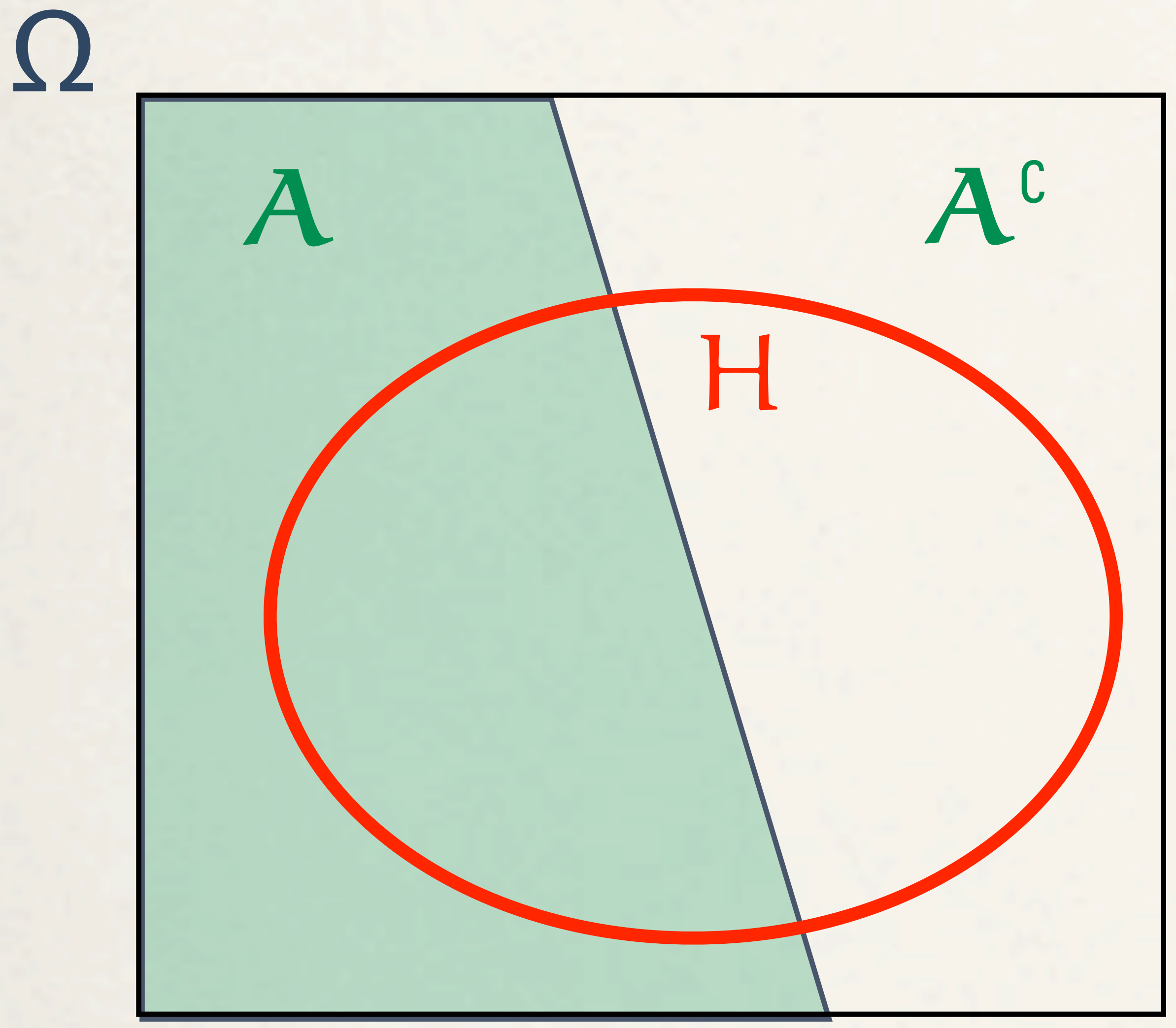
Additivity, reprised

$$\mathbf{P}(H \cap A) = \mathbf{P}(H \mid A)\,\mathbf{P}(A)$$

$$H = (H \cap A) \cup (H \cap A^c)$$



$$\mathbf{P}(H) = \mathbf{P}(H \cap A) + \mathbf{P}(H \cap A^c) = \mathbf{P}(H \mid A)\,\mathbf{P}(A) + \mathbf{P}(H \mid A^c)\,\mathbf{P}(A^c)$$

