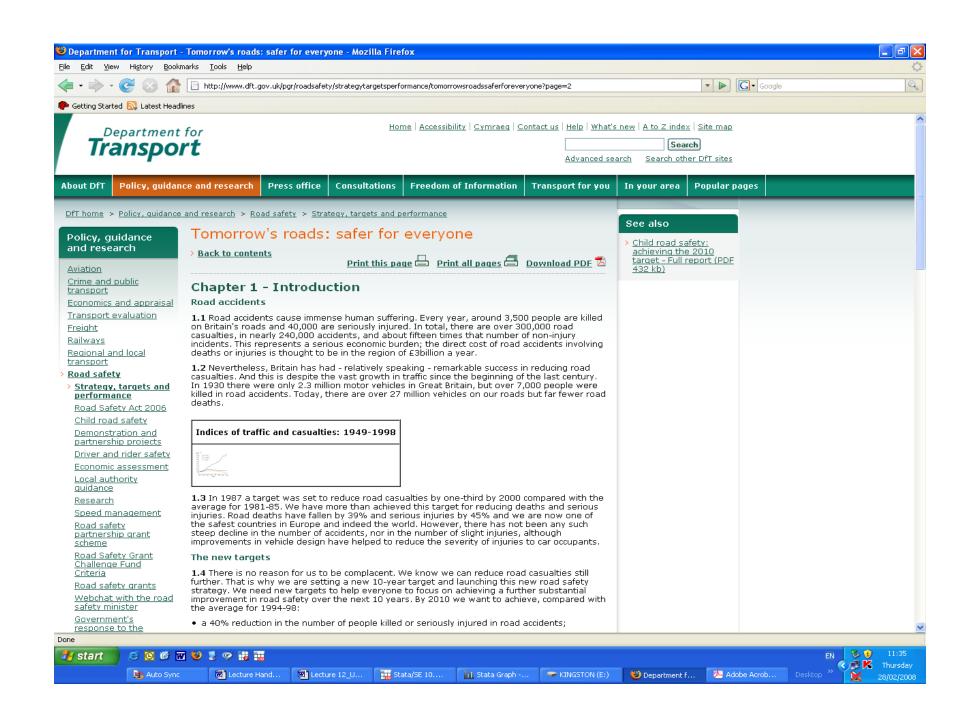
= T in year T

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then in the model above the trend term measures the annual change in the Y variable net of any seasonal influences

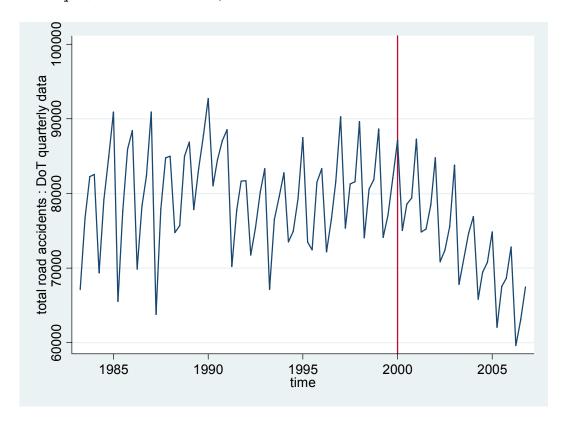


The new targets

- **1.4** There is no reason for us to be complacent. We know we can reduce road casualties still further. That is why we are setting a new 10-year target and launching this new road safety strategy. We need new targets to help everyone to focus on achieving a further substantial improvement in road safety over the next 10 years. By 2010 we want to achieve, compared with the average for 1994-98:
 - a 40% reduction in the number of people killed or seriously injured in road accidents;
 - a 50% reduction in the number of children killed or seriously injured; and
 - a 10% reduction in the slight casualty rate, expressed as the number of people slightly injured per 100 million vehicle kilometres.

The data set accidents.dta (on the course web site) contains quarterly information on the number of road accidents in the UK from 1983 to 2006

twoway (line acc time, xline(2000))



The graph shows that road accidents vary more within than between years

Can see seasonal influence from a regression of number of accidents on 3 dummy variables (1 for each quarter minus the default category – which is the 4th quarter)

. list acc year quart time Q1 Q2 Q3 Q4, clean

	acc	year	quart	time	Q1	Q2	Q3	Q4
1.	67135	1983	Q1	1983.25	1	0	0	0
2.	76622	1983	Q2	1983.5	0	1	0	0
3.	82277	1983	Q3	1983.75	0	0	1	0
4.	82550	1983	Q4	1984	0	0	0	1
5.	69362	1984	Q1	1984.25	1	0	0	0
6.	79124	1984	Q2	1984.5	0	1	0	0

A regression of road accident numbers on quarterly dummies (q4=winter is default given by constant term at 85249 accidents, on average in the 4th quarter) shows accidents are significantly less likely to happen outside the fourth quarter (October-December). On average there are 14,539 fewer accidents in the first quarter of the year than in the last

