2.4.1 From Linear Transformation to Matrix-Vector Multiplication

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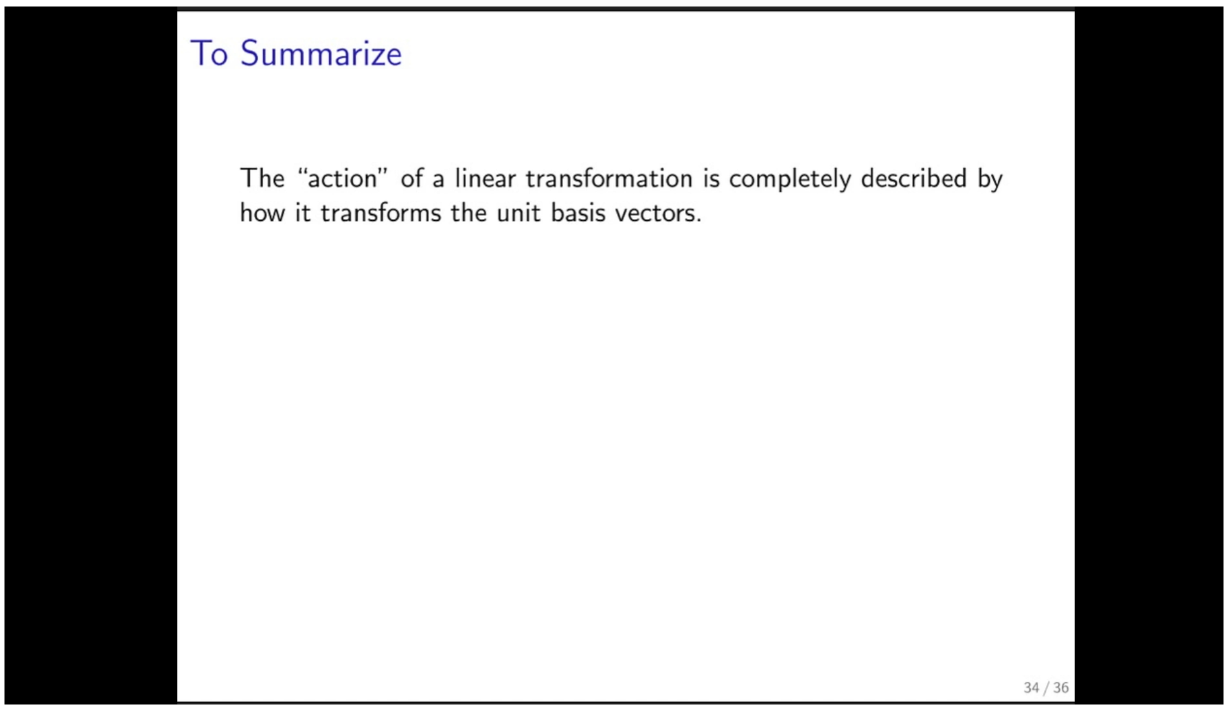
Week 2 due Oct 11, 2023 16:42 IST

2.4.1 From Linear Transformation to Matrix-Vector Multiplication



8:21 / 8:42 2.0x

And then for the final component, it's this times that plus this times that,

et cetera, plus this times that.

And that's summarized right here.

So these are just individual components of the result vector.

This is probably the definition of matrix vector multiplication that

you've seen before somewhere.

So to summarize, the action of a linear transformation is completely

described by how it transforms the unit basis vectors.

We can represent a linear transformation by a two-dimensional

array that we call a matrix.

And evaluating the linear transformation is equivalent to

performing the matrix vector multiplication as you know it.

Video

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Reading Assignment

0 points possible (ungraded)

Read Unit 2.4.1 of the notes. [LINK]

- [x] Done

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? L( ) is not necessarily equal to A though L(x)=Ax?

