Difference between Two Means (Independent Groups)

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Prerequisites
Sampling Distribution of Difference between Means, Confidence Intervals, Confidence Interval on the Difference between Means, Logic of Hypothesi Testing, Testing a Single Mean

Learning Objectives
1. State the assumptions for testing the difference between two means
2. Estimate the population variance assuming homogeneity of variance
3. Compute the standard error of the difference between means
4. Compute t and p for the difference between means
5. Format data for computer analysis

It is much more common for a researcher to be interested in the differen between means than in the specific values of the means themselves. This covers how to test for differences between means from two separate grou subjects. A later section describes how to test for differences between the of two conditions in designs where only one group of subjects is used and subject is tested in each condition.

We take as an example the data from the "Animal Research" case stu this experiment, students rated (on a 7-point scale) whether they though research is wrong. The sample sizes, means, and variances are shown se for males and females in Table 1.

Table 1. Means and Variances in Animal Research study.

Group	n	Mean	Variance
Females	17	5.353	2.743
Males	17	3.882	2.985

As you can see, the females rated animal research as more wrong than d males. This sample difference between the female mean of 5.35 and the mean of 3.88 is 1.47. However, the gender difference in this particular sa not very important. What is important is whether there is a difference in t *population* means.

In order to test whether there is a difference between population mea are going to make three assumptions:

1. The two populations have the same variance. This assumption i the assumption of *homogeneity of variance*.
2. The populations are *normally distributed*.
3. Each value is sampled *independently* from each other value. Thi assumption requires that each subject provide only one value. I subject provides two scores, then the scores are not independe analysis of data with two scores per subject is shown in the sec the correlated t test later in this chapter.

The consequences of violating the first two assumptions are investigated simulation in the next section. For now, suffice it to say that small-to-mo violations of assumptions 1 and 2 do not make much difference. It is imp not to violate assumption 3.

We saw the following general formula for significance testing in the se testing a single mean:

$$t = \frac{\text{statistic} - \text{hypothesized value}}{\text{estimated standard error of the statistic}}$$

In this case, our statistic is the difference between sample means anc hypothesized value is 0. The hypothesized value is the null hypothesis tha difference between population means is 0.

We continue to use the data from the "Animal Research" case study a

compute a significance test on the difference between the mean score of
females and the mean score of the males. For this calculation, we will ma
three assumptions specified above.

The first step is to compute the statistic, which is simply the differenc
between means.

$$M_1 - M_2 = 5.3529 - 3.8824 = 1.4705$$

Since the hypothesized value is 0, we do not need to subtract it from the

The next step is to compute the estimate of the standard error of the
In this case, the statistic is the difference between means, so the estimat
standard error of the statistic is ($s_{M_1-M_2}$). Recall from the relevant sectio
chapter on sampling distributions that the formula for the standard error
difference between means is:

$$\sigma_{M_1-M_2} = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} = \sqrt{\frac{\sigma^2}{n} + \frac{\sigma^2}{n}} = \sqrt{\frac{2\sigma^2}{n}}$$

In order to estimate this quantity, we estimate σ^2 and use that estimate i
of σ^2. Since we are assuming the two population variances are the same,
estimate this variance by averaging our two sample variances. Thus, our
of variance is computed using the following formula:

$$\text{MSE} = \frac{s_1^2 + s_2^2}{2}$$

where MSE is our estimate of σ^2. In this example,

$$\text{MSE} = (2.743 + 2.985)/2 = 2.864.$$

Since n (the number of scores **in each group**) is 17,

$$s_{M_1-M_2} = \sqrt{\frac{2MSE}{n}} = \sqrt{\frac{(2)(2.864)}{17}} = 0.5805.$$

The next step is to compute t by plugging these values into the formula:

$$t = 1.4705/.5805 = 2.533.$$

Finally, we compute the probability of getting a t as large or larger than 2
as small or smaller than -2.533. To do this, we need to know the *degrees
freedom*. The degrees of freedom is the number of independent estimates
variance on which MSE is based. This is equal to $(n_1 - 1) + (n_2 - 1)$, wher
the sample size of the first group and n_2 is the sample size of the second
For this example, $n_1 = n_2 = 17$. When $n_1 = n_2$, it is conventional to use "r
refer to the sample size of each group. Therefore, the degrees of freedom
16 = 32.

Once we have the degrees of freedom, we can use the t distribution c
to find the probability. Figure 1 shows that the probability value for a two
test is 0.0164. The two-tailed test is used when the null hypothesis can b
rejected regardless of the direction of the effect. As shown in Figure 1, it
probability of a t < -2.533 or a t > 2.533.

