The inclusion-exclusion theorem

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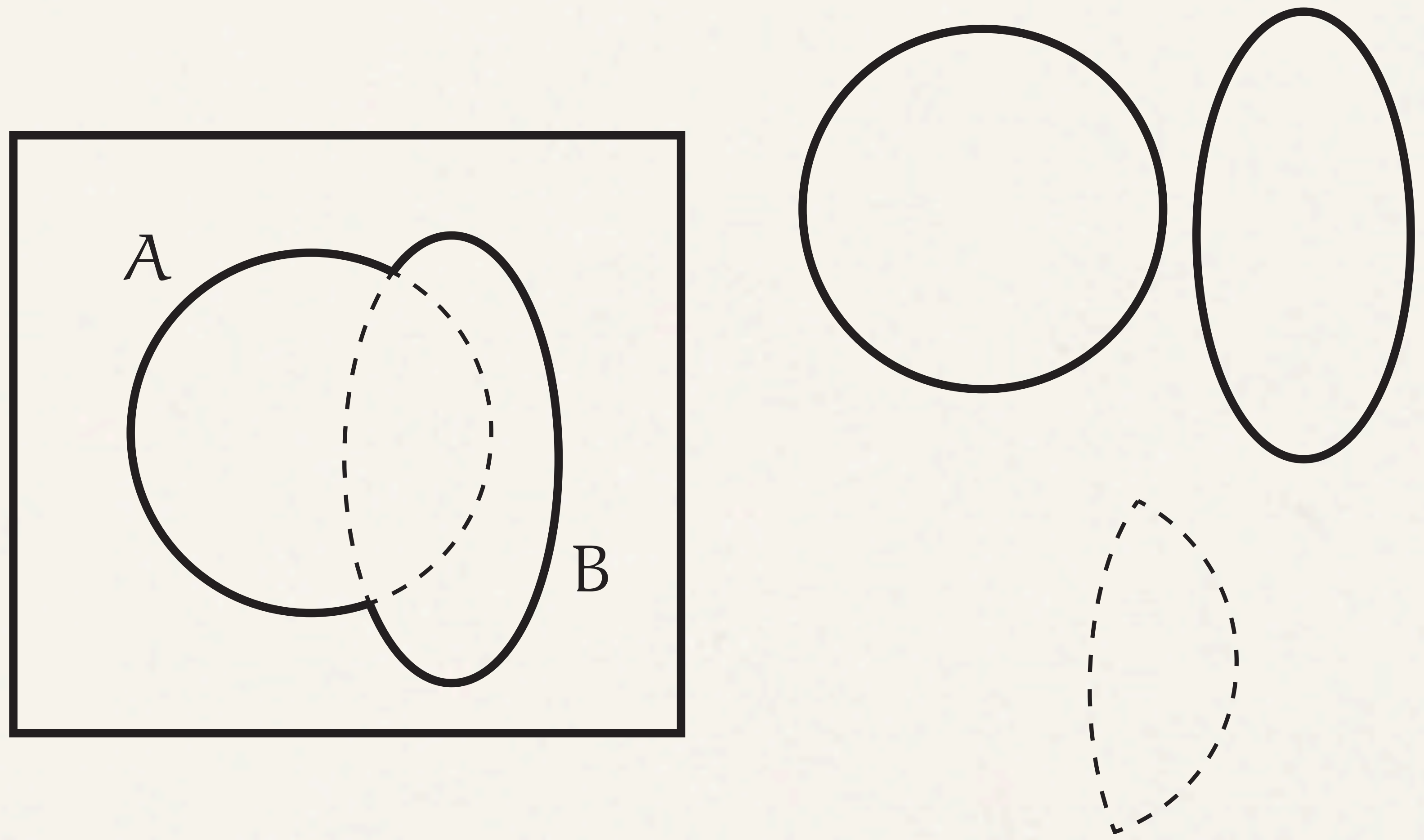

