10. Identifiability

Preparation: Injectivity

1/1 point (graded)

The notation $f : S \to T$ denotes that f is a function, also called a **map** , defined on all of a set S and whose outputs lie in a set T. A function $f : S \to T$ is **injective** if for all $x, y \in S$, $f(x) = f(y)$ implies that $x = y$.

Alternatively: a function is injective if we can **uniquely** recover some input x based on an output $f(x)$.

Which of the following functions are injective? (Choose all that apply.)

- [x] $f_1 : \mathbb{R} \to \mathbb{R}$, given by $f_1(x) = x$.
- [ ] $f_2 : \mathbb{R} \to \mathbb{R}$, given by $f_2(x) = x^2$.
- [ ] $f_3 : \mathbb{R} \to \mathbb{R}$, given by $f_3(x) = \sin(x)$.
- [x] $f_4 : [0,1] \to \{\text{probability distributions on } \{0,1\}\}$, given by $f_4(p) = \text{Ber}(p)$.

Solution:

The first choice $f_1(x) = x$ is the identity function, so if $f_1(x) = f_1(y)$, then $x = y$ by definition of f_1. So f_1 is injective.
The second choice $f_2(x) = x^2$ is not injective because, for example, both $+1$ and -1 map to the same value, 1, after applying f_2. In general, if $f_2(x) = c$ for some constant $c > 0$, then there are two possible choices for x: either $x = \sqrt{c}$ or $x = -\sqrt{c}$.
The third choice $f_3(x) = \sin(x)$ is not injective. In fact, there are infinitely many points x such that $f_3(x) = 0$. Recall from trigonometry that all values in the set $\{2\pi x : x \in \mathbb{Z}\}$ will map to 0 after applying f_3.
The fourth choice $f_4(p) = \text{Ber}(p)$ is injective: if $p \in [0,1]$, then $f_4(p) = \text{Ber}(p)$, so that p specifies the probability that $X \sim \text{Ber}(p)$ is equal to 1. Since a distribution on $\{0,1\}$ is uniquely determined by $P(X = 1)$, the map f_4 is injective.

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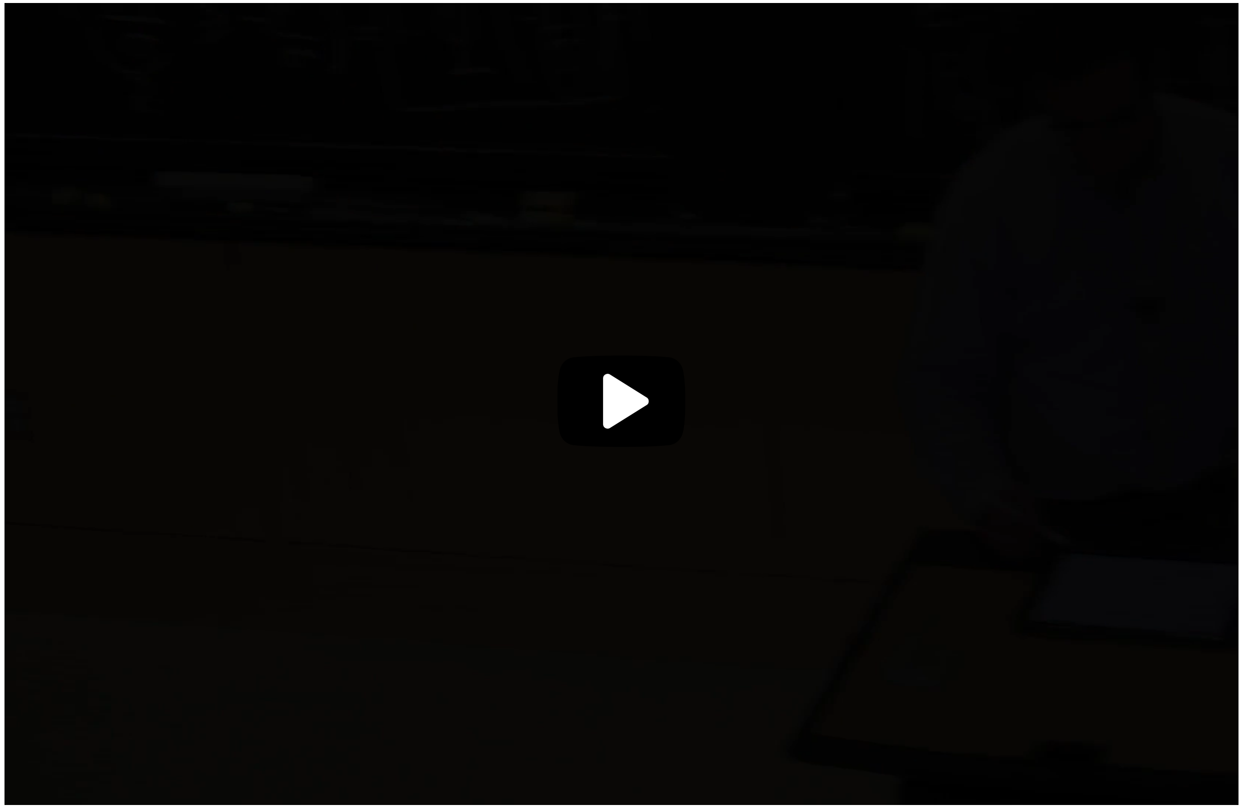
You have used 1 of 3 attempts

