

Exploiting independence!

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Trial	1	2	3	4	5	6	7	8	9	10

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Trials 2 through 9: $10 - 2 = 8$ trials yielding neither 6 nor 7

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A winning sequence after obtaining 6 on the first throw:

- *Additivity*: the probability that a trial yields neither 6 nor 7 is $1 - p_6 - p_7$.
- *Independent trials*: the probability of obtaining the given sequence from trials 2 through 10 is independent of the outcome of the first trial and given by $(1 - p_6 - p_7)^{10-2} p_6$.

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$$\mathbf{P}(W_n \mid F_k) = (1 - p_k - p_7)^{n-2} p_k \qquad (k \in \{4, 5, 6, 8, 9, 10\}; \, n \geq 2)$$

Progression after throwing 4, 5, 6, 8, 9, or 10

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A geometric series! If $-1 < x < 1$, then $1 + x + x^2 + \cdots = 1 / (1 - x)$.

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$$= \frac{p_k}{p_k + p_7} \qquad (k \in \{4, 5, 6, 8, 9, 10\})$$