Robotics

Estimation and Learning
with Dan Lee

Week 1.
Gaussian Model Learning

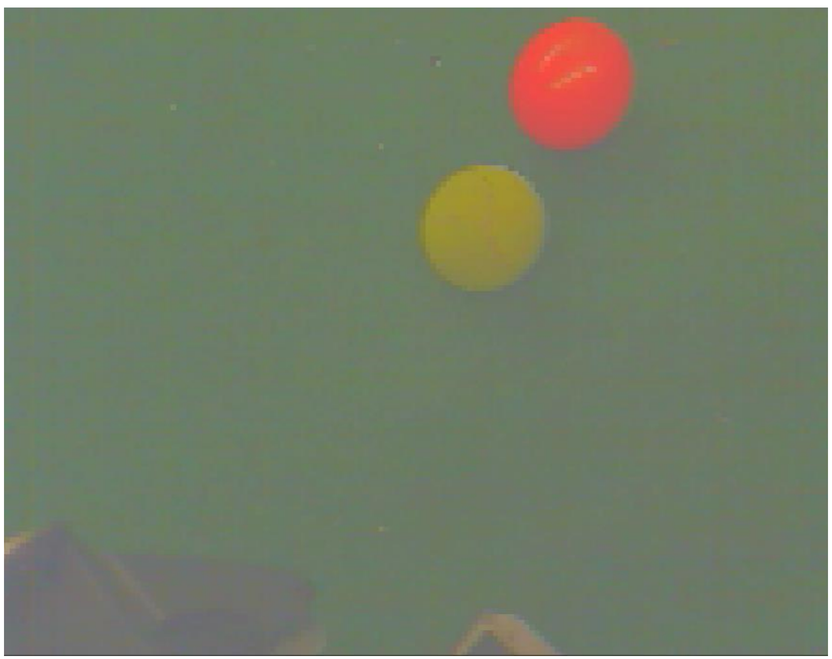
1.3.1 Multivariate Gaussian Distribution

