Georgia Institute of Technology

Computational Photography

Dr. Irfan Essa
Professor
School of Interactive Computing

Study the basics of computation and its impact on the entire workflow of photography, from capturing, manipulating and collaborating on, and sharing photographs.

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Making a Panorama

Stitching a Series of Images to construct a Wide FOV Panorama.

Lesson Objectives

- ★ Outline the five (5) steps needed to generate a Panorama and describe in your own works the first four (4) steps in detail.
- ★ Explain in your own words the concept of Image Re-projection needed for Panoramas
- ★ Describe in your own words the concept of Homography and how it is computed from a pair of images.
- ★ Explain the method used to deal with bad matches between images and how it is used by computing inliers and outliers.
- ★ Describe in your own words, some additional considerations needed to construct panoramas.

3

REVIEW: 5 Steps to Make a Panorama

(Lords Cricket Ground, London, UK, by I. Essa)

1. Capture Images
2. Detection and Matching
3. Warping ➔ Aligning Images
4. Blending, Fading, Cutting
5. Cropping (Optional)

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Align Images

Align Images

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Align Images

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L (on top)

R (on top)

Align Images

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Align Images

Translating Images is NOT Enough

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L (on top)

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5

A Bundle of Rays Contains all Views

Slide motivated by James Hays

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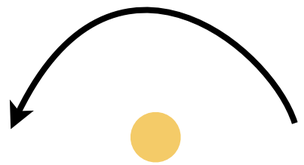

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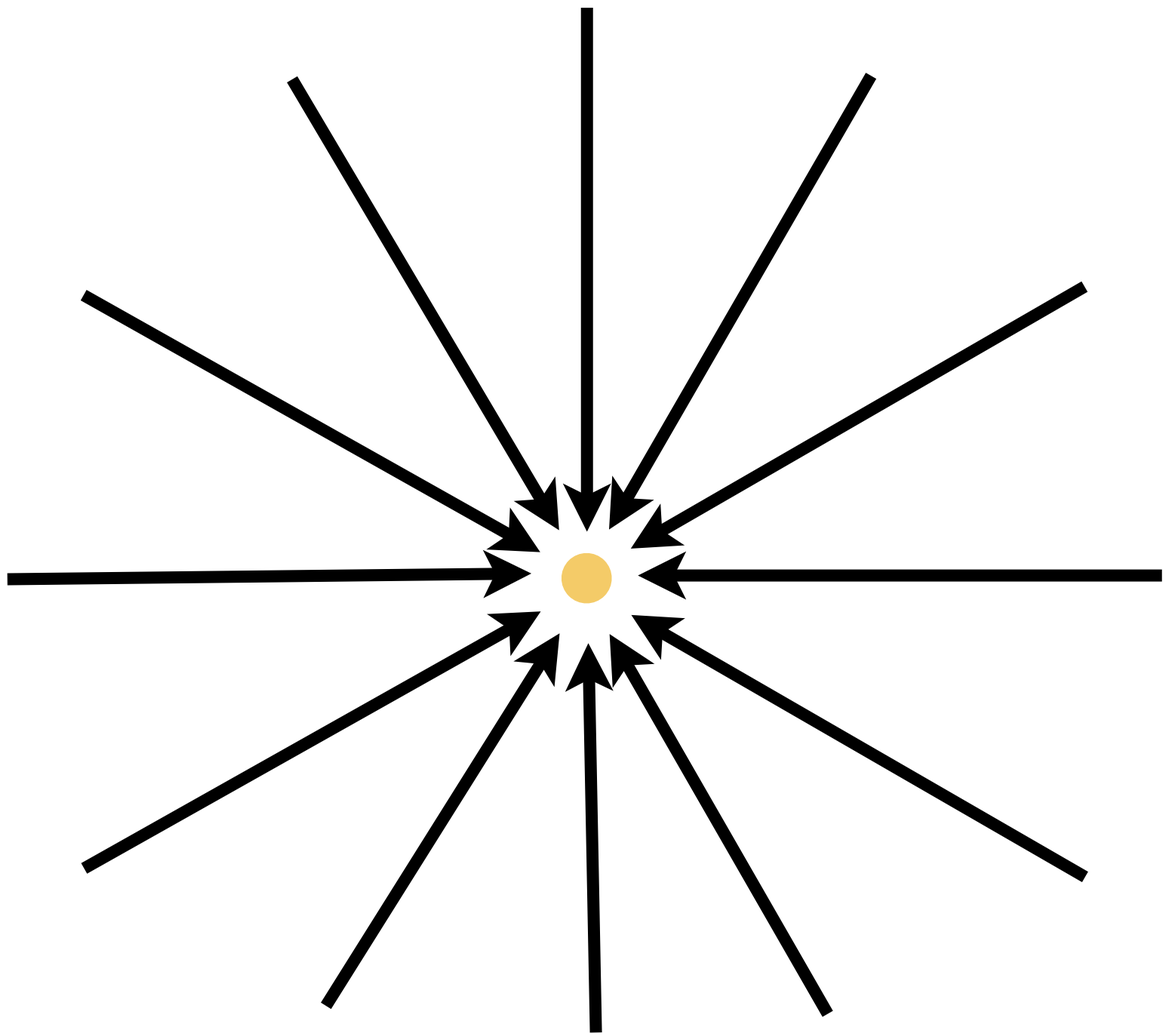

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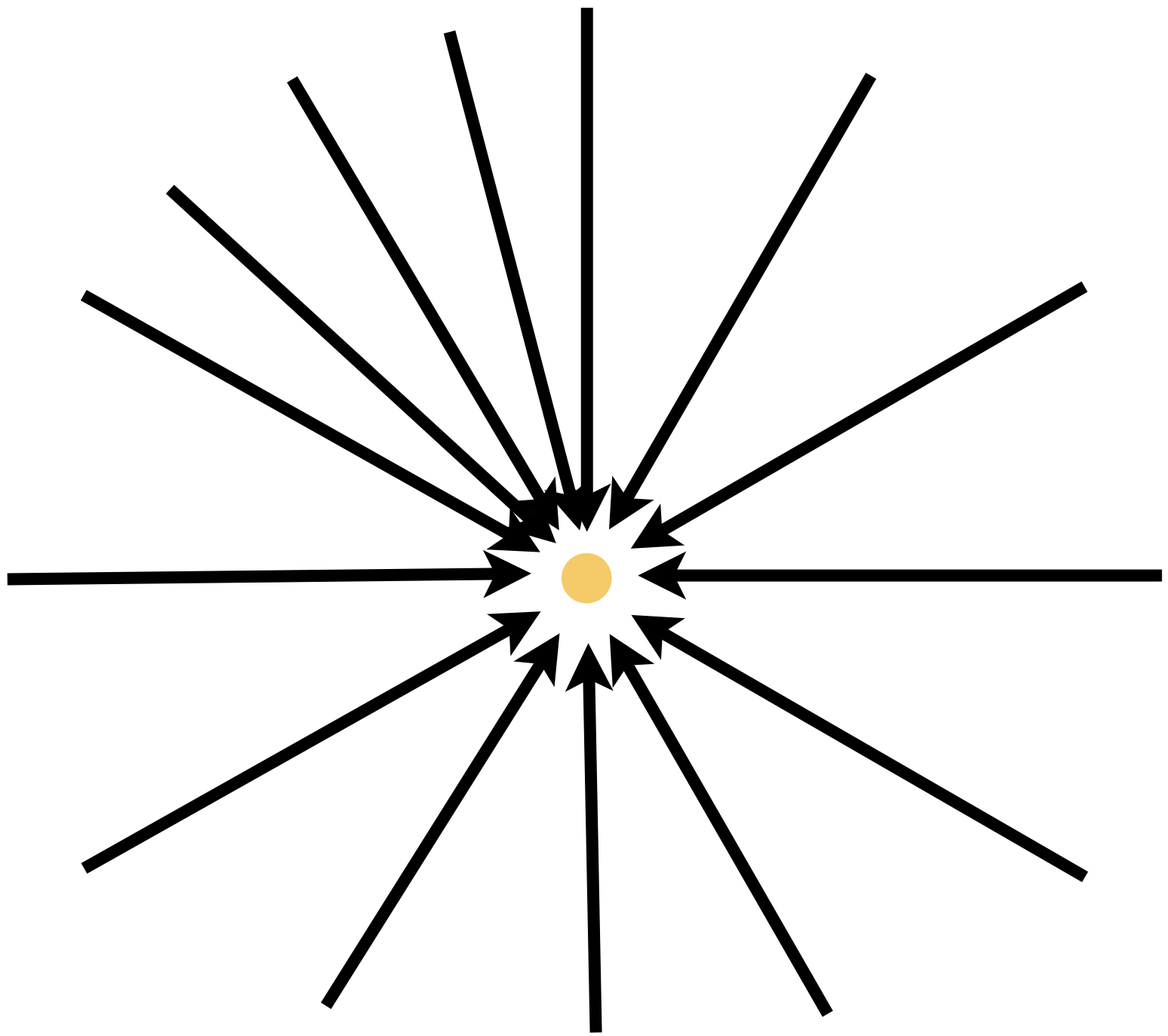

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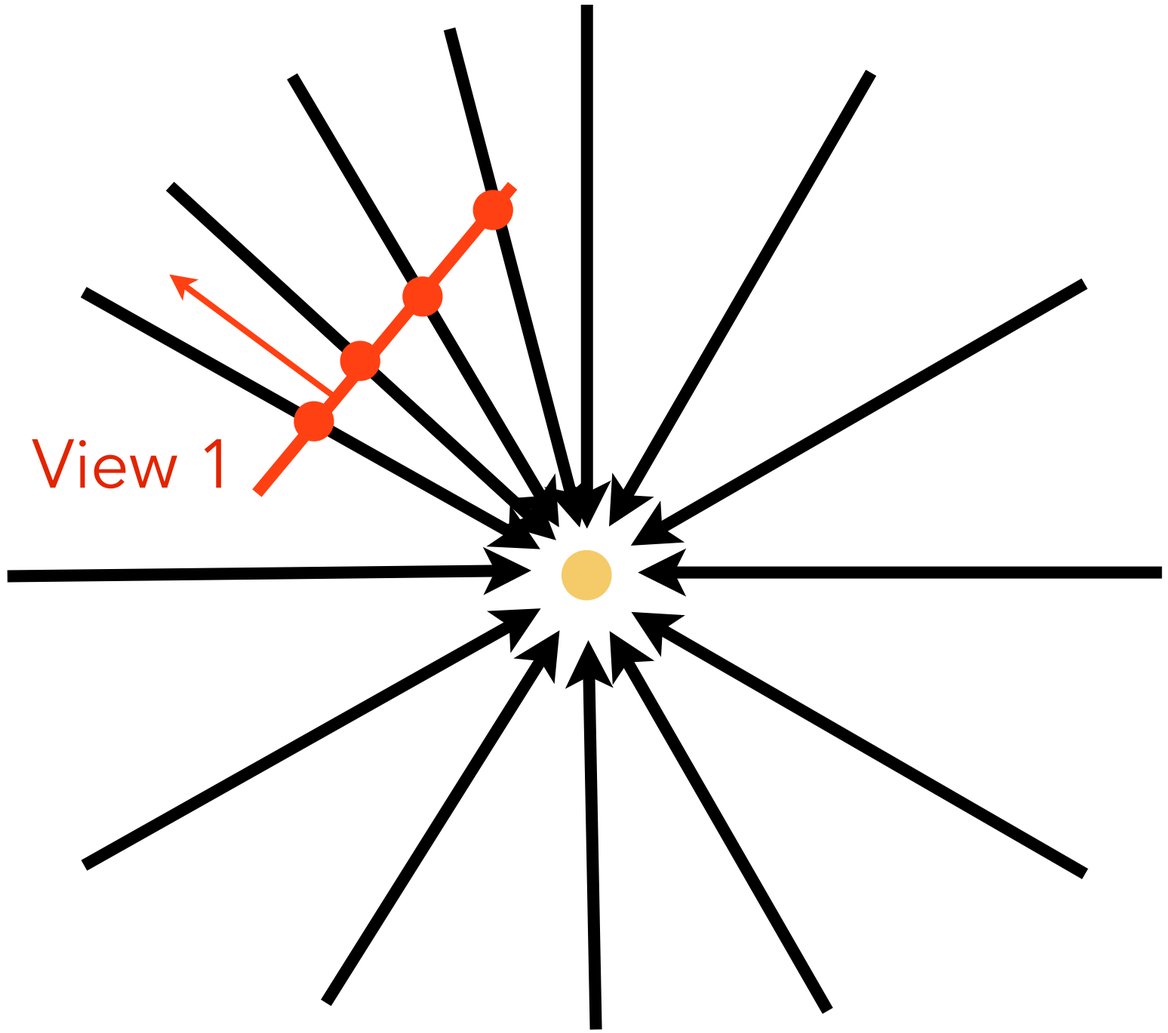

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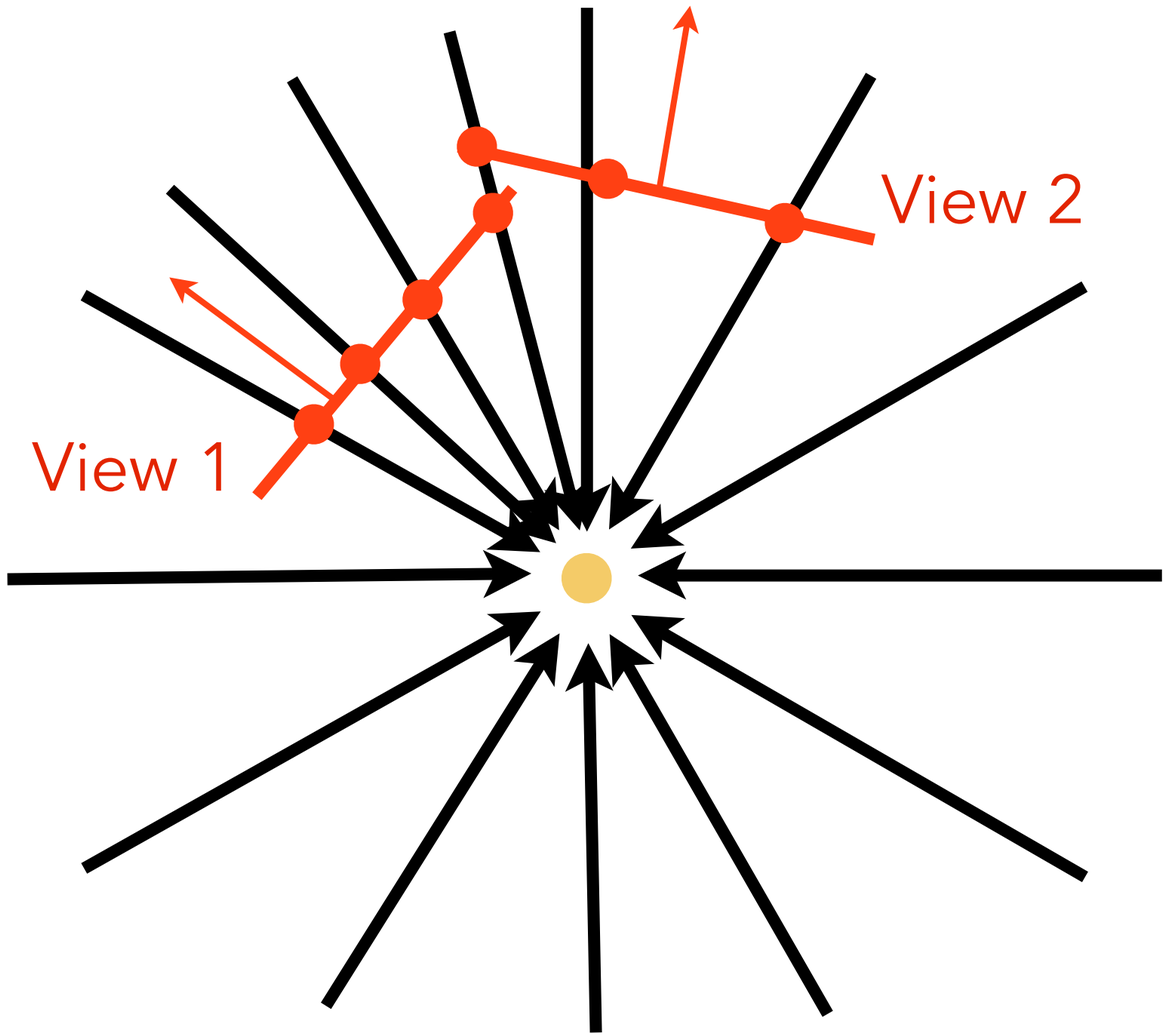

