

Computational Photography



Dr. Irfan Essa

Professor

School of Interactive Computing

Study the basics of computation and its impact on the entire workflow of photography, from capturing, manipulating and collaborating on, and sharing photographs.

Image Processing and Filtering, via Convolution and Cross-Correlation

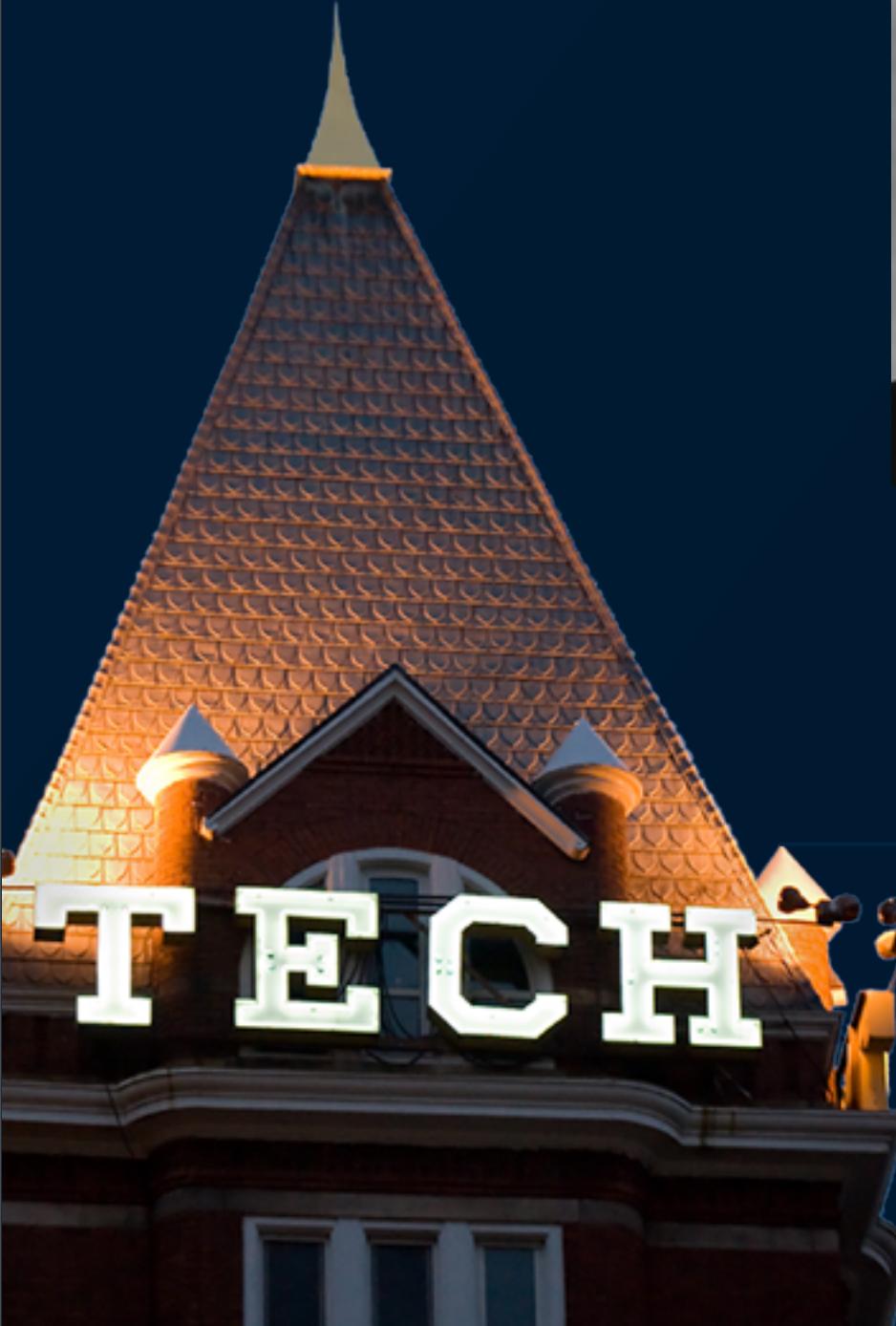


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Point-process and Neighboring Pixels
Computations on an Image using Cross-
Correlation and Convolution



Lesson Objectives

- ★ Describe in your own words the Cross-Correlation method for filtering images.
- ★ Describe in your own words the Convolution method for filtering images.
- ★ Identify at least two (2) differences between the Cross-Correlation and Convolution methods for filtering images.
- ★ Recall two (2) properties of Convolution methods for image filtering.



Review: Mathematical Representation for Smoothing

$$G[3,3] = \frac{1}{9}(A + B + C + D + E + F + G + H + I)$$

a	b	c
d	e	f
g	h	i

20	20	10	20	10	20	10	10	13
30	0	0	0	0	0	0	0	30
20	0	A	B	C	90	90	0	20
20	0	D	E	F	90	90	0	20
10	0	G	H	I	90	90	0	10
10	0	90	90	90	90	90	0	10
10	0	90	90	90	90	90	0	10
20	0	0	0	0	0	0	0	20
20	20	10	20	10	20	10	10	13

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$$G[i,j] = \frac{1}{(2k+1)^2} \sum_{u=-k}^k \sum_{v=-k}^k F[i+u, j+v]$$

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20	0	D	E	F	90	90	0	20
10	0	G	H	I	90	90	0	10
10	0	90	90	90	90	90	0	10
10	0	90	90	90	90	90	0	10
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Loop over all pixels in neighborhood
around image pixel $F[i,j]$

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10	0	90	90	90	90	90	0	10
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Attribute uniform weight to each pixel

Loop over all pixels in neighborhood around image pixel $F[i,j]$

a	b	c
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30	0	0	0	0	0	0	0	30
20	0	A	B	C	90	90	0	20
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10	0	90	90	90	90	90	0	10
10	0	90	90	90	90	90	0	10
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10	0	90	90	90	90	90	0	10
20	0	0	0	0	0	0	0	20
20	20	10	20	10	20	10	10	13

More Generally,

$$G[3,3] = a * A + b * B + c * C + d * D + e * E + f * F + h * H + i * I$$

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10	0	90	90	90	90	90	0	10
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20	0	0	0	0	0	0	0	20
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30	0	0	0	0	0	0	0	30
20	0	A	B	C	90	90	0	20
20	0	D	E	F	90	90	0	20
10	0	G	H	I	90	90	0	10
10	0	90	90	90	90	90	0	10
10	0	90	90	90	90	90	0	10
20	0	0	0	0	0	0	0	20
20	20	10	20	10	20	10	10	13

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$$G[i,j] = \sum_{u=-k}^k \sum_{v=-k}^k h[u,v]F[i+u, j+v]$$

