Hypothesis Testing for Homoscedasticity of the Coefficient of Restitution of Table Tennis Balls at Various Temperatures

Abstract

I made a project to determine whether or not a table tennis ball bounce is homoscedastic at different temperatures. Homoscedasticity is the idea that the variances or standard deviations are the same between the values or temperatures. I built a stand and placed a board on the ground for the balls to bounce on. I dropped ping pong balls from 90 cm and measured the bounce height with the help of a digital camera and a ruler. I dropped the balls at 6 different temperatures all measured with a CBL hooked up to a temperature sensor.

After the bounces were recorded into an excel spreadsheet I found the test statistics for the data like the variance, standard deviation, mean and a 98% confidence interval for the mean. I also converted all the values to the coefficient of restitution for a more useful approach. I created a linear regression for the values to see if the temperatures correlated to the coefficient of restitution. The temperatures had to be transformed as the data was not linear based on my residual plot, rather more logarithmic. When the data was transformed a better linear regression could be made to show the correlation between temperature and coefficient of restitution. I then proceeded to test my hypothesis and find the results. Using the variances and an F_{max} table I did a hypothesis F-Test, which is really the first step of an ANOVA test, analysis of variance test to determine whether or not the values are homoscedastic. My calculated F_{max} value was less than the table value which means I failed to reject H_o and that I am 95% sure that the coefficient of restitution is homoscedastic at my temperatures.

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Purpose/ Hypothesis

My research was to find if the bounce table tennis balls was homoscedastic at different temperatures. My null hypothesis is that the variances of the bounce heights were the same, which means they are homoscedastic.

Introduction

As an avid ping pong player, I like to be able to play some table tennis anywhere I go. Unfortunately I live in Fairbanks Alaska which means the cold temperatures in the winter restrict my outdoor table tennis playing. As a cold weather enthusiast I assumed that the temperature is most likely wrecking my game on my outdoor table. I wondered if the temperatures actually changed the bounce height of the balls and hurt my game. But playing the sport long enough I realized bounce height really isn't as big of a deal as consistency of the bounce height. Not knowing enough about ping pong balls and the standard deviation of their bounce height correlating to temperatures I decided it was important to learn about. I figured I would design a project that would test whether temperature truly affects the standard deviation of the bounciness of table tennis balls.

Literature Review

I started by researching what table tennis balls are made of. After many websites and published documents I found out that the normal ping pong ball is made out of celluloid or cellulose nitrate (Esquivel, n.d.). It was one of the first synthetic plastics ever created and has gone from uses in billiard balls and film strips to the common uses in ping pong balls and guitar picks (Wiley, n.d.). A table tennis ball weighs 2.7 grams, is 40 mm in diameter and is filled with air. When dropped from 30 cm onto a steel plate, a table tennis ball should rebound 23 cm (Esquivel, n.d.). Using those numbers and the equation for the coefficient of restitution, or "bounciness" a ping pong ball has the number of .88 on a steel plate.

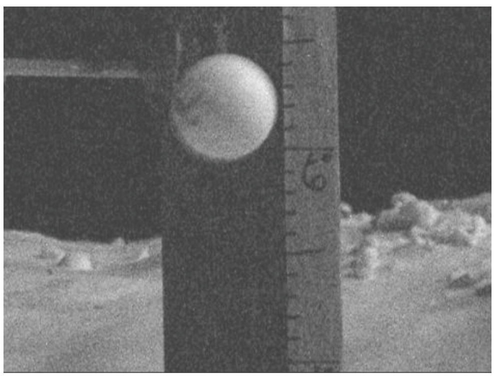
I also researched other papers related to my project and found one using a bouncy ball and correlating it to warmer temperatures. They hypothesized that the higher the temperature of the ball, the larger the coefficient of restitution. The project resulted in that there is an inverse exponential relationship between temperature and a rubber ball bouncing on linoleum. A rubber bouncy ball is much different than a cellulose air filled table tennis ball, but the idea of the project is similar to mine (Tamiya, 2010).

