Question 1

Dominance

1\ 2	x	y	z
a	1,2	2,2	5,1
b	4,1	3,5	3,3
c	5,2	4,4	7,0
d	2,3	0,4	3,0

Find the strictly dominant strategy:

Your Answer		Score	Explanation
3) c;	✔	1.00	
Total		1.00 / 1.00	

Question Explanation

(3) c is a strictly dominant strategy.

- Because when 2 plays x or y or z, playing c always gives 1 a strictly higher payoff than playing a, b or d.
- None of the strategies is always strictly best for player 2.

Question 2

Dominance

1\ 2	x	y	z
a	1,2	2,2	5,1
b	4,1	3,5	3,3
c	5,2	4,4	7,0
d	2,3	0,4	3,0

Find a very weakly dominant strategy that is not also strictly dominant.

Your Answer		Score	Explanation
6) y;	✔	1.00	
Total		1.00 / 1.00	

Question Explanation

(6) *y* is a very weakly dominant strategy that is not also strictly dominant.

- Because when 1 plays *a*, *b*, *c* or *d*, playing *y* always gives 2 a weakly higher payoff than playing *x* or *z*.
- Note that it is only weakly higher when 1 plays *a*, as then playing *x* and *y* gives 2 the same payoff.

Question 3

Dominance

1\ 2	x	y	z
a	1,2	2,2	5,1
b	4,1	3,5	3,3
c	5,2	4,4	7,0
d	2,3	0,4	3,0

When player 1 plays d, what is player 2's best response:

Your Answer		Score	Explanation
b) Only y	✔	1.00	
Total		1.00 / 1.00	

Question Explanation

(b) only y is a best response for player 2. When player 1 plays d, player 2 earns 3 from playing x, 4 from playing y and 0 from playing z. Thus only y is a best response.

Question 4

Dominance

1\ 2	x	y	z
a	1,2	2,2	5,1
b	4,1	3,5	3,3
c	5,2	4,4	7,0
d	2,3	0,4	3,0

Find all strategy profiles that form pure strategy Nash equilibria (there may be more than one, or none):

Your Answer		Score	Explanation
☐ 1) (a, x);	✔	0.08	
☐ 2) (b, x);	✔	0.08	
☐ 3) (c, x);	✔	0.08	
☐ 4) (d, x);	✔	0.08	
☐ 5) (a, y);	✔	0.08	
☐ 6) (b, y);	✔	0.08	
☑ 7) (c, y);	✔	0.08	

☐ 8) (d, y);	✔	0.08
☐ 9) (a, z);	✔	0.08
☐ 10) (b, z);	✔	0.08
☐ 11) (c, z);	✔	0.08
☐ 12) (d, z).	✔	0.08
Total		1.00 / 1.00

Question Explanation

(7) (c, y) is the only pure strategy Nash equilibria.

- Check that no one wants to deviate.
- Note that c is the strictly dominant strategy and so is the only possible strategy for player 1 in a pure strategy Nash equilibrium.
- When player 1 plays c, playing y gives player 2 the highest payoff.

Question 5

Nash Equilibrium - Bargaining

There are 2 players that have to decide how to split one dollar. The bargaining process works as follows. Players simultaneously announce the share they would like to receive s_1 and s_2, with $0 \leq s_1, s_2 \leq 1$. If $s_1 + s_2 \leq 1$, then the players receive the shares they named and if $s_1 + s_2 > 1$, then both players fail to achieve an agreement and receive zero.

Which of the following is a strictly dominant strategy?

Your Answer	Score	Explanation

d) None of the above.	✔	1.00
Total		1.00 / 1.00

Question Explanation

(d) is true.

- No player has any strictly dominant strategies. Any of the options given constitutes a best response to some strategy played by the other player, and so no strategy always strictly outperforms all other strategies.
- Strategies (a) and (c) are in the set of best responses of player i when player j's strategy is $s_j > 1$.
- Strategies (b) is the best response of player i when player j's strategy is $s_j = 0.5$.

Question 6

Nash Equilibrium - Bargaining

There are 2 players that have to decide how to split one dollar. The bargaining process works as follows. Players simultaneously announce the share they would like to receive s_1 and s_2, with $0 \leq s_1, s_2 \leq 1$. If $s_1 + s_2 \leq 1$, then the players receive the shares they named and if $s_1 + s_2 > 1$, then both players fail to achieve an agreement and receive zero.

Which of the following strategy profiles is a pure strategy Nash equilibrium?

Your Answer		Score	Explanation
d) All of the above	✔	1.00	
Total		1.00 / 1.00	

Question Explanation

(d) is true.

- Check that no one wants to deviate.

- Note that when player i plays $s_i<1$, player j's best response is $s_j=1-s_i$. This holds in a) and b). Thus, both players are best responding.
- .When player i plays $s_i=1$, player j's best response can be any number as she will get 0 no matter 1. Thus c) also forms a pure strategy NE.

Question 7

Bertrand Duopoly

- Two firms produce identical goods, with a production cost of c per unit.
- Each firm sets a nonnegative price (p_1 and p_2).
- All consumers buy from the firm with the lower price, if $p_i \neq p_j$. Half of the consumers buy from each firm if $p_i=p_j$.
- D is the total demand.
- Profit of firm i is:
  - 0 if $p_i>p_j$ (no one buys from firm i);
  - $D(p_i-c)/2$ if $p_i=p_j$(Half of customers buy from firm i);
  - $D(p_i-c)$ if $p_i<p_j$ (All customers buy from firm i);

Find the pure strategy Nash equilibrium:

Your Answer		Score	Explanation
c) Both firms set $p=c$.	✔	1.00	
Total		1.00 / 1.00	

Question Explanation

(c) is true.

- Notice than in a) and b) at least one firm i is making negative profits since $p_i<c$ and it sells a positive quantity. Thus, firm i would prefer to deviate to $p_i>p_j$ and earn a profit of 0.
- It is easy to verify that $p_1=p_2=c$ is an equilibrium by checking that no firm wants to deviate:
  - When $p_1=p_2=c$, both firms are earning null profits.
  - If firm 1 increases its price above c ($p_1>c$), it will still earn null profits.
  - If firm 2 decreases its price below c ($p_1<c$), it will earn strictly negative profits.

- In both cases, either the firm is indifferent or strictly worse off. Then, it does not have incentives to deviate given the other firm's strategy.

Question 8

Voting

- Three voters vote over two candidates (A and B), and each voter has two pure strategies: vote for A and vote for B.
- When A wins, voter 1 gets a payoff of 1, and 2 and 3 get payoffs of 0; when B wins, 1 gets 0 and 2 and 3 get 1. Thus, 1 prefers A, and 2 and 3 prefer B.
- The candidate getting 2 or more votes is the winner (majority rule).

Find all very weakly **dominant** strategies (there may be more than one, or none).

Your Answer		Score	Explanation
☑ a) Voter 1 voting for A.	✔	0.25	
☐ b) Voter 1 voting for B.	✔	0.25	
☐ c) Voter 2 (or 3) voting for A.	✔	0.25	
☑ d) Voter 2 (or 3) voting for B.	✔	0.25	
Total		1.00 / 1.00	

Question Explanation

(a) and (d) are (very weakly) dominant strategies.

- Check (b): for voter 1, voting for candidate A always results in at least as high a payoff as voting for candidate B and indeed is sometimes strictly better (when the other players vote for different candidates).

    - When voters 2 and 3 vote for B, voter 1 is indifferent between A or B (since B will win anyways).
    - When either 2 or 3 (or both) vote for A, voter 1 strictly prefers to vote for A than for B.
- Check (c): for voter 2, voting for candidate B is a very weakly dominant strategy.
    - When voters 1 and 3 vote for A, voter 2 is indifferent between A or B (since A will win anyways).
    - When either 1 or 3 (or both) vote for B, voter 2 strictly prefers to vote for B than for A.
- (b) and (c) can't be very weakly dominant strategies, since they sometimes do worse than the other strategy.

Question 9

Voting

- Three voters vote over two candidates (A and B), and each voter has two pure strategies: vote for A and vote for B.
- When A wins, voter 1 gets a payoff of 1, and 2 and 3 get payoffs of 0; when B wins, 1 gets 0 and 2 and 3 get 1. Thus, 1 prefers A, and 2 and 3 prefer B.
- The candidate getting 2 or more votes is the winner (majority rule).

Find **all** pure strategy Nash equilibria (there may be more than one, or none)?

Your Answer		Score	Explanation
☑ a) All voting for A.	✔	0.25	
☑ b) All voting for B.	✔	0.25	
☑ c) 1 voting for A, and 2 and 3 voting for B.	✔	0.25	
☐ d) 1 and 2 voting for A, and 3 voting for B.	✔	0.25	

Total	1.00 / 1.00

Question Explanation

(a), (b) and (c) are pure strategy Nash equilibria.

- It is easy to verify that (a), (b) and (c) are equilibria by checking that no voter wants to deviate:
  - When all voters vote for the same candidate, no single voter has any incentives to deviate because his/her individual vote can't modify the outcome of the election.
  - In (c), voter 1 is indifferent between candidates A and B, and voters 2 and 3 are best responding to the strategies played by the remaining voters (if voter 2 votes for A, candidate A wins; if voter 2 votes for B, candidate B wins).
- (d) is not an equilibrium, since voter 2 has incentives to deviate and vote for candidate B.

Question 1

Mixed Strategy Nash Equilibrium

1\ 2	Left	Right
Left	4,2	5,1
Right	6,0	3,3

Find a mixed strategy Nash equilibrium where player 1 randomizes over the pure strategy Left and Right with probability p for Left. What is p?

Your Answer		Score	Explanation
b) 3/4	✔	1.00	
Total		1.00 / 1.00	

Question Explanation

(b) is true.

- In a mixed strategy equilibrium in this game both players must mix and so 2 must be indifferent between Left and Right.
- Left gives 2 an expected payoff: $2p+0(1-p)$
- Right gives 2 an expected payoff: $1p+3(1-p)$

- Setting these two payoffs to be equal leads to $p=3/4$.

Question 2

Comparative Statics

1\ 2	Left	Right
Left	x,2	0,0
Right	0,0	2,2

In a mixed strategy Nash equilibrium where player 1 plays Left with probability p and player 2 plays Left with probability q. How do p and q change as X is increased ($X>1$)?

Your Answer		Score	Explanation
a) p is the same, q decreases.	✔	1.00	
Total		1.00 / 1.00	

Question Explanation

(a) is true.

- In a mixed strategy equilibrium, 1 and 2 are each indifferent between Left and Right.
- For p:
  - Left gives 2 an expected payoff: $2p$
  - Right gives 2 an expected payoff: $2(1-p)$
  - These two payoffs are equal, thus we have $p=1/2$.
- For q: setting the Left expected payoff equal to the Right leads to $Xq=2(1-q)$, thus $q=2/(X+2)$, which decreases in X.

Question 3

Employment

- There are 2 firms, each advertising an available job opening.
- Firms offer different wages: $w_1=4$ and $w_2=6$.
- There are two unemployed workers looking for jobs. They simultaneously apply to either of the firms.
  - If only one worker applies to a firm, then he/she gets the job

- If both workers apply to the same firm, the firm hires a worker at random and the other worker remains unemployed (and receives a payoff of 0).

Find a mixed strategy Nash Equilibrium where p is the probability that worker 1 applies to firm 1 and q is the probability that worker 2 applies to firm 1.

Your Answer		Score	Explanation
a) $p=q=1/2$;	✘	0.00	
Total		0.00 / 1.00	

Question Explanation

(d) is correct.

- In a mixed strategy equilibrium, worker 1 and 2 must be indifferent between applying to firm 1 and 2.
- For a given p, worker 2's indifference condition is given by $2p+4(1-p)=6p+3(1-p)$.
- Similarly, for a given q, worker 1's indifference condition is given by $2q+4(1-q)=6q+3(1-q)$.
- Both conditions are satisfied when $p=q=1/5$.

Question 4

Treasure

- A king is deciding where to hide his treasure, while a pirate is deciding where to look for the treasure.
- The payoff to the king from successfully hiding the treasure is 5 and from having it found is 2.
- The payoff to the pirate from finding the treasure is 9 and from not finding it is 4.
- The king can hide it in location X, Y or Z.

Suppose the pirate has two pure strategies: inspect both X and Y (they are close together), or just inspect Z (it is far away). Find a mixed strategy Nash equilibrium where p is the probability the treasure is hidden in X or Y and $1-p$ that it is hidden in Z (treat the king as having two strategies) and q is the probability that the pirate inspects X and Y:

Your Answer		Score	Explanation
a) $p=1/2$, $q=1/2$;	✔	1.00	
Total		1.00 / 1.00	

Question Explanation

(a) is true.

- There is no pure strategy equilibrium, so in a mixed strategy equilibrium, both players are indifferent among their strategies.
- For p:
  - Inspecting X \& Y gives pirate a payoff: $9p+4(1-p)$
  - Inspecting Z gives pirate a payoff: $4p+9(1-p)$
  - These two payoffs are equal, thus we have $p=1/2$.
- For q: indifference for the king requires that $5q+2(1-q)=2q+5(1-q)$, thus $q=1/2$.

Question 5

Treasure

- A king is deciding where to hide his treasure, while a pirate is deciding where to look for the treasure.
- The payoff to the king from successfully hiding the treasure is 5 and from having it found is 2.
- The payoff to the pirate from finding the treasure is 9 and from not finding it is 4.
- The king can hide it in location X, Y or Z.

Suppose instead that the pirate can investigate any two locations, so has three pure strategies: inspect XY or YZ or XZ. Find a mixed strategy Nash equilibrium where the king mixes over three locations (X, Y, Z) and the pirate mixes over (XY, YZ, XZ). The following probabilities (king), (pirate) form an equilibrium:

Your Answer		Score	Explanation
d) (1/3, 1/3, 1/3), (1/3, 1/3, 1/3);	✔	1.00	

Total	1.00 / 1.00

Question Explanation

(d) is true.

