Question 1

$|0\rangle=(00)$

Your Answer		Score	Explanation
False	✔	1.00	$|0\rangle=(10)$ and $|1\rangle=(01)$
Total		1.00 / 1.00	

Question Explanation

$|0\rangle=(10)$ and $|1\rangle=(01)$

Question 2

$|1\rangle=(11)$

Your Answer		Score	Explanation
False	✔	1.00	$|0\rangle=(10)$ and $|1\rangle=(01)$
Total		1.00 / 1.00	

Question Explanation

$|0\rangle=(10)$ and $|1\rangle=(01)$

Question 3

A quantum state is a unit vector in a complex vector space.

Your Answer		Score	Explanation
True	✔	1.00	
Total		1.00 / 1.00	

Question 4

Measurements can only be performed in the computational (standard) basis.

Your Answer	Score	Explanation

◉ False	✔	1.00	Measurements can be performed in any orthonormal basis.
Total		1.00 / 1.00	

Question Explanation

Measurements can be performed in any orthonormal basis.

Question 5

The probability amplitude of $|x\rangle$ is (directly) proportional to the probability that the outcome of a measurement is x.

Your Answer		Score	Explanation
◉ False	✔	1.00	The probability that the outcome of a measurement is x is proportional to the **square** of the magnitude of the probability amplitude of $|x\rangle$.
Total		1.00 / 1.00	

Question Explanation

The probability that the outcome of a measurement is x is proportional to the **square** of the magnitude of the probability amplitude of $|x\rangle$.

Question 6

The inner product of $a|0\rangle+b|1\rangle$ and $c|0\rangle+d|1\rangle$ is ac^*+bd^*.

Your Answer		Score	Explanation
◉ False	✔	1.00	Please see the announcement at the home page
Total		1.00 / 1.00	

Question Explanation

Please see the announcement at the home page.

Question 7

The inner product of $|+\rangle$ and $|-\rangle$ is 1.

Your Answer	Score	Explanation
False	1.00	$|+\rangle$ and $|-\rangle$ are orthogonal, so their inner product is 0. One can also verify it by explicitly computing the inner product.
Total	1.00 / 1.00	

Question Explanation

$|+\rangle$ and $|-\rangle$ are orthogonal, so their inner product is 0. One can also verify it by explicitly computing the inner product.

Question 8

In $C2$, how many **real** (unit) vectors are there whose projection onto $|1\rangle$ has length $3\sqrt{2}$?

Your Answer	Score	Explanation
4	1.50	There are four such vectors: $12|0\rangle+3\sqrt{2}|1\rangle$, $12|0\rangle-3\sqrt{2}|1\rangle$, $-12|0\rangle+3\sqrt{2}|1\rangle$, $-12|0\rangle-3\sqrt{2}|1\rangle$.
Total	1.50 / 1.50	

Question Explanation

There are four such vectors: $12|0\rangle+3\sqrt{2}|1\rangle$, $12|0\rangle-3\sqrt{2}|1\rangle$, $-12|0\rangle+3\sqrt{2}|1\rangle$, $-12|0\rangle-3\sqrt{2}|1\rangle$.

Question 9

In $C2$, how many complex (unit) vectors are there whose projection onto $|1\rangle$ has length $3\sqrt{2}$?

Your Answer	Score	Explanation
	1.50	There are infinitely many such vectors. For

Infinite		instance, $\frac{1}{2}|0\rangle+\frac{\sqrt{3}}{2}e^{\theta i}|1\rangle$ for any θ works.
Total	1.50 / 1.50	

Question Explanation

There are infinitely many such vectors. For instance, $\frac{1}{2}|0\rangle+\frac{\sqrt{3}}{2}e^{\theta i}|1\rangle$ for any θ works.

Question 10

In the double-slit experiment, consider the point at the middle of the final (detector) screen which is equidistant from the two slits. Suppose the intensity at that point is 1 when either slit is open. Now for each of the three cases (a) bullet (b) wave (c) quantum mechanics (photons or electrons) calculate the intensity at the same point when both slits are open. If the answers are 1, 2 and 3 respectively, fill in $1, 2, 3$ in the space below.

Answer for Question 10

Your Answer		Score	Explanation
2,4,4	✔	10.00	We have seen in Lecture 1 that the intensity simply adds up with bullets. In contrast, with waves or quantum particles, the intensity is defined as the square of the amplitude and it is the amplitudes that add/subtract. At the middle point, we have a completely constructive interference because it is equidistant from the two slits. Thus, Bullet : $1+1=2$ Wave : $(\sqrt{1}+\sqrt{1})^2=4$ Quantum particle : $(\sqrt{1}+\sqrt{1})^2=4$ Thus, the answer is $2, 4, 4$.
Total		10.00 / 10.00	

Question Explanation

We have seen in Lecture 1 that the intensity simply adds up with bullets. In contrast, with waves or quantum particles, the intensity is defined as the square of the amplitude and it is the amplitudes that add/subtract. At the middle point, we have a completely constructive interference because it is equidistant from the two slits. Thus,

Bullet : $1+1=2$
Wave : $(\sqrt{1}+\sqrt{1})^2=4$
Quantum particle : $(\sqrt{1}+\sqrt{1})^2=4$

Thus, the answer is $2, 4, 4$.

Question 11

Let $|\phi\rangle=\frac{1}{2}|0\rangle+\frac{1+\sqrt{2}i}{2}|1\rangle$ be the state of a qubit. What is the inner product of $|\phi\rangle$ and $|+\rangle$?

Your Answer		Score	Explanation
◉ $\frac{2-\sqrt{2}i}{2\sqrt{2}}$	✔	10.00	Recall that $\vert+\rangle=\frac{1}{\sqrt{2}}\vert 0\rangle+\frac{1}{\sqrt{2}}\vert 1\rangle$. In the mathematics convention, $(\vert\phi\rangle,\vert+\rangle)=\frac{1}{2}\cdot\frac{1}{\sqrt{2}}+\frac{1+\sqrt{2}i}{2}\cdot\frac{1}{\sqrt{2}}=\frac{2+\sqrt{2}i}{2\sqrt{2}}$. In the physics convention, $\langle\phi\vert+\rangle=\frac{1}{2}\cdot\frac{1}{\sqrt{2}}+\frac{1-\sqrt{2}i}{2}\cdot\frac{1}{\sqrt{2}}=\frac{2-\sqrt{2}i}{2\sqrt{2}}$.
Total		10.00 / 10.00	

Question Explanation

Recall that $|+\rangle=\frac{1}{\sqrt{2}}|0\rangle+\frac{1}{\sqrt{2}}|1\rangle$.

In the mathematics convention, $(|\phi\rangle,|+\rangle)=\frac{1}{2}\cdot\frac{1}{\sqrt{2}}+\frac{1+\sqrt{2}i}{2}\cdot\frac{1}{\sqrt{2}}=\frac{2+\sqrt{2}i}{2\sqrt{2}}$.
In the physics convention, $\langle\phi|+\rangle=\frac{1}{2}\cdot\frac{1}{\sqrt{2}}+\frac{1-\sqrt{2}i}{2}\cdot\frac{1}{\sqrt{2}}=\frac{2-\sqrt{2}i}{2\sqrt{2}}$.

Question 12

Let $|\phi\rangle=\frac{1}{2}|0\rangle+\frac{1+\sqrt{2}i}{2}|1\rangle$. Write $|\phi\rangle$ in the form $\alpha_0|+\rangle+\alpha_1|-\rangle$. What is α_0?

Your Answer		Score	Explanation
◉ $\frac{2+\sqrt{2}i}{2\sqrt{2}}$	✔	10.00	Use the relations $\vert 0\rangle=\frac{1}{\sqrt{2}}\vert+\rangle+\frac{1}{\sqrt{2}}\vert-\rangle$ and $\vert 1\rangle=\frac{1}{\sqrt{2}}\vert+\rangle-\frac{1}{\sqrt{2}}\vert-\rangle$. $\vert\phi\rangle=\frac{1}{2\sqrt{2}}\vert+\rangle+\frac{1}{2\sqrt{2}}\vert-\rangle+\frac{1+\sqrt{2}i}{2\sqrt{2}}\vert+\rangle-\frac{1+\sqrt{2}i}{2\sqrt{2}}\vert-\rangle=\frac{2+\sqrt{2}i}{2\sqrt{2}}\vert+\rangle-\frac{\sqrt{2}i}{2\sqrt{2}}\vert-\rangle$.

Total	10.00 / 10.00

Question Explanation

Use the relations $|0\rangle=\frac{1}{\sqrt{2}}|+\rangle+\frac{1}{\sqrt{2}}|-\rangle$ and $|1\rangle=\frac{1}{\sqrt{2}}|+\rangle-\frac{1}{\sqrt{2}}|-\rangle$.

$|\phi\rangle=\frac{1}{2\sqrt{2}}|+\rangle+\frac{1}{2\sqrt{2}}|-\rangle+\frac{1+\sqrt{2}i}{2\sqrt{2}}|+\rangle-\frac{1+\sqrt{2}i}{2\sqrt{2}}|-\rangle=\frac{2+\sqrt{2}i}{2\sqrt{2}}|+\rangle-\frac{\sqrt{2}i}{2\sqrt{2}}|-\rangle$.

Question 13

In Lecture 2, we have seen that $\frac{1}{\sqrt{2}}|0\rangle+\frac{1}{\sqrt{2}}|1\rangle$ can be distinguished from $\frac{1}{\sqrt{2}}|0\rangle-\frac{1}{\sqrt{2}}|1\rangle$ although they only differ by a phase in the $|1\rangle$ component. Note that this is a relative phase because they differ in the phase of $|1\rangle$ relative to that of $|0\rangle$. In contrast, $|0\rangle$ and $-|0\rangle$ differ by a ``global phase.'' Is it possible to detect the global phase via measurement? Suppose we perform a measurement in some arbitrary basis $|u\rangle,|u_\perp\rangle$. Clearly, we can write $|0\rangle=a|u\rangle+b|u_\perp\rangle$ for some complex numbers a and b.

If the qubit is initially in state $|0\rangle$, what is the probability that the outcome is u?

Your Answer		Score	Explanation
$|a|^2$	✔	5.00	The probability is the square of the magnitude of the inner product between $|0\rangle$ and $|u\rangle$, which is $|a|^2$.
Total		5.00 / 5.00	

Question Explanation

The probability is the square of the magnitude of the inner product between $|0\rangle$ and $|u\rangle$, which is $|a|^2$.

Question 14

In the above problem, if the qubit is initially in state $-|0\rangle$, what is the probability that the outcome is u?

Your	Score	Explanation

Answer			
◉ $\lvert a\rvert^2$	✔	5.00	The probability is the square of the magnitude of the inner product between $-\lvert 0\rangle$ and $\lvert u\rangle$, which is $\lvert a\rvert^2$. This tells us that there is no measurement that can distinguish $\lvert 0\rangle$ from $-\lvert 0\rangle$. In general, global phases cannot be detected by any physical means, and thus they are irrelevant. That is, $\lvert\phi\rangle$ and $e^{i\theta}\lvert\phi\rangle$ are considered to be identical!
Total		5.00 / 5.00	

Question Explanation

The probability is the square of the magnitude of the inner product between $-\lvert 0\rangle$ and $\lvert u\rangle$, which is $\lvert a\rvert^2$. This tells us that there is no measurement that can distinguish $\lvert 0\rangle$ from $-\lvert 0\rangle$. In general, global phases cannot be detected by any physical means, and thus they are irrelevant. That is, $\lvert\phi\rangle$ and $e^{i\theta}\lvert\phi\rangle$are considered to be identical!

Question 15

A qubit is either in the state $\lvert 0\rangle$ or $\lvert\phi\rangle=\cos\theta\lvert 0\rangle+\sin\theta\lvert 1\rangle$. Which of the following measurements best distinguishes between these two possibilities?

