1. $(a+b)'(a+b) = a'a + a'b + b'a + b'b = a'a + 2a'b + b'b$

$b'a$ = scalar

2. $$aa' = \begin{pmatrix} a_1a_1 & a_1a_2 & \cdots & a_1a_p \\ a_2a_1 & a_2a_2 & \cdots & a_2a_p \\ \cdots & \cdots & \ddots & \cdots \\ a_pa_1 & a_pa_2 & \cdots & a_pa_p \end{pmatrix}$$

$$tr(aa') = \sum_{i=1}^{p} (aa')_{ii} = \sum_{i=1}^{p} a_i^2 = a'a$$

3. $$tr(cA) = \sum_{i=1}^{p} (cA)_{ii} = \sum_{i=1}^{p} ca_{ii} = c\sum_{i=1}^{p} a_{ii} = c\,tr(A)$$

4. $cAB = I$ $(p \times p)$ identity matrix

$$cAB = I$$
$$AB = \frac{1}{c} I$$
$$A^{-1}AB = \frac{1}{c} A^{-1} I$$
$$B = \frac{1}{c} A^{-1}$$

5. $B(\rho) \cdot A = I$

$$B(\rho) \cdot A = \frac{1}{\rho}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \frac{1}{\rho}\begin{pmatrix} da-bc & db-bd \\ -ca+ac & -cb+ad \end{pmatrix} =$$

$$\frac{1}{\rho}\begin{pmatrix} ad-bc & 0 \\ 0 & ad-bc \end{pmatrix}$$

$\rho = ad - bc \qquad ad \neq bc$

6. $\iota'\iota = \sum_{i=1}^{p} \iota_i \iota_i = \sum_{i=1}^{p} 1 = p$

$(\iota'\iota)^2 = \iota\iota'\iota\iota' = \iota(\iota'\iota)\iota' = p\,\iota\iota'$