Georgia Institute of Technology

Computational Photography

Dr. Irfan Essa
Professor
School of Interactive Computing

Study the basics of computation and its impact on the entire workflow of photography, from capturing, manipulating and collaborating on, and sharing photographs.

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Image Processing and Filtering, via Convolution and Cross-Correlation

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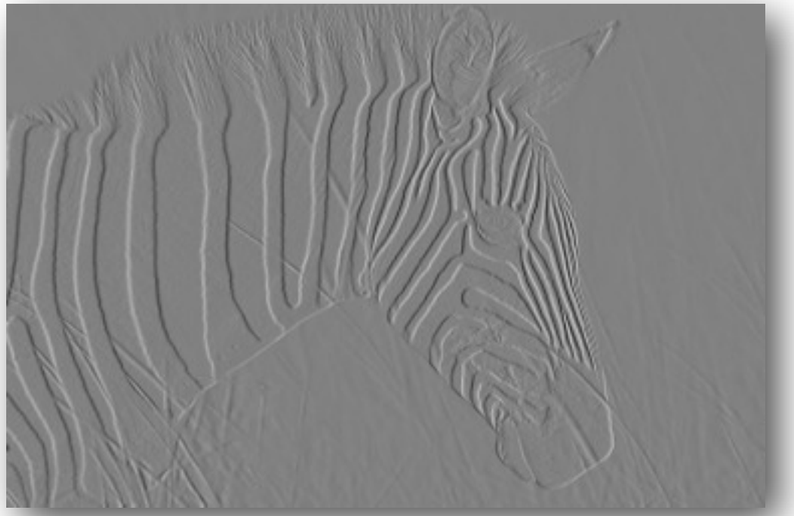
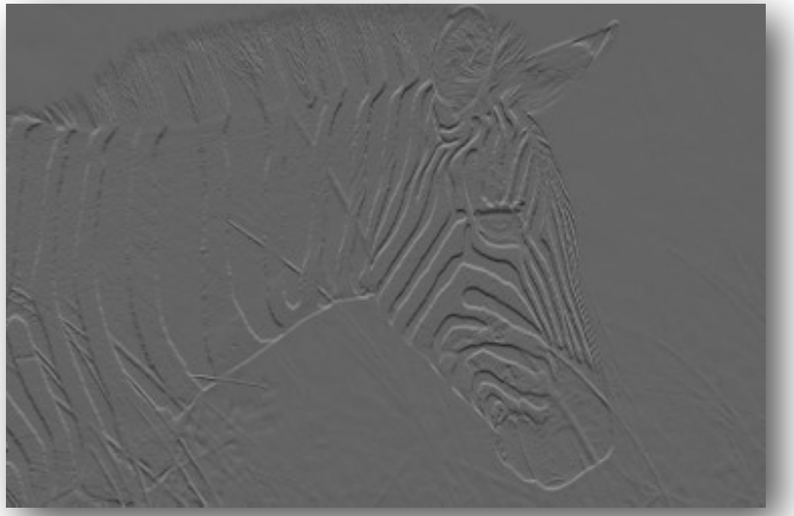
Towards Edge Detection: Computing Image Gradients

Lesson Objectives

- ★ Describe in your own terms how derivatives can be computed for an image using kernels and neighborhood operations.
- ★ Describe the three (3) methods for computing edges using kernels.
- ★ Recall why image noise can complicate the computation of gradients.
- ★ Explain the Canny Edge Detector extends the three (3) Edge detection methods.

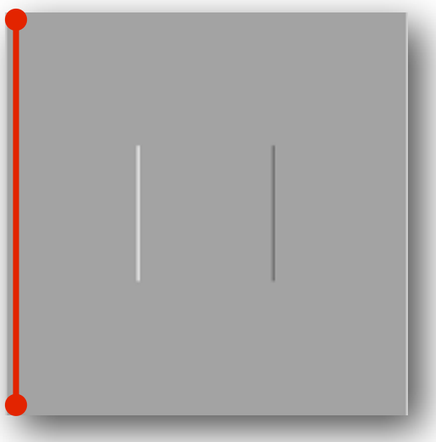
Review: Differentiating an Image in X and Y

