Bose–Einstein statistics

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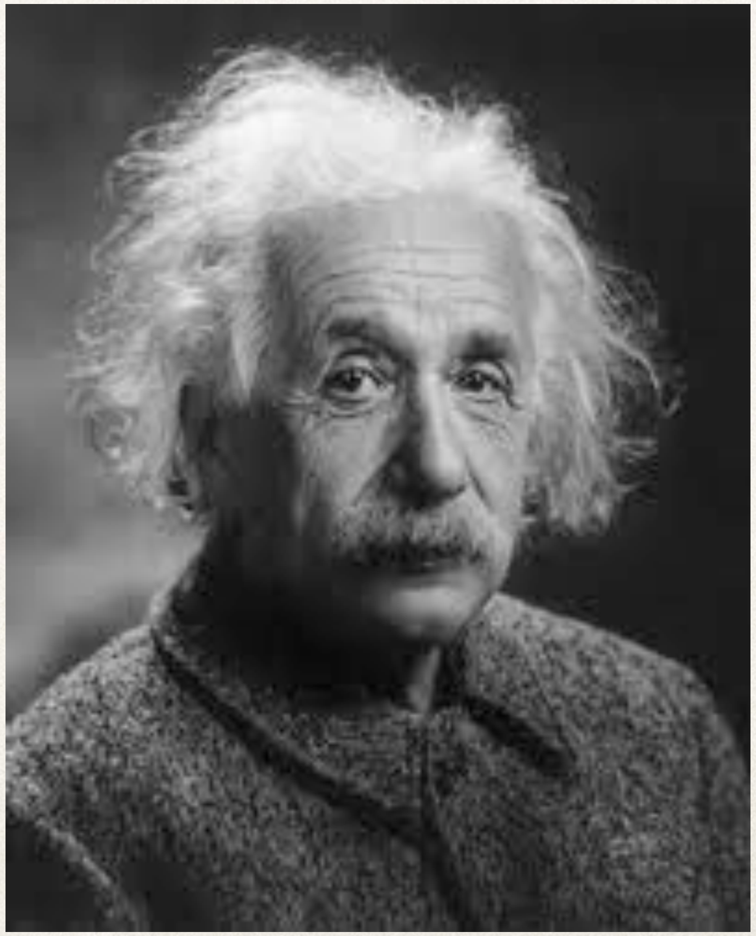