Experiment

To start my experiment I had to collect my materials. I went to a local grocery store and randomly chose a box of thirty 40 mm ping pong balls, using a less than random technique of mixing the three different boxes around on the floor and picking one. It was a cluster sampling technique as the balls come in "clusters" or boxes and I planned to test the entire cluster as representative of the population. I obtained a CBL, a Texas Instruments Calculator-Based Laboratory System, with a temperature sensor attached so I could accurately and consistently measures temperatures in different locations. I knew I needed a consistent way to drop and measure the bounce of the balls so I built a stand. Using some 2x4s and other wood scraps I built a tower that allowed me to drop a ball at 90 cm and had one cm increments up the side to measure the bounce. I chose a hard wood board for the ball to bounce on so I could transport it easily. Although the specifications for an official table tennis ball were made on a steel plate, I did not care what the height of the ball was and figured as long as the surface is consistent, the variance of the bounce heights should not be affected, wood of steel. A normal digital camera on a tripod was put the same distance and height away to record the bounce of the balls so I could see the height in the warmth of my house. The camera was not high speed, but has fast enough frames that when I went frame by frame the apex of the bounce could be recorded accurately.

The first data I recorded in my house to make sure it all worked. I dumped out the balls and jumbled them up to make it relatively random when I chose one to drop so that extraneous factors and confounding variables that have to do of which ball was dropped first like wind, sudden temperature changes and the such would be avoided. I dropped all thirty of the balls when the camera recorded their height. After viewing the playback of the bounces and recorded their heights into Excel I went to apply the other treatments. I went outside my house and repeated the experiment at -23.4° C and recorded the data.

Before I started dropping the balls outside, I left all the materials exposed to the cold temperature for over 20 minutes so they would become air temperature and dropped them randomly again to avoid various extraneous factors. I continued to do this for every single treatment. When putting up the board, stand, and tripod I used a level to make sure the balls bounced relatively straight. The next few weeks were spent driving around town to find the coldest temperatures possible, and respectively recorded -32.1°, -35.6°, -13.4°, and -5.34°. After all the data was recorded and entered into an Excel spreadsheet the interpretation had to begin.



