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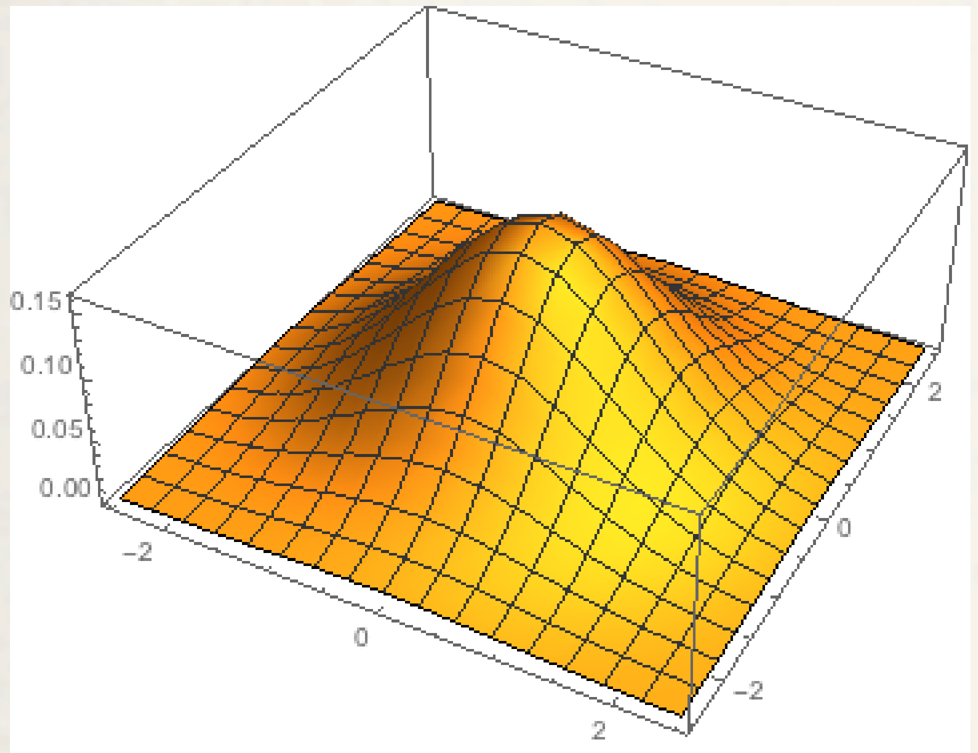

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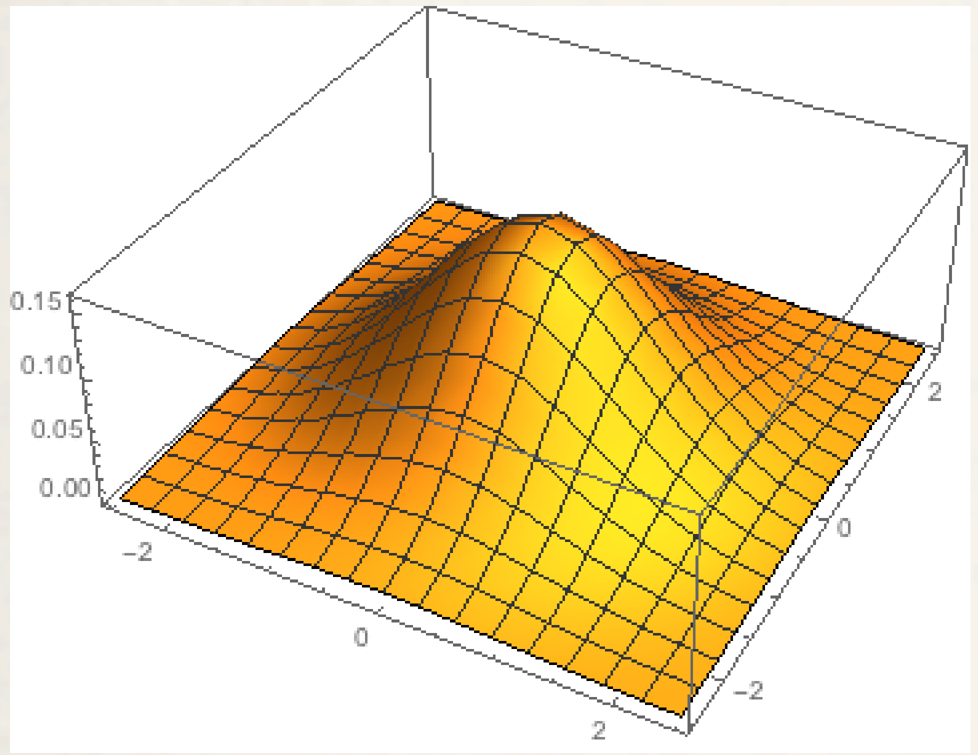

