Computational Photography

Dr. Irfan Essa
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Study the basics of computation and its impact on the entire workflow of photography, from capturing, manipulating and collaborating on, and sharing photographs.

Digital Images: Into the Frequency Domain

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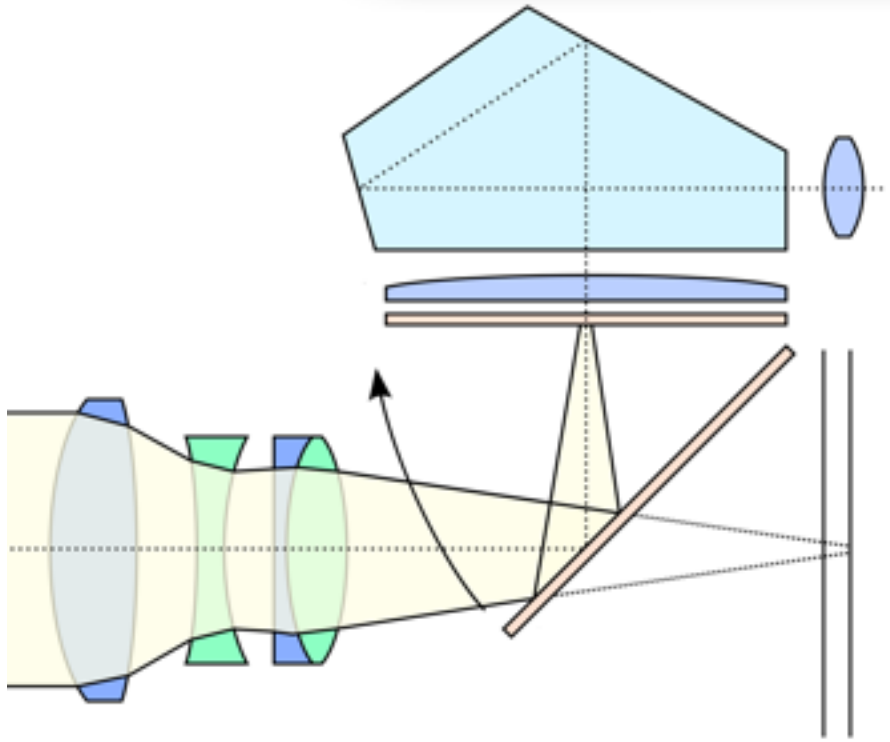
Images are samples of light information stored in array of pixels.

Lesson Objectives

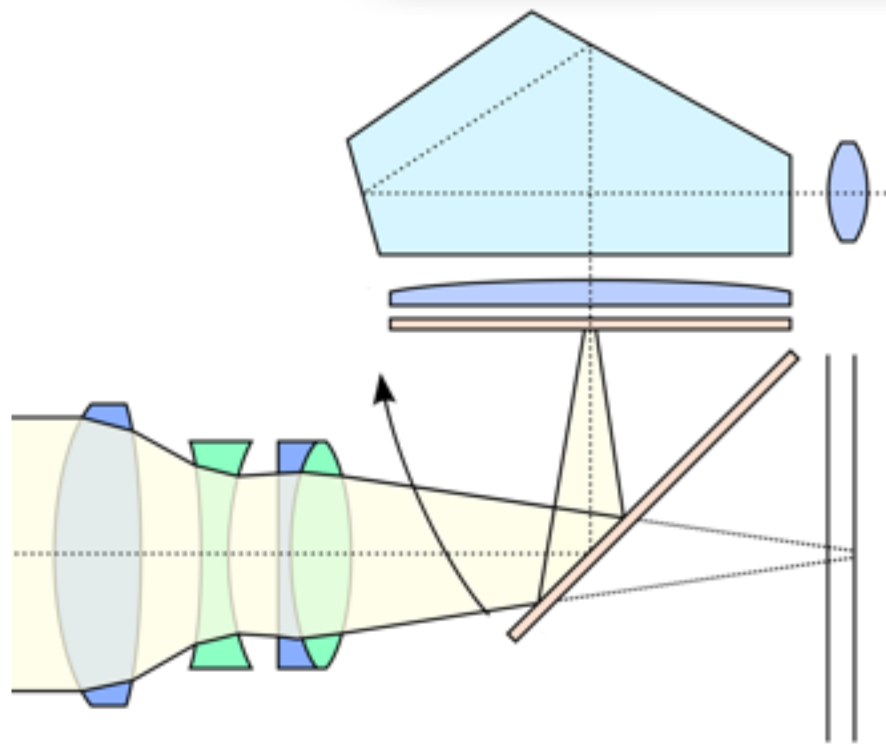
- ★ Describe in your own words how sines and cosines can be used to reconstruct a signal.
- ★ Describe in your own words the Fourier Transform using terms like Frequency, Amplitude and Phase.
- ★ Explain in your own words the value of using Frequency Domains for a Signal.
- ★ Recall the three (3) properties of Convolution as it is associated with the Fourier Transform.

REVIEW: Images and Camera



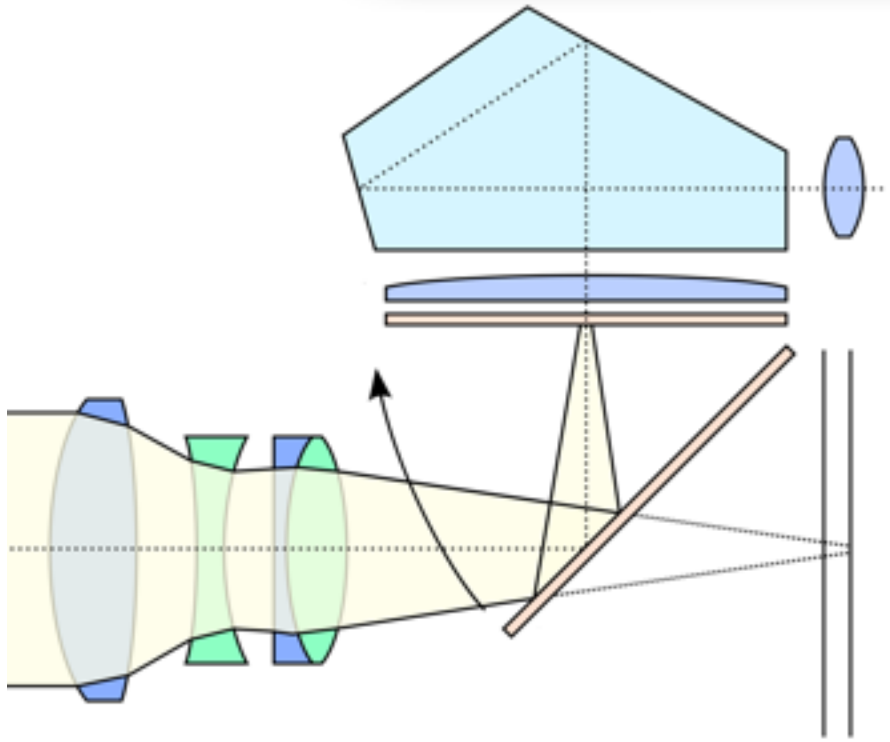
REVIEW: Images and Camera

- Rays of Light go through
  - a Camera
  - Optics/Lens



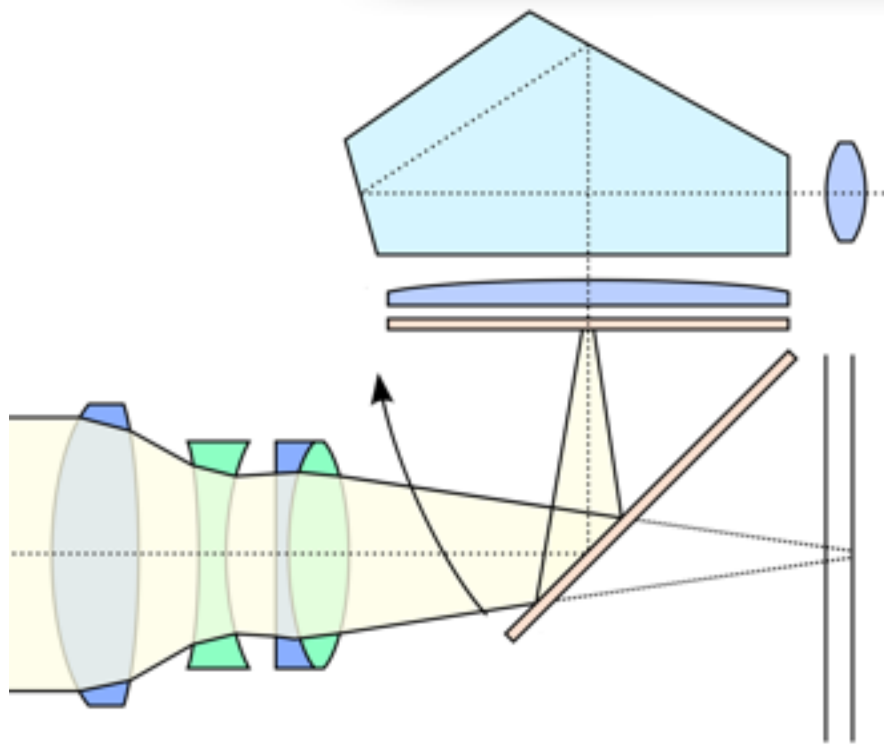
REVIEW: Images and Camera

- ★ Rays of Light go through
  - a Camera
  - Optics/Lens
- ★ Controlled by Aperture, Shutter, and Film Sensitivity



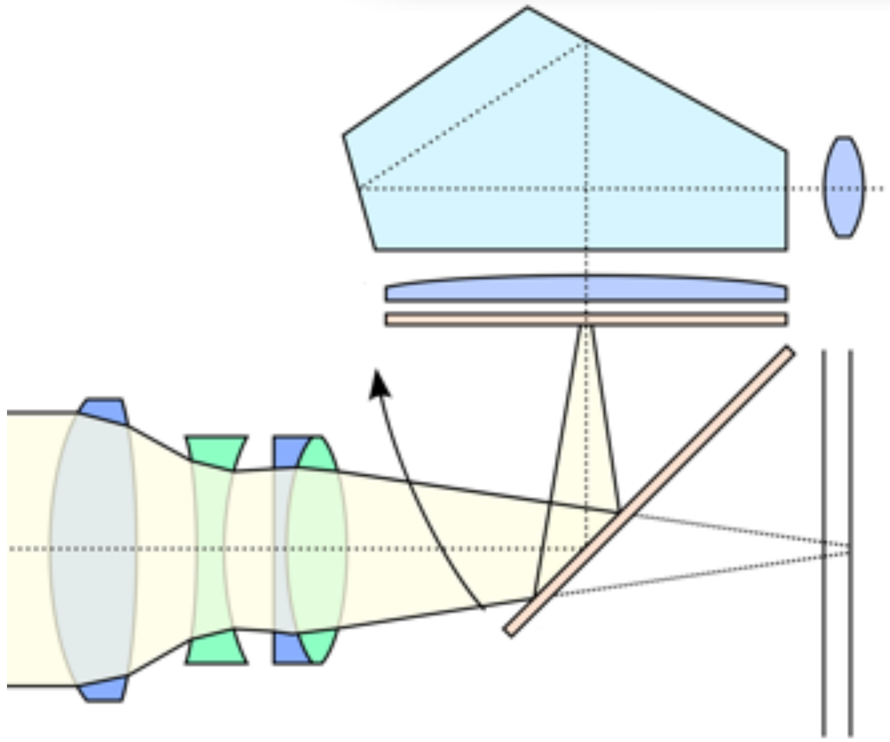
REVIEW: Images and Camera

- Rays of Light go through
  - a Camera
  - Optics/Lens
- Controlled by Aperture, Shutter, and Film Sensitivity
- Sensor to Convert Light to Digital Information



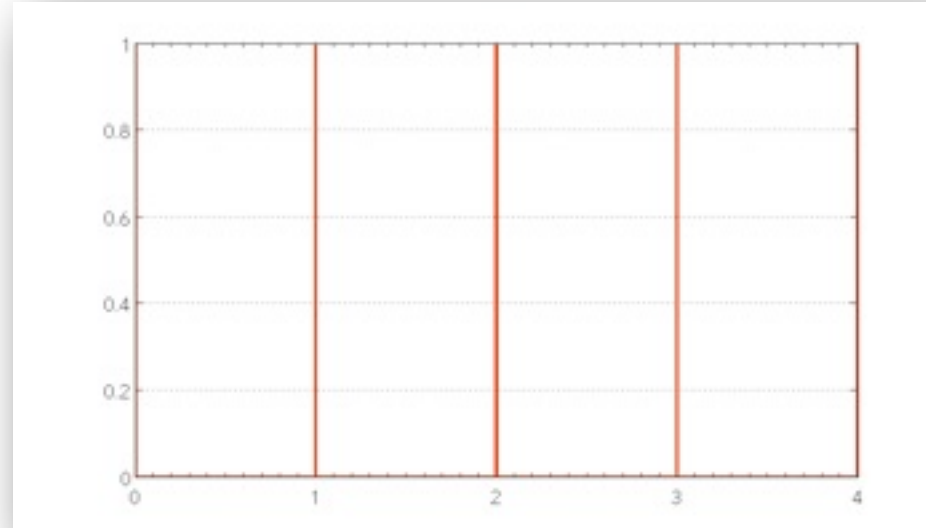
REVIEW: Images and Camera

- Rays of Light go through
  - a Camera
  - Optics/Lens
- Controlled by Aperture, Shutter, and Film Sensitivity
- Sensor to Convert Light to Digital Information
- Process the Image
  - Improve Quality
  - Find Relevant and Useful Information



Sum of Cosines

$f^T(t)$ Target Signal



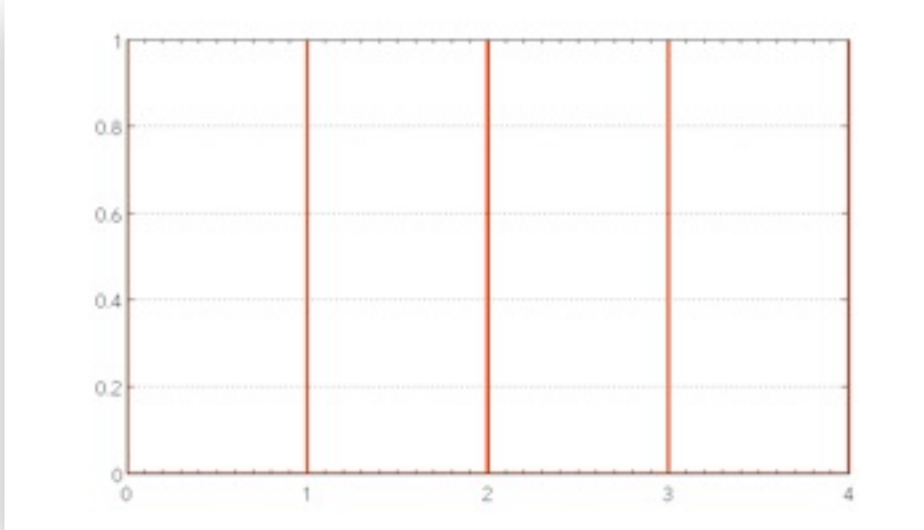
Repeating Impulse Function

Sum of Cosines

★ Using this as our basic building block

- $f(t) = A\cos(n\omega t)$

$f^T(t)$ Target Signal



Repeating Impulse Function

Sum of Cosines

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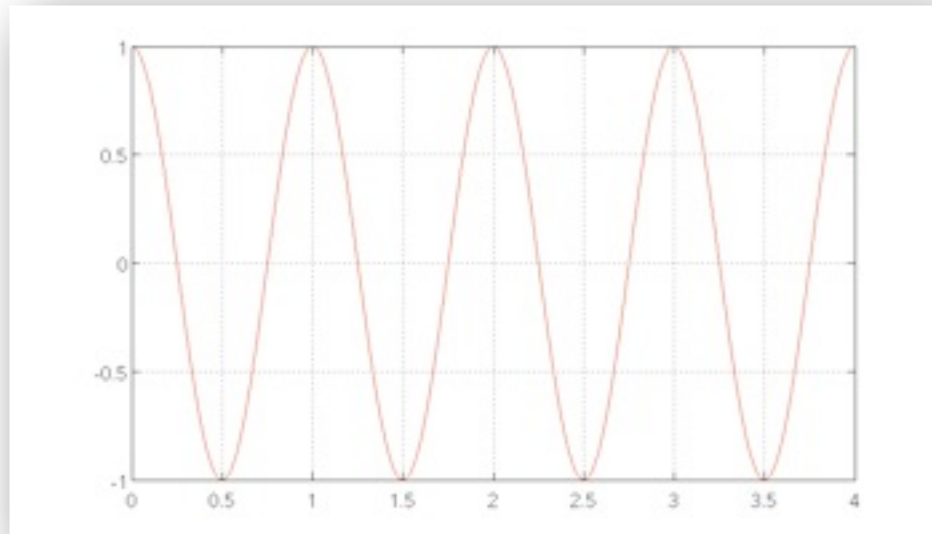
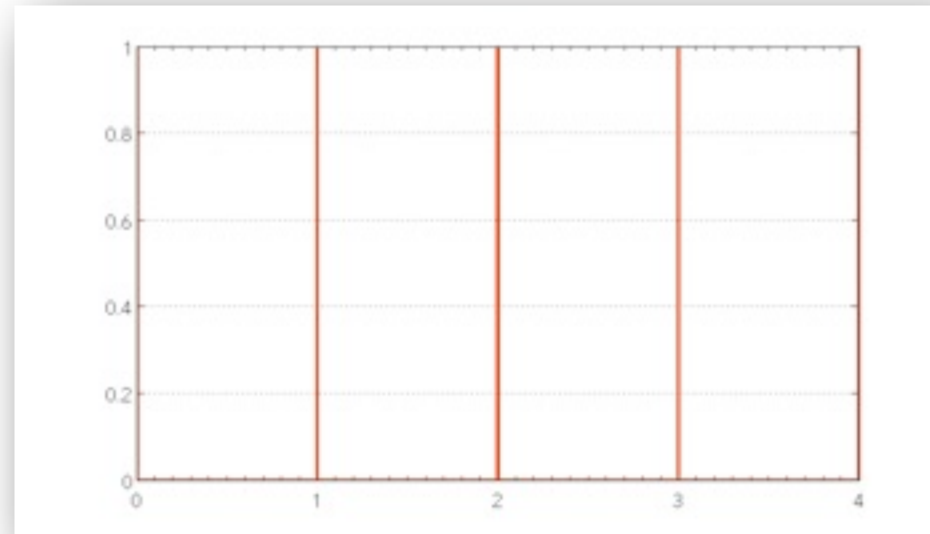
f_1



$f^T(t)$ Target Signal



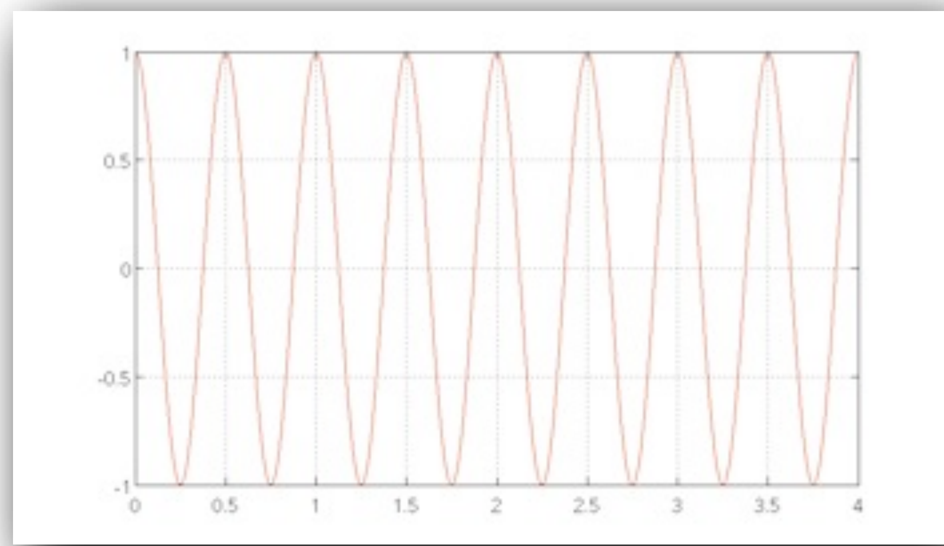
Repeating Impulse Function

Sum of Cosines

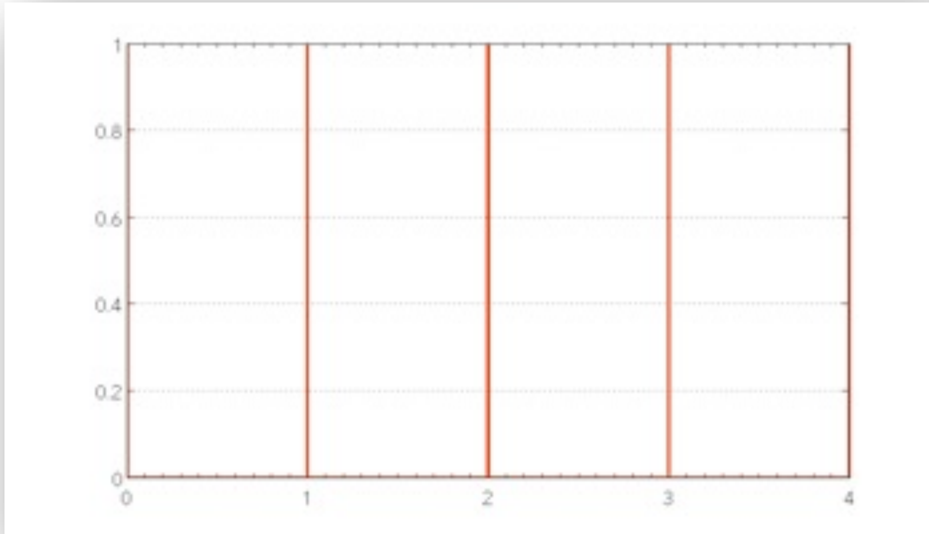
★ Using this as our basic building block

- $f(t) = A\cos(n\omega t)$

f_1 f_2



$f^T(t)$ Target Signal

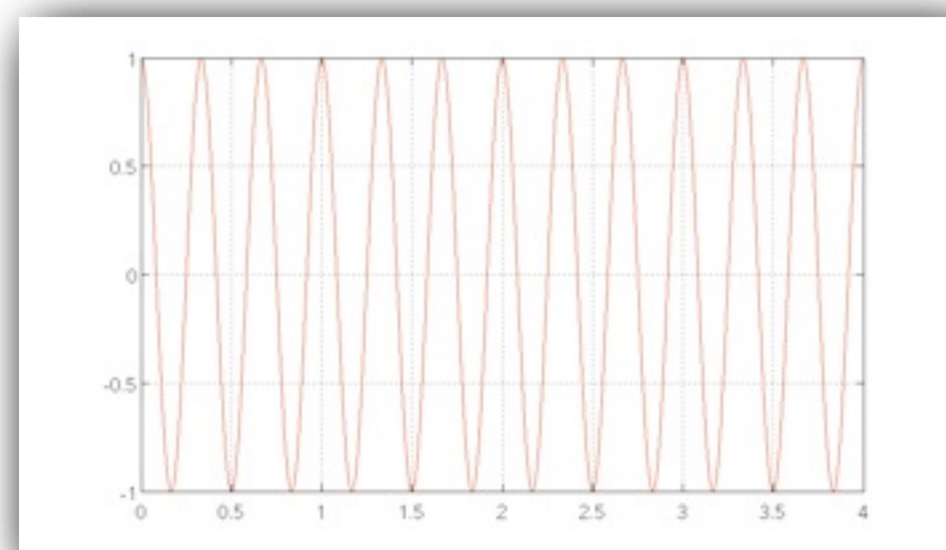


Repeating Impulse Function

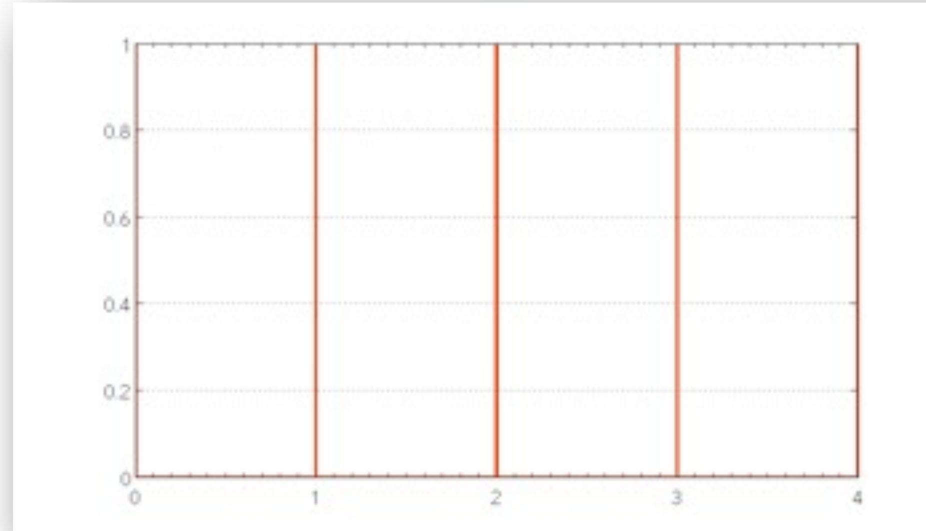
Sum of Cosines

- Using this as our basic building block
  - $f(t) = A\cos(n\omega t)$

f_1 f_2 f_3



$f^T(t)$ Target Signal



Repeating Impulse Function

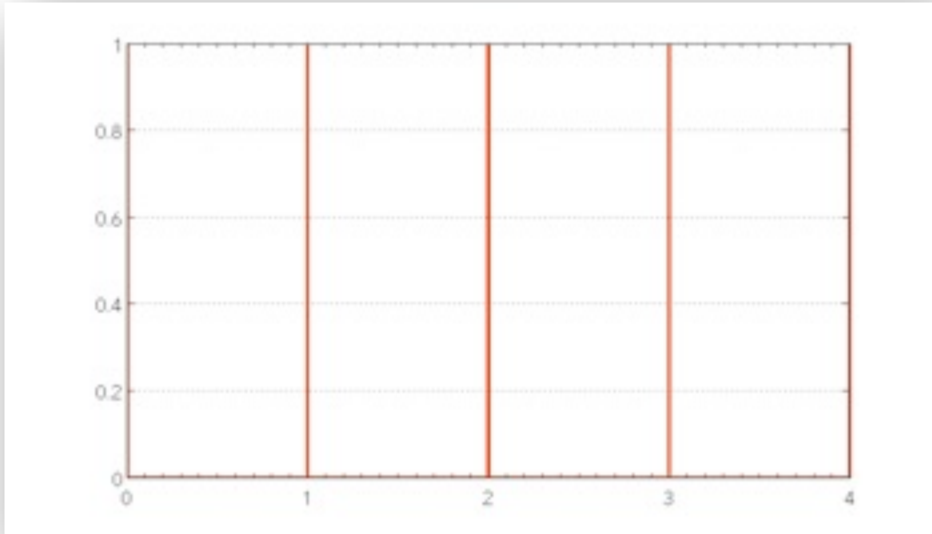
Sum of Cosines

★ Using this as our basic building block

- $f(t) = A\cos(n\omega t)$

$f_1 \quad f_2 \quad f_3 \quad f_4$

$f^T(t)$ Target Signal



Repeating Impulse Function

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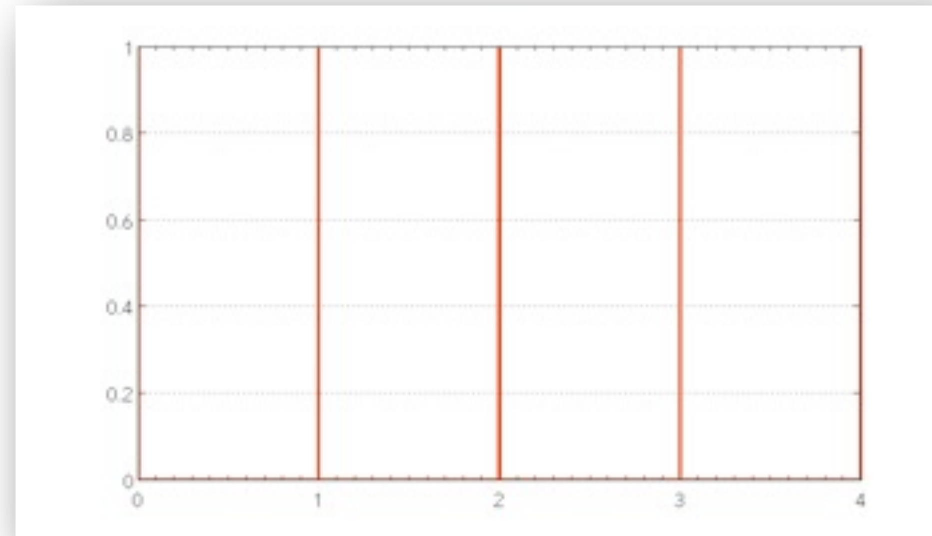
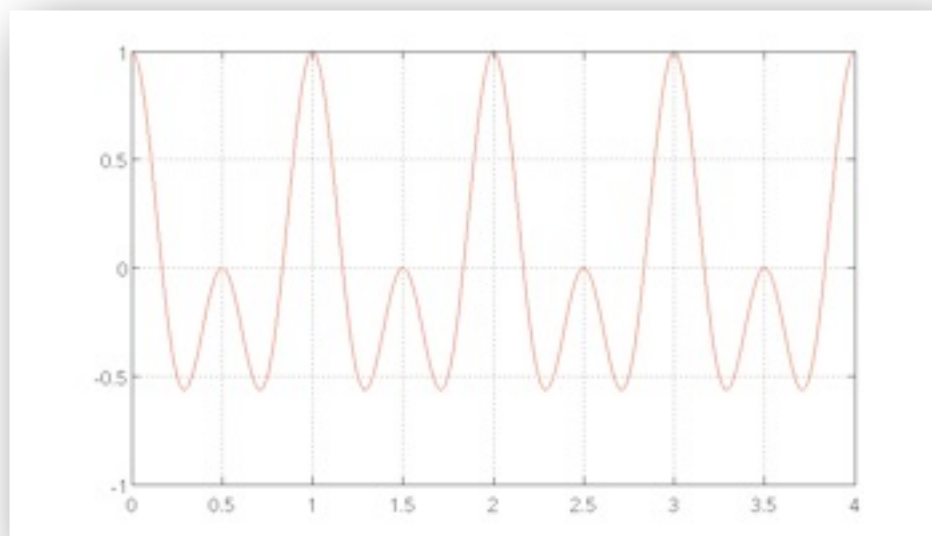


