# Maxwell–Boltzmann statistics

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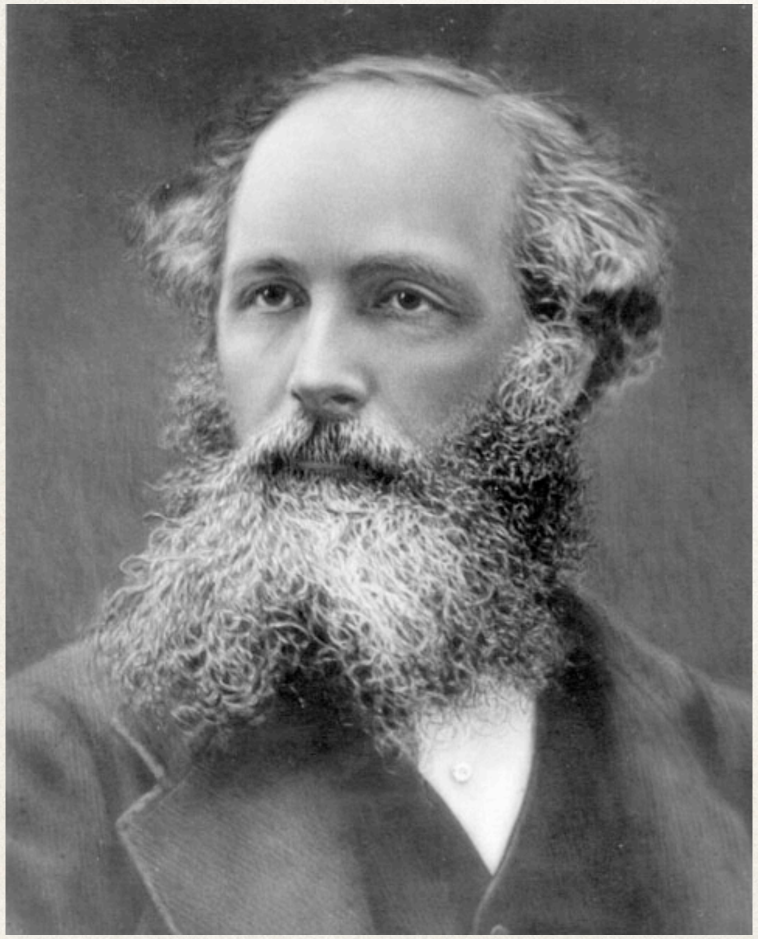
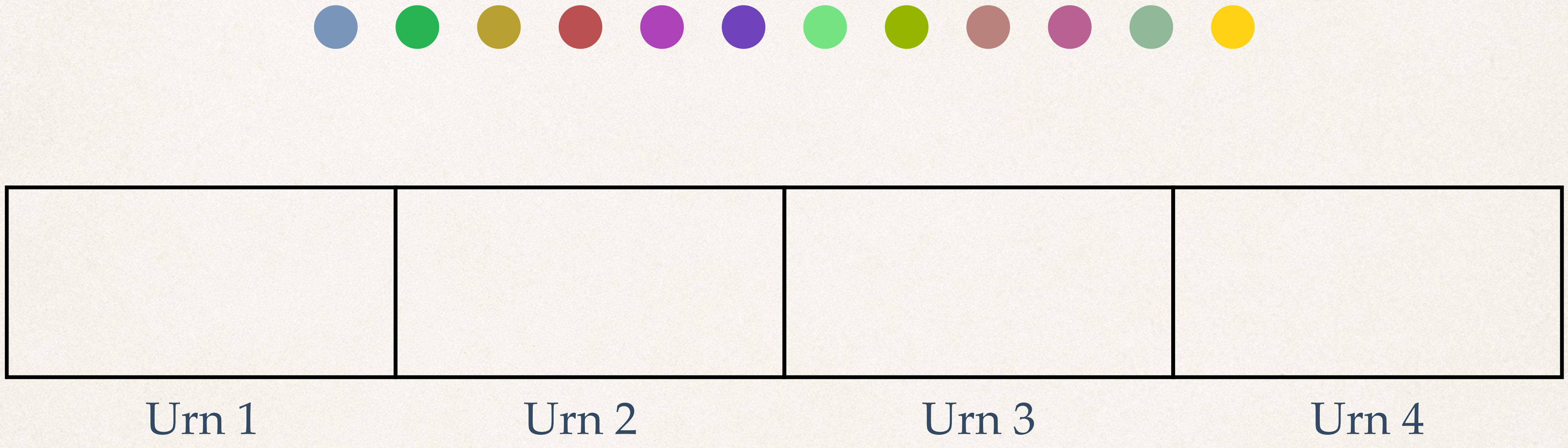


# A covey of binomial coefficients

In how many ways can we obtain a given occupancy configuration (4, 2, 0, 6)?

