Feedback — Assignment 3

You submitted this quiz on **Sat 16 Mar 2013 11:04 AM PDT -0700**. You got a score of **63.00** out of **63.00**.

Question 1

For answering the first three questions, consider the polyhedron defined here. Select the points that correspond to vertices of the polyhedron $\mathcal{P}$ defined by

$$\left\{(x,y)\in\mathbb{R}^2 \mid \begin{array}{l}(1)\ x\le 2\\ (2)\ x+2y\ge 2\\ (3)\ x+2y\le 6\\ (4)\ x-2y\ge -2\\ (5)\ 3x+6y\le 18\end{array}\right\}$$

where we have numbered the inequalities from 1 to 5.

Your Answer		Score	Explanation
☑ $(2,0)$	✓	0.75	
☑ $(0,1)$	✓	0.75	
☐ $(\frac{3}{2},\frac{1}{2})$	✓	0.75	
☐ $(2,1)$	✓	0.75	
Total		3.00 / 3.00	

Question Explanation

The points $(2,0)$ and $(0,1)$ are feasible and have exactly two independent active inequalities each, and hence form vertices of the polyhedron $\mathcal{P}$. For $(0,1)$ the active inequalities are $x-2y\ge -2$and $x+2y\ge 2$, whereas for $(2,0)$ the active inequalities are $x\le 2$and $x+2y\ge 2$ The remaining points don't satisfy this condition.

Question 2

Using the numbering of the inequalities from 1 to 5 in the polyhedron $\mathcal{P}$ from the previous question, which of the following are feasible bases representing the vertex $(2,2)$?

Your Answer		Score	Explanation
☐ $\{1,2\}$	✔	0.60	
☑ $\{1,5\}$	✔	0.60	
☑ $\{3,4\}$	✔	0.60	
☑ $\{1,4\}$	✔	0.60	
☐ $\{3,5\}$	✔	0.60	
Total		3.00 / 3.00	

Question Explanation

The point \$(2,2)\$ is active at all inequalities except \$2\$. So every pair of linearly independent inequalities from the set \$\\{1,3,4,5\\}\$ is correct. Since inequalities \$3\$ and \$5\$ are linearly dependent this is not a valid basis.

Question 3

Which of the following inequalities form a witness to the fact that the vertices $(2,2)$ and $(0,1)$ are adjacent?

Your Answer		Score	Explanation
☐ $x+2y \geq 2$	✔	0.25	
☑ $x-2y \geq -2$	✔	0.25	
☐ $x+2y \leq 6$	✔	0.25	

☐ $x \leq 2$	✓	0.25
Total		1.00 / 1.00

Question Explanation

As $\mathcal{P}$ lies in $\mathbb{R}^2$ we need exactly $2 - 1 = 1$ inequality that is active and independent at the two given vertices. Since $x - 2y \geq -2$is active at both $(2, 2)$ and $(0, 1)$, it forms a witness to adjacency of the two given vertices.

Question 4

Consider the unit cube $\mathcal{C}$ in $\mathbb{R}^3$ defined as $\left\{(x, y, z) \in \mathbb{R}^3 \mid \begin{matrix} x, y, z \geq 0 \\ x, y, z \leq 1 \end{matrix}\right\}$ Recall the following theorem about an alternate characterization of adjacent vertices: Two vertices u and v of a polyhedron $\mathcal{P} \subset \mathbb{R}^n$ are adjacent iff $\exists c \in \mathbb{R}^n$ such that the set of optimal solutions of $\max\{c^T x \mid x \in \mathcal{P}\}$ is $\{\lambda x_1 + (1 - \lambda)x_2 \mid \lambda \in \mathbb{R}, 0 \leq \lambda \leq 1\}$Using $c := (0, 0, 1)^T$ as the objective function vector for a maximization LP defined on $\mathcal{P}$ we see that every point on the line segment joining $(0, 0, 1)^T$ and $(1, 1, 1)^T$ has an objective function value of 1, which is optimal. Can we conclude that $(0, 0, 1)^T$ and $(1, 1, 1)^T$ are adjacent? And why?

