One-Way Analysis of Variance

> Note: Much of the math here is tedious but straightforward. We'll skim over it in class but you should be sure to ask questions if you don't understand it.

I. Overview

A. We have previously compared two populations, testing hypotheses of the form

H_0: $\mu_1 = \mu_2$
H_A: $\mu_1 \neq \mu_2$

But in many situations, we may be interested in more than two populations.

Examples:
✓ Compare the average income of blacks, whites, and others.
✓ Compare the educational attainment of Catholics, Protestants, Jews.

B. Q: Why not just compare pairwise - take each possible pairing, and see which are significant?

A: Because by chance alone, some contrasts would be significant. For example, suppose we had 7 groups. The number of pairwise combinations is ${}_7C_2 = 21$. If $\alpha = .05$, we expect one of the differences to be significant.

Therefore, you want to simultaneously investigate differences between the means of several populations.

C. To do this, you use ANOVA - Analysis of Variance. ANOVA is appropriate when

✓ You have a dependent, interval level variable
✓ You have 2 or more populations, i.e. the independent variable is categorical. In the 2 population case, ANOVA becomes equivalent to a 2-tailed T test (2 sample tests, Case II, σ's unknown but assumed equal).

D. Thus, with ANOVA you test

H_0: $\mu_1 = \mu_2 = \mu_3 = ... = \mu_J$
H_A: The means are not all equal.

E. Simple 1-factor model: Suppose we want to compare the means of J different populations. We have j samples of size N_j. Any individual score can be written as follows:

$y_{ij} = \mu + \tau_j + \varepsilon_{ij}$, where j = 1, J (# groups) and i = 1, 2, ..., N_j

That is, an observation is the sum of three components:

1. The <u>grand mean</u> μ of the combined populations. For example, the overall average income might be $15,000.

2. A treatment effect τ_j associated with the particular population from which the observation is taken; put another way, τ_j is the deviation of the group mean from the overall mean. For example, suppose the average White income is \$20,000. Then τ_{whites} = \$5,000.

3. A random error term ε_{ij}. This reflects variability within each population. Not everyone in the group will have the same value. For example, the average white income might be \$20,000, but some whites will make more, some will make less. (For a white who makes \$18,000, $\varepsilon_{ij} = -2{,}000$.)

F. An alternative way to write the model is

$$y_{ij} = \mu_j + \varepsilon_{ij},$$

where μ_j = mean of the jth population = $\mu + \tau_j$.

G. We are interested in testing the hypothesis

$$H_0: \quad \mu_1 = \mu_2 = \mu_3 = ... = \mu_J$$

But if the J means are equal, this means that $\mu_j = \mu$, which means that there are no treatment effects. That is, the above hypothesis is equivalent to

$$H_0: \quad \tau_1 = \tau_2 = \tau_3 = ... = \tau_J = 0$$

H. Estimating the treatment effects: As usual, we use sample information to estimate the population parameters. It is pretty simple to estimate the treatment effects:

$$\hat{\mu} = \bar{y} = \frac{\sum_{j=1}^{J}\sum_{i=1}^{N_j} y_{ij}}{N}, \quad \hat{\mu}_j = \bar{y}_j = \frac{\sum_{i=1}^{N_j} y_{ij}}{N_j} = \frac{T_{A_j}}{N_j}, \quad \hat{\tau}_j = \hat{\mu}_j - \hat{\mu} = \bar{y}_j - \bar{y}$$

Example: A firm wishes to compare four programs for training workers to perform a certain manual task. Twenty new employees are randomly assigned to the training programs, with 5 in each program. At the end of the training period, a test is conducted to see how quickly trainees can perform the task. The number of times the task is performed per minute is recorded for each trainee, with the following results:

Observation	*Program 1*	*Program 2*	*Program 3*	*Program 4*
1	9	10	12	9
2	12	6	14	8
3	14	9	11	11
4	11	9	13	7
5	13	10	11	8
$T_{Aj} = \Sigma y_{ij}$	59	44	61	43
$\hat{\mu}_j = T_{Aj}/N_j$	11.8	8.8	12.2	8.6

Estimate the treatment effects for the four programs.

