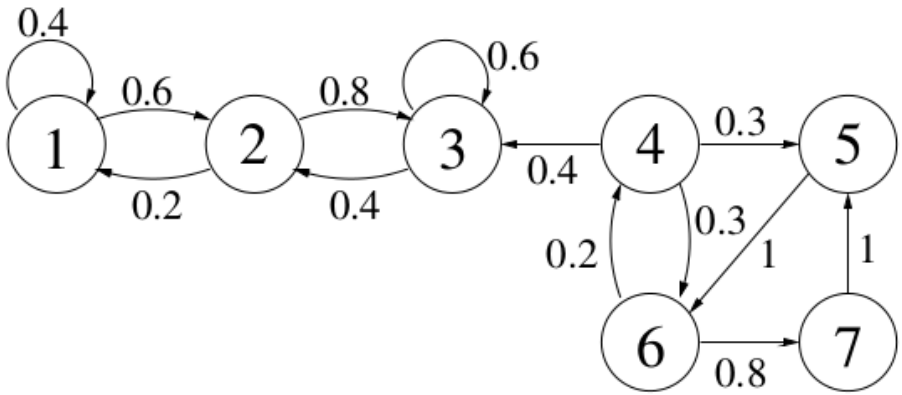
Exercise: Path calculation

(3/3 points)

Consider a Markov chain with the following transition probability graph:



1. $\mathbf{P}(X_1 = 6, X_2 = 4, X_3 = 3 \mid X_0 = 4) =$ 0.024 ✔

   Answer: 0.024

2.

- Unit 6: Further topics on random variables
- Unit 7: Bayesian inference
- Exam 2
- Unit 8: Limit theorems and classical statistics
- Unit 9: Bernoulli and Poisson processes
- **Unit 10: Markov chains**

Unit overview

Lec. 24: Finite-state Markov chains
Exercises 24 due May 18, 2016 at 23:59 UTC

Lec. 25: Steady-state behavior of Markov chains
Exercises 25 due May 18, 2016 at 23:59 UTC

$\mathbf{P}(X_{103} = 3 \mid X_{100} = 1) =$ 0.48 ✔ **Answer:** 0.48

Answer:

1. The desired probability corresponds to a unique path through the Markov chain. Hence, we can simply multiply one-step transition probabilities along the path:

$$\mathbf{P}(X_1 = 6, X_2 = 4, X_3 = 3 \mid X_0 = 4) = p_{46}p_{64}p_{43} = (0.3)(0.2)(0.4) = 0.024.$$

2. We are looking for the 3-step transition probability from state 1 to state 3, $r_{13}(3)$. We can always use the recursion formula to calculate this, but in this particular case, we can directly observe that there are only 2 possible paths: $1 \to 1 \to 2 \to 3$ and $1 \to 2 \to 3 \to 3$. Hence,

$$\begin{aligned}\mathbf{P}(X_{103} = 3 \mid X_{100} = 1) &= p_{11}p_{12}p_{23} + p_{12}p_{23}p_{33} \\ &= (0.4)(0.6)(0.8) + (0.6)(0.8)(0.6) \\ &= 0.48.\end{aligned}$$

You have used 1 of 2 submissions

Lec. 26: Absorption probabilities and expected time to absorption
Exercises 26 due May 18, 2016 at 23:59 UTC

Solved problems

Problem Set 10
Problem Set 10 due May 18, 2016 at 23:59 UTC