Your Answer		Score	Explanation
○ Yes because there exist no other points inside $\mathcal{C}$ but outside the line segment joining $(0, 0, 1)^T$ and $(1, 1, 1)^T$ that have an optimal objective function value.			
◉ No because there also exist points inside $\mathcal{C}$ but outside the line segment joining $(0, 0, 1)^T$ and $(1, 1, 1)^T$ that have an optimal objective function value.	✓	3.00	
○ Yes because there also exist points inside $\mathcal{C}$ but outside the line segment joining $(0, 0, 1)^T$ and $(1, 1, 1)^T$ that have an optimal objective function value.			
○ No because there exist no other points inside $\mathcal{C}$ but outside the line segment joining $(0, 0, 1)^T$ and			

$(1,1,1)^T$ that have an optimal objective function value.

Total		3.00 / 3.00

Question Explanation

With the given choice of c we note that any point lying on the face $z = 1$ of $\mathcal{C}$ has an objective function value 1. Since the line segment joining $(0,0,1)$and $(1,1,1)$is a proper subset of set of optimal solutions, we conclude that the correct answer is no.

Question 5

If for a particular objective function vector $c \in \mathbb{R}^n$ and a polyhedron $\mathcal{P} := \{x \in \mathbb{R}^n \mid Ax \leq b\}$it is true that the optimum for the maximization LP is attained at two distinct vertices u and v, then which of the following statements are true?

Your Answer		Score	Explanation
☐ The points $\{\lambda u + (1-\lambda)v \mid 0 \leq \lambda \leq 1, \lambda \in \mathbb{R}\}$ are infeasible	✔	0.75	
☑ The optimum is also attained along the points $\{\lambda u + (1-\lambda)v \mid 0 \leq \lambda \leq 1, \lambda \in \mathbb{R}\}$	✔	0.75	
☐ The optimum is not attained along the points $\{\lambda u + (1-\lambda)v \mid 0 < \lambda < 1, \lambda \in \mathbb{R}\}$	✔	0.75	
☑ The points $\{\lambda u + (1-\lambda)v \mid 0 \leq \lambda \leq 1, \lambda \in \mathbb{R}\}$ are feasible	✔	0.75	
Total		3.00 / 3.00	

Question Explanation

Let us denote the set $\{\lambda u + (1-\lambda)v \mid 0 \leq \lambda \leq 1, \lambda \in \mathbb{R}\}$by L. First, every point in L is feasible since any inequality of $\mathcal{P}$ of the form $a_i^T x \leq b$is satisfied by by a point $\lambda u + (1-\lambda)v \in L$which can be seen as follows:

$a_i^T(\lambda u + (1-\lambda)v) = \lambda a_i^T u + (1-\lambda)a_i^T v \leq \lambda b_i + (1-\lambda)b_i = b_i$. Secondly, the points in L are all optimal since $c^T(\lambda u + (1-\lambda)v) = \lambda c^T u + (1-\lambda)c^T v = c^T u$. (Note that $c^T u = c^T v$ since they are optimum objective function values)

Question 6

Of the following list of inequality descriptions of polyhedra, select the one that has a degenerate vertex that can be represented by 6 feasible bases.

Your Answer		Score	Explanation
$\mathcal{P}_3 = \left\{ (x,y) \in \mathbb{R}^2 \mid \begin{array}{c} x,y \geq 0 \\ x,y \leq 2 \\ x-y \geq 0 \\ x+y \leq 2 \end{array} \right\}$.			
$\mathcal{P}_1 = \left\{ (x,y) \in \mathbb{R}^2 \mid \begin{array}{c} x,y \geq 0 \\ x,y \leq 1 \\ x-y \geq -1 \\ x-y \leq 1 \\ x+y \leq 1 \end{array} \right\}$.	✓	5.00	
$\mathcal{P}_4 = \left\{ (x,y) \in \mathbb{R}^2 \mid \begin{array}{c} x,y \geq 0 \\ x-y \geq 0 \end{array} \right\}$			
$\mathcal{P}_2 = \left\{ (x,y,z) \in \mathbb{R}^3 \mid \begin{array}{c} x,y,z \geq 0 \\ x,y,z \leq 1 \\ x+y+z \leq 1 \end{array} \right\}$.			
Total		5.00 / 5.00	

Question Explanation

Drawing the feasible regions of the given polyhedra, we find that only the polyhedron $\mathcal{P}_1$ has a degenerate vertex with 4 active inequalities. For example, at the vertex $(0,1)$ we have the following set of active inequalities: $x \geq 0, y \leq 1, x+y \leq 1, x-y \geq -1$. Choosing any two of these four inequalities yields a feasible basis that describes the vertex $(0,1)$. Since $\binom{4}{2} = 6$, this is the correct answer.

