Erasmus
School of
Economics

MOOC Econometrics

Lecture 6.1 on Time Series: Motivation

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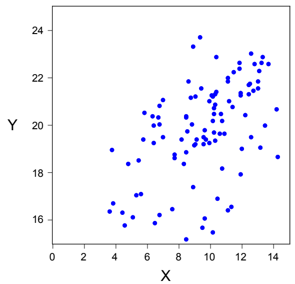
Introduction

- Time series: variable is observed at regular frequency, yearly, quarterly, monthly, weekly, daily, split-second.
- Past values often have predictive power for future.
- Can get spurious regression results if own past is neglected.
- Data: $x_t = 1 + 0.9x_{t-1} + \varepsilon_{x,t}$ and $y_t = 2 + 0.9y_{t-1} + \varepsilon_{y,t}$
  Two series completely uncorrelated: $E(\varepsilon_{x,t}\varepsilon_{y,s}) = 0$ for all t, s.

Spurious regression



Dependent variable: Y (sample size $n = 100$)						
	Coef.	t-Stat.	p-value	Coef.	t-Stat.	p-value
Constant	15.99	23.45	0.000	2.91	2.87	0.005
X	0.40	5.78	$\boxed{0.000}$	0.07	1.53	$\boxed{0.129}$
Y(-1)	-	-	-	0.82	14.01	0.000
R-squared	0.254			0.753		

Test question

Dependent variable: Y (sample size $n = 100$)						
	Coef.	t-Stat.	p-value	Coef.	t-Stat.	p-value
Constant	2.88	2.83	0.006	2.69	2.66	0.009
Y(-1)	0.83	14.02	0.000	0.86	17.03	0.000
X	0.15	1.61	0.110	-	-	-
X(-1)	-0.09	-0.99	0.324	-	-	-
R-squared	0.756			0.747		

Test

Is joint effect of X and $X(-1)$ on Y significant?
Note: The relevant 5% critical value is 3.1.

Answer test

- Use F-test (see Lecture 2): $F = \frac{(R_1^2 - R_0^2)/g}{(1-R_1^2)/(n-k)}$
- number of restrictions: $g = 2$

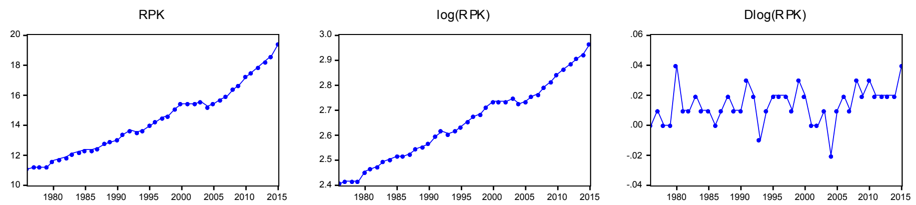
  number of observations: $n = 100$

  number of parameters unrestricted model: $k = 4$

  values of R-squared: $R_1^2 = 0.756$ and $R_0^2 = 0.747$
- Substitute these values in formula for F-test:

  $F = \frac{(0.756-0.747)/2}{(1-0.756)/(100-4)} = 1.8 < 3.1$
- Joint effect of X and $X(-1)$ on Y is not significant.

Example: RPK



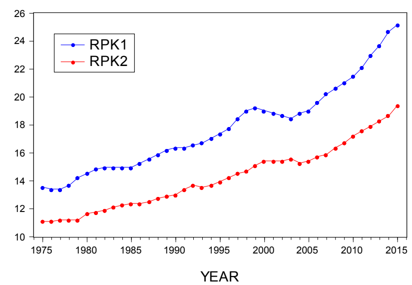
- RPK: Revenue Passenger Kilometers (in billions)

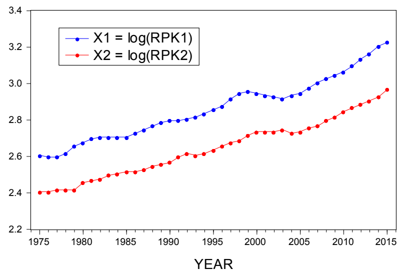
  yearly totals 1976-2015, trend somewhat exponential
- log(RPK): more linear trend

  $\text{Dlog(RPK)} = \log(\text{RPK}) - \log(\text{RPK})(\text{-}1) \approx \frac{\text{RPK}-\text{RPK(-1)}}{\text{RPK(-1)}}$

  yearly growth rate of RPK

Two airline companies



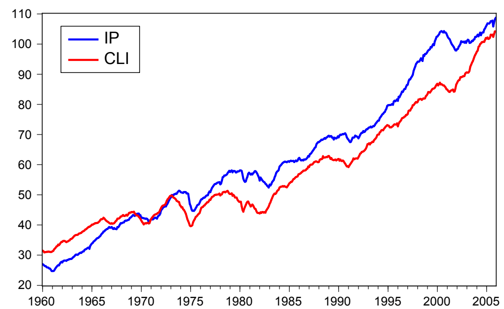


- After taking logs, seems common trend for series X1 and X2.
- Issues:

  → univariate time series: relate RPK to its own past

  → bivariate time series: relate two RPK series to own and others past

Macroeconomic example



- IP: Monthly index of Industrial Production for USA
- CLI: Monthly Composite Leading Index USA
- Question: Can we predict IP one quarter ahead?

  → Answers in Lecture 6.5

TRAINING EXERCISE 6.1

- Train yourself by making the training exercise (see the website).
- After making this exercise, check your answers by studying the webcast solution (also available on the website).

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Lecture 6.2 on Time Series: Representation

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Stationarity

- Time series: y_t, where $t = 1, \ldots, n$ is time index.
- y_t stationary if
  - $\rightarrow$ mean $E(y_t) = \mu$ is fixed (same for all t)
  - $\rightarrow$ autocovariance $E((y_t - \mu)(y_{t-k} - \mu)) = \gamma_k$ (same for all t)
- Special case: $\gamma_k = 0$ for all $k = 1, 2, \ldots$
  - $\rightarrow$ WHITE NOISE
- Recall Assumption A5 (Lectures 1 & 2): $E(\varepsilon_i \varepsilon_j) = 0$ for all $i \neq j$.
- White noise cannot be predicted from own past (by linear models).
  - $\rightarrow$ Purpose: Time series model such that residuals are white noise.

