

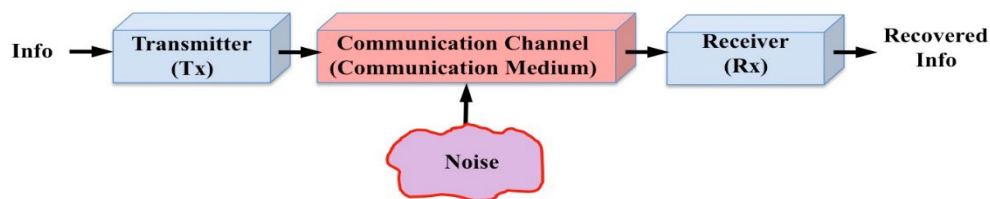
Lesson 18: Intro to Modulation

Objectives:

- (a) Define the term *baseband signal* and describe some potential limitations associated with transmitting baseband signals directly.
- (b) Discuss the role of modulation in signal transmission and the methods of modulating a sinusoidal carrier.
- (c) Using a plot of an AM signal, determine V_{max} , V_{min} , V_m , V_c and m .
- (d) Create a frequency plot for and determine the sidebands and bandwidth of an AM signal where the information signal consists of one or more distinct sinusoids.
- (e) Determine the condition for overmodulation in an AM signal, and the consequence of overmodulation.

Connection to Cyber Security

In Lesson 17, we defined a communications system and learned that the wireless section of EC312 will focus on communications in which the communication channel is free space.



We also learned that in order to send any information through a communications system it must be in the form of a signal (which is the name given to the function that conveys our information), and if our communication channel is free space, it means we're dealing with signals carried in the Electromagnetic Spectrum (EM). Finally, we learned that signals can be represented as a function of either time or frequency. Wireless channels have different vulnerabilities than we saw in the host section of the course, because of the frequency of transmission. It is possible that a wireless network can be attacked like we saw in the networks section of the course, but to see how such a cyber-attack can be carried out on a wireless network or a wireless communication in general, you must first understand how information is carried through the wireless channel.

1. Baseband Signals

If you're sitting in your EC312 classroom in the basement of Rickover Hall and you speak to the student next to you, will that person be able to hear you? Will you be heard across the room? How about at the end of that loooong Rickover passageway? Does anyone in Bancroft have a chance of hearing what you're grumbling about in your EC312 classroom? Why not? Your voice doesn't travel as far as you might like it to—your voice creates pressure waves in the air, and the strength of these waves attenuates over distance. The louder you yell the farther you'll be heard, but this of course has its limits. As an alternative to walking around screaming all day, you might consider speaking at a more normal volume into a microphone. A microphone is a device that transforms sound pressure waves into electrical signals. You could then send the electrical output of the microphone to an antenna. Then your voice frequencies would travel as electromagnetic waves ("radio waves"), and as long as you provided enough power to the antenna, you could presumably greatly extend the geographic range of your EC312 musings. In this chapter, we'll see that the latter approach is on the right track, but in order to be practical, it requires a bit more finesse.

In this example our voice signal, which you'll recall is comprised of frequencies roughly in the range between 300 Hz and 3 kHz, is what we call a *baseband signal*. Baseband signals are information signals at their original frequencies, typically low frequencies. To transmit a baseband signal directly as is, we use *baseband transmission*...as you'll see in shortly, communication systems typically will upshift the frequency spectrum of baseband signals to a higher range of frequencies to allow transmission through the atmosphere.

In general, before signals can be transmitted effectively, they must first be converted to a form that is compatible with the communication medium. One facet of this conversion is *transducing* the signal from its natural physical form into an electrical signal. For example,

- Microphones convert acoustic pressure waves (sound) into electrical signals.
- Video cameras convert light patterns into electrical signals.
- Computer keyboards convert physical input (typing) into an electrical signals.

But, as alluded to above, even after converting your voice signal to, say, a voltage signal using a microphone, attempting to transmit it over the air as a baseband signal is impractical. Why? Let's look at an example that will point us in the right direction.

Practice Problem 18.1

Physics dictates that antenna length is intrinsically tied to the wavelength of the signal it is transmitting or receiving. To transmit a signal through the atmosphere with an antenna efficiently, the length of the antenna must be at least a tenth of a wavelength long.

What is the approximate length of the antenna required to transmit the sound of a tuning fork (which creates musical note A = 440 Hz)? Note: this sound must be transduced into an electrical signal first before it is transmitted.

Solution:

Wait! To transmit that lousy tuning fork signal my antenna needs to be at *least* 68 km? That's over 42 miles! We'd need an antenna that extends into the upper parts of the atmosphere for that. Clearly, that's not going to work.

Well, I know that if I want to listen to the Navy game on the radio (because for some reason I avoided the mandatory fun), I can tune in to AM radio station 1430 WNAV.

