Erasmus School of Economics

MOOC Econometrics

Lecture 6.4 on Time Series: Evaluation and Illustration

Dick van Dijk, Philip Hans Franses, Christiaan Heij

Erasmus University Rotterdam

First evaluation step: Check for stationarity

- Take difference of time series until stationarity.
- Test equation: Augmented Dickey-Fuller

  $$\Delta y_t = \alpha + \beta t + \rho y_{t-1} + \gamma_1 \Delta y_{t-1} + \ldots + \gamma_L \Delta y_{t-L} + \varepsilon_t$$

  Critical value $t_{\hat{\rho}}$: -2.9 if $\beta = 0$, -3.5 if $\beta \neq 0$
- For stationary data:

  $\rightarrow$ OLS in AR: $y_t = \alpha + \sum_{j=1}^{p} \beta_j y_{t-j} + \varepsilon_t$

  with trend: $y_t = \alpha + \gamma t + \sum_{j=1}^{p} \beta_j y_{t-j} + \varepsilon_t$

  $\rightarrow$ OLS in ADL: $y_t = \alpha + \sum_{j=1}^{p} \beta_j y_{t-j} + \sum_{j=1}^{r} \gamma_j x_{t-j} + \varepsilon_t$

  with trend: $y_t = \alpha + \delta t + \sum_{j=1}^{p} \beta_j y_{t-j} + \sum_{j=1}^{r} \gamma_j x_{t-j} + \varepsilon_t$
- t- and F-tests as usual.

Check for cointegration

- If x_t and y_t are both non-stationary: check for cointegration.
- Test method: Engle-Granger two-step method

  $\rightarrow$ OLS in $y_t = \alpha + \beta x_t + \varepsilon_t \quad \rightarrow \quad b$ and OLS residuals e_t

  $\rightarrow$ OLS in $\Delta e_t = \alpha + \beta t + \rho e_{t-1} + \gamma_1 \Delta e_{t-1} + \ldots + \gamma_L \Delta e_{t-L} + \omega_t$

  Critical value $t_{\hat{\rho}}$: -3.4 if $\beta = 0$, -3.8 if $\beta \neq 0$
- If x_t and y_t are cointegrated, estimate ECM:

  $$\Delta y_t = \alpha + \beta t + \gamma_0 (y_{t-1} - b x_{t-1}) + \sum_{j=1}^{p} \gamma_{y,j} \Delta y_{t-j} + \sum_{j=1}^{r} \gamma_{x,j} \Delta x_{t-j} + \varepsilon_t$$

  (or with $\beta = 0$)
- t- and F-tests as usual.

Diagnostic tests

- Choice of lag lengths: BIC (see Lecture 3).
- Stability check: Chow tests (see Lecture 3).
- Normal residuals: Jarque-Bera (see Lecture 3), critical value: 6.0.
- Out-of-sample forecasting: Lecture 6.5.
- Model should in particular capture autocorrelation in time series.

  $\rightarrow$ Test if model residuals are uncorrelated: white noise.
- Two tests: ACF and Breusch-Godfrey.
- ACF rule-of-thumb: significant if $|\text{ACF}| > 2/\sqrt{n}$.

Test question

Test

Let y_t be white noise with variance σ^2. Show that OLS estimator b in $y_t = \alpha + \beta y_{t-1} + \varepsilon_t$ gives the first-order autocorrelation of y_t. Further show that $(-2/\sqrt{n},\ 2/\sqrt{n})$ is approximate 95% confidence interval for β. Hint: Use results of Lecture 1.

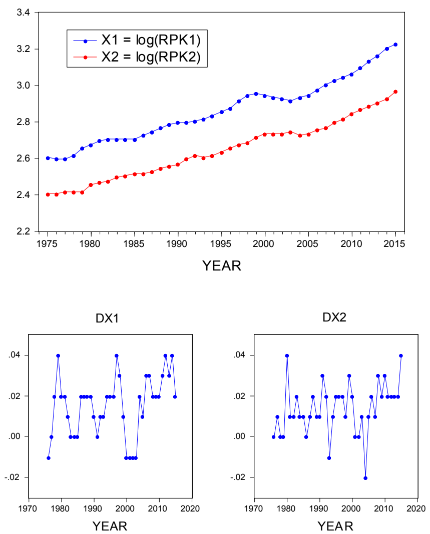
Answer:

- $y_t = \alpha = \beta x_t + \varepsilon_t$ where $x_t = y_{t-1}$, $t = 2, \ldots, n$, so
  $b = \sum_{t=2}^{n}(y_t - \bar{y})(y_{t-1} - \bar{y}) / \sum_{t=2}^{n}(y_{t-1} - \bar{y})^2$
- $\text{var}(b) = \sigma^2 / \sum_{t=2}^{n}(y_{t-1} - \bar{y})^2$, where
  $\sum_{t=2}^{n}(y_{t-1} - \bar{y})^2 = (n-1)\sum_{t=2}^{n}(y_{t-1} - \bar{y})^2/(n-1) \approx (n-1)\sigma^2$,
  $\text{var}(b) \approx \sigma^2/((n-1)\sigma^2) = 1/(n-1) \approx 1/n$
- If n large then $b \approx 0$ and $\text{SE}(b) \approx 1/\sqrt{n}$
  $b - 2\text{SE}(b) < \beta < b - 2\text{SE}(b) \quad \rightarrow -2/\sqrt{n} < \beta < 2/\sqrt{n}$

