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Two-Tailed Test of Population Mean with Unknown Variance

The null hypothesis of the **two-tailed test of the population mean** can be expressed as follows:

$$\mu = \mu_0$$

where μ_0 is a hypothesized value of the true population mean μ .

Let us define the test statistic t in terms of the **sample mean**, the sample size and the **sample standard deviation** s :

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$$

Then the null hypothesis of the two-tailed test is to be *rejected* if $t \leq -t_{\alpha/2}$ or $t \geq t_{\alpha/2}$, where $t_{\alpha/2}$ is the $100(1 - \alpha)$ **percentile** of the **Student t distribution** with $n - 1$ degrees of freedom.

Problem

Suppose the mean weight of King Penguins found in an Antarctic colony last year was 15.4 kg. In a sample of 35 penguins same time this year in the same colony, the mean penguin weight is 14.6 kg. Assume the sample standard deviation is 2.5 kg. At .05 significance level, can we reject the null hypothesis that the mean penguin weight does not differ from last year?

Solution

The null hypothesis is that $\mu = 15.4$. We begin with computing the test statistic.

```
> xbar = 14.6          # sample mean
> mu0 = 15.4           # hypothesized value
> s = 2.5              # sample standard deviation
> n = 35               # sample size
> t = (xbar-mu0)/(s/sqrt(n))
> t                    # test statistic
[1] -1.8931
```

We then compute the critical values at .05 significance level.

```
> alpha = .05
> t.half.alpha = qt(1-alpha/2, df=n-1)
> c(-t.half.alpha, t.half.alpha)
[1] -2.0322  2.0322
```

Answer

The test statistic -1.8931 lies between the critical values -2.0322, and 2.0322. Hence, at .05 significance level, we do *not* reject the null hypothesis that the mean penguin weight does not differ from last year.

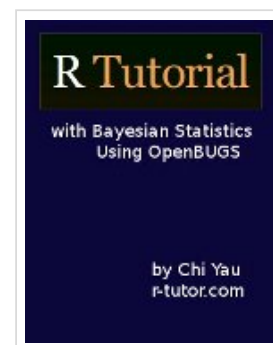
Alternative Solution

Instead of using the critical value, we apply the `pt` function to compute the two-tailed **p-value** of the test statistic. It doubles the *lower* tail p-value as the sample mean is *less* than the hypothesized value. Since it turns out to be greater than the .05 significance level, we do *not*

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reject the null hypothesis that $\mu = 15.4$.

```
> pval = 2 * pt(t, df=n-1) # lower tail
> pval                    # two-tailed p-value
[1] 0.066876
```

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