Autoregressive model

- Notation for white noise (uncorrelated series) with mean zero: ε_t
- AR(1): $y_t = \alpha + \beta y_{t-1} + \varepsilon_t$
- Stationary if $-1 < \beta < 1$

  $$y_t = \alpha + \beta y_{t-1} + \varepsilon_t = \alpha + \beta(\alpha + \beta y_{t-2} + \varepsilon_{t-1}) + \varepsilon_t$$
  $$= \alpha(1+\beta) + \varepsilon_t + \beta\varepsilon_{t-1} + \beta^2 y_{t-2} = \ldots$$
  $$= \alpha \sum_{j=0}^{t-2} \beta^j + \sum_{j=0}^{t-2} \beta^j \varepsilon_{t-j} + \beta^{t-1} y_1$$

  For $t \rightarrow \infty$ we get $\beta^{t-1} y_1 \rightarrow 0$ and $y_t = \alpha/(1-\beta) + \sum_{j=0}^{\infty} \beta^j \varepsilon_{t-j}$
- AR(2): $y_t = \alpha + \beta_1 y_{t-1} + \beta_2 y_{t-2} + \varepsilon_t$
- AR(p): $y_t = \alpha + \beta_1 y_{t-1} + \beta_2 y_{t-2} + \ldots + \beta_p y_{t-p} + \varepsilon_t$

Test question

- AR(1) $y_t = \alpha + \beta y_{t-1} + \varepsilon_t$,

  ε_t uncorrelated with y_{t-k} for all $k = 1, 2, \ldots$.

Test

If $\beta = 1$, then argue why y_t can not be stationary.

Answer:

- If $\alpha \neq 0$, then y_t can not have fixed mean:

  $E(\varepsilon_t) = 0$, so $\mu = E(y_t) = \alpha + E(y_{t-1}) + 0 = \alpha + \mu \neq \mu$
- And if $\alpha = 0$ then y_t can not have fixed variance:

  $y_t = y_{t-1} + \varepsilon_t$, so $(y_t - \mu) = (y_{t-1} - \mu) + \varepsilon_t$ (uncorrelated)

  $E((y_t - \mu)^2) = E((y_{t-1} - \mu)^2) + E(\varepsilon_t^2) > E((y_{t-1} - \mu)^2)$.

Moving average

- MA(1): $y_t = \alpha + \varepsilon_t + \gamma\varepsilon_{t-1}$
- As ε_t is uncorrelated with its own past and future, y_t is correlated with y_{t-1} but not with y_{t-k} for $k = 2, 3, \ldots$.
- MA(q): $y_t = \alpha + \varepsilon_t + \gamma_1\varepsilon_{t-1} + \ldots + \gamma_q\varepsilon_{t-q}$
- ARMA(1,1) : $y_t = \alpha + \beta y_{t-1} + \varepsilon_t + \gamma\varepsilon_{t-1}$
- ARMA(p, q): $y_t = \alpha + \beta_1 y_{t-1} + \ldots + \beta_p y_{t-p} + \varepsilon_t + \gamma_1\varepsilon_{t-1} + \ldots + \gamma_q\varepsilon_{t-q}$

Two autoregressive equations

- If two autoregressive processes are related, the univariate process becomes ARMA.

Test

Let $\varepsilon_{x,t}$ and $\varepsilon_{y,t}$ be two mutually independent white noise processes, and let $y_t = \gamma x_t + \varepsilon_{y,t}$ and $x_t = \delta x_{t-1} + \varepsilon_{x,t}$. Derive the orders p and q for the ARMA model for y_t (that does not include x_t).
Hint: Eliminate x_t by considering $y_t - \delta y_{t-1}$.

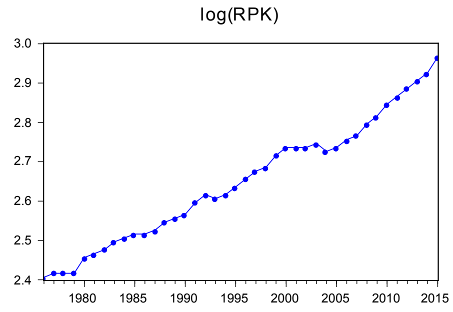
Answer:

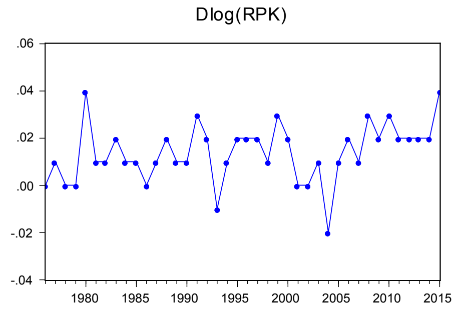
- $y_t - \delta y_{t-1} = \gamma(x_t - \delta x_{t-1}) + \varepsilon_{y,t} - \delta\varepsilon_{y,t-1}$
  $y_t = \delta y_{t-1} + \gamma\varepsilon_{x,t} + \varepsilon_{y,t} - \delta\varepsilon_{y,t-1}$
- AR-order $p = 1$, and error $\omega_t = \gamma\varepsilon_{x,t} + \varepsilon_{y,t} - \delta\varepsilon_{y,t-1}$ is MA(1): $E(\omega_t\omega_{t-1}) = -\delta \text{var}(\varepsilon_{y,t-1})$, $E(\omega_t\omega_{t-2}) = E(\omega_t\omega_{t-3}) = \ldots = 0$

(Partial) Autocorrelation Function - (P)ACF

- k-th order sample autocorrelation coefficient:
  $$\text{ACF}_k = \text{cor}(y_t, y_{t-k}) = \frac{\sum_{t=k+1}^{n}(y_t - \bar{y})(y_{t-k} - \bar{y})}{\sum_{t=k+1}^{n}(y_t - \bar{y})^2}$$
- If y_t is MA(q), then $\text{ACF}_k \approx 0$ for all $k > q$.
- k-th order sample partial autocorrelation coefficient:
  PACF_k is the OLS coefficient b_k in regression model
  $$y_t = \alpha + \beta_1 y_{t-1} + \ldots + \beta_{k-1} y_{t-k+1} + \beta_k y_{t-k} + \varepsilon_t$$
- If y_t is AR(p), then $\text{PACF}_k \approx 0$ for all $k > p$.
- 5% critical value: not significant if $-2/\sqrt{n} < \text{(P)ACF} < 2/\sqrt{n}$

