Probability densities in a continuum sample space

A generic probability density function

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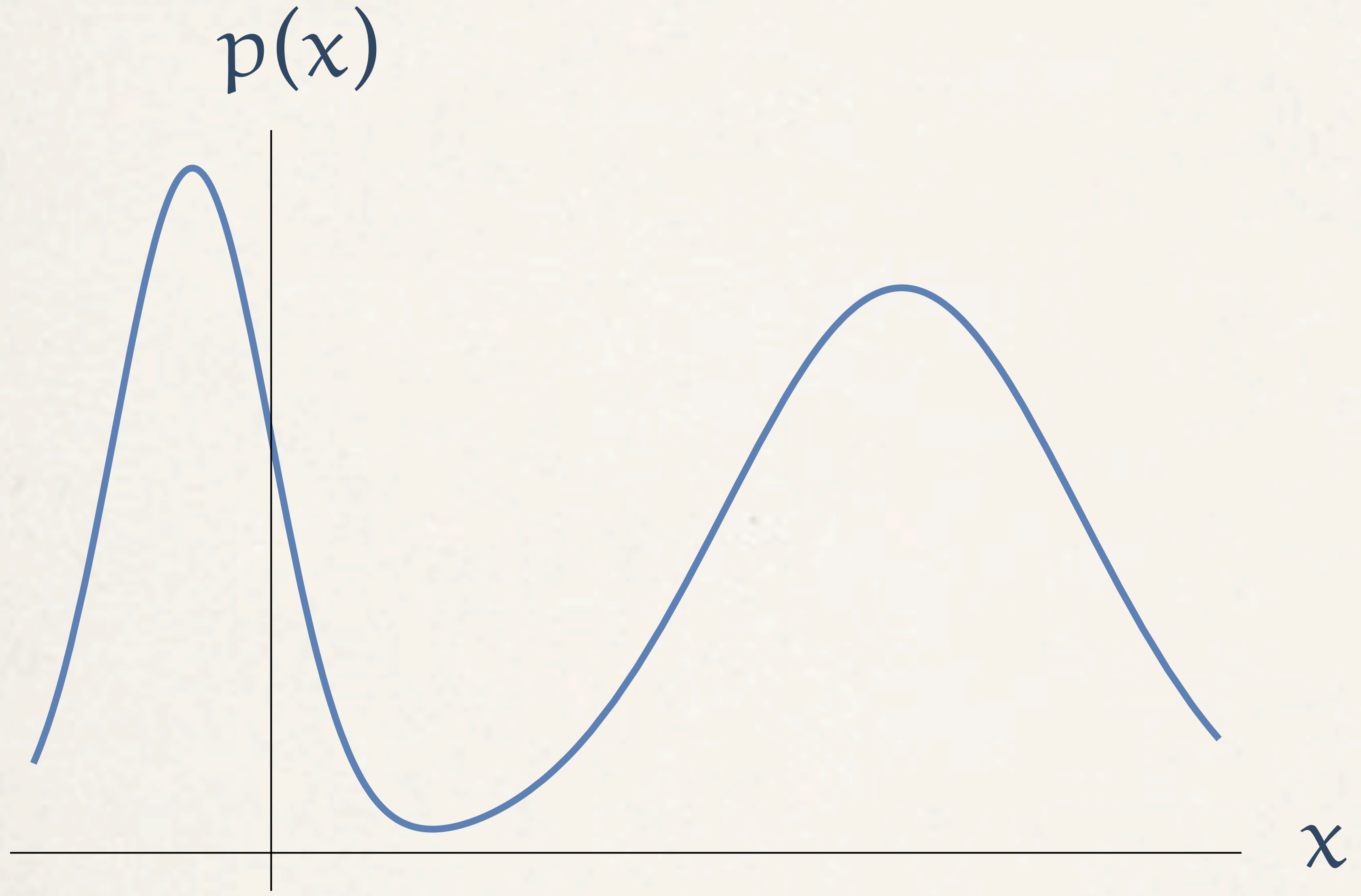

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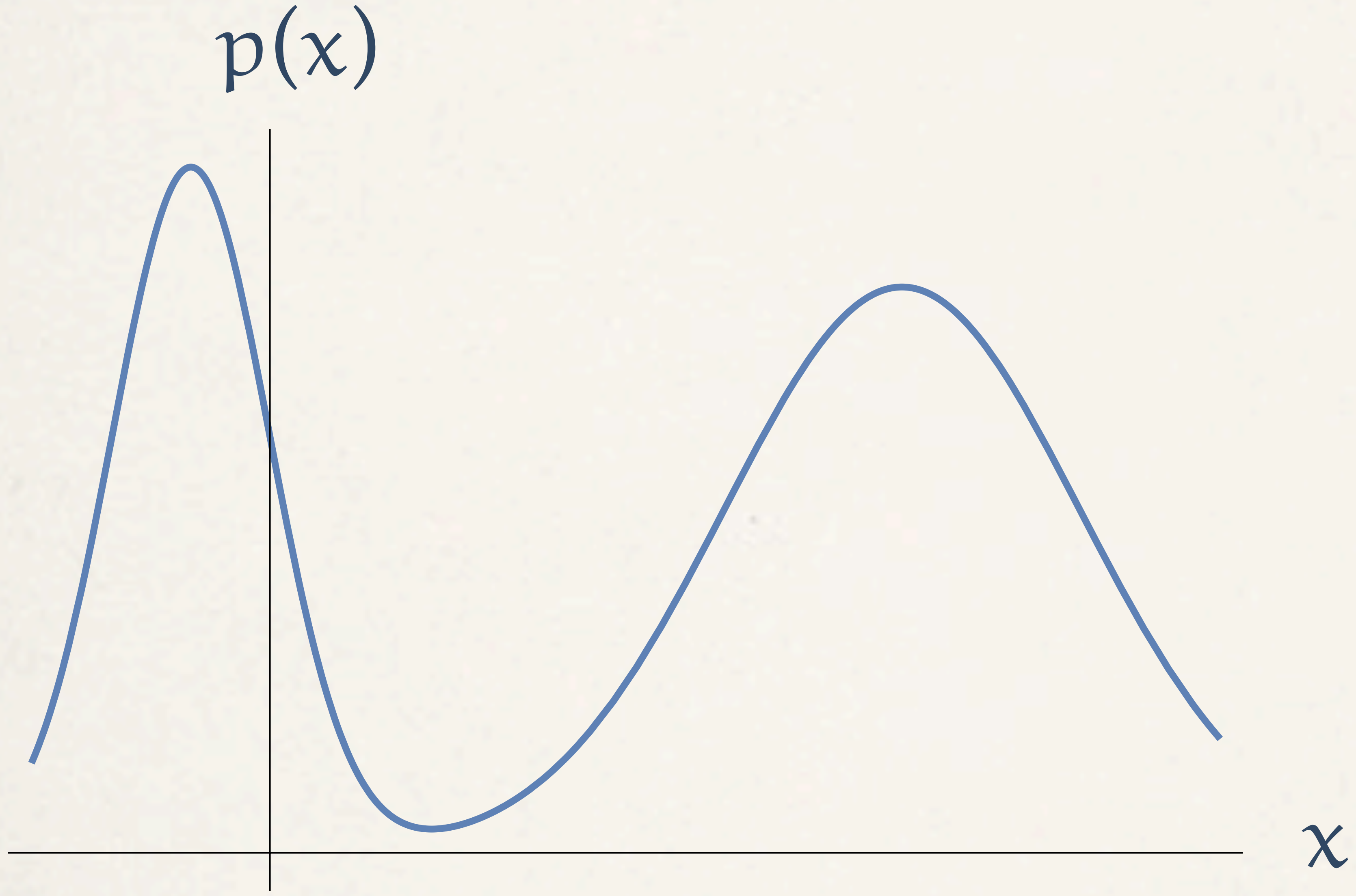