Random placement of n balls in r urns

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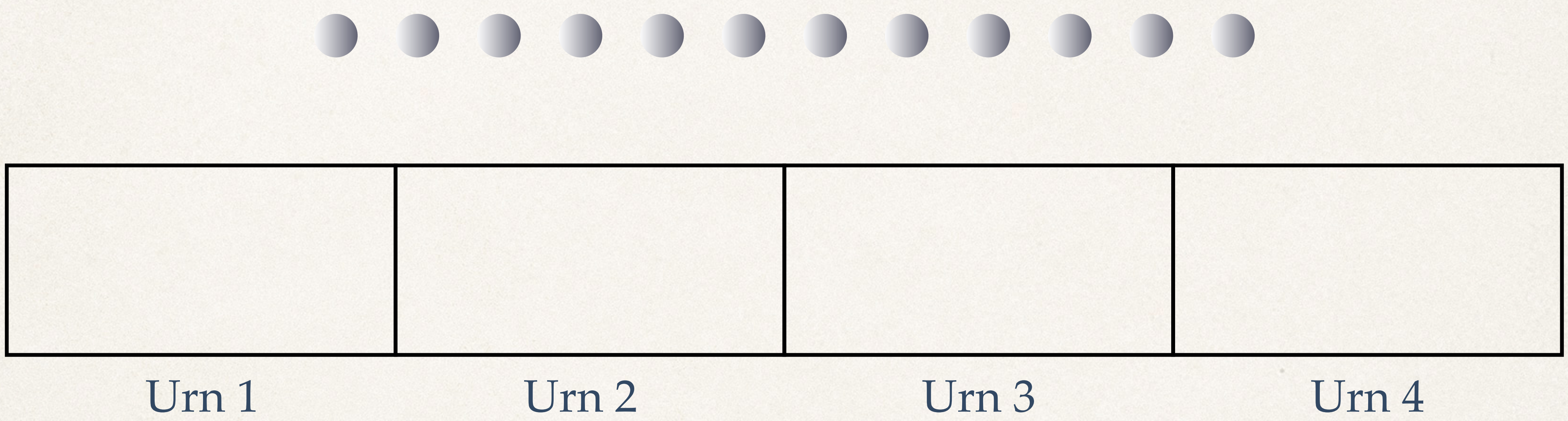
