DelftX: OT.1x Observation theory: Estimating the Unknown Help

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Module 4 Assessment - Part 1

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Distribution of estimated residuals

1/1 point (graded)

Assume normally distributed observables in $\underline{y}$. For the model $\mathrm{E}\{\underline{y}\} = Ax$ and $\mathrm{D}\{\underline{y}\} = Q_{yy}$, the estimator of the residual vector (i.e., $\underline{\hat{e}} = \underline{y} - A\underline{\hat{x}}_{\mathrm{BLU}}$) is always normally distributed.

Is this statement true or false?

- (x) True ✔
- ( ) False

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✔ Correct (1/1 point)

Finding the BLUE

1.0/1.0 point (graded)

For the model $\mathrm{E}\{\underline{y}\} = A\,x$ and $\mathrm{D}\{\underline{y}\} = Q_{yy} = \sigma^2 I$, where I is the identity matrix, the best linear unbiased estimator is given as:

$$\underline{\hat{x}}_{\mathrm{BLU}} = (A^{\mathrm{T}}A)^{-1}A^{\mathrm{T}}\underline{y}$$

Is this statement true or false?

- (selected) True ✔
- False

Feedback

$$\underline{\hat{x}}_{\mathrm{BLU}} = (A^{\mathrm{T}}Q_{yy}^{-1}A)^{-1}Q_{yy}^{-1}A^{\mathrm{T}}\underline{y} = (A^{\mathrm{T}}(\tfrac{1}{\sigma^2}IA)^{-1})A^{\mathrm{T}}(\tfrac{1}{\sigma^2}I)\underline{y} = \sigma^2(A^{\mathrm{T}}A)^{-1}A^{\mathrm{T}}(\tfrac{1}{\sigma^2}I)\underline{y} = (A^{\mathrm{T}}A)^{-1}A^{\mathrm{T}}$$

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Theory on weighted LS estimators

1.0/1.0 point (graded)

Assuming the linear model $\mathrm{E}\{\underline{y}\} = Ax$ is correct, a weighted least squares estimator of x is always unbiased, regardless of the distribution of the observations and the chosen weight matrix.

Is this statement true or false?

- (selected) True ✔

False

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Theory on weighted LS estimators (continued)

1/1 point (graded)

For the model $\mathrm{E}\{\underline{y}\} = A\,x$, the expectation of the least-squares estimation error is always zero (i.e. $\mathrm{E}\{\underline{\hat{x}} - x\} = 0$).

Is this statement true or false?

True ✔

False

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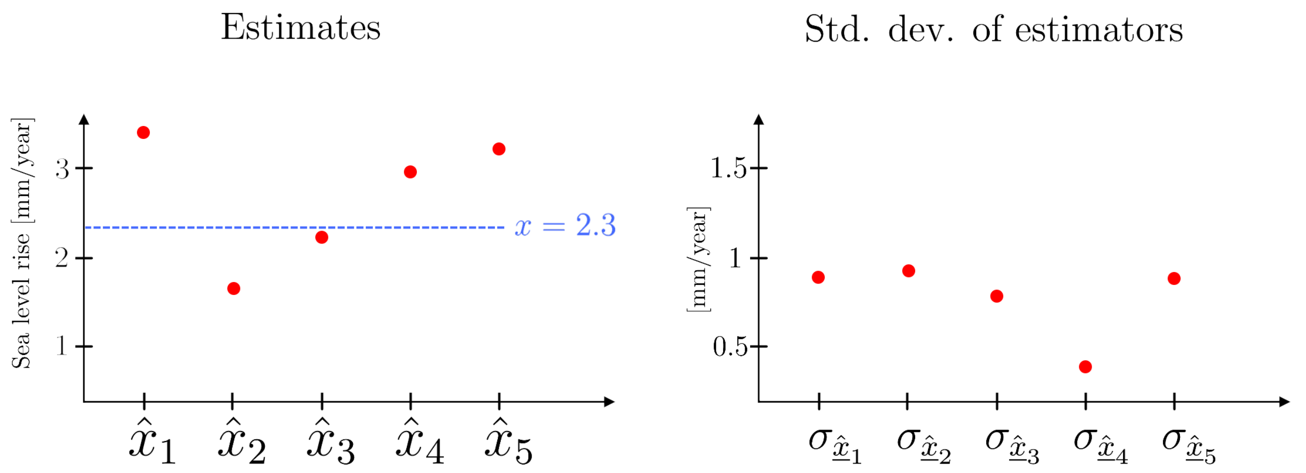
✔ Correct (1/1 point)

Choosing the best estimator

2.0/2.0 points (graded)

Let us consider the sea level rise rate, denoted by $\underline{x}$, somewhere along the Dutch coast. Assume we have the observations over 50 years and we want a good estimation of $\underline{x}$.

