Specification and Diagnostic Tests

Empirical research is usually an interactive process. The process begins with a specification of the relationship to be estimated. Selecting a specification usually involves several choices: the variables to be included, the functional form connecting these variables, and if the data are time series, the dynamic structure of the relationship between the variables.

Inevitably, there is uncertainty regarding the appropriateness of this initial specification. Once you estimate your equation, EViews provides tools for evaluating the quality of your specification along a number of dimensions. In turn, the results of these tests influence the chosen specification, and the process is repeated.

Here we describe the extensive menu of specification test statistics that are available as views or procedures of an equation object. While we attempt to provide you with sufficient statistical background to conduct the tests, practical considerations ensure that many of the descriptions are incomplete. We refer you to standard statistical and econometric references for further details.

Background

Each test procedure described below involves the specification of a null hypothesis, which is the hypothesis under test. Output from a test command consists of the sample values of one or more test statistics and their associated probability numbers (*p*-values). The latter indicate the probability of obtaining a test statistic whose absolute value is greater than or equal to that of the sample statistic if the null hypothesis is true. Thus, low *p*-values lead to the rejection of the null hypothesis. For example, if a *p*-value lies between 0.05 and 0.01, the null hypothesis is rejected at the 5 percent but not at the 1 percent level.

Bear in mind that there are different assumptions and distributional results associated with each test. For example, some of the test statistics have exact, finite sample distributions (usually t or F-distributions). Others are large sample test statistics with asymptotic χ^2 distributions. Details vary from one test to another and are given below in the description of each test.

Types of Tests

The **View** button on the equation toolbar gives you a choice among three categories of tests to check the specification of the equation. Additional tests are discussed elsewhere: these include <u>unit root tests</u>, the <u>Granger causality test</u>, tests specific to <u>binary</u>, <u>ordered</u>, <u>censored</u>, and <u>count models</u>, and the <u>Johansen's test for cointegration</u>.

Coefficient Tests

These tests evaluate restrictions on the estimated coefficients, including the special case of tests for omitted and redundant variables.

Wald Test — Coefficient Restrictions

The Wald test computes the test statistic by estimating the unrestricted regression without imposing the coefficient restrictions specified by the null hypothesis. The Wald statistic measures how close the unrestricted estimates come to satisfying the restrictions under the null hypothesis. If the restrictions are in fact true, then the unrestricted estimates should come close to satisfying the restrictions.

We first give a general formula for the computation of the Wald test statistic; users who are familiar with the formula or wish to skip the technical details can skip this subsection.

Details

Consider a general nonlinear regression model

$$y = x(\beta) + \epsilon$$

where β is a k vector of parameters to estimate. Any restrictions on the parameters can be written as

$$H_0$$
: $g(\beta) = 0$

where g is a smooth q dimensional vector imposing q restrictions on β . The Wald statistic is then

computed as

$$W = ng(b)' \left(\frac{\partial g}{\partial \beta} V \frac{\partial g}{\partial \beta'} \right)^{-1} g(b)$$

where *n* is the number of observations and *b* is the unrestricted parameter estimates. *V* is the estimated variance of b given by

$$V = ns^2 \left(\frac{\partial x}{\partial \beta} \frac{\partial x}{\partial \beta'} \right)^{-1}, \quad s^2 = \frac{u'u}{n-k}$$

where *u* is the unrestricted residuals.

More formally, under the null hypothesis H_0 , the Wald statistic has an asymptotic $\chi^2(q)$ distribution, where a is the number of restrictions under H_0 .

The Wald test can be applied to equations estimated by least squares, two-stage least squares, nonlinear least squares, binary, ordered, censored, truncated, and count models, and systems estimators. The Wald test is the only test described here that can be applied to equations estimated by system methods.

For the special case of a linear regression model

$$y = X\beta + \epsilon$$

and linear restrictions

$$H_0: R\beta - r = 0$$

where R is a known $q \times k$ matrix, and r is a q vector, respectively, the Wald statistic reduces to

$$W = (Rb - r)' \left(s^2 R(X'X)^{-1} R' \right)^{-1} (Rb - r)$$

which is asymptotically distributed as $\chi^2(q)$ under H_0 .

If we further assume that the errors ε are independent and identically normally distributed, we have an exact, finite sample *F*-statistic:

$$F = \frac{\left(\tilde{u}'\tilde{u} - u'u\right)/q}{u'u/(n-k)} = W/q$$

where \bar{a} is the vector of residuals from the restricted regression. The F-statistic compares the residual sum of squares computed with and without the restrictions imposed. If the restrictions are valid, there should be little difference in the two residual sum of squares and the F-value should be small.

EViews reports both the chi-square and the F-statistics and the associated p-values.

How to Perform Wald Coefficient Tests

To demonstrate how to perform Wald tests, we consider an example. Suppose a Cobb-Douglas production function has been estimated in the form:

$$\log Q = A + \alpha \log L + \beta \log K + \epsilon$$

where Q, K and L denote value-added output and the inputs of capital and labor respectively. The hypothesis of constant returns to scale is then tested by the restriction: $\alpha + \beta = 1$.

Estimation of the Cobb-Douglas production function using annual data from 1947 to 1971 provided the following result:

Dependent Variable: LOG(Q) Method: Least Squares Date: 08/11/97 Time: 16:56 Sample: 1947 1971

Included observations: 25

Variable	Coefficient	Std. Error	t-Statistic	Prob.
С	-2.327939	0.410601	-5.669595	0.0000
LOG(L)	1.591175	0.167740	9.485970	0.0000
LOG(K)	0.239604	0.105390	2.273498	0.0331

The sum of the coefficients on LOG(L) and LOG(K) appears to be in excess of one, but to determine whether the difference is statistically relevant, we will conduct the hypothesis test of constant returns.

To carry out a Wald test, choose **View/Coefficient Tests/Wald-Coefficient Restrictions...** from the equation toolbar. Enter the restrictions into the edit box, with multiple coefficient restrictions separated by commas. The restrictions should be expressed as equations involving the estimated coefficients and constants (you may not include series names). The coefficients should be referred to as C(1), C(2), and so on, unless you have used a different coefficient vector in estimation.

