

From Expectation to Probability

Problem. A lottery ticket costs \$10. A 40% of the lottery budget goes to prizes. Show that the chances to win \$500 or more are less than 1%.

Assume the contrary: the probability to win at least \$500 is at least 0.01. Let us denote the number of tickets sold by n. Then the budget of the lottery is 10n dollars. The amount spent on the prizes is $10n \cdot 0.4 = 4n$ dollars. By our assumption, at least $\frac{n}{100}$ tickets win at least \$500. In total, these tickets win $\frac{n}{100} \cdot 500 = 5n$ dollars. But this amount exceeds the budget 4n dollars spent on the prizes! We arrive at the contradiction, hence our assumption was wrong.

