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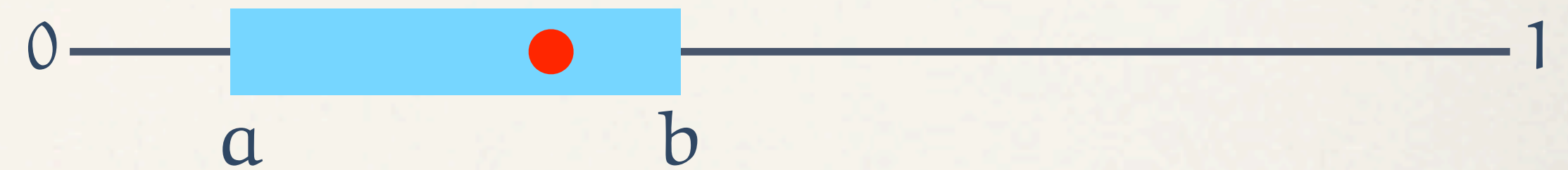
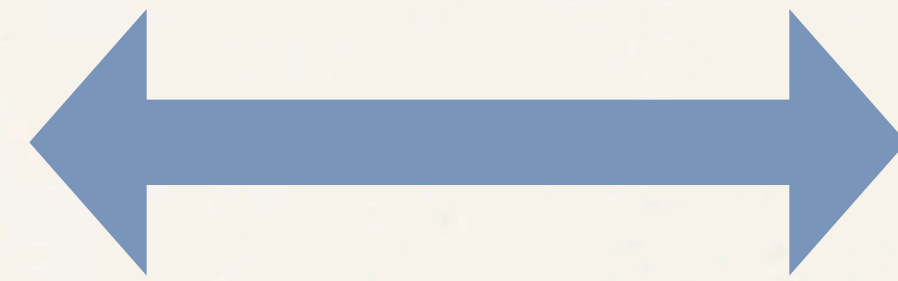
A continuous sample space:

$$\Omega = [0, 1] := \{x : 0 \leq x \leq 1\}$$

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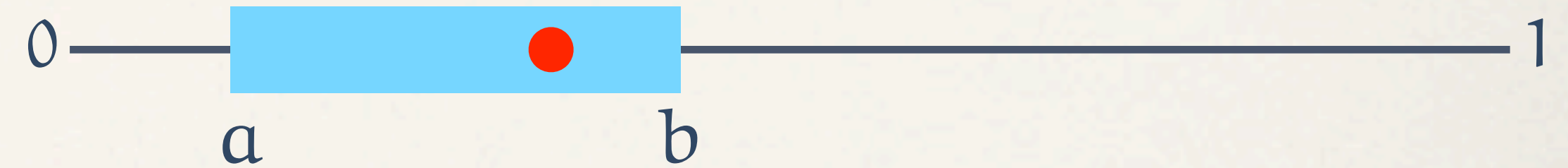
