Computational Photography

Dr. Irfan Essa
Professor
School of Interactive Computing

Study the basics of computation and its impact on the entire workflow of photography, from capturing, manipulating and collaborating on, and sharing photographs.

Image Processing and Filtering, via Convolution and Cross-Correlation

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Towards Edge Detection: Computing Image Gradients

Lesson Objectives

- ★ Describe in your own words two (2) reasons for the need to detect Features in an image.
- ★ Define what is an Edge of an image from the perspective of Information Theory.
- ★ Describe in your own words the concept of an Image Gradient as needed to compute Edges.
- ★ Explain the concept of an Image Gradient with respect to how it is computed:
    - in continuous form for a function; and
    - in a discrete form for an image.

Review: Convolution and Cross-Correlation Methods

- Can be applied to process images, *e.g.*, smoothing images with kernels, removing noise, *etc.*
- When *h* is symmetric, no difference, use either one?

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Cross-Correlation: $G = h \otimes F$ $$G[i,j] = \sum_{u=-k}^{k} \sum_{v=-k}^{k} h[u,v]F[i+u, j+v]$$

Convolution: $G = h * F$ $$G[i,j] = \sum_{u=-k}^{k} \sum_{v=-k}^{k} h[u,v]F[i-u, j-v]$$

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Using Filters to Find Features in Image

Slide adapted from Aaron Bobick

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- Filtering as a way to remove or reduce noise

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- Consider how filters will allow us to abstract higher-level "features".

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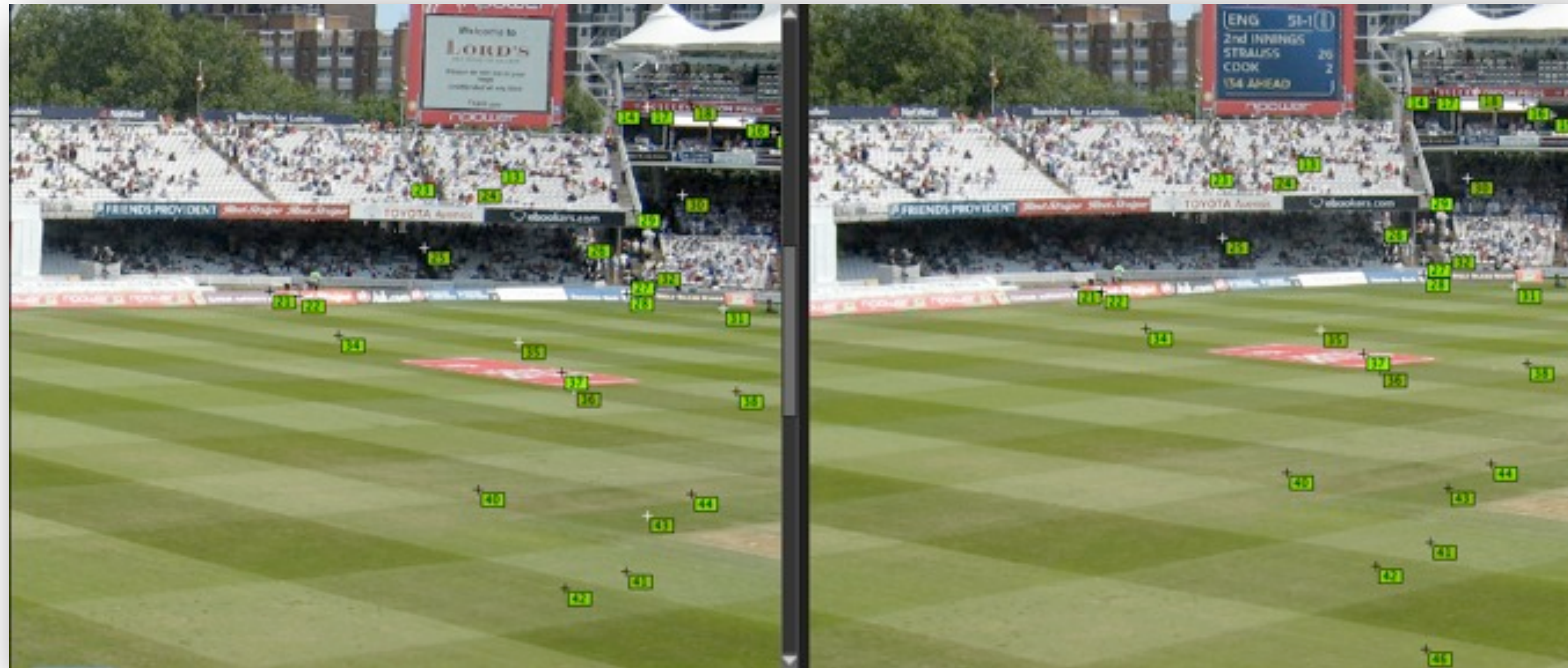

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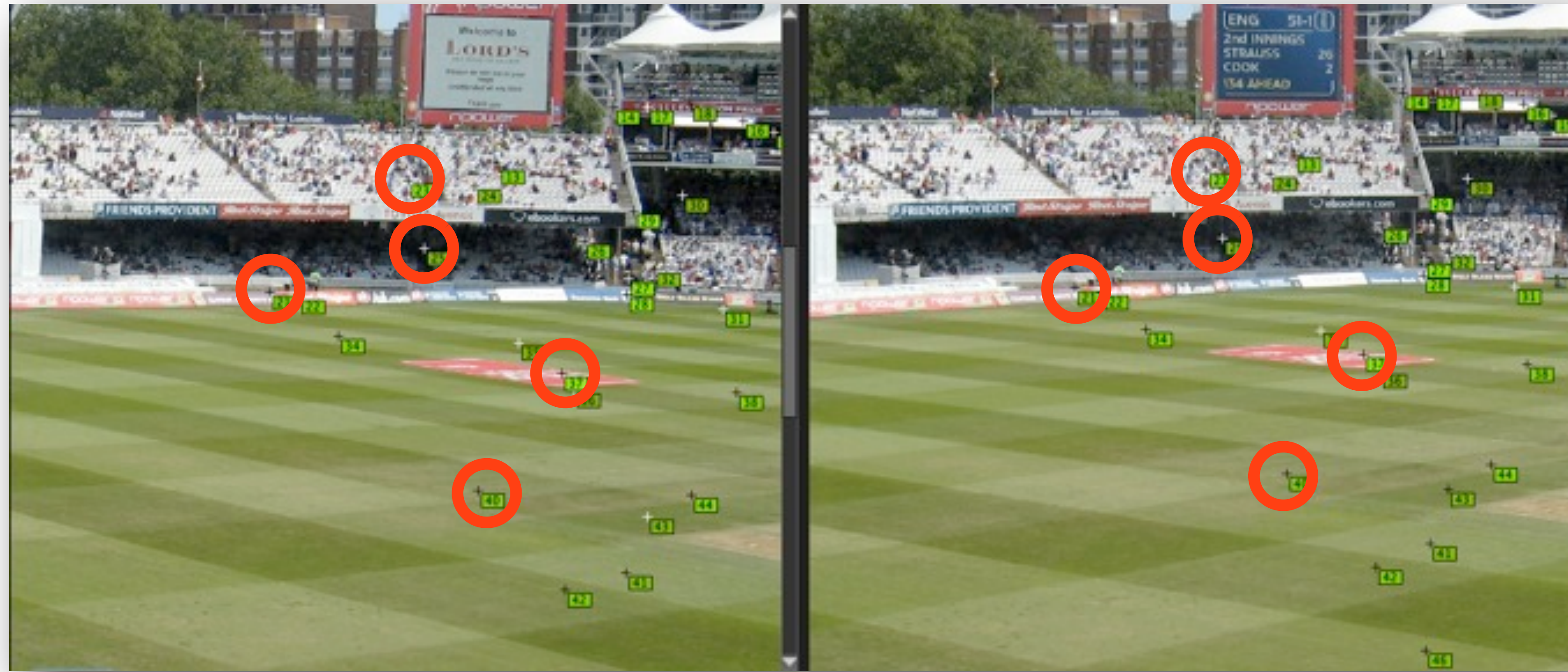

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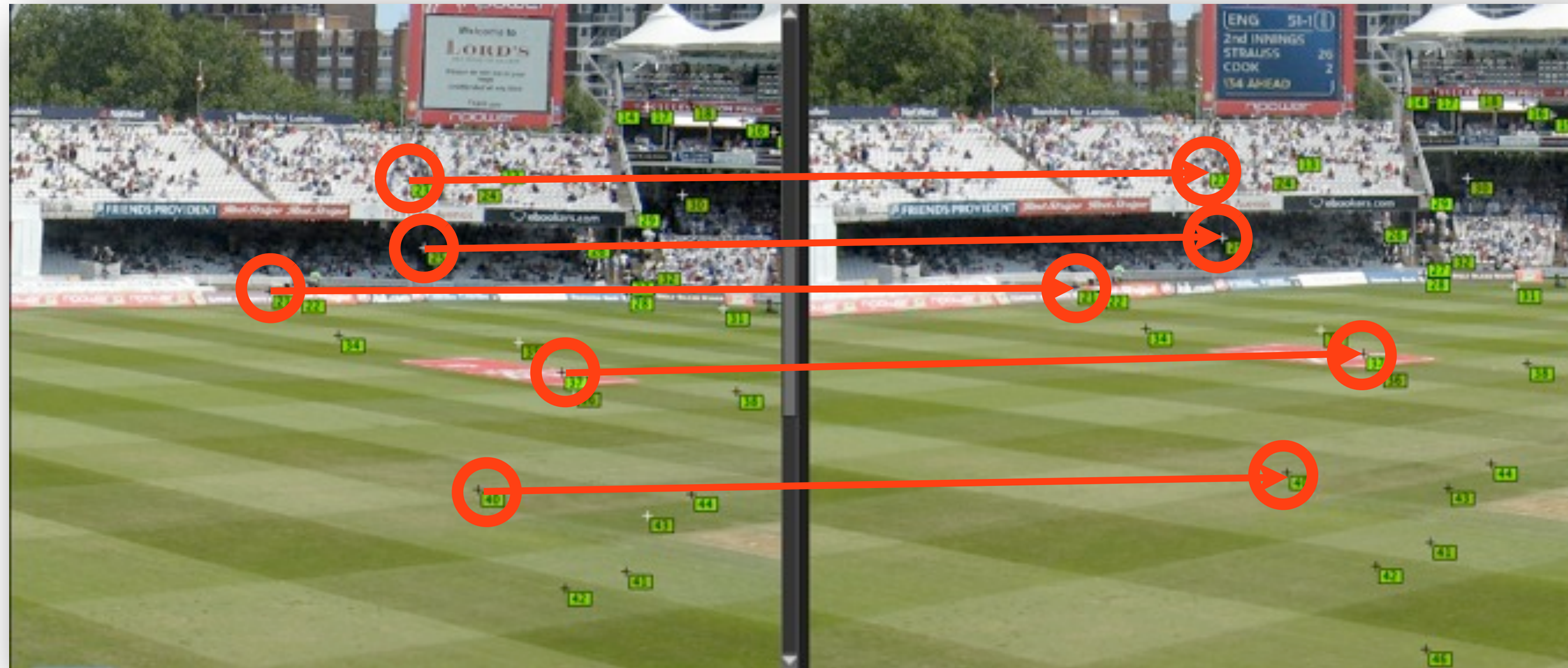

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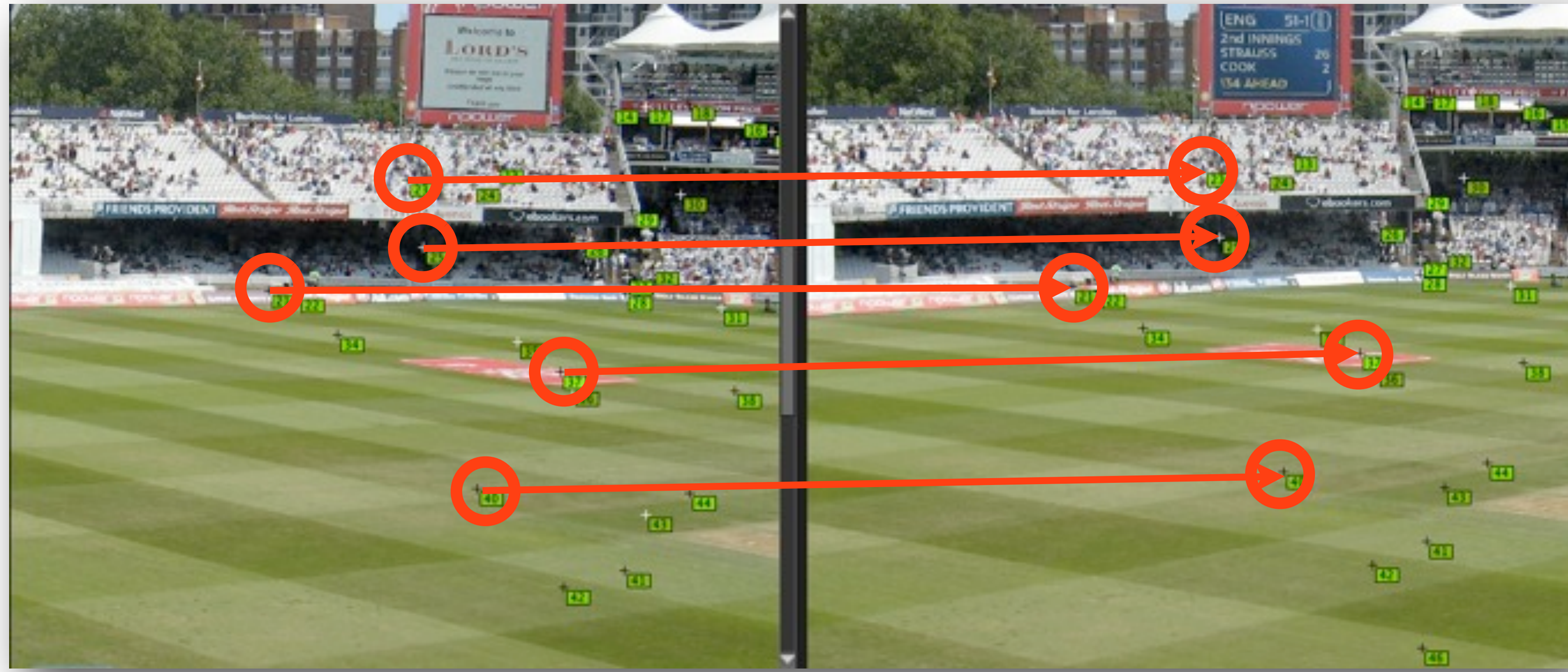
    - Map raw pixels to an intermediate representation that will be used for subsequent processing



