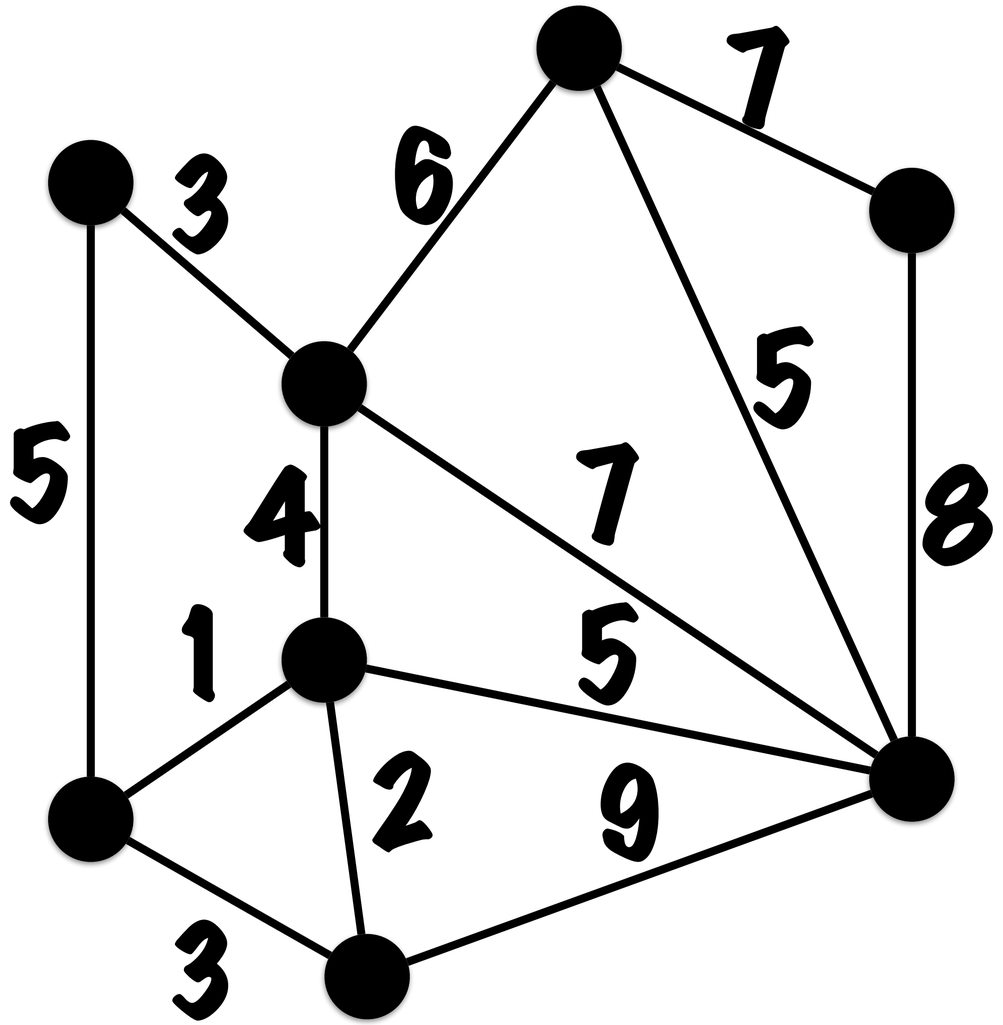
Steiner forest

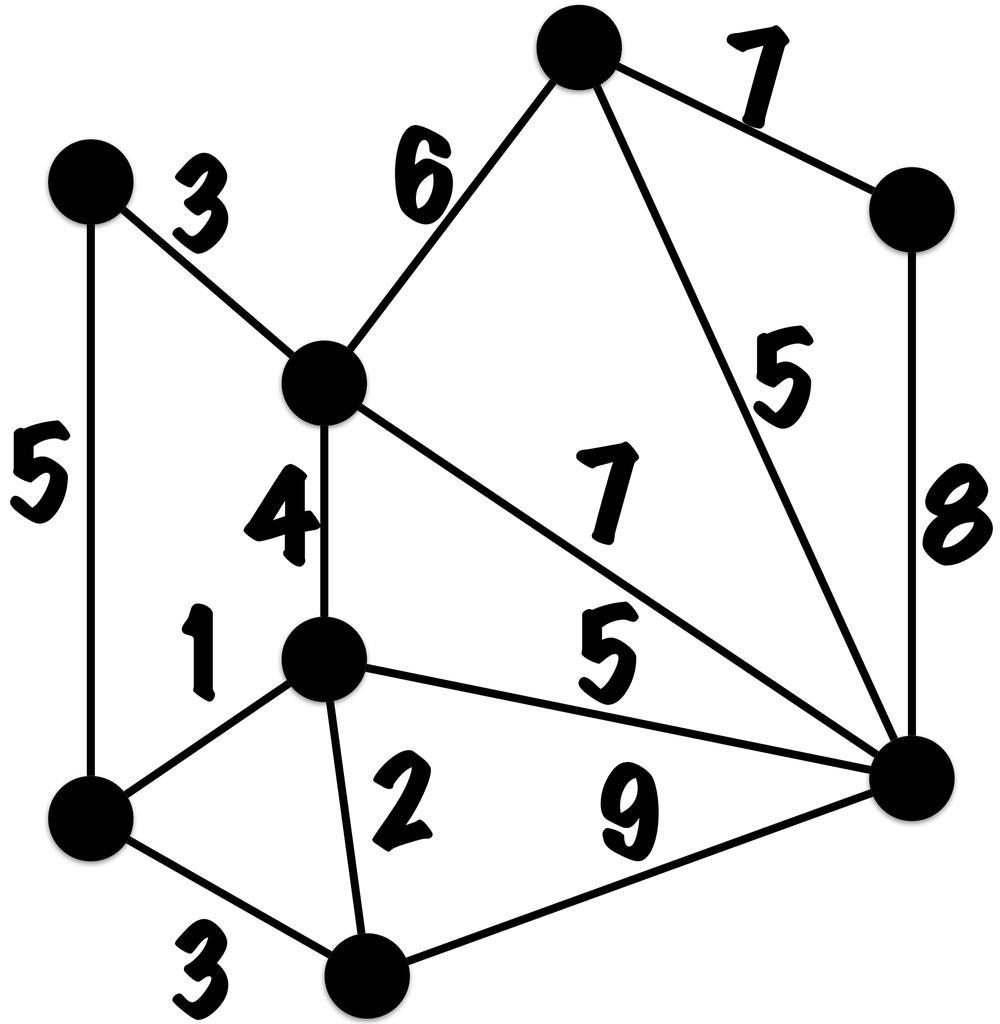
