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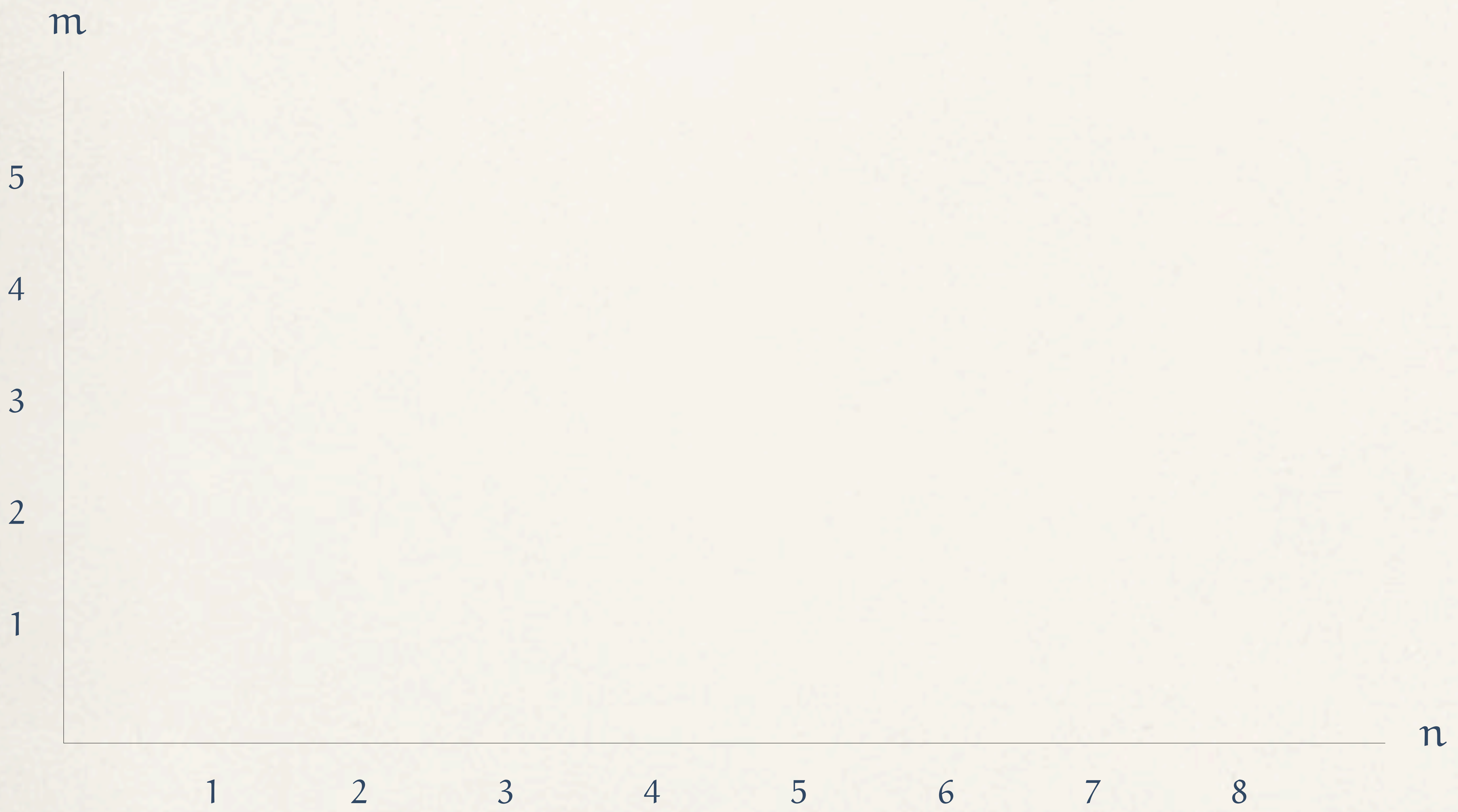