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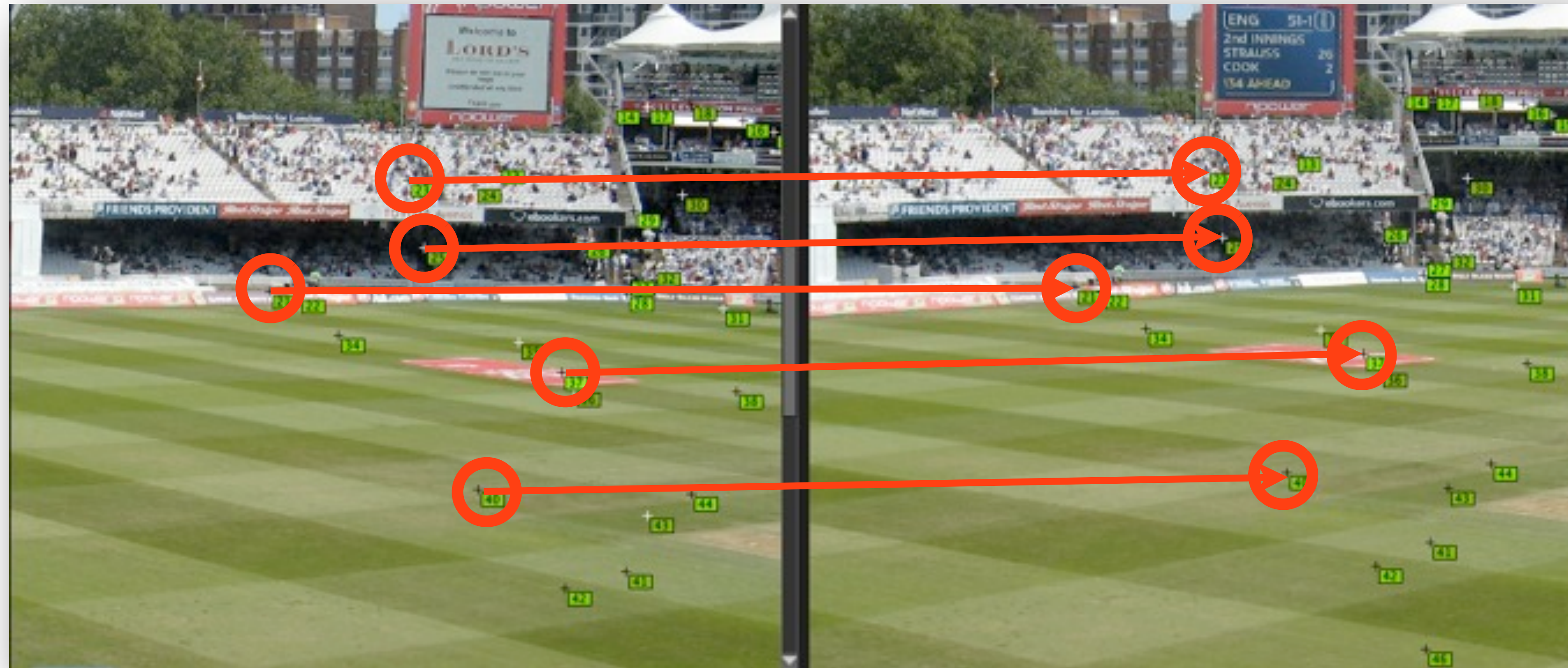

- ★ Filtering as a way to remove or reduce noise
- ★ Consider how filters will allow us to abstract higher-level "features".
  - Map raw pixels to an intermediate representation that will be used for subsequent processing
  - Reduce amount of data, discard redundancy, preserve what's useful

Slide adapted from Aaron Bobick

What are Good Features to Match between Images

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- ★ **Good Features**

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- Parts or properties of the image that encode it in a compact form

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- ★ **Good Features**
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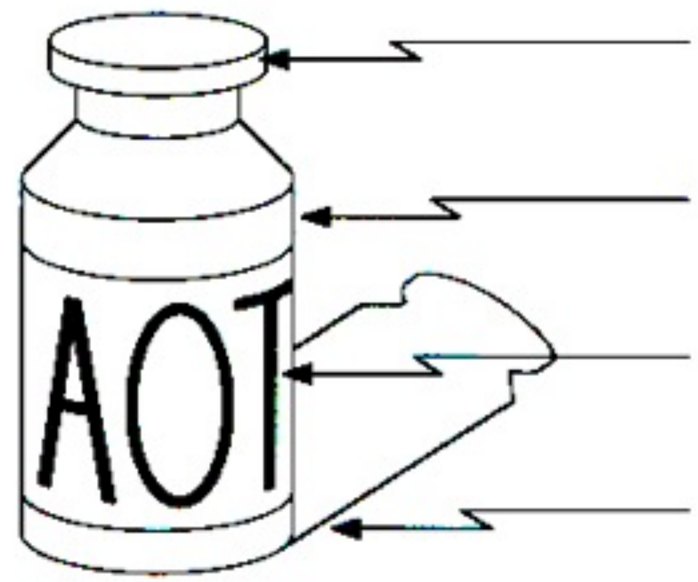
- ★ **Edges**



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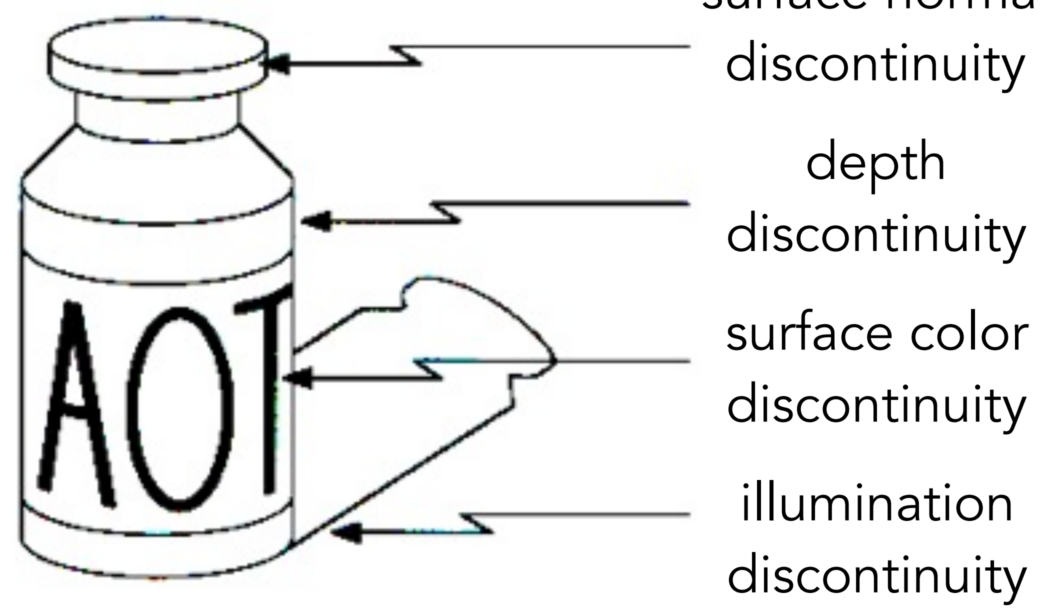
- Edges are caused by a variety of factors



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- **Good Features**
  - Parts or properties of the image that encode it in a compact form
- **Edges**
  - Edges are caused by a variety of factors
  - Information theory view: Edges encode change, therefore edges efficiently encode an image

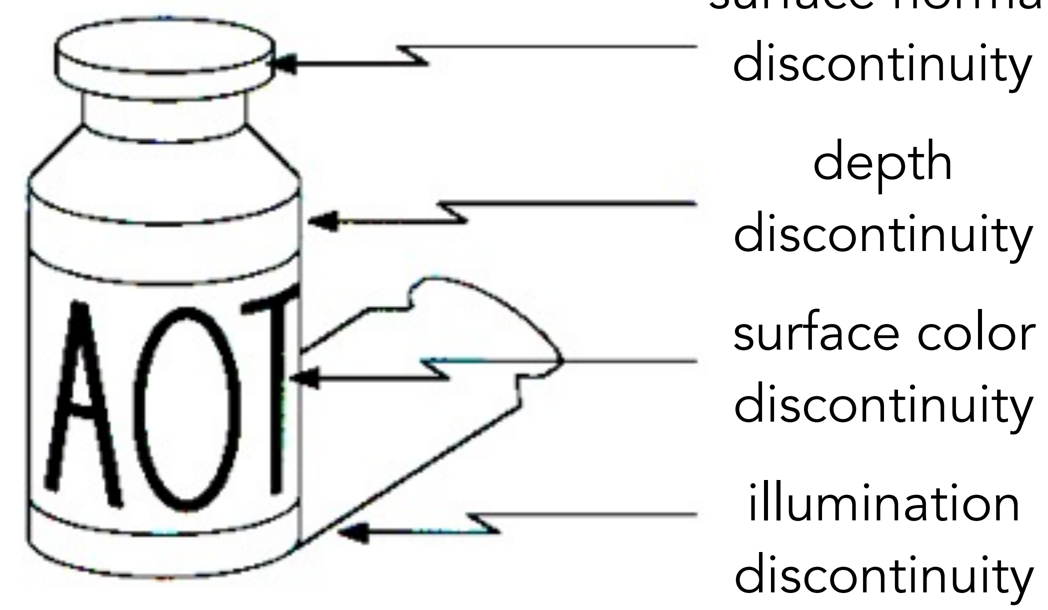

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Image Courtesy Professor Henrik Christensen

Cause of Edges in Real Images

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Images as Functions

- Edges appear as cliffs in the 3d height-filed view of an image
- Image: *F(x,y)*

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Edge Detection: Look for Changes

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9	15	12	84	84	88
12	14	10	82	88	89
11	17	16	12	88	90
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Edge Detection: Look for Changes

★ Basic Idea:

- Look for a neighborhood with strong signs of change

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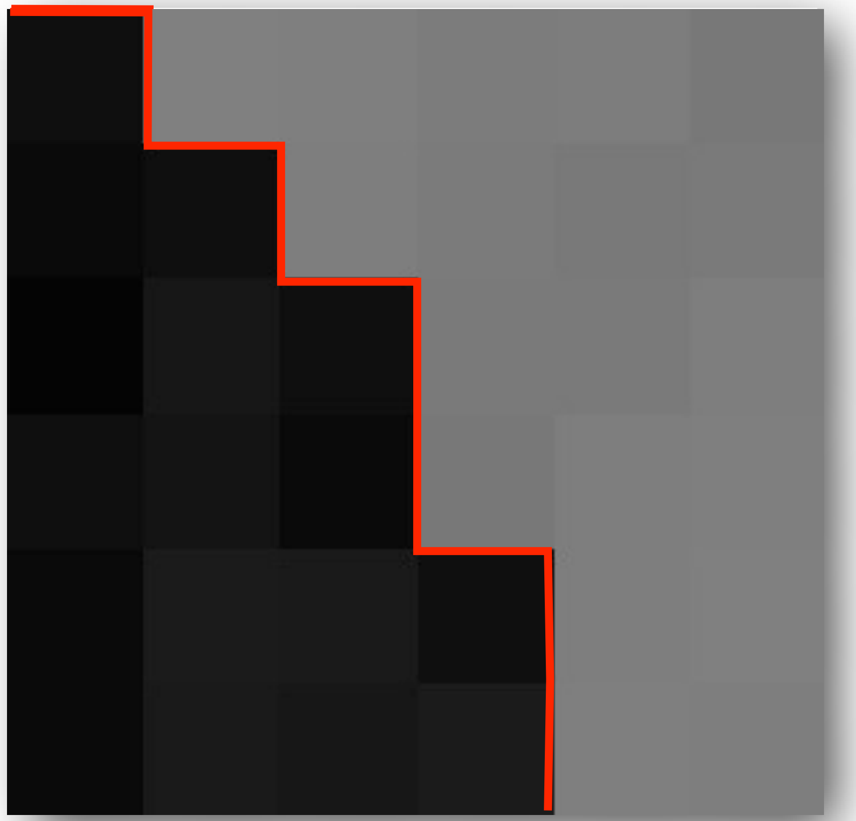

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- ★ Basic Idea:
  - Look for a neighborhood with strong signs of change
- ★ Issues to consider
  - Size of the neighborhood?
  - What metrics to consider to detect a “change”?