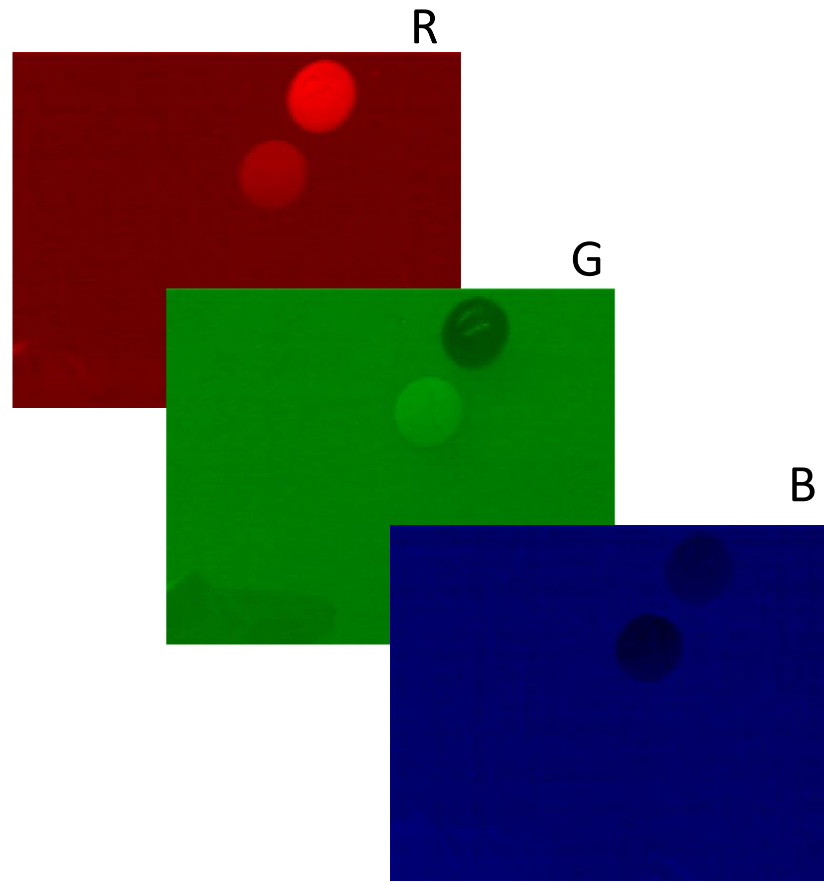
Penn Engineering

Multivariate Gaussian : Example

Ball color in multi-channels

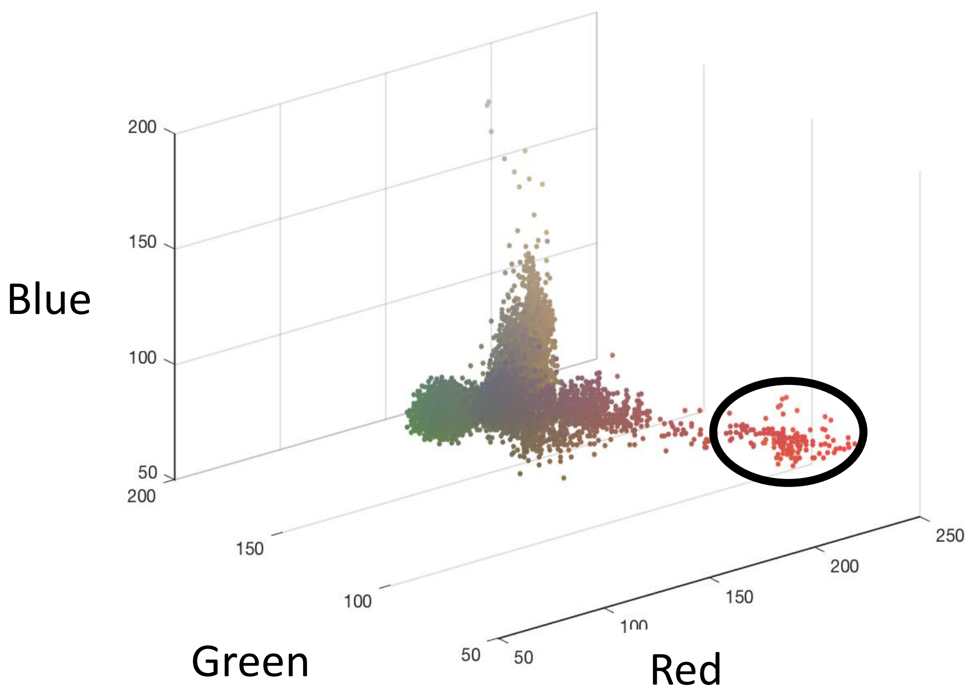
RGB Image

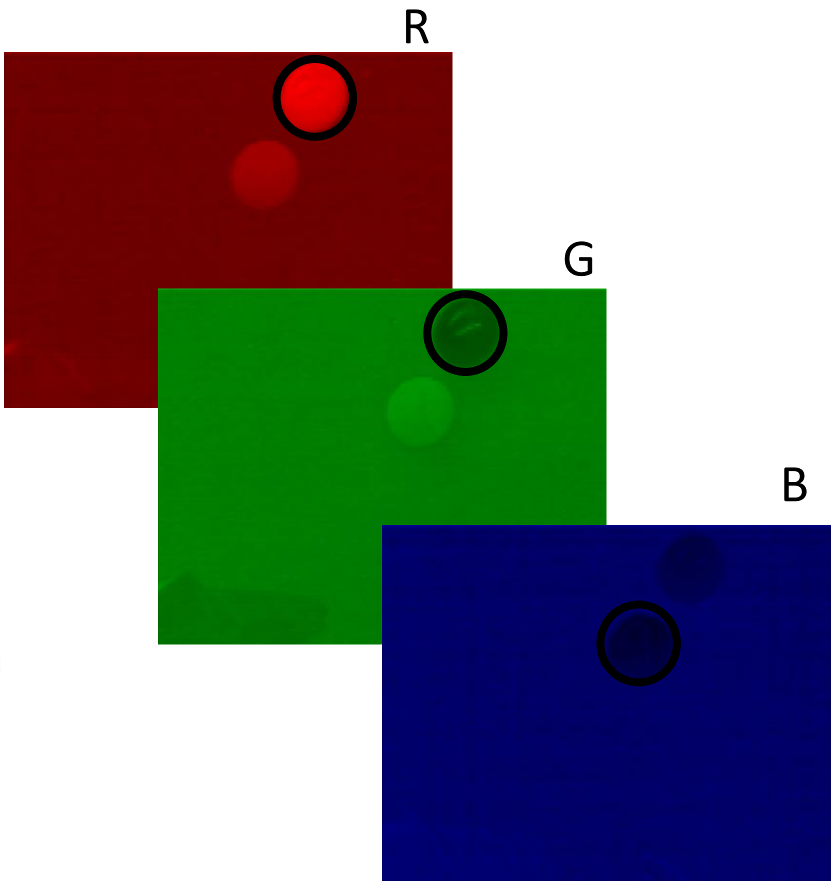




Multivariate Gaussian : Example

Ball color in multi-channels





Multivariate Gaussian

$$p(\mathbf{x}) = \frac{1}{(2\pi)^{D/2}|\Sigma|^{1/2}} \exp\left\{-\frac{1}{2}(\mathbf{x}-\boldsymbol{\mu})^{\boldsymbol{T}}\Sigma^{-1}(\mathbf{x}-\boldsymbol{\mu})\right\}$$

D	Number of Dimensions
$\mathbf{x}$	Variable
$\boldsymbol{\mu}$	Mean *vector*
Σ	Covariance *matrix*

(Dimension = 1)

$$\boxed{p(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left\{-\frac{(x-\mu)^2}{2\sigma^2}\right\}}$$

Multivariate Gaussian

