Quo vadis?

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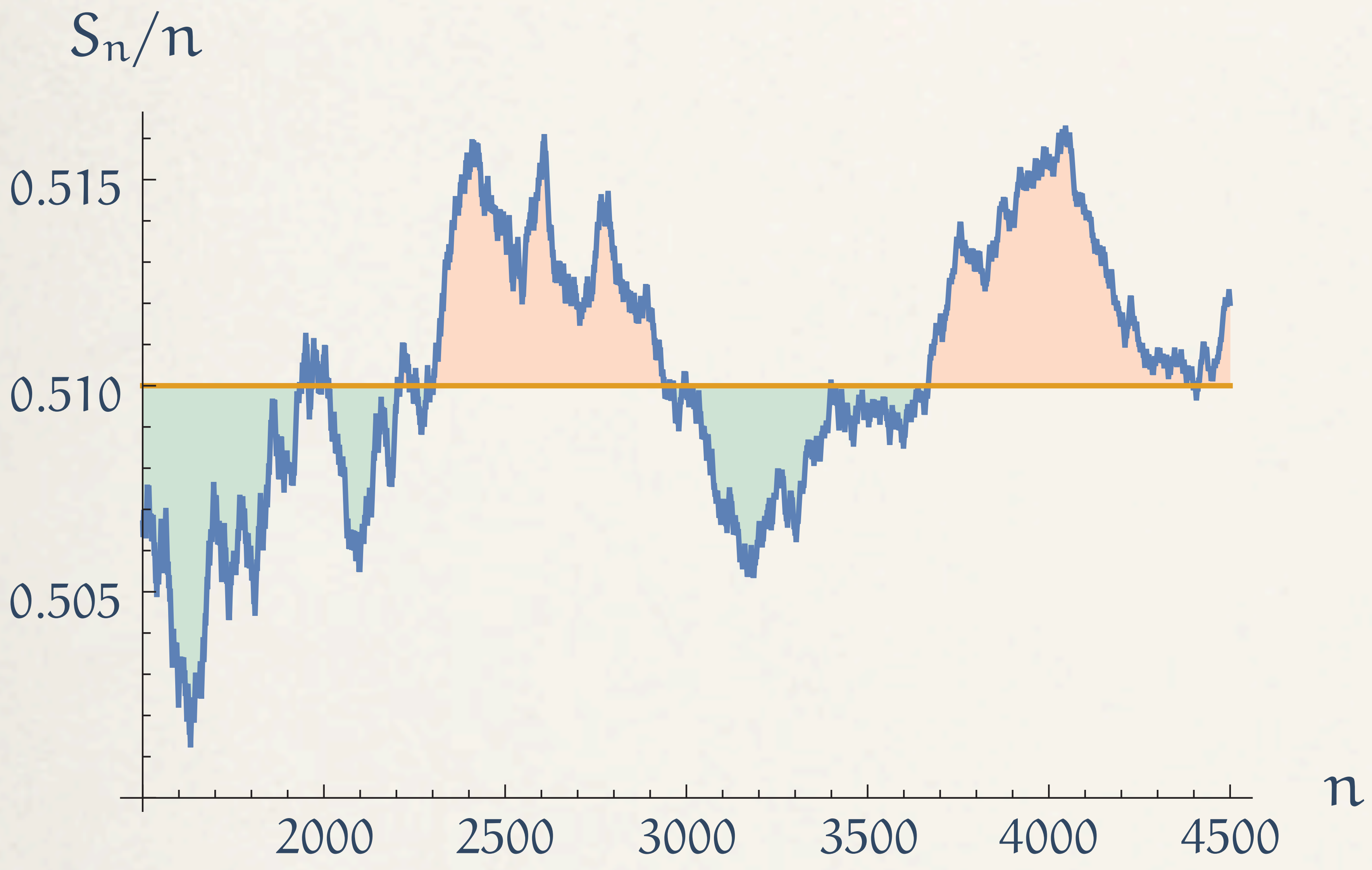

