1.3.5.8. Chi-Square Test for the Variance

Purpose:
Test if the variance is equal to a specified value

A chi-square test (Snedecor and Cochran, 1983) can be used to test if the variance of a population is equal to a specified value. This test can be either a two-sided test or a one-sided test. The two-sided version tests against the alternative that the true variance is either less than or greater than the specified value. The one-sided version only tests in one direction. The choice of a two-sided or one-sided test is determined by the problem. For example, if we are testing a new process, we may only be concerned if its variability is greater than the variability of the current process.

Definition

The chi-square hypothesis test is defined as:

H_0: $\sigma^2 = \sigma_0^2$

H_a: $\sigma^2 < \sigma_0^2$ for a lower one-tailed test

           $\sigma^2 > \sigma_0^2$ for an upper one-tailed test

           $\sigma^2 \neq \sigma_0^2$ for a two-tailed test

Test Statistic: $T = (N-1)(s/\sigma_0)^2$

where N is the sample size and s is the sample standard deviation. The key element of this formula is the ratio s/σ_0 which compares the ratio of the sample standard deviation to the target standard deviation. The more this ratio deviates from 1, the more likely we are to reject the null hypothesis.

Significance Level: α.

Critical Region: Reject the null hypothesis that the variance is a specified value, σ_0^2, if

$T > \chi^2_{1-\alpha,\ N-1}$ for an upper one-tailed alternative

$T < \chi^2_{\alpha,\ N-1}$ for a lower one-tailed alternative

$T < \chi^2_{\alpha/2,\ N-1}$ for a two-tailed alternative

      or

$$T > \chi^2_{1-\alpha/2,\ N-1}$$

where $\chi^2_{\cdot,\ N-1}$ is the [critical value](#) of the [chi-square distribution](#) with N - 1 degrees of freedom.

The formula for the hypothesis test can easily be converted to form an interval estimate for the variance:

$$\sqrt{\frac{(N-1)s^2}{\chi^2_{1-\alpha/2,\ N-1}}} \leq \sigma \leq \sqrt{\frac{(N-1)s^2}{\chi^2_{\alpha/2,\ N-1}}}$$

A confidence interval for the standard deviation is computed by taking the square root of the upper and lower limits of the confidence interval for the variance.

Chi-Square Test Example

A chi-square test was performed for the [GEAR.DAT](#) data set. The observed variance for the 100 measurements of gear diameter is 0.00003969 (the standard deviation is 0.0063). We will test the null hypothesis that the true variance is equal to 0.01.

H_0: σ^2 = 0.01
H_a: σ^2 ≠ 0.01

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Test statistic:    T = 0.3903
Degrees of freedom:  N - 1 = 99
Significance level:  α = 0.05
Critical values:   X²α/2,N-1 = 73.361
                   X²1-α/2,N-1 = 128.422
Critical region: Reject H₀ if T < 73.361 or T > 128.422
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The test statistic value of 0.3903 is much smaller than the lower critical value, so we reject the null hypothesis and conclude that the variance is not equal to 0.01.

Questions

The chi-square test can be used to answer the following questions:

1. Is the variance equal to some pre-determined threshold value?
2. Is the variance greater than some pre-determined threshold value?
3. Is the variance less than some pre-determined threshold value?

Related Techniques

[F Test](#)
[Bartlett Test](#)
[Levene Test](#)

Software

The chi-square test for the variance is available in many general purpose statistical software programs. Both [Dataplot code](#) and [R code](#) can be used to generate the analyses in this section. These scripts use the [GEAR.DAT](#) data file.