Lemma

$$\min\{7x_1 + x_2 + 5x_3 : x \text{ feasible}\} \geq \max\{10y_1 + 6y_2 : y \text{ feasible}\}$$

Linear programming duality Theorem

$$\min\{7x_1 + x_2 + 5x_3 : x \text{ feasible}\} = \max\{10y_1 + 6y_2 : y \text{ feasible}\}$$

In general

(P)

$$\min \mathbf{c} \cdot \mathbf{x} :$$
$$\mathbf{A}\mathbf{x} \geq \mathbf{b}$$
$$\mathbf{x} \geq \mathbf{0}$$

$\longleftrightarrow$ duality

(D)

$$\max \mathbf{b} \cdot \mathbf{y} :$$
$$\mathbf{A}^{\mathbf{T}}\mathbf{y} \leq \mathbf{c}$$
$$\mathbf{y} \geq \mathbf{0}$$

Strong duality Theorem in general

(P)

$$\min \mathbf{c}\cdot\mathbf{x}:$$
$$\mathbf{A}\mathbf{x} \geq \mathbf{b}$$
$$\mathbf{x} \geq 0$$

(D)

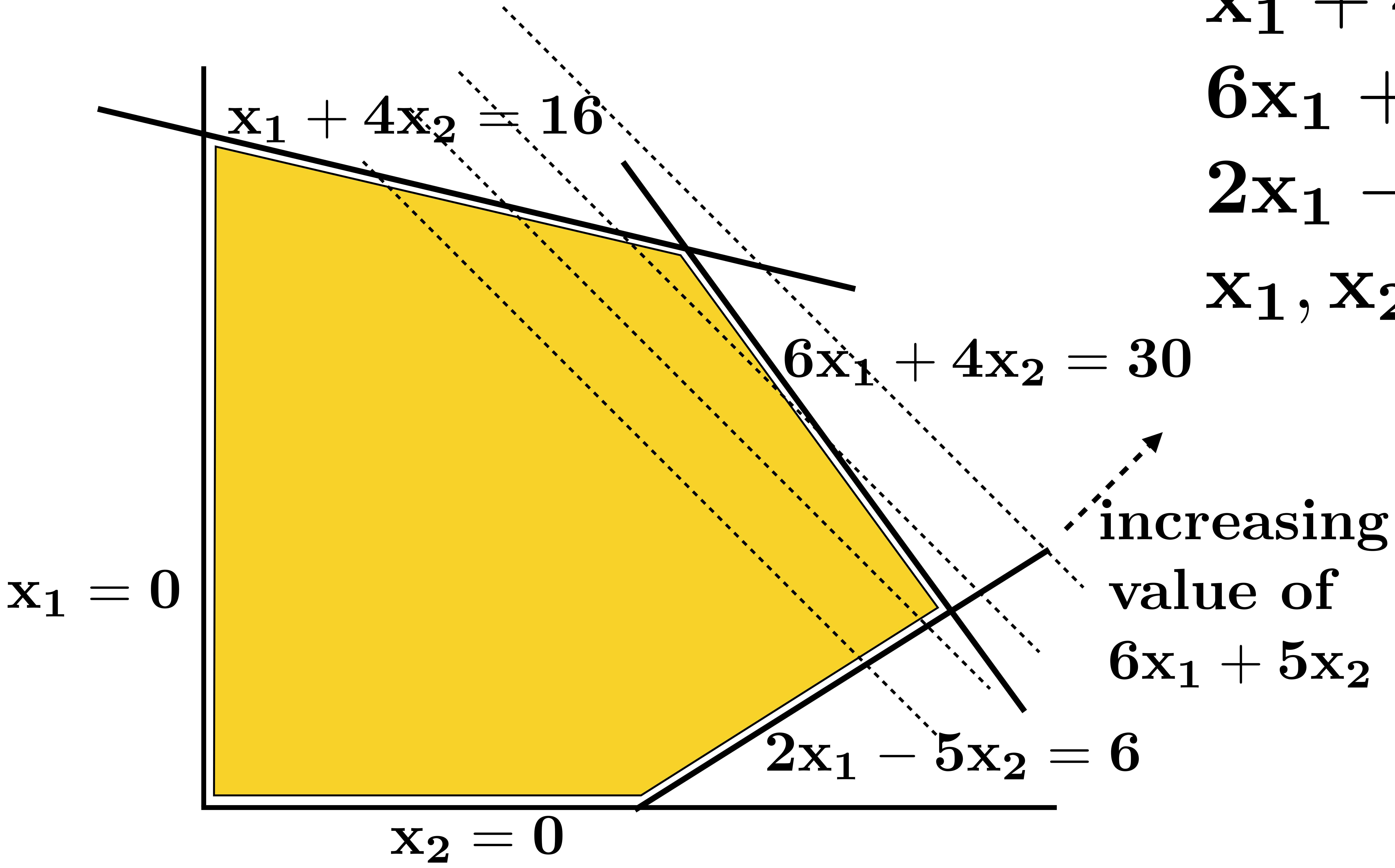
$$\max \mathbf{b}\cdot\mathbf{y}:$$
$$\mathbf{A}^{\mathbf{T}}\mathbf{y} \leq \mathbf{c}$$
$$\mathbf{y} \geq 0$$

