Tableau 3, Part 3

Chance in commonplace settings:
Beyond balls and urns — unequal probabilities and infinite spaces

The game of craps

The game of craps

What is the probability that you win on the first throw in craps?

The game of craps

What is the probability that you win on the first throw in craps?

- The rules of the game:

  - Throw a pair of dice and sum their face values. You win immediately if you obtain 7 or 11. You lose immediately if you obtain 2, 3, or 12.

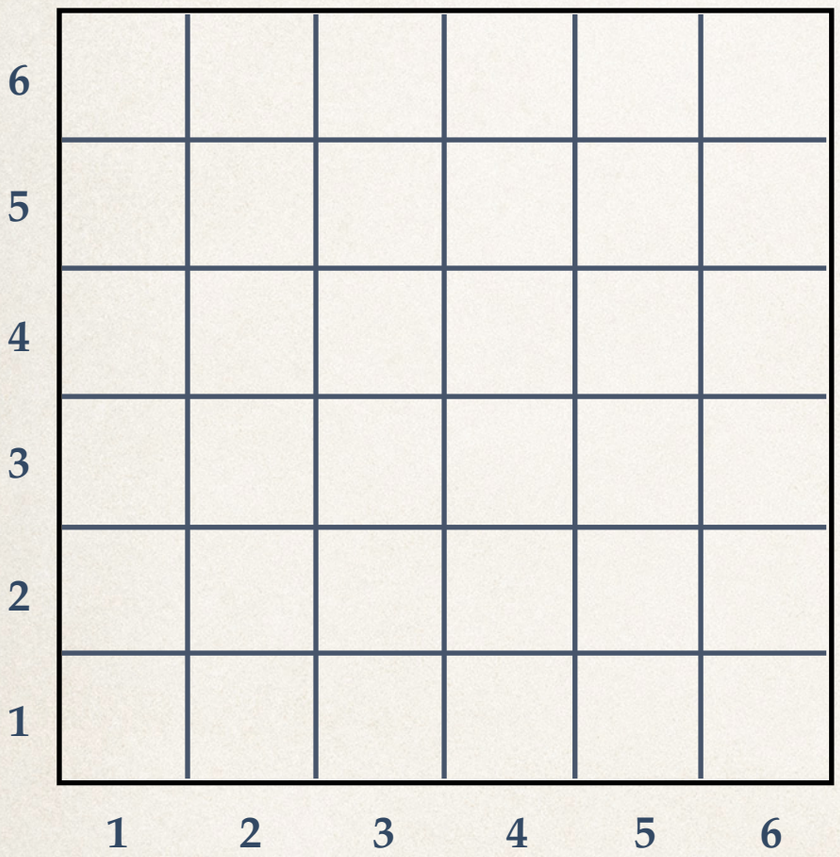
  - If you obtain 4, 5, 6, 8, 9, or 10 on the first throw proceed by repeatedly throwing the pair of dice and summing the face values. Stop at the first instant when the sum either replicates that of your original throw or is equal to 7. You win in the first case, lose in the second.

