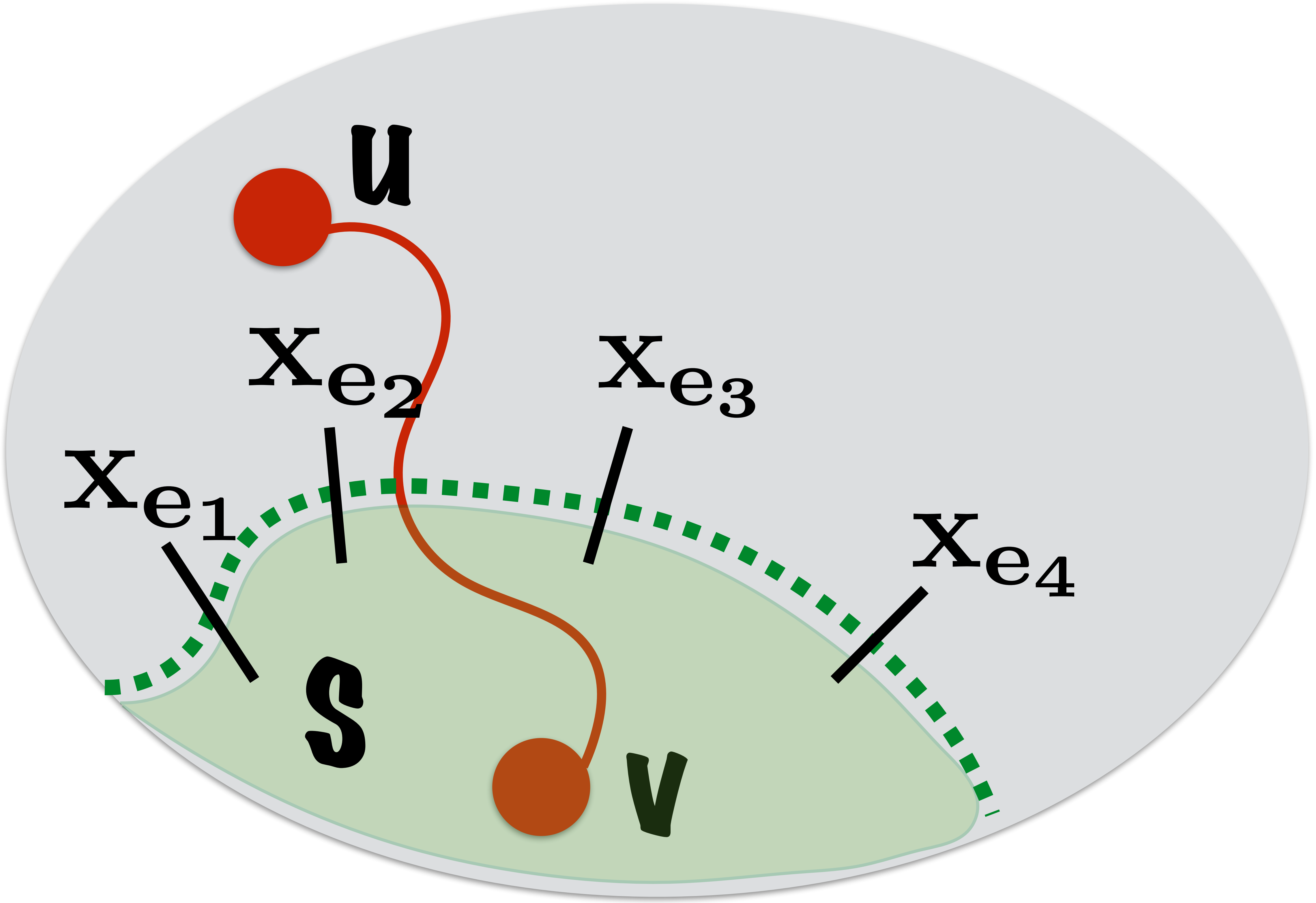
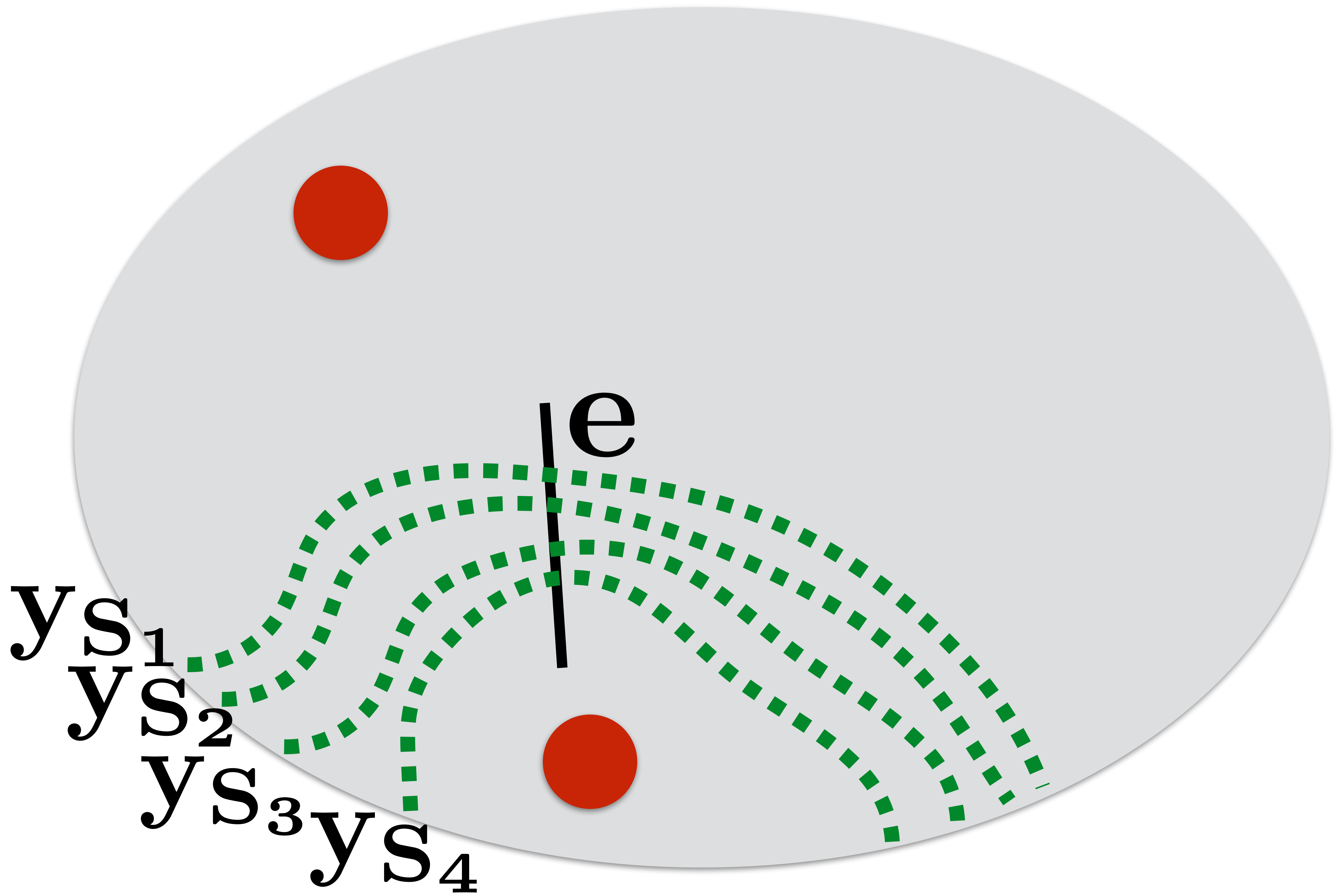
Steiner forest

primal-dual pair

$$\min \sum_{\mathbf{e}} \mathbf{c_e x_e} :$$
$$\sum_{\mathbf{e} \in \delta(\mathbf{S})} \mathbf{x_e} \geq \mathbf{1} \quad \forall \mathbf{S} \in \mathcal{S} \quad [\mathbf{y_S}]$$
$$\mathbf{x_e} \geq \mathbf{0} \quad \forall \mathbf{e} \in \mathbf{E}$$

$$\max \sum_{\mathbf{S}} \mathbf{y_S} :$$
$$\sum_{\mathbf{S}:\mathbf{e} \in \delta(\mathbf{S})} \mathbf{y_S} \leq \mathbf{c_e} \quad \forall \mathbf{e} \in \mathbf{E} \quad [\mathbf{x_e}]$$
$$\mathbf{y_S} \geq \mathbf{0} \quad \forall \mathbf{S} \in \mathcal{S}$$





$$\sum_{e \in \delta(S)} x_e \geq 1$$

$$\sum_{S : e \in \delta(S)} y_S \leq c_e$$

$$\min \sum_e c_e x_e :$$
$$\sum_{e \in \delta(S)} x_e \geq 1 \quad \forall S \in \mathcal{S}$$
$$x_e \geq 0 \quad \forall e \in E$$

$$\max \sum_S y_S :$$
$$\sum_{S: e \in \delta(S)} y_S \leq c_e \quad \forall e \in E$$
$$y_S \geq 0 \quad \forall S \in \mathcal{S}$$

Initialization:

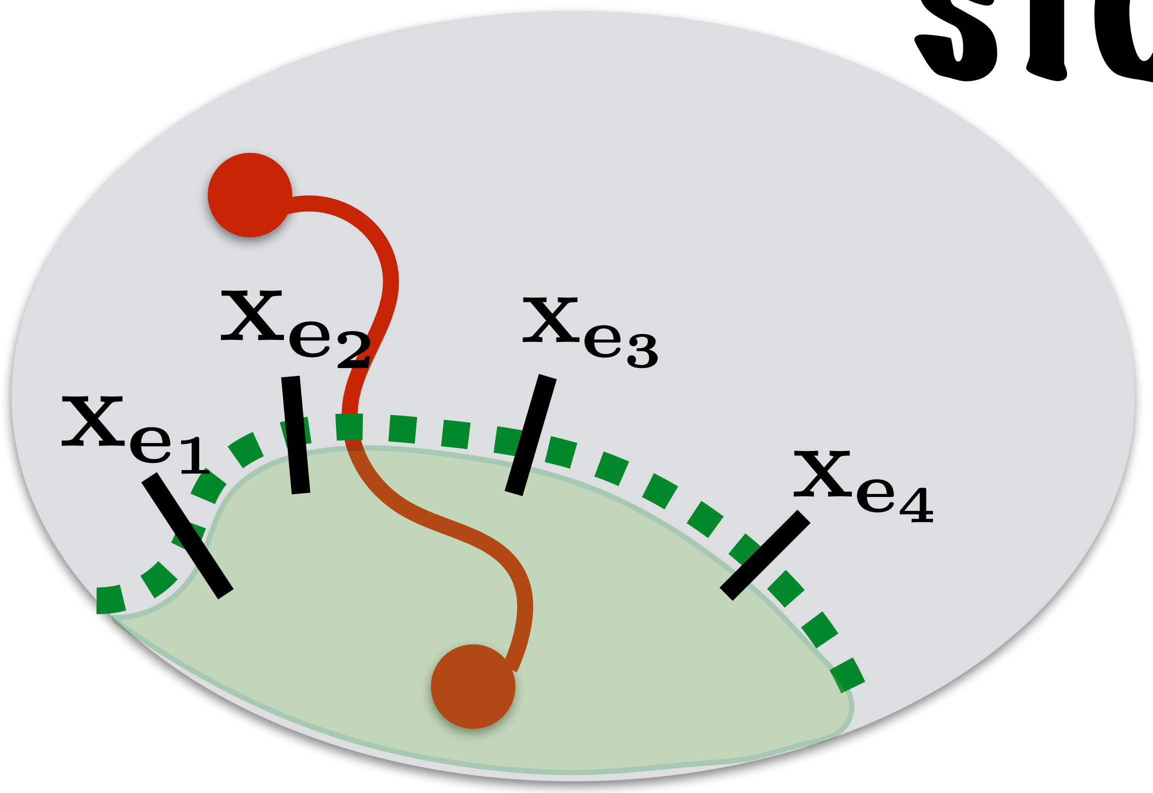
$$x \leftarrow 0, y \leftarrow 0$$

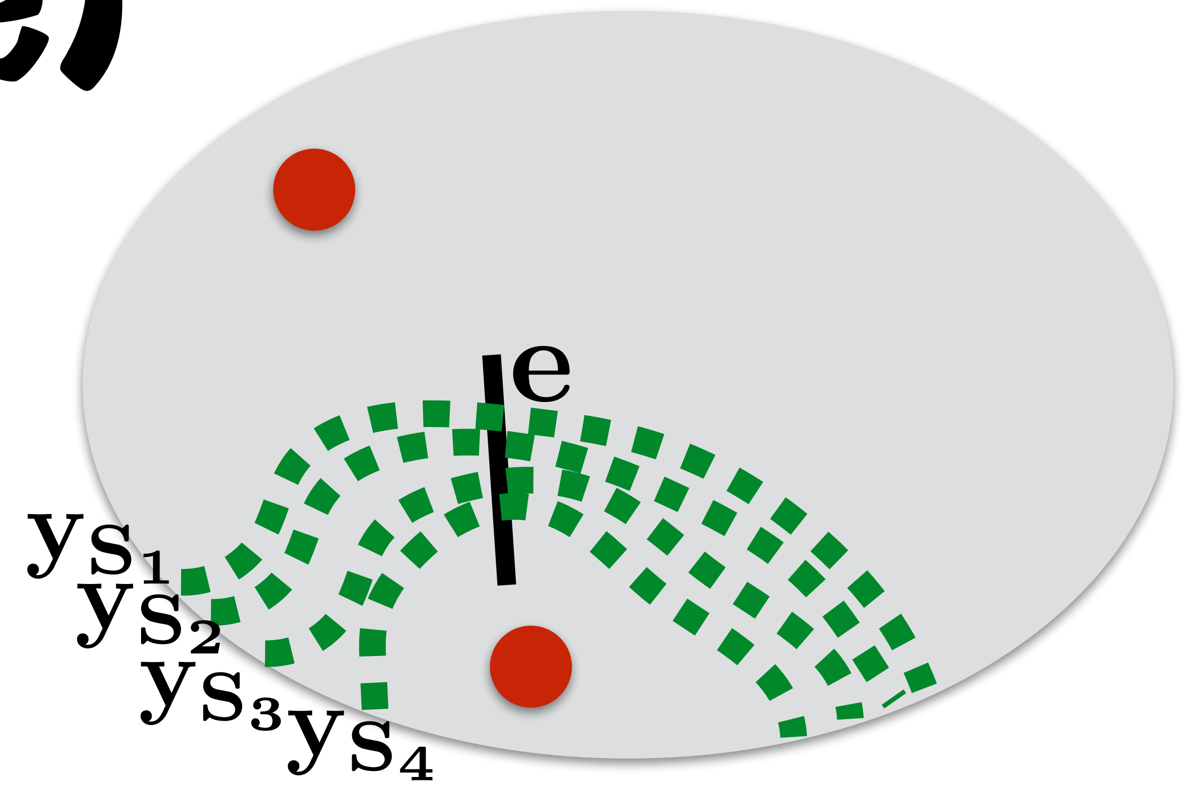
Iteration: while x not satisfiable

raise y as much as possible

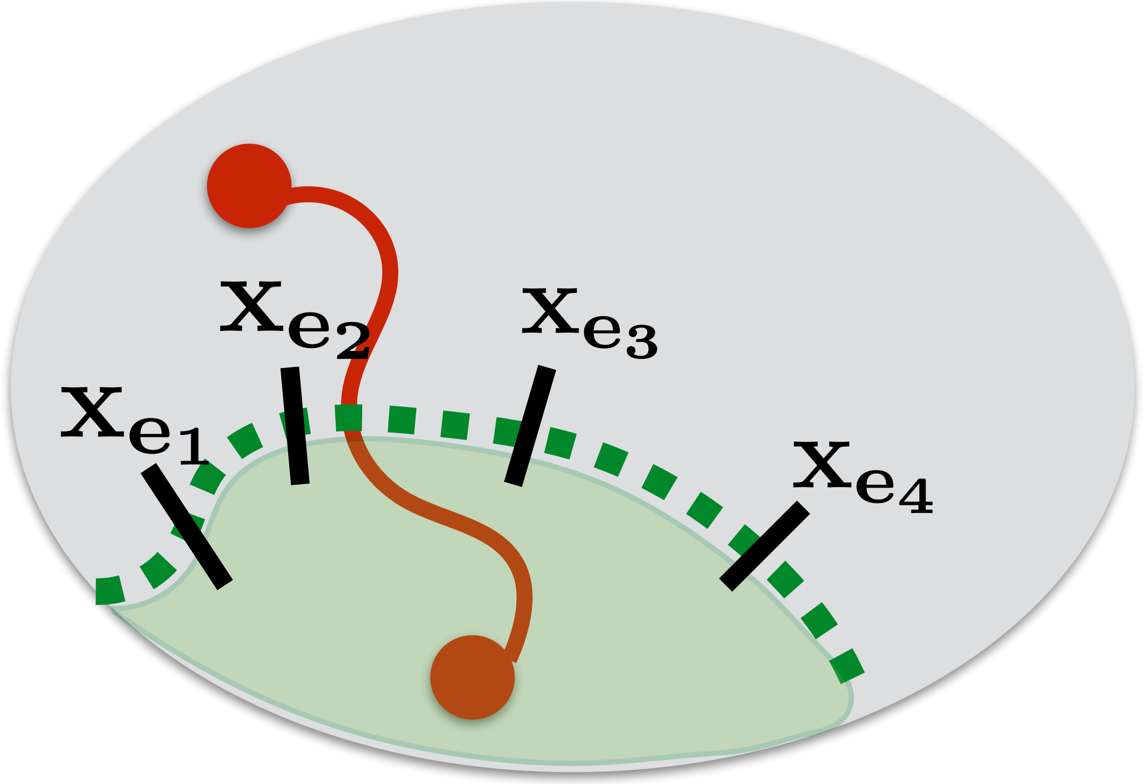
stopped by tight constraint (e)

