Problem 3: Splitting students into 3 classes

(2/2 points)

A group of 90 students is to be split at random into 3 classes of equal size. All partitions are equally likely. Joe and Jane are members of the 90-student group. Find the probability that Joe and Jane end up in the same class. **Note:** Your answer should be a number. Do not enter '!' or combinations in your answer.

0.3333333 ✔ **Answer:** 0.32584

Answer:
We label the classes A, B, and C. The probability that Joe and Jane will both be in class A is the number of possible combinations for class A that involve both Joe and Jane, divided by the total number of combinations for class A. There are $\binom{90}{30}$ ways to choose the 30 students for class A. Since we fix Joe and Jane to be in class A, there are 28 remaining spots in class A, to be filled by the remaining 88 students. There are $\binom{88}{28}$ ways of doing this. Therefore, the probability we are after is $\binom{88}{28} / \binom{90}{30}$.

Since there are three classes, the probability that Joe and Jane end up in the same class is simply three times the answer we found above: $3\binom{88}{28}/\binom{90}{30}$, which after cancellations simplifies to $\frac{29}{89}$.

Another way of looking at the problem is as follows. We think of the three classes as consisting of a total of 90 places, 30 places for each class. Suppose that one of the students, say Joe, first picks one of the available 90 places, with each place being equally likely to be picked. Then, Jane picks one of the remaining 89 places, with each place being equally likely. Regardless of which place Joe picked, there are exactly 29 places that would put Jane in the same class as Joe. Therefore, Jane has a probability of $29/89$ of picking a place in the same class as Joe.

You have used 2 of 2 submissions

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