Facility location

Primal

$$\min \sum_{\mathbf{i}} \mathbf{f_i y_i} + \sum_{\mathbf{i,j}} \mathbf{c_{ij} x_{ij}} :$$

$$\sum_{\mathbf{i}} \mathbf{x_{ij}} \geq 1 \quad \textbf{for all j}$$

$$\mathbf{x_{ij}} \leq \mathbf{y_i} \quad \textbf{for all i, j}$$

$$\mathbf{x_{ij}}, \mathbf{y_i} \geq \mathbf{0}$$

Dual

$$\max \sum_{\mathbf{j}} \alpha_{\mathbf{j}} :$$

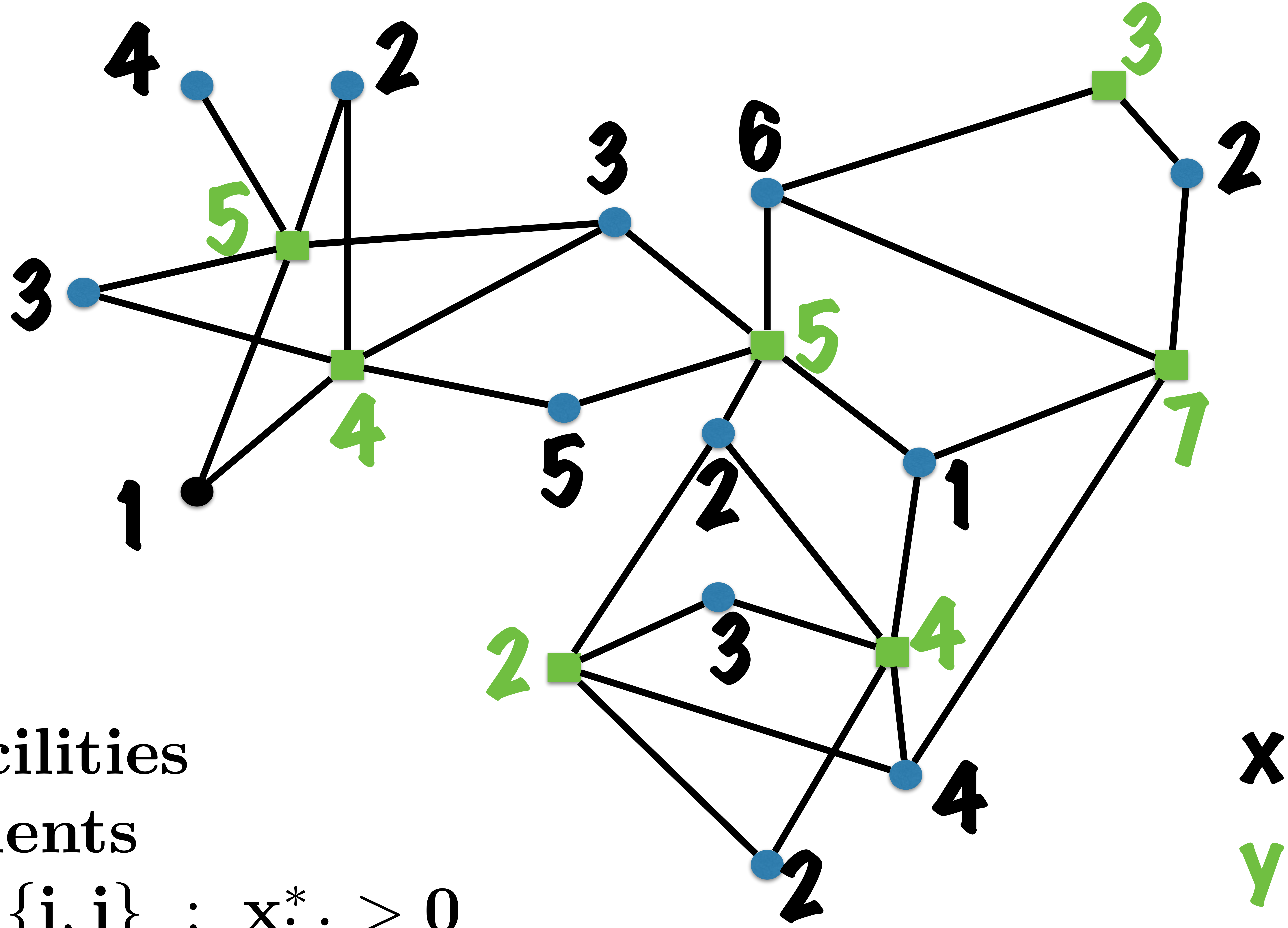
$$\sum_{\mathbf{j}} \beta_{\mathbf{ij}} \leq \mathbf{f_i} \quad \textbf{for all i}$$

$$\alpha_{\mathbf{j}} \leq \beta_{\mathbf{ij}} + \mathbf{c_{ij}} \quad \textbf{for all i, j}$$

$$\alpha_{\mathbf{j}}, \beta_{\mathbf{ij}} \geq \mathbf{0}$$

Algorithm

1. Solve the primal and dual LPs: $y_i^*, x_{ij}^*, \alpha_j^*, \beta_{ij}^*$
2. While some clients are unassigned
   j_C: unassigned client s.t. $\alpha_{j_C}^*$ is min
   i_C : cheapest facility s.t. $x_{i_C, j_C}^* > 0$
   open facility i_C
   assign to i_C all unassigned clients s.t.
   there is a facility with $x_{i, j_C} > 0$ and $x_{i,j} > 0$