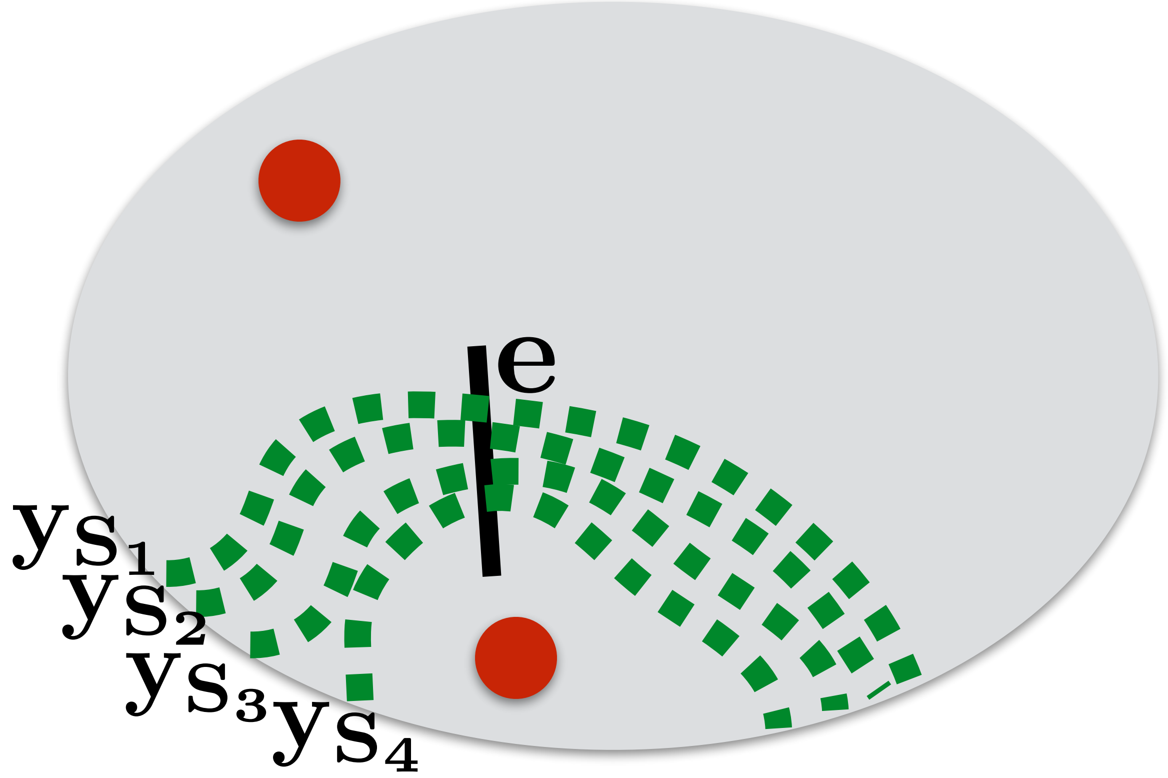
$$x_e \leftarrow 1$$



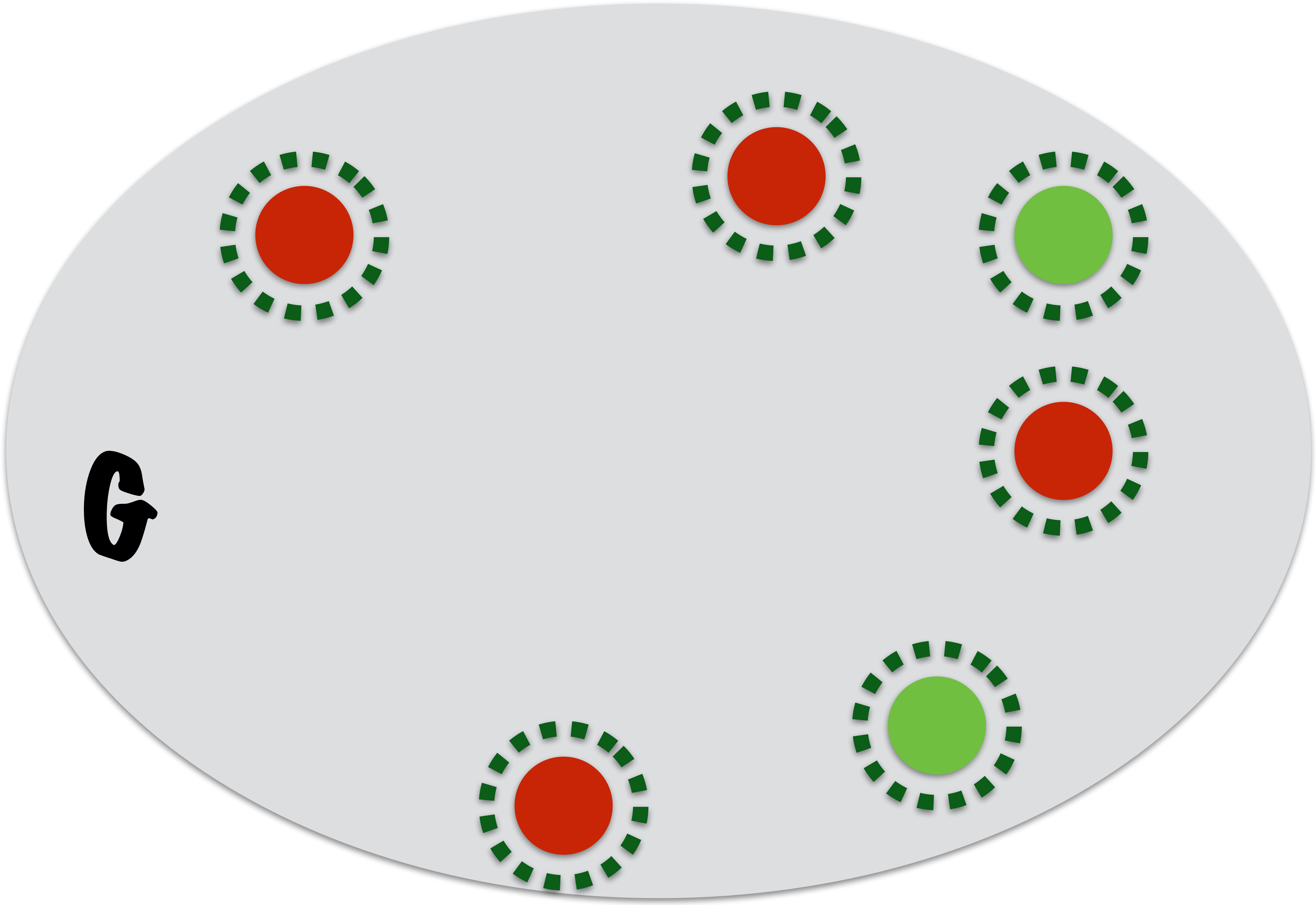


Q: How do we raise y?





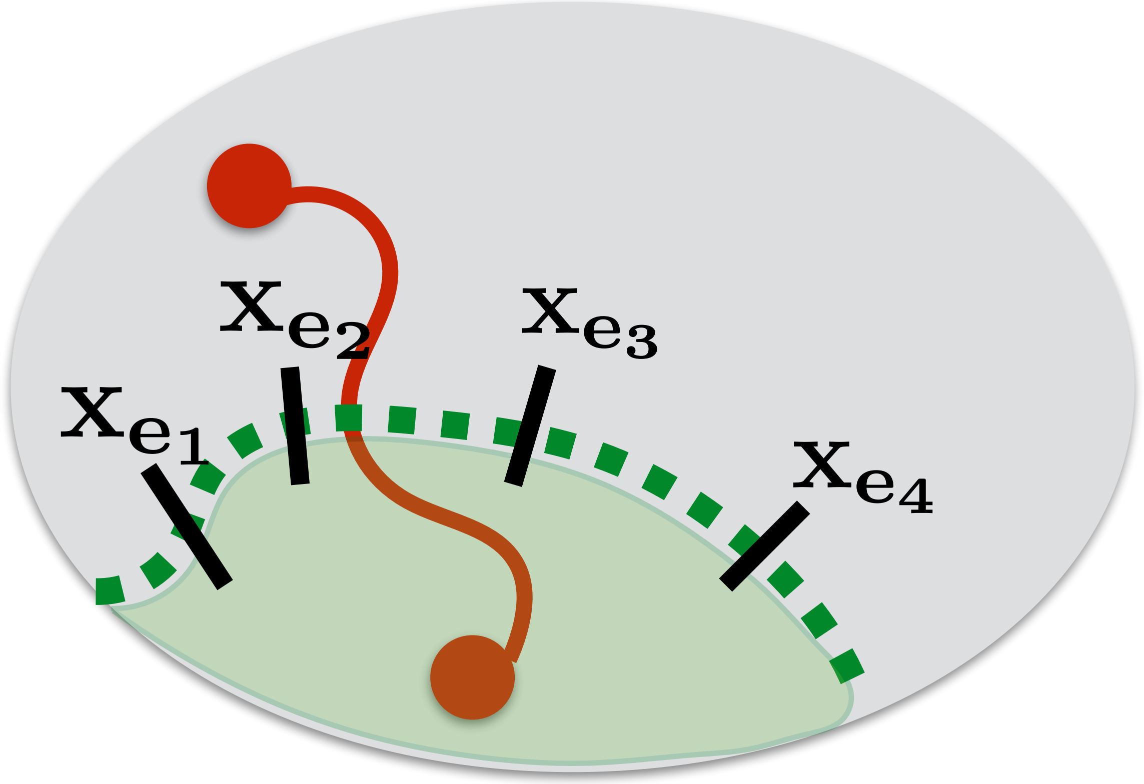
Initially...

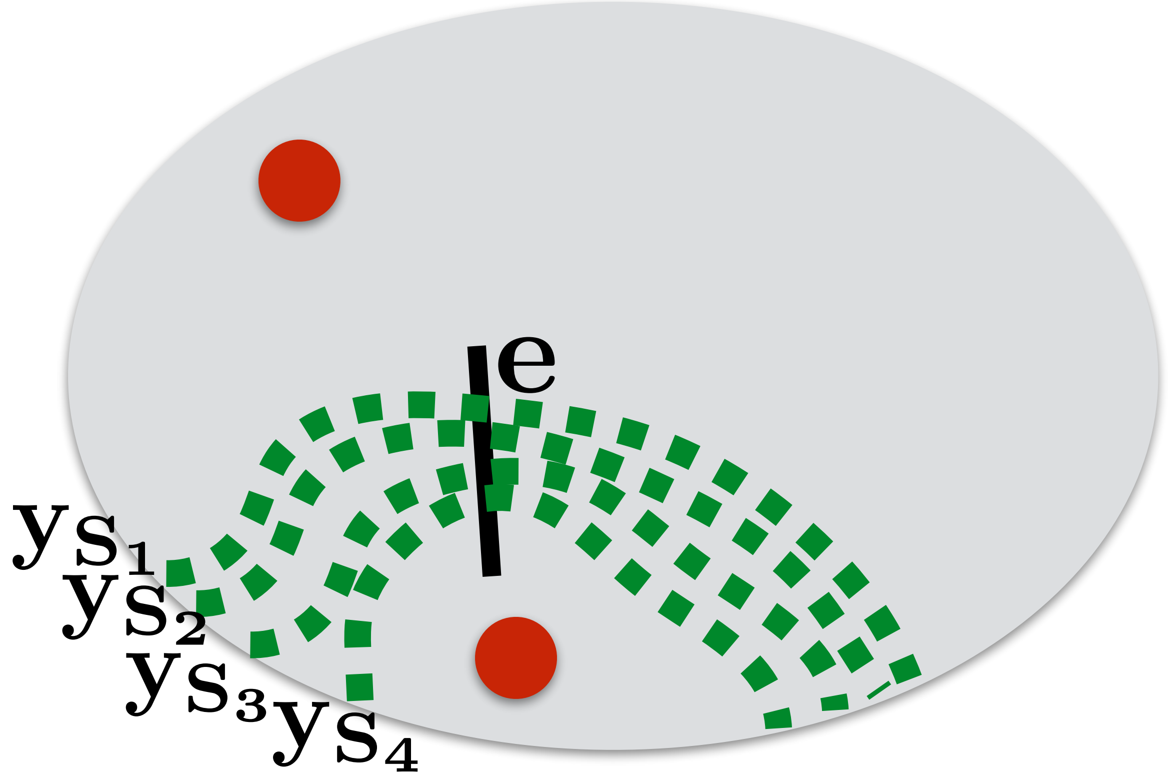


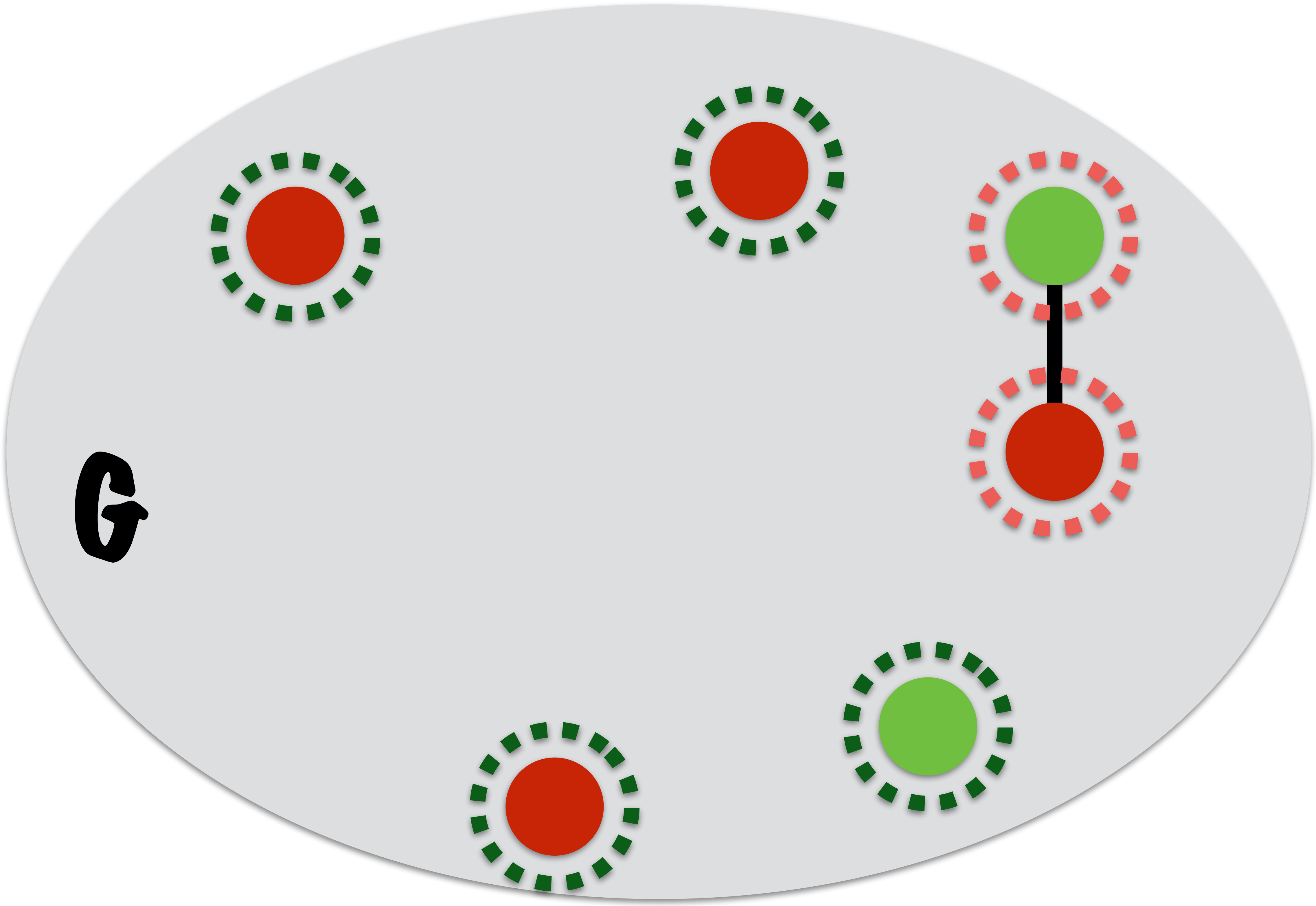
Raise all singleton terminals simultaneously

$$y_S \leftarrow y_S + \epsilon \;\; \forall S = \{u\}$$

Q: How do we raise y?

