Independent trials in the continuum

Two trials

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A pair of independent random variables: $X \sim$ density $p_1(x)$, $Y \sim$ density $p_2(y)$

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Product space and measure: $(X, Y) \sim$ (joint) density $p_1(x)p_2(y)$

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Verifying independence:

Two trials

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$\mathbf{P}\{X \in A, Y \in B\}$

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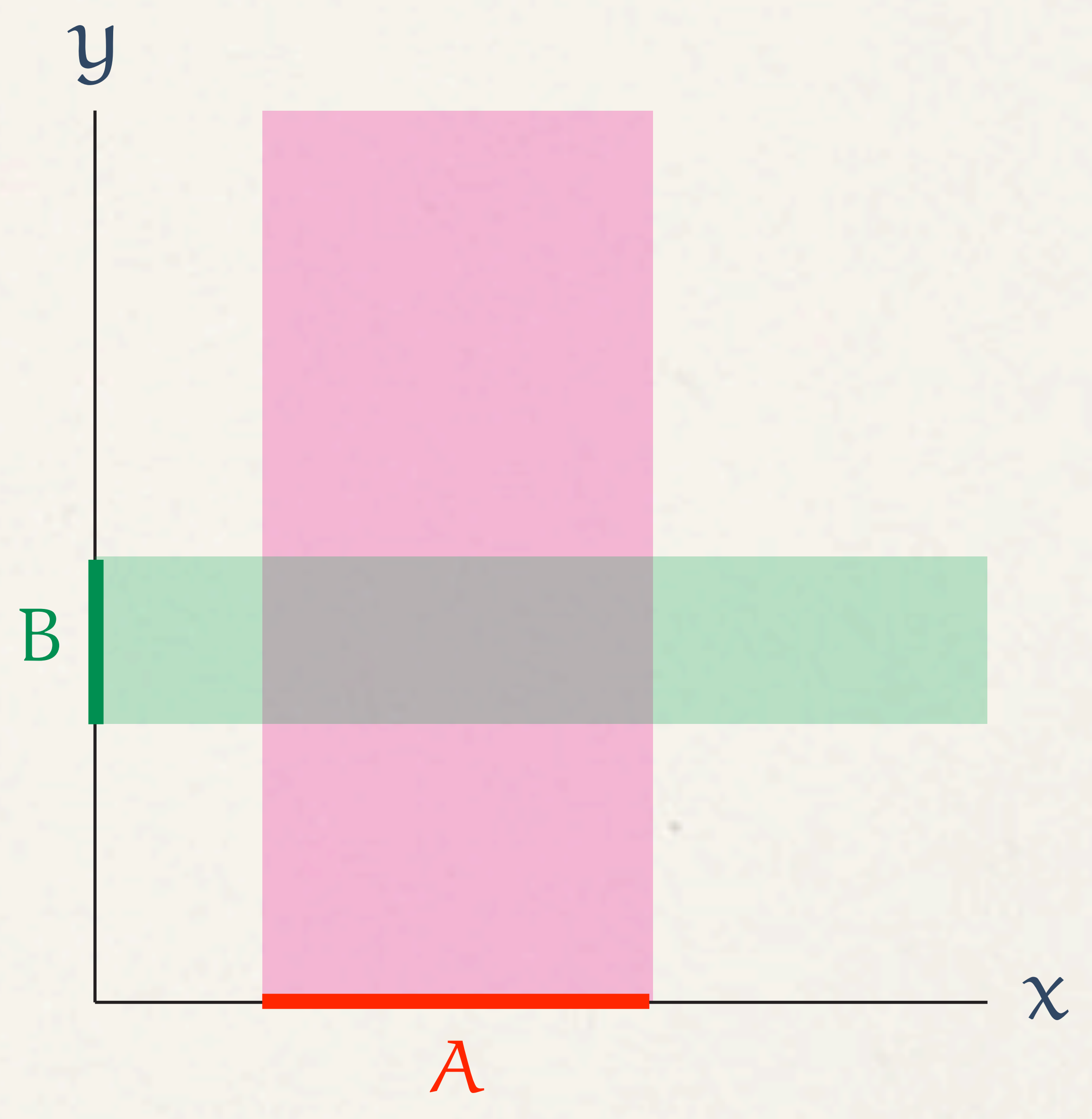

