The dance of error, confidence, and sample size

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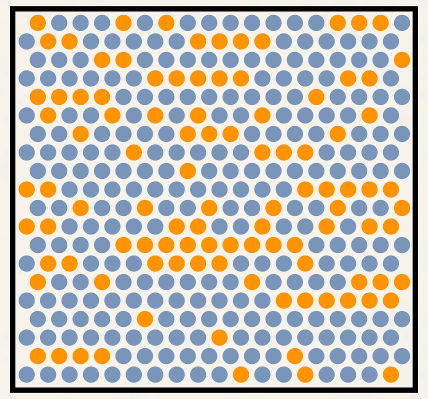
How well does S_n/n approximate p?

$p = 0.6$



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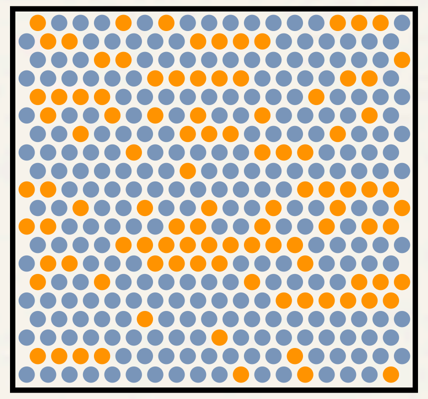
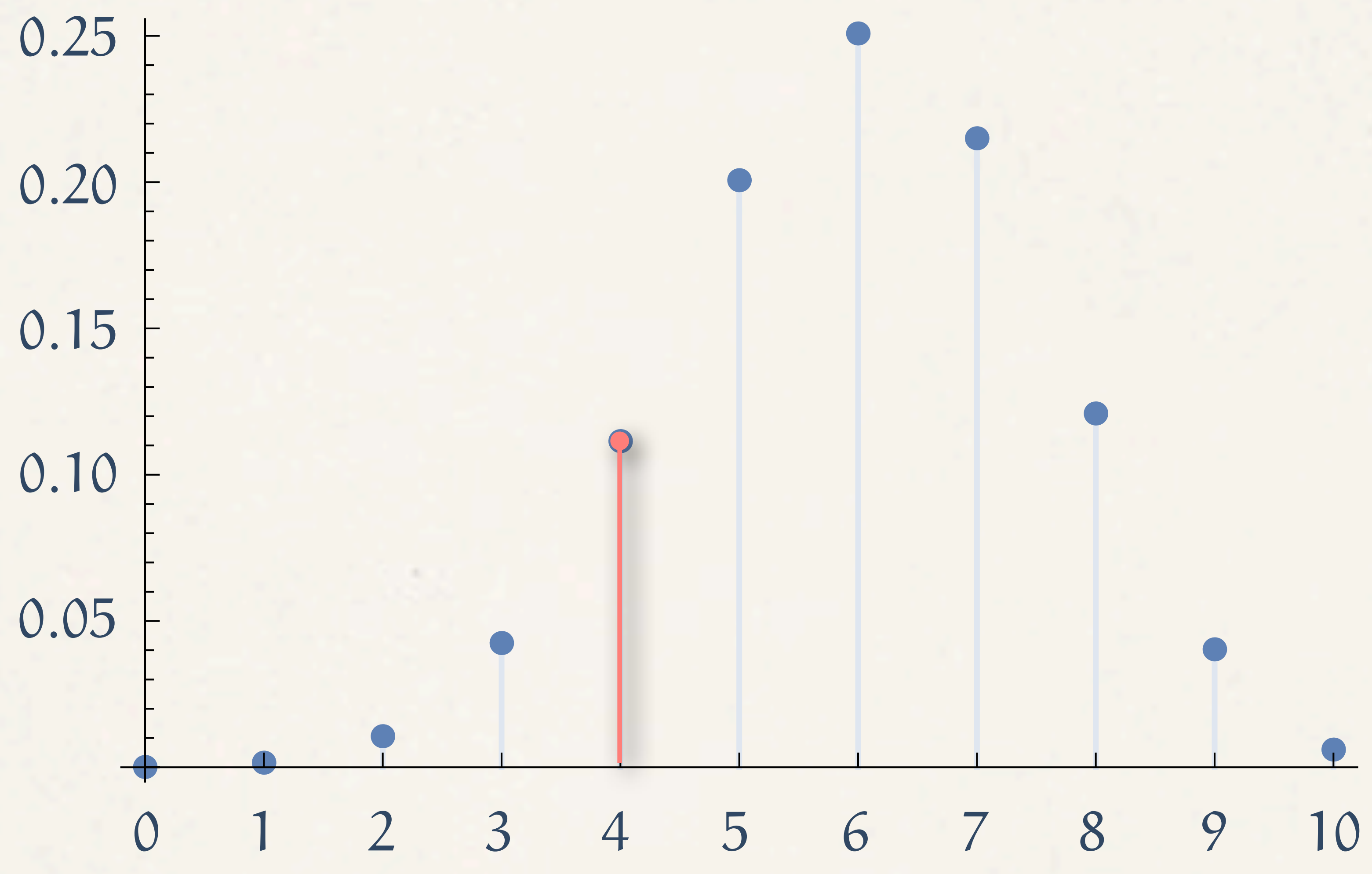


Random sample Bernoulli(0.6)										Accumulated successes Binomial(10, 0.6)
X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	X_{10}	S_{10}
1	0	0	1	1	1	0	0	0	0	4

$$\mathbf{P}\{S_{10} = k\} = b_{10}(k; 0.6) := \binom{10}{k} 0.6^k (1-0.6)^{10-k} \qquad (k = 0, 1, \ldots, 10)$$



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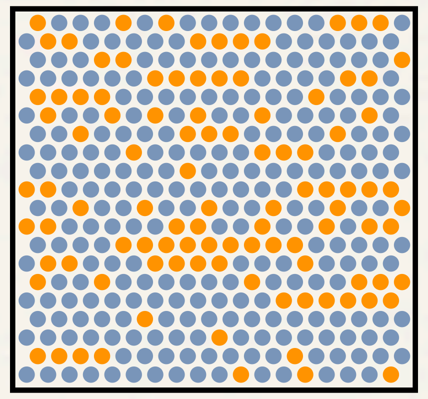

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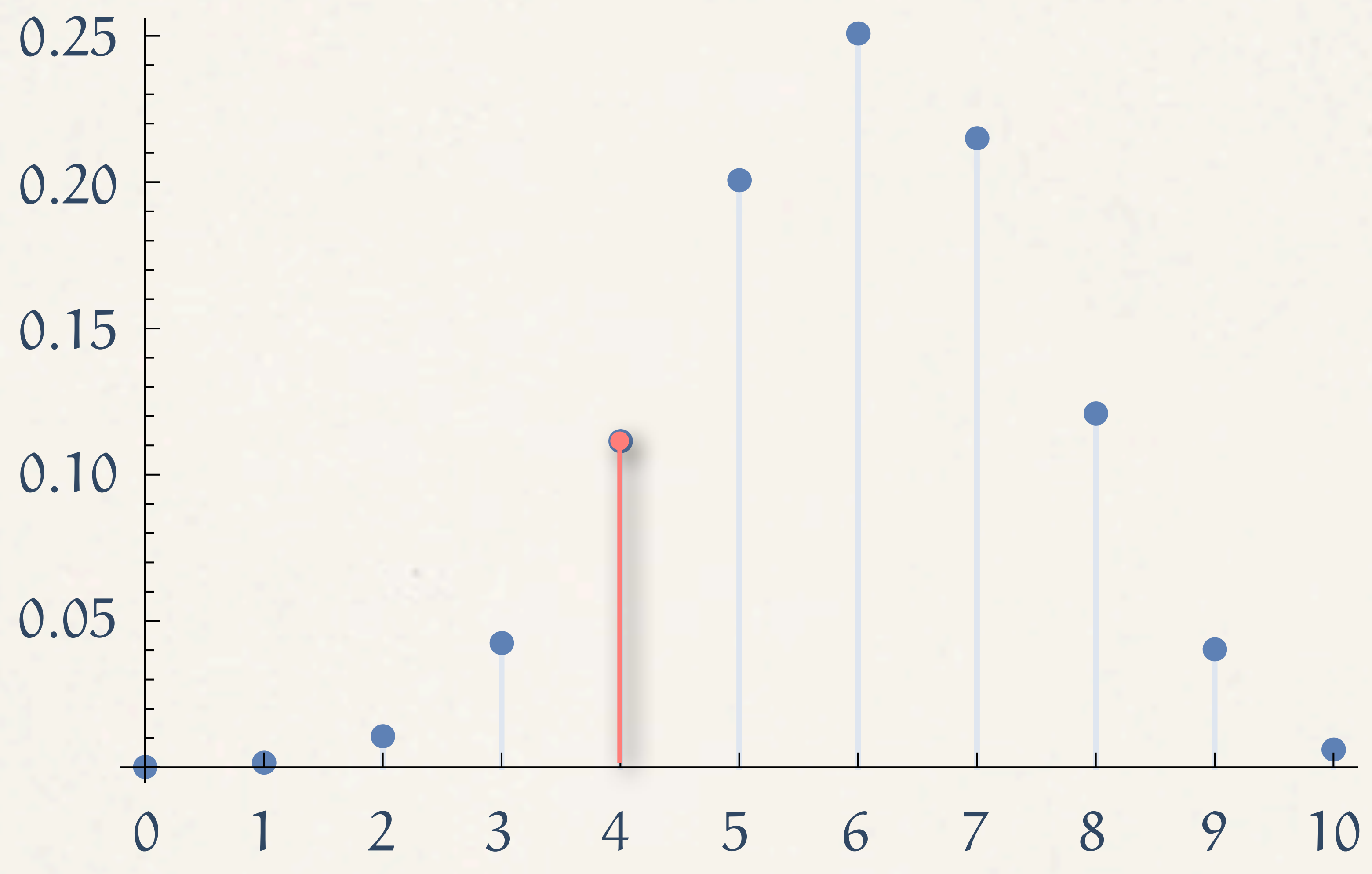

