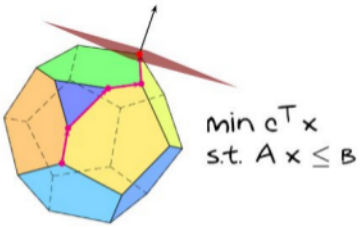
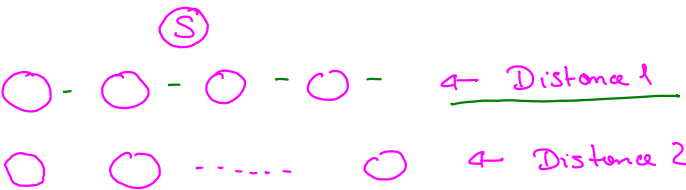
How efficient is the simplex method?

- Connected layer families
- The induction operation



Connected layer families

$G_p = (V, E)$, $V \subseteq \binom{[m]}{n}$ Diameter realized by $s, t \in V$

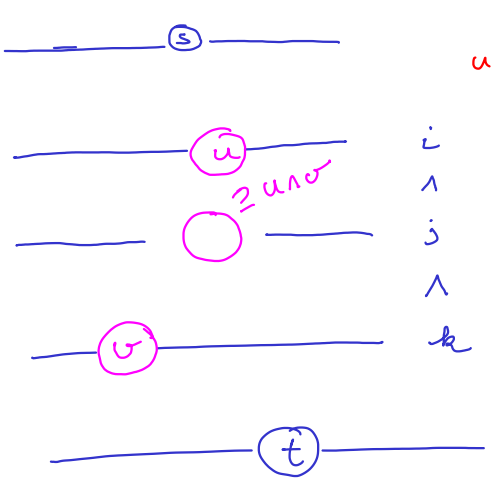


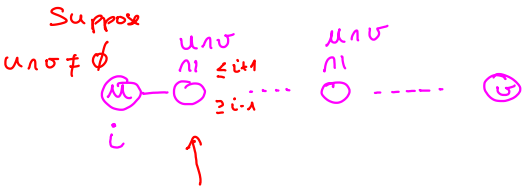
Partitions V into Layers

$\mathcal{L}_0 = \{s\}$, $\mathcal{L}_1 = \{v : d(s,v) = 1\}$

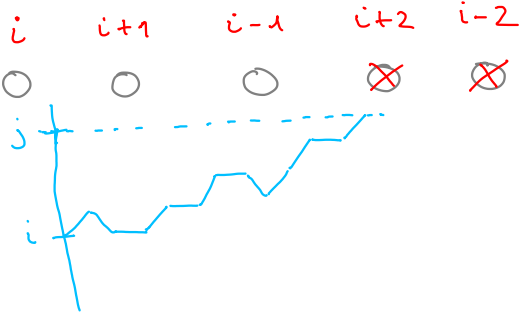
$\mathcal{L}_\Delta = \{\dots, t, \dots\}$

Connected layer families (cont.)





Which are impossible layers



Connected layer families (definition)

Connected layer family

A connected layer family is a sequence of sets $\mathcal{L}_i \subseteq \binom{[m]}{n}$ for $i = 1, \ldots, \ell$ where the sets $\mathcal{L}_i$ satisfy the following conditions.

a) *Disjointness*: for all $1 \leqslant i \neq j \leqslant \ell$, $\mathcal{L}_i \cap \mathcal{L}_j = \emptyset$;

b) *Connectivity*: for all $1 \leqslant i < j < k \leqslant \ell$ and $u \in \mathcal{L}_i$, $v \in \mathcal{L}_k$ there is a $w \in \mathcal{L}_j$ such that $u \cap v \subseteq w$.

Vertices: $\bigcup_{i=1}^{\ell} \mathcal{L}_i$,

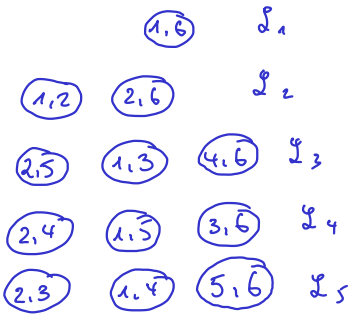
Symbols: $\{1, \ldots, m\}$

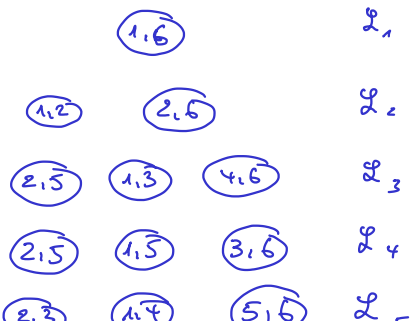
Dimension: n

$\mathcal{L}_i$: layers

Quiz

Which of the following is not a connected layer fam.

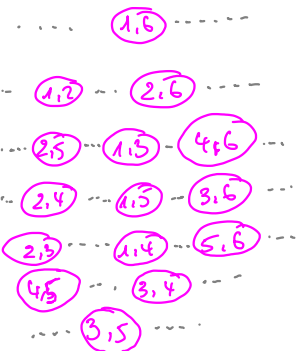




The induction operation

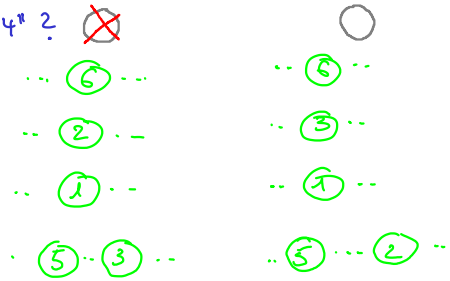
Let $\mathcal{L}_1, \ldots, \mathcal{L}_\ell$ be a connected layer family with m symbols in dimension n and let $a \in \{1, \ldots, m\}$ be a symbol. The *induction* on a is the following sequence of operations.

1. Remove all vertices from the connected layer family that do not contain a.
2. Remove a from all vertices.
3. Remove all empty layers.



Which is the result of ind.

on "4"?



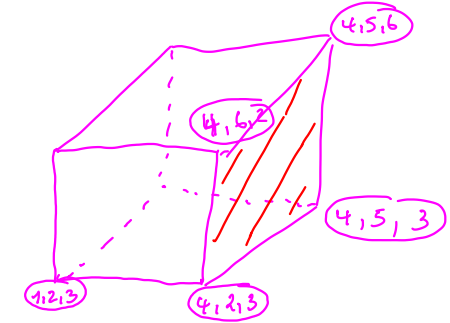
Induction results in connected layer family

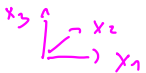
Lemma

Given some n-dimensional connected layer family with m symbols and suppose that the symbol a occurs in some vertex. Induction on a results in a $n-1$-dimensional connected layer family with $m-1$ symbols.

$n \geq 2, \quad m \geq n$

Geometric interpretation





$x_1 \geq 0$

$x_2 \geq 0$

$x_3 \geq 0$

$x_1 \leq 1$

$x_2 \leq 1$

$x_3 \leq 1$