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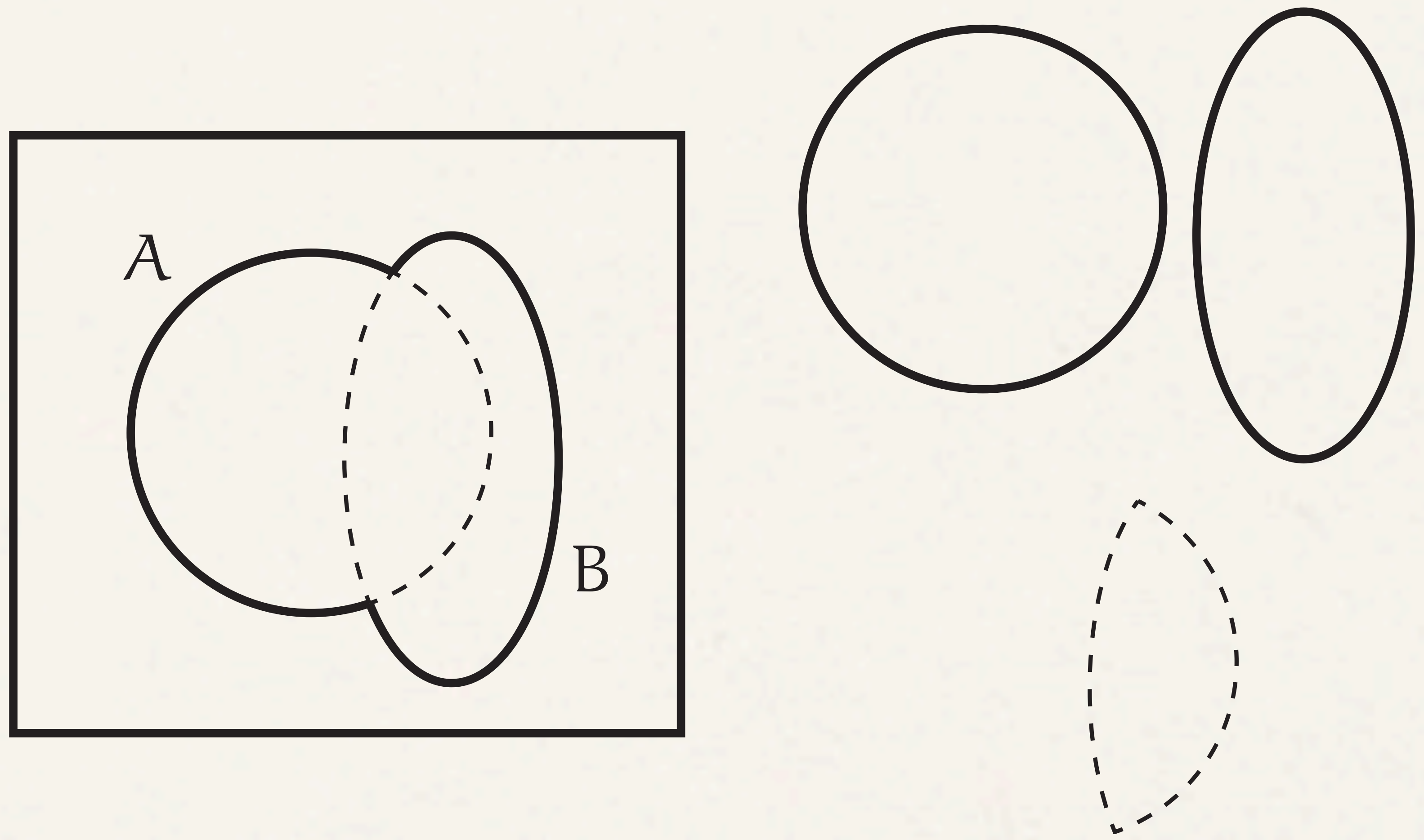

$$\mathbf{P}(A \cup B \cup C) = \mathbf{P}(A) + \mathbf{P}(B) + \mathbf{P}(C) - \mathbf{P}(A \cap B) - \mathbf{P}(A \cap C) - \mathbf{P}(B \cap C) + \mathbf{P}(A \cap B \cap C)$$

