

sandipan_dey >

Next >

<u>Course Progress Dates Discussion Syllabus Outline laff routines Community</u>

★ Course / Week 7: More Gaussian Elimination and Matrix Inversi... / 7.3 The Inverse Mat...

()

7.3.4 More Advanced (But Still Simple) Examples

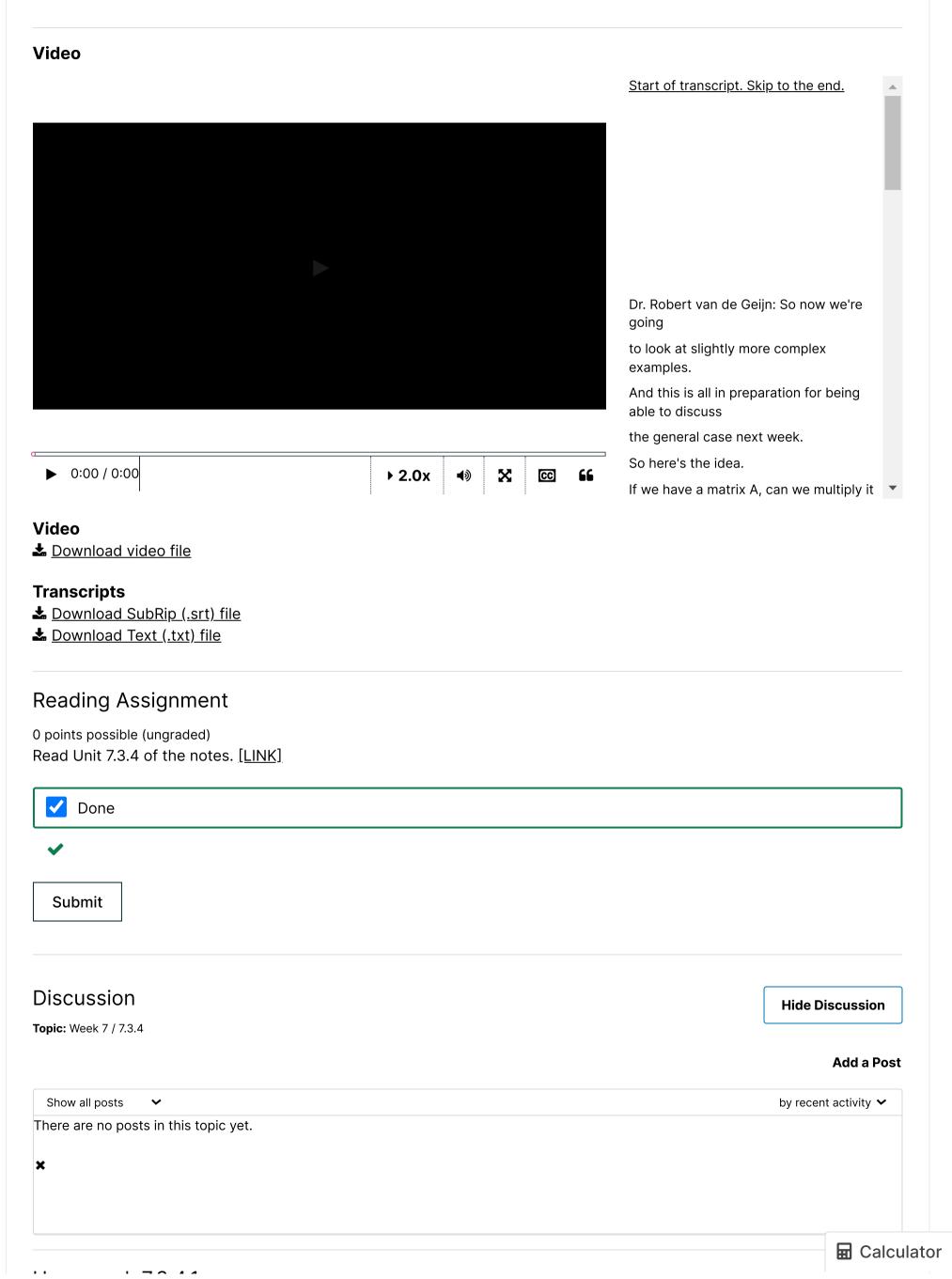
☐ Bookmark this page

Previous

■ Calculator

Week 7 due Nov 20, 2023 01:42 IST Completed

7.3.4 More Advanced (But Still Simple) Examples



Homework /.3.4.1

1/1 point (graded)

