Densities in two and more dimensions

Two dimensions

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$\frac{\text{mass}}{\text{area}} \times \text{area} = \text{mass}$

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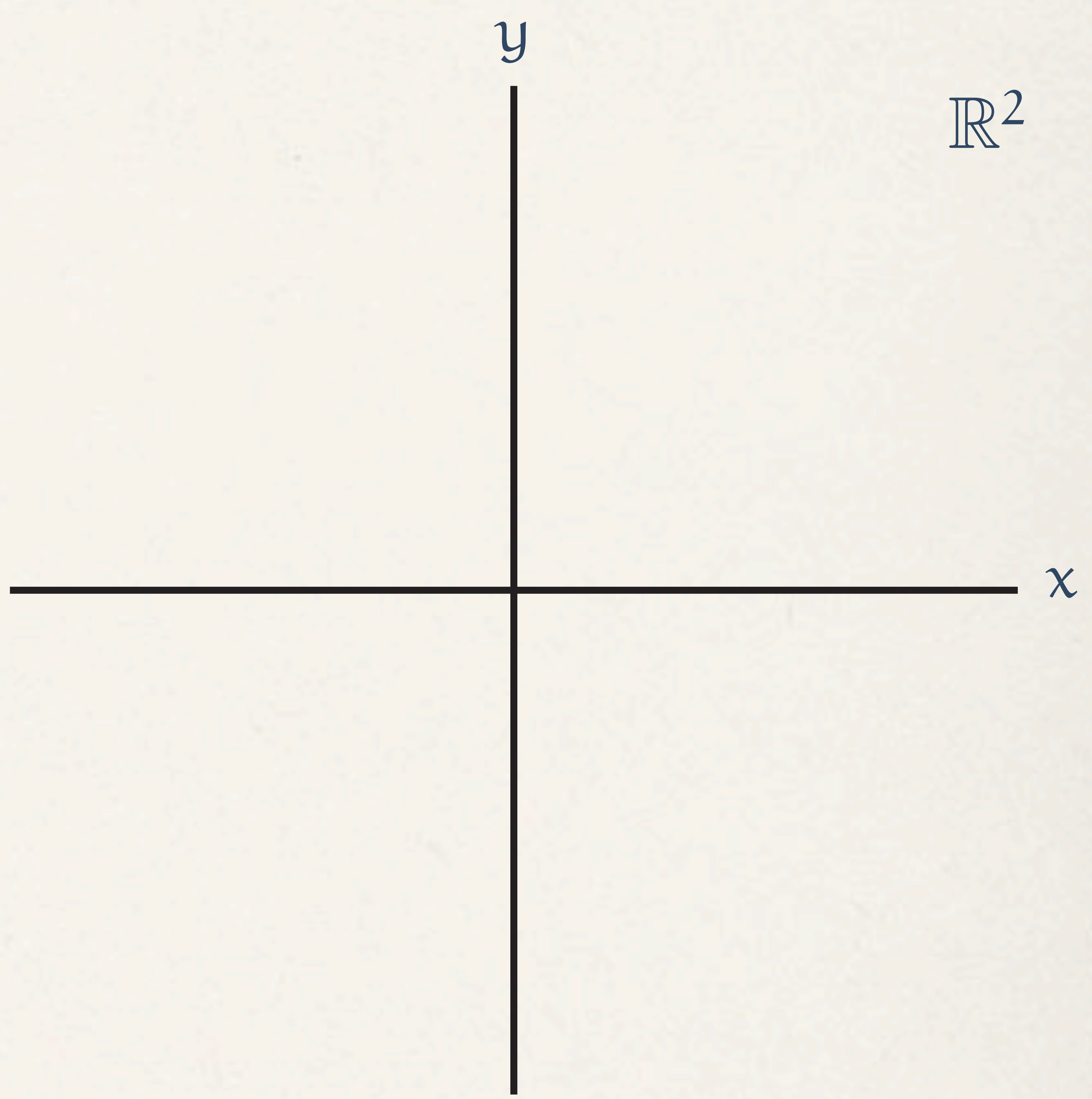
Euclidean plane sample space:

$$\Omega = \mathbb{R}^2 := \{ (x, y) : -\infty < x, y < \infty \}$$



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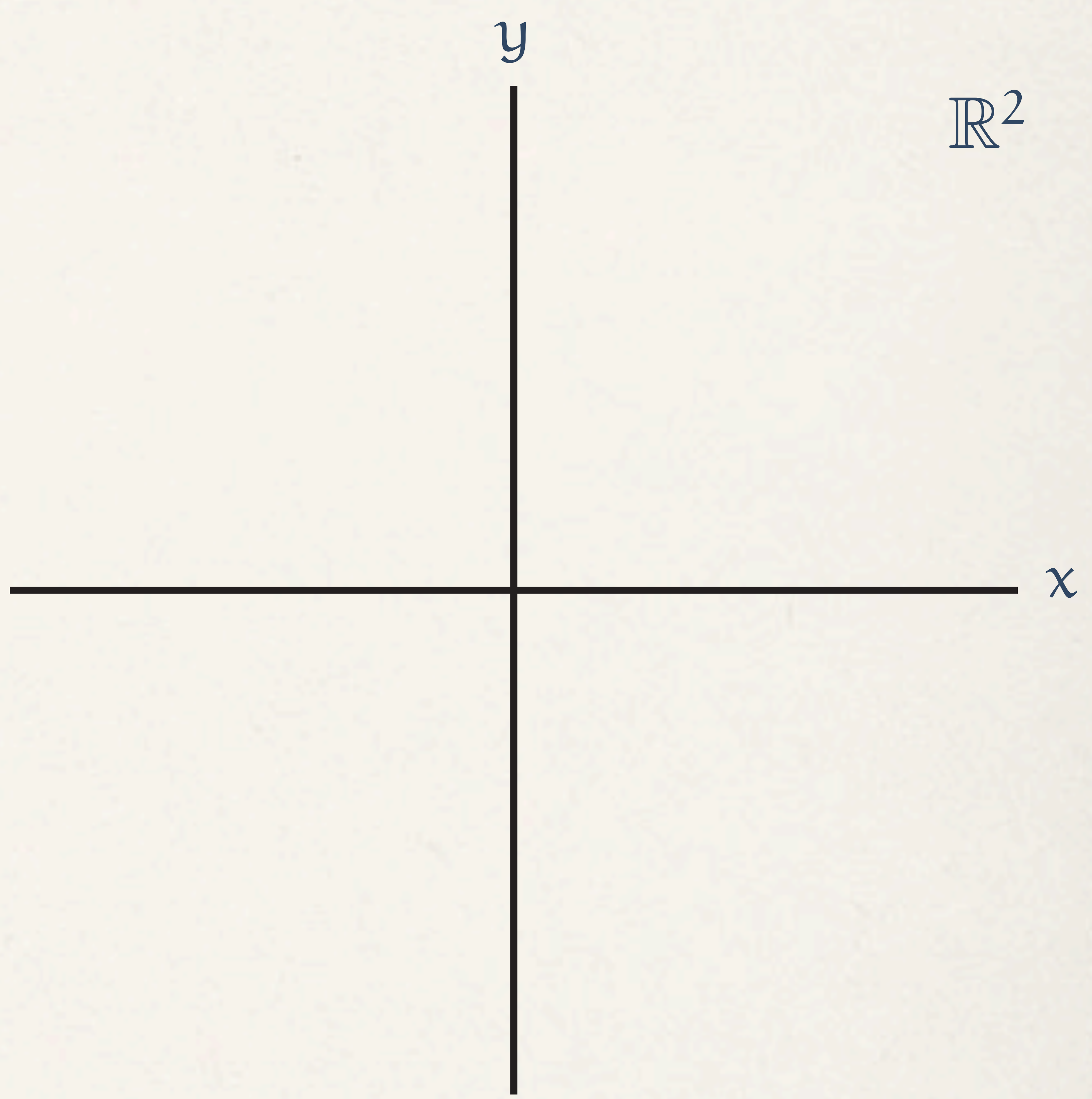

