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Gamma codes

Table 5.5: Some examples of unary and γ codes. Unary codes are only shown for the smaller numbers. Commas in γ codes are for readability only and are not part of the actual codes.

	number	unary code	length	offset	γ code	
	0	0				
	1	10	0		0	
	2	110	10	0	10,0	
	3	1110	10	1	10,1	
	4	11110	110	00	110,00	
	9	1111111110	1110	001	1110,001	
	13		1110	101	1110,101	
	24		11110	1000	11110,1000	
	511		111111110	11111111	111111110,11111111	
	1025		11111111110	0000000001	11111111110,0000000001	

VB codes use an adaptive number of *bytes* depending on the size of the gap. Bit-level codes adapt the length of the code on the finer grained *bit* level. The simplest bit-level code is *unary code* . The unary code of n is a string of n 1s followed by a 0 (see the first two columns of Table 5.5). Obviously, this is not a very efficient code, but it will come in handy in a moment.

How efficient can a code be in principle? Assuming the 2^n gaps G with $1 \leq G \leq 2^n$ are all equally likely, the optimal encoding uses n bits for each G. So some gaps ($G = 2^n$ in this case) cannot be encoded with fewer than $\log_2 G$ bits. Our goal is to get as close to this lower bound as possible.

A method that is within a factor of optimal is γ *encoding* . γ codes implement variable-length encoding by splitting the representation of a gap G into a pair of *length* and *offset*. *Offset* is G in binary, but with the leading 1 removed. For example, for 13 (binary 1101) *offset* is 101. *Length* encodes the length of *offset* in unary code. For 13, the length of *offset* is 3 bits, which is 1110 in unary. The γ code of 13 is therefore 1110101, the concatenation of length 1110 and offset 101. The right hand column of Table 5.5 gives additional examples of γ codes.

A γ code is decoded by first reading the unary code up to the 0 that terminates it, for example, the four bits 1110 when decoding 1110101. Now we know how long the offset is: 3 bits. The offset 101 can then be read correctly and the 1 that was chopped off in encoding is prepended: 101 $\rightarrow$ 1101 = 13.

The length of *offset* is $\lfloor \log_2 G \rfloor$ bits and the length of *length* is $\lfloor \log_2 G \rfloor + 1$ bits, so the length of the entire code is $2 \times \lfloor \log_2 G \rfloor + 1$ bits. γ codes are always of odd length and they are within a factor of 2 of what we claimed to be the optimal encoding length $\log_2 G$. We derived this optimum from the assumption that the 2^n gaps between 1 and 2^n are equiprobable. But this need not be the case. In general, we do not know the probability distribution over gaps a priori.

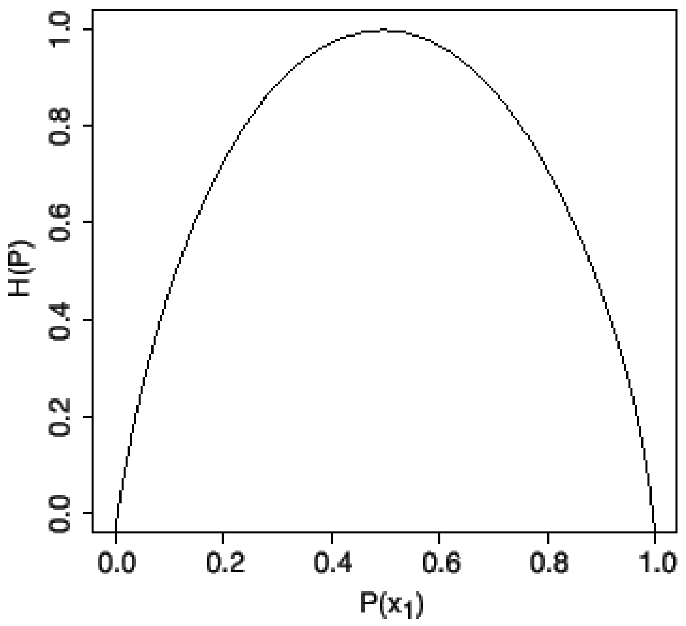


Figure 5.9: Entropy $H(P)$ as a function of $P(x_1)$ for a sample space with two outcomes x_1 and x_2.

