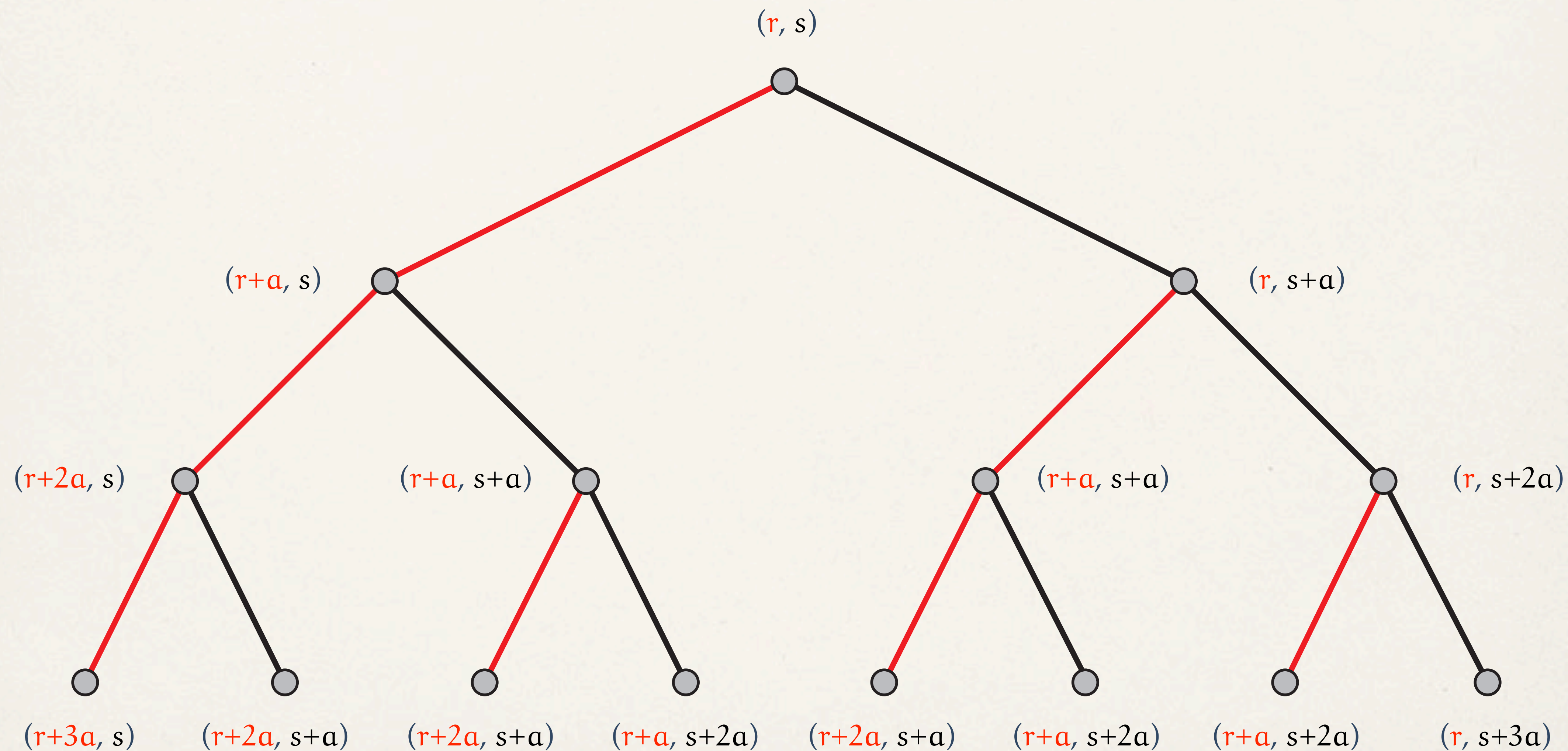


# A population growth tree

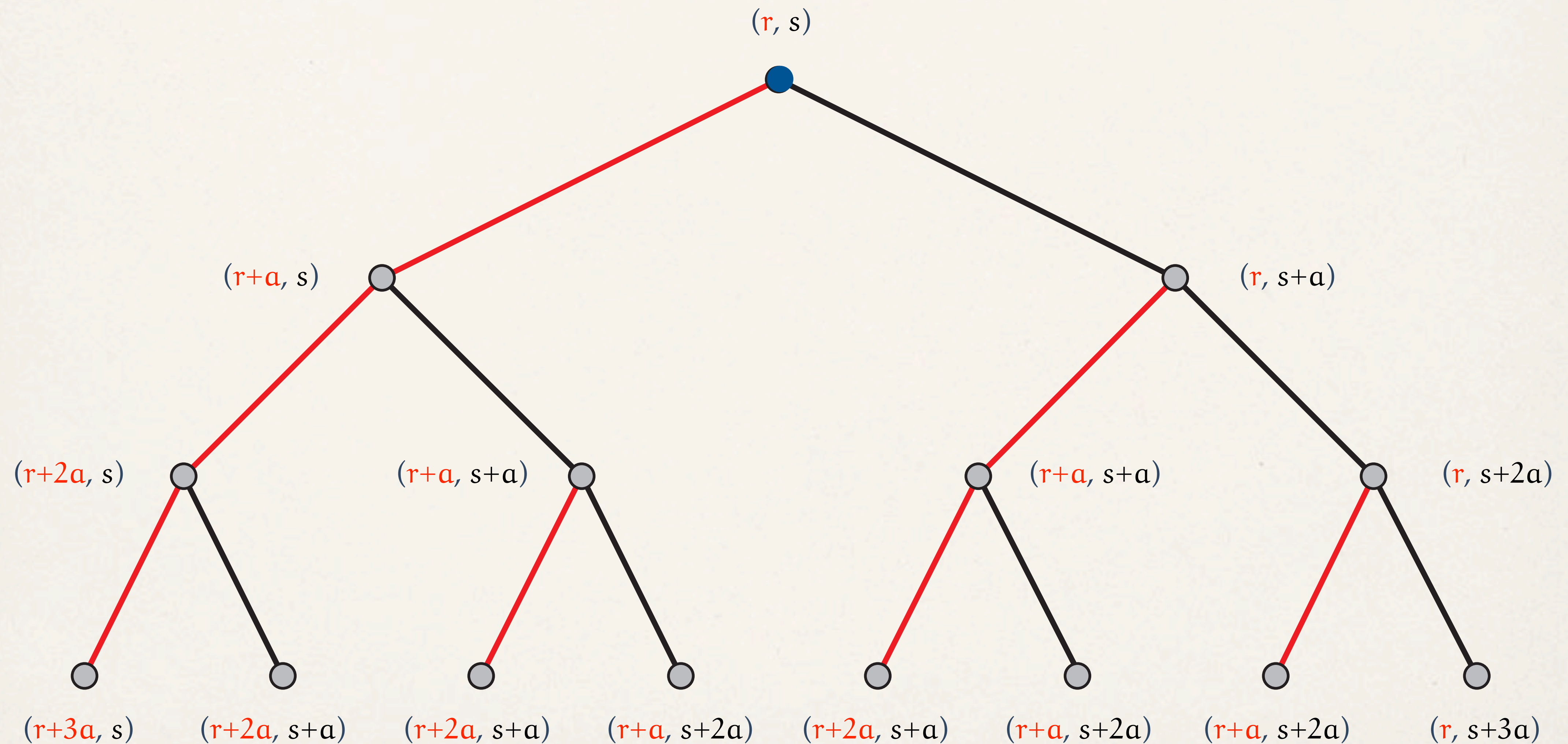
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# A population growth tree

