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## 7. Worked examples: Concavity in 1 dimension

### Worked Examples: Concavity in 1 dimensions

## Concave and convex functions



### Definition

A function twice differentiable function  $h : \Theta \subset \mathbb{R} \rightarrow \mathbb{R}$  is said to be **concave** if its second derivative satisfies

$$h''(\theta) \leq 0, \quad \forall \theta \in \Theta$$

It is said to be **strictly concave** if the inequality is strict:  $h''(\theta) < 0$

A function  $h$  is said to be (strictly) **convex** if  $h''(\theta) \geq 0$  (strictly  $> 0$ ).

(Caption will be displayed when you start playing the video.)

concave, i.e.  $h''(\theta) \leq 0$  ( $h''(\theta) < 0$ ).

Examples:

- ▶  $\Theta = \mathbb{R}, h(\theta) = -\theta^2,$
- ▶  $\Theta = (0, \infty), h(\theta) = \sqrt{\theta},$
- ▶  $\Theta = (0, \infty), h(\theta) = \log \theta,$
- ▶  $\Theta = [0, \pi], h(\theta) = \sin(\theta)$
- ▶  $\Theta = \mathbb{R}, h(\theta) = 2\theta - 3$

▶ 0:00 / 0:00

▶ 1.50x



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## Review: 1D Optimization via Calculus

4/4 points (graded)

(For this problem, you are welcome to use any computational tools that would be helpful.)

Let  $f(x) = \frac{1}{3}x^3 - x^2 - 3x + 10$  defined on the interval  $[-4, 4]$ .

Let  $x_1$  and  $x_2$  be the critical points of  $f$ , and let's impose that  $x_1 < x_2$ . Fill in the next two boxes with the values of  $x_1$  and  $x_2$ , respectively:  
(Recall that the **critical points** of  $f$  are those  $x \in \mathbb{R}$  such that  $f'(x) = 0$ .)

$x_1 =$   ✓ Answer: -1

$x_2 =$   ✓ Answer: 3

Fill in the next two boxes with the values of  $f''(x_1)$  and  $f''(x_2)$ , respectively:

$f''(x_1) =$   ✓ Answer: -4

$f''(x_2) =$   ✓ Answer: 4

**Solution:**

Observe that

$$f'(x) = x^2 - 2x - 3 = (x - 3)(x + 1).$$

Hence the **critical points** are  $x_1 = -1$  and  $x_2 = 3$ . The **second derivative** is

$$f''(x) = 2x - 2$$

so that

$$f''(x_1) = -4, \quad f''(x_2) = -4.$$

You have used 1 of 3 attempts

**i** Answers are displayed within the problem

## Review: 1D Optimization via Calculus (Continued)

4/4 points (graded)

(For this problem, you are welcome to use any computational tools that would be helpful.)

Recall that  $x_1$  and  $x_2$  are the critical points of the function  $f(x) = \frac{1}{3}x^3 - x^2 - 3x + 10$ .

According to the second derivative test,  $x_1$  is a ...

☒ Local Maximum☐ Local Minimum☐ None of the above

and  $x_2$  is a

☐ Local Maximum☒ Local Minimum

☐ None of the above

At what value of  $x$  is the (global) minimum value of  $f(x)$  attained on the interval  $[-4, 4]$ ?

✓ Answer: -4

At what value of  $x$  the (global) maximum value of  $f(x)$  attained on the interval  $[-4, 4]$ ?

✓ Answer: -1

### Solution:

The previous problem implies that  $f$  is concave at  $x_1$  and convex at  $x_2$ , so  $x_1$  is a **local maximum** and  $x_2$  is a **local minimum**. To figure out the *global* extrema, we need to test the critical points as well as the endpoints:  $-4$  and  $4$ . We compute that

$$f(x_1) = \frac{35}{3} \approx 11.6666, \quad f(x_2) = 1$$

$$f(-4) = -\frac{46}{3} \approx -15.3333, \quad f(4) = 10/3 \approx 3.3333$$

Hence the **maximum value** of  $f$  on  $[-4, 4]$  is  $\frac{35}{3} \approx 11.6666$  and the **minimum value** is  $-\frac{46}{3} \approx -15.3333$ .

**Remark:** It is very important to remember to test the endpoints when doing optimization.

You have used 1 of 2 attempts

**i** Answers are displayed within the problem

## Strict Concavity

1/1 point (graded)

Which of the following functions are strictly concave? (Choose all that apply.) (Recall that a twice-differentiable function  $f : I \rightarrow \mathbb{R}$ , where  $I$  is a subset of  $\mathbb{R}$ , is **strictly concave** if  $f''(x) < 0$  for all  $x \in I$ .)

☐  $f_1(x) = x$  on  $\mathbb{R}$

☒  $f_2(x) = -e^{-x}$  on  $\mathbb{R}$

☒  $f_3(x) = x^{0.99}$  on the interval  $(0, \infty)$

☐  $f_4(x) = x^2$  on  $\mathbb{R}$



### Solution:

- $f_1(x) = x$  is **not** strictly concave because  $f_1''(x) = 0$ .
- $f_2(x) = -e^{-x}$  is strictly concave because  $f_2''(x) = -e^{-x} < 0$  for all  $x \in \mathbb{R}$ .
- $f_3(x) = x^{0.99}$  is strictly concave because  $f_3''(x) = (0.99)(-.01)x^{-1.01} < 0$  for all  $x \in (0, \infty)$ .
- $f_4(x) = x^2$  is **not** strictly concave because  $f_4''(x) = 2 > 0$ . In fact, this function is strictly *convex*.

Submit

You have used 1 of 2 attempts

**i** Answers are displayed within the problem

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Be careful with the sentence Strictly Concave... as I unfortunately made the mistake twice of not realizing about it and put one that is concave but not strictly concave which g...

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