positivity: $p(x) \geq 0$

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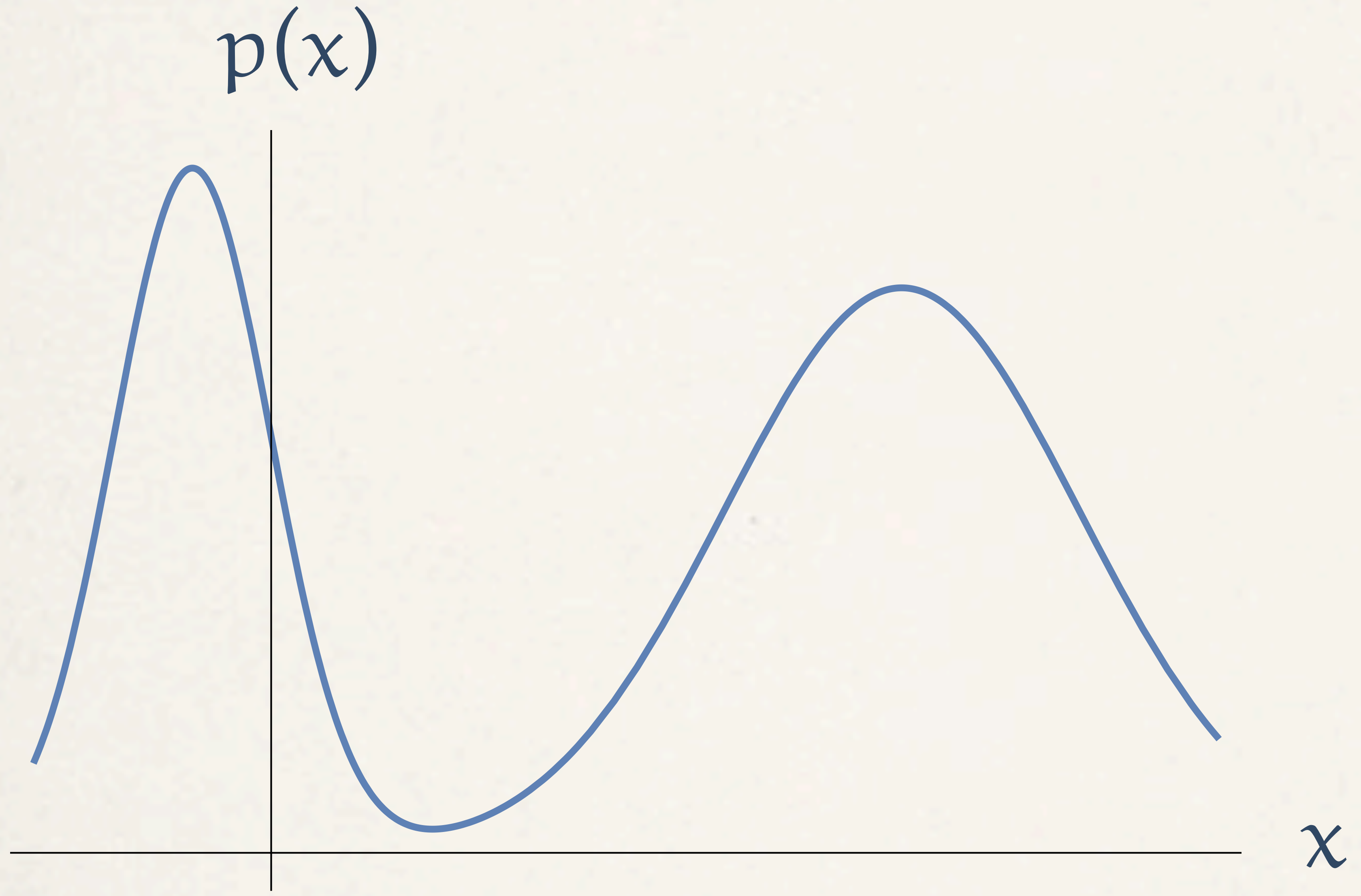

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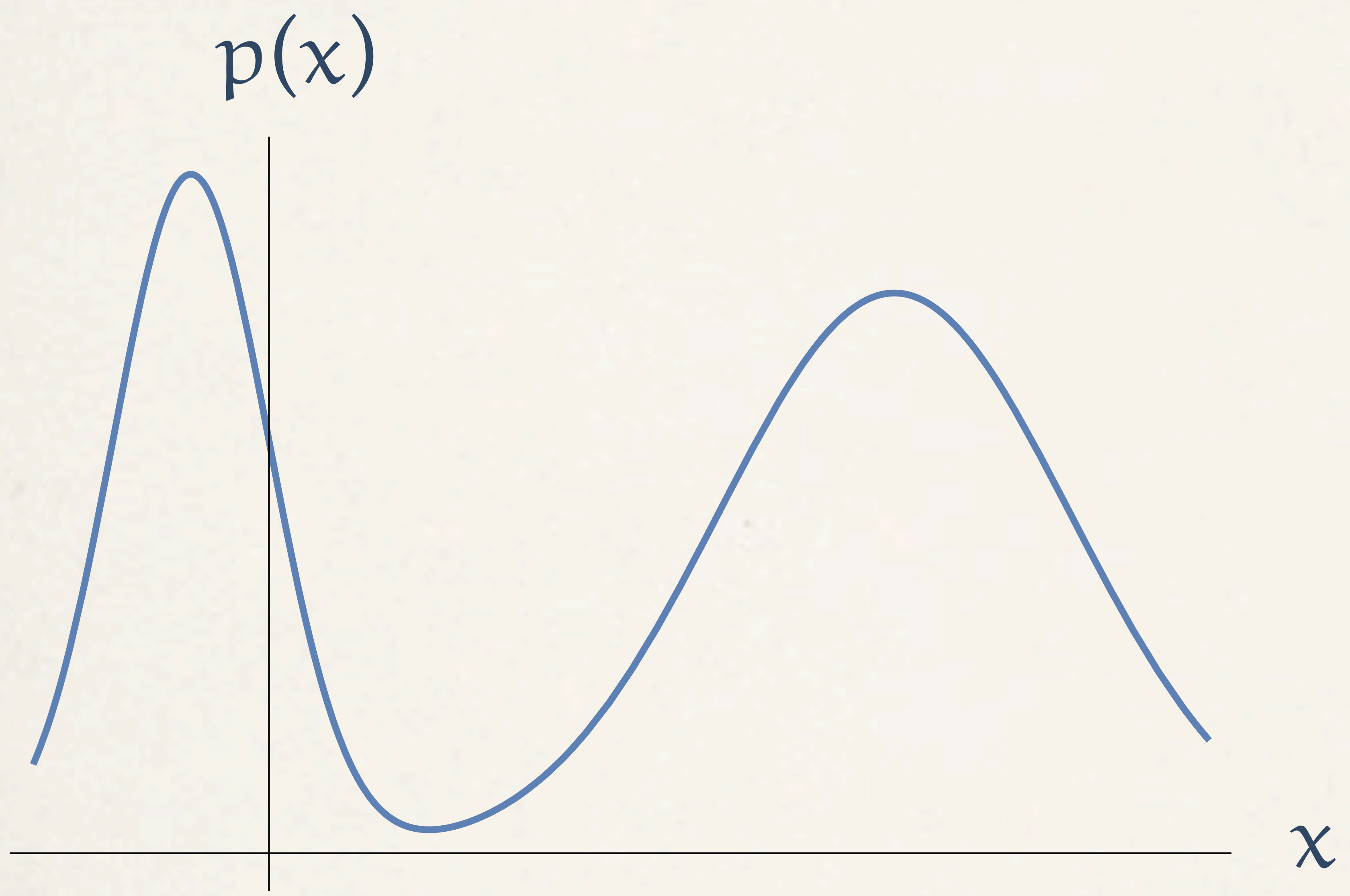

