Feedback — Assignment 1

You submitted this quiz on **Wed 27 Feb 2013 9:38 AM PST -0800**. You got a score of **24.00** out of **24.00**.

Question 1

Recall some basic definitions from linear algebra: The kernel of a matrix $A \in \mathbb{R}^{m \times n}$ is defined as $Ker(A) := \{x \in \mathbb{R}^n \mid Ax = 0\}$and the rowspace of A is defined as $\{y \in \mathbb{R}^n \mid \exists \lambda \in \mathbb{R}^m \text{ s.t. } A^T\lambda = y\}$ Is it true that every vector from the kernel of $A \in \mathbb{R}^{m \times n}$ is orthogonal to every vector from the rowspace of A?

Your Answer		Score	Explanation
Yes	✓	1.00	
No			
Total		1.00 / 1.00	

Question Explanation

Take a vector x from the kernel of A and a vector y from the rowspace of A. Then, $x^T y = x^T A^T \lambda = (Ax)^T \lambda = 0$where the first equality follows from the fact that there exists a λ such that $A^T\lambda = y$ (as y is in the rowspace of A) and the final inequality follows from the fact that x is in the kernel of A.

Question 2

Suppose we start with a matrix $A \in \mathbb{R}^{m \times n}$ and perform a sequence of elementary row operations on A to get a matrix B. Is $Ker(A) = Ker(B)$?

Your Answer	Score	Explanation
No		

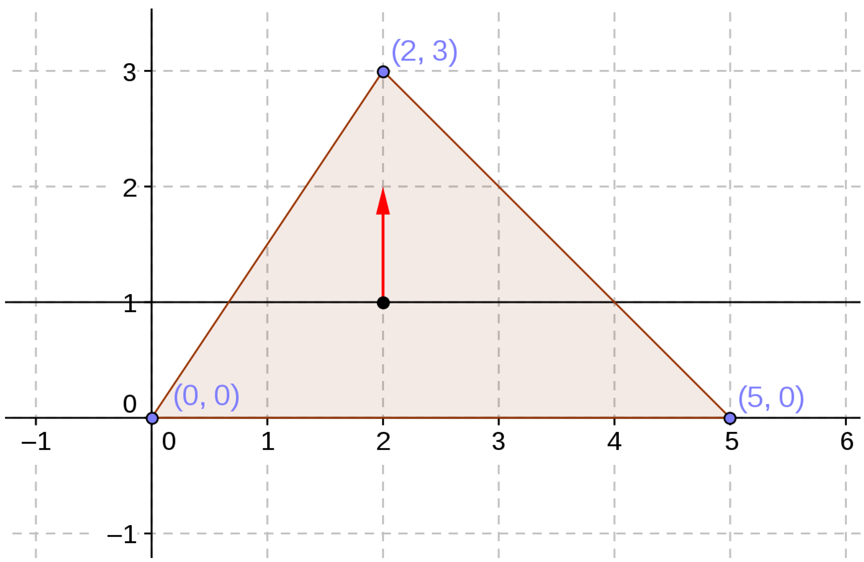
Yes	✓	1.00
Total		1.00 / 1.00

Question Explanation

Yes, because the elementary row operations (swapping two rows, multiplying a row by a non-zero scalar, and adding a row to another) preserve the set of solutions x to a system of equations of the form $Ax = 0$. Thus, the kernel of A and B are exactly identical.

Question 3

For the objective function in the diagram (indicated using the vector), what is the maximum value it can attain over the shaded region?