Four possible cases:

- (P) is empty, (D) has value $+\infty$
- (D) is empty, (P) has value $-\infty$
- value(P)=value(D)
- ((P) and (D) both empty)

What about the geometry?

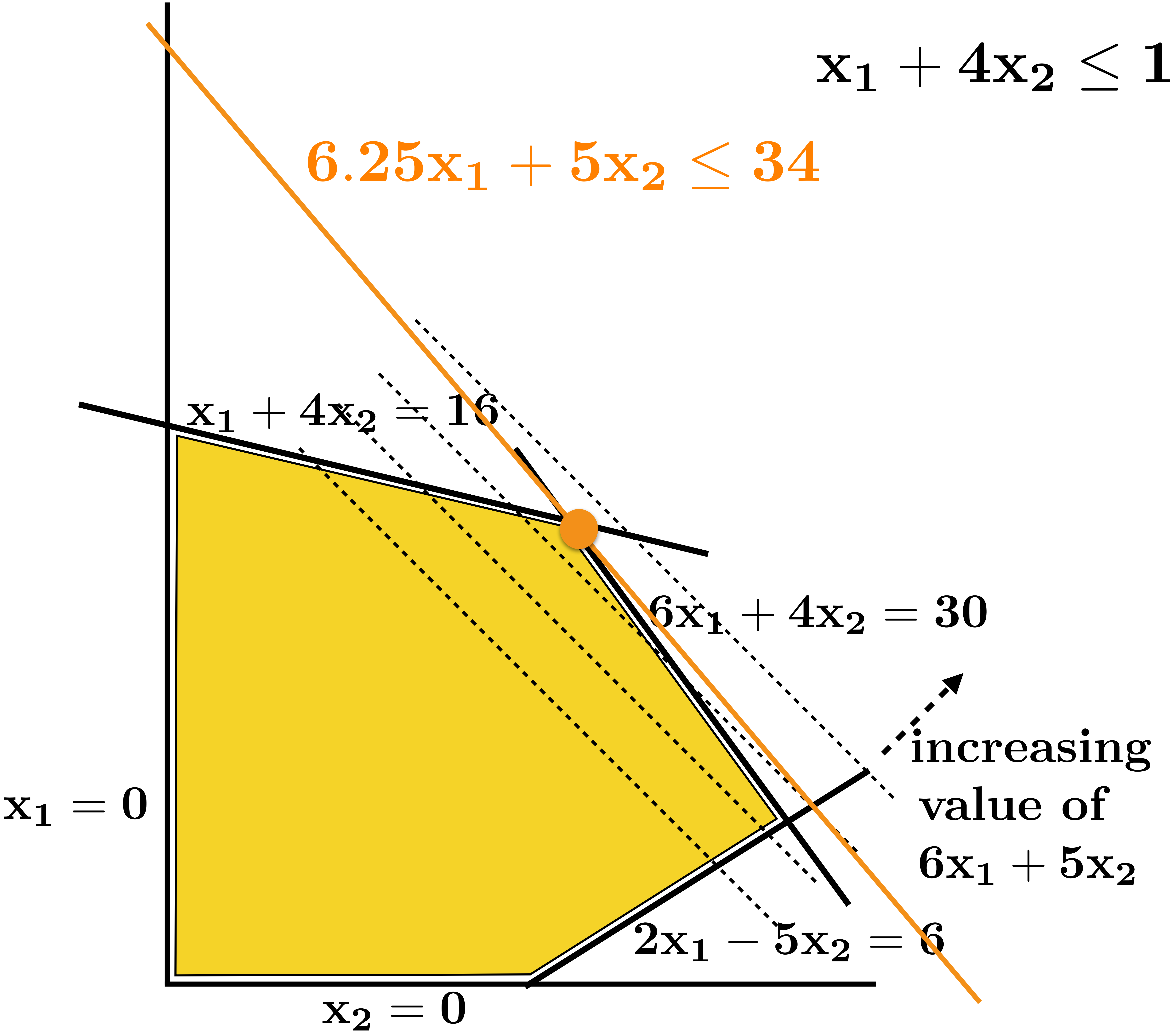
$$\max \mathbf{6x_1 + 5x_2} :$$
$$\mathbf{x_1 + 4x_2 \leq 16}$$
$$\mathbf{6x_1 + 4x_2 \leq 30}$$
$$\mathbf{2x_1 - 5x_2 \leq 6}$$
$$\mathbf{x_1, x_2 \geq 0}$$



