# The dance of additivity

# Two variables

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A pair of *independent* random variables X and Y:

$$X \sim p_1(x), \quad \mathsf{E}(X) = \mu_1, \quad \mathrm{Var}(X) = \sigma_1^2$$
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$$\mathsf{E}\big(f(X, Y)\big) = \iint_{-\infty}^{+\infty} f(x, y) \cdot \overbrace{p_1(x)p_2(y)}^{p(x,y)}\, dy\, dx$$

$$\mathrm{Var}\big(f(X, Y)\big) = \iint_{-\infty}^{+\infty} \Big[f(x, y) - \mathsf{E}\big(f(X, Y)\big)\Big]^2 \cdot \overbrace{p_1(x)p_2(y)}^{p(x,y)}\, dy\, dx$$

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$$= \iint_{-\infty}^{+\infty} x \cdot p_1(x)p_2(y) \, dy\,dx + \iint_{-\infty}^{+\infty} y \cdot p_1(x)p_2(y) \, dy\,dx$$

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$$= \int_{-\infty}^{\infty} xp_1(x)\,dx \cdot \int_{-\infty}^{\infty} p_2(y)\,dy + \int_{-\infty}^{\infty} p_1(x)\,dx \cdot \int_{-\infty}^{\infty} yp_2(y)\,dy$$

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$$= \underbrace{\int_{-\infty}^{\infty} xp_1(x)\, dx}_{E(X)} \cdot \underbrace{\int_{-\infty}^{\infty} p_2(y)\, dy}_{1} + \underbrace{\int_{-\infty}^{\infty} p_1(x)\, dx}_{1} \cdot \underbrace{\int_{-\infty}^{\infty} yp_2(y)\, dy}_{E(Y)}$$

# Sums $f(X,Y) = X + Y$

change of variable theorem for expectation

$$
\begin{aligned}
E(X+Y) &= \iint_{-\infty}^{+\infty} \overbrace{(x+y)}^{f(x,y)} \cdot \overbrace{p_1(x)p_2(y)}^{p(x,y)}\, dy\, dx \\
&= \iint_{-\infty}^{+\infty} x \cdot p_1(x)p_2(y)\, dy\, dx + \iint_{-\infty}^{+\infty} y \cdot p_1(x)p_2(y)\, dy\, dx \\
&= \underbrace{\int_{-\infty}^{\infty} x p_1(x)\, dx}_{E(X)} \cdot \underbrace{\int_{-\infty}^{\infty} p_2(y)\, dy}_{1} + \underbrace{\int_{-\infty}^{\infty} p_1(x)\, dx}_{1} \cdot \underbrace{\int_{-\infty}^{\infty} y p_2(y)\, dy}_{E(Y)} \\
&= E(X) + E(Y)
\end{aligned}
$$

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$$E(X+Y) = \iint_{-\infty}^{+\infty} \overbrace{(x+y)}^{f(x,y)} \cdot p(x,y)\, dy\, dx$$

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The expectation of a sum is the sum of expectations
(even if the variables are dependent)

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$$\mathbb{E}(X+Y) = \mathbb{E}(X) + \mathbb{E}(Y)$$

$$\mathrm{Var}(X+Y) = \iint_{-\infty}^{+\infty} \Big[\overbrace{(x+y)}^{f(x,y)} - \overbrace{\big(\mathbb{E}(X)+\mathbb{E}(Y)\big)}^{\mathbb{E}(f(x,y))}\Big]^2 \cdot \overbrace{p_1(x)p_2(y)}^{p(x,y)}\, dy\, dx$$

change of variable theorem for variance

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change of variable theorem for variance

$$= \iint_{-\infty}^{+\infty} \Big[\big(x - E(X)\big) + \big(y - E(Y)\big)\Big]^2 \cdot p_1(x)p_2(y)\, dy\, dx$$

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$$\mathrm{Var}(X+Y) = \iint\limits_{-\infty}^{+\infty} \Big[\overbrace{(x+y)}^{f(x,y)} - \overbrace{\big(E(X) + E(Y)\big)}^{E(f(x,y))}\Big]^2 \cdot \overbrace{p_1(x)p_2(y)}^{p(x,y)}\, dy\, dx$$

$$= \iint\limits_{-\infty}^{+\infty} \overset{[a+b]^2 = a^2 + 2ab + b^2}{\Big[\big(x - E(X)\big) + \big(y - E(Y)\big)\Big]^2} \cdot p_1(x)p_2(y)\, dy\, dx$$

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$[a+b]^2 = a^2 + 2ab + b^2$

$$= \iint_{-\infty}^{+\infty} \Big[\big(x - E(X)\big) + \big(y - E(Y)\big)\Big]^2 \cdot p_1(x)p_2(y)\, dy\,dx$$

$$= \iint_{-\infty}^{+\infty} \big(x - E(X)\big)^2 \cdot p_1(x)p_2(y)\, dy\,dx$$

$$+ 2 \iint_{-\infty}^{+\infty} \big(x - E(X)\big)\big(y - E(Y)\big) \cdot p_1(x)p_2(y)\, dy\,dx$$

$$+ \iint_{-\infty}^{+\infty} \big(y - E(Y)\big)^2 \cdot p_1(x)p_2(y)\, dy\,dx$$

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$$= \iint_{-\infty}^{+\infty} \big(x - E(X)\big)^2 \cdot p_1(x)p_2(y)\, dy\, dx \qquad A$$

$$+ 2 \iint_{-\infty}^{+\infty} \big(x - E(X)\big)\big(y - E(Y)\big) \cdot p_1(x)p_2(y)\, dy\, dx \qquad B$$

$$+ \iint_{-\infty}^{+\infty} \big(y - E(Y)\big)^2 \cdot p_1(x)p_2(y)\, dy\, dx \qquad C$$

$$A = \iint_{-\infty}^{+\infty} (x - \mathbf{E}(X))^2 \cdot p_1(x)p_2(y)\, dy\, dx$$

$$B = 2 \iint_{-\infty}^{+\infty} (x - \mathbf{E}(X))(y - \mathbf{E}(Y)) \cdot p_1(x)p_2(y)\, dy\, dx$$

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$$f(X, Y) = X + Y$$

$$\mathrm{Var}(X + Y)$$

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The variance of a sum is the sum of variances
(if the variables are independent)

Slogan 1

Expectation is additive.

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Slogan 2

Variance is additive if the summands are independent.