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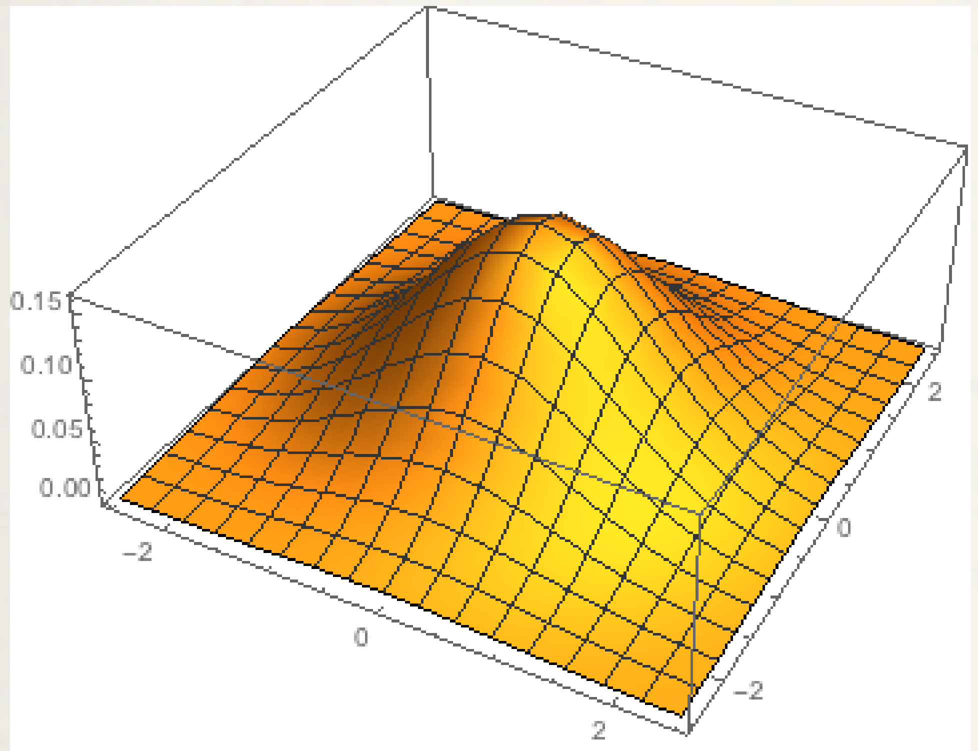

