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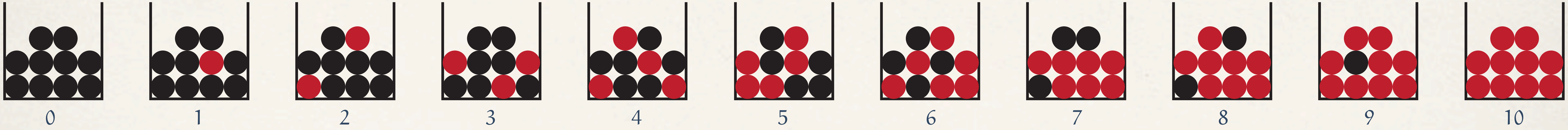

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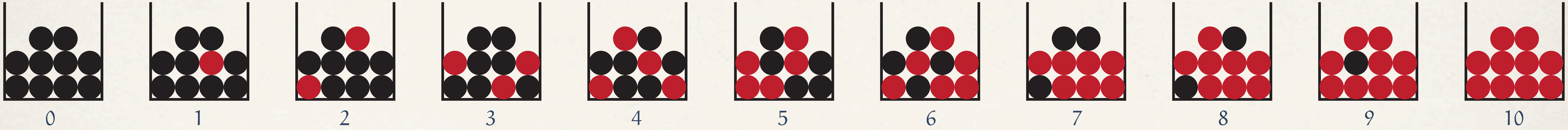

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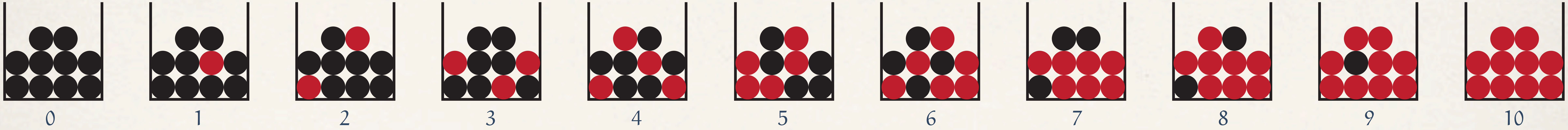

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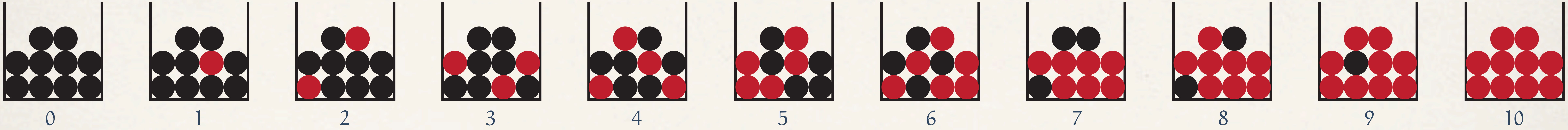

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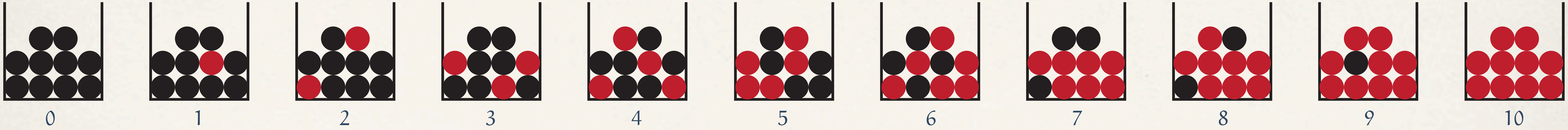

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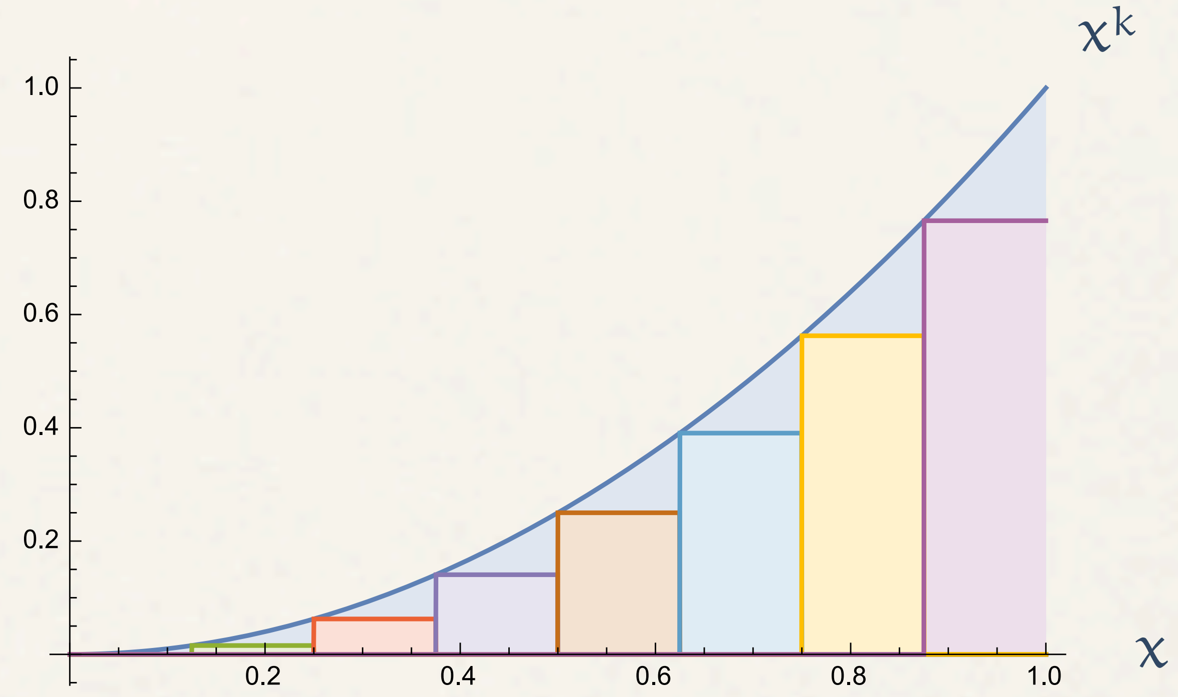