Attribute non-uniform weights

Cross-Correlation Method

$$G[i, j] = \sum_{u=-k}^k \sum_{v=-k}^k h[u, v] F[i + u, j + v]$$

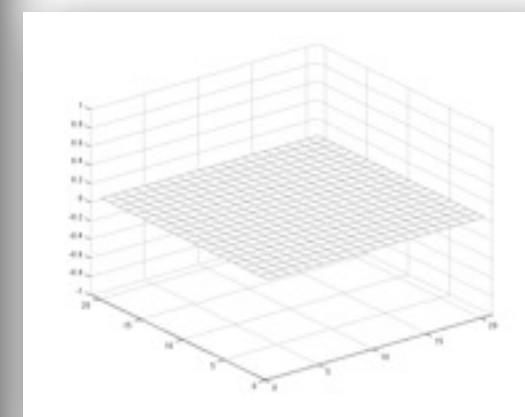
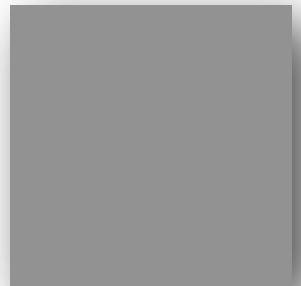
- ★ Denoted by $G = h \otimes F$
- ★ Filtering an image: replace each pixel with a linear combination of its neighbors.
- ★ The filter “kernel” or “mask” $h[u, v]$ is the prescription for the weights in the linear combination.

In signal processing, **cross-correlation** is a measure of similarity of two waveforms as a function of a time-lag applied to one of them. This is also known as a *sliding dot product* or *sliding inner-product*.

Example: Box Filter



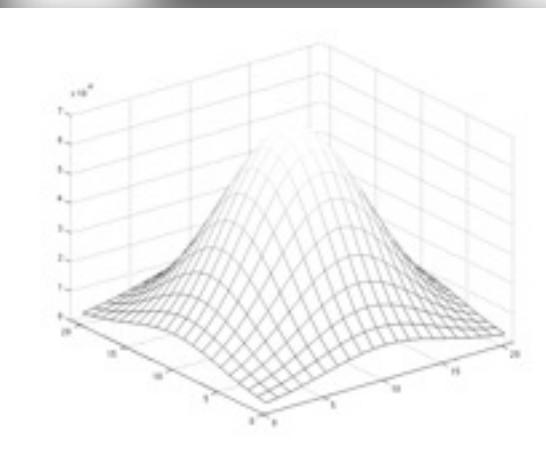
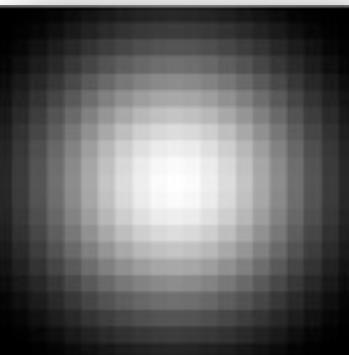
Box/Average
Filter
 21×21



Example: Gaussian Filter



Gaussian
Filter
 21×21



Filtering by a Kernel (defining Convolution)

0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	1	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0



a	b	c
d	e	f
g	h	i

$h[i,j]$

=

$F[i,j]$

$G[i,j]$

- ★ Impulse function
- ★ 'Filter' means to slide the kernel over the image which results in a reversed response.

Filtering by a Kernel (defining Convolution)

0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0



a	b	c
d	e	f
g	h	i

=

$h[i,j]$

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0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0



a	b	c
d	e	f
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$h[i,j]$

=

0								

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0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0



a	b	c
d	e	f
g	h	i

$h[i,j]$

=

0	f

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0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	1	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0



a	b	c
d	e	f
g	h	i

$h[i,j]$

=

0	f	e

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0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	1	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

\otimes

a	b	c
d	e	f
g	h	i

$h[i,j]$

=

0	f	e	d

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0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0



a	b	c
d	e	f
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$h[i,j]$

=

0	f	e	d	0

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0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0



a	b	c
d	e	f
g	h	i

$h[i,j]$

=

0	i	h	g	0	
0	f	e	d	0	
0	c	b	a	0	

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0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0



a	b	c
d	e	f
g	h	i

$h[i,j]$

=

0	i	h	g	0	
0	f	e	d	0	
0	c	b	a	0	

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0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0



a	b	c
d	e	f
g	h	i

$h[i,j]$

=

i	h	g
f	e	d
c	b	a

0	i	h	g	0
0	f	e	d	0
0	c	b	a	0

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0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0



a	b	c
d	e	f
g	h	i

$h[i,j]$

=

ə	q	ɔ
p	ə	ʃ
6	ɥ	!

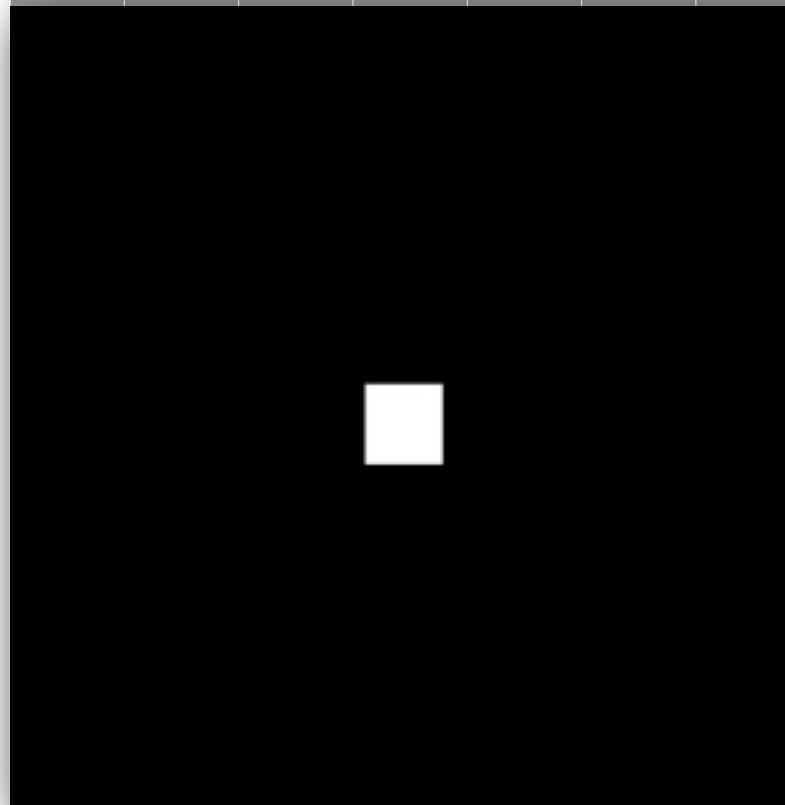
0	i	h	g	0
0	f	e	d	0
0	c	b	a	0

$F[i,j]$

$G[i,j]$

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Filtering by a Kernel (defining Convolution)


