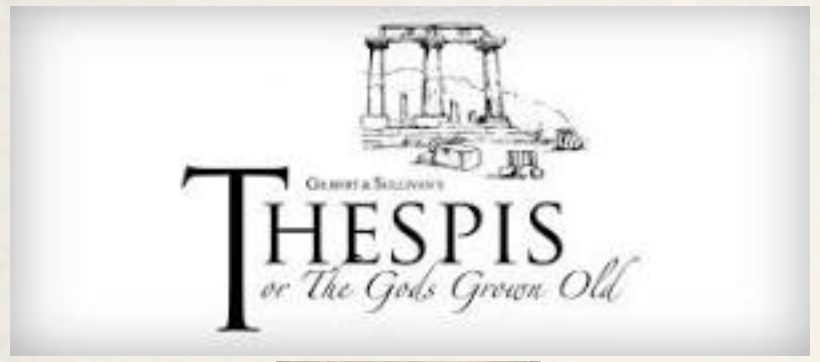
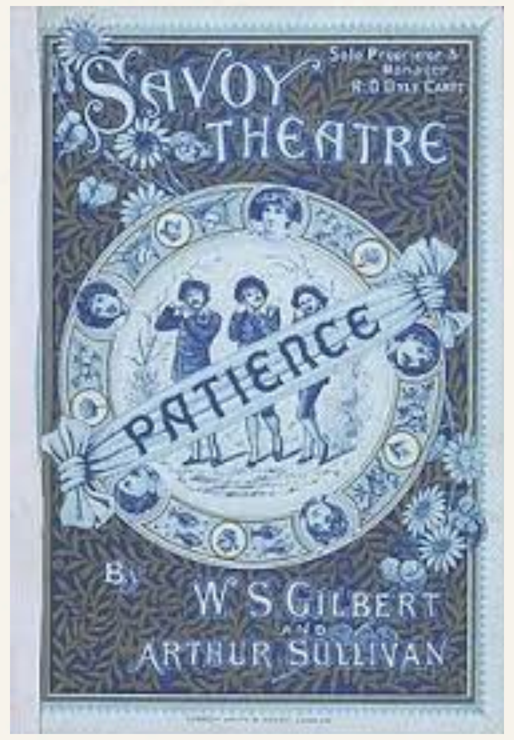

THESPIS
or The Gods Grown Old


PLAYBILL
H.M.S. PINAFORE


THE SORCERER
D'OYLY CARTE OPERA CO.


SAVOY THEATRE
PATIENCE
BY W. S. GILBERT AND ARTHUR SULLIVAN

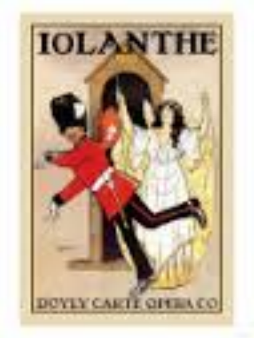

DAY PERFORMANCE
SATURDAY, MAY 24th.
MONDAY, MAY 19, 1884
D'OYLY CARTE'S OPERA COMPANY
PRINCESS IDA,
OR, CASTLE ADAMANT.