Recall that from Chapter 17, when referring to a commercial AM radio station, such as 1430 WNAV, that the 1430 refers to the center of its transmission frequency in kHz. So what size antenna does WNAV use?



Practice Problem 18.2

(a) What is the wavelength of an AM radio station whose transmission frequency is 1430 kHz?

Solution:

(b) What is the approximate antenna length if the station uses an antenna that is half the wavelength long?

Solution:

105 meters? Okay, that's still big – it's about 115 yards - almost an entire football field... but at least you don't need an antenna that reaches into outer space now. In reality, WNAV's antenna, pictured at right, is 117 meters. We're close!

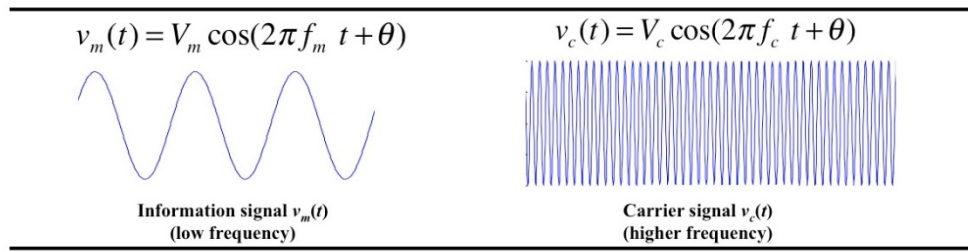
The purpose of those two examples was to demonstrate that we need to somehow get our *baseband* information to a higher frequency (shorter wavelength) in order to be able to transmit it across our channel. Higher frequencies give us reasonable antenna sizes plus some added benefits: first, signals will attenuate less quickly if the higher frequencies are well-chosen. Second, multiple people - whose voices all occupy the same baseband frequencies - can communicate without interfering if each transmits on a different higher frequency range. We can shift baseband information to higher frequencies for transmission using a process called *modulation*.



2. Modulation

To overcome limitations of the communications channel and permit multiple access, information signals are *impressed* upon a higher-frequency carrier signal for transmission. This process is called *modulation*. Now we're dealing with two signals:

1. Original ("baseband") information signal - frequency is too low to transmit efficiently
2. Higher frequency ("carrier") signal - we *can* transmit this efficiently, so we use it to **carry** our information

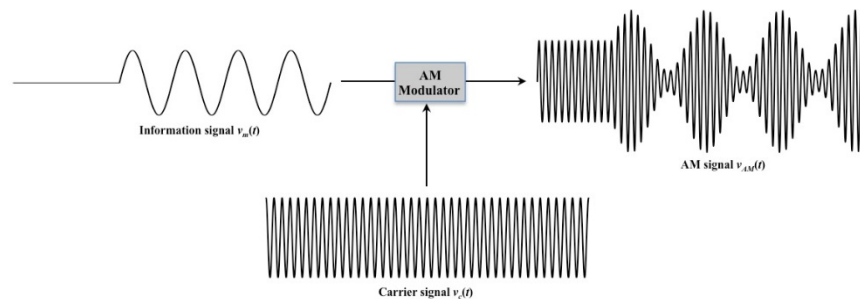


Mathematically, the cosine wave representing the higher-frequency carrier is given by:

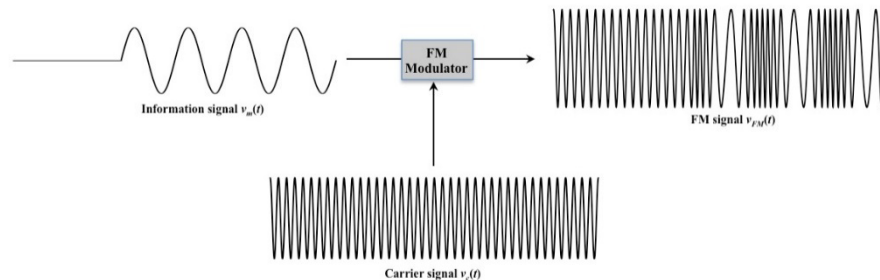
$$v_c(t) = V_c \cos(2\pi f_c t + \theta_c)$$

Modulation is the process of varying any of three properties (amplitude, frequency or phase, underlined above) of a high-frequency carrier using the lower-frequency information signal (baseband signal). A *modulator* is a component of a communication system which achieves modulation. The three types of modulation we will focus on are:

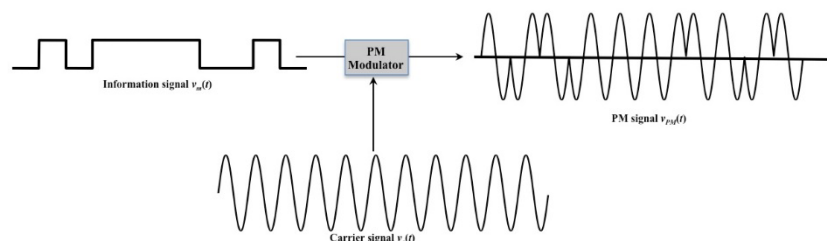
2.1 Amplitude modulation (AM) Varying the amplitude V_c of the carrier with the info signal.