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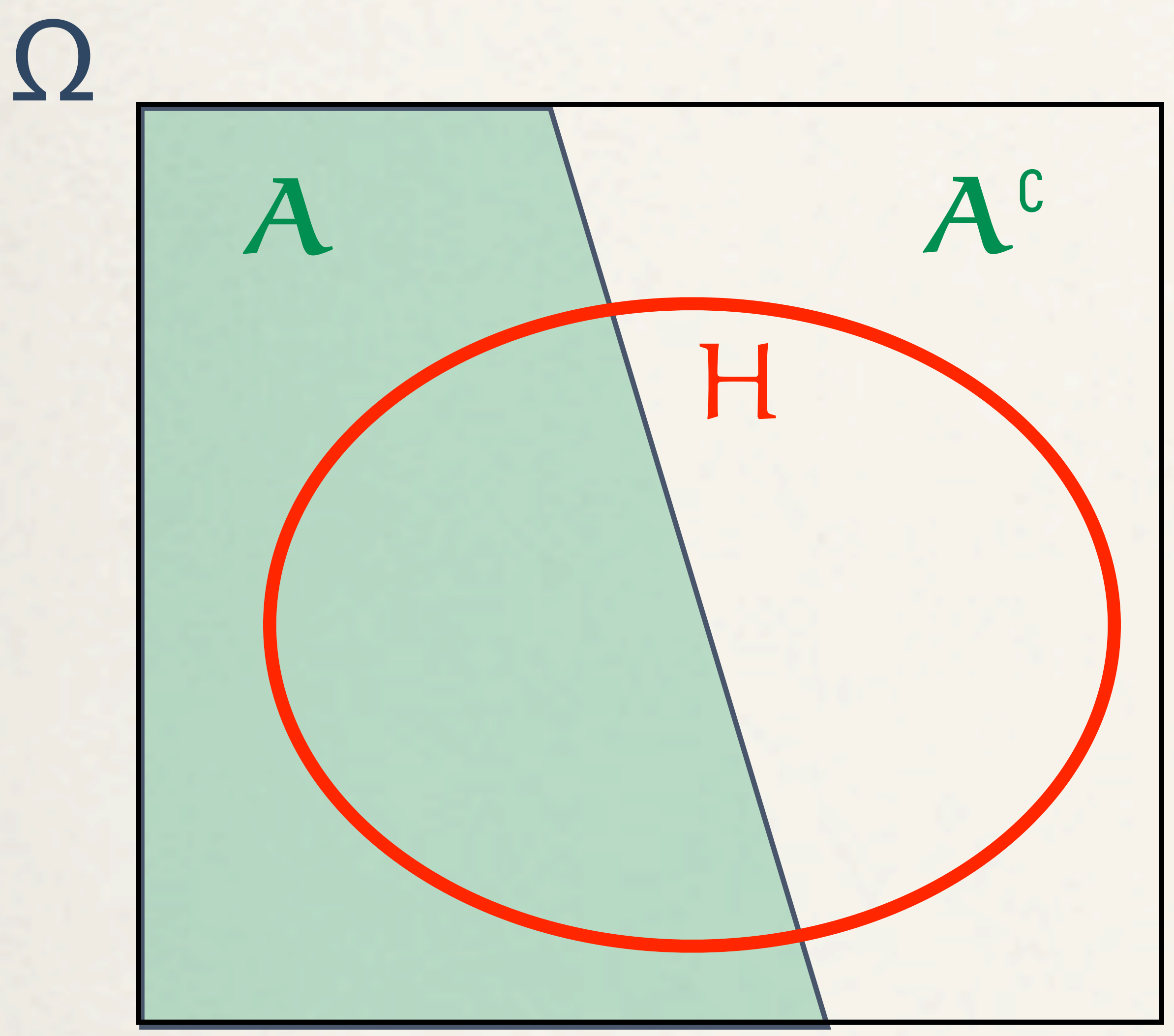

