Tableau 9, Part 1

Independence
A first look at independent events

Independent possibilities multiply

A multiplication table of possibilities

Given three letters and five numbers, how many (letter, number) possibilities are there?

$\mathfrak{A} \times \mathfrak{B}$		$\mathfrak{B}$				
		1	2	3	4	5
$\mathfrak{A}$	a	(a, 1)	(a, 2)	(a, 3)	(a, 4)	(a, 5)
	b	(b, 1)	(b, 2)	(b, 3)	(b, 4)	(b, 5)
	c	(c, 1)	(c, 2)	(c, 3)	(c, 4)	(c, 5)

If $\mathfrak{A} = \{a, b, c\}$ and $\mathfrak{B} = \{1, 2, 3, 4, 5\}$, the Cartesian product $\mathfrak{A} \times \mathfrak{B}$ of (letter, number) pairs has $3 \times 5 = 15$ elements.

A multiplication table of possibilities

Given three letters and five numbers, how many (letter, number) possibilities are there?

$\mathfrak{A} \times \mathfrak{B}$		$\mathfrak{B}$				
		1	2	3	4	5
$\mathfrak{A}$	a	(a, 1)	(a, 2)	(a, 3)	(a, 4)	(a, 5)
	b	(b, 1)	(b, 2)	(b, 3)	(b, 4)	(b, 5)
	c	(c, 1)	(c, 2)	(c, 3)	(c, 4)	(c, 5)

If $\mathfrak{A} = \{a, b, c\}$ and $\mathfrak{B} = \{1, 2, 3, 4, 5\}$, the Cartesian product $\mathfrak{A} \times \mathfrak{B}$ of (letter, number) pairs has $3 \times 5 = 15$ elements.

A basic principle of counting: Independent possibilities multiply!

Coin tosses

Independent possibilities multiply!

Coin tosses

Independent possibilities multiply!

- A coin is tossed twice: $\mathfrak{A} = \{\mathfrak{H}, \mathfrak{T}\}$, $\mathfrak{B} = \{\mathfrak{H}, \mathfrak{T}\}$.
  - *Sample space*: $\Omega = \mathfrak{A} \times \mathfrak{B} = \{\mathfrak{H}\mathfrak{H}, \mathfrak{H}\mathfrak{T}, \mathfrak{T}\mathfrak{H}, \mathfrak{T}\mathfrak{T}\}$.
  - *Events:*
    - A := First toss is heads = $\{\mathfrak{H}\mathfrak{H}, \mathfrak{H}\mathfrak{T}\}$.
    - B := Second toss is tails = $\{\mathfrak{H}\mathfrak{T}, \mathfrak{T}\mathfrak{T}\}$.
  - *Probability measure*: Combinatorial setting with mass function assigning equal probability 1/4 to each atom.

Coin tosses

Independent possibilities multiply!

- A coin is tossed twice: $\mathfrak{A} = \{\mathfrak{H}, \mathfrak{T}\}$, $\mathfrak{B} = \{\mathfrak{H}, \mathfrak{T}\}$.
  - *Sample space*: $\Omega = \mathfrak{A} \times \mathfrak{B} = \{\mathfrak{H}\mathfrak{H}, \mathfrak{H}\mathfrak{T}, \mathfrak{T}\mathfrak{H}, \mathfrak{T}\mathfrak{T}\}$.
  - *Events:*
    - A := First toss is heads $= \{\mathfrak{H}\mathfrak{H}, \mathfrak{H}\mathfrak{T}\}$.
    - B := Second toss is tails $= \{\mathfrak{H}\mathfrak{T}, \mathfrak{T}\mathfrak{T}\}$.
  - *Probability measure*: Combinatorial setting with mass function assigning equal probability $1/4$ to each atom.

$$\mathbf{P}(A) = \mathbf{P}\{\mathfrak{H}\mathfrak{H}, \mathfrak{H}\mathfrak{T}\} = \frac{2}{4} = \frac{1}{2}$$

Coin tosses

Independent possibilities multiply!

- A coin is tossed twice: $\mathfrak{A} = \{\mathfrak{H}, \mathfrak{T}\}$, $\mathfrak{B} = \{\mathfrak{H}, \mathfrak{T}\}$.
    - *Sample space*: $\Omega = \mathfrak{A} \times \mathfrak{B} = \{\mathfrak{H}\mathfrak{H}, \mathfrak{H}\mathfrak{T}, \mathfrak{T}\mathfrak{H}, \mathfrak{T}\mathfrak{T}\}$.
    - *Events:*
        - $A :=$ First toss is heads $= \{\mathfrak{H}\mathfrak{H}, \mathfrak{H}\mathfrak{T}\}$.
        - $B :=$ Second toss is tails $= \{\mathfrak{H}\mathfrak{T}, \mathfrak{T}\mathfrak{T}\}$.
    - *Probability measure*: Combinatorial setting with mass function assigning equal probability $1/4$ to each atom.

$$\mathbf{P}(A) = \mathbf{P}\{\mathfrak{H}\mathfrak{H}, \mathfrak{H}\mathfrak{T}\} = \frac{2}{4} = \frac{1}{2}$$

$$\mathbf{P}(B) = \mathbf{P}\{\mathfrak{H}\mathfrak{T}, \mathfrak{T}\mathfrak{T}\} = \frac{2}{4} = \frac{1}{2}$$

Coin tosses

Independent possibilities multiply!

