How to determine covariance matrix Q and R in Kalman-filter

I am implementing getting orientation from smartphone. I want to use Kalman filter and should determine process noise covariance matrix Q and measurement noise covariance matrix R. (newbie to Kalman filter)

I don't have any idea how to determine Q. What I think about R is as follows:

state vector : quaternion from (accelerometer + gyroscope)

(1) My phone is stand still. I get covaraince matrix from Matlab

1.0e-04 *

```
0.0000    0.0005    0.0035   -0.0000
0.0005    0.0063    0.0411   -0.0002
0.0035    0.0411    0.2881   -0.0014
```

-0.0000 -0.0002 -0.0014 0.0000

(2) My phone had been moved for 5 seconds.

covariance matrix is

```
0.0417   -0.0533   -0.0008   -0.0014
-0.0533    0.0784    0.0015    0.0018
```

```
-0.0008    0.0015    0.0001    0.0001
-0.0014    0.0018    0.0001    0.0001
```

Is there anyone to help?

(added)

Details are omitted.

case 1: Kalman Filter

The row data from my phone is p, q, r (angular velocity). I omit the conversion equation between angular velocity and quaternion.

$$\begin{aligned} x_{k+1} &= Ax_k + w_k \\ z_k &= Hx_k + \nu_k \\ Q &: \text{ covariance matrix for } w_k \\ R &: \text{ covariance matrix for } \nu_k \end{aligned}$$

$$\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \\ \dot{q}_4 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 0 & -p & -q & -r \\ p & 0 & r & -q \\ q & -r & 0 & p \\ r & q & -p & 0 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix}$$

$$\underbrace{\begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix}_{k+1}}_{x_{k+1}} = \underbrace{\left(I + \Delta t \cdot \frac{1}{2} \begin{bmatrix} 0 & -p & -q & -r \\ p & 0 & r & -q \\ q & -r & 0 & p \\ r & q & -p & 0 \end{bmatrix} \right)}_{A} \underbrace{\begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix}_{k}}_{x_k}$$

$$H = I$$

My guess for covariance matrix is as follows: (but I don' know how to infer..)

$$Q = 0.001I, \quad R = 10I.$$

case 2: Extended Kalman Filter

$$\begin{aligned} x_{k+1} &= f(x_k) + w_k \\ z_k &= h(x_k) + \nu_k \\ Q &: \text{ covariance matrix for } w_k \\ R &: \text{ covariance matrix for } \nu_k \end{aligned}$$

$$A = \left.\frac{\partial f}{\partial x}\right|_{x_k}, \quad H = \left.\frac{\partial h}{\partial x}\right|_{x_k}$$

$$\begin{aligned} \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\varphi} \end{bmatrix} &= \begin{bmatrix} 1 & \sin\phi\tan\theta & \cos\phi\tan\theta \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi\sec\theta & \cos\phi\sec\theta \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix} \\ &= \begin{bmatrix} p + q\sin\phi\tan\theta + r\cos\phi\tan\theta \\ q\cos\phi - r\sin\phi \\ q\sin\phi\sec\theta + r\cos\phi\sec\theta \end{bmatrix} \\ &= f(x) + w \end{aligned}$$

$$z = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \phi \\ \theta \\ \varphi \end{bmatrix} + \nu = Hx + \nu$$

$$A = \begin{bmatrix} \frac{\partial f_1}{\partial \phi} & \frac{\partial f_1}{\partial \theta} & \frac{\partial f_1}{\partial \varphi} \\ \frac{\partial f_2}{\partial \phi} & \frac{\partial f_2}{\partial \theta} & \frac{\partial f_2}{\partial \varphi} \\ \frac{\partial f_3}{\partial \phi} & \frac{\partial f_3}{\partial \theta} & \frac{\partial f_3}{\partial \varphi} \end{bmatrix}$$

(I emphasize that details are omitted.) In this case, also I don't know how to infere Q, R.

kalman-filters sensor

edited Jan 1 '15 at 12:20 asked Dec 24 '14 at 16:16

jakeoung
207 1 10

The direct use of a quaternion in a Kalman Filter is bad news - a quaternion is not a vector and the "states" are not independent, which essentially destroys the assumptions of the filter. Accordingly, the covariance is meaningless. – Damien Dec 27 '14 at 1:07

Instead, formulate the filter in terms of *error states*, or if you insist on using direct attitude terms, use an Extended Kalman Filter with Euler Angles. There's some serious maths here, but textbooks from Groves and Farrell are quite useful. – Damien Dec 27 '14 at 1:10

Thank for your comment. Actually, eve with that, how to determine Q and R? – jakeoung Dec 27 '14 at 7:06

You will need to post your process and measurement model (as $\LaTeX$, not code) before I can make an informed comment. – Damien Dec 27 '14 at 11:35

Okay, I've added equations. – jakeoung Dec 27 '14 at 12:56