Based on the same data, five different companies provide five different estimates of $\underline{x}$ (shown in the left figure). These five diferent estimates are the results of application of the WLS estimation with five different weight matrices. In the right figure, we see the standard deviations of the associated estimators of these five estimates. In reality, we never know the real/true value of the sea level rise. But for this hypothetical question, let's assume that the true value is 2.3 mm/year (indicated with the blue dashed line on the left figure).



Among these five estimates which one do you think is the best, based on how we have defined the "best" estimator in this module?

- $\hat{\underline{x}}_1$
- $\hat{\underline{x}}_2$

- ○ $\hat{\underline{x}}_3$
- ◉ $\hat{\underline{x}}_4$ ✔
- ○ $\hat{\underline{x}}_5$

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BLUE

4.0/4.0 points (graded)

An object is moving along a straight line with constant but unknown speed v. It started at the origin $y = 0$ at $t = 0$. Uncorrelated observations y_i of the object's distance from the origin $y = 0$ have been made at corresponding time instants $t_i = i$ seconds ($i = 1, 2, \ldots, m$). The precision of the observations is given by $\sigma_{y_i} = i$.

What is the BLUE of v ?

- ○ $\hat{v} = \frac{1}{m} \sum_i^m y_i$
- ◉ $\hat{v} = \frac{1}{m} \sum_i^m \frac{y_i}{i}$ ✔
- ○ $\hat{v} = \frac{1}{m} \sum_i^m \frac{y_i}{i^2}$

- The information in the question is not sufficient.

Feedback

$\hat{v} = (A^{\mathrm{T}} Q_{yy}^{-1} A)^{-1} Q_{yy}^{-1} A^{\mathrm{T}} \underline{y} =$

$$\left([1\ 2 \ldots\ m]^{\mathrm{T}} \begin{bmatrix} 1 & & & \\ & 4 & & \\ & & \ddots & \\ & & & m^2 \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 2 \\ \vdots \\ m \end{bmatrix} \right)^{-1} \left([1\ 2 \ldots\ m]^{\mathrm{T}} \begin{bmatrix} 1 & & & \\ & 4 & & \\ & & \ddots & \\ & & & m^2 \end{bmatrix}^{-1} \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_m \end{bmatrix} \right) =$$

$$\left((\textstyle\sum_i^m 1)^{-1} \right) \left([\tfrac{1}{1}\ \ \tfrac{1}{2}\ \ \cdots\ \ \tfrac{1}{m}] \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_m \end{bmatrix} \right) = \tfrac{1}{m} \textstyle\sum_i^m \tfrac{y_i}{i}$$

Submit

You have used 1 of 2 attempts

Biased or unbiased property

1.0/1.0 point (graded)

Assume the linear model $\mathrm{E}\{\underline{y}\} = Ax$, where $\underline{y}$ is the vector of observables for different measurments of the height of a building, and x is the true value of the bulding height. The observations have been taken with different techniques and with different precision. The ordinary least squares estimate of x has been given as

$$\hat{x} = (A^T A)^{-1} A^T y = 54\mathrm{m}$$

What is the expectation of the estimation error $\underline{\epsilon} = \underline{\hat{x}} - x$?

- (selected) 0 ✔
- 54
- Unknown

Submit You have used 1 of 1 attempt

BLUE of an angle

2.0/2.0 points (graded)

The angle α has been measured with three different instruments. The observations are normally distributed. The measurements and their associated standard deviation have been given as

$$y_1 = 45.3^\circ, \quad \sigma_{y_1} = 0.5^\circ$$

$$y_2 = 45.5^\circ, \quad \sigma_{y_2} = 0.01^\circ$$

$$y_3 = 44.2^\circ, \quad \sigma_{y_3} = 1.0^\circ$$

Give the most precise estimate of the angle α (upto 2 decimal places).

45.48 ✔ **Answer:** 45.5

45.48

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