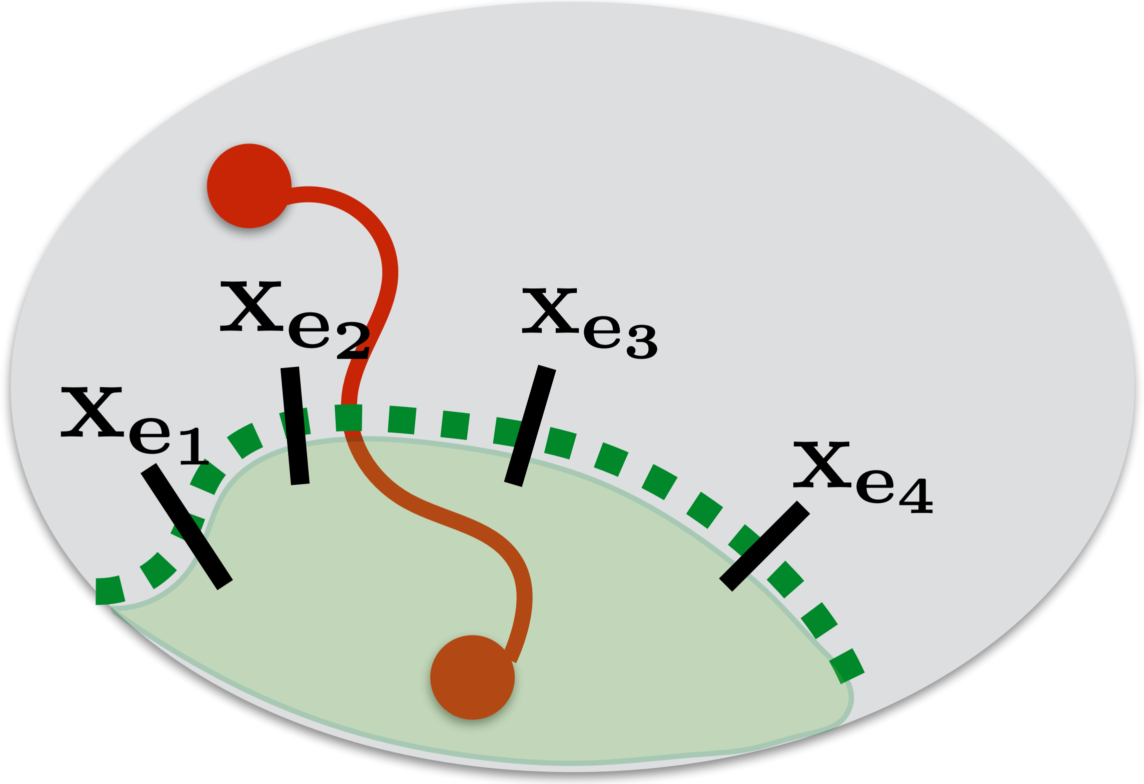


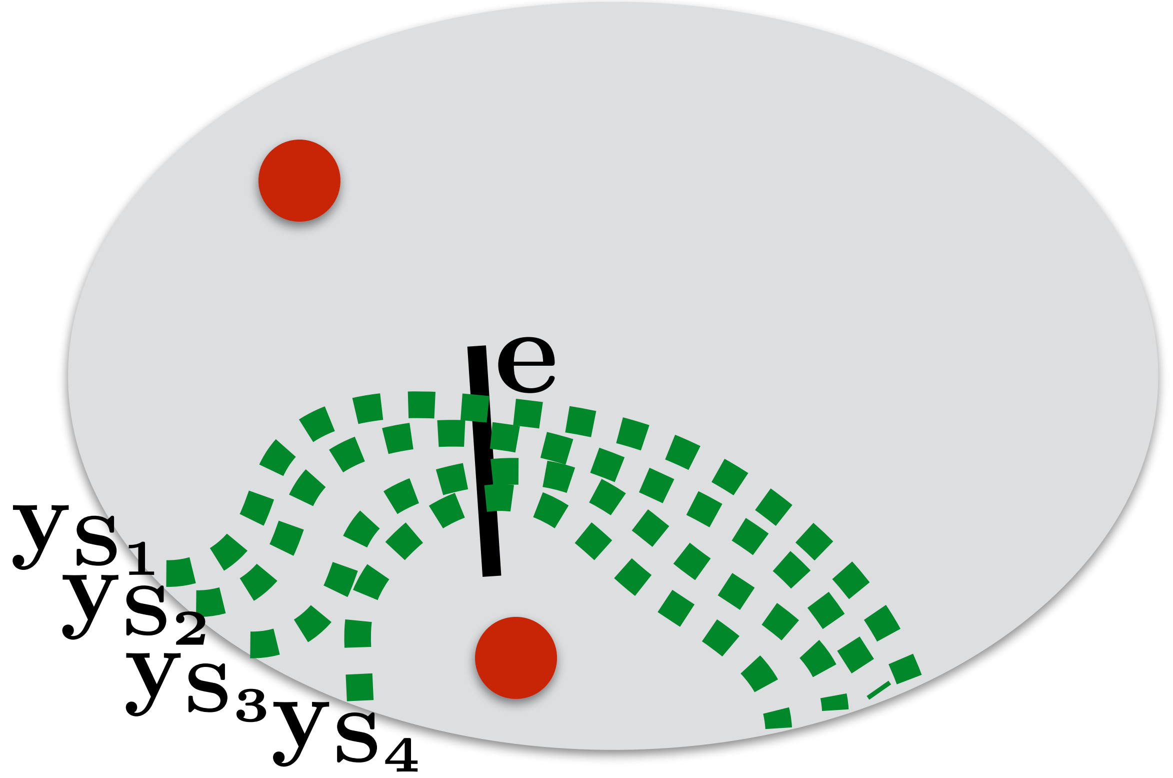




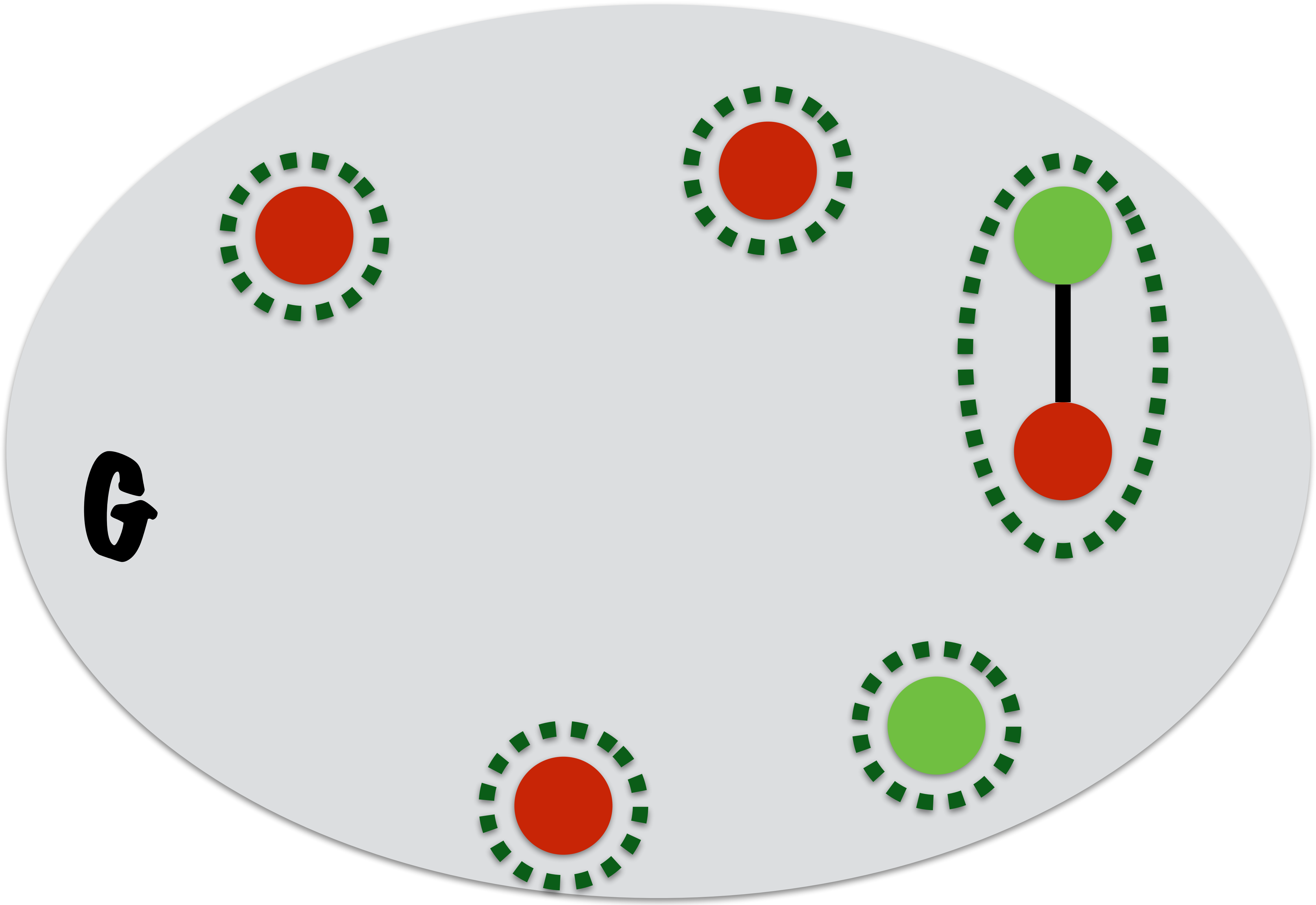
Tight constraint...

Q: How do we raise y?