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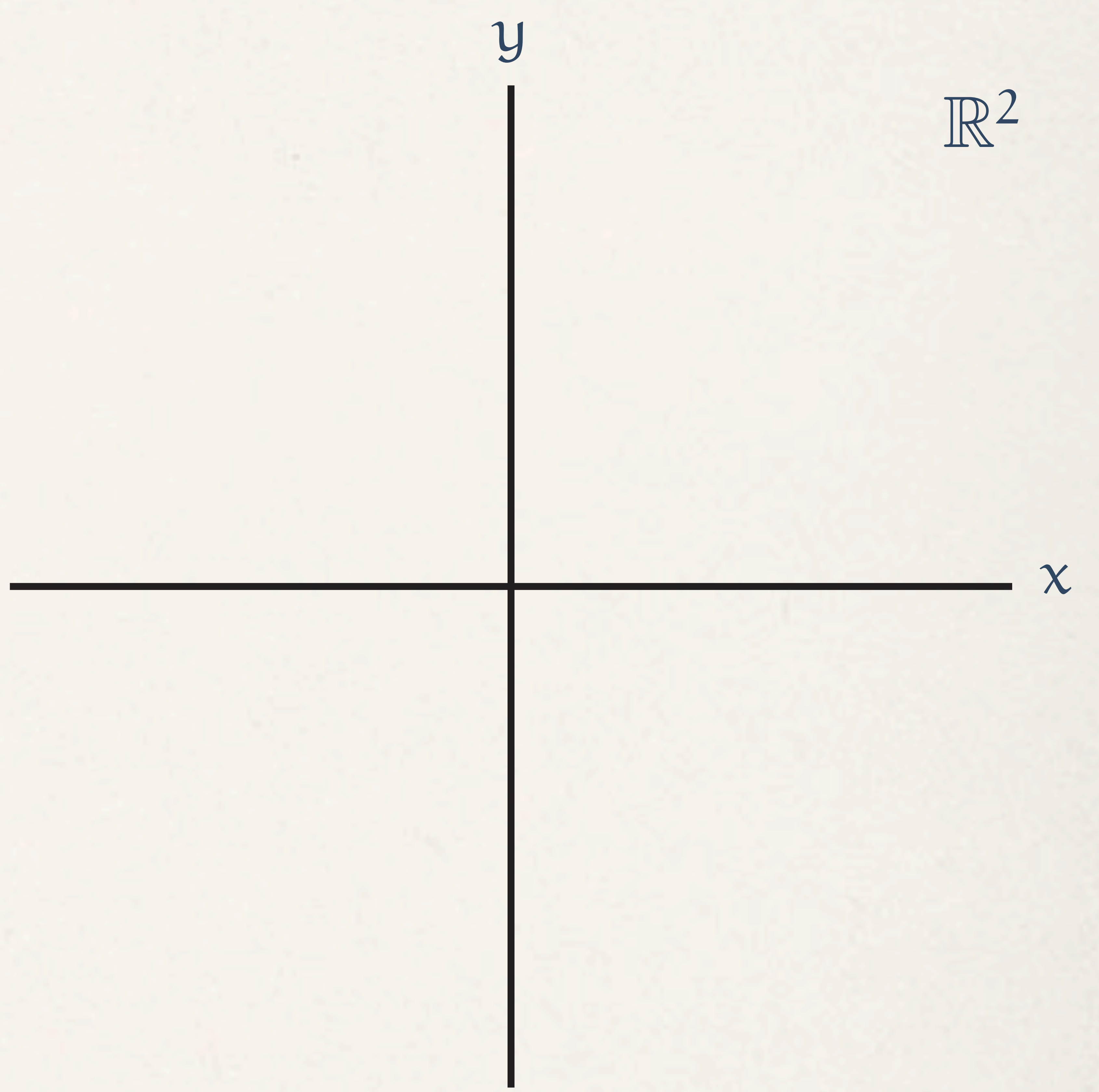

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