$n = 12$

$r = 4$



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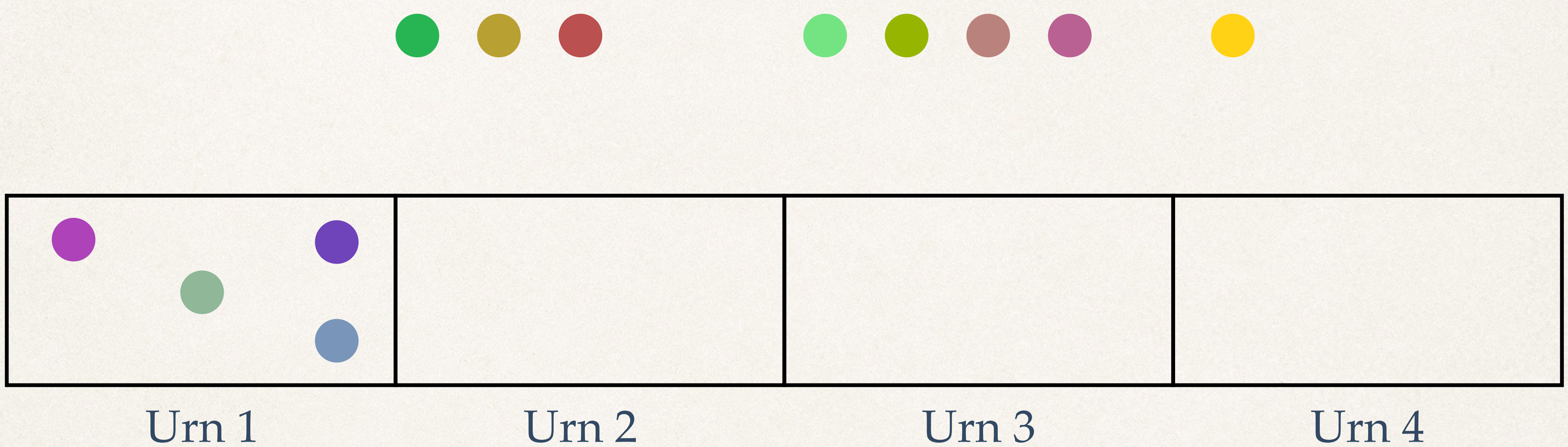


$$\binom{12}{4}$$

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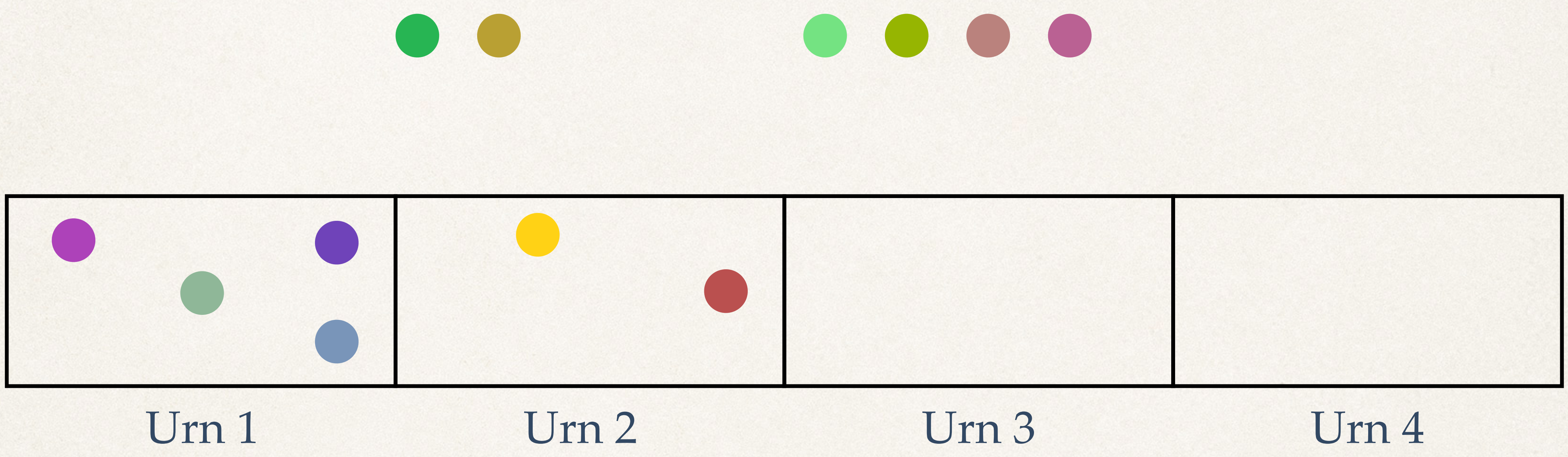