Your Answer		Score	Explanation
◉ $\cos(\frac{\theta}{2}-\frac{\pi}{4})\lvert 0\rangle+\sin(\frac{\theta}{2}-\frac{\pi}{4})\lvert 1\rangle,\sin(\frac{\theta}{2}-\frac{\pi}{4})\lvert 0\rangle-\cos(\frac{\theta}{2}-\frac{\pi}{4})\lvert 1\rangle$	✔	10.00	Suppose we measure in the $\lvert u\rangle,\lvert u^\perp\rangle$ basis where $\lvert u\rangle=\cos x\lvert 0\rangle+\sin x\lvert 1\rangle$. We will report $\lvert 0\rangle$ if the measurement outcome is u, and we will report $\lvert\phi\rangle$ otherwise. Note that the probability that we report $\lvert 0\rangle$ when the qubit was indeed in $\lvert\phi\rangle$ is given by $\cos^2(\theta-x)$. The probability that we report $\lvert\phi\rangle$ when the qubit was in $\lvert 0\rangle$ is $\cos^2(\frac{\pi}{2}+x)$. Thus, the probability of error of this procedure is $\max\{\cos^2(\theta-x), \cos^2(\frac{\pi}{2}$

		$+x)\}$. This is minimized when $\theta - x = \frac{\pi}{2} + x$, which is equivalent to $x = \frac{\theta}{2} - \frac{\pi}{4}$.
Total	10.00 / 10.00	

Question Explanation

Suppose we measure in the $|u\rangle, |u^\perp\rangle$ basis where $|u\rangle = \cos x|0\rangle + \sin x|1\rangle$. We will report $|0\rangle$ if the measurement outcome is u, and we will report $|\phi\rangle$ otherwise. Note that the probability that we report $|0\rangle$ when the qubit was indeed in $|\phi\rangle$ is given by $\cos^2(\theta - x)$. The probability that we report $|\phi\rangle$when the qubit was in $|0\rangle$ is $\cos^2(\frac{\pi}{2} + x)$. Thus, the probability of error of this procedure is $\max\{\cos^2(\theta - x), \cos^2(\frac{\pi}{2} + x)\}$. This is minimized when $\theta - x = \frac{\pi}{2} + x$, which is equivalent to $x = \frac{\theta}{2} - \frac{\pi}{4}$.

Question 1

The standard basis for a two-qubit quantum system is $|00\rangle, |01\rangle, |10\rangle, |11\rangle$.

Your Answer		Score	Explanation
True	✔	1.00	
Total		1.00 / 1.00	

Question 2

Any state of a two-qubit quantum system can be written in the form $(a|0\rangle + b|1\rangle) \otimes (c|0\rangle + d|1\rangle)$. $(a, b, c, d \in \mathbb{C})$

Your Answer		Score	Explanation
False	✔	1.00	Entangled states cannot be written in this form.
Total		1.00 / 1.00	

Question Explanation

Entangled states cannot be written in this form.

Question 3

Two qubits are entangled if their state can be written as $(a|0\rangle+b|1\rangle)\otimes(a|0\rangle+b|1\rangle)$. $(a,b\in\mathbb{C})$

Your Answer		Score	Explanation
◉ False	✔	1.00	Two qubits are entangled if they cannot be written as $(a|0\rangle+b|1\rangle)\otimes(c|0\rangle+d|1\rangle)$ for any $a,b,c,d\in\mathbb{C}$ that satisfy the normalization condition.
Total		1.00 / 1.00	

Question Explanation

Two qubits are entangled if they cannot be written as $(a|0\rangle+b|1\rangle)\otimes(c|0\rangle+d|1\rangle)$ for any $a,b,c,d\in\mathbb{C}$ that satisfy the normalization condition.

Question 4

$a|00\rangle+b|11\rangle=a|++\rangle+b|--\rangle$ for any $a,b\in\mathbb{C}$ that satisfy the normalization condition.

Your Answer		Score	Explanation
◉ False	✔	1.00	Let $a=1$ and $b=0$. It is clear that $|00\rangle\neq|++\rangle$.
Total		1.00 / 1.00	

Question Explanation

Let $a=1$ and $b=0$. It is clear that $|00\rangle\neq|++\rangle$.

Question 5

Bell's theorem shows that no local hidden variable theory can be correct.

Your Answer		Score	Explanation
◉ False	✔	1.00	Please see the announcement regarding this question.
Total		1.00 / 1.00	

Question 6

Pick one of the following four alternatives which is **not** an orthogonal basis for a two-qubit system, or options 5 or 6 if none or all are valid.

Your Answer	**Score**	**Explanation**
12√|00⟩+12√|11⟩, 12√|01⟩+12√|10⟩, 12√|++⟩+12√|−−⟩, 12√|+−⟩+12√|−+⟩	5.00	By the rotational invariance of the Bell state, we know that 12√|00⟩+12√|11⟩=12√|++⟩+12√|−−⟩, so clearly they cannot be orthogonal.
Total	5.00 / 5.00	

Question Explanation

By the rotational invariance of the Bell state, we know that 12√|00⟩+12√|11⟩=12√|++⟩+12√|−−⟩, so clearly they cannot be orthogonal.

Question 7

A vertically polarized photon goes through two polarizing filters, the first of which is vertically aligned and the second at 45 degrees. What is the probability that the photon is transmitted through both filters?

Your Answer	**Score**	**Explanation**
12	5.00	A vertically polarized photon will get transmitted through the first filter with probability 1, at which point it is still vertically aligned. It will get transmitted through a 45-degree filter with probability 12.
Total	5.00 / 5.00	

Question Explanation

A vertically polarized photon will get transmitted through the first filter with probability 1, at which point it is still vertically aligned. It will get transmitted through a 45-degree filter with probability 12.

Question 8

Now, you are allowed to place a polarizing filter between the two filters in the previous question. If you wish to maximize the probability that the photon is transmitted (though all three filters), what angle would you orient the additional filter? Here, assume that a 0° filter corresponds to a horizontal filter and 90° a vertical filter. Write your answer in degrees to the nearest tenth. (ex: 45.0)

Answer for Question 8

Your Answer		Score	Explanation
67.5	✔	5.00	Suppose we orient the additional filter at angle θ. Then, the probability that the photon goes through all three filters is $\Pr[\text{it goes through 1st filter}]\cdot\Pr[\text{it goes through 2nd filter}]\cdot\Pr[\text{it goes through 3rd filter}]=1\cdot\cos^2(90^\circ-\theta)\cdot\cos^2(\theta-45^\circ)$. This quantity is maximized when $\theta=67.5^\circ$.
Total		5.00 / 5.00	

Question Explanation

Suppose we orient the additional filter at angle θ. Then, the probability that the photon goes through all three filters is $\Pr[\text{it goes through 1st filter}]\cdot\Pr[\text{it goes through 2nd filter}]\cdot\Pr[\text{it goes through 3rd filter}]=1\cdot\cos^2(90^\circ-\theta)\cdot\cos^2(\theta-45^\circ)$. This quantity is maximized when $\theta=67.5^\circ$.

Question 9

In that case, what is the probability that the photon is transmitted through all three? Round your answer to the nearest thousandth. (ex: 0.182)

Answer for Question 9

Your Answer		Score	Explanation
0.729	✔	5.00	$1\cdot\cos^2(90^\circ-67.5^\circ)\cdot\cos^2(67.5^\circ-45^\circ)=0.72855\cdots$
Total		5.00 / 5.00	

Question Explanation

$1\cdot\cos^2(90^\circ-67.5^\circ)\cdot\cos^2(67.5^\circ-45^\circ)=0.72855\cdots$

Question 10

A two-qubit system was originally in the state $\frac{3}{4}|00\rangle-\frac{\sqrt{5}}{4}|01\rangle+\frac{1}{4}|10\rangle-\frac{1}{4}|11\rangle$, and then we measured the first qubit to be 0. Now, if we measure the second qubit in the standard basis, what is the probability that the outcome is 0? Write your answer as a fraction in simplest form. (ex: 2/3. Note that 4/6 will not be deemed correct if the answer was 2/3.)

Answer for Question 10

Your Answer		Score	Explanation
9/14	✔	10.00	If we measured the first qubit to be 0, the new state is now $(\frac{3}{4}|00\rangle-\frac{\sqrt{5}}{4}|01\rangle)/\sqrt{|\frac{3}{4}|^2+|-\frac{\sqrt{5}}{4}|^2}=\frac{3}{\sqrt{14}}|00\rangle-\sqrt{\frac{5}{14}}|01\rangle$. Thus, the probability that the second qubit is also 0 is $|\frac{3}{\sqrt{14}}|^2=\frac{9}{14}$.
Total		10.00 / 10.00	

Question Explanation

If we measured the first qubit to be 0, the new state is now $(\frac{3}{4}|00\rangle-\frac{\sqrt{5}}{4}|01\rangle)/\sqrt{|\frac{3}{4}|^2+|-\frac{\sqrt{5}}{4}|^2}=\frac{3}{\sqrt{14}}|00\rangle-\sqrt{\frac{5}{14}}|01\rangle$. Thus, the probability that the second qubit is also 0 is $|\frac{3}{\sqrt{14}}|^2=\frac{9}{14}$.

Question 11

(Extra credit) A two-qubit system was originally in the state 2√+6√6|00⟩+2+6√6|01⟩+2√−6√6|10⟩+2−6√6|11⟩, and then we measured the first qubit to be +. Now, if we measure the second qubit in the standard basis, what is the probability that the outcome is 0? Write your answer as a fraction in simplest form. (ex: 2/3. Note that 4/6 will not be deemed correct if the answer was 2/3.)

Answer for Question 11

Your Answer		Score	Explanation
1/3	✔	10.00	2√+6√6|00⟩+2+6√6|01⟩+2√−6√6|10⟩+2−6√6|11⟩ =2√+6√6(12√|+⟩+12√|−⟩)|0⟩+2+6√6(12√|+⟩+12√|−⟩)|1⟩+2√−6√6(12√|+⟩−12√|−⟩)|0⟩+2−6√6(12√|+⟩−12√|−⟩)|1⟩ =13|+0⟩+2√3|+1⟩+13√|−0⟩+13√|−1⟩ If we measure the first qubit to be +, the new state is 13√|+0⟩+2√3√|+1⟩. Thus, the probability that the second qubit is 0 is 13.
Total		10.00 / 10.00	

Question Explanation

2√+6√6|00⟩+2+6√6|01⟩+2√−6√6|10⟩+2−6√6|11⟩
=2√+6√6(12√|+⟩+12√|−⟩)|0⟩+2+6√6(12√|+⟩+12√|−⟩)|1⟩+2√−6√6(12√|+⟩−12√|−⟩)|0⟩+2−6√6(12√|+⟩−12√|−⟩)|1⟩
=13|+0⟩+2√3|+1⟩+13√|−0⟩+13√|−1⟩
If we measure the first qubit to be +, the new state is 13√|+0⟩+2√3√|+1⟩.
Thus, the probability that the second qubit is 0 is 13.

Question 12

Let |ϕ⟩=12√|01⟩−12√|10⟩. Which of the following is **not** equal to |ϕ⟩?

Your Answer	Score	Explanation

All of the above are equal to $\lvert\phi\rangle$.	✔	10.00	Since $\lvert\phi\rangle=\frac{1}{\sqrt{2}}\lvert 01\rangle-\frac{1}{\sqrt{2}}\lvert 10\rangle$ is invariant with respect to any unitary rotation, all of the given four states are equal to $\lvert\phi\rangle$.
Total		10.00 / 10.00	

Question Explanation

Since $\lvert\phi\rangle=\frac{1}{\sqrt{2}}\lvert 01\rangle-\frac{1}{\sqrt{2}}\lvert 10\rangle$ is invariant with respect to any unitary rotation, all of the given four states are equal to $\lvert\phi\rangle$.

Question 13

If the first qubit is in the state $\frac{1}{\sqrt{3}}\lvert 0\rangle+\frac{\sqrt{2}}{\sqrt{3}}\lvert 1\rangle$ and the second qubit is in the state $\frac{\sqrt{2}}{\sqrt{3}}\lvert 0\rangle+\frac{1}{\sqrt{3}}\lvert 1\rangle$, what is the amplitude of $\lvert 00\rangle$ in the composite state of the two qubit system?

Your Answer		Score	Explanation
$\frac{\sqrt{2}}{3}$	✔	5.00	$(\frac{1}{\sqrt{3}}\lvert 0\rangle+\frac{\sqrt{2}}{\sqrt{3}}\lvert 1\rangle)(\frac{\sqrt{2}}{\sqrt{3}}\lvert 0\rangle+\frac{1}{\sqrt{3}}\lvert 1\rangle)=\frac{\sqrt{2}}{3}\lvert 00\rangle+\frac{1}{3}\lvert 01\rangle+\cdots$
Total		5.00 / 5.00	

Question Explanation

$(\frac{1}{\sqrt{3}}\lvert 0\rangle+\frac{\sqrt{2}}{\sqrt{3}}\lvert 1\rangle)(\frac{\sqrt{2}}{\sqrt{3}}\lvert 0\rangle+\frac{1}{\sqrt{3}}\lvert 1\rangle)=\frac{\sqrt{2}}{3}\lvert 00\rangle+\frac{1}{3}\lvert 01\rangle+\cdots$

Question 14

If the first qubit is in the state $\frac{1}{\sqrt{3}}\lvert 0\rangle+\frac{\sqrt{2}}{\sqrt{3}}\lvert 1\rangle$ and the second qubit is in the state $\frac{\sqrt{2}}{\sqrt{3}}\lvert 0\rangle+\frac{1}{\sqrt{3}}\lvert 1\rangle$, what is the amplitude of $\lvert 11\rangle$ in the composite state of the two qubit system?

