stochastic vs deterministic trend/seasonality in time series forecasting

I have moderate background in time series forecasting. I have looked at several forecasting books, and I don't see the following questions addressed in any of them.

I have two questions:

1. How would I determine objectively (via statistical test) if a given time series has:
    - Stochastic Seasonality or a Deterministic Seasonality
    - Stochastic Trend or a Deterministic Trend
2. What would happen if i model my time series as a deterministic trend/seasonality when the series has a clearly stochastic component?

Any help addressing these questions would be greatly appreciated.

Example data for trend:

```
7,657
5,451
10,883
9,554
9,519
10,047
10,663
10,864
11,447
12,710
15,169
16,205
14,507
15,400
16,800
19,000
20,198
18,573
19,375
21,032
23,250
25,219
28,549
29,759
28,262
28,506
33,885
34,776
35,347
34,628
33,043
30,214
31,013
31,496
34,115
33,433
34,198
35,863
37,789
34,561
36,434
34,371
33,307
33,295
36,514
36,593
38,311
42,773
45,000
46,000
42,000
47,000
47,500
48,000
48,500
47,000
48,900
```

time-series forecasting arima stochastic-processes

edited Jun 14 '14 at 3:25

asked Jun 12 '14 at 21:16

forecaster
3,015 1 12 33

3 Answers

1) As regards your first question, some tests statistics have been developed and discussed in the literature to test the null of stationarity and the null of a unit root. Some of the many papers that were written on this issue are the following:

Related to the trend:

- Dickey, D. y Fuller, W. (1979a), Distribution of the estimators for autoregressive time series with a unit root, Journal of the American Statistical Association 74, 427-31.
- Dickey, D. y Fuller, W. (1981), Likelihood ratio statistics for autoregressive time series with a unit root, Econometrica 49, 1057-1071.
- Kwiatkowski, D., Phillips, P., Schmidt, P. y Shin, Y. (1992), Testing the null hypothesis of stationarity against the alternative of a unit root: How sure are we that economic time series have a unit root?, Journal of Econometrics 54, 159-178.
- Phillips, P. y Perron, P. (1988), Testing for a unit root in time series regression, Biometrika 75, 335-46.
- Durlauf, S. y Phillips, P. (1988), Trends versus random walks in time series analysis, Econometrica 56, 1333-54.

Related to the seasonal component:

- Hylleberg, S., Engle, R., Granger, C. y Yoo, B. (1990), Seasonal integration and cointegration, Journal of Econometrics 44, 215-38.
- Canova, F. y Hansen, B. E. (1995), Are seasonal patterns constant over time? a test for seasonal stability, Journal of Business and Economic Statistics 13, 237-252.
- Franses, P. (1990), Testing for seasonal unit roots in monthly data, Technical Report 9032, Econometric Institute.
- Ghysels, E., Lee, H. y Noh, J. (1994), Testing for unit roots in seasonal time series. some theoretical extensions and a monte carlo investigation, Journal of Econometrics 62, 415-442.

The textbook Banerjee, A., Dolado, J., Galbraith, J. y Hendry, D. (1993), Co-Integration,Error Correction, and the econometric analysis of non-stationary data, Advanced Texts in Econometrics. Oxford University Press is also a good reference.

2) Your second concern is justified by the literature. If there is a unit root test then the traditional t-statistic that you would apply on a linear trend does not follow the standard distribution. See for example, Phillips, P. (1987), Time series regression with unit root, Econometrica 55(2), 277-301.

If a unit root exists and is ignored, then the probability of rejecting the null that the coefficient of a linear trend is zero is reduced. That is, we would end up modelling a deterministic linear trend too often for a given significance level. In the presence of a unit root we should instead transform the data by taking regular differences to the data.