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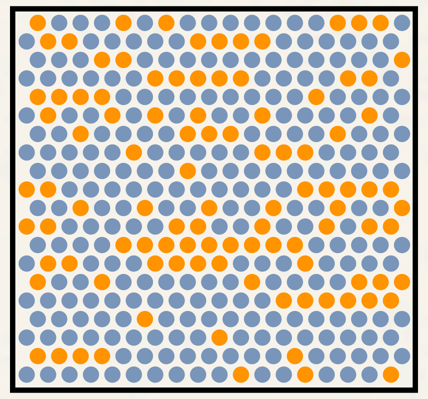

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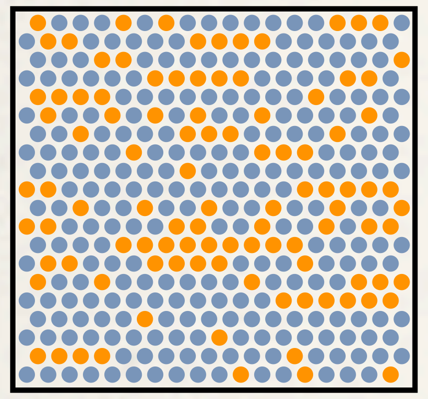

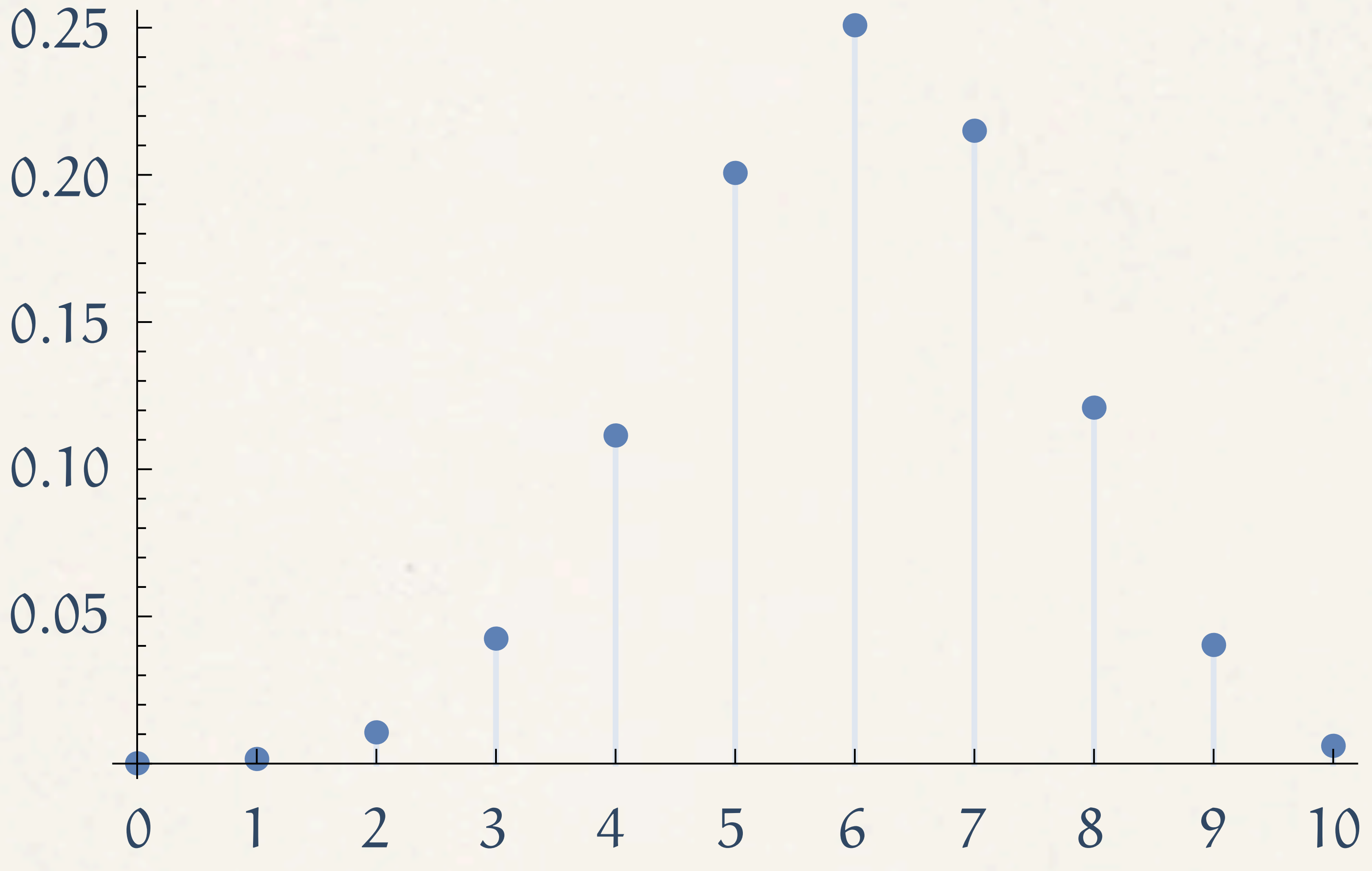

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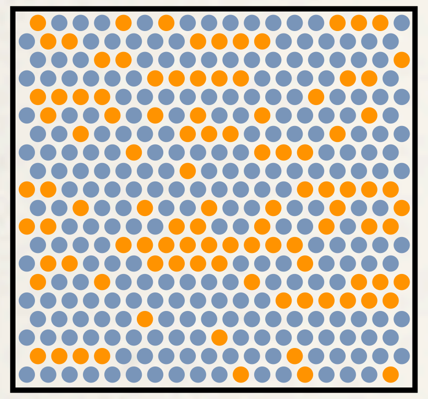

