A simple computation — and a dimensional tweak to the tale from physics

A problem in numerical integration

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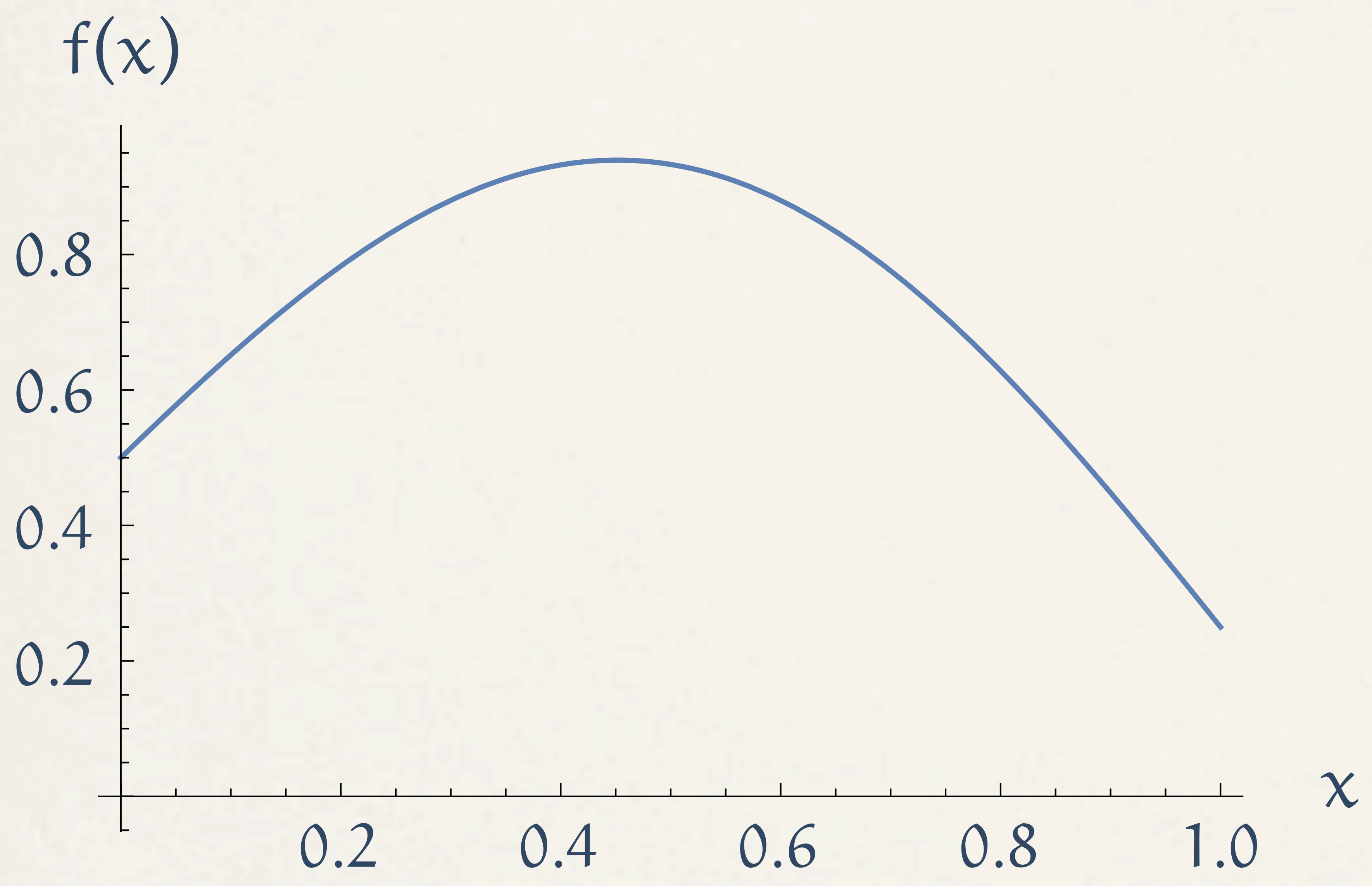

