Feedback — Assignment #1: Basics of Linear Programming

Help Center

You submitted this homework on **Sat 7 Sep 2013 9:56 AM PDT**. You got a score of **20.00** out of **20.00**.

The questions in this assignment concern the linear programming problem below.

$$
\begin{array}{lcccccc}
\max & 2x_1 & +3x_2 & & & \\
\text{s.t.} & x_1 & & & \leq & 5 \\
 & x_1 & -x_2 & +x_3 & \leq & 10 \\
 & & x_2 & & \leq & 15 \\
 & & & x_3 & \leq & 22 \\
 & x_1 & & & \geq & 0 \\
 & & x_2 & & \geq & 0 \\
 & & & x_3 & \geq & 0
\end{array}
$$

Answer the questions below for this fixed linear programming problem.

Question 1

How many decision variables does the LP have?

You entered:

3

Your Answer	Score	Explanation

3	✔	1.00	The decision variables in this problem are x_1, x_2, x_3
Total		1.00 / 1.00	

Question Explanation

The decision variables in a LP are a combination of variables that occur in the objective functions and the constraints.

Question 2

Which of the following statements are true about the **feasible region** of the LP (recalled below)?

$$\begin{array}{llllll} \max & 2x_1 & +3x_2 & & & \\ \text{s.t.} & x_1 & & & \leq & 5 \\ & x_1 & -x_2 & +x_3 & \leq & 10 \\ & & x_2 & & \leq & 15 \\ & & & x_3 & \leq & 22 \\ & x_1 & & & \geq & 0 \\ & & x_2 & & \geq & 0 \\ & & & x_3 & \geq & 0 \end{array}$$

Select all options that are correct and make sure that incorrect options are not selected.

Your Answer		Score	Explanation
☑ The point $(x_1 : 0, x_2 : 0, x_3 : 0)$ belongs to the feasible region of this LP.	✔	1.00	It does! Simply go through every inequality in the LP, and check that the origin satisfies it.
☐ The feasible region is a three dimensional paraboloid (a bowl shaped figure).	✔	1.00	This is incorrect. The feasible region is a three dimensional polyhedron.

☑ The point $(x_1 : \pi, x_2 : \pi, x_3 : \pi)$ belongs to the feasible region of this LP. Assume $\pi = 3.1415$ for your calculations.	✔	1.00	It does. Simply go through every inequality in the LP and check that the point (π, π, π) satisfies it.
☐ The feasible region is not bounded. It extends infinitely in some direction.	✔	1.00	The constraints clearly indicate that $x_1 \in [0, 5]$, $x_2 \in [0, 15]$ and $x_3 \in [0, 22]$. Clearly the feasible region cannot extend infinitely in some direction.
☐ The feasible region contains a point where the variable x_1 achieves a strictly negative value.	✔	1.00	The constraint $x_1 \geq 0$ ensures that no such point can be part of the feasible region.
Total		5.00 / 5.00	

Question Explanation

By default, if nothing is said about the valuations of variables in an LP, it is taken to be a **real** linear program. I.e, the variables are assumed to be real-valued.

The feasible region of the LP is the polyhedron that is defined by the inequalities of the LP.

Question 3

Which of the following options holds for the point $(x_1 : 5, x_2 : 15, x_3 : 22)$? Choose the correct answer.

For your convenience the problem is recalled below:

$$
\begin{array}{llllll}
\max & 2x_1 & +3x_2 & & & \\
\text{s.t.} & x_1 & & & \leq & 5 \\
& x_1 & -x_2 & +x_3 & \leq & 10 \\
& & x_2 & & \leq & 15 \\
& & & x_3 & \leq & 22 \\
& x_1 & & & \geq & 0 \\
& & x_2 & & \geq & 0 \\
& & & x_3 & \geq & 0
\end{array}
$$

Your Answer		Score	Explanation
This point is **not** an optimal solution of the LP.	✔	2.00	This is true. The point (5,15,22) is infeasible. It fails to satisfy the constraint $x_1 - x_2 + x_3 \leq 10$. An infeasible point cannot be optimal.
It satisfies more than a two-thirds majority of the constraints in the LP. Therefore, it can be treated as belonging to the feasible region.			
It is one of the optimal solutions to the LP.			
It is a feasible solution to the LP.			
Total		2.00 / 2.00	

Question Explanation

The point $(5, 15, 22)$ is infeasible. It fails to satisfy the constraint $\underbrace{x_1 - x_2 + x_3}_{=12} \leq 10$.

