Lesson 3.2-3.3

Back to Week 1

11/11 points earned
(100%)

Quiz passed!

1 / 1
points

1.
If continuous random variable X has probability density function (PDF) $f(x)$, what is the interpretation of the following integral: $\int_{-2}^{5} f(x)dx$?

- $P(X \geq -2 \cup X \leq 5)$
- $P(X \leq -2 \cap X \geq 5)$
-

$P(X \leq -2 \cap X \leq 5)$

$P(X \geq -2 \cap X \leq 5)$

Correct Response

This could also be written $P(-2 \leq X \leq 5)$.

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1 / 1 points

2. If $X \sim \text{Uniform}(0, 1)$, then what is the value of $P(-3 < X < 0.2)$?

0.2

Correct Response

$\int_{-3}^{0.2} f(x)dx = \int_{-3}^{0.2} I_{\{0<x<1\}}(x)dx = \int_{0}^{0.2} 1dx = 0.2.$

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1 / 1 points

3. If $X \sim \text{Exponential}(5)$, find the expected value $E(X)$.

0.2

Correct Response

With $X \sim \text{Exponential}(\lambda)$, we have $E(X) = 1/\lambda$.

1 / 1 points

4.
Which of the following scenarios could we most appropriately model using an exponentially distributed random variable?

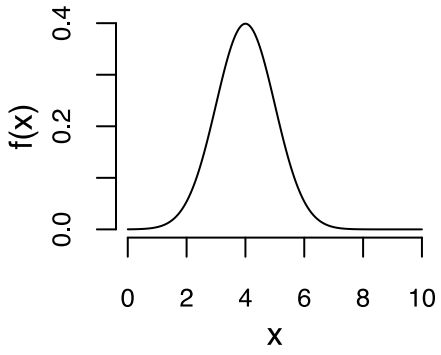
- The number of failed lightbulbs in a batch of 5000 after 100 hours in service
- The lifetime in hours of a particular lightbulb

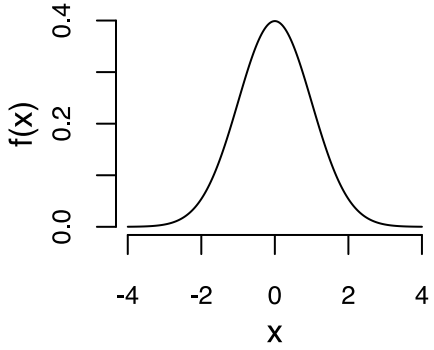
Correct Response
This is a positive, continuous quantity.

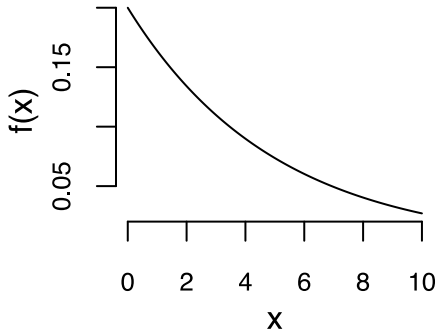
- The hours of service until all light bulbs in a batch of 5000 fail
- The probability of a light bulb failure before 100 hours in service

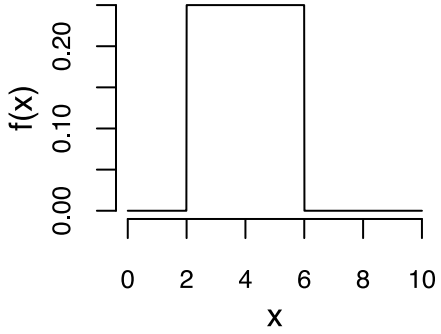
1 / 1 points

5.
If $X \sim \text{Uniform}(2, 6)$, which of the following is the PDF of X?


f(x)
0.0
0.2
0.4
0
2
4
6
8
10
x

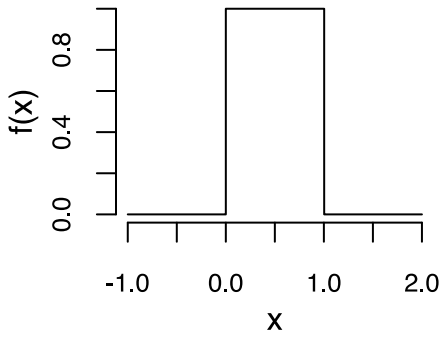

f(x)
0.0
0.2
0.4
-4
-2
0
2
4
x





Correct Response

This PDF has uniform value (1/4) over the interval $[2, 6]$ and is 0 everywhere else.



