4. Introduction to Graphs

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Exercises due Oct 20, 2021 17:29 IST Completed

Graph Model

A **graph** $G = (V, E)$ is a tuple of two sets V and E, where

- V is a set of **nodes** or **vertices** .
- E is a set of **edges** or **links** representing relationships between the nodes in V. Each element of this set is either
  - a set $\{i, j\}$ if the edge is undirected (as a set does not describe any order between the two vertices, so $\{i, j\} \in E$ implies $\{i, j\} = \{j, i\} \in E$);
  - a tuple (i, j), if the edge is directed from i to j (thus $(i, j) \in E$ does not imply $(j, i) \in E$).

For example, a transportation network of **cities** and **roads** connecting the cities is a graph. In this case, cities are nodes in the graph and the roads connecting the cities are the edges in the graph.

In this module on network analysis we will learn some basic properties of different types of **graphs** . These properties will help us analyze network data and make sense of such data.

Definitions

- A (directed) **walk** in a graph is a sequence of (directed) edges $(e_1, e_2, \ldots, e_k)$ such that every pair (e_i, e_{i+1}) of edges shares a node, v_i. The node (or vertex) sequence of this walk, $(v_0, v_1, \ldots, v_k)$, is such that every pair (v_{i-1}, v_i) of nodes are connected by the (directed) edge e_i.
- A (directed) **trail** is a walk where every edge in the sequence is unique.
- A (directed) **path** is a trail where every node in the node sequence is unique.
- A (directed) **cycle** is a (directed) trail that starts and terminates at the same node and such that **all other** nodes in the node sequence are unique.

Simple network	Undirected network with at most one edge between any pair of vertices, and no self-loops.
Multigraph	May contain self-loops or multiple links between vertices.
Weighted network	Edges have weights or vertices have attributes.
Tree	A graph with no cycles.
Acyclic network	Graph with no directed cycles.
Bipartite	Vertices can be divided into two classes where there are no edges between vertices in the same class (but there can exist edges between vertices in different classes).
Hypergraph	Generalized edges'' which connect more than two vertices together.

Types of Graphs

5/5 points (graded)

Identify the categories that each of the following graphs fall under. Drawing each graph on a piece of paper would be instructive.

1. $G = (V = \{1, 2, 3, 4, 5\}, E = \{(1, 2), (2, 3), (3, 4), (4, 5), (5, 2)\})$.

- [x] Directed

- [ ] Undirected
- [x] Cyclic
- [ ] Acyclic

✔

2. $G = (V = \{1,2,3,4,5\}, E = \{(1,2),(1,4),(3,4),(4,5),(5,2)\})$

- [x] Directed
- [ ] Undirected
- [ ] Cyclic
- [x] Acyclic

✔

3. $G = (V = \{1,2,3,4,5\}, E = \{\{1,2\},\{1,4\},\{3,4\},\{4,5\},\{5,2\}\})$

- [ ] Directed
- [x] Undirected
- [x] Cyclic
- [ ] Acyclic

✔

4. $G = (V = \{1,2,3,4,5\}, E = \{\{1,2\},\{1,4\},\{3,4\},\{4,5\},\{5,2\},\{3,3\}\})$

- [ ] Simple Graph
- [x] Multigraph (a simple graph is also multigraph)
- [ ] Hypergraph

✔

5. $G = (V = \{1,2,3,4,5\}, E = \{\{1,2\},\{1,4\},\{3,1\},\{4,5\},\{5,2\}\})$

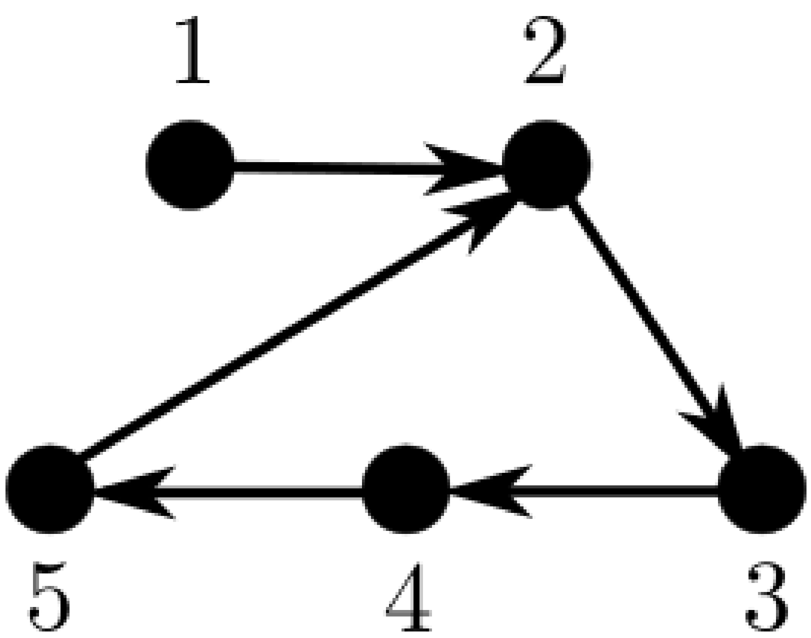
- [x] Bipartite Graph
- [ ] Multigraph (a simple graph is also multigraph) ✔
- [ ] Hypergraph

