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$$

$$
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$$

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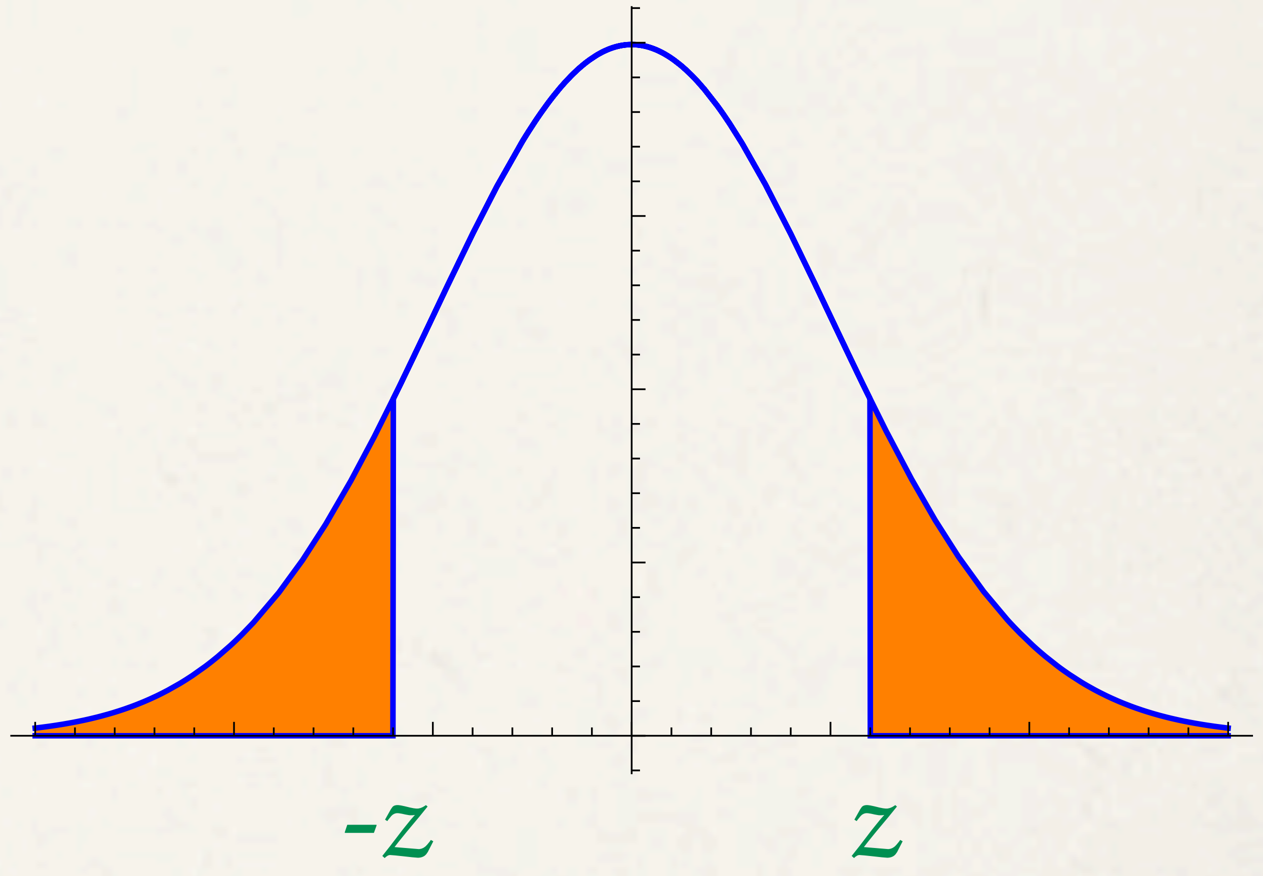
Normal approximation!



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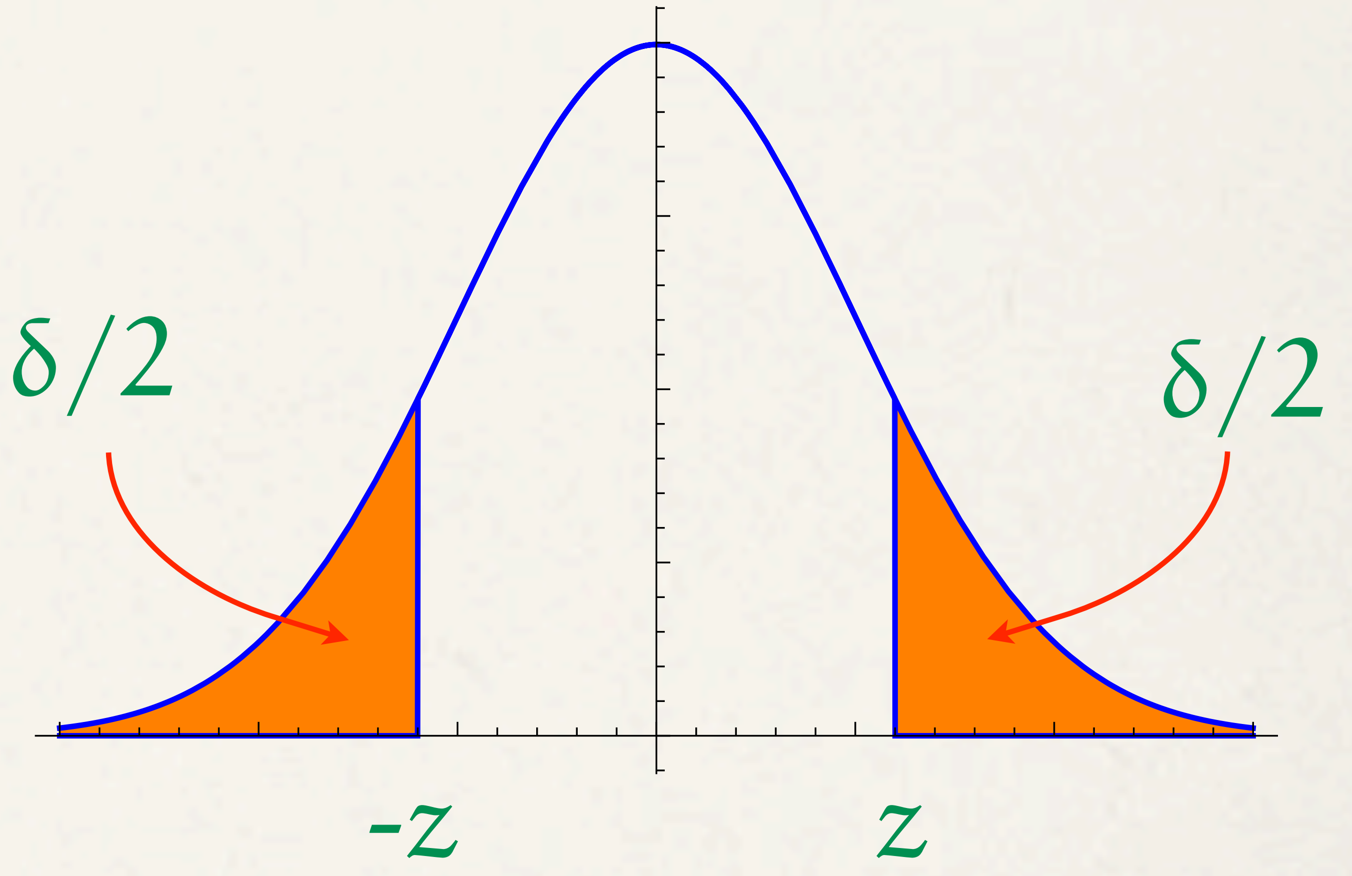

