Course > Unit 1: ... > 2 Nulls... > 14. Bas...

14. Basis and dimension of the nullspace

Formula for the dimension of the nullspace. Suppose that the result of putting a matrix $\mathbf{A}$ in row echelon form is $\mathbf{B}$. Then $\mathrm{NS}(\mathbf{A}) = \mathrm{NS}(\mathbf{B})$ (since row reductions do not change the solutions), and

$$\boxed{\dim \mathrm{NS}(\mathbf{A}) = \# \text{ non-pivot columns of } \mathbf{B}.}$$

(The boxed formula holds since it is the same as $\dim \mathrm{NS}(\mathbf{B}) = \#$ free variables.)

Steps to find a basis and the dimension of the space of solutions to a homogeneous linear system
$\mathbf{Ax} = \mathbf{0}$:

1. Perform Gaussian (Gauss–Jordan) elimination on $\mathbf{A}$ to convert it to a matrix $\mathbf{B}$ in (reduced) row echelon form.
2. Identify the pivots of $\mathbf{B}$.
3. Count the number of **non-pivot** columns of $\mathbf{B}$; that number is $\dim \mathrm{NS}(\mathbf{A})$.
4. Use back-substitution to find the general solution to $\mathbf{Bx} = \mathbf{0}$.
5. The general solution will be expressed as the general linear combination of a list of vectors; that list is a basis for $\mathrm{NS}(\mathbf{A})$.

Warning: You must put the matrix in row echelon form before counting non-pivot columns!

Nullspace concept check I

1/1 point (graded)

Which of the following is a basis for the nullspace of the matrix

$$\mathbf{A} = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 0 & 1 \end{pmatrix}?$$

- $\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$
- $\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$
- $\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ ✔
- $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

Solution:

The matrix is already in row echelon form. It has 2 pivots (in orange), therefore 2 pivot variables, and one free variable corresponding to the non-pivot column in blue:

$$\mathbf{A} = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 0 & 1 \end{pmatrix}.$$

Use back substitution to get the general solution $\mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$ for the nullspace. The matrix has one non-pivot column, so we expect a 1-dimensional nullspace. We start the back substitution with

$$x_3 = 0.$$

The variable x_2 is a free variable, so we set

$$x_2 = c \ \text{ for a parameter } \ c$$

Then we have from the first row

$$x_1 = 0.$$

The general solution is

$$\mathbf{x} = \begin{pmatrix} 0 \\ c \\ 0 \end{pmatrix} = c \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix},$$

so a basis for the nullspace of A is $\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$.

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Answers are displayed within the problem

Nullspace concept check II

1/1 point (graded)

What is the nullspace of the matrix $\mathbf{A} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$?

- The x-axis in $\mathbb{R}^3$.
- The xy-plane in $\mathbb{R}^3$.
- The xz-plane in $\mathbb{R}^3$.
- The yz-plane in $\mathbb{R}^3$. ✔
- All of $\mathbb{R}^3$.

Solution:

The nullspace of A is the yz-plane in $\mathbb{R}^3$.

The matrix is already in row echelon form. It has 1 pivot (in orange), therefore one pivot variable x, and two free variables y and z corresponding to the non-pivot columns in blue.

$$\mathbf{A} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

To find the nullspace, we set the free variables equal to parameters:

$$\begin{aligned} y &= c_1 \\ z &= c_2 \end{aligned}$$

Then the equation $\mathbf{Ax} = \mathbf{0}$ forces $x = 0$. Thus the nullspace is all vectors of the form $\begin{pmatrix} 0 \\ c_1 \\ c_2 \end{pmatrix}$ for c_1 and c_2 any real numbers. This is exactly the yz-plane.

Alternative solution: Notice that in this case, we can see that system $\mathbf{Ax} = \mathbf{0}$ has one equation, $x = 0$. This forces $x = 0$ and y and z can be anything. This is exactly a description of the yz-plane.

