Data Mining
Association Rules: Advanced Concepts and Algorithms

Lecture Notes for Chapter 7

Introduction to Data Mining

by

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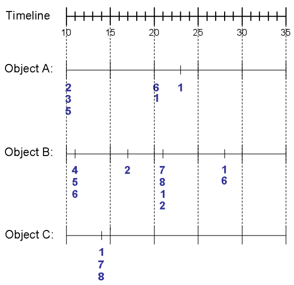
Frequent pattern mining

- It all started with frequent itemsets in supermarket transactions.
- Suppose people have a *bonuscard,* now we can track transactions of a customer through time.
- The pattern becomes more complex: from *itemset* to *sequence of itemsets*.
- More complex patterns: trees (e.g. XML data) and graphs (browsing patterns, chemical structures, etc).

Sequence Data

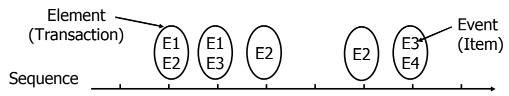
Sequence Database:

Object	Timestamp	Events
A	10	2, 3, 5
A	20	6, 1
A	23	1
B	11	4, 5, 6
B	17	2
B	21	7, 8, 1, 2
B	28	1, 6
C	14	1, 8, 7



Examples of Sequence Data

Sequence Database	Sequence	Element (Transaction)	Event (Item)
Customer	Purchase history of a given customer	A set of items bought by a customer at time t	Books, diary products, CDs, etc
Web Data	Browsing activity of a particular Web visitor	A collection of files viewed by a Web visitor after a single mouse click	Home page, index page, contact info, etc
Event data	History of events generated by a given sensor	Events triggered by a sensor at time t	Types of alarms generated by sensors
Genome sequences	DNA sequence of a particular species	An element of the DNA sequence	Bases A,T,G,C



Formal Definition of a Sequence

- A sequence is an ordered list of elements (transactions)

  $s = < e_1 e_2 e_3 \ldots >$

  – Each element contains a collection of events (items)

  $e_i = \{i_1, i_2, \ldots, i_k\}$

  – Each element is attributed to a specific time or location
- Length of a sequence, |s|, is given by the number of elements of the sequence
- A k-sequence is a sequence that contains k (not necessarily distinct) events (items)

Examples of Sequence

- Web sequence:

  < {Homepage} {Electronics} {Digital Cameras} {Canon Digital Camera} {Shopping Cart} {Order Confirmation} {Return to Shopping} >

- Sequence of initiating events causing the nuclear accident at 3-mile Island:
  (http://stellar-one.com/nuclear/staff_reports/summary_SOE_the_initiating_event.htm)

  < {clogged resin} {outlet valve closure} {loss of feedwater} {condenser polisher outlet valve shut} {booster pumps trip} {main waterpump trips} {main turbine trips} {reactor pressure increases}>

- Sequence of books checked out at a library:

  <{Fellowship of the Ring} {The Two Towers} {Return of the King}>

Formal Definition of a Subsequence

- A sequence <a_1 a_2 … a_n> is contained in another sequence <b_1 b_2 … b_m> ($m \geq n$) if there exist integers $i_1 < i_2 < \ldots < i_n$ such that $a_1 \subseteq b_{i1}$, $a_2 \subseteq b_{i2}$, …, $a_n \subseteq b_{in}$

Data sequence	Subsequence	Contain?
< {2,4} {3,5,6} {8} >	< {2} {3,5} >	Yes
< {1,2} {3,4} >	< {1} {2} >	No
< {2,4} {2,4} {2,5} >	< {2} {4} >	Yes

- The support of a subsequence w is defined as the fraction of data sequences that contain w
- A *sequential pattern* is a frequent subsequence (i.e., a subsequence whose support is ≥ *minsup*)

Sequential Pattern Mining: Definition

- Given:
  - a database of sequences
  - a user-specified minimum support threshold, *minsup*
- Task:
  - Find all subsequences with support ≥ *minsup*

Sequential Pattern Mining: Challenge

- Given a sequence: <{a b} {c d e} {f} {g h i}>
  - Examples of subsequences:
    - <{a} {c d} {f} {g} >, < {c d e} >, < {b} {g} >, etc.
- How many k-subsequences can be extracted from a given n-sequence?