$\{A, A^c\}$ partitions Ω

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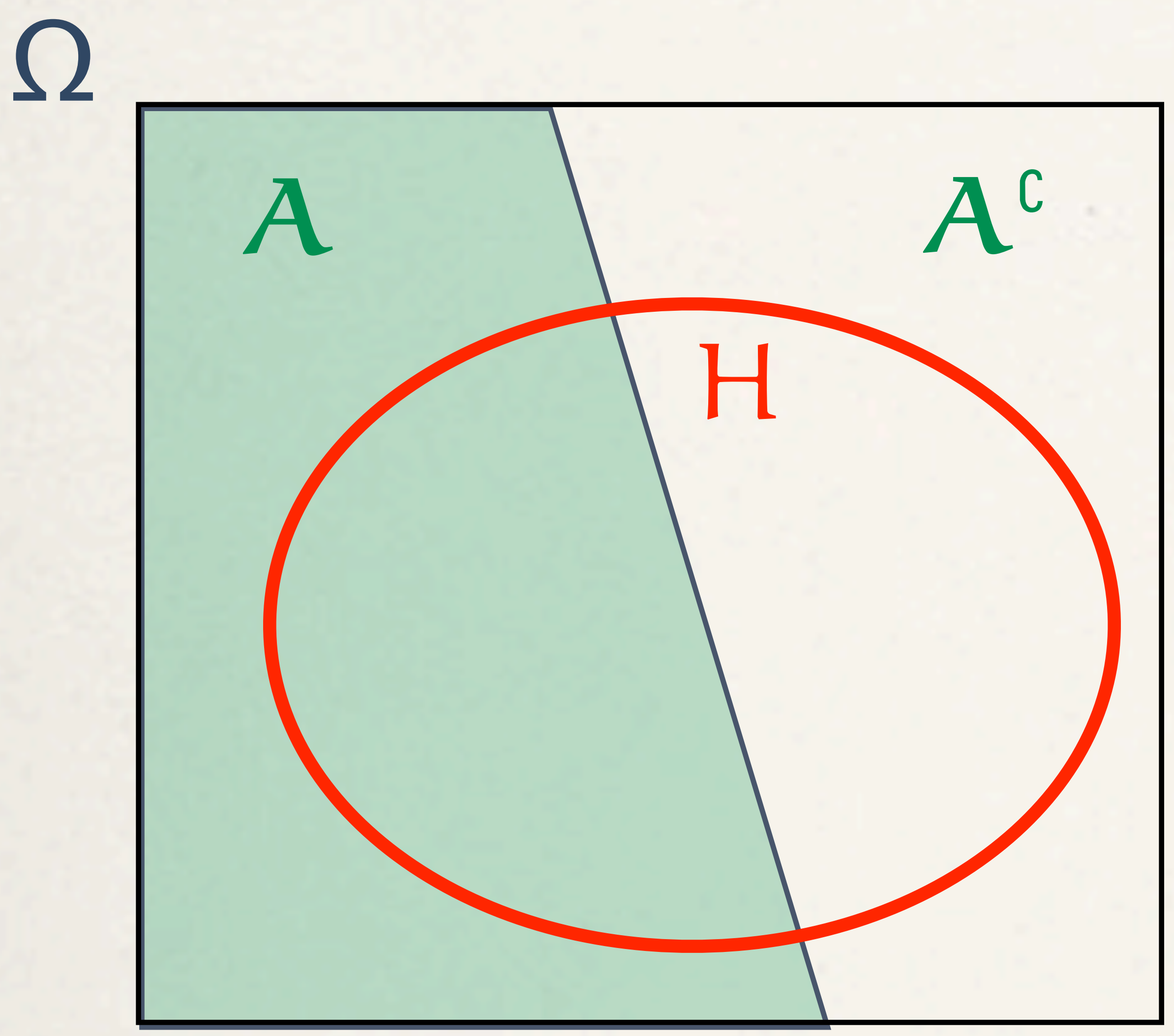

