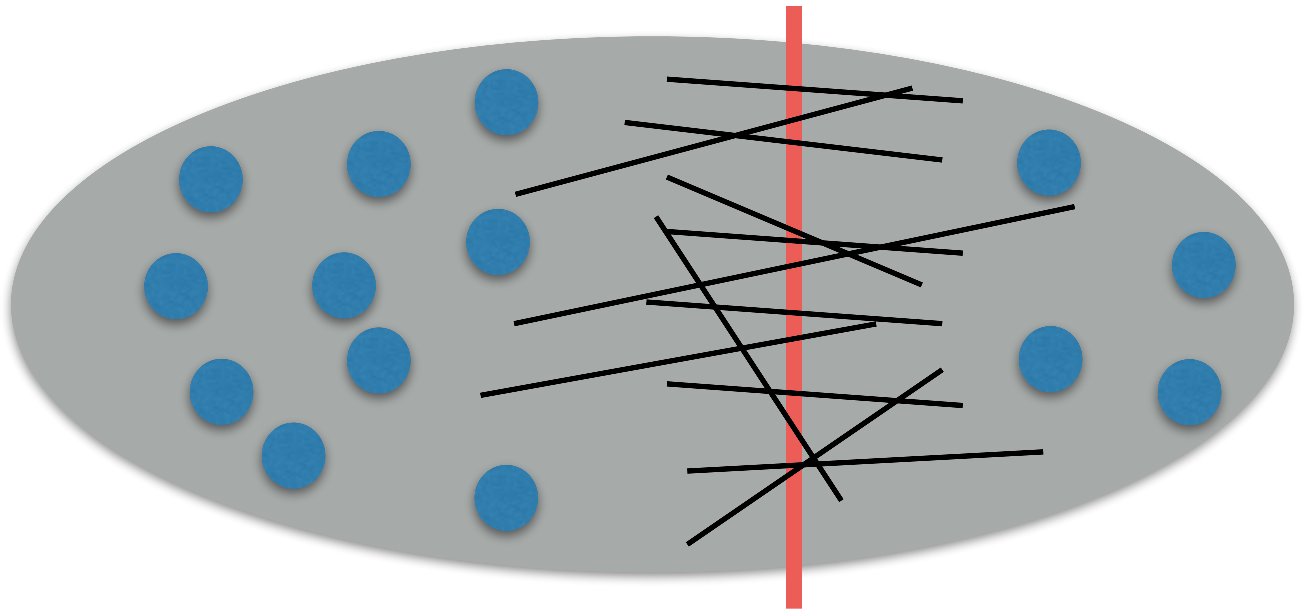
Maxcut

Partition graph
to maximize
edge weight across cut

Simple algorithm: Random



S = random subset
each vertex goes into S
independently w.pr. .5

Output cut (S, V-S)

Theorem: it's a 2-approximation

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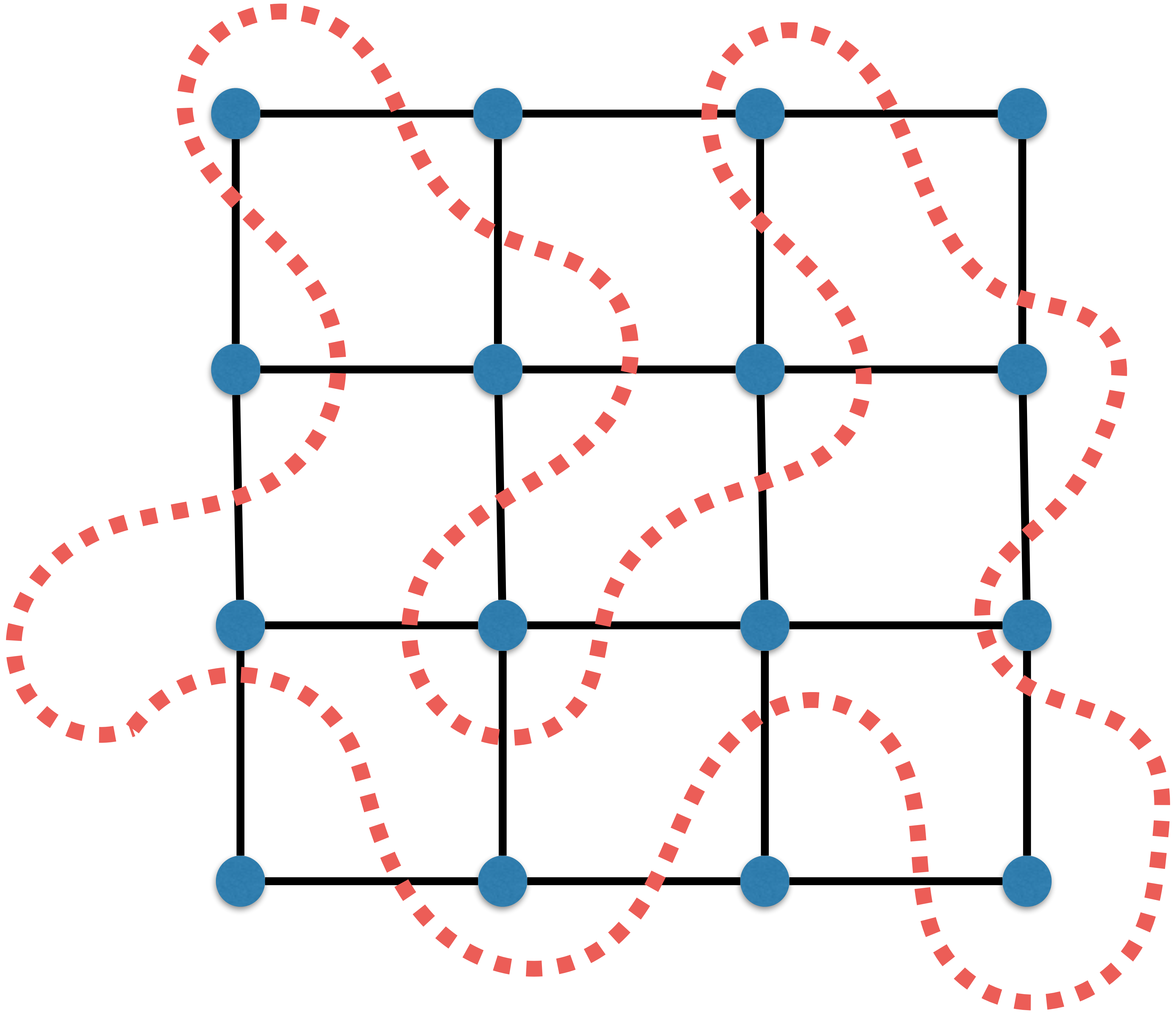
Analysis: bounding OPT