Repeating Impulse Function



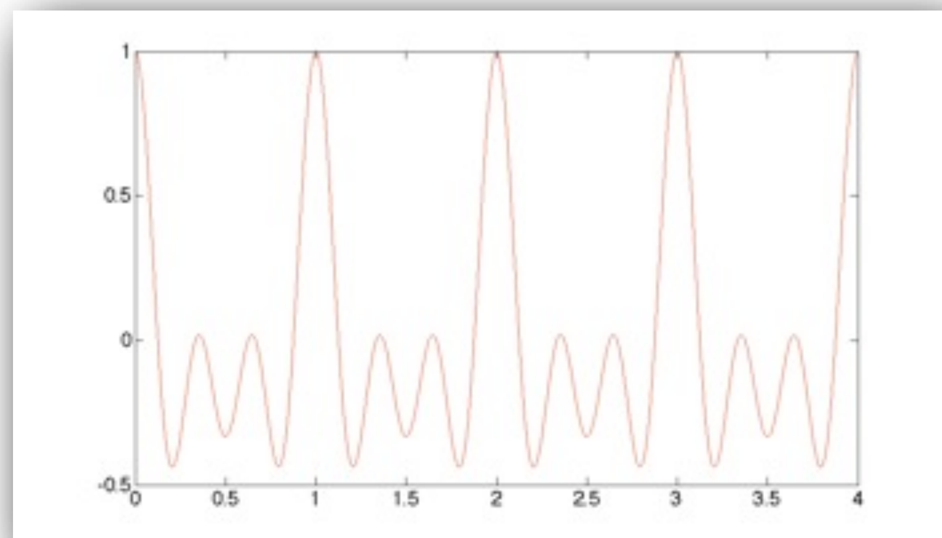
$f_1 + f_2$

Sum of Cosines

★ Using this as our basic building block

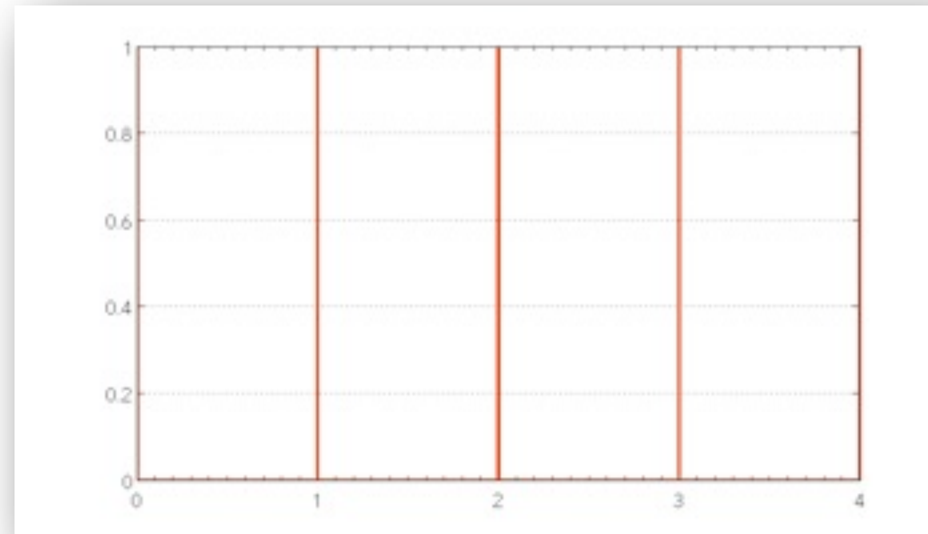
- $f(t) = A\cos(n\omega t)$

$f_1 \quad f_2 \quad f_3 \quad f_4$



$f_1 + f_2 + f_3$

$f^T(t)$ Target Signal



Repeating Impulse Function

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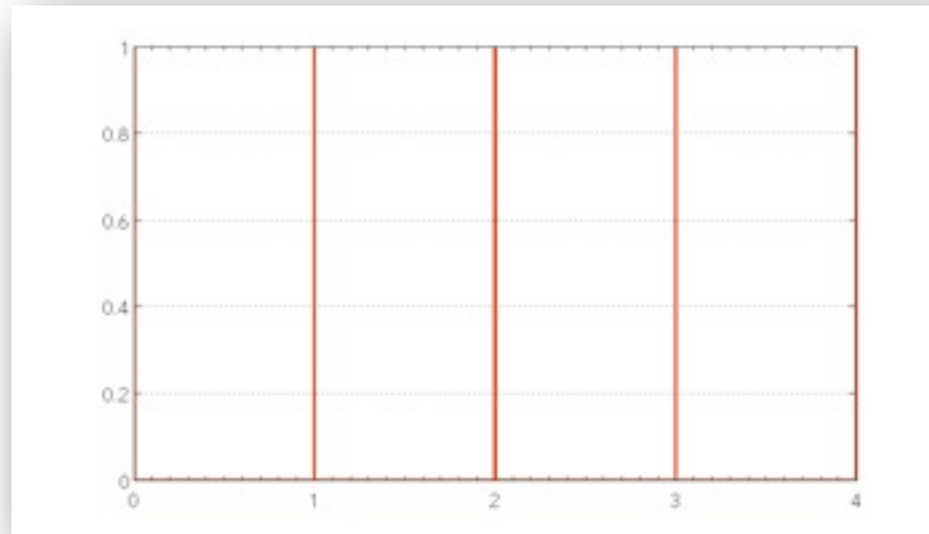
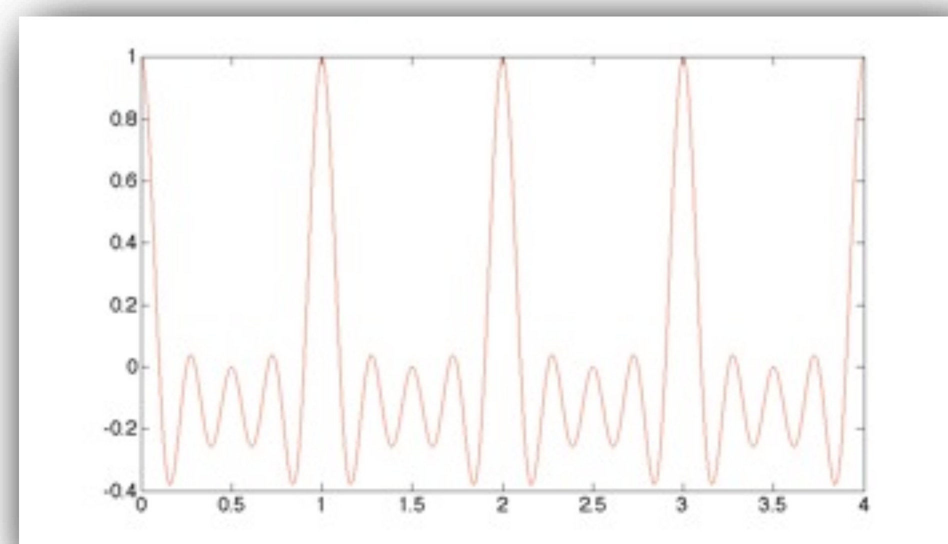


Repeating Impulse Function



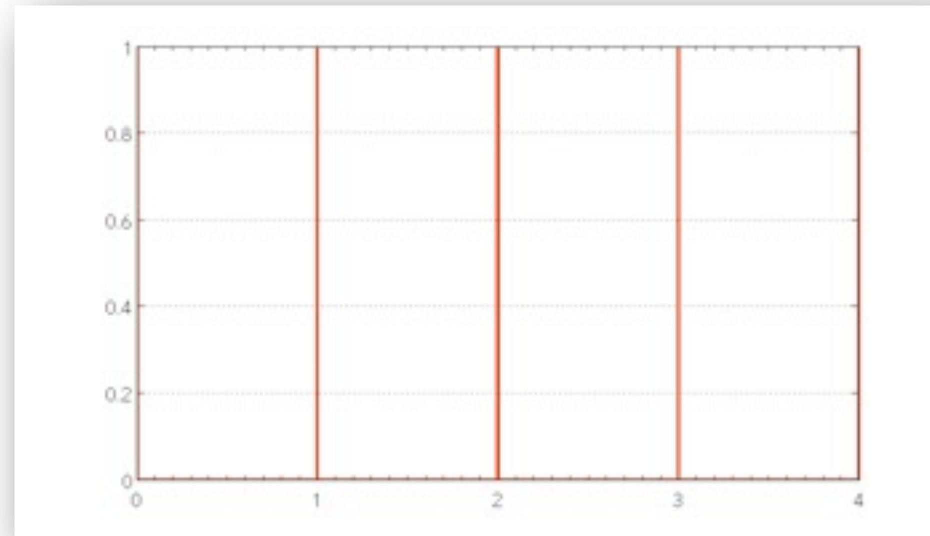
$f_1 + f_2 + f_3 + f_4$

Sum of Cosines

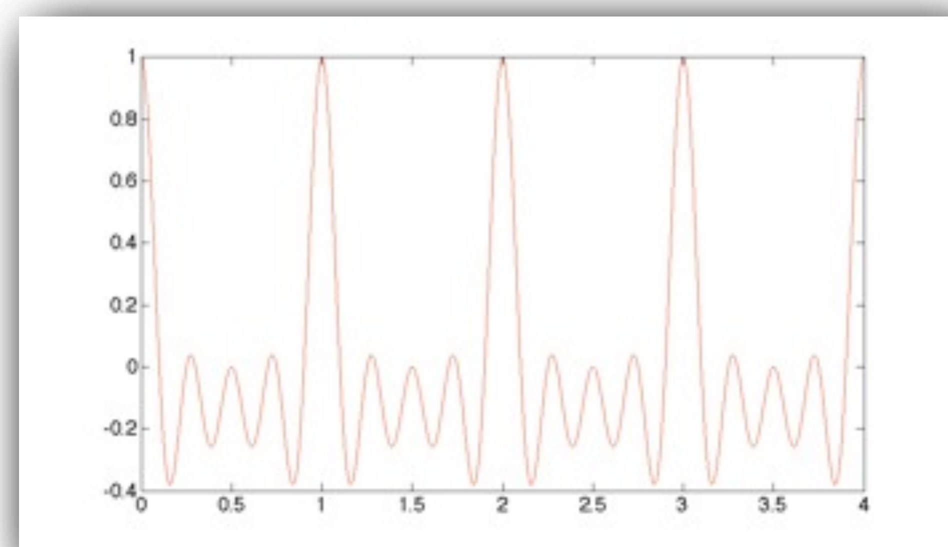
- Using this as our basic building block
  - $f(t) = A\cos(n\omega t)$
- We want to get the target signal by adding enough of them
  - $f^T(t) = \sum_{n=1}^{N} A\cos(n\omega t)$

$f_1 \quad f_2 \quad f_3 \quad f_4$

$f^T(t)$ Target Signal



Repeating Impulse Function



$f_1 + f_2 + f_3 + f_4$

Sum of Cosines

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  - $f(t) = A\cos(n\omega t)$
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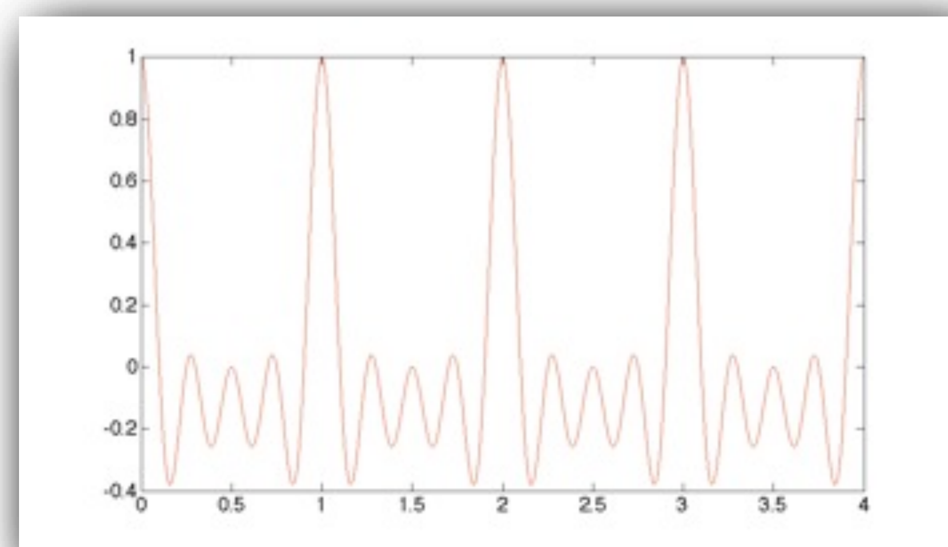
  - $f^T(t) = \sum_{n=1}^{N} A\cos(n\omega t)$
- ★ *N* is Max number of blocks needed, *A* is amplitude and *n* is frequency

$f_1 \quad f_2 \quad f_3 \quad f_4$



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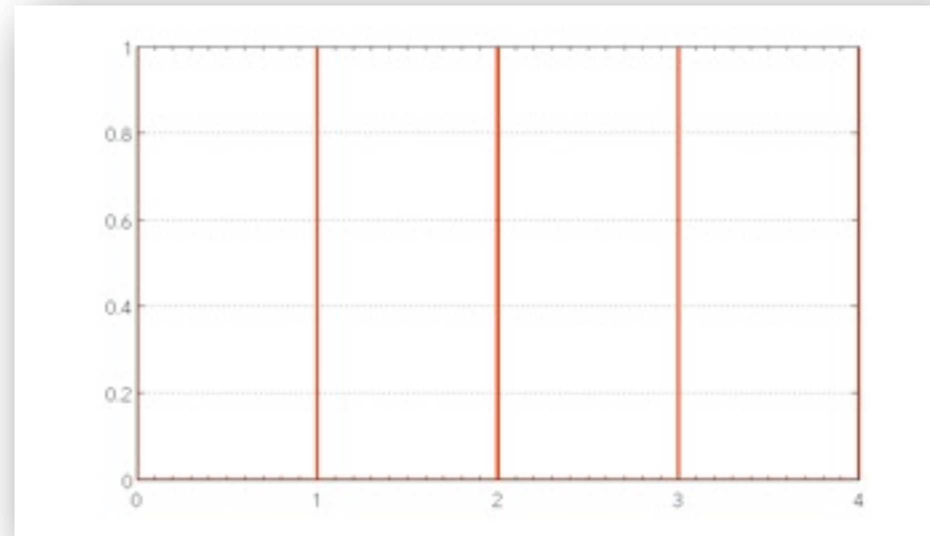

Repeating Impulse Function

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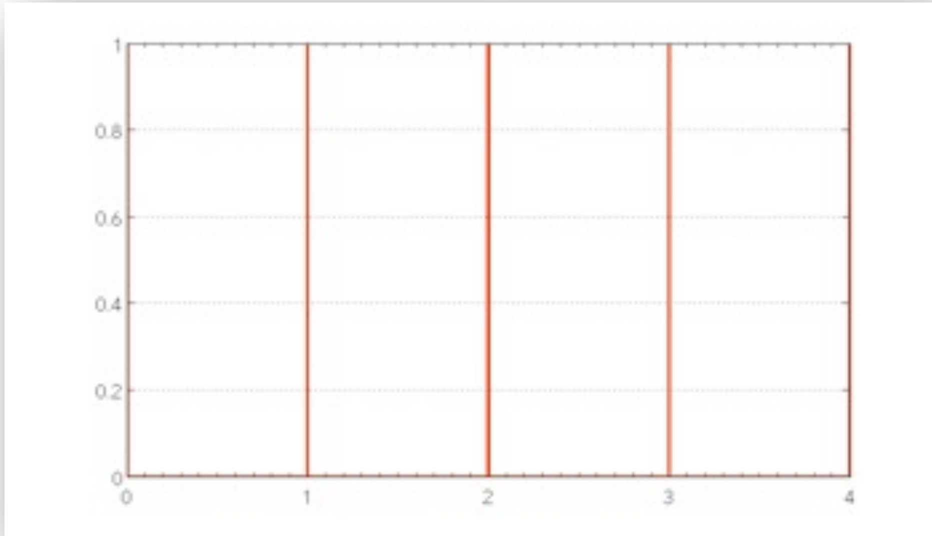
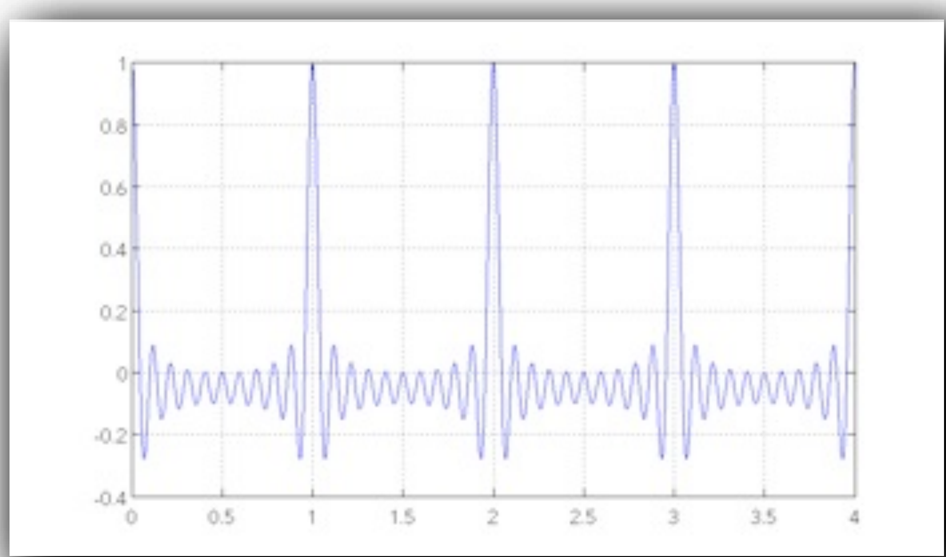
$f_1 \quad f_2 \quad f_3 \quad f_4$

$f^T(t)$ Target Signal



Repeating Impulse Function

N=10

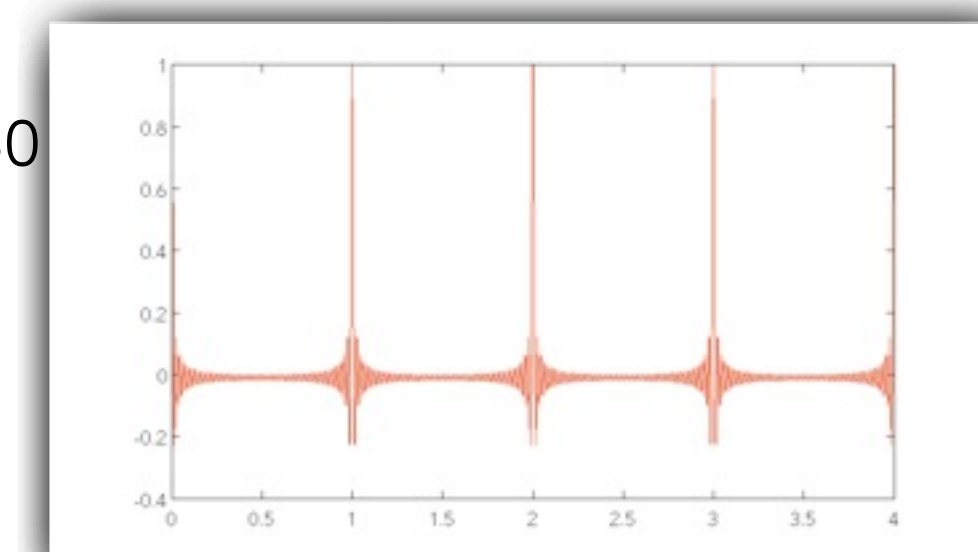


$f_1 + f_2 + f_3 + f_4$

Sum of Cosines

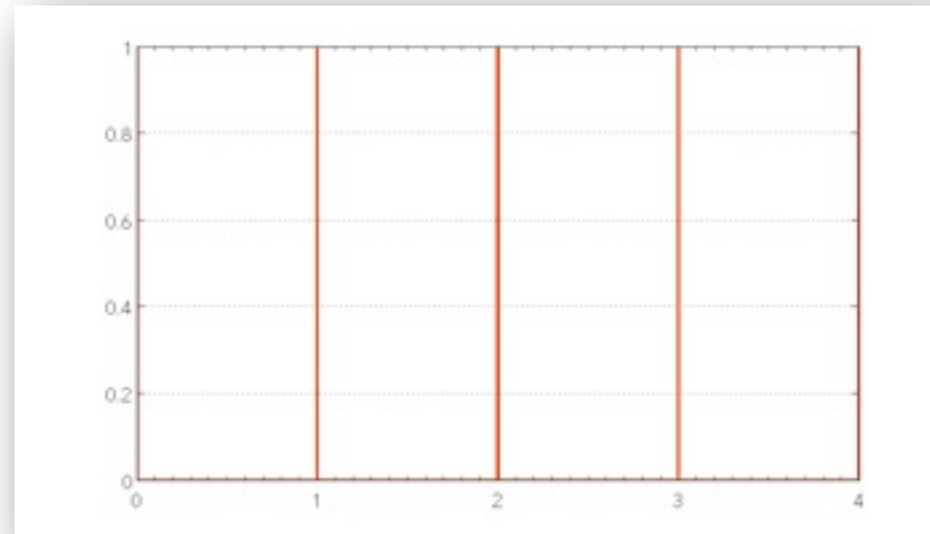
- Using this as our basic building block
  - $f(t) = A\cos(n\omega t)$
- We want to get the target signal by adding enough of them
  - $f^T(t) = \sum_{n=1}^{N} A\cos(n\omega t)$ N=50
- *N* is Max number of blocks needed, *A* is amplitude and *n* is frequency

$f_1 \quad f_2 \quad f_3 \quad f_4$



$f_1 + f_2 + f_3 + f_4$

$f^T(t)$ Target Signal



Repeating Impulse Function

A Fourier Transform

Jean Baptiste Joseph Fourier
(1768-1830)
http://en.wikipedia.org/wiki/File:Fourier2.jpg

A Fourier Transform

- ★ Any periodic function can be rewritten as a weighted sum of sines and cosines of different frequencies.

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- ★ Transforms a mathematical function of time, *f(t)*, into a new function, *F(ω)*, whose argument is frequency with units of cycles/sec (hertz) or radians per second.

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- ★ The Fourier transform is a reversible operation.

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- ★ The Fourier transform is a reversible operation.
- ★ For every ω from 0 to ∞ (infinity) *F(ω)* holds the Amplitude A and phase ϕ of a sine function:

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A Fourier Transform

★ Any periodic function can be rewritten as a weighted sum of sines and cosines of different frequencies.

★ Transforms a mathematical function of time, *f(t)*, into a new function, *F*(ω), whose argument is frequency with units of cycles/sec (hertz) or radians per second.

★ The new function is the Fourier transform and/or the frequency spectrum of the function *f*.

★ The Fourier transform is a reversible operation.

★ For every ω from 0 to ∞ (infinity) *F*(ω) holds the Amplitude A and phase ϕ of a sine function:

$$F(\omega) = A\cos(\omega t + \phi)$$

Jean Baptiste Joseph Fourier
(1768-1830)
http://en.wikipedia.org/wiki/File:Fourier2.jpg

Frequency Domain of a Signal

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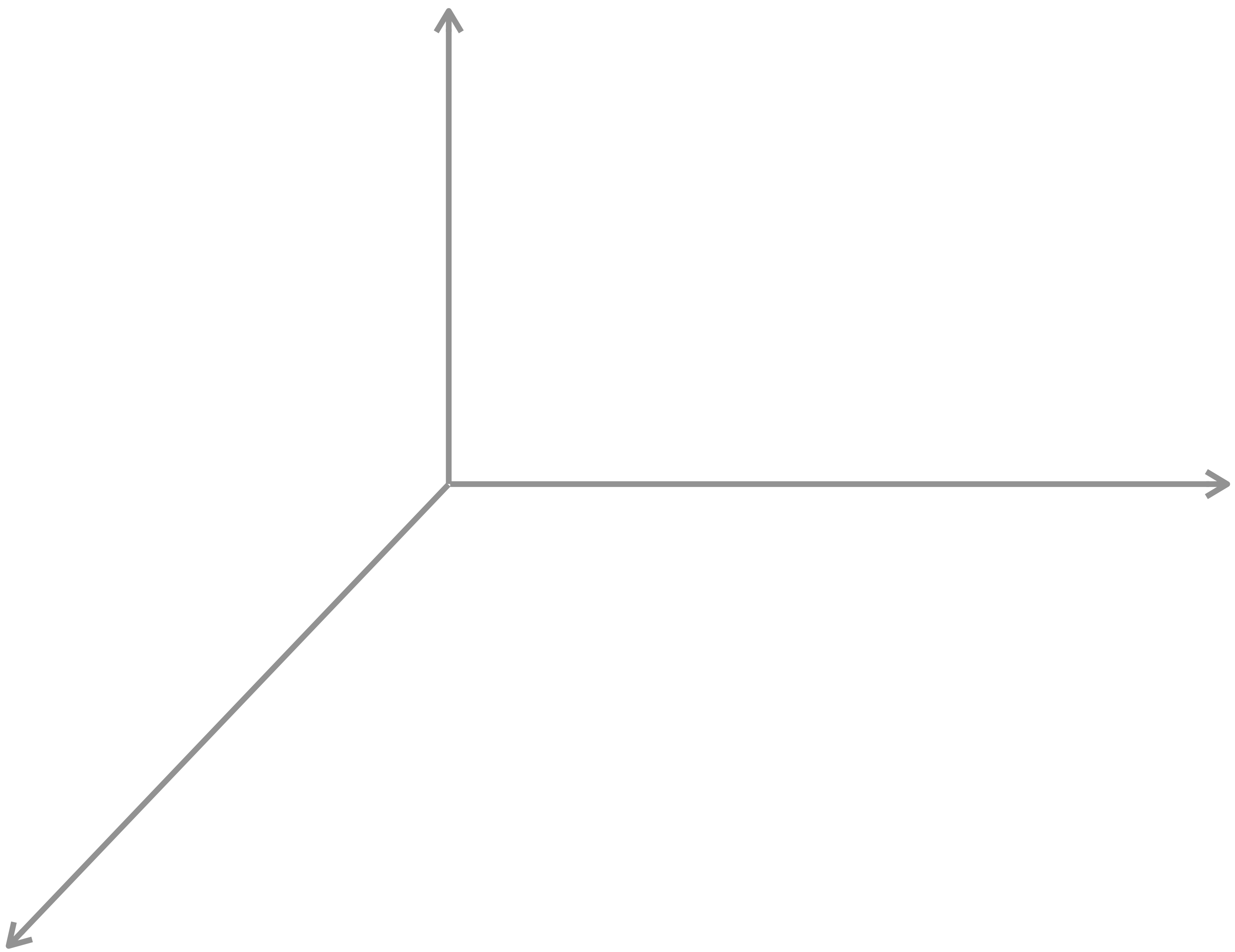
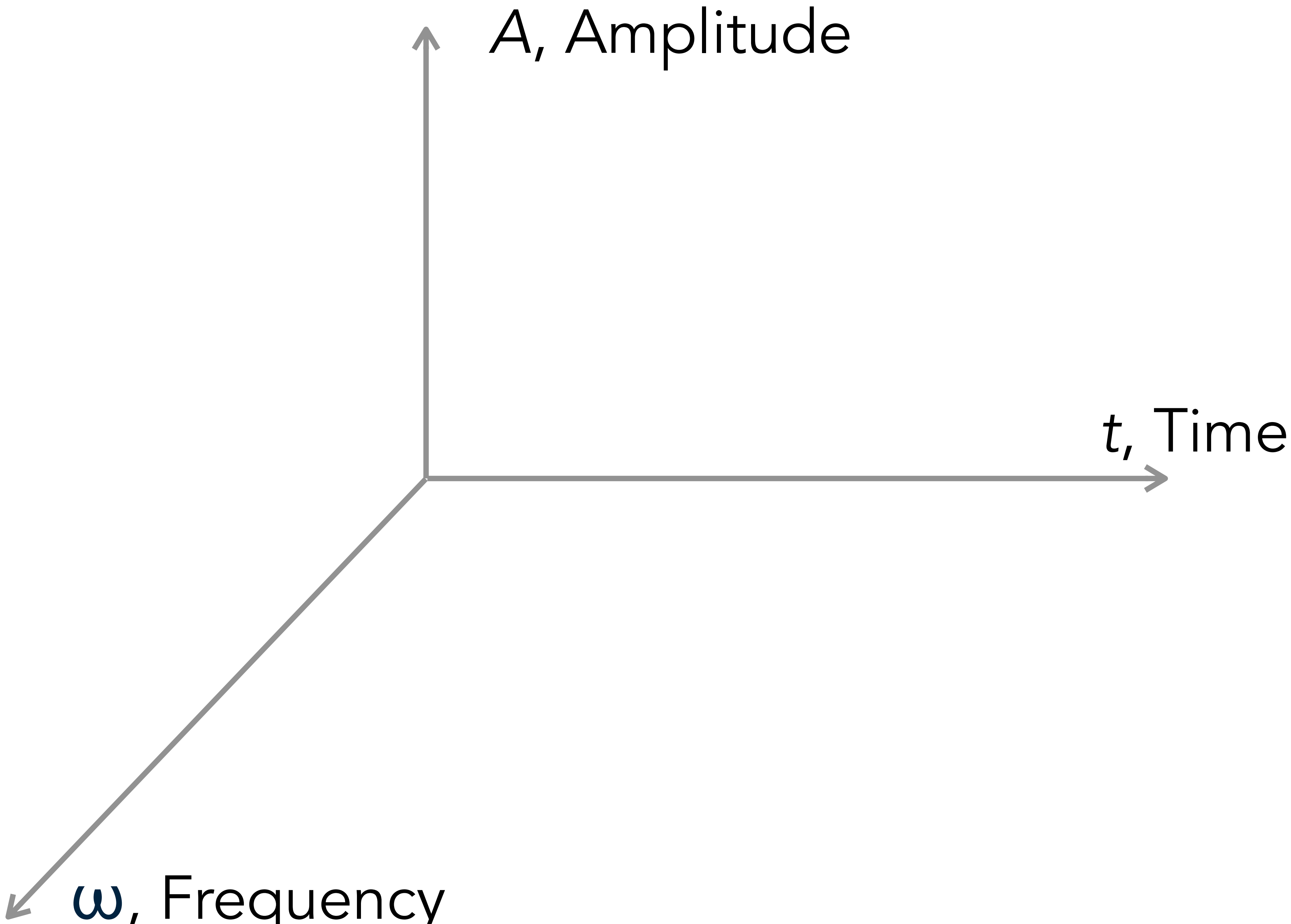
Frequency Domain of a Signal



Frequency Domain of a Signal



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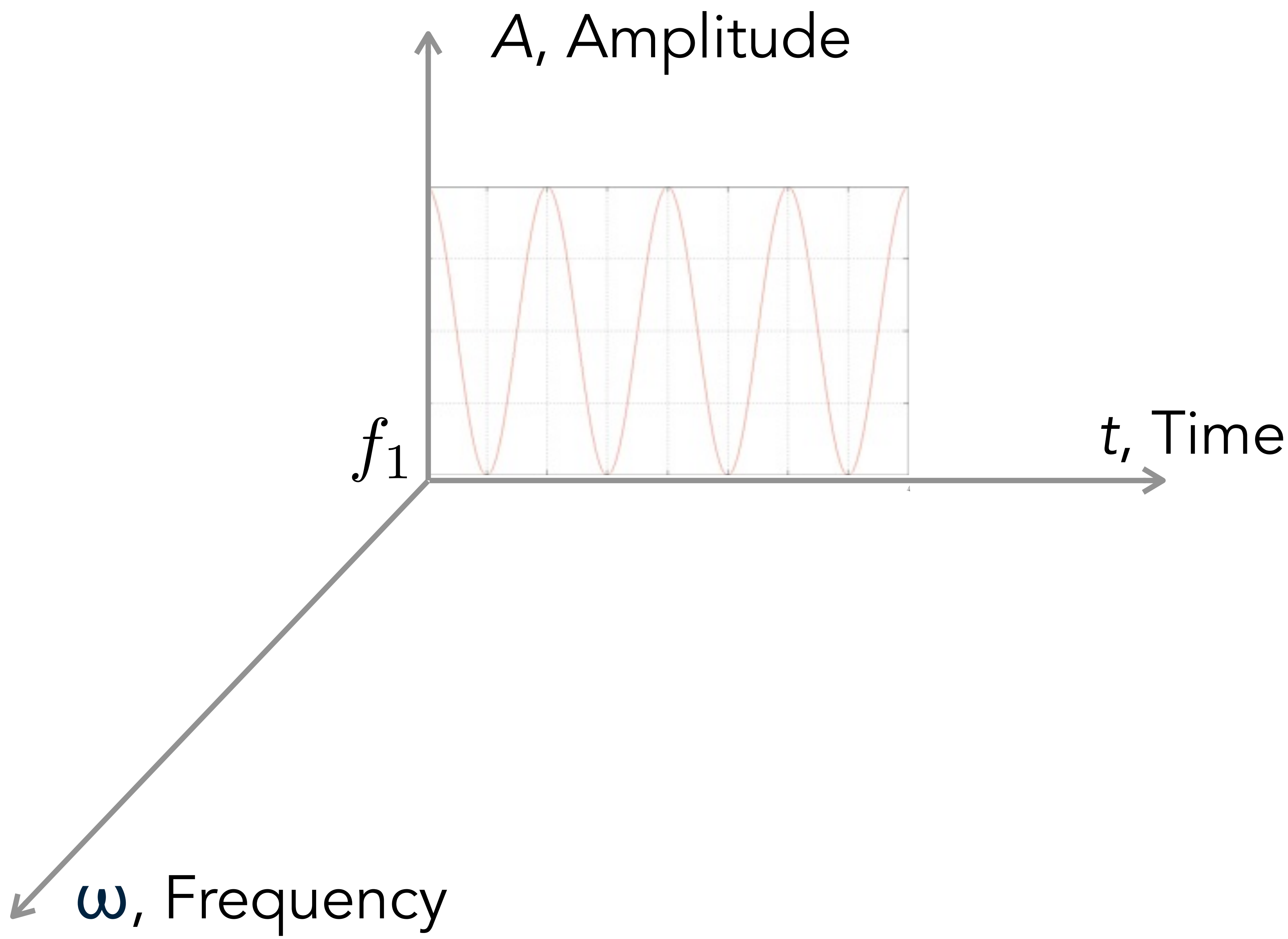

