Feedback — Problem Set 6

Help Center

You submitted this homework on **Sat 23 Feb 2013 10:05 PM PST**. You got a score of **9.00** out of **9.00**.

Question 1

Core

- Three players together can obtain 1 to share, any two players can obtain 0.8, and one player by herself can obtain zero.
- Then, $N = 3$ and $v(1) = v(2) = v(3) = 0$, $v(1,2) = v(2,3) = v(3,1) = 0.8$, $v(1,2,3) = 1$.

Which allocation is in the core of this coalitional game?

Your Answer		Score	Explanation
a) (0,0,0);			
b) (0.4, 0.4, 0);			
c) (1/3, 1/3, 1/3);			
d) The core is empty;	✔	1.00	
Total		1.00 / 1.00	

Question Explanation

(d) is true.

- By definition, the core of this game is formed by a triplet $(x_1, x_2, x_3) \in R^3_+$ that satisfies:
  - $x_i + x_j \geq 0.8$ for $i \neq j$

- $x_1 + x_2 + x_3 \geq 1$
- There is no triplet (x_1, x_2, x_3) that satisfies all inequalities. Then, the core is empty.

Question 2

Buyers and Sellers

- There is a market for an indivisible good with B buyers and S sellers.
- Each seller has only one unit of the good and has a reservation price of 0.
- Each buyer wants to buy only one unit of the good and has a reservation price of 1.
- Thus $v(C) = min(B_C, S_C)$ where B_C and S_C are the number of buyers and sellers in coalition C (and so, for instance, $v(i) = 0$ for any single player, and $v(i, j) = 1$ if i, j are a pair of a buyer and seller).

If the number of buyers and sellers is $B = 2$ and $S = 1$, respectively, which allocations are in the core? [There might be more than one]

Your Answer		Score	Explanation
☑ a) Each seller receives 1 and each buyer receives 0.	✔	0.33	
☐ b) Each seller receives 0 and each buyer receives 1.	✔	0.33	
☐ c) Each seller receives 1/2 and each buyer receives 1/2.	✔	0.33	
Total		1.00 / 1.00	

Question Explanation

(a) is true.

- By definition, the core of this game is formed by a vector of payoffs to buyers ($b1$ and $b2$) and to the seller (s) $(x_{b1}, x_{b2}, x_s) \in R^3_+$ that satisfies:
  - $x_{b1} + x_{b2} \geq 0$;
  - $x_{bi} + x_s \geq 1$ for $i = 1, 2$;

- $x_{b1} + x_{b2} + x_s \geq 1$;
- and the feasibility constraint $x_{b1} + x_{b2} + x_s \leq 1$.

- It is easy to verify that allocation (a) is the only one that satisfies the set of inequalities.

Question 3

Buyers and Sellers

- There is a market for an indivisible good with B buyers and S sellers.
- Each seller has only one unit of the good and has a reservation price of 0.
- Each buyer wants to buy only one unit of the good and has a reservation price of 1.
- Thus $v(C) = min(B_C, S_C)$ where B_C and S_C are the number of buyers and sellers in coalition C (and so, for instance, $v(i) = 0$ for any single player, and $v(i, j) = 1$ if i, j are a pair of a buyer and seller).

Now assume that competition among sellers increases, so that $B = 2$ and $S = 2$. Which allocations are in the core? [There might be more than one]

Your Answer		Score	Explanation
☑ a) Each seller receives 1 and each buyer receives 0.	✔	0.33	
☑ b) Each seller receives 0 and each buyer receives 1.	✔	0.33	
☑ c) Each seller receives 1/2 and each buyer receives 1/2.	✔	0.33	
Total		1.00 / 1.00	

Question Explanation

All are in the core.

- Again, the core of this game is formed by a vector of payoffs to buyers and sellers $(x_{b1}, x_{b2}, x_{s1}, x_{s2}) \in R^4_+$ that satisfies:
  - $x_{b1} + x_{b2} \geq 0$;

- $x_{s1} + x_{s2} \geq 0$;
- $x_{bi} + x_{sj} \geq 1$ for $i = 1, 2$ and $j = 1, 2$;
- $x_{b1} + x_{b2} + x_{s1} + x_{s2} \geq 2$;
- and the feasibility constraint $x_{b1} + x_{b2} + x_{s1} + x_{s2} \leq 2$.

- It is easy to verify that allocations (a), (b) and (c) satisfy the set of inequalities.
- In fact, any split of the surplus that gives α to all sellers and $1 - \alpha$ to all buyers (with $\alpha \in [0, 1]$) is in the core. That is, any split of the surplus is possible; the only restriction imposed by the increase in competition (i.e., increase in the number of sellers) is that all pairs must receive the same share of the surplus.

Question 4

Core and Shapley Value

- The instructor of a class allows the students to collaborate and write up together a particular problem in the homework assignment.
- Points earned by a collaborating team are divided among the students in any way they agree on.
- There are exactly three students taking the course, all equally talented, and they need to decide which of them if any should collaborate.
- The problem is so hard that none of them working alone would score any points. Any two of them can score 4 points together. If all three collaborate, they can score 6 points.

Which allocations are in the core of this coalitional game?

Your Answer		Score	Explanation
a) (0,0,0);			
b) (2, 2, 0);			
c) (2, 2, 2);	✔	1.00	
d) The core is empty;			
Total		1.00 / 1.00	

Question Explanation

(c) is true.

- By definition, the core of this game is formed by a vector of payoffs to each student $(x_1, x_2, x_3) \in R^3_+$ that satisfies:
  - $x_i + x_j \geq 4$ for $i \neq j$
  - $x_1 + x_2 + x_3 \geq 6$
  - (2,2,2) is the only option that satisfies these inequalities. Then, it belongs to the core.

