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Number representations in base 2

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$$x = x_1 \cdot 2^{-1} + x_2 \cdot 2^{-2} + x_3 \cdot 2^{-3} + \cdots = .x_1x_2x_3\cdots \qquad (0 \le x \le 1;\, x_n \in \{0, 1\},\, n \ge 1)$$

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Every real number x in the unit interval has a dyadic expansion $.x_1x_2x_3\ldots$, and conversely.