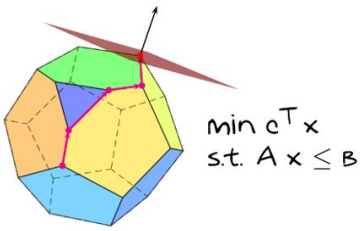
Paths, Cycles and Flows

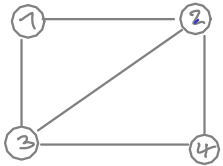
- Representing graphs



Adjacency matrix

$G = (V, E)$ *undirected* graph, $V = \{1, \ldots, n\}$
$M \in \{0,1\}^{|V| \times |V|}$ with

$$a_{ij} = \begin{cases} 1 & \text{if } ij \in E \\ 0 & \text{otherwise.} \end{cases}$$



$$\begin{pmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{pmatrix}$$

Symmetric

$\Theta(|V|^2)$

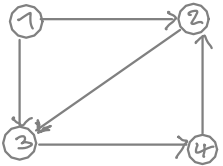
$|E| \approx |V|^2$

$|A| \approx |V|^2$

$\notin \Omega(|V|^2)$

$D = (V, A)$ *directed* graph, $V = \{1, \ldots, n\}$
$M \in \{0,1\}^{|V| \times |V|}$ with

$$a_{ij} = \begin{cases} 1 & \text{if } (i,j) \in A \\ 0 & \text{otherwise.} \end{cases}$$

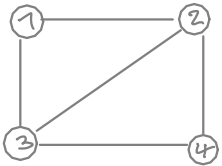


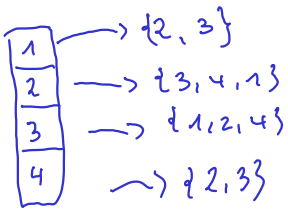
$$\begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

Adjacency list

$G = (V, E)$ *undirected* graph, $V = \{1, \ldots, n\}$

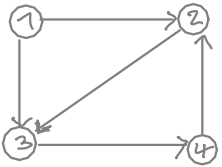
- *Array* V indexed by vertices/nodes
- $V[i]$ *points* to *list* of neighbors of i.

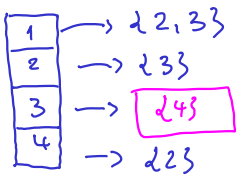




$D = (V, A)$ *directed* graph, $V = \{1, \ldots, n\}$

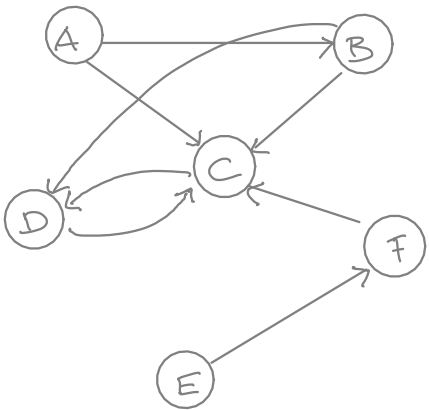
- *Array* V indexed by vertices/nodes
- $V[i]$ *points* to *list* of heads of outgoing arcs of i.





Adjacency list representation in Python

```
graph = {'A': ['B', 'C'],
         'B': ['C', 'D'],
         'C': ['D'],
         'D': ['C'],
         'E': ['F'],
         'F': ['C']}
```



FOR $(u, v) \in A$

DO

. ....

$O(|A| + |V|)$