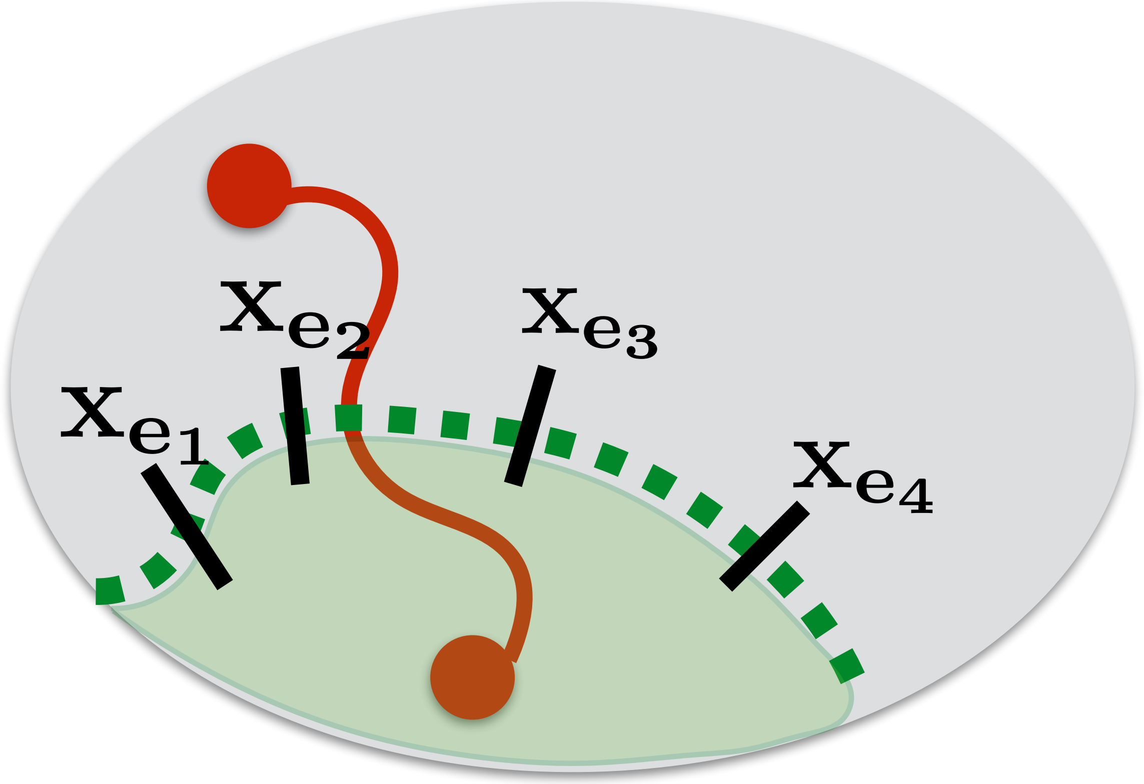


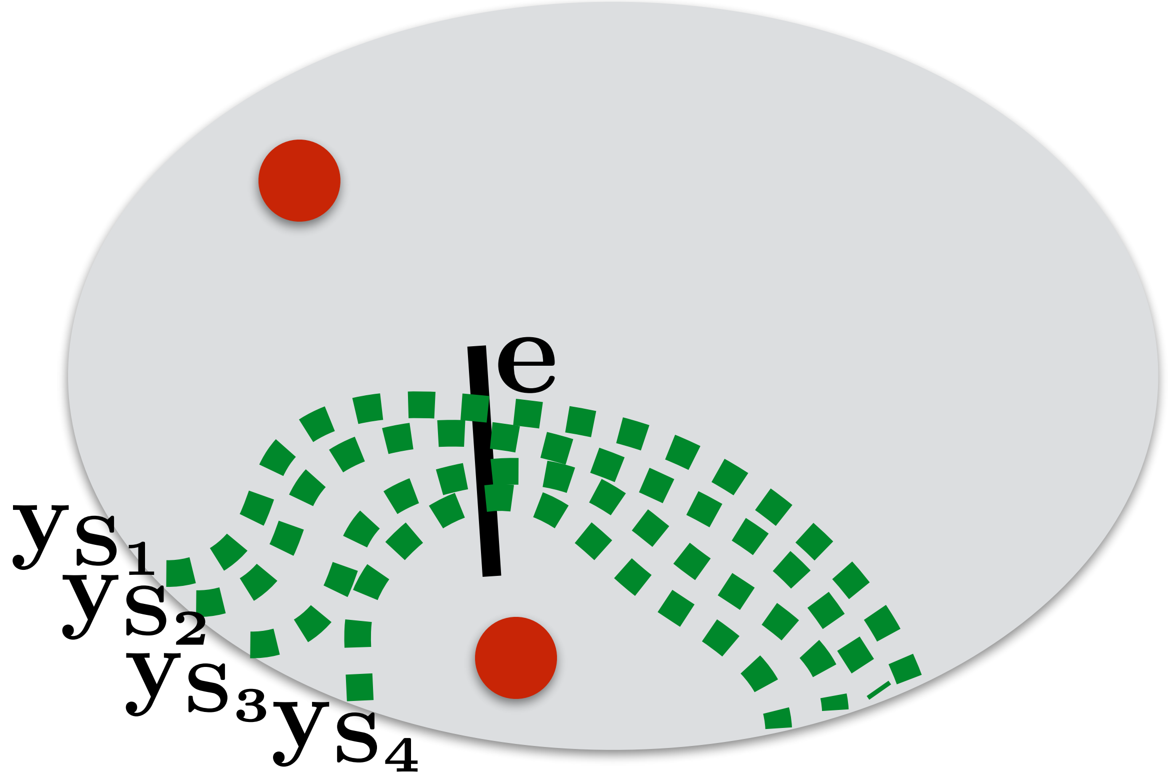


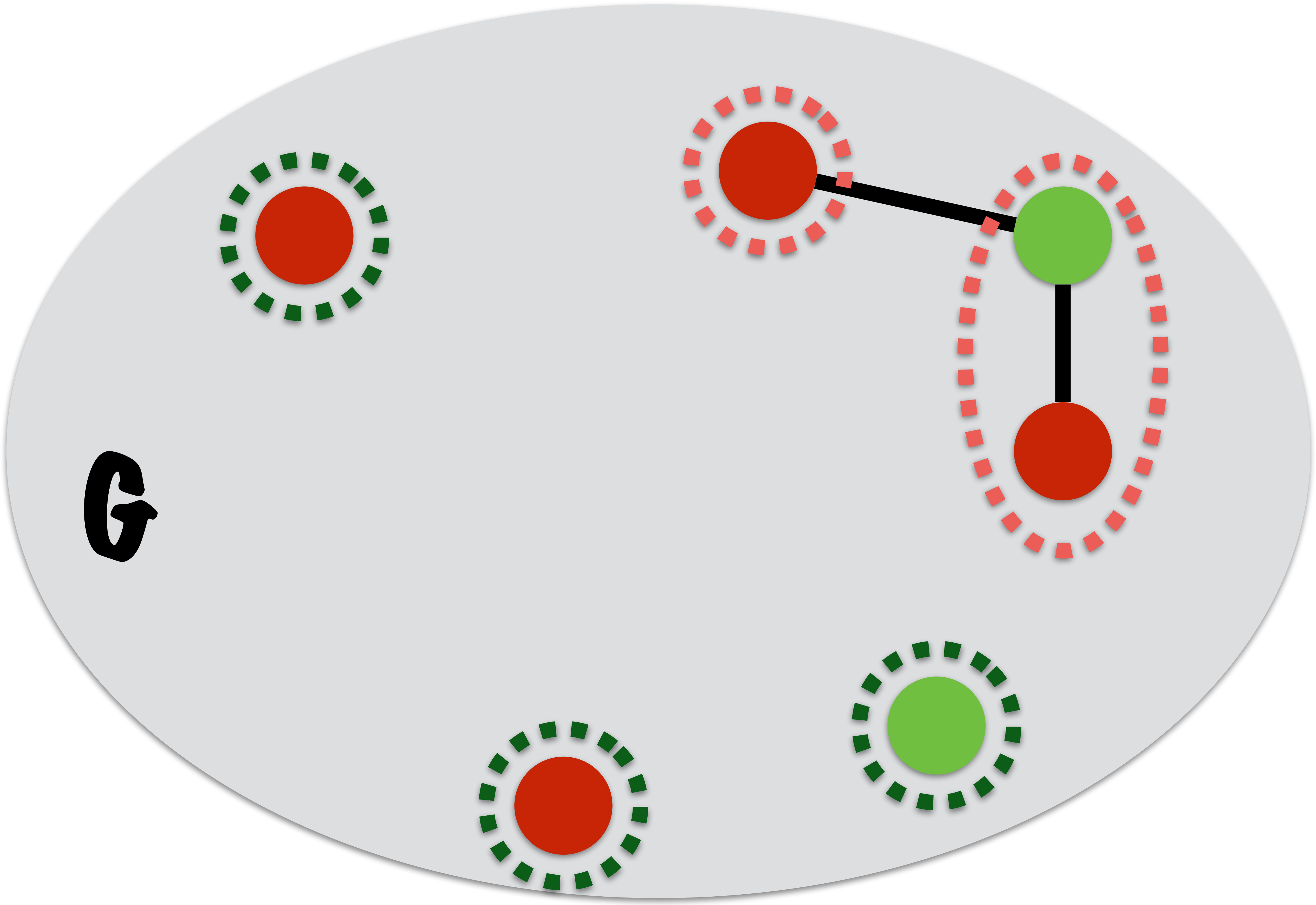
Then...



Q: How do we raise y?

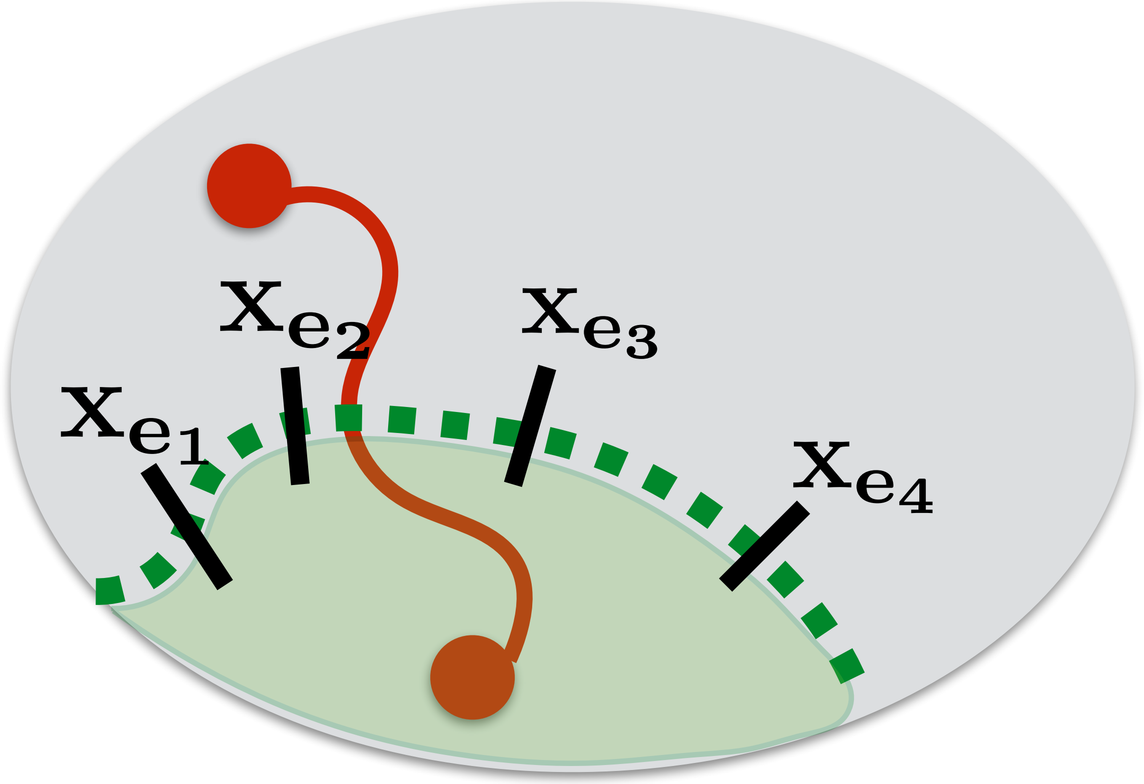


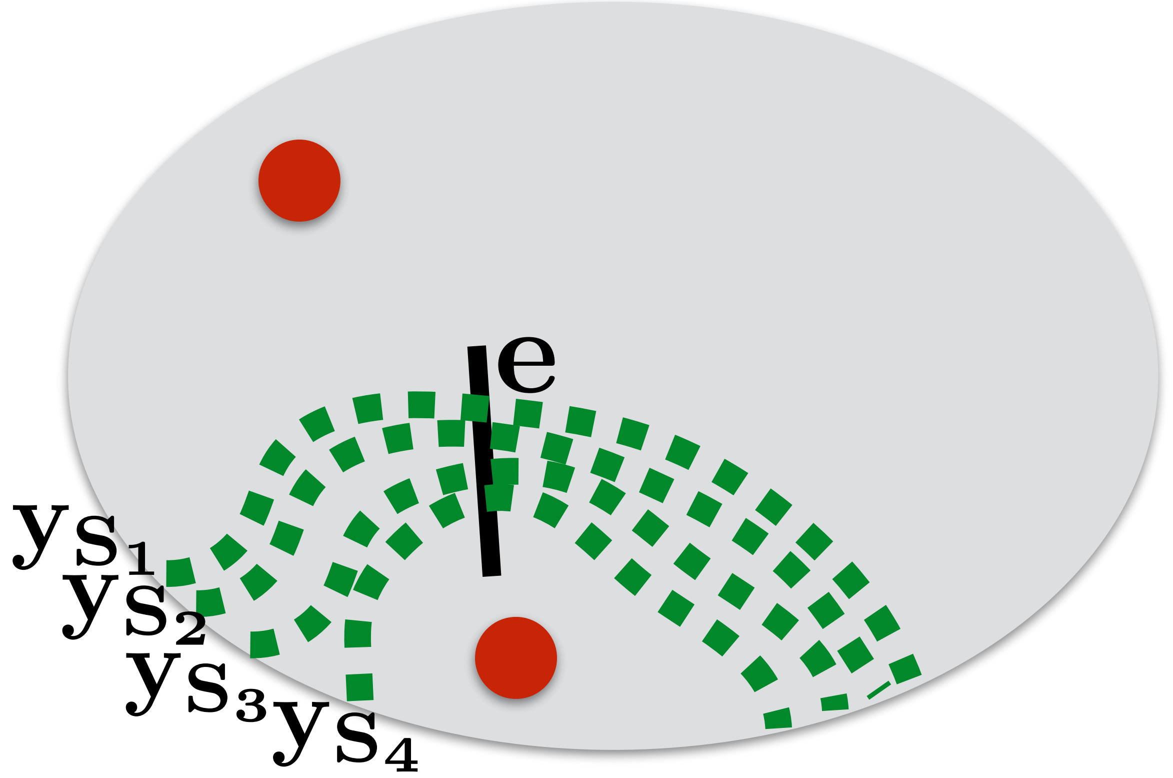


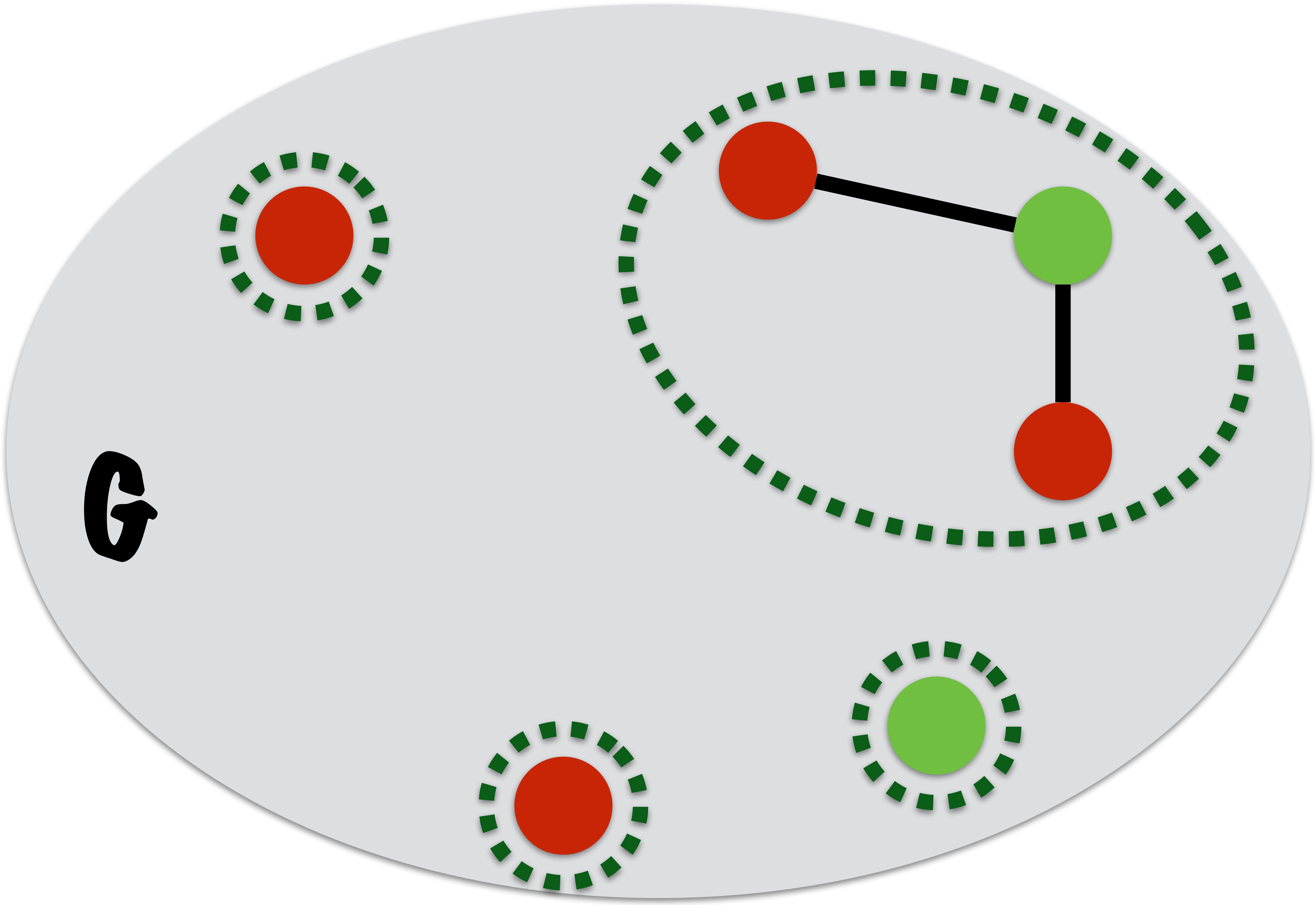


Tight constraint...

Q: How do we raise y?

