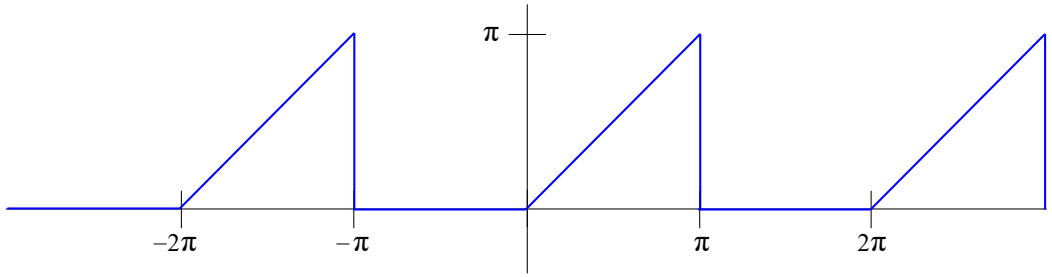
11. A worked example

Find the Fourier series of the 2π-periodic function

$$f(t) = \begin{cases} t & 0 < t < \pi \\ 0 & -\pi < t < 0 \end{cases}.$$

Solution: In this example, we will work out the terms a_n and leave it to you to find the coefficients b_n.

First, draw a picture.



Next, find the constant term:

$$\frac{a_0}{2} = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(t)\ dt = \frac{\text{area under one period of curve}}{\text{length of period}} = \frac{\pi^2/2}{2\pi} = \frac{\pi}{4}.$$

Now for the other terms a_n:

$$\begin{aligned} a_n &= \frac{1}{\pi}\int_{-\pi}^{\pi} f(t)\cos(nt)\ dt \\ &= \frac{1}{\pi}\int_{-\pi}^{0} 0\cos(nt)\ dt + \frac{1}{\pi}\int_{0}^{\pi} t\cos(nt)\ dt \\ &= \frac{1}{\pi}\int_{0}^{\pi} t\cos(nt)\ dt \end{aligned}$$

Evaluating this integral, we find

$$\frac{1}{\pi}\int_{0}^{\pi} t\cos(nt)\ dt = \frac{1}{\pi}\frac{\cos(n\pi)-1}{n^2} = \frac{1}{\pi}\frac{(-1)^n-1}{n^2} = \begin{cases} \frac{-2}{\pi n^2} & n \text{ odd} \\ 0 & n \text{ even} \end{cases}.$$

Therefore the Fourier series is given by

$$f(t) = \frac{\pi}{4} + \sum_{n \text{ odd}} \frac{-2}{\pi n^2} \cos(nt) + \sum_{n} b_n \sin(nt).$$

We leave the computation of the b_n as an exercise for you.

Finish the example

1/1 point (graded)

Find the coefficients b_n of the Fourier series for the 2π-periodic function

$$f(t) = \begin{cases} t & 0 < t < \pi \\ 0 & -\pi < t < 0 \end{cases}.$$

(Find a formula that holds for both n even and n odd.)

$b_n =$ -(-1)^n/n ✔ **Answer:** (-1)^(n+1)/n

Solution:

We can use computer assistance or integration by parts to see that

$$b_n = \frac{1}{\pi} \int_0^{\pi} t \sin(nt)\, dt = \frac{-\cos(n\pi)}{n} = \frac{(-1)^{n+1}}{n}.$$

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Answers are displayed within the problem

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