11.3.4 Orthogonal Bases (Alternative Explanation)

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Week 11 due Dec 22, 2023 21:12 IST Completed

11.3.4 Orthogonal Bases (Alternative Explanation)

Kindly note the following post on the discussion board about the below video:

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Video Slide 30 Error

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discussion posted a day ago by **encipher**

I believe there is an error on slide 30 in the video.

$\rho_{1,2} q_1$ should read $\rho_{1,k} q_1$

Best regards.

Related to: Week 11 / 11.3.4

Video

Start of transcript. Skip to the end.

Dr. Robert van de Geijn: We're now going to go

through an alternative explanation of the Gram-Schmidt process, which

will allow us to then link Gram-Schmidt orthogonalization to something called

the QR factorization in the next unit.

We're given n vectors, a0 through n

0:00 / 0:00 2.0x

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Reading Assignment

0 points possible (ungraded)

Read Unit 11.3.4 of the notes. [LINK]

- [x] Done

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Calculator

✔ Correct

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? Homework 11.3.4.3 - Can a negative length here be correct?
In Homework 11.3.4.3, the answer to "Compute the length of the component of a1 in the direction of q0" is shown to be "-8/sqrt(6)". The minus s... 3

Homework 11.3.4.1

13/13 points (graded)

Consider $A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{pmatrix}$.

- Is $a_0 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$ and $a_1 = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$ an orthonormal basis for $\mathcal{C}(A)$?

FALSE ✔ Answer: FALSE

- Compute the length of a_0:

sqrt(2) ✔ Answer: sqrt(2)

$\sqrt{2}$

$\rho_{0,0} = \|a_0\|_2 = \sqrt{a_0^T a_0} =$ (to enter the square root of n, say sqrt(n)).

- Normalize a_0 to length one:

$q_0 =$

1/sqrt(2) ✔ Answer: 1/sqrt(2)

$\frac{1}{\sqrt{2}}$

0 ✔ Answer: 0

0

1/sqrt(2) ✔ Answer: 1/sqrt(2)

$\frac{1}{\sqrt{2}}$

$q_0 = a_0 / \rho_{0,0} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$ (which can be put in standard form, but let's leave it alone...)

- Compute the length of the component of a_1 in the direction of q_0:

1/sqrt(2) ✔ Answer: 1/sqrt(2)

$\frac{1}{\sqrt{2}}$

Calculator

$$\rho_{0,1} = q_0^T a_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}^T \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} = \frac{1}{\sqrt{2}}.$$

- Compute the component of a_1 orthogonal to q_0:

$a_1^\perp =$

-1/2 ✔ Answer: -1/2

$-\frac{1}{2}$

1 ✔ Answer: 1

1

1/2 ✔ Answer: 1/2

$\frac{1}{2}$

$$a_1^\perp = a_1 - \rho_{0,1} q_0 = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} - \frac{1}{\sqrt{2}} \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} - \frac{1}{2} \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -\frac{1}{2} \\ 1 \\ \frac{1}{2} \end{pmatrix}.$$

- Compute the length of $a_1^\perp$:

sqrt(3/2) ✔ Answer: sqrt(3)/sqrt(2)

$\sqrt{\frac{3}{2}}$

$$\rho_{1,1} = \|a_1^\perp\|_2 = \sqrt{a_1^{\perp\,T} a_1^\perp} = \sqrt{\frac{1}{4} + 1 + \frac{1}{4}} = \frac{\sqrt{3}}{\sqrt{2}}.$$

- Normalize $a_1^\perp$ to have length one:

$q_1 =$

-1/sqrt(6) ✔ Answer: -sqrt(2)/(2*sqrt(3))

$-\frac{1}{\sqrt{6}}$

sqrt(2/3) ✔ Answer: sqrt(2)/sqrt(3)

$\sqrt{\frac{2}{3}}$

1/sqrt(6) ✔ Answer: sqrt(2)/(2*sqrt(3))

$\frac{1}{\sqrt{6}}$

$$q_1 = a_1^\perp / \rho_{1,1} = \frac{\sqrt{2}}{\sqrt{3}} \begin{pmatrix} -\frac{1}{2} \\ 1 \\ \frac{1}{2} \end{pmatrix}.$$ (which can be put in standard form, but let's not!)

Submit

Calculator

Answers are displayed within the problem

Homework 11.3.4.2

13/13 points (graded)

Consider $A = \begin{pmatrix} 1 & -1 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 2 \end{pmatrix}$.

- Is $a_0 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, a_1 = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$, and $a_2 = \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}$ an orthonormal basis for $\mathcal{C}(A)$.

FALSE

✔ Answer: FALSE

- Compute the length of a_0:

sqrt(3)

✔ Answer: sqrt(3)

$\sqrt{3}$

$\rho_{0,0} = \|a_0\|_2 = \sqrt{a_0^T a_0} = \sqrt{3}.$

- Normalize a_0 to length one:

$q_0 =$

1/sqrt(3)

✔ Answer: 1/sqrt(3)

$\frac{1}{\sqrt{3}}$

1/sqrt(3)

✔ Answer: 1/sqrt(3)

$\frac{1}{\sqrt{3}}$

1/sqrt(3)

✔ Answer: 1/sqrt(3)

$\frac{1}{\sqrt{3}}$

$q_0 = a_0 / \rho_{0,0} = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ (which can be put in standard form, but let's leave it alone...)

