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5. A confidence interval for Poisson variables

(a)

2/2 points (graded)

Let $X_1, \ldots, X_n$ be i.i.d. Poisson random variables with parameter $\lambda > 0$ and denote by $\overline{X}_n$ their empirical average,

$$\overline{X}_n = \frac{1}{n}\sum_{i=1}^{n} X_i.$$

Find two sequences $(a_n)_{n\geq 1}$ and $(b_n)_{n\geq 1}$ such that $a_n\left(\overline{X}_n - b_n\right)$ converges in distribution to a standard Gaussian random variable $Z \sim N(0,1)$.

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$a_n =$ sqrt(n/lambda) ✔

$\sqrt{\frac{n}{\lambda}}$

$b_n =$ lambda ✔

λ

STANDARD NOTATION

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✔ Correct (2/2 points)

(b)

1/1 point (graded)

Secondly, express $\mathbf{P}\left(|Z| \leq t\right)$ in terms of $\Phi\left(r\right) = \mathbf{P}\left(Z \leq r\right)$ for $t > 0$.

Write `Phi(t)` (with capital `P`)for $\Phi\left(t\right)$.

$\mathbf{P}\left(|Z| \leq t\right) =$ 2*Phi(t)-1 ✔

STANDARD NOTATION

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Generating Speech Output ct (1/1 point)

(c)

2/2 points (graded)

Using the previous questions, find an interval $\mathcal{I}_\lambda$ that **depends on** λ and that is centered around $\overline{X}_n$ such that

$$\mathbf{P}\left[\mathcal{I}_\lambda \ni \lambda\right] \to .95, \quad n \to \infty.$$

(In other words, the interval before applying any of the 3 methods.)

(Write barX_n for $\overline{X}_n$.)

(*Hint*: The 97.5% -quantile of the standard Gaussian distribution is 1.96.)

$\mathcal{I}_\lambda = [A, B]$ for

$A =$ barX_n -1.96*sqrt(lambc ✔ $B =$ barX_n +1.96*sqrt(lamb(✔

STANDARD NOTATION

Submit You have used 1 of 3 attempts

✔ Correct (2/2 points)

(d)

1/1 point (graded)

Which of the following is a confidence interval $\mathcal{J}$ that fulfills

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$$\mathbf{P}\left[\mathcal{J} \ni \lambda\right] \to .95, \quad n \to \infty.$$

(Choose all that apply.)

- [ ] $\mathcal{J} = [\overline{X}_n - 1.96\sqrt{\lambda/n}, \overline{X}_n + 1.96\sqrt{\lambda/n}]$
- [ ] $\mathcal{J} = [\overline{X}_n - 1.96\sqrt{\overline{X}_n/n^2}, \overline{X}_n + 1.96\sqrt{\overline{X}_n/n^2}]$
- [x] $\mathcal{J} = [\overline{X}_n - 1.96\sqrt{\overline{X}_n/n}, \overline{X}_n + 1.96\sqrt{\overline{X}_n/n}]$
- [ ] $\mathcal{J} = [\overline{X}_n - 1.96\sqrt{100/n}, \overline{X}_n + 1.96\sqrt{100/n}]$

✔

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✔ Correct (1/1 point)

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Topic: Unit 2 Foundation of Inference:Homework 1: Estimation, Confidence Interval, Modes of Convergence / 5. A confidence interval for Poisson variables

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Question(d), shall answer depend on λ or not

question posted 2 days ago by **Cool7**

Question(c) specifically asked for answer depend on λ, but question(d) doesn't. Does that mean we need to choose the option not depend on λ?

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1 response

Hryhorchuk
2 days ago

The constraint used by prof. in the lecture applies here as well.

Confidence interval?

- For a fixed $\alpha \in (0,1)$, if $q_{\alpha/2}$ is the $(1-\alpha/2)$-quantile of $\mathcal{N}(0,1)$, then with probability $\simeq 1-\alpha$ (if n is large enough !),

$$\bar{R}_n \in \left[p - \frac{q_{\alpha/2}\sqrt{p(1-p)}}{\sqrt{n}}, p + \frac{q_{\alpha/2}\sqrt{p(1-p)}}{\sqrt{n}}\right].$$

- It yields

$$\lim_{n\to\infty} \mathbb{P}\left(\left[\bar{R}_n - \frac{q_{\alpha/2}\sqrt{p(1-p)}}{\sqrt{n}}, \bar{R}_n + \frac{q_{\alpha/2}\sqrt{p(1-p)}}{\sqrt{n}}\right] \ni p\right) = 1-\alpha$$

- But this is **not** a confidence interval because it depends on p!
- To fix this, there are 3 solutions.

Thanks, missed this part.

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posted 2 days ago by **Cool7**

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