The throw of two dice

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Two dice are thrown. If the sum of the face values is 8, what is the chance that at least one of the two dice shows a 6 (an “ace”)?

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- *Probability measure* **P**:
  - Combinatorial setting: to each *atom* (singleton set) assign equal *probability mass*
  $$\mathbf{P}\{(i, j)\} = 1/36 \qquad (1 \leq i, j \leq 6)$$
  - Event probabilities via additivity:
  $$\mathbf{P}(A) = 11/36$$
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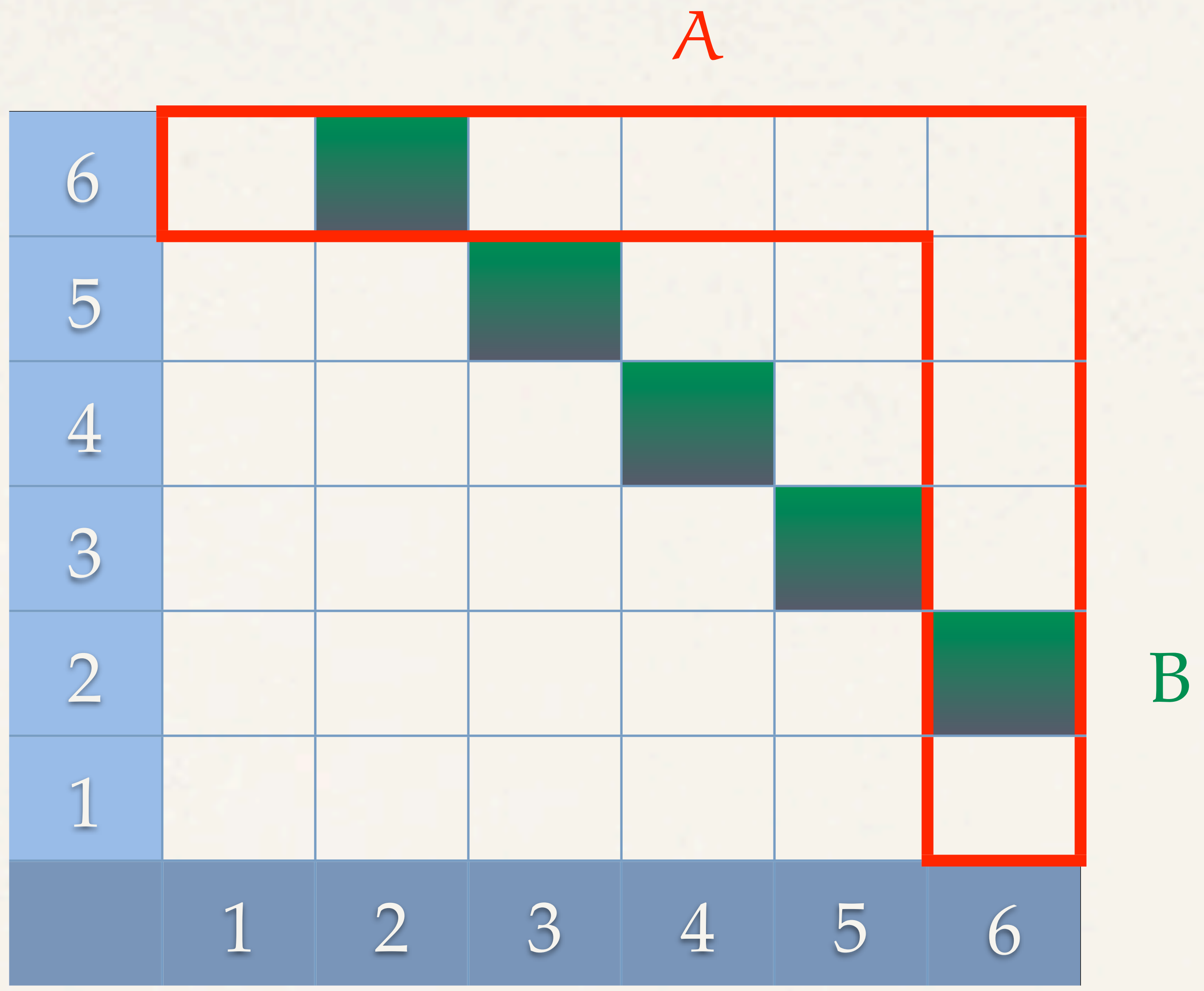

