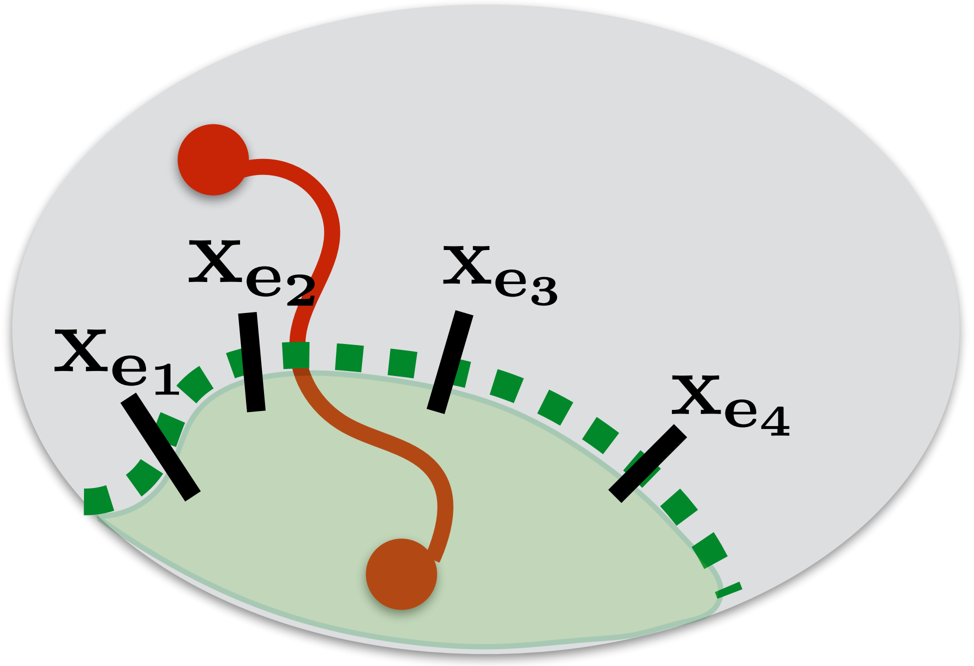
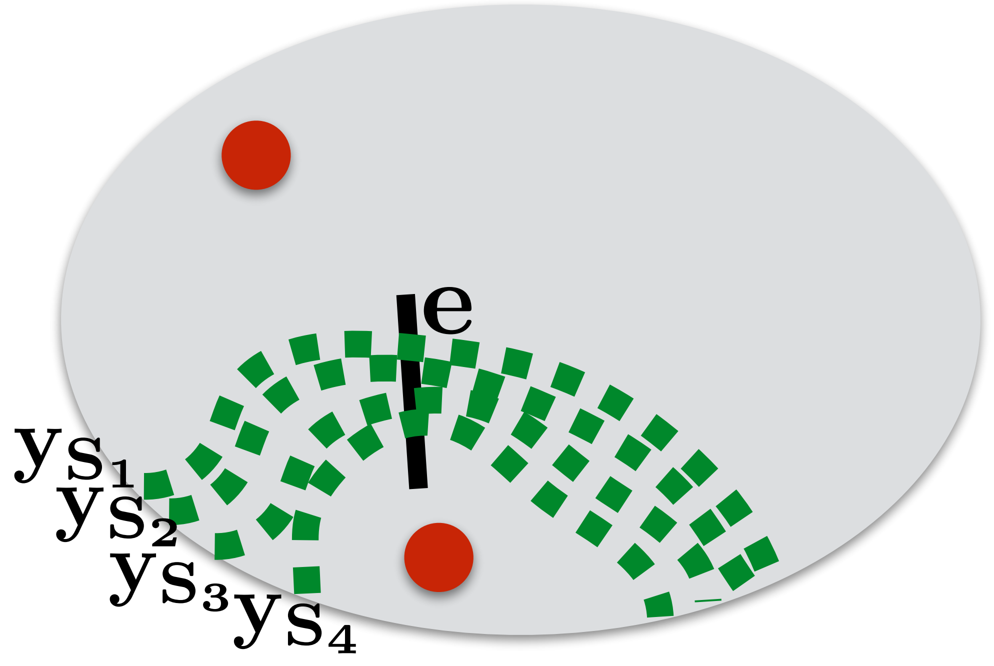
Steiner forest