Derivatives of *F(x,y)* to get Edges

Test Image

Slide adapted from Aaron Bobick and Svetlana Lazebnik

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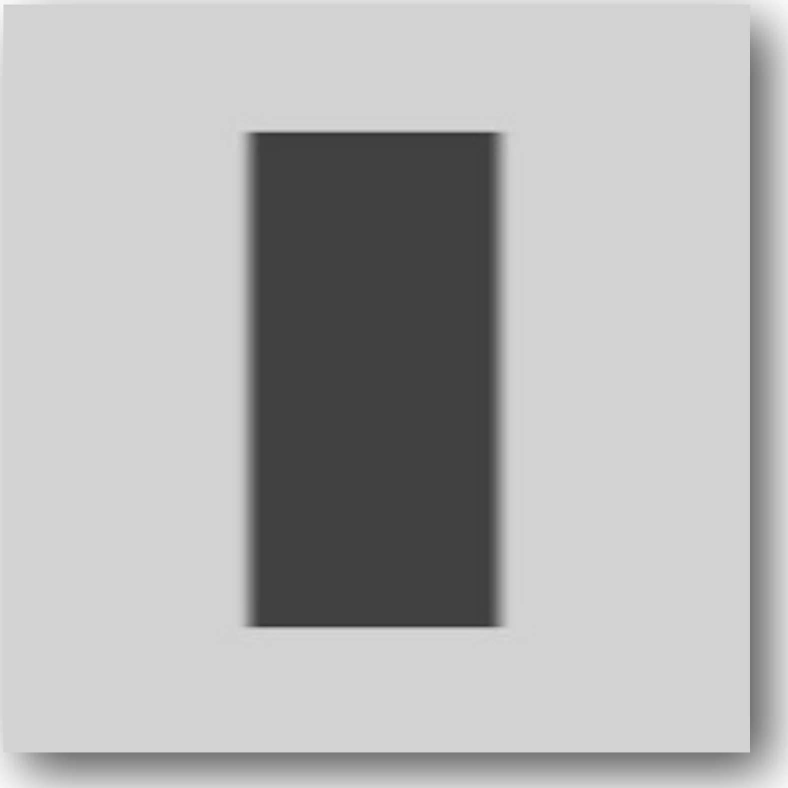

Test Image

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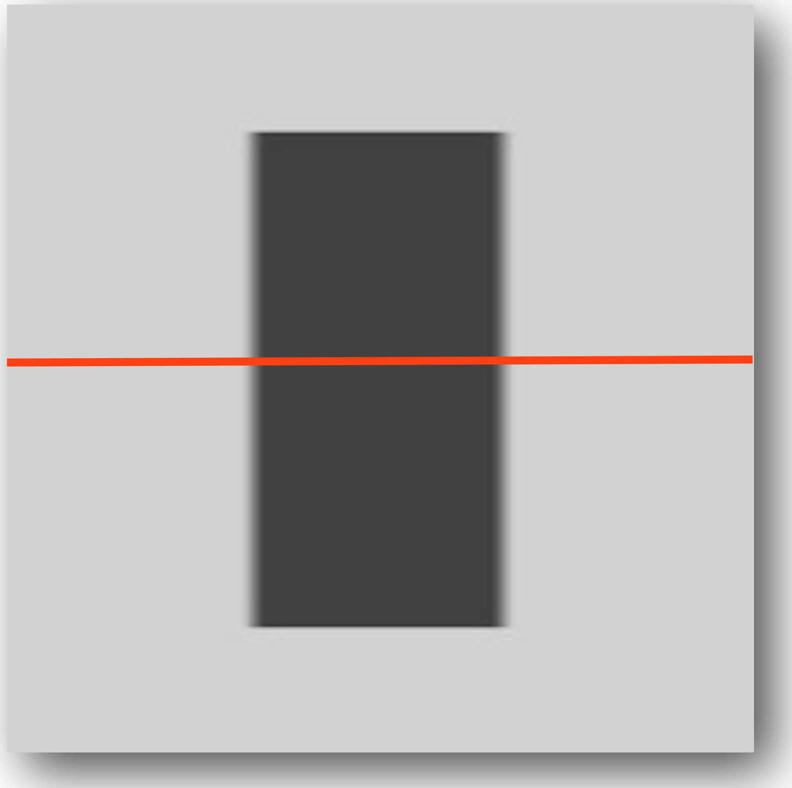

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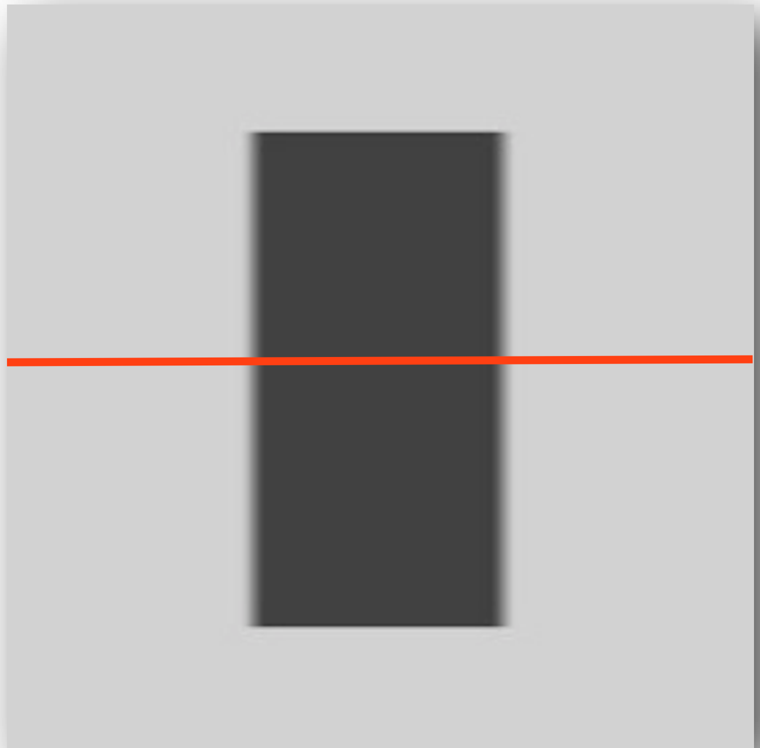

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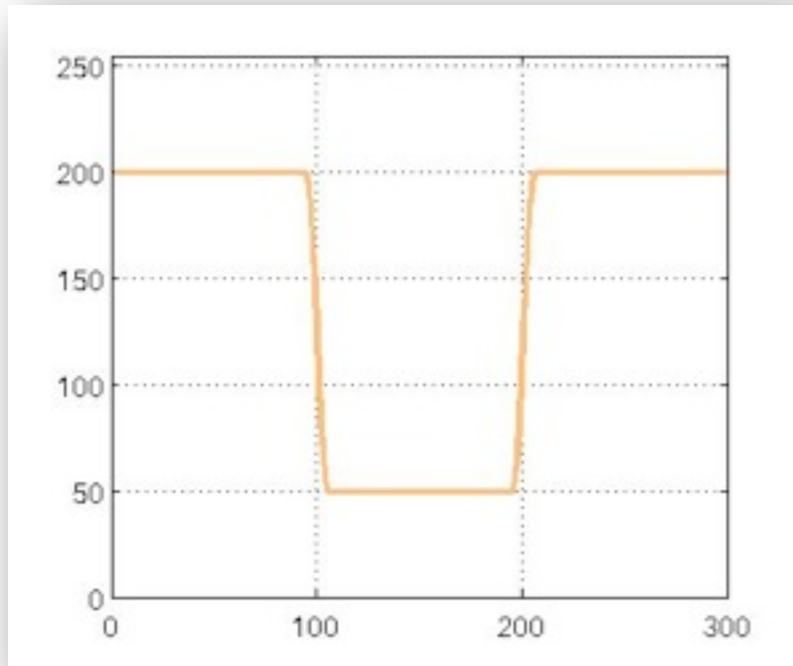

Intensity along
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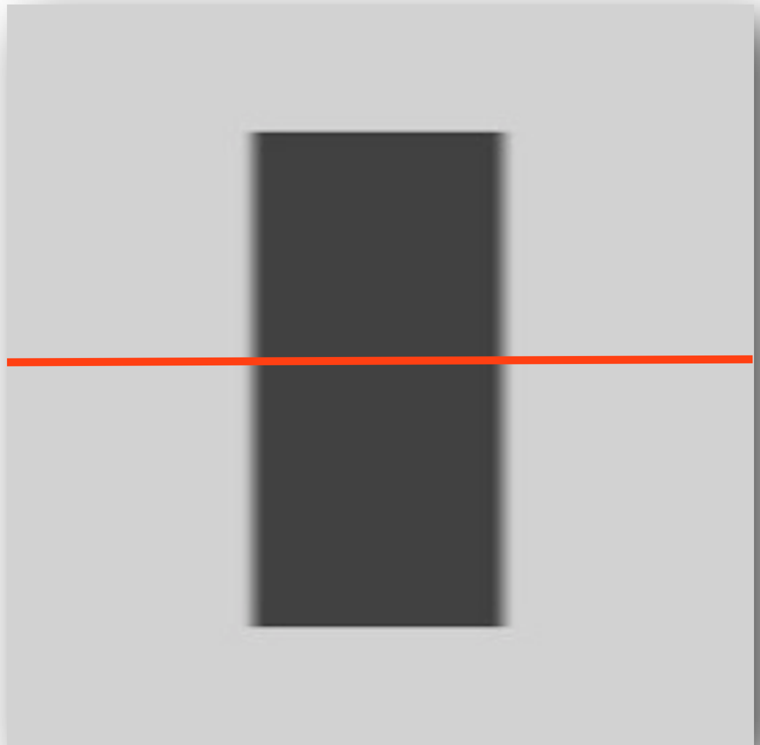

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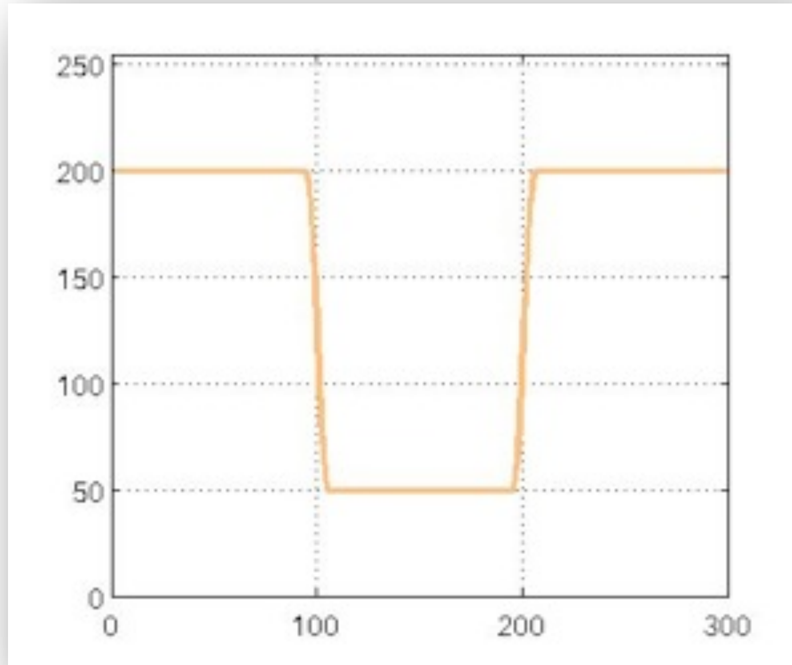

