Chevalier de Méré's paradox

Antoine Gombaud

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- A specious (and incorrect) argument:

  A) There is a one-in-six chance of an ace in a die throw. So in four throws the odds of an ace should be 4 : 6.

  B) There is a one-in-36 chance of a double ace when a pair of dice are thrown. So in twenty four throws of the pair, the odds of a double ace are in the proportionate ratio 24 : 36.

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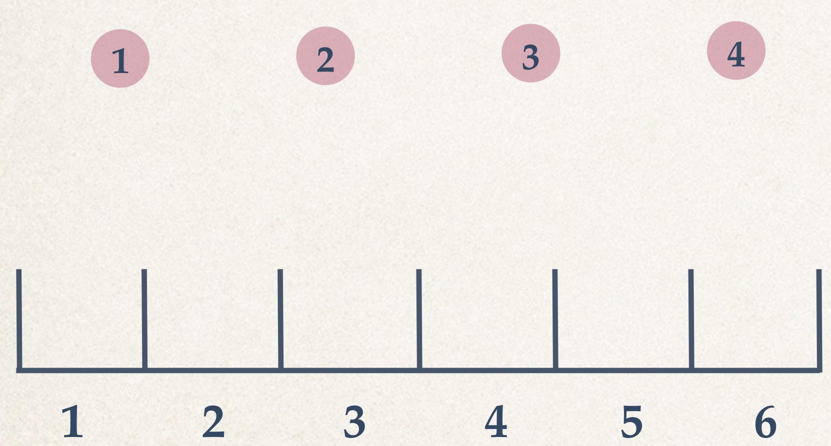
The argument implicitly adopts the working principle that *the chances increase proportionately with the number of trials*. Thus, if the reasoning in A) is taken at face value then the odds of (at least) one ace in six throws of a die should be 6 : 6 = 1, which is absurd. So something is wrong with the principle.

A balls and urns metaphor: one die

A) Four throws of a die: each throw represents a distinct ball, the face values that are possible represent six distinct urns.



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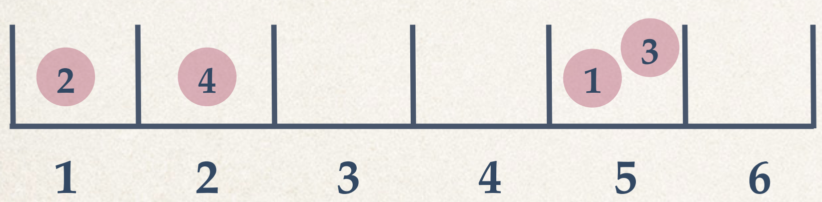

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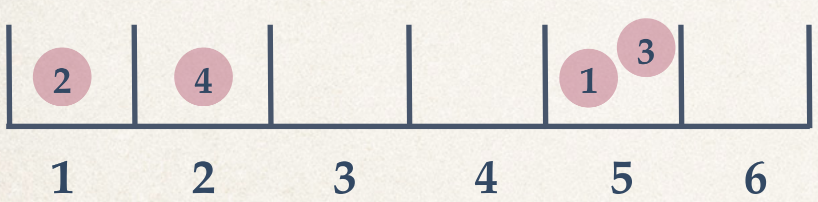

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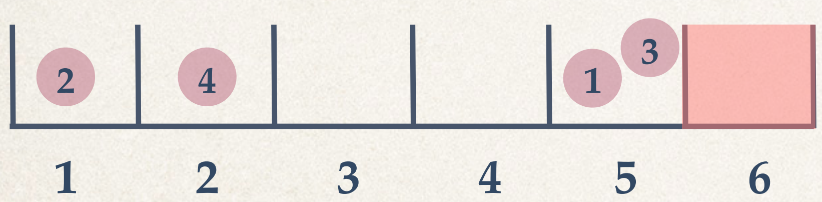

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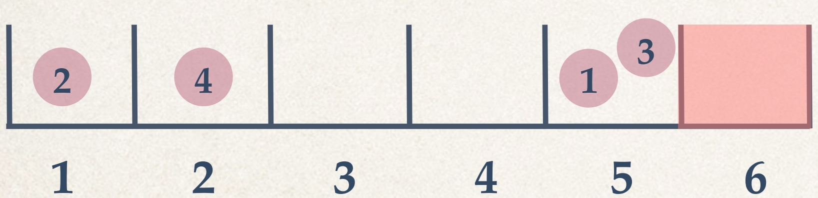

- Possible outcomes: (k_1, k_2, k_3, k_4). Ordered sample (of urns) with replacement. The number of such sequences is 6^4.