 \otimes

a	b	c
d	e	f
g	h	i

 $h[i,j]$

=

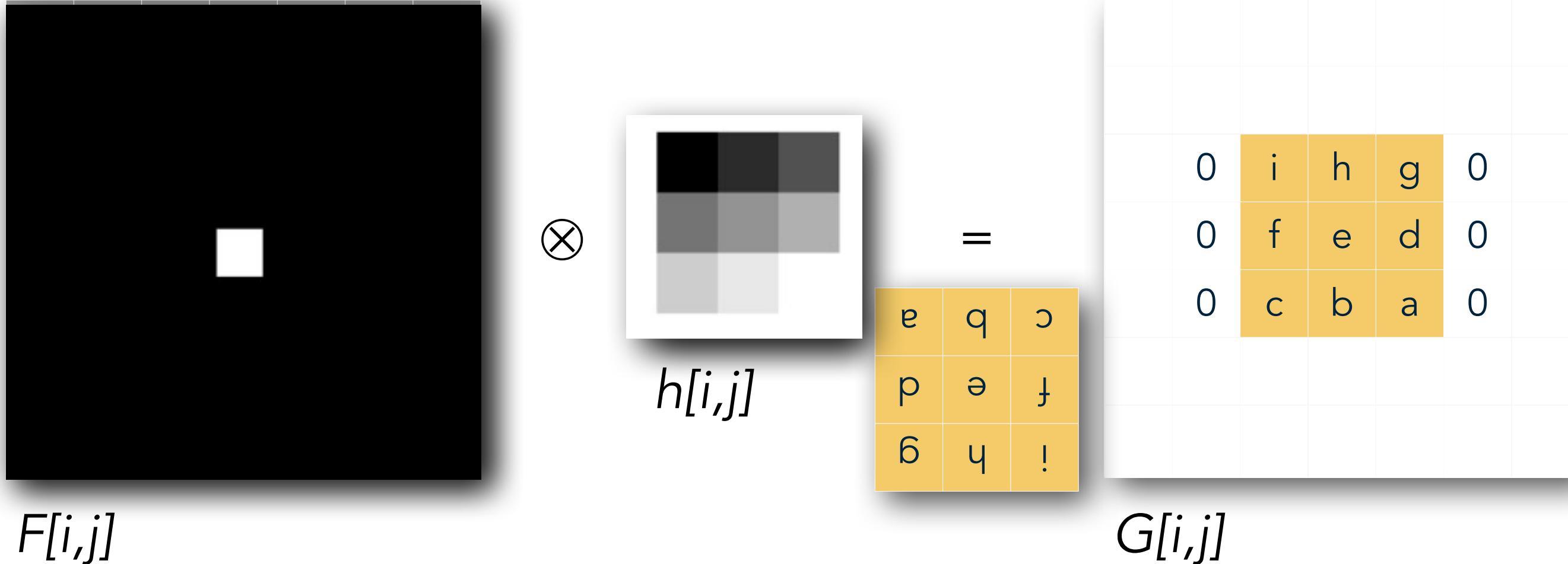
æ	ø	ɔ
p	ə	ʃ
ѓ	ɥ	!

0	i	h	g	0
0	f	e	d	0
0	c	b	a	0

 $F[i,j]$
 $G[i,j]$

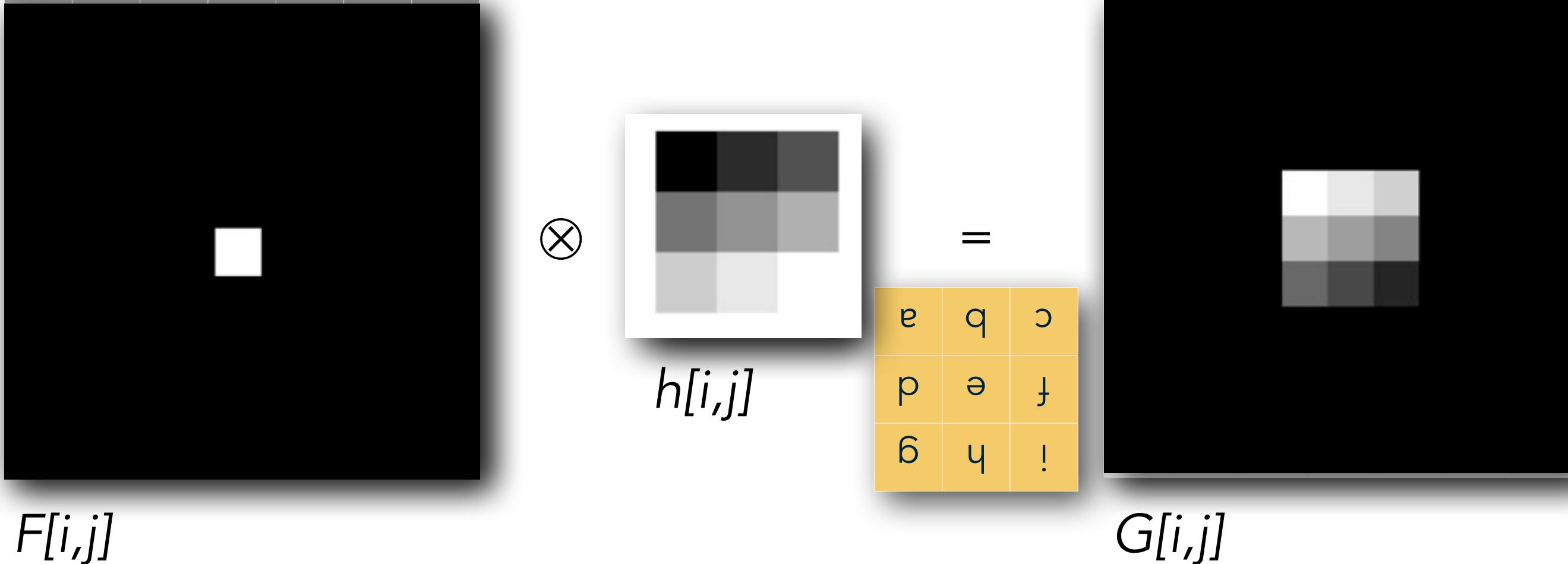
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Filtering by a Kernel (defining Convolution)



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Convolution Method

$$G[i, j] = \sum_{u=-k}^k \sum_{v=-k}^k h[u, v]F[i - u, j - v]$$

- ★ Denoted by $G = h * F$
- ★ Flip the filter in both dimensions (bottom to top, right to left)
- ★ Then apply cross-correlation

Convolution is a mathematical operation on two functions F and h , producing a third function that is typically viewed as a modified version of one of the original functions, giving the area overlap between the two functions as a function of the amount that one of the original functions is translated.