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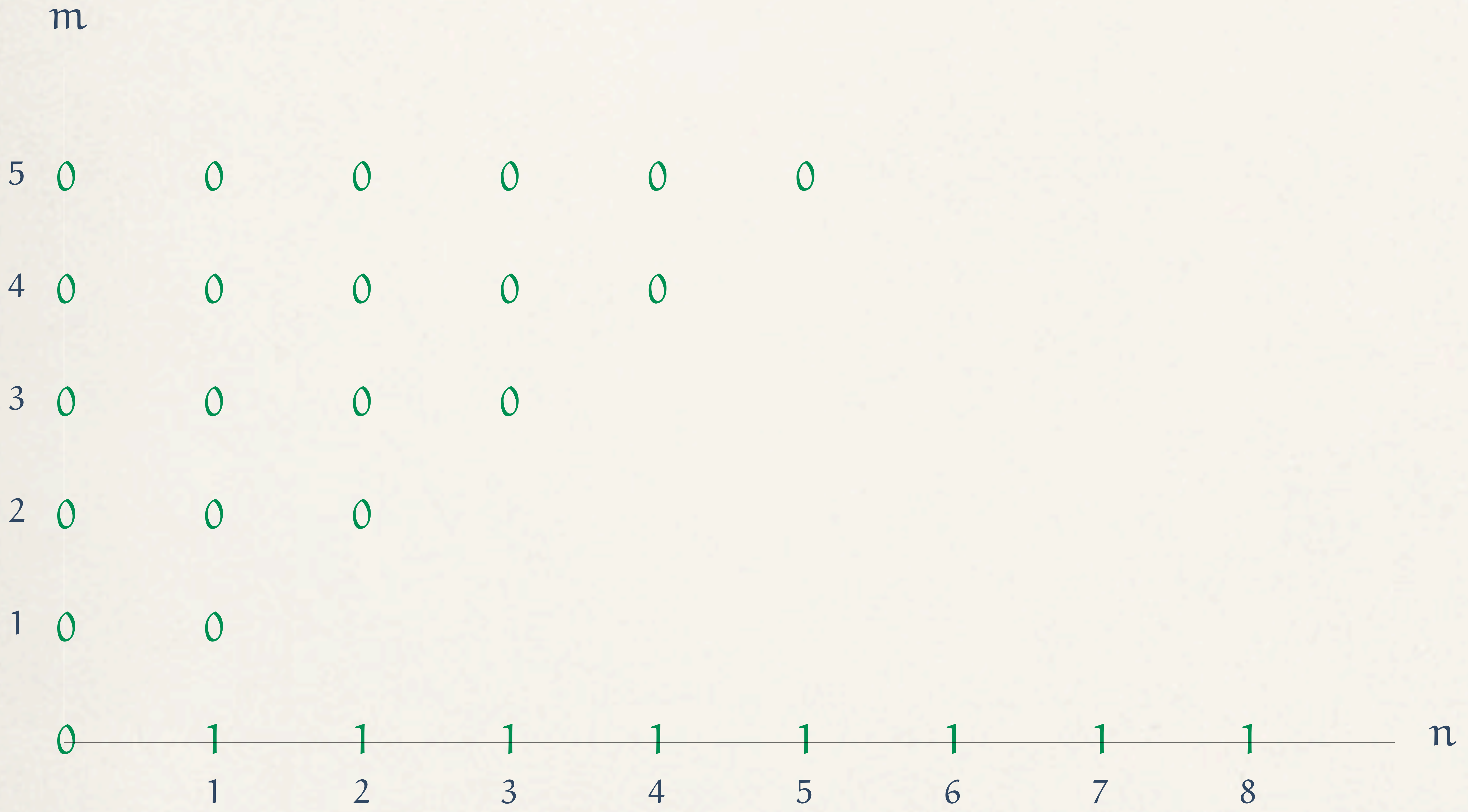

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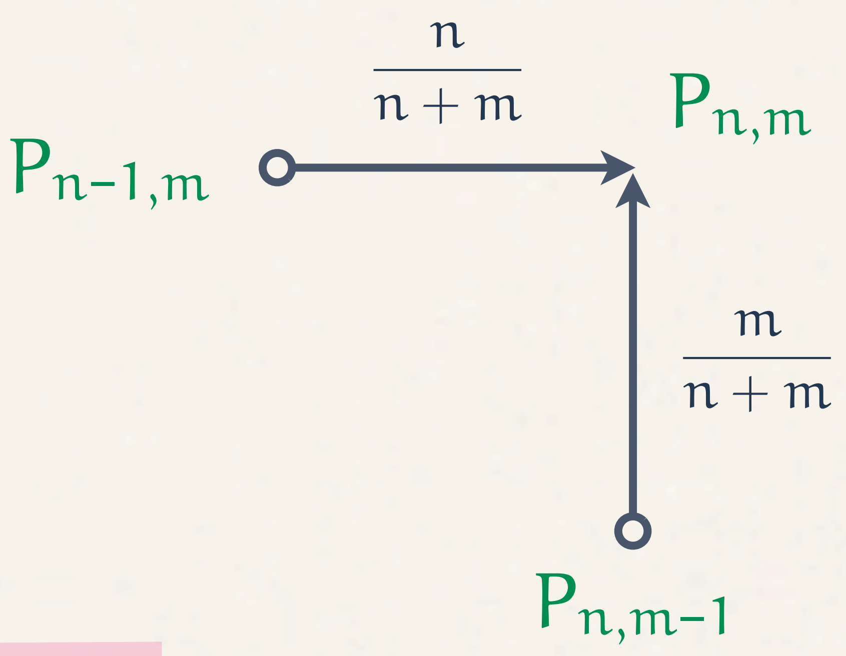