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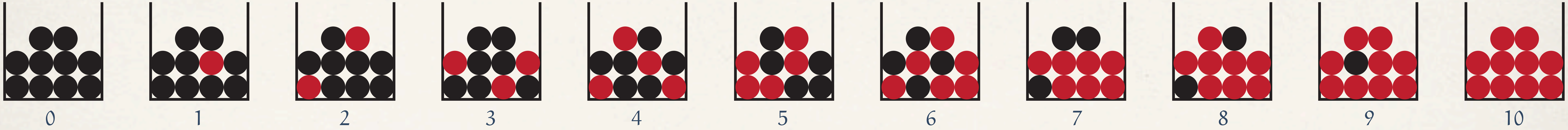

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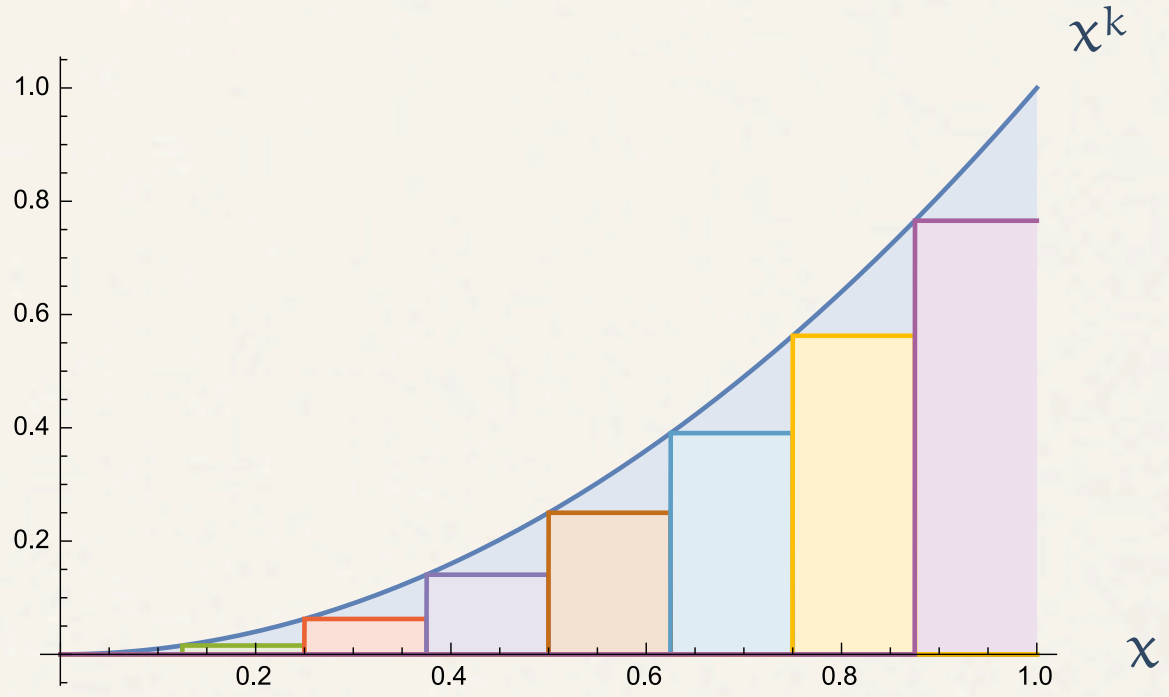

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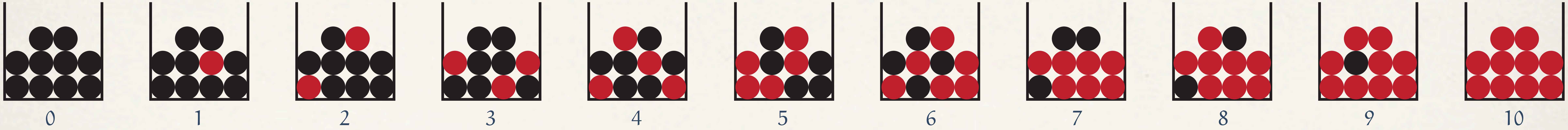

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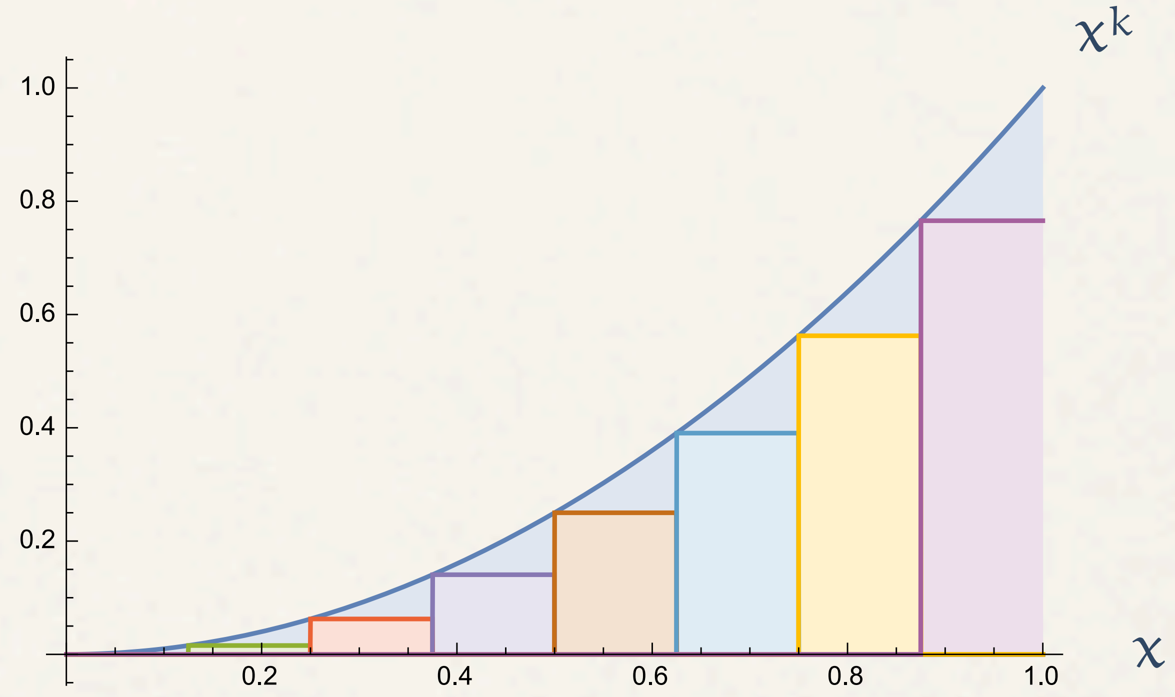