Initialization:

$$\mathbf{x} \leftarrow 0, \mathbf{y} \leftarrow 0$$

Iteration: while $\mathbf{x}$ not satisfiable

in parallel, raise every unfrozen y_S with

S minimal

stopped by tight constraint (e)

$$x_e \leftarrow 1$$

freeze y_S in tight constraints

Pruning: let F={edges defined by x}

for each edge e of F in reverse order,

remove e if unnecessary

Theorem:

It's a 2-approximation for Steiner forest

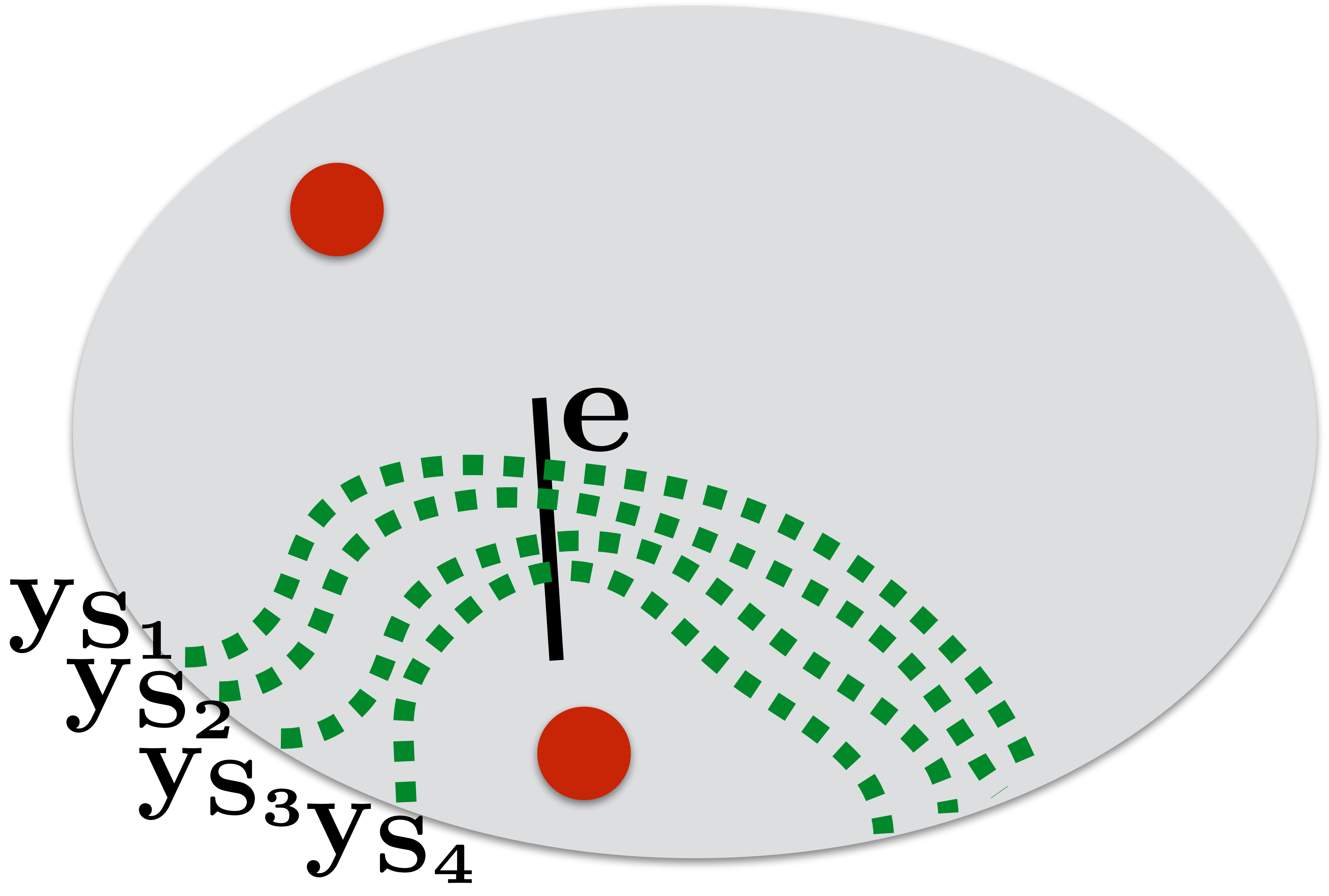
Observations:

- Final y is feasible
- Final x is a feasible forest
- Output F' is a forest
- its leaves are terminals
- slackness condition:

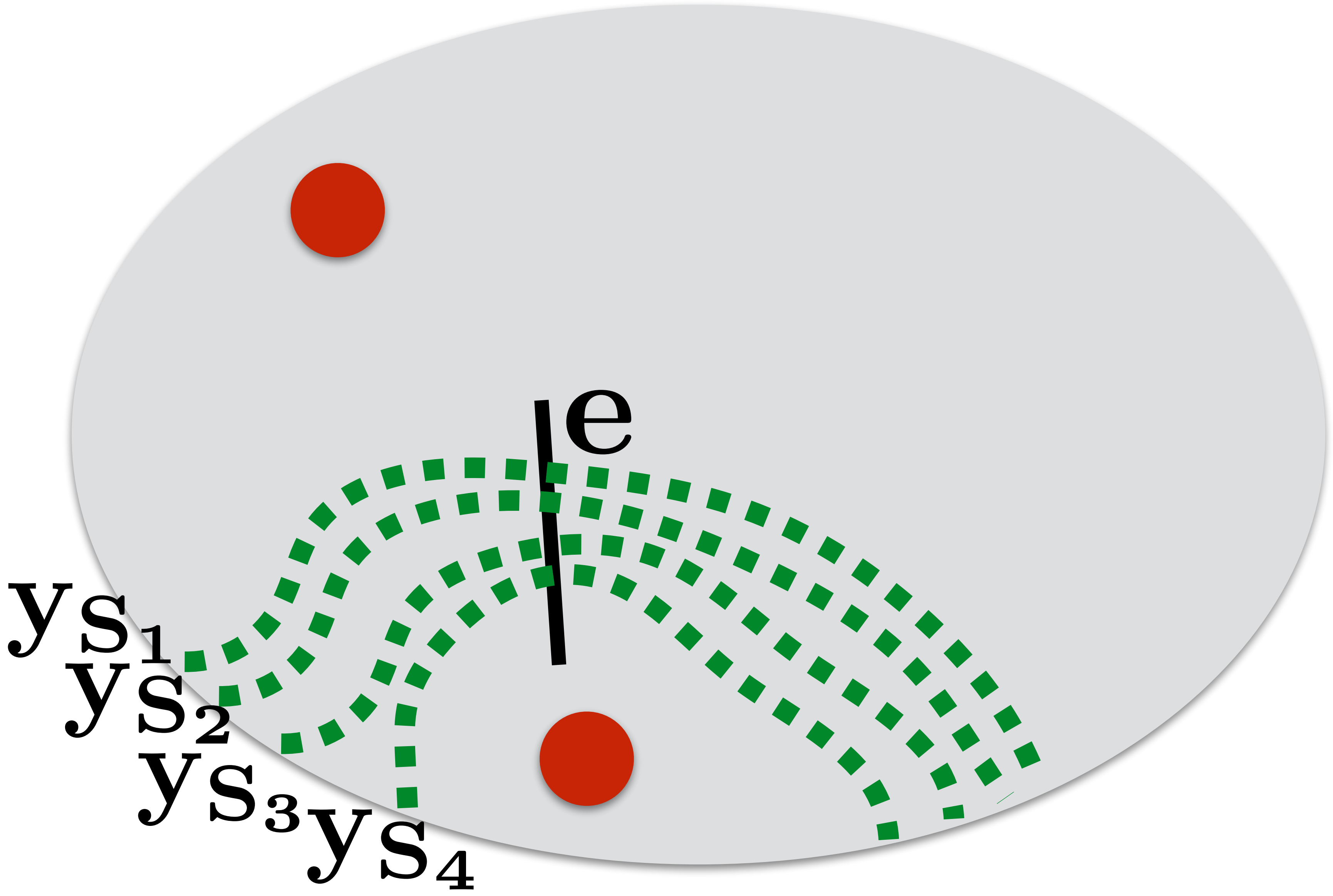
$$e \in F' \implies e \in F$$

$$\implies x_e = 1$$

$$\implies \sum_{S : e \in \delta(S)} y_S = c_e$$



slackness condition implies bound on cost of output by using dual variables:



$$\sum_{e \in F'} c_e = \sum_{e \in F'} \sum_{S: e \in \delta(S)} y_S$$

As usual, invert summations:

$$\sum_{\mathbf{e} \in \mathbf{F}'} \sum_{\mathbf{S}:\mathbf{e} \in \delta(\mathbf{S})} \mathbf{y}_{\mathbf{S}} = \sum_{\mathbf{S}} \mathbf{y}_{\mathbf{S}} |\mathbf{F}' \cap \delta(\mathbf{S})|$$

Q: How to upper bound

$$\sum_{\mathbf{S}} \mathbf{y_S} |\mathbf{F'} \cap \delta(\mathbf{S})|$$

A: Incrementally. Bound

$$\sum_{\textbf{time t}} \sum_{\textbf{S ``active'' between } t \textbf{ and } t+dt} \epsilon |\mathbf{F'} \cap \delta(\mathbf{S})|$$

S “active” if its dual variable is being raised

Definition of "active" at current time

Initialization:

$$\mathrm{x} \leftarrow 0, \mathrm{y} \leftarrow 0$$

Iteration: while x not satisfiable

in parallel, raise every unfrozen $\mathrm{y_S}$ with S minimal

stopped by tight constraint (e)

$$\mathrm{x_e} \leftarrow 1$$

freeze $\mathrm{y_S}$ in tight constraints

Pruning: let F={edges defined by x}

remove unnecessary edges from F

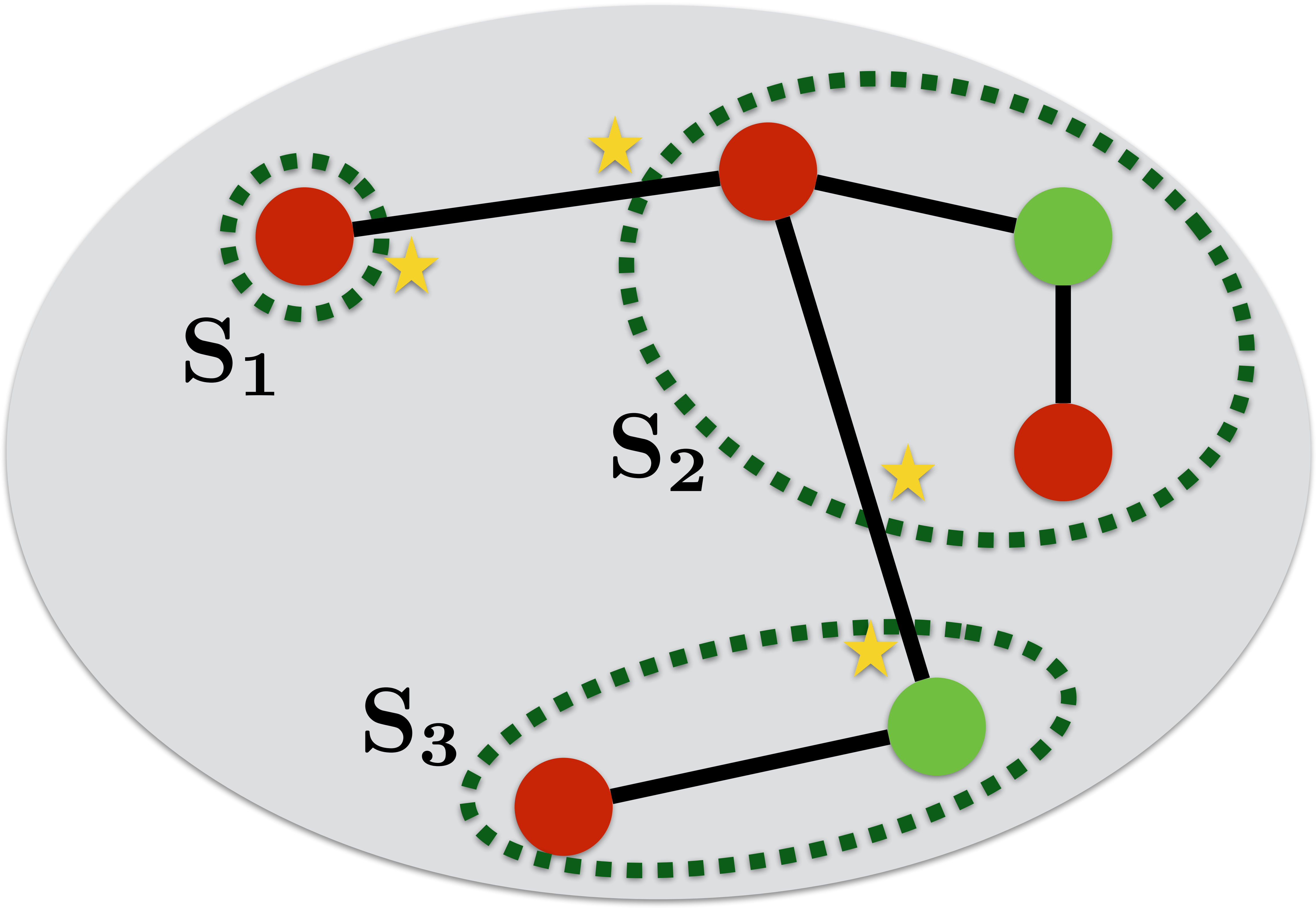
"active": unfrozen with S minimal

Q: How to upper bound

$$\sum_{\mathbf{S\ currently\ active}} |\mathbf{F}' \cap \delta(\mathbf{S})|$$

A: Let's try some examples

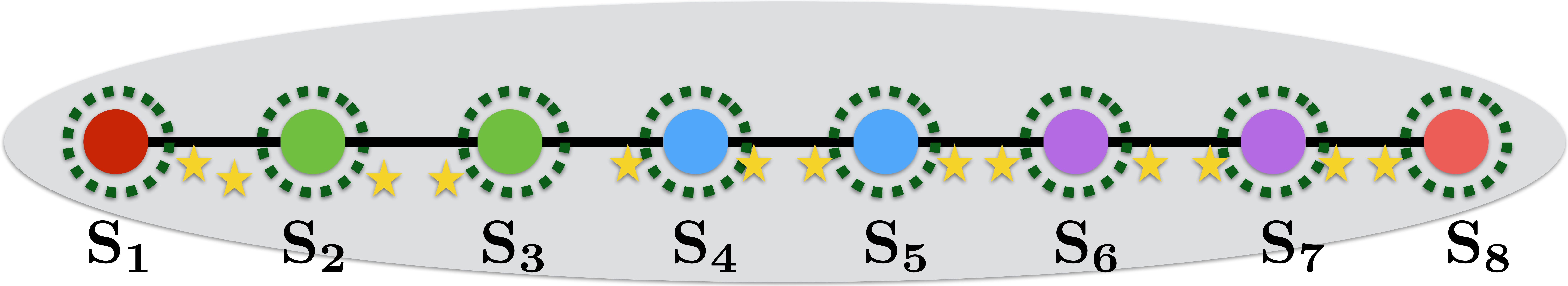
F'



