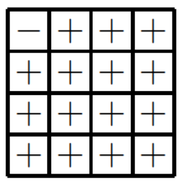
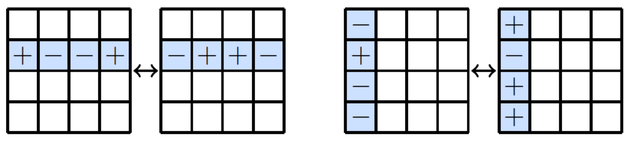
Switching Signs

Problem. All cells of a 4×4 table contain plus signs except for the top left cell that contains a minus sign.



In each step, one is allowed to switch all of the signs in any row or column.



Is it possible to switch all signs to `$+$'?

As you may guess, the parity is going to help here. In particular, the parity of the number of minuses never changes. Indeed, if we switch all signs in a row containing k minuses, we get $4 - k$ minuses. Hence, the number of minuses changes by $k - (4 - k) = 2k - 4$, an even number. (In the first example in the problem statement, the number of minuses does not change, in the second one, it changes by 2.) Hence, the parity of the number of minuses does not change. We conclude that it is impossible to get a configuration with no minuses.

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