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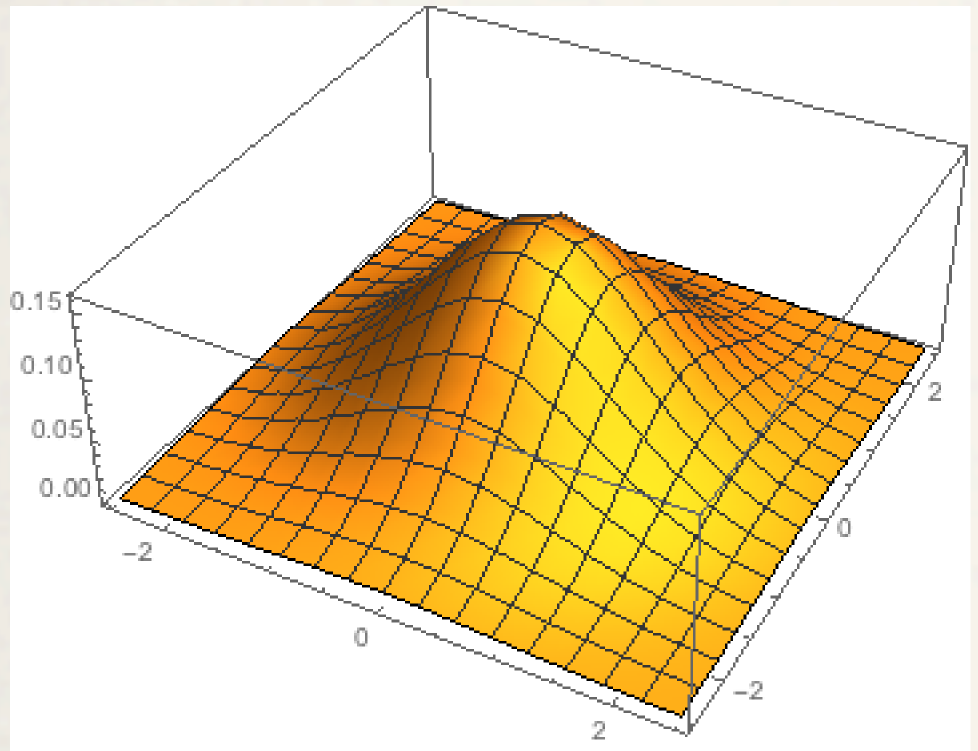

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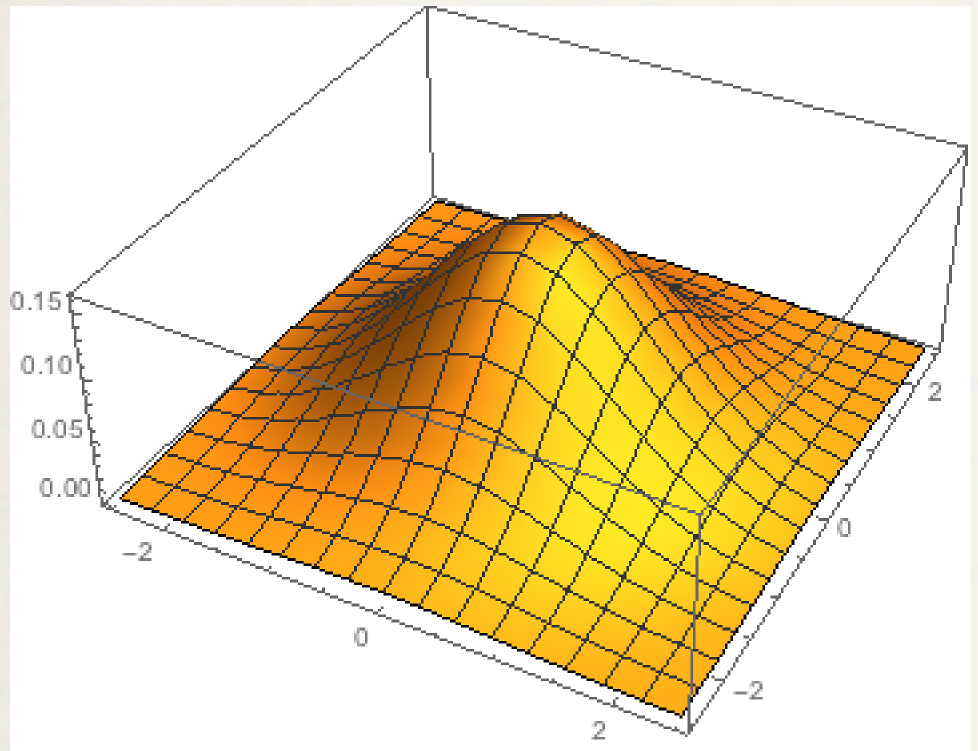

