Fractals from the Lorenz Equations

Determine the fractal that arises from using Newton's method to compute the fixed-point solutions of the Lorenz equations. Use the parameter values $r = 28$, $\sigma = 10$ and $\beta = 8/3$. Initial values (x_0, z_0) are taken on a grid in the x-z plane with always $y_0 = 3\sqrt{2}$. For assessment purposes, the computational grid and the graphics code will be given in the Learner Template. To pass the assessment, every pixel in your figure needs to be colored correctly.

(Hint: Some grid points may require as many as 33 Newton iterations to converge while others may require as few as three. Unfortunately, if you uniformly use 33 Newton iterations at every grid point, the MATLAB Grader may time out. You can accelerate your code by using a ***while*** loop instead of a ***for*** loop.)

Script

Save Reset MATLAB Documentation (https://www.mathworks.com/help/)

```
r=28; sigma=10; beta=8/3;
x1=0; y1=0; z1=0;
x2=sqrt(beta*(r-1)); y2=sqrt(beta*(r-1)); z2=r-1;
x3=-sqrt(beta*(r-1)); y3=-sqrt(beta*(r-1)); z3=r-1;
nx=500; nz=500;
xmin=-40; xmax=40; zmin=-40; zmax=40;
x_grid=linspace(xmin,xmax,nx); z_grid=linspace(zmin,zmax,nz);
[X,Z]=meshgrid(x_grid,z_grid);

% Write Newton's method using every grid point as the initial condition
% Perform enough iterations that every initial condition converges to a root
% Save the x-values of the converged roots in the matrix X
% To pass the assessment, every pixel in the figure must be correctly colored

%!!!!!!!!! Set initial value y=3*sqrt(2) for all values (x,z) on the grid !!!!!!!!!

for kk = 1:nz
    for nn = 1:nx
        x = X(kk,nn);
        z = Z(kk,nn);
        y=3*sqrt(2);
        RelTol=1.e-06; AbsTol=1.e-09;
        error=Inf;
        while error > max(RelTol*max(abs([x,y,z])),AbsTol)
            J= [-sigma sigma 0; r-z -1 -x; y x -beta]; % DEFINE THE JACOBIAN MATRIX
            rhs = -[sigma*(y-x); x*(r-z)-y; x*y-beta*z]; % DEFINE THE RIGHT-HAND SIDE
            delta_xyz=J\rhs;
            x = x + delta_xyz(1);
            y = y + delta_xyz(2);
            z = z + delta_xyz(3);
            error=max(abs(delta_xyz));
        end
        X(kk, nn) = x;
    end
end




eps=1.e-03;
X1 = abs(X-x1) < eps; X2 = abs(X-x2) < eps; X3 = abs(X-x3) < eps;
X4 = ~(X1+X2+X3);
figure;
map = [1 0 0; 0 1 0; 0 0 1; 0 0 0]; colormap(map); %[red;green;blue;black]
X=(X1+2*X2+3*X3+4*X4);
image([xmin xmax], [zmin zmax], X); set(gca,'YDir','normal');
xlabel('$x$', 'Interpreter', 'latex', 'FontSize',14);
ylabel('$z$', 'Interpreter', 'latex', 'FontSize',14);
title('Fractal from the Lorenz Equations', 'Interpreter', 'latex','FontSize', 16)

```

Run Script

Assessment: All Tests Passed

Submit

- Test red pixel values
- Test green pixel values
- Test blue pixel values

Output