Tableau 9, Part 2

Independence
Repeated independent trials, product spaces

Independent possibilities multiply

Extended multiplication tables

Extended multiplication tables

Alphabets

$\mathfrak{A} = \{a_1, \dots , a_L\}, \mathfrak{B} = \{b_1, \dots , b_M\}, \mathfrak{C} = \{c_1, \dots , c_N\}$

Extended multiplication tables

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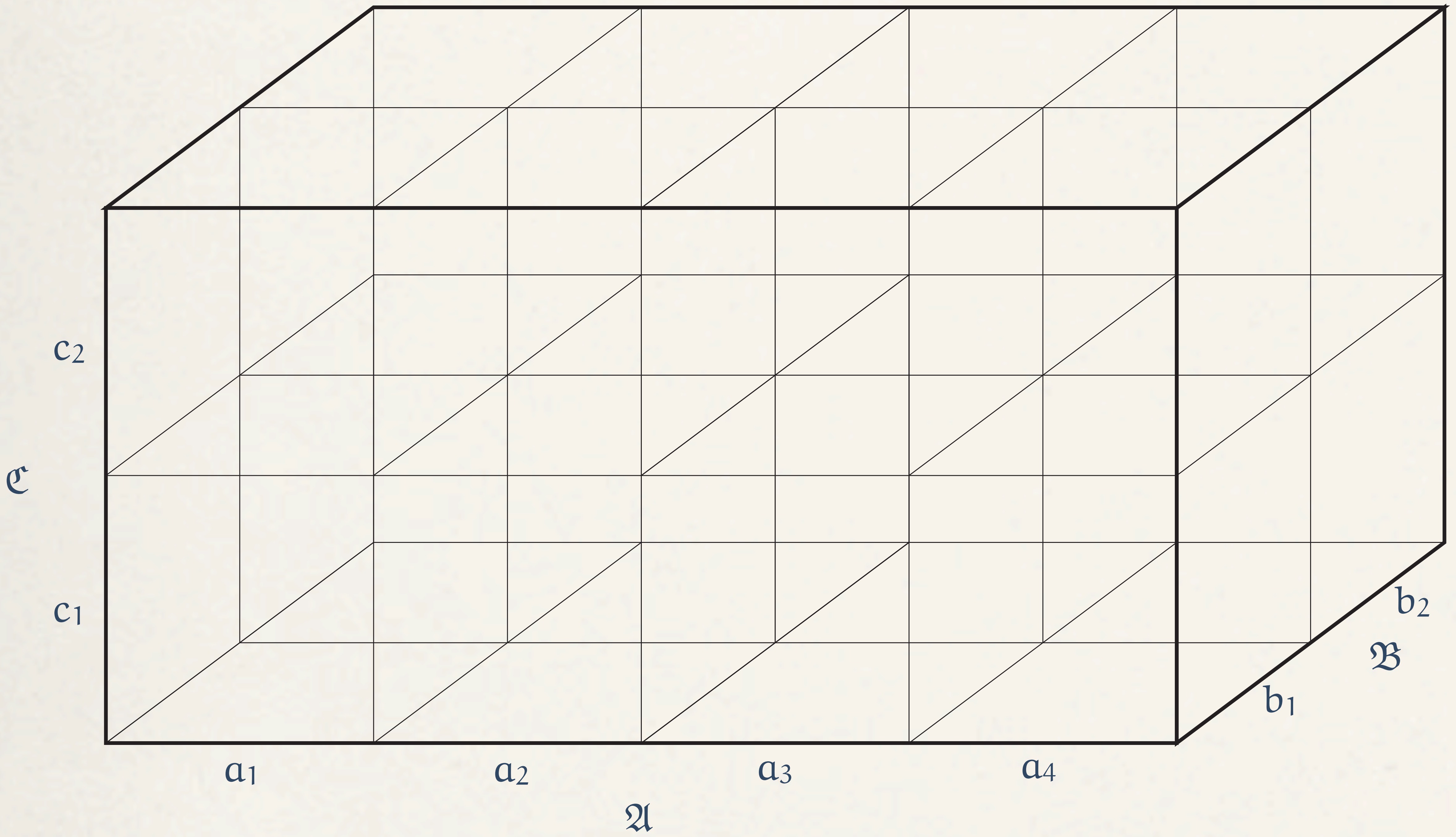