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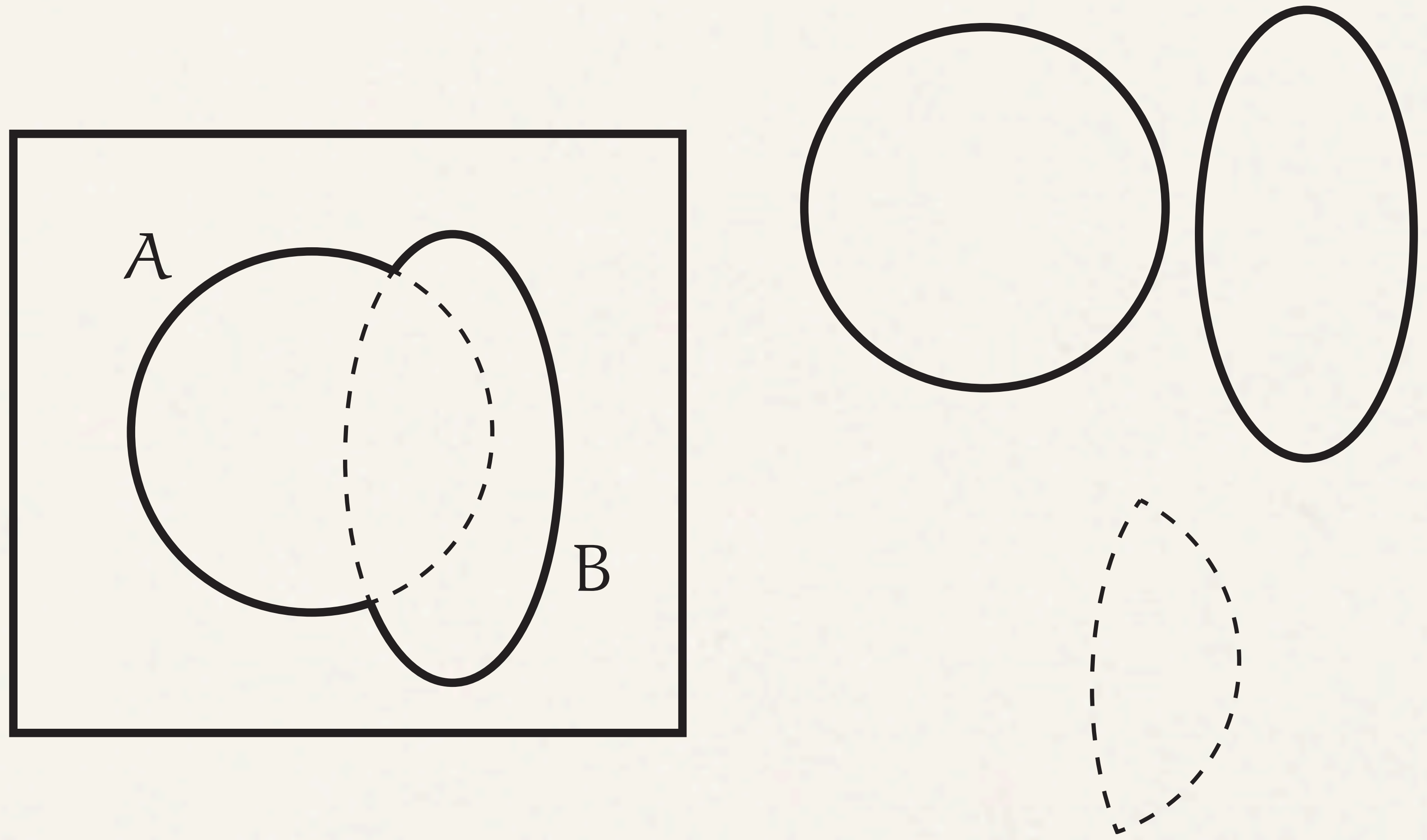

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$$S_k := \underbrace{\sum_{1 \leq j_1 < j_2 < \cdots < j_k \leq n} \mathbf{P}(A_{j_1} \cap A_{j_2} \cap \cdots \cap A_{j_k})}_{\text{sum of all k-wise intersections}} \qquad (1 \leq k \leq n)$$

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The inclusion–exclusion theorem (baby version)

$$\mathbf{P}(A_1 \cup A_2 \cup \cdots \cup A_n) = S_1 - S_2 + S_3 - \cdots + (-1)^{n-1} S_n$$