STAT 200

Elementary Statistics

9.4 - Comparing Two Independent Means

Two independent means are compared using an **independent t test.**

1. Check any necessary assumptions and write null and alternative hypotheses.

There are two assumptions for the following test of comparing two independent means: (1) the two samples are independent and (2) each sample is randomly sampled from a population that is approximately normally distributed.

Below are the possible null and alternative hypothesis pairs:

Research Question	**Are the means of group 1 and group 2 different?**	**Is the mean of group 1 greater than the mean of group 2?**	**Is the mean of group 1 less than the mean of group 2?**
Null Hypothesis, H_0	$\mu_1 - \mu_2 = 0$	$\mu_1 - \mu_2 = 0$	$\mu_1 - \mu_2 = 0$
Alternative Hypothesis, H_a	$\mu_1 - \mu_2 \neq 0$	$\mu_1 - \mu_2 > 0$	$\mu_1 - \mu_2 < 0$
Type of Hypothesis Test	Two-tailed, non-directional	Right-tailed, directional	Left-tailed, directional

2. Calculate an appropriate test statistic.

This will be a *t* test statistic. The calculations for these test statistics can get quite involved. Below you are presented with the formulas that are used, however, in real life these calculations are performed using statistical software (e.g., Minitab Express).

Recall that test statistics are typically a fraction with the numerator being the difference observed in the sample and the denominator being the standard error.

The standard error of the difference between two means is different depending on whether or not the standard deviations of the two groups are similar.

Pooled Standard Error Method (Similar Standard Deviations)

If the two standard deviations are similar (neither is more than twice of the other), then the pooled standard error is used:

Pooled standard error

$$s_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}$$

s_p = pooled standard deviation

Pooled standard deviation

$$s_p=\sqrt{\frac{(n_1-1)s_1^2+(n_2-1)s_2^2}{n_1+n_2-2}}$$

Test statistic for independent means (pooled)

$$t=\frac{\bar{x}_1-\bar{x}_2}{s_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}}$$

Degrees of freedom for independent means (pooled)

$$df=n_1+n_2-2$$

Unpooled Standard Error Method (Differing Standard Deviations)

If the two standard deviations are not similar (one is more than twice of the other), then the unpooled standard error is used:

Unpooled standard error

$$\sqrt{\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}}$$

Test statistic for independent means (unpooled)

$$t=\frac{\bar{x}_1-\bar{x}_2}{\sqrt{\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}}}$$

The degrees of freedom are found using a complicated approximation formula. You won't have to do that calculation "by hand" because Minitab Express will compute it for you, but is done by:

Degrees of freedom for independent means (unpooled)

$$df = \frac{(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2})^2}{\frac{1}{n_1-1}(\frac{s_1^2}{n_1})^2 + \frac{1}{n_2-1}(\frac{s_2^2}{n_2})^2}$$

NOTE: If one is performing hand calculations using the unpooled method, then choice of degrees of freedom can be made by choosing the smaller of $n_1 - 1$ and $n_2 - 1$.

3. Determine a *p* value associated with the test statistic.

The *t* test statistic found in Step 2 is used to determine the *p* value.

4. Decide between the null and alternative hypotheses.

If $p \leq \alpha$ reject the null hypothesis. If $p > \alpha$ fail to reject the null hypothesis.

5. State a "real world" conclusion.

Based on your decision in Step 4, write a conclusion in terms of the original research question.

- 9.4.1 - Video: Height by Biological Sex (Pooled Method) Example (/stat200/node/241)
- 9.4.2. - Studying (Pooled Method) Example (/stat200/node/242)
- 9.4.3 - Cholesterol (Unpooled Method) Example (/stat200/node/243)

‹ 9.3.2 – Same Sex Marriage Example (/stat200/node/245) up (/stat200/node/57) 9.4.1 - Video: Height by Biological Sex (Pooled Method) Example › (/stat200/node/241)