Test on serial correlation: Breusch-Godfrey

- Step 1: Estimate model and get residuals e_t.
- Step 2: Regress e_t on all variables of model and r lags of e_t.
- Step 3: $\text{BG} = nR^2$ of Step 2, and $\text{BG} \approx \chi^2(r)$ if e_t white noise.
- Example: Model $y_t = \alpha + \beta y_{t-1} + \gamma x_{t-1} + \varepsilon_t$
  - $\rightarrow$ Step 1: OLS residuals $e_t = y_t - a - b y_{t-1} - c x_{t-1}$.
  - $\rightarrow$ Step 2: OLS in $e_t = \alpha + \beta y_{t-1} + \gamma x_{t-1} + \delta_1 e_{t-1} + \delta_2 e_{t-2} + \omega_t$
  - $\rightarrow$ Step 3: $\text{BG} = nR^2 \approx \chi^2(2)$ if e_t white noise.
  - $\rightarrow$ Conclusion: Model not correctly specified if $\text{BG} > 6.0$.
  - $\rightarrow$ Should then adjust model, e.g. more lags of y_t and x_t.

Illustration: Revenue Passenger Kilometers (RPK)



- Graphs suggest: X_1 and X_2 non-stationary, ΔX_1 and ΔX_2 stationary.

Tests on stationarity

- Let y_t denote log(RPK), either X_{1t} or X_{2t}: trend
  ADF: $\Delta y_t = \alpha + \beta t + \rho y_{t-1} + \gamma \Delta y_{t-1} + \varepsilon_t$
  t-value of $\hat{\rho}$: $t = -2.8$ for X_1, $t = -1.2$ for X_2
- Let y_t denote either ΔX_{1t} or ΔX_{2t}: no trend
  ADF: $\Delta y_t = \alpha + \rho y_{t-1} + \gamma \Delta y_{t-1} + \varepsilon_t$
  t-value of $\hat{\rho}$: $t = -3.3$ for X_1, $t = -3.7$ for X_2

Test

What conclusions do you draw from these outcomes?

Answer:

- As $t > -3.5$, X_1 and X_2 not stationary.
- As $t < -2.9$, ΔX_1 and ΔX_2 are both stationary.

Granger causality tests

	ADL for ΔX_{1t}			ADL for ΔX_{2t}		
	Coef.	t-Stat.	p-value	Coef.	t-Stat.	p-value
Constant	0.01	1.85	0.07	0.01	2.86	0.01
$\Delta X_{1,t-1}$	0.87	4.96	0.00	0.18	1.29	$\boxed{0.21}$
$\Delta X_{1,t-2}$	-0.42	-2.02	0.05	0.61	3.68	$\boxed{0.00}$
$\Delta X_{2,t-1}$	0.35	1.74	$\boxed{0.09}$	-0.29	-1.81	0.08
$\Delta X_{2,t-2}$	-0.19	-1.27	$\boxed{0.21}$	-0.13	-1.05	0.30

- Company 1 Granger causal for company 2, not other way round.

  $\rightarrow$ See t-tests (confirmed by F-tests on two coefficients jointly).

Engle-Granger test and ECM

- Step 1: OLS: $X_{2t} = 0.01 + \underline{0.92} X_{1t} + e_t$.
- Step 2: ADF: $\Delta e_t = 0.00 - 0.50 e_{t-1} + 0.30 \Delta e_{t-1} + \text{res}_t$

  $\rightarrow$ t-value of coefficient e_{t-1}: $t = -3.5 < -3.4$

  $\rightarrow$ e_t stationary $\rightarrow$ X_{1t} and X_{2t} cointegrated.
- ECM (after removing insignificant coefficients):

  $\Delta X_{1t} = 0.00 + 1.02 \Delta X_{1t} + 0.46(X_{2,t-1} - \underline{0.92} X_{1,t-1}) + e_{1t}$

  $\Delta X_{2t} = 0.02 - 0.45(X_{2,t-1} - \underline{0.92} X_{1,t-1}) + e_{2t}$
- If $D_{t-1} = X_{2,t-1} - 0.92 X_{1,t-1}$ is positive, then

  $0.46 > 0 \rightarrow X_{1t} \uparrow \rightarrow D_t = X_{2t} - 0.92 X_{1t} \downarrow$

  $-0.45 < 0 \rightarrow X_{2t} \downarrow \rightarrow D_t = X_{2t} - 0.92 X_{1t} \downarrow$
- Error correction mechanism acts on both variables.

ECM: Check for serial correlation and normality

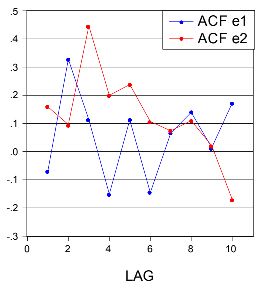
- ECM models for log(RPK) of airline companies 1 and 2 ($n = 39$):

  $\Delta X_{1t} = 0.00 + 1.02 \Delta X_{1t} + 0.46(X_{2,t-1} - 0.92 X_{1,t-1}) + e_{1t}$

  $\Delta X_{2t} = 0.02 - 0.45(X_{2,t-1} - 0.92 X_{1,t-1}) + e_{2t}$
- Jarque-Bera test: $\text{JB}_1 = 0.4 < 6$, $\text{JB}_2 = 1.8 < 6$.

  Breusch-Godfrey test (1 lag): $\text{BG}_1 = 0.3 < 3.9$, $\text{BG}_2 = 1.2 < 3.9$.

  ACF: $2/\sqrt{n} = 2/\sqrt{39} = 0.32$.



TRAINING EXERCISE 6.4

- Train yourself by making the training exercise (see the website).
- After making this exercise, check your answers by studying the webcast solution (also available on the website).