Problem 1

Assume you are the production manager for a lecture of an online course. For the production, n tasks have to be executed. A task j requires a working time of p_j hours to be completed. You have m employees at your disposal that can each, due to his or her qualifications, work on a subset of the tasks. Denote by S_i the set of jobs that employee i can work on.
As the production manager you want to create a work allocation plan that ensures that all tasks are completed. However, this allocation should also be fair. Consider the maximum number of working hours of each employee. You would like to minimize this quantity.
Model this problem as a linear program.

Problem 2

Let (1) be a linear program in inequality standard form, i.e.

$$\max\{c^{\mathrm{T}}x \mid Ax \leq b, x \in \mathbb{R}^n\} \tag{1}$$

where $A \in \mathbb{R}^{m\times n}$, $b \in \mathbb{R}^m$, and $c \in \mathbb{R}^n$.
Prove that there is an equivalent linear program (2) of the form

$$\min\{\tilde{c}^{\mathrm{T}}x \mid \tilde{A}x = \tilde{b}, x \geq 0, x \in \mathbb{R}^{\tilde{n}}\} \tag{2}$$

where $\tilde{A} \in \mathbb{R}^{\tilde{m}\times\tilde{n}}$, $\tilde{b} \in \mathbb{R}^{\tilde{m}}$, and $\tilde{c} \in \mathbb{R}^{\tilde{n}}$ are such that every optimal point of (1) corresponds to an optimal point of (2) and vice versa.
Linear programs of the form in (2) are said to be in *equality standard form.*