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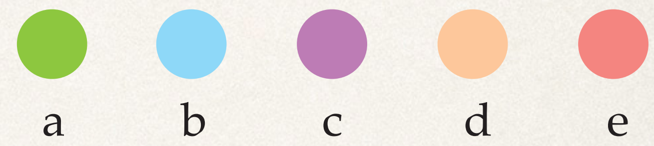

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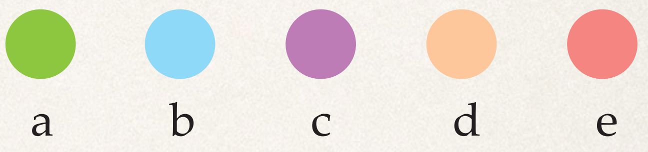

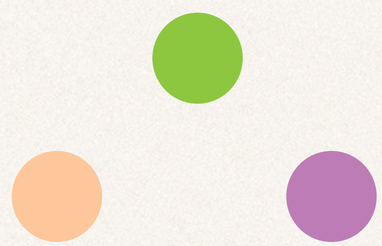

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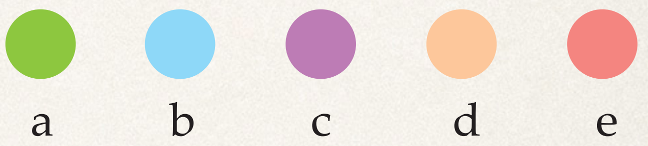

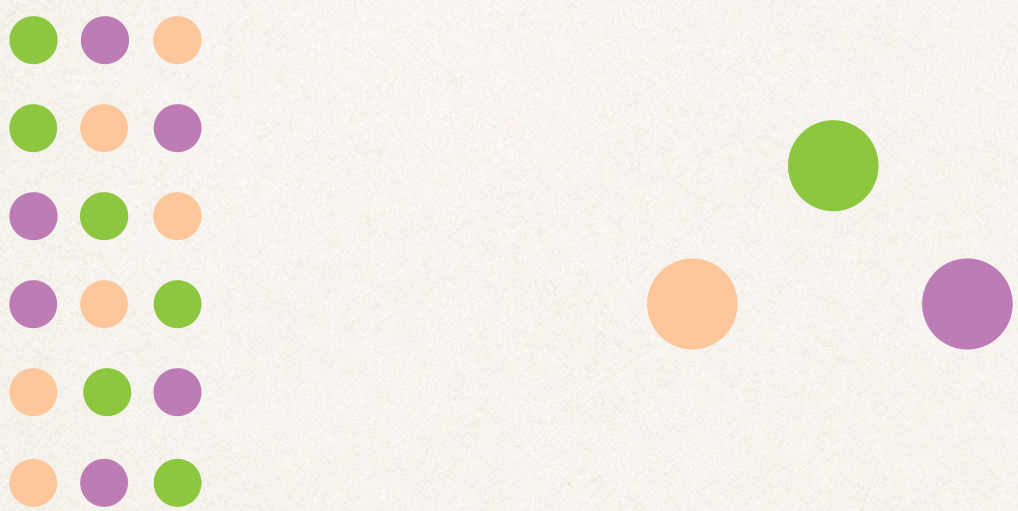

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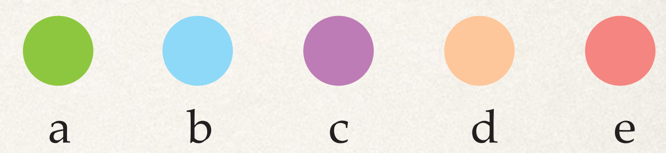

