



Bookmarks



Bookmark

▸ General Information

▸ Week 1

▸ Week 2

▸ Week 3

▼ Week 4

**Lecture**Lecture questions due Oct 04,  
2016 at 19:30 IST**Recitation****Problem Set 4**Homework 4 due Oct 04, 2016 at  
19:30 IST

Week 4 &gt; Recitation &gt; Practice Problem 3

**PART A**

Consider the problem:

$$\begin{array}{ll}
 \max & x_1 + x_2 \\
 \text{s.t.:} & \\
 & x_1 + x_2 \leq 8 \\
 & -x_1 + x_2 \leq 2 \\
 & x_1 - x_2 \leq 4 \\
 & x_2 \geq 0 \\
 & x_1 \in \{0, 1, 4, 6\}
 \end{array}
 \left. \vphantom{\begin{array}{l} \max \\ \text{s.t.:} \end{array}} \right\}$$

When you formulate this problem as an IP, how many binary variables need to be added (Assume that no non-binary variables are added)?

3



Answer: [3, 4]


3

**SOLUTION**

There are three resource constraints, and two additional logical constraints.

## PART B

Formulate the problem as an equivalent integer program

☒ I am ready to have the answer shown 

### SOLUTION

Everything except the condition that  $x_1$  can only be one of four values is already linear. We can use the standard way of doing this by introducing 4 binary variables  $w_1, \dots, w_4$  and imposing  $x_1 = 0$ .

$$\begin{aligned} w_1 + w_2 + 4w_3 + 6w_4 \\ w_1 + w_2 + w_3 + w_4 &\leq 1 \end{aligned}$$

However, it is easy to see that we need only three binary variables as follows


$$\begin{aligned} x_1 &= w_1 + 4w_2 + 6w_3 \\ w_1 + w_2 + w_3 &\leq 1 \end{aligned}$$

## PART C

How would your answer to the previous part change if the objective function were changed to:

$$\text{MAX } x_1^2 + 2x_2$$

*NOTE: It is possible to modify the problem in many different ways. See if you can find a way that adds a single decision variable  $y$  (which will be equal to  $(x_1)^2$ ) and only one additional constraint.*

☒ I am ready to have the answer shown 

### SOLUTION

Let  $y = x_1^2$ , then note that  $y$  is either **0, 1, 16** or 36, depending on whether  $x_1$  is **0, 1, 4** or 6 respectively. Hence we add the constraint  $y = w_1 + 16w_2 + 36w_3$  where  $w_1, w_2, w_3$  are as described above (in the solution with 3 binary variables). The objective and all constraints are now linear.

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