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An old friend … and a new vantage point

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An old friend … and a new vantage point

Indicator for the occurrence of the "bad" event

For $j = 1, \ldots, n$: $\quad X_j = \begin{cases} 1 & \text{if } A_j \text{ occurs,} \\ 0 & \text{otherwise.} \end{cases}$

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For $j = 1, \dots, n$: $$X_j = \begin{cases} 1 & \text{if } A_j \text{ occurs,} \\ 0 & \text{otherwise.} \end{cases}$$

$$M_n = X_1 + X_2 + \cdots + X_n$$

Accumulated sum of "bad" events

definition

$$S_0 := 1$$

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A recasting of the inclusion–exclusion theorem

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Accumulated sum of "bad" events

A recasting of the inclusion–exclusion theorem

$$\mathbf{P}(A_1^\complement \cap A_2^\complement \cap \cdots \cap A_n^\complement) = 1 - \mathbf{P}(A_1 \cup A_2 \cup \cdots \cup A_n)$$

definition

$$S_0 := 1$$

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An old friend … and a new vantage point

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Accumulated sum of "bad" events

A recasting of the inclusion–exclusion theorem

$$\mathbf{P}\{M_n = 0\} = \mathbf{P}(A_1^c \cap A_2^c \cap \cdots \cap A_n^c) = 1 - \mathbf{P}(A_1 \cup A_2 \cup \cdots \cup A_n) = S_0 - S_1 + S_2 - S_3 + \cdots + (-1)^n S_n$$

For $j = 1, \dots, n$: $$X_j = \begin{cases} 1 & \text{if } A_j \text{ occurs,} \\ 0 & \text{otherwise.} \end{cases}$$

$$M_n = X_1 + X_2 + \cdots + X_n$$

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The inclusion–exclusion theorem (industrial strength)

$$\mathbf{P}\{M_n = 0\} = S_0 - S_1 + S_2 - S_3 + \cdots + (-1)^n S_n$$

$$\mathbf{P}\{M_n = k\} = \sum_{j=0}^{n-k} (-1)^j \binom{j+k}{k} S_{j+k}$$

For $j = 1, \ldots, n$: $\quad X_j = \begin{cases} 1 & \text{if } A_j \text{ occurs,} \\ 0 & \text{otherwise.} \end{cases}$

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