Submission Phase

1. Do assignment ☑ (/maththink-005/human_grading/view/courses/972631/assessments/8/submissions)

Evaluation Phase

2. Evaluate peers ☑ (/maththink-005/human_grading/view/courses/972631/assessments/8/peerGradingSets)

3. Self-evaluate ☑ (/maththink-005/human_grading/view/courses/972631/assessments/8/selfGradingSets)

Results Phase

4. See results ☑ (/maththink-005/human_grading/view/courses/972631/assessments/8/results/mine)

Your effective grade is 239

Your unadjusted grade is 239, which was calculated based on a combination of the grade you received from your peers and the grade you gave yourself.

See below for details.

The submission deadline for this Problem Set is Sunday November 30 at 8:00 PM PST. Note that you can repeatedly submit your entries as you work through the problems, which means you can change them at any time prior to the final deadline. The final version of your submission is the one that will be graded. For each question, you may enter your answer into the appropriate entry field on this form (including TeX entry), or you may upload a file (e.g. JPEG, scanned PDF of handwritten solution, PDF from a Word file, etc.) Your answers will be peer evaluated according to the course rubric (http://spark-public.s3.amazonaws.com/maththink/readings/Evaluation_Rubric.pdf). Because the peer evaluation training phase is active immediately after the submission deadline, the system cannot accept any submission after that. If you think there may be any possible delay in submitting on the Sunday morning, you should make a final submit on the Saturday evening at the latest. You can download a PDF version of the Problem Set here (https://d396qusza40orc.cloudfront.net/maththink%2Fproblemsets%2FTFPS.pdf). **As always, you are expected to work alone on this Problem Set.**

1. Say whether the following is true or false and support your answer by a proof.
$(\exists m \in \mathcal{N})(\exists n \in \mathcal{N})(3m + 5n = 12)$

1. The statement is FALSE

Proof by Contradiction

Let's assume to the contrary

$\Rightarrow \exists\, m,n \in \mathbb{N} \mid 3m+5n=12$

$\Rightarrow \exists\, m,n \in \mathbb{N} \mid 5n = 12-3m = 3(4-m) \wedge m,n \geq 1$

$\Rightarrow$ 3 divides $5n \wedge (n \geq 1, m \geq 1)$

$\Rightarrow 3 \mid n \wedge n \geq 1, m \geq 1$

$\Rightarrow n \geq 3 \wedge m \geq 1$

$\Rightarrow 3m+5n \geq 3.1+5.3 = 18$,

a contradiction (proved)

Evaluation/feedback on the above work

Note: this section can only be filled out during the evaluation phase.

Logical correctness

Score from your peers: **4**

Score from yourself: **4**

Clarity

Score from your peers: **3**

Score from yourself: **4**

Opening

Score from your peers: **4**

Score from yourself: **4**

Stating the conclusion

Score from your peers: **4**

Score from yourself: **4**

Reasons given

Score from your peers: **4**

Score from yourself: **4**

Overall valuation

Score from your peers: **4**

Score from yourself: **4**

peer 1 → what if n= 0 and m=4

peer 2 → You go from 3 divides 5n to 3 divides n. OK I see what you've done here - either 3 divides 5 or 3 divides n. That leads to 5n = 15 so the contradiction can be seen at this point.

peer 3 → *[This area was left blank by the evaluator.]*

2. Say whether the following is true or false and support your answer by a proof: The sum of any five consecutive integers is divisible by 5 (without remainder).

2. For any $n \in \mathbb{N}$, five consecutive numbers are
$5n, 5n+1, 5n+2, 5n+3, 5n+4$.

Sum of the numbers $= 5n + 5n+1 + 5n+2 + 5n+3 + 5n+4$

$= 5 \cdot 5n + (1+2+3+4)$

$= 25n + \frac{4 \cdot (4+1)}{2}$

$= 25n + 10 = 5(5n+2)$,

which is divisible by 5 (proved)

Evaluation/feedback on the above work

Note: this section can only be filled out during the evaluation phase.

