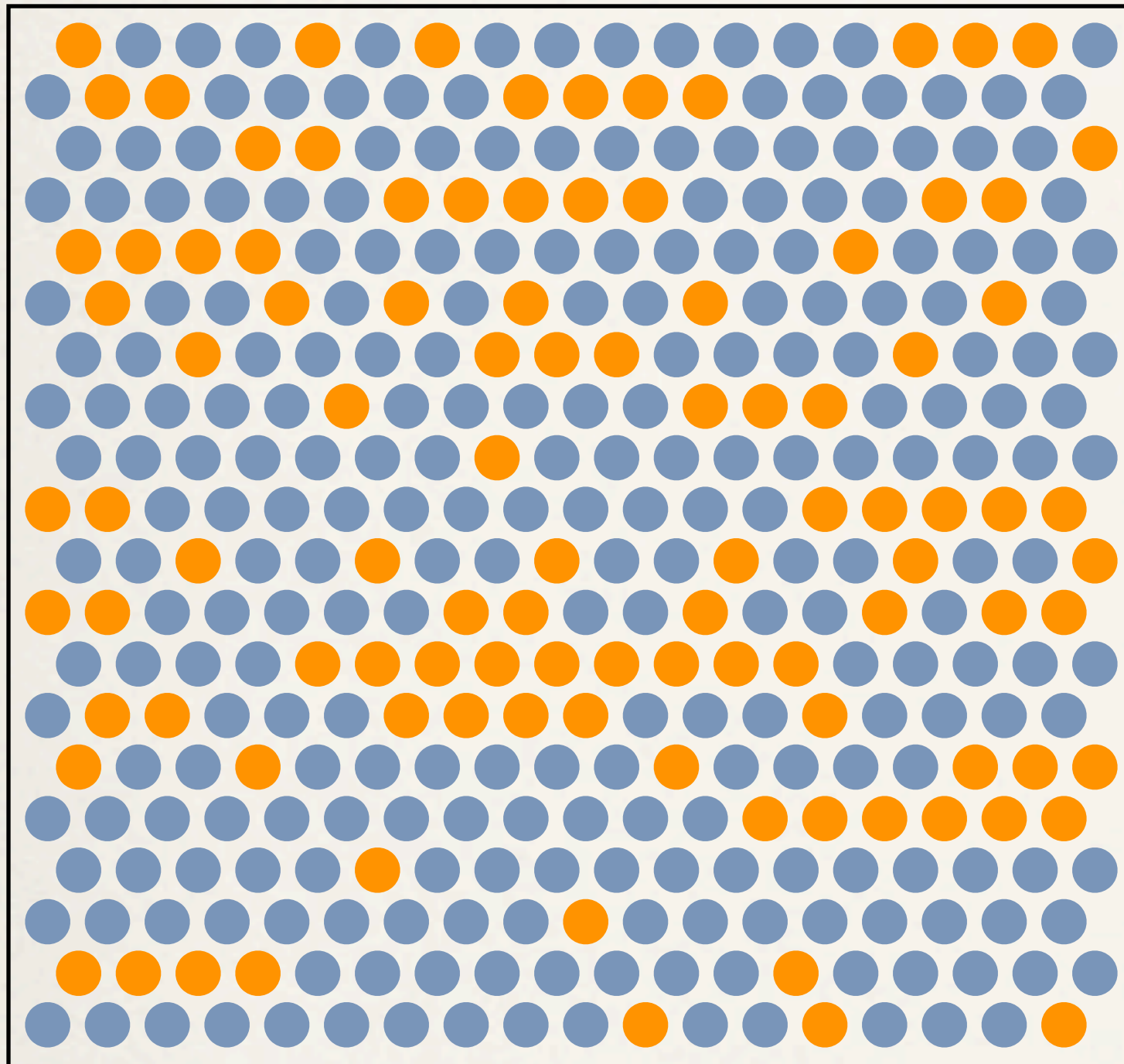
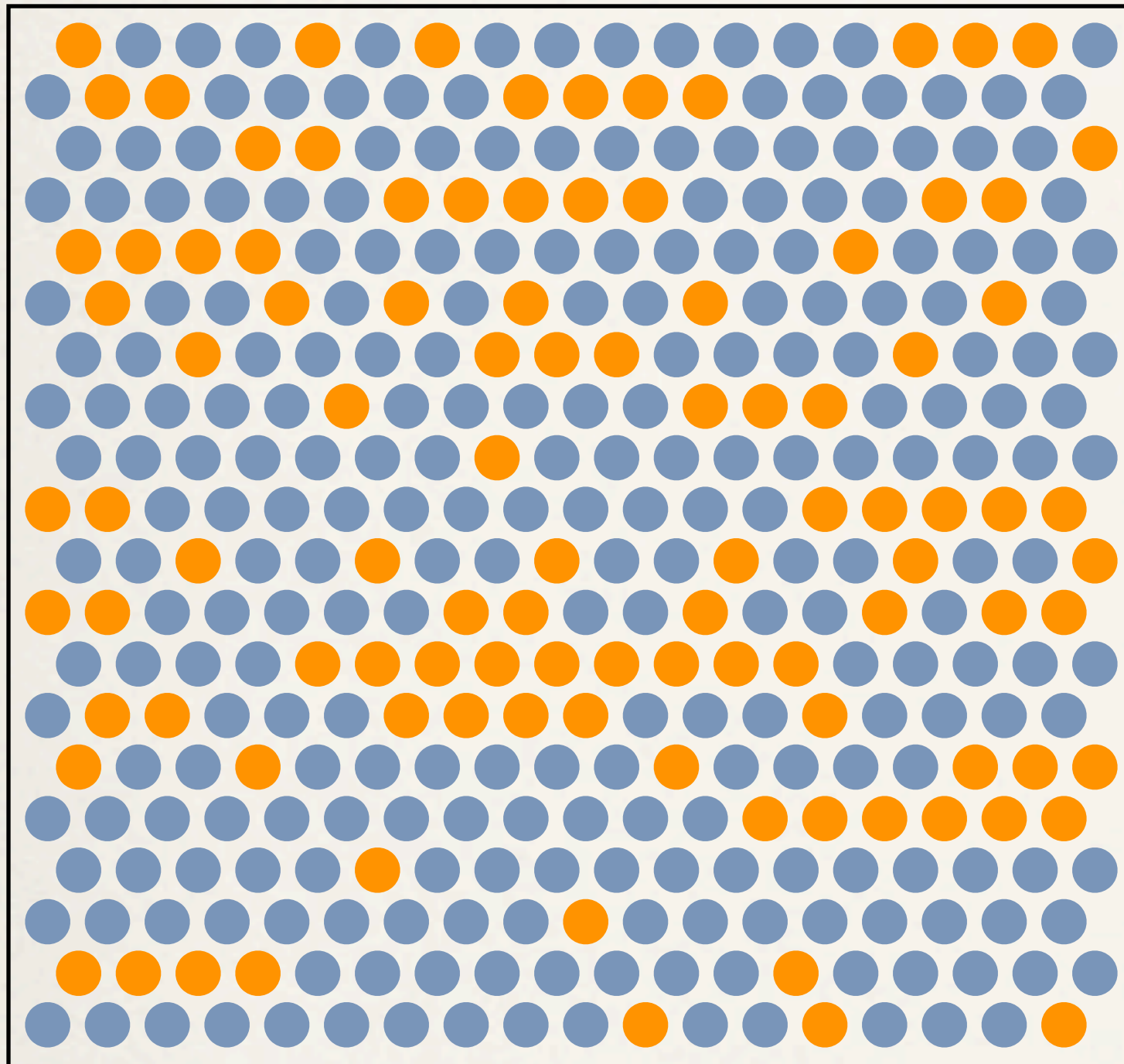


Sampling from a dichotomous population



What fraction p of the balls in the urn is orange?