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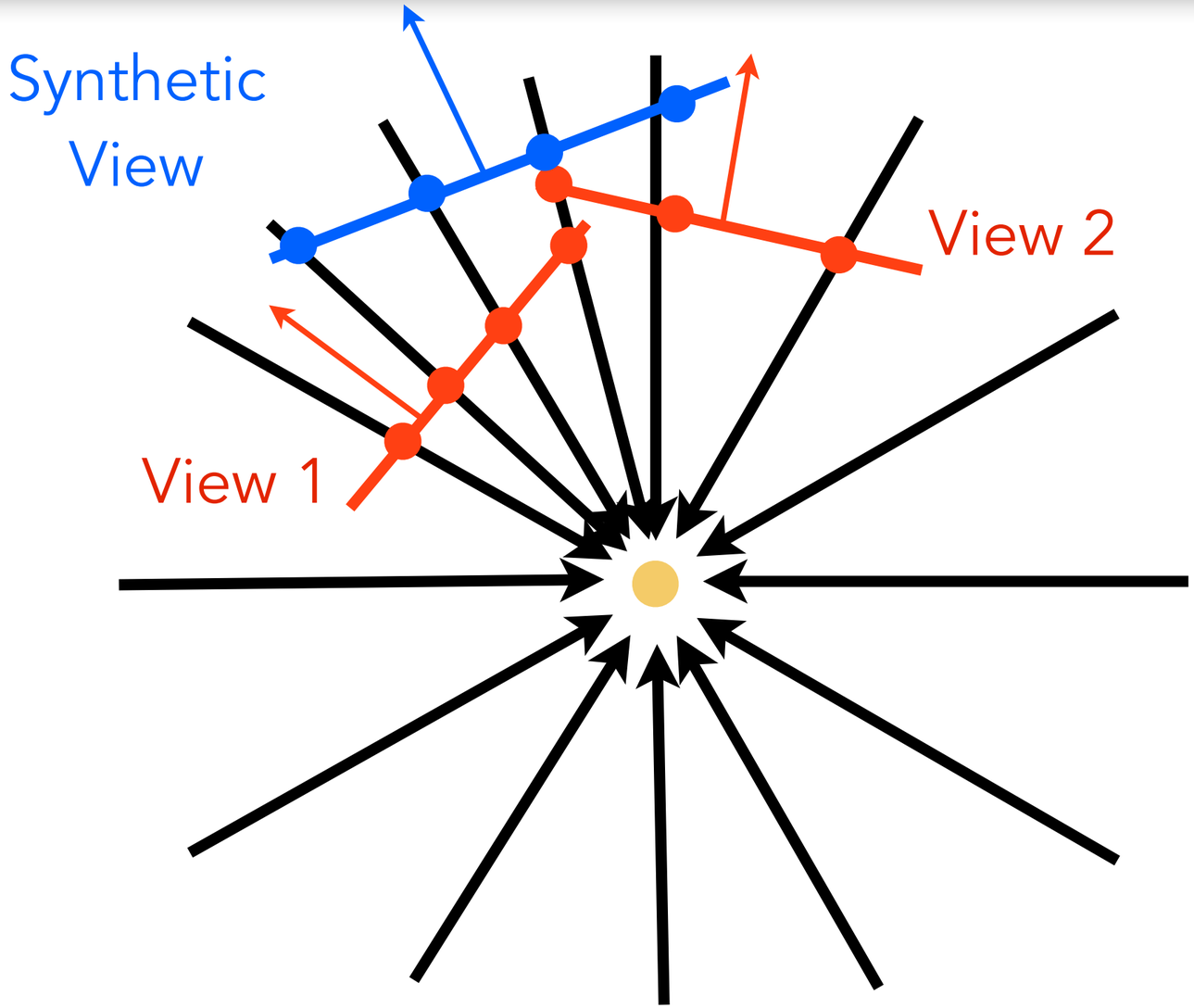

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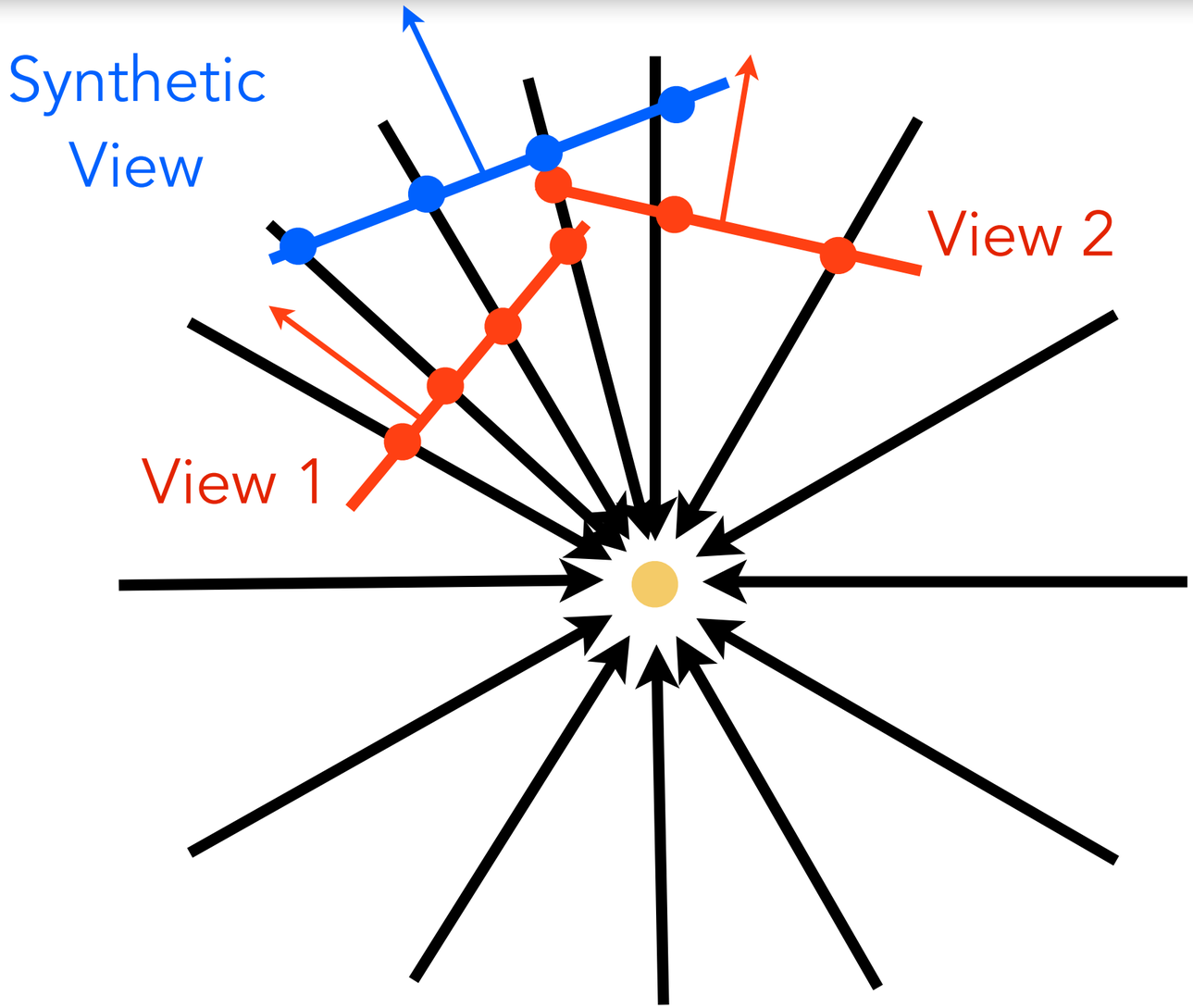

A Bundle of Rays Contains all Views

Possible to generate any synthetic camera view as long as it has **the same center of projection**!

Slide motivated by James Hays

Projection Plane

Image Re-projection to Panorama Projection Plane (PP)

7

Projection Plane

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Projection Plane

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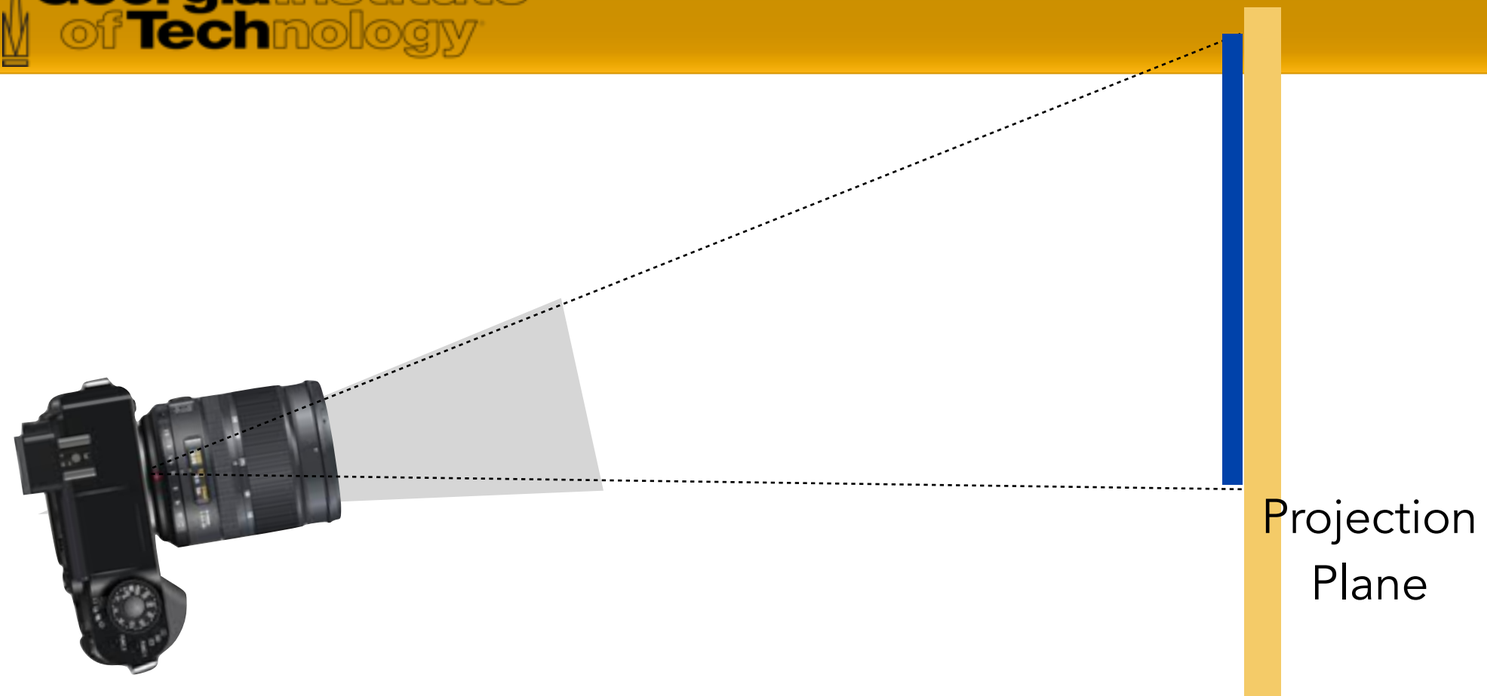

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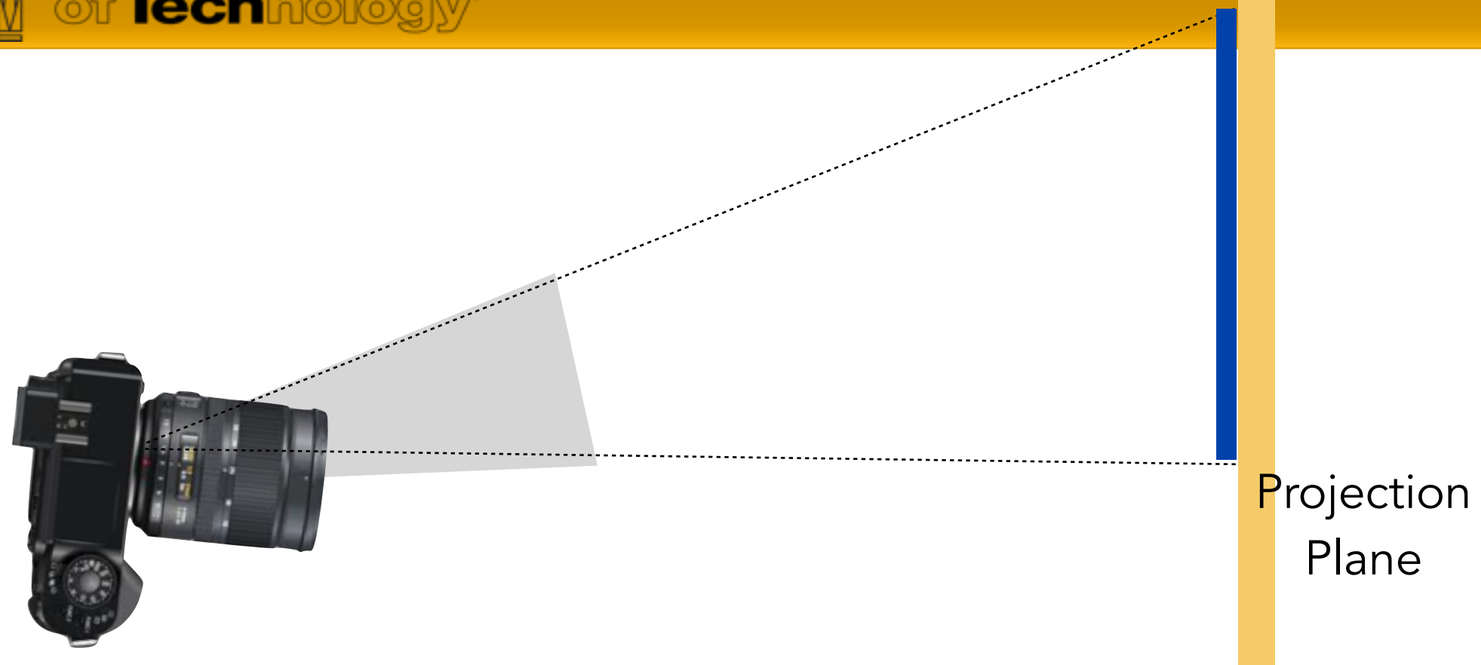

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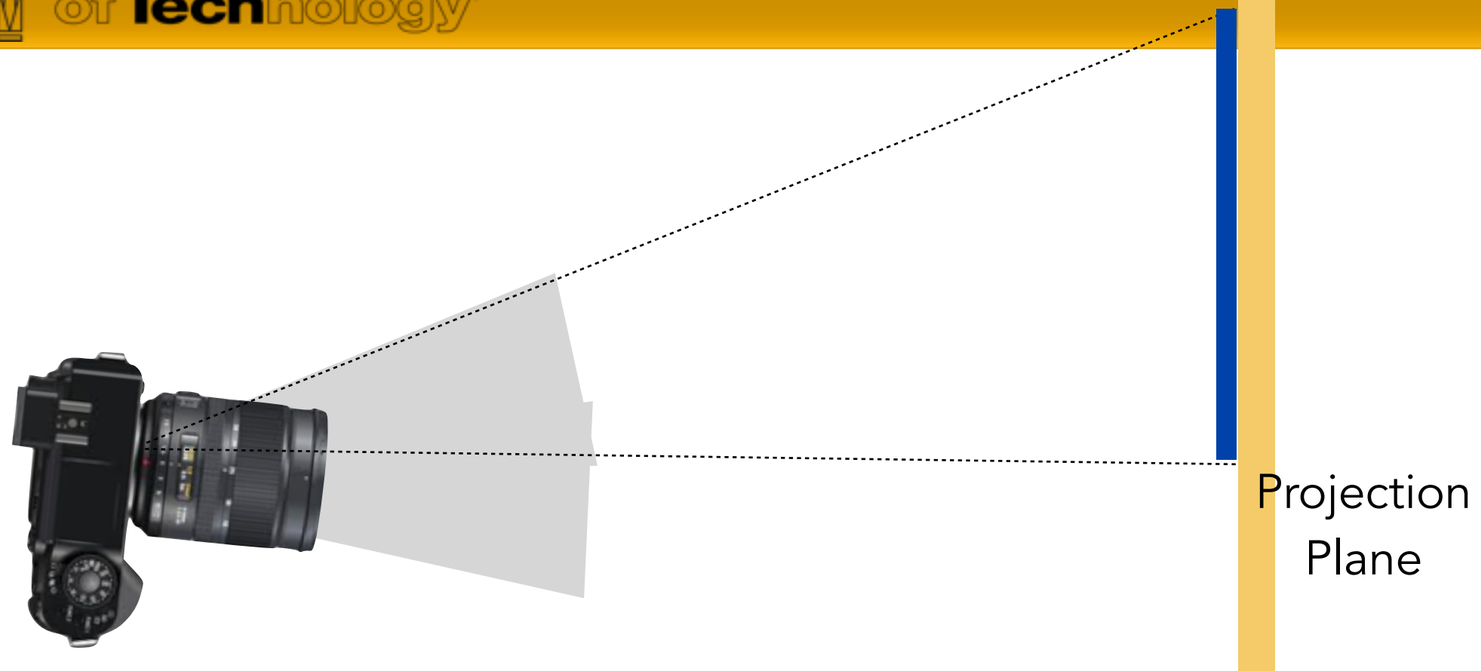

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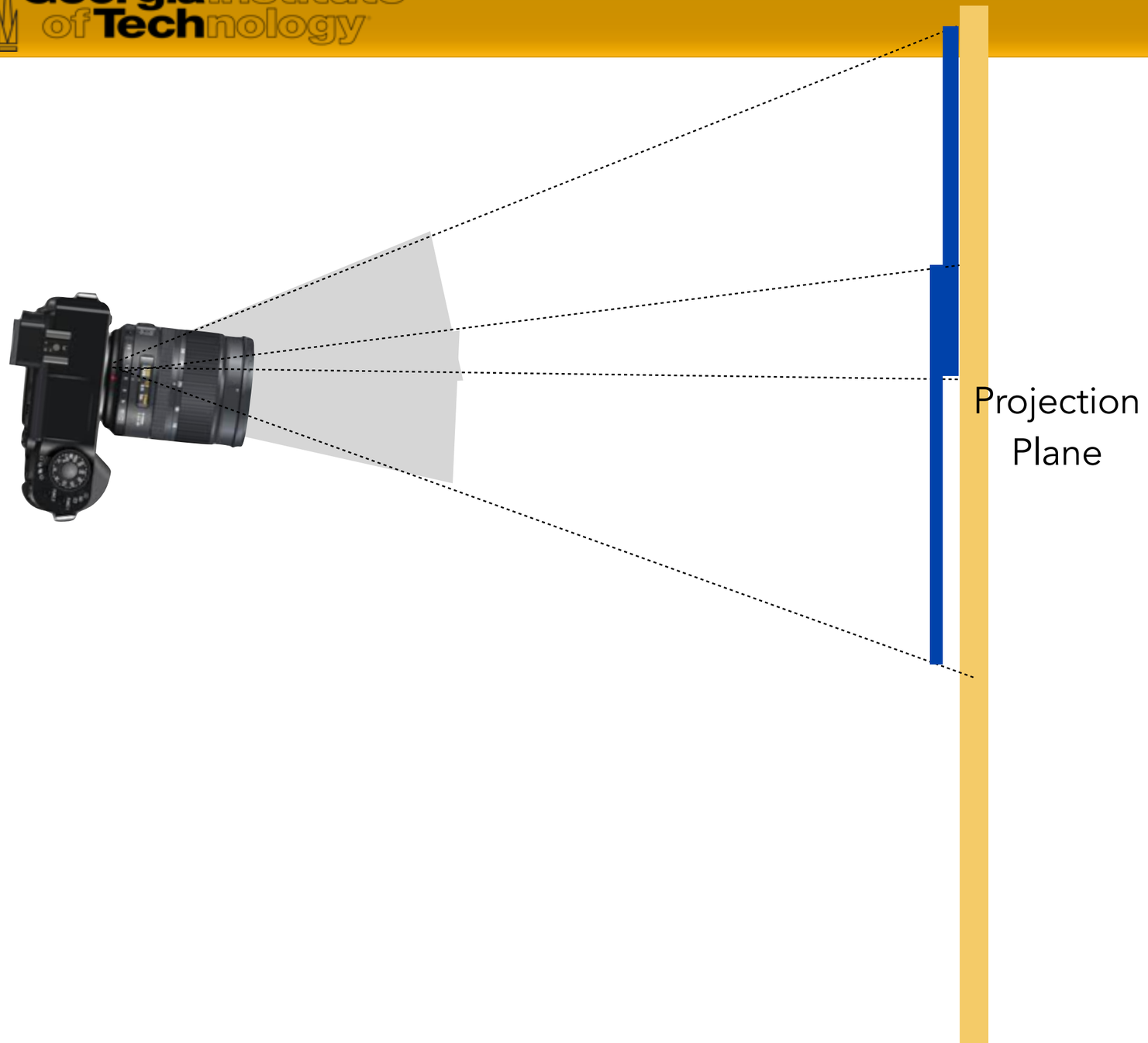

