Feedback — Week 4 Assignment (dualization)

Help Center

You submitted this homework on **Tue 15 Oct 2013 7:40 AM PDT**. You got a score of **26.00** out of **26.00**.

Dualization Homework

The questions in this homework assignment talk about the video lectures posted for week #4 on dualization.

Question 1

Consider the primal LP below:

$$
\begin{array}{lccccccl}
\max & -x_1 & -3x_2 & +x_3 & & & & \\
 & -x_1 & -x_2 & & \leq & 5 & \leftarrow & y_1 \\
 & a_1 x_1 & & -3x_3 & \leq & 2 & \leftarrow & y_2 \\
 & x_1 & +a_2 x_2 & +2x_3 & \leq & 2 & \leftarrow & y_3 \\
 & x_1 & & +a_3 x_3 & \leq & 0 & \leftarrow & y_4 \\
 & & x_2 & -x_3 & \leq & 0 & \leftarrow & y_5 \\
 & x_1, & x_2, & x_3 & \geq & 0 & &
\end{array}
$$

and the corresponding dual problem:

$$
\begin{array}{lrrrrrcl}
\min & \underline{c_1}\, y_1 & +\underline{c_2}\, y_2 & +\underline{c_3}\, y_3 & & & & \\
\text{s.t.} & -y_1 & -2y_2 & +y_3 & +y_4 & & \geq & \underline{b_1} \\
& -y_1 & & +y_3 & & +y_5 & \geq & \underline{b_2} \\
& & -3y_2 & +2y_3 & -y_4 & -y_5 & \geq & \underline{b_3} \\
& y_1, & y_2, & y_3, & y_4, & y_5 & \geq & 0
\end{array}
$$

Note the missing numbers c_1, c_2, c_3 in the dual.

Write down the values of $c_1,\ c_2,\ c_3$. Your answer should be three numbers separated by a space ( ).

You entered:

5 2 2

Your Answer		Score	Explanation
5	✔	1.67	
2	✔	1.67	
2	✔	1.67	
Total		5.00 / 5.00	

Question Explanation

The dual objective is defined by the primal RHS constants. Given that, you should be able to read off the dual objective at once.

Question 2

Consider the primal LP below:

$$
\begin{array}{lrrrcll}
\max & -x_1 & -3x_2 & +x_3 & & & \\
 & -x_1 & -x_2 & & \leq & 5 & \leftarrow \mathsf{y}_1 \\
 & \underline{\mathsf{a}_1}\, x_1 & & -3x_3 & \leq & 2 & \leftarrow \mathsf{y}_2 \\
 & x_1 & +\underline{\mathsf{a}_2}\, x_2 & +2x_3 & \leq & 2 & \leftarrow \mathsf{y}_3 \\
 & x_1 & & +\underline{\mathsf{a}_3}\, x_3 & \leq & 0 & \leftarrow \mathsf{y}_4 \\
 & & x_2 & -x_3 & \leq & 0 & \leftarrow \mathsf{y}_5 \\
 & x_1, & x_2, & x_3 & \geq & 0 &
\end{array}
$$

and the corresponding dual problem:

$$
\begin{array}{lrrrrrcl}
\min & \underline{\mathsf{c}_1}\, y_1 & +\underline{\mathsf{c}_2}\, y_2 & +\underline{\mathsf{c}_3}\, y_3 & & & & \\
\text{s.t.} & -y_1 & -2y_2 & +y_3 & +y_4 & & \geq & \underline{\mathsf{b}_1} \\
 & -y_1 & & +y_3 & & +y_5 & \geq & \underline{\mathsf{b}_2} \\
 & & -3y_2 & +2y_3 & -y_4 & -y_5 & \geq & \underline{\mathsf{b}_3} \\
 & y_1, & y_2, & y_3, & y_4, & y_5 & \geq & 0
\end{array}
$$

Note the missing numbers b_1, b_2, b_3 in the dual.

Write down the values of $b_1,\ b_2,\ b_3$. Your answer should be three numbers separated by a space ().

You entered:

-1 -3 1

Your Answer		Score	Explanation
-1	✔	1.00	
-3	✔	1.00	

1	✔	1.00
Total		3.00 / 3.00

Question Explanation

The primal objective becomes the RHS constants in the dual. Given that fact, you should be able to read the dual RHS constants directly.