Question 5

Core and Shapley Value

- The instructor of a class allows the students to collaborate and write up together a particular problem in the homework assignment.
- Points earned by a collaborating team are divided among the students in any way they agree on.
- There are exactly three students taking the course, all equally talented, and they need to decide which of them if any should collaborate.
- The problem is so hard that none of them working alone would score any points. Any two of them can score 4 points together. If all three collaborate, they can score 6 points.

What is the Shapley value of each player?

Your Answer		Score	Explanation
a) $\phi = (0, 0, 0)$			
b) $\phi = (2, 0, 2)$			
c) $\phi = (1/3, 1/3, 1/3)$			
d) $\phi = (2, 2, 2)$	✔	1.00	
Total		1.00 / 1.00	

Question Explanation

(d) is true.

- Use the definition of the Shapley Value to compute its value for each player.
- Another way to find the Shapley Value is to remember that:
  - by the axiom of symmetry, all agents should receive the same payoff.
  - the Shapley value divides the payoff to the grand coalition completely
  - Then, all agents will have a Shapley value of $6/3 = 2$.

Question 6

Production

- There is a single capitalist (c) and a group of 2 workers ($w1$ and $w2$).
- The production function is such that total output is 0 if the firm (coalition) is composed only of the capitalist or of the workers (a coalition between the capitalist and a worker is required to produce positive output).
- The production function satisfies:
  - $F(c \cup w1) = F(c \cup w2) = 3$
  - $F(c \cup w1 \cup w2) = 4$

Which allocations are in the core of this coalitional game? [There might be more than one]

Your Answer		Score	Explanation
☑ a) $x_c = 2, x_{w1} = 1, x_{w2} = 1$;	✔	0.33	
☑ b) $x_c = 2.5, x_{w1} = 0.5, x_{w2} = 1$;	✔	0.33	
☑ c) $x_c = 4, x_{w1} = 0, x_{w2} = 0$;	✔	0.33	
Total		1.00 / 1.00	

Question Explanation

(d) is true.

- It is easy to verify that allocations (a), (b) and (c) satisfy the definition of the core.
- It can be shown more generally that for any given number n of workers and any increasing and concave production function f, the core of this coalitional game is defined by:
  - $x_{wi} \leq f(c \cup w1 \ldots \cup wn) - f(c \cup w1 \ldots \cup w(n-1))$
  - $x_c + \sum_{i=1}^{n} x_{w1} \leq f(c \cup w1 \cup \ldots \cup wn)$
- Intuitively, the first equation requires each worker to receive less than the marginal product of the n^{th} worker. If this condition would not hold for worker i, then the rest of the workers and the capitalist could abandon him and get a higher value for the new coalition.
- The second condition is a feasibility condition (the sum of payoffs of the grand coalition is not greater than the resources available).

Question 7

Production

- There is a single capitalist (c) and a group of 2 workers ($w1$ and $w2$).
- The production function is such that total output is 0 if the firm (coalition) is composed only of the capitalist or of the workers (a coalition between the capitalist and a worker is required to produce positive output).
- The production function satisfies:
  - $F(c \cup w1) = F(c \cup w2) = 3$
  - $F(c \cup w1 \cup w2) = 4$

What is the Shapley value of the capitalist?

Your Answer		Score	Explanation
a) 3;			
b) 4;			
c) 7/3;	✔	1.00	

d) 7;

Total	1.00 / 1.00

Question Explanation

(c) is true.

- Use the definition of the Shapley Value to compute its value for the capitalist.

Question 8

Production

- There is a single capitalist (c) and a group of 2 workers ($w1$ and $w2$).
- The production function is such that total output is 0 if the firm (coalition) is composed only of the capitalist or of the workers (a coalition between the capitalist and a worker is required to produce positive output).
- The production function satisfies:
  - $F(c \cup w1) = F(c \cup w2) = 3$
  - $F(c \cup w1 \cup w2) = 4$

What is the Shapley value of each worker?

Your Answer		Score	Explanation
a) 1;			
b) 5/6;	✔	1.00	
c) 3/4;			
d) 1/2;			
Total		1.00 / 1.00	

Question Explanation

(b) is true.

- Use the definition of the Shapley Value to compute its value for each worker.
- Another way to find the Shapley Value is to remember that:
  - by the axiom of symmetry, all workers should receive the same payoff
  - the Shapley value divides the payoff to the grand coalition completely
  - Then, all agents will have a Shapley value of $(F(c \cup w1 \cup w2) - 7/3)/2 = (4 - 7/3)/2 = 5/6$.

Question 9

Production

- There is a single capitalist (c) and a group of 2 workers ($w1$ and $w2$).
- The production function is such that total output is 0 if the firm (coalition) is composed only of the capitalist or of the workers (a coalition between the capitalist and a worker is required to produce positive output).
- The production function satisfies:
  - $F(c \cup w1) = F(c \cup w2) = 3$
  - $F(c \cup w1 \cup w2) = 4$

True or False: If there was an additional 3^{rd} worker that is completely useless (i.e., his marginal contribution is 0 in every coalition), then the sum of the Shapley Values of the capitalist and the first two workers will remain unchanged.

Your Answer		Score	Explanation
a) True;	✔	1.00	
b) False;			
Total		1.00 / 1.00	

Question Explanation

(a) is correct.

- The Shapley Value satisfies the Dummy player Axiom:
  - if i is a dummy player, then he/she must have a Shapley Value of 0
- Since the 3^{rd} worker is a Dummy player (check the definition), his/her Shapley Value must be 0.
- Thus, the statement is true because the Shapley Value divides the payoff of the grand coalition completely.