Uniform distribution joint $\to$ marginal

Asked 6 years, 6 months ago Active 9 months ago Viewed 1k times

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Let vector (X, Y) have a uniform distribution on the set $N = \{(x, y) : x < 1, y < 1, 1 < x + y\}$. Determine distribution $X - Y$.

So far I've thought of this:

$$P[X|Y = y] \sim U(0, 1 - x)\ \forall y \in (0, 1)$$
$$P[Y|X = x] \sim U(0, 1 - y)\ \forall x \in (0, 1)$$

but honestly I don't know how to go about it.

Any hints?

probability statistics probability-distributions

edited May 25 '13 at 18:53 wolfies 4,474 2 11 23

asked May 25 '13 at 14:50 shimee 1,269 1 15 31

1 Draw the picture of N and draw a line $X - Y < C \Leftrightarrow Y > X - C$. And think carefully how you would interprete the areas. – newbie May 25 '13 at 19:00

1 You may want to discuss the conditions $C \leq -1, -1 < C \leq 0, 0 < C \leq 1$ and $C > 1$. – newbie May 25 '13 at 19:06

2 Answers

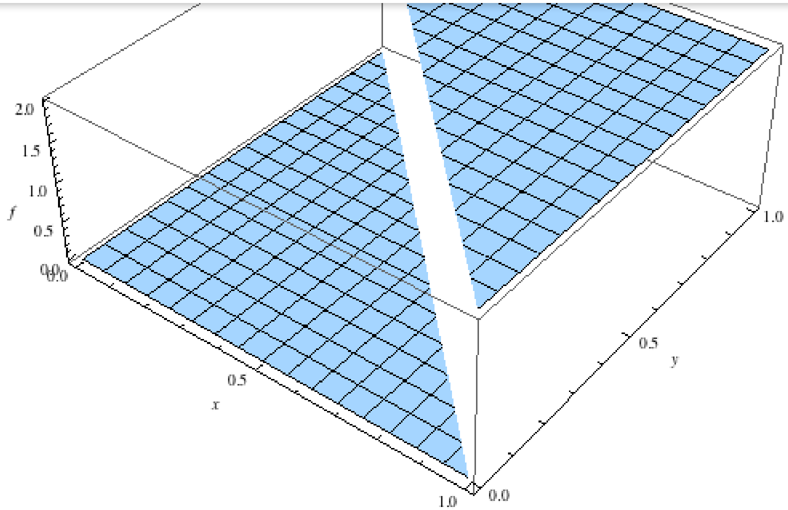
2

So, random variables X and Y have joint pdf $f(x, y)$:

```
In[1]:= f = { 2   1 < x + y ;      domain[f] = {{x, 0, 1}, {y, 0, 1}};
            { 0   True
```

(source: tri.org.au)

which appears thusly:



(source: tri.org.au)

Let $Z = X - Y$. Then, the cdf of Z is $P(Z < z) = P(X - Y < z)$:

In[2]:= `cdf = Prob[x - y < z, f]`

Out[2]= $\begin{cases} 1 & z > 1 \\ \frac{1}{2} + z - \frac{z^2}{2} & 0 \le z \le 1 \\ \frac{1}{2}(1+z)^2 & -1 < z < 0 \\ 0 & \text{True} \end{cases}$

(source: tri.org.au)

where `Prob` is a mathStatica function.

Simply differentiate to obtain the pdf ...

edited Feb 28 at 19:07
Glorfindel
3,474 9 19 30

answered May 25 '13 at 19:08
wolfies
4,474 2 11 23

There is actually a fully "visual" solution to this.

$z > 0$ *and at the point* $(1+z, 1)$ *on the boundary* $y = 1$ *if* $z < 0$.)

Thus the density of $X - Y$ is proportional to the function $f : z \mapsto (1 - |z|)\mathbf{1}_{|z| \leqslant 1}$. Since the integral of f is 1, f is the density of $X - Y$.

answered Jun 16 '13 at 21:11
Did
254k 25 233 483