11.2.1 Component in the Direction of ...

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Week 11 due Dec 22, 2023 21:12 IST Completed

11.2.1 Component in the Direction of ...

Video

Start of transcript. Skip to the end.

Dr. Robert van de Geijn: We're now going to look

at the mathematics behind the opener for this week,

and a lot of what we're going to look at is actually

a review of the end of last week when we talked about projection onto the column

0:00 / 0:00 2.0x

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Reading Assignment

0 points possible (ungraded)

Read Unit 11.2.1 of the notes. [LINK]

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HW 11.2.1.4 Q5

Hello. I saw your explanation 2 for homework 11.2.1.4 Q5, but I failed to figure out how [a(a^(T)a)^(-1)a^(T)][a(a^(T)a)^(-1)a^(T)] equals to [a(a^(T...

3

Homework 11.2.1.3

There's no explanation given for why those are the costs. Could someone explain?

Calculator

Meaning of the summary 2

11.2.1.2 how did we get the result 6

Homework 11.2.1.1

8/8 points (graded)

Let $a = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $P_a(x)$ and $P_a^{\perp}(x)$ be the projection of vector x onto $\mathrm{Span}(\{a\})$ and $\mathrm{Span}(\{a\})^{\perp}$, respectively.

Compute

Preparation: $(a^T a)^{-1} = (e_0^T e_0)^{-1} = 1^{-1} = 1$. So, $a(a^T a)^{-1} a^T = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix}^T$.

1. $P_a\left(\begin{pmatrix} 2 \\ 0 \end{pmatrix}\right) =$

   2 ✓ Answer: 2

   0 ✓ Answer: 0

   $$P_a\left(\begin{pmatrix} 2 \\ 0 \end{pmatrix}\right) = a(a^T a)^{-1} a^T x = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix}^T \begin{pmatrix} 2 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} 2 = \begin{pmatrix} 2 \\ 0 \end{pmatrix}.$$

   Now, you could have figured this out more simply: The vector x, in this case, is clearly just a multiple of vector a, and hence its projection onto the span of a is just the vector x itself.

2. $P_a^{\perp}\left(\begin{pmatrix} 2 \\ 0 \end{pmatrix}\right) =$

   0 ✓ Answer: 0

   0 ✓ Answer: 0

   $$P_a^{\perp}\left(\begin{pmatrix} 2 \\ 0 \end{pmatrix}\right) = \begin{pmatrix} 0 \\ 0 \end{pmatrix}.$$

   Since x is a multiple of a, the component of x perpendicular to a is clearly 0, the zero vector. Alternatively, can compute $x - P_a(x)$ using the result you computed for $P_a(x)$. Alternatively, you can compute $(I - a(a^T a)^{-1} a^T)\, x$.

3. $P_a\left(\begin{pmatrix} 4 \\ 2 \end{pmatrix}\right) =$

   4 ✓ Answer: 4

   0 ✓ Answer: 0

   $$P_a\left(\begin{pmatrix} 4 \\ 2 \end{pmatrix}\right) = \begin{pmatrix} 4 \\ 0 \end{pmatrix}.$$

4. $P_a^{\perp}\left(\begin{pmatrix} 4 \\ 2 \end{pmatrix}\right) =$

   0 ✓ Answer: 0

   2 ✓ Answer: 2

   $$P_a^{\perp}\left(\begin{pmatrix} 4 \\ 2 \end{pmatrix}\right) = \begin{pmatrix} 0 \\ 2 \end{pmatrix}.$$

Calculator

Submit

Answers are displayed within the problem

Homework 11.2.1.2

12/12 points (graded)

Let $a = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$ and $P_a(x)$ and $P_a^{\perp}(x)$ be the projection of vector x onto $\mathrm{Span}(\{a\})$ and $\mathrm{Span}(\{a\})^{\perp}$, respectively.

Compute

Preparation: $(a^T a)^{-1} = 2^{-1} = 1/2$. So, $a(a^T a)^{-1} a^T = \frac{1}{2} \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}^T$.

$P_a\left(\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}\right) =$

1/2 ✔ Answer: .5

1/2 ✔ Answer: .5

0 ✔ Answer: 0

1.

$$P_a\left(\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}\right) = a(a^T a)^{-1} a^T x = \frac{1}{2} \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}^T \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} (1) = \begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \\ 0 \end{pmatrix}.$$

Notice that we did not actually form $a(a^T a)^{-1} a^T$. Let's see if we had:

$$\frac{1}{2}\left(\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}^T\right)\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}\begin{pmatrix} 1 & 1 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$$

$$= \frac{1}{2}\begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \\ 0 \end{pmatrix}$$

That is a LOT more work!!!

-1/2 ✔ Answer: -.5

Calculator

$P_a^\perp\left(\begin{pmatrix}0\\1\\1\end{pmatrix}\right) =$ 1/2 ✔ Answer: .5 | 1 ✔ Answer: 1

2.

$$P_a^\perp\left(\begin{pmatrix}0\\1\\1\end{pmatrix}\right) = \begin{pmatrix}0\\1\\1\end{pmatrix} - \begin{pmatrix}\frac{1}{2}\\\frac{1}{2}\\0\end{pmatrix} = \begin{pmatrix}-\frac{1}{2}\\\frac{1}{2}\\1\end{pmatrix}$$

This time, had we formed $I - a(a^Ta)^{-1}a^T$, the work would have been even more.