- Check (a):
  - Pirate inspects (XY, YZ, XZ) with prob (4/9, 4/9, 1/9);
  - Y is inspected with prob 8/9 while X (or Z) is inspected with prob 5/9;
  - King prefers to hide in X or Z, which contradicts the fact that in a mixed strategy equilibrium, king should be indifferent.
- Similarly, you can verify that (b) and (c) are not equilibria in the same way.
- In (d), every place is chosen by king and inspected by pirate with equal probability and they are indifferent between all strategies.

Question 1

Iterated removal of strictly dominated strategies

Normal			
1 \ 2	L	M	R
U	3,8	2,0	1,2
D	0,0	1,7	8,2

We say that a game is *dominance solvable*, if iterative deletion of strictly dominated strategies yields a unique outcome. True or false: Is the previous game dominance solvable? Consider both pure strategies and mixed strategies.

Your Answer		Score	Explanation
◉ a) True;	✔	1.00	
Total		1.00 / 1.00	

Question Explanation

(a) is correct, the previous game is dominance solvable.

- For player 2, R is dominated by a mixed strategy between L and M (for example, play $L$with probability $1/2$ and M with probability $1/2$).
- Once R is removed, for player 1 D is dominated by U.
- Finally, once R and D are removed, M is dominated by L.
- Since the process of iterative deletion of strictly dominated strategies yields the unique outcome (U,L), the game is dominance solvable.

Question 2

Iterated removal of weakly dominated strategies

In order to illustrate the problem that arises when iteratively eliminating **weakly** dominated strategies, consider the following game:

Normal			
1 \ 2	L	M	R
U	4,3	3,5	3,5
D	3,4	5,3	3,4

True or false: in the above game the order of elimination of **weakly** dominated strategies does not matter (that is, the final outcome is the same regardless of the order in which weakly dominated strategies are eliminated.). [Hint: start the process of iterative elimination of **weakly** dominated strategies by eliminating different strategies at the beginning of the process.]

Your Answer		Score	Explanation
b) False.	✔	1.00	
Total		1.00 / 1.00	

Question Explanation

(b) is correct, in the previous game the order of elimination of weakly dominated strategies does matter.

- Consider the following attempts:
- If we start by eliminating the weakly dominated strategy L:
  - For 2, L is weakly dominated by R.
  - Once L is removed, for 1 U is weakly dominated by D.

- Once L and U are removed, M is strictly dominated by R.
- The outcome of this process of elimination is (D,R).

- If we start by eliminating the weakly dominated strategy M:
  - For 2, M is weakly dominated by R.
  - Once M is removed, for 1 D is weakly dominated by U.
  - Once M and D are removed, L is strictly dominated by R.
  - The outcome of this process of elimination is (U,R).
- Then, the order of elimination of **weakly** dominated strategies does affect the outcome of the process.

Question 3

Minimax

Consider the matching-pennies game:

1\ 2	Left	Right
Left	2,-2	-2,2
Right	-2,2	2,-2

Which is a maxmin strategy for player 1:

Your Answer		Score	Explanation
c) Play Left and Right with probability 1/2.	✔	1.00	
Total		1.00 / 1.00	

Question Explanation

(c) is true.

- Recall from lecture: $S_1=argmax_{s'_1\in S_1}\min_{s_2\in S_2}u_1(s'_1,s_2)$
- Given a strategy of 1: play Left with probability p and Right with $1-p$ $(0\le p\le 1)$:
  - If $p>1/2$, $s_2=Right$ leads 1 to earn $(-2)p+2(1-p)<0$;
  - If $p<1/2$, $s_2=Left$ leads 1 to earn $2p+(-2)(1-p)<0$;
  - If $p=1/2$, then regardless of 2's strategy 1 earns 0.
  - Thus $p=1/2$ is the maxmin strategy.

Question 4

Minimax

Consider the matching-pennies game:

1\ 2	Left	Right
Left	2,-2	-2,2
Right	-2,2	2,-2

Apply the Minimax theorem presented in lecture 3-4 to find the payoff that any player must receive in any Nash Equilibrium:

Your Answer		Score	Explanation
d) 0.	✔	1.00	
Total		1.00 / 1.00	

Question Explanation

(d) is true.

- Since the previous game is a (finite) two-player, zero-sum game, by the theorem presented in the lecture we know that in any Nash equilibrium each player receives a payoff that is equal to both his maximin value and his minimax value.
- From the previous question we know that each player's maximin strategy is to play Left and Right with probability 1/2, which gives an expected payoff (maximin value) of 0.

Question 5

Correlated Equilibrium

1\ 2	B	F
B	2,1	0,0
F	0,0	1,2

Consider the following assignment device (for example a fair coin):

- With probability 1/2 it tells players 1 and 2 to play B, and with probability 1/2 it tells them to play F.
- Both players know that the device will follow this rule.

What is the expected payoff of each player when both players follow the recommendations made by the device? If one of players follows the recommendation, does the other player have an incentive to follow the recommendation as well?

Your Answer		Score	Explanation
c) Expected payoff $=1.5$; player has an incentive to follow the recommendation.	✔	1.00	
Total		1.00 / 1.00	

Question Explanation

(c) is true.

- If both players follow the recommendation of the device, they will play (B,B) with probability $1/2$ and (F,F) with probability $1/2$. Then, the expected payoff is $1/2*2+1/2*1=1.5$.
- It is easy to check that if one of the players is following the recommendation, then the other player has an incentive to do the same:
  - Suppose that player 1 follows the recommendation of the device.
  - When player 2 is told to play B, he/she knows that player 1 was also told to play B (and that is the strategy that he/she will play).
  - Player's 2 best response to player 1 playing B is to also play B.
  - The same holds when player 2 is told to play F. Therefore, player 2 will follow what the device tells him/her to do.

Question 1

Splitting Coins

- Two players have to share 50 coins (of equal value).
- Players' payoffs are the number of coins they each get

- First, player 1 splits the coins into 2 piles.
- Second, player 2 chooses one pile for him/herself and gives the other pile to player 1

What is agent 1's strategy in a backward induction solution?

Your Answer		Score	Explanation
a) Splitting coins into 25/25.	✔	1.00	
Total		1.00 / 1.00	

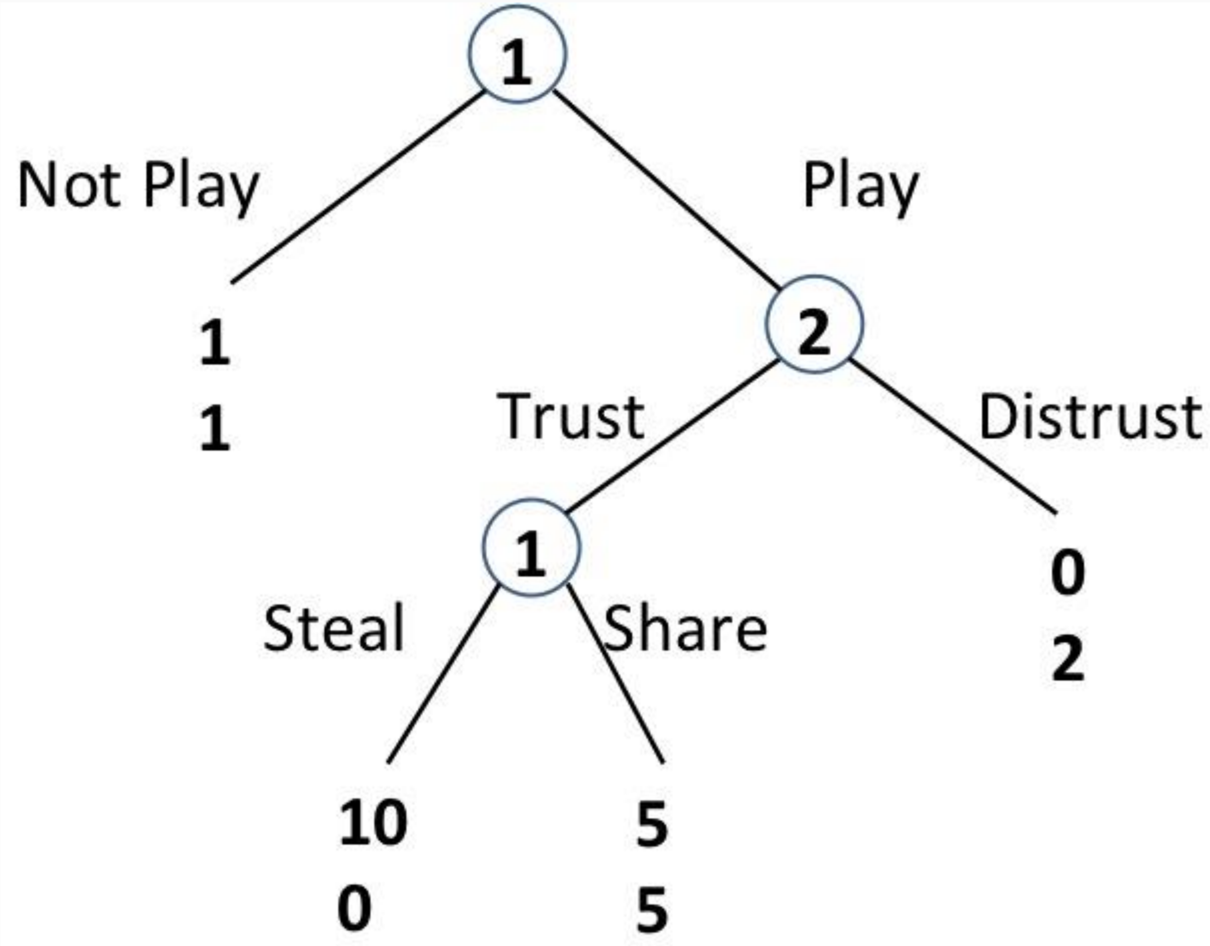
Question Explanation

(a) is true.

- When there are two piles for 2 to choose, 2 prefers to have the pile with more coins and gives 1 the pile with fewer coins.
- Thus, player 1 will never get more than 25 coins from any split. Thus, when 1 splits the coins, it is best to split them into 25/25 as any other split will lead to fewer coins for player 1.

Question 2

Trust Game



Find all of the pure strategy **Nash** Equilibria of this game. There can be more than one equilibrium. [Here ((Not Play,Steal),(Trust)) indicates that player 1 chooses Not Play at the first decision node and Steal at the second decision node, and 2 chooses Trust at his unique decision node.]

Your Answer		Score	Explanation
☑ a) ((Not play, Steal),(Distrust))	✔	0.20	
☑ b) ((Not play, Share), (Distrust))	✔	0.20	
☐ c) ((Not play, Steal), (Trust))	✔	0.20	

☐ d) ((Play, Steal), (Distrust))	✔	0.20
☐ e) ((Play, Share), (Trust))	✔	0.20
Total		1.00 / 1.00

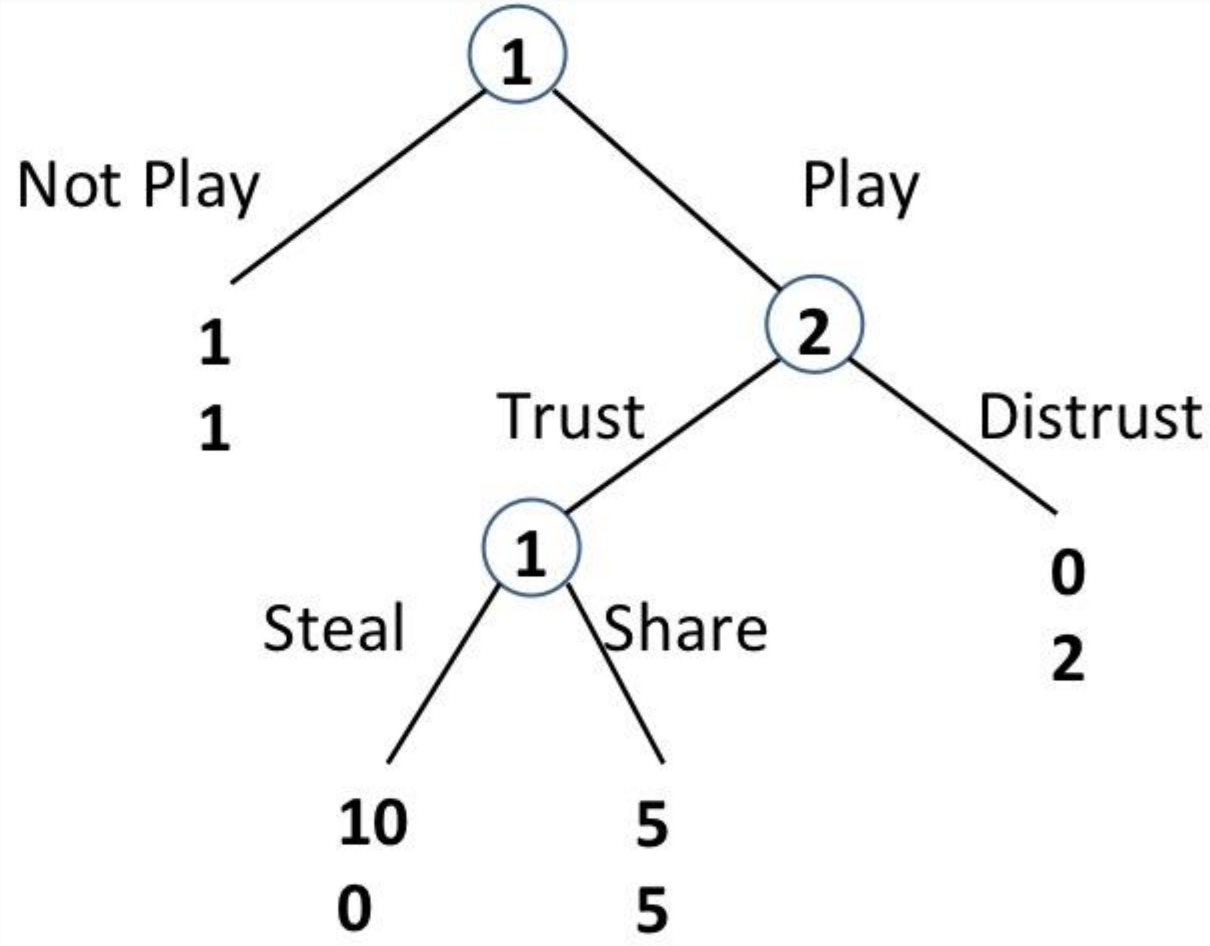
Question Explanation

(a) and (b) are correct.

