F.3.3 Final Questions 5-6

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F.3.3 Final Questions 5-6

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Question 5

10.0/10.0 points (graded)

Consider the matrix $A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \\ 2 & 1 \end{pmatrix}$ and vector $b = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$.

1. Compute the linear least-squares solution, $\hat{x}$, to $Ax = b$.

$\hat{x} =$ 1/3 ✔ Answer: 1/3

2/3 ✔ Answer: 2/3

- $A^T A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \\ 2 & 1 \end{pmatrix}^T \begin{pmatrix} 0 & 1 \\ 1 & 0 \\ 2 & 1 \end{pmatrix} = \begin{pmatrix} 5 & 2 \\ 2 & 2 \end{pmatrix}$
- $(A^T A)^{-1} = \begin{pmatrix} 5 & 2 \\ 2 & 2 \end{pmatrix}^{-1} = \frac{1}{(5)(2)-(2)(2)} \begin{pmatrix} 2 & -2 \\ -2 & 5 \end{pmatrix} = \frac{1}{6} \begin{pmatrix} 2 & -2 \\ -2 & 5 \end{pmatrix} = \begin{pmatrix} 1/3 & -1/3 \\ -1/3 & 5/6 \end{pmatrix}$
- $A^T b = \begin{pmatrix} 0 & 1 \\ 1 & 0 \\ 2 & 1 \end{pmatrix}^T \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$
- $(A^T A)^{-1} A^T b = \begin{pmatrix} 1/3 & -1/3 \\ -1/3 & 5/6 \end{pmatrix} \begin{pmatrix} 3 \\ 2 \end{pmatrix} = \begin{pmatrix} 1 - 2/3 \\ -1 + 5/3 \end{pmatrix} = \begin{pmatrix} 1/3 \\ 2/3 \end{pmatrix}$

2. Compute the projection of b onto the column space of A. Let us denote this with $\hat{b}$.

$\hat{b} =$ 2/3 ✔ Answer: 2/3

1/3 ✔ Answer: 1/3

4/3 ✔ Answer: 4/3

Calculator

$$A(A^TA)^{-1}A^Tb = \begin{pmatrix} 0 & 1 \\ 1 & 0 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} 1/3 \\ 2/3 \end{pmatrix} = \begin{pmatrix} 0+2/3 \\ 1/3+0 \\ 2/3+2/3 \end{pmatrix} = \begin{pmatrix} 2/3 \\ 1/3 \\ 4/3 \end{pmatrix}$$

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Answers are displayed within the problem

Question 6

10.0/10.0 points (graded)

Consider $A = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 0 & -2 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix}$.

1. Compute the QR factorization of A.

(Yes, you may use a IPython Notebook to perform the calculations... I did it by hand, and got it wrong the first time!)

$Q =$

0.8944272 ✔	0.1825742 ✔	0.3779645 ✔
Answer: 2 * sqrt(5)/5	Answer: 1 * sqrt(30)/30	Answer: 1 * sqrt(7)/7
0.4472136 ✔	-0.3651484 ✔	-0.7559289 ✔
Answer: sqrt(5)/5	Answer: -2 * sqrt(30)/30	Answer: -2 * sqrt(7)/7
0 ✔	0.9128709 ✔	-0.3779645 ✔
Answer: 0	Answer: 5 * sqrt(30)/30	Answer: -1 * sqrt(7)/7
0 ✔	0.0000000 ✔	0.3779645 ✔
Answer: 0	Answer: 0	Answer: 1 * sqrt(7)/7

$R =$

2.236068 ✔	0.8944272 ✔	0 ✔
Answer: sqrt(5)	Answer: 2 * sqrt(5)/5	Answer: 0
0 ✔	1.0954451 ✔	0 ✔
Answer: 0	Answer: sqrt(30)/5	Answer: 0
0 ✔	0 ✔	2.645751 ✔
Answer: 0	Answer: 0	Answer: sqrt(7)

$$Q = \left(\frac{\sqrt{5}}{5}\begin{pmatrix} 2 \\ 1 \\ 0 \\ 0 \end{pmatrix} \middle| \frac{\sqrt{30}}{30}\begin{pmatrix} 1 \\ -2 \\ 5 \\ 0 \end{pmatrix} \middle| \frac{\sqrt{282}}{282}\begin{pmatrix} 5 \\ -10 \\ -11 \\ 6 \end{pmatrix} \right)$$

$$R = \begin{pmatrix} \sqrt{5} & 2\sqrt{(5)}/5 & 0 \\ 0 & \sqrt{30}/5 & \sqrt{30}/6 \end{pmatrix}$$

Calculator

$$\begin{pmatrix} 0 & 0 & \sqrt{282}/6 \end{pmatrix}$$

$$Q = \left(\frac{\sqrt{5}}{5}\begin{pmatrix} 2 \\ 1 \\ 0 \\ 0 \end{pmatrix} \middle| \frac{\sqrt{30}}{30}\begin{pmatrix} 1 \\ -2 \\ 5 \\ 0 \end{pmatrix} \middle| \frac{\sqrt{7}}{7}\begin{pmatrix} 1 \\ -2 \\ -1 \\ 1 \end{pmatrix} \right)$$

$$R = \begin{pmatrix} \sqrt{5} & 2\sqrt{(5)}/5 & 0 \\ 0 & \sqrt{30}/5 & 0 \\ 0 & 0 & \sqrt{7} \end{pmatrix}$$

- $\rho_{0,0} = \|a_0\|_2 = \sqrt{4+1} = \sqrt{5}$

  $q_0 = a_0/\rho_{0,0} = \frac{\sqrt{5}}{5}\begin{pmatrix} 2 \\ 1 \\ 0 \\ 0 \end{pmatrix}$

- $\rho_{0,1} = q_0^T a_1 = \left[\frac{\sqrt{5}}{5}\begin{pmatrix} 2 \\ 1 \\ 0 \\ 0 \end{pmatrix}\right]^T \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix} = \frac{2\sqrt{5}}{5}$

$$\begin{aligned} a_1^\perp &= a_1 - \rho_{0,1}q_0 = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix} - \frac{2\sqrt{5}}{5}\left[\frac{\sqrt{5}}{5}\begin{pmatrix} 2 \\ 1 \\ 0 \\ 0 \end{pmatrix}\right] \\ &= \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix} - \frac{2}{5}\begin{pmatrix} 2 \\ 1 \\ 0 \\ 0 \end{pmatrix} \\ &= \begin{pmatrix} 1-4/5 \\ \end{pmatrix} \quad \begin{pmatrix} 1 \\ \end{pmatrix} \end{aligned}$$

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Calculator