<{a b} {c d e} {f} {g h i}> n = 9

k=4: Y _ _ Y Y _ _ _ Y

<{a} {d e} {i}>

Answer :

$$\binom{n}{k} = \binom{9}{4} = 126$$

Apriori property for sequences

Let D be a database that contains a collection of data sequences d. The support of a sequence t is the fraction of all data sequences that contain t:

$$s(t) = \frac{|\{d \in D : t \text{ is a subsequence of } d\}|}{|D|}$$

Apriori property:
If a data sequence d contains a sequence t, then it will also contain any subsequence of t.

Therefore:
If w is a subsequence of t, then s(w) ≥ s(t).

Sequential Pattern Mining: Example

Object	Timestamp	Events
A	1	1,2,4
A	2	2,3
A	3	5
B	1	1,2
B	2	2,3,4
C	1	1, 2
C	2	2,3,4
C	3	2,4,5
D	1	2
D	2	3, 4
D	3	4, 5
E	1	1, 3
E	2	2, 4, 5

Minsup = 50%

Examples of Frequent Subsequences:

< {1,2} >	s=60%
< {2,3} >	s=60%
< {2,4}>	s=80%
< {3} {5}>	s=80%
< {1} {2} >	s=80%
< {2} {2} >	s=60%
< {1} {2,3} >	s=60%
< {2} {2,3} >	s=60%
< {1,2} {2,3} >	s=60%

Generalized Sequential Pattern (GSP)

- **Step 1**:
  - Make the first pass over the sequence database D to yield all the 1-element frequent sequences
- **Step 2**:

  Repeat until no new frequent sequences are found
  - **Candidate Generation**:
    - Merge pairs of frequent subsequences found in the (k-1)*th* pass to generate candidate sequences that contain k items
  - **Candidate Pruning**:
    - Prune candidate *k*-sequences that contain infrequent (*k-1)*-subsequences
  - **Support Counting**:
    - Make a new pass over the sequence database D to find the support for these candidate sequences
  - **Candidate Elimination**:
    - Eliminate candidate *k*-sequences whose actual support is less than *minsup*

Candidate Generation

- Base case (k=2):
  - Merging two frequent 1-sequences <{i_1}> and <{i_2}> will produce three candidate 2-sequences: <{i_1} {i_2}>, <{i_2} {i_1}> and <{i_1 i_2}>
  - Each frequent 1-sequence <{i_m}> produces the candidate 2-sequence <{i_m} {i_m}>.

For example, if <{A}> and <{B}> are frequent, this produces the candidate 2-sequences: <{A} {B}>, <{B} {A}>, <{A,B}>, <{A} {A}> and <{B} {B}>.

Candidate Generation

- General case (k>2):
  - A frequent (*k-1*)-sequence w_1 is merged with frequent (*k-1*)-sequence w_2 to produce a candidate *k*-sequence if the subsequence obtained by removing the first (using the lexicographic order of events within each element) event in w_1 is the same as the subsequence obtained by removing the last event in w_2.
    - The resulting candidate after merging is given by the sequence w_1 extended with the last event of w_2.
      - If the last two events in w_2 belong to the same element, then the last event in w_2 becomes part of the last element in w_1
      - Otherwise, the last event in w_2 becomes a separate element appended to the end of w_1

Candidate Generation Examples

- Merging the sequences
  w_1=<{1} {2 3} {4}> and w_2 =<{2 3} {4 5}>
  will produce the candidate sequence < {1} {2 3} {4 5}> because the last two events in w_2 (4 and 5) belong to the same element
- Merging the sequences
  w_1=<{1} {2 3} {4}> and w_2 =<{2 3} {4} {5}>
  will produce the candidate sequence < {1} {2 3} {4} {5}> because the last two events in w_2 (4 and 5) do not belong to the same element
- We do not have to merge the sequences
  w_1 =<{1} {2 6} {4}> and w_2 =<{1} {2} {4 5}>
  to produce the candidate < {1} {2 6} {4 5}> because if the latter is a viable candidate, then it can be obtained by merging w_1 with
  < {2 6} {4 5}>

