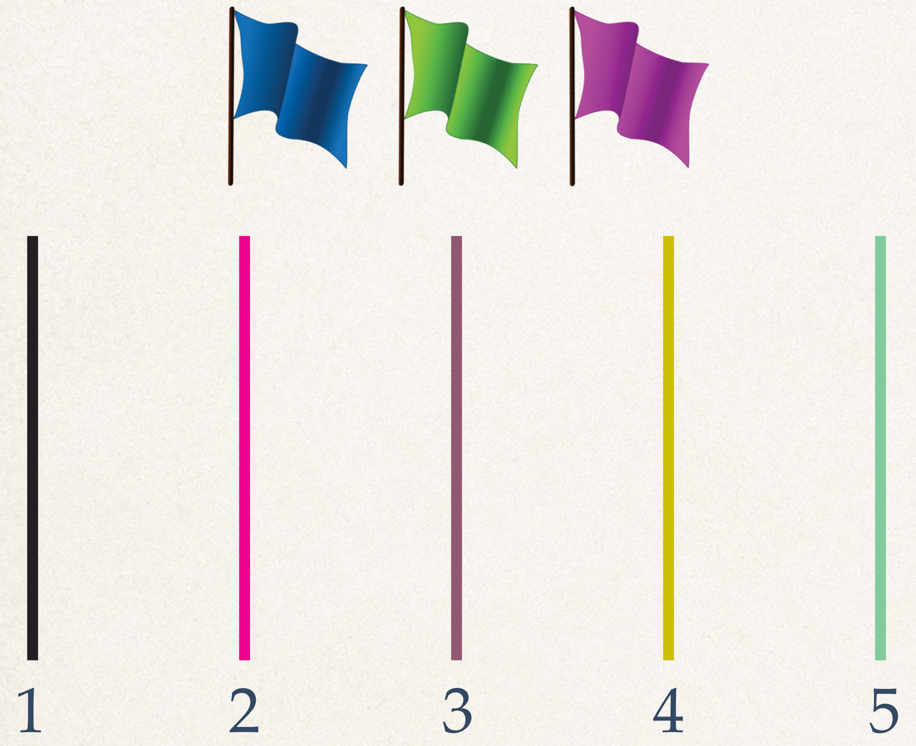
# Sampling without replacement

How many ways are there of distributing three flags upon five flagsticks?



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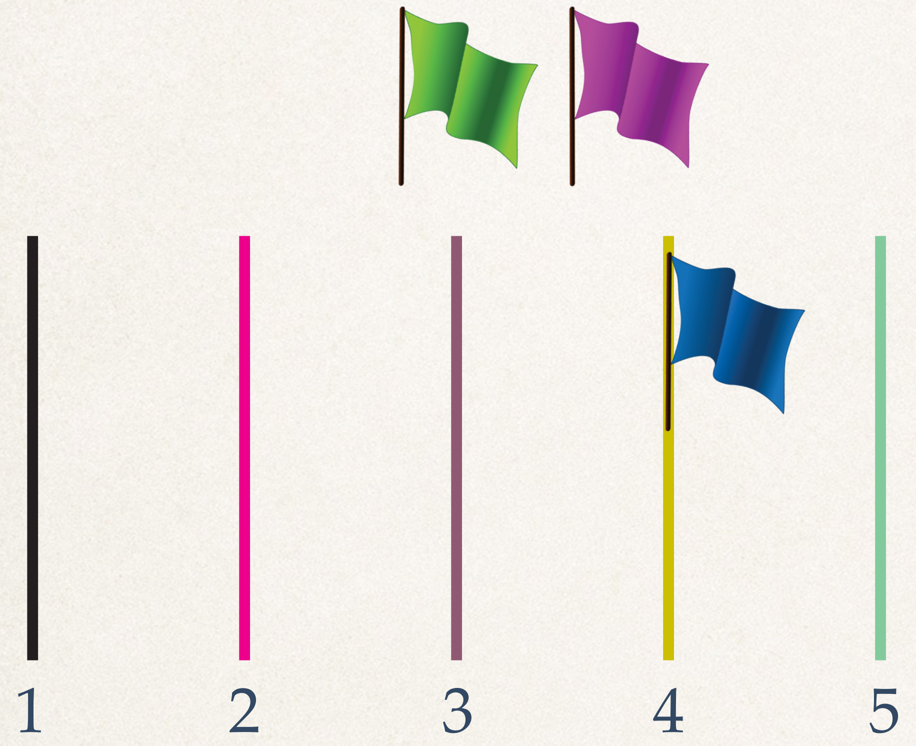

