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    - No. Positivity will be violated for any choice of C as $p(k)$ alternates sign for odd and even values of k.