MITx: 15.053x Optimization Methods in Business Analytics

Bookmarks

Week 2 > Problem Set 2 > Problem 4

Bookmark

PART A

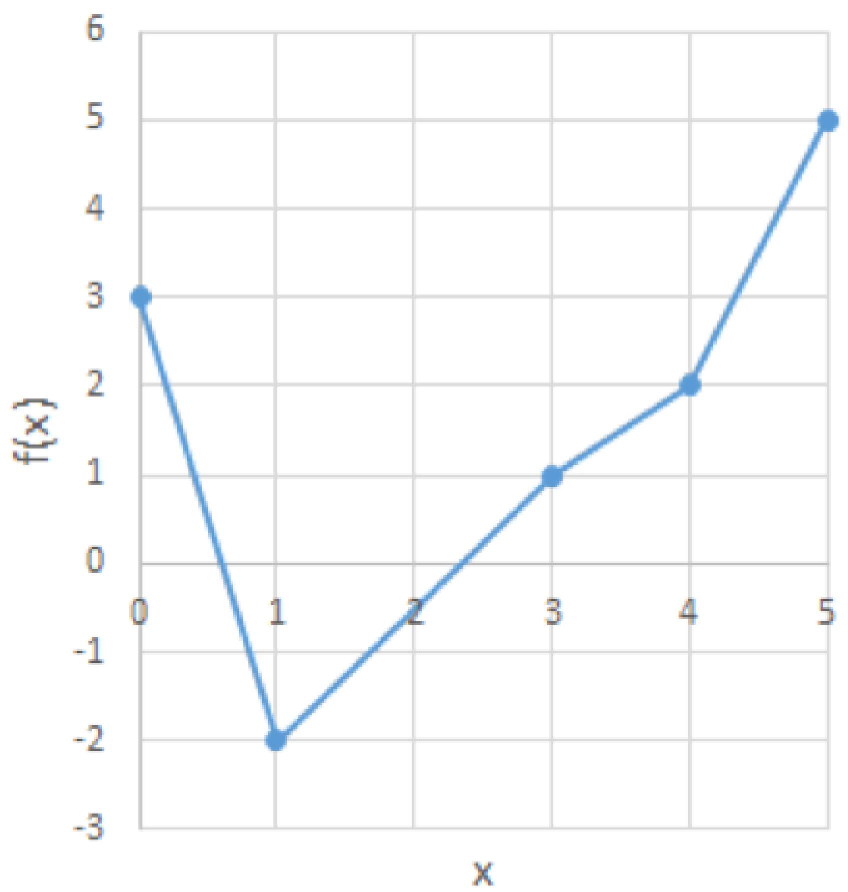
(1/1 point)
Consider

$f(x) : [0,5] \to \mathbb{R}$ a piecewise linear function such that the graph of the function corresponds to the segments between the points