The characteristic of a discrete probability distribution P that determines its coding properties (including whether a code is optimal) is its *entropy* $H(P)$, which is defined as follows:

$$H(P) = -\sum_{x \in X} P(x) \log_2 P(x) \tag{4}$$

where X is the set of all possible numbers we need to be able to encode (and therefore $\sum_{x \in X} P(x) = 1.0$). Entropy is a measure of uncertainty as shown in Figure 5.9 for a probability distribution P over two possible outcomes, namely, $X = \{x_1, x_2\}$. Entropy is maximized ($H(P) = 1$) for $P(x_1) = P(x_2) = 0.5$ when uncertainty about which x_i will appear next is largest; and minimized ($H(P) = 0$) for $P(x_1) = 1, P(x_2) = 0$ and for $P(x_1) = 0, P(x_2) = 1$ when there is absolute certainty.

It can be shown that the lower bound for the expected length $E(L)$ of a code L is $H(P)$ if certain conditions hold (see the references). It can further be shown that for $1 < H(P) < \infty$, γ encoding is within a factor of 3 of this optimal encoding, approaching 2 for large $H(P)$:

$$\frac{E(L_\gamma)}{H(P)} \leq 2 + \frac{1}{H(P)} \leq 3. \tag{5}$$

What is remarkable about this result is that it holds for any probability distribution P. So without knowing anything about the properties of the distribution of gaps, we can apply γ codes and be certain that they are within a factor of ≈ 2 of the optimal code for distributions of large entropy. A code like γ code with the property of being within a factor of optimal for an arbitrary distribution P is called *universal* .

In addition to universality, γ codes have two other properties that are useful for index compression. First, they are *prefix free* , namely, no γ code is the prefix of another. This means that there is always a unique decoding of a sequence of γ codes - and we do not need delimiters between them, which would decrease the efficiency of the code. The second property is that γ codes are *parameter free* . For many other efficient codes, we have to fit the parameters of a model (e.g., the binomial distribution) to the distribution of gaps in the index. This complicates the implementation of compression and decompression. For instance, the parameters need to be stored and retrieved. And in dynamic indexing, the distribution of gaps can change, so that the original parameters are no longer appropriate. These problems are avoided with a parameter-free code.

How much compression of the inverted index do γ codes achieve? To answer this question we use Zipf's law, the term distribution model introduced in Section 5.1.2 . According to Zipf's law, the collection frequency cf_i is proportional to the inverse of the rank i, that is, there is a constant c' such that:

$$\mathrm{cf}_i = \frac{c'}{i}. \tag{6}$$

We can choose a different constant c such that the fractions c/i are relative frequencies and sum to 1 (that is, $c/i = \mathrm{cf}_i/T$):

$$1 = \sum_{i=1}^{M} \frac{c}{i} = c \sum_{i=1}^{M} \frac{1}{i} \quad = \quad c\, H_M \tag{7}$$

$$c = \frac{1}{H_M} \tag{8}$$

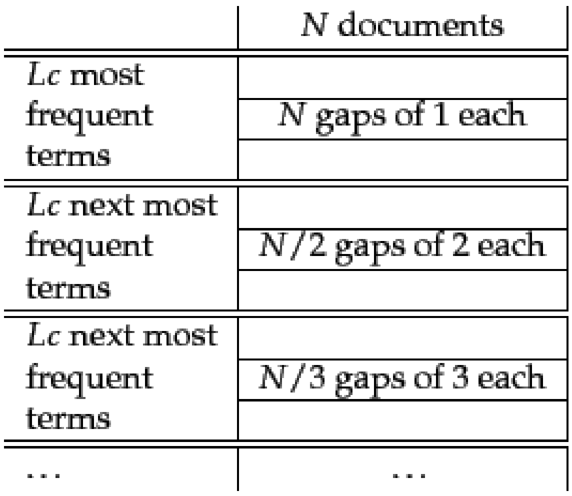
where M is the number of distinct terms and H_M is the Mth *harmonic number* . Reuters-RCV1 has $M = 400{,}000$ distinct terms and $H_M \approx \ln M$, so we have

$$c = \frac{1}{H_M} \approx \frac{1}{\ln M} = \frac{1}{\ln 400{,}000} \approx \frac{1}{13}. \tag{9}$$

Thus the ith term has a relative frequency of roughly $1/(13i)$, and the expected average number of occurrences of term i in a document of length L is:

$$L\frac{c}{i} \approx \frac{200 \times \frac{1}{13}}{i} \approx \frac{15}{i} \tag{10}$$

where we interpret the relative frequency as a term occurrence probability. Recall that 200 is the average number of tokens per document in Reuters-RCV1 (Table 4.2).



