



MITx: 6.041x Introduction to Probability - The Science of Uncertainty



Bookmarks

- ▶ Unit 0: Overview
- ▶ Entrance Survey
- ▶ Unit 1: Probability models and axioms
- ▶ Unit 2: Conditioning and independence
- ▶ Unit 3: Counting
- ▼ Unit 4: Discrete random variables

Unit overview

Lec. 5: Probability mass functions and expectations

Exercises 5 due Mar 02, 2016 at 23:59 UTC



Unit 4: Discrete random variables > Problem Set 4 > Problem 3 Vertical: PMF, expectation, and variance

Bookmark

Problem 3: PMF, expectation, and variance

(6/6 points)

The random variables X and Y have the joint PMF

$$p_{X,Y}(x,y) = \begin{cases} c \cdot (x+y)^2, & \text{if } x \in \{1, 2, 4\} \text{ and } y \in \{1, 3\}, \\ 0, & \text{otherwise.} \end{cases}$$

All answers in this problem should be numerical.


1. Find the value of the constant c .

$c =$ ✓ Answer: 0.00781


2. Find $\mathbf{P}(Y < X)$.

$\mathbf{P}(Y < X) =$ ✓ Answer: 0.64844

**Lec. 6: Variance;
Conditioning on an event;
Multiple r.v.'s**

Exercises 6 due Mar 02, 2016 at
23:59 UTC 


**Lec. 7: Conditioning on a
random variable;
Independence of r.v.'s**

Exercises 7 due Mar 02, 2016 at
23:59 UTC 

Solved problems

**Additional theoretical
material**

Problem Set 4

Problem Set 4 due Mar 02, 2016
at 23:59 UTC 

Unit summary

- ▶ Exam 1
- ▶ Unit 5: Continuous
random variables
- ▶ Unit 6: Further topics
on random variables
- ▶ Unit 7: Bayesian

3. Find $\mathbf{P}(Y = X)$.

$$\mathbf{P}(Y = X) = \boxed{1/32}$$

✓ Answer: 0.03125

4. Find the following probabilities.

$$\mathbf{P}(X = 1) =$$

$$\boxed{5/32}$$

✓ Answer: 0.15625

$$\mathbf{P}(X = 2) = \boxed{17/64}$$

✓ Answer: 0.26563

$$\mathbf{P}(X = 3) = \boxed{0}$$

✓ Answer: 0

$$\mathbf{P}(X = 4) = \boxed{37/64}$$

✓ Answer: 0.57813

5. Find the expectations $\mathbf{E}[X]$ and $\mathbf{E}[XY]$.

$$\mathbf{E}[X] =$$

$$\boxed{3}$$

✓ Answer: 3

$$\mathbf{E}[XY] = \boxed{227/32}$$

✓ Answer: 7.09375

6. Find the variance of \mathbf{X} .

inference

- ▶ Exam 2
- ▶ Unit 8: Limit theorems and classical statistics
- ▶ Unit 9: Bernoulli and Poisson processes
- ▶ Unit 10: Markov chains
- ▶ Exit Survey
- ▶ Final Exam

 $\text{var}(X) =$

1.46875



Answer: 1.46875

Answer:

1. From the joint PMF, there are six (x, y) pairs with nonzero probability mass. These pairs are $(1, 1)$, $(1, 3)$, $(2, 1)$, $(2, 3)$, $(4, 1)$, $(4, 3)$. Because the probability of the entire sample space must equal 1, we have:

$$c(1+1)^2 + c(1+3)^2 + c(2+1)^2 + c(2+3)^2 + c(4+1)^2 + c(4+3)^2 = 1.$$

Solving for c , we get $c = \frac{1}{128}$.

2. There are three possible outcomes for which $y < x$: $(2, 1)$, $(4, 1)$, $(4, 3)$.

$$\mathbf{P}(Y < X) = p_{X,Y}(2, 1) + p_{X,Y}(4, 1) + p_{X,Y}(4, 3) = \frac{9}{128} + \frac{25}{128} + \frac{49}{128} = \frac{83}{128}.$$

3. There is only one possible outcome for which $y = x$: $(1, 1)$.

$$\mathbf{P}(Y = X) = p_{X,Y}(1, 1) = \frac{4}{128}.$$

4. We use the formula $p_X(x) = \sum_y p_{X,Y}(x, y)$.

For example, $p_X(2) = p_{X,Y}(2, 1) + p_{X,Y}(2, 3) = \frac{34}{128}$. More generally, we find that

$$p_X(x) = \begin{cases} 20/128, & \text{if } x = 1, \\ 34/128, & \text{if } x = 2, \\ 74/128, & \text{if } x = 4, \\ 0, & \text{otherwise.} \end{cases}$$

5. We have

$$\mathbf{E}[X] = \sum_x xp_X(x) = 1 \cdot \frac{20}{128} + 2 \cdot \frac{34}{128} + 4 \cdot \frac{74}{128} = 3.$$

Using the expected value rule,

$$\mathbf{E}[XY] = \sum_x \sum_y xyp_{X,Y}(x, y)$$

$$\begin{aligned}
 &= 1 \cdot \frac{4}{128} + 2 \cdot \frac{9}{128} + 4 \cdot \frac{25}{128} + 3 \cdot \frac{16}{128} + 6 \cdot \frac{25}{128} + 12 \cdot \frac{49}{128} \\
 &= \frac{227}{32}.
 \end{aligned}$$

6. The variance of a random variable X can be computed as $\mathbf{E}[X^2] - (\mathbf{E}[X])^2$ or as $\mathbf{E}[(X - \mathbf{E}[X])^2]$. We use the second approach here. We have

$$\text{var}(X) = (1 - 3)^2 \frac{20}{128} + (2 - 3)^2 \frac{34}{128} + (4 - 3)^2 \frac{74}{128} = \frac{47}{32}.$$

You have used 1 of 4 submissions

DISCUSSION

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