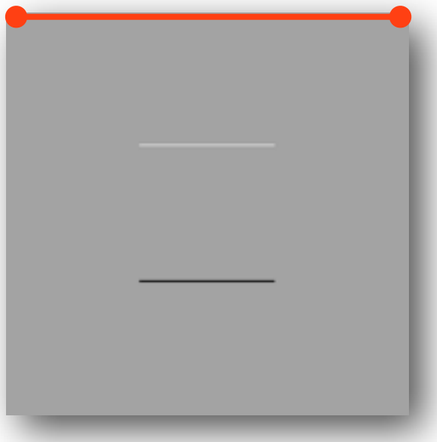


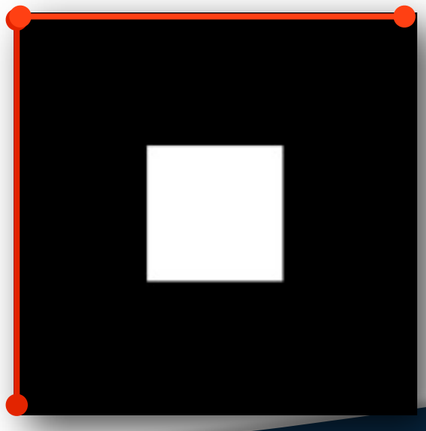


$$\frac{\delta F(x,y)}{\delta x} \approx F(x+1,y) - F(x,y)$$

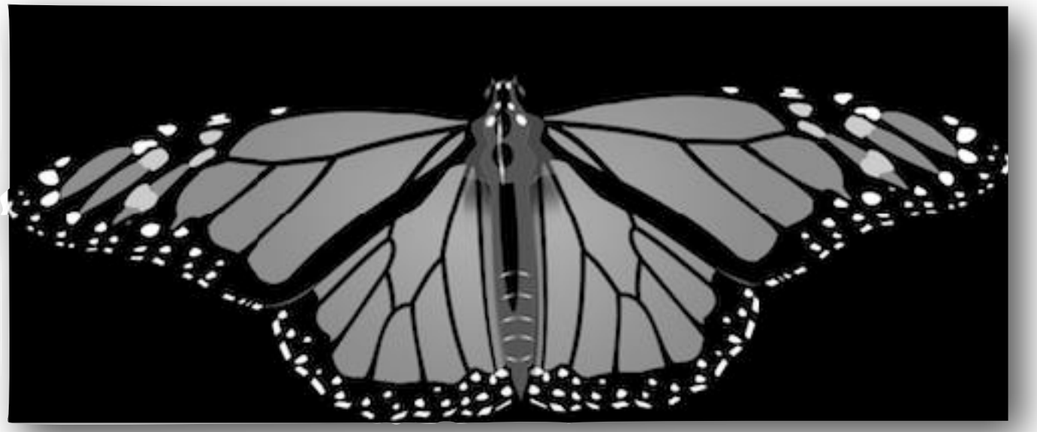
$$\frac{\delta F(x,y)}{\delta y} \approx F(x,y+1) - F(x,y)$$







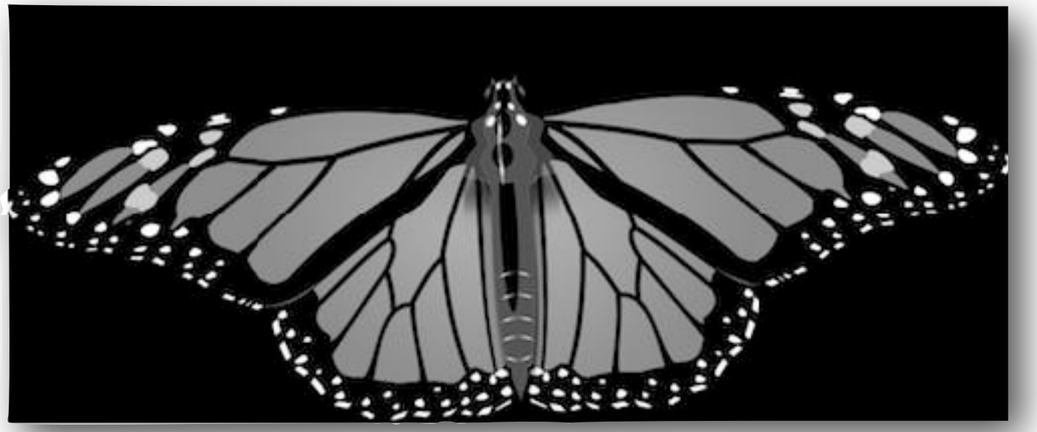
Derivatives of an Image using Correlation



Original Image

Derivatives of an Image using Correlation

$$\frac{\delta F(x,y)}{\delta x}$$



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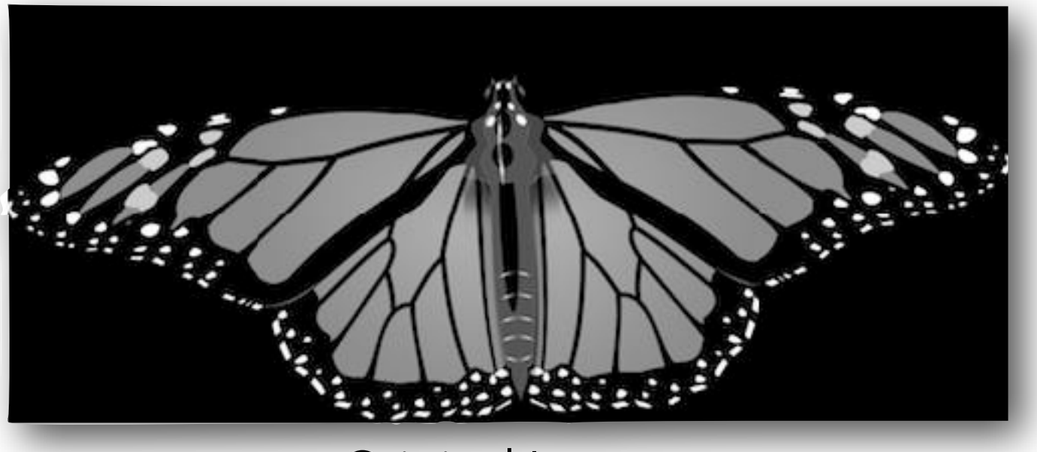