$$\operatorname{Find} \left(\begin{matrix} -2 & 0 \\ 4 & 2 \end{matrix} \right)^{-1} =$$

$$\left(\begin{array}{cc} -\frac{1}{2} & 1 \\ 0 & \frac{1}{2} \end{array} \right)$$

$$\bigcirc \left(\begin{array}{cc} \frac{1}{2} & 0 \\ 1 & \frac{1}{2} \end{array} \right)$$

$$\left(\begin{array}{cc} -\frac{1}{2} & 0 \\ 1 & \frac{1}{2} \end{array} \right)$$

$$egin{pmatrix} igcup_{-rac{1}{2}} & 0 \ 1 & -rac{1}{2} \end{pmatrix}$$

Homework 7.3.4.1 Compute
$$\begin{pmatrix} -2 & 0 \\ 4 & 2 \end{pmatrix}^{-1} = \begin{pmatrix} -\frac{1}{2} & 0 \\ 1 & \frac{1}{2} \end{pmatrix}$$
.

Answer: Here is how you can find the answer First, solve

$$\left(\begin{array}{cc} -2 & 0 \\ 4 & 2 \end{array}\right) \left(\begin{array}{c} \chi_0 \\ \chi_1 \end{array}\right) = \left(\begin{array}{c} 1 \\ 0 \end{array}\right).$$

If you do forward substitution, you see that the solution is $\begin{pmatrix} -\frac{1}{2} \\ 1 \end{pmatrix}$, which becomes the first column of A^{-1} .

Next, solve

$$\left(\begin{array}{cc} -2 & 0 \\ 4 & 2 \end{array}\right) \left(\begin{array}{c} \chi_0 \\ \chi_1 \end{array}\right) = \left(\begin{array}{c} 0 \\ 1 \end{array}\right).$$

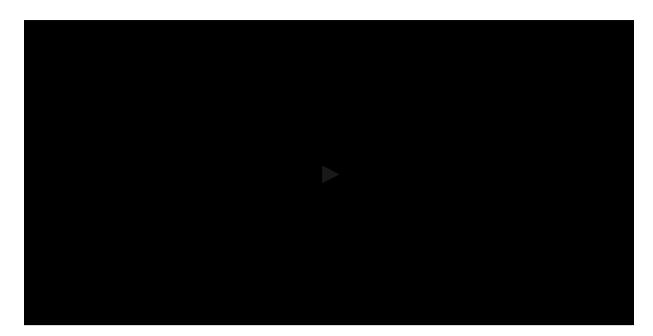
If you do forward substitution, you see that the solution is $\begin{pmatrix} 0 \\ \frac{1}{2} \end{pmatrix}$, which becomes the second column of A^{-1} .

Check:

$$\left(\begin{array}{cc} -2 & 0\\ 4 & 2 \end{array}\right) \left(\begin{array}{cc} -\frac{1}{2} & 0\\ 1 & \frac{1}{2} \end{array}\right) = \left(\begin{array}{cc} 1 & 0\\ 0 & 1 \end{array}\right)$$

Submit

Video



O:00 / 0:00

▶ 2.0x

X

CC

66

Start of transcript. Skip to the end.

Dr. Robert van de Geijn: OK, so the way to compute this inverse

is to note that the lower triangular matrix times its inverse partition

by columns must be equal to the identity partition by its columns.

That's what we talked about in the last video.