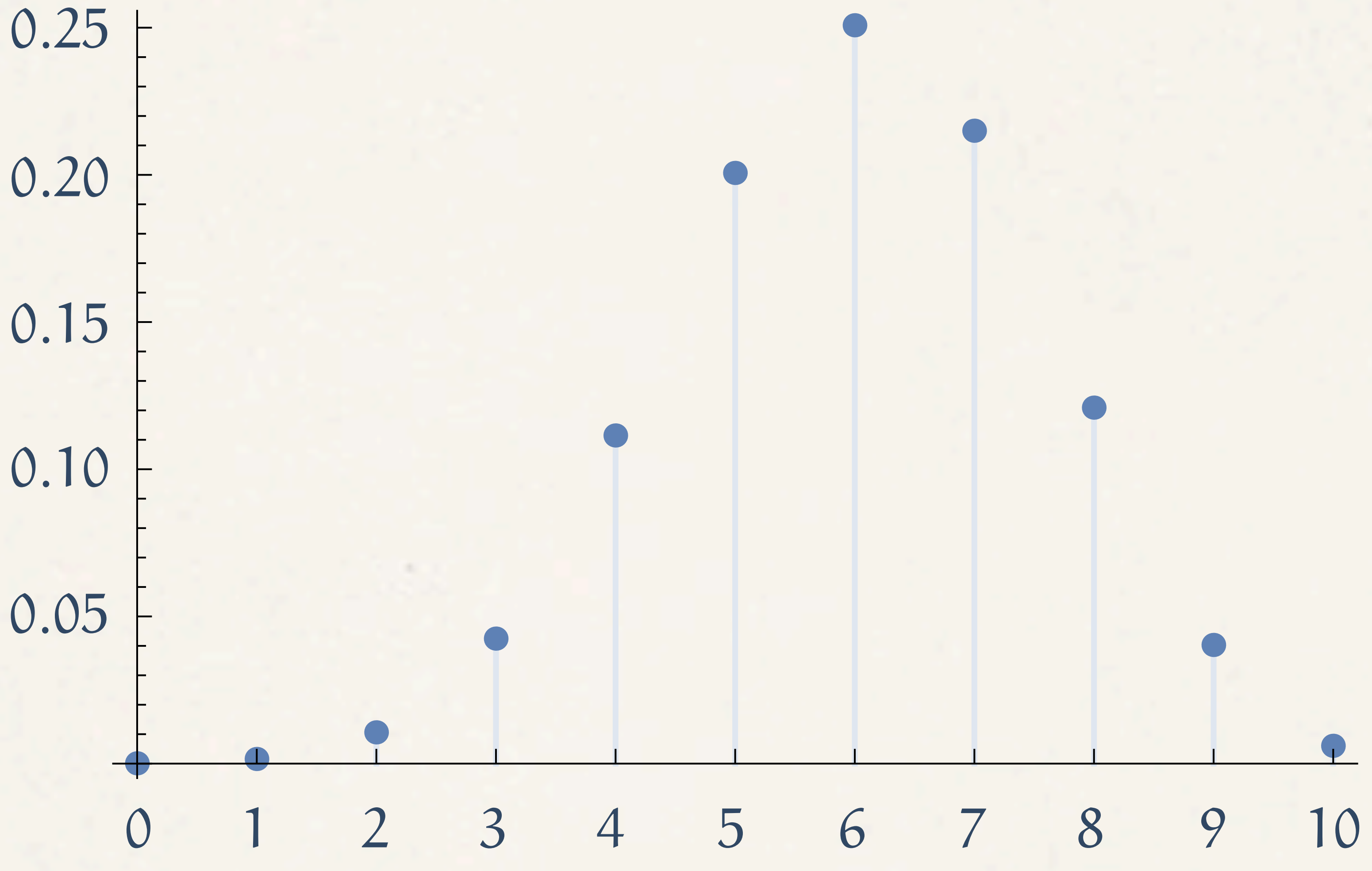

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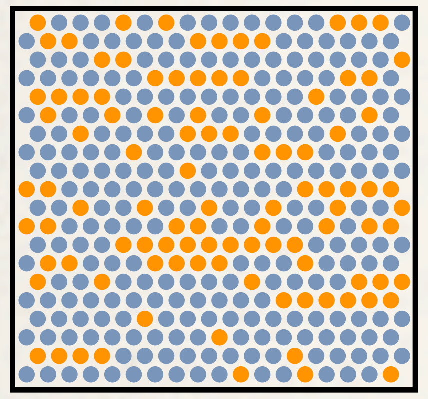

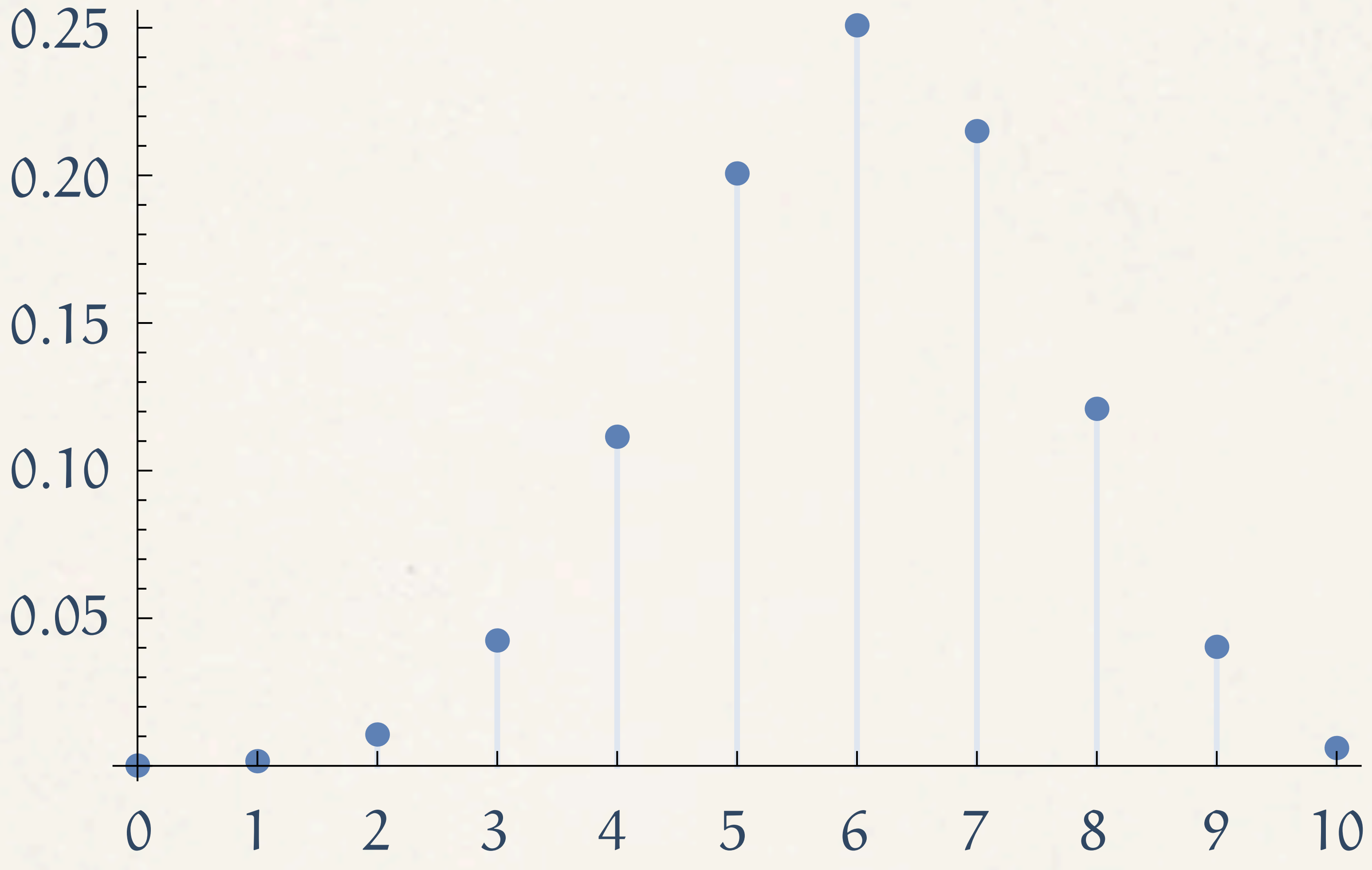

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$$\mathbf{P}\{0.6 - {\color{red}0.1} \le \tfrac{S_{10}}{10} \le 0.6 + {\color{red}0.1}\} = \mathbf{P}\{5 \le S_{10} \le 7\}$$

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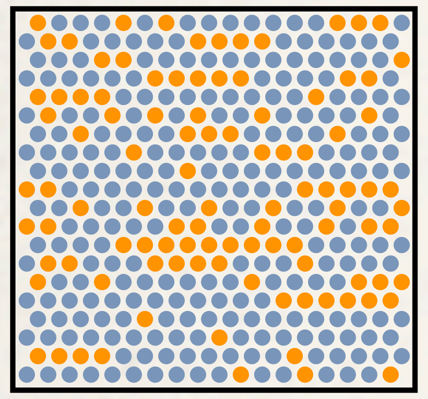

