3.09 Simple Regression: Exponential regression

In this video you'll learn how to recognize an exponential pattern in data and how to interpret a simple **exponential regression model**. Not all data follow a linear pattern. In some cases relationships between two variables are better described using for example a power function or a logistic function. There are several types of nonlinear regression that are commonly used; here we'll only discuss exponential regression to give you a general idea of how nonlinear relations can be modeled.

Take the example where we predicted popularity of cat videos - measured as number of video views - using the cat's age as the predictor. Suppose the scatterplot of the data looked like this. Obviously a linear model is inappropriate here. The decrease in popularity is not constant, it seems to become smaller and smaller as cat age increases.

This relationship can be modeled using and exponential regression equation, which - at the population level - looks like this: $\mu_y = \alpha \cdot \beta^x$.
Exponential models are useful to describe growth rates over time, such as interest rates in finances or for example growth or decline of animal populations in biology. Because in many of these cases the predictor is time, the symbol x is sometimes substituted with the symbol t. But we'll stick with x here.

If x equals zero, beta to the power of x equals one, so the equation reduces to $\mu_y = \alpha$. This means that alpha represents the intercept, or the starting value at x equals zero. When x equals one, the equation changes to
$\mu_y = \alpha \cdot \beta$.
So with an increase of one unit in x, the starting value is multiplied by beta. If we move up another unit in x, from one to two, the predicted value for μ_y changes from $\alpha \cdot \beta$ to $\alpha \cdot \beta \cdot \beta$. So the predicted value at x = 2 is the previous value at x = 1 times β. The same holds if we move from x = 2, to x = 3 etcetera, etcetera.
This is why we call the exponential model a multiplicative model, because for each increase in x, the predicted value increases at the percentage rate set by β. This is different from the additive, linear model, where the predicted value always increases by the exact same amount.
So in an exponential model the regression coefficient β indicates the percentage rate of change. β is larger than zero by necessity. If β is smaller than one the exponential curve will descend, as *x* increases the predicted value will decrease. In our example, as x increases by one unit, a β of 0.6 will result in a 60 percent decrease in the predicted value,

compared to the previous predicted value. If β is larger than one, the function increases. If β equals one, well it's no longer an exponential function.
So how do we obtain estimates for α and β, or *a* and *b* if we consider the regression model at the sample level? Well there's a trick we can use. If we take the logarithm on both sides of the equation, the equation transforms into a linear model[1].

You don't have to be familiar with the algebra rules for simplifying logarithms. You just have to understand that taking the logarithm of a variable shrinks large values to a large degree and shrinks smaller values to a lesser degree. So you can see a log transformation as pushing down hard on the large values and gently squeezing the smaller values. Here I've used the natural logarithm, with base number *e*, but you can use any base number.

If a variable is truly exponential, applying a log-transformation will change it into a linear variable, as you can see in the equation. I've replaced the log of *a* and the log of *b* with a capital *A* and *B* to make this even clearer.
We can now use the linear regression formulas to calculate the intercept and regression coefficient for this linearized version of the equation: $A = \bar{y} - B\bar{x}$, $B = r \cdot \frac{s_y}{s_x}$. Remember to use the variable $\boldsymbol{log\text{-}y}$ and not *y* when calculating the mean, standard deviation and correlation!
Once we've calculated the intercept capital *A* en regression coefficient capital *B* for the linearized model, we can determine what the values of the lowercase *a* and *b*, the intercept and regression coefficient of the exponential model are. We find *a* and *b* by doing the opposite of taking the log, we raise a base number to the power of *a* and *b*. What base number? Well that depends on the type of logarithm I used. If I had used 10 as a base number, I would calculate *a* and *b* by raising 10 to the power of capital *A* and capital *B*. If I used natural logs, like in the example, I find *a* and *b* by raising the number *e* to the power of capital *A* and capital *B*.

One last thing to remember: Just like in linear regression you should be very careful extrapolating values beyond the observed range of the predictor. Always ask yourself whether predicted values beyond the range for which you have observed values make sense. In our example the predictions for older cats of 10 to 15 years will approach zero. It remains to be seen whether the exponential relation truly holds in this age range. Older cats can still be pretty cute!

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[1] $y = a \cdot b^x$ -> $\ln(y) = \ln(a \cdot b^x) = \ln(a) + \ln(b^x) = \ln(a) + \ln(b) \cdot x = A + B \cdot x$