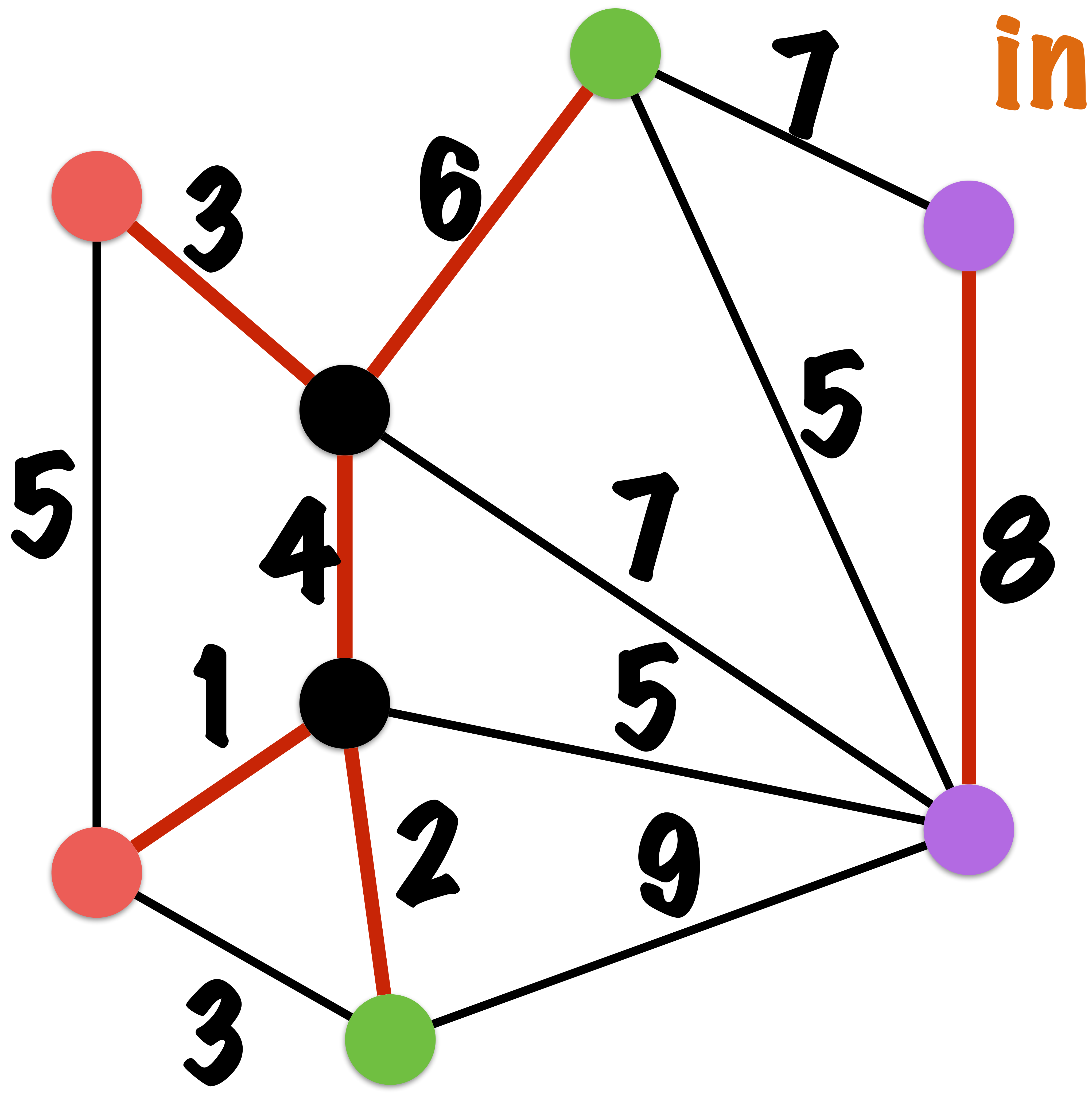
Steiner forest

Integer programming model



input: sets of terminals S1,S2,...

