3. Differential of f(x,y,z)

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Lecture due Oct 5, 2021 20:30 IST

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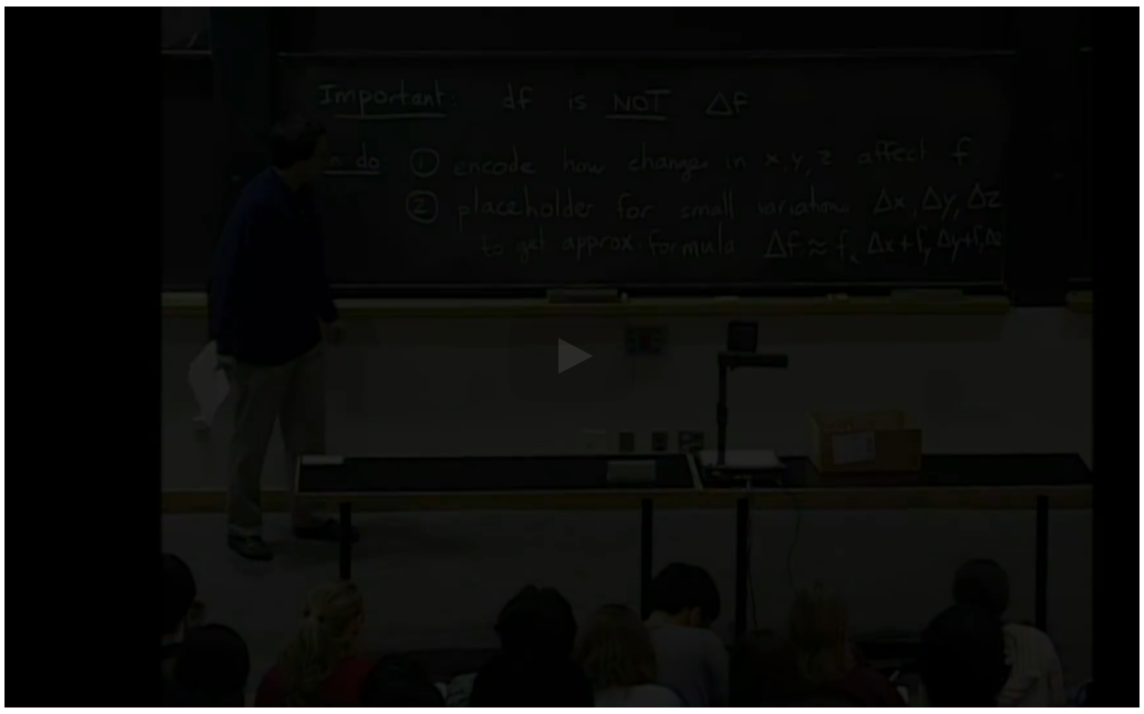
Total Differentials



4:40 / 4:40 2.0x

numbers,

then I will actually get a numerical quantity.

And that will be an approximation formula

for delta f.

It will be the linear approximation

or the tangent plane approximation.

So what we can do--

well, so let me start first with maybe something

even before that.

So the first thing that it does is

it can encode how changes in x, y, z affect the value of f.

I would say that's the more--

that's the most general answer to what is this formula?

What are these differentials?

It's a relation between x, y, z, and f.

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Differential of $f(x, y, z)$

The language of differentials is particularly useful for understanding functions of several variables.

Suppose we have a quantity f that depends on x, y and z, say $f = f(x, y, z)$. Then the "differential of f" is as follows:

$$df = f_x\, dx + f_y\, dy + f_z\, dz \tag{6.111}$$

How to understand this notation? The equation expresses the fact that if we change x, y, and z by small amounts $\Delta x, \Delta y$, and Δz then it will cause a change in f that is approximately equal to

$$\Delta f \approx f_x\, \Delta x + f_y\, \Delta y + f_z\, \Delta z \tag{6.112}$$

where f_x, f_y, f_z are the partial derivatives of f at the starting point. Furthermore, it expresses that the approximation gets better and better as $\Delta x, \Delta y$, and Δz shrink to 0.

Infinitesimal interpretation

Although the notion is not completely precise, it is sometimes helpful to think of each differential as representing an "infinitesimal change." If it were possible to change x, y, and z by infinitesimal amounts dx, dy, and dz, then, in some sense, it would cause an infinitesimal change in f in the amount of df.

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Placeholder interpretation

Perhaps a more useful interpretation is to think of these differentials as placeholders that track how changes in each input variable x, y, and z cause changes in the output variable f. When we need to do an actual approximation, we will replace the differentials with numerical values, and replace the $=$ sign with an $\approx$ sign.

Example 3.1 Suppose $f(x, y, z) = (x+2y)^2 + z$. At the point $(1,1,1)$ the value of f is 10. If we move (x, y, z) from $(1,1,1)$ to $(0.9, 1, 1.1)$, it will cause f to change a little bit. By how much? The differential of f can tell us.

By differentiation, we have

$$df = f_x\, dx + f_y\, dy + f_z\, dz \tag{6.113}$$

At $(1,1,1)$ the partial derivatives of f are:

$$f_x = 6 \tag{6.114}$$

$$f_y = 12 \tag{6.115}$$

$$f_z = 1 \tag{6.116}$$

So we have:

$$df = 6\, dx + 12\, dy + \ dz \tag{6.117}$$

This tells us that for small values of $\Delta x, \Delta y, \Delta z$ we will have

$$\Delta f \approx 6\, \Delta x + 12\, \Delta y + 1\, \Delta z \tag{6.118}$$

Therefore, moving (x, y, z) from $(1,1,1)$ to $(0.9, 1, 1.1)$ will change f by

$$\Delta f \approx 6\,(-0.1) + 12\,(0) + 1\,(0.1) = -0.5 \tag{6.119}$$

So we expect $f(0.9, 1, 1.1)$ to be near 9.5 (indeed it is very close).

When f is a function of several variables, df is known as a "total differential". The total differential can also be written as:

$$df = \frac{\partial f}{\partial x}\, dx + \frac{\partial f}{\partial y}\, dy + \frac{\partial f}{\partial z}\, dz \tag{6.120}$$

Note: Using differentials is just another way of thinking about the same linear approximation we have used throughout the course. It might feel needlessly abstract at this point, but once you get used to thinking in terms of differentials, you will have an easier time understanding more complicated formulas in calculus. For example, the notation gives a nice shortcut to understanding the multivariable chain rule, which is the subject of the following pages.

Practice

1/1 point (graded)
Suppose g is a quantity that depends on two variables u and v, say $g = g(u, v)$. Which of the following is the correct formula for dg?

- ○ $dg = du + dv$
- ◉ $dg = g_u\, du + g_v\, dv$
- ○ $dg = g_x\, dx + g_y\, dy + g_z\, dz$
- ○ $dg = g_u + g_v$
- ○ $dg = f_x\, dx + f_y\, dy$

✔

Solution:

The function g depends on variables u and v. So the differential dg should depend on the differentials du and dv.

If u changes by a small amount Δu, then it will cause g to change by $g_u \Delta u$. The same sentence is true replacing u with v. Therefore the total differential of g is given by

$$dg = g_u\, du + g_v\, dv \tag{6.121}$$

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Answers are displayed within the problem

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