Convolution vs. Cross-Correlation Methods

- ★ For both a Gaussian and a box filter, how will the outputs differ for each of the methods?
- ★ If the input is an impulse signal, how will the outputs differ for each of the methods?

Convolution vs. Cross-Correlation Methods

Cross-Correlation: $G = h \otimes F$
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Convolution: $G = h * F$
$$G[i, j] = \sum_{u=-k}^k \sum_{v=-k}^k h[u, v]F[i - u, j - v]$$

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Properties of Convolution

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- ★ Linear and Shift Invariants: Behaves the same everywhere (i.e., the value of the output depends on the pattern in the image neighborhood, not the position of the neighborhood).

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- ★ Identity: Unit Impulse $E = [....0,0,1,0,0,...]$, $F * E = F$
 - True of Cross-Correlation?

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- ★ Identity: Unit Impulse $E = [....0,0,1,0,0,...]$, $F * E = F$
 - True of Cross-Correlation?
- ★ Separable: If the filter is separable, convolve all rows, then convolve all columns.

Linear Filters

Linear Filters



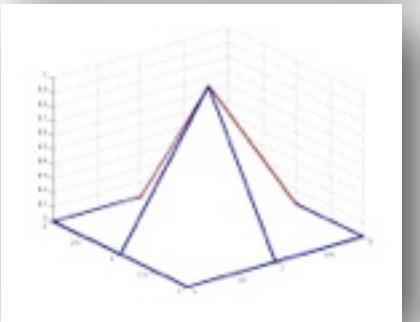
original, 64x64

Linear Filters



original, 64x64

0	0	0
0	1	0
0	0	0

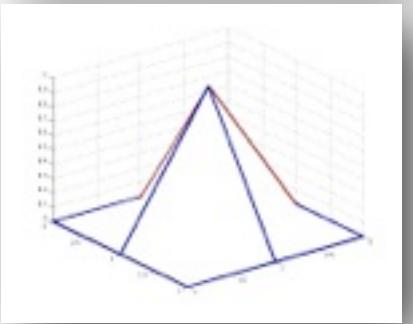


Linear Filters

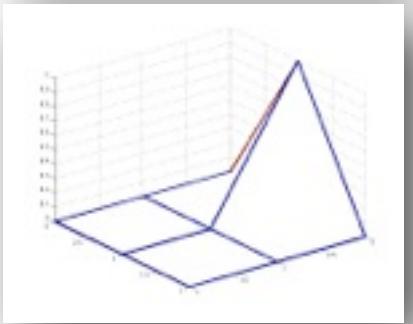


original, 64x64

0	0	0
0	1	0
0	0	0



0	0	0
0	0	1
0	0	0

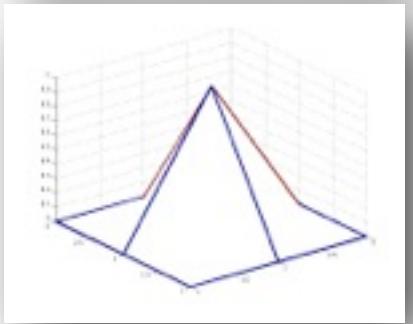


Linear Filters

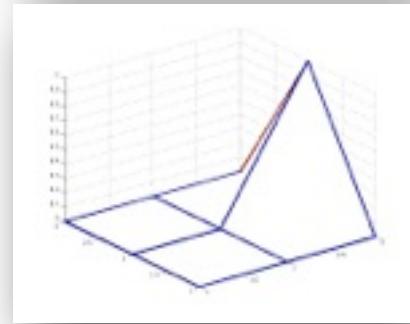


original, 64x64

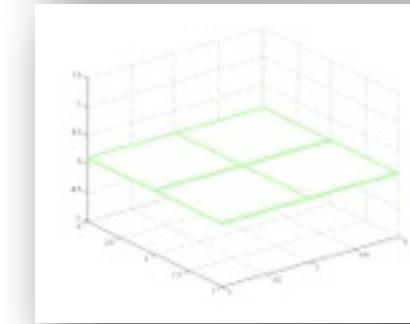
0	0	0
0	1	0
0	0	0



0	0	0
0	0	1
0	0	0



1/9	1/9	1/9
1/9	1/9	1/9
1/9	1/9	1/9

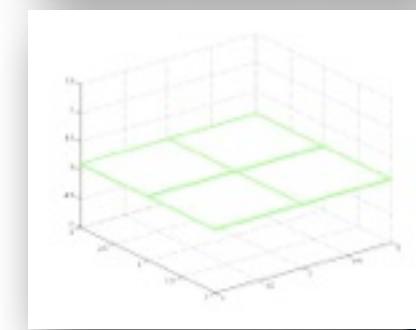
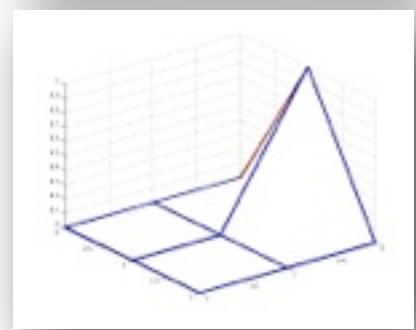
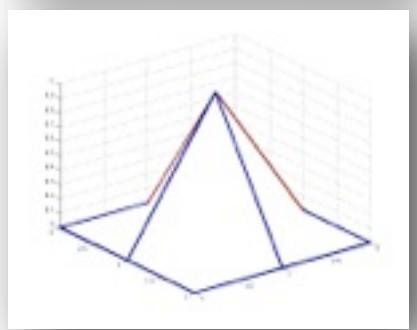


Linear Filters

0	0	0
0	1	0
0	0	0

0	0	0
0	0	1
0	0	0

1/9	1/9	1/9
1/9	1/9	1/9
1/9	1/9	1/9



original, 64x64

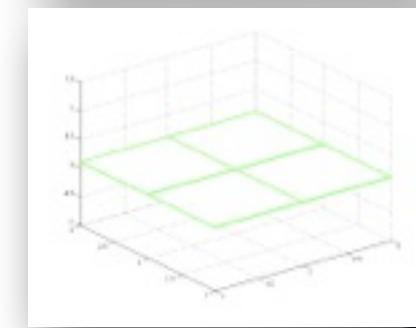
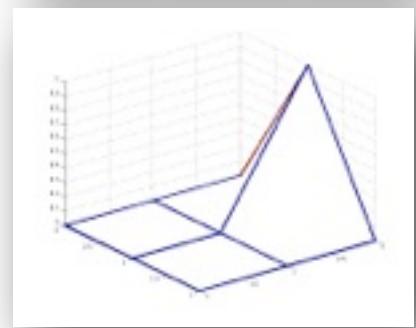
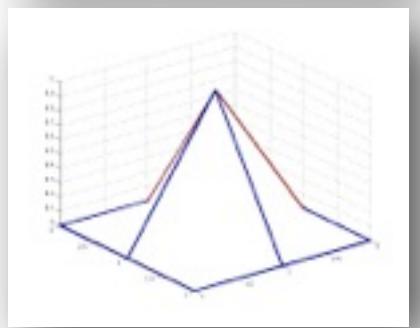
Linear Filters



