✕ Lessons

Neighbour swapping

Consider a string consisting of the first n natural numbers, $123\ldots n$.

(a) For each number in the string, allow it to either stay fixed or change places with one of its neighbours. Define a_n to be the number of different strings that can be formed. Here are the first four values of n:

n	strings	a_n
1	1	1
2	12, 21	2
3	123, 132, 213	3
4	1234, 1243, 1324, 2134, 2143	5

Prove that $a_n = F_{n+1}$.

(b) Consider a problem similar to that above, but now allow the first 1 to change places with the last n, as if the string lies on a circle. Suppose $n \geq 3$, and define b_n as the number of different strings that can be formed. Show that $b_n = L_n$, where L_n is the nth Lucas number.

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There are two ways you can submit your answer to this prompt:

1) Easier option: *You can write up the proof on a piece of paper, take a picture or scan it, and then post it as your reply by clicking on the Camera or Picture icon. Make sure the proof is legible so others can comment on it.*

2) Advanced option: *You can type in your answer using Mathjax input. Read more here - Mathjax Basic Tutorial.*
Important Notes: *Coursera requires you to use double $$ at the start and end of each expression (single $ does not work). HKUST cannot support any technical problems encountered with Mathjax input in Coursera.*

Participation is optional

Type your response here...

WK   IP   +2

6 learners have submitted a response.