$$u_i = \beta_1 + \sum_{j=2}^{k} \beta_j x_{ji} + \eta_i$$

a) $$\Pr[y_i = 1] = \Pr[u_i \geq 0] = \Pr[\beta_1 + \sum_{j=2}^{k} \beta_j x_{ji} + \eta_i \geq 0]$$
$$= \Pr[\eta_i \geq -\beta_1 - \sum_{j=2}^{k} \beta_j x_{ji}]$$
$$= \Pr[-\eta_i \leq \beta_1 + \sum_{j=2}^{k} \beta_j x_{ji}]$$

b) $$\Pr[y_i = 1] = \Pr[\eta_i \leq \beta_1 + \sum_{j=2}^{k} \beta_j x_{ji}] = \frac{\exp(\beta_1 + \sum_{j=2}^{k} \beta_j x_{ji})}{1 + \exp(\beta_1 + \sum_{j=2}^{k} \beta_j x_{ji})}$$

c) $$\frac{\frac{\partial \Pr[y_i = 1]}{\partial x_{ji}}}{\frac{\partial \Pr[y_i = 1]}{\partial x_{ei}}} = \frac{\Pr[y_i = 1]\Pr[y_i = 0]\beta_j}{\Pr[y_i = 1]\Pr[y_i = 0]\beta_e} = \frac{\beta_j}{\beta_e}$$

d) $$\frac{\partial \frac{\exp(\beta_1 + \sum_{j=2}^{k} \beta_j x_{ji})}{1 + \exp(\beta_1 + \sum_{j=2}^{k} \beta_j x_{ji})}}{\partial x_{ji}} =$$

$$(1 + \exp(\beta_1 + \sum_{j=2}^{k} \beta_j x_{ji}))^{-1} \frac{\partial \exp(\beta_1 + \sum_{j=2}^{k} \beta_j x_{ji})}{\partial x_{ji}} + \exp(\beta_1 + \sum_{j=2}^{k} \beta_j x_{ji}) \frac{\partial(\exp(\beta_1 + \sum_{j=2}^{k} \beta_j x_{ji}) + 1)^{-1}}{\partial x_{ji}} \quad (1)$$

$$\frac{\partial \exp(\beta_1 + \sum_{j=2}^{k} \beta_j x_{ji})}{\partial x_{ji}} = \exp(\beta_1 + \sum_{j=2}^{k} \beta_j x_{ji})\beta_j$$

$$\frac{\partial (1 + \exp(\beta_1 + \sum_{j=2}^{k} \beta_j x_{ji}))^{-1}}{\partial x_{ji}} = \frac{1}{(1 + \exp(\beta_1 + \sum_{j=2}^{k} \beta_j x_{ji}))^2} \frac{\partial \exp(\beta_1 + \sum_{j=2}^{k} \beta_j x_{ji})}{\partial x_{ji}}$$

$$(1) \quad \frac{\exp(\beta_1 + \sum_{j=2}^{k} \beta_j x_{ji})\beta_j}{1 + \exp(\beta_1 + \sum_{j=2}^{k} \beta_j x_{ji})} - \frac{\exp(\beta_1 + \sum_{j=2}^{k} \beta_j x_{ji}) \exp(\beta_1 + \sum_{j=2}^{k} \beta_j x_{ji})\beta_j}{(1 + \exp(\beta_1 + \sum_{j=2}^{k} \beta_j x_{ji}))^2}$$

$$\Pr[y_i = 1]\beta_j - \Pr[y_i = 1]\Pr[y_i = 1]\beta_j = \Pr[y_i = 1](1 - \Pr[y_i = 1])\beta_j = \Pr[y_i = 1]\Pr[y_i = 0]\beta_j$$