5.1.9 Exam: Improving on Forward Euler for a stiff system

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Exams due Aug 30, 2023 05:00 IST Completed

A Forward Euler method is used to solve a system of two linear differential equations of the following form:

$$\frac{\mathrm{d}\underline{u}}{\mathrm{d}t} = A\underline{u}, \qquad A = \begin{bmatrix} -1000 & -100 \\ 0 & -1 \end{bmatrix} \tag{5.14}$$

It works well, but there is a limit to how large the time step `dt` can be: Above `dt = 2e-3`, the numerical method is unstable. There is inherent stiffness in the differential equation.

Problem: Identify a better method

3.0/3.0 points (graded)

Which of the following methods will fix this issue, and enable the computation of a meaningful answer for Δt well above `2e-3`? Here, u^k denotes the k-th iterate (a vector of two components), and A is the matrix defined in the code above.

Method A

$$u^{k+1} = u^k + \Delta t\, Au^{k-1} \tag{5.15}$$

Method B

$$u^{k+1} = u^k + \Delta t\, Au^k \tag{5.16}$$

Method C

$$u^* = u^k + \frac{\Delta t}{2} Au^k \tag{5.17}$$

$$u^{k+1} = u^k + \Delta t\, Au^* \tag{5.18}$$

Method D

$$u^* = u^k + \Delta t\, Au^k \tag{5.19}$$

$$u^{k+1} = u^k + \Delta t\, A\frac{u^k + u^*}{2} \tag{5.20}$$

Method E

$$u^{k+1} = u^k + \Delta t\, Au^{k+1} \tag{5.21}$$

- ( ) Method A
- ( ) Method B
- ( ) Method C
- ( ) Method D
- (•) Method E

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Problem: Implementation of Method A

1.0/1.0 point (graded)

Which of the following best describes the implementation required for Method A.

- A loop over time, and an explicit update of the iterate(s), like in a Forward Euler code ✔
- A loop over time, and a call to a scalar root-finder like the bisection method in order to compute the next iterate
- A loop over time, and a call to a linear system solver (like Gaussian elimination) in order to compute the next iterate
- A loop over time, and a call to a nonlinear system root-finder like the Newton-Raphson method in order to compute the next iterate

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Problem: Implementation of Method C

1.0/1.0 point (graded)

Which of the following best describes the implementation required for Method C.

- A loop over time, and an explicit update of the iterate(s), like in a Forward Euler code
- A loop over time, and a call to a scalar root-finder like the bisection method in order to compute the next iterate
- A loop over time, and a call to a linear system solver (like Gaussian elimination) in order to compute the next iterate
- A loop over time, and a call to a nonlinear system root-finder like the Newton-Raphson method in order to compute the next iterate

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Problem: Implementation of Method E

1.0/1.0 point (graded)

Which of the following best describes the implementation required for Method E.

- [ ] A loop over time, and an explicit update of the iterate(s), like in a Forward Euler code
- [ ] A loop over time, and a call to a scalar root-finder like the bisection method in order to compute the next iterate
- [x] A loop over time, and a call to a linear system solver (like Gaussian elimination) in order to compute the next iterate
- [ ] A loop over time, and a call to a nonlinear system root-finder like the Newton-Raphson method in order to compute the next iterate

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