✔

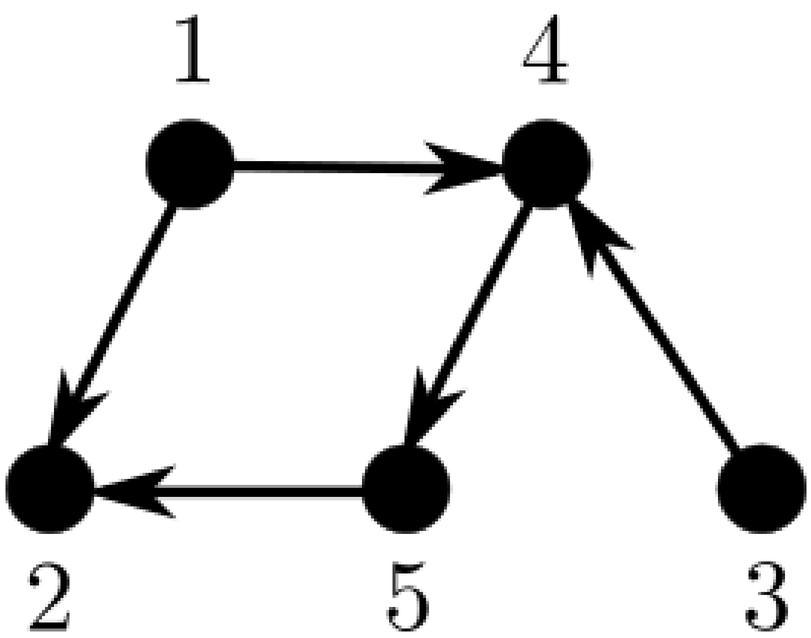
Grading note(Oct 18): Question 5 will be regraded after the due date to accept answers that either include or exclude the second choice “Multigraph".

Solution:

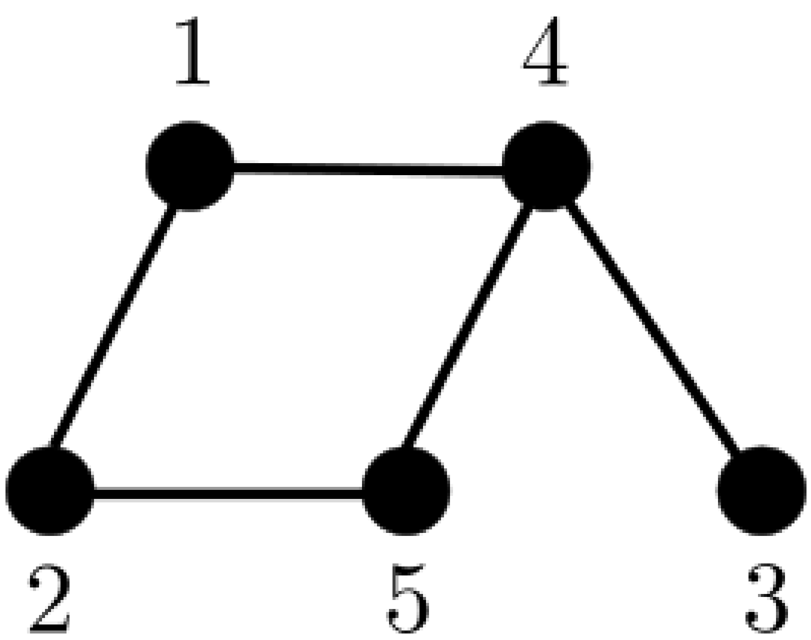
We can sketch the graphs to see what types they belong to.



