Distinct Eigenvalues and Linearly Independent Eigenvectors

I understand that if $v_1, \ldots, v_r$ are the eigenvectors that correspond to distinct eigenvalues then they are linearly independent (*)

However what if I have say two linearly independent eigenvectors corresponding to **one** eigenvalue and an eigenvector corresponding to another, with A a 3x3 matrix and $Av = \lambda v$. Are these three eigenvectors linearly independent? Does this follow from (*)?

(linear-algebra)

asked Aug 23 '15 at 21:40
usainlightning
164 2 15

1 Answer

The answer is yes. Let's assumme that v_1, v_2 are the eigenvectors that correspond to the same eigenvalue α_1. Observe that $\lambda_1 v_1 + \lambda_2 v_2$ is an eigenvector for the eigenvalues α_1 and is therefore linearly independent of our third vector v_3. This means that if $\lambda_1 v_1 + \lambda_2 v_2 + \lambda_3 v_3 = 0$ we necessarily have $\lambda_3 = 0$. Now this implies $\lambda_1 v_1 + \lambda_2 v_2 = 0$, which by assumption yields $\lambda_1 = \lambda_2 = 0$.

Note that this is nothing else than observing that the sum of eigenspaces to different eigenvalues is a direct sum.

edited Aug 23 '15 at 22:28

answered Aug 23 '15 at 21:50
Dominik
17.3k 1 17 44

Why is $\lambda_1 v_1 + \lambda_2 v_2$ linearly independent of v_3? – usainlightning Aug 23 '15 at 22:07

1 v_3 is meant to be an eigenvector to an eigenvalue $\alpha_2 \neq \alpha_1$, so this follows from the lemma (*) in your post. – Dominik Aug 23 '15 at 22:09

Where you wrote $\lambda_2 v_2 + \lambda_2 v_2 + \lambda_3 v_3 = 0$ I assume you meant something else? – usainlightning Aug 23 '15 at 22:27

Indeed, there were some mistakes in the indices. I've corrrected it now. – Dominik Aug 23 '15 at 22:29

I don't understand why setting $\lambda_1 v_1 + \lambda_2 v_2 + \lambda_3 v_3 = 0$ implies that $\lambda_3 = 0$ – usainlightning Aug 23 '15 at 23:00

It's a linear combination of two eigenvectors for different eigenvalues. – Dominik Aug 24 '15 at 6:30