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A generic ball and urn problem

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- There are nine possible outcomes for the experiment:
  - A visual representation of outcomes: the urns specified in order, balls are labelled a and b, order within an urn unimportant:

$$ab|-|-, \quad -|ab|-, \quad -|-|ab,$$
$$a|b|-, \quad a|-|b, \quad b|a|-, \quad b|-|a, \quad -|a|b, \quad -|b|a.$$

  - Another representation of outcomes: (i, j), where i and j identify urns 1, 2, 3 that are the locations of the balls specified in order:

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- The event the second urn is occupied is described by an aggregate of five outcomes:

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The probability the second urn is occupied is hence $5/9$.