

IF AN INTEGER  $N$  IS DIVISIBLE BY A PRIME  $P$ ,  
THEN  $N+1$  IS NOT DIVISIBLE BY  $P$ .

PROOF: SUPPOSE  $N$  IS DIVISIBLE BY  $P$ . THEN  
THERE IS AN INTEGER  $Q$  SUCH THAT  $N = PQ$ .  
SO  $N+1 = PQ+1$ . THEN

$$\frac{N+1}{P} = q + \frac{1}{P}.$$

BUT  $q + \frac{1}{P}$  IS NOT AN INTEGER, SO  $N+1$  IS NOT  
DIVISIBLE BY  $P$ .