Solution. Note that $\Sigma\Sigma y_{ij} = 207$, so $\hat{\mu} = 207/20 = 10.35$. Since $\hat{\tau}_j = \hat{\mu}_j - \hat{\mu}$, we get

$\hat{\tau}_1 = 11.8 - 10.35 = 1.45$,
$\hat{\tau}_2 = 8.8 - 10.35 = -1.55$,
$\hat{\tau}_3 = 12.2 - 10.35 = 1.85$,
$\hat{\tau}_4 = 8.6 - 10.35 = -1.75$

I. Computing the treatment effects is easy - but how do we test whether the differences in effects are significant???

Note the following:

$$s^2_{total} = s^2 = \frac{\Sigma\Sigma(y_{ij} - \bar{y})^2}{N - 1} = \frac{SS\ Total}{DF\ Total} = MS\ Total$$

where SS = Sum of squares (i.e. sum of the squared deviations from the mean), DF = degrees of freedom, and MS = Mean square. Also,

$$SS\ Total = SS\ Within + SS\ Between$$

Where

$$\Sigma\Sigma(y_{ij} - \bar{y}_j)^2 = \Sigma\Sigma\hat{\varepsilon}^2_{ij} = SS\ Within = SS\ Errors = SS\ Residual$$

$$\sum_j\sum_i(\bar{y}_j - \bar{y})^2 = \sum_j\sum_i\hat{\tau}^2_j = \sum_j N_j\hat{\tau}^2_j = SS\ Between = SS\ Explained$$

SS Within captures variability within each group. If all group members had the same score, SS Within would equal 0. It is also called SS Errors or SS Residual, because it reflects variability that cannot be explained by group membership. Note that there are N_j degrees of freedom associated with each individual sample, so the total number of degrees of freedom within = $\Sigma(N_j - 1) = N - J$.

SS Between captures variability between each group. If all groups had the same mean, SS Between would equal 0. The term SS Explained is also used because it reflects variability that is "explained" by group membership. Note that there are J samples, one grand mean, hence DF Between = J - 1.

We further define

$$MS\ Within = \frac{SS\ Within}{DF\ Within} = \frac{SS\ Within}{N - J},$$

$$MS\ Between = \frac{SS\ Between}{DF\ Between} = \frac{SS\ Between}{J - 1},$$

$$MS\ Total = \frac{SS\ Within + SS\ Between}{N - 1} = \frac{SS\ Total}{N - 1} = Total\ Variance$$

Proof (Optional): Note that

$$y_{ij} = y_{ij} - \bar{y}_j + \bar{y}_j, and$$

$$y_{ij} - \bar{y} = y_{ij} - \bar{y}_j + \bar{y}_j - \bar{y}$$

We simply add and subtract $\bar{y}_j$. Why do we do this? Note that $y_{ij} - \bar{y}_j$ = deviation of the individual's score from the group score $= \hat{\varepsilon}_{ij}$; and $\bar{y}_j - \bar{y}$ = deviation of the group score from the total score = $\hat{\tau}_j$. Hence,

$$SS\ Total = \sum\sum(y_{ij} - \bar{y})^2 = \sum\sum(y_{ij} - \bar{y}_j + \bar{y}_j - \bar{y})^2 = \sum\sum(\hat{\varepsilon}_{ij} + \hat{\tau}_j)^2 = \sum\sum\hat{\varepsilon}_{ij}^2 + \sum\sum\hat{\tau}_j^2 + 2\sum\sum\hat{\varepsilon}_{ij}\hat{\tau}_j$$

Let us deal with each term in turn:

$$\sum\sum(y_{ij} - \bar{y}_j)^2 = \sum\sum\hat{\varepsilon}_{ij}^2 = SS\ Within = SS\ Errors = SS\ Residual$$

SS Within captures variability within each group. If all group members had the same score, SS Within would equal 0. It is also called SS Errors or SS Residual, because it reflects variability

that cannot be explained by group membership. Note that there are N_j degrees of freedom associated with each individual sample, so the total number of degrees of freedom within = $\Sigma(N_j - 1) = N - J$.