$$p(\mathbf{x}) = \frac{1}{(2\pi)^{D/2}|\Sigma|^{1/2}} \exp\left\{-\frac{1}{2}(\mathbf{x}-\boldsymbol{\mu})^{T}\Sigma^{-1}(\mathbf{x}-\boldsymbol{\mu})\right\}$$

Σ Covariance *matrix*

* Diagonal terms: variance
* Off-diagonal terms: correlation

(Dimension = 2)

$$\boxed{\Sigma = \begin{bmatrix} \sigma_{x_1}^2 & \sigma_{x_1x_2}^2 \\ \sigma_{x_2x_1}^2 & \sigma_{x_2}^2 \end{bmatrix}} \quad (\sigma_{x_1x_2}^2 = \sigma_{x_2x_1}^2)$$

Multivariate Gaussian

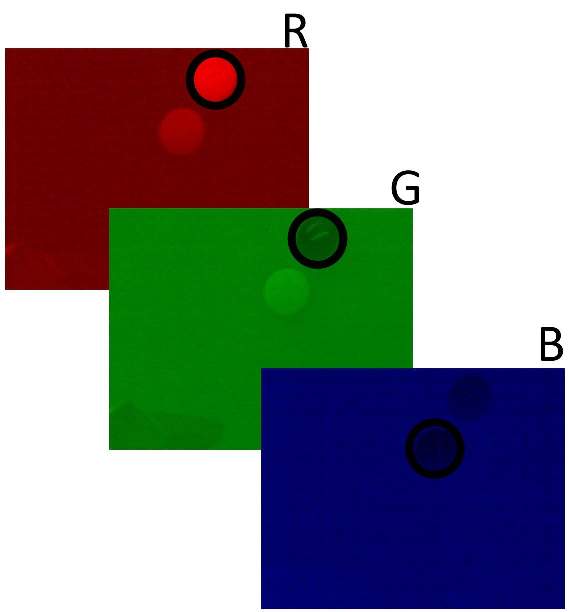
$$p(\mathbf{x}) = \frac{1}{(2\pi)^{D/2}|\Sigma|^{1/2}} \exp\left\{-\frac{1}{2}(\mathbf{x}-\boldsymbol{\mu})^{\boldsymbol{T}}\Sigma^{-1}(\mathbf{x}-\boldsymbol{\mu})\right\}$$

Determinant of Σ

Multivariate Gaussian

$$p(\mathbf{x}) = \frac{1}{(2\pi)^{D/2}|\Sigma|^{1/2}} \exp\left\{-\frac{1}{2}(\mathbf{x}-\boldsymbol{\mu})^{T}\Sigma^{-1}(\mathbf{x}-\boldsymbol{\mu})\right\}$$