Steiner tree

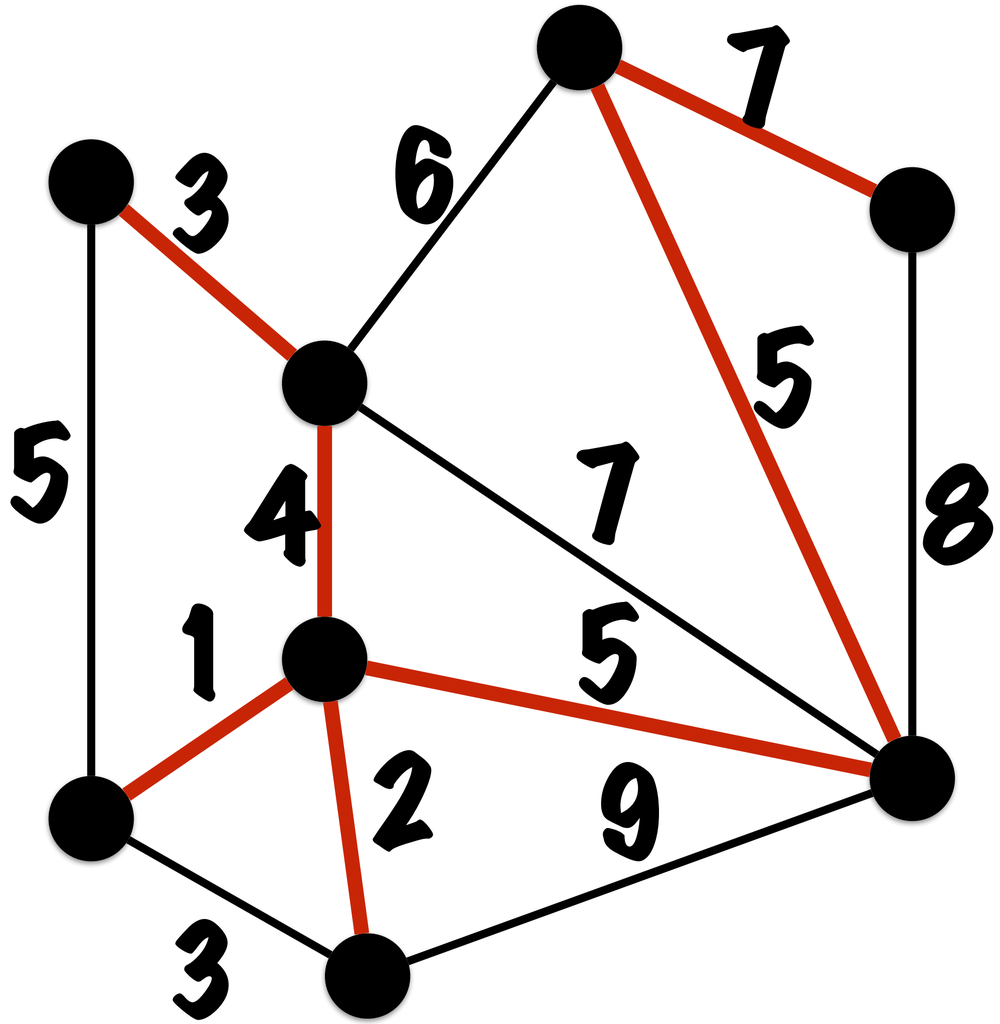
Observe:

if the subset = all nodes
then Steiner tree =
minimum spanning tree



MST

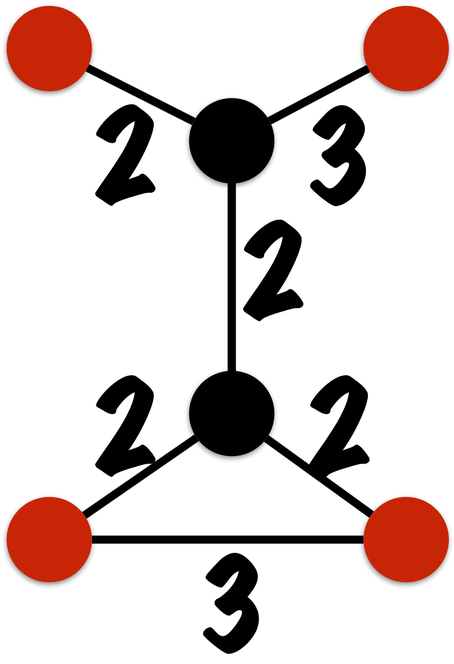


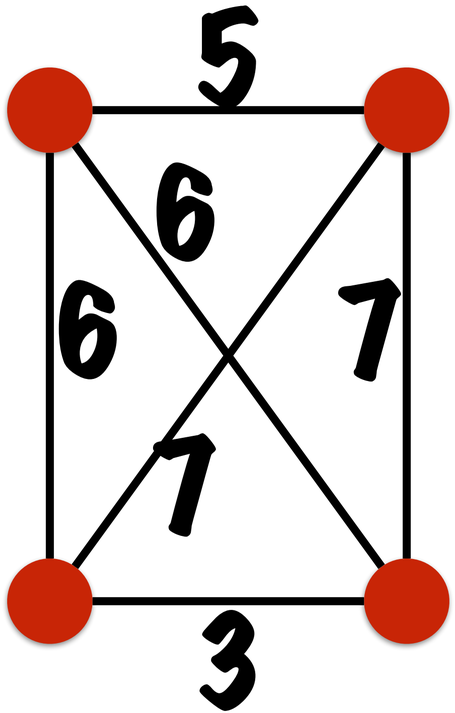


Steiner tree approx. algorithm

1. Define new graph:

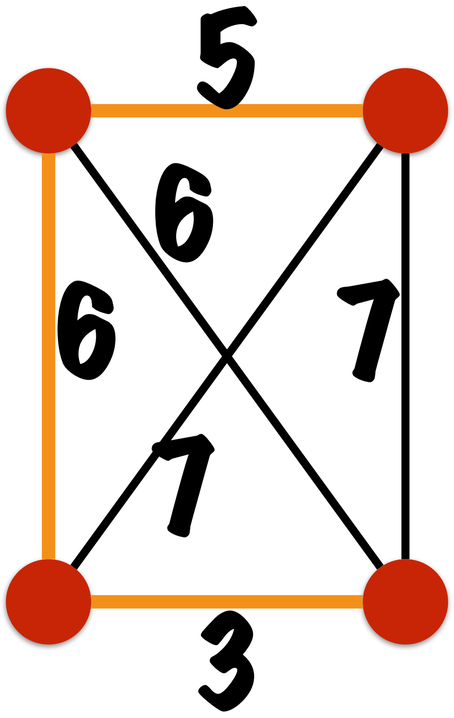
- Vertex set = S
- Edge set = complete graph
- Weight of {u,v} = shortest path length in G





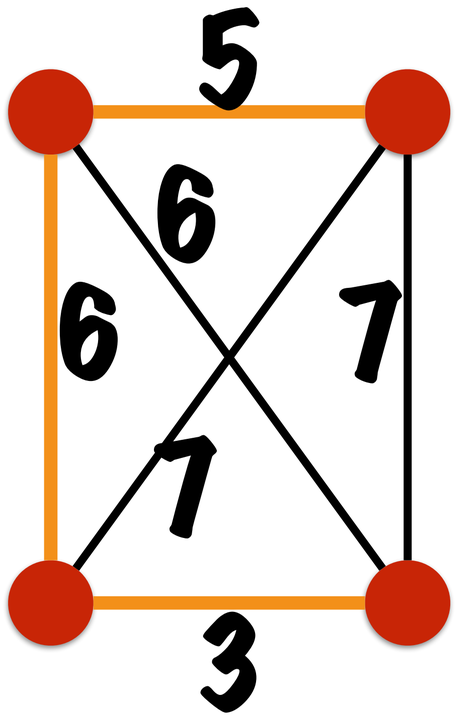
Steiner tree algorithm

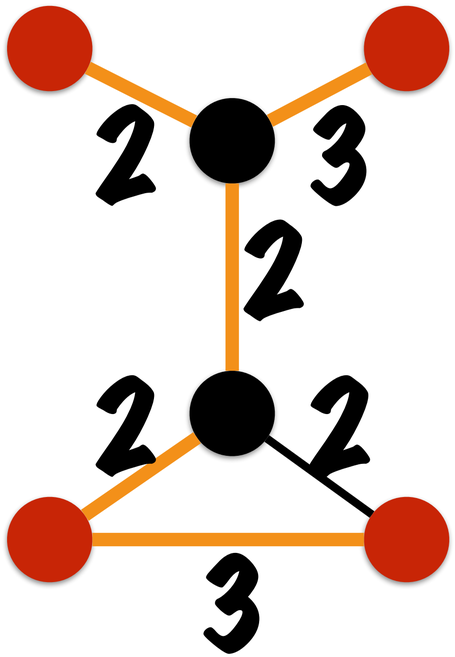
2. Compute minimum spanning tree on new graph