1 / 1 points

6. If $X \sim \text{Uniform}(2, 6)$, what is $P(2 < X \leq 3)$? Round your answer to two decimal places.

0.25

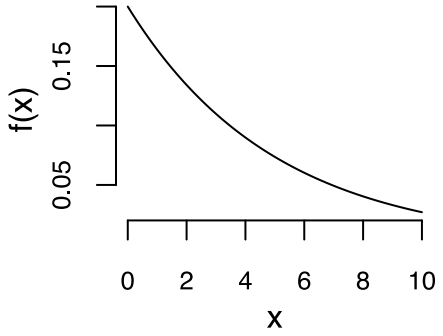
Correct Response

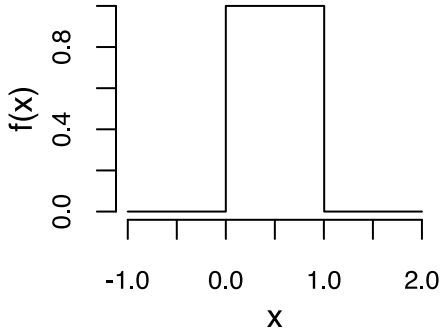
This is $\int_2^3 1/4 dx$.

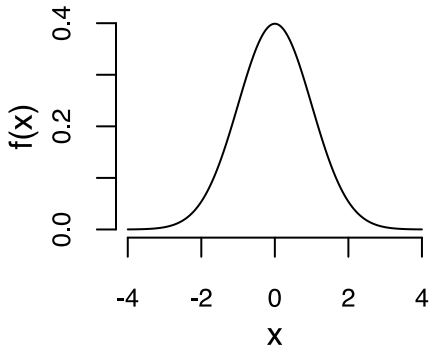
1 / 1 points

7.

If $X \sim \text{N}(0, 1)$, which of the following is the PDF of X?

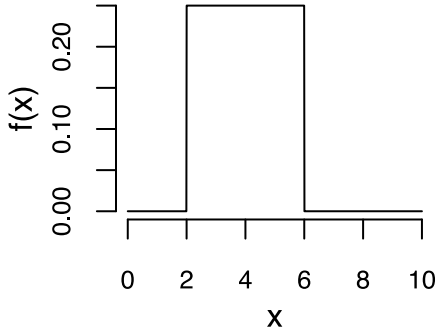

f(x)
0.05
0.15
0
2
4
6
8
10
x

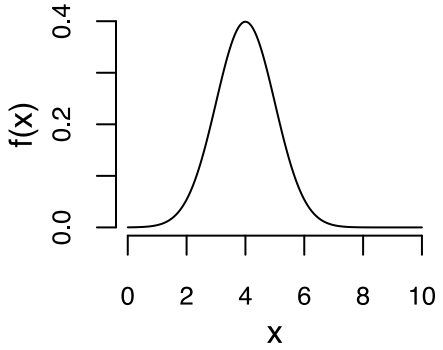

f(x)
0.0
0.4
0.8
-1.0
0.0
1.0
2.0
x



Correct Response

This is the standard normal distribution.





1 / 1 points

8. If $X \sim \mathrm{N}(2, 1)$, what is the expected value of $-5X$? This is denoted as $E(-5X)$.

-10

Correct Response

For any number c and any random variable with expectation $E(X)$, we have $E(cX) = cE(X)$.

1 / 1 points

9. Let $X \sim \mathrm{N}(1, 1)$ and $Y \sim \mathrm{N}(4, 3^2)$. What is the value of $E(X + Y)$?

5

Correct Response

For random variables X and Y with expectations $E(X)$ and $E(Y)$, we always have $E(X+Y) = E(X) + E(Y)$.

1 / 1 points

10.

The normal distribution is also linear in the sense that if $X \sim \mathrm{N}(\mu, \sigma^2)$, then for any real constants $a \neq 0$ and b, the distribution of $Y = aX + b$ is distributed $\mathrm{N}(a\mu + b, a^2\sigma^2)$.

Using this fact, what is the distribution of $Z = \frac{X-\mu}{\sigma}$?

- $\mathrm{N}(\mu, \sigma^2)$
- $\mathrm{N}(\mu, \sigma)$
- $\mathrm{N}(\mu/\sigma, 1)$
- $\mathrm{N}(1, \sigma^2)$
- $\mathrm{N}(0, 1)$

Correct Response

Here $a = 1/\sigma$ and $b = -\mu/\sigma$. Subtracting the mean and dividing by the standard deviation is referred to as standardizing a random variable.

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1 / 1 points

11.
Which of the following random variables would yield the highest value of $P(-1 < X < 1)$?

Hint: Random variables with larger variance are more dispersed.

- [x] $X \sim \mathrm{N}(0, 0.1)$

Correct Response
Of the four options, this is the least dispersed, meaning that most of the probability is associated with small values of X.

- [ ] $X \sim \mathrm{N}(0, 1)$
- [ ] $X \sim \mathrm{N}(0, 10)$
- [ ] $X \sim \mathrm{N}(0, 100)$