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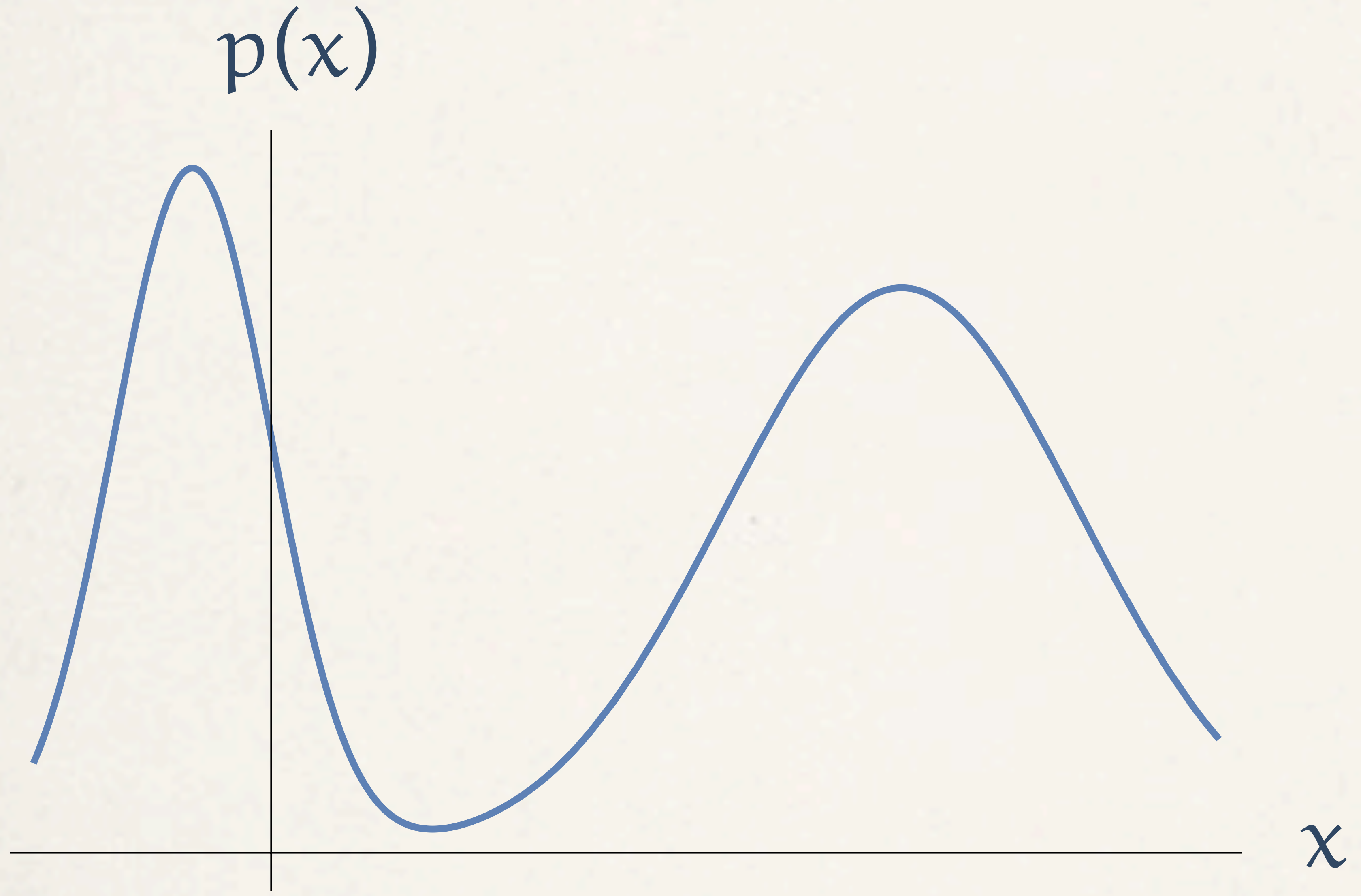

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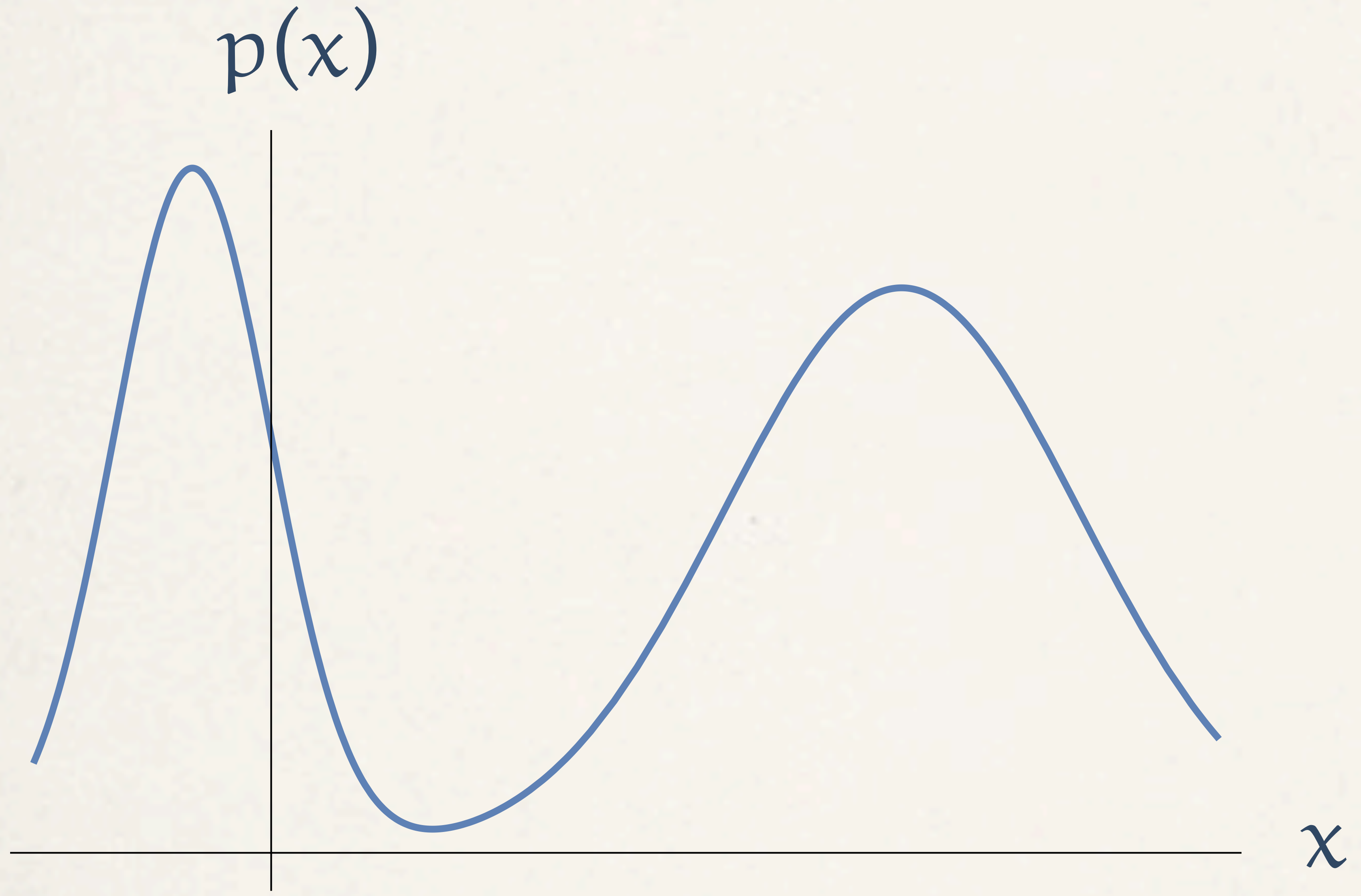