: facilities

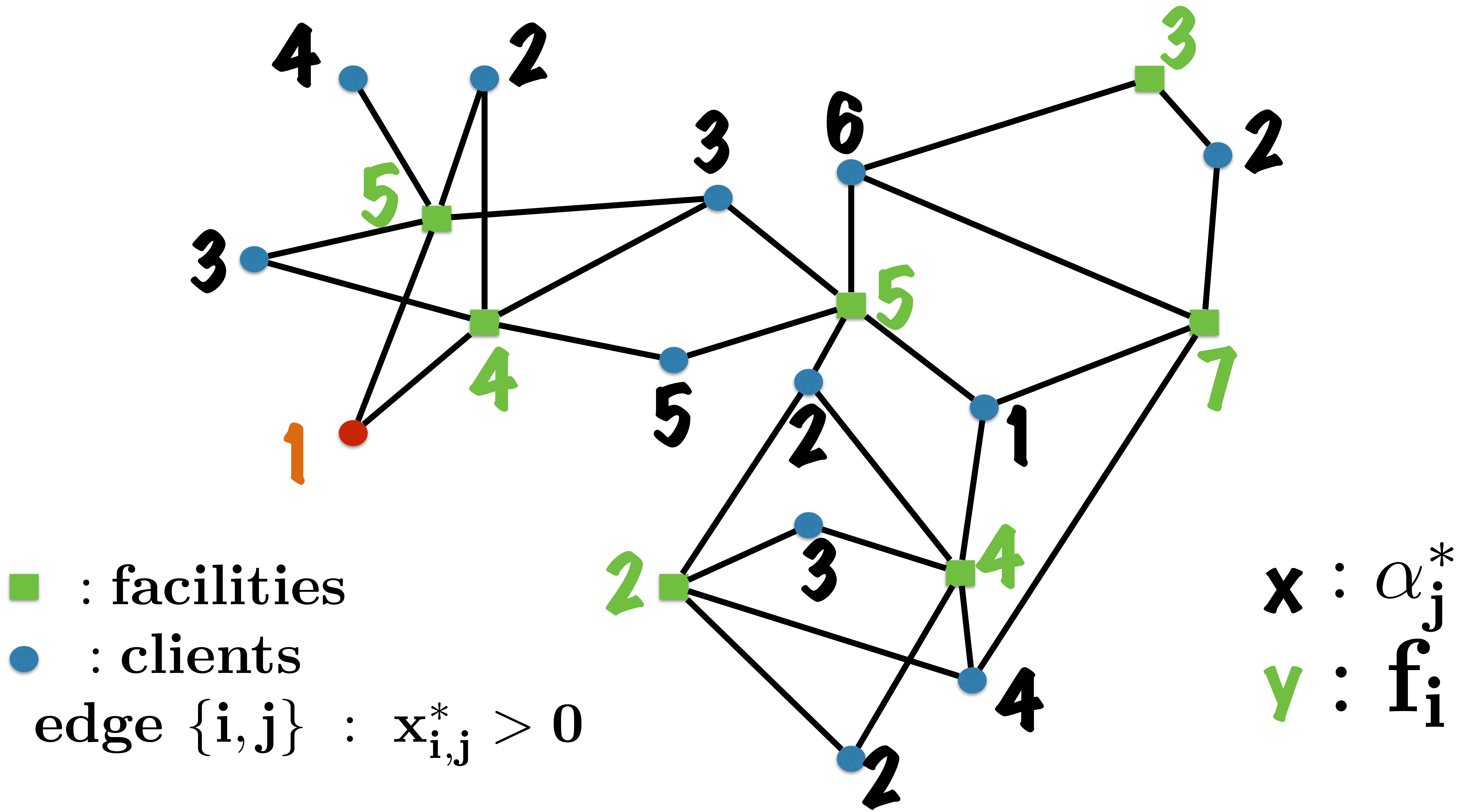
: clients

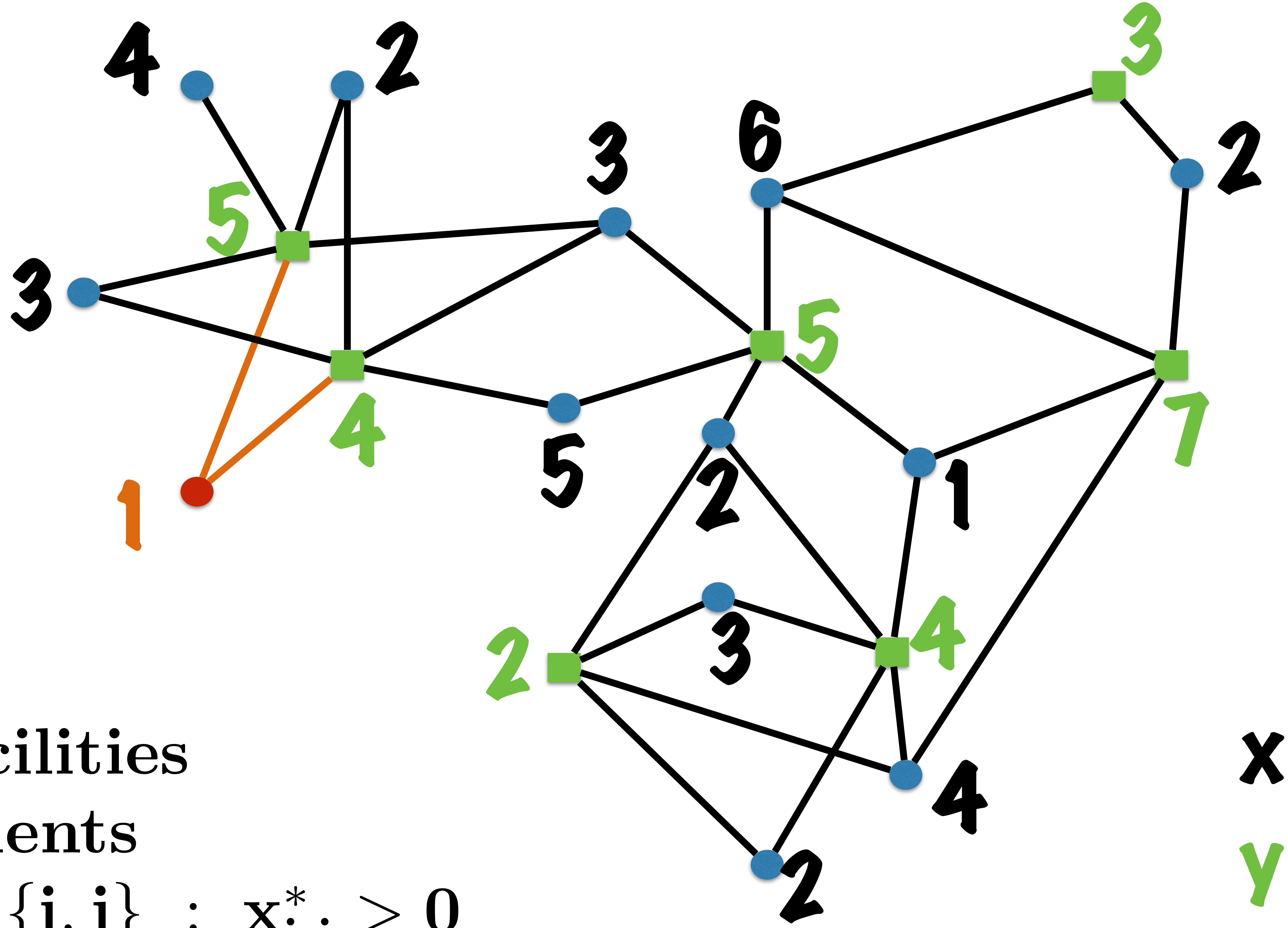
edge $\{i, j\}$: $x^*_{i,j} > 0$

$x : \alpha^*_j$

$y : f_i$

First iteration





