Two dice

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One die has five red faces and one white face. The other die has five white faces and one red face.
A die is chosen at random and thrown twice.
What is the chance that the second throw shows red given that the first throw shows red?

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  - Random selection of die: $\mathbf{P}(A) = \mathbf{P}(A^c) = 1/2$.

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$$\mathbf{P}(R_1 \cap R_2 \mid A) = 5^2/6^2 = 25/36, \qquad \mathbf{P}(R_1 \cap R_2 \mid A^c) = 1^2/6^2 = 1/36.$$

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$$= \frac{5}{6} \cdot \frac{1}{2} + \frac{1}{6} \cdot \frac{1}{2} = \frac{1}{2} = 0.5$$

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$$\mathbf{P}(R_2 \mid R_1) = \frac{\mathbf{P}(R_1 \cap R_2)}{\mathbf{P}(R_1)} = \frac{\frac{13}{18}\cdot\frac{1}{2}}{\frac{1}{2}} = \frac{13}{18} = 0.72\dot{2}$$

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What is the mass function for this problem?