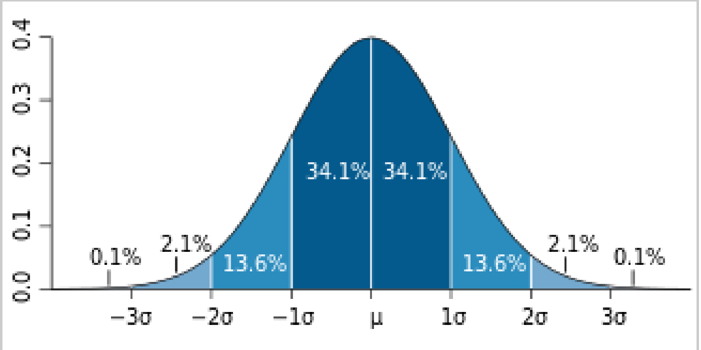
Standard error

From Wikipedia, the free encyclopedia

The **standard error** is the standard deviation of the sampling distribution of a statistic.[1] The term may also be used to refer to an estimate of that standard deviation, derived from a particular sample used to compute the estimate.



For a value that is sampled with an unbiased normally distributed error, the above depicts the proportion of samples that would fall between 0, 1, 2, and 3 standard deviations above and below the actual value.

For example, the sample mean is the usual estimator of a population mean. However, different samples drawn from that same population would in general have different values of the sample mean. The **standard error of the mean** (i.e., of using the sample mean as a method of estimating the population mean) is the standard deviation of those sample means over all possible samples (of a given size) drawn from the population. Secondly, the standard error of the mean can refer to an estimate of that standard deviation, computed from the sample of data being analyzed at the time.

In regression analysis, the term "standard error" is also used in the phrase standard error of the regression to mean the ordinary least squares estimate of the standard deviation of the underlying errors.[2][3]

Contents

Standard error of the mean

Further information: Variance#Sum of uncorrelated variables (Bienaymé formula)

The **standard error of the mean** (SEM) is the standard deviation of the sample-mean's estimate of a population mean. (It can also be viewed as the standard deviation of the error in the sample mean relative to the true mean, since the sample mean is an unbiased estimator). SEM is usually estimated by the sample estimate of the population standard deviation (sample standard deviation) divided by the square root of the sample size (assuming statistical independence of the values in the sample):

$$\mathrm{SE}_{\bar{x}} = \frac{s}{\sqrt{n}}$$

where

> s is the sample standard deviation (i.e., the sample-based estimate of the standard deviation of the population), and
> n is the size (number of observations) of the sample.

This estimate may be compared with the formula for the true standard deviation of the sample mean:

$$\mathrm{SD}_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

where

> σ is the standard deviation of the population.

This formula may be derived from what we know about the variance of a sum of independent random variables.[4]

- If $X_1, X_2, \ldots, X_n$ are n independent observations from a population that has a mean μ and standard deviation σ, then the variance of the total $T = (X_1 + X_2 + \cdots + X_n)$ is $n\sigma^2$.
- The variance of T/n must be $\frac{1}{n^2}n\sigma^2 = \frac{\sigma^2}{n}$.
- And the standard deviation of T/n must be $\sigma/\sqrt{n}$.
- Of course, T/n is the sample mean $\bar{x}$.

Note: the standard error and the standard deviation of small samples tend to systematically underestimate the population standard error and deviations: the standard error of the mean is a biased estimator of the population standard error. With n = 2 the underestimate is about 25%, but for n = 6 the underestimate is only 5%. Gurland and Tripathi (1971)[5] provide a correction and equation for this effect. Sokal and Rohlf (1981)[6] give an equation of the correction factor for small samples of $n < 20$. See unbiased estimation of standard deviation for further discussion.

A practical result: Decreasing the uncertainty in a mean value estimate by a factor of two requires acquiring four times as many observations in the sample. Or decreasing standard error by a factor of ten requires a hundred times as many observations.

Student approximation when σ value is unknown

In practical applications, the true value of the σ value is unknown. As a result, we need to use an approximation of the gaussian sample distribution. In this application the standard error is the standard deviation of a t-student distribution. T-distributions are slightly different from the gaussian, and vary depending on the size of the sample. To estimate the Standard error of a t-student distribution it is sufficient to use the sample standard deviation "s" instead of σ, and we could use this value to calculate confidential intervals.