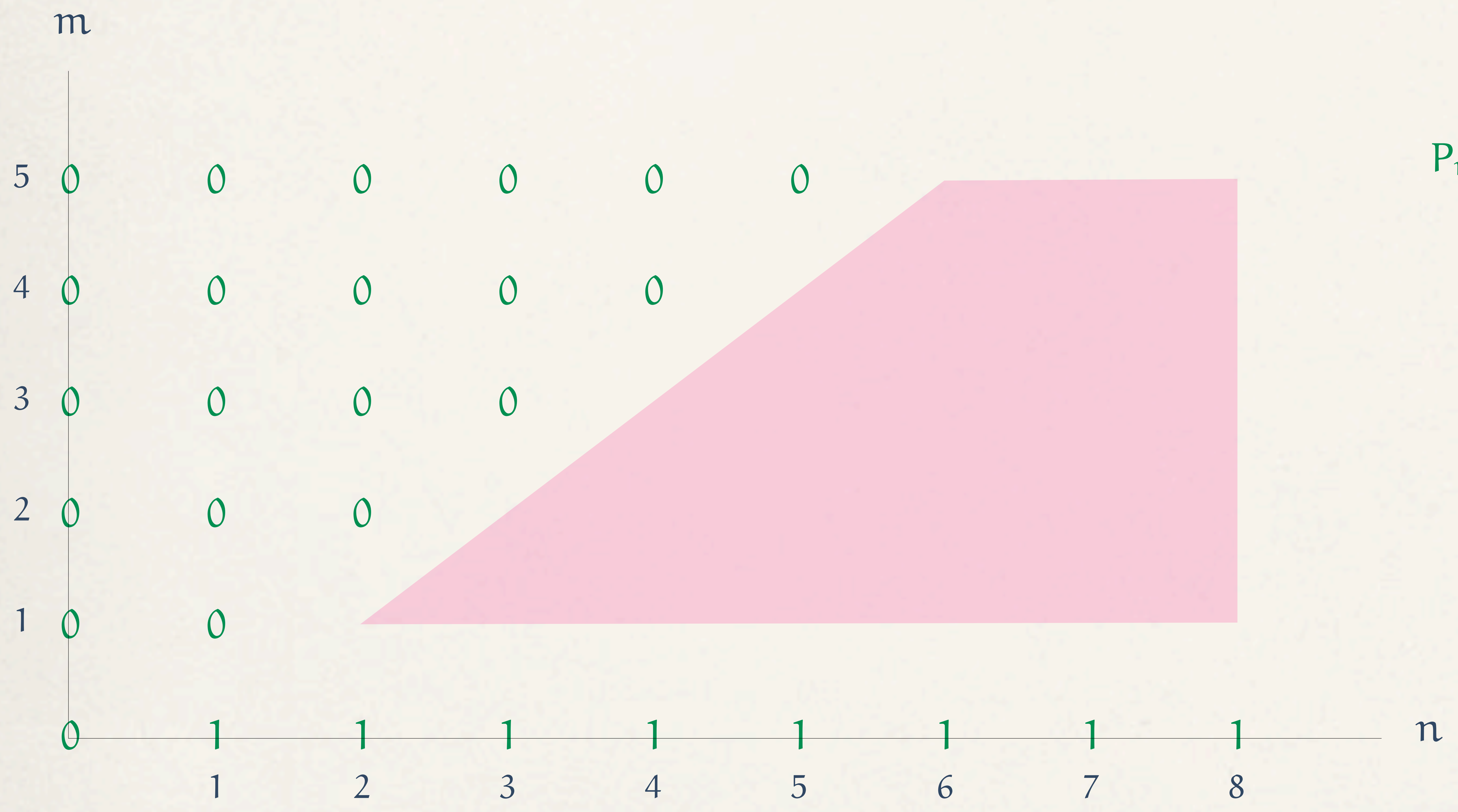

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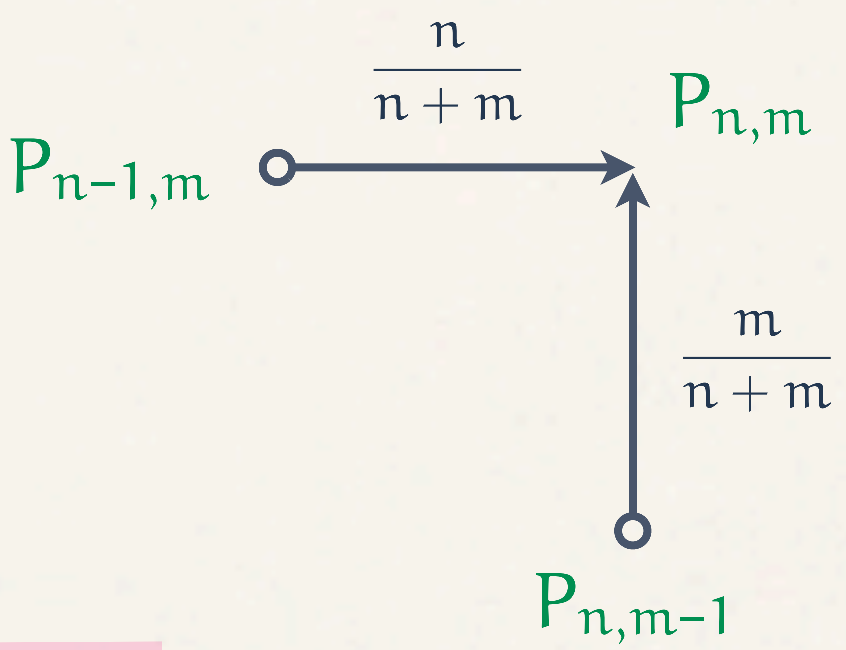

