Summary of Tableau 11, Part 1

The fabulous limit laws
Chebyshev’s enduring inequality, the magisterial law of large numbers

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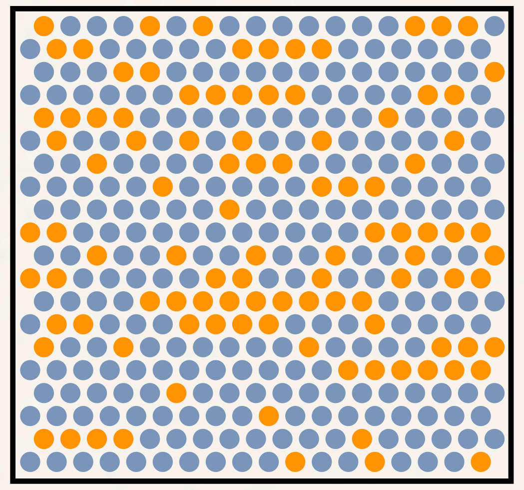
A model of a poll



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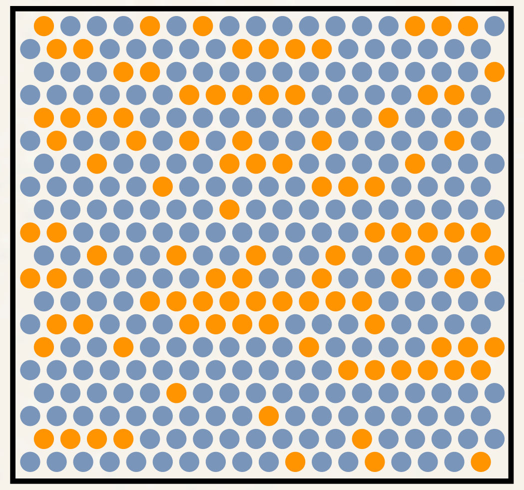

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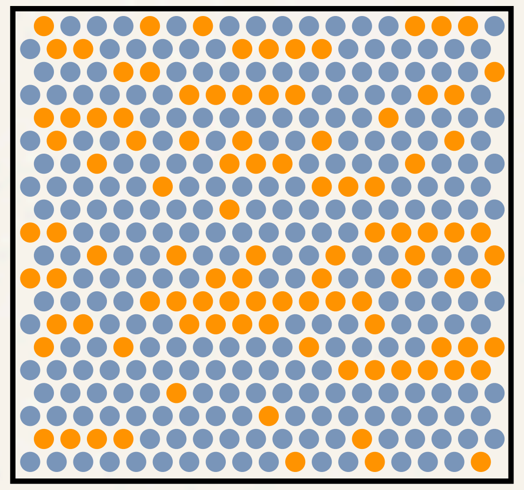

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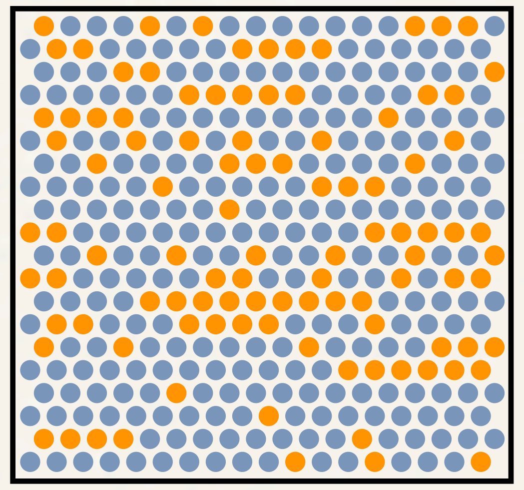

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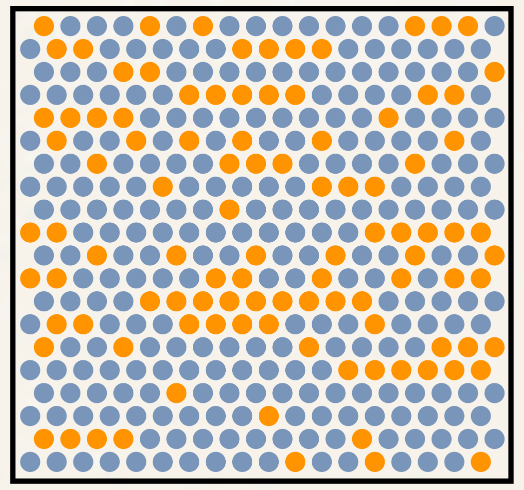

