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A solution to the Three Prisoners Puzzle

One strategy is for each prisoner is to follow these instructions:

- If you see that the other two prisoners have hats of the same color, answer the guard's question on the assumption that your hat is of the opposite color.

- If you see that the other two prisoners have hats of different colors, refrain from answering the guard's question.
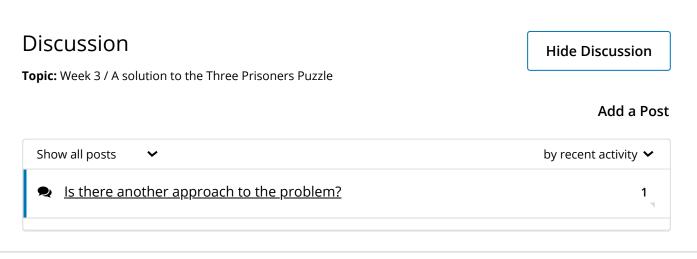
If all three prisoners follow this procedure, their chance of survival will be 75%. For (as shown below) there are eight possible hat distributions, all equally likely, and as long as they follow the strategy the prisoners are set free in six of those eight possibilities ($6/8 = 75\%$).

Prisoner A	Prisoner B	Prisoner C	Result of following Strategy
red	red	red	Everyone answers incorrectly
red	red	blue	C answers correctly
red	blue	red	B answers correctly
red	blue	blue	A answers correctly
blue	red	red	A answers correctly
blue	red	blue	B answers correctly
blue	blue	red	C answers correctly
blue	blue	blue	Everyone answers incorrectly

Note that although it is true that the prisoners increase their chance of collective of success to 75%, the chance that any given prisoner answers correctly, given that he offers an answer, remains fixed at 50%.

Suppose, for example, that you are prisoner A, and that you see two red hats before you. The agreed-upon strategy asks you to answer "Blue!". But because the color of each hat was chosen on the basis of an independent coin toss, a $\langle Blue, Red, Red \rangle$ hat distribution is exactly is likely as a $\langle Red, Red, Red \rangle$ hat distribution. So the probability that you will answer correctly is precisely 50%.

The group improves its chances of collective success not because it increases the chance of success of individual answers, but because it manages to coordinate individual successes and failures in a certain kind of way. To see this, go back to the figure above, and consider the eight different hat distributions that are possible. Each prisoner answers correctly in two of these eight hat distributions, answers incorrectly in two, and remains silent in four. So each prisoner has as many successes as failures (which is why his chances of answering correctly, given that he answers, are 50%). But the prisoners' individual successes and failures are distributed unevenly across the space of possible hat distributions. They are coordinated in such a way that only two of the eight possibilities involve failures (which is why the strategy gives the group a 75% chance of success).

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