# 12. Summary

Bookmark this page

**Summarize**

## Big Picture

When we are interested in finding the maximum or minimum value of a function, we look to see mathematically what properties of that function are necessary to have a maximum or minimum.

Local maxima and minima of a function $f(x,y)$ occur at points where the gradient is zero (or undefined). We call points where the gradient is zero **critical points.** A critical point can be a local maximum, a local minimum, or neither, which is called a saddle point.

The gradient or level curves of a function give us graphical information about the behavior of a function that allows us to determine the type of critical point we have.

## Mechanics

### Critical points

**Definition 12.1** Let $f(x,y)$ be a function of two variables. A **critical point** of $f(x,y)$ is a point $(x_0,y_0)$ at which $\nabla f(x_0,y_0)=\vec{0}$. In other words, when $f_x(x_0,y_0)=0$ and $f_y(x_0,y_0)=0$ simultaneously.

**Extension to higher dimension: Critical points**

Let $f(x_1,x_2,\ldots,x_n)$ be a function of $n$ variables. A **critical point** of $f(x_1,x_2,\ldots,x_n)$ is a point $(x_1^*,x_2^*,\ldots,x_n^*)$ at which $\nabla f(x_1^*,x_2^*,\ldots,x_n^*)=\vec{0}$. In other words, when $f_{x_1}(x_1^*,x_2^*,\ldots,x_n^*)=0$, $f_{x_2}(x_1^*,x_2^*,\ldots,x_n^*)=0$, $\ldots$, and $f_{x_n}(x_1^*,x_2^*,\ldots,x_n^*)=0$ simultaneously.

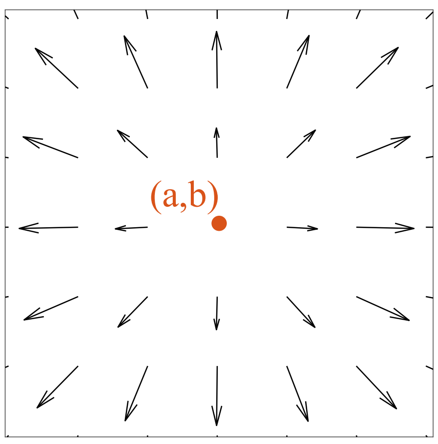
Hide

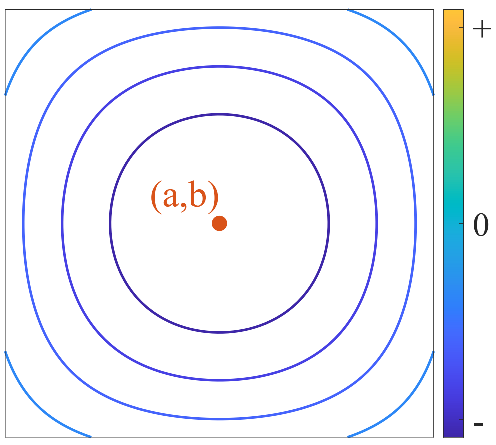
### Graphical methods

Suppose $(x,y)=(a,b)$ is a critical point of $f(x,y)$ (meaning $\nabla f(a,b)=\langle 0,0\rangle$).

**Case 1:** If the vectors representing $\nabla f(x,y)$ surrounding $(a,b)$ are pointing away from $(a,b)$, then $f(x,y)$ is decreasing as we approach $(a,b)$ from every direction. This means $(a,b)$ is a local minimum of $f(x,y)$.

The figure below on the left shows the gradient field near a local minimum. The figure below on the right shows the corresponding level curves.



