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- *Compound chance experiment, product space and measure*:
  - Sample space: $\Omega = \mathfrak{A}_1 \times \cdots \times \mathfrak{A}_n = \{(a_{1k_1}, \ldots, a_{nk_n}) : k_1 \geq 1, \ldots, k_n \geq 1\}$.
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- *Repeated independent trials*:
  - Common alphabet: $\mathfrak{A}_1 = \cdots = \mathfrak{A}_n = \mathfrak{A} := \{a_k, k \geq 1\}$.
  - Common atomic mass function: $\{a_k\} \mapsto p(k)$.
- *Product space and measure*:
  - $\Omega = \mathfrak{A}^n = \{(a_{k_1}, \ldots, a_{k_n}): k_1 \geq 1, \ldots, k_n \geq 1\}$.
  - $\mathbf{P}\{(a_{k_1}, \ldots, a_{k_n})\} := p(k_1) \times \cdots \times p(k_n)$.

Slogan

In the case of a finite or even countably infinite number of repeated independent trials, events in the compound experiment (product space) that are determined by distinct trials are independent.