Feedback — Problem Set 1

Help Center

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Question 1

Dominance

1\ 2	x	y	z
a	1,2	2,2	5,1
b	4,1	3,5	3,3
c	5,2	4,4	7,0
d	2,3	0,4	3,0

Find the strictly dominant strategy:

Your Answer		Score	Explanation
1) a;			
2) b;			
3) c;	✔	1.00	
4) d;			
5) x;			
6) y;			

7) z	
Total	1.00 / 1.00

Question Explanation

(3) c is a strictly dominant strategy.

- Because when 2 plays x or y or z, playing c always gives 1 a strictly higher payoff than playing a, b or d.
- None of the strategies is always strictly best for player 2.

Question 2

Dominance

1\ 2	x	y	z
a	1,2	2,2	5,1
b	4,1	3,5	3,3
c	5,2	4,4	7,0
d	2,3	0,4	3,0

Find a very weakly dominant strategy that is not also strictly dominant.

Your Answer		Score	Explanation
7) z			
5) x;			
1) a;			
6) y;	✔	1.00	

4) d;	
2) b;	
3) c;	
Total	1.00 / 1.00

Question Explanation

(6) y is a very weakly dominant strategy that is not also strictly dominant.

- Because when 1 plays a, b, c or d, playing y always gives 2 a weakly higher payoff than playing x or z.
- Note that it is only weakly higher when 1 plays a, as then playing x and y gives 2 the same payoff.

Question 3

Dominance

1\ 2	x	y	z
a	1,2	2,2	5,1
b	4,1	3,5	3,3
c	5,2	4,4	7,0
d	2,3	0,4	3,0

When player 1 plays d, what is player 2's best response:

Your Answer		Score	Explanation
a) Only x			
b) Only y	✔	1.00	

c) Only z

d) Both y and z

Total	1.00 / 1.00

Question Explanation

(b) only y is a best response for player 2. When player 1 plays d, player 2 earns 3 from playing x, 4 from playing y and 0 from playing z. Thus only y is a best response.

Question 4

Dominance

1\ 2	x	y	z
a	1,2	2,2	5,1
b	4,1	3,5	3,3
c	5,2	4,4	7,0
d	2,3	0,4	3,0

Find all strategy profiles that form pure strategy Nash equilibria (there may be more than one, or none):

Your Answer		Score	Explanation
1) (a, x);	✔	0.08	
2) (b, x);	✔	0.08	
3) (c, x);	✔	0.08	

☐ 4) (d, x);	✔	0.08
☐ 5) (a, y);	✔	0.08
☐ 6) (b, y);	✔	0.08
☑ 7) (c, y);	✔	0.08
☐ 8) (d, y);	✔	0.08
☐ 9) (a, z);	✔	0.08
☐ 10) (b, z);	✔	0.08
☐ 11) (c, z);	✔	0.08
☐ 12) (d, z).	✔	0.08
Total		1.00 / 1.00

Question Explanation

(7) (c, y) is the only pure strategy Nash equilibria.

- Check that no one wants to deviate.
- Note that c is the strictly dominant strategy and so is the only possible strategy for player 1 in a pure strategy Nash equilibrium.
- When player 1 plays c, playing y gives player 2 the highest payoff.

Question 5

Nash Equilibrium - Bargaining

There are 2 players that have to decide how to split one dollar. The bargaining process works as follows. Players simultaneously announce the

share they would like to receive s_1 and s_2, with $0 \leq s_1, s_2 \leq 1$. If $s_1 + s_2 \leq 1$, then the players receive the shares they named and if $s_1 + s_2 > 1$, then both players fail to achieve an agreement and receive zero.

Which of the following is a strictly dominant strategy?

Your Answer		Score	Explanation
a)1;			
b) 0.5;			
c) 0;			
d) None of the above.	✔	1.00	
Total		1.00 / 1.00	

Question Explanation

(d) is true.

- No player has any strictly dominant strategies. Any of the options given constitutes a best response to some strategy played by the other player, and so no strategy always strictly outperforms all other strategies.
- Strategies (a) and (c) are in the set of best responses of player i when player j's strategy is $s_j > 1$.
- Strategies (b) is the best response of player i when player j's strategy is $s_j = 0.5$.

Question 6

Nash Equilibrium - Bargaining

There are 2 players that have to decide how to split one dollar. The bargaining process works as follows. Players simultaneously announce the share they would like to receive s_1 and s_2, with $0 \leq s_1, s_2 \leq 1$. If $s_1 + s_2 \leq 1$, then the players receive the shares they named and if $s_1 + s_2 > 1$, then both players fail to achieve an agreement and receive zero.

Which of the following strategy profiles is a pure strategy Nash equilibrium?

Your Answer		Score	Explanation
a) (0.3, 0.7);			
b) (0.5, 0.5);			
c) (1.0, 1.0);			
d) All of the above	✔	1.00	
Total		1.00 / 1.00	

Question Explanation

(d) is true.