If you then say, well, I want to solve with _

Video

▲ Download video file

Transcripts

- <u>★ Download Text (.txt) file</u>

Homework 7.3.4.2

1/1 point (graded)

$$\operatorname{Find} \begin{pmatrix} 1 & -2 \\ 0 & 2 \end{pmatrix}^{-1} =$$

Answer:

$$\left(\begin{array}{cc} 1 & 1 \\ 0 & \frac{1}{2} \end{array}\right).$$

Here is how you can find this matrix: First, solve



⊞ Calculator

$$\left(\begin{array}{cc} 1 & -2 \\ 0 & 2 \end{array}\right) \left(\begin{array}{c} \chi_0 \\ \chi_1 \end{array}\right) = \left(\begin{array}{c} 1 \\ 0 \end{array}\right).$$

If you do back substitution, you see that the solution is $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$, which becomes the first column

of A^{-1} . Next, solve

$$\left(\begin{array}{cc} 1 & -2 \\ 0 & 2 \end{array}\right) \left(\begin{array}{c} \chi_0 \\ \chi_1 \end{array}\right) = \left(\begin{array}{c} 0 \\ 1 \end{array}\right).$$

If you do back substitution, you see that the solution is $\begin{pmatrix} 1 \\ \frac{1}{2} \end{pmatrix}$, which becomes the second column of A^{-1} .

Check:

$$\left(\begin{array}{cc} 1 & -2 \\ 0 & 2 \end{array}\right) \left(\begin{array}{cc} 1 & 1 \\ 0 & \frac{1}{2} \end{array}\right) = \left(\begin{array}{cc} 1 & 0 \\ 0 & 1 \end{array}\right)$$

Submit

• Answers are displayed within the problem

Homework 7.3.4.3

1/1 point (graded)

$$ext{Let } lpha_{0,0}
eq 0 ext{ and } lpha_{1,1}
eq 0. ext{ Then} inom{lpha_{0,0} & 0}{lpha_{1,0} & lpha_{1,1}} = inom{rac{1}{lpha_{0,0}} & 0}{-rac{lpha_{1,0}}{lpha_{0,0}lpha_{1,1}} & rac{1}{lpha_{1,1}}}$$

True ~

Answer: True

Answer: True

Here is how you can find the matrix: First, solve

$$\left(\begin{array}{cc} \alpha_{0,0} & 0\\ \alpha_{1,0} & \alpha_{1,1} \end{array}\right) \left(\begin{array}{c} \chi_0\\ \chi_1 \end{array}\right) = \left(\begin{array}{c} 1\\ 0 \end{array}\right).$$

If you solve the lower triangular system, you see that the solution is $\left(\begin{array}{c} \frac{1}{\alpha_{0,0}} \\ -\frac{\alpha_{1,0}}{\alpha_{0,0}\alpha_{1,1}} \end{array}\right)$, which

becomes the first column of A^{-1} .

Next, solve

$$\left(\begin{array}{cc} \alpha_{0,0} & 0 \\ \alpha_{1,0} & \alpha_{1,1} \end{array}\right) \left(\begin{array}{c} \chi_0 \\ \chi_1 \end{array}\right) = \left(\begin{array}{c} 0 \\ 1 \end{array}\right).$$

If you solve this system, you find the solution $\begin{pmatrix} 0 \\ \frac{1}{\alpha_{1,1}} \end{pmatrix}$, which becomes the second column of

 A^{-1} .

Check:

$$\begin{pmatrix} \alpha_{0,0} & 0 \\ \alpha_{1,0} & \alpha_{1,1} \end{pmatrix} \begin{pmatrix} \frac{1}{\alpha_{0,0}} & 0 \\ -\frac{\alpha_{1,0}}{\alpha_{0,0}\alpha_{1,1}} & \frac{1}{\alpha_{1,1}} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Submit

Answers are displayed within the problem

Homework 7.3.4.4

1/1 point (graded)

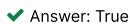
Partition lower triangular matrix $m{L}$ as

$$L = \left(egin{array}{c|c} L_{00} & 0 \ \hline l_{10}^T & \lambda_{11} \end{array}
ight)$$

