Module 1 Honors

Back to Week 1

8/8 points earned (100%)

Quiz passed!

1 / 1 points

1.
Which of the following (possibly more than one) must be true if random variable X is continuous with PDF $f(x)$?

- [ ] $\lim_{x\to\infty} f(x) = \infty$

Correct Response
This condition would cause $\int_{-\infty}^{\infty} f(x)dx = \infty$.

- [ ]

$f(x) \geq 0$ always

Correct Response

$\int_{-\infty}^{\infty} f(x)dx = 1$

Correct Response

$f(x)$ is a continuous function

Correct Response

One counter-example is the Uniform(0,1) PDF, which has jumps at 0 and 1.

$X >= 0$ always

Correct Response

Continuous random variables can take negative values.

$f(x)$ is an increasing function of x

Correct Response

This condition would cause $\int_{-\infty}^{\infty} f(x)dx = \infty$.

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1 / 1
points

2. If $X \sim \text{Exp}(3)$, what is the value of $P(X > 1/3)$? Round your answer to two decimal places.

0.37

Correct Response

This is $P(X > 1/3) = \int_{1/3}^{\infty} 3e^{-3x}\,dx$

$$= -e^{-3x}|_{1/3}^{\infty}$$

$$= 0 - (-e^{-3/3}) = e^{-1} = 0.368$$

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1 / 1
points

3. Suppose $X \sim \text{Uniform}(0, 2)$ and $Y \sim \text{Uniform}(8, 10)$. What is the value of $E(4X + Y)$?

13

Correct Response

This is $E(4X + Y) = 4E(X) + E(Y) = 4(1) + 9$.

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1 / 1
points

4.

For Questions 4-7, consider the following:

Suppose $X \sim \mathrm{N}(1, 5^2)$ and $Y \sim \mathrm{N}(-2, 3^2)$ and that X and Y are independent. We have $Z = X + Y \sim \mathrm{N}(\mu, \sigma^2)$ because the sum of normal random variables also follows a normal distribution.

- What is the value of μ?

-1

Correct Response

$\mu = E(Z) = E(X+Y) = E(X) + E(Y) = 1 + (-2)$

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1 / 1 points

Adding normals:

- What is the value of σ^2?

5. Hint: If two random variables are independent, the variance of their sum is the sum of their variances.

34

Correct Response

$\sigma^2 = Var(Z) = Var(X+Y) = Var(X) + Var(Y) = 25 + 9$.

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1 / 1
points

6.
Adding normals:

If random variables X and Y are not independent, we still have $E(X+Y) = E(X) + E(Y)$, but now $Var(X+Y) = Var(X) + Var(Y) + 2Cov(X,Y)$ where $Cov(X,Y) = E[(X - E[X])(Y - E[Y])]$ is called the covariance between X and Y.

- A convenient formula for calculating variance was given in the supplementary material: $Var(X) = E[(X - E[X])^2] = E[X^2] - (E[X])^2$. Which of the following is an analogous expression for the covariance of X and Y?

Hint: Expand the terms inside the expectation in the definition of $Cov(X,Y)$ and recall that $E(X)$ and $E(Y)$ are just constants.

- $E(XY) - E(X)E(Y)$

Correct Response

$$Cov(X,Y) = E[(X - E[X])(Y - E[Y])]$$
$$= E[XY - XE(Y) - E(X)Y + E(X)E(Y)]$$
$$= E[XY] - E[XE(Y)] - E[E(X)Y] + E[E(X)E(Y)]$$
$$= E[XY] - E(X)E(Y) - E(X)E(Y) + E(X)E(Y)$$

- $(E[X^2] - (E[X])^2) \cdot (E[Y^2] - (E[Y])^2)$
- $E[X^2] - (E[X])^2 + E[Y^2] - (E[Y])^2$
- $E[Y^2] - (E[Y])^2$

1 / 1
points

7.
Adding normals:

- Consider again $X \sim \mathrm{N}(1, 5^2)$ and $Y \sim \mathrm{N}(-2, 3^2)$, but this time X and Y are *not* independent. Then $Z = X + Y$ is still normally distributed with the same mean found in Question 4. What is the variance of Z if $E(XY) = -5$?

Hint: Use the formulas introduced in Question 6.

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Correct Response

$$Var(Z) = Var(X) + Var(Y) + 2Cov(X, Y) = 25 + 9 + 2Cov(X, Y)$$
$$= 34 + 2(E[XY] - E[X]E[Y])$$
$$= 34 + 2(-5 - 1(-2)) = 34 - 2(3)$$

1 / 1
points

8.
Free point:

1) Use the definition of conditional probability to show that for events A and B, we have
$P(A \cap B) = P(B|A)P(A) = P(A|B)P(B)$.

2) Show that the two expressions for independence $P(A|B) = P(A)$ and $P(A \cap B) = P(A)P(B)$ are equivalent.

☐ Solution (1)

Write $P(B|A) = \frac{P(A \cap B)}{P(A)}$ and multiply both sides by $P(A)$.

- [ ] Solution (2)

Plug these expressions into those from (1).