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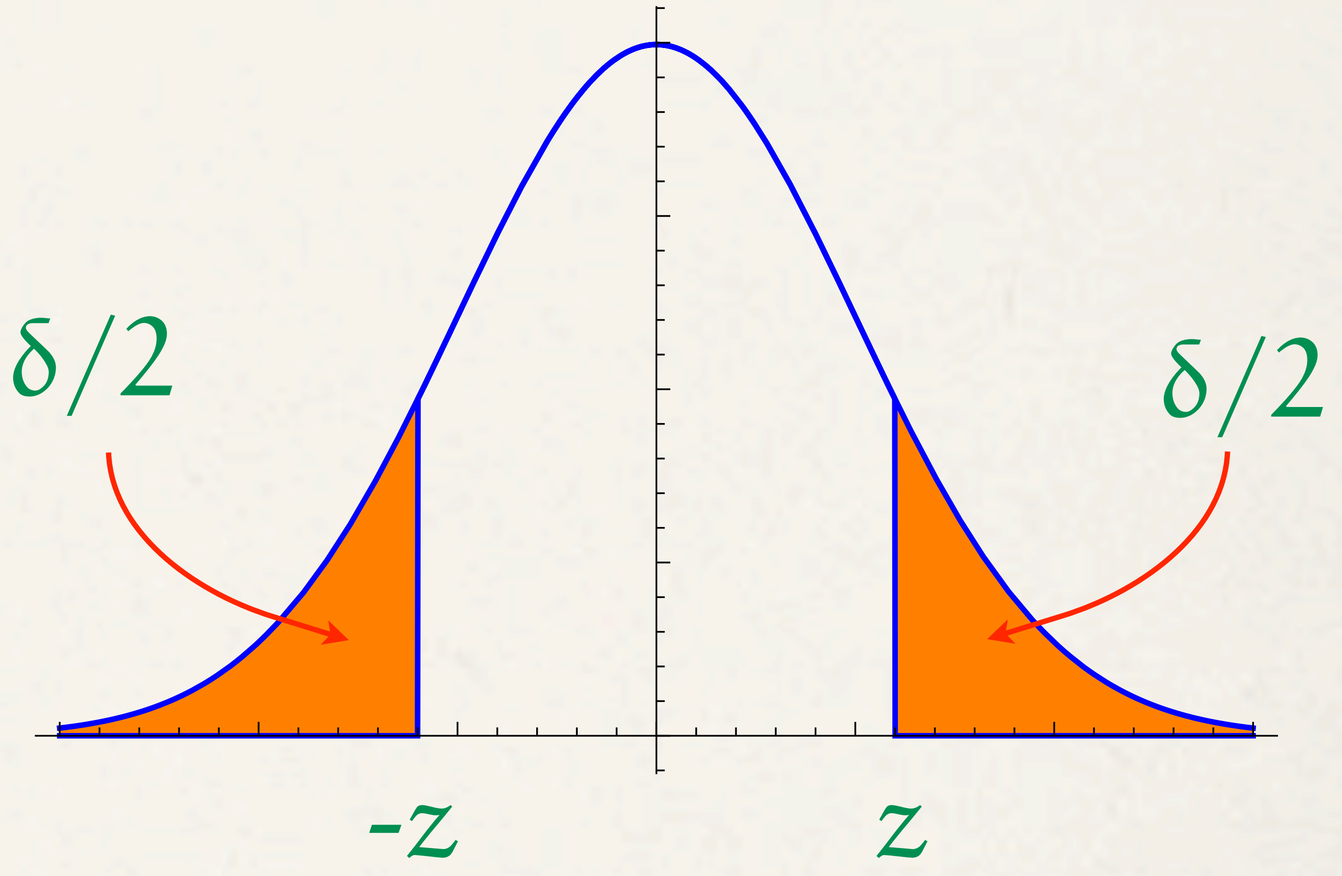

$$z = \frac{\epsilon\sqrt{n}}{\sqrt{pq}}$$

The operating principle!

Select the sample size n so that the quantile $z = z(\delta)$ is the unique real number for which the area under the right tail of the bell curve is $\delta/2$.

