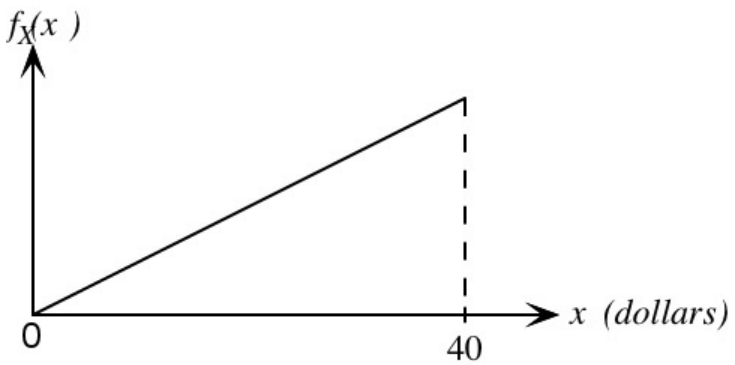
Problem 4: Paul goes to the casino

(7/7 points)

Paul is vacationing in Monte Carlo. On any given night, he takes X dollars to the casino and returns with Y dollars. The random variable X has the PDF shown in the figure. Conditional on $X = x$, the continuous random variable Y is uniformly distributed between zero and $2x$.



1. Determine the joint PDF $f_{X,Y}(x,y)$.

   If $0 < x < 40$ and $0 < y < 2x$,

   $f_{X,Y}(x,y) =$ 1/1600 ✔

   If $y < 0$ or $y > 2x$,

   $f_{X,Y}(x,y) =$ 0 ✔

2. On any particular night, Paul makes a profit of $Z = Y - X$ dollars. Find the probability that Paul makes a positive profit (i.e., $\mathbf{P}(Z > 0)$):

   1/2 ✔

3. Find the PDF of Z. Express your answers in terms of z using standard notation . *Hint:* Start by finding $f_{Z|X}(z \mid x)$.

If $0 < z < 40$, $f_Z(z) =$ 1/40-z/1600 ✔

If $-40 < z < 0$, $f_Z(z) =$ 1/40+z/1600 ✔

If $z < -40$ or $z > 40$, $f_Z(z) =$ 0 ✔

4. What is $\mathbf{E}[Z]$?

$\mathbf{E}[Z] =$ 0 ✔

You have used 3 of 3 submissions

DISCUSSION

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