Module 5 Peer Review Assignment

Problem 1

Roll two six-sided fair dice. Let X denote the larger of the two values. Let Y denote the smaller of the two values.

a) Construct a table that gives the joint probability mass function for X and Y. (Note: "X is the larger value and Y is the smaller value in a two dice roll" means that for any two dice roll, X will be greater than or equal to Y).

MY ANSWER HERE:

The joint probability mass function is constructed based on the sample space of throwing two dices - which is 36 uniqe outcomes. The outcomes where X and Y are equal (e.g. 1 and 1) happens with a probability of 1/36. The probability of a uniqe outcome where one dice is larger than the other happens with a probability of 2/36 (e.g. 4 and 3, and 3 and 4). The rest of the outcomes have zero probability (denoted - in the table).

X/Y	1	2	3	4	5	6
1	1/36	-	-	-	-	-
2	2/36	1/36	-	-	-	-
3	2/36	2/36	1/36	-	-	-
4	2/36	2/36	2/36	1/36	-	-
5	2/36	2/36	2/36	2/36	1/36	-
6	2/36	2/36	2/36	2/36	2/36	1/36

b) What is $P(X \geq 3, Y = 1)$?

MY ANSWER HERE

$P(X \geq 3, Y = 1)$

$=> P(X = 3, Y = 1) + P(X = 4, Y = 1) + P(X = 5, Y = 1) + P(X = 6, Y = 1)$

$= (2 + 2 + 2 + 2)/36$

$= 2/9$

c) What is $P(X \geq Y + 2)$?

MY ANSWER HERE

$P(X \geq Y + 2)$

$=> P(X \geq 3, Y = 1) + P(X \geq 4, Y = 2) + P(X \geq 5, Y = 3) + P(X = 6, Y = 4)$

$= 8/36 + 6/36 + 4/36 + 2/36$

$= 20/36$

$= 5/9$

d) Are X and Y independent? Explain.

MY ANSWER HERE

When two variable are independent, knowing the value of one of them does not affect the probability of the other. This is not the case here, becuase if we know the value of Y, then we know that X is at least as high as Y or higher. For instance if Y = 4, we know that X must be 4, 5 or 6. This violates the condition for independence.

Problem 2

Let (X, Y) be continuous random variables with joint PDF:

$$f(x, y) = \begin{cases} cxy^2 & \text{if } 0 \leq x \leq 1 \text{ and } 0 \leq y \leq 1 \\ 0 & \text{else} \end{cases}$$

Part a)

Solve for c. Show your work.

MY ANSWER HERE

The joint probability density function must integrate to 1. This happens only in the interval 0 to 1, for which the function has a value other than 0.

$\int_0^1 \int_0^1 cxy^2 \, dxdy = 1$

$c[(1/2)x^2]_0^1 \, [(1/3)x^3]_0^1 = 1$

$c[(1/2)(1^2 - 0^2)]_0^1 \, [(1/3)(1^3 - 0^3)]_0^1 = 1$

$c(1/2)(1/3) = 1$

$c = 6$

Part b)

Find the marginal distributions $f_X(x)$ and $f_Y(y)$. Show your work.

MY ANSWER HERE

$f_X(x) = \int_0^1 6xy^2 dy = 2x[(1^3 - 0^3)]_0^1 = 2x$, while x is in the interval $[0, 1]$

$f_Y(y) = \int_0^1 6xy^2 dx = 3y^2[(1^3 - 0^3)]_0^1 = 3y^2$, while y is in the interval $[0, 1]$

Part c)

Solve for $E[X]$ and $E[Y]$. Show your work.

MY ANSWER HERE

$E[X] = \int_0^1 x * 2x \, dx = \int_0^1 2x^2 \, dx = 2/3$

$E[Y] = \int_0^1 y * 3y^2 \, dy = \int_0^1 3y^3 \, dx = 3/4$

Part d)

Using the joint PDF, solve for $E[XY]$. Show your work.

MY ANSWER HERE

$E[XY] = \int_0^1 \int_0^1 xy * (6xy^2) \, dxdy$

$= 6 * \int_0^1 \int_0^1 x^2 y^3 \, dxdy$

$= 6[1/3 * x^3]_0^1 [1/4 * y^4]_0^1$

$= 1/2$

Part e)

Are X and Y independent?

MY ANSWER HERE

We can conclude that X and Y are independent. Using the results from above, we can show that $f(x, y) = f_X(x) * f_Y(y) => 6xy^2 = 2x * 3y^2$ for the interval $[0, 1]$. For any other interval f(x,y) and f_X(x) *f_Y(y) are both zero. Thereby the conditions for independence are satisfied, since \$ f(x,y) = f_X(x)* f_Y(y) for any value of x and y.

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