What Chebyshev's inequality has to say

$$\{|S_n - S'_n| > n\Delta\} \subseteq \overbrace{\{|S_n - np| > n\Delta/2\}}^{A} \cup \overbrace{\{|S'_n - np| > n\Delta/2\}}^{B}$$

Boole's inequality: $\mathbf{P}(A \cup B) \leq \mathbf{P}(A) + \mathbf{P}(B)$

$$\mathbf{P}\left\{\left|\frac{S_n}{n} - \frac{S_n}{n}\right| > \Delta\right\} = \mathbf{P}\{|S_n - S'_n| > n\Delta\}$$

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S_n and S'_n are statistical copies with the same (binomial) distribution

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Chebyshev's inequality with $\Delta/2$ playing the role of ϵ

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\end{aligned}$$

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A slogan

The smaller that $2/n\Delta^2$ is, the more likely it is that the drug in question exhibits significant side-effects of the type being tested.

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- If the sample size is $n = 100$, at what discrepancy Δ can you conclude, with 50% confidence that there are side-effects? What if you desire a confidence level of 90%
- What if the sample size is $n = 3000$?

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	δ	
n	0.5	0.1
100	0.2	0.45
3000	0.04	0.08