6. Probability tables

Gaussian probabilities

4/4 points (graded)

Let $X \sim N(1, 2.25)$. As a reminder, the 2.25 here represents the value of σ^2. Using the normal probability table below, compute the following probabilities:

Normal probability table

Show

$\mathbf{P}(X > 1) =$ 0.5 Answer: 0.5

$\mathbf{P}(|X-2| \leq 1) =$ 0.4087887802741321 Answer: 0.4082

$\mathbf{P}(X^2 > 4) =$ 0.2752426694951021 Answer: 0.2752427

$\mathbf{P}(X^2 - 2X - 1 > 0) =$ 0.3457785861511602 Answer: 0.3457786

STANDARD NOTATION

Solution:

First, note that for $Z \sim \mathcal{N}(0,1)$, $x > 0$, we have

$$\mathbf{P}(Z \leq -x) = \mathbf{P}(Z \geq x) = 1 - \mathbf{P}(Z \leq x),$$

and

$$\mathbf{P}(Z \geq x) = 1 - \mathbf{P}(Z \leq x).$$

Moreover, if $X \sim \mathcal{N}(1, 2.25)$, we can write it as $X = 1.5Z + 1$, where $Z \sim \mathcal{N}(0,1)$. This allows us to reduce all probabilities to the ones that are listed in the table.

In particular,

$$\begin{aligned} \mathbf{P}(X > 1) = \quad & \mathbf{P}(1.5Z + 1 > 1) = \mathbf{P}(1.5Z > 0) \\ = \quad & \mathbf{P}(Z \geq 0) = 1 - \mathbf{P}(Z \leq 0) = 1 - 0.5000 = 0.5000, \end{aligned}$$

$$\begin{aligned} \mathbf{P}(|X - 2| \leq 1) = \quad & \mathbf{P}(-1 \leq (X - 2) \leq 1) = \mathbf{P}(-1 \leq (1.5Z + 1 - 2) \leq 1) \\ = \quad & \mathbf{P}(0 \leq 1.5Z \leq 2) \\ \simeq \quad & \mathbf{P}(0 \leq Z \leq 1.33) \\ = \quad & \mathbf{P}(Z \leq 1.33) - \mathbf{P}(Z \leq 0) \simeq 0.9082 - 0.5000 = 0.4082 \end{aligned}$$

$$\begin{aligned} \mathbf{P}(X^2 > 4) = \quad & \mathbf{P}(|X| > 2) = \mathbf{P}(|1.5Z + 1| > 2) \\ = \quad & \mathbf{P}(1.5Z + 1 \leq -2) + \mathbf{P}(1.5Z + 1 \geq 2) \\ = \quad & \mathbf{P}(Z \leq -2) + \mathbf{P}\left(Z \geq \frac{2}{3}\right) \\ = \quad & 1 - \mathbf{P}(Z \leq 2) + 1 - \mathbf{P}\left(Z \leq \frac{2}{3}\right) \\ \simeq \quad & 2 - 0.9772 - 0.7486 = 0.2742 \end{aligned}$$

$$\begin{aligned} \mathbf{P}(X^2 - 2X - 1 > 0) = \quad & \mathbf{P}((X - 1)^2 - 2 > 0) = \mathbf{P}(|X - 1| > \sqrt{2}) \\ = \quad & \mathbf{P}(|1.5Z| > \sqrt{2}) \end{aligned}$$

$$\begin{aligned} &= \mathbf{P}\left(Z > \frac{\sqrt{2}}{1.5}\right) + \mathbf{P}\left(Z < -\frac{\sqrt{2}}{1.5}\right) \\ &= 2 - 2\mathbf{P}\left(Z < \frac{\sqrt{2}}{1.5}\right) \\ &\simeq 2 - 2\,(0.8264) \\ &= 0.3472. \end{aligned}$$

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Answers are displayed within the problem

Approximation of Binomial variables

1/1 point (graded)

Using the normal probability table, evaluate approximately $\mathbf{P}(X > 400)$, where X is a binomial random variable with parameters 1000 and $.3$.

Normal probability table

Show

$\mathbf{P}(X > 400) \simeq$ 0 ✔ **Answer:** 0.0002

STANDARD NOTATION

Solution:

A binomial distribution with parameters (n, p) has expectation np and variance $np(1-p)$. Hence, by the Central Limit Theorem, we have

$$\frac{1}{\sqrt{np(1-p)}}(X - np) \xrightarrow{(\mathrm{D})} Z \sim \mathcal{N}(0,1).$$

The probability in question can therefore be approximated by

$$\begin{aligned} \mathbf{P}(X > 400) = \quad & \mathbf{P}\left(\frac{1}{\sqrt{1000 \times 0.3 \times 0.7}}(X - 300) > \frac{100}{\sqrt{1000 \times 0.3 \times 0.7}}\right) \\ \simeq \quad & 1 - \mathbf{P}\left(Z \leq \frac{100}{\sqrt{1000 \times 0.3 \times 0.7}}\right) \\ \simeq \quad & 1 - \mathbf{P}(Z \leq 6.90) \\ \leq \quad & 1 - 0.9998 = 0.0002. \end{aligned}$$

Note: This is only an estimate, because the probability table ends here. In fact, the probability is approximately 7×10^{-12}.

Submit

You have used 1 of 3 attempts

Answers are displayed within the problem

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Topic: Unit 0. Course Overview, Syllabus, Guidelines, and Homework on Prerequisites:Homework 0: Probability and Linear algebra Review / 6. Probability tables

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grader answer doesn't match given solution

discussion posted 7 days ago by anonymous

it's slightly off - just wanted to make sure that a green tick is actually correct - there's been some discrepancies in other courses.

Also - in this course, is it the convention to round up when doing the table lookup? Thanks.

This post is visible to everyone.

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2 responses

Cool7

7 days ago

Grader answer looks like not using table lookup, but solution does use table lookup.

I think when we are told to use table, we'd better to use it. Sometimes grader accept both but more offen grader only accept answer using table lookup method.

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IvanZinDC

5 days ago

yeah, basically the table says that anything beyond a certain Z score is the same low probability.

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