QUESTION 3

The *Sandwich Theorem* (also sometimes called the *Squeeze Theorem*) says that if $\{a_n\}_{n=1}^{\infty}$, $\{b_n\}_{n=1}^{\infty}$, $\{c_n\}_{n=1}^{\infty}$ are sequences such that, from some point n_0 onwards,

$$a_n \leq b_n \leq c_n,$$

and if

$$\lim_{n\to\infty} a_n = L \ , \ \lim_{n\to\infty} c_n = L,$$

then $\{b_n\}_{n=1}^{\infty}$ is convergent and

$$\lim_{n\to\infty} b_n = L.$$

Taking the Sandwich Theorem to be correct (which it is), grade the following proof.

Theorem $\lim_{n\to\infty} \dfrac{\sin^2 n}{3^n} = 0$

Proof: For any n,

$$0 \leq \frac{\sin^2 n}{3^n} \leq \frac{1}{3^n}$$

Clearly, $\lim_{n\to\infty} \dfrac{1}{3^n} = 0$. Hence, by the Sandwich Theorem,

$$\lim_{n\to\infty} \frac{\sin^2 n}{3^n} = 0$$

as required.