Original Image

$$\frac{\delta F(x,y)}{\delta y}$$

-1	1

X-Correlate with Kernel

-1
1

Derivatives of an Image using Correlation

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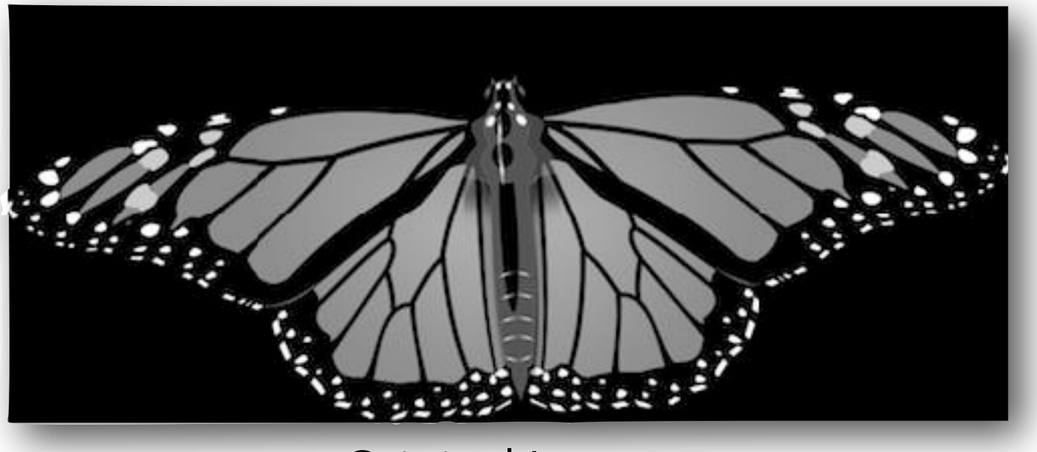

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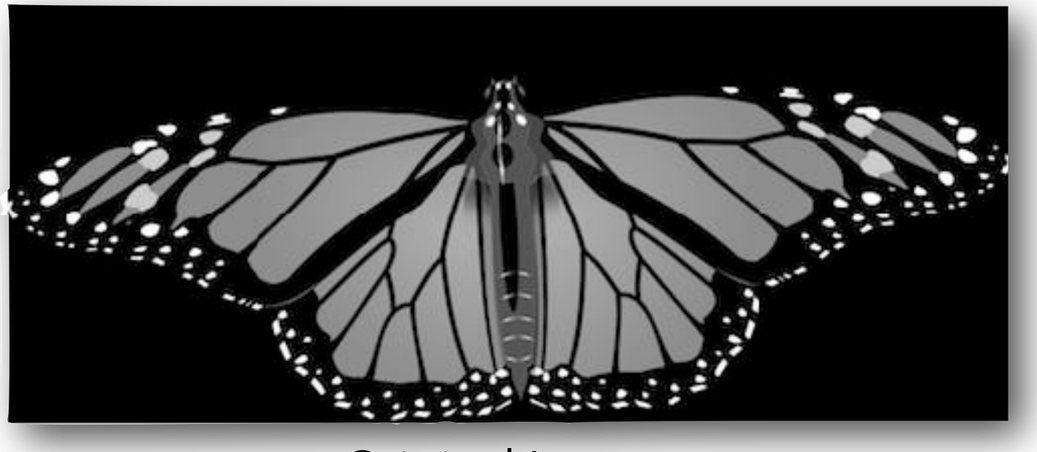

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-1	1

X-Correlate with Kernel

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Discrete Gradient, using Correlation

We want an "operator" (mask/kernel) that we can apply to the image as a cross-correlation that implements the discretized (using Finite Differences) derivative computations:

Discrete Gradient, using Correlation

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Gradient in x and y

$$\frac{\delta F(x,y)}{\delta y} \approx F(x,y+1) - F(x,y)$$

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H_x

0	0
-1	1
0	0

Symmetric about Image Point.
Which is the "middle" point?

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Average of "left" and "right" derivatives

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Rotated Kernel for y: H_y

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0	0
-1	1
0	0

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Which is the "middle" point?

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-1/2	0	1/2
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Average of "left" and "right" derivatives

Various Kernels for Computing Gradients

H_x H_y

Prewitt:

Sobel:

Roberts:

Slide adapted from Aaron Bobick

Various Kernels for Computing Gradients

Prewitt:

H_x

-1	0	1
-1	0	1
-1	0	1

H_y

-1	-1	-1
0	0	0
1	1	1

Sobel:

Roberts:

Slide adapted from Aaron Bobick

Various Kernels for Computing Gradients

	H_x	H_y
Prewitt:	-1 0 1 -1 0 1 -1 0 1	-1 -1 -1 0 0 0 1 1 1
Sobel:	-1 0 1 -2 0 2 -1 0 1	-1 -2 -1 0 0 0 1 2 1
Roberts:		

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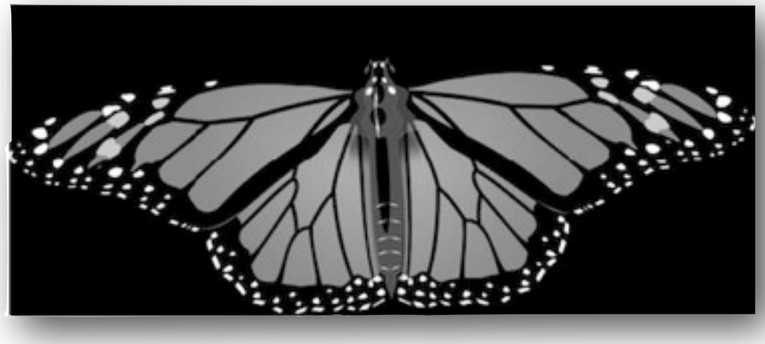
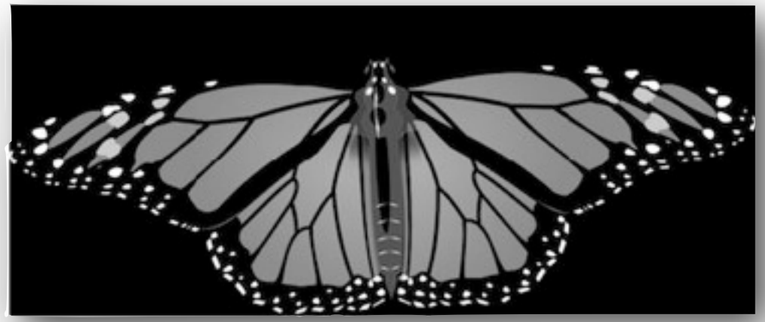
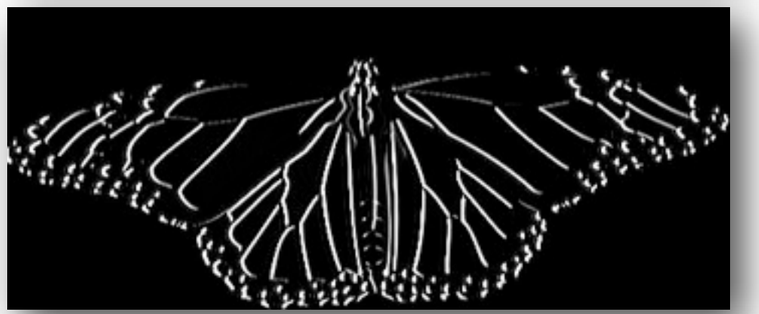
F



