

An urn problem

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 - ❖ What is the probability that the *first two* urns get k balls between them?

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 - ❖ $S \sim \text{Binomial}(n, 1/r)$.
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$$\mathbf{P}\{T = k\} = b_n(k; 2/r) = \binom{n}{k} \left(\frac{2}{r}\right)^k \left(1 - \frac{2}{r}\right)^{n-k}$$