What the law of large numbers has to say

Portfolio: $\mathbf{a} = (a_1, \ldots, a_m)$

Stock price relatives at close of day n: $\mathbf{X}^{(n)} = (X_1^{(n)}, \ldots, X_m^{(n)})$

$$\Delta_n = \frac{1}{n}\left[\log_2(S_1) + \log_2(S_2) + \cdots + \log_2(S_n)\right]$$

$$W_n = 2^{n \cdot \Delta_n}$$

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generic stock price relative

A stochastically stationary market: repeated independent trials $\mathbf{X}, \mathbf{X}^{(1)}, \mathbf{X}^{(2)}, \ldots, \mathbf{X}^{(n)}, \ldots$

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The sequence of log wealth relatives is independent with a common distribution!

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$$\Delta_n = \frac{1}{n}\left[\log_2(S_1) + \log_2(S_2) + \cdots + \log_2(S_n)\right] \text{ is concentrated at } \Delta(\mathbf{a})$$

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In short, we write: $\Delta_n \doteq \Delta(\mathbf{a})$

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$\mathbf{E}[\log_2(S)] = \mathbf{E}\left[\log_2(S(\mathbf{X}))\right] =: \Delta(\mathbf{a})$ doubling rate for portfolio $\mathbf{a}$

Negative $\Delta(\mathbf{a})$	−0.001	−0.1	−0.5
Positive $\Delta(\mathbf{a})$	0.001	0.1	0.5
# days to **double** (**halve**) wealth	1000	10	2

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Optimal portfolio selection

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Slogan

Select the portfolio $\mathbf{a}^*$ that maximises the doubling rate: $\mathbf{a}^* := \arg\max_{\mathbf{a}} \Delta(\mathbf{a})$

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optimal doubling rate of wealth