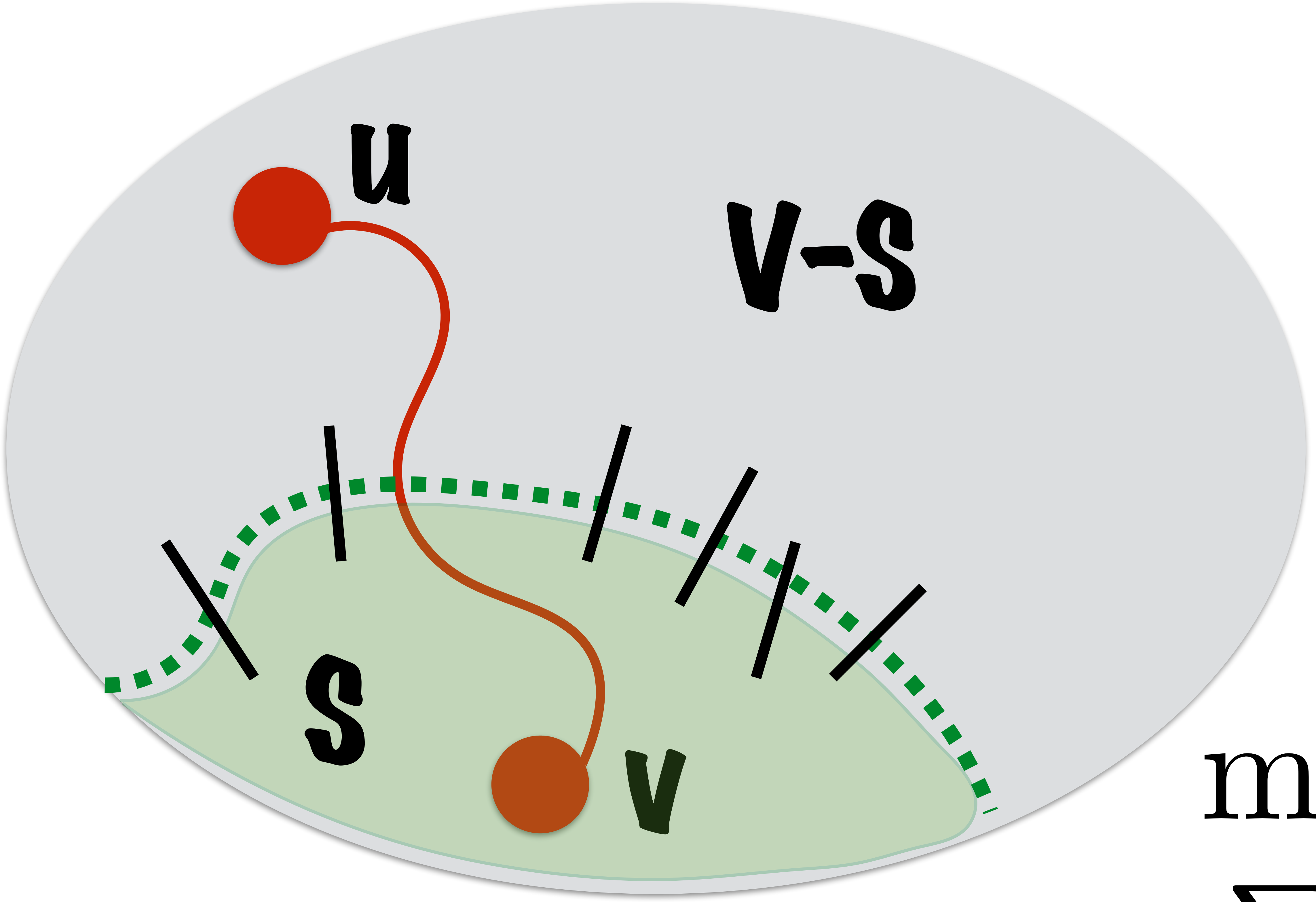
Steiner forest

Linear programming relaxation

$$\mathcal{S} = \{\mathbf{S} : \exists \mathbf{i} \exists \mathbf{u}, \mathbf{v} \in \mathbf{S_i} : |\mathbf{S} \cap \{\mathbf{u}, \mathbf{v}\}| = \mathbf{1}\}$$



