Independent trials, product spaces

The wisdom in the toss of a coin

- A fair coin ($\mathfrak{h}$ = heads, $\mathfrak{t}$ = tails) is tossed thrice.
  - *Sample space*: $\Omega = \{\mathfrak{hhh}, \mathfrak{hht}, \mathfrak{hth}, \mathfrak{htt}, \mathfrak{thh}, \mathfrak{tht}, \mathfrak{tth}, \mathfrak{ttt}\}$.
  - *Events:*
    - A := The first toss results in heads.
    - B := The second toss results in tails.
    - C := The third toss results heads.
  - *Probability measure*: Combinatorial setting with mass function assigning equal probability 1/8 to each atom.
- The events A,B, and C are independent.

Event		Probability
A	$\{\mathfrak{hhh}, \mathfrak{hht}, \mathfrak{hth}, \mathfrak{htt}\}$	1/2
B	$\{\mathfrak{hth}, \mathfrak{htt}, \mathfrak{tth}, \mathfrak{ttt}\}$	1/2
C	$\{\mathfrak{hhh}, \mathfrak{hth}, \mathfrak{thh}, \mathfrak{tth}\}$	1/2
$A \cap B$	$\{\mathfrak{hth}, \mathfrak{htt}\}$	$1/4 = \frac{1}{2} \times \frac{1}{2}$
$A \cap C$	$\{\mathfrak{hhh}, \mathfrak{hth}\}$	1/4
$B \cap C$	$\{\mathfrak{hth}, \mathfrak{tth}\}$	1/4
$A \cap B \cap C$	$\{\mathfrak{hth}\}$	$1/8 = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$

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What is the key behind independence in this setting?

Repeated independent tosses

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- *Individual chance experiments*:
  - Trial (toss) 1: $\mathfrak{A} = \{\mathfrak{h}, \mathfrak{t}\}$. Atomic mass $1/2$.
  - Trial (toss) 2: $\mathfrak{B} = \{\mathfrak{h}, \mathfrak{t}\}$. Atomic mass $1/2$.
  - Trial (toss) 3: $\mathfrak{C} = \{\mathfrak{h}, \mathfrak{t}\}$. Atomic mass $1/2$.

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  - Trial (toss) 3: $\mathfrak{C} = \{\mathfrak{h}, \mathfrak{t}\}$. Atomic mass $1/2$.

- *Compound chance experiment*:
  - $\Omega = \{\mathfrak{hhh}, \mathfrak{hht}, \mathfrak{hth}, \mathfrak{htt}, \mathfrak{thh}, \mathfrak{tht}, \mathfrak{tth}, \mathfrak{ttt}\} = \mathfrak{A} \times \mathfrak{B} \times \mathfrak{C}$. Atomic mass $1/8 = (1/2) \times (1/2) \times (1/2)$.

Repeated independent tosses

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  - Trial (toss) 1: $\mathfrak{A} = \{\mathfrak{h}, \mathfrak{t}\}$. Atomic mass $1/2$.
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  - Trial (toss) 3: $\mathfrak{C} = \{\mathfrak{h}, \mathfrak{t}\}$. Atomic mass $1/2$.
- *Compound chance experiment*:
  - $\Omega = \{\mathfrak{hhh}, \mathfrak{hht}, \mathfrak{hth}, \mathfrak{htt}, \mathfrak{thh}, \mathfrak{tht}, \mathfrak{tth}, \mathfrak{ttt}\} = \mathfrak{A} \times \mathfrak{B} \times \mathfrak{C}$. Atomic mass $1/8 = (1/2) \times (1/2) \times (1/2)$.
- *Events:*
  - A := The first toss is heads = $\{\mathfrak{hhh}, \mathfrak{hht}, \mathfrak{hth}, \mathfrak{htt}\}$ is completely determined by the subset $\{\mathfrak{h}\}$ of $\mathfrak{A}$.
  - B := The second toss is heads = $\{\mathfrak{hhh}, \mathfrak{hht}, \mathfrak{thh}, \mathfrak{tht}\}$ is completely determined by the subset $\{\mathfrak{t}\}$ of $\mathfrak{B}$.
  - C := The third toss is heads = $\{\mathfrak{hhh}, \mathfrak{hth}, \mathfrak{thh}, \mathfrak{tth}\}$ is completely determined by the subset $\{\mathfrak{h}\}$ of $\mathfrak{C}$.