- To see that (a) and (b) form two Nash Equilibria we have to check that no player wants to deviate:
  - Given that player 1 decides Not to play, player 2 is indifferent between Trust and Distrust since his/her node is not reached.
  - Given that player 2 decides to Distrust, player 1 prefers to Not play and get a payoff of 1rather than Play and getting a payoff of 0.
  - No player has an incentive to deviate.
- (c) cannot be a NE since 1 would prefer to deviate to (Play, Steal) and get 10 instead of (Not play, Steal) and getting a payoff of 1.
- (d) cannot be a NE since 1 would prefer to deviate to (Not play, Steal) and get 1 instead of (Play, Steal) and getting a payoff of 0.
- (e) cannot be a NE since 1 would prefer to deviate to (Play, Steal) and get 10 instead of (Play, Share) and getting a payoff of 5.

Question 3

Trust Game



Which is the Subgame Perfect Equilibrium of this game?

Your Answer		**Score**	**Explanation**
a) ((Not play, Steal),(Distrust))	✔	1.00	
Total		1.00 / 1.00	

Question Explanation

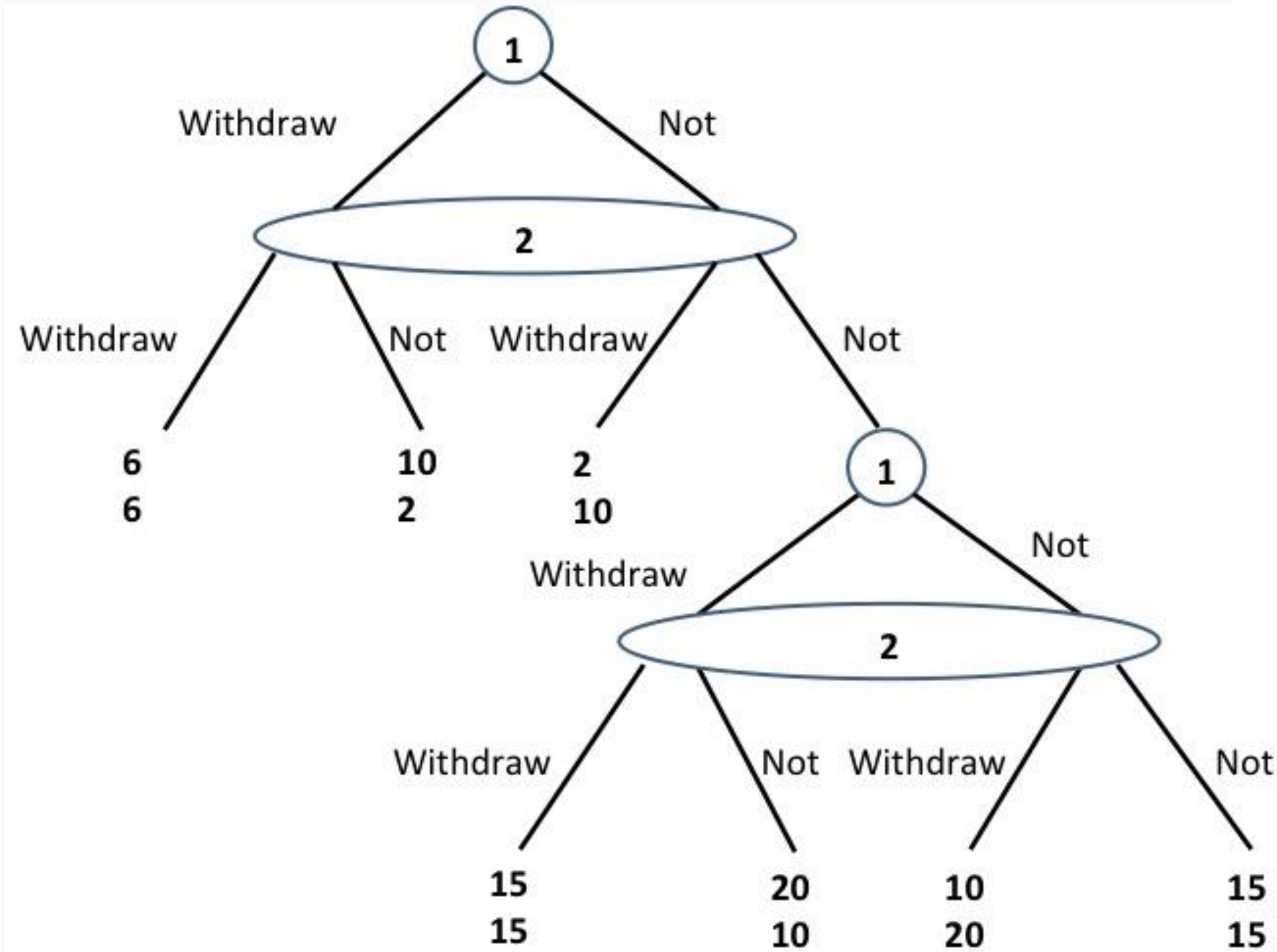
(a) is correct.

- Recall that the set of Subgame Perfect Equilibria is a subset of the set of Nash Equilibria (and sometimes a strict subset). Thus, given the answer to the previous question, we only need to check if (a) and (b) are SPNE.

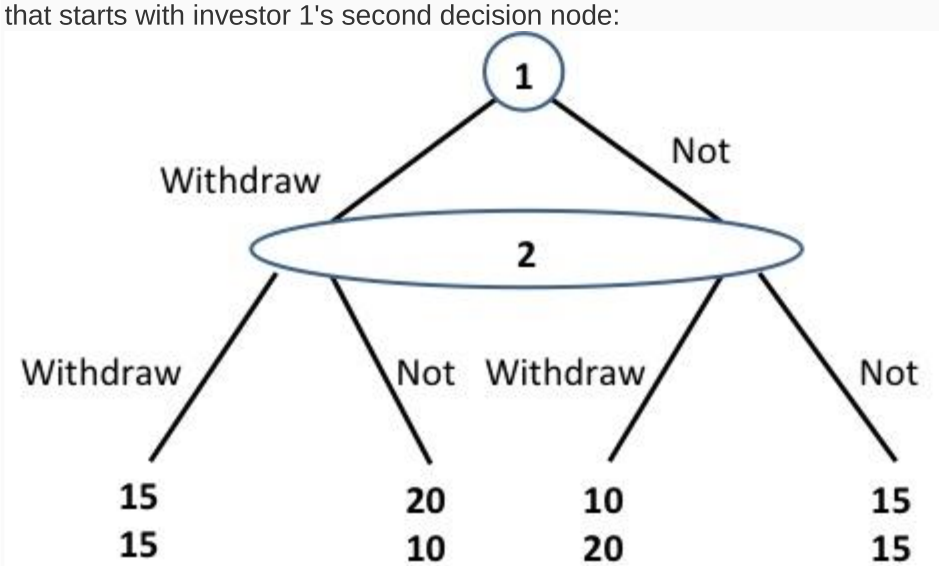
- Checking (a):
  - If 2 plays Trust, 1 prefers to Steal;
  - Given this, if 2 plays Trust he/she receives 0 and if 2 plays Distrust he/she receives 2. Player 2 will choose Distrust.
  - Since player 2 plays Distrust, 1 prefers Not Play (payoff 1) to Play (payoff 0).
  - ((Not Play, Steal), (Distrust)) is SPNE.
- Checking (b):
  - ((Not Play, Share), (Distrust)) cannot be a SPNE since 1 is not best responding in his/her second decision node (his/her best response is Steal).

Question 4

- There are 2 investors. Each has deposited $10 in the same bank.
- The bank invested both deposits in a single long-term project.
  - If the bank wants to end the project before its completion, a total of $12 can be recovered (out of the $20 invested).
  - If the bank waits until the project is completed, it will receive a total of $30.
- Investors can withdraw money from their bank accounts at only 2 periods: before the project is completed and after.
- The extensive from representation of the game between both investors is depicted below:



In order to find the subgame perfect Nash equilibrium of the whole game first focus on the subgame that starts with investor 1's second decision node:



Which is a pure strategy Nash equilibrium of this subgame?

Your Answer		Score	Explanation
a) (Withdraw, Withdraw)	✔	1.00	
Total		1.00 / 1.00	

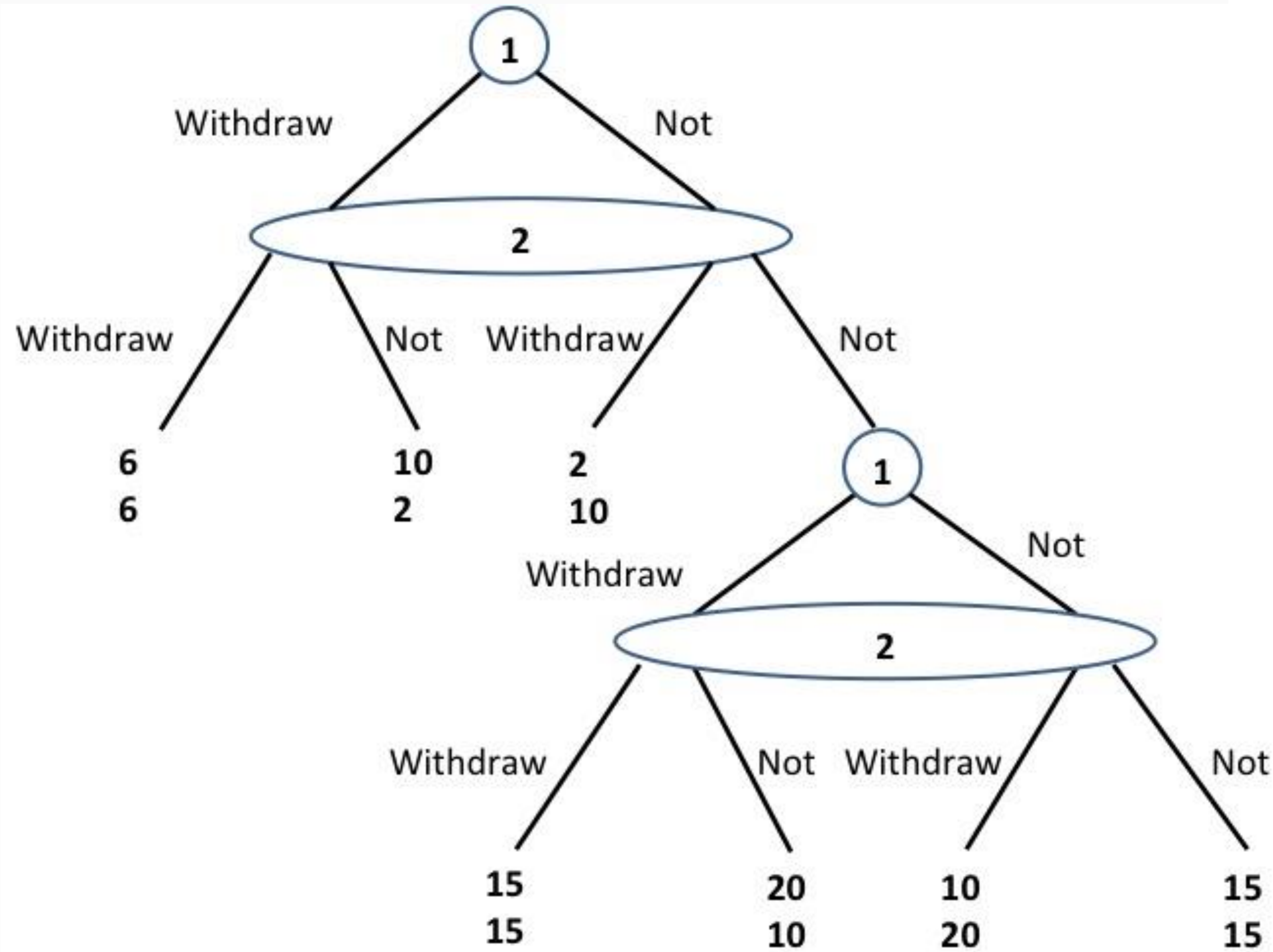
Question Explanation

(a) is correct.

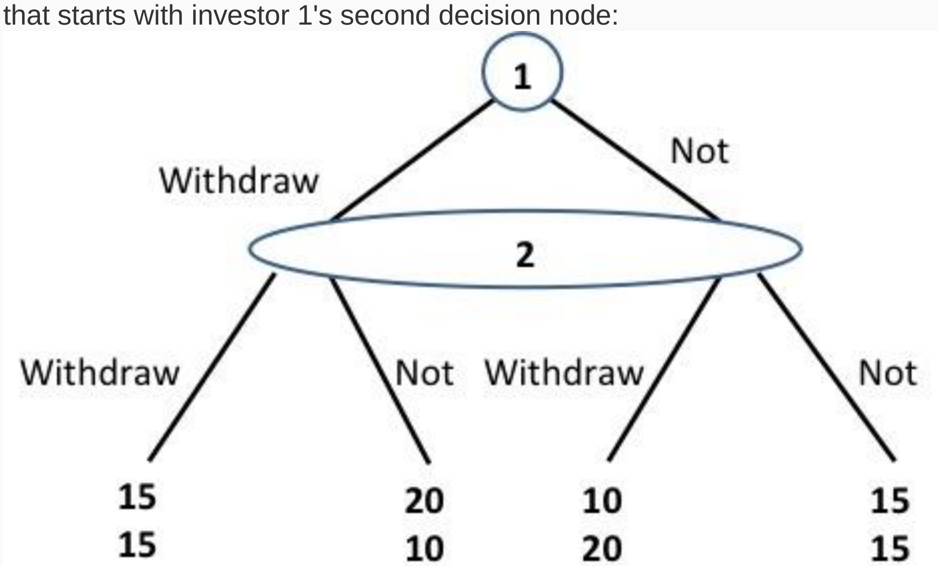
- It is easy to check that Withdraw is a strictly dominant strategy for both players.
- Therefore, they have to play Withdraw in equilibrium.

