3.

Setup:

Let X be a random variable with pdf given by

$$f_\mu(x) = \begin{cases} 0, & x \le 0 \\ \dfrac{1}{\sqrt{2\pi x^3}} \exp\left(-\dfrac{(x-\mu)^2}{2\mu^2 x}\right), & x > 0\,. \end{cases}$$

Canonical Form

2/2 points (graded)

Show that $\{f_\mu, \mu > 0\}$ belongs to the canonical exponential family of distributions by writing it in canonical form. Identify the canonical parameter θ, the function $b(\theta)$, and take $\phi = 1$ (no need to identify the function c).

$\theta =$ -1/(2*mu^2) ✓ **Answer:** -1/(2*mu^2)

$-\frac{1}{2\cdot\mu^2}$

$b(\theta) =$ -sqrt(-2*theta) ✓ **Answer:** -sqrt(-2*theta)

$-\sqrt{-2\cdot\theta}$

STANDARD NOTATION

Submit You have used 1 of 3 attempts

Answers are displayed within the problem

Canonical Link

1/1 point (graded)

What is the canonical link $g(\mu)$?

$g(\mu) =$ -1/(2*mu^2) ✓ **Answer:** -1/(2*mu^2)

$-\frac{1}{2 \cdot \mu^2}$

STANDARD NOTATION

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Answers are displayed within the problem

Expectation and Variance

2.0/2 points (graded)

Compute the expected value $\mathbb{E}[X]$ and the variance $\mathsf{Var}(X)$ of X.

(You may enter in terms of μ or the canonical parameter θ.)

$\mathbb{E}[X] =$ mu ✓ **Answer:** 1/sqrt(-2*theta)

Var $(X) =$ mu^3 ✓ **Answer:** (-2*theta)^(-3/2)

STANDARD NOTATION

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ℹ Answers are displayed within the problem

Fisher Information

0/1 point (graded)

Compute the Fisher information $I(\theta)$.

(You may enter in terms of θ or μ.)

$I(\theta) =$ 1/mu^3 ✗ **Answer:** (-2*theta)^(-3/2)

STANDARD NOTATION

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ℹ Answers are displayed within the problem

MLE

2.0/2 points (graded)

Let $X_1, \ldots, X_n$ be n i.i.d. copies of X.

Compute the maximum likelihood estimator $\hat{\mu}$ of μ and the maximum likelihood estimator $\hat{\theta}$ of θ.

(Enter **barX_n** for $\overline{X_n}$. If applicable, enter **bar(X_n^2)** for $\overline{X_n^2}$. Note “barX_n^2" represents $\left(\overline{X_n}\right)^2$, NOT $\overline{X_n^2}$.)

$\hat{\theta} =$ -1/(2*(barX_n)^2) ✓ **Answer:** -1/(2*barX_n^2)

$\hat{\mu} =$ barX_n ✓ **Answer:** barX_n

STANDARD NOTATION

Submit You have used 1 of 3 attempts

Answers are displayed within the problem

Asymptotic Distribution and Mean

2.0/2.0 points (graded)

In the next two problems, you will specify the asymptotic distribution of $\sqrt{n}\,(\hat{\theta} - \theta)$ and of $\sqrt{n}\,(\hat{\mu} - \mu)$.

0. What type of distribution does $\sqrt{n}\,(\hat{\theta} - \theta)$ converge in distribution to?

$\sqrt{n}\,(\hat{\theta} - \theta)$ converges in distribution to

○ a beta distribution

○ a Student t distribution

● a normal distribution

○ a χ^2 distribution

✓

What is the mean of this asymptotic distribution (in the question above)?

Mean: 0 ✓ **Answer:** 0

0

0. What type of distribution do $\sqrt{n}\,(\hat{\mu}-\mu)$ converge in distribution to?

$\sqrt{n}\,(\hat{\mu}-\mu)$ converges in distribution to

○ a beta distribution

○ a Student t distribution

● a normal distribution

○ a χ^2 distribution

✓

What is the mean of this asymptotic distribution (in the question above)?

Mean: 0 ✓ **Answer:** 0

0

STANDARD NOTATION

Submit You have used 1 of 3 attempts

Answers are displayed within the problem

Asymptotic Variance

2.0/4.0 points (graded)

Continuing from the problem above,

0. Find the asymptotic variance $V\left(\hat{\theta}\right)$ of $\hat{\theta}$, i.e. the variance of the asymptotic distribution of $\sqrt{n}\left(\hat{\theta}-\theta\right)$.