Ball color in multi-channels



$$D = 3$$

$$\mathbf{x} = \begin{bmatrix} x_R & x_G & x_B \end{bmatrix}$$

$$\boldsymbol{\mu} = \begin{bmatrix} \mu_R & \mu_G & \mu_B \end{bmatrix}$$

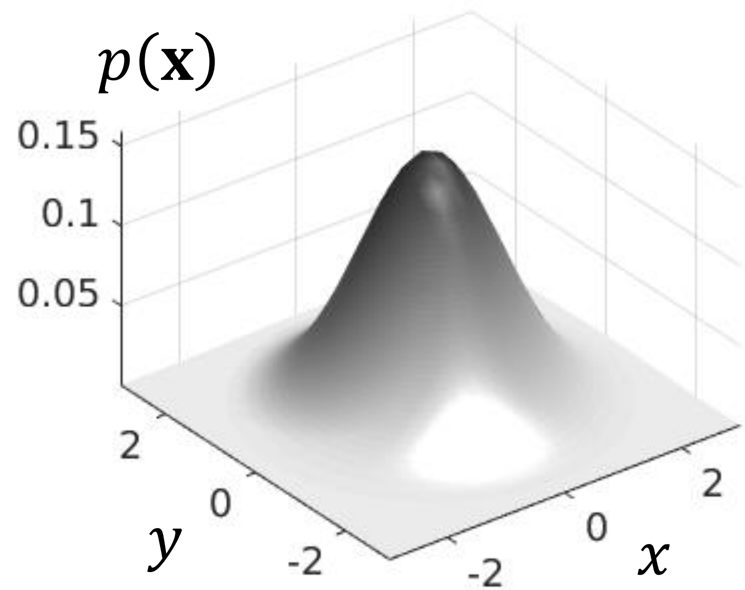
$$\Sigma = \begin{bmatrix} \sigma_{x_R}^2 & \sigma_{x_R x_G}^2 & \sigma_{x_R x_B}^2 \\ \sigma_{x_R x_G}^2 & \sigma_{x_G}^2 & \sigma_{x_G x_B}^2 \\ \sigma_{x_R x_B}^2 & \sigma_{x_G x_B}^2 & \sigma_{x_B}^2 \end{bmatrix}$$

Multivariate Gaussian: 2D

$$p(\mathbf{x}) = \frac{1}{2\pi}\exp\left\{-\frac{x^2+y^2}{2}\right\}$$

- 2D Zero-mean Spherical Case

$$D = 2$$
$$\mathbf{x} = [x \quad y]^T$$
$$\boldsymbol{\mu} = [0 \quad 0]^T$$
$$\Sigma = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$



Multivariate Gaussian: 2D

$$p(\mathbf{x}) = \frac{1}{2\pi} \exp\left\{-\frac{x^2 + y^2}{2}\right\}$$

- 2D Zero-mean Spherical Case

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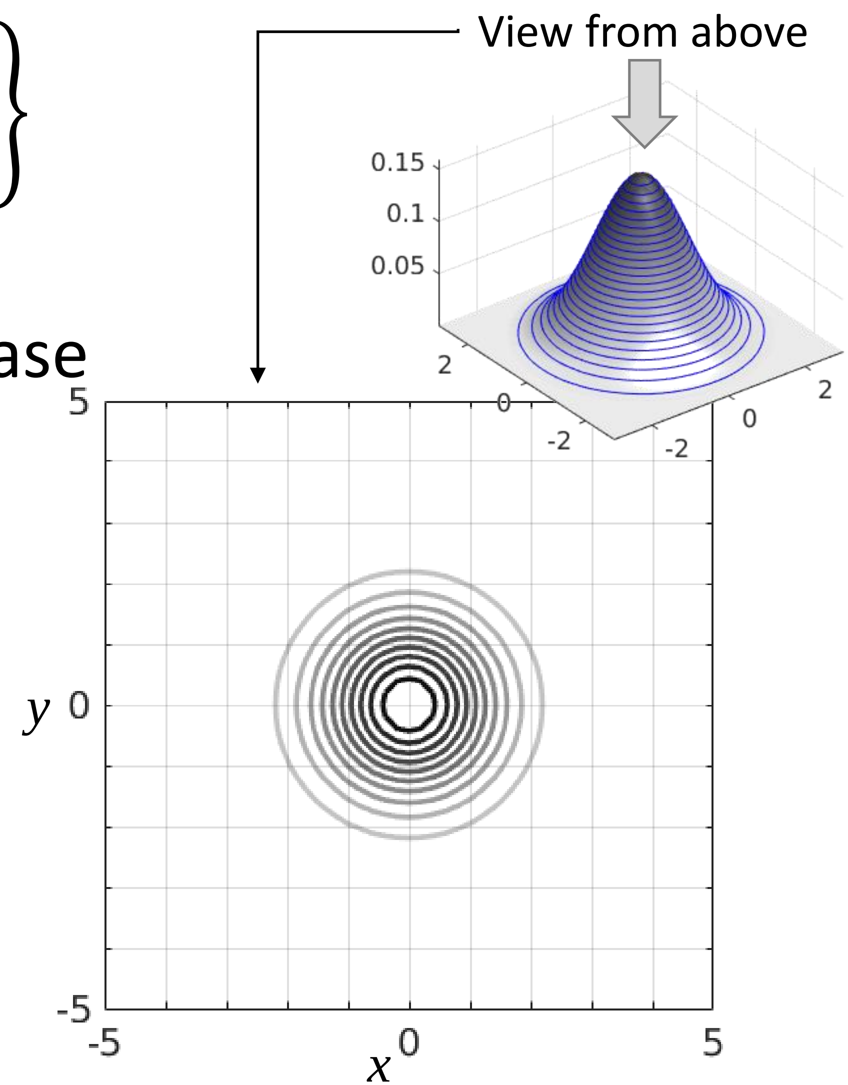