Question 5

- There are 2 investors. Each has deposited $10 in the same bank.
- The bank invested both deposits in a single long-term project.
  - If the bank wants to end the project before its completion, a total of $12 can be recovered (out of the $20 invested).
  - If the bank waits until the project is completed, it will receive a total of $30.
- Investors can withdraw money from their bank accounts at only 2 periods: before the project is completed and after.
- The extensive from representation of the game between both investors is depicted below:



In order to find the subgame perfect Nash equilibrium of the whole game first focus on the subgame that starts with investor 1's second decision node:



What are the subgame perfect Nash equilibria of the whole game? There might be more than one. [Hint: ((Withdraw, Not),(Not, Withdraw)) are the first and second investors' strategies in their first and second decision nodes, respectively. So, ((Withdraw, Not),(Not, Withdraw)) indicates that the first investor withdraws at her first decision node but not at her second, while the second invest does not withdraw at his first decision node but does at his second decision node.]

Your Answer		Score	Explanation
☑ a) ((Withdraw, Withdraw), (Withdraw, Withdraw))	✔	0.25	
☐ b) ((Withdraw, Withdraw), (Not, Withdraw))	✔	0.25	
☐ c) ((Not, Withdraw), (Withdraw, Withdraw))	✔	0.25	
☑ d) ((Not, Withdraw), (Not, Withdraw))	✔	0.25	
Total		1.00 / 1.00	

Question Explanation

(a) and (d) are correct.

- In a subgame perfect Nash equilibrium both investors must play Withdraw in the subgame analyzed above. Then, the payoff that they expect to receive when neither of them Withdraws in the first period is (15,15).
- Replacing the payoff of (Not, Not) in the first period to (15,15) simplifies the whole game to a one-period game. This simplified game has two equilibria: one in which both investors withdraw in the first period, and another in which both investors wait and withdraw the money in the second period.
    - Verify that the best response to the other investor withdrawing, is also to withdraw.
    - Verify that the best response to the other investor not withdrawing, is also not to withdraw.
- (b) and (c) are not subgame perfect Nash equilibrium because either of the investors has incentives to deviate in his/her first decision node.

Question 6

Pirates' Games

- Five pirates have obtained 100 gold coins and have to divide up the loot. The pirates are all extremely intelligent, treacherous and selfish (especially the captain) each wanting to maximize the number of coins that he gets.
- It is always the captain who proposes a distribution of the loot. All pirates vote on the proposal, and if half the crew or more go "Aye", the loot is divided as proposed.
- If the captain fails to obtain support of at least half his crew (which includes himself), all pirates turn against him and make him walk the plank. The pirates then start over again with the next most senior pirate as captain (the pirates have a strict order of seniority denoted by A, B, C, D and E).
- Pirates' preferences are ordered in the following way. First of all, each pirate wants to survive. Second, given survival, each pirate wants to maximize the number of gold coins he receives. Finally, each pirate would prefer to throw another overboard in the case of indifference.

What is the maximum number of coins that the original captain gets to keep across all subgame perfect equilibria of this game? (Hint, work by backward induction to reason what the split will be if three captains have been forced to walk the plank and there are only two pirates left. Just one vote is enough to approve the split among the two pirates. Then use that to solve for what happens when two have walked the plank and there are three pirates left, and so forth.)

Your Answer		Score	Explanation
d) 98;	✔	1.00	
Total		1.00 / 1.00	

Question Explanation

(d) is correct.

- If three captains have walked the plan and only pirates D and E remain, captain D will offer a split $D=100$ and $E=0$ since just one vote is enough to approve a split among two pirates.

- If two captains have walked the plan and only pirates C, D and E remain, captain C will offer a split $C=99$, $D=0$ and $E=1$ since $E=1$ is larger than what E would get if he rejects the proposal (remember that D would offer $E=0$). Thus, pirate E approves the split.
- If pirates B, C, D and E remain, captain B will offer a split $B=99$, $C=0$, $D=1$ and $E=0$ since only two votes are needed to approve a split among four pirates. Pirate D approves the split because otherwise he would receive 0.
- In the last case, pirate B would have not preferred to offer 1 coin to E. The reason is that when indifferent, pirate E would prefer to throw pirate B overboard.
- If all 5 pirates remain, in order to obtain enough votes, pirate A will offer a split $A=98$, $B=0$, $C=1$, $D=0$ and $E=1$.

Question 1

Repeated Games with single NE in stage game

Two players play the following normal form game.

1\ 2	Left	Middle	Right
Left	4,2	3,3	1,2
Middle	3,3	5,5	2,6
Right	2,1	6,2	3,3

Which is the pure strategy Nash equilibrium of this stage game (if it is played only once)?

Your Answer		Score	Explanation
i) (Right, Right).	✔	1.00	
Total		1.00 / 1.00	

Question Explanation

(i) is the unique Nash equilibrium of the stage game.

- (Right, Right) is a Nash equilibrium of the stage game because Right is the best response when the other player is playing Right.
- It is also the unique Nash equilibrium. To see this, check that in all other cases at least one player has an incentive to deviate.

Question 2

Repeated Games with single NE in stage game
Two players play the following normal form game.

1\ 2	Left	Middle	Right
Left	4,2	3,3	1,2
Middle	3,3	5,5	2,6
Right	2,1	6,2	3,3

Suppose that the game is repeated for two periods. What is the outcome from the subgame perfect Nash equilibrium of the whole game:

Your Answer		Score	Explanation
b) (Right, Right) is played in both periods.	✔	1.00	
Total		1.00 / 1.00	

Question Explanation

(b) is true.

- The stage game has a unique Nash equilibrium.
- In the second period, (Right, Right) must be played regardless of the outcome obtained in the first period.
- Then, it is optimal for both players to maximize the current payoff at the first period and play (Right, Right).

Question 3

Repeated Games with single NE in stage game
Two players play the following normal form game.

1\ 2	Left	Middle	Right
Left	4,2	3,3	1,2
Middle	3,3	5,5	2,6
Right	2,1	6,2	3,3

Suppose that there is a probability p that the game continues next period and a probability $(1-p)$ that it ends. What is the threshold p_* such that when $p \geq p_*$ (Middle, Middle) is sustainable as a subgame perfect equilibrium by grim trigger strategies, but when $p<p_*$ playing Middle in all periods is not a best response? [Here the grim strategy is: play Middle if the play in all previous periods was (Middle, Middle); play Right otherwise.]

Your Answer		Score	Explanation
b) 1/3;	✔	1.00	
Total		1.00 / 1.00	

Question Explanation

(b) is true.

- In the infinitely repeated game supporting (Middle, Middle):
  - Suppose player 1 uses the grim trigger strategy.
  - If player 2 deviates to the best response Right, player 2 earns 6-5=1 more in the current period, but loses 2 from all following periods, which is 2p/(1-p) in total.
  - Thus in order to support (Middle, Middle), the threshold is 1=2p/(1-p), which is p = 1/3.
  - It is easy to check that the threshold is the same for player 1.

Question 4

Repeated Games with multiple NE in stage game
Consider the following game:

1\ 2	Left	Middle	Right
Left	1,1	5,0	0,0
Middle	0,5	4,4	0,0
Right	0,0	0,0	3,3

Which are the pure strategy Nash equilibria of this stage game? There can be more than one.

Your Answer		Score	Explanation
a) (Left, Left);	✔	0.11	

☐	b) (Left, Middle);	✔	0.11
☐	c) (Left, Right);	✔	0.11
☐	d) (Middle, Left);	✔	0.11
☐	e) (Middle, Middle);	✔	0.11
☐	f) (Middle, Right);	✔	0.11
☐	g) (Right, Left);	✔	0.11
☐	h) (Right, Middle);	✔	0.11
☑	i) (Right, Right).	✔	0.11
Total			1.00 / 1.00

Question Explanation

(a) and (i) are pure strategy Nash equilibria of the stage game.

- (Left, Left) and (Right, Right) are Nash equilibria of the stage game because Right is the best response when the other player is playing Right, and Left is the best response when the other player is playing Left.
- There are no other Nash equilibria. To see this, check that in all other cases at least one player has an incentive to deviate.

Question 5

Repeated Games with multiple NE in stage game

Consider the following game:

1\ 2	Left	Middle	Right
Left	1,1	5,0	0,0
Middle	0,5	4,4	0,0
Right	0,0	0,0	3,3

Suppose that the game is repeated for two periods. Which of the following outcomes occur in some subgame perfect equilibrium?

Your Answer		Score	Explanation
d) All of the above.	✔	1.00	
Total		1.00 / 1.00	

Question Explanation

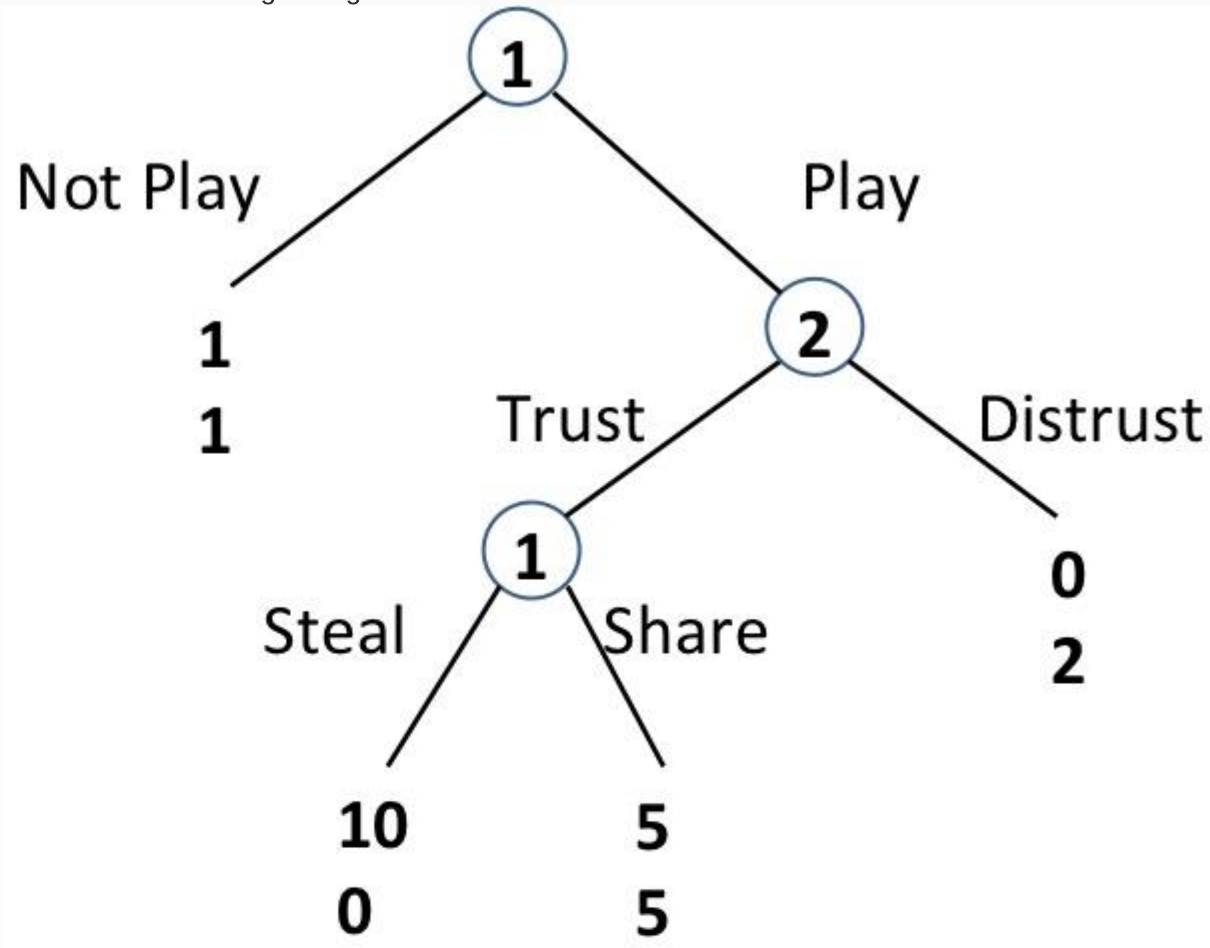
(d) is true.

- Recall that playing a Nash equilibrium of the stage game in each period forms a subgame perfect Nash equilibrium of the whole game. Then, (a) and (b) are subgame perfect Nash equilibria.
- Outcome (c) can be obtained when both players play the following strategy:
    - Play Middle in the first period.
    - If outcome in first period was (Middle, Middle) play Right in the second period; otherwise play Left.
- It is easy to check that this grim strategy forms a subgame perfect Nash equilibrium:
    - Suppose that player 1 plays this strategy.
    - If player 2 plays the same strategy, he/she will receive a total payoff of $4+3=7$(assume no discounting).
    - If player 2 deviates to (Left, Right), he/she will receive a total payoff of $5+1=6$ (which is lower than the payoff of following the grim strategy).

Question 6

Repeated Trust Game

Consider the following trust game:



There is a probability p that the game continues next period and a probability $(1-p)$ that it ends. The game is repeated indefinitely. Which statement is true? [Grim trigger in (c) and (d) is player 1 playing Not play and player 2 playing Distrust forever after a deviation from ((Play,Share), (Trust)).]

Your Answer		Score	Explanation
c) ((Play,Share), (Trust)) is sustainable as a subgame perfect equilibrium by grim trigger in the indefinitely repeated game with a probability of continuation of $p \geq 5/9$.	✔	1.00	
Total		1.00 / 1.00	

Question Explanation

(c) is true.

- There are only two Nash equilibria in the one-shot game: ((Not play, Steal),(Distrust)) and ((Not play, Share), (Distrust)). Both require player 2 playing Distrust.
- Since both equilibria lead to the same payoff for both players, there can't exist a subgame perfect equilibrium in the finely repeated game in which player 2 plays Trust in any period (verify this by backward induction).
- In the infinitely repeated game supporting ((Play,Share), (Trust)):
  - Suppose player 2 uses the grim trigger strategy: start playing Trust and play Distrust forever after a deviation from ((Play,Share), (Trust)).
  - If player 1 deviates and plays (Play, Steal), player 1 earns $10-5=5$ more in the current period, but loses 4 from all following periods, which is $4p/(1-p)$ in total.
  - Thus in order to support ((Play,Share), (Trust)), the threshold is $5=4p/(1-p)$, which is $p=5/9$.
  - Note that given player 1's strategy, player 2 has no incentive to deviate for any value of p.