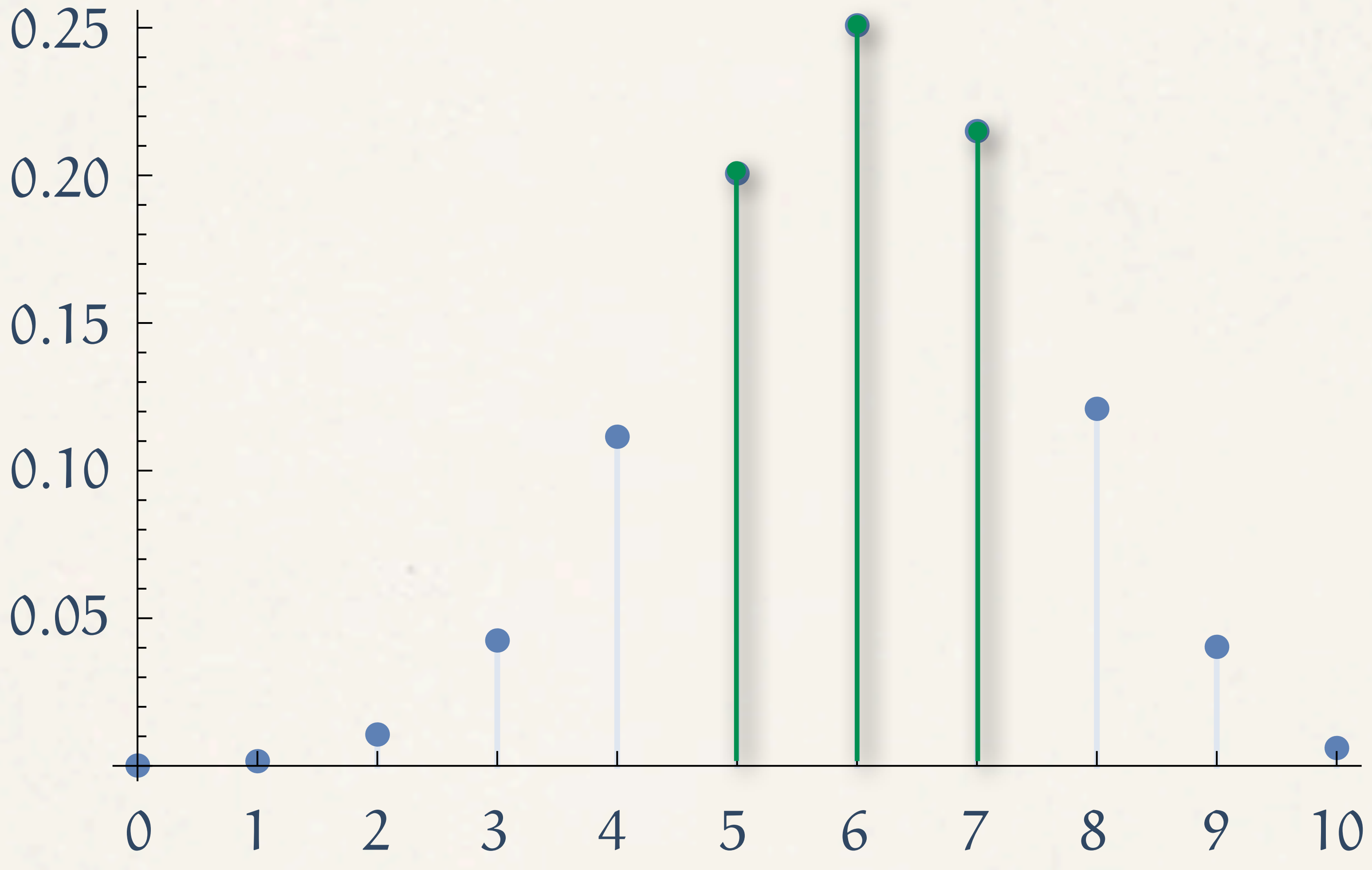

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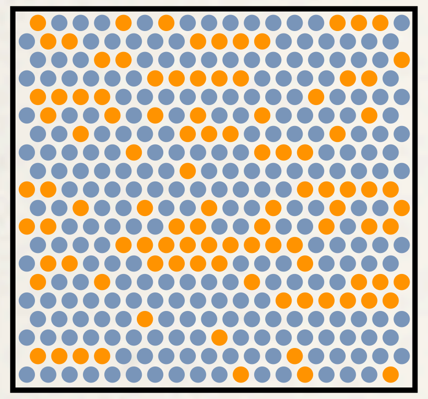

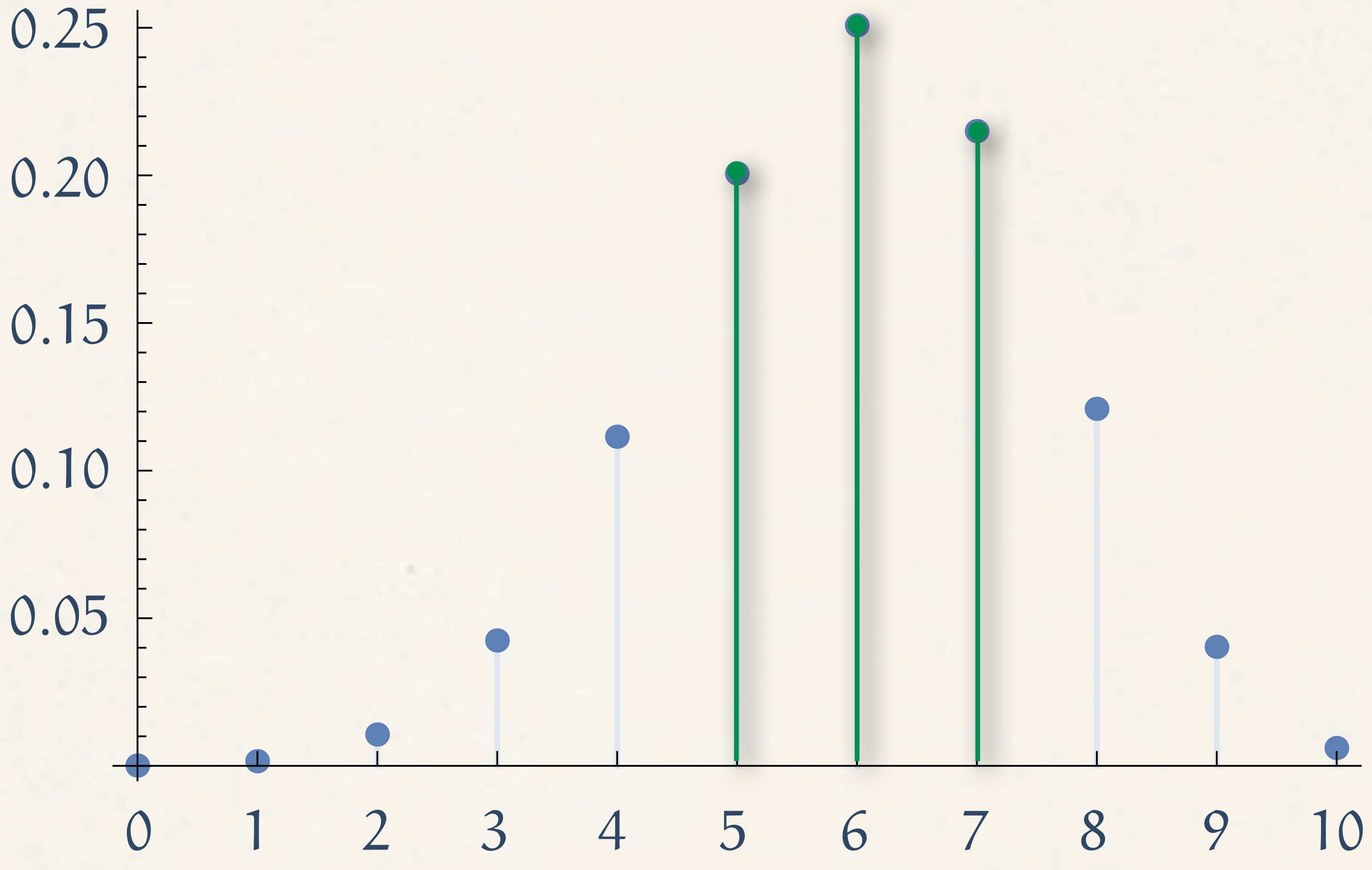

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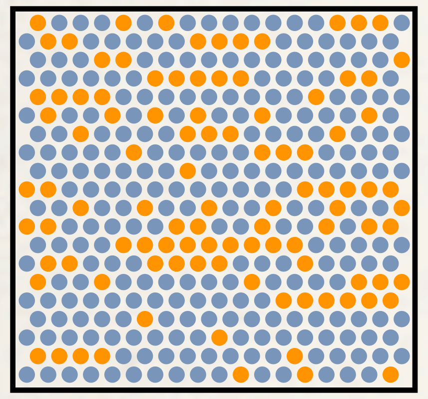

