Examples

Coin tosses, once more

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  - *Sample space*: Ω = {𝔥𝔥𝔥, 𝔥𝔥𝔱, 𝔥𝔱𝔥, 𝔥𝔱𝔱, 𝔱𝔥𝔥, 𝔱𝔥𝔱, 𝔱𝔱𝔥, 𝔱𝔱𝔱}.

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  - *Events:*
    - A := The first toss results in heads.

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  - *Events:*
    - A := The first toss results in heads.
    - B := The second toss results in tails.

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  - *Events:*
    - A := The first toss results in heads.
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    - C := The third toss results in heads.

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  - *Events:*
    - A := The first toss results in heads.
    - B := The second toss results in tails.
    - C := The third toss results in heads.
  - *Probability measure*: Combinatorial setting with mass function assigning equal probability 1/8 to each atom.

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  - *Events:*
    - A := The first toss results in heads.
    - B := The second toss results in tails.
    - C := The third toss results in heads.
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- Are A,B, and C independent events?

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  - *Events:*
    - A := The first toss results in heads.
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  - *Probability measure*: Combinatorial setting with mass function assigning equal probability 1/8 to each atom.
- Are A,B, and C independent events?

Event		Probability
A	{𝔥𝔥𝔥, 𝔥𝔥𝔱, 𝔥𝔱𝔥, 𝔥𝔱𝔱}	1/2
B	{𝔥𝔱𝔥, 𝔥𝔱𝔱, 𝔱𝔱𝔥, 𝔱𝔱𝔱}	1/2
C	{𝔥𝔥𝔥, 𝔥𝔱𝔥, 𝔱𝔥𝔥, 𝔱𝔱𝔥}	1/2
A ∩ B	{𝔥𝔱𝔥, 𝔥𝔱𝔱}	1/4
A ∩ C	{𝔥𝔥𝔥, 𝔥𝔱𝔥}	1/4
B ∩ C	{𝔥𝔱𝔥, 𝔱𝔱𝔥}	1/4
A ∩ B ∩ C	{𝔥𝔱𝔥}	1/8

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- A fair coin ($\mathfrak{h}$ = heads, $\mathfrak{t}$ = tails) is tossed thrice.
  - *Sample space*: $\Omega = \{\mathfrak{hhh}, \mathfrak{hht}, \mathfrak{hth}, \mathfrak{htt}, \mathfrak{thh}, \mathfrak{tht}, \mathfrak{tth}, \mathfrak{ttt}\}$.
  - *Events:*
    - A := The first toss results in heads.
    - B := The second toss results in tails.
    - C := The third toss results in heads.
  - *Probability measure*: Combinatorial setting with mass function assigning equal probability $1/8$ to each atom.
- Are A,B, and C independent events?

Event		Probability
A	$\{\mathfrak{hhh}, \mathfrak{hht}, \mathfrak{hth}, \mathfrak{htt}\}$	$1/2$
B	$\{\mathfrak{hth}, \mathfrak{htt}, \mathfrak{tth}, \mathfrak{ttt}\}$	$1/2$
C	$\{\mathfrak{hhh}, \mathfrak{hth}, \mathfrak{thh}, \mathfrak{tth}\}$	$1/2$
$A \cap B$	$\{\mathfrak{hth}, \mathfrak{htt}\}$	$1/4 = \frac{1}{2} \times \frac{1}{2}$
$A \cap C$	$\{\mathfrak{hhh}, \mathfrak{hth}\}$	$1/4 = \frac{1}{2} \times \frac{1}{2}$
$B \cap C$	$\{\mathfrak{hth}, \mathfrak{tth}\}$	$1/4 = \frac{1}{2} \times \frac{1}{2}$
$A \cap B \cap C$	$\{\mathfrak{hth}\}$	$1/8$

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  - *Events:*
    - A := The first toss results in heads.
    - B := The second toss results in tails.
    - C := The third toss results in heads.
  - *Probability measure*: Combinatorial setting with mass function assigning equal probability $1/8$ to each atom.
- Are A,B, and C independent events?

Event		Probability
A	$\{\mathfrak{hhh}, \mathfrak{hht}, \mathfrak{hth}, \mathfrak{htt}\}$	$1/2$
B	$\{\mathfrak{hth}, \mathfrak{htt}, \mathfrak{tth}, \mathfrak{ttt}\}$	$1/2$
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  - *Sample space*: $\Omega = \{\mathfrak{hhh}, \mathfrak{hht}, \mathfrak{hth}, \mathfrak{htt}, \mathfrak{thh}, \mathfrak{tht}, \mathfrak{tth}, \mathfrak{ttt}\}$.
  - *Events:*
    - A := The first toss results in heads.
    - B := The second toss results in tails.
    - C := The third toss results in heads.
  - *Probability measure*: Combinatorial setting with mass function assigning equal probability $1/8$ to each atom.
- Are A,B, and C independent events?

Event		Probability
A	$\{\mathfrak{hhh}, \mathfrak{hht}, \mathfrak{hth}, \mathfrak{htt}\}$	$1/2$
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$A \cap B$	$\{\mathfrak{hth}, \mathfrak{htt}\}$	$1/4 = \frac{1}{2} \times \frac{1}{2}$
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$A \cap B \cap C$	$\{\mathfrak{hth}\}$	$1/8 = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$

A, B, and C are independent.