2.2 Frequency modulation (FM) Varying the frequency f_c of the carrier with the info signal.



2.3 Phase modulation (PM) Varying the phase angle θ_c of the carrier with the info signal.



Since the intention of the "Wireless" section of EC312 is to give you a broad understanding of wireless communication techniques rather than to make you communication engineers, we're only going to go into more detail with amplitude

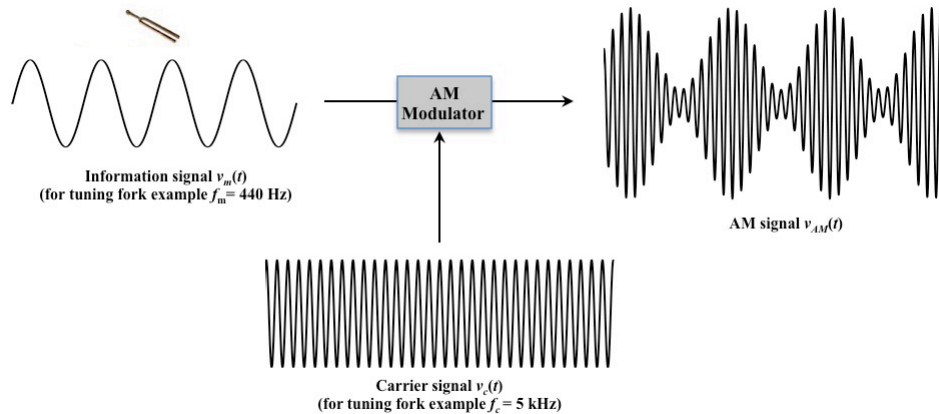
modulation (AM) in this chapter. This is NOT to say that frequency modulation and phase modulation are unimportant – they’re very important and very widely used. The fact is that there’s only so much modulation that can be reasonably covered in the last several chapters of this course, and AM is the easiest to visualize and demonstrate. Later, in Chapter 25: Digital Modulation, we will again address amplitude, frequency and phase modulation to some extent as they apply to digital communications.

3. Amplitude Modulation (AM)

In amplitude modulation, the information signal is used to vary the amplitude of the carrier sine wave. For simplicity, consider a cosine wave information signal, $v_m(t)$ (a 440 Hz tuning fork) and a cosine carrier, $v_c(t)$ (frequency 5000 Hz (or 5 kHz)).

$$v_m(t) = V_m \cos(2\pi f_m t) = V_m \cos(2\pi 440t)$$

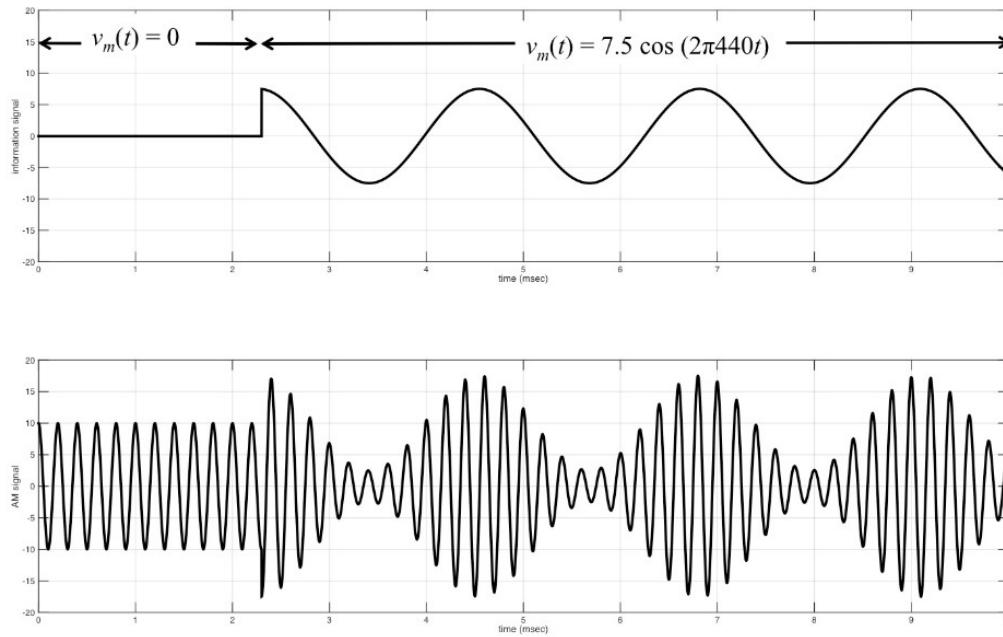
The diagram of an amplitude modulation system using this information signal follows.