Logical correctness

Score from your peers: **4**

Score from yourself: **4**

Clarity

Score from your peers: **4**

Score from yourself: **4**

Opening

Score from your peers: **4**

Score from yourself: **4**

Stating the conclusion

Score from your peers: **4**

Score from yourself: **4**

Reasons given

Score from your peers: **4**

Score from yourself: **4**

Overall valuation

Score from your peers: **4**

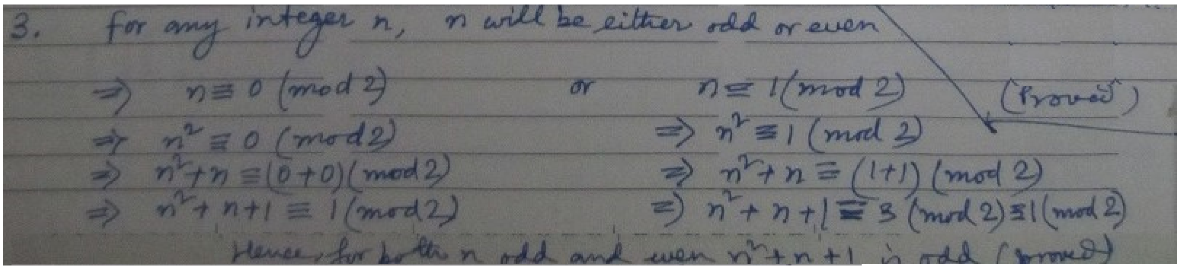
Score from yourself: **4**

peer 1 → you can not represent any integers by 5n

peer 2 → Not necessary to start with 5n but it still works.

peer 3 → *[This area was left blank by the evaluator.]*

3. Say whether the following is true or false and support your answer by a proof: For any integer n, the number $n^2 + n + 1$ is odd.



3. for any integer n, n will be either odd or even

$\Rightarrow n \equiv 0 \pmod 2$ or $n \equiv 1 \pmod 2$ (Proved)

$\Rightarrow n^2 \equiv 0 \pmod 2$ $\Rightarrow n^2 \equiv 1 \pmod 2$

$\Rightarrow n^2+n \equiv (0+0) \pmod 2$ $\Rightarrow n^2+n \equiv (1+1) \pmod 2$

$\Rightarrow n^2+n+1 \equiv 1 \pmod 2$ $\Rightarrow n^2+n+1 \equiv 3 \pmod 2 \equiv 1 \pmod 2$

Hence, for both n odd and even n^2+n+1 is odd (proved)

Evaluation/feedback on the above work

Note: this section can only be filled out during the evaluation phase.

Logical correctness

Score from your peers: **4**

Score from yourself: **4**

Clarity

Score from your peers: **4**

Score from yourself: **4**

Opening

Score from your peers: **4**

Score from yourself: **4**

Stating the conclusion

Score from your peers: **4**

Score from yourself: **4**

Reasons given

Score from your peers: **4**

Score from yourself: **4**

Overall valuation

Score from your peers: **4**

Score from yourself: **4**

peer 1 → *[This area was left blank by the evaluator.]*

peer 2 → OK I know how to use the modulo notation, but what about someone who doesn't? Hope they don't penalise you for using something that wasn't presented in the course.

peer 3 → *[This area was left blank by the evaluator.]*

4. Prove that every odd natural number is of one of the forms $4n + 1$ or $4n + 3$, where n is an integer.

4. Any odd number can be represented as $2m+1$ for any $m \in N$

Now, m can be even or odd.

Case-1 m even,

$\exists n \in N \mid m = 2n$

$\Rightarrow$ The odd number is $2m+1 = 4n+1$

Case-2 m odd,

$\exists n \in N \mid m = 2n+1$

$\Rightarrow$ The odd number is $2m+1 = 2(2n+1)+1$

$= 4n+2+1$

$= 4n+3$

(Proved)

Evaluation/feedback on the above work

Note: this section can only be filled out during the evaluation phase.

