# How to solve the Monty Hall problem using Bayes Theorem

Asked 4 years, 2 months ago Modified 2 years, 2 months ago Viewed 2k times

1

1

I've recently come across the Monty hall problem and while the reasoning behind switching doors makes sense intuitively to me I can't seem to understand the maths behind it.

I've seen many proofs online using Bayes Theorem and I manage to understand the majority of it aside from one thing.

In the classic setting of the Monty Hall problem let $A$ be the event that the car is behind the first door that I chose. Let $B$ the event that Monty reveals a goat behind door 2.

Then, using Bayes Theorem, we have

$$P(A \mid B) = \frac{P(B \mid A)P(A)}{P(B)}$$

Now, $P(B \mid A) = \frac{1}{2}$ because if the car is behind door 1 then Monty can choose either the second or third door. $P(A) = \frac{1}{3}$ because there's a one in three chance of the car being behind the first door.

This all makes sense to me - my struggle comes in finding $P(B)$.

I understand that there are 3 separate scenarios:

-**The car is behind door 1** As above in this case $P(B) = \frac{1}{2}$.

- **The car is behind door 2** Clearly, $P(B) = 0$ as Monty cannot reveal a goat behind door 2.
- **The car is behind door 3** If the car is behind door 3, then Monty is forced to open door 2 and so in this case $P(B) = 1$.

The way I see it, by combining these three scenarios

$$P(B) = \frac{1}{2} + 0 + 1 = \frac{3}{2}$$

But in every proof that I have seen they divide this by 3 for some unknown reason. I feel like I may be missing something blindingly obvious but I don't understand why this is true.

Could someone explain the reason that $P(B) = \frac{1}{2}$ as opposed to $\frac{3}{2}$?

probability bayes-theorem monty-hall

Share Cite Edit Follow Flag

asked Jul 25, 2018 at 19:44

Inspector gadget
**45** 4

3 A probability cannot exceed 1 – Peter Jul 25, 2018 at 19:56

The scenarios must be weighted by their probabilities to be combined. – user558317 Aug 28, 2018 at 3:28

## 3 Answers

Sorted by: Highest score (default)

1

To answer your question:

> The way I see it, by combining these three scenarios $P(B) = 1/2 + 0 + 1 = 3/2$

Let us denote scenarios one through three $S_1$, $S_2$, and $S_3$.

$P(B|S_1) = 1/2$

$P(B|S_2) = 0$

$P(B|S_3) = 1$

How to combine them:

$P(B) = P(B|S_1) * P(S_1) + P(B|S_2) * P(S_2) + P(B|S_3) * P(S_3)$

Given $P(S_1) = P(S_2) = P(S_3) = 1/3$

$P(B) = P(B|S_1) * 1/3 + P(B|S_2) * 1/3 + P(B|S_3) * 1/3$

$P(B) = \{P(B|S_1) * P(S_1) + P(B|S_2) * P(S_2) + P(B|S_3) * P(S_3)\} * (1/3)$

Substituting in $P(B|S)$:

$P(B) = (1/2 + 0 + 1) * (1/3) = 3/2 * 1/3 = 1/2$

(As for the proof, I am not familiar with it, and not certain that it is valid/relevant, only that it produces the correct answer.)

Share Cite Edit Follow Flag

answered Aug 28, 2018 at 3:26

user558317
**65** 5

Because each of the scenarios has probability $\frac{1}{}$. You're applying the law of total probability

Because each of the scenarios has probability $\frac{1}{3}$. You're applying the law of total probability, and each term contains a factor $\frac{1}{3}$.

0

Let's say it rains tomorrow with probability $\frac{1}{2}$. If it rains, I brush my teeth. If it doesn't rain, I also brush my teeth. Is the probability that I brush my teeth tomorrow 1 or 2?

You also have an error in your calculation of $P(B \mid A)$. This is $\frac{1}{3}$, not $\frac{1}{2}$. This follows by symmetry – conditioning on an event that doesn't distinguish one of the three doors from the others can't make the probabilities for the doors being opened non-uniform.

Share Cite Edit Follow Flag

answered Jul 25, 2018 at 20:13
joriki
**217k** 14 267 476

I thought $P(B \mid A)$ would be a half, simply for the reason that Monty knows what is behind each door and so if the car is behind the first door, he then has a choice of two doors to pick.
– Inspector gadget Jul 28, 2018 at 18:13

---

0

> In the classic setting of the Monty Hall problem let $A$ be the event that the car is behind the first door that I chose. Let $B$ the event that Monty reveals a goat behind door 2.

Ah! You want $B$ to be the event that Monty chooses door 2 (rather than door 3) since *you always select door 1*.

Let $A_n$ be the event that the car is behind door $n$. $A_1$ is the event that it is behind the door you choose.

$$\begin{aligned}\mathsf{P}(A_1 \mid B) &= \frac{\mathsf{P}(B \mid A_1)\mathsf{P}(A_1)}{\mathsf{P}(B \mid A_1)\mathsf{P}(A_1) + \mathsf{P}(B \mid A_2)\mathsf{P}(A_2) + \mathsf{P}(B \mid A_3)\mathsf{P}(A_3)} \\ &= \frac{\frac{1}{2}\frac{1}{3}}{\frac{1}{2}\frac{1}{3} + 0 + \frac{1}{1}\frac{1}{3}} \\ &= \frac{1}{3}\end{aligned}$$

Share Cite Edit Follow Flag

edited Jul 25, 2018 at 23:30

answered Jul 25, 2018 at 21:50
Graham Kemp
**122k** 6 51 114