

# Iterative Solution of the Laplace Equation

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Using the Jacobi method, solve the Laplace equation inside a unit square. Set the boundary conditions to be zero on the left and bottom sides, and to go from zero to one across the top, and from one to zero down the right side. Model these boundary conditions as

$\Phi = x(2 - x)$  for  $y = 1$ , and  $\Phi = y(2 - y)$  for  $x = 1$ .

## Script ?

Reference Solution

 Save

 Reset

 MATLAB Documentation (<https://www.mathworks.com/help/>)

```
1 %%%% Define the rectangle and grid parameters %%%%%%%%%%
2 Lx=1; Ly=1; %rectangle dimensions
3 Nx=100; Ny=100; %# of intervals
4 nx=Nx+1; ny=Ny+1; %# of gridpoints in x,y directions including boundaries
5 dx=Lx/Nx; dy=Ly/Ny; %grid size in x,y directions
6 x=(0:Nx)*dx; y=(0:Ny)*dy; %x,y values on the grid
7 %%%% Define the iteration parameters and initial condition %%%%%%%%%%
8 eps=1.e-6; %convergence criteria for each value of Phi
9 index_x=2:nx-1; index_y=2:ny-1; %internal grid points
10 Phi=zeros(nx,ny);%matrix with solution and boundary conditions
11 %%%% DEFINE THE BOUNDARY CONDITIONS %%%%%%%%%%
12 %set the boundary conditions
13 Phi(:,1)=0; %bottom
14 Phi(1,:)=0; %left
15 Phi(:,ny)=x.*(2-x); %top
16 Phi(nx,:)=y.*(2-y); %right
17 %%%% Jacobi iteration %%%%%%%%%%
18 Phi_old=Phi;
19 error=2*eps; ncount=0;
20 while (error > eps)
21     ncount=ncount+1;
22     Phi(index_x,index_y)=0.25*(Phi(index_x+1,index_y) ...
23         +Phi(index_x-1,index_y)+Phi(index_x,index_y+1)+Phi(index_x,index_y-1));
24     error=max(abs(Phi(:)-Phi_old(:)));
25     if any(isnan(Phi(:))) || any(isinf(Phi(:)))
26         fprintf('iterations diverge\n');
27         return;
28     end
29     Phi_old=Phi;
30     %fprintf('%g %e\n',ncount, error);
31 end
32 fprintf('%g\n',ncount);
33 %%%% graphics %%%%%%%%%%
34 [X,Y]=meshgrid(x,y);
35 v=[0.8 0.6 0.4 0.2 0.1 0.05 0.01]; %SET THE CONTOUR LEVELS
36 contour(X,Y,Phi,v,'ShowText','on');%requires transpose (read the notes)
37 axis equal;
38 set(gca, 'YTick', [0 0.2 0.4 0.6 0.8 1]);
39 set(gca, 'XTick', [0 0.2 0.4 0.6 0.8 1]);
40 xlabel('$x$', 'Interpreter','latex', 'FontSize',14 );
41 ylabel('$y$', 'Interpreter','latex', 'FontSize',14);
42 title('Solution of the Laplace equation', 'Interpreter','latex', 'FontSize',16);
43
```

 Run Script ?

## Assessment: All Tests Passed

Submit ?

 Check the value of Phi

## Output



