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$$\mathbf{P}(A) = \sum_{\omega \in A} \mathbf{P}\{\omega\} = p(k_1) + p(k_2) + \cdots$$

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Additivity

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- The atomic probabilities $p(k) := \mathbf{P}\{\omega_k\}$ determine a function called the mass function or distribution. This function satisfies *positivity*, $p(k) \geq 0$, and is properly *normalised*, $\sum_k p(k) = 1$.
- The probability measure $\mathbf{P}(\,\cdot\,)$ is determined from the distribution $p(\,\cdot\,)$ via *additivity*:

$$\mathbf{P}\{\omega_{k_1}, \omega_{k_2}, \omega_{k_3}, \dots\} = p(k_1) + p(k_2) + p(k_3) + \cdots .$$

Any *honest* mass function $p(k)$ induces a discrete probability measure.
All we have to do is verify that it is *positive* and properly *normalised*.