Independent Samples T-test

- With previous tests, we were interested in comparing a single sample with a population
- With most research, you do not have knowledge about the population -- you don't know the population mean and standard deviation

INDEPENDENT SAMPLES T-TEST:
- Hypothesis testing procedure that uses separate samples for each treatment condition (between subjects design)
- Use this test when the population mean and standard deviation are unknown, and 2 separate groups are being compared

Example: Do males and females differ in terms of their exam scores?
- Take a sample of males and a separate sample of females and apply the hypothesis testing steps to determine if there is a significant difference in scores between the groups

Formula:

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{s_{\bar{x}_1 - \bar{x}_2}}$$

- We are interested in a difference between 2 populations (females, μ_1, and males, μ_2) and we use 2 samples (females, $\bar{x}_1$, and males, $\bar{x}_2$) to estimate this difference

ESTIMATED STANDARD ERROR OF THE DIFFERENCE:

- Gives us the total amount of error involved in using 2 sample means to estimate 2 population means. It tells us the average distance between the sample difference ($\bar{x}_1$-$\bar{x}_2$) and the population difference (μ_1-μ_2)

- As we've done previously, we have to estimate the standard error using the sample standard deviation or variance and, since there are 2 samples, we must average the two sample variances.

POOLED VARIANCE: The average of the two sample variances, allowing the larger sample to weighted more heavily

Formulae:

$$s^2_{pooled} = \frac{(df_1)s^2{}_1 + (df_2)s^2{}_2}{df_1 + df_2} \quad \textbf{\textit{OR}} \quad s^2_{pooled} = \frac{SS_1 + SS_2}{df_1 + df_2}$$

df_1=df for 1st sample; n_1-1

df_2=df for 2nd sample; n_2-1

Estimated Standard Error of the Difference

$$s_{\bar{x}_1 - \bar{x}_2} = \sqrt{\frac{s^2_{pooled}}{n_1} + \frac{s^2_{pooled}}{n_2}}$$

$$s_{\bar{x}_1 - \bar{x}_2} = \sqrt{\left(\frac{SS_1 + SS_2}{n_1 + n_2 - 2}\right)\left(\frac{1}{n_1} + \frac{1}{n_2}\right)} \longleftarrow \text{book formula}$$

Degrees of freedom (df) for the Independent t statistic is $n_1 + n_2 - 2$ or $df_1 + df_2$

Hypothesis testing using an Independent Samples t-Test:

Example: Do males and females differ in their test scores for exam 2? The mean test score for females is 27.1 (s=2.57, n=19), and the mean test score for males is 26.7 (s=3.63, n=20)

Step 1: State the hypotheses

H_0: $\mu_1-\mu_2=0$ $(\mu_1=\mu_2)$

H_1: $\mu_1-\mu_2\neq0$ $(\mu_1 \neq\mu_2)$

- This is a two-tailed test (no direction is predicted)

Step 2: Set the criterion

- α=?
- df= n_1+n_2-2=?
- Critical value for the t-test ?

Step 3: Collect sample data, calculate *x* and s

From the example we know the mean test score for females is 27.1 (s=2.57, n=19), and the mean test score for males is 26.7 (s=3.63, n=20)

Step 4: Compute the t-statistic

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{s_{\bar{x}_1 - \bar{x}_2}}$$

where

$$s_{\bar{x}_1 - \bar{x}_2} = \sqrt{\frac{s^2_{pooled}}{n_1} + \frac{s^2_{pooled}}{n_2}}$$

- Calculate the estimated standard error of the difference

$$s^2_{pooled} = \frac{(df_1)s^2{}_1 + (df_2)s^2{}_2}{df_1 + df_2}$$

$$s^2_{pooled} = \frac{(18)2.57^2 + (19)3.63^2}{18 + 19}$$

$$= \frac{(18)6.61 + (19)13.18}{37}$$

$$= \frac{118.98 + 250.36}{37}$$

$$= \frac{369.34}{37} = \underline{9.98}$$

- Compute the standard error (continued)

$$s_{\bar{x}_1-\bar{x}_2} = \sqrt{\frac{s^2_{pooled}}{n_1} + \frac{s^2_{pooled}}{n_2}}$$

$$s_{\bar{x}_1-\bar{x}_2} = \sqrt{\frac{9.98}{19} + \frac{9.98}{20}} = \sqrt{.525 + .499} = 1.01$$

- Calculate the t statistic

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{s_{\bar{x}_1-\bar{x}_2}}$$

*This always defaults to 0

$$t = \frac{(27.1 - 26.7)}{1.01} = \frac{.4}{1.01} = .396$$

Step 5: Make a decision about the hypotheses

- The critical value for a two-tailed t-test with df=37 (approx. 40) and α=.05 is 2.021
- Will we reject or fail to reject the null hypothesis?

