Two Cells of Opposite Colors: Hints

Practice with Lab Sandbox

We encourage you to read the hints below only when you have already tried to solve the puzzle yourself.

First, let us focus on the following question: *Why are we sure that there is a pair of adjacent cells of opposite colors?* Informally, this is true because when moving from left to right, the color of a cell should switch from white to black at least once. More formally, consider the leftmost cell. It is white. If its neighbour cell is black, then we are done. If it is white, then consider the next cell to the right. If it is black, then, again, we are done. Otherwise proceed one step to the right. We know that this cannot continue forever as the rightmost cell is black.

Note three important things about this proof:

1. It does not rely on the fact that we have 20 cells. In other words, it works for *any number of cells.*

2. It doesn't prove that the sequence always contains a *unique* pair of adjacent cells. There may be many such pairs. What we proved above is that there is *at least one* such pair.

3. It *proves* that there *exists* a pair of adjacent cells, but does not give an *efficient method* for finding this pair (by revealing the color of a few cells). In particular, if we start checking the color of the cells one by one from the left, then we will eventually find a required pair of adjacent cells, but in the worst case it will require us to reveal the color of all cells.

To design an efficient method, recall that we are studying *recursion* in this module. To use recursion, let's reveal the color of the middle cell (i.e., cell number 10). Assume that its color is black. This means that somewhere in the left half of the initial sequence there must be a pair of adjacent cells of opposite colors! Indeed, the leftmost cell is white, the tenth cell is black, so by the argument above somewhere between them there must be a required pair. The same reasoning shows that if the tenth cell is white, then there must be a required pair in the right half of the initial sequence.

Do you see how to proceed?

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