1.2.2 How Can We Determine Solution Trajectories?

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We've just seen the phase plane for the situation of $\beta = \frac{1}{2}$. Wes said that the coexistence equilibrium in the first quadrant is stable meaning that regardless of where we start in the first quadrant, all solution trajectories will approach this point.

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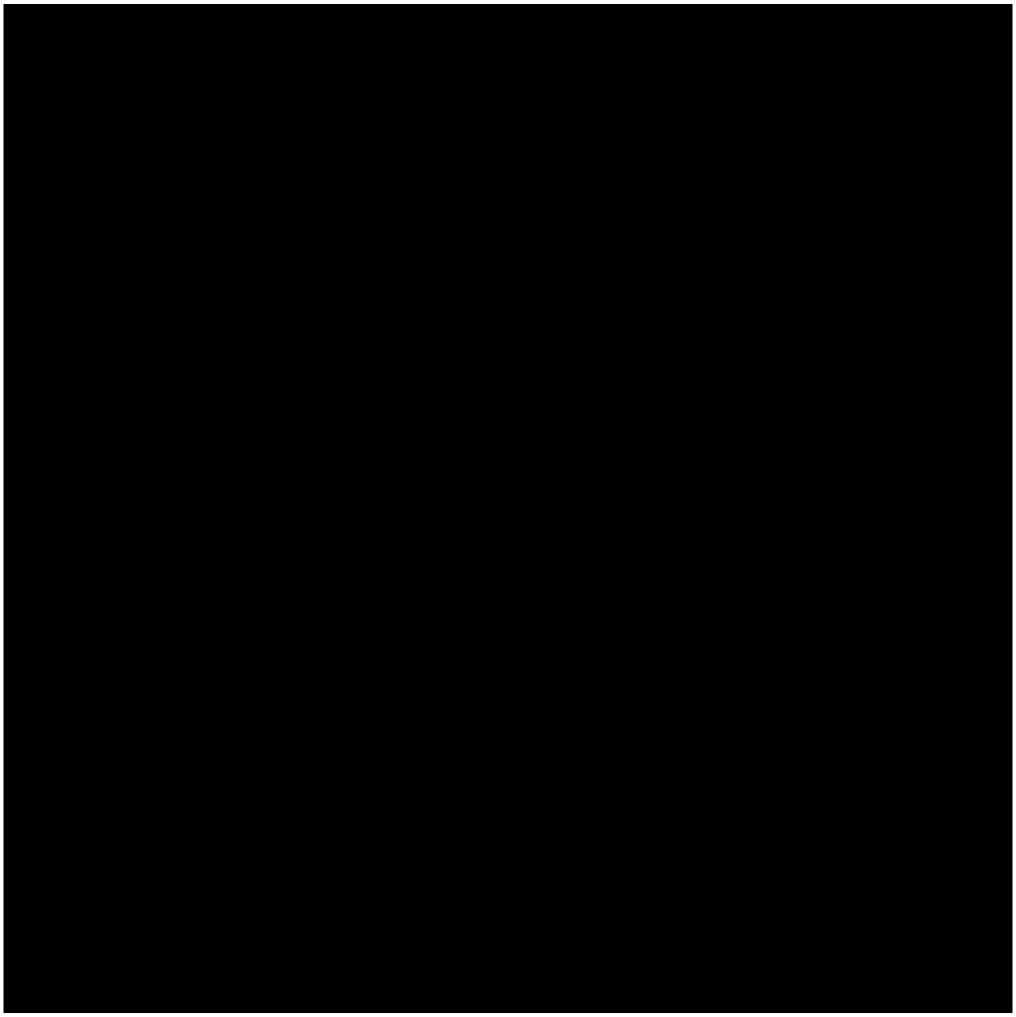
How do we know this? How can we use the arrows to determine the solution trajectories?

As in the marlin-sardine example from Population Dynamics I, we try to follow the arrows to see what happens over time. In the southwest region, arrows are headed up and to the right. By the orientation of the nullclines, this means they will either head directly toward the equilibrium point or they cross into the northwest or southeast triangular region. Similarly, in the northeast region, arrows are headed down and to the left, either to the point or into the northwest or southeast triangular region.

In the two triangular regions, the arrows are headed toward that point. Furthermore, because of the orientation of the nullclines, the trajectories cannot cross back out of those regions. Thus as time increases without bound, they must head toward the equilibrium point. (This last statement uses a big fact from theory of differential equations that if the limits of $x(t)$ and $y(t)$ both exist, then $(x(t), y(t))$ approaches an equilibrium point.)

Note this is a big difference from the marlin-sardine example where the solutions ended up being closed cycles around the equilibrium point. Here the trajectories are drawn toward the coexistence equilibrium point as shown in this computer-generated image of trajectories.



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