Logical correctness

Score from your peers: **4**

Score from yourself: **4**

Clarity

Score from your peers: **4**

Score from yourself: **4**

Opening

Score from your peers: **4**

Score from yourself: **4**

Stating the conclusion

Score from your peers: **4**

Score from yourself: **4**

Reasons given

Score from your peers: **4**

Score from yourself: **4**

Overall valuation

Score from your peers: **4**

Score from yourself: **4**

peer 1 → *[This area was left blank by the evaluator.]*

peer 2 → Didn't state you were using the division theorem in the opening or elsewhere.

peer 3 → *[This area was left blank by the evaluator.]*

5. Prove that for any integer n, at least one of the integers n, $n+2$, $n+4$ is divisible by 3.

5. Any integer n will ~~either have 0,1,2~~ have reminder as 0, 1 or 2 when divided by 3.

① If n is not divisible by 3,
$\exists m \in N \mid n = 3m+1$
or $n = 3m+2$

If $n = 3m+1$,
$n+2 = 3m+3 = 3(m+1)$
$n+4 = 3m+5$
in this case $n+2$ is divisible by 3.

If $n = 3m+2$,
$n+2 = 3m+4$,
$n+4 = 3m+6 = 3(m+2)$,
in this case $n+4$ is divisible by 3.

② Otherwise n is divisible by 3

Combining ① & ②,
at least one of n, $n+2$ and $n+4$ will be divisible by 3.

Evaluation/feedback on the above work

Note: this section can only be filled out during the evaluation phase.

Logical correctness

Score from your peers: **4**

Score from yourself: **4**

Clarity

Score from your peers: **4**

Score from yourself: **4**

Opening

Score from your peers: **4**

Score from yourself: **4**

Stating the conclusion

Score from your peers: **4**

Score from yourself: **4**

Reasons given

Score from your peers: **4**

Score from yourself: **4**

Overall valuation

Score from your peers: **4**

Score from yourself: **4**

peer 1 → *[This area was left blank by the evaluator.]*

peer 2 → The opening does state in a roundabout way that the Division theorem is being used.

peer 3 → *[This area was left blank by the evaluator.]*

6. A classic unsolved problem in number theory asks if there are infinitely many pairs of "twin primes", pairs of primes separated by 2, such as 3 and 5, 11 and 13, or 71 and 73. Prove that the only prime triple (i.e. three primes, each 2 from the next) is 3, 5, 7.

6. Any ~~primes~~ triple can be represented as $n, n+2, n+4$. From problem 5, at least one of these 3 integers will be divisible by 3 for any n.

When $n=3$, we have the triple $3, 5, 7$ which is a prime triple.

But for any $n > 3$, at least one of $n, n+2, n+4$ will be divisible by $3 \Rightarrow$ at least one will be composite (of the form $3k$, when $k \geq 2$)

(Proved)

Evaluation/feedback on the above work

Note: this section can only be filled out during the evaluation phase.

Logical correctness

Score from your peers: **4**

Score from yourself: **4**

Clarity

Score from your peers: **4**

Score from yourself: **4**

Opening

Score from your peers: **4**

Score from yourself: **4**

Stating the conclusion

Score from your peers: **4**

Score from yourself: **4**

Reasons given

Score from your peers: **4**

Score from yourself: **4**

Overall valuation

Score from your peers: **4**

Score from yourself: **4**

peer 1 → *[This area was left blank by the evaluator.]*

peer 2 → *[This area was left blank by the evaluator.]*

peer 3 → *[This area was left blank by the evaluator.]*

7. Prove that for any natural number n, $2 + 2^2 + 2^3 + \ldots + 2^n = 2^{n+1} - 2$

7. By induction on n

Basis: $n = 1$,

$2^1 = 2^2 - 2 = 2^{1+1} - 2$

The equality holds for $n = 1$.