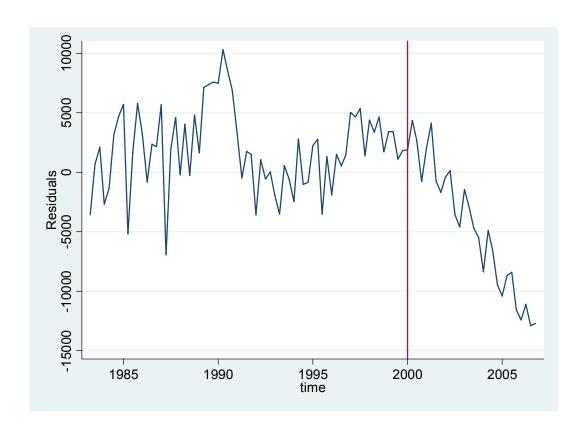
. req acc Q1 Q2 Q3

Source	SS	df	MS		Number of obs F(3, 91)	= 95 = 34.16
Model Residual	2.6976e+09 2.3957e+09		9214117 26242.3		Prob > F R-squared Adj R-squared	= 0.0000 = 0.5296
Total	5.0933e+09	94 541	84365.9		Root MSE	= 5130.9
acc	Coef.	Std. Err.	t	P> t	[95% Conf.	Interval]
Q1 Q2 Q3 _cons	-14539.44 -9292.567 -5074.609 85249.61	1497.179 1497.179 1497.179 1069.869	-9.71 -6.21 -3.39 79.68	0.000 0.000 0.001 0.000	-17513.4 -12266.53 -8048.572 83124.45	-11565.48 -6318.604 -2100.646 87374.77

Saving residual values after netting out the influence of the seasons is the basis for the production of "seasonally adjusted" data (better guide to underlying trend), used in many official government statistics.

Can get a sense of how this works with the following command after a regression

```
. predict rhat, resid
/* saves the residuals in a new variable with the name "rhat" */
```



Graph of the residuals is much smoother than the original series – it should be since much of the seasonality has been taken out by the dummy variables. The graph also shows that once seasonality accounted for, there is little evidence in a change in the number of road accidents over time until the year 2000

To model both seasonal and trend components of an economic series, simply include both seasonal dummies and a time trend in the regression model

$$Y_t = b_0 + b_1Q_1 + b_2Q_2 + b_3Q_3 + b_4TREND + u_t$$

MS

. reg acc Q1 Q2 Q3 year

Source |

SS

df

Number of obs =

95

Model Residual Total	3.4052e+09 1.6881e+09 5.0933e+09	90 1875	1308410 56630.6 		F(4, 90) Prob > F R-squared Adj R-squared Root MSE	= 0.0000 = 0.6686
acc	Coef.	Std. Err.	t	P> t	[95% Conf.	Interval]
Q1 Q2 Q3 year _cons	-14340.33 -9093.455 -4875.497 -398.2231 879306.5	1264.153 1264.153 1264.153 64.83547 129285.1	-11.34 -7.19 -3.86 -6.14 6.80	0.000 0.000 0.000 0.000 0.000	-16851.79 -11604.92 -7386.958 -527.0301 622459.1	-11828.87 -6581.995 -2364.037 -269.4161 1136154

Can see that there is a downward trend in road accidents (of around 400 a year over the whole sample period) net of any seasonality. Could also use dummy variable interactions to test whether this trend is stronger after 2000. How?

Can also use seasonal dummy variables to check whether an apparent association between variables is in fact caused by seasonality in the data

```
Source | SS | df | MS | Number of obs = 71

F( 1, 69) = 6.19

Model | 236050086 | 1 236050086 | Prob > F | = 0.0153

Residual | 2.6325e+09 | 69 38151620.6 | R-squared | = 0.0823

Adj R-squared | = 0.0690

Total | 2.8685e+09 | 70 40978741.5 | Root MSE | = 6176.7

acc | Coef. Std. Err. | t | P>|t | [95% Conf. Interval]

du | -4104.777 | 1650.228 | -2.49 | 0.015 | -7396.892 | -812.662

_cons | 79558.78 | 768.3058 | 103.55 | 0.000 | 78026.06 | 81091.51
```

. req acc du

The regression suggests a negative association between the change in the unemployment rate and the level of accidents (a 1 percentage point rise in the unemployment rate leads to a fall in the number of accidents by 4104 if this regression is to be believed)

Might this be in part because seasonal movements in both data series are influencing the results (the unemployment rate also varies seasonally, typically higher in q1 of each year)

. reg acc du q2-q4

Source	ss	df	MS		Number of obs	
Model Residual	2.1275e+09 741050172		31865433 .228032.9		F(4, 66) Prob > F R-squared Adj R-squared	= 0.0000 = 0.7417
Total	2.8685e+09	70 40	978741.5		Root MSE	= 3350.8
acc	Coef.		f. t	P> t	[95% Conf.	Interval]
du q2 q3 q4 _cons	-1030.818 5132.594 10093.64 14353.92 72488.21	1009.324 1266.59 1174.291 1212.479 834.607	-1.02 4.05 8.60 11.84	0.311 0.000 0.000 0.000 0.000	-3045.999 2603.766 7749.089 11933.13 70821.87	984.3627 7661.422 12438.18 16774.72 74154.56

Can see if add quarterly seasonal dummy variables then apparent effect of unemployment disappears.