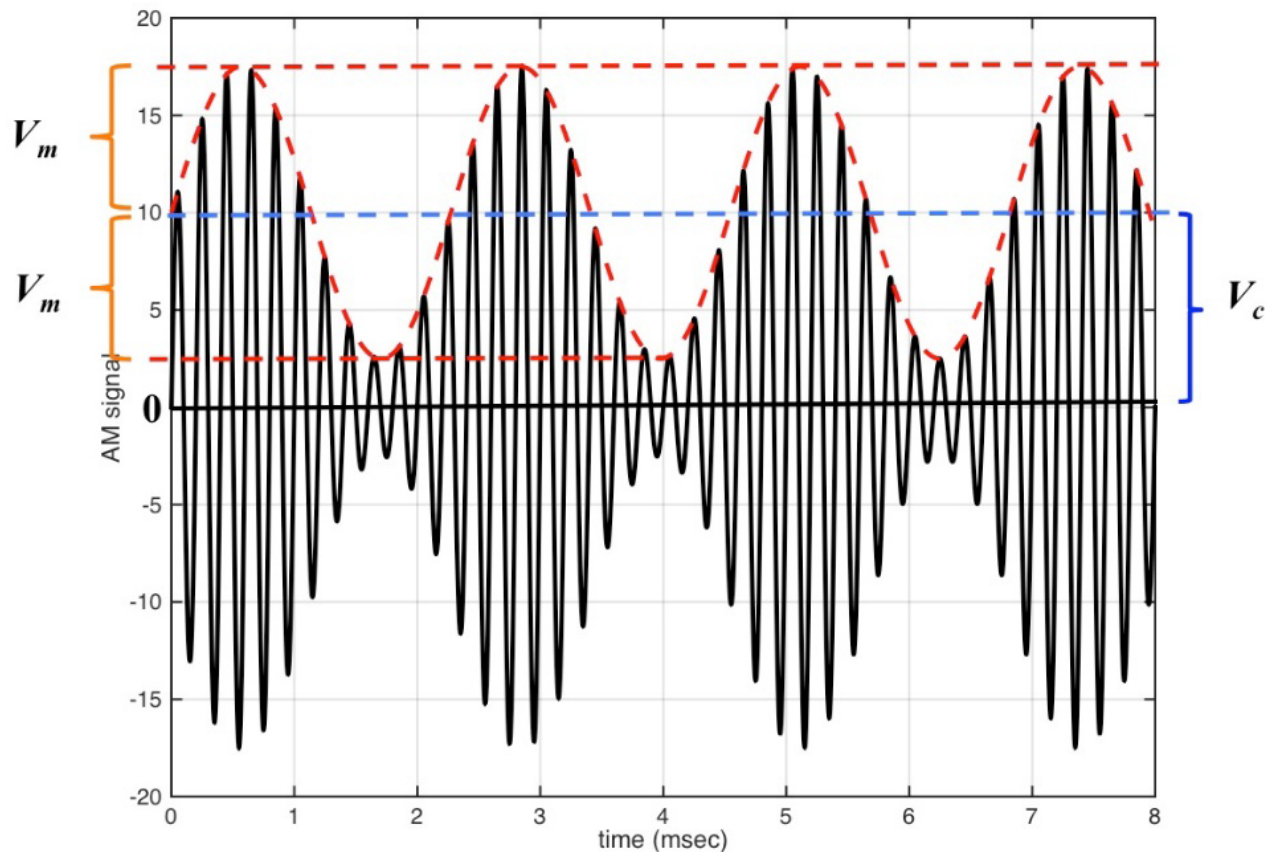
The AM wave ($v_{AM}(t)$) is the product of the carrier (with amplitude = 1) with a modulating signal. The modulating signal is the information signal $v_m(t)$ with an added offset, V_c . The AM signal is then given by:

$$v_{AM}(t) = \underbrace{(V_c + v_m(t))}_{\text{Modulating Signal}} \underbrace{\cos(2\pi f_c t)}_{\text{Carrier}} = \underbrace{(V_c + V_m \cos(2\pi f_m t))}_{\text{Modulating Signal}} \underbrace{\cos(2\pi f_c t)}_{\text{Carrier}}$$

In the figure below, the top plot is of the information signal and the bottom is the resulting AM signal. Note that the information signal starts at a value of zero (for approximately 2 msec), so the resulting AM signal is the *unmodulated* carrier signal (meaning that the carrier is not being modulated). When the information signal is no longer zero, it starts to modulate the carrier’s amplitude as shown. The horizontal axes are time in msec. The information signal is equal to zero at the beginning, then changes to the tuning fork cosine wave at approximately 2 msec. Here, the carrier and information signal parameters are: $f_c = 5$ kHz, $V_c = 10$ V, $f_m = 440$ Hz, $V_m = 7.5$ V.



