Unit 1: Quiz

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Unit 1: Quiz

The problems in the quiz will be automatically graded.

You are given only **ONE** attempt for each problem. You will be able to see the solution after you submit your answers.

Problem 1

4/4 points (graded)

Consider a collection of 9 bears. There is a family of red bears consisting of one father bear, one mother bear, and one baby bear. There is a similar green bear family, and a similar blue bear family. In each of the scenarios below, we draw 5 consecutive times from this collection, keeping track (in order) of the kind of bears that we get.

1a. If we draw with replacement (i.e., returning the bear after each draw), how many possible outcomes are in the sample space S? (An outcome is a 5-tuple of bears.)

59049

Answer: 59049

1b. If we draw without replacement (i.e., not returning the bear after each draw), how many possible outcomes are in the sample space S? (An outcome is a 5-tuple of bears.)

15120

✔ **Answer:** 15120

1c. If we draw with replacement, how many outcomes have no red bears?

7776

✔ **Answer:** 7776

1d. If we draw without replacement, how many outcomes have no red bears?

720

✔ **Answer:** 720

Solution

1a. There are 9 bears to choose from each time, so the number of possible outcomes is $9 \times 9 \times 9 \times 9 \times 9 = 9^5 = 59049$.

1b. There are 9 bears for the first choice, 8 bears remaining for the second choice, 7 bears remaining for the third choice, etc., so $9 \times 8 \times 7 \times 6 \times 5 = 15120$ possible outcomes.

An alternative view is this: There are $\binom{9}{5} = \frac{9!}{5!4!} = 126$ ways to select 5 out of the 9 bears, without regard to order, and then $5! = 120$ ways to order them, so there are $(126)(120) = 15120$ ways altogether, if you take order into account.

1c. Similar to (1a), there are 6 bears to choose from each time, so the number of possible outcomes is $6 \times 6 \times 6 \times 6 \times 6 = 6^5 = 7776$.

1d. Similar to (1b), there are $(6)(5)(4)(3)(2) = 720$ possible outcomes. Or, using the alternative view, there are $\binom{6}{5} = \frac{6!}{5!1!} = 6$ ways to select 5 out of the 6 bears, without regard to order, and then $5! = 120$ ways to order them, so there are $(6)(120) = 720$ outcomes.

Submit You have used 1 of 1 attempt

Correct (4/4 points)

Problem 2

4/4 points (graded)

2. Consider a collection of 4 suite-mates. They choose which one of them (exactly 1 of them) goes to the store on Wednesday night.

2a. How many outcomes are there?

4 **Answer:** 4

2b. How many possible events are there?

16 **Answer:** 16

2c. Suppose on Friday they need to buy a lot of food for the weekend, so they choose (exactly) two suite-mates to go together to the store on Friday. (You can completely ignore what happened on Wednesday.) How many outcomes are there for the pair of Friday shoppers?

6 **Answer:** 6

2d. Same scenario as (2c). How many possible events are there for the pair of Friday shoppers?

64

Answer: 64

solution

2ab. There are 4 outcomes, and thus, there are $2^4 = 16$ possible events.
2cd. There are $(4)(3)/2 = 6$ outcomes for the pair of people who go to the store, or equivalently, $\binom{4}{2} = \frac{4!}{2!2!} = 6$ outcomes. So there are $2^6 = 64$ possible events.

Submit You have used 1 of 1 attempt

Correct (4/4 points)

Problem 3

1/3 points (graded)

3. Consider 10 consecutive tosses of a coin.

3a. How many outcomes are there?

1

Answer: 1024

3b. How many events are there?

- 2^{10}

- 2^{100}
- 1024^2
- 2^{1024}

3c. In how many of the outcomes does the 3rd head occur on the 10th flip?

36

Answer: 36

Explanation

3a. Each outcome is a list of 10 coins, so there are $2^{10} = 1024$ possible outcomes.
3b. Since there are 1024 outcomes, there are 2^{1024} possible events.
3c. There are $(9)(8)/2 = 36$ ways to pick which two out of the first nine flips will be heads. This is also $\binom{9}{2} = \frac{9!}{2!7!} = 36$. So there are 36 possible outcomes.

Submit You have used 1 of 1 attempt

Problem 4

5/5 points (graded)

4. Consider a collection of 9 bears. There is a family of red bears consisting of one father bear, one mother bear, and one baby bear. There is a similar green bear family, and a similar blue bear family. We draw 5 consecutive times from this collection *without replacement* (i.e., not returning the bear after each draw). We keep track (in order) of the kind of bears that we get.

