Problem 1

Provide a proof or counterexample to the following statements:

1. An iteration of the simplex method cannot move the feasible solution by a strictly positive distance while leaving the objective function value unchanged.
2. An index that has just left the basis cannot re-enter in the very next iteration.
3. An index that has just entered the basis cannot leave in the very next iteration. Distinguish between the degenerate and non-degenerate case.
4. If B is an optimal basis, then all the components of λ_B are strictly positive.

Problem 2

Prove or give a counter-example for the following statements:

1. Let B be an optimal basis. If λ_B is strictly positive then the optimal solution is unique.
2. If the optimal solution is unique then λ_B is strictly positive for the optimal basis B.