Steiner tree algorithm

3. Output corresponding set of original edges





Here
Output=12
OPT=11

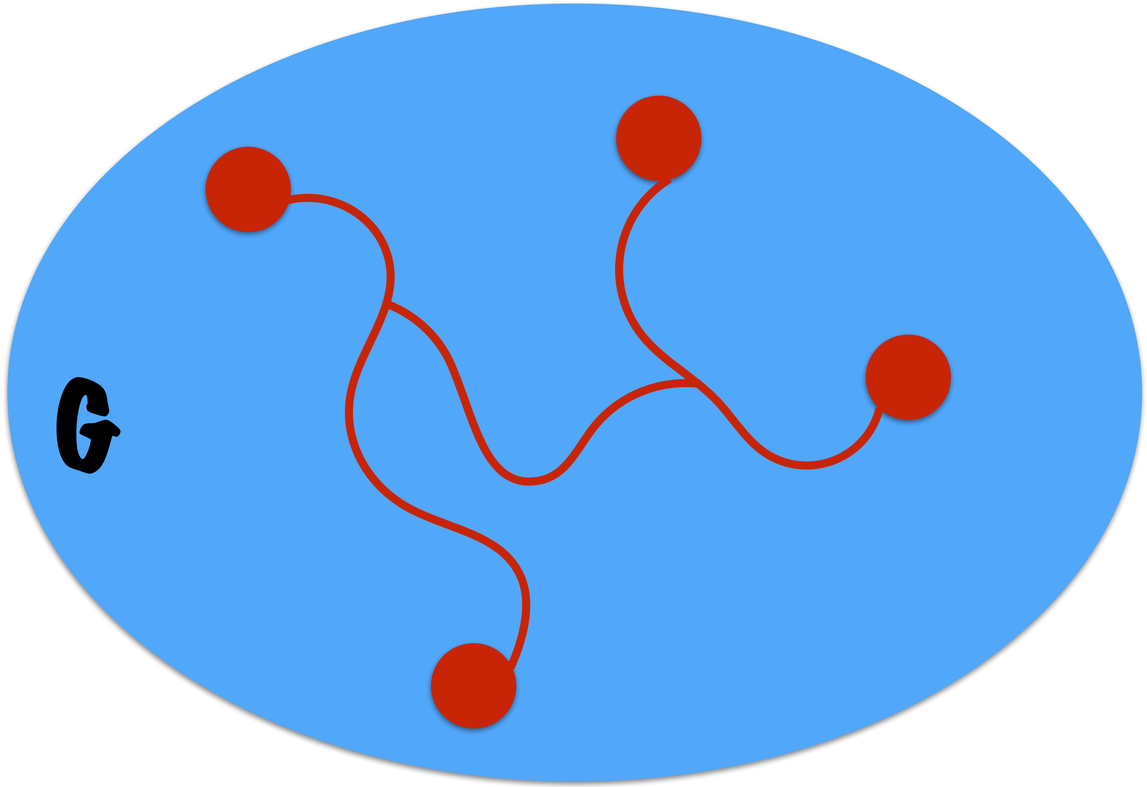
Steiner tree algorithm

1. new complete graph on subset
2. minimum spanning tree on that graph
3. output corresponding set of original edges

