Numerically solving Lotka-Volterra ODE in R

Asked 4 years, 5 months ago Modified today Viewed 558 times Part of R Language Collective

2

Disclaimer: Cross-post on Stack Computational Science

Aim: I am trying to numerically solve a Lotka-Volterra ODE in R, using de `sde.sim()` function in the sde package. I would like to use the `sde.sim()` function in order to eventually transform this system into an SDE. So initially, I started with an simple ODE system (Lotka Volterra model) without a noise term.

The Lotka-Volterra ODE system:

$$\begin{cases} \frac{dx}{dt} = \alpha x - \beta xy. \\ \frac{dy}{dt} = \delta xy - \gamma y \end{cases}$$

with initial values for x = 10 and y = 10.

The parameter values for alpha, beta, delta and gamma are 1.1, 0.4, 0.1 and 0.4 respectively (mimicking this example).

Attempt to solve problem:

```
library(sde)
d <- expression((1.1 * x[0] - 0.4 * x[0] * x[1]), (0.1 * x[0] * x[1] - 0.4 * x[1]))
s <- expression(0, 0)
X <- sde.sim(X0=c(10,10), T = 10, drift=d, sigma=s)
plot(X)
```

However, this does not seem to generate a nice cyclic behavior of the predator and prey population.

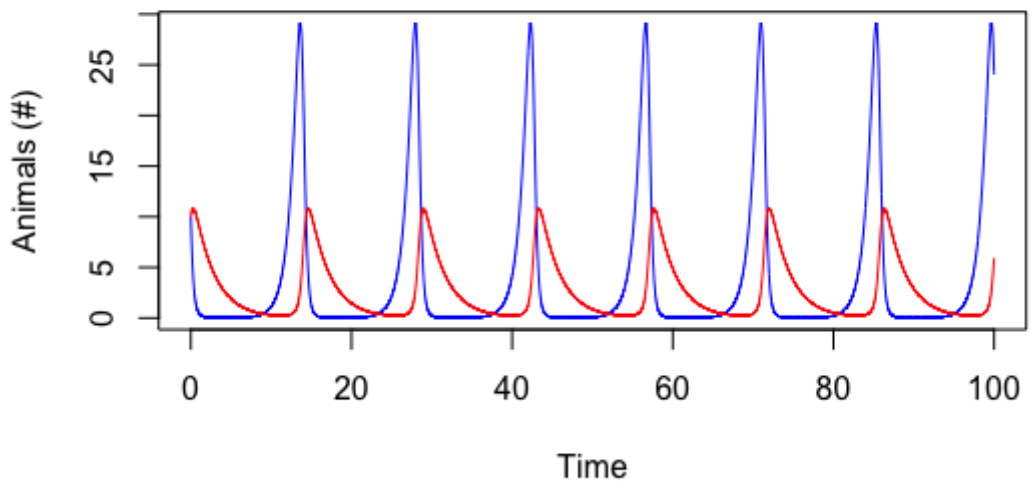
Expected Output

I used the `deSolve` package in R to generate the expected output.

```
library(deSolve)
alpha <-1.1
beta <- 0.4
gamma <- 0.1
delta <- 0.4

yini <- c(X = 10, Y = 10)
Lot_Vol <- function (t, y, parms) {
  with(as.list(y), {
    dX <- alpha * X - beta * X * Y
    dY <- 0.1 * X * Y - 0.4 * Y
    list(c(dX, dY))
  }) }
times <- seq(from = 0, to = 100, by = 0.01)
out   <- ode(y = yini, times = times, func = Lot_Vol, parms = NULL)
plot(y=out[, "X"], x = out[, "time"], type = 'l', col = "blue", xlab = "Time", ylab =
```

```
"Animals (#)")
lines(y=out[, "Y"], x = out[, "time"], type = 'l', col = "red")
```



Question

I think something might be wrong the the drift function, however, I am not sure what. What is going wrong in the attempt to solve this system of ODEs in `sde.sim()` ?

r ggplot2 numerical-methods ode runge-kutta

Share Edit Follow Close Flag

edited 27 secs ago
Sandipan Dey
21.3k 2 49 62

asked Feb 12, 2019 at 9:39
user213544
2,026 3 21 51

1 Shouldn't it be `0.1 * x[0] * x[1] - 0.4 * x[1]` for the second argument of `d <- expression(...)` ? – Maurits Evers Feb 12, 2019 at 10:20

Yes, sorry (sloppy mistake). I updated the code. – user213544 Feb 12, 2019 at 10:22

So does this solve the issue? If not please include your expected output (plot). – Maurits Evers Feb 12, 2019 at 10:47

I included the expected output and code (using the `deSolve` package). – user213544 Feb 12, 2019 at 11:43

2 Answers

Sorted by:
Reset to default

Date modified (newest first)

