Source Flow

Consider the velocity field of a fluid given by

$$\boldsymbol{u}(x, y, z) = \frac{\Lambda(x\boldsymbol{i} + y\boldsymbol{j} + z\boldsymbol{k})}{4\pi(x^2 + y^2 + z^2)^{3/2}}.$$

(a) Using spherical coordinates, show that

$$\boldsymbol{u}(r) = \frac{\Lambda\hat{\boldsymbol{r}}}{4\pi r^2}.$$

(b) Using spherical coordinates, show that

$$\boldsymbol{\nabla} \cdot \boldsymbol{u} = 0$$

provided $r \neq 0$.

(c) Using the divergence theorem, show that

$$\int_V \boldsymbol{\nabla} \cdot \boldsymbol{u}\, dV = \Lambda,$$

provided that the volume V contains the origin, and is zero otherwise. You have therefore shown that the divergence of the velocity field is given by

$$\boldsymbol{\nabla} \cdot \boldsymbol{u} = \Lambda\delta(\boldsymbol{r}),$$

where $\delta(\boldsymbol{r})$ is the three-dimensional Dirac delta function. This velocity field is called a source flow.