$$\sum_j \sum_i (\bar{y}_j - \bar{y})^2 = \sum_j \sum_i \hat{\tau}_j^2 = \sum_j N_j \hat{\tau}_j^2 = SS\ Between = SS\ Explained$$

(The third equation is valid because all cases within a group have the same value for $\bar{y}_j$.) SS Between captures variability <u>between</u> each group. If all groups had the same mean, SS Between would equal 0. The term SS Explained is also used because it reflects variability that is "explained" by group membership. Note that there are J samples, one grand mean, hence DF Between = J - 1.

$$2\sum_j \sum_i (y_{ij} - \bar{y}_j)(\bar{y}_j - \bar{y}) = 2\sum_j \sum_i \hat{\varepsilon}_{ij} \hat{\tau}_j = 2\sum_j \hat{\tau}_j \sum_i \hat{\varepsilon}_{ij} = 2\sum_j \hat{\tau}_j * 0 = 0$$

(The latter is true because the deviations from the mean must sum to 0). Hence,

$$SS\ Total = SS\ Within + SS\ Between$$

J. Now that we have these, what do we do with them? For hypothesis testing, we have to make certain assumptions. Recall that $y_{ij} = \mu + \tau_j + \varepsilon_{ij}$. ε_{ij} is referred to as a "random error term" or "disturbance." If we assume:

(1) $\varepsilon_{ij} \sim N(0, \sigma^2)$,

(2) σ^2 is the same for all samples,

(3) the random error terms are independent (Note that these assumptions basically mean that the ε's are iid, independent and identically distributed);

Then, if H_0 is true,

$$F = \frac{MS\ Between}{MS\ Within} \sim \mathrm{F(J - 1, N - J),\ and\ E(F) = 1}$$

That is, if H_0 is true, then the test statistic F has an **F** distribution with J - 1 and N - J degrees of Freedom.

See Appendix E, Table V (Hayes, pp. 935-941), for tables on the F distribution. See especially tables 5-3 (Q = .05) and 5-5 (Q = .01).

K. Rationale:

✓ The basic idea is to determine whether all of the variation in a set of data is attributable to random error (chance) or whether some of the variation is attributable to chance and some is attributable to differences in the means of the J populations of interest.

✓ First, the sample variance for the entire set of data is computed and is seen to be composed of two parts: the numerator, which is a sum of squares, and the denominator, which is the degrees of freedom.

✓ The total sum of squares can be partitioned into SS Between and SS Within, and the total degrees of freedom can be partitioned into d.f. between and d.f. Within.

✓ By dividing each sum of squares by the respective d.f., MS between and MS within are determined; these represent the sample variability between the different samples and the sample variability within all the samples, respectively.

✓ But the variability *within* the samples must be due to random error alone, according to the assumptions of the one-factor model.

✓ The variability *between* the samples, on the other hand, may be attributable both to chance and to any differences in the J population means.

✓ Thus, if MS Between is significantly greater than MS Within (as measured by the F-test), then the null hypothesis of zero treatment effects must be rejected.

L. Comments on the F distribution:

✓ There are two sets of d.f., rather than 1.

✓ F is not symmetric. All values are positive.

✓ Like χ^2, we are only interested in values in the right-hand side of the tail.

✓ In the tables, columns give the d.f. for MS Between (J - 1), while the rows give the d.f. for MS Within (N - J).

✓ Look at Table 5-3, column 1; compare with Table 3 for the T distribution, the column labeled 2Q = .05. Note that $F = T^2$. A two sample test, case II, $\sigma_1 = \sigma_2 = \sigma$, with a 2-tailed alternative hypothesis, can also be tested using ANOVA.

