Product space and measure

Product space and measure

- *Individual chance experiments, independent trials*:

Product space and measure

- *Individual chance experiments, independent trials*:
    - Trial 1: $\mathfrak{A} = \{a_j, j \geq 1\}$. Atomic mass function $\{a_j\} \mapsto p_j$.

Product space and measure

- *Individual chance experiments, independent trials*:
  - Trial 1: $\mathfrak{A} = \{a_j, j \geq 1\}$. Atomic mass function $\{a_j\} \mapsto p_j$.
  - Trial 2: $\mathfrak{B} = \{b_k, k \geq 1\}$. Atomic mass function $\{b_k\} \mapsto q_k$.

Product space and measure

- *Individual chance experiments, independent trials*:
  - Trial 1: $\mathfrak{A} = \{a_j,\ j \geq 1\}$. Atomic mass function $\{a_j\} \mapsto p_j$.
  - Trial 2: $\mathfrak{B} = \{b_k,\ k \geq 1\}$. Atomic mass function $\{b_k\} \mapsto q_k$.
- *Compound chance experiment, product space and measure*:

Product space and measure

- *Individual chance experiments, independent trials*:
  - Trial 1: $\mathfrak{A} = \{a_j, j \geq 1\}$. Atomic mass function $\{a_j\} \mapsto p_j$.
  - Trial 2: $\mathfrak{B} = \{b_k, k \geq 1\}$. Atomic mass function $\{b_k\} \mapsto q_k$.
- *Compound chance experiment, product space and measure*:
  - $\Omega = \mathfrak{A} \times \mathfrak{B} = \{(a_j, b_k)\colon j \geq 1, k \geq 1\}$. Atomic measure $\mathbf{P}\{(a_j, b_k)\} := p_j q_k$.

Product space and measure

- *Individual chance experiments, independent trials*:
  - Trial 1: $\mathfrak{A} = \{a_j, j \geq 1\}$. Atomic mass function $\{a_j\} \mapsto p_j$.
  - Trial 2: $\mathfrak{B} = \{b_k, k \geq 1\}$. Atomic mass function $\{b_k\} \mapsto q_k$.
- *Compound chance experiment, product space and measure*:
  - $\Omega = \mathfrak{A} \times \mathfrak{B} = \{(a_j, b_k): j \geq 1, k \geq 1\}$. Atomic measure $\mathbf{P}\{(a_j, b_k)\} := p_j q_k$.
- Suppose that $\mathbb{J}$ and $\mathbb{K}$ are any two subsets of indices. Then:

Product space and measure

- *Individual chance experiments, independent trials*:
  - Trial 1: $\mathfrak{A} = \{a_j,\ j \geq 1\}$. Atomic mass function $\{a_j\} \mapsto p_j$.
  - Trial 2: $\mathfrak{B} = \{b_k,\ k \geq 1\}$. Atomic mass function $\{b_k\} \mapsto q_k$.
- *Compound chance experiment, product space and measure*:
  - $\Omega = \mathfrak{A} \times \mathfrak{B} = \{(a_j, b_k) : j \geq 1, k \geq 1\}$. Atomic measure $\mathbf{P}\{(a_j, b_k)\} := p_j q_k$.
- Suppose that $\mathbb{J}$ and $\mathbb{K}$ are any two subsets of indices. Then:
  - The event $A := \{(a_j, b_k) : j \in \mathbb{J}, k \geq 1\}$ is completely determined by the subset $\mathfrak{S} = \{a_j : j \in \mathbb{J}\}$ of $\mathfrak{A}$;

