Exercise: Conditional expectation

(1/1 point)

Let X and Y be zero-mean independent random variables. Which one of the following statements is correct? *Hint:* You can take for granted the intuitive fact that $\mathbf{E}[X \mid X = x] = x$.

- ○ $\mathbf{E}[X + Y \mid X] = 0.$
- ○ $\mathbf{E}[X + Y \mid X] = x.$
- ◉ $\mathbf{E}[X + Y \mid X] = X.$ ✔
- ○ $\mathbf{E}[X + Y \mid X] = X + Y.$

Answer:
Using linearity of expectations, and then the independence assumption, we have

$$\mathbf{E}[X + Y \mid X = x] = \mathbf{E}[X \mid X = x] + \mathbf{E}[Y \mid X = x] = x + \mathbf{E}[Y] = x.$$

Translating this statement into abstract notation, we obtain $\mathbf{E}[X + Y \mid X] = X.$

You have used 1 of 2 submissions