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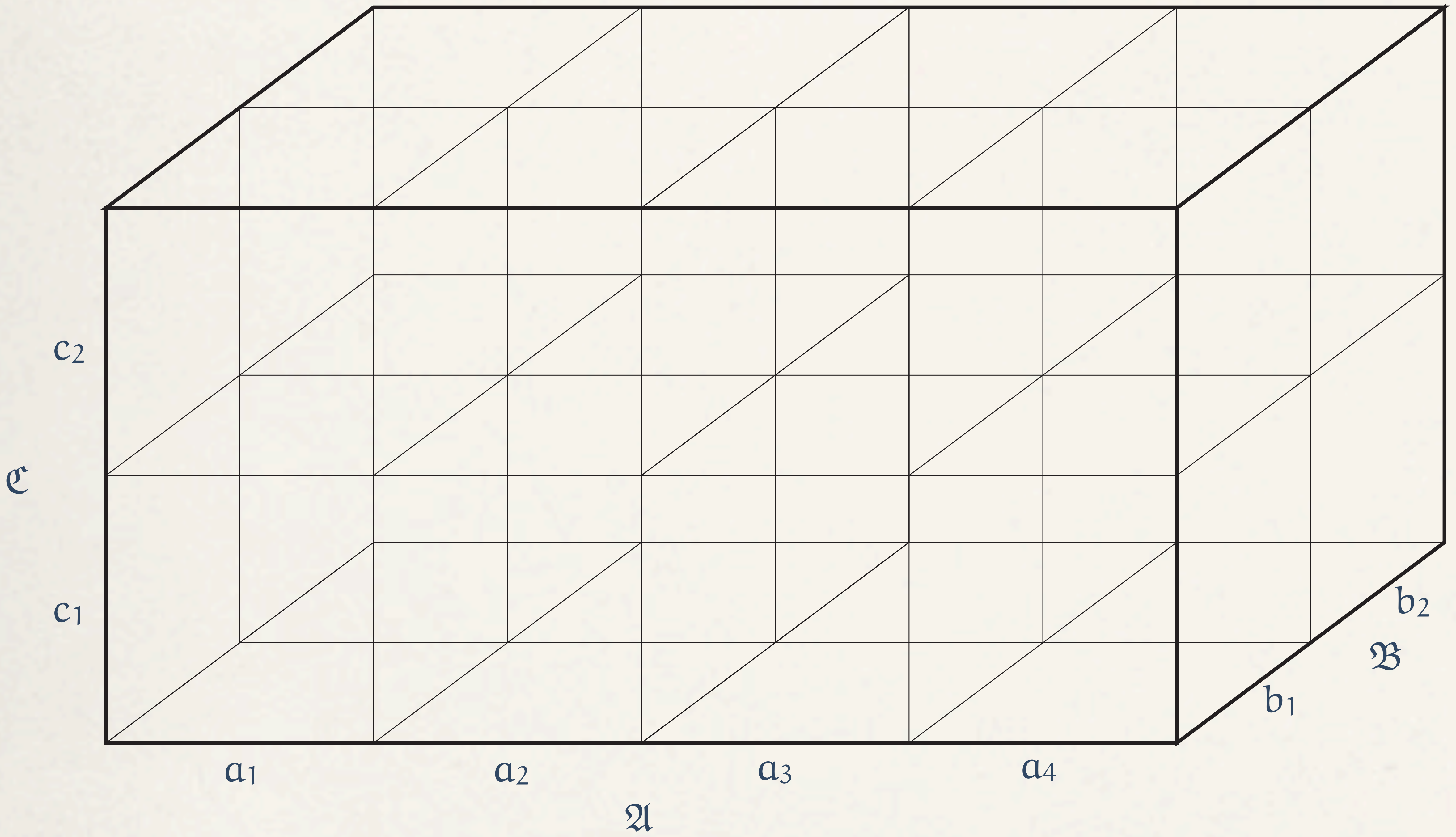

Cartesian products

$$\mathfrak{A} \times \mathfrak{B} = \{(a, b): a \in \mathfrak{A}, b \in \mathfrak{B}\}$$

$$\mathfrak{A} \times \mathfrak{C} = \{(a, c): a \in \mathfrak{A}, c \in \mathfrak{C}\}$$

$$\mathfrak{B} \times \mathfrak{C} = \{(b, c): b \in \mathfrak{B}, c \in \mathfrak{C}\}$$

$$\mathfrak{A} \times \mathfrak{B} \times \mathfrak{C} = \{(a, b, c): a \in \mathfrak{A}, b \in \mathfrak{B}, c \in \mathfrak{C}\}$$