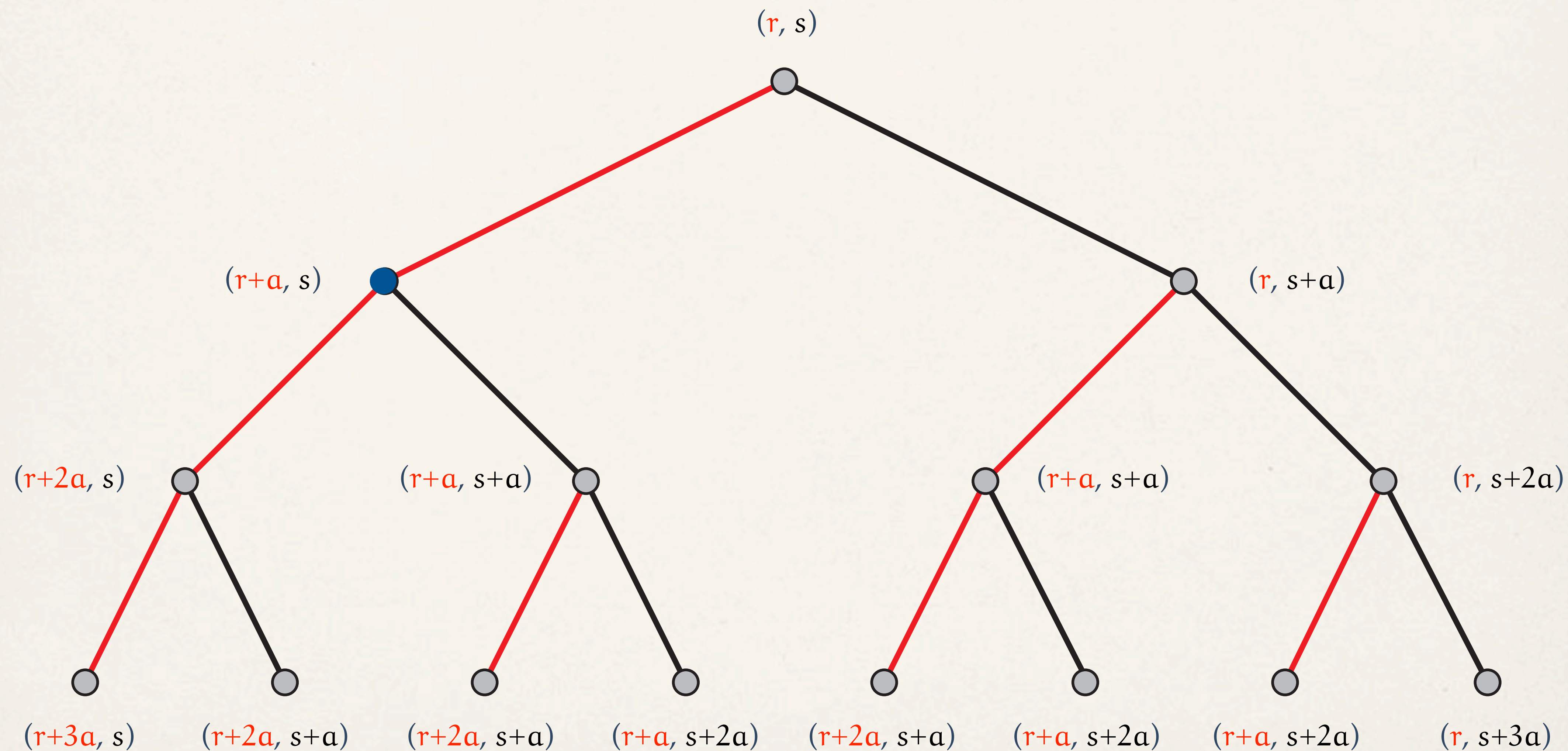
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# A population growth tree

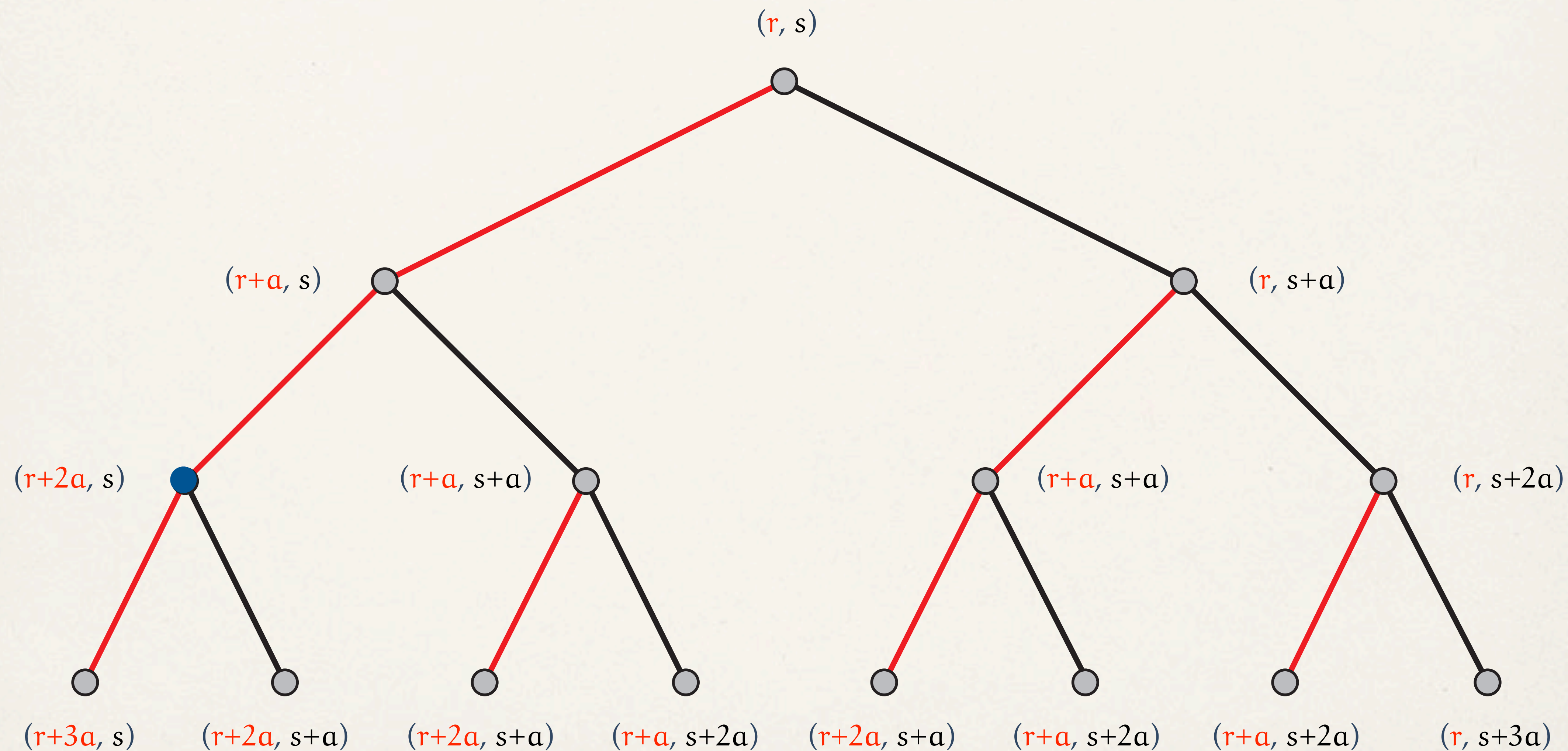
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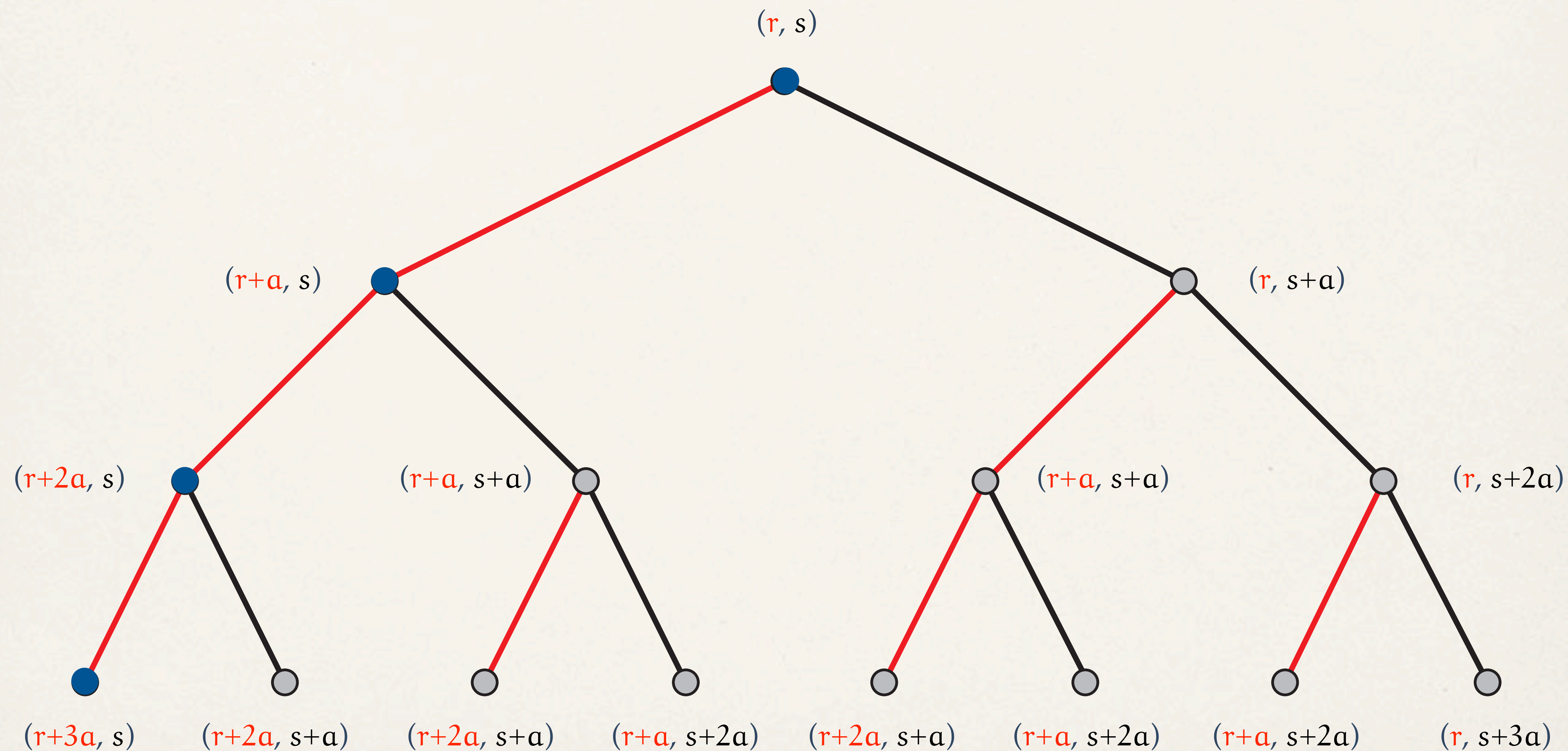