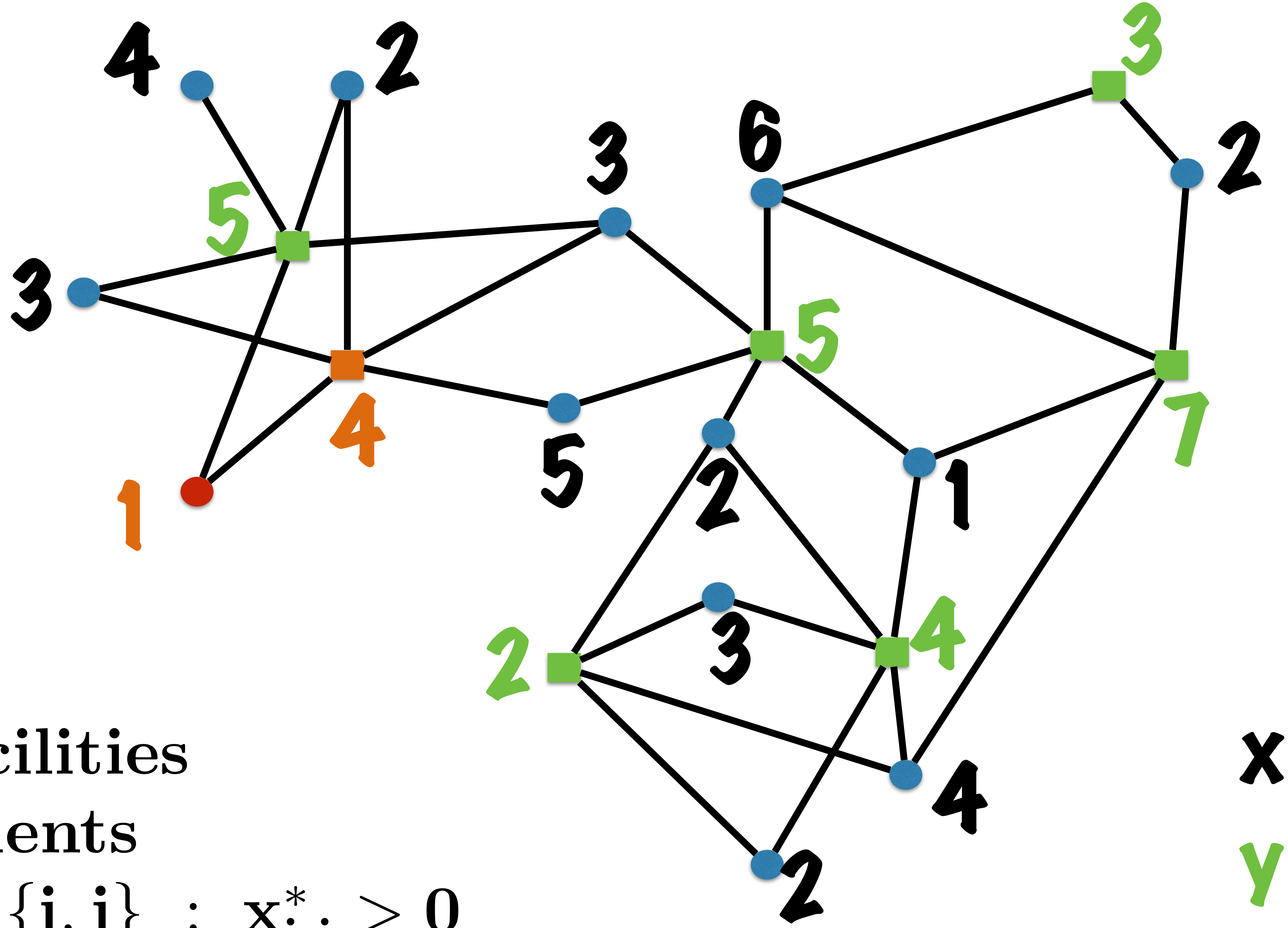
: facilities

: clients

edge $\{i,j\}$: $x^*_{i,j} > 0$

$\mathbf{x} : \alpha^*_j$

$\mathbf{y} : f_i$



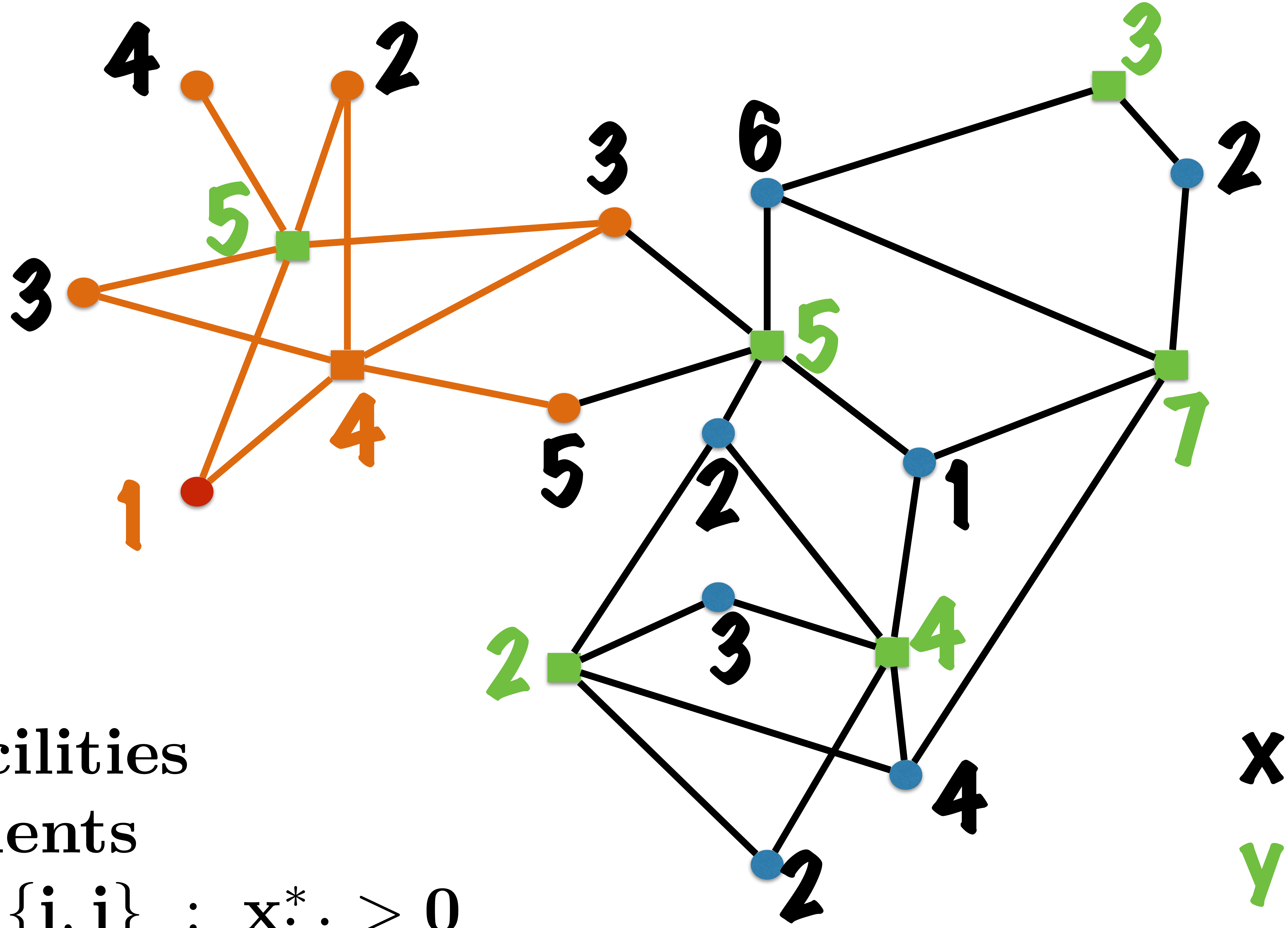
