Advice from a magician — a view from a sample point

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$$S_k := \sum_{1 \le j_1 < j_2 < \cdots < j_k \le n} \mathbf{P}(A_{j_1} \cap A_{j_2} \cap \cdots \cap A_{j_k}) \qquad (1 \le k \le n)$$

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Inclusion–exclusion: The distribution of the number of occurrences of $A_1, \dots, A_n$ is given by

$$\mathbf{P}\{X_1 + \cdots + X_n = k\} = \binom{k}{k} S_k - \binom{k+1}{k} S_{k+1} + \cdots + (-1)^j \binom{k+j}{k} S_{k+j} + \cdots + (-1)^{n-k} S_n$$

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- Identify the sets: suppose ω is in $A_{j_1}, A_{j_2}, \ldots, A_{j_l}$.
- Then $X_1 + X_2 + \cdots + X_n = l$.
- Three cases: (1) $l < k$. (2) $l = k$. (3) $l > k$.