The basic events are intervals:

$$A = [a, b] := \{x : a < x < b\}$$

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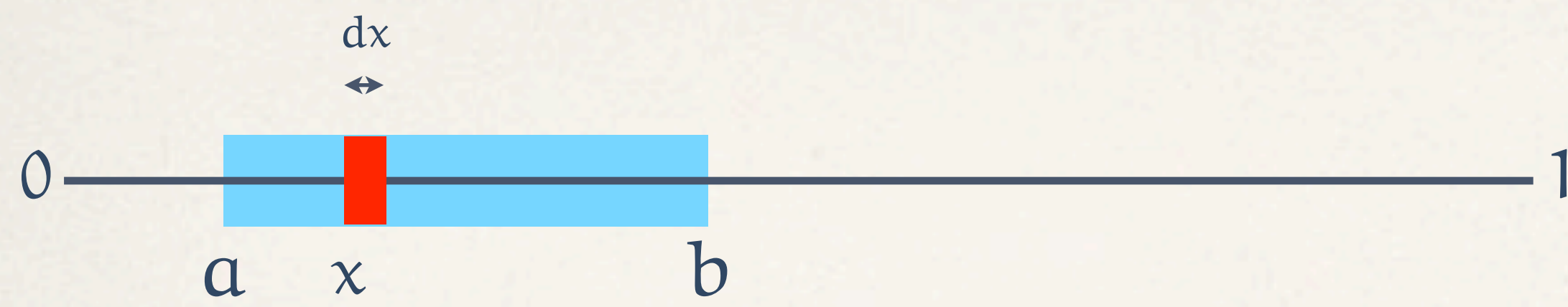
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Probabilities identified with lengths: $\mathbf{P}([a, b]) = b - a$

