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Problem 1: Independent normal random variables

(3/3 points)

Let U , V , and W be independent standard normal random variables (that is, independent normal random variables, each with mean 0 and variance 1), and let $X = 3U + 4V$ and $Y = U + W$. Give a numerical answer for each part below. You may want to refer to the standard normal table .

1. What is the probability that $X \geq 8$?



$\mathbf{P}(X \geq 8) =$ Answer: 0.0548

2.



$\mathbf{E}[XY] =$ Answer: 3

3.

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Exam 2

Exam 2 due Apr 20, 2016 at 23:59 UTC



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$\text{var}(X + Y) =$ Answer: 33

Answer:

1. Since X is a sum of independent normal random variables, X is also a normal random variable. Its mean and variance are

$$\begin{aligned}\mathbf{E}[X] &= \mathbf{E}[3U + 4V] \\ &= 3 \cdot \mathbf{E}[U] + 4 \cdot \mathbf{E}[V] \\ &= 0, \\ \text{var}(X) &= \text{var}(3U + 4V) \\ &= 9 \cdot \text{var}(U) + 16 \cdot \text{var}(V) \\ &= 25.\end{aligned}$$

Therefore,

$$\begin{aligned}\mathbf{P}(X \geq 8) &= \mathbf{P}\left(\frac{X - 0}{5} \geq \frac{8 - 0}{5}\right) \\ &= 1 - \Phi(1.6) \\ &\approx 1 - 0.9452 \\ &= 0.0548.\end{aligned}$$

2. Since U , V , and W are zero-mean and independent, we have

$$\begin{aligned}\mathbf{E}[XY] &= \mathbf{E}[(3U + 4V)(U + W)] \\ &= \mathbf{E}[3U^2 + 3UW + 4UV + 4VW] \\ &= 3 \cdot \mathbf{E}[U^2] + 3 \cdot \mathbf{E}[U]\mathbf{E}[W] + 4 \cdot \mathbf{E}[U]\mathbf{E}[V] + 4 \cdot \mathbf{E}[V]\mathbf{E}[W] \\ &= 3 \cdot \mathbf{E}[U^2] \\ &= 3 \cdot (\text{var}(U) + (\mathbf{E}[U])^2) \\ &= 3.\end{aligned}$$

3. Since U , V , and W are independent with variance equal to 1 , we have

$$\begin{aligned}\text{var}(X + Y) &= \text{var}(3U + 4V + U + W) \\ &= \text{var}(4U + 4V + W) \\ &= \text{var}(4U) + \text{var}(4V) + \text{var}(W) \\ &= 16 \cdot 1 + 16 \cdot 1 + 1 \\ &= 33.\end{aligned}$$

You have used 2 of 2 submissions



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