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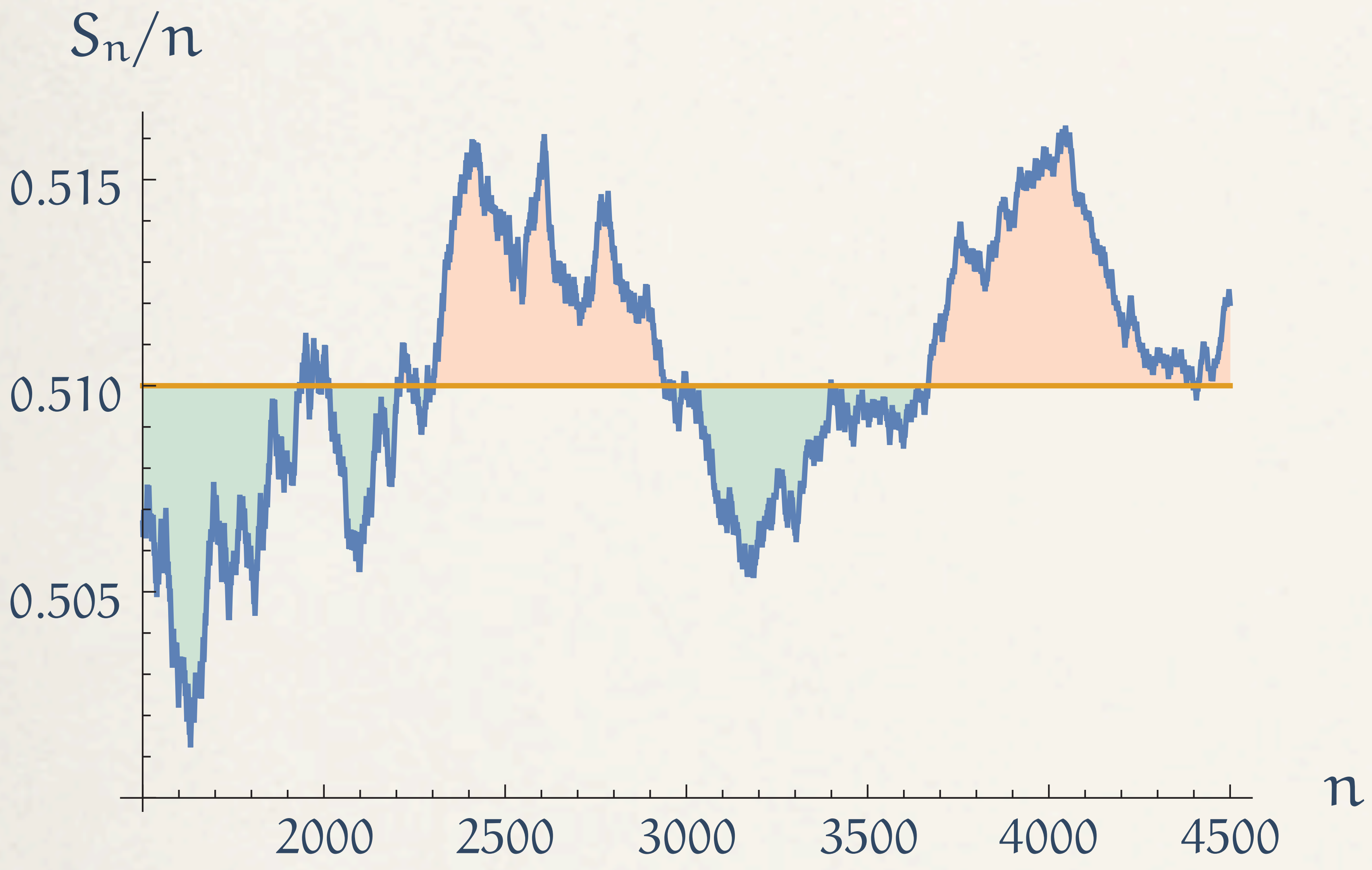

What are the chances that the sample mean repeatedly drifts away from 0.5 by more than 10%? By more than ϵ?

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The strong law of large numbers
(Kolmogorov, 1933)

$$\mathbf{P}\left\{\left|\frac{S_n}{n} - \mu\right| > \epsilon \text{ i.o.}\right\} = 0$$