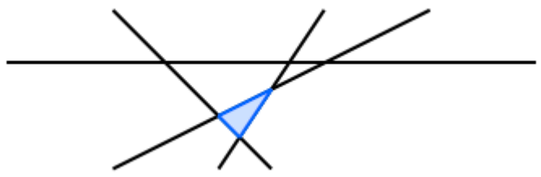
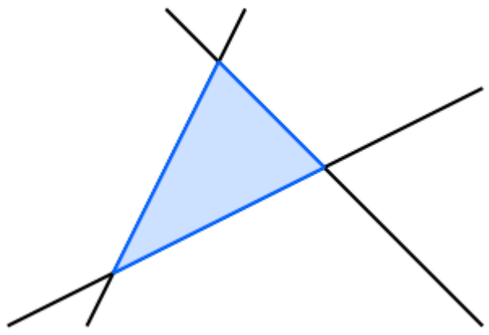
Triangular Piece

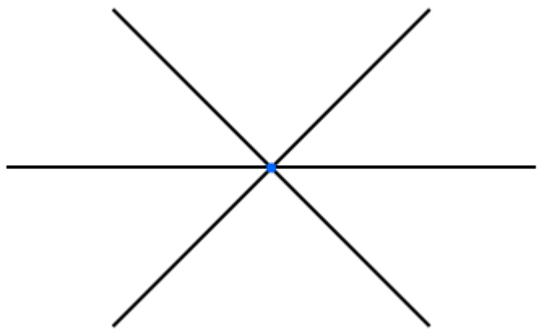
Problem:
Several straight lines (at least three) cut a plane into pieces. Each line intersects with every other line, and all the intersection points are different. Prove that there is at least one triangular piece.



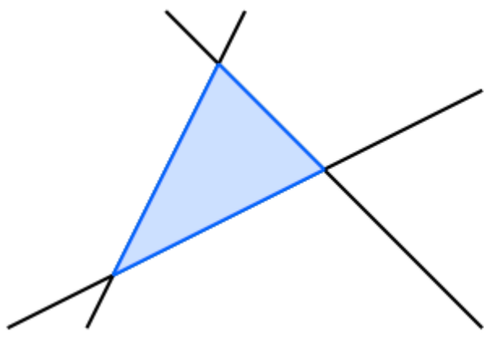
We will prove this statement by induction on the number of lines. The base case is when there are only three lines. Since the lines intersect at three different points, they must form a triangle.

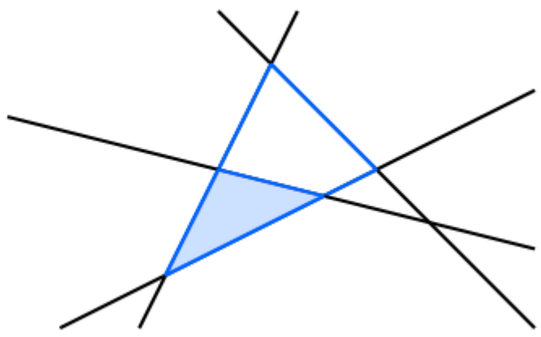


Indeed, the only case when three lines do not form a triangle - when they intersect at one point - is forbidden.

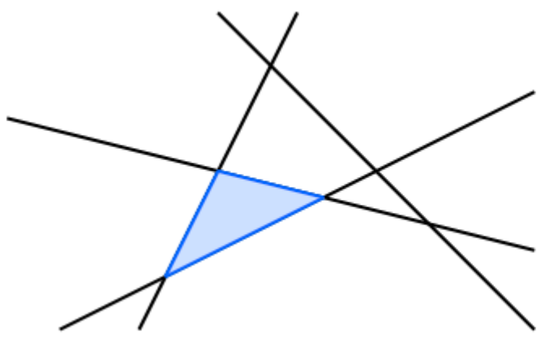


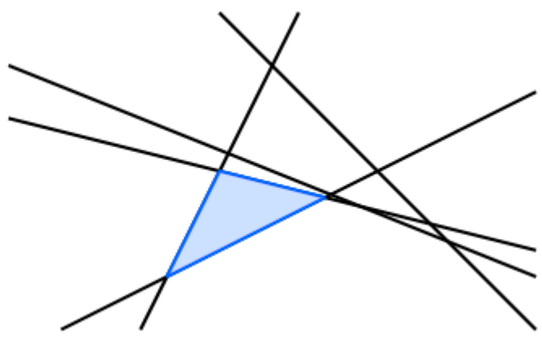
In order to prove the induction step, we start with $n \geq 3$ lines (which, by the induction hypothesis, already form a triangle T) and add one more line L. Consider two cases:
Case 1: L intersects T. In this case, L must intersect two sides of T (L cannot intersect T in a vertex, because no triple of lines intersects at one point). When a line intersects two sides of a triangle, it creates a new triangle.





Case 2: L does not intersect T. In this case, T remains intact, so the new picture still has the triangle T.





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