The first throw in craps

The first throw in craps

Sum of face values
2
3
4
5
6
7
8
9
10
11
12

The first throw in craps



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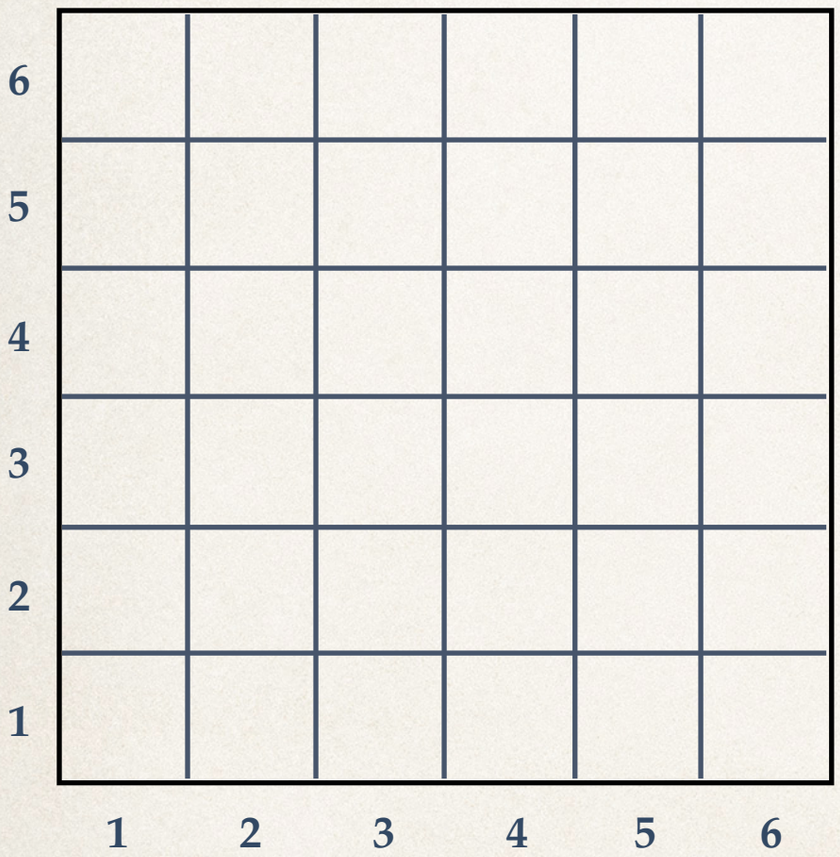
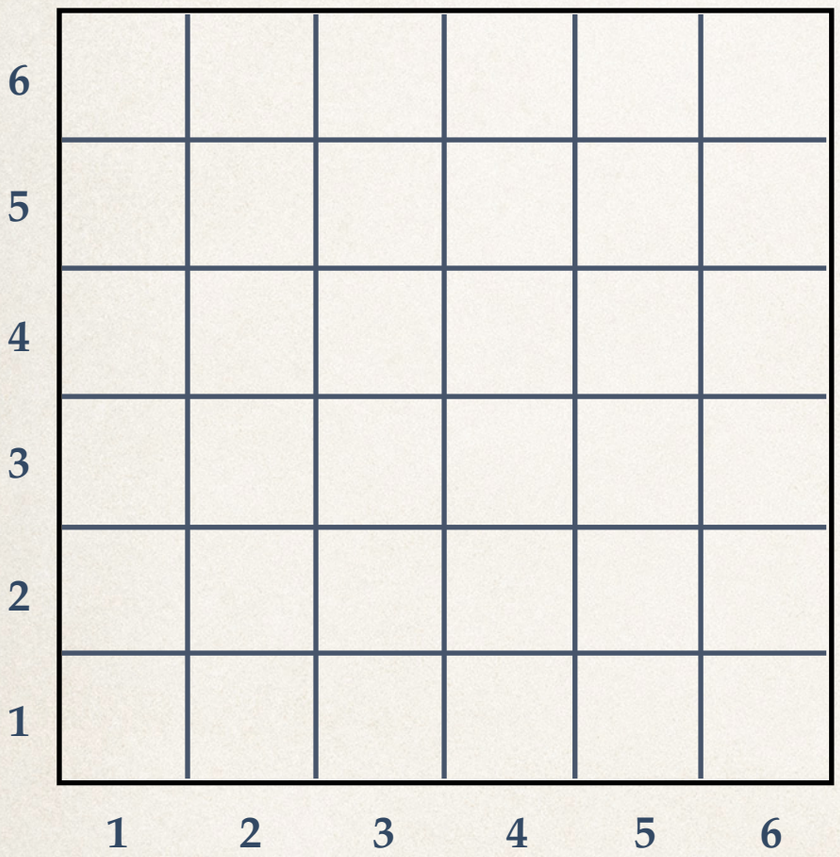


Sum of face values	Aggregate pairs
2	(1,1)
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	

