7.4.3.3. The ANOVA table and tests of hypotheses about means

Sums of Squares help us compute the variance estimates displayed in ANOVA Tables

The sums of squares **SST** and **SSE** previously computed for the one-way ANOVA are used to form two mean squares, one for *treatments* and the second for *error*. These mean squares are denoted by MST and MSE, respectively. These are typically displayed in a tabular form, known as an *ANOVA Table*. The ANOVA table also shows the statistics used to test hypotheses about the population means.

Ratio of MST and MSE

When the null hypothesis of equal means is true, the two mean squares estimate the same quantity (error variance), and should be of approximately equal magnitude. In other words, their ratio should be close to 1. If the null hypothesis is false, MST should be larger than MSE.

Divide sum of squares by degrees of freedom to obtain mean squares

The mean squares are formed by dividing the sum of squares by the associated degrees of freedom.

Let $N = \sum n_i$. Then, the degrees of freedom for treatment are

$$DFT = k - 1\,,$$

and the degrees of freedom for error are

$$DFE = N - k\,.$$

The corresponding *mean squares* are:

$$MST = SST/DFT$$

$$MSE = SSE/DFE.$$

The F-test

The test statistic, used in testing the equality of treatment means is: $F = MST/MSE$.

The critical value is the tabular value of the F distribution, based on the chosen α level and the degrees of freedom DFT and DFE.

The calculations are displayed in an ANOVA table, as follows:

ANOVA table

Source	SS	DF	MS	F
Treatments	SST	$k-1$	$SST/(k-1)$	MST/MSE
Error	SSE	$N-k$	$SSE/(N-k)$	
Total (corrected)	SS	$N-1$		

The word "source" stands for source of variation. Some authors prefer to use "between" and "within" instead of "treatments" and "error", respectively.

ANOVA Table Example

A numerical example

The data below resulted from measuring the difference in resistance resulting from subjecting identical resistors to three different temperatures for a period of 24 hours. The sample size of each group was 5. In the language of design of experiments, we have an experiment in which each of three treatments was replicated 5 times.

	Level 1	Level 2	Level 3
	6.9	8.3	8.0
	5.4	6.8	10.5
	5.8	7.8	8.1
	4.6	9.2	6.9
	4.0	6.5	9.3
means	5.34	7.72	8.56

The resulting ANOVA table is

Example ANOVA table

Source	SS	DF	MS	F
Treatments	27.897	2	13.949	9.59
Error	17.452	12	1.454	
Total (corrected)	45.349	14		
Correction Factor	779.041	1		

Interpretation

The test statistic is the F value of 9.59. Using an α of 0.05, *of the ANOVA table*

we have $F_{0.05;\,2,\,12} = 3.89$ (see the F distribution table in Chapter 1). Since the test statistic is much larger than the critical value, we reject the null hypothesis of equal population means and conclude that there is a (statistically) significant difference among the population means. The p-value for 9.59 is 0.00325, so the test statistic is significant at that level.

Techniques for further analysis

The populations here are resistor readings while operating under the three different temperatures. What we do **not** know at this point is whether the three means are all different or which of the three means is different from the other two, and by how much.

There are several techniques we might use to further analyze the differences. These are:

- constructing confidence intervals around the difference of two means,
- estimating combinations of factor levels with confidence bounds
- multiple comparisons of combinations of factor levels tested simultaneously.

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