A simple explanation of the Lasso and Least Angle Regression

Give a set of input measurements x1, x2 ...xp and an outcome measurement y, the lasso fits a linear model

yhat=b0 + b1*x1+ b2*x2 + ... bp*xp

The criterion it uses is:

Minimize sum((y-yhat)^2) subject to sum[absolute value(bj)] <= s

The first sum is taken over observations (cases) in the dataset. The bound "s" is a tuning parameter. When "s" is large enough, the constraint has no effect and the solution is just the usual multiple linear least squares regression of y on x1, x2, ...xp.

However when for smaller values of s (s>=0) the solutions are shrunken versions of the least squares estimates. Often, some of the coefficients bj are zero. Choosing "s" is like choosing the number of predictors to use in a regression model, and cross-validation is a good tool for estimating the best value for "s".

Computation of the Lasso solutions

The computation of the lasso solutions is a quadratic programming problem, and can be tackled by standard numerical analysis algorithms. But the least angle regression procedure is a better approach. This algorithm exploits the special structure of the lasso problem, and provides an efficient way to compute the solutions simulataneously for all values of "s".

Least angle regression is like a more "democratic" version of forward stepwise regression. Recall how forward stepwise regression works:

Forward stepwise regression algorithm:

- Start with all coefficients bj equal to zero.
- Find the predictor xj most correlated with y, and add it into the model. Take residuals r= y-yhat.
- Continue, at each stage adding to the model the predictor most correlated with r.
- Until: all predictors are in the model

The least angle regression procedure follows the same general scheme, but doesn't add a predictor fully into the model. The coefficient of that predictor is increased only until that predictor is no longer the one most correlated with the residual r. Then some other competing predictor is invited to "join the club".

Least angle regression algorithm:

- Start with all coefficients bj equal to zero.
- Find the predictor xj most correlated with y
- Increase the coefficient bj in the direction of the sign of its correlation with y. Take residuals r=y-yhat along the way. Stop when some other predictor xk has as much correlation with r as xj has.
- Increase (bj, bk) in their joint least squares direction, until some other predictor xm has as much correlation with the residual r.
- Continue until: all predictors are in the model

Surprisingly it can be shown that, with one modification, this procedure gives the entire path of lasso solutions, as s is varied from 0 to infinity. The modification needed is: if a non-zero coefficient hits zero, remove it from the active set of predictors and recompute the joint direction.