



MITx: 6.041x Introduction to Probability - The Science of Uncertainty



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Unit overview

Lec. 5: Probability mass functions and expectations

Exercises 5 due Mar 02, 2016 at 23:59 UTC

Lec. 6: Variance; Conditioning on an event; Multiple r.v.'s

Exercises 6 due Mar 02, 2016 at 23:59 UTC

Lec. 7: Conditioning on a random variable; Independence of r.v.'s

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Bookmark

Problem 5: Indicator variables

(6/6 points)

Consider a sequence of independent tosses of a biased coin at times $k = 0, 1, 2, \dots, n$. On each toss, the probability of Heads is p , and the probability of Tails is $1 - p$.

A reward of one unit is given at time k , for $k \in \{1, 2, \dots, n\}$, if the toss at time k resulted in Tails and the toss at time $k - 1$ resulted in Heads. Otherwise, no reward is given at time k .

Let R be the sum of the rewards collected at times $1, 2, \dots, n$.

We will find $\mathbf{E}[R]$ and $\mathbf{var}(R)$ by carrying out a sequence of steps. Express your answers below in terms of p and/or n using standard notation. Remember to write '*' for all multiplications and to include parentheses where necessary.

We first work towards finding $\mathbf{E}[R]$.

1. Let I_k denote the reward (possibly 0) given at time k , for $k \in \{1, 2, \dots, n\}$. Find $\mathbf{E}[I_k]$.

$$\mathbf{E}[I_k] = (1-p)*p$$



2. Using the answer to part 1, find $\mathbf{E}[R]$.

$$\mathbf{E}[R] = n*(1-p)*p$$



The variance calculation is more involved because the random variables I_1, I_2, \dots, I_n are not independent. We begin by computing the following values.

3. If $k \in \{1, 2, \dots, n\}$, then

$$\mathbf{E}[I_k^2] = (1-p)*p$$



4. If $k \in \{1, 2, \dots, n - 1\}$, then

Exercises 7 due Mar
02, 2016 at 23:59 UTC

Solved problems

Additional
theoretical
material

Problem Set 4

Problem Set 4 due Mar
02, 2016 at 23:59 UTC

Unit summary

- ▶ Unit 5:
Continuous
random
variables

$$\mathbf{E}[I_k I_{k+1}] = \boxed{0} \quad \checkmark$$

5. If $k \geq 1$, $\ell \geq 2$, and $k + \ell \leq n$, then

$$\mathbf{E}[I_k I_{k+\ell}] = \boxed{(1-p)^2 p^2} \quad \checkmark$$

6. Using the results above, calculate the numerical value of $\text{var}(R)$ assuming that $p = 3/4$, $n = 10$.

$$\text{var}(R) = \boxed{0.890625} \quad \checkmark$$

You have used 1 of 3 submissions

Your answers have been saved but not graded. Click 'Check' to grade them.

DISCUSSION

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