Input Image

Slide adapted from Aaron Bobick

Various Kernels for Computing Gradients

	H_x	H_y	$H_x \otimes F$	F
Prewitt:	-1 0 1 -1 0 1 -1 0 1	-1 -1 -1 0 0 0 1 1 1		
Sobel:	-1 0 1 -2 0 2 -1 0 1	-1 -2 -1 0 0 0 1 2 1		Input Image
Roberts:	0 1 -1 0	1 0 0 -1		

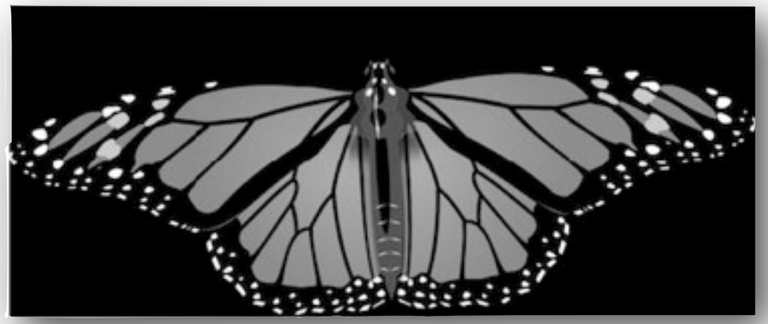
Slide adapted from Aaron Bobick

Various Kernels for Computing Gradients

Edges

$H_x \otimes F$

F



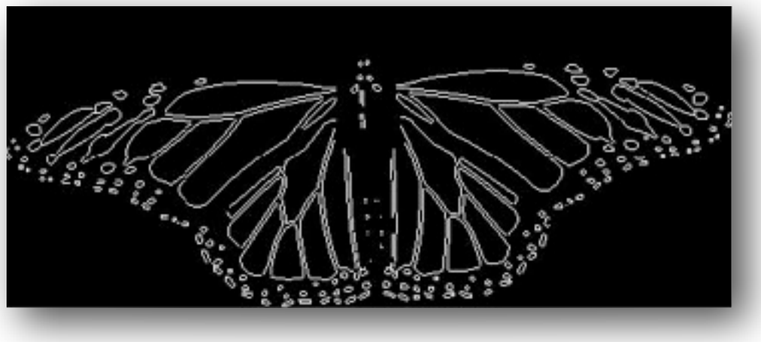
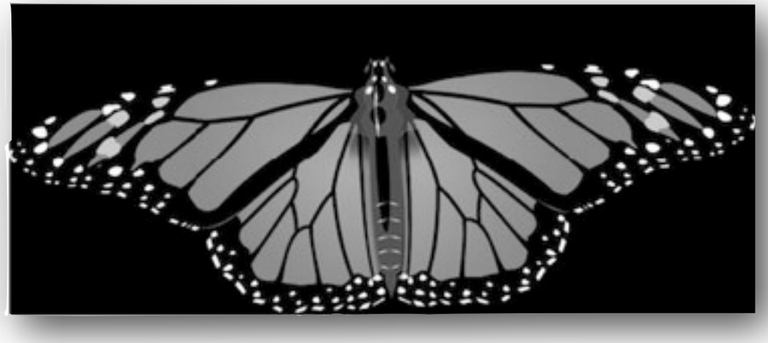
Input Image

Prewitt:

Sobel:

Roberts:

Various Kernels for Computing Gradients

Edges	$H_x \otimes F$	F
Prewitt:			
Sobel:			Input Image
Roberts:			

