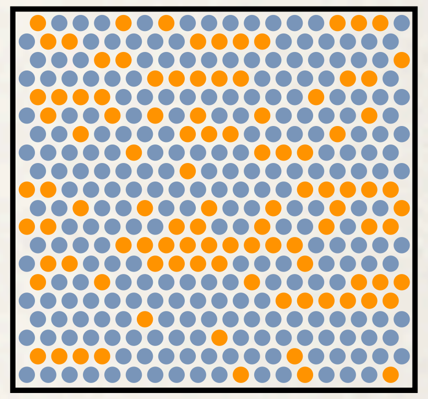
The distribution of accumulated successes



Is $\frac{S_n}{n}$ a good approximation to p?

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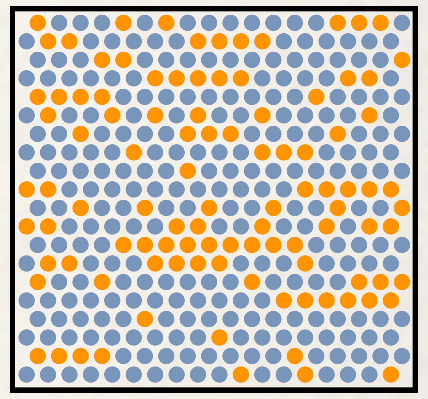

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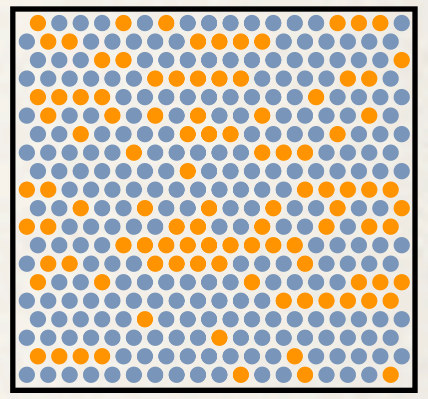

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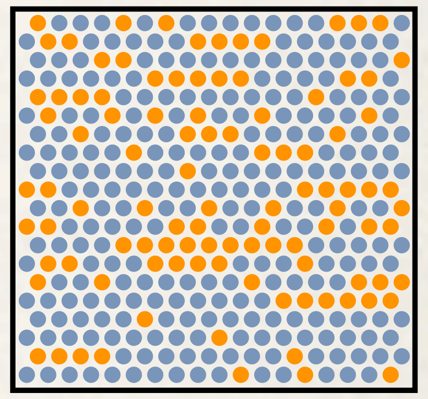

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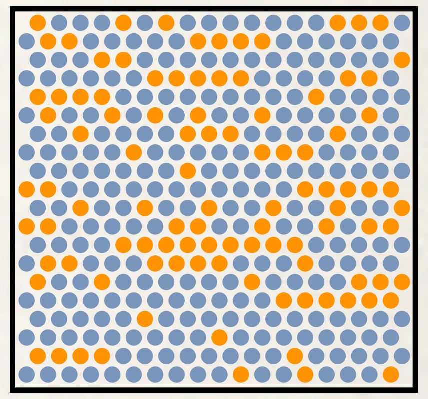

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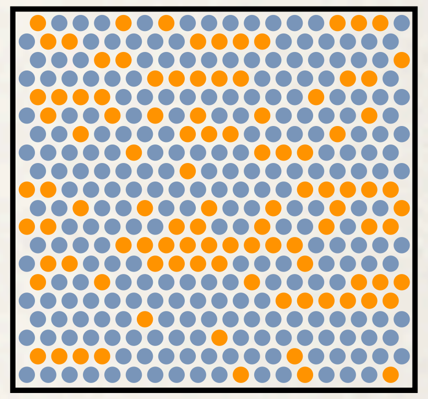

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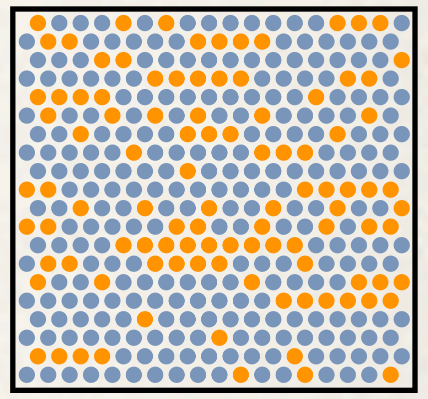

