A general setting

Independent families

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A finite or countably infinite collection of events $\{A_j,\ j \geq 1\}$ in a probability space is independent if (and only if), for *every finite* subset $\mathbb{J}$ of indices (positive integers), we have a rule of products

$$\mathbf{P}\Bigl(\bigcap_{j\in\mathbb{J}} A_j\Bigr) = \prod_{j\in\mathbb{J}} \mathbf{P}(A_j).$$

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This means:
for *every* integer $k \geq 2$ and *every* selection of integer indices $j_1, \ldots, j_k$,
we have $\mathbf{P}(A_{j_1} \cap \cdots \cap A_{j_k}) = \mathbf{P}(A_{j_1}) \times \cdots \times \mathbf{P}(A_{j_k})$.

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