Expectation of negative binomial distribution

Asked 1 year, 9 months ago Modified 1 year, 9 months ago Viewed 5k times

2

Given $X \sim \text{NBin}(n, p)$, I've seen two different calculations for $\mathbb{E}(X)$:

$$1.\mathbb{E}(X) = \frac{n}{p}, \quad \text{or}$$
$$2.\mathbb{E}(Y) = \frac{n(1-p)}{p}$$

Proof for 1.: Proof for the calculation of mean in negative binomial distribution

Proof for 2: Although I can't find a concrete proof on stackexchange, this is the expected value used in the wikipedia article for negative binomials, and I have also seen this value used in some questions here.

I've heard someone say that both are valid depending on the way you define the negative binomial, but I still don't quite understand the difference between the set-ups for the two different $\mathbb{E}(X)$.

Could someone explain their differences? Thank you!

probability-distributions expected-value negative-binomial

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edited Nov 10, 2020 at 9:45
tommik
31.8k 4 14 31

asked Nov 10, 2020 at 9:36
punypaw
425 3 9

You can find definitions of $NBin(r,p)$ here: en.wikipedia.org/wiki/Negative_binomial_distribution – Kavi Rama Murthy Nov 10, 2020 at 9:40

1 Answer

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The negative binomial distribution is the sum of n i.i.d. geometric distributions.

As for the Geometric, alse for the NBinomial you have 2 kinds of parametrizations

1. The variable counting the total trials to get n successes
2. The variable counting the total failures to get n successes

Thus you can prove your expectations in the following way:

1. Start from the Geometric distribution that counts how many trials you need to get the first success:

$$P(X = x) = q^{x-1}p$$

$x = 1, 2, 3, \ldots$. and $q = 1 - p$

2. Calculate $\mathbb{E}[X]$

$$\mathbb{E}[X] = p\sum_{x=0}^{\infty} q^{x-1} = p\sum_{x=0}^{\infty} \frac{d}{q} q^x = p\frac{d}{dq}\frac{q}{1-q} = \cdots = \frac{1}{p}$$

Hence the expectation of the NBinomial counting how many trials you need to get k successes is simply

$$\boxed{\mathbb{E}[\Sigma_i X_i] = k\frac{1}{p} \qquad (1)}$$

3. note that the geometric distribution conunting the failures before the first success is

$$Y = X - 1$$

Thus its mean is $\mathbb{E}[Y] = \frac{1}{p} - 1 = \frac{q}{p}$

Hence the Expectation of the NBinomial counting the number of failures before you get k successes is

$$\boxed{\mathbb{E}[\Sigma_i Y_i] = k\frac{q}{p} \qquad (2)}$$

...that's all!

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edited Nov 10, 2020 at 10:06

answered Nov 10, 2020 at 9:46
tommik
31.8k 4 14 31

1 Oh I see. So the first definition is the expected number of trials (successes + failures) before n (or k) successes, while the second is the expected number of *failures* before n (or k) successes? – punypaw Nov 11, 2020 at 0:32

@punypaw : Yes, absolutely correct – tommik Nov 11, 2020 at 0:37