

MITx: 6.008.1x Computational Probability and Inference

Heli

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Exercise: The Sum-Product Algorithm - A Numerical Calculation

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Exercise: The Sum-Product Algorithm - A Numerical Calculation

3/3 points (graded)

Consider a chain graph $X_1 \leftrightarrow X_2 \leftrightarrow X_3$, with the following edge potentials:

| ψ_{12} | | X | ζ_2 | ψ_{23} | | X | ζ_3 |
|-------------|---|---|-----------|-------------|---|---|-----------|
| | | 0 | 1 | | | 0 | 1 |
| V_{\perp} | 0 | 5 | 1 | V | 0 | 0 | 1 |
| Λ_1 | 1 | 1 | 5 | X_2 | 1 | 1 | 0 |

The node potentials are just functions that always output 1.

• (a) Compute p_{X_1,X_3} . What is $p_{X_3|X_1}(0|0)$? For this part, do the calculation without using the sumproduct algorithm, using what you learned from the first part of the course. Note that at times, you may have unnormalized quantities that you normalize (to sum to 1) at the end, which is fine.

(Please be precise with at least 3 decimal places, unless of course the answer doesn't need that many decimal places. You could also put a fraction.)

Exercises due Oct 27, 2016 at 02:30 IST

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G.

Week 6: Special Case: Marginalization in Hidden Markov Models

Exercises due Oct 27, 2016 at 02:30 IST

Week 6: Homework 5

<u>Homework due Oct 27, 2016 at 02:30 IST</u>

Weeks 6 and 7: Mini-project on Robot Localization

Mini-projects due Nov 03, 2016 at 02:30 IST

ullet (b) Let's use the sum-product algorithm to show what the probability of $X_3=0$ is given that $X_1=0$.

Start this problem by incorporating the observation to create a new graphical model that only has 2 nodes. Then compute the message $m_{2\rightarrow 3}$ (remember that this is a table). What is the table? In providing your answer, please normalize the message table so that its entries sum to 1.

(Please be precise with at least 3 decimal places, unless of course the answer doesn't need that many decimal places. You could also put a fraction.)

$$m_{2 o 3}(0)=$$
 1/6 \checkmark Answer: 1/6

Use the last part of the sum-product algorithm that computes the marginal distribution using incoming messages. You should verify for yourself that the marginal distribution that you get for X_3 (in the graph that already accounts for conditioning on $X_1=0$) agrees with your answer to part (a) for the value of $p_{X_3|X_1}(0|0)$.

Solution:

• (a) Compute p_{X_1,X_3} . What is $p_{X_3|X_1}(0|0)$? For this part, do the calculation without using the sumproduct algorithm, using what you learned from the first part of the course. Note that at times, you may have unnormalized quantities that you normalize (to sum to 1) at the end, which is fine.

Solution: We have

$$egin{align} p_{X_1,X_3}(x_1,x_3) &=& \sum_{x_2} p_{X_1,X_2,X_3}(x_1,x_2,x_3) \propto \overbrace{\sum_{x_2} \psi_{12}(x_1,x_2) \psi_{23}(x_2,x_3)}^{ ext{this is matrix multiplication}} \ &=& \psi_{12}(x_1,0) \psi_{23}(0,x_3) + \psi_{12}(x_1,1) \psi_{23}(1,x_3). \end{array}$$

Thus, p_{X_1,X_3} is proportional to the following table (you could get this from matrix multiplication or by brute force):

| | | | X_3 | | |
|---|-------------|---|-----------------------------|---|--|
| | | | 0 | 1 | |
| | Y | 0 | $5 \cdot 0 + 1 \cdot 1 = 1$ | $5 \cdot 1 + 1 \cdot 0 = 5$ $1 \cdot 1 + 5 \cdot 0 = 1$ | |
| - | Λ_1 | 1 | $1 \cdot 0 + 5 \cdot 1 = 5$ | $1 \cdot 1 + 5 \cdot 0 = 1$ | |

Upon normalization, we procure p_{X_1,X_3} :

| p_{X_1,X_3} | | X | -3 |
|--------------------|---|------|------|
| | | 0 | 1 |
| $oldsymbol{V}_{-}$ | 0 | 1/12 | 5/12 |
| Λ_1 | 1 | 5/12 | 1/12 |

Thus,

$$p_{X_3|X_1}(0|0) = rac{1/12}{1/12 + 5/12} = 1/6.$$

ullet (b) Let's use the sum-product algorithm to show what the probability of $X_3=0$ is given that $X_1=0$.

