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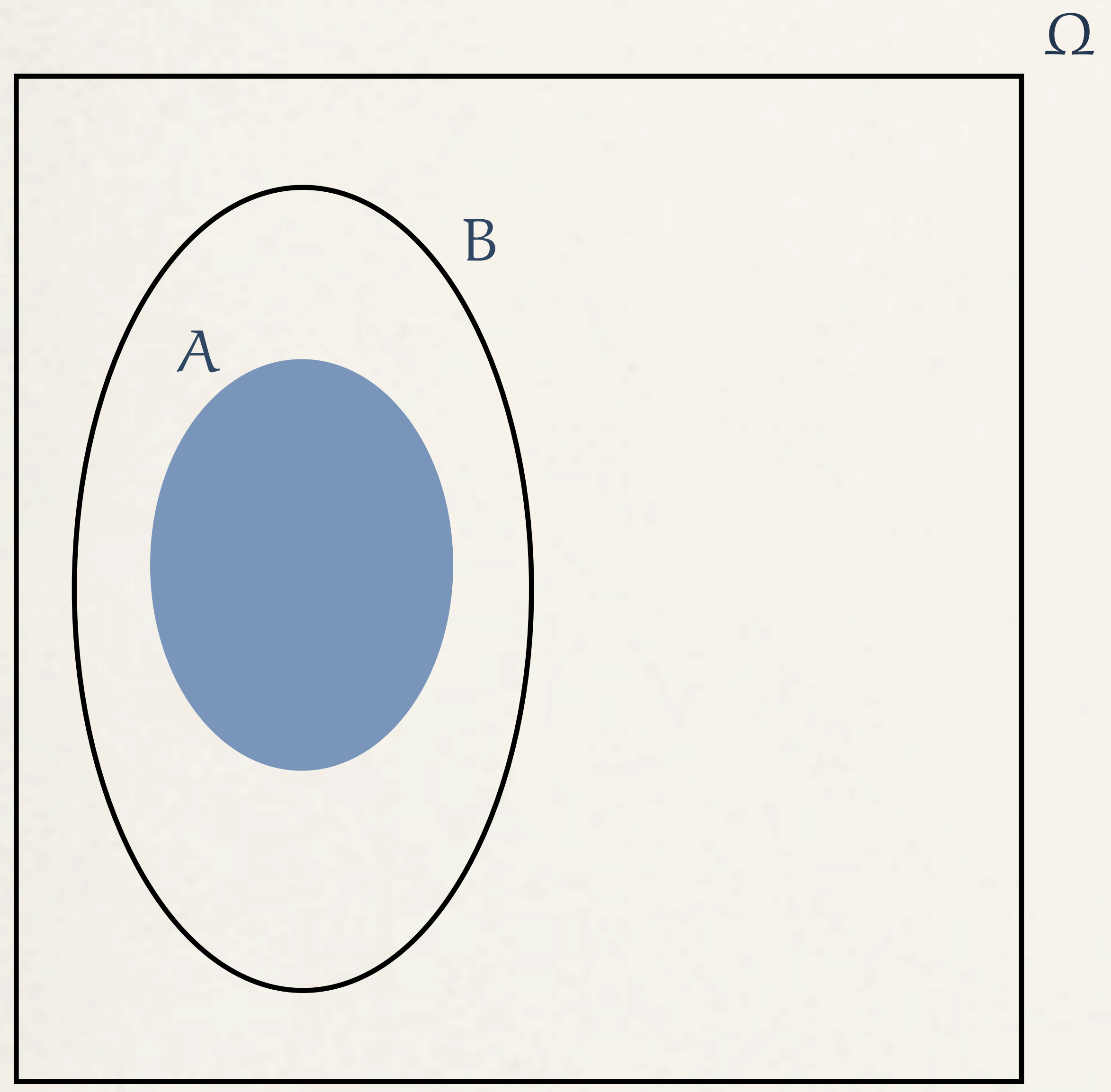

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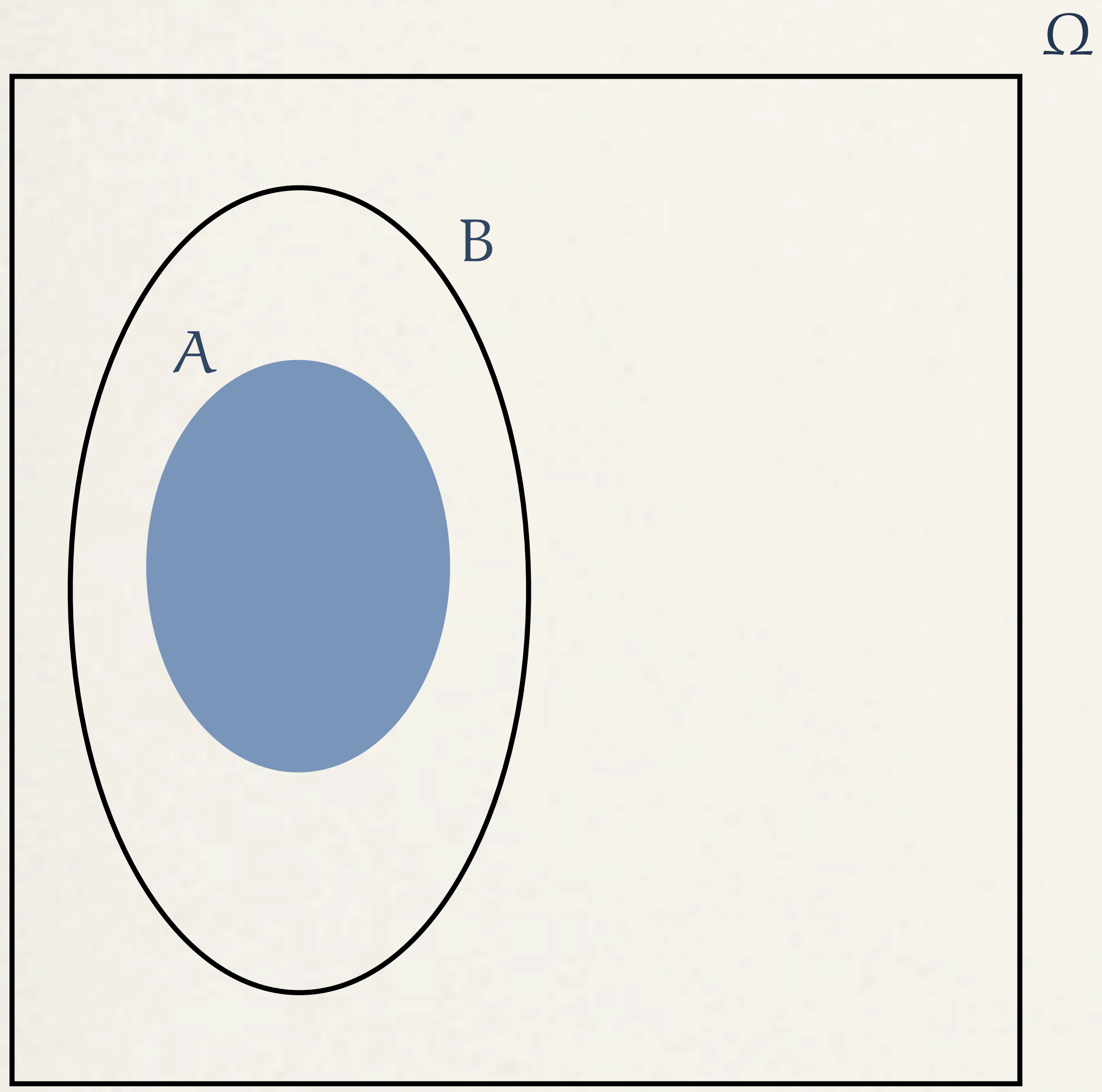