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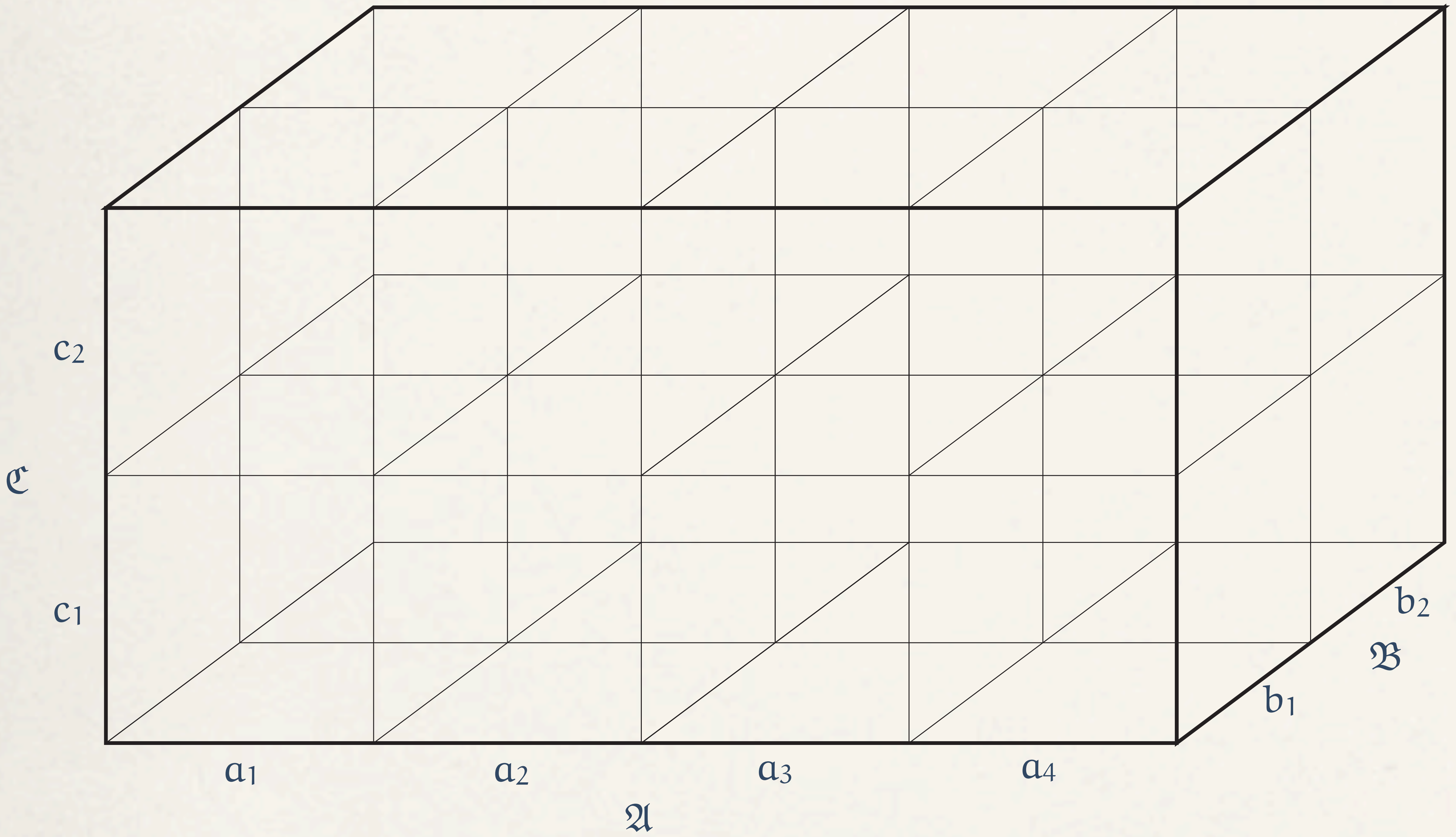

Cartesian products	# elements
$\mathfrak{A} \times \mathfrak{B} = \{(a, b): a \in \mathfrak{A}, b \in \mathfrak{B}\}$	LM
$\mathfrak{A} \times \mathfrak{C} = \{(a, c): a \in \mathfrak{A}, c \in \mathfrak{C}\}$	LN
$\mathfrak{B} \times \mathfrak{C} = \{(b, c): b \in \mathfrak{B}, c \in \mathfrak{C}\}$	MN
$\mathfrak{A} \times \mathfrak{B} \times \mathfrak{C} = \{(a, b, c): a \in \mathfrak{A}, b \in \mathfrak{B}, c \in \mathfrak{C}\}$	LMN