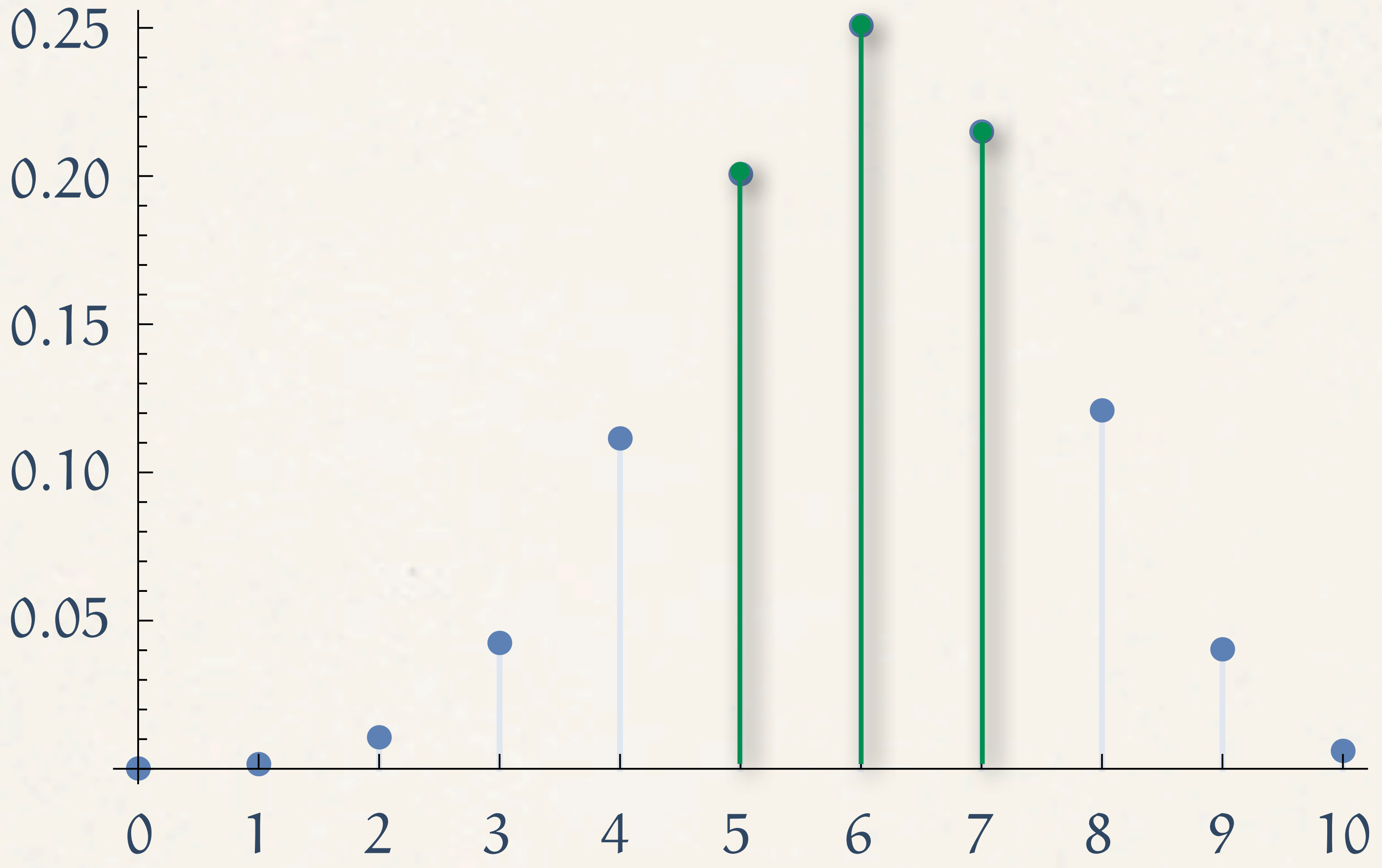

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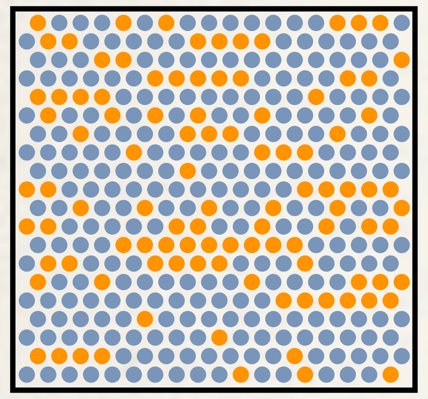

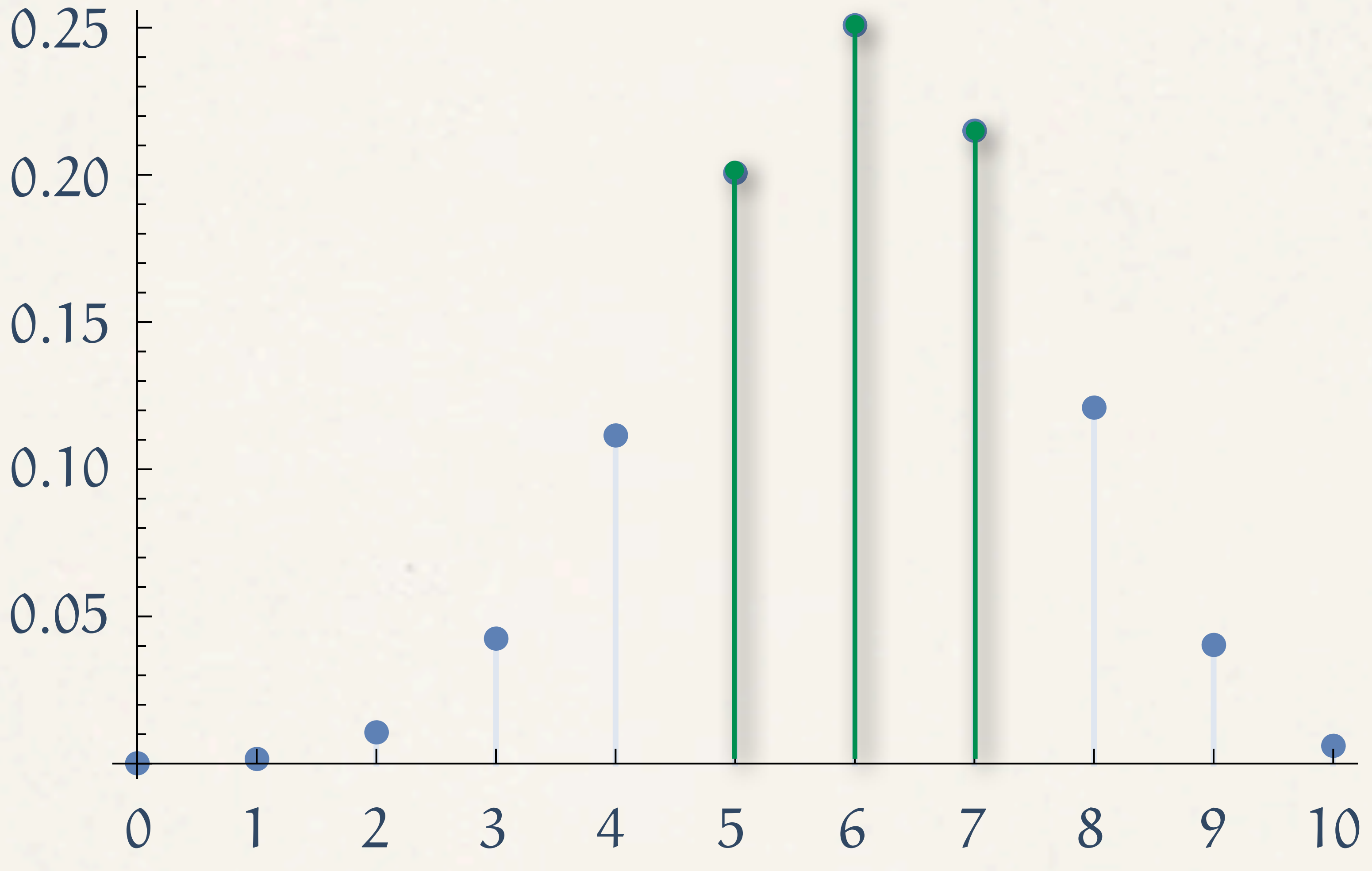

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error

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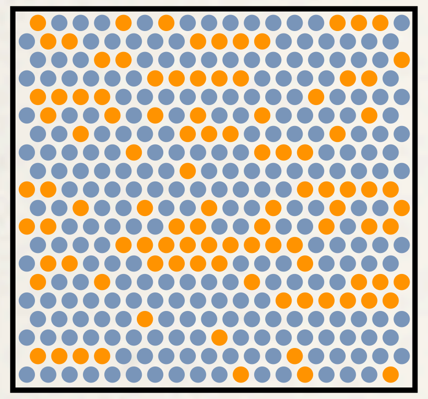

