# 2. Lecture 4

The following exercises can be done after lecture 4.

## 4-1

1/1 point (graded)

The image of the unit square (the set of x and y such that $0 \le x \le 1$ and $0 \le y \le 1$) under matrix multiplication by

$$\begin{pmatrix} 3 & -1 \\ -6 & 2 \end{pmatrix}$$

is a line segment. The rightmost endpoint of this segment is a point of the form $\begin{pmatrix} a \\ b \end{pmatrix}$. Find $a$.

$a =$ 3 ✔ **Answer:** 3

**Solution:**

The image of $\begin{pmatrix} x \\ y \end{pmatrix}$ under this transformation is

$$\begin{pmatrix} 3 & -1 \\ -6 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3x - y \\ -6x + 2y \end{pmatrix}.$$

The first coordinate is maximized by letting $x = 1$ and $y = 0$. Hence $a = 3$.

Submit You have used 1 of 4 attempts

Answers are displayed within the problem

## 4-2

0/5 points (graded)

True or false: If $\mathbf{A}$ is an $n \times n$ matrix, and $\mathbb{C}^n$ has a basis consisting of eigenvectors of $\mathbf{A}$, then $\mathbf{A}$ has $n$ distinct eigenvalues.

- True ✖
- False ✔

**Solution:**

False.

If $\mathbf{A}$ has $n$ distinct eigenvalues, then it is guaranteed that $\mathbb{C}^n$ has a basis consisting of eigenvectors. But there are also some matrices $\mathbf{A}$ with repeated eigenvalues (such as $\mathbf{I}$) such that $\mathbb{C}^n$ still has a basis consisting of eigenvectors.

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## 4-3

5/5 points (graded)

What is the characteristic polynomial of $\begin{pmatrix} 1 & 4 \\ 5 & 9 \end{pmatrix}$?

(Type $L$ to denote the variable $\lambda$ in the polynomial.)

L^2-10*L-11 ✔ **Answer:** L^2-10*L-11

$L^2 - 10 \cdot L - 11$

FORMULA INPUT HELP

**Solution:**

The characteristic polynomial is $\lambda^2 - 10\lambda - 11$.

The characteristic polynomial of a $2 \times 2$ matrix $\mathbf{A}$ is

$$\lambda^2 - \mathrm{tr}(\mathbf{A})\lambda + \det(\mathbf{A})$$

For the given matrix $\mathbf{A}$, the trace is $10$ and the determinant is $-11$.
Alternative solution: The characteristic polynomial is

$$\det(\lambda\mathbf{I} - \mathbf{A}) = \det\begin{pmatrix} \lambda - 1 & -4 \\ -5 & \lambda - 9 \end{pmatrix} = (\lambda - 1)(\lambda - 9) - (-4)(-5) = \lambda^2 - 10\lambda - 11.$$

(One could also use $\det(\mathbf{A} - \lambda\mathbf{I})$; there is no need to change the sign when the size of the matrix is even.)

Submit You have used 1 of 7 attempts

Answers are displayed within the problem

## 4-4

5/5 points (graded)

What is the characteristic polynomial of $\begin{pmatrix} 0 & 438 & 691 \\ 0 & 1 & 300 \\ 0 & 0 & 2 \end{pmatrix}$?

(Type $L$ to denote the variable $\lambda$ in the polynomial.)

L^3-3*L^2+2*L

**Answer:** L^3-3*L^2+2*L

$L^3 - 3 \cdot L^2 + 2 \cdot L$

**Solution:**

The characteristic polynomial is $\lambda(\lambda - 1)(\lambda - 2) = \lambda^3 - 3\lambda^2 + 2\lambda$.

The matrix $\lambda I - A$ is upper triangular with diagonal entries $\lambda$, $\lambda - 1$, $\lambda - 2$, so its determinant is $\lambda(\lambda - 1)(\lambda - 2)$.

Submit You have used 2 of 10 attempts

Answers are displayed within the problem

## 2. Lecture 4

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