A rule of products

Independent events

Definition

We say that events A and B in a probability space are independent if (and only if)

$$\mathbf{P}(A \cap B) = \mathbf{P}(A) \times \mathbf{P}(B).$$

The relationship with conditional probabilities

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If A and B are independent events of positive probability then

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Slogan

Mathematical independence captures the intuitive idea that if A and B are independent then the occurrence of one does not affect the chances of the other.

If A and B are independent does this imply that A and B^c are also independent?

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Additivity

$$\mathbf{P}(A \cap B^c) = \mathbf{P}(A) - \mathbf{P}(A \cap B)$$

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Each of the following statements implies the other three:

- A and B are independent.
- A and B^c are independent.
- A^c and B are independent.
- A^c and B^c are independent.