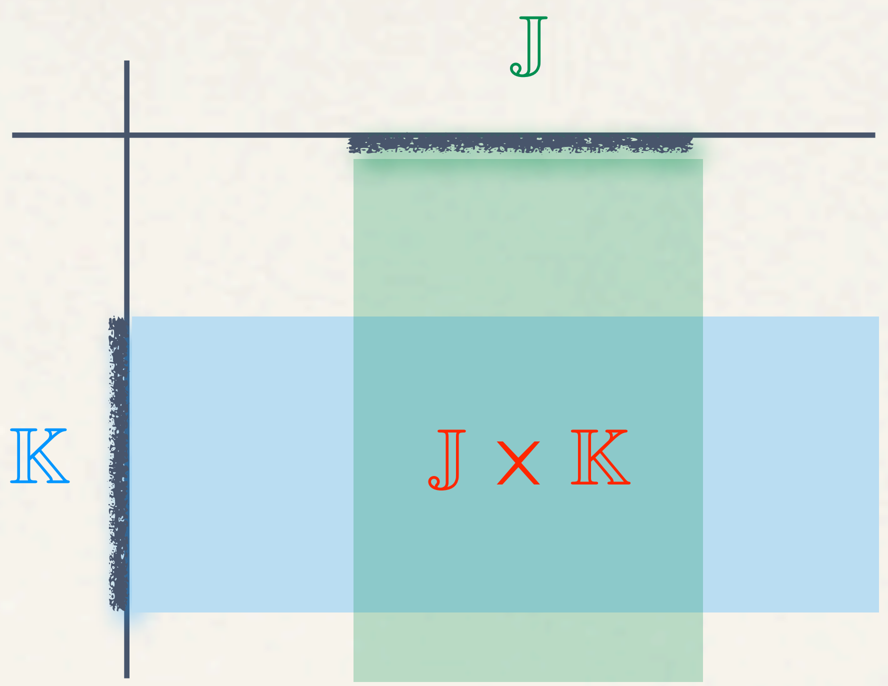
Product space and measure

- *Individual chance experiments, independent trials*:
  - Trial 1: $\mathfrak{A} = \{a_j, j \geq 1\}$. Atomic mass function $\{a_j\} \mapsto p_j$.
  - Trial 2: $\mathfrak{B} = \{b_k, k \geq 1\}$. Atomic mass function $\{b_k\} \mapsto q_k$.
- *Compound chance experiment, product space and measure*:
  - $\Omega = \mathfrak{A} \times \mathfrak{B} = \{(a_j, b_k): j \geq 1, k \geq 1\}$. Atomic measure $\mathbf{P}\{(a_j, b_k)\} := p_j q_k$.
- Suppose that $\mathbb{J}$ and $\mathbb{K}$ are any two subsets of indices. Then:
  - The event $A := \{(a_j, b_k): j \in \mathbb{J}, k \geq 1\}$ is completely determined by the subset $\mathfrak{S} = \{a_j: j \in \mathbb{J}\}$ of $\mathfrak{A}$;
  - The event $B := \{(a_j, b_k): j \geq 1, k \in \mathbb{K}\}$ is completely determined by the subset $\mathfrak{T} = \{b_k: k \in \mathbb{K}\}$ of $\mathfrak{B}$.

Product space and measure

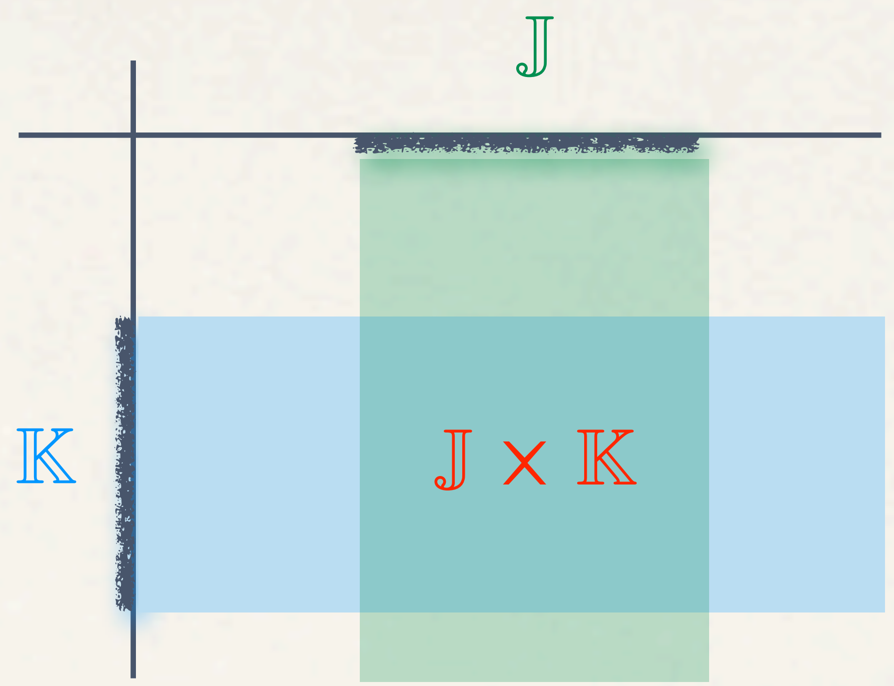
- *Individual chance experiments, independent trials*:
  - Trial 1: $\mathfrak{A} = \{a_j, j \geq 1\}$. Atomic mass function $\{a_j\} \mapsto p_j$.
  - Trial 2: $\mathfrak{B} = \{b_k, k \geq 1\}$. Atomic mass function $\{b_k\} \mapsto q_k$.
- *Compound chance experiment, product space and measure*:
  - $\Omega = \mathfrak{A} \times \mathfrak{B} = \{(a_j, b_k): j \geq 1, k \geq 1\}$. Atomic measure $\mathbf{P}\{(a_j, b_k)\} := p_j q_k$.
- Suppose that $\mathbb{J}$ and $\mathbb{K}$ are any two subsets of indices. Then:
  - The event $A := \{(a_j, b_k): j \in \mathbb{J}, k \geq 1\}$ is completely determined by the subset $\mathfrak{S} = \{a_j: j \in \mathbb{J}\}$ of $\mathfrak{A}$;
  - The event $B := \{(a_j, b_k): j \geq 1, k \in \mathbb{K}\}$ is completely determined by the subset $\mathfrak{T} = \{b_k: k \in \mathbb{K}\}$ of $\mathfrak{B}$.
  - "Rectangular" intersection: $A \cap B = \{(a_j, b_k): j \in \mathbb{J}, k \in \mathbb{K}\} = \mathfrak{S} \times \mathfrak{T}$.

