UNIVERSITY OF LONDON

# Probability and Statistics: To $p$, or not to $p$?

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## 6.2 Risk

The decision tree analysis in Section 6.1 was solved by choosing the option with the maximum expected profit. As such, we only considered the mean (average) outcome.

In practice people care about **risk** and tend to factor it into their decision making. Of course, different people have different **attitudes to risk** so we can profile people's risk appetite as follows.

### Degrees of risk aversion

A decision-maker is **risk-averse** if s/he prefers the certain outcome of £$x$ over a risky project with a mean (EMV) of £$x$.

A decision-maker is **risk-loving** (also known as risk-seeking) if s/he prefers a risky project with a mean (EMV) of £$x$ over the certain outcome of £$x$.

A decision-maker is **risk-neutral** if s/he is indifferent between a sure payoff and an uncertain outcome with the same expected monetary value.

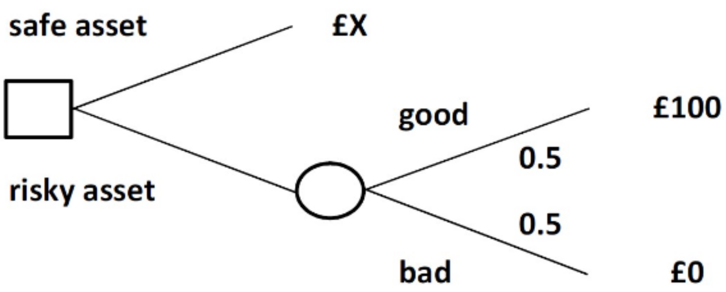
The **certainty equivalent (CE)** of a risky project is the amount of money which makes the decision-maker indifferent between receiving this amount for sure and the risky project.

How might we model risk?

### Example

A risky project pays out £100 with a probability of 0.5 and £0 with a probability of 0.5. The certainty equivalent, $X$, makes the decision-maker indifferent between the certain outcome $X$ and the risky project.

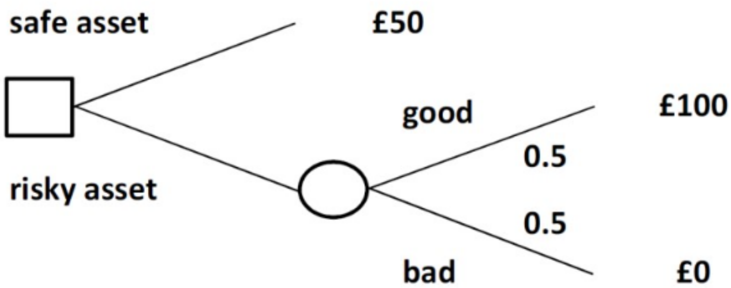
Consider the following decision tree:




Let $X$ be the value which makes you *indifferent* between the safe and risky assets. Immediately we see that:

$$\text{E(risky)} = 0.5 \times £100 + 0.5 \times £0 = £50.$$

Consider a risk-neutral individual, for whom $X = 50$:

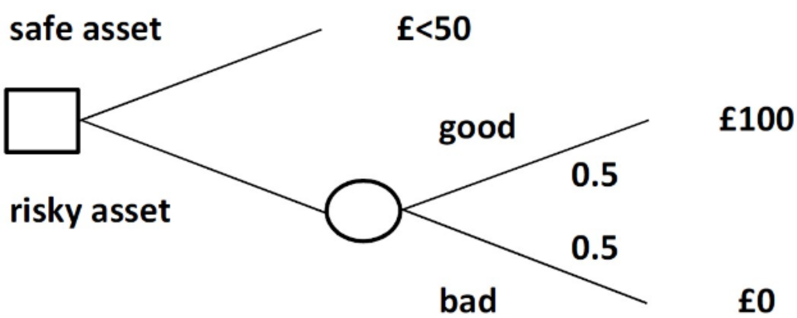



Hence:

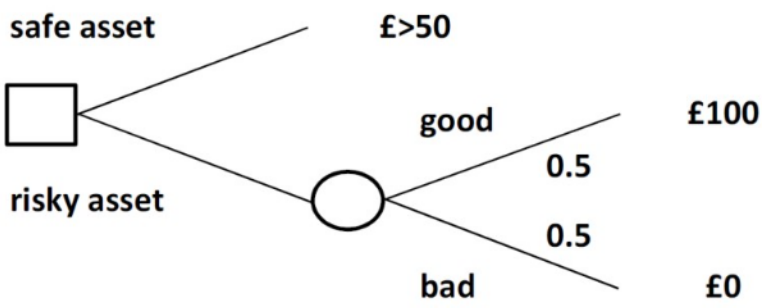
$$\text{E(safe)} = 1 \times £50 = £50$$

so this individual does not care about risk and only focuses on the expected return. Therefore, for $X = 50$, such an individual sees no difference between the safe and risky assets, even though the safe asset is risk-free while the risky asset is risky – hence the name!

The decision tree for a risk-averse individual would be:

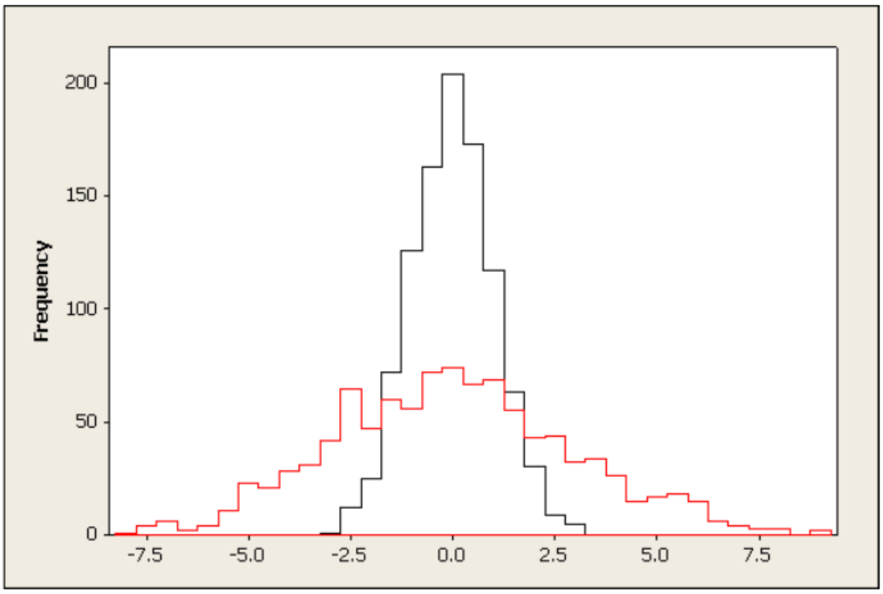


and for a risk-loving-individual:




## Example

In Section 3.4 you saw the following diagram used to illustrate the returns of two stocks:




At the time it was noted that the black stock was the safer stock due to its smaller variation. Hence depending on an investor's risk appetite, they would invest in the black or red stock accordingly.

## Risk premium

The **risk premium** (of a risky project) is defined as:

$$\text{EMV} - \text{CE}.$$

Interpretation: The amount of money the decision-maker is willing to pay to receive a safe payoff of $X$ rather than face the risky project with an expected payoff of $X$.

Risk profiles can be determined using the following:

$$\begin{aligned} \textbf{CE} &< \textbf{EMV} \Rightarrow \textbf{risk-averse} \\ \textbf{CE} &= \textbf{EMV} \Rightarrow \textbf{risk-neutral} \\ \textbf{CE} &> \textbf{EMV} \Rightarrow \textbf{risk-loving.} \end{aligned}$$

**If risk-neutral, the decision-maker uses the EMV criterion as seen in Section 6.1.**