Computation of the Feigenbaum delta

Compute the Feigenbaum delta from the logistic map. The logistic map is given by

$x_{i+1} = \mu x_i(1 - x_i)$,

and the Feigenbaum delta is defined as

$\delta = \lim_{n\to\infty} \delta_n$, where $\delta_n = \frac{m_{n-1} - m_{n-2}}{m_n - m_{n-1}}$,

and where m_n is the value of μ for which $x_0 = 1/2$ is in the orbit of the period-N cycle with $N = 2^n$.

Here is a resonable outline:

Loop 1 Start at period-2^n with $n = 2$, and increment n with each iteration

Compute initial guess for m_n using m_{n-1}, m_{n-2} and δ_{n-1}.

Loop 2 Iterate Newton's method, either a fixed number of times or until convergence

Initialize logistic map

Loop 3 Iterate the logistic map 2^n times

Compute x and x'

Loop 3 (end)

One step of Newton's method

Loop 2 (end)

Save m_n and compute δ_n

Loop 1 (end)

Grading will be done on the converged values of δ_n up to $n = 11$. Set $\delta_1 = 5$.

Script

Save Reset MATLAB Documentation (https://www.mathworks.com/help/)

```matlab
% Compute the Feigenbaum delta
% Store approximate values in the row vector delta for assessment, where length(delta)= num_doublings and
% delta(2:num_doublings) are computed from the algorithm described in Lectures 21-23.
num_doublings=11; delta=zeros(1,num_doublings); delta(1)=5;
% Write your code here
m = zeros(1,num_doublings);
m(1) = 1 + sqrt(5);
for n=2:num_doublings
    if n == 2
        m_prev = 2;
    else
        m_prev = m(n-2);
    end
    m(n) = m(n-1) + (m(n-1) - m_prev) / delta(n-1);
    for k = 1:25
        x_N = 1/2; %x_0 = 1/2;
        dx_N = 0;  %dx_0 = 0;
        for i = 1:2^n
            dx_N = x_N*(1-x_N) + m(n)*dx_N*(1-2*x_N);
            x_N = m(n)*x_N*(1-x_N);
        end
        m(n) = m(n) - (x_N - 1/2) / dx_N;
    end
    delta(n) = (m(n-1) - m_prev) / (m(n) - m(n-1));
end










% Output your results
fprintf('n        delta(n)\n');
for n=1:num_doublings
    fprintf('%2g %18.15f\n',n,delta(n));
end

```

Run Script

Assessment: All Tests Passed

Submit

Test delta variable

Output

```
n        delta(n)
 1  5.000000000000000
 2  4.708943013540505
 3  4.680770998010699
 4  4.662959611113936
 5  4.668403925918192
 6  4.668953740967305
 7  4.669157181381530
 8  4.669191002146906
 9  4.669199473291195
```

```
10  4.669201139368885
11  4.669201286732338
```