A familial paradox

A family with two children is known to have (at least) one boy. What are the chances the other child is a boy?

A familial paradox

A family with two children is known to have (at least) one boy. What are the chances the other child is a boy?

• The events of interest:

The events of interest:A := both children are boys = {\mathbb{B}}

• The events of interest:

 $A := both children are boys = \{\mathfrak{BB}\}\$

 $B := (at least) one child is a boy = {\mathfrak{BB}, \mathfrak{BG}, \mathfrak{GB}}$

• The events of interest:

 $A := both children are boys = \{\mathfrak{BB}\}\$

 $B := (at least) one child is a boy = {\mathfrak{BB}, \mathfrak{BG}, \mathfrak{GB}}$

- **Probability** measure **P**:
 - Combinatorial setting: to each *atom* (singleton set) assign equal *probability mass*

$$\mathbf{P}\{\mathfrak{BB}\} = \mathbf{P}\{\mathfrak{BB}\} = \mathbf{P}\{\mathfrak{BB}\} = \mathbf{P}\{\mathfrak{BB}\} = 1/4$$

$$P(A) = P\{\mathfrak{BB}\} = 1/4$$

$$P(B) = P\{\mathfrak{BB}\} + P\{\mathfrak{BB}\} + P\{\mathfrak{BB}\} = 3/4$$

 33
 35

 53
 55

 54
 55

Sample space: it is natural to list the gender of the child by age $\Omega = \{\mathfrak{BB}, \mathfrak{BG}, \mathfrak{GB}, \mathfrak{GB}\}$

• The events of interest:

 $A := both children are boys = \{\mathfrak{BB}\}\$

 $B := (at least) one child is a boy = {\mathfrak{BB}, \mathfrak{BG}, \mathfrak{GB}}$

- *Probability measure* **P**:
 - Combinatorial setting: to each *atom* (singleton set) assign equal *probability mass*

$$\mathbf{P}\{\mathfrak{BB}\} = \mathbf{P}\{\mathfrak{BB}\} = \mathbf{P}\{\mathfrak{BB}\} = \mathbf{P}\{\mathfrak{BB}\} = 1/4$$

$$P(A) = P\{\mathfrak{BB}\} = 1/4$$

$$P(B) = P\{\mathfrak{BB}\} + P\{\mathfrak{BB}\} + P\{\mathfrak{BB}\} = 3/4$$

The chance of A (without side information)
$$P(A) = 1/4$$

• The events of interest:

 $A := both children are boys = \{\mathfrak{BB}\}\$

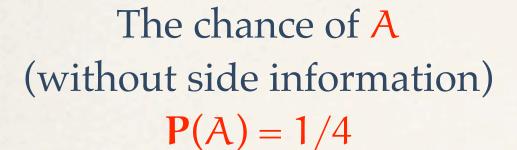
 $B := (at least) one child is a boy = {\mathfrak{BB}, \mathfrak{BG}, \mathfrak{GB}}$

- Probability measure P:
 - Combinatorial setting: to each *atom* (singleton set) assign equal *probability mass*

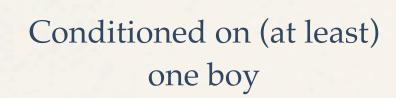
$$P\{\mathfrak{BB}\} = P\{\mathfrak{BB}\} = P\{\mathfrak{BB}\} = P\{\mathfrak{BB}\} = 1/4$$

$$P(A) = P\{\mathfrak{BB}\} = 1/4$$

$$P(B) = P\{\mathfrak{BB}\} + P\{\mathfrak{BB}\} + P\{\mathfrak{BB}\} = 3/4$$











• The events of interest:

 $A := both children are boys = {\mathfrak{BB}}$ $B := (at least) one child is a boy = {\mathfrak{BB}, \mathfrak{BG}, \mathfrak{GB}}$

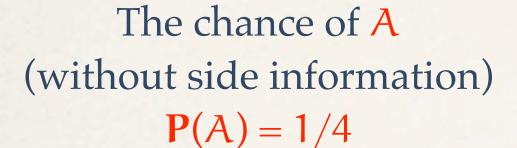
Probability measure **P**:

• Combinatorial setting: to each *atom* (singleton set) assign equal *probability mass*

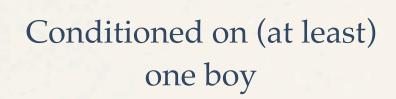
$$P\{\mathfrak{BB}\} = P\{\mathfrak{BB}\} = P\{\mathfrak{BB}\} = P\{\mathfrak{BB}\} = 1/4$$

$$P(A) = P\{\mathfrak{BB}\} = 1/4$$

$$P(B) = P\{\mathfrak{BB}\} + P\{\mathfrak{BB}\} + P\{\mathfrak{BB}\} = 3/4$$









The chance of A (given that B occurs)

$$\frac{\mathbf{P}(A \cap B)}{\mathbf{P}(B)} = \frac{1/4}{3/4} = \frac{1}{3}$$

B BB BB GB Sample space: it is natural to list the gender of the child by age $\Omega = \{\mathfrak{BB}, \mathfrak{BG}, \mathfrak{GB}, \mathfrak{GB}\}$

• The events of interest:

 $A := both children are boys = \{\mathfrak{BB}\}\$

 $B := (at least) one child is a boy = {\mathfrak{BB}, \mathfrak{BG}, \mathfrak{GB}}$

- Probability measure P:
 - Combinatorial setting: to each *atom* (singleton set) assign equal *probability mass*

$$\mathbf{P}\{\mathfrak{BB}\} = \mathbf{P}\{\mathfrak{BB}\} = \mathbf{P}\{\mathfrak{BB}\} = \mathbf{P}\{\mathfrak{BB}\} = 1/4$$

$$P(A) = P\{\mathfrak{BB}\} = 1/4$$

$$P(B) = P\{\mathfrak{BB}\} + P\{\mathfrak{BB}\} + P\{\mathfrak{BB}\} = 3/4$$