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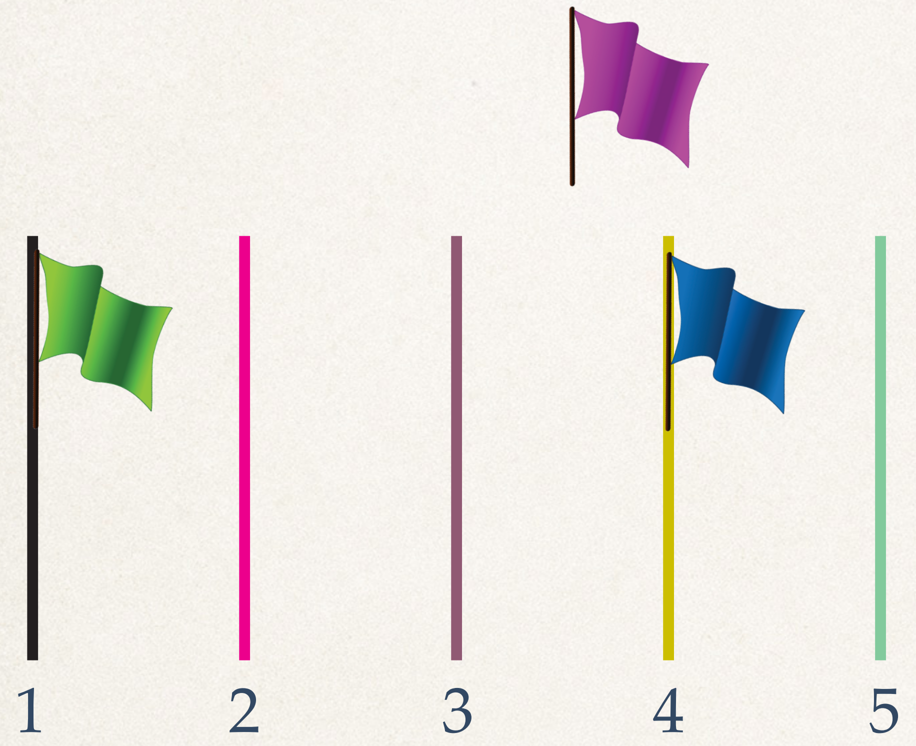

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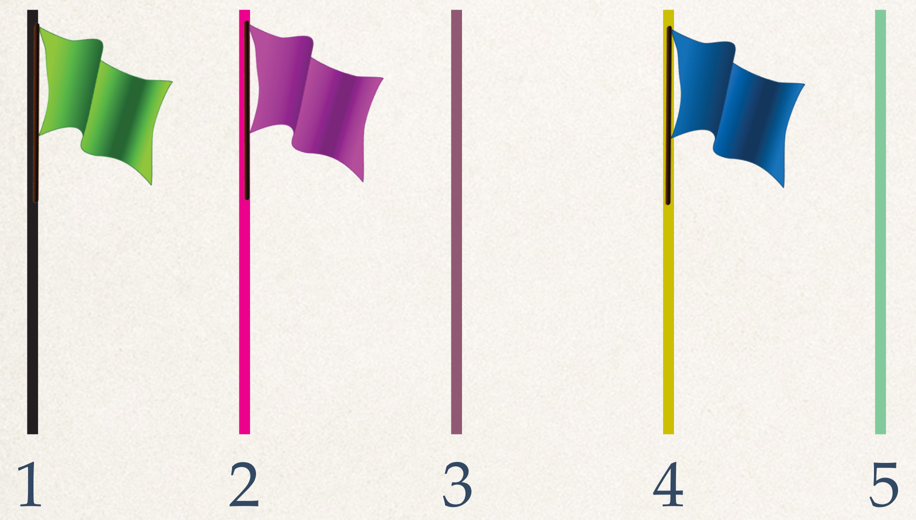

