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Lesson 8

Back to Week 3

10/10 points earned (100%)

Quiz passed!

1 / 1 points

1.

For Questions 1-8, consider the chocolate chip cookie example from the lesson.

As in the lesson, we use a Poisson likelihood to model the number of chips per cookie, and a conjugate gamma prior on λ, the expected number of chips per cookie. Suppose your prior expectation for λ is 8.

- The conjugate prior with mean 8 and effective sample size of 2 is $\mathrm{Gamma}(a, 2)$. Find the value of a.

16

Correct Response

The expected value is $a/2 = 8$, so $a = 16$.

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1 / 1
points

Cookies:

2. • The conjugate prior with mean 8 and standard deviation 1 is $\mathrm{Gamma}(a, 8)$. Find the value of a.

64

Correct Response

The prior mean is $a/8 = 64/8 = 8$ and prior standard deviation is $\sqrt{a}/8 = \sqrt{64}/8 = 1$.

Note that this is not the prior standard deviation for number of chips per cookie. It is the prior standard deviation for λ the *expected* number of chips per cookie. It represents our level of confidence in the prior for λ.

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1 / 1
points

3.
Cookies:

- Suppose you are not very confident in your prior guess of 8, so you want to use a prior effective sample size of 1/100 cookies. Then the conjugate prior is $\mathrm{Gamma}(a, 0.01)$. Find the value of a. Round your answer to two decimal places.

0.08

Correct Response
The expected value is $\frac{a}{0.01} = 8$, so $a = 0.08$.

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1 / 1
points

4.
Cookies:

Suppose you decide on the prior Gamma(8, 1), which has prior mean 8 and effective sample size of one cookie.

We collect data, sampling five cookies and counting the chips in each. We find 9, 12, 10, 15, and 13 chips.

- What is the posterior distribution for λ?

Gamma(67, 6)

Correct Response
The chip total is 59 in five cookies, so we have posterior $\alpha = 8 + 59$ and $\beta = 1 + 5$.

Gamma(1, 8)

- Gamma(5, 59)
- Gamma(6, 67)
- Gamma(8, 1)
- Gamma(59, 5)

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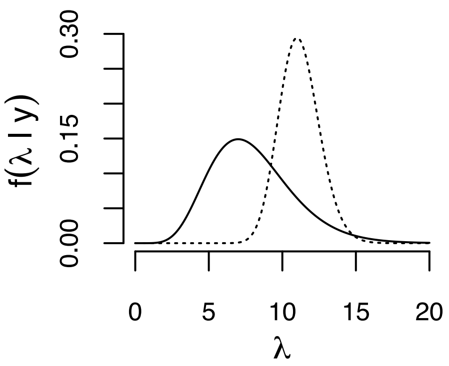
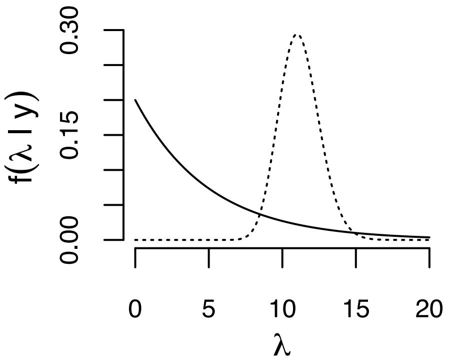
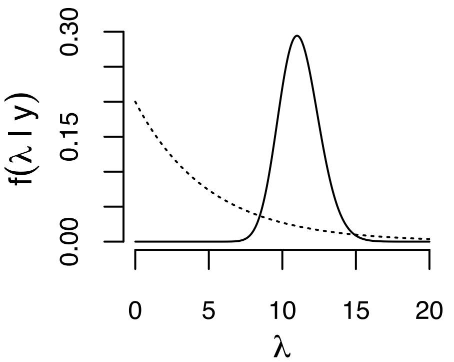
1 / 1
points

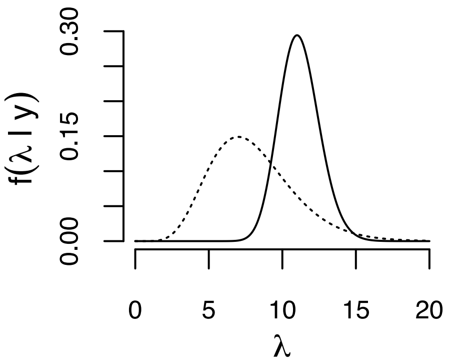
5.
Cookies:

- Continuing the previous question, what of the following graphs shows the prior density (dotted line) and posterior density (solid line) of λ?




f(λ | y)
0.00
0.15
0.30
0
5
10
15
20
λ


f(λ | y)
0.00
0.15
0.30
0
5
10
15
20
λ



Correct Response

The data sample mean (11.8) is greater than the prior mean, and consequently the posterior mean is greater than the prior mean also.

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1 / 1 points

Cookies:

6. • Continuing Question 4, what is the posterior mean for λ? Round your answer to one decimal place.

11.2

Correct Response

This is $\alpha/\beta = 67/6$ where α and β are the posterior gamma parameters.