$\{A, A^c\}$ *partitions* $\boldsymbol{\Omega}$

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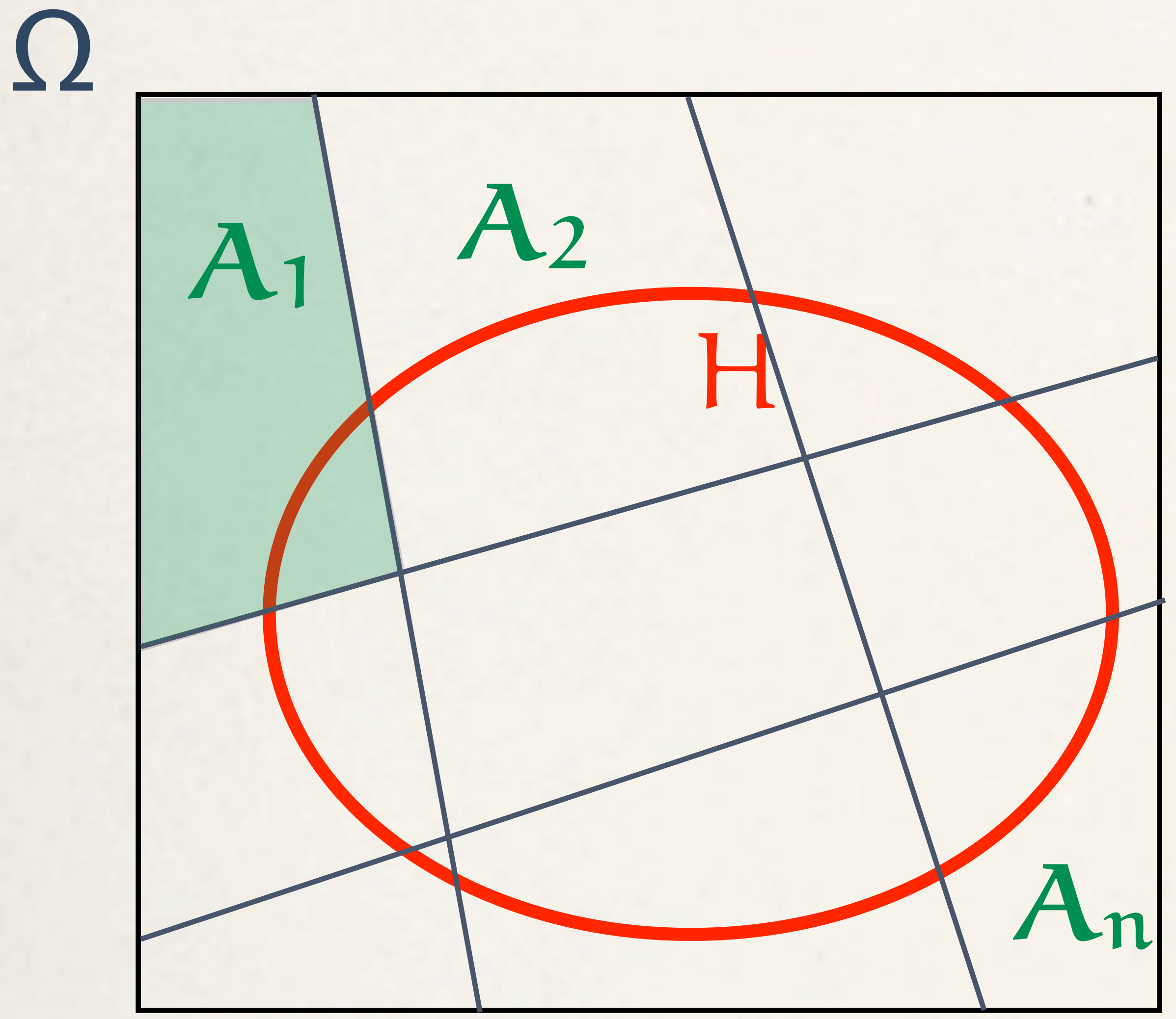