Question 7

Consider the two dimensional polyhedron $\mathcal{P} = \left\{ x \in \mathbb{R}^2 \mid \begin{bmatrix} 0 & -1 \\ -1 & 1 \\ 1 & 1 \end{bmatrix} x \leq \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix} \right\}$ Suppose we maximized the objective function vector $c := [2\ 1]^T$ over this polyhedron. Then, the vertex $(1, 1)$ can be described by the feasible basis $B := \{2, 3\}$ What is the value of λ_B corresponding to this feasible basis? Is B an optimal basis?

Your Answer		Score	Explanation
$\lambda_B = \begin{bmatrix} \frac{1}{2} \\ \frac{3}{2} \end{bmatrix}$ and the basis B is not optimal			
$\lambda_B = \begin{bmatrix} -\frac{1}{2} \\ \frac{3}{2} \end{bmatrix}$ and the basis B is not optimal	✓	5.00	
$\lambda_B = \begin{bmatrix} \frac{1}{2} \\ -\frac{3}{2} \end{bmatrix}$ and the basis B is not optimal			
$\lambda_B = \begin{bmatrix} \frac{1}{2} \\ \frac{3}{2} \end{bmatrix}$ and the basis B is optimal			
Total		5.00 / 5.00	

Question Explanation

We can solve for the multipliers λ_B by solving the system $\lambda_B^T A_B = c^T$ where A is the matrix from the inequality description of $\mathcal{P}$. Hence, $\lambda_B^T = [2 \quad 1] \begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} -\frac{1}{2} & \frac{3}{2} \end{bmatrix}$. Since λ_B is not non-negative, we conclude that the basis B is not optimal.

Question 8

Suppose x_1 and x_2 are two variables of a linear program $\max\ c^T x$ subject to $Ax \leq b$, which of the following pairs of LPs would you need to solve to incorporate an additional constraint of the form $|x_1 - 2| = |x_2|$?

Your Answer		Score	Explanation
◉	✓	5.00	

$$\begin{aligned} \text{LP1: maximize } & c^T x \\ \text{subject to } & Ax \leq b \\ & x_1 - x_2 \leq 2 \\ & -x_1 + x_2 \leq -2 \end{aligned}$$

$$\begin{aligned} \text{LP2: maximize } & c^T x \\ \text{subject to } & Ax \leq b \\ & x_1 + x_2 \leq 2 \\ & -x_1 - x_2 \leq -2 \end{aligned}$$

○

$$\begin{aligned} \text{LP1: maximize } & c^T x \\ \text{subject to } & Ax \leq b \\ & x_1 + x_2 \leq 2 \\ & -x_1 + x_2 \leq -2 \end{aligned}$$

$$\begin{aligned} \text{LP2: maximize } & c^T x \\ \text{subject to } & Ax \leq b \\ & x_1 - x_2 \leq 2 \\ & -x_1 - x_2 \leq -2 \end{aligned}$$

○

$$\begin{aligned} \text{LP1: maximize } & c^T x \\ \text{subject to } & Ax \leq b \\ & x_1 - x_2 \leq -2 \\ & -x_1 + x_2 \leq 2 \end{aligned}$$

$$\begin{aligned} \text{LP2: maximize } & c^T x \\ \text{subject to } & Ax \leq b \\ & x_1 + x_2 \leq -2 \\ & -x_1 - x_2 \leq 2 \end{aligned}$$

○

$$\begin{aligned} \text{LP1: maximize } & c^T x \\ \text{subject to } & Ax \leq b \\ & x_1 - x_2 \leq -2 \\ & -x_1 + x_2 \leq 2 \end{aligned}$$

$$\begin{aligned} \text{LP2: maximize } & c^T x \\ \text{subject to } & Ax \leq b \\ & x_1 + x_2 \leq 2 \\ & -x_1 - x_2 \leq -2 \end{aligned}$$

