A familial paradox

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1/2?

- *Sample space*: it is natural to list the gender of the child by age

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$$\Omega = \{\mathfrak{B}\mathfrak{B}, \mathfrak{B}\mathfrak{G}, \mathfrak{G}\mathfrak{B}, \mathfrak{G}\mathfrak{G}\}$$

- The *events of interest:*
  A := both children are boys = $\{\mathfrak{B}\mathfrak{B}\}$
  B := (at least) one child is a boy = $\{\mathfrak{B}\mathfrak{B}, \mathfrak{B}\mathfrak{G}, \mathfrak{G}\mathfrak{B}\}$

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- *Probability measure* **P**:
  - Combinatorial setting: to each *atom* (singleton set) assign equal *probability mass*

    $$\mathbf{P}\{\mathfrak{BB}\} = \mathbf{P}\{\mathfrak{BG}\} = \mathbf{P}\{\mathfrak{GB}\} = \mathbf{P}\{\mathfrak{GG}\} = 1/4$$

  - Event probabilities via additivity:

    $$\mathbf{P}(A) = \mathbf{P}\{\mathfrak{BB}\} = 1/4$$

    $$\mathbf{P}(B) = \mathbf{P}\{\mathfrak{BB}\} + \mathbf{P}\{\mathfrak{BG}\} + \mathbf{P}\{\mathfrak{GB}\} = 3/4$$

Ω

$\mathfrak{B}\mathfrak{B}$	$\mathfrak{B}\mathfrak{G}$
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The chance of A
(without side information)
$\mathbf{P}(A) = 1/4$

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Conditioned on (at least)
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B

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Conditioned on (at least) one boy

The chance of A
(given that B occurs)

$$\frac{\mathbf{P}(A \cap B)}{\mathbf{P}(B)} = \frac{1/4}{3/4} = \frac{1}{3}$$

B

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