Relationship between F and t statistics

It might initially appear that the analysis of variance and t-tests are different for the hypotheses

$$H_0 : \mu = k$$
$$H_A : \mu \neq k$$

Recall that the t-test is based on the test statistic

$$t = \frac{\overline{y} - k}{s/\sqrt{n}}$$

The F ratio that underlies the analysis of variance test is the square of this,

$$F = \frac{n(\overline{y} - k)^2}{s^2} = t^2$$

The p-value for the t-test is the probability of recording a t-value as far from zero, and this equals the probability of getting as large an F-ratio as the one evaluated from the data.

The t-test and F-test therefore result in identical p-values.

Why use the analysis of variance table?

Nothing has been gained from testing the mean of a normal sample with an analysis of variance table and test, so you may wonder why we have bothered to described it here.

The analysis of variance approach can be extended to analyse and test more complex models.

We have only described the analysis of variance approach to testing the mean of a single sample as a simple illustration of this general methodology -- you should use a t-test in practice.

Examples

In the examples below, a hypothesis test about the population mean is performed both with a t-test and an F-test. Note in all examples that:

$$F = t^2$$

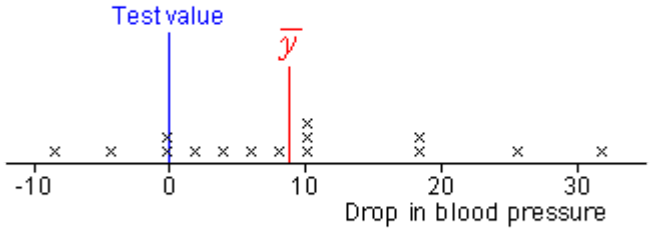
The p-values and therefore the conclusions are identical for both tests.

Blood pressure and the pill ▼

Data: The blood pressure of 15 college-aged women was measured before starting to take the pill and also after 6 months of use. The stacked dot plot below shows the decrease in blood pressure over the 6 months.

Question:

Has the mean blood pressure changed?



Hypotheses:

$H_0 : \mu = 0$ $H_A : \mu \neq 0$

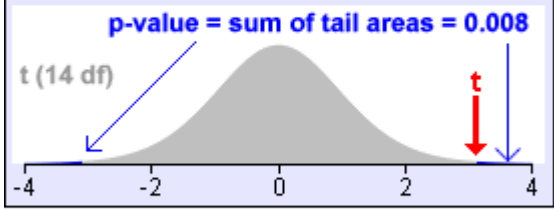
t test

$n = 15$

$\bar{y} = 8.80$

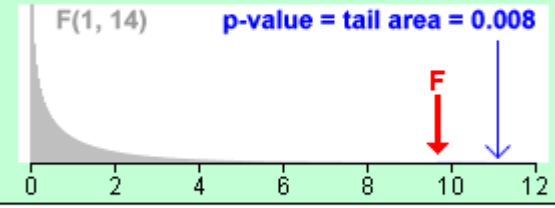
$s = 10.98$

$$t = \frac{\bar{y} - 0}{s/\sqrt{n}} = 3.11$$



F test

Source	SS	df	MSS	F
Mean	1161.6	1	1161.6	9.64
About mean	1686.4	14	120.5	
Total	2848.0	15	189.9	



Conclusion: There is strong evidence that mean blood pressure is lower 6 months after use of the pill. There would be only 0.8% chance of such an unusual F (or t) value if the mean decrease was really zero.