$(0,3),(1,a),(3,1),(4,2),(5,5).$

$a \in \mathbb{R}$ **is a constant with a fixed value.**

Below is a graph for the function under $a = -2$,



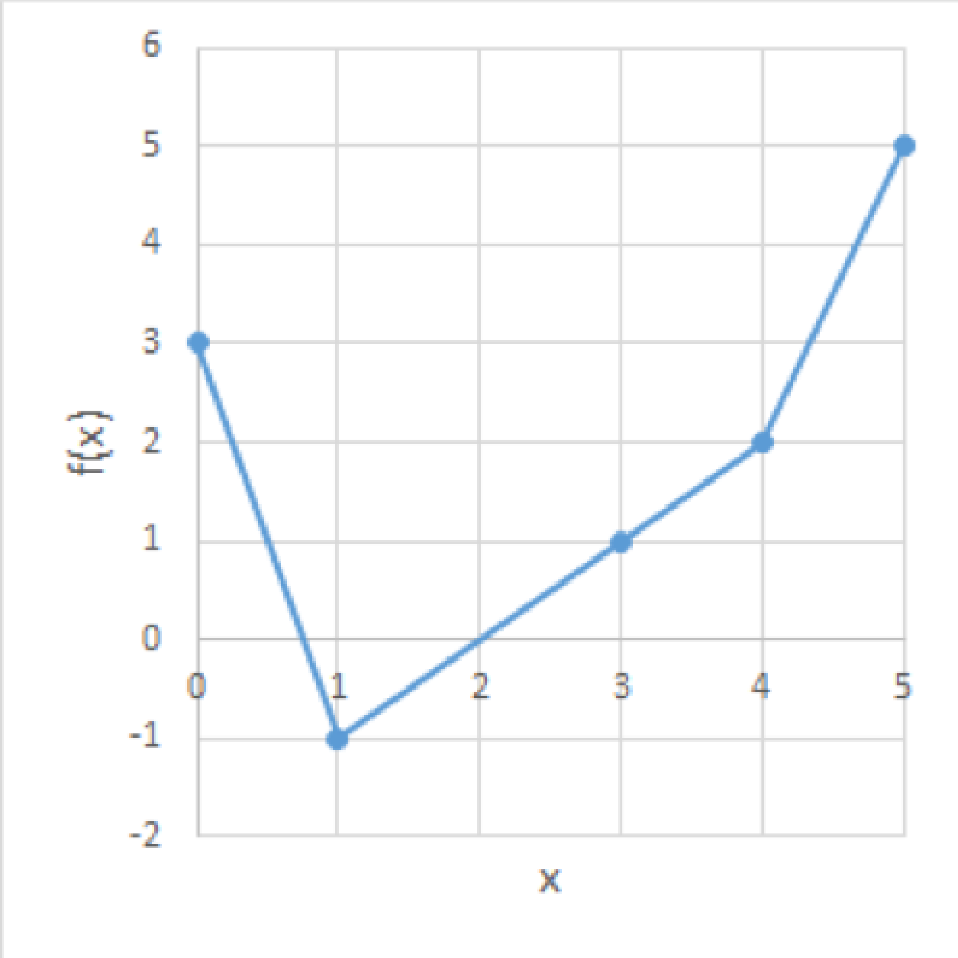
Is this function convex (assuming that $a = -2$) ?

- Convex
- Not convex ✔

You have used 1 of 1 submissions

PART B

(1/1 point)
Below is a graph for the function under $a = -1$,



Is this function convex?

