

Question 7

Proof by induction.

Let $n = 1$: LHS = 2

$$\text{RHS} = 2^2 - 2 = 2$$

LHS = RHS therefore true.

Assume true for $n = k$ i.e. $2 + 2^2 + 2^3 + \dots + 2^n = 2^{n+1} - 2$

Let $n = k + 1$

$$2 + 2^2 + 2^3 + \dots + 2^k + 2^{k+1} = 2^{((k+1)+1)} - 2$$

$$= 2^{k+2} - 2 \dots \dots \dots \text{goal}$$

$$2 + 2^2 + 2^3 + \dots + 2^k + 2^{k+1} = [2 + 2^2 + 2^3 + \dots + 2^k] + 2^{k+1}$$

$$= 2^{k+1} - 2 + 2^{k+1}$$

$$= 2 \cdot 2^{k+1} - 2$$

$$= 2^{k+2} - 2 \dots \dots \dots \text{goal.}$$

Since true for $n = 1$, $n = k$ and $n = k + 1$ then by induction the statement is true for all n .