

The probability of conviction

G occurs if $Z = 1$

C occurs if $\begin{cases} S_{12} \leq 5 & \text{when } Z = 1, \\ S_{12} \geq 7 & \text{when } Z = 0. \end{cases}$

$$\mathbf{P(C)} = \mathbf{P(C \mid G)} \mathbf{P(G)} + \mathbf{P(C \mid G^0)} \mathbf{P(G^0)}$$

Additivity: The theorem of total probability!

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$$Z \sim \text{Bernoulli}(p), \quad S_{12} \sim \text{Binomial}(12, p)$$

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Diagram illustrating the binomial probability formula with red arrows indicating the correspondence between terms: p^k in the first term corresponds to p^k in the second term, and $(1-p)^{12-k}$ in the first term corresponds to $(1-p)^{12-k}$ in the second term.

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The equation shows the symmetry of the binomial distribution. Red curved arrows indicate the mapping of k to $12-k$ in the binomial coefficient and the corresponding swap of p and $1-p$ in the probability terms.

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Probability of conviction with a dissenting minority vote of exactly k