7.4.3.5. Confidence intervals for the difference of treatment means

Confidence intervals for the difference between two means

This page shows how to construct a confidence interval around $(\mu_i - \mu_j)$ for the one-way ANOVA by continuing the example shown on a previous page.

Formula for the confidence interval

The formula for a $100(1-\alpha)$ % confidence interval for the difference between two treatment means is:

$$(\hat{\mu}_i - \hat{\mu}_j) \pm t_{1-\alpha/2,\, N-k} \sqrt{\hat{\sigma}^2_\epsilon \left(\frac{1}{n_i} + \frac{1}{n_j}\right)},$$

where $\hat{\sigma}^2_\epsilon = MSE$.

Computation of the confidence interval for $\mu_3 - \mu_1$

For the example, we have the following quantities for the formula.

- $\bar{y}_3 = 8.56$
- $\bar{y}_1 = 5.34$
- $\sqrt{1.454(1/5 + 1/5)} = 0.763$
- $t_{0.975,\, 12} = 2.179$

Substituting these values yields (8.56 - 5.34) ± 2.179(0.763) or 3.22 ± 1.616.

That is, the confidence interval is (1.604, 4.836).

Additional 95 % confidence intervals

A 95 % confidence interval for $\mu_3 - \mu_2$ is: (-1.787, 3.467).

A 95 % confidence interval for $\mu_2 - \mu_1$ is: (-0.247, 5.007).

Contrasts discussed later

Later on the topic of estimating more general linear combinations of means (primarily contrasts) will be discussed, including how to put confidence bounds around

contrasts.