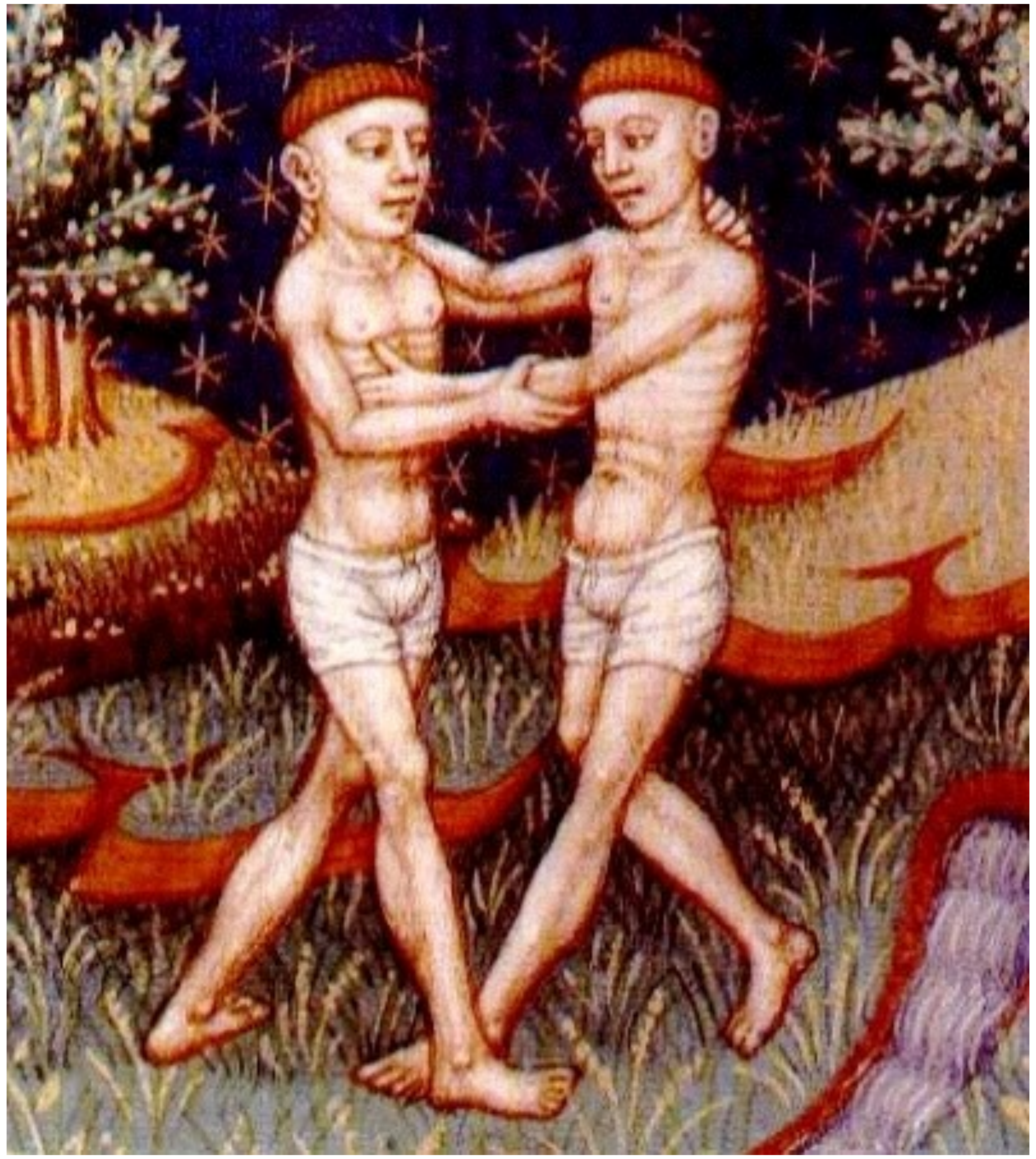

# Putting a linear program in form appropriate for taking dual

$$\max \mathbf{c} \cdot \mathbf{x} :$$
$$\mathbf{A}\mathbf{x} \leq \mathbf{b}$$
$$\mathbf{x} \geq \mathbf{0}$$