■ : facilities

● : clients

edge $\{i,j\}$: $x^*_{i,j} > 0$

x : α^*_j

y : f_i



: facilities

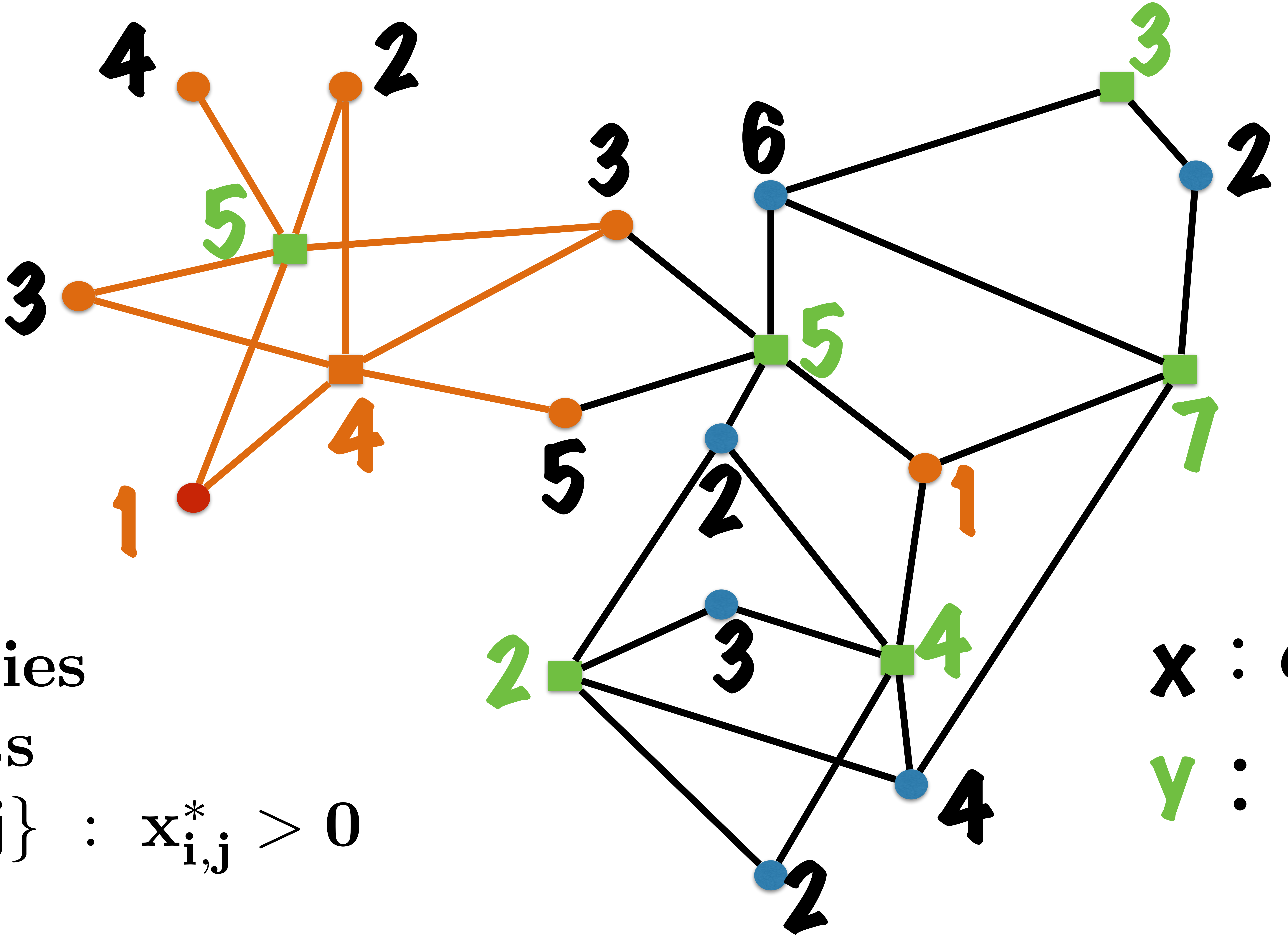
: clients

edge $\{i,j\}$: $x^*_{i,j} > 0$