To test the hypothesis of constant returns to scale, type the following restriction in the dialog box:

```
c(2) + c(3) = 1
```

and click **OK**. EViews reports the following result of the Wald test:

 Wald Test:

 Equation: EQ1

 Null Hypothesis:
 C(2)+C(3)=1

 F-statistic
 120.0177
 Probability
 0.000000

 Chi-square
 120.0177
 Probability
 0.000000

EViews reports an F-statistic and a Chi-square statistic with associated p-values. The Chi-square statistic is equal to the F-statistic times the number of restrictions under test. In this example, there is only one restriction and so the two test statistics are identical with the p-values of both statistics indicating that we can decisively reject the null hypothesis of constant returns to scale.

To test more than one restriction, separate the restrictions by commas. For example, to test the hypothesis that the elasticity of output with respect to labor is 2/3 and with respect to capital is 1/3, type the restrictions as

```
c(2)=2/3, c(3)=1/3
```

and EViews reports

Wald Test:

Equation: DEMAND

Null Hypothesis: C(2)=2/3
C(1)=1/3

F-statistic 385 6769 Proba

F-statistic 385.6769 Probability 0.000000 Chi-square 771.3538 Probability 0.000000

As an example of a nonlinear model with a nonlinear restriction, we estimate a production function of the form

$$\log Q = \beta_1 + \beta_2 \log(\beta_3 K^{\beta_4} + (1-\beta_3) L^{\beta_4})$$

and test the constant elasticity of substitution (CES) production function restriction $\beta_2 = 1/\beta_4$. This is an example of a nonlinear restriction. To estimate the (unrestricted) nonlinear model, you should select **Quick/Estimate Equation...** and then enter the following specification:

```
\log(q) = c(1) + c(2)*\log(c(3)*k^c(4)+(1-c(3))*l^c(4))
```

To test the nonlinear restriction, choose **View/Coefficient Tests/Wald-Coefficient Restrictions...** from the equation toolbar and type the following restriction in the Wald Test dialog box:

```
c(2)=1/c(4)
```

The results are presented below:

```
      Wald Test:

      Equation: EQ2

      Null Hypothesis:
      C(2)=1/C(4)

      F-statistic
      0.028507
      Probability
      0.867539

      Chi-square
      0.028507
      Probability
      0.865923
```

Since this is a nonlinear test, we focus on the Chi-square statistic which fails to reject the null hypothesis.

It is well-known that nonlinear Wald tests are not invariant to the way that you specify the nonlinear restrictions. In this example, the nonlinear restriction $\beta_2 = 1/\beta_4$ can equivalently be written as $\beta_2 \beta_4 = 1$ or $\beta_4 = 1/\beta_2$. For example, entering the restriction as

```
c(2)*c(4)=1
```

yields:

The test now decisively rejects the null hypothesis. We hasten to add that this is not a situation that is unique to EViews, but is a more general property of the Wald test. Unfortunately, there does not seem to be a general solution to this problem; see <u>Davidson and MacKinnon (1993</u>, Chapter 13) for further discussion and references.

An alternative is to use the LR test, which is invariant to reparameterization. To carry out the LR test, you compute both the unrestricted model (as estimated above) and the restricted model (imposing the nonlinear restrictions) and compare the log likelihood values. We will estimate the two specifications using the command form. The restricted model can be estimated by the entering the following command in the command window:

```
equation eq_ces0.ls log(q) = c(1) + c(2)*log(c(3)*k^{(1/c(2))}+(1-c(3))*l^{(1/c(2))})
```

while the unrestricted model is estimated using the command:

```
equation eq_ces1.ls log(q) = c(1) + c(2)*log(c(3)*k^c(4)+(1-c(3))*l^c(4))
```

Note that we save the two equations in the named equation objects EQ0 and EQ1. The LR test statistic can then be computed as

```
scalar lr=-2*(eq_ces0.@logl-eq_ces1.@logl)
```

To see the value of LR, double click on LR; the value will be displayed in the status line at the bottom of the EViews window. You can also compute the *p*-value of LR by the command

```
scalar lr_pval=1-@cchisq(lr,1)
```

Omitted Variables

This test enables you to add a set of variables to an existing equation and to ask whether the set makes a significant contribution to explaining the variation in the dependent variable. The null hypothesis H_0 is that the additional set of regressors are not jointly significant.

The output from the test is an F-statistic and a likelihood ratio (LR) statistic with associated p-values, together with the estimation results of the unrestricted model under the alternative. The F-statistic is based on the difference between the residual sums of squares of the restricted and unrestricted

regressions. The LR statistic is computed as

$$LR = -2(\ell^r - \ell^s)$$

where $\ell^{\mathfrak{s}}$ and $\ell^{\mathfrak{r}}$ are the maximized values of the (Gaussian) log likelihood function of the unrestricted and restricted regressions, respectively. Under H_0 , the LR statistic has an asymptotic χ^2 distribution with degrees of freedom equal to the number of restrictions, i.e. the number of added variables.

Bear in mind that:

- The omitted variables test requires that the same number of observations exist in the original and test equations. If any of the series to be added contain missing observations over the sample of the original equation (which will often be the case when you add lagged variables), the test statistics cannot be constructed.
- The omitted variables test can be applied to equations estimated with linear LS, TSLS, ARCH (mean equation only), binary, ordered, censored, truncated, and count models. The test is available only if you specify the equation by listing the regressors, not by a formula.

To perform an LR test in these settings, you can estimate a separate equation for the unrestricted and restricted models over a common sample, and evaluate the LR statistic and *p*-value using scalars and the <code>@cchisq</code> function, as described above.

