What is the mass function for this problem?

$$\mathbf{P}(H) = \mathbf{P}(H \mid A)\,\mathbf{P}(A) + \mathbf{P}(H \mid A^c)\,\mathbf{P}(A^c)$$

- *Sample space* Ω: set of triples (coin toss; first face, second face); generic sample point $\omega = (x; y_1, y_2)$, $x \in \{1, 2\}$, $y_1, y_2 \in \{R, W\}$.

- The *events of interest:*
  $R_1 :=$ first throw shows red $= \{\, (x; y_1, y_2) : y_1 = R \,\}$.
  $R_2 :=$ second throw shows red $= \{\, (x; y_1, y_2) : y_2 = R \,\}$.
  $A :=$ first die is chosen $= \{\, (x; y_1, y_2) : x = 1 \,\}$.

- Implicit *probability measure* $\mathbf{P}$:
  - Random selection of die: $\mathbf{P}(A) = \mathbf{P}(A^c) = 1/2$.
  - Conditional probabilities for a given die:

$$\mathbf{P}(R_1 \mid A) = 5/6, \qquad \mathbf{P}(R_1 \mid A^c) = 1/6,$$

$$\mathbf{P}(R_1 \cap R_2 \mid A) = 5^2/6^2 = 25/36, \qquad \mathbf{P}(R_1 \cap R_2 \mid A^c) = 1^2/6^2 = 1/36.$$

What is the mass function for this problem?

$$\mathbf{P}(H) = \mathbf{P}(H \mid A)\,\mathbf{P}(A) + \mathbf{P}(H \mid A^c)\,\mathbf{P}(A^c)$$

- *Sample space* Ω: set of triples (coin toss; first face, second face); generic sample point $\omega = (x; y_1, y_2)$, $x \in \{1, 2\}$, $y_1, y_2 \in \{R, W\}$.

- The *events of interest:*
  $R_1 :=$ first throw shows red $= \{ (x; y_1, y_2) : y_1 = R \}$.
  $R_2 :=$ second throw shows red $= \{ (x; y_1, y_2) : y_2 = R \}$.
  $A :=$ first die is chosen $= \{ (x; y_1, y_2) : x = 1 \}$.

- Implicit *probability measure* $\mathbf{P}$:
  - Random selection of die: $\mathbf{P}(A) = \mathbf{P}(A^c) = 1/2$.
  - Conditional probabilities for a given die:

$$\mathbf{P}(R_1 \mid A) = 5/6, \qquad \mathbf{P}(R_1 \mid A^c) = 1/6,$$

$$\mathbf{P}(R_1 \cap R_2 \mid A) = 5^2/6^2 = 25/36, \qquad \mathbf{P}(R_1 \cap R_2 \mid A^c) = 1^2/6^2 = 1/36.$$

$$\mathbf{P}\{(1; R, R)\} = \mathbf{P}(A \cap R_1 \cap R_2)$$

What is the mass function for this problem?

$$\mathbf{P}(H) = \mathbf{P}(H \mid A)\,\mathbf{P}(A) + \mathbf{P}(H \mid A^c)\,\mathbf{P}(A^c)$$

- *Sample space* Ω: set of triples (coin toss; first face, second face); generic sample point $\omega = (x; y_1, y_2)$, $x \in \{1, 2\}$, $y_1, y_2 \in \{R, W\}$.

- The *events of interest:*
  $R_1 :=$ first throw shows red $= \{\, (x; y_1, y_2) : y_1 = R \,\}$.
  $R_2 :=$ second throw shows red $= \{\, (x; y_1, y_2) : y_2 = R \,\}$.
  $A :=$ first die is chosen $= \{\, (x; y_1, y_2) : x = 1 \,\}$.

- Implicit *probability measure* $\mathbf{P}$:
  - Random selection of die: $\mathbf{P}(A) = \mathbf{P}(A^c) = 1/2$.
  - Conditional probabilities for a given die:

$$\mathbf{P}(R_1 \mid A) = 5/6, \qquad \mathbf{P}(R_1 \mid A^c) = 1/6,$$

$$\mathbf{P}(R_1 \cap R_2 \mid A) = 5^2/6^2 = 25/36, \qquad \mathbf{P}(R_1 \cap R_2 \mid A^c) = 1^2/6^2 = 1/36.$$

$$\mathbf{P}\{(1; R, R)\} = \mathbf{P}(A \cap R_1 \cap R_2) = \mathbf{P}(R_1 \cap R_2 \mid A)\,\mathbf{P}(A)$$

What is the mass function for this problem?

$$\mathbf{P}(H) = \mathbf{P}(H \mid A)\,\mathbf{P}(A) + \mathbf{P}(H \mid A^c)\,\mathbf{P}(A^c)$$

- *Sample space* Ω: set of triples (coin toss; first face, second face); generic sample point $\omega = (x; y_1, y_2)$, $x \in \{1, 2\}$, $y_1, y_2 \in \{R, W\}$.

- The *events of interest:*
  $R_1 :=$ first throw shows red $= \{ (x; y_1, y_2) : y_1 = R \}$.
  $R_2 :=$ second throw shows red $= \{ (x; y_1, y_2) : y_2 = R \}$.
  $A :=$ first die is chosen $= \{ (x; y_1, y_2) : x = 1 \}$.

