Problem 3

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PART A

2/2 points (graded)

In the facility location problem, the goal is to optimally place facilities so as to minimize transportation costs from the facilities to the customers. In practice, one rarely knows the demand of customers with a high degree of accuracy. In addition, the transportation costs themselves may vary with time and are subject to uncertainty. In this version of the facility location problem, we assume that all data is known.

In the first part of this problem, the goal is to locate one or more facilities out of five possible sites, which we designated as $F_1, F_2, F_3, F_4,$ and F_5. The cost of selecting a facility is \$40. Coincidentally, there are also five customers that need to be serviced. We refer to them as C_1, C_2, C_3, C_4 and C_5. The delivery cost from each possible facility site to each of the five customers is known. The cost of satisfying (all of) the demand of Customer C_j from site F_i is d_{ij}. (Note that the first index is for the site and the second is for the customer.) The delivery cost from all possible sites to all customers are given in Table 1 below.

	Customer C_1	Customer C_2	Customer C_3	Customer C_4	Customer C_5
Site F_1	30	15	59	78	27

- Exit Survey

Site F_2	50	42	25	30	53
Site F_3	64	14	30	20	62
Site F_4	46	19	66	48	11
Site F_5	19	40	60	31	27

Table 1: Delivery Cost from All Possible Sites to All Customers

The decision variables for this problem are as follows. We let $y_j = 1$if a facility is located at site F_j. Otherwise, $y_j = 0$. We let $0 \leq x_{ij} \leq 1$ represent the fraction of demand from $C_j$satisfied by facility F_i. The cost associated with customer C_1 would be

$$d_{11}x_{11} + d_{21}x_{21} + d_{31}x_{31} + d_{41}x_{41} + d_{51}x_{51}$$

One would also need to add constraints that ensures that $x_{ij} = 0, \forall j = 1, \ldots, 5$ whenever $y_i = 0$, which means we cannot use site F_i to serve any customer C_j if we don't locate a facility at site i; (2) all the customers' demand need to be satisfied.

Formulate this problem as a deterministic optimization problem in which the objective is to minimize the expected total cost which is the sum of the facility opening cost and the expected delivery cost.

Choose the correct objection function from below.

- $\text{MIN } \sum_{i=1}^{5} 40y_i + \sum_{i=1}^{5} \sum_{j=1}^{5} x_{ij}$

- $\text{MAX } \sum_{i=1}^{5} 40y_i - \sum_{i=1}^{5}\sum_{j=1}^{5} x_{ij}$
- $\text{MAX } \sum_{i=1}^{5} 40y_i + \sum_{i=1}^{5}\sum_{j=1}^{5} d_{ij}x_{ij}$
- $\text{MIN } \sum_{i=1}^{5} 40y_i + \sum_{i=1}^{5}\sum_{j=1}^{5} d_{ij}x_{ij}$ ✔

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PART B

2/2 points (graded)

Choose the correct constraint(s) from below:

Assume that a facility i is either used to serve a customer j or it is not, rather than a fractional amount.

- [ ] $\sum_{i=1}^{5} y_i = 1, \forall j = 1, \ldots, 5$
- [x] $\sum_{i=1}^{5} x_{ij} = 1, \forall j = 1, \ldots, 5$
- [ ] $\sum_{i=1}^{5}\sum_{j=1}^{5} x_{ij} = 1, \forall i = 1, \ldots, 5, \forall j = 1, \ldots, 5$

- [x] $x_{ij} \leq y_i, \forall i = 1, \ldots, 5, \forall j = 1, \ldots, 5$
- [ ] $x_{ij} \geq y_i, \forall i = 1, \ldots, 5, \forall j = 1, \ldots, 5$
- [x] $x_{ij} \in \{0, 1\}, \forall i = 1, \ldots, 5, \forall j = 1, \ldots, 5$
- [ ] $x_{ij} \geq 1, \forall i = 1, \ldots, 5, \forall j = 1, \ldots, 5$
- [x] $y_i \in \{0, 1\}, \forall i = 1, \ldots, 5$
- [ ] $y_i \geq 0, \forall i = 1, \ldots, 5$

✔

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PART C

2/2 points (graded)

Solve the deterministic facility location you formulated above using Julia/JuMP. Which facilities are open? Select both of them.