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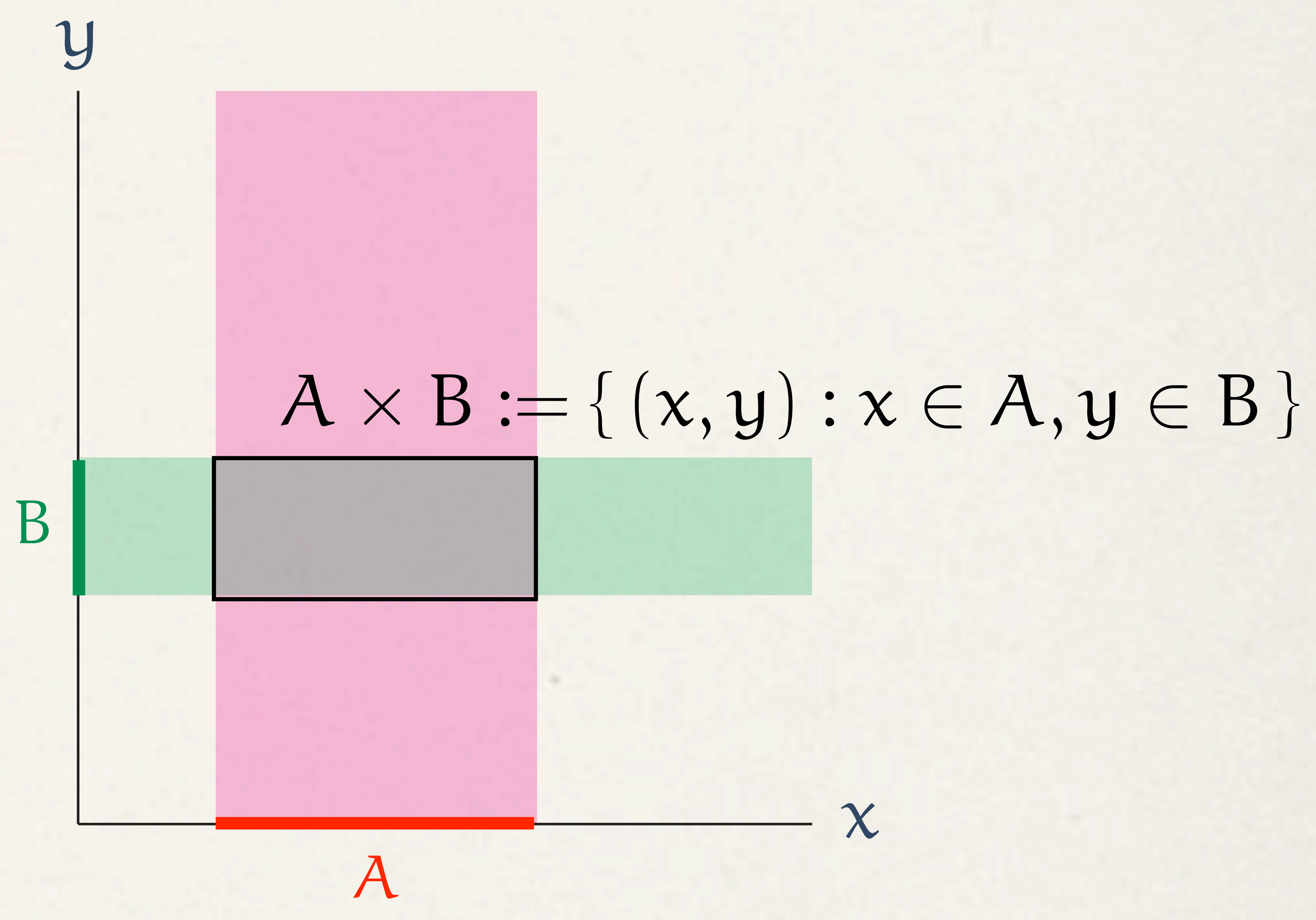

