A pyramid of binomial coefficients

Pascal's triangle

$$
\begin{array}{ccccccccccccc}
\binom{6}{0} & & \binom{6}{1} & & \binom{6}{2} & & \binom{6}{3} & & \binom{6}{4} & & \binom{6}{5} & & \binom{6}{6} \\
& \binom{5}{0} & & \binom{5}{1} & & \binom{5}{2} & & \binom{5}{3} & & \binom{5}{4} & & \binom{5}{5} & \\
& & \binom{4}{0} & & \binom{4}{1} & & \binom{4}{2} & & \binom{4}{3} & & \binom{4}{4} & & \\
& & & \binom{3}{0} & & \binom{3}{1} & & \binom{3}{2} & & \binom{3}{3} & & & \\
& & & & \binom{2}{0} & & \binom{2}{1} & & \binom{2}{2} & & & & \\
& & & & & \binom{1}{0} & & \binom{1}{1} & & & & & \\
& & & & & & \binom{0}{0} & & & & & &
\end{array}
$$

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$$\binom{6}{0}^{1} \quad \binom{6}{1}^{6} \quad \binom{6}{2}^{15} \quad \binom{6}{3}^{20} \quad \binom{6}{4}^{15} \quad \binom{6}{5}^{6} \quad \binom{6}{6}^{1}$$

$$\binom{5}{0}^{1} \quad \binom{5}{1}^{5} \quad \binom{5}{2}^{10} \quad \binom{5}{3}^{10} \quad \binom{5}{4}^{5} \quad \binom{5}{5}^{1}$$

$$\binom{4}{0}^{1} \quad \binom{4}{1}^{4} \quad \binom{4}{2}^{6} \quad \binom{4}{3}^{4} \quad \binom{4}{4}^{1}$$

$$\binom{3}{0}^{1} \quad \binom{3}{1}^{3} \quad \binom{3}{2}^{3} \quad \binom{3}{3}^{1}$$

$$\binom{2}{0}^{1} \quad \binom{2}{1}^{2} \quad \binom{2}{2}^{1}$$

$$\binom{1}{0}^{1} \quad \binom{1}{1}^{\mathbf{1}}$$

$$\binom{0}{0}^{1}$$

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$$\binom{4}{0}^{1} \quad \binom{4}{1}^{4} \quad \binom{4}{2}^{6} \quad \binom{4}{3}^{4} \quad \binom{4}{4}^{1}$$

$$\binom{3}{0}^{1} \quad \binom{3}{1}^{3} \quad \binom{3}{2}^{3} \quad \binom{3}{3}^{1}$$

$$\binom{2}{0}^{1} \quad \binom{2}{1}^{2} \quad \binom{2}{2}^{1}$$

$$\binom{1}{0}^{1} \quad \binom{1}{1}^{\mathbf{1}}$$

$$\binom{0}{0}^{1}$$

$$\binom{5}{2} = \binom{4}{1} + \binom{4}{2}$$

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$$\binom{3}{0}^{1} \quad \binom{3}{1}^{3} \quad \binom{3}{2}^{3} \quad \binom{3}{3}^{1}$$

$$\binom{2}{0}^{1} \quad \binom{2}{1}^{2} \quad \binom{2}{2}^{1}$$

$$\binom{1}{0}^{1} \quad \binom{1}{1}^{1}$$

$$\binom{0}{0}^{1}$$

$$\binom{5}{2} = \binom{4}{1} + \binom{4}{2}$$

$$\binom{6}{4} = \binom{5}{3} + \binom{5}{4}$$

$$\binom{4}{1} = \binom{3}{0} + \binom{3}{1}$$

$$\binom{2}{1} = \binom{1}{0} + \binom{1}{1}$$

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$$\binom{3}{0}^{1} \quad \binom{3}{1}^{3} \quad \binom{3}{2}^{3} \quad \binom{3}{3}^{1}$$

$$\binom{2}{0}^{1} \quad \binom{2}{1}^{2} \quad \binom{2}{2}^{1}$$

$$\binom{1}{0}^{1} \quad \binom{1}{1}^{1}$$

$$\binom{0}{0}^{1}$$

$$\binom{5}{2} = \binom{4}{1} + \binom{4}{2}$$

$$\binom{6}{4} = \binom{5}{3} + \binom{5}{4}$$

$$\binom{4}{1} = \binom{3}{0} + \binom{3}{1}$$

$$\binom{2}{1} = \binom{1}{0} + \binom{1}{1}$$

$$\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$$

Verifying Pascal's triangle

$$\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$$

Verifying Pascal's triangle

$$\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$$

$$\binom{n-1}{k-1} + \binom{n-1}{k} = \frac{(n-1)^{\underline{k-1}}}{(k-1)!} + \frac{(n-1)^{\underline{k}}}{k!}$$

Verifying Pascal's triangle

$$\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$$

$$\binom{n-1}{k-1} + \binom{n-1}{k} = \frac{(n-1)^{\underline{k-1}}}{(k-1)!} + \frac{(n-1)^{\underline{k}}}{k!}$$

$$= \frac{(n-1)^{\underline{k-1}}}{k!} \cdot k + \frac{(n-1)^{\underline{k-1}}}{k!} \cdot \big((n-1) - (k-1)\big)$$

Verifying Pascal's triangle

$$\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$$

$$\begin{aligned}
\binom{n-1}{k-1} + \binom{n-1}{k} &= \frac{(n-1)^{\underline{k-1}}}{(k-1)!} + \frac{(n-1)^{\underline{k}}}{k!} \\
&= \frac{(n-1)^{\underline{k-1}}}{k!} \cdot k + \frac{(n-1)^{\underline{k-1}}}{k!} \cdot \big((n-1)-(k-1)\big) \\
&= \frac{(n-1)^{\underline{k-1}}}{k!} \left[k + \big((n-1)-(k-1)\big)\right]
\end{aligned}$$

Verifying Pascal's triangle

$$\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$$

$$\begin{aligned}
\binom{n-1}{k-1} + \binom{n-1}{k} &= \frac{(n-1)^{\underline{k-1}}}{(k-1)!} + \frac{(n-1)^{\underline{k}}}{k!} \\
&= \frac{(n-1)^{\underline{k-1}}}{k!} \cdot k + \frac{(n-1)^{\underline{k-1}}}{k!} \cdot \big((n-1) - (k-1)\big) \\
&= \frac{(n-1)^{\underline{k-1}}}{k!} \Big[k + \big((n-1) - (k-1)\big)\Big] \\
&= \frac{(n-1)^{\underline{k-1}}}{k!} \cdot n = \frac{n^{\underline{k}}}{k!} = \binom{n}{k}
\end{aligned}$$