Reversing the direction of conditioning: Bayes's rule for events

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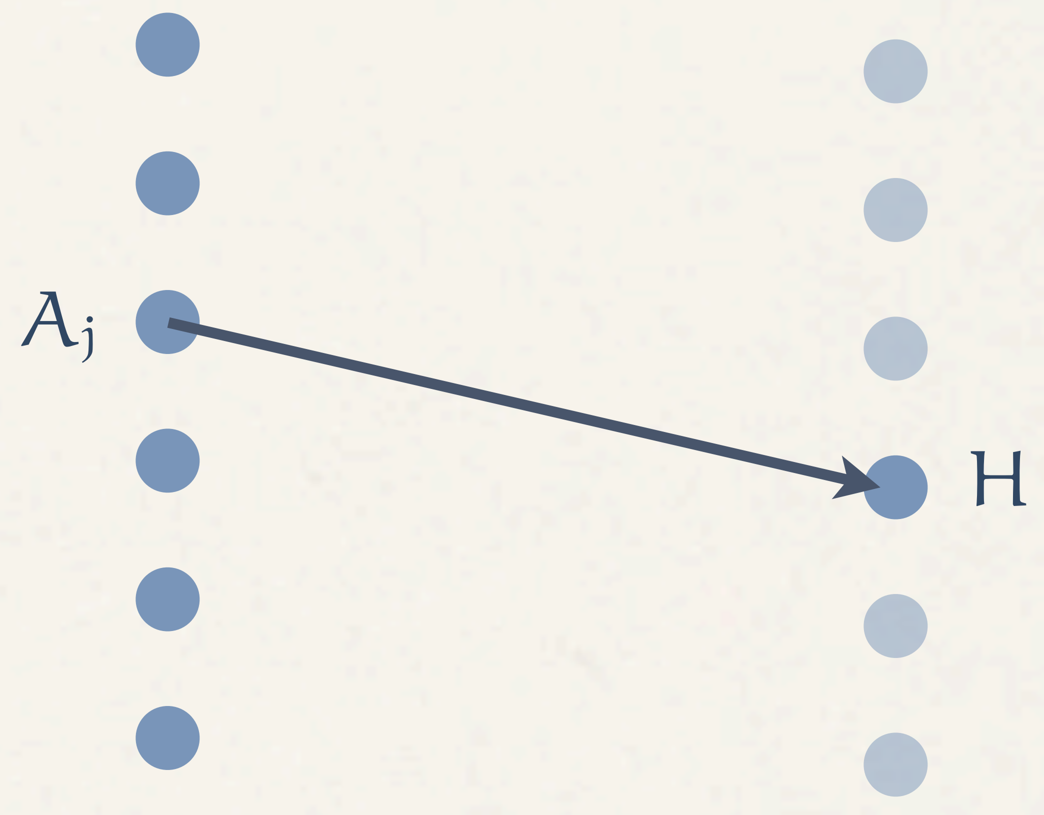