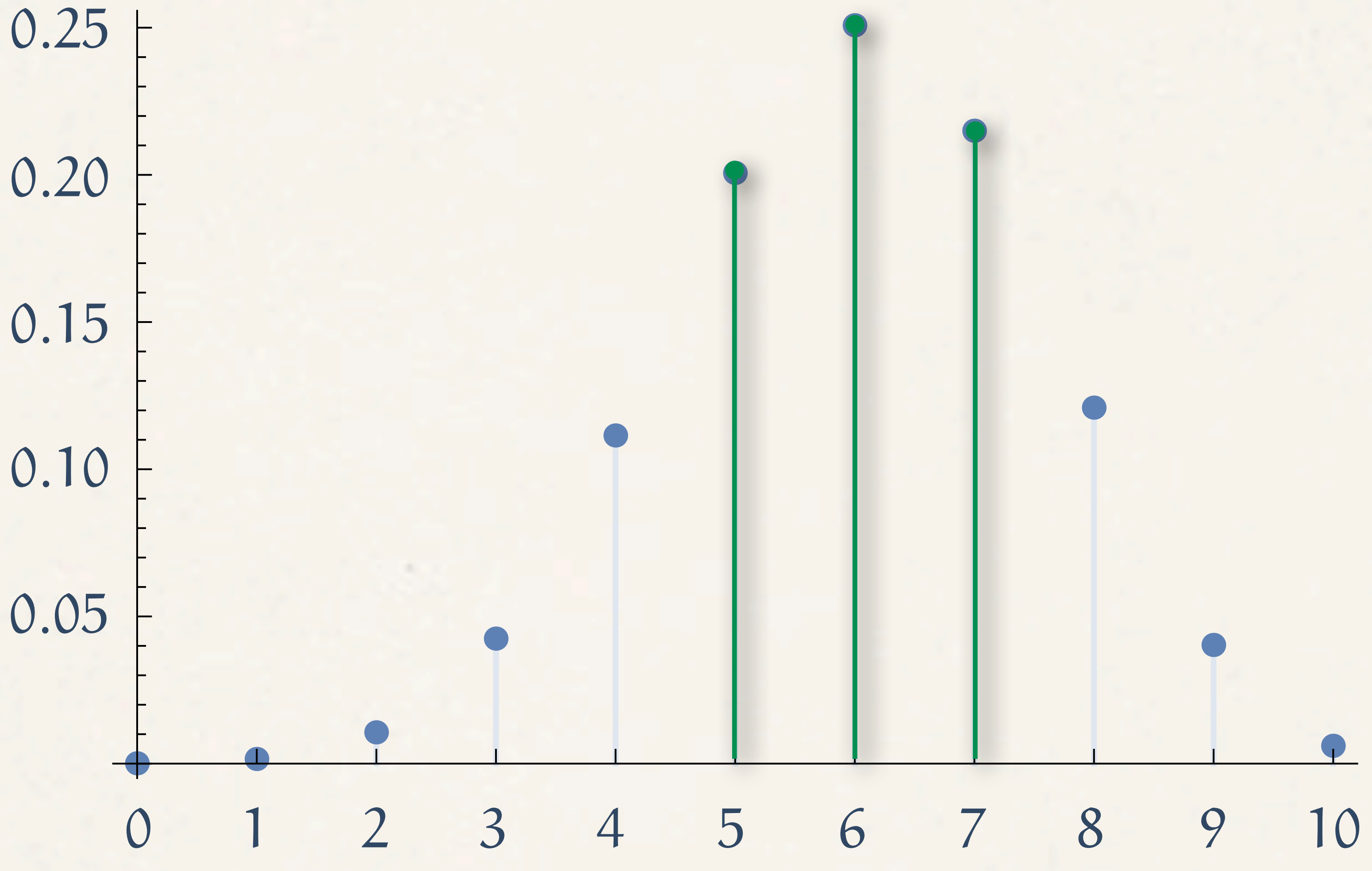

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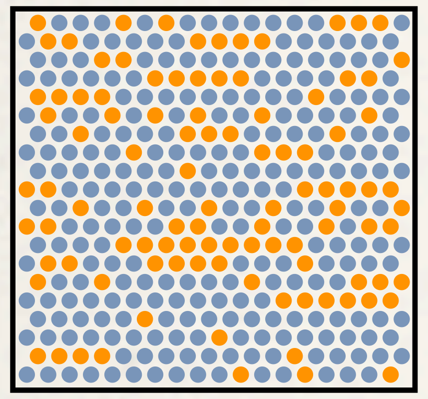

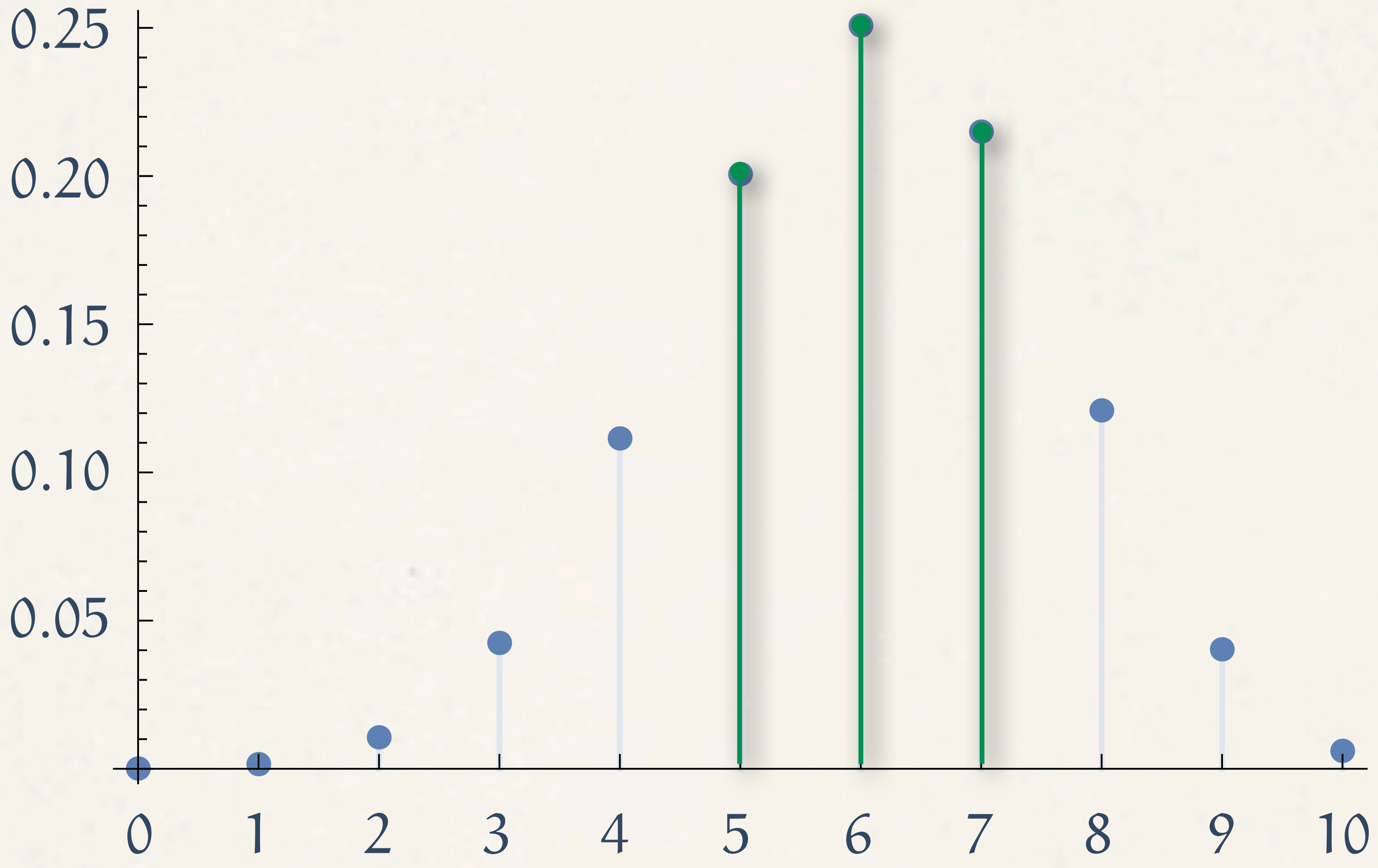

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confidence

For a given error, how does the confidence vary with the sample size n?

What is the probability that the relative frequency of successes in the sample (the estimate S_n/n) differs from the actual population proportion (the true bias $p = 0.6$) by no more than 10%?