Solution: We have

which is actually just a two node graph over X_2 and X_3 . The node potential for X_2 is $\widetilde{\phi}_2$, the edge potential is ψ_{23} , and the node potential for X_3 is identically 1. Computing a single message is sufficient:

$$m_{2 o 3}(x_3) = \sum_{x_2} \underbrace{\psi_{12}(0,x_2)}_{ riangle ilde{\phi}_2(x_2)} \psi_{23}(x_2,x_3) = 5 \psi_{23}(0,x_3) + 1 \psi_{23}(1,x_3),$$

which, in vector form, corresponds to 5 times the first row of table ψ_{23} (corresponding to $X_2=0$) plus 1 times the second row of table ψ_{23} (corresponding to $X_2=1$):

$$m_{2\to 3} = 5 \cdot (0 \quad 1) + (1 \quad 0) = (1 \quad 5).$$

Thus, normalizing so the message table sums to 1:

•
$$m_{2 o 3}(0)=$$
 $\boxed{1/6}$

•
$$m_{2 o 3}(1)=\left\lceil 5/6 \right
ceil$$

Note that we can actually always normalize messages in this way because at the very end when we compute the marginal distributions, we will re-normalize anyways!

Then the node marginal at $X_{\mathbf{3}}$ is

$$p_{X_3}(x_3) \propto \phi_3(x_3) m_{2 o 3}(x_3) = m_{2 o 3}(x_3) = (1/6 \quad 5/6)\,,$$

from which we conclude that $p_{X_3|X_1}(0|0)=1/6$ (here we re-introduce X_1 into the notation, remembering that the 2 node graphical model we are working with actually was conditioned on $X_1=0$).

Note that if we did not normalize the message table, then that is fine too since in the last step, we would instead have

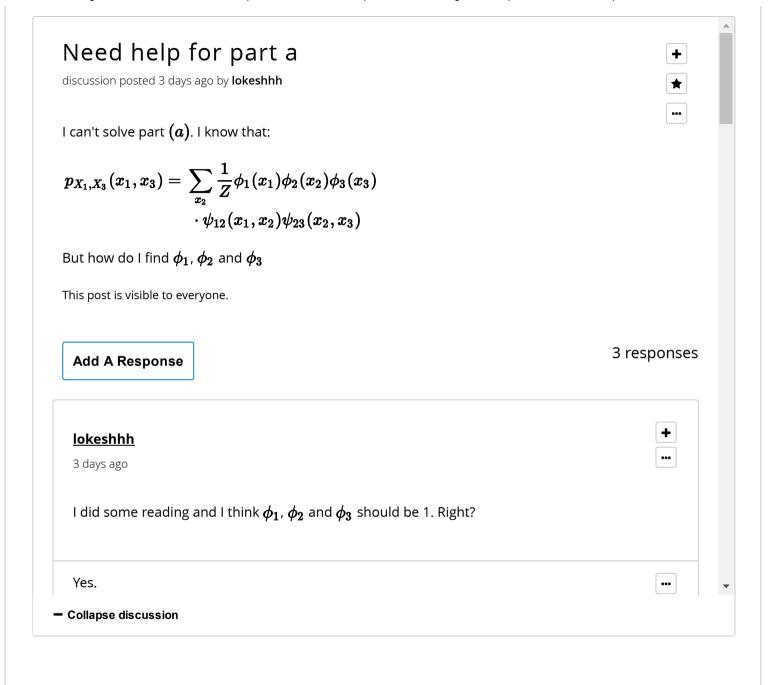
$$p_{X_3}(x_3) \propto \phi_3(x_3) m_{2 o 3}(x_3) = m_{2 o 3}(x_3) = (1 \quad 5) \, ,$$

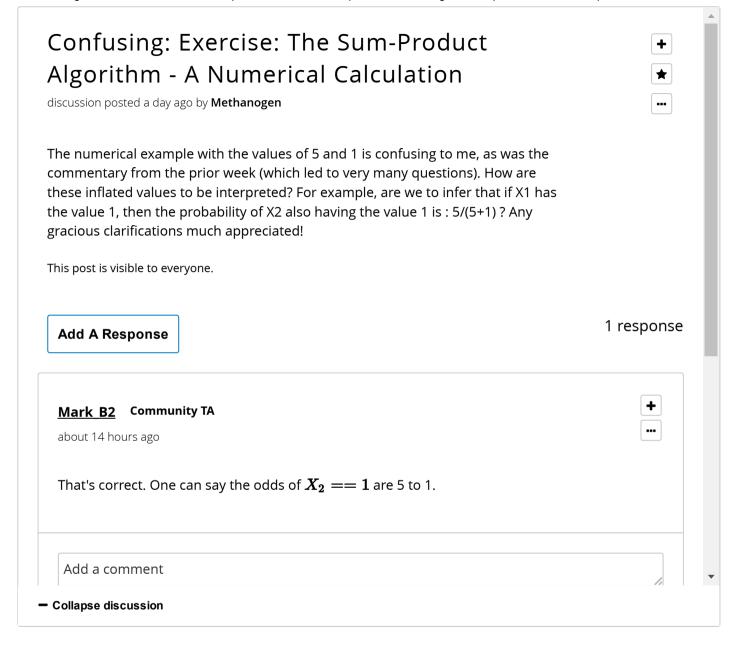
at which point we would normalize anyways to get the same answer for what the marginal p_{X_3} at X_3 is, again where it's actually the posterior distribution $p_{X_3|X_1}(\cdot \mid 0)$.