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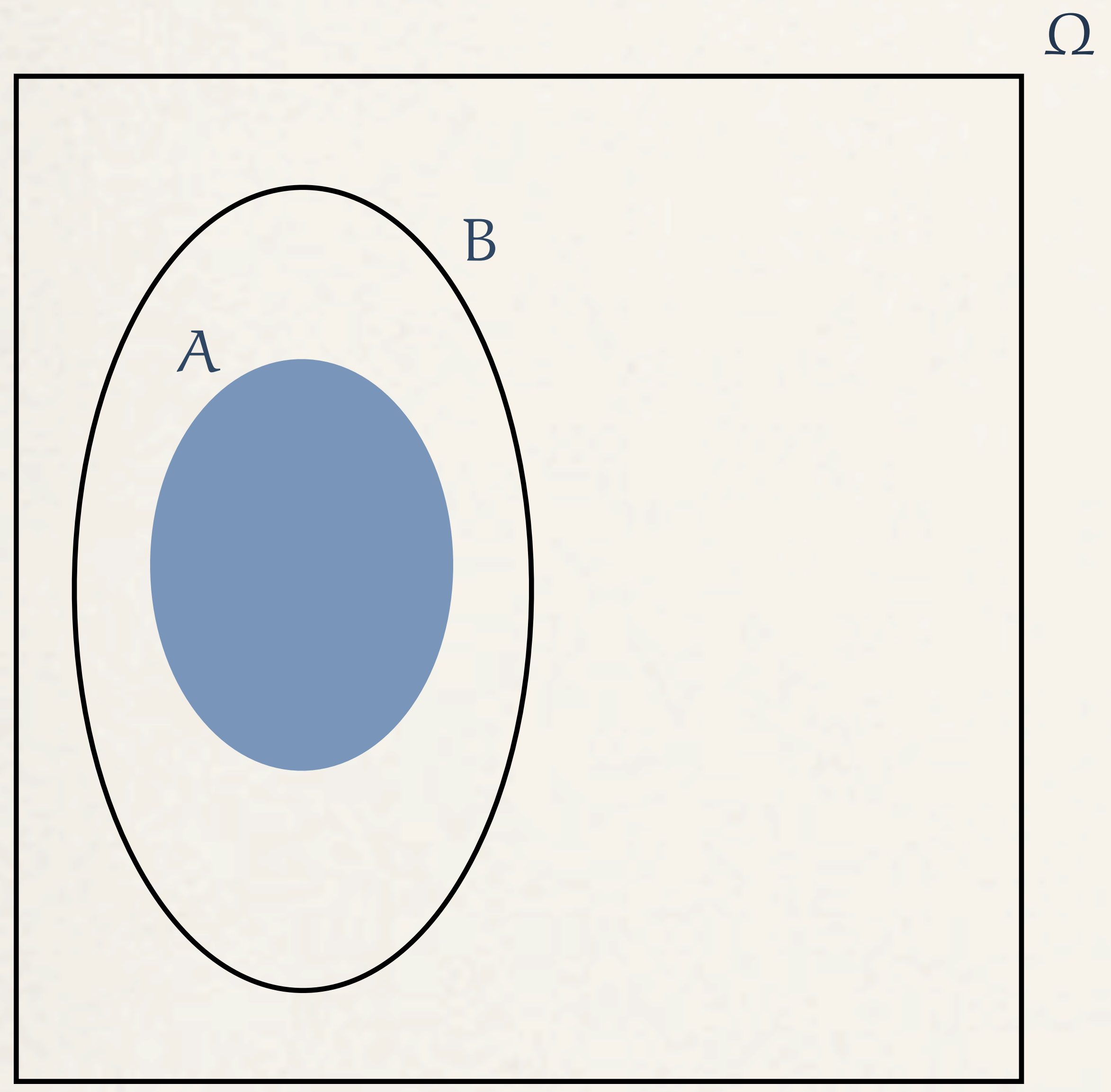

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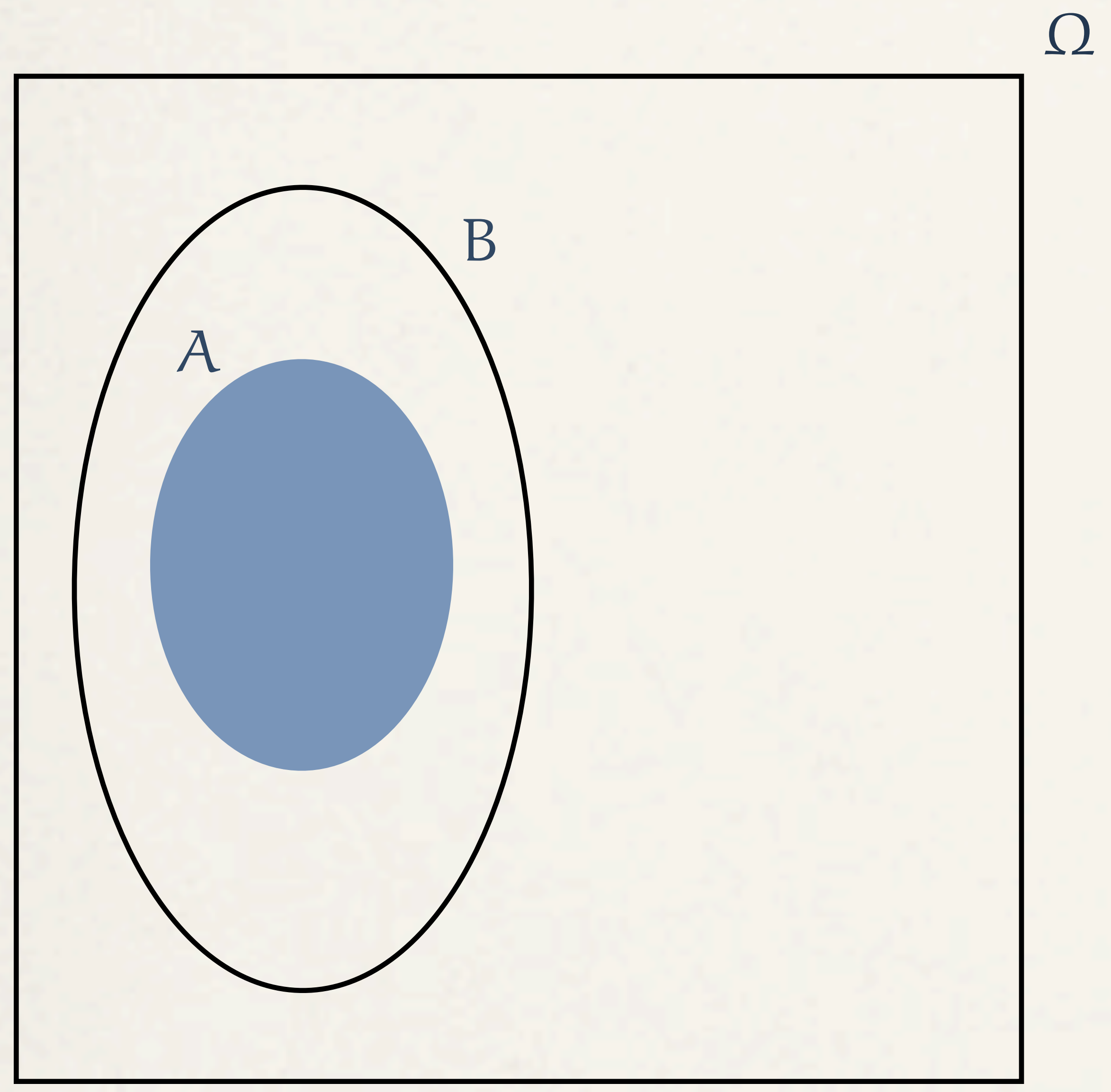

