3.2.4 Triangular Matrices

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Week 3 due Oct 18, 2023 06:12 IST Completed

3.2.4 Triangular Matrices

0:00 / 0:00 2.0x

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Reading Assignment

0 points possible (ungraded)

Read Unit 3.2.4 of the notes. [LINK]

- [x] Done

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? How do I Generalize Tiny Timmy example?
I'm having hard time understanding how to manipulate the tiny timmy example mathematically? What am I supposed to do to figure out how ti 2

Calculator

Homework 3.2.3.3 Flame API does not generate code for x vector
I am trying to do Homework 3.2.3.3. I choose unblocked. I choose the following options for operand 2: Tag: X, type: vector, direction: t->b, input/... 5

Question about Homework 3.2.4.1
Why is alpha-11: 0 by itself incorrect? I don't know what I'm missing from this lesson. 2

Homework 3.2.4.1

1/1 point (graded)

Let $L_U : \mathbb{R}^3 \to \mathbb{R}^3$ be defined as $L_U \left(\begin{pmatrix} \chi_0 \\ \chi_1 \\ \chi_2 \end{pmatrix} \right) = \begin{pmatrix} 2\chi_0 - \chi_1 + \chi_2 \\ 3\chi_1 - \chi_2 \\ -2\chi_2 \end{pmatrix}$

We have proven for similar functions that they are linear transformations, so we will skip that part. What matrix, U, represents this linear transformation?

- [ ] $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$
- [x] $\begin{pmatrix} 2 & -1 & 1 \\ 0 & 3 & -1 \\ 0 & 0 & -2 \end{pmatrix}$
- [ ] $\begin{pmatrix} -2 & 1 & -1 \\ 0 & 3 & 1 \\ 0 & 0 & -2 \end{pmatrix}$
- [ ] $\begin{pmatrix} 2 & -1 & 1 \\ 1 & -3 & -1 \\ 1 & 1 & -2 \end{pmatrix}$
- [ ] $\begin{pmatrix} -2 & 1 & -1 \\ 1 & 3 & 1 \\ 1 & 1 & -2 \end{pmatrix}$

✔

Explanation

$$U = \begin{pmatrix} 2 & -1 & 1 \\ 0 & 3 & -1 \\ 0 & 0 & -2 \end{pmatrix}.$$

(You can either evaluate $L(e_0)$, $L(e_1)$, $L(e_2)$, or figure this out by examination.)

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Homework 3.2.4.2

1/1 point (graded)

Calculator

A matrix that is both lower and upper triangular is, in fact, a diagonal matrix.

Always ✔ Answer: Always

Answer: Always
Let A be both lower and upper triangular. Then $\alpha_{i,j} = 0$ if $i < j$ and $\alpha_{i,j} = 0$ if $i > j$ so that

$$\alpha_{i,j} = \begin{cases} 0 & \text{if } i < j \\ 0 & \text{if } i > j. \end{cases}$$

But this means that $\alpha_{i,j} = 0$ if $i \neq j$, which means A is a diagonal matrix.

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Homework 3.2.4.3

1/1 point (graded)
A matrix that is both strictly lower and strictly upper triangular is, in fact, a zero matrix.

Always ✔ Answer: Always

Explanation
Let A be both strictly lower and strictly upper triangular. Then

$$\alpha_{i,j} = \begin{cases} 0 & \text{if i} \leq \text{j} \\ 0 & \text{if i} \geq \text{j} \end{cases}$$

But this means that $\alpha_{i,j} = 0$ for all i and j, which means A is a zero matrix.

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Homework 3.2.4.4

1/1 point (graded)

Algorithm: $[A] := \text{SET\_TO\_LOWER\_TRIANGULAR\_MATRIX}(A)$

Partition $A \rightarrow \left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right)$

where A_{TL} is 0×0

while $m(A_{TL}) < m(A)$ **do**

Repartition

$$\left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c|c|c} A_{00} & a_{01} & A_{02} \\ \hline a_{10}^T & \alpha_{11} & a_{12}^T \\ \hline A_{20} & a_{21} & A_{22} \end{array}\right)$$

where α_{11} is 1×1

set the elements of the current column above the diagonal to zero

Calculator

$a_{01} := 0$ set a_{01}'s components to zero

Continue with

$$\left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c|c|c} A_{00} & a_{01} & A_{02} \\ \hline a_{10}^T & \alpha_{11} & a_{12}^T \\ \hline A_{20} & a_{21} & A_{22} \end{array}\right)$$

endwhile

In the above algorithm you could have replaced $a_{01} = 0$ with $a_{12}^T = 0$.

Always ✔ Answer: Always

Explanation

•$a_{01} = 0$ sets the elements above the diagonal to zero, one column at a time.
•$a_{12}^T = 0$ sets the elements to the right of the diagonal to zero, one row at a time.
Both of these result in the upper triangular part being set to zero. Therefore you can replace $a_{01} = 0$ with $a_{12}^T = 0$.

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Homework 3.2.4.5

5/5 points (graded)

Consider the following algorithm.

Algorithm: $[A] := \text{SET\_TO\_???\_TRIANGULAR\_MATRIX}(A)$

Partition $A \rightarrow \left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right)$

where A_{TL} is 0×0

while $m(A_{TL}) < m(A)$ **do**

Repartition

$$\left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c|c|c} A_{00} & a_{01} & A_{02} \\ \hline a_{10}^T & \alpha_{11} & a_{12}^T \\ \hline A_{20} & a_{21} & A_{22} \end{array}\right)$$

where α_{11} is 1×1

?????

