Notes: Integration by Substitution

The Substitution Rule

Substitution Rule for Indefinite Integrals. If $u = g(x)$ is a differentiable function whose range is an interval I and f is continuous on I, then $\boxed{\int f(g(x))g'(x)\,dx = \int f(u)\,du}$

Substitution Rule for Definite Integrals. If $u = g(x)$ is a differentiable function whose range is an interval I and f is continuous on I, and g' is continuous on $[a, b]$, then

$$\boxed{\int_a^b f(g(x))g'(x)\,dx = \int_{g(a)}^{g(b)} f(u)\,du}$$

Process for Integration by Substitution:

1. Choose the expression to be substituted for, and set it equal to u.
2. Take the derivative of this term with respect to x to find du, in terms of dx.
3. Substitute u and du into the integral.
4. Integrate with respect to u.
5. Substitute x back in to the result.

Sample Problems

Sample Problem 1. Evaluate the indefinite integral $\displaystyle\int \frac{16x}{(4x^2+4)^2}\,dx$.

Solution. Let $u = 4x^2 + 4$. Then $du = 8x\,dx$, so

$$\int \frac{16x}{(4x^2+4)^2}dx = \int \frac{2\,du}{u^2} = \int 2u^{-2}\,du$$

Evaluate this integral using the rule for power functions:

$$\int 2u^{-2}\,du = 2\,\frac{u^{-2+1}}{\quad} + C$$