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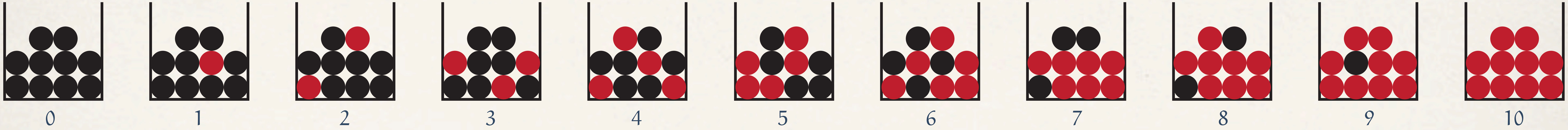

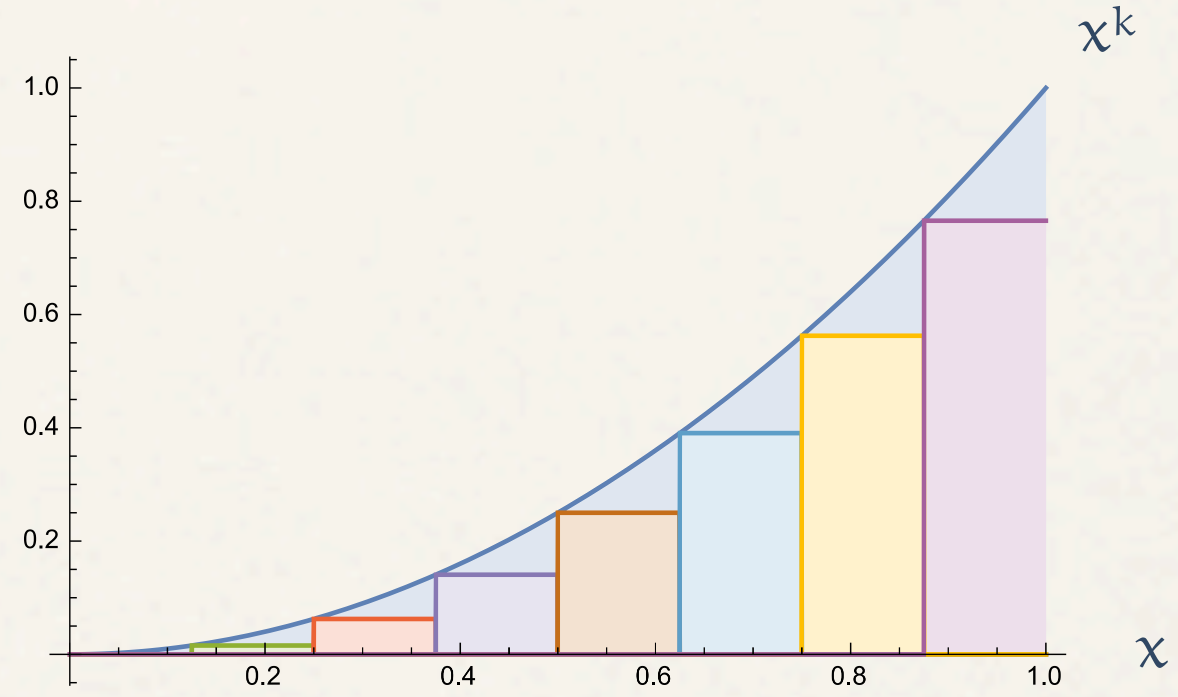

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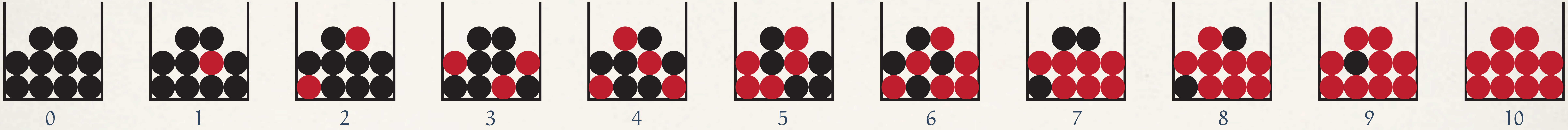

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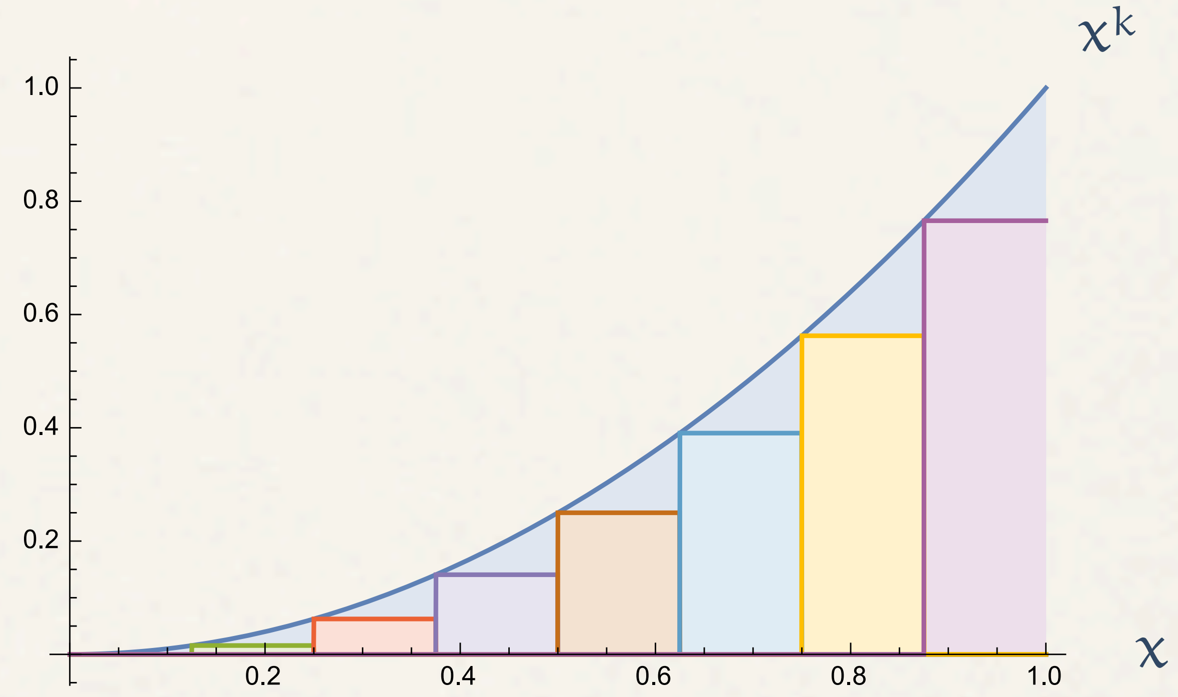