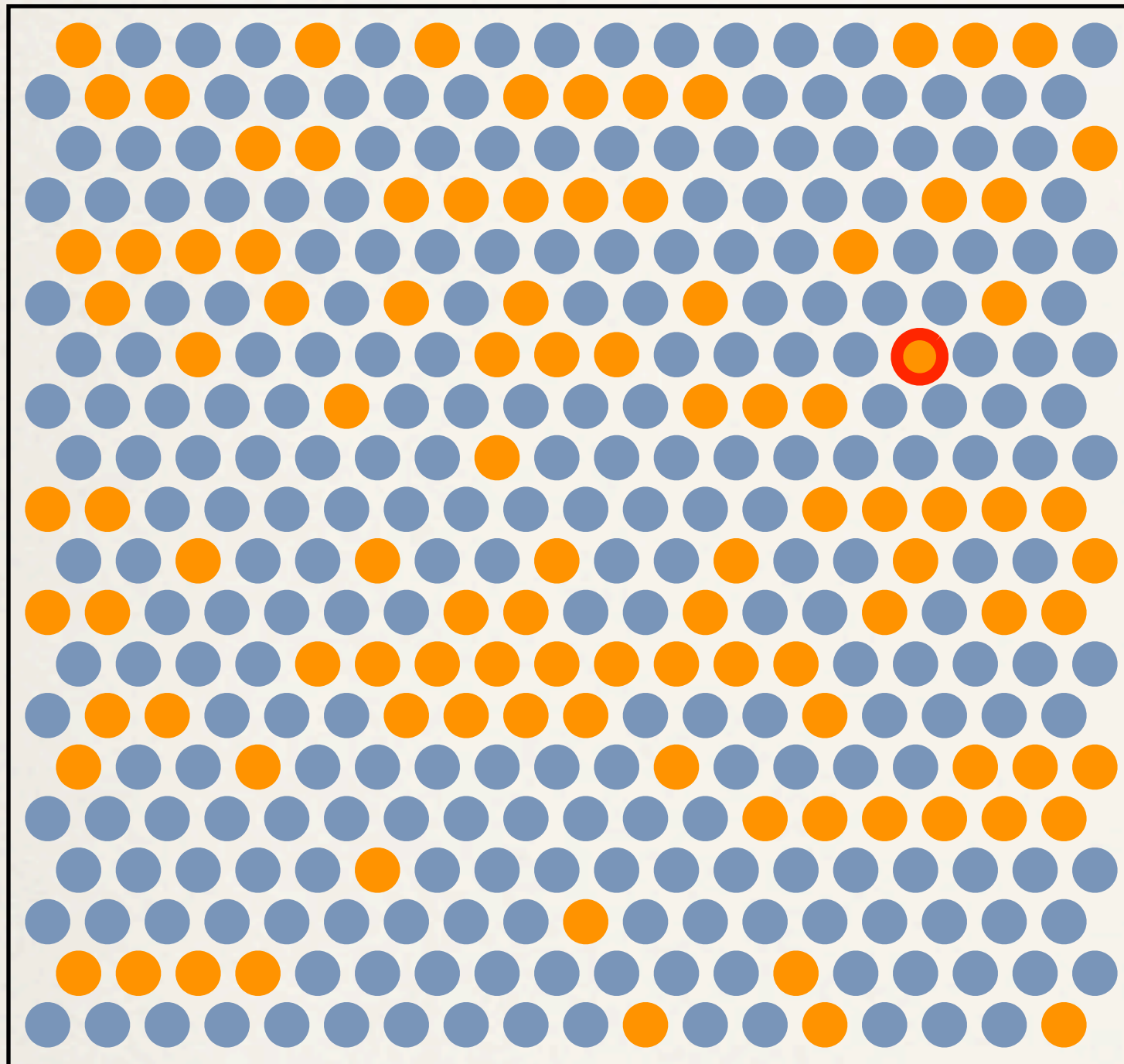
Sampling from a dichotomous population



What fraction p of the balls in the urn is orange?

A random selection: X

Sampling from a dichotomous population

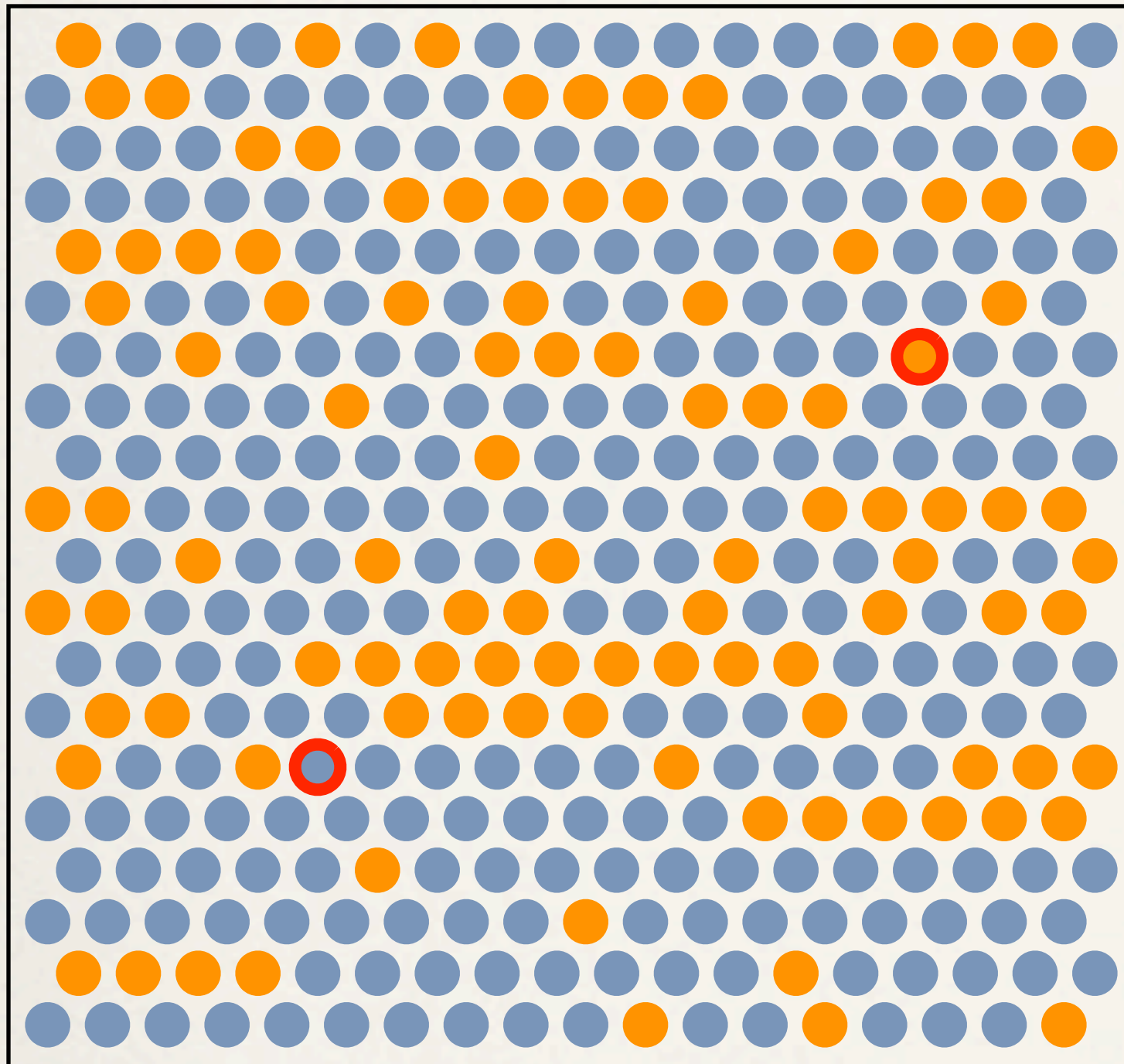


What fraction p of the balls in the urn is orange?

