

Theorem: If the sequence $\{a_n\}_{n=1}^{\infty}$ tends to limit L as $n \rightarrow \infty$

then for any fixed number $M > 0$ the sequence $\{Ma_n\}_{n=1}^{\infty}$ tends to the limit ML

Proof: We are given $\lim_{n \rightarrow \infty} \{a_n\} = L$

We can find an integer n_1 such that $n \geq n_1 \Rightarrow |a_n - L| < \frac{\epsilon}{M}$

Because $M > 0$ we can say

$$M|a_n - L| < \epsilon$$

$$|Ma_n - ML| < \epsilon$$

By the definition of limit we can say that sequence $\{Ma_n\}_{n=1}^{\infty}$ tends to the limit ML

This proves the theorem