Coin tosses, redux

Toss a coin repeatedly without end

- *Sample space* Ω: each sample point ω is an infinite sequence of heads and tails.
    - Represent heads by 1 and tails by 0.
    - $\Omega = \{ x_1x_2x_3 \dots : \ x_n \in \{0, 1\} \text{ for each } n = 1, 2, 3, \dots \}$
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    - Event A that the first head occurs on the fourth toss.
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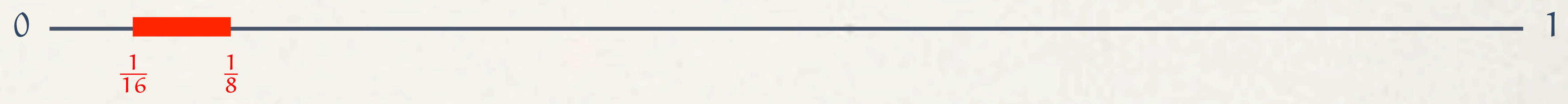

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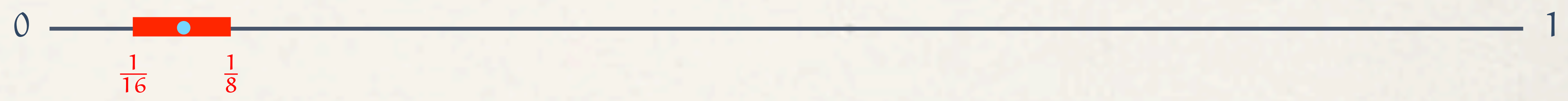

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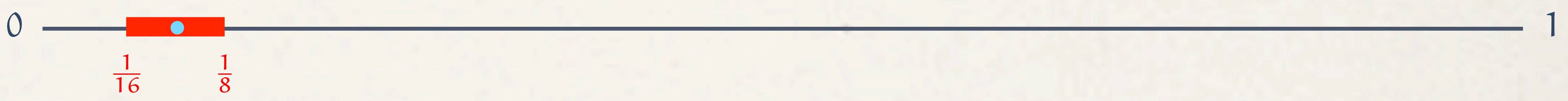

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  - Appeal to symmetry: the first quartile is as likely as the last quartile.

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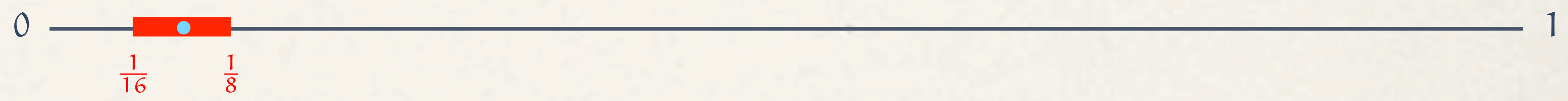

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  - The probability of an interval is proportional to its length:

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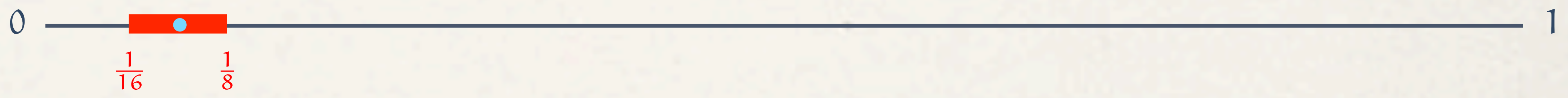

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The sample space forms a continuum of points.

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Basic events may be identified with intervals.

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Intervals are the carriers of mass in the continuum.

The uniform density