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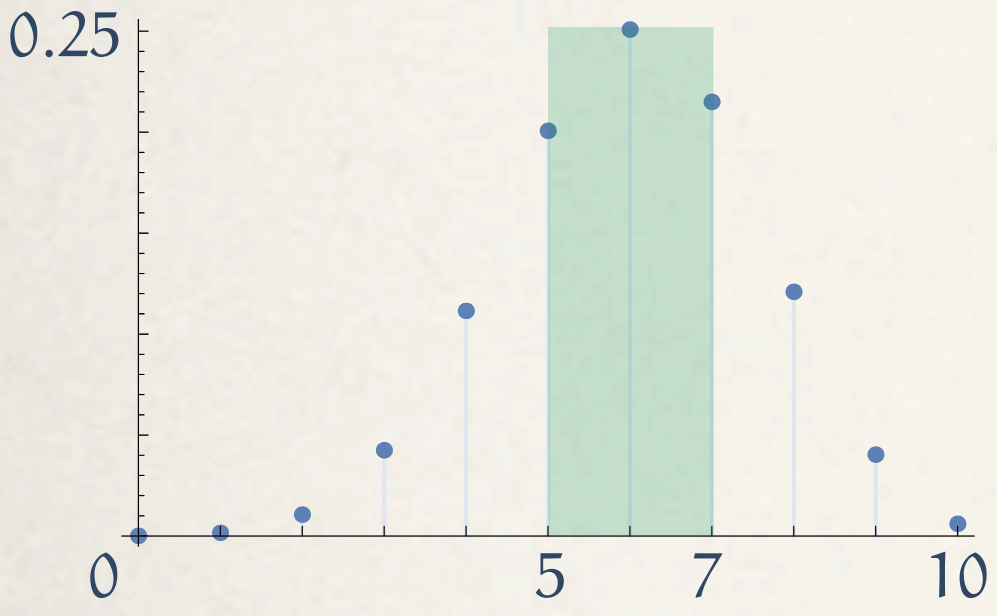

error

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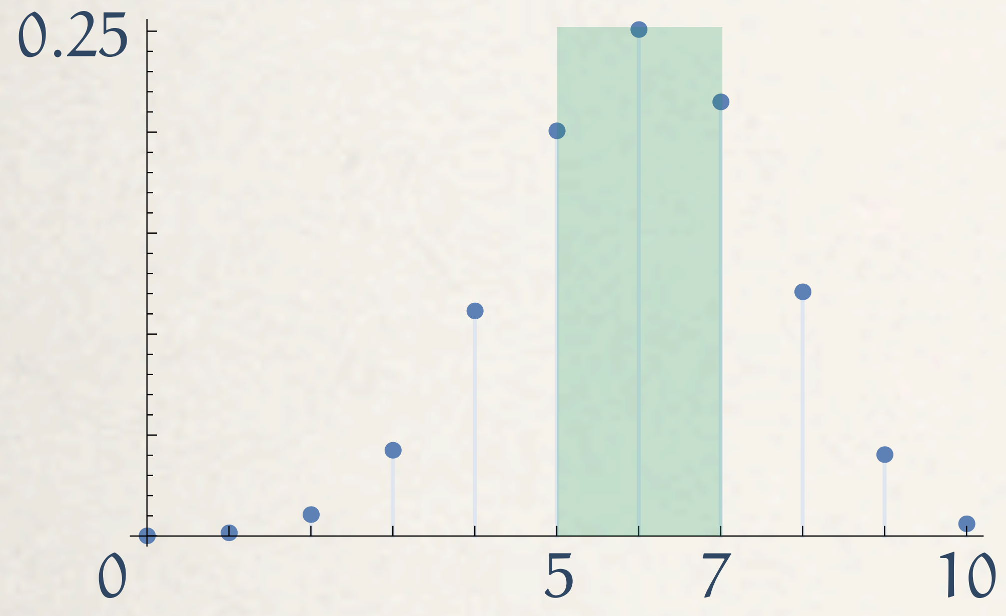

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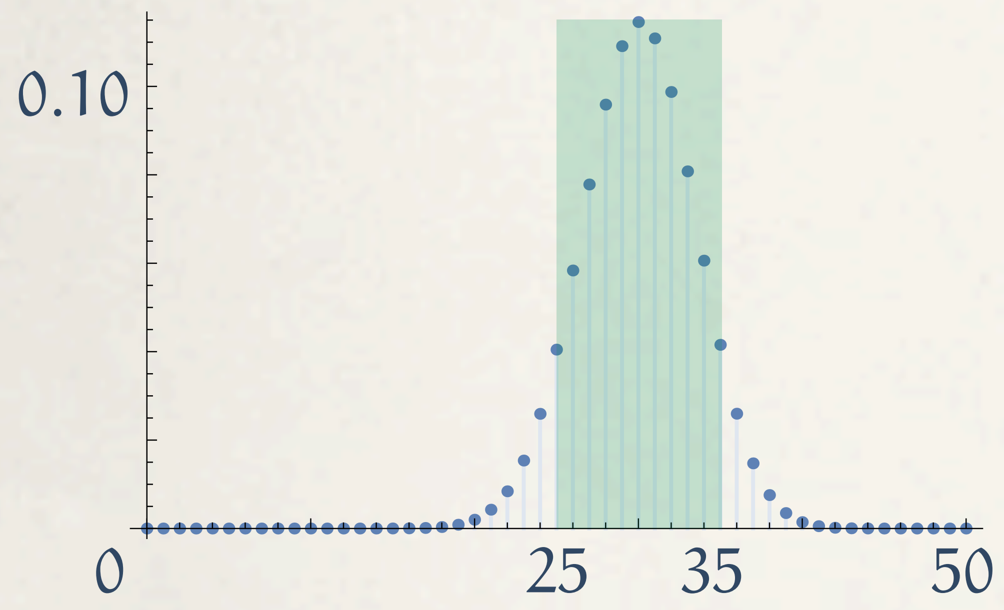

$$\mathbf{P}\{0.6 - 0.1 \leq \tfrac{S_{50}}{50} \leq 0.6 + 0.1\} = b_{50}(25; 0.6) + b_{50}(26; 0.6) + \cdots + b_{50}(35; 0.6)$$

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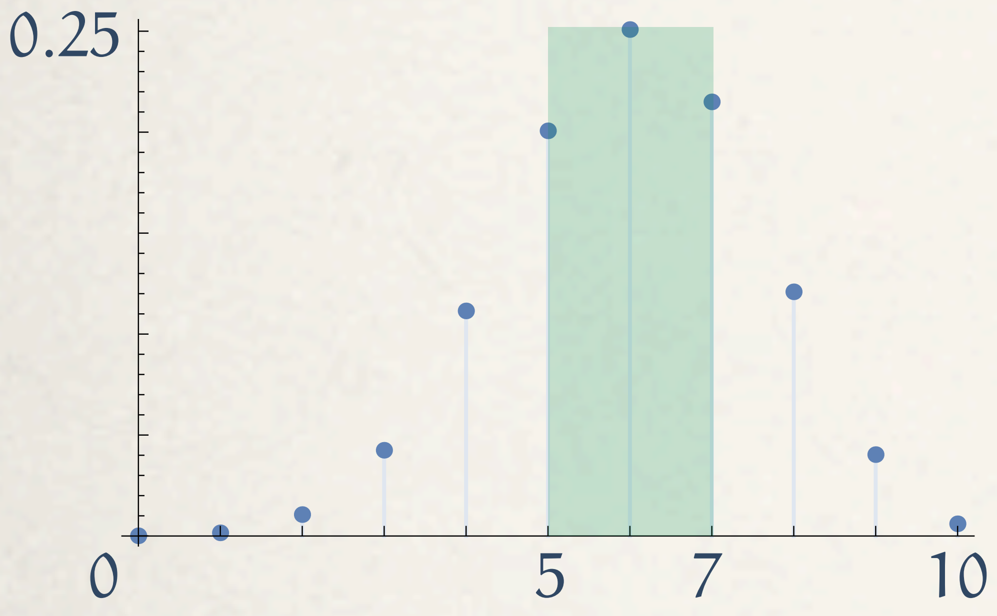
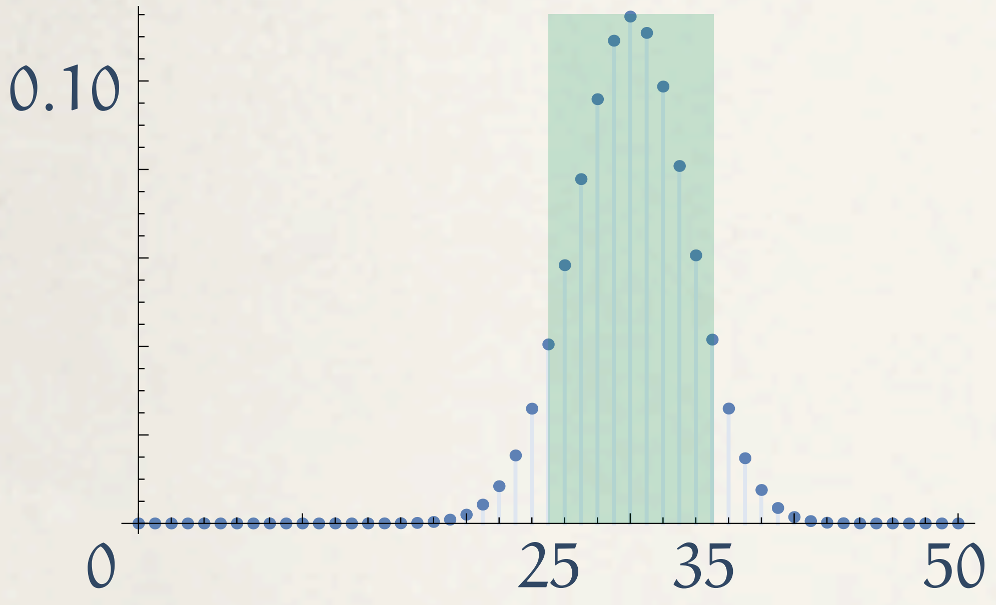


error

$$\mathbf{P}\{0.6 - {\color{red}0.1} \le \tfrac{S_{10}}{10} \le 0.6 + {\color{red}0.1}\} = b_{10}(5; 0.6) + b_{10}(6; 0.6) + b_{10}(7; 0.6)$$



