Algebra

Applied Mathematics

Calculus and Analysis

Discrete Mathematics

Foundations of Mathematics

Geometry

History and Terminology

Number Theory

Probability and Statistics

Recreational Mathematics

Topology

Alphabetical Index

New in MathWorld

Applied Mathematics > Numerical Methods > Root-Finding >

Hermite-Gauss Quadrature

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Hermite-Gauss quadrature, also called Hermite quadrature, is a Gaussian quadrature over the interval $(-\infty, \infty)$ with weighting function $W(x) = e^{-x^2}$ (Abramowitz and Stegun 1972, p. 890). The abscissas for quadrature order n are given by the roots x_i of the Hermite polynomials $H_n(x)$, which occur symmetrically about 0. The weights are

$$w_i = -\frac{A_{n+1} \gamma_n}{A_n H'_n(x_i) H_{n+1}(x_i)} \tag{1}$$

$$=\frac{A_n}{A_{n-1}}\frac{\gamma_{n-1}}{H_{n-1}(x_i)H'_n(x_i)},$$
(2)

where A_n is the coefficient of x^n in $H_n(x)$. For Hermite polynomials,

$$A_n = 2^n, (3)$$

so

$$\frac{A_{n+1}}{A_n} = 2. ag{4}$$

Additionally,

$$\gamma_n = \sqrt{\pi} \ 2^n \, n!, \tag{5}$$

so

$$w_i = -\frac{2^{n+1} n! \sqrt{\pi}}{H_{n+1}(x_i) H_n'(x_i)}$$
(6)

$$=\frac{2^{n}(n-1)!\sqrt{\pi}}{H_{n-1}(x_{i})H'_{n}(x_{i})}$$
(7)

$$=\frac{2^{n+1} n! \sqrt{\pi}}{[H'_n(x_i)]^2}$$
 (8)

$$=\frac{2^{n+1} n! \sqrt{\pi}}{[H_{n+1}(x_i)]^2} \tag{9}$$

$$= \frac{2^{n-1} n! \sqrt{\pi}}{n^2 [H_{n-1}(x_i)]^2},\tag{10}$$

where (8) and (9) follow using the recurrence relation

$$H'_{n}(x) = 2 n H_{n-1}(x) = 2 x H_{n}(x) - H_{n+1}(x)$$
(11)

to obtain

$$H'_n(x_i) = 2 n H_{n-1}(x_i) = -H_{n+1}(x_i),$$
 (12)

and (10) is from Abramowitz and Stegun (1972 p. 890).

The error term is

$$E = \frac{n! \sqrt{\pi}}{2^n (2n)!} f^{(2n)}(\xi). \tag{13}$$

Beyer (1987) gives a table of abscissas and weights up to n = 12.

n	x_i	w_i
2	±0.707107	0.886227
3	0	1.18164
	±1.22474	0.295409
4	± 0.524648	0.804914
	±1.65068	0.0813128
5	0	0.945309
	±0.958572	0.393619
	±2.02018	0.0199532

The abscissas and weights can be computed analytically for small n.

n	x_i	w_i
2	$\pm \frac{1}{2} \sqrt{2}$	$\frac{1}{2} \sqrt{\pi}$
3	0	$\frac{2}{3} \sqrt{\pi}$
	$\pm \frac{1}{2} \sqrt{6}$	$\frac{1}{6} \sqrt{\pi}$
4	$\pm\sqrt{\frac{3-\sqrt{6}}{2}}$	$\frac{\sqrt{\pi}}{4(3-\sqrt{6})}$
	$\pm\sqrt{\frac{3+\sqrt{6}}{2}}$	$\frac{\sqrt{\pi}}{4\left(3+\sqrt{6}\right)}$

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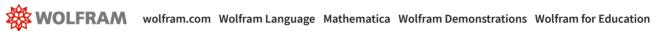
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approximate zero

More things to try:	= approximate zero = area between y=sinc(x) and the x-axis from x=-4pi to 4pi
	= factoradic form of the permutation (3 1 2 5 4)
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