**Real Statistics Using Excel**
*Everything you need to do real statistical analysis using Excel*

## Comparing two means when variances are known

**Theorem 1**: Let $\bar{x}$ and $\bar{y}$ be the means of two samples of size $n_x$ and $n_y$ respectively. If $x$ and y are normal or $n_x$ and $n_y$ are sufficiently large for the Central Limit Theorem to hold, then $\bar{x} - \bar{y}$ has normal distribution with mean $\mu_x - \mu_y$ and standard deviation

$$\sqrt{\sigma_x^2/n_x + \sigma_y^2/n_y}$$

Proof: Since the samples are random, $\bar{x}$ and $\bar{y}$ are normally and independently distributed. By the Central Limit Theorem and Property 1 and 2 of Basic Characteristics of the Normal Distribution, we know that $\bar{x} - \bar{y}$ is normally distributed with mean

$$\mu_{\bar{x}-\bar{y}} = \mu_{\bar{x}} - \mu_{\bar{y}} = \mu_x - \mu_y$$

and standard deviation

$$\sigma_{\bar{x}-\bar{y}} = \sqrt{\sigma_{\bar{x}}^2 + \sigma_{\bar{y}}^2} = \sqrt{\sigma_x^2/n_x + \sigma_y^2/n_y}$$

**Hypothesis Testing**: When the population is normal or the sample sizes are sufficiently large, we can use the above theorem to compare two population means. The theorem requires that the population standard deviations be known, which is usually not the case. Often, especially with large samples, the standard deviation of the samples can be used as an approximation for the population standard deviations. We can also employ the t-test (see Two Sample t-Test with Equal Variances and Two Sample t-Test with Unequal Variances) which doesn't require that the variances be known, and is especially useful when the sample sizes are small.

**Excel Tools**: Excel provides a data analysis tool called **z-Test: Two Sample for Means** to automate the hypothesis testing process (as shown in Example 1).

**Example 1**: The average height of 5 year old boys in a certain country is known to be normally distributed with mean 95 cm and standard deviation 16 cm. A firm is selling a nutrient which it claims will significantly increase the height of children. In order to demonstrate its claim it selects a random sample of 60 four year old boys, half of whom are given the nutrient for one year and half of whom are not. Given that the heights of the boys at 5 years of age are as in the Figure 1, determine whether the nutrient is effective in increasing height.

| | A | B | C | D | E | F | G | H | I | J | K |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | Two-sample testing of the mean | | | | | | | | | | |
| 2 | | | | | | | | | | | |
| 3 | | Control | | | Nutrient | | | | Control | Nutrient | |
| 4 | 82.67 | 89.03 | 109.13 | 106.18 | 95.47 | 117.51 | | sample size | 30 | 30 | |
| 5 | 90.11 | 94.51 | 81.59 | 100.86 | 108.66 | 115.64 | | sample mean | 95.74 | 106.69 | |
| 6 | 89.20 | 93.32 | 94.99 | 129.85 | 83.32 | 97.22 | | | | | |
| 7 | 119.15 | 89.26 | 101.34 | 100.30 | 117.64 | 131.04 | | pop variance | 256 | | |
| 8 | 83.01 | 110.36 | 104.82 | 87.56 | 96.90 | 101.58 | | alpha | 0.05 | | |
| 9 | 93.61 | 92.52 | 106.92 | 96.87 | 66.46 | 103.80 | | pooled variance | 17.06667 | | =I7/I4+I7/J4 |
| 10 | 88.42 | 112.87 | 80.50 | 112.57 | 87.80 | 111.99 | | pooled std dev | 4.131182 | | =SQRT(I9) |
| 11 | 97.02 | 64.05 | 106.31 | 148.36 | 115.52 | 119.34 | | z-score | -2.65146 | | =(I5-J5)/I10 |
| 12 | 126.11 | 80.06 | 85.46 | 131.62 | 102.34 | 95.10 | | p-value | 0.008014 | | =2*NORMSDIST(I11) |
| 13 | 127.96 | 74.13 | 103.69 | 114.60 | 97.01 | 107.62 | | z-crit | -1.95996 | | =NORMSINV(I8/2) |
| 14 | | | | | | | | sig | yes | | =IF(I12<I8,"yes","no") |

**Figure 1 – Two sample test using z-scores**

In addition to the raw data, Figure 1 shows how to calculate the z-score for the difference between the sample means based on a normal population with a known standard deviation of 16 (i.e. a known variance of $16^2$ = 256). Here the null hypothesis $H_0$ is

$$\mu_{control} = \mu_{nutrient}$$

or equivalently

$$\mu_{control} - \mu_{nutrient} = 0$$

This is a two-tail test, which is why the p-value (in cell I12) is doubled. Since p-value = .008 < .05 = $\alpha$, we reject the null hypothesis, and conclude there is a significant difference between the boys that take the nutrients and those that don't.