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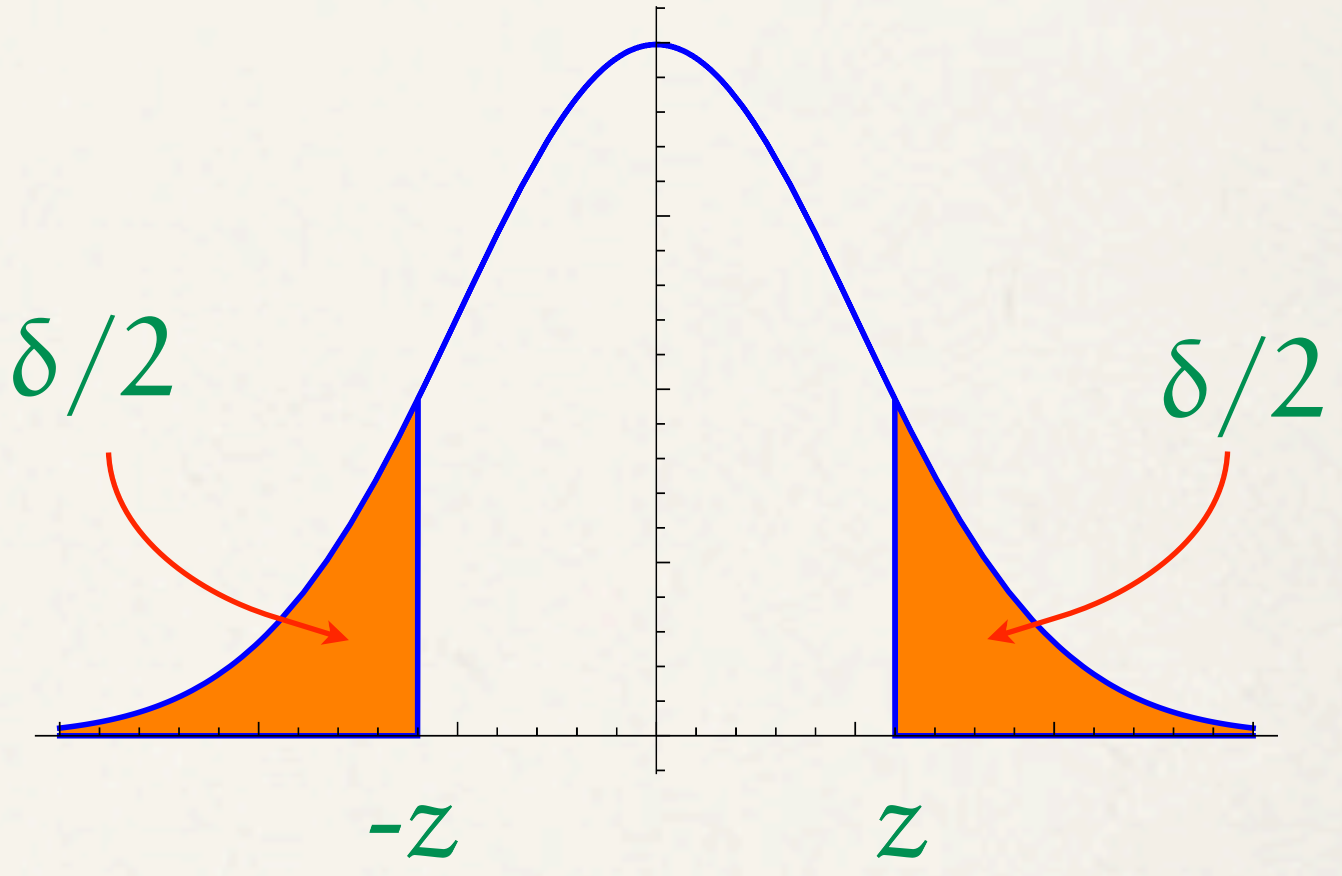

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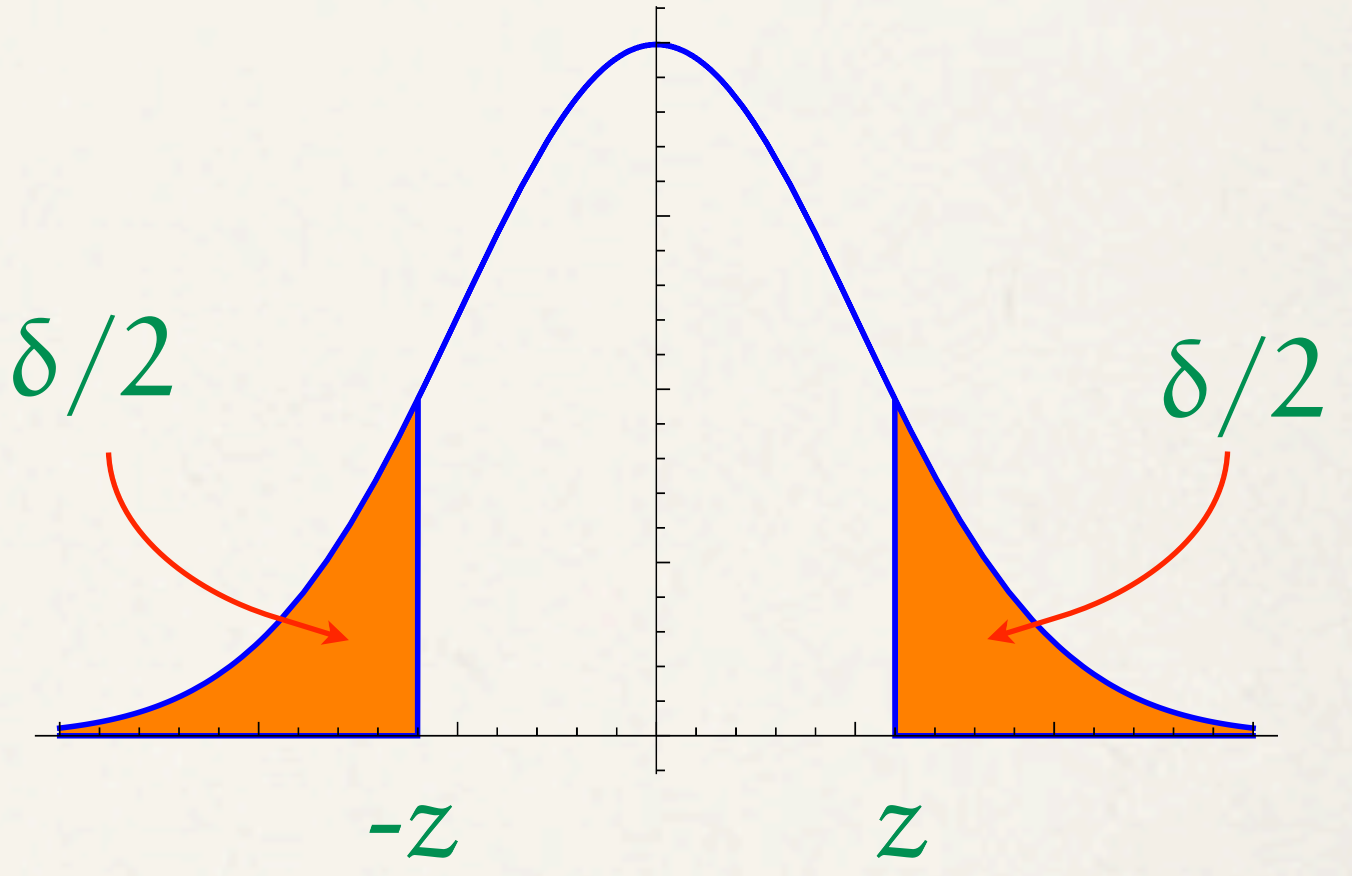
$$=\delta$$