Total	5.00 / 5.00

Question Explanation

$|x_1 - 2| = |x_2|$can be resolved only in two ways: either it must be the case that $x_1 - 2 = x_2$ or $x_1 - 2 = -x_2$. So we solve the two LPs corresponding to each of the two choices and then take the best of the two solutions.

$$\begin{aligned} \text{LP1: maximize } & c^T x \\ \text{subject to } & Ax \leq b \\ & x_1 - x_2 \leq 2 \\ & -x_1 + x_2 \leq -2 \end{aligned}$$

$$\begin{aligned} \text{LP2: maximize } & c^T x \\ \text{subject to } & Ax \leq b \\ & x_1 + x_2 \leq 2 \\ & -x_1 - x_2 \leq -2 \end{aligned}$$

Question 9

For questions 9 through 15 that follow, we will consider this linear program:

$$\begin{array}{ll} \text{maximize} & x+y \\ \text{subject to} & (1)\ 4x-y\geq 4 \\ & (2)\ x+2y\leq 10 \\ & (3)\ 2x+y\leq 11 \\ & (4)\ 5x-2y\leq 23 \\ & (5)\ 4x+2y\geq 4 \end{array}$$

We will also assume the numbering of the inequalities as listed above. We will solve this LP in a step-by-step manner simulating the different steps of the simplex algorithm over the course of the next few questions. To begin, we use $(1,0)$ as the initial vertex. First, what is the feasible basis for the initial vertex?

Your Answer		Score	Explanation
○ $\{1,2\}$			
◉ $\{1,5\}$	✓	1.00	
○ $\{1,3\}$			
○ $\{2,5\}$			
Total		1.00 / 1.00	

Question Explanation

The first and the fifth inequalities in the description of the LP are independent and active at the point $(1,0)$. Hence the basis for the first iteration of the simplex method is $B=\{1,5\}$. Remark: In principle, we could've had more than one possible basis for the vertex, but this is not the case for this particular example.

Question 10

Using the feasible basis B for the initial vertex $(1,0)$, determine λ_B (as described in the simplex method from the lecture)

Your Answer	Score	Explanation

$\begin{bmatrix} \frac{1}{6} \\ -\frac{5}{6} \end{bmatrix}$		
$\begin{bmatrix} \frac{1}{6} \\ -\frac{5}{12} \end{bmatrix}$	✓	1.00
$\begin{bmatrix} -\frac{1}{3} \\ \frac{5}{6} \end{bmatrix}$		
$\begin{bmatrix} -\frac{1}{6} \\ \frac{5}{6} \end{bmatrix}$		
Total		1.00 / 1.00

Question Explanation

To find λ_B for $B = \{1,5\}$we need to solve the system of equations $\lambda_B^T A_B = c^T$.
Hence, $\lambda_B^T = [1 \quad 1] \begin{bmatrix} -4 & 1 \\ -4 & -2 \end{bmatrix}^{-1} = \begin{bmatrix} \frac{1}{6} & -\frac{5}{12} \end{bmatrix}$.

Question 11

What is the direction $d \in \mathbb{R}^2$ in which we should move from the initial vertex $(1,0)$ in order to increase the value of the objective function?

Your Answer		Score	Explanation
$\begin{bmatrix} -\frac{1}{6} \\ -\frac{1}{3} \end{bmatrix}$			
$\begin{bmatrix} \frac{1}{12} \\ \frac{1}{3} \end{bmatrix}$	✓	3.00	
$\begin{bmatrix} -\frac{1}{2} \\ -\frac{1}{3} \end{bmatrix}$			

$\begin{bmatrix} -\frac{1}{6} \\ \frac{1}{12} \end{bmatrix}$

Total		3.00 / 3.00

Question Explanation

Having computed $\lambda_B = \begin{bmatrix} \frac{1}{6} \\ -\frac{5}{12} \end{bmatrix}$ we can deduce that the leaving index is 5 since the second component of λ_B is negative. Since we need to choose a direction d that is orthogonal to the first row of A_B and such that the product of the second row of A_B with d is -1, we simply pick the second column of $-A_B^{-1}$. As
$-A_B^{-1} = -\begin{bmatrix} -4 & 1 \\ -4 & -2 \end{bmatrix}^{-1} = \begin{bmatrix} \frac{1}{6} & \frac{1}{12} \\ -\frac{1}{3} & \frac{1}{3} \end{bmatrix} \implies d = \begin{bmatrix} \frac{1}{12} \\ \frac{1}{3} \end{bmatrix}.$

Question 12

What is the largest ϵ that you can travel in the computed direction d starting from the initial vertex $(1, 0)$ while still remaining in the feasible region?

