Puzzle solved?

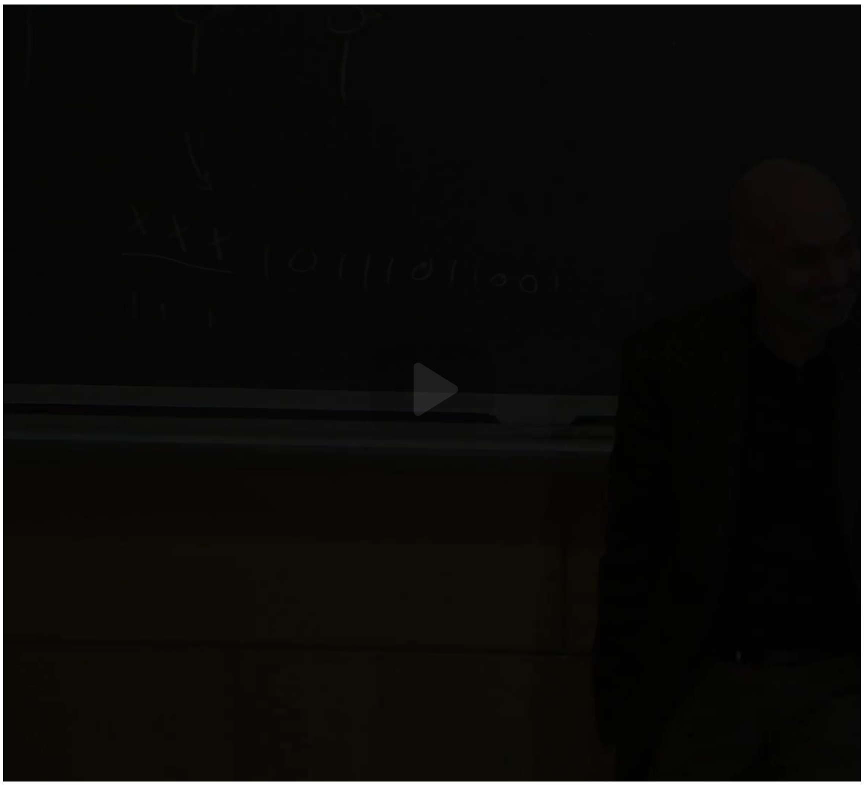
We have succeeded in identifying a strategy that guarantees that at most finitely many people will guess incorrectly.

Unfortunately, our success leads to paradox. For there is a seemingly compelling argument for the conclusion that it should be *impossible* for there to be such a strategy:

> We know that a random process will be used to decide what kind of hat to place on each person. Let us imagine that it works as follows. Each person has an "assistant". While the person closes his or her eyes, the assistant flips a coin. If the coin lands Heads, the assistant places a red hat on the person's head. If the coin lands Tails, the assistant places a blue hat on the person's head.
>
> We shall assume that assistants always use fair coins. Accordingly, the probability that a coin toss lands Heads is exactly 50%, independently of how other coin tosses might have landed. This means, in particular, that knowing the colors of the hats ahead of you gives you *no information whatsoever* about the color of your own hat. So even after you've seen the colors of the hats of everyone in front of you, you should assign a probability of 50% to the proposition that your own hat is red.
>
> This seems to entail that that none of the $P_1, P_2, P_3, \ldots$ could have a better than 50% chance of correctly guessing the color of their hat. But if this is so, it should be impossible for there be a strategy that guarantees that the vast majority of $P_1, P_2, P_3, \ldots$ answers correctly.

Since we have found such a strategy, we know that this argument must go wrong somewhere. To solve the paradox, we must understand where the argument goes wrong.

Video Review: There is Still Something Puzzling



1:33 / 1:33 1.50x

one

could raise the probability of guessing correctly beyond 50%.

And if they don't, how could they possibly

guarantee that most people are going to be saved?

All but a tiny fragment-- so it would

be infinitely many people.

All but a finite number are going to be saved, guaranteed.

So what's going on?

Video

Download video file

Transcripts

Download SubRip (.srt) file

Download Text (.txt) file

Towards a Solution

Let me begin with a preliminary observation. There is a difference between saying that the group increases its probability of *collective* success and saying that each member of the group increases her own probability of success. To see this, imagine that the group has two options:

Strategy each member of the group follows the strategy outlined above

Coin Toss each member of the group makes her decision by tossing a coin

Of the uncountably many outcomes that Coin Toss allows, only countably many are such that at most finitely many people answer incorrectly. So one should expect that following Strategy will lead to a better collective outcome than following Coin Toss.

On the other hand, it is not at all clear that choosing Strategy over Coin Toss would increase the probability that any individual member of the group will answer correctly. To see this, suppose that the group decides to follow Strategy, and imagine yourself in P_k's position. Although you can be confident that the number of people who will die is finite, you have no idea how large that number will be. All you know is that there is some number n such that, for any $m > n$, P_m will survive. Notice, moreover, that the vast majority of values that n could take---all except for the first k---are too large to offer you much comfort. So it's not clear that you should expect to be better off following Strategy than following Coin Toss.

The core of the paradox we are considering is that it seems hard to reconcile two facts. On the one hand, there are reasons for thinking that none of the $P_1, P_2, P_3, \ldots$ could have a better than 50% chance of correctly guessing the color of their hat. On the other, we have found a strategy that guarantees that the vast majority of $P_1, P_2, P_3, \ldots$ answers correctly. This would seem very puzzling indeed if the upshot of the strategy was that each individual member of the group was able to increase her probability of success 50%. But we have now seen that it is not clear that this is so. All we know is that the group increases its probability of *collective* success.

To fully answer the paradox, however, we need an understanding of how it is that the group is able to increase its probability of collective success. To this we now turn.

Problem 1

1/1 point (ungraded)

P_k is feeling pretty good about her chances of surviving because of the following thought:

> Of the infinitely many people in the group, there are only finitely many—an incredibly small minority—who will die.

Is P_k right to feel optimistic on the basis of this thought?

Absolutely!

It's more complicated than that...

Explanation

It is not at all clear that she should feel optimistic. For consider the following argument:

> Although it is certainly true that the number of people who will die is finite, there is also no limit to the size of that number. We know that there is some number n such that, for any $m > n$, P_m will survive. But from the point of view of any given member of the group, the vast majority of values that n could take—all except for the first k—are too large to offer much comfort.

Submit

Answers are displayed within the problem

Discussion

Hide Discussion

Topic: Week 3 / Puzzle solved?

Add a Post

Show all posts by recent activity

Video: minute 1:03

"But because they acquire no additional information ....". Maybe I got it wrong, but we were pr... 1