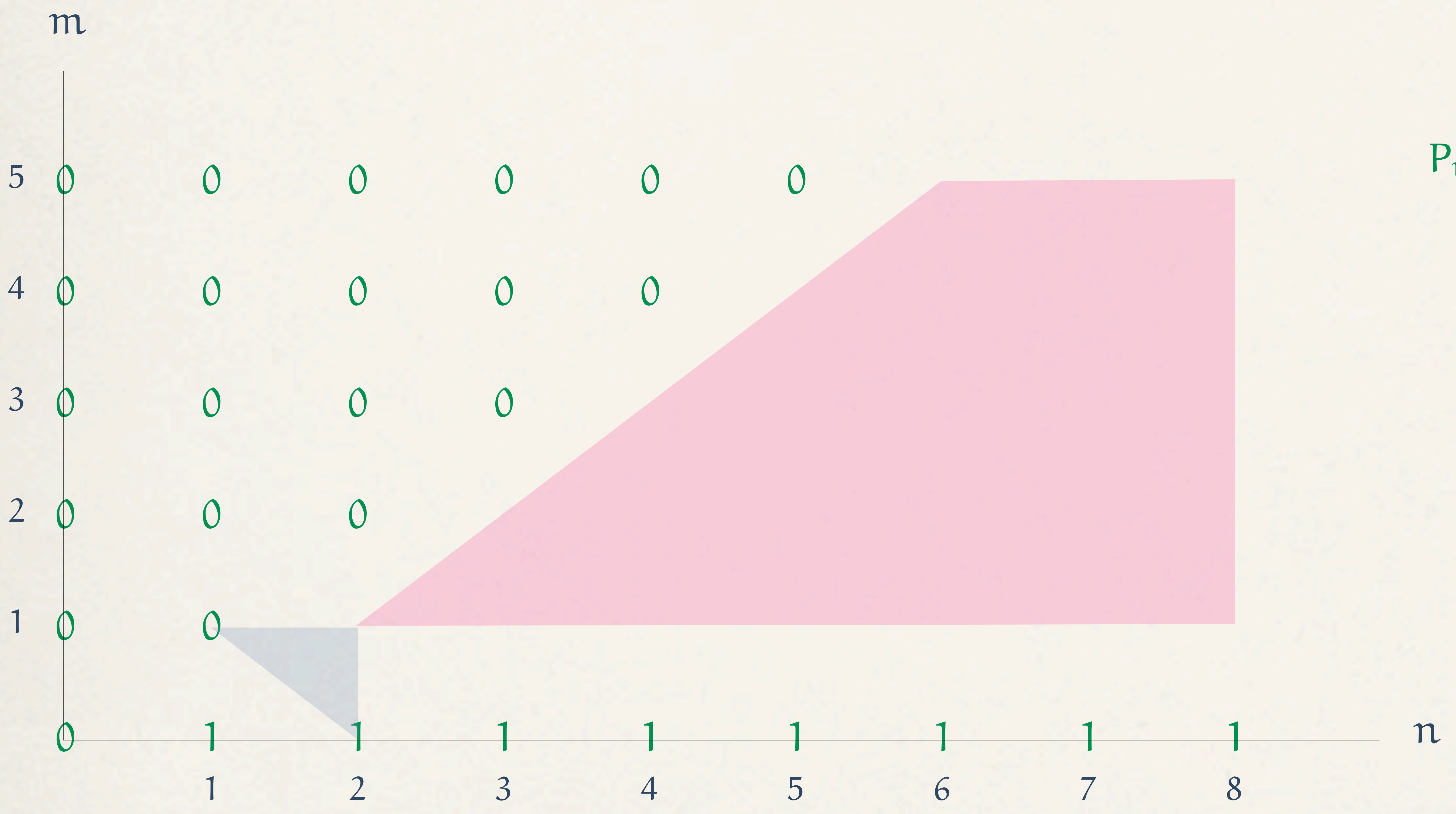

$$P_{2,1} = P_{1,1}\cdot\tfrac{2}{3} + P_{2,0}\cdot\tfrac{1}{3} = 0\cdot\tfrac{2}{3} + 1\cdot\tfrac{1}{3} = \tfrac{1}{3}$$

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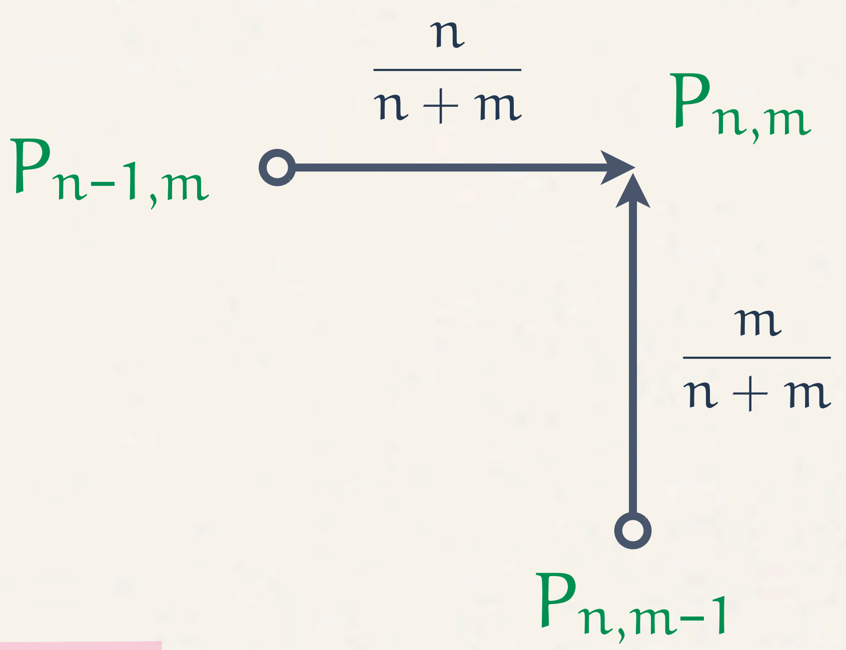

