Test Exercise 5 – MOOC Econometrics – Binary Choice

Question 1 – Sum of the Partial derivatives of the Probabilities are equal to zero

From slide 12 from Lecture 5.5, we are given the partial derivative for when probability of the response is positive ($resp_i$ =1).

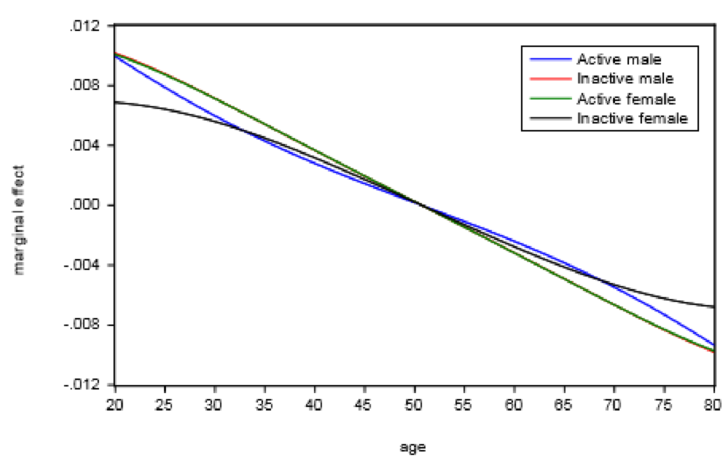
$$
\begin{aligned}
\frac{\partial \Pr[\text{resp}_i = 1]}{\partial age_i} &= \Pr[\text{resp}_i = 1] \Pr[\text{resp}_i = 0](\beta_3 + 2\beta_4(\text{age}_i/10)^2) \\
&\approx \Pr[\text{resp}_i = 1] \Pr[\text{resp}_i = 0](0.07 - 2 \times 0.07\text{age}_i/100)
\end{aligned}
$$

Marginal effect depends on

- age_i
- $\Pr[\text{resp}_i = 1]$ and $\Pr[\text{resp}_i = 0]$ and hence also on male and active dummy.

When this relationship is graphed, we find that that there are a series of negatively sloped lines.

Marginal effect of age



When the probability of the response is negative ($\Pr(rep_i=0)$, the lines of these results would be a mirrored reflection of the graph above. Therefore, at each age point the marginal effects of would cancel each other out. For example, at age 40 for active males, the marginal effect appears to be approximately 0.004 for $\Pr(rep_1=1)$. The expected marginal effect for age 40 for the $\Pr(resp_1=0)$ would be equal to -0.004. Since these two results cancel each other out, the sum would be equal to 0.

Question B – Recoding Dependent Variable

The result from our original model is as follows.

```
Model 3: Logit, using observations 1-925
Dependent variable: response
QML standard errors

             coefficient   std. error     z        slope
  ---------------------------------------------------------
  const      -2.48836     0.905783    -2.747
  male        0.953694    0.157805     6.043    0.232178
  activity    0.913748    0.185231     4.933    0.219249
  age         0.0699453   0.0359327    1.947    0.0174817
  age010squ  -0.0686920   0.0344304   -1.995   -0.0171684

Mean dependent var   0.508108   S.D. dependent var   0.500205
McFadden R-squared   0.061115   Adjusted R-squared   0.053315
Log-likelihood      -601.8624   Akaike criterion     1213.725
Schwarz criterion    1237.874   Hannan-Quinn         1222.938

Number of cases 'correctly predicted' = 570 (61.6%)
f(beta'x) at mean of independent vars = 0.250
Likelihood ratio test: Chi-square(4) = 78.3543 [0.0000]

           Predicted
              0     1
  Actual 0  196   259
         1   96   374
```

If we are to transform the dependent variable, where $resp_i^{new} = -resp_i + 1$, the results for the coefficients would be as follows.

```
Model 4: Logit, using observations 1-925
Dependent variable: response_new
Standard errors based on Hessian

             coefficient   std. error     z        slope
  ---------------------------------------------------------
  const       2.48836     0.889992     2.796
  male       -0.953694    0.158183    -6.029   -0.232178
  activity   -0.913748    0.184779    -4.945   -0.219249
  age        -0.0699453   0.0356054   -1.964   -0.0174817
  age010squ   0.0686920   0.0340963    2.015    0.0171684

Mean dependent var   0.491892   S.D. dependent var   0.500205
McFadden R-squared   0.061115   Adjusted R-squared   0.053315
Log-likelihood      -601.8624   Akaike criterion     1213.725
Schwarz criterion    1237.874   Hannan-Quinn         1222.938

Number of cases 'correctly predicted' = 570 (61.6%)
f(beta'x) at mean of independent vars = 0.250
Likelihood ratio test: Chi-square(4) = 78.3543 [0.0000]

           Predicted
              0     1
  Actual 0  374    96
         1  259   196
```

The results show that except for the constant value, the coefficient values change their signs (positive to negative, and negative to positive).

If we were to illustrate this change using the Odds ratio, we had the original odds ratio equation as follows.

$$\frac{\Pr(response_i = 1)}{\Pr(response_i) = 0} = \exp\left[(\beta_0 + \beta_1 male + \beta_2 active + \beta_3 age + \beta_4 \left(\frac{age}{10}\right)^2\right]$$

If you inverse the formula where

$$\left[\frac{\Pr(response_i) = 1)}{\Pr(response_i) = 0}\right]^{-1} = \frac{\Pr(response_i) = 0)}{\Pr(response_i) = 1)}$$

Since you are changing the probability values by multiplying the exponent values by -1, you will get a change in the correspondent coefficient values.

Question C – Extending Logit Model

Lecture 5.2 discussed how to adjust the Logit Model by changing the parameters for either β_1 and/or β_0, and this would either make the Logit model graph steeper (where $\beta_1 > 1$) or shifted to either the right or left of the average value. To shift the logit model further to the right would mean to have the $\beta_0 < 0$.