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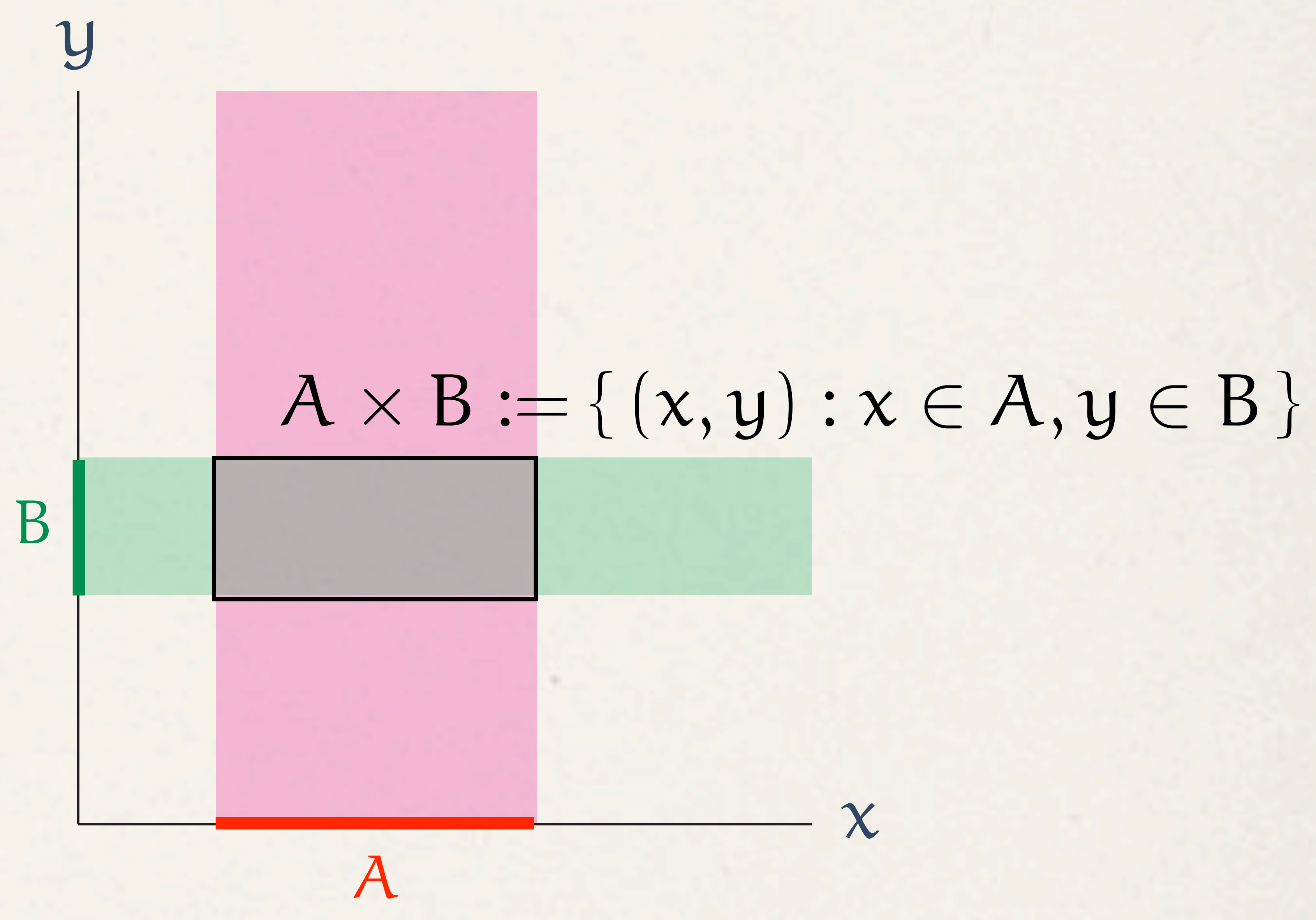
$$\mathbf{P}\{X \in A, Y \in B\} = \iint_{A \times B} p_1(x)p_2(y)\,dy\,dx$$