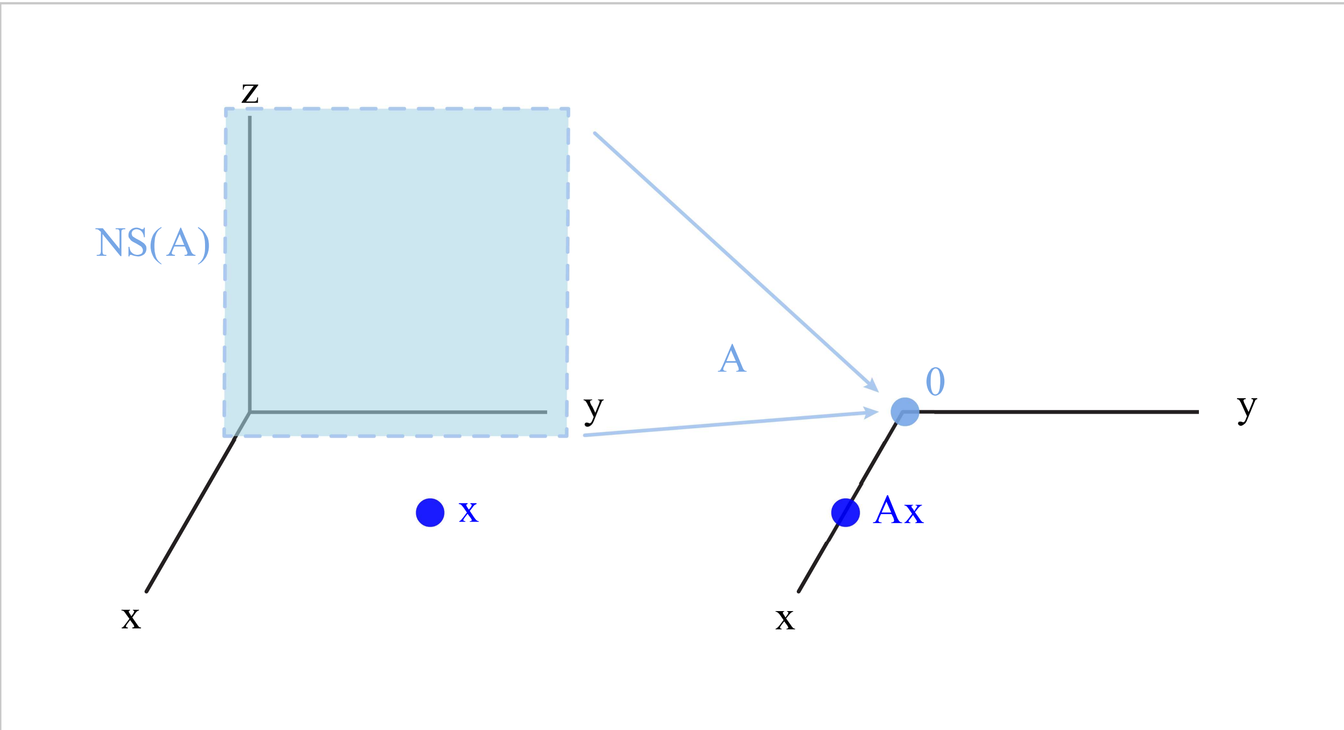
To see this we notice that

$$A\begin{pmatrix}1\\0\\0\end{pmatrix} = \begin{pmatrix}1\\0\end{pmatrix},$$

as well as that

$$A\begin{pmatrix}0\\1\\0\end{pmatrix} = A\begin{pmatrix}0\\0\\1\end{pmatrix} = \mathbf{0}.$$

So under A a vector $\mathbf{x} = \begin{pmatrix}x\\y\\z\end{pmatrix}$ in $\mathbb{R}^3$ gets sent to $A\mathbf{x} = \begin{pmatrix}x\\0\end{pmatrix}$ in $\mathbb{R}^2$. Hence the nullspace consists of all vectors in $\mathbb{R}^3$ with the x component equal to zero: this is the yz-plane.



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Nullity (A) 1