$V\left(\hat{\theta}\right) =$ (1/sqrt(-2*theta))^3 ✗ **Answer:** (-2*theta)^(3/2)

$\left(\frac{1}{\sqrt{-2\cdot\theta}}\right)^3$

0. Find the asymptotic variance $V\left(\hat{\mu}\right)$ of $\hat{\theta}$, i.e. the variance of the asymptotic distribution of $\sqrt{n}\left(\hat{\mu}-\mu\right)$.

$V\left(\hat{\mu}\right) =$ mu^3 ✓ **Answer:** mu^3

μ^3

STANDARD NOTATION

Submit You have used 1 of 3 attempts

Answers are displayed within the problem

One-sided Test

2.0/2.0 points (graded)

Construct a test for $H_0\ :\ \mu = 1$, vs $H_1\ :\ \mu < 1$. Give the formula of the (asymptotic) p-value of this test in terms of $\hat{\mu}$.

(Different reasonable answers will be accepted.)

(Enter **hatmu** for $\hat{\mu}$.
To avoid double jeopardy, you may enter **V** for the asymptotic variance of $\hat{\mu}$ evaluated at $\mu = \hat{\mu}$.
If applicable, enter **Phi(z)** for the cdf $\Phi\left(z\right) = \mathbf{P}\left(Z \le z\right)$ of the standard normal variable, e.g. enter **Phi(0.1)** for $\Phi\left(0.1\right)$; enter **q(alpha)** for the $1-\alpha$ quantile q_α of the standard normal distribution, i.e. $\mathbf{P}\left(Z \le q_\alpha\right) = 1-\alpha$. For, example enter **q(0.01)** for $q_{0.01}$)

p-value= Phi((hatmu-1)/sqrt(V)) ✓ **Answer:** Phi(sqrt(n/V)*(hatmu-1))

Submit You have used 1 of 3 attempts

ℹ Answers are displayed within the problem

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💬	[Staff] Grader is incorrect? I think there are some errors in grader!		65
💬	[Staff] Please double check my answers 1,2,6 (In the order the questions) [Staff] Please double check my answers 1,2,6 (In the order the questions) Thanks a lot!		1
💬	Asymptotic Variance as a function of mu - GRADER ISSUE if theta = -(1/(2*(mu^2))) (grader's answer) and V(theta) = (-2*theta)^(3/2) (again, grader's answer), then V(mu) = mu^(-3), and NOT mu^(+3) (SIGN ISSUE HERE). Please clarify.		3
?	[Staff] bar(X_n) and barX_n In Question for MLE, I submitted "bar(X_n)" for the estimator, while the answer shown in grader is "barX_n". I thought I was kind of confused by the instruction about "bar(...		1
?	[STAFF] Asymptotic Variance at p-value I use the asymptotic variance V = mu^3 = 1^3 = 1 for p-value calculation. So my answer was Phi(sqrt(n)*(hatmu-1)). Is this wrong ?	3 new_	8
?	Interpreting the differences in Asymptotic Variances between theta and mu		4
☑	Question 4: Fisher Infrormation I see all grader issues almost resolved but still have one answer marked incorrectly. I have Fisher information in terms of mu and it is 1/(mu^3). It looks correct to me. Is th...		3
?	[STAFF]Asymptotic variance of theta Dear staff, I have answered the asymptotic variance of theta as 1/mu^3 instead of (-2*theta)^(3/2) as both mu or theta can be used.		3
💬	[Staff] MLE question, additional brackets marks answer as incorrect Dear Staff, I erroneously entered -1/(2*bar(X_n)^2) for the MLE of theta, instead of -1/(2*barX_n^2), which is marked as the correct anwer. The former was accepted and al...		7
💬	[staff] the answer of the last question(One-sided Test) The answer given is "1-Phi(sqrt(n)*(hatmu-1)/V)", isn't it sqrt(V) instead of (V)?		5

The answer given is "1-Phi(sqrt(n)*(hatmu-1)/V)", isn't it sqrt(V) instead of (V)?

[Staff] Something is wrong with the grader

Dear Staff, Please check the following: 1. My answer of variance is $Var\left[X\right] = \mu^3$ which is equivatent to $Var\left[X\right] = \mu^3 = (-2 * \theta)^{3/2}$ 2. My answer to Fisher's informati...

1 new_

? [Staff] My inputs of Var(X), I(theta) and V(hattheta) are equivalent to the answer provided but marked incorrect.

1 new_ **5**

? [Staff] Grader Accepts only one form of answer?