Typical events: visions of centrality

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The central limit theorem S_n^* is asymptotically normally distributed

$$\mathbf{P}\{a < S_n^* \le b\} \to \int_a^b \phi(x)\,dx \qquad (n \to \infty)$$