E(X)/E(Y) compared to E(X/Y)

Asked 4 years, 4 months ago Modified 4 years, 4 months ago Viewed 2k times

1

Is there any sort of inequality stating the relationship between the two?

1) if X and Y are independent they are equal, I think. Since 1/Y will be independent to X too as well right?

2) but what if two are dependent?

1

probability probability-theory inequality random-variables expectation

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asked Mar 25, 2018 at 6:26

james black 1,775 10 24

2 If X, Y are independent then $E[X/Y] = E[X]E[1/Y]$. But $E[1/Y]$ is not the same as $1/E[Y]$.
– Michael Mar 25, 2018 at 6:42

1 On the other hand, if $Y = aX$ then $E[X]/E[Y] = E[X/Y]$, assuming no divide by zero issues.
– Michael Mar 25, 2018 at 6:44

1 Answer

Sorted by: Highest score (default)

3

If X, Y are independent then

$$E\left[\frac{X}{Y}\right] = E[X]E\left[\frac{1}{Y}\right]$$

In general $E[\frac{1}{Y}]$ is not the same as $\frac{1}{E[Y]}$. Let's assume $E[X]$, $E[\frac{1}{Y}]$ are finite.

The function $1/y$ is strictly convex over the domain $y > 0$. So if $Y > 0$ with prob 1, then by Jensen's inequality we have:

$$E\left[\frac{1}{Y}\right] \geq \frac{1}{E[Y]}$$

with equality if and only if $Var(Y) = 0$. So if X, Y independent and if $Y > 0$ with prob 1 then

- $E[X] = 0 \implies E\left[\frac{X}{Y}\right] = 0 = \frac{E[X]}{E[Y]}$.

- $E[X] > 0 \implies E\left[\frac{X}{Y}\right] \geq \frac{E[X]}{E[Y]}$ with equality if and only if $Var(Y) = 0$.

- $E[X] < 0 \implies E\left[\frac{X}{Y}\right] \leq \frac{E[X]}{E[Y]}$ with equality if and only if $Var(Y) = 0$.

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edited Mar 25, 2018 at 14:46

answered Mar 25, 2018 at 14:34

Michael
20.7k 2 27 46

btw why is 1/Y and X independent if X and Y are independent? linearity of exp doesnt apply here – james black Mar 26, 2018 at 7:59

@jamesblack : Linearity of expectation means $E[X+Y] = E[X] + E[Y]$ and that holds regardless of whether or not X and Y are independent. If X and Y are independent, intuitively it means that information about Y tells you nothing about X, so information about some function of Y (such as e^Y, Y^2, or $1/Y$) also tells you nothing about X. More formally (and avoiding divide-by-zero issues) we get $P[X \leq x, 1/Y \leq z] = P[X \leq x, Y \geq 1/z] = P[X \leq x]P[Y \geq 1/z] = P[X \leq x]P[1/Y \leq z]$ where the second equality holds because X, Y are independent. – Michael Mar 26, 2018 at 13:00

thank you and why is $E[X] > 0 \implies E\left[\frac{X}{Y}\right] \geq \frac{E[X]}{E[Y]}$ with equality if and only if $Var(Y) = 0$? what makes it greated? like what does E(X/Y) specifically equal to if possible – james black Mar 28, 2018 at 10:31

also do you mean if X,Y independent and if Y>0 with prob 1 or dependent? since if independent, then we have the equality there is no point in estbliashing inequality – james black Mar 28, 2018 at 10:38

If $a > 0$ and $r \leq s$ then $ar \leq as$. My answer above indeed only considers the case when X, Y are independent. As I described in my answer, it is *not true* that if X, Y are independent then $E[X/Y] = E[X]/E[Y]$. On the other hand, note that I have already given an exact equality for $E[X/Y]$, that is, I have already given what it is specifically equal to. – Michael Mar 28, 2018 at 15:42