MITx: 15.053x Optimization Methods in Business Analytics

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Lecture questions due Sep 27, 2016 at 19:30 IST

Recitation

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Homework 3 due Sep 27, 2016 at 19:30 IST

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PART A

(1/1 point)
Integer Programming Formulation, eHarmony

eHarmony is an online dating site focused on long term relationships. It takes a scientific approach to love and marriage. About nearly 4% of US marriages in 2012 are a result of eHarmony. The company has generated over $1 billion in cumulative revenue from 2000, the year it was founded. Unlike other online dating websites, eHarmony does not have users browse others' profiles. Instead, eHarmony computes a compatibility score between two people and uses optimization algorithms to determine their users' best matches. In this problem, we are going to see how eHamony uses integer programming to find good matches. The compatibility scores in the table below indicates how compatible a match is. A higher number indicates that the compatibility is greater.

Table 1: Compatibility Scores

	Woman 1	**Woman 2**	**Woman 3**
Man 1	1	3	5
Man 2	4	2	2

Man 3	1	5	3

Decision variables:

- $x_{ij} \in \{0,1\}$, indicator variable. $x_{ij} = 1$ is man i is matched with woman j; 0 if man i is not matched with woman j

Objective Function:

- $x_{11} + 3x_{12} + 5x_{13} + 4x_{21} + 2x_{22} + 2x_{23} + x_{31} + 5x_{32} + 3x_{33}$

Should the objective function be maximized or minimized?

● MAX ✓

○ MIN

You have used 1 of 3 submissions

PART B

(1/1 point)

Which of the following constraints corresponds to the constraint "Each man should be matched to exactly one woman"? There are three constraints that should be selected.

Check all that apply

☑ $x_{11} + x_{12} + x_{13} = 1.$

☑ $x_{21} + x_{22} + x_{23} = 1$

☑ $x_{31} + x_{32} + x_{33} = 1$

☐ $x_{11} + x_{21} + x_{31} = 1$

☐ $x_{12} + x_{22} + x_{32} = 1$

☐ $x_{13} + x_{23} + x_{33} = 1$

✓

You have used 1 of 3 submissions

PART C

(1/1 point)

What is the female assignment constraint "Each woman should be matched to exactly one man"?

Check all that apply

☐ $x_{11} + x_{12} + x_{13} = 1$

☐ $x_{21} + x_{22} + x_{23} = 1$

☐ $x_{31} + x_{32} + x_{33} = 1$

☑ $x_{11} + x_{21} + x_{31} = 1$

☑ $x_{12} + x_{22} + x_{32} = 1$

☑ $x_{13} + x_{23} + x_{33} = 1$

✓

You have used 1 of 3 submissions

PART D

(1/1 point)

Solve the integer program above. What is the optimal objective function value?

14 ✓

14

You have used 1 of 3 submissions

PART D II

(1/1 point)

Now solve the problem as a linear program by relaxing the integrality/binary constraints. What is the optimal objective function value now?

14 ✓

14

You have used 1 of 3 submissions

PART E

(1/1 point)

Consider the following two logic constraints eHarmony wants to impose

- If man 1 matches woman 3, then man 2 must match woman 2
- If man 3 matches woman 3, then man 2 must match woman 2

Write two more constraints on top of part (a) to achieve these two logic requirements

● $x_{13} \leq x_{22}$ ✓
$x_{33} \leq x_{22}$

○ $x_{13} \geq x_{22}$
$x_{33} \geq x_{22}$

○ $x_{13} \geq x_{22}$
$x_{33} \leq x_{22}$

○ $x_{13} \leq x_{22}$
$x_{33} \geq x_{22}$

○ $x_{13} = x_{22}$
$x_{33} = x_{22}$

You have used 1 of 3 submissions

PART F

(1/1 point)

Solve the IP, and include the constraints from part E. What is the optimal objective function value?

8 ✓

8

You have used 1 of 3 submissions

PART F II

(1/1 point)

Solve the linear problem, and include the constraints from part E. Remove the integrality constraints. What is the optimal objective function value?

10 ✓

10

You have used 1 of 3 submissions

PART G

(1/1 point)

Suppose we have n men and n women. Denote c_{ij} as the compatibility score between man i and woman j. Define appropriate decision variables and write an algebraic integer programming formulation to match pairs that maximizes the total compatibility. Each man (woman) should be matched to exactly one woman (man).

Decision variables:

- $x_{ij} \in \{0, 1\}$, indicator variable. $x_{ij} = 1$ is man i is matched with woman j; 0 if man i is not matched with woman j

Formulation:

$$\left.\begin{array}{lr}
\textbf{min} & \sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij}x_{ij} \\
\textbf{s.t.:} & \\
(1) & \sum_{j=1}^{n} x_{ij} = 1, \forall i = 1,\ldots,n \\
(2) & \sum_{i=1}^{n} x_{ij} = 1, \forall j = 1,\ldots,n \\
(3) & x_{ij} \in \{0,1\}, \forall i,j = 1,\ldots,n
\end{array}\right\}$$

You are concerned that some of the inequalities have been reversed. Which of the following is the best response? Select the best answer.

- ☐ The linear program is correct
- ☑ The objective should be maximized
- ☐ Constraints (1), (2) should have $\leq$ inequalities
- ☐ Constraints (1), (2) should have $\geq$ inequalities
- ☐ The objective should be maximized and constraints (1), (2) should have $\leq$ inequalities
- ☐ The objective should be maximized and constraints (1), (2) should have $\geq$ inequalities

✓

You have used 1 of 3 submissions

PART H. This part is optional.

This problem and the following are optional. If you are interested in using Julia and JuMP in practice, we encourage you to try it out. This exercise and the following one are similar to PART D and PART D II. Instead of solving a problem using a 3 x 3 matrix, this problem includes a 1000 x 1000 matrix. If you solve the problem, you can honestly say that you have solved a million variable integer program.

Till now, eHarmony has 33 million members. Spreadsheet optimization is too cumbersome to use for day-to-day operations. Instead they need to rely on "modeling languages." In this part, we are going to use Julia/JuMP to solve a large-scale matching problem for eHarmony. In the material of problem set 3, you will find a csv (comma-separated values) file compatibility_score_integers.csv, which represents compatibility score among 1000 men and 1000 women. The numbers in "compatibility_score.csv" constitutes a 1000 x 1000 matrix, where each row corresponds to a man, each column corresponds to a woman. Your task here is to solve this 1000 x 1000 matching problem based on the integer programming formulation in PART G using Julia/JuMP. The "readcsv" function in Julia will be helpful. We also provide a smaller data set 20 x 20 compatibility_score_debug_integers.csv in the problem set materials. You may debug your model with this smaller data set first before try the 1000 x 1000 version. Any bug in the 1000 x 1000 version could possibly take an absurdly long time to fix. (If you have correctly solved the 1000 x 1000 instance of this problem, then you have solved your first linear (integer) program with 1 million variables.)

What is the optimal (integral) objective value? Error checking hint: the optimal value is between 99970 and 99989.

99984 ✓

99984

You have used 1 of 5 submissions

PART I. This part is optional.

Solve the problem from PART H without integral constraints (i.e. replacing $x_{ij} \in \{0, 1\}$ with $0 \leq x_{ij} \leq 1, \forall i, j = 1, \ldots, n$) using data in "compatibility_score_integers.csv". What is the optimal objective value in the output? HINT: It should be the same as for the integer program.

99984 ✓

99984

You have used 1 of 10 submissions