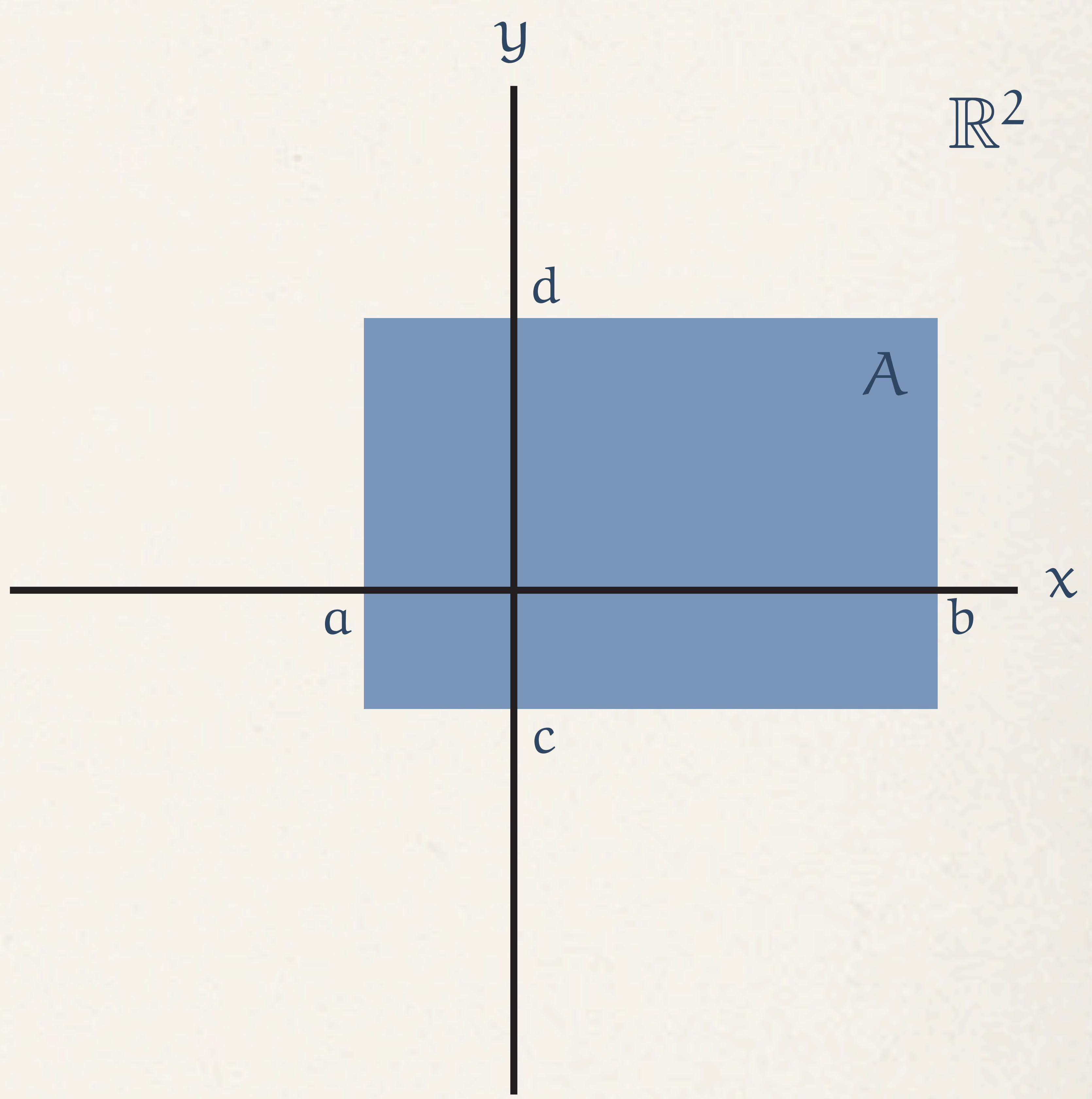

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basic region: rectangle

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