Reference Solution to "Two-Dimensional Diffusion Equation"

```
%%%%% Define the square and grid parameters %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
L=1;  %square is 2L x 2L
N=100; %# of intervals in x and y directions
n=N+1; %# of gridpoints in x,y directions including boundaries
h=2*L/N;  %grid size in x,y directions
x=-L + (0:N)*h; %x values on the grid
y=-L + (0:N)*h; %y values on the grid
[X,Y]=meshgrid(x,y);
%%%%% Define the indices associated with the boundaries %%%%%%%%%%%%%%%%%%%%
% boundary_index = [bottom, left, top, right]
boundary_index=[       1:n,     1:n:1+(n-1)*n, ...
            1+(n-1)*n:n*n,   n:n:n*n        ];
%%%%% Diffusion constant and time-step parameters
D=1;
dt=h^2/(2*D); %borderline stability of FTCS scheme
alpha=dt*D/h^2; %equation parameter
nsteps=1000; %number of time steps
%%%%% Construct the matrix and compute lu
decomposition%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
diagonals = [2*(1+2*alpha)*ones(n^2,1), -alpha*ones(n^2,4)];
A=spdiags(diagonals,[0 -1 1 -n n], n^2, n^2); %use sparse matrices
I=speye(n^2);
A(boundary_index,:)=I(boundary_index,:);
[PL, U]=lu(A);
%%%%% Define initial conditions %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
u=zeros(n,n,nsteps);
sigma=L/4;
u(:,:,1)=1/(2*pi*sigma^2)*exp(-0.5*(X.^2+Y.^2)/sigma^2);
u(1,:,1)=0; u(n,:,1)=0; u(:,1,1)=0; u(:,n,1)=0;
%%%%% Advance solution %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
for m=2:nsteps
   b=zeros(n,n);
   b(2:n-1,2:n-1)=2*(1-2*alpha)*u(2:n-1,2:n-1,m-1) ...
      + alpha*(u(1:n-2,2:n-1,m-1)+u(3:n,2:n-1,m-1) ...
      + u(2:n-1,1:n-2,m-1)+u(2:n-1,3:n,m-1));
   b=reshape(b,n^2,1);
   u(:,:,m)=reshape(U\(PL\b),n,n);
end
%%%%% Plot with animation:  UNCOMMENT TO RUN ON MATLAB ONLINE OR DESKTOP
%%%%%%%%%%%%%%%%%%%%%%
% figure('units','normalized','outerposition',[0 0 1 1])
% s=surf(X,Y,u(:,:,1)); zlim([0, 2.6]);
% xlabel('$x$','Interpreter','latex','FontSize',14);
% ylabel('$y$','Interpreter','latex','FontSize',14);
% zlabel('$u(x,y,t)$','Interpreter','latex','FontSize',14);
% title('Solution of the 2D diffusion equation','Interpreter','latex','FontSize',16);
% pause(1)
% for j=2:nsteps
%     s.ZData=u(:,:,j); pause(0.01);
% end
```

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