



[Lecture 14: Wald's Test, Likelihood
Ratio Test, and Implicit Hypothesis](#)

[Course](#) > [Unit 4 Hypothesis testing](#) > [Test](#)

2. Worked Example: Two-Sample T-
> test with Small Sample Sizes

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2. Worked Example: Two-Sample T-test with Small Sample Sizes

Note: At around 5:15 mark in the following video, Prof. Rigollet says "So clearly, you reject (the null hypothesis)" while the correct assertion should have been "So clearly, you fail to reject (the null hypothesis)".

Worked Example: Two-Sample T-test with Small Sample Sizes

Non-asymptotic test

- ▶ Example $n = 70, m = 50, \bar{X}_n = 156.4, \bar{Y}_m = 132.7,$
 $\hat{\sigma}_d^2 = 5198.4, \hat{\sigma}_c^2 = 3867.0,$

$$\frac{156.4 - 132.7}{\sqrt{\frac{5198.4}{70} + \frac{3867.0}{50}}} = 1.57$$

- ▶ Using the shorthand formula $N = \min(n, m) = 50$, we get
 $q_{5\%} = 1.68$ and

$$\text{p-value} = P[t_{50} > 1.57] = 0.0614$$

- ▶ Using the W-S formula

some data from a control group of size m equals 50

$$N = \frac{\left(\frac{5198.4}{70} + \frac{3867.0}{50}\right)}{\frac{5198.4^2}{70^2(70-1)} + \frac{3867.0^2}{50^2(50-1)}} = 113.78$$

we round to 113.

- ▶ We get

$$\text{p-value} = P[t_{113} > 1.57] = 0.0596$$

0:26 / 9:21

1.50x

Video

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Non-asymptotic Two-Sample Test using t-statistic

Assume

- $X_1, \dots, X_n \stackrel{iid}{\sim} \mathcal{N}(\mu_X, \sigma_X^2),$
- $Y_1, \dots, Y_m \stackrel{iid}{\sim} \mathcal{N}(\mu_Y, \sigma_Y^2),$
- $X_1, \dots, X_n, Y_1, \dots, Y_m$ are independent.

Then, for any n and m , the distribution of the test statistic below is approximated by a t -distribution:

$$\frac{\bar{X}_n - \bar{Y}_m - (\mu_X - \mu_Y)}{\sqrt{\hat{\sigma}_X^2/n + \hat{\sigma}_Y^2/m}} \underset{\sim}{\text{approx.}} t_N$$

where the degrees of freedom N is given by the **Welch-Satterthwaite formula** :

$$\min(n, m) \leq N = \frac{(\hat{\sigma}_X^2/n + \hat{\sigma}_Y^2/m)^2}{\frac{\hat{\sigma}_X^4}{n^2(n-1)} + \frac{\hat{\sigma}_Y^4}{m^2(m-1)}} \leq n + m$$

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Worked Example: Two-Sample T-test with Small Sample Sizes

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