- The complement of the event that an ace is thrown occurs if none of k_1, k_2, k_3, k_4 is equal to six. There are 5^4 sequences favourable to this event. Under random selection this event hence has probability $5^4/6^4 \approx 0.482$.

- The event that there is an ace hence has probability $\mathbf{P}(A) = 1 - \frac{5^4}{6^4} \approx 0.518$.

A balls and urns metaphor: two dice

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B) Twenty four throws of a pair of dice: each throw represents a distinct ball, the face values that are possible represent six distinct urns.

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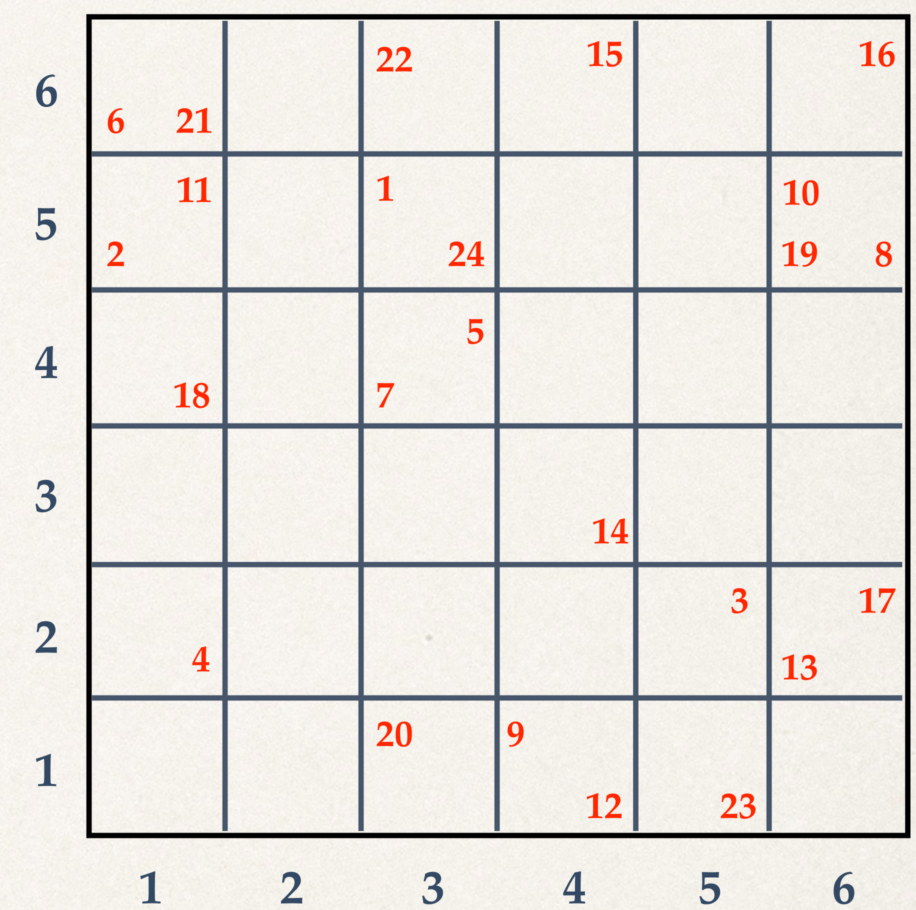