# A population growth tree

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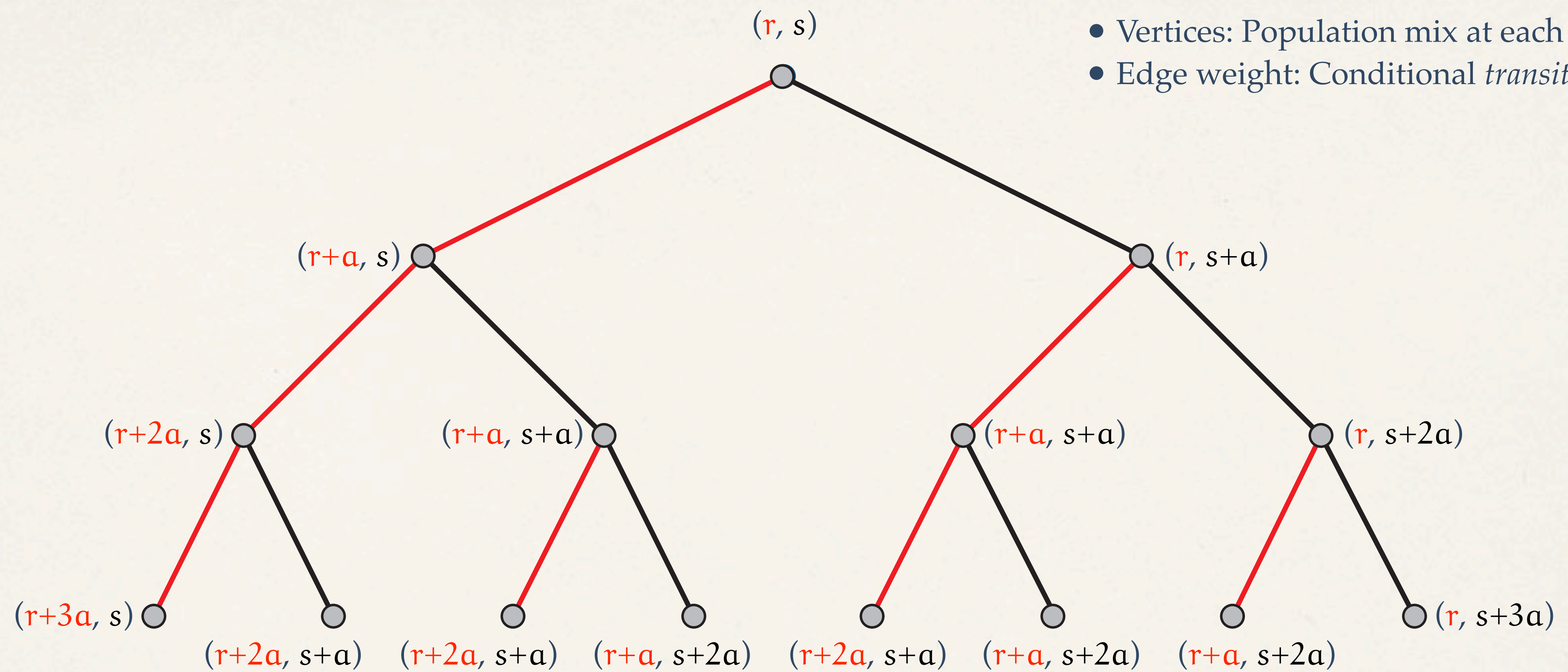




# A population growth tree

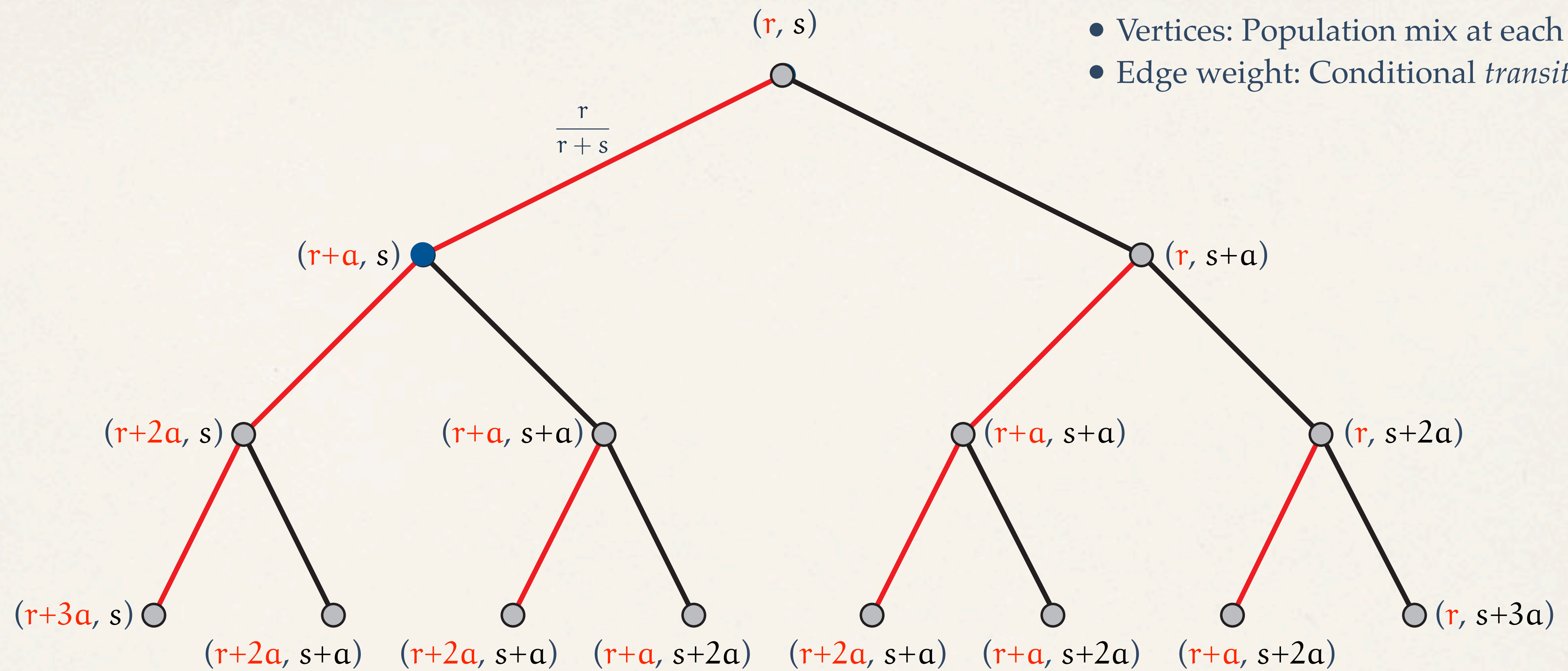






- \* Sample space  $\Omega$ : each sequence of ball colours drawn leads to a distinct outcome  $\omega = (x_1, x_2, x_3, \dots)$  where  $x_1, x_2, x_3, \dots \in \{\text{red, black}\}$ .