0

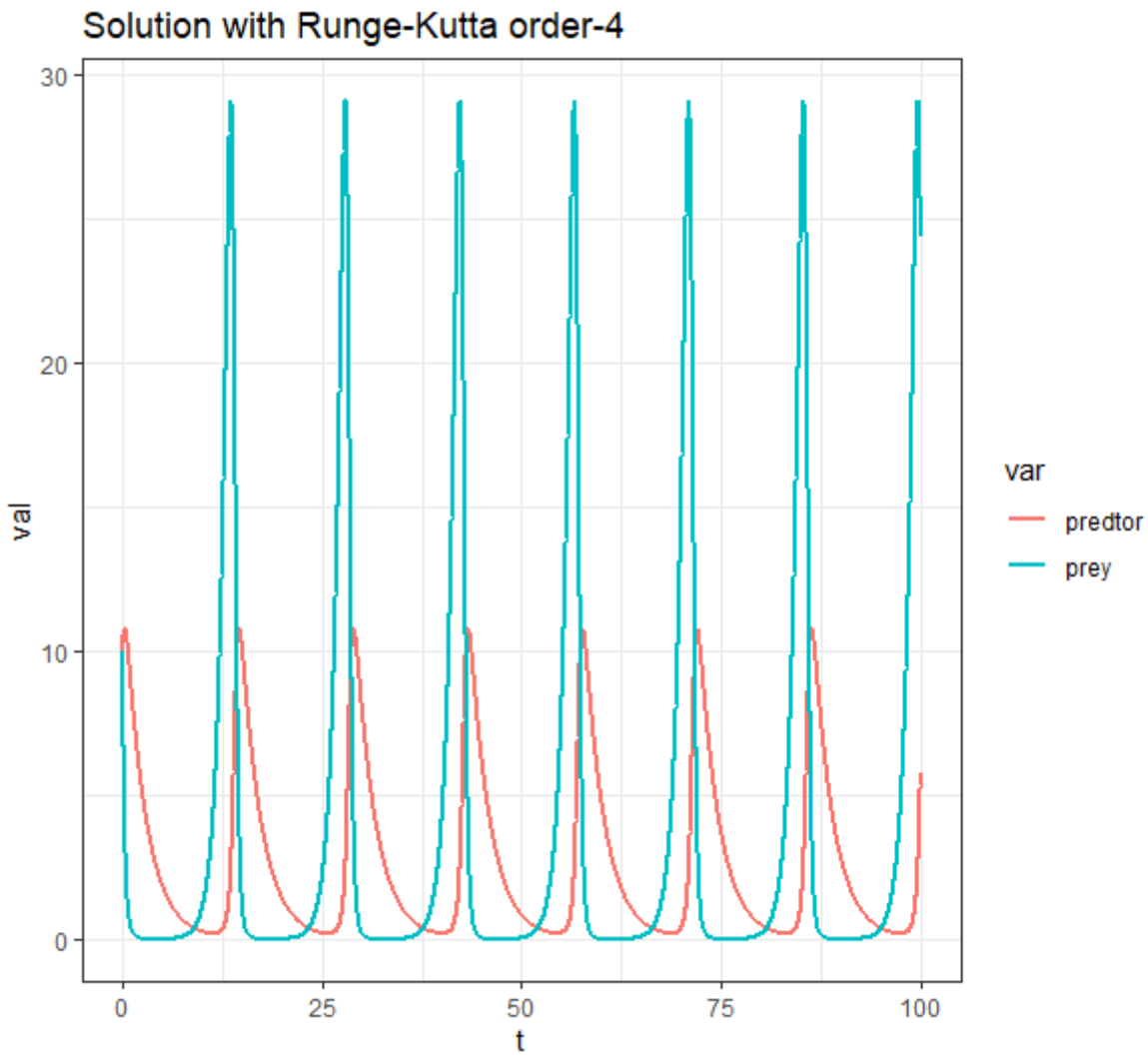
Solving Lotka-Volterra system of ODEs with Runge-Kutta order 4 (RK4) method with the following code, we can get the desired plot using the following code:

```
Runge_Kutta_4_update <- function(xn, delta_t, f) {
  xn <- as.numeric(xn)
  k1 <- f(xn) * delta_t
  k2 <- f(xn + k1/2) * delta_t
  k3 <- f(xn + k2/2) * delta_t
  k4 <- f(xn + k3) * delta_t
  return (xn + 1/6 * (k1 + 2*k2 + 2*k3 + k4))
}

f <- function(v) {
  x <- v[1]
  y <- v[2]
  return (c(alpha * x - beta * x * y, delta * x * y - gamma * y))
}

solve_Lotka_Volterra <- function(v0, delta_t=0.01, niter=10000) {
  ts <- c(0)
  vs <- data.frame(prey=v0[1], predtor=v0[2])
  for (i in seq(2, niter)) {
    ts <- c(ts, ts[i-1] + delta_t)
    v <- Runge_Kutta_4_update(vs[i-1, ], delta_t, f)
    vs <- rbind(vs, data.frame(prey=v[1], predtor=v[2]))
  }
  vs['t'] <- ts
  return (vs)
}

alpha <- 1.1
beta <- 0.4
delta <- 0.1
gamma <- 0.4
solution_df <- solve_Lotka_Volterra(c(10,10))
library(tidyverse)
solution_df %>% gather('var', 'val', -t) %>% ggplot(aes(t, val, col=var)) +
  geom_line(lwd=1) + theme_bw() + ggtitle('Solution with Runge-Kutta order-4')
```



