Confidence Intervals For Proportions and Choosing the Sample Size

A Large Sample Confidence Interval for a Population Proportion

Recall that a confidence interval for a population mean is given by

Confidence Interval for a Population Mean

$$\bar{x} \pm \frac{z_c\, s}{\sqrt{n}}$$

We can make a similar construction for a confidence interval for a population proportion. Instead of $\bar{x}$, we can use p and instead of s, we use $\sqrt{p(1-p)}$, hence, we can write the confidence interval for a large sample proportion as

Confidence Interval Margin of Error for a Population Proportion

$$E = z_c\sqrt{\frac{p(1-p)}{n}}$$

Example

1000 randomly selected Americans were asked if they believed the minimum wage should be raised. 600 said yes. Construct a 95% confidence interval for the proportion of Americans who believe that the minimum wage should be raised.

Solution:

We have

$p = 600/1000 = .6$ $\quad z_c = 1.96$ $\quad$ and $\quad n = 1000$

We calculate:

$$0.6 \pm 1.96\sqrt{\frac{(0.6)(0.4)}{1000}} = 0.6 \pm 0.03$$

Hence we can conclude that between 57 and 63 percent of all Americans agree with the proposal. In other words, with a margin of error of .03 , 60% agree.

Calculating n for Estimating a Mean

Example

Suppose that you were interested in the average number of units that students take at a two year college to get an AA degree. Suppose you wanted to find a 95% confidence interval with a margin of error of .5 for μ knowing $\sigma = 10$. How many people should we ask?

Solution

Solving for n in

$$\text{Margin of Error} = E = \pm z_c \sigma / \sqrt{n}$$

we have

$$E\sqrt{n} = z_c\sigma$$

$$\sqrt{n} = \frac{z_c \sigma}{E}$$

Squaring both sides, we get

$$\boxed{n = \left(\frac{z_c\sigma}{E}\right)^2}$$

We use the formula:

$$n = \left(\frac{1.96(10)}{0.5}\right)^2 = 1{,}536$$

Example

A Subaru dealer wants to find out the age of their customers (for advertising purposes). They want the margin of error to be 3 years old. If they want a 90% confidence interval, how many people do they need to know about?

Solution:

We have

$E = 3, \quad z_c = 1.65$

but there is no way of finding sigma exactly. They use the following reasoning: most car customers are between 16 and 68 years old hence the range is

Range = 68 - 16 = 52

The range covers about four standard deviations hence one standard deviation is about

$\sigma \cong 52/4 = 13$

We can now calculate n:

$$n = \left(\frac{1.65(13)}{3}\right)^2 = 51.1$$

Hence the dealer should survey at least 52 people.

Finding n to Estimate a Proportion

Example

Suppose that you are in charge to see if dropping a computer will damage it. You want to find the proportion of computers that break. If you want a 90% confidence interval for this proportion, with a margin of error of $\pm$ 4%, How many computers should you drop?

Solution

The formula states that

$$E = z_c\sqrt{\frac{p(1-p)}{n}}$$

Squaring both sides, we get that

$$E^2 = \frac{z_c^2\, p(1 - p)}{n}$$

Multiplying by n, we get

$$nE^2 = z_c^2[p(1 - p)]$$

$$\boxed{n = p(1-p)\left(\frac{z_c}{E}\right)^2}$$

This is the formula for finding n.

Since we do not know p, we use .5 (A conservative estimate)

$$n = \frac{1}{4}\left(\frac{z_c}{E}\right)^2$$

$$n = \frac{1}{4}\left(\frac{1.645}{0.04}\right)^2 \approx 425.4$$

We round 425.4 up for greater accuracy

We will need to drop at least 426 computers. This could get expensive.

[Handout of more examples and exercises on finding the sample size]

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