☰ Item Navigation

What have we learned?

In this lesson, we have studied in greater details the discrete Fourier transform (DFT), the Fourier analysis tool for finite-length signals.

The DFT is a simple change of basis. The analysis formula (moving from the time domain to the frequency domain) is given by the inner product of the signal with each of the Fourier basis vectors:

$$\begin{aligned} X[k] &= \langle \mathbf{w}^{(k)}, \mathbf{x} \rangle \\ &= \sum_{n=0}^{N-1} x[n] e^{-j\frac{2\pi}{N}nk}. \end{aligned}$$

The synthesis formula (moving from the frequency domain to the time domain) is a linear combination of the basis functions, scaled by the coefficients we found in the analysis formula:

$$\begin{aligned} x[n] &= \frac{1}{N} \sum_{k=0}^{N-1} X[k] \mathbf{w}^{(k)} \\ &= \frac{1}{N} \sum_{k=0}^{N-1} X[k] e^{j\frac{2\pi}{N}nk} \end{aligned}$$

In the most general case, the DFT coefficients $X[k]$ are complex numbers and thus have a real and imaginary part. In order to interpret these coefficients, it is convenient to represent their magnitude $|X[k]|$ and phase $\angle X[k]$, wrapped between $-\pi$ and π. In a DFT plot, frequencies between 0 and $N/2$ (resp. $N/2$ and $N-1$) correspond to digital frequencies smaller than π (resp. larger than π), in other words counterclockwise rotations (resp. clockwise rotations). Frequencies close to 0 and $N-1$ corresponds to the low frequencies and those close to $N/2$ to the high frequencies.

Finally, the DFT of a real signal is symmetric in magnitude, mathematically

$|X[k]| = |X[N-k]|, k = 1, \ldots, \lfloor N/2 \rfloor.$

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