*Note:*The student's probability distribution is a perfect approximation of the gaussian when the sample size is over 100.

Assumptions and usage

If the data are assumed to be normally distributed, quantiles of the normal distribution and the sample mean and standard error can be used to calculate approximate confidence intervals for the mean. The following expressions can be used to calculate the upper and lower 95% confidence limits, where $\bar{x}$ is equal to the sample mean, SE is equal to the standard error for the sample mean, and 1.96 is the 0.975 quantile of the normal distribution:

Upper 95% limit $= \bar{x} + (\mathrm{SE} \times 1.96),$
Lower 95% limit $= \bar{x} - (\mathrm{SE} \times 1.96).$

In particular, the standard error of a sample statistic (such as sample mean) is the estimated standard deviation of the error in the process by which it was generated. In other words, it is the standard deviation of the sampling distribution of the sample statistic. The notation for standard error can be any one of SE, SEM (for standard error of *measurement* or *mean*), or S_E.

Standard errors provide simple measures of uncertainty in a value and are often used because:

- If the standard error of several individual quantities is known then the standard error of some function of the quantities can be easily calculated in many cases;
- Where the probability distribution of the value is known, it can be used to calculate a good approximation to an exact confidence interval; and
- Where the probability distribution is unknown, relationships like Chebyshev's or the Vysochanskiï–Petunin inequality can be used to calculate a conservative confidence interval
- As the sample size tends to infinity the central limit theorem guarantees that the sampling distribution of the mean is asymptotically normal.

Standard error of mean versus standard deviation

In scientific and technical literature, experimental data is often summarized either using the mean and standard deviation or the mean with the standard error. This often leads to confusion about their interchangeability. However, the mean and standard deviation are descriptive statistics, whereas the standard error of the mean describes bounds on a random sampling process. Despite the small difference in equations for the standard deviation and the standard error, this small difference changes the meaning of what is being reported from a description of the variation in measurements to a probabilistic statement about how the number of samples will provide a better bound on estimates of the population mean, in light of the central limit theorem.

Put simply, the **standard error** of the sample is an estimate of how far the sample mean is likely to be from the population mean, whereas the **standard deviation** of the sample is the degree to which individuals within the sample differ from the sample mean. If the population standard deviation is finite, the standard error of the sample will tend to zero with increasing sample size, because the estimate of the population mean will improve, while the standard deviation of the sample will tend to the population standard deviation as the sample size increases.

Correction for finite population

The formula given above for the standard error assumes that the sample size is much smaller than the population size, so that the population can be considered to be effectively infinite in size. This is usually the case even with finite populations, because most of the time, people are primarily interested in managing the processes that created the existing finite population; this is called an analytic study, following W. Edwards Deming. If people are interested in managing an existing finite population that will not change over time, then it is necessary to adjust for the population size; this is called an enumerative study.

When the sampling fraction is large (approximately at 5% or more) in an enumerative study, the estimate of the error must be corrected by multiplying by a "finite population correction"[7]

$$\mathrm{FPC} = \sqrt{\frac{N-n}{N-1}}$$

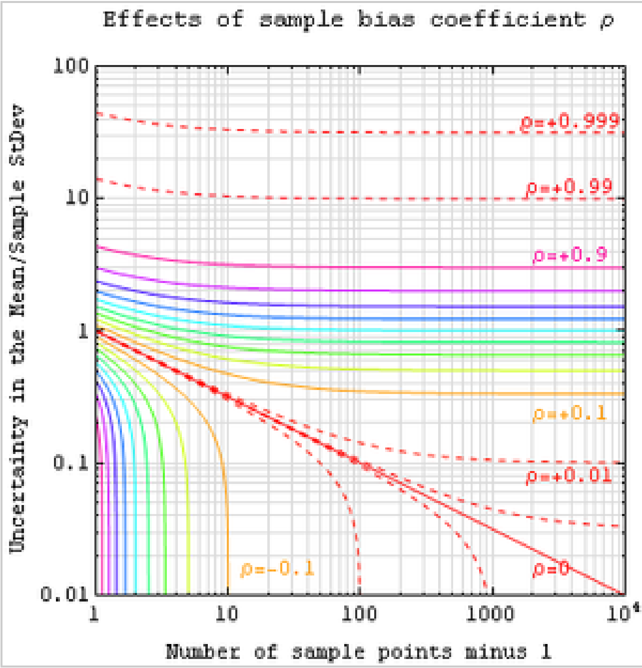
to account for the added precision gained by sampling close to a larger percentage of the population. The effect of the FPC is that the error becomes zero when the sample size n is equal to the population size N.

