Direct Solution of the Laplace Equation

My Solutions ›

Using the direct method, solve the Laplace equation inside a unit square. Set the boundary conditions to be zero on the left and bottom sides, and to go from zero to one across the top, and from one to zero down the right side. Model these boundary conditions as

$\Phi = x(2-x)$ for $y = 1$, and $\Phi = y(2-y)$ for $x = 1$.

Script

Reference Solution | Save | Reset | MATLAB Documentation (https://www.mathworks.com/help/)

```
%%%%% Define the rectangle and grid parameters %%%%%%%%%%%%%%%%%%%%%%%%%%%
Lx=1; Ly=1; %rectangle dimensions
Nx=100; Ny=100; %# of intervals
nx=Nx+1; ny=Ny+1; %# of gridpoints in x,y directions including boundaries
dx=Lx/Nx; dy=Ly/Ny; %grid size in x,y directions
x=(0:Nx)*dx; y=(0:Ny)*dy; %x,y values on the grid
%%%%% Define the indices associated with the boundaries %%%%%%%%%%%%%%%%%%
% boundary_index = [bottom, left, top, right]
boundary_index=[          1:nx,   1:nx:1+(ny-1)*nx, ...
             1+(ny-1)*nx:nx*ny,   nx:nx:nx*ny           ];
%%%%% Set up matrix %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
diagonals = [4*ones(nx*ny,1), -ones(nx*ny,4)];
A=spdiags(diagonals,[0 -1 1 -nx nx], nx*ny, nx*ny); %use sparse matrices
I=speye(nx*ny);
A(boundary_index,:)=I(boundary_index,:);
%%%%% SET-UP RIGHT HAND SIDE %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
b=zeros(nx,ny);
b(:,1)=0;          %bottom
b(1,:)=0;          %left
b(:,ny)=x.*(2-x);  %top
b(nx,:)=y.*(2-y);  %right
b=reshape(b,nx*ny,1); %make column vector
%%%%% Solve the Laplace equation using Gaussian elimination %%%%%%%%%%%%%%%
Phi=A\b; %solution step (all the computational time is here)
Phi=reshape(Phi,nx,ny); %make matrix
%%%%% Graphics %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
[X,Y]=meshgrid(x,y);
v=[0.8 0.6 0.4 0.2 0.1 0.05 0.01];
contour(X,Y,Phi',v,'ShowText','on');%requires transpose (read the notes)
axis equal;
set(gca, 'YTick', [0 0.2 0.4 0.6 0.8 1]);
set(gca, 'XTick', [0 0.2 0.4 0.6 0.8 1]);
xlabel('$x$','Interpreter','latex','FontSize',14 );
ylabel('$y$','Interpreter','latex','FontSize',14);
title('Solution of the Laplace equation','Interpreter','latex','FontSize',16);

```

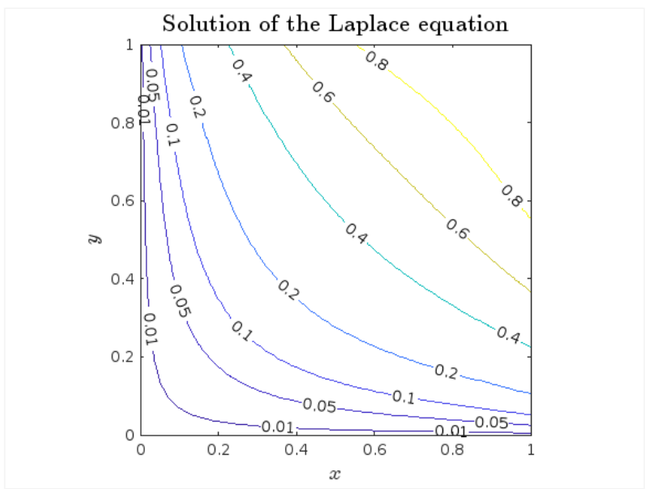
Run Script

Assessment: All Tests Passed

Submit

✓ **Check the value of Phi**

Output



© 2022 The MathWorks, Inc.