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Supertask

In philosophy, a **supertask** is a countably infinite sequence of operations that occur sequentially within a finite interval of time.[1] Supertasks are called "hypertasks" when the number of operations becomes uncountably infinite. A hypertask that includes one task for each ordinal number is called an "ultratask".[2] The term *supertask* was coined by the philosopher James F. Thomson, who devised Thomson's lamp. The term *hypertask* derives from Clark and Read in their paper of that name.[3]

Contents

History

Zeno

Motion

The origin of the interest in supertasks is normally attributed to Zeno of Elea. Zeno claimed that motion was impossible. He argued as follows: suppose our burgeoning "mover", Achilles say, wishes to move from A to B. To achieve this he must traverse half the distance from A to B. To get from the midpoint of AB to B, Achilles must traverse half *this* distance, and so on and so forth. However many times he performs one of these "traversing" tasks, there is another one left for him to do before he arrives at B. Thus it follows, according to Zeno, that motion (travelling a non-zero distance in finite time) is a supertask. Zeno further argues that supertasks are not possible (how can this sequence be completed if for each traversing there is another one to come?). It follows that motion is impossible.

Zeno's argument takes the following form:

1. Motion is a supertask, because the completion of motion over any set distance involves an infinite number of steps
2. Supertasks are impossible
3. Therefore, motion is impossible

Most subsequent philosophers reject Zeno's bold conclusion in favor of common sense. Instead, they turn his argument on its head (assuming it is valid) and take it as a proof by contradiction where the possibility of motion is taken for granted. They accept the possibility of motion and apply *modus tollens* (contrapositive) to Zeno's argument to reach the conclusion that either motion is not a supertask or not all supertasks are impossible.

Achilles and the tortoise

Zeno himself also discusses the notion of what he calls "Achilles and the tortoise". Suppose that Achilles is the fastest runner, and moves at a speed of 1 m/s. Achilles chases a tortoise, an animal renowned for being slow, that moves at 0.1 m/s. However, the tortoise starts 0.9 metres ahead. Common sense seems to decree that Achilles will catch up with the tortoise after exactly 1 second, but Zeno argues that this is not the case. He instead suggests that Achilles must inevitably come up to the point where the tortoise has started from, but by the time he has accomplished this, the tortoise will already have moved on to another point. This continues, and every time Achilles reaches the mark where the tortoise was, the tortoise will have reached a new point that Achilles will have to catch up with; while it begins with 0.9 metres, it becomes an additional 0.09 metres, then 0.009 metres, and so on, infinitely. While these distances will grow very small, they will remain finite, while Achilles' chasing of the tortoise will become an unending supertask. Much commentary has been made on this particular paradox; many assert that it finds a loophole in common sense.[4]

Thomson

James F. Thomson believed that motion was not a supertask, and he emphatically denied that supertasks are possible. The proof Thomson offered to the latter claim involves what has probably become the most famous example of a supertask since Zeno. Thomson's lamp may either be on or off. At time t = 0 the lamp is off, at time t = 1/2 it is on, at time t = 3/4 (= 1/2 + 1/4) it is off, t = 7/8 (= 1/2 + 1/4 + 1/8) it is on, etc. The natural question arises: at t = 1 is the lamp on or off? There does not seem to be any non-arbitrary way to decide this question. Thomson goes further and claims this is a contradiction. He says that the lamp cannot be on for there was never a point when it was on where it was not immediately switched off again. And similarly he claims it cannot be off for there was never a point when it was off where it was not immediately switched on again. By Thomson's reasoning the lamp is neither on nor off, yet by stipulation it must be either on or off – this is a contradiction. Thomson thus believes that supertasks are impossible.