Question 7

Tit for tat

In an infinitely repeated Prisoner's Dilemma, a version of what is known as a "tit for tat" strategy of a player i is described as follows:

- There are two "statuses" that player i might be in during any period: "normal" and "revenge";
- In a normal status player i cooperates;
- In a revenge status player i defects;
- From a normal status, player i switches to the revenge status in the next period only if the other player defects in this period;
- From a revenge status player i automatically switches back to the normal status in the next period regardless of the other player's action in this period.

Consider an indefinitely repeated game so that with probability p that the game continues to the next period and with probability $(1-p)$ it ends.

	Cooperate (C)	Defect (D)
Cooperate (C)	4,4	0,5
Defect (D)	5,0	1,1

True or False: When player 1 uses the above-described "tit for tat" strategy and starts the first period in a revenge status (thus plays defect for sure), an infinite payoff maximizing strategy has player 2 defect in the first period.

Your Answer		Score	Explanation
True.	✔	1.00	
Total		1.00 / 1.00	

Question Explanation

True.

- If player 1 uses "tit for tat" strategy and starts in a revenge status, the payoff in the first period is higher for player 2 from defection than cooperation.
- Moreover, the action played by 2 in the first period when 1 begins in revenge status doesn't affect the remaining periods since 1 switches to normal status in the second period regardless of what player 2 does in the first period.

Question 8

Tit for tat

In an infinitely repeated Prisoner's Dilemma, a version of what is known as a "tit for tat" strategy of a player i is described as follows:

- There are two "statuses" that player i might be in during any period: "normal" and "revenge";
- In a normal status player i cooperates;
- In a revenge status player i defects;
- From a normal status, player i switches to the revenge status in the next period only if the other player defects in this period;
- From a revenge status player i automatically switches back to the normal status in the next period regardless of the other player's action in this period.

Consider an indefinitely repeated game so that with probability p that the game continues to the next period and with probability $(1-p)$ it ends.

	Cooperate (C)	Defect (D)
Cooperate (C)	4,4	0,5
Defect (D)	5,0	1,1

What is the payoff for player 2 from always cooperating when player 1 uses this tit for tat strategy and begins in a normal status? How about always defecting when 1 begins in a normal status?

Your Answer		Score	Explanation
b) $4+4p+4p^2+4p^3+\ldots$; $5+p+5p^2+p^3+\ldots$	✔	1.00	
Total		1.00 / 1.00	

Question Explanation

(b) is true.

- If 2 always cooperates, then 1 stays `normal' and cooperates always as well, and the payoff to each player is 4 in each period.
- If 2 always defects, then 1 is normal in odd periods and switches to revenge in even periods (because 2 defects). 1 cooperates in odd periods and defects in even periods, thus 2 earns 5 in odd periods and 1 in even periods.

Question 9

Tit for tat

In an infinitely repeated Prisoner's Dilemma, a version of what is known as a "tit for tat" strategy of a player i is described as follows:

- There are two "statuses" that player i might be in during any period: "normal" and "revenge";
- In a normal status player i cooperates;
- In a revenge status player i defects;
- From a normal status, player i switches to the revenge status in the next period only if the other player defects in this period;
- From a revenge status player i automatically switches back to the normal status in the next period regardless of the other player's action in this period.

Consider an indefinitely repeated game so that with probability p that the game continues to the next period and with probability $(1-p)$ it ends.

	Cooperate (C)	Defect (D)
Cooperate (C)	4,4	0,5
Defect (D)	5,0	1,1

What is the threshold p_* such that when $p \geq p_*$ always cooperating by player 2 is a best response to player 1 playing tit for tat and starting in a normal status, but when $p < p_*$ always cooperating is not a best response?

Your Answer		Score	Explanation
b) 1/3	✔	1.00	
Total		1.00 / 1.00	

Question Explanation

(b) is true.

- From part (2), in order to sustain cooperation, we need $4+4p+4p^2+4p^3+\ldots \geq 5+p+5p^2+p^3+\ldots$, which is $4+4p \geq 5+p$, thus $p \geq 1/3$.
- p* = 1/3.
- Note that this just checks always cooperating against always defecting. However, you can easily check that if player 2 wants to defect in the first period, then s/he should also do so in the second period (our answer from part (1)). Then the third period looks just like we are starting the game over, so player 2 would want to defect again...

Question 1

War Game

- Two opposed armies are poised to seize an island.
- Each army can either "attack" or "not-attack".
- Also, Army 1 is either "weak" or "strong" with probability p and $(1-p)$, respectively. Army 2 is always "weak".
- Army's 1 type is known only to its general.
- An army can capture the island either by attacking when its opponent does not or by attacking when its rival does if it is strong and its rival is weak. If two armies of equal strength both attack, neither captures the island.
- The payoffs are as follows
  - The island is worth M if captured.
  - An army has a "cost" of fighting, which is equal to $s>0$ if it is strong and $w>0$ if it is weak (where $s<w<M$).
  - There is no cost of attacking if its rival does not attack.
- These payoffs are pictured in the payoff matrices below:

Weak		
1 \ 2	Attack	Not-attack
Attack	-w,-w	M,0
Not-attack	0,M	0,0

with probability p

Strong		
1 \ 2	Attack	Not-attack
Attack	M-s,-w	M,0
Not-attack	0,M	0,0

with probability $1-p$. When $p=1/2$, which is a pure strategy Bayesian equilibrium (there could be other equilibria that are not listed as one of the options): (1's type - 1's strategy; 2's strategy)

Your Answer		Score	Explanation
a) (Weak - Not-Attack, Strong - Attack; Attack);	✔	1.00	
Total		1.00 / 1.00	

Question Explanation

(a) is true.

- Check (a): If 2 chooses Attack, indeed the Weak type prefers Not-Attack and Strong type prefers Attack. Thus, with probability $1/2$, player 1 is a Weak type who chooses Not-Attack and with probability $1/2$, player 1 is a Strong type who chooses Attack. Thus, 2 prefers Attack with a payoff $(M-w)/2$, while Not-Attack gives a lower payoff of 0 (since $w<M$).
- (b) is not a Bayesian equilibrium because when Attack and Not-Attack are chosen by 1 (depending on the type) with $1/2$ probability, player 2 prefers to Attack instead Not-Attack.
- (c) is not a Bayesian equilibrium because when 2 chooses Attack, the Weak type prefers Not-Attack instead of Attack.

Question 2

Rock, Paper, Scissors

Consider the following variation to the Rock (R), Paper (P),Scissors (S) game:

- Suppose that with probability p player 1 faces a Normal opponent and with probability $1-p$, he faces a Simple opponent that will always play P.
- Player 2 knows whether he is Normal or Simple, but player 1 does not.
- The payoffs are pictured in the payoff matrices below:

Normal			
1 \ 2	R	P	S
R	0,0	-1,1	1,-1
P	1,-1	0,0	-1,1
S	-1,1	1,-1	0,0

with probability p

Simple	
1 \ 2	P
R	-1,1
P	0,0
S	1,-1

with probability $1-p$.

Suppose $p=1/3$, select all pure strategy Bayesian equilibria (there may be more than one): (Form: 1's strategy; 2's type - 2's strategy)

Your Answer		Score	Explanation
☐ a) (S; Normal - P, Simple - P)	✔	0.25	
☑ b) (S; Normal - R, Simple - P)	✔	0.25	
☐ c) (R; Normal - P, Simple - P)	✔	0.25	
☐ d) (P; Normal - P, Simple - P)	✔	0.25	
Total		1.00 / 1.00	

Question Explanation

(b) is true.

- Check (b): If 1 chooses S, Normal type prefers R and Simple type plays P. If 2 chooses R with 1/3 and and P with 2/3 probability (depending on the type), 1 is indifferent between P (with payoff =1/3*1) and S (with payoff =1/3*(-1)+2/3*1=1/3) and prefers P or S to R (with payoff = 2/3*(-1)).
- It is easy to check by similar calculations that for each of the other answers (a), (c) and (d) some player would like to deviate.

Question 3

Rock, Paper, Scissors

Consider the following variation to the Rock (R), Paper (P),Scissors (S) game:

- Suppose that with probability p player 1 faces a Normal opponent and with probability $1-p$, he faces a Simple opponent that will always play P.
- Player 2 knows whether he is Normal or Simple, but player 1 does not.
- The payoffs are pictured in the payoff matrices below:

Normal			
1 \ 2	R	P	S
R	0,0	-1,1	1,-1
P	1,-1	0,0	-1,1
S	-1,1	1,-1	0,0

with probability p

Simple	
1 \ 2	P
R	-1,1
P	0,0
S	1,-1

with probability $1-p$.

Suppose $p=2/3$, select all pure strategy Bayesian equilibria (there may be more than one): (Form: 1's strategy; 2's type - 2's strategy)

Your Answer	Score	Explanation

☐ a) (R; Normal - P, Simple - P)	✔	0.33
☐ b) (P; Normal - S, Simple - P)	✔	0.33
☐ c) (S; Normal - R, Simple - P)	✔	0.33
Total		1.00 / 1.00

Question Explanation

There is no pure strategy Bayesian equilibria.

- Check (a): If 1 chooses R, Normal type prefers P and Simple type plays P. If 2 chooses P with 2/3 and and P with 1/3 probability (depending on the type), 1 prefers S (with payoff =1) instead of R (with payoff = -1) or P (with payoff=0).
- Check (b): If 1 chooses P, Normal type prefers S and Simple type plays P. If 2 chooses S with 2/3 and and P with 1/3 probability (depending on the type), 1 is indifferent between R (with payoff =2/3*1+1/3*(-1)=1/3) and S (with payoff = 2/3*(0)+1/3*1=1/3) and prefers R or S to P (with payoff = 2/3*(-1)+1/3*(0)=-2/3).
- Check (c): If 1 chooses S, Normal type prefers R and Simple type plays P. If 2 chooses R with 2/3 and and P with 1/3 probability (depending on the type), 1 prefers P (with payoff =2/3*(1)+1/3*0=2/3) instead of R (with payoff = 2/3*(0)+1/3*(-1)=-1/3) or S (with payoff=2/3*(-1)+1/3*(1)=-1/3).
- Thus it doesn't exist, as (a), (b) and (c) are the only possible pure equilibria given 2's best responses.

Question 4

Work or Startup

- An engineer has a talent t in {1,2} with equal probability (prob$=1/2$), and the value of t is private information to the engineer.
- The engineer's pure strategies are applying for a job or being an entrepreneur and doing a startup.
- The company's pure strategies are either hiring or not hiring the engineer.

- ***If the engineer applies for the job and the company does not hire, then the engineer becomes an entrepreneur and does a startup.***
- The utility of the engineer is t (talent) from being an entrepreneur, and w (wage) from being hired.
- The utility of the company is $(t-w)$ from hiring the engineer and 0 otherwise.
- These are pictured in the payoff matrices below, with the engineer being the row player and the company being the column player.

t=2	Hire	Not
Startup	2,0	2,0
Work	w,2-w	2,0

t=1	Hire	Not
Startup	1,0	1,0
Work	w,1-w	1,0

Suppose $w=2$, which of the below are pure strategy Bayesian equilibria, there may be more than one and check all that apply. (Form: Engineer's strategy, company's strategy)

Your Answer		Score	Explanation
☐ a) ($t=2$ Startup, $t=1$ Work, Hire);	✔	0.25	
☑ b) ($t=2$ Startup, $t=1$ Work, Not);	✔	0.25	
☐ c) ($t=2$ Work, $t=1$ Work, Hire);	✔	0.25	
☑ d) ($t=2$ Work, $t=1$ Work, Not);	✔	0.25	
Total		1.00 / 1.00	

Question Explanation

(b) and (d) are true.

- Because $w=2$, type $t=1$ prefers to work if the company hires and type $t=2$ is indifferent between work and startup.
- Given that type $t=1$ prefers to work, the company prefers not to hire since it loses money from type $t=1$ and only breaks even from $t=2$.
- Thus (b) and (d) are true.

Question 5

Work or Startup

- An engineer has a talent t in {1,2} with equal probability (prob$=1/2$), and the value of t is private information to the engineer.
- The engineer's pure strategies are applying for a job or being an entrepreneur and doing a startup.
- The company's pure strategies are either hiring or not hiring the engineer.
- ***If the engineer applies for the job and the company does not hire, then the engineer becomes an entrepreneur and does a startup.***
- The utility of the engineer is t (talent) from being an entrepreneur, and w (wage) from being hired.
- The utility of the company is $(t-w)$ from hiring the engineer and 0 otherwise.
- These are pictured in the payoff matrices below, with the engineer being the row player and the company being the column player.

t=2	Hire	Not
Startup	2,0	2,0
Work	w,2-w	2,0

t=1	Hire	Not
Startup	1,0	1,0
Work	w,1-w	1,0

Suppose $w=1$, which of the below are pure strategy Bayesian equilibria, there may be more than one and check all that apply. (Form: Engineer's strategy, company's strategy)

Your Answer		Score	Explanation
☐ a) ($t=2$ Work, $t=1$ Startup, Hire);	✔	0.25	
☐ b) ($t=2$ Work, $t=1$ Startup, Not);	✔	0.25	
☑ c) ($t=2$ Startup, $t=1$ Work, Hire);	✔	0.25	
☑ d) ($t=2$ Startup, $t=1$ Work, Not);	✔	0.25	
Total		1.00 / 1.00	

Question Explanation

(c) and (d) are true.

- Because $w=1$, $t=1$ is indifferent between work and startup and $t=2$ prefers to startup.
- Given $t=1$ is indifferent and $t=2$ prefers not to work, the company is indifferent between hire or not since $w-t=1-1=0$.
- Thus (c) and (d) are true.

Question 6

Battle of Sexes

Modify the Battle of Sexes to have incomplete information:

- There are two possible types of player 2 (column):
  - "Meet" player 2 wishes to be at the same movie as player 1, just as in the usual game. (This type has probability p)
  - "Avoid" 2 wishes to avoid player 1 and go to the other movie. (This type has probability $1-p$)
- 2 knows her type, and 1 does not.
- They simultaneously choose P or L.
- These payoffs are shown in the matrices below.