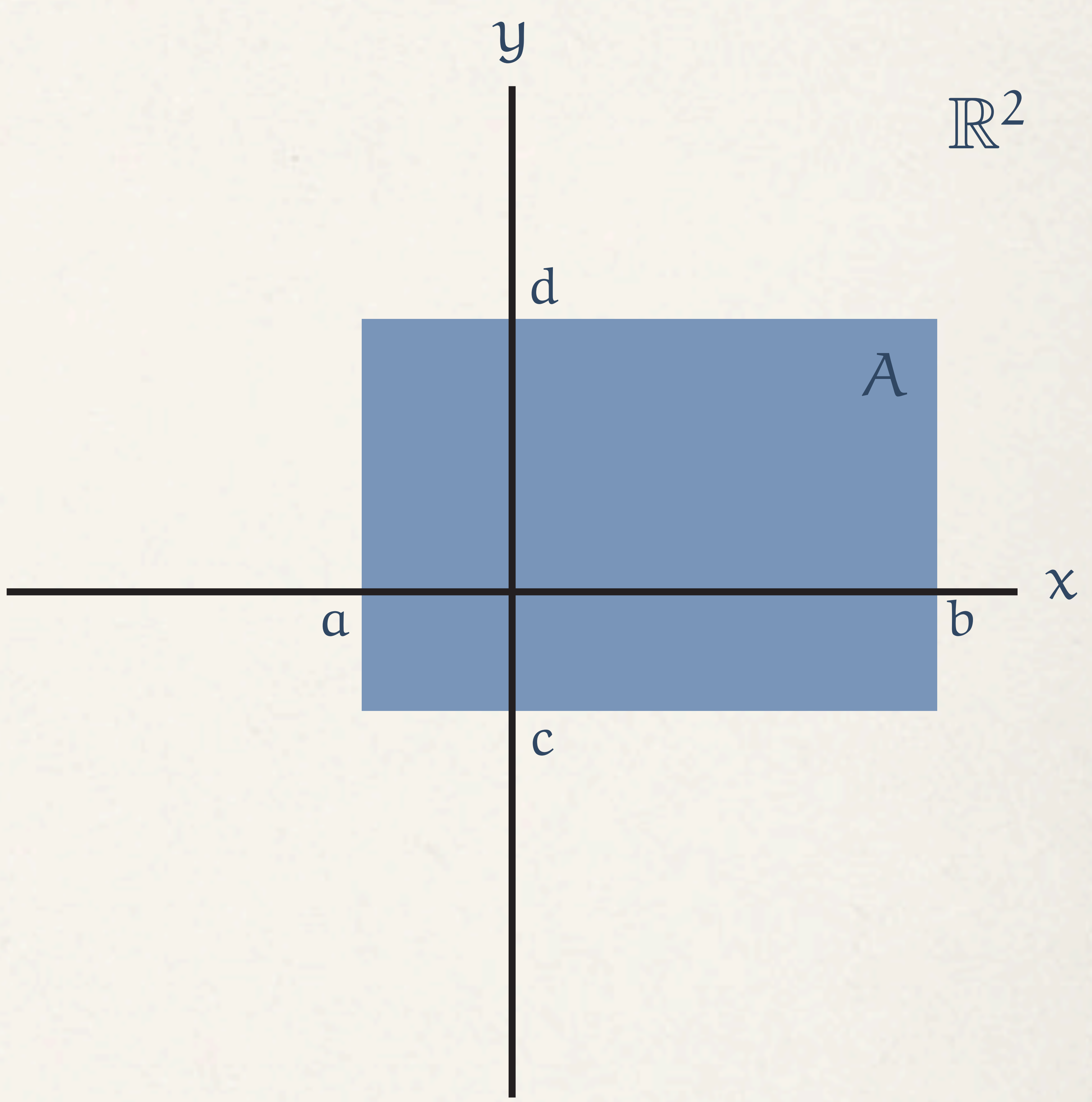

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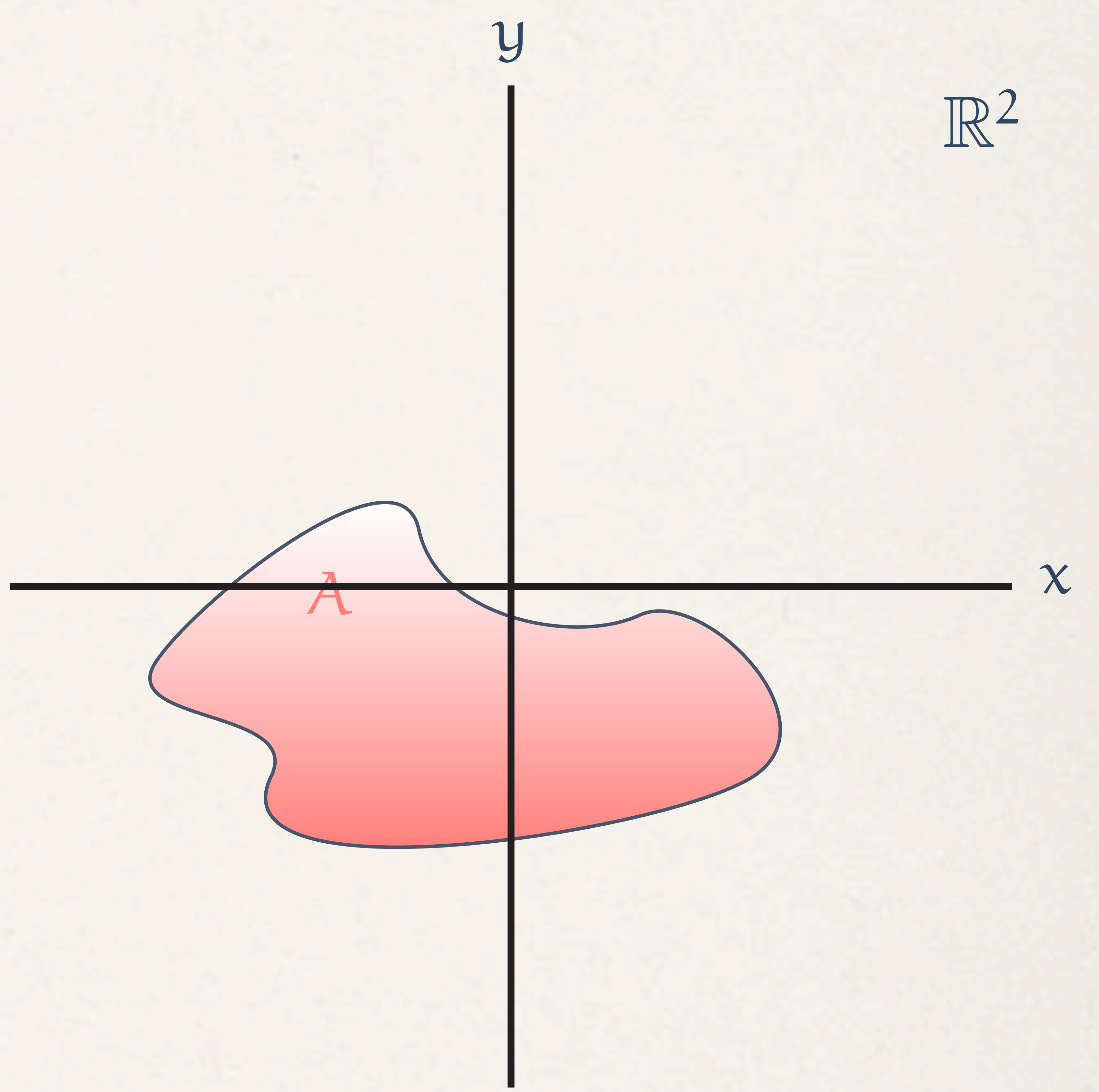

