What is the difference between Singular Value and Eigenvalue?

I am trying to prove some statements about singular value decomposition, but I am not sure what the difference between singular value and eigenvalue is. Is singular value just another name for eigenvalue?

(definition) (eigenvalues-eigenvectors) (spectral-theory)

edited Apr 3 '12 at 8:08
Emre
2,358 1 11 20

asked Apr 3 '12 at 3:19
Ramon
179 1 3 3

2 They agree in finite dimensions, but not necessarily for infinite-dimensional operators. I've heard the term "singular value" applied to any value for which $(A-\lambda I)^{-1}$ either does not exist or is not continuous, while eigenvalues refer only to those values for which $(A-\lambda I)^{-1}$ does not exist. – Alex Becker Apr 3 '12 at 3:30

1 The singular value is a nonnegative scalar of a square or rectangular matrix while an eigenvalue is a scalar (any scalar) of a square matrix. – Hassan Muhammad Apr 3 '12 at 3:37

^Note that I was addressing square matrices specifically, or in the infinite-dimensional case, endomorphisms. – Alex Becker Apr 3 '12 at 3:56

1 My guess is that the question is about the singular value decomposition for matrices of finite-dimensional operators. – yep Apr 3 '12 at 4:02

1 They are not the same thing at all, and has nothing to do with dimension. They only agree in the special case where the matrix is symmetric. This agreement also extends (in a sense) for infinite dimensional compact operators. – Nick Alger Sep 30 '12 at 2:46

5 Answers

The singular values of a $M \times N$ matrix X are the square roots of the eigenvalues of the $N \times N$ matrix X^*X (where $*$ stands for the transpose-conjugate matrix if it has complex coefficients, or the transpose if it has real coefficients).

Thus, if X is $N \times N$ real symmetric matrix with non-negative eigenvalues, then eigenvalues and singular values coincide, but it is not generally the case !

edited Oct 1 '12 at 23:34

answered Apr 3 '12 at 4:47

Student

1,158 6 13

Correction: "positive semidefinite", not "with non-negative coefficients". For example, $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ has an eigenvalue -1, which is not a singular value. – user31373 Sep 30 '12 at 4:49

Typo corrected, thx. – Student Oct 1 '12 at 23:33

what about the case in which X is square but not symmetric. The eigenvalues of X could be negative correct? And in that case how do we define the singular values? – Matteo Apr 9 '14 at 18:42

@matteo: I don't understand your question. Whatever matrix X you choose (square or not), the matrix X^*X is Hermitian (or symmetric is the entries are real) positive definite, and the definition I provided makes sense. If X is a square matrix with a negative eigenvalue, then its eigenvalues and singular values are just not the same. – Student Apr 21 '14 at 16:07

I guess you're right, I wasn't really thinking of the fact that they're simply different. Just to make sure about one last thing, is X^*X always hermitian and positive definite? – Matteo Apr 21 '14 at 16:33

> is singular value just another name for eigenvalue?

No, singular values & eigenvalues are different.

> What is the difference between Singular Value and Eigenvalue?

There are many possible answers to this question. Since I don't know what you're trying to prove, I'd recommend carefully comparing definitions between the two: eigendecomposition, singular value decomposition

[EDIT: You might find the first several chapters of the book "Numerical Linear Algebra" by Trefethen and Bau more useful than the Wikipedia article. They're available here.]

Two important points:

- Notice in particular that the SVD is defined for any matrix, while the eigendecomposition is defined only for square matrices (and more specifically, normal matrices).
- Notice that singular values are always real, while eigenvalues need not be real.

edited Apr 3 '12 at 3:58 answered Apr 3 '12 at 3:42

yep
441 2 7

Given a matrix A, if the eigenvalues of $A^H A$ are $\lambda_i \geq 0$, then $\sqrt{\lambda_i}$ are the singular values of A. If t is an eigenvalue of A, then $|t|$ is a singular value of A. And here is an example should be noticed,

$$A = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{pmatrix},$$

the eigenvalues of A are $1, 1, 0$ while the singular values of A are $\sqrt{3}, 1, 0$.

answered Dec 14 '13 at 13:00

Eden Harder
293 2 11

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1 That's a very nice example. Another example is

$$\mathbf{A} = \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}.$$

The eigenvalues are 1 and 0, the singular values are $\sqrt{2}$ and 0. – hbp Oct 22 '15 at 6:18

"If t is an eivenvalue of A, then $|t|$ is a singular value of A" - this is not true, though it does convey part of the (admittedly vague, but still useful) intuition that the eigenvalues and singular values are "the same size" – stochastic Apr 12 at 22:50

Also Very Good pdf by Matlab.....

http://www.mathworks.com/moler/eigs.pdf

answered Jan 23 '13 at 3:28

Enoch
9 1

6 While this link may answer the question, it is better to include the essential parts of the answer here and provide the link for reference. Link-only answers can become invalid if the linked page changes. – user53153 Jan 23 '13 at 4:33

Singular values of the SVD decomposition of the matrix A is the square root of the eigenvalues of the matrix (A multiplied by A transpose) or(A transpose multplied by A), the two ar identical with positive eigenvalues.

answered Jul 15 '14 at 13:51

Harald Trasti
1