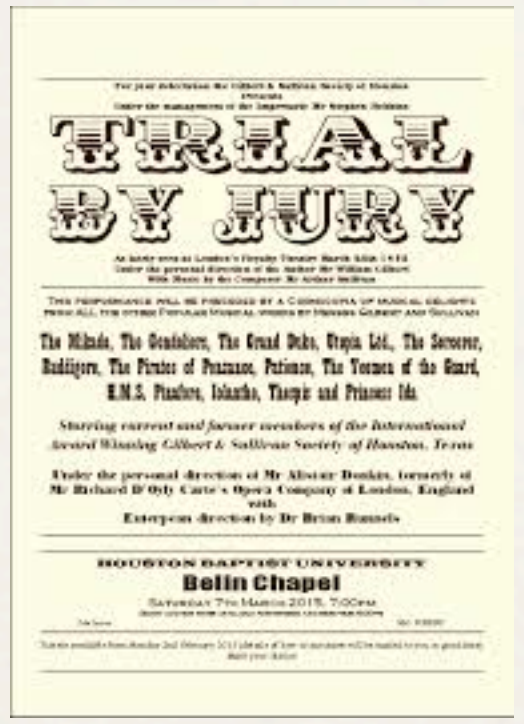

D'OYLY CARTE'S OPERA COMPANY
THE MIKADO
W. S. GILBERT & ARTHUR SULLIVAN

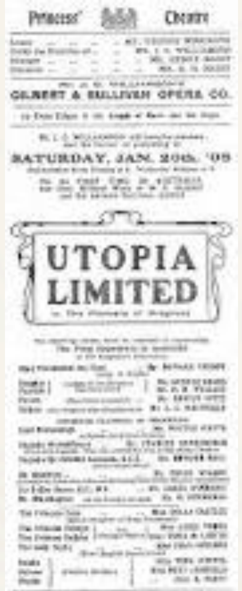

Royal Lyceum Theatre
D'OYLY CARTE'S OPERA COMPANY
RUDDIGORE
BY W. S. GILBERT & ARTHUR SULLIVAN

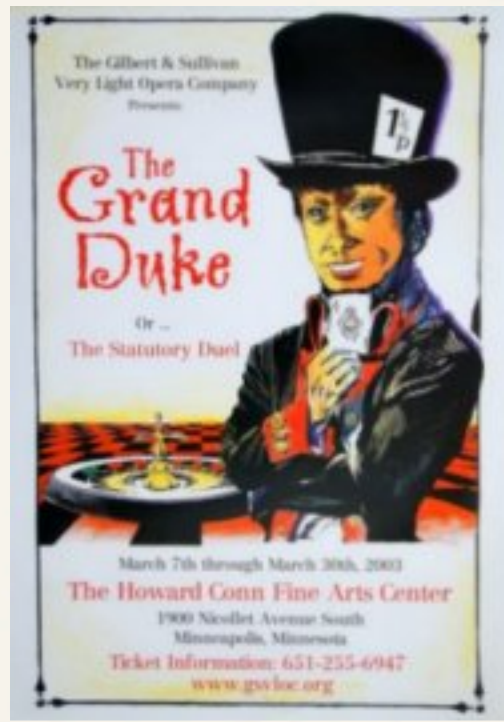

PLAYBILL
The Pirates of Penzance

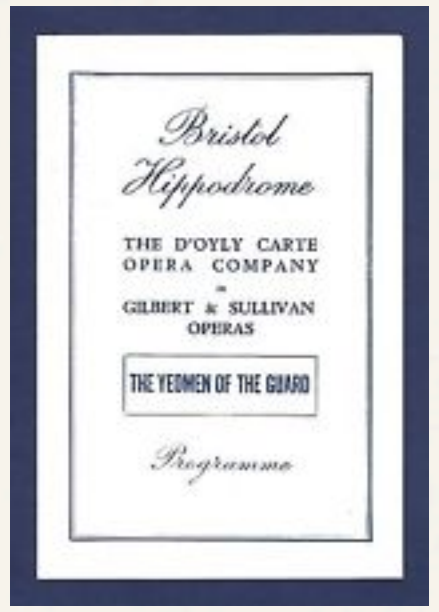

Mr D'OYLY CARTE'S
Opera Company
COMIC OPERA
"THE GONDOLIERS"
AS PERFORMED AT
THE SAVOY THEATRE, LONDON.


IOLANTHE
D'OYLY CARTE OPERA CO


TRIAL BY JURY
HOUSTON BAPTIST UNIVERSITY
Belin Chapel


GILBERT & SULLIVAN OPERA CO.
UTOPIA LIMITED


The Gilbert & Sullivan
Very Light Opera Company
Presents
The Grand Duke
Or ...
The Statutory Duel
March 7th through March 30th, 2003
The Howard Conn Fine Arts Center
1900 Nicollet Avenue South
Minneapolis, Minnesota
Ticket Information: 651-255-6947
www.gvloc.org


Bristol Hippodrome
THE D'OYLY CARTE OPERA COMPANY
GILBERT & SULLIVAN OPERAS
THE YEOMEN OF THE GUARD
Programme

The coupon collector's problem

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$A_1^c \cap \cdots \cap A_n^c = \{X_1 + \cdots + X_n = 0\}$ $\{X_1 + \cdots + X_n = k\}$

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The inclusion–exclusion sums

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The 1st term $\mathbf{P}(A_1) = \mathbf{P}\{\,(k_1, k_2, \ldots, k_n) : k_i \neq 1 \text{ (each i)}\,\}$

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desired confidence

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$$\lim_{n\to\infty}\left(1 - \tfrac{x}{n}\right)^n = e^{-x}$$