You entered:

12

Your Answer		Score	Explanation
12	✓	5.00	
Total		5.00 / 5.00	

Question Explanation

To compute the largest ϵ that we can travel in the direction $d = \begin{bmatrix} \frac{1}{12} \\ \frac{1}{3} \end{bmatrix}$ starting from $(1, 0)$ while still remaining in the feasible region, we first need to determine the set K, which is the set of indices of rows of A whose inner product with the direction d is positive. Since $Ad = \begin{bmatrix} -4 & 1 \\ 1 & 2 \\ 2 & 1 \\ 5 & -2 \\ -4 & -2 \end{bmatrix} \begin{bmatrix} \frac{1}{12} \\ \frac{1}{3} \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{3}{4} \\ \frac{1}{2} \\ -\frac{1}{4} \\ -1 \end{bmatrix}$, we set $K := \{2, 3\}$. Now, for each $k \in K$ we compute the value $(b_k - a_k x_0)/(a_k d)$ where a_k denotes the kth row of A, and pick the k^* where the minimum is attained. In this case, the minimum of 12 is attained at the index 2 (the value corresponding to $k = 3$ is 18). Thus, the required distance is 12. The new vertex is then $\begin{bmatrix} 1 \\ 0 \end{bmatrix} + 12 \begin{bmatrix} \frac{1}{12} \\ \frac{1}{3} \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$.

Question 13

What is the new basis B?

Your Answer		Score	Explanation
○ $\{3, 5\}$			
◉ $\{1, 2\}$	✓	5.00	
○ $\{2, 5\}$			
○ $\{1, 3\}$			
Total		5.00 / 5.00	

Question Explanation

Having computed the set $K := \{2, 3\}$ from the solution to the previous question, and having found that the minimum of $(b_k - a_k x_0)/(a_k d)$ over all $k \in K$ is attained at the index 2, we find that the entering index is 2. Hence the new basis is $B = \{1, 2\}$.

Question 14

Using the new basis B, compute λ_B. Following the steps in the simplex method as before, compute the leaving and entering indices and enter them in that order in the text box below separated by a single space.

You entered:

1 3

Your Answer		Score	Explanation
1	✓	2.50	
3	✓	2.50	
Total		5.00 / 5.00	

Question Explanation

To find λ_B for $B = \{1, 2\}$we need to solve the system of equations $\lambda_B^T A_B = c^T$. Hence, $\lambda_B^T = \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} -4 & 1 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} -\frac{1}{9} & \frac{5}{9} \end{bmatrix}$. Thus we immediately see that the leaving index is 1. Since we need to choose a direction d that is orthogonal to the second row of A_B and such that the product of the first row of A_B with d is -1, we simply pick the first column of $-A_B^{-1}$. As $-A_B^{-1} = -\begin{bmatrix} -4 & 1 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} \frac{2}{9} & -\frac{1}{9} \\ -\frac{1}{9} & -\frac{4}{9} \end{bmatrix} \implies d = \begin{bmatrix} \frac{2}{9} \\ -\frac{1}{9} \end{bmatrix}$. Now, to compute the maximum distance e that we can travel in the direction $d = \begin{bmatrix} \frac{2}{9} \\ -\frac{1}{9} \end{bmatrix}$ starting from $(2, 4)$ while still remaining in the feasible region, we first need to determine the set K, which is the set of indices of rows of A whose inner product with the direction d is positive. Since $Ad = \begin{bmatrix} -4 & 1 \\ 1 & 2 \\ 2 & 1 \\ 5 & -2 \\ -4 & -2 \end{bmatrix} \begin{bmatrix} \frac{2}{9} \\ -\frac{1}{9} \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \\ -\frac{2}{3} \\ \frac{1}{3} \\ \frac{4}{3} \end{bmatrix}$, we set $K := \{3, 4\}$. Now, for each $k \in K$ we compute the value $(b_k - a_k x_0)/(a_k d)$where a_k denotes the kth row of A, and pick the k^* where the minimum is attained. In this case, the minimum of 9 is attained at the index 3 (the value corresponding to $k = 4$is $\frac{63}{4}$). Thus, the required distance is 9. Hence, the entering index is 3 so that the new basis is $B = \{2, 3\}$.