GSP Example

Frequent 3-sequences

< {1} {2} {3} >
< {1} {2 5} >
< {1} {5} {3} >
< {2} {3} {4} >
< {2 5} {3} >
< {3} {4} {5} >
< {5} {3 4} >

Candidate Generation

< {1} {2} {3} {4} >
< {1} {2 5} {3} >
< {1} {5} {3 4} >
< {2} {3} {4} {5} >
< {2 5} {3 4} >

Candidate Pruning

< {1} {2 5} {3} >

For example: <{2 5} {3}> + <{5} {3 4}> : <{2 5} {3 4}>
<{1} {5} {3}> + <{5} {3 4}> : <{1} {5} {3 4}>
<{2 5} {3 4}> is pruned because 3-subsequence <{2} {3 4}> is not frequent.

GSP Exercise

Number	Frequent 3-sequences
1	<{1,2,3}>
2	<{1,2} {3}>
3	<{1} {2,3}>
4	<{1,2} {4}>
5	<{1,3} {4}>
6	<{1,2,4}>
7	<{2,3} {3}>
8	<{2,3} {4}>
9	<{2} {3} {3}>
10	<{2} {3} {4}>

Generate the candidate 4-sequences.

Which ones are pruned?

GSP Exercise

Number	Frequent 3-sequences
1	<{1,2,3}>
2	<{1,2} {3}>
3	<{1} {2,3}>
4	<{1,2} {4}>
5	<{1,3} {4}>
6	<{1,2,4}>
7	<{2,3} {3}>
8	<{2,3} {4}>
9	<{2} {3} {3}>
10	<{2} {3} {4}>

1 + 7: < {1,2,3} {3}>

GSP Exercise

Number	Frequent 3-sequences
1	<{1,2,3}>
2	<{1,2} {3}>
3	<{1} {2,3}>
4	<{1,2} {4}>
5	<{1,3} {4}>
6	<{1,2,4}>
7	<{2,3} {3}>
8	<{2,3} {4}>
9	<{2} {3} {3}>
10	<{2} {3} {4}>

1 + 7: <{1,2,3} {3}>
1 + 8: <{1,2,3} {4}>

GSP Exercise

Number	Frequent 3-sequences
1	<{1,2,3}>
2	<{1,2} {3}>
3	<{1} {2,3}>
4	<{1,2} {4}>
5	<{1,3} {4}>
6	<{1,2,4}>
7	<{2,3} {3}>
8	<{2,3} {4}>
9	<{2} {3} {3}>
10	<{2} {3} {4}>

1 + 7: <{1,2,3} {3}>
1 + 8: <{1,2,3} {4}>
2 + 9: <{1,2} {3} {3}>

GSP Exercise

Number	Frequent 3-sequences
1	<{1,2,3}>
2	<{1,2} {3}>
3	<{1} {2,3}>
4	<{1,2} {4}>
5	<{1,3} {4}>
6	<{1,2,4}>
7	<{2,3} {3}>
8	<{2,3} {4}>
9	<{2} {3} {3}>
10	<{2} {3} {4}>

1 + 7: <{1,2,3} {3}>
1 + 8: <{1,2,3} {4}>
2 + 9: <{1,2} {3} {3}>
2+10: <{1,2} {3} {4}>

GSP Exercise

Number	Frequent 3-sequences
1	<{1,2,3}>
2	<{1,2} {3}>
3	<{1} {2,3}>
4	<{1,2} {4}>
5	<{1,3} {4}>
6	<{1,2,4}>
7	<{2,3} {3}>
8	<{2,3} {4}>
9	<{2} {3} {3}>
10	<{2} {3} {4}>

1 + 7: <{1,2,3} {3}>
1 + 8: <{1,2,3} {4}>
2 + 9: <{1,2} {3} {3}>
2+10: <{1,2} {3} {4}>
3 + 7: <{1} {2,3} {3}>