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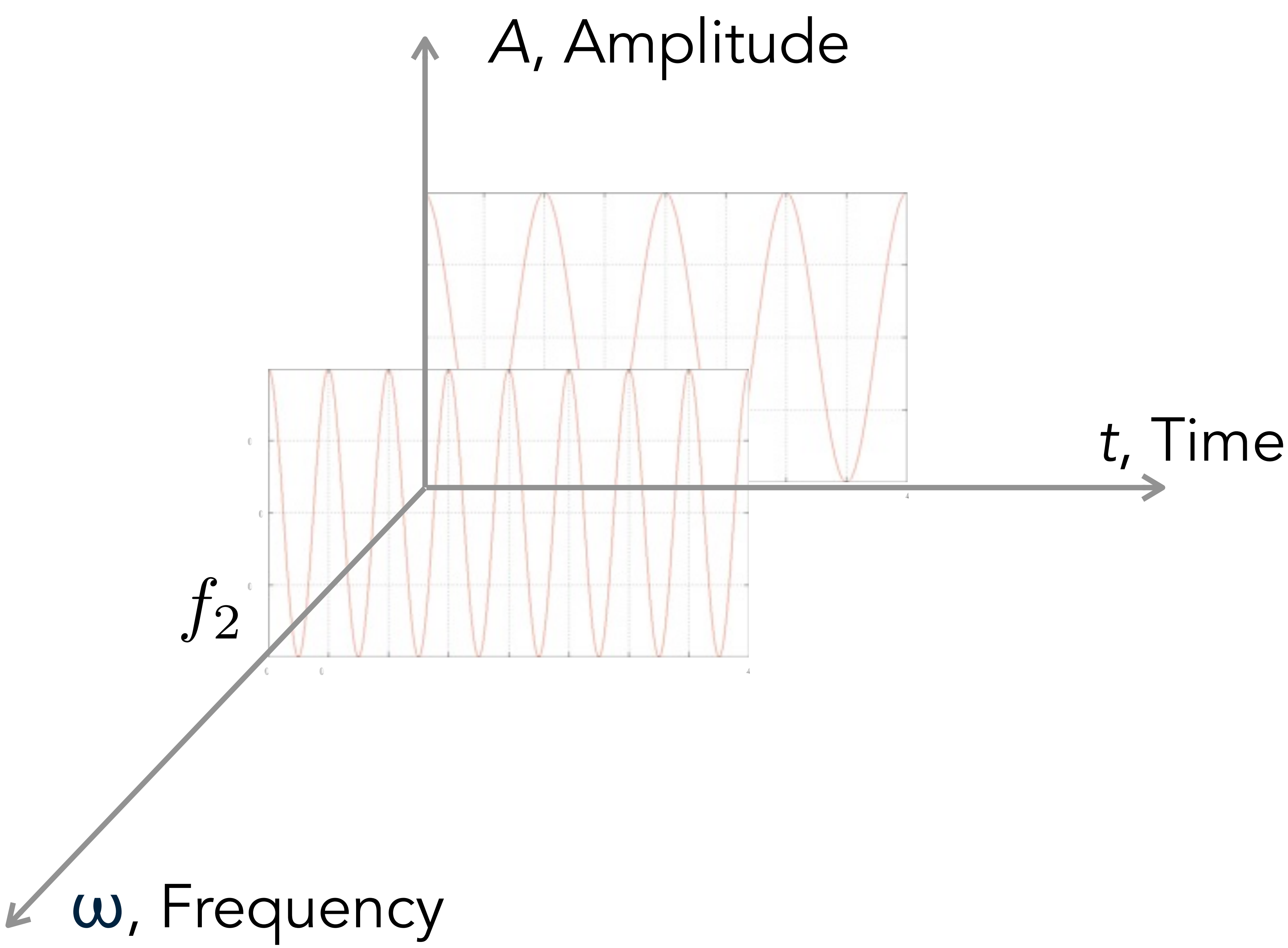

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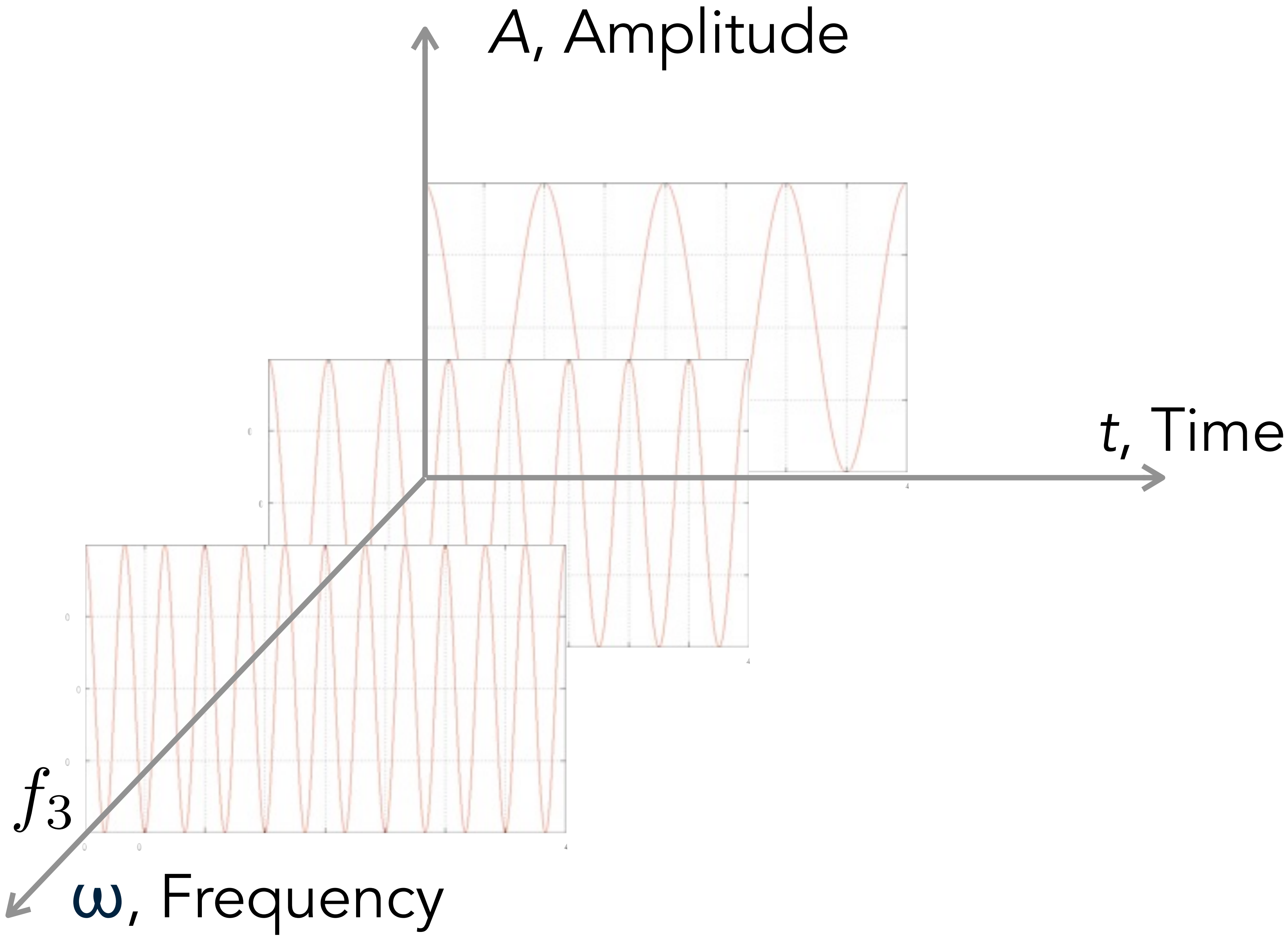

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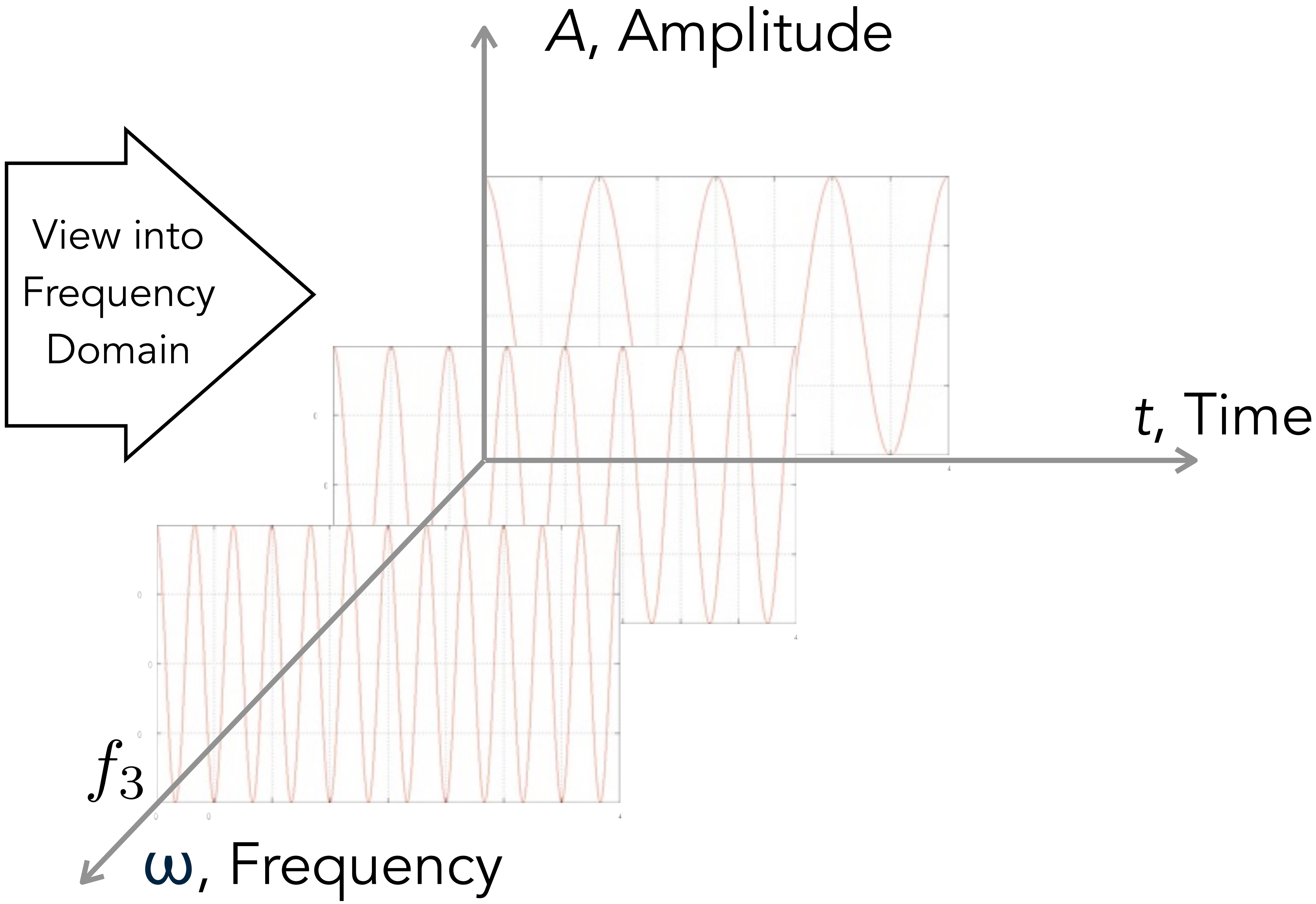
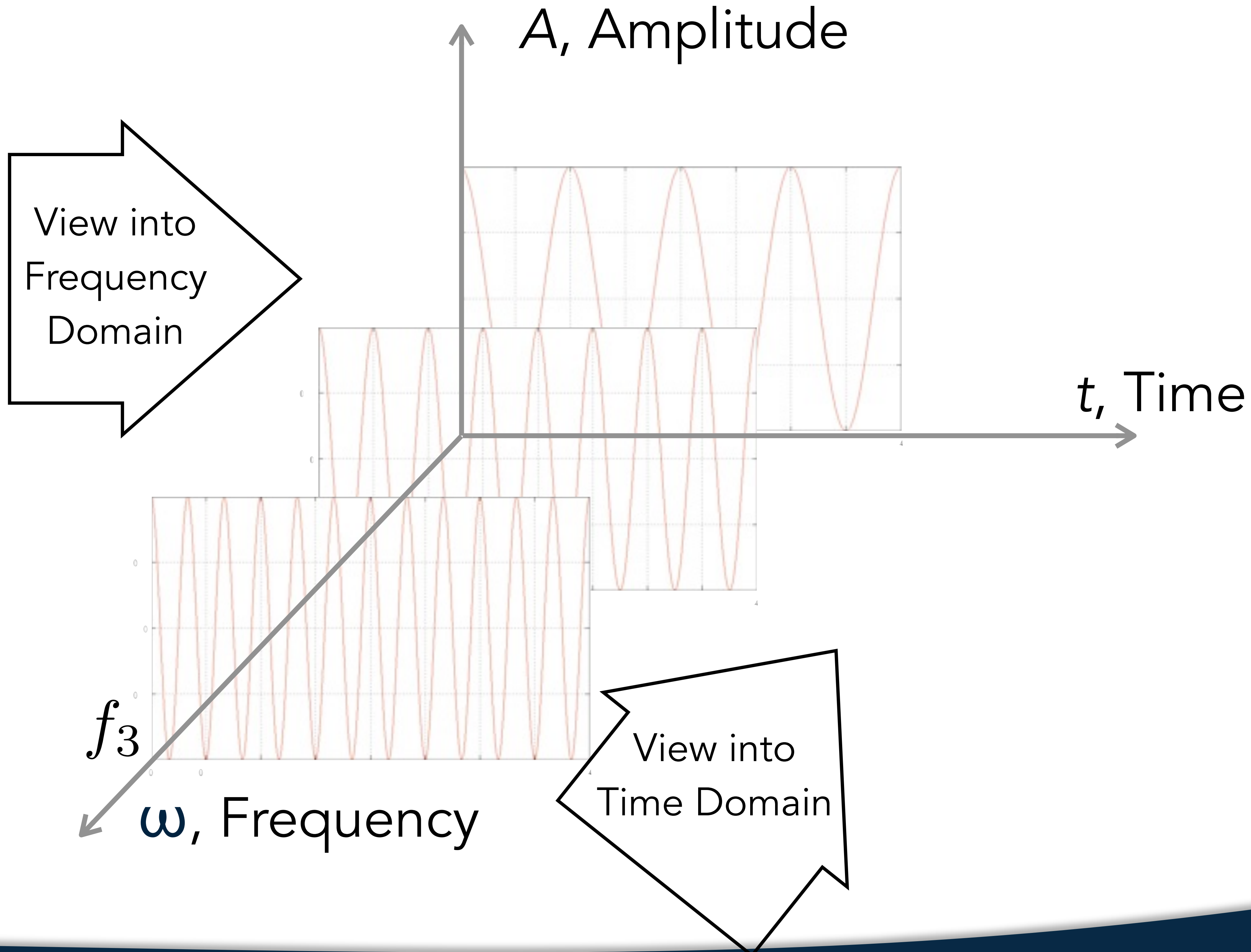
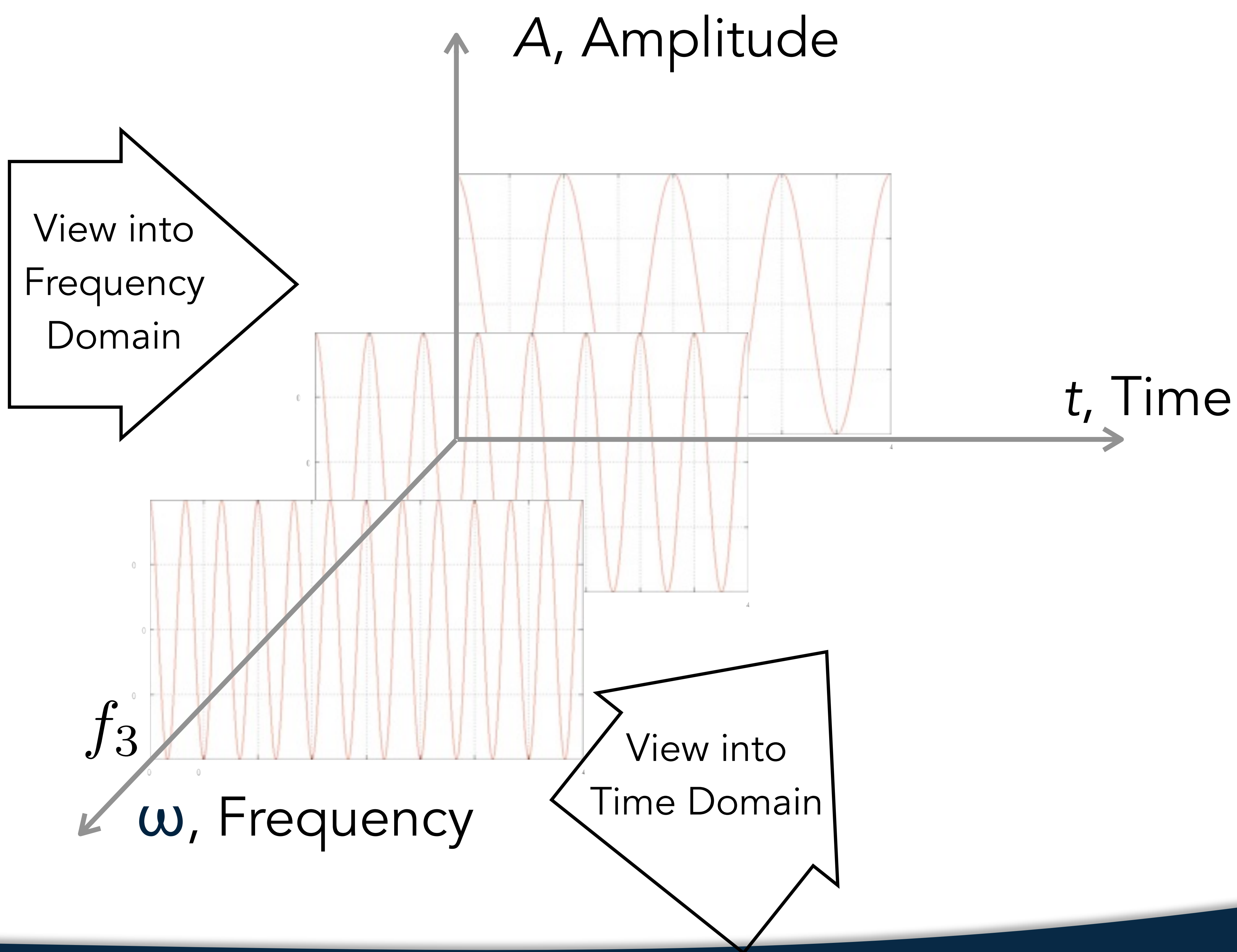


Frequency Domain of a Signal



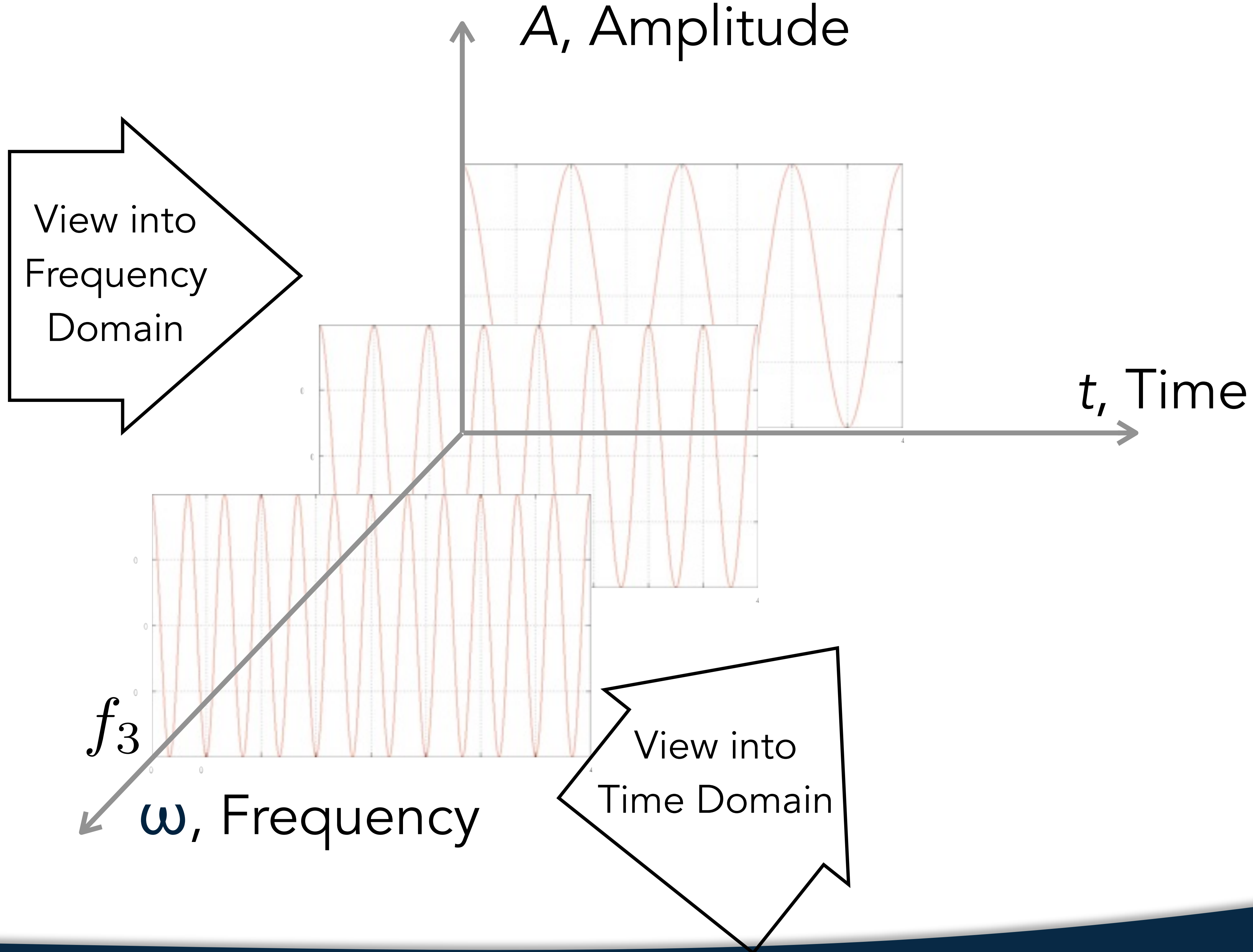
Frequency Domain of a Signal

1. How many N?

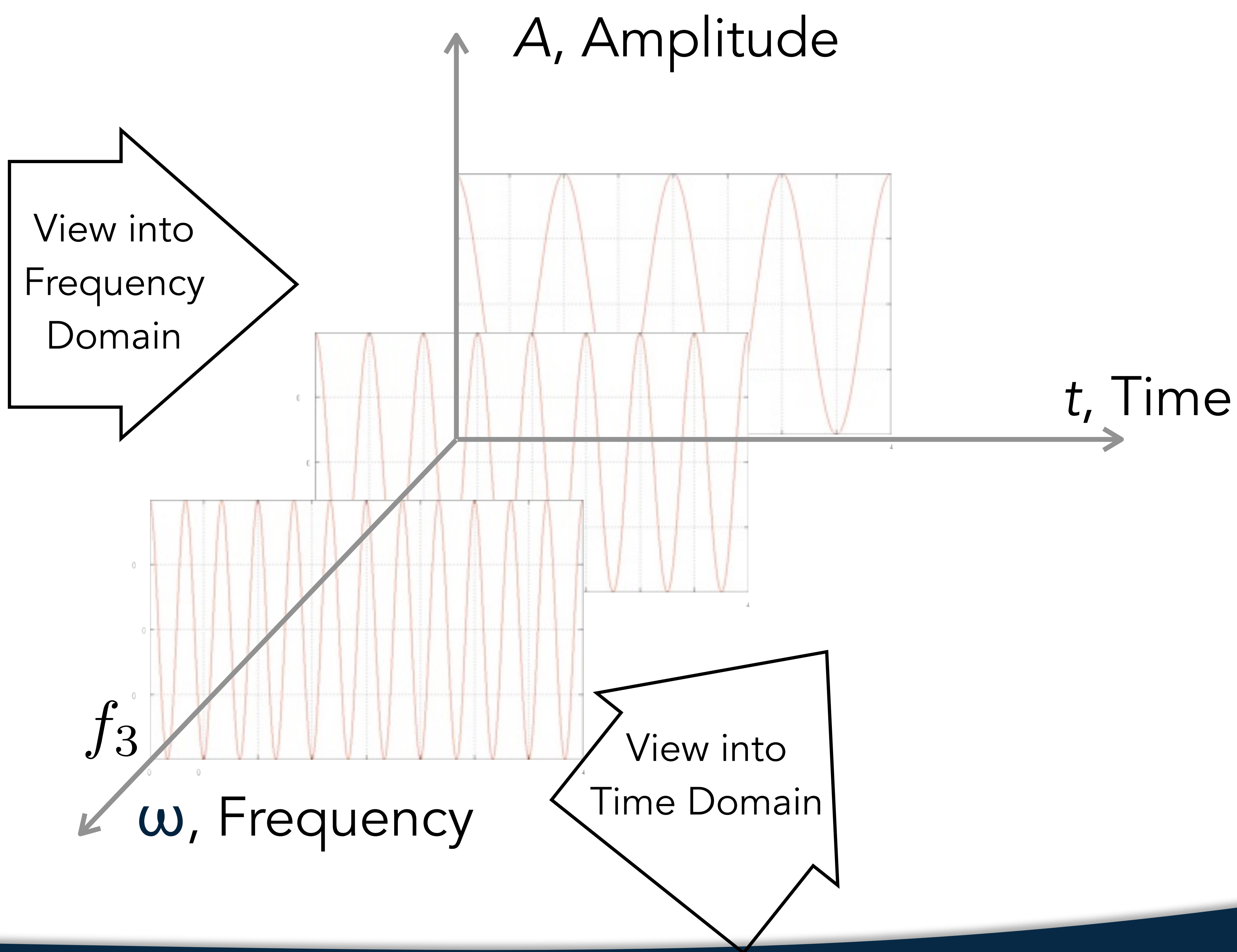


Frequency Domain of a Signal

1. How many N?
2. What does each control?

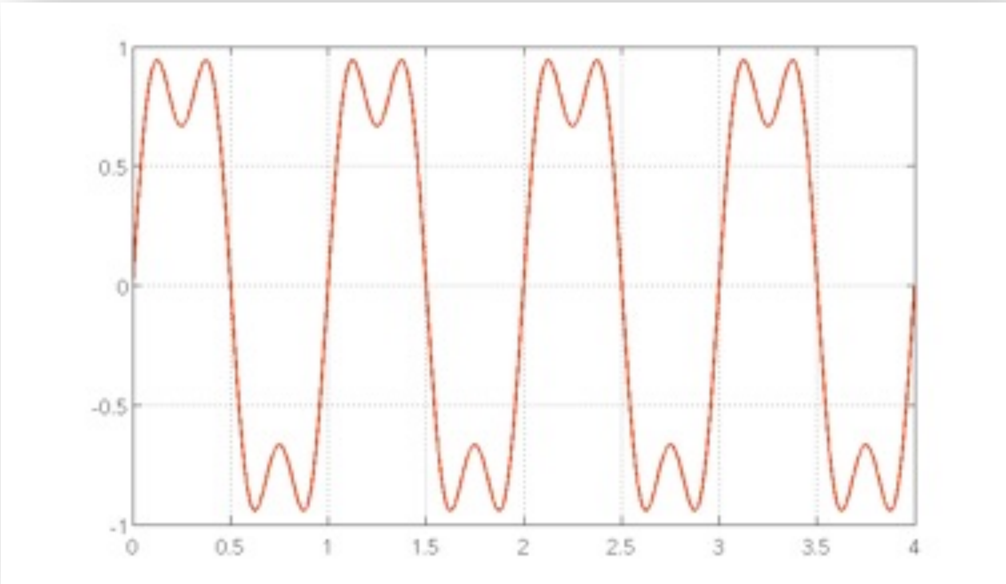


Frequency Domain of a Signal

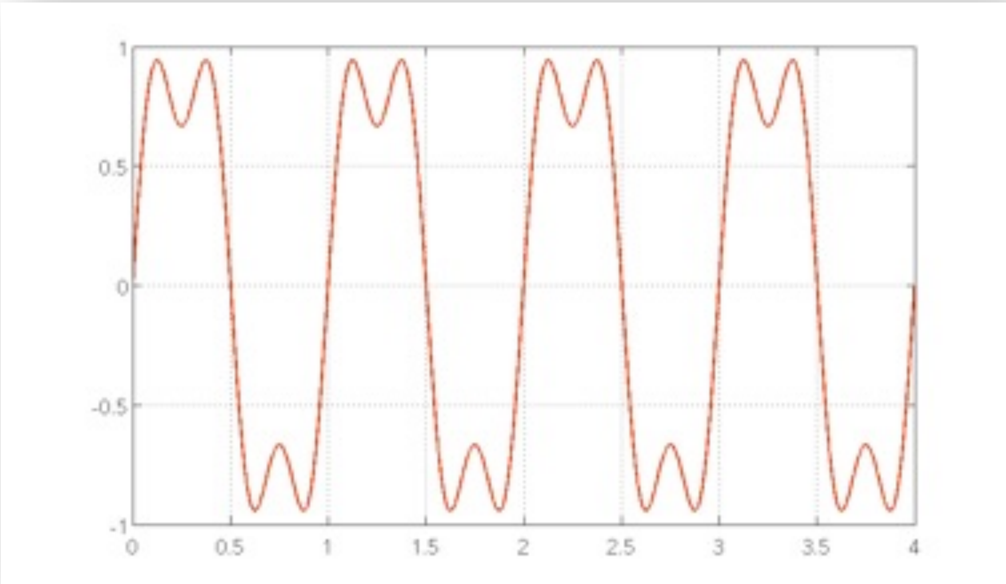


1. How many N?
2. What does each control?
3. Which one encodes the coarse vs. fine structure of the signal?

Time, Frequency, and Frequency Spectra



Time, Frequency, and Frequency Spectra



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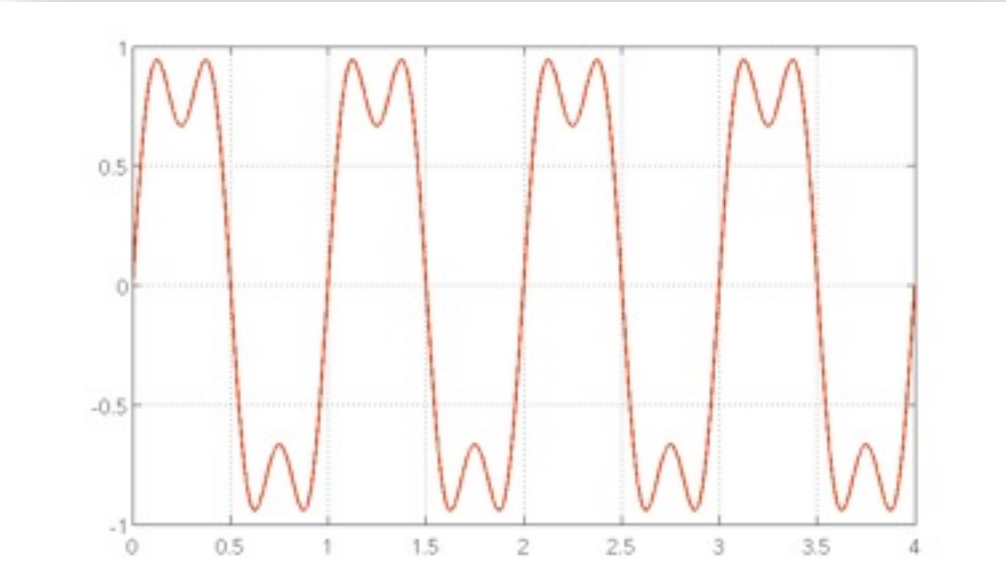

