Balls in Boxes

Problem. Is it possible to place 30 balls in ten boxes so that all boxes contain a different number of balls?

Perhaps you see an obstacle: in a sense, we have too few balls. To make this formal, assume that the challenge is possible. Let us place the boxes in order of increasingnumber of balls and let us denote these numbers by $b_0, \ldots, b_9$ (this 0-based indexing, used in programming constantly, will prove useful in this problem):

$$b_0 < b_1 < \cdots < b_9 \,.$$

What do we know about b_0? Almost nothing: b_0 is non-negative (that is, $b_0 \geq 0$), and that is all we can say. However, for b_1 we can already say something non-trivial: it is greater than b_0, hence $b_1 \geq 1$. Next, $b_2 \geq 2$, since $b_2 > b_1 \geq 1$. Proceeding in the same fashion, we conclude that (b_i \ge i\) for every $i = 0, 1, \ldots, 9$. This means that

$$b_0 + b_1 + \cdots + b_9 \geq 0 + 1 + \cdots + 9 = 45 \,,$$

contradicting the fact that we have 30 balls. We have found that our task with ten boxes is only possible when we have 45 or more balls to place.