M. Computational procedures for ANOVA. The above formulas are, in practice, a little awkward to deal with. When doing computations by hand, the following procedure is generally easier:

One Way Anova: Computational Procedures	
Formula	Explanation
$T_{A_j} = \sum_{i}^{N_j} y_{ij}$	T_{Aj} = the sum of the scores in group A_j, where A_1 = first group, A_2 = second group, etc. Add up the values for the observations for group A_1, then A_2, etc. Also sometimes called just T_j.
$(1) = \frac{(\sum\sum y_{ij})^2}{N} = N\overline{Y}^2$	Sum all the observations. Square the result. Divide by the total number of observations.
$(2) = \sum\sum y_{ij}^2$	Square each observation. Sum the squared observations.
$(3) = \sum_j \frac{T_{A_j}^{\ 2}}{N_{A_j}}$	Square T_{A1}, and divide by N_{A1}. Repeat for each of the J groups, and add the results together.
SS Total = (2) - (1)	Total Sum of Squares
SS Between = (3) - (1). Or, if treatment effects have been computed, use $\sum N_j \hat{\tau}_j^2$	Between Sum of Squares. This is also sometimes called SS_A, SS Treatment, or SS Explained
SS Within = (2) - (3)	Within sum of squares. Also called SS error, or SS Residual
MS Total = SS Total / (N - 1)	Mean square total. Same as s^2, the sample variance.
MS Between = SS Between / (J - 1)	Mean square between. Also called MS_A, MS Treatment, or MS Explained
MS Within = SS Within / (N - J)	Mean Square Within. Also called MS error or MS Residual
F = MS Between / MS Within	Test statistic. d.f. = (J - 1, N - J)

N. The ANOVA Table. The results of an analysis of variance are often presented in a table that looks something like the following (with the appropriate values filled in):

Source	SS	D.F.	Mean Square	F
A (or Treatment, or Explained)	SS Between	J - 1	SS Between/ (J - 1)	$\frac{\text{MS Between}}{\text{MS Within}}$
Error (or Residual)	SS Within	N - J	SS Within / (N - J)	
Total	SS Total	N - 1	SS Total / (N - 1)	

O. Hypothesis testing using ANOVA. As usual, we determine the critical value of the test statistic for a given value of α. If the test statistic is less than the critical value, we accept H_0, if it is greater than the critical value we reject H_0.

EXAMPLES:

1. Again consider this problem: A firm wishes to compare four programs for training workers to perform a certain manual task. Twenty new employees are randomly assigned to the training programs, with 5 in each program. At the end of the training period, a test is conducted to see how quickly trainees can perform the task. The number of times the task is performed per minute is recorded for each trainee, with the following results:

Program 1: 9, 12, 14, 11, 13
Program 2: 10, 6, 9, 9, 10
Program 3: 12, 14, 11, 13, 11
Program 4: 9, 8, 11, 7, 8

(a) Construct the ANOVA table
(b) Using α = .05, determine whether the treatments differ in their effectiveness.

Solution.

(a) As we saw before, $T_{A1} = 59$, $T_{A2} = 44$, $T_{A3} = 61$, $T_{A4} = 43$. Also,

$$(1) = \frac{(\sum\sum y_{ij})^2}{N} = \frac{207^2}{20} = 2142.45$$

$$(2) = \sum\sum y_{ij}^2 = 9^2 + 10^2 + 12^2 + \ldots + 8^2 = 2239$$

$$(3) = \sum_j \frac{T_{A_j}^{\;2}}{N_{A_j}} = \frac{59^2}{5} + \frac{44^2}{5} + \frac{61^2}{5} + \frac{43^2}{5} = 2197.4$$

SS Total = (2) - (1) = 2239 - 2142.45 = 96.55,
SS Between = (3) - (1) = 2197.4 - 2142.45 = 54.95; or,
SS Between = $\sum N_j \hat{\tau}_j^2 = 5 * 1.45^2 + 5 * 1.55^2 + 5 * 1.85^2 + 5 * 1.75^2 = 54.95$
SS Within = (2) - (3) = 2239 - 2197.4 = 41.6,

MS Total = SS Total/ (N - 1) = 96.55 / 19 = 5.08,
MS Between = SS Between/ (J - 1) = 54.95/3 = 18.32,
MS Within = SS Within/ (N - J) = 41.6/16 = 2.6,
F = MS Between / MS Within = 18.32 / 2.6 = 7.04

