Why does the rank of the design matrix X equal the rank of X'X?

Why does the rank of the design matrix $\boldsymbol{X}$ equal the rank of $\boldsymbol{X}'\boldsymbol{X}$? Is this true in all circumstances?

If X is not linearly independent, what would the rank of X'X be?

regression mathematical-statistics linear-model linear-algebra

edited Feb 13 '13 at 4:48

asked Feb 13 '13 at 3:15
kurt
43 1 5

Welcome to the site, @kurt. I tried to edit your Q for greater clarity. Please make sure it's still asking what you want to know. In addition, I couldn't make sense of your 2nd paragraph; did you mean "If X is not *linearly independent*, what would the rank of X'X be"? – gung ♦ Feb 13 '13 at 4:02

1 Answer

For any matrix X, $R(X'X) = R(X)$. Where R() is the rank function.

You could prove this using null space. If $Xz = 0$ for some z, then clearly $X'Xz = 0$. Conversely, if $X'Xz = 0$, then $z'X'Xz = 0$, and it follows that $Xz = 0$. This implies X and $X'X$ have the same null space. Hence the result.

answered Feb 13 '13 at 9:15
vinux
2,663 1 14 18

Really appreciate it !!! – kurt Feb 14 '13 at 5:13