Theorem: it's a 2-approximation

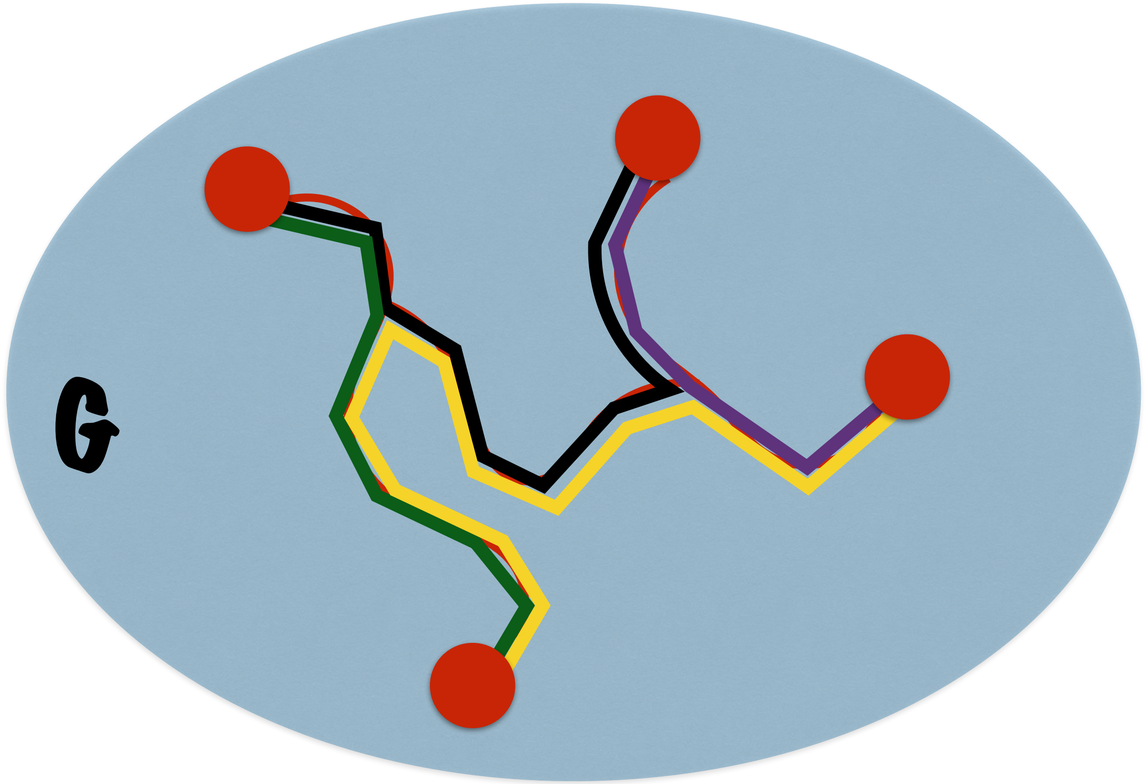
Theorem: it's a 2-approximation

Consider unknown OPT tree



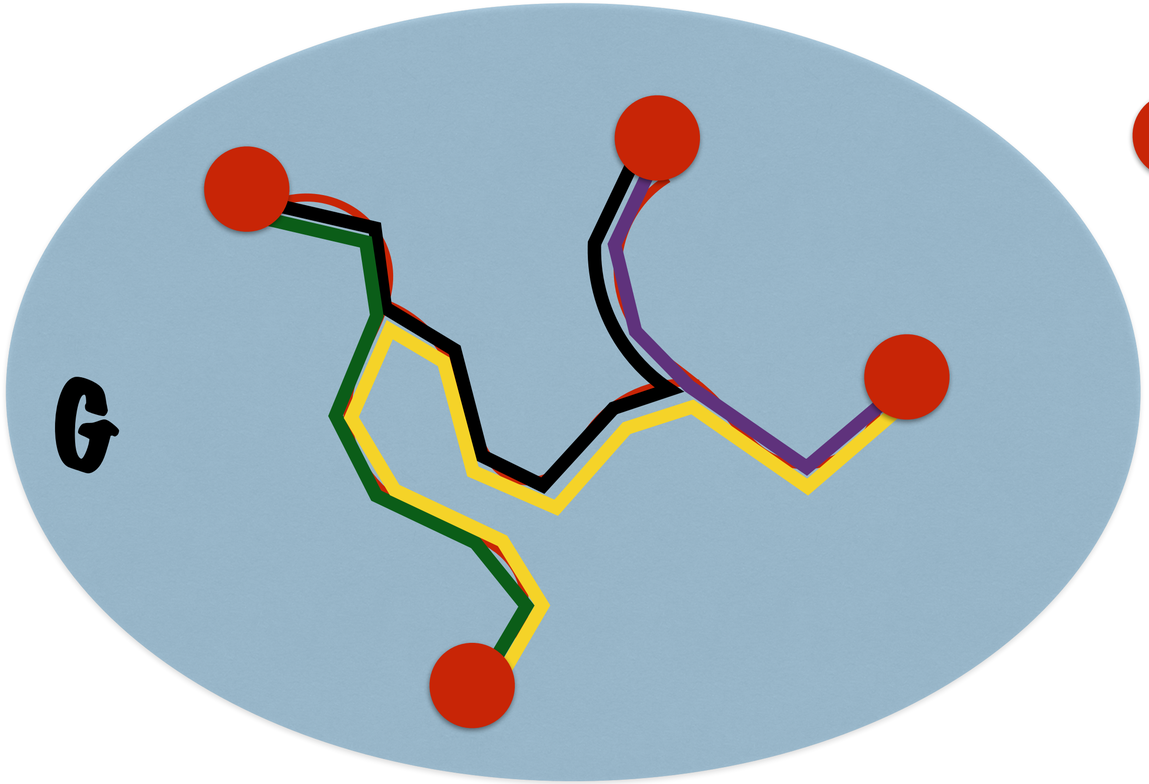
Theorem: it's a 2-approximation

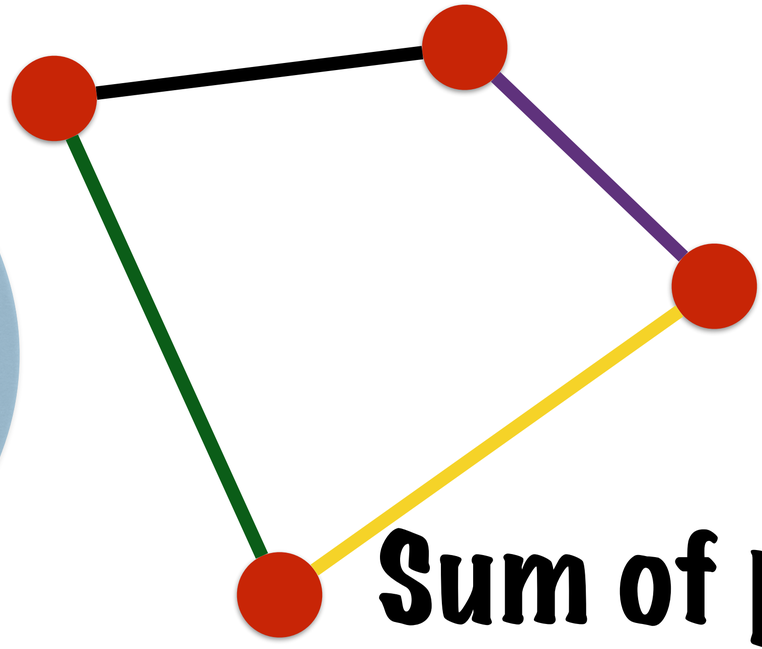
Go around the OPT tree to define terminal-to-terminal paths