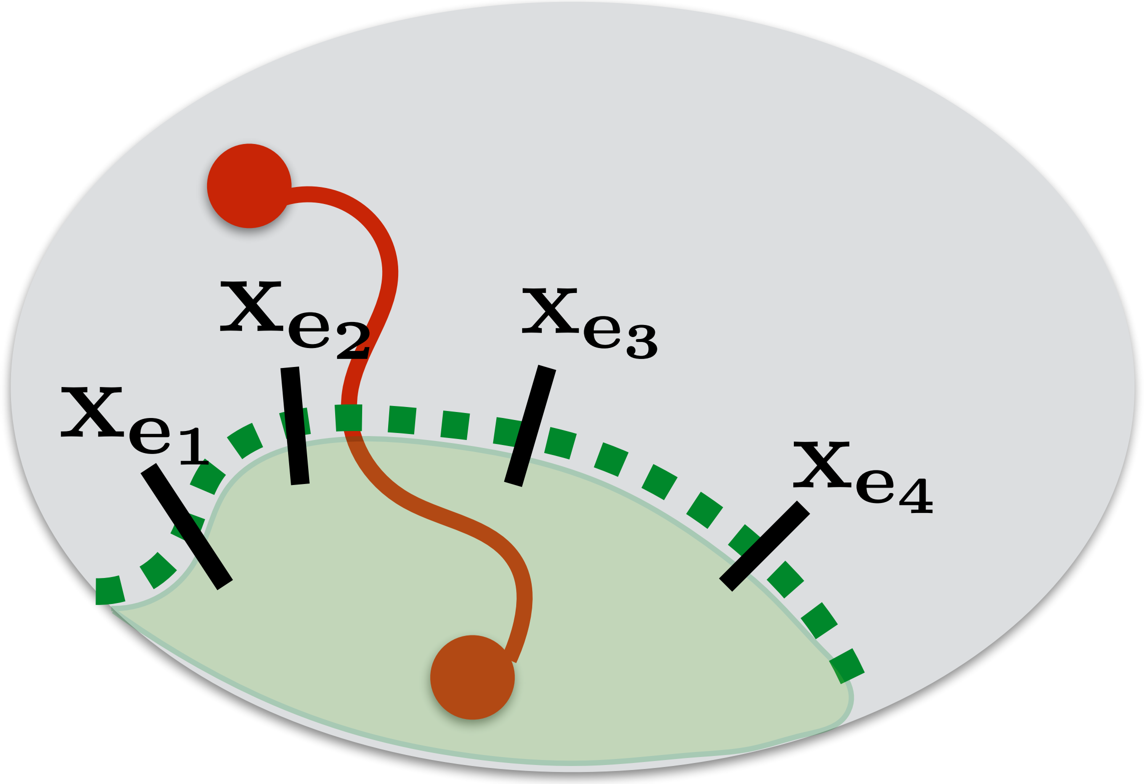


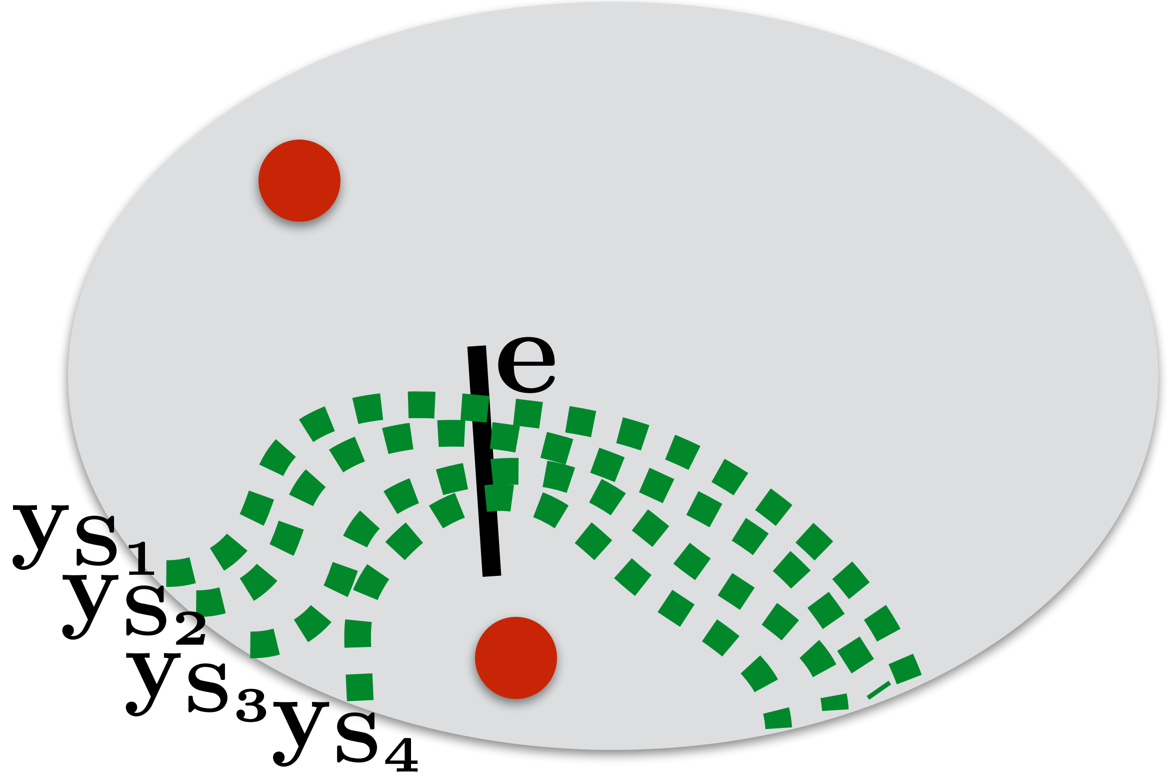


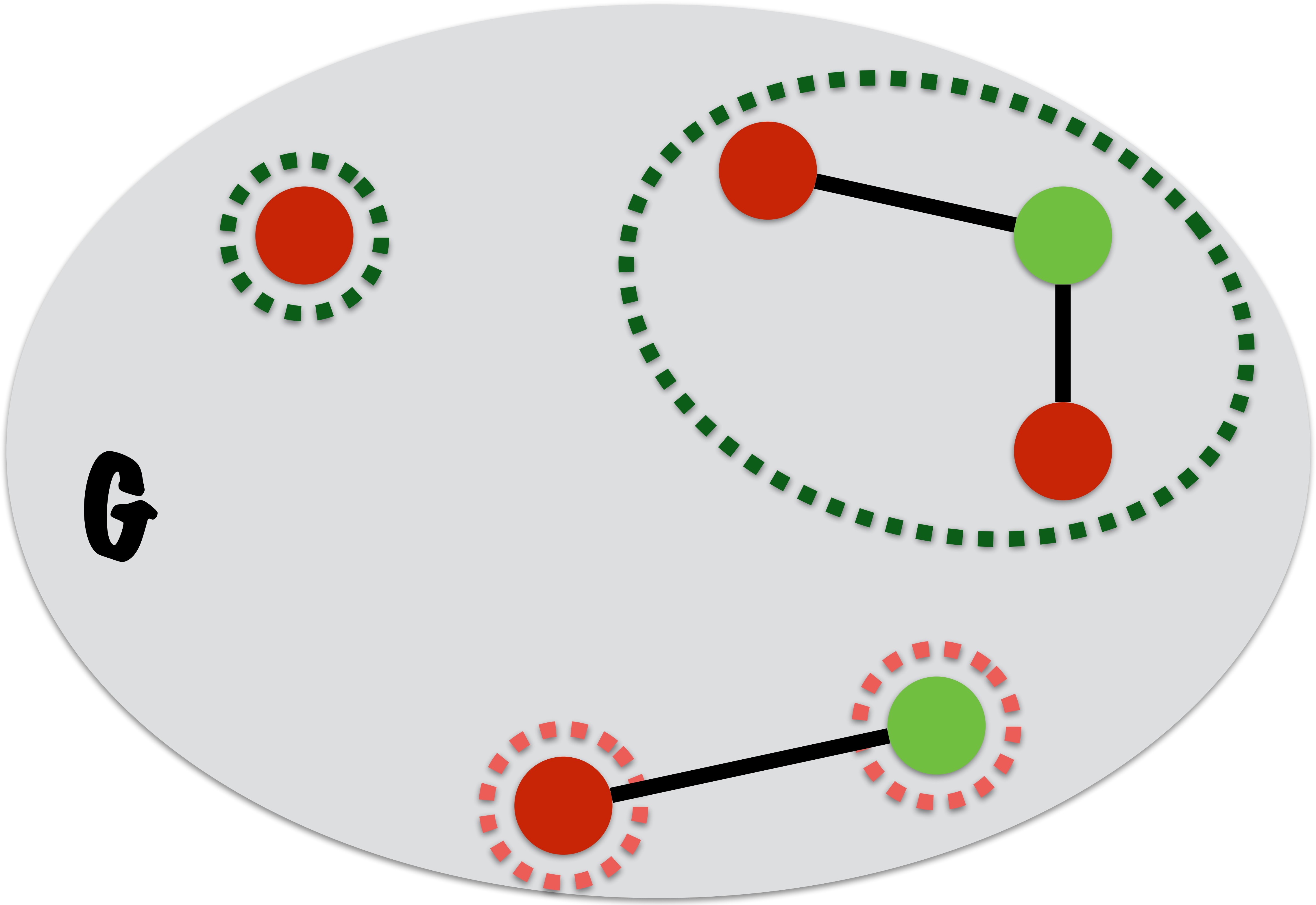


Then…

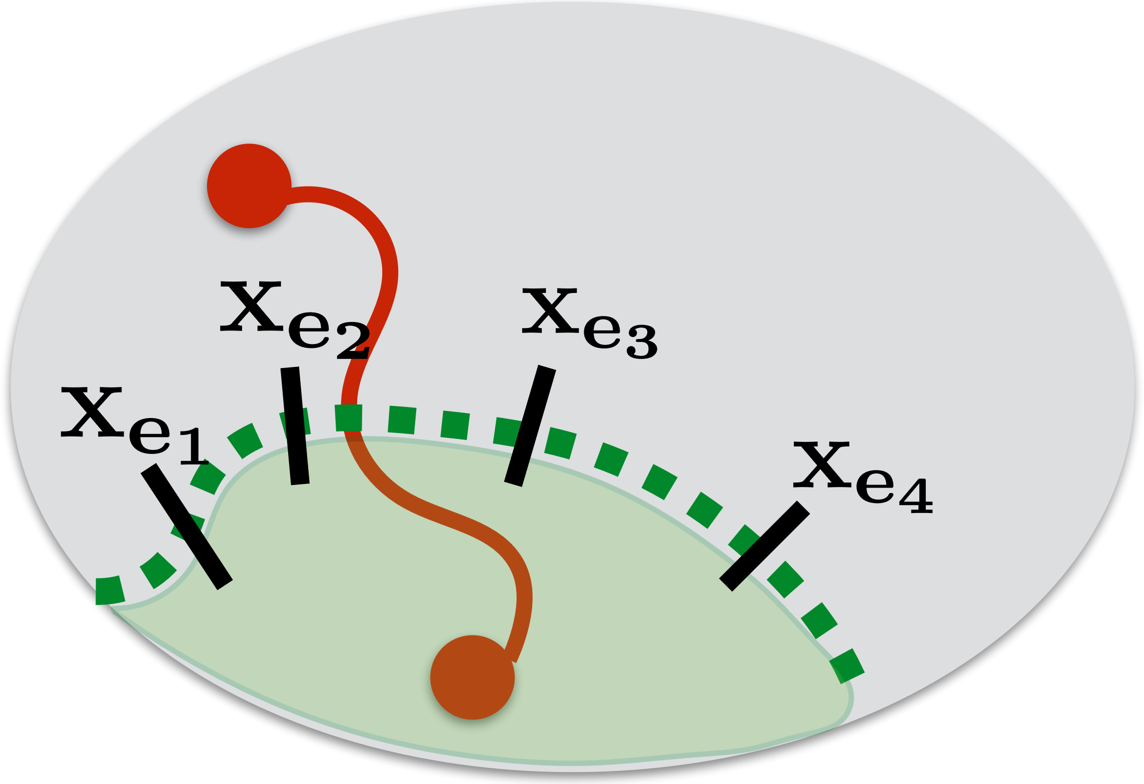
Q: How do we raise y?



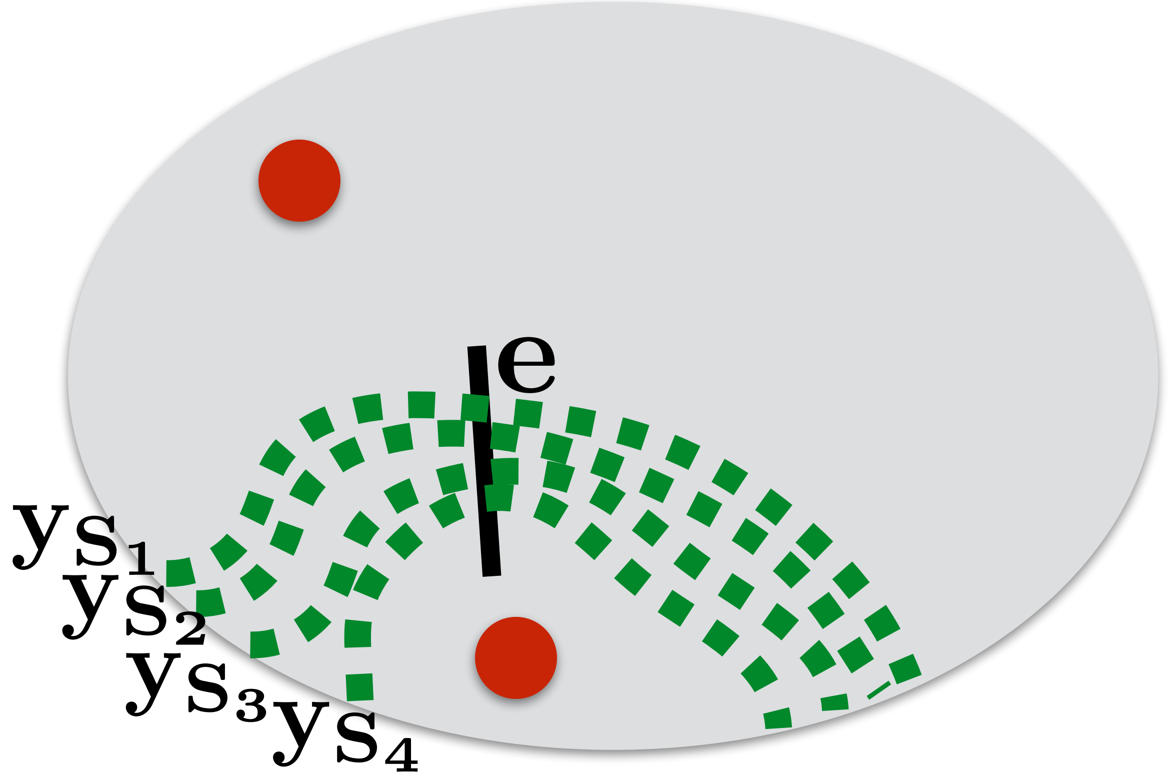


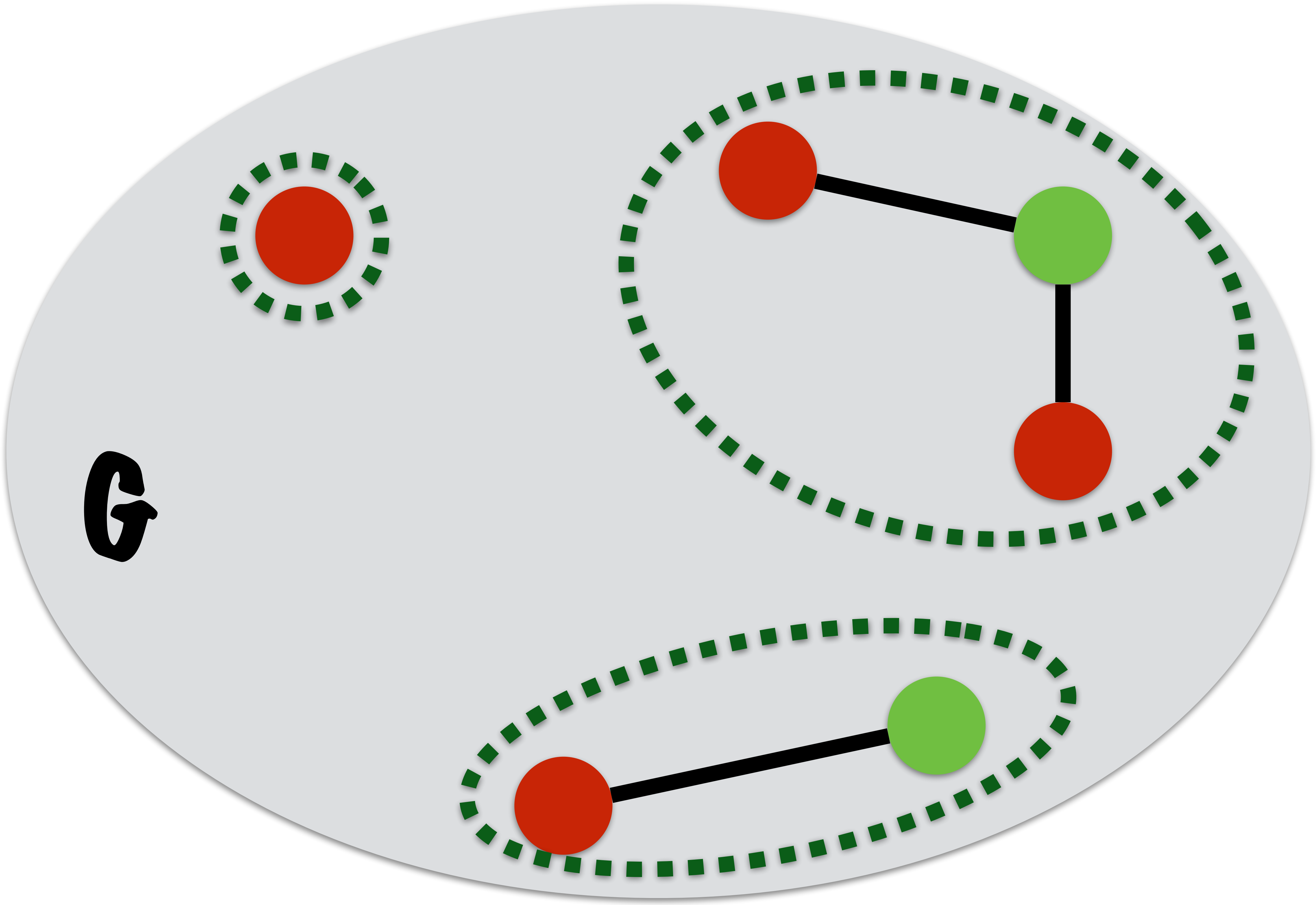


Tight constraint...



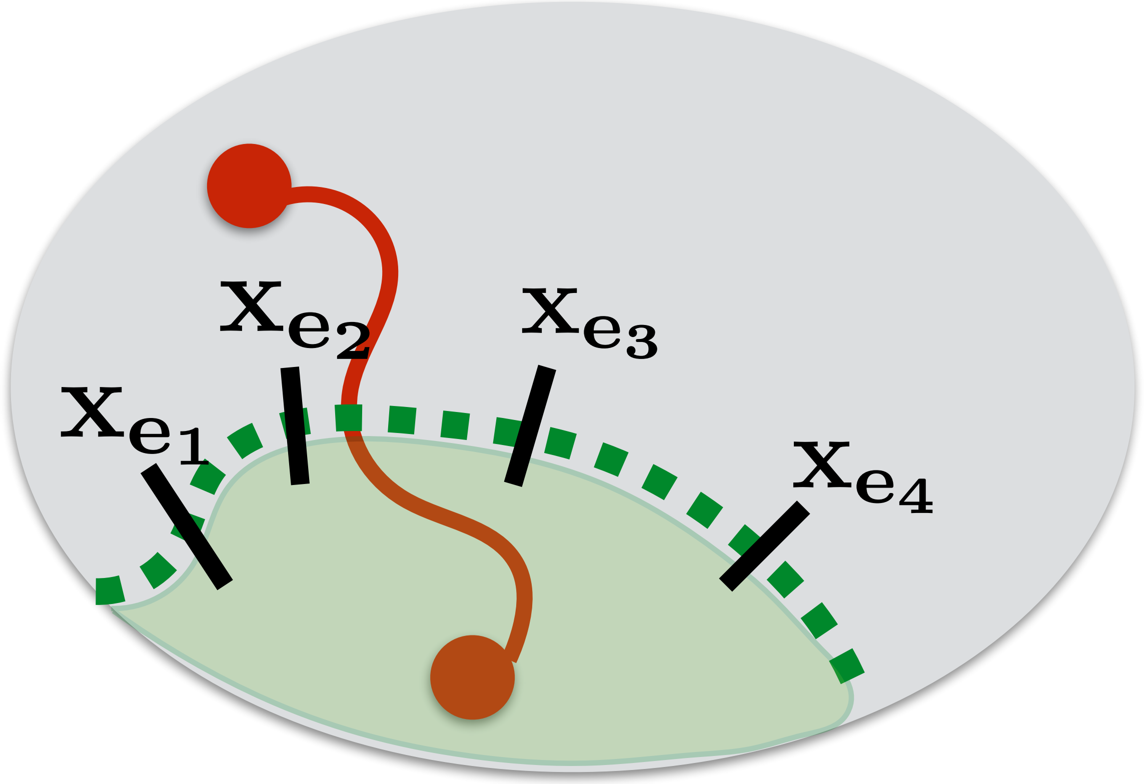
Q: How do we raise y?