Assume that $oldsymbol{L}$ has no zeroes on its diagonal. Then

$$L^{-1} = \left(egin{array}{c|c} L_{00}^{-1} & 0 \ \hline -rac{l_{10}^T L_{00}^{-1}}{\lambda_{11}} & rac{1}{\lambda_{11}} \end{array}
ight)$$

True



Answer: True Stictly speaking, one needs to show that L_{00} has an inverse... This would require a proof by induction. We'll skip that part. Instead, we'll just multiply out:

$$\left(\begin{array}{c|c|c} L_{00} & 0 \\ \hline l_{10}^T & \lambda_{11} \end{array}\right) \left(\begin{array}{c|c|c} L_{00}^{-1} & 0 \\ \hline -\frac{l_{10}^T L_{00}^{-1}}{\lambda_{11}} & \frac{1}{\lambda_{11}} \end{array}\right) = \left(\begin{array}{c|c|c} L_{00} L_{00}^{-1} & L_{00} + 0 \frac{1}{\lambda_{11}} \\ \hline l_{10}^T L_{00}^{-1} - \lambda_{11} \frac{l_{10}^T L_{00}^{-1}}{\lambda_{11}} & l_{10}^T \times 0 + \lambda_{11} \frac{1}{\lambda_{11}} \end{array}\right) \\
= \left(\begin{array}{c|c|c} I & 0 \\ \hline 0 & 1 \end{array}\right).$$

Submit

Answers are displayed within the problem

Video

0:00 / 0:00

▶ 2.0x

X

66 CC

Start of transcript. Skip to the end.

Dr. Robert van de Geijn: So now we get to a more general case.

And the exercise is to show that the inverse of this lower triangular matrix partitioned like that is given by this.

The way to verify that that is indeed the inverse

is a matter of just multiplying the two matrices together.

Video

▲ Download video file

Transcripts

- **▲** Download Text (.txt) file

Homework 7.3.4.5

1/1 point (graded)

The inverse of a lower triangular matrix with no zeroes on its diagonal is a lower triangular matrix.

TRUE ~

✓ Answer: TRUE

Answer: True

Proof by induction on n, the size of the square matrix.

Let L be the lower triangular matrix.

Base case: n=1. Then $L=(\lambda_{11})$, with $\lambda_{11}\neq 0$. Clearly, $L^{-1}=(1/\lambda_{11})$.

Inductive step: Inductive Hypothesis: Assume that the inverse of any $n \times n$ lower triangular matrix with no zeroes on its diagonal is a lower triangular matrix.

We need to show that the inverse of any $(n+1) \times (n+1)$ lower triangular matrix, L, with no zeroes on its diagonal is a lower triangular matrix.

Partition

$$L = \left(\begin{array}{c|c} L_{00} & 0 \\ \hline l_{10}^T & \lambda_{11} \end{array}\right)$$

We know then that L_{00} has no zeroes on its diagonal and $\lambda_{11} \neq 0$. We also saw that then

$$L^{-1} = \begin{pmatrix} L_{00}^{-1} & 0 \\ \hline -\frac{1}{\lambda_{11}} l_{10}^T L_{00}^{-1} & \frac{1}{\lambda_{11}} \end{pmatrix}$$

Hence, the matrix has an inverse, and it is lower triangular.

By the Principle of Mathematical Induction, the result holds.

Submit

Answers are displayed within the problem

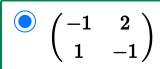
The answer to the last exercise suggests an algorithm for inverting a lower triangular matrix. See if you can implement it!

Homework 7.3.4.7

1/1 point (graded)

Find
$$\begin{pmatrix} 1 & 2 \\ 1 & 1 \end{pmatrix}^{-1} =$$

 $\begin{array}{c} \bigcirc & \begin{pmatrix} 1 & 2 \\ 1 & 1 \end{pmatrix}$



 $\begin{pmatrix} -1 & \frac{1}{2} \\ 1 & -1 \end{pmatrix}$

■ Calculator

$$\bigcirc \begin{pmatrix} 1 & \frac{1}{2} \\ 1 & 1 \end{pmatrix}$$



Answer:

$$\left(\begin{array}{cc} -1 & 2 \\ 1 & -1 \end{array}\right).$$

Here is how you can find this matrix: First, you compute the LU factorization. Since there is only one step, this is easy:

$$\left(\begin{array}{cc} 1 & 0 \\ -1 & 1 \end{array}\right) \left(\begin{array}{cc} 1 & 2 \\ 1 & 1 \end{array}\right) = \left(\begin{array}{cc} 1 & 2 \\ 0 & -1 \end{array}\right).$$