Product space and measure



- *Individual chance experiments, independent trials*:
  - Trial 1: $\mathfrak{A} = \{a_j,\ j \geq 1\}$. Atomic mass function $\{a_j\} \mapsto p_j$.
  - Trial 2: $\mathfrak{B} = \{b_k,\ k \geq 1\}$. Atomic mass function $\{b_k\} \mapsto q_k$.
- *Compound chance experiment, product space and measure*:
  - $\Omega = \mathfrak{A} \times \mathfrak{B} = \{(a_j, b_k) \colon j \geq 1,\ k \geq 1\}$. Atomic measure $\mathbf{P}\{(a_j, b_k)\} := p_j q_k$.
- Suppose that $\mathbb{J}$ and $\mathbb{K}$ are any two subsets of indices. Then:
  - The event $A := \{(a_j, b_k) \colon j \in \mathbb{J},\ k \geq 1\}$ is completely determined by the subset $\mathfrak{S} = \{a_j \colon j \in \mathbb{J}\}$ of $\mathfrak{A}$;
  - The event $B := \{(a_j, b_k) \colon j \geq 1,\ k \in \mathbb{K}\}$ is completely determined by the subset $\mathfrak{T} = \{b_k \colon k \in \mathbb{K}\}$ of $\mathfrak{B}$.
  - "Rectangular" intersection: $A \cap B = \{(a_j, b_k) \colon j \in \mathbb{J},\ k \in \mathbb{K}\} = \mathfrak{S} \times \mathfrak{T}$.

Product space and measure



- *Individual chance experiments, independent trials*:
  - Trial 1: $\mathfrak{A} = \{a_j, j \geq 1\}$. Atomic mass function $\{a_j\} \mapsto p_j$.
  - Trial 2: $\mathfrak{B} = \{b_k, k \geq 1\}$. Atomic mass function $\{b_k\} \mapsto q_k$.

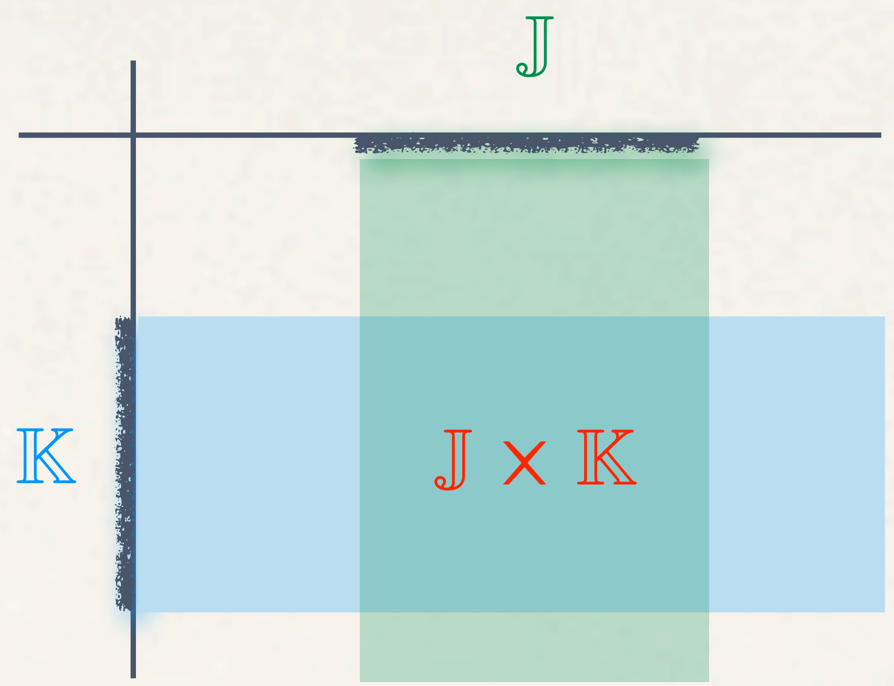
$$p_j \geq 0, \qquad \sum_{j \geq 1} p_j = 1$$

$$q_k \geq 0, \qquad \sum_{k \geq 1} q_k = 1$$

- *Compound chance experiment, product space and measure*:
  - $\Omega = \mathfrak{A} \times \mathfrak{B} = \{(a_j, b_k) \colon j \geq 1, k \geq 1\}$. Atomic measure $\mathbf{P}\{(a_j, b_k)\} := p_j q_k$.