$$\binom{12}{4}\binom{12-4}{2}$$

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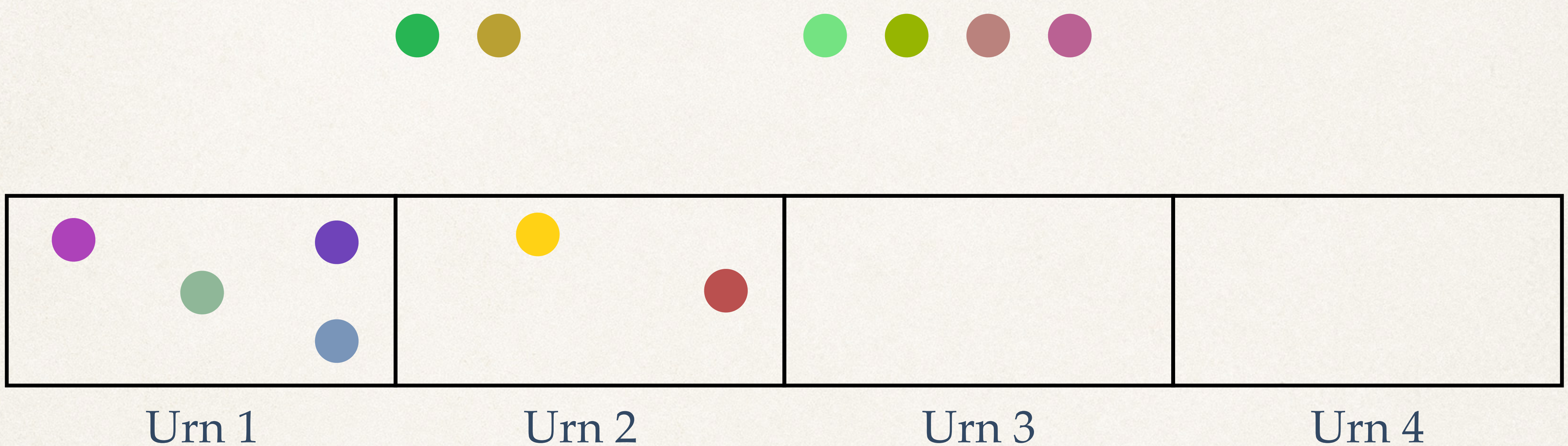


$$\binom{12}{4}\binom{12-4}{2}\binom{12-4-2}{0}$$

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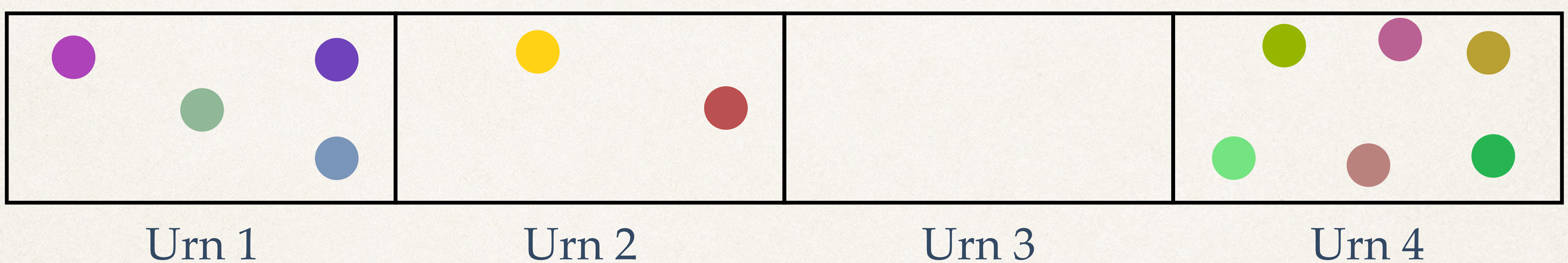


