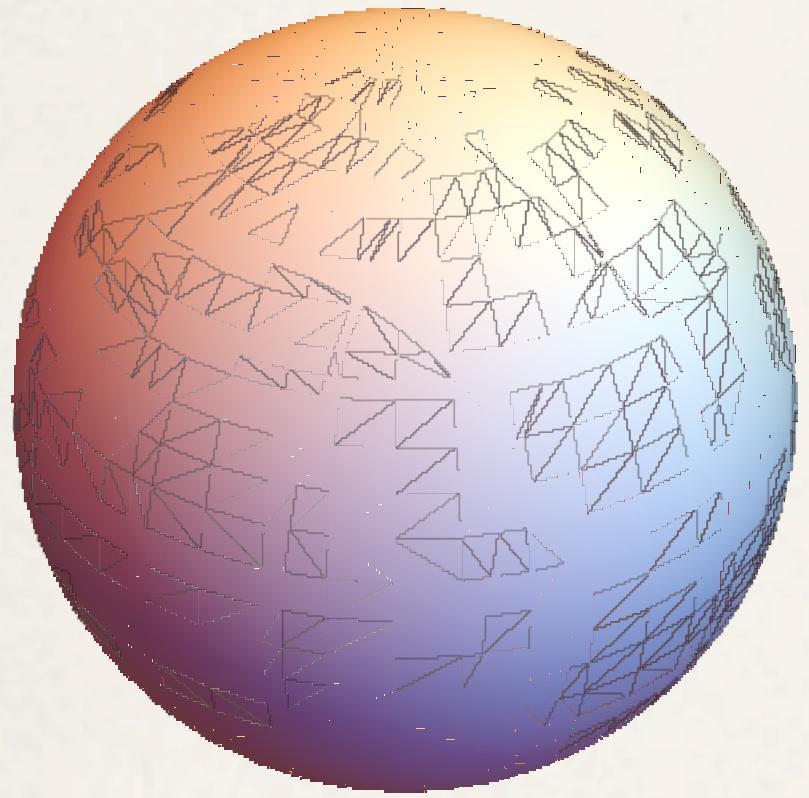
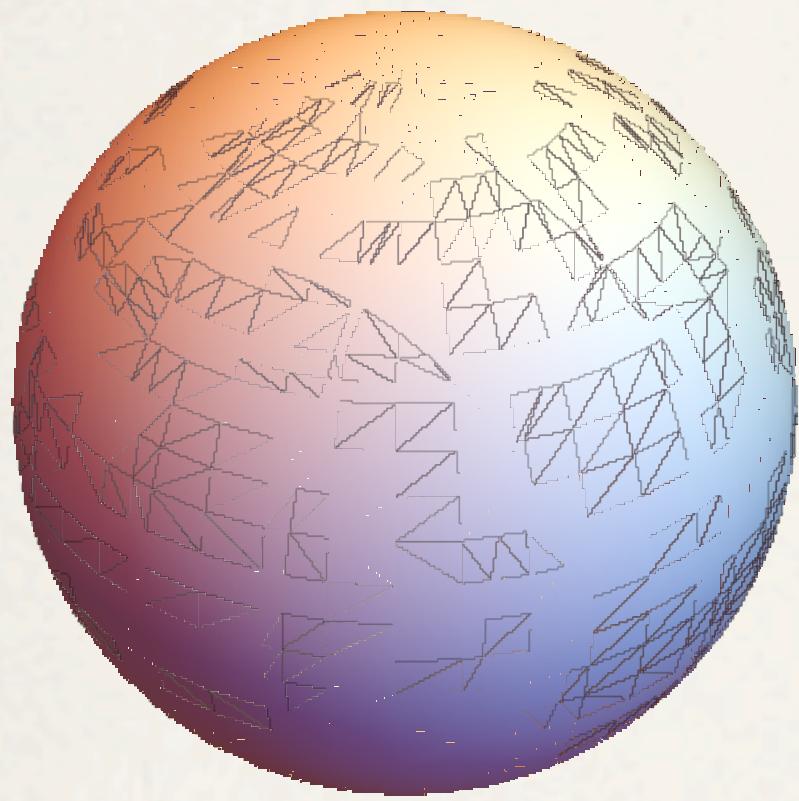


Embedding a cube in a two-coloured sphere

Embedding a cube in a two-coloured sphere



Embedding a cube in a two-coloured sphere



For a given two-coloured sphere, is it always possible to embed a cube so that all vertices have the same colour?

We are given a sphere coloured 50% red and 50% blue.

Is it *always* possible to embed a cube in it so that every vertex is blue?

We are given a sphere coloured 50% red and 50% blue.