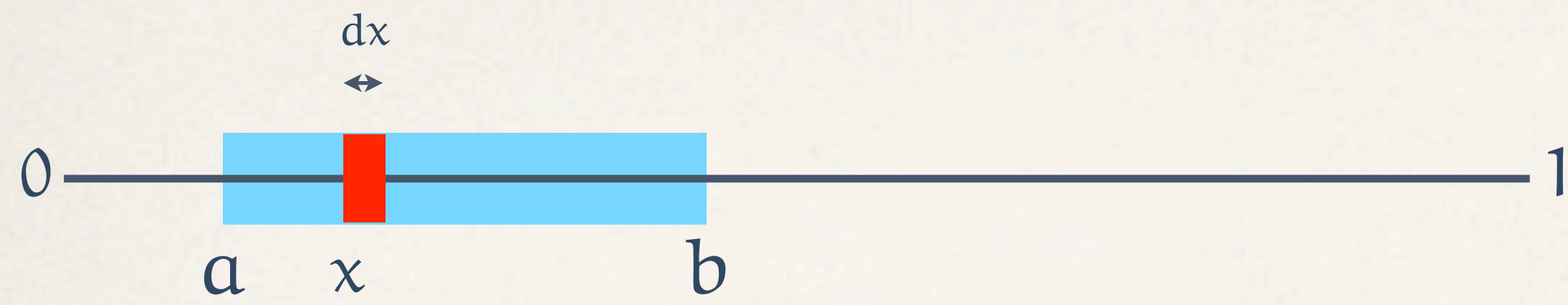


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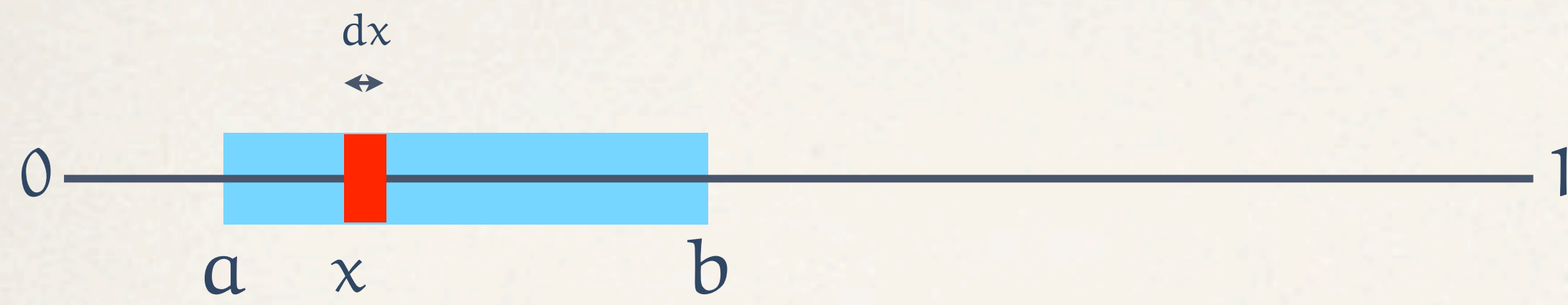
The probability of an infinitesimal interval $[x, x + dx]$:



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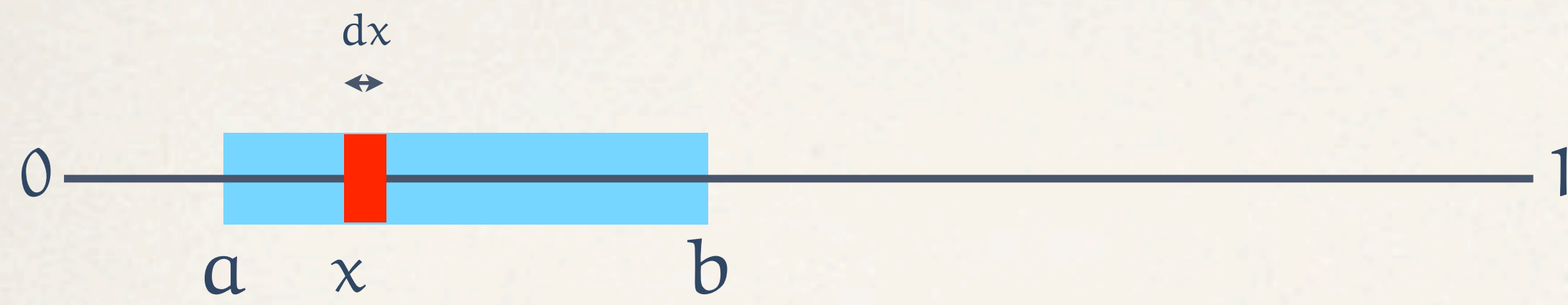