Multivariate Gaussian: 2D

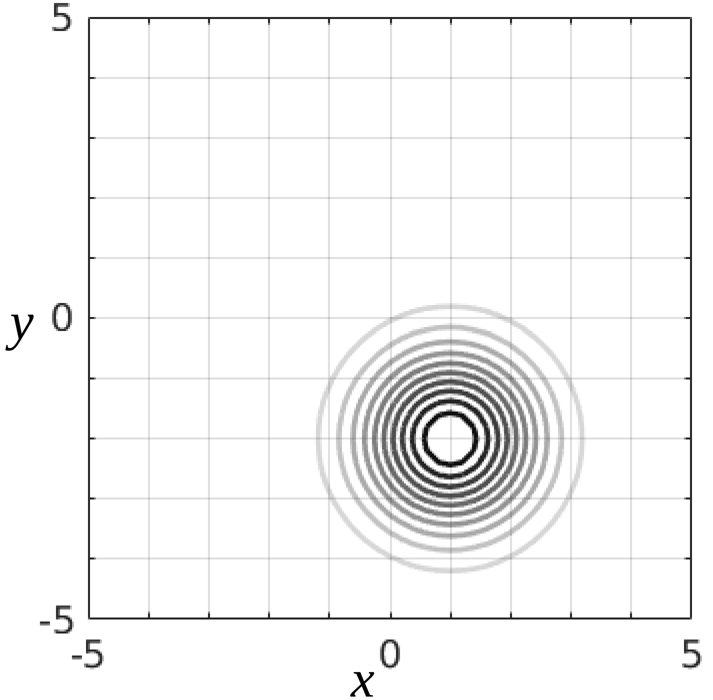
$$p(\mathbf{x}) = \frac{1}{2\pi} \exp\left\{-\frac{(x-\mu_x)^2 + (y-\mu_y)^2}{2}\right\}$$