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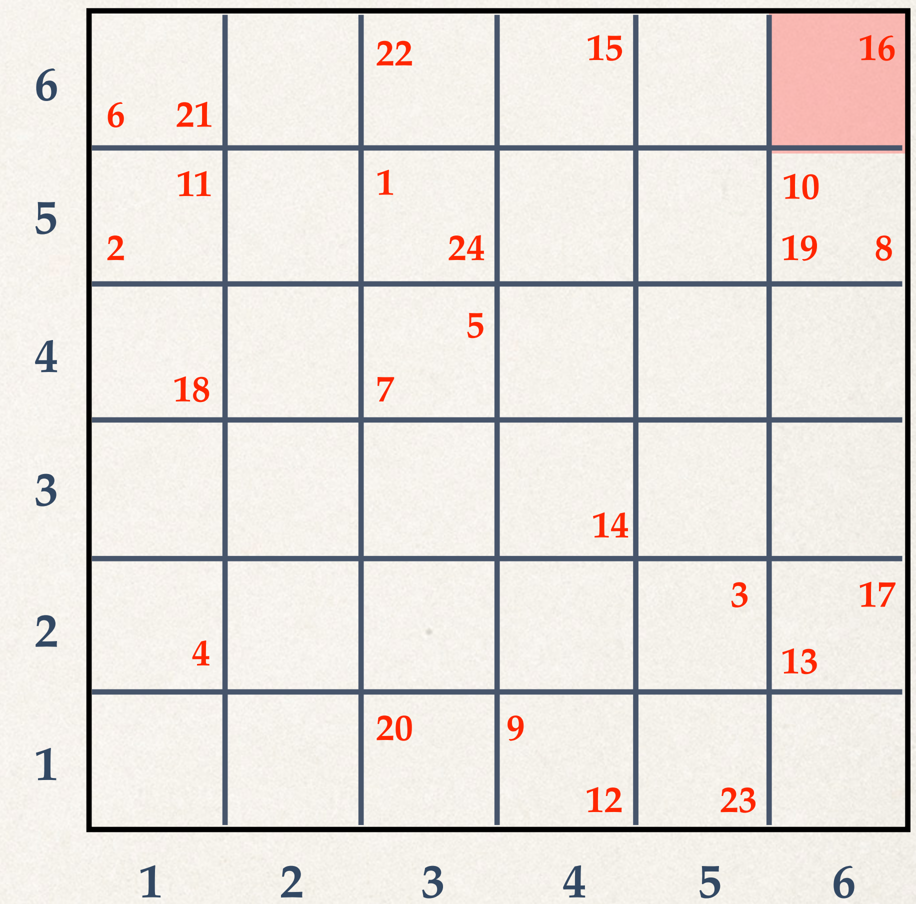

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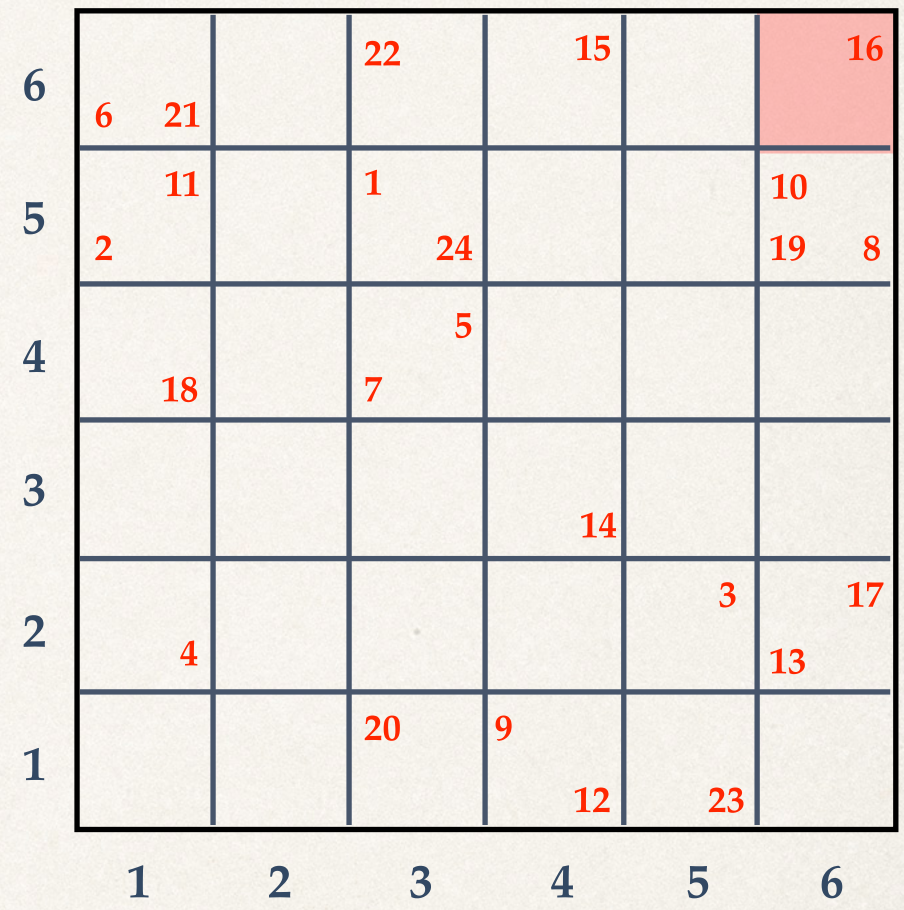
- Possible outcomes: $(j_1, k_1), \ldots, (j_{24}, k_{24})$. Ordered sample (of pairs of urns) with replacement. The number of such sequences is 36^{24}.