Ball Bounce recorded by Camera

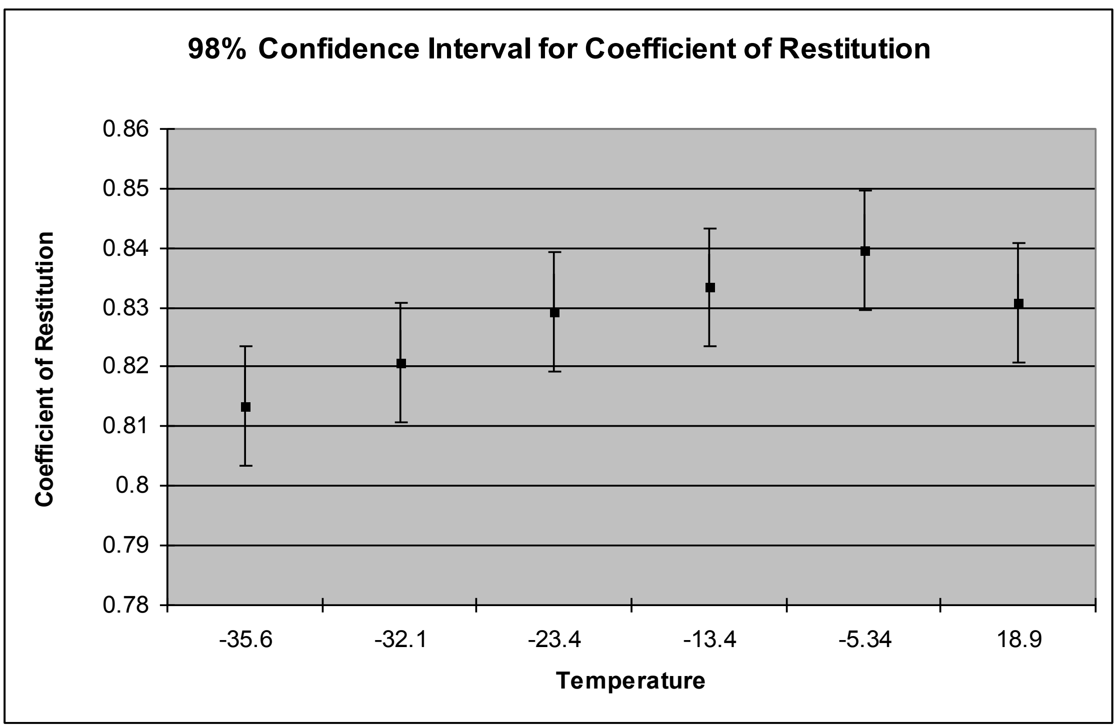
Set up of ball bouncing apparatus

Results

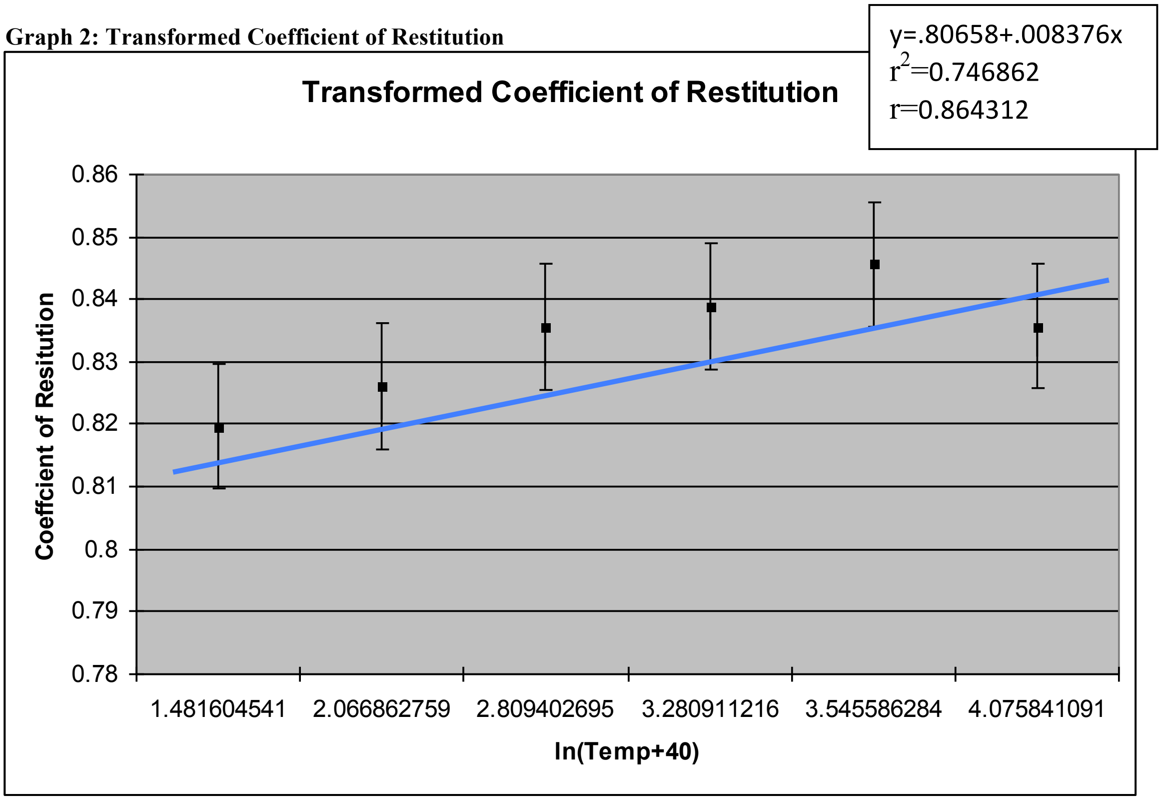
To start interpreting the data I found the mean, standard deviation, variance, and a 98% confidence interval for each mean, shown in Table 2 and Table 3 (see appendix). I also converted all the bounce heights to the coefficient of restitution with the formula, $C_R = \sqrt{\frac{h_b}{h_i}}$ (Tamiya, 2010) where h_b is the height of the bounce and h_i is the initial height of the object.

To get an understanding of the regression I was dealing with I put the confidence intervals into a regression plot. After looking at the residual plot and realizing it was not linear, as shown in Graph 4 in the appendix, I did a transformation and got an R squared value of .747 which means 74.7% of the variability can be attributed to a linear relationship between the natural log of the temperatures plus 40 and the coefficient of restitution of ping pong balls.. I added 40 to all the y-values (temperatures) to make them positive so I could log them and get a much more linear graph. The resulting regression lines for coefficient of restitution and transformed are shown in Graph 1 and Graph 2 respectively. The graphs of bounce height and the residual plot for the Graph 2 are shown in the appendix as Graph 3 and 4. As interesting as the regression lines were I had to begin with my hypothesis test.