Meet	L	P
L	2,1	0,0
P	0,0	1,2

with probability p

Avoid	L	P
L	2,0	0,2
P	0,1	1,0

with probability $1-p$.

When $p=1/2$, which is a pure strategy Bayesian equilibrium: (1's strategy; 2's type - 2's strategy)

Your Answer		Score	Explanation
a) (L; Meet - L, Avoid - P);	✔	1.00	

Total	1.00 / 1.00

Question Explanation

(a) is true.

- Check (a): If 1 chooses L, indeed the Meet type prefers L and Avoid type prefers P. Thus with probability=1/2, 2 is a Meet type who chooses L and with probability=1/2, 2 is a Avoid types who chooses P. Thus, 1 prefers L with a payoff of 1/2*2, while P gives a lower payoff of 1/2*1.
- (b) is not a Bayesian equilibrium because when L and P are chosen by 2 (depending on the type) with 1/2 probability, 1 prefers L instead of P.
- (c) is not a Bayesian equilibrium because when 1 chooses L, the Meet type prefers L instead of P.

Question 7

Battle of Sexes

Modify the Battle of Sexes to have incomplete information:

- There are two possible types of player 2 (column):
  - "Meet" player 2 wishes to be at the same movie as player 1, just as in the usual game. (This type has probability p)
  - "Avoid" 2 wishes to avoid player 1 and go to the other movie. (This type has probability $1-p$)
- 2 knows her type, and 1 does not.
- They simultaneously choose P or L.
- These payoffs are shown in the matrices below.

Meet	L	P
L	2,1	0,0
P	0,0	1,2

with probability p

Avoid	L	P
L	2,0	0,2
P	0,1	1,0

with probability $1-p$.

When $p=1/4$, which is a pure strategy Bayesian equilibrium : (1's strategy; 2's type - 2's strategy)

Your Answer		Score	Explanation
d) It does not exist.	✔	1.00	
Total		1.00 / 1.00	

Question Explanation

(d) is true.

- Check (a): if 1 chooses L, Meet type prefers L and Avoid type prefers P. If 2 chooses L with 1/4 and P with 3/4 probability (depending on the type), 1 prefers P (with payoff = 3/4*1) instead of L (with payoff = 1/4*2).
- Check (b): if 1 chooses P, Meet type prefers P and Avoid type prefers L. If 2 chooses L with 3/4 and P with 1/4 probability (depending on the type), 1 prefers L (with payoff 3/4*2) instead of P (with payoff = 1/4*1).
- (c) is not a Bayesian equilibrium because when 1 chooses L, Meet type prefers L instead of P.
- Thus it doesn't exist, as (a) and (b) are the only possible pure equilibria given 2's best responses.

Question 1

Core

- Three players together can obtain 1 to share, any two players can obtain 0.8, and one player by herself can obtain zero.
- Then, $N=3$ and $v(1)=v(2)=v(3)=0$, $v(1,2)=v(2,3)=v(3,1)=0.8$, $v(1,2,3)=1$.

Which allocation is in the core of this coalitional game?

Your Answer	Score	Explanation
a) (0,0,0);		
b) (0.4, 0.4, 0);		

c) (1/3, 1/3, 1/3);

d) The core is empty;	✔	1.00
Total		1.00 / 1.00

Question Explanation

(d) is true.

- By definition, the core of this game is formed by a triplet $(x_1,x_2,x_3)\in R^3_+$ that satisfies:
  - $x_i+x_j\geq 0.8$ for $i\neq j$
  - $x_1+x_2+x_3\geq 1$
  - There is no triplet (x_1,x_2,x_3) that satisfies all inequalities. Then, the core is empty.

Question 2

Buyers and Sellers

- There is a market for an indivisible good with B buyers and S sellers.
- Each seller has only one unit of the good and has a reservation price of 0.
- Each buyer wants to buy only one unit of the good and has a reservation price of 1.
- Thus $v(C)=min(B_C,S_C)$ where B_C and S_C are the number of buyers and sellers in coalition C (and so, for instance, $v(i)=0$ for any single player, and $v(i,j)=1$ if i,j are a pair of a buyer and seller).

If the number of buyers and sellers is $B=2$ and $S=1$, respectively, which allocations are in the core? [There might be more than one]

Your Answer		Score	Explanation
☑ a) Each seller receives 1 and each buyer receives 0.	✔	0.33	

☐ b) Each seller receives 0 and each buyer receives 1.	✔	0.33
☐ c) Each seller receives 1/2 and each buyer receives 1/2.	✔	0.33
Total		1.00 / 1.00

Question Explanation

(a) is true.

- By definition, the core of this game is formed by a vector of payoffs to buyers ($b1$ and $b2$) and to the seller (s) $(x_{b1},x_{b2},x_s) \in R^3_+$ that satisfies:
  - $x_{b1}+x_{b2} \geq 0$;
  - $x_{bi}+x_s \geq 1$ for $i=1,2$;
  - $x_{b1}+x_{b2}+x_s \geq 1$;
  - and the feasibility constraint $x_{b1}+x_{b2}+x_s \leq 1$.
- It is easy to verify that allocation (a) is the only one that satisfies the set of inequalities.

Question 3

Buyers and Sellers

- There is a market for an indivisible good with B buyers and S sellers.
- Each seller has only one unit of the good and has a reservation price of 0.
- Each buyer wants to buy only one unit of the good and has a reservation price of 1.
- Thus $v(C)=min(B_C,S_C)$ where B_C and S_C are the number of buyers and sellers in coalition C (and so, for instance, $v(i)=0$ for any single player, and $v(i,j)=1$ if i,j are a pair of a buyer and seller).

Now assume that competition among sellers increases, so that $B=2$ and $S=2$. Which allocations are in the core? [There might be more than one]

Your Answer	Score	Explanation

☑	a) Each seller receives 1 and each buyer receives 0.	✔	0.33
☑	b) Each seller receives 0 and each buyer receives 1.	✔	0.33
☑	c) Each seller receives 1/2 and each buyer receives 1/2.	✔	0.33
Total			1.00 / 1.00

Question Explanation

All are in the core.

- Again, the core of this game is formed by a vector of payoffs to buyers and sellers $(x_{b1},x_{b2},x_{s1},x_{s2})\in R^4_+$ that satisfies:
  - $x_{b1}+x_{b2}\geq 0$;
  - $x_{s1}+x_{s2}\geq 0$;
  - $x_{bi}+x_{sj}\geq 1$ for $i=1,2$ and $j=1,2$;
  - $x_{b1}+x_{b2}+x_{s1}+x_{s2}\geq 2$;
  - and the feasibility constraint $x_{b1}+x_{b2}+x_{s1}+x_{s2}\leq 2$.
- It is easy to verify that allocations (a), (b) and (c) satisfy the set of inequalities.
- In fact, any split of the surplus that gives α to all sellers and $1-\alpha$ to all buyers (with $\alpha\in[0,1]$) is in the core. That is, any split of the surplus is possible; the only restriction imposed by the increase in competition (i.e., increase in the number of sellers) is that all pairs must receive the same share of the surplus.

Question 4

Core and Shapley Value

- The instructor of a class allows the students to collaborate and write up together a particular problem in the homework assignment.
- Points earned by a collaborating team are divided among the students in any way they

  agree on.

- There are exactly three students taking the course, all equally talented, and they need to decide which of them if any should collaborate.
- The problem is so hard that none of them working alone would score any points. Any two of them can score 4 points together. If all three collaborate, they can score 6 points.

Which allocations are in the core of this coalitional game?

Your Answer		Score	Explanation
a) (0,0,0);			
b) (2, 2, 0);			
c) (2, 2, 2);	✔	1.00	
d) The core is empty;			
Total		1.00 / 1.00	

Question Explanation

(c) is true.

- By definition, the core of this game is formed by a vector of payoffs to each student $(x_1,x_2,x_3) \in R^3_+$ that satisfies:
  - $x_i+x_j \geq 4$ for $i \neq j$
  - $x_1+x_2+x_3 \geq 6$
  - (2,2,2) is the only option that satisfies these inequalities. Then, it belongs to the core.

Question 5

Core and Shapley Value

- The instructor of a class allows the students to collaborate and write up together a particular problem in the homework assignment.

- Points earned by a collaborating team are divided among the students in any way they agree on.
- There are exactly three students taking the course, all equally talented, and they need to decide which of them if any should collaborate.
- The problem is so hard that none of them working alone would score any points. Any two of them can score 4 points together. If all three collaborate, they can score 6 points.

What is the Shapley value of each player?

Your Answer		Score	Explanation
○ a) $\phi=(0,0,0)$			
○ b) $\phi=(2,0,2)$			
○ c) $\phi=(1/3,1/3,1/3)$			
● d) $\phi=(2,2,2)$	✔	1.00	
Total		1.00 / 1.00	

Question Explanation

(d) is true.

- Use the definition of the Shapley Value to compute its value for each player.
- Another way to find the Shapley Value is to remember that:
  - by the axiom of symmetry, all agents should receive the same payoff.
  - the Shapley value divides the payoff to the grand coalition completely
  - Then, all agents will have a Shapley value of $6/3=2$.

Question 6

Production

- There is a single capitalist (c) and a group of 2 workers ($w1$ and $w2$).
- The production function is such that total output is 0 if the firm (coalition) is composed only of the capitalist or of the workers (a coalition between the capitalist and a worker is required to produce positive output).

- The production function satisfies:
  - $F(c \cup w1) = F(c \cup w2) = 3$
  - $F(c \cup w1 \cup w2) = 4$

Which allocations are in the core of this coalitional game? [There might be more than one]

Your Answer		Score	Explanation
☑ a) $x_c=2$, $x_{w1}=1$, $x_{w2}=1$;	✔	0.33	
☑ b) $x_c=2.5$, $x_{w1}=0.5$, $x_{w2}=1$;	✔	0.33	
☑ c) $x_c=4$, $x_{w1}=0$, $x_{w2}=0$;	✔	0.33	
Total		1.00 / 1.00	

Question Explanation

(d) is true.

- It is easy to verify that allocations (a), (b) and (c) satisfy the definition of the core.
- It can be shown more generally that for any given number n of workers and any increasing and concave production function f, the core of this coalitional game is defined by:
  - $x_{wi} \leq f(c \cup w1 \ldots \cup wn) - f(c \cup w1 \ldots \cup w(n-1))$
  - $x_c + \sum_{i=1}^{n} x_{w1} \leq f(c \cup w1 \cup \ldots \cup wn)$
- Intuitively, the first equation requires each worker to receive less than the marginal product of the n_{th} worker. If this condition would not hold for worker i, then the rest of the workers and the capitalist could abandon him and get a higher value for the new coalition.
- The second condition is a feasibility condition (the sum of payoffs of the grand coalition is not greater than the resources available).

Question 7

Production

- There is a single capitalist (c) and a group of 2 workers ($w1$ and $w2$).

- The production function is such that total output is 0 if the firm (coalition) is composed only of the capitalist or of the workers (a coalition between the capitalist and a worker is required to produce positive output).
- The production function satisfies:
  - $F(c \cup w1) = F(c \cup w2) = 3$
  - $F(c \cup w1 \cup w2) = 4$

What is the Shapley value of the capitalist?

Your Answer		Score	Explanation
a) 3;			
b) 4;			
c) 7/3;	✔	1.00	
d) 7;			
Total		1.00 / 1.00	

Question Explanation

(c) is true.

- Use the definition of the Shapley Value to compute its value for the capitalist.

Question 8

Production

- There is a single capitalist (c) and a group of 2 workers ($w1$ and $w2$).
- The production function is such that total output is 0 if the firm (coalition) is composed only of the capitalist or of the workers (a coalition between the capitalist and a worker is required to produce positive output).

- The production function satisfies:
  - $F(c \cup w1) = F(c \cup w2) = 3$
  - $F(c \cup w1 \cup w2) = 4$

What is the Shapley value of each worker?

Your Answer		Score	Explanation
a) 1;			
b) 5/6;	✔	1.00	
c) 3/4;			
d) 1/2;			
Total		1.00 / 1.00	

Question Explanation

(b) is true.

- Use the definition of the Shapley Value to compute its value for each worker.
- Another way to find the Shapley Value is to remember that:
  - by the axiom of symmetry, all workers should receive the same payoff
  - the Shapley value divides the payoff to the grand coalition completely
  - Then, all agents will have a Shapley value of $(F(c \cup w1 \cup w2) - 7/3)/2 = (4 - 7/3)/2 = 5/6$.

Question 9

Production

- There is a single capitalist (c) and a group of 2 workers ($w1$ and $w2$).

- The production function is such that total output is 0 if the firm (coalition) is composed only of the capitalist or of the workers (a coalition between the capitalist and a worker is required to produce positive output).
- The production function satisfies:
  - $F(c \cup w1) = F(c \cup w2) = 3$
  - $F(c \cup w1 \cup w2) = 4$

True or False: If there was an additional 3_{rd} worker that is completely useless (i.e., his marginal contribution is 0 in every coalition), then the sum of the Shapley Values of the capitalist and the first two workers will remain unchanged.

Your Answer		Score	Explanation
a) True;	✔	1.00	
b) False;			
Total		1.00 / 1.00	

Question Explanation

(a) is correct.

- The Shapley Value satisfies the Dummy player Axiom:
  - if i is a dummy player, then he/she must have a Shapley Value of 0
- Since the 3_{rd} worker is a Dummy player (check the definition), his/her Shapley Value must be 0.
- Thus, the statement is true because the Shapley Value divides the payoff of the grand coalition completely.

Question 1

1-3 Defining Games

Consider the following normal form:

- N={1, 2}

- A_i={Movie, Theater} Each player chooses an action of either going to a movie or going to the theater.
- Player 1 prefers to see a movie with Player 2 over going to the theater with Player 2.
- Player 2 prefers to go to the theater with Player 1 over seeing a movie with Player 1.
- Players get a payoff of 0 if they end up at a different place than the other player.

Player 1\ Player 2	Movie	Theater
Movie	a,b	0,0
Theater	0,0	c,d

Which restrictions should a, b, c and d satisfy?