Theorem: it's a 2-approximation

Map terminal-to-terminal paths to edges in complete graph of terminals

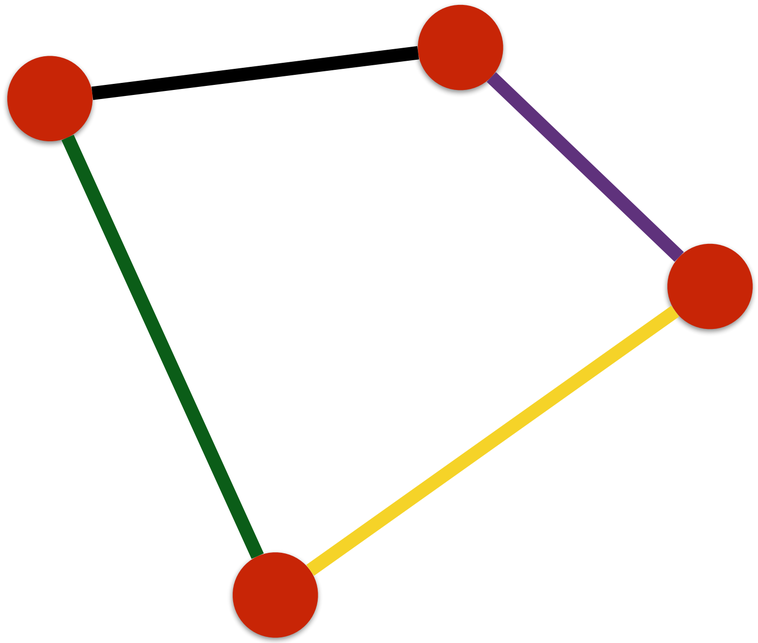


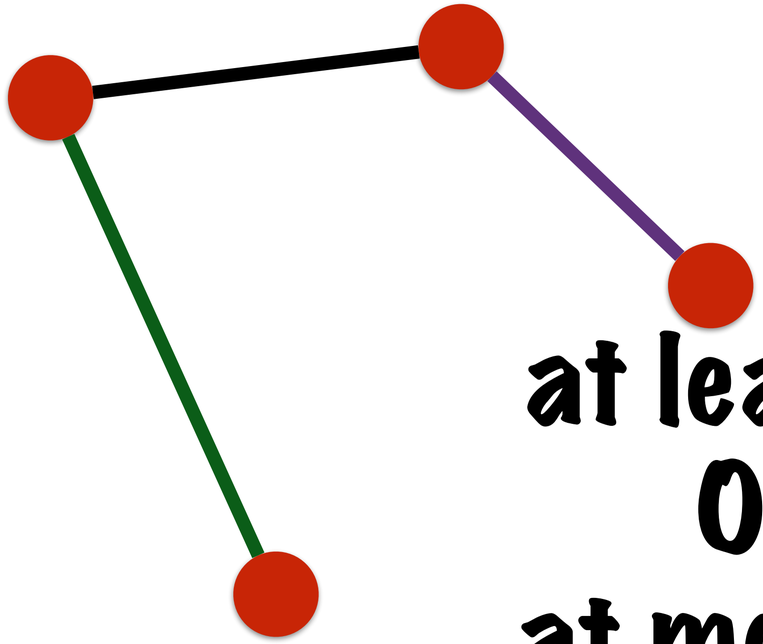


Sum of path lengths
= 2 OPT

Theorem: it's a 2-approximation