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1 / 1
points

7.
Cookies:

- Continuing Question 4, use R or Excel to find the lower end of a 90% equal-tailed credible interval for λ. Round your answer to one decimal place.

9.0

Correct Response
The interval is (9.02, 13.50).

In R:

```
qgamma(p=0.05, shape=67, rate=6)
```

In Excel:

```
= GAMMA.INV(0.05, 67, 1/6)
```

Where probability=0.05, alpha=67, and beta=1/6. Note that the beta in Excel's parameterization of the gamma distribution (the shape parameter) is the reciprocal of the parameter used in this course (the rate parameter).

1 / 1
points

8.

Cookies:

- Continuing Question 4, suppose that in addition to the five cookies reported, we observe an additional ten cookies with 109 total chips. What is the new posterior distribution for λ, the expected number of chips per cookie?

Hint: You can either use the posterior from the previous analysis as the prior here, or you can start with the original Gamma(8,1) prior and update with all fifteen cookies. The result will be the same.

- Gamma(11, 109)
- Gamma(176, 16)

Correct Response

This is $\mathrm{Gamma}(\alpha, \beta)$ with $\alpha = 8 + 59 + 109$ and $\beta = 1 + 5 + 10$.

The posterior mean is now 176/16=11. The data suggest there are more than 8 chips per cookie on average.

- Gamma(16, 176)
- Gamma(109, 10)
- Gamma(10, 109)

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1 / 1
points

9.

For Questions 9-10, consider the following scenario:

A retailer notices that a certain type of customer tends to call their customer service hotline more often than other customers, so they begin keeping track. They decide a Poisson process model is appropriate for counting calls, with calling rate θ calls per customer per day.

The model for the total number of calls is then $Y \sim \text{Poisson}(n \cdot t \cdot \theta)$ where n is the number of customers in the group and t is the number of days. That is, if we observe the calls from a group with 24 customers for 5 days, the expected number of calls would be $24 \cdot 5 \cdot \theta = 120 \cdot \theta$.

The likelihood for Y is then $f(y \mid \theta) = \frac{(nt\theta)^y e^{-nt\theta}}{y!} \propto \theta^y e^{-nt\theta}$.

This model also has a conjugate gamma prior $\theta \sim \text{Gamma}(a, b)$ which has density (PDF) $f(\theta) = \frac{b^a}{\Gamma(a)} \theta^{a-1} e^{-b\theta} \propto \theta^{a-1} e^{-b\theta}$.

- Following the same procedure outlined in the lesson, find the posterior distribution for θ.

( ) $\text{Gamma}(a + y, b + nt)$

Correct Response
If we multiply the likelihood and the prior, we get $f(\theta \mid y) \propto \theta^y e^{-nt\theta} \theta^{a-1} e^{-b\theta} = \theta^{a+y-1} e^{-(b+nt)\theta}$, which is proportional to a gamma PDF.

( ) $\text{Gamma}(y, nt)$

( ) $\text{Gamma}(a + 1, b + y)$

( ) $\text{Gamma}(a + y - 1, b + 1)$

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1 / 1
points

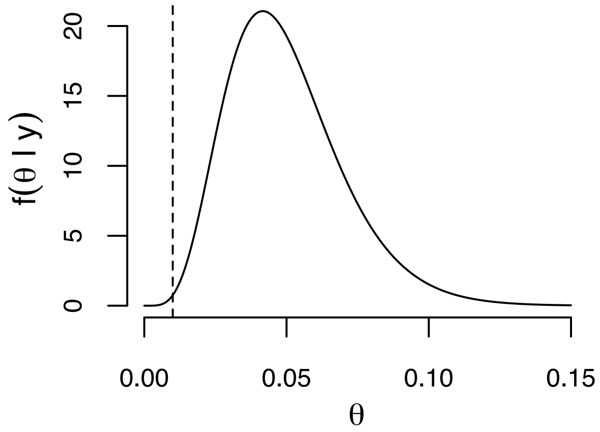
10.

Poisson process:

On average, the retailer receives 0.01 calls per customer per day. To give this group the benefit of the doubt, they set the prior mean for θ at 0.01 with standard deviation 0.5. This yields a $\mathrm{Gamma}(\frac{1}{2500}, \frac{1}{25})$ prior for θ.

Suppose there are $n = 24$ customers in this particular group of interest, and the retailer monitors calls from these customers for $t = 5$ days. They observe a total of $y = 6$ calls from this group.

The following graph shows the resulting $\mathrm{Gamma}(6.0004, 120.04)$ posterior for θ, the calling rate for this group. The vertical dashed line shows the average calling rate of 0.01.



- Does this posterior inference for θ suggest that the group has a higher calling rate than the average of 0.01 calls per customer per day?

○ Yes, the posterior mean for θ is twice the average of 0.01.

○ Yes, most of the posterior mass (probability) is concentrated on values of θ greater than 0.01.

Correct Response

The posterior probability that $\theta > 0.01$ is 0.998

The posterior probability that $\theta > 0.01$ is 0.998.

- No, the posterior mean is exactly 0.01.
- No, most of the posterior mass (probability) is concentrated on values of θ less than 0.01.