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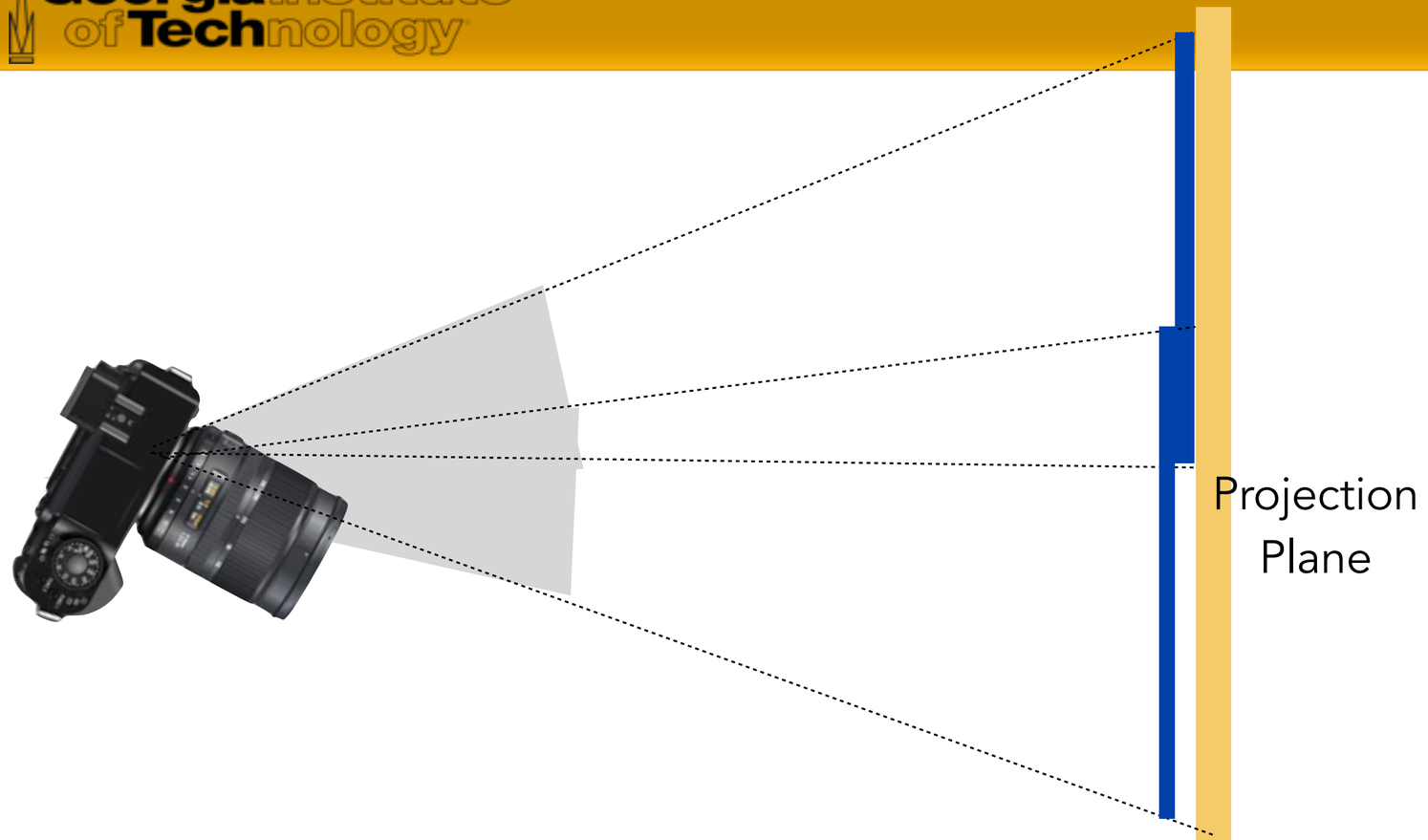

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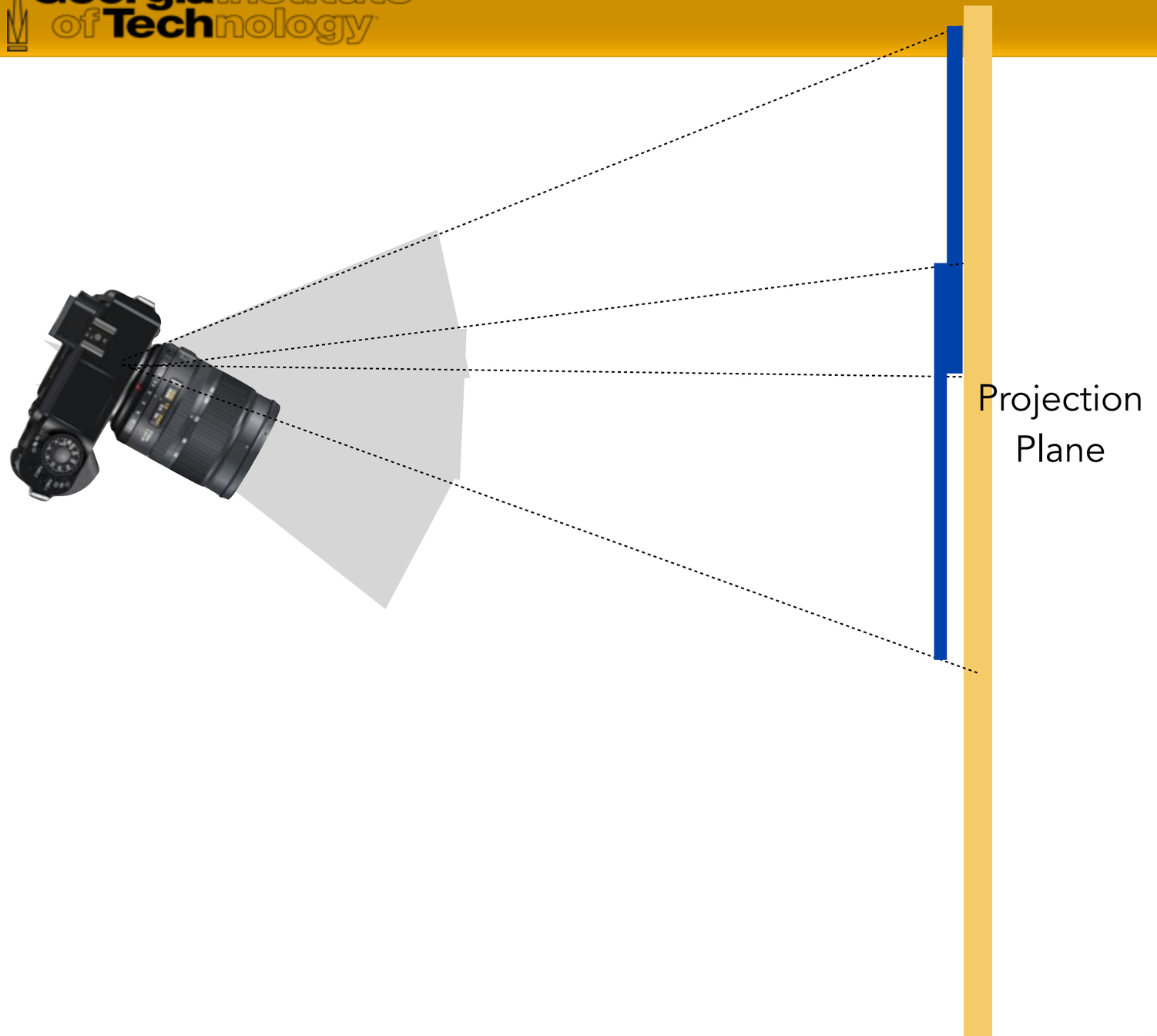

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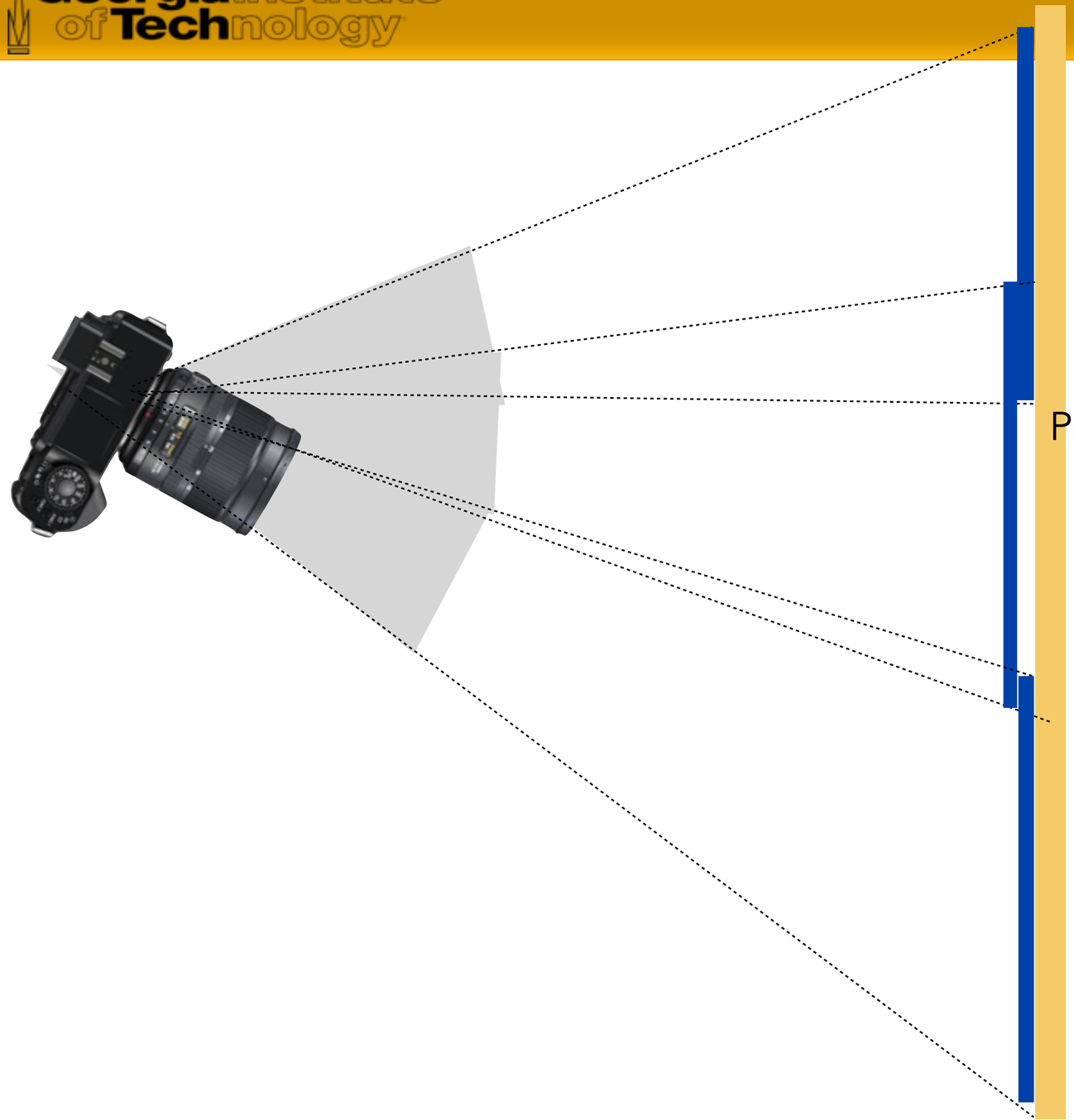

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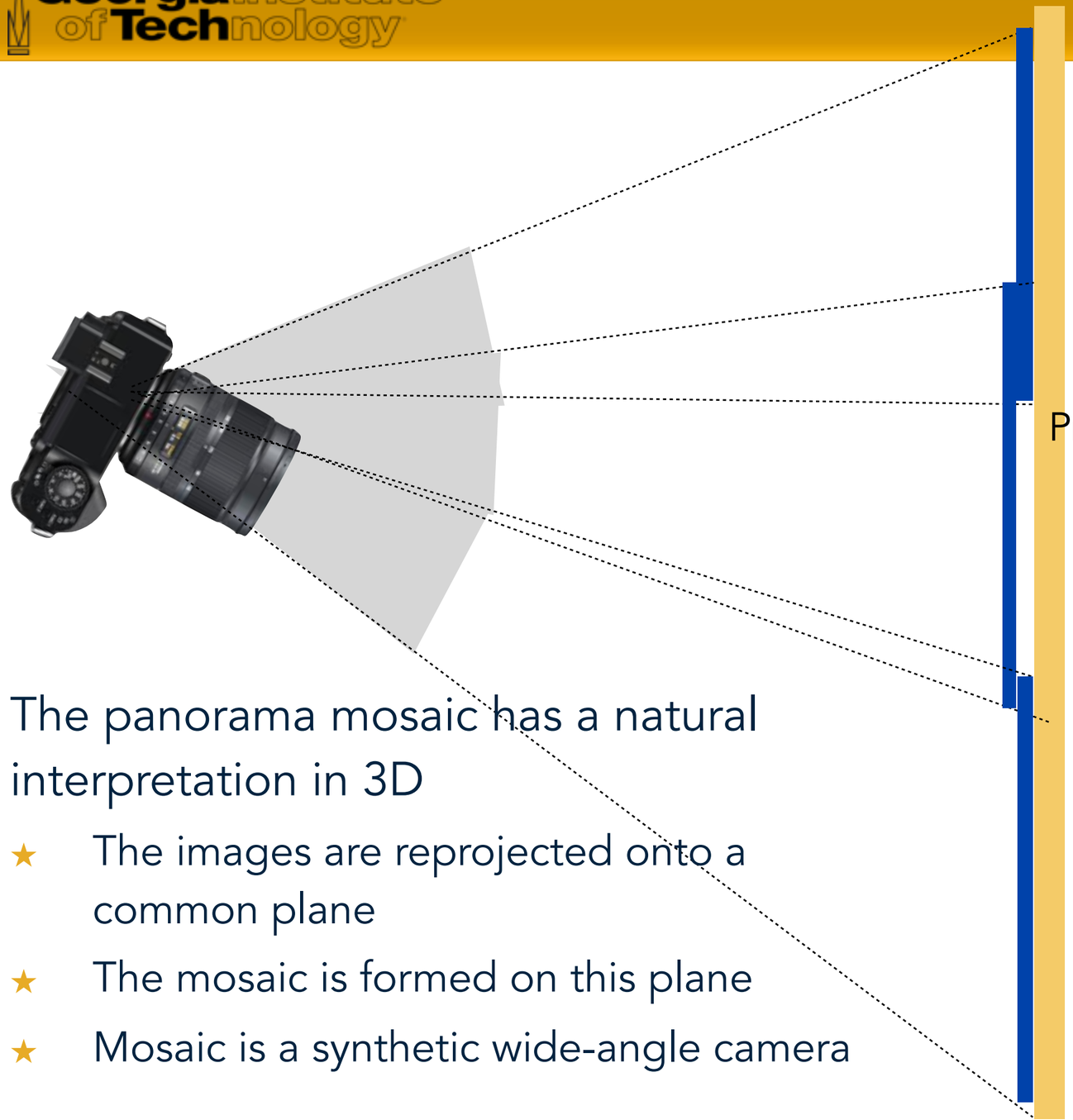

Image Re-projection to Panorama Projection Plane (PP)

- The panorama mosaic has a natural interpretation in 3D
  - The images are reprojected onto a common plane
  - The mosaic is formed on this plane
  - Mosaic is a synthetic wide-angle camera

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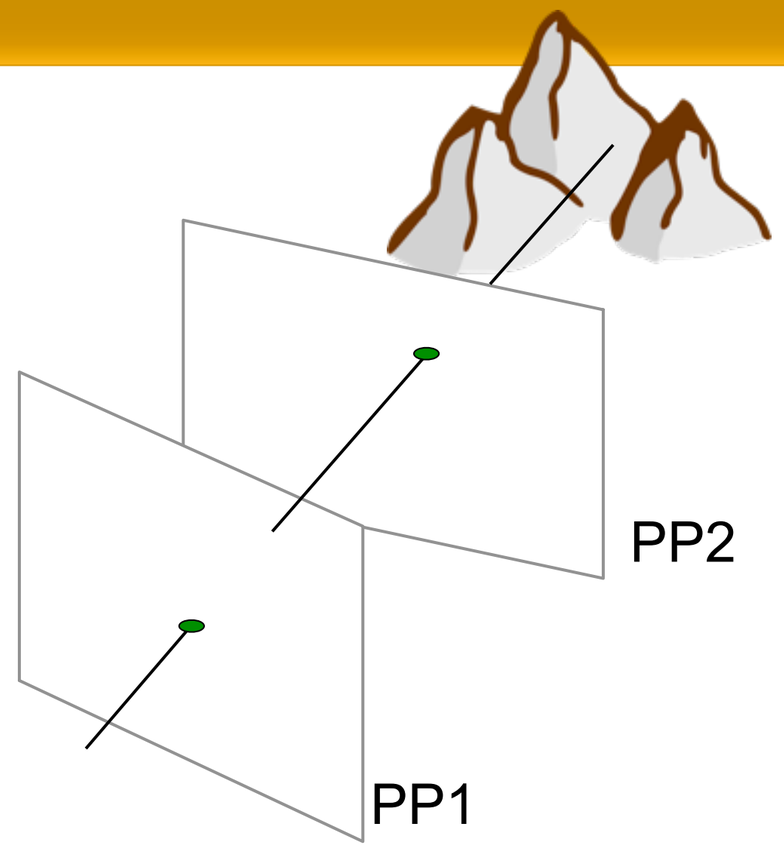
Image Re-Projection

- ★ Need to relate two images from the same camera center and map a pixel from PP1 to PP2
  - Cast a ray through each pixel in PP1
  - Draw the pixel where that ray intersects PP2
  - Rather than thinking of this as a 3D re-projection,
- ★ Think of it as a 2D image warp from one image to another
- ★ Do not need to know the geometry of the two planes with respect to the eye?