Induction hypothesis:

Let's assume $\forall n \leq m$,

$2 + 2^2 + 2^3 + \cdots + 2^n = 2^{n+1} - 2$

Induction Step:

for $n = m+1$,

$(2 + 2^2 + 2^3 + \cdots + 2^m) + 2^{m+1}$

$= (2^{m+1} - 2) + 2^{m+1}$, by induction hypothesis

$= 2 \cdot 2^{m+1} - 2$

$= 2^{m+2} - 2 = 2^{(m+1)+1} - 2$,

the equality holds for $n = m+1$ as well

(Prov

$\Rightarrow \forall n \in \mathbb{N}$,

$2 + 2^2 + 2^3 + \cdots + 2^n = 2^{n+1} - 2$

Evaluation/feedback on the above work

Note: this section can only be filled out during the evaluation phase.

Logical correctness

Score from your peers: **4**

Score from yourself: **4**

Clarity

Score from your peers: **4**

Score from yourself: **4**

Opening

Score from your peers: **4**

Score from yourself: **4**

Stating the conclusion

Score from your peers: **4**

Score from yourself: **4**

Reasons given

Score from your peers: **4**

Score from yourself: **4**

Overall valuation

Score from your peers: **4**

Score from yourself: **4**

peer 1 → *[This area was left blank by the evaluator.]*

peer 2 → *[This area was left blank by the evaluator.]*

peer 3 → *[This area was left blank by the evaluator.]*

8. Prove (from the definition of a limit of a sequence) that if the sequence $\{a_n\}_{n=1}^{\infty}$ tends to limit L as $n \to \infty$, then for any fixed number $M > 0$, the sequence $\{Ma_n\}_{n=1}^{\infty}$ tends to the limit ML.

8. By condition,

$$\lim_{n\to\infty} a_n = L$$

Let $\epsilon > 0$ be given,
then we can choose N large enough
s.t. $|a_n - L| < \epsilon$
whenever $n \geq N$.

Now, for a fixed $M > 0$,

$$n \geq N \Rightarrow$$
$$|Ma_n - ML| = |M||a_n - L|$$
$$= M|a_n - L| < M\epsilon = \epsilon_0$$

Hence, $\lim_{n\to\infty} Ma_n = ML$.

(Proved)

Evaluation/feedback on the above work

Note: this section can only be filled out during the evaluation phase.

Logical correctness

Score from your peers: **4**

Score from yourself: **3**

Clarity

Score from your peers: **4**

Score from yourself: **4**

Opening

Score from your peers: **4**

Score from yourself: **4**

Stating the conclusion

Score from your peers: **4**

Score from yourself: **3**

Reasons given

Score from your peers: **4**

Score from yourself: **3**

Overall valuation

Score from your peers: **4**

Score from yourself: **3**

peer 1 → *[This area was left blank by the evaluator.]*

peer 2 → It's close but look at KDs video 3 for the correct way to use the epsilon.

peer 3 → *[This area was left blank by the evaluator.]*

9. Given an infinite collection $A_n, n = 1, 2, \ldots$ of intervals of the real line, their *intersection* is defined to be $\bigcap_{n=1}^{\infty} A_n = \{x \mid (\forall n)(x \in A_n)\}$ Give an example of a family of intervals $A_n, n = 1, 2, \ldots$, such that $A_{n+1} \subset A_n$ for all n and $\bigcap_{n=1}^{\infty} A_n = \emptyset$. Prove that your example has the stated property.

9. Let $A_n = (0, \frac{1}{n})$, an open interval.

$A_1 = (0,1)$, $A_2 = (0, 1/2)$, $A_3 = (0, 1/3)$, ...

$A_{n+1} \subset A_n \subset \cdots \subset A_3 \subset A_2 \subset A_1$

where $\bigcap_{n=1}^{\infty} A_n = \phi$.

$$A_n \cap A_{n+1} = (0, \tfrac{1}{n}) \cap (0, \tfrac{1}{n+1})$$
$$= (0, \tfrac{1}{n+1}) = A_{n+1}$$

$A_1 \cap A_2 = (0, 1/2)$

$$A_1 \cap A_2 \cap A_3 = (0, 1/2) \cap (0, 1/3)$$
$$= (0, 1/3)$$

$\vdots$

$A_1 \cap A_2 \cap \cdots \cap A_n = (0, 1/n)$

$\Leftrightarrow \bigcap_{k=1}^{n} A_k = (0, \frac{1}{n})$

when $n \to \infty$, LUB of the interval is 0

$\bigcap_{k=1}^{\infty} A_n = \emptyset$, since LUB is outside the intersection set

(Proved)

Evaluation/feedback on the above work

Note: this section can only be filled out during the evaluation phase.