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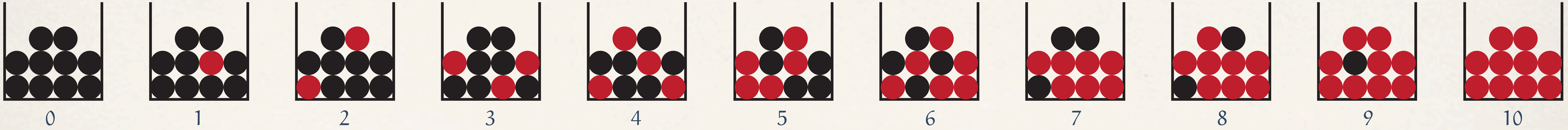

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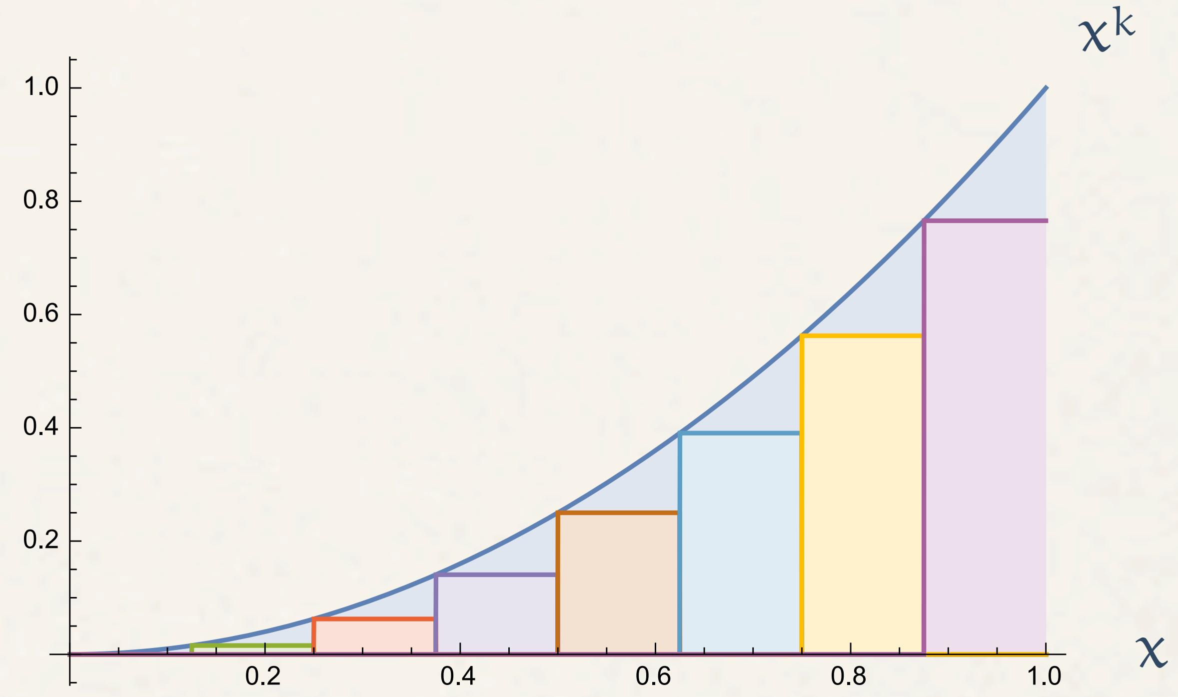

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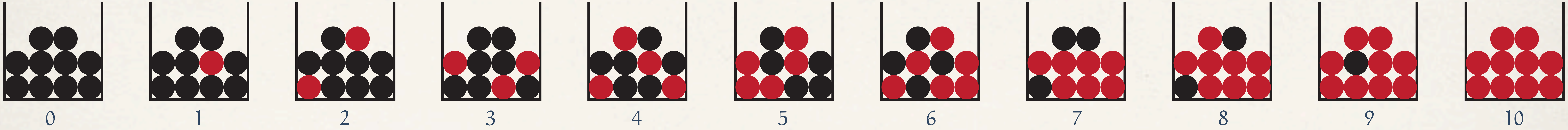

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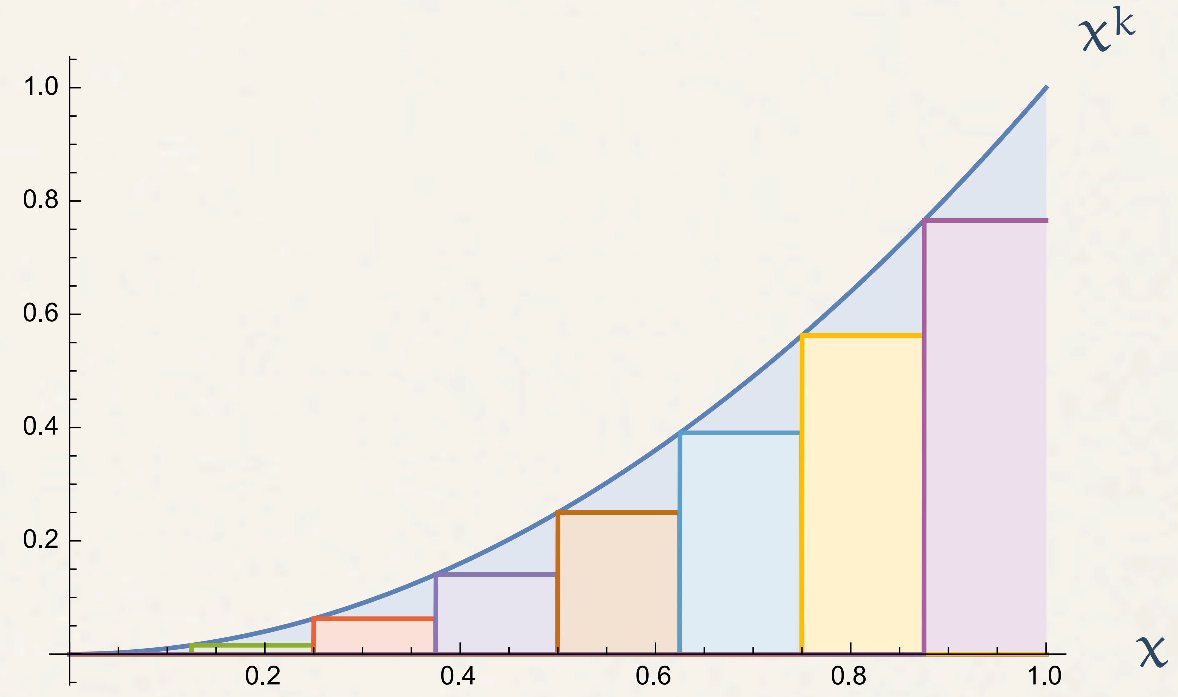

