Feedback — Least-Squares Assignment

Help Center

You submitted this homework on **Sat 9 Nov 2013 12:03 PM PST**. You got a score of **13.00** out of **13.00**.

Question 1

Which of the following (A, b) pair solve the least squares problem (J = $||Ax - b||_2^2$) with the cost function
$J = (x_1 - 2x_2 + x_3 + 1)^2 + (x_2 - 2x_3 + x_4)^2 + (x_3 - 5)^2$

Your Answer		Score	Explanation
$A = \begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}$ and $b = \begin{bmatrix} 1 \\ 0 \\ -5 \end{bmatrix}$			
$A = \begin{bmatrix} 1 & -2 & 1 & 0 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$ and $b = \begin{bmatrix} -1 \\ 0 \\ 5 \end{bmatrix}$	✔	4.00	Correct Answer
$A = \begin{bmatrix} 1 & -2 & 1 & 0 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$ and $b = \begin{bmatrix} 1 \\ 0 \\ -5 \end{bmatrix}$			
$A = \begin{bmatrix} 1 & -2 & 1 & 0 \\ 0 & 1 & -2 & 1 \end{bmatrix}$ and $b = \begin{bmatrix} 1 \\ -5 \end{bmatrix}$			
Total		4.00 / 4.00	

Question Explanation

As an example, the cost function $J=(x_2-3x_3+3)^2+(x_1-x_2)^2$ would correspond to $A=\begin{bmatrix}0 & 1 & -3\\ 1 & -1 & 0\end{bmatrix}$ and $b=\begin{bmatrix}-3\\ 0\end{bmatrix}$

Question 2

The following cost function is related to the smoothing (de-noising) problem shown in lecture: $J=||x-x_{\text{cor}}||^2+\mu\sum_{k=2}^{n-1}(x_{k-1}-2x_k+x_{k+1})^2$. In this cost function, the smoothing term ($(\mu\sum_{k=2}^{n-1}(x_{k-1}-2x_k+x_{k+1})^2$) is using a slightly different representation of the derivative than the problem presented in lecture (center-difference vs. backward-difference). Please choose the appropriate D matrix that will correctly represent this version of the smoothing term:

Your Answer		Score	Explanation
$D=\mu\begin{bmatrix}1 & -2 & 1 & 0 & \cdots & 0\\ 0 & 1 & -2 & 1 & \cdots & 0\\ & & \ddots & \ddots & \ddots & \\ 0 & \cdots & 0 & 1 & -2 & 1\end{bmatrix}$			
$D=\sqrt{\mu}\begin{bmatrix}1 & -2 & 1 & 0 & \cdots & 0\\ 0 & 1 & -2 & 1 & \cdots & 0\\ & & \ddots & \ddots & \ddots & \\ 0 & \cdots & 0 & 1 & -2 & 1\end{bmatrix}$	✔	4.00	
$D=\sqrt{\mu}\begin{bmatrix}-1 & 2 & -1 & 0 & \cdots & 0\\ 0 & -1 & 2 & -1 & \cdots & 0\\ & & \ddots & \ddots & \ddots & \\ 0 & \cdots & 0 & -1 & 2 & -1\end{bmatrix}$			

$$D = \sqrt{\mu}\begin{bmatrix} 2 & -1 & 2 & 0 & \cdots & 0 \\ 0 & 2 & -1 & 2 & \cdots & 0 \\ & & \ddots & \ddots & \ddots & \\ 0 & \cdots & 0 & 2 & -1 & 2 \end{bmatrix}$$

Total	4.00 / 4.00

Question Explanation

If we write out the smoothing terms we would have $(\sqrt{\mu}(x_1 - 2x_2 + x_3))^2$ as our first term. Try to continue writing by hand a another few terms.

Question 3

In this problem, we want to best-fit a cubic polynomial $y(t) = c_0 + c_1 t + c_2 t^2 + c_3 t^3$ given a set of n noisy data points (t_k, y_k). This is similar to the classic line-fitting least-squares problem where the residual is $r_i = y(t_i) - y_i$ but our variables are ($x = [c_0\,, c_1\,, c_2\,, c_3]^T$) rather than the classic ($x = [m, b]^T$). We are still minimizing $||r||^2 = ||Ax - b||^2$. Pick the appropriate A and b matrices that represent this problem:

Your Answer		Score	Explanation
$A = \begin{bmatrix} 1 & t_1 & t_1^2 & t_1^3 \\ 1 & t_2 & t_2^2 & t_2^3 \\ & & \vdots & \\ 1 & t_n & t_n^2 & t_n^3 \end{bmatrix}; b = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}$	✔	4.00	

$$A = \begin{bmatrix} 1 & t_1 & t_1^2 & t_1^3 \\ 1 & t_2 & t_2^2 & t_2^3 \\ & & \vdots & \\ 1 & t_{n-1} & t_{n-1}^2 & t_{n-1}^3 \end{bmatrix} ; b = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_{n-1} \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & t_1 & t_1^2 \\ 1 & t_2 & t_2^2 \\ & \vdots & \\ 1 & t_n & t_n^2 \end{bmatrix} ; b = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}$$

$$A = \begin{bmatrix} t_1^3 & t_1^2 & t_1 & 1 \\ t_2^3 & t_2^2 & t_2 & 1 \\ & & \vdots & \\ t_n^3 & t_n^2 & t_n & 1 \end{bmatrix} ; b = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}$$

Total 4.00 / 4.00

Question Explanation

If we write out the first term of the cost function we get $||r||^2 = (c_0 + c_1 t_i + c_2 t_i^2 + c_3 t_i^3 - y_i)^2 \cdots$ which should then be reformulated to be $||r||^2 = (a_1^T x - b_1)^2$ where a_1^T is the first row of A and b_1 is the first term of b.

Question 4

In this problem, we want to best-fit a quartic polynomial $y(t) = c_0 + c_1 t + c_2 t^2 + c_3 t^3 + c_4 t^4$ to data. The data (found here) has 4001 points where the first column of the data represents t_i and the second column of the data represents y_i. Please enter the value you found for c_1 to 2 decimal places

You entered:

6.86

Your Answer		Score	Explanation
6.86	✔	1.00	The solution found using Matlab is: Sol = [3.1052 6.8643 3.9999 -0.9754 -0.0047] where $c_1 = 6.8643$
Total		1.00 / 1.00	

Question Explanation

the A and b matrices can be formulated very similarly to the previous question but with an extra column since the fitting function is a quartic rather than a cubic.