Normal approximation!



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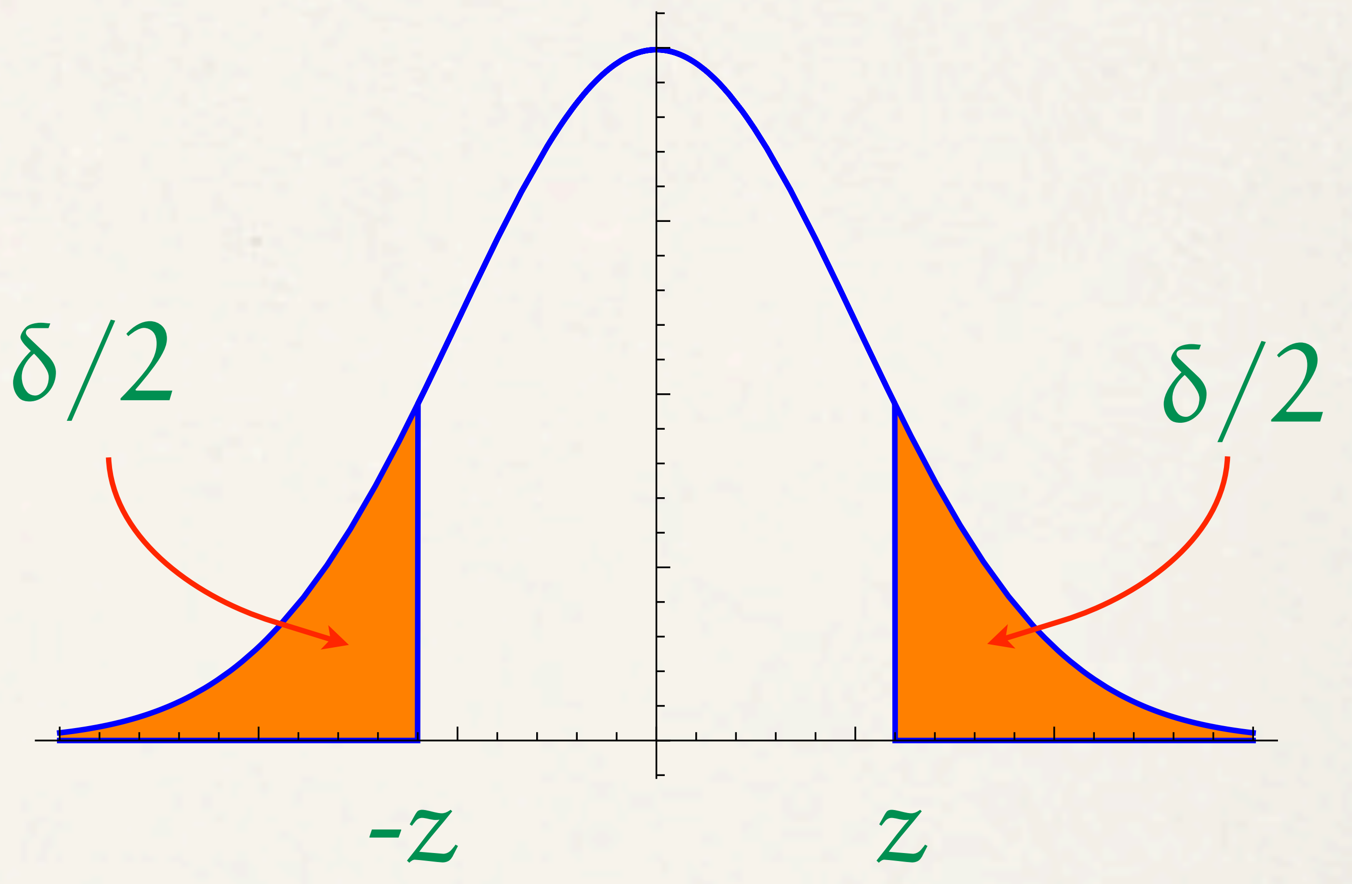