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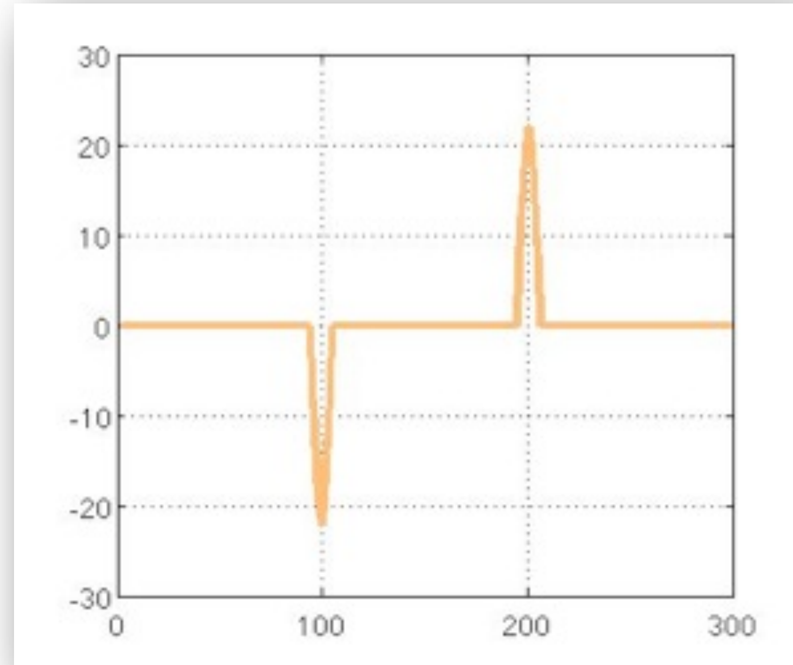

First Derivative

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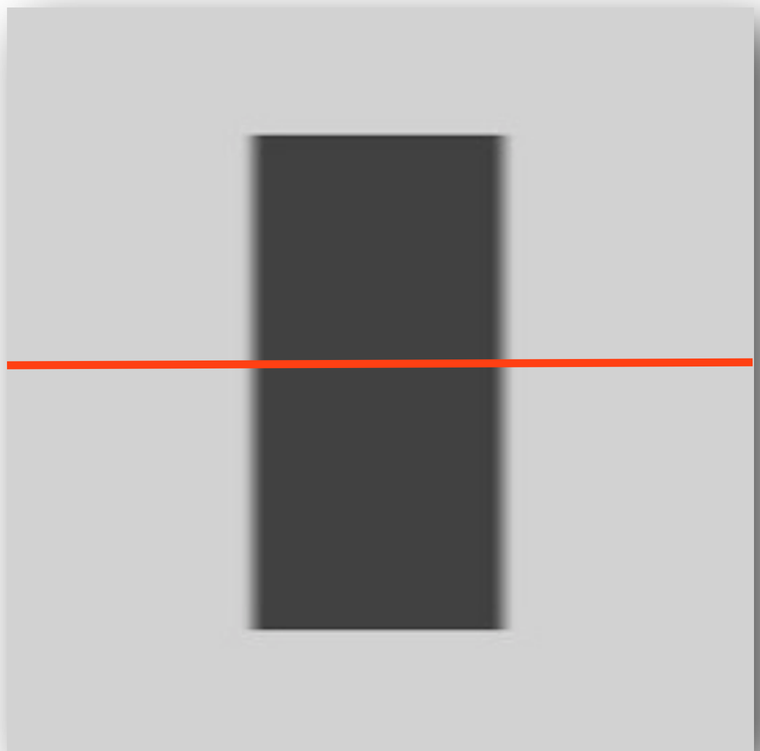

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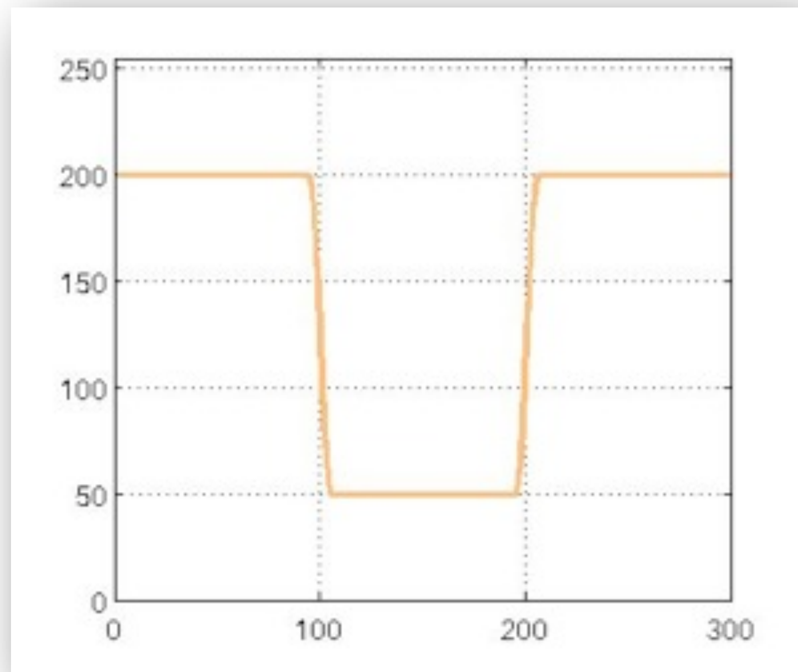

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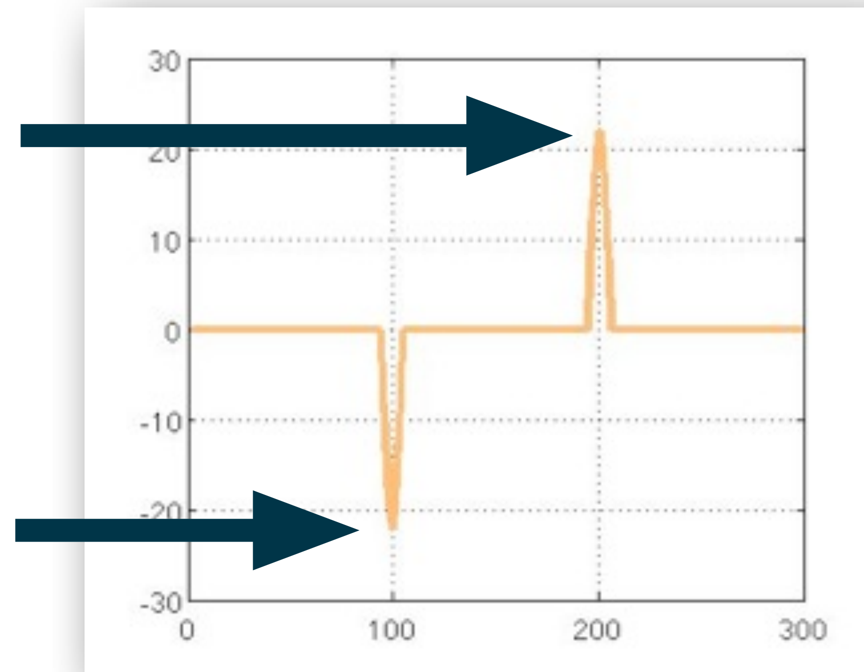

First Derivative

- ★ An edge is where there is rapid change in the image intensity function
- ★ Extrema of the derivative is where the vertical edges are (as we just looked for derivative in *x*)

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Differential Operators for Images

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★ Need an operation that when applied to an image returns its derivatives: Differential Operators

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- ★ Model these "operators" as masks/kernels: When applied to an image yield a new function that is the image *gradient function*.

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- Then "threshold" this *gradient function* to select the edge pixels.

Differential Operators for Images

- ★ Need an operation that when applied to an image returns its derivatives: Differential Operators
- ★ Model these "operators" as masks/kernels: When applied to an image yield a new function that is the image *gradient function*.
- ★ Then "threshold" this *gradient function* to select the edge pixels.
- ★ Need to define a *gradient*!

Gradient of an image = Measure of change in an Image (function, *F*) in *x* (rows) and *y* (columns)

Definition: Image Gradient (Mathematically)

Gradient of an Image is: $\nabla F = \left[\frac{\delta F}{\delta x}, \frac{\delta F}{\delta y} \right]$

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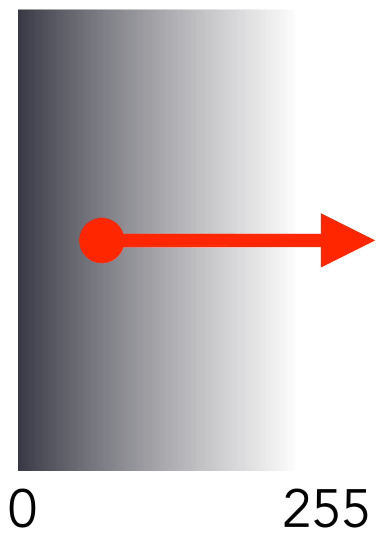

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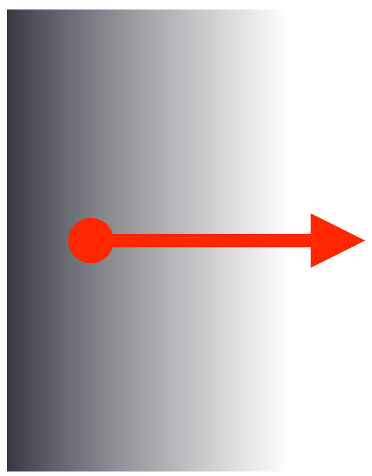

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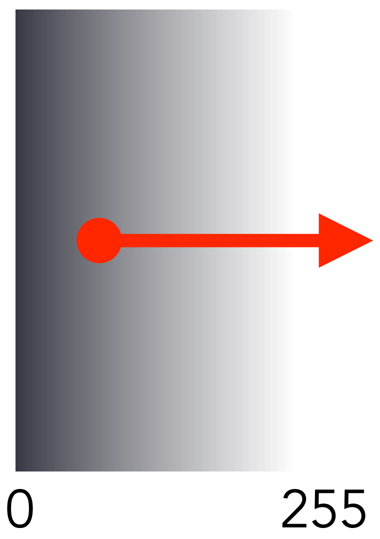

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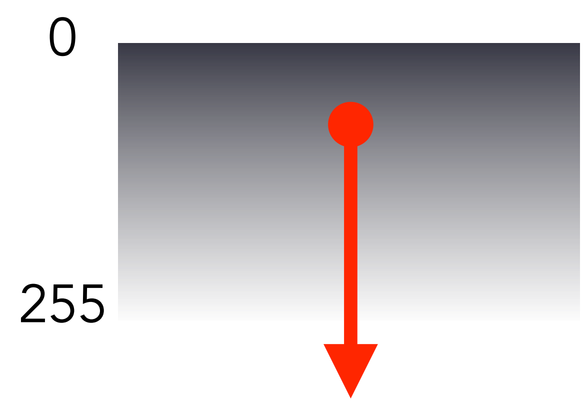

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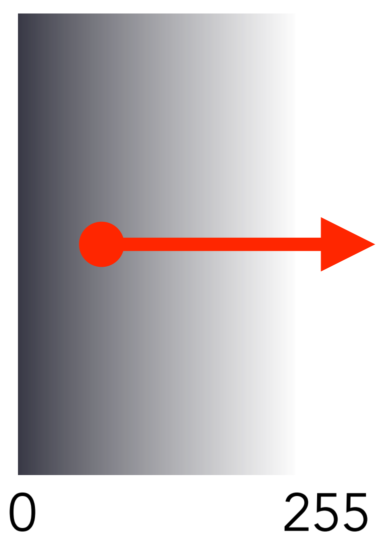

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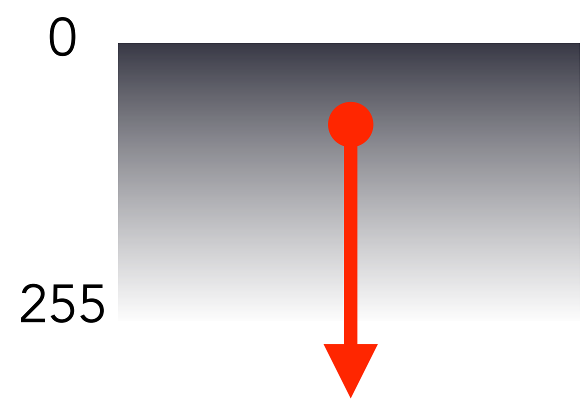

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Georgia Institute of Technology

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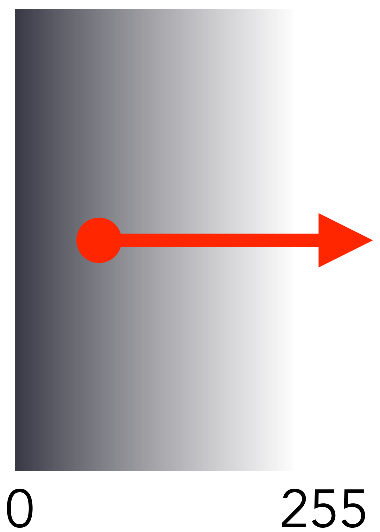

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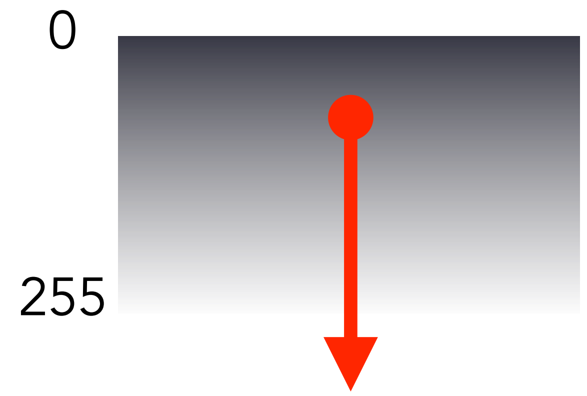

