## Observation Theory

## Script V12D – Precision and Covariance Matrix

You have now seen some examples of covariance matrices, all with covariances equal to zero.

But what does it mean if the covariances are not equal to zero?

In order to understand that we need to look at how the covariance of two variables is defined.

As you can see from the formula in blue, it is equal to the correlation coefficient of the two variables, multiplied by their standard deviations.

The correlation coefficient is a value between -1 and 1, and the following examples explain how to interpret its value.

Let's assume that the standard deviation of 2 random variables are 1 and 2 respectively, and the correlation coefficient is equal to rho.

Then from the definition, we know that the covariance is equal to 2 times the correlation coefficient.

The first value for rho we choose is zero.

This plot shows then 10,000 outcomes for the random vector with these given standard deviations and correlation coefficient.

The horizontal axis refers e1, the vertical axis to e2.

Note that the mean values are indicated along those axis.

Since the standard deviation of the second variable is twice that of the first variable, also the spread in that direction is twice as large.

But since the two variables are uncorrelated, rho=0, knowing the outcome of e.g. E2 does not give a clue on the expected outcome of e1; it may take a quite large range of values, as visualised by the red line.

Now let's look at another extreme, where the 2 variables are FULLY correlated, rho = 1.

The range of values that the individual variables can take is still the same.

But now the 2 variables have a linear relation.

Knowing the outcome of one variable will give you the exact value of the other variable.

Now an example with rho between 0 and 1; then you would get this.

We can see that knowing the outcome of e2 limits the range of values that e1 can take, indicated again by the red line.

Moreover, in this case e2 is larger than the mean – this gives you a high likelihood that e1 will be larger than its mean as well.

This is because of the positive correlation indicated by rho = 0.7

If on the other hand, the two variables would have a negative correlation

We would get the opposite, as seen in the plot on the left-hand side.

A value larger than the mean for e1 will more likely give you a value smaller than the mean for e2 and vice versa.

Now let’s look at the distribution of the random errors.

As we saw before, a single random error follows the normal distribution

With mean zero and a certain standard deviation.

This is the univariate normal distribution.

How about the multivariate case, where we are interested in the distribution of a vector of m random errors?

This vector will follow the multivariate normal distribution, which depends on the vector with mean values, in this case the zero-vector , and the covariance matrix

In the 2-dimensional case we can still visualise the normal PDF, for higher dimensions we cannot.

Therefore we will look at this example:

Where we collect 1 laser distance measurement with high precision and 1 rope measurement with much lower precision.

The corresponding PDF looks like this

On the left you can see a surface plot, on the right the top view.

The larger standard deviation of the rope measurement results in a larger spread in the vertical direction of this plot.

In the exercises you will look at different examples of covariance matrices and the multivariate normal PDF for the 2-dimensional case.

In the next lecture we will go back to our estimation problem and identify the different elements of this problem.