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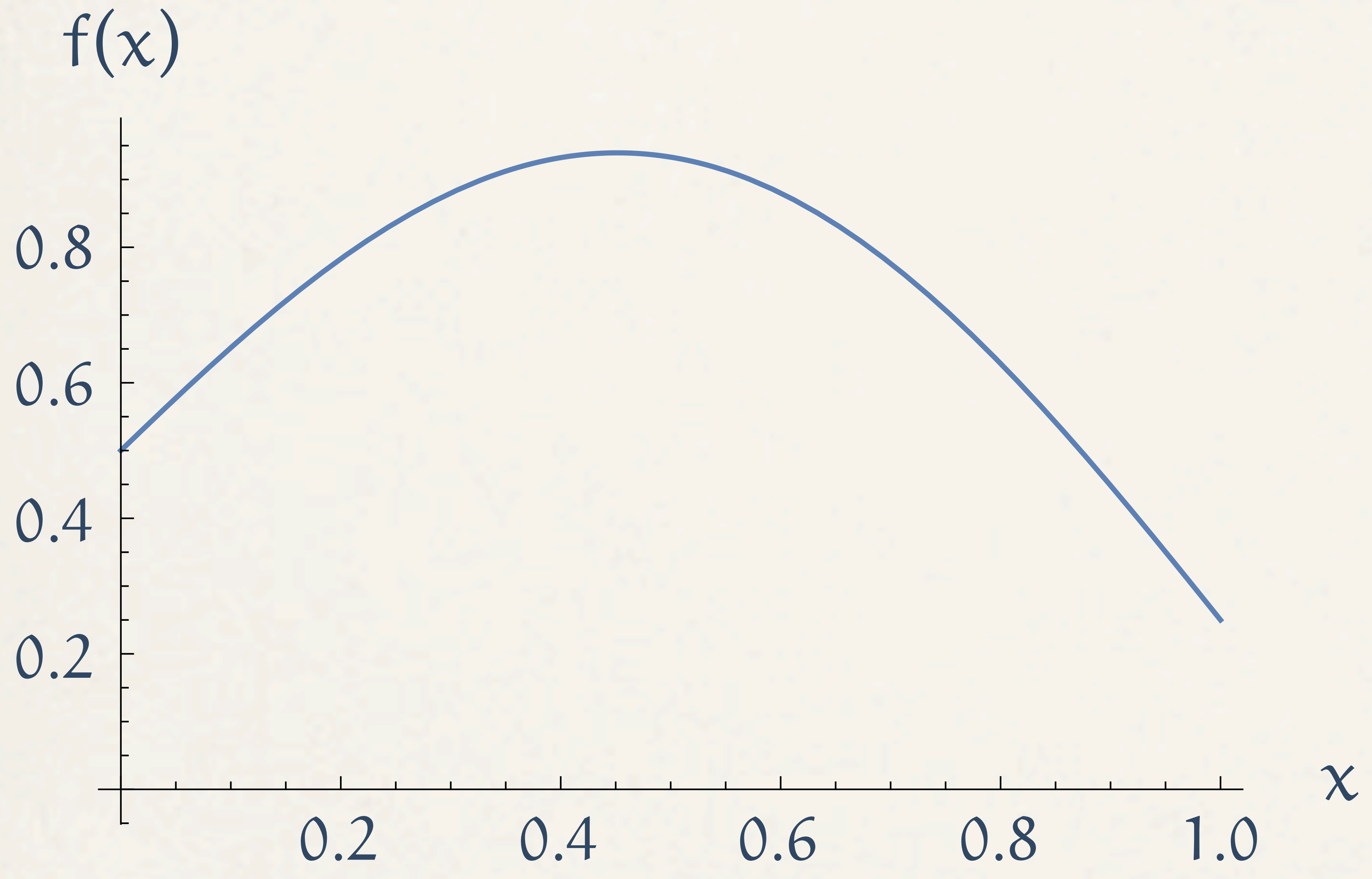