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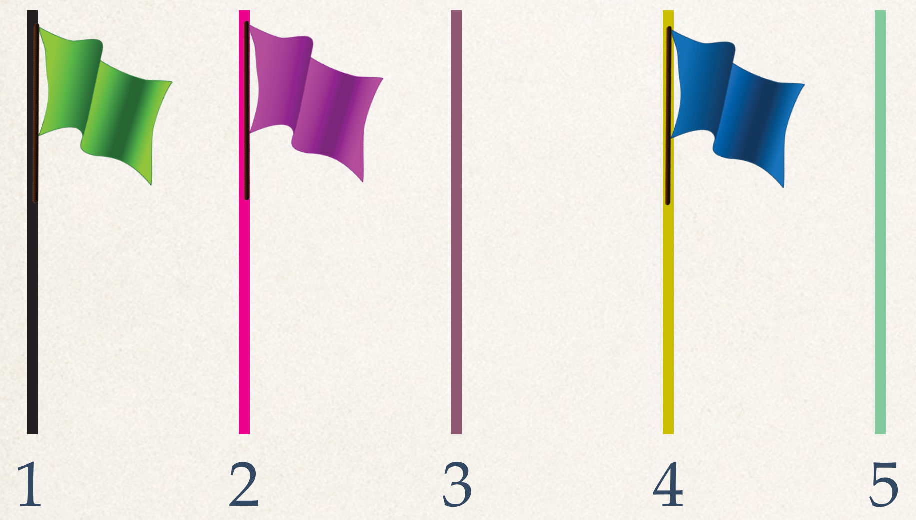
(5, 1, 2)



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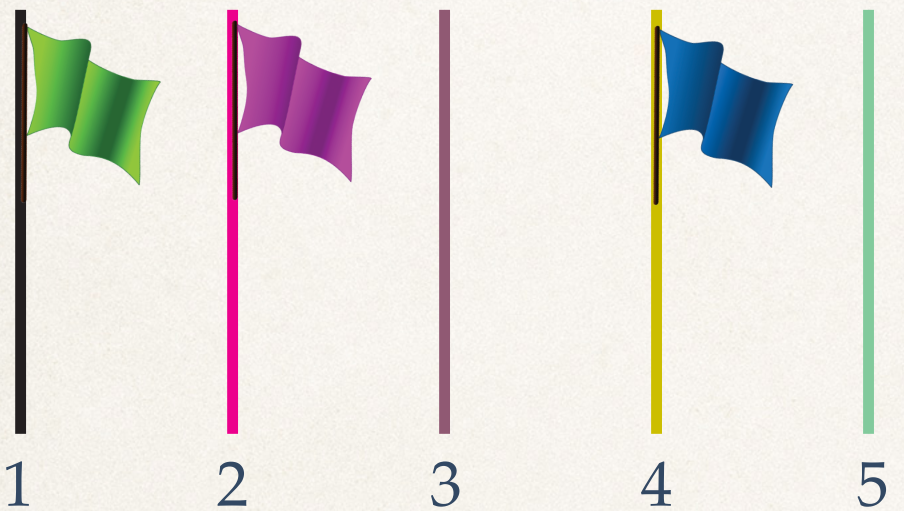


With the flags placed in a specified sequence, this is equivalent to specifying an ordered triple of sticks, say, $(j_1, j_2, j_3)$, where sampling is without replacement.

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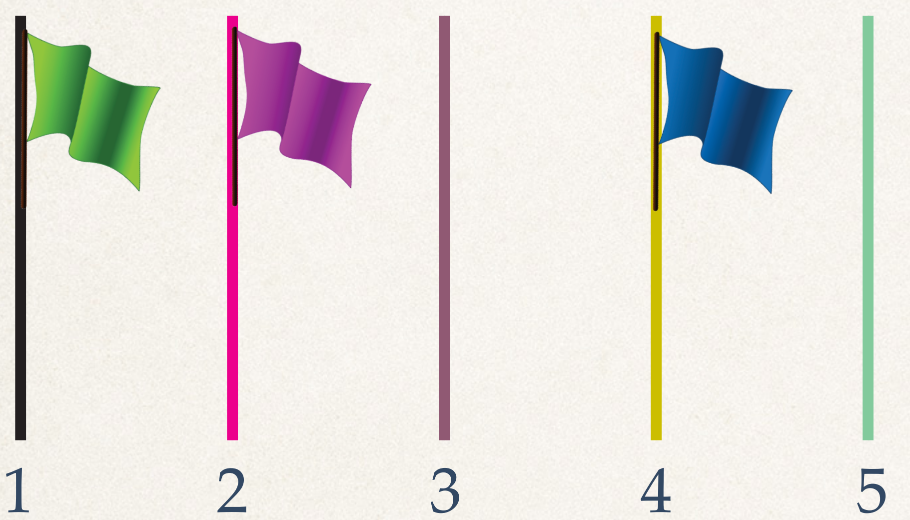


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Falling factorial, Pochhammer's symbol: "5-to-3-falling"

