Trees

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TREES

A *tree* is a graph for which there are no loops, and we can reach from any node to any other node (moving along edges in the graph). We'll be seeing trees quite a bit so here are some basics of trees.

Example:

This graph is not a tree since there is no path from X_1 to X_2.

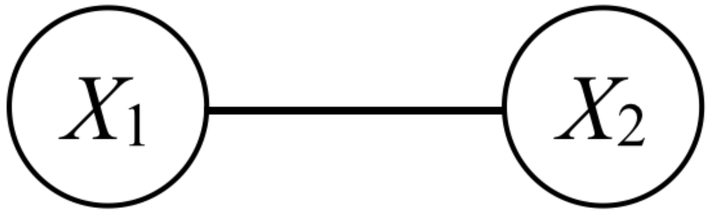
Example:

Exercises due Oct 27, 2016 at 02:30 IST

Week 6: Special Case: Marginalization in Hidden Markov Models
Exercises due Oct 27, 2016 at 02:30 IST

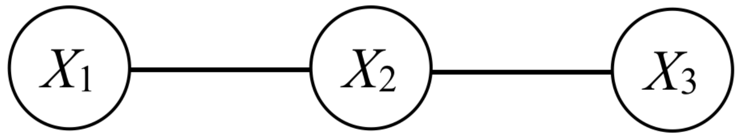
Week 6: Homework 5
Homework due Oct 27, 2016 at 02:30 IST

Weeks 6 and 7: Mini-project on Robot Localization (to be posted)



This graph is a tree since there are no loops and we can reach from any node to any other node.

Example:



This graph is a tree since there are no loops and we can reach from any node to any other node.

Theorem: For any graph that has n nodes, if the graph is a tree, then it will always have exactly $n-1$ edges.

Proof: We use induction.

Base case $n=1$: There is only 1 node so there are no edges, so the claim clearly holds.

Inductive step: Suppose the claim holds for every tree of size (i.e., number of nodes) up to k. Thus, every tree of size k nodes has $k-1$ edges. Now consider a tree T with $k+1$ nodes. Take a leaf node v from T and note that the tree T with v removed is a tree T' of size k, which by the inductive hypothesis has $k-1$ edges. Since v is a leaf node though, it has exactly 1 neighbor, which means that the tree T has 1 more edge than the tree T', i.e., T has k edges. This finishes the inductive step. $\square$