

# General Least Squares Quiz

**10/10 points (100%)**

Quiz, 10 questions

 **Congratulations! You passed!**

Next Item


 1 / 1  
points

1.

If  $I$  is an identity matrix and  $H$  is an idempotent matrix,  $I - H$  is necessarily idempotent?



True


**Correct**

$$(I - H)^2 = I^2 - IH - HI - H^2 = I - H$$



False


 1 / 1  
points

2.

If a design matrix,  $X$ , is orthonormal (so that  $X'X = I$ ), the least squares estimate for the coefficients for outcome vector  $Y$  simplifies to (check all that apply):


 $X'Y$ 

**Correct**

$$(X'X)^{-1} = I$$



It can't be simplified from the typical estimate without further information.



Un-selected is correct

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The vector with  $i^{th}$  element  $\langle X_i, Y \rangle$  where  $X_i$  is the column  $i$  from  $X$ .



Correct

If  $X = [X_1 \ X_2 \ \dots \ X_p]$  then

$$X'Y = (\langle X_1, Y \rangle \ \langle X_2, Y \rangle \ \dots \ \langle X_p, Y \rangle)'$$



1 / 1  
points

3.

Let  $X$  be a design matrix.  $X$ ,  $X'$ ,  $X'X$  and  $XX'$  all have the same matrix rank. (Figure out why or why not.)



True



Correct

This is a standard linear algebra result.



False



Un-selected is correct



1 / 1  
points

4.

Let  $J_a$  be a vector of ones of length  $a$ , let  $\otimes$  be the Kronecker product (see wikipedia for definition) and let  $I_b$  be a  $b \times b$  identity matrix. Let  $X = J_a \otimes I_b$  be a design matrix and  $Y$  be a  $ab \times 1$  response vector. What does the least squares estimate of the coefficients simplify to? (Check all that apply.)



The empirical means of the  $b$  means of the elements of  $Y$  in groups of size  $a$ .



**Correct**

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See the next lecture on least squares examples. The Kronecker product is just a convenient way to write out ANOVA type examples.

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Quiz, 10 questions



The empirical standard deviations of the elements of  $Y$  in groups of size  $a$ .

**Un-selected is correct**

One over (the inverse of ) the empirical means of the  $b$  means of the elements of  $Y$  in groups of size  $a$ .

**Un-selected is correct**

1 / 1  
points

5.

For matrices  $A$  and  $B$  satisfying the matrix dimensions, let  $e_{A|B} = (I - B(B'B)^{-1}B')A$  be the matrix of residuals. Let  $X = [X_1 \ X_2]$  be a design matrix comprised of submatrices  $X_1$  and  $X_2$  and let  $Y$  be a response vector. The least squares estimates with  $X$  as a design matrix and  $Y$  as the response vector for the components of  $\beta$  corresponding to  $X_1$  are equal to?



A least squares fit of  $e_{Y|X_2}$  as the outcome and  $e_{X_1|X_2}$  as the design matrix.

**Correct**

A least squares fit of  $e_{Y|X_1}$  as the outcome and  $e_{X_2|X_1}$  as the design matrix.



A least squares fit of  $Y$  as the outcome and  $X_1$ .



A least squares of  $e_{Y|X_1}$  as the outcome and  $e_{X_1|X_2}$  as the predictor.

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1 / 1  
points

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Quiz, 10 questions

6.

Let  $X$  be a design matrix ( $n \times p$ ) and  $Y$  ( $n \times 1$ ) be the outcome vector. Let  $x_{new}$  be a  $p \times 1$  vector of new values of  $X$  that we would like to predict at. What would be the predicted value using the least squares fit (check all that apply)?

☒  $x'_{new}(X'X)^{-1}X'Y$



Correct

☐  $X(X'X)^{-1}X'Y$



Un-selected is correct

☐  $x'_{new}\hat{\beta}$  where  $\hat{\beta} = X(X'X)^{-1}X'Y$



Correct

1 / 1  
points

7.

Fit the swiss dataset with Fertility as the outcome and the rest of the variables as predictors. What is the Agriculture coefficient?

☐ 0.1942

☒ -0.1721



Correct

☐ 60.3044



points

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Quiz, 10 questions

Fit the swiss dataset with Fertility as the outcome and the rest of the variables as predictors. Predict Fertility in a province with the following data

```
1 newData = data.frame(
2   Agriculture = 27.7,
3   Examination = 22,
4   Education = 29,
5   Catholic = 58.33,
6   Infant.Mortality = 19.3)
7
```

☐ 65.68☐ 82.51☐ 74.62☒ 58.07**Correct**1 / 1  
points

9.

Let  $Y$  be an  $n \times 1$  response vector. Let  $U = XW$  where  $W$  is a non-identity invertible  $p \times p$  matrix and  $U$  and  $X$  are  $n \times p$ . What can be said about the fitted values from the least squares fit of  $U$  as a design versus  $X$  as a design matrix with  $Y$  as the outcome (check all that apply)?

The fitted values,  $\hat{Y}$ , will be identical.**Correct**

Note the the space spanned by the rows  $X$  and  $XW$  are the same.

$$XW(W'X'XW)^{-1}W'X'Y = XWW^{-1}(X'X)^{-1}(W')^{-1}W'X'Y = \hat{Y}$$

.

The fitted values,  $\hat{Y}$ , will be different.

Un-selected is correct

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Quiz, 10 questions



The slope estimates will always be the same.



Un-selected is correct



Nothing definitive can be determined from the information given.



Un-selected is correct

1 / 1  
points

10.

Suppose that a design vector  $X$  contains an intercept column. What can be said about the fitted regression surface  $y = x\hat{\beta}$ ?



It satisfies  $\bar{Y} = \bar{X}'\hat{\beta}$ , where  $\bar{Y}$  is the average of the  $Y$  and  $\bar{X}$  is the columnwise average of  $X$ . In other words, it passes through the average of the  $Y$  and  $X$  exactly like in linear regression.



Correct

$$e'J_n = (Y - X\hat{\beta})'J_n = 0 \text{ so that } n\bar{Y} - n\bar{X}'\hat{\beta} = 0.$$



It doesn't necessarily satisfy  $\bar{Y} = \bar{X}'\hat{\beta}$ , where  $\bar{Y}$  is the average of the  $Y$  and  $\bar{X}$  is the columnwise average of  $X$ .



Un-selected is correct



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