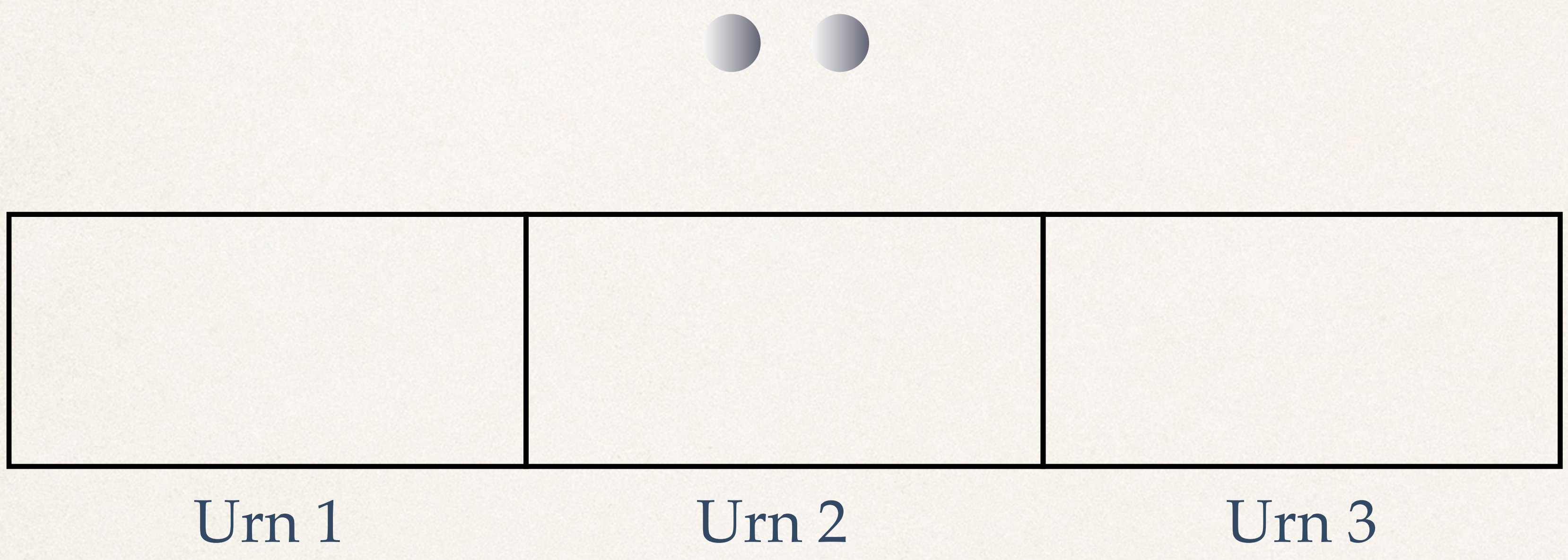
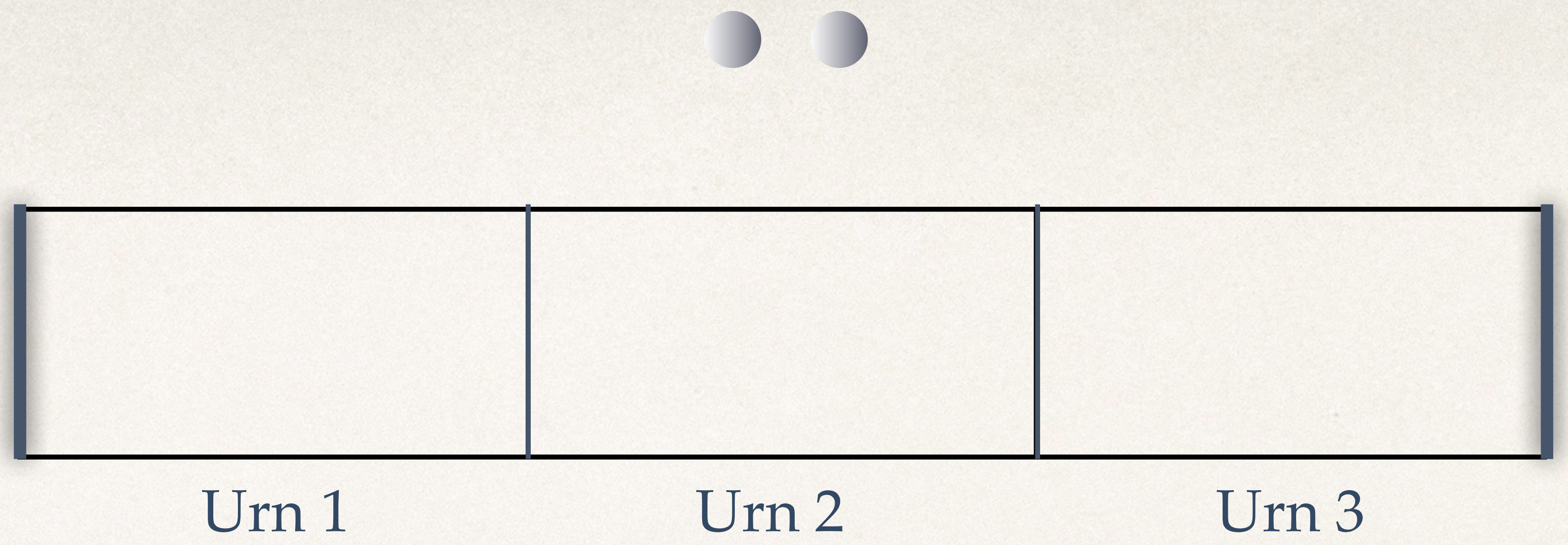
What if the balls are not distinguishable?



