

Summary of Tableau 11, Part 2

The fabulous limit laws

The law of large numbers in continuous spaces, computation à la Monte Carlo

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The Background

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Expectation and variance in continuous settings

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$$X \sim p(x)$$

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$$X \sim p(x) \qquad \mathbf{E}(X) := \mu = \int_{-\infty}^{\infty} x \cdot p(x) \, dx$$

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✦ Expectation: $\mathbf{E}(X + Y) = \mathbf{E}(X) + \mathbf{E}(Y)$

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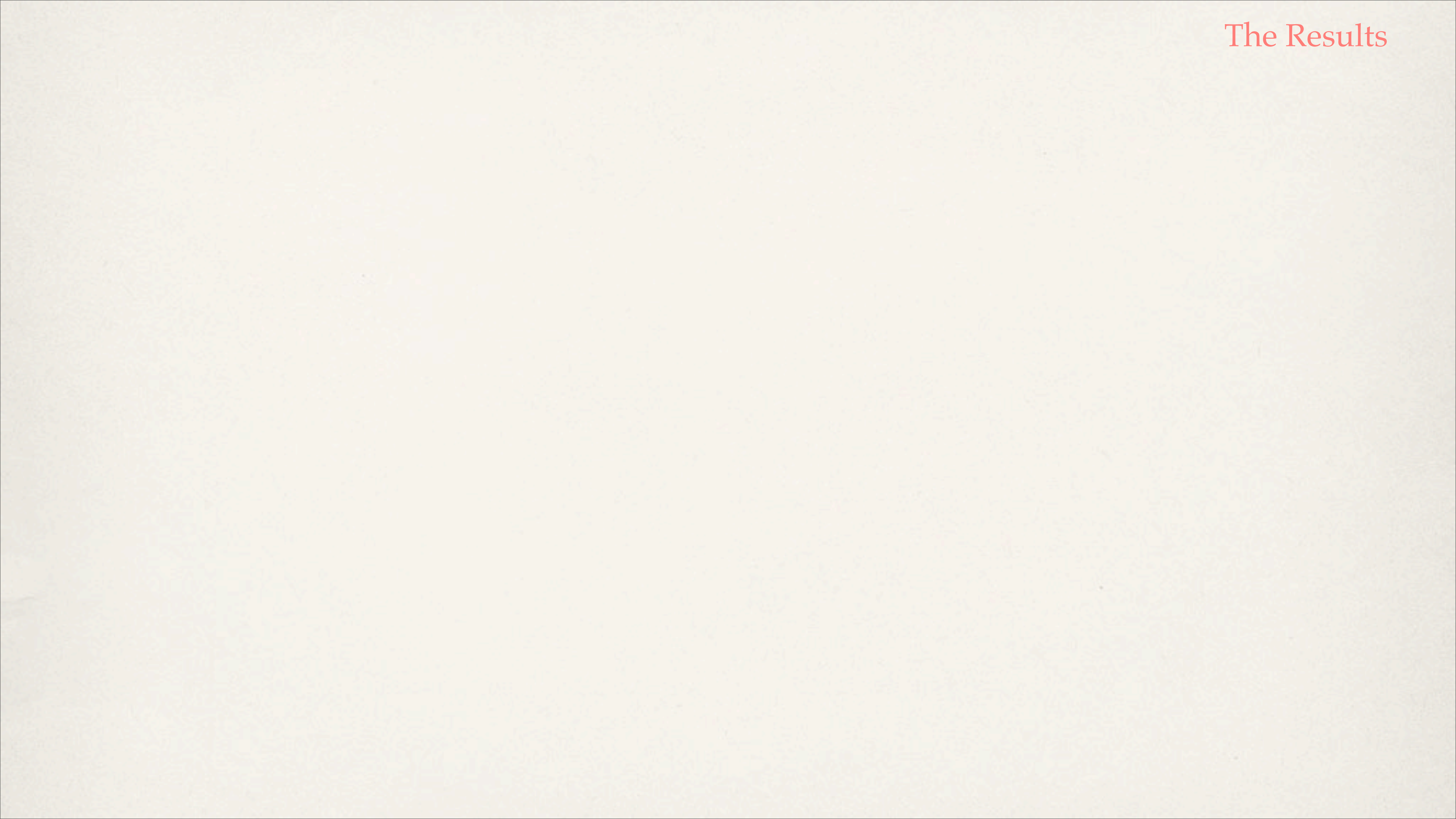
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Additivity

- * Expectation: $\mathbf{E}(X + Y) = \mathbf{E}(X) + \mathbf{E}(Y)$
- * Variance (if summands are independent): $\text{Var}(X + Y) = \text{Var}(X) + \text{Var}(Y)$



Random sample

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$$X, X_1, \dots, X_n, \dots$$

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The weak law of large numbers

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$$\mathbf{P}\left\{\left|\frac{S_n}{n} - \mu\right| > \epsilon\right\} \leq \frac{\sigma^2}{n\epsilon^2} \rightarrow 0 \quad (n \rightarrow \infty)$$

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The sample mean S_n/n is concentrated at its expected value μ .

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Applications

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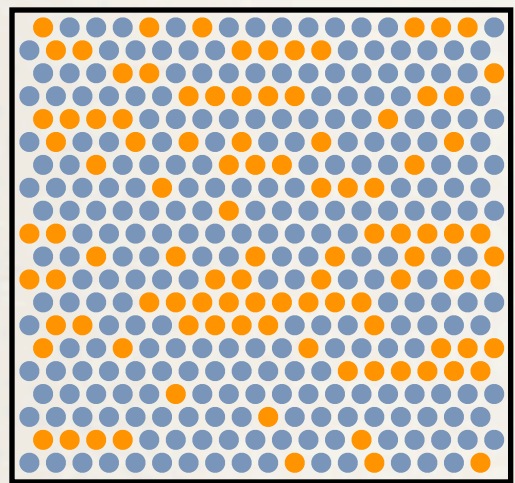
$$\text{Var}(S_n) = n\sigma^2$$

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Applications



- ✧ Estimation of population parameters, polls
- ✧ Testing sera and vaccines, drug approval
- ✧ Statistical estimates of a mean
- ✧ Quality testing, time to failure
- ✧ Actuarial models, risk assessment
- ✧ Theory of fair games
- ✧ Monte Carlo methods
- ✧ Stock portfolio selection, the horse race