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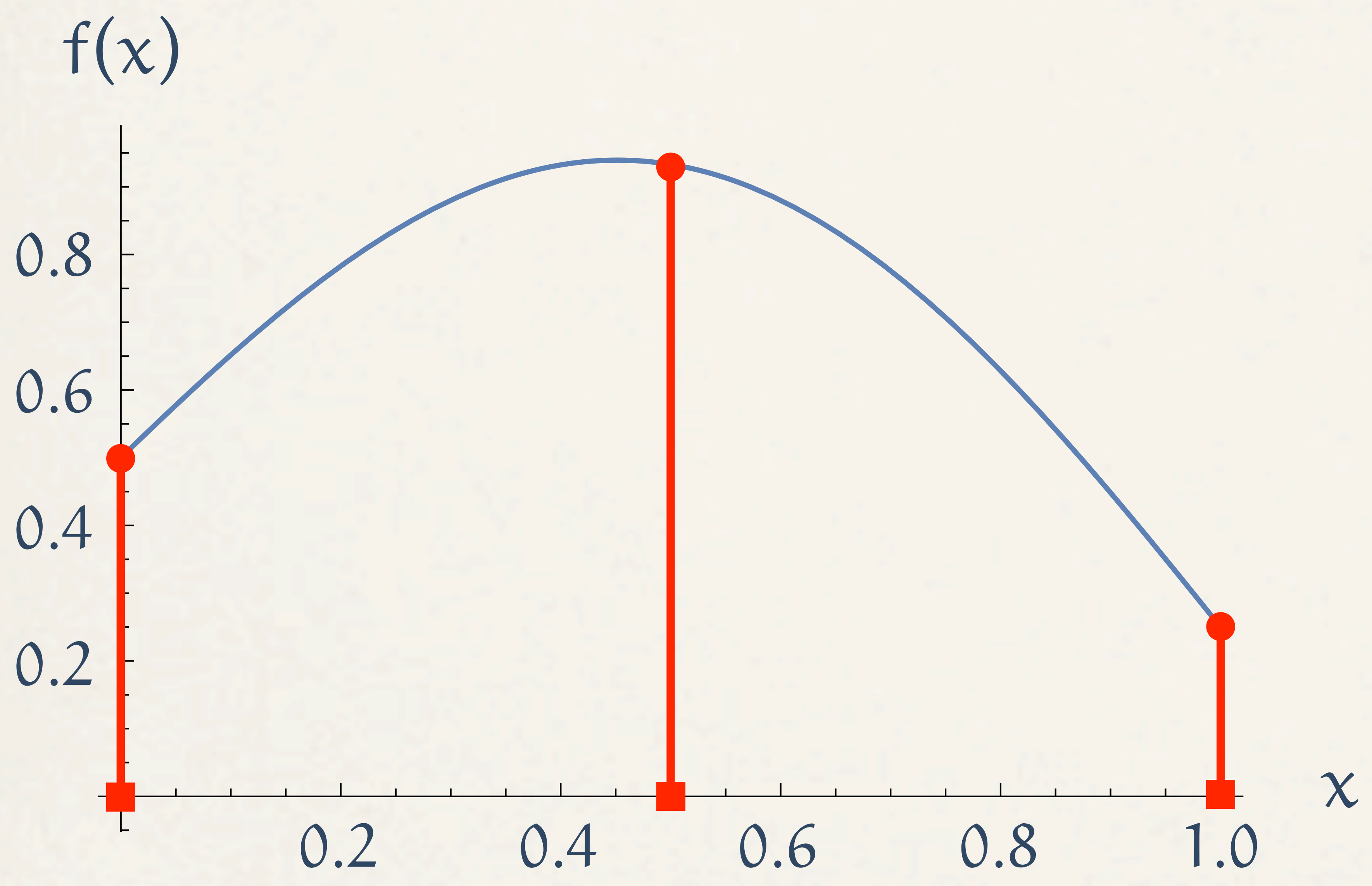