3) For illustration, if you use R you can do the following analysis with your data.

```
x <- structure(c(7657, 5451, 10883, 9554, 9519, 10047, 10663, 10864,
  11447, 12710, 15169, 16205, 14507, 15400, 16800, 19000, 20198,
  18573, 19375, 21032, 23250, 25219, 28549, 29759, 28262, 28506,
  33885, 34776, 35347, 34628, 33043, 30214, 31013, 31496, 34115,
  33433, 34198, 35863, 37789, 34561, 36434, 34371, 33307, 33295,
  36514, 36593, 38311, 42773, 45000, 46000, 42000, 47000, 47500,
  48000, 48500, 47000, 48900), .Tsp = c(1, 57, 1), class = "ts")
```

First, you can apply the Dickey-Fuller test for the null of a unit root:

```
require(tseries)
adf.test(x, alternative = "explosive")
#   Augmented Dickey-Fuller Test
#   Dickey-Fuller = -2.0685, Lag order = 3, p-value = 0.453
#   alternative hypothesis: explosive
```

and the KPSS test for the reverse null hypothesis, stationarity against the alternative of stationarity around a linear trend:

```
kpss.test(x, null = "Trend", lshort = TRUE)
#   KPSS Test for Trend Stationarity
#   KPSS Trend = 0.2691, Truncation lag parameter = 1, p-value = 0.01
```

Results: ADF test, at the 5% significance level a unit root is not rejected; KPSS test, the null of stationarity is rejected in favour of a model with a linear trend.

Aside note: using `lshort=FALSE` the null of the KPSS test is not rejected at the 5% level, however, it selects 5 lags; a further inspection not shown here suggested that choosing 1-3 lags is appropriate for the data and leads to reject the null hypothesis.

In principle, we should guide ourselves by the test for which we were able to the reject the null hypothesis (rather than by the test for which we did not reject (we accepted) the null). However, a regression of the original series on a linear trend turns out to be not reliable. On the one hand, the R-square is high (over 90%) which is pointed in the literature as an indicator of spurious regression.

```
fit <- lm(x ~ 1 + poly(c(time(x))))
summary(fit)
#Coefficients:
#                 Estimate Std. Error t value Pr(>|t|)
#(Intercept)       28499.3      381.6   74.69   <2e-16 ***
#poly(c(time(x)))  91387.5     2880.9   31.72   <2e-16 ***
#---
#Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
#
#Residual standard error: 2881 on 55 degrees of freedom
#Multiple R-squared:  0.9482,   Adjusted R-squared:  0.9472
#F-statistic:  1006 on 1 and 55 DF,  p-value: < 2.2e-16
```

On the other hand, the residuals are autocorrelated:

```
acf(residuals(fit)) # not displayed to save space
```

Moreover, the null of a unit root in the residuals cannot be rejected.

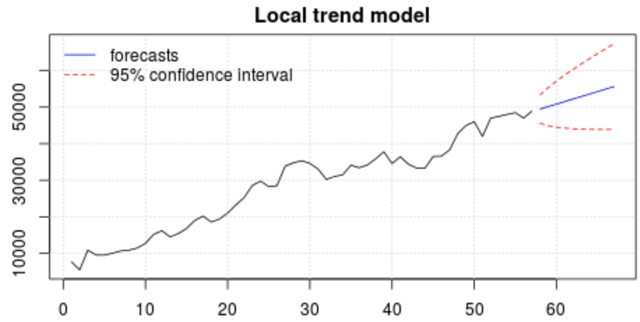
```
adf.test(residuals(fit))
#   Augmented Dickey-Fuller Test
#Dickey-Fuller = -2.0685, Lag order = 3, p-value = 0.547
#alternative hypothesis: stationary
```

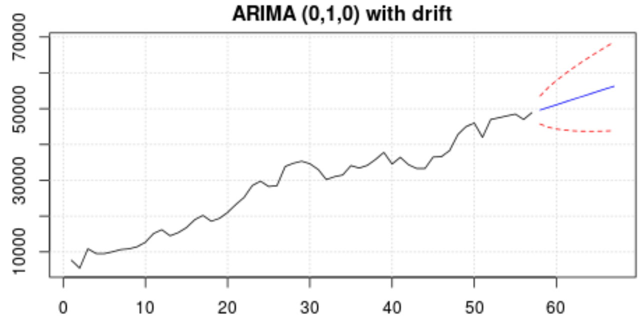
At this point, you can choose a model to be used to obtain forecasts. For example, forecasts based on a structural time series model and on an ARIMA model can be obtained as follows.

```
# StructTS
fit1 <- StructTS(x, type = "trend")
fit1
#Variances:
# level    slope  epsilon
#2982955        0   487180
#
# forecasts
p1 <- predict(fit1, 10, main = "Local trend model")
p1$pred
# [1] 49466.53 50150.56 50834.59 51518.62 52202.65 52886.68 53570.70 54254.73
# [9] 54938.76 55622.79

# ARIMA
require(forecast)
fit2 <- auto.arima(x, ic="bic", allowdrift = TRUE)
fit2
#ARIMA(0,1,0) with drift
#Coefficients:
#         drift
#      736.4821
#s.e.  267.0055
#sigma^2 estimated as 3992341:  log likelihood=-495.54
#AIC=995.09   AICc=995.31   BIC=999.14
#
# forecasts
p2 <- forecast(fit2, 10, main = "ARIMA model")
p2$mean
# [1] 49636.48 50372.96 51109.45 51845.93 52582.41 53318.89 54055.37 54791.86
# [9] 55528.34 56264.82
```