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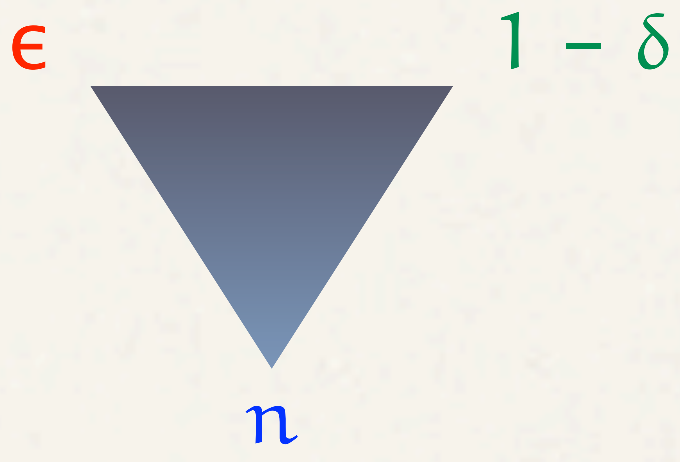
The dance of error, confidence, and sample size



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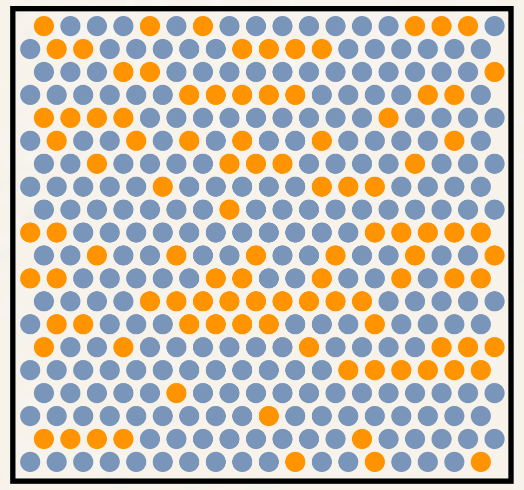

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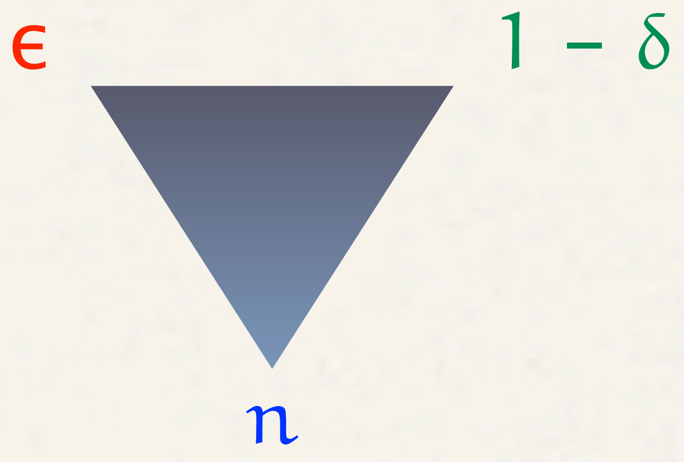

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$$\mathbf{P}\left\{\left|\frac{S_n}{n} - p\right| > \epsilon\right\} \leq \delta$$

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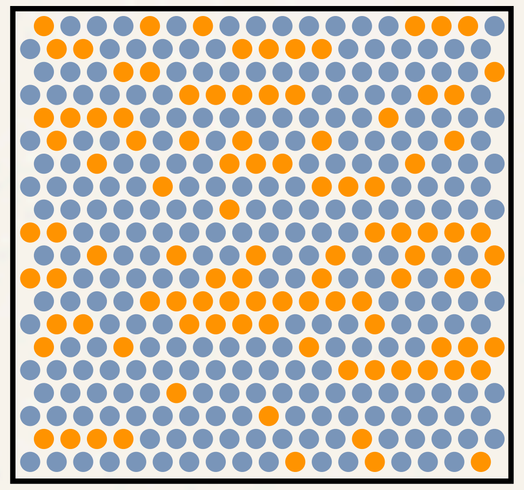

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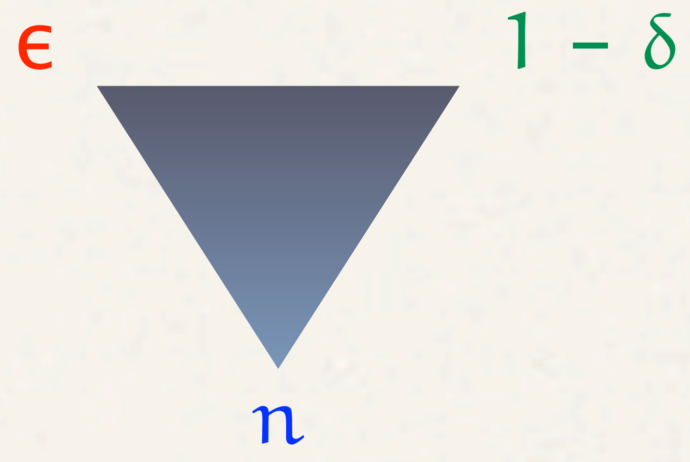

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Chebyshev's inequality, the law of large numbers

$$\mathbf{P}\left\{\left|\frac{S_n}{n} - p\right| > \epsilon\right\} \le \frac{1}{4n\epsilon^2} \to 0 \qquad (n \to \infty)$$

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  - A doubling of sample size for a given confidence.