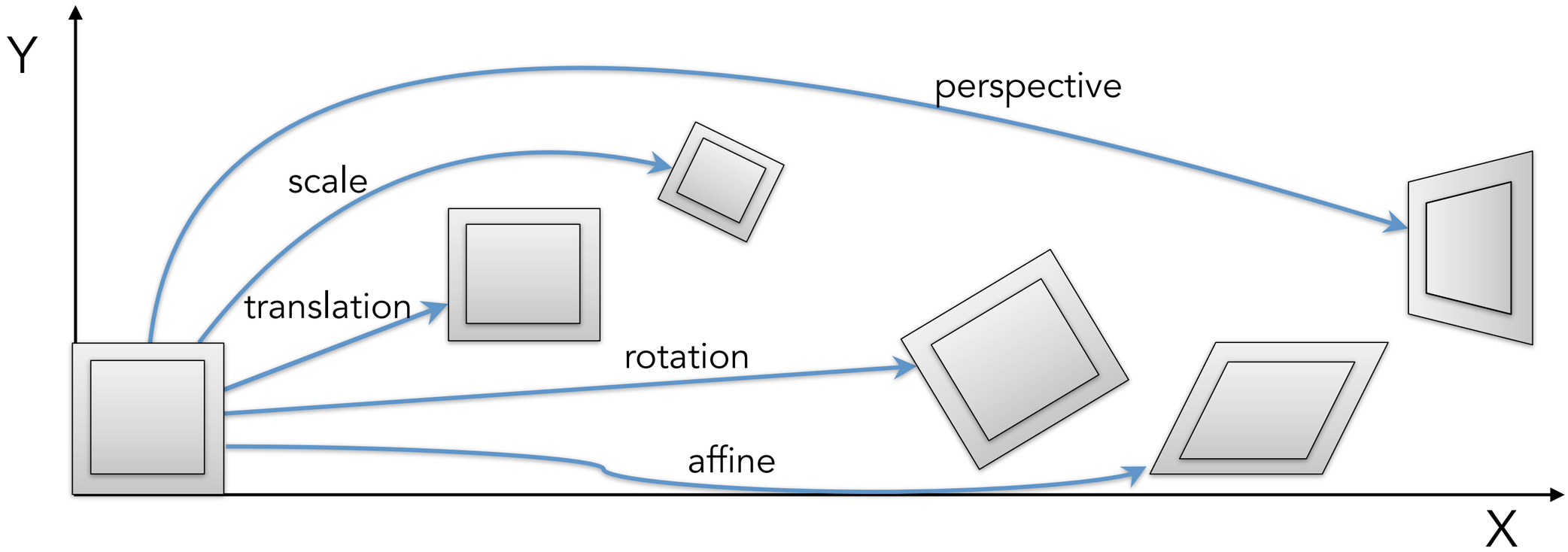
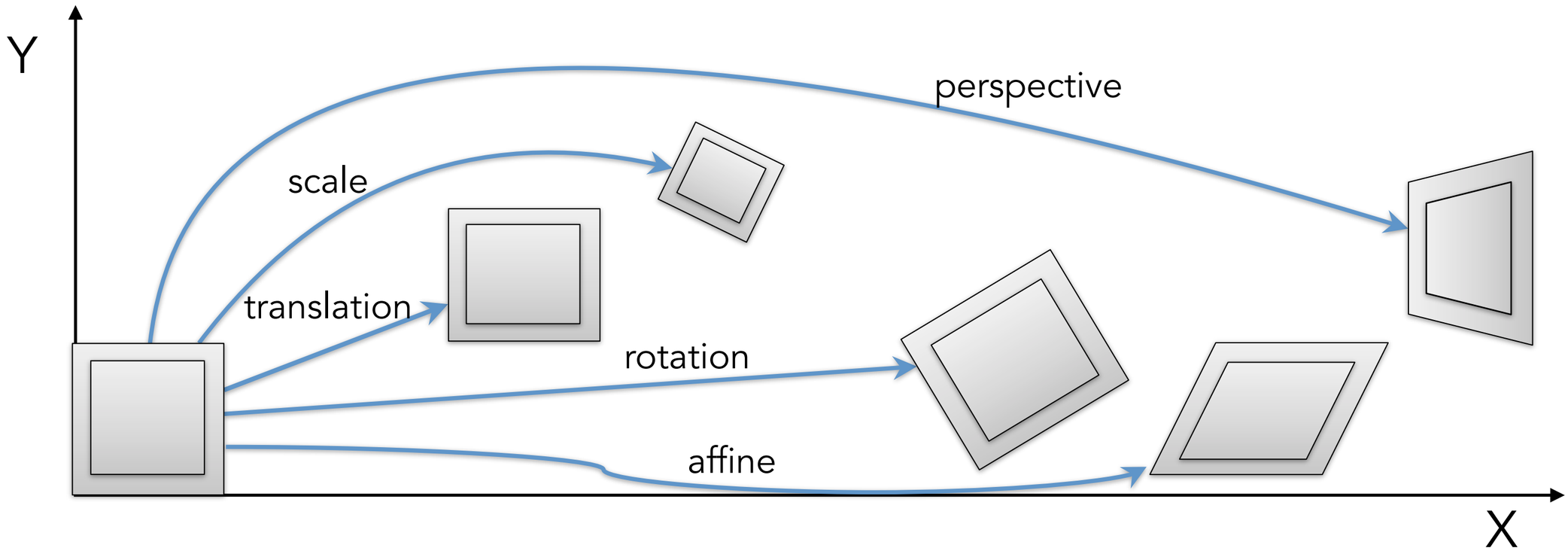
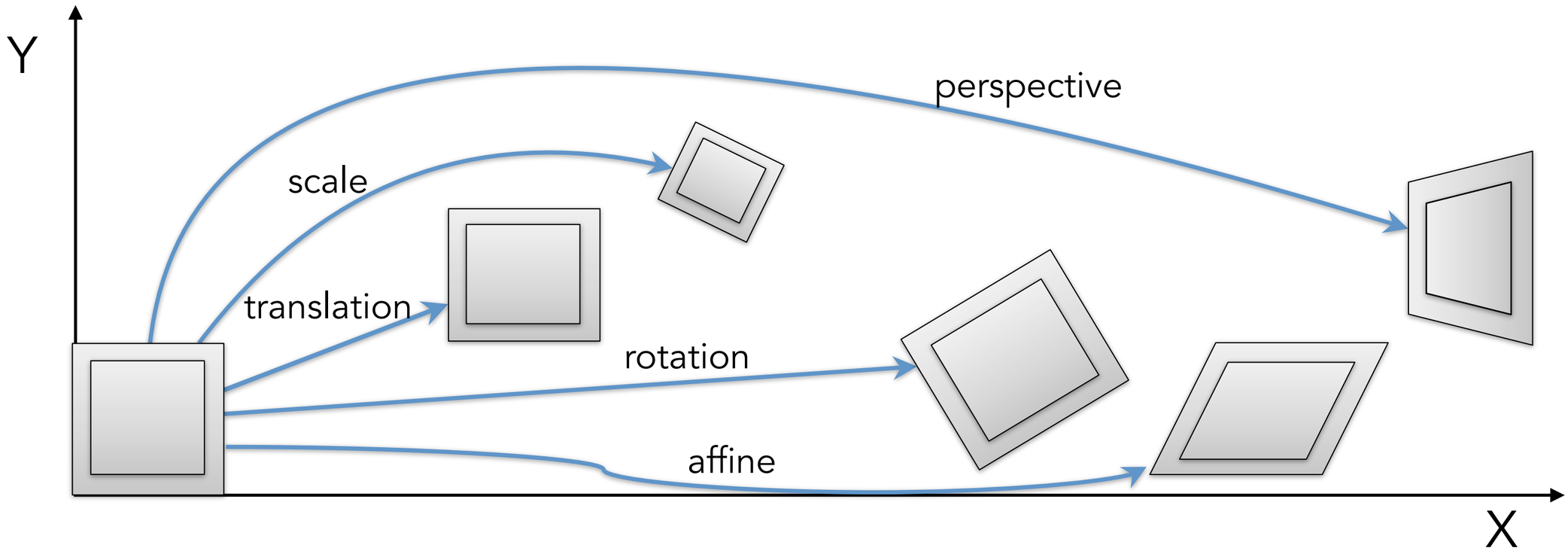


Image Warping



Which transform is the right one for warping PP1 into PP2?

Image Warping



Which transform is the right one for warping PP1 into PP2?

e.g. translation, Euclidean, affine, projective

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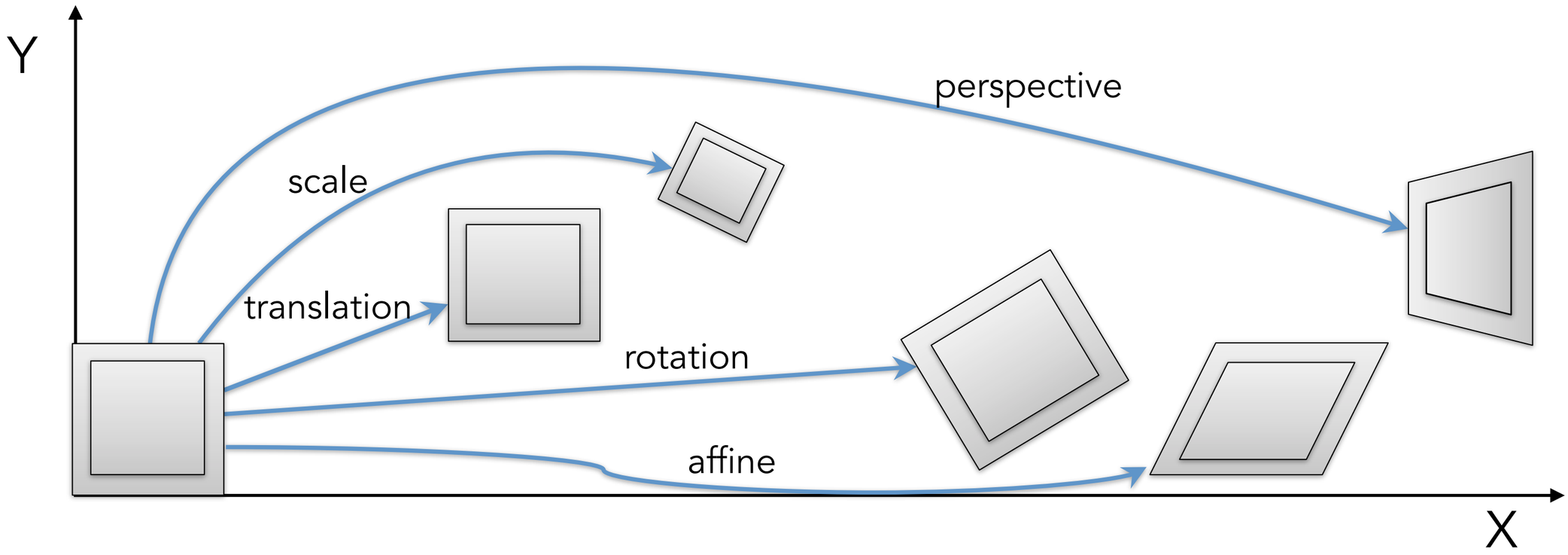

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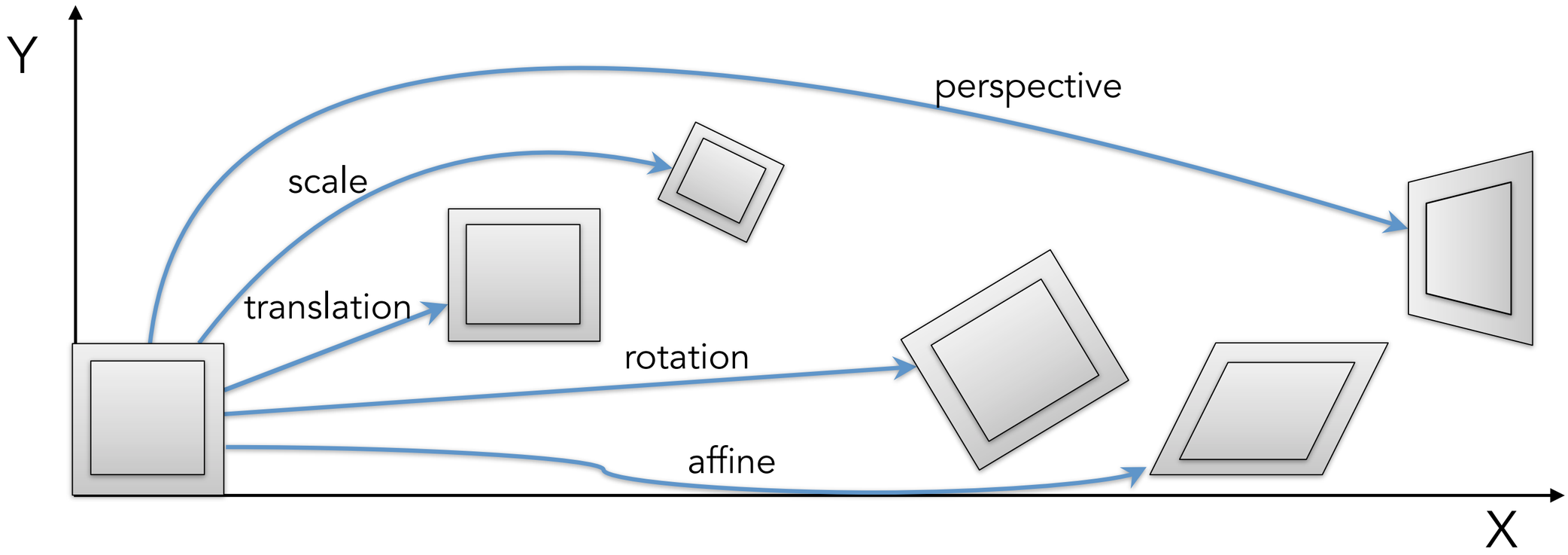
Translation: 2 unknowns

Image Warping



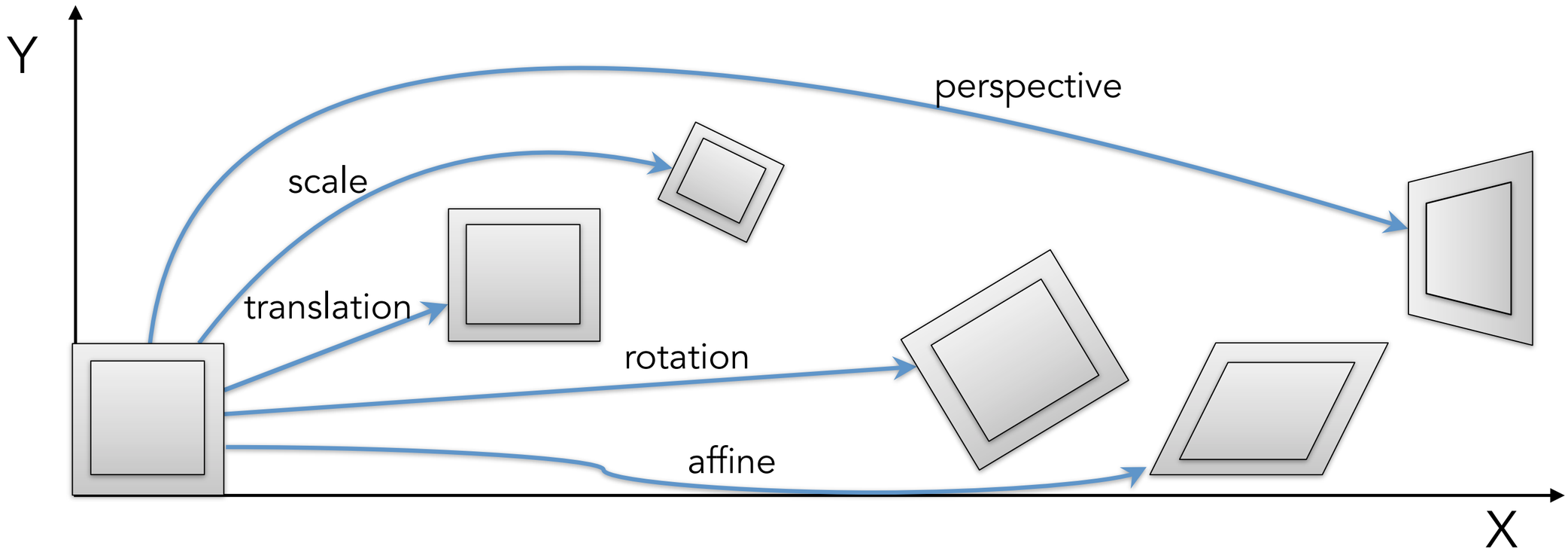
Which transform is the right one for warping PP1 into PP2?

e.g. translation, Euclidean, affine, projective

Translation: 2 unknowns

Affine: 6 unknowns

Image Warping



Which transform is the right one for warping PP1 into PP2?