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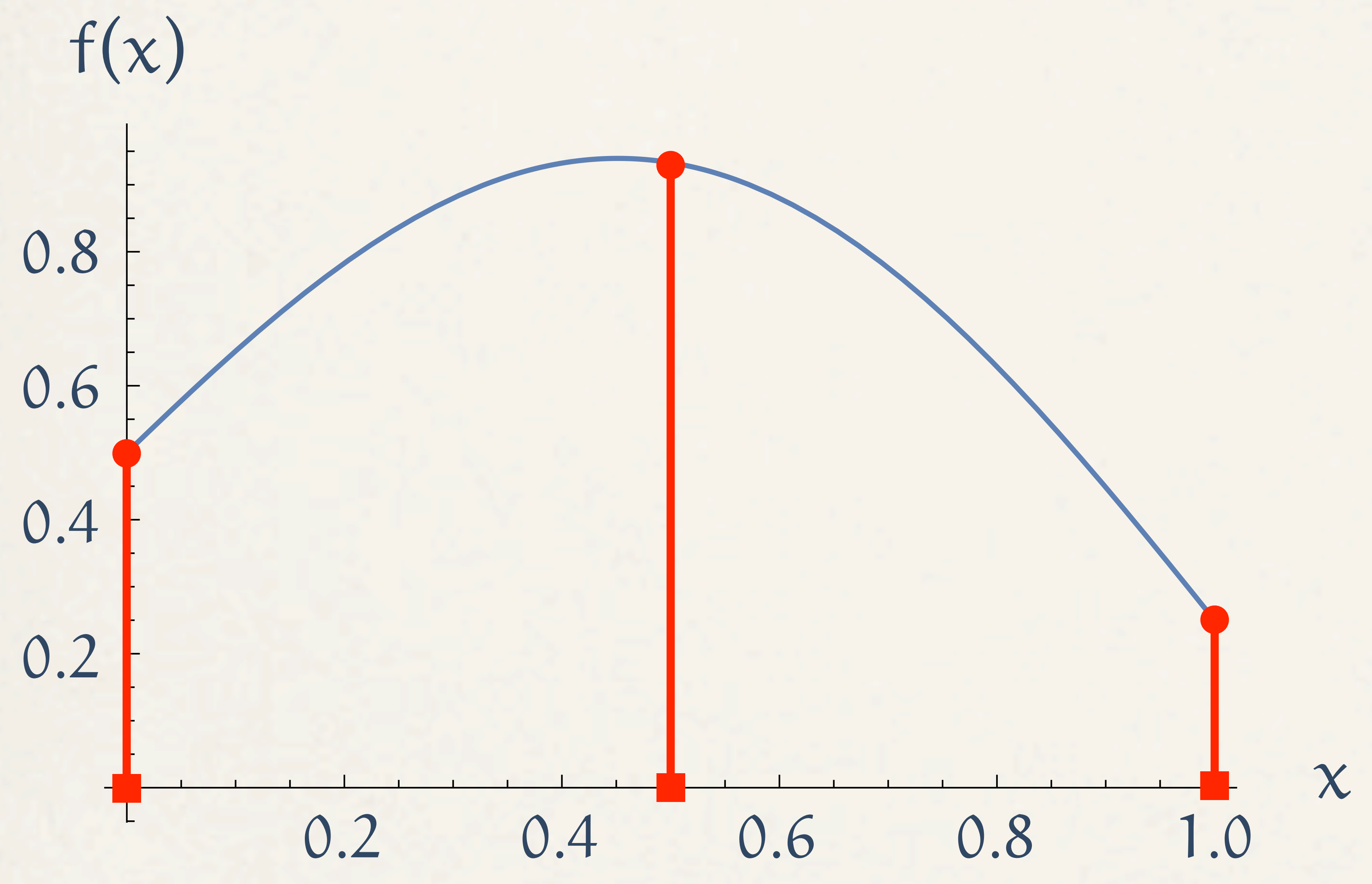

$$J = \int_0^1 f(x)\,dx \approx \frac{1}{6}\left[f(0) + 4f\left(\tfrac{1}{2}\right) + f(1)\right]$$

Simpson's rule

Scaling with dimension

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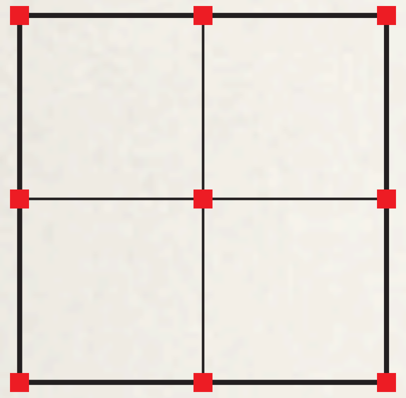