- Suppose that $\mathbb{J}$ and $\mathbb{K}$ are any two subsets of indices. Then:
  - The event $A := \{(a_j, b_k) \colon j \in \mathbb{J}, k \geq 1\}$ is completely determined by the subset $\mathfrak{S} = \{a_j \colon j \in \mathbb{J}\}$ of $\mathfrak{A}$;
  - The event $B := \{(a_j, b_k) \colon j \geq 1, k \in \mathbb{K}\}$ is completely determined by the subset $\mathfrak{T} = \{b_k \colon k \in \mathbb{K}\}$ of $\mathfrak{B}$.
  - "Rectangular" intersection: $A \cap B = \{(a_j, b_k) \colon j \in \mathbb{J}, k \in \mathbb{K}\} = \mathfrak{S} \times \mathfrak{T}$.

Product space and measure



- *Individual chance experiments, independent trials*:
  - Trial 1: $\mathfrak{A} = \{a_j, j \geq 1\}$. Atomic mass function $\{a_j\} \mapsto p_j$.
  - Trial 2: $\mathfrak{B} = \{b_k, k \geq 1\}$. Atomic mass function $\{b_k\} \mapsto q_k$.

$$p_j \geq 0, \qquad \sum_{j \geq 1} p_j = 1$$

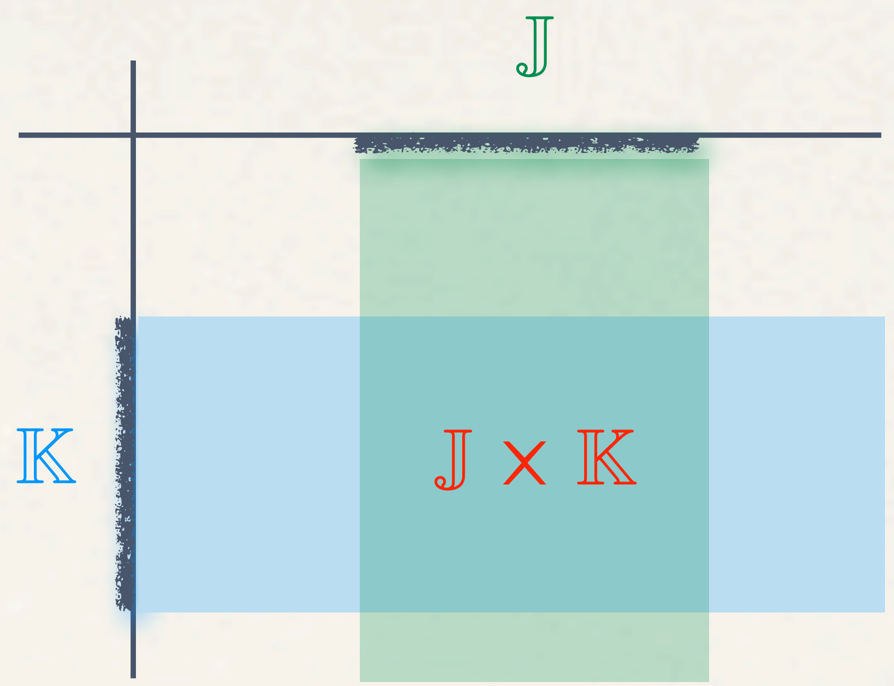
$$q_k \geq 0, \qquad \sum_{k \geq 1} q_k = 1$$

- *Compound chance experiment, product space and measure*:
  - $\Omega = \mathfrak{A} \times \mathfrak{B} = \{(a_j, b_k): j \geq 1, k \geq 1\}$. Atomic measure $\mathbf{P}\{(a_j, b_k)\} := p_j q_k$.

Is the product measure suitably normalised?

- Suppose that $\mathbb{J}$ and $\mathbb{K}$ are any two subsets of indices. Then:
  - The event $A := \{(a_j, b_k): j \in \mathbb{J}, k \geq 1\}$ is completely determined by the subset $\mathfrak{S} = \{a_j: j \in \mathbb{J}\}$ of $\mathfrak{A}$;
  - The event $B := \{(a_j, b_k): j \geq 1, k \in \mathbb{K}\}$ is completely determined by the subset $\mathfrak{T} = \{b_k: k \in \mathbb{K}\}$ of $\mathfrak{B}$.
  - “Rectangular” intersection: $A \cap B = \{(a_j, b_k): j \in \mathbb{J}, k \in \mathbb{K}\} = \mathfrak{S} \times \mathfrak{T}$.

Product space and measure



- *Individual chance experiments, independent trials*:
  - Trial 1: $\mathfrak{A} = \{a_j, j \geq 1\}$. Atomic mass function $\{a_j\} \mapsto p_j$.
  - Trial 2: $\mathfrak{B} = \{b_k, k \geq 1\}$. Atomic mass function $\{b_k\} \mapsto q_k$.

