The ancient art of sieves


RWI

Boole's inequality

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- `Bad' events: A_j
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$\bigcup_j A_j :=$ a bad event occurs

$\bigcap_j A_j^c :=$ no bad event occurs

When Boole's bound is near zero

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If $\sum_j \mathbf{P}(A_j) < \epsilon$ then $\mathbf{P}\big(\bigcap_j A_j^c\big) = 1 - \mathbf{P}\big(\bigcup_j A_j\big) > 1 - \epsilon$.

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Slogan

If Boole's bound is near zero then *most* outcomes are good.

When Boole's bound is less than one

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If $\sum_j \mathbf{P}(A_j) < 1$ then $\mathbf{P}\big(\bigcap_j A_j^c\big) = 1 - \mathbf{P}\big(\bigcup_j A_j\big) > 0.$

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Some colourful terminology:

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If $\sum_j \mathbf{P}(A_j) < 1$ then $\mathbf{P}\left(\bigcap_j A_j^c\right) = 1 - \mathbf{P}\left(\bigcup_j A_j\right) > 0$.

Slogan: Boole's sieve

If Boole's bound is less than one then there *exist* good outcomes.

Embedding a cube in a two-coloured sphere