$$\min \sum_{\mathbf{e}} \mathbf{c_e x_e} :$$

$$\sum_{\mathbf{e} \in \delta(\mathbf{S})} \mathbf{x_e} \geq \mathbf{1} \quad \forall \mathbf{S} \in \mathcal{S}$$

$$\mathbf{x_e} \geq \mathbf{0} \quad \forall \mathbf{e} \in \mathbf{E}$$

Taking the dual

$$\min \sum_{\mathbf{e}} \mathbf{c_e x_e} :$$
$$\sum_{\mathbf{e} \in \delta(\mathbf{S})} \mathbf{x_e} \geq \mathbf{1} \quad \forall \mathbf{S} \in \mathcal{S} \quad [\mathbf{y_S}]$$
$$\mathbf{x_e} \geq \mathbf{0} \quad \forall \mathbf{e} \in \mathbf{E}$$

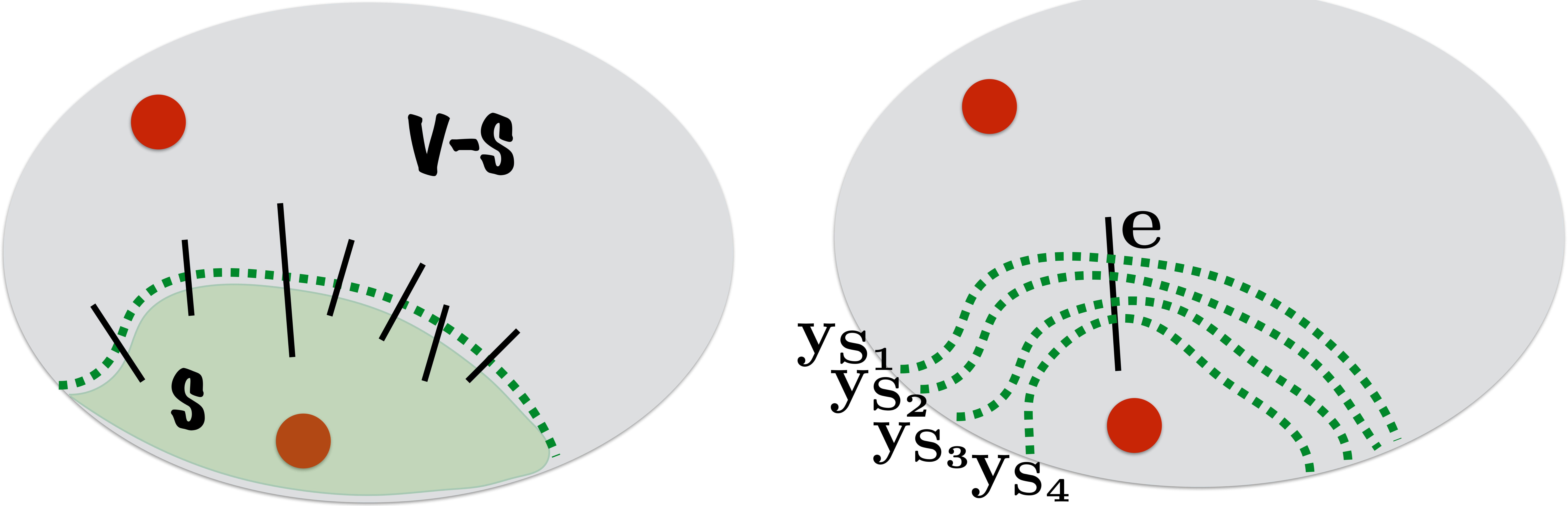
$$\max \sum_{\mathbf{S}} \mathbf{y_S} :$$
$$\sum_{\mathbf{S}:\mathbf{e} \in \delta(\mathbf{S})} \mathbf{y_S} \leq \mathbf{c_e} \quad \forall \mathbf{e} \in \mathbf{E} \quad [\mathbf{x_e}]$$
$$\mathbf{y_S} \geq \mathbf{0} \quad \forall \mathbf{S} \in \mathcal{S}$$