Answers are displayed within the problem

Identifiability



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And we know that if I write this,

I end up with an identifiable model--

check.

But if I take this one, I don't.

So this one is not the one you want.

So it's just a matter of how you want to write your model.

There's many models that are possible.

And some are identifiable.

And some are not.

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Identifiability of Statistical Models

1/1 point (graded)

Let $\{P_\theta\}_{\theta\in\Theta}$ denote a family of distributions that depends on an unknown parameter $\theta \in \Theta$.

Recall that the parameter θ is **identifiable** if the map $\theta \mapsto P_\theta$ is injective. Here, the notation $\theta \mapsto P_\theta$ denotes a function that takes as input $\theta \in \Theta$ and outputs a probability distribution P_θ. In other words, if $\theta \neq \theta'$ (and both in Θ), then $P_\theta \neq P_{\theta'}$

Which of the following families of distributions has an identifiable parameter? (Choose all that apply.)

- [x] $\{\mathrm{Ber}\,(p)\}_{p\in[0,1]}$ ✔
- [ ] $\{\mathrm{Ber}\,(p^2)\}_{p\in[-1,1]}$
- [x] $\{\mathrm{Ber}\,(\sin\,(p))\}_{p\in[0,\frac{\pi}{2}]}$ ✔
- [ ] $\{\mathrm{Ber}\,(\sin\,(p))\}_{p\in[0,\pi]}$

✔

Solution:

Remark: A family of distributions $\{\mathrm{Ber}\,(f\,(p))\}_{p\in S}$ (here $S \subset \mathbb{R}$ is a set where the parameter p lives) has the parameter p identified if and only if the function $f\,(p)$ is injective.
The function $f\,(p) = p$ is injective on the interval $[0, 1]$, so the first choice $\{\mathrm{Ber}\,(p)\}_{p\in[0,1]}$ is correct. However, the function

$f(p) = p^2$ on the interval $[-1, 1]$ is not injective, so the second choice $\{\mathrm{Ber}(p^2)\}_{p\in[-1,1]}$ is incorrect.

Let's look more carefully at the last two choices, $\{\mathrm{Ber}(\sin(p))\}_{p\in[0,\frac{\pi}{2}]}$ and $\{\mathrm{Ber}(\sin(p))\}_{p\in[0,\pi]}$. Observe that the function $f(p) = \sin(p)$ *is* injective on the interval $[0, \frac{\pi}{2}]$ but *is not* injective on the interval $[0, \pi]$. Hence, $\{\mathrm{Ber}(\sin(p))\}_{p\in[0,\frac{\pi}{2}]}$ has an identified parameter, but $\{\mathrm{Ber}(\sin(p))\}_{p\in[0,\pi]}$ does not have an identified parameter.

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Answers are displayed within the problem

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? Only 1 attempt for part (2)?

Just making sure that 1 attempt was intentional, given that most other problems allow 2 or 3 attempts?

2

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