Various Kernels for Computing Gradients

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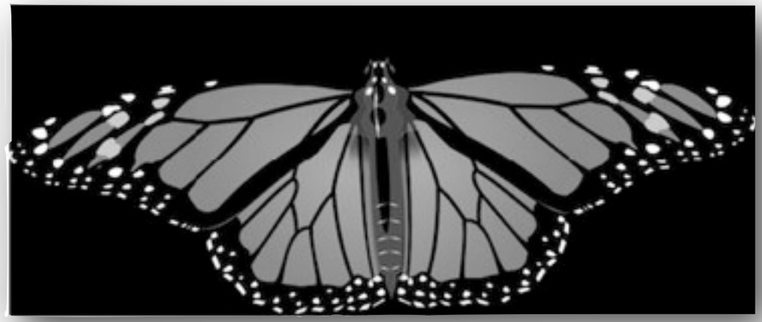
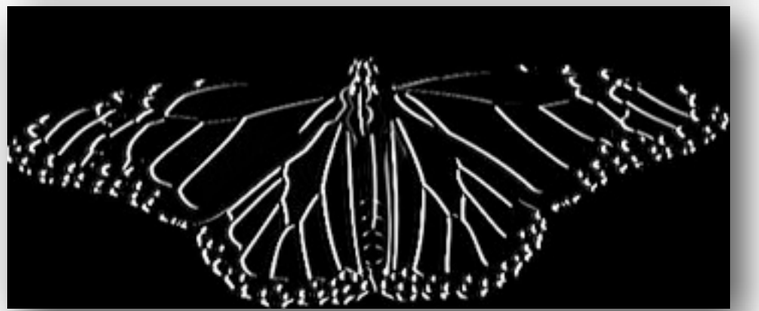
Prewitt:			
Sobel:			Input Image
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Various Kernels for Computing Gradients

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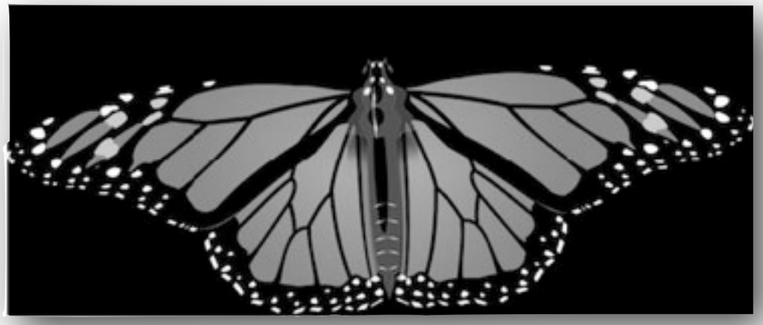
Prewitt:			
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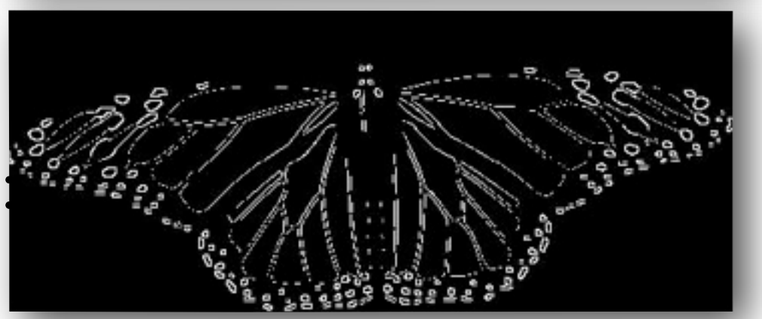
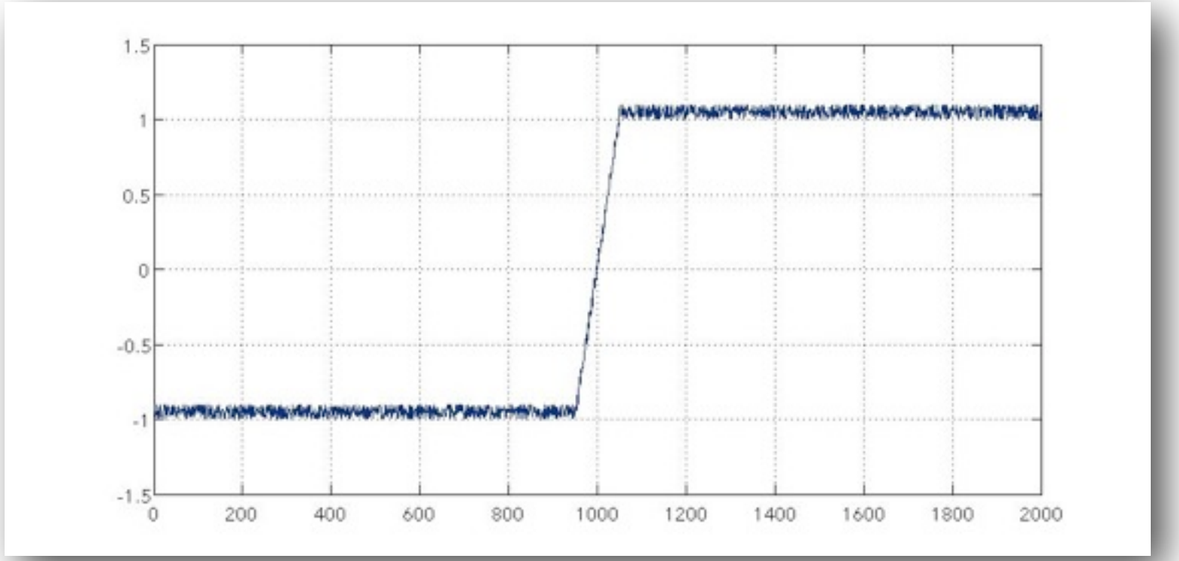
Roberts:			

Impact of Noise on Gradients (1D Example)

$f(x)$



Impact of Noise on Gradients (1D Example)

