11. Review of the rank nullity theorem, preparation for invertibility

When the rank is equal to the number of columns

4/4 points (graded)

Consider the matrix of rank 2

$$\mathbf{A} = \begin{pmatrix} 1 & 3 \\ 2 & 1 \\ 7 & 1 \\ 4 & 1 \end{pmatrix}.$$

What is the dimension of the nullspace $\mathrm{NS}(\mathbf{A})$?

- ● 0 ✓
- ○ 1
- ○ 2
- ○ 3
- ○ 4

What is the dimension of the column space $\mathrm{CS}(\mathbf{A})$?

- ○ 0
- ○ 1
- ● 2 ✓
- ○ 3
- ○ 4

How many solutions can an equation $\mathbf{A}\mathbf{x} = \mathbf{b}$ have? (Choose the option that works in all possible cases.)

○ 0

○ 1

○ infinitely many

● 0 or 1 ✓

○ 0 or infinitely many

○ 1 or infinitely many

What is $\mathtt{rref}(\mathbf{A})$?

○ $\begin{pmatrix} 1 & 3 \\ 0 & -5 \\ 0 & -20 \\ 0 & -11 \end{pmatrix}$

● $\begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}$ ✓

○ $\begin{pmatrix} 1 & 3 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}$

○ $\begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}$

Solution:

This matrix has rank 2, which means it has 2 pivot columns, and no free columns. Therefore the dimension of the nullspace is 0.

The dimension of the column space is equal to the rank, which is 2. Therefore the column space is a 2 dimensional plane inside of $\mathbb{R}^4$.

The equation $\mathbf{Ax} = \mathbf{b}$ has no solution if $\mathbf{b}$ is not in the column space, which is possible since the column space is not all of $\mathbb{R}^4$. However, for any vector $\mathbf{b}$ in the column space, there is only one solution since the nullspace only contains the zero vector. Thus the answer is that $\mathbf{Ax} = \mathbf{b}$ can have 0 or 1 solutions.

Note that the first option is not in reduced echelon form, so we can eliminate that as a possible answer. Performing the elimination steps, we find that the reduced row echelon form is $\begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}$, which has two pivot columns.

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You have used 1 of 3 attempts

Answers are displayed within the problem

When the rank is equal to the number of rows

4/4 points (graded)

Consider the matrix of rank 2

$$\mathbf{A} = \begin{pmatrix} 1 & 2 & 7 & 4 & 4 \\ 3 & 1 & 1 & 1 & 0 \end{pmatrix}.$$

What is the dimension of the nullspace $\mathrm{NS}(\mathbf{A})$?

- ○ 0
- ○ 1
- ○ 2
- ● 3 ✓
- ○ 4
- ○ 5

What is the dimension of the column space $\mathrm{CS}(\mathbf{A})$?

- ○ 0
- ○ 1
- ● 2 ✓
- ○ 3
- ○ 4
- ○ 5

How many solutions can an equation $\mathbf{Ax} = \mathbf{b}$ have? (Choose the option that works in all possible cases.)

- ○ 0
- ○ 1
- ● infinitely many ✓
- ○ 0 or 1
- ○ 0 or infinitely many
- ○ 1 or infinitely many

What is $\mathtt{rref}(\mathbf{A})$?

○ $\begin{pmatrix} 1 & 2 & 7 & 4 & 4 \\ 0 & -5 & -20 & -11 & -12 \end{pmatrix}$

○ $\begin{pmatrix} 1 & 2 & 7 & 4 & 4 \\ 0 & 1 & 4 & 2.2 & 2.4 \end{pmatrix}$

● $\begin{pmatrix} 1 & 0 & -1 & -0.4 & -0.8 \\ 0 & 1 & 4 & 2.2 & 2.4 \end{pmatrix}$ ✓

○ $\begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{pmatrix}$

Solution:

This matrix has rank 2, which means it has 2 pivot columns, and 3 free columns. Therefore the dimension of the nullspace is 3.

The dimension of the column space is equal to the rank, which is 2. Therefore the column space is a 2 dimensional subspace of $\mathbb{R}^2$ which is all of $\mathbb{R}^2$.

The equation $\mathbf{A}\mathbf{x} = \mathbf{b}$ has at least one solution for all vectors $\mathbf{b}$ since the column space is all of $\mathbb{R}^2$. Because the nullspace is 3 dimensional, every equation has infinitely many solutions.

Note that the first two options are not in reduced echelon form, so we can eliminate those as possible answers. Performing the elimination steps, we find that the reduced row echelon form is $\begin{pmatrix} 1 & 0 & -1 & -0.4 & -0.8 \\ 0 & 1 & 4 & 2.2 & 2.4 \end{pmatrix}$, which has a 2 by 2 identity matrix (the two pivot columns) followed by a matrix of consisting of three free columns.

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When a square matrix has full rank

4/4 points (graded)

Consider the matrix of rank 2

$$\mathbf{A} = \begin{pmatrix} 1 & 2 \\ 3 & 1 \end{pmatrix}.$$

What is the dimension of the nullspace $\mathrm{NS}(\mathbf{A})$?

- ● 0 ✓
- ○ 1
- ○ 2
- ○ 3
- ○ 4

What is the dimension of the column space $\mathrm{CS}(\mathbf{A})$?

- ○ 0
- ○ 1
- ● 2 ✓
- ○ 3
- ○ 4

How many solutions can an equation $\mathbf{Ax} = \mathbf{b}$ have? (Choose the option that works in all possible cases.)

- ○ 0
- ● 1 ✓
- ○ infinitely many

○ 0 or 1

○ 0 or infinitely many

○ 1 or infinitely many

What is $\mathtt{rref}(\mathbf{A})$?

● $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ ✓

○ $\begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}$

○ $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

Solution:

This matrix has rank 2, which means it has 2 pivot columns, and no free columns. Therefore the dimension of the nullspace is 0.

The dimension of the column space is equal to the rank, which is 2. Therefore the column space is a 2 dimensional subspace of $\mathbb{R}^2$ which is all of $\mathbb{R}^2$.

The equation $\mathbf{Ax} = \mathbf{b}$ has at least one solution for all vectors $\mathbf{b}$ since the column space is all of $\mathbb{R}^2$. Because the nullspace is 0 dimensional, every equation has only one solution. Matrices with one unique solution for every possible right hand side $\mathbf{b}$ are precisely the matrices that are called invertible.

Note that the row reduced echelon form of this matrix is the identity matrix.

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