	N documents
Lc most frequent terms	N gaps of 1 each
Lc next most frequent terms	$N/2$ gaps of 2 each
Lc next most frequent terms	$N/3$ gaps of 3 each
...	...

Figure 5.10: Stratification of terms for estimating the size of a γ encoded inverted index.

Now we have derived term statistics that characterize the distribution of terms in the collection and, by extension, the distribution of gaps in the postings lists. From these statistics, we can calculate the space requirements for an inverted index compressed with γ encoding. We first stratify the vocabulary into blocks of size $Lc = 15$. On average, term i occurs $15/i$ times per document. So the average number of occurrences $\overline{f}$ per document is $1 \leq \overline{f}$ for terms in the first block, corresponding to a total number of N gaps per term. The average is $\frac{1}{2} \leq \overline{f} < 1$ for terms in the second block, corresponding to $N/2$ gaps per term, and $\frac{1}{3} \leq \overline{f} < \frac{1}{2}$ for terms in the third block, corresponding to $N/3$ gaps per term, and so on. (We take the lower bound because it simplifies subsequent calculations. As we will see, the final estimate is too pessimistic, even with this assumption.) We will make the somewhat unrealistic assumption that all gaps for a given term have the same size as shown in Figure 5.10. Assuming such a uniform distribution of gaps, we then have gaps of size 1 in block 1, gaps of size 2 in block 2, and so on.

Encoding the N/j gaps of size j with γ codes, the number of bits needed for the postings list of a term in the jth block (corresponding to one row in the figure) is:

$$\begin{aligned} \mathit{bits\text{-}per\text{-}row} &= \frac{N}{j} \times (2 \times \lfloor \log_2 j \rfloor + 1) \\ &\approx \frac{2N \log_2 j}{j}. \end{aligned}$$

To encode the entire block, we need $(Lc) \cdot (2N \log_2 j)/j$ bits. There are $M/(Lc)$ blocks, so the postings file as a whole will take up:

$$\sum_{j=1}^{\frac{M}{Lc}} \frac{2NLc \log_2 j}{j}. \qquad (11)$$

For Reuters-RCV1, $\frac{M}{Lc} \approx$ 400,000 $/15 \approx 27{,}000$ and

$$\sum_{j=1}^{27{,}000} \frac{2 \times 10^6 \times 15 \log_2 j}{j} \approx 224 \text{ MB}. \qquad (12)$$

So the postings file of the compressed inverted index for our 960 MB collection has a size of 224 MB, one fourth the size of the original collection.

When we run γ compression on Reuters-RCV1, the actual size of the compressed index is even lower: 101 MB, a bit more than one tenth of the size of the collection. The reason for the discrepancy between predicted and actual value is that (i) Zipf's law is not a very good approximation of the actual distribution of term frequencies for Reuters-RCV1 and (ii) gaps are not uniform. The Zipf model predicts an index size of 251 MB for the unrounded numbers from Table 4.2 . If term frequencies are generated from the Zipf model and a compressed index is created for these artificial terms, then the compressed size is 254 MB. So to the extent that the assumptions about the distribution of term frequencies are accurate, the predictions of the model are correct.

Table: Index and dictionary compression for Reuters-RCV1. The compression ratio depends on the proportion of actual text in the collection. Reuters-RCV1 contains a large amount of XML markup. Using the two best compression schemes, γ encoding and blocking with front coding, the ratio compressed index to collection size is therefore especially small for Reuters-RCV1: $(101+7.9)/3600 \approx 0.03$.

$(101+5.9)/3600 \approx 0.03$.

	data structure	size in MB
	dictionary, fixed-width	19.211.2
	dictionary, term pointers into string	10.8 7.6
	$\sim$, with blocking, $k=4$	10.3 7.1
	$\sim$, with blocking & front coding	7.9 5.9
	collection (text, xml markup etc)	3600.0
	collection (text)	960.0
	term incidence matrix	40,000.0
	postings, uncompressed (32-bit words)	400.0
	postings, uncompressed (20 bits)	250.0
	postings, variable byte encoded	116.0
	postings, γ encoded	101.0

Table 5.6 summarizes the compression techniques covered in this chapter. The term incidence matrix (Figure 1.1 , page 1.1) for Reuters-RCV1 has size $400{,}000 \times 800{,}000 = 40 \times 8 \times 10^9$ bits or 40 GB.