We can also use Excel's data analysis tool to automatically calculate the z-score from the sample data (although we must first reorganize the data in the form of either a single row or single column). Figure 2 shows the output of the data analysis tool for Example 1.

| z-Test: Two Sample for Means | | |
|---|---|---|
| | | |
| | *Control* | *Nutrient* |
| Mean | 95.73733 | 106.691 |
| Known Variance | 256 | 256 |
| Observations | 30 | 30 |
| Hypothesized Mean Difference | 0 | |
| z | -2.65146 | |
| P(Z<=z) one-tail | 0.004007 | |
| z Critical one-tail | 1.644854 | |
| P(Z<=z) two-tail | 0.008014 | |
| z Critical two-tail | 1.959964 | |

**Figure 2 – Output of z-Test: Two Sample for Means data analysis tool**

Looking at the two-tail results, we see once again that .008 < .05 (or alternatively $|z|$ = 2.65 > 1.96 = $z$-crit), and so we reject the null hypothesis.

### 7 Responses to *Comparing two means when variances are known*

**Isaac Hayford** *says:*
November 2, 2015 at 8:59 pm

Can you please help me solve the ff question: In Norway, the distribution of birth weights for full-term infants whose gestational age is 40 weeks and whose mothers did not smoke during pregnancy is approximately normal with mean 3500 grams and standard deviation 430 grams (Bellinger et al., 1995; New England Journal of Medicine 332:549-555). An investigator plans to conduct a study to determine whether or not the mean birth weight of full-term babies whose mother smoked throughout pregnancy is different from that of the non-smoking mothers.

Suppose the investigator believes that the true mean birthweight for the infants from smoking mothers could be as low as 3200 grams or as high as 3800 grams (i.e. he anticipates conducting a two-tailed test) with the true variability being the same within each of the two groups. He intends to design a balanced CRD (i.e. equal sample sizes) in weighing babies from randomly selected mothers from each of the two groups.

a) Now, the investigator wants to risk a 10% or less chance of failing to detect a mean difference between the two groups of mothers. Suppose the investigator intends to eventually analyze the data assuming that the variance(s) are known. What sample sizes per each of the two groups would needed for this study?

b) Obviously, the investigator will not able to assume the variance(s) as known when he analyzes the data and intends to publish the results. Readdress the question in (a) given this more normal circumstance.

c) What power would be afforded from sample sizes of 10 babies per each of the two groups if a conventional t-test was going to be used to analyze the data?

d) What should be the sample sizes for the two groups if the investigator desires the 95% t-based CI on the mean difference to be no greater than 50 grams?

Reply

**Charles** *says:*
November 3, 2015 at 3:44 pm

Isaac,

If I understand your question properly, the four questions you are asking can be resolved as follows:

a) What is the sample size required for a test using the normal distribution? See the following webpages:
Statistical Power and Sample Size
Power and Sample Size using Real Statistics

b) and d) What is the sample size required for a t test?
Sample Size Requirements for t test

c) What is the power of a t test?
Power for t test

You can also use the Real Statistics Statistical Power and Sample Size data analysis tool to answer these sorts of questions, as described on the following webpage:
Real Statistics Statistical Power Analysis Tool

Charle

Reply

**Isaac Hayford** *says:*
November 3, 2015 at 5:25 pm

Thank you very much Charle!

Reply

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**Anurag** *says:*
July 21, 2015 at 2:01 pm

Hi, I want to know how did you calculate population variance??

Reply

**Charles** *says:*
July 21, 2015 at 2:08 pm

I used Excel's VARP (or VAR.P) function. Please see the webpage Measures of Variability for details.

Charles

Reply

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**Celina** *says:*
March 3, 2015 at 3:36 pm

Hello,

Can I follow example 1 even if the sample sizes are different?

Basically I want to compare the mean of two samples with different sample sizes (in Excel). I have the mean, the variance and the sample size for both.

Thanks

Reply

**Charles** *says:*
March 3, 2015 at 4:09 pm

Celina,
Yes, the sample sizes can be different.
Charles

Reply

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