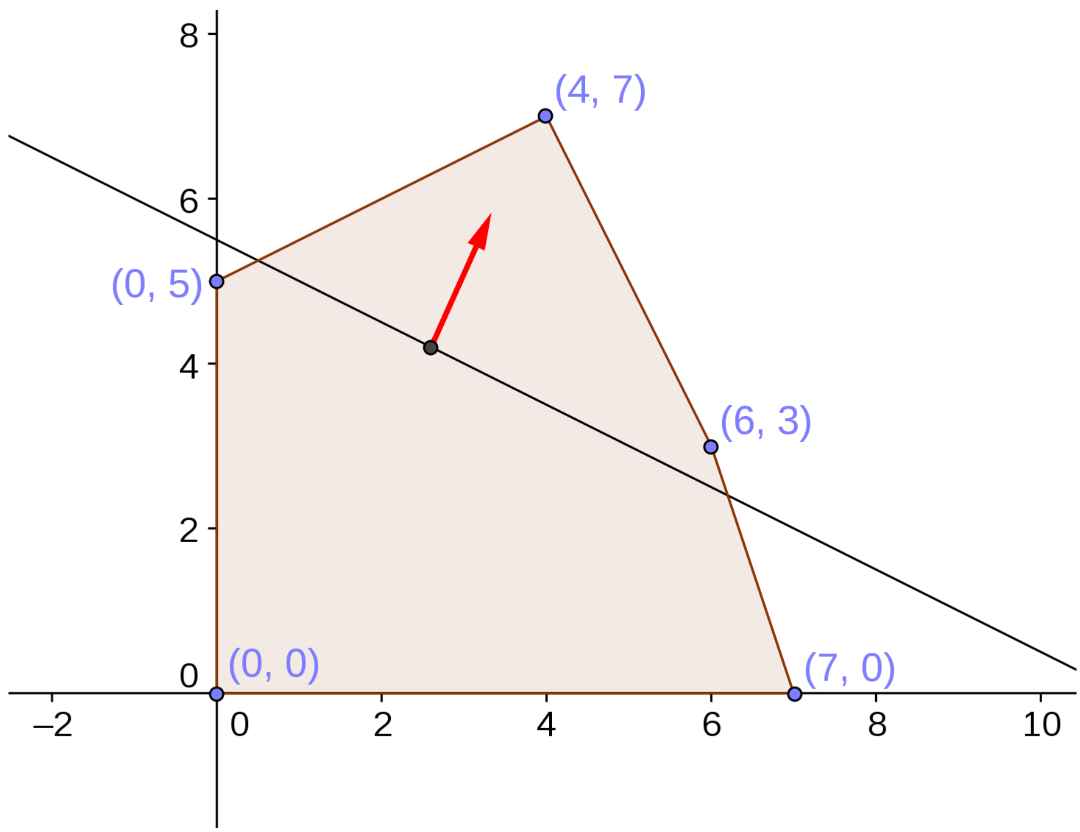
Your Answer		Score	Explanation
3	✓	1.00	
5			
1			
2			
Total		1.00 / 1.00	

Question Explanation

From the diagram we can infer that the objective function vector is $c = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ since the vector points in the direction of the positive y-axis. Clearly, the maximum y-value over the shaded region is attained at the corner $(2, 3)$ of the triangle, and this is equal to 3.

Question 4

The objective function vector $c = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ is indicated in the diagram. The line drawn in the diagram corresponds to $x + 2y = 11$ Using the diagram deduce the maximum possible value that the objective function can attain in the shaded region.



