What have we learned?

Consider a signal in which frequency components begin and end at different points in time. If we apply a simple DFT on the whole signal, the frequency representation completely obfuscates time information. A solution to this problem is called the short-time Fourier transform (STFT).

The STFT is nothing more than a repeated application of the DFT to subsequent segments of length L of a given signal. In this way, we obtain simultaneously time and frequency information. The STFT is visually depicted using a spectrogram, i.e., a plot of the magnitude of the DFT coefficients in dB. The spectrogram is indexed by two quantities, the portion of the signal under scrutiny and the frequency index k. Various colors are used to represent various magnitudes and darker values correspond to smaller ones.

When we increase the length L of the analysis window we can detect more frequencies, i.e, the resolution in frequency is increased. But, as L grows larger, our timing detection is less precise, that is, the resolution in time is decreased. This observation shows a fundamental property of the STFT: we cannot increase simultaneously time and frequency resolution. There is a fundamental uncertainty principle between time and frequency.