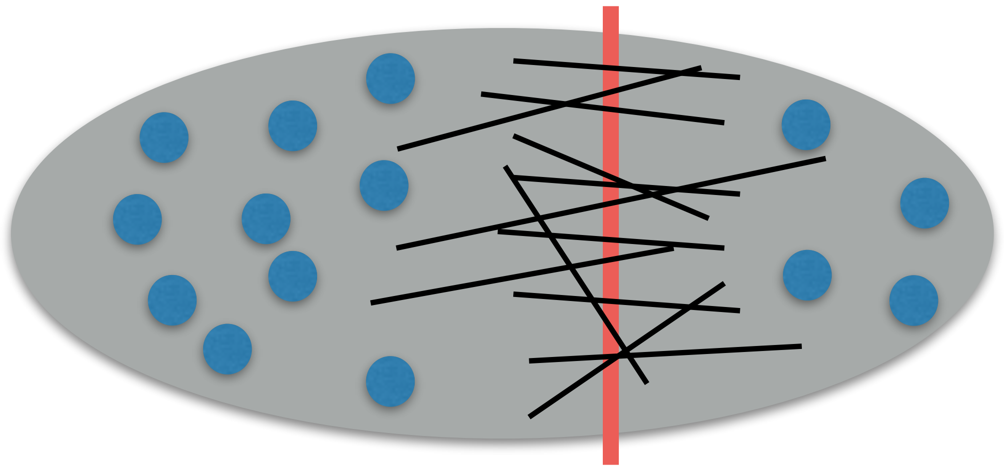
Upper bound on OPT:

$$\mathrm{OPT} \leq \sum_{\{i,j\} \in E} w_{ij}$$



Partition graph to maximize edge weight across cut

Analysis: bounding output



$$\mathrm{Output} = \sum_{\{i,j\}\in E} w_{ij} \cdot 1(\{i,j\} \text{ crosses cut})$$

$$E(\mathrm{Output}) = \sum_{\{i,j\}\in E} w_{ij} \cdot \Pr(\{i,j\} \text{ crosses cut})$$

whether {i,j} crosses & probability	i in S .5	i not in S .5
j in S .5	no p=.25	yes p=.25
j not in S .5	yes p=.25	no p=.25

$$\Pr(\{i,j\} \text{ crosses cut}) = .25 + .25 = .5$$

$$E(\mathrm{Output}) = (1/2) \sum_{\{i,j\}\in E} w_{ij}$$

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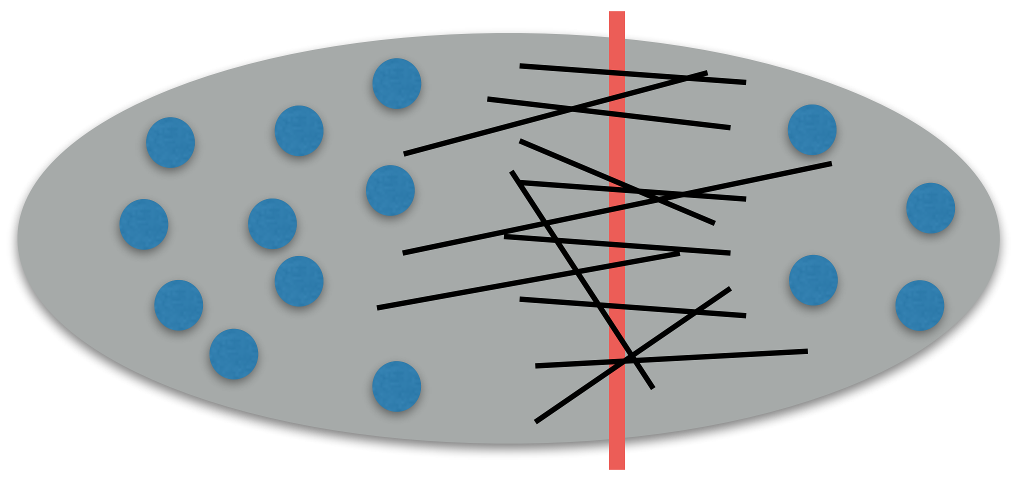
Together



$$\mathbf{OPT} \leq \sum_{\{\mathbf{i},\mathbf{j}\}\in\mathbf{E}} \mathbf{w_{ij}}$$

$$\mathbf{E(Output)} = \mathbf{(1/2)} \sum_{\{\mathbf{i},\mathbf{j}\}\in\mathbf{E}} \mathbf{w_{ij}}$$

$$\mathbf{E(Output)} \geq \mathbf{(1/2)OPT}$$

QED

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