Your Answer		Score	Explanation
18	✓	3.00	
15			
24			

12

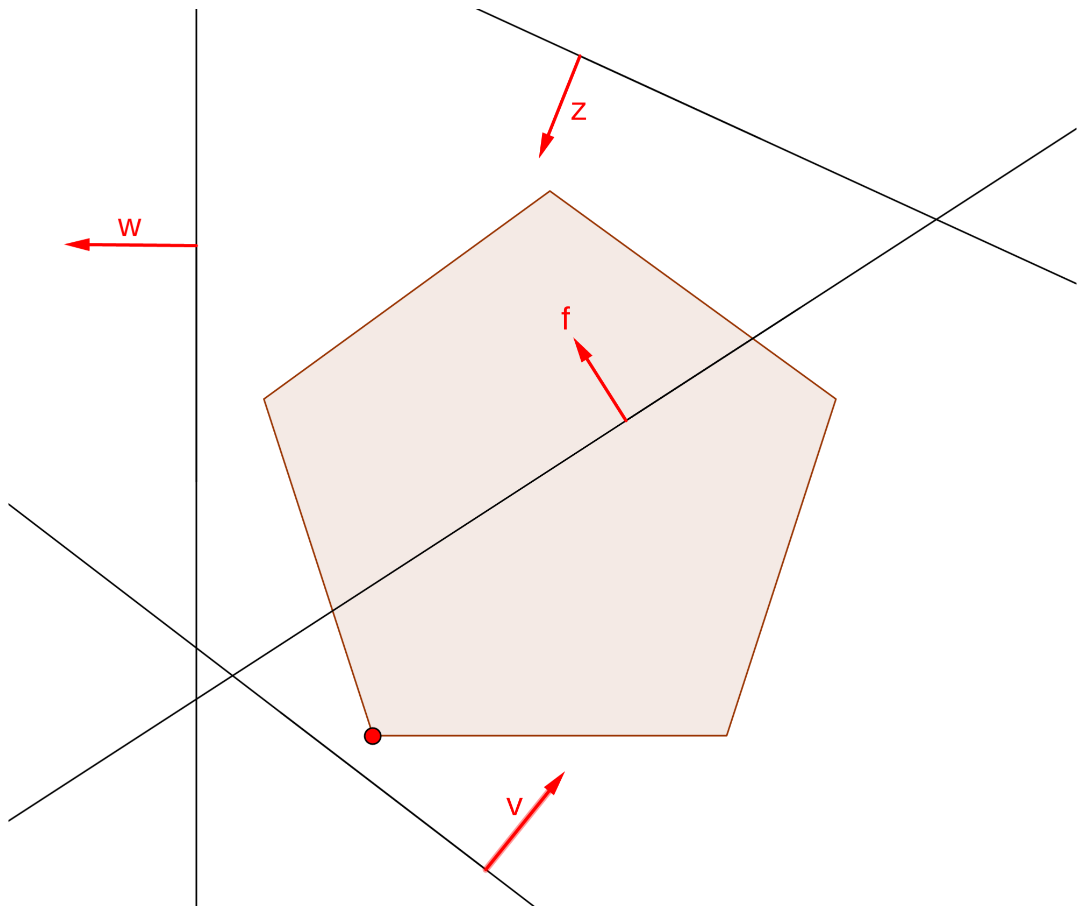
Total	3.00 / 3.00

Question Explanation

Moving the line in the direction of the objective function vector we see graphically that the maximum should be attained at the corner $(4, 7)$ of the polygon. At this point, the objective function has the value $4 + 2 \times 7 = 18$

Question 5

Which of the indicated objective functions attain their maximum over the shaded region at the encircled vertex in red? Check all options that apply.



Your Answer		Score	Explanation
z	✓	0.75	
v	✓	0.75	

w	✓	0.75
f	✓	0.75
Total		3.00 / 3.00

Question Explanation

We graphically move the lines that are normal to the indicated objective function vectors in the indicated direction as much as possible while still staying in the shaded region. Doing this for each of the vectors, we find that only the objective function indicated by the vector z attains the maximum at the encircled red vertex. Note that the objective function indicated by v attains its minimum at the red vertex and is hence not a correct answer.

Question 6

Given the multipliers 4 and 3 for the two inequalities $3x + 2y \leq 4$and $x + 5y \leq 10$ respectively, it is possible to upper bound the value of the objective function $15x + 23y$ subject to the two constraints. What is this upper bound?

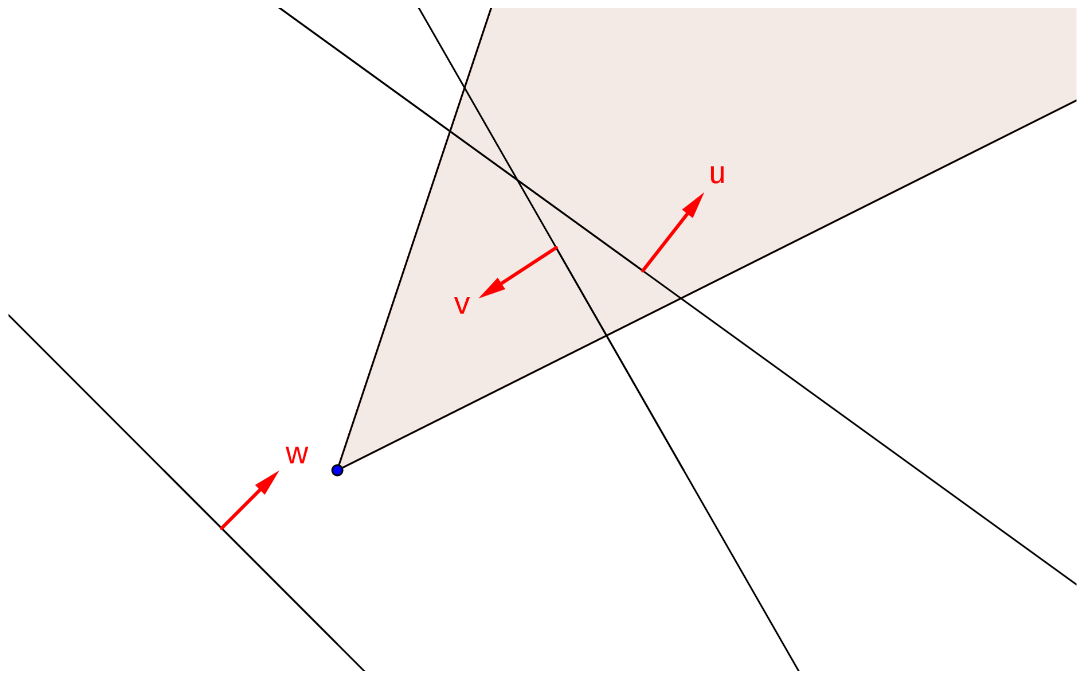
Your Answer		Score	Explanation
21			
46	✓	3.00	
125			
69			
Total		3.00 / 3.00	