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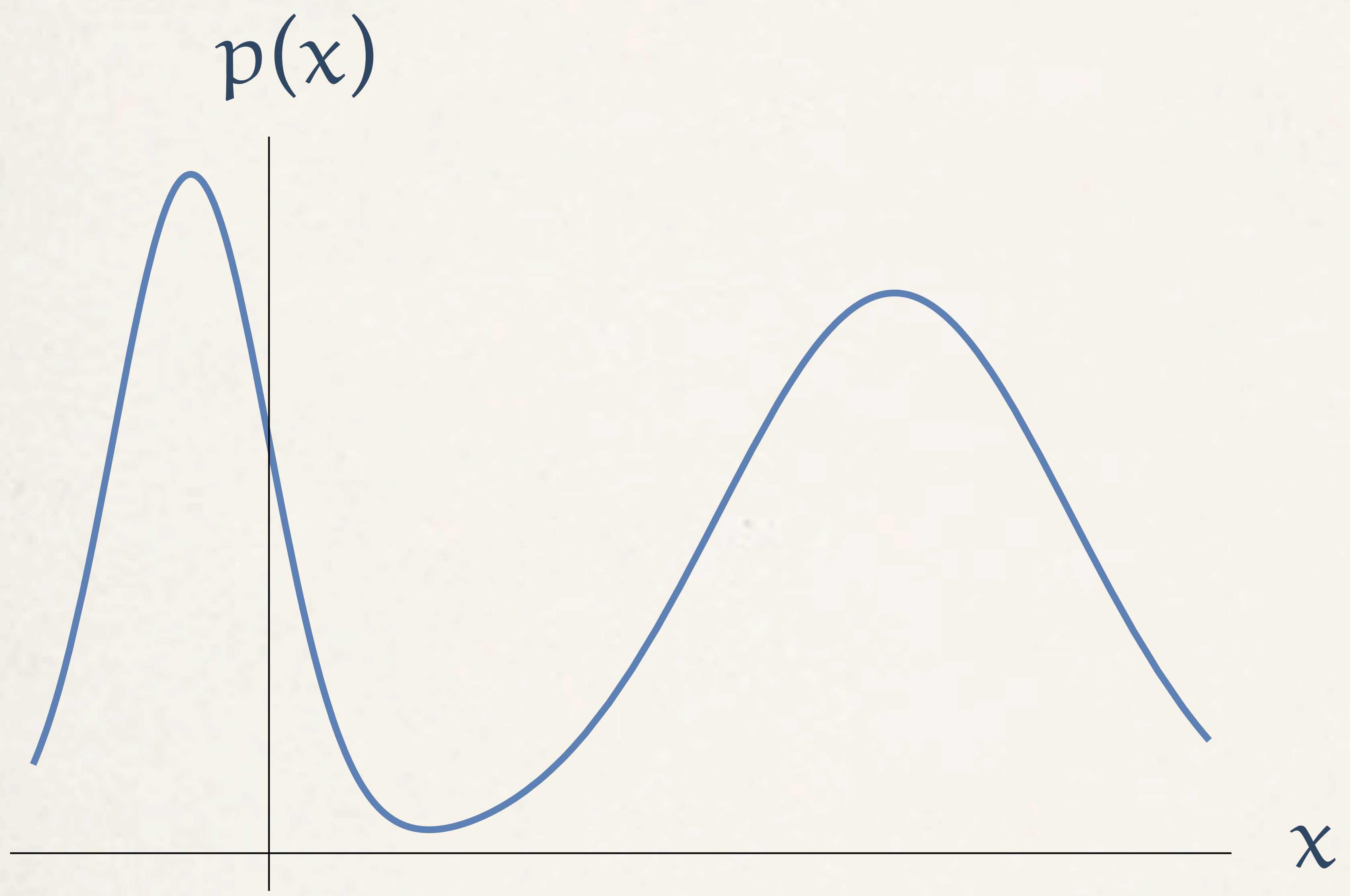

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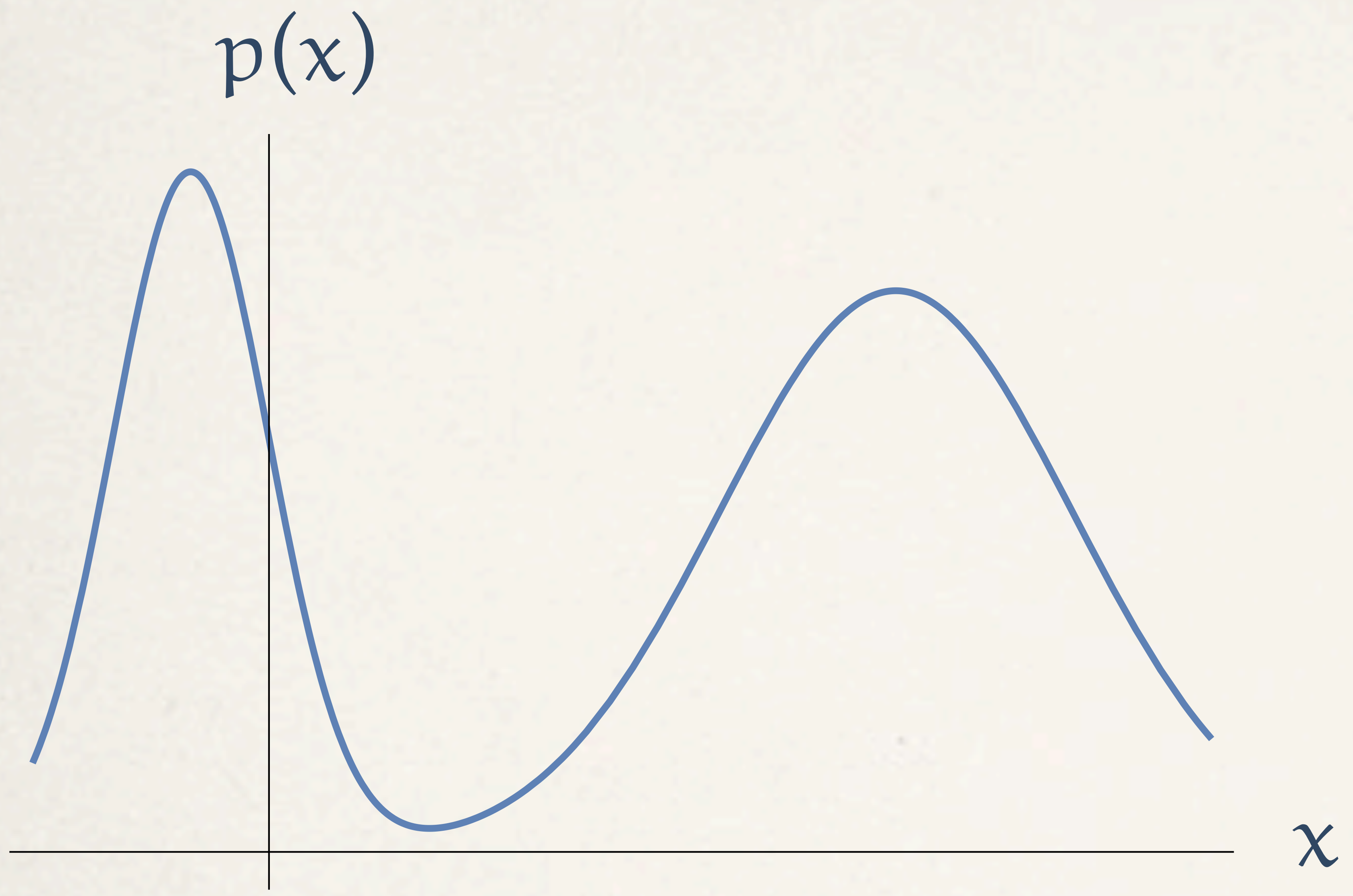
a density is any non-negative function with unit area under the curve