## Proof by example

$$\min \mathbf{2x_1 - 3x_2 + x_3} :$$
$$\mathbf{x_1 + x_2 = 4}$$
$$\mathbf{x_2 - 4x_3 \geq 5}$$
$$\mathbf{x_2 \geq 0}$$

1. Transform min into max

$$\begin{aligned} \max\ & \mathbf{c} \cdot \mathbf{x} : \\ & \mathbf{A}\mathbf{x} \leq \mathbf{b} \\ & \mathbf{x} \geq \mathbf{0} \end{aligned}$$

$$\begin{aligned} \min\ & \mathbf{2x_1 - 3x_2 + x_3} : \\ & \mathbf{x_1 + x_2 = 4} \\ & \mathbf{x_2 - 4x_3 \geq 5} \\ & \mathbf{x_2 \geq 0} \end{aligned}$$

$$\Downarrow$$

$$\begin{aligned} \max\ & \mathbf{-2x_1 + 3x_2 - x_3} : \\ & \mathbf{x_1 + x_2 = 4} \\ & \mathbf{x_2 - 4x_3 \geq 5} \\ & \mathbf{x_2 \geq 0} \end{aligned}$$

2. Transform equalities into inequalities

$$\max \mathbf{c} \cdot \mathbf{x} :$$
$$\mathbf{A}\mathbf{x} \leq \mathbf{b}$$
$$\mathbf{x} \geq \mathbf{0}$$

$$\max -\mathbf{2x_1} + \mathbf{3x_2} - \mathbf{x_3} :$$
$$\mathbf{x_1} + \mathbf{x_2} = \mathbf{4}$$
$$\mathbf{x_2} - \mathbf{4x_3} \geq \mathbf{5}$$
$$\mathbf{x_2} \geq \mathbf{0}$$

$$\Downarrow$$

$$\max -\mathbf{2x_1} + \mathbf{3x_2} - \mathbf{x_3} :$$
$$\mathbf{x_1} + \mathbf{x_2} \geq \mathbf{4}$$
$$\mathbf{x_1} + \mathbf{x_2} \leq \mathbf{4}$$
$$\mathbf{x_2} - \mathbf{4x_3} \geq \mathbf{5}$$
$$\mathbf{x_2} \geq \mathbf{0}$$

## 3. Make inequalities in the correct direction

$$\max \mathbf{c} \cdot \mathbf{x} :$$
$$\mathbf{A}\mathbf{x} \leq \mathbf{b}$$
$$\mathbf{x} \geq \mathbf{0}$$

$$\max -2\mathbf{x_1} + 3\mathbf{x_2} - \mathbf{x_3} :$$
$$\mathbf{x_1} + \mathbf{x_2} \geq 4$$
$$\mathbf{x_1} + \mathbf{x_2} \leq 4$$
$$\mathbf{x_2} - 4\mathbf{x_3} \geq 5$$
$$\mathbf{x_2} \geq 0$$

$$\Downarrow$$

$$\max -2\mathbf{x_1} + 3\mathbf{x_2} - \mathbf{x_3} :$$
$$-\mathbf{x_1} - \mathbf{x_2} \leq -4$$
$$\mathbf{x_1} + \mathbf{x_2} \leq 4$$
$$-\mathbf{x_2} + 4\mathbf{x_3} \leq -5$$
$$\mathbf{x_2} \geq 0$$

## 4. Reduce to non-negative variables

$$\max \mathbf{c} \cdot \mathbf{x} :$$
$$\mathbf{A}\mathbf{x} \leq \mathbf{b}$$
$$\mathbf{x} \geq \mathbf{0}$$

$$\max -2x_1 + 3x_2 - x_3 :$$
$$-x_1 - x_2 \leq -4$$
$$x_1 + x_2 \leq 4$$
$$-x_2 + 4x_3 \leq -5$$
$$x_2 \geq 0$$

$$\Downarrow$$

$$\max -2(x_1^+ - x_1^-) + 3x_2 - (x_3^+ - x_3^-) :$$
$$-(x_1^+ - x_1^-) - x_2 \leq -4$$
$$(x_1^+ - x_1^-) + x_2 \leq 4$$
$$-x_2 + 4(x_3^+ - x_3^-) \leq -5$$
$$x_2, x_1^+, x_1^-, x_3^+, x_3^- \geq 0$$

Done!