Interpreting the dual

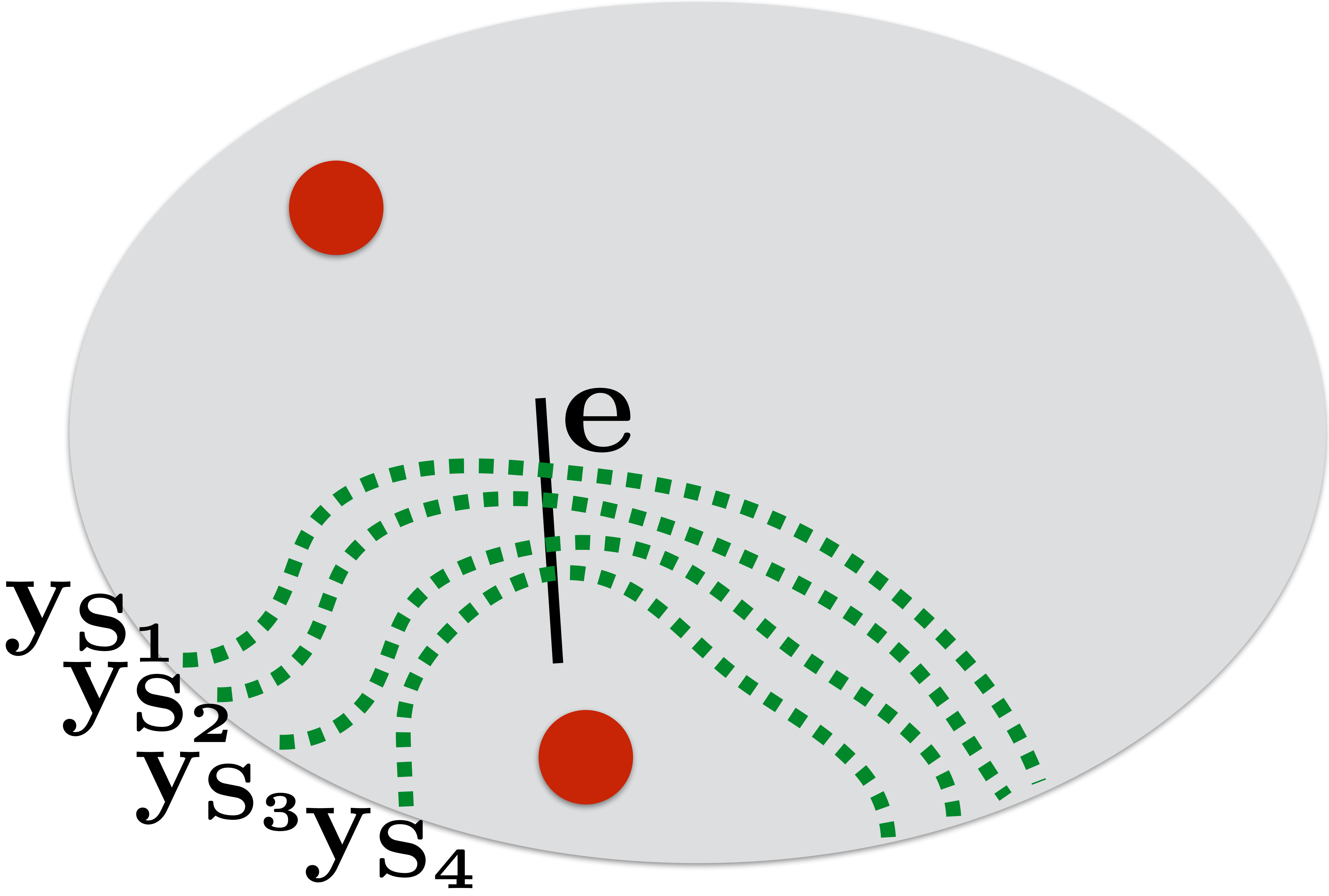
$$\max \sum_{\mathbf{S}} \mathbf{y_S} :$$

$$\sum_{\mathbf{S}:\mathbf{e}\in\delta(\mathbf{S})} \mathbf{y_S} \le \mathbf{c_e} \quad \forall \mathbf{e} \in \mathbf{E} \quad [\mathbf{x_e}]$$

$$\mathbf{y_S} \ge \mathbf{0} \quad \forall \mathbf{S} \in \mathcal{S}$$



$$\max \sum_S y_S :$$
$$\sum_{S: e \in \delta(S)} y_S \leq c_e \quad \forall e \in E \quad [x_e]$$
$$y_S \geq 0 \quad \forall S \in \mathcal{S}$$



Sooo many cuts containing e!!

Q: How can we hope for feasibility?

A: most variables will be 0

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