The ANOVA Table therefore looks like this:

<table>
<tr><th>Source</th><th>SS</th><th>D.F.</th><th>Mean Square</th><th>F</th></tr>
<tr><td>A (or Treatment, or Explained)</td><td>SS Between = 54.95</td><td>J - 1 = 3</td><td>SS Between/ (J - 1) = 18.32</td><td rowspan="3">$\frac{\text{MS Between}}{\text{MS Within}}$ =

7.04</td></tr>
<tr><td>Error (or Residual)</td><td>SS Within = 41.6</td><td>N - J = 16</td><td>SS Within / (N - J) = 2.6</td></tr>
<tr><td>Total</td><td>SS Total = 96.55</td><td>N - 1 = 19</td><td>SS Total / (N - 1) = 5.08</td></tr>
</table>

NOTE: Most computer programs would not be nice enough to spell out "SS Between =", etc.; that is, you would have to know from the location of the number in the table whether it was SS Between, MS Within, or whatever. See the SPSS examples below.

(b) For $\alpha = .05$, the critical value for an F with d.f. (3, 16) is 3.24. Ergo, we reject the null hypothesis. More formally,

Step 1:
H_0: $\mu_1 = \mu_2 = \mu_3 = \mu_4$, i.e. treatments are equally effective
H_A: The means are not all equal.

Step 2: An F statistic is appropriate, since the dependent variable is continuous and there are 2 or more groups.

Step 3: Since $\alpha = .05$ and d.f. = 3, 16, accept H_0 if $F_{3,16} \le 3.24$

Step 4: The computed value of the F statistic is 7.04

Step 5: Reject H_0. The treatments are not equally effective.

There are several SPSS routines that can do one-way Anova. These include ANOVA (which, alas, requires that you enter the syntax directly rather than use menus; but it will give you the MCA table if you want it), MEANS, and ONEWAY. Which you use depends on any additional information you might like as well as the format you happen to like best. I'll use ONEWAY but feel free to try the others. If using the SPSS pull-down menus, after entering the data select ANALYZE/ COMPARE MEANS/ ONE WAY ANOVA.

```
* Problem 1.  Employee training.

DATA LIST FREE / program score.
BEGIN DATA.
1   9
1  12
1  14
1  11
1  13
2  10
2   6
2   9
2   9
2  10
3  12
3  14
3  11
3  13
3  11
4   9
4   8
4  11
4   7
4   8
END DATA.
ONEWAY
  score BY program
  /STATISTICS DESCRIPTIVES
  /MISSING ANALYSIS .
```

Descriptives

SCORE

	N	Mean	Std. Deviation	Std. Error	95% Confidence Interval for Mean Lower Bound	95% Confidence Interval for Mean Upper Bound	Minimum	Maximum
1.00	5	11.8000	1.9235	.8602	9.4116	14.1884	9.00	14.00
2.00	5	8.8000	1.6432	.7348	6.7597	10.8403	6.00	10.00
3.00	5	12.2000	1.3038	.5831	10.5811	13.8189	11.00	14.00
4.00	5	8.6000	1.5166	.6782	6.7169	10.4831	7.00	11.00
Total	20	10.3500	2.2542	.5041	9.2950	11.4050	6.00	14.00

ANOVA

SCORE

	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	54.950	3	18.317	7.045	.003
Within Groups	41.600	16	2.600		
Total	96.550	19			

---

2. For each of the following, indicate whether H_0 should be accepted or rejected.

a. A researcher has collected data from 21 Catholics, 21 Protestants, and 21 Jews. She wants to see whether the groups significantly differ at the .05 level in their incomes. Her computed F = 3.0.