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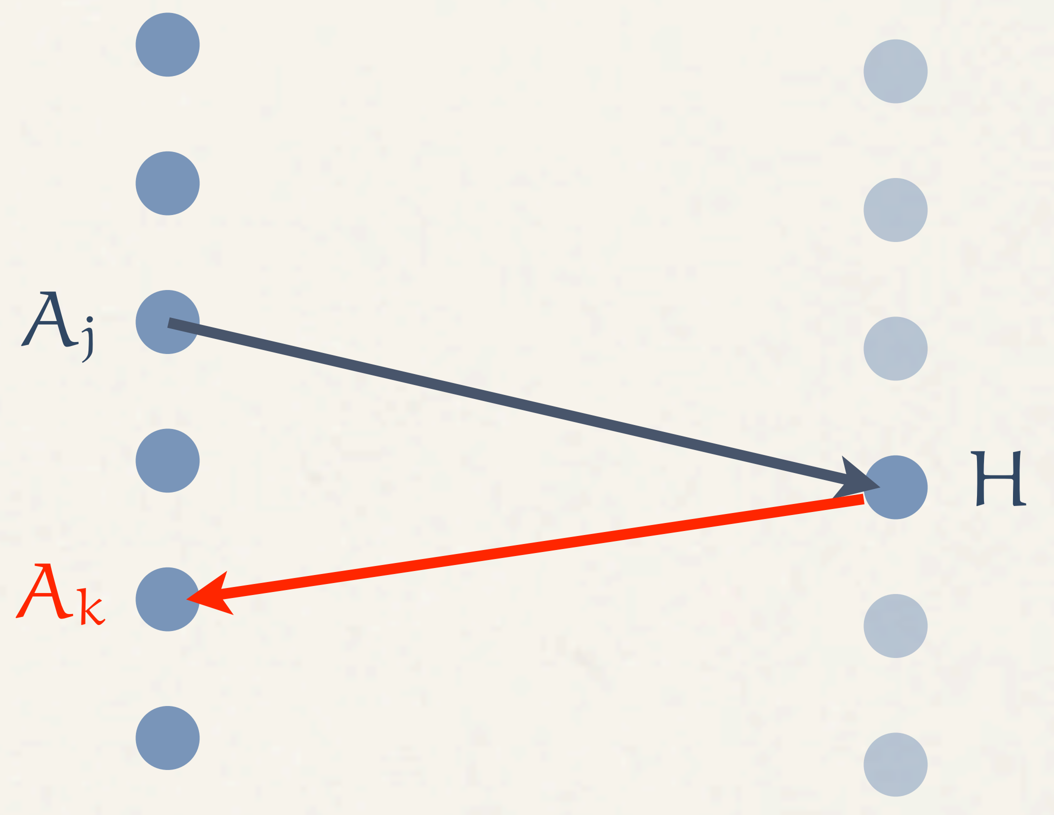

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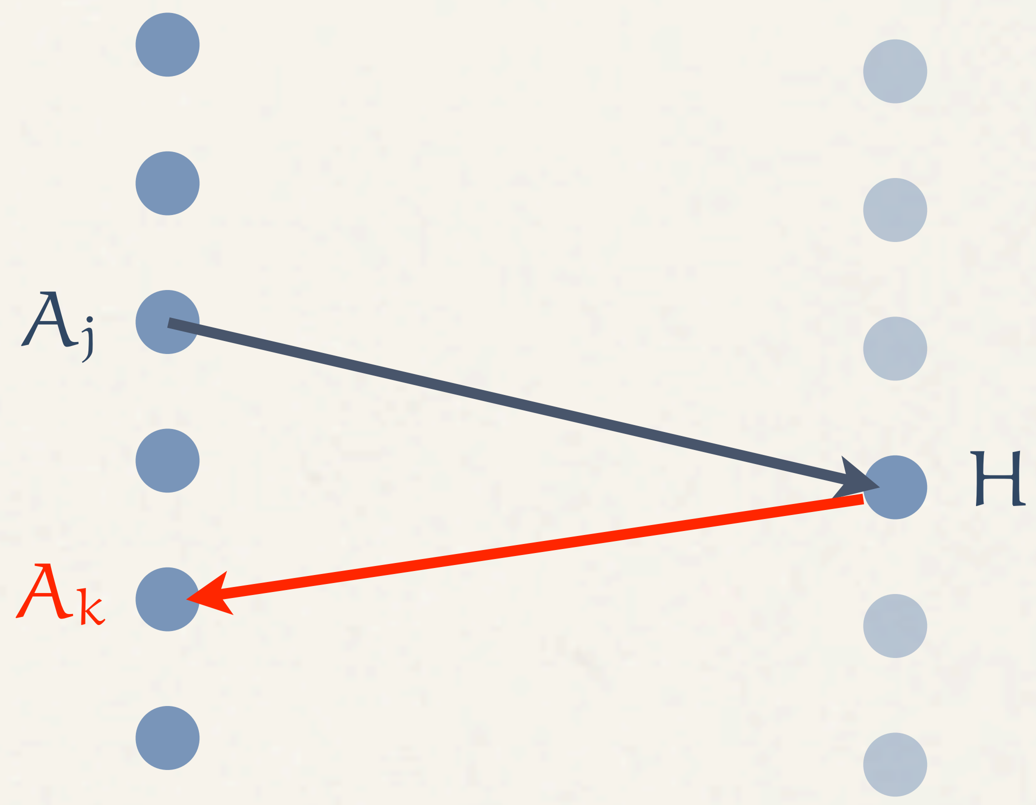