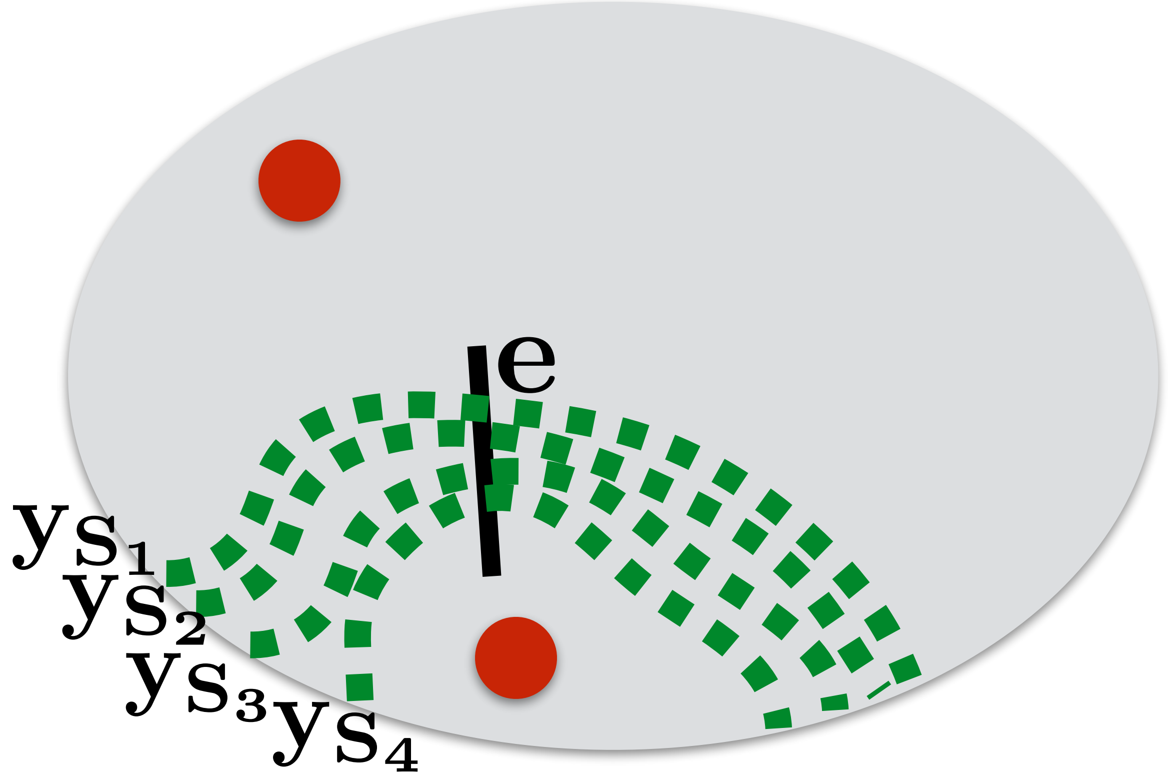


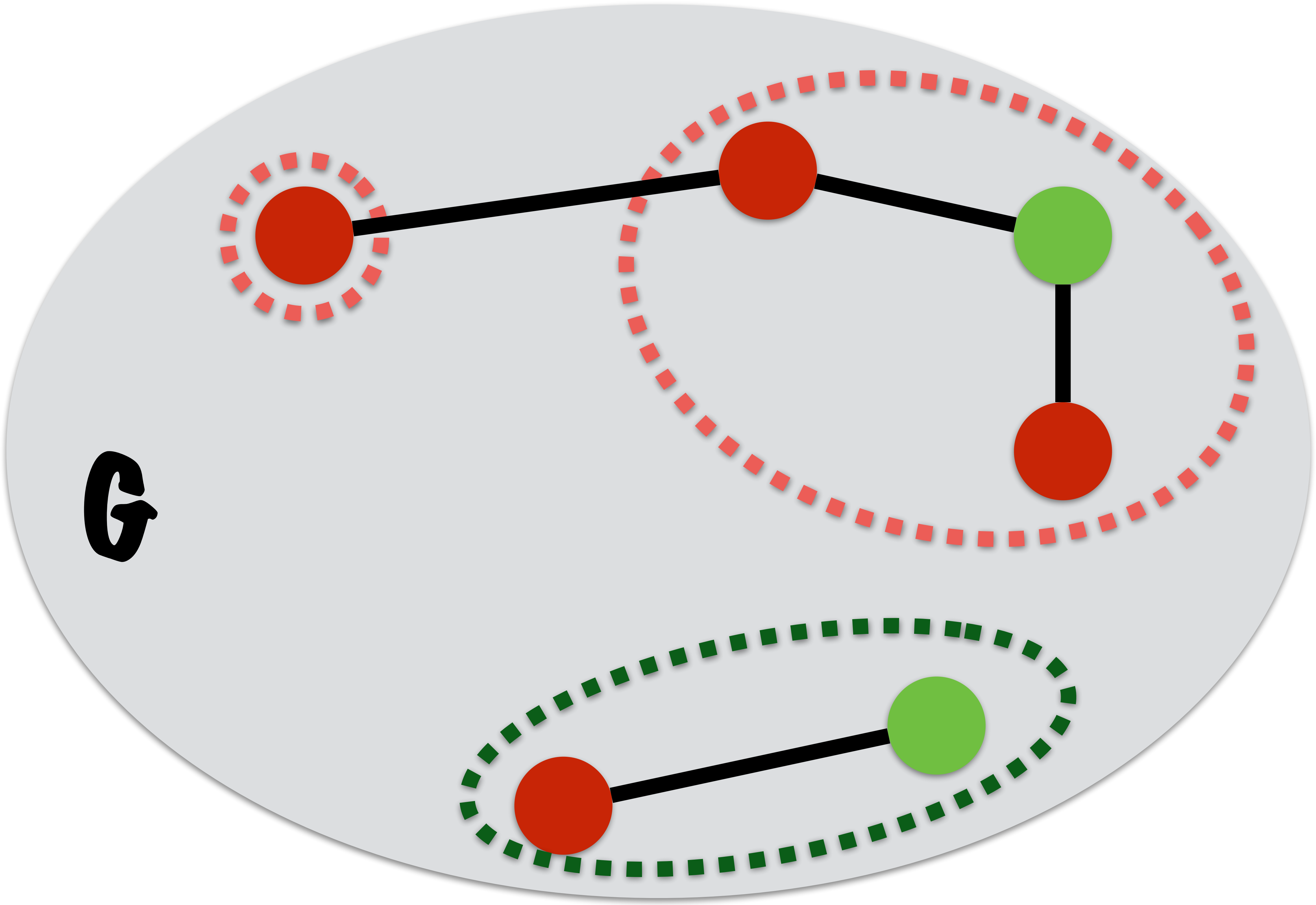


Then...

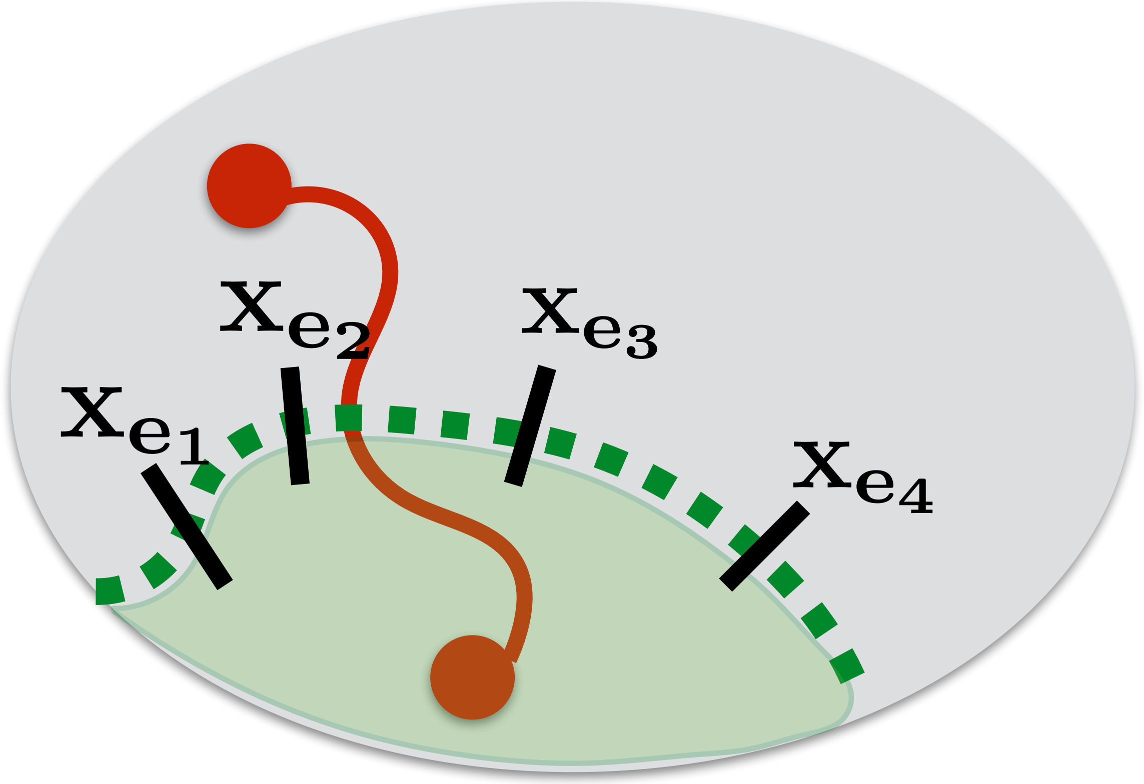
Q: How do we raise y?



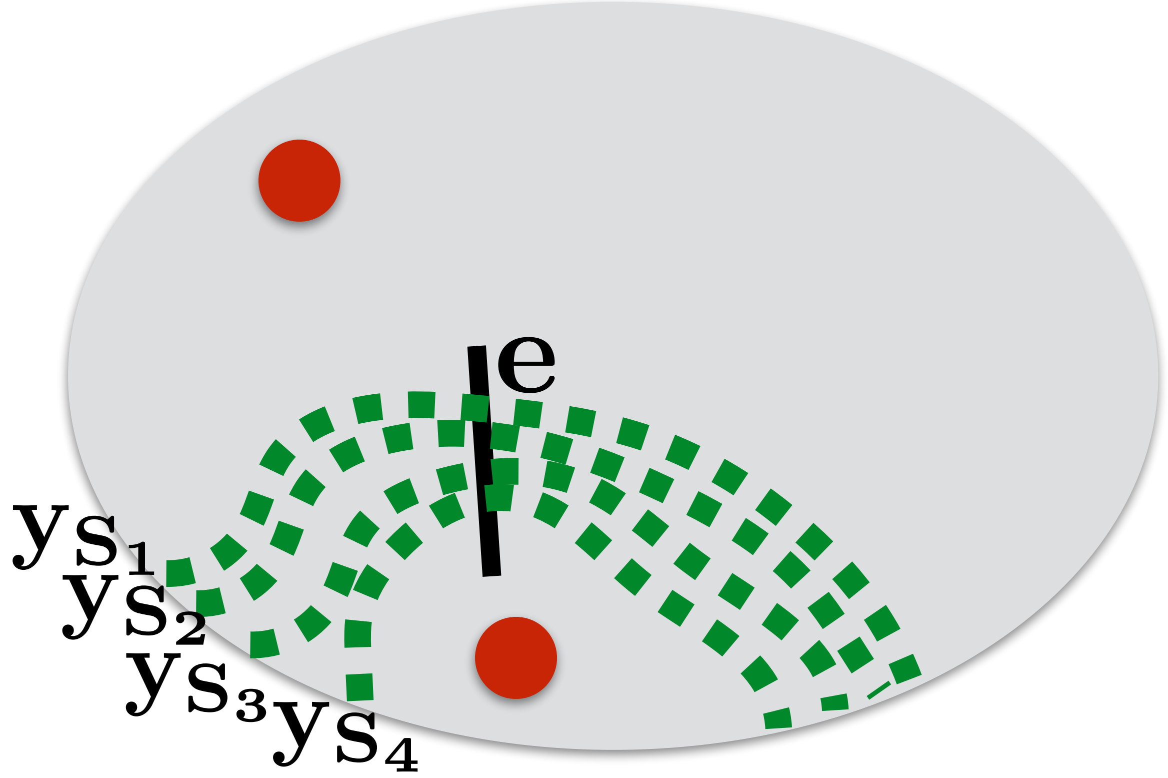


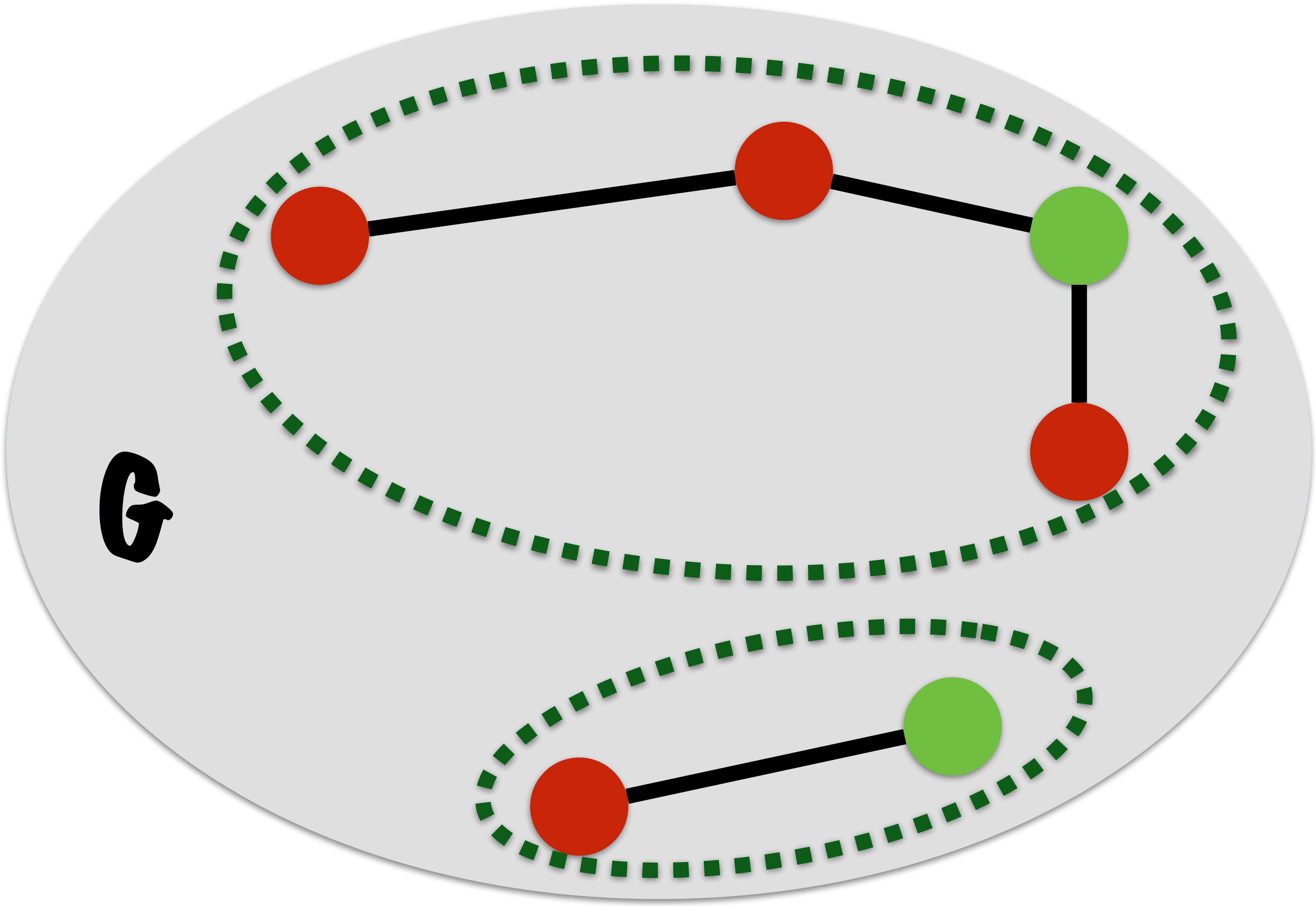


Tight constraint...

