Maxcut

Solve the quadratic relaxation for Maxcut

$$\max \sum_{\{\mathbf{i},\mathbf{j}\}\in \mathbf{E}} \mathbf{w_{ij}} \frac{-\mathbf{v_i}\cdot\mathbf{v_j}+\mathbf{1}}{\mathbf{2}} \quad :$$

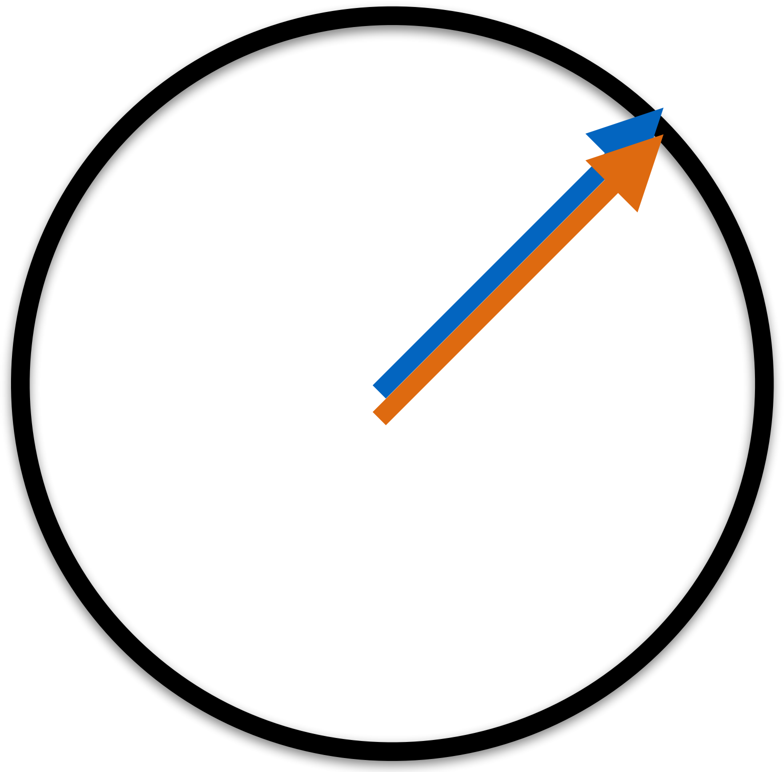
$$\mathbf{v_i}\cdot\mathbf{v_i} = \mathbf{1}$$

How do we round?

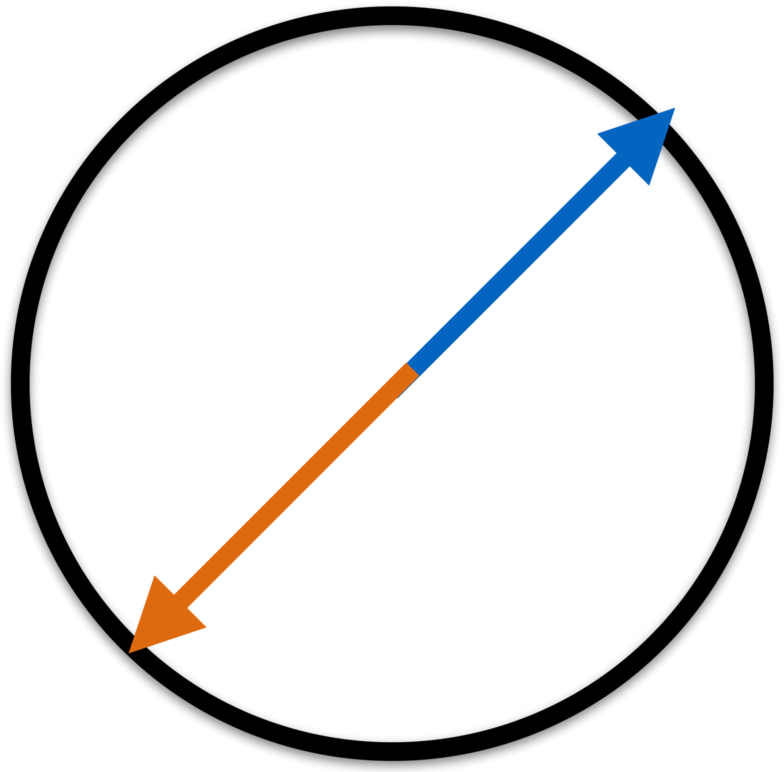
$$\mathbf{v_i} \mapsto \mathbf{x_i} \in \{-1, 1\}$$