GSP Exercise

Number	Frequent 3-sequences
1	<{1,2,3}>
2	<{1,2} {3}>
3	<{1} {2,3}>
4	<{1,2} {4}>
5	<{1,3} {4}>
6	<{1,2,4}>
7	<{2,3} {3}>
8	<{2,3} {4}>
9	<{2} {3} {3}>
10	<{2} {3} {4}>

1 + 7: <{1,2,3} {3}>
1 + 8: <{1,2,3} {4}>
2 + 9: <{1,2} {3} {3}>
2+10: <{1,2} {3} {4}>
3 + 7: <{1} {2,3} {3}>
3 + 8: <{1} {2,3} {4}>

GSP Exercise

Number	Frequent 3-sequences
1	<{1,2,3}>
2	<{1,2} {3}>
3	<{1} {2,3}>
4	<{1,2} {4}>
5	<{1,3} {4}>
6	<{1,2,4}>
7	<{2,3} {3}>
8	<{2,3} {4}>
9	<{2} {3} {3}>
10	<{2} {3} {4}>

1 + 7: <{1,2,3} {3}>
1 + 8: <{1,2,3} {4}>
2 + 9: <{1,2} {3} {3}>
2+10: <{1,2} {3} {4}>
3 + 7: <{1} {2,3} {3}>
3 + 8: <{1} {2,3} {4}>

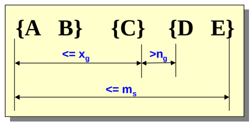
GSP Exercise: candidate pruning

Number	Frequent 3-sequences
1	<{1,2,3}>
2	<{1,2} {3}>
3	<{1} {2,3}>
4	<{1,2} {4}>
5	<{1,3} {4}>
6	<{1,2,4}>
7	<{2,3} {3}>
8	<{2,3} {4}>
9	<{2} {3} {3}>
10	<{2} {3} {4}>

1 + 7: <{1,2,3} {3}>
1 + 8: <{1,2,3} {4}>
2 + 9: <{1,2} {3} {3}>
2+10: <{1,2} {3} {4}>
3 + 7: <{1} {2,3} {3}>
3 + 8: <{1} {2,3} {4}>

<{1,2,3}{4}> is not pruned because <{1,2,3}>, <{1,2}{4}> <{1,3}{4}> and <{2,3}{4}> are all frequent.

Timing Constraints (I)



x_g: max-gap
n_g: min-gap
m_s: maximum span

$x_g = 2$, $n_g = 0$, $m_s = 4$

Data sequence	Subsequence	Contain?
< {2,4} {3,5,6} {4,7} {4,5} {8} >	< {6} {5} >	Yes
< {1} {2} {3} {4} {5}>	< {1} {4} >	No
< {1} {2,3} {3,4} {4,5}>	< {2} {3} {5} >	Yes
< {1,2} {3} {2,3} {3,4} {2,4} {4,5}>	< {1,2} {5} >	No

Timing Constraints: Formal Definition

A data sequence $d = \langle d_1 \ldots d_m \rangle$ contains a sequence $w = \langle w_1 \ldots w_n \rangle$ if there exist integers $i_1 < \ldots < i_n$ such that $w_j \subseteq d_{i_j}$ and

1. maxgap: $time(d_{i_j}) - time(d_{i_{j-1}}) \le maxgap$
2. mingap: $time(d_{i_j}) - time(d_{i_{j-1}}) > mingap$
3. maxspan: $time(d_{i_n}) - time(d_{i_1}) \le maxspan$

Note: this definition applies if the window size = 0, otherwise things get more complicated (see timing constraints II for definition of window size).

Mining Sequential Patterns with Timing Constraints

- Approach 1:
  - Mine sequential patterns without timing constraints
  - Postprocess the discovered patterns
- Approach 2:
  - Modify GSP to directly prune candidates that violate timing constraints
  - Question:
    - Does Apriori principle still hold?