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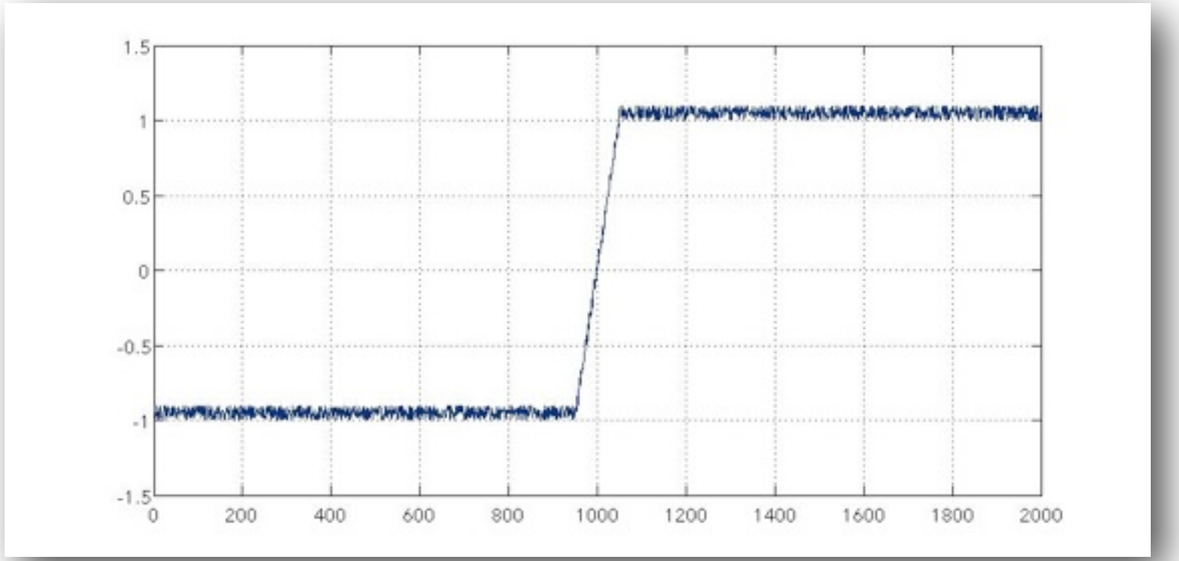
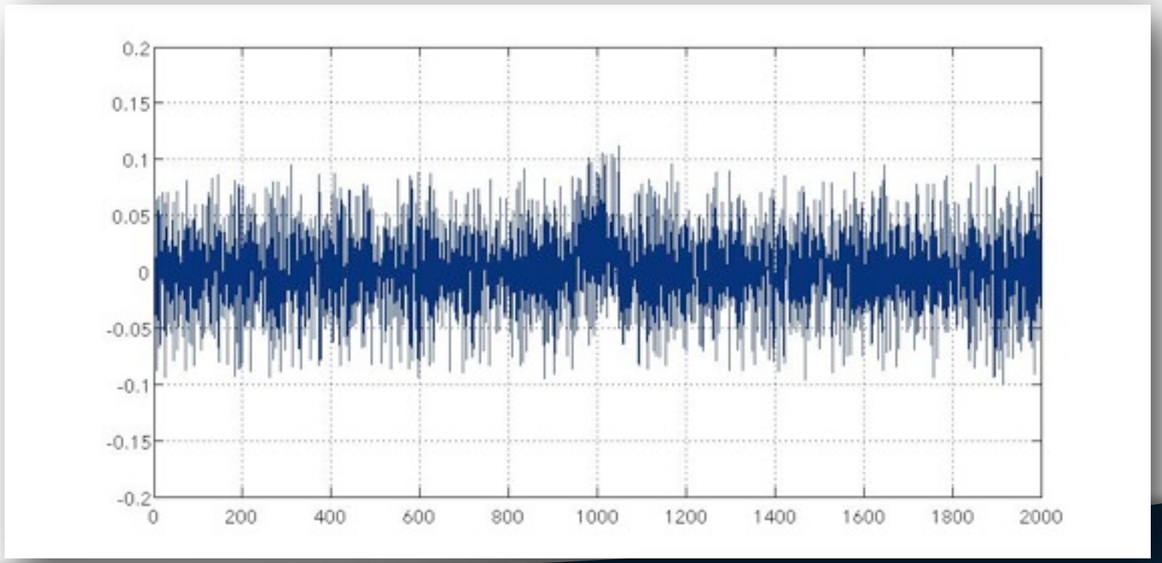


$$\frac{\delta f(x)}{\delta x}$$



It gets harder to detect an edge when there is significant noise in the signal.

Impact of Noise on Gradients

Original Image +
Increasing Gaussian
Noise

Gradient

Increasing Noise

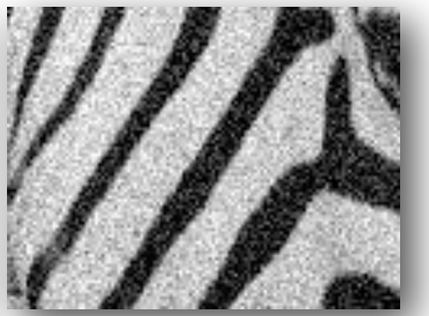
Impact of Noise on Gradients

Original Image + Increasing Gaussian Noise

Gradient

Increasing Noise

Impact of Noise on Gradients



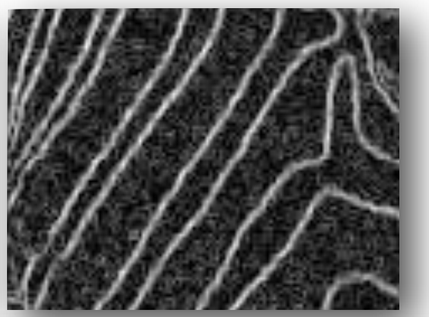
Original Image + Increasing Gaussian Noise