0	0	0
0	1	0
0	0	0

0	0	0
0	0	1
0	0	0

1/9	1/9	1/9
1/9	1/9	1/9
1/9	1/9	1/9



original, 64x64

Linear Filters

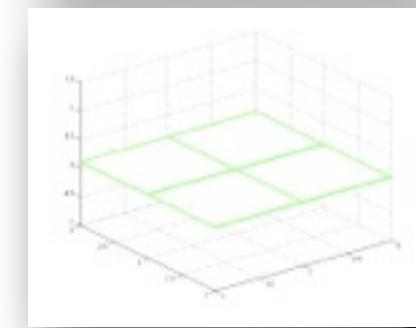
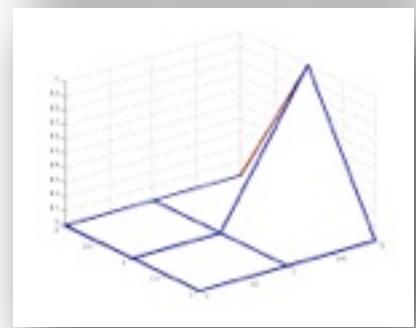
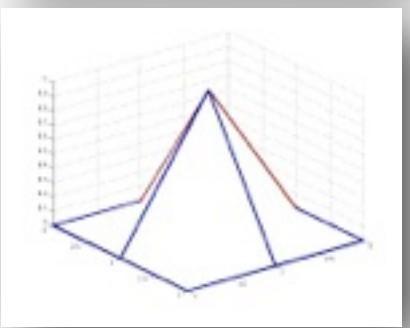


original, 64x64

0	0	0
0	1	0
0	0	0

0	0	0
0	0	1
0	0	0

1/9	1/9	1/9
1/9	1/9	1/9
1/9	1/9	1/9



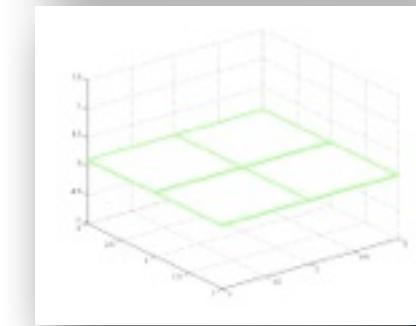
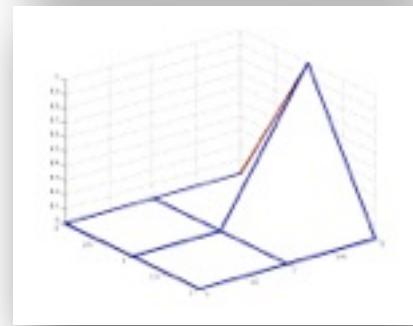
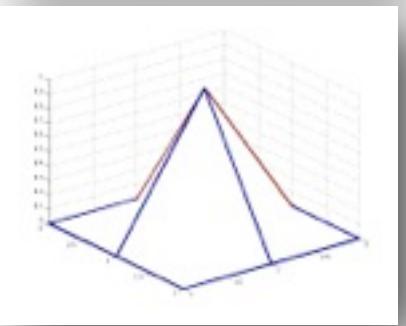
Linear Filters



0	0	0
0	1	0
0	0	0

0	0	0
0	0	1
0	0	0

1/9	1/9	1/9
1/9	1/9	1/9
1/9	1/9	1/9



Linear Filters

Linear Filters



original, 64x64

Linear Filters

0	0	0
0	1	0
0	0	0

$$X_2 - \begin{array}{|c|c|c|} \hline 1/9 & 1/9 & 1/9 \\ \hline 1/9 & 1/9 & 1/9 \\ \hline 1/9 & 1/9 & 1/9 \\ \hline \end{array} =$$



original, 64x64

Linear Filters

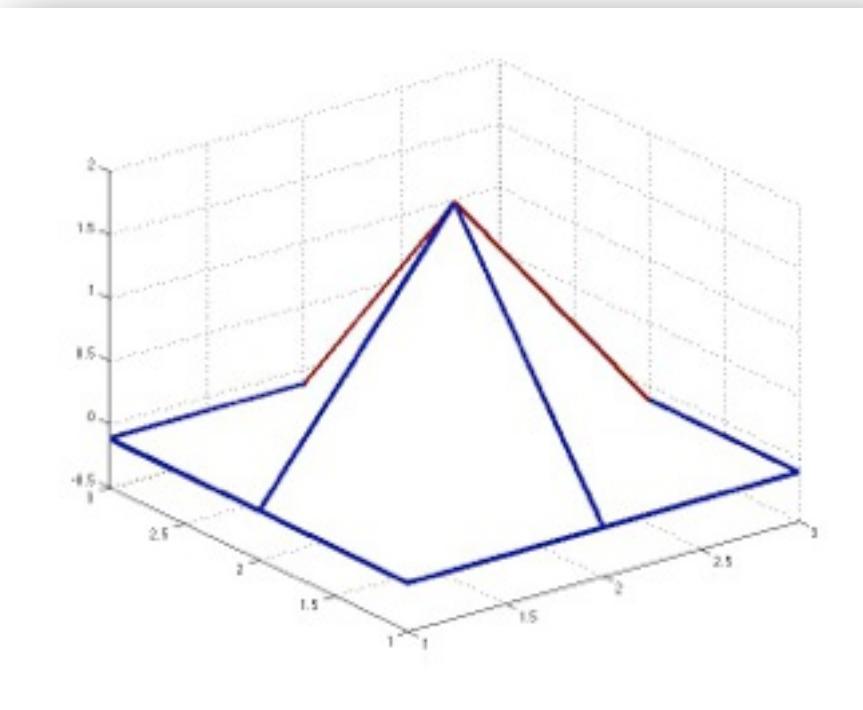
0	0	0
0	1	0
0	0	0

$$X_2 - \begin{array}{|c|c|c|} \hline 1/9 & 1/9 & 1/9 \\ \hline 1/9 & 1/9 & 1/9 \\ \hline 1/9 & 1/9 & 1/9 \\ \hline \end{array} =$$



