Iterative Solution of the Laplace Equation

Using the Jacobi method, solve the Laplace equation inside a unit square. Set the boundary conditions to be zero on the left and bottom sides, and to go from zero to one across the top, and from one to zero down the right side. Model these boundary conditions as

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\Phi = x(2-x) for y = 1, and \Phi = y(2-y) for x = 1.
```

Script @

```
2 Lx=1; Ly=1; %rectangle dimensions
3 Nx=100; Ny=100; %# of intervals
4 | nx=Nx+1; ny=Ny+1; %# of gridpoints in x,y directions including boundaries
5 dx=Lx/Nx; dy=Ly/Ny; %grid size in x,y directions
6 x=(0:Nx)*dx; y=(0:Ny)*dy; %x,y values on the grid
8 eps=1.e-6; %convergence criteria for each value of Phi
9 index_x=2:nx-1; index_y=2:ny-1; %internal grid points
10 Phi=zeros(nx,ny); matrix with solution and boundary conditions
12 %set the boundary conditions
13 Phi(:,1)=0;
                  %bottom
14 Phi(1,:)=0;
                  %left
15 Phi(:,ny)=x.*(2-x);
                       %top
16 Phi(nx,:)=y.*(2-y);
                       %right
18 Phi_old=Phi;
19 error=2*eps; ncount=0;
20 while (error > eps)
21
    ncount=ncount+1;
22
    Phi(index_x,index_y)=0.25*(Phi(index_x+1,index_y) ...
23
     +Phi(index_x-1,index_y)+Phi(index_x,index_y+1)+Phi(index_x,index_y-1));
24
    error=max(abs(Phi(:)-Phi_old(:)));
    if any(isnan(Phi(:))) || any(isinf(Phi(:)))
25
26
        fprintf('iterations diverge\n');
27
        return;
28
29
     Phi_old=Phi;
30
     %fprintf('%g %e\n',ncount, error);
31 end
32 fprintf('%g\n',ncount);
[X,Y]=meshgrid(x,y);
35 v=[0.8 0.6 0.4 0.2 0.1 0.05 0.01]; %SET THE CONTOUR LEVELS
36 contour(X,Y,Phi',v,'ShowText','on');%requires transpose (read the notes)
37 axis equal;
38 set(gca, 'YTick', [0 0.2 0.4 0.6 0.8 1]);
39 set(gca, 'XTick', [0 0.2 0.4 0.6 0.8 1]);
40 | xlabel('$x$','Interpreter','latex','FontSize',14 );
41 | ylabel('$y$','Interpreter','latex','FontSize',14);
42 title('Solution of the Laplace equation', 'Interpreter', 'latex', 'FontSize', 16);
43
```

► Run Script

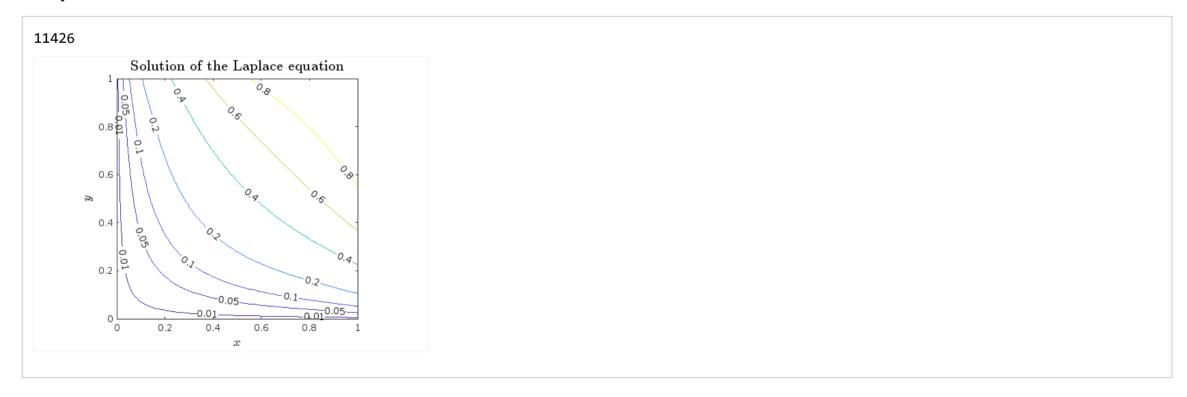
Assessment: All Tests Passed

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Check the value of Phi

Output



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