Exercise: Design of a phone system

(2/2 points)

A telephone company establishes a direct connection between two cities expecting Poisson traffic with rate 30 calls/min. The durations of calls are independent and exponentially distributed with mean 3 min. Inter-arrival times are independent of call durations. What is the fewest number of circuits that the company should provide to ensure that all circuits are in use with probability less than or equal to 0.05? It is assumed that blocked calls are lost (i.e., a blocked call is not attempted again). (Hint: Simply look at the previous video, and do a numerical trial and error to find the answer.)

95

Answer: 95

Answer:
The given parameters for the Poisson processes are $\lambda = 30$ and $\mu = 1/3$. Let B be the number of circuits. Following the analysis presented in the previous video, we have that

$$\pi_B \;= \pi_0 \cdot \frac{\lambda^B}{\mu^b B!}$$

$$= \left(\frac{1}{\sum_{i=0}^{B} \frac{\lambda^i}{\mu^i i!}}\right)\left(\frac{\lambda^B}{\mu^B B!}\right)$$

$$= \left(\frac{1}{\sum_{i=0}^{B} \frac{30^i}{(1/3)^i i!}}\right)\left(\frac{30^B}{(1/3)^B B!}\right).$$

Our goal is to find the smallest B such that $\pi_B \leq 0.05$.

Since π_B is a function of B, we can evaluate it for various values of B. It turns out that for $B = 94$ we have $\pi_B \approx 0.05481$ and for $B = 95$ we have $\pi_B \approx 0.04936$. Therefore, the smallest B that meets the desired criterion is 95. Note that fewer circuits are required here than in the previous video where the desired probability was more strict ($\pi_B \leq 0.01$ instead of $\pi_B \leq 0.05$).

You have used 1 of 2 submissions

Lec. 26: Absorption probabilities and expected time to absorption
Exercises 26 due May 18, 2016 at 23:59 UTC

Solved problems

Problem Set 10
Problem Set 10 due May 18, 2016 at 23:59 UTC

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