Deviance (statistics)

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In statistics, **deviance** is a quality of fit statistic for a model that is often used for statistical hypothesis testing. It is a generalization of the idea of using the sum of squares of residuals in ordinary least squares to cases where model-fitting is achieved by maximum likelihood.

Contents

Definition

The deviance for a model M_0, based on a dataset y, is defined as:[1][2]

$$D(y) = -2\Big(\log\left(p(y \mid \hat{\theta}_0)\right) - \log\left(p(y \mid \hat{\theta}_s)\right)\Big).$$

Here $\hat{\theta}_0$ denotes the fitted values of the parameters in the model M_0, while $\hat{\theta}_s$ denotes the fitted parameters for the "full model" (or "saturated model"): both sets of fitted values are implicitly functions of the observations y. Here the **full model** is a model with a parameter for every observation so that the data are fitted exactly. This expression is simply −2 times the log-likelihood ratio of the reduced model compared to the full model. The deviance is used to compare two models – in particular in the case of generalized linear models where it has a similar role to residual variance from ANOVA in linear models (RSS).

Suppose in the framework of the GLM, we have two nested models, M_1 and M_2. In particular, suppose that M_1 contains the parameters in M_2, and k additional parameters. Then, under the null hypothesis that M_2 is the true model, the difference between the deviances for the two models follows an approximate chi-squared distribution with k-degrees of freedom.[2]

Some usage of the term "deviance" can be confusing. According to Collett:[3]

> "the quantity $-2\log\left(p(y \mid \hat{\theta}_0)\right)$ is sometimes referred to as a *deviance*. This is [...] inappropriate, since unlike the deviance used in the context of generalized linear modelling, $-2\log\left(p(y \mid \hat{\theta}_0)\right)$ does not measure deviation from a model that is a perfect fit to the data." However, since the principal use is in the form of the difference of the deviances of two models, this confusion in definition is unimportant.

See also

- Pearson's chi-squared test, an alternative quality of fit statistic for generalized linear models for count data.
- Hosmer–Lemeshow test, a quality of fit statistic that can be used for binary data.
- Akaike information criterion
- Deviance information criterion
- Peirce's criterion
- Discrepancy function

Notes

1. Nelder, J.A.; Wedderburn, R.W.M. (1972). "Generalized Linear Models". *Journal of the Royal Statistical Society. Series A (General)* **135** (3): 370–384. doi:10.2307/2344614 (https://dx.doi.org/10.2307%2F2344614). JSTOR 2344614 (https://www.jstor.org/stable/2344614).
2. McCullagh and Nelder (1989)
3. Collett (2003)Template:Page 76

References

- McCullagh, Peter; Nelder, John (1989). *Generalized Linear Models, Second Edition*. Chapman & Hall/CRC. ISBN 0-412-31760-5.

- Collett, David (2003). *Modelling Survival Data in Medical Research, Second Edition*. Chapman & Hall/CRC. ISBN 1-58488-325-1.

External links

- Generalized Linear Models (http://userwww.sfsu.edu/~efc/classes/biol710/Glz/Generalized%20Linear%20Models.htm) - Edward F. Connor
- Lectures notes on Deviance (http://www.unc.edu/courses/2006spring/ecol/145/001/docs/lectures/lecture22.htm)

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