e.g. translation, Euclidean, affine, projective

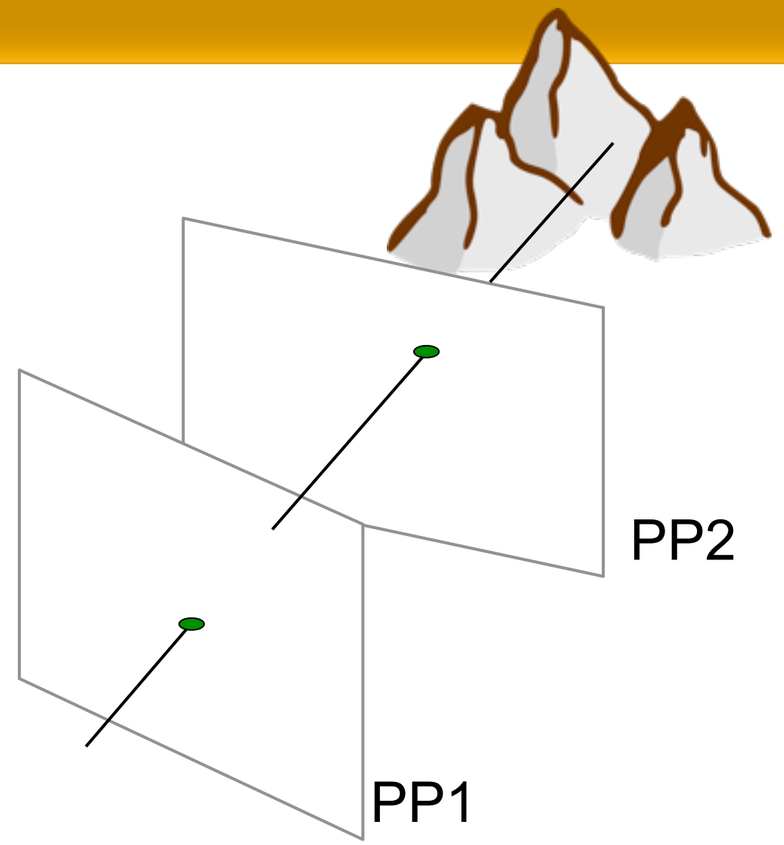
Translation: 2 unknowns

Affine: 6 unknowns

Projective: 8 unknowns

Image Warping

Introducing Homography



- Need to relate two images from the same camera center and map a pixel from PP1 to PP2
  - rectangle should map to arbitrary quadrilateral
  - parallel lines aren't parallel
  - straight lines must be straight
- Referred to as Homography

$$\begin{bmatrix} wx' \\ wy' \\ w \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \qquad p' = Hp$$

http://en.wikipedia.org/wiki/Homography

Computing Homography

$$\begin{bmatrix} wx' \\ wy' \\ w \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \qquad p' = Hp$$

11

Computing Homography

$$\begin{bmatrix} wx' \\ wy' \\ w \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \qquad p' = Hp$$

11

Zoomed

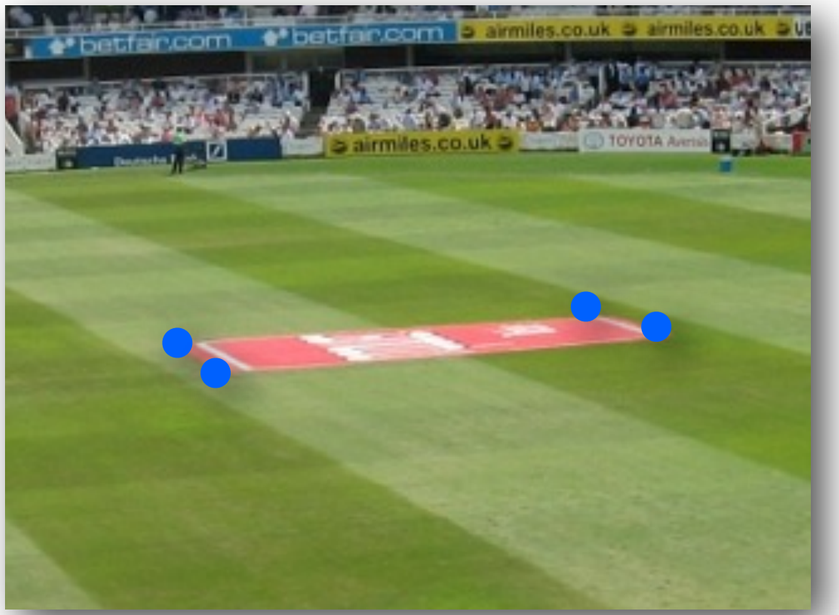
Zoomed and Left Pan

Computing Homography

$$\begin{bmatrix} wx' \\ wy' \\ w \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \qquad p' = Hp$$

11

Zoomed



Zoomed and Left Pan

Computing Homography

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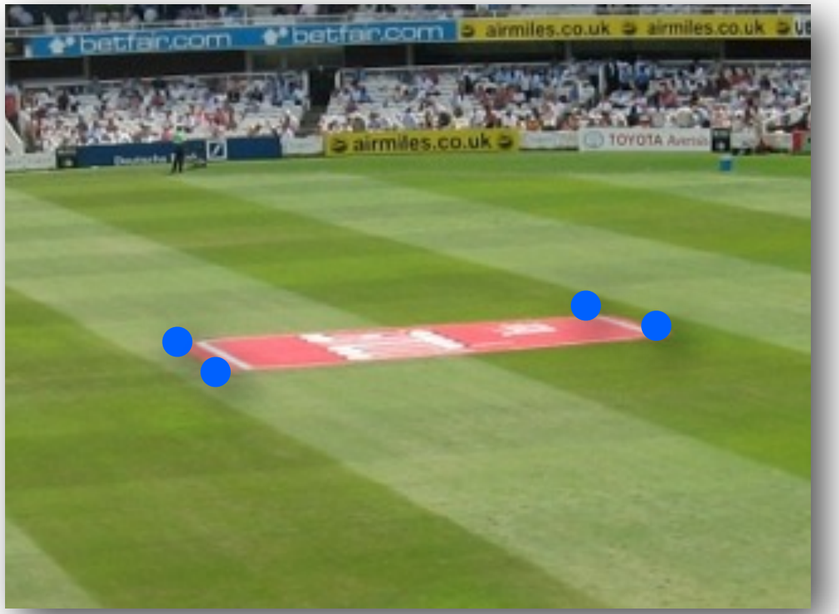
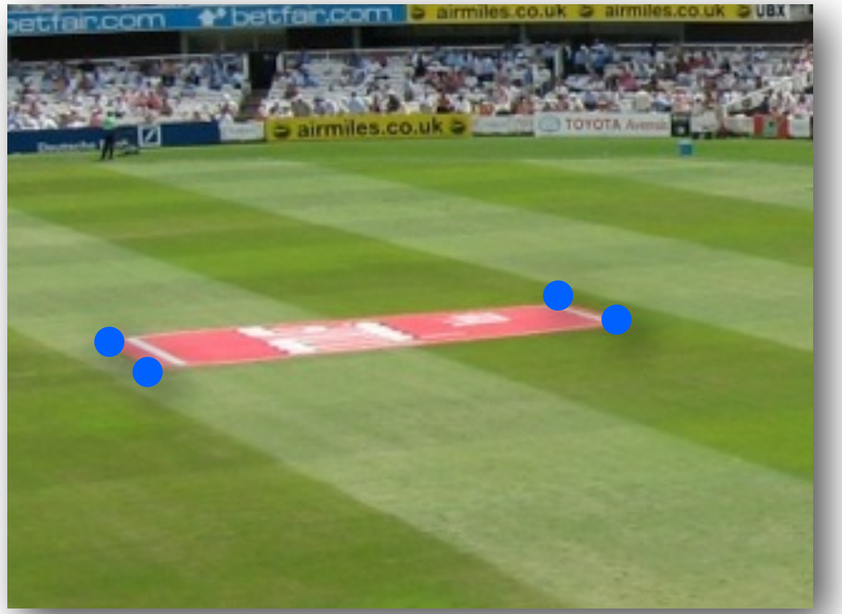
11

Computing Homography

Zoomed



Zoomed and Left Pan



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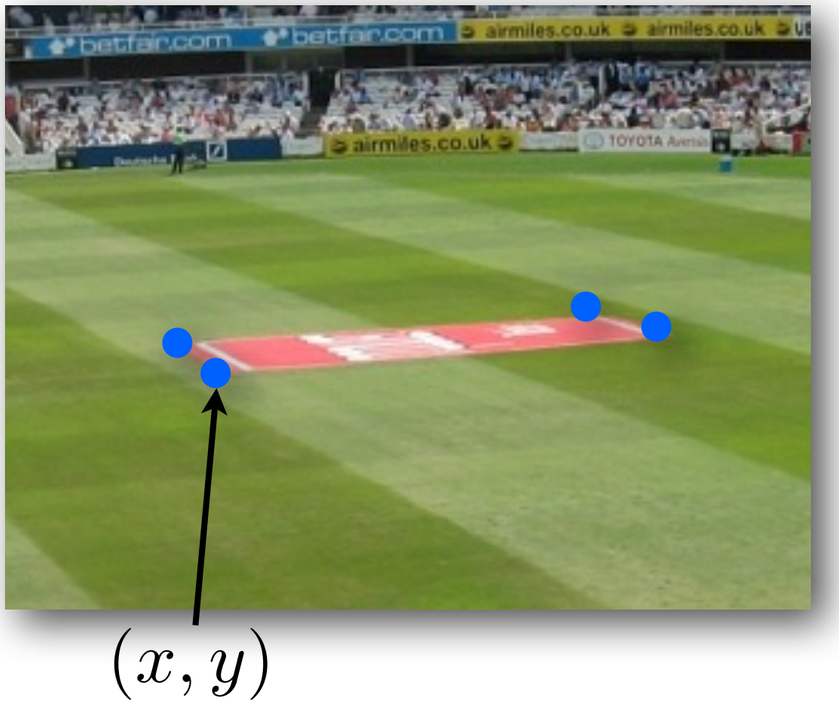
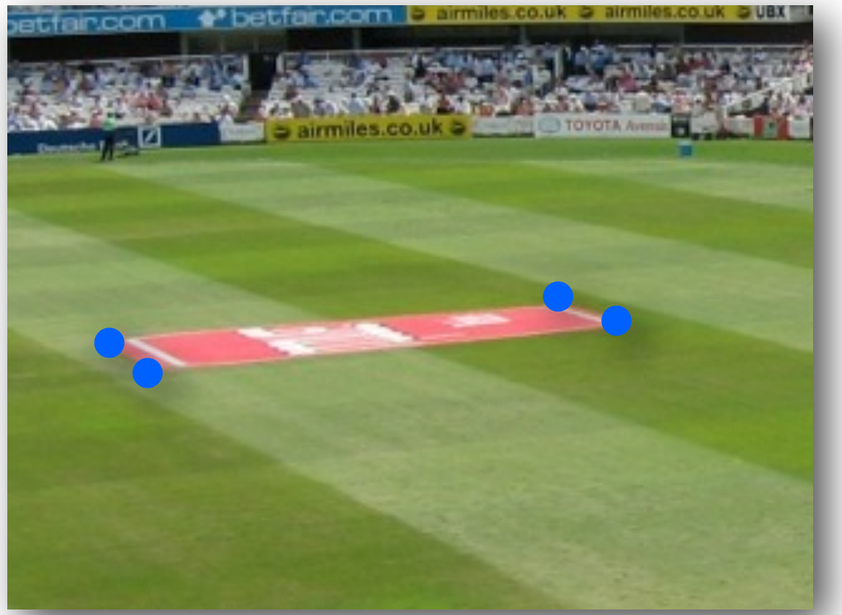
Computing Homography

Zoomed



Zoomed and Left Pan



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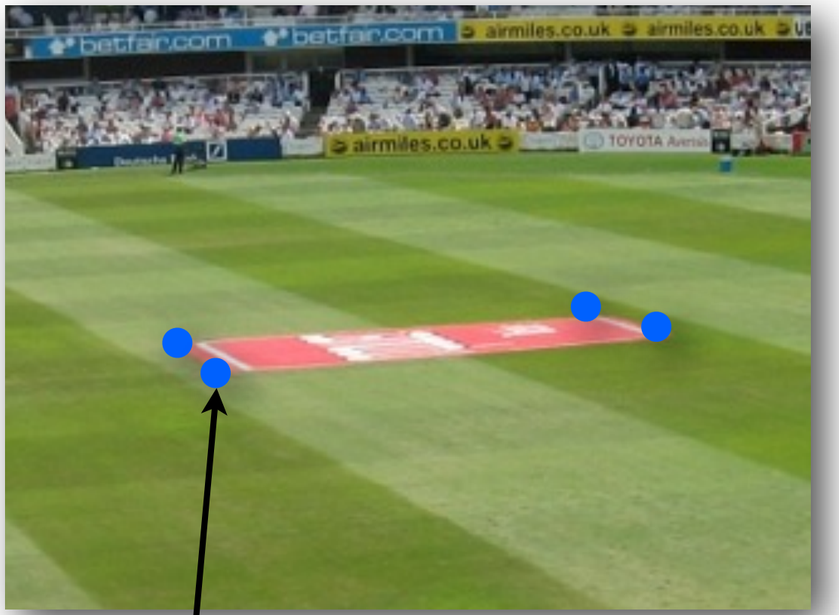
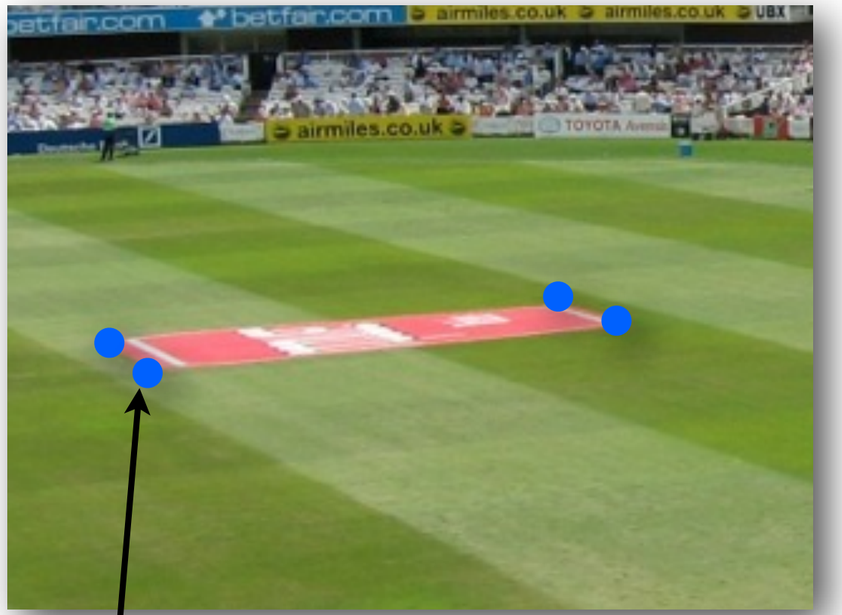
Computing Homography

Zoomed



(x, y)

Zoomed and Left Pan



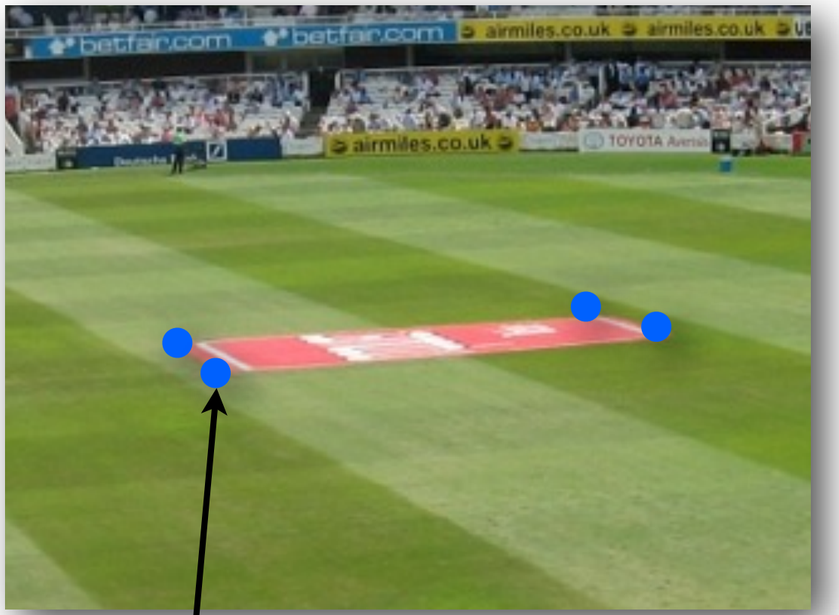
$(wx^{'}/w, wy^{'}/w) = (x^{'}, y^{'})$

$$\begin{bmatrix} wx^{'} \\ wy^{'} \\ w \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \qquad p^{'} = Hp$$

11

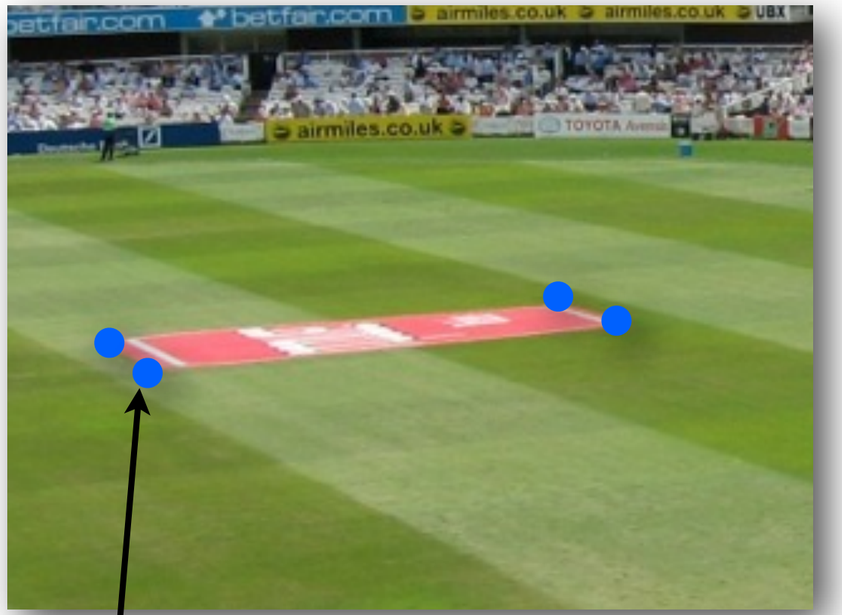
Computing Homography

Zoomed



(x, y)

Zoomed and Left Pan



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$p_1, p_2, \ldots, p_n$

$$\begin{bmatrix} wx^{'} \\ wy^{'} \\ w \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \qquad p^{'} = Hp$$

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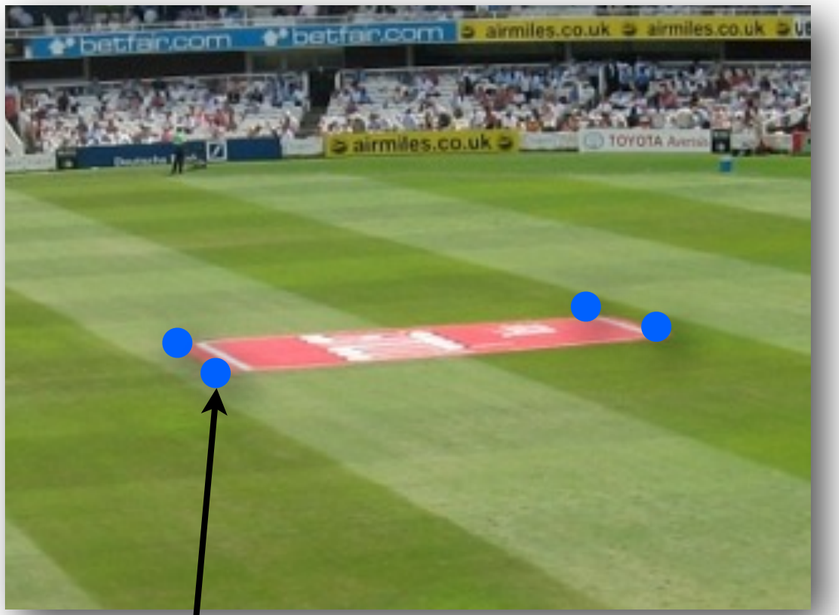
Computing Homography