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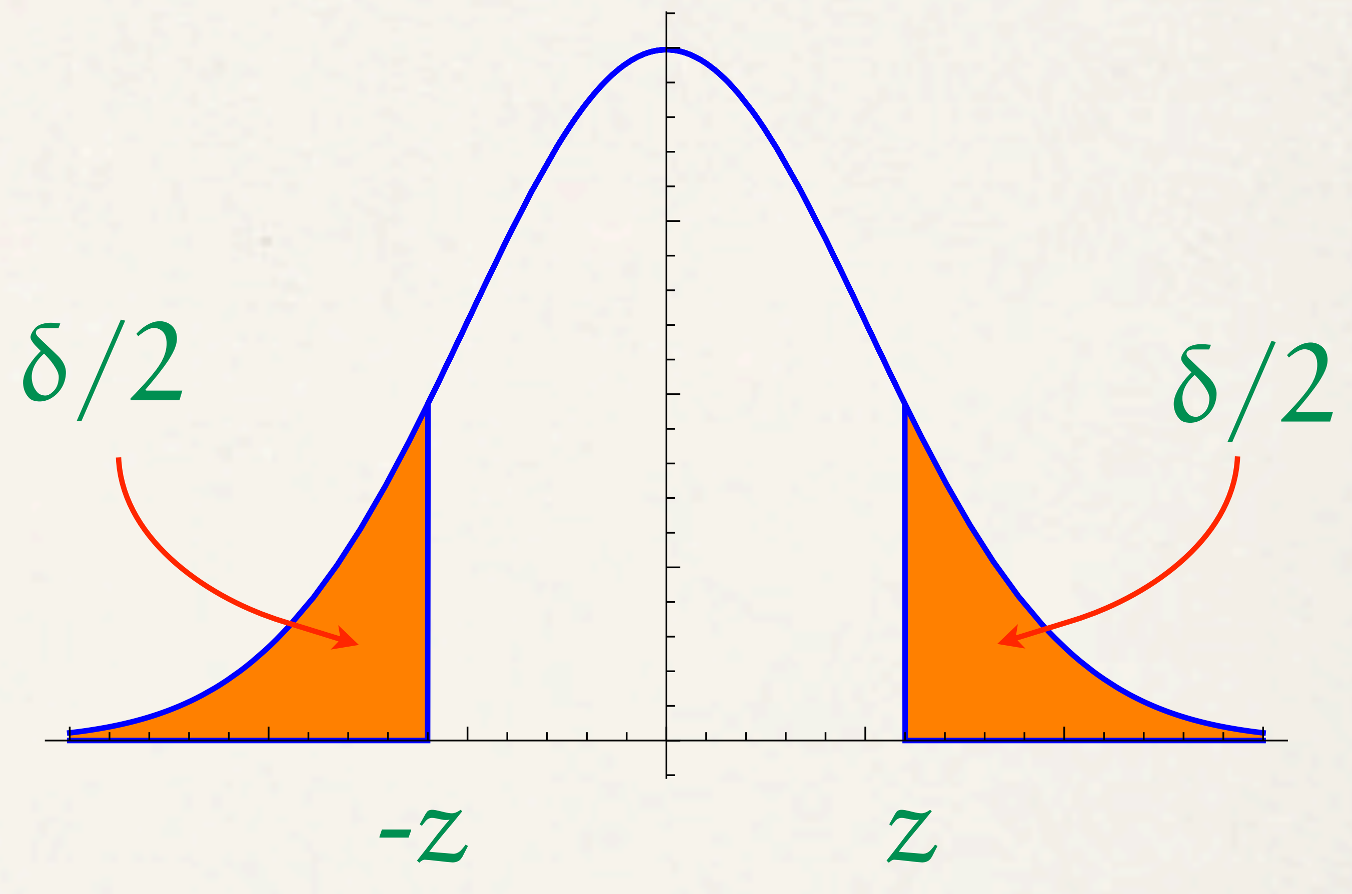

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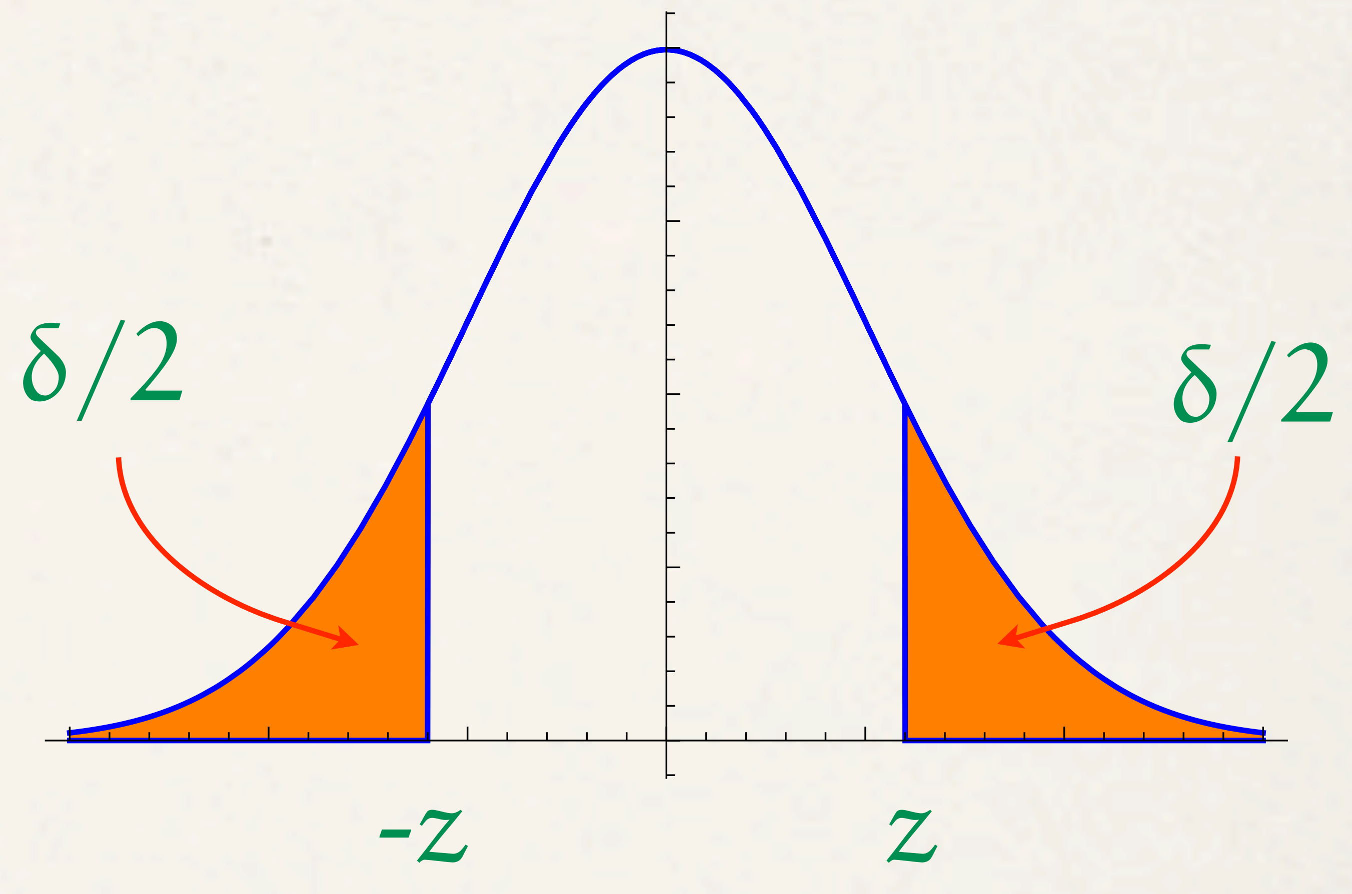