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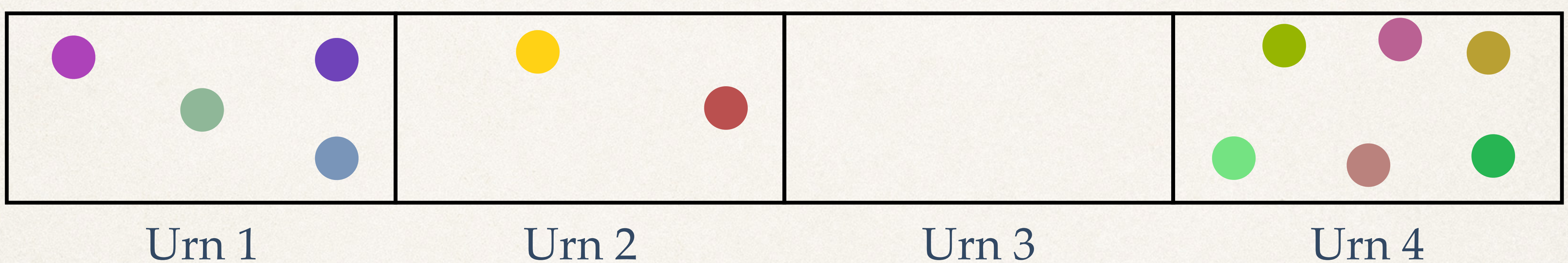


$$\binom{12}{4}\binom{12-4}{2}\binom{12-4-2}{0}\binom{12-4-2-0}{6} = \frac{(12)!}{4!\,8!}\cdot\frac{8!}{2!\,6!}\cdot\frac{6!}{0!\,6!}\cdot\frac{6!}{6!\,0!}$$

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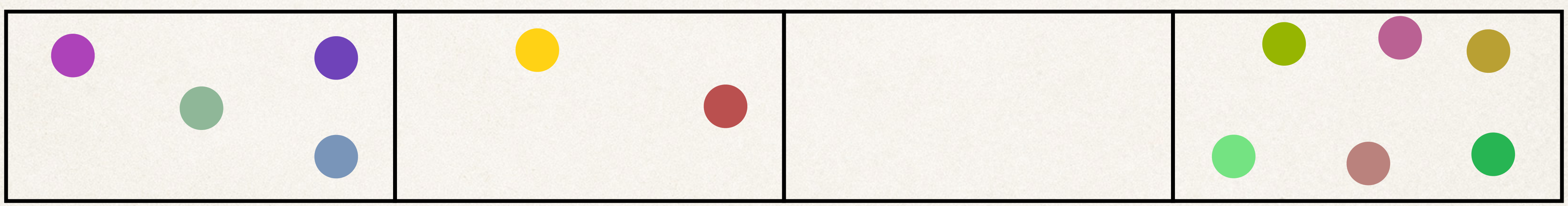

Urn 1 Urn 2 Urn 3 Urn 4

$$\binom{12}{4}\binom{12-4}{2}\binom{12-4-2}{0}\binom{12-4-2-0}{6} = \frac{(12)!}{4!\,\cancel{8!}} \cdot \frac{\cancel{8!}}{2!\,\cancel{6!}} \cdot \frac{\cancel{6!}}{0!\,\cancel{6!}} \cdot \frac{\cancel{6!}}{6!\,0!} = \frac{(12)!}{4!\,2!\,0!\,6!}$$