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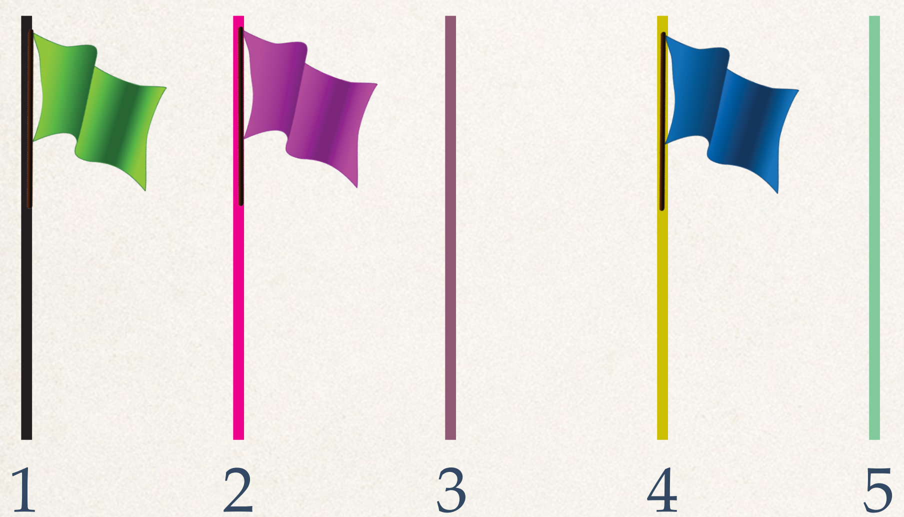


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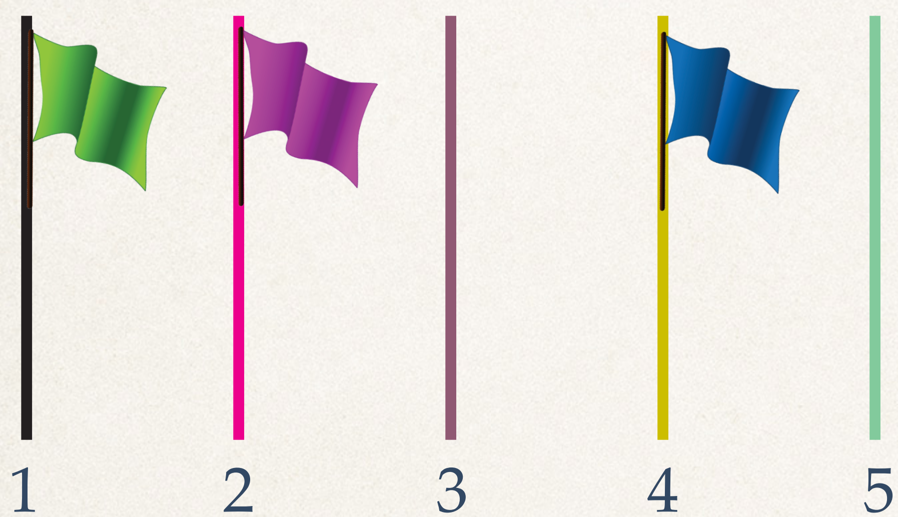


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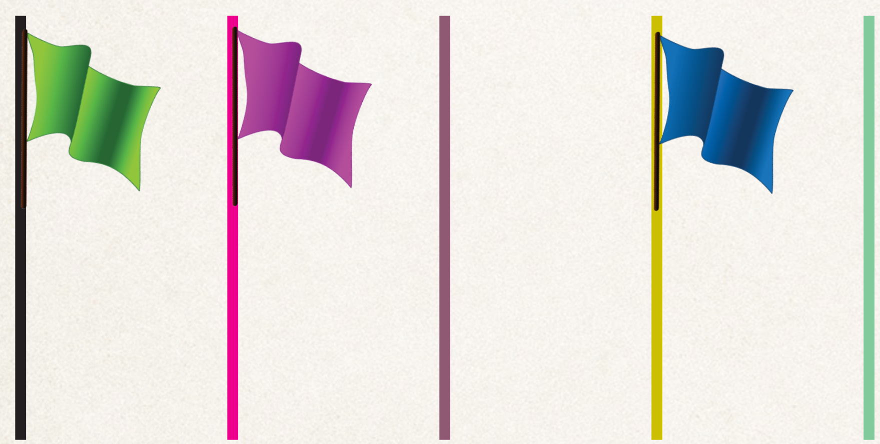

