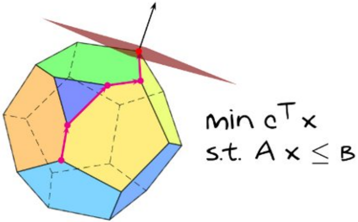
Linear programming

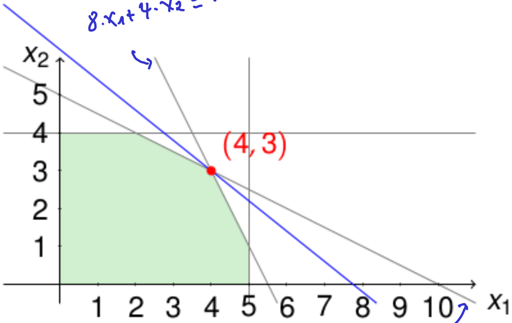
- Example: Proving optimality



Proving optimality

$\beta = 775$

$8\cdot x_1 + 4\cdot x_2 \le 44$ $\left(* \frac{25}{4}\right)$



$3\cdot x_1 + 6\cdot x_2 \le 30$ $(* 50/3)$

$$50\cdot x_1 + 100\, x_2 \le 500$$
$$50 x_1 + 25\, x_2 \le 275$$
$+$

Each feasible (x_1, x_2) satisfies

$$\underbrace{100 x_1 + 125 x_2}_{\text{Profit}} \le 775$$

Profit of $(4,3)$ IS 775

$\Rightarrow$ $(4,3)$ is optimal

Quiz

$$\begin{array}{lrcl}
\text{maximize} & 94 \cdot x_1 + 128 \cdot x_2 & & \\
\text{such that:} & 3 \cdot x_1 + 6 \cdot x_2 & \leqslant & 30 \ (\times 18) \\
 & 8 \cdot x_1 + 4 \cdot x_2 & \leqslant & 44 \ (\times 5) \\
 & x_1 & \leqslant & 5 \\
 & x_2 & \leqslant & 4 \\
 & x_1 & \geqslant & 0 \\
 & x_2 & \geqslant & 0
\end{array}$$

Profit of $(4, 3)$: $94 \cdot 4 + 128 \cdot 3 = 760$

Is $(4, 3)$ optimal production plan?

- No, because there is a feasible production plan yielding more profit.
- Yes, because the first two inequalities yield an upper bound of 760 on the profit of any feasible production plan.
  This can be seen by multiplying those inequalities by 11 and 6 respectively and by adding up the resulting inequalities.
- Yes, because the first two inequalities yield an upper bound of 760 on the profit of any feasible production plan.
  This can be seen by multiplying those inequalities by 18 and 5 respectively and by adding up the resulting inequalities.

$$\underbrace{(3 \cdot 18 + 8 \cdot 5)}_{=94} x_1 + \underbrace{(6 \cdot 18 + 4 \cdot 5)}_{=128} x_2 \leq \underbrace{30 \cdot 18 + 44 \cdot 5}_{=760}$$

Question: Is this principle general?