- The complement of the event that an ace pair is obtained occurs if none of the pairs $(j_1, k_1), \ldots, (j_{24}, k_{24})$ is equal to $(6, 6)$. There are 35^{24} sequences favourable to this event. Under random selection this event hence has probability $35^{24}/36^{24} \approx 0.509$.

- The event that there is an ace hence has probability $\mathbf{P}(B) = 1 - \frac{35^{24}}{36^{24}} \approx 0.491$.



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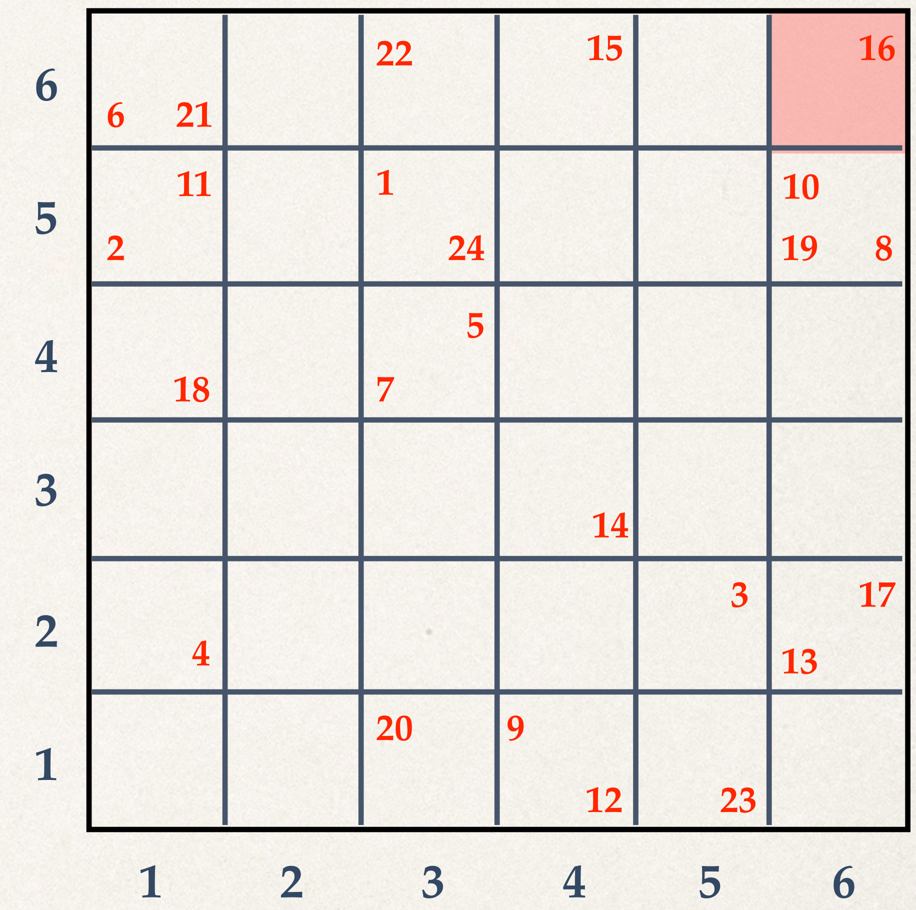

It is (slightly) more likely to obtain an ace in 4 throws of a die than it is to obtain a double ace in 24 throws of a pair of dice.