- A coin is tossed twice: $\mathfrak{A} = \{\mathfrak{H}, \mathfrak{T}\}$, $\mathfrak{B} = \{\mathfrak{H}, \mathfrak{T}\}$.
  - *Sample space*: $\Omega = \mathfrak{A} \times \mathfrak{B} = \{\mathfrak{H}\mathfrak{H}, \mathfrak{H}\mathfrak{T}, \mathfrak{T}\mathfrak{H}, \mathfrak{T}\mathfrak{T}\}$.
  - *Events:*
    - $A :=$ First toss is heads $= \{\mathfrak{H}\mathfrak{H}, \mathfrak{H}\mathfrak{T}\}$.
    - $B :=$ Second toss is tails $= \{\mathfrak{H}\mathfrak{T}, \mathfrak{T}\mathfrak{T}\}$.
  - *Probability measure*: Combinatorial setting with mass function assigning equal probability $1/4$ to each atom.

$$\mathbf{P}(A) = \mathbf{P}\{\mathfrak{H}\mathfrak{H}, \mathfrak{H}\mathfrak{T}\} = \frac{2}{4} = \frac{1}{2}$$

$$\mathbf{P}(B) = \mathbf{P}\{\mathfrak{H}\mathfrak{T}, \mathfrak{T}\mathfrak{T}\} = \frac{2}{4} = \frac{1}{2}$$

$$\mathbf{P}(A \cap B) = \mathbf{P}\{\mathfrak{H}\mathfrak{T}\} = \frac{1}{4}$$

Coin tosses

Independent possibilities multiply!

- A coin is tossed twice: $\mathfrak{A} = \{\mathfrak{H}, \mathfrak{T}\}$, $\mathfrak{B} = \{\mathfrak{H}, \mathfrak{T}\}$.
  - *Sample space*: $\Omega = \mathfrak{A} \times \mathfrak{B} = \{\mathfrak{H}\mathfrak{H}, \mathfrak{H}\mathfrak{T}, \mathfrak{T}\mathfrak{H}, \mathfrak{T}\mathfrak{T}\}$.
  - *Events:*
    - $A :=$ First toss is heads $= \{\mathfrak{H}\mathfrak{H}, \mathfrak{H}\mathfrak{T}\}$.
    - $B :=$ Second toss is tails $= \{\mathfrak{H}\mathfrak{T}, \mathfrak{T}\mathfrak{T}\}$.
  - *Probability measure*: Combinatorial setting with mass function assigning equal probability $1/4$ to each atom.

$$\mathbf{P}(A) = \mathbf{P}\{\mathfrak{H}\mathfrak{H}, \mathfrak{H}\mathfrak{T}\} = \frac{2}{4} = \frac{1}{2}$$

$$\mathbf{P}(B) = \mathbf{P}\{\mathfrak{H}\mathfrak{T}, \mathfrak{T}\mathfrak{T}\} = \frac{2}{4} = \frac{1}{2}$$

$$\mathbf{P}(A \cap B) = \mathbf{P}\{\mathfrak{H}\mathfrak{T}\} = \frac{1}{4} = \frac{1 \times 1}{2 \times 2} = \frac{1}{2} \cdot \frac{1}{2} = \mathbf{P}(A) \times \mathbf{P}(B)$$

Cards

Independent possibilities multiply!

Cards

Independent possibilities multiply!

- A card is selected at random from a standard deck of 52 cards: four suits $\mathfrak{A} = \{\spadesuit, \heartsuit, \diamondsuit, \clubsuit\}$, thirteen ranks $\mathfrak{B} = \{2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K, A\}$.
  - *Sample space*: $\Omega = \mathfrak{A} \times \mathfrak{B}$; each card is a (suit, rank) pair.
  - *Events:*
    - A := Card drawn is a $\spadesuit = \{(\spadesuit, 2), (\spadesuit, 3), (\spadesuit, 4), (\spadesuit, 5), (\spadesuit, 6), (\spadesuit, 7), (\spadesuit, 8), (\spadesuit, 9), (\spadesuit, 10), (\spadesuit, J), (\spadesuit, Q), (\spadesuit, K), (\spadesuit, A)\}$.
    - B := Card drawn is an ace $= \{(\spadesuit, A), (\heartsuit, A), (\diamondsuit, A), (\clubsuit, A)\}$.
  - *Probability measure*: Combinatorial setting with mass function assigning equal probability $1/52$ to each atom.

Cards

Independent possibilities multiply!

