MITx: 6.041x Introduction to Probability - The Science of Uncertainty

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Problem 5: Arrivals during overlapping time intervals

(8/8 points)

Consider a Poisson process with rate λ. Let N be the number of arrivals in $(0, t]$ and M be the number of arrivals in $(0, t+s]$, where $t > 0, s \geq 0$.

In each part below, your answers will be algebraic expressions in terms of λ, t, s, m and/or n. Enter 'lambda' for λ and use 'exp()' for exponentials. Do **not** use 'fac()' or '!' for factorials. Follow standard notation .

1. For $0 \leq n \leq m$, the conditional PMF $p_{M|N}(m \mid n)$ of M given N is of the form $\frac{a}{b!}$ for suitable algebraic expressions in place of a and b.

   $a =$ (lambda*s)^(m-n)*exp(-lambda*s)

   Answer: lambda^(m-n)*s^(m-n)*exp(-lambda*s)

   $b =$ m-n **Answer:** m-n

2.

For $0 \leq n \leq m$, the joint PMF $p_{N,M}(n,m)$ of N and M is of the form $\frac{c}{n!d!}$ for suitable algebraic expressions in place of c and d.

$c =$ lambda^m*t^n*s^(m-n)*exp(-lambda*(s+t)) ✔

Answer: lambda^m*s^(m-n)*t^n*exp(-lambda*(t+s))

$d =$ m-n ✔ **Answer:** m-n

3. For $0 \leq n \leq m$, the conditional PMF $p_{N|M}(n|m)$ of N given M is of the form $f \cdot \frac{g!}{n!h!}$ for suitable algebraic expressions in place of f, g, and h.

$f =$ t^n*s^(m-n)/(t+s)^m ✔ **Answer:** s^(m-n)*t^n/(t+s)^m

$g =$ m ✔ **Answer:** m

$h =$ m-n ✔ **Answer:** m-n

4.

$E[NM] =$ lambda*t + lambda^2*t*s + lambda^2*t^2 ✔

Answer: lambda*t*lambda*s+lambda*t+(lambda*t)^2

Answer:

Solved problems

Additional theoretical material

Problem Set 9
Problem Set 9 due May 11, 2016 at 23:59 UTC

Unit summary

- Unit 10: Markov chains
- Exit Survey

1. We are given that there are n arrivals in the first t time units, so we are looking for the probability that there are $m-n$ arrivals in the subsequent s time units, which follows a Poisson distribution:

$$p_{M|N}(m \mid n) = \frac{(\lambda s)^{m-n} e^{-\lambda s}}{(m-n)!}, \quad \text{for } m \geq n \geq 0.$$

2.

$$\begin{aligned} p_{N,M}(n,m) &= p_{M|N}(m \mid n) p_N(n) \\ &= \left(\frac{(\lambda s)^{m-n} e^{-\lambda s}}{(m-n)!}\right)\left(e^{-\lambda t}\frac{(\lambda t)^n}{n!}\right) \\ &= \frac{\lambda^m s^{m-n} t^n e^{-\lambda(s+t)}}{(m-n)!n!}, \quad \text{for } m \geq n \geq 0. \end{aligned}$$

3.

$$\begin{aligned} p_{N|M}(n \mid m) &= \frac{p_{N,M}(n,m)}{p_M(m)} \\ &= \frac{p_{N,M}(n,m)}{e^{-\lambda(t+s)}(\lambda(t+s))^m} m! \\ &= \frac{m!}{(m-n)!n!}\frac{s^{m-n}t^n}{(s+t)^m}, \quad \text{for } m \geq n \geq 0. \end{aligned}$$

4. We can rewrite the expectation as

$$\begin{aligned}
\mathbf{E}[NM] &= \mathbf{E}[N(M-N)+N^2] \\
&= \mathbf{E}[N]\mathbf{E}[M-N]+\mathbf{E}[N^2] \\
&= (\lambda t)(\lambda s)+\left(\mathbf{var}(N)+(\mathbf{E}[N])^2\right) \\
&= (\lambda t)(\lambda s)+\lambda t+(\lambda t)^2,
\end{aligned}$$

where the second equality is obtained because of the independence of the number of arrivals, N and $M-N$, during disjoint time intervals.

You have used 2 of 3 submissions

DISCUSSION

Click “Show Discussion" below to see discussions on this problem.