5. Partial Modeling, Regression Function, and Conditional Quantiles

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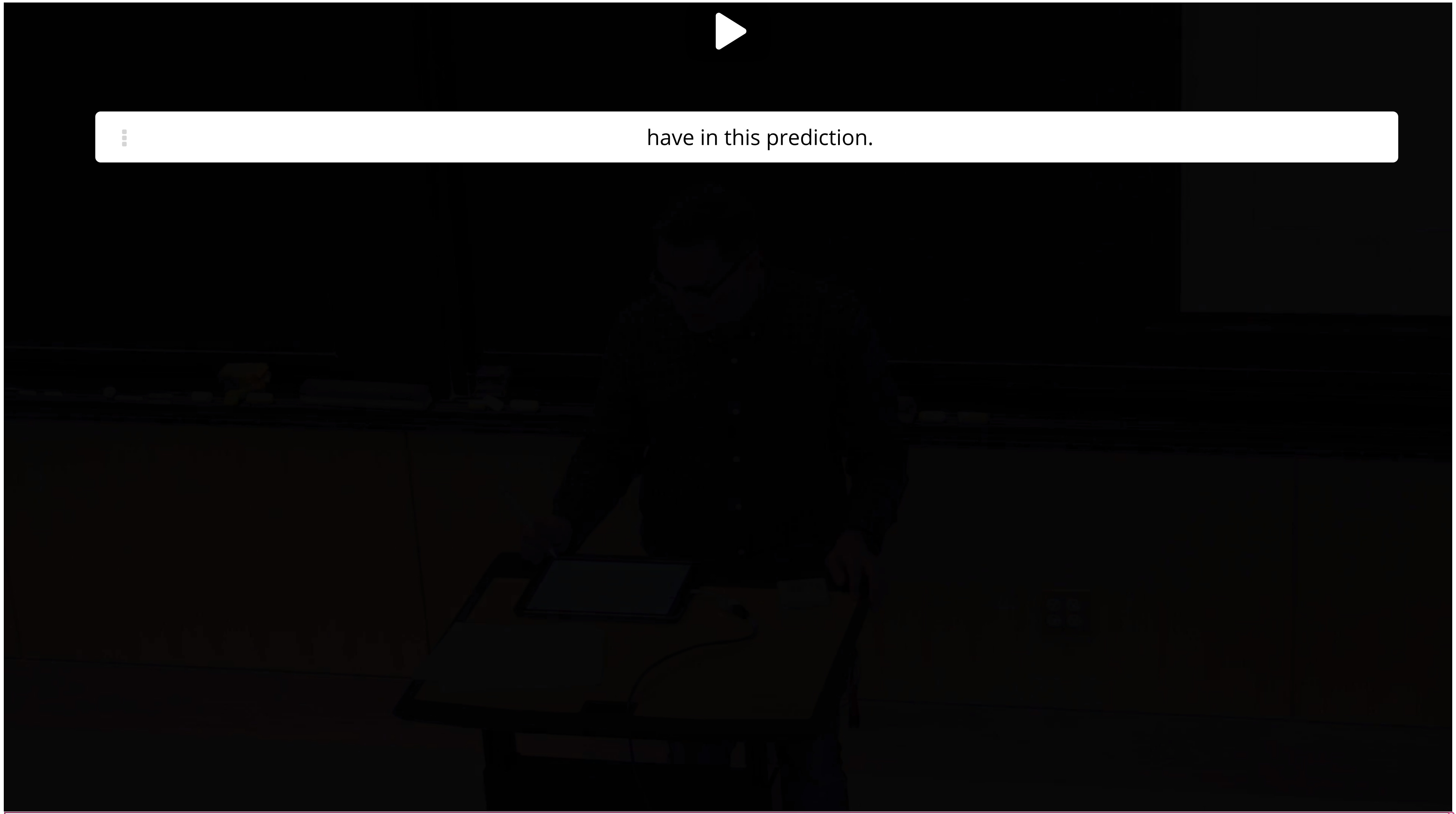

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Given a joint probability distribution $\mathbf{P}$ for the random pair (X, Y), the **regression function** of Y with respect to X is defined as

$$\nu(x) = \mathbb{E}[Y|X=x] = \sum_{\Omega_Y} y \cdot \mathbf{P}(Y=y \mid X=x)$$

which tells us the average value of Y given the knowledge that $X=x$. In the case of continuous distributions where we can compute the conditional density $f(y|x)$, the expression on the right hand side is replaced with an integral:

$$\mathbb{E}[Y|X=x] = \int_{\Omega_Y} yf(y|x)\,dy$$

A Linear Model

1/1 point (graded)
Assume (X,Y) is a pair such that $Y = 3X+5+\varepsilon$ where $\varepsilon \sim \mathcal{N}(0,1)$, independent of X. What is $\mathbb{E}[Y|X=x]$?

3*x+5

✔ **Answer:** 3*x+5

$3 \cdot x + 5$

STANDARD NOTATION

Solution:

From the definition:

$$\begin{aligned}\nu(x) &= \mathbb{E}[Y|X=x] \\ &= \mathbb{E}[3X+5+\varepsilon \mid X=x] \\ &= \mathbb{E}[3x+5+\varepsilon] \\ &= 3x+5+\mathbb{E}[\varepsilon] \qquad \text{(linearity of expectation)} \\ &= 3x+5.\end{aligned}$$

Linear models of the type $Y = a + bX + \varepsilon$ – hence the name **Linear Regression** – are the main focus of this chapter.

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Answers are displayed within the problem

Note: The notions of conditional expectation and conditional variance as random variables, $\mathbb{E}\left[Y|X\right]$ and $\mathsf{Var}\left(Y \mid X\right)$, are not to be confused with the expectation and variance of Y given $X = x$. In this lecture, we are interested in these quantites not as random variables, but as constants for each $X = x$.

Concept Check: Conditional Quantile

1/1 point (graded)

Let (X, Y) be a pair of RVs with joint density $f(x, y) = x + y$, over the sample space $\Omega = [0,1]^2$.

For a given x, what is the value $q_\alpha(x)$ such that $P\left[Y \leq q_\alpha(x) | X = x\right] = 1 - \alpha$? That is, what is the conditional $(1-\alpha)$-quantile function (of x) of $Y|X = x$?

-x+sqrt(x^2+(2*x+1)*(1-ε **Answer:** (1/2)*(-2*x + sqrt(4*x^2 + (1-alpha)*8*(x+1/2)))

$-x + \sqrt{x^2 + (2 \cdot x + 1) \cdot (1 - \alpha)}$

STANDARD NOTATION

Solution:

We know from a previous problem that for this joint distribution on (X, Y), the conditional pdf $h(y|x), 0 \leq x \leq 1$ is given as

$$h(y|x) = \frac{x+y}{x+\frac{1}{2}}, \quad 0 \leq y \leq 1.$$

In order to find the $(1-\alpha)$-quantile value for each x, we need to solve for z (hiding the dependency on x for simplicity) in

$$\int_0^z \frac{x+y}{x+\frac{1}{2}} \mathrm{d}y = (1-\alpha),$$

from which we can obtain that $q_\alpha(x) = z = \frac{1}{2}\left(-2x + \sqrt{4x^2 + 8(1-\alpha)(x+0.5)}\right)$.

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