QUESTION 5

Theorem $\lim_{n\to\infty} \dfrac{n+1}{2n+1} = \dfrac{1}{2}$.

Proof: Let $\epsilon > 0$ be given. Choose N large enough so that $N \geq \dfrac{1}{2\epsilon}$.

Then, for $n \geq N$,

$$\begin{aligned}
\left|\frac{n+1}{2n+1} - \frac{1}{2}\right| &= \left|\frac{2(n+1)-(2n+1)}{2(2n+1)}\right| \\
&= \left|\frac{1}{2(2n+1)}\right| \\
&= \frac{1}{2(2n+1)} \\
&< \frac{1}{2n+1} \\
&< \frac{1}{2n} \leq \frac{1}{2N} \leq \epsilon
\end{aligned}$$

By the definition of a limit, this proves the theorem.