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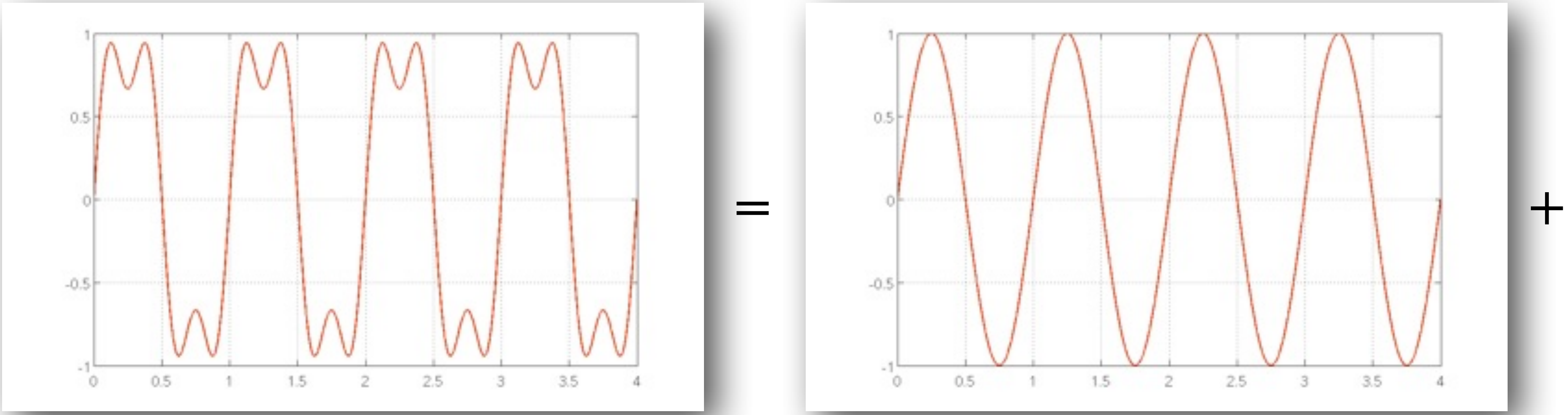

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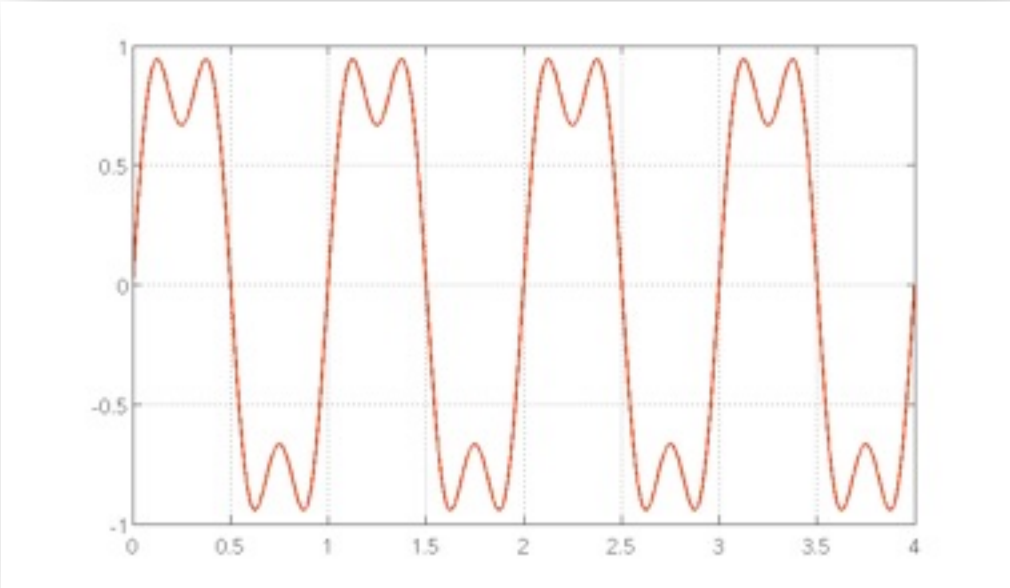

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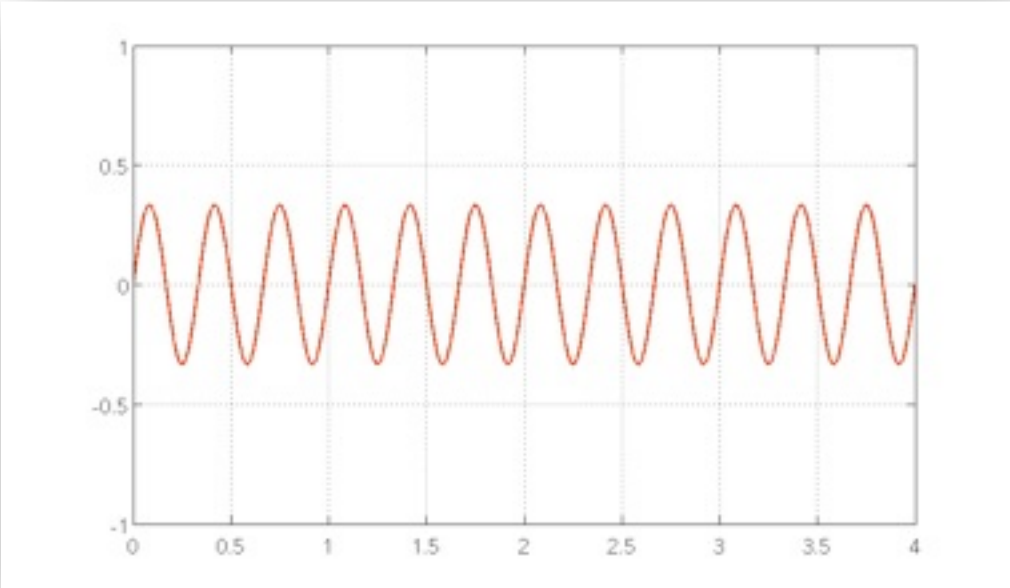

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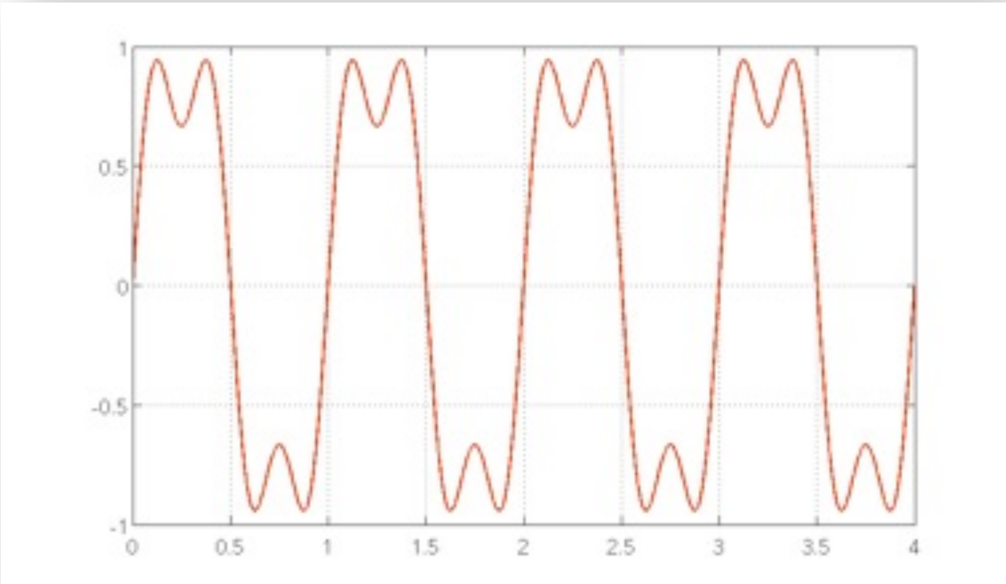

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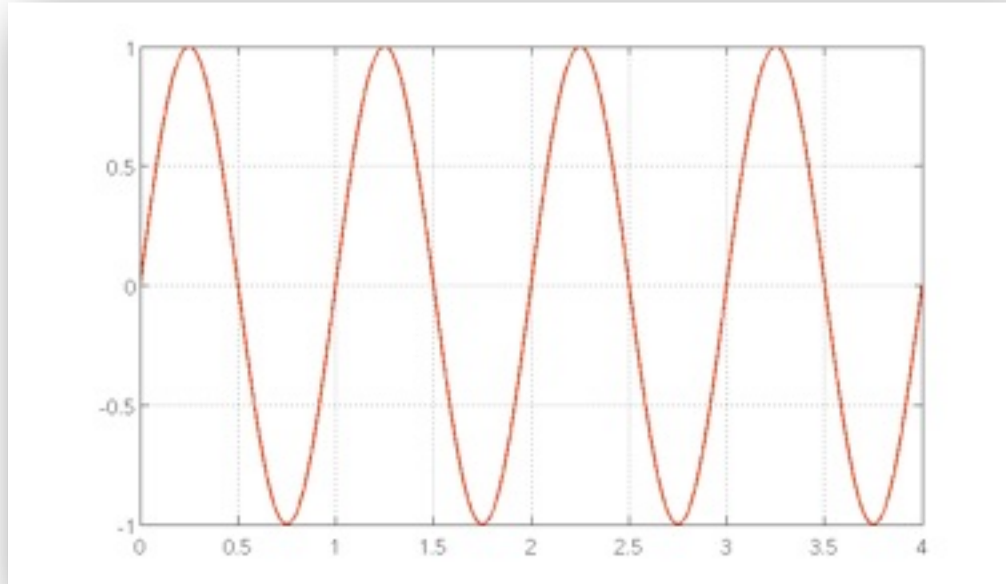

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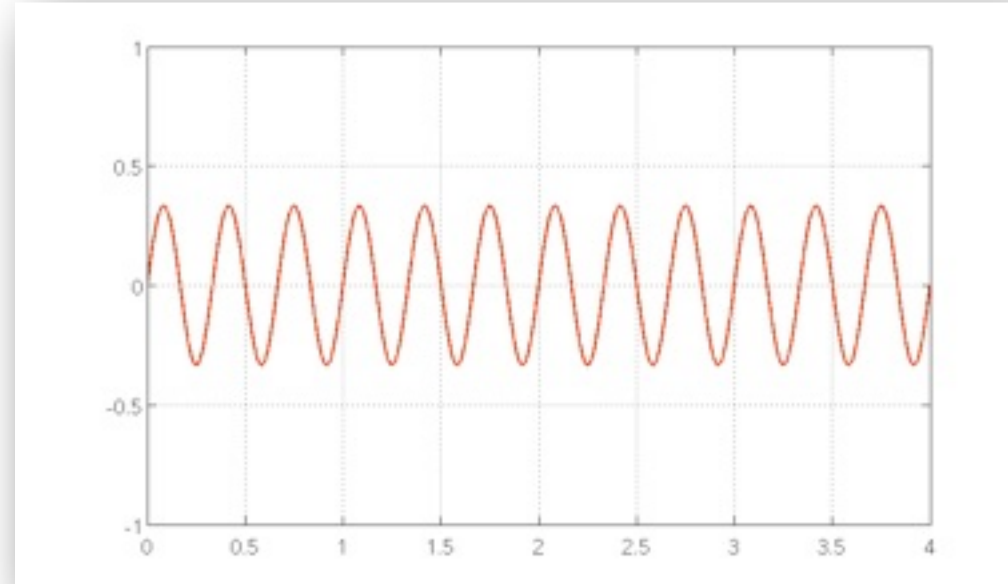
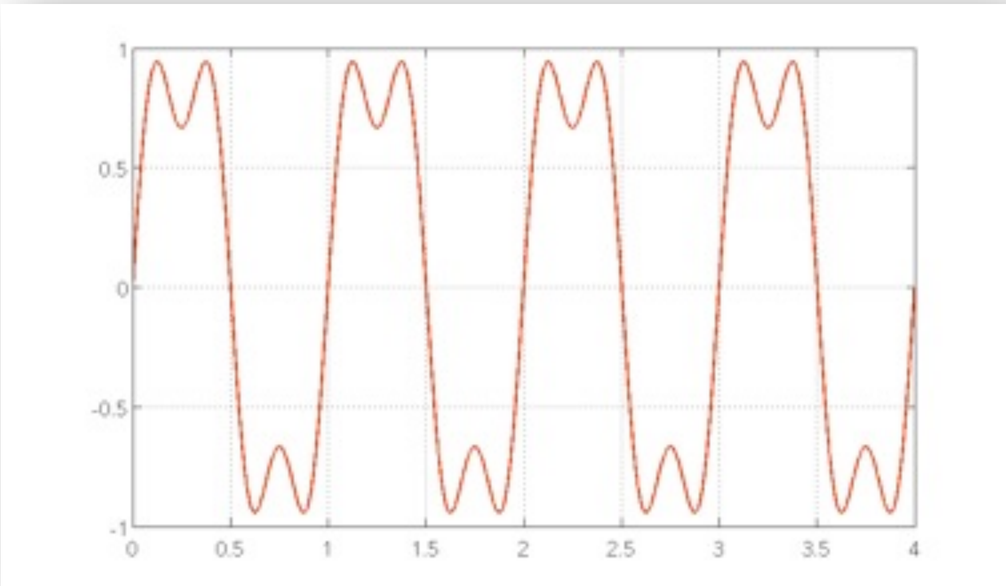


$$g(t) = \sin(2p\omega t) + \frac{1}{3}\sin(2p(3\omega)t)$$

Time, Frequency, and Frequency Spectra



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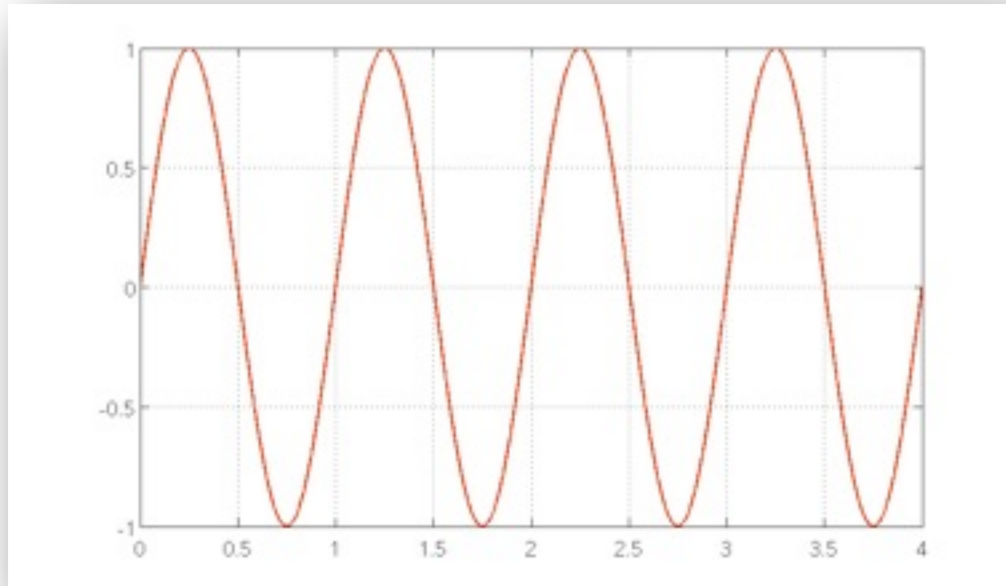

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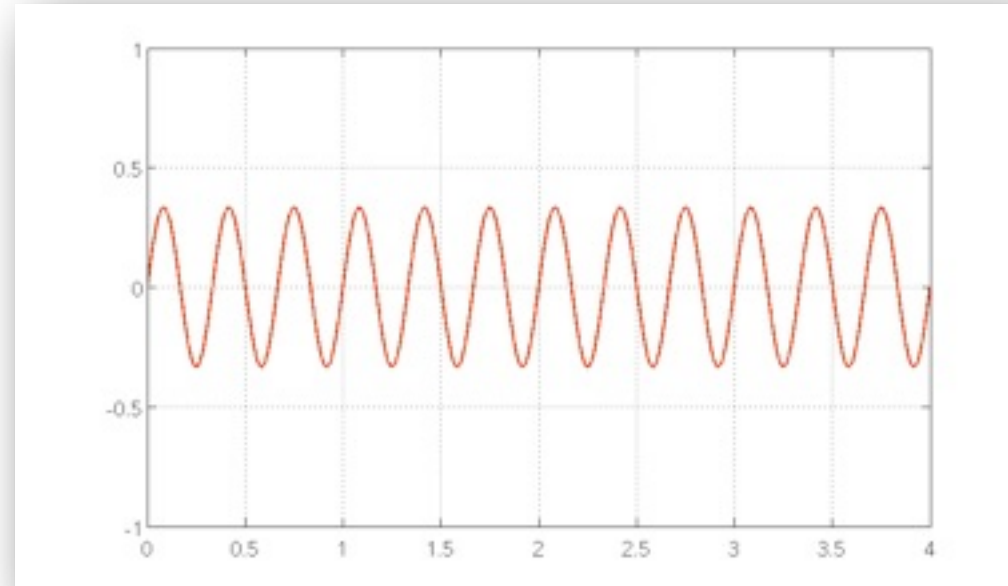

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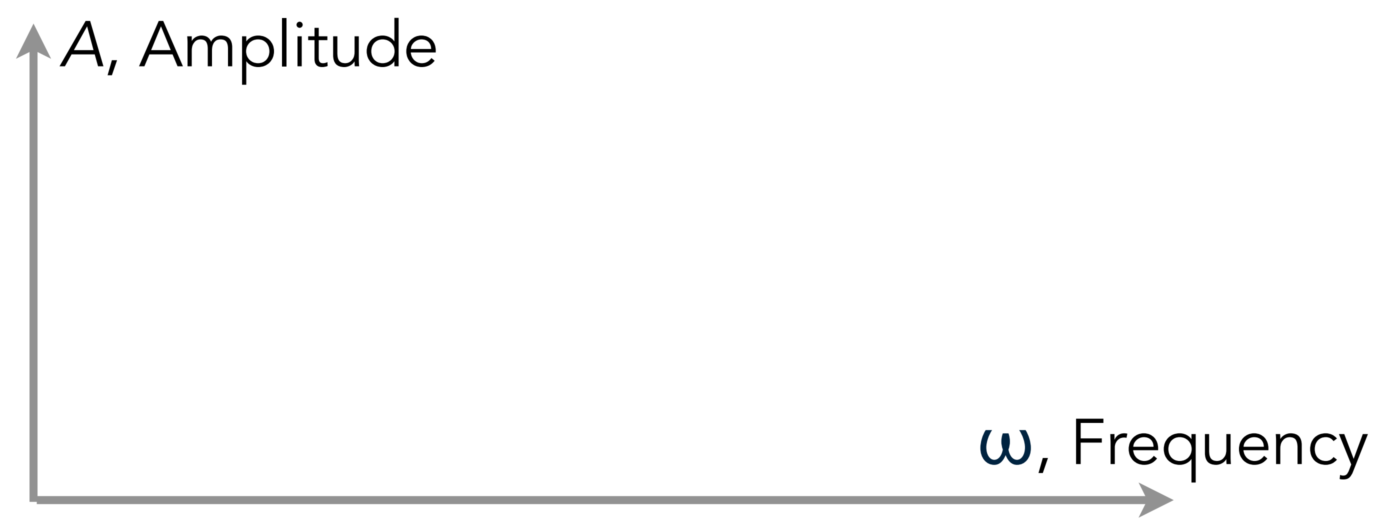
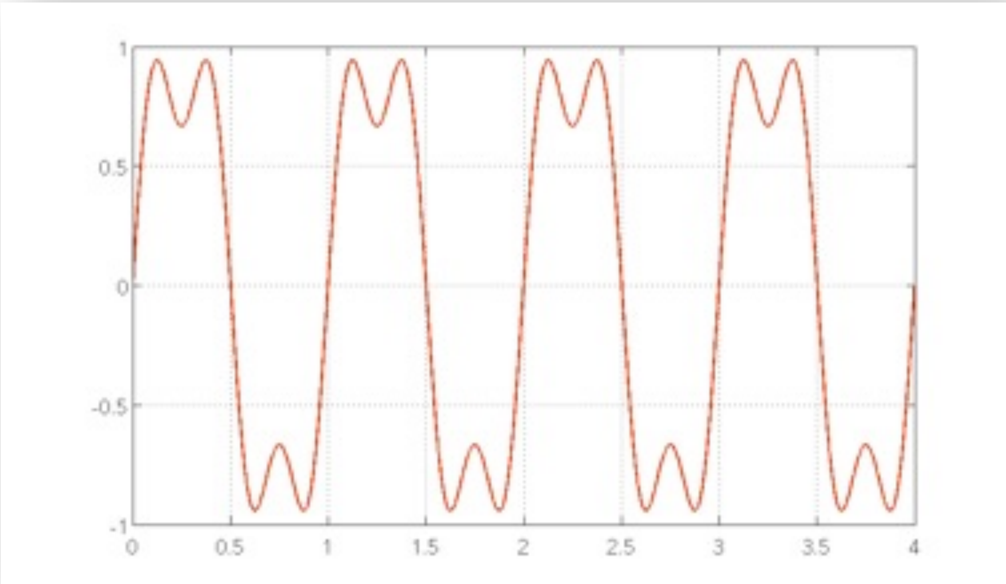


Time, Frequency, and Frequency Spectra



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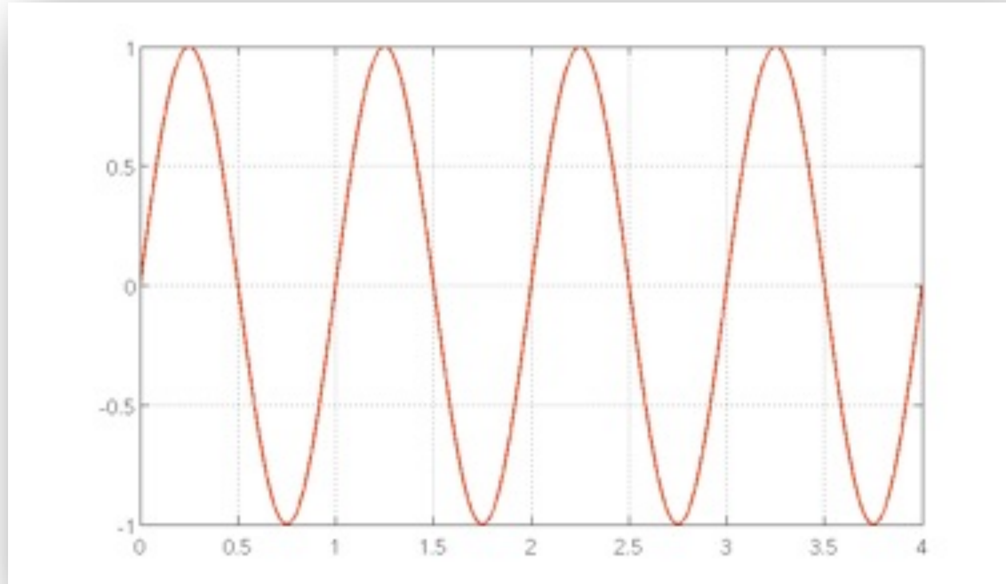

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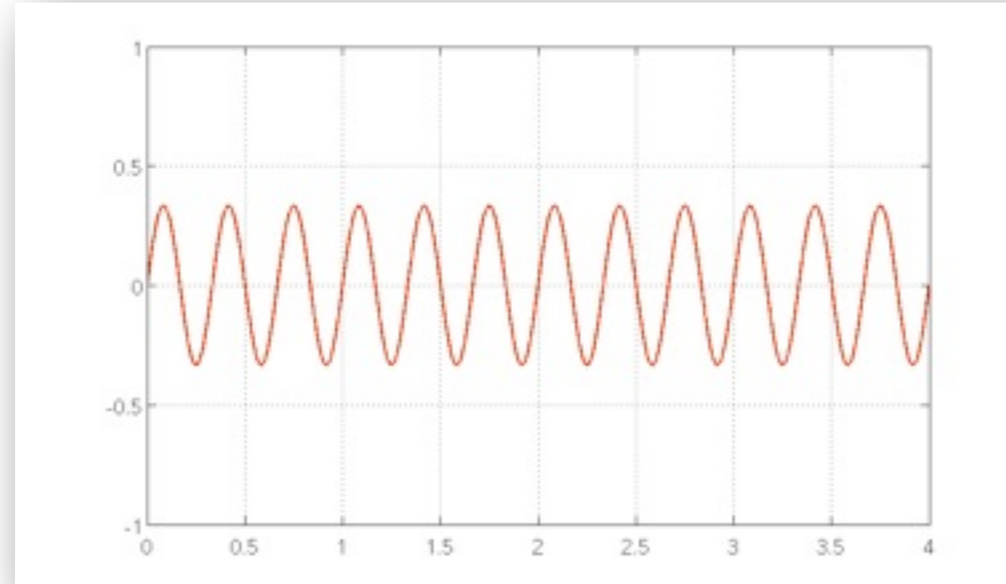
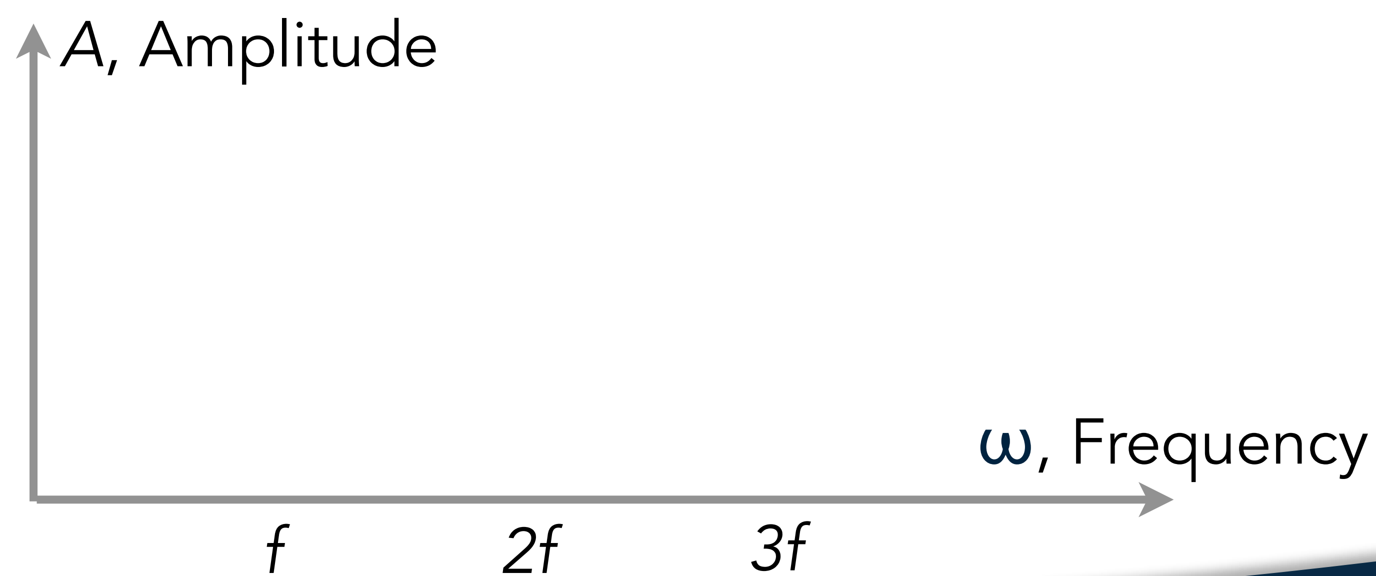


$$g(t) = \sin(2p\omega t) + \frac{1}{3}\sin(2p(3\omega)t)$$



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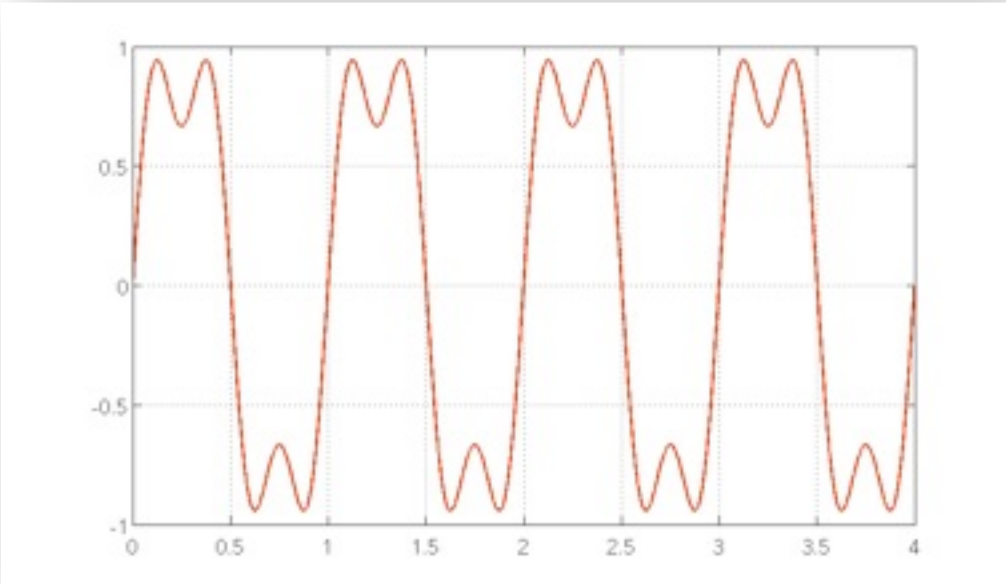

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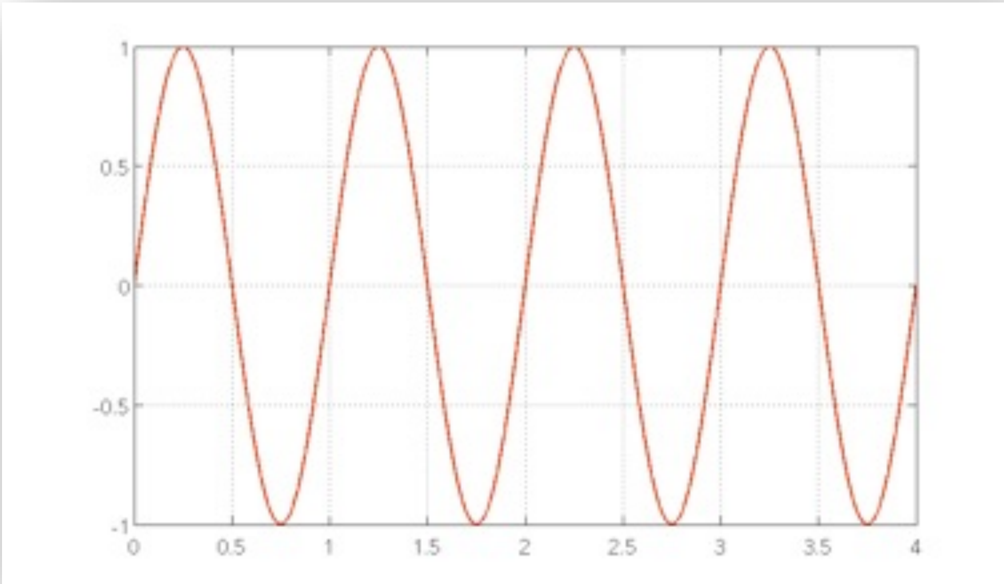