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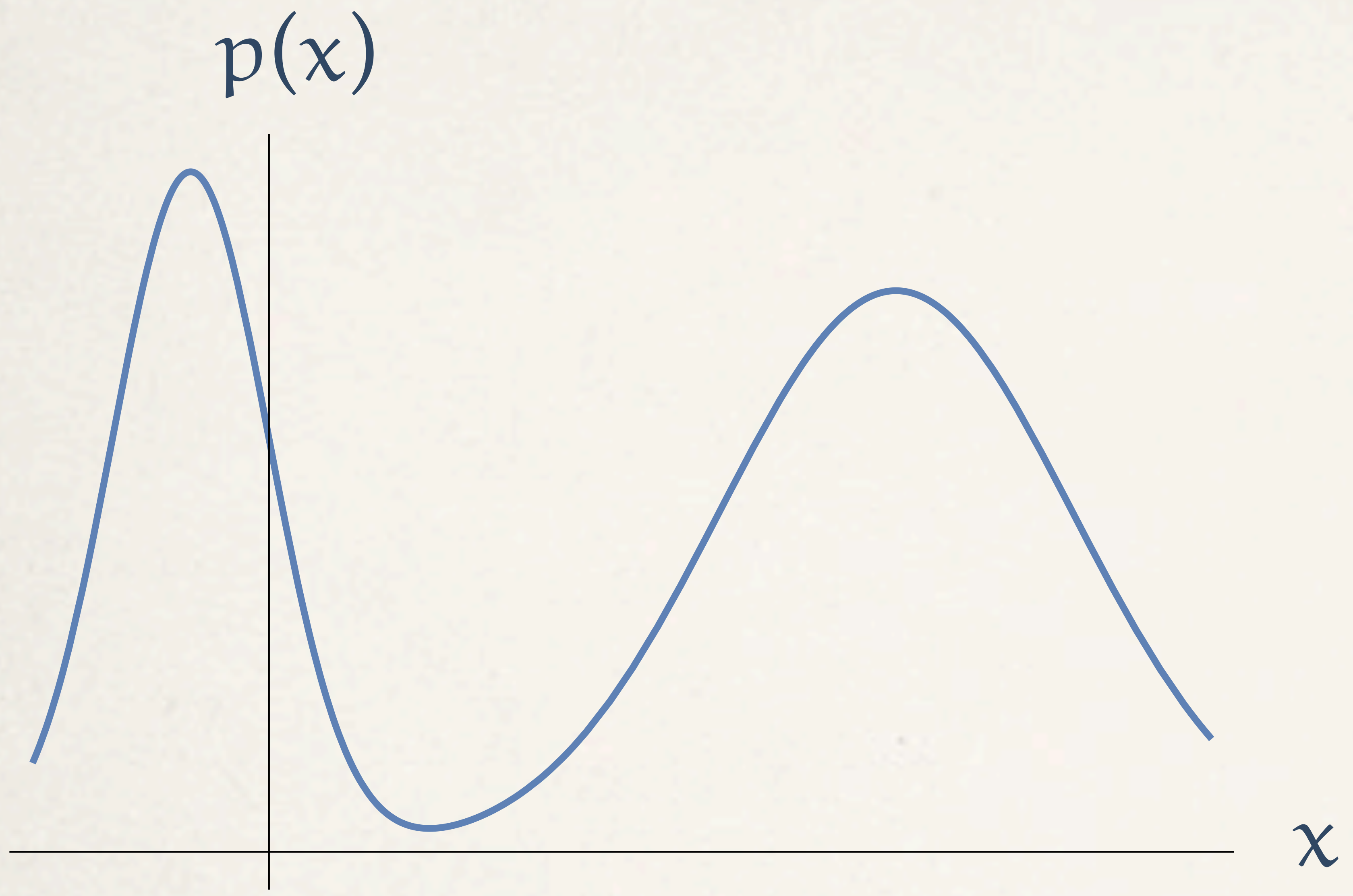

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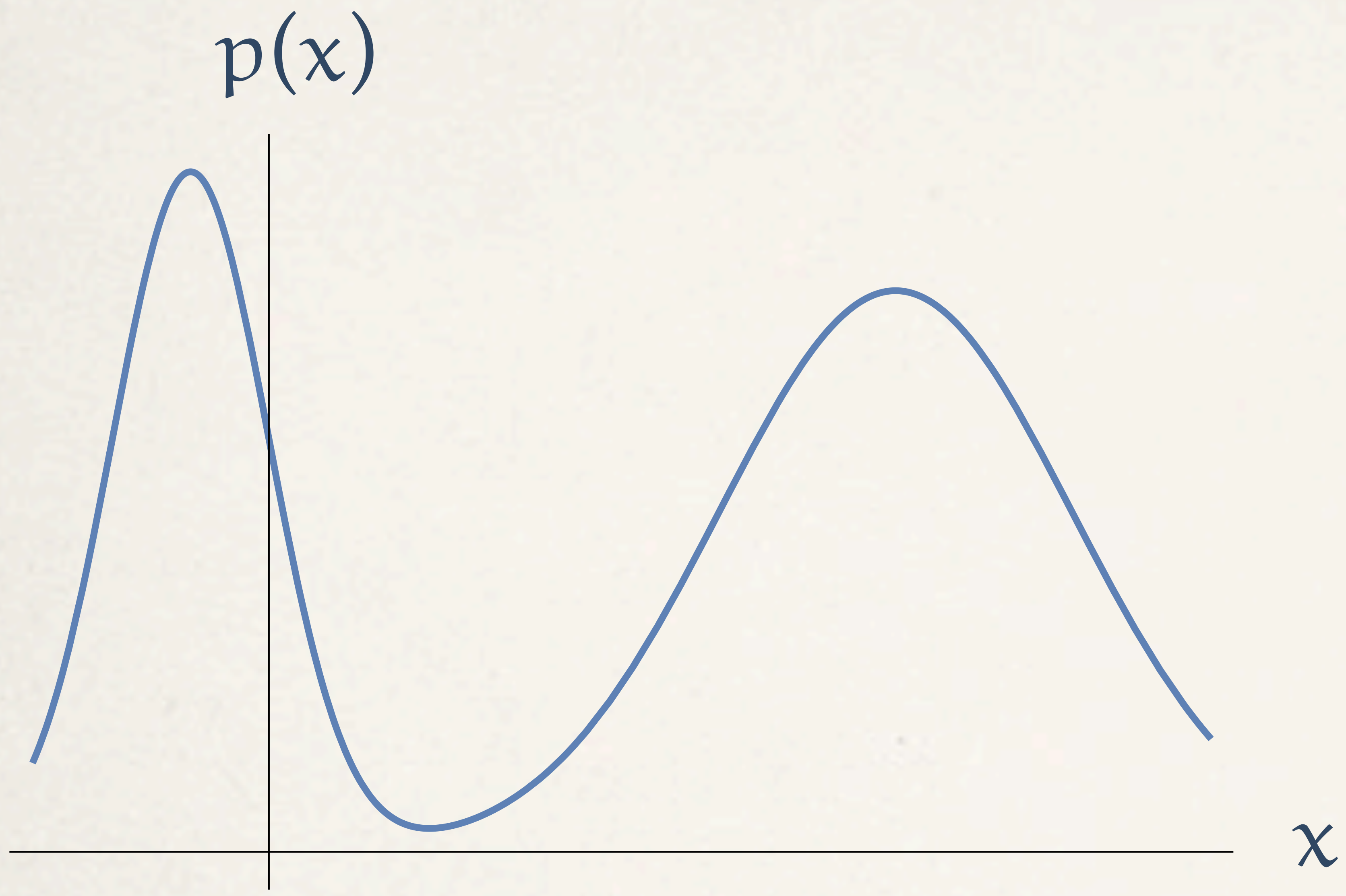
The continuum sample space: $\Omega = \mathbb{R} = (-\infty, \infty)$



