Notes: Sequences and Series

Sequences

Definitions

A **sequence** is an infinite ordered list of numbers, $\{a_n\} = \{a_n\}_{n=1}^{\infty} = a_1, a_2, \ldots, a_n, \ldots$

A sequence is **convergent** if the terms a_n get close to a **limit** L when n is sufficiently large. If this is the case, we write $\lim\limits_{n\to\infty} a_n = L$.

A sequence that is not convergent is called **divergent**.

A sequence $\{a_n\}$ is **increasing** if $a_n < a_{n+1}$ for all n, and **decreasing** if $a_n > a_{n+1}$ for all n. A sequence is called **monotonic** if it is either increasing or decreasing.

Limit Laws

Limits of sequences follow all of the same limit laws defined for functions: If $\{a_n\}$ and $\{b_n\}$ are convergent and c is a constant, then

- $\lim\limits_{n\to\infty} (a_n \pm b_n) = \lim\limits_{n\to\infty} a_n \pm \lim\limits_{n\to\infty} b_n$
- $\lim\limits_{n\to\infty} (ca_n) = c \lim\limits_{n\to\infty} a_n$
- $\lim\limits_{n\to\infty} (a_n b_n) = \lim\limits_{n\to\infty} a_n \cdot \lim\limits_{n\to\infty} b_n$
- $\lim\limits_{n\to\infty} \dfrac{a_n}{b_n} = \dfrac{\lim_{n\to\infty} a_n}{\lim_{n\to\infty} b_n}$ if $\lim\limits_{n\to\infty} b_n \neq 0$
- $\lim\limits_{n\to\infty} {a_n}^p = \left(\lim\limits_{n\to\infty} a_n\right)^p$ if $p > 0$ and $a_n > 0$.

Examples of Sequences

The **harmonic sequence** has terms of the form $a_n = \dfrac{1}{n}$. The harmonic sequence converges to 0.

A **geometric sequence** is a sequence where each term is found by multiplying the previous term by a