output: set of edges

min output cost
s.t.
for all i
for all u,v in Si
u and v are connected

Integer programming model

output: set of edges $\quad x_e \in \{0,1\}$

min output cost $\quad \min \sum_e c_e x_e$

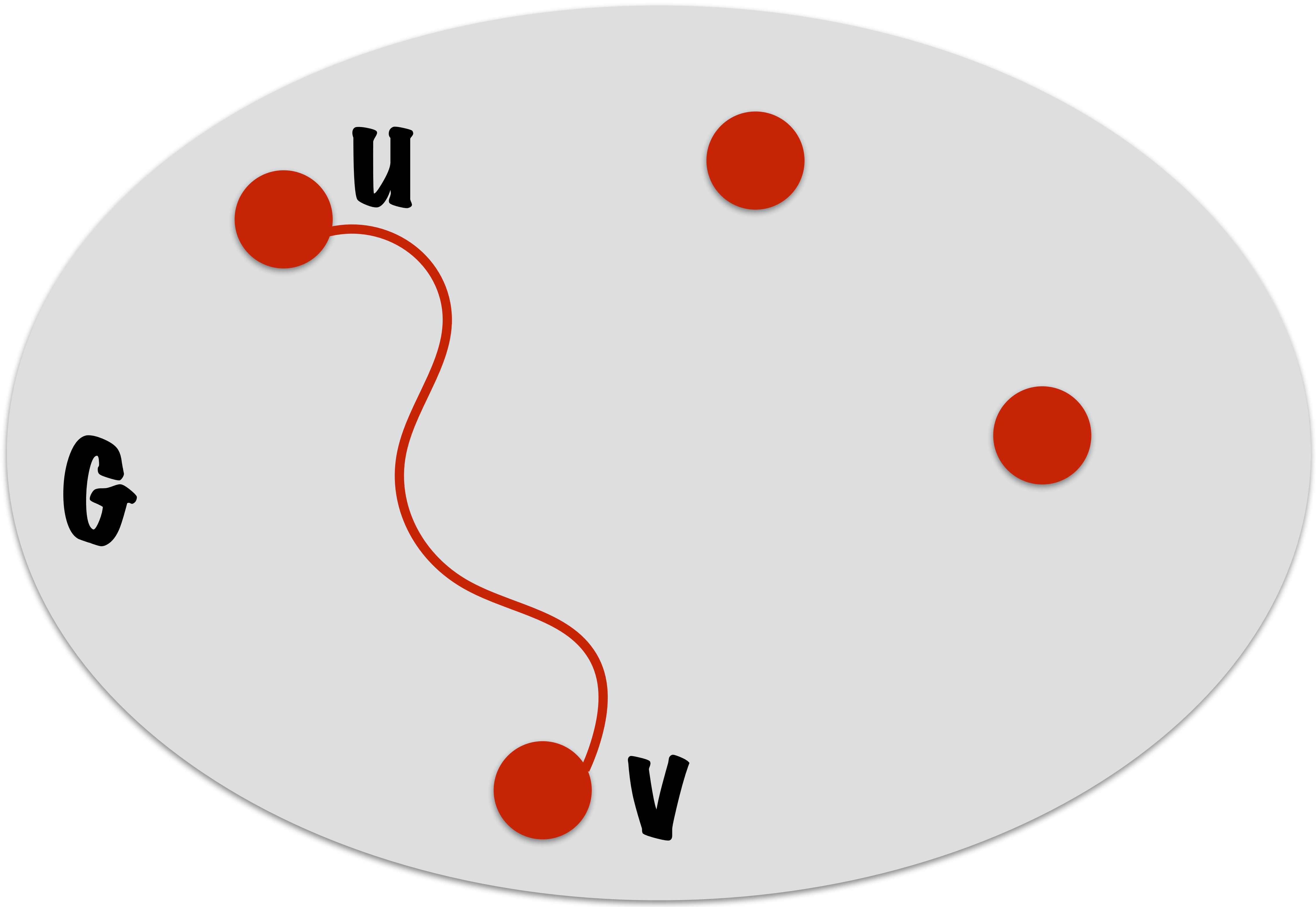
s.t.