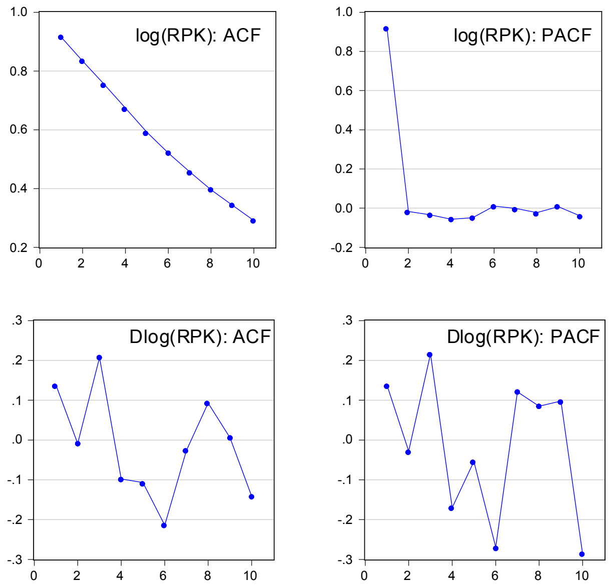
Example: RPK of airline - time series





- log(RPK) is not stationary
- first difference of log(RPK) (yearly growth rate) is stationary

Example: RPK of airline - ACF and PACF



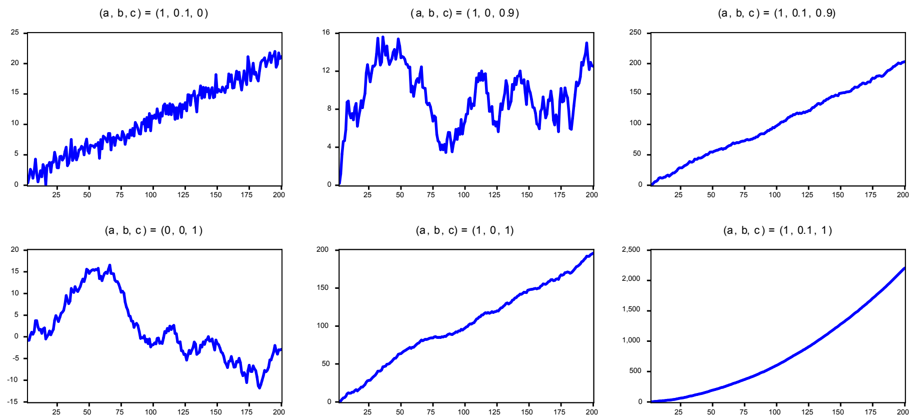
Trends: stochastic and deterministic

- $y_t = y_{t-1} + \varepsilon_t$: random walk, stochastic trend, no clear direction
- $y_t = \alpha + y_{t-1} + \varepsilon_t$ $\quad(\alpha \neq 0)$: stochastic trend
- $y_t = \alpha + \beta t + y_{t-1} + \varepsilon_t$ $\quad(\beta \neq 0)$: stochastic (explosive) trend
- $y_t = \alpha + \beta t + \varepsilon_t$ $\quad(\beta \neq 0)$: deterministic trend
- $y_t = \alpha + \beta t + \gamma y_{t-1} + \varepsilon_t$ $(\beta \neq 0, |\gamma| < 1)$: deterministic trend
- Stochastic trend can be removed by taking first difference:

  Example: $y_t = \alpha + y_{t-1} + \varepsilon_t$, then $\Delta y_t = y_t - y_{t-1} = \alpha + \varepsilon_t$

Examples of deterministic and stochastic trends

- DGP: $y_t = a + bt + cy_{t-1} + \varepsilon_t$
- Stochastic trend: $c = 1$ (bottom row)



Cointegration

- Sometimes: x_t and y_t each have stochastic trend,

  but $y_t - cx_t$ is stationary for some value of c.
- Cointegration (common stochastic trend)

Test

Suppose that $z_t = z_{t-1} + \varepsilon_{z,t}$ is unobserved, whereas $x_t = \alpha_1 + \gamma_1 z_t + \varepsilon_{x,t}$ and $y_t = \alpha_2 + \gamma_2 z_t + \varepsilon_{y,t}$ are observed, where $\varepsilon_{z,t}, \varepsilon_{x,t}, \varepsilon_{y,t}$ are white noise processes. Show that x_t and y_t are cointegrated, and find the value of c for which $y_t - cx_t$ is stationary.

Answer:

- $\gamma_1 y_t - \gamma_2 x_t = (\gamma_1\alpha_2 - \gamma_2\alpha_1) + (\gamma_1\varepsilon_{y,t} - \gamma_2\varepsilon_{x,t})$,

  where $\varepsilon_t = \gamma_1\varepsilon_{y,t} - \gamma_2\varepsilon_{x,t}$ is white noise $\quad\rightarrow\quad$ stationary

  $\gamma_1 y_t - \gamma_2 x_t = \gamma_1(y_t - \gamma_2/\gamma_1 x_t)$, so $c = \gamma_2/\gamma_1$.

TRAINING EXERCISE 6.2

- Train yourself by making the training exercise (see the website).
- After making this exercise, check your answers by studying the webcast solution (also available on the website).

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Lecture 6.3 on Time Series: Specification and Estimation

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Forecasting

- Past values of time series → Model → Forecast future values
- Notation:

  y_t: time series of interest ($t = 1, \ldots, n$)

  x_t: time series possible explanatory factor (restrict to one)

  $PY_{t-1} = \{y_{t-1}, y_{t-2}, \ldots, y_1\}$: past information on y at time t

  $PX_{t-1} = \{x_{t-1}, x_{t-2}, \ldots, x_1\}$

  Univariate time series forecast model: $\widehat{y}_t = F(PY_{t-1})$

  Forecast model with explanatory factor: $\widehat{y}_t = F(PY_{t-1}, PX_{t-1})$
- Aim: Optimal use of past information to get best forecasts.
- Wish: Forecast error $\varepsilon_t = y_t - \widehat{y}_t$ uncorrelated with past information.