$$x_1 + 4x_2 \le 16 \Rightarrow .25x_1 + x_2 \le 4$$

$$\frac{\quad 6x_1 + 4x_2 \le 30 \quad}{6.25x_1 + 5x_2 \le 34}$$

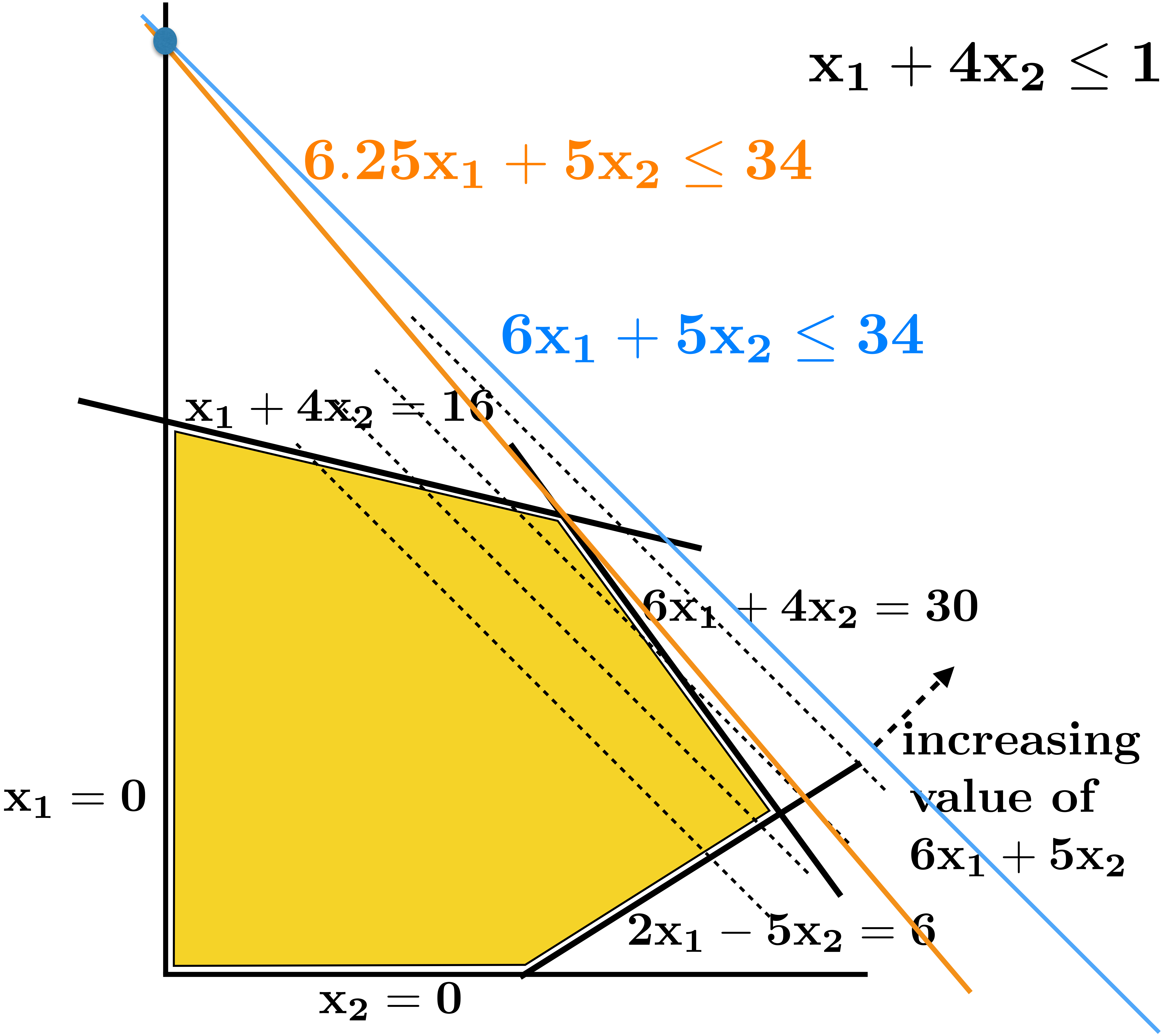
$$\Rightarrow 6x_1 + 5x_2 \le 34$$



$$x_1 + 4x_2 \le 16 \Rightarrow .25x_1 + x_2 \le 4$$

$$\begin{array}{r} 6x_1 + 4x_2 \le 30 \\ \hline 6.25x_1 + 5x_2 \le 34 \end{array}$$