Question Explanation

Multiplying $3x + 2y \leq 4$by 4 and $x + 5y \leq 10$by 3, we have $12x + 8y \leq 16$and $3x + 15y \leq 30$respectively. Adding the two inequalities we have $15x + 23y \leq 46$ Thus the upper bound is 46.

Question 7

For the feasible region shown in the diagram, we have a choice of three different objective functions each corresponding to a different linear program. Of the three linear programs, how many are unbounded?



Your Answer		Score	Explanation
2	✓	3.00	
0			
1			
3			
Total		3.00 / 3.00	

Question Explanation

The objective function vectors u and w define an LP on the feasible region that are unbounded since we can indefinitely increase the value of the objective function by moving in the direction indicated by the vectors, while still retaining feasibility (the corresponding parallely translated lines intersect the shaded region). The objective function vector v however attains its maximum over the shaded region at the blue point in the diagram, and hence the corresponding LP is bounded.

Question 8

Click on all the assertions whose conjunction makes this statement correct: A linear program $\max\{c^T x : x \in \mathbb{R}^n,\ Ax = b\}$ is unbounded if and only if

Your Answer		Score	Explanation
☑ $b \in Im(A)$	✓	1.00	
☑ $\exists d \in Ker(A)$ such that $c^T d \neq 0$	✓	1.00	
☐ $c \in Ker(A)$	✓	1.00	
Total		3.00 / 3.00	

Question Explanation

First the linear program has to be feasible. Hence, we require that $b \in Im(A)$. Secondly, for the LP to be unbounded there must exist a direction $d \in \mathbb{R}^n$ such that, if x_0 is a feasible point for the LP, then so is $x_0 + \lambda d$ for every $\lambda > 0$. Since it is feasible, it must be the case that $A(x_0 + \lambda d) = b \implies Ax_0 + \lambda Ad = b$. And since x_0 is feasible and hence $Ax_0 = b$ we have $Ad = 0$. Hence, $d \in Ker(A)$. Also, we require that $c^T d \neq 0$ since we require either d or $-d$ to be a direction in which the objective function value increases i.e., $c^T d > 0$ or $c^T(-d) > 0$.

Question 9

Choose the pairs of inequalities that are equivalent. (Two inequalities $a^T x \leq \beta$ and $c^T x \leq \delta$ are equivalent if $\{x \in \mathbb{R}^n : a^T x \leq \beta\} = \{x \in \mathbb{R}^n : c^T x \leq \delta\}$)

Your Answer		Score	Explanation
☐ $7x_1 + 3x_2 \leq 3,\ 14x_1 + 6x_2 \leq 7$	✓	1.00	
☑ $3x_1 + 6x_2 \leq 0,\ x_1 + 2x_2 \leq 0$	✓	1.00	
☐ $3x_1 + 4x_2 \leq 5,\ -6x_1 - 8x_2 \geq 10$	✓	1.00	
Total		3.00 / 3.00	