The first throw in craps



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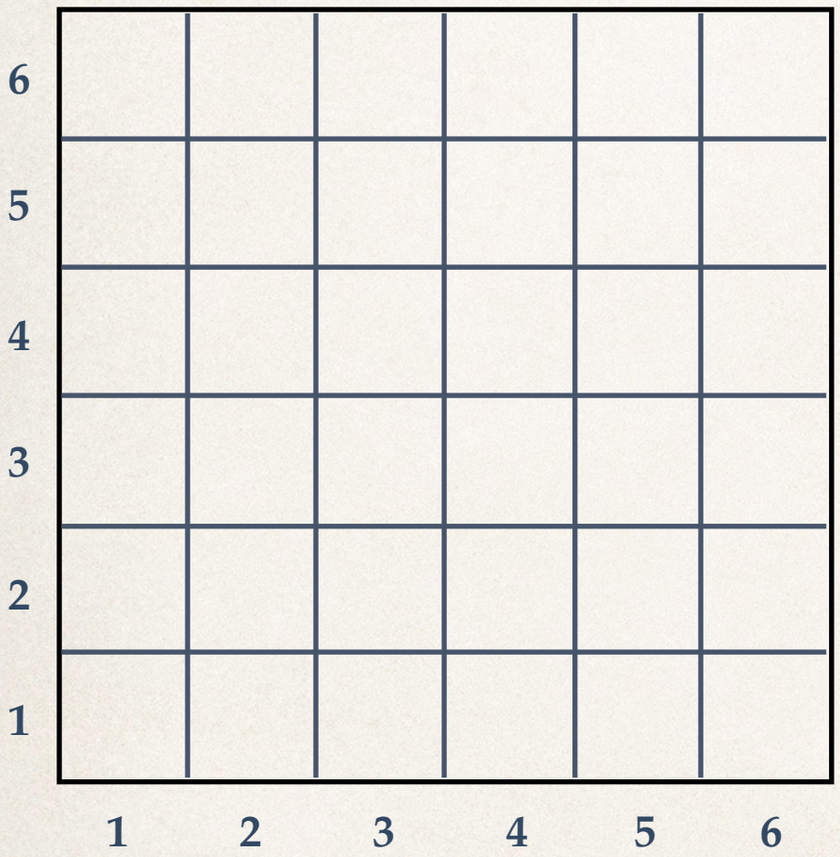

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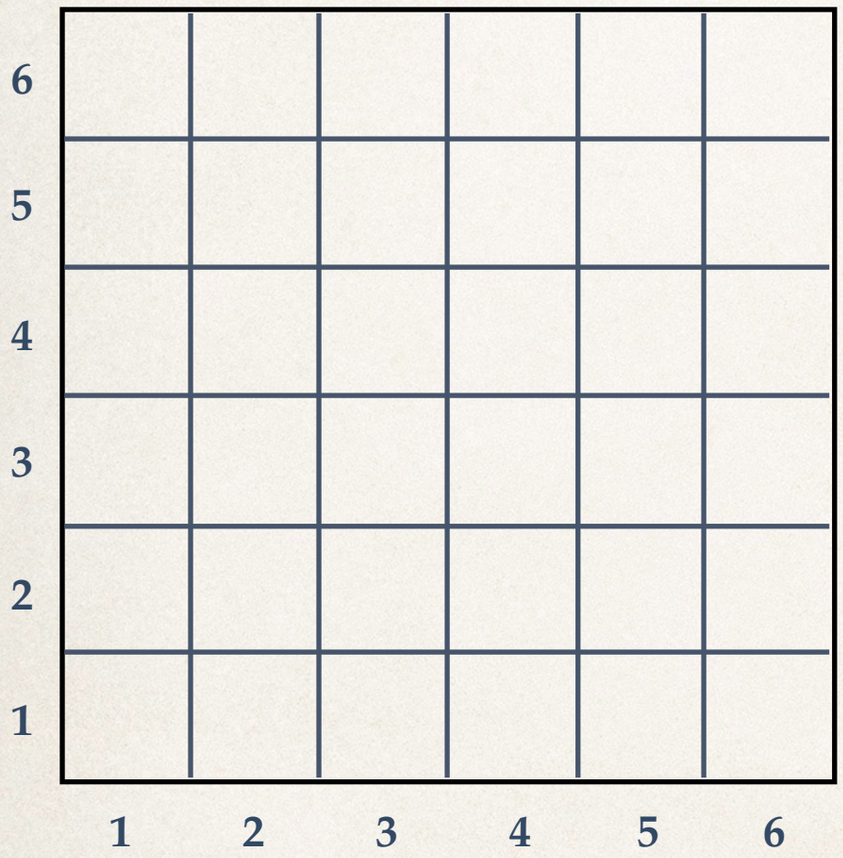