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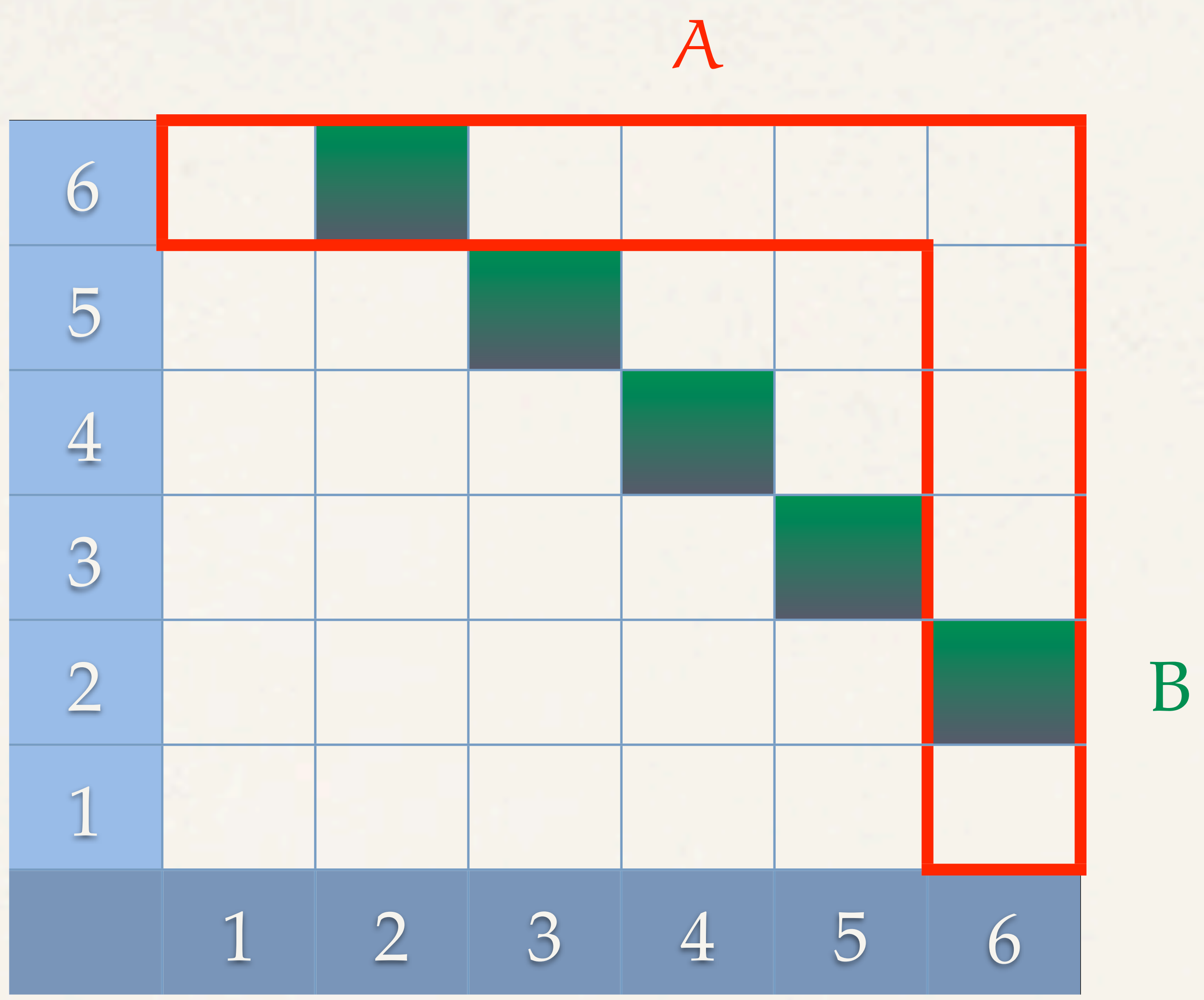

