<u>Z-TEST / Z-STATISTIC:</u> used to test hypotheses about μ when the population standard deviation is <u>known</u>

- and population distribution is normal or sample size is large

<u>T-TEST / T-STATISTIC:</u> used to test hypotheses about μ when the population standard deviation is <u>unknown</u>

- Technically, requires population distributions to be normal, but is robust with departures from normality
- Sample size can be small

The only difference between the *z*- and *t*-tests is that the *t*-statistic <u>estimates</u> standard error by using the sample standard deviation, while the *z*-statistic utilizes the population standard deviation

<u>One Sample T-test</u>

<u>Formula</u>:

$$t = \frac{\bar{x} - \mu}{s_{\bar{x}}} \quad \text{where} \quad s_{\bar{x}} = \frac{s}{\sqrt{n}}$$

- $S_{\bar{x}}$ = estimated standard error of the mean
- Because we're using sample data, we have to correct for sampling error. The method for doing this is by using what's called <u>degrees of freedom</u>

Degrees of Freedom

- degrees of freedom (df) are defined as the number of scores in a sample that are free to vary
- we know that in order to calculate variance we must know the mean ($\overline{X}$)

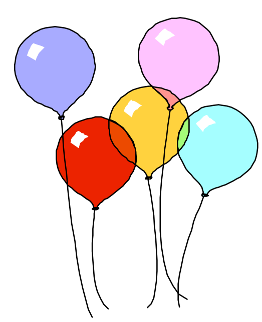
$$s = \sqrt{\frac{\sum (x_i - \bar{x})}{n-1}}$$

- this limits the number of scores that are free to vary
- $df = n - 1$ where n is the number of scores in the sample

Degrees of Freedom Cont. Picture Example

•There are five balloons: one blue, one red, one yellow, one pink, & one green.

•If 5 students (n=5) are each to select one balloon only 4 will have a choice of color (df=4). The last person will get whatever color is left.



- The particular t-distribution to use depends on the number of degrees of freedom(df) there are in the calculation
- Degrees of freedom (df)
  - df for the t-test are related to sample size
  - For single-sample t-tests, df= n-1
  - df count how many observations are free to vary in calculating the statistic of interest
- For the single-sample t-test, the limit is determined by how many observations can vary in calculating **s** in

$$t_{obt} = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$$

Z-test vs. T-test

$$z_{obt} = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

The z-test assumes that:

- the numerator varies from one sample to another
- the denominator is constant

Thus, the sampling distribution of z derives from the sampling distribution of the mean

$$t_{obt} = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$$

The z-test assumes that:

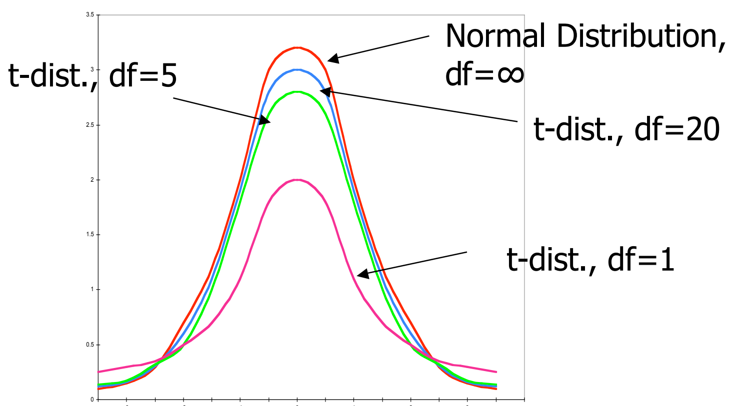
- the numerator varies from one sample to another
- the denominator varies from one sample to another

- **Therefore the sampling distribution is broader than it otherwise would be**
- **The sampling distribution changes with *n***
- **It approaches normal as *n* increases**

Characteristics of the t-distribution:

- The t-distribution is a family of distributions -- a slightly different distribution for each sample size (degrees of freedom)
- It is flatter and more spread out than the normal z-distribution
- As sample size increases, the t-distribution approaches a normal distribution

Introduction to the *t*-statistic



When df are large the curve approximates the normal distribution. This is because as n is increased the estimated standard error will not fluctuate as much between samples.

- Note that the t-statistic is analogous to the z-statistic, except that both the sample mean and the sample s.d. must be calculated
- Because there is a different distribution for each df, we need a different table for each df
  - Rather than actually having separate tables for each t-distribution, Table D in the text provides the critical values from the tables for df= 1 to df= 120
  - As df increases, the t-distribution becomes increasingly normal
  - For df=∞, the t-distribution is

Procedures in doing a t-test

1. Determine H_0 and H_1
2. Set the criterion
   - Look up $\boldsymbol{t_{crit}}$, which depends on alpha and df
3. Collect sample data, calculate $\boldsymbol{\bar{x}}$ and $\boldsymbol{s}$
4. Calculate the test statistic

$$t_{obt} = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$$

5. Reject H_0 if $\boldsymbol{t_{obt}}$ is more extreme than $\boldsymbol{t_{crit}}$

Example:
A population of heights has a $\mu=68$. What is the probability of selecting a sample of size $n=25$ that has a mean of 70 or greater and a $s=4$?