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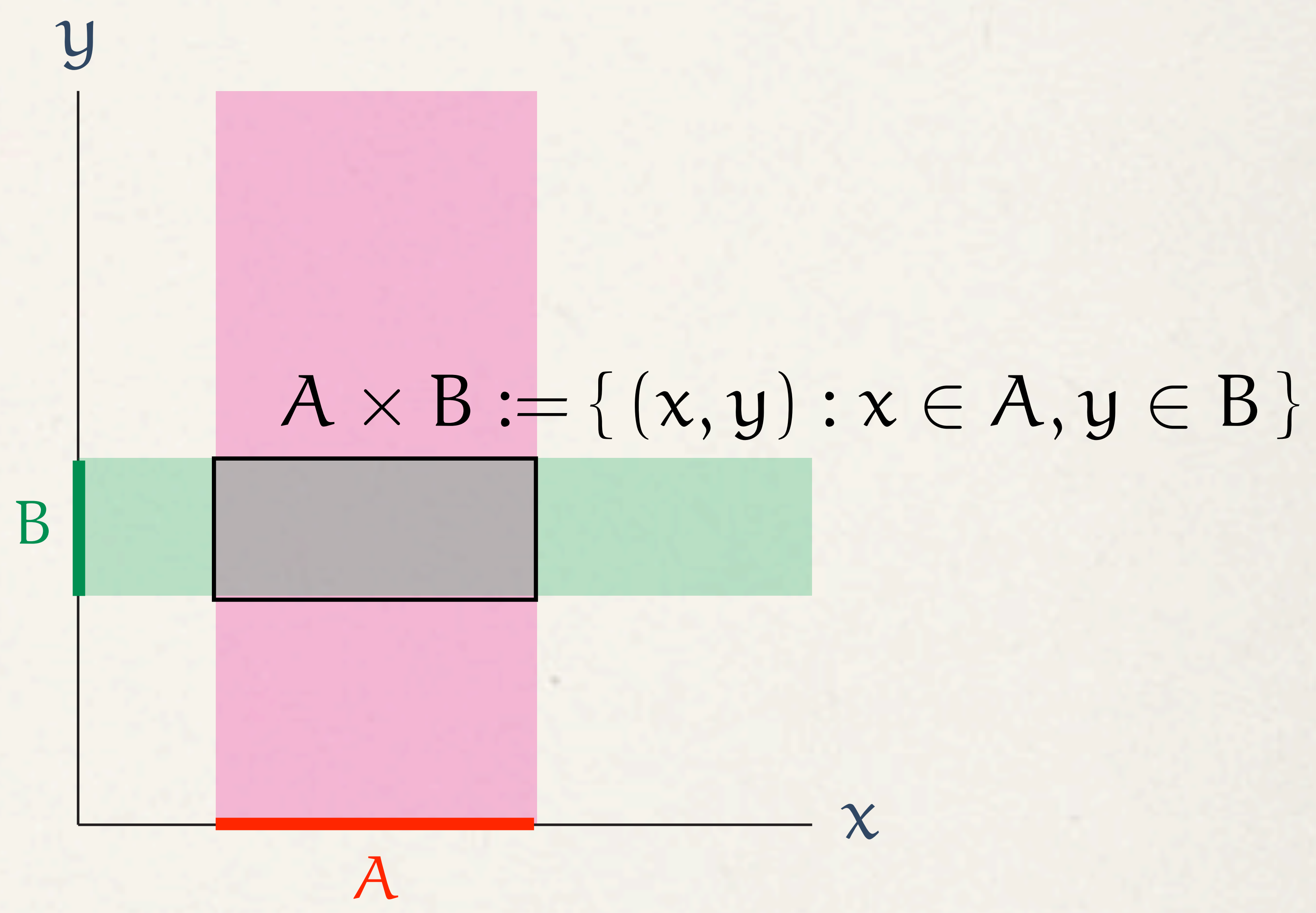