A plot of the forecasts:

```
par(mfrow = c(2, 1), mar = c(2.5,2.2,2,2))
plot((cbind(x, p1$pred)), plot.type = "single", type = "n",
      ylim = range(c(x, p1$pred + 1.96 * p1$se)), main = "Local trend model")
    grid()
    lines(x)
    lines(p1$pred, col = "blue")
lines(p1$pred + 1.96 * p1$se, col = "red", lty = 2)
lines(p1$pred - 1.96 * p1$se, col = "red", lty = 2)
legend("topleft", legend = c("forecasts", "95% confidence interval"),
  lty = c(1,2), col = c("blue", "red"), bty = "n")
plot((cbind(x, p2$mean)), plot.type = "single", type = "n",
      ylim = range(c(x, p2$upper)), main = "ARIMA (0,1,0) with drift")
grid()
lines(x)
lines(p2$mean, col = "blue")
    lines(ts(p2$lower[,2], start = end(x)[1] + 1), col = "red", lty = 2)
lines(ts(p2$upper[,2], start = end(x)[1] + 1), col = "red", lty = 2)
```



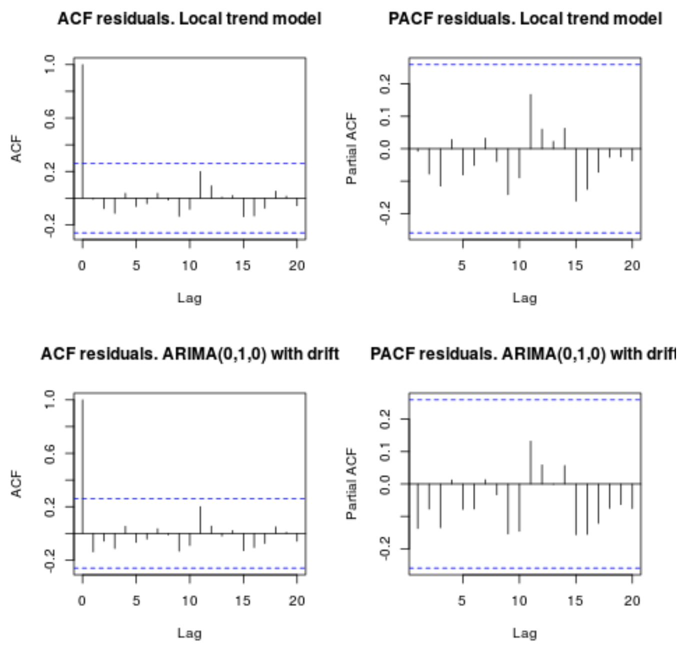


The forecasts are similar in both cases and look reasonable. Notice that the forecasts follow a relatively deterministic pattern similar to a linear trend, but we did not modelled explicitly a linear trend. The reason is the following: i) in the local trend model, the variance of the slope component is estimated as zero. This turns the trend component into a drift that has the effect of a linear trend. ii) ARIMA(0,1,1), a model with a drift is selected in a model for the differenced series.The effect of the constant term on a differenced series is a linear trend. This is discussed in this post.

You may check that if a local model or an ARIMA(0,1,0) without drift are chosen, then the forecasts are a straight horizontal line and, hence, would have no resemblance with the observed dynamic of the data. Well, this is part of the puzzle of unit root tests and deterministic components.