Calculator

? 2.4.1.1 true for k = K +1?
In the proof by induction, I'm unclear why the result, when you assume k = K+1, includes both the k-1 components and the k components? Shoul... 4

Homework 2.4.1.1

1/1 point (graded)
In the video and the text the following theorem is given:

Theorem Let $v_o, v_1, \ldots, v_{n-1} \in \mathbb{R}^n$, $\alpha_0, \alpha_1, \ldots, \alpha_{n-1} \in \mathbb{R}$, and let $L : \mathbb{R}^n \to \mathbb{R}^m$ be a linear transformation. Then

$$L\left(\alpha_0 v_0 + \alpha_1 v_1 + \cdots + \alpha_{n-1} v_{n-1}\right) = \alpha_0 L\left(v_0\right) + \alpha_1 L\left(v_1\right) + \cdots + \alpha_{n-1} L\left(v_{n-1}\right).$$

Give an alternative proof for this theorem that mimics the proof by induction for the lemma that states that $L\left(v_0 + \cdots + v_{n-1}\right) = L\left(v_0\right) + \cdots + L\left(v_{n-1}\right)$.

Done

Explanation

Answer: Proof by induction on k.

Base case: $k = 1$. For this case, we must show that $L(\alpha_0 v_0) = \alpha_0 L(v_0)$. This follows immediately from the definition of a linear transformation.

Inductive step: Inductive Hypothesis (IH): Assume that the result is true for $k = K$ where $K \geq 1$:

$$L(\alpha_0 v_0 + \alpha_1 v_1 + \cdots + \alpha_{K-1} v_{K-1}) = \alpha_0 L(v_0) + \alpha_1 L(v_1) + \cdots + \alpha_{K-1} L(v_{K-1}).$$

We will show that the result is **then** also true for $k = K + 1$. In other words, that

$$L(\alpha_0 v_0 + \alpha_1 v_1 + \cdots + \alpha_{K-1} v_{K-1} + \alpha_K v_K) = \alpha_0 L(v_0) + \alpha_1 L(v_1) + \cdots + \alpha_{K-1} L(v_{K-1}) + \alpha_K L(v_K).$$

Assume that $K \geq 1$ and $k = K + 1$. Then

$$\begin{aligned}
& L(\alpha_0 v_0 + \alpha_1 v_1 + \cdots + \alpha_{k-1} v_{k-1}) \\
= \quad & \langle k - 1 = (K+1) - 1 = K \rangle \\
& L(\alpha_0 v_0 + \alpha_1 v_1 + \cdots + \alpha_K v_K) \\
= \quad & \langle \text{expose extra term -- We know we can do this, since } K \geq 1 \rangle \\
& L(\alpha_0 v_0 + \alpha_1 v_1 + \cdots + \alpha_{K-1} v_{K-1} + \alpha_K v_K) \\
= \quad & \langle \text{associativity of vector addition} \rangle \\
& L((\alpha_0 v_0 + \alpha_1 v_1 + \cdots + \alpha_{K-1} v_{K-1}) + \alpha_K v_K) \\
= \quad & \langle L \text{ is a linear transformation)} \rangle \\
& L(\alpha_0 v_0 + \alpha_1 v_1 + \cdots + \alpha_{K-1} v_{K-1}) + L(\alpha_K v_K) \\
= \quad & \langle \text{Inductive Hypothesis} \rangle \\
& \alpha_0 L(v_0) + \alpha_1 L(v_1) + \cdots + \alpha_{K-1} L(v_{K-1}) + L(\alpha_K v_K) \\
= \quad & \langle \text{Definition of a linear transformation} \rangle \\
& \alpha_0 L(v_0) + \alpha_1 L(v_1) + \cdots + \alpha_{K-1} L(v_{K-1}) + \alpha_K L(v_K)
\end{aligned}$$

By the Principle of Mathematical Induction the result holds for all k.

Submit

Calculator

Answers are displayed within the problem

Homework 2.4.1.2

1/1 point (graded)

Let L be the linear transformation such that

$$L\left(\begin{pmatrix}1\\0\end{pmatrix}\right)=\begin{pmatrix}3\\5\end{pmatrix} \text{ and } L\left(\begin{pmatrix}0\\1\end{pmatrix}\right)=\begin{pmatrix}2\\-1\end{pmatrix}$$

$$L\left(\begin{pmatrix}2\\3\end{pmatrix}\right)=$$

- ( ) $\begin{pmatrix}6\\10\end{pmatrix}$
- ( ) $\begin{pmatrix}6\\-3\end{pmatrix}$
- ( ) $\begin{pmatrix}0\\13\end{pmatrix}$
- (●) $\begin{pmatrix}12\\7\end{pmatrix}$
- ( ) Not enough information

