

A pyramid of binomial coefficients

Pascal's triangle

$$\begin{array}{cccccccc} \binom{6}{0} & & \binom{6}{1} & & \binom{6}{2} & & \binom{6}{3} & & \binom{6}{4} & & \binom{6}{5} & & \binom{6}{6} \\ & \binom{5}{0} & & \binom{5}{1} & & \binom{5}{2} & & \binom{5}{3} & & \binom{5}{4} & & \binom{5}{5} \\ & & \binom{4}{0} & & \binom{4}{1} & & \binom{4}{2} & & \binom{4}{3} & & \binom{4}{4} \\ & & & \binom{3}{0} & & \binom{3}{1} & & \binom{3}{2} & & \binom{3}{3} \\ & & & & \binom{2}{0} & & \binom{2}{1} & & \binom{2}{2} \\ & & & & & \binom{1}{0} & & \binom{1}{1} \\ & & & & & & \binom{0}{0} \end{array}$$

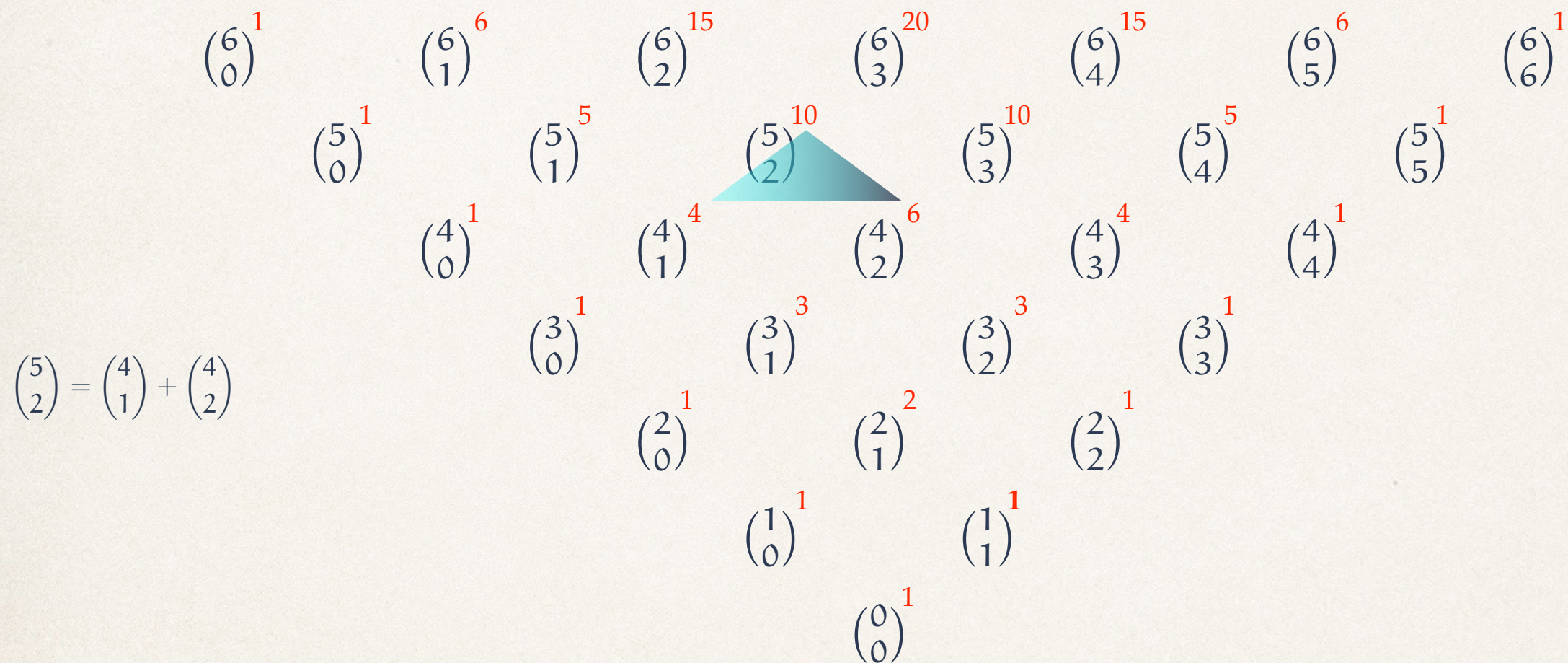
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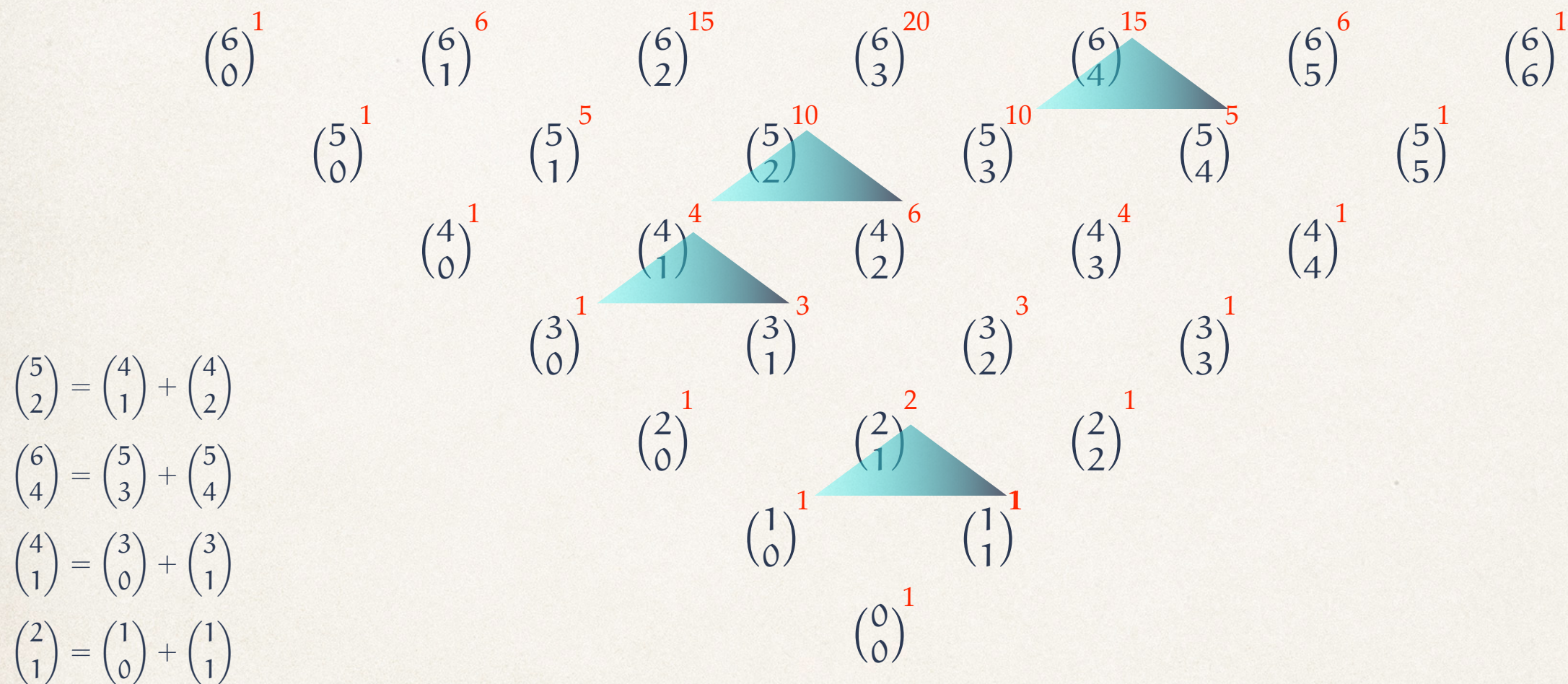
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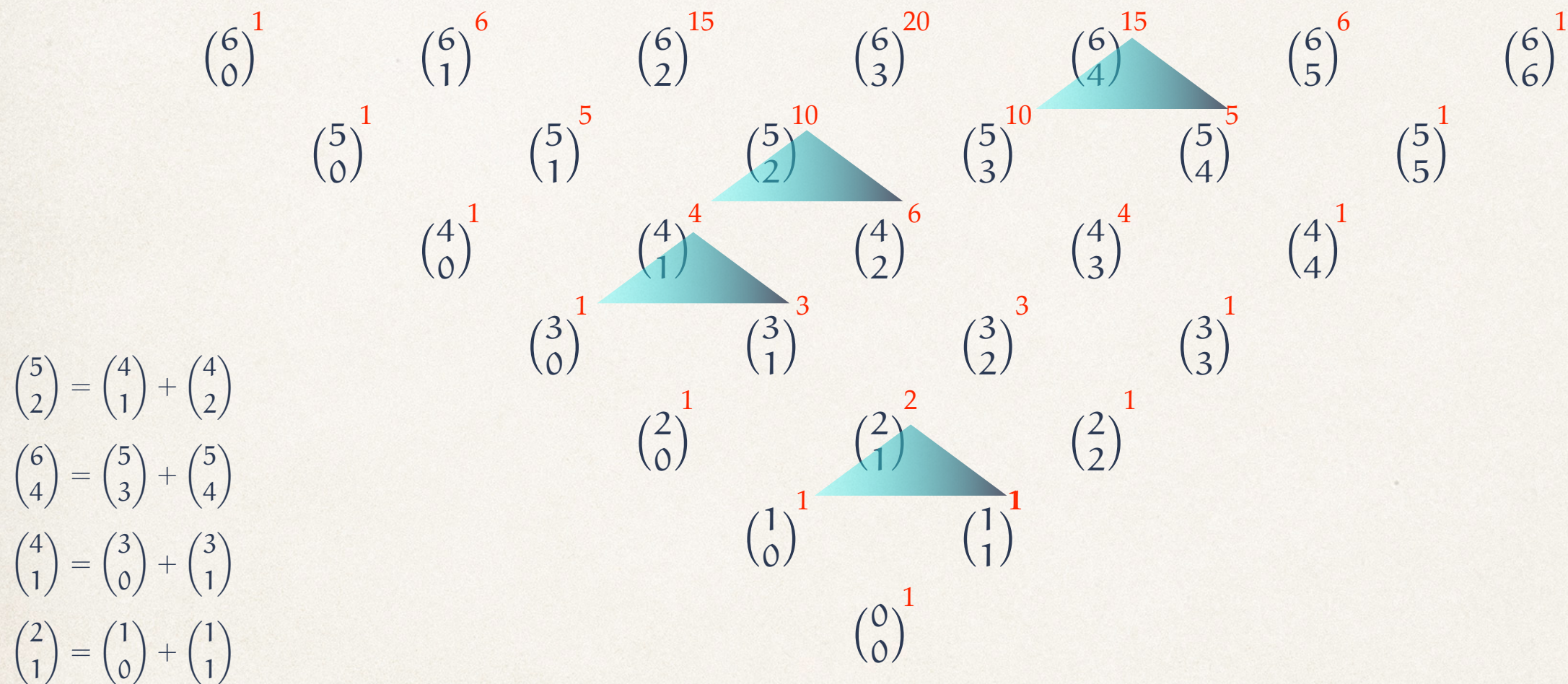
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$$\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$$

Verifying Pascal's triangle

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