$$p_j \geq 0, \qquad \sum_{j\geq 1} p_j = 1$$

$$q_k \geq 0, \qquad \sum_{k\geq 1} q_k = 1$$

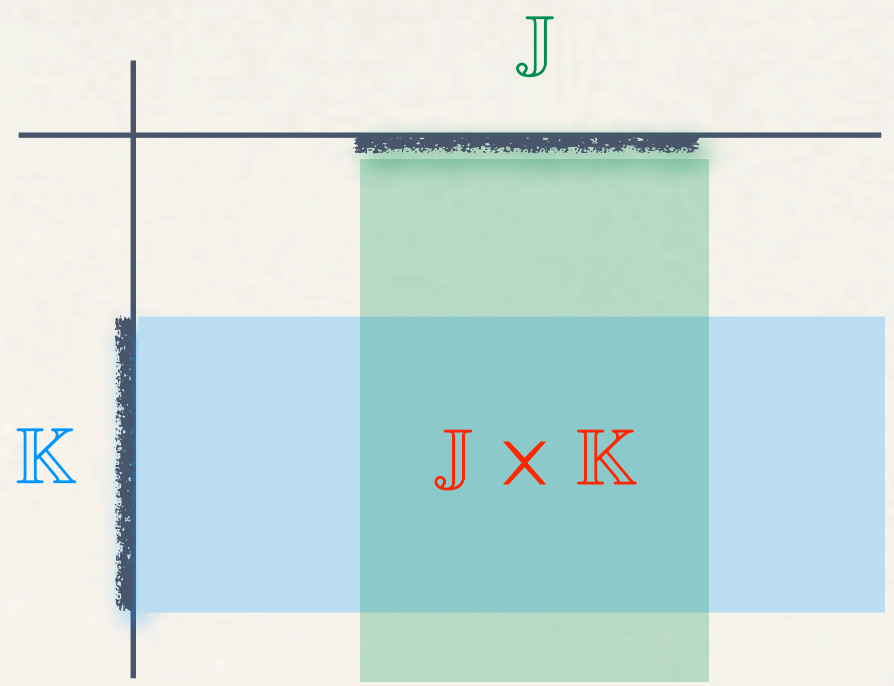
- *Compound chance experiment, product space and measure*:
  - $\Omega = \mathfrak{A} \times \mathfrak{B} = \{(a_j, b_k)\colon j \geq 1, k \geq 1\}$. Atomic measure $\mathbf{P}\{(a_j, b_k)\} := p_j q_k$.

Is the product measure suitably normalised?

$$\sum_{j\geq 1}\sum_{k\geq 1} p_j q_k = \Big(\sum_{j\geq 1} p_j\Big)\Big(\sum_{k\geq 1} q_k\Big) = 1$$

- Suppose that $\mathbb{J}$ and $\mathbb{K}$ are any two subsets of indices. Then:
  - The event $A := \{(a_j, b_k)\colon j \in \mathbb{J}, k \geq 1\}$ is completely determined by the subset $\mathfrak{S} = \{a_j\colon j \in \mathbb{J}\}$ of $\mathfrak{A}$;
  - The event $B := \{(a_j, b_k)\colon j \geq 1, k \in \mathbb{K}\}$ is completely determined by the subset $\mathfrak{T} = \{b_k\colon k \in \mathbb{K}\}$ of $\mathfrak{B}$.
  - "Rectangular" intersection: $A \cap B = \{(a_j, b_k)\colon j \in \mathbb{J}, k \in \mathbb{K}\} = \mathfrak{S} \times \mathfrak{T}$.

Product space and measure



- *Individual chance experiments, independent trials*:
  - Trial 1: $\mathfrak{A} = \{a_j, j \geq 1\}$. Atomic mass function $\{a_j\} \mapsto p_j$.
  - Trial 2: $\mathfrak{B} = \{b_k, k \geq 1\}$. Atomic mass function $\{b_k\} \mapsto q_k$.

$$p_j \geq 0, \qquad \sum_{j \geq 1} p_j = 1$$

$$q_k \geq 0, \qquad \sum_{k \geq 1} q_k = 1$$

- *Compound chance experiment, product space and measure*:
  - $\Omega = \mathfrak{A} \times \mathfrak{B} = \{(a_j, b_k): j \geq 1, k \geq 1\}$. Atomic measure $\mathbf{P}\{(a_j, b_k)\} := p_j q_k$.

