Test your understanding

$$A, B, C \text{ independent} \iff \begin{cases} \mathbf{P}(A \cap B) = \mathbf{P}(A)\,\mathbf{P}(B) \\ \mathbf{P}(A \cap C) = \mathbf{P}(A)\,\mathbf{P}(C) \\ \mathbf{P}(B \cap C) = \mathbf{P}(B)\,\mathbf{P}(C) \\ \mathbf{P}(A \cap B \cap C) = \mathbf{P}(A)\,\mathbf{P}(B)\,\mathbf{P}(C) \end{cases}$$

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What if we complement one (or more) of the events?

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What if we complement one (or more) of the events?

$$\mathbf{P}(A \cap B \cap C^{\complement}) \overset{\text{Additivity}}{=} \mathbf{P}(A \cap B) - \mathbf{P}(A \cap B \cap C)$$

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What if we complement one (or more) of the events?

$$\mathbf{P}(A \cap B \cap C^{c}) \overset{\text{Additivity}}{=} \mathbf{P}(A \cap B) - \mathbf{P}(A \cap B \cap C)$$

$$\overset{\text{Independence}}{=} \mathbf{P}(A)\,\mathbf{P}(B) - \mathbf{P}(A)\,\mathbf{P}(B)\,\mathbf{P}(C)$$

$$A, B, C \text{ independent} \iff \begin{cases} \mathbf{P}(A \cap B) = \mathbf{P}(A)\,\mathbf{P}(B) \\ \mathbf{P}(A \cap C) = \mathbf{P}(A)\,\mathbf{P}(C) \\ \mathbf{P}(B \cap C) = \mathbf{P}(B)\,\mathbf{P}(C) \\ \mathbf{P}(A \cap B \cap C) = \mathbf{P}(A)\,\mathbf{P}(B)\,\mathbf{P}(C) \end{cases}$$

What if we complement one (or more) of the events?

$$\begin{aligned} \mathbf{P}(A \cap B \cap C^{\complement}) &\overset{\text{Additivity}}{=} \mathbf{P}(A \cap B) - \mathbf{P}(A \cap B \cap C) \\ &\overset{\text{Independence}}{=} \mathbf{P}(A)\,\mathbf{P}(B) - \mathbf{P}(A)\,\mathbf{P}(B)\,\mathbf{P}(C) \\ &\overset{\text{Factorisation}}{=} \mathbf{P}(A)\,\mathbf{P}(B)\big(1 - \mathbf{P}(C)\big) \end{aligned}$$

$$A, B, C \text{ independent} \iff \begin{cases} \mathbf{P}(A \cap B) = \mathbf{P}(A)\,\mathbf{P}(B) \\ \mathbf{P}(A \cap C) = \mathbf{P}(A)\,\mathbf{P}(C) \\ \mathbf{P}(B \cap C) = \mathbf{P}(B)\,\mathbf{P}(C) \\ \mathbf{P}(A \cap B \cap C) = \mathbf{P}(A)\,\mathbf{P}(B)\,\mathbf{P}(C) \end{cases}$$

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$$\begin{aligned} \mathbf{P}(A \cap B \cap C^{\complement}) &\overset{\text{Additivity}}{=} \mathbf{P}(A \cap B) - \mathbf{P}(A \cap B \cap C) \\ &\overset{\text{Independence}}{=} \mathbf{P}(A)\,\mathbf{P}(B) - \mathbf{P}(A)\,\mathbf{P}(B)\,\mathbf{P}(C) \\ &\overset{\text{Factorisation}}{=} \mathbf{P}(A)\,\mathbf{P}(B)\big(1 - \mathbf{P}(C)\big) \\ &\overset{\text{Additivity}}{=} \mathbf{P}(A)\,\mathbf{P}(B)\,\mathbf{P}(C^{\complement}) \end{aligned}$$

$$
A, B, C \text{ independent} \iff \begin{cases} \mathbf{P}(A \cap B) = \mathbf{P}(A)\,\mathbf{P}(B) \\ \mathbf{P}(A \cap C) = \mathbf{P}(A)\,\mathbf{P}(C) \\ \mathbf{P}(B \cap C) = \mathbf{P}(B)\,\mathbf{P}(C) \\ \mathbf{P}(A \cap B \cap C) = \mathbf{P}(A)\,\mathbf{P}(B)\,\mathbf{P}(C) \end{cases}
$$

What if we complement one (or more) of the events?

$$
\begin{aligned}
\mathbf{P}(A \cap B \cap C^{c}) &\overset{\text{Additivity}}{=} \mathbf{P}(A \cap B) - \mathbf{P}(A \cap B \cap C) \\
&\overset{\text{Independence}}{=} \mathbf{P}(A)\,\mathbf{P}(B) - \mathbf{P}(A)\,\mathbf{P}(B)\,\mathbf{P}(C) \\
&\overset{\text{Factorisation}}{=} \mathbf{P}(A)\,\mathbf{P}(B)\big(1 - \mathbf{P}(C)\big) \\
&\overset{\text{Additivity}}{=} \mathbf{P}(A)\,\mathbf{P}(B)\,\mathbf{P}(C^{c})
\end{aligned}
$$

Intersection probabilities factor into products for any combination of independent events or their complements.