Logical correctness

Score from your peers: **4**

Score from yourself: **4**

Clarity

Score from your peers: **4**

Score from yourself: **4**

Opening

Score from your peers: **4**

Score from yourself: **4**

Stating the conclusion

Score from your peers: **4**

Score from yourself: **3**

Reasons given

Score from your peers: **4**

Score from yourself: **4**

Overall valuation

Score from your peers: **4**

Score from yourself: **4**

peer 1 → *[This area was left blank by the evaluator.]*

peer 2 → *[This area was left blank by the evaluator.]*

peer 3 → *[This area was left blank by the evaluator.]*

10. Give an example of a family of intervals $A_n, n = 1, 2, \ldots$, such that $A_{n+1} \subset A_n$ for all n and $\bigcap_{n=1}^{\infty} A_n$ consists of a single real number. Prove that your example has the stated property.

10. Let $A_n = [0, \frac{1}{n})$, an closed-open interval

$A_1 = [0, 1)$, $A_2 = [0, \frac{1}{2})$, $A_3 = [0, \frac{1}{3})$, ...

$A_{n+1} \subset A_n \subset \cdots \subset A_3 \subset A_2 \subset A_1$

where $\bigcap_{n=1}^{\infty} A_n = \{0\}$

$A_n \cap A_{n+1} = [0, \frac{1}{n}) \cap [0, \frac{1}{n+1})$

$= [0, \frac{1}{n+1}) = A_{n+1}$

$A_1 \cap A_2 = [0, \frac{1}{2})$

$A_1 \cap A_2 \cap A_3 = [0, \frac{1}{2}) \cap [0, \frac{1}{3})$

$= [0, \frac{1}{3})$

$\vdots$

$A_1 \cap A_2 \cap \cdots \cap A_n = [0, \frac{1}{n})$

$\Leftrightarrow \bigcap_{k=1}^{n} A_k = [0, \frac{1}{n})$

when $n \to \infty$, LUB of the interval is 0

$\bigcap_{k=1}^{\infty} A_n = \{0\}$, since GLB is inside the intersection set

(Proved)

Evaluation/feedback on the above work

Note: this section can only be filled out during the evaluation phase.

Logical correctness

Score from your peers: **4**

Score from yourself: **4**

Clarity

Score from your peers: **4**

Score from yourself: **4**

Opening

Score from your peers: **4**

Score from yourself: **4**

Stating the conclusion

Score from your peers: **4**

Score from yourself: **4**

Reasons given

Score from your peers: **4**

Score from yourself: **4**

Overall valuation

Score from your peers: **4**

Score from yourself: **4**

peer 1 → *[This area was left blank by the evaluator.]*

peer 2 → *[This area was left blank by the evaluator.]*

peer 3 → *[This area was left blank by the evaluator.]*

Overall evaluation/feedback

Note: this section can only be filled out during the evaluation phase.

In this section you should provide the person submitting the work with a concise summary of your evaluation of their entire submission.

FIRST, enter the ***total*** of the *Overall valuation* scores you gave for each of the ten solutions (i.e., enter a single number between 0 and 40, inclusive).

THEN provide a short textual commentary.

The goal is to be as helpful and informative as possible. Anyone who has stayed in the course until now must be smart. Your goal now is to provide feedback to help them become even better. If you did not fully understand their solutions, tell them so. Since you are an intended reader of their work, it is valuable feedback to them to know that you did not understand it.

peer 1 → the total is 216 good job

peer 2 → 38 The answers were generally very good - Q1 riquired a bit of thought but I got it eventually.

peer 3 → 40 I have given you full marks as I could not evaluate your work due to some personal issues. I wish you good luck.