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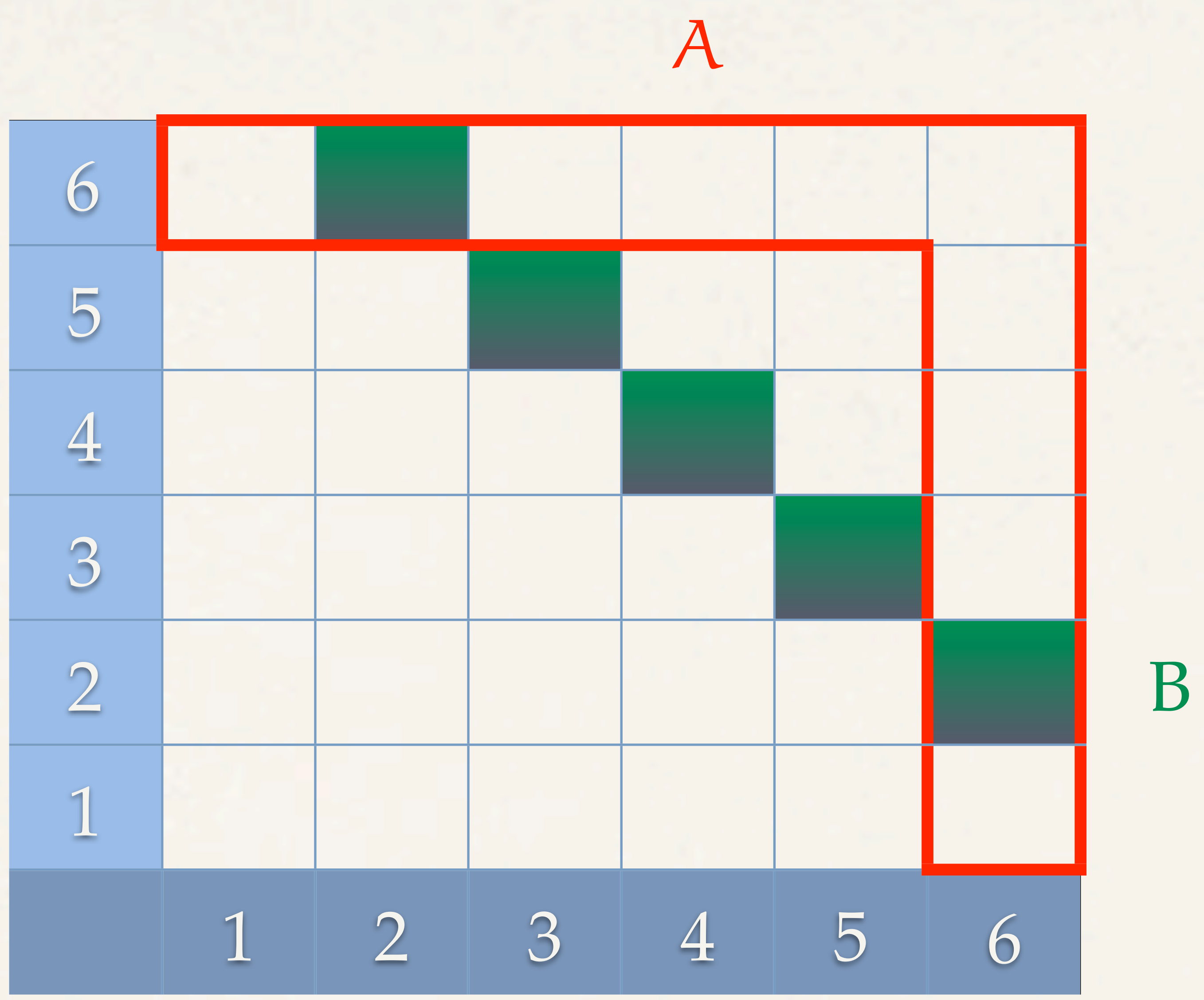
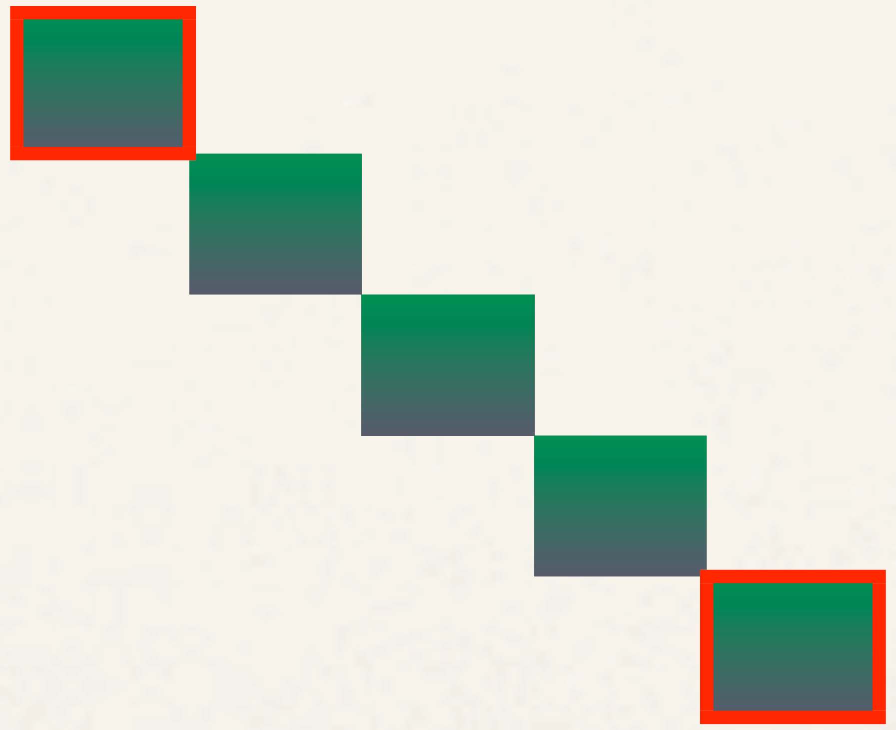