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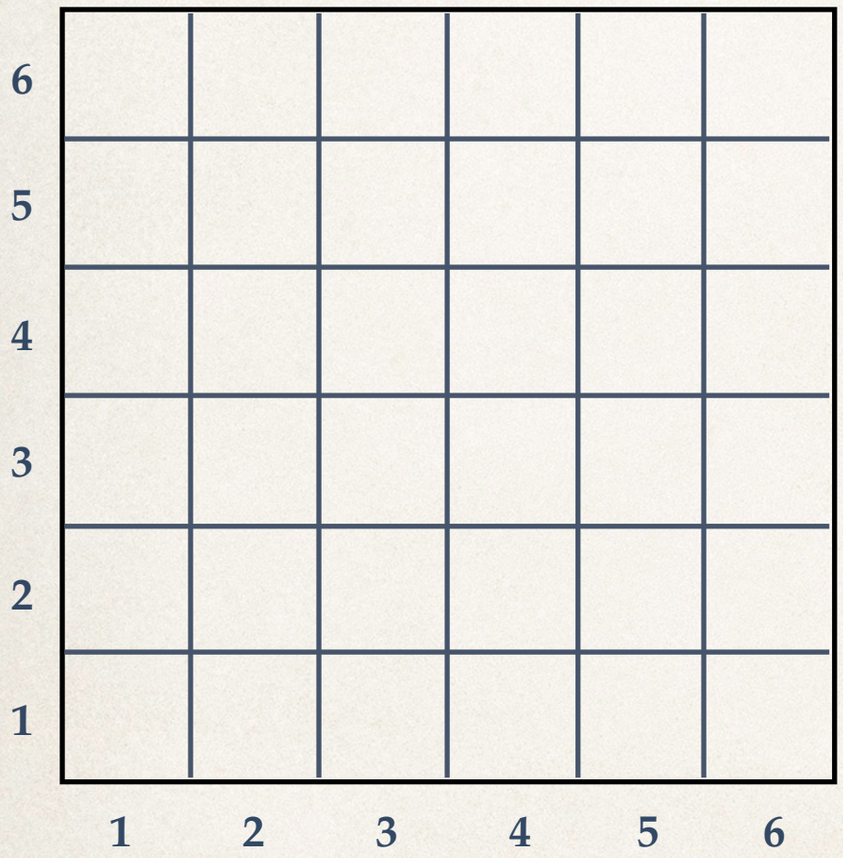

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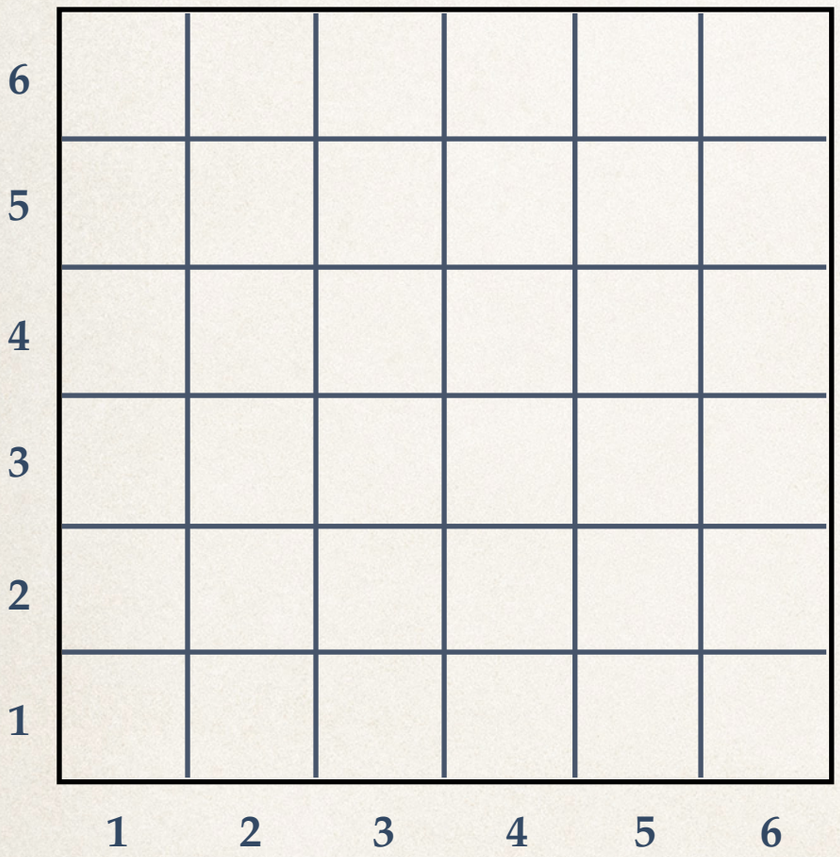