A random selection: X

● with probability $p = 101/360 = 0.28$

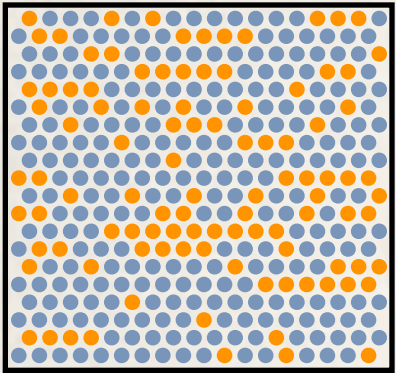
Sampling from a dichotomous population



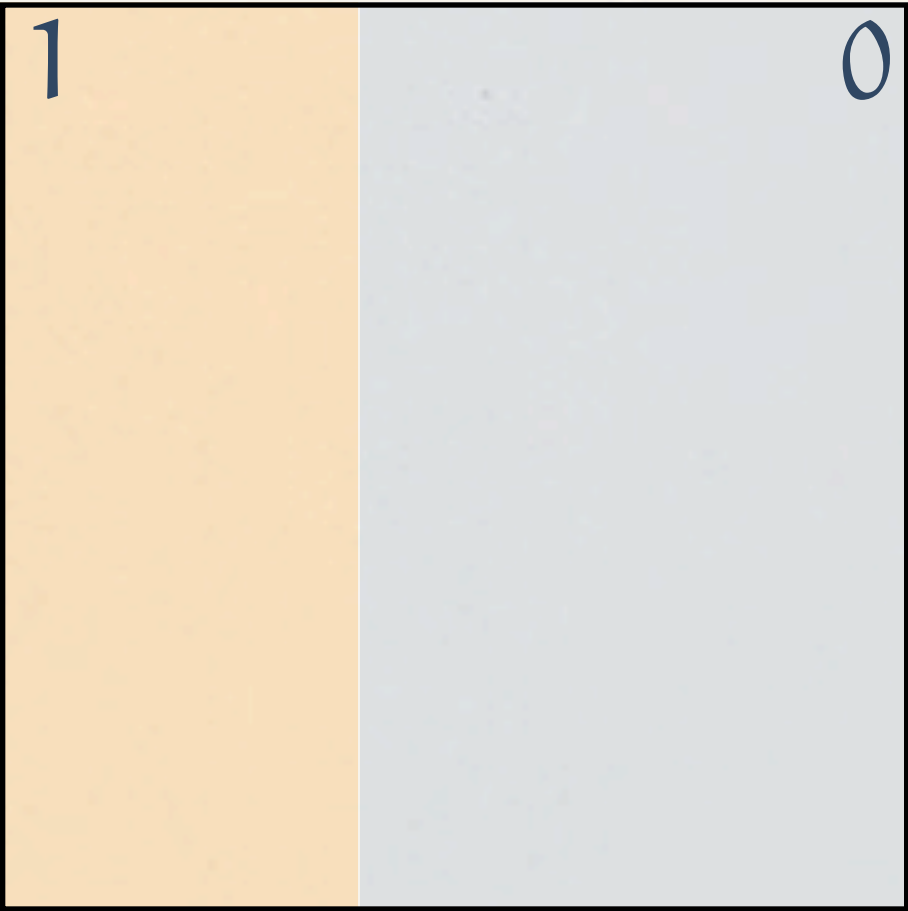
What fraction p of the balls in the urn is orange?

A random selection: X

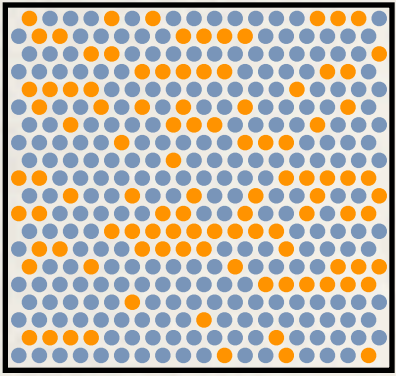
- with probability $p = 101/360 = 0.28$
- with probability $q = 259/360 = 0.72$



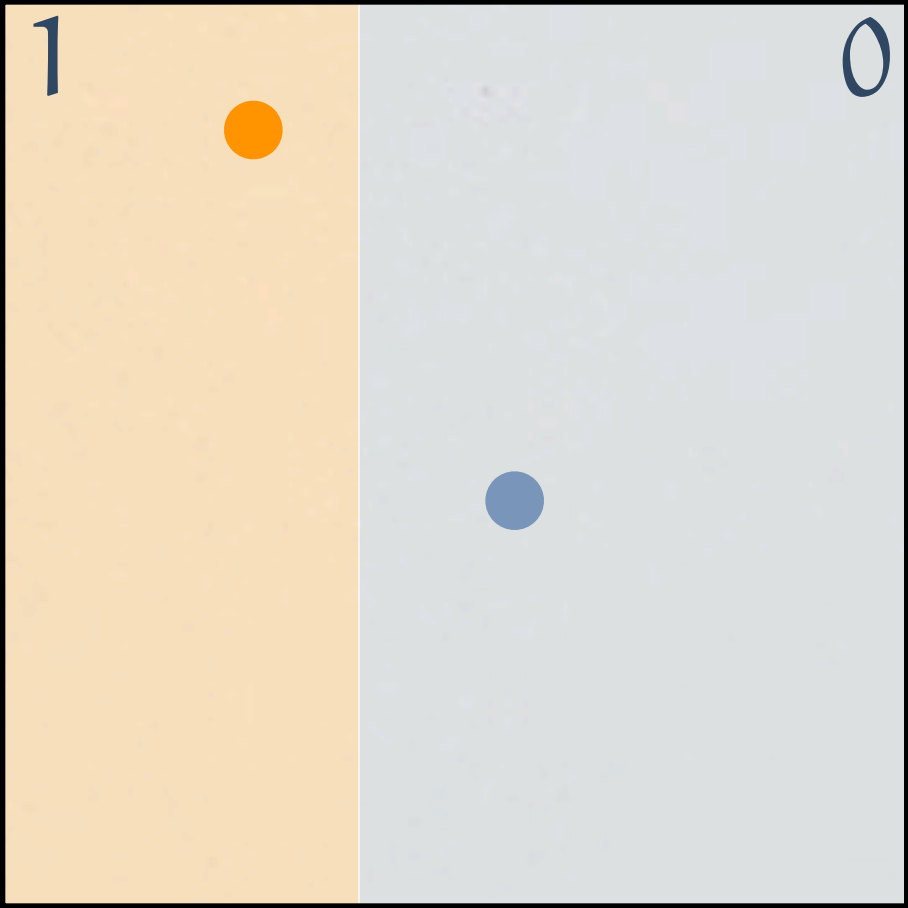
Fixed but unknown
proportion p



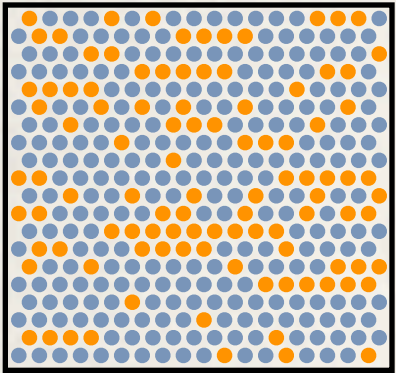
Fixed but unknown
proportion $q = 1 - p$



