Optimal Strategy

After going through the example of the previous section, let us try to come up with an optimal strategy for the game by answering the following questions.

Problem. You see a pool of dice on a table. Would you like to choose a dice first? If you would, which dice would you like to choose?

To summarize our setting, the game is specified by a pool of dice. The first player picks a dice, next the second player picks one of the remaining dice. They throw their dice and whoever gets the larger outcome wins. Each players wants to win in the long run, or in other words, wants to have a better winning probability in each round. Our goal is to determine who of the players has a winning strategy and to find this strategy.

To find the player with a winning strategy, we first need to describe in reasonably simple terms how to determine the winner. It turns out that the following simple criteria holds.

Proposition. The first player has a winning strategy if and only if there is a dice that is better than all others.

Proof.

1. If *there is* a dice that is better than all others, then *the first player wins*. Indeed, if there is the best dice, then the first player can just pick this dice. Whatever the second player takes is worse, so the first player wins.

2. If there is *no* dice that is better than all others, then *the first player does not win*. Indeed, this means that for any dice, there is another dice that is better. Then, the second player wins: whatever the first player picks, the second player can just pick a better dice.

Now we are ready to implement the solution! You can do it in the next quiz.

✓ **Completed**

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