Apriori Principle for Sequence Data

Object	Timestamp	Events
A	1	1,2,4
A	2	2,3
A	3	5
B	1	1,2
B	2	2,3,4
C	1	1, 2
C	2	2,3,4
C	3	2,4,5
D	1	2
D	2	3, 4
D	3	4, 5
E	1	1, 3
E	2	2, 4, 5

Suppose:

$x_g = 1$ (max-gap)
$n_g = 0$ (min-gap)
$m_s = 5$ (maximum span)
minsup = 60%

<{2} {5}> support = 40%
but
<{2} {3} {5}> support = 60%

Problem exists because of max-gap constraint
No such problem if max-gap is infinite

Maxgap and the apriori property

- Due to the maxgap constraint, the support of a sequence can be increased by inserting an element between two elements.
- Not by inserting an element at the beginning or the end of the sequence.
- This spoils the apriori property that the support of a sequence is never bigger than the support of any of its subsequences.
- No pruning possible anymore?

Contiguous Subsequences

- s is a contiguous subsequence of
  w = <e_1>< e_2>…< e_k>
  if any of the following conditions hold:
  1. s is obtained from w by deleting an item from either e_1 or e_k
  2. s is obtained from w by deleting an item from any element e_i that contains more than 2 items
  3. s is a contiguous subsequence of s' and s' is a contiguous subsequence of w (recursive definition)
- Examples: s = < {1} {2} >
  – is a contiguous subsequence of
  < {1} {2 3}>, < {1 2} {2} {3}>, and < {3 4} {1 2} {2 3} {4} >
  – is not a contiguous subsequence of
  < {1} {3} {2}> and < {2} {1} {3} {2}>

Apriori principle with maxgap constraint

Apriori property:
If a data sequence d contains a sequence t, then it will also contain any *contiguous* subsequence of t.

Therefore:
If w is a *contiguous* subsequence of t, then s(w) ≥ s(t).

Apriori principle with maxgap constraint

Example:

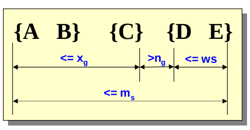
Suppose t = <{A},{A,B},{C},{B,C}> is contained in a data sequence d (meaning it also satisfies the maxgap constraint), then any contiguous subsequence of t also satisfies the maxgap constraint with respect to d.

We can only violate the maxgap constraint by removing {A,B} or {C} but that would result in a non-contiguous subsequence.

Modified Candidate Pruning Step

- Without maxgap constraint:
  – A candidate k-sequence is pruned if at least one of its (k-1)-subsequences is infrequent
- With maxgap constraint:
  – A candidate *k*-sequence is pruned if at least one of its *contiguous* (*k*-*1*)-subsequences is infrequent
- So with a maxgap constraint we can do less pruning.

Timing Constraints (II)



x_g: max-gap
n_g: min-gap
ws: window size
m_s: maximum span

x_g = 2, n_g = 0, ws = 1, m_s= 5

Data sequence	Subsequence	Contain?
< {2,4} {3,5,6} {4,7} {4,6} {8} >	< {3} {5} >	No
< {1} {2} {3} {4} {5}>	< {1,2} {3} >	Yes
< {1,2} {2,3} {3,4} {4,5}>	< {1,2} {3,4} >	Yes

Modified Support Counting Step

- Given a candidate pattern: <{a, c}>
  – Any data sequences that contain

  <… {a c} … >,
  <… {a} … {c}…> (where time({c}) – time({a}) ≤ ws)
  <…{c} … {a} …> (where time({a}) – time({c}) ≤ ws)

  will contribute to the support count of candidate pattern

Exercise: timing constraints

Data sequence: d=<{1,2,3} {2,4} {2,4,5} {3,5} {6}> where elements occur on consecutive time points 1…5.

Given are the following sequences w = <$e_1 e_2 \dots e_i e_{i+1} \dots e_n$ >

1. <{1} {2} {3}>
2. <{1,2,3,4} {5,6}>
3. <{2,4} {2,4} {6}>
4. <{1} {2,4} {6}>
5. <{1,2} {3,4} {5,6}>

Are they subsequences of d, subject to the following timing constraints?

mingap=0 (interval between last event in e_i, and first event in e_{i+1} >0)

maxgap=3 (interval between first event in e_i, and last event in $e_{i+1} \leq 3$)

maxspan=5 (interval between first event in e_1, and last event in $e_n \leq 5$)

ws=1 (time between first and last events in $e_i \leq 1$)

