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MLE for a homogeneous Poisson process?

Asked 3 years, 1 month ago Active 1 year, 7 months ago Viewed 2k times

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If we have a data set consisting of event times $\{t_1, t_2, \ldots, t_N\}$ and would like to model this as a Poisson process with intensity λ, how do we do it? Intuitively, I would expect that we can calculate the average waiting time $w = \frac{1}{N-1}\sum_{i=2}^{N}(t_i - t_{i-1})$ and then set $\hat{\lambda} = 1/w$?

Is this correct? If so, how is it justified? Do we have to consider fitting a Poisson distribution to the number of events after doing some sort of binning?

poisson-distribution poisson-process

Share Cite Edit Follow Flag edited Aug 5 '18 at 13:35 asked Aug 5 '18 at 13:25

theQman
507 4 12

Look at interarrival times $\{t_1, t_2 - t_1, \ldots, t_N - t_{N-1}\}$ and fit exponential distribution with parameter λ to them. – stans Aug 5 '18 at 13:43

Thanks. Can we do something similar if the process is non-homogeneous? – theQman Aug 5 '18 at 14:36

Similar: yes. But not exactly the same. Naturally, if λ changes with time, you need to make up your mind on the exact form of time dependence and the number of parameters involved. The best course of action would be for you to read up on non-homogeneous Poisson process. Ross is a good reference. – stans Aug 5 '18 at 16:15

1 Answer

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You can use Maximum Likelihood Estimation, either with synchronous data (time-binned data) or asynchronous data (time-stamped data). The likelihood function changes accordingly.

For time-binned (or synchronous) data, you can simply use the joint Poisson probability mass function for your observed counts as the likelihood function:

$$L = \prod_{i=1}^{K} \frac{\lambda^{x_i}}{x_i!}\exp(-\lambda),$$

where K is the number of bins, x_i the count of events in bin i, and λ the constant intesity that you want to estimate.

For asynchronous data, the likelihood is specified as follows:

$$L = \left[\prod_{i=1}^{N(T)} \lambda^*(t_i)\right] \exp\left[-\int_0^T \lambda^*(s)ds\right],$$

where $N(T)$ is the number of points at end-of-sample time T, and $\lambda^*(t)$ is the conditional intensity function, which is simply the constant $\lambda^*(t) = \lambda$ for the homogeneous Poisson process.

In some cases including the homogeneous Poisson process, there are closed-form solutions for both cases (take logs, set derivative with respect to λ equal to zero, and solve for λ). Otherwise the log-likelihood can be optimised numerically.

For more background on theory and estimation, these are good references:

- Lecture notes on temporal point processes by Rassmussen
- *Daley, D. J.; Vere-Jones, D.*, **An introduction to the theory of point processes. Vol. I: Elementary theory and methods.**, Probability and Its Applications. New York, NY: Springer. xxi, 469 p. (2003). ZBL1026.60061.

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edited Feb 11 '20 at 15:54
Glorfindel
674 1 9 18

answered Aug 6 '18 at 11:40
mloning
388 2 12

"For synchronous data (i.e. counts in regular time bins), you can simply use the joint Poisson probability mass function for your observed counts as the likelihood function." Can you explain this a bit more? If time is divided into bins, then what are fitting to a Poisson distribution? The number of times k events occurs in a bin? – theQman Aug 7 '18 at 0:24

For each bin, we count the number of events and fit \lambda to that joint distribution of counts. Please also see updated question. I hope this makes it clearer! – mloning Aug 7 '18 at 6:33