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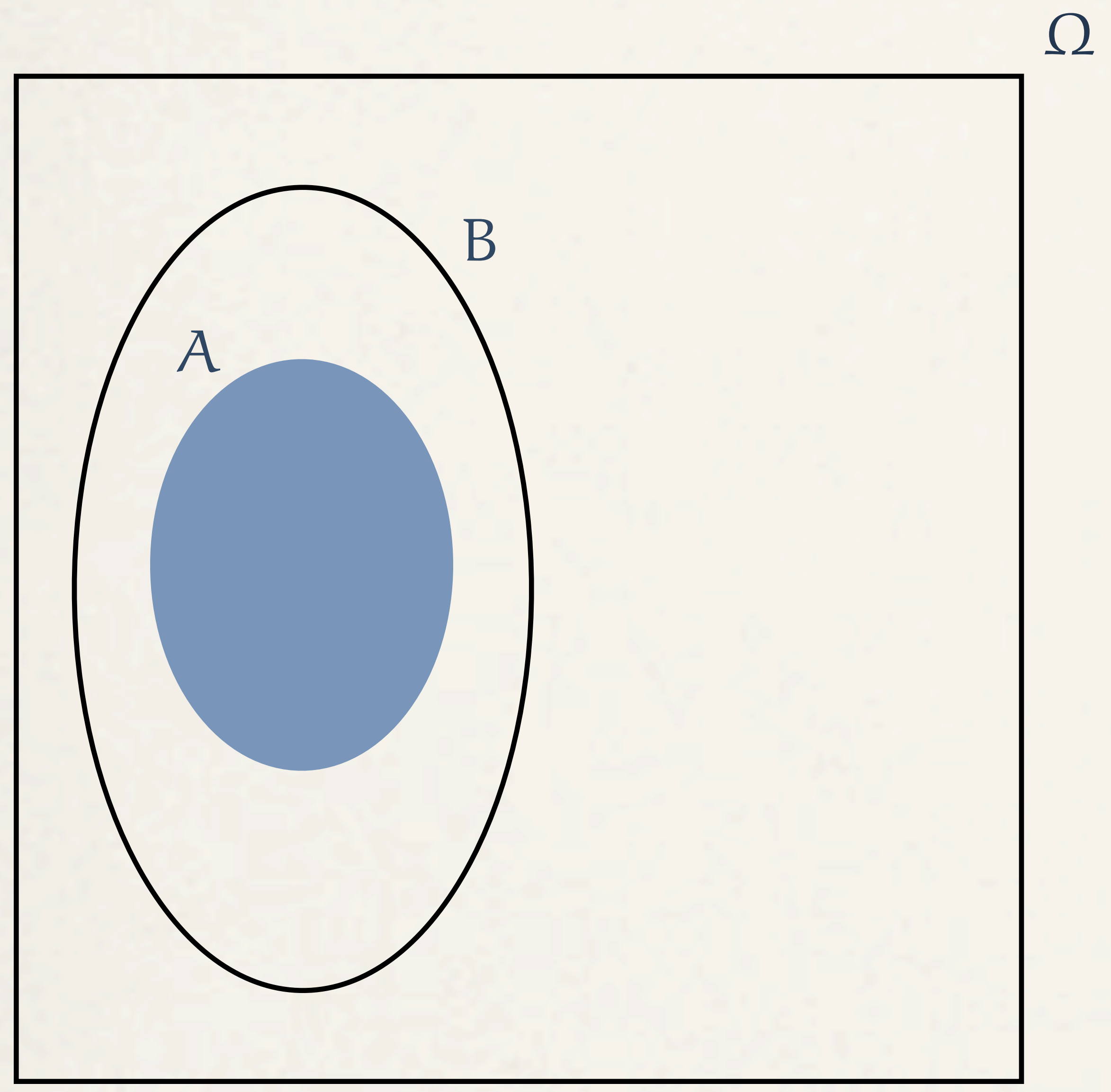

