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Source Flow

Consider the velocity field of a fluid given by

$$oldsymbol{u}(x,y,z) = rac{\Lambda(xoldsymbol{i} + yoldsymbol{j} + zoldsymbol{k})}{4\pi(x^2 + y^2 + z^2)^{3/2}}.$$

(a) Using spherical coordinates, show that

$$oldsymbol{u}(r) = rac{\Lambda \hat{oldsymbol{r}}}{4\pi r^2}.$$

(b) Using spherical coordinates, show that

$$\nabla \cdot \boldsymbol{u} = 0$$

provided r
eq 0.

(c) Using the divergence theorem, show that

$$\int_{V} \mathbf{\nabla} \cdot \mathbf{u} \ dV = \Lambda,$$

provided that the volume V contains the origin, and is zero otherwise. You have therefore shown that the divergence of the velocity field is given by

$$oldsymbol{
abla}\cdotoldsymbol{u}=\Lambda\delta(oldsymbol{r})$$
,

where $\delta(m{r})$ is the three-dimensional Dirac delta function. This velocity field is called a source flow.

✓ Completed

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