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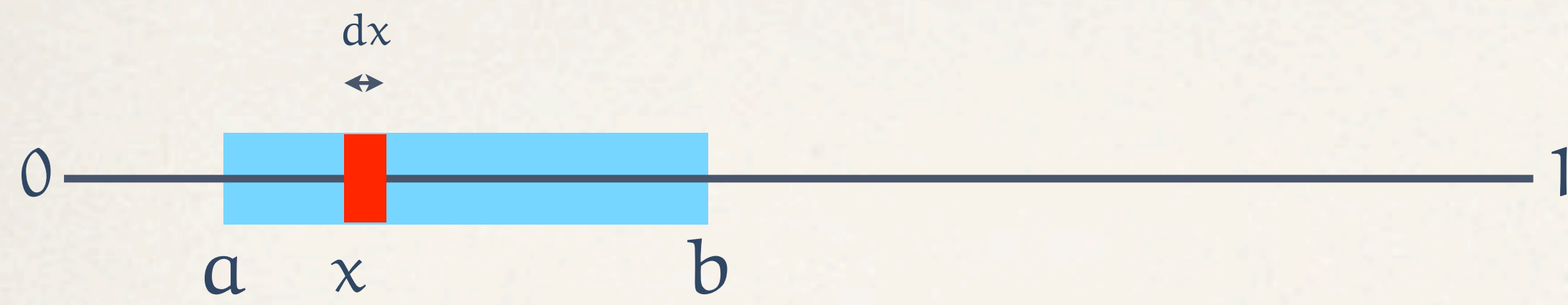


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uniform mass **density** per unit length at the point x



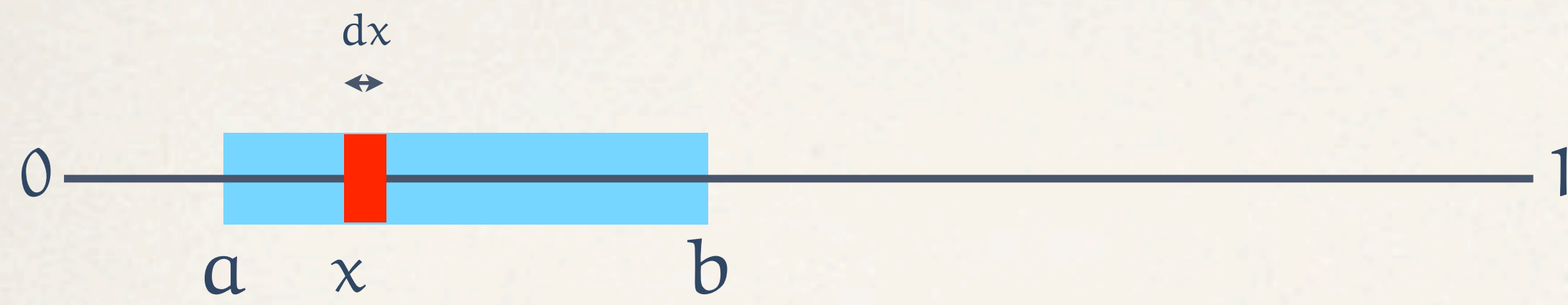