$x : \alpha^*_j$

$y : f_i$

Second iteration



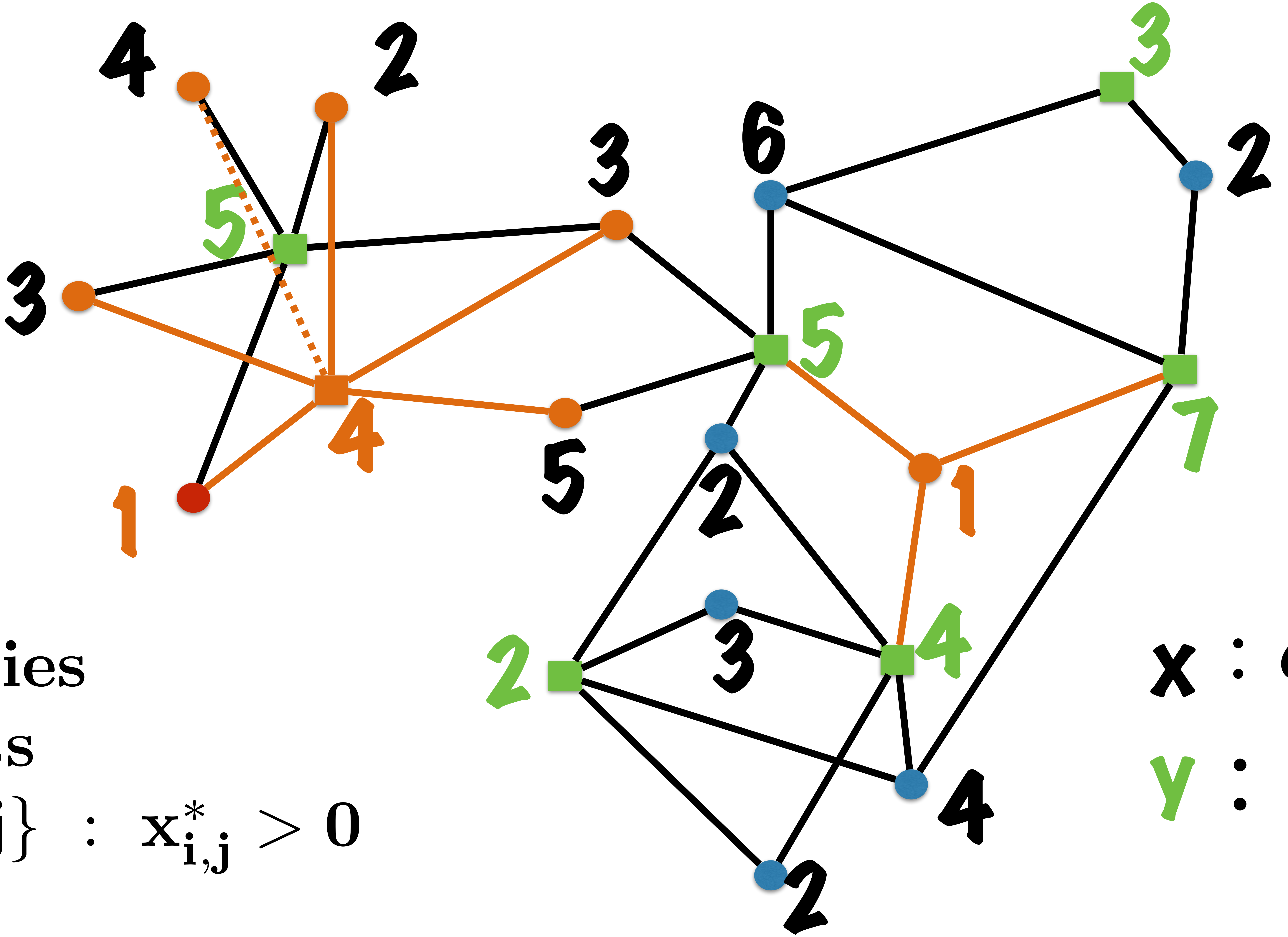
■ : facilities

● : clients

edge $\{\mathbf{i},\mathbf{j}\}$: $\mathbf{x}^*_{\mathbf{i},\mathbf{j}} > 0$

