A theorem of de Moivre and Laplace

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The de Moivre–Laplace limit theorem

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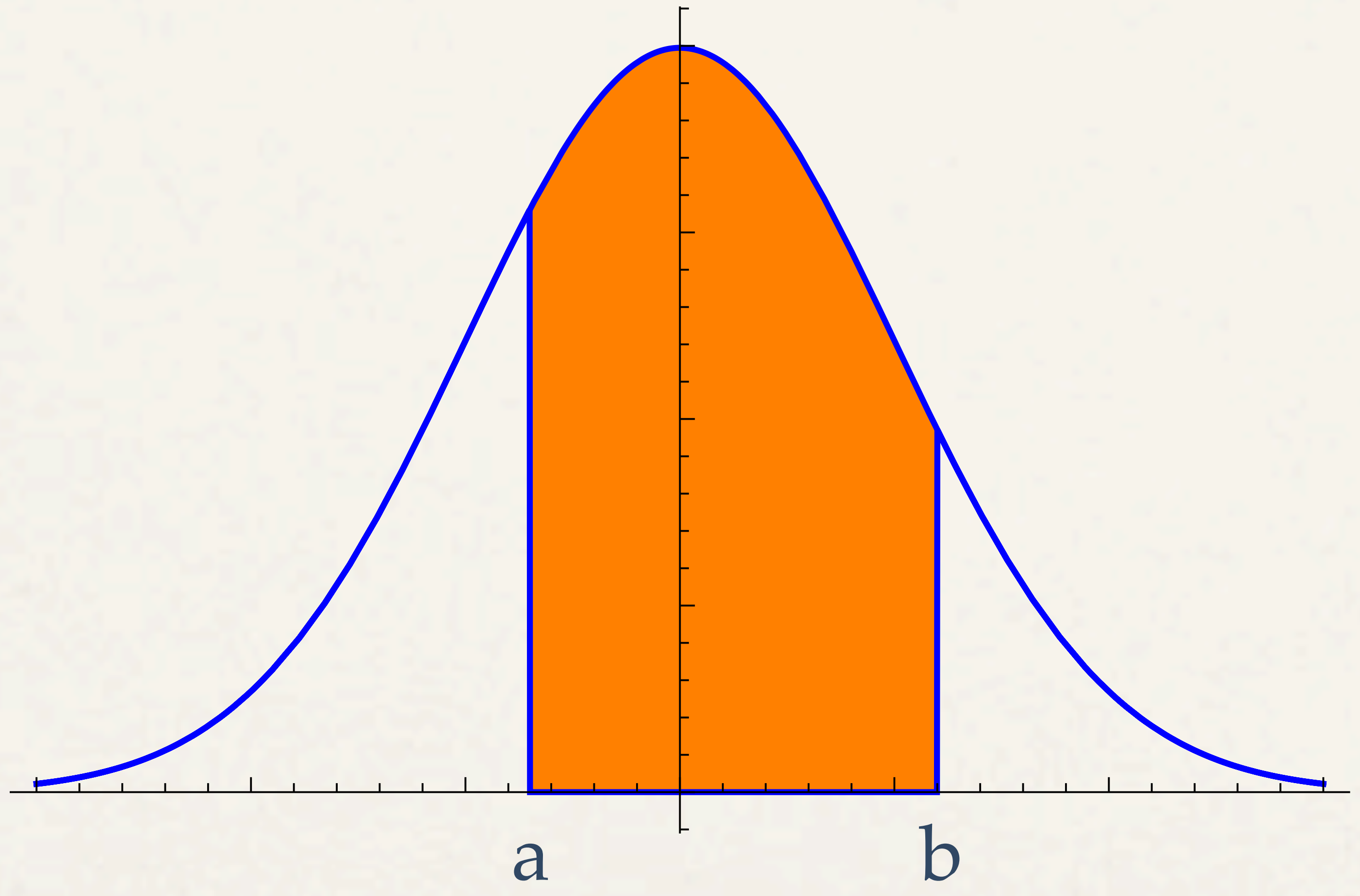