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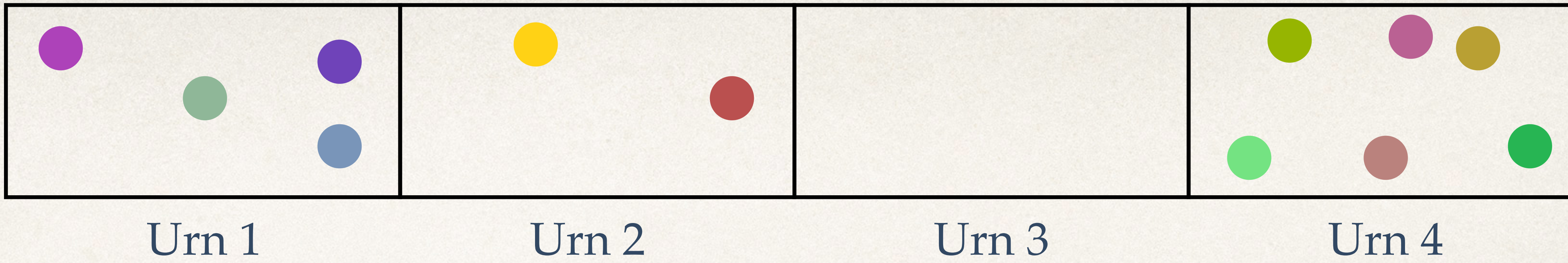
$(4, 2, 0, 6)$




- Distribute $n$ balls in $r$ urns:
  - In how many ways can we obtain a given occupancy configuration $(k_1, k_2, \ldots, k_r)$?
  - How many ways can the balls be distributed in the urns in total?

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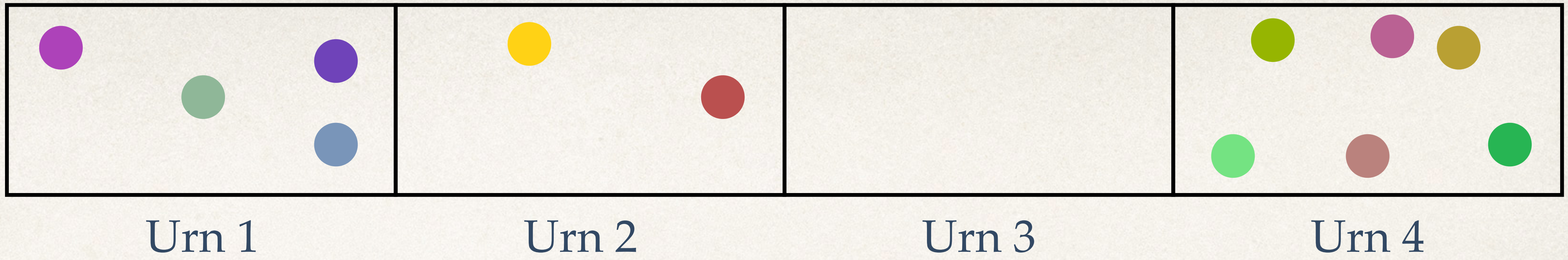


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$$\binom{n}{k_1}\binom{n-k_1}{k_2}\binom{n-k_1-k_2}{k_3}\cdots\binom{n-k_1-\cdots-k_{r-1}}{k_r}$$

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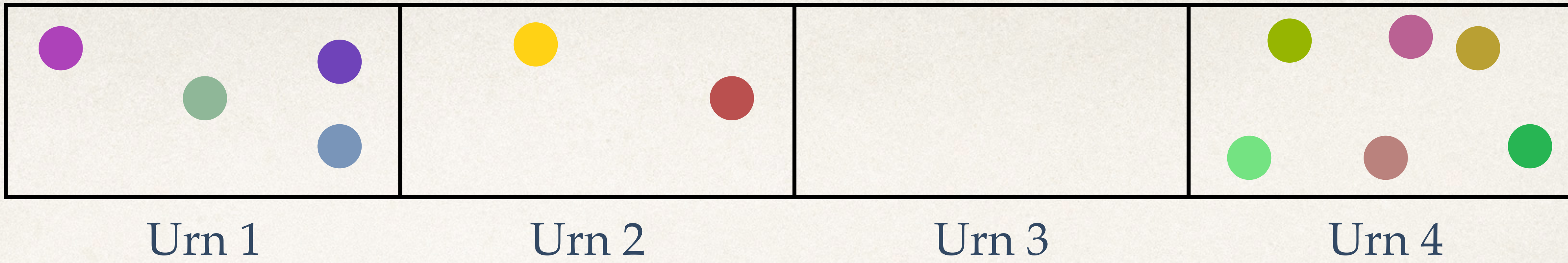