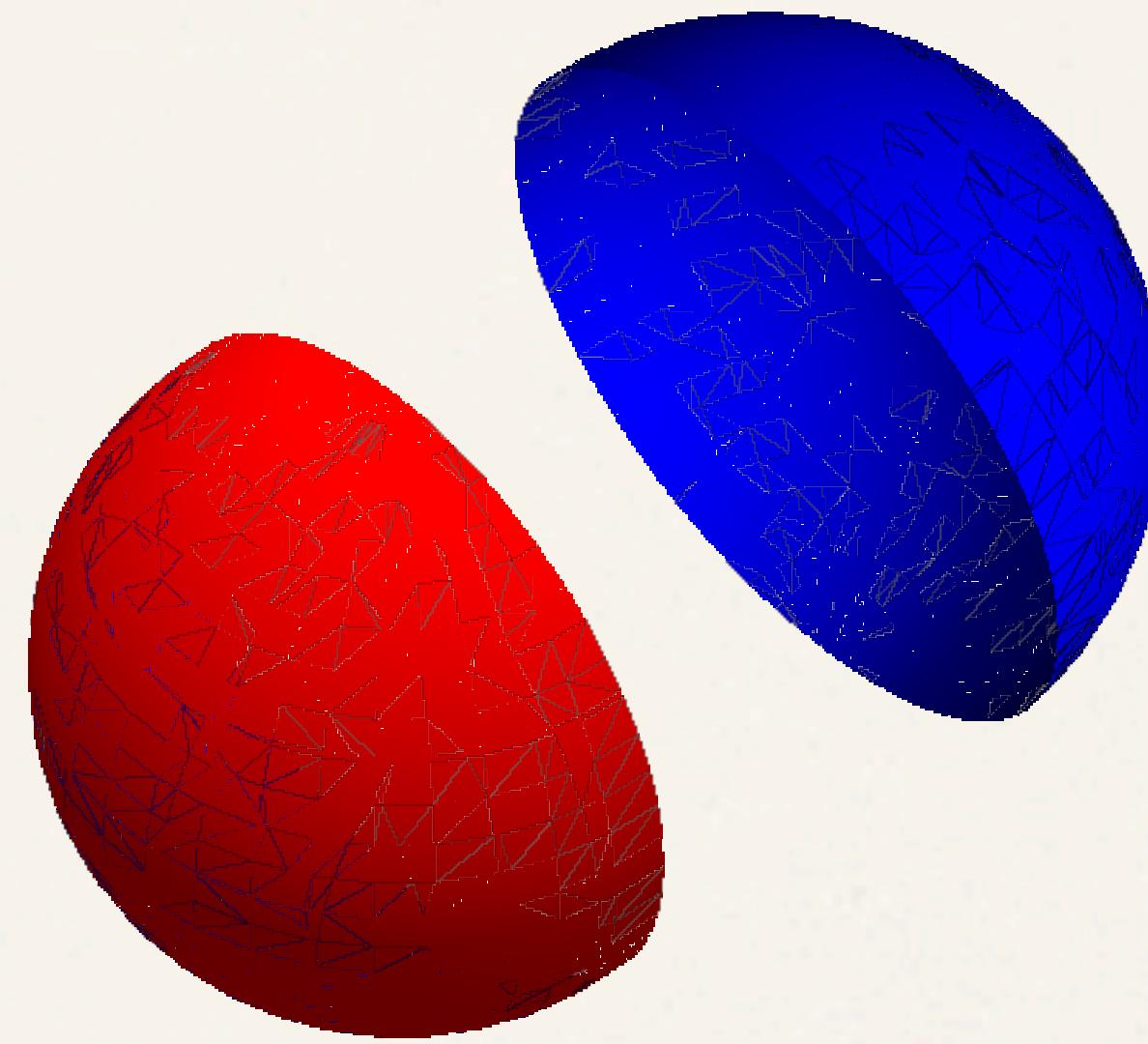
Is it *always* possible to embed a cube in it so that every vertex is blue?

No!

We are given a sphere coloured 50% red and 50% blue.

Is it *always* possible to embed a cube in it so that every vertex is blue?