Exercise: timing constraints

d=<{1,2,3} {2,4} {2,4,5} {3,5} {6}>

Given are the following sequences w = <$e_1 e_2 \dots e_i e_{i+1} \dots e_n$ >

1. <{1} {2} {3}> Yes
2. <{1,2,3,4} {5,6}>
3. <{2,4} {2,4} {6}>
4. <{1} {2,4} {6}>
5. <{1,2} {3,4} {5,6}>

Are they subsequences of d, subject to the following timing constraints?

mingap=0 (interval between last event in e_i, and first event in e_{i+1} >0)

maxgap=3 (interval between first event in e_i, and last event in $e_{i+1} \leq 3$)

maxspan=5 (interval between first event in e_1, and last event in $e_n \leq 5$)

ws=1 (time between first and last events in $e_i \leq 1$)

Exercise: timing constraints

d=<{1,2,3} {2,4} {2,4,5} {3,5} {6}>

Given are the following sequences w = <$e_1 e_2 \dots e_i e_{i+1} \dots e_n$ >

1. <{1} {2} {3}> Yes
2. <{1,2,3,4} {5,6}> No (maxgap constraint violated)
3. <{2,4} {2,4} {6}>
4. <{1} {2,4} {6}>
5. <{1,2} {3,4} {5,6}>

Are they subsequences of d, subject to the following timing constraints?

mingap=0 (interval between last event in e_i, and first event in e_{i+1} >0)

maxgap=3 (interval between first event in e_i, and last event in $e_{i+1} \leq 3$)

maxspan=5 (interval between first event in e_1, and last event in $e_n \leq 5$)

ws=1 (time between first and last events in $e_i \leq 1$)

Exercise: timing constraints

d=<{1,2,3} {2,4} {2,4,5} {3,5} {6}>

Given are the following sequences w = <$e_1 e_2 \dots e_i e_{i+1} \dots e_n$ >

1. <{1} {2} {3}> Yes
2. <{1,2,3,4} {5,6}> No
3. <{2,4} {2,4} {6}> Yes
4. <{1} {2,4} {6}>
5. <{1,2} {3,4} {5,6}>

Are they subsequences of d, subject to the following timing constraints?

mingap=0 (interval between last event in e_i, and first event in e_{i+1} >0)

maxgap=3 (interval between first event in e_i, and last event in $e_{i+1} \leq 3$)

maxspan=5 (interval between first event in e_1, and last event in $e_n \leq 5$)

ws=1 (time between first and last events in $e_i \leq 1$)

Exercise: timing constraints

d=<{1,2,3} {2,4} {2,4,5} {3,5} {6}>

Given are the following sequences w = <$e_1 e_2 \dots e_i e_{i+1} \dots e_n$ >

1. <{1} {2} {3}> Yes
2. <{1,2,3,4} {5,6}> No
3. <{2,4} {2,4} {6}> Yes
4. <{1} {2,4} {6}> Yes
5. <{1,2} {3,4} {5,6}>

Are they subsequences of d, subject to the following timing constraints?

mingap=0 (interval between last event in e_i, and first event in e_{i+1} >0)

maxgap=3 (interval between first event in e_i, and last event in $e_{i+1} \leq 3$)

maxspan=5 (interval between first event in e_1, and last event in $e_n \leq 5$)

ws=1 (time between first and last events in $e_i \leq 1$)

Exercise: timing constraints

d=<{1,2,3} {2,4} {2,4,5} {3,5} {6}>

Given are the following sequences w = <$e_1 e_2 \dots e_i e_{i+1} \dots e_n$ >

1. <{1} {2} {3}> Yes
2. <{1,2,3,4} {5,6}> No
3. <{2,4} {2,4} {6}> No
4. <{1} {2,4} {6}> Yes
5. <{1,2} {3,4} {5,6}> No (violates mingap and maxgap constraint)

Are they subsequences of d, subject to the following timing constraints?

mingap=0 (interval between last event in e_i, and first event in e_{i+1} >0)

maxgap=3 (interval between first event in e_i, and last event in $e_{i+1} \leq 3$)

maxspan=5 (interval between first event in e_1, and last event in $e_n \leq 5$)

ws=1 (time between first and last events in $e_i \leq 1$)