3 active sets: S_1, S_2, S_3

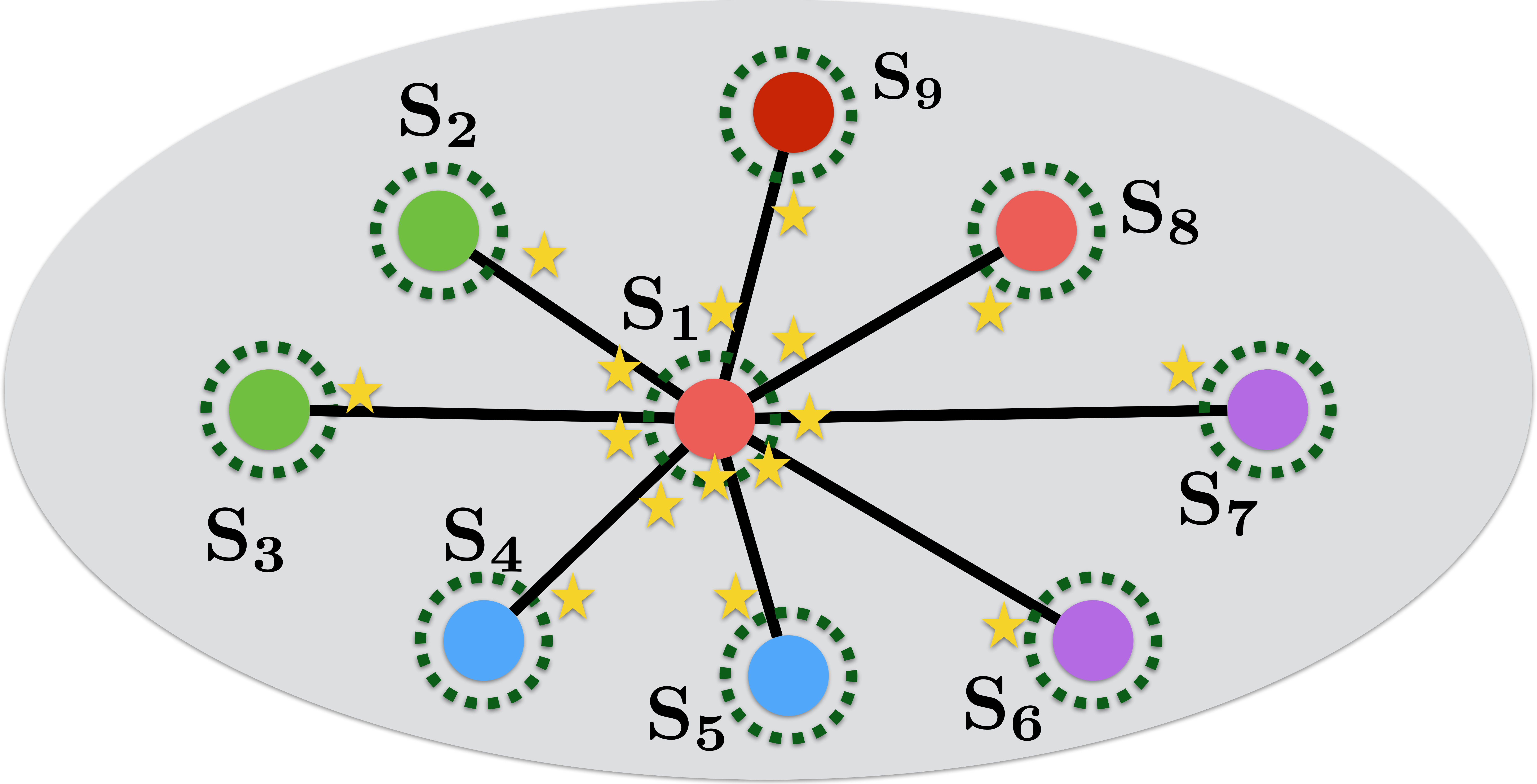
$$\sum_{S\ \text{active}} |F' \cap \delta(S)| = \#(\text{stars}) = 1 + 2 + 1$$

F’