Your Answer		Score	Explanation
a) $a>c, b>d$;			
b) $a>d, b<c$;			
c) $a>c, b<d$;	✔	1.00	
d) $a<c, b<d$;			
Total		1.00 / 1.00	

Question Explanation

(c) is true.

- Since Player 1 prefers to seeing a movie over going to the theater, then Player 1's payoff under (Movie, Movie) has to be larger than the payoff under (Theater, Theater). Thus, $a>c$.
- Since Player 2 prefers to go to the theater over seeing a movie, then Player 2's payoff under (Theater, Theater) has to be larger than the payoff under (Movie, Movie). Thus, $b<d$.

Question 2

1-4 Examples of Games

Consider the following constant-sum game:

	H	T

H	1,-1	
T		0,?

What should be filled in ?:

Your Answer		Score	Explanation
a) -1;			
b) 0;	✔	1.00	
c) 1;			
d) 2.			
Total		1.00 / 1.00	

Question Explanation

(b) is true.

- In a constant-sum game, there is a constant k such that $u_1(a_1,a_2)+u_2(a_1,a_2)=k$, for all possible (a_1,a_2).
- We know $u_1(H,H)=1$ and $u_2(H,H)=-1$, thus $k=1+(-1)=0$.
- Thus $?=u_2(T,T)=k-u_1(T,T)=0-0=0$.

Question 3

1-6 Strategic Reasoning

n people guess an integer between 1 and 100, and the winner is the player whose guess is closest to the mean of the guesses + 1 (ties broken randomly). Which of the following is an equilibrium:

Your Answer	Score	Explanation

a) All announce 1.

b) All announce 50.

c) All announce 75.

d) All announce 100. ✔ 1.00

Total 1.00 / 1.00

Question Explanation

(d) is true.

- Each player's best response is to announce a number closest to the average + 1, subject to the constraint of the 100.
- So, each person wants to name a number above average, and so nothing is stable except all saying 100.
- They cannot announce more than 100, and that is then an equilibrium.

Question 4

1-7 Best Response and Nash Equilibrium

Consider the collective-action game:

Player 1\ Player 2	Revolt	Not
Revolt	2,2	-1,1
Not	1,-1	0,0

When player 1 plays "Not", for player 2

Your Answer	Score	Explanation

a) "Revolt" is a best response.

b) "Not" is a best response. ✔ 1.00

c) "Revolt" and "Not" are both best responses.

d) There is no best response.

Total 1.00 / 1.00

Question Explanation

(b) is true.

- When player 1 plays "Not", player 2 gets -1 from "Revolt" and 0 from "Not". Thus "Not" is a best response.
- No strategy is a dominant strategy:
  - When the other player plays "Not", it is strictly better to play "Not";
  - When the other player plays "Revolt", it is strictly better to play "Revolt";
  - No strategy always dominates the other strategy.

Question 5

1-8 Nash Equilibrium of Example Games

Consider the following game in which two firms must decide whether to open a new plant or not:

Firm 1\ Firm 2	Build	Not
Build	1,1	3,0
Not	0,3	2,2

Find all pure strategy Nash equilibrium:

Your Answer	**Score**	**Explanation**

a) Only (Build, Not).

b) Only (Not, Not).

c) Only (Build, Build). ✔ 1.00

d) Only (Not, Build).

Total 1.00 / 1.00

Question Explanation

(c) is true.

- (Build, Build) is a pure strategy Nash equilibrium:
  - When firm 1 chooses Build, firm 2 gets 1 from Build and 0 from Not, so firm 2 has no incentive to deviate from Build.
  - When firm 2 chooses Build, firm 1 gets 1 from Build and 0 from Not, so firm 1 has no incentive to deviate from Build.
- (Not, Not) is not a pure strategy Nash equilibrium:
  - When firm 1 chooses Not, firm 2 gets 3 from Build and 2 from Not. So firm 2 would gain by deviating to Build.
- Similarly, you can check that from each of the other combinations of pure strategies some player would strictly benefit from deviating.

Question 6

1-9 Dominant Strategies

Consider the game:

Player 1\ Player 2	Left	Right
Up	2,1	1,1
Down	0,1	0,2

Which of the players has a strictly dominant strategy?

Your Answer	**Score**	**Explanation**

a) Player 1.	✔	1.00
b) Player 2.		
c) Both players.		
d) Neither player.		
Total		1.00 / 1.00

Question Explanation

(a) is true.

- "Up" is a strictly dominant strategy for player 1 because
  - When player 2 plays Left, player 1 gets 2 from Up and 0 from Down (Up is strictly better); When player 2 plays Right, player 1 gets 1 from Up and 0 from Down (Up is strictly better).
- Player 2 does not have a strictly dominant strategy, only a very weakly dominant strategy.
  - When player 1 plays Up, player 2 gets 1 from either Left or Right (so is indifferent); When player 1 plays Down, player 2 gets 1 from Left and 2 from Right (Right is strictly better.).

Question 7

1-10 Pareto Optimality

Consider the game:

Player 1\ Player 2	Left	Right
Left	3,3	1,1
Right	1,4	1,1

Which of the following outcomes is Pareto-optimal? (There might be more than one, or none.)

Your Answer	Score	Explanation

☑ a) (3,3);	✔	0.33	
☑ b) (1,4);	✔	0.33	
☐ c) (1,1);	✔	0.33	
Total		1.00 / 1.00	

Question Explanation

(a) and (b) are Pareto-optimal.

- Checking that (a) and (b) are Pareto-optimal:
  - Neither outcome is Pareto-dominated by (1,1).
  - Also, (a) does not Pareto-dominate (b) and vice versa (in (a) one player is strictly better off and the other player is strictly worse off than in (b)).
- (c) can't be Pareto-optimal since is it Pareto-dominated by (a) and (b)
  - At least one player is strictly better off and the remaining player is at least indifferent between both outcomes.

Question 1

2-3 Computing Mixed Nash Equilibrium (I)

Consider the predator/prey game with a mixed strategy:

mixed		p	1-p
	Pred\ Prey	Active	Passive
q	Active	2,-5	3,-6
1-q	Passive	3,-2	-1,0

What are p and q in a mixed-strategy equilibrium? (Hint: payoff of the predator when playing active is $2p+3(1-p)$; when playing passive is $3p-(1-p)$. Payoffs should be equal since the predator should be indifferent.)

Your Answer	Score	Explanation

a) 2/3 ; 1/2.

b) 2/5; 1/3.

c) 4/5; 3/5.

d) 4/5; 2/3. ✔ 1.00

Total 1.00 / 1.00

Question Explanation

(d) is true.

- For p, following the hint we have, $2p+3(1-p)=3p-(1-p)$, implies $p=4/5$.
- For q, payoff of the prey when playing active is $-5q-2(1-q)$; when playing passive it is $-6q$.
- These two payoffs should be equal: $-5q-2(1-q)=-6q$, implies $q=2/3$.

Question 2

2-5 Example: Mixed Strategy Nash

Consider the penalty kick game with a very accurate kicker:

mixed		p	1-p
	Kicker\ Goalie	Left	Right
q	Left	0,1	1,0
1-q	Right	1,0	0,1

What are p and q in a mixed-strategy equilibrium? (Hint: payoff of the kicker when playing left is $0p+(1-p)$; when playing right is $p+0(1-p)$. Payoffs should be equal since the kicker should be indifferent.)

Your Answer	Score	Explanation

a) 1/2 ; 3/4.

b) 3/4; 1.

c) 1/2; 1/2. ✔ 1.00

d) 3/4; 3/4.

Total 1.00 / 1.00

Question Explanation

(c) is true.

- For p, following the hint we have, $0p+1(1-p)=p+0(1-p)$, implies $p=1/2$.
- For q, payoff of the goalie when playing left is $q+0(1-q)$; when playing right it is $0q+(1-q)$.
- These two payoffs should be equal: $q+0(1-q)=0q+(1-q)$, implies $q=1/2$.

Question 3

2-6 Data: Professional Sports and Mixed Strategies

Consider the following game:

1\ 2	L	R
T	2,2	0,2
B	1,2	3,3

Find all pure-strategy and mixed-strategy Nash equilibria:

Your Answer	**Score**	**Explanation**

a) (T, L);

b) (B, R);

c) Player 1 plays T with prob $q=1$, player 2 plays L with prob $p=3/4$;

d) All of above. ✔ 1.00

Total 1.00 / 1.00

Question Explanation

(d) is true.

- You can verify that (T, L) and (B, R) are pure strategy Nash equilibria by showing no single player would be better off by deviating from the prescribed strategy, taking the other's as given.
- Mixed equilibrium where player 1 plays T with $prob=q$ and player 2 plays L with $prob=p$:
    - For p, payoff of 1 when playing T is $2p+0(1-p)$, and when playing B is $1p+3(1-p)$. These payoffs should be equal implying $p=3/4$.
    - For q, you can check this in the same way yourself.
- Notice that the mixed-strategy equilibrium requires one of the players to be playing deterministically, that is, not randomizing at all.

Question 1

3-2 Strictly Dominated Strategies & Iterative Removal

Consider the game:

Player 1\ Player 2	u	m	d
U	2,1	5,3	3,1
M	6,7	2,10	0,0
D	5,0	1,1	2,4

Which pair of strategies survives the process of iterative removal of strictly dominated strategies?

Your Answer	Score	Explanation

a) (U,u);

◉ b) (U,m);	✔	1.00
○ c) (M,u);		
○ d) (D,d);		
Total		1.00 / 1.00

Question Explanation

(b) is true.

- Check (a):
  - u is dominated by m.
  - M and D are dominated by U, once u is removed.
  - d is dominated by m, once M and D are removed.

Question 2

3-4 Maxmin Strategies

Consider the game:

Player 1\ Player 2	Movie	Home
Movie	3,0	1,2
Home	2,1	0,3

Which is a maxminstrategy for player 1:

Your Answer		Score	Explanation
◉ a) Play Movie;	✔	1.00	
○ b) Play Home;			

c) Play Movie with 1/2 and Home with 1/2;

d) Play Movie with 1/3 and Home with 2/3.

Total 1.00 / 1.00

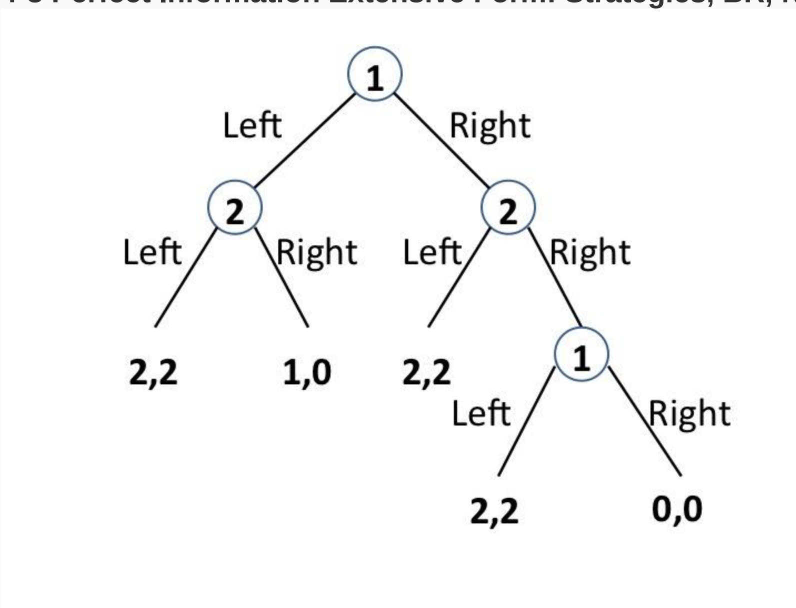
Question Explanation

(a) is true.

- Recall from lecture: $S_1 = argmax_{s'_1 \in S_1} \min_{s_2 \in S_2} u_1(s'_1, s_2)$
- Regardless of player 1's strategy, choosing Home by player 2 minimizes 1's payoff:
  - If 1 plays Movie, 1 gets 3 when 2 plays Movie and 1 when 2 plays Home;
  - If 1 plays Home, 1 gets 2 when 2 plays Movie and 0 when 2 plays Home;
- Given 2 plays Home to minimize 1's payoff, 1 plays Movie to maximize the minimized payoff.

Question 1

4-3 Perfect Information Extensive Form: Strategies, BR, NE



What is the number of pure strategies that each player has:

Your Answer		Score	Explanation
a) Both have 2 strategies.			
b) Both have 4 strategies.	✔	1.00	
c) Player 1 has 2, and player 2 has 4.			
d) Player 1 has 3, and player 2 has 4.			
Total		1.00 / 1.00	

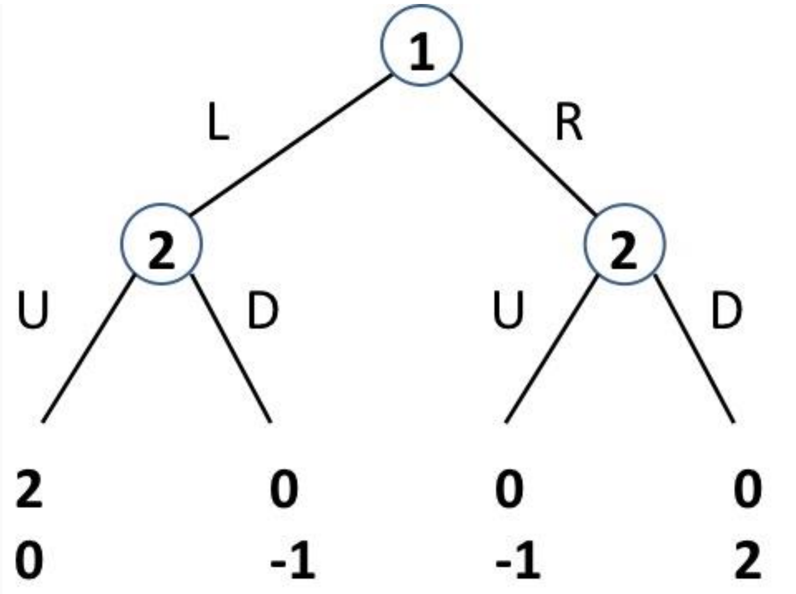
Question Explanation

(b) is true.

