10.3.2 Orthogonal Spaces

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Week 10 due Dec 16, 2023 07:42 IST Completed

10.3.2 Orthogonal Spaces

10.3.2 Part 1

Start of transcript. Skip to the end.

Dr. Robert van de Geijn: Now that we know the two vectors are orthogonal,

if the dot product between the two vectors is equal to 0,

we can talk about orthogonal spaces.

This is a little bit harder to visualize,

because we have a hard time visualizing spaces.

But hopefully the opener for this week

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Reading Assignment

0 points possible (ungraded)
Read Unit 10.3.2 of the notes. [LINK]

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Sudden change of vector space notation 4

Zero vector

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? Is the zero of size n orthogonal to all other vectors in R superscript n?
Homework 10.3.2.1 seems to imply that this is true. It seems odd because, if true, then that is a property that is unique to the zero vector. I think ... 2

Homework 10.3.2.1

1/1 point (graded)
Let $\mathbf{V} = \{0\}$ where 0 denotes the zero vector of size n. Then $\mathbf{V} \perp \mathbb{R}^n$.

Always ✔ Answer: Always

Let $x \in \mathbf{V}$ and $y \in \mathbb{R}^n$. Then $x = 0$ since that is the only element in set (subspace) $\mathbf{V}$. Hence $x^T y = 0^T y = 0$ and therefore x and y are orthogonal.

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Answers are displayed within the problem

Homework 10.3.2.2

1/1 point (graded)
Let

$$\mathbf{V} = \mathrm{Span}\left(\left\{\begin{pmatrix}1\\0\\0\end{pmatrix}, \begin{pmatrix}0\\1\\0\end{pmatrix}\right\}\right) \quad \text{and} \quad \mathbf{W} = \mathrm{Span}\left(\left\{\begin{pmatrix}0\\0\\1\end{pmatrix}\right\}\right)$$

Then $\mathbf{V} \perp \mathbf{W}$.

TRUE ✔ Answer: TRUE

Let $x \in \mathbf{V}$ and $y \in \mathbf{W}$. Then

$$x = \chi_0 \begin{pmatrix}1\\0\\0\end{pmatrix} + \chi_1 \begin{pmatrix}0\\1\\0\end{pmatrix} = \begin{pmatrix}\chi_0\\\chi_1\\0\end{pmatrix} \quad \text{and} \quad y = \psi_2 \begin{pmatrix}0\\0\\1\end{pmatrix} = \begin{pmatrix}0\\0\\\psi_2\end{pmatrix}.$$

But then $x^T y = \chi_0 \times 0 + \chi_1 \times 0 + 0 \times \psi_2 = 0$. Hence x and y are orthogonal.

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Homework 10.3.2.3

1/1 point (graded)
Let $\mathbf{V}, \mathbf{W} \subset \mathbb{R}^m$ be subspaces. If $\mathbf{V} \perp \mathbf{W}$ then $\mathbf{V} \cap \mathbf{W} = \{0\}$, the zero vector.

Always ✔ Answer: Always

- $\mathbf{V} \cap \mathbf{W} \subset \{0\}$:

  Let $x \in \mathbf{V} \cap \mathbf{W}$. We will show that $x = 0$ and hence $x \in \{0\}$.

$$x \in \mathbf{V} \cap \mathbf{W}$$

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$$\begin{array}{l} \quad \Rightarrow \quad < \text{Definition of } S \cap T > \\ x \in \mathbf{V} \wedge x \in \mathbf{W} \\ \quad \Rightarrow \quad < \mathbf{V} \perp \mathbf{W} > \\ x^T x = 0 \\ \quad \Rightarrow \quad < x^T x = 0 \text{ iff } x = 0 > \\ x = 0 \end{array}$$

- $\{0\} \subset \mathbf{V} \cap \mathbf{W}$:

  Let $x \in \{0\}$. We will show that then $x \in \mathbf{V} \cap \mathbf{W}$.

$$\begin{array}{l} x \in \{0\} \\ \quad \Rightarrow \quad < 0 \text{ is the only element of } \{0\} > \\ x = 0 \\ \quad \Rightarrow \quad < 0 \in \mathbf{V} \text{ and } 0 \in \mathbf{W} > \\ x \in \mathbf{V} \wedge x \in \mathbf{W} \\ \quad \Rightarrow \quad < \text{Definition of } S \cap T > \\ x \in \mathbf{V} \cap \mathbf{W} \end{array}$$

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Homework 10.3.2.4

1/1 point (graded)

If $\mathbf{V} \in \mathbb{R}^m$ is a subspace, then $\mathbf{V}^\perp$ is a subspace.

TRUE ✔ Answer: TRUE

- $0 \in \mathbf{V}^\perp$: Let $x \in \mathbf{V}$. Then $0^T x = 0$ and hence $0 \in \mathbf{V}^\perp$.

- If $x, y \in \mathbf{V}^\perp$ then $x + y \in \mathbf{V}^\perp$: Let $x, y \in \mathbf{V}^\perp$ and let $z \in \mathbf{V}$. We need to show that $(x+y)^T z = 0$.

$$\begin{array}{l} (x+y)^T z \\ \quad = \quad < \text{property of dot} > \\ x^T z + y^T z \\ \quad = \quad < x, y \in \mathbf{V}^\perp \text{ and } z \in \mathbf{V} > \\ 0 + 0 \\ \quad = \quad < \textit{algebra} > \\ 0 \end{array}$$

  Hence $x + y \in \mathbf{V}^\perp$.

- If $\alpha \in \mathbb{R}$ and $x \in \mathbf{V}^\perp$ then $\alpha x \in \mathbf{V}^\perp$: Let $\alpha \in \mathbb{R}$, $x \in \mathbf{V}^\perp$ and let $z \in \mathbf{V}$. We need to show that $(\alpha x)^T z = 0$.

$$\begin{array}{l} (\alpha x)^T z \\ \quad = \quad < \text{algebra} > \\ \alpha x^T z \\ \quad = \quad < x \in \mathbf{V}^\perp \text{ and } z \in \mathbf{V} > \\ \alpha 0 \\ \quad = \quad < \text{algebra} > \end{array}$$

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$$0$$

Hence $\alpha x \in \mathbf{V}^{\perp}$.

Hence $\mathbf{V}^{\perp}$ is a subspace.

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Answers are displayed within the problem

10.3.2 Part 2

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Now let's work through that.

First thing we do is distribute.

And then we recognize that since x and y are in Vperp,

and z is in V, each of these equals 0.

But what that means is that x plus y quantity dot product with z

is also equal to 0 for all vector z in V, and, therefore, x plus y

must be in Vperp.

We're not quite done.

We also need to show that if you take an arbitrary scalar and an arbitrary

vector x in Vperp, then alpha times x is also in Vperp.

Again take an arbitrary vector z in V.

What we need to show is that alpha times x is perpendicular to z.

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