✔

Explanation

Transcripted in final section of this week

Answer:

$$\begin{pmatrix}2\\3\end{pmatrix}=2\begin{pmatrix}1\\0\end{pmatrix}+3\begin{pmatrix}0\\1\end{pmatrix}.$$

Hence

$$\begin{aligned}L(\begin{pmatrix}2\\3\end{pmatrix}) &= L(2\begin{pmatrix}1\\0\end{pmatrix}+3\begin{pmatrix}0\\1\end{pmatrix})=2L(\begin{pmatrix}1\\0\end{pmatrix})+3L(\begin{pmatrix}0\\1\end{pmatrix})\\ &= 2\begin{pmatrix}3\\5\end{pmatrix}+3\begin{pmatrix}2\\-1\end{pmatrix}=\begin{pmatrix}2\times3+3\times2\\2\times5+3\times(-1)\end{pmatrix}=\begin{pmatrix}12\\7\end{pmatrix}.\end{aligned}$$

Click to PDF of answer in video

Calculator

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Answers are displayed within the problem

Homework 2.4.1.3-5

3/3 points (graded)

Let L be the linear transformation such that

$L\left(\begin{pmatrix}1\\0\end{pmatrix}\right) = \begin{pmatrix}3\\5\end{pmatrix}$ and $L\left(\begin{pmatrix}1\\1\end{pmatrix}\right) = \begin{pmatrix}5\\4\end{pmatrix}$

$L\left(\begin{pmatrix}3\\3\end{pmatrix}\right) =$

- ( ) $\begin{pmatrix}9\\15\end{pmatrix}$
- ( ) $\begin{pmatrix}-9\\15\end{pmatrix}$
- (•) $\begin{pmatrix}15\\12\end{pmatrix}$
- ( ) $\begin{pmatrix}15\\15\end{pmatrix}$
- ( ) Not enough information

✔

$L\left(\begin{pmatrix}-1\\0\end{pmatrix}\right) =$

- ( ) $\begin{pmatrix}-3\\5\end{pmatrix}$
- ( ) $\begin{pmatrix}-3\\0\end{pmatrix}$
- (•) $\begin{pmatrix}-3\\-5\end{pmatrix}$
- ( ) $\begin{pmatrix}5\\-4\end{pmatrix}$
- ( ) Not enough information

✔

$L\left(\begin{pmatrix}2\\3\end{pmatrix}\right) =$

Calculator

- ( ) $\begin{pmatrix} 6 \\ 10 \end{pmatrix}$
- ( ) $\begin{pmatrix} 6 \\ -3 \end{pmatrix}$
- ( ) $\begin{pmatrix} 0 \\ 13 \end{pmatrix}$
- (•) $\begin{pmatrix} 12 \\ 7 \end{pmatrix}$
- ( ) Not enough information

✔

Explanation

2.4.1.3

Answer:

$$\begin{pmatrix} 3 \\ 3 \end{pmatrix} = 3 \begin{pmatrix} 1 \\ 1 \end{pmatrix}.$$

Hence

$$L(\begin{pmatrix} 3 \\ 3 \end{pmatrix}) = 3L(\begin{pmatrix} 1 \\ 1 \end{pmatrix}) = 3 \begin{pmatrix} 5 \\ 4 \end{pmatrix} = \begin{pmatrix} 15 \\ 12 \end{pmatrix}.$$

2.4.1.4

Answer:

$$\begin{pmatrix} -1 \\ 0 \end{pmatrix} = (-1) \begin{pmatrix} 1 \\ 0 \end{pmatrix}.$$

Hence

$$L(\begin{pmatrix} -1 \\ 0 \end{pmatrix}) = (-1)L(\begin{pmatrix} 1 \\ 0 \end{pmatrix}) = (-1) \begin{pmatrix} 3 \\ 5 \end{pmatrix} = \begin{pmatrix} -3 \\ -5 \end{pmatrix}.$$

2.4.1.5

Answer:

$$\begin{pmatrix} 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 3 \\ 3 \end{pmatrix} + \begin{pmatrix} -1 \\ 0 \end{pmatrix}.$$