4a. Let A_j denote the event that exactly j of the red bears are chosen during the 5 draws. Do the events A_0, A_1, A_2, A_3 constitute a partition of the sample space? (As always, be sure to justify your answer.)

Yes ▾ ✔ **Answer:** Yes

4b. Find the probabilities of each of these four events.

$P(A_0)$ = 0.04761905 ✔ **Answer:** 0.0476

$P(A_1)$ = 0.35714286 ✔ **Answer:** 0.3571

$P(A_2)$ = 0.47619048 ✔ **Answer:** 0.4762

$P(A_3)$ = 0.11904762 ✔ **Answer:** 0.1190

Solution

4a. Yes, the events are a partition of the sample space. Each outcome has either 0, 1, 2, or 3 bears, so $A_0 \cup A_1 \cup A_2 \cup A_3$ is the whole sample space, and the events A_0, A_1, A_2, A_3 are disjoint, so the events do constitute a sample space.

4b. The probabilities are:

$$P(A_0) = \frac{\binom{3}{0}\binom{6}{5}}{\binom{9}{5}} = \frac{1}{21}; \quad P(A_1) = \frac{\binom{3}{1}\binom{6}{4}}{\binom{9}{5}} = \frac{5}{14};$$
$$P(A_2) = \frac{\binom{3}{2}\binom{6}{3}}{\binom{9}{5}} = \frac{10}{21}; \quad P(A_3) = \frac{\binom{3}{3}\binom{6}{2}}{\binom{9}{5}} = \frac{5}{42}.$$

Submit You have used 1 of 1 attempt

✔ Correct (5/5 points)

Problem 5

2/2 points (graded)

5a. Flip a fair coin ten times. Find the probability that there are at least three heads among the ten flips.

0.9453125 ✔ **Answer:** 0.9453

5b. Flip a fair coin until the third head appears, and then stop right after that flip. What is the probability that it took you ten or more flips to accomplish this?

0.08984375 ✔ **Answer:** 0.0898

Explanation

5a. The probability of 0 heads is $(1/2)^{10}$. The probability of 1 head is $(10)(1/2)^{10}$. The probability of 2 heads is $\binom{10}{2}(1/2)^{10}$. So the desired probability is the probability of the complement, i.e., $1-((1/2)^{10}+(10)(1/2)^{10}+\binom{10}{2}(1/2)^{10}) = 121/128$.

5b. The probability it takes j flips is $\binom{j-1}{2}(1/2)^j$. So we use the complement to get the desired probability, namely $1-\sum_{j=3}^{9}\binom{j-1}{2}(1/2)^j = 23/256$.

Submit You have used 2 of 3 attempts

Problem 6

4/4 points (graded)

6. Consider events A_1, A_2, A_3 with the following properties:

$$P(A_1) = P(A_2) = P(A_3) = 1/4$$

$$P(A_1 \cap A_2) = P(A_1 \cap A_3) = P(A_2 \cap A_3) = 1/8$$

$$P(A_1 \cap A_2 \cap A_3) = 1/16$$

6a. Find the probability $P(A_1 \cup A_2 \cup A_3)$.

0.4375 ✔ **Answer:** 0.4375

6b. Do the events A_1, A_2, A_3 constitute a partition of the sample space? (As always, be sure to justify your answer.)

- ○ Yes, the events A_1, A_2, A_3 constitute a partition of the sample space.
- ◉ No, the events A_1, A_2, A_3 do not constitute a partition of the sample space. ✔

6c. Let $A_4 = (A_1 \cup A_2 \cup A_3)^c$. What is the probability of the event A_4?

0.5625

✔ **Answer:** 0.5625

6d. Do the events A_1, A_2, A_3, A_4 constitute a partition of the sample space? (As always, be sure to justify your answer.)

- ○ Yes, the events A_1, A_2, A_3, A_4 constitute a partition of the sample space.
- ◉ No, the events A_1, A_2, A_3, A_4 do not constitute a partition of the sample space. ✔

Solution

6a. The probability is $P(A_1 \cup A_2 \cup A_3) = 1/4 + 1/4 + 1/4 - 1/8 - 1/8 - 1/8 + 1/16 = 7/16$.
6b. The events A_1, A_2, A_3 do not constitute a partition of the sample space because the probability of their union is only $7/16$, and moreover, the intersections of the events are not empty.
6c. The probability is $P(A_4) = P((A_1 \cup A_2 \cup A_3)^c) = 1 - 7/16 = 9/16$.
6d. The events A_1, A_2, A_3, A_4 do not constitute a partition of the sample space because (even though the probability of their union is 1), the intersections of the events are not empty.

Submit You have used 1 of 1 attempt