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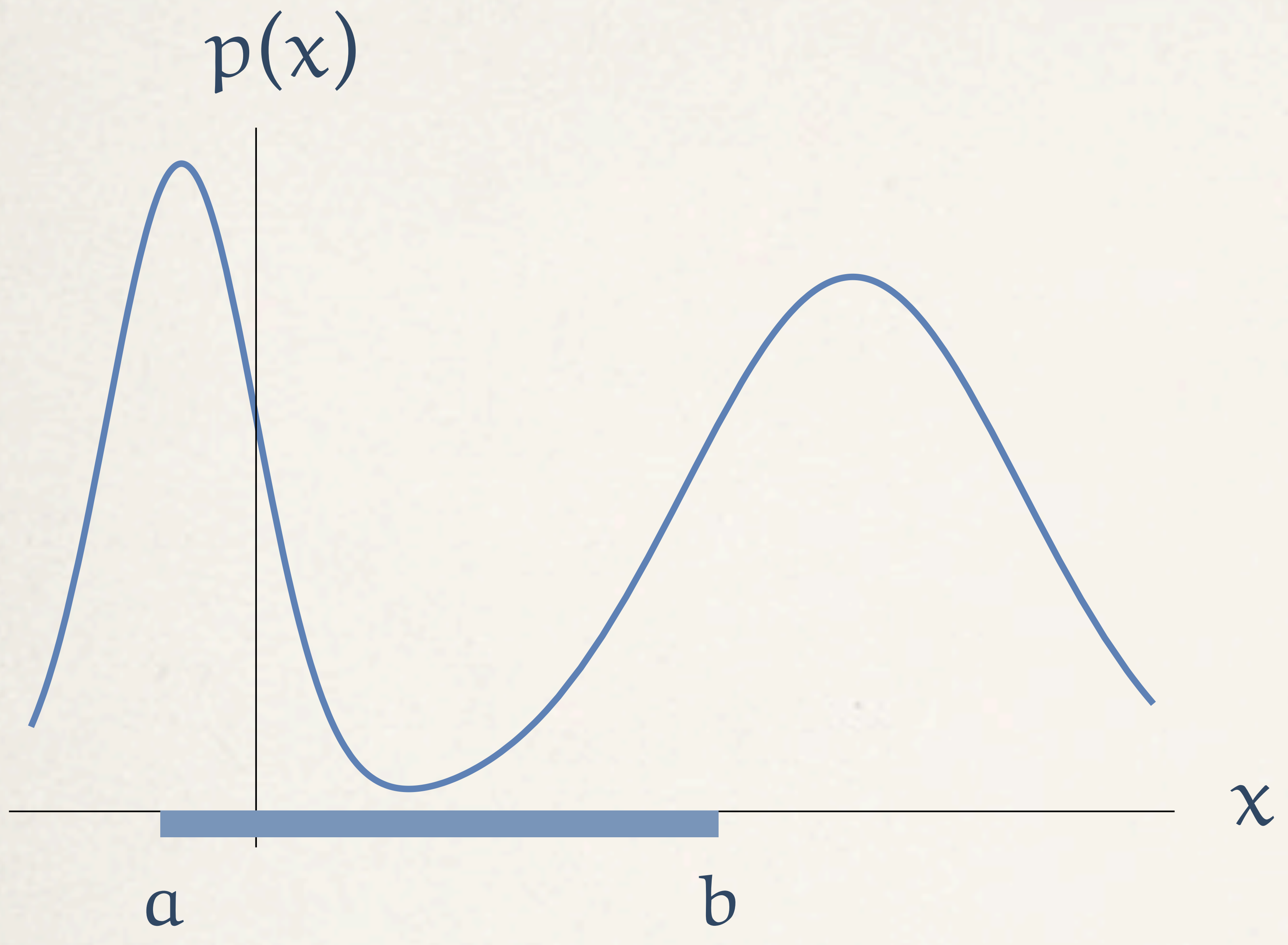
Convention: upper case symbols R, …, Z at the end of the alphabet represent the sample points; generic sample point (random variable) $\omega = X$.



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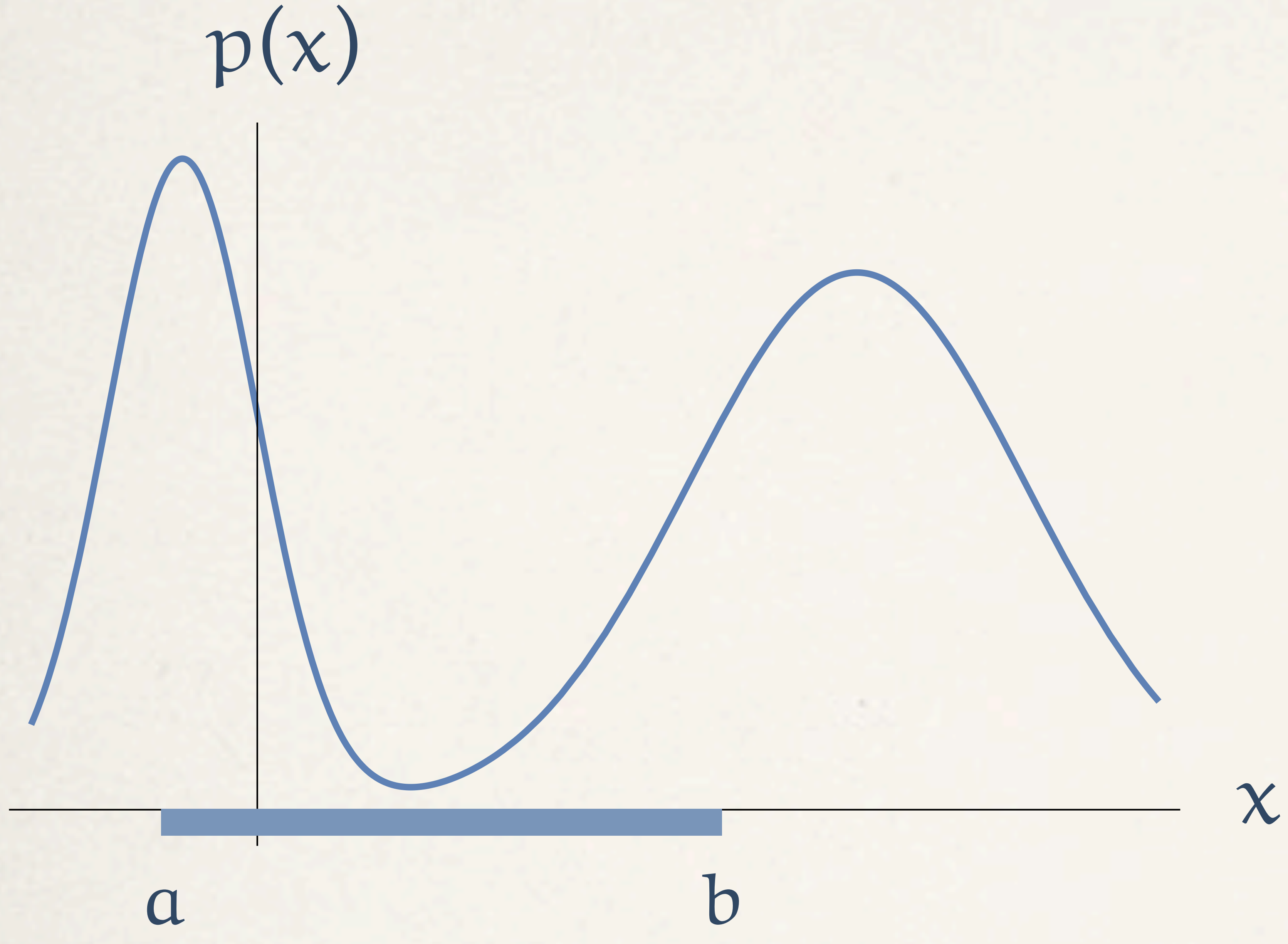
The basic events are intervals: $A = [a, b]$



