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A, B, and C are pairwise independent but not (jointly) independent.

1) $\mathbf{P}(A \cap B) = \mathbf{P}(A) \times \mathbf{P}(B)$,

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Independence implies pairwise independence but pairwise independence does not imply independence.