Point Vortex

Consider the velocity field of a fluid given by

$$\boldsymbol{u}(x,y)=\frac{\Gamma}{2\pi}\left(\frac{-y\boldsymbol{i}+x\boldsymbol{j}}{x^2+y^2}\right).$$

(a) Using polar coordinates, show that

$$\boldsymbol{u}(r,\theta)=\frac{\Gamma\hat{\boldsymbol{\theta}}}{2\pi r}.$$

(b) The vorticity field of the fluid is defined as

$$\boldsymbol{\omega}=\boldsymbol{\nabla}\times\boldsymbol{u}.$$

Using polar coordinates, show that $\boldsymbol{\omega}=0$ provided $r\neq 0$.

(c) Using Stokes theorem, show that the integral of the vorticity field over a small area in the x-y plane containing the origin is equal to Γ and therefore that the vorticity is given by

$$\boldsymbol{\omega}=\Gamma\delta(\boldsymbol{r}),$$

where $\delta(\boldsymbol{r})$ is the two-dimensional Dirac delta function. This is the definition of a point vortex of strength Γ.

Completed

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