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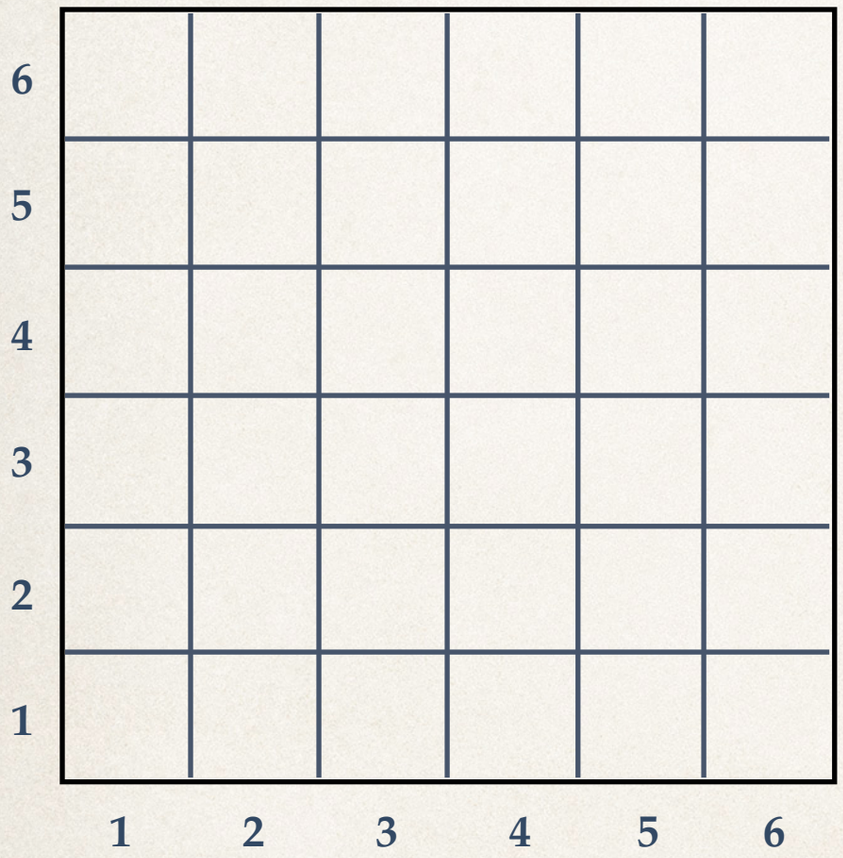

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An induced space of chance-driven outcomes

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An induced space of chance-driven outcomes

Sum of face values	Aggregate pairs	Probability
2	(1,1)	1/36
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4	(3,1), (2,2), (1,3)	3/36
5	(4,1), (3,2), (2,3), (1,4)	4/36
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Event	Aggregate outcomes
Win	{7, 11}
Lose	{2, 3, 12}
Continue	{4, 5, 6, 8, 9, 10}

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The hugely important principle of additivity: possibilities add when they are mutually exclusive.

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Probabilities:

$$\mathbf{P}\{\text{Win}\} = \mathbf{P}\{7\} + \mathbf{P}\{11\} = \tfrac{6}{36} + \tfrac{2}{36} = \tfrac{2}{9}$$

$$\mathbf{P}\{\text{Lose}\} = \mathbf{P}\{2\} + \mathbf{P}\{3\} + \mathbf{P}\{12\} = \tfrac{1}{36} + \tfrac{2}{36} + \tfrac{1}{36} = \tfrac{1}{9}$$

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$$\mathbf{P}\{\text{Continue}\} = 2\left(\tfrac{3}{36} + \tfrac{4}{36} + \tfrac{5}{36}\right) = \tfrac{2}{3}$$