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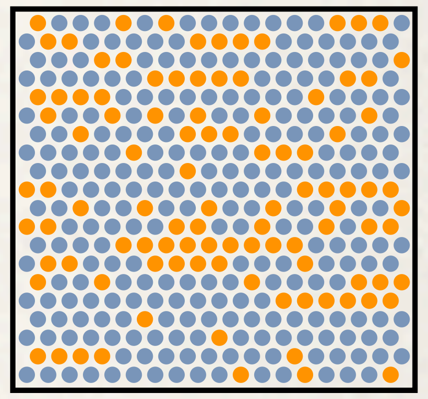

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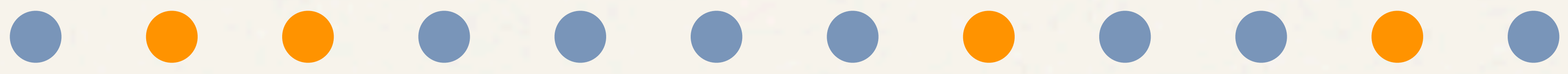

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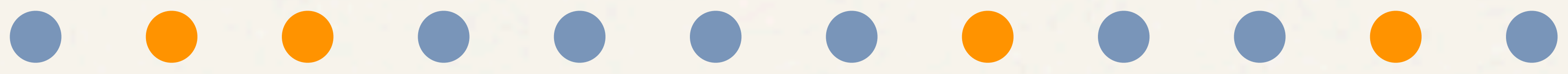

Probability of observing a particular sequence of 4 successes in 12 trials: p^4q^{12-4}

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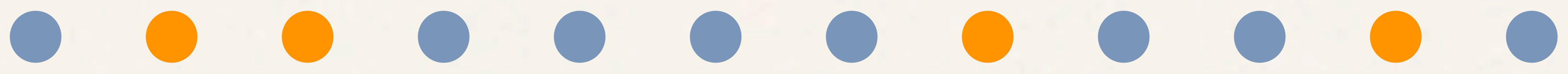

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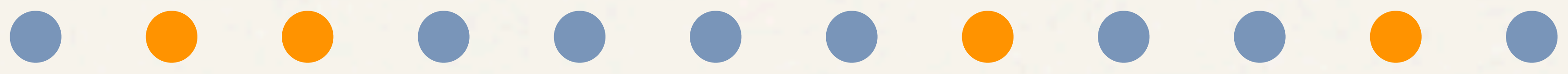

Probability of observing a particular sequence of k successes in n trials: $p^k q^{n-k}$

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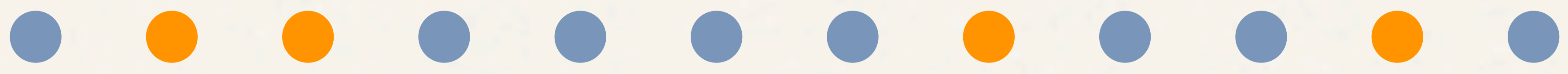

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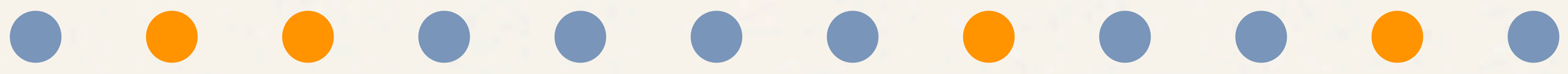

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The binomial distribution

- Accumulated successes: New chance variables from old.
  - The (new) sample space: $\Omega = \{0, 1, \ldots, n\}$.
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Positivity ✓ $b_n(k;p) \geq 0$ for $k = 0, 1, \ldots, n$

Normalisation ✓ $\sum_k b_n(k;p) = \sum_{k=0}^{n} \binom{n}{k} p^k q^{n-k} = (p+q)^n = 1$

The binomial theorem!

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