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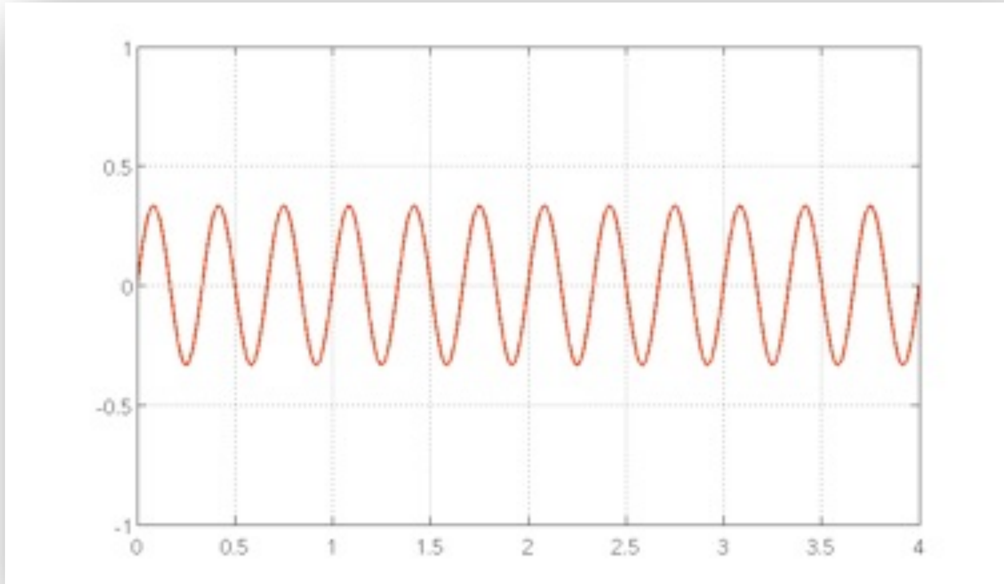

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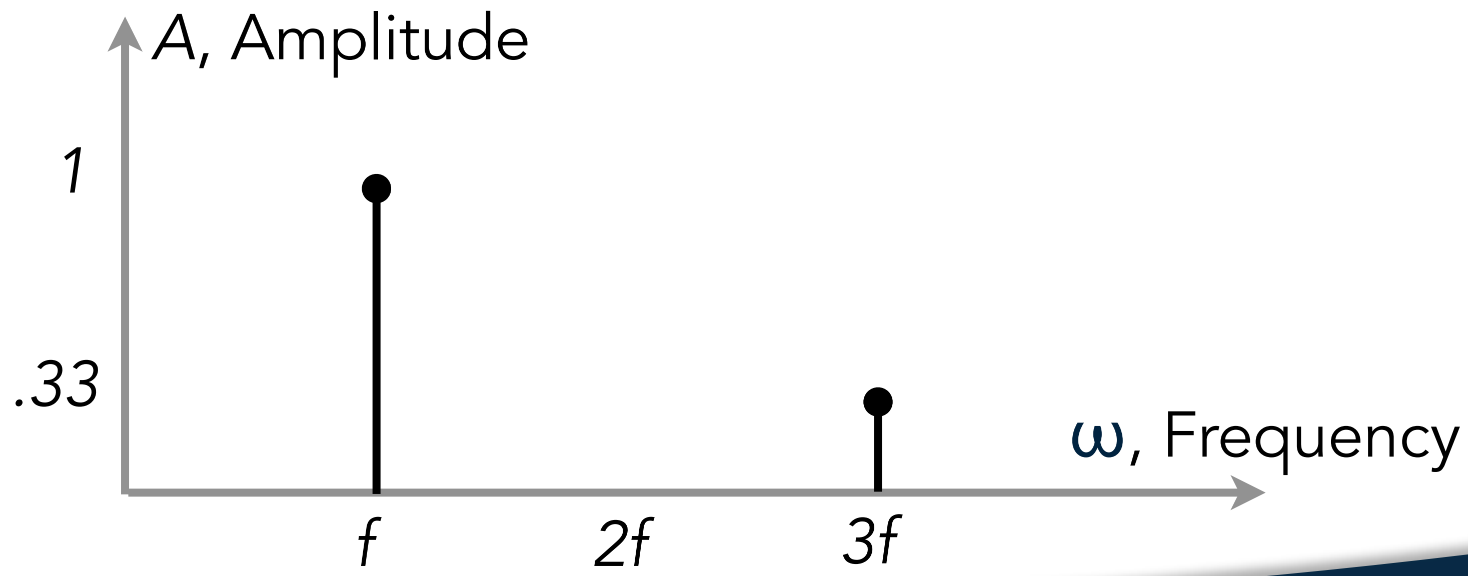
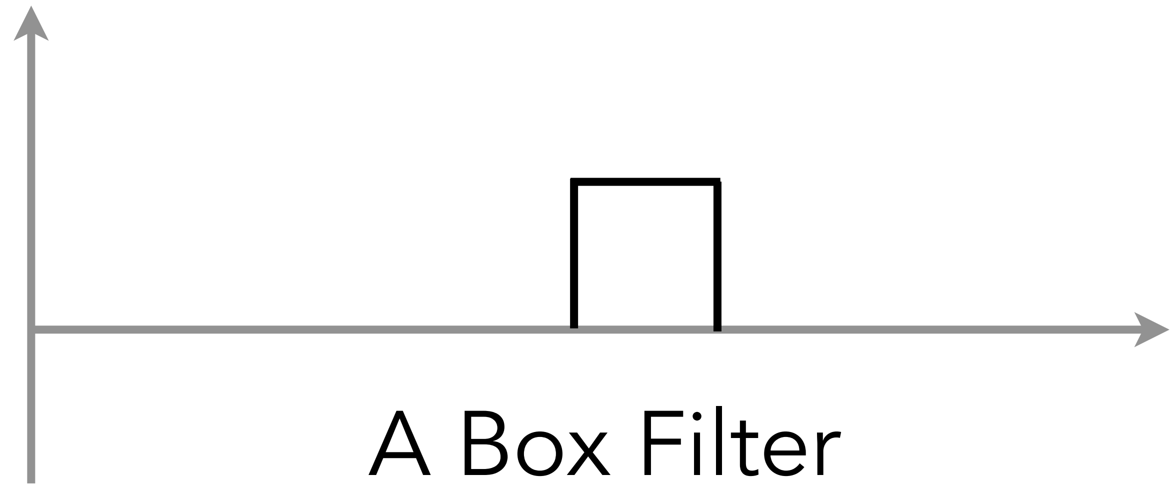


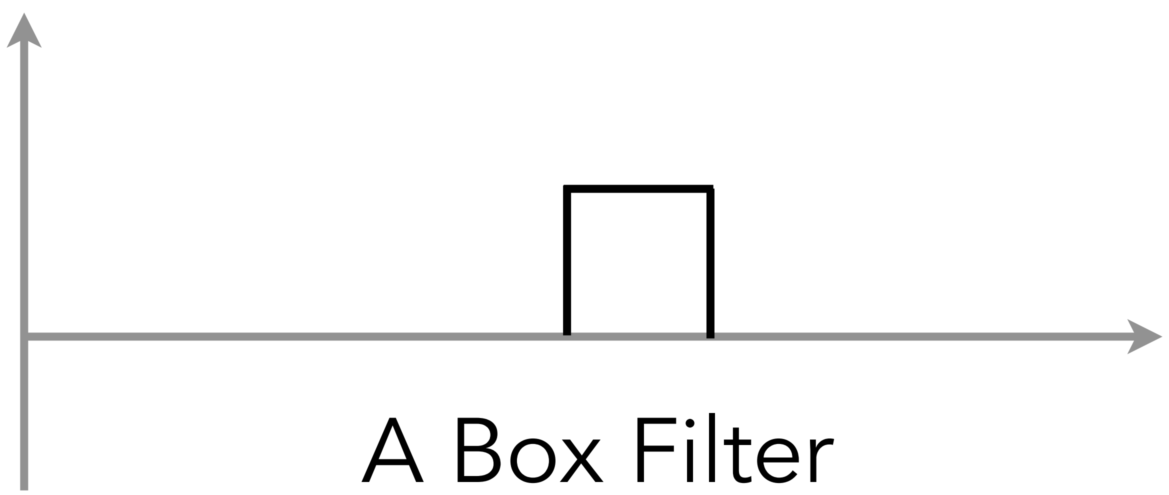
Frequency Spectra



A Box Filter

$$A \sum_{k=1}^{\infty} \frac{1}{k} \sin(2\pi k t)$$

Frequency Spectra



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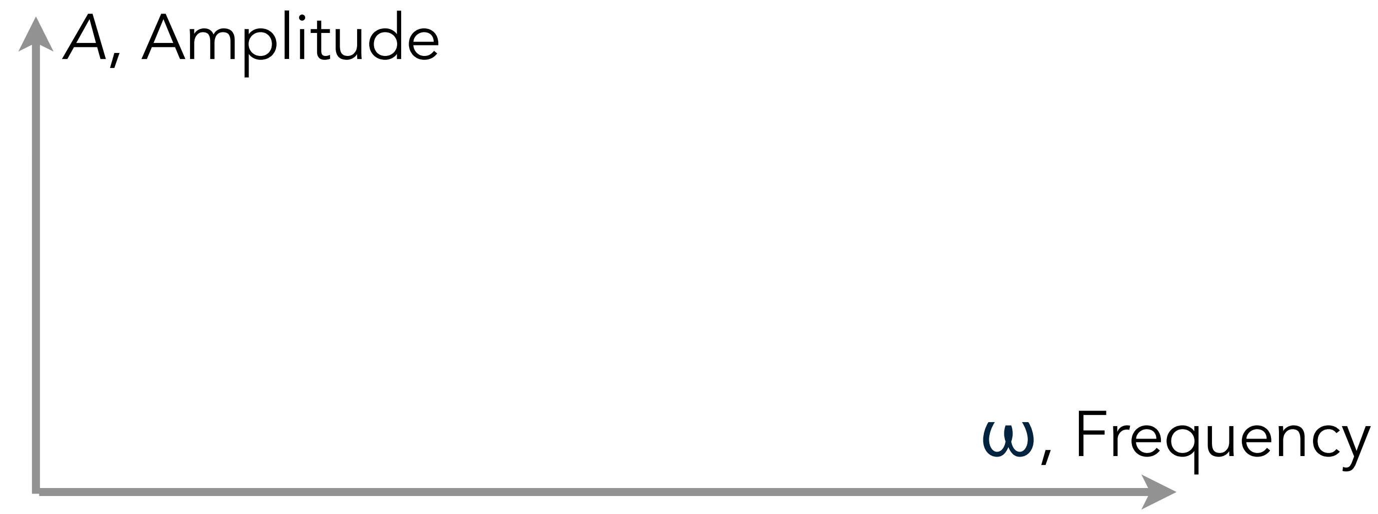

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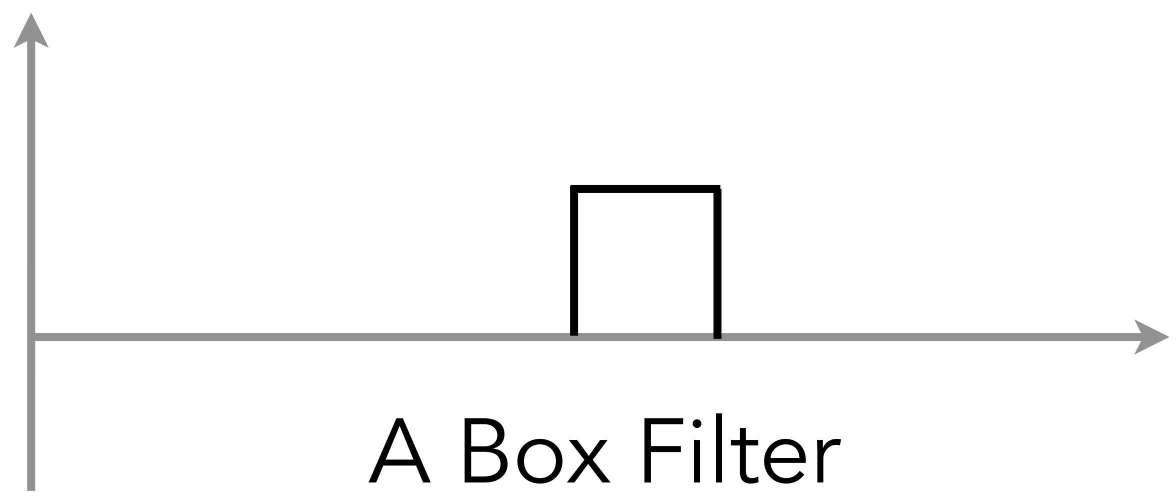

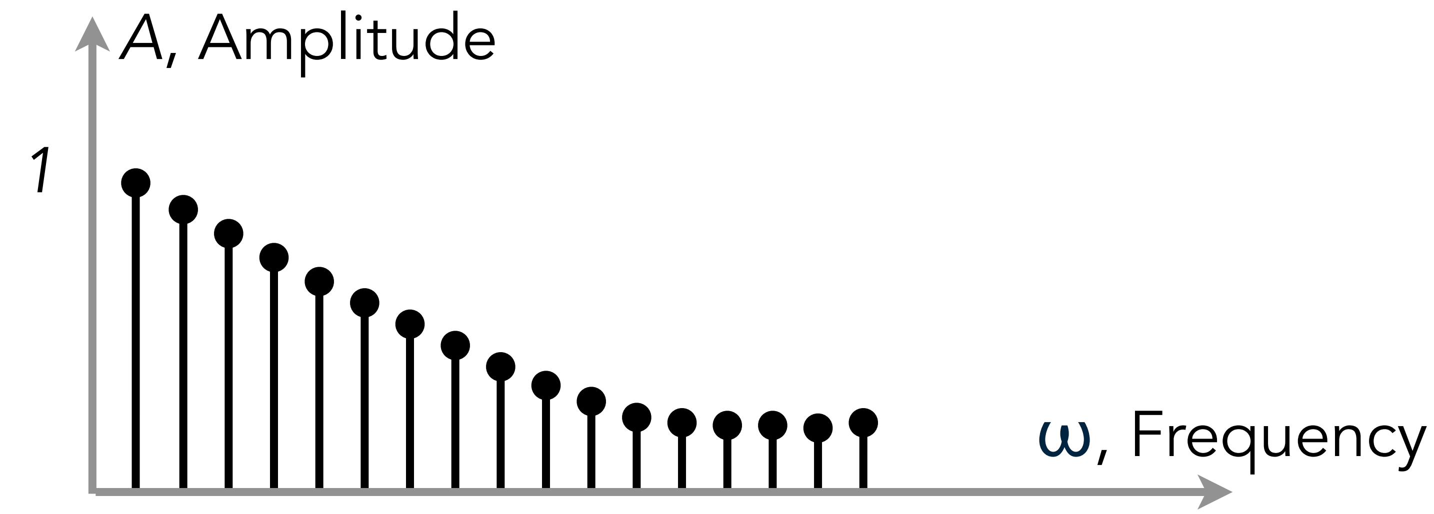

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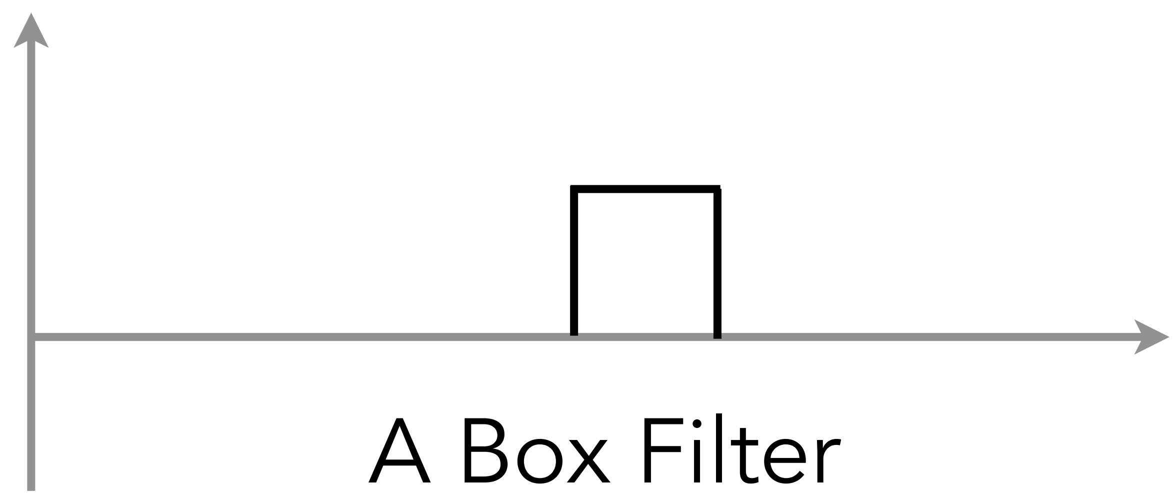

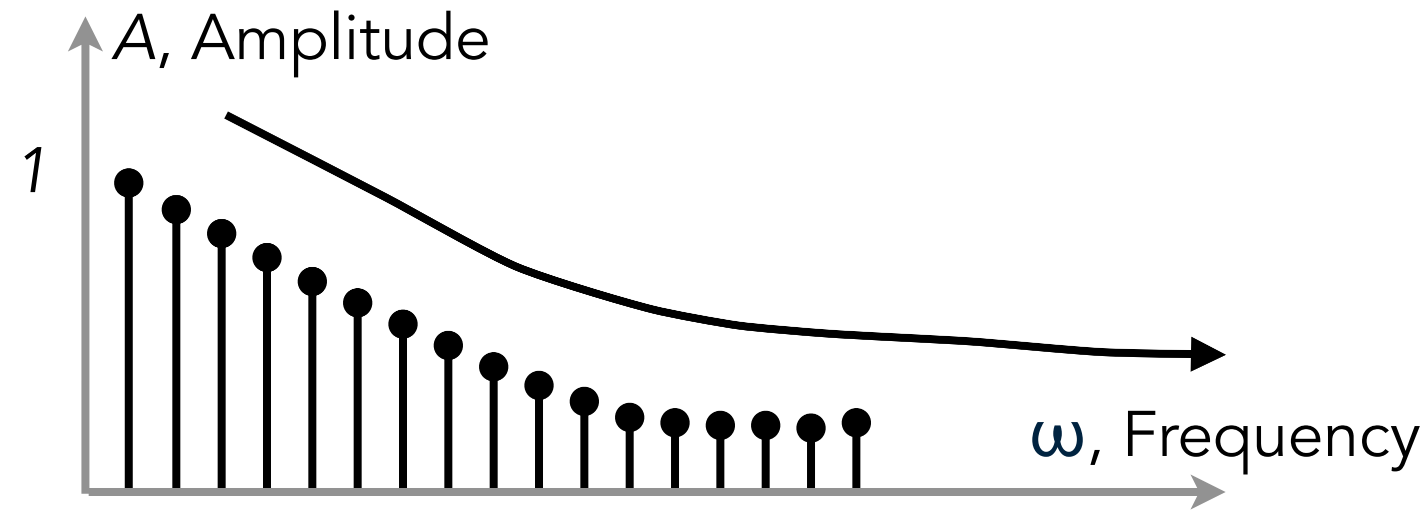

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Convolution Theorem and the Fourier Transform

10

Convolution Theorem and the Fourier Transform

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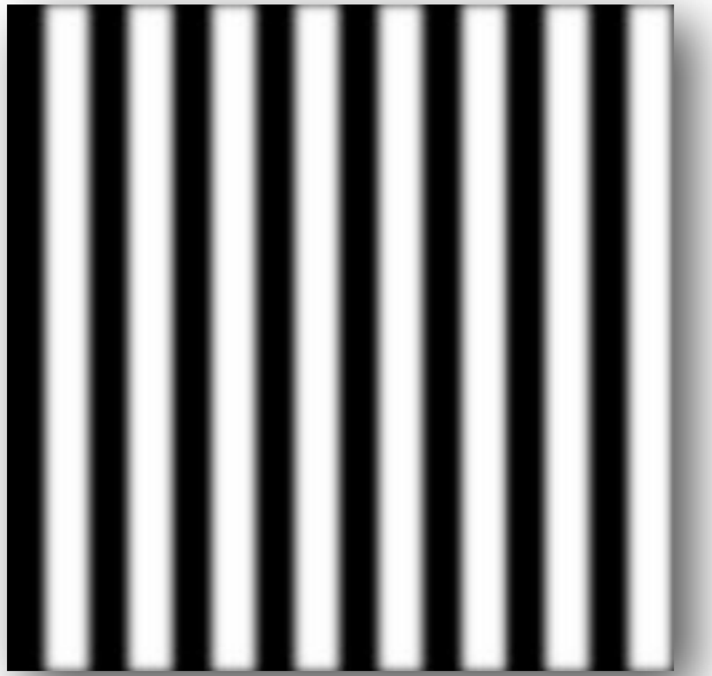
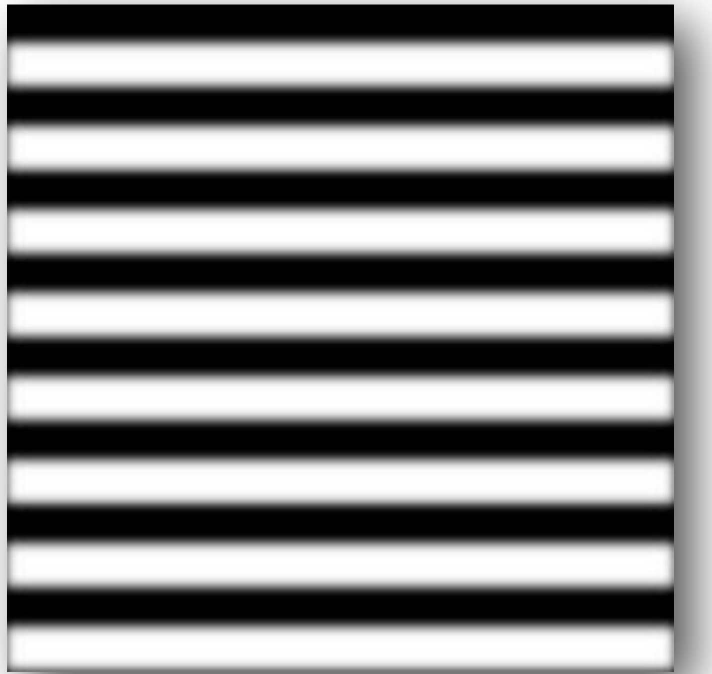
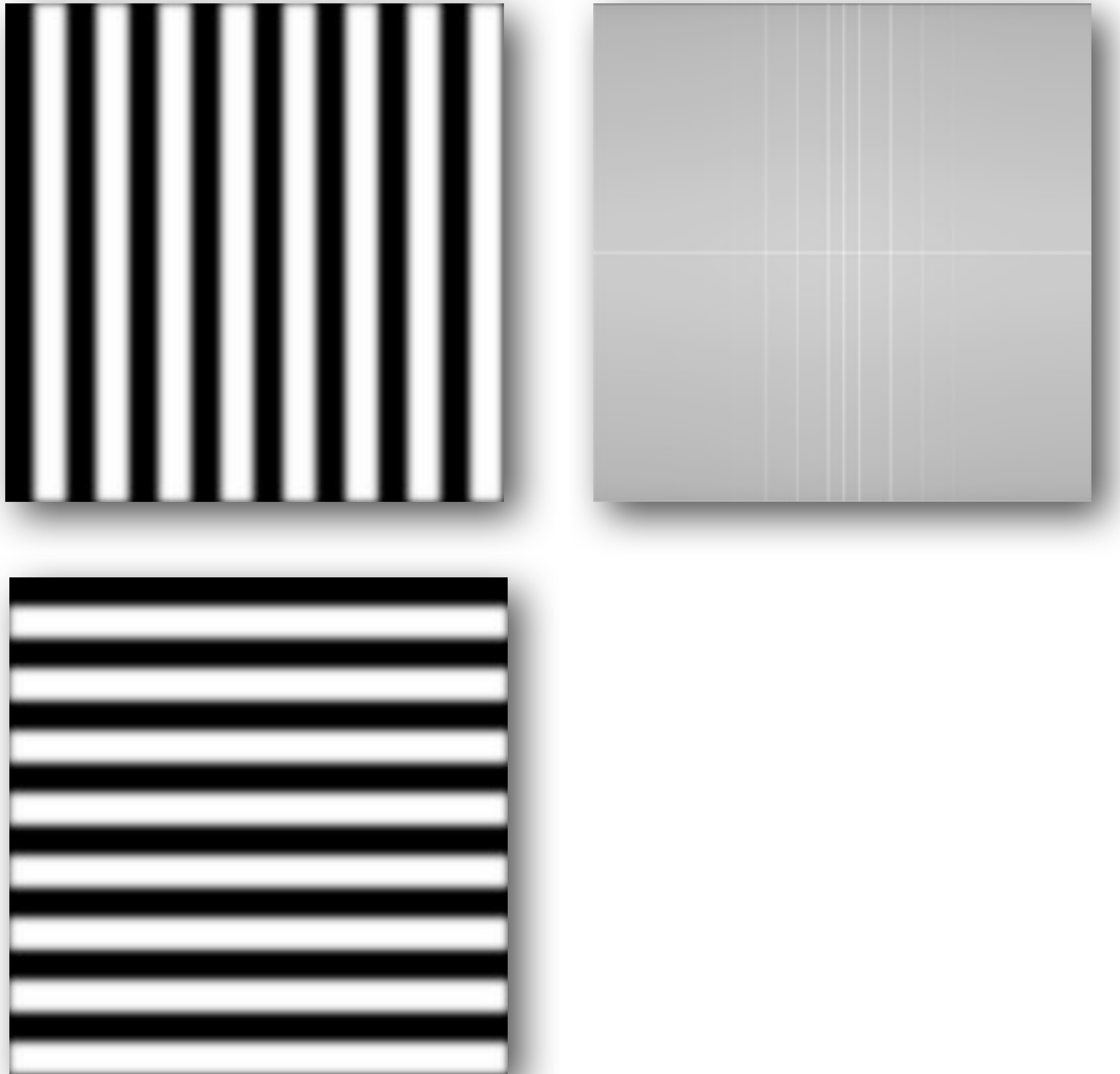
Now, Frequency Spectra for Images





Now, Frequency Spectra for Images



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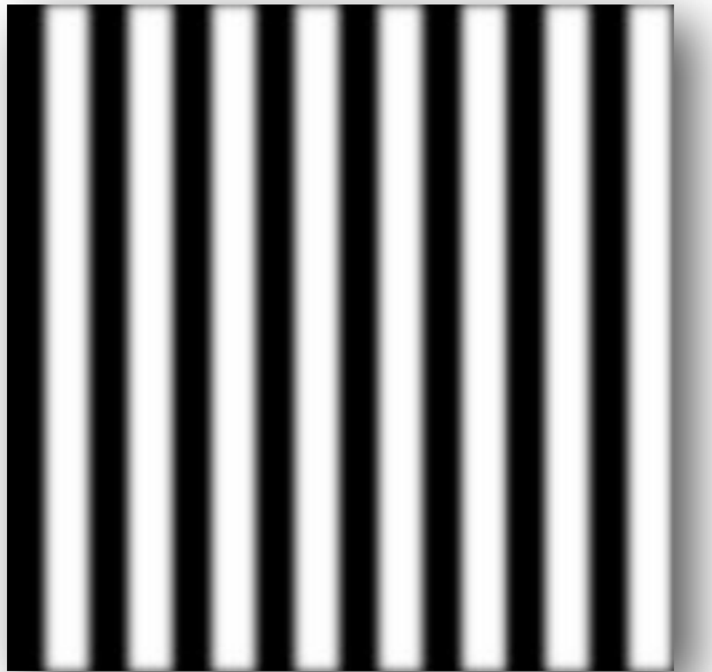
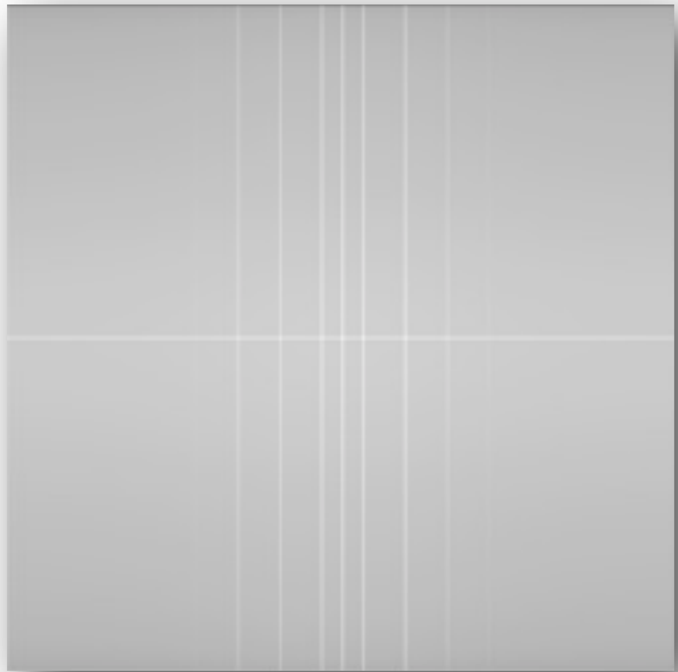
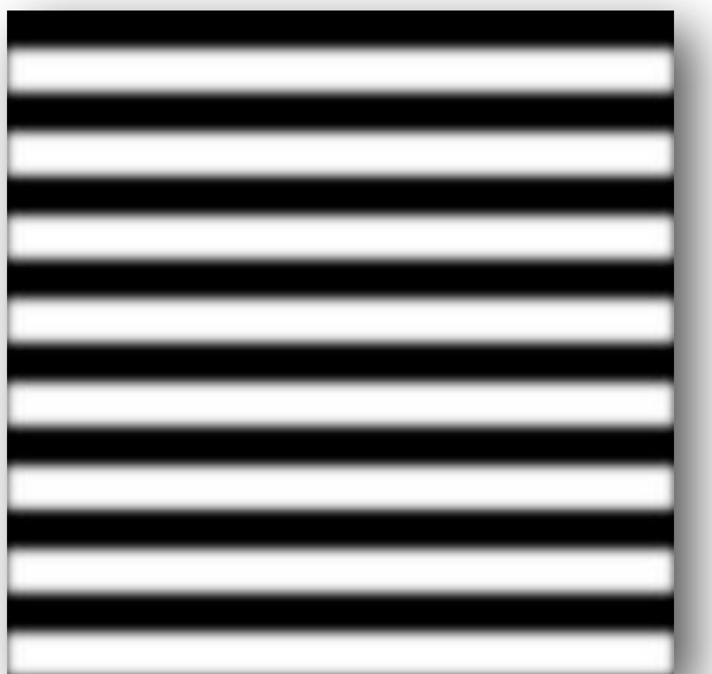