Your Answer		Score	Explanation
$\frac{\sqrt{2}}{3}$	✔	5.00	$(\frac{1}{\sqrt{3}}\lvert 0\rangle+\frac{\sqrt{2}}{\sqrt{3}}\lvert 1\rangle)(\frac{\sqrt{2}}{\sqrt{3}}\lvert 0\rangle+\frac{1}{\sqrt{3}}\lvert 1\rangle)=\cdots+\frac{2}{3}\lvert 10\rangle+\frac{\sqrt{2}}{3}\lvert 11\rangle$
Total		5.00 / 5.00	

Question Explanation

$(\frac{1}{\sqrt{3}}\lvert 0\rangle+\frac{\sqrt{2}}{\sqrt{3}}\lvert 1\rangle)(\frac{\sqrt{2}}{\sqrt{3}}\lvert 0\rangle+\frac{1}{\sqrt{3}}\lvert 1\rangle)=\cdots+\frac{2}{3}\lvert 10\rangle+\frac{\sqrt{2}}{3}\lvert 11\rangle$

Question 15

Factor $\frac{1}{2\sqrt{2}}|00\rangle-\frac{1}{2\sqrt{2}}|01\rangle+\frac{\sqrt{3}}{2\sqrt{2}}|10\rangle-\frac{\sqrt{3}}{2\sqrt{2}}|11\rangle$ into $(a|0\rangle+b|1\rangle)\otimes(c|0\rangle+d|1\rangle)$, where $|a|^2+|b|^2=1$ and $|c|^2+|d|^2=1$. What is the value of $|a|$? Write your answer as a fraction in simplest form. (ex: 2/3. Note that 4/6 will not be deemed correct if the answer was 2/3.)

Answer for Question 15

Your Answer		Score	Explanation
1/2	✔	10.00	A factorization of the given state is $(\frac{1}{2}|0\rangle+\frac{\sqrt{3}}{2}|1\rangle)\otimes(\frac{1}{\sqrt{2}}|0\rangle-\frac{1}{\sqrt{2}}|1\rangle)$.
Total		10.00 / 10.00	

Question Explanation

A factorization of the given state is $(\frac{1}{2}|0\rangle+\frac{\sqrt{3}}{2}|1\rangle)\otimes(\frac{1}{\sqrt{2}}|0\rangle-\frac{1}{\sqrt{2}}|1\rangle)$.

Question 16

How do we write $|\psi\rangle=\frac{1}{\sqrt{2}}|0\rangle+\frac{e^{i\theta}}{\sqrt{2}}|1\rangle$ in the sign basis?

Your Answer		Score	Explanation
◉ $\frac{1+e^{i\theta}}{2}|+\rangle+\frac{1-e^{i\theta}}{2}|-\rangle$	✔	5.00	$|\psi\rangle=\frac{1}{\sqrt{2}}|0\rangle+\frac{e^{i\theta}}{\sqrt{2}}|1\rangle=\frac{1}{\sqrt{2}}(\frac{1}{\sqrt{2}}|+\rangle+\frac{1}{\sqrt{2}}|-\rangle)+\frac{e^{i\theta}}{\sqrt{2}}(\frac{1}{\sqrt{2}}|+\rangle-\frac{1}{\sqrt{2}}|-\rangle)=\frac{1+e^{i\theta}}{2}|+\rangle+\frac{1-e^{i\theta}}{2}|-\rangle$
Total		5.00 / 5.00	

Question Explanation

$|\psi\rangle=\frac{1}{\sqrt{2}}|0\rangle+\frac{e^{i\theta}}{\sqrt{2}}|1\rangle=\frac{1}{\sqrt{2}}(\frac{1}{\sqrt{2}}|+\rangle+\frac{1}{\sqrt{2}}|-\rangle)+\frac{e^{i\theta}}{\sqrt{2}}(\frac{1}{\sqrt{2}}|+\rangle-\frac{1}{\sqrt{2}}|-\rangle)=\frac{1+e^{i\theta}}{2}|+\rangle+\frac{1-e^{i\theta}}{2}|-\rangle$

Question 17

Consider the state $|\psi\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{e^{i\theta}}{\sqrt{2}}|1\rangle$ from the previous question. To estimate the phase θ, we measure $|\psi\rangle$ in the sign basis. What is the probability that the outcome of the measurement is +? Recall that $e^{i\theta} = \cos\theta + i\sin\theta$.

Your Answer		Score	Explanation
$\frac{1+\cos\theta}{2}$	✔	5.00	$|\psi\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{e^{i\theta}}{\sqrt{2}}|1\rangle = \frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}|+\rangle + \frac{1}{\sqrt{2}}|-\rangle\right) + \frac{e^{i\theta}}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}|+\rangle - \frac{1}{\sqrt{2}}|-\rangle\right) = \frac{1+e^{i\theta}}{2}|+\rangle + \frac{1-e^{i\theta}}{2}|-\rangle$ $\Pr[+] = \left\|\frac{1+e^{i\theta}}{2}\right\|^2 = \frac{1+e^{i\theta}}{2}\cdot\frac{1+e^{-i\theta}}{2} = \frac{1}{4}(1+e^{i\theta}+e^{-i\theta}+1) = \frac{1}{4}(2+\cos\theta+i\sin\theta+\cos(-\theta)+i\sin(-\theta)) = \frac{1+\cos\theta}{2}$
Total		5.00 / 5.00	

Question Explanation

$|\psi\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{e^{i\theta}}{\sqrt{2}}|1\rangle = \frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}|+\rangle + \frac{1}{\sqrt{2}}|-\rangle\right) + \frac{e^{i\theta}}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}|+\rangle - \frac{1}{\sqrt{2}}|-\rangle\right) = \frac{1+e^{i\theta}}{2}|+\rangle + \frac{1-e^{i\theta}}{2}|-\rangle$

$\Pr[+] = \left\|\frac{1+e^{i\theta}}{2}\right\|^2 = \frac{1+e^{i\theta}}{2}\cdot\frac{1+e^{-i\theta}}{2} = \frac{1}{4}(1+e^{i\theta}+e^{-i\theta}+1) = \frac{1}{4}(2+\cos\theta+i\sin\theta+\cos(-\theta)+i\sin(-\theta)) = \frac{1+\cos\theta}{2}$

Question 1

(5 points) Let $U = \begin{pmatrix} 1 & 3i \\ 3 & -i \end{pmatrix}$. What is $U^\dagger$?

Your Answer		Score	Explanation
$\begin{pmatrix} 1 & 3 \\ -3i & i \end{pmatrix}$	✔	5.00	
Total		5.00 / 5.00	

Question 2

(5 points) Which of the following unitary matrices is **not** self-inverse? (An invertible matrix U is self-inverse if $U^{-1} = U$.)

Your Answer		Score	Explanation
$R_\theta = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$ where $\theta = \frac{\pi}{4}$	✔	5.00	A matrix A is self-inverse if and only if $AA = I$. Note that $R_{\frac{\pi}{4}} \cdot R_{\frac{\pi}{4}} = R_{\frac{\pi}{2}}$, which

	is clearly not equal to I.
Total	5.00 / 5.00

Question Explanation

A matrix A is self-inverse if and only if $AA=I$. Note that $R_{\frac{\pi}{4}} \cdot R_{\frac{\pi}{4}} = R_{\frac{\pi}{2}}$, which is clearly not equal to I.

Question 3

(5 points) What is ZX applied to $|0\rangle$?

Your Answer		Score	Explanation
$-|1\rangle$	✔	5.00	$ZX|0\rangle=Z|1\rangle=-|1\rangle$
Total		5.00 / 5.00	

Question Explanation

$ZX|0\rangle=Z|1\rangle=-|1\rangle$

Question 4

(5 points) What is ZX applied to $H|0\rangle$?

Your Answer		Score	Explanation
$|-\rangle$	✔	5.00	$ZXH|0\rangle=ZX|+\rangle=ZX(\frac{1}{\sqrt{2}}|0\rangle+\frac{1}{\sqrt{2}}|1\rangle)=Z(\frac{1}{\sqrt{2}}|0\rangle+\frac{1}{\sqrt{2}}|1\rangle)=Z|+\rangle=|-\rangle$
Total		5.00 / 5.00	

Question Explanation

$ZXH|0\rangle=ZX|+\rangle=ZX(\frac{1}{\sqrt{2}}|0\rangle+\frac{1}{\sqrt{2}}|1\rangle)=Z(\frac{1}{\sqrt{2}}|0\rangle+\frac{1}{\sqrt{2}}|1\rangle)=Z|+\rangle=|-\rangle$

Question 5

In Questions 5-7, we seek to understand measurements in arbitrary bases in terms of unitary transformations. Suppose we have a qubit $|\psi\rangle=\alpha|+\rangle+\beta|-\rangle$.

(1 point) If we measure the qubit in the sign basis, what is the probability that the outcome is $-$?

Your Answer		Score	Explanation
$\lvert\beta\rvert^2$	✔	1.00	
Total		1.00 / 1.00	

Question 6

(5 points) Instead of measuring directly in the sign basis, what if we apply a Hadamard gate to the qubit and then measure it in the standard basis? What is the probability that the outcome is 1?

Your Answer		Score	Explanation
$\lvert\beta\rvert^2$	✔	5.00	$H(\alpha\lvert+\rangle+\beta\lvert-\rangle)=\alpha\lvert 0\rangle+\beta\lvert 1\rangle$. The probability of measuring 1 is $\lvert\beta\rvert^2$.
Total		5.00 / 5.00	

Question Explanation

$H(\alpha\lvert+\rangle+\beta\lvert-\rangle)=\alpha\lvert 0\rangle+\beta\lvert 1\rangle$. The probability of measuring 1 is $\lvert\beta\rvert^2$.

Question 7

(5 points) We want to measure a qubit in the sign basis, but we only know how to perform standard basis measurements. We want to implement a sign basis measurement by applying some unitary rotation to the qubit and then measuring it in the standard basis. Which unitary do we need to apply? (We will regard the outcome 0 as $+$ and 1 as $-$.)

Your Answer		Score	Explanation
H	✔	5.00	$H\lvert\psi\rangle=H(\alpha\lvert+\rangle+\beta\lvert-\rangle)=\alpha\lvert 0\rangle+\beta\lvert 1\rangle$. Thus, the sign basis measurement

		of $\|\psi\rangle$ and the standard basis measurement of $H\|\psi\rangle$ have the same measurement probabilities.
Total	5.00 / 5.00	

Question Explanation

$H|\psi\rangle = H(\alpha|+\rangle + \beta|-\rangle) = \alpha|0\rangle + \beta|1\rangle$. Thus, the sign basis measurement of $|\psi\rangle$ and the standard basis measurement of $H|\psi\rangle$ have the same measurement probabilities.

Question 8

(5 points) Which of the following pairs of quantum gates commute? Select all that apply. (Matrices A and B are said to commute if $AB=BA$.)

Your Answer		Score	Explanation
☑ I and X	✔	1.00	Clearly, I commutes with any other matrix.
☑ $CNOT$ and X applied to the target qubit	✔	1.00	Note that $CNOT$ maps $\|00\rangle$ to $\|00\rangle$, $\|01\rangle$ to $\|01\rangle$, $\|10\rangle$ to $\|11\rangle$, and $\|11\rangle$ to $\|10\rangle$. If we then apply X on the target qubit, the net effect will be that $\|00\rangle$ gets mapped to $\|01\rangle$, $\|01\rangle$ to $\|00\rangle$, $\|10\rangle$ to $\|10\rangle$, and $\|11\rangle$ to $\|11\rangle$. Check that you get the same mapping if you apply X first and then the $CNOT$.
☐ X and Z	✔	1.00	
☐ H and X	✔	1.00	
☑ Real rotation by 30° and real rotation by 45°	✔	1.00	Rotations with common rotation axes commute. Observe that regardless of the order you perform these rotations, the net effect is real rotation by 75°.
Total		5.00 / 5.00	

Question Explanation

Clearly, I commutes with any other matrix.

To see that $CNOT$ and X on the target qubit commute, note that $CNOT$ maps $|00\rangle$ to $|00\rangle$, $|01\rangle$ to $|10\rangle$, $|10\rangle$ to $|11\rangle$, and $|11\rangle$ to $|10\rangle$. If we then apply X on the target qubit, the net effect will be that $|00\rangle$ gets mapped to $|01\rangle$, $|01\rangle$ to $|00\rangle$, $|10\rangle$ to $|10\rangle$, and $|11\rangle$ to $|11\rangle$. Check that you get the same mapping if you apply X first and then the $CNOT$.

Rotations with common rotation axes commute. Observe that regardless of the order you perform these rotations, the net effect is real rotation by 75°.

