Typical events: visions of centrality



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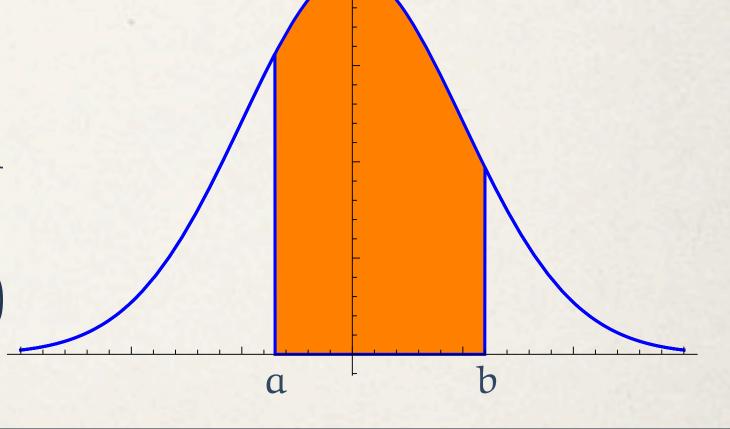
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The central limit theorem

 S_n^* is asymptotically normally distributed

$$\mathbf{P}\{a < S_n^* \le b\} \to \int_0^b \phi(x) \, dx \qquad (n \to \infty)$$



infinitely often