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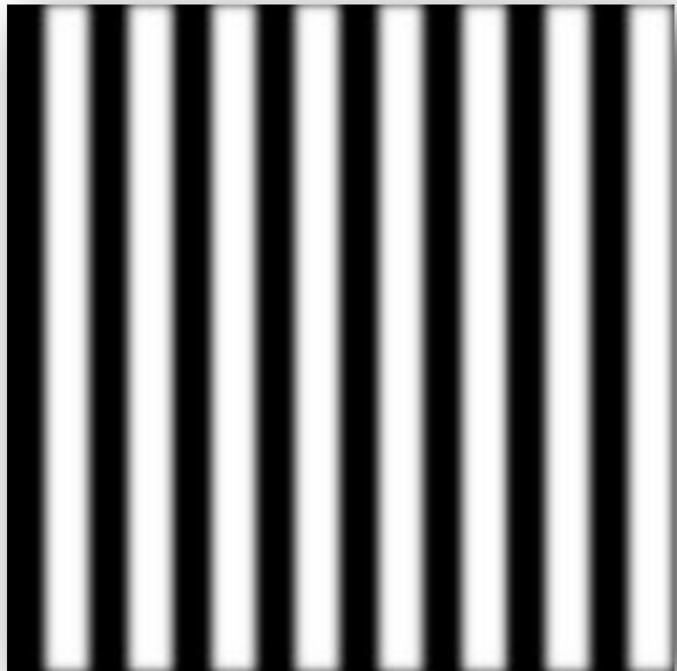
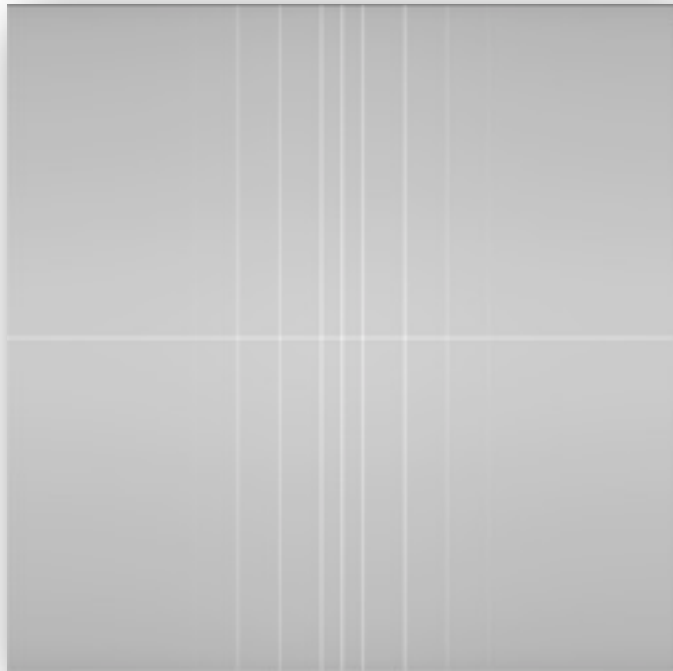
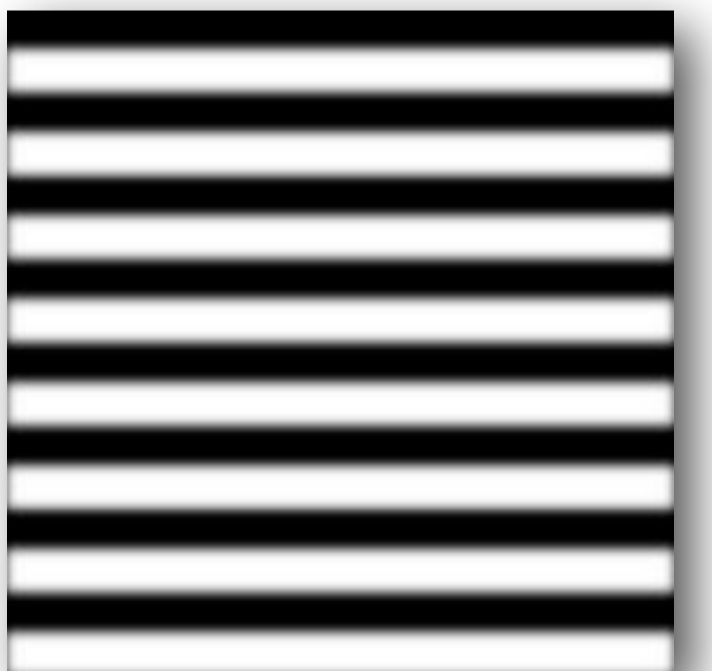
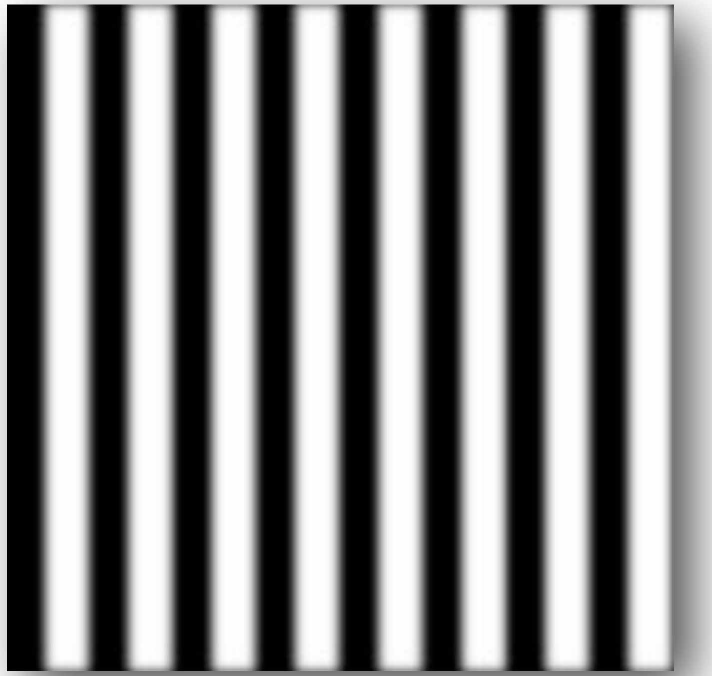
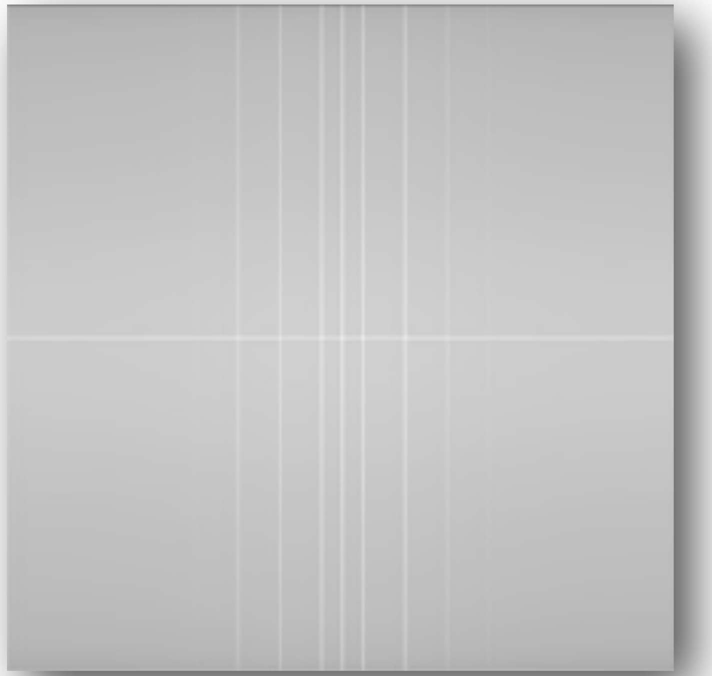
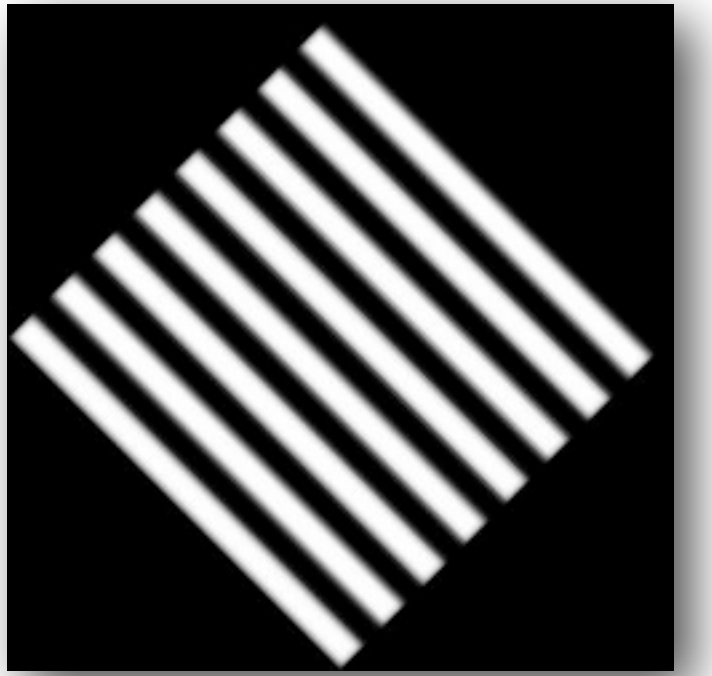
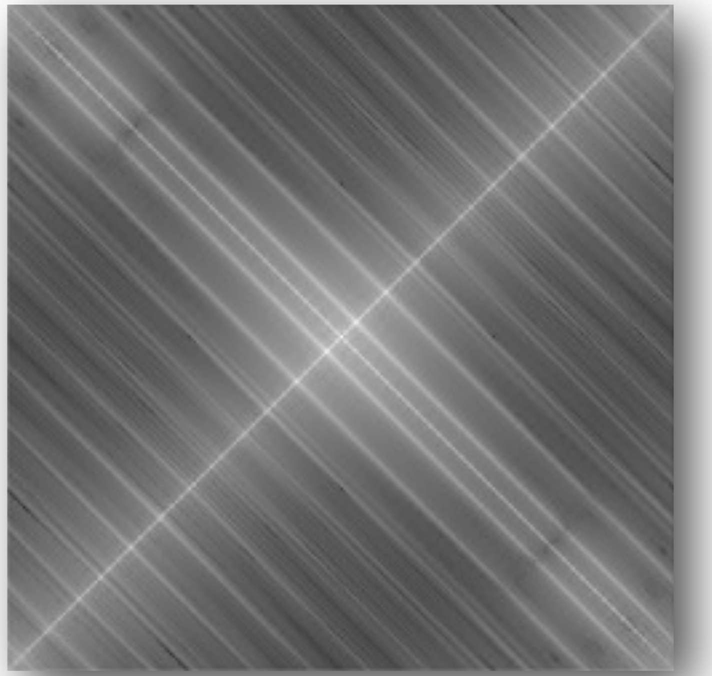
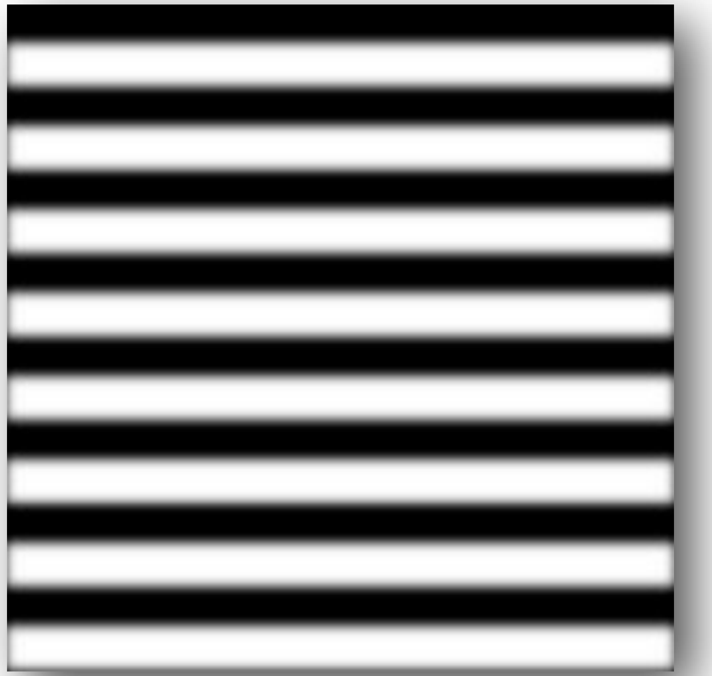






Now, Frequency Spectra for Images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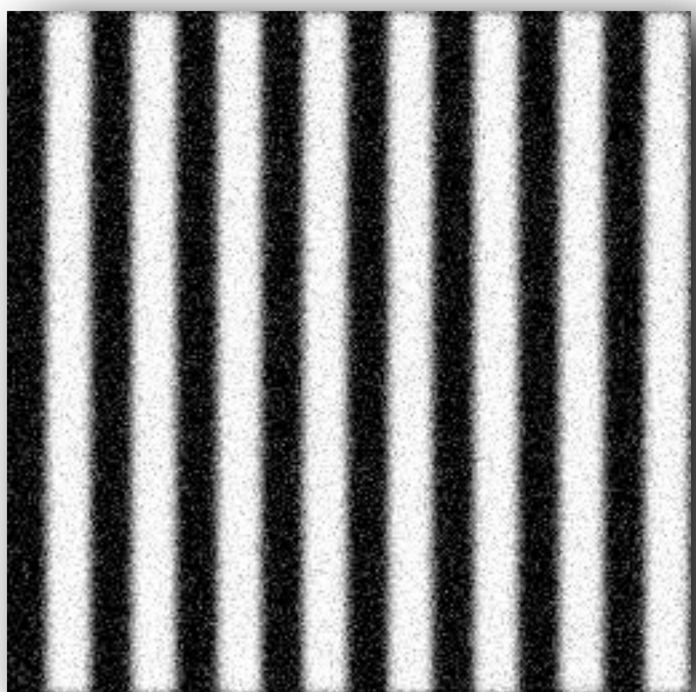
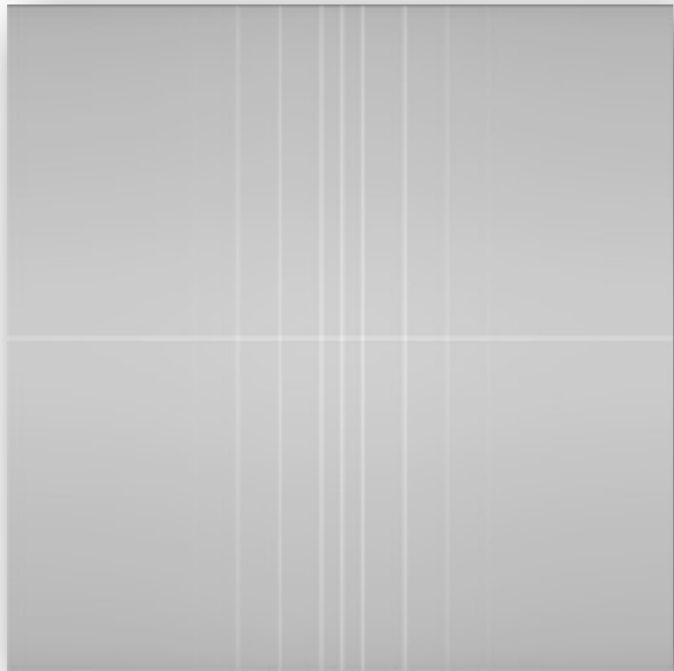
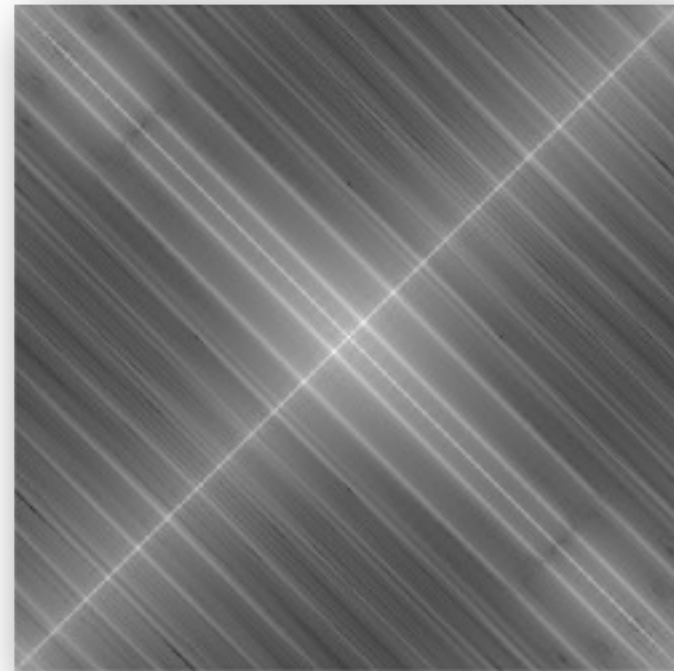
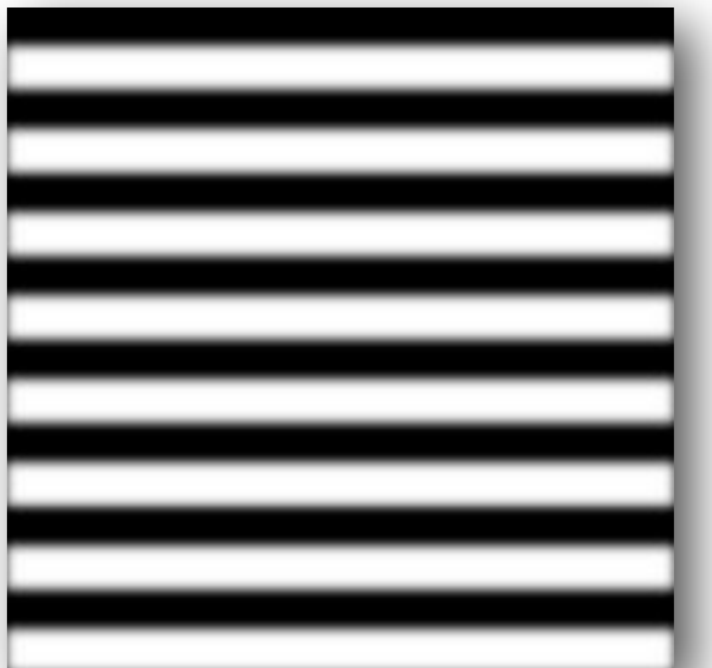


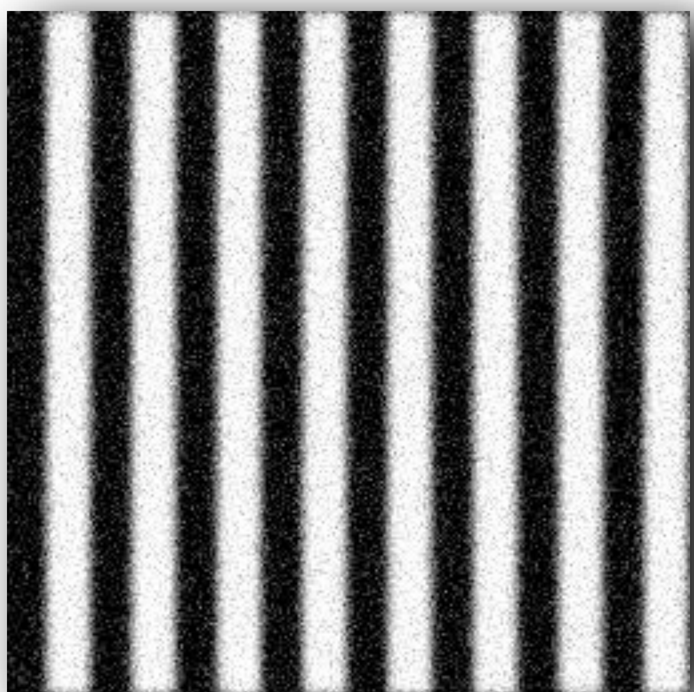
Noise Added







Now, Frequency Spectra for Images



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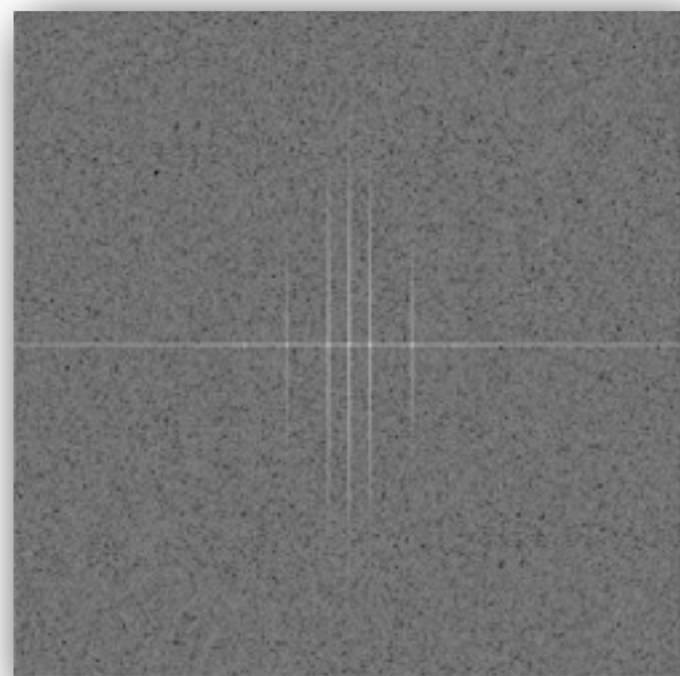
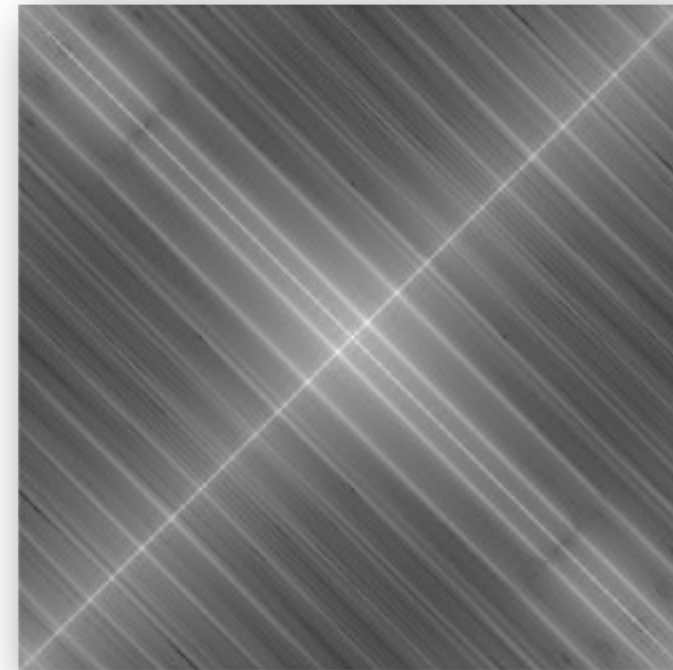
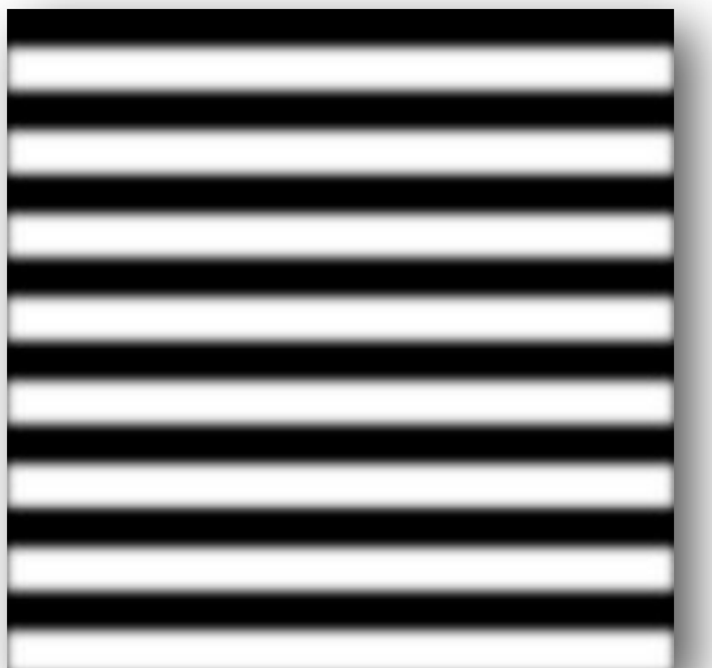
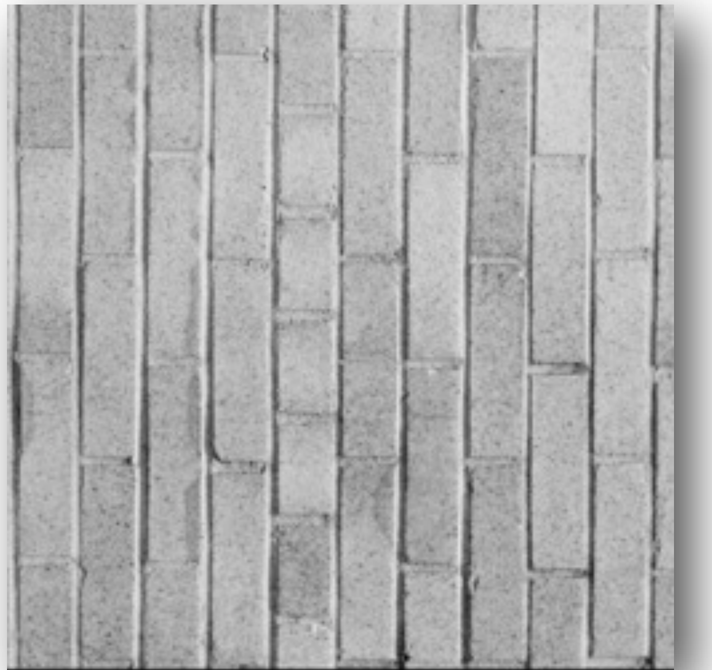
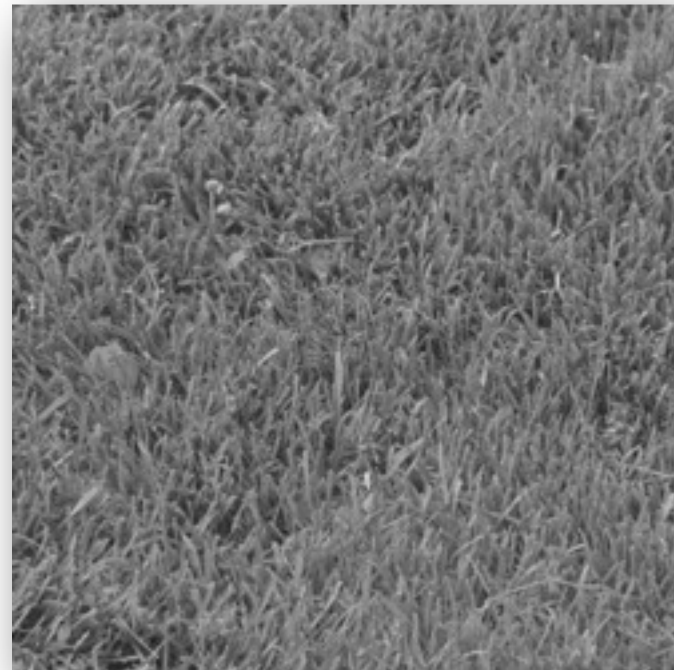






Frequency Spectra for Real Images





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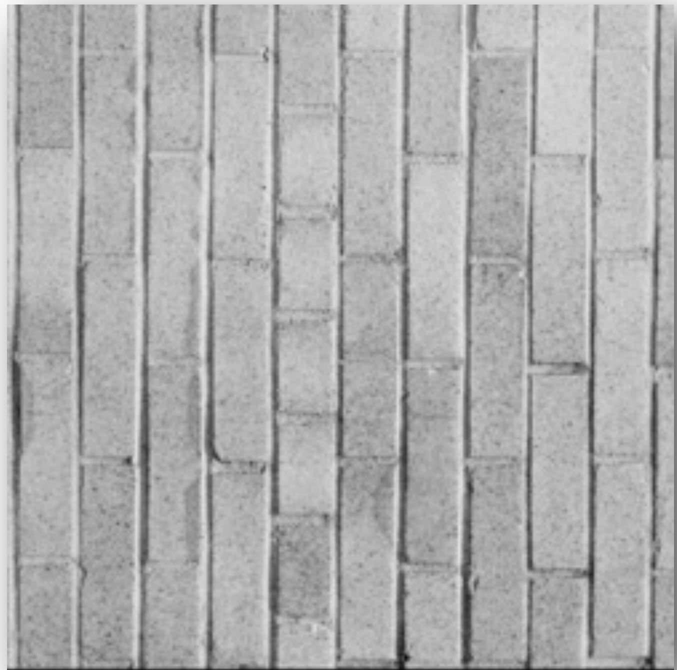

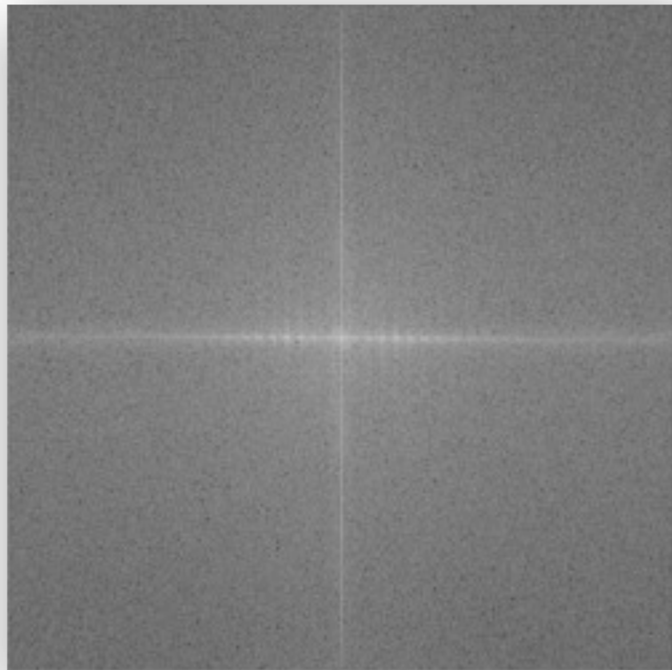

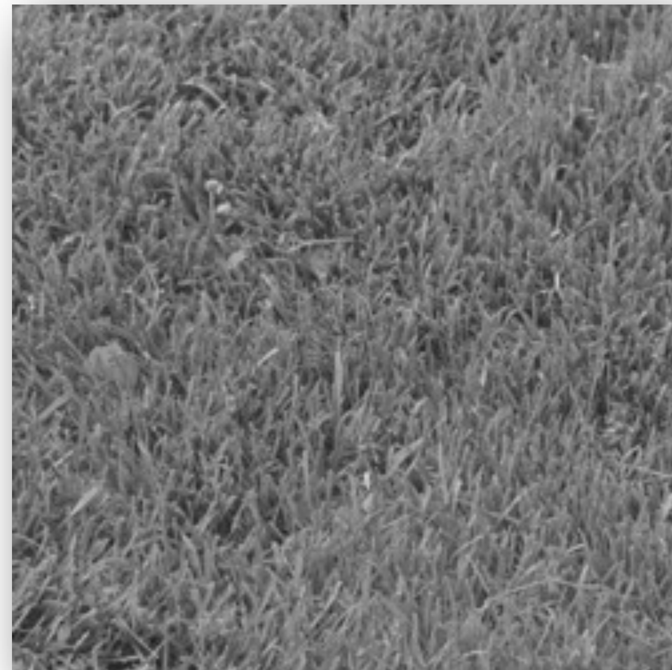

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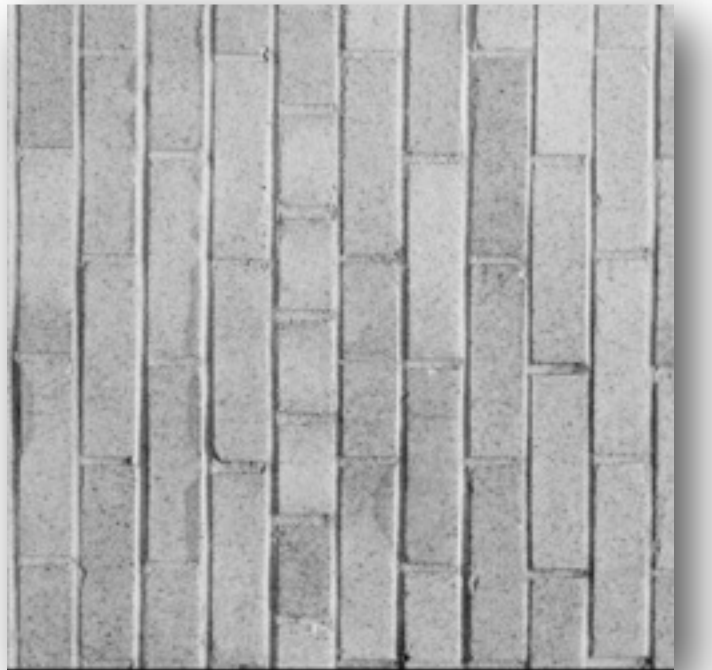
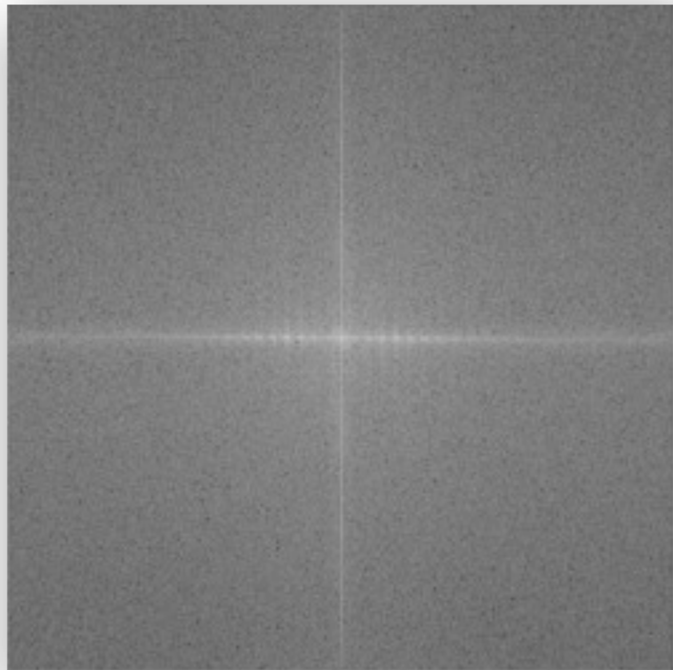


