MITx: 6.041x Introduction to Probability - The Science of Uncertainty

Exercise: Independence and variances

(3/3 points)

The pair of random variables (X, Y) is equally likely to take any of the four pairs of values $(0,1)$, $(1,0)$, $(-1,0)$, $(0,-1)$. Note that X and Y each have zero mean.

a) Find $\mathbf{E}[XY]$.

$\mathbf{E}[XY] =$ 0 ✔ **Answer:** 0

b) For this pair of random variables (X, Y), is it true that $\mathrm{var}(X+Y) = \mathrm{var}(X) + \mathrm{var}(Y)$?

Yes ✔ **Answer:** Yes

c) We know that if X and Y are independent, then $\mathrm{var}(X+Y) = \mathrm{var}(X) + \mathrm{var}(Y)$. Is the converse true? That is, does the condition $\mathrm{var}(X+Y) = \mathrm{var}(X) + \mathrm{var}(Y)$ imply independence?

No ✔ **Answer:** No

Answer:
a) At each possible outcome, we have $XY = 0$, and therefore $\mathbf{E}[XY] = 0$.

b) Since the random variables have zero mean, $\mathbf{E}[X+Y] = 0$, $\mathrm{var}(X) = \mathbf{E}[X^2]$, and $\mathrm{var}(Y) = \mathbf{E}[Y^2]$. Combining this with the result from part (a), we conclude that

$$\begin{aligned}\mathrm{var}(X+Y) &= \mathbf{E}[(X+Y)^2] - (\mathbf{E}[X+Y])^2\\ &= \mathbf{E}[(X+Y)^2]\\ &= \mathbf{E}[X^2] + 2\mathbf{E}[XY] + \mathbf{E}[Y^2]\\ &= \mathbf{E}[X^2] + \mathbf{E}[Y^2]\\ &= \mathrm{var}(X) + \mathrm{var}(Y).\end{aligned}$$

c) We have here an example of two random variables that satisfy the condition $\mathrm{var}(X+Y)=\mathrm{var}(X)+\mathrm{var}(Y)$. But these random variables are not independent. For example, the information that $X=1$ tells us that the value of Y must be zero.

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