Graph 1: 98% Confidence Interval for Coefficient of Restitution



Graph 2: Transformed Coefficient of Restitution



The hypothesis test is an F-Test to test Homoscedasticity. All the assumptions are met as the sample size is sufficiently large, $n \geq 30$. The samples are also independent as one bounce did not affect the next bounce of a ball. The box I used is a cluster of the population and is not more than 10% of the population to hypothesis testing can give statistically significant results.

My Null Hypothesis is that the variances of the data are equal, the data is homoscedastic. The Alternative Hypothesis that the variances are not equal, or the data is heteroscedastic. The alpha I used is .05, or .05 significance level.

To test homoscedasticity variances are looked at to see if they are relatively the same. Looking at the ratio between the maximum and minimum variance in the group gives a probability distribution for equal variances, an F_{max} value. If the variances were all the same, when the max and min were divided they would equal 1.00. As the world is not perfect, a table was created that gives a number to compare the F_{max} value to and reject or fail to reject H_o. The data has 30 data points, or thirty coefficient of restitution values, so it has 29 degrees of freedom as d.f. is calculated by n-1. There are six different samples, six temperatures, so the k value is 6. Looking at Table 1 the highlighted value is what the

F_{max} value is tested against, 30 d.f, 6 groups – d.f.=29 was unobtainable for me but 30 should give close enough results.

The hypothesis test is carried out as follows:

$S_a{}^2 = variance\ of\ Coefficient\ of\ Resititution\ for\ -35.6℃$

$S_b{}^2 = variance\ of\ Coefficient\ of\ Resititution\ for\ -32.1℃$

$S_c{}^2 = variance\ of\ Coefficient\ of\ Resititution\ for\ -23.4℃$

$S_d{}^2 = variance\ of\ Coefficient\ of\ Resititution\ for\ -13.4℃$

$S_e{}^2 = variance\ of\ Coefficient\ of\ Resititution\ for\ -5.34℃$

$S_f{}^2 = variance\ of\ Coefficient\ of\ Resititution\ for\ 18.9℃$

$$H_o: S_a{}^2 = S_b{}^2 = S_c{}^2 = S_d{}^2 = S_e{}^2 = S_f{}^2$$

$$H_a: Variances\ are\ not\ equal$$

$F_{max} = \frac{s_{max}{}^2}{s_{min}{}^2}$ $\quad F_{max} = \frac{.007210854}{.005374423}$ $\quad F_{max} = 1.80015$

$$F_{max} = F_{\max\alpha\,[k,n-1]}$$

$$1.80015 < 2.91$$

Table 1: F_{max} Table

F_{max} Table for α=.05					
d.f.	Number of Levels (Groups, Categories) (k)				
	2	3	4	5	6
2	39	87.5	142	202	266
3	15.4	27.8	39.2	50.7	62
4	9.6	15.5	20.6	25.2	29.5
5	7.15	10.8	13.7	16.3	18.7
6	5.82	9.38	10.4	12.1	13.7
7	4.99	6.94	8.44	9.7	10.8
8	4.43	6	7.18	8.12	9.03
9	4.03	5.34	6.31	7.11	7.8
10	3.72	4.85	5.67	6.34	6.92
12	3.28	4.16	4.79	5.3	5.72
15	2.86	3.54	4.01	4.37	4.68
20	2.46	2.95	3.29	3.54	3.76
30	2.07	2.4	2.61	2.78	2.91
∞	1.00	1.00	1.00	1.00	1.00

Conclusion

As the calculated F_{max} value is less than the table value I fail to reject H_o. That means that I am 95% sure that the coefficient of restitution of ping pong balls is homoscedastic at the different temperatures I measured. The variances of the bounce heights at the different temperatures are statistically the same, so the bounces have the same consistency at the temperatures.

Variances and their squares, standard deviations, are used widely in statistics and are sometimes assumed to be the same or not a problem in hypothesis tests. However, many variances are different between two categories which can affect the results like in some types of Chi-squared testing. When making statistical justifications it is important to note standard deviations and consider how they affect the real life situation of the problem. My experiment was to prove something that many people automatically assume, that the balls bounce pretty consistently. The fact they are homoscedastic shows how exact the machinery used to create them and the type of plastic that is used. Doing a simple linear regression or even various 2-sample T tests would have been interesting as well. Using a hypothesis test against the claimed "ping pong ball standard height" could have given me interesting results. However the F_{max} part of an ANOVA test uses more statistical knowledge and seems much more interesting to me. Variances are essential to many parts of inferential statistics and an experiment based around them is important.