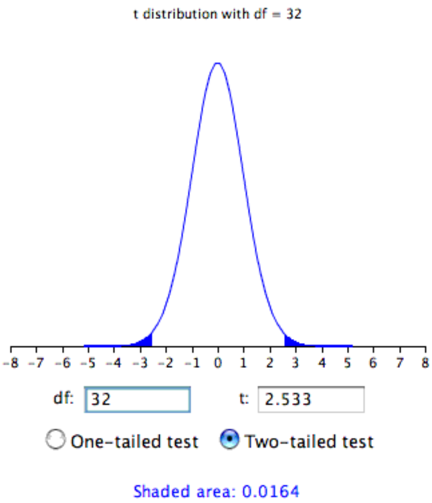

Figure 1. The two-tailed probability.

The results of a one-tailed test are shown in Figure 2. As you can see, the probability value of 0.0082 is half the value for the two-tailed test.

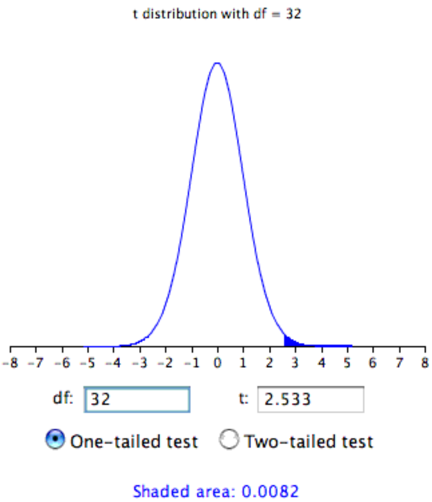

Figure 2. The one-tailed probability.

Online Calculator: t distribution

Formatting Data for Computer Analysis

Most computer programs that compute t tests require your data to be in a form. Consider the data in Table 2.

Table 2. Example Data.

Group 1	Group 2
3	2
4	6
5	8

Here there are two groups, each with three observations. To format these a computer program, you normally have to use two variables: the first sp the group the subject is in and the second is the score itself. The reforma version of the data in Table 2 is shown in Table 3.

Table 3. Reformatted Data.

G	Y
1	3

1	4
1	5
2	2
2	6
2	8

To use Analysis Lab to do the calculations, you would copy the data and tl

1. Click the "Enter/Edit Data" button. (You may be warned that for secu reasons you must use the keyboard shortcut for pasting data.)
2. Paste your data.
3. Click "Accept Data."
4. Set the Dependent Variable to Y.
5. Set the Grouping Variable to G.
6. Click the "t-test/confidence interval" button.

The t value is -0.718, the df = 4, and p = 0.512.

Computations for Unequal Sample Sizes (optional)

The calculations are somewhat more complicated when the sample sizes equal. One consideration is that MSE, the estimate of variance, counts th group with the larger sample size more than the group with the smaller size. Computationally, this is done by computing the sum of squares erro as follows:

$$SSE = \sum (X - M_1)^2 + \sum (X - M_2)^2$$

where M_1 is the mean for group 1 and M_2 is the mean for group 2. Consi following small example:

Table 4. Unequal n.

Group 1	Group 2
3	2
4	4
5	

$M_1 = 4$ and $M_2 = 3$.

$SSE = (3-4)^2 + (4-4)^2 + (5-4)^2 + (2-3)^2 + (4-3)^2 = 4$

Then, MSE is computed by: MSE = SSE/df

where the degrees of freedom (df) is computed as before:
$df = (n_1 - 1) + (n_2 - 1) = (3 - 1) + (2 - 1) = 3$.
MSE = SSE/df = 4/3 = 1.333.

The formula

$$s_{M_1-M_2} = \sqrt{\frac{2MSE}{n}}$$

is replaced by

$$s_{M_1-M_2} = \sqrt{\frac{2MSE}{n_h}}$$

where n_h is the harmonic mean of the sample sizes and is computed as 1

$$n_h = \frac{2}{\frac{1}{n_1} + \frac{1}{n_2}} = \frac{2}{\frac{1}{3} + \frac{1}{2}} = 2.4.$$

and

$$s_{M_1-M_2} = \sqrt{\frac{(2)(1.333)}{2.4}} = 1.054.$$

Therefore,

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t = (4-3)/1.054 = 0.949
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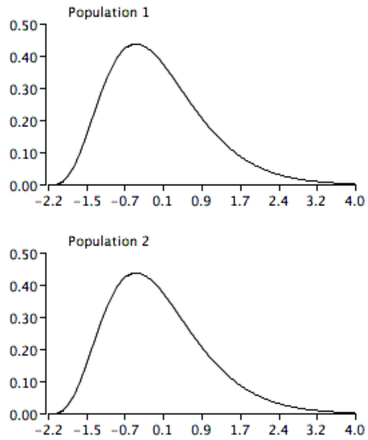
and the two-tailed p = 0.413.

Question 1 out of 9.
The graphs show a violation of the assumption of (check all that apply)



- [ ] normality
- [ ] homogeneity of variance

Check Answer | Previous Question | Next Question

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