original, 64x64

-0.11	-0.11	-0.11
-0.11	1.9	-0.11
-0.11	-0.11	-0.11



Linear Filters

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$\times 2 -$

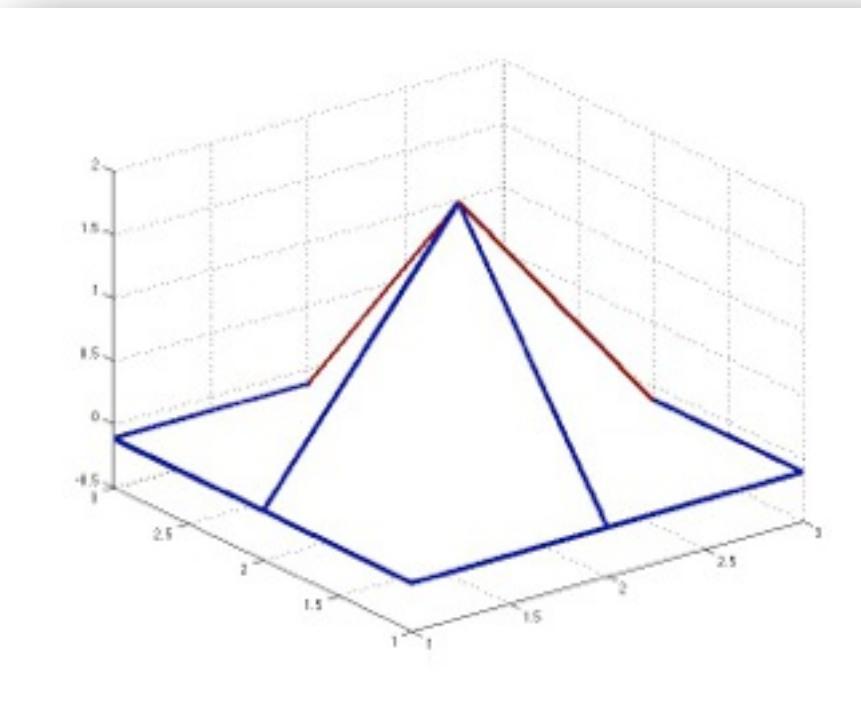
$$\begin{bmatrix} 1/9 & 1/9 & 1/9 \\ 1/9 & 1/9 & 1/9 \\ 1/9 & 1/9 & 1/9 \end{bmatrix}$$

=



original, 64x64

$$\begin{bmatrix} -0.11 & -0.11 & -0.11 \\ -0.11 & 1.9 & -0.11 \\ -0.11 & -0.11 & -0.11 \end{bmatrix}$$

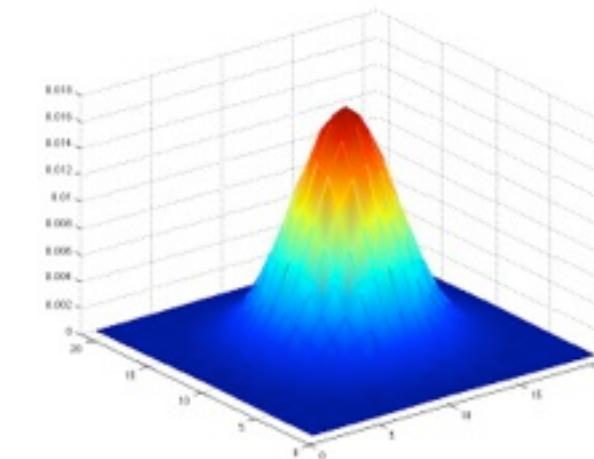


Using Gaussian Filters?

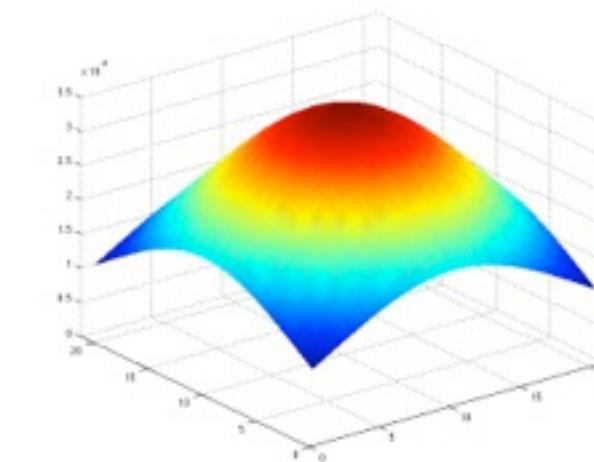
- ★ Square kernels are NOT smooth
- ★ Smoothing with an average actually doesn't compare well with a defocussed lens
- ★ Most obvious difference: A single point of light viewed in a defocussed lens looks like a fuzzy blob; the averaging process is square.
- ★ Gaussian function in 2D, with σ as the Variance of the Gaussian (centered at $(0,0)$) is:

$$h(u, v) = \frac{1}{2\pi\sigma^2} \exp^{-\frac{(u^2+v^2)}{2\sigma^2}}$$

2 was missing in the video



21X21, $\sigma=3$



21X21, $\sigma=9$

The Equation for the Gaussian in the Video is missing a 2 in the numerator of the Exponent. Fixed here. Also added the statement that this is the function for a Gaussian centered at 0,0 and the reference to wikipedia for a Gaussian. Thanks to the student who found this typo

For more information on Gaussian see http://en.wikipedia.org/wiki/Gaussian_function