How to Perform an Omitted Variables Test

To test for omitted variables, select View/Coefficient Tests/Omitted Variables-Likelihood Ratio... In the dialog that opens, list the names of the test variables, each separated by at least one space. Suppose, for example, that the initial regression is

```
ls log(q) c log(l) log(k)
```

If you enter the list

in the dialog, then EViews reports the results of the unrestricted regression containing the two additional explanatory variables, and displays statistics testing the hypothesis that the coefficients on the new variables are jointly zero. The top part of the output depicts the test results:

Omitted Variables: LOG(M) LOG(E)

 F-statistic
 4.267478
 Probability
 0.028611

 Log likelihood ratio
 8.884940
 Probability
 0.011767

The F-statistic has an exact finite sample F-distribution under H_0 if the errors are independent and identically distributed normal random variables. The numerator degrees of freedom is the number of additional regressors and the denominator degrees of freedom is the number of observations less the total number of regressors. The log likelihood ratio statistic is the LR test statistic and is asymptotically distributed as a χ^2 with degrees of freedom equal to the number of added regressors.

In our example, the tests reject the null hypothesis that the two series do not belong to the equation at a 5% significance level, but cannot reject the hypothesis at a 1% significance level.

Redundant Variables

The redundant variables test allows you to test for the statistical significance of a subset of your included variables. More formally, the test is for whether a subset of variables in an equation all have zero coefficients and might thus be deleted from the equation. The redundant variables test can be applied to equations estimated by linear LS, TSLS, ARCH (mean equation only), binary, ordered, censored, truncated, and count methods. The test is available only if you specify the equation by listing the regressors, not by a formula.

How to Perform a Redundant Variables Test

To test for redundant variables, select View/Coefficient Tests/Redundant Variables-Likelihood

Ratio... In the dialog that appears, list the names of each of the test variables, separated by at least one space. Suppose, for example, that the initial regression is

```
ls log(q) c log(l) log(k) log(m) log(e)
```

If you type the list

```
log(m) log(e)
```

in the dialog, then EViews reports the results of the restricted regression dropping the two regressors, followed by the statistics associated with the test of the hypothesis that the coefficients on the two variables are jointly zero.

The test statistics are the F-statistic and the Log likelihood ratio. The F-statistic has an exact finite sample F-distribution under H_0 if the errors are independent and identically distributed normal random variables. The numerator degrees of freedom are given by the number of coefficient restrictions in the null hypothesis. The denominator degrees of freedom are given by the total regression degrees of freedom. The LR test is an asymptotic test, distributed as a χ^2 with degrees of freedom equal to the number of excluded variables under H_0 . In this case, there are two degrees of freedom.

Residual Tests

EViews provides tests for serial correlation, normality, heteroskedasticity, and autoregressive conditional heteroskedasticity in the residuals from your estimated equation. Not all of these tests are available for every specification.

Correlograms and Q-statistics

This view displays the autocorrelations and partial autocorrelations of the equation residuals up to the specified number of lags. Further details on these statistics and the Ljung-Box Q-statistics that are also computed are provided in Series Views, <u>Correlogram</u>.

This view is available for the residuals from least squares, two-stage least squares, nonlinear least squares and binary, ordered, censored, and count models. In calculating the probability values for the Q-statistics, the degrees of freedom are adjusted to account for estimated ARMA terms.

To display the correlograms and Q-statistics, push View/Residual Tests/Correlogram-Q-statistics on the equation toolbar. In the Lag Specification dialog box, specify the number of lags you wish to use in computing the correlogram.

Correlograms of Squared Residuals

This view displays the autocorrelations and partial autocorrelations of the squared residuals up to any specified number of lags and computes the Ljung-Box *Q*-statistics for the corresponding lags. The correlograms of the squared residuals can be used to check autoregressive conditional heteroskedasticity (ARCH) in the residuals; see also <u>ARCH LM Test</u> below.

If there is no ARCH in the residuals, the autocorrelations and partial autocorrelations should be zero at all lags and the *Q*-statistics should not be significant; see Series Views, <u>Correlogram</u> for a discussion of the correlograms and *Q*-statistics.

This view is available for equations estimated by least squares, two-stage least squares, and nonlinear least squares estimation. In calculating the probability for *Q*-statistics, the degrees of freedom are adjusted for the inclusion of ARMA terms.

To display the correlograms and *Q*-statistics of the squared residuals, push **View/Residual Tests/Correlogram Squared Residuals** on the equation toolbar. In the Lag Specification dialog box that opens, specify the number of lags over which to compute the correlograms.

Histogram and Normality Test

This view displays a histogram and descriptive statistics of the residuals, including the Jarque-Bera statistic for testing normality. If the residuals are normally distributed, the histogram should be bell-shaped and the Jarque-Bera statistic should not be significant; see Series Views, <u>Jarque-Bera test</u> for further details. This view is available for residuals from least squares, two-stage least squares, nonlinear least squares, and binary, ordered, censored, and count models.

To display the histogram and Jarque-Bera statistic, select View/Residual

Tests/Histogram-Normality. The Jarque-Bera statistic has a χ^2 distribution with two degrees of freedom under the null hypothesis of normally distributed errors.

Serial Correlation LM Test

This test is an alternative to the Q-statistics for testing serial correlation. The test belongs to the class of asymptotic (large sample) tests known as Lagrange multiplier (LM) tests.

Unlike the Durbin-Watson statistic for AR(1) errors, the LM test may be used to test for higher order ARMA errors, and is applicable whether or not there are lagged dependent variables. Therefore, we recommend its use whenever you are concerned with the possibility that your errors exhibit autocorrelation.

The null hypothesis of the LM test is that there is no serial correlation up to lag order p, where p is a pre-specified integer. The local alternative is ARMA(r,q) errors, where the number of lag terms $p = \max\{r,q\}$. Note that the alternative includes both AR(p) and MA(p) error processes, and that the test may have power against a variety of autocorrelation structures. See <u>Godfrev (1988)</u> for a discussion.

