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9. Gradients and steepest increase

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Problem Set A due Aug 18, 2021 20:30 IST Completed

2A-11(a)

1/1 point (graded)

Suppose that $f(x,y) = x^3 + xy^2 + y^3$. It's straightforward to compute that $f(1,1) = 3$, $f_x(1,1) = 4$, and $f_y(1,1) = 5$, and so $\nabla f(1,1) = \langle 4, 5 \rangle$.

Suppose that we want to find the closest point to $(1,1)$ where $f(x,y) = 3.01$. Suppose that this closest point is $\langle 1 + \Delta x, 1 + \Delta y \rangle$. Decide which of the following is true and carefully think about your reasoning.

- ○ $\langle \Delta x, \Delta y \rangle$ is almost perpendicular to $\langle 4, 5 \rangle$.
- ◉ $\langle \Delta x, \Delta y \rangle$ is almost parallel to $\langle 4, 5 \rangle$.
- ○ $\langle 1 + \Delta x, 1 + \Delta y \rangle$ is almost perpendicular to $\langle 4, 5 \rangle$.
- ○ $\langle 1 + \Delta x, 1 + \Delta y \rangle$ is almost parallel to $\langle 4, 5 \rangle$.
- ○ None of the above.

✔

Solution:

The closest point will be in the direction of steepest increase, i.e. along the gradient. Thus the correct option is almost parallel to the gradient vector $\langle 4, 5 \rangle$.

Submit You have used 1 of 1 attempt

Answers are displayed within the problem

2A-11(b)

2/2 points (graded)

Use the linear approximation to find a good approximation of $\langle \Delta x, \Delta y \rangle$ based on the answer above.

(Give answer approximate up to 5 decimal places.)

$\Delta x =$ 0.0009756098 ✔ **Answer:** 0.00098

$\Delta y =$ 0.001219512 ✔ **Answer:** 0.00122

Solution:

We look for a solution of the form $\langle \Delta x, \Delta y \rangle = a\langle 4, 5 \rangle$. Plugging into the formula for the linear approximation we get:

$$f(1 + \Delta x, 1 + \Delta y) \approx f(1,1) + f_x(1,1)\,\Delta x + f_y(1,1)\,\Delta y \quad (3.132)$$

$$3.01 \approx 3 + 4\Delta x + 5\Delta y \quad (3.133)$$

$$4\Delta x + 5\Delta y \approx 0.01$$

$$4(4a) + 5(5a) \approx 0.01$$

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$$4(4a) + 5(5a) \approx 0.01 \quad (3.135)$$

$$16a + 25a \approx 0.01 \quad (3.136)$$

$$a \approx 0.01/41 \approx 0.00024 \quad (3.137)$$

Therefore

$$\Delta x \approx 0.00098 \quad (3.138)$$

$$\Delta y \approx 0.00122 \quad (3.139)$$

Plugging in 1.00098 for x and 1.00122 for y into our formula for f does give $3.01003\ldots$ which has an error of 0.001%.

Submit You have used 1 of 5 attempts

Answers are displayed within the problem

9. Gradients and steepest increase

Topic: Unit 2: Geometry of Derivatives / 9. Gradients and steepest increase

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- bad question 2A-11(b) — 6
  i spent 3 hours trying to get the approximation. i ran out of time on the next problem set (27% grade) partly because of this question...
- 2A-11(b)?? — 10
  This question looks analogous to lecture 6, page 8, first question asking for (delx,dely). However, the analogous calculation is giving ...
- Excellent problem — 2
  I loved it! I feel bad for not thinking through it correctly :-)
- percentage error due to approximation — 2
- [Staff] 2A-11a — 9
- [Staff] Typo 2a-11(a) answers — 2

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