

Additivity: conditioning on the first trial

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Given: $\mathbf{P}(F_k) = p_k \qquad (k = 2, 3, \dots, 12)$

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 $\mathbf{P}(W \mid F_7) = \mathbf{P}(W \mid F_{11}) = 1$

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$$\mathbf{P}(W) = \sum_{k=2}^{12} \mathbf{P}(W \mid F_k) \mathbf{P}(F_k) = p_7 + p_{11} + \sum_{k \in \{4, 5, 6, 8, 9, 10\}} \mathbf{P}(W \mid F_k) p_k$$

Additivity, once more: conditioning on the number of trials

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$$\mathbf{P}(W \mid F_k) = \mathbf{P}\left(\bigcup_{n \geq 1} W_n \mid F_k\right) \quad \text{(decomposition: you win by winning on *some* throw n)}$$

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$$\begin{aligned} \mathbf{P}(W \mid F_k) &= \mathbf{P}\left(\bigcup_{n \geq 1} W_n \mid F_k\right) \\ &= \sum_{n \geq 1} \mathbf{P}(W_n \mid F_k) \end{aligned}$$

(decomposition: you win by winning on *some* throw n)

(additivity: the events W_n are mutually exclusive)

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(additivity: the events W_n are mutually exclusive)

(domain knowledge: you cannot win on the first throw unless you throw 7 or 11)

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$$\begin{aligned} \mathbf{P}(W \mid F_k) &= \mathbf{P}\left(\bigcup_{n \geq 1} W_n \mid F_k\right) && \text{(decomposition: you win by winning on \textit{some} throw } n\text{)} \\ &= \sum_{n \geq 1} \mathbf{P}(W_n \mid F_k) && \text{(additivity: the events } W_n \text{ are mutually exclusive)} \\ &= \sum_{n \geq 2} \mathbf{P}(W_n \mid F_k) && \text{(domain knowledge: you cannot win on the first throw unless you throw 7 or 11)} \end{aligned}$$

Evaluate $\mathbf{P}(W_n \mid F_k)$ when $k = 4, 5, 6, 8, 9$, or 10 .