$$D = 2$$

$$\mathbf{x} = [x \quad y]^T$$

$$\boxed{\boldsymbol{\mu} = [\mu_x \quad \mu_y]^T}$$

$$\Sigma = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$



Multivariate Gaussian: 2D

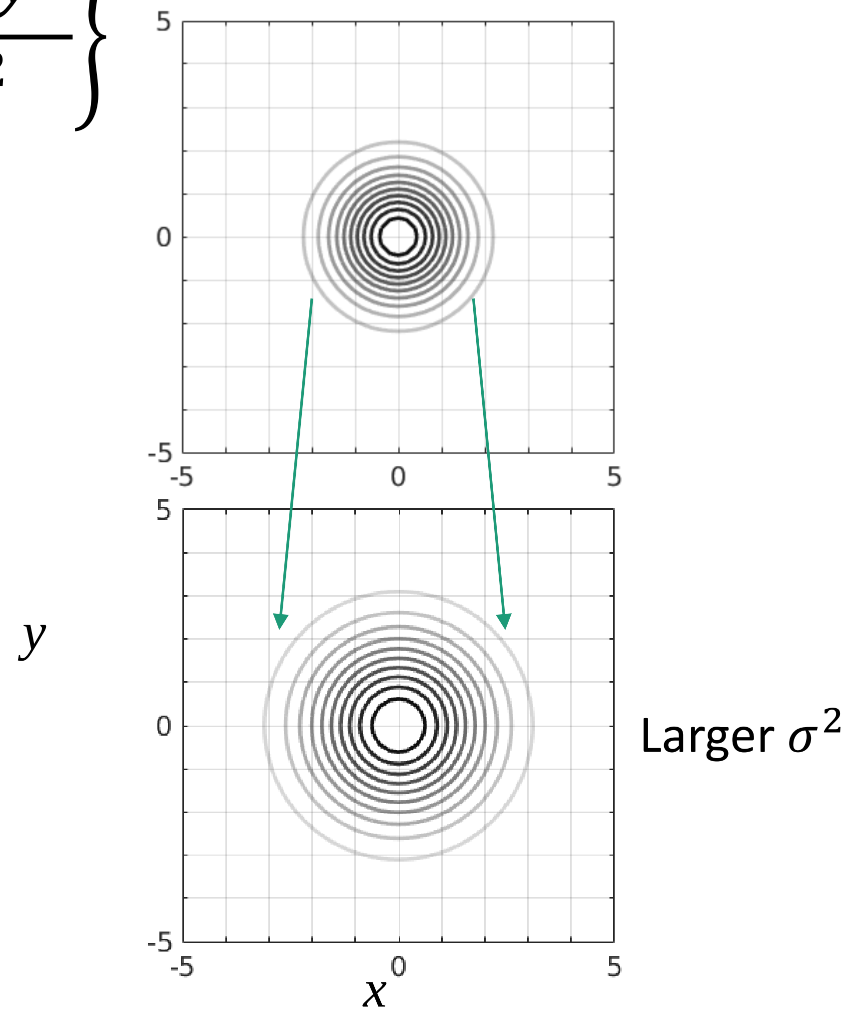
$$p(\mathbf{x}) = \frac{1}{2\pi\sigma^2}\exp\left\{-\frac{x^2+y^2}{2\sigma^2}\right\}$$

$$D = 2$$

$$\mathbf{x} = [x \quad y]^T$$

$$\boldsymbol{\mu} = [0 \quad 0]^T$$

$$\boxed{\Sigma = \begin{bmatrix} \sigma^2 & 0 \\ 0 & \sigma^2 \end{bmatrix}}$$



Multivariate Gaussian: 2D

$$p(\mathbf{x}) = \frac{1}{2\pi\sigma^2} \exp\left\{-\frac{x^2 + y^2}{2\sigma^2}\right\}$$

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$$\mathbf{x} = [x \quad y]^T$$

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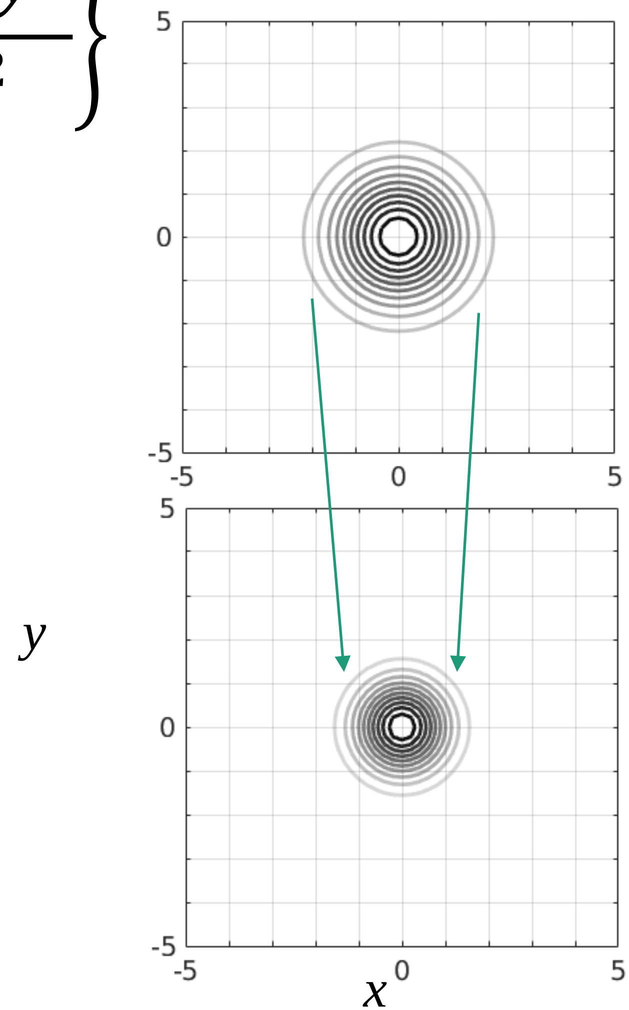
$$\boxed{\Sigma = \begin{bmatrix} \sigma^2 & 0 \\ 0 & \sigma^2 \end{bmatrix}}$$



