A little calculus

$$f(x) = x^r\,(1-x)^{1-r}$$

$$\text{Power rule}: \quad \frac{d}{dx}x^a = ax^{a-1} \qquad \text{(constant } a\text{)}$$

$$\text{Chain rule}: \quad \frac{d}{dx}g(x)h(x) = g'(x)h(x) + g(x)h'(x)$$

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\frac{d}{dx}x^r(1-x)^{1-r} &= rx^{r-1}\cdot(1-x)^{1-r} + x^r\cdot(1-r)(1-x)^{1-r-1}(-1) \\
&= x^{r-1}(1-x)^{1-r-1}\left[r(1-x)-(1-r)x\right] \\
&= x^{r-1}(1-x)^{-r}\left[r-rx-x+rx\right]
\end{aligned}$$

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$$\begin{aligned}
\frac{d}{dx} x^r (1-x)^{1-r} &= r x^{r-1} \cdot (1-x)^{1-r} + x^r \cdot (1-r)(1-x)^{1-r-1}(-1) \\
&= x^{r-1}(1-x)^{1-r-1}\left[r(1-x) - (1-r)x\right] \\
&= x^{r-1}(1-x)^{-r}\left[r - rx - x + rx\right] \\
&= x^{r-1}(1-x)^{-r}\left[r - x\right]
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\frac{d}{dx}x^r(1-x)^{1-r} &= rx^{r-1}\cdot(1-x)^{1-r} + x^r\cdot(1-r)(1-x)^{1-r-1}(-1)\\
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&= x^{r-1}(1-x)^{-r}\left[r-rx-x+rx\right]\\
&= x^{r-1}(1-x)^{-r}\left[r-x\right]
\end{aligned}
$$

$f'(x) = 0$ if, and only if, $x = r$. Conclude that $f(x)$ achieves its maximum at $x = r$.