

Two-Dimensional Diffusion Equation

Solve the two-dimensional diffusion equation on a square with u equal zero on the boundaries.

Script

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MATLAB Documentation (https://www.mathworks.com/help/)

```
1 %%%% Define the square and grid parameters %%%%%%%%%%
2 L=1; %square is 2L x 2L
3 N=100; %% of intervals in x and y directions
4 n=N+1; %% of gridpoints in x,y directions including boundaries
5 h=2*L/N; %grid size in x,y directions
6 x=-L + (0:N)*h; %x values on the grid
7 y=-L + (0:N)*h; %y values on the grid
8 [X,Y]=meshgrid(x,y);
9 %%%% Define the indices associated with the boundaries %%%%%%%%%%
10 % boundary_index = [bottom, left, top, right]
11 boundary_index=[      1:n,      1:n:1+(n-1)*n, ...
12                  1+(n-1)*n:n*n,    n:n:n*n      ];
13 %%%% Diffusion constant and time-step parameters
14 D=1;
15 dt=h^2/(2*D); %borderline stability of FTCS scheme
16 alpha=dt*D/h^2; %equation parameter
17 nsteps=1000; %number of time steps
18 %%%% CONSTRUCT THE MATRIX AND COMPUTE LU DECOMPOSITION %%%%%%%%%%
19 diagonals = [2*(1+2*alpha)*ones(n*n,1), -alpha*ones(n*n,4)];
20 A = spdiags(diagonals,[0 -1 1 -n n], n*n, n*n);
21 I = speye(n*n);
22 A(boundary_index,:) = I(boundary_index,:); %boundaries
23 [PL, U] = lu(A);
24 indices = setdiff(1:n*n,boundary_index);
25
26 %%%% Define initial conditions %%%%%%%%%%
27 u=zeros(n,n,nsteps);
28 sigma=L/4;
29 u(:, :,1)=1/(2*pi*sigma^2)*exp(-0.5*(X.^2+Y.^2)/sigma^2);
30 u(1, :,1)=0; u(n, :,1)=0; u(:,1,1)=0; u(:,n,1)=0; %b.c.
31 %%%% ADVANCE SOLUTION u %%%%%%%%%%
32 for m=2:nsteps
33     b = zeros(n*n, 1);
34     for index=1:numel(indices)
35         k = indices(index);
36         i = mod(k, n);
37         j = (k - i) / n + 1;
38         b(k) = alpha*(u(i,j-1,m-1)+u(i,j+1,m-1)+u(i+1,j,m-1)+u(i-1,j,m-1)) + 2*(1-2*alpha)*u(i,j,m-1);
39     end
40     s = U\ (PL\b); %A \ b;
41     u(:, :,m) = reshape(s, n, n);
42 end
43 % %%%% Plot with animation: UNCOMMENT TO RUN ON MATLAB ONLINE OR DESKTOP %%%
44 % figure('units','normalized','outerposition',[0 0 1 1])
45 % s=surf(X,Y,u(:, :,1)); zlim([0, 2.6]);
46 % xlabel('$x$', 'Interpreter','latex','FontSize',14);
47 % ylabel('$y$', 'Interpreter','latex','FontSize',14);
48 % zlabel('$u(x,y,t)$', 'Interpreter','latex','FontSize',14);
49 % title('Solution of the 2D diffusion equation', 'Interpreter','latex','FontSize',16);
50 % pause(1)
51 % for j=2:nsteps
52 %     s.ZData=u(:, :,j); pause(0.01);
53 % end
54
```

Run Script

Assessment: All Tests Passed

Submit

✔ Solution of $u=u(x,y,t)$

Output

Code ran without output.