Solution. Note that n = 63, j = 3. Hence, d.f. = 3 - 1, 63 - 3 = 2, 60. Looking at table V, we see that for $\alpha = .05$ we should accept H_0 if $F \le 3.15$. Since the researcher got an F of 3.0, she should accept H_0.

b. A manager wants to test (using $\alpha = .025$) whether the mean delivery time of components supplied by 5 outside contractors is the same. He draws a random sample of 5 delivery times for each of the 5 contractors. He computes the following:

SS Between = 4
SS Within = 50

Solution. Note that n = 25 (5 delivery times for each of 5 contractors) and J = 5 (5 contractors). Hence

MS Between = SS Between/(J - 1) = 4/4 = 1
MS Within = SS Within/(N - J) = 50/20 = 2.5
F = MS Between/MS Within = 1/2.5 = .4
D.F. = (J - 1, N - J) = (4, 20)

For $\alpha = .025$, accept H_0 if $F \le 3.51$.
Therefore, accept H_0.

---

3. An economist wants to test whether mean housing prices are the same regardless of which of 3 air-pollution levels typically prevails. A random sample of house purchases in 3 areas yields the price data below.

MEAN HOUSING PRICES (THOUSANDS OF DOLLARS):

	Pollution Level		
Observation	Low	Mod	High
1	120	61	40
2	68	59	55
3	40	110	73
4	95	75	45
5	83	80	64
Σ	406	385	277

(a) Compute the treatment effects
(b) Construct the ANOVA Table
(c) At the .025 level of significance, test whether housing prices differ by level of pollution.

Solution.

(a)
$$\hat{\mu}_1 = 81.2, \hat{\mu}_2 = 77, \hat{\mu}_3 = 55.4, \hat{\mu} = 71.2$$
$$\hat{\tau}_1 = 81.2 - 71.2 = 10$$
$$\hat{\tau}_2 = 77.0 - 71.2 = 5.8$$
$$\hat{\tau}_3 = 55.4 - 71.2 = -15.8$$

(b) $T_{A1} = 406$, $T_{A2} = 385$, $T_{A3} = 277$,

$$(1) = \frac{(\sum\sum y_{ij})^2}{N} = \frac{1068^2}{15} = 76041.6$$

$$(2) = \sum\sum y_{ij}^2 = 120^2 + 61^2 + ... + 64^2 = 83940$$

$$(3) = \sum_j \frac{T_{A_j}^{\;2}}{N_{A_j}} = \frac{406^2}{5} + \frac{385^2}{5} + \frac{277^2}{5} = 77958$$

SS Total = (2) - (1) = 83940 - 76041.6 = 7898.4,
SS Between = (3) - (1) = 77958 - 76041.6 = 1916.4; or,
SS Between = $\sum N_j \hat{\tau}_j^2$ = 5 * 10^2 + 5 * 5.8^2 + 5 * -15.8^2 = 1916.4,
SS Within = (2) - (3) = 83940 - 77958 = 5982,
MS Total = SS Total/ (N - 1) = 7898.4 / 14 = 564.2,
MS Between = SS Between/ (J - 1) = 1916.4 / 2 = 958.2,
MS Within = SS Within / (N - J) = 5982 / 12 = 498.5,
F = MS Between / MS Within = 958.2 / 498.5 = 1.92

Source	SS	D.F.	Mean Square	F
A (or Treatment, or Explained)	SS Between = 1916.4	J - 1 = 2	SS Between/ (J - 1) = 958.2	MS Between = MS Within 1.92
Error (or Residual)	SS Within = 5982.0	N - J = 12	SS Within / (N - J) = 498.5	
Total	SS Total = 7898.4	N - 1 = 14	SS Total / (N - 1) = 564.2	

(c) For $\alpha = .025$ and df = 2, 12, accept H_0 if the computed F is $\leq$ 5.10. Since F = 1.92, do not reject H_0. More formally,

Step 1.

H_0: The τ's all = 0 (i.e. prices are the same in each area)
H_A: The τ's are not all equal (prices not all the same)

Step 2. Appropriate stat is

F = MS Between/ MS Within.

Since n = 15 and j = 3, d.f. = 2, 12.

