Shapiro–Wilk test

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The **Shapiro–Wilk test** is a test of normality in frequentist statistics. It was published in 1965 by Samuel Sanford Shapiro and Martin Wilk.[1]

Contents

Theory

The Shapiro–Wilk test utilizes the null hypothesis principle to check whether a sample $x_1, ..., x_n$ came from a normally distributed population. The test statistic is:

$$W = \frac{\left(\sum_{i=1}^n a_i x_{(i)}\right)^2}{\sum_{i=1}^n (x_i - \overline{x})^2}$$

where

- $x_{(i)}$ (with parentheses enclosing the subscript index i) is the ith order statistic, i.e., the ith-smallest number in the sample;
- $\overline{x} = \left(x_1 + \cdots + x_n\right) / n$ is the sample mean;
- the constants a_i are given by[1]

  $$(a_1, \dots, a_n) = \frac{m^\mathsf{T} V^{-1}}{(m^\mathsf{T} V^{-1} V^{-1} m)^{1/2}}$$

  where

  $$m = (m_1, \dots, m_n)^\mathsf{T}$$

  and $m_1, \dots, m_n$ are the expected values of the order statistics of independent and identically distributed random variables sampled from the standard normal distribution, and V is the covariance matrix of those order statistics.

The user may reject the null hypothesis if W is below a predetermined threshold.

Interpretation

The null-hypothesis of this test is that the population is normally distributed. Thus if the *p*-value is less than the chosen alpha level, then the null hypothesis is rejected and there is evidence that the data tested are not from a normally distributed population. In other words, the data are not normal. On the contrary, if the *p*-value is greater than the chosen alpha level, then the null hypothesis that the data came from a normally distributed population cannot be rejected. E.g. for an alpha level of 0.05, a data set with a *p*-value of 0.02 rejects the null hypothesis that the data are from a normally distributed population.[2] However, since the test is biased by sample size,[3] the test may be statistically significant from a normal distribution in any large samples. Thus a Q–Q plot is required for verification in addition to the test.

Power analysis

Monte Carlo simulation has found that Shapiro–Wilk has the best power for a given significance, followed closely by Anderson–Darling when comparing the Shapiro–Wilk, Kolmogorov–Smirnov, Lilliefors, and Anderson–Darling tests.[4]

Approximation

Royston proposed an alternative method of calculating the coefficients vector by providing an algorithm for calculating values, which extended the sample size to 2000.[5] This technique is used in several software packages including R,[6] Stata,[7][8] SPSS and SAS.[9] Rahman and Govidarajulu extended the sample size further up to 5000.[10]

See also

- Anderson–Darling test
- Cramér–von Mises criterion
- Kolmogorov–Smirnov test
- Normal probability plot
- Ryan–Joiner test
- Watson test
- Lilliefors test
- D'Agostino's K-squared test

References

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9. Park, Hun Myoung (2002–2008). "Univariate Analysis and Normality Test Using SAS, Stata, and SPSS" (PDF). *[working paper]*. Retrieved 26 February 2014.
10. Rahman und Govidarajulu (1997). "A modification of the test of Shapiro and Wilk for normality". *Journal of Applied Statistics* **24** (2): 219–236. doi:10.1080/02664769723828.

External links

- Samuel Sanford Shapiro (http://www.answers.com/topic/samuel-sanford-shapiro)
- Algorithm AS R94 (Shapiro Wilk) FORTRAN code (http://lib.stat.cmu.edu/apstat/R94)
- Exploratory analysis using the Shapiro–Wilk normality test in R (http://cran.us.r-project.org/doc/manuals/R-intro.html#Examining-the-distribution-of-a-set-of-data)

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