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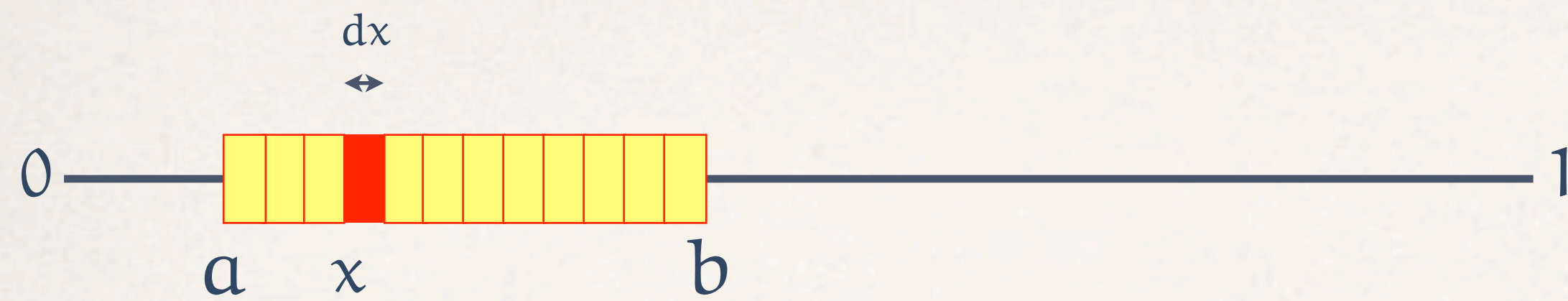
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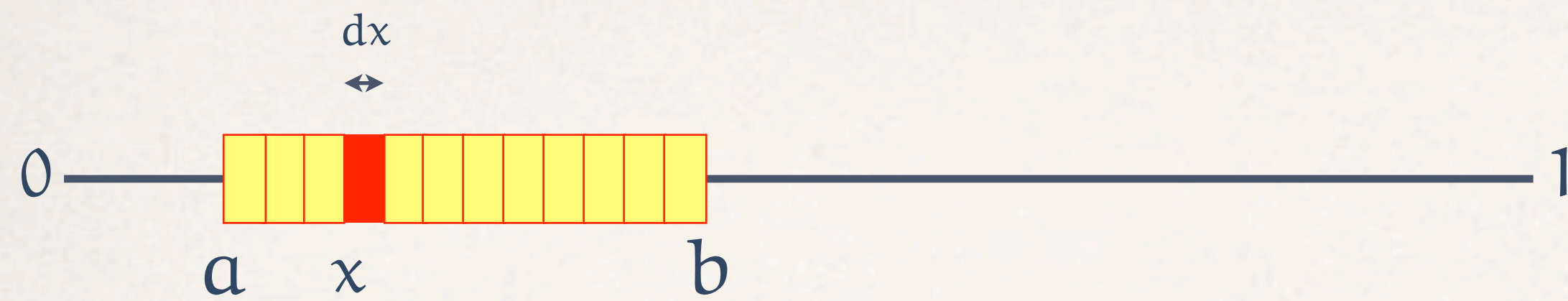
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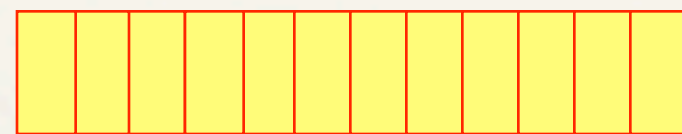