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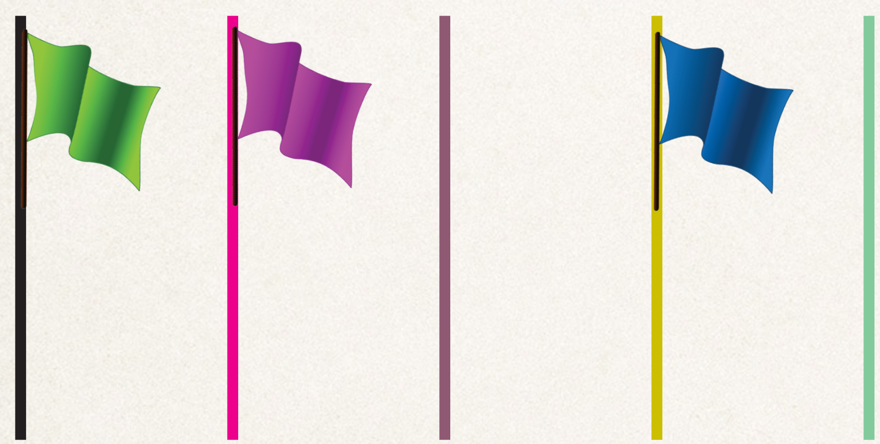

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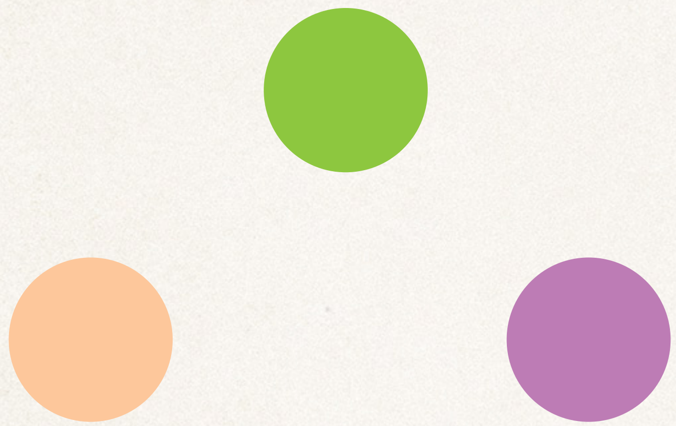
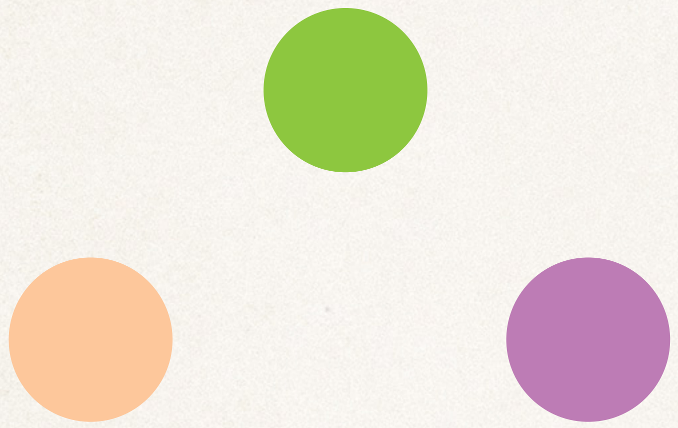
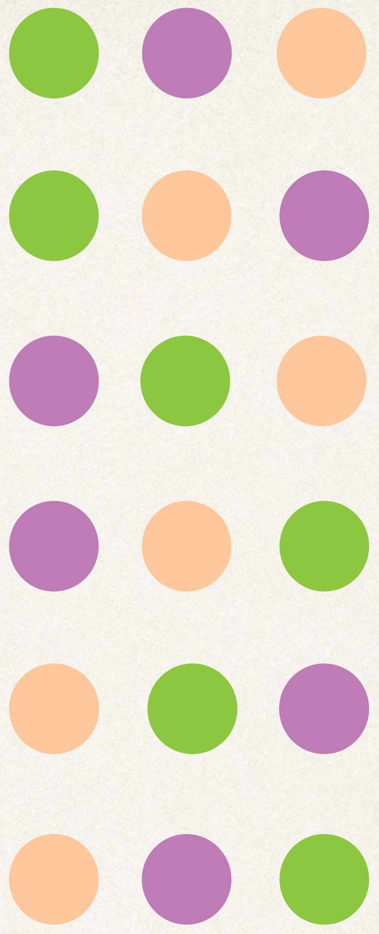
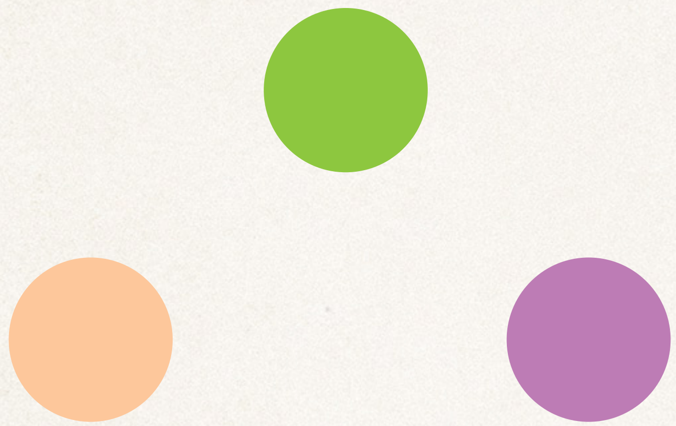
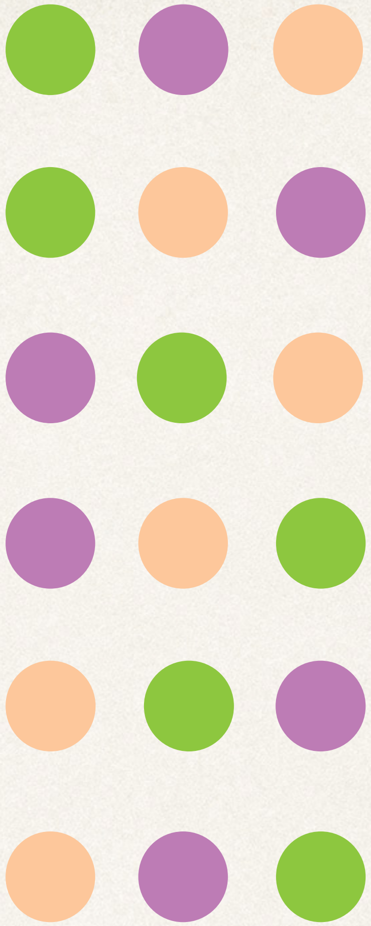