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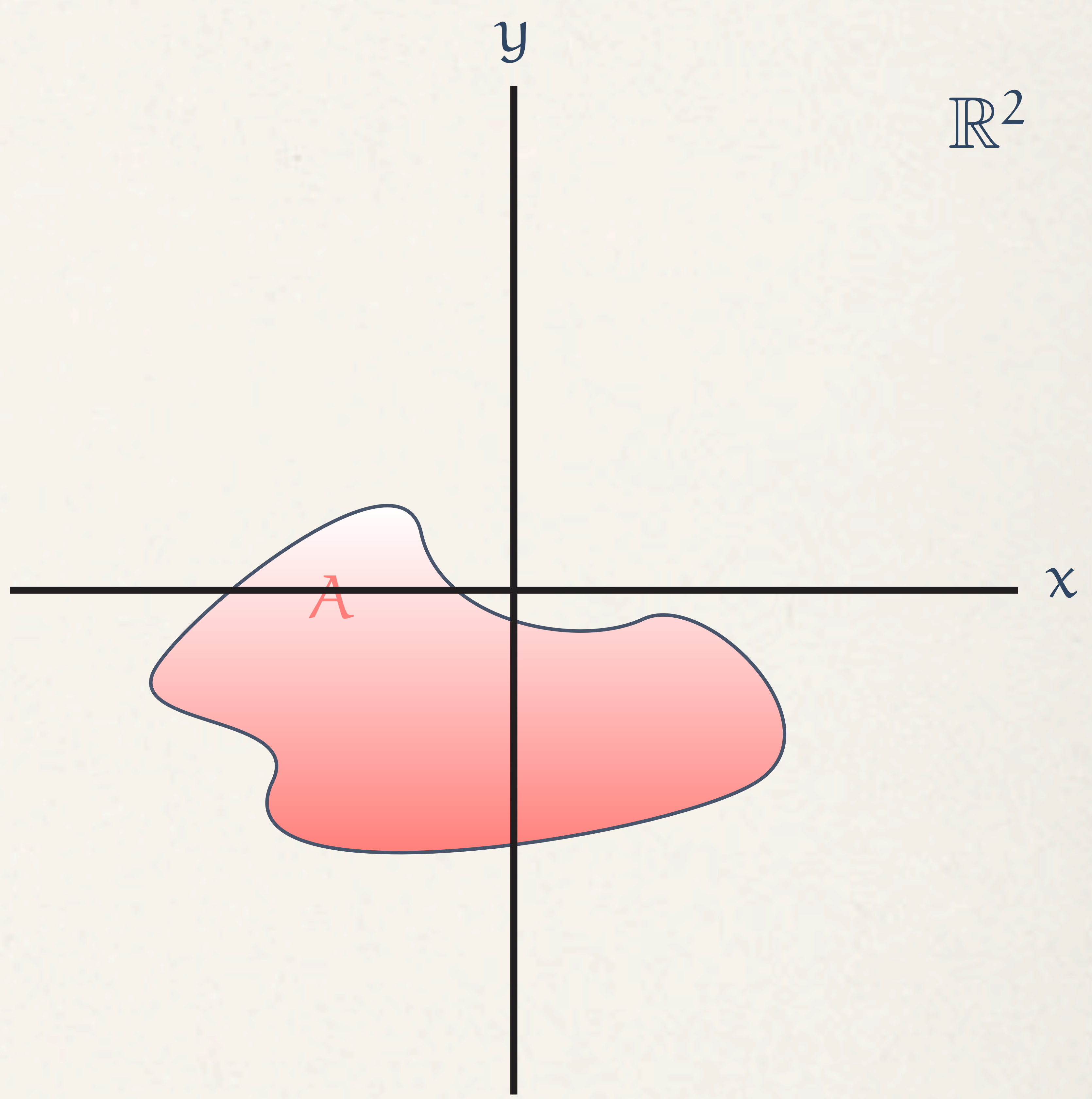

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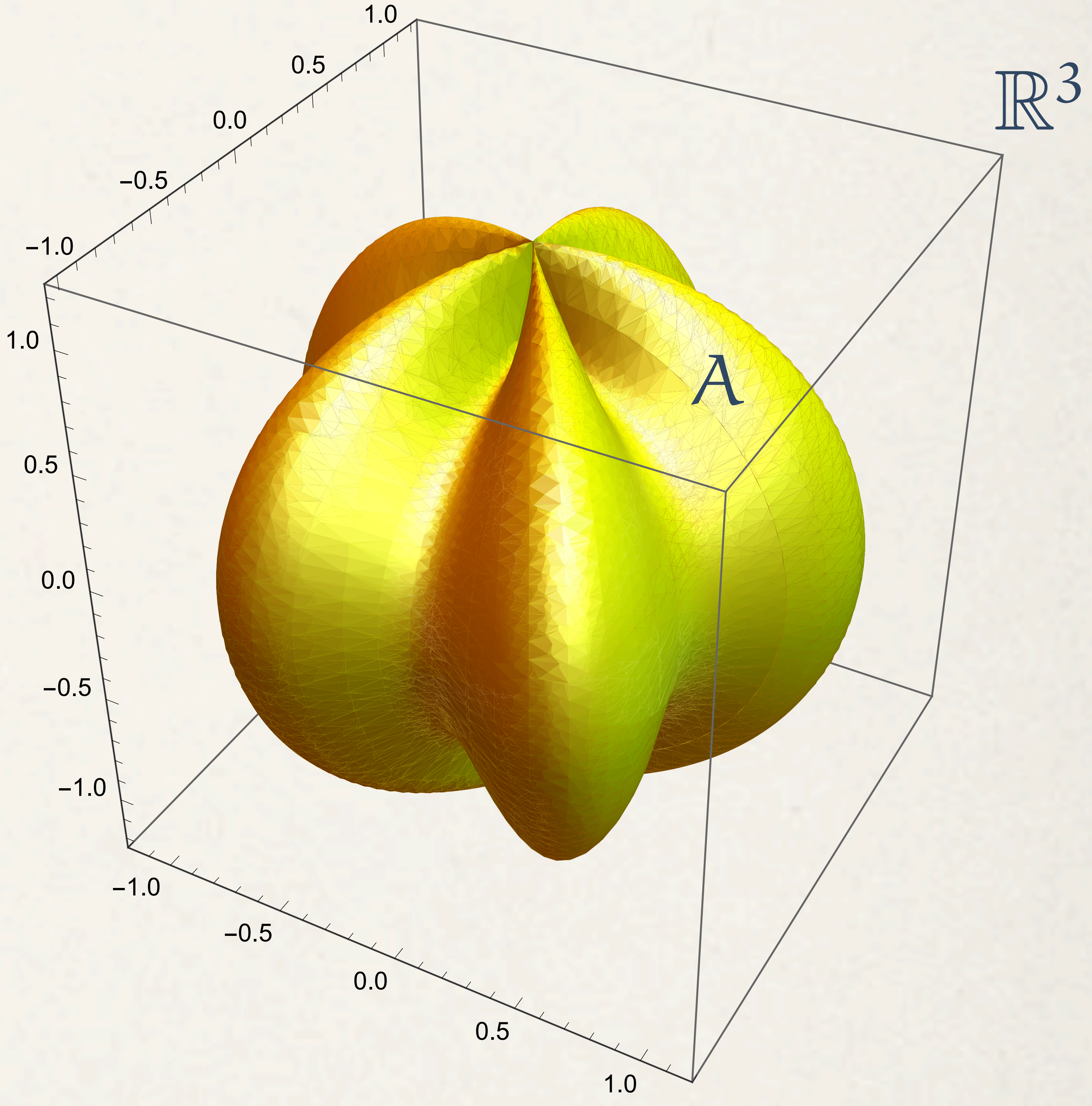

