13. Partial derivatives: geometric meaning

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Partial derivatives: geometric meaning

Recall that an approximation for the slope of the tangent line near a point x_0 is the slope of the secant line between the points $(x_0, f(x_0))$ and $(x_0 + \Delta x, f(x_0 + \Delta x))$. This means that

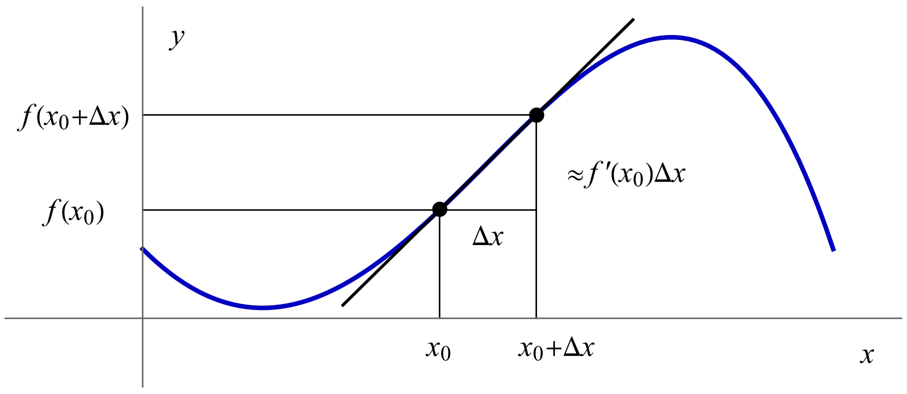
$$\underbrace{f'(x_0)}_{\text{Slope of tangent line}} \approx \underbrace{\frac{f(x_0+\Delta x) - f(x_0)}{(x_0+\Delta x) - x_0}}_{\text{Slope of secant line}} = \frac{f(x_0+\Delta x) - f(x_0)}{\Delta x}. \tag{2.28}$$

(To review this idea in general, you may want to look at the Secant Approximation mathlet.)

Solving for the term $f(x_0 + \Delta x)$ gives the linear approximation for the function $f(x)$ near x_0:

$$f(x_0 + \Delta x) \approx f(x_0) + f'(x_0)\,\Delta x. \tag{2.29}$$

This means that the derivative measures how $f(x)$ changes if we increase x by a small amount Δx.



The same idea can be applied to functions of more than one variable. Just as we saw in the single variable case, the partial derivative with respect to x evaluated at a point (x_0, y_0) can be approximated by

$$f_x(x_0, y_0) \approx \frac{f(x_0+\Delta x, y_0) - f(x_0, y_0)}{\Delta x}. \tag{2.30}$$

Solving for the term $f(x_0 + \Delta x)$ gives

$$f(x_0 + \Delta x, y_0) \approx f(x_0, y_0) + f_x(x_0, y_0)\,\Delta x. \tag{2.31}$$

We can apply the same argument for f_y to obtain

$$f(x_0, y_0 + \Delta y) \approx f(x_0, y_0) + f_y(x_0, y_0)\,\Delta y. \tag{2.32}$$

So the partial derivative with respect to x measures how f changes if we increase x by a small amount. Similarly, the partial derivative with respect to y measures how f changes if we increase y by a small amount.

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