Conditioned on
sum being 8



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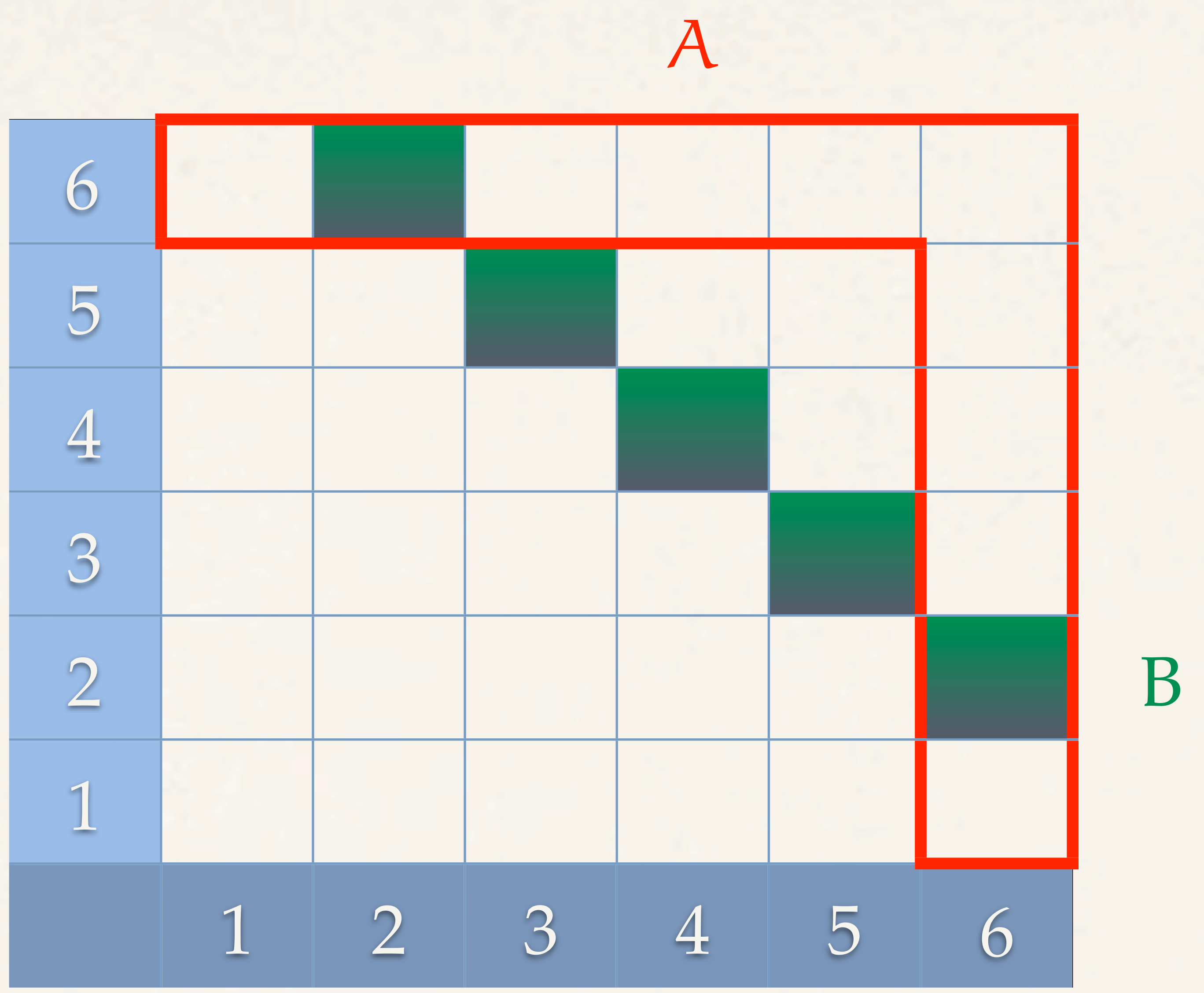