There are other extraneous factors and confounding variables that I encountered that could affect the results of this experiment. The balls did not always bounce straight up for many reason creating a type of measurement bias, the way I helped get around this is using the camera. Because the camera was always at the same angle the inconsistent bounces were recorded for each temperature, which means each temperature had error, but about the same amount of error when I recorded it. The quality of the balls may have degraded throughout the experiment but there was not much I could do about that less than doing the whole thing in one day. The temperature order I did was random so the worn out or dirty balls didn't confound with the warmer/ colder temperatures. Other confounding variables I had to ignore were things like moisture in the air that may have affected ball rebound. If I were to do this project again I would probably have a type of dispenser that dropped the balls evenly and without spin. That way the extraneous factors would be limited, and my hands would not freeze off in the temperatures around 30 degrees below zero Fahrenheit. The experiment was far from perfect as far as recording and dropping the balls, however it was still good to use statistics in something related to the real world.

Appendix

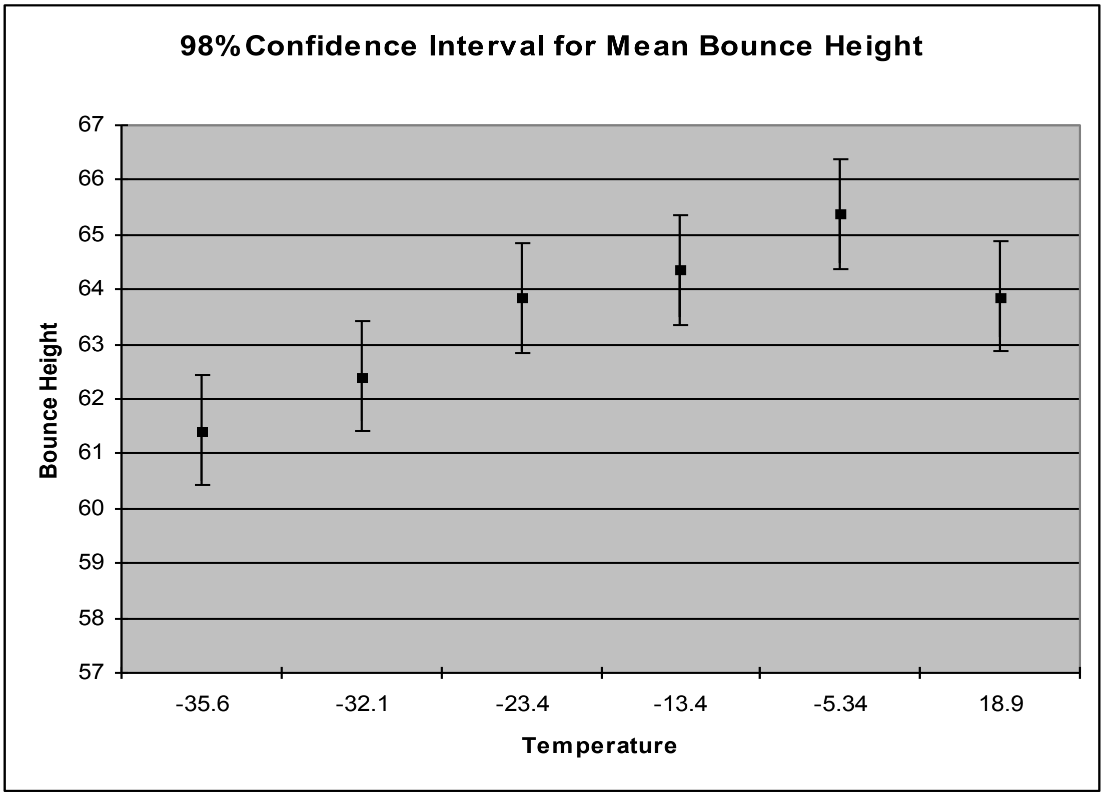
Table 2: Bounce Heights in Cm

Temperature in ° C						
-35.6	-32.1	-23.4	-13.4	-5.34	18.9	
63	60	64	63	64	62	
61	63	64	62	65	63	
61	61	63	64	65	64	
61	62	63	64	66	64	
61	61	65	63	66	63	
60	63	64	63	63	64	
60	61	63	64	65	62	
61	62	64	64	65	63	
62	62	64	64	66	64	
62	62	64	64	65	64	
61	63	64	64	65	65	
61	63	64	62	65	64	
61	62	63	65	65	63	
61	63	64	65	66	63	
60	62	63	63	65	65	
59	63	63	64	63	63	
62	62	64	65	64	65	
61	62	65	65	65	63	
62	62	63	65	64	63	
58	63	62	66	63	63	
61	62	62	64	66	64	
62	61	65	64	65	63	
62	62	63	64	64	63	
61	64	62	64	66	63	
60	61	64	63	65	65	
62	62	62	65	63	63	
60	61	62	63	66	63	
62	61	64	65	66	64	
60	61	63	64	66	64	
61	63	60	63	66	63	
1.033351872	0.90971765	1.09334455	0.94443318	1.01483253	0.82000841	Sx
1.067816092	0.82758621	1.1954023	0.89195402	1.02988506	0.67241379	Var.
60.96666667	62	63.3333333	63.9333333	64.9333333	63.5	Mean
(60.5,61.43)	(61.59,62.41)	(62.89,63.85)	(63.51,64.36)	(64.48,65.39)	(63.13,63.87)	98% CI

