Problem 1

A polyhedron $P = \{x \in \mathbb{R}^n : Ax \leq b\}$ *contains a line*, if there exists a nonzero $v \in \mathbb{R}^n$ and an $x^* \in \mathbb{R}^n$ such that for all $\lambda \in \mathbb{R}$, the point $x^* + \lambda \cdot v \in P$. Show that a nonempty polyhedron P contains a line if and only if A does not have full column-rank.

Problem 2 (2.7 in [BT])

Suppose you are given two representations of the same non-empty polyhedron

$$P = \{x \in \mathbb{R}^n | a_i^{\mathrm{T}} x \leq b_i, i = 1, \ldots, m\} = \{x \in \mathbb{R}^n | g_j^{\mathrm{T}} x \leq h_j, j = 1, \ldots, k\}.$$

Show that if the vectors $a_1, \ldots, a_m$ span $\mathbb{R}^n$ then so do $g_1, \ldots, g_k$.