Hence

$$L(\begin{pmatrix} 2 \\ 3 \end{pmatrix}) = L(\begin{pmatrix} 3 \\ 3 \end{pmatrix}) + L(\begin{pmatrix} -1 \\ 0 \end{pmatrix}) = \text{(from the previous two exercises)}$$
$$\begin{pmatrix} 15 \\ 12 \end{pmatrix} + \begin{pmatrix} -3 \\ -5 \end{pmatrix} = \begin{pmatrix} 12 \\ 7 \end{pmatrix}.$$

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Answers are displayed within the problem

Homework 2.4.1.6

Calculator

1/1 point (graded)
Let L be the linear transformation such that

$$L\left(\begin{pmatrix} 1 \\ 1 \end{pmatrix}\right) = \begin{pmatrix} 5 \\ 4 \end{pmatrix}$$

$$L\left(\begin{pmatrix} 3 \\ 2 \end{pmatrix}\right) =$$

- $\begin{pmatrix} 6 \\ 10 \end{pmatrix}$
- $\begin{pmatrix} 6 \\ -3 \end{pmatrix}$
- $\begin{pmatrix} 0 \\ 13 \end{pmatrix}$
- $\begin{pmatrix} 12 \\ 7 \end{pmatrix}$
- Not enough information (selected)

✓

Explanation

Answer: The problem is that you can't write $\begin{pmatrix} 3 \\ 2 \end{pmatrix}$ as a linear combination (scalar multiple in this case) of $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$. So, there isn't enough information.

Submit

Answers are displayed within the problem

Homework 2.4.1.7

1/1 point (graded)
Let L be the linear transformation such that

$L\left(\begin{pmatrix} 1 \\ 1 \end{pmatrix}\right) = \begin{pmatrix} 5 \\ 4 \end{pmatrix}$ and $L\left(\begin{pmatrix} 2 \\ 2 \end{pmatrix}\right) = \begin{pmatrix} 10 \\ 8 \end{pmatrix}$

$$L\left(\begin{pmatrix} 3 \\ 2 \end{pmatrix}\right) =$$

- $\begin{pmatrix} 6 \\ 10 \end{pmatrix}$
- $\begin{pmatrix} 6 \\ -3 \end{pmatrix}$
- $\begin{pmatrix} 0 \\ \end{pmatrix}$

Calculator

$\begin{pmatrix} \\ 13 \end{pmatrix}$

- $\begin{pmatrix} 12 \\ 7 \end{pmatrix}$
- Not enough information

Explanation

Answer: The problem is that you can't write $\begin{pmatrix} 3 \\ 2 \end{pmatrix}$ as a linear combination of $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 2 \\ 2 \end{pmatrix}$. So, there isn't enough information.

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Answers are displayed within the problem

Homework 2.4.1.8

4/4 points (graded)

Give the matrix that corresponds to the linear transformation

$$f\left(\begin{pmatrix} \chi_0 \\ \chi_1 \end{pmatrix}\right) = \begin{pmatrix} 3\chi_0 - \chi_1 \\ \chi_1 \end{pmatrix}$$

3	Answer: 3	-1	Answer: -1
0	Answer: 0	1	Answer: 1

Submit

Answers are displayed within the problem

Homework 2.4.1.9

6/6 points (graded)

Give the matrix that corresponds to the linear transformation

$$f\left(\begin{pmatrix} \chi_0 \\ \chi_1 \\ \chi_2 \end{pmatrix}\right) = \begin{pmatrix} 3\chi_0 - \chi_1 \\ \chi_2 \end{pmatrix}$$

3	Answer: 3	-1	Answer: -1	0	Answer: 0
0	Answer: 0	0	Answer: 0	1	Answer: 1

Explanation

Answer:

Calculator

- $f(\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}) = \begin{pmatrix} 3-0 \\ 0 \end{pmatrix}. = \begin{pmatrix} 3 \\ 0 \end{pmatrix}.$
- $f(\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}) = \begin{pmatrix} -1 \\ 0 \end{pmatrix}.$
- $f(\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}) = \begin{pmatrix} 0 \\ 1 \end{pmatrix}.$

Hence $\begin{pmatrix} 3 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

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Answers are displayed within the problem

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