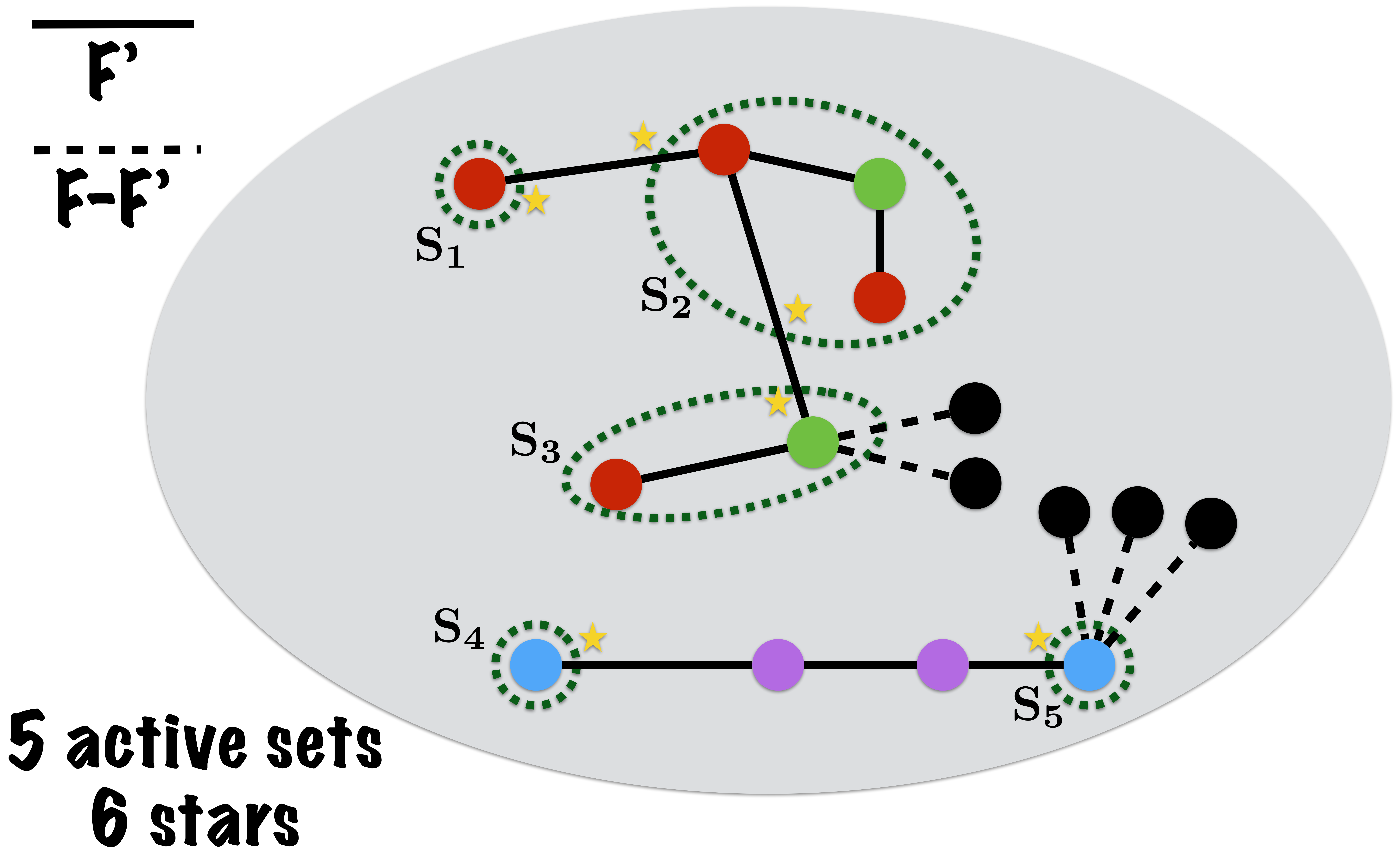


8 active sets
14 stars

F'