The test statistic is computed by an auxiliary regression as follows: suppose you have estimated the regression

$$y_t = X_t b + e_t$$

where e are the residuals. The test statistic for lag order p is based on the regression

$$e_t = X_t \gamma + \alpha_1 e_{t-1} + \alpha_2 e_{t-2} + \dots + \alpha_p e_{t-p} + v_t$$

This is a regression of the residuals on the original regressors X and lagged residuals up to order p. EViews reports two test statistics from this test regression. The F-statistic is an omitted variable test for the joint significance of all lagged residuals. Because the omitted variables are residuals and not independent variables, the exact finite sample distribution of the F-statistic under H_0 is not known, but we still present the F-statistic for comparison purposes.

The Obs*R-squared statistic is the Breusch-Godfrey LM test statistic. This LM statistic is computed as the number of observations, times the (uncentered) \mathbb{R}^2 from the test regression. Under quite general conditions, the LM test statistic is asymptotically distributed as a χ^2 (p).

The serial correlation LM test is available for residuals from least squares or two-stage least squares. The original regression may include AR and MA terms, in which case the test regression will be modified to take account of the ARMA terms.

To carry out the test, push **View/Residual Tests/Serial Correlation LM Test...** on the equation toolbar and specify the highest order of the AR or MA process that might describe the serial correlation. If the test indicates serial correlation in the residuals, LS standard errors are invalid and should not be used for inference.

ARCH LM Test

This is a Lagrange multipler (LM) test for autoregressive conditional heteroskedasticity (ARCH) in the residuals (<u>Engle 1982</u>). This particular specification of heteroskedasticity was motivated by the observation that in many financial time series, the magnitude of residuals appeared to be related to the magnitude of recent residuals. ARCH in itself does not invalidate standard LS inference. However, ignoring ARCH effects may result in loss of efficiency; see <u>ARCH and GARCH Models</u> for a discussion of estimation of ARCH models in EViews.

The ARCH LM test statistic is computed from an auxiliary test regression. To test the null hypothesis that there is no ARCH up to order q in the residuals, we run the regression

$$e_t^2 = \beta_0 + \beta_1 e_{t-1}^2 + \beta_2 e_{t-2}^2 + \dots + \beta_q e_{t-q}^2 + v_t$$

where e is the residual. This is a regression of the squared residuals on a constant and lagged squared residuals up to order q. EViews reports two test statistics from this test regression. The F-statistic is an omitted variable test for the joint significance of all lagged squared residuals. The Obs*R-squared statistic is Engle's LM test statistic, computed as the number of observations times

the R^2 from the test regression. The exact finite sample distribution of the F-statistic under H_0 is not known but the LM test statistic is asymptotically distributed $\chi^2(q)$ under quite general conditions. The ARCH LM test is available for equations estimated by least squares, two-stage least squares, and nonlinear least squares.

To carry out the test, push **View/Residual Tests/ARCH LM Test...** on the equation toolbar and specify the order of ARCH to be tested against.

White's Heteroskedasticity Test

This is a test for heteroskedasticity in the residuals from a least squares regression (White, 1980). Ordinary least squares estimates are consistent in the presence heteroskedasticity, but the conventional computed standard errors are no longer valid. If you find evidence of heteroskedasticity, you should either choose the robust standard errors option to correct the standard errors (see <u>HAC</u>) or you should model the heteroskedasticity to obtain more efficient estimates using weighted least squares.

White's test is a test of the null hypothesis of no heteroskedasticity against heteroskedasticity of some unknown general form. The test statistic is computed by an auxiliary regression, where we regress the squared residuals on all possible (nonredundant) cross products of the regressors. For example, suppose we estimated the following regression:

$$y_t = b_1 + b_2 x_t + b_3 z_t + e_t$$

The test statistic is then based on the auxiliary regression:

$$e_t^2 = \alpha_0 + \alpha_1 x_t + \alpha_2 z_t + \alpha_2 x_t^2 + \alpha_4 z_t^2 + \alpha_5 x_t z_t + v_t$$

EViews reports two test statistics from the test regression. The *F*-statistic is an omitted variable test for the joint significance of all cross products, excluding the constant. It is presented for comparison purposes.

The Obs*R-squared statistic is White's test statistic, computed as the number of observations times the centered R^2 from the test regression. The exact finite sample distribution of the *F*-statistic under H_0 is not known, but White's test statistic is asymptotically distributed as a χ^2 with degrees of freedom equal to the number of slope coefficients (excluding the constant) in the test regression.

White also describes this approach as a general test for model misspecification, since the null hypothesis underlying the test assumes that the errors are both homoskedastic and independent of the regressors, and that the linear specification of the model is correct. Failure of any one of these conditions could lead to a significant test statistic. Conversely, a non-significant test statistic implies that none of the three conditions is violated.

When there are redundant cross-products, EViews automatically drops them from the test regression. For example, the square of a dummy variable is the dummy variable itself, so that EViews drops the squared term to avoid perfect collinearity.

To carry out White's heteroskedasticity test, select **View/Residual Tests/White Heteroskedasticity**. EViews has two options for the test: cross terms and no cross terms. The cross terms version of the test is the original version of White's test that includes all of the cross product terms. However, with many right-hand side variables in the regression, the number of possible cross product terms becomes very large so that it may not be practical to include all of them. The no cross terms option runs the test regression using only squares of the regressors.

Specification and Stability Tests

EViews provides a number of test statistic views that examine whether the parameters of your model are stable across various subsamples of your data.

One recommended empirical technique is to split the T observations in your data set of observations into T_1 observations to be used for estimation, and $T_2 = T_1 - T$ observations to be used for testing and evaluation. Using all available sample observations for estimation promotes a search for a specification that best fits that specific data set, but does not allow for testing predictions of the model against data that have not been used in estimating the model. Nor does it allow one to test for parameter

constancy, stability and robustness of the estimated relationship. In time series work you will usually take the first T_1 observations for estimation and the last T_2 for testing. With cross section data you may wish to order the data by some variable, such as household income, sales of a firm, or other indicator variables and use a sub-set for testing.

There are no hard and fast rules for determining the relative sizes of T_1 and T_2 . In some cases there may be obvious points at which a break in structure might have taken place—a war, a piece of legislation, a switch from fixed to floating exchange rates, or an oil shock. Where there is no reason a priori to expect a structural break, a commonly used rule-of-thumb is to use 85 to 90 percent of the observations for estimation and the remainder for testing.

