The Material Acceleration

Consider the vector differential equation

$$\frac{d\boldsymbol{r}}{dt} = \boldsymbol{u}(t, \boldsymbol{r}(t)),$$

where

$$\boldsymbol{r} = x_1\boldsymbol{i} + x_2\boldsymbol{j} + x_3\boldsymbol{k}, \qquad \boldsymbol{u} = u_1\boldsymbol{i} + u_2\boldsymbol{j} + u_3\boldsymbol{k}.$$

(a) Write down the differential equations for dx_1/dt, dx_2/dt and dx_3/dt;

(b) Use the chain rule to determine formulas for d^2x_1/dt^2, d^2x_2/dt^2 and d^2x_3/dt^2;

(c) Write your solution for $d^2\boldsymbol{r}/dt^2$ as a vector equation using the ∇ differential operator.