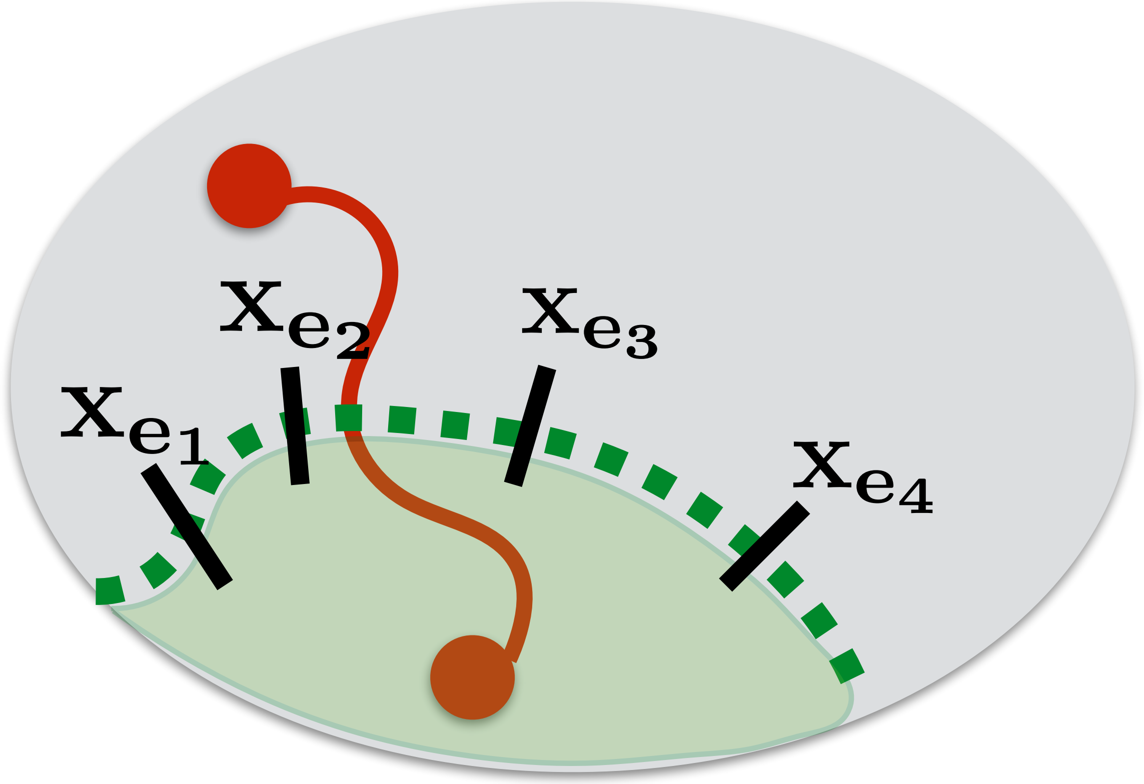


Q: How do we raise y?

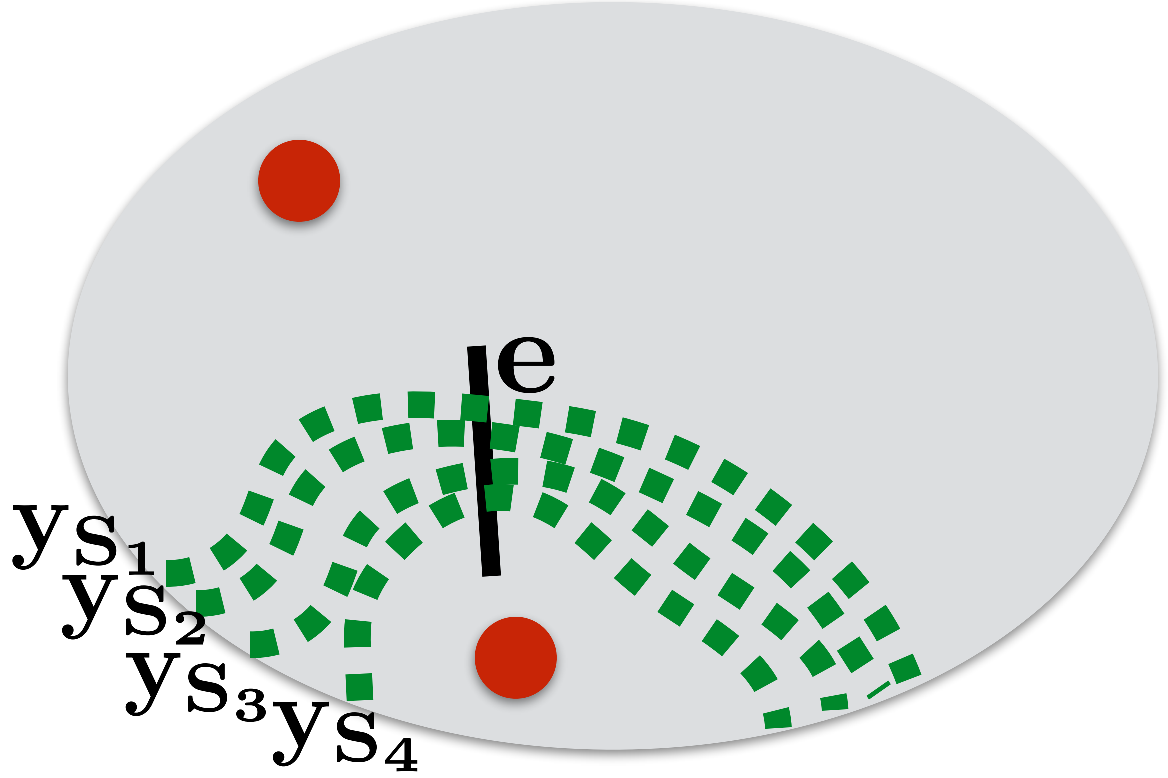


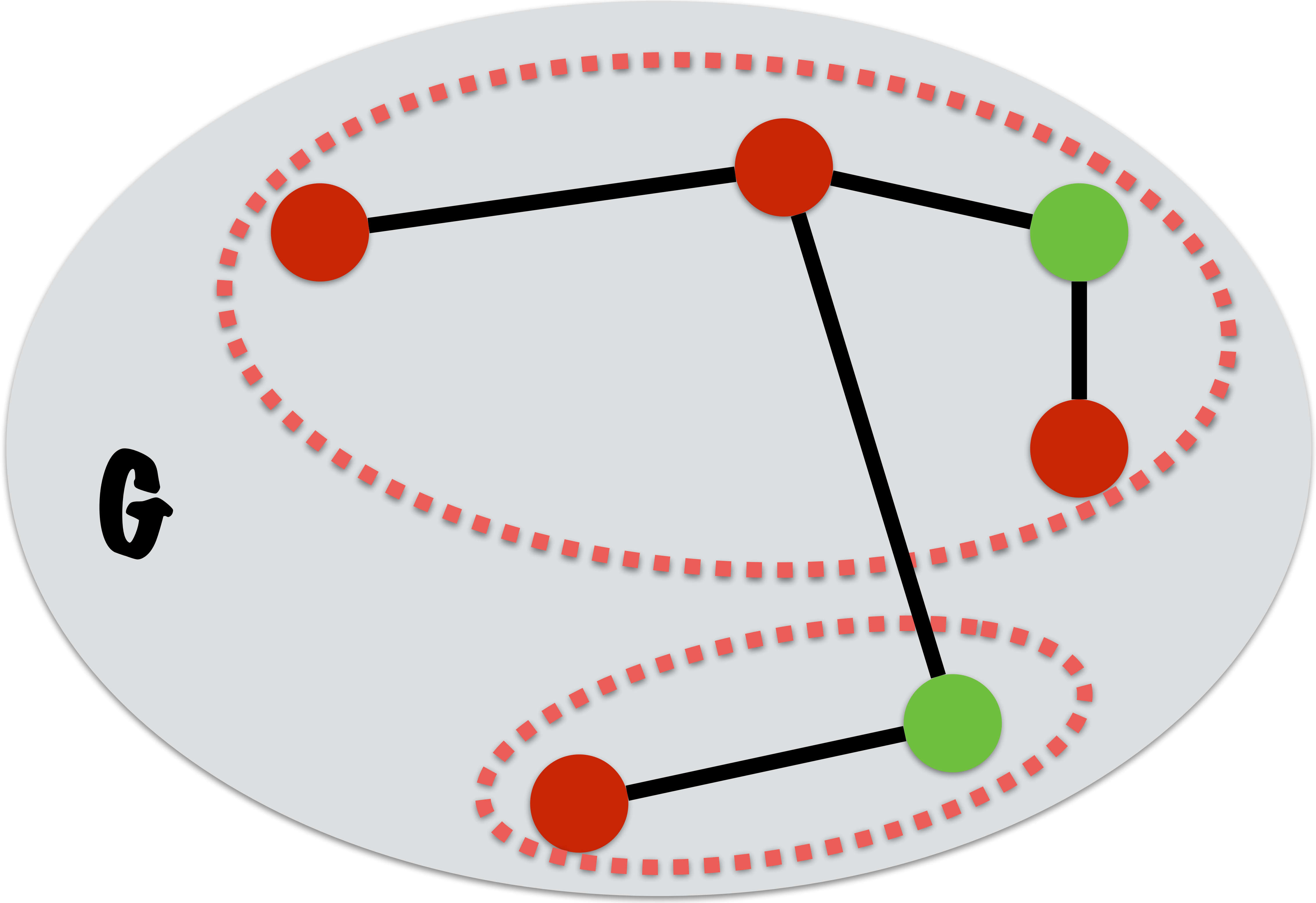


Then...

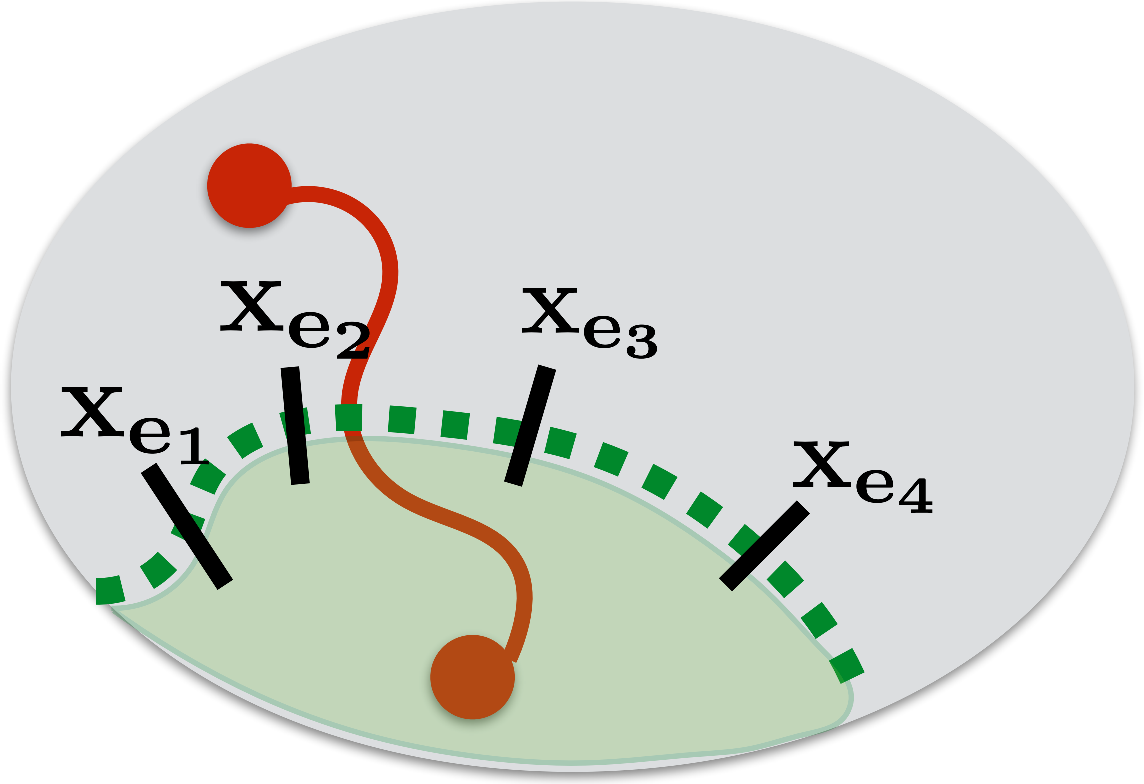


Q: How do we raise y?

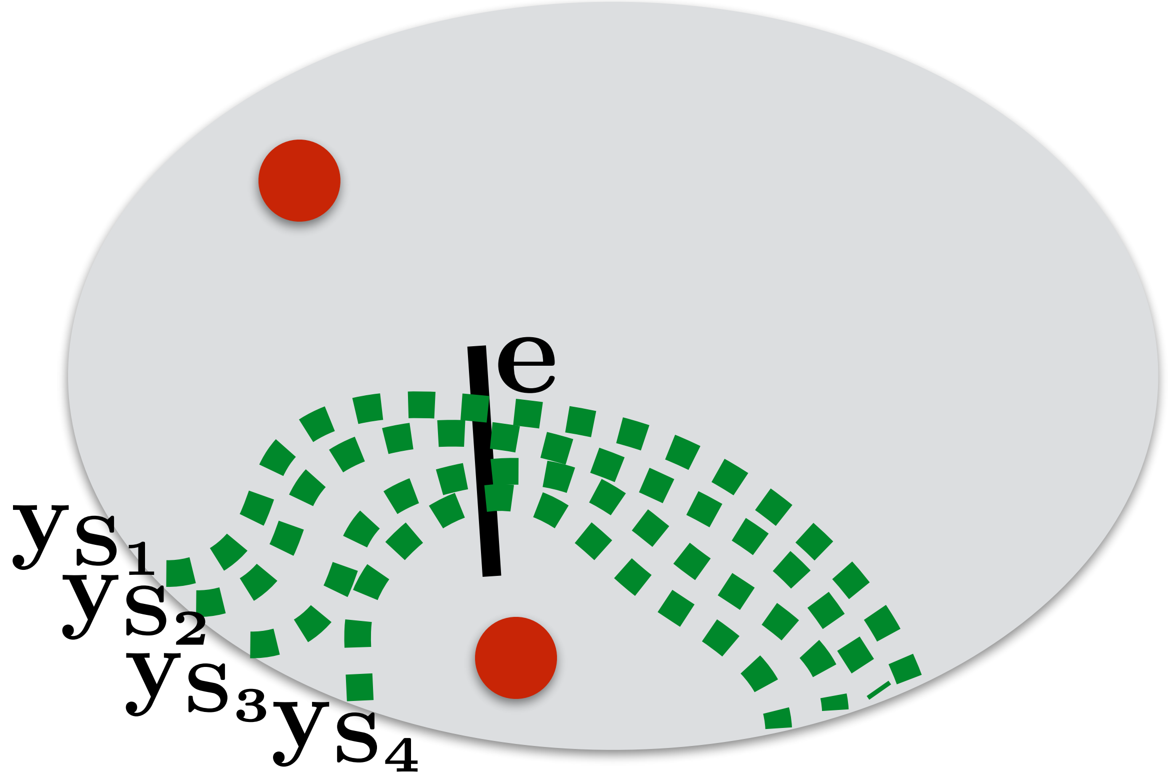


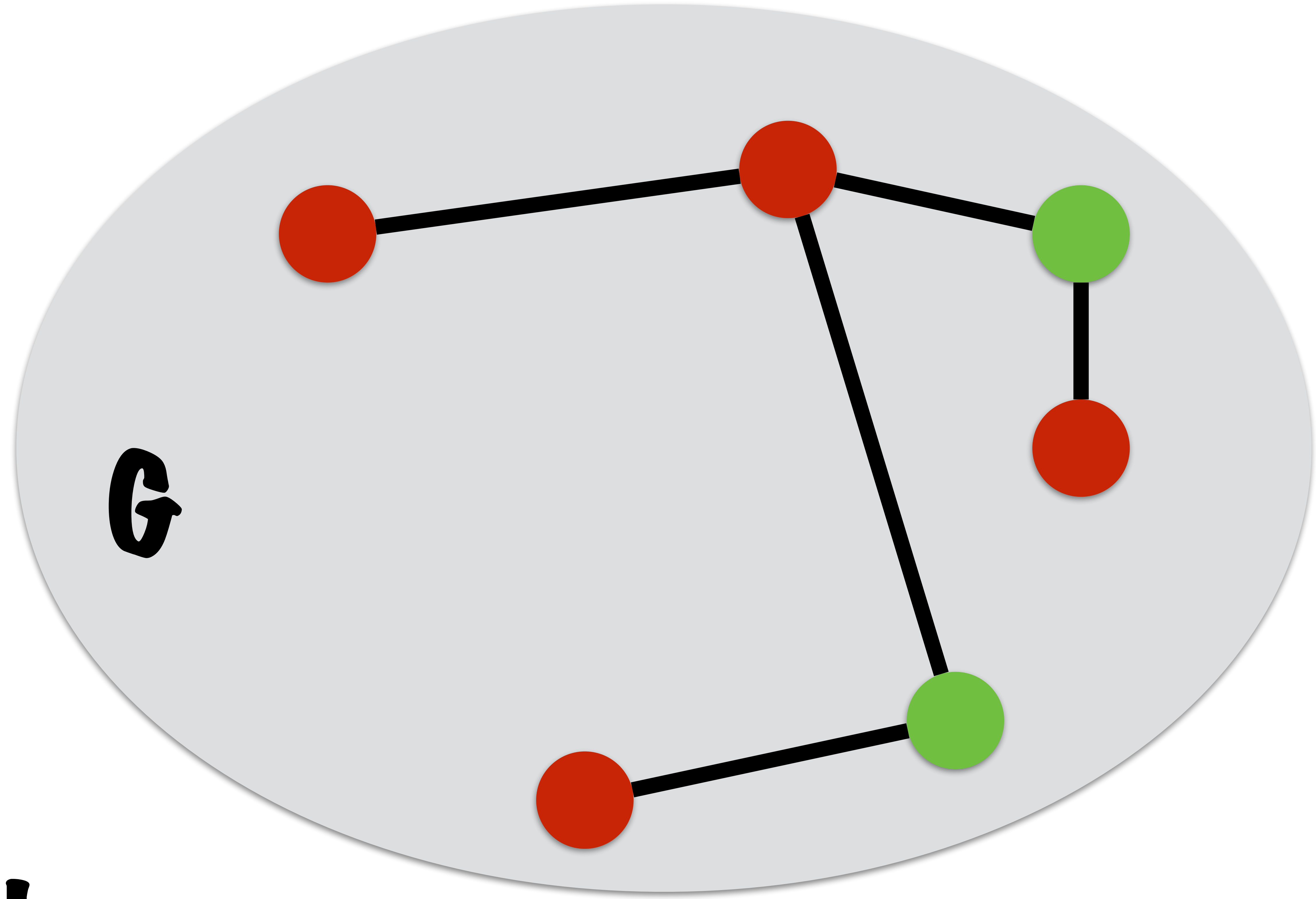


Tight constraint...