Question 9

(7 points) Suppose we have two qubits in the state $(a|0\rangle+b|1\rangle)|0\rangle$. We first apply a CNOT gate with the first qubit as the control and the second qubit as the target, and then we apply another CNOT with the first qubit as the target and the second qubit as the control. What is the resulting state?

Your Answer	Score	Explanation
◉ $|0\rangle(a|0\rangle+b|1\rangle)$ ✔	7.00	Initial state: $a|00\rangle+b|10\rangle$ After the first CNOT: $a|00\rangle+b|11\rangle$ After the second CNOT: $a|00\rangle+b|01\rangle$
Total	7.00 / 7.00	

Question Explanation

Initial state: $a|00\rangle+b|10\rangle$
After the first CNOT: $a|00\rangle+b|11\rangle$
After the second CNOT: $a|00\rangle+b|01\rangle$

Question 10

(7 points) Suppose we have a two-qubit system where the first qubit is in the state $|\psi\rangle$ which we know to be either $|+\rangle$ or $|-\rangle$ and the second qubit is $|0\rangle$. We wish to make a copy of the first qubit, i.e., we want to end up in the state $|\psi\rangle|\psi\rangle$. Which of the following circuits achieves it?

Your Answer	Score	Explanation
◉ Apply H on the first qubit, then ✔	7.00	First,

apply $CNOT$ from the first qubit (control) to the second qubit (target), and finally apply H on both qubits.		suspose $\|\psi\rangle$ was $\|+\rangle$. Initial state: $\|+0\rangle$ After the first H: $\|00\rangle$ After $CNOT$: $\|00\rangle$ After H on both qubits: $\|++\rangle$, as desired. Now, suppose $\|\psi\rangle$ was $\|-\rangle$. Initial state: $\|-0\rangle$ After the first H: $\|10\rangle$ After $CNOT$: $\|11\rangle$ After H on both qubits: $\|--\rangle$, as desired.
Total	7.00 / 7.00	

Question Explanation

First, suppose $|\psi\rangle$ was $|+\rangle$.
Initial state: $|+0\rangle$
After the first H: $|00\rangle$
After $CNOT$: $|00\rangle$
After H on both qubits: $|++\rangle$, as desired.

Now, suppose $|\psi\rangle$ was $|-\rangle$.
Initial state: $|-0\rangle$
After the first H: $|10\rangle$
After $CNOT$: $|11\rangle$
After H on both qubits: $|--\rangle$, as desired.

Question 11

(10 points) Alice and Bob share a state $a|++\rangle+b|--\rangle$, where the first qubit is Alice's and the second qubit is Bob's. Alice measures her qubit in the standard basis and sends the measurement outcome to Bob. What does Bob have to do to make his qubit in $a|0\rangle+b|1\rangle$?

Your Answer		Score	Explanation
◉ If Alice	✔	10.00	Note that $a|++\rangle+b|--\rangle=\frac{1}{\sqrt{2}}a(|0\rangle+|1\rangle)|+\rangle+\frac{1}{\sqrt{2}}b(|0\rangle-|1\rangle)|-\rangle$

sends him 0, apply H on his qubit. If Alice sends him 1, apply ZH on his qubit.		$=\frac{1}{\sqrt{2}}|0\rangle(a|+\rangle+b|-\rangle)+\frac{1}{\sqrt{2}}|1\rangle(a|+\rangle-b|-\rangle)$. If Alice measures 0, Bob's qubit will be in $a|+\rangle+b|-\rangle$. Thus, if he applies H, he ends up with $a|0\rangle+b|1\rangle$, as desired. On the other hand, if Alice measures 1, Bob's qubit will be in $a|+\rangle-b|-\rangle$. If he applies H, the qubit is in $a|0\rangle-b|1\rangle$. To correct the phase, he also has to apply Z.
Total	10.00 / 10.00	

Question Explanation

Note that $a|++\rangle+b|--\rangle=\frac{1}{\sqrt{2}}a(|0\rangle+|1\rangle)|+\rangle+\frac{1}{\sqrt{2}}b(|0\rangle-|1\rangle)|-\rangle$
$=\frac{1}{\sqrt{2}}|0\rangle(a|+\rangle+b|-\rangle)+\frac{1}{\sqrt{2}}|1\rangle(a|+\rangle-b|-\rangle)$.
If Alice measures 0, Bob's qubit will be in $a|+\rangle+b|-\rangle$. Thus, if he applies H, he ends up with $a|0\rangle+b|1\rangle$, as desired. On the other hand, if Alice measures 1, Bob's qubit will be in $a|+\rangle-b|-\rangle$. If he applies H, the qubit is in $a|0\rangle-b|1\rangle$. To correct the phase, he also has to apply Z.

Question 1

(1 point) A matrix U is unitary if and only if $U=U^\dagger$.

Your Answer		Score	Explanation
False	✔	1.00	U is unitary if and only if $UU^\dagger=U^\dagger U=I$.
Total		1.00 / 1.00	

Question Explanation

U is unitary if and only if $UU^\dagger=U^\dagger U=I$.

Question 2

(1 point) If matrices U and V are both unitary, so is UV.

Your Answer		Score	Explanation
	✔	1.00	If $UU^\dagger=U^\dagger U=I$ and $VV^\dagger=V^\dagger V=I$,

True		then $(UV)(UV)^\dagger=UVV^\dagger U^\dagger=UIU^\dagger=UU^\dagger=I$ and $(UV)^\dagger(UV)=V^\dagger U^\dagger UV=V^\dagger IV=V^\dagger V=I$.
Total	1.00 / 1.00	

Question Explanation

If $UU^\dagger=U^\dagger U=I$ and $VV^\dagger=V^\dagger V=I$,
then $(UV)(UV)^\dagger=UVV^\dagger U^\dagger=UIU^\dagger=UU^\dagger=I$ and $(UV)^\dagger(UV)=V^\dagger U^\dagger UV=V^\dagger IV=V^\dagger V=I$.

Question 3

(1 point) If we measure a qubit $|\psi\rangle$ in the standard basis, the probability that the outcome is 0 is $|\langle 0|\psi\rangle|^2$.

Your Answer		Score	Explanation
True	✓	1.00	
Total		1.00 / 1.00	

Question 4

(1 point) If we measure a qubit $|\psi\rangle$ in the sign basis, the probability that the outcome is + is $|\langle +|\psi\rangle|^2$.

Your Answer		Score	Explanation
True	✓	1.00	
Total		1.00 / 1.00	

Question 5

(1 point) Unitary rotations preserve inner products, i.e., the inner product of $|\phi\rangle$ and $|\psi\rangle$ is equal to the inner product of $U|\phi\rangle$ and $U|\psi\rangle$.

Your Answer		Score	Explanation
True	✓	1.00	The inner product of $U|\phi\rangle$ and $U|\psi\rangle$ is $\langle\phi|U^\dagger U|\psi\rangle=\langle\phi|\psi\rangle$.

Total	1.00 / 1.00

Question Explanation

The inner product of $U|\phi\rangle$ and $U|\psi\rangle$ is $\langle\phi|U^\dagger U|\psi\rangle = \langle\phi|\psi\rangle$.

Question 6

(5 points) Suppose we have a qubit in the state $|0\rangle$. We apply a unitary U on it and then measure it in the standard basis. What is the probability of seeing a 1?

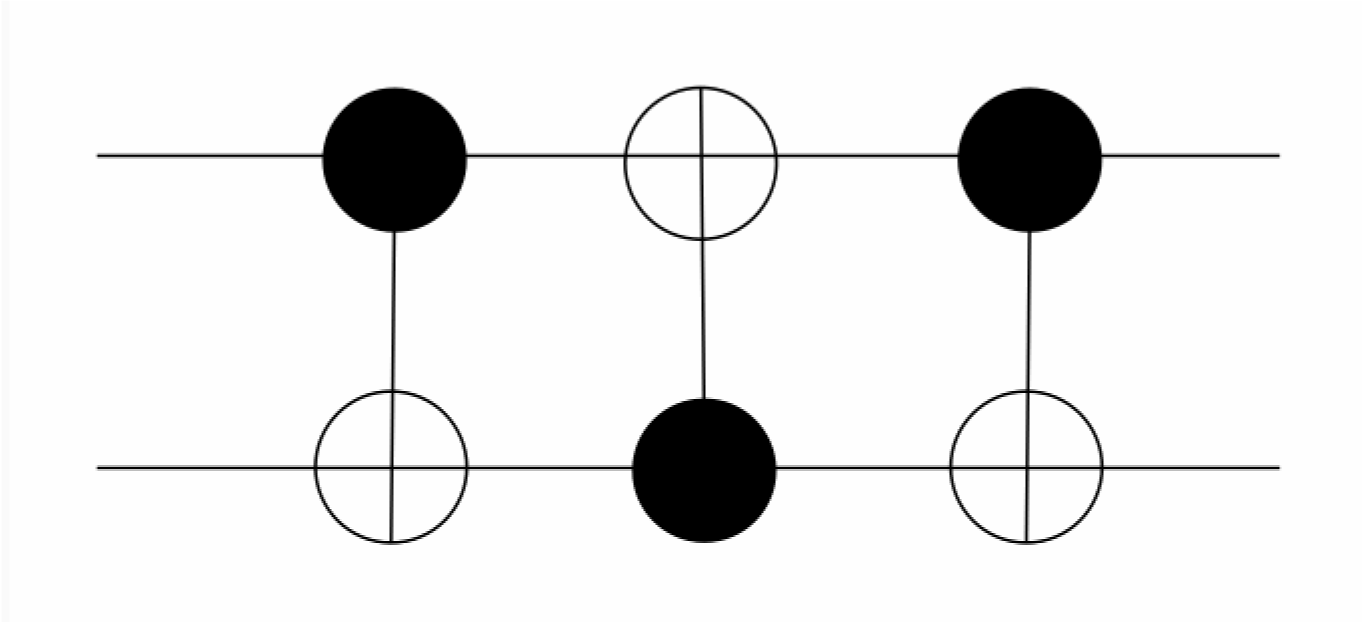
Your Answer		Score	Explanation
◉ $\|\langle 1\|U\|0\rangle\|^2$	✔	5.00	The state of the system after we apply U is $U\|0\rangle$. The probability that the outcome of the measurement is 1 is the square of the inner product of $\|1\rangle$ and $U\|0\rangle$, which is $\|\langle 1\|U\|0\rangle\|^2$.
Total		5.00 / 5.00	

Question Explanation

The state of the system after we apply U is $U|0\rangle$. The probability that the outcome of the measurement is 1 is the square of the inner product of $|1\rangle$ and $U|0\rangle$, which is $|\langle 1|U|0\rangle|^2$.

Question 7

(10 points) Which of the following matrices represents the quantum circuit shown below?



Your Answer	Score	Explanation
$\begin{pmatrix}1&0&0&0\\0&0&1&0\\0&1&0&0\\0&0&0&1\end{pmatrix}$	10.00	We know that the matrix for CNOT is $\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&0&1\\0&0&1&0\end{pmatrix}$ and the target qubit are swapped, is represented by the fo Then, $\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&0&1\\0&0&1&0\end{pmatrix}$ … Looking at the final matrix for this, it becomes clear what
Total	10.00 / 10.00	

Question Explanation

We know that the matrix for CNOT is $\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&0&1\\0&0&1&0\end{pmatrix}$. Convince yourself that the second CNOT in the circuit diagram, for which the control qubit and the target qubit are swapped, is represented by the following matrix: $\begin{pmatrix}1&0&0&0\\0&0&0&1\\0&0&1&0\\0&1&0&0\end{pmatrix}$.

Then, $\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&0&1\\0&0&1&0\end{pmatrix}\begin{pmatrix}1&0&0&0\\0&0&0&1\\0&0&1&0\\0&1&0&0\end{pmatrix}\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&0&1\\0&0&1&0\end{pmatrix}=\begin{pmatrix}1&0&0&0\\0&0&1&0\\0&1&0&0\\0&0&0&1\end{pmatrix}$.

Looking at the final matrix for this, it becomes clear what this circuit does. It swaps the positions of qubits, and is for this reason known as the SWAP gate.

Question 8

(5 points) Which of the following are equivalent ways of writing $\alpha|00\rangle+\beta|01\rangle$? Select all that apply.

