Reductio ad Absurdum

A natural way to show that a statement is true is to show that its opposite is impossible. This approach is known as *reductio ad absurdum* or *proof by contradiction*. This basic idea is used constantly in combination with other methods.

Reductio ad absurdum is known from ancient times. A variation of it was used in the *Socratic method* of argument (described by Plato around 400 BC). Socrates, in dialogues with his students, revealed contradictions in their beliefs by asking them questions one by one and showing that their believes are inconsistent.

Problem. There are boys and girls in a class. Some of them study French, while others study German. Prove that there are a boy and a girl studying different languages.

Let us start by observing that the problem statement assumes that the following conditions hold:

1. There is at least one boy in the class.

2. There is at least one girl in the class.

3. At least one of the students studies French.

4. At least one of the students studies German.

All four conditions are essential: say, if there were no girls in the class, one would not be able to find a girl and a boy studying different languages. The task would also be impossible if one of these languages was not studied by anyone in the class.

To solve the problem, assume that *one cannot find a boy and a girl studying different languages*. This is the opposite of the statement we need to prove. We are going to derive a contradiction from this assumption. This will imply that the statement holds.

To do this, take any girl (there must be at least one!), call her Alice, and assume that she studies French (if she studies German, the argument is analogous). According to our assumption, this must mean that all of the boys in the class study French too. Since there is at least one boy and he studies French, all of the girls must also be studying French, contradicting the assumption that somebody studies German. We assumed that there are no boy and girl studying different languages and reached a contradiction, this implies that our assumption was wrong, that is, there are a boy and a girl studying different languages.