Two-Tailed Test of Population Proportion

The null hypothesis of the **two-tailed test about population proportion** can be expressed as follows:

$$p = p_0$$

where p_0 is a hypothesized value of the true population proportion p.

Let us define the test statistic z in terms of the sample proportion and the sample size:

$$z = \frac{\bar{p} - p_0}{\sqrt{p_0(1 - p_0)/n}}$$

Then the null hypothesis of the two-tailed test is to be *rejected* if $z \leq -z_{\alpha/2}$ or $z \geq z_{\alpha/2}$, where $z_{\alpha/2}$ is the $100(1 - \alpha)$ percentile of the standard normal distribution.

Problem

Suppose a coin toss turns up 12 heads out of 20 trials. At .05 significance level, can one reject the null hypothesis that the coin toss is fair?

Solution

The null hypothesis is that $p = 0.5$. We begin with computing the test statistic.

```
> pbar = 12/20            # sample proportion
> p0 = .5                 # hypothesized value
> n = 20                  # sample size
> z = (pbar−p0)/sqrt(p0*(1−p0)/n)
> z                       # test statistic
[1] 0.89443
```

We then compute the critical values at .05 significance level.

```
> alpha = .05
> z.half.alpha = qnorm(1−alpha/2)
> c(−z.half.alpha, z.half.alpha)
[1] −1.9600  1.9600
```

Answer

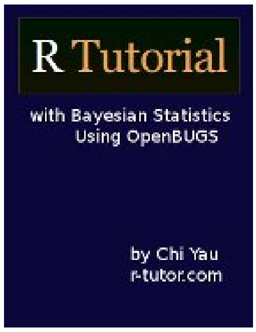
The test statistic 0.89443 lies between the critical values -1.9600 and 1.9600. Hence, at .05 significance level, we do *not* reject the null hypothesis that the coin toss is fair.

Alternative Solution 1

Instead of using the critical value, we apply the pnorm function to compute the two-tailed **p-value** of the test statistic. It doubles the *upper* tail p-value as the sample proportion is *greater* than the hypothesized value. Since it turns out to be greater than the .05 significance level, we do not reject the null hypothesis that $p = 0.5$.

```
> pval = 2 * pnorm(z, lower.tail=FALSE)  # upper tail
> pval                    # two−tailed p−value
[1] 0.37109
```

Alternative Solution 2



We apply the prop.test function to compute the p-value directly. The Yates continuity correction is disabled for pedagogical reasons.

```
> prop.test(12, 20, p=0.5, correct=FALSE)

        1-sample proportions test without continuity
        correction

data:  12 out of 20, null probability 0.5
X-squared = 0.8, df = 1, p-value = 0.3711
alternative hypothesis: true p is not equal to 0.5
95 percent confidence interval:
  0.38658 0.78119
sample estimates:
  p
0.6
```

‹ Upper Tail Test of Population Proportion up Type II Error ›

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