How Not to Use the Rule of Sum

Consider the following problem.

Problem. What is the number of integers from 1 to 10 that are divisible by 2 or 3?

Well, this is easy. There are five integers divisible by 2 ($2, 4, 6, 8, 10$) and three integers divisible by 3 ($3, 6, 9$). Hence, by the sum rule, the answer is $5 + 3 = 8$.

Do you see a flaw in this argument?

Let's just find these numbers.

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print([n for n in range(1, 11)
        if n % 2 == 0 or n % 3 == 0])
```

Run

Reset

We see that the correct answer is seven. Looking back at our argument, we can see that it is the number 6 that causes problems: it is divisible by both 2 and 3 and because of that we counted it twice. The lesson learned here is: *when applying the sum rule, one should ensure that no object belongs to both classes*.

Mathematicians formulate the sum rule as follows: the cardinality of the union of two disjoint sets is the sum of their cardinalities. As you may guess, the sets are formed by apples and oranges, the union is what happens when we put them in the same bag, and disjointness means that we do not count the same object twice (as we first did in our last example).

Philosophers would add that the sum rule is not a \emph{rule}, but a \emph{definition}: the definition of addition operation.