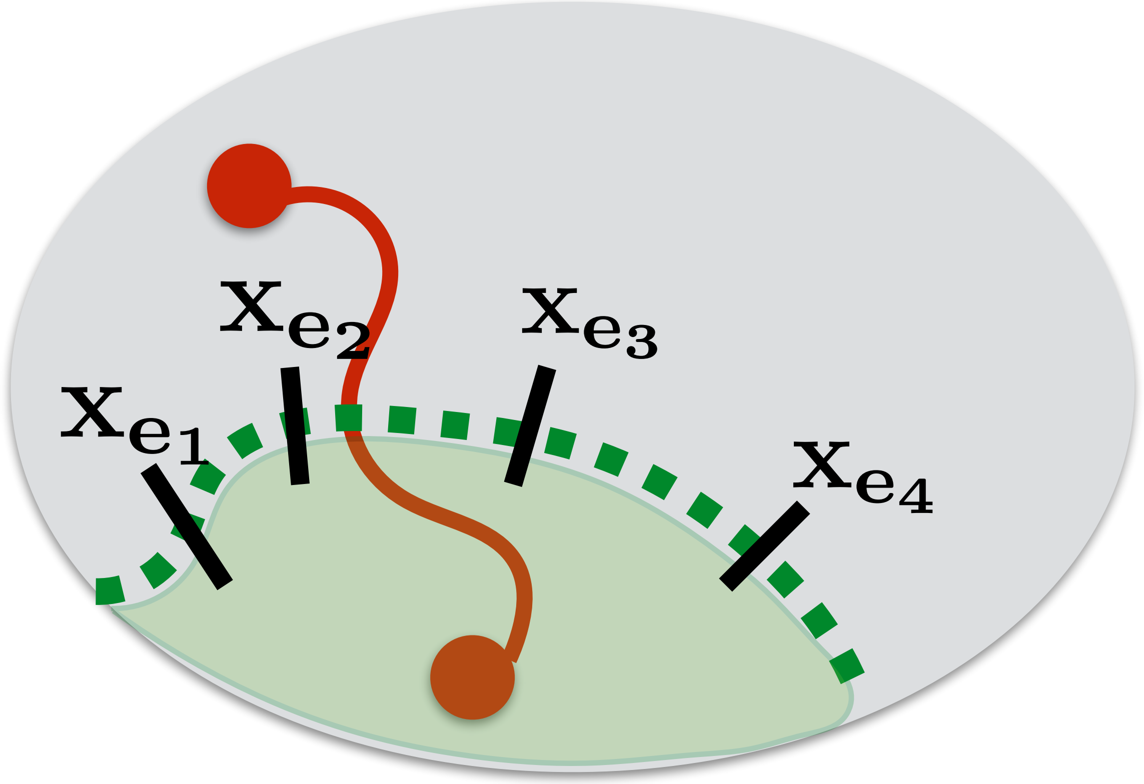
Q: How do we raise y?

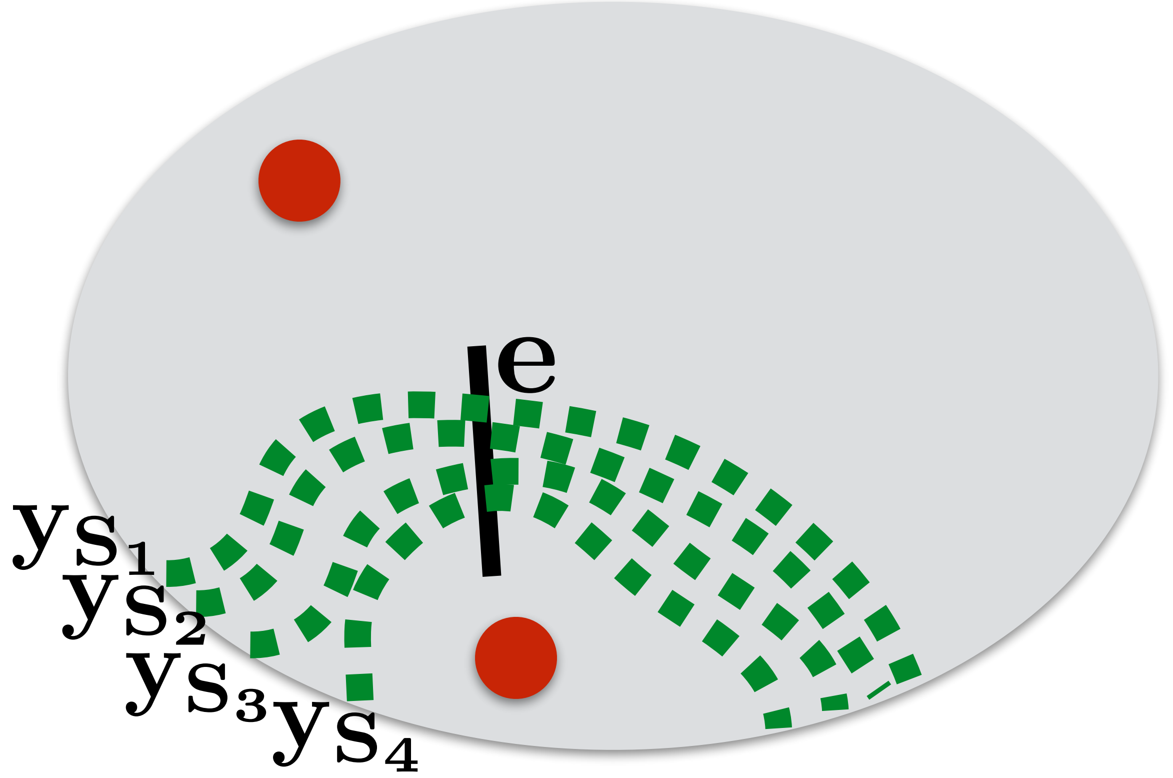


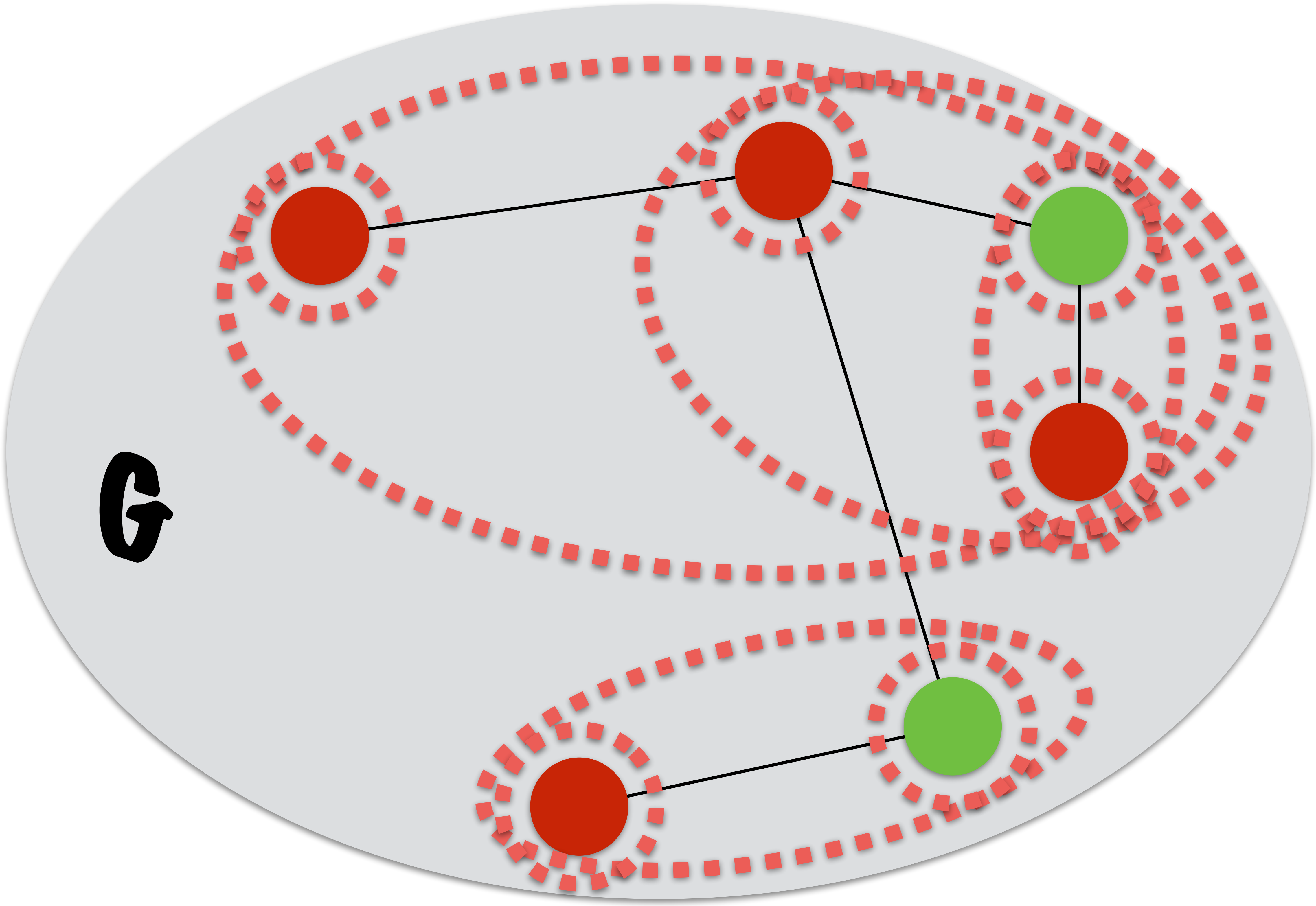


x is feasible

Q: How did we raise y?





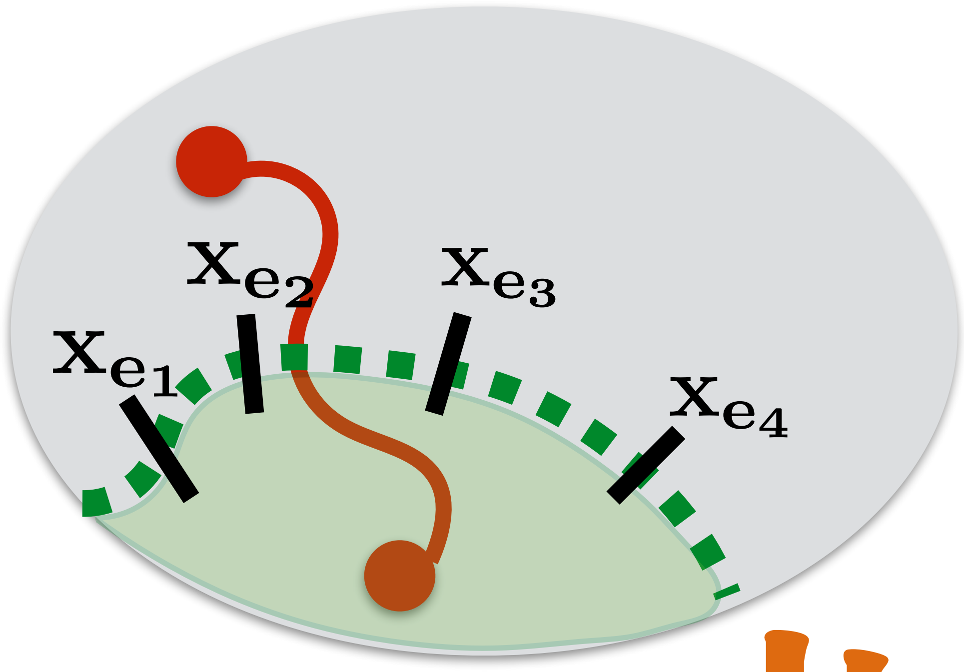


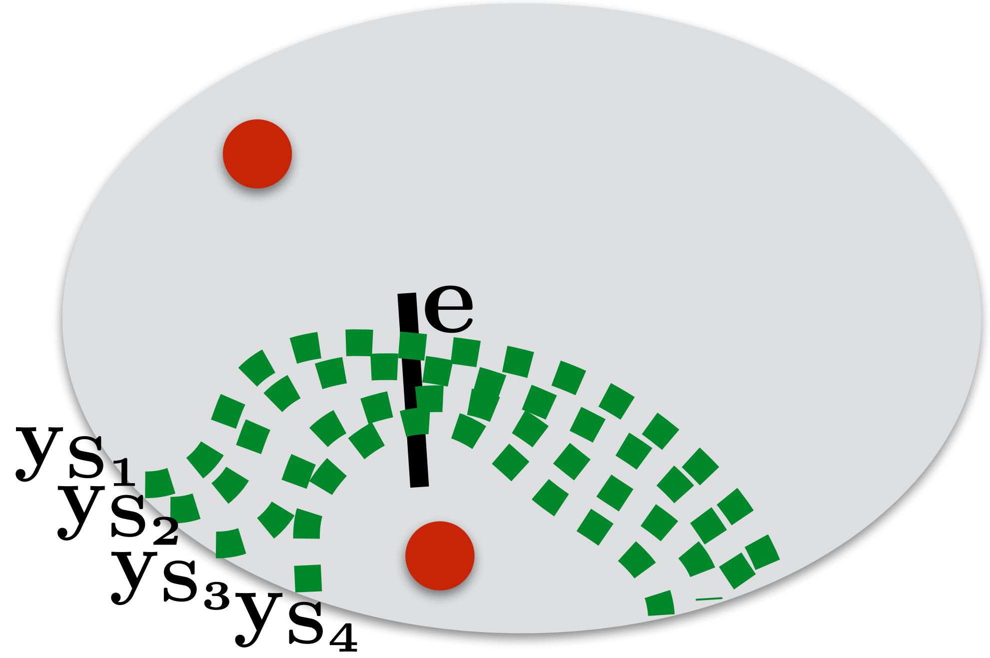
$$\min \sum_e c_e x_e :$$
$$\sum_{e \in \delta(S)} x_e \geq 1 \quad \forall S \in \mathcal{S}$$
$$x_e \geq 0 \quad \forall e \in E$$

$$\max \sum_S y_S :$$
$$\sum_{S: e \in \delta(S)} y_S \leq c_e \quad \forall e \in E$$
$$y_S \geq 0 \quad \forall S \in \mathcal{S}$$

Initialization:

$$x \leftarrow 0, y \leftarrow 0$$





Iteration: while x not satisfiable

in parallel, raise every unfrozen y_S with minimal S

stopped by tight constraint (e)

$$x_e \leftarrow 1$$

freeze y_S in tight constraints

Steiner forest