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$A \cup B$ and C are independent.

Slogan

Events determined by disjoint subsets of independent events are also independent.