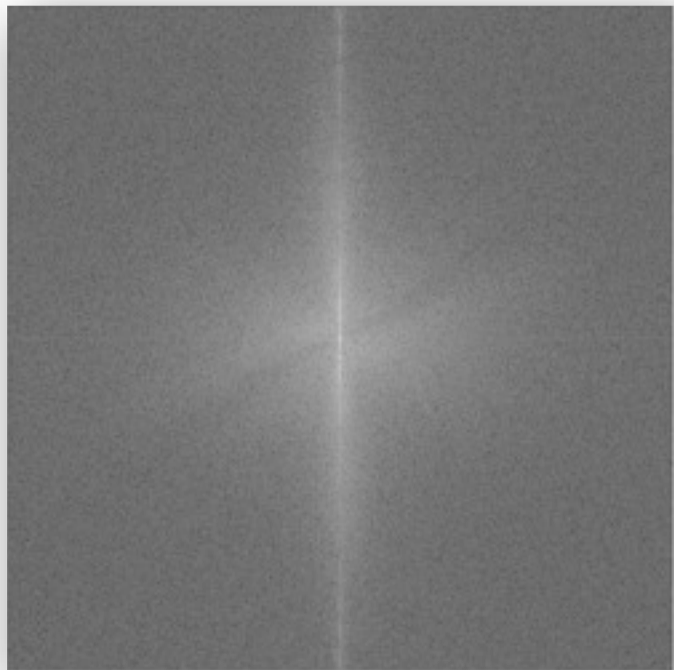

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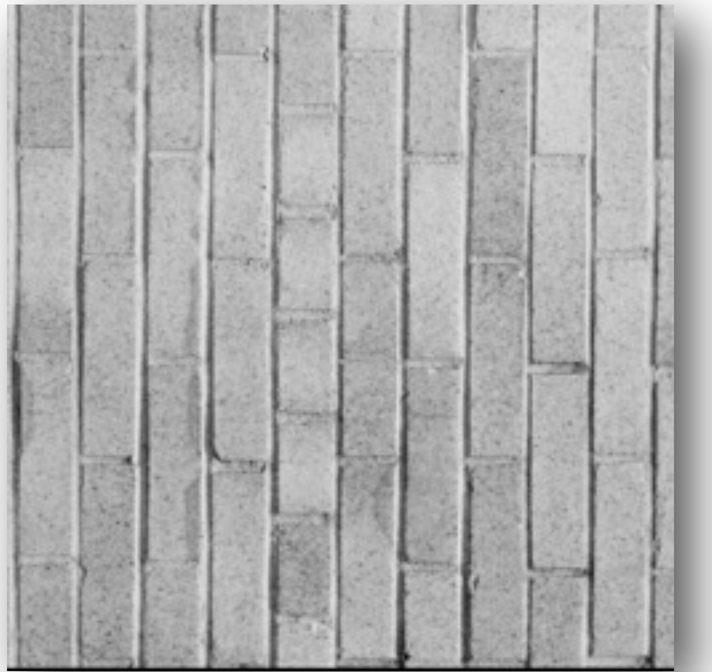
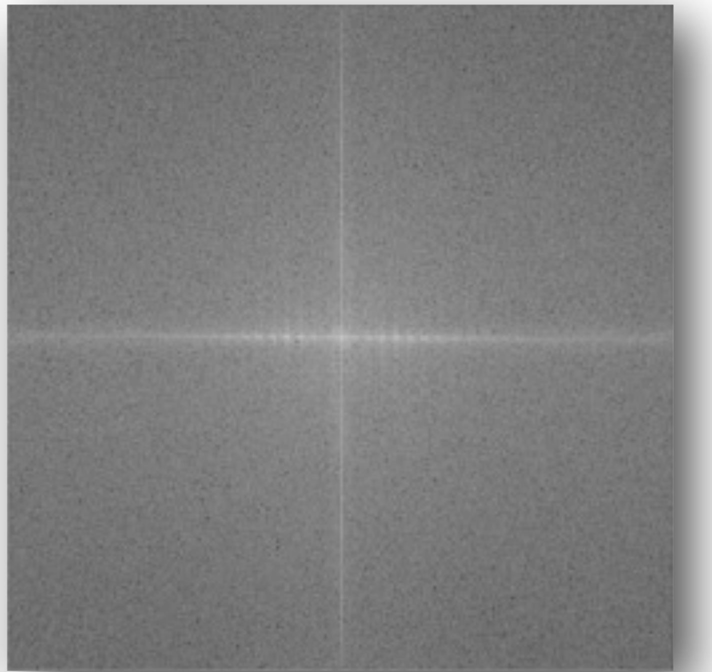
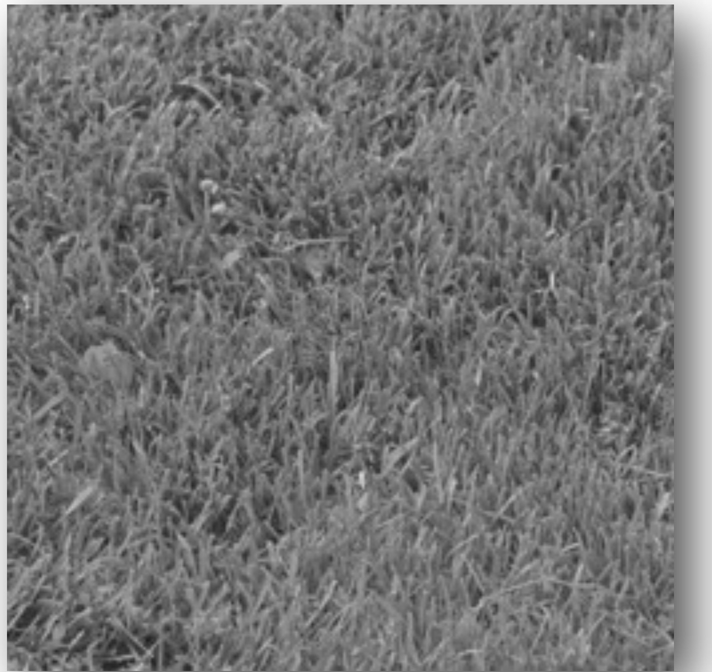
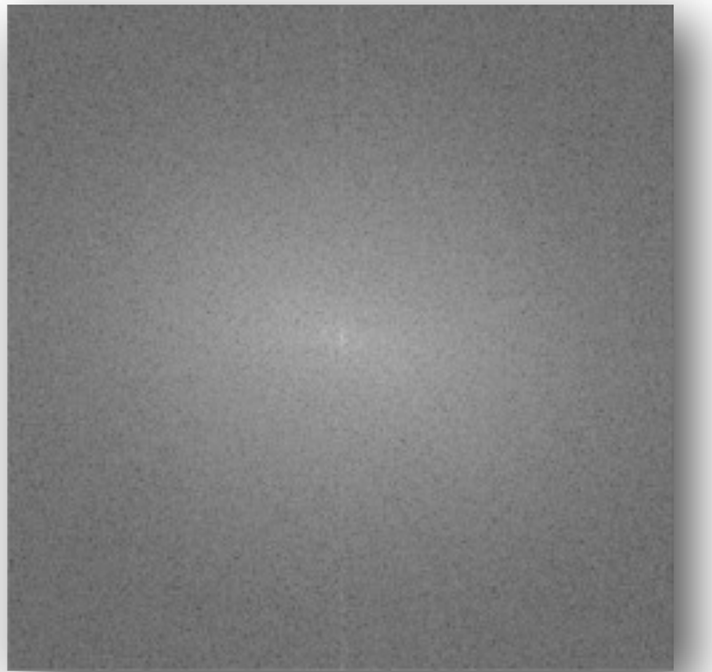
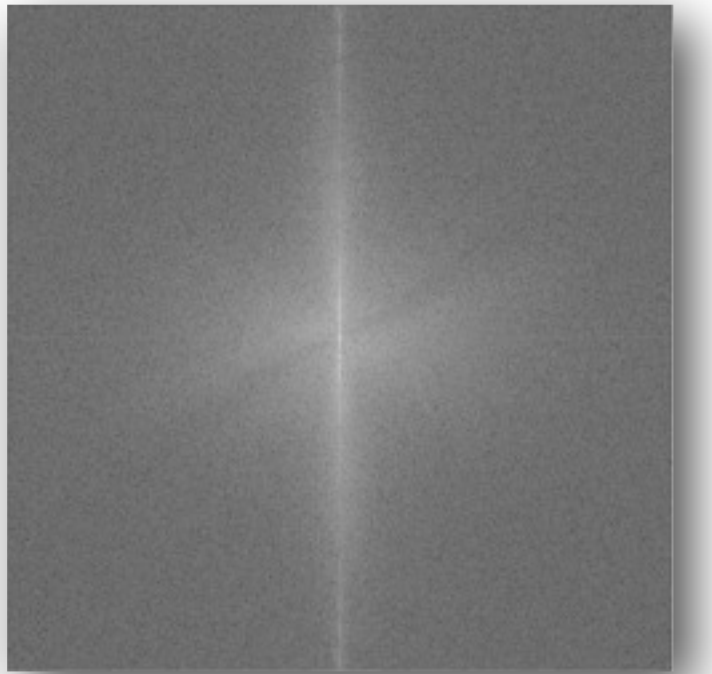




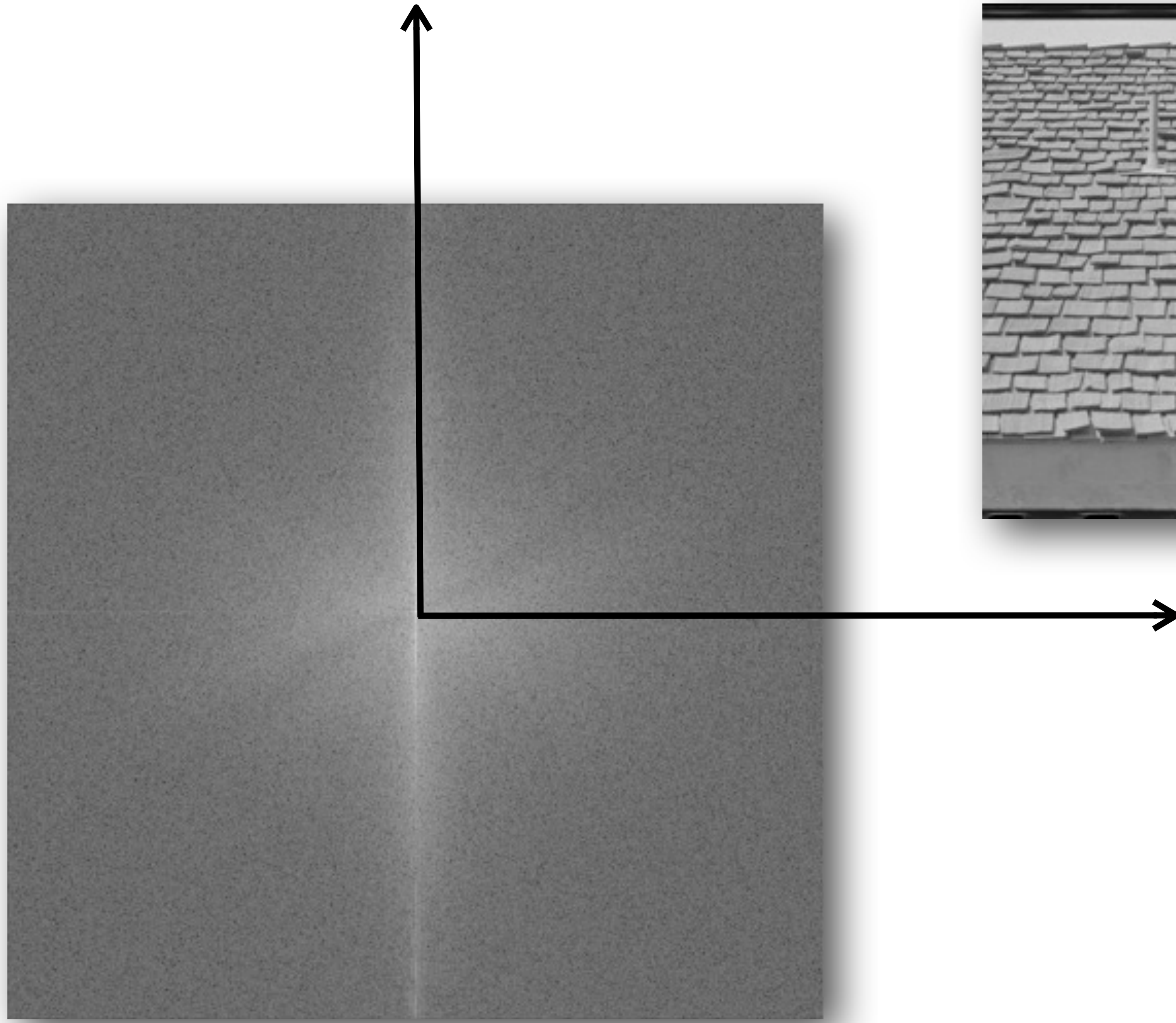






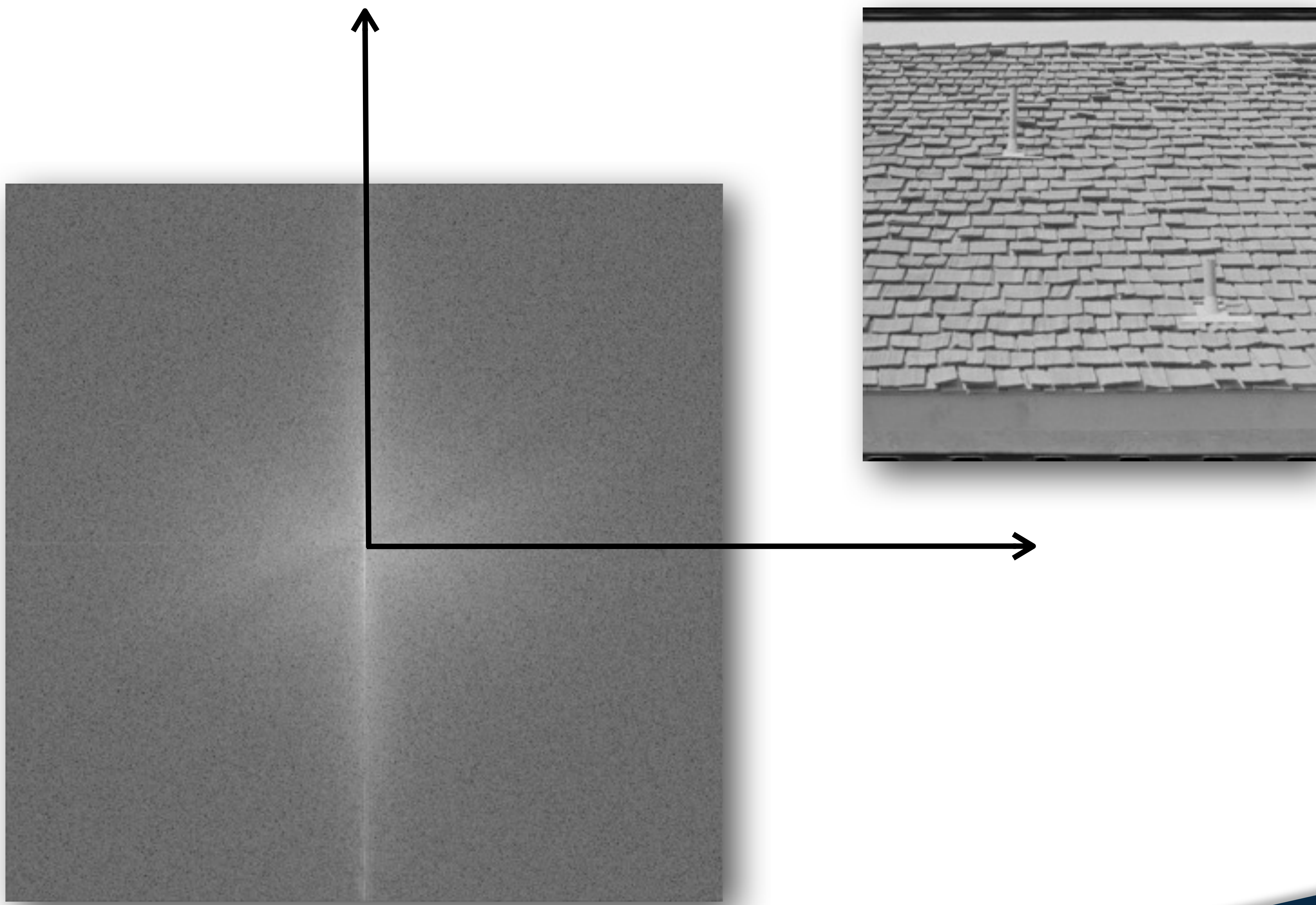
12

Fourier Transform: Some Observations



Fourier Transform: Some Observations

★ For every ω from 0 to ∞ (infinity) $F(\omega)$ holds the Amplitude A and phase ϕ of a sine function.



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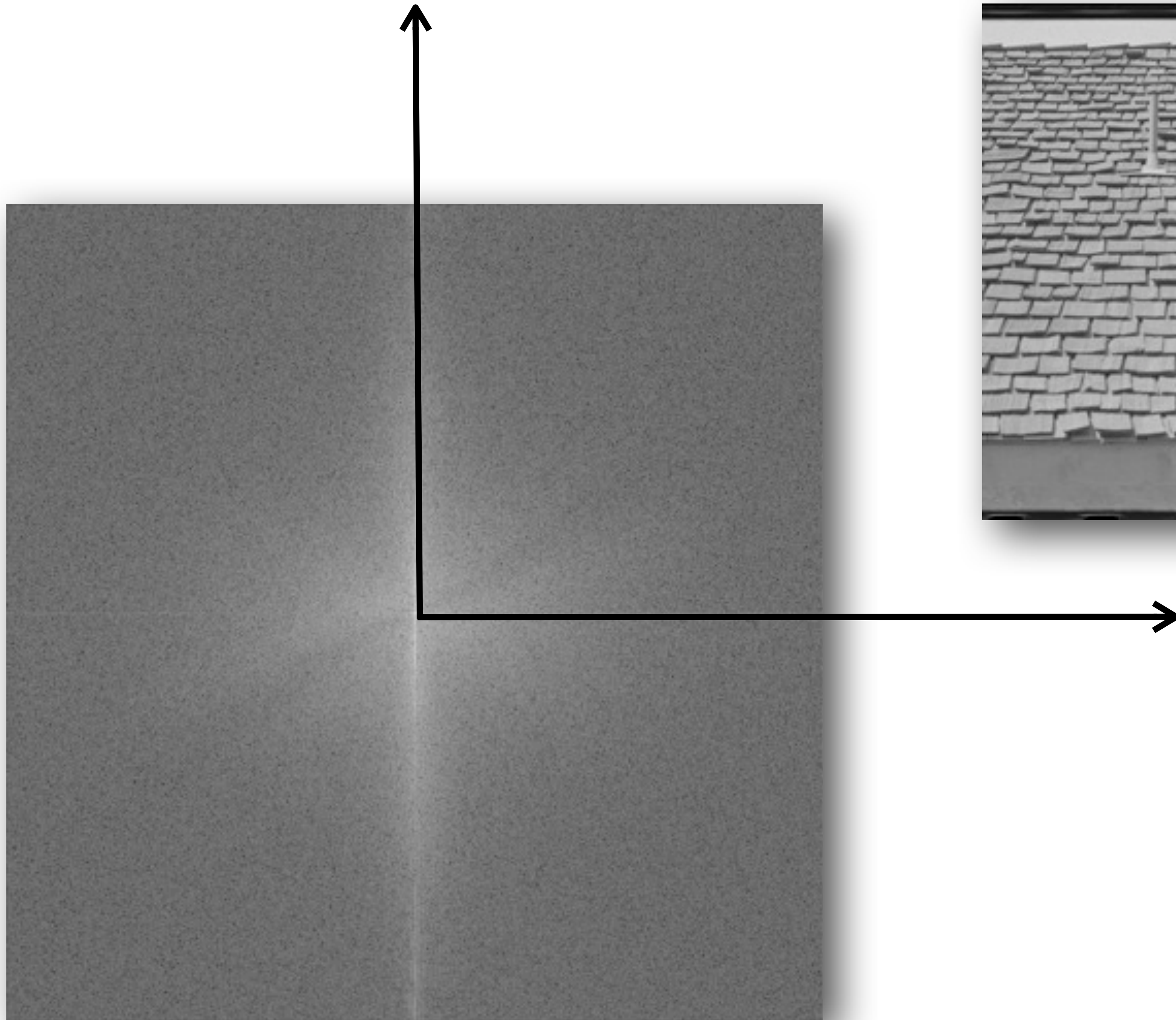
- It uses Real and Complex Numbers to achieve this



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- For every ω from 0 to ∞ (infinity) *F*(ω) holds the Amplitude A and phase ϕ of a sine function.
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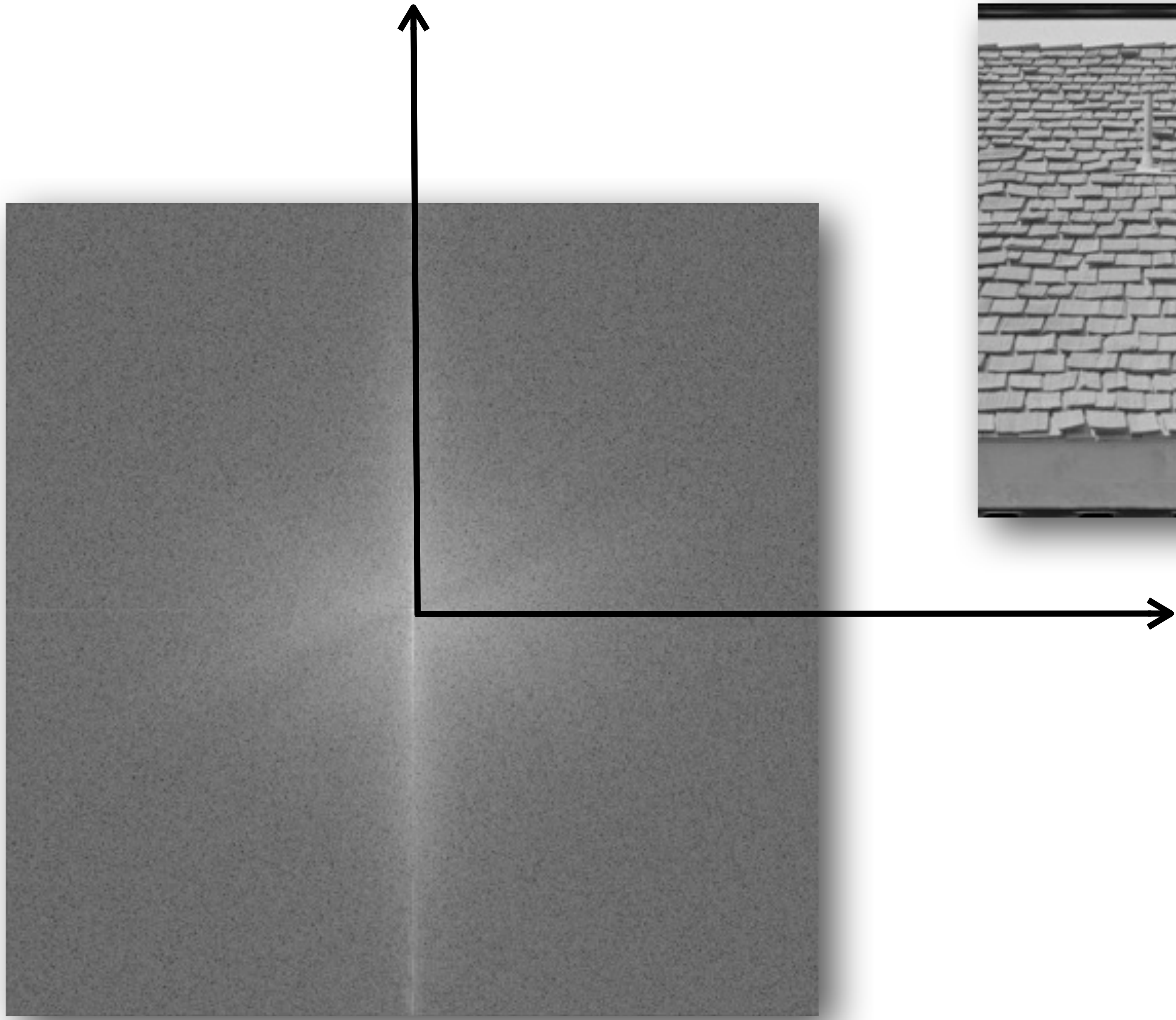
$$A\sin(\omega t + \phi)$$



13

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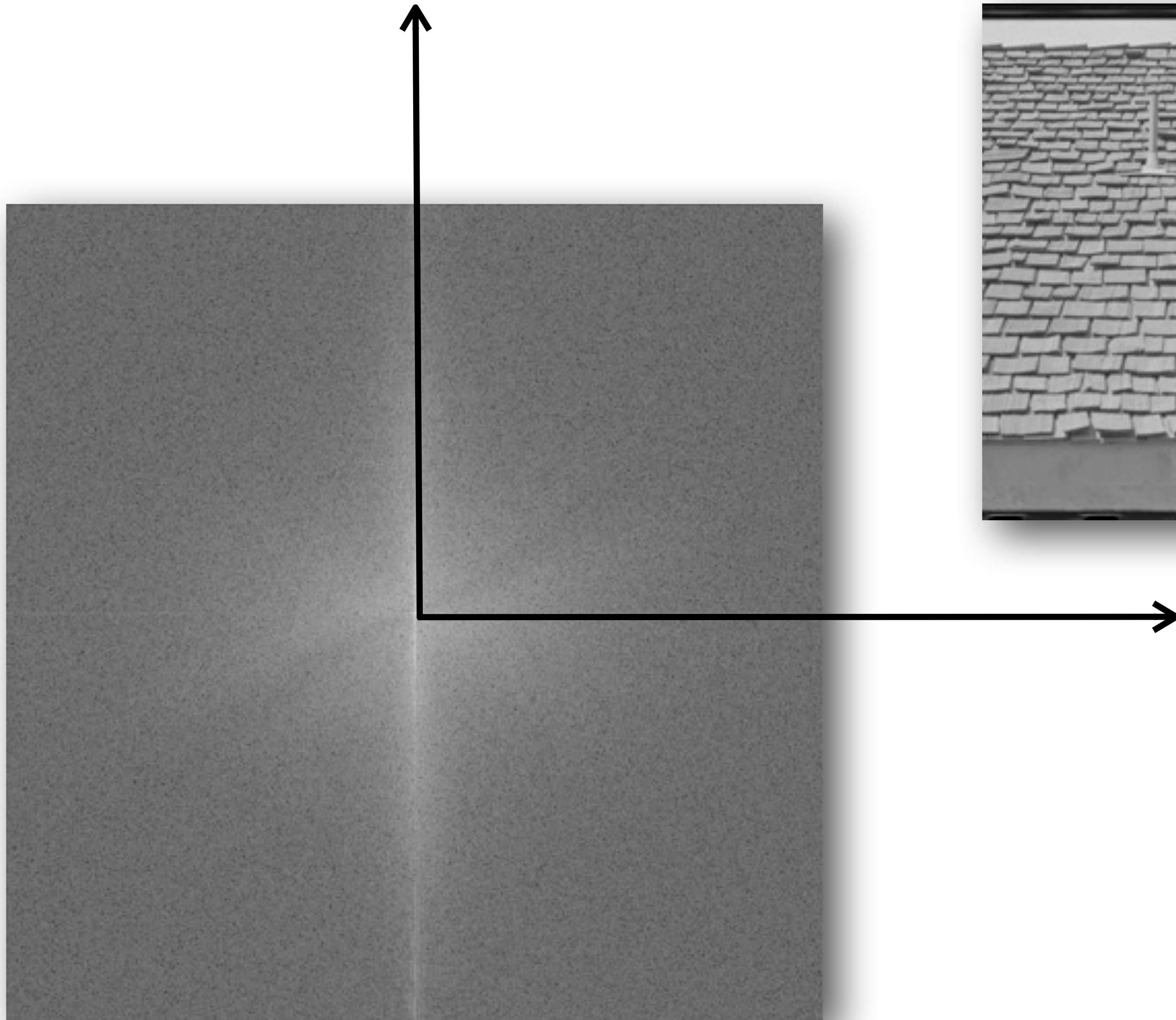

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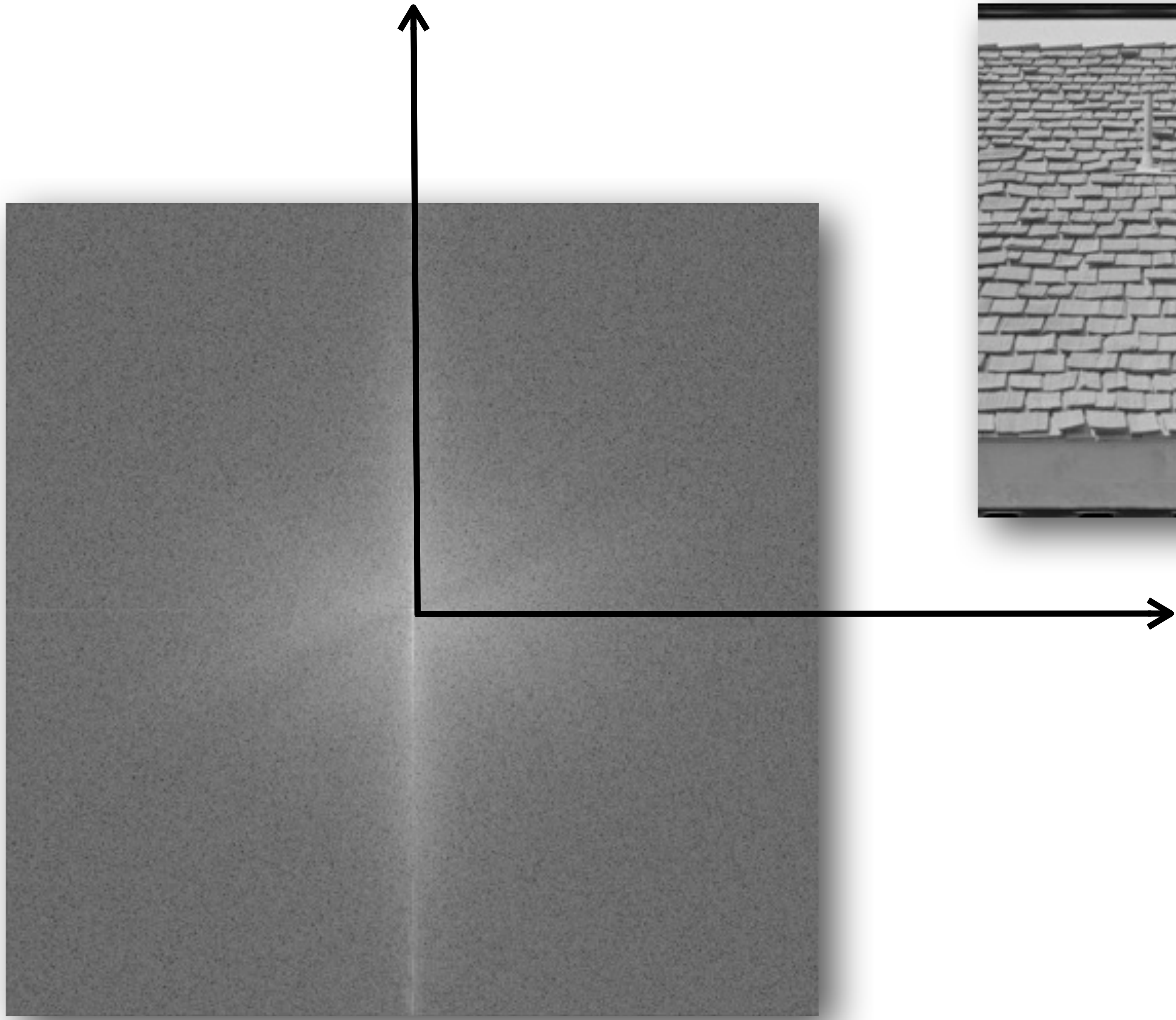