Question 4

Exactly one of the solutions given below is claimed to be an optimal solution to this problem. Assuming this, choose the correct optimal solution.

For your convenience the problem is recalled below:

$$\begin{array}{lrrrcl}
\max & 2x_1 & +3x_2 & & & \\
\text{s.t.} & x_1 & & & \leq & 5 \\
& x_1 & -x_2 & +x_3 & \leq & 10 \\
& & x_2 & & \leq & 15 \\
& & & x_3 & \leq & 22 \\
& x_1 & & & \geq & 0 \\
& & x_2 & & \geq & 0 \\
& & & x_3 & \geq & 0
\end{array}$$

Your Answer		Score	Explanation
◉ $(x_1: 5, x_2: 15, x_3: 20)$	✔	6.00	Feasible and achieves objective value of 55
○ $(x_1: 5, x_2: 15, x_3: 22)$			
○ $(x_1: 5, x_2: 10, x_3: 0)$			
○ $(x_1: 0, x_2: 0, x_3: 0)$			
○ $(x_1: 200, x_2: 300, x_3: 0)$			
○ $(x_1: \pi, x_2: \pi, x_3: \pi)$, take $\pi = 3.1415$			
Total		6.00 / 6.00	

Question Explanation

The optimal solution should be feasible and achieve the largest value of the objective since the problem is a maximization.

Question 5

Assume that optimal solution obtained from a correct LP solver, sets x_1, x_2, x_3 to integer values yielding an objective value of 55.

Suppose we turn the problem into an ILP by further constraining the decision variables to integers, what is the optimal objective value of the ILP?

You entered:

55

Your Answer		Score	Explanation
55	✔	2.00	The LP and the ILP have to exhibit the same answer since the LP solver yields an integer optimum.
Total		2.00 / 2.00	

Question Explanation

If a (real) LP solver yields an integer optimal solution, then the ILP has the same optimum as the LP.

Question 6

Suppose we add a fresh inequality (different from all the previously existing ones) to the (old) LP (**LP-A**) above to obtain a *new* LP (**LP-B**), which of the following possibilities are true?

Assume that the fresh inequality involves x_1, x_2 and x_3 and in particular, does not introduce any new decision variables.

Please ensure that all the correct answers are selected and the incorrect answers remain unselected.

Your Answer		Score	Explanation
☐ The optimal solution of the **LP-B** (new LP), if it exists, will always be *greater than or equal to* that of **LP-A** (old LP), regardless of what the new inequality is.	✔	0.80	This is not necessarily true. In fact, the optimal solution (if it exists) of the new LP will be less than or equal to the old LP since the problem is a maximization problem.
☐ **LP-B** (new LP) will always be feasible, no matter what inequality is added.	✔	0.80	Not necessary. Suppose we added the inequality $x_1 \leq -1$, the new LP will become infeasible.
☑ The optimal solution of the **LP-B** (new LP) , if it exists, will always be *less than or equal to* that of the **LP-A** (old LP), regardless of what the new inequality is.	✔	0.80	The feasible region of the new LP is a subset of the old one. Therefore, the optimal solution of the new LP (if exists) will be less than or equal to that of the old LP.
☑ **LP-B** (new LP) can become infeasible for some choice of the new inequality.	✔	0.80	Yes, for instance $x_1 \leq -1$, or any new inequality that contradicts the constraints of the old LP.
☐ The feasible region of **LP-B** (new LP) can become unbounded for some choice of the new inequality.	✔	0.80	The feasible region for the old LP is bounded, therefore the new LP will also have a bounded feasible region. As a result, the answer to the optimization problem can never be unbounded.
Total		4.00 / 4.00	

Question Explanation

When a new constraint is added, there are two possibilities:

1. The feasible region for **LP-B** is now empty, or in other words **LP-B** is infeasible or,
2. The feasible region of the **LP-B** is a non-empty subset of the feasible region for **LP-A** .

Naturally, if the optimal solution to the new problem exists, its objective value has to be less than or equal to that of the old problem (since we are maximizing).