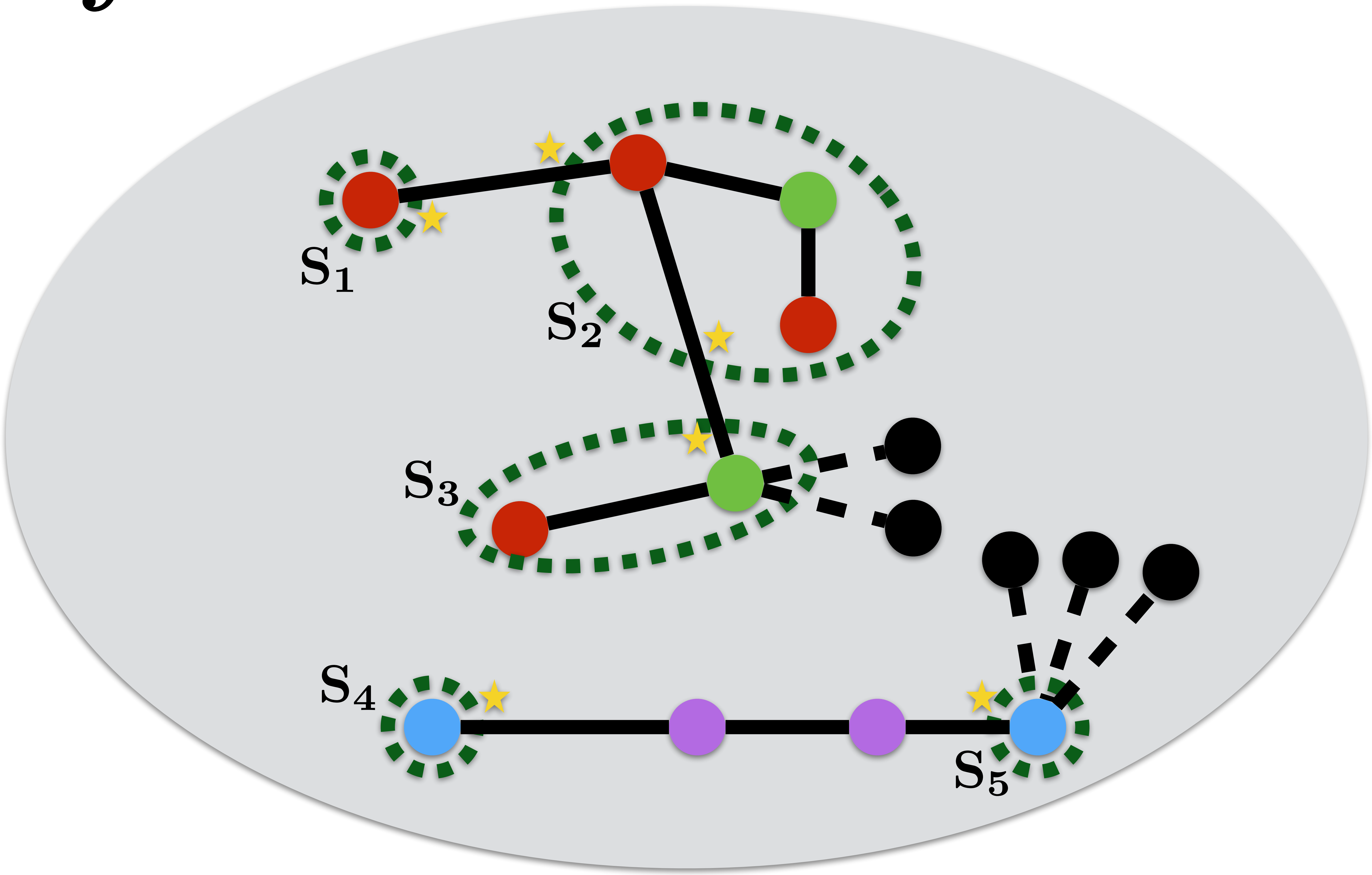


9 active sets
16 stars


F'
F-F'
S_1
S_2
S_3
S_4
S_5
5 active sets
6 stars

Q: How to upper bound

$$\sum_{\mathbf{S}\ \text{currently active}} |\mathbf{F}' \cap \delta(\mathbf{S})|$$



Lemma: $$\frac{\sum_{\mathbf{S}\ \text{currently active}} |\mathbf{F}' \cap \delta(\mathbf{S})|}{\#(\text{currently active sets})} \leq \mathbf{2}$$

Analysis assuming the lemma holds

Cost(output)=

$$\sum_{e \in F'} c_e = \sum_{e \in F'} \sum_{S: e \in \delta(S)} y_S$$

$$= \sum_S y_S |F' \cap \delta(S)|$$

$$= \sum_t \sum_{S \text{ active}} \epsilon |F' \cap \delta(S)|$$

$$\leq \sum_t 2\epsilon \cdot \#(S \text{ active})$$

$$= 2 \sum_S y_S$$

$$\leq 2 \cdot \text{OPT}$$

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