Univariate time series model

- Forecast: $\widehat{y}_t = F(PY_{t-1})$ where $PY_{t-1} = \{y_{t-1}, y_{t-2}, \ldots, y_1\}$.
- Find forecast model F so that $\varepsilon_t = y_t - \widehat{y}_t$ uncorrelated with PY_{t-1}.
- Popular choice: F linear function of p past values:

  $\widehat{y}_t = \alpha + \beta_1 y_{t-1} + \ldots + \beta_p y_{t-p}$.
- $y_t = \widehat{y}_t + \varepsilon_t = \alpha + \beta_1 y_{t-1} + \ldots + \beta_p y_{t-p} + \varepsilon_t$.
- AR(p) model, because ε_t is white noise.

Test question

- Forecast: $\widehat{y}_t = \alpha + \beta_1 y_{t-1} + \ldots + \beta_p y_{t-p}$.
- Forecast error $\varepsilon_t = y_t - \widehat{y}_t$ uncorrelated with y_s for all $s < t$.

Test

Show that ε_t is white noise, i.e., ε_t is uncorrelated with ε_s for all $t \neq s$.

Answer:

- Without loss of generality, consider case $s < t$.
- $\varepsilon_s = y_s - \alpha - \sum_{j=1}^{p} \beta_j y_{s-j}$ linear function of y_r, $r \leq s < t$.
- ε_t is uncorrelated with y_r for all $r < t$, so also uncorrelated with ε_s.

Estimation

- Forecast error: $\varepsilon_t = y_t - \alpha - \sum_{j=1}^{p} \beta_j y_{t-j}$.
- Minimize sum of squared forecast errors: $\sum_{t=p+1}^{n} \varepsilon_t^2$.
- OLS!
- Estimation of ARMA models: Maximum Likelihood.

Time series model with explanatory factor

- Forecast: $\widehat{y}_t = F(PY_{t-1}, PX_{t-1})$.
- Find F such that $\varepsilon_t = y_t - \widehat{y}_t$ uncorrelated with PY_{t-1} and PX_{t-1}.
- Popular choice: linear F:

  $\widehat{y}_t = \alpha + \beta_1 y_{t-1} + \ldots + \beta_p y_{t-p} + \gamma_1 x_{t-1} + \ldots + \gamma_r x_{t-r}$.
- $y_t = \widehat{y}_t + \varepsilon_t = \alpha + \sum_{j=1}^{p} \beta_j y_{t-j} + \sum_{j=1}^{r} \gamma_j x_{t-j} + \varepsilon_t$.

  Autoregressive Distributed Lag model: ADL(p, r).
- Estimation: minimize $\sum_{t=m+1}^{n} \varepsilon_t^2$, where $m = \max(p, r) \to$ OLS!

Granger causality

- Two variables of interest: y_t and x_t.
- Make ADL model for each variable:

  $y_t = \alpha + \sum_{j=1}^{p} \beta_j y_{t-j} + \sum_{j=1}^{r} \gamma_j x_{t-j} + \varepsilon_t$.

  $x_t = \alpha^* + \sum_{j=1}^{p^*} \beta_j^* x_{t-j} + \sum_{j=1}^{r^*} \gamma_j^* y_{t-j} + \varepsilon_t^*$.
- x_t helps to predict y_t if $\gamma_j \neq 0$ for some j

  y_t helps to predict x_t if $\gamma_j^* \neq 0$ for some j
- x_t is Granger causal for y_t if it helps to predict y_t, whereas y_t does not help to predict x_t.
- Test $H_0 : \gamma_j^* = 0$ for all $j = 1, \ldots, r^*$ by means of F-test.
- Note: Two ADL equations are estimated by OLS per equation.

Consequences of non-stationarity

- Regression assumption A2 not satisfied: regressors y_{t-j} are random.
- Standard OLS t- and F-tests hold true in large enough samples provided all variables in equation are stationary.
- So: First test for non-stationarity before any estimation.
- AR(1): $y_t = \alpha + \beta y_{t-1} + \varepsilon_t$, test $H_0 : \beta = 1$ against $H_1 : -1 < \beta < 1$.
- Rewrite: $\Delta y_t = y_t - y_{t-1} = \alpha + (\beta - 1) y_{t-1} + \varepsilon_t = \alpha + \rho y_{t-1} + \varepsilon_t$

  where $\rho = \beta - 1$
- So: $\Delta y_t = \alpha + \rho y_{t-1} + \varepsilon_t$, test $H_0 : \rho = 0$ against $H_1 : \rho < 0$.
- Reject H_0 of non-stationarity if $t_{\widehat{\rho}} < -2.9$ (not conventional -1.65!).

Test question

Test

Rewrite the AR(2) model $y_t = \alpha + \beta_1 y_{t-1} + \beta_2 y_{t-2} + \varepsilon_t$ as $\Delta y_t = \delta + \rho y_{t-1} + \gamma \Delta y_{t-1} + \varepsilon_t$, and express the parameters (δ, ρ, γ) in terms of $(\alpha, \beta_1, \beta_2)$.

Answer:

- $$\begin{aligned} \Delta y_t &= y_t - y_{t-1} \\ &= \alpha + (\beta_1 - 1)y_{t-1} + \beta_2 y_{t-2} + \varepsilon_t \\ &= \alpha + (\beta_1 + \beta_2 - 1)y_{t-1} - \beta_2 y_{t-1} + \beta_2 y_{t-2} + \varepsilon_t \\ &= \alpha + (\beta_1 + \beta_2 - 1)y_{t-1} - \beta_2(y_{t-1} - y_{t-2}) + \varepsilon_t \\ &= \alpha + (\beta_1 + \beta_2 - 1)y_{t-1} - \beta_2 \Delta y_{t-1} + \varepsilon_t \end{aligned}$$
- So: $\delta = \alpha$, $\rho = \beta_1 + \beta_2 - 1$, and $\gamma = -\beta_2$.

Augmented Dicky-Fuller test

- Two types of test equations: with or without deterministic trend.
- Test without deterministic trend if data no clear trend direction:
  $$\Delta y_t = \alpha + \rho y_{t-1} + \gamma_1 \Delta y_{t-1} + \ldots + \gamma_L \Delta y_{t-L} + \varepsilon_t$$
- Reject H_0 of non-stationarity if $\boxed{t_{\hat{\rho}} < -2.9}$
- Test with deterministic trend if data clear trend direction:
  $$\Delta y_t = \alpha + \beta t + \rho y_{t-1} + \gamma_1 \Delta y_{t-1} + \ldots + \gamma_L \Delta y_{t-L} + \varepsilon_t$$
- Reject H_0 of non-stationarity if $\boxed{t_{\hat{\rho}} < -3.5}$
- Choice lag L: serial correlation check, or AIC/BIC (see Lecture 3).