- A card is selected at random from a standard deck of 52 cards: four suits $\mathfrak{A} = \{\spadesuit, \heartsuit, \diamondsuit, \clubsuit\}$, thirteen ranks $\mathfrak{B} = \{2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K, A\}$.
  - *Sample space*: $\Omega = \mathfrak{A} \times \mathfrak{B}$; each card is a (suit, rank) pair.
  - *Events:*
    - A := Card drawn is a $\spadesuit = \{(\spadesuit, 2), (\spadesuit, 3), (\spadesuit, 4), (\spadesuit, 5), (\spadesuit, 6), (\spadesuit, 7), (\spadesuit, 8), (\spadesuit, 9), (\spadesuit, 10), (\spadesuit, J), (\spadesuit, Q), (\spadesuit, K), (\spadesuit, A)\}$.
    - B := Card drawn is an ace $= \{(\spadesuit, A), (\heartsuit, A), (\diamondsuit, A), (\clubsuit, A)\}$.
  - *Probability measure*: Combinatorial setting with mass function assigning equal probability $1/52$ to each atom.

$$\mathbf{P}(A) = \frac{13}{52} = \frac{1}{4}$$

Cards

Independent possibilities multiply!

- A card is selected at random from a standard deck of 52 cards: four suits $\mathfrak{A} = \{\spadesuit, \heartsuit, \diamondsuit, \clubsuit\}$, thirteen ranks $\mathfrak{B} = \{2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K, A\}$.
  - *Sample space*: $\Omega = \mathfrak{A} \times \mathfrak{B}$; each card is a (suit, rank) pair.
  - *Events:*
    - $A :=$ Card drawn is a $\spadesuit = \{(\spadesuit, 2), (\spadesuit, 3), (\spadesuit, 4), (\spadesuit, 5), (\spadesuit, 6), (\spadesuit, 7), (\spadesuit, 8), (\spadesuit, 9), (\spadesuit, 10), (\spadesuit, J), (\spadesuit, Q), (\spadesuit, K), (\spadesuit, A)\}$.
    - $B :=$ Card drawn is an ace $= \{(\spadesuit, A), (\heartsuit, A), (\diamondsuit, A), (\clubsuit, A)\}$.
  - *Probability measure*: Combinatorial setting with mass function assigning equal probability $1/52$ to each atom.

$$\mathbf{P}(A) = \frac{13}{52} = \frac{1}{4}$$

$$\mathbf{P}(B) = \frac{4}{52} = \frac{1}{13}$$

Cards

Independent possibilities multiply!

- A card is selected at random from a standard deck of 52 cards: four suits $\mathfrak{A} = \{\spadesuit, \heartsuit, \diamondsuit, \clubsuit\}$, thirteen ranks $\mathfrak{B} = \{2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K, A\}$.
  - *Sample space*: $\Omega = \mathfrak{A} \times \mathfrak{B}$; each card is a (suit, rank) pair.
  - *Events:*
    - $A :=$ Card drawn is a $\spadesuit = \{(\spadesuit, 2), (\spadesuit, 3), (\spadesuit, 4), (\spadesuit, 5), (\spadesuit, 6), (\spadesuit, 7), (\spadesuit, 8), (\spadesuit, 9), (\spadesuit, 10), (\spadesuit, J), (\spadesuit, Q), (\spadesuit, K), (\spadesuit, A)\}$.
    - $B :=$ Card drawn is an ace $= \{(\spadesuit, A), (\heartsuit, A), (\diamondsuit, A), (\clubsuit, A)\}$.
  - *Probability measure*: Combinatorial setting with mass function assigning equal probability $1/52$ to each atom.

$$\mathbf{P}(A) = \frac{13}{52} = \frac{1}{4}$$

$$\mathbf{P}(B) = \frac{4}{52} = \frac{1}{13}$$

$$\mathbf{P}(A \cap B) = \mathbf{P}\{(\spadesuit, A)\} = \frac{1}{52}$$

Cards

Independent possibilities multiply!

- A card is selected at random from a standard deck of 52 cards: four suits $\mathfrak{A} = \{\spadesuit, \heartsuit, \diamondsuit, \clubsuit\}$, thirteen ranks $\mathfrak{B} = \{2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K, A\}$.
  - *Sample space*: $\Omega = \mathfrak{A} \times \mathfrak{B}$; each card is a (suit, rank) pair.
  - *Events:*
    - $A :=$ Card drawn is a $\spadesuit = \{(\spadesuit, 2), (\spadesuit, 3), (\spadesuit, 4), (\spadesuit, 5), (\spadesuit, 6), (\spadesuit, 7), (\spadesuit, 8), (\spadesuit, 9), (\spadesuit, 10), (\spadesuit, J), (\spadesuit, Q), (\spadesuit, K), (\spadesuit, A)\}$.
    - $B :=$ Card drawn is an ace $= \{(\spadesuit, A), (\heartsuit, A), (\diamondsuit, A), (\clubsuit, A)\}$.
  - *Probability measure*: Combinatorial setting with mass function assigning equal probability $1/52$ to each atom.

$$\mathbf{P}(A) = \frac{13}{52} = \frac{1}{4}$$

$$\mathbf{P}(B) = \frac{4}{52} = \frac{1}{13}$$

$$\mathbf{P}(A \cap B) = \mathbf{P}\{(\spadesuit, A)\} = \frac{1}{52} = \frac{1 \times 1}{4 \times 13} = \frac{1}{4} \cdot \frac{1}{13} = \mathbf{P}(A) \times \mathbf{P}(B)$$