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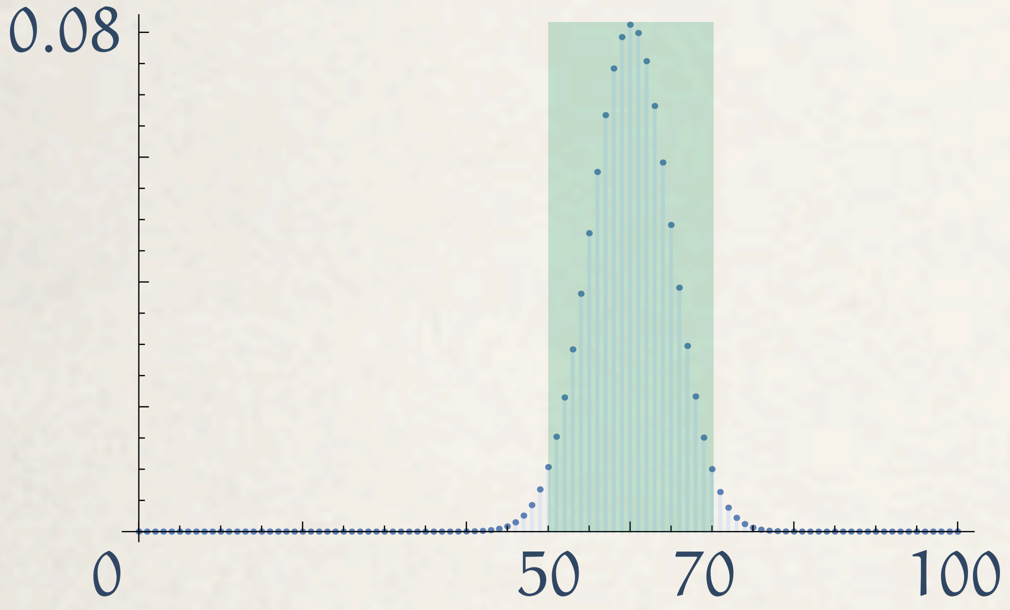

$$\mathbf{P}\{0.6 - {\color{red}0.1} \le \tfrac{S_{100}}{100} \le 0.6 + {\color{red}0.1}\} = b_{100}(50; 0.6) + b_{100}(51; 0.6) + \cdots + b_{100}(70; 0.6)$$

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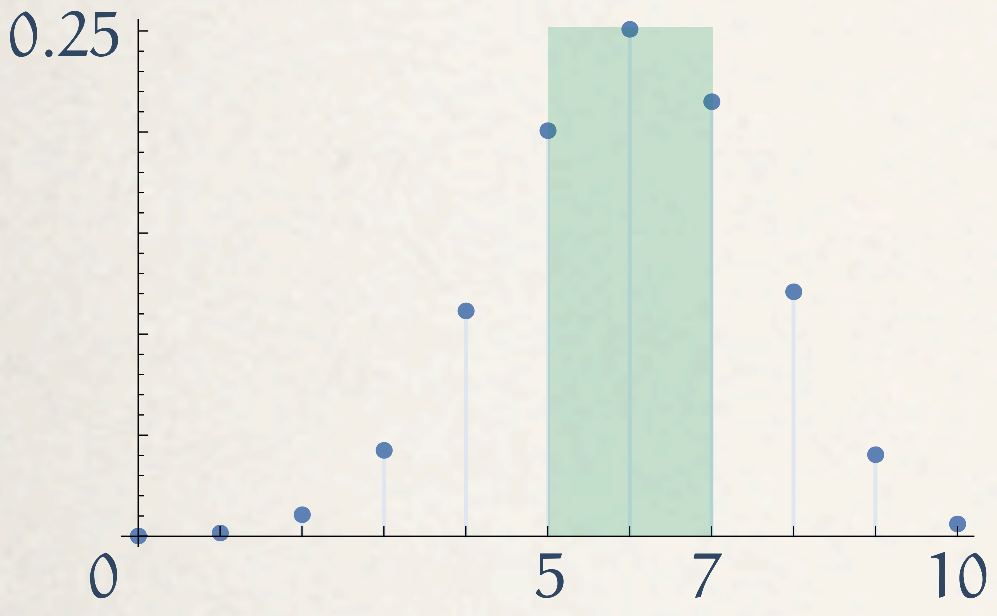

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confidence

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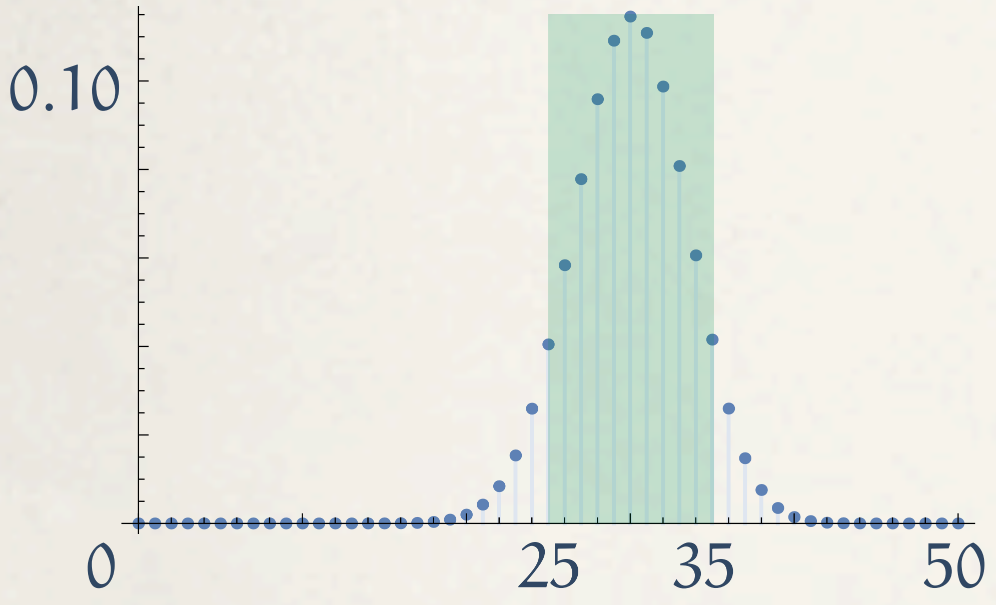

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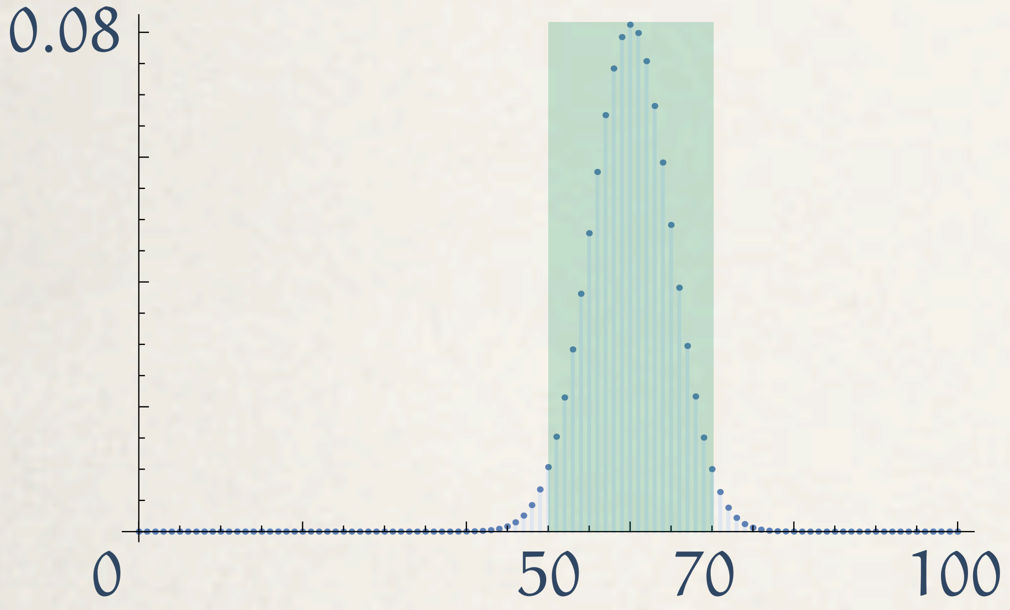

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