EViews provides built-in procedures which facilitate variations on this type of analysis.

Chow's Breakpoint Test

The idea of the breakpoint Chow test is to fit the equation separately for each subsample and to see whether there are significant differences in the estimated equations. A significant difference indicates a structural change in the relationship. For example, you can use this test to examine whether the demand function for energy was the same before and after the oil shock. The test may be used with least squares and two-stage least squares regressions.

To carry out the test, we partition the data into two or more subsamples. Each subsample must contain more observations than the number of coefficients in the equation so that the equation can be estimated using each subsample. The Chow breakpoint test is based on a comparison of the sum of squared residuals obtained by fitting a single equation to the entire sample with the sum of squared residuals obtained when separate equations are fit to each subsample of the data.

EViews reports two test statistics for the Chow breakpoint test. The *F*-statistic is based on the comparison of the restricted and unrestricted sum of squared residuals and in the simplest case involving a single breakpoint, is computed as

$$F = \frac{(\tilde{u}'\tilde{u} - u_1'u_1 - u_2'u_2) / k}{(u_1'u_1 + u_2'u_2) / (T - 2k)}$$

where $\bar{u}'\bar{u}$ is the restricted sum of squared residuals, $u'_{\bar{u}}u_{\bar{u}}$ is the sum of squared residuals from subsample i, T is the total number of observations, and k is the number of parameters in the equation. This formula can be generalized naturally to more than one breakpoint. The F-statistic has an exact finite sample F-distribution if the errors are independent and identically distributed normal random variables.

The log likelihood ratio statistic is based on the comparison of the restricted and unrestricted maximum of the (Gaussian) log likelihood function. The LR test statistic has an asymptotic χ^2 distribution with degrees of freedom equal to (m-1)k under the null hypothesis of no structural change, where m is the number of subsamples.

One major drawback of the breakpoint test is that each subsample requires at least as many observations as the number of estimated parameters. This may be a problem if, for example, you want to test for structural change between wartime and peacetime where there are only a few observations in the wartime sample. The <u>Chow forecast test</u>, discussed below, should be used in such cases.

To apply the Chow breakpoint test, push **View/Stability Tests/Chow Breakpoint Test...** on the equation toolbar. In the dialog that appears, list the dates or observation numbers for the breakpoints. For example, if your original equation was estimated from 1950 to 1994, entering

1960

in the dialog specifies two subsamples, one from 1950 to 1959 and one from 1960 to 1994. Typing

1960 1970

specifies three subsamples, 1950 to 1959, 1960 to 1969, and 1970 to 1994.

Chow's Forecast Test

The Chow forecast test estimate sthe model for a subsample comprised of the first T_1 observations. The estimated model is then used to predict the values of the dependent variable in the remaining T_2 data points. A large difference between the actual and predicted values casts doubt on the stability of the estimated relation over the two subsamples. The Chow forecast test can be used with least squares and two-stage least squares regressions.

EViews reports two test statistics for the Chow forecast test. The F-statistic is computed as

$$F = \frac{\left(\bar{u}'\bar{u} - u'u\right) / T_2}{u'u / (T_1 - k)}$$

where $\bar{u}'\bar{u}$ is the residual sum of squares when the equation is fitted to all T sample observations, u'u is the residual sum of squares when the equation is fitted to T_1 observations, and k is the number of estimated coefficients. This F-statistic has an exact finite sample F-distribution only if the errors are independent, and identically,normally distributed.

The log likelihood ratio statistic is based on the comparison of the restricted and unrestricted maximum of the (Gaussian) log likelihood function. Both the restricted and unrestricted log likelihood are obtained by estimating the regression using the whole sample. The restricted regression uses the original set of regressors, while the unrestricted regression adds a dummy variable for each forecast point. The LR test statistic has an asymptotic χ^2 distribution with degrees of freedom equal to the number of forecast points T_2 under the null hypothesis of no structural change.

To apply Chow's forecast test, push **View/Stability Tests/Chow Forecast Test...** on the equation toolbar and specify the date or observation number for the beginning of the forecasting sample. The date should be within the current sample of observations.

As an example, suppose we estimate a consumption function using quarterly data from 1947:1 to 1994:4 and specify 1973:1 as the first observation in the forecast period. The test reestimates the equation for the period 1947:1 to 1972:4, and uses the result to compute the prediction errors for the remaining quarters, and reports the following results:

 Chow Forecast Test: Forecast from 1973:1 to 1994:4
 0.708348
 Probability
 0.951073

 Log likelihood ratio
 91.57088
 Probability
 0.376108

Neither of the forecast test statistics reject the null hypothesis of no structural change in the consumption function before and after 1973:1.

If we test the same hypothesis using the Chow breakpoint test, the result is

 Chow Breakpoint Test: 1973:1
 38.39198
 Probability
 0.000000

 Log likelihood ratio
 65.75468
 Probability
 0.000000

Note that both of the breakpoint test statistics decisively reject the hypothesis from above. This example illustrates the possibility that the two Chow tests may yield conflicting results.

Ramsey's RESET Test

RESET stands for *Regression Specification Error Test* and was proposed by <u>Ramsey (1969)</u>. The classical normal linear regression model is specified as

$$y = X\beta + \epsilon$$

where the disturbance vector ε is presumed to have the multivariate normal distribution $N(0, \sigma^2 I)$. Specification error is an omnibus term which covers any departure from the assumptions of the maintained model. Serial correlation, heteroskedasticity, or non-normality of ε all violate the assumption that the disturbances are distributed $N(0, \sigma^2 I)$. Tests for these specification errors have been described above. In contrast, RESET is a general test for the following types of specification

errors:

- Omitted variables; X does not include all relevant variables.
- Incorrect functional form; some or all of the variables in *y* and *X* should be transformed to logs, powers, reciprocals, or in some other way.
- Correlation between X and ε , which may be caused by measurement error in X, simultaneous equation considerations, combination of lagged y values and serially correlated disturbances.

