a) $y_t^* = \gamma + \delta x_t$ in the PA model, this gives

$$y_t = y_{t-1} + \lambda(\gamma + \delta x_{t-1} - y_{t-1}) + \varepsilon_t$$

$$= y_{t-1} + \lambda\gamma + \lambda\delta x_{t-1} - \lambda y_{t-1} + \varepsilon_t$$

$$= \lambda\gamma + (1-\lambda)y_{t-1} + \lambda\delta x_{t-1} + \varepsilon_t$$

ADL(p, r) model

AR lag $p=1$

DL lag $r=1$

b) $x_t^* = x_{t-1}^* + \lambda(x_{t-1} - x_{t-1}^*) + \varepsilon_t$

$= (1-\lambda)x_{t-1}^* + \lambda x_{t-1} + \varepsilon_t$

So $\underline{x_t^* - (1-\lambda)x_{t-1}^*} = \lambda x_{t-1} + \varepsilon_t \quad (*)$

$y_t = \gamma + \delta x_t^*$, or $\delta x_t^* = y_t - \gamma$

we get $\delta(x_t^* - (1-\lambda)x_{t-1}^*) = (y_t - \gamma) - (1-\lambda)(y_{t-1} - \gamma)$

and from $(*)$ $\delta(x_t^* - (1-\lambda)x_{t-1}^*) = \delta\lambda x_{t-1} + \delta\varepsilon_t$

Hence $y_t - \gamma - (1-\lambda)(y_{t-1} - \gamma) = \delta\lambda x_{t-1} + \delta\varepsilon_t$

Or $y_t = \gamma - (1-\lambda)\gamma + (1-\lambda)y_{t-1} + \delta\lambda x_{t-1} + \delta\varepsilon_t$

$= \gamma\lambda + (1-\lambda)y_{t-1} + \delta\lambda x_{t-1} + \delta\varepsilon_t$

ADL model with $p=1$, $r=1$

c) Condition is that $-1 < 1-\lambda < 1$, that is $0 < \lambda < 2$.

We already assumed that $0 \leq \lambda \leq 1$. So, the condition is that $\lambda \neq 0$. Indeed, if $\lambda = 0$:

- production is not adjusted in any systematic way in PA model
- expectations are not adjusted with realized demand in AE model

d) rewrite AE model

$$x_t^* = x_{t-1}^* + \lambda_1(x_{t-1} - x_{t-1}^*) + \lambda_2(x_{t-2} - x_{t-2}^*) + \varepsilon_t$$

$$= (1-\lambda_1)x_{t-1}^* - \lambda_2 x_{t-2}^* + \lambda_1 x_{t-1} + \lambda_2 x_{t-2} + \varepsilon_t$$

$$\underline{x_t^* - (1-\lambda_1)x_{t-1}^* + \lambda_2 x_{t-2}^*} = \lambda_1 x_{t-1} + \lambda_2 x_{t-2} + \varepsilon_t$$

from $y_t = \gamma + \delta x_t^*$, or $\delta x_t^* = y_t - \gamma$

Hence: $\delta x_t^* = y_t - \gamma$

$$-(1-\lambda_1)\delta x_{t-1}^* = -(1-\lambda_1)(y_{t-1} - \gamma)$$

$$\lambda_2 \delta x_{t-2}^* = \lambda_2(y_{t-2} - \gamma) \qquad +$$

$$\delta(\underbrace{x_t^* - (1-\lambda_1)x_{t-1}^* + \lambda_2 x_{t-2}^*}_{\lambda_1 x_{t-1} + \lambda_2 x_{t-2} + \varepsilon_t}) = y_t - \gamma - (1-\lambda_1)(y_{t-1} - \gamma) + \lambda_2(y_{t-2} - \gamma)$$

So $\delta\lambda_1 x_{t-1} + \delta\lambda_2 x_{t-2} + \delta\varepsilon_t = y_t - \gamma - (1-\lambda_1)y_{t-1} + (1-\lambda_1)\gamma + \lambda_2 y_{t-2} - \lambda_2\gamma$

Rewrite $y_t = \gamma - (1-\lambda_1)\gamma + \lambda_2\gamma + (1-\lambda_1)y_{t-1} - \lambda_2 y_{t-2} + \delta\lambda_1 x_{t-1} + \delta\lambda_2 x_{t-2} + \delta\varepsilon_t$

$$= (\lambda_1 + \lambda_2)\gamma + (1-\lambda_1)y_{t-1} - \lambda_2 y_{t-2} + \delta\lambda_1 x_{t-1} + \delta\lambda_2 x_{t-2} + \delta\varepsilon_t$$

ADL with 2 lags for y and 2 lags for x.

ADL (p, r), with $p = r = 2$