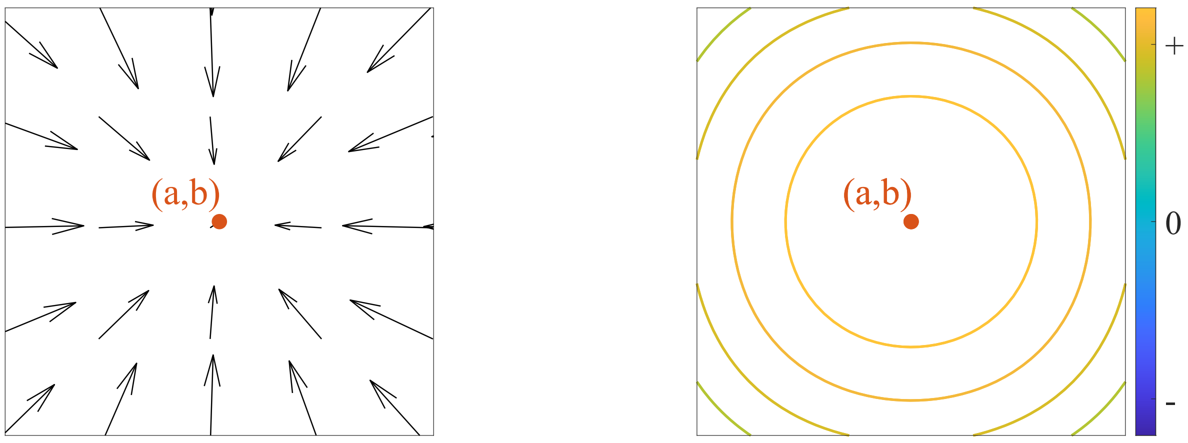




**Case 2:** If the vectors representing $\nabla f(x,y)$ surrounding $(a,b)$ are pointing towards

Calculator Hide Notes

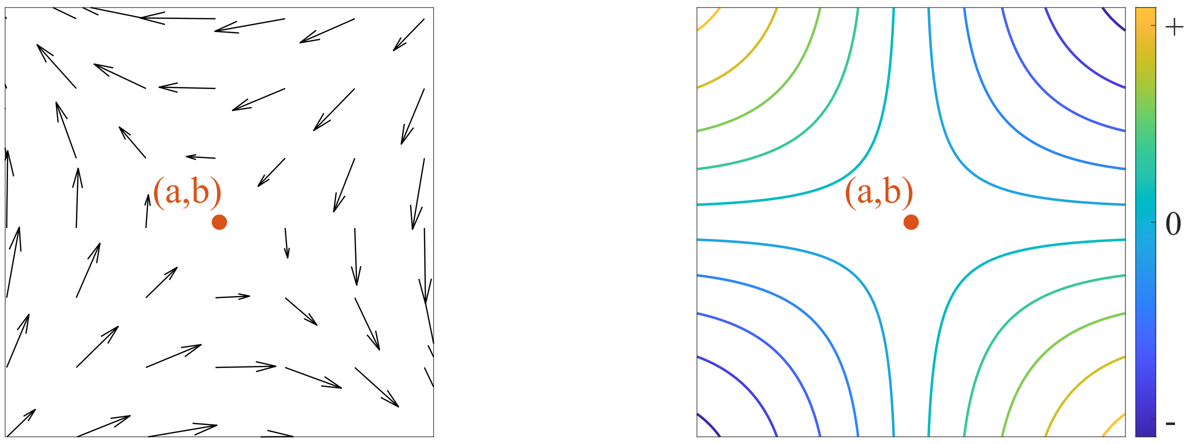
increasing as we approach $(a, b)$ from every direction. This means $(a, b)$ is a local maximum of $f(x, y)$.

The figure below on the left shows the gradient field near a local maximum. The figure below on the right shows the corresponding level curves.




**Case 3:** If some vectors representing $\nabla f(x, y)$ near $(a, b)$ point towards $(a, b)$ and some point away from $(a, b)$, then $f(x, y)$ is increasing as we approach $(a, b)$ from some directions and decreasing as we approach $(a, b)$ from other directions. This means $(a, b)$ is a saddle point of $f(x, y)$.

The figure below on the left shows the gradient field near a saddle point. The figure below on the right shows the corresponding level curves.




## Ask Yourself

**What does "finding critical points" have to do with "maximizing a function of two variables"?**

The critical points give us candidates for local maxima and minima. Having a list of the local maxima and minima gives a lot of insight when studying a function of two variables.

Hide

**What does it mean to "maximize a function of two variables"?**

It means to find a point $(x, y)$ where the value $f(x, y)$ is highest. For example, $f(x, y)$ could be the temperature of an ocean at position $(x, y)$. Then "maximizing $f$" would tell us where the ocean has the highest temperature.

Hide

## 12. Summary

Hide Discussion

**Topic:** Unit 3: Optimization / 12. Summary

**Add a Post**

Show all posts

Calculator

Hide Notes

[STAFF] Tiny typo
"Local maxima and minima of a function occur at at points where the gradient is zero (or undefined). We call points where the gradie...
1

< Previous

Next Up: Recitation 8: Critical point practice >
9 min + 4 activities

© All Rights Reserved

## edX

About
Affiliates
edX for Business
Open edX
Careers
News

## Legal

Terms of Service & Honor Code
Privacy Policy
Accessibility Policy
Trademark Policy
Sitemap

## Connect

Blog
Contact Us
Help Center
Media Kit
Donate

GET IT ON Google play
Download on the App Store

© 2021 edX Inc. All rights reserved.
深圳市恒宇博科技有限公司 粤ICP备17044299号-2

Calculator
Hide Notes