Gradient

Increasing Noise

Impact of Noise on Gradients

Original Image + Increasing Gaussian Noise



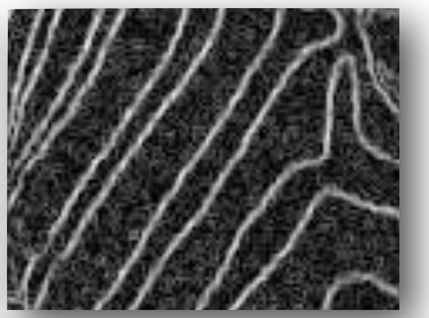
Gradient

Increasing Noise

Impact of Noise on Gradients



Original Image + Increasing Gaussian Noise

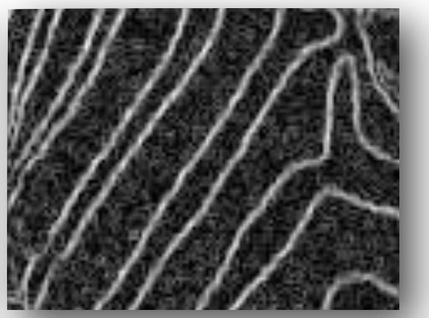
Gradient

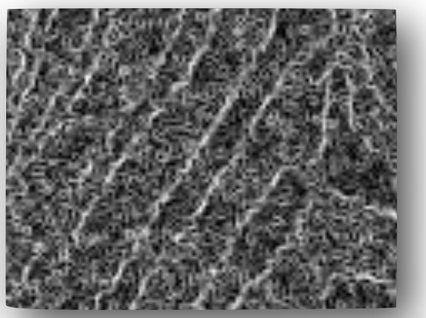
Increasing Noise

10

Impact of Noise on Gradients

Original Image + Increasing Gaussian Noise





Gradient

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Impact of Noise on Gradients




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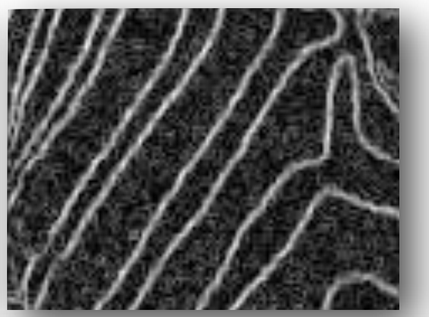
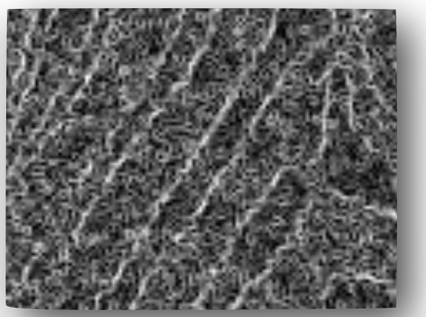
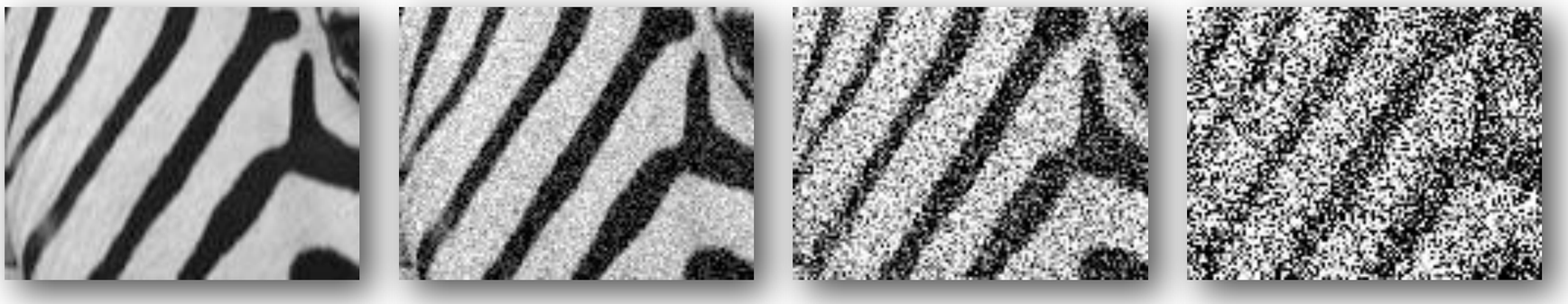