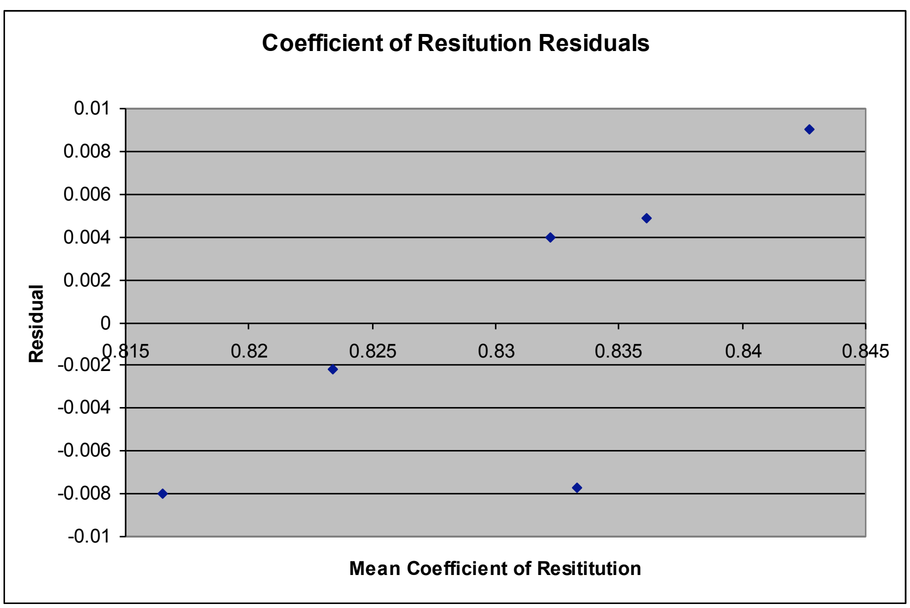
Table 3: Coefficient of Restitution

Temperature in °C						
-35.6	-32.1	-23.4	-13.4	-5.34	18.9	
0.83004601	0.81004196	0.83660774	0.83004601	0.83660774	0.823431992	
0.81676442	0.83004601	0.83660774	0.82343199	0.843118404	0.83004601	
0.81676442	0.81676442	0.83004601	0.83660774	0.843118404	0.83660774	
0.81676442	0.82343199	0.83004601	0.83660774	0.849579176	0.83660774	
0.81676442	0.81676442	0.843118404	0.83004601	0.849579176	0.83004601	
0.81004196	0.83004601	0.83660774	0.83004601	0.83004601	0.83660774	
0.81004196	0.81676442	0.83004601	0.83660774	0.843118404	0.823431992	
0.81676442	0.82343199	0.83660774	0.83660774	0.843118404	0.83004601	
0.82343199	0.82343199	0.83660774	0.83660774	0.849579176	0.83660774	
0.82343199	0.82343199	0.83660774	0.83660774	0.843118404	0.83660774	
0.81676442	0.83004601	0.83660774	0.83660774	0.843118404	0.843118404	
0.81676442	0.83004601	0.83660774	0.82343199	0.843118404	0.83660774	
0.81676442	0.82343199	0.83004601	0.8431184	0.843118404	0.83004601	
0.81676442	0.83004601	0.83660774	0.8431184	0.849579176	0.83004601	
0.81004196	0.82343199	0.83004601	0.83004601	0.843118404	0.843118404	
0.80326325	0.83004601	0.83004601	0.83660774	0.83004601	0.83004601	
0.82343199	0.82343199	0.83660774	0.8431184	0.83660774	0.843118404	
0.81676442	0.82343199	0.843118404	0.8431184	0.843118404	0.83004601	
0.82343199	0.82343199	0.83004601	0.8431184	0.83660774	0.83004601	