Random placement of n balls in r urns

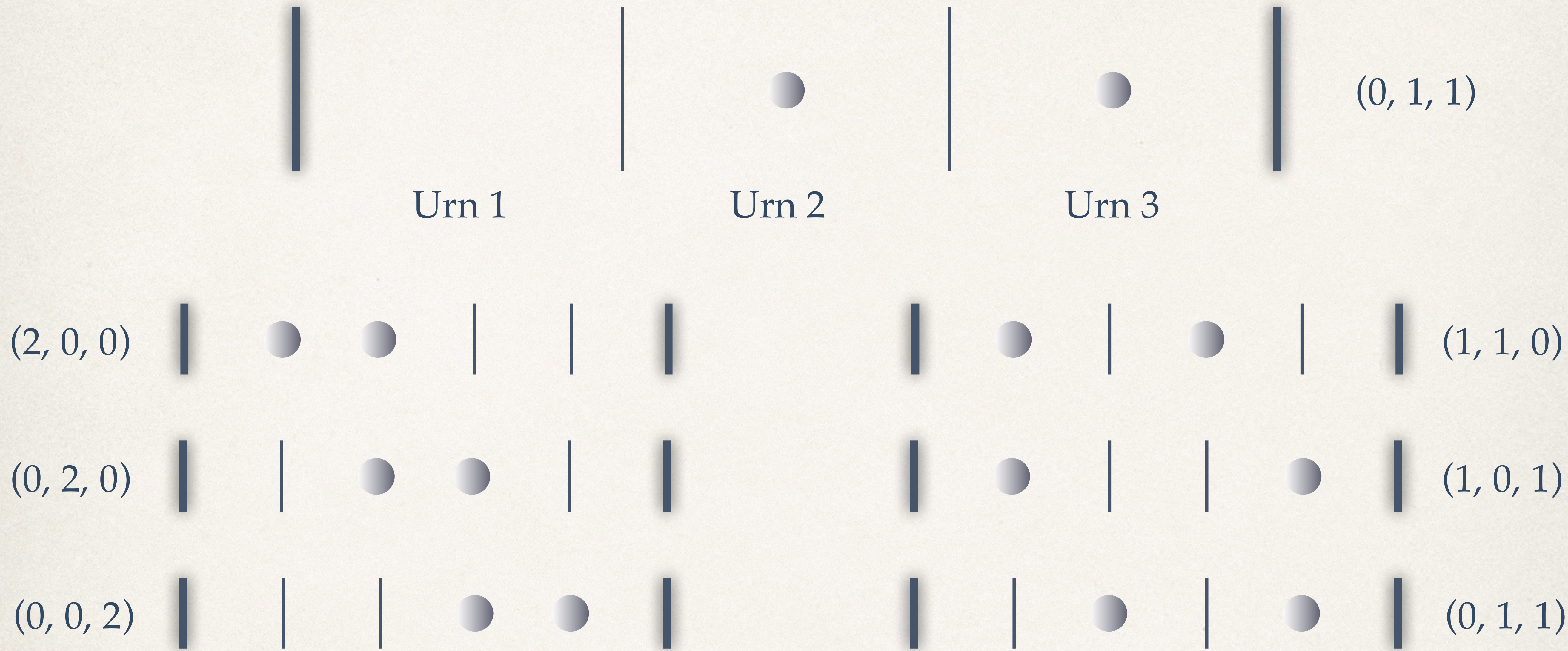
What if the balls are not distinguishable?