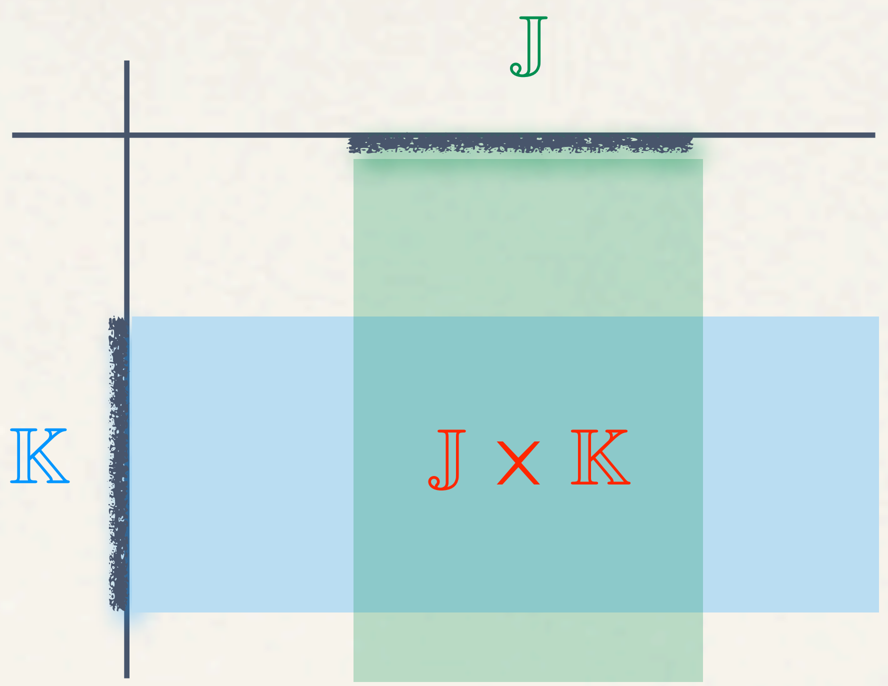
Is the product measure suitably normalised?

$$\sum_{j \geq 1} \sum_{k \geq 1} p_j q_k = \Bigl(\sum_{j \geq 1} p_j\Bigr)\Bigl(\sum_{k \geq 1} q_k\Bigr) = 1$$

- Suppose that $\mathbb{J}$ and $\mathbb{K}$ are any two subsets of indices. Then:
  - The event $A := \{(a_j, b_k): j \in \mathbb{J}, k \geq 1\}$ is completely determined by the subset $\mathfrak{S} = \{a_j: j \in \mathbb{J}\}$ of $\mathfrak{A}$;
  - The event $B := \{(a_j, b_k): j \geq 1, k \in \mathbb{K}\}$ is completely determined by the subset $\mathfrak{T} = \{b_k: k \in \mathbb{K}\}$ of $\mathfrak{B}$.
  - "Rectangular" intersection: $A \cap B = \{(a_j, b_k): j \in \mathbb{J}, k \in \mathbb{K}\} = \mathfrak{S} \times \mathfrak{T}$.

$$\mathbf{P}(A) = \sum_{j \in \mathbb{J}} \sum_{k \geq 1} p_j q_k = \Bigl(\sum_{j \in \mathbb{J}} p_j\Bigr)\Bigl(\sum_{k \geq 1} q_k\Bigr) = \sum_{j \in \mathbb{J}} p_j$$

Product space and measure



- *Individual chance experiments, independent trials*:
  - Trial 1: $\mathfrak{A} = \{a_j, j \geq 1\}$. Atomic mass function $\{a_j\} \mapsto p_j$.
  - Trial 2: $\mathfrak{B} = \{b_k, k \geq 1\}$. Atomic mass function $\{b_k\} \mapsto q_k$.

$$p_j \geq 0, \qquad \sum_{j \geq 1} p_j = 1$$

$$q_k \geq 0, \qquad \sum_{k \geq 1} q_k = 1$$

- *Compound chance experiment, product space and measure*:
  - $\Omega = \mathfrak{A} \times \mathfrak{B} = \{(a_j, b_k): j \geq 1, k \geq 1\}$. Atomic measure $\mathbf{P}\{(a_j, b_k)\} := p_j q_k$.

Is the product measure suitably normalised?

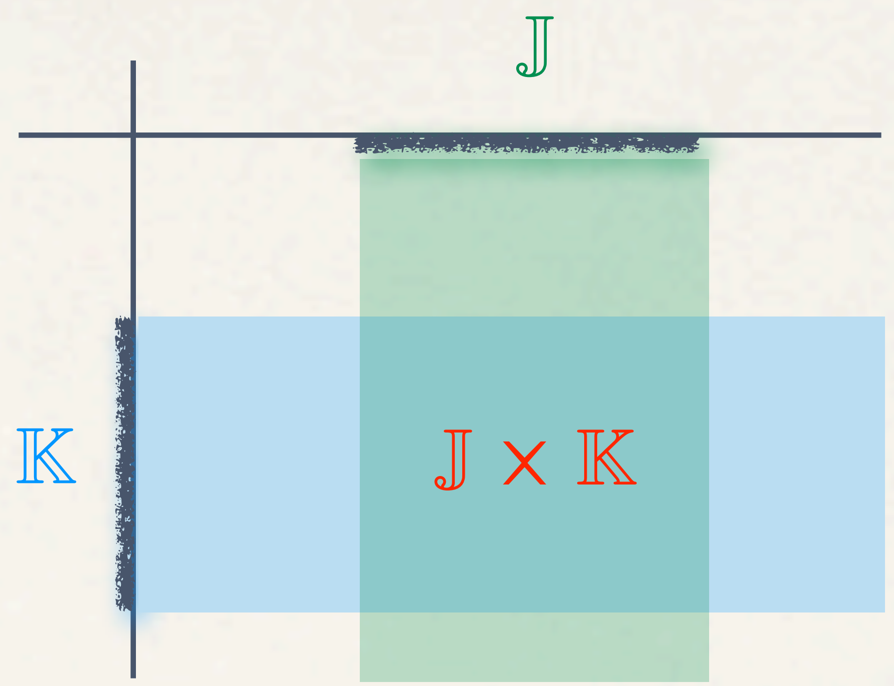
$$\sum_{j \geq 1} \sum_{k \geq 1} p_j q_k = \Big(\sum_{j \geq 1} p_j\Big)\Big(\sum_{k \geq 1} q_k\Big) = 1$$