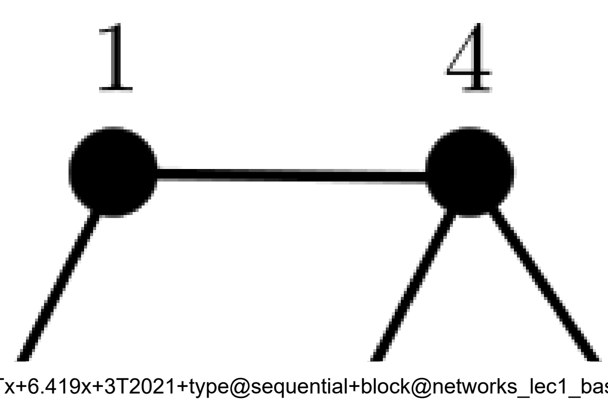
1: Graph for 1. The graph is directed since the edges are written as tuples and not as sets and the graph is cyclic since there is a directed cycle $(2, 3, 4, 5, 2)$.

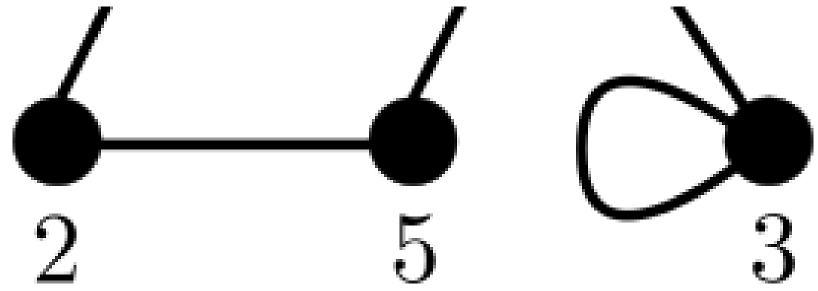


2: Graph for 2. The graph is directed since the edges are written as tuples and not as sets and the graph is acyclic since there is no directed path that starts and ends at the same node.

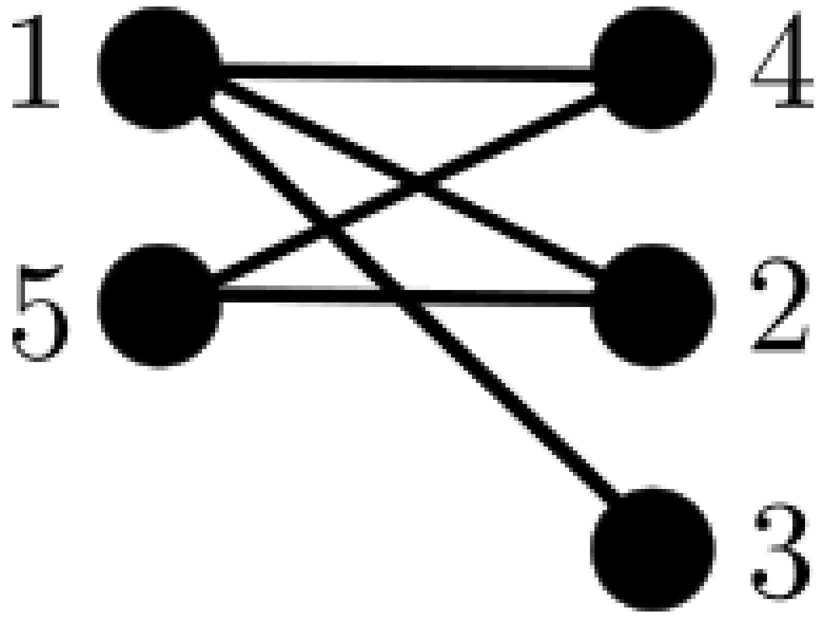


3: Graph for 3. The graph is undirected since the edges are written as sets and the graph is cyclic since there is a cycle $(1, 4, 5, 2, 1)$.





4: Graph for 4. The graph is a multigraph since there is a self loop.



5: Graph for 5. The graph is bipartite as it can be seen that the vertex set can be partitioned into two disjoint subsets such that both ends of any edge do not belong to the same vertex subset.

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You have used 1 of 4 attempts

Answers are displayed within the problem

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Aren't all graphs also hypergraphs?

discussion posted 3 months ago by **Sience-Guy**

From Wikipedia: "In mathematics, a hypergraph is a generalization of a graph in which an edge can join any number of vertices. In contrast, in an ordinary graph, an edge connects exactly two vertices.". Doesn't this imply that all of the graphs are also hypergraphs?

This post is visible to everyone.

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1 response

Dmitrii_Ivanov (Community TA)
2 months ago

Hi!

In a hypergraph hyperedges in the form {a,b,c} should present.

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