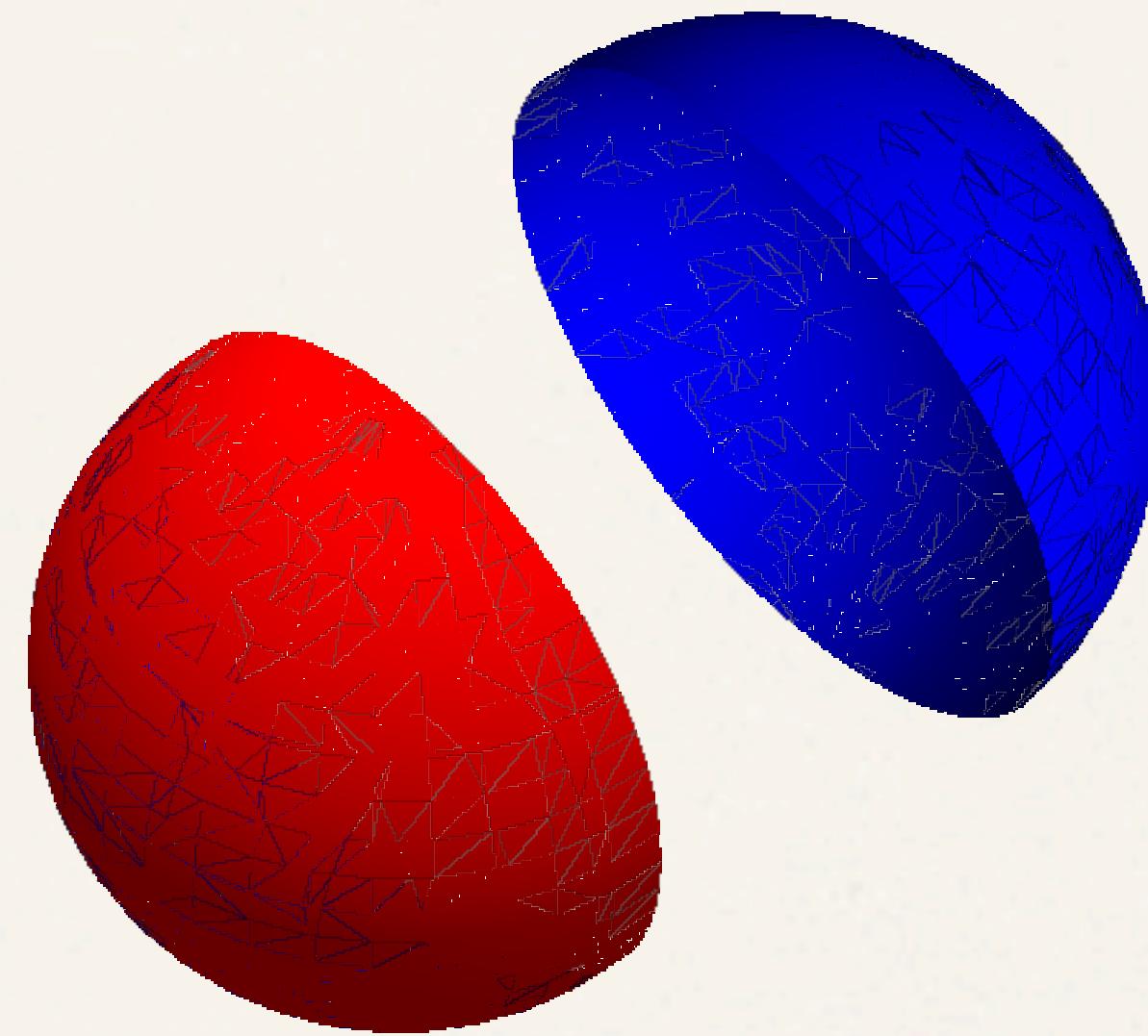
No!



We are given a sphere coloured 50% red and 50% blue.

Is it *always* possible to embed a cube in it so that every vertex is blue?

No!



What if the sphere is coloured 10% red and 90% blue?

Now, is it *always* possible to embed a cube in it so that every vertex is blue?

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A new question

Embed a cube with vertices in a *random orientation* in the sphere.

What are the chances that every vertex is blue?

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Bad events $A_j := j$ th vertex is red

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$$P(A_1) = \dots = P(A_8) = 0.1$$

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$$P(A_1) = \dots = P(A_8) = 0.1$$

$$P(A_1 \cup A_2 \cup \dots \cup A_8) \leq P(A_1) + P(A_2) + \dots + P(A_8) \leq 8 \times 0.1 = 0.8$$

What if the sphere is coloured 10% **red** and 90% **blue**?

Now, is it *always* possible to embed a cube in it so that every vertex is **blue**?

A new question

Embed a cube with vertices in a *random orientation* in the sphere.

What are the chances that every vertex is **blue**?

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$$P(A_1^c \cap A_2^c \cap \dots \cap A_8^c) = 1 - P(A_1 \cup A_2 \cup \dots \cup A_8) \geq 1 - 0.8 = 0.2$$

What if the sphere is coloured 10% red and 90% blue?

Now, is it *always* possible to embed a cube in it so that every vertex is blue?

A new question

Embed a cube with vertices in a *random orientation* in the sphere.

What are the chances that every vertex is blue?

Bad events $A_j := j\text{th vertex is red}$

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It is *always* possible to embed a cube in a sphere coloured 10% red and 90% blue
in such a way that every vertex is coloured blue!