Continue with

$$\left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c|c|c} A_{00} & a_{01} & A_{02} \\ \hline a_{10}^T & \alpha_{11} & a_{12}^T \\ \hline A_{20} & a_{21} & A_{22} \end{array}\right)$$

endwhile

Change the ????? in the above algorithm so that it sets A to its

Calculator

(Check all that apply)

Upper triangular part.

- [x] $a_{21} := 0$
- [ ] $\alpha_{11} := 0$
- [ ] $a_{12}^T := 0$
- [x] $a_{10}^T := 0$
- [ ] $a_{01} := 0$

✓

Strictly upper triangular part.

- [x] $a_{21} := 0; \ \alpha_{11} := 0$
- [ ] $\alpha_{11} := 0$
- [ ] $a_{12}^T := 0; \ \alpha_{11} := 0$
- [x] $a_{10}^T := 0; \ \alpha_{11} := 0$
- [ ] $a_{01} := 0; \ \alpha_{11} := 0$

✓

Unit upper triangular part.

- [x] $a_{21} := 0; \ \alpha_{11} := 1$
- [ ] $\alpha_{11} := 1$
- [ ] $a_{12}^T := 0; \ \alpha_{11} := 0$
- [ ] $a_{10}^T := 0; \ \alpha_{11} := 0$
- [ ] $a_{01} := 0; \ \alpha_{11} := 1$

✓

Strictly lower triangular part.

- [ ] $a_{21} := 0; \ \alpha_{11} := 0$
- [ ] $\alpha_{11} := 0$
- [x] $a_{12}^T := 0; \ \alpha_{11} := 0$
- [ ] $a_{10}^T := 0; \ \alpha_{11} := 0$
- [x] $a_{..} := 0; \ \alpha_{..} := 0$

Calculator

- [x] $a_{01} := 0; \alpha_{11} := 0$

Unit lower triangular part.

- [ ] $a_{21} := 0; \alpha_{11} := 1$
- [ ] $\alpha_{11} := 1$
- [ ] $a_{12}^T := 0; \alpha_{11} := 0$
- [ ] $a_{10}^T := 0; \alpha_{11} := 0$
- [x] $a_{01} := 0; \alpha_{11} := 1$

Explanation

- Upper triangular part. (`Set_to_upper_triangular_matrix`)

  Answer: $a_{21} := 0$ or $a_{10}^T := 0$.

- Strictly upper triangular part. (`Set_to_strictly_upper_triangular_matrix`)

  Answer: $a_{21} := 0; \alpha_{11} := 0$ or $\alpha_{11} := 0; a_{10}^T := 0$.

- Unit upper triangular part. (`Set_to_unit_upper_triangular_matrix`) (This means also setting the diagonal elements to "1").

  Answer: $a_{21} := 0; \alpha_{11} := 1$ or $\alpha_{11} := 1; a_{10}^T := 0$.

- Strictly lower triangular part. (`Set_to_strictly_lower_triangular_matrix`)

  Answer: $a_{01} := 0; \alpha_{11} := 0$ or $\alpha_{11} := 0; a_{12}^T := 0$.

- Unit lower triangular part. (`Set_to_unit_lower_triangular_matrix`)

  Answer: $a_{01} := 0; \alpha_{11} := 1$ or $\alpha_{11} := 1; a_{12}^T := 0$.

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Homework 3.2.4.6

1/1 point (graded)

Implement functions for each of the algorithms from the last homeworks. In other words, implement functions that, given a square matrix `A`, return a matrix equal to

- the upper triangular part. (`Set_to_upper_triangular_matrix_unb`)
- the strictly upper triangular part. (`Set_to_strictly_upper_triangular_matrix_unb`)
- the unit upper triangular part. (`Set_to_unit_upper_triangular_matrix_unb`)
- strictly lower triangular part. (`Set_to_strictly_lower_triangular_matrix_unb`)
- unit lower triangular part. (`Set_to_unit_lower_triangular_matrix_unb`)

Calculator

(Implement as many as you enjoy implementing and/or until you "get the point". Then move on. We suggest you implement at least one of these.)

Some links that will come in handy:

- Spark (alternatively, open the file `LAFF-2.0xM/Spark/index.html`)
- PictureFLAME (alternatively, open the file `LAFF-2.0xM/PictureFLAME/PictureFLAME.html`)

You will need these in many future exercises. Bookmark them!

☑ Done/Skip

✔

Answer:

- View a document that we put together that has most algorithms and MATLAB implementations that are homework problems in this week:
  Week 3 algorithms and implementations .
  This document is best viewed two pages, side by side, so that you can see the algorithm on the left and its implementation on the right. (Only some of the algorithms for this homework are given.)

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Homework 3.2.4.7

1/1 point (graded)
In MATLAB try this:

```
A = [ 1,2,3;4,5,6;7,8,9 ]
tril( A )
tril( A, -1 )
tril( A, -1 ) + eye( size( A ) )
triu( A )
triu( A, 1 )
triu( A, 1 ) + eye( size( A ) )
```

☑ Done

✔

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Homework 3.2.4.8

1/1 point (graded)

Apply the triangular matrix $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ to Timmy Two Space. What happens?

(Check all that apply)

☐ Timmy shifts off the grid.

☐ Timmy is flipped with respect to the vertical axis.

Calculator

- [ ] Timmy is stretched by a factor of two in the vertical direction.
- [x] Timmy is skewed to the right.
- [ ] Timmy doesn't change at all.

✔

Just plug it into the Timmy webpage .

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Calculator