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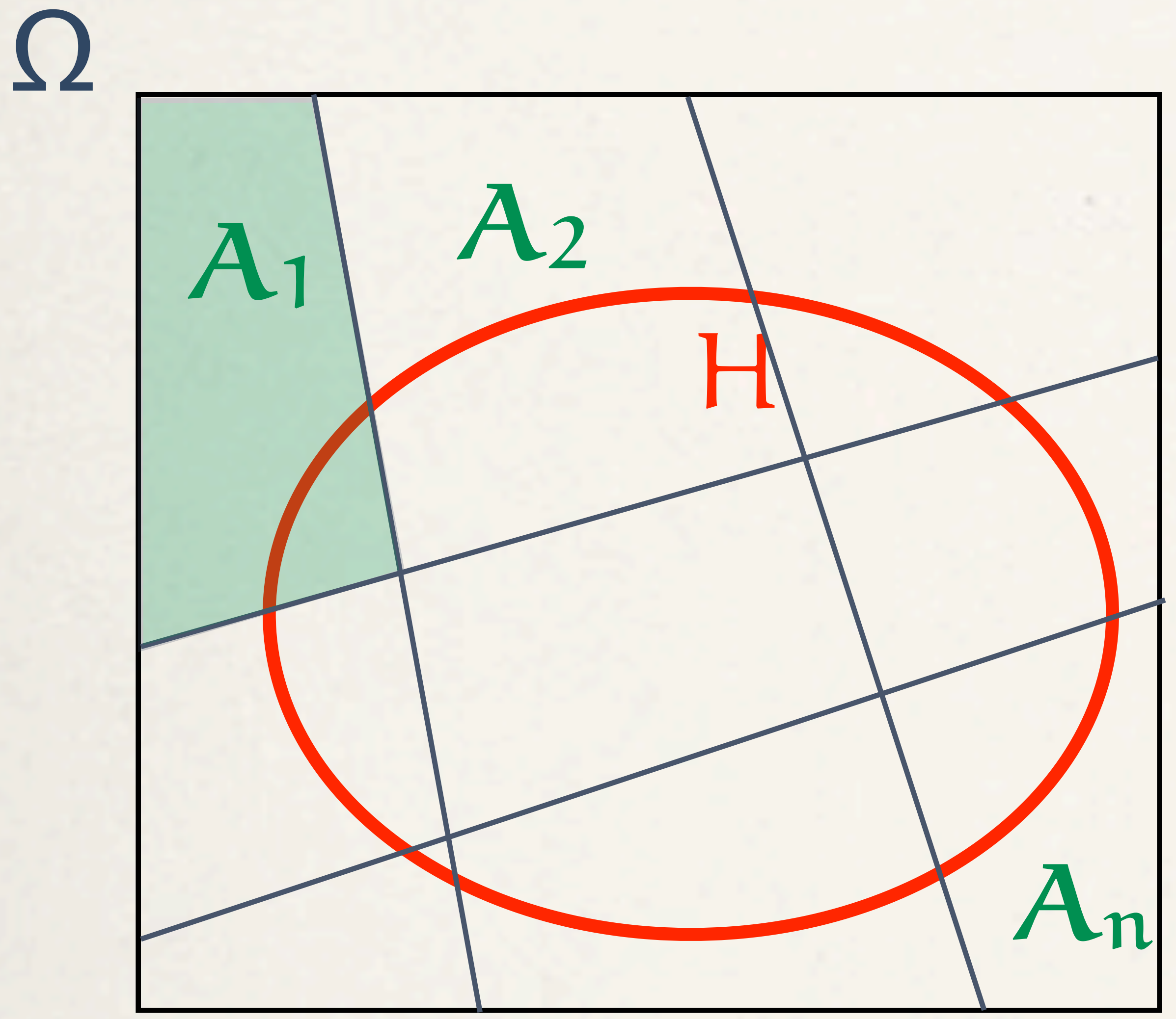

$\{A_1, A_2, \ldots, A_n\}$ *partitions* Ω

$$\mathbf{P}(H) = \mathbf{P}(H \mid A)\,\mathbf{P}(A) + \mathbf{P}(H \mid A^c)\,\mathbf{P}(A^c)$$