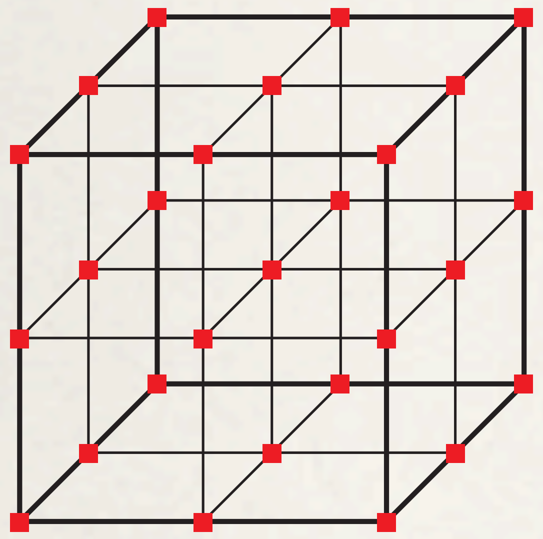

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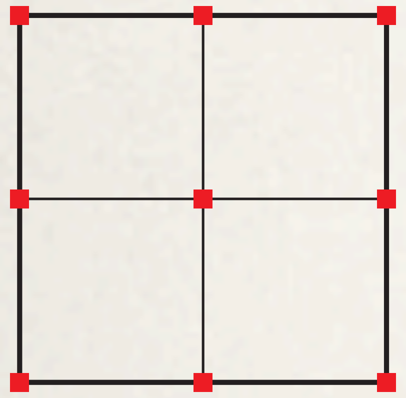

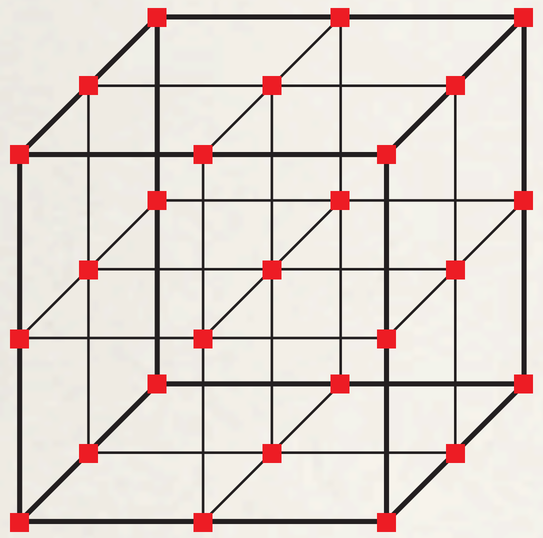

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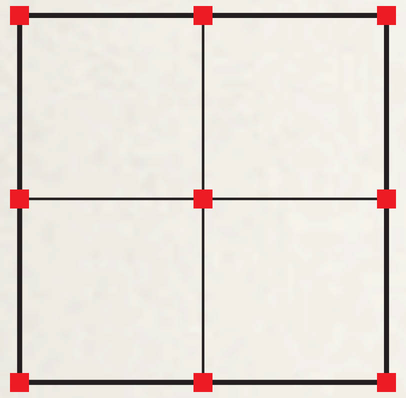
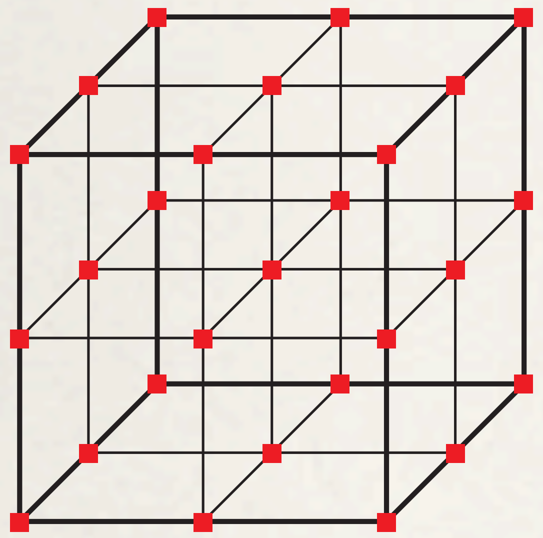


dimension	# computations



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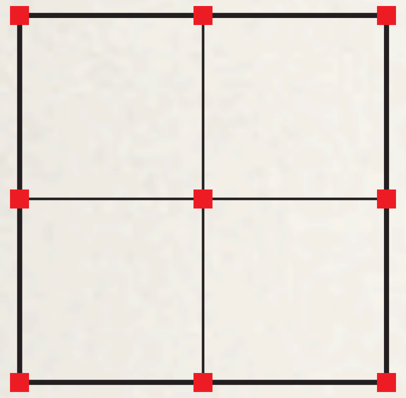
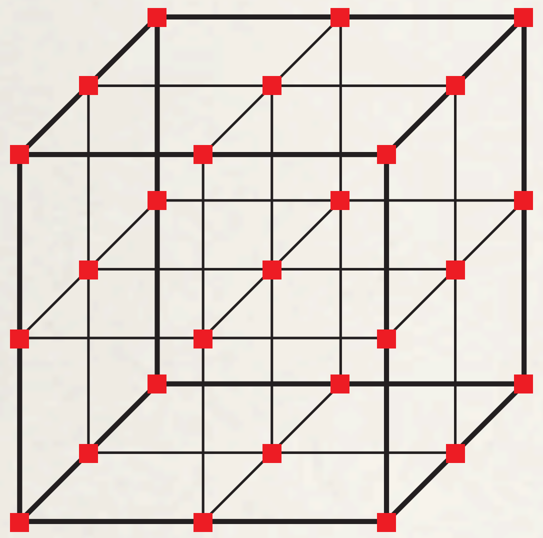


