Exercise: Bias and MSE

(2/2 points)

We estimate the unknown mean θ of a random variable X with unit variance by forming the sample mean $M_n = (X_1 + \cdots + X_n)/n$ of n i.i.d. samples X_i and then forming the estimator

$$\widehat{\Theta}_n = \frac{1}{3} \cdot M_n.$$

Your answers below can be functions of θ and n. Follow standard notation and use 'theta' to indicate θ.

The bias $\mathbf{E}[\widehat{\Theta}_n] - \theta$ of this estimator is:

-2*theta/3 ✔

Answer: -2*(theta)/3

The mean squared error of this estimator is:

1/(9*n)+4*theta^2/9 ✔

Answer: 1/(9*n)+4*(theta)^2/9

Answer:
Since $\mathbf{E}[M_n] = \theta$, we have $\mathbf{E}[\widehat{\Theta}_n] = \theta/3$, and the bias is $-2\theta/3$.

The variance of $\widehat{\Theta}_n$ is $1/9$ times the variance of M_n, which is $1/n$. The mean squared error is the sum of the variance and the square of the bias: $1/(9n) + (4\theta^2/9)$.

You have used 1 of 2 submissions