- Suppose that $\mathbb{J}$ and $\mathbb{K}$ are any two subsets of indices. Then:
  - The event $A := \{(a_j, b_k): j \in \mathbb{J}, k \geq 1\}$ is completely determined by the subset $\mathfrak{S} = \{a_j: j \in \mathbb{J}\}$ of $\mathfrak{A}$;
  - The event $B := \{(a_j, b_k): j \geq 1, k \in \mathbb{K}\}$ is completely determined by the subset $\mathfrak{T} = \{b_k: k \in \mathbb{K}\}$ of $\mathfrak{B}$.
  - "Rectangular" intersection: $A \cap B = \{(a_j, b_k): j \in \mathbb{J}, k \in \mathbb{K}\} = \mathfrak{S} \times \mathfrak{T}$.

$$\mathbf{P}(A) = \sum_{j \in \mathbb{J}} \sum_{k \geq 1} p_j q_k = \Big(\sum_{j \in \mathbb{J}} p_j\Big)\Big(\sum_{k \geq 1} q_k\Big) = \sum_{j \in \mathbb{J}} p_j$$

$$\mathbf{P}(B) = \sum_{j \geq 1} \sum_{k \in \mathbb{K}} p_j q_k = \Big(\sum_{j \geq 1} p_j\Big)\Big(\sum_{k \in \mathbb{K}} q_k\Big) = \sum_{k \in \mathbb{K}} q_k$$

Product space and measure



- *Individual chance experiments, independent trials*:
  - Trial 1: $\mathfrak{A} = \{a_j, j \geq 1\}$. Atomic mass function $\{a_j\} \mapsto p_j$.
  - Trial 2: $\mathfrak{B} = \{b_k, k \geq 1\}$. Atomic mass function $\{b_k\} \mapsto q_k$.

$$p_j \geq 0, \qquad \sum_{j\geq 1} p_j = 1$$

$$q_k \geq 0, \qquad \sum_{k\geq 1} q_k = 1$$

- *Compound chance experiment, product space and measure*:
  - $\Omega = \mathfrak{A} \times \mathfrak{B} = \{(a_j, b_k) \colon j \geq 1, k \geq 1\}$. Atomic measure $\mathbf{P}\{(a_j, b_k)\} := p_j q_k$.

Is the product measure suitably normalised?

$$\sum_{j\geq 1}\sum_{k\geq 1} p_j q_k = \Bigl(\sum_{j\geq 1} p_j\Bigr)\Bigl(\sum_{k\geq 1} q_k\Bigr) = 1$$

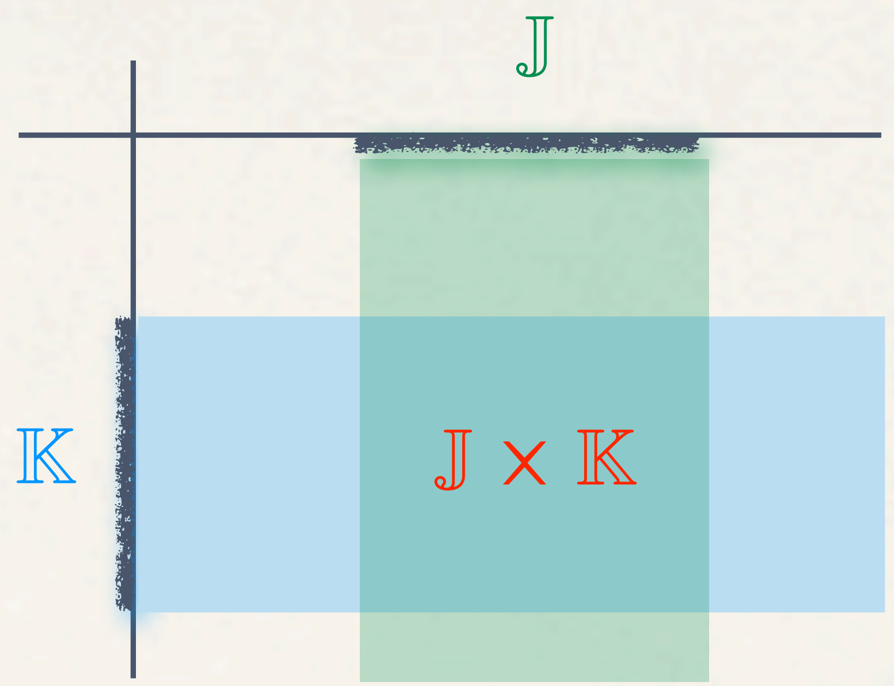
- Suppose that $\mathbb{J}$ and $\mathbb{K}$ are any two subsets of indices. Then:
  - The event $A := \{(a_j, b_k) \colon j \in \mathbb{J}, k \geq 1\}$ is completely determined by the subset $\mathfrak{S} = \{a_j \colon j \in \mathbb{J}\}$ of $\mathfrak{A}$;
  - The event $B := \{(a_j, b_k) \colon j \geq 1, k \in \mathbb{K}\}$ is completely determined by the subset $\mathfrak{T} = \{b_k \colon k \in \mathbb{K}\}$ of $\mathfrak{B}$.
  - "Rectangular" intersection: $A \cap B = \{(a_j, b_k) \colon j \in \mathbb{J}, k \in \mathbb{K}\} = \mathfrak{S} \times \mathfrak{T}$.