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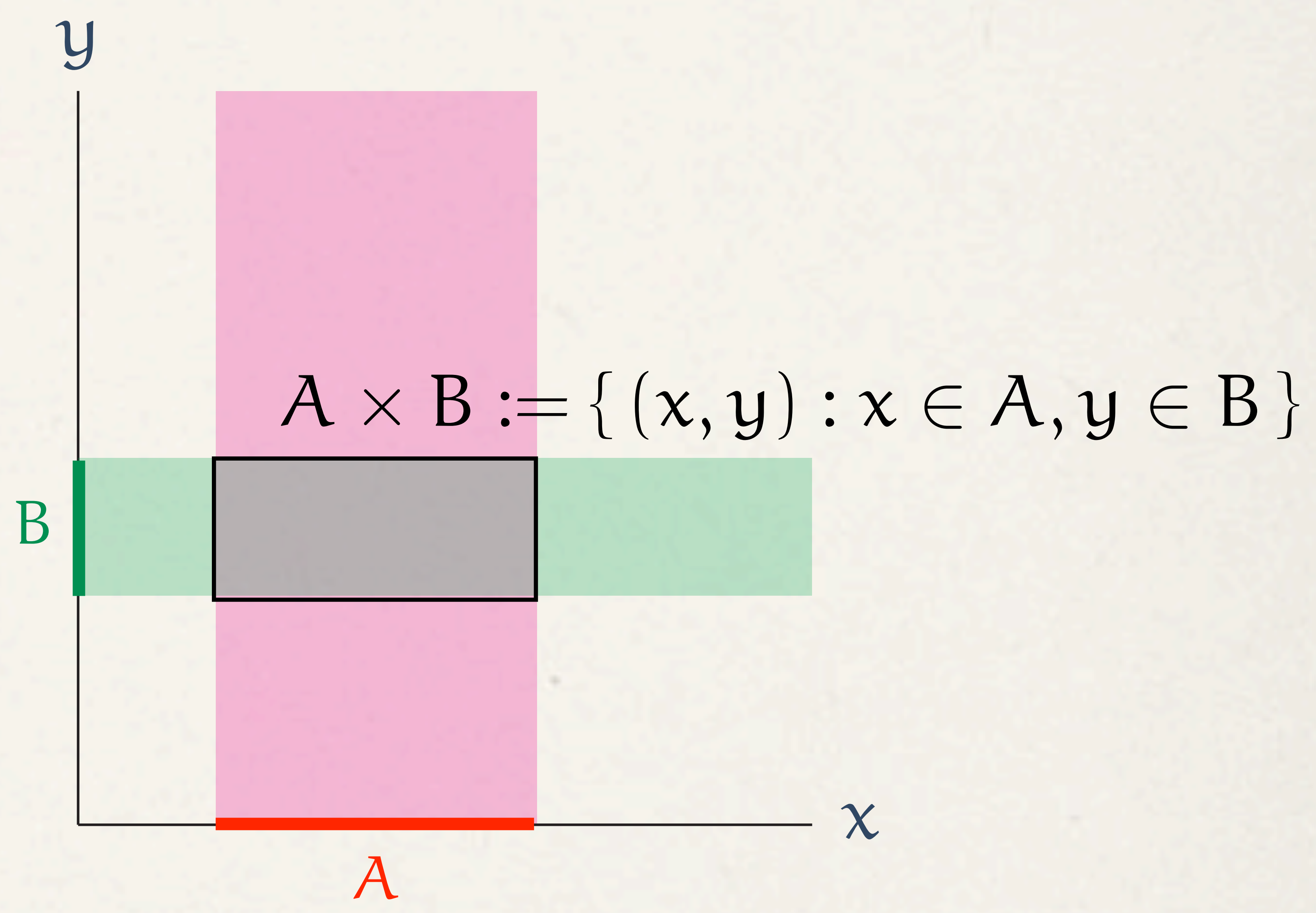
$$= \int_{\{x:x\in A\}} p_1(x)\,dx \cdot \int_{\{y:y\in B\}} p_2(y)\,dy$$