Summary of Specification and Estimation

- AR model for y_t:

  Step 1: Perform ADF test on y_t.
  → Non-stationarity rejected → model y_t
  → Non-stationarity not rejected → take Δy_t and perform ADF test on Δy_t

  Step 2: Estimate AR model for stationary series by OLS.
- ADL model for y_t with explanatory factor x_t:

  Step 1: Perform ADF tests on y_t and x_t.

  → Take difference until non-stationarity is rejected.

  Step 2: Estimate ADL model for stationary series by OLS.
- One exception: if x_t and y_t are cointegrated.

Cointegration and error correction model

- x_t and y_t are cointegrated if both series are non-stationary, but a linear combination (say $y_t - cx_t$) is stationary.
- $y_t = cx_t$: long-run equilibrium.
- Engle-Granger test for cointegration:
  → Step 1: OLS in $y_t = \alpha + \beta x_t + \varepsilon_t \quad \rightarrow \quad b$ and residuals e_t
  → Step 2: Cointegrated if ADF test on e_t rejects non-stationarity
  $$\Delta e_t = \alpha + \rho e_{t-1} + \gamma_1 \Delta e_{t-1} + \ldots + \gamma_L \Delta e_{t-L} + \omega_t$$
  Critical value $t_{\hat{\rho}}$: $\underline{-3.4}$ (if extra term βt: $\underline{-3.8}$)
- Error Correction Model (ECM): if x_t and y_t cointegrated, estimate
  $$\Delta y_t = \alpha + \beta_1(y_{t-1} - bx_{t-1}) + \beta_2 \Delta y_{t-1} + \beta_3 \Delta x_{t-1} + \varepsilon_t$$
  (or more lags for Δy_t and Δx_t)

TRAINING EXERCISE 6.3

- Train yourself by making the training exercise (see the website).
- After making this exercise, check your answers by studying the webcast solution (also available on the website).

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Lecture 6.4 on Time Series: Evaluation and Illustration

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First evaluation step: Check for stationarity

- Take difference of time series until stationarity.
- Test equation: Augmented Dickey-Fuller

  $\Delta y_t = \alpha + \beta t + \rho y_{t-1} + \gamma_1 \Delta y_{t-1} + \ldots + \gamma_L \Delta y_{t-L} + \varepsilon_t$

  Critical value $t_{\widehat{\rho}}$: -2.9 if $\beta = 0$, -3.5 if $\beta \neq 0$
- For stationary data:

  $\rightarrow$ OLS in AR: $y_t = \alpha + \sum_{j=1}^{p} \beta_j y_{t-j} + \varepsilon_t$

  with trend: $y_t = \alpha + \gamma t + \sum_{j=1}^{p} \beta_j y_{t-j} + \varepsilon_t$

  $\rightarrow$ OLS in ADL: $y_t = \alpha + \sum_{j=1}^{p} \beta_j y_{t-j} + \sum_{j=1}^{r} \gamma_j x_{t-j} + \varepsilon_t$

  with trend: $y_t = \alpha + \delta t + \sum_{j=1}^{p} \beta_j y_{t-j} + \sum_{j=1}^{r} \gamma_j x_{t-j} + \varepsilon_t$
- t- and F-tests as usual.

Check for cointegration

- If x_t and y_t are both non-stationary: check for cointegration.
- Test method: Engle-Granger two-step method

  $\rightarrow$ OLS in $y_t = \alpha + \beta x_t + \varepsilon_t \quad \rightarrow \quad b$ and OLS residuals e_t

  $\rightarrow$ OLS in $\Delta e_t = \alpha + \beta t + \rho e_{t-1} + \gamma_1 \Delta e_{t-1} + \ldots + \gamma_L \Delta e_{t-L} + \omega_t$

  Critical value $t_{\widehat{\rho}}$: -3.4 if $\beta = 0$, -3.8 if $\beta \neq 0$
- If x_t and y_t are cointegrated, estimate ECM:

  $\Delta y_t = \alpha + \beta t + \gamma_0 (y_{t-1} - b x_{t-1}) + \sum_{j=1}^{p} \gamma_{y,j} \Delta y_{t-j} + \sum_{j=1}^{r} \gamma_{x,j} \Delta x_{t-j} + \varepsilon_t$

  (or with $\beta = 0$)
- t- and F-tests as usual.

Diagnostic tests

- Choice of lag lengths: BIC (see Lecture 3).
- Stability check: Chow tests (see Lecture 3).
- Normal residuals: Jarque-Bera (see Lecture 3), critical value: 6.0.
- Out-of-sample forecasting: Lecture 6.5.
- Model should in particular capture autocorrelation in time series.

  $\rightarrow$ Test if model residuals are uncorrelated: white noise.
- Two tests: ACF and Breusch-Godfrey.
- ACF rule-of-thumb: significant if $|\text{ACF}| > 2/\sqrt{n}$.

Test question

Test

Let y_t be white noise with variance σ^2. Show that OLS estimator b in $y_t = \alpha + \beta y_{t-1} + \varepsilon_t$ gives the first-order autocorrelation of y_t. Further show that $(-2/\sqrt{n},\ 2/\sqrt{n})$ is approximate 95% confidence interval for β. Hint: Use results of Lecture 1.