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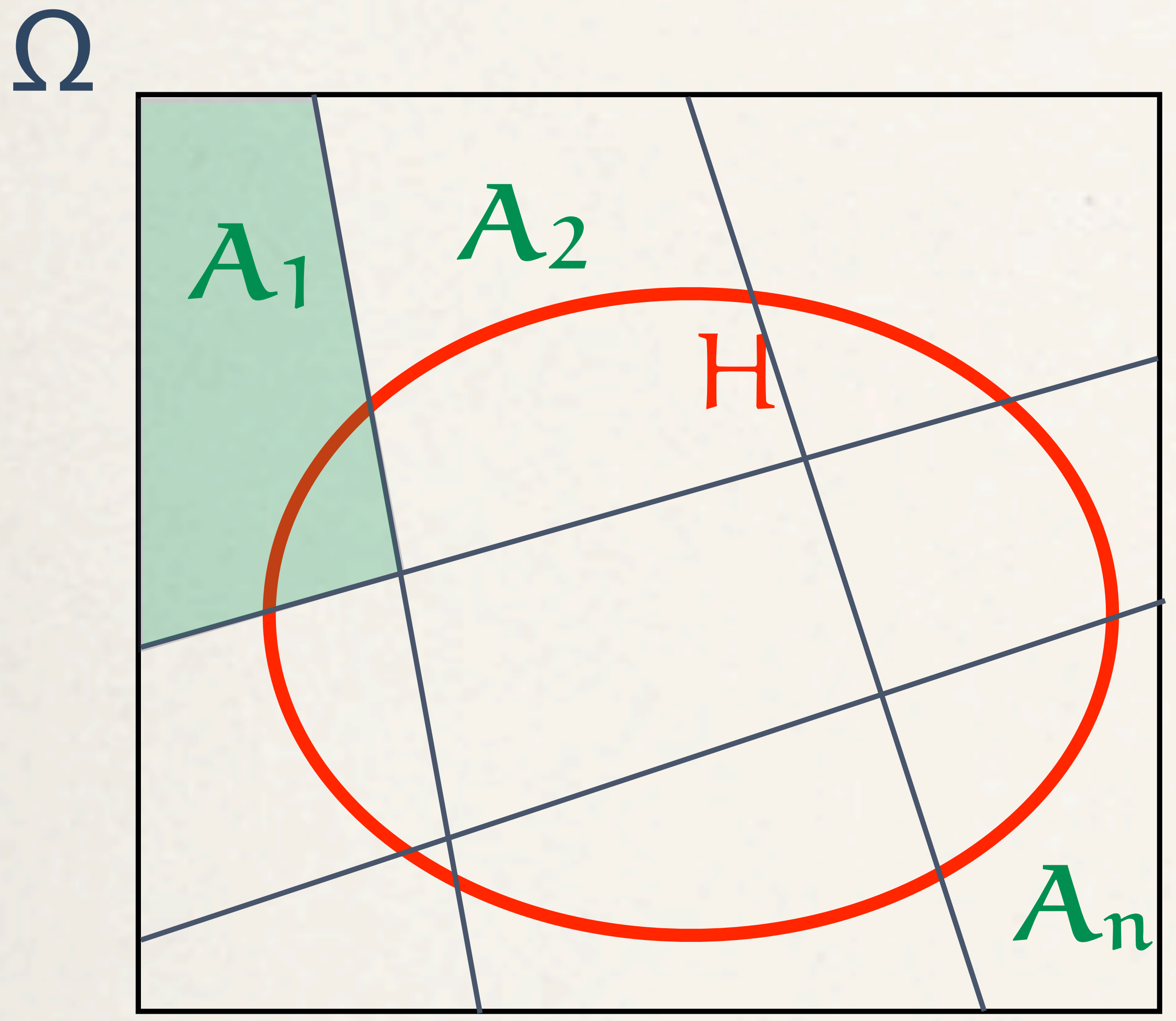