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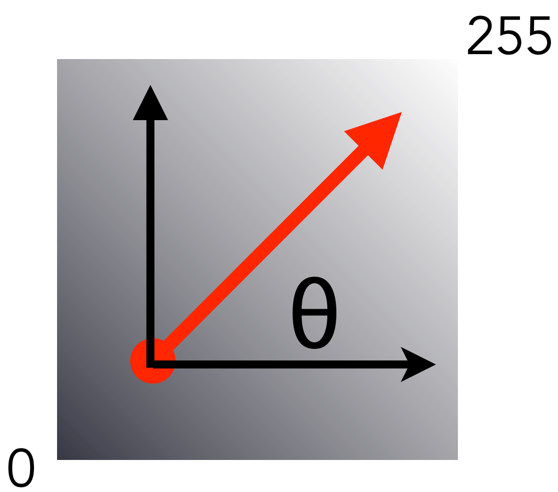

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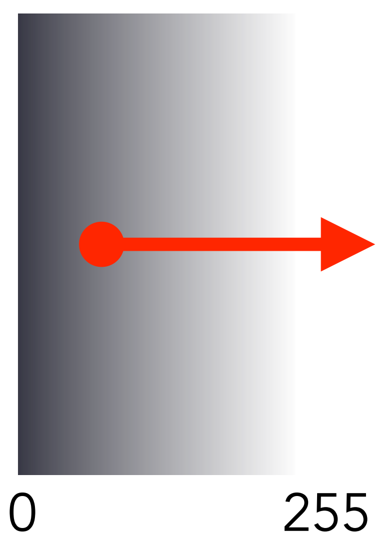

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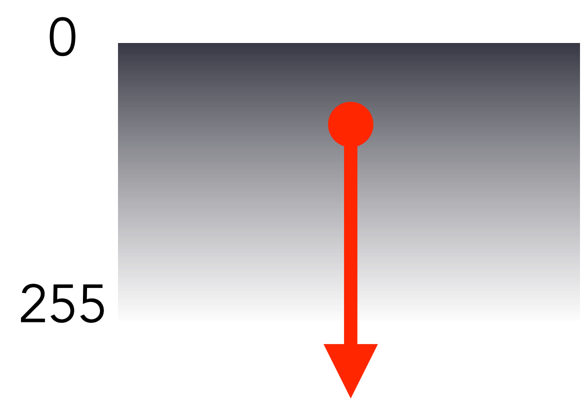

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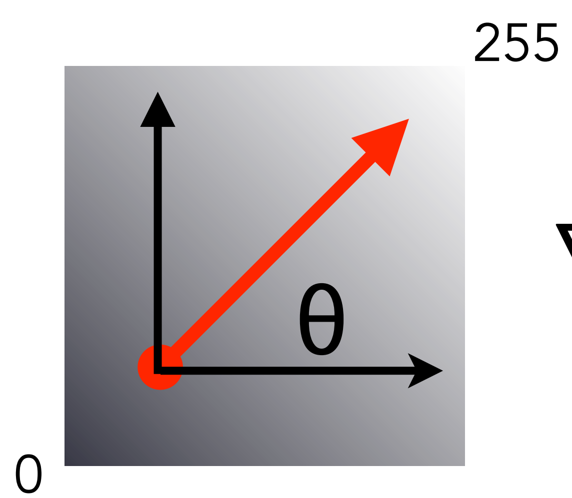

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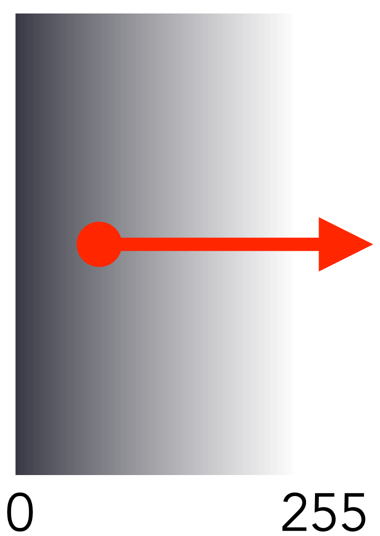

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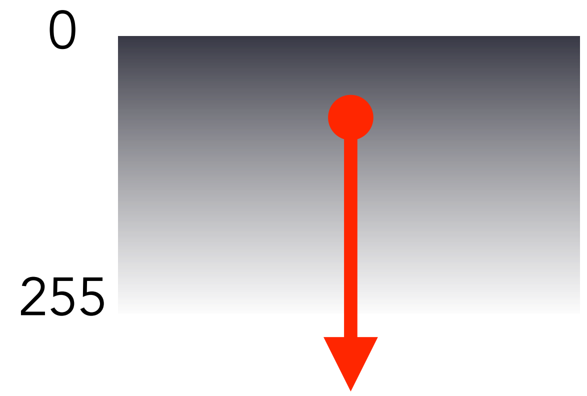

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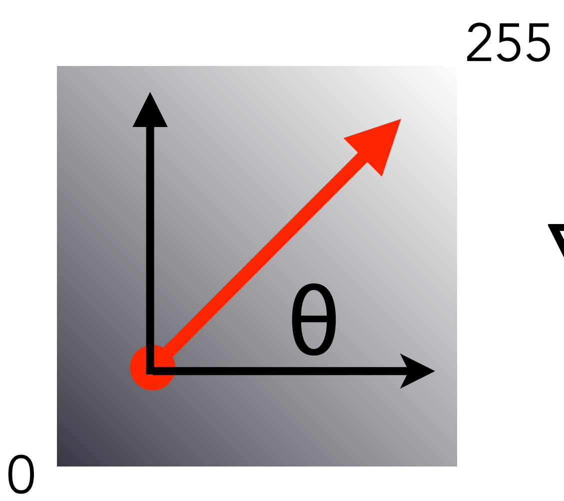

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The gradient points in the direction of most rapid increase in intensity

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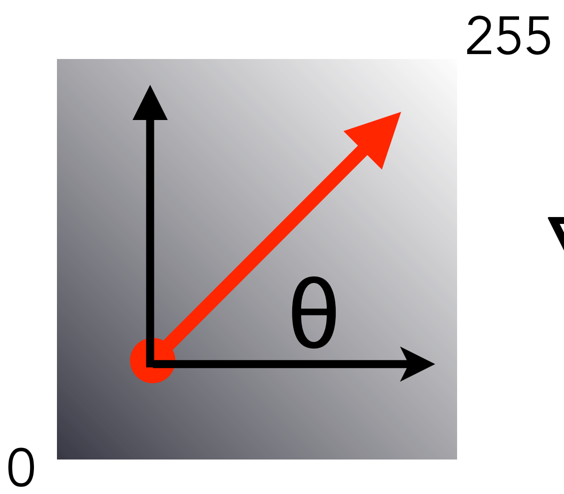

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Slide adapted from Aaron Bobick

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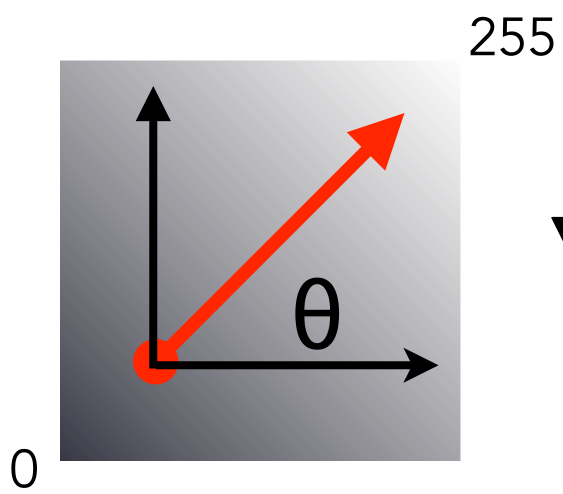

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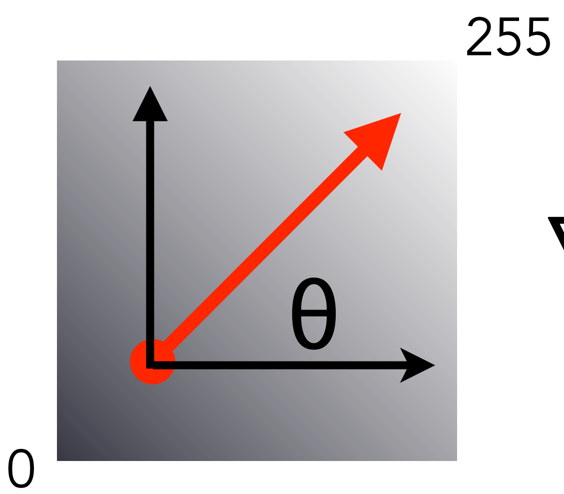

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Gradient of an image = Measure of change in an Image (function, *F*) in *x* (rows) and *y* (columns)

Gradient Direction is: $\theta = \tan^{-1}\left[\frac{\delta F}{\delta y} / \frac{\delta F}{\delta x} \right]$

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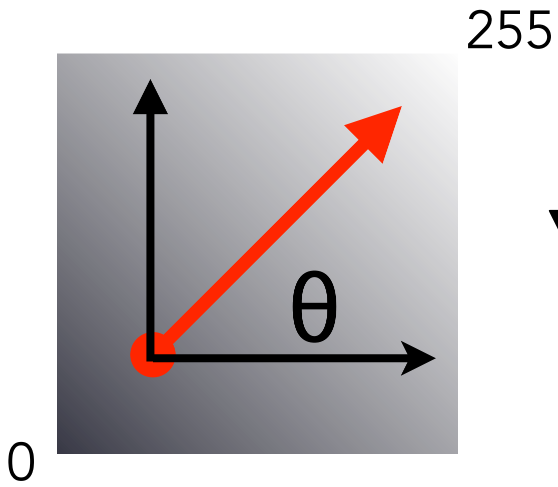

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How does this direction relate to edge direction?

Slide adapted from Aaron Bobick

Definition: Image Gradient (Mathematically)

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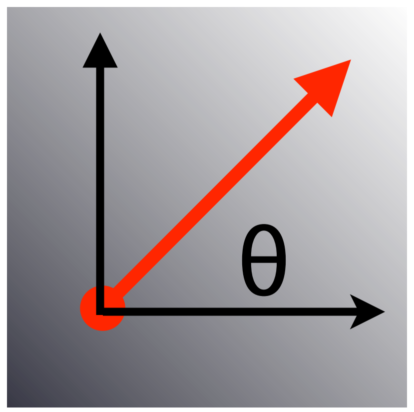

0

$$\nabla F = \left[\frac{\delta F}{\delta x}, \frac{\delta F}{\delta y} \right]$$

The gradient points in the direction of most rapid increase in intensity

Gradient of an image = Measure of change in an Image (function, *F*) in *x* (rows) and *y* (columns)

Gradient Direction is: $\theta = \tan^{-1} \left[\frac{\delta F}{\delta y} / \frac{\delta F}{\delta x} \right]$

How does this direction relate to edge direction?

Gradient Magnitude provides edge strength:

$$\| \nabla F \| = \sqrt{ \left(\frac{\delta F}{\delta x} \right)^2 + \left(\frac{\delta F}{\delta y} \right)^2 }$$

Slide adapted from Aaron Bobick

Definition: Image Gradient (Discrete)

For 2D function, *F(x,y)*, the partial derivative is:

$$\frac{\delta F(x,y)}{\delta x} = \lim_{\epsilon \to 0} \frac{F(x+\epsilon, y) - F(x,y)}{\epsilon}$$

For discrete data, we can approximate using finite differences:

Slide adapted from Aaron Bobick

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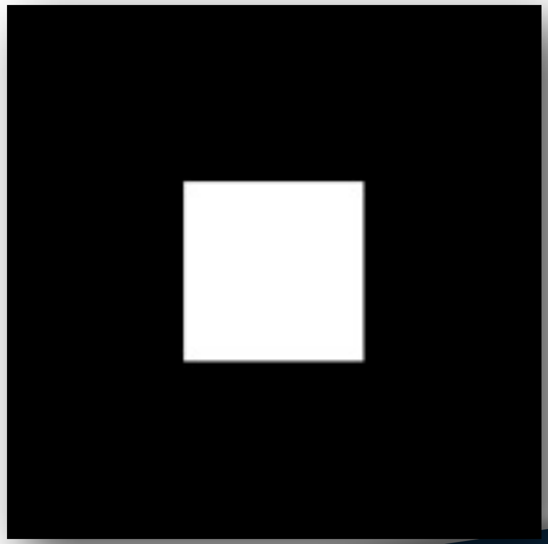
Slide adapted from Aaron Bobick

Differentiating an Image in X and Y



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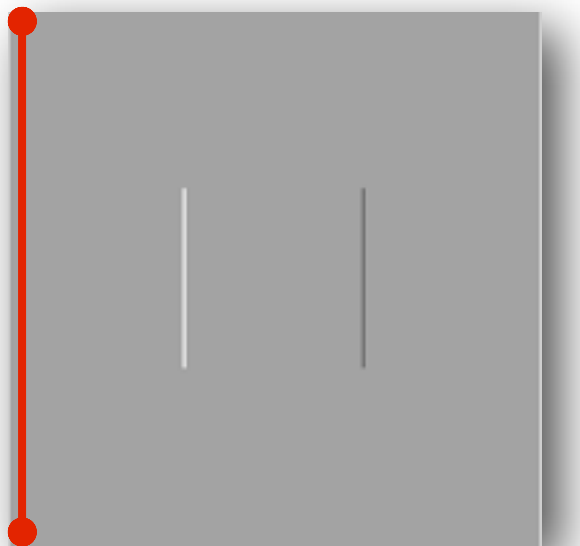