Answer:

- $y_t = \alpha = \beta x_t + \varepsilon_t$ where $x_t = y_{t-1}$, $t = 2, \ldots, n$, so
  $b = \sum_{t=2}^{n}(y_t - \bar{y})(y_{t-1} - \bar{y}) / \sum_{t=2}^{n}(y_{t-1} - \bar{y})^2$
- $\text{var}(b) = \sigma^2 / \sum_{t=2}^{n}(y_{t-1} - \bar{y})^2$, where
  $\sum_{t=2}^{n}(y_{t-1} - \bar{y})^2 = (n-1)\sum_{t=2}^{n}(y_{t-1} - \bar{y})^2/(n-1) \approx (n-1)\sigma^2$,
  $\text{var}(b) \approx \sigma^2/((n-1)\sigma^2) = 1/(n-1) \approx 1/n$
- If n large then $b \approx 0$ and $\text{SE}(b) \approx 1/\sqrt{n}$
  $b - 2\text{SE}(b) < \beta < b - 2\text{SE}(b) \quad \rightarrow -2/\sqrt{n} < \beta < 2/\sqrt{n}$

Test on serial correlation: Breusch-Godfrey

- Step 1: Estimate model and get residuals e_t.
- Step 2: Regress e_t on all variables of model and r lags of e_t.
- Step 3: $\text{BG} = nR^2$ of Step 2, and $\text{BG} \approx \chi^2(r)$ if e_t white noise.
- Example: Model $y_t = \alpha + \beta y_{t-1} + \gamma x_{t-1} + \varepsilon_t$

  $\rightarrow$ Step 1: OLS residuals $e_t = y_t - a - b y_{t-1} - c x_{t-1}$.

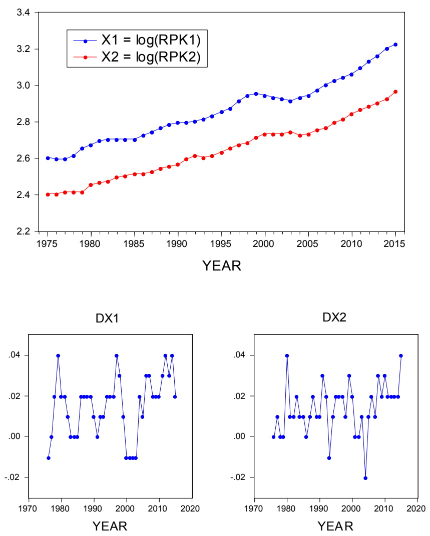
  $\rightarrow$ Step 2: OLS in $e_t = \alpha + \beta y_{t-1} + \gamma x_{t-1} + \delta_1 e_{t-1} + \delta_2 e_{t-2} + \omega_t$

  $\rightarrow$ Step 3: $\text{BG} = nR^2 \approx \chi^2(2)$ if e_t white noise.

  $\rightarrow$ Conclusion: Model not correctly specified if $\text{BG} > 6.0$.

  $\rightarrow$ Should then adjust model, e.g. more lags of y_t and x_t.

Illustration: Revenue Passenger Kilometers (RPK)



- Graphs suggest: X_1 and X_2 non-stationary, ΔX_1 and ΔX_2 stationary.

Tests on stationarity

- Let y_t denote log(RPK), either X_{1t} or X_{2t}: trend
  ADF: $\Delta y_t = \alpha + \beta t + \rho y_{t-1} + \gamma \Delta y_{t-1} + \varepsilon_t$
  t-value of $\hat{\rho}$: $t = -2.8$ for X_1, $t = -1.2$ for X_2
- Let y_t denote either ΔX_{1t} or ΔX_{2t}: no trend
  ADF: $\Delta y_t = \alpha + \rho y_{t-1} + \gamma \Delta y_{t-1} + \varepsilon_t$
  t-value of $\hat{\rho}$: $t = -3.3$ for X_1, $t = -3.7$ for X_2

Test

What conclusions do you draw from these outcomes?

Answer:

- As $t > -3.5$, X_1 and X_2 not stationary.
- As $t < -2.9$, ΔX_1 and ΔX_2 are both stationary.

Granger causality tests

	ADL for ΔX_{1t}			ADL for ΔX_{2t}		
	Coef.	t-Stat.	p-value	Coef.	t-Stat.	p-value
Constant	0.01	1.85	0.07	0.01	2.86	0.01
$\Delta X_{1,t-1}$	0.87	4.96	0.00	0.18	1.29	0.21
$\Delta X_{1,t-2}$	-0.42	-2.02	0.05	0.61	3.68	0.00
$\Delta X_{2,t-1}$	0.35	1.74	0.09	-0.29	-1.81	0.08
$\Delta X_{2,t-2}$	-0.19	-1.27	0.21	-0.13	-1.05	0.30

- Company 1 Granger causal for company 2, not other way round.

  $\rightarrow$ See t-tests (confirmed by F-tests on two coefficients jointly).

Engle-Granger test and ECM

- Step 1: OLS: $X_{2t} = 0.01 + 0.92X_{1t} + e_t$.
- Step 2: ADF: $\Delta e_t = 0.00 - 0.50e_{t-1} + 0.30\Delta e_{t-1} + \text{res}_t$

  $\rightarrow$ t-value of coefficient e_{t-1}: $t = -3.5 < -3.4$

  $\rightarrow$ e_t stationary $\rightarrow$ X_{1t} and X_{2t} cointegrated.
- ECM (after removing insignificant coefficients):

  $\Delta X_{1t} = 0.00 + 1.02\Delta X_{1t} + 0.46(X_{2,t-1} - 0.92X_{1,t-1}) + e_{1t}$

  $\Delta X_{2t} = 0.02 - 0.45(X_{2,t-1} - 0.92X_{1,t-1}) + e_{2t}$
- If $D_{t-1} = X_{2,t-1} - 0.92X_{1,t-1}$ is positive, then

  $0.46 > 0 \rightarrow X_{1t} \uparrow \rightarrow D_t = X_{2t} - 0.92X_{1t} \downarrow$

  $-0.45 < 0 \rightarrow X_{2t} \downarrow \rightarrow D_t = X_{2t} - 0.92X_{1t} \downarrow$
- Error correction mechanism acts on both variables.

ECM: Check for serial correlation and normality

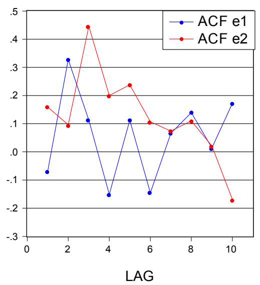
- ECM models for log(RPK) of airline companies 1 and 2 ($n = 39$):

  $\Delta X_{1t} = 0.00 + 1.02\Delta X_{1t} + 0.46(X_{2,t-1} - 0.92X_{1,t-1}) + e_{1t}$

  $\Delta X_{2t} = 0.02 - 0.45(X_{2,t-1} - 0.92X_{1,t-1}) + e_{2t}$
- Jarque-Bera test: $\text{JB}_1 = 0.4 < 6$, $\text{JB}_2 = 1.8 < 6$.

