

Double Counting

- ✔

Quiz: Puzzle: Sums of Rows and Columns  
1 question
- ✔

Reading: Double Counting  
10 min
- ✔

Quiz: 'Homework Assignment' Problem  
1 question
- Ⓜ

Reading: `Homework Assignment' Problem  
10 min
- ✔

Quiz: 'Homework Assignment' Problem 2  
1 question
- Ⓜ

Practice Quiz: Girls and Boys  
1 question
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Quiz: Chess Tournaments  
1 question

Invariants

Termination

Even and Odd Numbers

Double Counting

"There are things that never change." Apart from being just a philosophical statement, this phrase turns out to be an important idea in discrete mathematics and computer science. A property that is preserved during a process is called an *invariant*. Invariants are widely used in analyzing the behavior of algorithms, programs, and other processes. In this week, we will develop the important skill of finding the right invariant for a problem. We will start with double counting.

Looking at the same object from various angles can help not only in your everyday life, but also in analyzing mathematical structures.

**Problem: Sums of Rows and Columns.** Fill a  $3 \times 5$  table with integers so that the sum of each row is equal to 20 and the sum of each column is equal to 10.

Let us see whether we are lucky enough to do this on the first try. To do this, let us fill the first two rows with arbitrary numbers so that the sum in each of them is equal to 20.

2	4	7	3	4	→ 20
5	5	1	6	3	→ 20

Since we want the sum in every column to be equal to 10, we are forced to fill in the last row as follows.

2	4	7	3	4	→ 20
5	5	1	6	3	→ 20
3	1	2	1	3	
↓	↓	↓	↓	↓	
10	10	10	10	10	

This makes the sum of the last row equal 10. That is, from the eight constraints (three for rows and five for columns) we were able to satisfy all but one. Thus, our first attempt is unsuccessful, but this does *not* mean that the task is impossible: perhaps, if we started with some other numbers in the first two rows, we would be able to fill in the table as needed.

Try filling the first two rows with various numbers so that the sums of the first and the second rows are 20. Then fill the third row so that the sum of each column is 10. What is the sum of the last row in the resulting table?

A closer look at the table reveals that it is not a coincidence that the sum in the last row is 10. Indeed, if the sum of every column is equal to 10, then the sum of all elements in the table is equal to 50. Since the sum of the first two rows is equal to 20, the sum of the last row is  $50 - 20 - 20 = 10$ . And this is why a filling of the table with the required properties does not exist.

Our invariant in this problem is the sum of all elements in the table. And the trick that we used is known as *double counting*: if you count the elements of a set in two different ways, the two results should be the same. (In other contexts, double counting could also refer to a wrong argument where the same object is counted more than once.)

✔ Completed

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