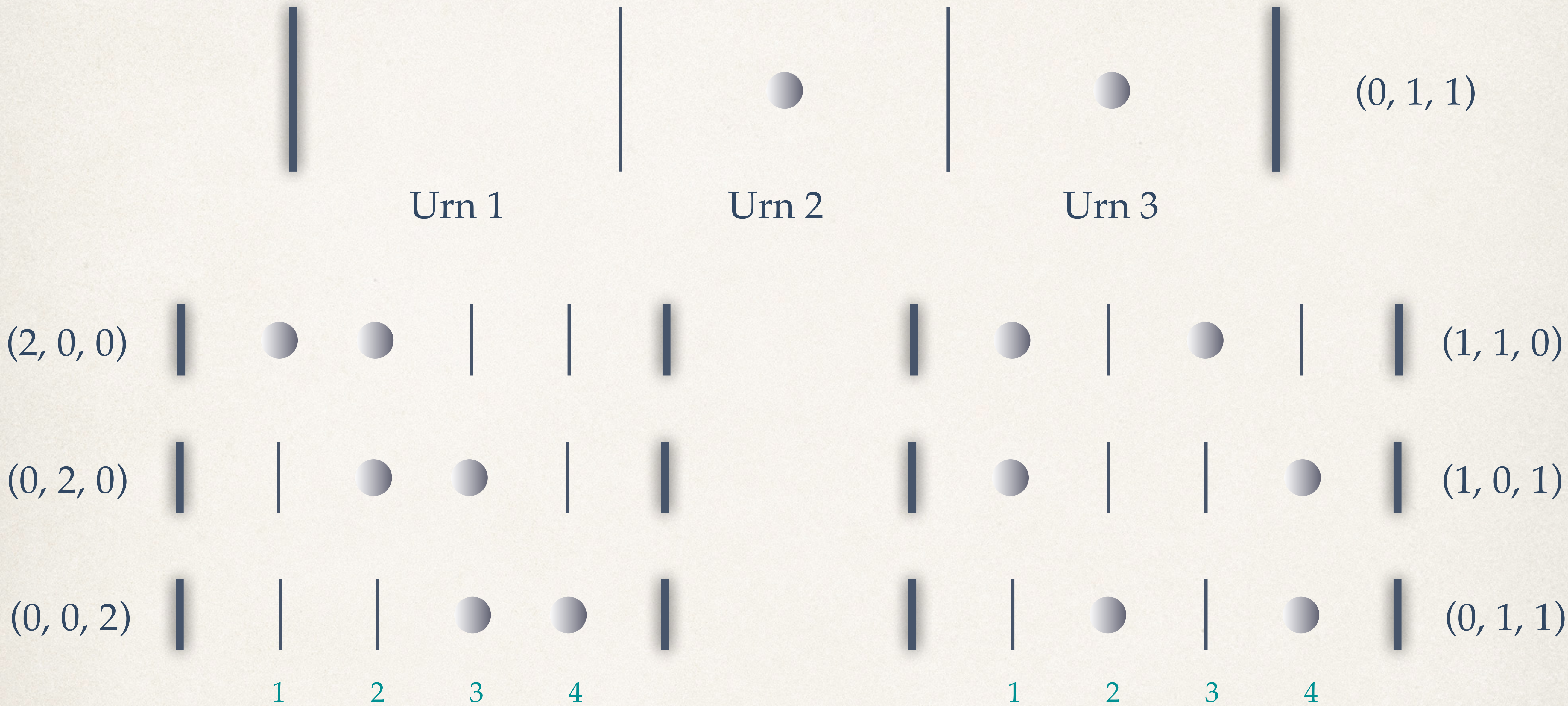


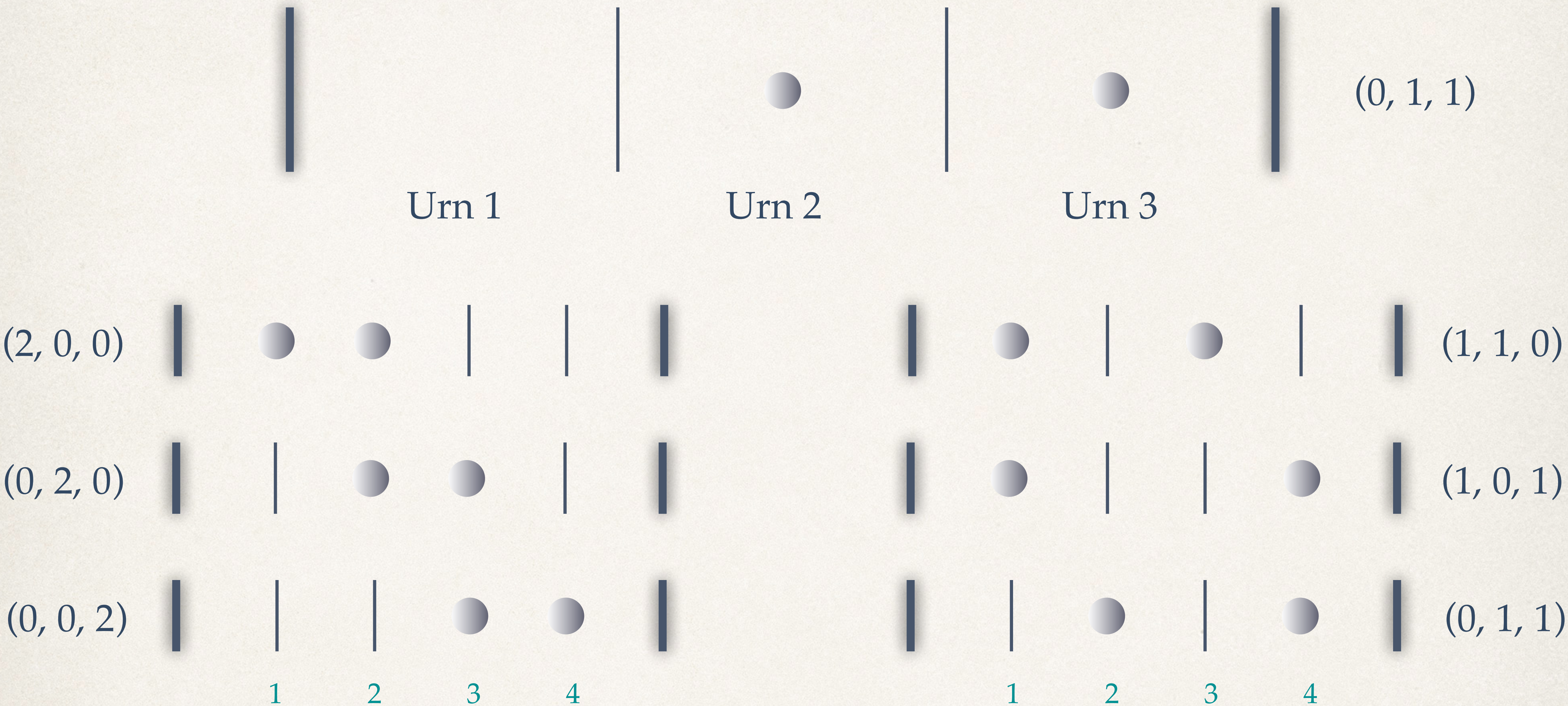

Urn 1
Urn 2
Urn 3


Urn 1
Urn 2
Urn 3

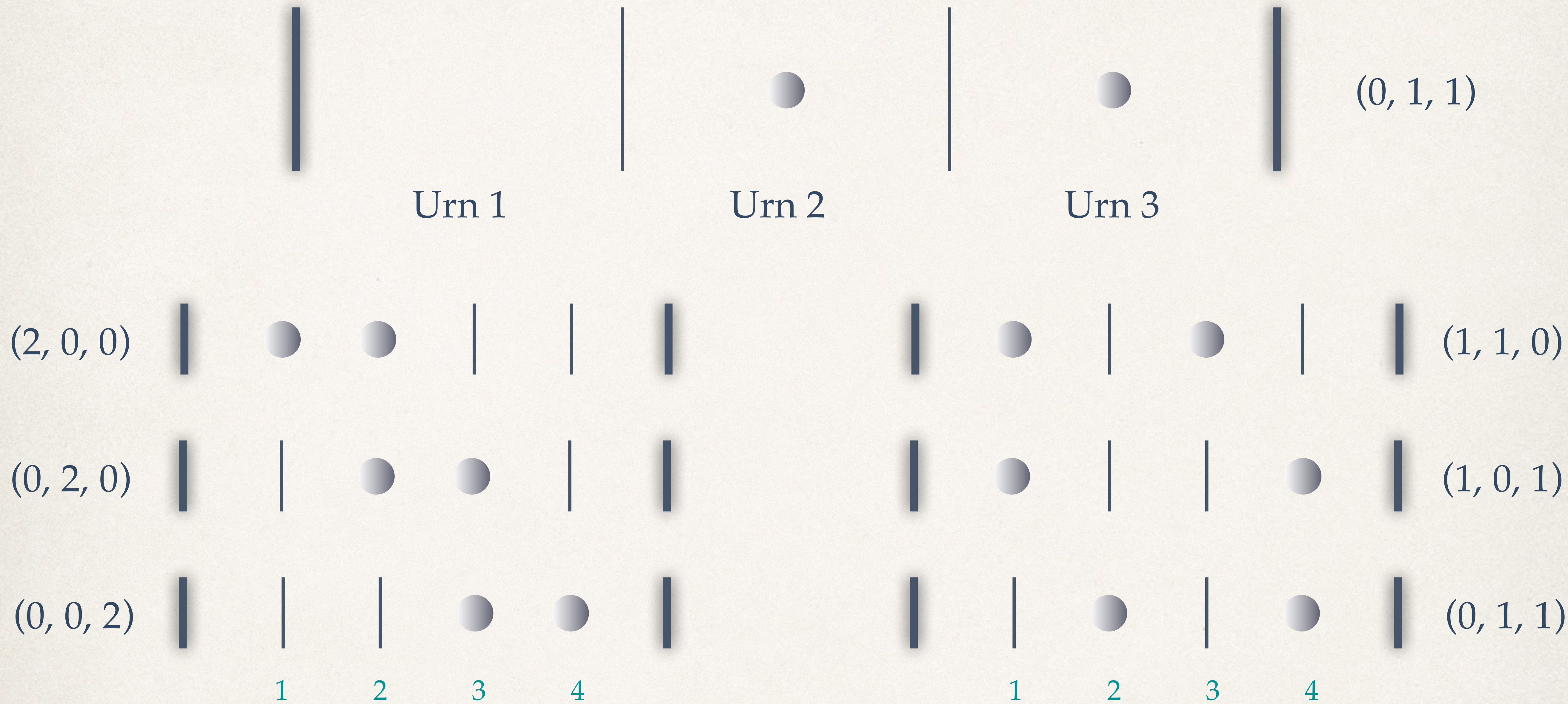

(0, 1, 1)
Urn 1
Urn 2
Urn 3


(0, 1, 1)
Urn 1
Urn 2
Urn 3
(2, 0, 0)
(1, 1, 0)
(0, 2, 0)
(1, 0, 1)
(0, 0, 2)
(0, 1, 1)


(0, 1, 1)
Urn 1
Urn 2
Urn 3
(2, 0, 0)
(1, 1, 0)
(0, 2, 0)
(1, 0, 1)
(0, 0, 2)
(0, 1, 1)
1 2 3 4
1 2 3 4



Number of stones (indistinguishable balls)	2
Number of sticks (separators; urn walls)	$3 - 1 = 2$
