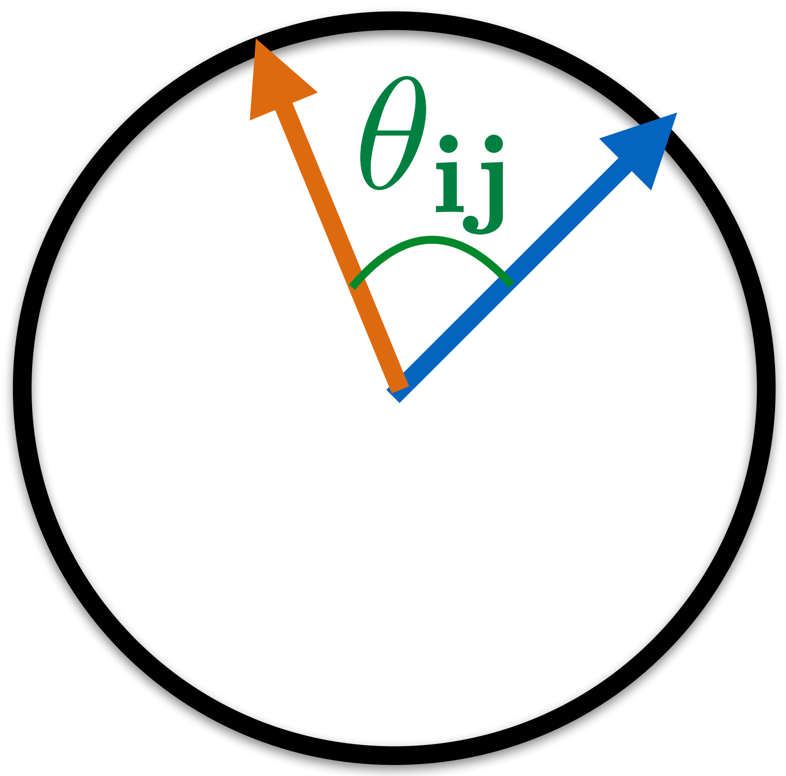
Maxcut

Analysis

$$\textbf{OPT:} \leq \sum_{\{\mathbf{i},\mathbf{j}\}\in \mathbf{E}} \mathbf{w_{ij}} \frac{-\mathbf{v_i}\cdot\mathbf{v_j}+\mathbf{1}}{\mathbf{2}}$$

$$= \sum_{\{\mathbf{i},\mathbf{j}\}\in \mathbf{E}} \mathbf{w_{ij}} \frac{-\cos\theta_{\mathbf{ij}}+\mathbf{1}}{\mathbf{2}}$$



Analysis

E(Value(Output)): $\sum_{\{i,j\}\in E} w_{ij}\ E\left(\frac{-x_i x_j + 1}{2}\right)$

