

Today we will learn two different methods for testing hypotheses

- The P -value Method
- The Traditional method - Using Rejection Regions (critical value approach)

The P -value Method of Hypothesis Testing

Decision Rule Based on P -value

To use a P -value to make a conclusion in a hypothesis test, compare the P -value with α

1. If $P \leq \alpha$, then reject H_0 .
2. If $P > \alpha$, then fail to reject H_0 .

Recall:

A P -value (or probability value) is the probability of getting a value of the the sample test statistic that is *at least as extreme* as the one found from the sample data, assuming that the null hypothesis is true. By *extreme* we mean: far from what we would expect to observe if the null hypothesis is true.

In other words, a small P -value indicates that observation of the test statistic would be unlikely if the null hypothesis is true. The lower the P -value, the more evidence there is in favor of rejecting the null hypothesis.

z -Test for a Mean μ

The **z -test for a mean** is a statistical test for a population mean. The z -test can be used when the population is normal and σ is known, or for any population when the sample size n is at least 30. The **test statistic** is the sample mean \bar{x} and the **standardized test statistic** is z .

$$z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

Recall that $\frac{\sigma}{\sqrt{n}} = \sigma_x$ (standard error).

When $n \geq 30$, you can use the sample standard deviation s in place of σ .

GUIDELINES

Using P -values for a z -test for a Mean μ

1. Write the null hypothesis H_0 and alternate hypothesis H_a ; then identify the claim and decide whether the test is left-tailed, right-tailed, or two-tailed.
2. Identify the level of significance, α .
3. Determine the standardized test statistic z .
4. Find the area that corresponds to z (as before - use Table).
5. Find the P -value.
 - left-tailed test:* $P = (\text{Area in left tail})$
 - right-tailed test:* $P = (\text{Area in right tail})$
 - two-tailed test:* $P = 2(\text{Area in tail of test statistic})$
6. Make a decision to reject H_0 or fail to reject H_0 .
 - If $P \leq \alpha$, then reject H_0
 - If $P > \alpha$, then fail to reject H_0
7. Interpret the decision in the context of the original claim.

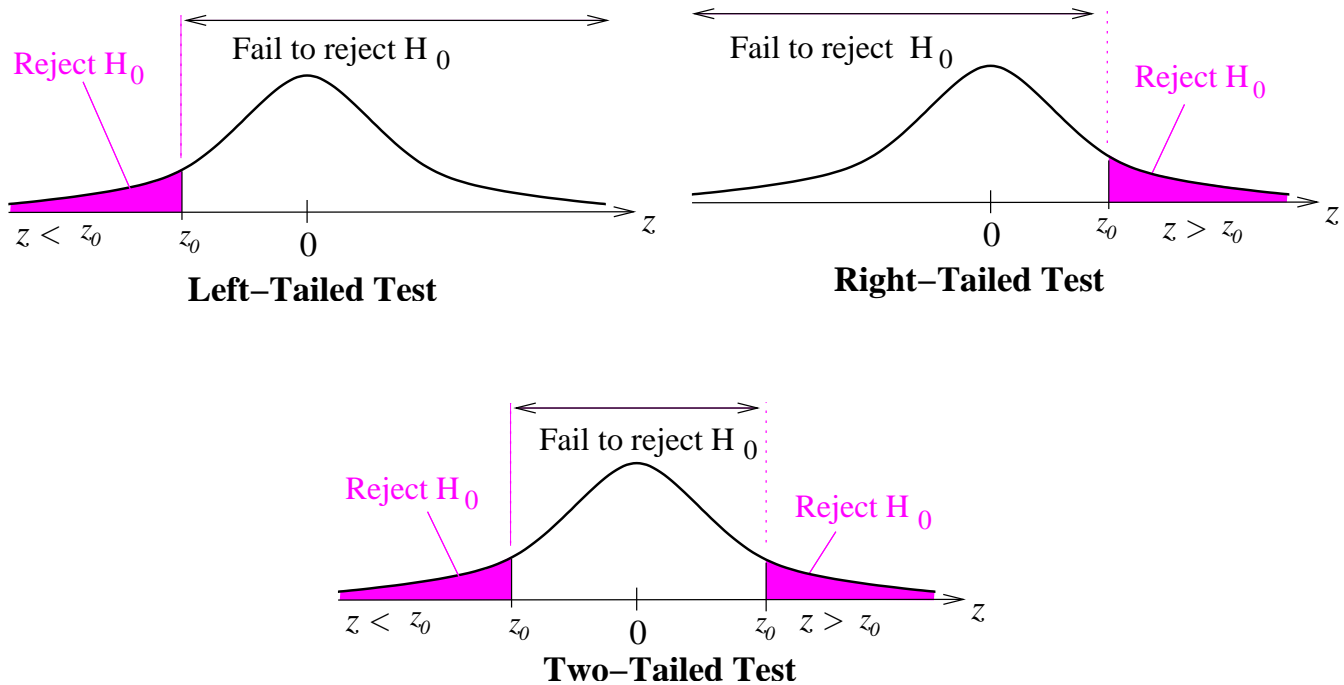
The Rejection Region Method of Hypothesis Testing

DEFINITION

The traditional method of hypothesis testing. A **rejection region** (or **critical region**) of the sampling distribution is the range of values for which the null hypothesis is not probable. If a test statistic falls in this region, the null hypothesis is rejected. A **critical value** z_0 separates the rejection region from the non-rejection region.

Remarks

1. The rejection region is the set of all values of the test statistic that cause us to reject the null hypothesis.
2. The significance level (α) is the probability that the test statistic will fall in the rejection region when the null hypothesis is actually true. If the test statistic falls in the rejection region, we will reject the null hypothesis, so α is the probability of making the mistake of rejecting the null hypothesis when it is true.



GUIDELINES

Finding Critical Values in a Normal Distribution

1. Specify the level of significance α .
2. Decide whether the test is left-tailed, right-tailed, or two-tailed.
3. Find the critical value(s) z_0 . If the hypothesis test is
 - (a) *left-tailed*, find the z -score that corresponds to an area of α .
 - (b) *right-tailed*, find the z -score that corresponds to an area of $1 - \alpha$.
 - (c) *two-tailed*, find the z -score that corresponds to an area of $\frac{1}{2}\alpha$ and $1 - \frac{1}{2}\alpha$.
4. Sketch the standard normal distribution.

Remember that, failing to reject the null hypothesis does not mean that you have accepted the null hypothesis as true (just that there is not enough evidence to reject the null hypothesis).

We use a z -test for the mean when the population is normal and σ is known, or for any population when the sample size n is at least 30 (then we may use: $\sigma \approx s$).

GUIDELINES

Using Rejection Regions for a z -test for a Mean μ

1. Write the null hypothesis H_0 and alternative hypothesis H_a ; then identify the claim.
2. Specify the level of significance α .
3. Determine the critical value(s).
4. Determine the rejection region(s).
5. Determine the standardized test statistic z .
6. Sketch the sampling distribution, including test statistic, critical value(s) and rejection region(s).
7. Make a decision to reject H_0 or fail to reject H_0 .
 - (a) If z is in the rejection region - reject H_0 .
 - (b) If z is not in the rejection region - fail to reject H_0 .
8. Interpret the decision in the context of the original claim.

	Claim	
Decision	Claim is H_0 .	Claim is H_a
Reject H_0 .	There is enough evidence to reject the claim	There is enough evidence to support the claim
Fail to Reject H_0 .	There is Not enough evidence to reject the claim	There is Not enough evidence to support the claim