Thus,

$$\left(\begin{array}{cc} 1 & 2 \\ 1 & 1 \end{array}\right) = \left(\begin{array}{cc} 1 & 0 \\ 1 & 1 \end{array}\right) \left(\begin{array}{cc} 1 & 2 \\ 0 & -1 \end{array}\right).$$

Next, you solve

$$\left(\begin{array}{cc} 1 & 2 \\ 1 & 1 \end{array}\right) \left(\begin{array}{c} \chi_0 \\ \chi_1 \end{array}\right) = \left(\begin{array}{c} 1 \\ 0 \end{array}\right)$$

by solving
$$\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} \zeta_0 \\ \zeta_1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$
 followed by $\begin{pmatrix} 1 & 2 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} \chi_0 \\ \chi_1 \end{pmatrix} = \begin{pmatrix} \zeta_0 \\ \zeta_1 \end{pmatrix}$. If you do

this right, you get $\begin{pmatrix} -1 \\ 1 \end{pmatrix}$, which becomes the first column of the inverse.

You solve

$$\left(\begin{array}{cc} 1 & 2 \\ 1 & 1 \end{array}\right) \left(\begin{array}{c} \chi_0 \\ \chi_1 \end{array}\right) = \left(\begin{array}{c} 0 \\ 1 \end{array}\right)$$

in a similar manner, yielding the second column of the inverse, $\begin{pmatrix} 2 \\ -1 \end{pmatrix}$.

Check:

$$\left(\begin{array}{cc} 1 & 2 \\ 1 & 1 \end{array}\right) \left(\begin{array}{cc} -1 & 2 \\ 1 & -1 \end{array}\right) = \left(\begin{array}{cc} 1 & 0 \\ 0 & 1 \end{array}\right).$$

Submit

Answers are displayed within the problem

Homework 7.3.4.8

1/1 point (graded)

If
$$\alpha_{0,0}\alpha_{1,1}-\alpha_{1,0}\alpha_{0,1}\neq 0$$
 then

$$\left(egin{array}{cccc} lpha_{0,0} & lpha_{0,1} \end{array}
ight)^{-1} = \underbrace{\qquad \qquad } \left(egin{array}{cccc} lpha_{1,1} & -lpha_{0,1} \end{array}
ight)$$

■ Calculator

$$igl(lpha_{1,0} \quad lpha_{1,1}igr) \quad igr- \quad lpha_{0,0}lpha_{1,1} - lpha_{1,0}lpha_{0,1} igl(-lpha_{1,0} \quad lpha_{0,0}igr)$$

True ✓ ✓ Answer: True

Answer: True

Check:

$$\begin{pmatrix} \alpha_{0,0} & \alpha_{0,1} \\ \alpha_{1,0} & \alpha_{1,1} \end{pmatrix} \frac{1}{\alpha_{0,0}\alpha_{1,1} - \alpha_{1,0}\alpha_{0,1}} \begin{pmatrix} \alpha_{1,1} & -\alpha_{0,1} \\ -\alpha_{1,0} & \alpha_{0,0} \end{pmatrix}$$

$$= \frac{1}{\alpha_{0,0}\alpha_{1,1} - \alpha_{1,0}\alpha_{0,1}} \begin{pmatrix} \alpha_{0,0} & \alpha_{0,1} \\ \alpha_{1,0} & \alpha_{1,1} \end{pmatrix} \begin{pmatrix} \alpha_{1,1} & -\alpha_{0,1} \\ -\alpha_{1,0} & \alpha_{0,0} \end{pmatrix}$$