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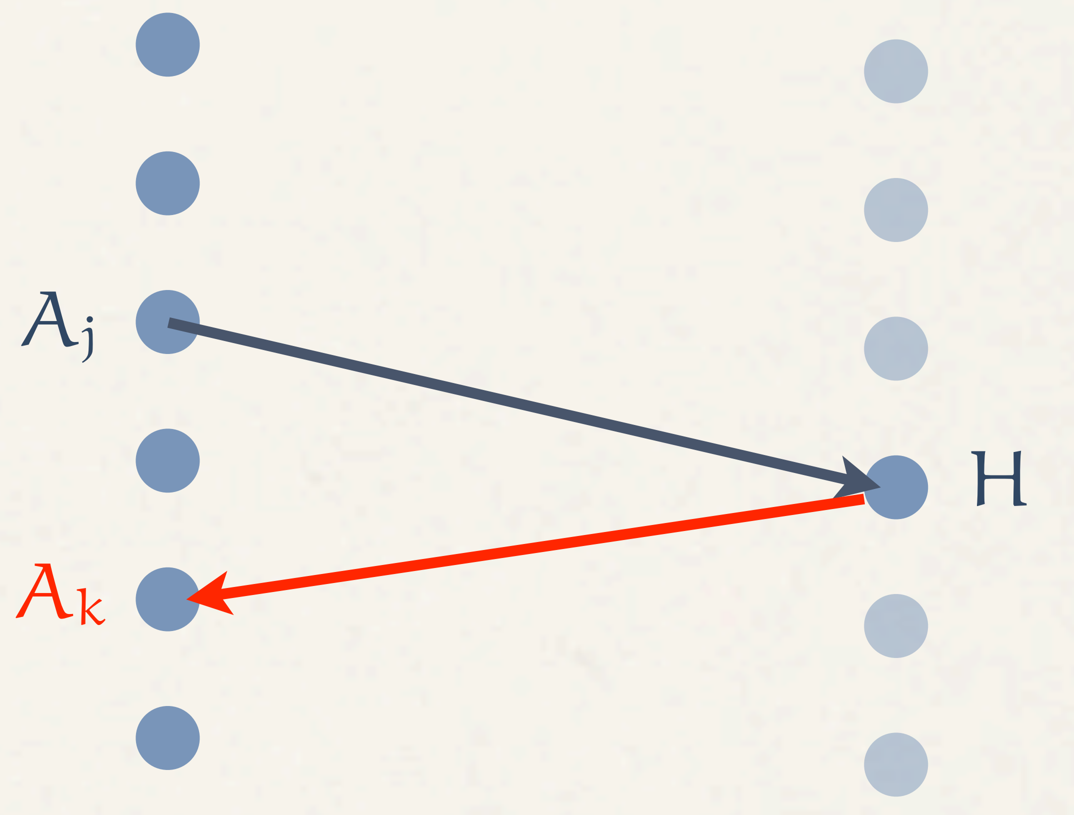