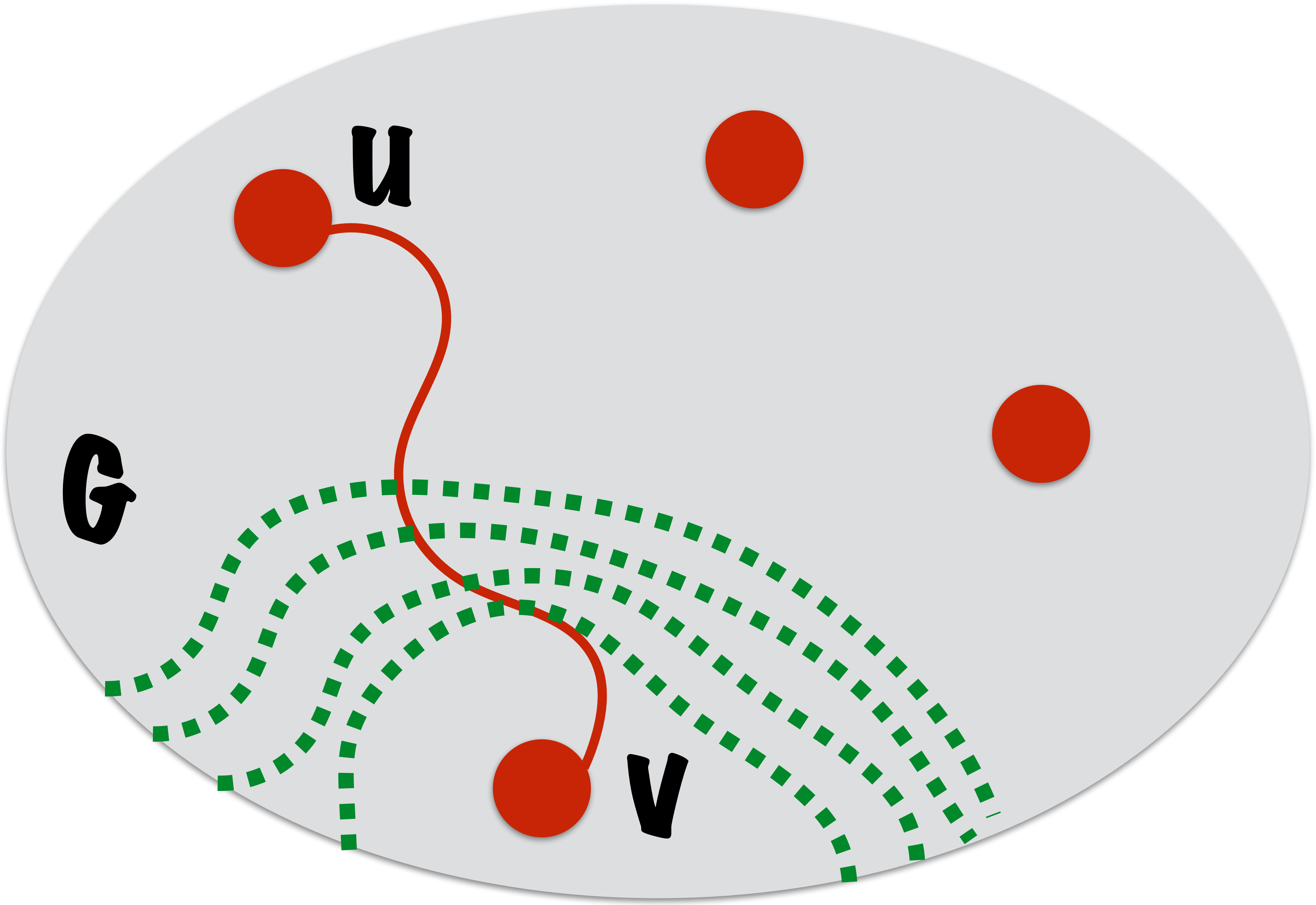
for all i $\quad \forall i \forall u, v \in S_i$