$$= \frac{1}{\alpha_{0,0}\alpha_{1,1} - \alpha_{1,0}\alpha_{0,1}} \begin{pmatrix} \alpha_{0,0}\alpha_{1,1} - \alpha_{0,1}\alpha_{1,0} & -\alpha_{0,0}\alpha_{0,1} + \alpha_{0,1}\alpha_{0,0} \\ \alpha_{1,0}\alpha_{1,1} - \alpha_{1,1}\alpha_{1,0} & -\alpha_{1,0}\alpha_{0,1} + \alpha_{1,1}\alpha_{0,0} \end{pmatrix}$$

$$= \frac{1}{\alpha_{0,0}\alpha_{1,1} - \alpha_{1,0}\alpha_{0,1}} \begin{pmatrix} \alpha_{0,0}\alpha_{1,1} - \alpha_{0,1}\alpha_{1,0} & 0 \\ 0 & \alpha_{1,1}\alpha_{0,0} - \alpha_{1,0}\alpha_{0,1} \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Submit

Answers are displayed within the problem

Homework 7.3.4.9

1/1 point (graded)

The 2 imes 2 matrix $A=egin{pmatrix} lpha_{0,0} & lpha_{0,1} \ lpha_{1,0} & lpha_{1,1} \end{pmatrix}$ has an inverse if and only if $lpha_{0,0}lpha_{1,1}-lpha_{1,0}lpha_{0,1}
eq 0.$

True ~

Answer: True

Answer: True

This is an immediate consequence of the last exercise.

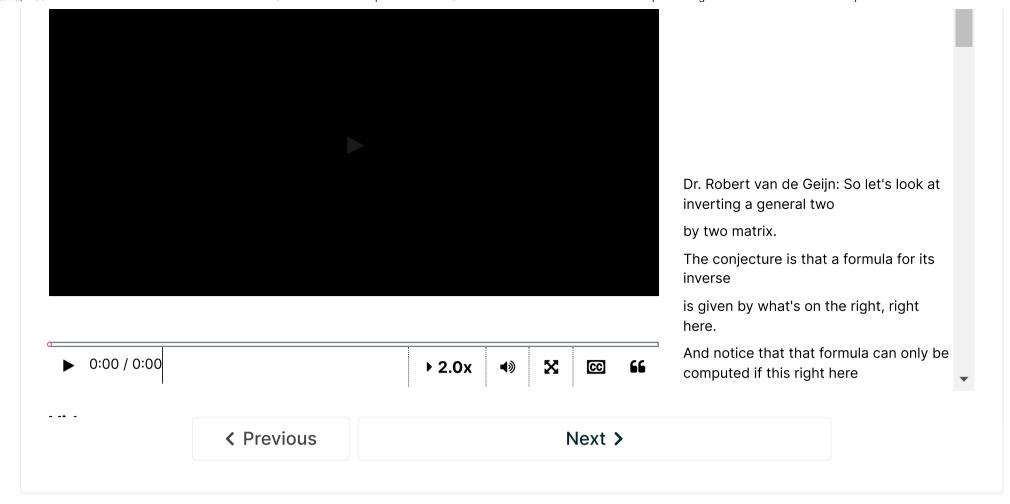
Submit

Answers are displayed within the problem

In the below video, towards the end, the instructor misspeaks and says that the inverse exists if and only if the determinant is zero. The correct statement is that the inverse exists if and only if the determinant is **not** zero.

Video

Start of transcript. Skip to the end.



© All Rights Reserved



edX

About

Affiliates

edX for Business

Open edX

<u>Careers</u>

<u>News</u>

Legal

Terms of Service & Honor Code

Privacy Policy

Accessibility Policy

<u>Trademark Policy</u>

<u>Sitemap</u>

Cookie Policy

Your Privacy Choices

Connect

<u>Idea Hub</u>

Contact Us

Help Center

Security

Media Kit

















© 2023 edX LLC. All rights reserved.

深圳市恒宇博科技有限公司 <u>粤ICP备17044299号-2</u>