dimension	# computations
1	3



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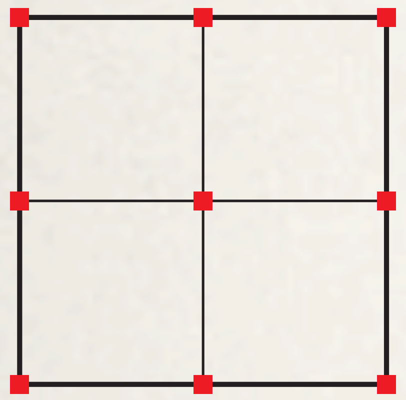
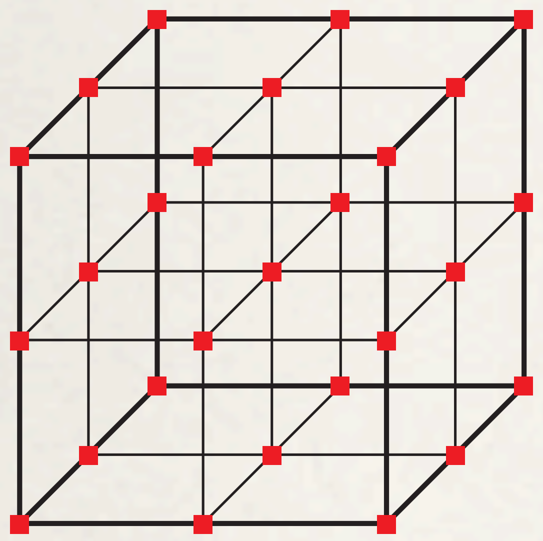


dimension	# computations
1	3
2	9



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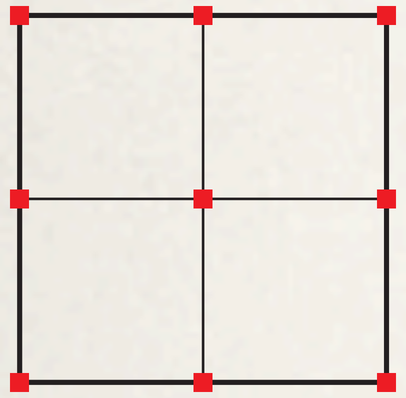
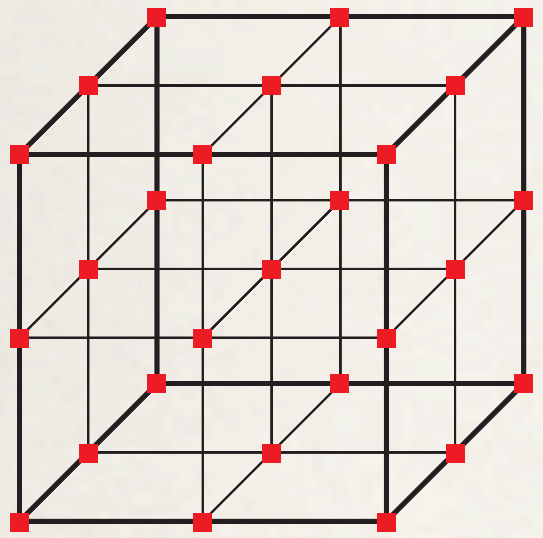


dimension	# computations
1	3
2	9
3	27



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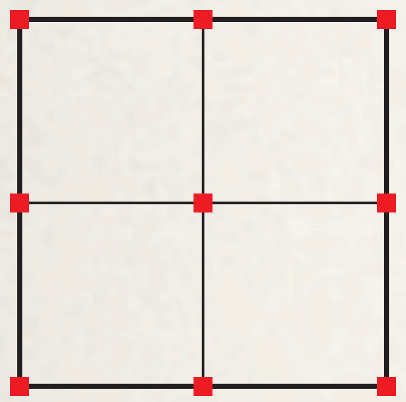