- \* The events of interest:  $R_k := \text{kth draw shows red} = \{ (x_1, x_2, \dots, x_k, \dots) : x_k = \text{red} \}$ ;  $B_k := (R_k)^c = \{ (x_1, x_2, \dots, x_k, \dots) : x_k = \text{black} \}$ .
- \* The implicit probability measure  $P$ , from initial selection through conditional evolution:

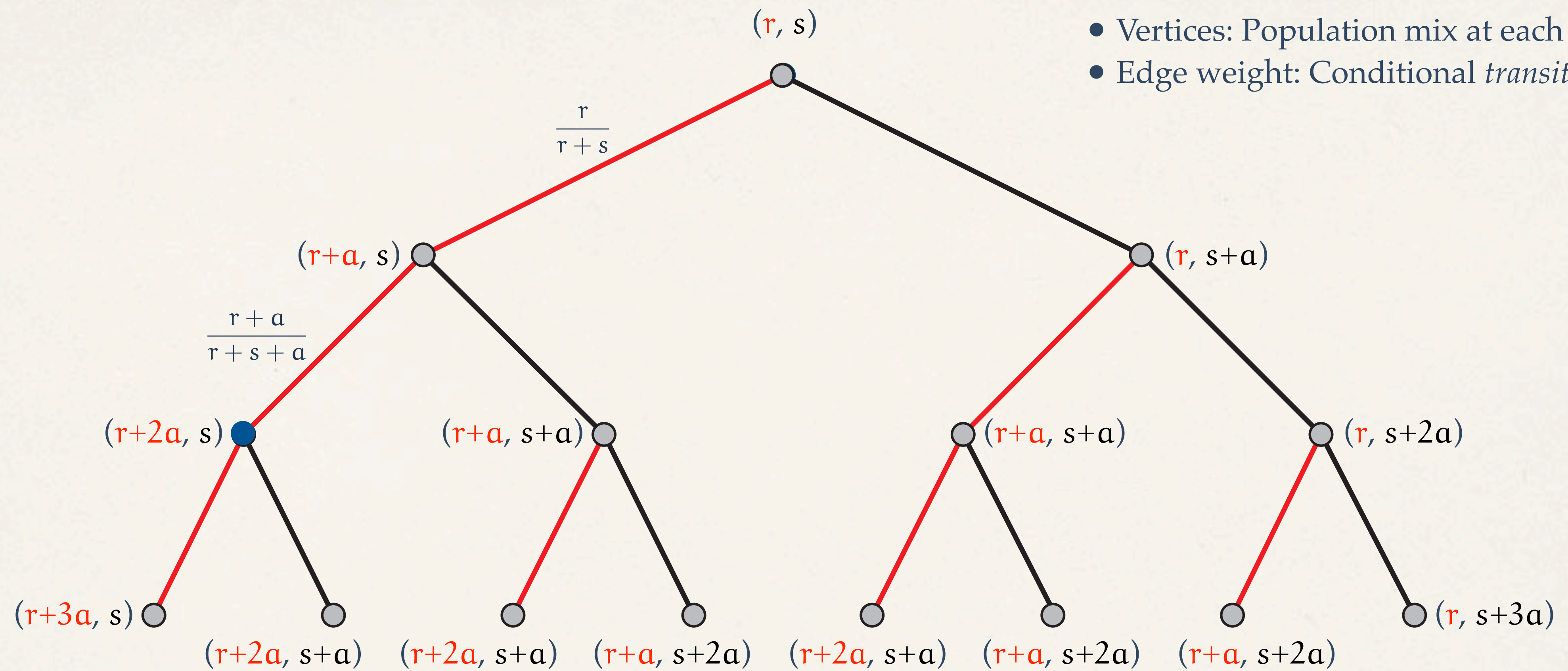




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$$P(R_1) = \frac{r}{r+s};$$

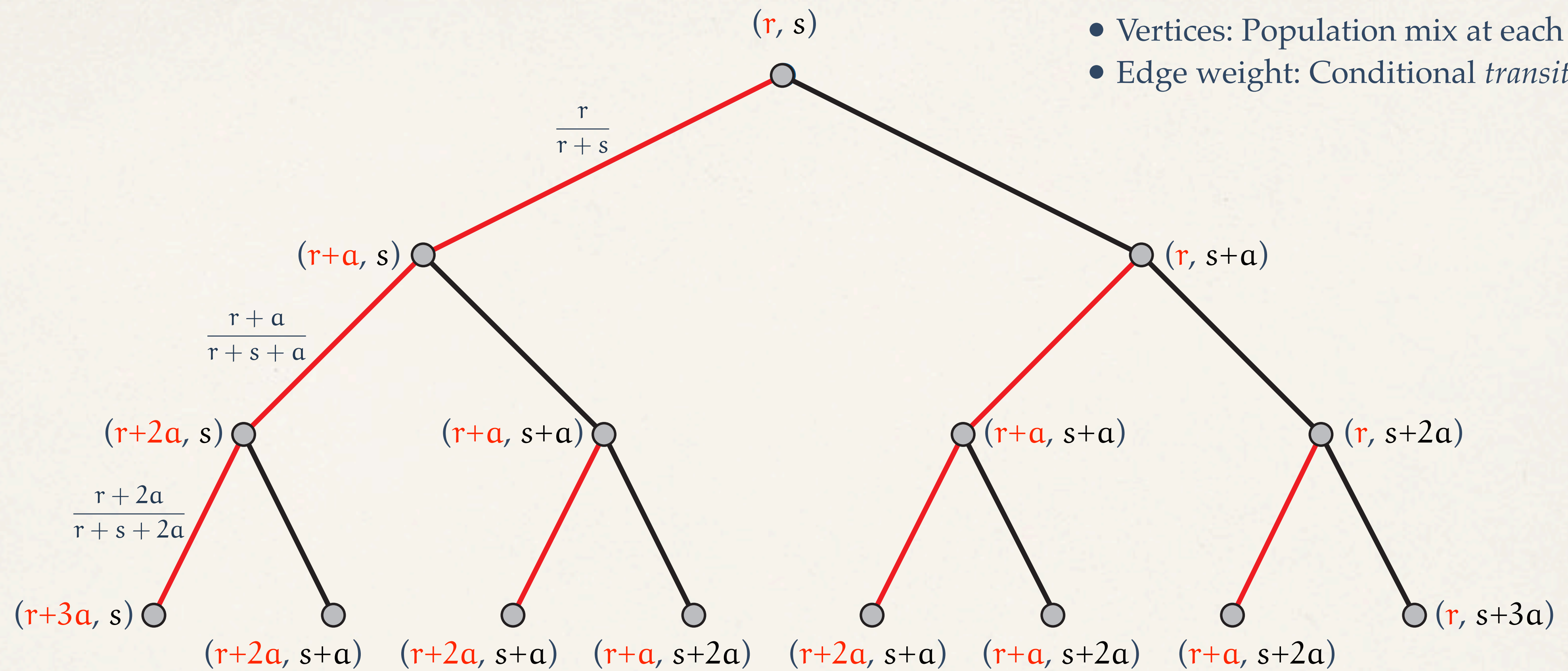




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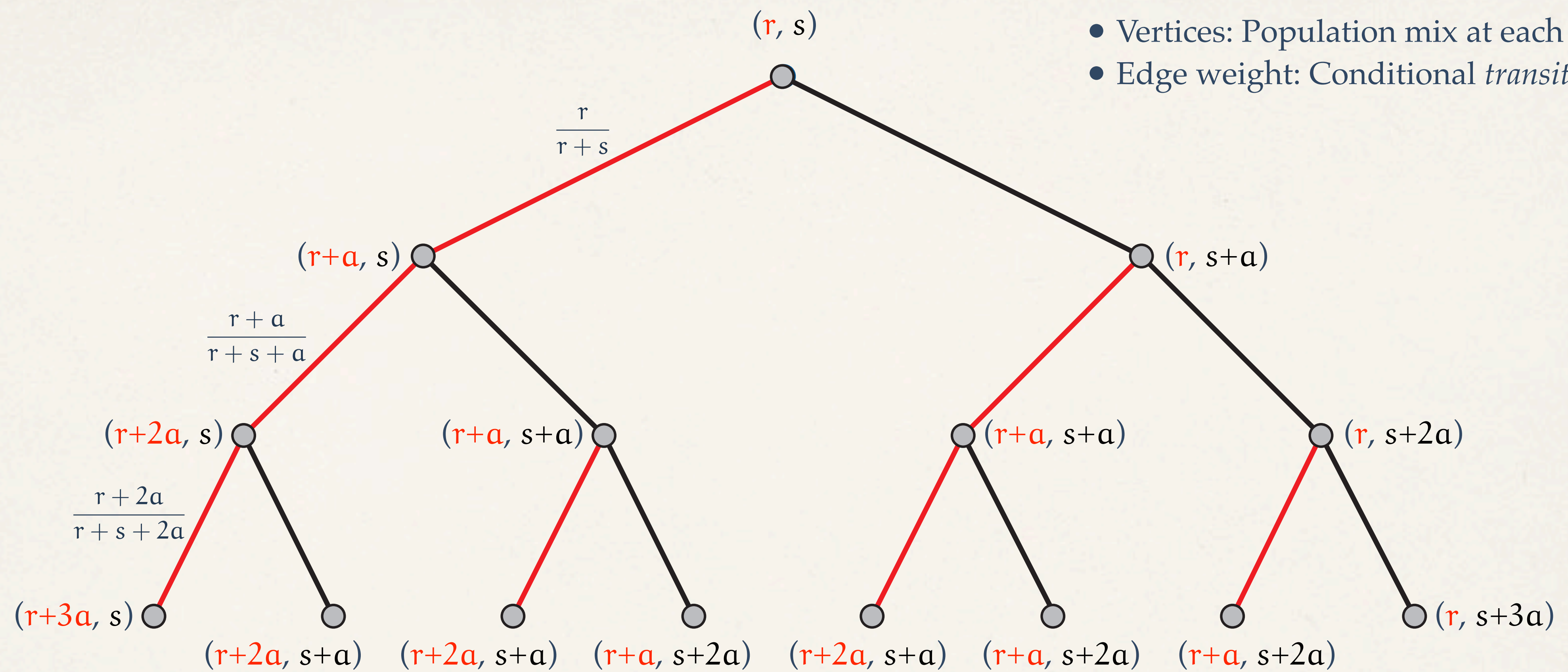
