Elliptical Planetary Orbits

My Solutions ›

Consider a planet in an elliptical orbit about the sun. Assume the sun is fixed in space and at the origin of the coordinate system, and the planet is located at pos $\mathbf{r}(t) = x(t)\mathbf{i} + y(t)\mathbf{j}$. Let $T = 2\pi/\omega$ be the period of the planet's orbit, let a and b be one-half the major and minor axes of the ellipse, and let $e = \sqrt{1 - b^2/a^2}$ b of the ellipse, with $0 \le e < 1$. Then the coordinates of the planet at time t are given by

$$x(t) = a(e - \cos E), \qquad y(t) = b \sin E\,.$$

The eccentric anomaly, $E = E(t)$, is a solution of Kepler's equation, a transcendental equation given by

$$E = \omega t + e \sin E.$$

By solving Kepler's equation (using fzero.m), you will compute and plot several different elliptical orbits of a planet.

The closest approach to the sun occurs when $t = 0$ and $E = 0$. To construct your plots, choose units so that the distance of closest approach is one. At the clo $y = 0$ and $x = a(e - 1)$, so choose the unit of length such that $a(e - 1) = -1$. Solving for a and then b in terms of e, we find

$$a = \frac{1}{1 - e}, \qquad b = \sqrt{\frac{1 + e}{1 - e}}.$$

If we also choose units such that the period of an orbit is one, then $\omega = 2\pi$. On a single plot, show the four orbits corresponding to $e = 0,\ 1/4,\ 1/2$ and $3/4$.

Script

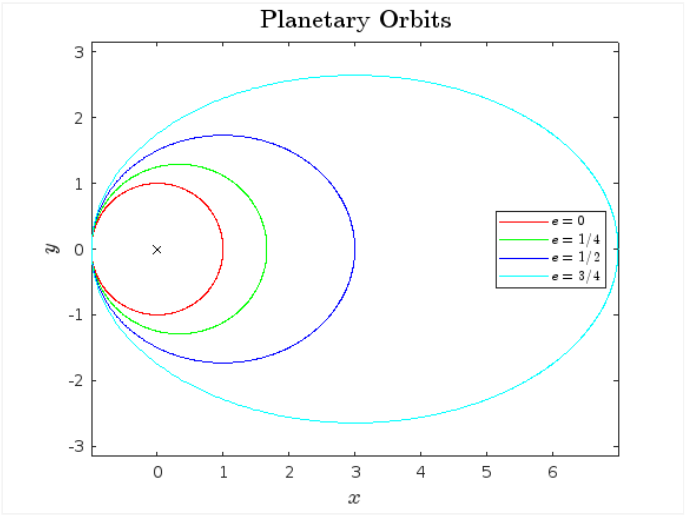
Reference Solution | Save | Reset | MATLAB Documentation (https://www.mathworks.com/help/)

```
period=1;  omega=2*pi/period;
e=[0,1/4,1/2,3/4]; color=['r','g','b','c'];
a=1./(1-e); b=sqrt((1+e)./(1-e));
t=linspace(0,period,1000);
x=zeros(length(t),length(e)); y=zeros(length(t),length(e));
for j=1:length(e)
    for i=1:length(t)
        E=fzero(@(E) omega*t(i) + e(j)*sin(E) - E, 0); % add anonymous function for root finding.  Make use of the variables e
        x(i,j)= a(j)*(e(j)-cos(E)); % assign x-coordinate.  Make use of the variables a(j), e(j) and E.
        y(i,j)= b(j)*sin(E); % assign y-coordinate.  Make use of the variables b(j) and E.
    end
end
for j=1:length(e)
    plot(x(:,j),y(:,j),color(j)); axis equal; hold on;
end
plot(0,0,'xk') %mark the origin
xlabel('$x$', 'Interpreter', 'latex', 'FontSize',14)
ylabel('$y$', 'Interpreter', 'latex', 'FontSize',14)
legend('$e=0$','$e=1/4$','$e=1/2$','$e=3/4$','Interpreter','latex','Location','East')
title('Planetary Orbits','Interpreter','latex','FontSize',16)

```

▶ Run Script

Output



Previous Assessment: All Tests Passed

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✅ **Test the x-coordinate of the orbits**

✅ **Test the y-coordinate of the orbits**