Your Answer		Score
☑ $|0\rangle\otimes\alpha|0\rangle+|0\rangle\otimes\beta|1\rangle$	✔	1.00
☑ $|0\rangle\otimes(\alpha|0\rangle+\beta|1\rangle)$	✔	1.00
☐ $(\alpha+\beta)\otimes(|00\rangle+|01\rangle)$	✔	1.00
☐ $\alpha|0\rangle\otimes\alpha|0\rangle+\beta|0\rangle\otimes\beta|1\rangle$	✔	1.00
☑ $\alpha|0\rangle\otimes|0\rangle+\beta|0\rangle\otimes|1\rangle$	✔	1.00
Total		5.00 / 5.00

Question Explanation

$\alpha|0\rangle\otimes|0\rangle+\beta|0\rangle\otimes|1\rangle=\alpha|00\rangle+\beta|01\rangle$.
$|0\rangle\otimes\alpha|0\rangle+|0\rangle\otimes\beta|1\rangle=\alpha|00\rangle+\beta|01\rangle$.
$\alpha|0\rangle\otimes\alpha|0\rangle+\beta|0\rangle\otimes\beta|1\rangle=\alpha^2|00\rangle+\beta^2|01\rangle$.
$|0\rangle\otimes(\alpha|0\rangle+\beta|1\rangle)=\alpha|00\rangle+\beta|01\rangle$.
$(\alpha+\beta)\otimes(|00\rangle+|01\rangle)$ is an invalid expression because $\alpha+\beta$ is a scalar and $|00\rangle+|01\rangle$ is a vector.

Question 9

(10 points) Which is the following matrices is equal to $Z\otimes X$?

Your Answer		Score	Explanation
◉ $\begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \end{pmatrix}$	✔	10.00	$Z\otimes X=\begin{pmatrix} 1\cdot X & 0\cdot X \\ 0\cdot X & -1\cdot X \end{pmatrix}=$
Total		10.00 / 10.00	

Question Explanation

$Z\otimes X=\begin{pmatrix} 1\cdot X & 0\cdot X \\ 0\cdot X & -1\cdot X \end{pmatrix}=\begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \end{pmatrix}$.

Question 10

(5 points) Which of the following observables correspond to a standard basis measurement?

Your Answer		Score	Explanation
Z	✔	5.00	The eigenvectors of Z are $\|0\rangle$ and $\|1\rangle$ with distinct eigenvalues, so Z correspo
Total		5.00 / 5.00	

Question Explanation

The eigenvectors of Z are $|0\rangle$ and $|1\rangle$ with distinct eigenvalues, so Z corresponds to a standard basis measurement.

Question 11

(5 points) Which of the following observables correspond to a sign basis measurement?

Your Answer		Score	Explanation
X	✔	5.00	The eigenvectors of X are $\|+\rangle$ and $\|-\rangle$ with distinct eigenvalues, so X corres
Total		5.00 / 5.00	

Question Explanation

The eigenvectors of X are $|+\rangle$ and $|-\rangle$ with distinct eigenvalues, so X corresponds to a sign basis measurement.

Question 12

(5 points) Suppose we measure a qubit $\alpha|0\rangle+\beta|1\rangle$ with respect to the observable $I=(1001)$.

What is the outcome of the measurement?

Your Answer		Score	Explanation
It is always $1$and the new state is $\alpha\|0\rangle+\beta\|1\rangle$	✔	5.00	I only has one eigenvalue 1, and thus the outcome of the measurement w of this eigenvalue, i.e. the subspace spanned by the eigenvectors correspo space $C2$. Thus, the new state will be $\alpha\|0\rangle+\beta\|1\rangle$ projected onto $C2$, whic
Total		5.00 / 5.00	

Question Explanation

I only has one eigenvalue 1, and thus the outcome of the measurement will always be 1. Moreover, the eigenspace of this eigenvalue, i.e. the subspace spanned by the eigenvectors corresponding to the eigenvalue 1, is the whole space $C2$. Thus, the new state will be $\alpha|0\rangle+\beta|1\rangle$ projected onto $C2$, which is simply $\alpha|0\rangle+\beta|1\rangle$.

Question 13

For questions 13-14, consider a qubit subject to the Hamiltonian (1221).

(5 points) Calculate the states of definite energy. What is the energy of each of these states? List them from lowest to highest, separated by a comma. (ex: $-2,4$)

Answer for Question 13

Your Answer		Score	Explanation
-1,3	✔	5.00	The states of definite energy and their energy are given by the eigenvectors and eigenvalues of the Hamiltonian. In the case of the given Hamiltonian, the eigenvectors are $|+\rangle$ and $|-\rangle$ with eigenvalues 3 and -1 respectively.
Total		5.00 / 5.00	

Question Explanation

The states of definite energy and their energy are given by the eigenvectors and eigenvalues of the Hamiltonian. In the case of the given Hamiltonian, the eigenvectors are $|+\rangle$ and $|-\rangle$ with eigenvalues 3 and -1 respectively.

Question 14

(10 points) Now suppose the state of the qubit at time 0 is $|\psi(0)\rangle=|0\rangle$. What is the probability that a standard basis measurement at time t will still find it in state $|0\rangle$?

Your Answer	Score	Explanation

◉ $\cos^2\frac{2t}{\hbar}$	✔	10.00	Note that $\lvert\psi(0)\rangle=\lvert 0\rangle=\frac{1}{\sqrt{2}}(\lvert+\rangle+\lvert-\rangle)$. Since the eigenvectors of the given Hamilt eigenvalues 3 and -1, $\lvert\psi(t)\rangle=\frac{e^{-3it/\hbar}}{\sqrt{2}}\lvert+\rangle+\frac{e^{it/\hbar}}{\sqrt{2}}\lvert-\rangle=\frac{e^{-3it/\hbar}}{2}(\lvert 0\rangle+\lvert 1\rangle)+\frac{e^{it/\hbar}}{2}(\lvert 0\rangle-\lvert 1\rangle)$. Thus, $\lvert\langle 0\lvert\psi(t)\rangle\rvert^2=\left\lvert\frac{e^{-3it/\hbar}+e^{it/\hbar}}{2}\right\rvert^2=\left\lvert e^{-it/\hbar}\cdot\frac{e^{-2it/\hbar}+e^{2it/\hbar}}{2}\right\rvert^2=\left\lvert\frac{e^{-2it/\hbar}+e^{2it/\hbar}}{2}\right\rvert^2 =\frac{1}{4}\left\lvert\cos(\frac{2t}{\hbar})+\cos(-\frac{2t}{\hbar})+i(\sin(\frac{2t}{\hbar})+\sin(-\frac{2t}{\hbar}))\right\rvert^2=\frac{1}{4}\lvert 2\cos\frac{2t}{\hbar}\rvert^2=\cos^2\frac{2t}{\hbar}$.
Total		10.00 / 10.00	

Question Explanation

Note that $|\psi(0)\rangle=|0\rangle=\frac{1}{\sqrt{2}}(|+\rangle+|-\rangle)$. Since the eigenvectors of the given Hamiltonian H is $|+\rangle$ and $|-\rangle$ with respective eigenvalues 3 and -1,
$|\psi(t)\rangle=\frac{e^{-3it/\hbar}}{\sqrt{2}}|+\rangle+\frac{e^{it/\hbar}}{\sqrt{2}}|-\rangle=\frac{e^{-3it/\hbar}}{2}(|0\rangle+|1\rangle)+\frac{e^{it/\hbar}}{2}(|0\rangle-|1\rangle)$. Thus,
$|\langle 0|\psi(t)\rangle|^2=\left|\frac{e^{-3it/\hbar}+e^{it/\hbar}}{2}\right|^2=\left|e^{-it/\hbar}\cdot\frac{e^{-2it/\hbar}+e^{2it/\hbar}}{2}\right|^2=\left|\frac{e^{-2it/\hbar}+e^{2it/\hbar}}{2}\right|^2$
$=\frac{1}{4}\left|\cos(\frac{2t}{\hbar})+\cos(-\frac{2t}{\hbar})+i(\sin(\frac{2t}{\hbar})+\sin(-\frac{2t}{\hbar}))\right|^2=\frac{1}{4}|2\cos\frac{2t}{\hbar}|^2=\cos^2\frac{2t}{\hbar}$.

Question 15

(10 points) Write down the observable M that corresponds to performing a measurement on a qubit in the sign basis $|+\rangle,|-\rangle$, with measurement value 0if the outcome is $+$ and value 2 if the outcome is $-$. You should write M in the standard basis. List the entries of M separated by commas. Do NOT include white spaces. (If $M=\begin{pmatrix}1&2\\3&-4\end{pmatrix}$, you should write 1,2,3,-4)

Answer for Question 15

Your Answer		Score	Explanation
1,-1,-1,1	✔	10.00	$M=0\lvert+\rangle\langle+\rvert+2\lvert-\rangle\langle-\rvert=2\begin{pmatrix}\frac{1}{\sqrt{2}}\\-\frac{1}{\sqrt{2}}\end{pmatrix}\begin{pmatrix}\frac{1}{\sqrt{2}}&-\frac{1}{\sqrt{2}}\end{pmatrix}=(1$
Total		10.00 / 10.00	

Question Explanation

$M=0|+\rangle\langle+|+2|-\rangle\langle-|=2\begin{pmatrix}\frac{1}{\sqrt{2}}\\-\frac{1}{\sqrt{2}}\end{pmatrix}\begin{pmatrix}\frac{1}{\sqrt{2}}&-\frac{1}{\sqrt{2}}\end{pmatrix}=\begin{pmatrix}1&-1\\-1&1\end{pmatrix}$.

Question 16

(Extra credit. 10 points) Which of the following is the observable corresponding to a standard basis measurement of the first qubit in a two-qubit system?

Your Answer		Score	Explanation
$\begin{pmatrix} 0&0&0&0 \\ 0&0&0&0 \\ 0&0&1&0 \\ 0&0&0&1 \end{pmatrix}$	✔	10.00	Note that $\vert 00\rangle$, $\vert 01\rangle$, $\vert 10\rangle$, and $\vert 11\rangle$ are orthonormal ei of $\begin{pmatrix} 0&0&0&0 \\ 0&0&0&0 \\ 0&0&1&0 \\ 0&0&0&1 \end{pmatrix}$ with eigen measurement outcome 0 will correspond to the case wh measurement outcome 1 will correspond to the case wh
Total		10.00 / 10.00	

Question Explanation

Note that $|00\rangle$, $|01\rangle$, $|10\rangle$, and $|11\rangle$ are orthonormal eigenvectors of $\begin{pmatrix} 0&0&0&0 \\ 0&0&0&0 \\ 0&0&1&0 \\ 0&0&0&1 \end{pmatrix}$ with eigenvalues 0,0,1, and 1 respectively. Thus, the measurement outcome 0 will correspond to the case where the first qubit is in $|0\rangle$ and the measurement outcome 1 will correspond to the case where the first qubit is $|1\rangle$ as desired.

Question 1

(1 point) Even though the free particle in 1D is a continuous system, it has discrete energy levels.

Your Answer		Score	Ex
False	✔	1.00	
Total		1.00 / 1.00	

Question 2

(1 point) Even though the particle in a box is a continuous system, it has discrete energy levels.

Your Answer		Score	Ex
True	✔	1.00	

Total	1.00 / 1.00

Question 3

(1 point) If A is an observable then $\int_{-\infty}^{\infty} g^*(x)Af(x)dx=\int_{-\infty}^{\infty} f^*(x)Ag(x)dx$.

Your Answer		Score	Explanation
False	✔	1.00	If A is an observable, then it is Hermitian. However, the Hermicity condition is $\int_{-\infty}^{\infty} g^*(x)Af(x)dx=\overline{\int_{-\infty}^{\infty} f^*(x)Ag(x)dx}$, not
Total		1.00 / 1.00	

Question Explanation

If A is an observable, then it is Hermitian. However, the Hermicity condition is $\int_{-\infty}^{\infty} g^*(x)Af(x)dx=\overline{\int_{-\infty}^{\infty} f^*(x)Ag(x)dx}$, not the one given in the problem.

Question 4

(1 point) The uncertainty principle says that we cannot simultaneously know the value of a pair of observables whose commutator is 0.

Your Answer		Score	Explanation
False	✔	1.00	The uncertainty principle says that we cannot simultaneously know the value of nonzero.
Total		1.00 / 1.00	

Question Explanation

The uncertainty principle says that we cannot simultaneously know the value of a pair of observables whose commutator is nonzero.

Question 5

(1 point) The position and momentum operators are self-adjoint.

Your Answer		Score	Explanation
True		1.00	All observables are self-adjoint.
Total		1.00 / 1.00	

Question Explanation

All observables are self-adjoint.

Question 6

For questions 6-9, consider a particle whose wavefunction at some fixed time t is represented by

$$\Psi(x)=\begin{cases}-A & \text{if } -2\le x\le 0,\\ A & \text{if } 0\le x\le 3,\\ 0 & \text{otherwise.}\end{cases}$$

(5 points) What is the normalization constant A? Round your answer to the nearest hundrenth. (For example, if your answer is 0.8592, you should write 0.86)

Answer for Question 6

Your Answer		Score	Explanation
0.45		5.00	$\int_{-\infty}^{\infty}|\Psi(x)|^2dx=5A^2=1$. Thus, $A=\frac{1}{\sqrt{5}}=0.447213\cdots\approx$
Total		5.00 / 5.00	

Question Explanation

$\int_{-\infty}^{\infty}|\Psi(x)|^2dx=5A^2=1$. Thus, $A=\frac{1}{\sqrt{5}}=0.447213\cdots\approx0.45$.

