

This means finite or countably infinite

Probability measure in discrete spaces

If a die is thrown four times, what are the chances that no ace is seen?

Sample space Ω : each sample point ω is of the form (k_1, k_2, k_3, k_4) where each of k_1 , k_2 , k_3 , k_4 is an integer from 1 to 6.

$$\Omega = \left\{ (k_1, k_2, k_3, k_4) : 1 \leq k_1, k_2, k_3, k_4 \leq 6 \right\} = \left\{ 1, 2, 3, 4, 5, 6 \right\}^4$$

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• Event A that no ace is seen:

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- Probability measure P:
 - To each atom (singleton set) {(k₁, k₂, k₃, k₄)} assign *probability* mass

$$P\{(k_1, k_2, k_3, k_4)\} = \frac{1}{\text{card }\Omega} = \frac{1}{6^4}$$

To the event A assign probability

$$\mathbf{P}(A) = \sum_{\substack{(k_1, k_2, k_3, k_4) \in A}} \mathbf{P}\{(k_1, k_2, k_3, k_4)\} = \frac{\operatorname{card} A}{\operatorname{card} \Omega} = \frac{5^4}{6^4}$$

What is the chance of losing on the first throw in craps?

Sample space Ω: each sample point ω is an integer in the range from 2 through 12.

$$\Omega = \{2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\} = \{k : 2 \le k \le 12\}$$

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 - To each atom (singleton set) {k} assign probability mass

k											
P {k}	1/36	2/36	3/36	4/36	5/36	6/36	5/36	4/36	3/36	2/36	1/36

• To the event A assign probability

$$P(A) = P{2, 3, 12} = P{2} + P{3} + P{12} = \frac{1}{36} + \frac{2}{36} + \frac{1}{36} = \frac{1}{9}$$

Toss a coin repeatedly until two successive tosses show the same face.

What is the chance that the coin is tossed four or more times?

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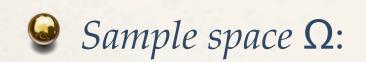
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We may identify the sample space with the set of integers \mathbb{Z}

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Event A that the coin is tossed four or more times at termination:

Toss a coin repeatedly until two successive tosses show the same face. What is the chance that the coin is tossed four or more times?

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	1	-1	2	-2	3	-3	4	-4	0 0 0

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Event A that the coin is tossed four or more times at termination:

A = { k : integer k,
$$|k| \ge 3$$
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- Probability measure P:
 - To each atom (singleton set) {k} assign probability mass $p(k) := P(k) = 2^{-|k|-1}$ (integer $k \neq 0$)
 - To the event A assign probability

$$P(A) = \sum_{\text{integer k with } |k| \ge 3} p(k) = 2\left(\frac{1}{2^4} + \frac{1}{2^5} + \frac{1}{2^6} + \cdots\right) = \frac{2 \cdot 2^{-4}}{1 - 2^{-1}} = \frac{1}{4}$$