2.

Setup:

The lifetime (in months) of a battery is modeled by a random variable X that has pdf

$$f_\theta(x) = K\theta^x \mathbf{1}(x > 0) \qquad \text{where } K = \ln(1/\theta)$$

for an unknown parameter $\theta \in (0,1)$. (Here $\mathbf{1}(x > 0)$ is the indicator variable that takes value 1 when its argument is true, i.e. when $x > 0$.)

Assume that we have n independent observations $X_1, \ldots, X_n$ of the lifetime of n batteries. We want to use these observations to estimate $\theta \in (0,1)$.

Note (October 24): Assume that the observations are of the same type of batteries.

Input instructions: For all problems below, when inputting the natural log function, use $\ln$, e.g. enter **ln(theta)** for $\ln(\theta)$.

Expectation and Variance

2/2 points (graded)

Compute the expected value $\mathbb{E}[X_i]$ and the variance $\mathsf{Var}[X_i]$ of X_i.

(Enter your answer in terms of θ **only**.)

Hint: Note that the given pdf is equivalent to the pdf of a common distribution except reparametrized with a different parameter.

$\mathbb{E}[X_i] =$ -1/ln(theta)

$-\frac{1}{\ln(\theta)}$

✔ **Answer:** 1/(-ln(theta))

$\mathsf{Var}[X_i] =$ 1/(ln(theta))^2

$\frac{1}{(\ln(\theta))^2}$

✔ **Answer:** 1/(ln(theta))^2

STANDARD NOTATION

Solution:

f is the pdf of an exponential reparametrized in the paramter θ instead of the usual λ where $\theta = e^{-\lambda}$, or equivalently $\lambda = \ln(1/\theta)$. Hence, $X_i \sim \mathsf{Exp}(\ln(1/\theta))$ and

$$\mathbb{E}[X_i] = \frac{1}{\lambda} = (\ln(1/\theta))^{-1}$$
$$\mathsf{Var}[X_i] = \frac{1}{\lambda^2} = (\ln(1/\theta))^{-2}.$$

Submit

You have used 1 of 3 attempts

Answers are displayed within the problem

Maximum Likelihood Estimator

3.0/3.0 points (graded)

Compute the maximum likelihood estimator $\hat{\theta}$ of θ.

(Enter **barX_n** for the sample average $\bar{X}_n = \frac{1}{n}\sum_{i=1}^{n} X_i$.)

$\hat{\theta} =$ e^(-1/barX_n) ✔ **Answer:** e^(-(1/barX_n))

STANDARD NOTATION

Submit You have used 1 of 3 attempts

Answers are displayed within the problem

Asymptotic Distribution

1/1 point (graded)

What kind of distribution does $\sqrt{n}\,(\hat{\theta} - \theta)$ converge in distribution to?

- (●) normal
- ( ) nonparametric
- ( ) Cauchy
- ( ) Binomial
- ( ) unknown

✔

Submit You have used 1 of 3 attempts

Answers are displayed within the problem

Asymptotic Variance

3.0/3.0 points (graded)

Find the asymptotic variance $V(\theta)$ of $\hat{\theta}$.

$V(\theta) =$ (theta*ln(theta))^2 ✔ **Answer:** theta^2*(ln(theta))^2

$(\theta \cdot \ln(\theta))^2$

STANDARD NOTATION

Solution:

Use either the Fisher information or the Delta Method. $I(\theta) = \frac{1}{\theta^2 (\ln \theta)^2}$

Submit You have used 1 of 3 attempts

Answers are displayed within the problem

Confidence Interval

2.0/2.0 points (graded)

Find the one-sided confidence interval of the form $\mathcal{I}_{\text{plug-in}} = (0, B]$ at asymptotic (confidence) level 95% for θ given by the plug-in method. Specify B below in terms of $n, \hat{\theta}$ and $V(\hat{\theta})$.

(Enter **hattheta** for $\hat{\theta}$, **V** for $V(\hat{\theta})$.
If applicable, for any numerical α, enter **q(alpha)** for the $1-\alpha$ quantile q_α of the standard normal distribution, i.e. $\mathbf{P}(Z \leq q_\alpha) = 1-\alpha)$. For, example enter **q(0.01)** for $q_{0.01}$;

Note (Oct 19): Earlier verion contained an error in the input instructions, and stated "If applicable, for any numerical α, enter **q(alpha)** for the $1-\alpha$ quantile q_α of the standard normal distribution, i.e. $\mathbf{P}(Z \leq q_\alpha) = \alpha)$." You may also use numerical values for q_α as long as they are accurate to at least 4 decimal places.)

$B =$ hattheta+q(0.05)*sqrt(V/n) ✔ **Answer:** hattheta+q(0.05)*sqrt(V/n)

STANDARD NOTATION

Submit You have used 1 of 3 attempts

Answers are displayed within the problem

Confidence Interval Continued

1/1 point (graded)
We observe $\hat{\theta} = 0.62$ for $n = 100$.

(Give an answer accurate to at least 3 decimal places.)

$B =$ 0.6687505331016123 ✔ **Answer:** 0.6687

STANDARD NOTATION

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Answers are displayed within the problem

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?	Algebra bump Now that the exam is over could I get a quick Algebra refresher -- for the life of me I couldn't figure out how to raise and drop my exponents to get where I needed to go.	9
💬	[Staff] please check my answer on the confidence interval Dear Staff, could you please check on my answer, I'm pretty sure that it is correct, since the previous and the following questions were all marked correct, I only used the a...	1
💬	[Staff] Confidence interval answer Hi, Can anyone please check my answer for confidence interval (second last question), it is correct, however instead of V I have used the plugin value of V from previous q...	3
?	[Staff] Please review my answer to one question. I placed an extra bracket by mistake but my answer is correct. Dear TAs, I have been busy with urgent office work so couldn't complete my exam but got almost all of the questions I attempted correct. However, my answer to proble...	8
?	[Staff] Confidence Interval Questions	5
?	[Staff] Confidence Interval answer	6
💬	[Staff] Please review my answer, plugged in the formula instead of V into the confidence interval.	3
💬	Conversion to an Exponential Distribution (Neat Trick)	1
💬	[STAFF] Asymptotic Variance answer Please, can you provide details about asymptotic variance answer?	6
💬	To what to aply the delta method	4
💬	3rd answer contradicts with the 4th?	2
?	Confidence interval continued Not sure how I managed to get the last part wrong but everything else right... it's just plugging in numbers into the answer for the previous part, so why is 0.62+1.65*0.62...	10
?	[Staff] My answer for last 2nd Question "Confidence Interval" is correct by isn't graded, neither correct nor incorrect same.	1

Confidence Interval 3

? [Staff] removed by staff 2

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