

QUESTION 5

The *Fibonacci sequence* is defined iteratively by setting  $F_1 = F_2 = 1$  and thereafter letting  $F_{n+2} = F_n + F_{n+1}$ .

**Theorem** For any natural number  $n$ ,

$$\sum_{k=1}^n F_k = F_{n+2}$$

*Proof:* By induction.

For  $n = 1$ , the left-hand side is  $F_1 = 1$  and the right-hand side is  $F_2 = 1$ , so the identity is valid for  $n = 1$ .

Assume the identity holds for  $n$ . Then:

$$\begin{aligned} \sum_{k=1}^{n+1} F_k &= \sum_{k=1}^n F_k + F_{n+1} \\ &= F_{n+2} + F_{n+1}, \text{ by the induction hypothesis} \\ &= F_{n+3}, \text{ by the definition of } F_{n+3} \end{aligned}$$

which is the identity for  $n + 1$ . The proof is complete.