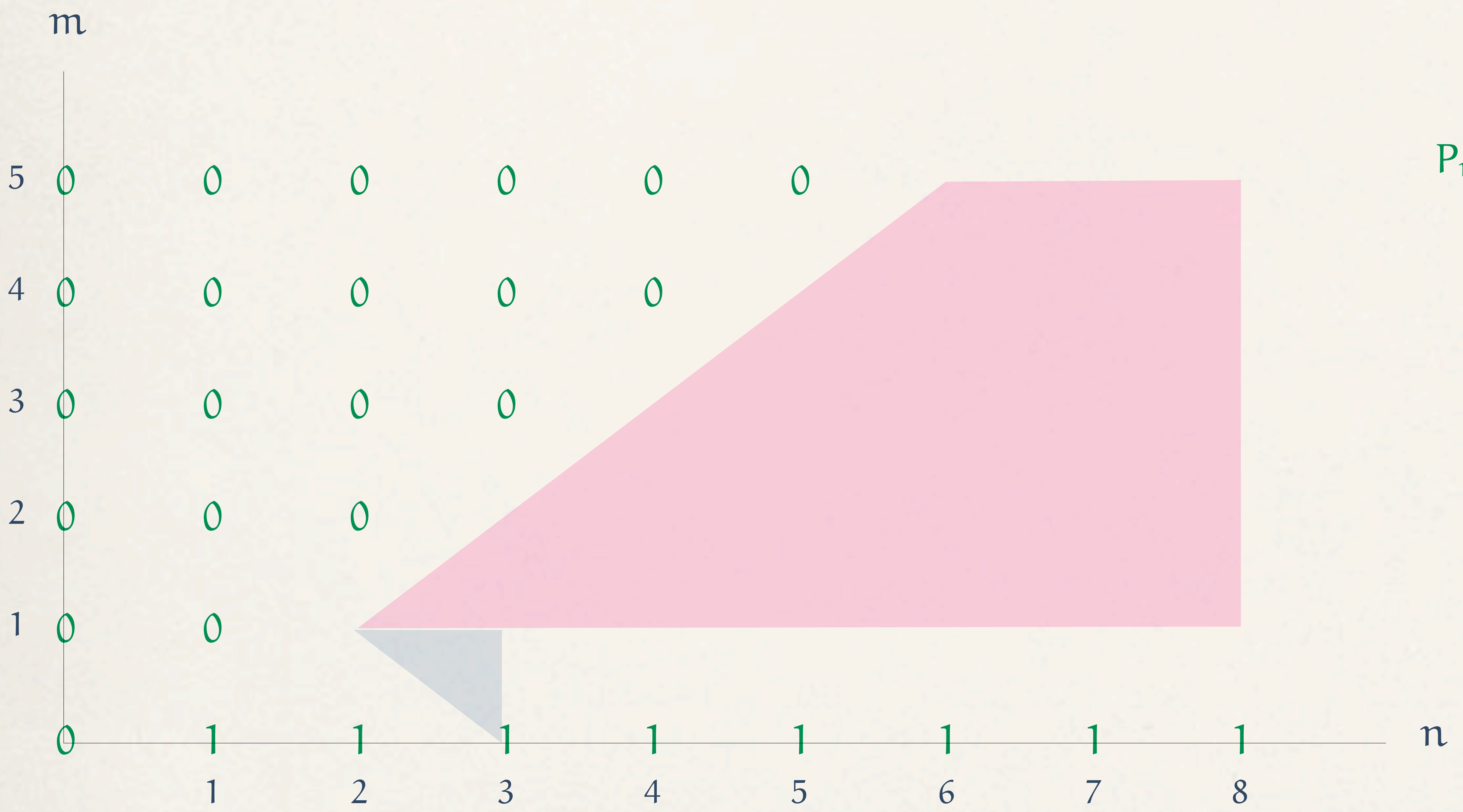

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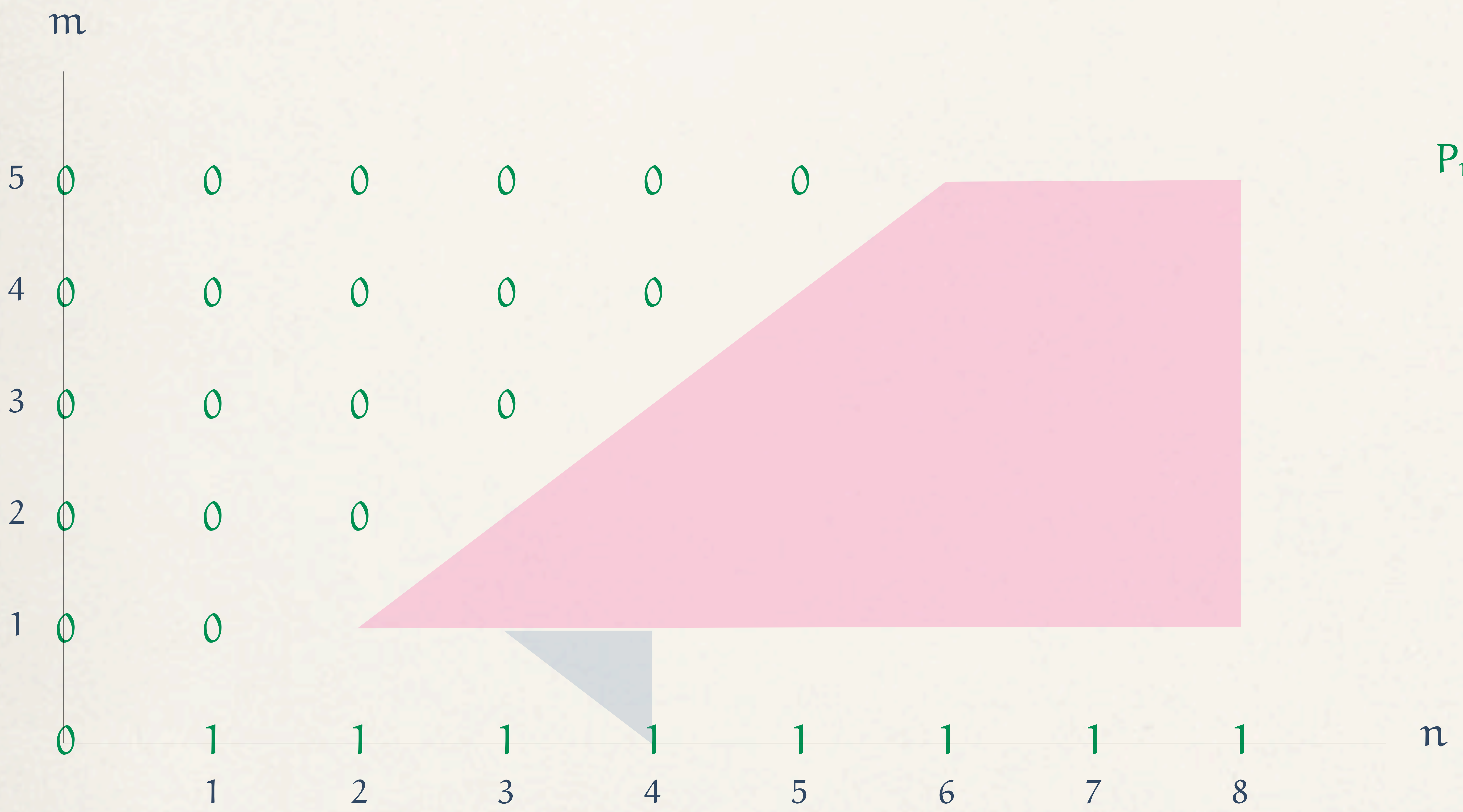
$$P_{3,1} = P_{2,1}\cdot\tfrac{3}{4} + P_{3,0}\cdot\tfrac{1}{4} = \tfrac{1}{3}\cdot\tfrac{3}{4} + 1\cdot\tfrac{1}{4} = \tfrac{2}{4}$$

A recurrence: $P_{n,m} = P_{n-1,m}\dfrac{n}{n+m} + P_{n,m-1}\dfrac{m}{n+m} \qquad (1 \le m < n)$

Boundary conditions: $P_{n,m} = \begin{cases} 0 & \text{if } m \ge n, \\ 1 & \text{if } 0 = m < n. \end{cases}$

