Swing a Pendulum to the Top

The dimensionless, unforced pendulum equation is given by

 $\ddot{\theta} + \alpha \dot{\theta} + \sin \theta = 0,$

where α is the only free parameter.

Consider initial conditions with the mass at the bottom, $\theta(0) = 0$. Using the shooting method, determine the smallest positive value of $\dot{\theta}(0)$ such that the mass becomes exactly balanced at the top ($\theta = \pi$). Plot this value of $\dot{\theta}(0)$ versus α for $0 \le \alpha \le 2$.

Script @



Your Solution

Reference Solution

```
1 theta0=0; u0=2; %initial ode conditions. u0 is initial guess for root.
2 inf=8*pi; %inf is a large number. Takes a long time to get to top.
3 tspan=[0 inf];
4 options = odeset('RelTol',1.e-6);
5 %rootfind u0 such that theta(inf)=pi
6 alpha_i=linspace(0, 2, 100);
7 u0_i=zeros(100,1);
8 for i=1:length(alpha_i)
       alpha=alpha_i(i);
      u0_i(i) = fzero(@(u0) F(tspan,theta0,u0,alpha,options), u0);
10
11 end
12 plot(alpha_i, u0_i);
13 xlabel('$\alpha$','Interpreter','latex','FontSize',14);
14 | ylabel('$d \theta/dt$','Interpreter','latex','FontSize',14);
15 title('Shooting to the Pendulum Top','Interpreter','latex','FontSize',16);
16
| function y=F(tspan,theta0,u0,alpha,options)
18 % use ode45 to define the root-finding problem
19
      [t,theta_u]=ode45(@(t,theta_u) pendulum(theta_u,alpha),tspan,[theta0;u0],options);
      theta=theta_u(:,1); u=theta_u(:,2);
20
21
      y=theta(end)-pi;
22 end
23
24 | function d_theta_u_dt = pendulum(theta_u,alpha)
25 % define the differential equation here
26
      theta=theta_u(1); u=theta_u(2);
27
      d_theta_u_dt=[u;-alpha*u-sin(theta)];
28 end
```

► Run Script

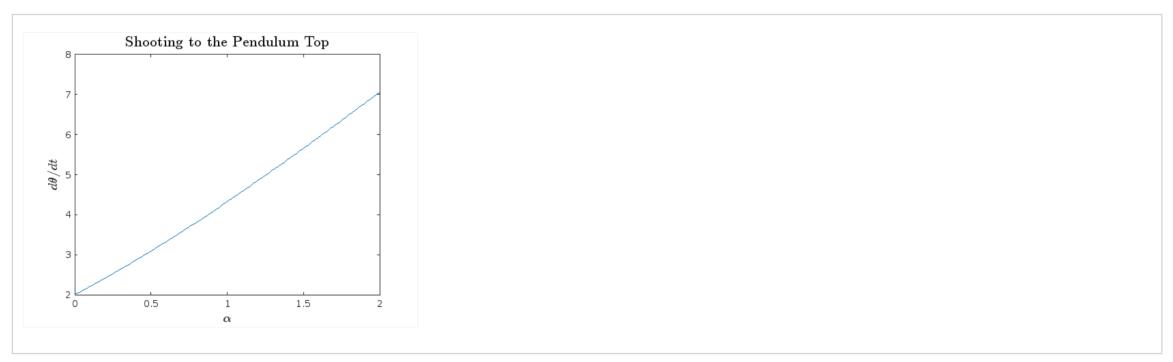
Assessment: All Tests Passed

Submit

?

Test u0_i

Output



© 2022 The MathWorks, Inc.