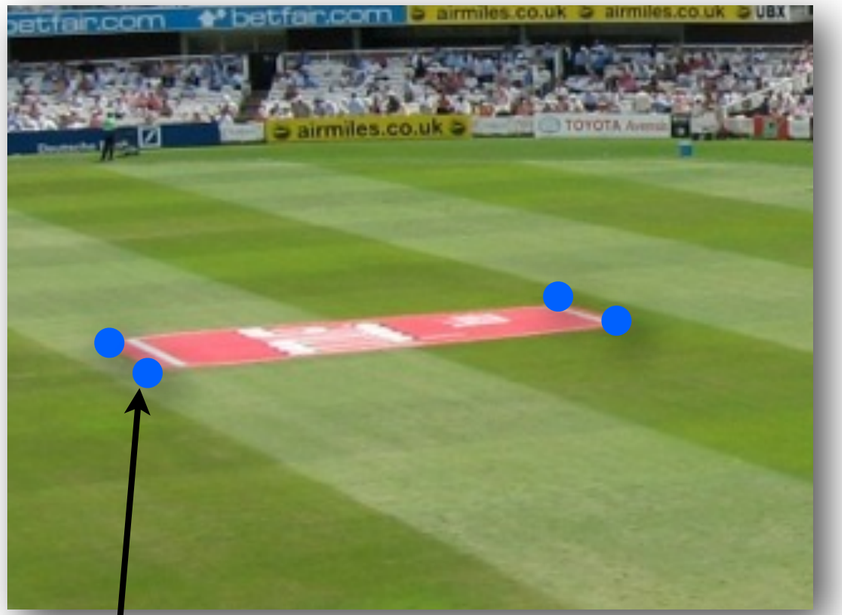
Zoomed



(x, y)

$p_1, p_2, \ldots, p_n$

Zoomed and Left Pan



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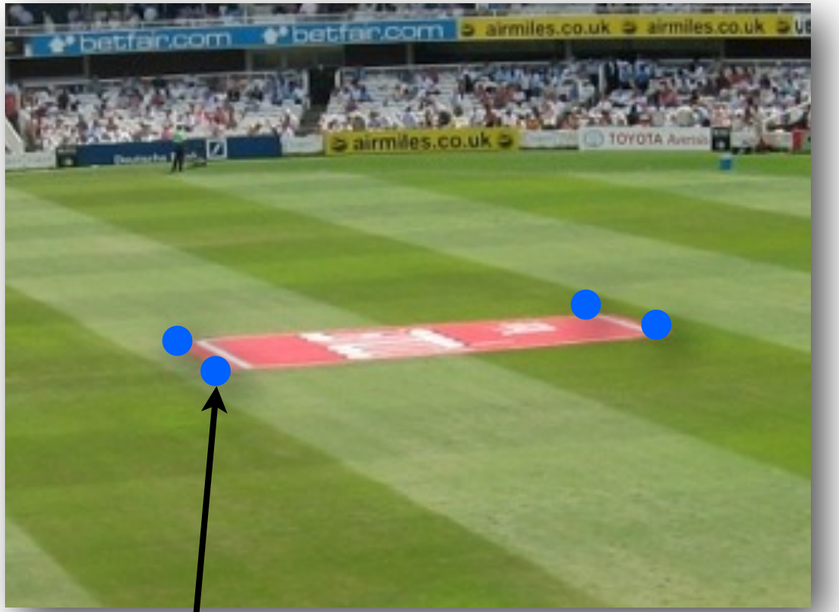
$p_1^{'}, p_2^{'}, \ldots, p_n^{'}$

$$\begin{bmatrix} wx^{'} \\ wy^{'} \\ w \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \qquad p^{'} = Hp$$

11

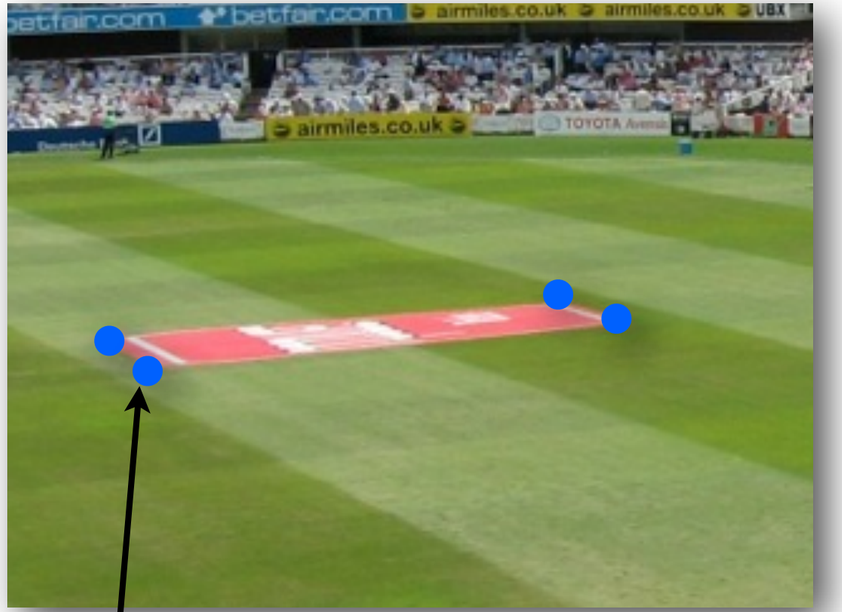
Zoomed



(x, y)

$p_1, p_2, \ldots, p_n$

Zoomed and Left Pan



$(wx^{'}/w, wy^{'}/w) = (x^{'}, y^{'})$

$p_1, p_2, \ldots, p_n$

$$\begin{bmatrix} wx^{'} \\ wy^{'} \\ w \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \qquad p^{'} = Hp$$

Computing Homography

To **compute** the homography given pairs of corresponding points in the images, we need to set up an equation where the parameters of **H** are the unknowns…

$$\begin{bmatrix} wx' \\ wy' \\ w \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \qquad p' = Hp$$

Solving for Homographies

- ★ Set scale factor *i*=1. So, there are 8 unknowns.
- ★ Set up a system of linear equations:
  - $Ah = b$
  - where vector of unknowns h = [a,b,c,d,e,f,g,h]^T
- ★ Need at least 8 equations, but the more the better…
- ★ Solve for h. If over-constrained (ie. more data), solve using least-squares:

$$\min \|Ah - b\|^2$$

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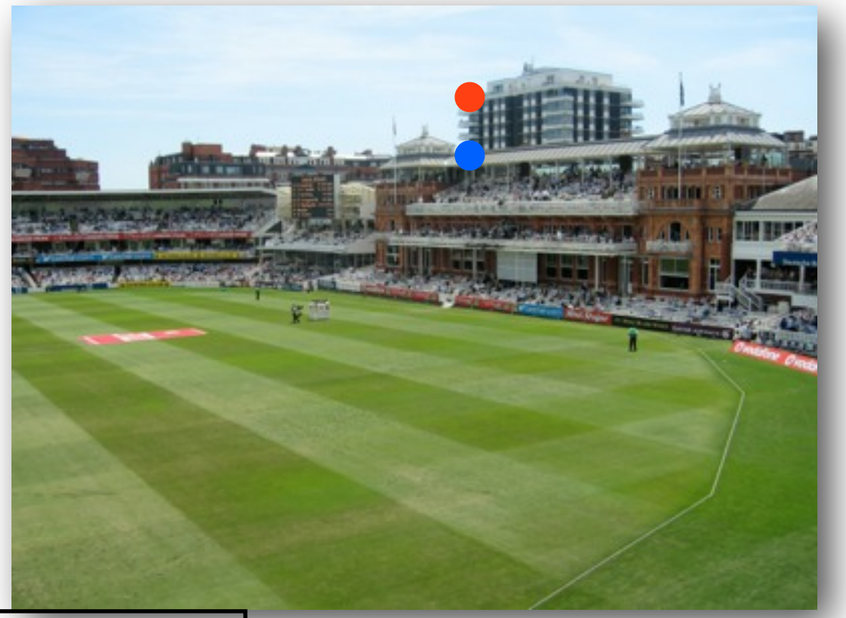
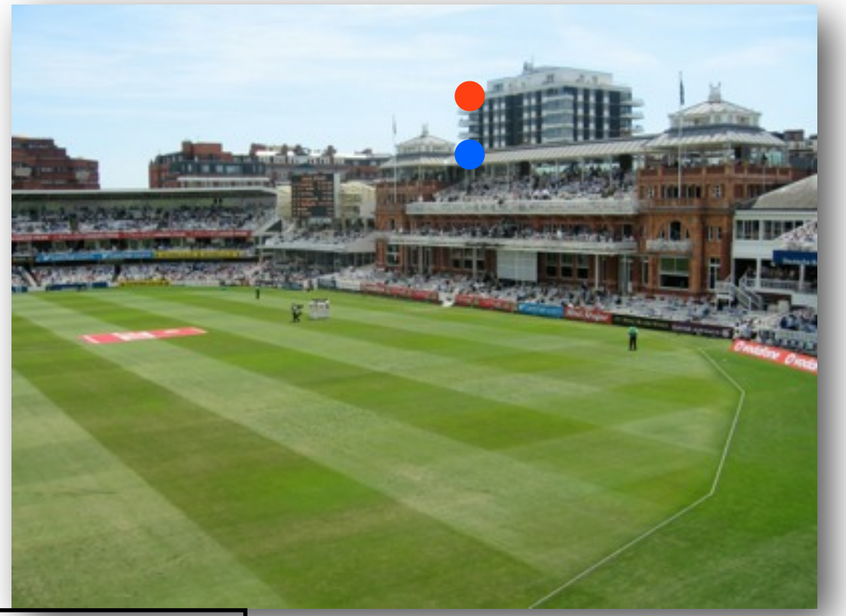
Warp into a shared Image Coordinates

Warp into a shared Image Coordinates

Warp into a shared Image Coordinates



Warp into a shared Image Coordinates



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Warp into a shared Image Coordinates

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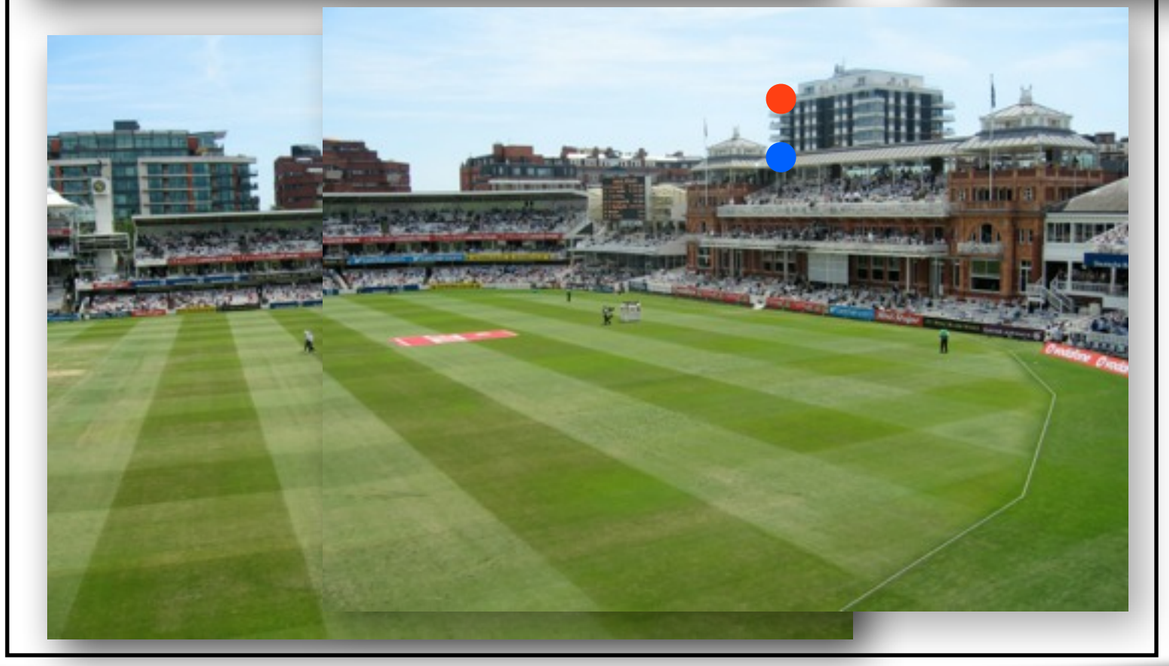


Warping and Interpolation

Warping and Interpolation

Warping and Interpolation

Warping and Interpolation

Warping and Interpolation

Warping and Interpolation

14

L 

R

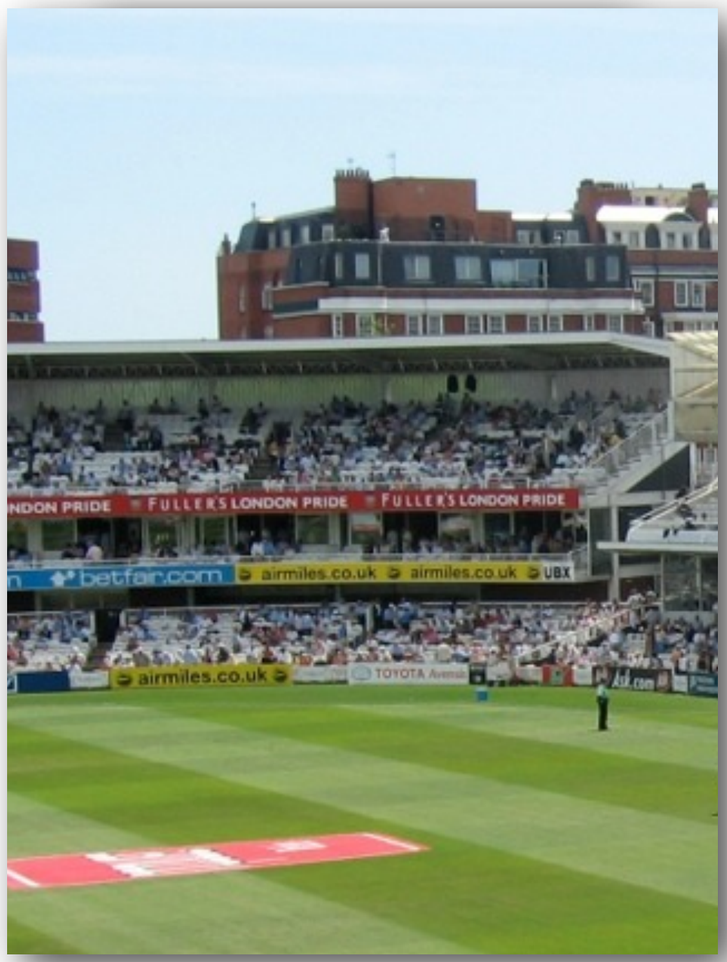
Dealing with BAD Matches

L

R

Dealing with BAD Matches

L



R

Dealing with BAD Matches

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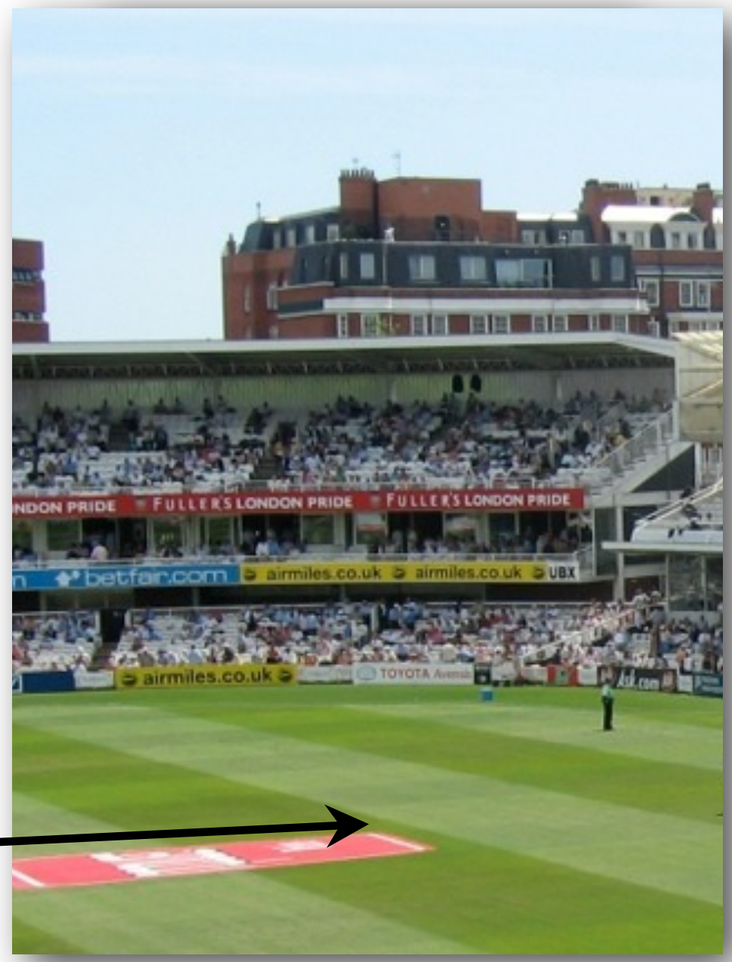