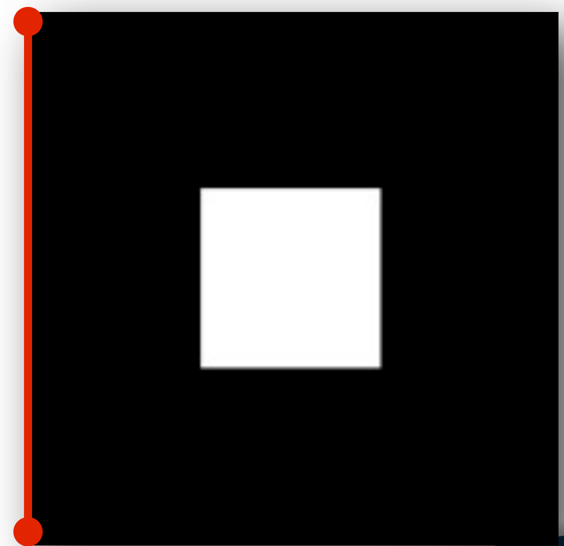

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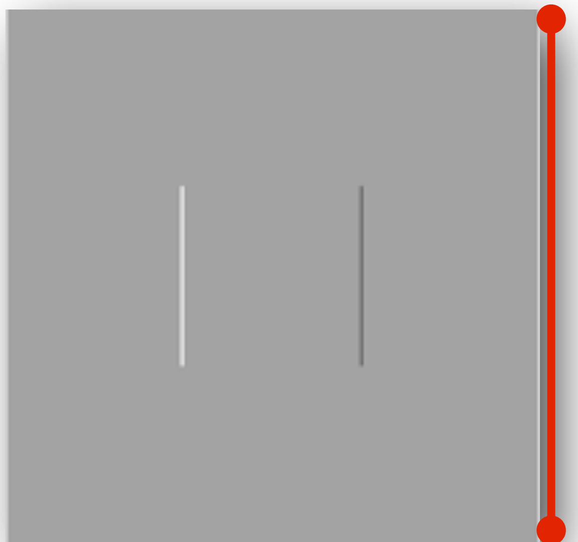

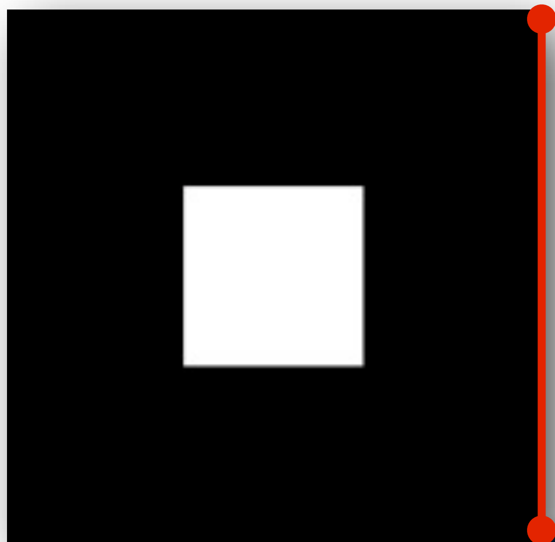

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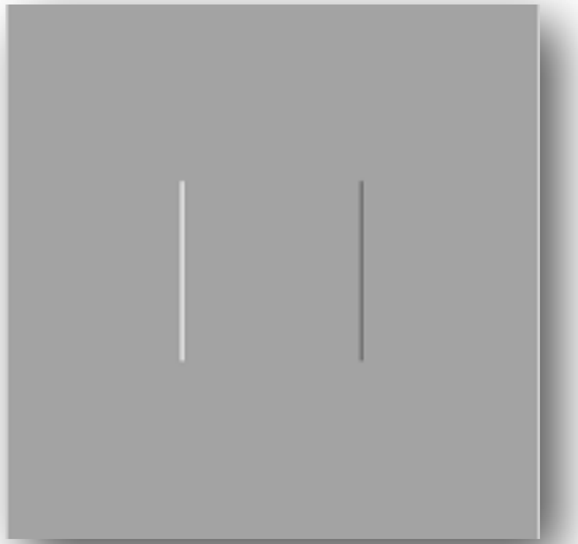

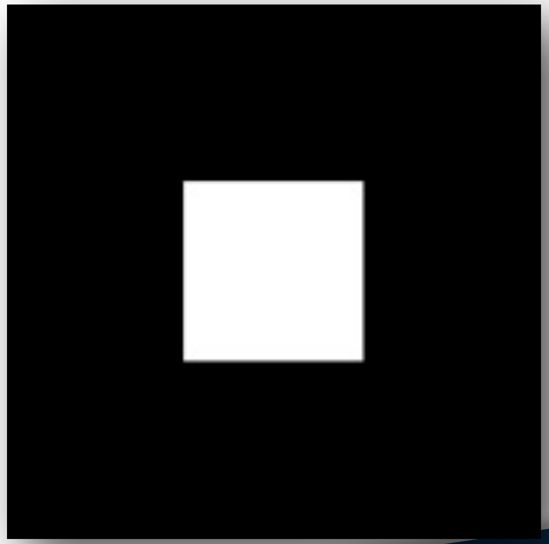

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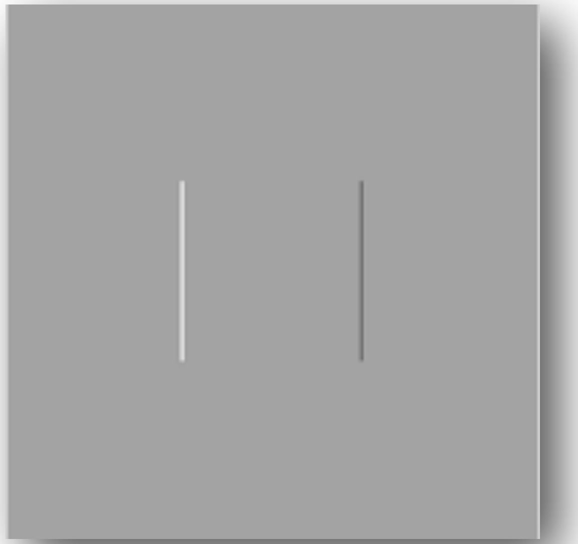

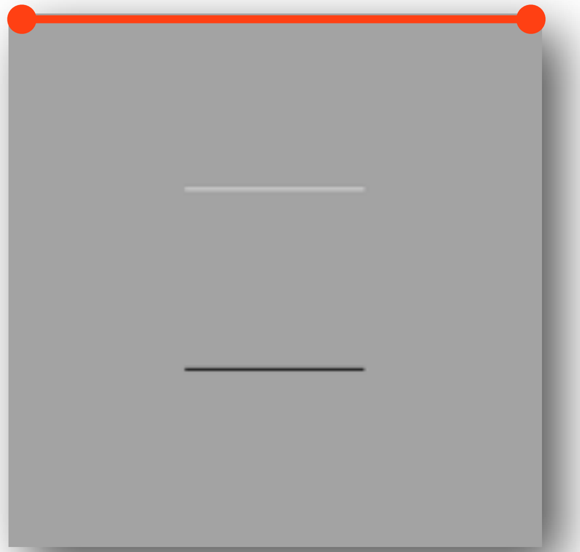

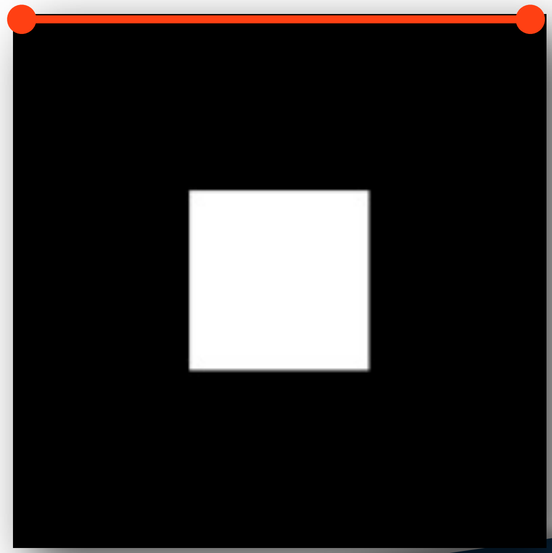

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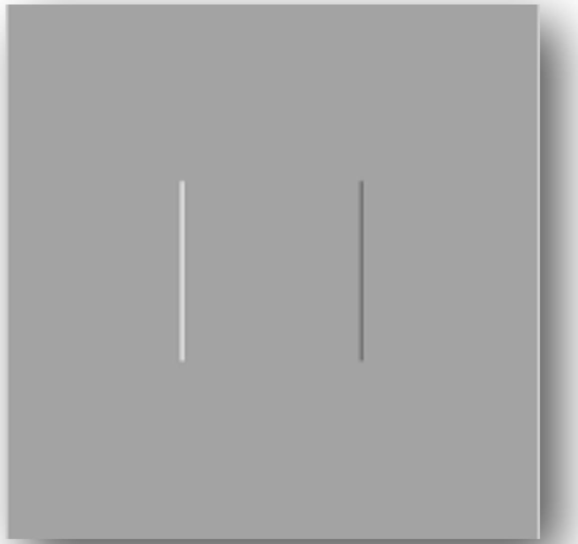

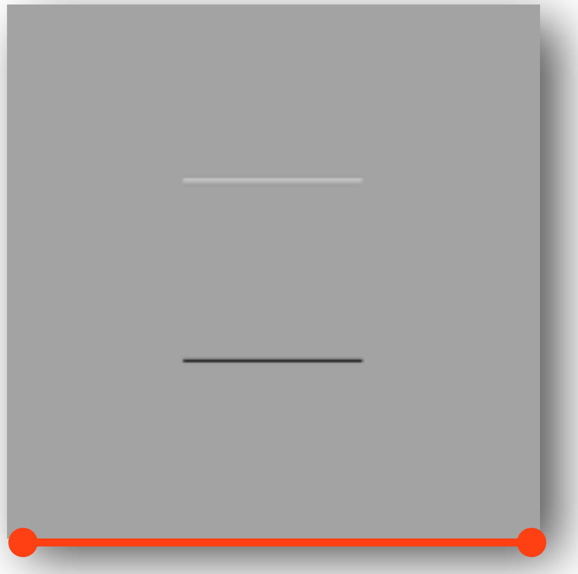

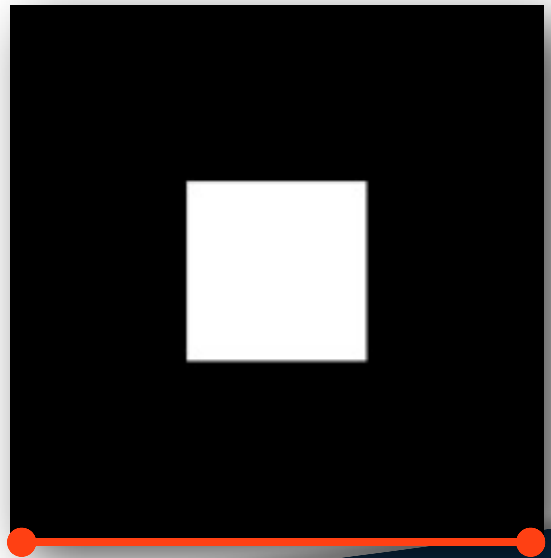

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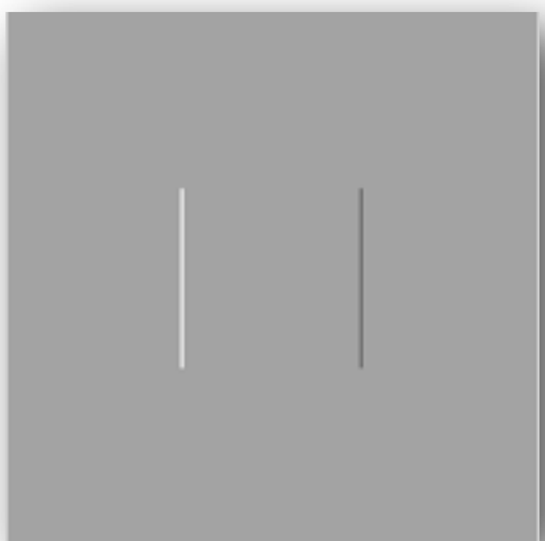