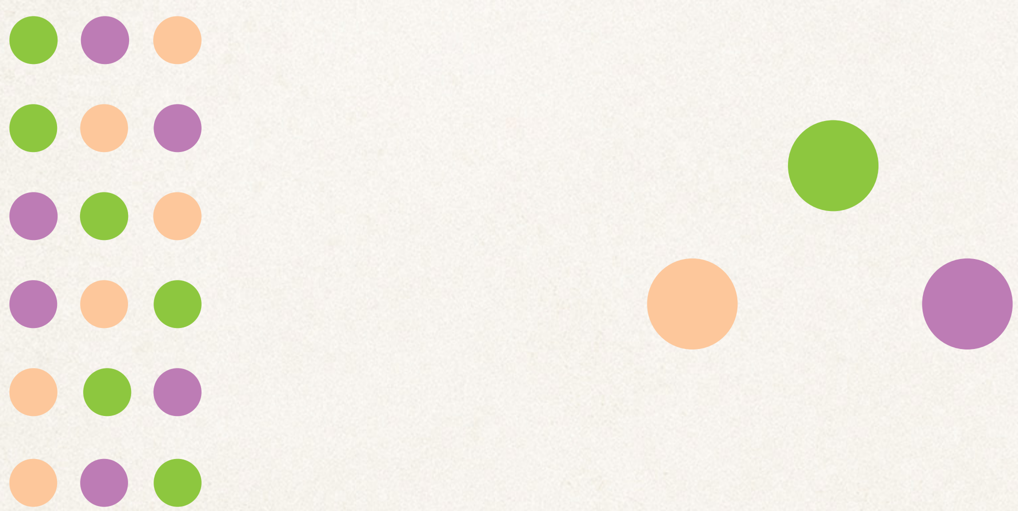

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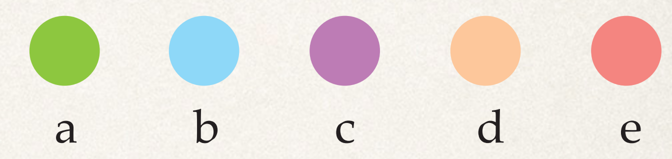

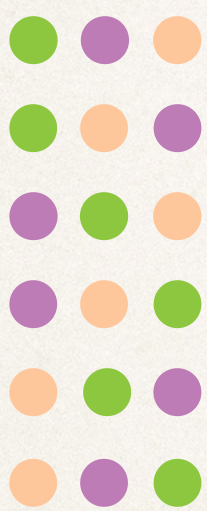

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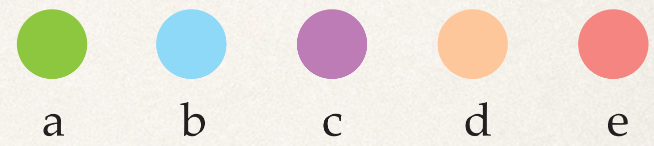

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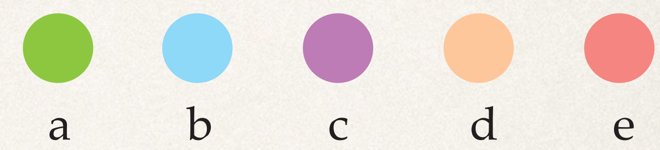

Ordered sample	Subset (subpopulation)
(a, c, d)	{a, c, d}
(a, d, c)	
(c, a, d)	
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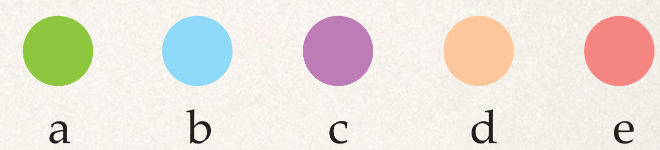

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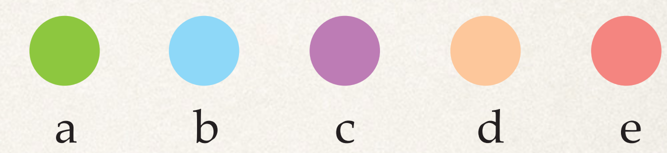

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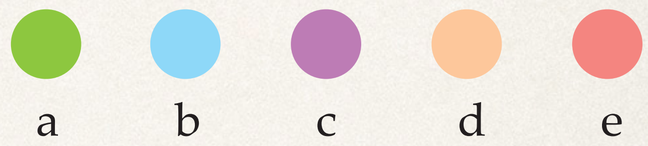

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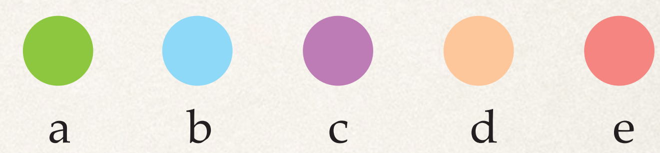

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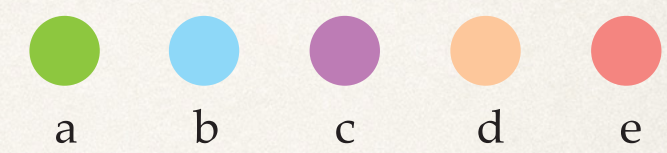

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Convention: $\binom{n}{0} = 1$ for $n \geq 0$.