Question 3

Consider the primal LP below:

$$
\begin{array}{lccccccl}
\max & -x_1 & -3x_2 & +x_3 & & & & \\
& -x_1 & -x_2 & & \leq & 5 & \leftarrow & y_1 \\
& a_1 x_1 & & -3x_3 & \leq & 2 & \leftarrow & y_2 \\
& x_1 & +a_2 x_2 & +2x_3 & \leq & 2 & \leftarrow & y_3 \\
& x_1 & & +a_3 x_3 & \leq & 1 & \leftarrow & y_4 \\
& & x_2 & -x_3 & \leq & -1 & \leftarrow & y_5 \\
& x_1, & x_2, & x_3 & \geq & 0 & &
\end{array}
$$

and the corresponding dual problem:

$$
\begin{array}{lccccccc}
\min & c_1 y_1 & +c_2 y_2 & +c_3 y_3 & & & & \\
\text{s.t.} & -y_1 & -2y_2 & +y_3 & +y_4 & & \geq & b_1 \\
& -y_1 & & +y_3 & & +y_5 & \geq & b_2 \\
& & -3y_2 & +2y_3 & -y_4 & -y_5 & \geq & b_3 \\
& y_1, & y_2, & y_3, & y_4, & y_5 & \geq & 0
\end{array}
$$

Note the missing numbers a_1, a_2, a_3 in the primal.

Write down the values of $a_1,\ a_2,\ a_3$. Your answer should be three numbers separated by a space ().

You entered:

-2 1 -1

Your Answer		Score	Explanation
-2	✔	2.00	
1	✔	2.00	
-1	✔	2.00	
Total		6.00 / 6.00	

Question Explanation

Note that the matrix of the primal is transposed in the dual. We have provided the matrix for the primal in full. Transpose it and read off $a_1, \ldots, a_3$

Question 4

Consider the dual dictionary:

$$
\begin{array}{c|ccccc}
y_1 & -1 & +2y_2 & -3y_4 & +y_5 & +y_7 \\
y_3 & 2 & -y_2 & +2y_4 & -y_5 & \\
y_6 & -1 & -y_2 & -y_4 & -y_5 & \\
\hline
\xi & -10 & -2y_2 & -3y_4 & -y_5 & +0y_7
\end{array}
$$

The complementary pairs are as follows:

Primal	Dual
x_1	y_5
x_2	y_6
x_3	y_7
x_4	y_1
x_5	y_2
x_6	y_3
x_7	y_4

Answer the following questions about the **complementary primal dictionary** and the **primal, dual** problems that give rise to these dictionary, which are not shown. Select all the correct answers and ensure that incorrect answers are not selected.

It will help you immensely to first write down the primal complement dictionary in advance on a sheet of paper to answer each of the 10 parts below. In doing so, focus on (a) the basic and non-basic variables in this dictionary, (b) the constant column, objective row, and finally (c) the matrix (rows) of the dictionary.

Your Answer		Score	Explanation
☑ If x_2 enters there is no leaving variable, therefore the **primal problem** is unbounded.	✔	2.18	
☐ x_6 is an entering variable in the complementary **primal dictionary**	✔	1.09	Reconsider what the objective coefficient corresponding to x_6 will be.
☐ The **primal problem** has 4 variables and 3 constraints	✔	0.55	Note: # constraints in dual = # variables in primal and # constraints in primal = # variables in dual.

☑ x_5 is a basic variable in the complementary **primal dictionary**	✔	1.09	
☑ The complementary **primal dictionary** is degenerate	✔	1.09	
☐ The objective value of the complementary **primal dictionary** is -10	✔	1.09	Note that we construct the dual as a minimization and to form a dictionary, turn it into a maximization problem.
☑ x_4 is an entering variable in the complementary **primal dictionary**	✔	1.09	
☐ The complementary **primal dictionary** is final	✔	1.09	The dual dictionary is infeasible. Therefore, this cannot be.
☑ The **primal problem** has 3 variables and 4 constraints	✔	0.55	
☐ The **dual problem** has a feasible solution.	✔	2.18	Reconstruct the primal dictionary and let x_2 be the entering variable. You will immediately conclude an important fact about the primal.
Total		12.00 / 12.00	