Feedback — Assignment on ILP solvers

Help Center

You submitted this homework on **Sun 10 Nov 2013 5:53 AM PST**. You got a score of **26.00** out of **26.00**.

Question 1

This assignment takes you through the process of "binarizing" an ILP to convert it into a 0-1 ILP. Consider the ILP shown below:

$$\begin{array}{llll} \max & x_1 + x_2 & & \\ \text{s.t.} & x_1 - 3x_2 & \leq & 10 \\ & 2x_1 + 3x_2 & \leq & 15 \\ 0 \leq & x_1 & \leq & 7 \\ 0 \leq & x_2 & \leq & 13 \\ & x_1, x_2 & \mathbb{Z} & \end{array}$$

We represent x_1 by a three bit binary number $b_{1,3} b_{1,2} b_{1,1}$ ($b_{1,3}$ is the most significant bit and $b_{1,1}$ is the least significant bit). Which of the following expressions replaces x_1 in the original ILP?

Your Answer		Score	Explanation
◉ $x_1 : 4b_{1,3} + 2b_{1,2} + b_{1,1}$	✔	3.00	
○ $x_1 : b_{1,1} + b_{1,2} + b_{1,3}$			
○ $x_1 : 8b_{1,1} + 4b_{1,2} + 2b_{1,1}$			
○ $x_1 : 1000b_{1,3} + 100b_{1,2} + 10b_{1,1}$			
Total		3.00 / 3.00	

Question Explanation

What is the expression for the number represented by $(b_{1,3}b_{1,2}b_{1,1})_2$ in binary?

Question 2

Continuing with problem 1, what is the minimum number of bits needed to represent x_2 in the binarized problem? Hint: you may have to solve an LP to find out.

Your Answer		Score	Explanation
5			
2			
3	✔	3.00	
4			
Total		3.00 / 3.00	

Question Explanation

Try to maximize the objective function x_2 and you will find your answer.

Question 3

In our lecture we considered pure ILPs where all the variables in the problem are integers. In this problem, we consider mixed integer programs

(MIPs), wherein a subset of the decision variables are integer variables and the remaining variables are considered real-valued.

Which of the following modifications to branch-and-bound procedure covered will help is solve mixed integer programs? Select all the correct answers. Assume that the problem is maximizing the objective function.

Your Answer		Score	Explanation
☐ It is possible to branch on any of the decision variables in the problem.	✔	0.83	No, only on the integer variables
☐ We can branch on any real-valued variable x_j whose LP relaxation optimal value is s_j, the branch constraints will be $x_j \leq s_j$ and $x_j \geq s_j$	✔	0.83	Such a branch will not achieve anything. So it is completely unnecessary.
☑ Branching is considered when the LP relaxation yields a fractional solution for an integer decision variable.	✔	0.83	
☑ A node can be converted into a leaf whenever its solution satisfies the integrality constraints for the integral decision variables.	✔	0.83	
☐ We cannot prune a node simply because its LP relaxation yields an optimum that is less than or equal to the best objective.	✔	0.83	Not true: the LP relaxation optimum for a feasible MIP will be an upper bound on its actual solution. So optimal pruning remains valid.
☐ Branching is considered when the LP relaxation yields a fractional solution for some decision variable.	✔	0.83	Note that branching on a real-valued decision variable is not correct.
Total		5.00 / 5.00	

Question Explanation

In fact, the branch and bound method readily extends to MIPs. The two main changes to note are that LP relaxation optimum needs to be integer only for the integer decision variables in the problem, branch constraints are also added only for integer variables that violate the integrality constraint.

Question 4

We apply branch-and-bound on a 0-1 (binary) ILP. Select all the true facts below. Assume that the problem is a maximization problem with n decision variables and m constraints.

Your Answer		Score	Explanation
☑ The maximum depth of the tree is given by the number of decision variables n.	✔	1.25	Yes: each branching step fixes the value of branch variable on either branch.
☐ We should explore the branch $x_i \geq 1$ first and the $x_i \leq 0$ since it is guaranteed to yield a larger value.	✔	1.25	Not really, we can never make a blanket statement like this. Wish one could ;-)
☑ For a branch variable x_i, the branch constraints are always $x_i \leq 0$ and $x_i \geq 1$.	✔	1.25	Correct since we already have $x_i \geq 0$ and $x_i \leq 1$, the LP relaxation has $x_i \in (0, 1)$.
☑ The total number of nodes in the final branch-and-bound tree will be less than 2^{n+1}	✔	1.25	Correct, since the depth of the tree is at most n
Total		5.00 / 5.00	

Question 5

Consider the following final dictionary encountered while solving an ILP given below (we assume all problem and slack variables are integers).

$$
\begin{array}{c|ccc}
x_2 & \frac{8}{3} & +\frac{2}{3}x_6 & -\frac{10}{3}x_3 \\
x_1 & \frac{5}{3} & -\frac{5}{3}x_6 & +\frac{1}{3}x_3 \\
x_4 & 6 & +\frac{14}{3}x_6 & -\frac{4}{3}x_3 \\
x_5 & \frac{4}{3} & +\frac{5}{6}x_6 & -\frac{1}{3}x_3 \\
\hline
z & -\frac{17}{3} & -\frac{7}{3}x_6 & -\frac{1}{3}x_3
\end{array}
$$

Select all the valid cutting plane constraints from the list below.

Your Answer		Score	Explanation
☐ $\frac{2}{3}x_6 - \frac{2}{3}x_3 \geq \frac{2}{3}$ corr. to x_2	✔	1.25	
☑ $\frac{1}{3}x_6 + \frac{1}{3}x_3 \geq \frac{2}{3}$ corresponding to x_2	✔	1.25	
☐ $\frac{1}{3}x_6 + \frac{1}{3}x_3 \geq 0$ corresponding to x_4	✔	1.25	Note that x_4 will not yield a cutting plane constraint.
☑ $\frac{1}{6}x_6 + \frac{1}{3}x_3 \geq \frac{1}{3}$ corresponding to x_5	✔	1.25	
☐ $\frac{5}{6}x_6 - \frac{1}{3}x_3 \geq \frac{4}{3}$ corr. to x_5	✔	1.25	
☐ $\frac{2}{3}x_6 - \frac{1}{3}x_3 \geq \frac{2}{3}$ corr. to x_1 .	✔	1.25	
☐ $\frac{5}{3}x_6 - \frac{1}{3}x_3 \geq \frac{5}{3}$ corr. to x_1	✔	1.25	

☑ $\frac{2}{3}x_6 + \frac{2}{3}x_3 \geq \frac{2}{3}$ corresponding to x_1	✔	1.25
Total		10.00 / 10.00

Question Explanation

Note that the cutting plane constraint for a row in the dictionary

$$x_{Bj}:\ b_j + a_{j1}x_1 + \cdots + a_{jn}x_n$$

requires that b_j is fractional and is given by

$$\mathrm{frac}(-a_{j1})x_1 + \cdots + \mathrm{frac}(-a_{jn})x_n \geq \mathrm{frac}(b_j)$$

where $\mathrm{frac}(x) = x - \lfloor x \rfloor$.
As an example, x_2 will yield the cut

$$\mathrm{frac}(-\frac{2}{3})x_6 + \mathrm{frac}(\frac{10}{3})x_3 \geq \mathrm{frac}(\frac{8}{3})$$

This yields the cut $\frac{1}{3}x_6 + \frac{1}{3}x_3 \geq \frac{2}{3}$