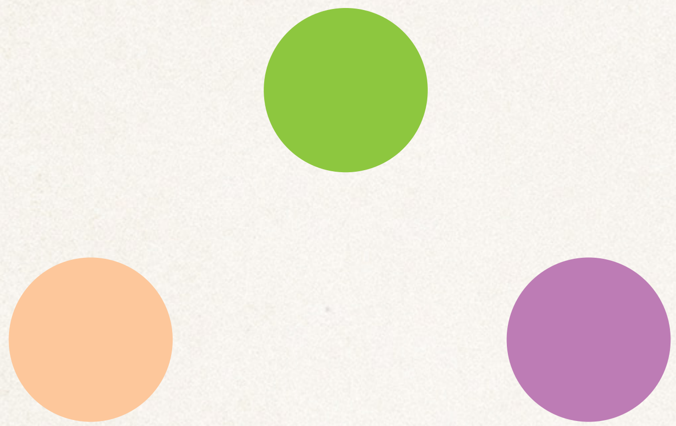




Lexicographic
arrangement



# How many ways can n distinct objects be arranged?



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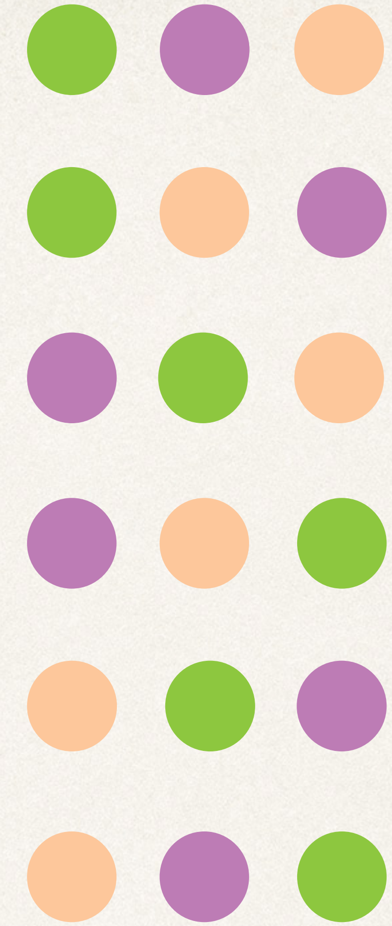