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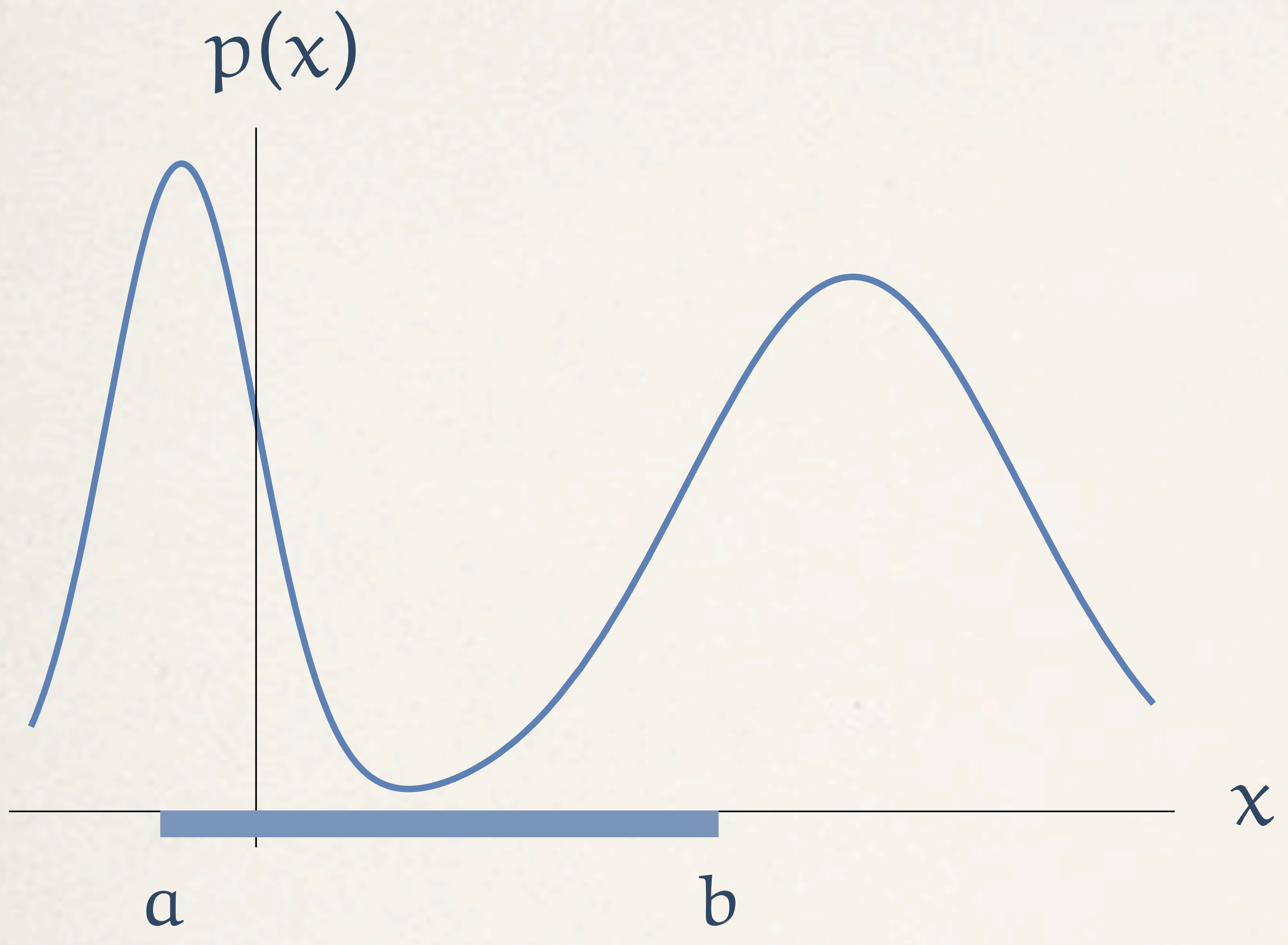

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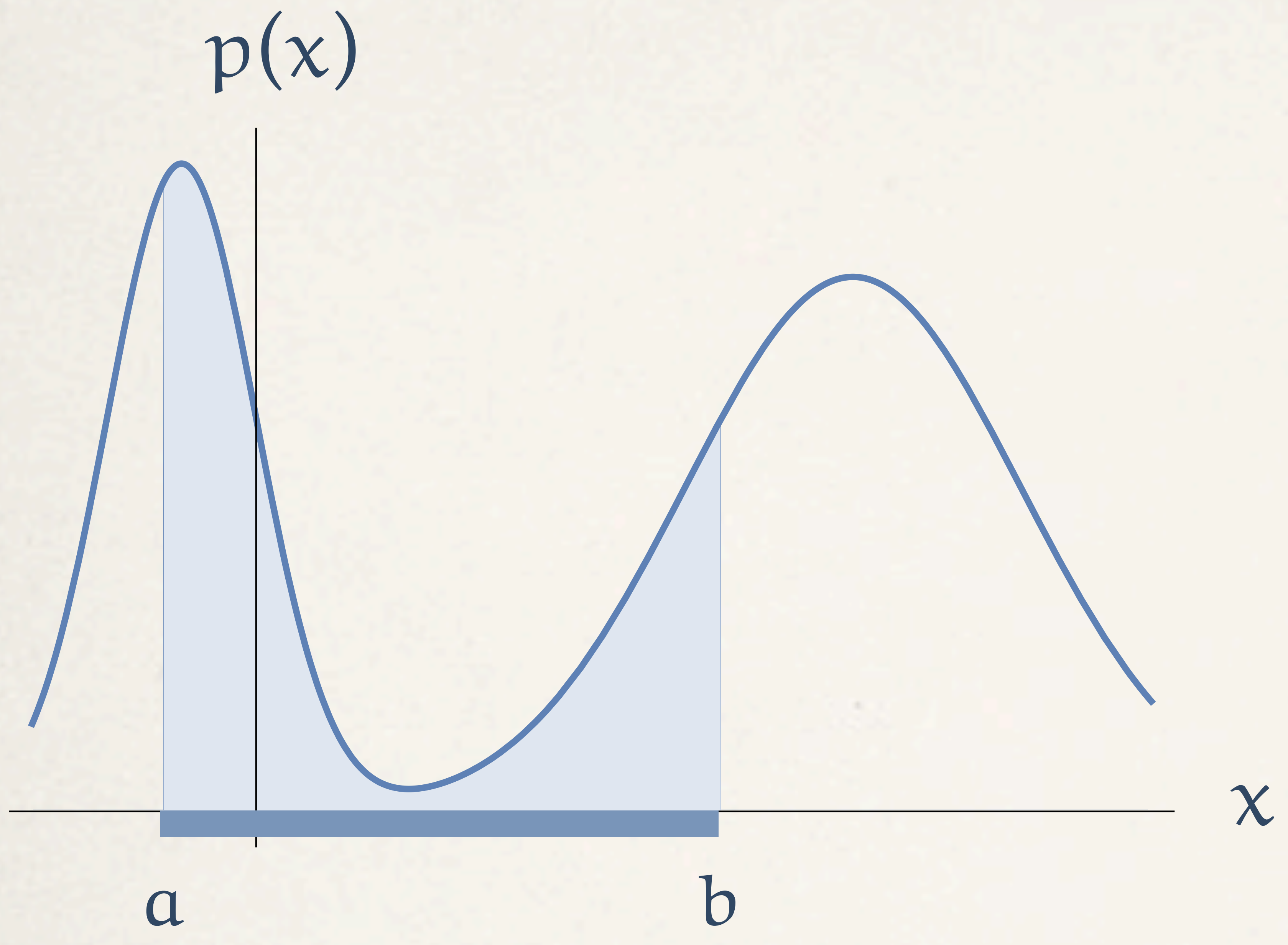
Probabilities identified with areas: $\mathbf{P}\{a \leq X \leq b\} := \mathbf{P}\big([a, b]\big)$



