Erasmus School of Economics

MOOC Econometrics

Training Exercise S.1

Notes:

- This exercise uses the data file `TrainExerS1` and requires a computer.
- The data set `TrainExerS1` is available on the website.

Questions

1. You want to investigate the precision of the estimates of the mean return on the stock market. You have a simulated sample of 1000 yearly return observations $y_j \sim NID(\mu, \sigma^2)$, available in data file `TrainExerS1`.
   (a) Construct a series of mean estimates m_i, where you use the first i observations, so $m_i = \frac{1}{i}\sum_{j=1}^{i} y_j$. Calculate the standard error for each estimate m_i. Make a graph of m_i and its 95% confidence interval, using the rule of thumb of the lecture. Start with $i = 5$.
   (b) Suppose that the standard deviation of the returns equals 15%. How many years of observations would you need to get the 95% confidence interval smaller than 1%?
2. Consider a sample y of n observations of random variables y_i, $i = 1, 2, \ldots, n$. The sample consists of two groups, 1 and 2, with n_1 and n_2 observations per group. The variables are independent, and follow a normal distribution with group dependent mean and group independent variance,
$$y_i \sim \begin{cases} N(\mu_1, \sigma^2), & \text{if } y_i \text{ belongs to group 1} \\ N(\mu_2, \sigma^2), & \text{if } y_i \text{ belongs to group 2.} \end{cases}$$
   The sample has been ordered such that the first n_1 observations belong to group 1, and the remaining n_2 observations belong to group 2. Derive an unbiased estimator of σ^2 by the following steps.
   (a) Define the $(n \times 2)$ matrix $H = \begin{pmatrix} \iota_{n_1} & 0_{n_1} \\ 0_{n_2} & \iota_{n_2} \end{pmatrix}$, where ι_k is the $(k \times 1)$ unit vector, and 0_k the $(k \times 1)$ zero vector. Show that $E[y] = H\mu$, with $\mu = (\mu_1, \mu_2)'$. What is the covariance matrix of y?
   (b) Define the (2×2) matrix $T = \begin{pmatrix} n_1 & 0 \\ 0 & n_2 \end{pmatrix}$. Show that $H'H = T$ and that $T^{-1} = \begin{pmatrix} \frac{1}{n_1} & 0 \\ 0 & \frac{1}{n_2} \end{pmatrix}$.
   (c) Show that $m = T^{-1}H'y$ is an unbiased estimator of the mean vector μ.
   (d) Define the random vector $z = y - Hm$. Show that $E[z] = 0_n$ and that we can write $z = My$ with $M = I_n - HT^{-1}H'$, where I_n is the $(n \times n)$ identity matrix.

(e) Show that we can write M as $\begin{pmatrix} M_1 & 0_{n_1,n_2} \\ 0_{n_2,n_1} & M_2 \end{pmatrix}$, with $M_j = \frac{1}{n_1}\iota_{n_j}\iota_{n_j}'$, $j = 1, 2$ and $0_{p,q}$ the $(p \times q)$ matrix with zeros. Show that M is symmetric, that $M^2 = M$, and calculate the trace of M. [Hint: Use that M_j corresponds with the matrix M in the lecture slides.]

(f) Calculate the covariance matrix of z and the expectation $E[z'z]$.

(g) Derive an unbiased estimator of σ^2.

3. Consider a sample of n observations $y_i \sim NID(\mu, \sigma^2)$. The biased variance estimator is given by $\tilde{s}^2 = \frac{1}{n}\sum_{i=1}^{n}(y_i - m)^2$, with m the sample mean. Derive its variance. Show whether this estimator is consistent.