Order of Convergence of the Secant Method

Take the following steps to determine the order of convergence of the secant method, given by

$$x_{n+1} = x_n - \frac{(x_n - x_{n-1})f(x_n)}{f(x_n) - f(x_{n-1})}.$$

(a) Let $\epsilon_n = r - x_n$, and subtract both sides of the secant method from r to obtain

$$\epsilon_{n+1} = \epsilon_n + \frac{(\epsilon_{n-1} - \epsilon_n)f(r - \epsilon_n)}{f(r - \epsilon_n) - f(r - \epsilon_{n-1})}.$$

(b) Taylor series expand $f(r - \epsilon_n)$ and $f(r - \epsilon_{n-1})$ for small ϵ using $f(r) = 0$. Obtain

$$\epsilon_{n+1} = \epsilon_n + \frac{-\epsilon_n f'(r) + \frac{1}{2}\epsilon_n^2 f''(r) + \dots}{f'(r) - \frac{1}{2}(\epsilon_{n-1} + \epsilon_n)f''(r) + \dots}.$$

(c) For small ϵ, use the Taylor series expansion

$$\frac{1}{1-\epsilon} = 1 + \epsilon + \epsilon^2 + \dots$$

to obtain

$$|\epsilon_{n+1}| = \frac{1}{2}\left|\frac{f''(r)}{f'(r)}\right| |\epsilon_n||\epsilon_{n-1}|.$$

(d) Try $|\epsilon_{n+1}| = k|\epsilon_n|^p$ and $|\epsilon_n| = k|\epsilon_{n-1}|^p$ to obtain the equation $p^2 = p + 1$. Determine p.

✓ **Completed**

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