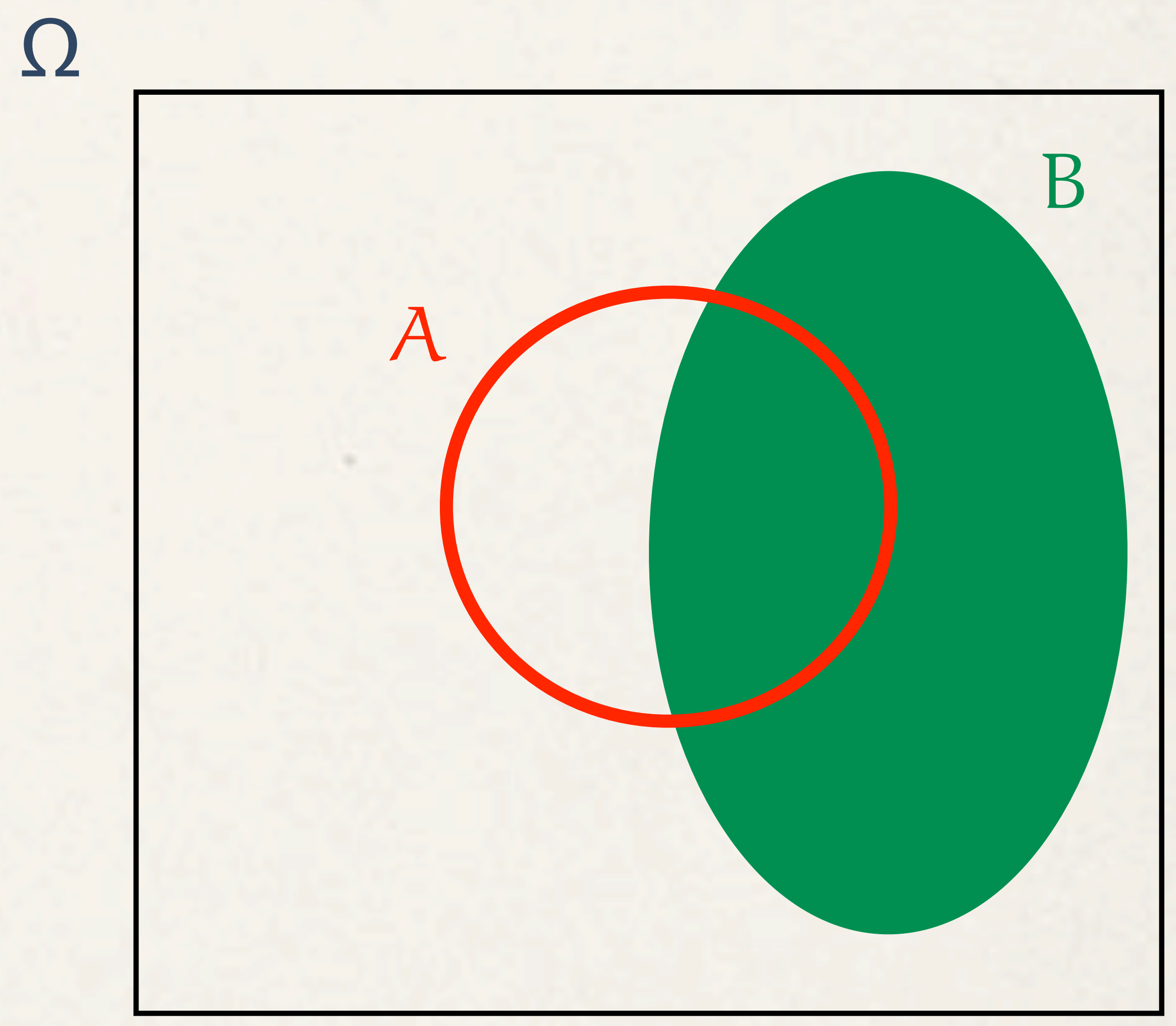
Conditional probability

The conditional probability of an event A given that an event B of positive probability has occurred (in short, the probability of A given B) is denoted $\mathbf{P}(A \mid B)$ and defined by

$$\mathbf{P}(A \mid B) = \frac{\mathbf{P}(A \cap B)}{\mathbf{P}(B)}$$

- The conditional probability $\mathbf{P}(A \mid B)$ is *undefined* if $\mathbf{P}(B) = 0$.
- The event B may be a *composite event* constructed via unions, intersections, and other set operations from other events.
- Conditional probability is *not symmetric*: in general, $\mathbf{P}(A \mid B) \neq \mathbf{P}(B \mid A)$.



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$$\mathbf{P}(A_1 \cap A_2 \cap \cdots \cap A_n) = \mathbf{P}(A_1 \mid A_2 \cap \cdots \cap A_n)\,\mathbf{P}(A_2 \mid A_3 \cap \cdots \cap A_n) \cdots \mathbf{P}(A_{n-1} \mid A_n)\,\mathbf{P}(A_n)$$

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$$\mathbf{P}(A_1 \cap A_2 \cap \cdots \cap A_n) = \prod_{j=1}^{n} \mathbf{P}(A_j \mid A_{j+1} \cap \cdots \cap A_n)$$