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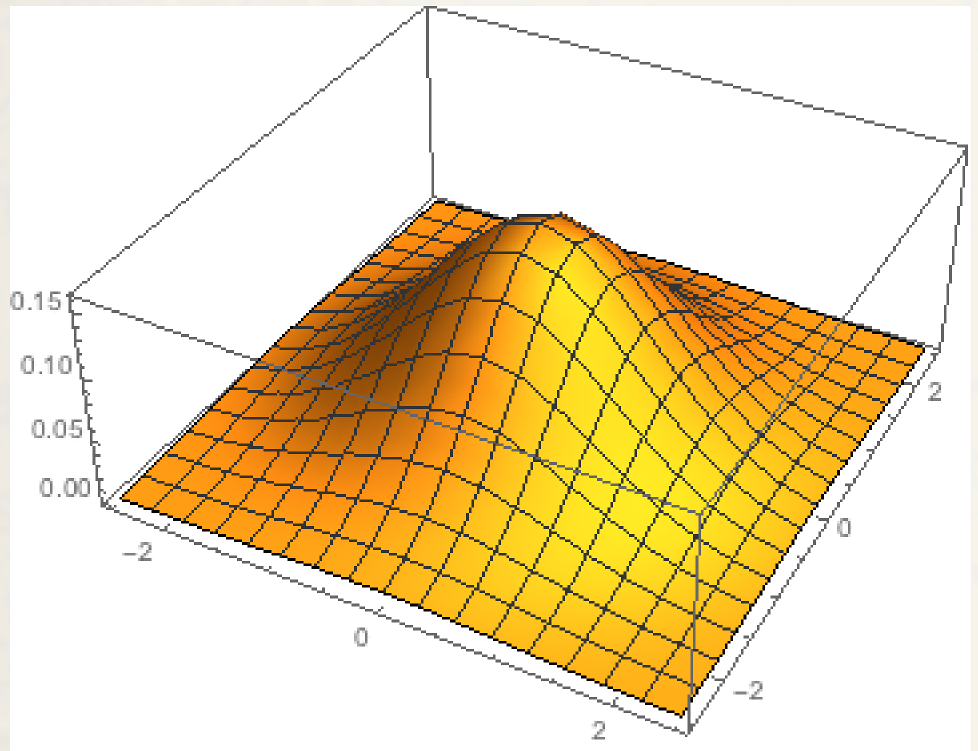

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$$= \left(\int_{-\infty}^{\infty} \phi(x)\,dx\right)\left(\int_{-\infty}^{\infty} \phi(y)\,dy\right) = 1 \cdot 1 = 1$$

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This is a two-dimensional normal density

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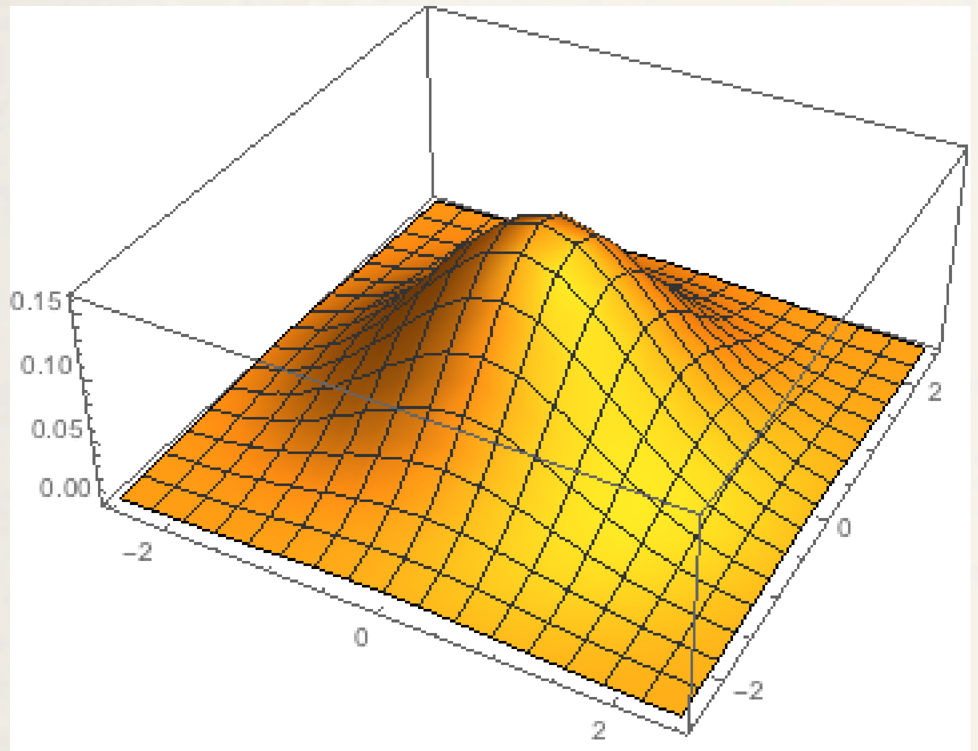

