

Recursion

Induction

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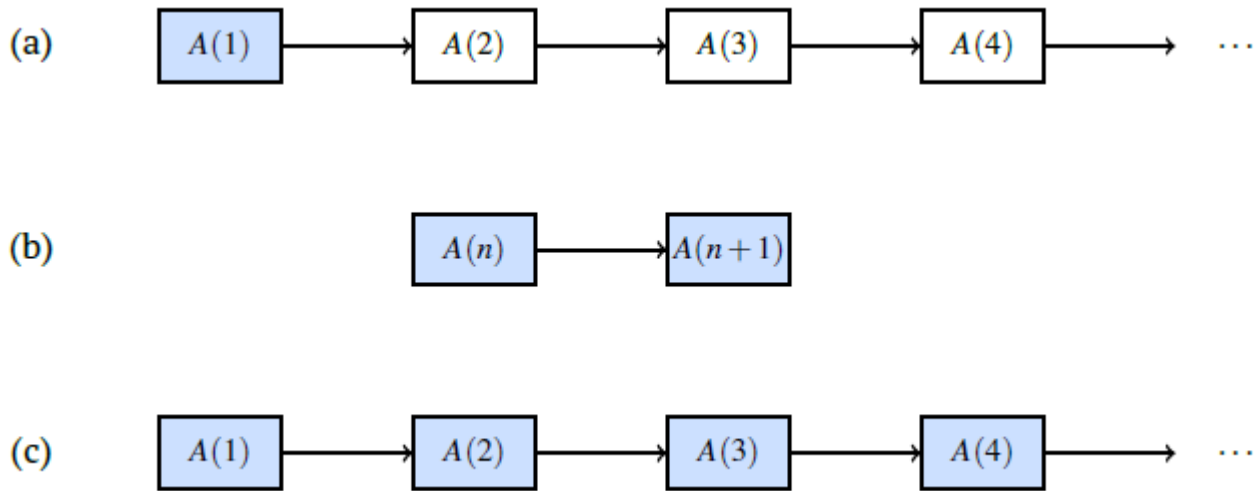
✓ Quiz: Induction  
9 questions

What is Induction?

A mathematical induction proof of the fact that all of the statements  $A(1), A(2), A(3) \dots$  are true, consists of two parts:

- 1. *The base case:* First, we prove that  $A(1)$  is true.
- 2. *The induction step:* Then, we prove that for every  $n \geq 1$ , the statement  $A(n)$  implies the statement  $A(n + 1)$  (that is, if  $A(n)$  is true, then  $A(n + 1)$  is true).

Mathematical induction assures that once we (i) proved the base case and (ii) proved the induction step, we have actually proved *all* statements  $A(n)$  for  $n \geq 1$ . See the figure:



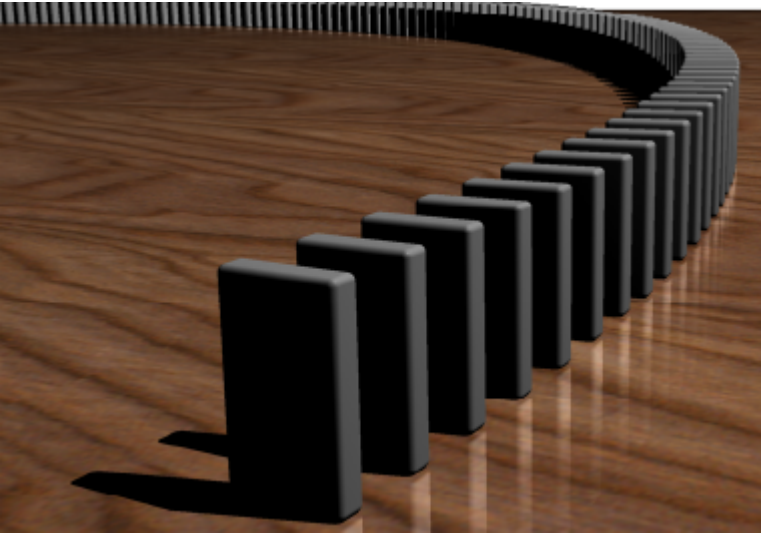
- (a) the base case ensures that the first statement is true;
- (b) the induction step lets us move from any statement to the following one;
- (c) from this, we conclude that all the statements are true.

Thus, a typical induction proof consists of two parts. The first part(usually the easiest one) is to prove the base case. Of course, there is nothing special about  $n = 1$ , and we could start at any other number  $n = c$  (say,  $c = 0$ , or  $c = 15$ , or  $c = -3$ ). Either way, we will prove all statements  $A(c), A(c + 1), A(c + 2), \dots$

The second part of an induction proof is to prove the induction step. Here, we assume that we already know that the statement  $A(n)$  is true. This assumption is called the *induction hypothesis*. Assuming the induction hypothesis, we prove the statement  $A(n + 1)$ , and this finishes the proof. Sometimes it comes in handy to use a stronger induction hypothesis : assume that all the previous statements  $A(1), \dots, A(n)$  are true, and use them to prove correctness of  $A(n + 1)$ . This variation of induction proofs is often called *strong induction*.

Let us recall a strong induction proof that we have seen. Consider the following statement  $A(n)$ :  $n$  can be represented as  $3k + 5l$  where  $k$  and  $l$  are non-negative integers. Problem [Coins](#) states that  $A(n)$  is true for all  $n \geq 8$ . Indeed, for  $n = 8, 9, 10, A(n)$  is true:  $8 = 3 \cdot 1 + 5 \cdot 1, 9 = 3 \cdot 3 + 5 \cdot 0, 10 = 3 \cdot 0 + 5 \cdot 2$  This is the base case. For the induction step, assume that  $A(8), A(9), \dots, A(n)$  are all true for some  $n \geq 11$ . Since  $n \geq 11, n - 3 \geq 8$  and hence  $A(n - 3)$  is true. Therefore,  $n - 3 = 3 \cdot k + 5 \cdot l$  for some  $k, l$ . Thus,  $n = 3k + 5l + 3 = 3(k + 1) + 5l$ .

Mathematical induction can be visualized as a domino effect:given an infinitely long chain of dominoes, each separated by a small distance, it suffices to push the first domino to knock down all dominoes in the chain (see Figure). In fact, this holds due to mathematical induction! Indeed, for every  $n \geq 1$ , let  $A(n)$  be the proposition that the domino number  $n$  falls down. Then, we ensure  $A(1)$  by pushing the first domino. Since the dominoes are too close to one another,  $A(n)$  implies  $A(n + 1)$ : each domino falls after it is knocked over by the previous one.



The domino effect : pushing the first domino knocks down all other dominoes. (Source : [Wikipedia](#)).

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