Edit 1 (inspection of residuals): The autocorrelation and partial ACF do not suggest a structure in the residuals.

```
resid1 <- residuals(fit1)
resid2 <- residuals(fit2)
par(mfrow = c(2, 2))
acf(resid1, lag.max = 20, main = "ACF residuals. Local trend model")
pacf(resid1, lag.max = 20, main = "PACF residuals. Local trend model")
acf(resid2, lag.max = 20, main = "ACF residuals. ARIMA(0,1,0) with drift")
pacf(resid2, lag.max = 20, main = "PACF residuals. ARIMA(0,1,0) with drift")
```



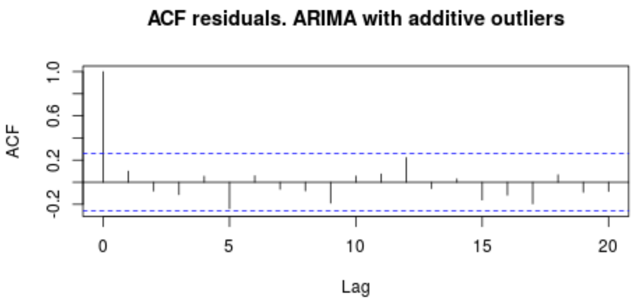
As IrishStat suggested, checking for the presence of outliers is also advisable. Two additive outliers are detected using the package `tsoutliers`.

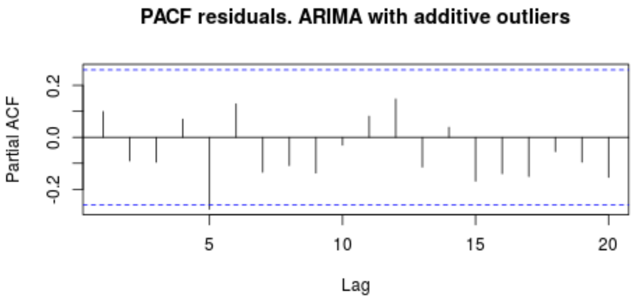
```
require(tsoutliers)
resol <- tsoutliers(x, types = c("AO", "LS", "TC"),
```

```
  remove.method = "bottom-up",
  args.tsmethod = list(ic="bic", allowdrift=TRUE))
resol
#ARIMA(0,1,0) with drift
#Coefficients:
#         drift        AO2       AO51
#      736.4821  -3819.000  -4500.000
#s.e.  220.6171   1167.396   1167.397
#sigma^2 estimated as 2725622:  log likelihood=-485.05
#AIC=978.09   AICc=978.88   BIC=986.2
#Outliers:
#  type ind time coefhat  tstat
#1   AO   2    2   -3819 -3.271
#2   AO  51   51   -4500 -3.855
```

Looking at the ACF, we can say that, at the 5% significance level, the residuals are random in this model as well.

```
par(mfrow = c(2, 1))
acf(residuals(resol$fit), lag.max = 20, main = "ACF residuals. ARIMA with additive
outliers")
pacf(residuals(resol$fit), lag.max = 20, main = "PACF residuals. ARIMA with additive
outliers")
```

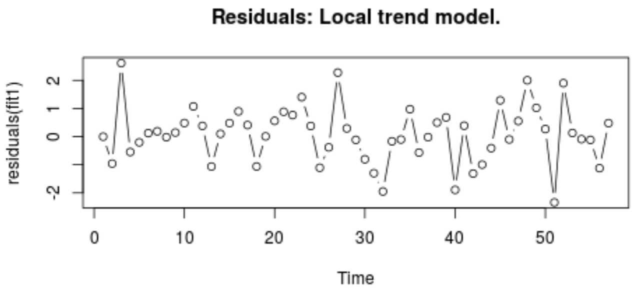


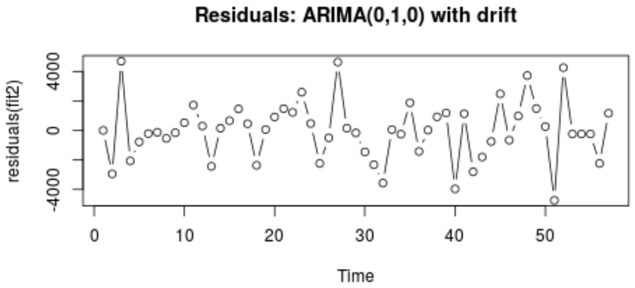


In this case, the presence of potential outliers does not appear to distort the performance of the models. This is supported by the Jarque-Bera test for normality; the null of normality in the residuals from the initial models (`fit1` , `fit2`) is not rejected at the 5% significance level.

```
jarque.bera.test(resid1)[[1]]
# X-squared = 0.3221, df = 2, p-value = 0.8513
jarque.bera.test(resid2)[[1]]
#X-squared = 0.426, df = 2, p-value = 0.8082
```