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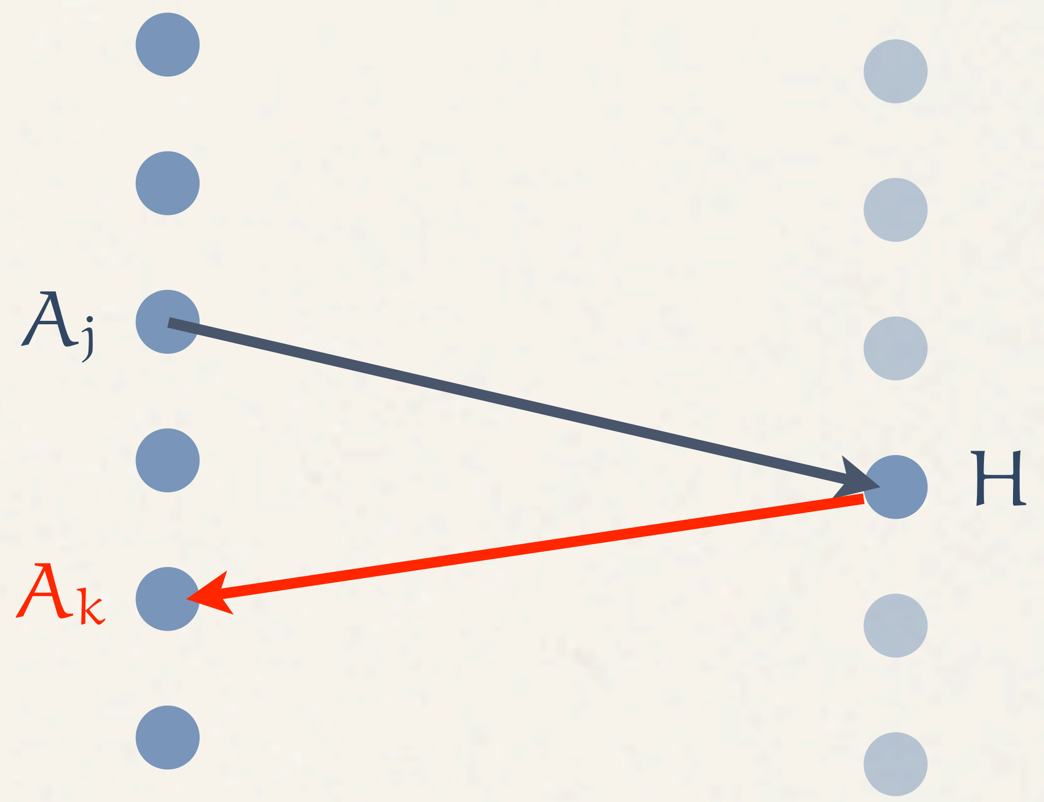

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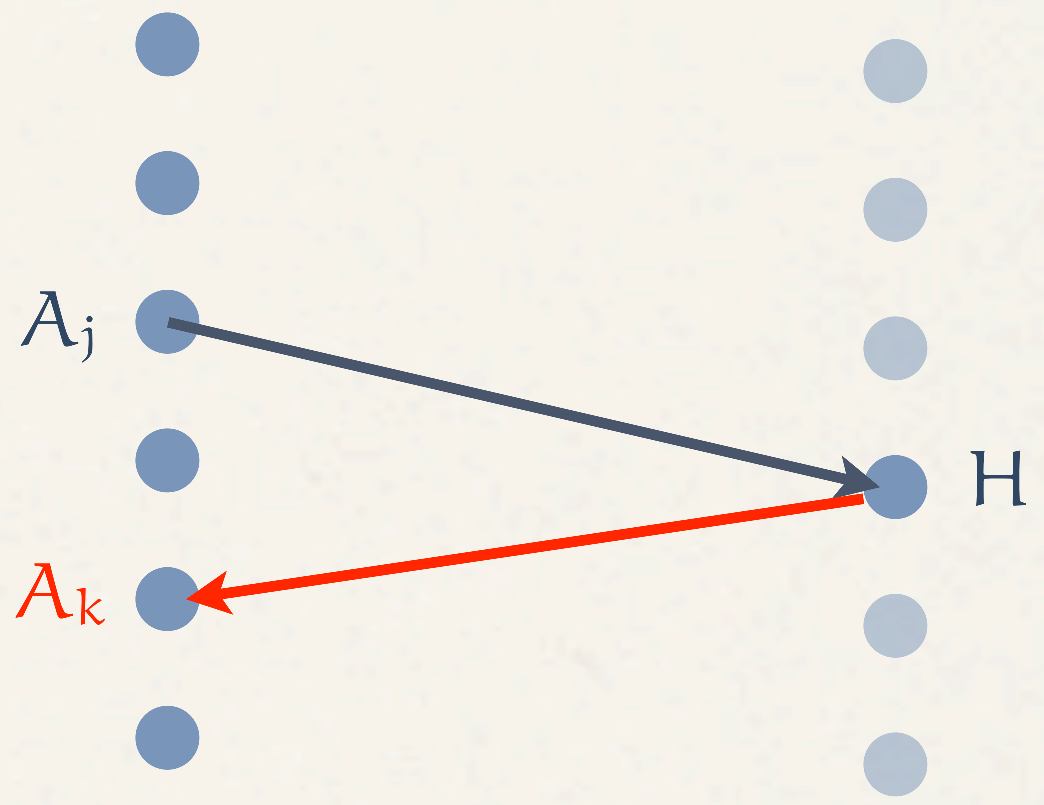

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Bayes's rule