Exercise: The posterior of a coin's bias

(3/3 points)

Let Θ be a continuous random variable that represents the unknown bias (i.e., the probability of Heads) of a coin.

a) The prior PDF f_Θ for the bias of a coin is of the form

$$f_\Theta(\theta) = a\theta^9(1-\theta), \quad \text{for } \theta \in [0,1],$$

where a is a normalizing constant. This indicates a prior belief that the bias Θ of the coin is

High ▾ ✔ **Answer:** High

b) We flip the coin 10 times independently and observe 1 Heads and 9 Tails. The posterior PDF of Θ will be of the form $c\theta^m(1-\theta)^n$, where c is a normalizing constant and where

$m =$ 10 ✔ **Answer:** 10

$n =$ 10 ✔ **Answer:** 10

Answer:

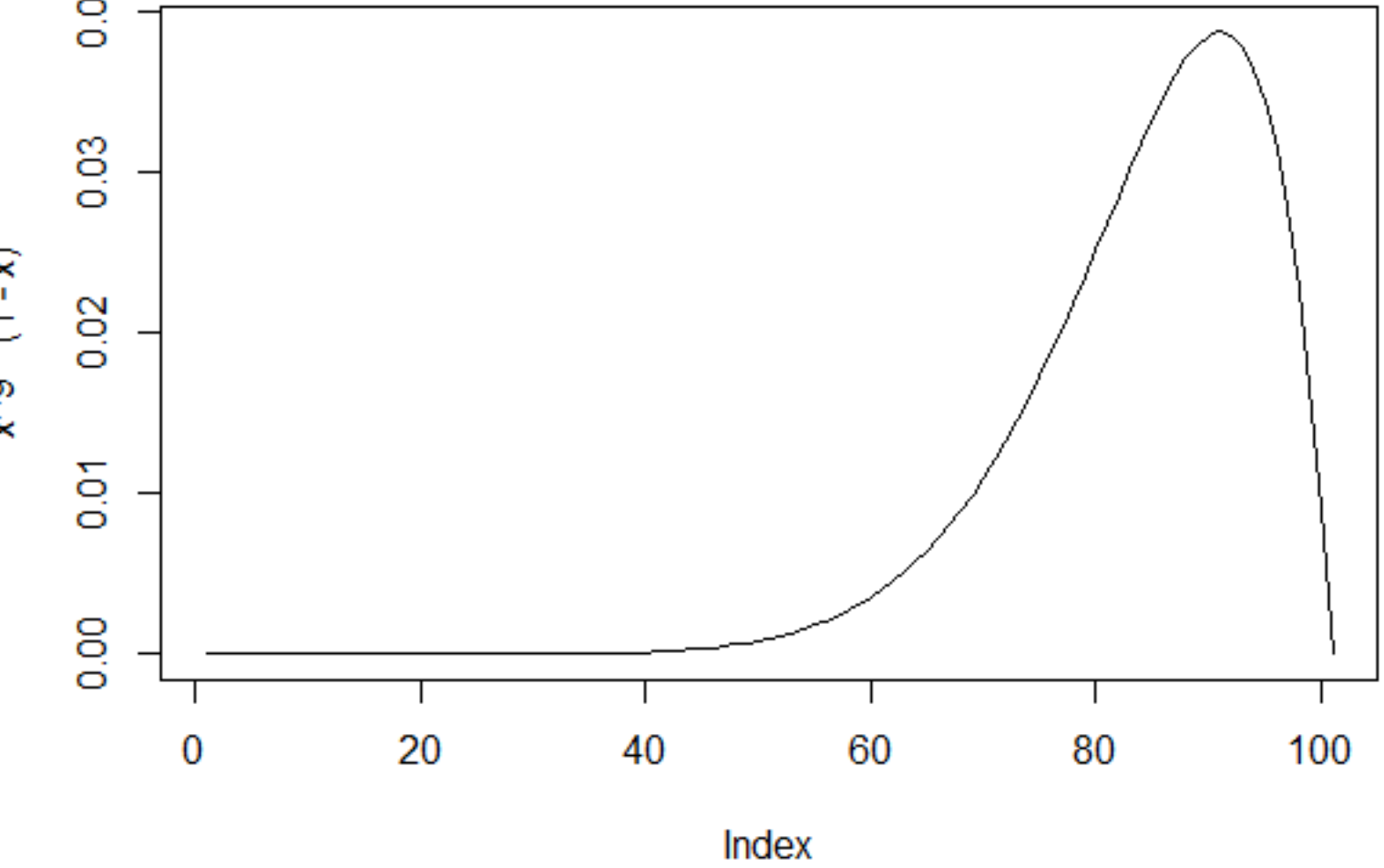
a) Because of the high exponent, the term θ^9 is very small when θ is small. This prior, as can also be seen by plotting it, is concentrated on high values of θ and indicates a prior belief in favor of large values.

b) As we saw in the last video, the power to which θ (respectively, $1-\theta$) is raised needs to be incremented by the number of Heads (respectively, Tails) observed, leading to $m = 9+1 = 10$ and $n = 1+9 = 10$. Notice that the resulting posterior is symmetric around 0.5.

This exercise indicates that the strength of the "evidence" incorporated in a prior with $\alpha = 9$ and $\beta = 1$ is exactly counterbalanced by observing 1 Heads and 9 Tails. Differently said, a

prior with $\alpha = 9$ and $\beta = 1$ can be thought of as equivalent to prior "evidence" based on 9 Heads and 1 Tails.

You have used 1 of 1 submissions


x^9 * (1 − x)
0.00
0.01
0.02
0.03
0
20
40
60
80
100
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