Matrix Exponential

From calculus, the exponential function is sometimes defined from the power series

$$e^x = 1 + x + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + \dots .$$

In analogy, the matrix exponential of an n-by-n matrix A can be defined by

$$e^{\mathrm{A}} = \mathrm{I} + \mathrm{A} + \frac{1}{2!}\mathrm{A}^2 + \frac{1}{3!}\mathrm{A}^3 + \dots .$$

If A is diagonalizable, show that

$$e^{\mathrm{A}} = \mathrm{S}e^{\Lambda}\mathrm{S}^{-1},$$

where

$$e^{\Lambda} = \begin{pmatrix} e^{\lambda_1} & 0 & \dots & 0 \\ 0 & e^{\lambda_2} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & e^{\lambda_n} \end{pmatrix}.$$