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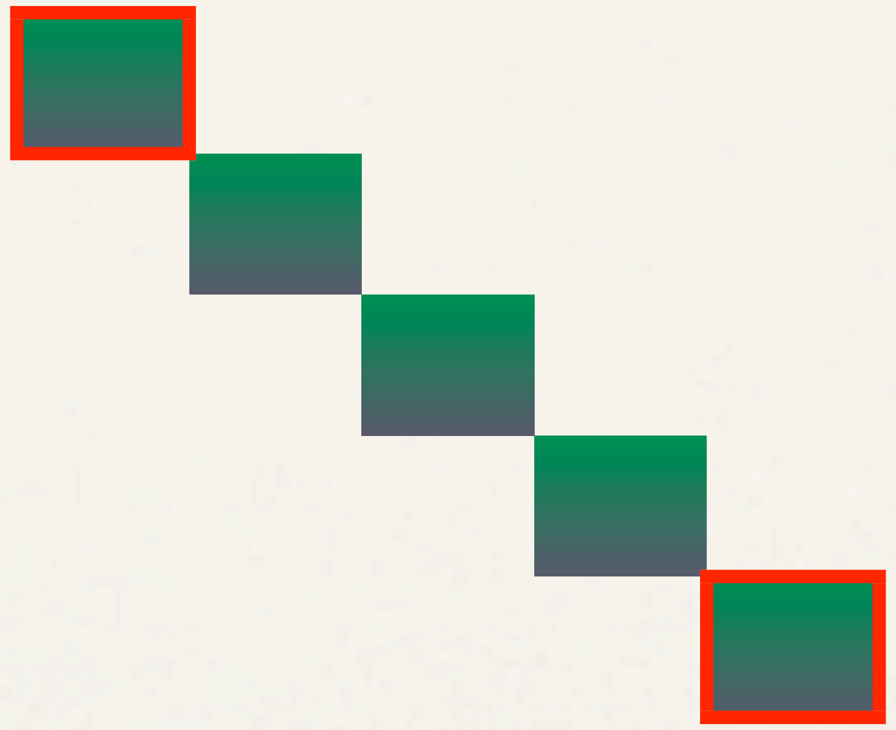

The chance of A
(given that B occurs)

$$\frac{\mathbf{P}(A \cap B)}{\mathbf{P}(B)} = \frac{2/36}{5/36} = \frac{2}{5}$$

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