## General Least Squares Quiz

10/10 points (100%)

Quiz, 10 questions

## **✓** Congratulations! You passed!

Next Item



1/1 points

1.

If I is an identity matrix and H is an idempotent matrix, I-H is necessarily idempotent?



True

Correct

$$(I-H)^2 = I^2 - IH - HI - H^2 = I - H$$



False



1/1 points

2.

If a design matrix, X, is orthonormal (so that  $X^\prime X = I$ ), the least squares estimate for the coefficients for outcome vector Y simplifies to (check all that apply):



X'Y

Correct

$$(X'X)^{-1} = I$$

It can't be simplified from the typical estimate without further information.

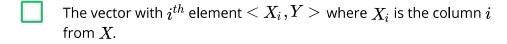


### **Un-selected** is correct

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If 
$$X = [X_1 \; X_2 \; \dots X_p]$$
 then  $X'Y = (< X_1, Y> \; < X_2, Y> \; \dots < X_p, Y>)'$ 



1/1 points

3.

Let X be a design matrix. X, X', X'X and XX' all have the same matrix rank. (Figure out why or why not.)



True

### Correct

This is a standard linear algebra result.



False

**Un-selected** is correct



1/1 points

4.

Let  $J_a$  be a vector of ones of length a,let  $\otimes$  be the Kronecker product (see wikipedia for definition) and let  $I_b$  be a  $b \times b$  identity matrix. Let  $X = J_a \otimes I_b$  be a design matrix and Y be a  $ab \times 1$  response vector. What does the least squares estimate of the coefficients simplify to? (Check all that apply.)

The empirical means of the b means of the elements of Y in groups of size a.

#### Correct

General Least Squares louise on least squares examples. The Kronecker product is just a convenient way to write out ANOVA type examples.

10/10 points (100%)

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The empirical standard deviations of the elements of $Y$ in groups
of size $a.$

### **Un-selected** is correct

One over (the inverse of ) the empirical means of the b means of the elements of Y in groups of size a.

### **Un-selected** is correct



1/1 points

5.

For matrices A and B satisfying the matrix dimensions, let  $e_{A|B} = (I - B(B'B)^{-1}B')A'$  be the matrix of residuals. Let  $X = [X_1 \mid X_2]$ be a design matrix comprised of submatrices  $X_1$  and  $X_2$  and let Y be a response vector. The least squares estimates with  $\boldsymbol{X}$  as a design matrix and Y as the response vector for the components of eta corresponding to  $X_1$  are equal to?

A least squares fit of  $e_{Y\mid X_2}$  as the outcome and  $e_{X_1\mid X_2}$  as the design matrix.

### Correct

A least squares fit of $e_{Y\mid X_1}$ as the outcome and $e_{X_2\mid X_1}$ as the design matrix.
A least squares fit of $Y$ as the outcome and $X_1.$
A least squares of $e_{Y\mid X_1}$ as the outcome and $e_{X_1\mid X_2}$ as the

predictor.

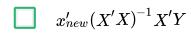
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6.

Let X be a design matrix  $(n \times p)$  and Y  $(n \times 1)$  be the outcome vector. Let  $x_{new}$  be a  $p \times 1$  vector of new values of X that we would like to predict at. What would be the predicted value using the least squares fit (check all that apply)?



Correct

**Un-selected** is correct

$$x_{new}'\hat{eta}$$
 where  $\hat{eta}=X(X'X)^{-1}X'Y$ 

Correct



1/1 points

7.

Fit the swiss dataset with Fertility as the outcome and the rest of the variables as predictors. What is the Agriculture coefficient?

0.1942

-0.1721

Correct

60.3044



points

## General Least Squares Quiz

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Quiz, 10 questions

Fit the swiss dataset with Fertility as the outcome and the rest of the variables as predictors. Predict Fertility in a province with the following data

```
1  newData = data.frame(
2  Agriculture = 27.7,
3  Examination = 22,
4  Education = 29,
5  Catholic = 58.33,
6  Infant.Mortality = 19.3)
7
```

65.68

82.51

74.62

58.07

**Correct** 



1/1 points

9.

Let Y be an  $n \times 1$  response vector. Let U = XW where W is a non-identity invertible  $p \times p$  matrix and U and X are  $n \times p$ . What can be said about the fitted values from the least squares fit of U as a design versus X as a design matrix with Y as the outcome (check all that apply)?

The fitted values,  $\hat{Y}$ , will be identical.

### Correct

Note the the space spanned by the rows X and XW are the same.  $XW(W'X'XW)^{-1}W'X'Y=XWW^{-1}(X'X)^{-1}(W')^{-1}W'X'Y=\hat{Y}$ 

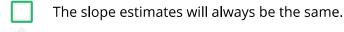
The fitted values,  $\hat{Y}$ , will be different.

### **Un-selected** is correct

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### **Un-selected** is correct

Nothing definitive can be determined from the information given.

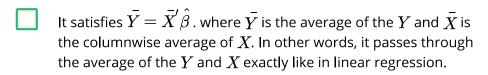
#### **Un-selected** is correct



1 / 1 points

10.

Suppose that a design vector X contains an intercept column. What can be said about the fitted regression surface  $y=x\hat{\beta}$ ?



### Correct

$$e'J_n=(Y-X\hat{eta})'J_n=0$$
 so that  $nar{Y}-nar{X}_1'eta=0$  .

It doesn't necessarily satisfy  $ar{Y}=ar{X}'\hat{eta}$  . where  $ar{Y}$  is the average of the Y and  $ar{X}$  is the columnwise average of X.

### **Un-selected is correct**





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