Correction for correlation in the sample

If values of the measured quantity A are not statistically independent but have been obtained from known locations in parameter space $\mathbf{x}$, an unbiased estimate of the true standard error of the mean (actually a correction on the standard deviation part) may be obtained by multiplying the calculated standard error of the sample by the factor f:

$$f = \sqrt{\frac{1+\rho}{1-\rho}},$$

where the sample bias coefficient ρ is the widely used Prais-Winsten estimate of the autocorrelation-coefficient (a quantity between −1 and +1) for all sample point pairs. This approximate formula is for moderate to large sample sizes; the reference gives the exact formulas for any sample size, and can be applied to heavily autocorrelated time series like Wall Street stock quotes. Moreover this formula works for positive and negative ρ alike.[8] See also unbiased estimation of standard deviation for more discussion.



Expected error in the mean of A for a sample of n data points with sample bias coefficient ρ. The unbiased **standard error** plots as the ρ=0 diagonal line with log-log slope -½.

Relative standard error

The relative standard error (RSE) is simply the standard error divided by the mean and expressed as a percentage. For example, consider two surveys of household income that both result in a sample mean of $50,000. If one survey has a standard error of $10,000 and the other has a standard error of $5,000, then the relative standard errors are 20% and 10% respectively. The survey with the lower relative standard error has a more precise measurement since there is less variance around the mean. In fact, data organizations often set reliability standards that their data must reach before publication. For example, the U.S. National Center for Health Statistics typically does not report an estimate if the relative standard error exceeds 30%. (NCHS also typically requires at least 30 observations - if not more - for an estimate to be reported.)[9]

See also

- Coefficient of variation
- Illustration of the central limit theorem
- Probable error
- Sample mean and sample covariance
- Variance

References

1. ^ Everitt, B.S. (2003) *The Cambridge Dictionary of Statistics*, CUP. ISBN 0-521-81099-X
2. ^ Kenney, J. and Keeping, E.S. (1963) *Mathematics of Statistics*, van Nostrand, p. 187
3. ^ Zwillinger D. (1995), *Standard Mathematical Tables and Formulae*, Chapman&Hall/CRC. ISBN 0-8493-2479-3 p. 626
4. ^ T.P. Hutchinson, Essentials of statistical methods in 41 pages
5. ^ Gurland, J; Tripathi RC (1971). "A simple approximation for unbiased estimation of the standard deviation". *American Statistician* (American Statistical Association) **25** (4): 30–32. doi:10.2307/2682923 (http://dx.doi.org/10.2307%2F2682923). JSTOR 2682923 (//www.jstor.org/stable/2682923).
6. ^ Sokal and Rohlf (1981) *Biometry: Principles and Practice of Statistics in Biological Research* , 2nd ed. ISBN 0-7167-1254-7 , p 53
7. ^ Isserlis, L. (1918). "On the value of a mean as calculated from a sample". *Journal of the Royal Statistical Society* (Blackwell Publishing) **81** (1): 75–81. doi:10.2307/2340569 (http://dx.doi.org/10.2307%2F2340569). JSTOR 2340569 (//www.jstor.org/stable/2340569). (Equation 1)
8. ^ James R. Bence (1995) Analysis of short time series: Correcting for autocorrelation. *Ecology* 76(2): 628 – 639.
9. ^ Klein RJ, Proctor SE, Boudreault MA, Turczyn KM. Healthy People 2010 criteria for data suppression. Statistical Notes, no 24. Hyattsville, Maryland: National Center for Health Statistics. June 2002. [1] (http://www.cdc.gov/nchs/data/statnt/statnt24.pdf)

Retrieved from "http://en.wikipedia.org/w/index.php?title=Standard_error&oldid=597783020"
Categories: Statistical deviation and dispersion
