Test your understanding

- *Sample space*: $\Omega = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$.
- *Mass function* (random selection): $\mathbf{P}(\{i\}) = \mathbf{P}\{i\} = 1/10$ for $1 \le i \le 10$.
- *Events*:
  - $A = \{2, 3, 4, 5, 6\}$.
  - $B = \{4, 5, 6, 7, 8, 9\}$, $C = \{2, 4, 6, 8, 10\}$, $D = \{1, 7, 8, 9\}$.
- *Evaluate*: $\mathbf{P}(A \mid B)$, $\mathbf{P}(A \mid C)$, $\mathbf{P}(A \mid D)$, and $\mathbf{P}(A \mid B \cap C)$.

Event		Probability
Ω	{1, 2, 3, 4, 5, 6, 7, 8, 9, 10}	1
A	{2, 3, 4, 5, 6}	5/10 = 1/2
B	{4, 5, 6, 7, 8, 9}	6/10 = 3/5
C	{2, 4, 6, 8, 10}	5/10 = 1/2
D	{1, 7, 8, 9}	4/10 = 2/5
$B \cap C$	{4, 6, 8}	3/10
$A \cap B$	{4, 5, 6}	3/10
$A \cap C$	{2, 4, 6}	3/10
$A \cap D$	$\varnothing$	0
$A \cap B \cap C$	{4, 6}	2/10 = 1/5

Event		Probability
Ω	{1, 2, 3, 4, 5, 6, 7, 8, 9, 10}	1
A	{2, 3, 4, 5, 6}	5/10 = 1/2
B	{4, 5, 6, 7, 8, 9}	6/10 = 3/5
C	{2, 4, 6, 8, 10}	5/10 = 1/2
D	{1, 7, 8, 9}	4/10 = 2/5
$B \cap C$	{4, 6, 8}	3/10
$A \cap B$	{4, 5, 6}	3/10
$A \cap C$	{2, 4, 6}	3/10
$A \cap D$	$\varnothing$	0
$A \cap B \cap C$	{4, 6}	2/10 = 1/5

$$\mathbf{P}(A \mid B) = \frac{\mathbf{P}(A \cap B)}{\mathbf{P}(B)} = \frac{3/10}{6/10} = \frac{1}{2}$$

$$\mathbf{P}(A \mid C) = \frac{\mathbf{P}(A \cap C)}{\mathbf{P}(C)} = \frac{3/10}{5/10} = \frac{3}{5}$$

$$\mathbf{P}(A \mid D) = \frac{\mathbf{P}(A \cap D)}{\mathbf{P}(D)} = \frac{0}{4/10} = 0$$

$$\mathbf{P}(A \mid B \cap C) = \frac{\mathbf{P}(A \cap B \cap C)}{\mathbf{P}(B \cap C)} = \frac{2/10}{3/10} = \frac{2}{3}$$

Event		Probability
Ω	{1, 2, 3, 4, 5, 6, 7, 8, 9, 10}	1
A	{2, 3, 4, 5, 6}	5/10 = 1/2
B	{4, 5, 6, 7, 8, 9}	6/10 = 3/5
C	{2, 4, 6, 8, 10}	5/10 = 1/2
D	{1, 7, 8, 9}	4/10 = 2/5
$B \cap C$	{4, 6, 8}	3/10
$A \cap B$	{4, 5, 6}	3/10
$A \cap C$	{2, 4, 6}	3/10
$A \cap D$	$\varnothing$	0
$A \cap B \cap C$	{4, 6}	2/10 = 1/5

$$\mathbf{P}(A \mid B) = \frac{\mathbf{P}(A \cap B)}{\mathbf{P}(B)} = \frac{3/10}{6/10} = \frac{1}{2}$$

$$\mathbf{P}(A \mid C) = \frac{\mathbf{P}(A \cap C)}{\mathbf{P}(C)} = \frac{3/10}{5/10} = \frac{3}{5}$$

$$\mathbf{P}(A \mid D) = \frac{\mathbf{P}(A \cap D)}{\mathbf{P}(D)} = \frac{0}{4/10} = 0$$

$$\mathbf{P}(A \mid B \cap C) = \frac{\mathbf{P}(A \cap B \cap C)}{\mathbf{P}(B \cap C)} = \frac{2/10}{3/10} = \frac{2}{3}$$

$$\mathbf{P}(A \mid D) < \mathbf{P}(A \mid B) = \mathbf{P}(A) < \mathbf{P}(A \mid C) < \mathbf{P}(A \mid B \cap C)$$

Event		Probability
Ω	$\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$	1
A	$\{2, 3, 4, 5, 6\}$	$5/10 = 1/2$
B	$\{4, 5, 6, 7, 8, 9\}$	$6/10 = 3/5$
C	$\{2, 4, 6, 8, 10\}$	$5/10 = 1/2$
D	$\{1, 7, 8, 9\}$	$4/10 = 2/5$
$B \cap C$	$\{4, 6, 8\}$	$3/10$
$A \cap B$	$\{4, 5, 6\}$	$3/10$
$A \cap C$	$\{2, 4, 6\}$	$3/10$
$A \cap D$	$\varnothing$	0
$A \cap B \cap C$	$\{4, 6\}$	$2/10 = 1/5$

$$\mathbf{P}(A \mid B) = \frac{\mathbf{P}(A \cap B)}{\mathbf{P}(B)} = \frac{3/10}{6/10} = \frac{1}{2}$$

$$\mathbf{P}(A \mid C) = \frac{\mathbf{P}(A \cap C)}{\mathbf{P}(C)} = \frac{3/10}{5/10} = \frac{3}{5}$$

$$\mathbf{P}(A \mid D) = \frac{\mathbf{P}(A \cap D)}{\mathbf{P}(D)} = \frac{0}{4/10} = 0$$

$$\mathbf{P}(A \mid B \cap C) = \frac{\mathbf{P}(A \cap B \cap C)}{\mathbf{P}(B \cap C)} = \frac{2/10}{3/10} = \frac{2}{3}$$

$$\mathbf{P}(A \mid D) < \mathbf{P}(A \mid B) = \mathbf{P}(A) < \mathbf{P}(A \mid C) < \mathbf{P}(A \mid B \cap C)$$

Slogan: Conditioning provides information that can effect event probabilities in unexpected ways.