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																					1	1	1	1	1	1	1	1	1
1	2	3	4	5	6																			$n-5$	$n-4$	$n-3$	$n-2$	$n-1$	n

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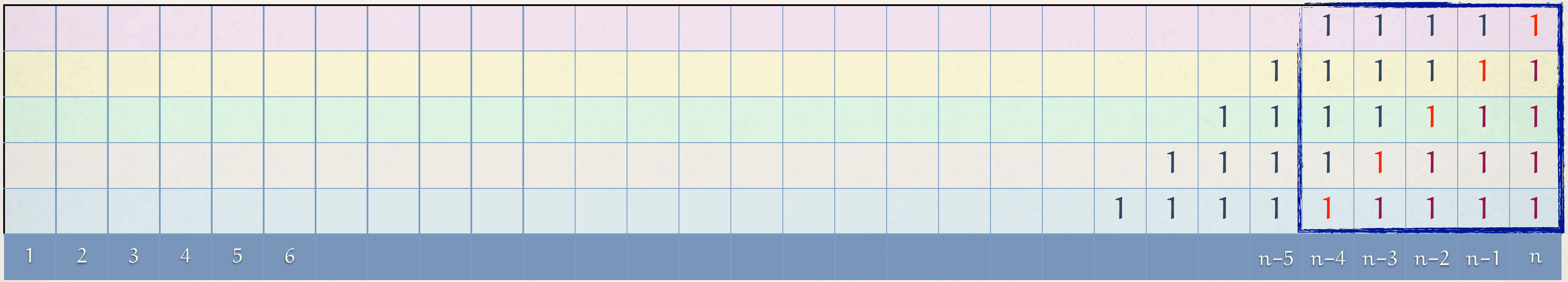

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Slogan: If you know u_1, u_2, …, u_{n-1} then you know u_n.