- [ ] 1

- [ ] 2
- [x] 3
- [ ] 4
- [x] 5

✔

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PART D

2/2 points (graded)

What will be the total minimum cost?

190.0 ✔

190.0

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PART E

3/3 points (graded)

In the second part of the problem, we will consider the stochastic version. The delivery cost of satisfying customers from the facilities is uncertain. It depends on which future scenario occurs. There are five possible scenarios. Below is the probability of each scenario: $p_1, p_2, \ldots, p_5$.

	Scenario 1	Scenario 2	Scenario 3	Scenario 4	Scenario 5
Probability	0.15	0.15	0.25	0.2	0.25

We will model this as a 2-stage stochastic optimization problem. In the first stage, the decision is which of the five facilities to select. Similarly, we let $y_j = 1$ if a facility is located at site F_i. Otherwise, $y_i = 0.$ The cost of opening up a facility is $40.

After the facilities are selected, the decision maker learns which of the five scenarios occurs. We let the scenarios be designated as S_1, S_2, S_3, S_4 and S_5. We let d_{ij}^k denote the cost of satisfying demand of customer C_j using facility F_i under scenario S_k. Note that the first subscript corresponds to the facility and the second subscript corresponds to the customer. The data of d_{ij}^k is stored in pset6_p1_data.xlsx

We let $x_{ij}^k = 1$ if customer $C_j$is served by facility F_i under scenario S_k. Otherwise, it is 0.

Model the problem of minimizing the following sum: the cost of opening facilities plus the expected cost of satisfying demand for customers from the open facilities

Choose the correct objection function from below.

- MIN $\sum_{k=1}^{5} \sum_{i=1}^{5} \sum_{j=1}^{5} d_{ij}^{k} x_{ij}^{k}$
- MIN $\sum_{i=1}^{5} 40 y_i + \sum_{k=1}^{5} \sum_{i=1}^{5} \sum_{j=1}^{5} d_{ij}^{k} x_{ij}^{k}$
- MIN $\sum_{i=1}^{5} 40 y_i + \sum_{k=1}^{5} p_k \sum_{i=1}^{5} \sum_{j=1}^{5} d_{ij}^{k} x_{ij}^{k}$ ✔
- MIN $\sum_{i=1}^{5} 40 y_i + \sum_{k=1}^{5} p_k \sum_{i=1}^{5} \sum_{j=1}^{5} x_{ij}^{k}$

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PART F

3/3 points (graded)

Choose all necessary constraint(s) from below.

- [x] $\sum_{i=1}^{5} x_{ij}^{k} = 1, \forall j = 1, \ldots, 5, \forall k = 1, \ldots, 5$
- [ ] $\sum_{j=1}^{5} x_{ij}^{k} = 1, \forall i = 1, \ldots, 5, \forall k = 1, \ldots, 5$

- [x] $x_{ij}^k \leq y_i, \forall i = 1, \ldots, 5, \forall j = 1, \ldots, 5, \forall k = 1, \ldots, 5$
- [ ] $x_{ij}^k \geq y_i, \forall i = 1, \ldots, 5, \forall j = 1, \ldots, 5, \forall k = 1, \ldots, 5$
- [x] $x_{ij}^k \in \{0, 1\}, \forall i = 1, \ldots, 5, \forall j = 1, \ldots, 5$
- [ ] $x_{ij}^k \geq 1, \forall i = 1, \ldots, 5, \forall j = 1, \ldots, 5$
- [x] $y_j \in \{0, 1\}, \forall j = 1, \ldots, 5$

✔

Submit You have used 1 of 3 attempts

PART G

3/3 points (graded)

Solve the stochastic facility location problem you formulated above using Julia/JuMP. Complete PS6_p1_partfg.jl to solve the problem. We already provide data in it.

Which facilities will be opened under the optimal solution?

- [x] 1

- [x] 2
- [ ] 3
- [ ] 4
- [ ] 5

✔

Submit You have used 2 of 3 attempts

PART H

3/3 points (graded)

What will be the total minimum cost?

173.4 ✔

173.4

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✔ Correct (3/3 points)