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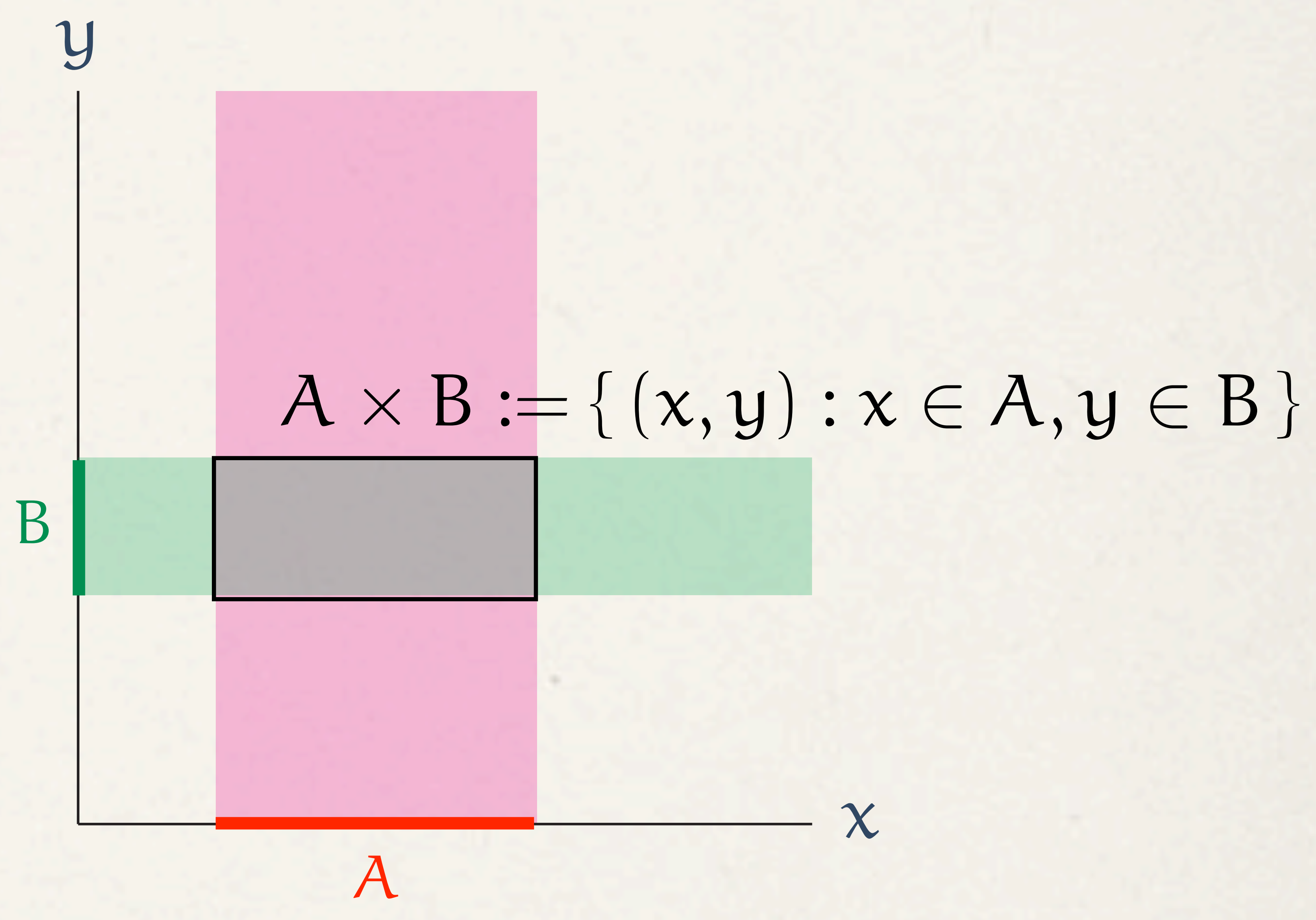
\mathbf{P}\{X \in A, Y \in B\} &= \iint_{A \times B} p_1(x)p_2(y)\,dy\,dx \\
&= \int_{\{x:x\in A\}} \int_{\{y:y\in B\}} p_1(x)p_2(y)\,dy\,dx \\
&= \int_{\{x:x\in A\}} p_1(x)\,dx \cdot \int_{\{y:y\in B\}} p_2(y)\,dy \\
&= \mathbf{P}\{X \in A\} \cdot \mathbf{P}\{Y \in B\}
\end{aligned}$$