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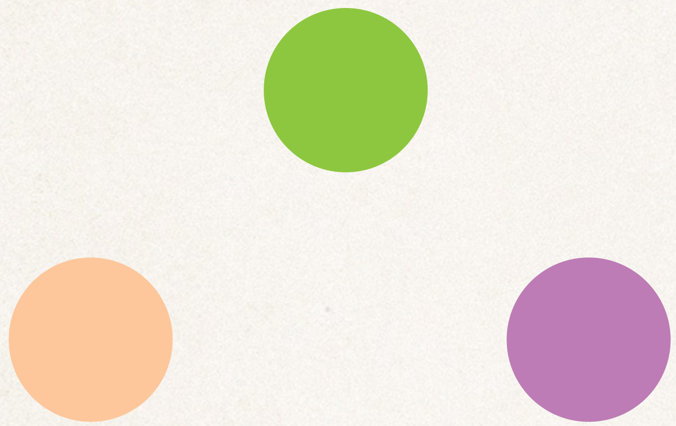

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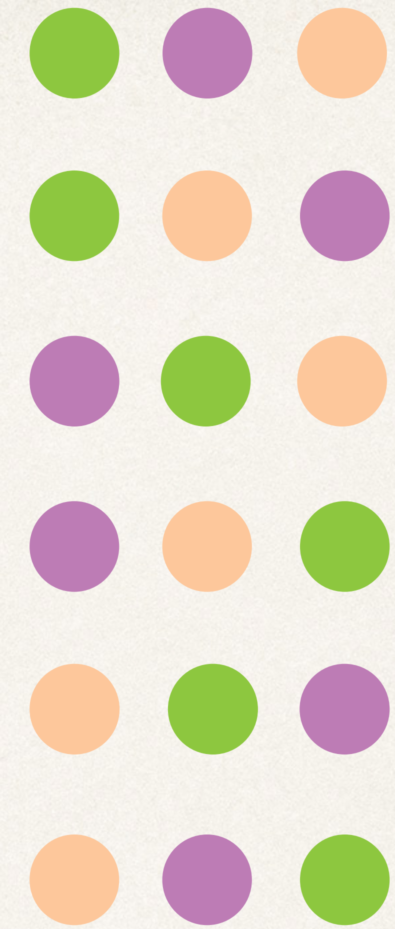

This results in an ordered sample of $n$ elements selected without replacement with each object selected.

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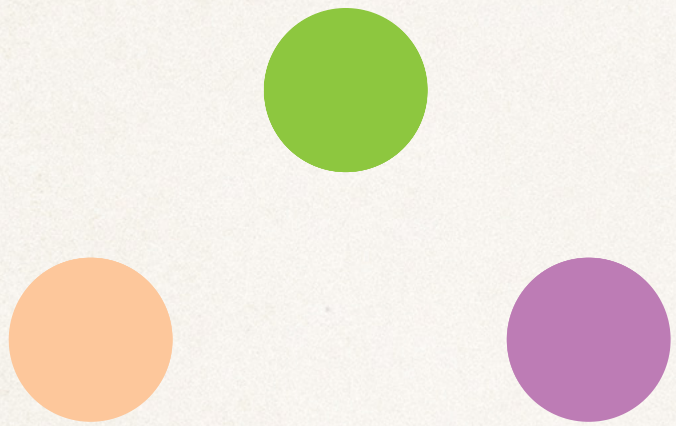

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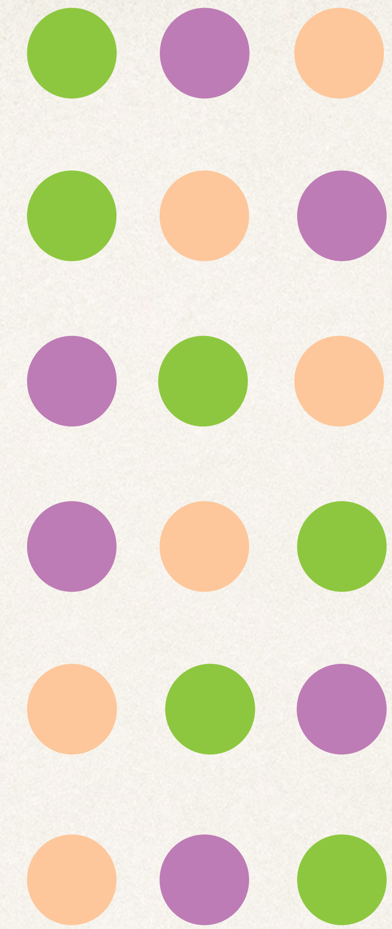