- Check that no one wants to deviate.
- Note that when player i plays $s_i < 1$, player j's best response is $s_j = 1 - s_i$. This holds in a) and b). Thus, both players are best responding.
- .When player i plays $s_i = 1$, player j's best response can be any number as she will get 0 no matter 1. Thus c) also forms a pure strategy NE.

Question 7

Bertrand Duopoly

- Two firms produce identical goods, with a production cost of c per unit.
- Each firm sets a nonnegative price (p_1 and p_2).
- All consumers buy from the firm with the lower price, if $p_i \neq p_j$. Half of the consumers buy from each firm if $p_i = p_j$.
- D is the total demand.

- Profit of firm i is:
  - 0 if $p_i > p_j$ (no one buys from firm i);
  - $D(p_i - c)/2$ if $p_i = p_j$ (Half of customers buy from firm i);
  - $D(p_i - c)$ if $p_i < p_j$ (All customers buy from firm i);

Find the pure strategy Nash equilibrium:

Your Answer		Score	Explanation
a) Both firms set $p = 0$.			
b) Firm 1 sets $p = 0$, and firm 2 sets $p = c$.			
c) Both firms set $p = c$.	✔	1.00	
d) No pure strategy Nash equilibrium exists.			
Total		1.00 / 1.00	

Question Explanation

(c) is true.

- Notice than in a) and b) at least one firm i is making negative profits since $p_i < c$ and it sells a positive quantity. Thus, firm i would prefer to deviate to $p_i > p_j$ and earn a profit of 0.
- It is easy to verify that $p_1 = p_2 = c$ is an equilibrium by checking that no firm wants to deviate:
  - When $p_1 = p_2 = c$, both firms are earning null profits.
  - If firm 1 increases its price above c ($p_1 > c$), it will still earn null profits.
  - If firm 2 decreases its price below c ($p_1 < c$), it will earn strictly negative profits.
  - In both cases, either the firm is indifferent or strictly worse off. Then, it does not have incentives to deviate given the other firm's strategy.

Question 8

Voting

- Three voters vote over two candidates (A and B), and each voter has two pure strategies: vote for A and vote for B.
- When A wins, voter 1 gets a payoff of 1, and 2 and 3 get payoffs of 0; when B wins, 1 gets 0 and 2 and 3 get 1. Thus, 1 prefers A, and 2 and 3 prefer B.
- The candidate getting 2 or more votes is the winner (majority rule).

Find all very weakly **dominant** strategies (there may be more than one, or none).

Your Answer		Score	Explanation
☑ a) Voter 1 voting for A.	✔	0.25	
☐ b) Voter 1 voting for B.	✔	0.25	
☐ c) Voter 2 (or 3) voting for A.	✔	0.25	
☑ d) Voter 2 (or 3) voting for B.	✔	0.25	
Total		1.00 / 1.00	

Question Explanation

(a) and (d) are (very weakly) dominant strategies.

- Check (b): for voter 1, voting for candidate A always results in at least as high a payoff as voting for candidate B and indeed is sometimes strictly better (when the other players vote for different candidates).
  - When voters 2 and 3 vote for B, voter 1 is indifferent between A or B (since B will win anyways).
  - When either 2 or 3 (or both) vote for A, voter 1 strictly prefers to vote for A than for B.
- Check (c): for voter 2, voting for candidate B is a very weakly dominant strategy.
  - When voters 1 and 3 vote for A, voter 2 is indifferent between A or B (since A will win anyways).
  - When either 1 or 3 (or both) vote for B, voter 2 strictly prefers to vote for B than for A.

- (b) and (c) can't be very weakly dominant strategies, since they sometimes do worse than the other strategy.

Question 9

Voting

- Three voters vote over two candidates (A and B), and each voter has two pure strategies: vote for A and vote for B.
- When A wins, voter 1 gets a payoff of 1, and 2 and 3 get payoffs of 0; when B wins, 1 gets 0 and 2 and 3 get 1. Thus, 1 prefers A, and 2 and 3 prefer B.
- The candidate getting 2 or more votes is the winner (majority rule).

Find **all** pure strategy Nash equilibria (there may be more than one, or none)?

Your Answer		Score	Explanation
☑ a) All voting for A.	✔	0.25	
☑ b) All voting for B.	✔	0.25	
☑ c) 1 voting for A, and 2 and 3 voting for B.	✔	0.25	
☐ d) 1 and 2 voting for A, and 3 voting for B.	✔	0.25	
Total		1.00 / 1.00	

Question Explanation

(a), (b) and (c) are pure strategy Nash equilibria.

- It is easy to verify that (a), (b) and (c) are equilibria by checking that no voter wants to deviate:
  - When all voters vote for the same candidate, no single voter has any incentives to deviate because his/her individual vote can't modify the outcome of the election.
  - In (c), voter 1 is indifferent between candidates A and B, and voters 2 and 3 are best responding to the strategies played by the

remaining voters (if voter 2 votes for A, candidate A wins; if voter 2 votes for B, candidate B wins).

- (d) is not an equilibrium, since voter 2 has incentives to deviate and vote for candidate B.