Assumptions for the Independent t-Test:

- Independence: Observations within each sample must be independent (they don't influence each other)
- Normal Distribution: The scores in each population must be normally distributed
- Homogeneity of Variance: The two populations must have equal variances (the degree to which the distributions are spread out is approximately equal)

Repeated Measures T-test

- Uses the same sample of subjects measured on two different occasions (within-subjects design)
- Use this when the population mean and standard deviation are unknown and you are comparing the means of a sample of subjects before and after a treatment
- We are interested in finding out how much difference exists between subjects' scores before the treatment and after the treatment

DIFFERENCE SCORE (or D)

- The difference between subjects' scores before the treatment and after the treatment
- It is computed as x_2-x_1, where x_2 is the subjects' score after the treatment and x_1 is the subjects' score before the treatment
- We use the sample of difference scores to estimate the population of difference scores (μ_D)

Example:

Does alcohol affect a person's ability to drive?

A researcher selects a sample of 5 people and sets up an obstacle course.

- Each subject drives the course and the number of cones he or she knocks over is counted.
- Next, the researcher has each subject drink a six-pack of beer, then drive the course again, counting the number of cones each subject knocks over.

NOTE: Theory has shown that alcohol decreases motor and cognitive skills

<u>Step 1</u>: State the hypotheses

H_0: $\mu_D=0$

H_1: $\mu_D \neq 0$

<u>Step 2</u>: Set the criterion

- One-tail test or two-tail test?
- α=?
- df=n-1
- Critical value for t ?

<u>Step 3</u>: Collect sample data, calculate D

- Once the difference scores are obtained, all further statistics are calculated using these scores instead of the pretest / posttest or before / after scores

Subject	Before (x_1)	After (x_2)	D (x_2 - x_1)
1	2	8	6
2	0	4	4
3	4	11	7
4	2	5	3
5	3	8	5
			$\sum D = 25$

- Find the mean (average) difference score (D)

$$\overline{D} = \frac{\sum D}{n}$$

$$\overline{D} = \frac{6+4+7+3+5}{5} = \frac{25}{5} = 5$$

- The average difference of the number of cones knocked down from before drinking to after drinking is 5 cones. Remember, we are hypothesizing the difference to be zero.

<u>Step 4</u>: Calculate the t-statistic

<u>Formula</u>:

estimated std. deviation of diff. scores

$$t = \frac{\overline{D}}{s_{\overline{D}}} \qquad \text{where} \qquad s_{\overline{D}} = \frac{s_D}{\sqrt{n}} \qquad \text{and} \qquad \overline{D} = \frac{\sum D}{n}$$

estimated std.error of mean diff. scores

the mean difference score

- Compute the estimated standard deviation of the difference scores (s_D)

D	$D-\overline{D}$	$(D-\overline{D})^2$
6	1	1
4	-1	1
7	2	4
3	-2	4
5	0	0
		$SS_{\overline{D}} = 10$

$$s_D = \sqrt{\frac{SS}{n-1}} = \sqrt{\frac{10}{4}} = 1.58$$

- The average deviation of the difference scores (D) about the mean difference score (D) is 1.58 cones

- Compute the estimated standard error of the mean difference scores

$$s_{\overline{D}} = \frac{s_D}{\sqrt{n}} \qquad s_{\overline{D}} = \frac{1.58}{\sqrt{5}} = .707$$

  – The average deviation of the sample mean difference scores (D) from the population mean difference score (μ_D) is .707 cones

- Compute the t-statistic

$$t = \frac{\overline{D}}{s_{\overline{D}}} \quad t = \frac{5}{.707} = 7.07$$

Step 5: Make a decision

- The critical value for a one-tailed t-test with df=4 and α=.05 is 2.132
- Will we reject or fail to reject the null hypothesis?

Advantages and Disadvantages of the Repeated Measures t-Test:

Advantages:

- Controls for pre-existing individual differences between samples (because only 1 sample of people are being used
- More economical (fewer subjects are needed)

Disadvantages:

- Subject to practice effects - the subjects are performing the measurement task (i.e. driving the obstacle course, taking an exam) twice - scores may improve due to the practice

Assumptions of the Repeated Measures t-Test:

Independent Observations: The scores from before and after the treatment must not be related (no practice effects)

Normal Distribution: The population of difference scores must be normally distributed

Summary of Hypothesis Testing through *t*-statistic

- We have looked at four inferential statistics:
  - z-score statistic
  - single sample *t*-statistic
  - independent samples *t*-statistic
  - repeated measures *t*-statistic, or matched subjects
- the generic formula for these statistics is:

$$z \text{ or } t = \frac{\text{sample statistic - population parameter}}{\text{standard error}}$$

Summary of Hypothesis Testing through *t*-statistic

- z-score statistic compares a sample to a population when the population s.d. is known
- *t*-statistic compares a sample to a population when the population s.d. is <u>un</u>known
- independent samples *t*-statistic compares 2 independent samples
- repeated measures *t*-statistic compares 1 sample measured on 2 occasions