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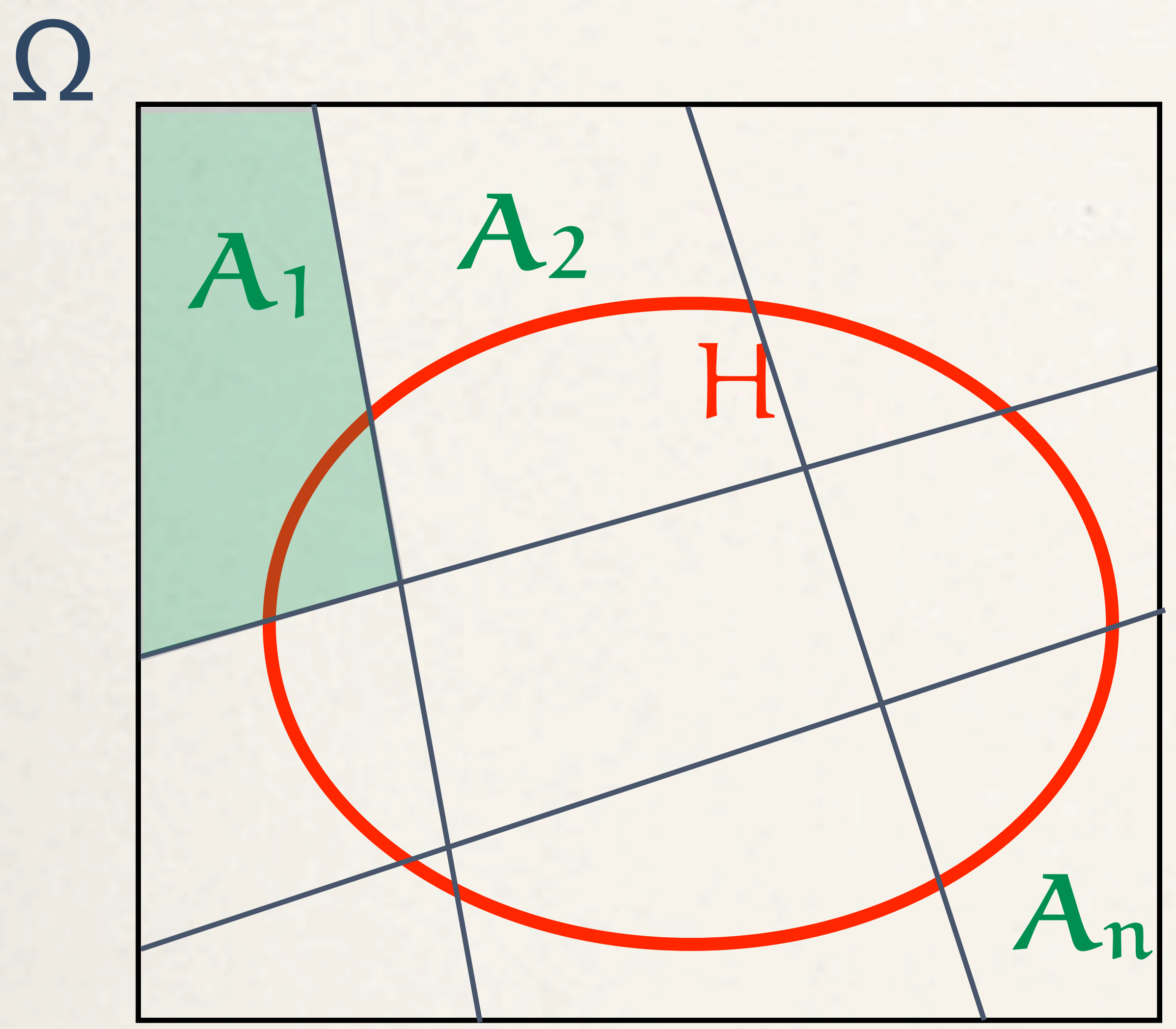

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$\{A_1, A_2, \ldots, A_n\}$ *partitions* Ω