x : $\alpha^*_{\mathbf{j}}$

y : $\mathbf{f_i}$



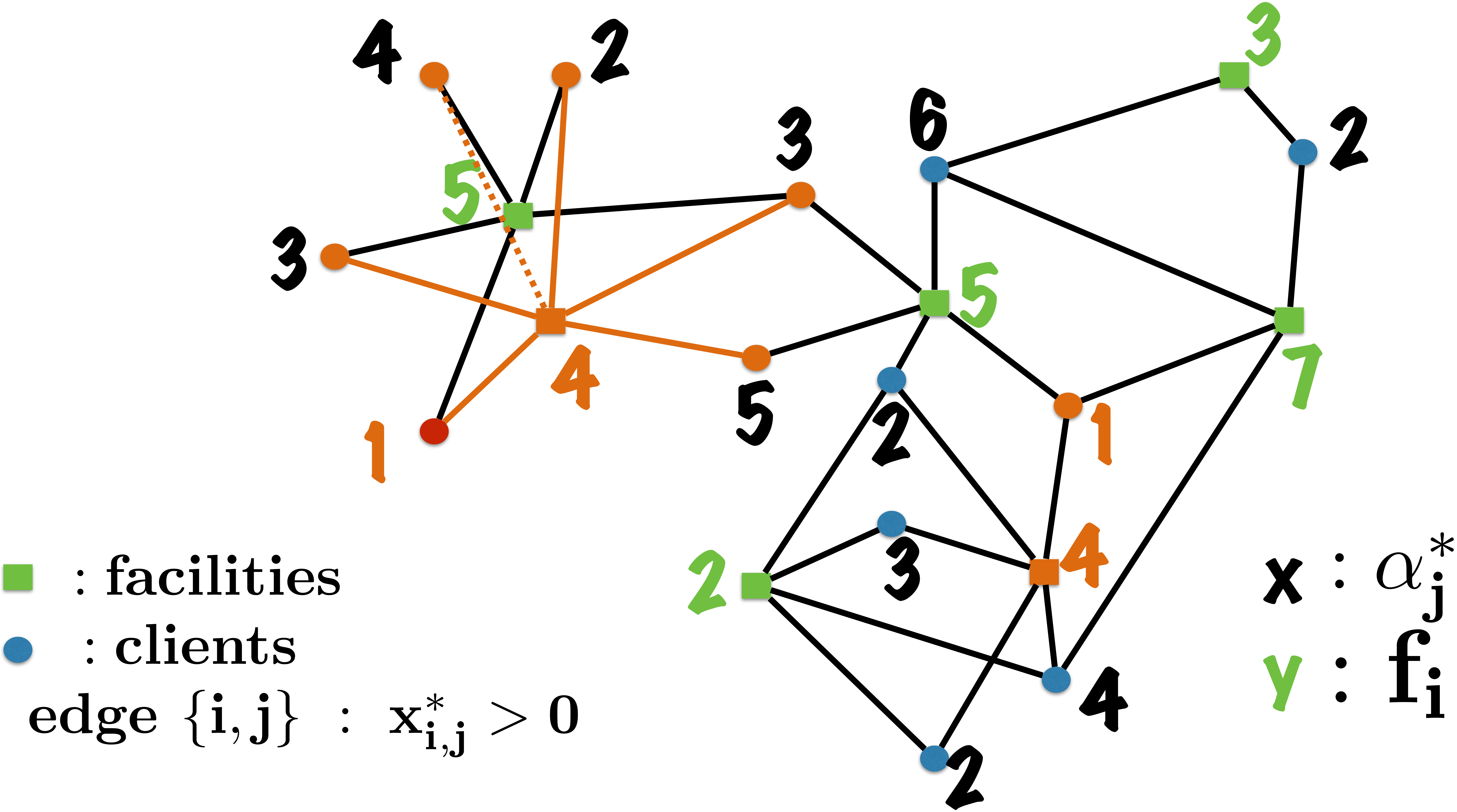
$\blacksquare$: facilities

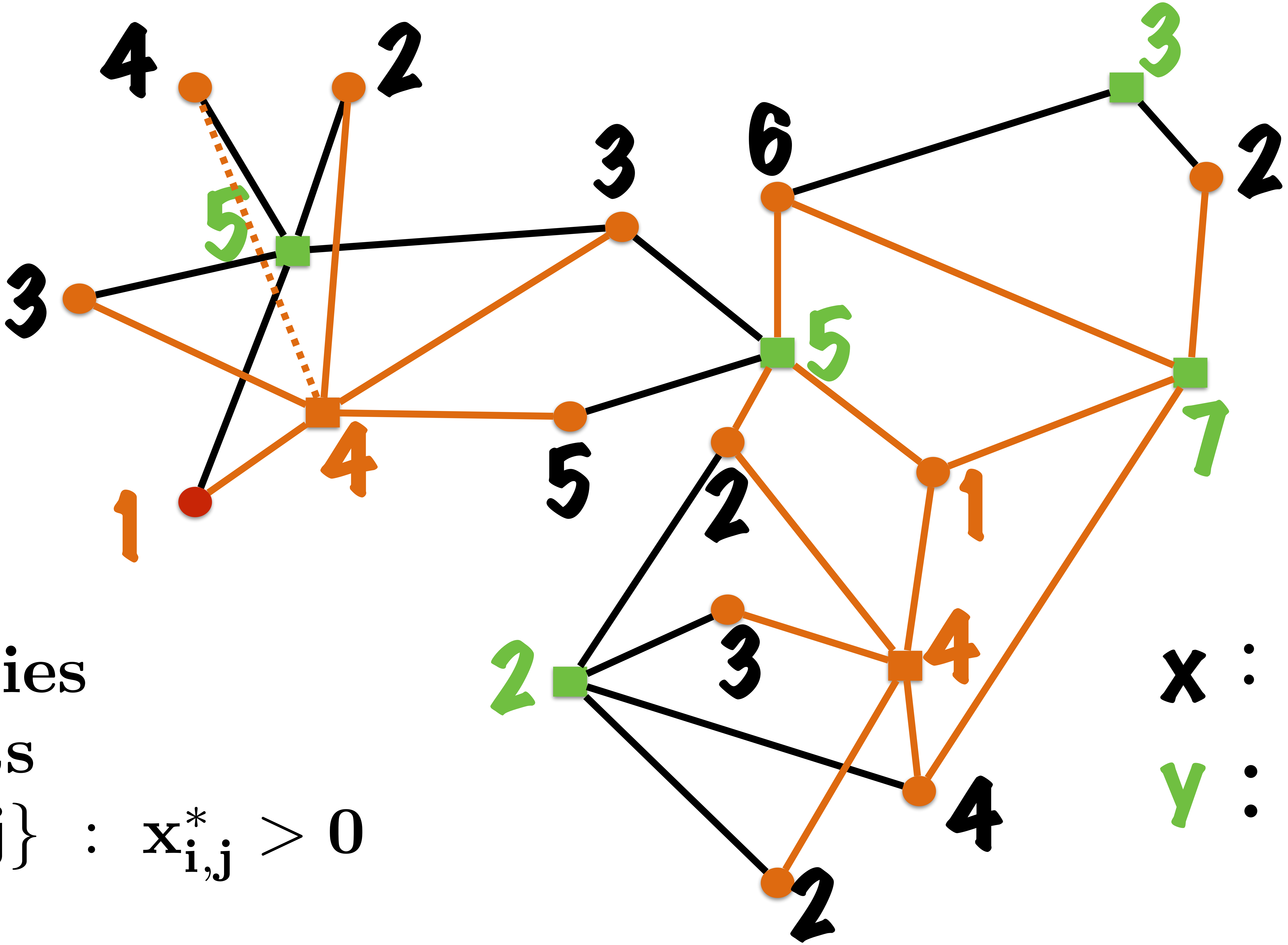
