Building the mathematical model

Last week, we looked at estimation problems from a rather abstract point of view.

We identified the problem of linking and unlinking a mathematical model to a real-life situation.

We also discussed how to systematically discriminate various components in the problem.

Now it is time to dive into this mathematical model.

This lecture is about the model formulation.

There are different types of mathematical models.

The one particular type we will focus on in this course is the "Model of Observation Equations".

This means that the observations are central in this model, and that we really try to use them to estimate some parameters.

There are also other models, such as the "Model of Condition Equations", which we will not discuss in this course.

The Model of Observation Equations consists of two parts, which are related to the first and second statistical moment of the observations.

We refer to these as the Functional Model, and the Stochastic model.

Both models each play a very specific role.

The Functional model relates the observations to the unknowns, or the parameters. And the stochastic model describes the quality of the observations.

In this video, we will focus only on the functional model.

We need to introduce a common notation.

If estimation theory could be seen as a story, let's introduce the main characters of that story.

In the Delft School, we generally denote the observations with the letter y. It is always written as a small letter. 'y' can be one single observation, but it can also be used to indicate a number of observations.

If there are more observations, we typically list them in a vector, with length m. You can remember that we use the letter m, as it is the first letter in the word 'Measurements'.

Now, there are two ways in which we can deal with observations. One way, is that we treat them as a list of numbers. If the measurements have already been taken, they are nothing more than a bunch of numbers, organized in a vector, as in this example. These numbers are by definition deterministic: they cannot change any more.

However, we can also consider observations in the case the measurements have not been taken yet. In that case, we generally refer to them as 'observables'.

Since they have not been acquired yet, their values should be interpreted as stochastic (or random) variables.

In our notation, we indicate this stochasticity by underlining the letter.

Again, if we are dealing with more than one observation, we lump them together in a vector, which would have the length m.

It is important to stress that the underline under a letter is not intended to indicate a vector, as in some other fields of mathematics.

We only use it to show that we are dealing with a stochastic variable.

Similarly, when the underline is absent, it always means that the variable is considered to be deterministic.

The unknown parameter is indicated by the letter x.

As discussed before, it is always considered to be deterministic in the traditional Delft School, so it is not underlined.

The reason for this is perhaps a little bit philosophical: we believe that there is always a correct value for this unknown parameter, even if we may never find it, so therefore is deterministic.

Similar like with the observations, we may consider more than one unknown.

In that case, we combine all these unknowns in a vector.

And the length of this vector is n, as in November.

The functional model itself gets its name from the Function that relates the unknown parameters to the observations. Remember that we called that the forward Model.

The notation for this functional relation is A and then brackets behind it, A(), where we can add the unknown parameters within these brackets.

The functional relation can be a non-linear relation, but in some cases we can also deal with it as a linear relation.

In that case, A will become a matrix, as we will see later on.

In any case, this functional relation converts the n unknowns to the m measurements.

The last characters in our story are needed to make the functional model complete.

Since all measurements will be affected by errors (either they are small or big), we need to include these errors also in the model.

Each measurement will have such an error, which is why the length of the vector of errors is equal to the length of the vector of observables.

So, now we have introduced the main characters of the story. In the following videos we will describe how they act together.