Under such specification errors, LS estimators will be biased and inconsistent, and conventional inference procedures will be invalidated. Ramsey (1969) showed that any or all of these specification errors produce a non-zero mean vector for e. Therefore, the null and alternative hypotheses of the RESET test are

$$H_0$$
: $\epsilon \sim N(0, \sigma^2 I)$
 H_1 : $\epsilon \sim N(\mu, \sigma^2 I)$, $\mu \neq 0$

The test is based on an augmented regression

$$y = X\beta + Z\gamma + \epsilon$$

The test of specification error evaluates the restriction $\gamma = 0$. The crucial question in constructing the test is to determine what variables should enter the Z matrix. Note that the Z matrix may, for example, be comprised of variables that are not in the original specification, so that the test of $\gamma = 0$ is simply the omitted variables test described above.

In testing for incorrect functional form, the nonlinear part of the regression model may be some function of the regressors included in *X*. For example, if a linear relation

$$y=\beta_1+\beta_2X+\epsilon$$

is specified instead of the true relation

$$y = \beta_1 + \beta_2 X + \beta_2 X^2 + \epsilon$$

the augmented model has $Z = X^2$ and we are back to the omitted variable case. A more general example might be the specification of an additive relation

$$y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \epsilon$$

instead of the (true) multiplicative relation

$$y = \beta_0 X_1^{\beta_1} X_2^{\beta_2} + \epsilon$$

A Taylor series approximation of the multiplicative relation would yield an expression involving powers and cross-products of the explanatory variables. Ramsey's suggestion is to include powers of the predicted values of the dependent variable (which are, of course, linear combinations of powers and cross-product terms of the explanatory variables) in Z. Specifically, Ramsey suggests

$$Z = [\hat{y}^2 \quad \hat{y}^3 \quad \hat{y}^4 \quad \cdots]$$

where \hat{y} is the vector of fitted values from the regression of y on X. The superscripts indicate the powers to which these predictions are raised. The first power is not included since it is perfectly collinear with the X matrix.

Output from the test reports the test regression and the *F*-statistic and Log likelihood ratio for testing the hypothesis that the coefficients on the powers of fitted values are all zero. A study by <u>Ramsey and Alexander (1984)</u> showed that the RESET test could detect specification error in an equation which was known *a priori* to be misspecified but which nonetheless gave satisfactory values for all the more traditional test criteria—goodness of fit, test for first order serial correlation, high *t*-ratios.

To apply the test, select **View/Stability Tests/Ramsey RESET Test...** and specify the number of fitted terms to include in the test regression. The fitted terms are the powers of the fitted values from the original regression, starting with the square or second power. For example, if you specify 1, then

the test will add $\hat{\mathcal{Y}}^2$ in the regression and if you specify 2, then the test will add $\hat{\mathcal{Y}}^2$ and $\hat{\mathcal{Y}}^3$ in the regression and so on. If you specify a large number of fitted terms, EViews may report a near singular matrix error message since the powers of the fitted values are likely to be highly collinear. The Ramsey RESET test is applicable only to an equation estimated by least squares.

Recursive Least Squares

More formally, let X_{t-1} denote the t-1 by k matrix of the regressors from period 1 to period t-1, and y_{t-1} the corresponding vector of observations on the dependent variable. These data up to period t-1 give an estimated coefficient vector, denoted by b_{t-1} . This coefficient vector gives you a forecast of the dependent variable in period t. The forecast is $x_t'b_{t-1}$, where x_t' is the row vector of observations on the regressors in period t. The forecast error is $y_{t-1}x_t'b_{t-1}$, and the forecast variance is given by:

$$\sigma^2 \Big(1 + x_t' (X_{t-1}' X_{t-1})^{-1} x_t \Big)$$

The recursive residual w_t is then defined as

$$w_t = \frac{y_t - x_t' b_{t-1}}{\left(1 + x_t' (X_{t-1}' X_{t-1})^{-1} x_t\right)^{1/2}}$$

These residuals can be computed for t = k+1,...,T. If the maintained model is valid, the recursive residuals will be independently and normally distributed with zero mean and constant variance σ^2 .

To calculate the recursive residuals, press View/Stability Tests/Recursive Estimates (OLS only)... on the equation toolbar. The Recursive Estimation dialog opens. There are six options available for the recursive estimates view. The recursive estimates view is only available for equations estimated by ordinary least squares without AR and MA terms. The Save Results as Series option allows you to save the recursive residuals and recursive coefficients as named series in the workfile; see Save Results as Series below.

Recursive Residuals

This option shows a plot of the recursive residuals about the zero line. Plus and minus two standard errors are also shown at each point. Residuals outside the standard error bands suggest instability in the parameters of the equation.

CUSUM Test

The CUSUM test (<u>Brown, Durbin, and Evans, 1975</u>) is based on the cumulative sum of the recursive residuals. This option plots the cumulative sum together with the 5% critical lines. The test finds parameter instability if the cumulative sum goes outside the area between the two critical lines.

The CUSUM test is based on the statistic

$$W_t = \sum\limits_{\tau = k+1}^t w_\tau / s, ~~ t = k+1, \ldots, T$$

where w is the recursive residual defined above, and s is the standard error of the regression fitted to all T sample points. If the b vector remains constant from period to period, $E[W_t] = 0$, but if β changes, W_t will tend to diverge from the zero mean value line. The significance of any departure from the zero line is assessed by reference to a pair of 5% significance lines, the distance between which increases with t. The 5% significance lines are found by connecting the points

$$[k, \pm 0.948(T-k)^{1/2}]$$
 and $[T, \pm 3 \times 0.948(T-k)^{1/2}]$

Movement of W_t outside the critical lines is suggestive of coefficient instability.