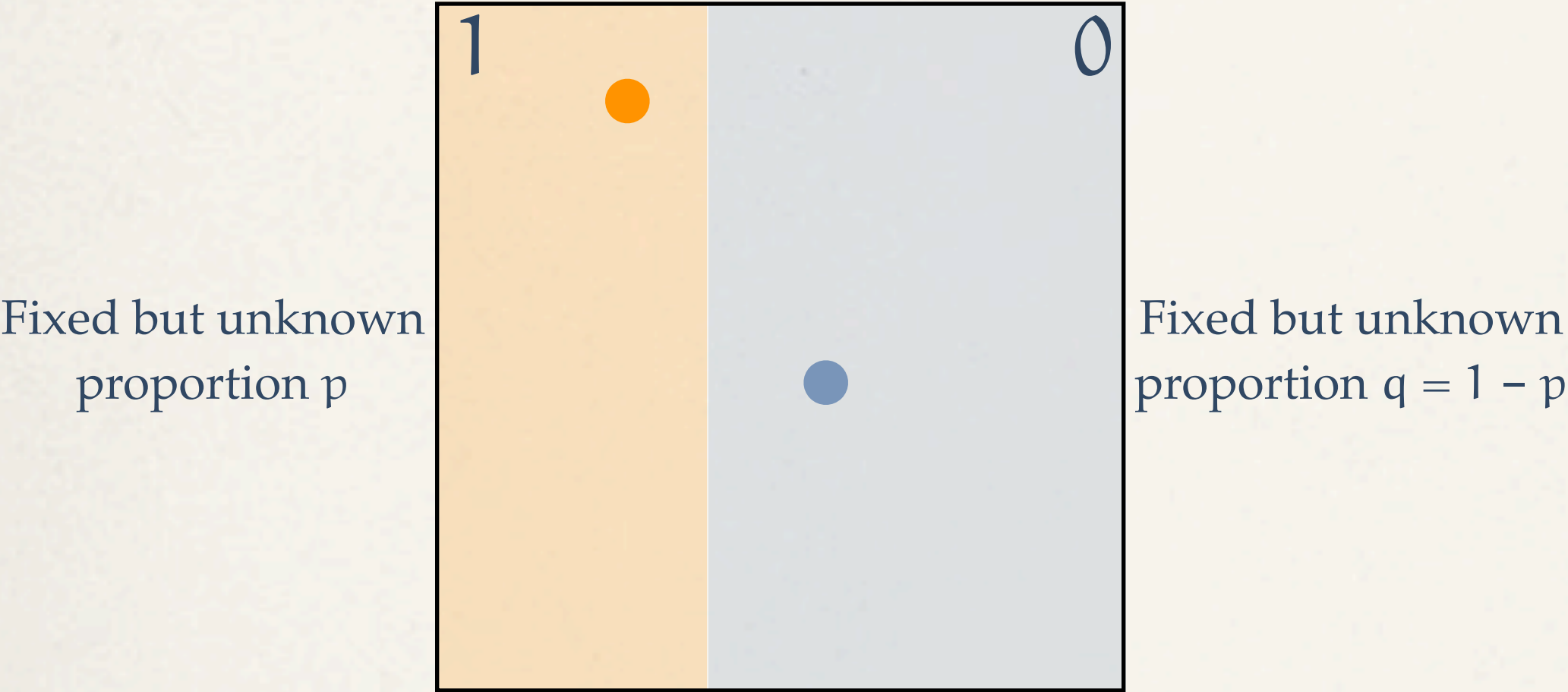
Fixed but unknown
proportion p

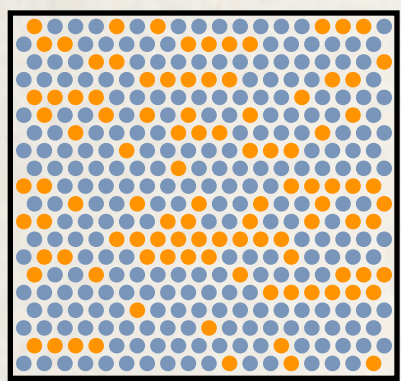


Fixed but unknown
proportion $q = 1 - p$

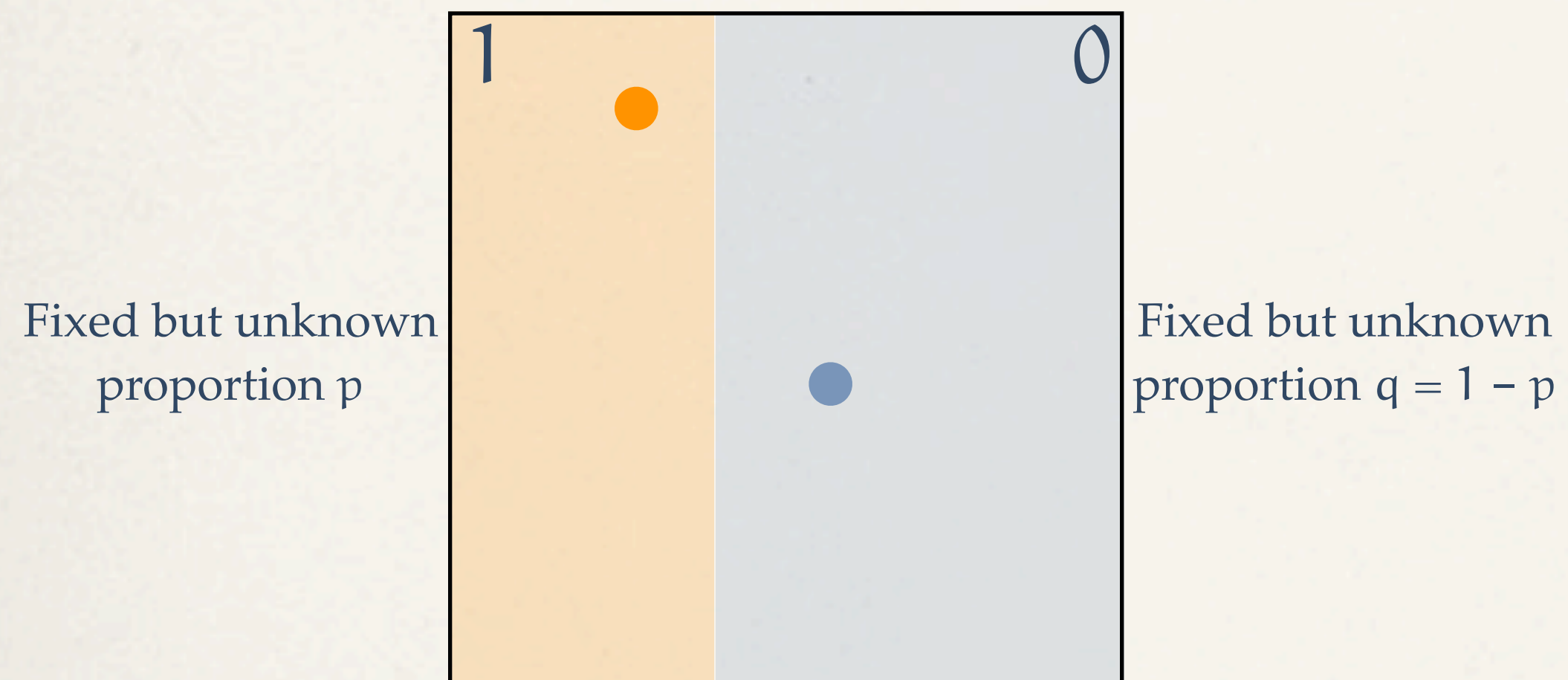


The toss of a coin





The toss of a coin



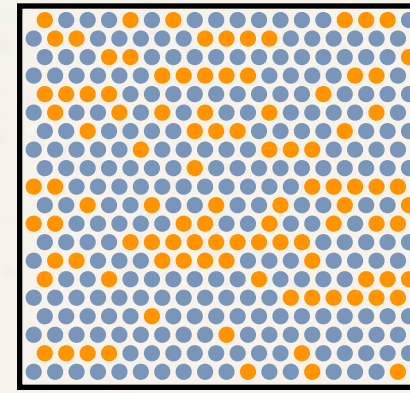
Bernoulli trial with success probability p

