12. Inverse matrices
Inverse matrices

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When a square matrix $\mathbf{A}$ has the property that for every vector $\mathbf{b}$, the system $\mathbf{Ax} = \mathbf{b}$ has a unique solution, then $\mathbf{A}$ has an inverse matrix.

Definition 12.1 The **inverse** of an $n \times n$ matrix $\mathbf{A}$ is another $n \times n$ matrix $\mathbf{A}^{-1}$ such that

$$\mathbf{A}\mathbf{A}^{-1} = \mathbf{I} \qquad \text{and} \qquad \mathbf{A}^{-1}\mathbf{A} = \mathbf{I}.$$

Suppose that $\mathbf{A}$ represents the linear transformation $\mathbf{f}$. Then $\mathbf{A}^{-1}$ exists if and only if an inverse function $\mathbf{f}^{-1}$ exists; in that case, $\mathbf{A}^{-1}$ represents $\mathbf{f}^{-1}$.

Problem 12.2 Does the rotation matrix $\mathbf{R} := \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$ have an inverse? If so, what is it?

Solution: The inverse linear transformation is rotation by $-\theta$, so

$$\mathbf{R}^{-1} = \begin{pmatrix} \cos(-\theta) & -\sin(-\theta) \\ \sin(-\theta) & \cos(-\theta) \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}.$$

(As a check, try multiplying $\mathbf{R}$ by this matrix, in either order.)

Problem 12.3 Does the projection matrix $\mathbf{A} := \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ have an inverse? If so, what is it?

Solution: The associated linear transformation $\mathbf{f}$ is not a 1-to-1 correspondence, because it maps more than one vector to $\mathbf{0}$ (it maps the whole z-axis to $\mathbf{0}$). Thus $\mathbf{f}^{-1}$ does not exist, so $\mathbf{A}^{-1}$ does not exist.

Algorithm to find the inverse of a matrix

Let $\mathbf{A}$ by an $n \times n$ matrix.

1. Form the $n \times 2n$ augmented matrix $[\mathbf{A}|\mathbf{I}]$.

2. Convert to RREF; the result will be $[\mathbf{I}|\mathbf{B}]$ for some $n \times n$ matrix $\mathbf{B}$.

3. Then $\mathbf{A}^{-1} = \mathbf{B}$.

This is a special case of solving a matrix equation $\mathbf{AX} = \mathbf{B}$since $\mathbf{A}^{-1}$ is the solution to $\mathbf{AX} = \mathbf{I}$.

Algorithm in practice

That's the Gauss--Jordan idea.

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Inverses concept check

1/1 point (graded)

If $\mathbf{A}$ and $\mathbf{B}$ are two $n \times n$ matrices, what is $(\mathbf{AB})^{-1}$? (Hint: Think of $\mathbf{A}$ and $\mathbf{B}$ as functions from $\mathbb{R}^n$ to $\mathbb{R}^n$.)

- ○ $A^{-1}B^{-1}$
- ○ $A^{-1}B$
- ◉ $B^{-1}A^{-1}$ ✔
- ○ $B^{-1}A$
- ○ AB^{-1}
- ○ BA^{-1}.

Solution:

$(AB)^{-1} = B^{-1}A^{-1}$. We can see this by computing:

$$\begin{aligned}(B^{-1}A^{-1})\,(AB) &= B^{-1}(A^{-1}A)B = B^{-1}IB \\ &= B^{-1}B = I,\end{aligned}$$

and

$$\begin{aligned}(AB)\,(B^{-1}A^{-1}) &= A(BB^{-1})A = AIA^{-1} \\ &= AA^{-1} = I.\end{aligned}$$

(The key rule is that **taking inverses reverses the order of matrix multiplication** . Similarly, $(ABC)^{-1} = C^{-1}B^{-1}A^{-1}$, and so on.)

Submit You have used 1 of 3 attempts

Answers are displayed within the problem

Worked examples: inverses

See you next time.

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