$P_a\left(\begin{pmatrix}0\\0\\1\end{pmatrix}\right) =$ 0 ✔ Answer: 0 | 0 ✔ Answer: 0 | 0 ✔ Answer: 0

3.

$$P_a\left(\begin{pmatrix}0\\0\\1\end{pmatrix}\right) = \begin{pmatrix}0\\0\\0\end{pmatrix}.$$

$P_a^\perp\left(\begin{pmatrix}0\\0\\1\end{pmatrix}\right) =$ 0 ✔ Answer: 0 | 0 ✔ Answer: 0 | 1 ✔ Answer: 1

4.

$$P_a^\perp\left(\begin{pmatrix}0\\0\\1\end{pmatrix}\right) = \begin{pmatrix}0\\0\\1\end{pmatrix}.$$

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Answers are displayed within the problem

Homework 11.2.1.3

2/2 points (graded)

Let $a, v, b \in \mathbb{R}^m$.

1. What is the approximate cost of computing $(av^T)\,b$, obeying the order indicated by the parentheses?

- [x] $3m^2$
- [ ] $3m$
- [ ] $2m^2 + 4m$

✔

2. What is the approximate cost of computing $(v^Tb)\,a$, obeying the order indicated by the parentheses?

Calculator

- ○ $m^2 + 2m$
- ◉ $3m$
- ○ $2m^2 + 4m$

✔

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Answers are displayed within the problem

Homework 11.2.1.4

6/6 points (graded)

Given $a, x \in \mathbb{R}^m$, let $P_a(x)$ and $P_a^{\perp}(x)$ be the projection of vector x onto $\text{Span}(\{a\})$ and $\text{Span}(\{a\})^{\perp}$, respectively. Then which of the following are true:

1. $P_a(a) = a$.

   TRUE ✔ Answer: TRUE

   - **Explanation 1:** a is in $\text{Span}(\{a\})$, hence it is its own projection.
   - **Explanation 2:** $\left(a(a^T a)^{-1} a^T\right) a = a(a^T a)^{-1}(a^T a) = a$.

2. $P_a(\chi a) = \chi a$.

   TRUE ✔ Answer: TRUE

   - **Explanation 1:** χa is in $\text{Span}(\{a\})$, hence it is its own projection.
   - **Explanation 2:** $\left(a(a^T a)^{-1} a^T\right) \chi a = \chi a(a^T a)^{-1}(a^T a) = \chi a$.

3. $P_a^{\perp}(\chi a) = 0$ (the zero vector).

   TRUE ✔ Answer: TRUE

   - **Explanation 1:** χa is in $\text{Span}(\{a\})$, and has no component orthogonal to a.
   - **Explanation 2:**

$$\begin{aligned} P_a^{\perp}(\chi a) &= \left(I - a(a^T a)^{-1} a^T\right) \chi a = \chi a - a(a^T a)^{-1} a^T \chi a \\ &= \chi a - \chi a(a^T a)^{-1} a^T a = \chi a - \chi a = 0. \end{aligned}$$

4. $P_a(P_a(x)) = P_a(x)$.

   TRUE ✔ Answer: TRUE

   - **Explanation 1:** $P_a(x)$ is in $\text{Span}(\{a\})$, hence it is its own projection.
   - **Explanation 2:**

Calculator

$$\begin{aligned} P_a(P_a(x)) &= (a(a^T a)^{-1} a^T)(a(a^T a)^{-1} a^T)x \\ &= a(a^T a)^{-1}(a^T a)(a^T a)^{-1} a^T)x = a(a^T a)^{-1} a^T)x = P_a(x). \end{aligned}$$

5. $P_a^\perp(P_a^\perp(x)) = P_a^\perp(x)$.

TRUE ✔ Answer: TRUE

- **Explanation 1:** $P_a^\perp(x)$ is in $\mathrm{Span}(\{a\})^\perp$, hence it is its own projection.
- **Explanation 2:**

$$\begin{aligned} P_a^\perp(P_a^\perp(x)) &= (I - a(a^T a)^{-1} a^T)(I - a(a^T a)^{-1} a^T)x \\ &= (I - a(a^T a)^{-1} a^T - (I - a(a^T a)^{-1} a^T)a(a^T a)^{-1} a^T)x \\ &= (I - a(a^T a)^{-1} a^T - a(a^T a)^{-1} a^T + a(a^T a)^{-1} a^T a(a^T a)^{-1} a^T)x \\ &= (I - a(a^T a)^{-1} a^T - a(a^T a)^{-1} a^T + a(a^T a)^{-1} a^T)x \\ &= (I - a(a^T a)^{-1} a^T)x = P_a^\perp(x). \end{aligned}$$

6. $P_a(P_a^\perp(x)) = 0$ (the zero vector).

TRUE ✔ Answer: TRUE

- **Explanation 1:** $P_a^\perp(x)$ is in $\mathrm{Span}(\{a\})^\perp$, hence orthogonal to $\mathrm{Span}(\{a\})$.
- **Explanation 2:**

$$\begin{aligned} P_a(P_a^\perp(x)) &= (a(a^T a)^{-1} a^T)(I - a(a^T a)^{-1} a^T)x \\ &= (a(a^T a)^{-1} a^T - a(a^T a)^{-1} a^T a(a^T a)^{-1} a^T)x \\ &= (a(a^T a)^{-1} a^T - a(a^T a)^{-1} a^T)x = 0. \end{aligned}$$

(Hint: Draw yourself a picture.)

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Answers are displayed within the problem

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