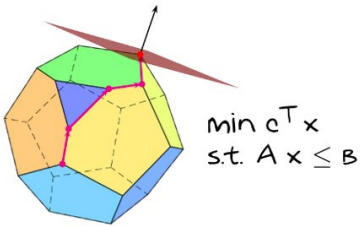
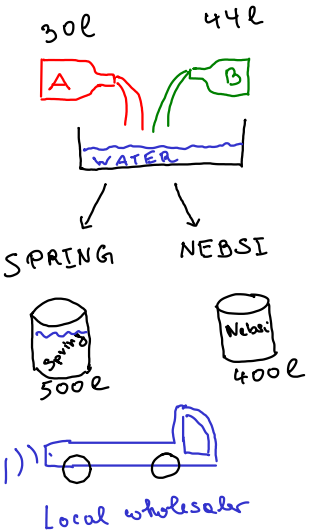
Linear programming

- Example



Softdrink production



100ℓ

	A	B	PROFIT
SPRING	3ℓ	8ℓ	100 CHF
NEBSI	6ℓ	4ℓ	125 CHF

Goal: Maximize Profit!

Quizz: Can we produce 500ℓ of Spring and 400ℓ of Nebsi ?

YES ○ NO ⊗

USAGE of A: $3 \cdot 5 + 4 \cdot 6 = 15 + 24 = 39$

The optimization problem

- Usage of A and B and profit per 100*l*:

	A	B	Profit
Spring	3*l*	8*l*	100 CHF
Nebsi	6*l*	4*l*	125 CHF

- On Stock:
  30*l* of A and 44*l* of *B*
- Capacity of barrels:
  Spring 500*l*, Nebsi 400*l*

Production plan:

$(x_1, x_2) \in \mathbb{R}^2$ $x_1 \cdot 100\,\ell$ Spring

$x_2 \cdot 100\,\ell$ Nebsi

(5,4) infeasible production plan

$$\max \quad 100x_1 + 125x_2$$

$$3 \cdot x_1 + 6 \cdot x_2 \leq 30 \quad (A)$$

$$8\,x_1 + 4\,x_2 \leq 44 \quad (B)$$

$$x_1 \leq 5$$

$$x_2 \leq 4$$

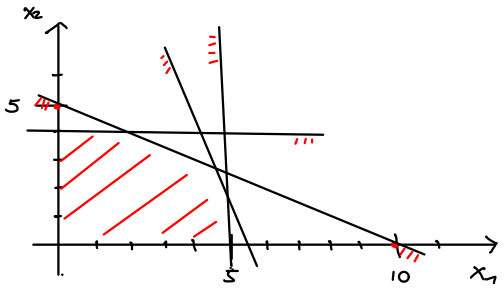
$$x_1 \geq 0$$

$$x_2 \geq 0$$

linear optimisation problem

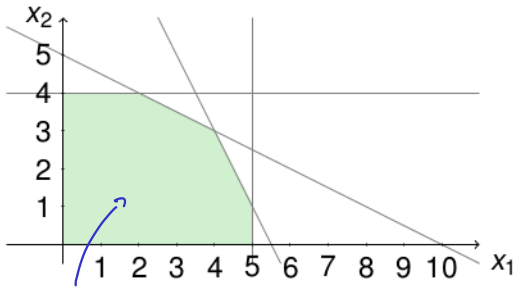
Feasible production plans

$$
\begin{array}{lrcl}
\text{max.} & 100 \cdot x_1 + 125 \cdot x_2 & & \\
\text{s.t.:} & 3 \cdot x_1 + 6 \cdot x_2 & \leq & 30 \\
& 8 \cdot x_1 + 4 \cdot x_2 & \leq & 44 \\
& x_1 & \leq & 5 \\
& x_2 & \leq & 4 \\
& x_1 & \geq & 0 \\
& x_2 & \geq & 0
\end{array}
$$



line $3 \cdot x_1 + 6 \cdot x_2 = 30$

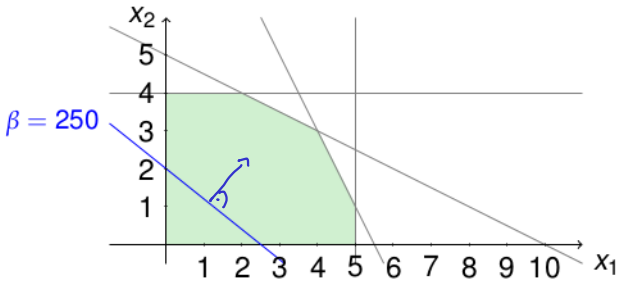
An optimal solution



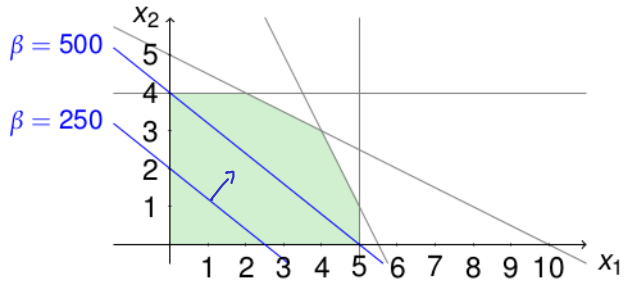
feasible production plans.

$\{(x_1, x_2) \in \mathbb{R}^2 : 100 \cdot x_1 + 125 \cdot x_2 = 250\} \cap$ Feas. prod. Plans.

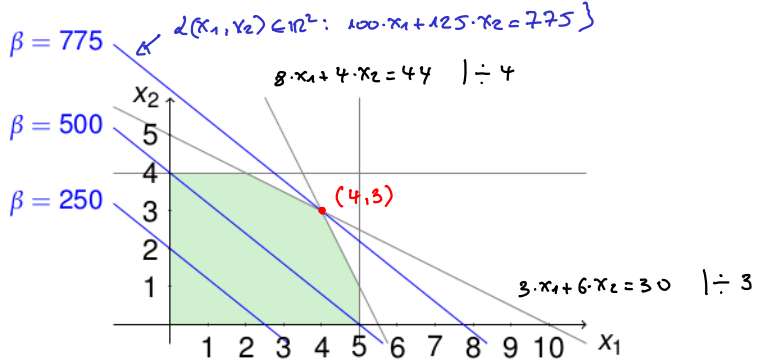
An optimal solution



An optimal solution



An optimal solution



$$\begin{cases} 2x_1 + x_2 = 11 \\ x_1 + 2 \cdot x_2 = 10 \end{cases} \quad \begin{matrix} \\ 1 \leftrightarrow 2 \end{matrix}$$

$$\Leftrightarrow \begin{cases} 2x_1 + x_2 = 11 \\ -3x_1 = -12 \end{cases}$$

$x_1 = 4$
$x_2 = 3$