Feedback — Problem Set 6

Help

You submitted this homework on **Sun 9 Nov 2014 12:04 PM PST**. You got a score of **16.00** out of **16.00**.

This problem set focuses primarily on material covered in Week 6 (Lecture 8), so I recommend you to watch the lecture and attempt Assignment 8 before submitting your answers. The deadline for completing (and submitting) the problem set is Monday November 10 at 9:00AM US-PST. Note that you can save your entries as you work through the problems, and can change them at any time prior to submission, but once you submit your answers no further changes are possible. Note: A downloadable PDF file of this problem set is supplied as an asset to Lecture 8B.

Question 1

Is the following proof valid or not? [3 points]

Theorem: For any natural number n, $2^n > 2n$.

Proof: By induction. The case $n = 1$ is obviously true, so assume the inequality holds for n.

That is, assume $2^n > 2n$. Then?

$2^{n+1} = 2 \cdot 2^n > 2 \cdot 2n$(by the induction hypothesis) $= 4n = 2n + 2n \geq 2n + 2$(since $n \geq 1$) $= 2(n + 1)$

This establishes the inequality for $n + 1$. Hence, by induction, the inequality holds for all n.

Your Answer		Score	Explanation
○ Valid			
◉ Invalid	✔	3.00	Correct. Note that the theorem is true if we amend it to say ``For every natural number $n > 2$ ..."
Total		3.00 / 3.00	

Question 2

Is the following proof valid or not? [3 points]

Theorem: If a nonempty finite set X has n elements, then X has exactly 2^n distinct subsets.

Proof: By induction on n.

The case $n = 1$ is true, since if X is a set with exactly one element, say $X = \{a\}$, then X has the two subsets $\emptyset$ and X itself.

Assume the theorem is true for n. Let X be a set of $n + 1$ elements. Let $a \in X$ and let $Y = X - \{a\}$(i.e., obtain Y by removing a from X). Then Y is a set with n elements. By the induction hypothesis, Y has 2^n subsets. List them as $Y_1, \ldots, Y_{2^n}$. Then all the subsets of X are $Y_1, \ldots, Y_{2^n}, Y_1 \cup \{a\}, \ldots, Y_{2^n} \cup \{a\}$(i.e., the subsets of Y together with the subsets of Y with a added to each one). There are $2 \cdot 2^n = 2^{n+1}$ sets in this list. This establishes the theorem for $n + 1$. Hence, by induction, it is true for all n.

Your Answer		Score	Explanation
◉ Valid	✔	3.00	Yes this is correct.
○ Invalid			
Total		3.00 / 3.00	

Question 3

True or false? If p is a prime number, then $\sqrt{p}$ is irrational. (Before entering your answer, you should either construct a proof of truth or find a counter-example, so you are sure. After you have completed the problem set, you should write up your proof or counter-example and share it with your study group for feedback. You can assume that if p is prime, then whenever p divides a product ab, p divides at least one of a, b.) [3 points]

Your Answer		Score	Explanation
◉ True	✔	3.00	Correct. But are you sure your reasoning is correct? If possible, show your argument to others for their feedback.
○			

False	
Total	3.00 / 3.00

Question 4

Evaluate this purported proof, and grade it according to the course rubric. Enter your grade (which should be a whole number between 0 and 24, inclusive) in the box. You should come within 4 points of the instructor's grade for full marks [5 points], within 6 points for partial marks [3 points].].

You should read the website section "Using the rubric" (it includes a short explanatory video) before attempting this question.

You entered:

21

Your Answer		Score	Explanation
21	✔	5.00	Good grade. This is a correct proof, expressed well, but missing explanations. Instructor grading: 4, 4, 4, 4, 0, 3. Total 19. SEE THE TUTORIAL VIDEO.
Total		5.00 / 5.00	

Question 5

This theorem is obviously false. On the drop-down menu, select the line number of the (false) statement where the proof logically breaks down. [2 points]

Your Answer	Score	Explanation
Line 1		
Line 2		

Line 3			
Line 4			
Line 5			
Line 6			
Line 7			
Line 8			
Line 9			
Line 10			
Line 11			
Line 12			
Line 13	✔	2.00	Correct! The set G might have only one member, in which case there is no such c.
Line 14			
Line 15			
Line 16			
Line 17			
Line 18			
Line 19			
Total		2.00 / 2.00	