Number of available positions	$2 + 3 - 1 = 4$
Number of distinct configurations	$\binom{2+3-1}{2} = 6$



Number of stones (indistinguishable balls)	2	n
Number of sticks (separators; urn walls)	$3-1=2$	$r-1$
Number of available positions	$2+3-1=4$	$n+r-1$
Number of distinct configurations	$\binom{2+3-1}{2}=6$	$\binom{n+r-1}{n}$

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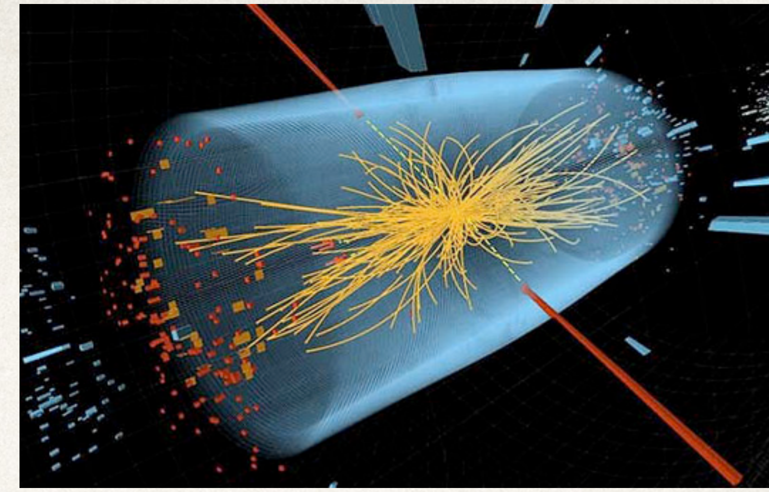
Given a random deployment of n indistinguishable balls into r distinguishable urns, the probability of obtaining a given occupancy configuration $(k_1, k_2, \ldots, k_r)$ is given by

$$P(k_1, k_2, \ldots, k_r) = \frac{1}{\binom{n+r-1}{n}} \qquad \begin{pmatrix} k_1, k_2, \ldots, k_r \geq 0 \\ k_1 + k_2 + \cdots + k_r = n \end{pmatrix}.$$

Bose–Einstein statistics



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Bosons—particles with integer spin: Photons/gluons/W/Z/Higgs Bosons, ^{4}He, ^{12}C