Dealing with BAD Matches

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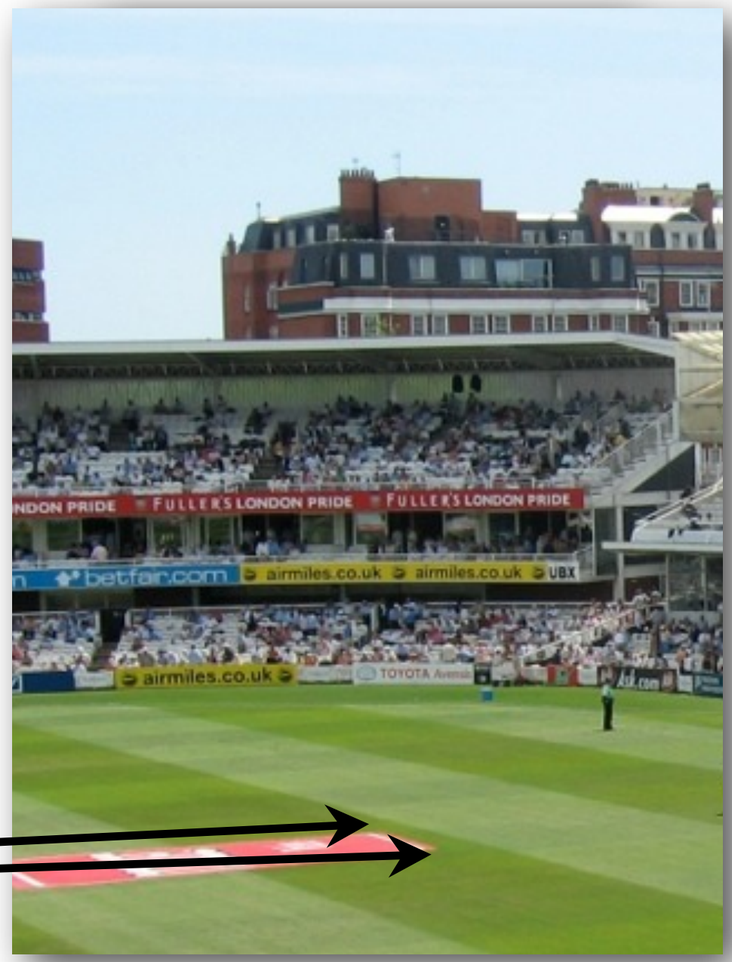

Dealing with BAD Matches

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Dealing with BAD Matches

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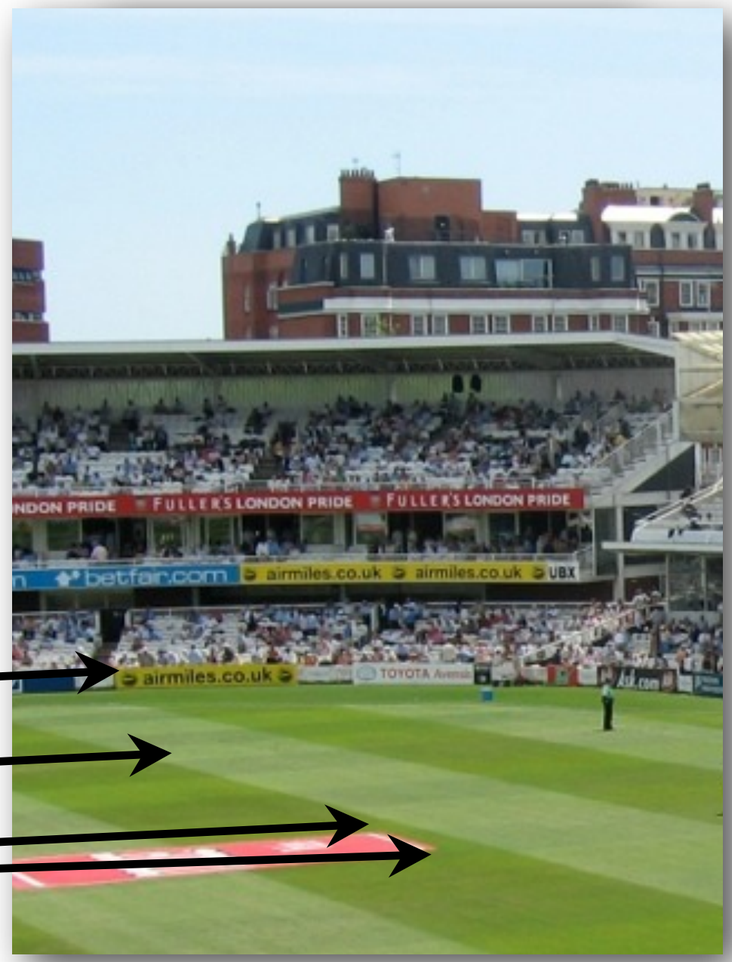

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Dealing with BAD Matches

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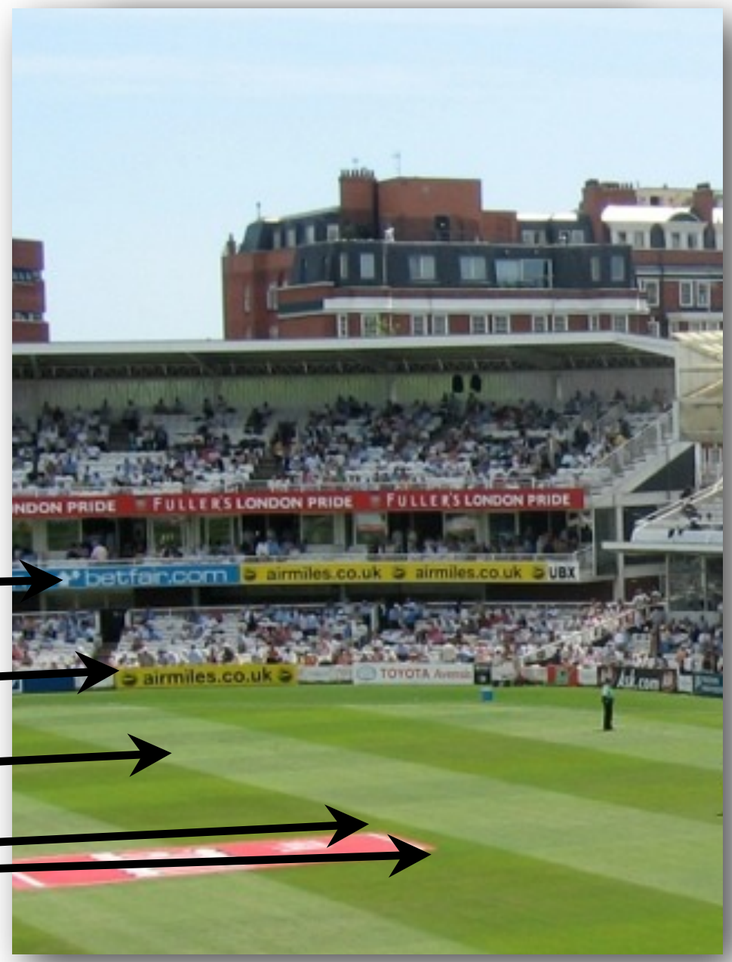

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Dealing with BAD Matches

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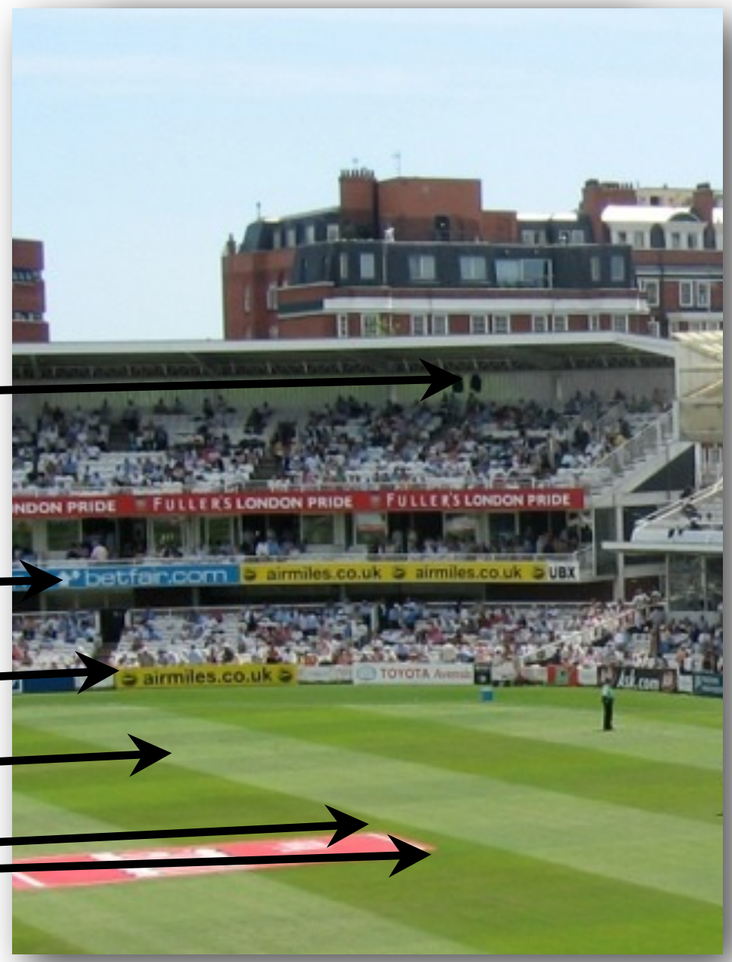

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Dealing with BAD Matches

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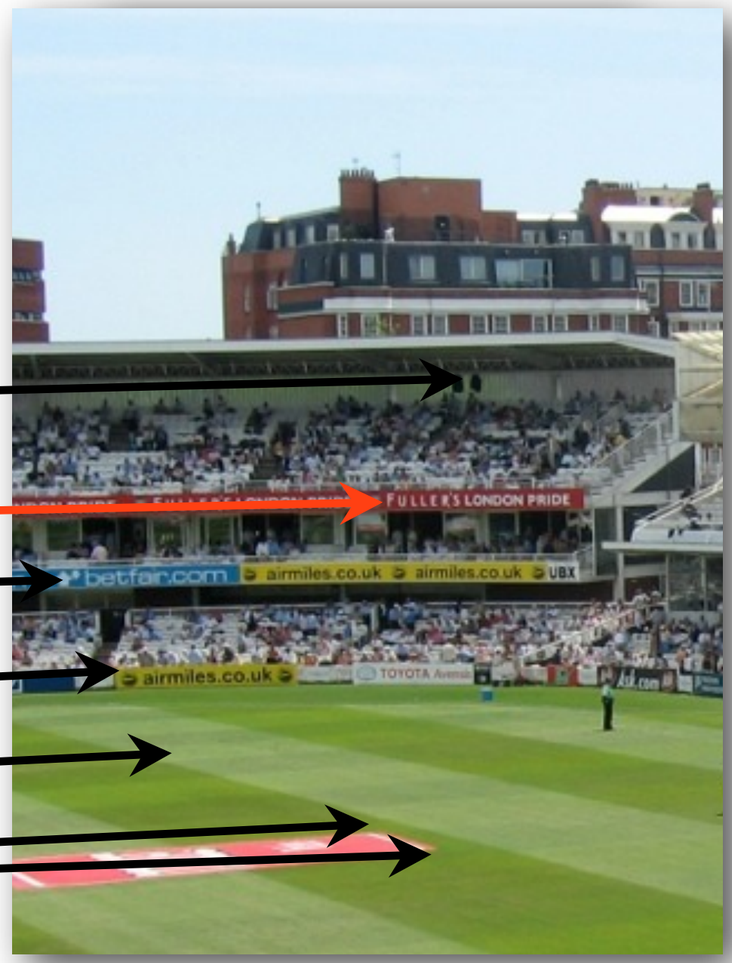

Dealing with BAD Matches

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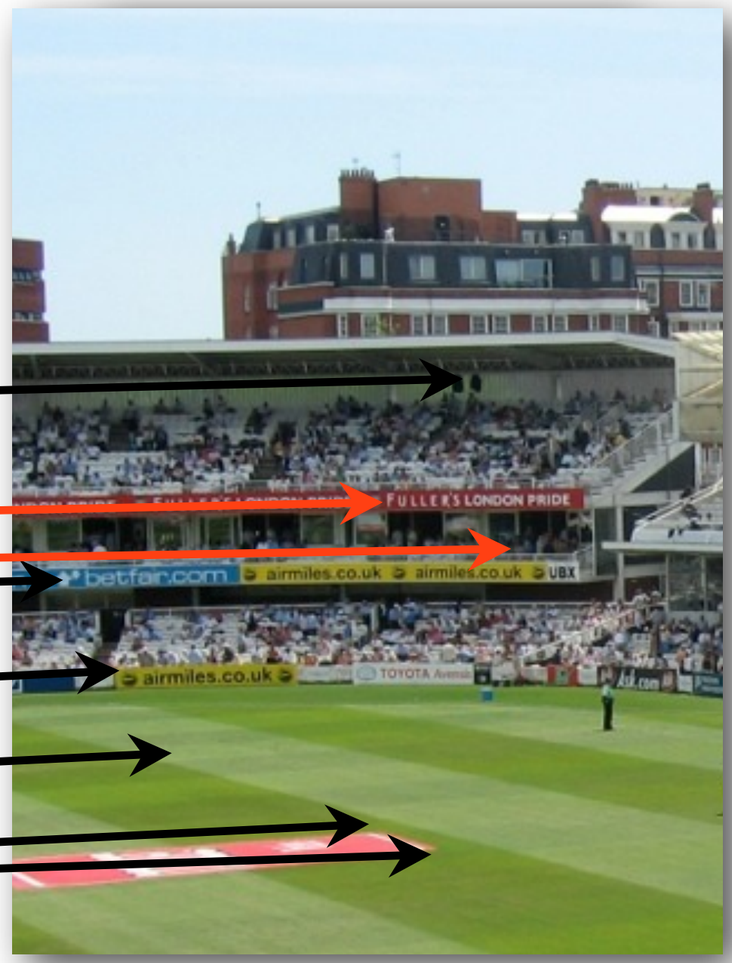

Dealing with BAD Matches

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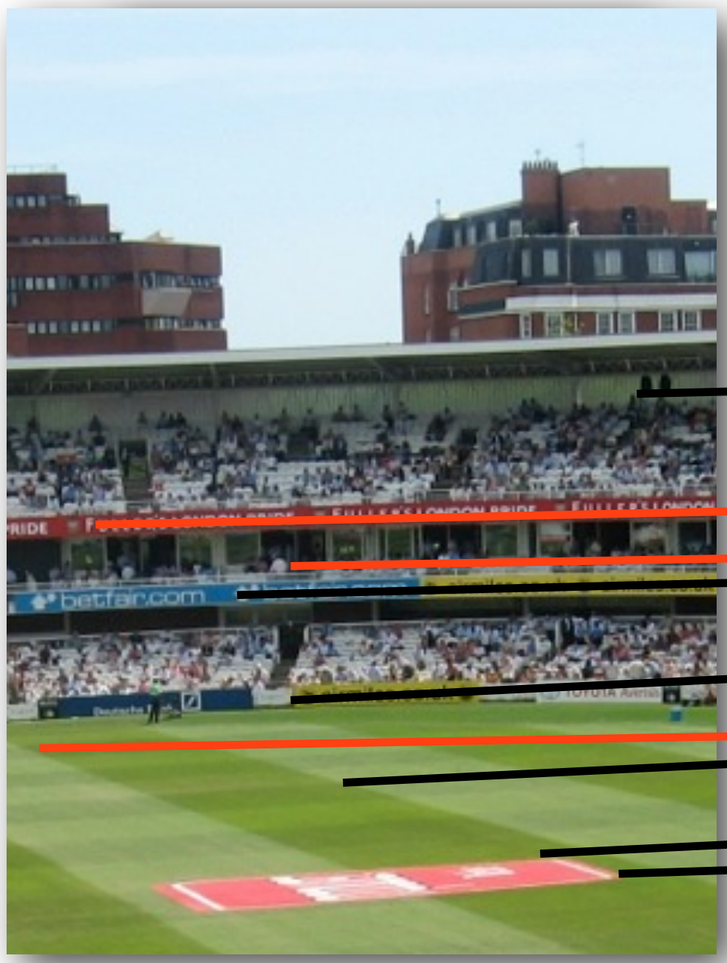

R

Dealing with BAD Matches

L

R

RANdom SAmple Consensus (RANSAC)

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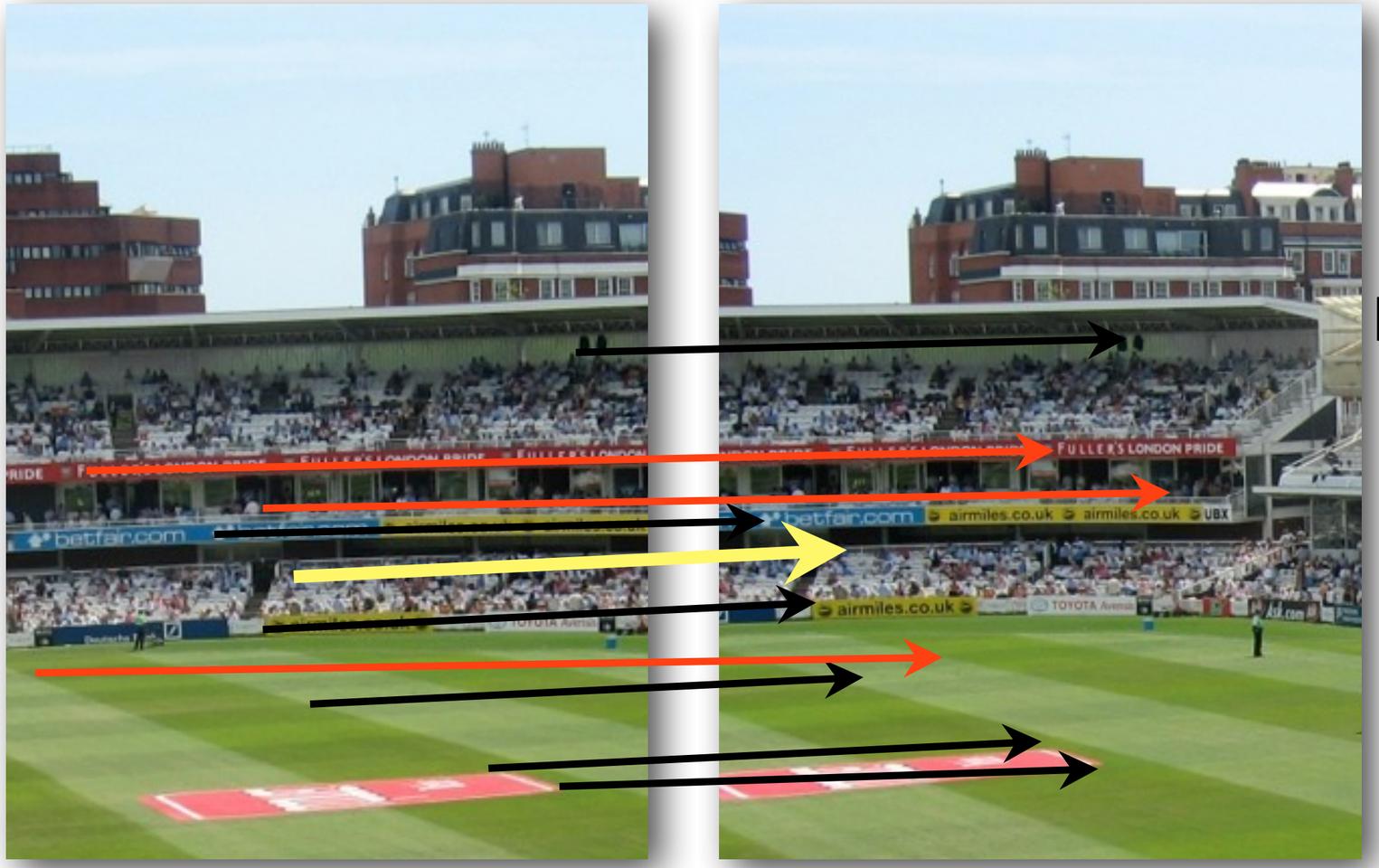
L

R

- Select ONE match, Count INLIERS
- Find “average” translation vector

RANdom SAmple Consensus (RANSAC)

L



R

- ★ Select ONE match, Count INLIERS
- ★ Find “average” translation vector

RANdom SAmple Consensus (RANSAC)

- RANSAC
  - LOOP
    - 1. Select four feature pairs (at random)
    - 2. Compute homography H (exact)
    - 3. Compute inliers where $SSD(p_i', H\,p_i) < \varepsilon$
      - SSD: Sum of Squared Differences
  - 4. Keep largest set of inliers
  - 5. Re-compute least-squares *H* estimate on all of the inliers
- The key idea is not that there are more inliers than outliers, but that the outliers are wrong in different ways.

