

Problem 7.6

Given $\exists y. (\forall x. (h(x) \Rightarrow f(x, y)) \wedge \forall z. (r(z) \Rightarrow f(y, z)))$ [there exists something that is slower than all horses and faster than all rabbits] and $f(x, y) \wedge f(y, z) \Rightarrow f(x, z)$ [faster is transitive], use the Fitch system to prove $\forall x. \forall z. (h(x) \wedge r(z) \Rightarrow f(x, z))$ [every horse is faster than every rabbit].

To apply a rule of inference, check the lines you wish to use as premises and click the button for the rule of inference. Reiteration allows you to repeat an earlier item. To delete one or more lines from a proof, check the desired lines and click Delete.

Whenever entering expressions, use Ascii characters only. Use \sim for \neg ; use $\&$ for \wedge ; use $|$ for \vee ; use \Rightarrow for \Rightarrow ; use \forall for \forall ; use \exists for \exists ; and use $:$ for $.$ in quantified sentences. Also, for variables use strings of alphanumeric characters that begin with a capital letter. For example, to write the sentence $\forall x. \exists y. (p(x) \wedge q(y) \Rightarrow r(y) \wedge \neg s(y))$, write $\forall x:\exists y:(p(x) \&q(y) \Rightarrow r(y) | \sim s(y))$.

Proof Editor		
1.	$f(X,Y) \& f(Y,Z) \Rightarrow f(X,Z)$	Premise
2.	$\exists Y:(\forall X:(h(X) \Rightarrow f(X,Y)) \& \forall Z:(r(Z) \Rightarrow f(Y,Z)))$	Premise
3.	$\forall X:(h(X) \Rightarrow f(X,Y)) \& \forall Z:(r(Z) \Rightarrow f(Y,Z))$	Assumption
4.	$\forall X:(h(X) \Rightarrow f(X,Y))$	And Elimination: 3
5.	$\forall Z:(r(Z) \Rightarrow f(Y,Z))$	And Elimination: 3
6.	$h(X) \Rightarrow f(X,Y)$	Universal Elimination: 4
7.	$r(Z) \Rightarrow f(Y,Z)$	Universal Elimination: 5
8.	$h(X) \& r(Z)$	Assumption
9.	$h(X)$	And Elimination: 8
10.	$r(Z)$	And Elimination: 8
11.	$f(X,Y)$	Implication Elimination: 6, 9

12.	$f(Y,Z)$	Implication Elimination: 7, 10
13.	$f(X,Y) \ \& \ f(Y,Z)$	And Introduction: 11, 12
14.	$f(X,Z)$	Implication Elimination: 1, 13
15.	$h(X) \ \& \ r(Z) \Rightarrow f(X,Z)$	Implication Introduction: 14
16.	$AZ:(h(X) \ \& \ r(Z) \Rightarrow f(X,Z))$	Universal Introduction: 15
17.	$AX:AZ:(h(X) \ \& \ r(Z) \Rightarrow f(X,Z))$	Universal Introduction: 16
18.	$AX:(h(X) \Rightarrow f(X,Y)) \ \& \ AZ:(r(Z) \Rightarrow f(Y,Z)) \Rightarrow AX:AZ:$ $(h(X) \ \& \ r(Z) \Rightarrow f(X,Z))$	Implication Introduction: 17
19.	$AY:(AX:(h(X) \Rightarrow f(X,Y)) \ \& \ AZ:(r(Z) \Rightarrow f(Y,Z)) \Rightarrow$ $AX:AZ:(h(X) \ \& \ r(Z) \Rightarrow f(X,Z)))$	Universal Introduction: 18
20.	$AX:AZ:(h(X) \ \& \ r(Z) \Rightarrow f(X,Z))$	Existential Elimination: 2, 19

Goal	$AX:AZ:(h(X) \ \& \ r(Z) \Rightarrow f(X,Z))$	Complete Submit
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Assumption

Reiteration

Delete

Negation Introduction

Negation Elimination

And Introduction

And Elimination

Or Introduction

Or Elimination

Implication Introduction

Implication Elimination

Biconditional Introduction

Biconditional Elimination

Universal Introduction

Universal Elimination

Existential Introduction

Existential Elimination

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