


Introduction to Week Five


Gradient


Divergence


Curl


Applications

 **Video:** Meaning of the Divergence and the Curl | Lecture 52
10 min

 **Reading:** The Navier-Stokes Equation
20 min

 **Video:** Maxwell's Equations | Lecture 53
11 min

 **Reading:** Electric Field of a Point Charge
10 min

 **Reading:** Magnetic Field of a Wire
10 min

Quiz

Farewell

The Navier-Stokes Equation

The incompressible Navier-Stokes equation governing fluid flow is given by

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} = -\frac{1}{\rho} \nabla p + \nu \nabla^2 \mathbf{u},$$

with $\nabla \cdot \mathbf{u} = 0$. Here, ρ and ν are fluid density and viscosity.

(a) By taking the divergence of the Navier-Stokes equation, derive the following equation for the pressure in terms of the velocity field:

$$\nabla^2 p = -\rho \frac{\partial u_i}{\partial x_j} \frac{\partial u_j}{\partial x_i}.$$

(b) By taking the curl of the Navier-Stokes equation, and defining the vorticity as $\boldsymbol{\omega} \equiv \nabla \times \mathbf{u}$, derive the vorticity equation

$$\frac{\partial \boldsymbol{\omega}}{\partial t} + (\mathbf{u} \cdot \nabla) \boldsymbol{\omega} = (\boldsymbol{\omega} \cdot \nabla) \mathbf{u} + \nu \nabla^2 \boldsymbol{\omega}.$$

You can use all the vector identities presented in these lecture notes, but you will need to prove that

$$\mathbf{u} \times (\nabla \times \mathbf{u}) = \frac{1}{2} \nabla (\mathbf{u} \cdot \mathbf{u}) - (\mathbf{u} \cdot \nabla) \mathbf{u}.$$

 **Completed** [Go to next item](#)

 Like  Dislike  Report an issue