Slide Adapted from Aaron Bobick and David Forsyth

Using Gaussian Filters for Smoothing



Original, 256x256

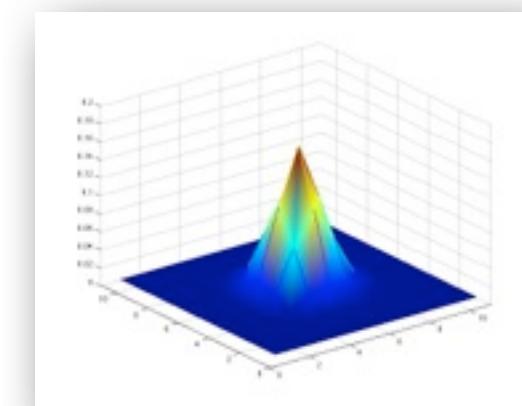
σ determines extent of smoothing

Using Gaussian Filters for Smoothing



Original, 256x256

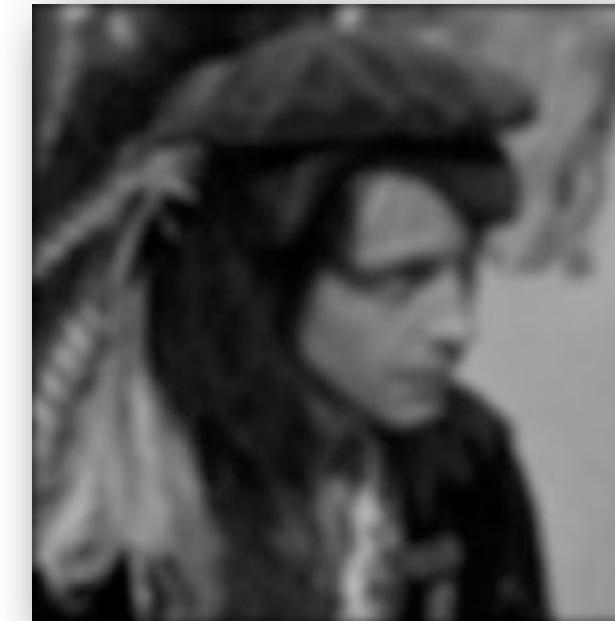
Kernel = 11x11



$\sigma=1$

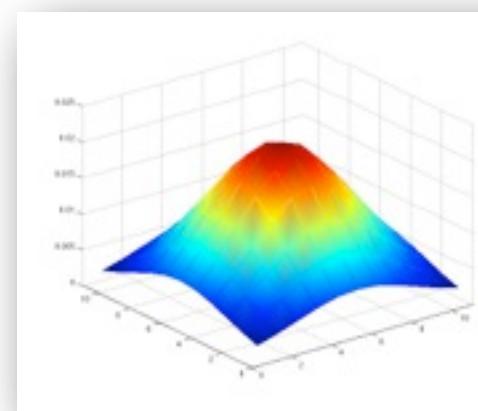
σ determines extent of smoothing

Using Gaussian Filters for Smoothing

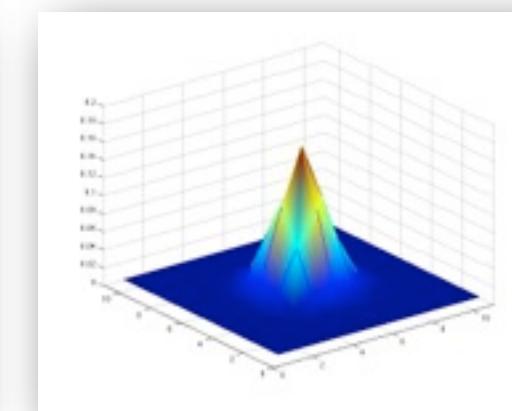


Original, 256x256

Kernel = 11x11



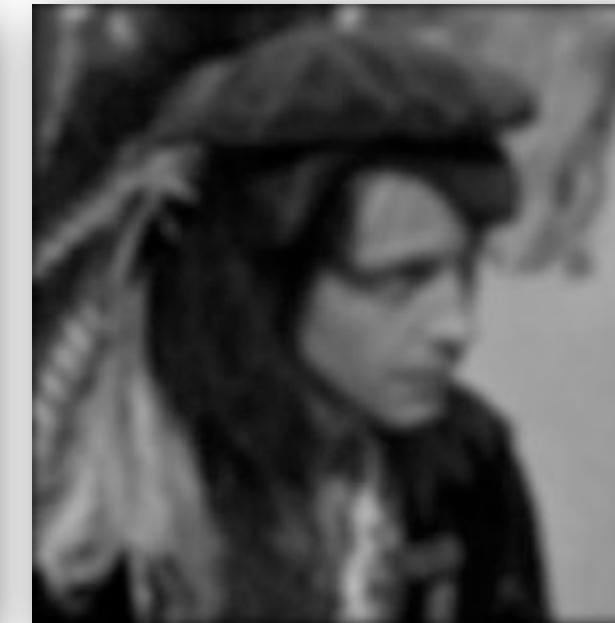
$\sigma=3$



$\sigma=1$

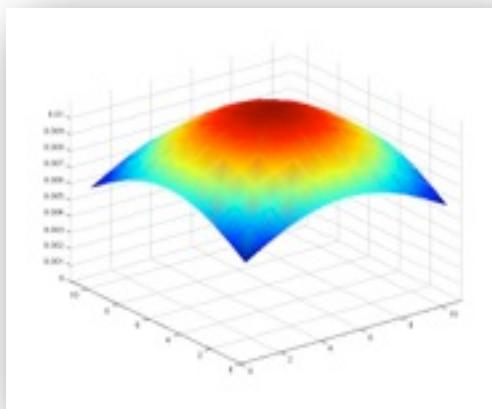
σ determines extent of smoothing

Using Gaussian Filters for Smoothing

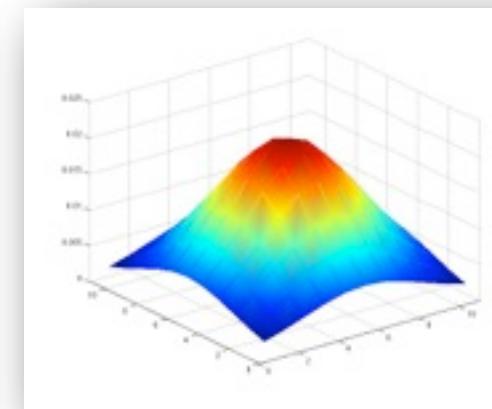


Original, 256x256

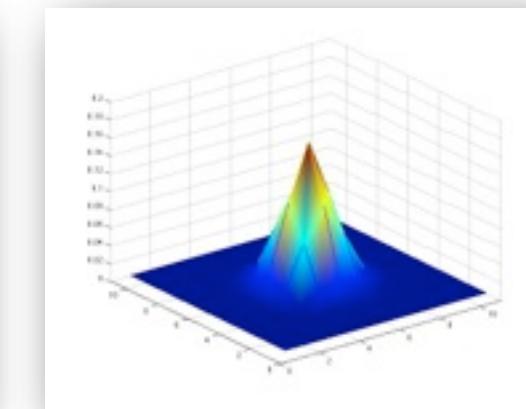
Kernel = 11x11



$\sigma=6$



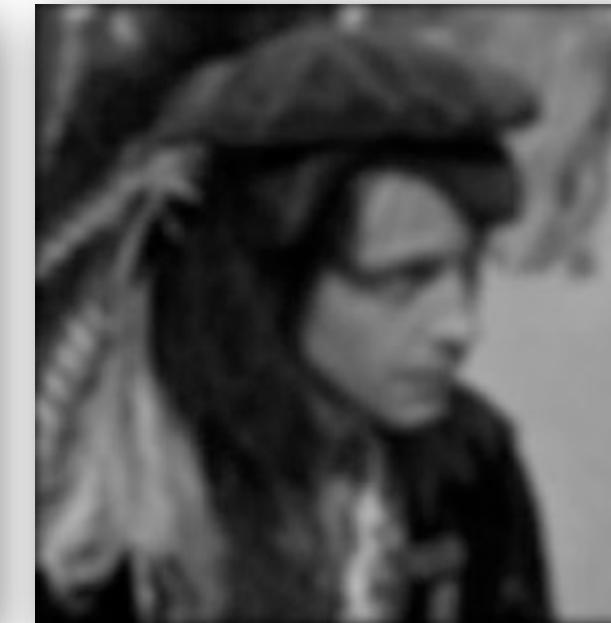
$\sigma=3$



$\sigma=1$

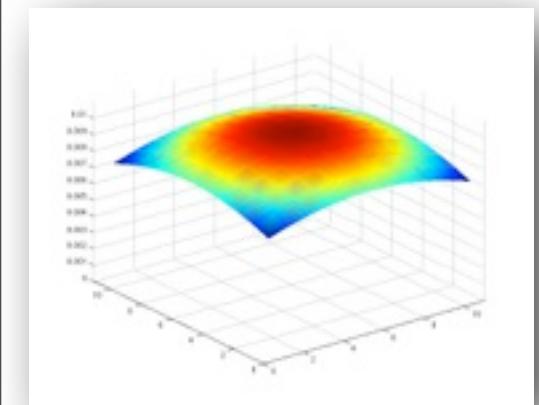
σ determines extent of smoothing

Using Gaussian Filters for Smoothing

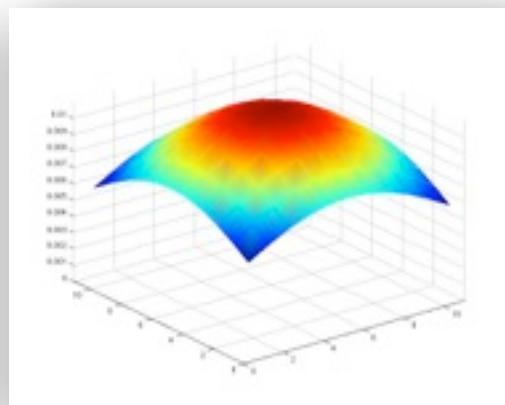