$\bullet$: clients

edge $\{i, j\}$: $x^*_{i,j} > 0$

x : α^*_j

y : f_i


: facilities
: clients
edge $\{i, j\}$: $x^*_{i,j} > 0$
x : α^*_j
y : f_i



: facilities

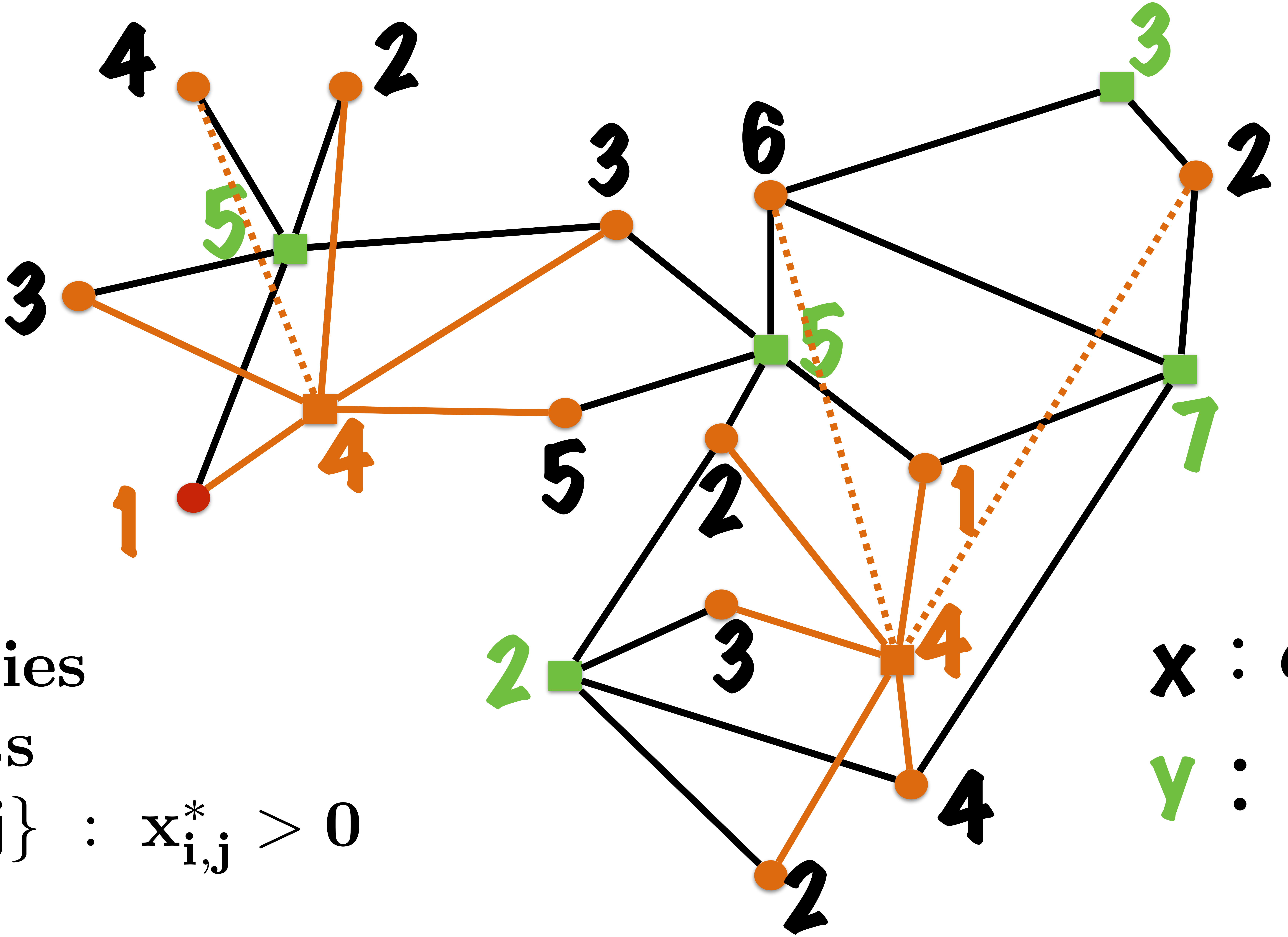
: clients

edge $\{i,j\}$: $x^*_{i,j} > 0$

x : α^*_j

y : f_i

Output



■ : facilities

● : clients

edge $\{i, j\}$: $x^*_{i,j} > 0$

x : α^*_j

y : f_i

Facility location