Step 3. For $\alpha = .025$, accept H_0 if F $\leq$ 5.10

Step 4. Compute test stat. As shown above, F = 1.92

Step 5. Do not reject H_0 [NOTE: the SPSS solutions follows later]

Here is how you could solve this problem using SPSS. If using the SPSS pull-down menus, after entering the data select ANALYZE/ COMPARE MEANS/ ONE WAY ANOVA.

```
* Problem 3.  Housing Prices.

DATA LIST FREE / plevel price.
BEGIN DATA.
1  120
1   68
1   40
1   95
1   83
2   61
2   59
2  110
2   75
2   80
3   40
3   55
3   73
3   45
3   64
END DATA.
ONEWAY
  price BY plevel
  /STATISTICS DESCRIPTIVES
  /MISSING ANALYSIS .
```

Oneway

Descriptives

PRICE

	N	Mean	Std. Deviation	Std. Error	95% Confidence Interval for Mean		Minimum	Maximum
					Lower Bound	Upper Bound		
1.00	5	81.2000	29.8781	13.3619	44.1015	118.2985	40.00	120.00
2.00	5	77.0000	20.5061	9.1706	51.5383	102.4617	59.00	110.00
3.00	5	55.4000	13.5019	6.0382	38.6352	72.1648	40.00	73.00
Total	15	71.2000	23.7523	6.1328	58.0464	84.3536	40.00	120.00

ANOVA

PRICE

	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	1916.400	2	958.200	1.922	.189
Within Groups	5982.000	12	498.500		
Total	7898.400	14			

Comment: Some Anova routines would also report that $R^2 = .243$. Note that R^2 = SS Between / SS Total = 1916.4/7898.4 = .243. That is, R^2 = Explained Variance divided by total variance. We will talk more about R^2 later.

F Test versus T Test. Finally, for good measure, we will do an F-Test vs. T-Test comparison. We will do a modified version of problem 1, combining treatments 1 and 3 (the most effective), and 2 and 4 (the least effective). We'll let SPSS do the work.

```
* F test versus T-test comparison.

DATA LIST FREE / program score.
BEGIN DATA.
1   9
1  12
1  14
1  11
1  13
2  10
2   6
2   9
2   9
2  10
3  12
3  14
3  11
3  13
3  11
4   9
4   8
4  11
4   7
4   8
END DATA.
RECODE PROGRAM (1, 3 = 1) (2, 4 = 2).
ONEWAY
  score BY program
  /STATISTICS DESCRIPTIVES
  /MISSING ANALYSIS .
```

Oneway

Descriptives

SCORE

	N	Mean	Std. Deviation	Std. Error	95% Confidence Interval for Mean: Lower Bound	95% Confidence Interval for Mean: Upper Bound	Minimum	Maximum
1.00	10	12.0000	1.5635	.4944	10.8816	13.1184	9.00	14.00
2.00	10	8.7000	1.4944	.4726	7.6309	9.7691	6.00	11.00
Total	20	10.3500	2.2542	.5041	9.2950	11.4050	6.00	14.00

ANOVA

SCORE

	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	54.450	1	54.450	23.280	.000
Within Groups	42.100	18	2.339		
Total	96.550	19			

Note that the F value is 23.28.

```
T-TEST / GROUPS PROGRAM (1, 2) / VARIABLES SCORE.
```

T-Test

Group Statistics

	PROGRAM	N	Mean	Std. Deviation	Std. Error Mean
SCORE	1.00	10	12.0000	1.5635	.4944
	2.00	10	8.7000	1.4944	.4726

Independent Samples Test

		Levene's Test for Equality of Variances		t-test for Equality of Means						
									95% Confidence Interval of the Difference	
		F	Sig.	t	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	Lower	Upper
SCORE	Equal variances assumed	.010	.921	4.825	18	.000	3.3000	.6839	1.8631	4.7369
	Equal variances not assumed			4.825	17.963	.000	3.3000	.6839	1.8629	4.7371

COMMENT: Note that $4.82^2 = 23.28$ (approximately), i.e. $t^2 = F$. When you only have two groups, both the F test and the T-Test are testing

H_0: $\mu_1 = \mu_2$
H_A: $\mu_1 \neq \mu_2$

Not surprisingly, then, both tests yield the same conclusion.