$$\Rightarrow 6x_1 + 5x_2 \le 34$$

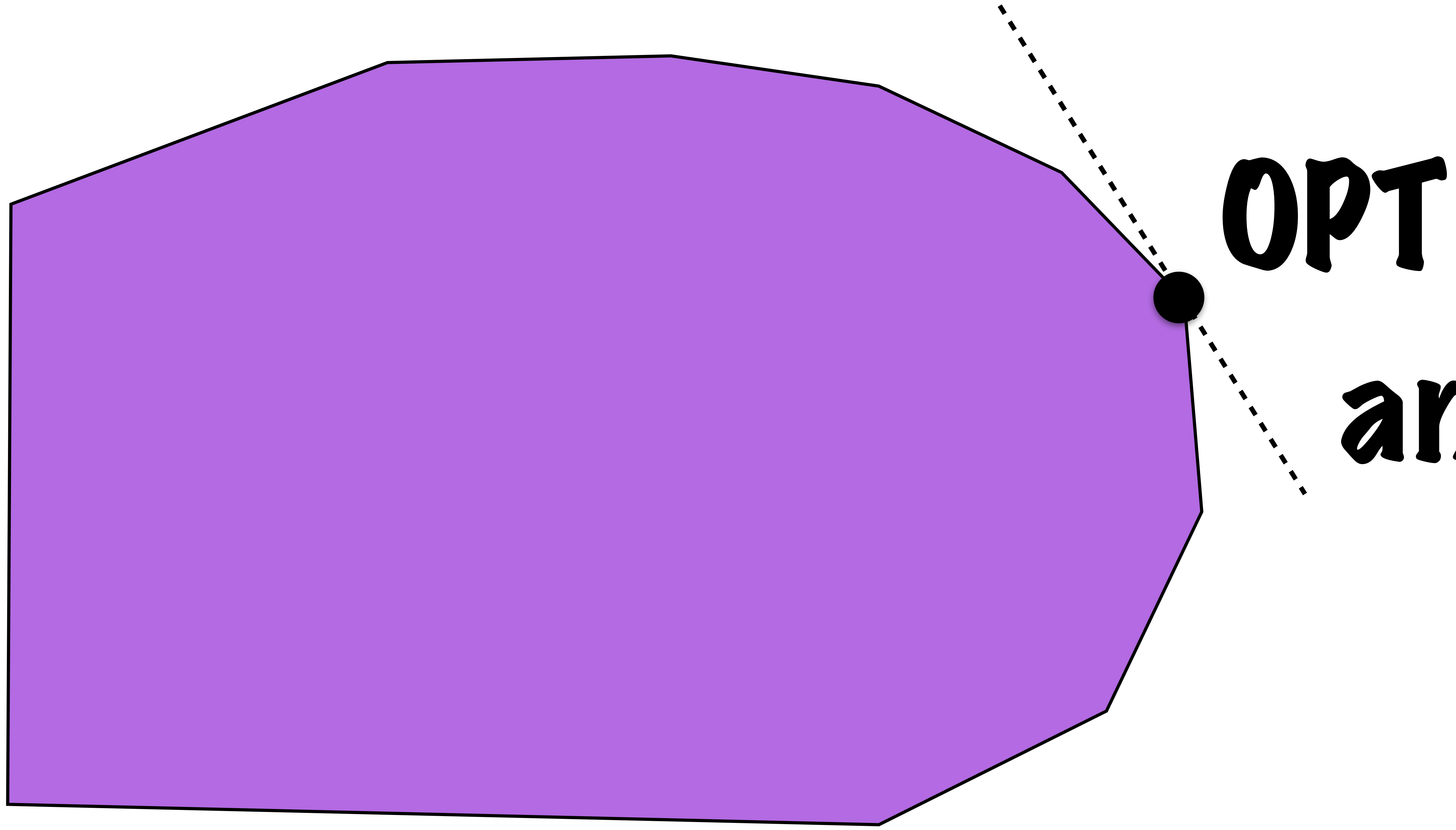


Linear programming duality Theorem

(P) = (D)

$$\max \mathbf{c} \cdot \mathbf{x} : \quad \mathbf{A}\mathbf{x} \leq \mathbf{b} \quad \mathbf{x} \geq 0$$

$$\min \mathbf{b} \cdot \mathbf{y} : \quad \mathbf{A}^T\mathbf{y} \geq \mathbf{c} \quad \mathbf{y} \geq 0$$



There exist constraints of (P) and a convex combination that imply exactly the right upper bound