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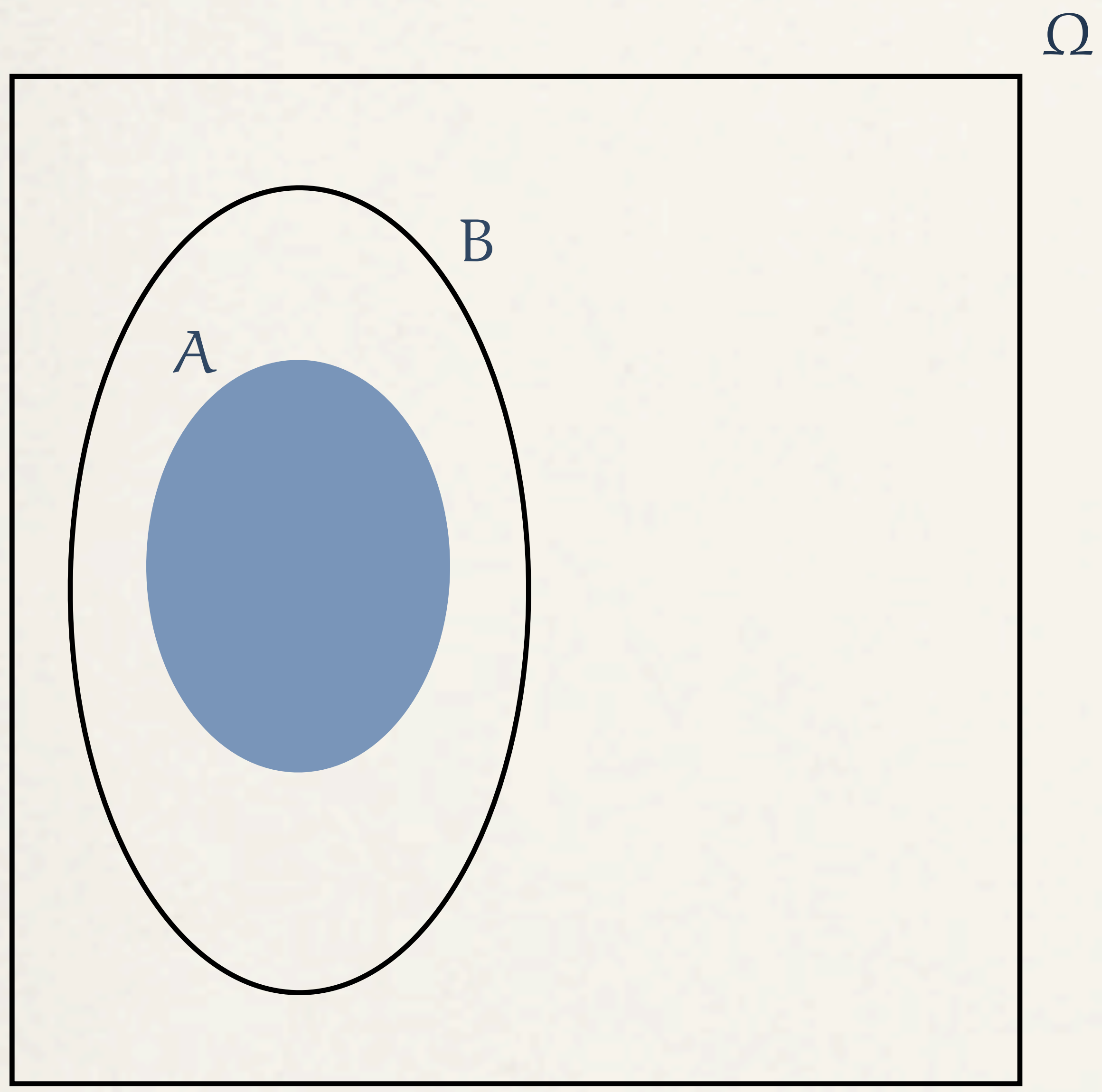

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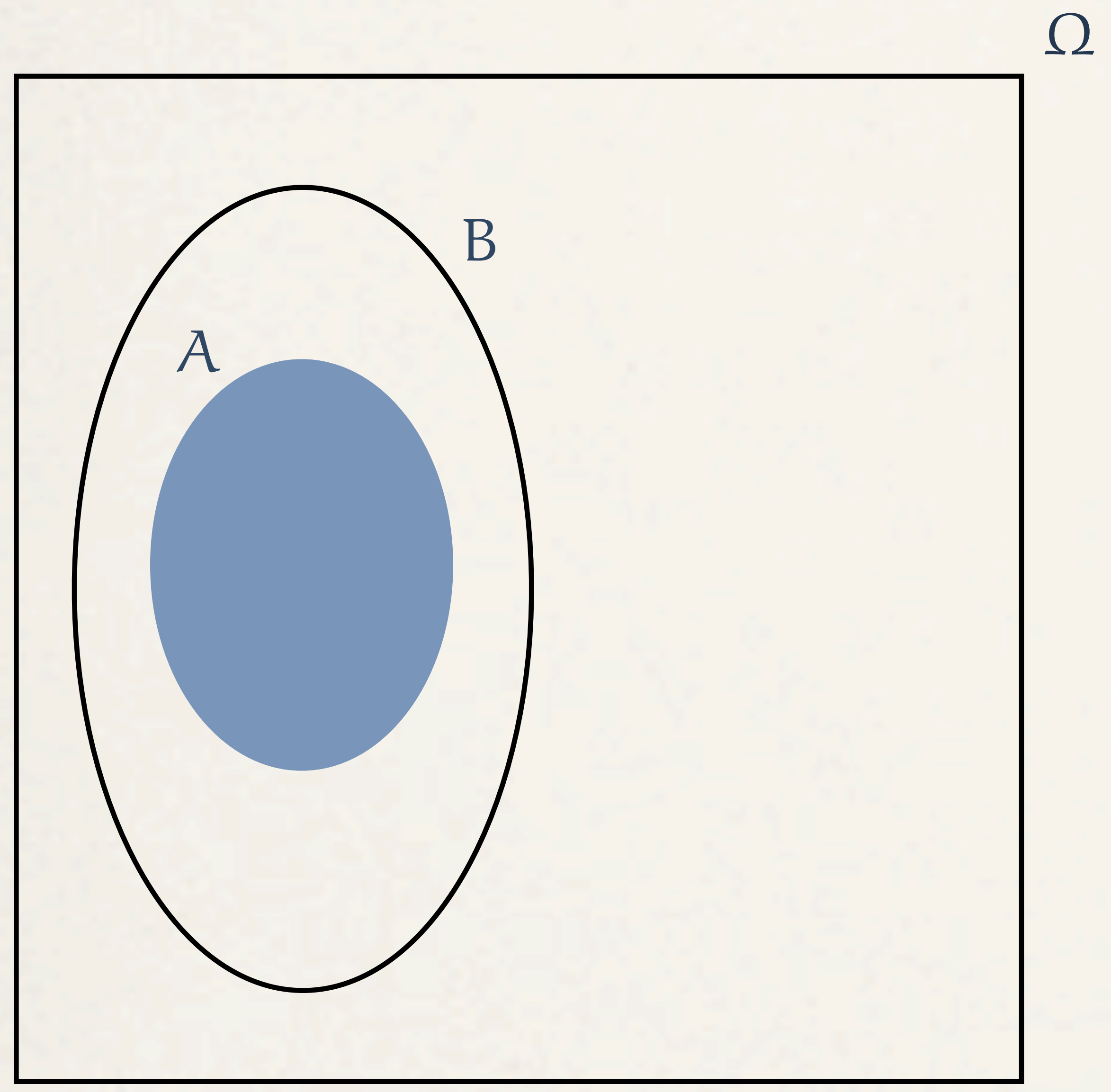

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Probability measure is bounded

Boole's inequality (the union bound)

