Summary of Tableau 3, Part 2

Chance in commonplace settings: Urn models in statistical physics

- Maxwell–Boltzmann statistics: distinguishable balls and urns. Classical theory from statistical mechanics; does not apply to quantum mechanical particles.

$$P(k_1, k_2, \ldots, k_r) = \frac{n!}{k_1!k_2!\cdots k_r!} \Big/ r^n$$

- Bose–Einstein statistics: indistinguishable balls; distinguishable urns. Applies to bosons: particles with integer spin.

$$P(k_1, k_2, \ldots, k_r) = \frac{1}{\binom{n+r-1}{n}}$$

- Fermi–Dirac statistics: indistinguishable balls obeying the Pauli exclusion principle; distinguishable urns. Applies to fermions: particles with half-integer spin.

$$P(k_1, k_2, \ldots, k_r) = \frac{1}{\binom{r}{n}}$$

Summary of Tableau 3, Part 2

Chance in commonplace settings: Urn models in statistical physics

Given a random placement of n balls in r urns, what is the probability $P(k_1, k_2, \ldots, k_r)$ of observing a given occupancy configuration $(k_1, k_2, \ldots, k_r)$?

- Maxwell–Boltzmann statistics: distinguishable balls and urns. Classical theory from statistical mechanics; does not apply to quantum mechanical particles.

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