Tableau 9, Part 3

Independence!
The warp and the woof of the fabric of chance

An application in gambling: the game of craps

The game of craps

* *The rules of the game*:

  * Throw a pair of dice and sum their face values. You win immediately if you obtain 7 or 11. You lose immediately if you obtain 2, 3, or 12.

  * If you obtain 4, 5, 6, 8, 9, or 10 on the first throw, take note of the sum and proceed by repeatedly throwing the pair of dice. Stop at the *first* instant when the sum of the face values of the two dice either replicates that of your first throw or is equal to 7. You win in the first case, lose in the second.

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  What is the probability that you win (eventually) in a game of craps?

Repeated independent trials

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Alphabet	Mass function
Sum of face values k	Atomic probability p_k
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❖ What is the sample space of a game of craps?

   ❖ The *sample points* of the experiment are the single numbers 2, 3, 7, 11, 12, together with sequences of numbers of the form $k_1 k_2 \ldots k_n$ for $n \geq 2$, where $k_1 \in \{4, 5, 6, 8, 9, 10\}$, $k_2, \ldots, k_{n-1} \notin \{k_1, 7\}$, and $k_n \in \{k_1, 7\}$.

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  ❖ The *event W* that you win at craps consists of the single letters 7 and 11, together with sequences of the form $k_1 k_2 .... k_n$ for $n \geq 2$, where $k_1 \in \{4, 5, 6, 8, 9, 10\}$, k_2, ..., $k_{n-1} \notin \{k_1, 7\}$, and $k_n = k_1$.

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  ❖ The first throw determines the nature of the sequence. The *events* $F_k :=$ the sum of face values on the *first* throw is k become of interest; we are given the atomic probabilities. $p_k = \mathbf{P}(F_k)$. The events F_2, F_3, ..., F_{12} partition the space. Additivity beckons!

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  ❖ The sample space is partitioned by the number of throws before the game terminates. The *events* W_n that the game terminates with a win on the nth throw (for $n = 1, 2, ...$) become interesting.