Question 15

Using your answer to the previous question, you should now have computed the basis for the third iteration of the simplex method. Computing λ_B you should also decide that this basis is optimal (i.e., if you have performed all the previous steps correctly then all the elements of λ_B will be non-negative). Enter the vertex (x^*, y^*) corresponding to the optimal solution in the text box below separated by a space.

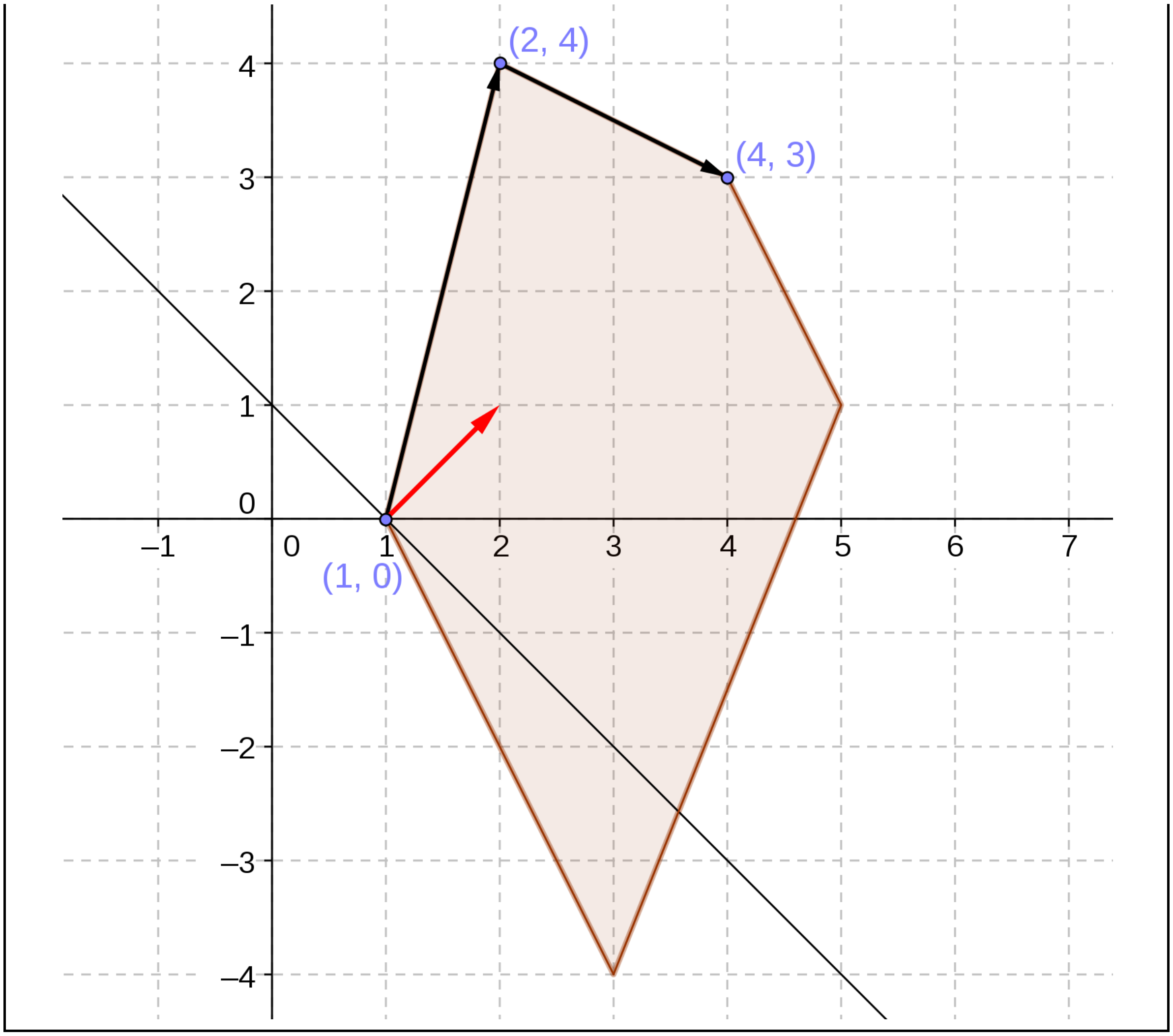
You entered:

4 3

Your Answer		Score	Explanation
4	✓	2.50	
3	✓	2.50	
Total		5.00 / 5.00	

Question Explanation

Continuing the calculations from where we left off in the last question, it follows then that the new vertex is $\begin{bmatrix} 2 \\ 4 \end{bmatrix} + 9 \begin{bmatrix} \frac{2}{9} \\ -\frac{1}{9} \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$, which has as the feasible basis $B = \{2, 3\}$. To find λ_B we need to solve the system of equations $\lambda_B^T A_B = c^T$. Hence, $\lambda_B^T = \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} \frac{1}{3} & \frac{1}{3} \end{bmatrix}$. Since all the components of λ_B are positive, we conclude that the vertex $(4, 3)$ is an optimal solution for this LP.
Look at the following figure for a graphic illustration of the two iterations of the simplex method. The objective function vector is shown in red, the feasible region for the LP is shaded, and the visited vertices are shown in blue.



Question 16

Choose the options that apply for the polyhedron $\mathcal{P}$ defined by

$$\left\{ (x_1, x_2, x_3) \in \mathbb{R}^3 \;\middle|\; \begin{array}{l} x_1 + x_2 + 2x_3 \le 8 \\ x_2 + 6x_3 \le 12 \\ x_1 \le 4 \\ x_2 \le 6 \\ x_1, x_2, x_3 \ge 0 \end{array} \right\}$$

Recall that a basic solution x^* is degenerate if there are more than n inequalities that are active at x^*.

Your Answer		Score	Explanation
☐ $(2, 6, 0)$ is a degenerate basic feasible solution.	✔	1.25	
☑ $(2, 6, 0)$ is a nondegenerate basic feasible	✔	1.25	

solution.

☐ $(4, 0, 2)$is a nondegenerate basic feasible solution	✓	1.25	
☑ $(4, 0, 2)$is a degenerate basic feasible solution	✓	1.25	
Total		5.00 / 5.00	

Question Explanation

$(2, 6, 0)$is a nondegenerate basic feasible solution becase there are exactly three active and linearly independent constraints at this point: $x_1 + x_2 + 2x_3 \leq 8, x_2 \leq 6, x_3 \geq 0$. However, $(4, 0, 2)$is a degenerate basic feasible solution because there are four active constraints at this point of which only three are linearly independent: $x_1 + x_2 + 2x_3 \leq 8, x_2 + 6x_3 \leq 12, x_1 \leq 4, x_2 \geq 0$

Question 17

Consider the simplex method applied to a linear program $\max\ c^T x$ subject to $Ax \leq b$ where A has full column rank. For each of the statements below, decide whether they are true or false and select the statements that are true.

Your Answer		Score	Explanation
☐ An iteration of the simplex method may move the feasible solution by a strictly positive distance while leaving the objective function value unchanged.	✓	1.25	The simplex algorithm terminates as soon as all the components of λ_B are non-negative for the current basis B. Since it has performed an iteration we infer that at least one component of λ_B is negative. We then know that the simplex algorithm must have computed a feasible direction d in which the objective function value strictly improves, so hence the it cannot remain the same.
☑ An index that has just entered the basis cannot leave in the very next iteration	✓	1.25	If an index that has just entered leaves again, then we walk along $-d$, where d was the previously computed direction. But the objective function strictly decreases along $-d$ and we

			know that this is not possible since the simplex method never moves in a direction that strictly decreases the objective function value.
☐ If B is an optimal basis, then all the components of λ_B are strictly positive	✓	1.25	As some components of λ_B could be zero even when B is an optimal basis
☐ An index that has just left the basis cannot re-enter in the very next iteration	✓	1.25	An index that has left the basis can enter in the very next iteration. An example is a triangle in the plane. Maybe the simplex method does not decide to walk to the neighboring optimal vertex in one step but makes a detour (while improving) via the other vertex. In this case, the inequality that has just left re-enters again.
Total		5.00 / 5.00	