$$X \sim \text{Bernoulli}(p)$$

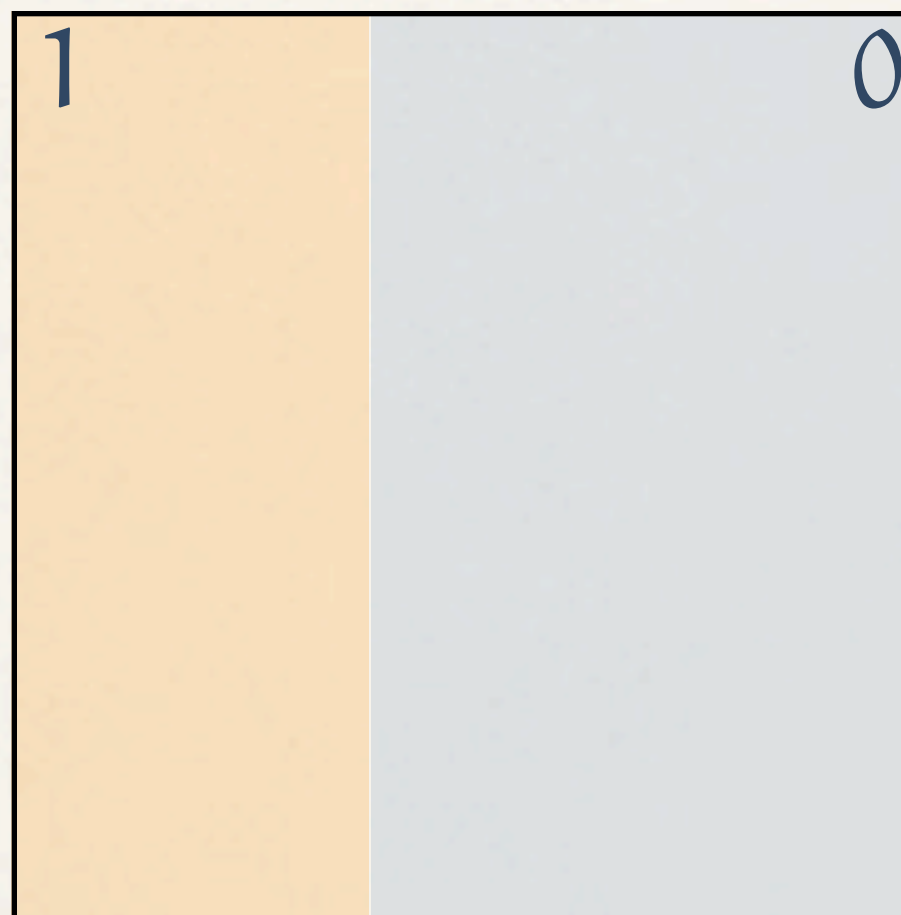
$$X = \begin{cases} 1 & \text{with probability } p, \\ 0 & \text{with probability } q. \end{cases}$$



A model for a poll



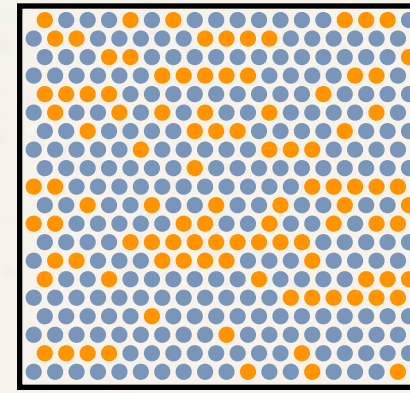
Random sample: repeated independent trials