$$\begin{tikzcd} P_{n-1,m} \arrow[r, "\frac{n}{n+m}"] & P_{n,m} \\ & P_{n,m-1} \arrow[u, "\frac{m}{n+m}"'] \end{tikzcd}$$



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$P_{4,1} = P_{3,1}\cdot\frac{4}{5} + P_{4,0}\cdot\frac{1}{5} = \frac{2}{4}\cdot\frac{4}{5} + 1\cdot\frac{1}{5} = \frac{3}{5}$

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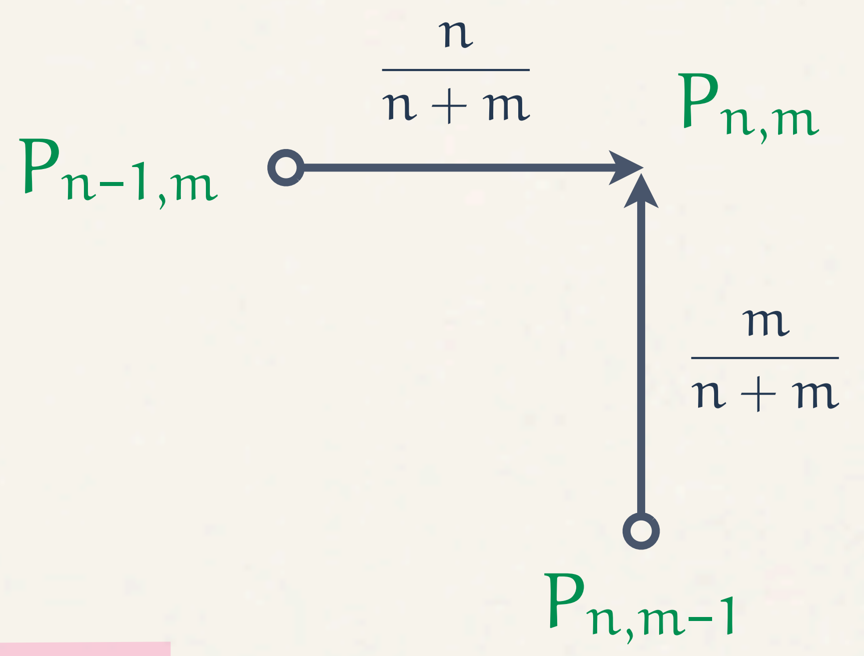