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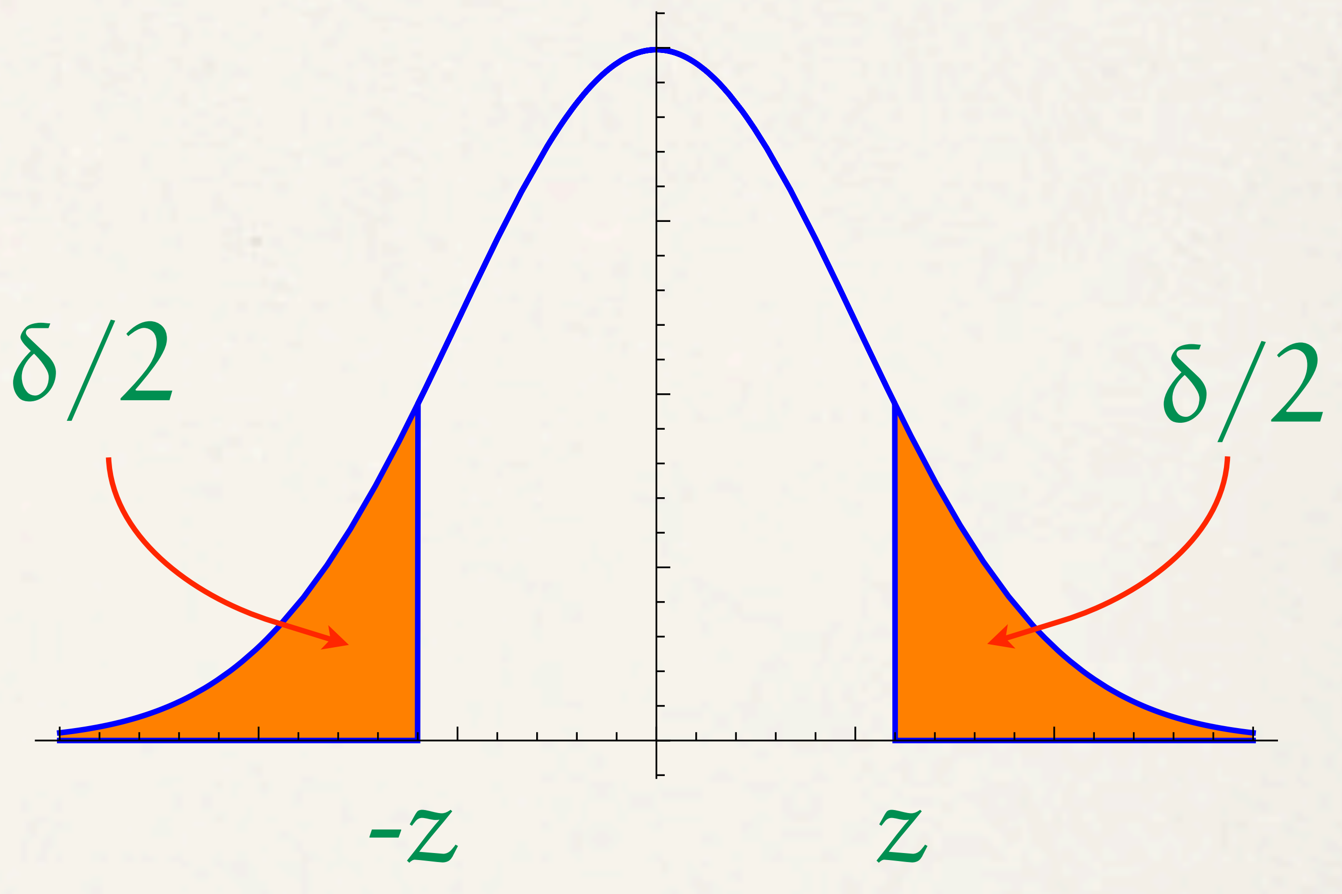