- Each player has two decision nodes and in each decision node there are two possible actions: Left or Right.
- Thus, players 1 and 2 both have 4 pure strategies:
    - Left, Left;
    - Left, Right;
    - Right, Left;
    - Right, Right;

Question 2

4-4 Subgame Perfection



How many subgames are in this game? Which is a subgame perfect equilibrium?

Your Answer		Score	Explanation
a) There are 1 subgames; (L), (U,D);			
b) There are 1 subgames; (L), (U,U);			
c) There are 3 subgames; (L), (U,D);	✔	1.00	
d) There are 3 subgames; (L), (U,U).			
Total		1.00 / 1.00	

Question Explanation

(c) is true.

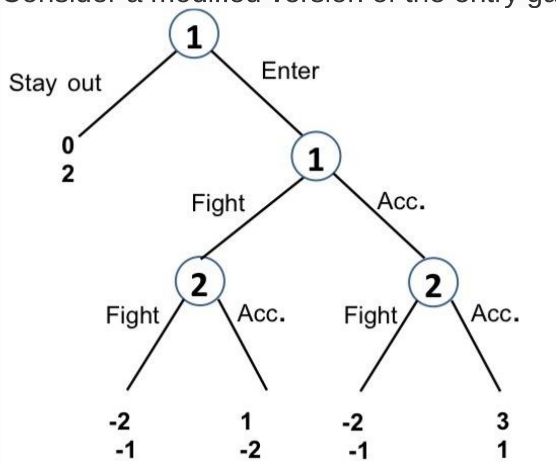
- There are 3 subgames: the original game and two single-player subgames (both nodes in which 2 has to decide between U and D).

- In the subgame following 1 choosing L, it is (uniquely) optimal for 2 to choose U; in the subgame such that 1 chooses R, it is (uniquely) optimal for 2 to choose D.
- Then 1 prefers L leading to (2, 0) to R leading to (0, 2).

Question 3

4-5 Backward Induction

Consider a modified version of the entry game:



Which is the backward induction solution of this game? [Here (Enter,Fight), (Fight, Acc.)) indicates that player 1 chooses Enter at the first decision node and Fight at the second decision node, and 2 chooses Fight at the left node and Accommodate at the right node.]

Your Answer		Score	Explanation
a) (Enter, Acc.), (Fight, Fight).			
b) (Enter, Fight), (Acc., Acc.).			
c) (Stay out, Acc.), (Fight, Acc.).			
d) (Enter, Acc.), (Fight, Acc.).	✔	1.00	

Total	1.00 / 1.00

Question Explanation

(d) is true.

- (a) and (b) cannot be the answer:
  - If 1 plays Fight, 2 prefers to Fight;
  - If 1 plays Acc., 2 prefers to Acc.;
  - Thus, the backward induction solution requires 2 playing (Fight, Acc.)
- Since 2 plays (Fight, Acc.), 1 prefers to Acc. than Fight (payoff of Acc. is 3 and payoff of Fight is -2).
- If 1 enters, he knows that by backward induction he will receive 3. This is better than 0, the outcome of staying out.

Question 4

4-6 Subgame Perfect Application: Ultimatum Bargaining

Consider the modified game:

- Player A makes an offer x in $0,1,...10$ to player B;
- Player B can accept or reject;
- A gets $10-x$ and B gets x if accepted;
- If rejected, player A gets 0 and player B gets a punishment of -1.

Which is a possible outcome (payoff to players A,B) from backward induction?

Your Answer	Score	Explanation
a) (9, 1).		
b) (5, 5).		
c) (0, -1).		

d) (10, 0).	✔	1.00
Total		1.00 / 1.00

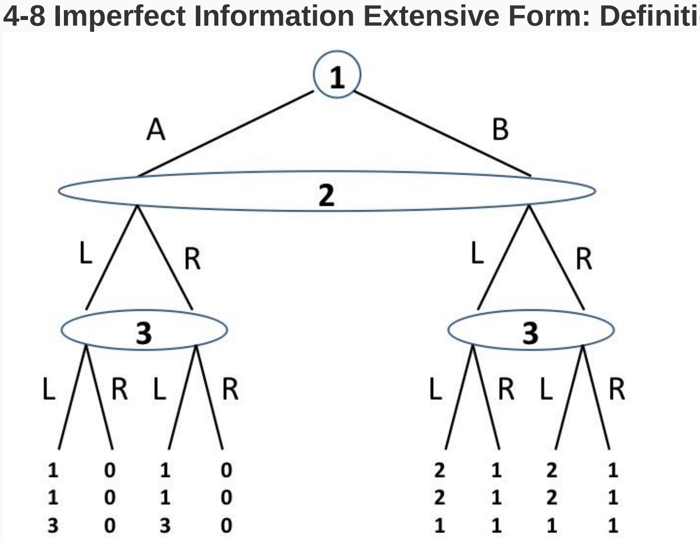
Question Explanation

(d) is true.

- In the subgame, it is optimal for B to accept always since by accepting B guarantees a payoff of at least 0, which is larger than the payoff of rejecting (-1).
- (a) and (b) cannot be backward induction outcomes, because A could offer 0 and get a payoff of 10 (since B always accepts).
- (c) cannot be a backward induction outcome since it corresponds to the outcome when B rejects.
- Thus, (d) is the **only** backward induction outcome.

Question 5

4-8 Imperfect Information Extensive Form: Definition, Strategies



What is player 3's knowledge of player 1's choice:

Your Answer	Score	Explanation
a) Player 3 knows nothing.		

b) Player 3 knows only player 2's choice, but not player 1's choice.

c) Player 3 knows whether it is A or not. ✔ 1.00

Total 1.00 / 1.00

Question Explanation

c) is true.

- From the figure, after 1 makes a choice, 3 knows whether the choice is A or not, but cannot distinguish player 2's choice, whether it was L or R, since they lead to the same information set of player 3.

Question 1

5-2 Infinitely Repeated Games: Utility

- Consider a repeated game such that with probability p the game continues to the next period and with probability $(1-p)$ it ends.
- The game starts in period 1 and in odd periods both players play L and in even periods both players play R. The stage game payoffs are listed below

1 \ 2	L	R
L	3,3	-1,4
R	4,-1	1,1

What is the expected total future payoff (starting at the beginning of the game) for each player, when the game is forecast to be played as described as above:

Your Answer	Score	Explanation
a) $3+3p+3p^2+3p^3+\dots$		
b) $4+-1p+4p^2+-1p^3+\dots$		
c) $3+1p+3p^2+1p^3+\dots$ ✔	1.00	

d) $4+3p+4p^2+3p^3+\ldots$

Total	1.00 / 1.00

Question Explanation

(c) is true.

- In odd periods, both players play L so that each earns 3 in those periods.
- In even periods, both players play R such that each earns 1 in those periods.
- Thus the total ex ante expected payoff for each player is $3+1p+3p^2+1p^3+\ldots$, as p is the probability that the second period is reached, p^2 is the probability that the third period is reached and so forth.

Question 2

5-6 Discounted Repeated Games

Consider the rock-paper-scissors game:

1\ 2	Rock	Paper	Scissors
Rock	0,0	-1,1	1,-1
Paper	1,-1	0,0	-1,1
Scissors	-1,1	1,-1	0,0

How many elements are there in H^2 (the set of histories of two plays of the game):

Your Answer		Score	Explanation
a) 2^3.			
b) 9^2.	✔	1.00	
c) 3^2.			
d) 3^3.			
Total		1.00 / 1.00	

Question Explanation

(b) is true.

- H_1 has 9 elements: (R,R), (R,P), (R,S), (P,R), (P,P), (P,S), (S,R), (S,P), (S,S).
- Then H_2 has 9x9 elements of the form (h_1,h_2) where h_1 and h_2 each has 9 possible values (the same as those in H_1).

Question 3

5-7 A Folk Theorem for Discounted Repeated Games

Player 1\ Player 2	Movie	Home
Movie	3,0	1,2
Home	2,1	0,3

Which per period payoff is not enforceable:

Your Answer		Score	Explanation
a) (0,3)			
b) (3,0)			
c) (2,1)			
d) All of above.	✔	1.00	
Total		1.00 / 1.00	

Question Explanation

(d).

- The minmax value of player 1 is 1 and of player 2 is 2.
- Thus (0,3), (3,0) and (2,1) are not enforceable since in each case they give to a player an expected value lower than her minmax value.

Question 1

6-2 Coalitional Game Theory: Definitions

Suppose $N=3$ and $v(1)=v(2)=v(3)=1$. Which of the following payoff functions is superadditive?

Your Answer		Score	Explanation
a) $v(1,2)=3$, $v(1,3)=4$, $v(2,3)=5$, $v(1,2,3)=5$;			
b) $v(1,2)=3$, $v(1,3)=4$, $v(2,3)=5$, $v(1,2,3)=7$;	✔	1.00	
c) $v(1,2)=0$, $v(1,3)=4$, $v(2,3)=5$, $v(1,2,3)=7$;			
d) None of the above.			
Total		1.00 / 1.00	

Question Explanation

(b) is true.

- Use the definition of superadditivity to check that (b) is the answer.
- (a) is not supperaditive because $5=v(2,3\cup 1)<v(2,3)+v(1)=5+1$.
- (c) is not supperaditive because $0=v(1\cup 2)<v(1)+v(2)=1+1$.

Question 2

6-3 The Shapley Value

Suppose $N=2$ and $v(1)=0$, $v(2)=2$, $v(1,2)=2$. What is the Shapley Value of both players?

Your Answer		Score	Explanation
a) $\phi_1(N,v)=1$, $\phi_2(N,v)=0$			
b) $\phi_1(N,v)=1/2$, $\phi_2(N,v)=1/2$			
c) $\phi_1(N,v)=1/3$, $\phi_2(N,v)=2/3$			
d) $\phi_1(N,v)=0$, $\phi_2(N,v)=2$	✔	1.00	
Total		1.00 / 1.00	

Question Explanation

(d) is true.

- Use the definition of the Shapley Value to compute its value for each player.
- Another way to find the Shapley Value is to notice that player 1 is a dummy player:
  - when added to the unique coalition $1,2$, player 1's contribution is 0.
  - By the theorem presented in the lecture, the Shapley Value satisfies the Dummy player axiom. Then, $\phi_1(N,v)$ must be 0.

Question 3

6-4 The Core

- Suppose $N=3$ and $v(1)=v(2)=v(3)=0$, $v(1,2)=v(2,3)=v(3,1)=2/3$, $v(1,2,3)=1$.

Which allocation is in the core of this coalitional game?

Your Answer		Score	Explanation
○ a) (0,0,0);			
○ b) (1/3, 1/3, 0);			
● c) (1/3, 1/3, 1/3);	✔	1.00	
○ d) None of the above.			
Total		1.00 / 1.00	

Question Explanation

(c) is true.

- By definition, the core of this game is formed by a triplet $(x_1,x_2,x_3)\in R^3_+$ that satisfies:
  - $x_i+x_j\geq 2/3$ for $i\neq j$

- $x_1+x_2+x_3 \geq 1$
- Then, the core is a singleton with $(x_1,x_2,x_3)=(1/3,1/3,1/3)$.

Question 1

7-4 Analyzing Bayesian Games

In the following two-player Bayesian game, the payoffs to player 2 depend on whether 2 is a friendly player (with probability p) or a foe (with probability $1-p$). See the following payoff matrices for details.

Friend	Left	Right
Left	3,1	0,0
Right	2,1	1,0

with probability p.

Foe	Left	Right
Left	3,0	0,1
Right	2,0	1,1

with probability $1-p$.

Player 2 knows if he/she is a friend or a foe, but player 1 doesn't know. If player 2 uses a strategy of Left when a friend and Right when a foe, what is true about player 1's expected utility?

Your Answer		Score	Explanation
a) It is 3 when 1 chooses Left;			
b) It is $3p$ when 1 chooses Left;	✔	1.00	
c) It is $2p$ when 1 chooses Right;			
d) It is 1 when 1 chooses Right;			
Total		1.00 / 1.00	

Question Explanation

(b) is true.

- If 1 chooses Left, with probability p player 2 is a friend and chooses Left and then 1 earns 3, and with probability $(1-p)$ player 2 is a foe and chooses Right and then 1 earns 0. Thus, the expected payoff is $3p+0(1-p)=3p$.

Question 2

7-5 Analyzing Bayesian Games: Another Example

Consider the conflict game:

Strong	Fight	Not
Fight	1,-2	2,-1
Not	-1,2	0,0

with probability p

Weak	Fight	Not
Fight	-2,1	2,-1
Not	-1,2	0,0

with probability $1-p$

Assume that player 1 plays fight when strong and not when weak. Given this strategy of player 1, there is a certain $p*$ such that player 2 will prefer 'fight' when Misplaced & , and 'not' when $p>p*$. For instance, in the lecture $p*$ was 1/3.

What is $p*$ in this modified game? (Hint: Write down the payoff of 2 when choosing Fight and Not Fight. Equalize these two payoffs to get $p*$):

Your Answer		Score	Explanation
a) 3/4			
b) 1/3			
c) 2/3	✔	1.00	
d) 1/2			
Total		1.00 / 1.00	

Question Explanation

(c) is true.

- Conditional on 1fighting when strong and not fighting when weak, the payoff of 2 when choosing Not is $-1p+0(1-p)$ and the payoff of 2 when choosing Fight is $(-2)p+2(1-p)$.

- Comparing these two payoffs, 2 is just indifferent when $-1p+0(1-p)=(-2)p+2(1-p)$, thus $p_*=2/3$, above which 2 prefers Not and below which 2 prefers to Fight.

http://gametheory.cs.ubc.ca/tcpbackoff
http://gametheory.cs.ubc.ca/matchingpennies
http://gametheory.cs.ubc.ca/coordination
http://gametheory.cs.ubc.ca/battleofthesexes
http://gametheory.cs.ubc.ca/twothirdsavg
http://gametheory.cs.ubc.ca/centipede
http://gametheory.cs.ubc.ca/ultimatum
http://gametheory.cs.ubc.ca/rainbowwarship
http://gametheory.cs.ubc.ca/repeatedprisoners
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