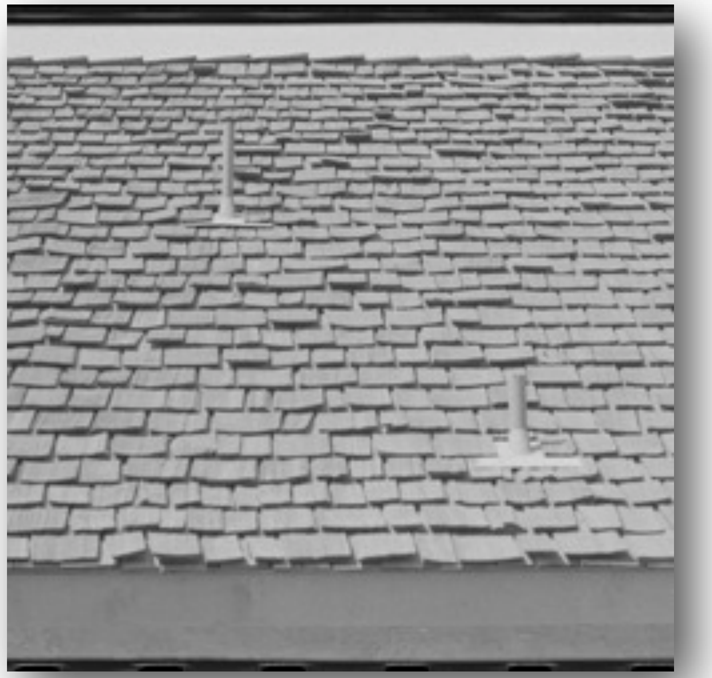

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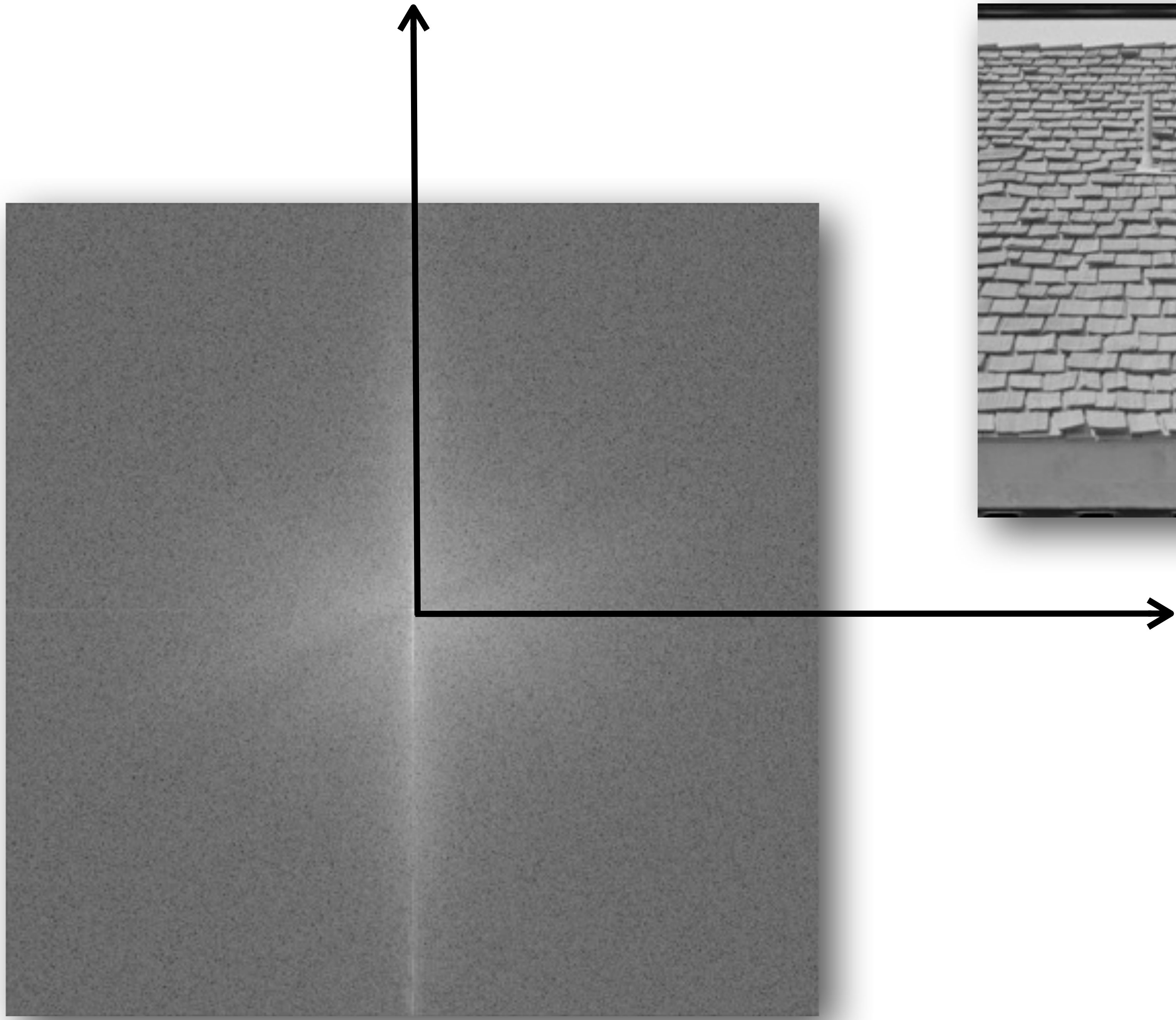
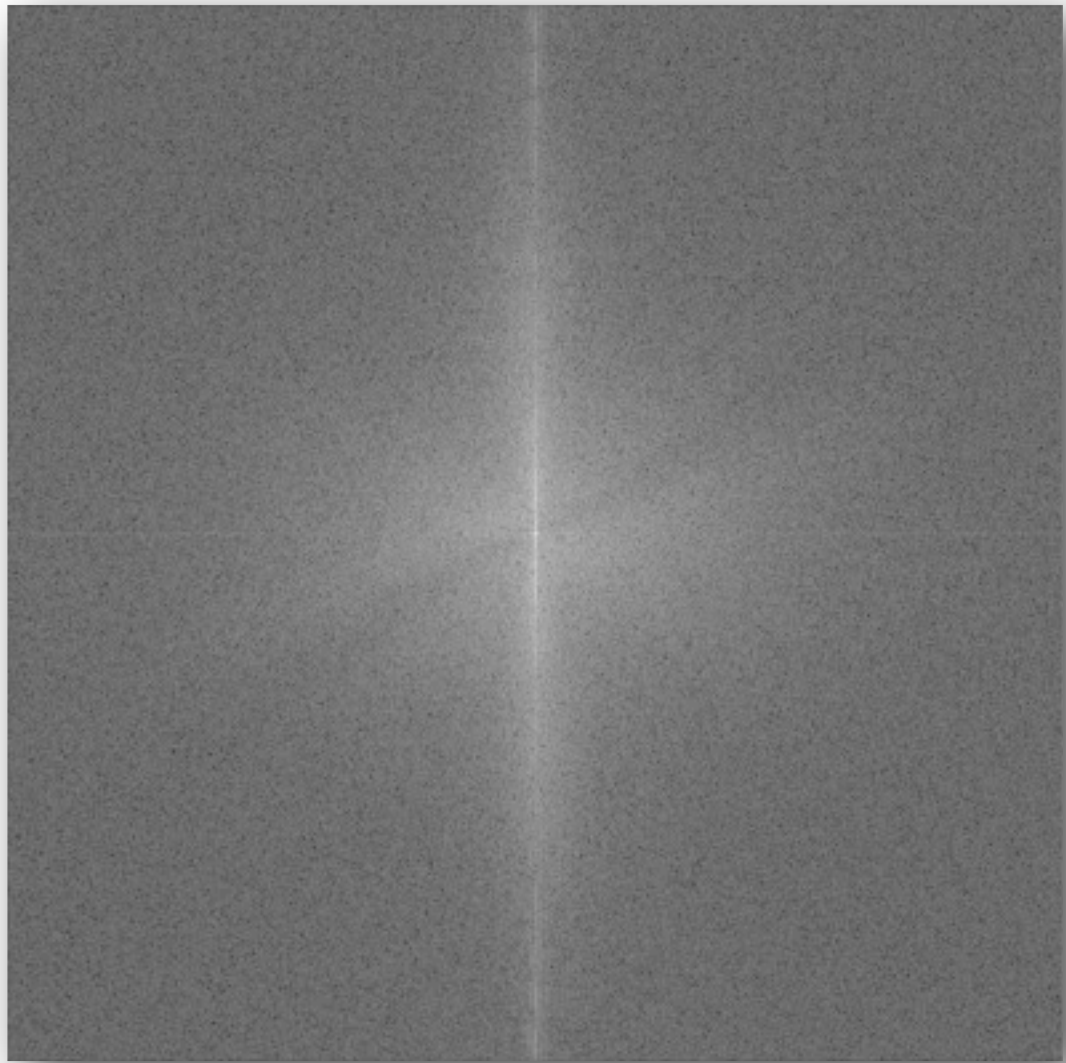
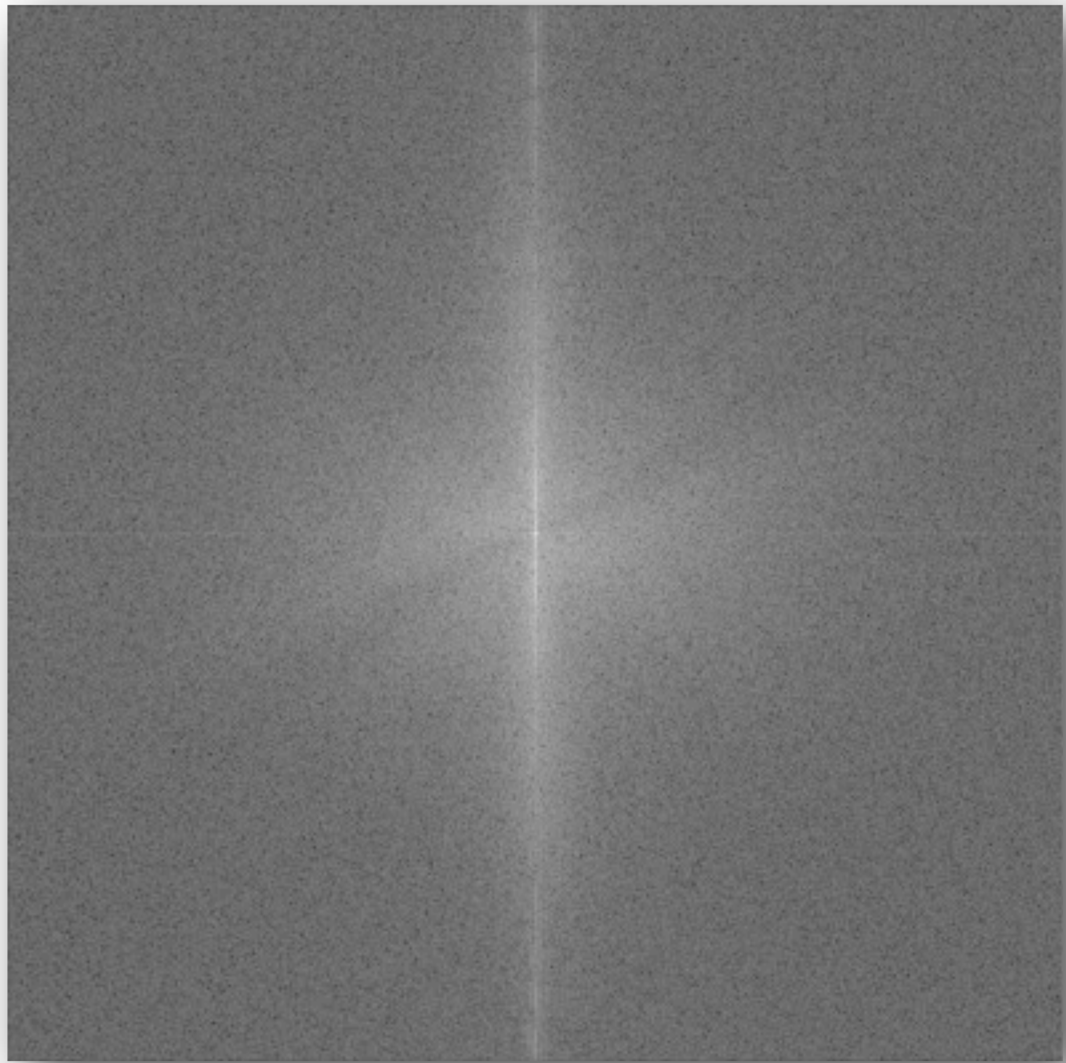
$$\phi = \tan^{-1}\frac{I(\omega)}{R(\omega)}$$



Using the Frequency Spectra

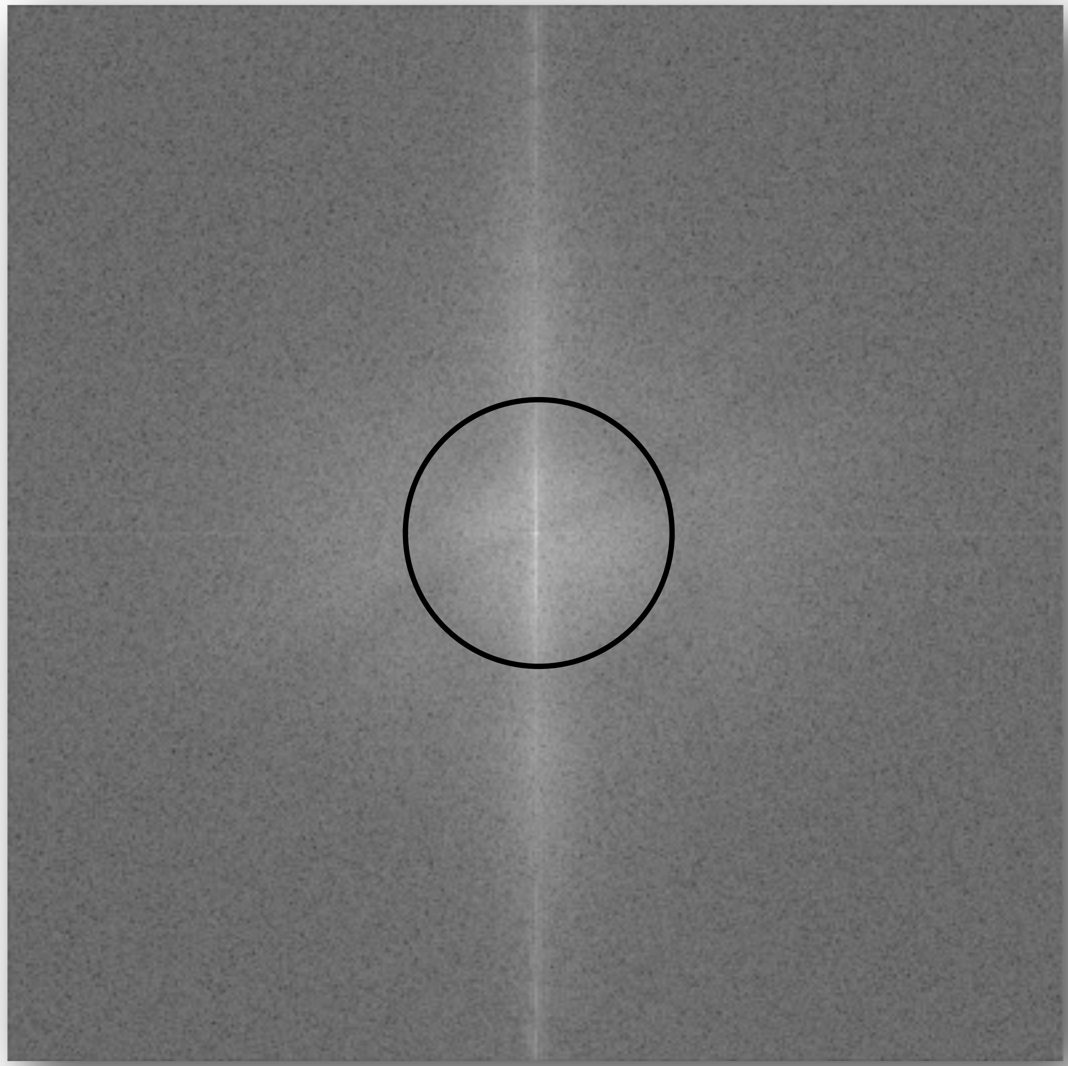


Using the Frequency Spectra



- Low-pass,

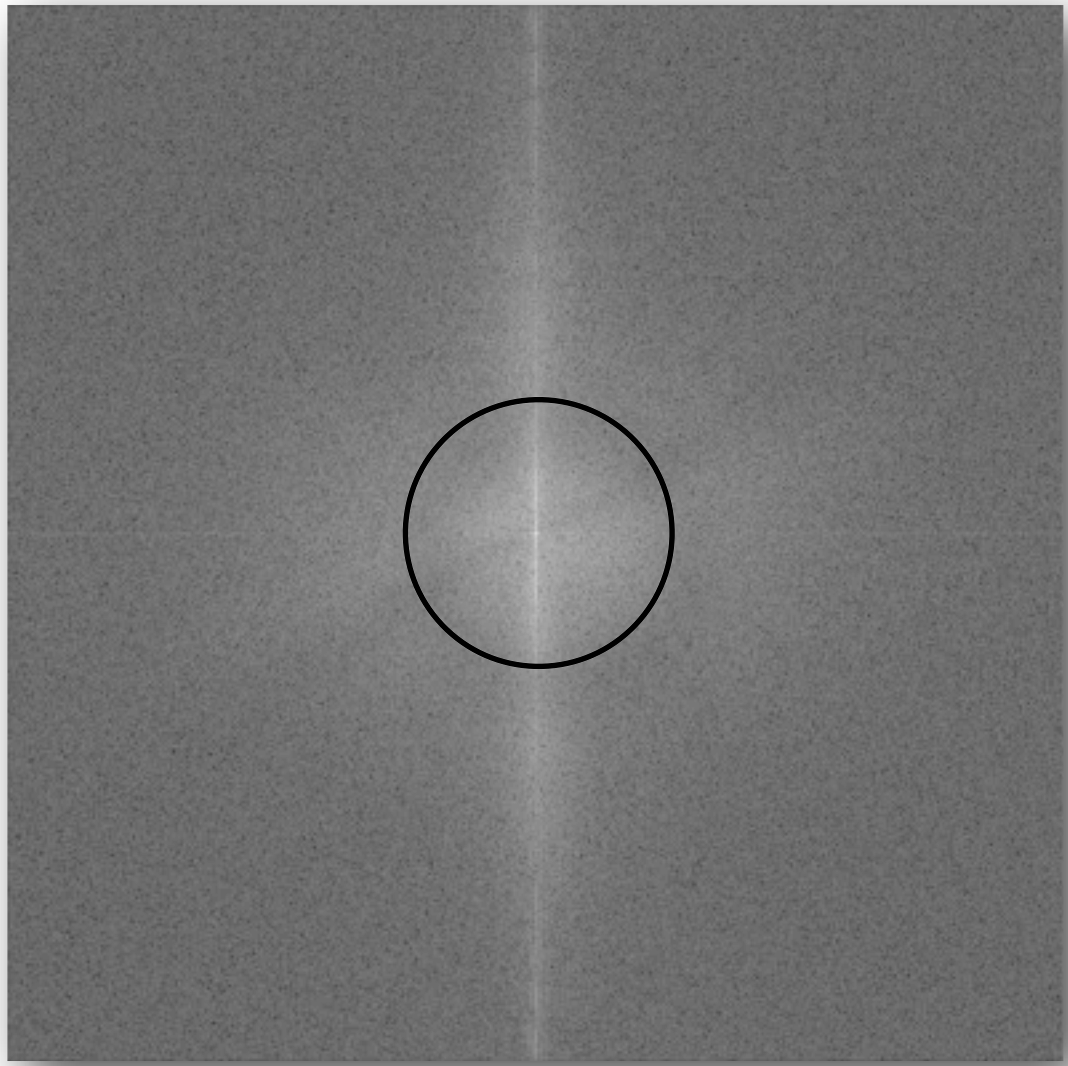
Using the Frequency Spectra



★ Low-pass,

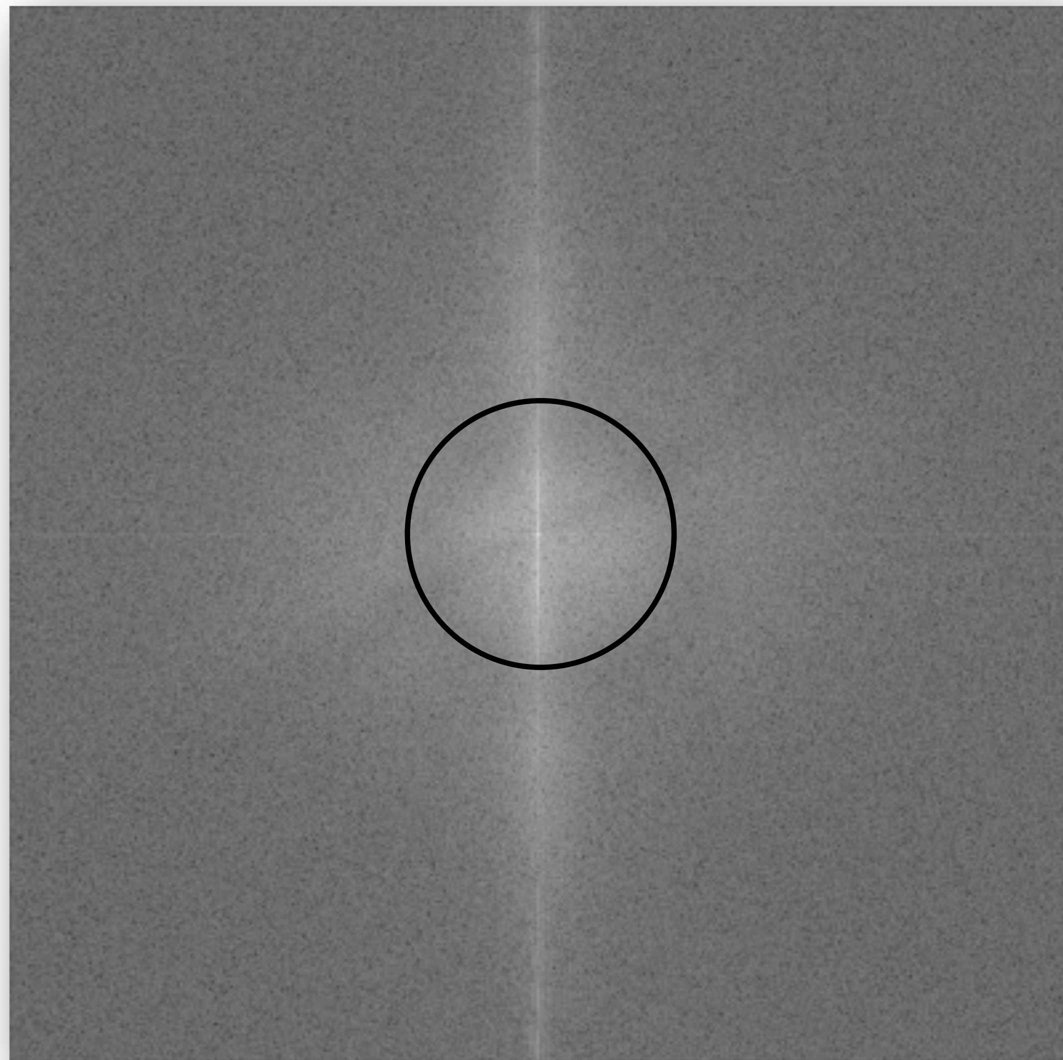
Using the Frequency Spectra

- Low-pass,
- High-Pass,



Using the Frequency Spectra

- ★ Low-pass,
- ★ High-Pass,
- ★ Band-pass Filtering



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Using the Frequency Spectra

- ★ Low-pass,
- ★ High-Pass,
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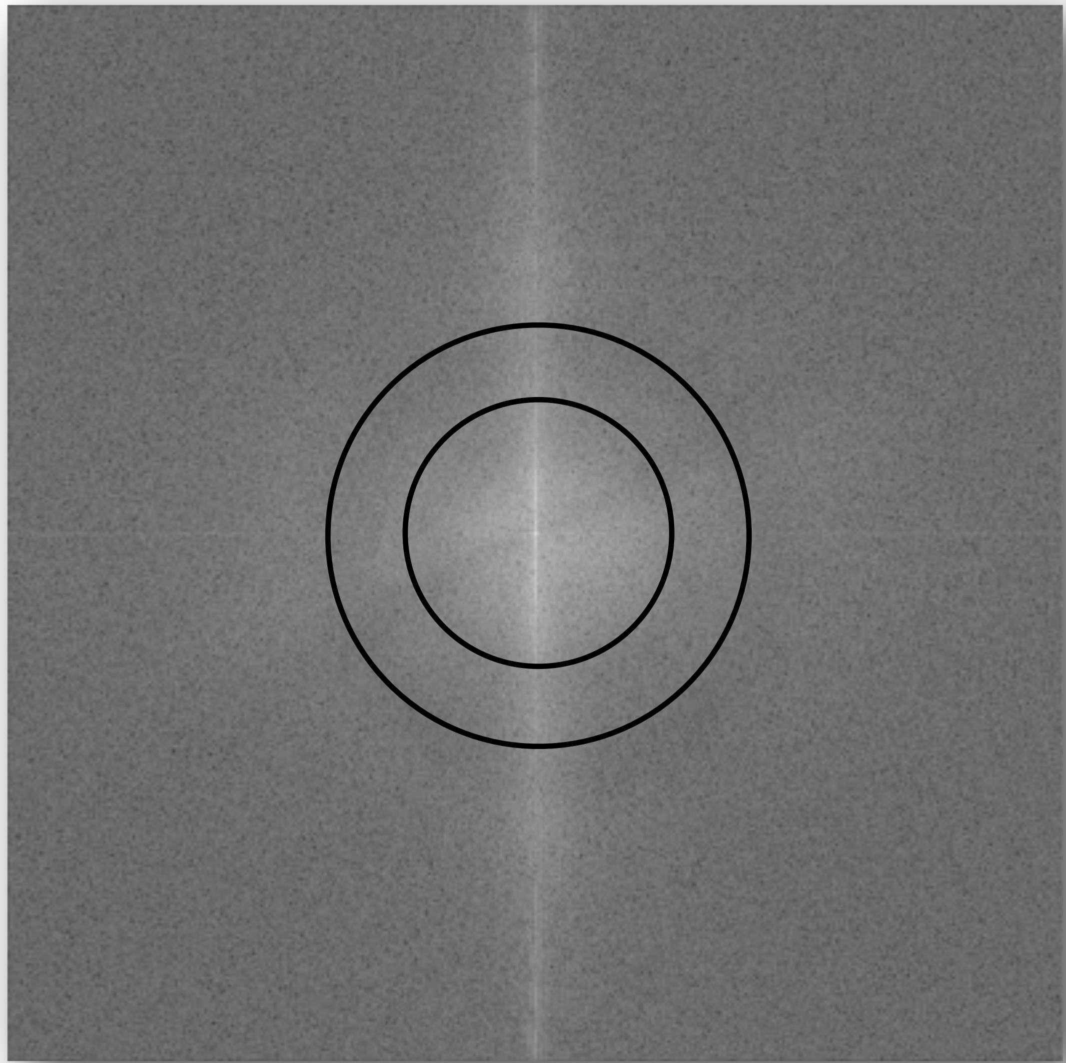
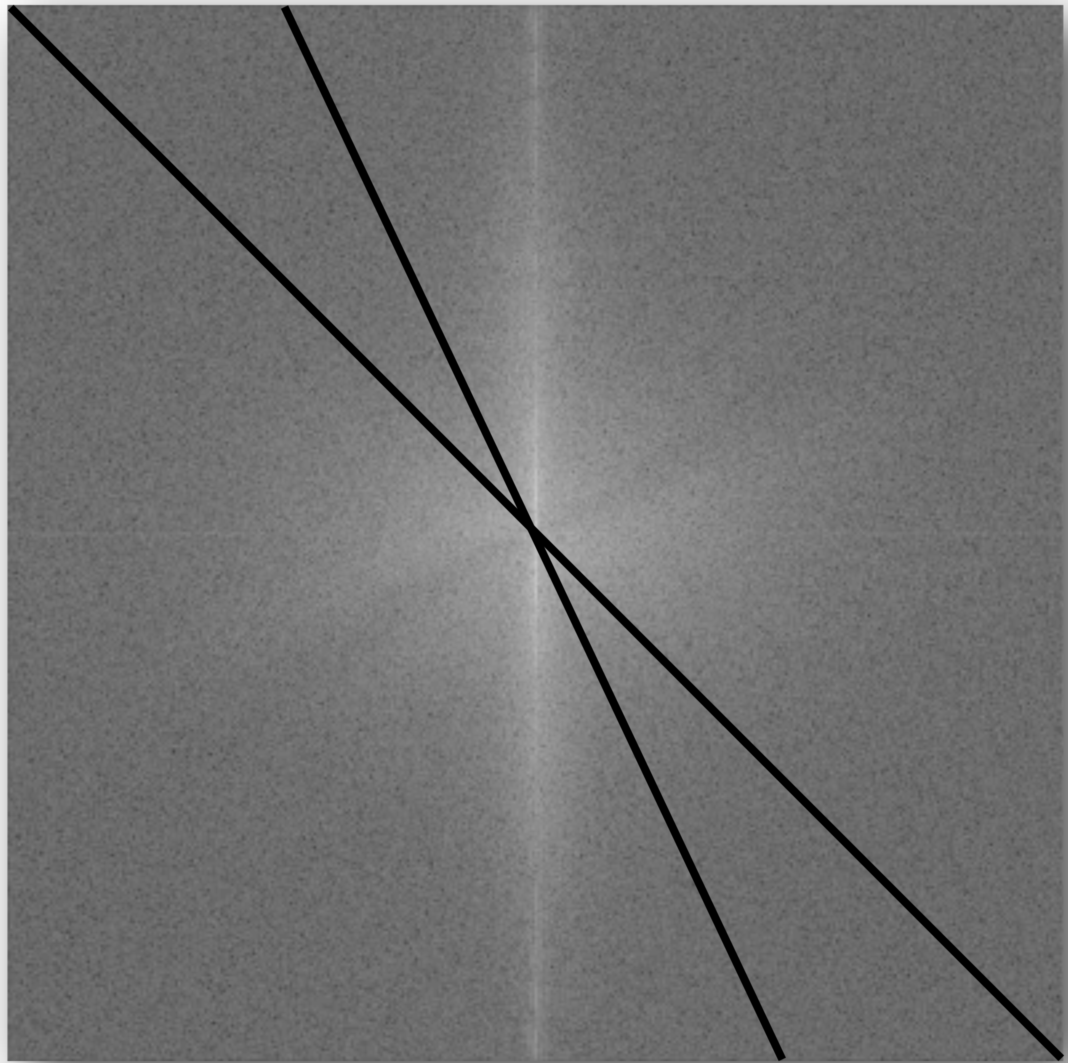


Using the Frequency Spectra

- Low-pass,
- High-Pass,
- Band-pass Filtering
- Change the spectrum and reconstruct



Blurring and Frequencies


Original Image


Gaussian 5x5 Blur


Smooth - Original

Summary

- ★ Introduced how sines and cosines can be used to reconstruct a signal.
- ★ Characterized the Fourier Transform using terms like Frequency, Amplitude and Phase.
- ★ Introduced the Frequency Domains for a Signal.
- ★ Identified the three (3) properties of Convolution as it is associated with the Fourier Transform.

Next Class

★ Merging and Blending of Images

Credits

- For more information, see
  - Richard Szeliski (2010) Computer Vision: Algorithms and Applications, Springer.
  - Forsyth & Ponce (2012), Computer Vision: A Modern Approach, Pearson.
- Some concepts in slides motivated by similar slides by A. Efros and J. Hays.
- Some images retrieved from
  - http://commons.wikimedia.org/.
  - List will be available on website.

www.flickr.com/photos/neneonline/231886965/

Computational Photography

Dr. Irfan Essa
Professor
School of Interactive Computing

Study the basics of computation and its impact on the entire workflow of photography, from capturing, manipulating and collaborating on, and sharing photographs.