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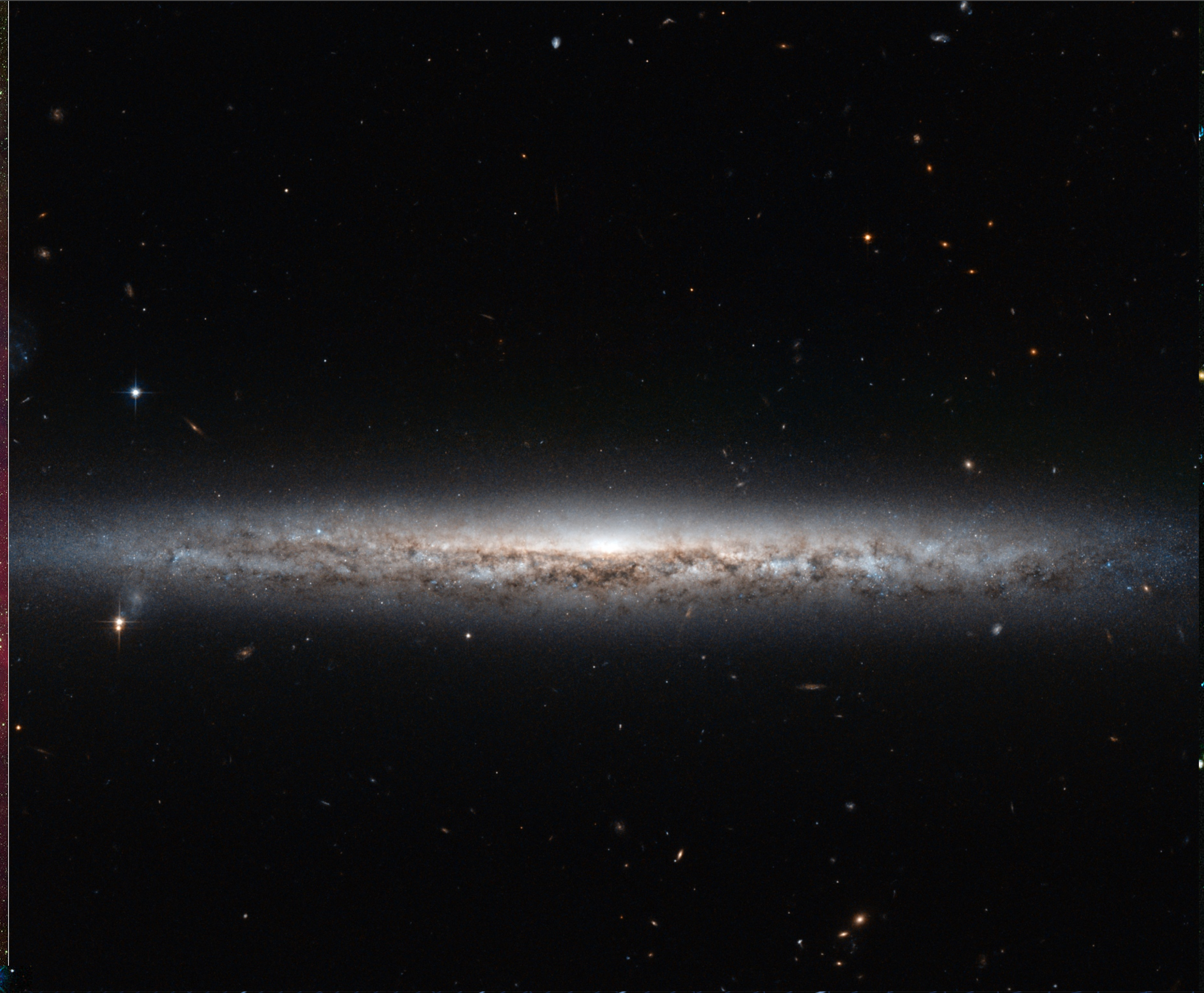
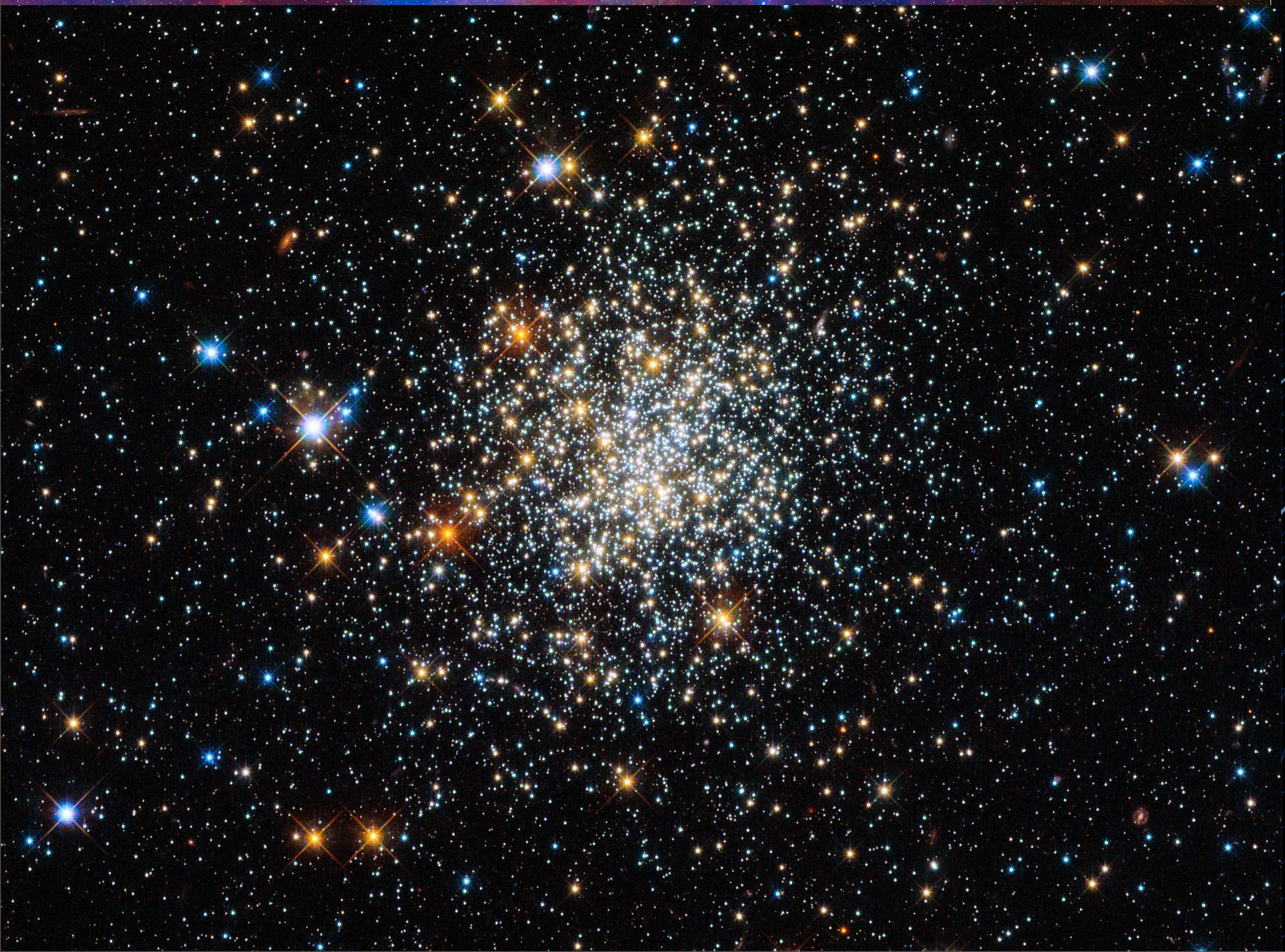
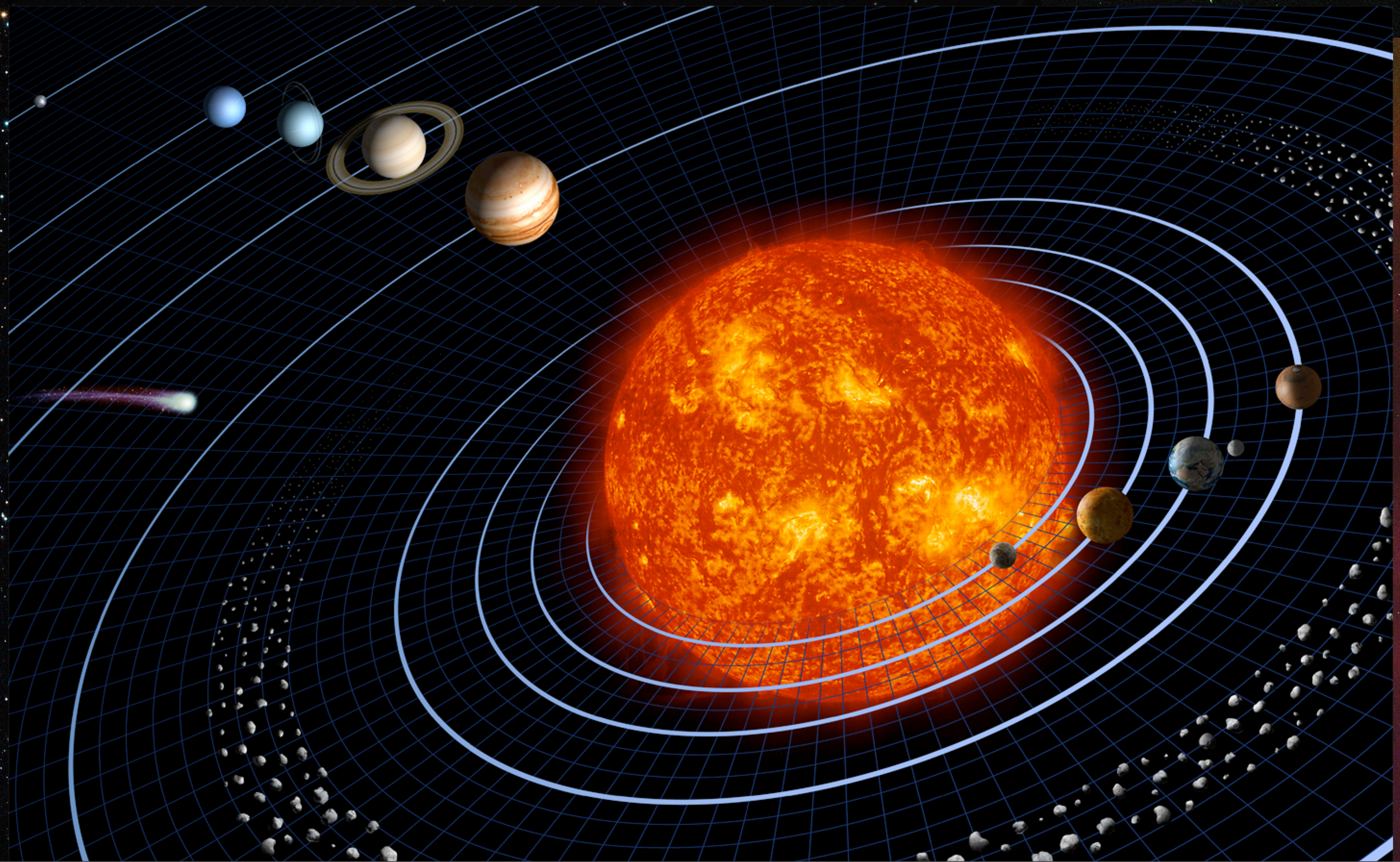
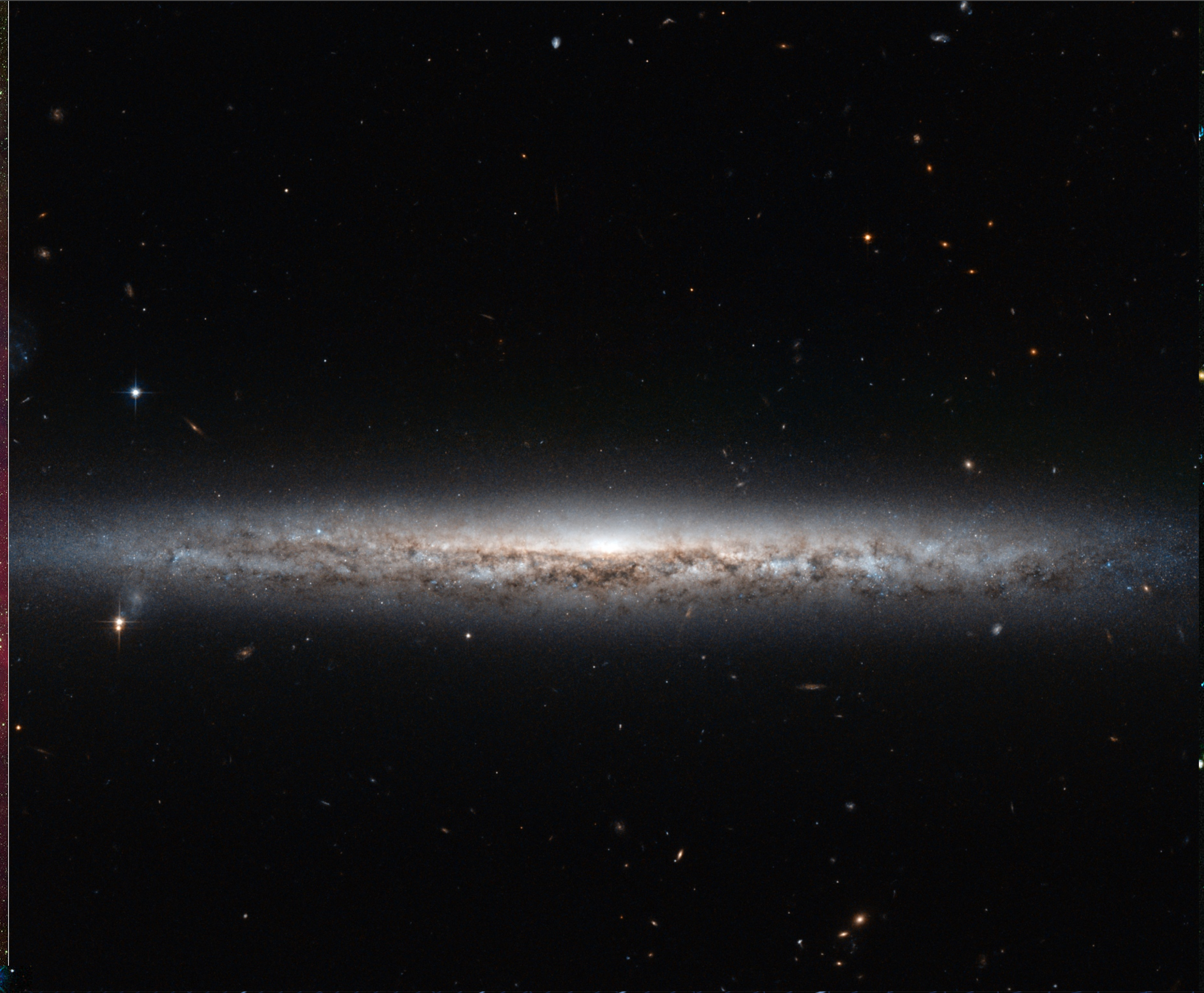
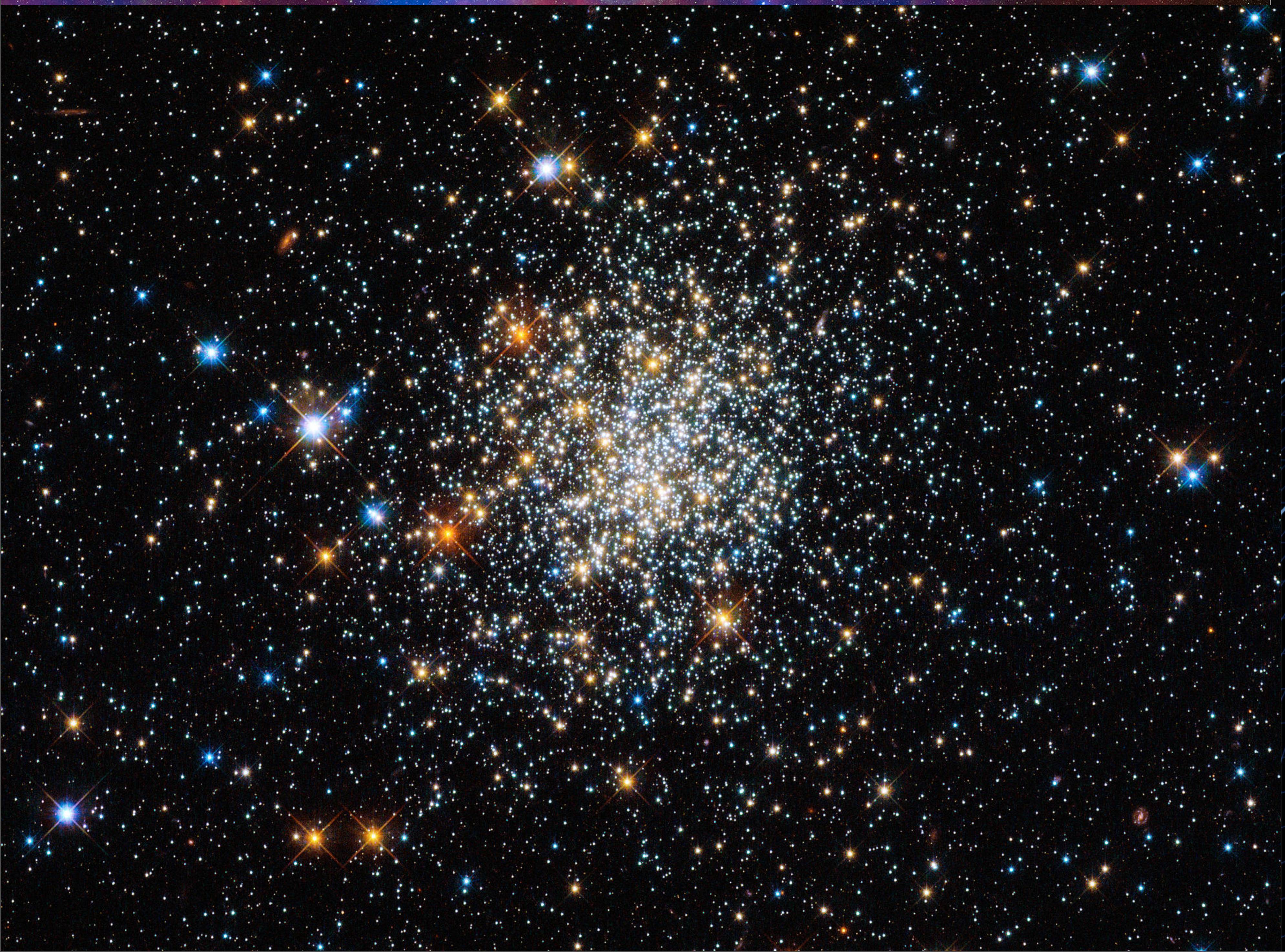