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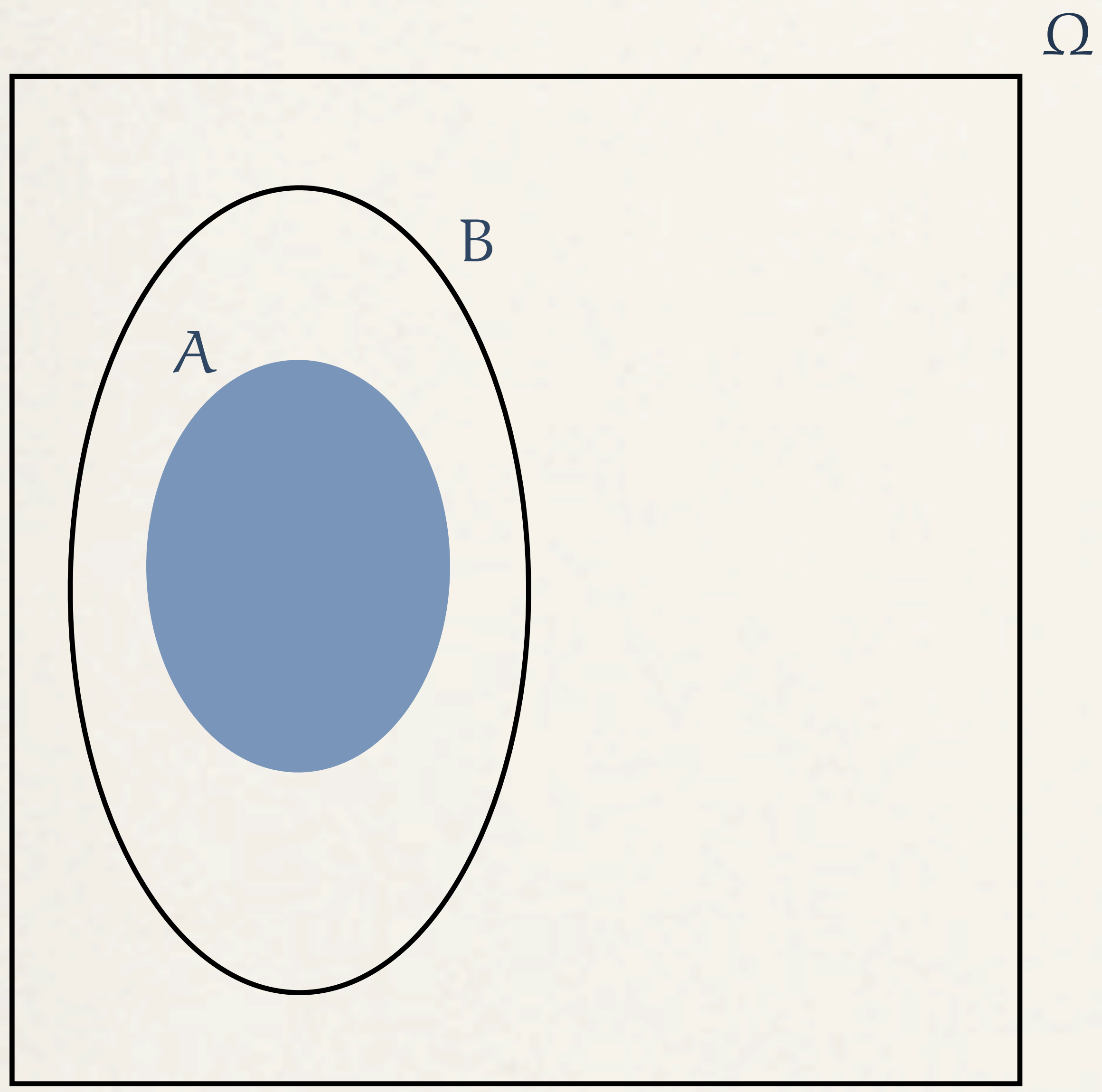

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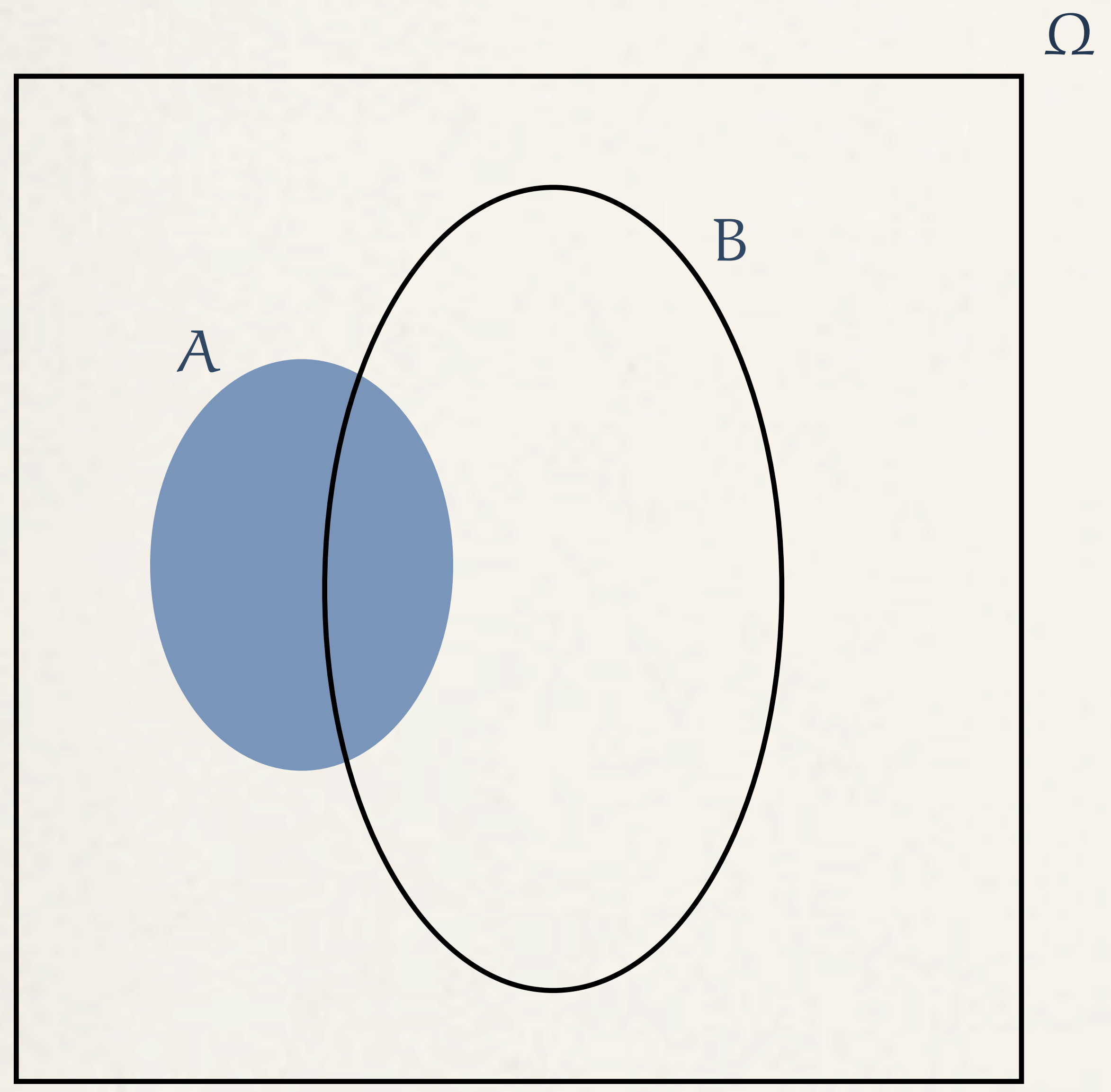