for all u,v in Si

u and v are connected $\quad ?$

Integer programming model

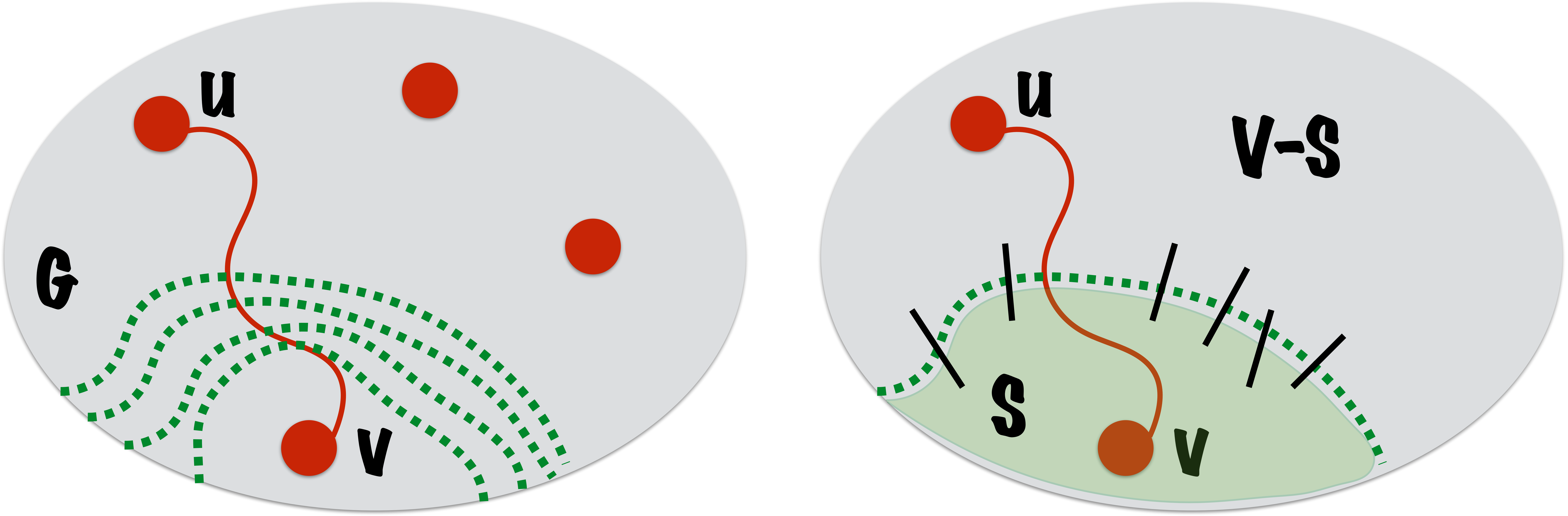




u and v are connected

every u-v cut is crossed

Integer programming model



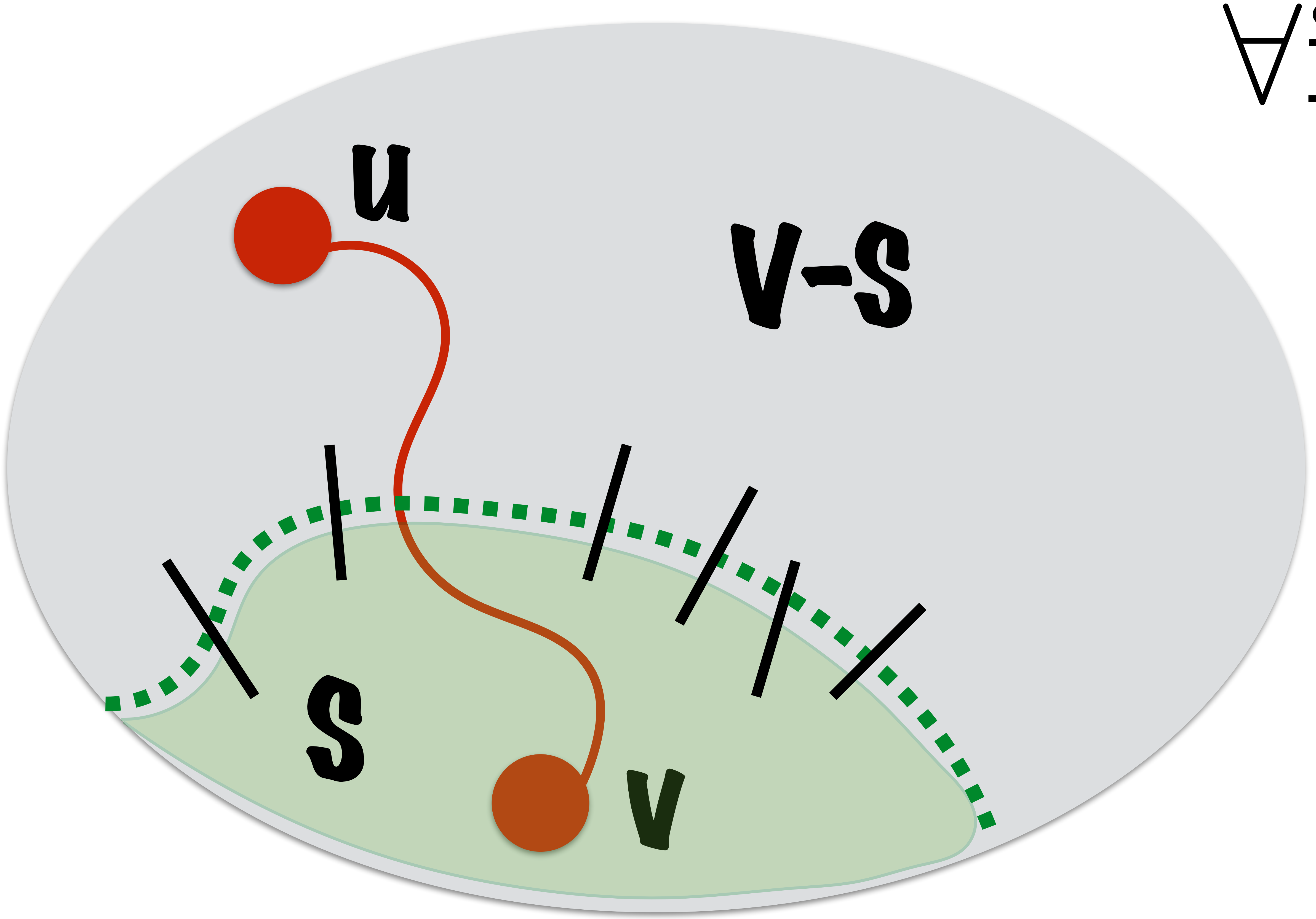
every u-v cut is crossed

$$\mathrm{S} \subseteq \mathrm{V} : |\mathrm{S} \cap \{\mathrm{u}, \mathrm{v}\}| = 1$$

$$\delta(\mathrm{S}) = \{\mathrm{e} \in \mathrm{S} \times \mathrm{V} \setminus \mathrm{S}\}$$