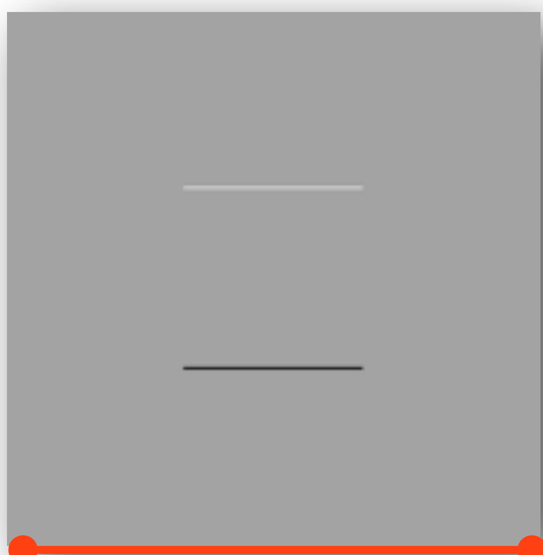

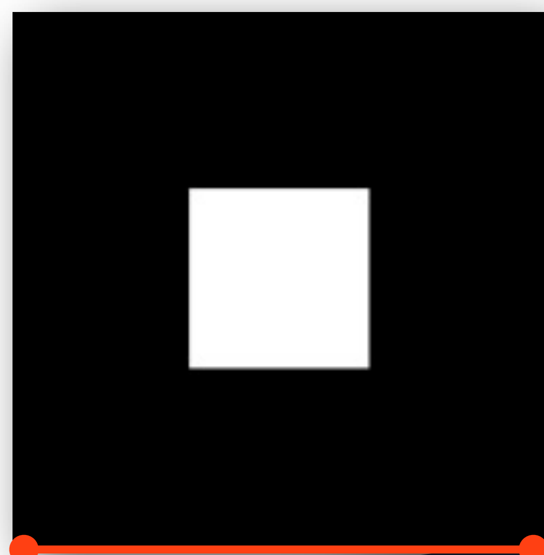

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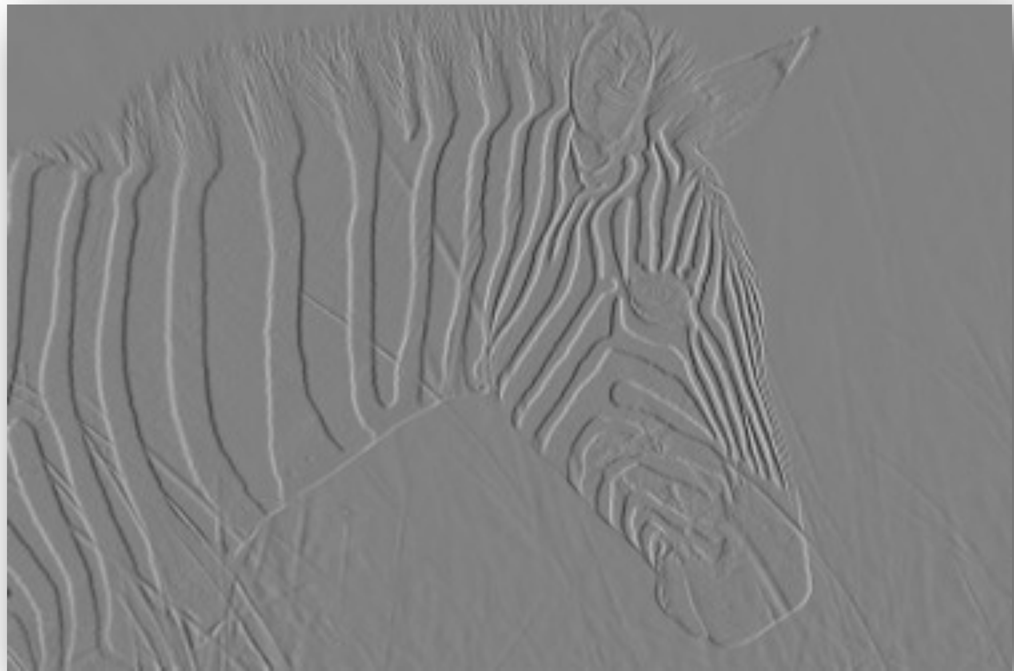

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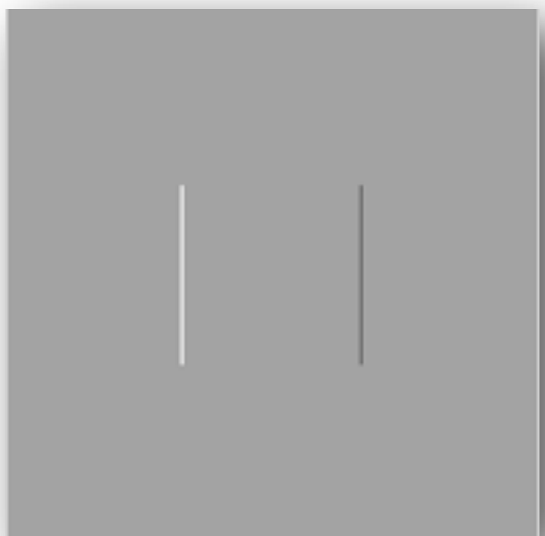

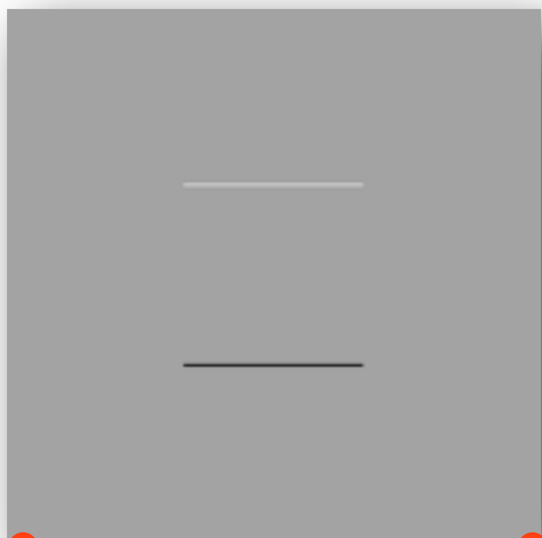

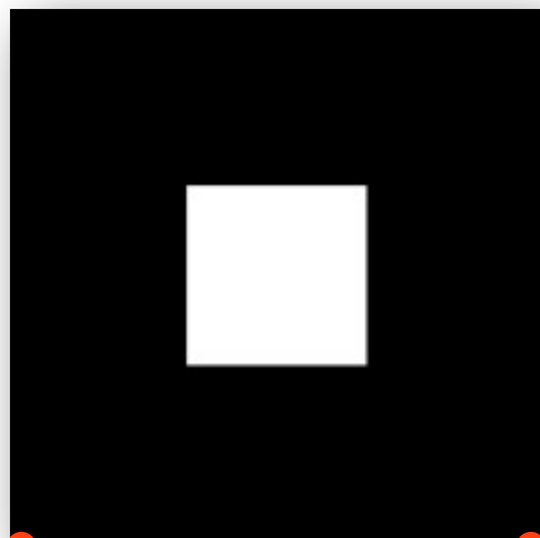

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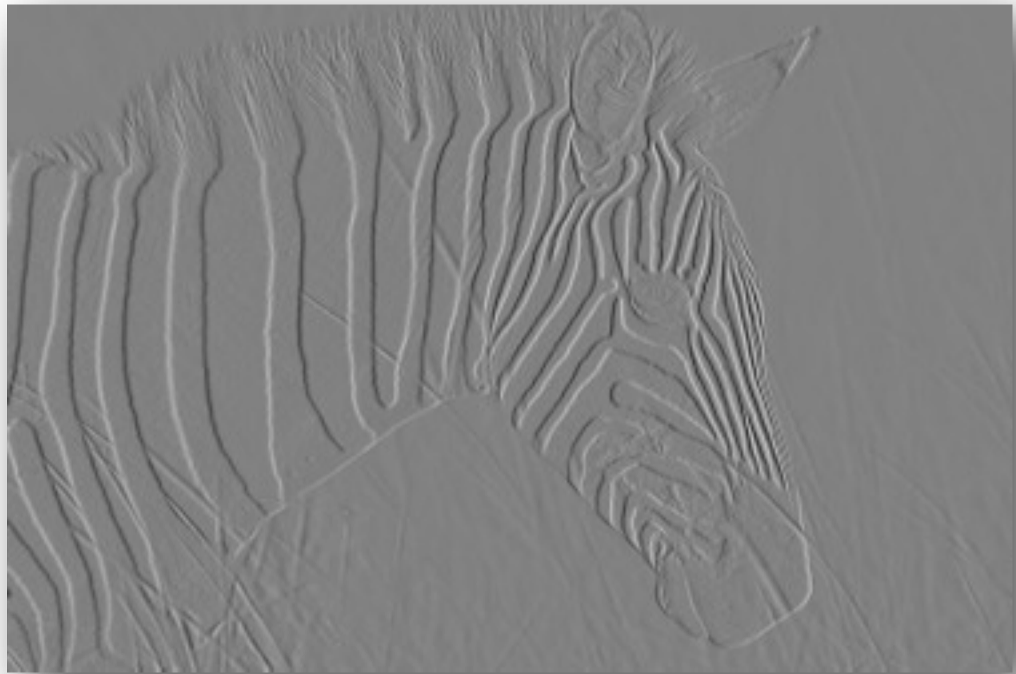

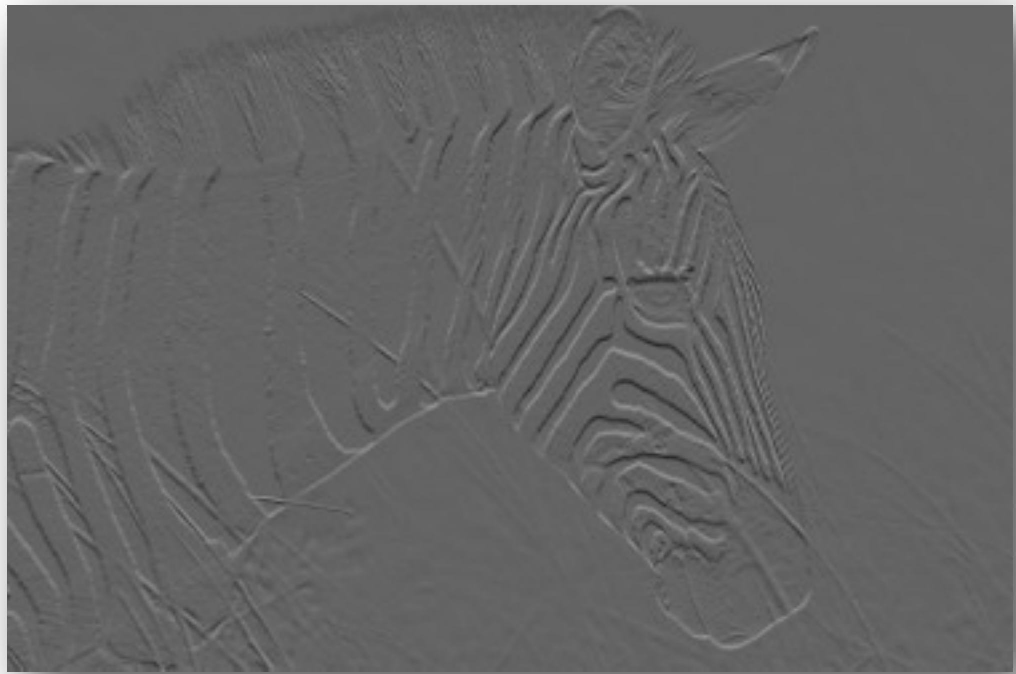

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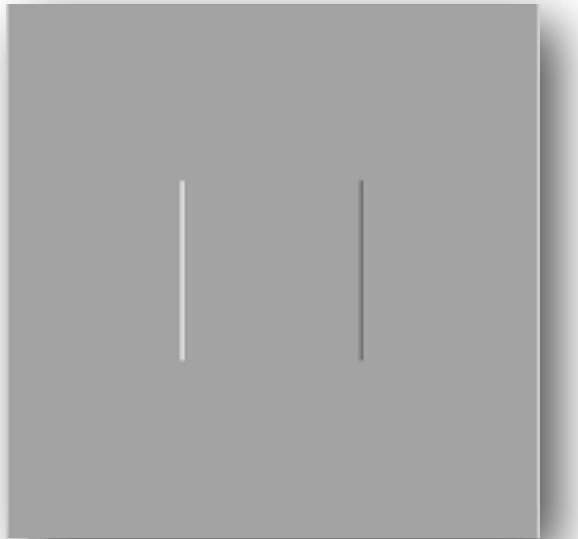

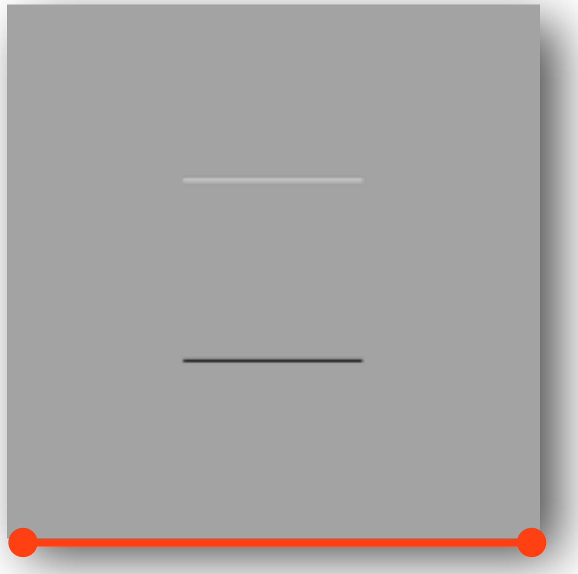

Gradient Images

$$\| \nabla F \| = \sqrt{\left(\frac{\delta F}{\delta x}\right)^2 + \left(\frac{\delta F}{\delta y}\right)^2} \qquad \theta = \tan^{-1}\left[\frac{\delta F}{\delta y} / \frac{\delta F}{\delta x}\right]$$