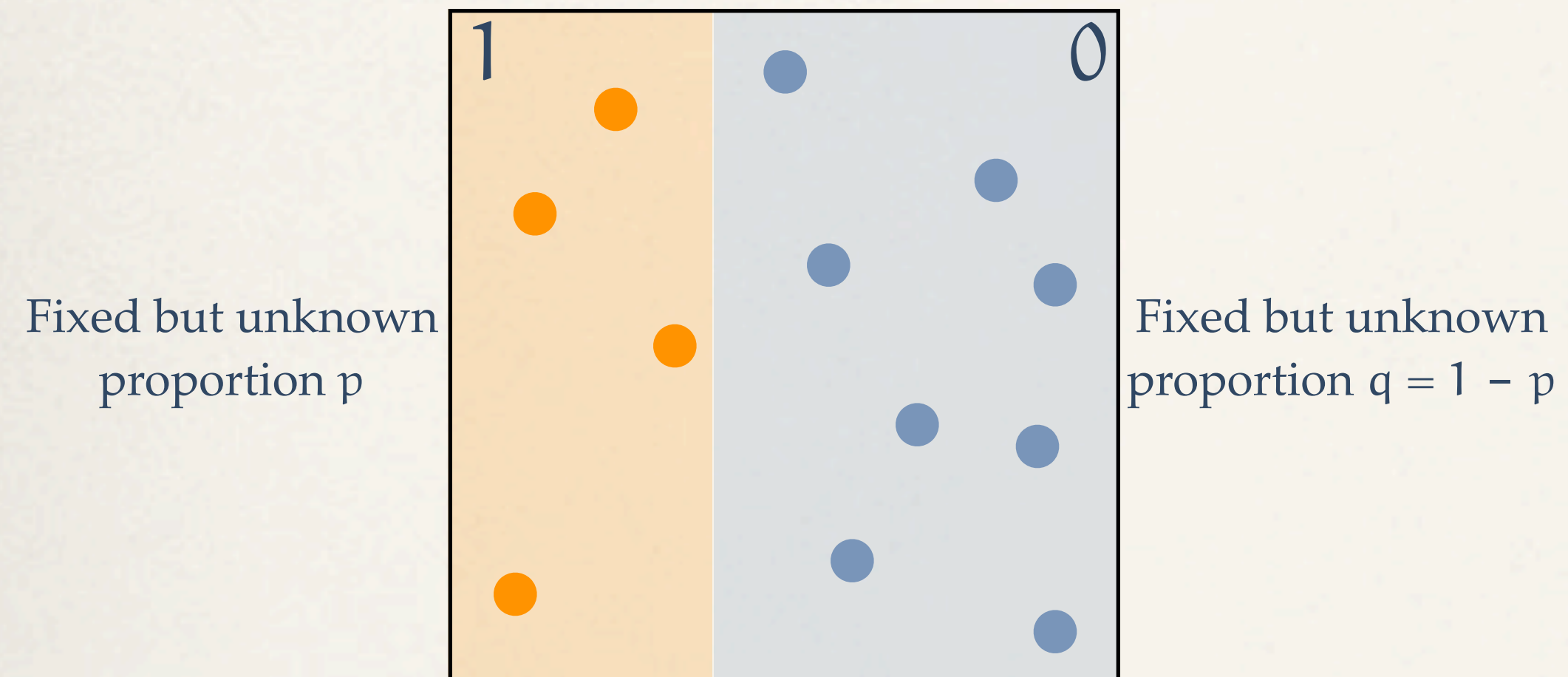
Fixed but unknown
proportion p

Fixed but unknown
proportion $q = 1 - p$

A model for a poll

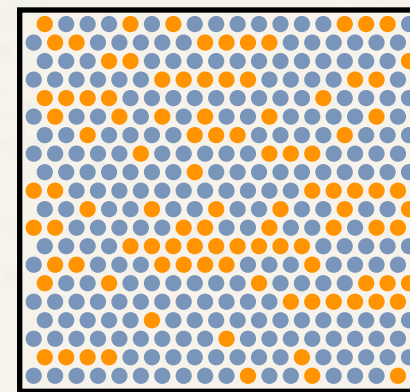


Random sample: repeated independent trials

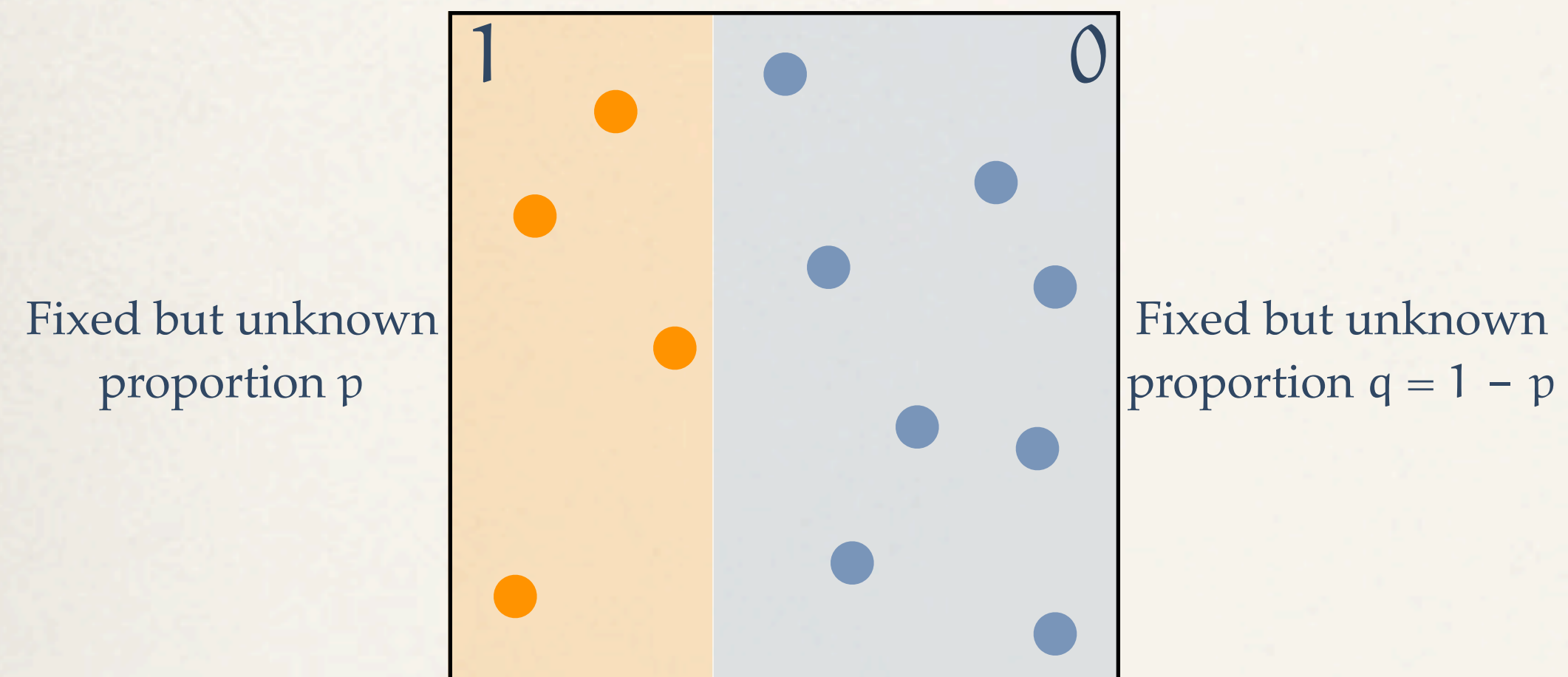


X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	X_{10}	X_{11}	X_{12}
0	1	1	0	0	0	0	1	0	0	1	0

A model for a poll



Random sample: repeated independent trials

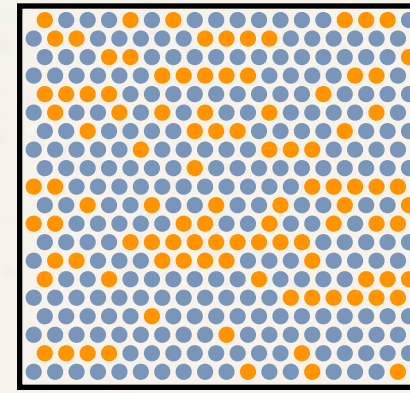


X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	X_{10}	X_{11}	X_{12}
0	1	1	0	0	0	0	1	0	0	1	0

