10. Eigenvalues, and trace and determinant

Recall that for any 2×2 matrix $\mathbf{A},$ the characteristic polynomial can also be written in terms of the trace and determinant as follows:

$$\lambda^2 - (\mathbf{tr}\mathbf{A})\lambda + (\mathbf{det}\mathbf{A}).$$

Proof

Show

For an $n \times n$ matrix $\mathbf{A},$ where $n \geq 2,$ it turns out that the characteristic polynomial, multiplied by ± 1 to make the leading coefficient 1, takes the form

$$\lambda^n - (\mathbf{tr}\mathbf{A})\lambda^{n-1} + \cdots \pm \mathbf{det}\mathbf{A}.$$

where the $\pm$ is $+$ if n is even, and $-$ if n is odd. So knowing $\mathbf{tr}\mathbf{A}$ and $\mathbf{det}\mathbf{A}$ determines 2 coefficients of the characteristic polynomial. More importantly, since

$$\lambda^n - (\mathbf{tr}\mathbf{A})\lambda^{n-1} + \cdots \pm \mathbf{det}\mathbf{A} \quad = \quad (\lambda - \lambda_1)(\lambda - \lambda_2)\cdots(\lambda - \lambda_n)$$

where $\lambda_1, \cdots, \lambda_n$ are the n (not necessarily distinct) eigenvalues. Comparing coefficients gives

$$\begin{aligned} \mathbf{tr}(\mathbf{A}) &= \lambda_1 + \lambda_2 + \cdots + \lambda_n \\ \mathbf{det}(\mathbf{A}) &= (\lambda_1)(\lambda_2)\cdots(\lambda_n). \end{aligned}$$

In other words, the trace is the **sum** of all n (not necessarily distinct) eigenvalues; the determinant is the **product** of all n eigenvalues.

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