for all i
for all u,v in Si
u and v are connected

$$\mathbf{x_e} \in \{0, 1\}$$

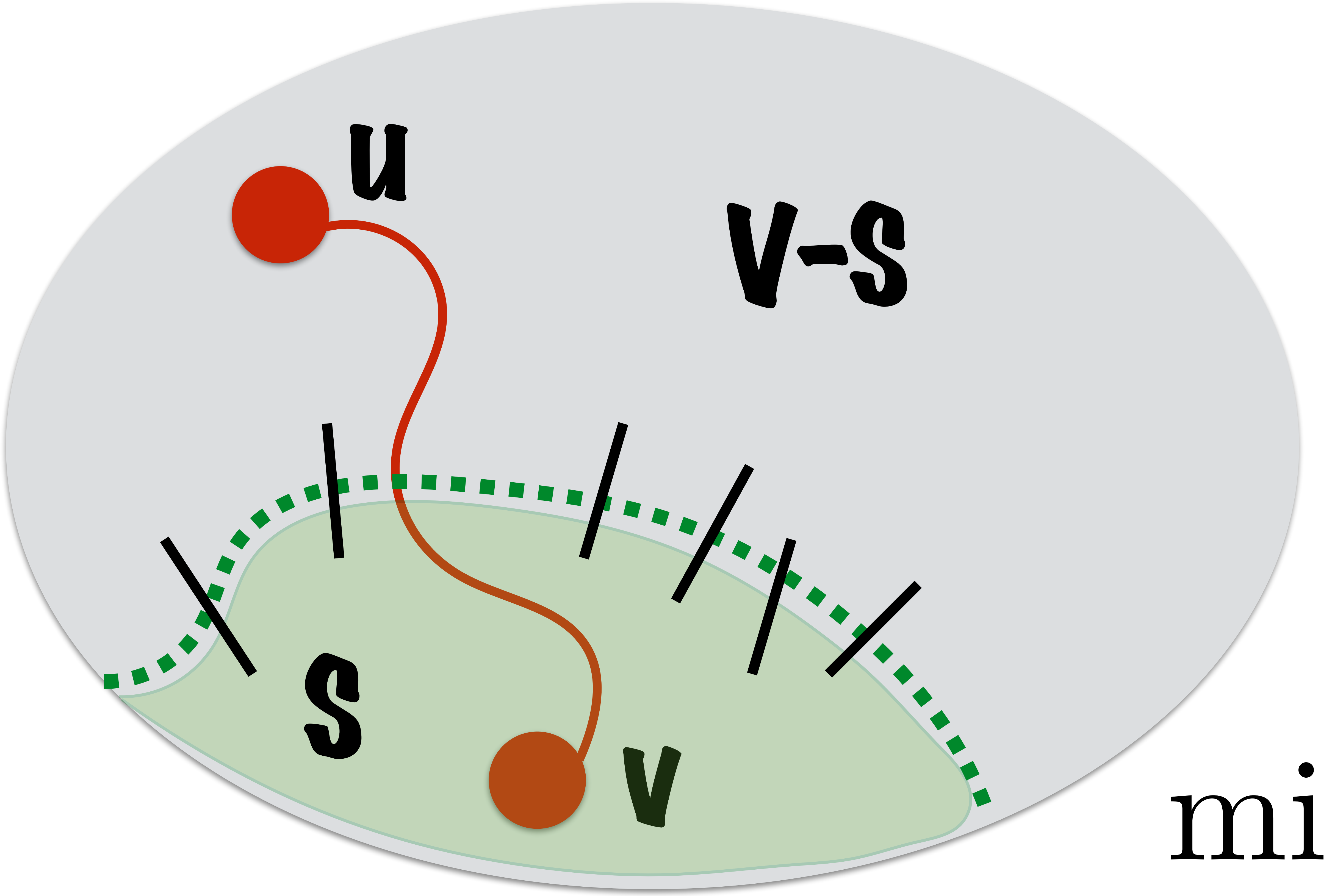


$$\forall \mathbf{i} \forall \mathbf{u}, \mathbf{v} \in \mathbf{S_i}$$

$$\forall \mathrm{S} \text{ s.t. } |\mathrm{S} \cap \{\mathbf{u}, \mathbf{v}\}| = 1$$

$$\sum_{\mathbf{e} \in \delta(\mathbf{S})} \mathbf{x_e} \geq 1$$

$$\mathcal{S} = \{\mathbf{S} : \exists \mathbf{i} \exists \mathbf{u}, \mathbf{v} \in \mathbf{S_i} : |\mathbf{S} \cap \{\mathbf{u}, \mathbf{v}\}| = \mathbf{1}\}$$



