Module 5 Peer Review Assignment

Problem 1

Roll two six-sided fair dice. Let X denote the larger of the two values. Let Y denote the smaller of the two values.

a) Construct a table that gives the joint probability mass function for X and Y. (Note: "X is the larger value and Y is the smaller value in a two dice roll" means that for any two dice roll, X will be greater than or equal to Y).

x/y	1	2	3	4	5	6
1	$1/36$	0	0	0	0	0
2	$2/36$	$1/36$	0	0	0	0
3	$2/36$	$2/36$	$1/36$	0	0	0
4	$2/36$	$2/36$	$2/36$	$1/36$	0	0
5	$2/36$	$2/36$	$2/36$	$2/36$	$1/36$	0
6	$2/36$	$2/36$	$2/36$	$2/36$	$2/36$	$1/36$

Since in all but the diagonal elements in the above array, we have couple of choices, corresponding to the permutation of the values in two dice.

b) What is $P(X \geq 3, Y = 1)$?

$P(X \geq 3, Y = 1) = \sum_{x=3}^{6} P(X = x, Y = 1) = \frac{8}{36} = \frac{2}{9}$

c) What is $P(X \geq Y + 2)$?

$$\begin{aligned} P(X \geq Y+2) = \quad & P(X=3,Y=1) + P(X=4,Y=1) + P(X=4,Y=2) + P(X=5,Y=1) + P(X=5,Y=2) + \\ & P(X=5,Y=3) + P(X=6,Y=1) + P(X=6,Y=2) + P(X=6,Y=3) + P(X=6,Y=4) \\ & = \frac{20}{36} = \frac{5}{9} \end{aligned}$$

d) Are X and Y independent? Explain.

No, they are dependent. For example, $P(X = 1) = \frac{1}{36}$ and $P(Y = 1) = \frac{11}{36}$ and $P(X = 1, Y = 1) = \frac{1}{36}$, Clearly, $P(X = 1, Y = 1) \neq P(X = 1)P(Y = 1)$.

Problem 2

Let (X, Y) be continuous random variables with joint PDF:

$$f(x,y) = \begin{cases} cxy^2 & \text{if } 0 \leq x \leq 1 \text{ and } 0 \leq y \leq 1 \\ 0 & \text{else} \end{cases}$$

Part a)

Solve for c. Show your work.

Since f is a pdf, we have $\int_0^1 \int_0^1 cxy^2 dxdy = c\int_0^1 xdx \int_0^1 y^2 dy = c\left[\frac{x^2}{2}\right]_0^1 \left[\frac{y^3}{3}\right]_0^1 = \frac{c}{6} = 1 \implies c = 6$

Part b)

Find the marginal distributions $f_X(x)$ and $f_Y(y)$. Show your work.

$$f_X(x) = 6\int_0^1 xy^2 dy = 6x\left[\frac{y^3}{3}\right]_0^1 = 2x, \quad 0 \le x \le 1$$

$$f_Y(y) = 6\int_0^1 xy^2 dx = 6y^2\left[\frac{x^2}{2}\right]_0^1 = 3y^2, \quad 0 \le y \le 1$$

Part c)

Solve for $E[X]$ and $E[Y]$. Show your work.

$$E[X] = \int_0^1 x f_X(x)dx = \int_0^1 2x^2 dx = 2\left[\frac{x^3}{3}\right]_0^1 = \frac{2}{3}$$

$$E[Y] = \int_0^1 y f_Y(y)dy = \int_0^1 3y^3 dy = 3\left[\frac{y^4}{4}\right]_0^1 = \frac{3}{4}$$

Part d)

Using the joint PDF, solve for $E[XY]$. Show your work.

$$E[XY] = \int_0^1\int_0^1 xyf(x,y)dxdy = \int_0^1\int_0^1 xy.6xy^2 dxdy = 6\int_0^1 x^2 dx \int_0^1 y^3 dy = 6\left[\frac{x^3}{3}\right]_0^1\left[\frac{y^4}{4}\right]_0^1 = \frac{1}{2}$$

Part e)

Are X and Y independent?

Yes, they are.

We have $E[XY] = \frac{1}{2} = \frac{2}{4} = \frac{2}{3}.\frac{3}{4} = E[X]E[Y]$, i.e., $cov(X,Y) = E[XY] - E[X]E[Y] = 0$, i.e., X and Y are uncorrelated.

Also, $f(x,y) = 6xy^2 = 2x.3y^2 = f_X(x)f_Y(y)$, for all $0 \le x, y \le 1$ which implies that they are independent.