

# Working backwards: $f_n$

$$s_n = f_1 + f_2 + \cdots + f_n$$

$$f_n = u_n - f_1 u_{n-1} - f_2 u_{n-2} - \cdots - f_j u_{n-j} - \cdots - f_{n-1} u_1$$

$$u_n = 2^{-5} - u_{n-1}2^{-1} - u_{n-2}2^{-2} - u_{n-3}2^{-3} - u_{n-4}2^{-4}$$

n	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
$u_n$	0	0	0	0	$\frac{1}{32}$	$\frac{1}{64}$	$\frac{1}{64}$	$\frac{1}{64}$	$\frac{1}{64}$	$\frac{17}{1024}$	$\frac{33}{2048}$	$\frac{33}{2048}$	$\frac{33}{2048}$	$\frac{33}{2048}$	$\frac{529}{32768}$	$\frac{1057}{65536}$	$\frac{1057}{65536}$	$\frac{1057}{65536}$	$\frac{1057}{65536}$	$\frac{16913}{1048576}$



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