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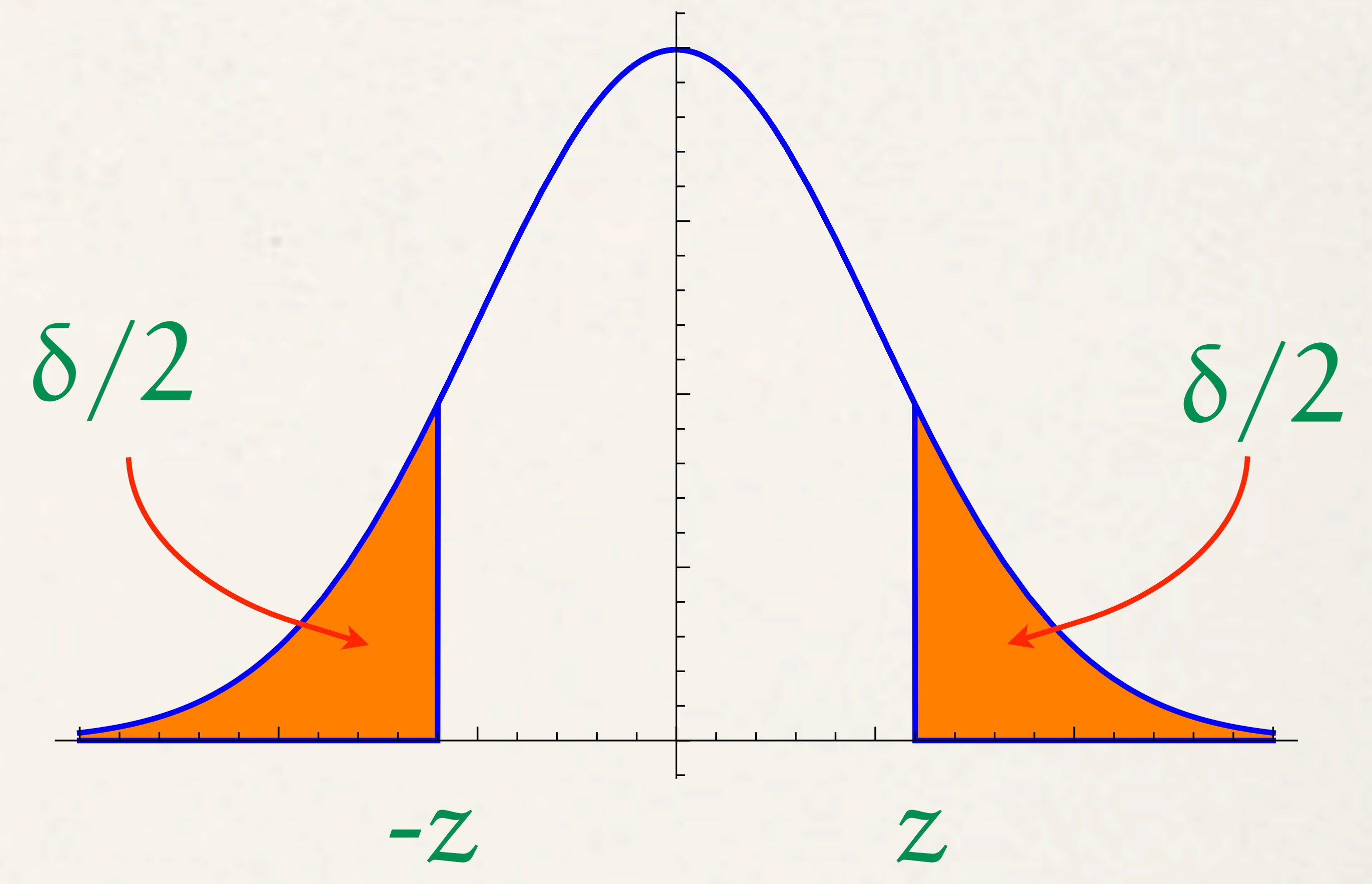

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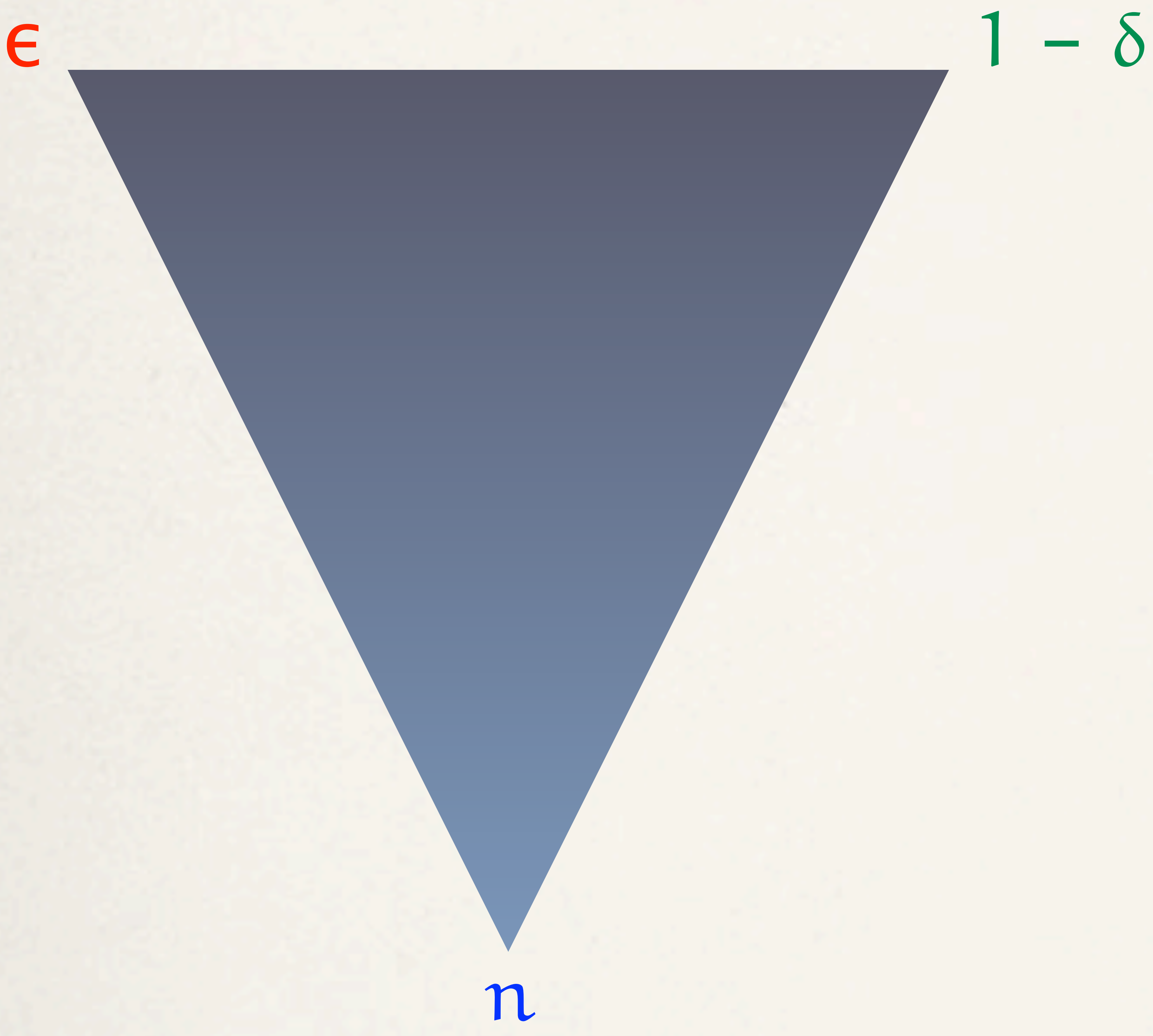
An estimate of the requisite sample size under a normal approximation:

To guarantee that the estimate error is no larger than ϵ with confidence at least $1 - \delta$, for any value p, we will require a sample size of at least

$$n \geq \frac{z(\delta)^2}{4\epsilon^2}$$

How are the error ϵ, the confidence $1 - \delta$, and the sample size n related? $n \geq \frac{z(\delta)^2}{4\epsilon^2}$



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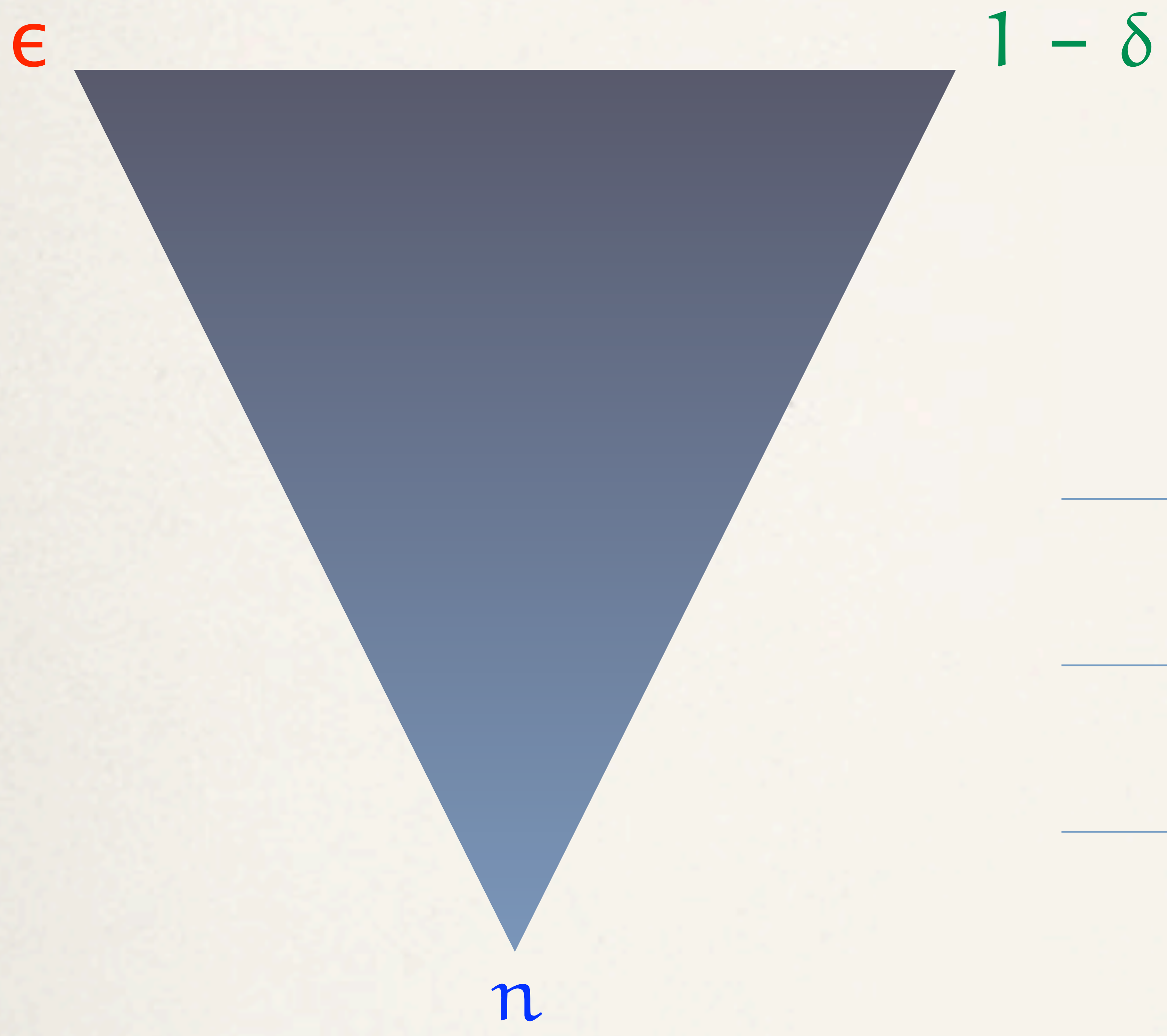

Error ϵ	Confidence $1 - \delta$	Quantile $z(\delta)$	Sample size (Chebyshev) n	Sample size (normal approximation) n
0.10	0.90	1.65	250	68
0.05	0.95	1.96	2000	385
0.03	0.95	1.96	5556	1068

The requisite statistical guarantee:

$$\mathbf{P}\left\{\left|\frac{S_n}{n} - p\right| > \epsilon\right\} \leq \delta$$

Slogan

A relatively small, honest, random sample *whose size does not depend upon the size of the underlying population or its composition* gives a good estimate of the underlying population proportions (sentiments).