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1	3
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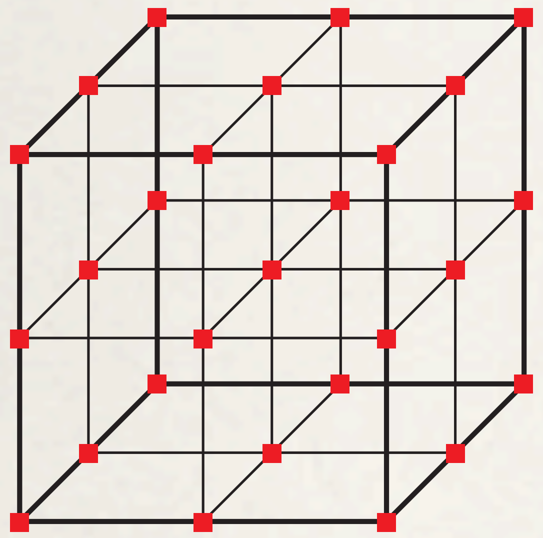
$\vdots$	$\vdots$
D	3^D



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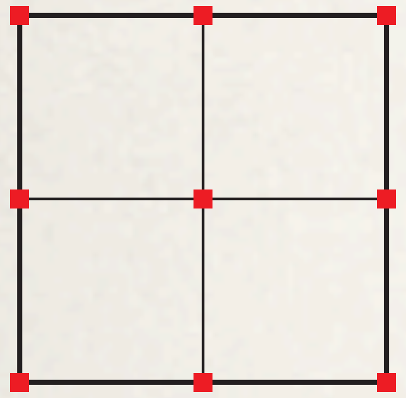

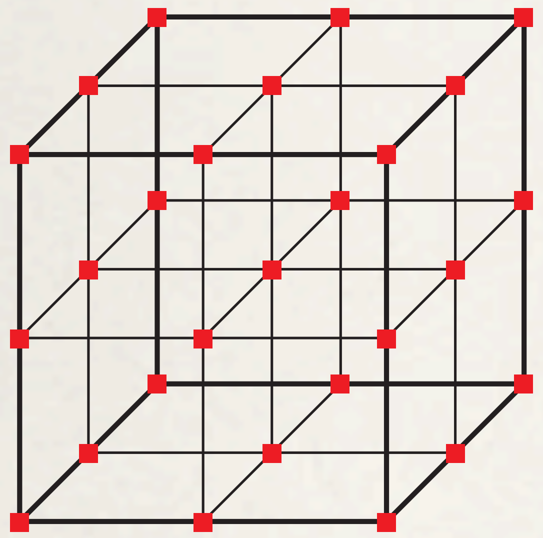

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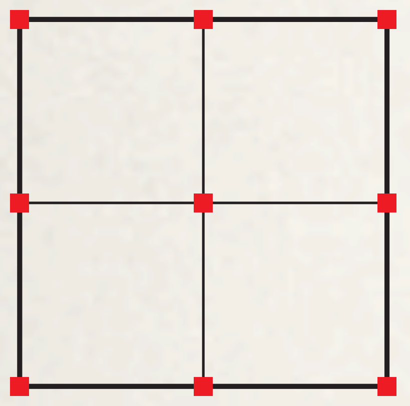

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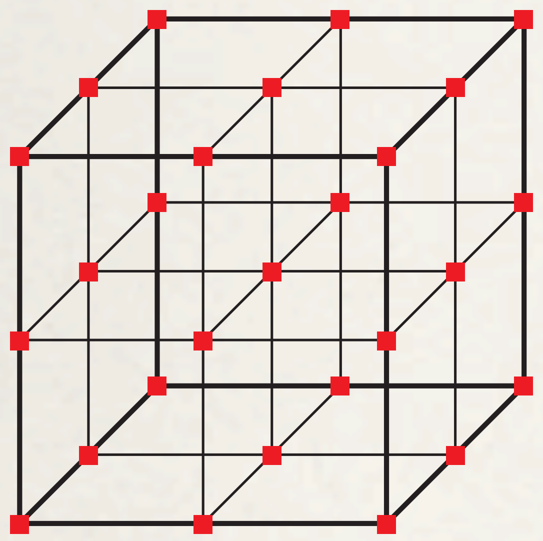

This is impossible! Even approximately.