Edit 2 (plot of residuals and their values) This is how the residuals look like:





And these are their values in a csv format:

```
0;6.9205
-0.9571;-2942.4821
2.6108;4695.5179
-0.5453;-2065.4821
-0.2026;-771.4821
0.1242;-208.4821
0.1909;-120.4821
-0.0179;-535.4821
0.1449;-153.4821
0.484;526.5179
1.0748;1722.5179
0.3818;299.5179
-1.061;-2434.4821
0.0996;156.5179
0.4805;663.5179
0.8969;1463.5179
0.4111;461.5179
-1.0595;-2361.4821
0.0098;65.5179
0.5605;920.5179
0.8835;1481.5179
0.7669;1232.5179
1.4024;2593.5179
0.3785;473.5179
-1.1032;-2233.4821
-0.3813;-492.4821
2.2745;4642.5179
0.2935;154.5179
-0.1138;-165.4821
-0.8035;-1455.4821
-1.2982;-2321.4821
-1.9463;-3565.4821
-0.1648;62.5179
-0.1022;-253.4821
0.9755;1882.5179
-0.5662;-1418.4821
-0.0176;28.5179
0.5;928.5179
0.6831;1189.5179
-1.8889;-3964.4821
0.3896;1136.5179
-1.3113;-2799.4821
-0.9934;-1800.4821
-0.4085;-748.4821
1.2902;2482.5179
-0.0996;-657.4821
0.5539;981.5179
2.0007;3725.5179
1.0227;1490.5179
0.27;263.5179
-2.336;-4736.4821
1.8994;4263.5179
0.1301;-236.4821
-0.0892;-236.4821
-0.1148;-236.4821
-1.1207;-2236.4821
0.4801;1163.5179
```

edited Jun 26 '14 at 9:37

answered Jun 26 '14 at 0:42

javlacalle
5,458 7 23

1 Did you verify that the residuals from your models were random i.e. no outliers or ARIMA structure which is required for test of significance of the estimated coefficiients to be meaningful. Note that if you have outliers in the residuals the ACF is meaningless as the bloated error variance leads to an underestimated ACF. Can you please provide plots of the errors which prove/suggest randomness otherwise your conclusions about the residuals being uncorrelated may be possibly false. – IrishStat Jun 26 '14 at 1:36

Definitely a complete analysis requires inspecting the residuals. I confined my answer to name some tools that can be used to apply the tests that "forecaster" was asking about and illustrated their usage. I am glad to see that you are interested in more details, I have edited my question. – javlacalle Jun 26 '14 at 7:21

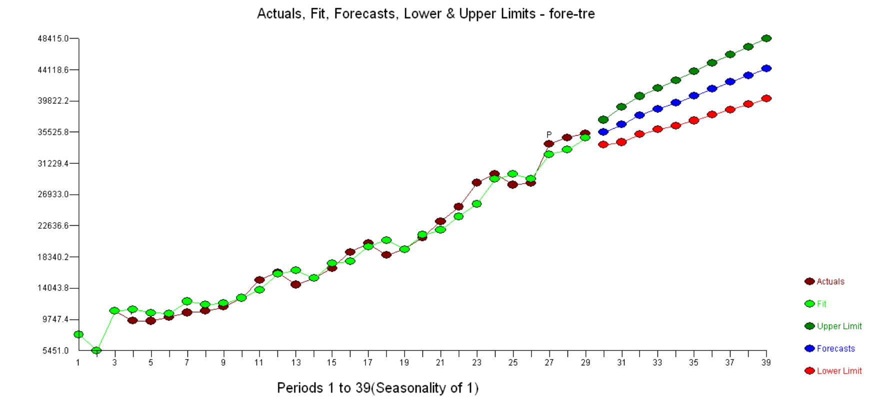
I had asked for a time plot of the residuals . Can you please provide them and also provide the residuals themselves so I can process them with AUTOBOX to confirm that they are free of evidented structure. The JB test is not preferred when testing for Pulses,Level Shifts, Seasonal Pulses and/or Local time trends in a data set although the presence of these kinds of structure could trigger a rejection of the normality assumption. The idea that if the null is not rejected it is proof of it's acceptabce can be dangerous. Please see unc.edu/~jbhill/tsay.pdf – IrishStat Jun 26 '14 at 9:13

