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Linear Transformations of Random Variables - Quiz

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Question 1

1/1 point (graded)

Suppose X is a continuous random variable. Let $Y = aX + b$, where $a \neq 0$ and b are constants. Then which of the following is **not** true about the density of Y?

a. $f_Y(y) = \frac{dF_Y(y)}{dy}$

b. $f_Y(y) = P(aX + b \leq y)$

c. $f_Y(y) = \frac{1}{a} f_x(\frac{y-b}{a})$ ✔

d. $f(x) = \begin{cases} \frac{1}{a} f_x(\frac{y-b}{a}), \text{if } a \geq 0 \\ \frac{-1}{a} f_x(\frac{y-b}{a}), \text{if } a \leq 0 \end{cases}$

e. $f_Y(y) = \frac{1}{|a|} f_x(\frac{y-b}{a})$

Explanation
All of the statements above, except c, are mathematically true. However, recall that we needed to consider two cases, the distinction between the sign of a. To see this, recall that if $a > 0$,

$$F_Y(y) = P(aX \leq y - b) = P(X \leq \frac{y-b}{a})$$

However, if $a < 0$,

$$F_Y(y) = P(aX \leq y - b) = P(X > \frac{y-b}{a}) = 1 - P(X \leq \frac{y-b}{a})$$

Hence, the expression given in c is correct only if $a > 0$, but not in the case where $a < 0$.

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✔ Correct (1/1 point)

Question 2

1/1 point (graded)
Suppose X is a continous random variable, distributed uniformly over the unit interval $[0, 1]$. Let $Y = 3X + 1$ What is the density of Y, $f_Y(y)$ evaluated at $y = 4$.

Please round your answer to the 2 decimal points. For example, if your answer is 0.567, please round to 0.57 and if it is 0.561, please round to 0.56.

0.33 ✔ **Answer:** 0.33

0.33

Explanation

From the formula Professor Ellison derived in lecture, we have that:

$$f_Y(y) = \frac{1}{|a|} f_x\left(\frac{y-b}{a}\right)$$

We know that the PDF of a uniform random variable distributed on an interval $[c, d]$ is given by $\frac{1}{d-c}$ for $x \in [c, d]$. Plugging in the numbers, we get:

$$f_Y(y) = \frac{1}{|a|} f_x\left(\frac{y-b}{a}\right) = \frac{1}{|3|} f_x\left(\frac{4-1}{3}\right) = \frac{1}{|3|}\frac{3}{3}$$

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