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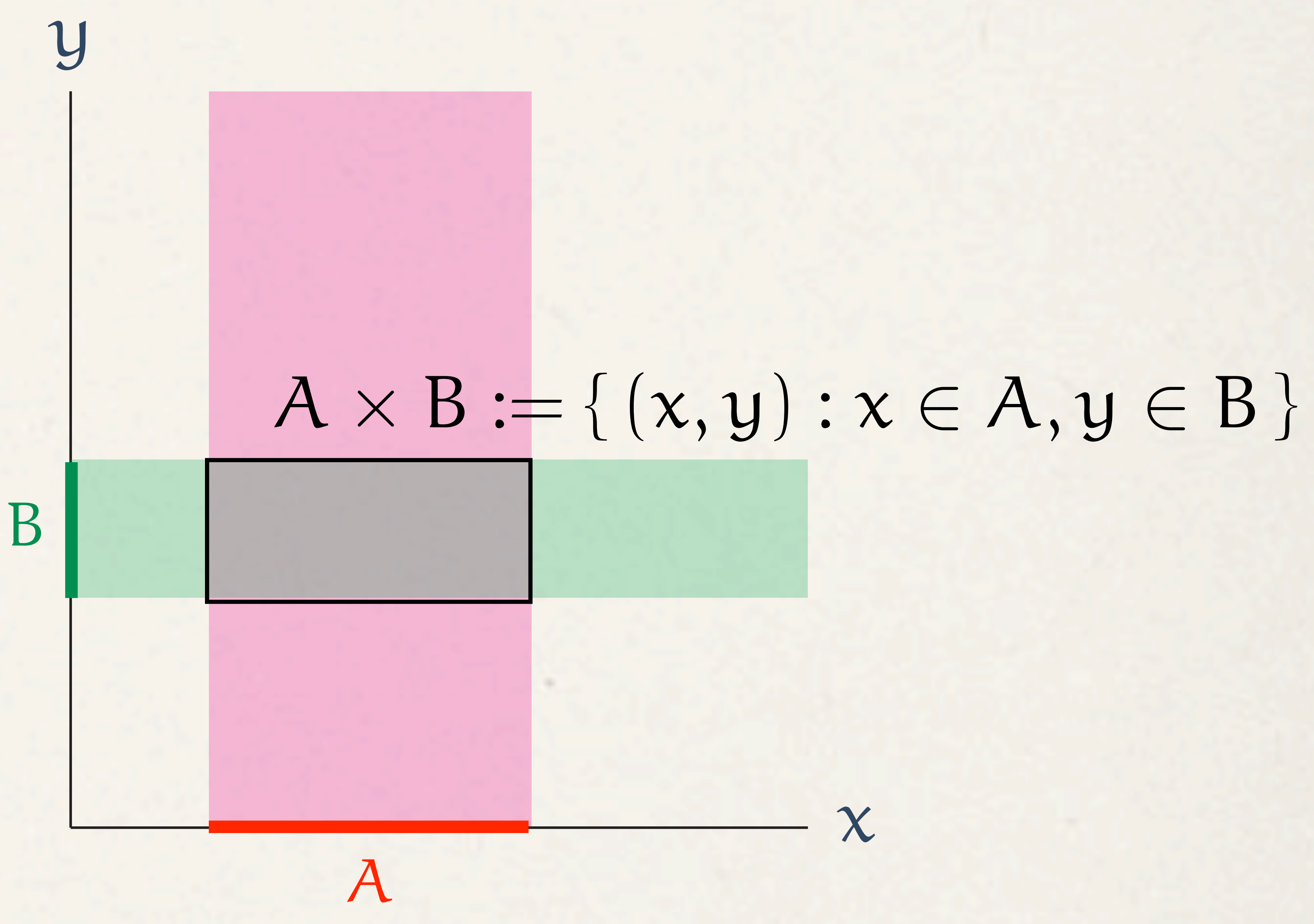