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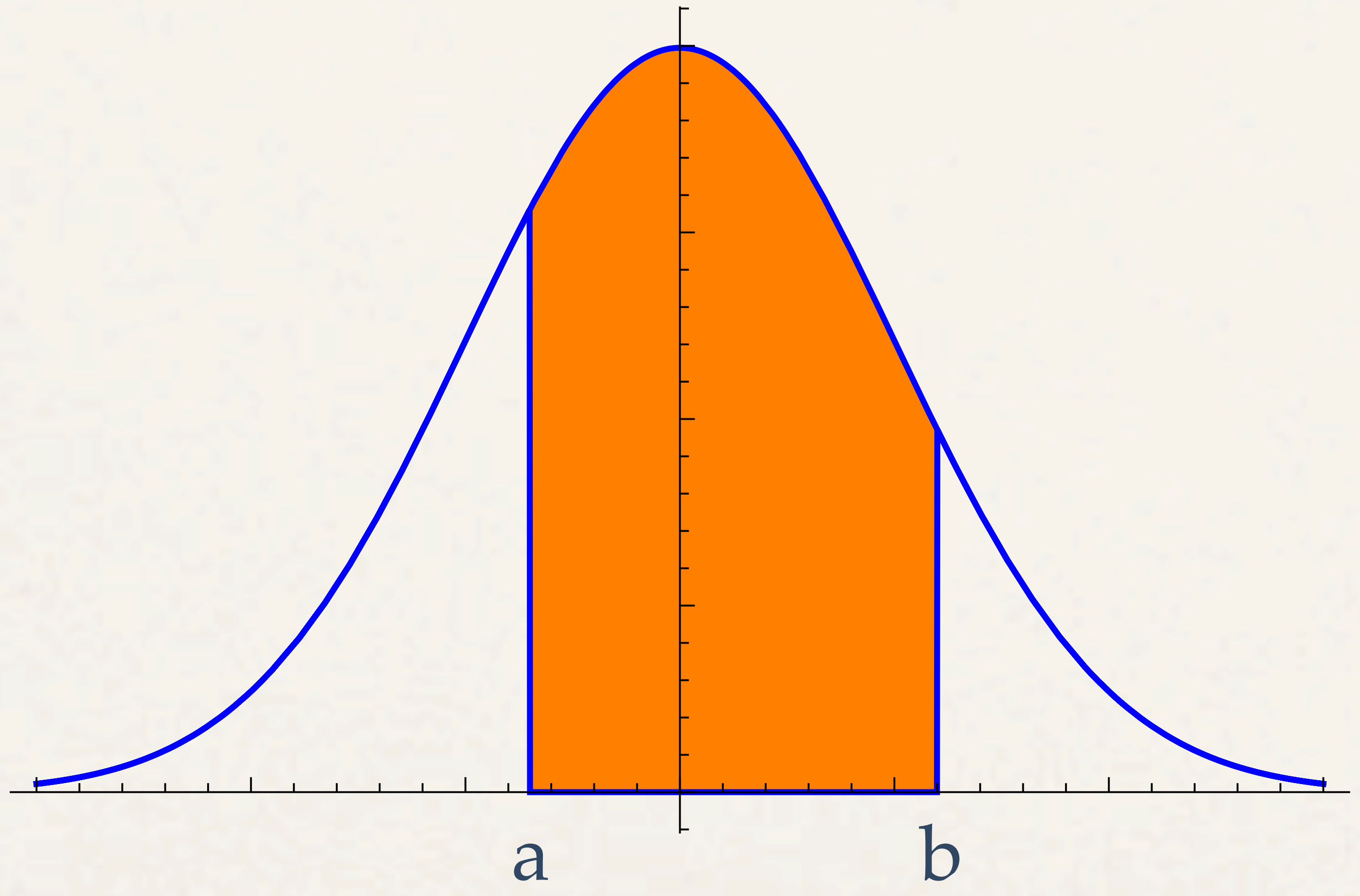
$$\mathbf{P}\{a < S_n^* \leq b\} \to \int_a^b \phi(x)\,dx = \Phi(b) - \Phi(a)$$



Slogan

Binomial probabilities (viewed in the proper scale)
are governed approximately by the area under the bell curve.