

Expectation and variance in continuous spaces

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The variance of X :

$$\sigma^2 = \int_{-\infty}^{\infty} (x - \mu)^2 \cdot p(x) dx =: \text{Var}(X)$$

Coordinate transformations: one dimension

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$$m = \int_{-\infty}^{\infty} f(x) \cdot p(x) \, dx \quad \text{change of variable formula} \quad =: \mathbf{E}(f(X))$$

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change of variable formula

$$m = \int_{-\infty}^{\infty} f(x) \cdot p(x) \, dx =: \mathbf{E}(f(X))$$

$$s^2 = \int_{-\infty}^{\infty} (f(x) - m)^2 \cdot p(x) \, dx$$

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Coordinate transformations: two and more dimensions

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$$m = \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} f(x_1, \dots, x_n) \cdot p(x_1, \dots, x_n) dx_n \cdots dx_1$$

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$$s^2 = \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} (f(x_1, \dots, x_n) - m)^2 \cdot p(x_1, \dots, x_n) dx_n \cdots dx_1$$

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$$\mathfrak{m} = \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} f(x_1, \dots, x_n) \cdot p(x_1, \dots, x_n) \, dx_n \cdots dx_1 =: \mathbf{E}(f(X_1, \dots, X_n))$$

$$s^2 = \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} (f(x_1, \dots, x_n) - \mathfrak{m})^2 \cdot p(x_1, \dots, x_n) \, dx_n \cdots dx_1 =: \text{Var}(f(X_1, \dots, X_n))$$