Events specified by X are independent of those specified by Y

n trials

n trials

A sequence of independent random variables: $X_1 \sim p_1(x_1)$, $X_2 \sim p_2(x_2)$, …, $X_n \sim p_n(x_n)$

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Verifying independence:

$$\mathbf{P}\{X_1 \in A_1, X_2 \in A_2, \ldots, X_n \in A_n\} = \int_{A_1 \times A_2 \times \cdots \times A_n} \!\!\!\! \cdots \int p_1(x_1) p_2(x_2) \cdots p_n(x_n)\, dx_n \cdots dx_2\, dx_1$$

n trials

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Verifying independence:

$$\begin{aligned}
\mathbf{P}\{X_1 \in A_1, X_2 \in A_2, \ldots, X_n \in A_n\} &= \int \cdots \int_{A_1 \times A_2 \times \cdots \times A_n} p_1(x_1) p_2(x_2) \cdots p_n(x_n)\, dx_n \cdots dx_2\, dx_1 \\
&= \int_{A_1} p_1(x_1)\, dx_1 \int_{A_2} p_2(x_2)\, dx_2 \cdots \int_{A_n} p_n(x_n)\, dx_n
\end{aligned}$$

n trials

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$$\begin{aligned}
\mathbf{P}\{X_1 \in A_1, X_2 \in A_2, \ldots, X_n \in A_n\} &= \underset{A_1 \times A_2 \times \cdots \times A_n}{\int \cdots \int} p_1(x_1) p_2(x_2) \cdots p_n(x_n)\, dx_n \cdots dx_2\, dx_1 \\
&= \int_{A_1} p_1(x_1)\, dx_1 \int_{A_2} p_2(x_2)\, dx_2 \cdots \int_{A_n} p_n(x_n)\, dx_n \\
&= \mathbf{P}\{X_1 \in A_1\} \times \mathbf{P}\{X_2 \in A_2\} \times \cdots \times \mathbf{P}\{X_n \in A_n\}
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&= \int_{A_1} p_1(x_1)\, dx_1 \int_{A_2} p_2(x_2)\, dx_2 \cdots \int_{A_n} p_n(x_n)\, dx_n \\
&= \mathbf{P}\{X_1 \in A_1\} \times \mathbf{P}\{X_2 \in A_2\} \times \cdots \times \mathbf{P}\{X_n \in A_n\}
\end{aligned}$$

Events specified by the variables X_1, X_2, …, X_n individually, or in disjoint groups, are independent