A zoomed-in plot of the resulting modulated AM signal is as follows, showing the graphical relation between V_m and V_c :

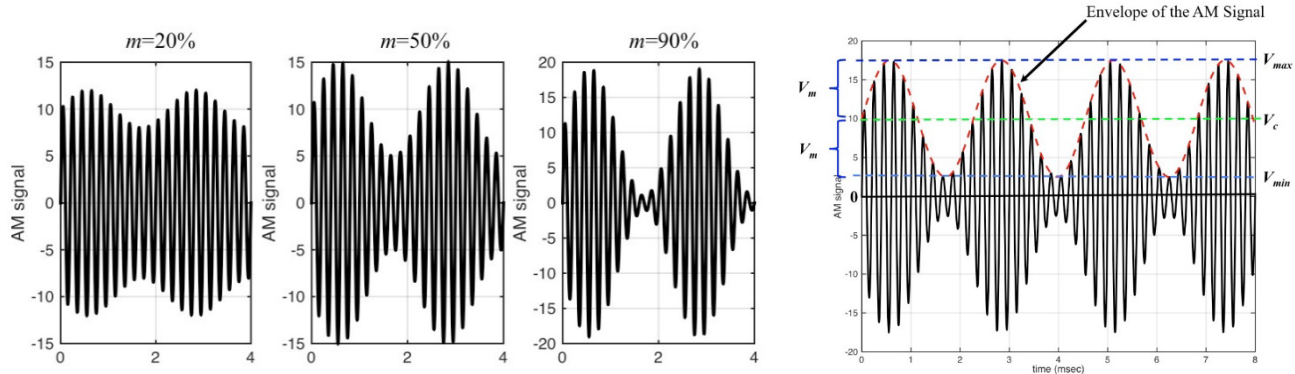


The *envelope* of the modulating signal (which is drawn onto the AM signal below in a dashed red line) varies above and below the unmodulated carrier amplitude, V_c . It is the envelope that carries the information signal; the receiver must separate the envelope from the received AM signal to recover the information that was transmitted. In this case, the envelope is in the shape of a sine wave, which is the same as the information signal. The values of V_m and V_c are related by the *modulation index* (m).

3.1 Modulation Index The relationship between the information signal amplitude, V_m , and the unmodulated carrier amplitude, V_c , is expressed as a ratio called the *modulation index* (m), defined as:

$$m = \frac{V_m}{V_c}$$

Sometimes m is expressed as a percentage: percent modulation = $m \times 100\%$. The following figure shows the AM signal at three different values of percent modulation: 20%, 50% and 90%. Overall, the greater the value of m , the closer the envelope gets to the horizontal (time) axis.



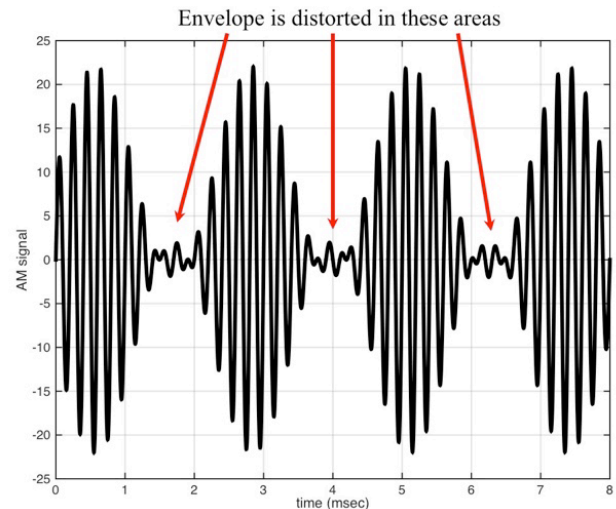
We can also mathematically determine the modulation index m from the maximum and minimum values of the envelope of $v_{AM}(t)$ as follows, where V_{max} is the maximum value of the envelope and V_{min} is the minimum value:

$$V_m = \frac{V_{max} - V_{min}}{2}$$

$$V_c = \frac{V_{max} + V_{min}}{2}$$

$$m = \frac{V_m}{V_c} = \frac{V_{max} - V_{min}}{V_{max} + V_{min}}$$

In order for the AM signal to convey the original signal accurately and prevent distortion, the information signal amplitude (V_m) must be less than the unmodulated carrier signal amplitude (V_c). Here again, the unmodulated carrier refers to the AM signal if the information signal amplitude is equal to 0 ($V_m = 0$), in which case, $v_m(t) = V_m \cos(2\pi f_m t)$. The maximum usable modulation index is $m = 1.0$, corresponding to 100% modulation, when V_m is equal to V_c . When V_m is greater than V_c (that is, $m > 1$), *overmodulation* occurs. Overmodulation, depicted below, results in distortion of the AM signal's envelope, and since the envelope holds the information, the recovered information signal is also distorted.