- We hypothesized about a population of heights with a mean of 68 inches. However, we do not know the population standard deviation. This tells us we must use a t-test instead of a z-test

Step 1: State the hypotheses

H_0: $\mu=68$

H_1: $\mu \geq 68$

<u>Step 2</u>: Set the criterion

- one-tail test or two-tail test?
- α=?
- df = n-1 = ?
- See table for critical t-value

<u>Step 3</u>: Collect sample data, calculate *x* and s

From the example we know the sample mean is 70, with a standard deviation (s) of 4.

<u>Step 4</u>: Calculate the test statistic

- Calculate the estimated standard error of the mean

$$s_{\bar{x}} = \frac{s}{\sqrt{n}} = \frac{4}{\sqrt{25}} = 0.8$$

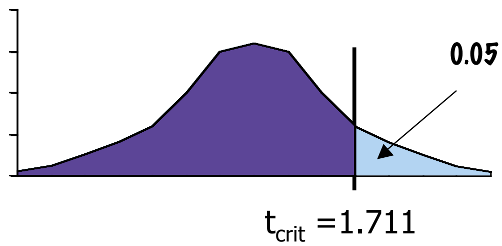
- Calculate the t-statistic for the sample

$$t = \frac{\bar{x} - \mu}{s_{\bar{x}}}$$

$$t = \frac{70 - 68}{0.8} = 2.5$$

<u>Step 5</u>: Reject H_0 if t_{obt} is more extreme than t_{crit}

- The critical value for a one-tailed t-test with df=24 and α=.05 is 1.711
- Will we reject or fail to reject the null hypothesis?



<u>Example</u>:

A researcher is interested in determining whether or not review sessions affect exam performance.

The independent variable, a review session, is administered to a sample of students (n=9) in an attempt to determine if this has an effect on the dependent variable, exam performance.

Based on information gathered in previous semesters, I know that the population mean for a given exam is 24.

The sample mean is 25, with a standard deviation (s) of 4.

- We hypothesized about a population mean for students who get a review based on the information from the population who didn't get a review ($\mu=24$). However, we do not know the population standard deviation. This tells us we must use a t-test instead of a z-test

Step 1: State the hypotheses

H_0: $\mu=24$

H_1: $\mu\geq24$

Step 2: Set the criterion

- one-tail test or two-tail test?
- α=?
- df = n-1 = ?
- See table for critical t-value

Step 3: Collect sample data, calculate *x* and s

From the example we know the sample mean is 25, with a standard deviation (s) of 4.

Step 4: **Calculate the test statistic**

- Calculate the estimated standard error of the mean

$$s_{\bar{x}} = \frac{s}{\sqrt{n}} = \frac{4}{\sqrt{9}} = \frac{4}{3} = 1.33$$

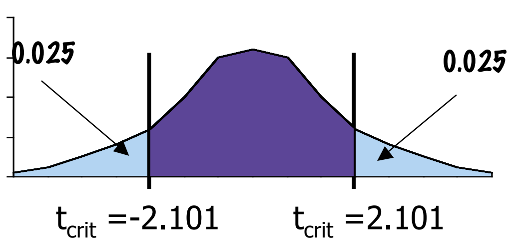
- Calculate the t-statistic for the sample

$$t = \frac{\bar{x} - \mu}{s_{\bar{x}}}$$

$$t = \frac{26 - 24}{1.33} = \frac{2}{1.33} = \underline{1.503}$$

Step 5: **Reject H_0 if t_{obt} is more extreme than t_{crit}**

- The critical value for a one-tailed t-test with df=8 and α=.05 is 1.86
- Will we reject or fail to reject the null hypothesis?



Assumptions of the t-Test:

- Independent Observations: Each person's score in the sample is not affected by other scores; if, for example, 2 subjects cheated from one another on the exam, the independence assumption would be violated
- Normality: The population sampled must be normally distributed
- Need to know only the population mean
- Need sample mean and standard deviation

Confidence Intervals

- Often, one's interest is not in testing a hypothesis, but in estimating a population mean or proportion
  - This cannot be done precisely, but only to some extent
  - Thus, one estimates an interval, not a point value
  - The interval contains the true value with a probability
  - The wider the interval, the greater the probability that it contains the true value
    - Thus there is a precision/confidence trade-off
    - The intervals are called confidence intervals(CI)
- Typical CIs contain the true value with probability .95 (95% CI) and with probability .99 (99% CI)
- CI is calculated with either t or z, as appropriate