Question 7

(5 points) What is the probability of finding the particle at a position $x\le 0$? Round your answer to the nearest hundreth.

Answer for Question 7

Your Answer		Score	Explanation
0.40	✔	5.00	$\int_{-\infty}^{0}|\Psi(x)|^2 dx = 2A^2 = \frac{2}{5} = 0.4$
Total		5.00 / 5.00	

Question Explanation

$\int_{-\infty}^{0}|\Psi(x)|^2 dx = 2A^2 = \frac{2}{5} = 0.4$

Question 8

(10 points) What is the expectation value of position x? Round your answer to the nearest hundreth.

Answer for Question 8

Your Answer		Score	Explanation
0.50	✔	10.00	$\int_{-\infty}^{\infty}\Psi(x)^* x \Psi(x) dx = \int_{-2}^{3}|\Psi(x)|^2 x dx = A^2\left[\frac{1}{2}x^2\right]_{-2}^{3} = $
Total		10.00 / 10.00	

Question Explanation

$\int_{-\infty}^{\infty}\Psi(x)^* x \Psi(x) dx = \int_{-2}^{3}|\Psi(x)|^2 x dx = A^2\left[\frac{1}{2}x^2\right]_{-2}^{3} = \frac{2.5}{5} = 0.5$

Question 9

(10 points) What is the expectation value of momentum p? Round your answer to the nearest hundreth. (Recall that the momentum operator is given by $\hat{p} = -i\hbar\frac{\partial}{\partial x}$. Assume that we are working in units such that $\hbar = 1$.)

Answer for Question 9

Your Answer		Score	Explanation
0.00	✔	10.00	$\int_{-\infty}^{\infty}\Psi(x)^*(-i\hbar\frac{\partial}{\partial x})\Psi(x)dx=\int_{-\infty}^{\infty}\Psi(x)^*(-i\hbar)\cdot 0$
Total		10.00 / 10.00	

Question Explanation

$\int_{-\infty}^{\infty}\Psi(x)^*(-i\hbar\frac{\partial}{\partial x})\Psi(x)dx=\int_{-\infty}^{\infty}\Psi(x)^*(-i\hbar)\cdot 0dx=0$

Question 10

For questions 10-12, evaluate the commutators $[A,B]$ for the given pairs of operators A and B.

(5 points) $A=\frac{d}{dx}$, $B=x$

Your Answer		Score	Explanation
1	✔	5.00	$[\frac{d}{dx},x]f(x)=\frac{d}{dx}xf(x)-x\frac{d}{dx}f(x)=f(x)+x\frac{d}{dx}f(x)-x\frac{d}{dx}f(x)$ Thus, $[\frac{d}{dx},x]=1$.
Total		5.00 / 5.00	

Question Explanation

$[\frac{d}{dx},x]f(x)=\frac{d}{dx}xf(x)-x\frac{d}{dx}f(x)=f(x)+x\frac{d}{dx}f(x)-x\frac{d}{dx}f(x)=f(x)$.
Thus, $[\frac{d}{dx},x]=1$.

Question 11

(5 points) $A=\frac{d^2}{dx^2}$, $B=x$

Your Answer		Score	Explanation
$2\frac{d}{dx}$	✔	5.00	We use the following commutator identity whose proof is left as exercise: $[K$ Then, $[\frac{d^2}{dx^2},x]=[\frac{d}{dx}\frac{d}{dx},x]=\frac{d}{dx}[\frac{d}{dx},x]+[\frac{d}{dx},x]\frac{d}{dx}=2\frac{d}{dx}$.

Total	5.00 / 5.00

Question Explanation

We use the following commutator identity whose proof is left as exercise: $[KL,M]=KLM-MKL=K[L,M]+[K,M]L$.
Then,
$[\frac{d^2}{dx^2},x]=[\frac{d}{dx}\frac{d}{dx},x]=\frac{d}{dx}[\frac{d}{dx},x]+[\frac{d}{dx},x]\frac{d}{dx}=2\frac{d}{dx}$.

Question 12

(5 points) $A=\frac{d}{dx}-x$, $B=\frac{d}{dx}+x$

Your Answer		Score	Explanation
◉ 2	✔	5.00	We use the linearity of commutator $[K+L,M]=[K,M]+[L,M]$ and $[K,M+L]=[$ exercise. Then, $[\frac{d}{dx}-x,\frac{d}{dx}+x]=[\frac{d}{dx},\frac{d}{dx}]-[x,\frac{d}{dx}]+[\frac{d}{dx},x]-[x,x]=0-[x,\frac{d}{dx}]+[\frac{d}{dx},x]-0=2[\frac{d}{dx},$
Total		5.00 / 5.00	

Question Explanation

We use the linearity of commutator $[K+L,M]=[K,M]+[L,M]$ and $[K,M+L]=[K,M]+[K,L]$, whose proof is left as exercise. Then,
$[\frac{d}{dx}-x,\frac{d}{dx}+x]=[\frac{d}{dx},\frac{d}{dx}]-[x,\frac{d}{dx}]+[\frac{d}{dx},x]-[x,x]=0-[x,\frac{d}{dx}]+[\frac{d}{dx},x]-0=2[\frac{d}{dx},x]=2$.

Question 13

For problems 13-16, consider the observable M that corresponds to performing a measurement on a qubit in the basis $|u\rangle=\frac{1}{\sqrt{2}}|0\rangle+(\frac{1}{2}+\frac{i}{2})|1\rangle$, and$|u_\perp\rangle=(\frac{1}{2}-\frac{i}{2})|0\rangle-\frac{1}{\sqrt{2}}|1\rangle$, with measurement value 1 if the outcome is $|u\rangle$ and value 5 if the outcome is $|u_\perp\rangle$.

(5 points) Write down M in the $|u\rangle,|u_\perp\rangle$ basis. List the entries of M separated by commas. Do NOT include white spaces. (If $M=\begin{pmatrix}1 & 2\\ 3 & -4\end{pmatrix}$, you should write $1,2,3,-4$)

Answer for Question 13

Your Answer		Score
3,-1.41+1.41,-1.41-1.41i,3	✖	0.00
Total		0.00 / 5.00

Question Explanation

In the $|u\rangle,|u\perp\rangle$ basis, M is just (1005).

Question 14

(10 points) If M is the Hamiltonian acting on a qubit which is in the state $|\psi(0)\rangle=|0\rangle$, what is the state of the qubit at time t?

Your Answer		Score	Explanation
◉ $12\sqrt{}e^{-it/\hbar}|u\rangle+(12+i2)e^{-5it/\hbar}|u\perp\rangle$	✔	10.00	Using the solution to the Schrödinger's equation, $|\psi(t)\rangle$ Since $|0\rangle=12\sqrt{}|u\rangle+(12+i2)|u\perp\rangle$, $|\psi(t)\rangle=e^{-iMt/\hbar}(12\sqrt{}|u\rangle+(12+i2)|u\perp\rangle)=e^{-it/\hbar}2\sqrt{}|u\rangle+e^{-5it/}$
Total		10.00 / 10.00	

Question Explanation

Using the solution to the Schrödinger's equation, $|\psi(t)\rangle=e^{-iMt/\hbar}|0\rangle$.
Since $|0\rangle=12\sqrt{}|u\rangle+(12+i2)|u\perp\rangle$,
$|\psi(t)\rangle=e^{-iMt/\hbar}(12\sqrt{}|u\rangle+(12+i2)|u\perp\rangle)=e^{-it/\hbar}2\sqrt{}|u\rangle+e^{-5it/\hbar}(12+i2)|u\perp\rangle$.

Question 15

(5 points) Now you perform an $X=(0110)$ measurement on the qubit at time t. What is the expected value of your measurement at time $t=0$? Round your answer to the nearest hundredth.

Answer for Question 15

Your Answer	Score	Explanation

0.00	✔	5.00	The expected value of the X measurement at time $t=($ $\langle\psi(0)\|X\|\psi(0)\rangle=\langle 0\|X\|0\rangle=(10)(0110)(10)=0.$
Total		5.00 / 5.00	

Question Explanation

The expected value of the X measurement at time $t=0$ is $\langle\psi(0)|X|\psi(0)\rangle=\langle 0|X|0\rangle=(10)(0110)(10)=0.$

Question 16

(5 points) What about at time $t=\pi/2$? Assume you are working in units such that $\hbar=1$.

Round your answer to the nearest hundredth.
Answer for Question 16

Your Answer		Score	Explanation
0.00	✔	5.00	To calculate the expected value at time $t=\pi/2$, first note that $\|\psi(\pi/2)\rangle=e^{-\pi i/2\hbar}2\sqrt{}\|u\rangle+e^{-5\pi i/2\hbar}(12+i2)\|u\perp\rangle$ $=e^{-\pi i/2}2\sqrt{}\|u\rangle+e^{-5\pi i/2}(12+i2)\|u\perp\rangle$ $=-i2\sqrt{}\|u\rangle-i(12+i2)\|u\perp\rangle$ $=-i(12\sqrt{}\|u\rangle+(12+i2)\|u\perp\rangle)=-i\|0\rangle.$ However, the global phase $-i$ is irrelevant, so the system is still in $\|0\rangle$ and the e still 0.
Total		5.00 / 5.00	

Question Explanation

To calculate the expected value at time $t=\pi/2$, first note that
$|\psi(\pi/2)\rangle=e^{-\pi i/2\hbar}2\sqrt{}|u\rangle+e^{-5\pi i/2\hbar}(12+i2)|u\perp\rangle$
$=e^{-\pi i/2}2\sqrt{}|u\rangle+e^{-5\pi i/2}(12+i2)|u\perp\rangle$
$=-i2\sqrt{}|u\rangle-i(12+i2)|u\perp\rangle$
$=-i(12\sqrt{}|u\rangle+(12+i2)|u\perp\rangle)=-i|0\rangle.$
However, the global phase $-i$ is irrelevant, so the system is still in $|0\rangle$ and the expected value of the measurement is still 0.

Question 17

(10 points) Suppose Alice has two entangled qubits in state $\alpha|00\rangle+\beta|11\rangle$, and then she sends the first qubit to Bob using quantum teleportation. What is the state of the qubit that Bob receives?

Your Answer		Score	Explanation
◉ Bob receives a qubit that is entangled with Alice's second qubit, and those two qubits are in the state $\alpha|00\rangle+\beta|11\rangle$	✔	10.00	We will investigate only the case in which both measurements of the quantum teleportation protocol are 0. Other cases are left as exercise. Note that the initial state of the system is $(\alpha|00\rangle+\beta|11\rangle)(\frac{1}{\sqrt{2}}|00\rangle+\frac{1}{\sqrt{2}}|11\rangle)$, where the first two qubits are Alice's initial qubits and the last two qubits are the Bell state used in the quantum teleportation protocol. After Alice applies CNOT from the first qubit to the third qubit, the state of the system is $\frac{\alpha}{\sqrt{2}}|0000\rangle+\frac{\alpha}{\sqrt{2}}|0011\rangle+\frac{\beta}{\sqrt{2}}|1110\rangle+\frac{\beta}{\sqrt{2}}|1101\rangle$. Now, Alice measures the third qubit, and suppose the outcome was 0. The new state is $\alpha|0000\rangle+\beta|1101\rangle$. Now Alice applies a Hadamard gate on the first qubit. The resulting state of the system is $\frac{\alpha}{\sqrt{2}}|0000\rangle+\frac{\alpha}{\sqrt{2}}|1000\rangle+\frac{\beta}{\sqrt{2}}|0101\rangle-\frac{\beta}{\sqrt{2}}|1101\rangle$. If Alice now measures the first qubit and finds it to be 0, we see that the resulting state of the system is $\alpha|0000\rangle+\beta|0101\rangle$, as desired.
Total		10.00 / 10.00	

Question Explanation

We will investigate only the case in which both measurements of the quantum teleportation protocol are 0. Other cases are left as exercise. Note that the initial state of the system is $(\alpha|00\rangle+\beta|11\rangle)(\frac{1}{\sqrt{2}}|00\rangle+\frac{1}{\sqrt{2}}|11\rangle)$, where the first two qubits are Alice's initial qubits and the last two qubits are the Bell state used in the quantum teleportation protocol. After Alice applies CNOT from the first qubit to the third qubit, the state of the system is $\frac{\alpha}{\sqrt{2}}|0000\rangle+\frac{\alpha}{\sqrt{2}}|0011\rangle+\frac{\beta}{\sqrt{2}}|1110\rangle+\frac{\beta}{\sqrt{2}}|1101\rangle$. Now, Alice measures the third qubit, and suppose the outcome was 0. The new state is $\alpha|0000\rangle+\beta|1101\rangle$. Now Alice applies a Hadamard gate on the first qubit. The resulting state of the system is $\frac{\alpha}{\sqrt{2}}|0000\rangle+\frac{\alpha}{\sqrt{2}}|1000\rangle+\frac{\beta}{\sqrt{2}}|0101\rangle-\frac{\beta}{\sqrt{2}}|1101\rangle$. If Alice now measures the first qubit and finds it to be 0, we see that the resulting state of the system is $\alpha|0000\rangle+\beta|0101\rangle$, as desired.

