A little algebraic spadework

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The basic binomial identity: $k \cdot b_n(k;p) = np \cdot b_{n-1}(k-1;p)$

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