$$\mathbf{P}(A) = \sum_{j\in\mathbb{J}}\sum_{k\geq 1} p_j q_k = \Bigl(\sum_{j\in\mathbb{J}} p_j\Bigr)\Bigl(\sum_{k\geq 1} q_k\Bigr) = \sum_{j\in\mathbb{J}} p_j$$

$$\mathbf{P}(B) = \sum_{j\geq 1}\sum_{k\in\mathbb{K}} p_j q_k = \Bigl(\sum_{j\geq 1} p_j\Bigr)\Bigl(\sum_{k\in\mathbb{K}} q_k\Bigr) = \sum_{k\in\mathbb{K}} q_k$$

$$\mathbf{P}(A\cap B) = \sum_{j\in\mathbb{J}}\sum_{k\in\mathbb{K}} p_j q_k = \Bigl(\sum_{j\in\mathbb{J}} p_j\Bigr)\Bigl(\sum_{k\in\mathbb{K}} q_k\Bigr) = \mathbf{P}(A)\,\mathbf{P}(B)$$

Product space and measure



- *Individual chance experiments, independent trials*:
  - Trial 1: $\mathfrak{A} = \{a_j, j \geq 1\}$. Atomic mass function $\{a_j\} \mapsto p_j$.
  - Trial 2: $\mathfrak{B} = \{b_k, k \geq 1\}$. Atomic mass function $\{b_k\} \mapsto q_k$.

$$p_j \geq 0, \qquad \sum_{j\geq 1} p_j = 1$$

$$q_k \geq 0, \qquad \sum_{k\geq 1} q_k = 1$$

- *Compound chance experiment, product space and measure*:
  - $\Omega = \mathfrak{A} \times \mathfrak{B} = \{(a_j, b_k): j \geq 1, k \geq 1\}$. Atomic measure $\mathbf{P}\{(a_j, b_k)\} := p_j q_k$.

Is the product measure suitably normalised?

$$\sum_{j\geq 1}\sum_{k\geq 1} p_j q_k = \Bigl(\sum_{j\geq 1} p_j\Bigr)\Bigl(\sum_{k\geq 1} q_k\Bigr) = 1$$

- Suppose that $\mathbb{J}$ and $\mathbb{K}$ are any two subsets of indices. Then:
  - The event $A := \{(a_j, b_k): j \in \mathbb{J}, k \geq 1\}$ is completely determined by the subset $\mathfrak{S} = \{a_j: j \in \mathbb{J}\}$ of $\mathfrak{A}$;
  - The event $B := \{(a_j, b_k): j \geq 1, k \in \mathbb{K}\}$ is completely determined by the subset $\mathfrak{T} = \{b_k: k \in \mathbb{K}\}$ of $\mathfrak{B}$.
  - "Rectangular" intersection: $A \cap B = \{(a_j, b_k): j \in \mathbb{J}, k \in \mathbb{K}\} = \mathfrak{S} \times \mathfrak{T}$.

$$\mathbf{P}(A) = \sum_{j\in\mathbb{J}}\sum_{k\geq 1} p_j q_k = \Bigl(\sum_{j\in\mathbb{J}} p_j\Bigr)\Bigl(\sum_{k\geq 1} q_k\Bigr) = \sum_{j\in\mathbb{J}} p_j$$

$$\mathbf{P}(B) = \sum_{j\geq 1}\sum_{k\in\mathbb{K}} p_j q_k = \Bigl(\sum_{j\geq 1} p_j\Bigr)\Bigl(\sum_{k\in\mathbb{K}} q_k\Bigr) = \sum_{k\in\mathbb{K}} q_k$$

$$\mathbf{P}(A\cap B) = \sum_{j\in\mathbb{J}}\sum_{k\in\mathbb{K}} p_j q_k = \Bigl(\sum_{j\in\mathbb{J}} p_j\Bigr)\Bigl(\sum_{k\in\mathbb{K}} q_k\Bigr) = \mathbf{P}(A)\,\mathbf{P}(B)$$

Any such events A and B determined by separate trials are independent.