  Breusch-Godfrey test (1 lag): $\text{BG}_1 = 0.3 < 3.9$, $\text{BG}_2 = 1.2 < 3.9$.

  ACF: $2/\sqrt{n} = 2/\sqrt{39} = 0.32$.



TRAINING EXERCISE 6.4

- Train yourself by making the training exercise (see the website).
- After making this exercise, check your answers by studying the webcast solution (also available on the website).

Erasmus
School of
Economics

MOOC Econometrics

Lecture 6.5 on Time Series: Application

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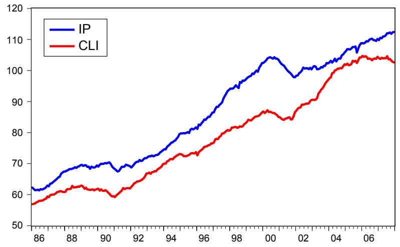
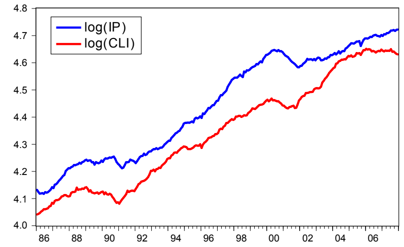
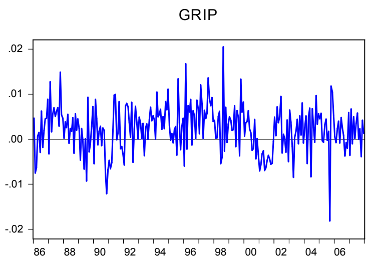
Industrial Production and Composite Leading Index

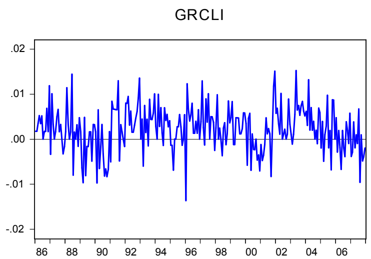




- IP: Industrial production USA (monthly data 1986 - 2007, $n = 264$)
- CLI: Composite Leading Index USA (Conference Board)
- Goal: Forecast IP one quarter (three months) ahead

Monthly growth rates: GRIP and GRCLI



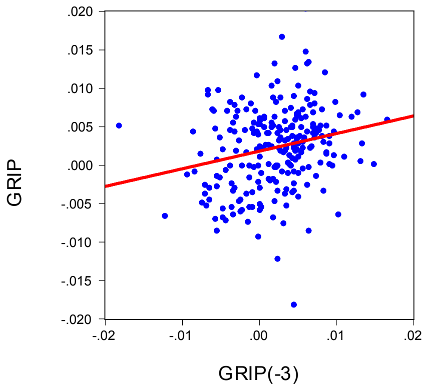


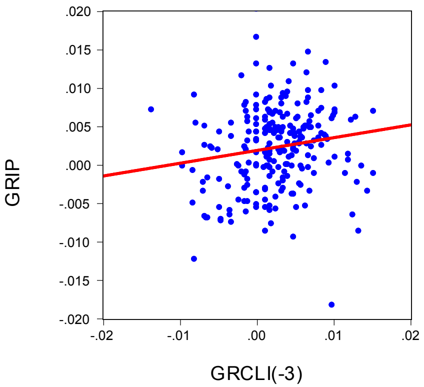
- Monthly growth rates: GRIP $= \Delta\log(\text{IP})$, GRCLI $= \Delta\log(\text{CLI})$
- Estimation sample: 1986 - 2005 ($n = 240$)
- Hold-out forecast sample: 2006 - 2007 ($n = 24$)

Tests on stationarity

- Let y_t denote log(IP) or log(CLI): trend
  ADF: $\Delta y_t = \alpha + \beta t + \rho y_{t-1} + \sum_{j=1}^{3} \gamma_j \Delta y_{t-j} + \varepsilon_t$
  $t_{\hat{\rho}} = -1.6$ for log(IP), $t_{\hat{\rho}} = -1.8$ for log(CLI) $\rightarrow$ not stationary
- Let y_t denote GRIP $= \Delta\log(\text{IP})$ or GRCLI $= \Delta\log(\text{CLI})$: no trend
  ADF: $\Delta y_t = \alpha + \rho y_{t-1} + \sum_{j=1}^{3} \gamma_j \Delta y_{t-j} + \varepsilon_t$
  $t_{\hat{\rho}} = -5.2$ for GRIP, $t_{\hat{\rho}} = -5.6$ for GRCLI $\rightarrow$ stationary
- Engle-Granger test on cointegration:
  Step 1: OLS: $\log(\text{IP}_t) = 0.08 + 1.01\log(\text{CLI}_t) + e_t$
  Step 2: ADF: $\Delta e_t = 0.00 + 0.00t - 0.01e_{t-1} + 0.04\Delta e_{t-1} + \text{res}_t$
  t-value e_{t-1} is $-0.6 > -3.8$ $\rightarrow$ not cointegrated

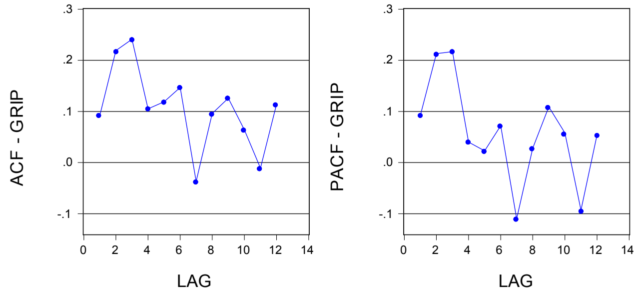
Forecast IP growth rate 3 months ahead





- Forecast GRIP_t with information $\{\text{GRIP}_{t-j}, \text{GRCLI}_{t-j}, j = 3, 4, \ldots\}$.
- Two models: AR for GRIP, and ADL in terms of GRIP and GRCLI.

AR model for GRIP



- $2/\sqrt{n} = 2/\sqrt{240} = 0.13 \rightarrow \text{AR}(3)$
- GRIP_{t-1} and GRIP_{t-2} may not be used
  $\rightarrow$ Start with lags 3-12 and reduce (down-testing).

