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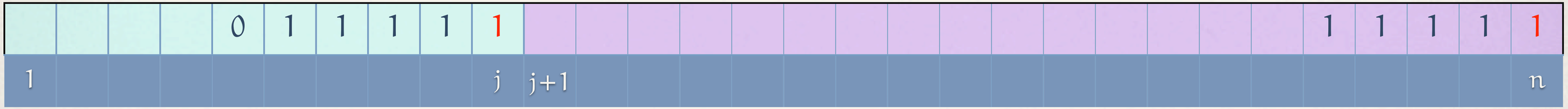

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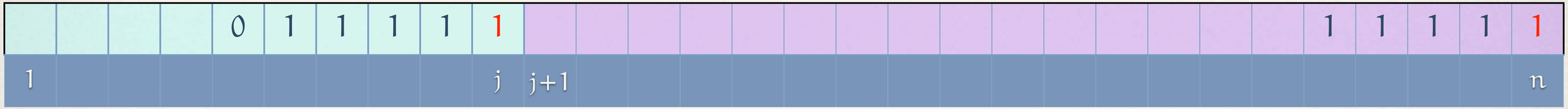

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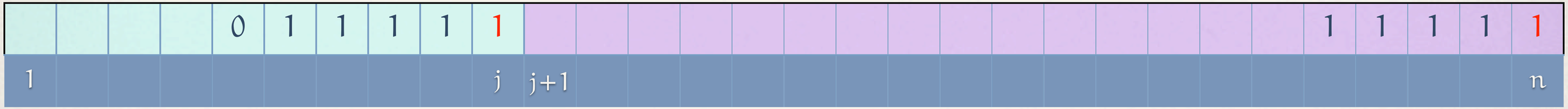

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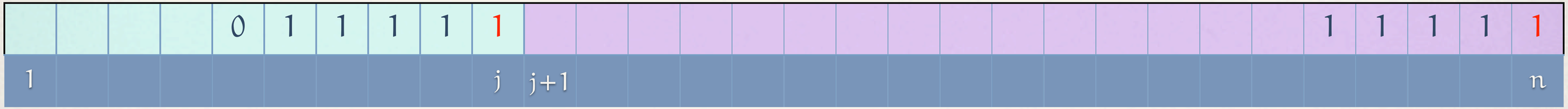

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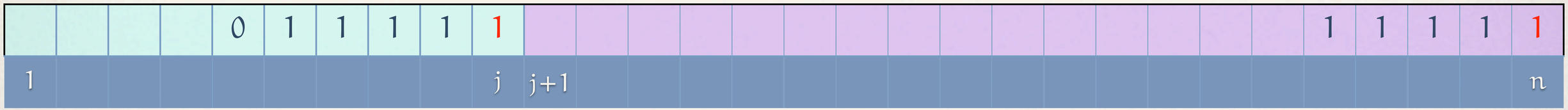

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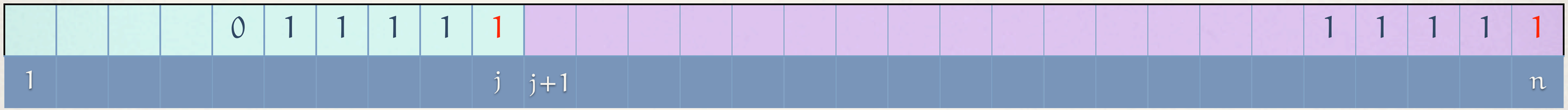

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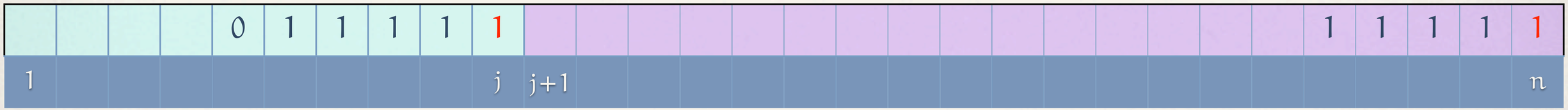

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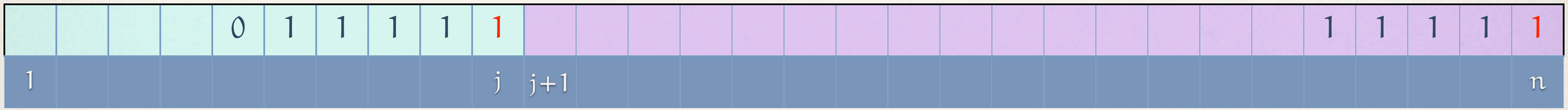

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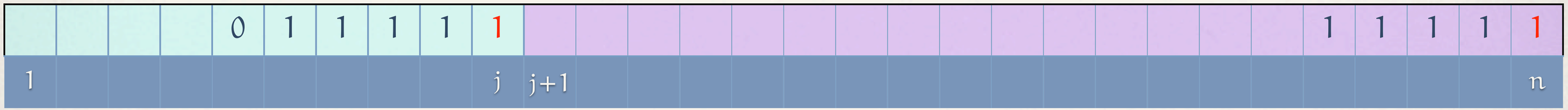

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Slogan: If you know $u_1, u_2, \ldots, u_n$ and $f_1, f_2, \ldots, f_{n-1}$ then you know f_n.