$= \sum_{\{i,j\}\in E} w_{ij}\ \Pr(x_i \neq x_j)$

Random line (hyperplane) H

above H: $v_i \mapsto 1$

below H: $v_i \mapsto -1$

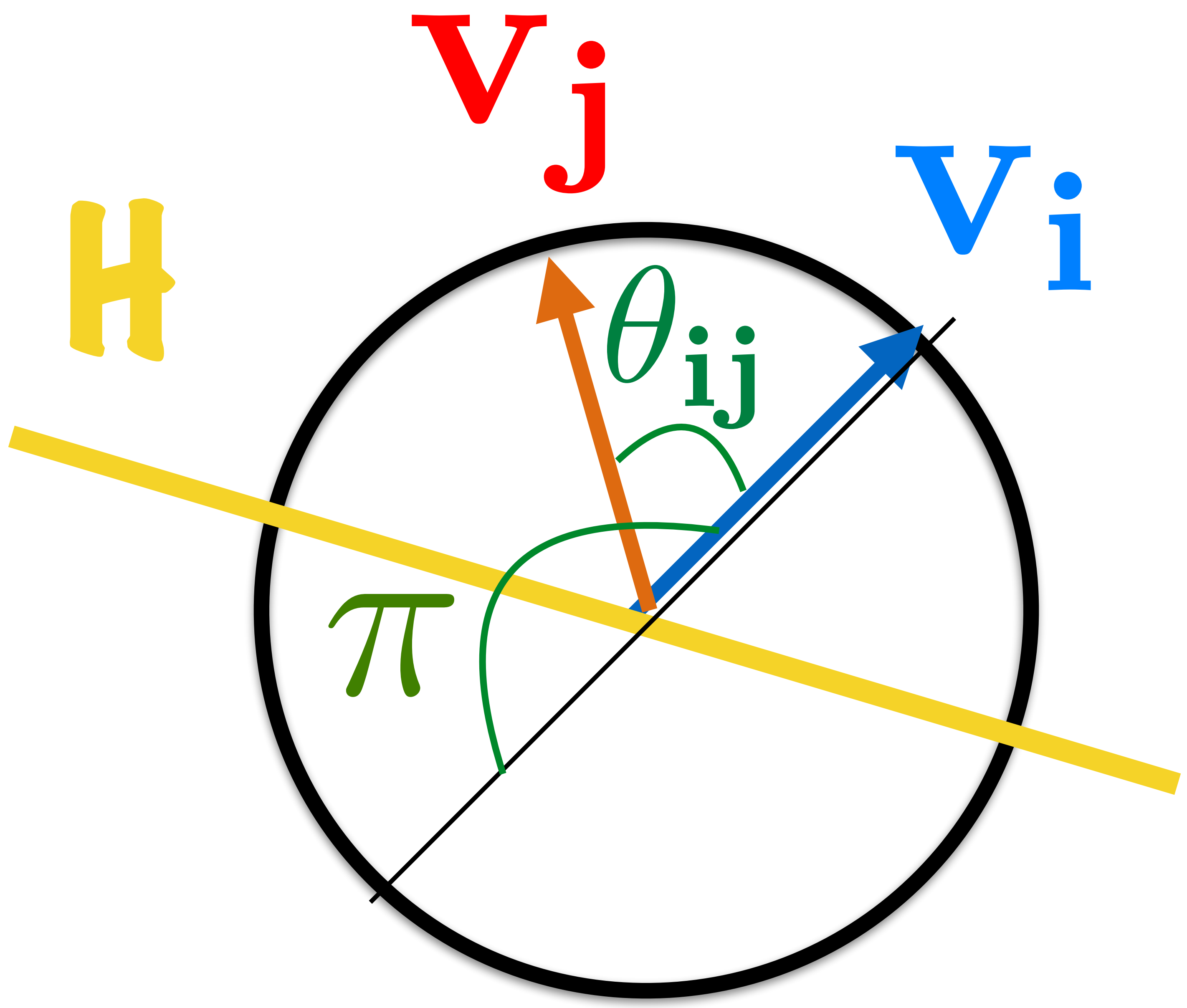
$$\Pr(x_i \neq x_j) = \Pr(\text{H separates } v_i \text{ from } v_j)$$

Analysis

Random line (hyperplane) H

above H: $\mathbf{v_i} \mapsto 1$

below H: $\mathbf{v_i} \mapsto -1$



$$\Pr(\mathbf{H\ separates\ v_i\ from\ v_j}) = \theta_{\mathbf{ij}}/\pi$$

Analysis

E(Value(Output)): $\sum_{\{i,j\}\in E} w_{ij} \frac{\theta_{ij}}{\pi}$

OPT: $\leq \sum_{\{i,j\}\in E} w_{ij} \frac{-\cos\theta_{ij}+1}{2}$

Lemma: $\forall \theta : \quad \frac{\theta}{\pi} \geq .878\ldots \frac{-\cos\theta+1}{2}$