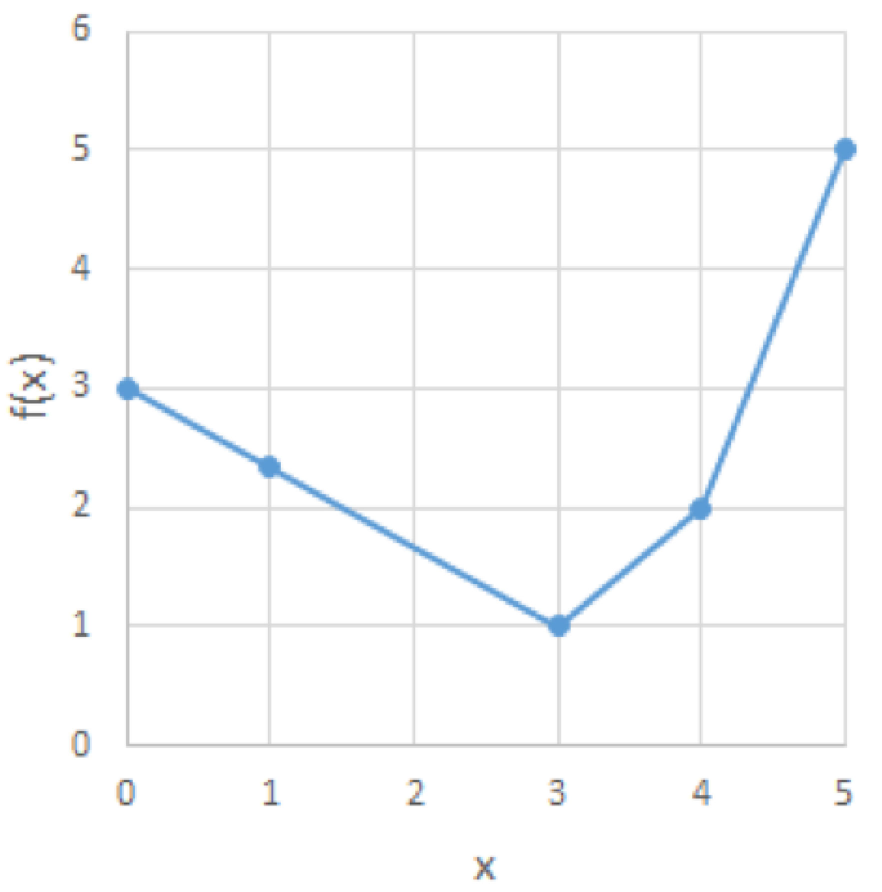
- Convex ✔
- Not convex

You have used 1 of 1 submissions

PART C

(1/1 point)

Below is a graph for the function under $a = \frac{7}{3}$,



Is this function convex?

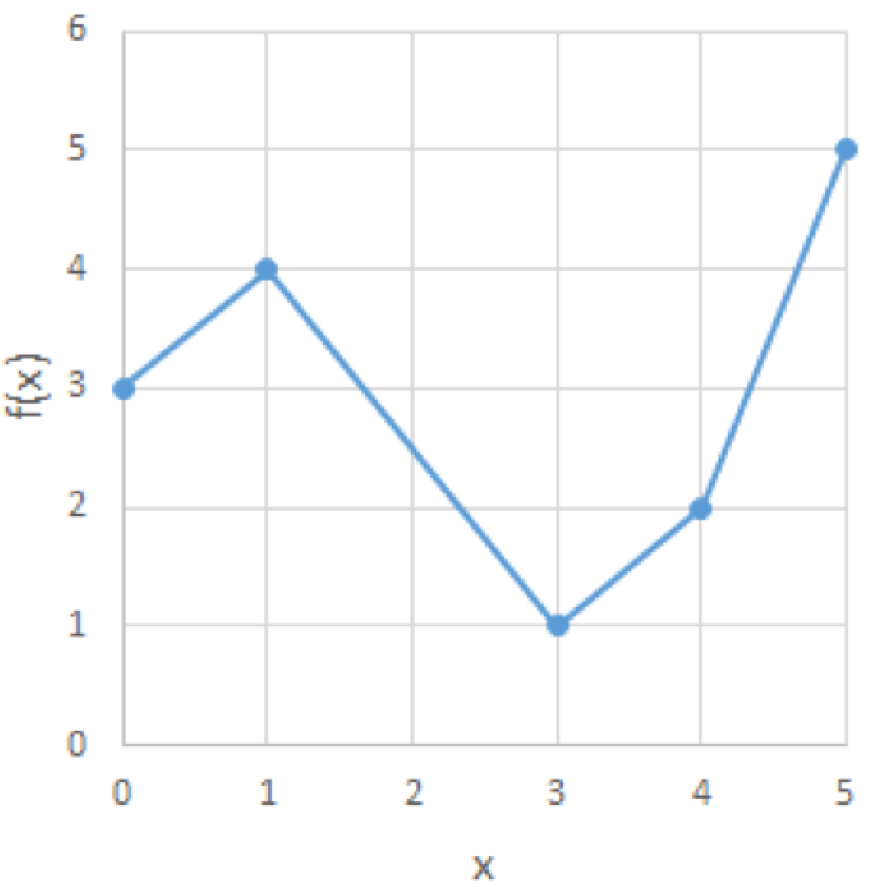
- (selected) Convex ✔
- Not convex

You have used 1 of 1 submissions

PART D

(1/1 point)

Below is a graph for the function under $a = 4$,



Is this function convex?

- ○ Convex
- ◉ Not convex ✔

You have used 1 of 1 submissions

PART E

(1/1 point)

Based on the previous question, what is the range of a for $f(x)$ to be a convex function?