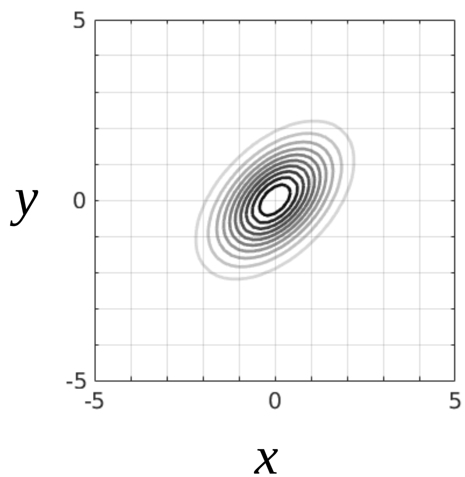
Multivariate Gaussian: 2D

$$p(\mathbf{x}) = \frac{1}{(2\pi)^{D/2}|\Sigma|^{1/2}} \exp\left\{-\frac{1}{2}(\mathbf{x}-\boldsymbol{\mu})^{\boldsymbol{T}}\Sigma^{-1}(\mathbf{x}-\boldsymbol{\mu})\right\}$$

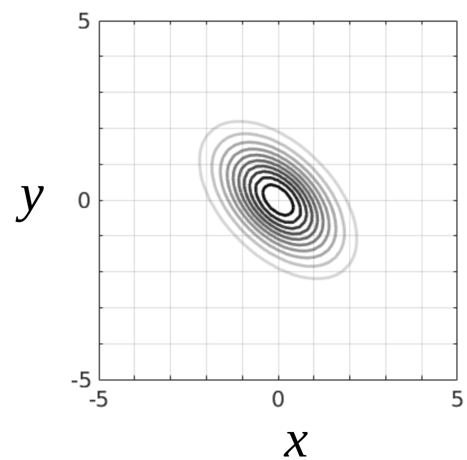
- 2D General Case

$$\Sigma = \begin{bmatrix} \sigma_x^2 & \sigma_{xy}^2 \\ \sigma_{xy}^2 & \sigma_y^2 \end{bmatrix}$$

$\sigma_{xy}^2 > 0$



$\sigma_{xy}^2 < 0$



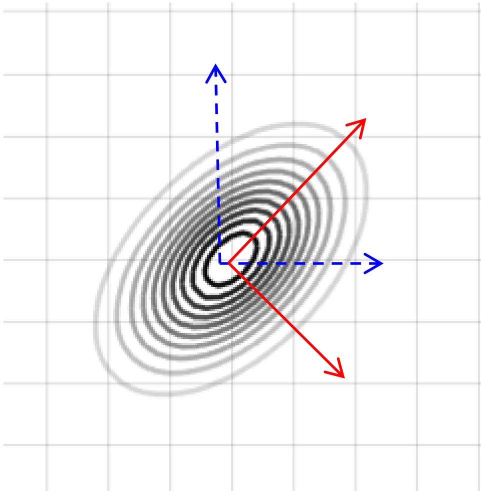
Multivariate Gaussian: 2D

$$p(\mathbf{x}) = \frac{1}{(2\pi)^{D/2}|\Sigma|^{1/2}} \exp\left\{-\frac{1}{2}(\mathbf{x}-\boldsymbol{\mu})^{\boldsymbol{T}}\Sigma^{-1}(\mathbf{x}-\boldsymbol{\mu})\right\}$$

- Properties of Covariance Matrix Σ

1) Σ is Symmetric and Positive Definite.

2) Diagonalization: Σ can be decomposed in the form of UDU^T.
(D is a Diagonal matrix.)



Multivariate Gaussian: 2D

$$p(\mathbf{x}) = \frac{1}{(2\pi)^{D/2}|\Sigma|^{1/2}} \exp\left\{-\frac{1}{2}(\mathbf{x}-\boldsymbol{\mu})^T\Sigma^{-1}(\mathbf{x}-\boldsymbol{\mu})\right\}$$