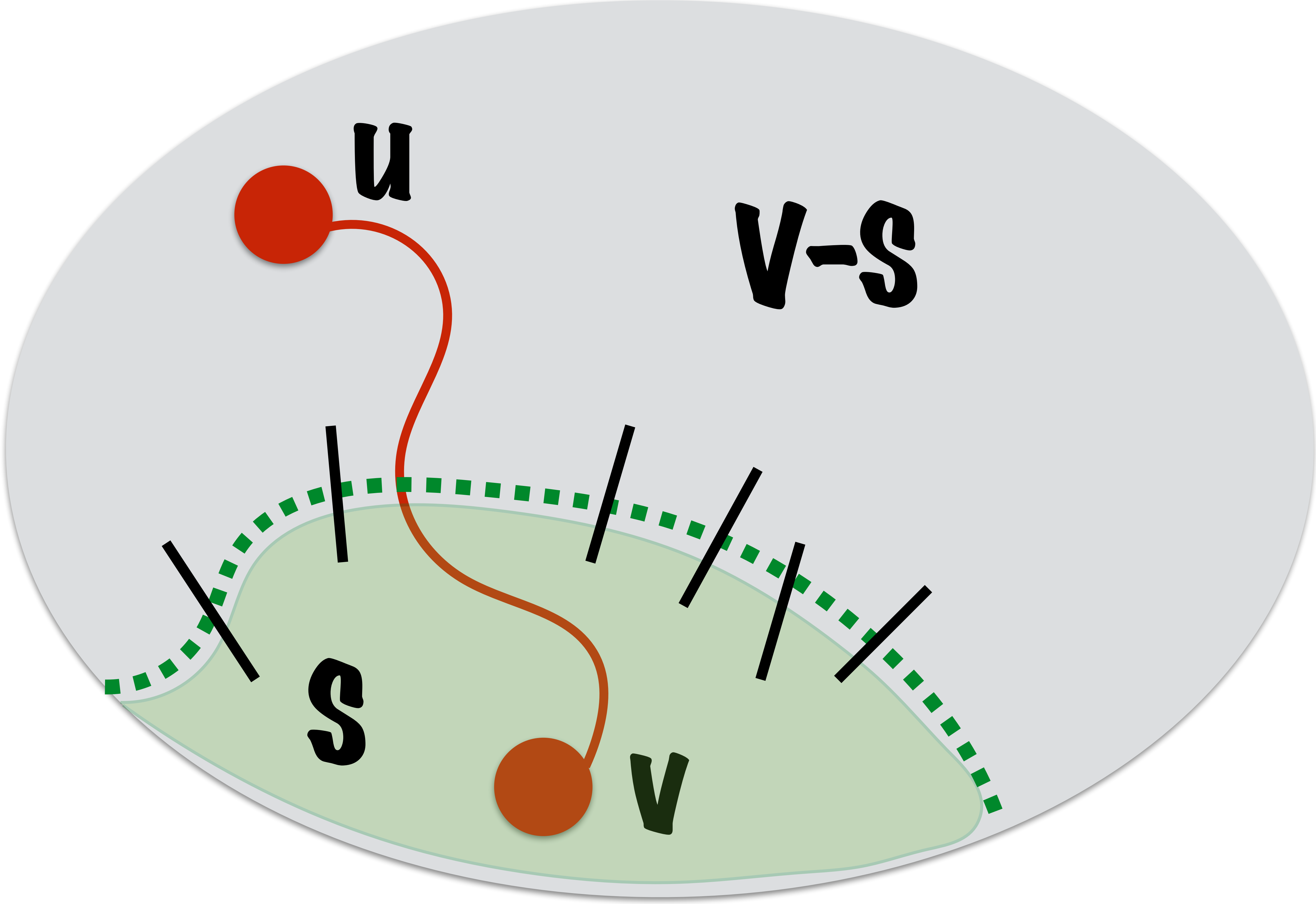
$$\min \textstyle\sum_{\mathbf{e}} \mathbf{c_e x_e} :$$

$$\forall \mathbf{S} \in \mathcal{S} \textstyle\sum_{\mathbf{e} \in \delta(\mathbf{S})} \mathbf{x_e} \geq \mathbf{1}$$

