Vector Derivative Identities

Use the Kronecker delta, the Levi-Civita symbol and the Einstein summation convention, and the identities

$$\boldsymbol{a} \cdot \boldsymbol{b} = \delta_{ij} a_i b_j, \quad (\boldsymbol{a} \times \boldsymbol{b})_i = \epsilon_{ijk} a_j b_k, \quad \epsilon_{ijk}\epsilon_{ilm} = \delta_{jl}\delta_{km} - \delta_{jm}\delta_{kl},$$

to prove the following identities:

(a) $\boldsymbol{\nabla} \cdot (f\boldsymbol{u}) = \boldsymbol{u} \cdot \boldsymbol{\nabla} f + f\boldsymbol{\nabla} \cdot \boldsymbol{u}$;

(b) $\boldsymbol{\nabla} \times (\boldsymbol{\nabla} \times \boldsymbol{u}) = \boldsymbol{\nabla}(\boldsymbol{\nabla} \cdot \boldsymbol{u}) - \nabla^2 \boldsymbol{u}$.