Coffee with Milk

In the previous lesson, we considered quantities that were invariant for straightforward reasons: no matter how one counts the elements of a set, the result is the same. In this lesson, we will consider problems where discovering the right invariant is a more challenging task.

Problem. There are two cups: cup 1 contains coffee and cup 2 contains milk. We take a spoon of coffee from cup 1 and pour it into cup 2. After that, we take one spoon of the drink in cup 2 and pour it into cup 1. What is larger, the amount of milk in cup 1 or the amount of coffee in cup 2?

It might seem that one needs to know the volumes of both cups as well as the volume of the spoon to answer the question, but it is not the case! What is important here is that the amount of drink in both cups remains the same (this is our invariant!). This means that the amount of coffee missing in cup 1 is the same as the amount of milk added to cup 1. And the amount of coffee in cup 2 is the amount of coffee missing in cup 1. Hence, the two quantities from the problem statement are equal.