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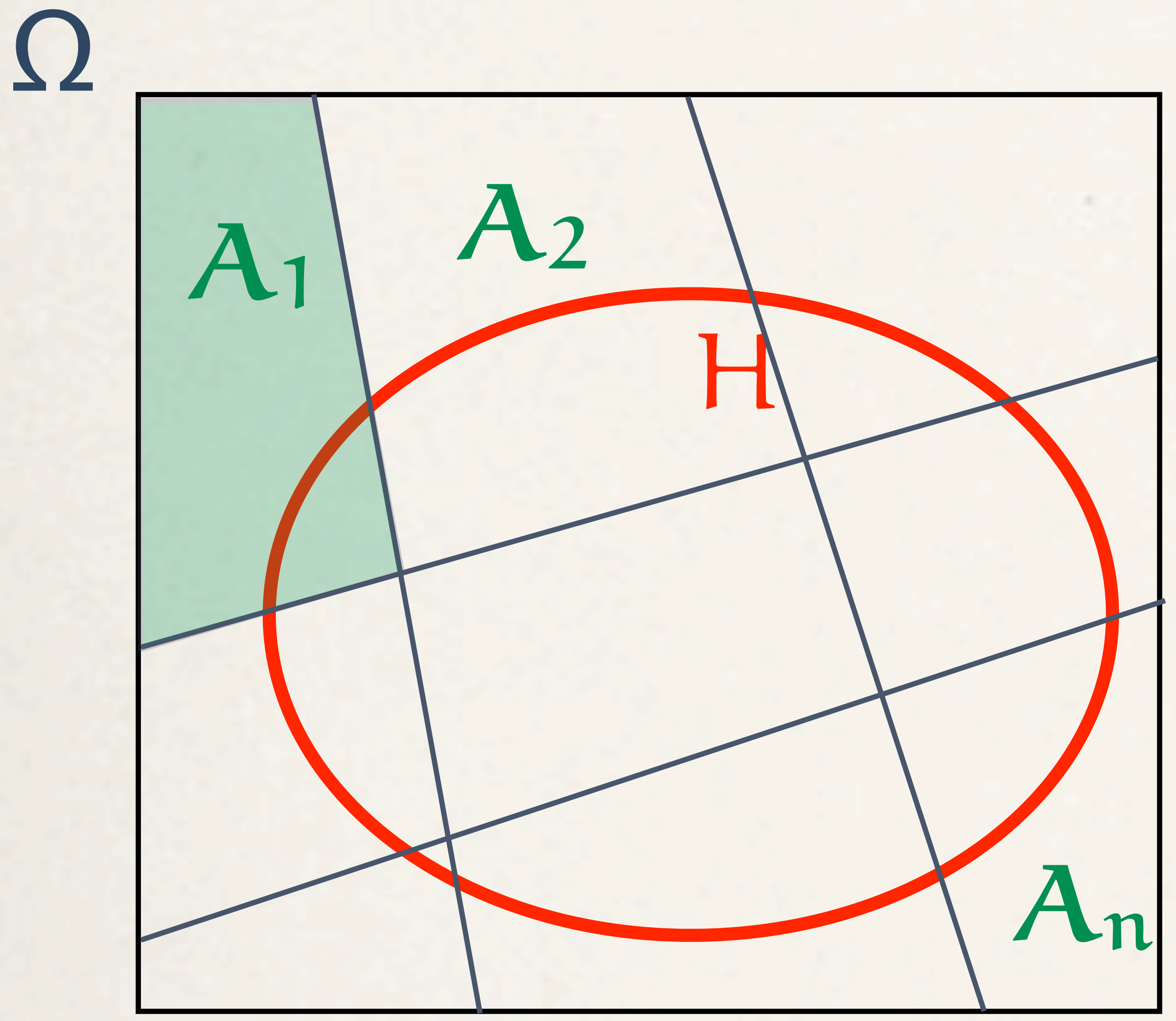

$$\begin{aligned}\mathbf{P}(H) &= \mathbf{P}(H \cap A_1) + \mathbf{P}(H \cap A_2) + \cdots + \mathbf{P}(H \cap A_n) \\ &= \mathbf{P}(H \mid A_1)\,\mathbf{P}(A_1) + \mathbf{P}(H \mid A_2)\,\mathbf{P}(A_2) + \cdots + \mathbf{P}(H \mid A_n)\,\mathbf{P}(A_n)\end{aligned}$$

$\{A_1, A_2, \ldots, A_n\}$ *partitions* Ω

The theorem of total probability

If $\{ A_j, j \geq 1 \}$ partitions Ω into a finite or countably infinite collection of events of positive probability, then we may decompose the probability of any event H in the form

$$\mathbf{P}(H) = \sum_j \mathbf{P}(H \mid A_j)\, \mathbf{P}(A_j)$$