Question 1

(1 point) Any classical circuit can be converted to a reversible circuit.

Your Answer		Score	Ex
True	✔	1.00	
Total		1.00 / 1.00	

Question 2

(1 point) Any classical reversible circuit can be implemented as a quantum circuit if we allow small error.

Your Answer		Score	Ex
True	✔	1.00	
Total		1.00 / 1.00	

Question 3

(1 point) For any $x \in \{0,1\}^n$, $H^{\otimes n}|x\rangle$ can be written as $\frac{1}{2^{n/2}}\sum_{y\in\{0,1\}^n}\beta_y|y\rangle$ where $\beta_y \in \{-1,1\}$ for all y.

Your Answer		Score	Ex
True	✔	1.00	
Total		1.00 / 1.00	

Question 4

(2 points) Simon's algorithm achieves an exponential speedup over any classical algorithm in the black box model.

Your Answer		Score	Ex
True	✔	2.00	

Total	2.00 / 2.00

Question 5

(5 points) Consider the state $|\psi\rangle=\frac{1}{\sqrt{2}}(|00\rangle+|11\rangle)$. What is the state that results from applying the 2-qubit Hadamard transform to $|\psi\rangle$?

Your Answer		Score	Explanation
◉ $\frac{1}{\sqrt{2}}(|00\rangle+|11\rangle)$	✔	5.00	$H^{\otimes 2}(\frac{1}{\sqrt{2}}(|00\rangle+|11\rangle))=\frac{1}{\sqrt{2}}(|++\rangle+|--\rangle)=\frac{1}{\sqrt{2}}(|0$
Total		5.00 / 5.00	

Question Explanation

$H^{\otimes 2}(\frac{1}{\sqrt{2}}(|00\rangle+|11\rangle))=\frac{1}{\sqrt{2}}(|++\rangle+|--\rangle)=\frac{1}{\sqrt{2}}(|00\rangle+|11\rangle)$

Question 6

(5 points) Consider the state $|\psi\rangle=\frac{1}{\sqrt{2}}(|01\rangle+|10\rangle)$. What is the state that results from applying the 2-qubit Hadamard transform to $|\psi\rangle$?

Your Answer		Score	Explanation
◉ $\frac{1}{\sqrt{2}}(|00\rangle-|11\rangle)$	✔	5.00	$H^{\otimes 2}(\frac{1}{\sqrt{2}}(|01\rangle+|10\rangle))=\frac{1}{\sqrt{2}}(|+-\rangle+|-+\rangle)=\frac{1}{\sqrt{2}}(|0$
Total		5.00 / 5.00	

Question Explanation

$H^{\otimes 2}(\frac{1}{\sqrt{2}}(|01\rangle+|10\rangle))=\frac{1}{\sqrt{2}}(|+-\rangle+|-+\rangle)=\frac{1}{\sqrt{2}}(|00\rangle-|11\rangle)$

Question 7

(5 points) Suppose we are given a reversible circuit for computing $x \mapsto f(x)$ for some bijection f, which consists of m gates. Which of the following statements is true about computing $f(x) \mapsto x$? (Note: in all statements we make about reversible circuits, we may be suppressing some zero work bits.)

Your Answer	Score	Explanati

(•) We can implement a circuit for computing $f(x) \mapsto x$ which only uses $O(m)$ gates.	✔	5.00	It suffices inverse.
Total		5.00 / 5.00	

Question Explanation

It suffices to replace each reversible gate by its inverse.

Question 8

(5 points) Suppose we are given a function $f:\{0,1\}^n \to \{0,1\}^n$. Which of the following statements is (are) correct? Select all that apply.

Your Answer

- [x] We can implement a quantum circuit that computes $\sum_x \alpha_x |x\rangle|0^n\rangle \mapsto \sum_x \alpha_x |x\rangle|f(x)\rangle$.
- [ ] We can implement a quantum circuit that computes $\sum_x \alpha_x |x\rangle \mapsto \sum_x \alpha_x |f(x)\rangle$.
- [ ] We can implement a quantum circuit that computes $\sum_x \alpha_x |f(x)\rangle \mapsto \sum_x \alpha_x |x\rangle$.
- [x] We can implement a quantum circuit that computes $\sum_x \alpha_x |x\rangle|f(x)\rangle \mapsto \sum_x \alpha_x |x\rangle|0^n\rangle$.

Total

Question Explanation

The second option is not true, because for example if f is a constant function, i.e., $f(x)=0^n$ for all x, then the given transformation cannot be unitary. The third option is not true for the same reason.

Question 9

(5 points) Given circuits for f and f^{-1}, we can create classical reversible circuits R_f, R^{-1}_f, $R_{f^{-1}}$, $R^{-1}_{f^{-1}}$. (R_f denotes the circuit that on input x outputs x and $f(x)$.) Which of the following sequences implements a circuit that on input x outputs $f(x)$ reversibly? (Assume that f is a bijection.)

Your Answer	**Score**

R_f followed by R^{-1}_f	✖	0.00
Total		0.00 / 5.00

Question Explanation

R_f takes as input x and produces x and $f(x)$. Also, $R_{f^{-1}}$ takes as input $f(x)$ and produces $f(x)$ and x. That is, $R^{-1}_{f^{-1}}$ is the circuit that on input $f(x)$and x produces $f(x)$. Thus, we can implement the desired circuit by concatenating R_f and $R^{-1}_{f^{-1}}$.

Question 10

For questions 10-12, let $|\beta\rangle=\sum_{x\in\{0,1,\}^n}\beta_x|x\rangle$ be the Hadamard transform of the superposition $|\alpha\rangle=\sum_{x\in\{0,1\}^n}\alpha_x|x\rangle$. Then, consider the `shift' by u of the superposition $|\alpha\rangle$: $|\alpha'\rangle=\sum_{x\in\{0,1\}^n}\alpha_x|x+u\rangle$ (where $x+u$ means bitwise sum modulo 2) and its Hadamard transform $|\beta'\rangle=\sum_{x\in\{0,1\}^n}\beta_{x'}|x\rangle$.

(5 points) If $n=2$ and $u=10$, what are the values of $\beta_{x'}$?

Your Answer		Score
$\beta_{x'}=\beta_x$ for $x=00,01$ and $\beta_{x'}=-\beta_x$ for $x=10,11$	✔	5.00
Total		5.00 / 5

Question 11

(5 points) If $n=3$ and $u=101$, what are the values of $\beta_{x'}$?

Your Answer	
$\beta_{x'}=\beta_x$ for $x=000,010,101,111$ and $\beta_{x'}=-\beta_x$ for all other x.	✔
Total	

Question 12

(5 points) In general, what is $\beta_{x'}$ as a function of x, β_x, and u?

Your Answer	Score	Explanation

$\beta_{x'}=(-1)^{u\cdot x}\beta_x$	✔	5.00	Recall that $H^{\otimes n}\lvert x\rangle=\frac{1}{\sqrt{2^n}}\sum_y(-1)^{x\cdot y}\lvert y\rangle$. Thus, $\lvert\beta\rangle=H^{\otimes n}\lvert\alpha\rangle=H^{\otimes n}(\sum_x\alpha_x\lvert x\rangle)=\frac{1}{\sqrt{2^n}}$ Thus, $\beta_y=\frac{1}{\sqrt{2^n}}\sum_x\alpha_x(-1)^{x\cdot y}$. Similarly, $\lvert\beta'\rangle=H^{\otimes n}\lvert\alpha'\rangle=\frac{1}{\sqrt{2^n}}\sum_x\sum_y\alpha_x(-1)^{(x+u)\cdot y}\lvert y$ thus $\beta_{y'}=\frac{1}{\sqrt{2^n}}\sum_x\alpha_x(-1)^{(x+u)\cdot y}=(-1)^{u\cdot y}\frac{1}{\sqrt{2^n}}\sum_x\alpha_x(-1)^{x\cdot y}=(-1)^{u\cdot y}\beta_y$.
Total		5.00 / 5.00	

Question Explanation

Recall that $H^{\otimes n}\lvert x\rangle=\frac{1}{\sqrt{2^n}}\sum_y(-1)^{x\cdot y}\lvert y\rangle$.
Thus, $\lvert\beta\rangle=H^{\otimes n}\lvert\alpha\rangle=H^{\otimes n}(\sum_x\alpha_x\lvert x\rangle)=\frac{1}{\sqrt{2^n}}\sum_x\sum_y\alpha_x(-1)^{x\cdot y}\lvert y\rangle$. Thus, $\beta_y=\frac{1}{\sqrt{2^n}}\sum_x\alpha_x(-1)^{x\cdot y}$.
Similarly, $\lvert\beta'\rangle=H^{\otimes n}\lvert\alpha'\rangle=\frac{1}{\sqrt{2^n}}\sum_x\sum_y\alpha_x(-1)^{(x+u)\cdot y}\lvert y\rangle$ and
thus $\beta_{y'}=\frac{1}{\sqrt{2^n}}\sum_x\alpha_x(-1)^{(x+u)\cdot y}=(-1)^{u\cdot y}\frac{1}{\sqrt{2^n}}\sum_x\alpha_x(-1)^{x\cdot y}=(-1)^{u\cdot y}\beta_y$.

Question 13

Questions 13-15 concern your understanding of Simon's algorithm. Suppose $n=4$ and the secret string s is 1101.

(5 points) Recall that in the first phase of Simon's algorithm, we first run Hadamard transform on the first n qubits and then run the quantum circuit for computing f, i.e. $\lvert x\rangle\lvert 0^n\rangle\mapsto\lvert x\rangle\lvert f(x)\rangle$. What is the state of the system at this point?

Your Answer		**Score**
$\frac{1}{\sqrt{2^n}}\sum_{x\in\{0,1\}^n}\lvert x\rangle\lvert f(x)\rangle$	✔	5.00
Total		5.00 / 5.00

Question 14

(5 points) Now we measure the second register (the last n qubits) and obtain 0101. If we know that $f(1001)=0101$, what is the state of the system at this point?

Your Answer		**Score**
$\frac{1}{\sqrt{2}}(\lvert 0100\rangle+\lvert 1001\rangle)\lvert 0101\rangle$	✔	5.00
Total		5.00 / 5.00

Question 15

(5 points) Finally, we apply a Hadamard transform on the first register (the first n qubits) again and measure the first register. Which of the following CANNOT be the outcome of the measurement?

Your Answer		Score	Explanation
◉ 1111	✔	5.00	Recall that we can only observe a y such that $s\cdot y=$
Total		5.00 / 5.00	

Question Explanation

Recall that we can only observe a y such that $s\cdot y=0$.

Question 16

(5 points) Let $f:\{0,1\}^n\rightarrow\{0,1\}^n$ be a 4-to-1 function with the following structure: there are n-bit strings $a,b\in\{0,1\}^n$: $f(x)=f(x+a)=f(x+b)=f(x+a+b)$ for every x. Suppose you compute the superposition $\sum_{x\in\{0,1\}^n}2^{-n/2}|x\rangle|f(x)\rangle$ and then measure the 2nd register as in Simon's algorithm. What is the resulting state of the first register?

Your Answer		Score	Explanation
◉ $\frac{1}{2}(|z\rangle+|z\oplus a\rangle+|z\oplus b\rangle+|z\oplus a\oplus b\rangle)$ for some random n-bit string z	✔	5.00	Loosely speaking, the action of a measurement is to project the state under measurement onto the subspace compatible with the measurement result (followed, obviously, by renormalization). So when we measure the second register, we get the value of the function computed for some input. Since f is 4-to-1, that means that there are four states $|x\rangle$ that correspond to the same function value $f(x)$. So the output state becomes $\frac{1}{2}(|z\rangle+|z\oplus a\rangle+|z\oplus b\rangle+|z\oplus a\oplus b\rangle)$ where z is a random n-bit string.
Total		5.00 / 5.00	

Question Explanation

Loosely speaking, the action of a measurement is to project the state under measurement onto the subspace compatible with the measurement result (followed, obviously, by renormalization). So when we measure the second register, we get the value of the function computed for some input. Since f is 4-to-1, that means that there are four states $|x\rangle$ that correspond to the same function value $f(x)$. So the output state becomes $\frac{1}{2}(|z\rangle+|z\oplus a\rangle+|z\oplus b\rangle+|z\oplus a\oplus b\rangle)$ where z is a random n-bit string.