Value(Output): $\sum_{\{i,j\}\in E} w_{ij} \frac{-x_i x_j + 1}{2}$

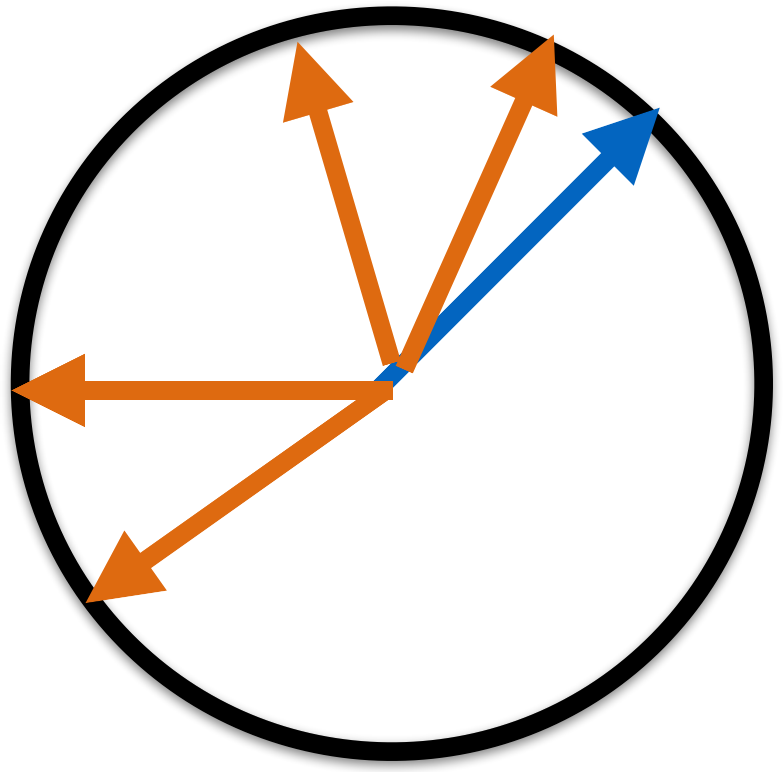
OPT: $\leq \sum_{\{i,j\}\in E} w_{ij} \frac{-v_i \cdot v_j + 1}{2}$



Want: $x_i = x_j$



Want: $x_i = -x_j$



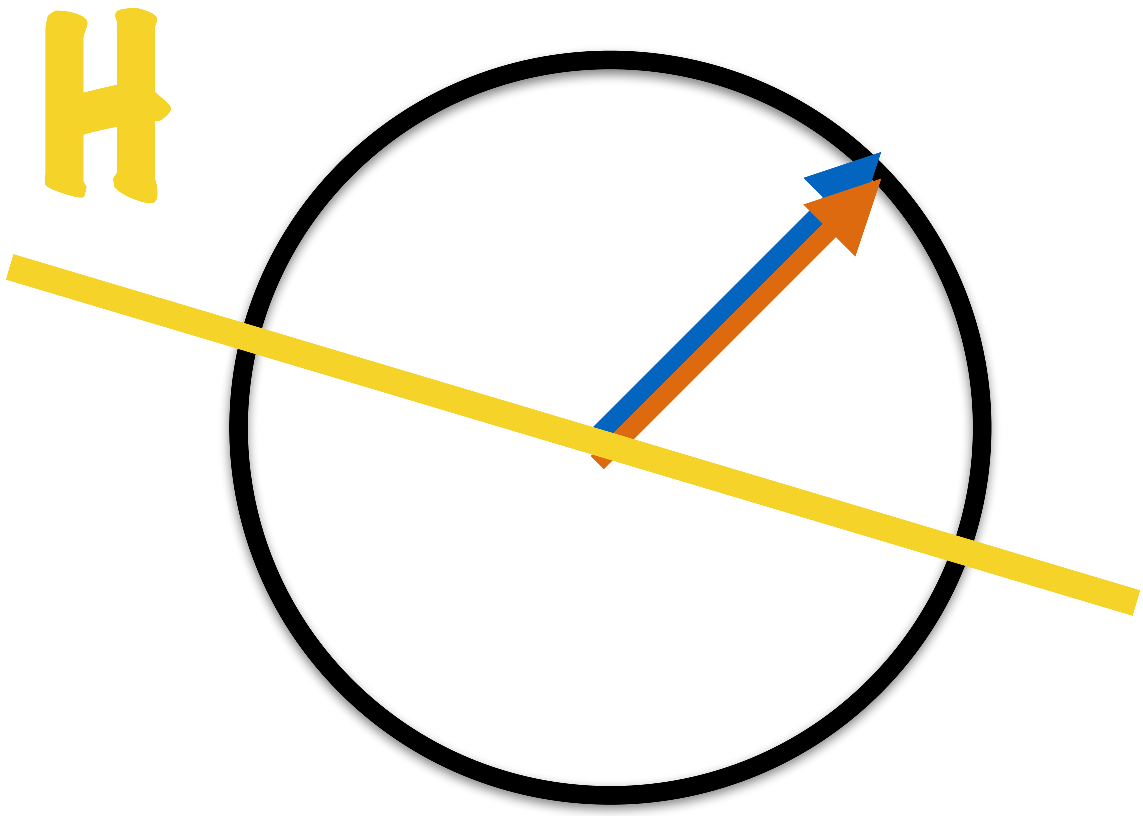
Want: ?