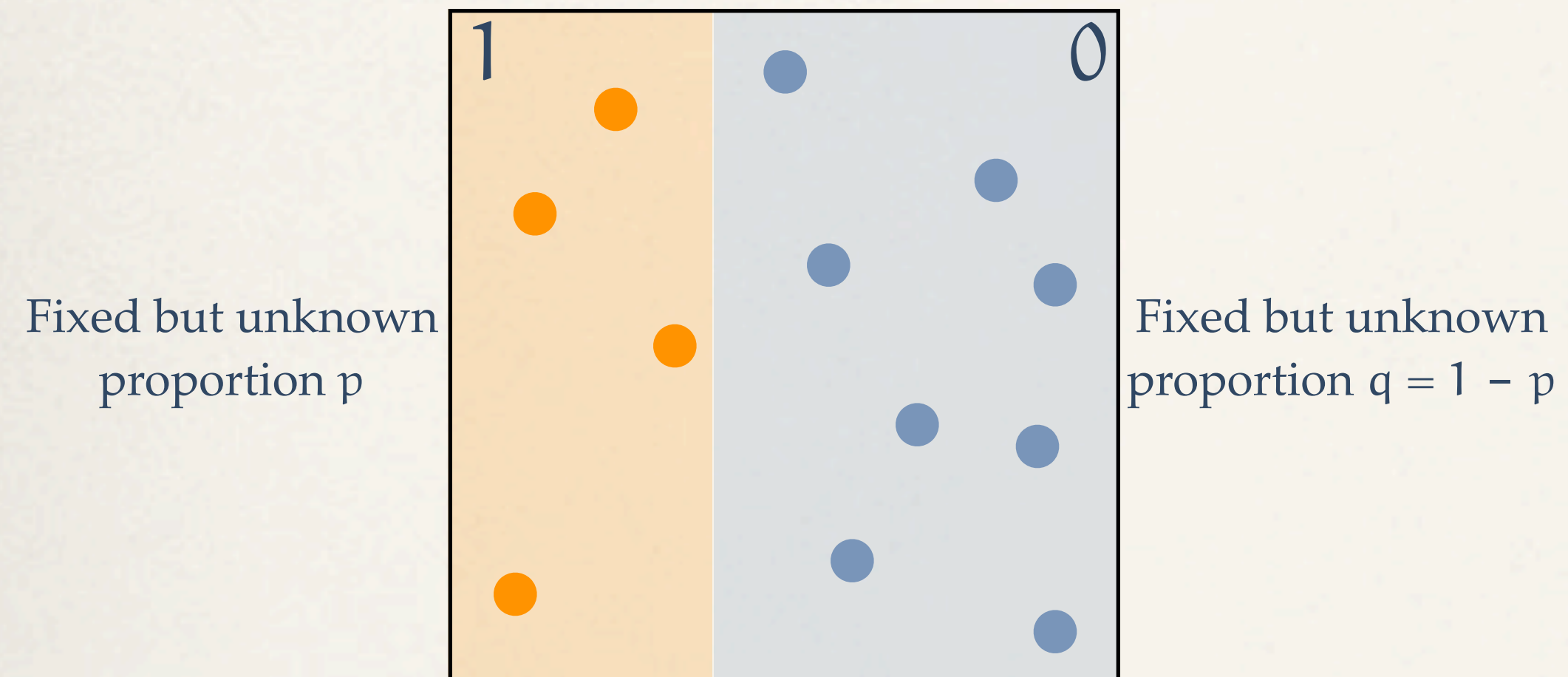
type 1 in sample: 4

frequency of type 1 in sample: $4/12 = 1/3$

A model for a poll



Random sample: repeated independent trials



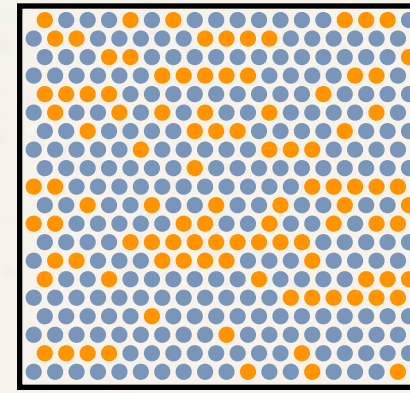
X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	X_{10}	X_{11}	X_{12}
0	1	1	0	0	0	0	1	0	0	1	0

type 1 in sample: 4

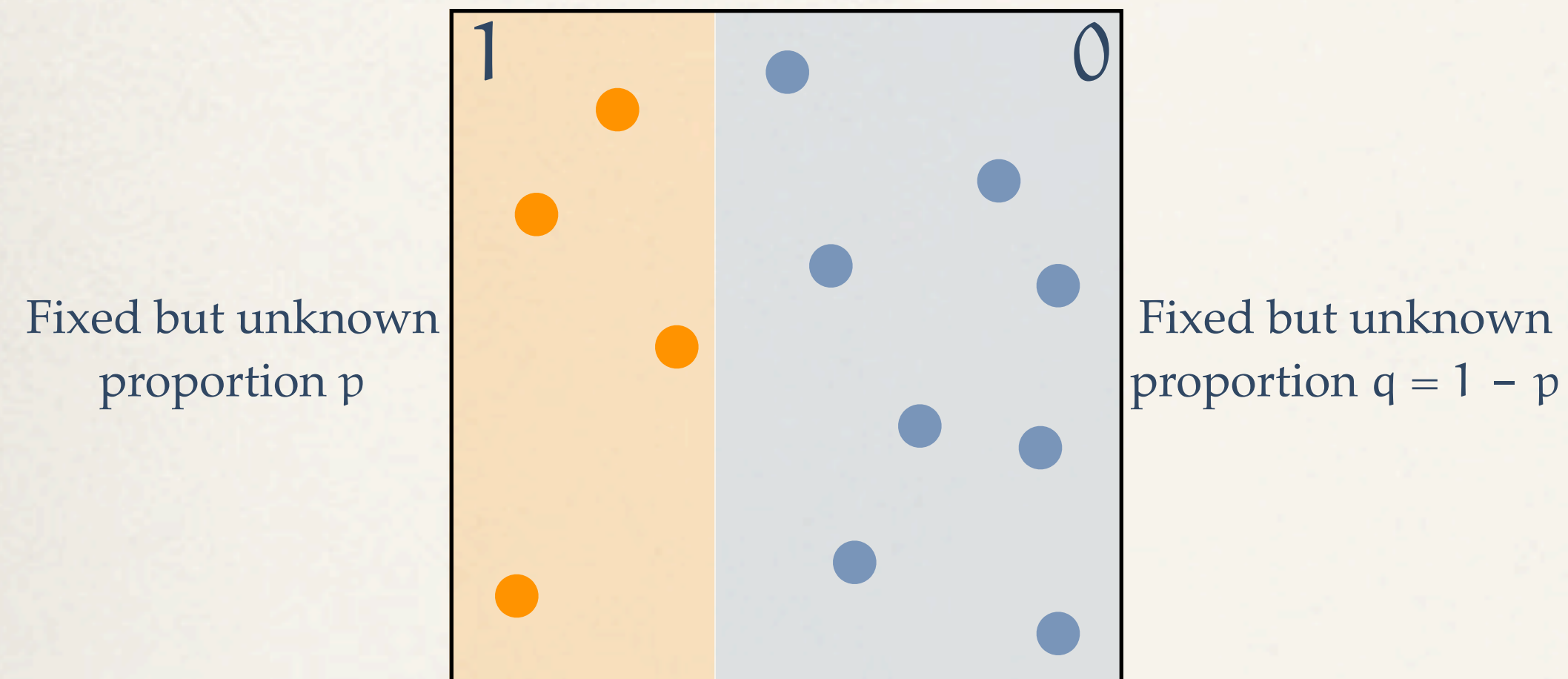
frequency of type 1 in sample: $4/12 = 1/3$

Are the subpopulation proportions in the sample representative of the proportions in the entire population?

A model for a poll



Random sample: repeated independent trials



X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	X_{10}	X_{11}	X_{12}
0	1	1	0	0	0	0	1	0	0	1	0

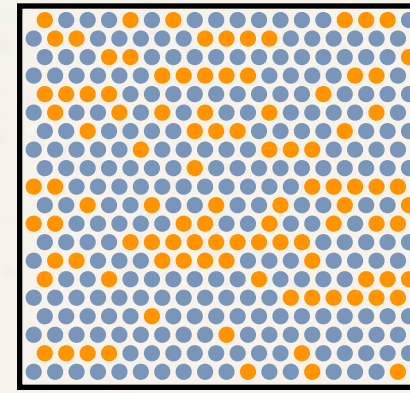
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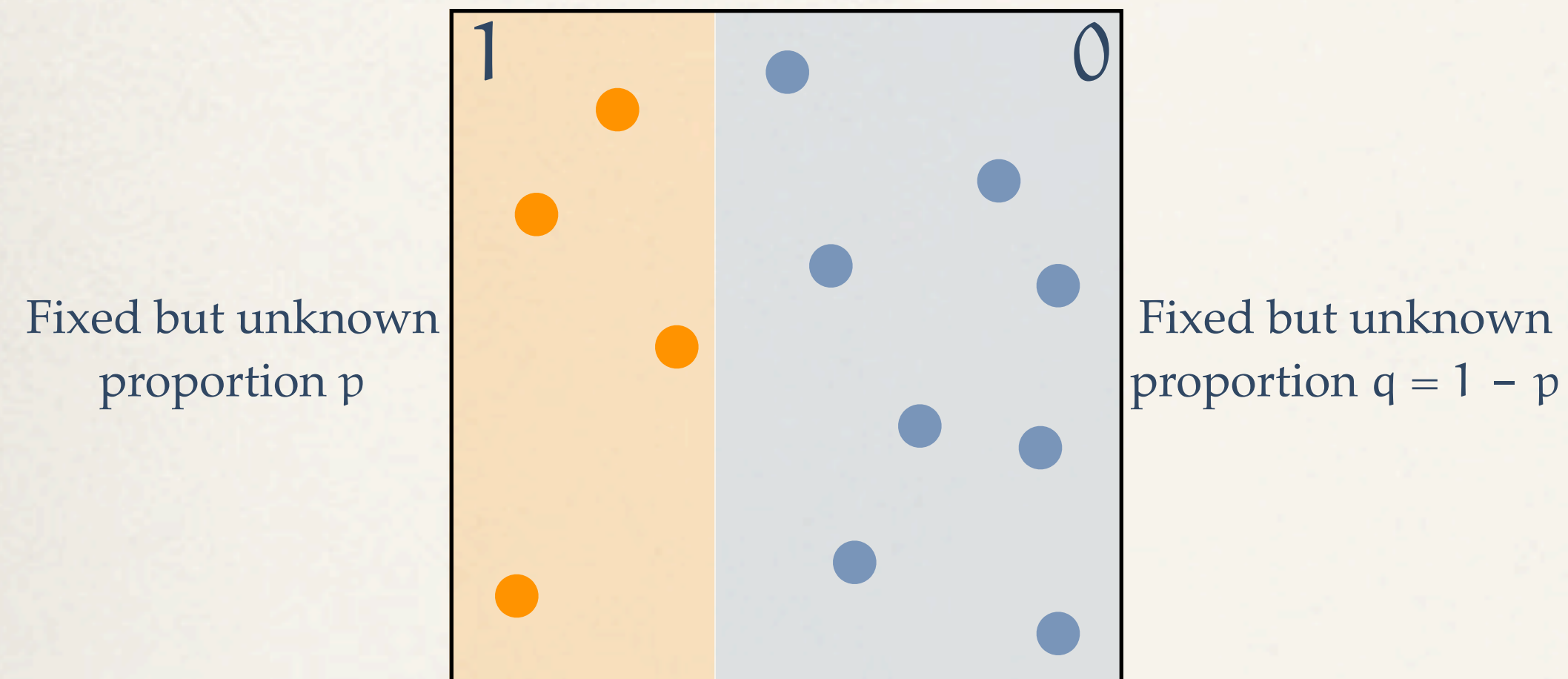
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Bernoulli(p) trials: $X_1, X_2, \dots, X_n = \begin{cases} 1 & \text{with probability } p, \\ 0 & \text{with probability } q. \end{cases}$