Remove one cycle edge

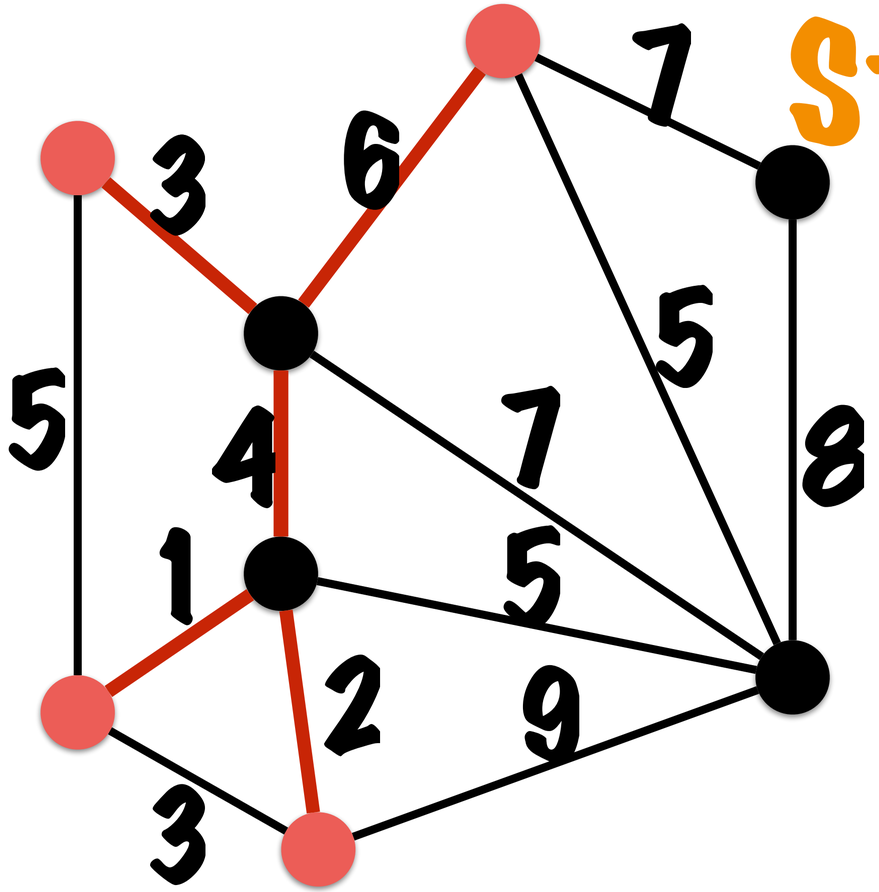




2OPT is
at least MST cost
Output cost is
at most MST cost

QED

Steiner tree



Henry Pollak

H.N. Gilbert

Jakob Steiner

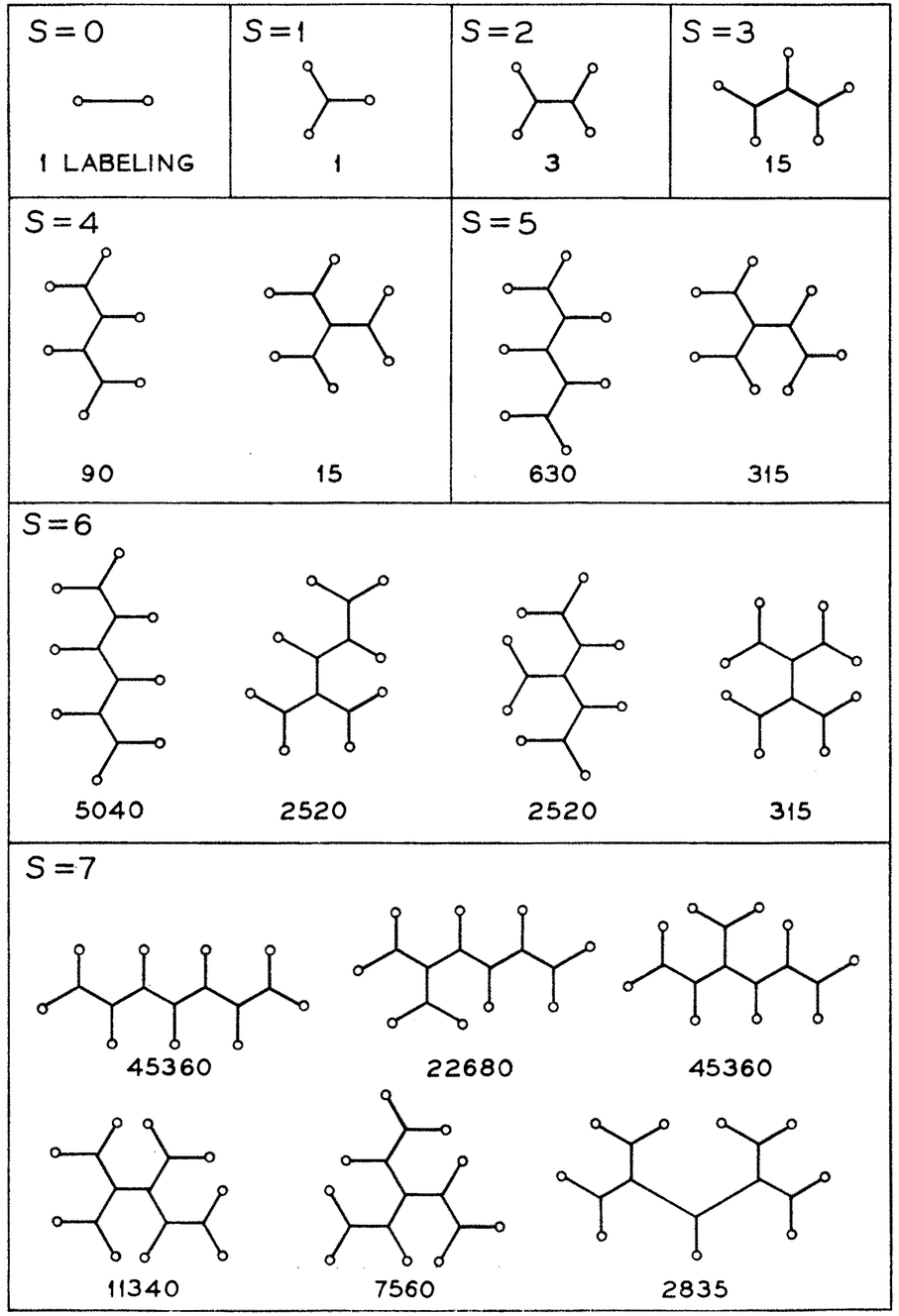
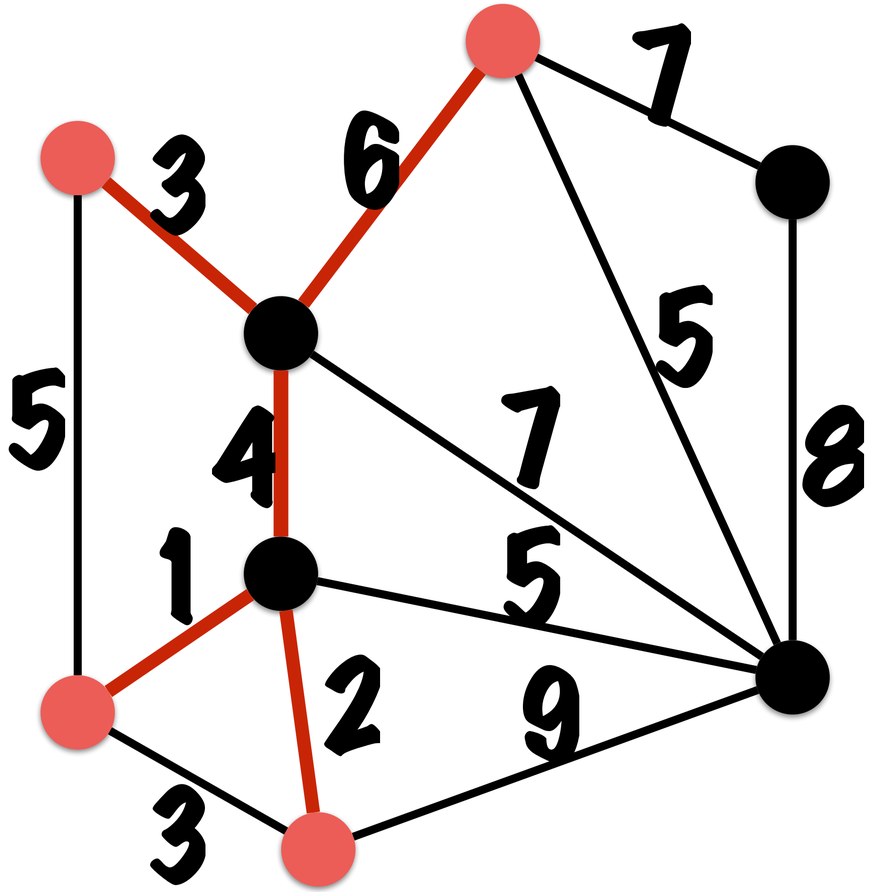