Benacerraf

Paul Benacerraf believes that supertasks are at least logically possible despite Thomson's apparent contradiction. Benacerraf agrees with Thomson insofar as that the experiment he outlined does not determine the state of the lamp at t = 1. However he disagrees with Thomson that he can derive a contradiction from this, since the state of the lamp at t = 1 need not be logically determined by the preceding states. Logical implication does not bar the lamp from being on, off, or vanishing completely to be replaced by a horse-drawn pumpkin. There are possible worlds in which Thomson's lamp finishes on, and worlds in which it finishes off not to mention countless others where weird and wonderful things happen at t = 1. The seeming arbitrariness arises from the fact that Thomson's experiment does not contain enough information to determine the state of the lamp at t = 1, rather like the way nothing can be found in Shakespeare's play to determine whether Hamlet was right- or left-handed. So what about the contradiction? Benacerraf showed that Thomson had committed a mistake. When he claimed that the lamp could not be on because it was never on without being turned off again – this applied only to instants of time *strictly less than 1*. It does not apply to 1 because 1 does not appear in the sequence {0, 1/2, 3/4, 7/8, ...} whereas Thomson's experiment only specified the state of the lamp for times in this sequence.

Modern literature

Most of the modern literature comes from the descendants of Benacerraf, those who tacitly accept the possibility of supertasks. Philosophers who reject their possibility tend not to reject them on grounds such as Thomson's but because they have qualms with the notion of infinity itself. Of course there are exceptions. For example, McLaughlin claims that Thomson's lamp is inconsistent if it is analyzed with internal set theory, a variant of real analysis.

Philosophy of mathematics

If supertasks are possible, then the truth or falsehood of unknown propositions of number theory, such as Goldbach's conjecture, or even undecidable propositions could be determined in a finite amount of time by a brute-force search of the set of all natural numbers. This would, however, be in contradiction with the Church-Turing thesis. Some have argued this poses a problem for intuitionism, since the intuitionist must distinguish between things that cannot in fact be proven (because they are too long or complicated; for example Boolos's "Curious Inference"[5]) but nonetheless are considered "provable", and those which *are* provable by infinite brute force in the above sense.

Physical possibility

Some have claimed Thomson's lamp is physically impossible since it must have parts moving at speeds faster than the speed of light (e.g., the lamp switch). Adolf Grünbaum suggests that the lamp could have a strip of wire which, when lifted, disrupts the circuit and turns off the lamp; this strip could then be lifted by a smaller distance each time the lamp is to be turned off, maintaining a constant velocity. However, such a design would ultimately fail, as eventually the distance between the contacts would be so small as to allow electrons to jump the gap, preventing the circuit from being broken at all. Still, for either a human or any device, to perceive or act upon the state of the lamp some measurement has to be done, for example the light from the lamp would have to reach an eye or a sensor. Any such measurement will take a fixed frame of time, no matter how small and, therefore, at some point measurement of the state will be impossible. Since the state at t=1 can not be determined even in principle, it is not meaningful to speak of the lamp being either on or off.

Other physically possible supertasks have been suggested. In one proposal, one person (or entity) counts upward from 1, taking an infinite amount of time, while another person observes this from a frame of reference where this occurs in a finite space of time. For the counter, this is not a supertask, but for the observer, it is. (This could theoretically occur due to time dilation, for example if the observer were falling into a black hole while observing a counter whose position is fixed relative to the singularity.) Gustavo E. Romero in the paper 'The collapse of supertasks'[6] maintains that any attempt to carry out a supertask will result in the formation of a black hole, making supertasks physically impossible.

Super Turing machines

The impact of supertasks on theoretical computer science has triggered some new and interesting work, for example Hamkins and Lewis – "Infinite Time Turing Machine".

Prominent supertasks

Ross–Littlewood paradox

Suppose there is a jar capable of containing infinitely many marbles and an infinite collection of marbles labelled 1, 2, 3, and so on. At time $t = 0$, marbles 1 through 10 are placed in the jar and marble 1 is taken out. At $t = 0.5$, marbles 11 through 20 are placed in the jar and marble 2 is taken out; at $t = 0.75$, marbles 21 through 30 are put in the jar and marble 3 is taken out; and in general at time $t = 1 - 0.5^n$, marbles $10n + 1$ through $10n + 10$ are placed in the jar and marble $n + 1$ is taken out. How many marbles are in the jar at time $t = 1$?

