Unit 2: Boundary value problems

Course > and PDEs

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## 2. Solve the boundary value problem

Find eigenvalues, eigenfunctions, and normal modes

3/3 points (graded)

Consider a cylinder with two open ends of length L. Longitudinal air waves/ pressure waves along the midline of the cylinder satisfy the PDE

$$rac{\partial^2 p}{\partial t^2} = c^2 rac{\partial^2 p}{\partial x^2}, \qquad 0 < x < L, \; t > 0.$$

$$rac{\partial^2 u}{\partial t^2} = c^2 rac{\partial^2 u}{\partial x^2}, \qquad 0 < x < L, \ t > 0.$$

where p is the pressure, u is the horizontal displacement of air molecules, and c is the speed of sound in the ambient air.

Let us find solutions for u.

Separating variables and looking for solutions of the form u(x,t) = v(x)w(t) leads to solving the ODEs

$$\frac{d^2w}{d\hat{x}^2} = \lambda c^2w, \qquad t>0$$

Solving leads to a family of solutions  $u_n\left(x,t\right)=v_n\left(x\right)w_n\left(t\right)$  for different values  $\lambda_n$ , which all satisfy the boundary conditions.

Find  $\lambda_n$ ,  $v_n$ , and  $w_n$  for  $n=1,2,3\ldots$ 

(Let the unknown constant in front of the cosine term be a, and the unknown term in front of the sine term be b.)

FORMULA INPUT HELP

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**1** Answers are displayed within the problem

## Solve the initial value problem

5/5 points (graded)

Suppose that the initial displacement takes the form

$$u\left( x,0 
ight) = rac{2x}{L} - 1, \qquad 0 < x < L.$$

Let the initial velocity of the displacement be zero (for convenience).

The general solution takes the form

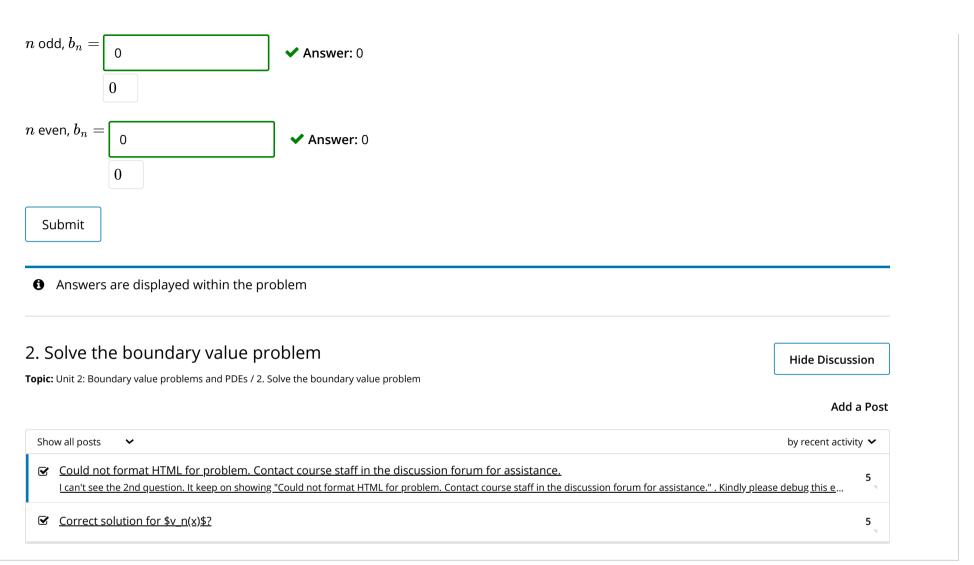
$$u\left(x,t
ight)=a_{0}/2+\sum_{n=1}^{\infty}\left(a_{n}\cos\left(\omega_{n}t
ight)+b_{n}\sin\left(\omega_{n}t
ight)
ight)\cos\left(lpha_{n}x
ight)$$

Determine what type of periodic extension is needed of this initial condition to solve for the coefficients in the Fourier series. Then use the initial condition to find the coefficients.

Hint: The triangle wave T(z) of period  $2\pi$  has Fourier series  $T(z)=\frac{\pi}{2}-\frac{4}{\pi}\sum_{n\,\mathrm{odd}}\frac{\cos{(nz)}}{n^2}$ .

The sawtooth wave  $W\left(z\right)$  of period  $2\pi$  has Fourier series  $W\left(z\right)=2\sum_{n=1}^{\infty}\frac{\left(-1\right)^{n+1}\sin\left(nz\right)}{n}.$ 

$$n$$
 even,  $a_n = \bigcirc{0}$   $\checkmark$  Answer: 0



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