

Pepys's problem

Pepys's problem

Is it more probable to obtain at least one ace (= 6) in six throws of a die,
or to obtain at least two aces in twelve throws?

Pepys's problem

Is it more probable to obtain at least one ace (= 6) in six throws of a die,
or to obtain at least two aces in twelve throws?

Analysis: Each throw of a die is a Bernoulli trial with success (ace) probability $p = 1/6$.
aces in n throws: $S_n \sim \text{Binomial}(n, 1/6)$.

Pepys's problem

Is it more probable to obtain at least one ace (= 6) in six throws of a die,
or to obtain at least two aces in twelve throws?

Analysis: Each throw of a die is a Bernoulli trial with success (ace) probability $p = 1/6$.
aces in n throws: $S_n \sim \text{Binomial}(n, 1/6)$.

$$\mathbf{P}\{S_6 \geq 1\} = 1 - \mathbf{P}\{S_6 = 0\} = 1 - b_6(0; 1/6)$$

Pepys's problem

Is it more probable to obtain at least one ace (= 6) in six throws of a die,
or to obtain at least two aces in twelve throws?

Analysis: Each throw of a die is a Bernoulli trial with success (ace) probability $p = 1/6$.
aces in n throws: $S_n \sim \text{Binomial}(n, 1/6)$.

$$\mathbf{P}\{S_6 \geq 1\} = 1 - \mathbf{P}\{S_6 = 0\} = 1 - b_6(0; 1/6) = 1 - \binom{6}{0} \left(\frac{1}{6}\right)^0 \left(1 - \frac{1}{6}\right)^{6-0} = 1 - \left(\frac{5}{6}\right)^6 = 0.665 \dots$$

Pepys's problem

Is it more probable to obtain at least one ace (= 6) in six throws of a die,
or to obtain at least two aces in twelve throws?

Analysis: Each throw of a die is a Bernoulli trial with success (ace) probability $p = 1/6$.
aces in n throws: $S_n \sim \text{Binomial}(n, 1/6)$.

$$\mathbf{P}\{S_6 \geq 1\} = 1 - \mathbf{P}\{S_6 = 0\} = 1 - b_6(0; 1/6) = 1 - \binom{6}{0} \left(\frac{1}{6}\right)^0 \left(1 - \frac{1}{6}\right)^{6-0} = 1 - \left(\frac{5}{6}\right)^6 = 0.665 \dots$$

$$\mathbf{P}\{S_{12} \geq 2\} = 1 - \mathbf{P}\{S_{12} = 0\} - \mathbf{P}\{S_{12} = 1\} = 1 - b_{12}(0; 1/6) - b_{12}(1; 1/6)$$

Pepys's problem

Is it more probable to obtain at least one ace (= 6) in six throws of a die,
or to obtain at least two aces in twelve throws?

Analysis: Each throw of a die is a Bernoulli trial with success (ace) probability $p = 1/6$.
aces in n throws: $S_n \sim \text{Binomial}(n, 1/6)$.

$$\mathbf{P}\{S_6 \geq 1\} = 1 - \mathbf{P}\{S_6 = 0\} = 1 - b_6(0; 1/6) = 1 - \binom{6}{0} \left(\frac{1}{6}\right)^0 \left(1 - \frac{1}{6}\right)^{6-0} = 1 - \left(\frac{5}{6}\right)^6 = 0.665 \dots$$

$$\begin{aligned} \mathbf{P}\{S_{12} \geq 2\} &= 1 - \mathbf{P}\{S_{12} = 0\} - \mathbf{P}\{S_{12} = 1\} = 1 - b_{12}(0; 1/6) - b_{12}(1; 1/6) \\ &= 1 - \binom{12}{0} \left(\frac{1}{6}\right)^0 \left(1 - \frac{1}{6}\right)^{12-0} - \binom{12}{1} \left(\frac{1}{6}\right)^1 \left(1 - \frac{1}{6}\right)^{12-1} \end{aligned}$$

Pepys's problem

Is it more probable to obtain at least one ace (= 6) in six throws of a die,
or to obtain at least two aces in twelve throws?

Analysis: Each throw of a die is a Bernoulli trial with success (ace) probability $p = 1/6$.
aces in n throws: $S_n \sim \text{Binomial}(n, 1/6)$.

$$\mathbf{P}\{S_6 \geq 1\} = 1 - \mathbf{P}\{S_6 = 0\} = 1 - b_6(0; 1/6) = 1 - \binom{6}{0} \left(\frac{1}{6}\right)^0 \left(1 - \frac{1}{6}\right)^{6-0} = 1 - \left(\frac{5}{6}\right)^6 = 0.665 \dots$$

$$\begin{aligned} \mathbf{P}\{S_{12} \geq 2\} &= 1 - \mathbf{P}\{S_{12} = 0\} - \mathbf{P}\{S_{12} = 1\} = 1 - b_{12}(0; 1/6) - b_{12}(1; 1/6) \\ &= 1 - \binom{12}{0} \left(\frac{1}{6}\right)^0 \left(1 - \frac{1}{6}\right)^{12-0} - \binom{12}{1} \left(\frac{1}{6}\right)^1 \left(1 - \frac{1}{6}\right)^{12-1} = 1 - \left(\frac{5}{6}\right)^{12} - 12 \left(\frac{5}{6}\right)^{11} = 0.618 \dots \end{aligned}$$