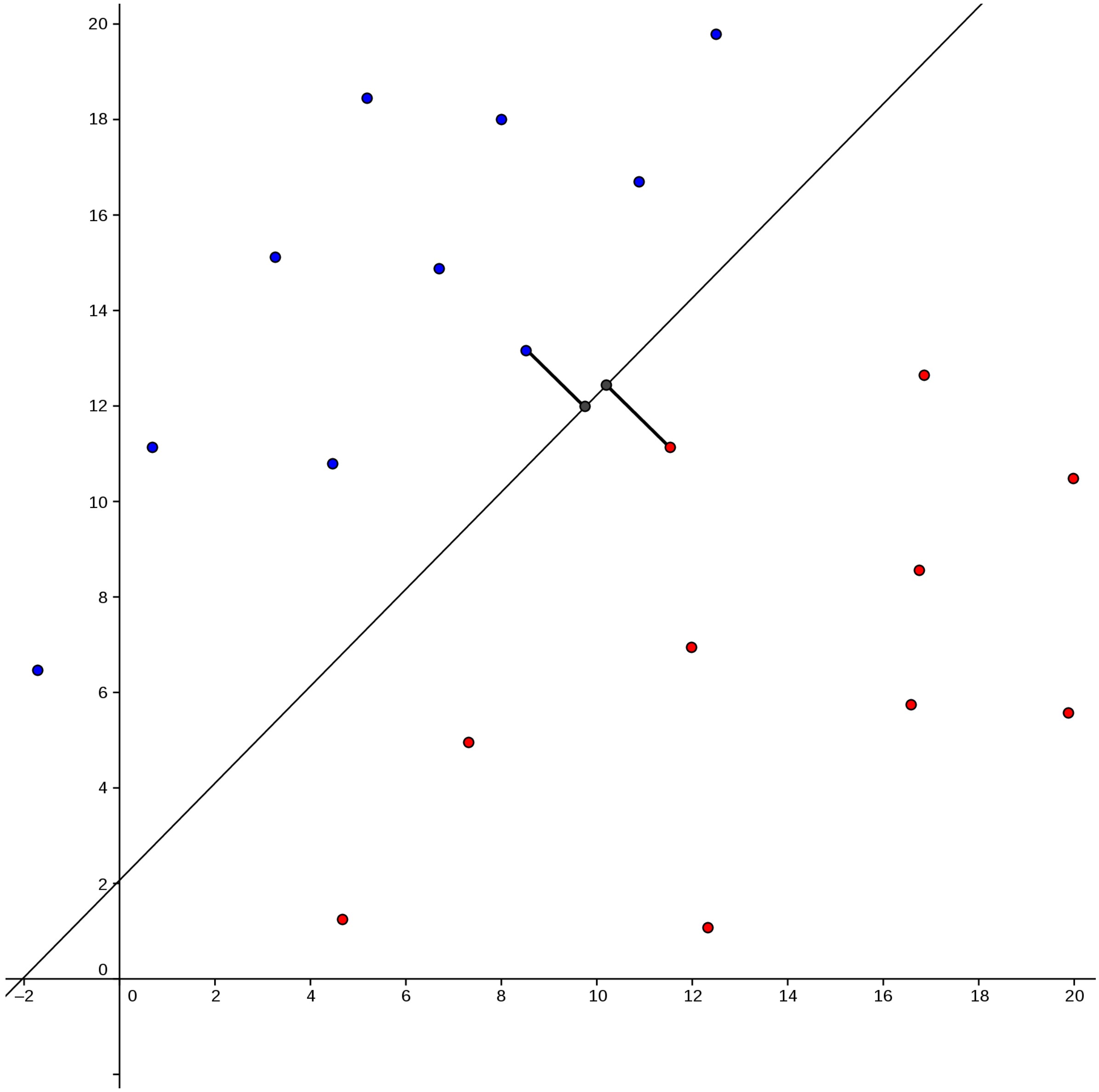
Question Explanation

The pair $3x_1 + 6x_2 \le 0$, $x_1 + 2x_2 \le 0$ is equivalent because we can derive the second from the first by multiplying by $\frac{1}{3}$. The pair $3x_1 + 4x_2 \le 5$, $-6x_1 - 8x_2 \ge 10$ is not equivalent since $x_1 = 1, x_2 = -1$ is feasible for the first but not for the second inequality. The pair $7x_1 + 3x_2 \le 3$, $14x_1 + 6x_2 \le 7$ is not equivalent because $x_1 = 1, x_2 = -\frac{7}{6}$ is feasible for the second but not for the first inequality.

