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  - *Probability measure*: Combinatorial setting with mass function assigning equal probability $1/8$ to each atom.
- Are A and B independent events?
- What if the family has n children?

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Checking independence:

$$\frac{n}{2^n} \stackrel{?}{=} \frac{1+n}{2^n}\left(1 - \frac{1}{2^{n-1}}\right)$$

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n	$1+n$	2^{n-1}
2	3	2
3	4	4
4	5	8
5	6	16
6	7	32

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A and B are independent if, and only if, $n = 3$.

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