7. Properties of Total Variation Distance

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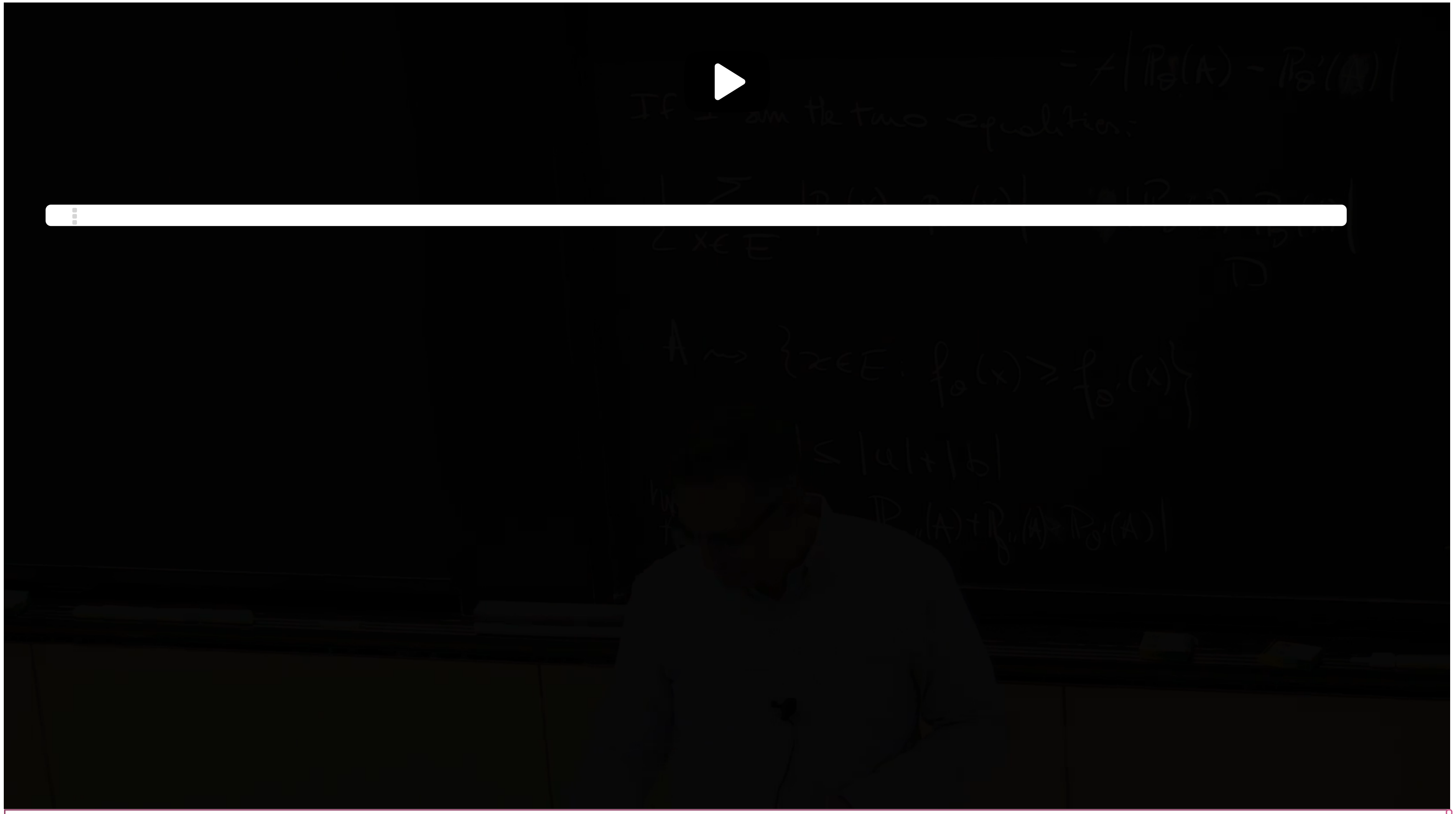

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Let d be a function that takes two probability measures $\mathbf{P}$ and $\mathbf{Q}$ and maps them to a real number $d(\mathbf{P}, \mathbf{Q})$. Then d is a **distance** on probability measures if the following four axioms hold. (Here, $\mathbf{P}, \mathbf{Q}$, and $\mathbf{V}$ are all probability measures.)