1 Thanks. I submitted the 57 residuals and 5 of them were tentatively flagged as exceptional. In order of importance (51,3,27,52 and 48). Your graph visually supports these point. The resultant errors exhibit no violation of randomness and subsequently no significant ACF. To adjust your observed values to accomodate the anomaly detection please use the following: +[X1(T)][(- 4494.5)] :PULSE 51 +[X2(T)][(+ 4937.5)] :PULSE 3 +[X3(T)][(+ 4884.5)] :PULSE 27 +[X4(T)][(+ 4505.5)] :PULSE 52 +[X5(T)][(+ 3967.5)] :PULSE 48 – IrishStat Jun 26 '14 at 10:51

1 @B_Miner Usually you will start by looking at the autocorrelation function of the residuals. If the autocorrelations are significant and large for large orders (i.e. the ACF does not decay exponentially to zero) then you may consider applying a unit root test on the residuals. If the analysis of the residuals suggests that there is unit root, that would mean that you should probably take first differences twice on the original data (i.e. take differences again in the differenced series). – javlacalle Jul 12 '14 at 15:36

With respect to your non-seasonal data ...Trends can be of two forms $y(t)=y(t-1)+\theta 0$ (A) Stochastic Trend or $Y(t)=a+bx1+cx2$ (B) Deterministic Trend etc where x1=1,2,3,4....t and x2=0,0,0,0,0,1,2,3,4 thus one trend applies to observations 1−t and a second trend applies to observations 6 to t.

Your non-seasonal series contained 29 values. I used AUTOBOX a piece of software that I had helped develop in a totally automatic fashion. AUTOBOX is a transparent procedure as it details each step in the modeling process. A graph of the series/fitted values/forecasts are presented here



. Using AUTOBOX to form a type A model led to the following

```
 AUTOMATIC FORECASTING SYSTEMS
        HATBORO PA 19040
          215-675-0652
     VERSION: 06/14/2014 09:27


MODELLING OUTPUT SERIES:fore-tre


[(1-B**1)]Y(T) =  1321.7                               fore-tre
                 +[X1(T)][(1-B**1)][(+  2015.7     )]   :PULSE           27
                 +[X2(T)][(1-B**1)][(-  2000.3     )]   :PULSE            4
                +     [(1+  .409B** 2)]**-1  [A(T)]
```

. The equation is presented again here

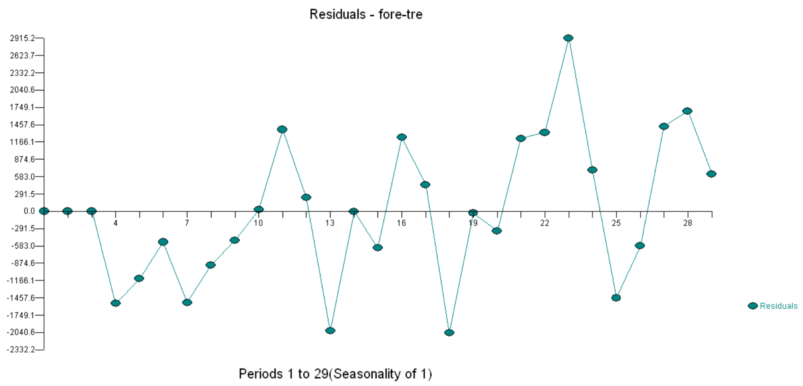
```
        MODEL COMPONENT        LAG    COEFF      STANDARD      P       T
 #                            (BOP)               ERROR      VALUE   VALUE

     Differencing                 1
    1CONSTANT                         .132E+04   285.        .0001    4.64
    2Autoregressive-Factor #  1   2  -.409       .159        .0164   -2.57

 INPUT SERIES X1   I~P00027          PULSE           27

     Differencing                 1
    3Omega (input) -Factor #  2   0   .202E+04   843.        .0247    2.39

 INPUT SERIES X2   I~P00004          PULSE            4

     Differencing                 1
    4Omega (input) -Factor #  3   0  -.200E+04   924.        .0401   -2.17
```