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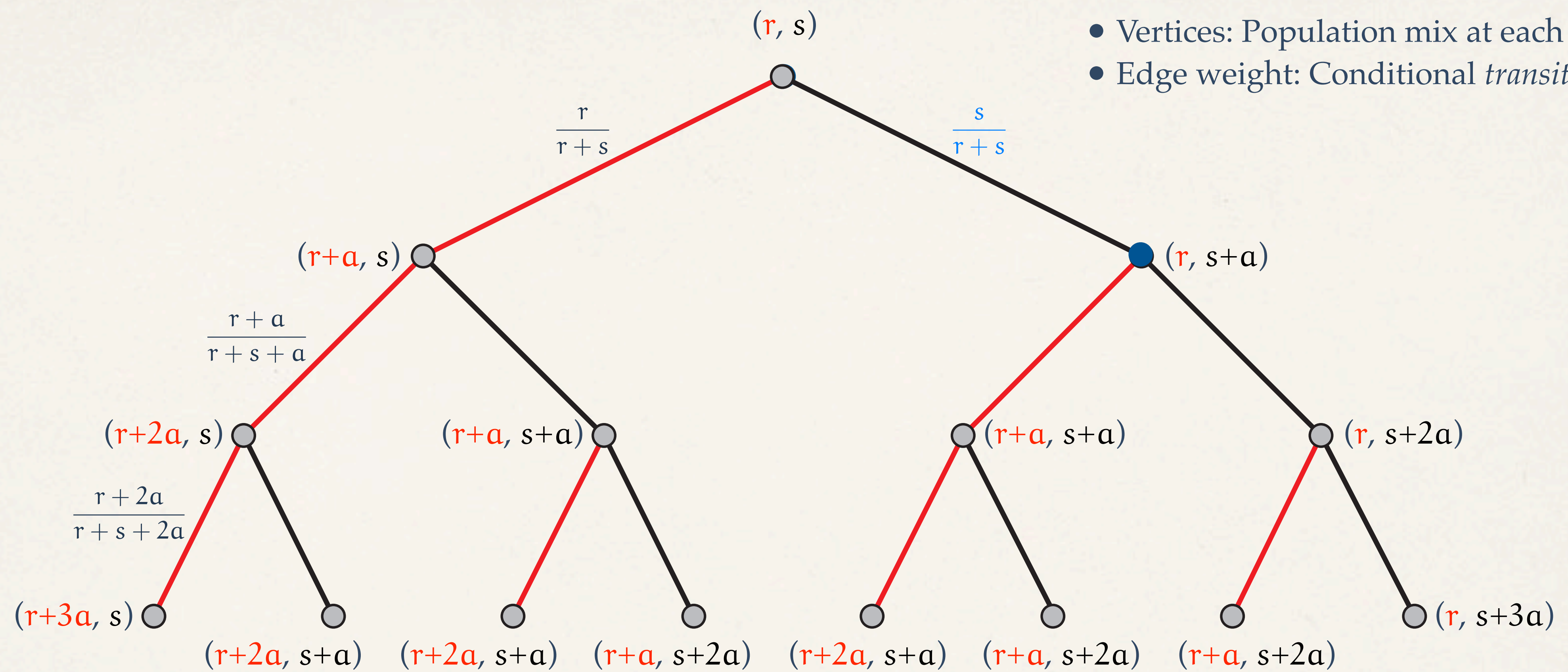
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Evaluate  $\mathbf{P}(B_1)$ ,  $\mathbf{P}(R_2 \mid B_1)$ , and  $\mathbf{P}(R_3 \mid R_2 \cap B_1)$ .





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