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10. Partial derivatives: definitions

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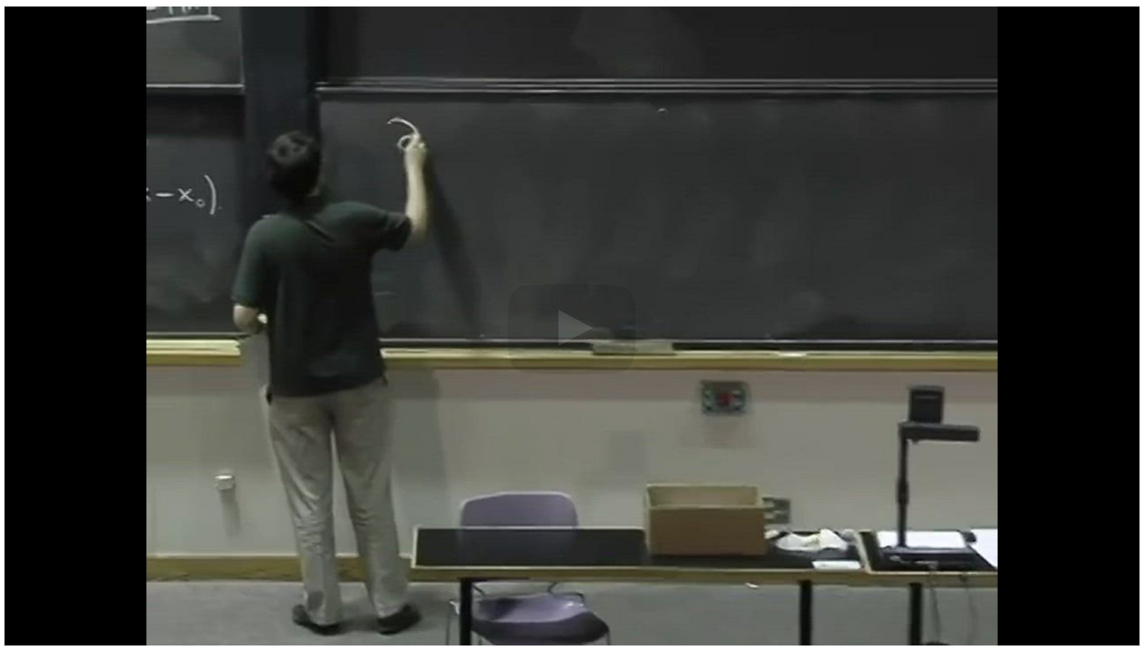
Partial derivatives



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PROFESSOR: So we have a notation.

This is a curly d.

And it is not a straight d, and it is not a delta.

It's a d that kind of curls backwards like that.

And the way you have this symbol is "partial".

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In single variable calculus, the derivative of a function $f(x)$ is the slope of the tangent line at x, which is defined by the following limit:

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}. \tag{2.8}$$

Note: The notation $f'(x)$ and the notation $\frac{df}{dx}$ both describe the derivative of f with respect to x.

Question: How do we define a derivative for a function of more than one variable?

For a function $f(x,y)$, we can take what are called **partial derivatives** of the function with respect to each variable. For example, to compute the derivative of $f(x,y)$ with respect to x, we treat y as a constant and differentiate each term with respect to x only. Formally, the definition is as follows:

Definition 10.1 The **partial derivative of $f(x,y)$ with respect to x** is defined by

$$f_x(x,y) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x, y) - f(x,y)}{\Delta x}. \tag{2.9}$$

The Leibniz notation for this is $\frac{\partial f}{\partial x}$.

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Definition 10.2 The **partial derivative of $f(x,y)$ with respect to y** is defined by

$$f_y(x,y) = \lim_{\Delta y \to 0} \frac{f(x, y+\Delta y) - f(x,y)}{\Delta y}. \tag{2.10}$$

The Leibniz notation for this is $\dfrac{\partial f}{\partial y}$.

Spoiler: Extension to higher dimension

We can extend the definition of partial derivatives to functions of more than two variables as follows.

Definition 10.3 For a function in n dimensions $f(x_1, x_2, \ldots, x_n)$, the **partial derivative with respect to the variable x_k** is defined by

$$f_{x_k} = \lim_{\Delta x_k \to 0} \frac{f(x_1, \ldots, x_k + \Delta x_k, \ldots, x_n) - f(x_1, \ldots, x_k, \ldots, x_n)}{\Delta x_k} \tag{2.11}$$

for $1 \leq k \leq n$. The Leibniz notation for this is $\dfrac{\partial f}{\partial x_k}$.

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Thanks for the Spoiler!
I had always heard it was easy to extend to higher dimensions beyond 3 but this is the first time I have seen a clear explanation. Tha... 1

Thank you for the visual approach
I did Prof. Auroux's class on OCW, and I loved it. It is very analytical in nature though. Not sure about this entire class yet, but so far I ... 3

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