5. Prove that for any integer n, at least one of the integers n, $n+2$, $n+4$ is divisible by 3

Theorem: Prove that for any integer n, at least one of the integers n, n+2, n+4 is divisible by 3.

Proof: By induction.

n=1 n, 1 is not divisible by 3

n+2, 3 is divisible by 3

n+4, 5 is not divisible by 3

Assume n is true. Then n, n+2 or n+4 is divisible by 3. You can write on of them as that as 3m, where m is an integer. Now, our number n must leave a remainder of either 0, 1, or 2 when divided by 3, so it can be written as either 3m, 3m+1, or 3m+2 for some m (the quotient)

So we have three cases:

n	n+2	n+4
3m	3m+2	3m+4
3m+1	3m+3	3m+5
3m+2	3m+4	3m+6

At least one of the integers n, n+2 , n+4 is divisible by 3.

So, let's proof now that n+1 is true

(n+1)	(n+1)+2	(n+1)+4
3m+1	3m+3	3m+5
3m+2	3m+4	3m+6
3m+3	3m+5	3m+7

For n+1 at least one of the integers (n+1), (n+1)+2, (n+1)+4 is divisible by 3, so n+1 is true . This proves the theorem.