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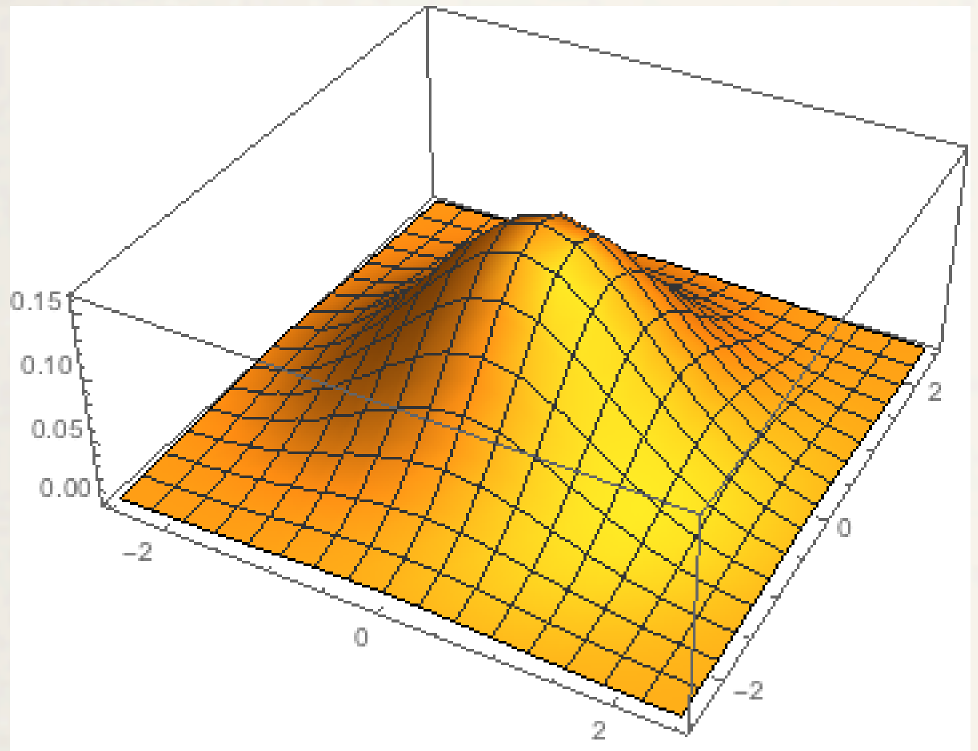

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Key observation: $p(x, y) = \phi(x) \cdot \phi(y)$

If $p_1(x_1), p_2(x_2), \ldots, p_n(x_n)$ are (one-dimensional) densities,
then $p(x_1, x_2, \ldots, x_n) = p_1(x_1)\, p_2(x_2) \cdots p_n(x_n)$ is a (n-dimensional) density.

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Slogan
The product of probability densities is a probability density.