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You have used 2 of 5 attempts

| Correct (3/3 points) | |
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part b: Incorporating the observation?

discussion posted 3 days ago by ripande

The problem says: "Start this problem by incorporating the observation to create a new graphical model that only has 2 nodes." I am not sure...

This post is visible to everyone.

+ Expand discussion

Need help in understanding this

discussion posted 2 days ago by ripande

$$P(X2, X3|X1 = 0) \propto \psi_{1,2} (X1 = 0, X2) * \psi_{2,3} (X2, X3)$$

But is it not the case that it is P(X2,X3,X1=0) that is proportional...

This post is visible to everyone.

+ Expand discussion

Numerical example for Sum-Product algorithms

discussion posted 4 days ago by nar3k

Hey guys!

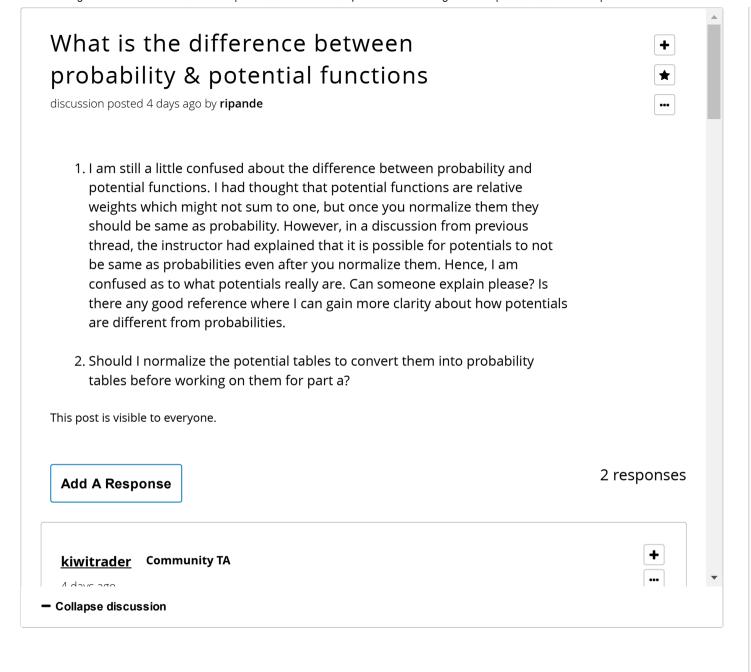
For people who like to see examples there is a presentations where you can find it

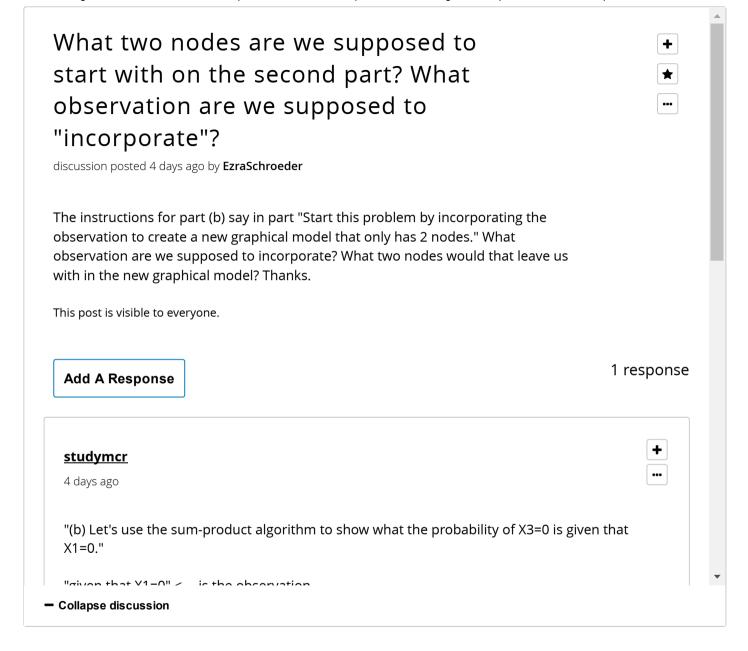
http://www.cse.psu.edu/~rtc12/CSE586/lectures/cse586GMplusMP_6pp.pdf

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