



MITx: 6.041x Introduction to Probability - The Science of Uncertainty



Bookmarks

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- ▶ Entrance Survey
- ▶ Unit 1: Probability models and axioms
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Unit overview

Lec. 5: Probability mass functions and expectations

Exercises 5 due Mar 02, 2016 at 23:59 UTC

Lec. 6: Variance; Conditioning on an event; Multiple r.v.'s

Exercises 6 due Mar 02, 2016 at 23:59 UTC

Lec. 7: Conditioning on a random variable; Independence of r.v.'s

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Bookmark

Exercise: A criterion for independence

(1/1 point)

Suppose that the conditional PMF of \mathbf{X} , given $\mathbf{Y} = \mathbf{y}$, is the same for every \mathbf{y} for which $p_Y(\mathbf{y}) > 0$. Is this enough to guarantee independence?

Yes Answer: Yes

Answer:

The condition given means that when I tell you the value of \mathbf{Y} , the conditional PMF of \mathbf{X} will be the same. Thus, the value of \mathbf{Y} makes no difference, and, intuitively, we have independence.

For a formal argument, let $c(\mathbf{x}) = p_{\mathbf{X}|\mathbf{Y}}(\mathbf{x} | \mathbf{y})$; we can define $c(\mathbf{x})$ this way (without a dependence on \mathbf{y}) since we are assuming that $p_{\mathbf{X}|\mathbf{Y}}(\mathbf{x} | \mathbf{y})$ is the same for all \mathbf{y} . Now,

$$p_{\mathbf{X},\mathbf{Y}}(\mathbf{x}, \mathbf{y}) = p_Y(\mathbf{y})p_{\mathbf{X}|\mathbf{Y}}(\mathbf{x} | \mathbf{y}) = p_Y(\mathbf{y})c(\mathbf{x}).$$

Summing over all \mathbf{y} , we obtain

$$p_X(\mathbf{x}) = \sum_{\mathbf{y}} p_{\mathbf{X},\mathbf{Y}}(\mathbf{x}, \mathbf{y}) = \sum_{\mathbf{y}} p_Y(\mathbf{y})c(\mathbf{x}) = c(\mathbf{x}).$$

Therefore, $c(\mathbf{x}) = p_X(\mathbf{x})$. It follows that

$$p_{\mathbf{X},\mathbf{Y}}(\mathbf{x}, \mathbf{y}) = p_{\mathbf{X}|\mathbf{Y}}(\mathbf{x} | \mathbf{y})p_Y(\mathbf{y}) = c(\mathbf{x})p_Y(\mathbf{y}) = p_X(\mathbf{x})p_Y(\mathbf{y}),$$

which establishes independence.

You have used 1 of 1 submissions

Exercises 7 due Mar
02, 2016 at 23:59 UTC

Solved problems

**Additional
theoretical
material**

Problem Set 4

Problem Set 4 due Mar
02, 2016 at 23:59 UTC

Unit summary

- ▶ Unit 5:
Continuous
random
variables

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