Complementary slackness conditions

Weak duality proof in one line

$$\sum_i c_i x_i \le \sum_{i=1}^{n} (A^T y)_i x_i = \sum_{j=1}^{m} (Ax)_j y_j \le \sum_j b_j y_j$$

for optimal solutions: x opt for (P), y opt for (D):

$$\sum_i c_i x_i = \sum_j b_j y_j$$

So in above proof, all inequalities are equalities

$$\forall i : c_i x_i = (A^T y)_i x_i$$
$$\forall j : b_j y_j = (Ax)_j y_j$$

$$c_i x_i = \left(\sum_j a_{ij} y_j\right) x_i$$
$$c_i \leq \left(\sum_j a_{ij} y_j\right)$$ **(constraint of (D))**

so: either $c_i = \sum_j a_{ij} y_j$
or $x_i = 0$

$$b_j y_j = \left(\sum_i a_{ij} xi\right) y_j$$
$$b_j \leq \left(\sum_i a_{ij} xi\right)$$ **(constraint of (P))**

so: either $b_j = \sum_i a_{ij} xi$
or $y_j = 0$

Complementary slackness conditions

If x is optimal for (P) and y optimal for (D)
then for every i:

$$c_i = \sum_j a_{ij} y_j \text{ or } x_i = 0$$

and for every j:

$$b_j = \sum_i a_{ij} xi \text{ or } y_j = 0$$