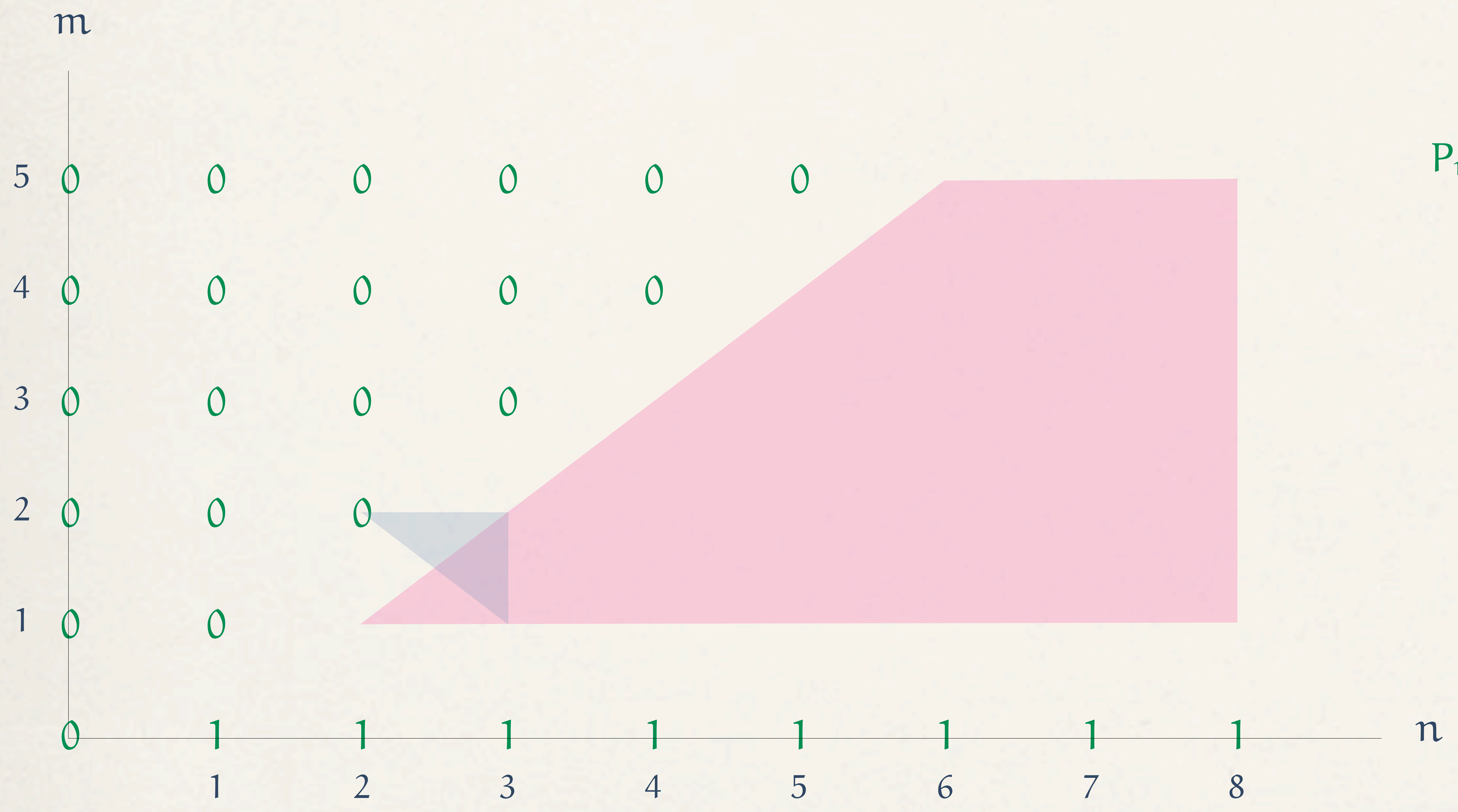

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$$P_{3,2} = P_{2,2}\cdot\tfrac{3}{5} + P_{3,1}\cdot\tfrac{2}{5} = 0\cdot\tfrac{3}{5} + \tfrac{2}{4}\cdot\tfrac{2}{5} = \tfrac{1}{5}$$