Question 10

Let us revisit the problem to find a classifier. Given m red points $x_1, \dots, x_m \in \mathbb{R}^k$ and n blue points $y_1, \dots, y_n \in \mathbb{R}^k$, determine $a \in \mathbb{R}^k$ and $\beta \in \mathbb{R}$ such that $a^T x_i > \beta,\ i = 1, \dots, m$ and $a^T y_j < \beta,\ j = 1, \dots, n$. The following optimization problem determines a classifier such that the minimum distance of the red points to the hyperplane is equal to the minimum distance of the blue points to the hyperplane and such that this minimum distance ($\frac{1}{\|a\|}$) is maximized.

$$\begin{aligned} \min\ & \|a\| \\ & a^T x_i \ge \beta + 1 \quad i = 1, \dots, m \\ & a^T y_j \le \beta - 1 \quad j = 1, \dots, n \\ & a \in \mathbb{R}^k,\ \beta \in \mathbb{R} \end{aligned}$$



In the diagram shown above, line segments join the classifier to the nearest red and blue points. The two line segments are of equal length.

However, the above formulation does not correspond to a linear program. But, if the objective function above is replaced by $\min \|a\|_1$, where $\|a\|_1$ is the ℓ_1-norm $\|a\|_1 = \sum_{i=1}^{k} |a_i|$, then, one can formulate a linear program to solve the corresponding optimization problem

$$\begin{aligned} \min\ & \|a\|_1 \\ & a^T x_i \geq \beta + 1 \quad i = 1, \ldots, m \\ & a^T y_j \leq \beta - 1 \quad j = 1, \ldots, n \\ & a \in \mathbb{R}^k,\ \beta \in \mathbb{R} \end{aligned}$$

Which of the following is a valid reformulation of the optimization problem above?

Your Answer		Score	Explanation
○			
◉	✓	3.00	

○

$$\min \sum_{i=1}^{k} h_i$$
$$h_i \geq a_i \quad i = 1, \ldots, k$$
$$a^T x_i \geq \beta + 1 \quad i = 1, \ldots, m$$
$$a^T y_j \leq \beta - 1 \quad j = 1, \ldots, n$$
$$a \in \mathbb{R}^k,\ \beta \in \mathbb{R}$$

◉ ✓ 3.00

$$\min \sum_{i=1}^{k} h_i$$
$$h_i \geq a_i \quad i = 1, \ldots, k$$
$$h_i \geq -a_i \quad i = 1, \ldots, k$$
$$a^T x_i \geq \beta + 1 \quad i = 1, \ldots, m$$
$$a^T y_j \leq \beta - 1 \quad j = 1, \ldots, n$$
$$a \in \mathbb{R}^k,\ \beta \in \mathbb{R}$$

○

$$\min \sum_{i=1}^{k} h_i$$
$$h_i \geq a_i \quad i = 1, \ldots, k$$
$$h_i \leq -a_i \quad i = 1, \ldots, k$$
$$a^T x_i \geq \beta + 1 \quad i = 1, \ldots, m$$
$$a^T y_j \leq \beta - 1 \quad j = 1, \ldots, n$$
$$a \in \mathbb{R}^k,\ \beta \in \mathbb{R}$$

○

$$
\begin{aligned}
\min h \\
& h \geq a_i \quad i = 1, \ldots, k \\
& h \geq -a_i \quad i = 1, \ldots, k \\
& a^T x_i \geq \beta + 1 \quad i = 1, \ldots, m \\
& a^T y_j \leq \beta - 1 \quad j = 1, \ldots, n \\
& a \in \mathbb{R}^k, \ \beta \in \mathbb{R}
\end{aligned}
$$

Total	3.00 / 3.00

Question Explanation

The objective $\|a\|_1 = \sum_{i=1}^{k} |a_i|$is not a linear function due to the presence of the absolute value functions $|\cdot|$. To remedy this, we will replace each of the $|a_i|$ by a new variable h_i. Now, we are trying to minimize the sum $\sum_{i=1}^{k} h_i$ with the understanding that each h_i represents the absolute value of a_i. So, we include the constraints $h_i \geq a_i$ and $h_i \geq -a_i$, since if h_i must be a valid replacement for $|a_i|$, then it must be larger than both a_i and $-a_i$. The equivalence follows since our objective function minimizes the sum of the h_i, and so each h_i will be exactly equal to the absolute value of a_i.