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$$= \frac{n!}{k_1!\cancel{(n-k_1)!}} \cdot \frac{\cancel{(n-k_1)!}}{k_2!\cancel{(n-k_1-k_2)!}} \cdot \frac{\cancel{(n-k_1-k_2)!}}{k_3!\cancel{(n-k_1-k_2-k_3)!}} \cdots \frac{\cancel{(n-k_1-\cdots-k_{r-1})!}}{k_r!(n-k_1-\cdots-k_r)!}$$

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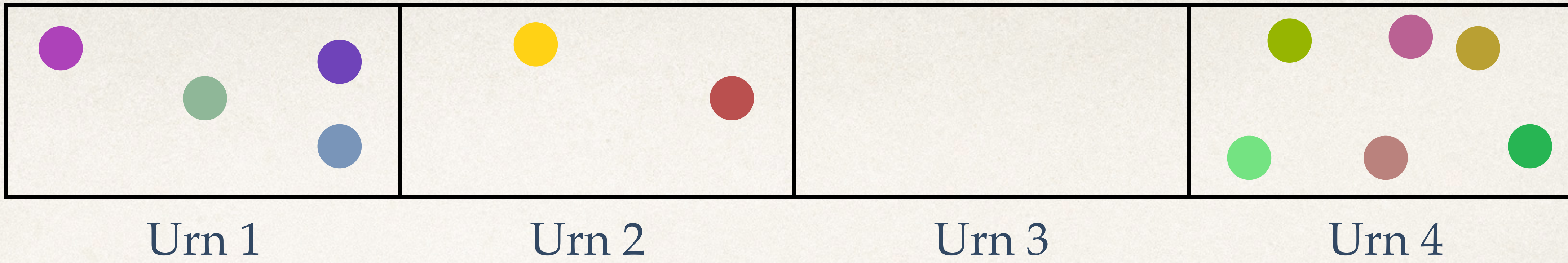


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$$= \frac{n!}{k_1!k_2!\cdots k_r!}$$ (recall $k_1 + \cdots + k_r = n$ and $0! = 1$)

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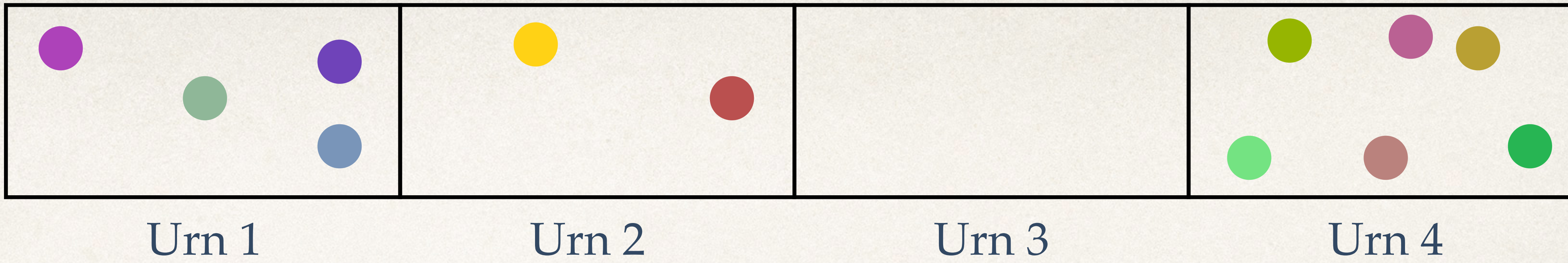


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  - How many ways can the balls be distributed in the urns in total?

Number of ordered samples of size $n$ (with replacement) from a population of $r$ urns: $r^n$

# Maxwell–Boltzmann statistics

Given a random deployment of $n$ balls into $r$ urns, the probability of obtaining a given occupancy configuration $(k_1, k_2, \ldots, k_r)$ is given by

$$P(k_1, k_2, \ldots, k_r) = \frac{n!}{k_1!k_2!\cdots k_r!}\bigg/ r^n \qquad \left(\begin{array}{c} k_1, k_2, \ldots, k_r \geq 0 \\ k_1 + k_2 + \cdots + k_r = n \end{array}\right).$$

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An unexpected poser: no known physical particles follow this "natural" law!