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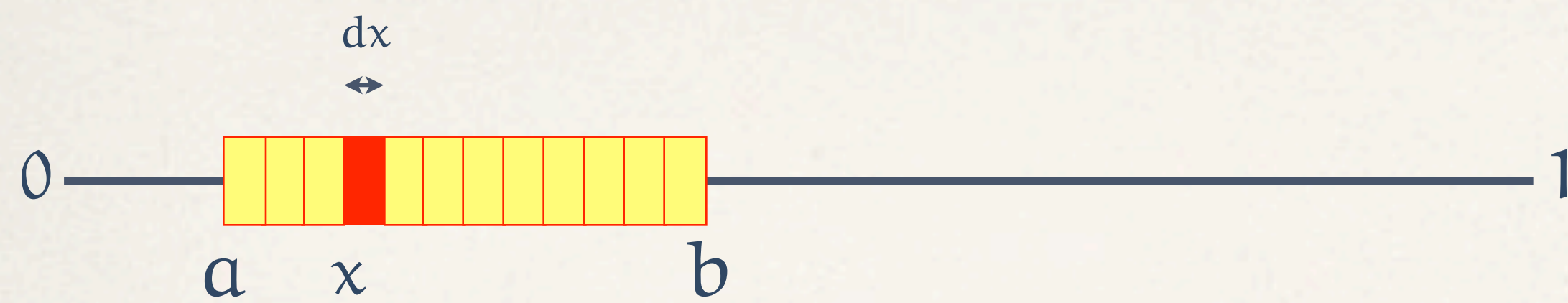
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Additivity:

$$\sum_x \mathbf{P}([x, x + dx])$$





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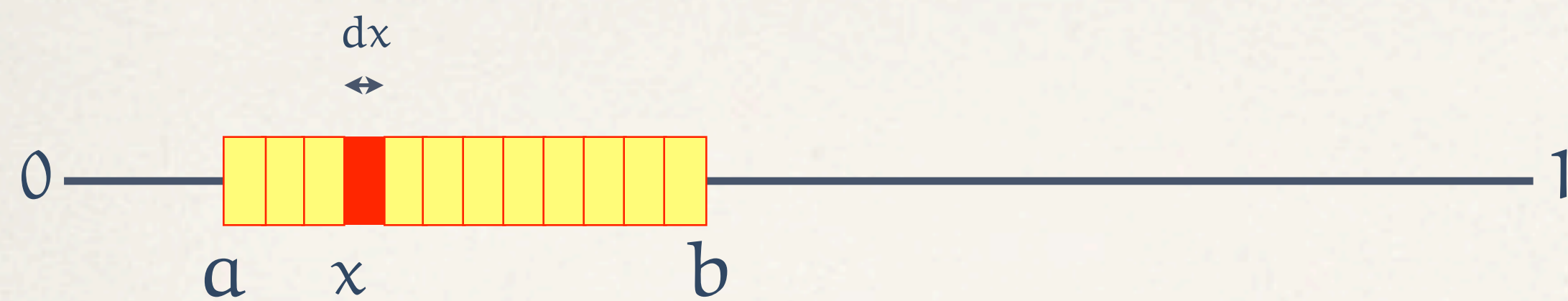
