Divisibility Tests

In some cases, one can check whether a is divisible by b without trying to divide by b.

Problem. What is the remainder and the quotient of 3756 when divided by 10?

Decimal system is really convenient here. Note that

$$3756 = 3750 + 6 = 375 \cdot 10 + 6.$$

Thus the quotient is 375 and the remainder is 6.

Clearly, this argument can be generalized to give the following.

Lemma. The remainder of an integer a when divided by 10 is the last digit of a, the quotent is an integer resulting from a by dropping its last digit off.

Corollary. An integer a is divisible by 10 if and only if its last digit is 0.

Problem. Is 7347 divisible by 5?

Let us try to use the same trick:

$$7347 = 734 \cdot 10 + 7 = 734 \cdot 2 \cdot 5 + 5 + 2.$$

We see that the remainder of 7347 when divided by 5 is 2 and thus 7347 is not divisible by 5.

We can generalize this to the following criteria for divisibility by 5.

Lemma. An integer a is divisible by 5 if and only if its last digit is either 0 or 5.

Indeed, let us denote the last digit of a by b. Then the last digit of $a - b$ is 0. Thus, $a - b$ is divisible by 5. We have shown above that this means that a and b have the same remainder when divided by 5. This remainder is 0 iff $b = 0$ or $b = 5$.

We can prove a similar statement for divisibility by 2.

Lemma. An integer a is divisible by 2 if and only if its last digit is either 0, 2, 4, 6, or 8.