CUSUM of Squares Test

The CUSUM of squares test (Brown, Durbin, and Evans, 1975) is based on the test statistic

$$S_t = \sum_{r=k+1}^{t} w_r^2 / \sum_{r=k+1}^{T} w_r^2$$

The expected value of S under the hypothesis of parameter constancy is

$$\mathbb{E}[S_t] = (t-k)/(T-k)$$

which goes from zero at t=k to unity at t=T. The significance of the departure of S from its expected value is assessed by reference to a pair of parallel straight lines around the expected value. See <u>Brown, Durbin, and Evans (1975)</u> or <u>Johnston and DiNardo (1997, Table D.8)</u> for a table of significance lines for the CUSUM of squares test.

The CUSUM of squares test provides a plot of S_t against t and the pair of 5 percent critical lines. As with the CUSUM test, movement outside the critical lines is suggestive of parameter or variance instability.

One-Step Forecast Test

If you look back at the <u>definition of the recursive residuals</u> given above, you will see that each recursive residual is the error in a one-step ahead forecast. To test whether the value of the dependent variable at time *t* might have come from the model fitted to all the data up to that point, each error can be compared with its standard deviation from the full sample.

The **One-Step Forecast Test** option produces a plot of the recursive residuals and standard errors and the sample points whose probability value is at or below 15 percent. The plot can help you spot the periods when your equation is least successful.

The upper portion of the plot (right vertical axis) repeats the recursive residuals and standard errors displayed by the **Recursive Residuals** option. The lower portion of the plot (left vertical axis) shows the probability values for those sample points where the hypothesis of parameter constancy would be rejected at the 5, 10, or 15 percent levels. The points with *p*-values less the 0.05 correspond to those points where the recursive residuals go outside the two standard error bounds.

N-Step Forecast Test

This test uses the recursive calculations to carry out a sequence of Chow Forecast tests. In contrast to the single Chow Forecast test described earlier, this test does not require the specification of a forecast period— it automatically computes all feasible cases, starting with the smallest possible sample size for estimating the forecasting equation and then adding one observation at a time. The plot from this test shows the recursive residuals at the top and significant probabilities in the lower portion of the diagram.

Recursive Coefficient Estimates

This view enables you to trace the evolution of estimates for any coefficient as more and more of the sample data are used in the estimation. The view will provide a plot of selected coefficients in the equation for all feasible recursive estimations. Also shown are the two standard error bands around the estimated coefficients.

If the coefficient displays significant variation as more data is added to the estimating equation, it is a strong indication of instability. Coefficient plots will sometimes show dramatic jumps as the postulated equation tries to digest a structural break.

To view the recursive coefficient estimates, click the **Recursive Coefficients** option and list the coefficients you want to plot in the **Coefficient Display List** field of the dialog box.

Save Results as Series

The **Save Results as Series** checkbox will do different things depending on the plot you have asked to be displayed. When paired with the **Recursive Coefficients** option, **Save Results as Series** will instruct EViews to save all recursive coefficients and their standard errors in the workfile as named series. EViews will name the coefficients using the next available name of the form, R_C1, R_C2, ..., and the corresponding standard errors as R_C1SE, R_C2SE, and so on.

If you check the **Save Results as Series** box with any of the other options, EViews saves the recursive residuals and the recursive standard errors as named series in the workfile. EViews will name the residual and standard errors as R_RES and R_RESSE, respectively.

Note that you can use the recursive residuals to reconstruct the CUSUM and CUSUM of squares series.

Applications

In this section, we show how to carry out other specification tests in EViews. For brevity, the discussion is based on commands, but most of these procedures can also be carried out using the menu system.

A Wald test of structural change with unequal variance

The *F*-statistics reported in the Chow tests have an *F*-distribution only if the errors are independent and identically normally distributed. This restriction implies that the residual variance in the two subsamples must be equal.

Suppose now that we wish to compute a Wald statistic for structural change with unequal subsample variances. Denote the parameter estimates and their covariance matrix in subsample i as b_i and V_i for i=1,2. Under the assumption that b_1 and b_2 are independent normal, the difference $b_1 - b_2$ has mean zero and variance $V_1 + V_2$. Therefore a Wald statistic for the null hypothesis of no structural change and independent samples can be constructed as

$$W = (b_1 - b_2)'(V_1 + V_2)^{-1}(b_1 - b_2)$$

which has an asymptotic χ^2 distribution with degrees of freedom equal to the number of estimated parameters in the *b* vector.

To carry out this test in EViews, we estimate the model in each subsample and save the estimated coefficients and their covariance matrix. For example, suppose we have a quarterly sample of 1947:1–1994:4 and wish to test whether there was a structural change in the consumption function in 1973:1. First estimate the model in the first sample and save the results by the commands

```
coef(2) b1
smpl 1947:1 1972:4
equation eq_1.ls log(cs)=b1(1)+b1(2)*log(gdp)
sym v1=eq_1.@cov
```

The first line declares the coefficient vector, B1, into which we will place the coefficient estimates in the first sample. Note that the equation specification in the third line explicitly refers to elements of this coefficient vector. The last line saves the coefficient covariance matrix as a symmetric matrix named V1. Similarly, estimate the model in the second sample and save the results by the commands

```
coef(2) b2
smpl 1973.1 1994.4
equation eq_2.ls log(cs)=b2(1)+b2(2)*log(gdp)
sym v2=eq_2.@cov
```

To compute the Wald statistic, use the command

```
matrix wald=@transpose(b1-b2)*@inverse(v1+v2) *(b1-b2)
```

The Wald statistic is saved in the 1 \times 1 matrix named WALD. To see the value, either double click on WALD or type show wald. You can compare this value with the critical values from the χ^2 distribution with 2 degrees of freedom. Alternatively, you can compute the p-value in EViews using the command

```
scalar wald_p=1-@cchisq(wald(1,1),2)
```

The p-value is saved as a scalar named WALD_P. To see the p-value, double click on WALD_P or type $show\ wald_p$. The p-value will be displayed in the status line at the bottom of the EViews window.

A Hausman test

A widely used class of tests in econometrics is the Hausman test. The underlying idea of the Hausman test is to compare two sets of estimates, one of which is consistent under both the null and the alternative and another which is consistent only under the null hypothesis. A large difference between the two sets of estimates is taken as evidence in favor of the alternative hypothesis.