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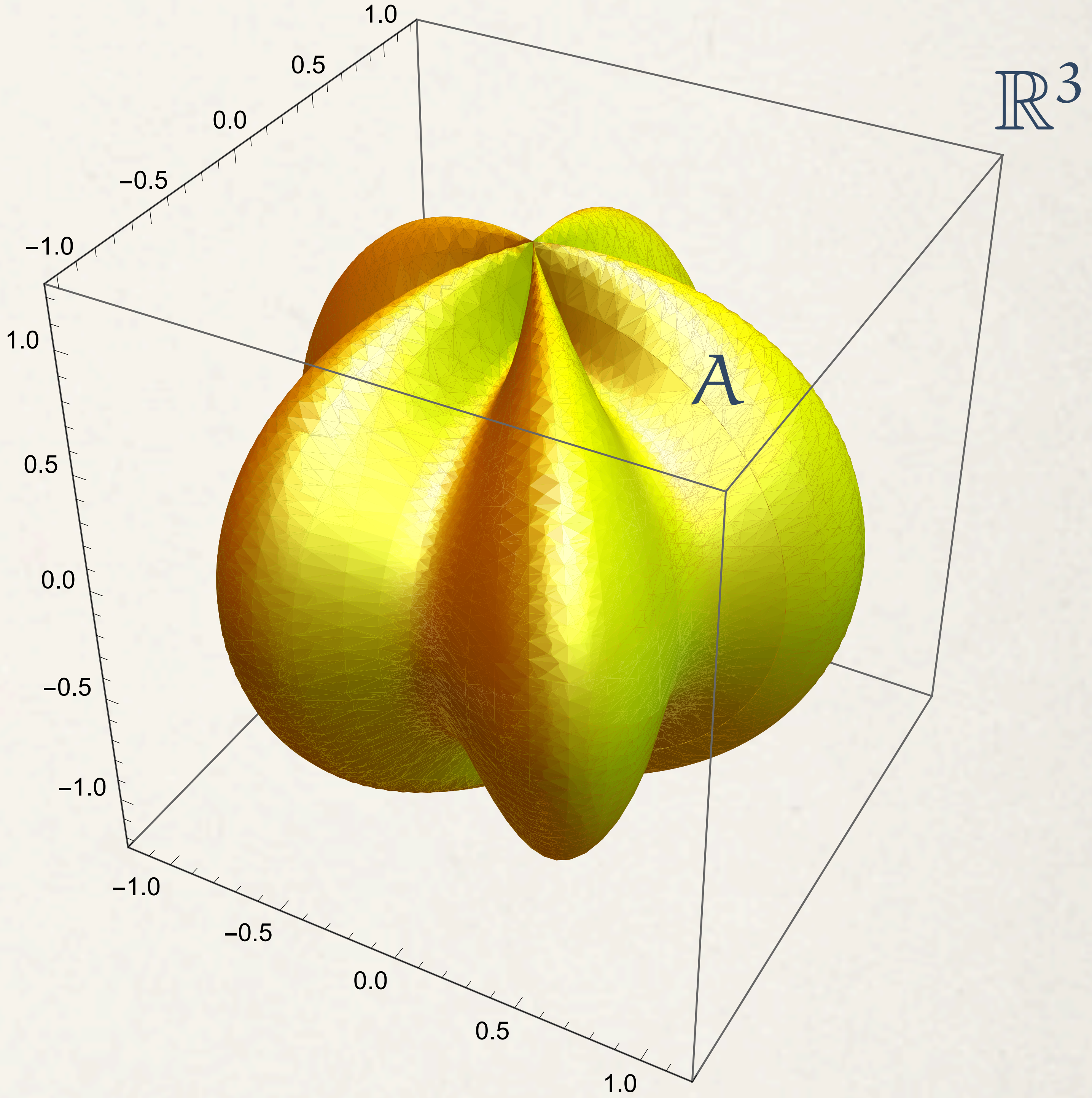

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