Randomized rounding

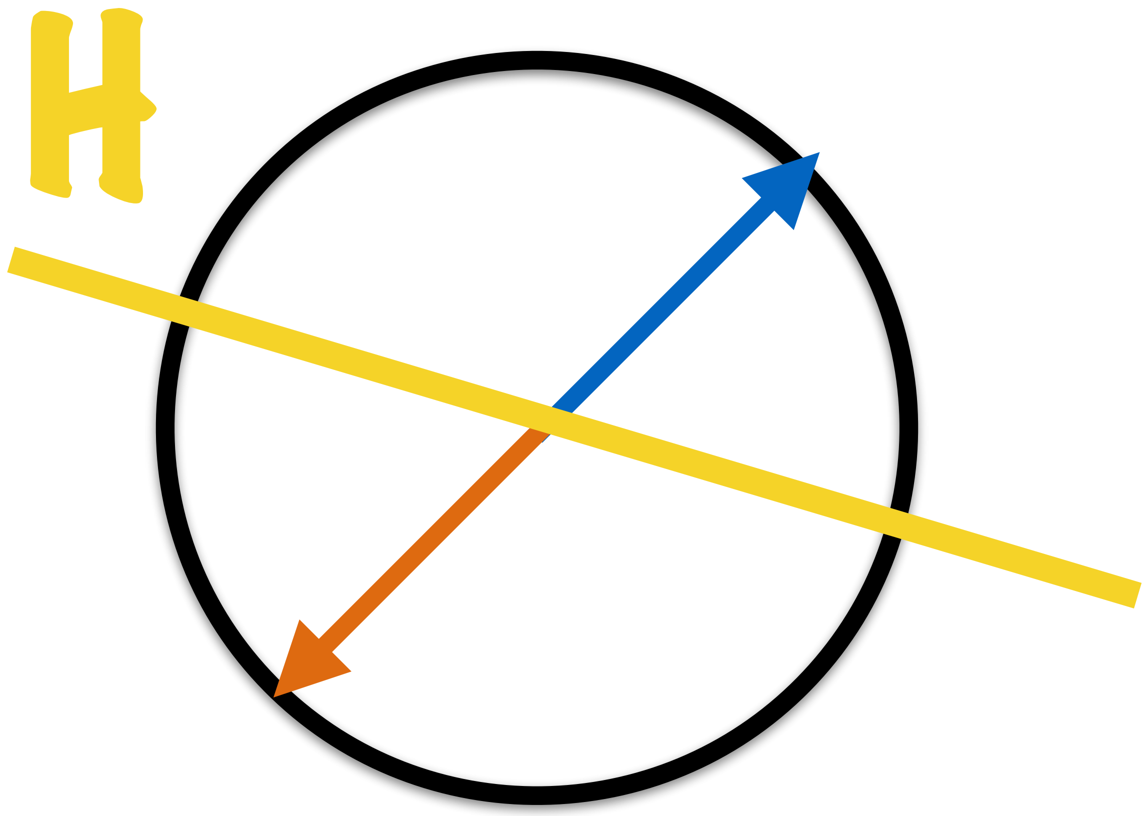
Random line (hyperplane) H

above H: $\mathrm{v_i} \mapsto 1$

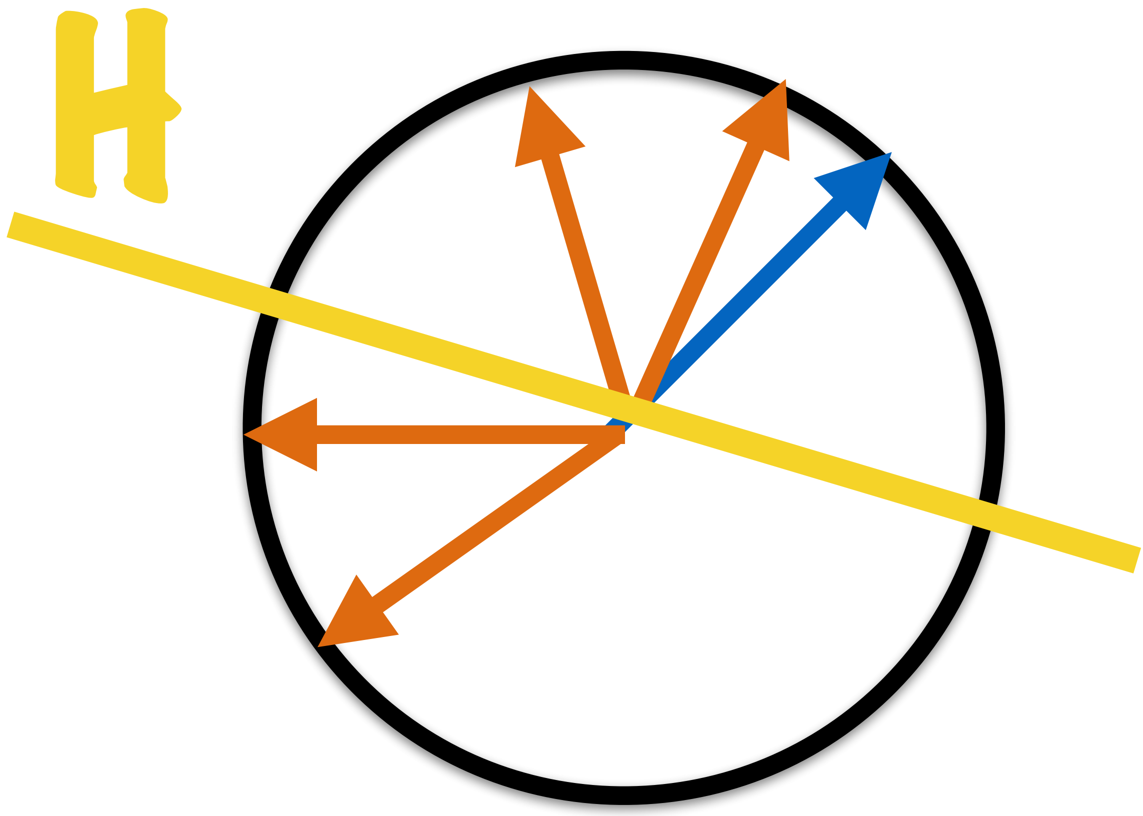
below H: $\mathrm{v_i} \mapsto -1$



Want: $\mathrm{x_i} = \mathrm{x_j}$



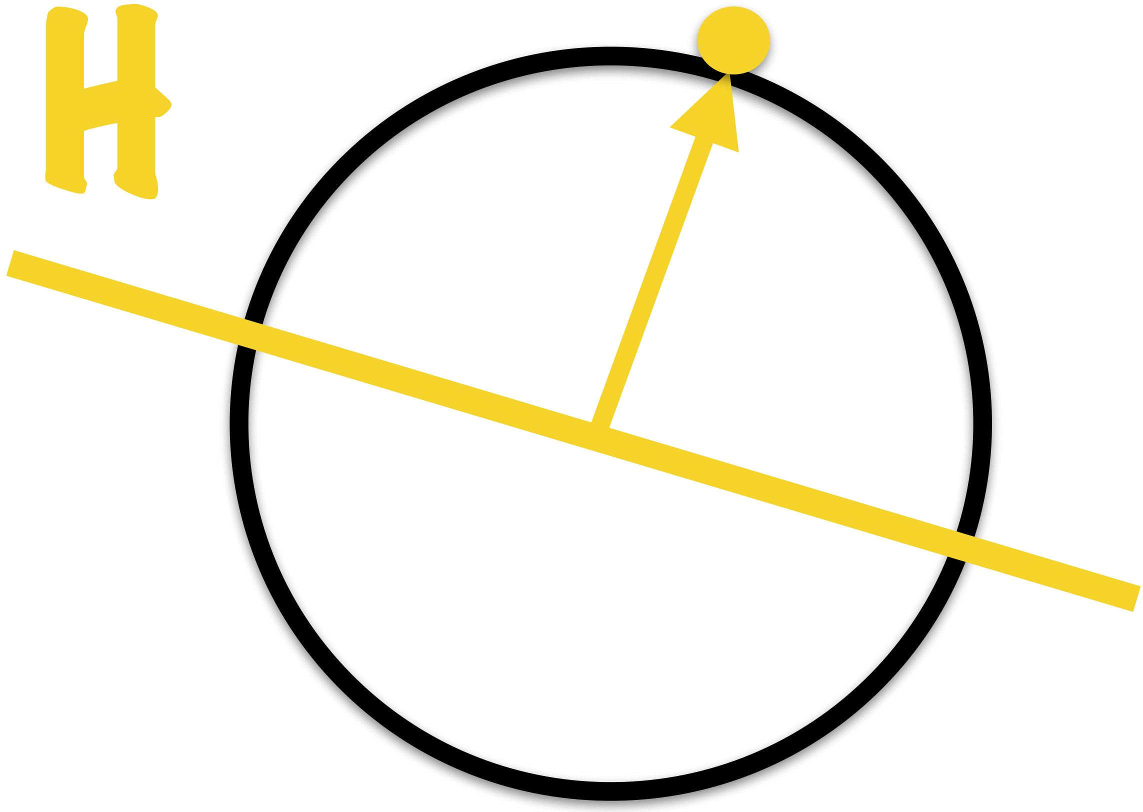
