Why Induction?

You are reviewing your friend's python code. The code contains a function that, given a positive integer n, computes the sum of the first n positive integers: $1+2+\cdots+n$.

```
def sum_of_integers(n):
    assert n > 0
    return sum(range(1, n + 1))
```

You suggest your friend to improve the code by using the *formula for arithmetic series*: for every positive integer n,

$$\sum_{i=1}^{n} i = 1+2+\cdots+n = \frac{n(n+1)}{2}\,.$$

A code that uses a formula $\frac{n(n+1)}{2}$ instead of summing up all integers from 1 to n is more efficient in practice: already for $n=10^9$, computing the sum takes noticeable time, whereas computing $\frac{n(n+1)}{2}$ takes almost nothing.

How could you convince your friend that the formula $\frac{n(n+1)}{2}$ is true for *every* positive integer n?

One possibility is to check that it holds for all $1 \le n \le 100$.

```
print(all(sum(range(1, n + 1)) == n * (n + 1) // 2
          for n in range(1, 101)))
```

Run

Reset

```
True
```

```
True
```

This code snippet ensures that the formula above is true for all $1 \le n \le 100$.

But it only convinces us that the formula holds *sometimes*. Moreover, we have no possibility to check every positive integer n for a simple reason: there are infinitely many of them! Thus, to be sure that the formula holds for *all values of* n, we need a rigorous mathematical proof.

This is reminiscent of software testing.Whereas
one often can run a program on several tests, it is usually impossible to cover all cases with tests.This is why it is important to \emph{ prove } correctness of a program(for all input values).

Mathematical induction is a general way of proving statements of the form:

for all integer $n \ge c$, the statement $A(n)$ is true.

Here, $A(n)$ is a statement (that might be true of false) that depends on n. In our warm-up example above, $A(n)$ asserts that $1+2+\cdots+n$ is equal to $\frac{n(n+1)}{2}$. In the next section, we introduce the method of mathematical induction and give many examples.

In general, checking a few first values of n to verify the correctness of $A(n)$ is a bad idea. Later we'll encounter several examples where $A(n)$ is true in the beginning, but eventually it becomes false. Moreover, there are examples of simple statements $A(n)$ that are true for all $1 \le n \le 10^{80}$, but are false in general! That is,
there exists n such that $A(n)$ is false, but the minimum such n is so huge that it cannot be found by a brute force search.

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