- Compute the length of the component of a_1 in the direction of q_0:

0

✔ Answer: 0

0

$$\rho_{0,1} = q_0^T a_1 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}^T \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} = 0.$$

Ah! This means that a_1 is orthogonal to q_1.

- Compute the component of a_1 orthogonal to q_0:

Calculator

-1 ✔ Answer: -1

-1

$a_1^\perp =$

0 ✔ Answer: 0

0

1 ✔ Answer: 1

1

$$a_1^\perp = a_1 - \rho_{0,1} q_0 = a_1 - 0 q_0 = a_1 = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}.$$

- Compute the length of $a_1^\perp$:

sqrt(2) ✔ Answer: sqrt(2)

$\sqrt{2}$

$\rho_{1,1} = \|a_1^\perp\|_2 = \sqrt{a_1^{\perp T} a_1^\perp} = \sqrt{2}.$

- Normalize $a_1^\perp$ to have length one:

$q_1 =$

-1/sqrt(2) ✔ Answer: -1/sqrt(2)

$-\frac{1}{\sqrt{2}}$

0 ✔ Answer: 0

0

1/sqrt(2) ✔ Answer: 1/sqrt(2)

$\frac{1}{\sqrt{2}}$

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Answers are displayed within the problem

Homework 11.3.4.3

13/13 points (graded)

Consider $A = \begin{pmatrix} 1 & 1 \\ 1 & -1 \\ -2 & 4 \end{pmatrix}$.

- Is $a_0 = \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix}$ and $a_1 = \begin{pmatrix} 1 \\ -1 \\ 4 \end{pmatrix}$ an orthonormal basis for $\mathcal{C}(A)$.

FALSE ✔ Answer: FALSE

Calculator

- Compute the length of a_0:

sqrt(6) ✔ Answer: sqrt(6)

$\sqrt{6}$

$$\rho_{0,0} = \|a_0\|_2 = \sqrt{a_0^T a_0} = \sqrt{6}.$$

- Normalize a_0 to length one:

$q_0 =$

1/sqrt(6) ✔ Answer: 1/sqrt(6)

$\frac{1}{\sqrt{6}}$

1/sqrt(6) ✔ Answer: 1/sqrt(6)

$\frac{1}{\sqrt{6}}$

-2/sqrt(6) ✔ Answer: -2/sqrt(6)

$-\frac{2}{\sqrt{6}}$

$$q_0 = a_0 / \rho_{0,0} = \frac{1}{\sqrt{6}} \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix}$$ (which can be put in standard form, but let's leave it alone...)

- Compute the length of the component of a_1 in the direction of q_0:

-8/sqrt(6) ✔ Answer: -8/sqrt(6)

$-\frac{8}{\sqrt{6}}$

$$\rho_{0,1} = q_0^T a_1 = \frac{1}{\sqrt{6}} \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix}^T \begin{pmatrix} 1 \\ -1 \\ 4 \end{pmatrix} = \frac{-8}{\sqrt{6}}.$$

- Compute the component of a_1 orthogonal to q_0:

$a_1^\perp =$

7/3 ✔ Answer: 7/3

$\frac{7}{3}$

1/3 ✔ Answer: 1/3

$\frac{1}{3}$

4/3 ✔ Answer: 4/3

$\frac{4}{3}$

$$a_1^\perp = a_1 - \rho_{0,1} q_0 = \begin{pmatrix} 1 \\ -1 \\ 4 \end{pmatrix} - \frac{-8}{\sqrt{6}} \frac{1}{\sqrt{6}} \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \\ 4 \end{pmatrix} + \frac{4}{3} \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix} = \begin{pmatrix} \frac{7}{3} \\ \frac{1}{3} \\ \frac{4}{3} \end{pmatrix}.$$

Calculator

- Compute the length of $a_1^{\perp}$:

sqrt(66)/3

$\frac{\sqrt{66}}{3}$

✔ Answer: sqrt(66)/3

$$\rho_{1,1} = \|a_1^{\perp}\|_2 = \sqrt{a_1^{\perp T} a_1^{\perp}} = \left\| \begin{pmatrix} \frac{7}{3} \\ \frac{1}{3} \\ \frac{4}{3} \end{pmatrix} \right\|_2 = \frac{1}{3} \left\| \begin{pmatrix} 7 \\ 1 \\ 4 \end{pmatrix} \right\|_2 = \frac{\sqrt{49+1+16}}{3} = \frac{\sqrt{66}}{3}.$$

- Normalize $a_1^{\perp}$ to have length one:

$q_1 =$

7/sqrt(66)

$\frac{7}{\sqrt{66}}$

✔ Answer: 7/sqrt(66)

1/sqrt(66)

$\frac{1}{\sqrt{66}}$

✔ Answer: 1/sqrt(66)

4/sqrt(66)

$\frac{4}{\sqrt{66}}$

✔ Answer: 4/sqrt(66)

$$q_1 = a_1^{\perp} / \rho_{1,1} = \frac{3}{\sqrt{66}} \begin{pmatrix} \frac{7}{3} \\ \frac{1}{3} \\ \frac{4}{3} \end{pmatrix}.$$ (which can be put in standard form, but let's not!)

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Answers are displayed within the problem

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