- Implicit *probability measure* $\mathbf{P}$:
  - Random selection of die: $\mathbf{P}(A) = \mathbf{P}(A^c) = 1/2$.
  - Conditional probabilities for a given die:

$$\mathbf{P}(R_1 \mid A) = 5/6, \qquad \mathbf{P}(R_1 \mid A^c) = 1/6,$$

$$\mathbf{P}(R_1 \cap R_2 \mid A) = 5^2/6^2 = 25/36, \qquad \mathbf{P}(R_1 \cap R_2 \mid A^c) = 1^2/6^2 = 1/36.$$

$$\mathbf{P}\{(1; R, R)\} = \mathbf{P}(A \cap R_1 \cap R_2) = \mathbf{P}(R_1 \cap R_2 \mid A)\,\mathbf{P}(A) = \frac{25}{36} \cdot \frac{1}{2} = \frac{25}{72}$$

What is the mass function for this problem?

$$\mathbf{P}(H) = \mathbf{P}(H \mid A)\,\mathbf{P}(A) + \mathbf{P}(H \mid A^c)\,\mathbf{P}(A^c)$$

- *Sample space* Ω: set of triples (coin toss; first face, second face); generic sample point $\omega = (x; y_1, y_2)$, $x \in \{1, 2\}$, $y_1, y_2 \in \{R, W\}$.

- The *events of interest:*
  $R_1 :=$ first throw shows red $= \{\, (x; y_1, y_2) : y_1 = R \,\}$.
  $R_2 :=$ second throw shows red $= \{\, (x; y_1, y_2) : y_2 = R \,\}$.
  $A :=$ first die is chosen $= \{\, (x; y_1, y_2) : x = 1 \,\}$.

- Implicit *probability measure* $\mathbf{P}$:
  - Random selection of die: $\mathbf{P}(A) = \mathbf{P}(A^c) = 1/2$.
  - Conditional probabilities for a given die:

$$\mathbf{P}(R_1 \mid A) = 5/6, \qquad \mathbf{P}(R_1 \mid A^c) = 1/6,$$

$$\mathbf{P}(R_1 \cap R_2 \mid A) = 5^2/6^2 = 25/36, \qquad \mathbf{P}(R_1 \cap R_2 \mid A^c) = 1^2/6^2 = 1/36.$$

$$\mathbf{P}\{(1; R, R)\} = \mathbf{P}(A \cap R_1 \cap R_2) = \mathbf{P}(R_1 \cap R_2 \mid A)\,\mathbf{P}(A) = \frac{25}{36} \cdot \frac{1}{2} = \frac{25}{72}$$

$$\mathbf{P}\{(2; R, W)\} = \mathbf{P}(A^c \cap R_1 \cap R_2^c) = \mathbf{P}(R_1 \cap R_2^c \mid A^c)\,\mathbf{P}(A^c) = \frac{1 \cdot 5}{36} \cdot \frac{1}{2} = \frac{5}{72}$$

What is the mass function for this problem?

$$\mathbf{P}(H) = \mathbf{P}(H \mid A)\,\mathbf{P}(A) + \mathbf{P}(H \mid A^{c})\,\mathbf{P}(A^{c})$$

Atom	Probability
$\{(1; R, R)\}$	25/72
$\{(1; R, W)\}$	5/72
$\{(1; W, R)\}$	5/72
$\{(1; W, W)\}$	1/72
$\{(2; R, R)\}$	1/72
$\{(2; R, W)\}$	5/72
$\{(2; W, R)\}$	5/72
$\{(2; W, W)\}$	25/72

$$\mathbf{P}\{(1; R, R)\} = \mathbf{P}(A \cap R_1 \cap R_2) = \mathbf{P}(R_1 \cap R_2 \mid A)\,\mathbf{P}(A) = \tfrac{25}{36} \cdot \tfrac{1}{2} = \tfrac{25}{72}$$

$$\mathbf{P}\{(2; R, W)\} = \mathbf{P}(A^{c} \cap R_1 \cap R_2^{c}) = \mathbf{P}(R_1 \cap R_2^{c} \mid A^{c})\,\mathbf{P}(A^{c}) = \tfrac{1 \cdot 5}{36} \cdot \tfrac{1}{2} = \tfrac{5}{72}$$

- *Sample space* Ω: set of triples (coin toss; first face, second face); generic sample point $\omega = (x; y_1, y_2)$, $x \in \{1, 2\}$, $y_1, y_2 \in \{R, W\}$.

- The *events of interest:*
  $R_1 :=$ first throw shows red $= \{\, (x; y_1, y_2) : y_1 = R \,\}$.
  $R_2 :=$ second throw shows red $= \{\, (x; y_1, y_2) : y_2 = R \,\}$.
  $A :=$ first die is chosen $= \{\, (x; y_1, y_2) : x = 1 \,\}$.

- Implicit *probability measure* $\mathbf{P}$:
  - Random selection of die: $\mathbf{P}(A) = \mathbf{P}(A^{c}) = 1/2$.
  - Conditional probabilities for a given die:

$$\mathbf{P}(R_1 \mid A) = 5/6, \qquad \mathbf{P}(R_1 \mid A^{c}) = 1/6,$$

$$\mathbf{P}(R_1 \cap R_2 \mid A) = 5^2/6^2 = 25/36, \qquad \mathbf{P}(R_1 \cap R_2 \mid A^{c}) = 1^2/6^2 = 1/36.$$