Feedback — Quiz 1: Bayesian Analysis (manual)

Help Center

You submitted this quiz on **Wed 24 Jul 2013 10:56 AM PDT**. You got a score of **8.00** out of **8.00**.

- **Overview:**

  In this exercise you will use Bayesian statistics to learn about model parameters. The example is the same that was previously examined in the lecture on maximum likelihood.

- **Data:**

  Coin flipped *[Math Processing Error]* times, heads came up *[Math Processing Error]* times

- **Model:**

  In any flip of the coin, there is probability *[Math Processing Error]* of getting heads and *[Math Processing Error]* of getting tails. The general formula for the probability of getting *[Math Processing Error]* heads out of *[Math Processing Error]* flips is given by:

  [Math Processing Error]

  The first part of this expression gives the number of ways one can get *[Math Processing Error]* heads in *[Math Processing Error]* flips (the "binomial coefficient"). The last part gives the probability of getting *[Math Processing Error]* heads, each with probability *[Math Processing Error]*, and *[Math Processing Error]* tails each with probability *[Math Processing Error]*.

  Recall that *[Math Processing Error]* (the factorial of N) is defined as follows:

  [Math Processing Error]

- **Goal:**

  We want to estimate *[Math Processing Error]* from the observed data using Bayesian statistics. To simplify matters a bit, we will work in a limited parameter space where we only consider the following discrete values for *[Math Processing Error]*:

  [Math Processing Error]

- **Prepare table for keeping track of computations:**

Prepare a table similar to the one shown below. You can either download and print the table using the link, or you can construct your own on a piece of paper, or in a computer file (for instance, you could use a spreadsheet program, and also do the computations there). Note that we have already specified values for the prior probabilities that you should use for the different possible values of *[Math Processing Error]*.

Table for computations: [Download link]

Model (p)	Prior Probability	Likelihood	Prior x Likelihood	Posterior Probability
0.1	0.05			
0.3	0.20			
0.5	0.50			
0.7	0.20			
0.9	0.05			
Sum	1.00			

Question 1

- **Compute likelihoods:**

  Using the equation above, compute the likelihood for each possible model. Recall that the possible models are *[Math Processing Error]*. The likelihood of one of these models is the probability of getting the observed data (*[Math Processing Error]*) given the model. Thus, the likelihood of *[Math Processing Error]* is *[Math Processing Error]*. Enter the results in column 3 in the table (labeled "Likelihood"). Also compute the sum of the likelihoods and enter the value below the column.

- **Question:**

  Based on this column, what is the maximum likelihood estimate of p?

You entered:

0.7

Your Answer		Score	Explanation
0.7	✔	1.00	
Total		1.00 / 1.00	

Question Explanation

The maximum likelihood estimate of a parameter value, is the parameter value having the highest likelihood.

Question 2

What is the sum of the likelihoods (use at least two significant digits)?

You entered:

0.4504215

Your Answer		Score	Explanation
0.4504215	✔	1.00	
Total		1.00 / 1.00	

Question Explanation

Note that the likelihoods do not have to sum to 1. Each likelihood is the probability of getting the data (7 heads) for one specific possible value of p. If we were to compute the probabilities of all possible outcomes (0 to 10 heads) for any given value of p, then those would sum to 1 (it is a probability distribution), but the likelihoods are essentially single values from a set of different distributions, and therefore do not have to sum to anything meaningful.

Question 3

- **Compute the product of priors and likelihoods:**

  For each row in the table, compute the product of the prior and the likelihood for that value of *[Math Processing Error]* (multiply column 2 by column 3). Enter the value in column 4 (labeled "Prior x Likelihood"). Also compute the sum of column 4.

- **Compute posterior probabilities:**

  For each row in the table, compute the posterior probability by dividing the number in column 4 (product of prior and likelihood) by the sum of column 4. Enter the values in column 5 (labeled "Posterior probabilities"). Also compute the sum of this column.

- **Question:**

  What is the sum of the posterior probabilities?

You entered:

1.0

Your Answer		Score	Explanation
1.0	✔	1.00	
Total		1.00 / 1.00	

Question Explanation

The posterior probabilities form a probability distribuiton, so they have to sum to the same value that all probability distributions sum to.

Question 4

- **Use posterior distribution to learn about parameter value:**

  In frequentist statistics, the goal is to obtain good point estimates of parameter values. In Bayesian statistics, the goal is instead to obtain a probability distribution over all possible parameter values. This is the posterior probability distribution. It shows how uncertain we are about the parameter value, and can be used as the basis for asking many different questions. Use the posterior to answer the following questions

- **Question:**

  What is the "maximum a posteriori" (MAP) estimate of the parameter value? (The MAP is simply the value that has the highest posterior probability).

You entered:

0.5

Your Answer		Score	Explanation
0.5	✔	1.00	
Total		1.00 / 1.00	

Question 5

What is the posterior probability that *[Math Processing Error]* (use at least two significant digits)?

You entered:

0.5023904951929066

Your Answer		Score	Explanation
0.5023904951929066	✔	1.00	
Total		1.00 / 1.00	

Question 6

What is the posterior probability that *[Math Processing Error]* (use at least two significant digits)?

You entered:

0.0154400878232

Your Answer		Score	Explanation
0.0154400878232	✔	1.00	
Total		1.00 / 1.00	

Question Explanation

The posterior probability that *[Math Processing Error]* can be found as the sum of the posterior probabilities for those possible parameter

values that are less than 0.5 (i.e., *[Math Processing Error]* and *[Math Processing Error]* in this case)

Question 7

What is the posterior probability that *[Math Processing Error]* (use at least two significant digits)?

You entered:

0.975390367672

Your Answer		Score	Explanation
0.975390367672	✔	1.00	
Total		1.00 / 1.00	

Question Explanation

The posterior probability that *[Math Processing Error]* can be found as the sum of the posterior probabilities for those possible parameter values that are greater than or equal to 0.3, and less than or equal to 0.7 (i.e., *[Math Processing Error]*, *[Math Processing Error]*, and *[Math Processing Error]* in this case)

Question 8

- **Find a 95% credible set:**

  Order the parameter values according to decreasing posterior probability. (The *[Math Processing Error]* with the highest probability is number 1, etc.). Include parameter values until the cumulated posterior probability is *[Math Processing Error]*. The values of *[Math Processing Error]* that were included make up a 95% credible set - the Bayesian version of a confidence interval.

- **Question:**

  List the p values that are included in your 95% credible set, separated by spaces, in order of decreasing posterior probability.

You entered:

0.5 0.7

Your Answer		Score	Explanation
0.5	✔	0.50	
0.7	✔	0.50	
Total		1.00 / 1.00	