If $ne^{-t/n} = \lambda$ or, equivalently, $t = n \log \frac{n}{\lambda}$, then $S_j \to \frac{\lambda^j}{j!}$

$$\mathbf{P}(A_1^c \cap A_2^c \cap \cdots \cap A_n^c) = S_0 - S_1 + S_2 - \cdots + (-1)^n S_n \to e^{-\lambda}$$

desired confidence

$$\mathbf{P}\{X_1 + X_2 + \cdots + X_n = k\} = S_k - \binom{k+1}{k} S_{k+1} + \binom{k+2}{k} S_{k+2} - \cdots + (-1)^{n-k}\binom{n}{k} S_n \to e^{-\lambda}\frac{\lambda^k}{k!}$$

The number of purchases needed to acquire all 14 Gilbert and Sullivan playbills:

$$A_1^\complement \cap \cdots \cap A_n^\complement = \{X_1 + \cdots + X_n = 0\} \qquad \{X_1 + \cdots + X_n = k\}$$

The inclusion–exclusion sums

$$S_j = \frac{n^{\underline{j}}}{j!}\left(1 - \frac{j}{n}\right)^t = \frac{n^{\underline{j}}}{j!}\left(1 - \frac{j}{n}\right)^{n \cdot \frac{t}{n}} \approx \frac{n^j \cdot e^{-j \cdot t/n}}{j!} = \frac{(ne^{-t/n})^j}{j!}$$

$$\lim_{n \to \infty} \left(1 - \tfrac{x}{n}\right)^n = e^{-x}$$

If $ne^{-t/n} = \lambda$ or, equivalently, $t = n \log \frac{n}{\lambda}$, then $S_j \to \frac{\lambda^j}{j!}$

$$\mathbf{P}(A_1^\complement \cap A_2^\complement \cap \cdots \cap A_n^\complement) = S_0 - S_1 + S_2 - \cdots + (-1)^n S_n \to e^{-\lambda}$$ desired confidence

$$\mathbf{P}\{X_1 + X_2 + \cdots + X_n = k\} = S_k - \binom{k+1}{k} S_{k+1} + \binom{k+2}{k} S_{k+2} - \cdots + (-1)^{n-k} \binom{n}{k} S_n \to e^{-\lambda} \frac{\lambda^k}{k!}$$

The number of purchases needed to acquire all 14 Gilbert and Sullivan playbills:

For 90% confidence $\lambda = -\log(0.9) = 0.10536$ and $t = \lceil 14 \log \frac{14}{0.10536} \rceil = 69$.

$A_1^c \cap \cdots \cap A_n^c = \{X_1 + \cdots + X_n = 0\}$ $\qquad$ $\{X_1 + \cdots + X_n = k\}$

The inclusion–exclusion sums

$$S_j = \frac{n^{\underline{j}}}{j!}\left(1 - \frac{j}{n}\right)^t = \frac{n^{\underline{j}}}{j!}\left(1 - \frac{j}{n}\right)^{n \cdot \frac{t}{n}} \approx \frac{n^j \cdot e^{-j \cdot t/n}}{j!} = \frac{(ne^{-t/n})^j}{j!}$$

$$\lim_{n\to\infty}\left(1 - \tfrac{x}{n}\right)^n = e^{-x}$$

If $ne^{-t/n} = \lambda$ or, equivalently, $t = n \log \frac{n}{\lambda}$, then $S_j \to \frac{\lambda^j}{j!}$

$$\mathbf{P}(A_1^c \cap A_2^c \cap \cdots \cap A_n^c) = S_0 - S_1 + S_2 - \cdots + (-1)^n S_n \to e^{-\lambda}$$ desired confidence

$$\mathbf{P}\{X_1 + X_2 + \cdots + X_n = k\} = S_k - \binom{k+1}{k} S_{k+1} + \binom{k+2}{k} S_{k+2} - \cdots + (-1)^{n-k}\binom{n}{k} S_n \to e^{-\lambda}\frac{\lambda^k}{k!}$$

The number of purchases needed to acquire all 14 Gilbert and Sullivan playbills:

For 90% confidence $\lambda = -\log(0.9) = 0.10536$ and $t = \lceil 14 \log \frac{14}{0.10536} \rceil = 69$.

For 99% confidence $\lambda = -\log(0.99) = 0.01005$ and $t = \lceil 14 \log \frac{14}{0.01005} \rceil = 102$.