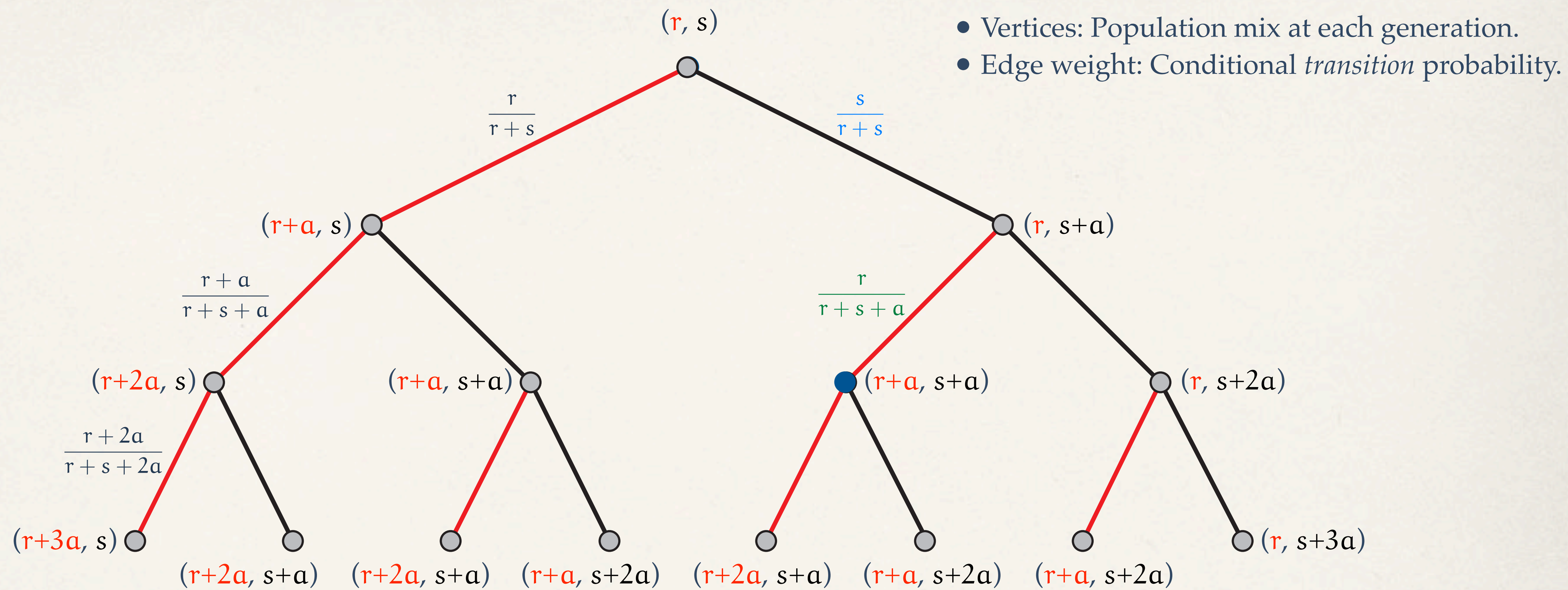
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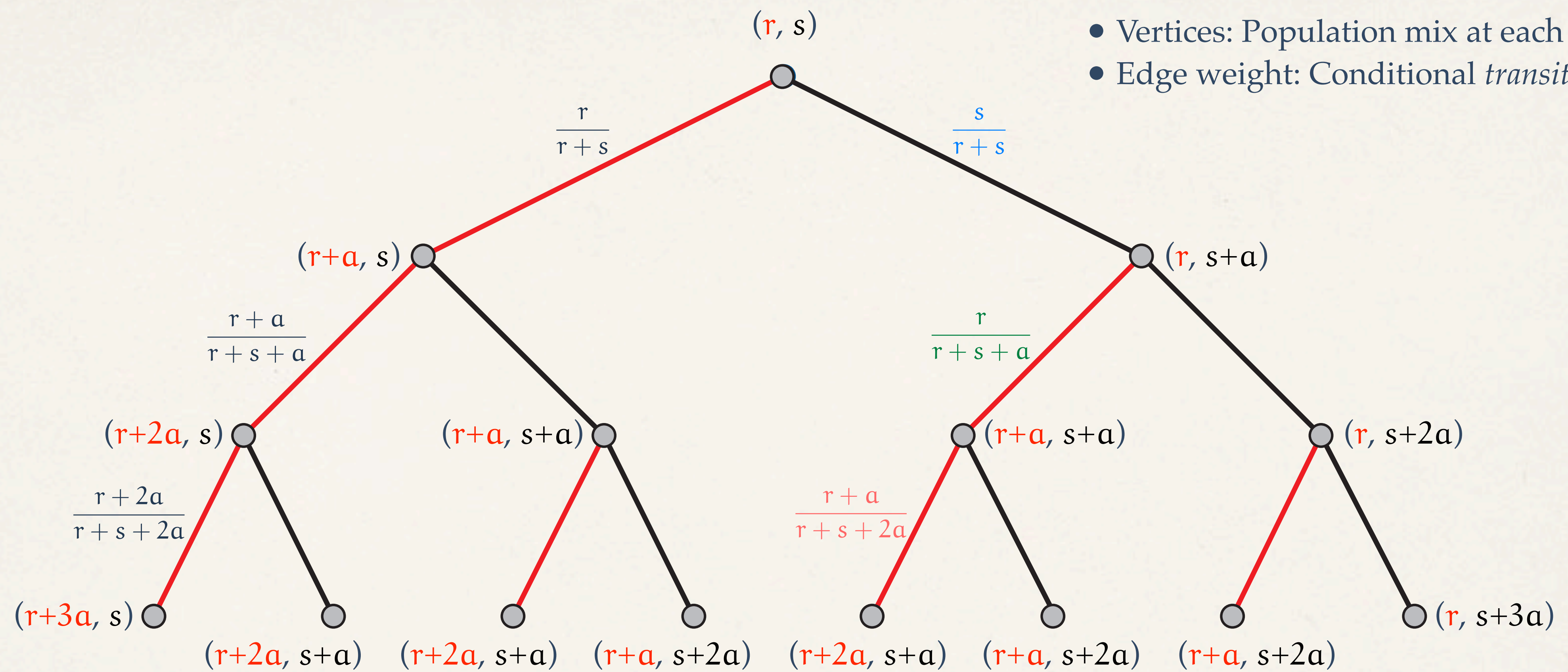
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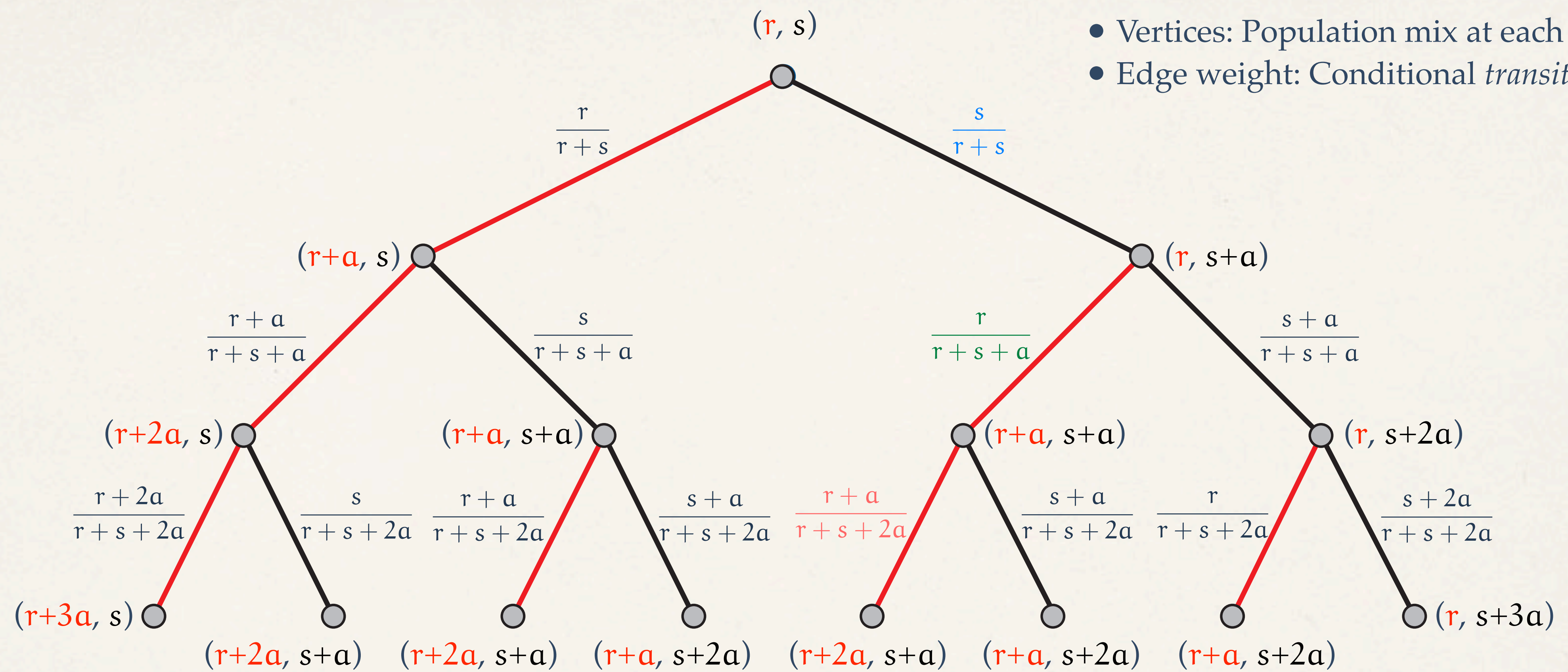
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