One argument states that there should be infinitely many marbles in the jar, because at each step before $t = 1$ the number of marbles increases from the previous step and does so unboundedly. A second argument, however, shows that the jar is empty. Consider the following argument: if the jar is non-empty, then there must be a marble in the jar. Let us say that that marble is labeled with the number n. But at time $t = 1 - 0.5^{n-1}$, the nth marble has been taken out, so marble n cannot be in the jar. This is a contradiction, so the jar must be empty. The Ross–Littlewood paradox is that here we have two seemingly perfectly good arguments with completely opposite conclusions.

Further complications are introduced by the following variant. Suppose that we follow the same process as above, but instead of taking out marble 1 at $t = 0$, one takes out marble 2. And, at $t = 0.5$ one takes out marble 3, at $t = 0.75$ marble 4, etc. Then, one can use the same logic from above to show that while at $t = 1$, marble 1 is still in the jar, no other marbles can be left in the jar. Similarly, one can construct scenarios where in the end, 2 marbles are left, or 17 or, of course, infinitely many. But again this is paradoxical: given that in all these variations the same number of marbles are added or taken out at each step of the way, how can the end result differ?

It is argued that the end result does depend on which marbles are taken out at each instant. However, one immediate problem with that view is that one can think of the thought experiment as one where none of the marbles are actually labeled, and thus all the above variations are simply different ways of describing the same process; it seems unreasonable to say that the end result of the one actual process depends on the way we describe what happens.

Moreover, Allis and Koetsier offer the following variation on this thought experiment: at $t = 0$, marbles 1 to 9 are placed in the jar, but instead of taking a marble out they scribble a 0 after the 1 on the label of the first marble so that it is now labeled "10". At $t = 0.5$, marbles 11 to 19 are placed in the jar, and instead of taking out marble 2, a 0 is written on it, marking it as 20. The process is repeated ad infinitum. Now, notice that the end result at each step along the way of this process is the same as in the original experiment, and indeed the paradox remains: Since at every step along the way, more marbles were added, there must be infinitely marbles left at the end, yet at the same time, since every marble with number n was taken out at $t = 1 - 0.5^{n-1}$, no marbles can be left at the end. However, in this experiment, no marbles are ever taken out, and so any talk about the end result 'depending' on which marbles are taken out along the way is made impossible.

A simpler variation goes as follows: at $t = 0$, there is one marble in the jar with the number 0 scribbled on it. At $t = 0.5$, the number 0 on the marble gets replaced with the number 1, at $t = 0.75$, the number gets changed to 2, etc. Now, no marbles are ever added to or removed from the jar, so at t = *1*, there should still be exactly that one marble in the jar. However, since we always replaced the number on that marble with some other number, it should have some number n on it, and that is impossible because we know precisely when that number was replaced, and never repeated again later. In other words, we can also reason that no marble can be left at the end of this process, which is quite a paradox.

Of course, it would be wise to heed Benacerraf's words that the states of the jars before $t = 1$ do not logically determine the state at $t = 1$. Thus, neither Ross's or Allis's and Koetsier's argument for the state of the jar at $t = 1$ proceeds by logical means only. Therefore, some extra premise must be introduced in order to say anything about the state of the jar at $t = 1$. Allis and Koetsier believe such an extra premise can be provided by the physical law that the marbles have continuous space-time paths, and therefore from the fact that for each n, marble n is out of the jar for $t < 1$, it must follow that it must still be outside the jar at $t = 1$ by continuity. Thus, the contradiction, and the paradox, remains.

One obvious solution to all these conundrums and paradoxes is to say that supertasks are impossible. If supertasks are impossible, then the very assumption that all of these scenarios had some kind of 'end result' to them is mistaken, preventing all of the further reasoning (leading to the contradictions) to go through.