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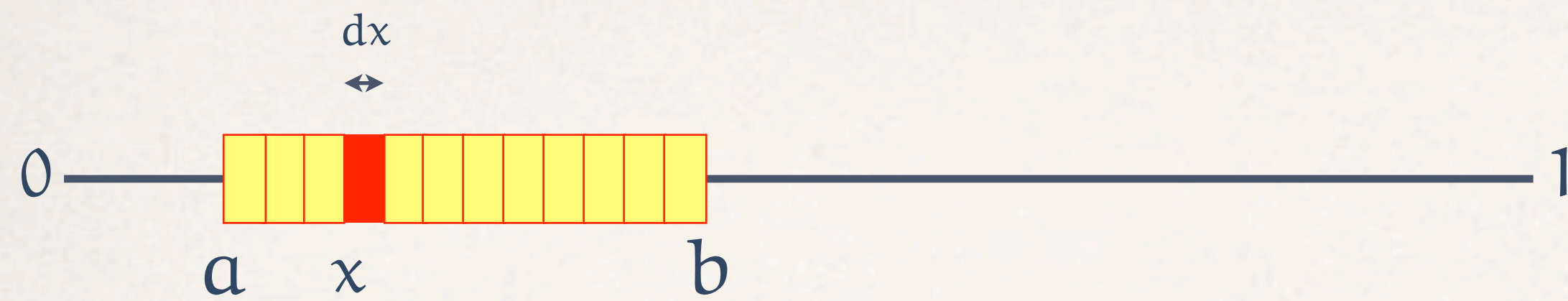
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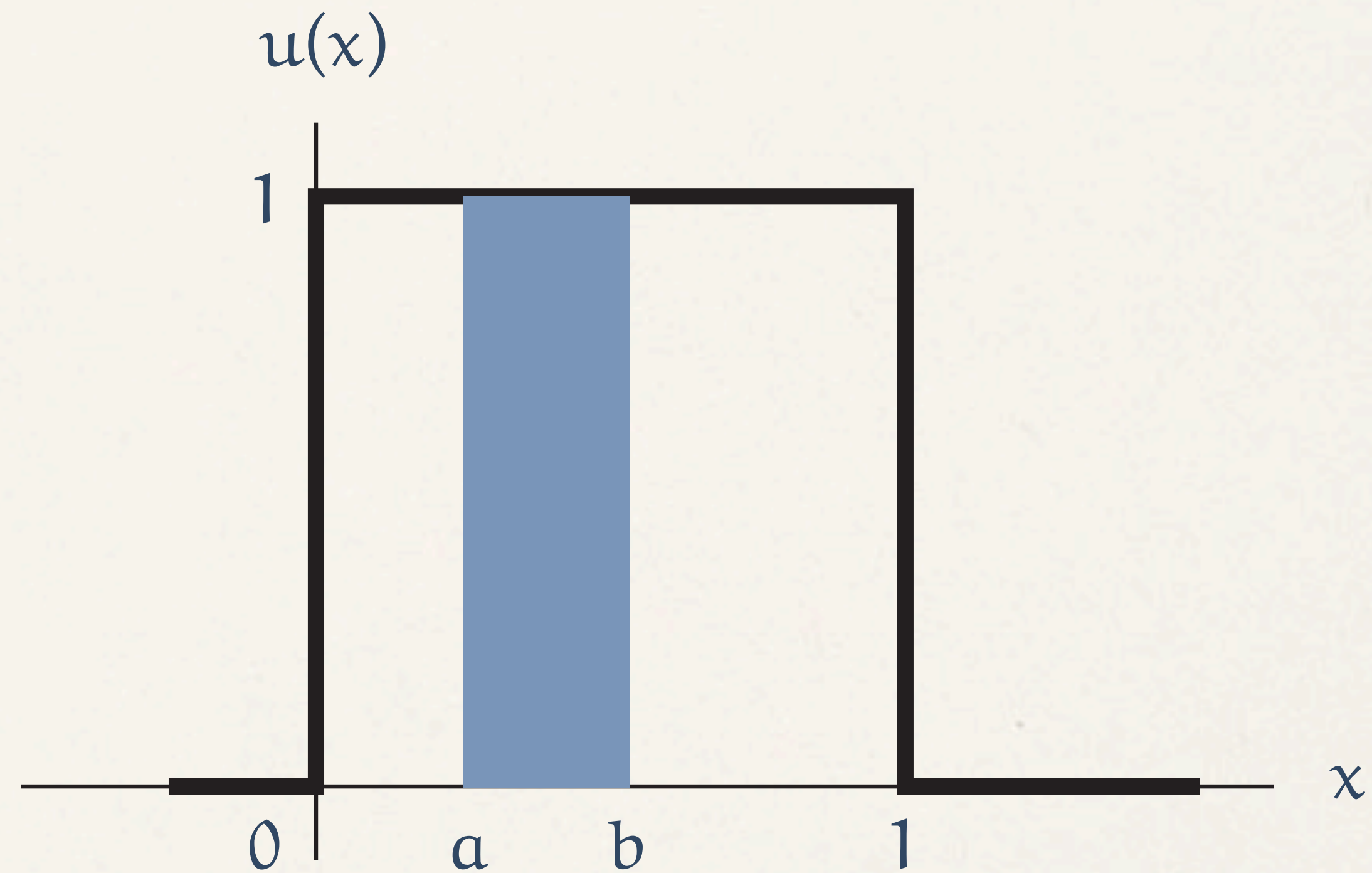
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The uniform density in the unit interval