- $d(\mathbf{P},\mathbf{Q}) = d(\mathbf{Q},\mathbf{P})$ (symmetric)
- $d(\mathbf{P},\mathbf{Q}) \geq 0$ (nonnegative)
- $d(\mathbf{P},\mathbf{Q}) = 0 \iff \mathbf{P} = \mathbf{Q}$ (definite)
- $d(\mathbf{P},\mathbf{V}) \leq d(\mathbf{P},\mathbf{Q}) + d(\mathbf{Q},\mathbf{V})$ (triangle inequality)

In the above, $\mathbf{P} = \mathbf{Q}$ means $\mathbf{P}(A) = \mathbf{Q}(A)$ for $A \subset E$, where E is the common sample space of $\mathbf{P}$ and $\mathbf{Q}$.

The total variation distance (TV) is a distance on probability measures.

Symmetry and Definiteness of Total Variation Distance

1/1 point (graded)
Let $\mathbf{P}$ be a probability measure. Which of the following is (are) true?

- [ ] One can find a measure $\mathbf{Q} \neq \mathbf{P}$ such that $\mathrm{TV}(\mathbf{P},\mathbf{Q}) = 0$.
- [x] $\mathrm{TV}(\mathbf{P},\mathbf{Q}) = \mathrm{TV}(\mathbf{Q},\mathbf{P})$.

✔

Solution:

Choice 1 is not true because of the following: By definition, $\mathbf{Q} \neq \mathbf{P}$ means that there is some set A of non-zero measure over which the measures $\mathbf{Q}$ and $\mathbf{P}$ are not the same. Therefore, over this set A, $|\mathbf{P}(A) - \mathbf{Q}(A)| > 0$, which implies that $\mathrm{TV}(\mathbf{P},\mathbf{Q}) \neq 0$.

Choice 2 (symmetry) is true because for any set A, $|\mathbf{P}(A) - \mathbf{Q}(A)| = |\mathbf{Q}(A) - \mathbf{P}(A)|$.

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Answers are displayed within the problem

Triangle Inequality

1/1 point (graded)

Which of the following quantities is greater than or equal to $\mathrm{TV}\left(\mathrm{Ber}\left(.5\right), \mathrm{Ber}\left(0.3\right)\right)$?
(Choose all that apply.)

- [x] $\mathrm{TV}\left(\mathrm{Ber}\left(0.5\right), \mathrm{Ber}\left(0.1\right)\right) + \mathrm{TV}\left(\mathrm{Ber}\left(0.1\right), \mathrm{Ber}\left(0.3\right)\right)$
- [x] $\mathrm{TV}\left(\mathrm{Ber}\left(0.5\right), \mathrm{Poiss}\left(5\right)\right) + \mathrm{TV}\left(\mathrm{Ber}\left(0.3\right), \mathrm{Poiss}\left(5\right)\right)$
- [x] $\mathrm{TV}\left(\mathrm{Bin}\left(7, 0.4\right), \mathrm{Ber}\left(0.5\right)\right) + \mathrm{TV}\left(\mathrm{Ber}\left(0.3\right), \mathrm{Bin}\left(7, 0.4\right)\right)$

Solution:

Recall the triangle inequality states that for distributions $\mathbf{P}$, $\mathbf{Q}$, and $\mathbf{V}$:

$$\mathrm{TV}\left(\mathbf{P}, \mathbf{V}\right) \leq \mathrm{TV}\left(\mathbf{P}, \mathbf{Q}\right) + \mathrm{TV}\left(\mathbf{Q}, \mathbf{V}\right).$$

- If we set $\mathbf{P} = \mathrm{Ber}\left(0.5\right)$, $\mathbf{V} = \mathrm{Ber}\left(0.3\right)$, and $\mathbf{Q} = \mathrm{Ber}\left(0.1\right)$, then applying the triangle inequality above gives the first upper bound.
- In the second choice, set $\mathbf{P} = \mathrm{Ber}\left(0.5\right)$, $\mathbf{V} = \mathrm{Ber}\left(0.3\right)$, and $\mathbf{Q} = \mathrm{Poiss}\left(5\right)$ and apply the triangle inequality.
- In the third choice, set $\mathbf{P} = \mathrm{Ber}\left(0.5\right)$, $\mathbf{V} = \mathrm{Ber}\left(0.3\right)$, and $\mathbf{Q} = \mathrm{Bin}\left(7, 0.4\right)$ and apply the triangle inequality.

Remark: Implicitly we are also using the symmetry property of total variation: $\mathrm{TV}\left(\mathbf{P}, \mathbf{Q}\right) = \mathrm{TV}\left(\mathbf{Q}, \mathbf{P}\right)$.

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? Symmetry and Definiteness of Total Variation Distance
I guess the question should be: Let P and Q be probability measures... - To avoid confusion. 2