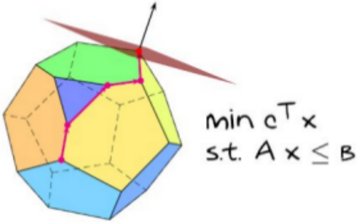
How efficient is the simplex method?

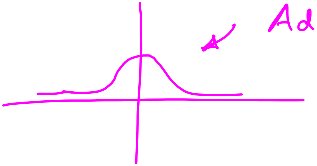
- The number of iterations



Bad news and somewhat good news

- The number of vertices can increase exponentially with the number of variables and constraints.
- For many popular pivoting rules, exponential lower bound on number of iterations has been established (Klee & Minty 1972).
- However, in practice, the simplex method typically performs $O(m)$ iterations.
- The *average running time* of the simplex method (in a certain natural probabilistic model) is polynomial (Borgwardt 1982).
- If an arbitrary linear program is perturbed (each coefficient with $N(0, \sigma)$), then the expected number of iterations of the simplex method is polynomial in $1/\sigma$, n and m (Spielman and Teng 2004).

Add to each coeffi



A famous open problem in optimization

Open problem

Is there a pivoting rule for the simplex algorithm that yields a polynomial number of iterations?

See also Problem 9 in Smale's list of *Mathematical Problems for the Next Century* (Smale 1998)

Best known so far:

- Kalai (1992, 1997) and Matoušek, Sharir & Welzl (1996) provide *randomized* pivoting rules with *expected number* of $2^{O(\sqrt{m})}$ iterations.
- Almost matching lower bounds for those provided by Friedmann, Hansen & Zwick (2011).

Linear programming in polynomial time

Suppose all coefficients of LP $\max\{c^T x\colon Ax \leqslant b\}$ (components of A, b and c) have size bounded by φ.

Open Problem:

Ellipsoid method (Khachiyan 1979)

A linear program $\max\{c^T x\colon x \in \mathbb{R}^n,\ Ax \leqslant b\}$ can be solved with a polynomial (in $m + n + \varphi$) number of elementary operations. ??

Furthermore, all numbers in the course of the algorithm have polynomial size in $m + n + \varphi$.

Polynomial time algorithm ! ??

Strongly polynomial !

See also (Grötschel, Lovász & Schrijver 1984)