# Swing a Pendulum to the Top

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The dimensionless, unforced pendulum equation is given by

$\ddot{\theta} + \alpha\dot{\theta} + \sin\theta = 0,$

where $\alpha$ is the only free parameter.

Consider initial conditions with the mass at the bottom, $\theta(0) = 0$. Using the shooting method, determine the smallest positive value of $\dot{\theta}(0)$ such that the mass becomes exactly balanced at the top ($\theta = \pi$). Plot this value of $\dot{\theta}(0)$ versus $\alpha$ for $0 \le \alpha \le 2$.

## Script

Reference Solution | Save | Reset | MATLAB Documentation (https://www.mathworks.com/help/)

Your Solution | Reference Solution

```
theta0=0; u0=2; %initial ode conditions. u0 is initial guess for root.
inf=8*pi; %inf is a large number.  Takes a long time to get to top.
tspan=[0 inf];
options = odeset('RelTol',1.e-6);
%rootfind u0 such that theta(inf)=pi
alpha_i=linspace(0, 2, 100);
u0_i=zeros(100,1);
for i=1:length(alpha_i)
    alpha=alpha_i(i);
    u0_i(i) = fzero(@(u0) F(tspan,theta0,u0,alpha,options), u0);
end
plot(alpha_i, u0_i);
xlabel('$\alpha$','Interpreter','latex','FontSize',14);
ylabel('$d \theta/dt$','Interpreter','latex','FontSize',14);
title('Shooting to the Pendulum Top','Interpreter','latex','FontSize',16);

function y=F(tspan,theta0,u0,alpha,options)
% use ode45 to define the root-finding problem
    [t,theta_u]=ode45(@(t,theta_u) pendulum(theta_u,alpha),tspan,[theta0;u0],options);
    theta=theta_u(:,1); u=theta_u(:,2);
    y=theta(end)-pi;
end

function d_theta_u_dt = pendulum(theta_u,alpha)
% define the differential equation here
    theta=theta_u(1); u=theta_u(2);
    d_theta_u_dt=[u;-alpha*u-sin(theta)];
end
```

Run Script

## Assessment: All Tests Passed

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Test u0_i

## Output