FIG. 3. *Full Steiner trees with s Steiner points*



Steiner tree

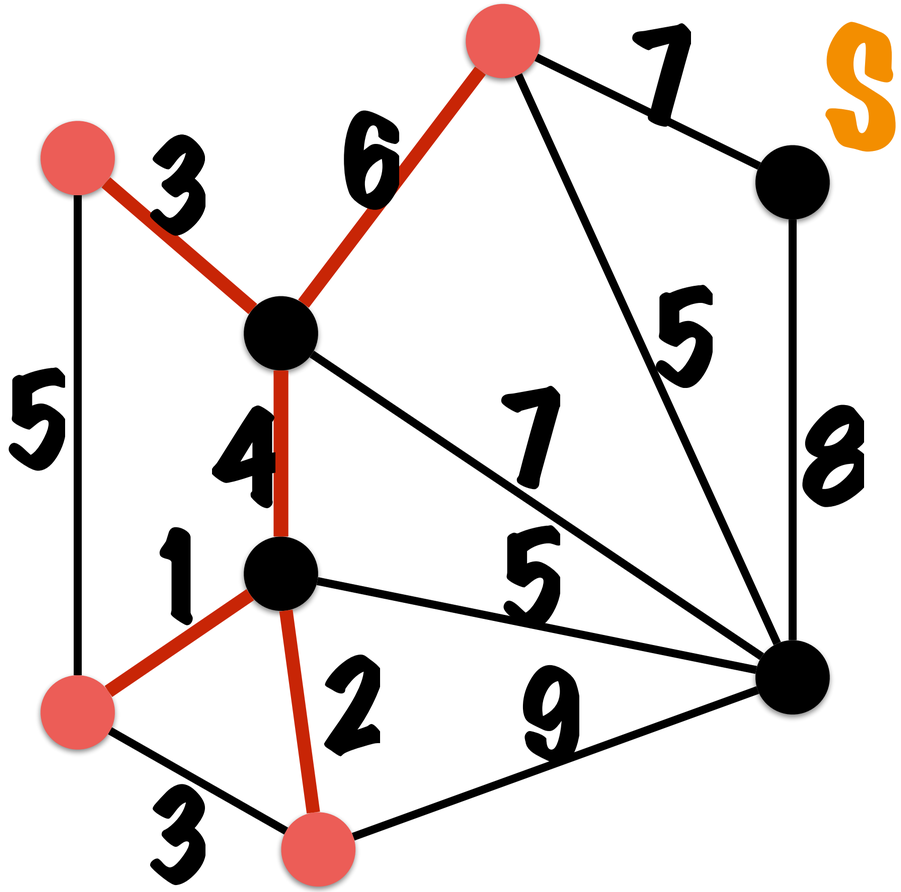
Richard Karp

Jakob Steiner

Marshall Bern

Paul Plassmann

Steiner tree 1.39



Jaroslaw Byrka

Thomas Rothvoss

Fabrizio Grandoni

Laura Sanita

Steiner forest