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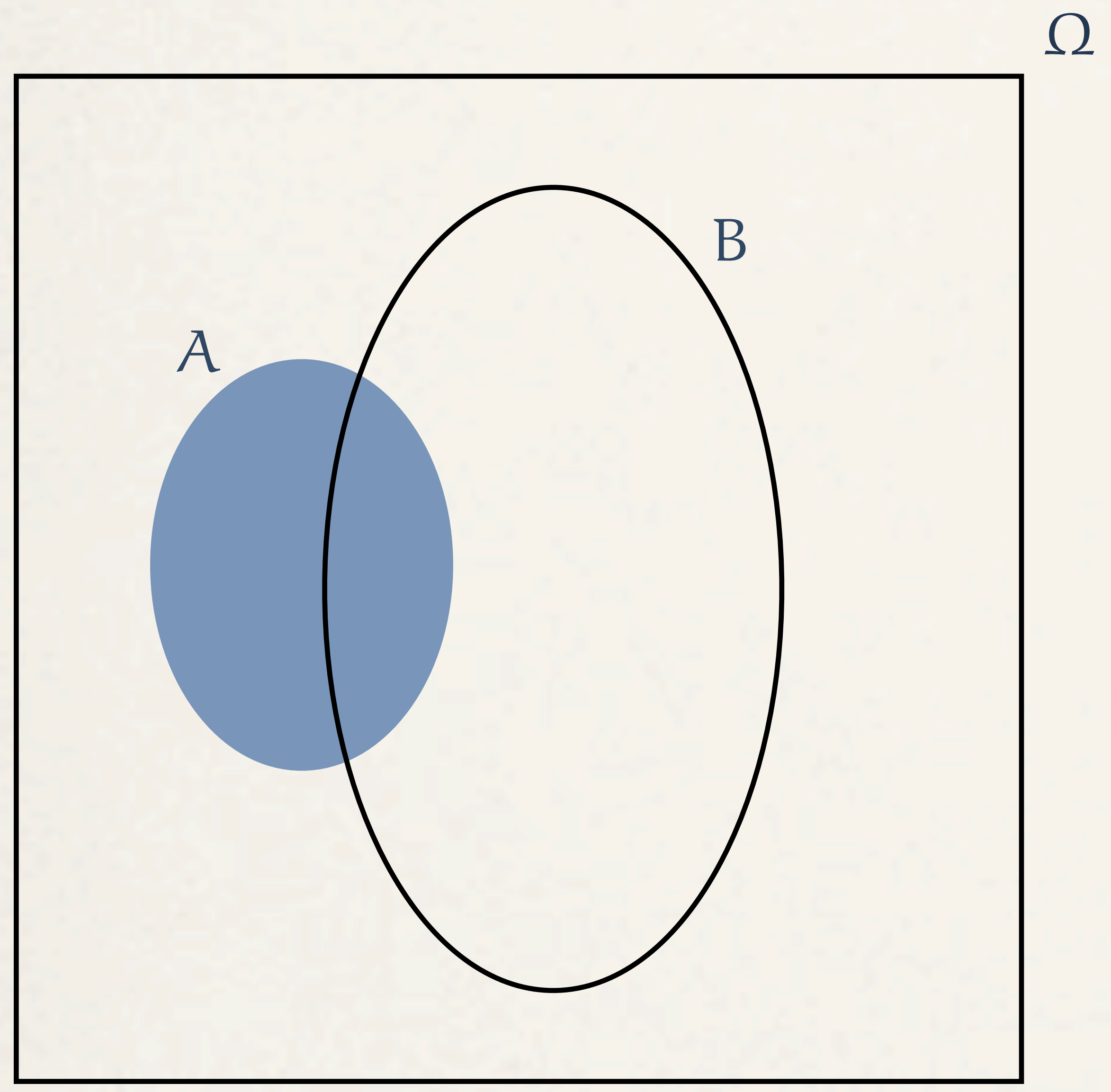

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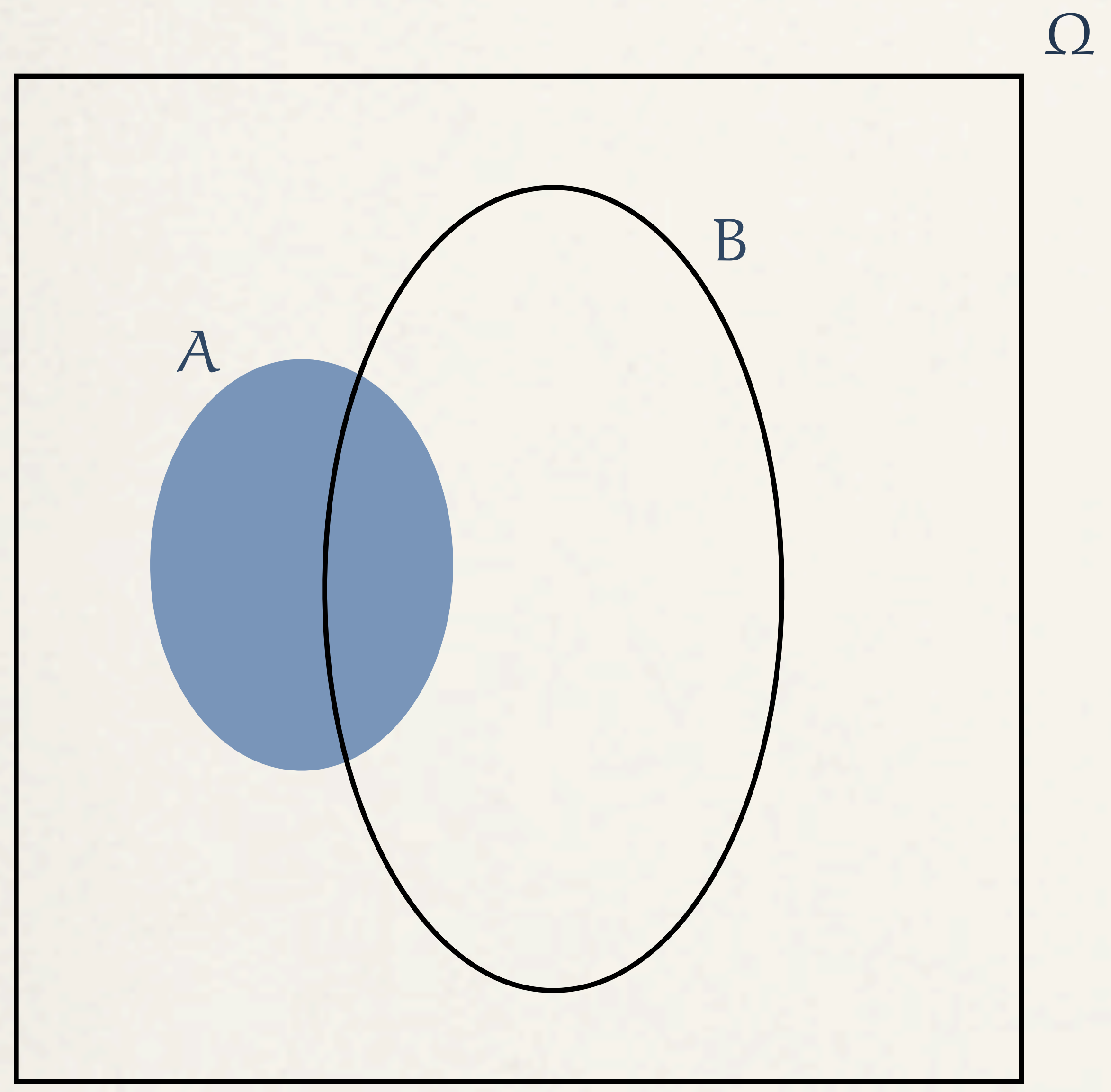