<u>Hausman (1978)</u> originally proposed a test statistic for endogeneity based upon a direct comparison of coefficient values. Here we illustrate a version of the Hausman test proposed by Davidson and MacKinnon (1989, 1993), which carries out the test by running an auxiliary regression.

The following equation was estimated by OLS:

```
Dependent Variable: LOG(M1)
Method: Least Squares
Date: 08/13/97 Time: 14:12
Sample(adjusted): 1959:02 1995:04
Included observations: 435 after adjusting endpoints
       Variable
                     Coefficient Std. Error
                                                  t-Statistic
                                                                Prob.
                        -0.022699
                                     0.004443
                                                  -5.108528
                                                               0.0000
      LOG(IP)
                       0.011630
                                     0.002585
                                                 4.499708
                                                               0.0000
     DLOG(PPI)
                        -0.024886
                                     0.042754
                                                 -0.582071
                                                               0.5608
        TB3
                       -0.000366
                                     9.91E-05
                                                 -3.692675
                                                               0.0003
     LOG(M1(-1))
                       0.996578
                                     0.001210
                                                 823.4440
                                                               0.0000
R-squared
                        0.999953 Mean dependent var
                                                             5.844581
Adjusted R-squared
                       0.999953 S.D. dependent var
                                                             0.670596
                       0.004601 Akaike info criterion
S.E. of regression
                                                            -7 913714
Sum squared resid
                        0.009102 Schwarz criterion
                                                            -7 866871
                        1726.233 F-statistic
                                                             2304897.
Log likelihood
Durbin-Watson stat
                        1.265920 Prob(F-statistic)
                                                             0.000000
```

Suppose we are concerned that industrial production (IP) is endogenously determined with money (M1) through the money supply function. If this were the case, then OLS estimates will be biased and inconsistent. To test this hypothesis, we need to find a set of instrumental variables that are correlated with the "suspect" variable IP but not with the error term of the money demand equation. The choice of the appropriate instrument is a crucial step. Here we take the unemployment rate (URATE) and Moody's AAA corporate bond yield (AAA) as instruments.

To carry out the Hausman test by artificial regression, we run two OLS regressions. In the first regression, we regress the suspect variable (log) IP on all exogenous variables and instruments and retrieve the residuals:

```
ls log(ip) c dlog(ppi) tb3 log(m1(-1)) urate aaa
series res_ip=resid
```

Then in the second regression, we re-estimate the money demand function including the residuals from the first regression as additional regressors. The result is:

Dependent Variable: LOG(M1) Method: Least Squares Date: 08/13/97 Time: 15:28 Sample(adjusted): 1959:02 1995:04

Included observations: 435 after adjusting endpoints

Variable	Coefficient	Std. Error	t-Statistic	Prob.
С	-0.007145	0.007473	-0.956158	0.3395
LOG(IP)	0.001560	0.004672	0.333832	0.7387
DLOG(PPI)	0.020233	0.045935	0.440465	0.6598
TB3	-0.000185	0.000121	-1.527775	0.1273
LOG(M1(-1))	1.001093	0.002123	471.4894	0.0000
RES_IP	0.014428	0.005593	2.579826	0.0102

If the OLS estimates are consistent, then the coefficient on the first stage residuals should not be significantly different from zero. In this example, the test (marginally) rejects the hypothesis of consistent OLS estimates. (To be more precise, this is an asymptotic test and you should compare the *t*-statistic with the critical values from the standard normal.)

Non-nested Tests

Most of the tests discussed above are nested tests in which the null hypothesis is obtained as a special case of the alternative hypothesis. Now consider the problem of choosing between the following two specifications of a consumption function:

$$\begin{split} H_1: \quad &CS_t = \alpha_1 + \alpha_2 GDP_t + \alpha_3 GDP_{t-1} + \epsilon_t \\ H_2: \quad &CS_t = \beta_1 + \beta_2 GDP_t + \beta_2 CS_{t-1} + \epsilon_t \end{split}$$

These are examples of non-nested models since neither model may be expressed as a restricted version of the other.

The *J*-test proposed by <u>Davidson and MacKinnon (1993)</u> provides one method of choosing between two non-nested models. The idea is that if one model is the correct model, then the fitted values from the other model should not have explanatory power when estimating that model. For example, to test model H_1 against model H_2 , we first estimate model H_2 and retrieve the fitted values:

```
equation eq_cs2.ls cs c gdp cs(-1) eq cs2.fit cs2
```

The second line saves the fitted values as a series named CS2. Then estimate model H_1 including the fitted values from model H_2 . The result is:

```
Dependent Variable: CS
Method: Least Squares
Date: 08/13/97 Time: 15:28
Sample(adjusted): 1947:2 1994:4
```

Included observations: 191 after adjusting endpoints

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	7.313232	4.391305	1.665389	0.0975
GDP	0.278749	0.029278	9.520694	0.0000
GDP(-1)	-0.314540	0.029287	-10.73978	0.0000
CS2	1.048470	0.019684	53.26506	0.0000

The fitted values from model H_2 enter significantly in model H_1 and we reject model H_1 .

We must also test model H_2 against model H_1 . Estimate model H_1 , retrieve the fitted values, and estimate model H_2 including the fitted values from model H_1 . The result of this "reverse" test are given by:

```
Dependent Variable: CS
Method: Least Squares
Date: 08/13/97 Time: 15:28
Sample(adjusted): 1947:2 1994:4
```

Included observations: 191 after adjusting endpoints

Variable	Coefficient	Std. Error	t-Statistic	Prob.
С	-1427.716	132.0349	-10.81318	0.0000
GDP	5.170543	0.476803	10.84419	0.0000
CS(-1)	0.977296	0.018348	53.26506	0.0000
CS1	-7.292771	0.679043	-10.73978	0.0000

The fitted values are again statistically significant and we reject model $\,H_{2}\,.$

In this example, we reject both specifications, against the alternatives, suggesting that another model

for the data is needed. It is also possible that we fail to reject both models, in which case the data not provide enough information to discriminate between the two models.	do