

MITx: 6.008.1x Computational Probability and Inference

Heli



- ▶ Introduction
- ▼ 1. Probability and Inference

Introduction to Probability (Week 1)

Exercises due Sep 22, 2016 at 02:30 IST

Probability Spaces and Events (Week 1)

Exercises due Sep 22, 2016 at 02:30 IST

Random Variables (Week 1)

Exercises due Sep 22, 2016 at 02:30 IST

Jointly Distributed Random Variables (Week 2)

Exercises due Sep 29, 2016 at 02:30 IST

Conditioning on Events (Week 2)

Exercises due Sep 29, 2016 at 02:30 IST

1. Probability and Inference > Inference with Bayes' Theorem for Random Variables (Week 3) > Exercise: Bayes' Theorem for Random Variables - Medical Diagnosis, Continued

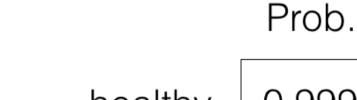
■ Bookmark

Exercise: Bayes' Theorem for Random Variables - Medical Diagnosis, Continued

(4/4 points)

(d)

Recall the medical diagnosis setup from before, summarized in these tables:



healthy X infected

0.999

0.001

Homework 1 (Week 2)

Homework due Sep 29, 2016 at 02:30 IST

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Inference with Bayes' Theorem for Random Variables (Week 3)

Exercises due Oct 06, 2016 at 02:30 IST

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Independence Structure (Week 3)

Exercises due Oct 06, 2016 at 02:30 IST

Homework 2 (Week 3)

Homework due Oct 06, 2016 at 02:30 IST

Notation Summary (Up Through Week 3)

Mini-project 1: Movie Recommendations (Week 3)

Mini-projects due Oct 13, 2016 at 02:30 IST

	$p_{Y X}$	X	
		healthy	infected
Y	positive	0.01	0.99
	negative	0.99	0.01

Recall that Bayes' theorem is given by

$$p_{X\mid Y}(x\mid y) = rac{p_X(x)p_{Y\mid X}(y\mid x)}{\sum_{x'}p_X(x')p_{Y\mid X}(y\mid x')}$$

for all values $oldsymbol{x}$ that random variable $oldsymbol{X}$ can take on.

Use Bayes' theorem to compute the following probabilities: (Please be precise with at least 3 decimal places, unless of course the answer doesn't need that many decimal places. You could also put a fraction.)

What is the MAP estimate for X given Y =positive?

healthy

infected

What is the MAP estimate for $m{X}$ given $m{Y} = \mathbf{negative}$?

healthy

infected

Solution:

Use Bayes' theorem to compute the following probabilities:

$$p_{X|Y}(\text{healthy} \mid \text{positive})$$

$$= \frac{p_{X|Y}(\text{positive} \mid \text{healthy})p_X(\text{healthy})}{p_{X|Y}(\text{positive} \mid \text{healthy})p_X(\text{healthy}) + p_{X|Y}(\text{positive} \mid \text{infected})p_X(\text{infected})}$$

$$= \frac{0.01 \times 0.999}{0.01 \times 0.999 + 0.99 \times 0.001}$$

$$\approx \boxed{0.9098360656}.$$

$$p_{X|Y}(\text{healthy} \mid \text{negative})$$

$$= \frac{p_{X|Y}(\text{negative} \mid \text{healthy})p_X(\text{healthy})}{p_{X|Y}(\text{negative} \mid \text{healthy})p_X(\text{healthy}) + p_{X|Y}(\text{negative} \mid \text{infected})p_X(\text{infected})}$$

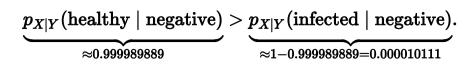
$$= \frac{0.99 \times 0.999}{0.99 \times 0.999 + 0.01 \times 0.001}$$

$$\approx \boxed{0.999989889}.$$

Note that if $Y = \mathbf{positive}$, then the probability that $X = \mathbf{infected}$ is just 1 minus the first probability computed. In this case, it is clear that **healthy** is the MAP estimate of X given $Y = \mathbf{positive}$ because

$$\underbrace{p_{X|Y}(\text{healthy} \mid \text{positive})}_{\approx 0.9098360656} > \underbrace{p_{X|Y}(\text{infected} \mid \text{positive})}_{\approx 1-0.9098360656=0.0901639344}.$$

Similarly, if $Y = \mathbf{negative}$, the MAP estimate for X is still **healthy** since



You have used 1 of 5 submissions

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