A model for a poll



Random sample: repeated independent trials



X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	X_{10}	X_{11}	X_{12}
0	1	1	0	0	0	0	1	0	0	1	0

type 1 in sample: 4

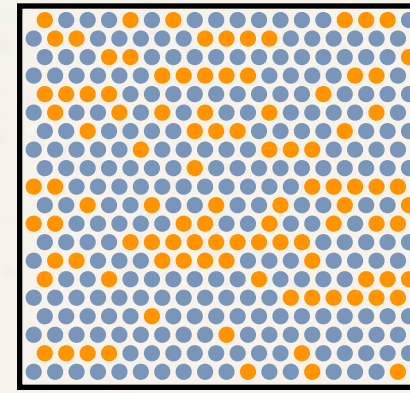
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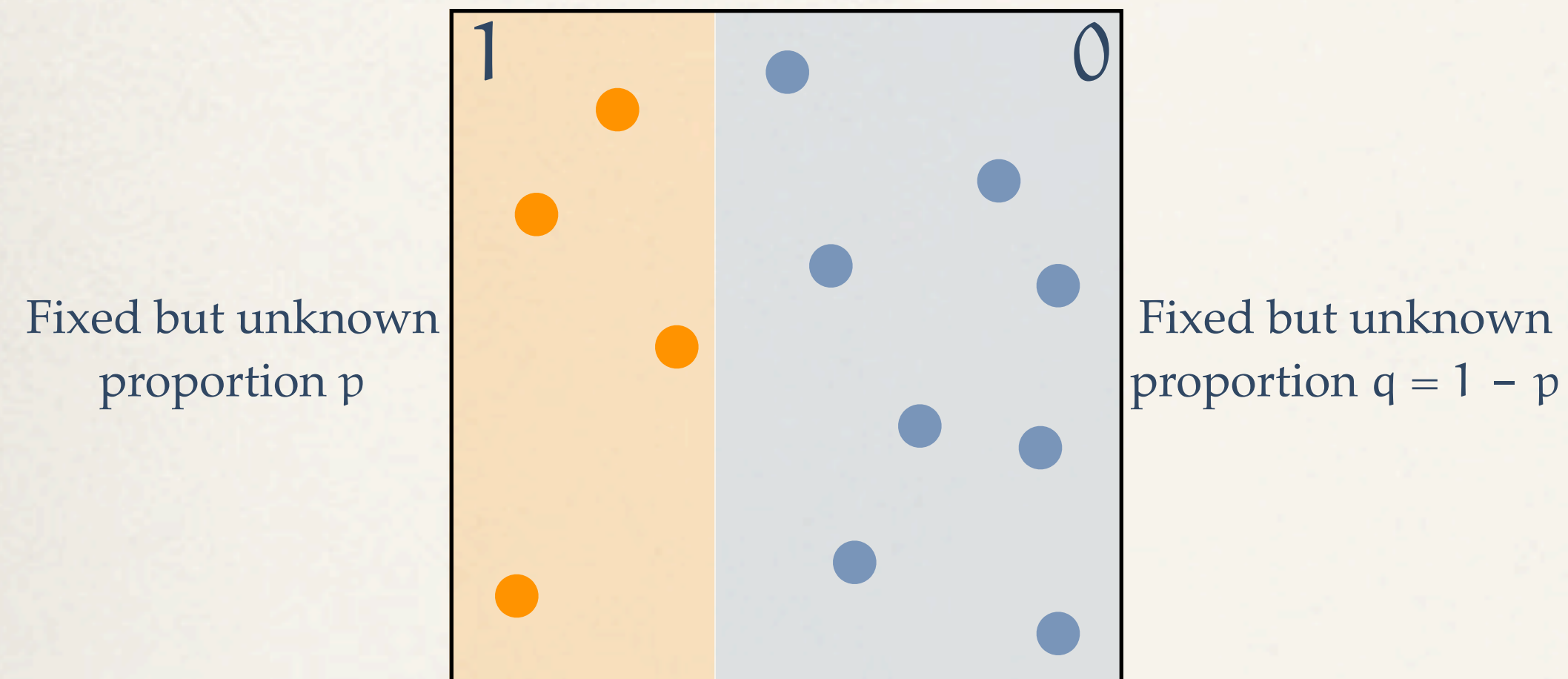
Bernoulli(p) trials: $X_1, X_2, \dots, X_n = \begin{cases} 1 & \text{with probability } p, \\ 0 & \text{with probability } q. \end{cases}$

Accumulated successes: $S_n = X_1 + X_2 + \dots + X_n$

A model for a poll



Random sample: repeated independent trials



X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	X_{10}	X_{11}	X_{12}
0	1	1	0	0	0	0	1	0	0	1	0

type 1 in sample: 4

frequency of type 1 in sample: $4/12 = 1/3$

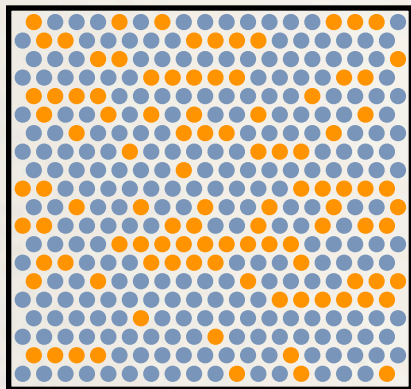
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Bernoulli(p) trials: $X_1, X_2, \dots, X_n = \begin{cases} 1 & \text{with probability } p, \\ 0 & \text{with probability } q. \end{cases}$

Accumulated successes: $S_n = X_1 + X_2 + \dots + X_n$

Is $\frac{S_n}{n}$ a good approximation to p ?

Enter the binomial



Bernoulli(p) trials: $X_1, X_2, \dots, X_n = \begin{cases} 1 & \text{with probability } p, \\ 0 & \text{with probability } q. \end{cases}$

Accumulated successes: $S_n = X_1 + X_2 + \dots + X_n$

Is $\frac{S_n}{n}$ a good approximation to p ?