0.79642684	0.83004601	0.823431992	0.84957918	0.83004601	0.83004601	
0.81676442	0.82343199	0.823431992	0.83660774	0.849579176	0.83660774	
0.82343199	0.81676442	0.843118404	0.83660774	0.843118404	0.83004601	
0.82343199	0.82343199	0.83004601	0.83660774	0.83660774	0.83004601	
0.81676442	0.83660774	0.823431992	0.83660774	0.849579176	0.83004601	
0.81004196	0.81676442	0.83660774	0.83004601	0.843118404	0.843118404	
0.82343199	0.82343199	0.823431992	0.8431184	0.83004601	0.83004601	
0.81004196	0.81676442	0.823431992	0.83004601	0.849579176	0.83004601	
0.82343199	0.81676442	0.83660774	0.8431184	0.849579176	0.83660774	
0.81004196	0.81676442	0.83004601	0.83660774	0.849579176	0.83660774	
0.81676442	0.83004601	0.810041961	0.83004601	0.83004601	0.83004601	
0.00694113	0.00604151	0.007210854	0.00617917	0.006855146	0.005374423	Sx
4.8179E-05	3.65E-05	5.19964E-05	3.8182E-05	4.6993E-05	2.88844E-05	Var.
0.81651271	0.82341057	0.832208803	0.83614982	0.842009815	0.83331658	Mean
(.8134,.8196)	(.8207,.8261)	(.8293,8356)	(.8334,.8389)	(.8397,.8456)	(.8309,.8357)	98% CI

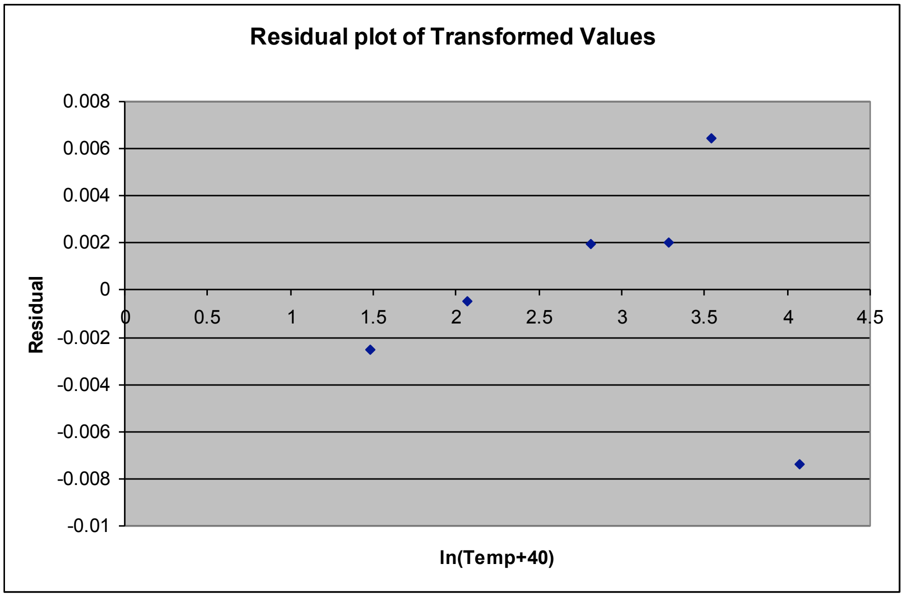
Graph 3: 98% Confidence Interval for Mean Bounce Height



Graph 4 : Residual Plot of Coefficient of Restitution



Graph 5: Residual plot of Transformed Values



As the residual plot does not a have a convincing pattern to it I can assume that a linear regression was the correct regression type.

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