Introduction to Week Six

Numerical Solutions of PDEs

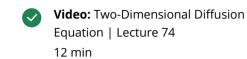
Direct Solution of Boundary Value Problems

Iterative Solution of Boundary Value Problems

Time-stepping Methods for Initial Value Problems

Quiz

Programming Assignment: Two- dimensional Diffusion Equation



Ungraded External Tool: Two-Dimensional Diffusion Equation (audit)

Reading: Reference Solution to "Two-Dimensional Diffusion Equation (audit)"

Graded External Tool: Two-Dimensional Diffusion Equation 1h

Reading: Reference Solution to "Two-Dimensional Diffusion Equation"

1 min

Farewell

Reference Solution to "Two-Dimensional Diffusion Equation (audit)"

L=1; %square is 2L x 2L N=100; %# of intervals in x and y directions n=N+1; %# of gridpoints in x,y directions including boundaries h=2*L/N; %grid size in x,y directions x=-L + (0:N)*h; %x values on the grid y=-L + (0:N)*h; %y values on the grid [X,Y]=meshgrid(x,y); % boundary_index = [bottom, left, top, right] boundary_index=[1:n, 1:n:1+(n-1)*n, ... 1+(n-1)*n:n*n, n:n:n*n %%%%% Diffusion constant and time-step parameters D=1; dt=h^2/(2*D); %borderline stability of FTCS scheme alpha=dt*D/h^2; %equation parameter nsteps=1000; %number of time steps %%%%% Construct the matrix and compute lu diagonals = $[2*(1+2*alpha)*ones(n^2,1), -alpha*ones(n^2,4)];$ A=spdiags(diagonals,[0 -1 1 -n n], n^2, n^2); %use sparse matrices I=speye(n^2); A(boundary_index,:)=I(boundary_index,:); [PL, U]=lu(A);u=zeros(n,n,nsteps); sigma=L/4; $u(:,:,1)=1/(2*pi*sigma^2)*exp(-0.5*(X.^2+Y.^2)/sigma^2);$ u(1,:,1)=0; u(n,:,1)=0; u(:,1,1)=0; u(:,n,1)=0; for m=2:nsteps b=zeros(n,n); b(2:n-1,2:n-1)=2*(1-2*alpha)*u(2:n-1,2:n-1,m-1) ... + alpha*(u(1:n-2,2:n-1,m-1)+u(3:n,2:n-1,m-1) ... + u(2:n-1,1:n-2,m-1)+u(2:n-1,3:n,m-1)); b=reshape(b,n^2,1); $u(:,:,m)=reshape(U\setminus(PL\setminus b),n,n);$ %%%%% Plot with animation: UNCOMMENT TO RUN ON MATLAB ONLINE OR DESKTOP % figure('units','normalized','outerposition',[0 0 1 1]) % s=surf(X,Y,u(:,:,1)); zlim([0, 2.6]); % xlabel('\$x\$','Interpreter','latex','FontSize',14); % ylabel('\$y\$','Interpreter','latex','FontSize',14); % zlabel('\$u(x,y,t)\$','Interpreter','latex','FontSize',14);



% title('Solution of the 2D diffusion equation','Interpreter','latex','FontSize',16);

% pause(1)

% end

% for j=2:nsteps

% s.ZData=u(:,:,j); pause(0.01);