,

The statistics of the model are

```
 Number of Residuals (R)          =n                           26
 Number of Degrees of Freedom     =n-m                         22
 Residual Mean                    =Sum R / n                 .000000
 Error/Residual Sum of Squares    =Sum R**2                  .397893E+08
 Variance                         =SOS/(n)                   .137204E+07
 Adjusted Variance                =SOS/(n-m)                 .180860E+07
 Standard Deviation RMSE          =SQRT(Adj Var)             1344.84
 Standard Error of the Mean       =Standard Dev/ (n-m)       286.721
 Mean / its Standard Error        =Mean/SEM                  .000000
 Mean Absolute Deviation          =Sum(ABS(R))/n             1016.81
 AIC Value ( Uses var )           =nln  +2m                  375.427
 SBC Value ( Uses var )           =nln  +m*lnn               380.459
 BIC Value ( Uses var )           =see Wei p153              394.262
 R Square                         =                          .976760
 Durbin-Watson Statistic          =[-A(T-1)]**2/A**2         1.31747

     D-W STATISTIC IS INCONCLUSIVE.
```

. A plot of the residuals is here



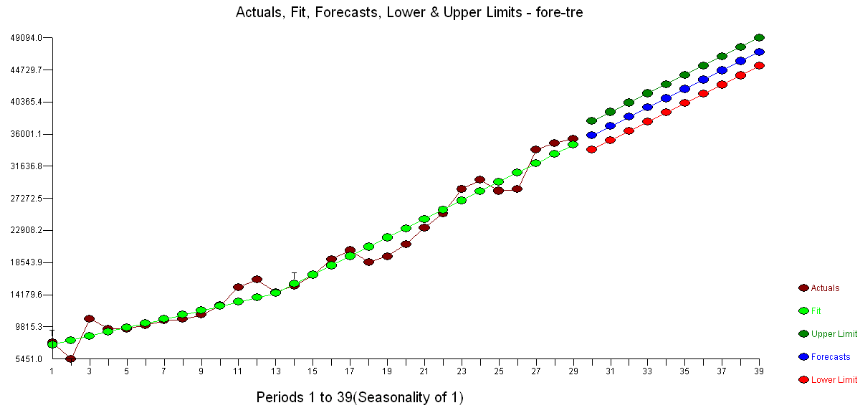
while the table of forecasted values are here

Historical Data | Auxiliaries | Graph | Reports | Interventio

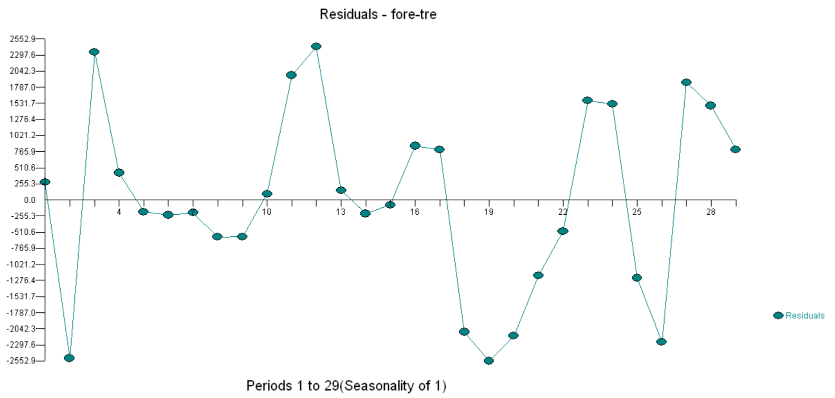
Forecast Data

Period/Major,Minor	Lower 80%	fore-tre	Upper 80%
30 30	33759.0000	35480.0000	37202.0000
31 31	34134.0000	36568.0000	39003.0000
32 32	35197.0000	37836.0000	40474.0000
33 33	35884.0000	38712.0000	41540.0000
34 34	36401.0000	39516.0000	42631.0000
35 35	37102.0000	40479.0000	43856.0000
36 36	37892.0000	41472.0000	45052.0000
37 37	38628.0000	42400.0000	46172.0000
38 38	39347.0000	43316.0000	47285.0000
39 39	40101.0000	44258.0000	48415.0000