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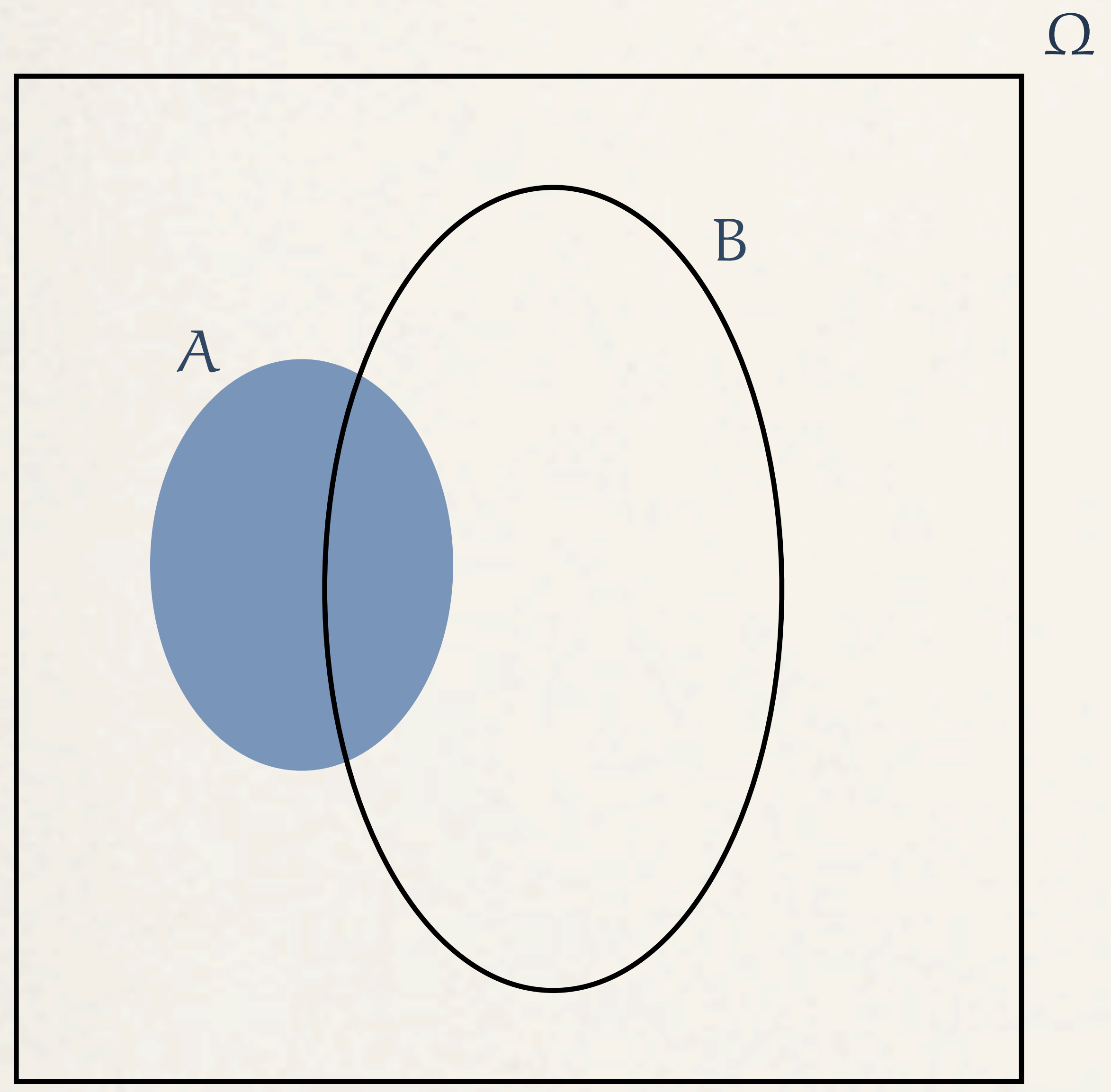