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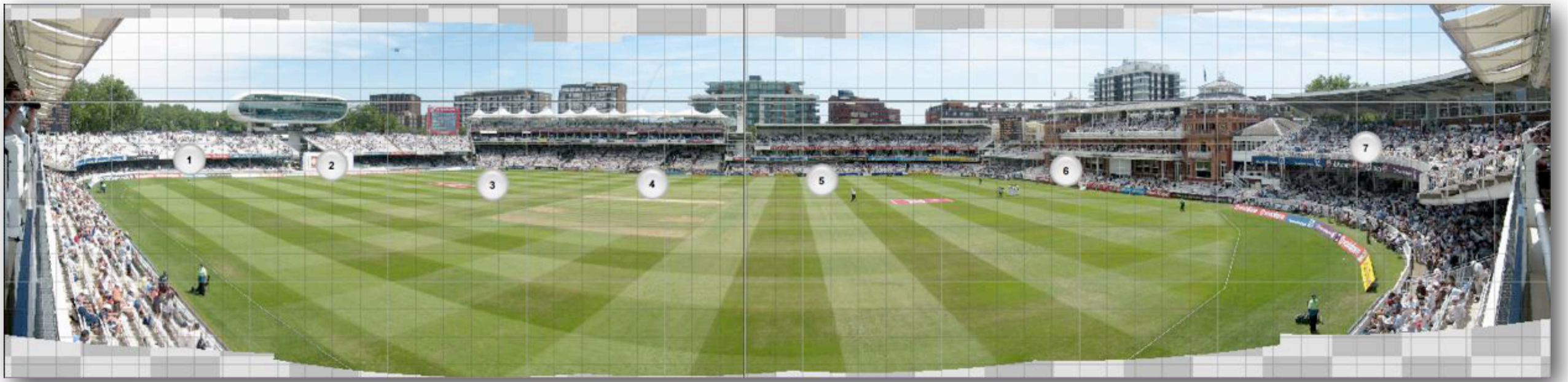
Using kolor autopano giga™ v3

The Final Result



Using kolor autopano giga™ v3

The Final Result

Image Re-projection to not Just a Plane

Projection Plane

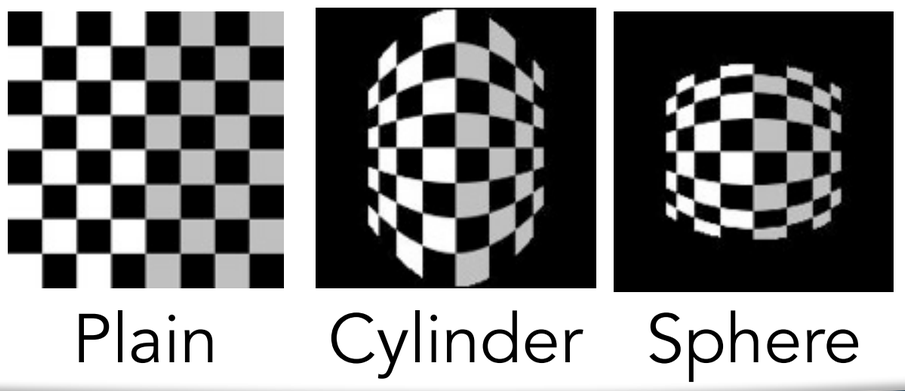
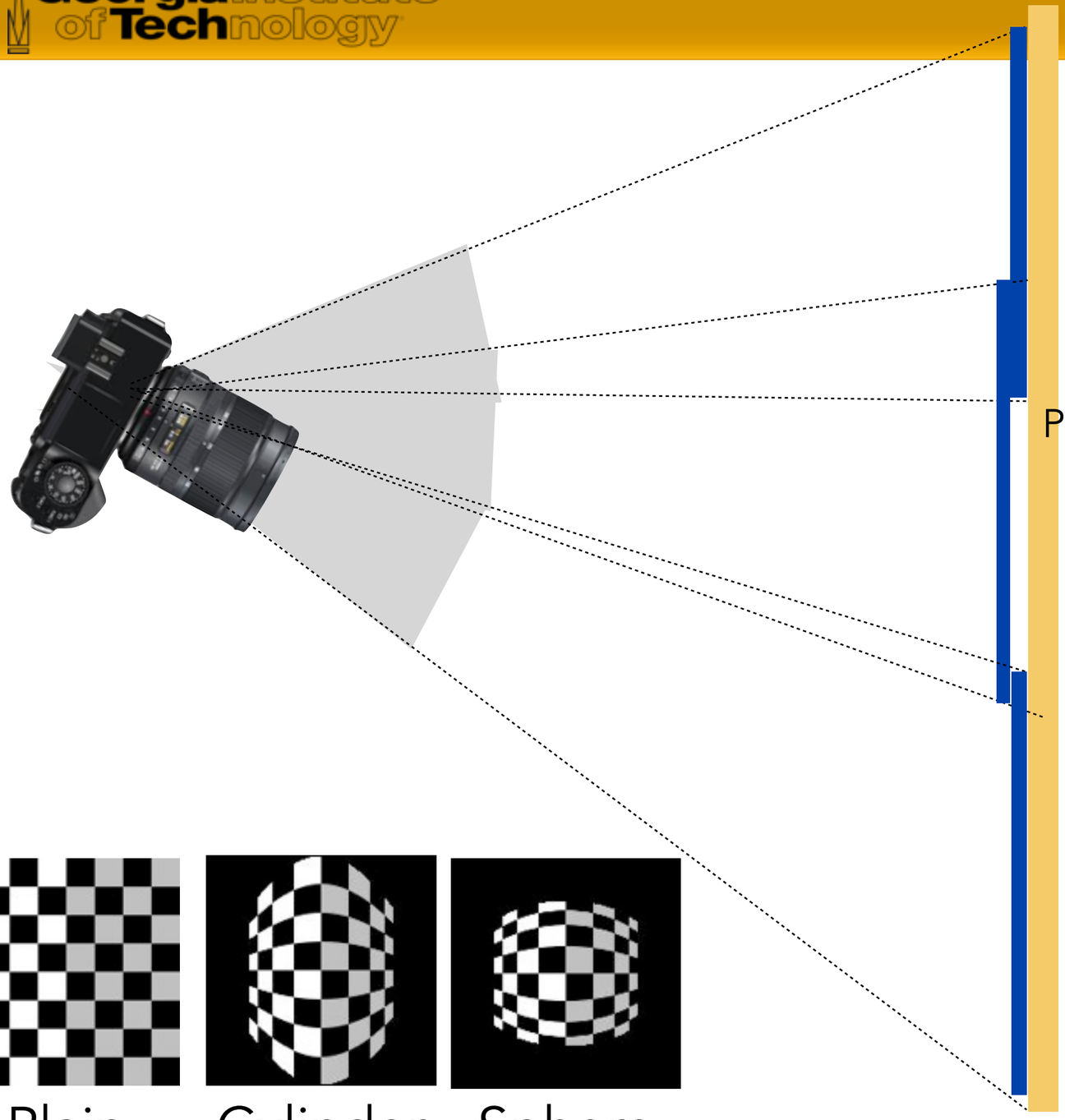


Image Re-projection to not Just a Plane



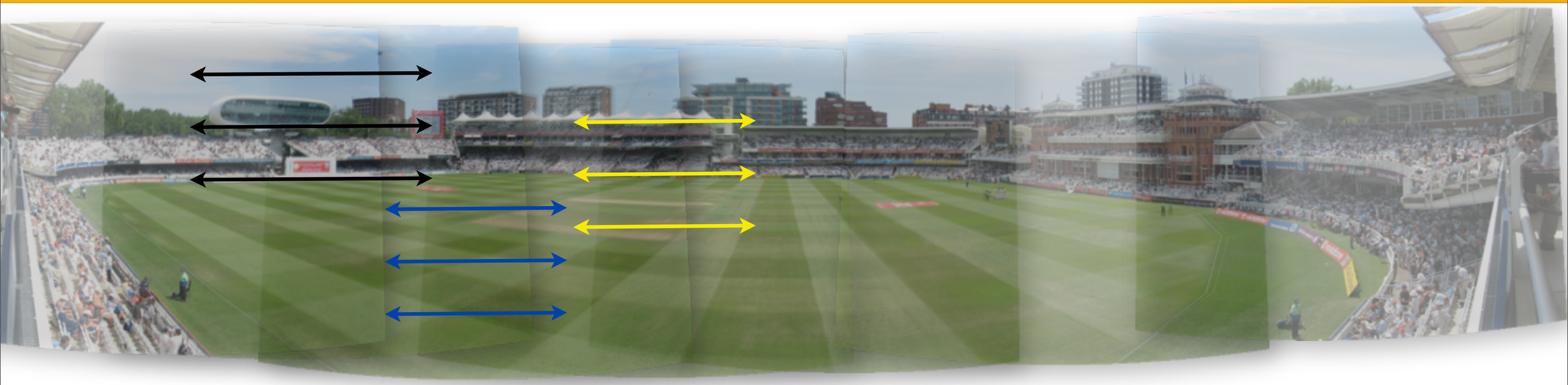
Planar, Spherical, Cylindrical, Panoramas

“Finding Panoramas”

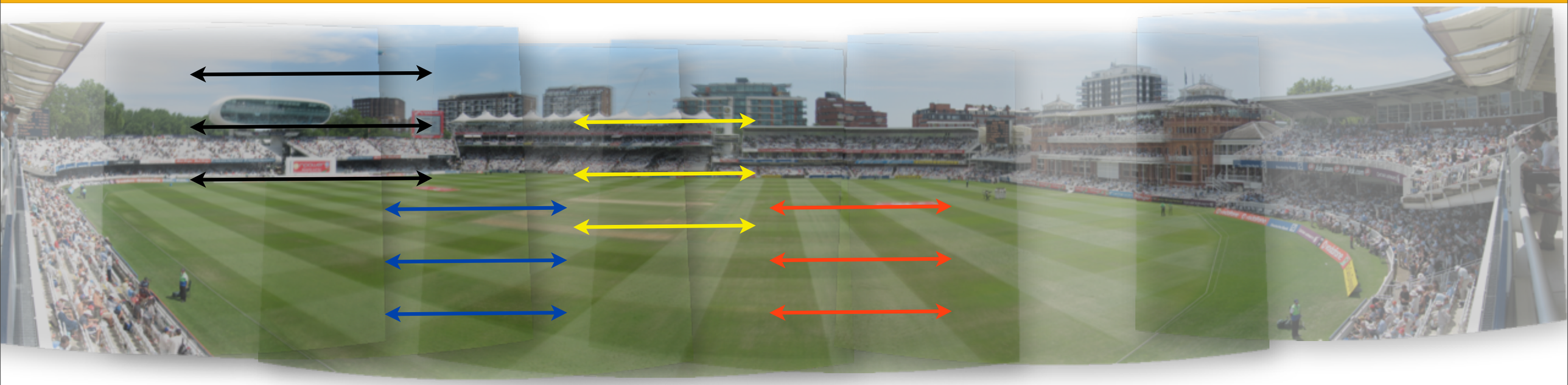
"Finding Panoramas"

"Finding Panoramas"

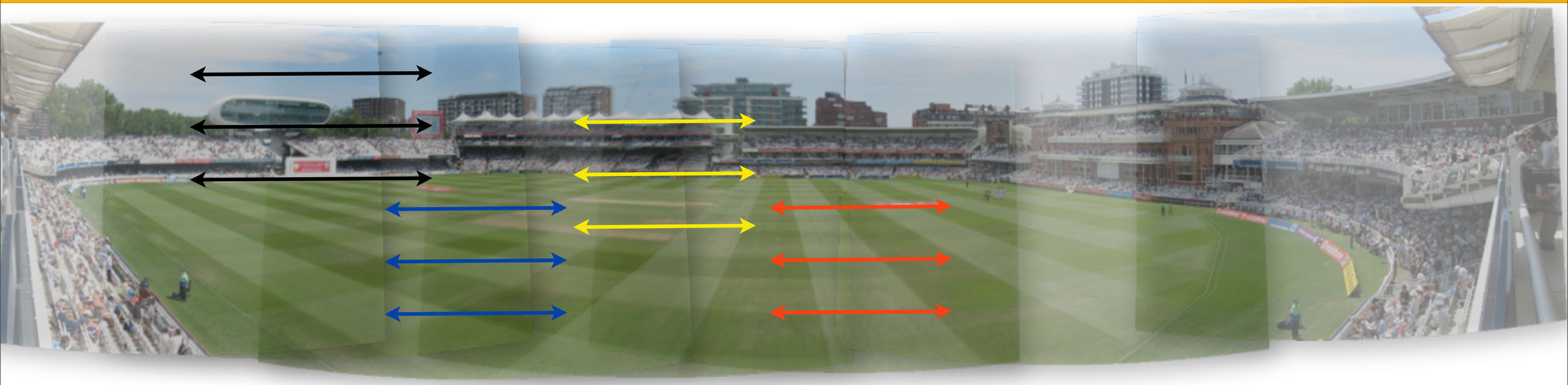
"Finding Panoramas"



"Finding Panoramas"

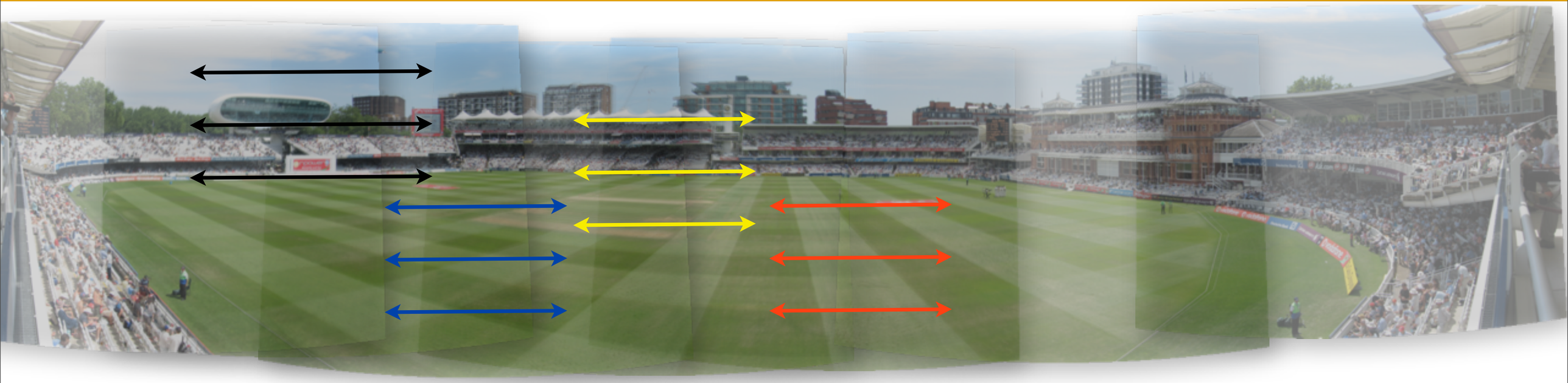


"Finding Panoramas"



Using RANSAC and related matching techniques, we can find images next to each other that form a panorama. So we don't have to take pictures in a sequence.

"Finding Panoramas"



Using RANSAC and related matching techniques, we can find images next to each other that form a panorama. So we don't have to take pictures in a sequence.

"Finding Panoramas"

Brown and Lowe (2003).

Summary

- ★ Provided details on one of the five (5) steps needed to generate a Panorama
- ★ Introduced the concept of Image Re-projection needed for Panoramas
- ★ Introduced the concept of Homography and how it is computed from a pair of images.
- ★ Introduced RANSAC as an approach to deal with Bad Matches across images.
- ★ Discussed additional considerations for Panoramas

Further Information

- ★ Brown and Lowe (2003). “Recognising Panoramas.” International Conference on Computer Vision (ICCV2003) (pdf | bib | ppt)
- ★ Microsoft Research Image Composite Editior (ICE)
- ★ Panorama Tools Graphical User Interface (PTGui)
- ★ Hugin Panorama Photo Stitcher

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Next Class

- ★ High-Dynamic Range Imaging.

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Credits

- ★ Softwares used
  - Autopano Giga™ 3.0 by kolor for MacOS
- ★ For more information, see
  - Richard Szeliski (2010) Computer Vision: Algorithms and Applications, Springer.
- ★ Some concepts in slides motivated by similar slides by J. Hays.
- ★ Some images retrieved from
  - http://commons.wikimedia.org/.
  - List will be available on website.

www.flickr.com/photos/neneonline/231886965/

Georgia Institute of Technology

Computational Photography

Dr. Irfan Essa
Professor
School of Interactive Computing

Study the basics of computation and its impact on the entire workflow of photography, from capturing, manipulating and collaborating on, and sharing photographs.