Summary of Tableau 11, Part 2

The fabulous limit laws
The law of large numbers in continuous spaces, computation à la Monte Carlo

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The Background

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Additivity

- Expectation: $\mathbf{E}(X+Y) = \mathbf{E}(X) + \mathbf{E}(Y)$
- Variance (if summands are independent): $\mathrm{Var}(X+Y) = \mathrm{Var}(X) + \mathrm{Var}(Y)$

Random sample

Random sample $X, X_1, \ldots, X_n, \ldots$

Random sample $X, X_1, \ldots, X_n, \ldots$ $\mathbf{E}(X) = \mu$

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The weak law of large numbers

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$$\mathbf{P}\left\{\left|\frac{S_n}{n} - \mu\right| > \epsilon\right\} \leq \frac{\sigma^2}{n\epsilon^2} \to 0 \qquad (n \to \infty)$$

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Applications

The Results

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Applications

- Estimation of population parameters, polls
- Testing sera and vaccines, drug approval
- Statistical estimates of a mean
- Quality testing, time to failure
- Actuarial models, risk assessment
- Theory of fair games
- Monte Carlo methods
- Stock portfolio selection, the horse race