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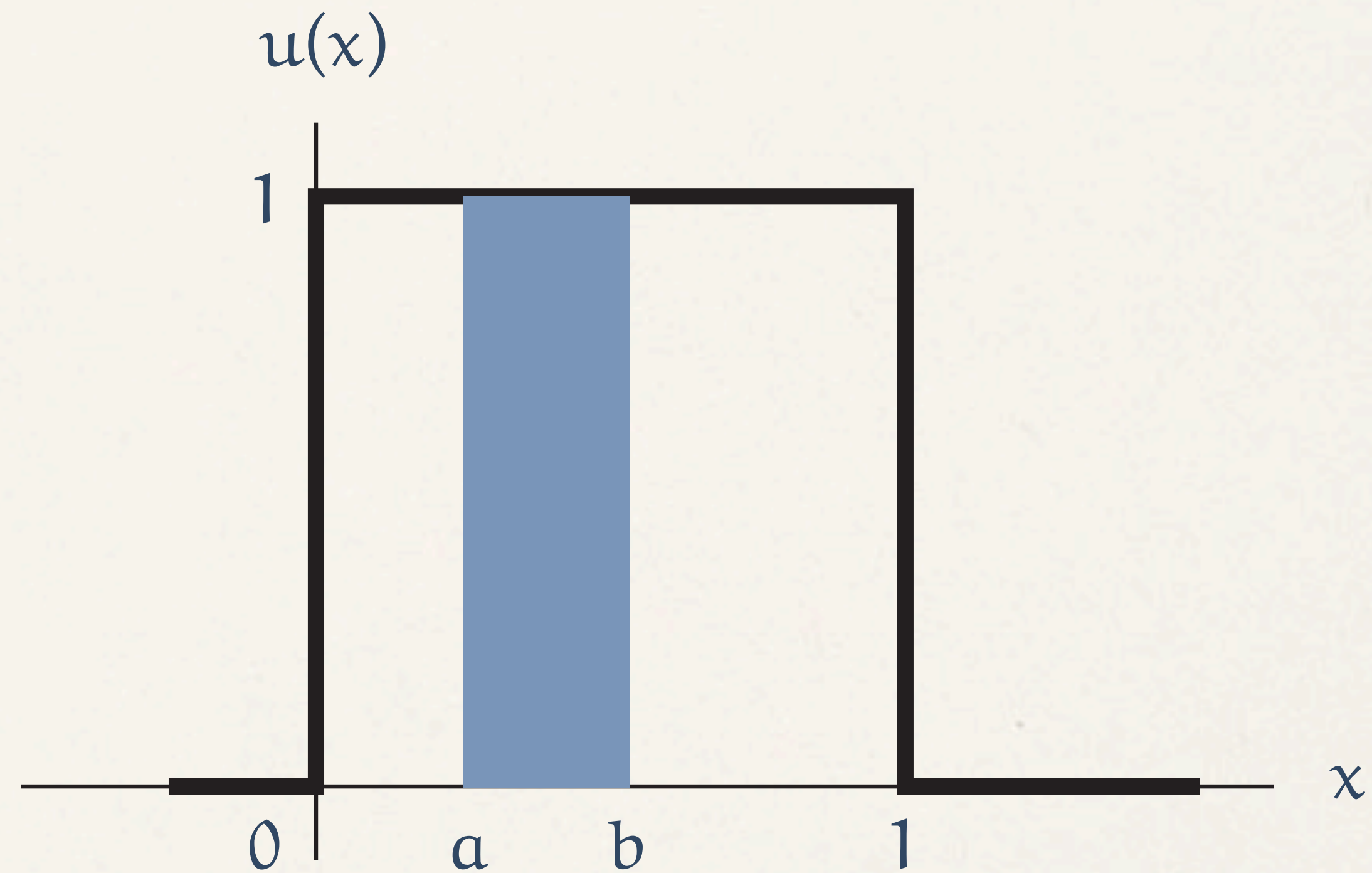
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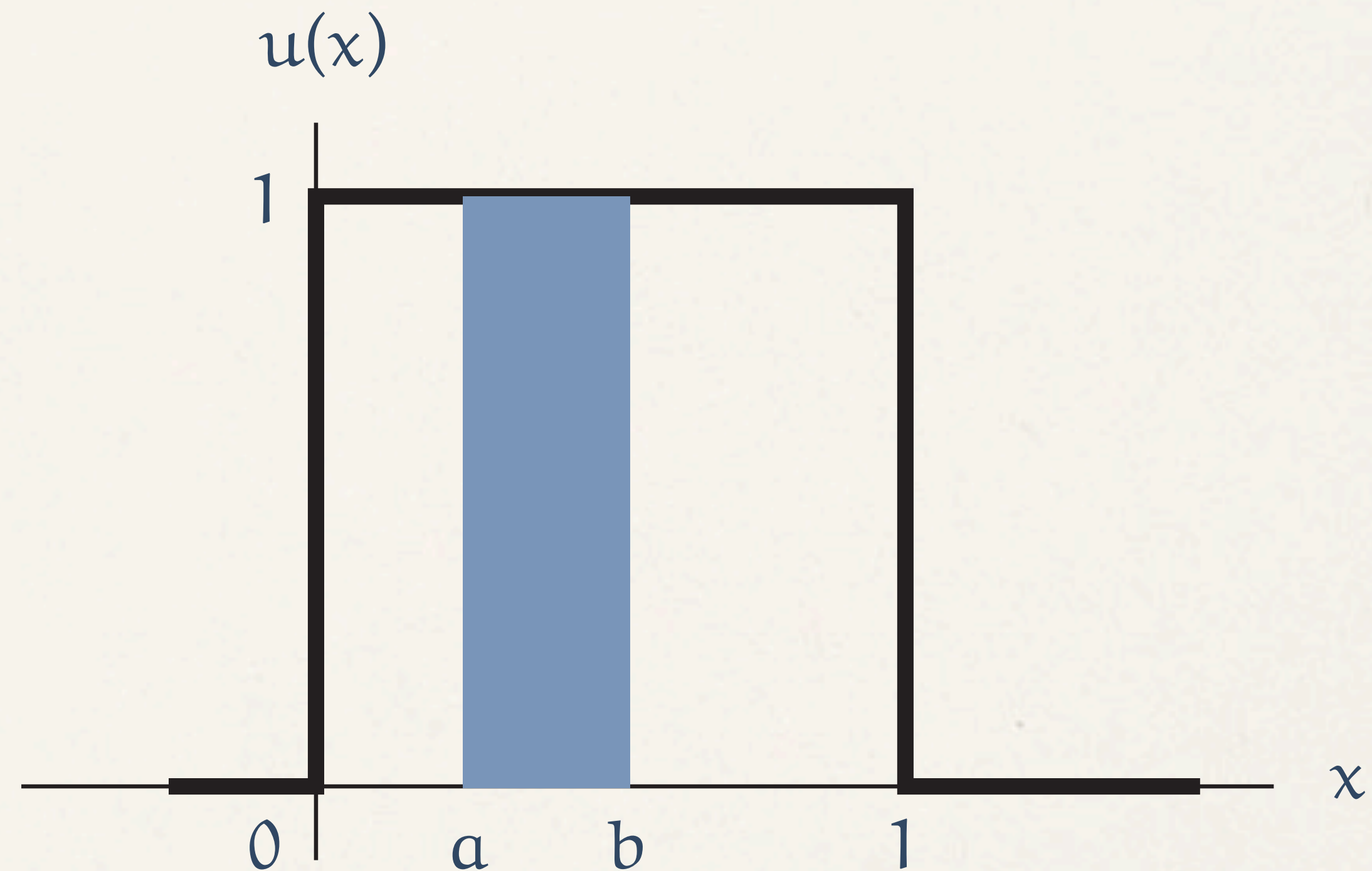
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We identify probabilities with areas under a density curve