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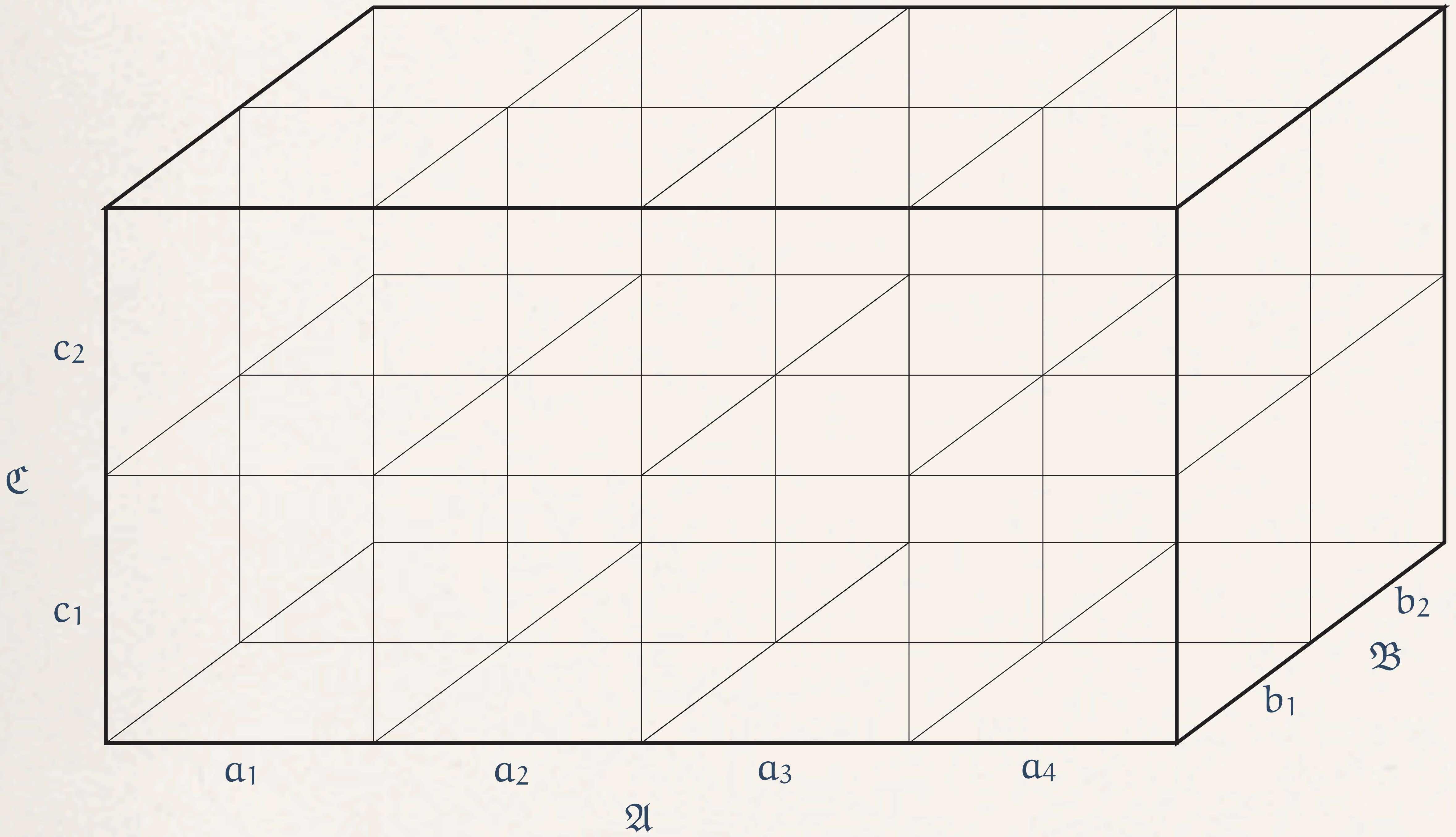

Cartesian products	# elements
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$\mathfrak{A} \times \mathfrak{C} = \{(a, c) : a \in \mathfrak{A}, c \in \mathfrak{C}\}$	LN
$\mathfrak{B} \times \mathfrak{C} = \{(b, c) : b \in \mathfrak{B}, c \in \mathfrak{C}\}$	MN
$\mathfrak{A} \times \mathfrak{B} \times \mathfrak{C} = \{(a, b, c) : a \in \mathfrak{A}, b \in \mathfrak{B}, c \in \mathfrak{C}\}$	LMN

A basic principle of counting: Independent possibilities multiply!

Three independent events

Independent possibilities multiply!

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Definition

Events A, B, and C in a probability space are independent if (and only if) each of the following four conditions is satisfied:

1) $\mathbf{P}(A \cap B) = \mathbf{P}(A) \times \mathbf{P}(B)$,
2) $\mathbf{P}(A \cap C) = \mathbf{P}(A) \times \mathbf{P}(C)$,
3) $\mathbf{P}(B \cap C) = \mathbf{P}(B) \times \mathbf{P}(C)$,
4) $\mathbf{P}(A \cap B \cap C) = \mathbf{P}(A) \times \mathbf{P}(B) \times \mathbf{P}(C)$.

The events A, B, and C are said to be pairwise independent if the first three conditions are satisfied.