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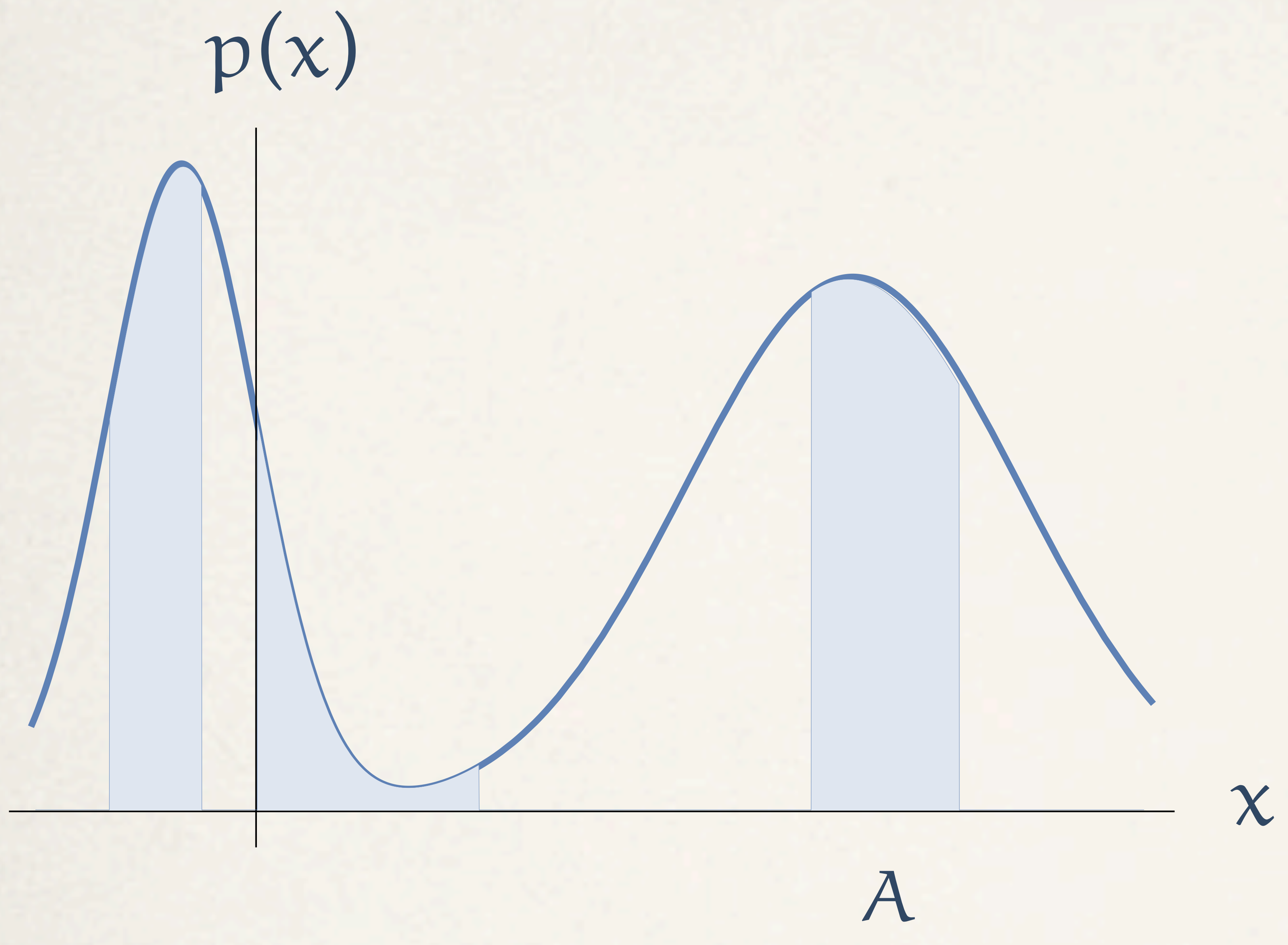

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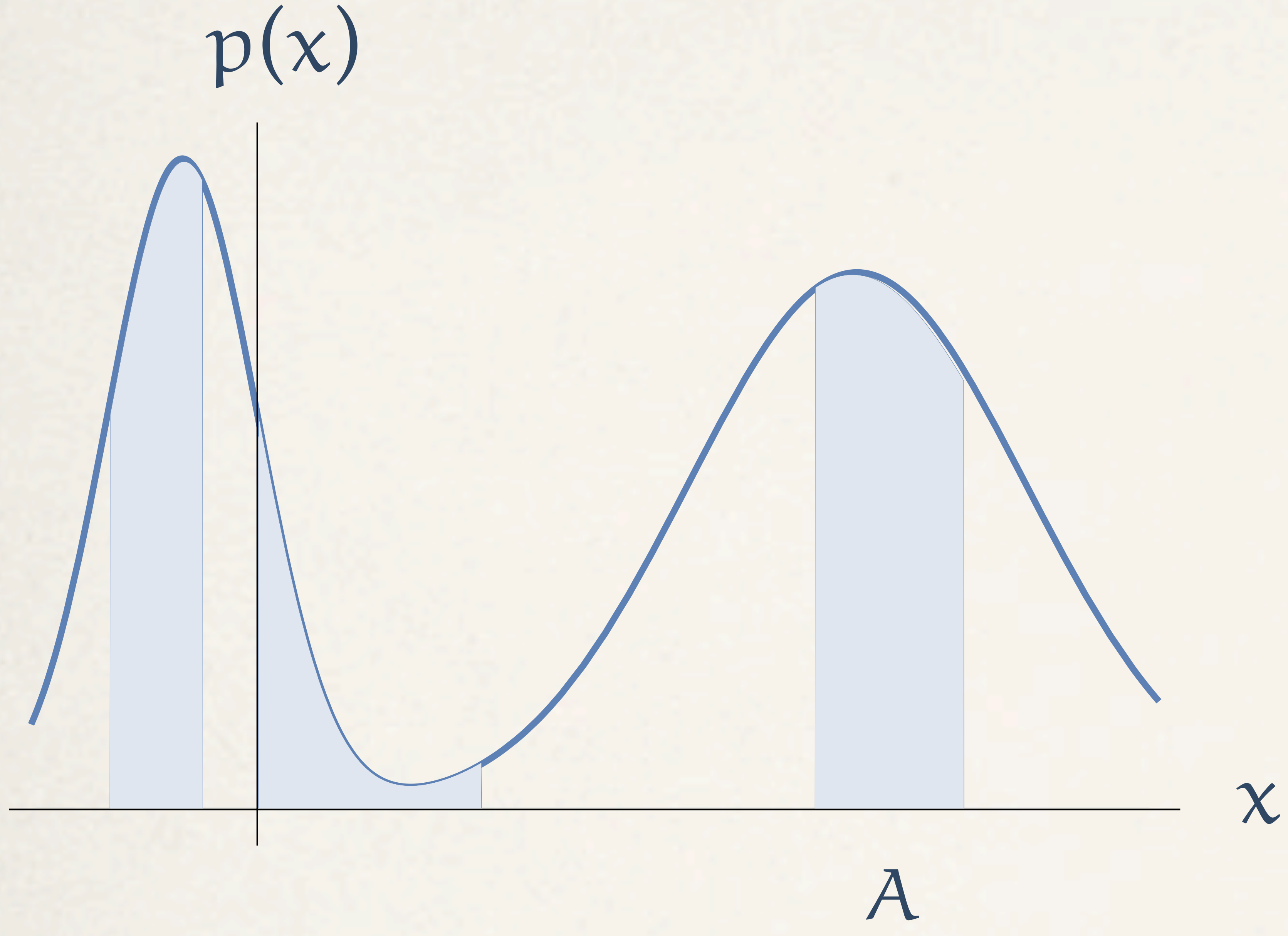

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