Original, 256x256

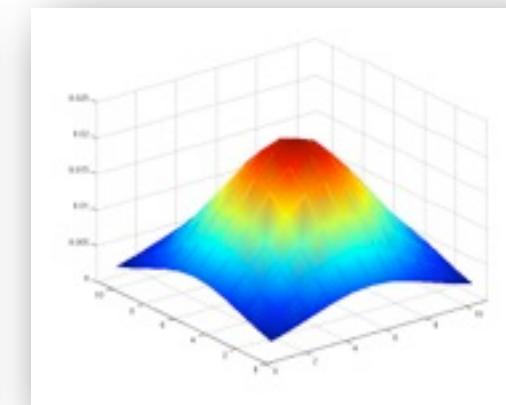
Kernel = 11x11



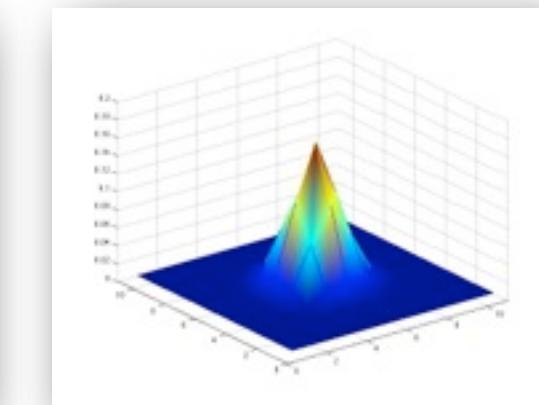
$\sigma=9$



$\sigma=6$



$\sigma=3$



$\sigma=1$

σ determines extent of smoothing

Summary

- ★ Presented Cross-Correlation method for filtering images.
- ★ Presented Convolution method for filtering images.
- ★ Discussed differences between the Cross-Correlation and Convolution methods for filtering images.
- ★ Discussed Properties of the Convolution method for filtering images.



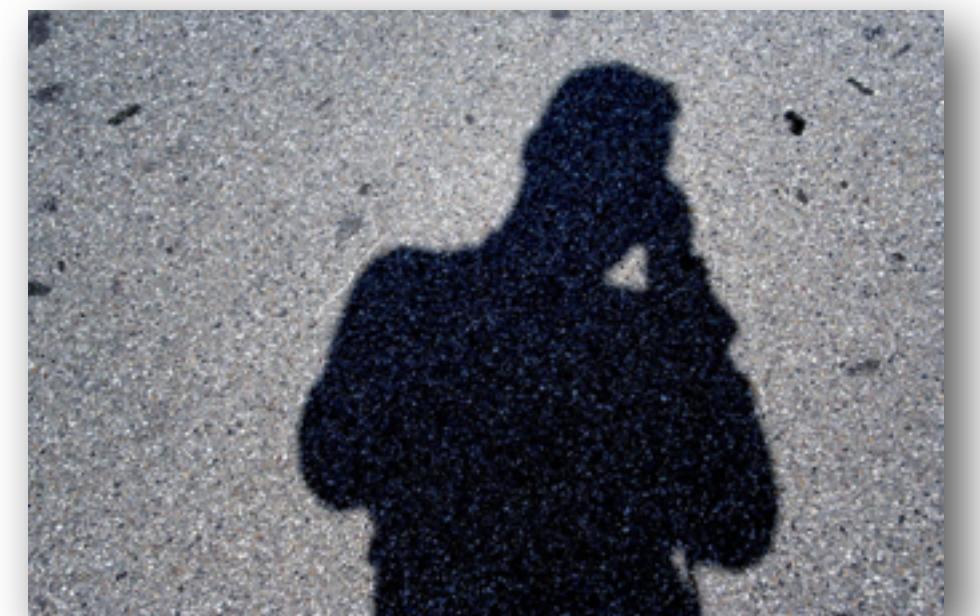
Next Class

★ Image Analysis: Edge
Detection



Credits

- ★ Matlab™ software by Mathworks Inc.
- ★ Some Slides adapted from Aaron Bobick,
Steve Seitz, Steve Marschner.
- ★ Images used from USC's Signal and Image
Processing Institute's Image Database



Computational Photography



Dr. Irfan Essa

Professor

School of Interactive Computing



Study the basics of computation and its impact on the entire workflow of photography, from capturing, manipulating and collaborating on, and sharing photographs.