Question 1

In question 1-5, we will work through an example of QFT_M for $M=6$.

(2 points) What is ω?

Your Answer		Score	Explanation
◉ $e^{i\pi/3}$	✔	2.00	Recall that $\omega=e^{2\pi}$
Total		2.00 / 2.00	

Question Explanation

Recall that $\omega=e^{2\pi i/M}$.

Question 2

(4 points) What is QFT_6 of $\frac{1}{\sqrt{2}}(|0\rangle+|3\rangle)$?

Your Answer		Score
◉ $\frac{1}{\sqrt{3}}(|0\rangle+|2\rangle+|4\rangle)$	✔	4.00
Total		4.00 / 4.00

Question Explanation

Note that $\omega^{3+j}=-\omega^j$ for $j=0,1,2$. Then we have the following relations:

$QFT_6|0\rangle=\frac{1}{\sqrt{6}}(|0\rangle+|1\rangle+|2\rangle+|3\rangle+|4\rangle+|5\rangle)$

$QFT_6|1\rangle=\frac{1}{\sqrt{6}}(|0\rangle+\omega|1\rangle+\omega^2|2\rangle-|3\rangle-\omega|4\rangle-\omega^2|5\rangle)$

$QFT_6|2\rangle=\frac{1}{\sqrt{6}}(|0\rangle+\omega^2|1\rangle-\omega|2\rangle+|3\rangle+\omega^2|4\rangle-\omega|5\rangle)$

$QFT_6|3\rangle=\frac{1}{\sqrt{6}}(|0\rangle-|1\rangle+|2\rangle-|3\rangle+|4\rangle-|5\rangle)$

$QFT_6|4\rangle=\frac{1}{\sqrt{6}}(|0\rangle-\omega|1\rangle+\omega^2|2\rangle+|3\rangle-\omega|4\rangle+\omega^2|5\rangle)$

$QFT_6|5\rangle = \frac{1}{\sqrt{6}}(|0\rangle - \omega^2|1\rangle - \omega|2\rangle - |3\rangle + \omega^2|4\rangle + \omega|5\rangle)$
Then, solutions to questions 2-5 can easily be calculated.

Question 3

(4 points) What is QFT_6 of $\frac{1}{\sqrt{2}}(|1\rangle + |4\rangle)$?

Your Answer		Score
◉ $\frac{1}{\sqrt{3}}(|0\rangle + \omega^2|2\rangle - \omega|4\rangle)$	✔	4.00
Total		4.00 / 4.00

Question 4

(5 points) What is QFT_6 of $\frac{1}{\sqrt{3}}(|0\rangle + |2\rangle + |4\rangle)$?

Your Answer		Score
◉ $\frac{1}{\sqrt{2}}(|0\rangle + |3\rangle)$	✔	5.00
Total		5.00 / 5.00

Question 5

(5 points) What is QFT_6 of $\frac{1}{\sqrt{3}}(|1\rangle + |3\rangle + |5\rangle)$?

Your Answer		Score
◉ $\frac{1}{\sqrt{2}}(|0\rangle - |3\rangle)$	✔	5.00
Total		5.00 / 5.00

Question 6

(10 points) Let $|\alpha\rangle = \frac{1}{\sqrt{M}}\sum_{j=0}^{M-1}\alpha_j|j\rangle$ and let $|\beta\rangle = \frac{1}{\sqrt{M}}\sum_{j=0}^{M-1}\beta_j|j\rangle$ be its QFT_M. Consider the shift of the superposition $|\alpha\rangle$: $|\alpha'\rangle = \frac{1}{\sqrt{M}}\sum_{j=0}^{M-1}\alpha_j|j+1 \pmod{M}\rangle$. Let $|\beta'\rangle = \frac{1}{\sqrt{M}}\sum_{j=0}^{M-1}\beta'_j|j\rangle$ be its QFT_M. Derive an expression for β'_j as a function of β_j.

Your Answer	Score	Explanation

◉ $e^{2\pi ji/M}\beta_j$	✔	10.00	Note that $QFT_M|\alpha\rangle=\sum_{j=0}^{M-1}\alpha_j\left(\frac{1}{\sqrt{M}}\sum_{l=0}^{M-1}\omega^{jl}|l\rangle\right)=\frac{1}{\sqrt{M}}\sum_{l=0}^{M-1}\sum_{j=0}^{M-1}\alpha_j\alpha$ Similarly, $QFT_M|\alpha'\rangle=\sum_{j=0}^{M-1}\alpha_j\left(\frac{1}{\sqrt{M}}\sum_{l=0}^{M-1}\omega^{(j+1)l}|l\rangle\right)=\frac{1}{\sqrt{M}}\sum_{l=0}^{M-1}\sum_{j=}$ Thus, $\beta_l'=\frac{1}{\sqrt{M}}\sum_{j=0}^{M-1}\alpha_j\omega^{(j+1)l}=\omega^l\beta_l=e^{2\pi li/M}\beta_l$.
Total		10.00 / 10.00	

Question Explanation

Note that $QFT_M|\alpha\rangle=\sum_{j=0}^{M-1}\alpha_j\left(\frac{1}{\sqrt{M}}\sum_{l=0}^{M-1}\omega^{jl}|l\rangle\right)=\frac{1}{\sqrt{M}}\sum_{l=0}^{M-1}\sum_{j=0}^{M-1}\alpha_j\omega^{jl}|l\rangle$.
Thus, $\beta_l=\frac{1}{\sqrt{M}}\sum_{j=0}^{M-1}\alpha_j\omega^{jl}$.
Similarly, $QFT_M|\alpha'\rangle=\sum_{j=0}^{M-1}\alpha_j\left(\frac{1}{\sqrt{M}}\sum_{l=0}^{M-1}\omega^{(j+1)l}|l\rangle\right)=\frac{1}{\sqrt{M}}\sum_{l=0}^{M-1}\sum_{j=0}^{M-1}\alpha_j\omega^{(j+1)l}|l\rangle$.
Thus, $\beta_l'=\frac{1}{\sqrt{M}}\sum_{j=0}^{M-1}\alpha_j\omega^{(j+1)l}=\omega^l\beta_l=e^{2\pi li/M}\beta_l$.

Question 7

In questions 7-9, consider the periodic superposition $|\alpha\rangle=\sqrt{\frac{k}{M}}\sum_{j=0}^{M/k-1}|jk\rangle$.

Let $\beta=\sum_j\beta_j|j\rangle$ be its QFT_M.

(5 points) Derive an expression for β_j.

Your Answer		Score	Explanation
◉ $\frac{\sqrt{k}}{M}\sum_{l=0}^{M/k-1}e^{2\pi jlki/M}$	✔	5.00	If $|\alpha\rangle=\sum_{j=0}^{M-1}\alpha_j|j\rangle$, then $QFT_M|\alpha\rangle=\sum_{j=0}^{M-1}\alpha_j\left(\frac{1}{\sqrt{M}}\sum_{l=0}^{M-1}\omega^{jl}|l\rangle\right.$ Thus, $\beta_l=\frac{1}{\sqrt{M}}\sum_{j=0}^{M-1}\alpha_j\omega^{jl}$. Since $\alpha_j=\sqrt{\frac{k}{M}}$ if k divides j and $\alpha_j=0$ otherwise, $\beta_l=\frac{\sqrt{k}}{M}\sum_{j=}^{M/k-1}$
Total		5.00 / 5.00	

Question Explanation

If $|\alpha\rangle=\sum_{j=0}^{M-1}\alpha_j|j\rangle$,
then $QFT_M|\alpha\rangle=\sum_{j=0}^{M-1}\alpha_j\left(\frac{1}{\sqrt{M}}\sum_{l=0}^{M-1}\omega^{jl}|l\rangle\right)=\frac{1}{\sqrt{M}}\sum_{l=0}^{M-1}\sum_{j=0}^{M-1}\alpha_j\omega^{jl}|l\rangle$.
Thus, $\beta_l=\frac{1}{\sqrt{M}}\sum_{j=0}^{M-1}\alpha_j\omega^{jl}$.
Since $\alpha_j=\sqrt{\frac{k}{M}}$ if k divides j and $\alpha_j=0$ otherwise, $\beta_l=\frac{\sqrt{k}}{M}\sum_{j=0}^{M/k-1}\omega^{jkl}$.

Question 8

(5 points) If j is a multiple of M/k, what is the value of β_j?

Your Answer	Score	Ex

◉ $\frac{1}{\sqrt{k}}$	✔	5.00	
Total		5.00 / 5.00	

Question 9

(5 points) If j is not a multiple of M/k, what is the value of β_j?

Your Answer		Score	Ex
◉ 0	✔	5.00	
Total		5.00 / 5.00	

Question 10

(10 points) What do you get if you apply QFT_M to the superposition $|\alpha\rangle=\sqrt{\frac{k}{M}}\sum_{j=0}^{M/k-1}|jk+1\rangle$? You may want to use your answer from question 6.

Your Answer		Score
◉ $|\beta\rangle=\frac{1}{\sqrt{k}}\sum_{l=0}^{k-1}e^{2\pi li/k}|\frac{lM}{k}\rangle$	✔	10.00
Total		10.00 / 10.00

Question Explanation

From question 7-9, we see that $QFT_M\sqrt{\frac{k}{M}}\sum_{j=0}^{M/k-1}|jk\rangle=\frac{1}{\sqrt{k}}\sum_{l=0}^{k-1}|\frac{lM}{k}\rangle$. Question 6 then tells us that $QFT_M\sqrt{\frac{k}{M}}\sum_{j=0}^{M/k-1}|jk+1\rangle=\frac{1}{\sqrt{k}}\sum_{l=0}^{k-1}\omega^{lM/k}|\frac{lM}{k}\rangle$.

Question 11

In questions 11-13, we will carry out some steps of the quantum factoring algorithm for $N=15$.

(5 points) What is the period k of the periodic superposition set up by quantum algorithm if it chooses $x=2$?

Answer for Question 11

Your Answer		Score	Explanation
4	✔	5.00	Since $2^1 \equiv 2 \pmod{15}$, $2^2 \equiv 4 \pmod{15}$, $2^3 \equiv 8 \pmod{15}$, $2^4 \equiv 1 \pmod{15}$, the
Total		5.00 / 5.00	

Question Explanation

Since $2^1 \equiv 2 \pmod{15}$, $2^2 \equiv 4 \pmod{15}$, $2^3 \equiv 8 \pmod{15}$, $2^4 \equiv 1 \pmod{15}$, the order of $2 \bmod 15$ is 4 and hence $k=4$.

Question 12

(5 points) Use k to find a non-trivial square root of $1 \pmod{15}$. Write your answer as an integer between 0 and 15.

Answer for Question 12

Your Answer		Score	Explanation
4	✔	5.00	Recall that x is a nontrivial square root of $1 \pmod{N}$ if $x^2 \equiv 1 \pmod{N}$ and $x \not\equiv \pm 1 \pmod{\text{m}}$ square root of $1 \pmod{15}$, since $4^2 = 2^k \equiv 1 \pmod{15}$, but $4 \not\equiv \pm 1 \pmod{15}$.
Total		5.00 / 5.00	

Question Explanation

Recall that x is a nontrivial square root of $1 \pmod{N}$ if $x^2 \equiv 1 \pmod{N}$ and $x \not\equiv \pm 1 \pmod{N}$. In this case, $2^{k/2} = 4$ is a non-trivial square root of $1 \pmod{15}$, since $4^2 = 2^k \equiv 1 \pmod{15}$, but $4 \not\equiv \pm 1 \pmod{15}$.

Question 13

(5 points) Then, the algorithm proceeds by computing $\gcd(x,y)$ for some integers x and y. List these two numbers in the ascending order, separated by a comma. Do NOT use white spaces. (If the two numbers are 12 and 3, you should write 3,12) If there are multiple combinations of numbers that can be a correct answer, provide any one of them.

Answer for Question 13

Your Answer		Score	Explanation
3,15	✔	5.00	If x is a non-trivial square root of $1 \pmod{15}$, the algorithm finds non-trivial div solving $\gcd(x+1,15)$ or $\gcd(x-1,15)$.
Total		5.00 / 5.00	

Question Explanation

If x is a non-trivial square root of $1 \pmod{15}$, the algorithm finds non-trivial divisors of 15 by solving $\gcd(x+1,15)$ or $\gcd(x-1,15)$.