Summary of Tableau 9, Part 2

Independence
Repeated independent trials, product spaces

- *Informally*: Independence captures the intuitive idea that if events are independent then the occurrence of one does not affect the chances of the others.

- The *formal definition*: A finite or countably infinite collection of events $\{A_j, j \geq 1\}$ in a probability space is independent if (and only if), for *every finite* subset $\mathbb{J}$ of indices (positive integers), we have a rule of products

$$\mathbf{P}\Big(\bigcap_{j \in \mathbb{J}} A_j\Big) = \prod_{j \in \mathbb{J}} \mathbf{P}(A_j).$$

- From independent trials to product spaces and product measure:
  - *Individual chance experiments, independent trials*:
    - Trials: $\mathfrak{A}_1 = \{a_{1k}, k \geq 1\}, \ldots, \mathfrak{A}_n = \{a_{nk}, k \geq 1\}$.
    - Atomic mass functions: $\{a_{1k}\} \mapsto p_1(k), \ldots, \{a_{nk}\} \mapsto p_n(k)$.
  - *Compound chance experiment, product space and measure*:
    - Sample space: $\Omega = \mathfrak{A}_1 \times \cdots \times \mathfrak{A}_n = \{(a_{1k_1}, \ldots, a_{nk_n}): k_1 \geq 1, \ldots, k_n \geq 1\}$.
    - Atomic measure: $\mathbf{P}\{(a_{1k_1}, \ldots, a_{nk_n})\} := p_1(k_1) \times \cdots \times p_n(k_n)$.

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Slogan
In the case of a finite or even countably infinite number of repeated independent trials, events in the compound experiment (product space) that are determined by non-overlapping groups of trials are independent.