fzero

Root of nonlinear function

Syntax

```
x = fzero(fun,x0)
x = fzero(fun,x0,options)

x = fzero(problem)

[x,fval,exitflag,output] = fzero( __ )
```

Description

`x = fzero(fun,x0)` tries to find a point `x` where `fun(x) = 0`. This solution is where `fun(x)` changes sign—`fzero` cannot find a root of a function such as `x^2`. example

`x = fzero(fun,x0,options)` uses `options` to modify the solution process. example

`x = fzero(problem)` solves a root-finding problem specified by `problem`. example

`[x,fval,exitflag,output] = fzero( __ )` returns `fun(x)` in the `fval` output, `exitflag` encoding the reason `fzero` stopped, and an output structure containing information on the solution process. example

Examples

collapse all

Root Starting From One Point

Calculate π by finding the zero of the sine function near `3`.

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```
fun = @sin; % function
x0 = 3; % initial point
x = fzero(fun,x0)
```

```
x = 3.1416
```

Root Starting from an Interval

Find the zero of cosine between `1` and `2`.

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```
fun = @cos; % function
x0 = [1 2]; % initial interval
x = fzero(fun,x0)
```

```
x = 1.5708
```

Note that $\cos(1)$ and $\cos(2)$ differ in sign.

Root of a Function Defined by a File

Find a zero of the function $f(x) = x^3 - 2x - 5$.

First, write a file called `f.m`.

```
function y = f(x)
y = x.^3 - 2*x - 5;
```

Save `f.m` on your MATLAB® path.

Find the zero of $f(x)$ near `2`.

```
fun = @f; % function
x0 = 2; % initial point
z = fzero(fun,x0)
```

```
z =
    2.0946
```

Since `f(x)` is a polynomial, you can find the same real zero, and a complex conjugate pair of zeros, using the `roots` command.

```
roots([1 0 -2 -5])
```

```
   ans =
   2.0946
  -1.0473 + 1.1359i
  -1.0473 - 1.1359i
```

Root of Function with Extra Parameter

Find the root of a function that has an extra parameter.

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```
myfun = @(x,c) cos(c*x);  % parameterized function
c = 2;                    % parameter
fun = @(x) myfun(x,c);    % function of x alone
x = fzero(fun,0.1)
```

```
x = 0.7854
```

Nondefault Options

Plot the solution process by setting some plot functions.

Define the function and initial point.

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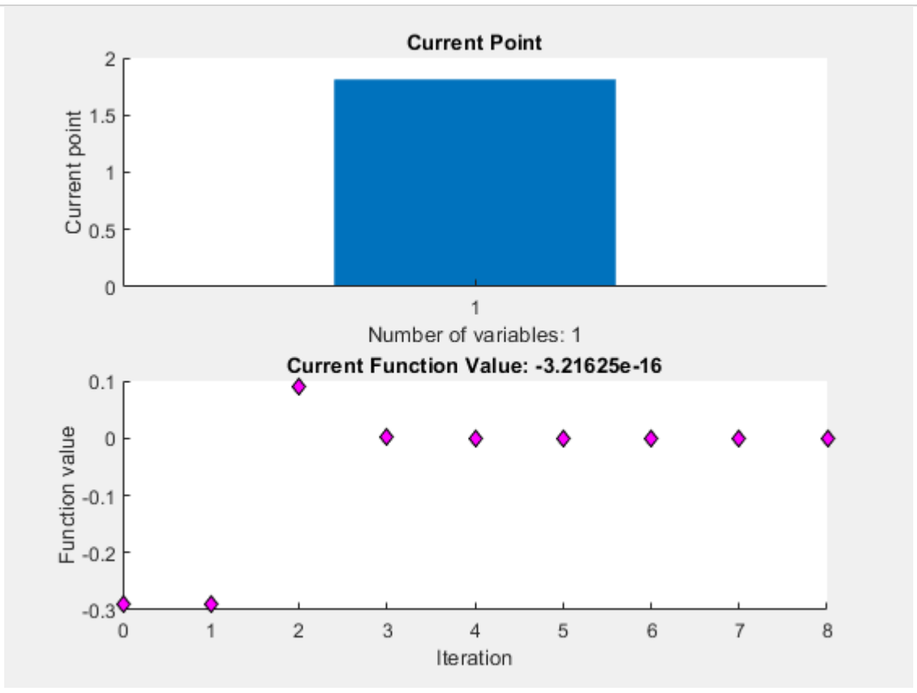
```
fun = @(x)sin(cosh(x));
x0 = 1;
```

Examine the solution process by setting options that include plot functions.

```
options = optimset('PlotFcns',{@optimplotx,@optimplotfval});
```

Run `fzero` including `options`.

```
x = fzero(fun,x0,options)
```




```
x = 1.8115
```

Solve Problem Structure

Solve a problem that is defined by a problem structure.

Define a structure that encodes a root-finding problem.

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```
problem.objective = @(x)sin(cosh(x));
problem.x0 = 1;
problem.solver = 'fzero'; % a required part of the structure
problem.options = optimset(@fzero); % default options
```

Solve the problem.

```
x = fzero(problem)
```

```
x = 1.8115
```

More Information from Solution

Find the point where `exp(-exp(-x)) = x`, and display information about the solution process.