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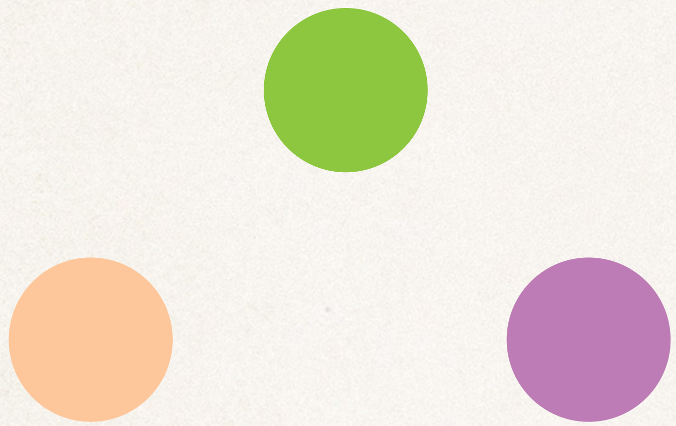

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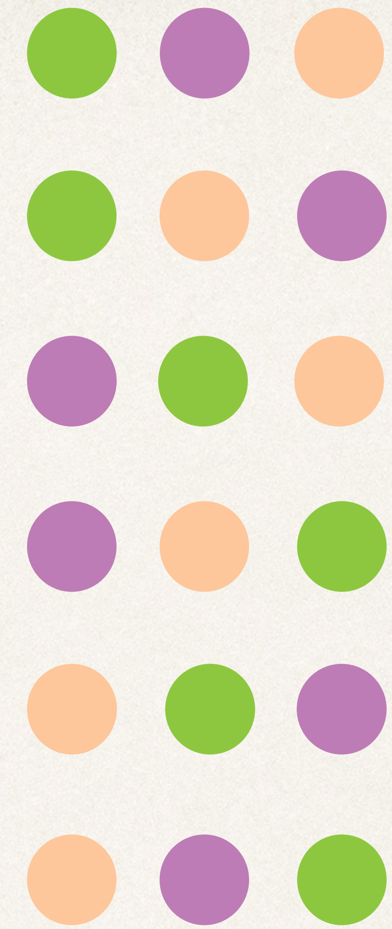

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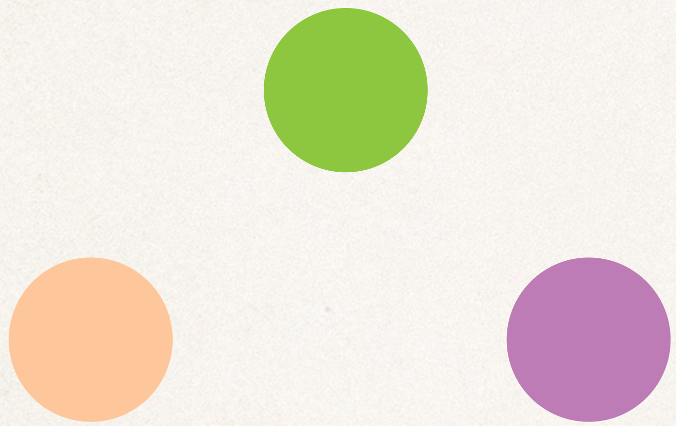

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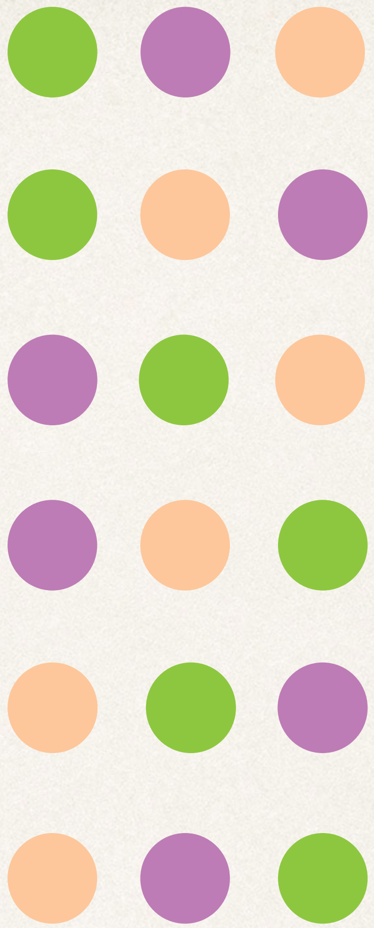

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Convention: $0! = 1$.