## How to solve the Monty Hall problem using Bayes Theorem

Asked 4 years, 2 months ago Modified 2 years, 2 months ago Viewed 2k times



I've recently come across the Monty hall problem and while the reasoning behind switching doors makes sense intuitively to me I can't seem to understand the maths behind it.



I've seen many proofs online using Bayes Theorem and I manage to understand the majority of it aside from one thing.



In the classic setting of the Monty Hall problem let A be the event that the car is behind the first door that I chose. Let B the event that Monty reveals a goat behind door 2.



Then, using Bayes Theorem, we have

$$P(A \mid B) = \frac{P(B \mid A)P(A)}{P(B)}$$

Now,  $P(B \mid A) = \frac{1}{2}$  because if the car is behind door 1 then Monty can choose either the second or third door.  $P(A) = \frac{1}{3}$  because there's a one in three chance of the car being behind the first door.

This all makes sense to me - my struggle comes in finding P(B).

I understand that there are 3 separate scenarios:

- -The car is behind door 1 As above in this case  $P(B) = \frac{1}{2}$ .
  - The car is behind door 2 Clearly, P(B) = 0 as Monty cannot reveal a goat behind door 2.
  - The car is behind door 3 If the car is behind door 3, then Monty is forced to open door 2 and so in this case P(B) = 1.

The way I see it, by combining these three scenarios

$$P(B) = \frac{1}{2} + 0 + 1 = \frac{3}{2}$$

But in every proof that I have seen they divide this by 3 for some unknown reason. I feel like I may be missing something blindingly obvious but I don't understand why this is true.

Could someone explain the reason that  $P(B) = \frac{1}{2}$  as opposed to  $\frac{3}{2}$ ?



3 A probability cannot exceed 1 – Peter Jul 25, 2018 at 19:56

The scenarios must be weighted by their probabilities to be combined. – user558317 Aug 28, 2018 at 3:28

## 3 Answers

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To answer your question:

1

The way I see it, by combining these three scenarios P(B) = 1/2 + 0 + 1 = 3/2



Let us denote scenarios one through three  $S_1$ ,  $S_2$ , and  $S_3$ .

$$P(B|S_1)=1/2$$

$$P(B|S_2) = 0$$

$$P(B|S_3) = 1$$

How to combine them:

$$P(B) = P(B|S_1) * P(S_1) + P(B|S_2) * P(S_2) + P(B|S_3) * P(S_3)$$

Given 
$$P(S_1) = P(S_2) = P(S_3) = 1/3$$

$$P(B) = P(B|S_1) * 1/3 + P(B|S_2) * 1/3 + P(B|S_3) * 1/3$$

$$P(B) = \{P(B|S_1) * P(S_1) + P(B|S_2) * P(S_2) + P(B|S_3) * P(S_3)\} * (1/3)$$

Substituting in P(B|S):

$$P(B) = (1/2 + 0 + 1) * (1/3) = 3/2 * 1/3 = 1/2$$

(As for the proof, I am not familiar with it, and not certain that it is valid/relevant, only that it produces the correct answer.)

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answered Aug 28, 2018 at 3:26



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because each of the scenarios has probability  $\frac{1}{3}$ . Tou ie applying the <u>law of total probability</u>, and each term contains a factor  $\frac{1}{3}$ .



**(**)

Let's say it rains tomorrow with probability  $\frac{1}{2}$ . If it rains, I brush my teeth. If it doesn't rain, I also brush my teeth. Is the probability that I brush my teeth tomorrow 1 or 2?

You also have an error in your calculation of  $P(B \mid A)$ . This is  $\frac{1}{3}$ , not  $\frac{1}{2}$ . This follows by symmetry – conditioning on an event that doesn't distinguish one of the three doors from the others can't make the probabilities for the doors being opened non-uniform.

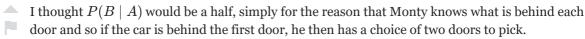
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answered Jul 25, 2018 at 20:13



joriki

**c** 14 267 476



- Inspector gadget Jul 28, 2018 at 18:13



0

In the classic setting of the Monty Hall problem let A be the event that the car is behind the first door that I chose. Let B the event that Monty reveals a goat behind door 2.



Ah! You want B to be the event that Monty chooses door 2 (rather than door 3) since you always select door 1.

Let  $A_n$  be the event that the car is behind door n.  $A_1$  is the event that it is behind the door you choose.

$$egin{aligned} \mathsf{P}(A_1 \mid B) &= rac{\mathsf{P}(B \mid A_1)\mathsf{P}(A_1)}{\mathsf{P}(B \mid A_1)\mathsf{P}(A_1) + \mathsf{P}(B \mid A_2)\mathsf{P}(A_2) + \mathsf{P}(B \mid A_3)\mathsf{P}(A_3)} \ &= rac{rac{1}{2}rac{1}{3}}{rac{1}{2}rac{1}{3} + 0 + rac{1}{1}rac{1}{3}} \ &= rac{1}{3} \end{aligned}$$

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edited Jul 25, 2018 at 23:30

answered Jul 25, 2018 at 21:50



**Graham Kemp 122k** 6 51

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