Lesson 5.3-5.4

Back to Week 2

8/8 points earned
(100%)

Quiz passed!

1 / 1
points

1.
We use the continuous version of Bayes' theorem if:

θ is continuous

Correct Response
If θ is continuous, we use a probability density for the prior.

- Y is continuous
- $f(y \mid \theta)$ is continuous
- All of the above
- None of the above

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1 / 1
points

2.
Consider the coin-flipping example from the lesson. Recall that the likelihood for this experiment was Bernoulli with unknown probability of heads, i.e., $f(y \mid \theta) = \theta^y (1-\theta)^{1-y} I_{\{0 \le \theta \le 1\}}$, and we started with a uniform prior on the interval $[0, 1]$.

After the first flip resulted in heads $(Y_1 = 1)$, the posterior for θ became $f(\theta \mid Y_1 = 1) = 2\theta I_{\{0 \le \theta \le 1\}}$.

Now use this posterior as your prior for θ before the next (second) flip. Which of the following represents the posterior PDF for θ after the second flip also results in heads $(Y_2 = 1)$?

- $f(\theta \mid Y_2 = 1) = \frac{(1-\theta) \cdot 2\theta}{\int_0^1 (1-\theta) \cdot 2\theta d\theta} I_{\{0 \le \theta \le 1\}}$
- $f(\theta \mid Y_2 = 1) = \frac{\theta(1-\theta) \cdot 2\theta}{\int_0^1 \theta(1-\theta) \cdot 2\theta d\theta} I_{\{0 \le \theta \le 1\}}$
- $f(\theta \mid Y_2 = 1) = \frac{\theta \cdot 2\theta}{\int_0^1 \theta \cdot 2\theta d\theta} I_{\{0 \le \theta \le 1\}}$

Correct Response
This simplifies to the posterior PDF $f(\theta \mid Y_2 = 1) = 3\theta^2 I_{\{0 \le \theta \le 1\}}$.

Incidentally, if we assume that the two coin flips are independent, we would have arrived at the same posterior if we had again started with a uniform prior and performed a single update using $Y_1 = 1$ and $Y_2 = 1$.

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1 / 1
points

3.

Consider again the coin-flipping example from the lesson. Recall that we used a Uniform(0,1) prior for θ. Which of the following is a correct interpretation of $P(0.3 < \theta < 0.9) = 0.6$?

- (0.3, 0.9) is a 60% credible interval for θ before observing any data.

Correct Response

The probability statement came from our prior, so the prior probability that θ is in this interval is 0.6.

- (0.3, 0.9) is a 60% credible interval for θ after observing $Y = 1$.
- (0.3, 0.9) is a 60% confidence interval for θ.
- The posterior probability that $\theta \in (0.3, 0.9)$ is 0.6.

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1 / 1
points

4.

Consider again the coin-flipping example from the lesson. Recall that the posterior PDF for θ, after observing $Y = 1$, was $f(\theta \mid Y = 1) = 2\theta I_{\{0 \le \theta \le 1\}}$. Which of the following is a correct interpretation of $P(0.3 < \theta < 0.9 \mid Y = 1) = \int_{0.3}^{0.9} 2\theta d\theta = 0.72$?

- (0.3, 0.9) is a 72% credible interval for θ before observing any data.
- (0.3, 0.9) is a 72% credible interval for θ after observing $Y = 1$.

Correct Response

The probability statement came from the posterior, so the posterior probability that θ is in this interval is 0.72.

- (0.3, 0.9) is a 72% confidence interval for θ.
- The prior probability that $\theta \in (0.3, 0.9)$ is 0.72.

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1 / 1
points

5.

Which two quantiles are required to capture the middle 90% of a distribution (thus producing a 90% equal-tailed interval)?

- .025 and .975
- 0 and .9
- .10 and .90
- .05 and .95

Correct Response

90% of the probability mass is contained between the .05 and .95 quantiles (or equivalently, the 5th and 95th percentiles). 5% of the probability lies on either side of this interval.

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1 / 1
points

6.
Suppose you collect measurements to perform inference about a population mean θ. Your posterior distribution after observing data is $\theta \mid \mathbf{y} \sim \mathrm{N}(0, 1)$.

Report the upper end of a 95% equal-tailed interval for θ. Round your answer to two decimal places.

1.96

Correct Response
The 95% equal-tailed interval for a standard normal distribution is (-1.96, 1.96).

Because the normal distribution is symmetric and unimodal (has only one peak), the equal-tailed interval is also the highest posterior density (HPD) interval.

In R:

```
qnorm(p=0.975, mean=0, sd=1)
```

In Excel:

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= NORM.INV(0.975, 0, 1)

```

where probability=0.975, mean=0, standard_dev=1.

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1 / 1
points

7.
What does "HPD interval" stand for?

- Highest precision density interval
- Highest point distance interval
- Highest posterior density interval

Correct Response

- Highest partial density interval

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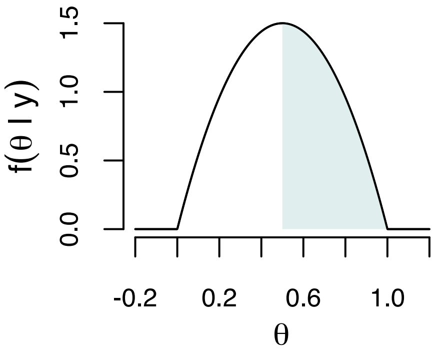
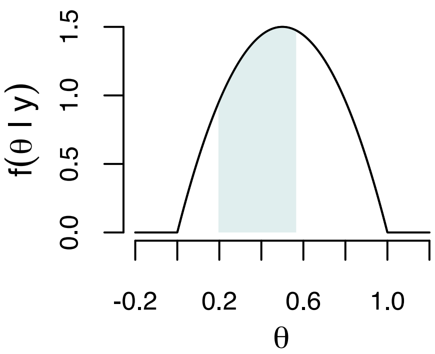
1 / 1
points

8.
Each of the following graphs depicts a 50% credible interval from a posterior distribution. Which of the intervals represents the HPD interval?

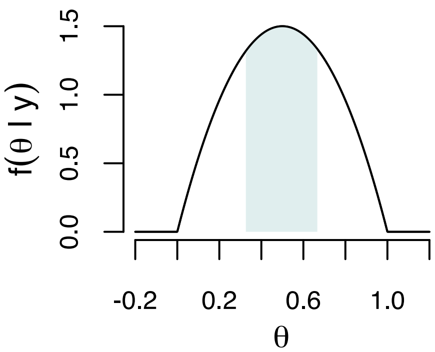
- 50% interval: $\theta \in (0.500, 1.000)$



○ 50% interval: $\theta \in (0.196, 0.567)$



○ 50% interval: $\theta \in (0.326, 0.674)$



Correct Response

This is the 50% credible interval with the highest posterior density values. It is the shortest possible interval containing 50% of the probability under this posterior distribution.

50% interval: $\theta \in (0.400, 0.756)$