Benardete's paradox

There has been considerable interest in J. A. Benardete's "Paradox of the Gods":[7]

> A man walks a mile from a point α. But there is an infinity of gods each of whom, unknown to the others, intends to obstruct him. One of them will raise a barrier to stop his further advance if he reaches the half-mile point, a second if he reaches the quarter-mile point, a third if he goes one-eighth of a mile, and so on ad infinitum. So he cannot even get started, because however short a distance he travels he will already have been stopped by a barrier. But in that case no barrier will rise, so that there is nothing to stop him setting off. He has been forced to stay where he is by the mere unfulfilled intentions of the gods.[8]
>
> — M. Clark, Paradoxes from A to Z

Grim Reaper paradox

Inspired by J. A. Benardete's paradox regarding an infinite series of assassins[9], David Chalmers describes the paradox as follows:

> There are countably many grim reapers, one for every positive integer. Grim reaper 1 is disposed to kill you with a scythe at 1pm, if and only if you are still alive then (otherwise his scythe remains immobile throughout), taking 30 minutes about it. Grim reaper 2 is disposed to kill you with a scythe at 12:30 pm, if and only if you are still alive then, taking 15 minutes about it. Grim reaper 3 is disposed to kill you with a scythe at 12:15 pm, and so on. You are still alive just before 12pm, you can only die through the motion of a grim reaper's scythe, and once dead you stay dead. On the face of it, this situation seems conceivable — each reaper seems conceivable individually and intrinsically, and it seems reasonable to combine distinct individuals with distinct intrinsic properties into one situation. But a little reflection reveals that the situation as described is contradictory. I cannot survive to any moment past 12pm (a grim reaper would get me first), but I cannot be killed (for grim reaper n to kill me, I must have survived grim reaper n+1, which is impossible).[10]

It has gained significance in philosophy via its use in arguing for a finite past, thereby bearing relevance to the kalam cosmological argument.[11][12][13][14]

Laraudogoitia’s supertask

This supertask, proposed by J. P. Laraudogoitia, is an example of indeterminism in Newtonian mechanics. The supertask consists of an infinite collection of stationary point masses. The point masses are all of mass m and are placed along a line AB that is a meters in length at positions B, AB / 2, AB / 4, AB / 8, and so on. The first particle at B is accelerated to a velocity of one meter per second towards A. According to the laws of Newtonian mechanics, when the first particle collides with the second, it will come to rest and the second particle will inherit its velocity of 1 m/s. This process will continue as an infinite amount of collisions, and after 1 second, all the collisions will have finished since all the particles were moving at 1 meter per second. However no particle will emerge from A, since there is no last particle in the sequence. It follows that all the particles are now at rest, contradicting conservation of energy. Now the laws of Newtonian mechanics are time-reversal-invariant; that is, if we reverse the direction of time, all the laws will remain the same. If time is reversed in this supertask, we have a system of stationary point masses along A to AB / 2 that will, at random, spontaneously start colliding with each other, resulting in a particle moving away from B at a velocity of 1 m/s. Alper and Bridger have questioned the reasoning in this supertask invoking the distinction between actual and potential infinity.

Davies' super-machine

Proposed by E. B. Davies,[15] this is a machine that can, in the space of half an hour, create an exact replica of itself that is half its size and capable of twice its replication speed. This replica will in turn create an even faster version of itself with the same specifications, resulting in a supertask that finishes after an hour. If, additionally, the machines create a communication link between parent and child machine that yields successively faster bandwidth and the machines are capable of simple arithmetic, the machines can be used to perform brute-force proofs of unknown conjectures. However, Davies also points out that – due to fundamental properties of the real universe such as quantum mechanics, thermal noise and information theory – his machine can't actually be built.

See also

- Actual infinity
- NP (complexity)
- Transcomputational problem
- Transfinite number
- Zeno machine
- Thomson's lamp

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External links

- Article on Supertasks in Stanford Encyclopedia of Philosophy (http://plato.stanford.edu/entries/spacetime-supertasks/)
- Cooke, Martin C. (2003). "Infinite Sequences: Finitist Consequence" (https://docs.google.com/document/edit?id=1D86gx8yjYXVBTBNEbtPCe4VEQdso6bwAA9ot8sYEY4M&hl=en&pref=2&pli=1). *Br. J. Philos. Sci*. **54** (4): 591–599. doi:10.1093/bjps/54.4.591 (https://doi.org/10.1093%2Fbjps%2F54.4.591).
- Supertasks - Vsauce (YouTube) (https://www.youtube.com/watch?v=ffUnNaQTfZE)