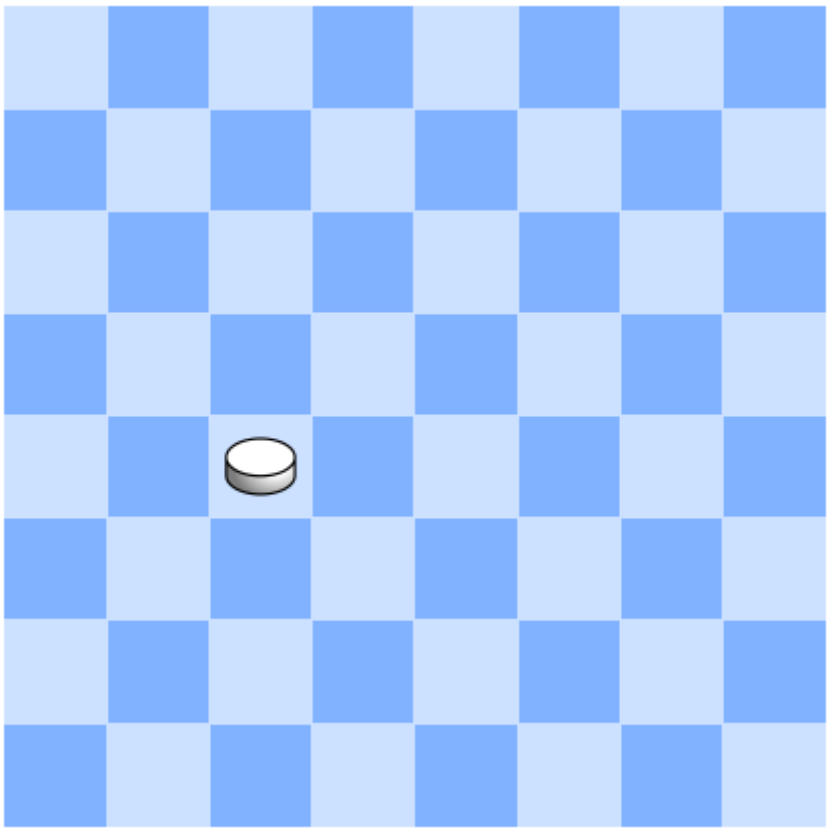
Even and Odd Numbers

Recall that integers divisible by 2 are called *even* (examples: $6, -8, 0$), while all others are called *odd* (examples: $-3, 7, 19$). This basic property of integers, called *parity*, finds applications in many problems.

Problem. A piece on a chessboard can move up, down, left, or right. Can it return to the original position after 17 moves? What about 18 moves?



The first thing to note is that the piece can return to its original position in principle: just move one step (say, to the right) and then move back. This means that returning back in 18 steps is also possible: repeat the previous strategy nine times.

For the case of 17 steps the same argument does not work: 17 is an odd number. As usual, this *does not* mean that this is impossible in general: it is still not excluded that there exists some other strategy.

You may notice that with each move, the color of the current cell changes. If the piece starts on a light cell, in one step it will be on a dark cell, in two steps it will be on a light cell again, and so on. Hence