Want: $\mathrm{x_i} = -\mathrm{x_j}$



Want: ?

How do we pick a random hyperplane?

Pick point uniformly at random on unit sphere
It defines a vector
H: normal hyperplane through the origin.

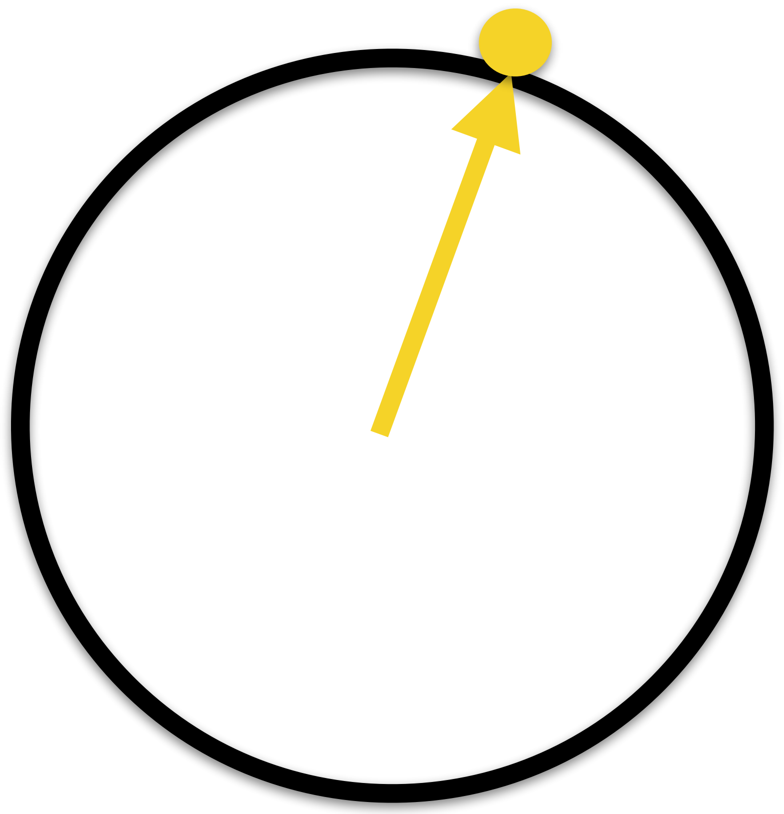


How to pick point on unit sphere?

Coordinate i: $\mathrm{x_i} \leftarrow \mathcal{N}(0,1)$

Unit vector $\mathbf{r} \leftarrow \mathbf{x}/||\mathbf{x}||$

Or: spherical coordinates
radius = 1
angles = independent uniform



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