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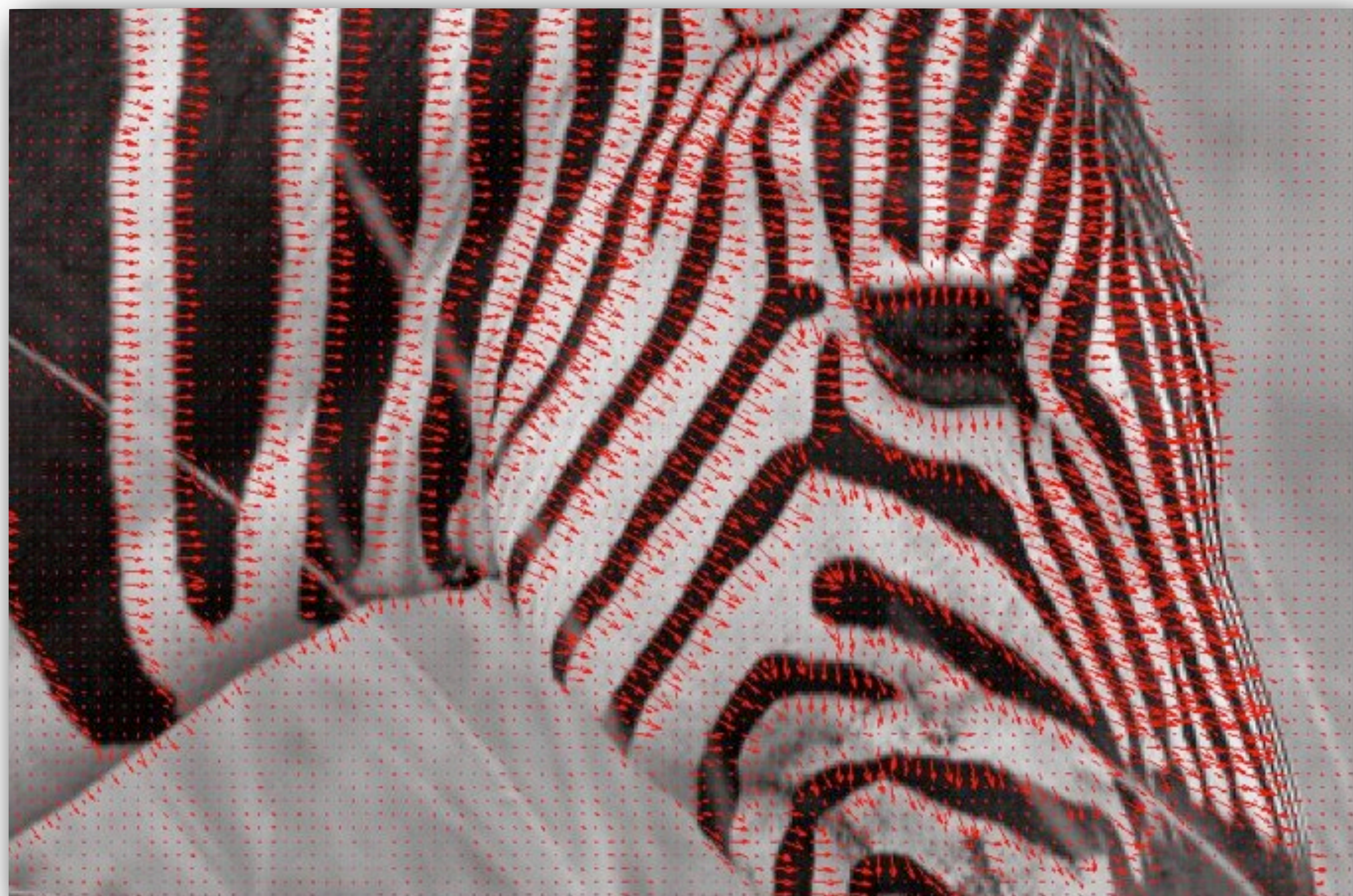

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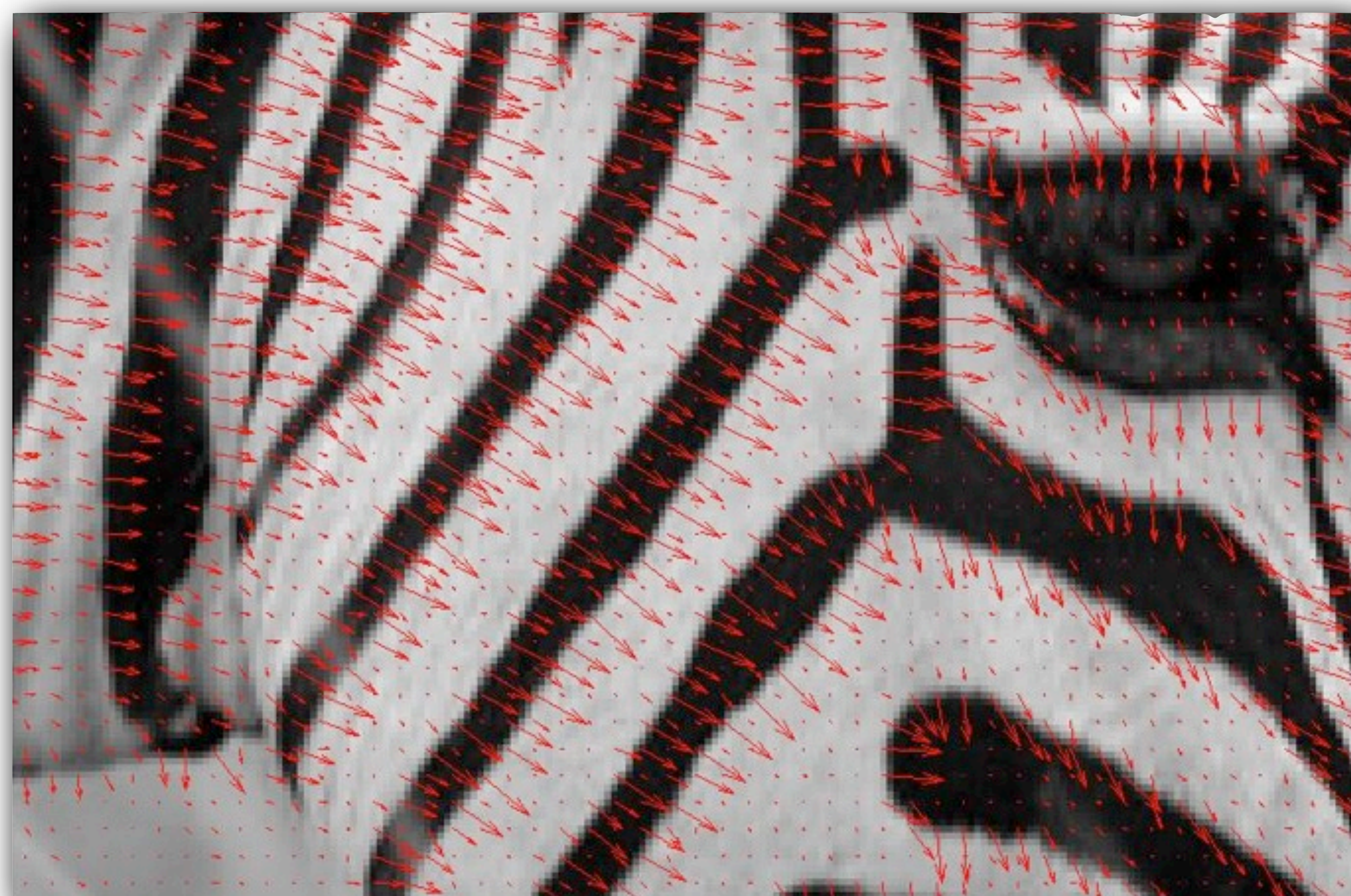

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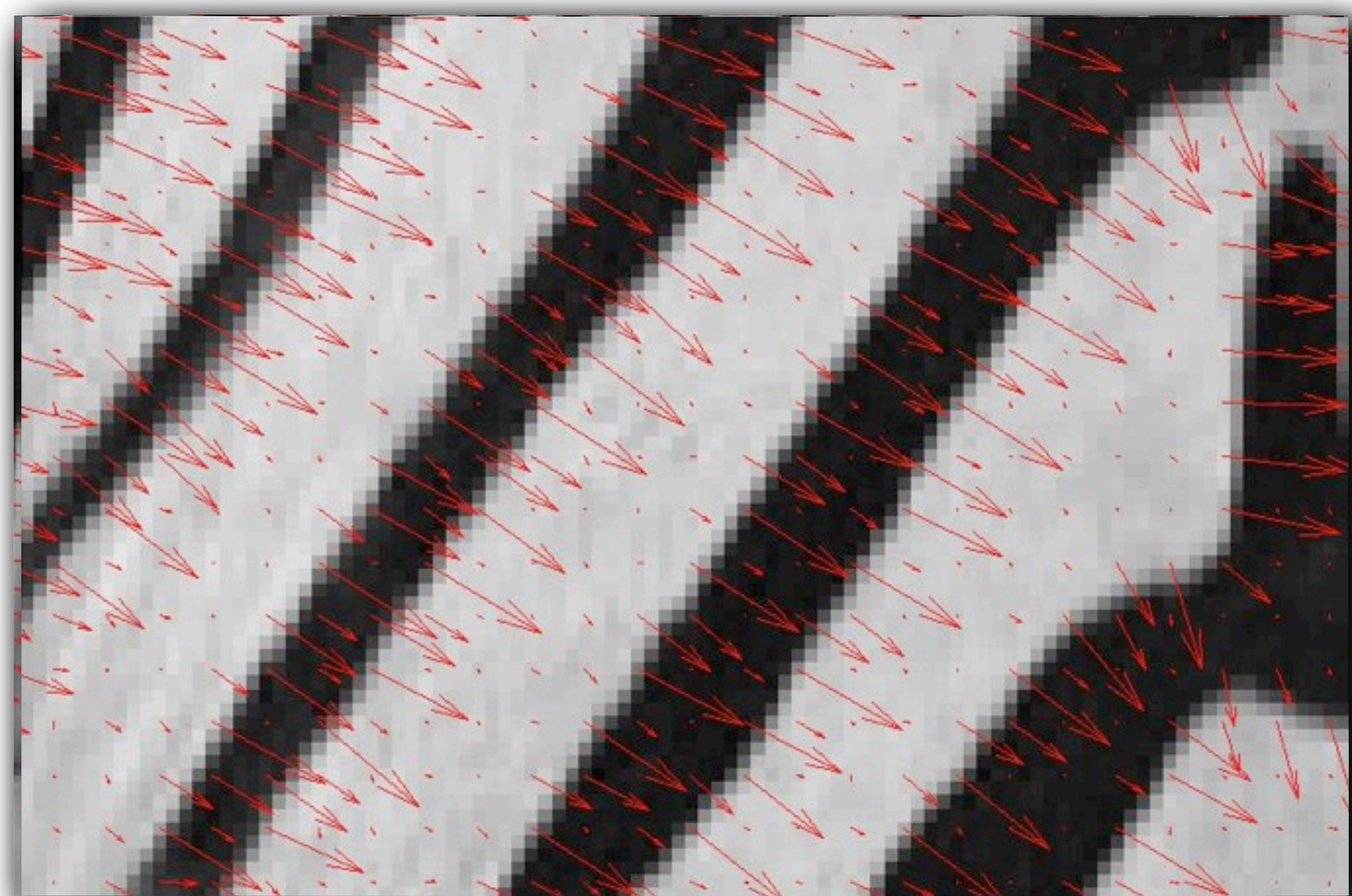

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Summary

- ★ Introduced the concept of finding Good Features of an image to support matching across images.
- ★ Presented Edges as one such feature, that are determined by measuring changes across an image.
- ★ Presented the Concept of Gradients and introduced the use of derivatives over images to compute the gradient over an image.

Next Class

- ★ Image Analysis: Edge Detection using Gradients

Credits

- ★ Matlab™ software by Mathworks Inc.
- ★ Some Slides adapted from Aaron Bobick
- ★ For more information, see Szeliski OR Forsyth & Ponce Text Book.
- ★ Images
  - Image Courtesy Professor Henrik Christensen
  - Images used from USC's Signal and Image Processing Institute's Image Database
  - Creative Commons Search
  - http://www.flickr.com/photos/lipkee/2904603582/

Computational Photography

Dr. Irfan Essa
Professor
School of Interactive Computing

Study the basics of computation and its impact on the entire workflow of photography, from capturing, manipulating and collaborating on, and sharing photographs.