Share Edit Delete Flag

answered 1 min ago

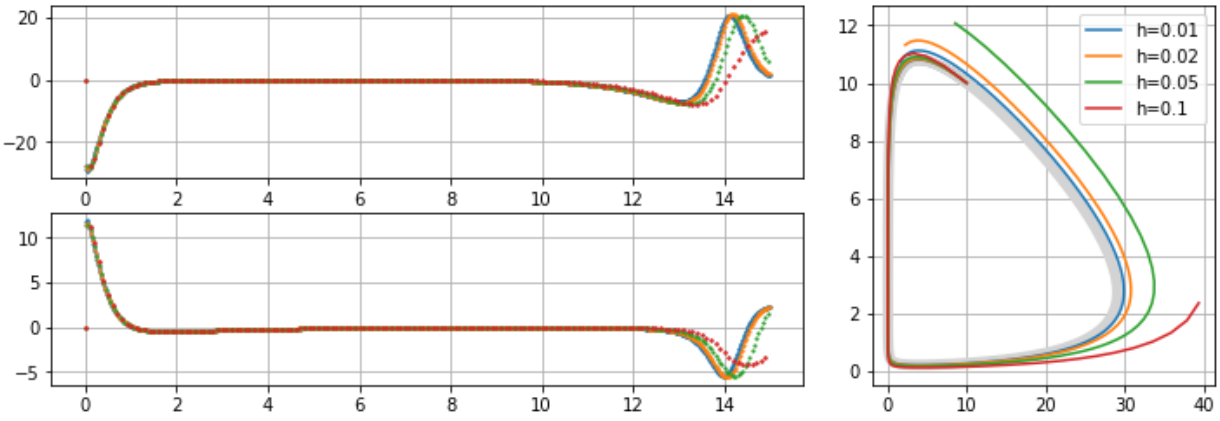
Sandipan Dey
21.3k 2 49 62

3

Assuming that not specifying a method takes the first in the list, and that all other non-specified parameters take default values, you are performing the Euler method with step size `h=0.1`.

As is known on a function that has convex concentric trajectories, the Euler method will produce an outward spiral. As a first order method, the error should grow to size about `T*h=10*0.1=1`. Or if one wants to take the more pessimistic estimate, the error has size `(exp(LT)-1)*h/L`, with `L=3` in some adapted norm this gives a scale of `3.5e11`.

Exploring the actual error `e(t)=c(t)*h` of the Euler method, one gets the following plots. Left are the errors of the components and right the trajectories for various step sizes in the Euler method. The error coefficient the function `c(t)` in the left plots is scaled down by the factor `(exp(L*t)-1)/L` to get comparable values over large time intervals, the value `L=0.06` gave best balance.



One can see that the actual error

```
abs(e(t))<30*h*(exp(L*t)-1)/L
```

is in-between the linear and exponential error models, but closer to the linear one.

To reduce the error, you have to decrease the step size. In the call of `SDE.sim`, this is achieved by setting the parameter `N=5000` or larger to get a step size `h=10/5000=0.002` so that you can hope to be correct in the first two digits with an error bound of `30*h*T=0.6`. In the SDE case you accumulate Gaussian noise of size `sqrt(h)` in every step, so that the truncation error of `O(h^2)` is a rather small perturbation of the random number.

Share Edit Follow Flag

edited Feb 12, 2019 at 12:36

answered Feb 12, 2019 at 10:59

Lutz Lehmann
25.1k 2 22 51

I think I do not understand it completely. Does that mean that the Euler method is not suitable to solve a Lotka Volterra model? Or am I unknowingly omitting essential parameters/code? – user213544 Feb 12, 2019 at 11:17

AFAIK, the best you can expect from an SDE solver used as ODE solver is an explicit order 2 RK method. These are not very nice to conserved quantities, like the level of a first integral. To reduce the error, you have to decrease the step size. So yes, you need to give the correct parameters, at least additionally `N=5000` to get a step size `h=10/5000=0.002` so that you can hope to be correct in the first two digits. In the SDE case you accumulate noise of size `sqrt(h)` in every step, so that the truncation error is not that important. – Lutz Lehmann Feb 12, 2019 at 11:38