Advice from a magician — a view from a sample point

Events $A_1, A_2, \ldots, A_n$

$$X_j = \begin{cases} 1 & \text{if } A_j \text{ occurs,} \\ 0 & \text{otherwise.} \end{cases} \qquad S_k := \sum_{1 \le j_1 < j_2 < \cdots < j_k \le n} \mathbf{P}(A_{j_1} \cap A_{j_2} \cap \cdots \cap A_{j_k}) \qquad (1 \le k \le n)$$

Inclusion–exclusion: The distribution of the number of occurrences of $A_1, \ldots, A_n$ is given by

$$\mathbf{P}\{X_1 + \cdots + X_n = k\} = \binom{k}{k} S_k - \binom{k+1}{k} S_{k+1} + \cdots + (-1)^j \binom{k+j}{k} S_{k+j} + \cdots + (-1)^{n-k} S_n$$

Fix any k and select any sample point ω.
How much does ω contribute to the left and to the right of the equation?

The selected sample point ω will lie in a certain number, say, l of the sets A_j:

- Identify the sets: suppose ω is in $A_{j_1}, A_{j_2}, \ldots, A_{j_l}$.
- Then $X_1 + X_2 + \cdots + X_n = l$.
- Three cases: (1) $l < k$. (2) $l = k$. (3) $l > k$.