The growth of wealth

Portfolio: $\mathbf{a} = (a_1, \ldots, a_m)$

Stock price relatives at close of day n: $\mathbf{x}^{(n)} = \left(x_1^{(n)}, \ldots, x_m^{(n)}\right)$

Wealth relative at close of day n: $S_n := S\left(\mathbf{x}^{(n)}\right) = a_1 x_1^{(n)} + \cdots + a_m x_m^{(n)}$

The growth of wealth: $W_n = W_{n-1} S_n$

Portfolio: $\mathbf{a} = (a_1, \ldots, a_m)$ Stock price relatives at close of day n: $\mathbf{x}^{(n)} = \left(x_1^{(n)}, \ldots, x_m^{(n)}\right)$

Wealth relative at close of day n: $s_n := s\left(\mathbf{x}^{(n)}\right) = a_1 x_1^{(n)} + \cdots + a_m x_m^{(n)}$

$$W_n = W_{n-1} s_n$$

Portfolio: $\mathbf{a} = (a_1, \ldots, a_m)$ Stock price relatives at close of day n: $\mathbf{X}^{(n)} = \left(X_1^{(n)}, \ldots, X_m^{(n)}\right)$

Wealth relative at close of day n: $S_n := S\left(\mathbf{X}^{(n)}\right) = a_1 X_1^{(n)} + \cdots + a_m X_m^{(n)}$

$$\begin{aligned} W_n &= W_{n-1} S_n \\ &= W_{n-2} S_{n-1} S_n \end{aligned}$$

Portfolio: $\mathbf{a} = (a_1, \ldots, a_m)$ Stock price relatives at close of day n: $\mathbf{x}^{(n)} = \left(x_1^{(n)}, \ldots, x_m^{(n)}\right)$

Wealth relative at close of day n: $s_n := s\left(\mathbf{x}^{(n)}\right) = a_1 x_1^{(n)} + \cdots + a_m x_m^{(n)}$

$$\begin{aligned} W_n &= W_{n-1} s_n \\ &= W_{n-2} s_{n-1} s_n \\ &= W_{n-3} s_{n-2} s_{n-1} s_n \end{aligned}$$

Portfolio: $\mathbf{a} = (a_1, \ldots, a_m)$ Stock price relatives at close of day n: $\mathbf{x}^{(n)} = \left(x_1^{(n)}, \ldots, x_m^{(n)}\right)$

Wealth relative at close of day n: $s_n := s\left(\mathbf{x}^{(n)}\right) = a_1 x_1^{(n)} + \cdots + a_m x_m^{(n)}$

$$
\begin{aligned}
W_n &= W_{n-1} s_n \\
&= W_{n-2} s_{n-1} s_n \\
&= W_{n-3} s_{n-2} s_{n-1} s_n \\
&= \cdots \\
&= W_0 s_1 s_2 \cdots s_n
\end{aligned}
$$

Portfolio: $\mathbf{a} = (a_1, \ldots, a_m)$ Stock price relatives at close of day n: $\mathbf{x}^{(n)} = \left(x_1^{(n)}, \ldots, x_m^{(n)}\right)$

Wealth relative at close of day n: $s_n := s\left(\mathbf{x}^{(n)}\right) = a_1 x_1^{(n)} + \cdots + a_m x_m^{(n)}$

$$
\begin{aligned}
W_n &= W_{n-1} s_n \\
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&= W_{n-3} s_{n-2} s_{n-1} s_n \\
&= \cdots \\
&= W_0 s_1 s_2 \cdots s_n \\
&= s_1 s_2 \cdots s_n
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$$

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Wealth relative at close of day n: $s_n := s\left(\mathbf{x}^{(n)}\right) = a_1 x_1^{(n)} + \cdots + a_m x_m^{(n)}$

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\begin{aligned}
W_n &= W_{n-1} s_n \\
&= W_{n-2} s_{n-1} s_n \\
&= W_{n-3} s_{n-2} s_{n-1} s_n \\
&= \cdots \\
&= W_0 s_1 s_2 \cdots s_n \\
&= s_1 s_2 \cdots s_n \\
&= 2^{\log_2 (s_1 s_2 \cdots s_n)}
\end{aligned}
$$

Portfolio: $\mathbf{a} = (a_1, \ldots, a_m)$ Stock price relatives at close of day n: $\mathbf{x}^{(n)} = \left(x_1^{(n)}, \ldots, x_m^{(n)}\right)$

Wealth relative at close of day n: $s_n := s\left(\mathbf{x}^{(n)}\right) = a_1 x_1^{(n)} + \cdots + a_m x_m^{(n)}$

$$
\begin{aligned}
W_n &= W_{n-1} s_n \\
&= W_{n-2} s_{n-1} s_n \\
&= W_{n-3} s_{n-2} s_{n-1} s_n \\
&= \cdots \\
&= W_0 s_1 s_2 \cdots s_n \\
&= s_1 s_2 \cdots s_n \\
&= 2^{\log_2(s_1 s_2 \cdots s_n)} \\
&= 2^{\log_2(s_1) + \log_2(s_2) + \cdots + \log_2(s_n)}
\end{aligned}
$$

Portfolio: $\mathbf{a} = (a_1, \ldots, a_m)$ Stock price relatives at close of day n: $\mathbf{x}^{(n)} = \left(x_1^{(n)}, \ldots, x_m^{(n)}\right)$

Wealth relative at close of day n: $S_n := S\left(\mathbf{x}^{(n)}\right) = a_1 x_1^{(n)} + \cdots + a_m x_m^{(n)}$

$$
\begin{aligned}
W_n &= W_{n-1} S_n \\
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&= \cdots \\
&= W_0 S_1 S_2 \cdots S_n \\
&= S_1 S_2 \cdots S_n \\
&= 2^{\log_2(S_1 S_2 \cdots S_n)} \\
&= 2^{\log_2(S_1) + \log_2(S_2) + \cdots + \log_2(S_n)} \\
&= 2^{n \cdot \frac{1}{n}[\log_2(S_1) + \log_2(S_2) + \cdots + \log_2(S_n)]}
\end{aligned}
$$

Portfolio: $\mathbf{a} = (a_1, \ldots, a_m)$ Stock price relatives at close of day n: $\mathbf{x}^{(n)} = \left(x_1^{(n)}, \ldots, x_m^{(n)}\right)$

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$$
\begin{aligned}
W_n &= W_{n-1} S_n \\
&= W_{n-2} S_{n-1} S_n \\
&= W_{n-3} S_{n-2} S_{n-1} S_n \\
&= \cdots \\
&= W_0 S_1 S_2 \cdots S_n \\
&= S_1 S_2 \cdots S_n \\
&= 2^{\log_2(S_1 S_2 \cdots S_n)} \\
&= 2^{\log_2(S_1) + \log_2(S_2) + \cdots + \log_2(S_n)} \\
&= 2^{n \cdot \frac{1}{n}[\log_2(S_1) + \log_2(S_2) + \cdots + \log_2(S_n)]} \\
&= 2^{n \cdot \Delta_n}
\end{aligned}
$$

$$
\Delta_n = \frac{1}{n}\left[\log_2(S_1) + \log_2(S_2) + \cdots + \log_2(S_n)\right]
$$