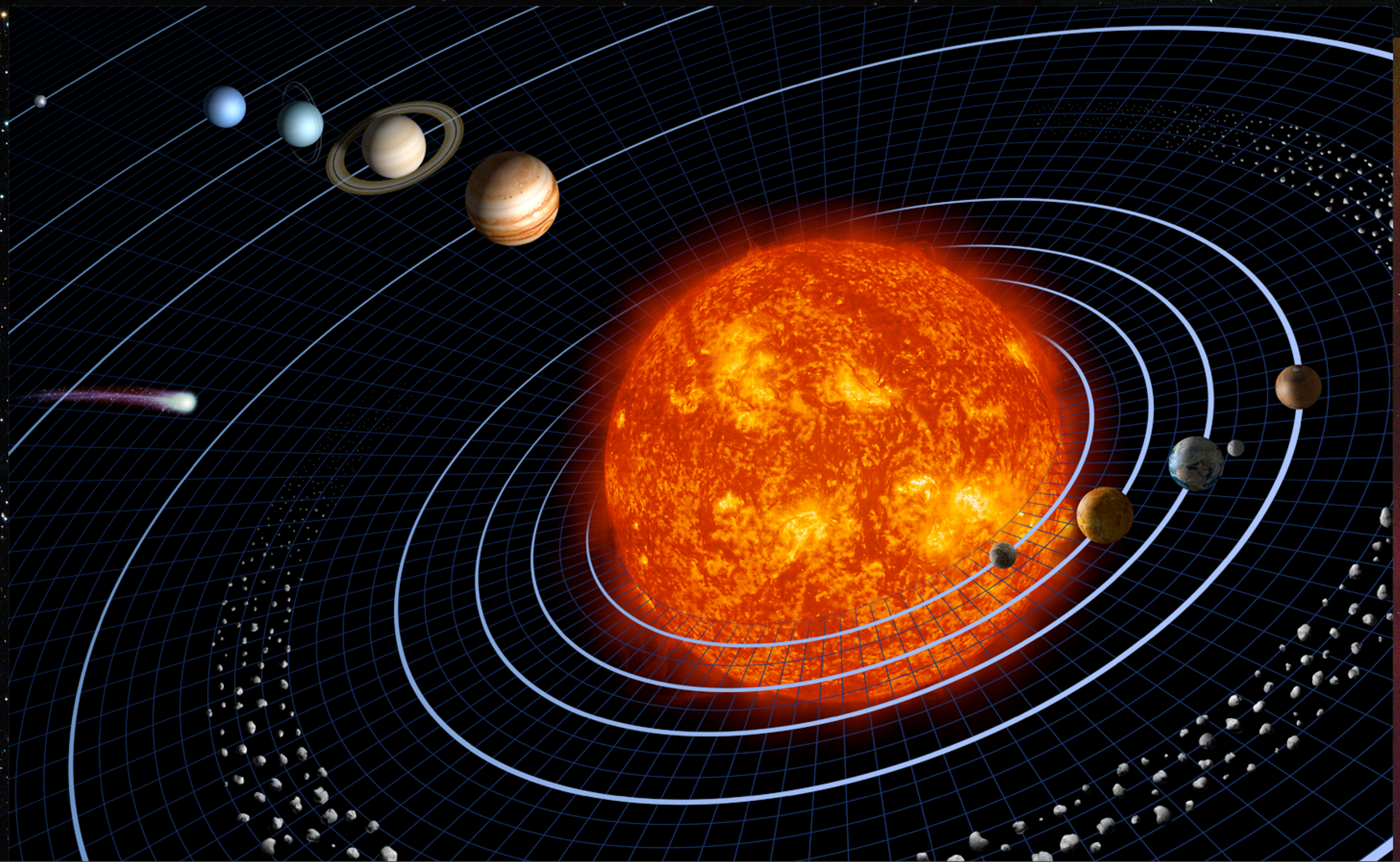
Evaluate: $\mathbf{P}(H_{r+s} \mid H_r) \approx \dfrac{r+1}{s+r+1}$



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What is the chance the sun will rise tomorrow?

Placing the most ancient epoch of history at five thousand years ago, or at 1,826,213 days, and the sun having risen constantly in the interval at each revolution of twenty-four hours, it is a bet of 1,826,214 to 1 that it will rise again tomorrow …

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But this number is incomparably greater for him who, recognising in the totality of phenomena the principal regulator of days and seasons, sees that nothing at the present moment can arrest the course of it.

— P. S. Laplace, *Théorie Analytique des Probabilités* (1812).