Module 3 Assessment - Part 1

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The following questions are part of the graded assignments for the assessment of module 3. The number of points per question is indicated. The total number of points you can earn in this module is 13.

Note, the second part of the assessment includes a MATLAB exercise (next unit).

Truth about least squares

0/2 points (graded)

Which of the following statements is correct? *Select all correct statements*

- [ ] If $\boldsymbol{y} = \boldsymbol{A}\boldsymbol{x}$, then the normal equation pre-multiplies both sides with the inverse of the design matrix $\boldsymbol{A}$.
- [x] The least-squares solution $\hat{\boldsymbol{x}}_{\mathrm{LS}}$ is a linear function of the observation vector $\boldsymbol{y}$.
- [ ] $\hat{\boldsymbol{y}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{e}$

- [ ] $\hat{e} = A\hat{x} - \hat{y}$

Explanation

The normal equation is given by $A^T y = A^T A x$

$\hat{x} = (A^T A)^{-1} A^T y$

$\hat{y} = A\hat{x}$

$\hat{e} = y - \hat{y} = y - A\hat{x}$

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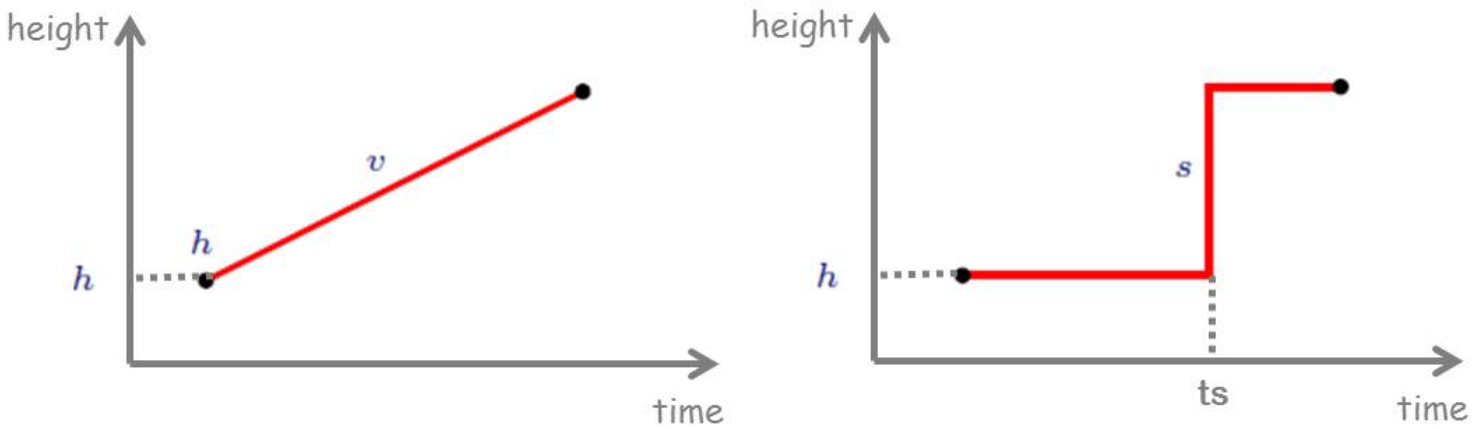
✖ Incorrect (0/2 points)

Airborne laser scanning

4/4 points (graded)

Consider a beach on the coast of the Netherlands. Airborne Laser Scanning is used every year to measure the beach's topography.



After processing, the data set from each acquistion is comprised of a grid of height measurements in meters. For this exercise, let's focus on one particular grid cell (i.e. one location on the beach). This particular grid cell's data set has the height observations from four consecutive years. The goal is to assess what is happening to the location's height over time. There are two scenarios, illustrated in the figure above.

Scenario 1 (left figure): The height at this location may have been changing with constant velocity.

Scenario 2 (right figure): The height may have been constant and then suddenly increased due to a sand suppletion between the second and third year (sand was dumped on the location to preserve the beach).

The height observations (in meters) are given by: $y = [5.2 \;\; 4.6 \;\; 5.5 \;\; 5.4]^T$.

Apply ordinary least squares, and calculate the sum of square residuals for both scenarios.

For scenario 1 (constant velocity) $\hat{e}^T \hat{e}$ = [answer to 3 decimal places]

0.375 ✔

0.375

For scenario 2 (suppletion at t_s) $\hat{e}^T \hat{e} =$ [answer to 3 decimal places]

0.185 ✔

0.185

Assuming that indeed the observations should be given equal weight, which scenario is more likely to be true?

Scenario 2 ✔

What is the estimated initial height h with scenario 2 if we apply weighted least squares with weights $w_i = i$ for $i = 1, 2, 3, 4$?

- $\hat{h} = 4.95$ m
- $\hat{h} = 4.85$ m
- $\hat{h} = 4.80$ m ✔
- $\hat{h} = 4.90$ m

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Truth about weighted least squares

2.0/2.0 points (graded)

Select all correct statements.

- [x] Ordinary least squares estimation is also a form of weighted least squares.
- [ ] Residuals will generally be smaller if we assign a smaller weight and apply weighted least squares.
- [ ] We will always have $\hat{e}^T W \hat{e} \leq \hat{e}^T \hat{e}$.
- [ ] The weighted least squares estimate is equal to the weighted average of the observations.
- [ ] The weighted least squares solution provides the minimum $\hat{e}^T \hat{e}$.

✔

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