Practice Problem 18.3

If a carrier signal $v_c(t) = 9 \cos(2\pi 5000t)$ Volts is modulated by a cosine wave $v_m(t) = 7.5 \cos(2\pi 440t)$ V, what is the percentage modulation of the resulting AM signal?

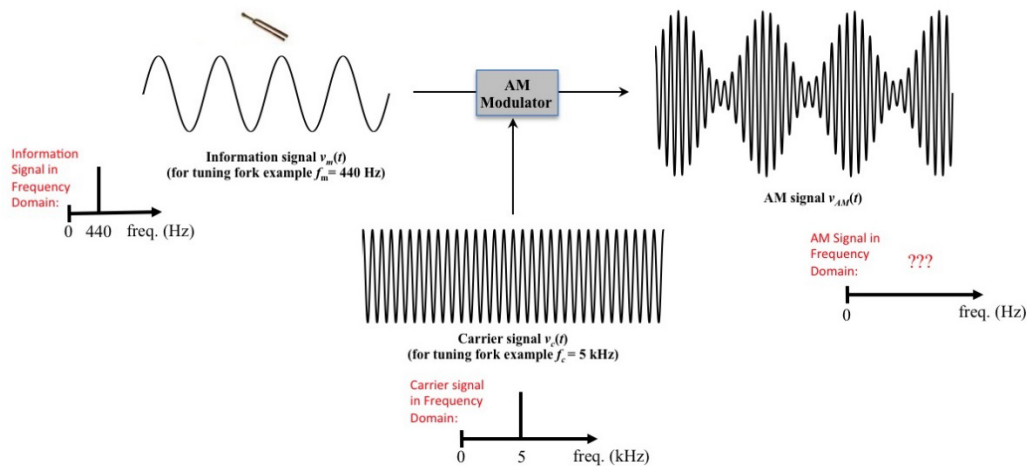
Solution:

Now that we have a basic understanding of how Amplitude Modulation works in the time domain, let's look at AM in the frequency domain.

4. AM in Frequency Domain

Recall the equation for the amplitude modulated waveform if the information signal is a single sine wave is given by: .

$v_{AM}(t) = (V_c + V_m \cos(2\pi f_m t)) \cos(2\pi f_c t)$. We already know the frequency domain representations of the modulating signal ($v_m(t)$) and the carrier signal ($v_c(t)$), but how does the amplitude modulated signal look in the frequency domain?



To answer this question, recall the trig identity for the product of two sine waves:

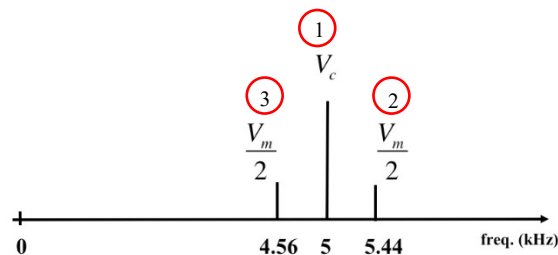
$$\cos A \cos B = \frac{1}{2} \cos(A + B) + \frac{1}{2} \cos(A - B)$$

Applying this trig identity for product of two sine waves to the AM signal results in:

$$\begin{aligned} v_{AM}(t) &= (V_c + V_m \cos(2\pi f_m t)) \cos(2\pi f_c t) \\ &= V_c \cos(2\pi f_c t) + V_m \cos(2\pi f_c t) \cos(2\pi f_m t) \\ &= \underbrace{V_c \cos(2\pi f_c t)}_1 + \underbrace{\frac{V_m}{2} \cos(2\pi(f_c + f_m)t)}_2 + \underbrace{\frac{V_m}{2} \cos(2\pi(f_c - f_m)t)}_3 \end{aligned}$$

This means that when a single sine wave information signal is used to modulate the carrier in AM, the resulting AM signal contains three sinusoids (labeled above): one at the carrier frequency (1), one f_m Hz above the carrier frequency (2), and one f_m Hz below the carrier frequency (3). For the tuning fork example, we have: $f_c = 5$ kHz, $f_c - f_m = 4.560$ kHz and $f_c + f_m = 5.440$ kHz. The trig identity puts the amplitudes at frequencies $f_c - f_m$ and $f_c + f_m$ at one half that of V_m .

