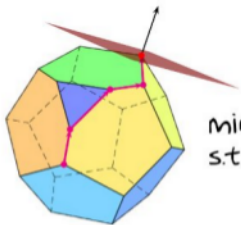


How efficient is the simplex method?

- Proving the Kalai & Kleitman bound



$$\begin{array}{ll} \min & c^T x \\ \text{s.t.} & A x \leq b \end{array}$$

How many layers has a connected layer family?

$h(n, m)$: Maximum number of layers of a n -dimensional connected layer family with m symbols ($m \geq n \geq 1$).

Goal: Prove $h(n, m) \leq m^{1+\log_2 n}$

Induction on m

$m=1, m=2$ (✓)

$m > 2, n=1$ (✓)

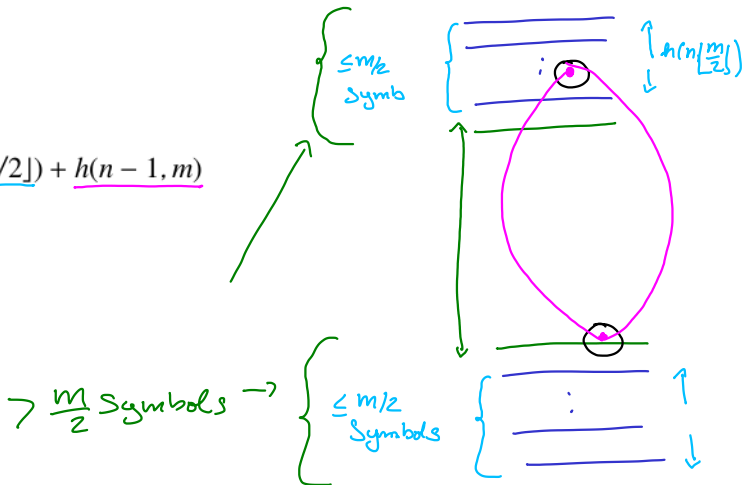
$m > 2, n \geq 2$

How many layers has a connected layer family?

$h(n, m)$: Maximum number of layers of a n -dimensional connected layer family with m symbols ($m \geq n \geq 1$).

Goal: Prove $h(n, m) \leq m^{1+\log n}$

$$n \geq 2: \quad h(n, m) \leq 2 \cdot \underbrace{h(n, \lfloor m/2 \rfloor)} + \underbrace{h(n-1, m)}$$



Polynomial Hirsch conjecture

Prominent open problem

Is $\Delta(n, m)$ bounded by a polynomial in n and m ?

- ▶ Hirsch conjecture: $\Delta_b(n, m) \leq m - n$ (Warren Hirsch (1957))
- ▶ Counterexample found by Santos (2010)
- ▶ Hähnle's Conjecture (2010): $\Delta(n, m) \leq n \cdot (m - 1)$ (even in abstract setting, see polymath 3)