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Copy Command

```
fun = @(x) exp(-exp(-x)) - x; % function
x0 = [0 1]; % initial interval
options = optimset('Display','iter'); % show iterations
[x fval exitflag output] = fzero(fun,x0,options)
```

```
 Func-count    x          f(x)             Procedure
    2              1     -0.307799        initial
    3       0.544459     0.0153522        interpolation
    4       0.566101    0.00070708        interpolation
    5       0.567143  -1.40255e-08        interpolation
    6       0.567143   1.50013e-12        interpolation
    7       0.567143             0        interpolation

Zero found in the interval [0, 1]
x = 0.5671
fval = 0
exitflag = 1
output = struct with fields:
    intervaliterations: 0
            iterations: 5
             funcCount: 7
             algorithm: 'bisection, interpolation'
               message: 'Zero found in the interval [0, 1]'
```

`fval` = 0 means `fun(x) = 0`, as desired.

Input Arguments

collapse all

`fun` — Function to solve

function handle | function name

Function to solve, specified as a handle to a scalar-valued function or the name of such a function. `fun` accepts a scalar `x` and returns a scalar `fun(x)`.

`fzero` solves `fun(x) = 0`. To solve an equation `fun(x) = c(x)`, instead solve `fun2(x) = fun(x) - c(x) = 0`.

To include extra parameters in your function, see the example Root of Function with Extra Parameter and the section Parameterizing Functions.

Example: `'sin'`

Example: `@myFunction`

Example: `@(x)(x-a)^5 - 3*x + a - 1`

Data Types: `char` | `function_handle` | `string`

`x0` — Initial value

scalar | 2-element vector

Initial value, specified as a real scalar or a 2-element real vector.

- Scalar — `fzero` begins at `x0` and tries to locate a point `x1` where `fun(x1)` has the opposite sign of `fun(x0)`. Then `fzero` iteratively shrinks the interval where `fun` changes sign to reach a solution.
- 2-element vector — `fzero` checks that `fun(x0(1))` and `fun(x0(2))` have opposite signs, and errors if they do not. It then iteratively shrinks the interval where `fun` changes sign to reach a solution. An interval `x0` must be finite; it cannot contain `±Inf`.

> **Tip**
>
> Calling `fzero` with an interval (`x0` with two elements) is often faster than calling it with a scalar `x0`.

Example: 3

Example: [2,17]

Data Types: `double`

`options` — Options for solution process

structure, typically created using `optimset`

Options for solution process, specified as a structure. Create or modify the `options` structure using `optimset`. `fzero` uses these `options` structure fields.

`Display`	Level of display: • `'off'` displays no output. • `'iter'` displays output at each iteration. • `'final'` displays just the final output. • `'notify'` (default) displays output only if the function does not converge.
`FunValCheck`	Check whether objective function values are valid. • `'on'` displays an error when the objective function returns a value that is `complex`, `Inf`, or `NaN`. • The default, `'off'`, displays no error.
`OutputFcn`	Specify one or more user-defined functions that an optimization function calls at each iteration, either as a function handle or as a cell array of function handles. The default is none (`[]`). See Optimization Solver Output Functions.
`PlotFcns`	Plot various measures of progress while the algorithm executes. Select from predefined plots or write your own. Pass a function handle or a cell array of function handles. The default is none (`[]`). • `@optimplotx` plots the current point. • `@optimplotfval` plots the function value. For information on writing a custom plot function, see Optimization Solver Plot Functions.
`TolX`	Termination tolerance on `x`, a positive scalar. The default is `eps`, 2.2204e–16.

Example: `options = optimset('FunValCheck','on')`

Data Types: `struct`

`problem` — Root-finding problem

structure

Root-finding problem, specified as a structure with all of the following fields.

`objective`	Objective function
`x0`	Initial point for `x`, real scalar or 2-element vector
`solver`	`'fzero'`
`options`	Options structure, typically created using `optimset`

For an example, see Solve Problem Structure.

Data Types: `struct`

Output Arguments

collapse all

`x` — Location of root or sign change

real scalar

Location of root or sign change, returned as a scalar.

`fval` — Function value at `x`

real scalar

Function value at `x`, returned as a scalar.

`exitflag` — Integer encoding the exit condition

integer

Integer encoding the exit condition, meaning the reason `fzero` stopped its iterations.

`1`	Function converged to a solution `x`.
`-1`	Algorithm was terminated by the output function or plot function.
`-3`	`NaN` or `Inf` function value was encountered while searching for an interval containing a sign change.
`-4`	Complex function value was encountered while searching for an interval containing a sign change.
`-5`	Algorithm might have converged to a singular point.
`-6`	`fzero` did not detect a sign change.

`output` — Information about root-finding process

structure

Information about root-finding process, returned as a structure. The fields of the structure are:

`intervaliterations`	Number of iterations taken to find an interval containing a root
`iterations`	Number of zero-finding iterations
`funcCount`	Number of function evaluations
`algorithm`	`'bisection, interpolation'`
`message`	Exit message

Algorithms

The `fzero` command is a function file. The algorithm, created by T. Dekker, uses a combination of bisection, secant, and inverse quadratic interpolation methods. An Algol 60 version, with some improvements, is given in [1]. A Fortran version, upon which `fzero` is based, is in [2].

Alternative Functionality

App

The **Optimize** Live Editor task provides a visual interface for `fzero`.

References

[1] Brent, R., *Algorithms for Minimization Without Derivatives*, Prentice-Hall, 1973.

[2] Forsythe, G. E., M. A. Malcolm, and C. B. Moler, *Computer Methods for Mathematical Computations*, Prentice-Hall, 1976.

Extended Capabilities

C/C++ Code Generation

Generate C and C++ code using MATLAB® Coder™.

Thread-Based Environment

Run code in the background using MATLAB® `backgroundPool` or accelerate code with Parallel Computing Toolbox™ `ThreadPool`.

Version History

Introduced before R2006a

See Also

`fminbnd` | `optimset` | `roots` | Optimize

Topics

Roots of Scalar Functions

Optimize Live Editor Task

Parameterizing Functions