E(Value(Output)) $\geq .878\ldots$ OPT

Better than .5

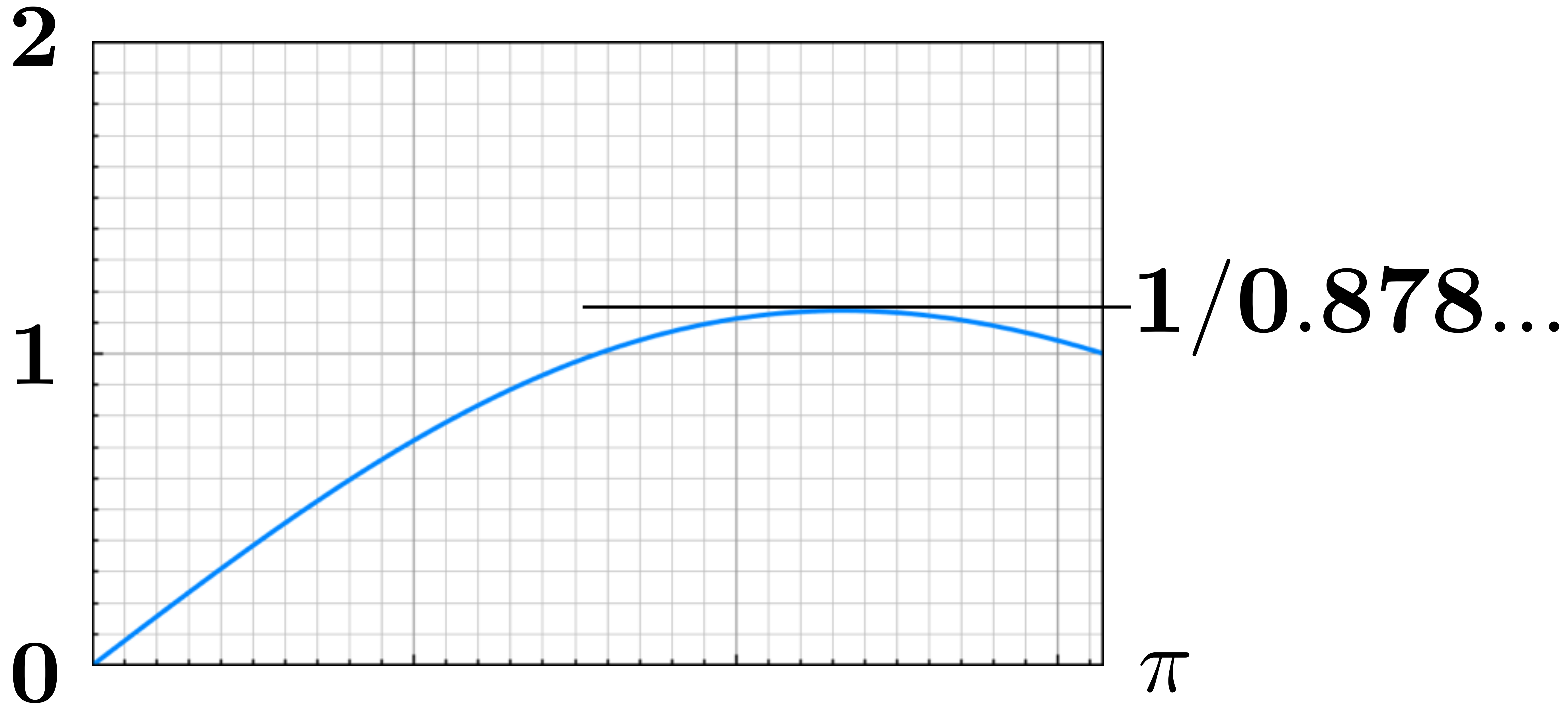
QED

Proof of Lemma

$$\forall\theta : \quad \frac{\theta}{\pi} \geq .878\ldots\frac{-\cos\theta+1}{2}$$

$$f : \theta \mapsto \frac{\pi(1-\cos\theta)}{2\theta}$$

What is the maximum of f?



QED

Recap

1. Solve SDP relaxation:

$$\max \sum_{\{i,j\}\in E} w_{ij} \frac{-v_i \cdot v_j + 1}{2} \quad : \quad v_i \cdot v_i = 1$$

2. Rounding:

Random line (hyperplane) H

above H: $v_i \mapsto 1$

below H: $v_i \mapsto -1$

3. Output resulting cut

Theorem: E(Value(Output)) $\geq .878\ldots$ OPT

Maxcut