AR model for GRIP

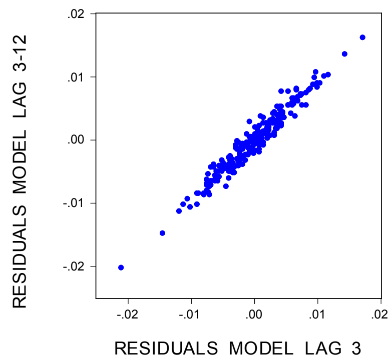
- $\text{GRIP}_t = \alpha + \sum_{j=3}^{L} \beta_j \text{GRIP}_{t-j} + \varepsilon_t$
- $L = 12$: lags 4-12 individually not significant.
- $L = 12$ has $R^2 = 0.0988$, and $L = 3$ gives $R^2 = 0.0519$

Test

Test if model with lags 3-12 can be simplified to one with lag 3 only.
Note: The relevant 5% critical value is 1.9.

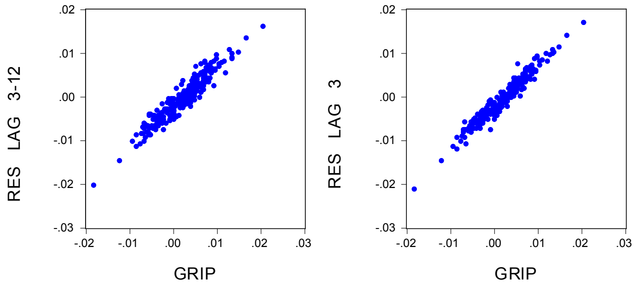
- F-test with $n = 240$, $k = 11$, and $g = 9$.
- $F = \frac{(0.0988-0.0519)/9}{(1-0.0988)/229} = 1.3 < 1.9$.
- Yes, use lag 3 only.

AR model for GRIP



- Both models nearly identical residuals.
- Also nearly identical diagnostics:
  $\rightarrow$ p-value Breusch-Godfrey (6 lags): $p_{12} = 0.03$, $p_3 = 0.03$
  $\rightarrow$ p-value Jarque-Bera: $p_{12} = 0.03$, $p_3 = 0.01$

AR model for GRIP



- Four outliers GRIP cause four associated large residuals.

  High growth: Feb 1996 (1.7%) and Aug 1998 (2.1%)

  Large negative growth: Nov 1990 (-1.2%) and Sep 2005 (-1.8%)

- Our forecast model: $\text{GRIP}_t = 0.0018 + 0.2288\text{GRIP}_{t-3} + e_t$

  ($t_b = 3.6$, $R^2 = 0.052$)

ADL model for GRIP

- Does Composite Leading Index help to predict GRIP 3 months ahead?

- If CLI is 'leading', by how many months?

- ADL: $\text{GRIP}_t = \alpha + \sum_{j=3}^{p} \beta_j \text{GRIP}_{t-j} + \sum_{j=3}^{r} \gamma_j \text{GRCLI}_{t-j} + \varepsilon_t$

- Start with $p = r = 6$ and reduce (down-testing).

- Model: $\text{GRIP}_t = 0.001 + 0.193\text{GRIP}_{t-3} + 0.219\text{GRCLI}_{t-6} + e_t$

  ($t_{b3} = 3.1$, $t_{b6} = 3.2$, $R^2 = 0.092$) $\rightarrow$ CLI leads IP by 6 months

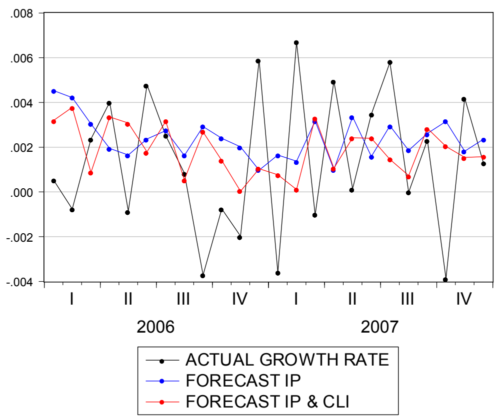
- p-values : Breusch-Godfrey (6 lags): 0.36, no serial correlation

  Jarque-Bera 0.04 (same 4 outliers as before)

Out-of-sample forecast of monthly growth rate IP

- AR (lag 3) and ADL (lags 3 and 6) estimated from data 1986-2005.

- Forecast monthly GRIP for Jan 2006 - Dec 2007 ($n = 24$) and the annual growth rates of IP for the years 2006 and 2007.



Test question

Test

Monthly growth rate of y_t is $g_t^m = \Delta \log(y_t)$, and annual growth rate is $g_t^y = \log(y_t) - \log(y_{t-12})$.
Show that the annual growth rate is simply obtained by adding monthly growth rates over the previous 12 months.

Answer:

- $$\begin{aligned} g_t^y &= \log(y_t) - \log(y_{t-12}) \\ &= (\log(y_t) - \log(y_{t-1})) + (\log(y_{t-1}) - \log(y_{t-2})) + \ldots \\ &\qquad + \ldots + (\log(y_{t-11}) - \log(y_{t-12})) \\ &= g_t^m + g_{t-1}^m + \ldots + g_{t-11}^m. \end{aligned}$$

Out-of-sample forecast of monthly growth rate IP

- Monthly growth rate IP much fluctuation, not easy to predict.
- Evaluation criteria: RMSE and MAE (see Lecture 3)

  SUM: sum of forecast errors $\sum_{t=1}^{24}(y_t - \widehat{y}_t)$
- Table shows forecast errors for the 24 months in 2006 and 2007.
- CLI improves the monthly IP growth forecast for 3-months ahead.

Model (lags)	AR(3-12)	AR(3)	ADL(AR 3, CLI 6)
RMSE ($\times$100)	0.369	0.367	0.350
MAE ($\times$100)	0.322	0.315	0.290
SUM ($\times$100)	5.240	5.731	4.518

Out-of-sample forecast of annual growth rate IP

- Table shows actual anunal IP growth rate (in %) and forecasts.
- CLI improves annual IP growth forecast considerably.
- Such long-term forecasts are important for firms and investors.

	Actual	Forecast		
		AR(3-12)	AR(3)	ADL(AR 3, CLI 6)
2006	1.288	2.859	3.042	2.492
2007	2.037	2.382	2.689	2.025
2006 and 2007	3.325	5.240	5.731	4.518

TRAINING EXERCISE 6.5

- Train yourself by making the training exercise (see the website).
- After making this exercise, check your answers by studying the webcast solution (also available on the website).