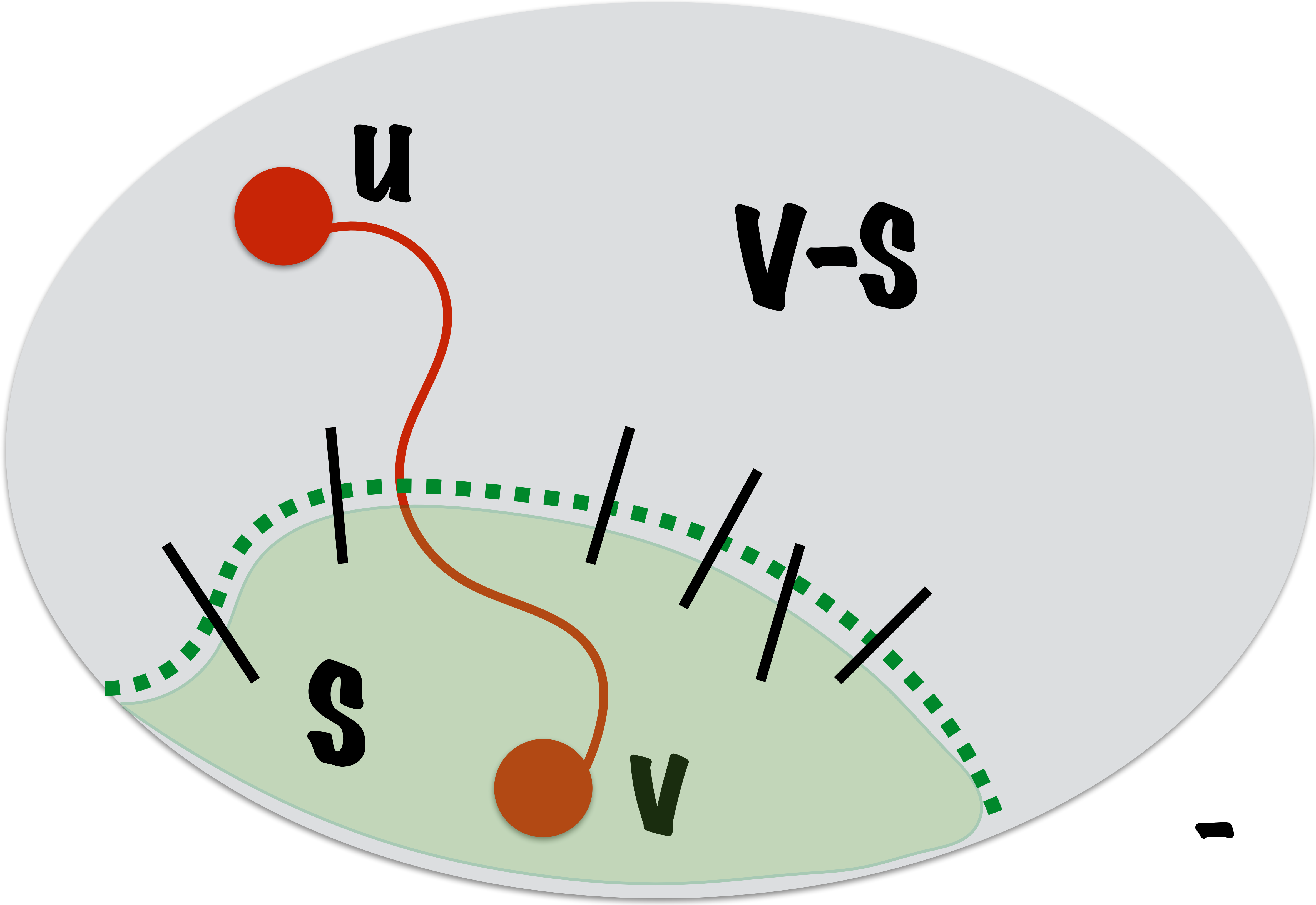
$$\mathbf{x_e} \in \{\mathbf{0}, \mathbf{1}\}$$

$$\min \sum_{\mathbf{e}} \mathbf{c_e x_e} :$$
$$\forall \mathbf{S} \in \mathcal{S} \sum_{\mathbf{e} \in \delta(\mathbf{S})} \mathbf{x_e} \geq \mathbf{1}$$
$$\mathbf{x_e} \in \{\mathbf{0}, \mathbf{1}\}$$

$$\min \sum_{\mathbf{e}} \mathbf{c_e x_e} :$$
$$\forall \mathbf{S} \in \mathcal{S} \sum_{\mathbf{e} \in \delta(\mathbf{S})} \mathbf{x_e} \geq \mathbf{1}$$
$$\mathbf{x_e} \geq \mathbf{0}$$



$$\min \sum_e c_e x_e :$$
$$\forall S \in \mathcal{S} \sum_{e \in \delta(S)} x_e \geq 1$$
$$x_e \geq 0$$



Exponential
number of constraints
but
- still solvable
- does not need to be solved

Steiner forest