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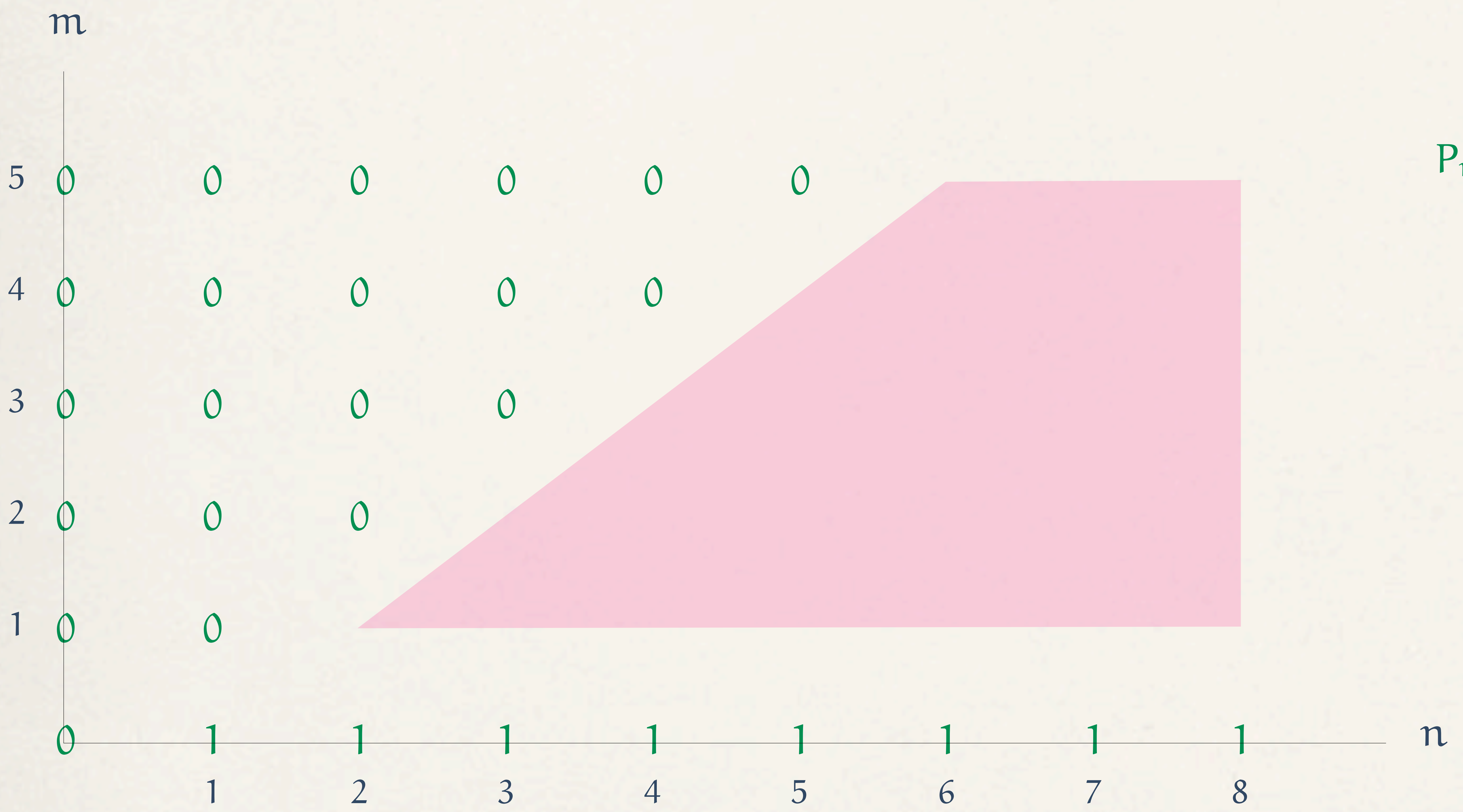

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$$P_{4,2} = P_{3,2} \cdot \tfrac{4}{6} + P_{4,1} \cdot \tfrac{2}{6} = \tfrac{1}{5} \cdot \tfrac{4}{6} + \tfrac{3}{5} \cdot \tfrac{2}{6} = \tfrac{2}{6}$$

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m \ n		1	2	3	4	5	6	7	8
5	0	0	0	0	0	0	1/11	2/12	3/13
4	0	0	0	0	0	1/9	2/10	3/11	4/12
3	0	0	0	0	1/7	2/8	3/9	4/10	5/11
2	0	0	0	1/5	2/6	3/7	4/8	5/9	6/10
1	0	0	1/3	2/4	3/5	4/6	5/7	6/8	7/9
	0	1	1	1	1	1	1	1	1

$$P_{2,1} = P_{1,1}\cdot\tfrac{2}{3} + P_{2,0}\cdot\tfrac{1}{3} = 0\cdot\tfrac{2}{3} + 1\cdot\tfrac{1}{3} = \tfrac{1}{3}$$

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$$P_{4,2} = P_{3,2}\cdot\tfrac{4}{6} + P_{4,1}\cdot\tfrac{2}{6} = \tfrac{1}{5}\cdot\tfrac{4}{6} + \tfrac{3}{5}\cdot\tfrac{2}{6} = \tfrac{2}{6}$$

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m \ n		1	2	3	4	5	6	7	8
5	0	0	0	0	0	0	1/11	2/12	3/13
4	0	0	0	0	0	1/9	2/10	3/11	4/12
3	0	0	0	0	1/7	2/8	3/9	4/10	5/11
2	0	0	0	1/5	2/6	3/7	4/8	5/9	6/10
1	0	0	1/3	2/4	3/5	4/6	5/7	6/8	7/9
	0	1	1	1	1	1	1	1	1

$$P_{2,1} = P_{1,1}\cdot\tfrac{2}{3} + P_{2,0}\cdot\tfrac{1}{3} = 0\cdot\tfrac{2}{3} + 1\cdot\tfrac{1}{3} = \tfrac{1}{3}$$

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Verification: $$\frac{n-m}{n+m} \stackrel{?}{=} \frac{(n-1)-m}{(n-1)+m} \cdot \frac{n}{n+m} + \frac{n-(m-1)}{n+(m-1)} \cdot \frac{m}{n+m}$$

$$P_{n,m} = \frac{n-m}{n+m} \qquad (0 \le m < n)$$

$$P_{n,m} = P_{n-1,m}\frac{n}{n+m} + P_{n,m-1}\frac{m}{n+m}$$

Verification: $$\frac{n-m}{n+m} \overset{\checkmark}{=} \frac{(n-1)-m}{(n-1)+m} \cdot \frac{n}{n+m} + \frac{n-(m-1)}{n+(m-1)} \cdot \frac{m}{n+m}$$

$$P_{n,m} = \frac{n-m}{n+m} \qquad (0 \le m < n)$$

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If 51% of the votes go to Jane and 49% to Bob there is a 2% chance that Jane will lead throughout the count.