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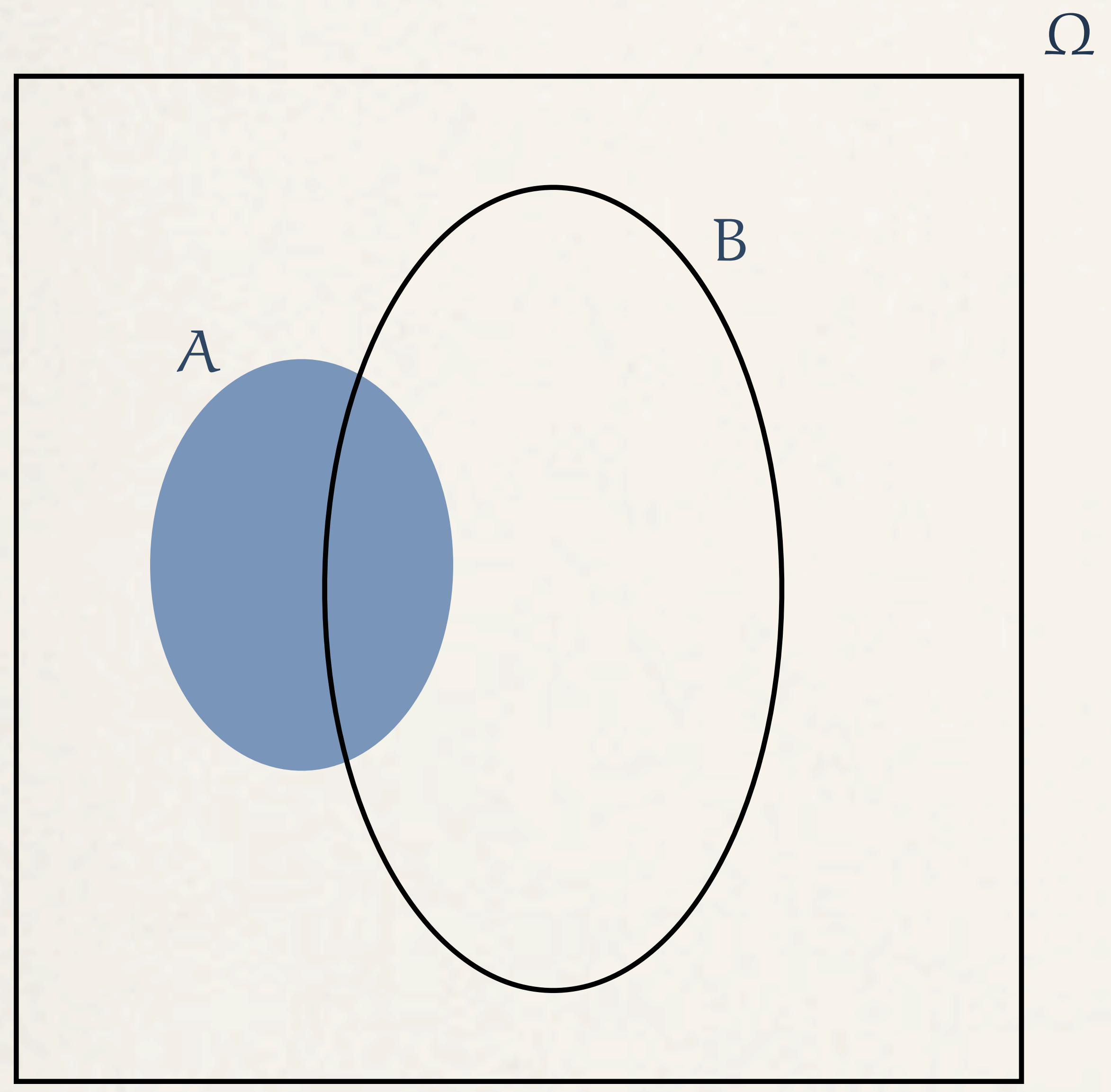

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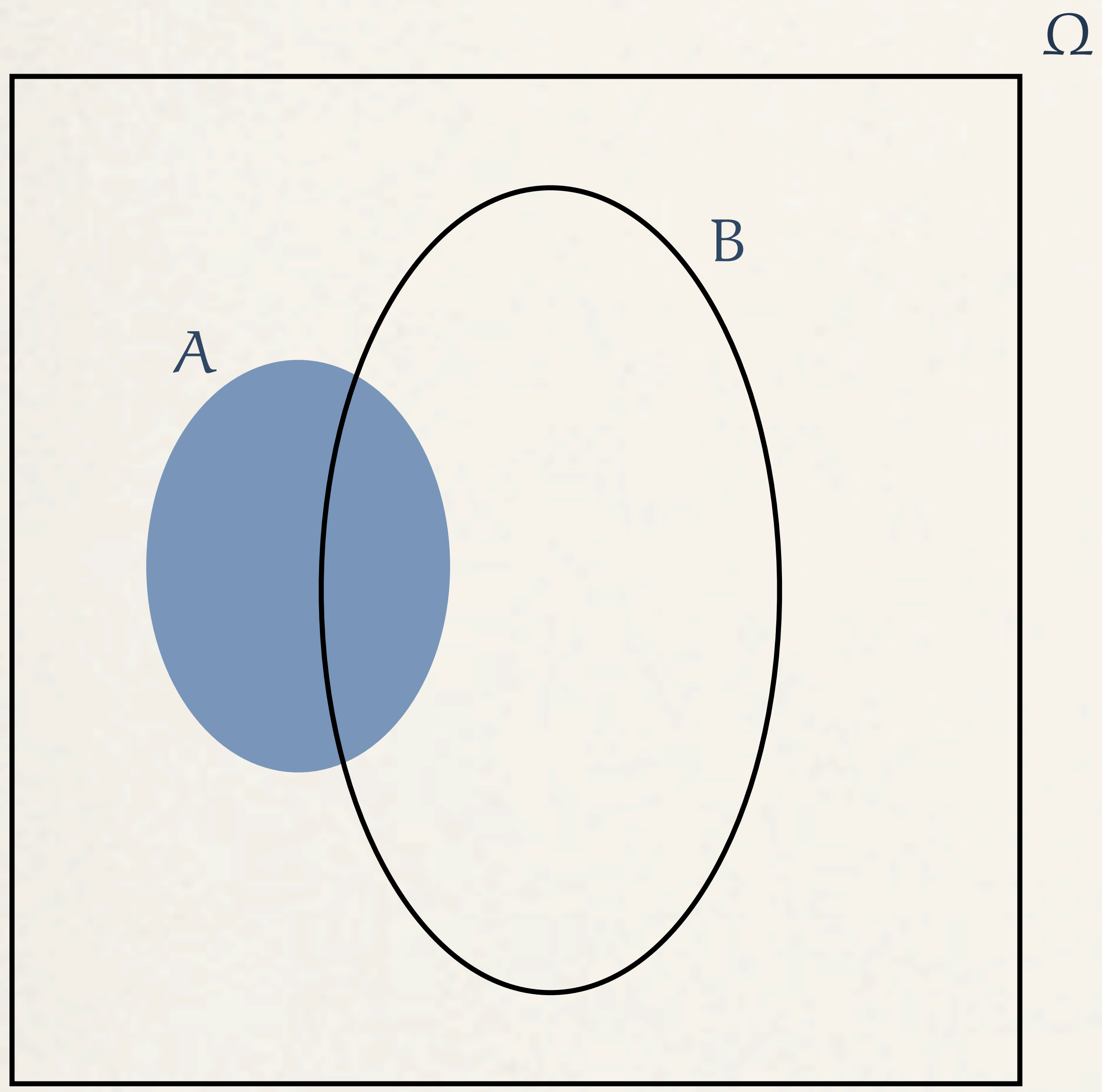

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$\mathbf{P}(A \cup B) \leq \mathbf{P}(A) + \mathbf{P}(B)$

What can you say about $\mathbf{P}(A \cup B \cup C)$ in terms of $\mathbf{P}(A)$, $\mathbf{P}(B)$, and $\mathbf{P}(C)$?

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$$\mathbf{P}(A_1 \cup A_2 \cup \cdots \cup A_n) \leq \mathbf{P}(A_1) + \mathbf{P}(A_2) + \cdots + \mathbf{P}(A_n)$$

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Slogan

The probability of a union of events is no larger than the sum of the event probabilities.