. Restricting AUTOBOX to a type B model led to AUTOBOX detecting an increased trend at period 14:.



```
  AUTOMATIC FORECASTING SYSTEMS
        HATBORO PA 19040
          215-675-0652
     VERSION: 06/14/2014 09:27


 MODELLING OUTPUT SERIES:fore-tre


 Y(T)  =   6798.2                                    fore-tre
        +[X1(T)][(+   581.44     )]          :TIME TREND        1
        +[X2(T)][(+   680.40     )]          :TIME TREND       14
       +                     +    [A(T)]


         MODEL COMPONENT        LAG    COEFF      STANDARD      P        T
  #                            (BOP)               ERROR       VALUE    VALUE

   1CONSTANT                          .680E+04    786.         .0000     8.65

  INPUT SERIES X1   I~T00001            TIME              1

   2Omega (input) -Factor #  1     0   581.       84.7         .0000     6.87

  INPUT SERIES X2   I~T00014            TIME             14

   3Omega (input) -Factor #  2     0   680.       130.         .0000     5.23
                                                                                !

  Number of Residuals (R)         =n                          29
  Number of Degrees of Freedom    =n-m                        26
  Residual Mean                   =Sum R / n                 .000000
  Error/Residual Sum of Squares   =Sum R**2                  .590977E+08
  Variance                        =SOS/(n)                   .203785E+07
  Adjusted Variance               =SOS/(n-m)                 .227299E+07
  Standard Deviation RMSE         =SQRT(Adj Var)             1507.64
  Standard Error of the Mean      =Standard Dev/  (n-m)      295.673
  Mean / its Standard Error       =Mean/SEM                  .000000
  Mean Absolute Deviation         =Sum(ABS(R))/n             1144.96
  AIC Value ( Uses var )          =nln  +2m                  427.295
  SBC Value ( Uses var )          =nln  +m*lnn               431.397
  BIC Value ( Uses var )          =see Wei p153              437.245
  R Square                        =                          .972007
  Durbin-Watson Statistic         =[-A(T-1)]**2/A**2         1.46396

     D-W STATISTIC SUGGESTS NO SIGNIFICANT AUTOCORRELATION for lag1.
```



Historical Data | Auxiliaries | Graph | Reports

Forecast Data

Period/Major,Minor	Lower 80%	fore-tre	Upper 80%
30 30	33878.0000	35808.0000	37738.0000
31 31	35140.0000	37070.0000	39000.0000
32 32	36402.0000	38332.0000	40262.0000
33 33	37664.0000	39594.0000	41523.0000
34 34	38926.0000	40855.0000	42785.0000
35 35	40188.0000	42117.0000	44047.0000
36 36	41449.0000	43379.0000	45309.0000
37 37	42711.0000	44641.0000	46571.0000
38 38	43973.0000	45903.0000	47833.0000
39 39	45235.0000	47165.0000	49094.0000

In terms of comparing models: Since the number of fitted observations differ (26 and 29 respectively) it is not possible to use standard metrics (i.e. r-square,error standard dev, AIC etc) to determine dominance although in this case the nod would go to A. The residuals from A are better due to the AR(2) structure. The forecasts from B are a tad aggressive while the pattern of the A forecasts are more intuitive. One could hold back say 4 observations and evaluate forecast accuracy for a 1 period out forecast from 4 distinct origins(25,26,27 and 28).

answered Jun 14 '14 at 18:42

IrishStat
11.8k 1 11 23

2Irish stat stanks for excellect response. I have read some were that we would combine oth stochastic and deterministic trends that is yt = y(t-1)+a+bt=ct ? would that be helpful – forecaster Jun 14 '14 at 19:08

The model form y(t)=B0 + B1*t + a(t)[thetha/phi] collapses if phi is say [1-B] since clearing fractions essentially differencing the t variable yielding a constant colliding with B0. In other words ARIMA structure joined with time indicators can create havoc. The model you specified is estimable but definitely is not a favored approach (lack of endogeneity perhaps !). Someone else reading this can comment might help on this. It is not a proper subset of a Transfer Function i.imgur.com/dv4bAts.png – IrishStat Jun 14 '14 at 20:13

There are 4 possible states of nature. There is no analytical solition to this question since the model sample space is relatively unlimited. To empirically answer this vexing question I have helped develop AUTOBOX http://www.autobox.com/cms/ . AUTOBOX runs a tournament to examine all 4 of these cases and assesses the quality of the 4 resultant models in terms of necessity and sufficiency. Why don't you post an example time series of your choice and I will post the 4 results showing how this problem has been solved .

answered Jun 12 '14 at 22:03

IrishStat
11.8k 1 11 23

Whoa, that would be awesome. Please do @forecaster. – JEquihua Jun 12 '14 at 22:30

thanks @irishstat, I have added an example for trend, I'll add seasonal data in the future. – forecaster Jun 14 '14 at 3:33