This means the resulting frequency domain plot for this tuning fork example looks like the following (the labels follow the ones above):



The process of modulating a carrier creates an upper and a lower *sideband* that is apparent in the frequency plot. The *lower sideband* (or LSB) is that portion of the transmitted signal that has frequency content less than the carrier frequency, and the *upper sideband* (or USB) has frequency content greater than the carrier frequency. For the tuning fork example, the USB is the 5440 Hz cosine, and the LSB is the 4560 Hz cosine. On a frequency plot of an AM signal, the lower sideband is a mirror image of the upper sideband centered about the carrier frequency.

What is the AM signal's bandwidth? Since bandwidth is the highest transmitted frequency minus the lowest frequency transmitted, it is $(f_c + f_m) - (f_c - f_m) = 2f_m = 880$ Hz. This is twice the bandwidth of the information signal we started with...if we didn't modulate the information signal, the transmission bandwidth would have only been f_m (440 Hz in this case)¹. This means that by transmitting with AM, we have doubled the required bandwidth to transmit the signal. Why is this a concern? *Bandwidth* is the #2 limiting factor in communications systems, and can be expensive to use... so we're going to want to send as much information as possible while occupying the minimum amount of bandwidth possible. (The #1 limiting factor in communications systems is *noise*, but that's a subject for a future lesson.)

Let's look at a slightly more complex example – suppose the information signal was comprised of two cosine waves:

¹ The bandwidth of a baseband signal is considered to be its maximum frequency content. In this case, if the message is a single sinusoid at a frequency of f_m Hz, we say its bandwidth is f_m Hz.

$$v_m(t) = V_1 \cos(2\pi f_1 t) + V_2 \cos(2\pi f_2 t)$$

What does the AM signal look like? Again, we apply the same trig identity to each sine in the information signal, resulting in:

$$\begin{aligned} v_{AM}(t) &= (V_c + v_m(t)) \cos(2\pi f_c t) \\ &= (V_c + V_1 \cos(2\pi f_1 t) + V_2 \cos(2\pi f_2 t)) \cos(2\pi f_c t) \\ &= V_c \cos(2\pi f_c t) + V_1 \cos(2\pi f_c t) \cos(2\pi f_1 t) + V_2 \cos(2\pi f_c t) \cos(2\pi f_2 t) \\ &= V_c \cos(2\pi f_c t) + \frac{V_1}{2} \cos(2\pi(f_c + f_1)t) + \frac{V_1}{2} \cos(2\pi(f_c - f_1)t) + \frac{V_2}{2} \cos(2\pi(f_c + f_2)t) + \frac{V_2}{2} \cos(2\pi(f_c - f_2)t) \end{aligned}$$

The net result is that for each cosine in the message, we will wind up with two cosines in the AM signal: one will have a frequency greater than the carrier frequency, one will have a frequency less than the carrier frequency. The bandwidth is still equal to the highest frequency in the AM signal minus the lowest frequency. This is equal to two times the max frequency in the information signal. That is, if f_{max} is the maximum of the two frequencies in the information signal (either f_1 or f_2), then the AM bandwidth is $BW = 2f_{max}$.

Practice Problem 18.4

Suppose we want to transmit the sound of a two chime doorbell ($f_1=349$ Hz, $f_2=440$ Hz) using VLF (very low frequency) communications (let $f_c = 20$ kHz). Each of the chimes has an amplitude of 10V, and the carrier's amplitude is 20V. Sketch the frequency domain representation of the transmitted signal and determine the bandwidth.

Which of the two chime frequencies determines the bandwidth?

Solution:

