Credible interval

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In Bayesian statistics, a **credible interval** is an interval in the domain of a posterior probability distribution or predictive distribution used for interval estimation.[1] The generalisation to multivariate problems is the **credible region**. Credible intervals are analogous to confidence intervals in frequentist statistics,[2] although they differ on a philosophical basis;[3] Bayesian intervals treat their bounds as fixed and the estimated parameter as a random variable, whereas frequentist confidence intervals treat their bounds as random variables and the parameter as a fixed value. Also, Bayesian credible intervals use (and indeed, require) knowledge of the situation-specific prior distribution, while the frequentist confidence intervals do not.

For example, in an experiment that determines the uncertainty distribution of parameter μ, if the subjective probability that μ lies between 35 and 45 is 0.95, then $35 \leq \mu \leq 45$ is a 95% credible interval.

Choosing a credible interval

Credible intervals are not unique on a posterior distribution. Methods for defining a suitable credible interval include:

- Choosing the narrowest interval, which for a unimodal distribution will involve choosing those values of highest probability density including the mode. This is sometimes called the **highest posterior density interval**.
- Choosing the interval where the probability of being below the interval is as likely as being above it. This interval will include the median. This is sometimes called the **equal-tailed interval**.
- Assuming that the mean exists, choosing the interval for which the mean is the central point.

It is possible to frame the choice of a credible interval within decision theory and, in that context, an optimal interval will always be a highest probability density set.[4]

Contrasts with confidence interval

A frequentist 95% confidence interval means that with a large number of repeated samples, 95% of such calculated confidence intervals would include the true value of the parameter. In frequentist terms, the parameter is *fixed* (cannot be considered to have a distribution of possible values) and the confidence interval is *random* (as it depends on the random sample).

Bayesian credible intervals can be quite different from frequentist confidence intervals for two reasons:

- credible intervals incorporate problem-specific contextual information from the prior distribution whereas confidence intervals are based only on the data;
- credible intervals and confidence intervals treat nuisance parameters in radically different ways.

For the case of a single parameter and data that can be summarised in a single sufficient statistic, it can be shown that the credible interval and the confidence interval *will* coincide if the unknown parameter is a location parameter (i.e. the forward probability function has the form $\Pr(x|\mu) = f(x - \mu)$), with a prior that is a uniform flat distribution;[5] and also if the unknown parameter is a scale parameter (i.e. the forward probability function has the form $\Pr(x|s) = f(x/s)$), with a Jeffreys' prior $\Pr(s|I) \propto 1/s$ [5] — the latter following because taking the logarithm of such a scale parameter turns it into a location parameter with a uniform distribution. But these are distinctly special (albeit important) cases; in general no such equivalence can be made.

References

1. Edwards, Ward, Lindman, Harold, Savage, Leonard J. (1963) "Bayesian statistical inference in psychological research". *Psychological Review*, **70**, 193-242
2. Lee, P.M. (1997) *Bayesian Statistics: An Introduction*, Arnold. ISBN 0-340-67785-6
3. "Frequentism and Bayesianism" (http://jakevdp.github.io/blog/2014/06/12/frequentism-and-bayesianism-3-confidence-credibility/).
4. O'Hagan, A. (1994) *Kendall's Advanced Theory of Statistics, Vol 2B, Bayesian Inference*, Section 2.51. Arnold, ISBN 0-340-52922-9
5. Jaynes, E. T. (1976). "Confidence Intervals vs Bayesian Intervals (http://bayes.wustl.edu/etj/articles/confidence.pdf)", in *Foundations of Probability Theory, Statistical Inference, and Statistical Theories of Science*, (W. L. Harper and C. A. Hooker, eds.), Dordrecht: D. Reidel, pp. 175 *et seq*

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