The numbers were the collection (3600 MB and 960 MB) are for the encoding of RCV1 of CD, which uses one byte per character, not Unicode.

γ codes achieve great compression ratios - about 15% better than variable byte codes for Reuters-RCV1. But they are expensive to decode. This is because many bit-level operations - shifts and masks - are necessary to decode a sequence of γ codes as the boundaries between codes will usually be somewhere in the middle of a machine word. As a result, query processing is more expensive for γ codes than for variable byte codes. Whether we choose variable byte or γ encoding depends on the characteristics of an application, for example, on the relative weights we give to conserving disk space versus maximizing query response time.

The compression ratio for the index in Table 5.6 is about 25%: 400 MB (uncompressed, each posting stored as a 32-bit word) versus 101 MB (γ) and 116 MB (VB). This shows that both γ and VB codes meet the objectives we stated in the beginning of the chapter. Index compression substantially improves time and space efficiency of indexes by reducing the amount of disk space needed, increasing the amount of information that can be kept in the cache, and speeding up data transfers from disk to memory.

Exercises.

- Compute variable byte codes for the numbers in Tables 5.3 5.5 .
- Compute variable byte and γ codes for the postings list $\langle 777, 17743, 294068, 31251336 \rangle$. Use gaps instead of docIDs where possible. Write binary codes in 8-bit blocks.
- Consider the postings list $\langle 4, 10, 11, 12, 15, 62, 63, 265, 268, 270, 400 \rangle$ with a corresponding list of gaps $\langle 4, 6, 1, 1, 3, 47, 1, 202, 3, 2, 130 \rangle$. Assume that the length of the postings list is stored separately, so the system knows when a postings list is complete. Using variable byte encoding: (i) What is the largest gap you can encode in 1 byte? (ii) What is the largest gap you can encode in 2 bytes? (iii) How many bytes will the above postings list require under this encoding? (Count only space for encoding the sequence of numbers.)
- A little trick is to notice that a gap cannot be of length 0 and that the stuff left to encode after shifting cannot be 0. Based on these observations: (i) Suggest a modification to variable byte encoding that allows you to encode slightly larger gaps in the same amount of space. (ii) What is the largest gap you can encode in 1 byte? (iii) What is the largest gap you can encode in 2 bytes? (iv) How many bytes will the postings list in Exercise 5.3.2 require under this encoding? (Count only space for encoding the sequence of numbers.)
- From the following sequence of γ-coded gaps, reconstruct first the gap sequence and then the postings sequence: 1110001110101011111101101111011.
- γ codes are relatively inefficient for large numbers (e.g., 1025 in Table 5.5) as they encode the length of the offset in inefficient unary code. δ *codes* differ from γ codes in that they encode the first part of the code (*length*) in γ code instead of unary code. The encoding of *offset* is the same. For example, the δ code of 7 is 10,0,11 (again, we add commas for readability). 10,0 is the γ code for *length* (2 in this case) and the encoding of *offset* (11) is unchanged. (i) Compute the δ codes for the other numbers in Table 5.5 . For what range of numbers is the δ code shorter than the γ code? (ii) γ code beats variable byte code in Table 5.6 because the index contains stop words and thus many small gaps. Show that variable byte code is more compact if larger gaps dominate. (iii) Compare the compression ratios of δ code and variable byte code for a distribution of gaps dominated by large gaps.
- Go through the above calculation of index size and explicitly state all the approximations that were made to arrive at Equation 11.
- For a collection of your choosing, determine the number of documents and terms and the average length of a document. (i) How large is the inverted index predicted to be by Equation 11? (ii) Implement an indexer that creates a γ-compressed inverted index for the

collection. How large is the actual index? (iii) Implement an indexer that uses variable byte encoding. How large is the variable byte encoded index?

Table: Two gap sequences to be merged in blocked sort-based indexing

γ encoded gap sequence of run 1	1110110111111001011111111110100011111001
γ encoded gap sequence of run 2	111110100000111111000100011111110010000011111010101

- To be able to hold as many postings as possible in main memory, it is a good idea to compress intermediate index files during index construction. (i) This makes merging runs in blocked sort-based indexing more complicated. As an example, work out the γ-encoded merged sequence of the gaps in Table 5.7 . (ii) Index construction is more space efficient when using compression. Would you also expect it to be faster?
- (i) Show that the size of the vocabulary is finite according to Zipf's law and infinite according to Heaps' law. (ii) Can we derive Heaps' law from Zipf's law?