- ○ $-2 \leq a \leq \frac{7}{3}$
- ○ $-1 \leq a \leq 4$
- ◉ $-1 \leq a \leq \frac{7}{3}$ ✔
- ○ $-2 \leq a \leq 4$

You have used 1 of 2 submissions

PART F

(1/1 point)

We are interested in solving the optimization problem: $min_{x \in [0,5]} f(x)$ in the case that the function is convex. (This technique does not necessarily work if the function is not convex.) To do this, we first break up f into four segments and and find the linear functions corresponding to each segment. We denote the four segments as $f_1(x), f_2(x), f_3(x), f_4(x)$ respectively from left to right.

Which of the following describe the segment going through points $(0,1)$ to $(1,a)$?

- $f_1(x) = (a+1)x + 1$
- $f_1(x) = (a-1)x + 1$ ✔
- $f_1(x) = (a-1)x - 1$
- $f_1(x) = (a+1)x - 1$

You have used 1 of 2 submissions

PART G

(3/3 points)

Which of the following describe the segment going through points $(1,a)$ to $(3,1)$?

- $f_2(x) = \frac{1+a}{2}x + \frac{3a-1}{2}$

- $f_2(x) = \frac{1-a}{2}x + \frac{3a+1}{2}$
- $f_2(x) = \frac{1+a}{2}x + \frac{3a+1}{2}$
- $f_2(x) = \frac{1-a}{2}x + \frac{3a-1}{2}$ ✔

You have used 1 of 3 submissions

PART H

(1/1 point)
Which of the following describe the segment going through points $(3, 1)$ to $(4, 2)$?

- $f_3(x) = x - 2$ ✔
- $f_3(x) = x + 2$
- $f_3(x) = -x + 2$
- $f_3(x) = -x - 2$

You have used 1 of 2 submissions

PART I

(1/1 point)

Which of the following describe the segment going through points $(4, 2)$ to $(5, 5)$?

- $f_4(x) = 3x + 10$
- $f_4(x) = -3x + 10$
- $f_4(x) = 3x - 10$ ✔
- $f_4(x) = -3x - 10$

You have used 1 of 2 submissions

PART J

(1/1 point)

Suppose a is under the range which make $f(x)$ convex. We can then turn $min_{x \in [0,5]}\ f(x)$ into the following min-max problem

min max $\{f_1(x), f_2(x), f_3(x), f_4(x)\}$

$$\left.\begin{array}{ll} \textbf{min max} & \{(a-1)x+1, \frac{1-a}{2}x+\frac{3a-1}{2}, x-2, 3x-10\} \\ \textbf{s.t.:} & \\ & x \le 5 \\ & x \ge 0 \end{array}\right\}$$

Make this min-max problem into a linear programming model

- ( )

$$\left.\begin{array}{lr} \textbf{min} & y \\ \textbf{s.t.:} & \\ & x \le 5 \\ & y \ge (a-1)x+1 \\ & y \ge \frac{1-a}{2}x+\frac{3a-1}{2}x \\ & y \le x-2 \\ & y \ge 3x-10 \\ & x \ge 0 \end{array}\right\}$$

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$$\left.\begin{array}{lr} \min & y \\ \text{s.t.:} & \\ & y \geq (a-1)x+1 \\ & y \geq \frac{1-a}{2}x + \frac{3a-1}{2}x \\ & y \geq x-2 \\ & y \geq 3x-10 \\ & x \geq 0 \end{array}\right\}$$

$$\left.\begin{array}{lr} \min & y \\ \text{s.t.:} & \\ & x \leq 5 \\ & y \geq (a-1)x+1 \\ & y \geq \frac{1-a}{2}x + \frac{3a-1}{2}x \\ & y \geq x-2 \\ & y \geq 3x-10 \\ & x \geq 0 \end{array}\right\}$$

$$\left.\begin{array}{lr} \min & y \\ \text{s.t.:} & \\ & x \leq 5 \\ & y \geq (a-1)x+1 \\ & y \geq \frac{1-a}{2}x - \frac{3a-1}{2}x \\ & y \geq x+2 \\ & y \geq 3x-10 \\ & x \geq 0 \end{array}\right\}$$

You have used 1 of 2 submissions