Gradient

Increasing Noise

10

Impact of Noise on Gradients



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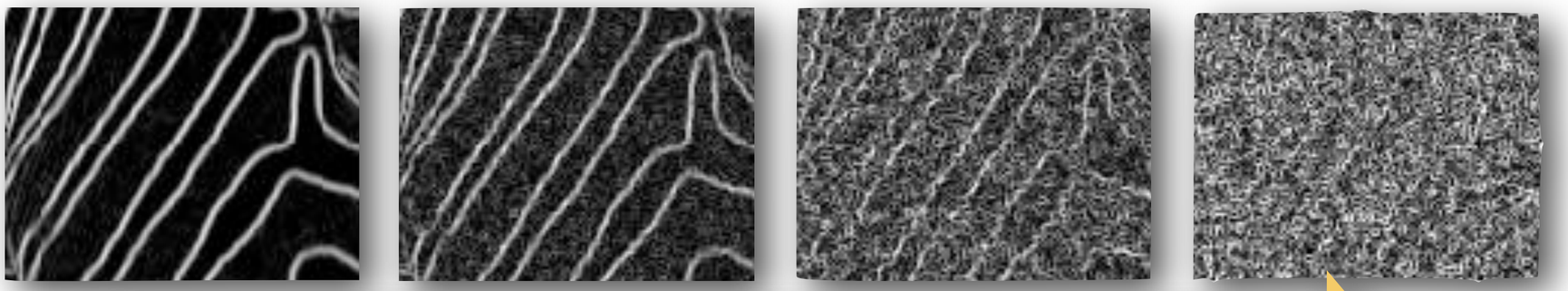
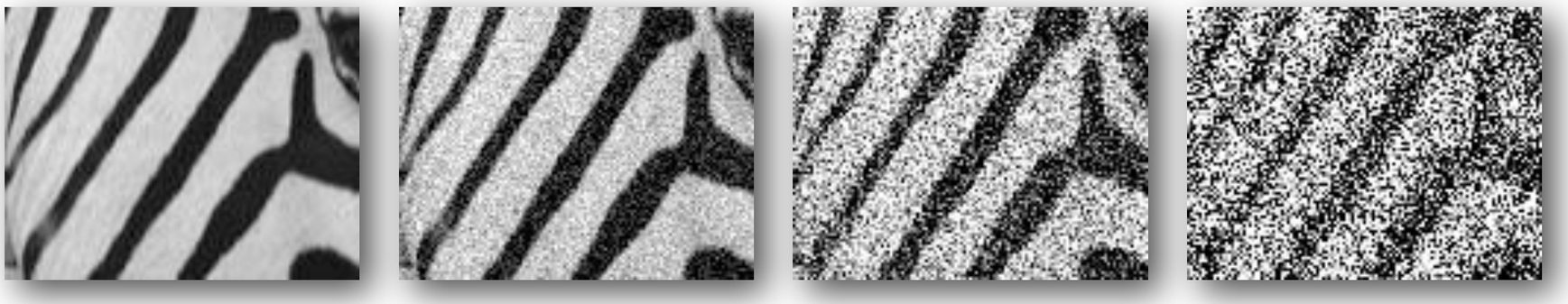


Gradient

Increasing Noise

Impact of Noise on Gradients



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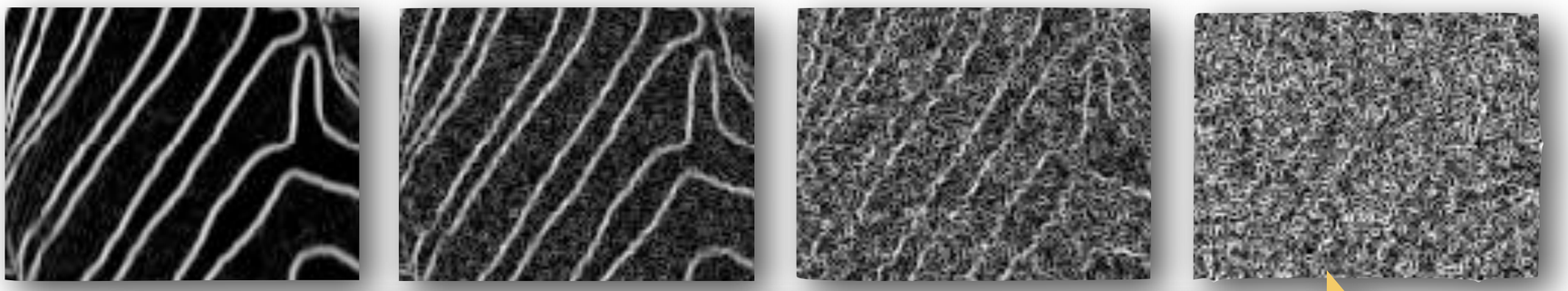

Gradient

Increasing Noise

Smooth the Image, before applying the Gradient Operations.

Convolution and Gradients

Convolution and Gradients

- Recall, Convolution is:

$$G = h * F$$

Convolution and Gradients

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- Derivative of a Convolution is

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- In notation of Convolution, using properties of Convolution, if D is kernel for derivatives and H is the Kernel for smoothing.

$$D * (H * F) = (D * H) * F$$

Convolution and Gradients

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- Derivative of a Convolution is

$$\frac{\delta G}{\delta x} = \frac{\delta}{\delta x}(h * F)$$

- In notation of Convolution, using properties of Convolution, if D is kernel for derivatives and H is the Kernel for smoothing.

$$D * (H * F) = (D * H) * F$$

- So we could define kernels with derivative and smoothing in one

Gradient to Edges

Image

Gradient Image

Edge Image

1. Smoothing: suppress noise
2. Compute Gradient
3. Apply Edge “enhancement”: filter for contrast
4. Edge localization: Determine which local maxima from filter output are actually edges vs. noise
5. Threshold, Thin

Canny Edge Detector

1. Filter image with derivative of Gaussian
2. Find magnitude and orientation of gradient
3. Non-maximum suppression:
   a. Thin multi-pixel wide “ridges” down to single pixel width
4. Linking and thresholding (hysteresis):
   a. Define two thresholds: low and high
   b. Use the high threshold to start edge curves and the low threshold to continue them

Slide adapted from Aaron Bobick

Summary

- ★ Brought together the concepts of Convolution and Correlation with Image Gradient computation.
- ★ Discussed how Edges are computed.
- ★ Presented the process of smoothing prior to Gradient computation.
- ★ Presented four (4) different methods for Edge computation.

Next Class

- ★ Camera, Optics, Lenses,..
- ★ We will return to using the concepts of Edge detection when we go back to the topic of Feature matching.

15

Credits

- Matlab™ software by Mathworks Inc.
- Some slides adapted from Aaron Bobick
- For more information, see Szeliski OR Forsyth & Ponce Text Book.
- Images
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