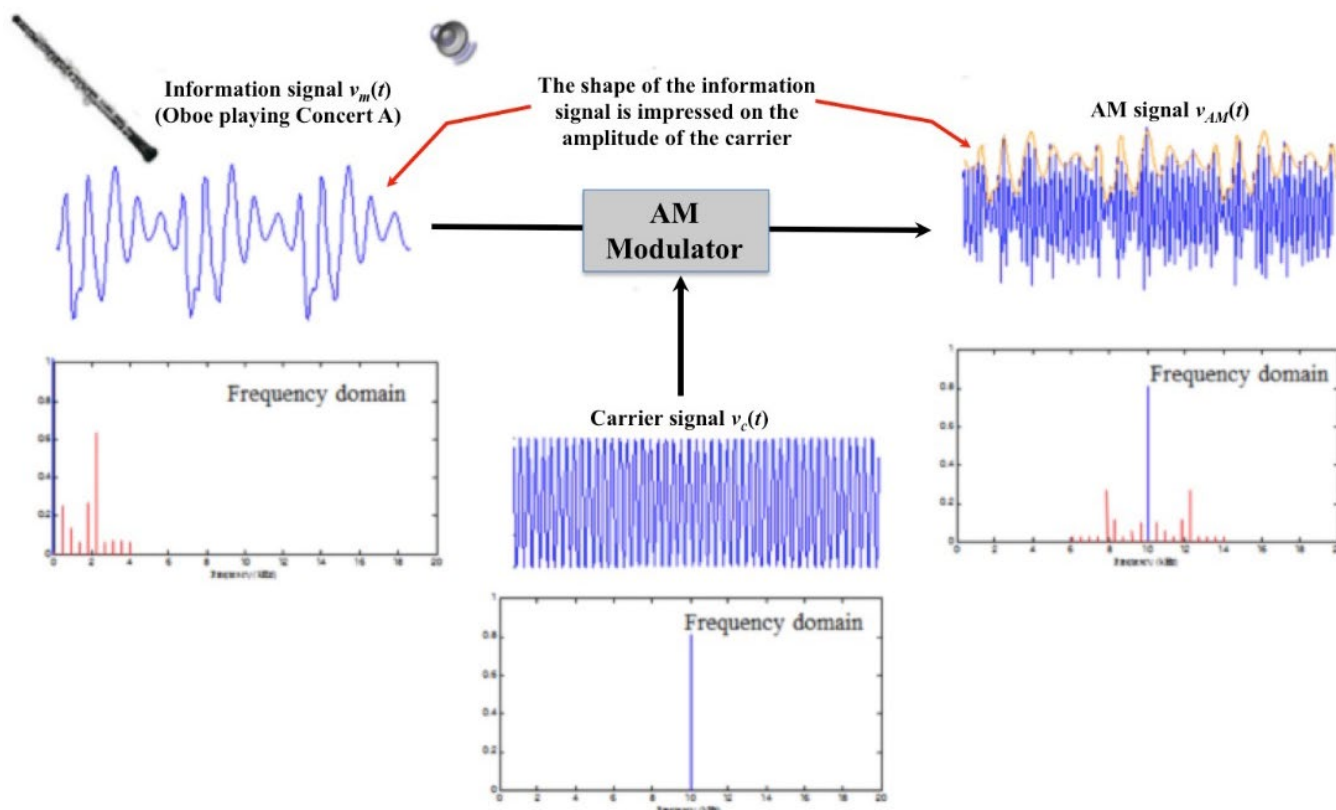
Practice Problem 18.5

If a carrier signal $v_c(t) = 20 \sin(2\pi 5000t)$ Volts is amplitude modulated by information signal

$v_m(t) = 4 \sin(2\pi 200t) - 6 \cos(2\pi 400t + \frac{4\pi}{11})$, sketch the frequency plot for the resulting AM signal and calculate the transmission bandwidth.

Solution:

An example of an even more complicated signal is the signal created by recording an oboe (the musical instrument) playing a single note. When someone blows into the mouthpiece of an oboe to play a note, their fingers are placed over certain holes to create the note. Because of the structure of the oboe (its length, diameter and placement of the holes for example), the sound the instrument makes is actually a combination of a number of tones (sinusoids) with different amplitudes. Different instruments can all play the same note, but each instrument will sound differently because the structure of the instrument produces different sinusoids with different amplitudes. Amplitude modulation of an oboe playing the note Concert A is demonstrated on the next figure. In this figure, the maximum frequency present in the note is approximately 4 kHz, so the bandwidth of the AM signal is $BW = 2f_{max} = 2(4 \text{ kHz}) = 8 \text{ kHz}$.



Other common information signals, such as voice or music, are composed of many different frequencies. AM modulation still works the same way, but in order to compute transmission bandwidth, we again compute it as $BW = 2f_{max}$. Again, here, f_{max} is the maximum frequency content present in the information signal.

5. Demodulation

Modulation is used to upshift the frequency content of a baseband signal, to facilitate transmission (e.g., to allow a smaller antenna). Demodulation is performed in the receiver to downshift that frequency content back to its baseband frequency. For example, if the 440 Hz tuning fork signal were transmitted on Annapolis AM radio station WYRE 810AM ($f_c = 810$ kHz), the transmitted signal is at a frequency of approximately 810 kHz, which is well outside our hearing (we can hear signals with frequency content up to approximately 20 kHz). In order to hear the tuning fork signal, our car's radio receiver must shift the frequency content back down to its original range (440 Hz). This is demodulation; it basically "undoes" what modulation did to the information signal. How demodulation works is beyond the scope of this course, but you should be aware of its importance in a communication system.

What's the point? AM is by no means the only form of modulation (though it's probably the easiest to work through and visualize). The intent of this course is not to make you all communications engineers, but you do need to have enough background in modulation to understand the implications (especially with regard to bandwidth) moving forward. You'll see this again in a few lessons, with digital applications.