The Strategy

Our strategy relies on getting $P_1, P_2, \ldots$ to agree in advance on a "representative" from each cell.

So, for instance, they might agree that $\langle 0, 1, 0, 0 \ldots \rangle$ is to be the representative for the cell that contains $\langle 0, 0, 0, 0 \ldots \rangle$. (The ability to pick a representative from each cell presupposes the Axiom of Choice, which we'll discuss further in Lectures 7 and 8.)

Imagine that $P_1, P_2, \ldots$ are all lined up, and that they've agreed on a choice of representatives. Each of them knows her position in line and can see the hat colors of everyone ahead of her.

Now consider person P_k. She can see the colors of the hats of $P_{k+1}, P_{k+2}, P_{k+3}, \ldots$, but not of $P_1 \ldots P_k$. So she doesn't know exactly what the actual assignment of hats to persons is. But, crucially, she is in a position to determine which *cell* contains the sequence of zeroes and ones corresponding to that assignment. To see this, suppose that P_k sees the following:

$$\begin{array}{ccccccccc} P_1 & P_2 & \ldots & P_k & P_{k+1} & P_{k+2} & P_{k+3} & P_{k+4} & \ldots \\ \downarrow & \downarrow & \ldots & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \ldots \\ ? & ? & \ldots & ? & \text{red} & \text{blue} & \text{red} & \text{red} & \ldots \end{array}$$

Let her fill in the missing information arbitrarily—using "blue", for example:

$$\begin{array}{ccccccccc} P_1 & P_2 & \ldots & P_k & P_{k+1} & P_{k+2} & P_{k+3} & P_{k+4} & \ldots \\ \downarrow & \downarrow & \ldots & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \ldots \\ \mathit{blue} & \mathit{blue} & \ldots & \mathit{blue} & \text{red} & \text{blue} & \text{red} & \text{red} & \ldots \end{array}$$

Since the resulting assignment of hat-colors differs from the actual assignment at most in the first k positions, the corresponding sequences of zeroes and ones must belong to the same cell.

So P_k has all the information she needs to identify the relevant cell.

And, of course, everyone else is in a position to use a similar technique.

So even though each of $P_1, P_2, \ldots$ has incomplete information about the distribution of hat-colors, they are each in a position to know which cell contains the ω-sequence representing the actual hat distribution. Call this cell O, and let r_O be the representative that had been previously agreed upon for O. Let the group agree on the following strategy:

> Everyone is to answer their question on the assumption that the actual sequence of hats is correctly described by r_O.

In other words, P_k will cry out "Red!" if r_O contains a zero in its kth position, and she will cry out "Blue!" if r_O contains a one in its kth position. Because r_O and the sequence corresponding to the actual hat distribution are members of the same cell, we know they differ in at most finitely many positions. So, as long as everyone conforms to the agreed-upon strategy, at most finitely many people will guess incorrectly. QED.

(It goes without saying that this strategy presupposes that every prisoner has super-human capabilities. For instance, they must be able to absorb information about infinitely many hat colors in a finite amount of time, and they must be able to pick representatives from sets with no natural ordering. This entails that no actual human could implement this strategy. But what that matters for present purposes is that the strategy exists. Recall that we are interested in logical possibility, not medical possibility.)

Video Review: The Strategy

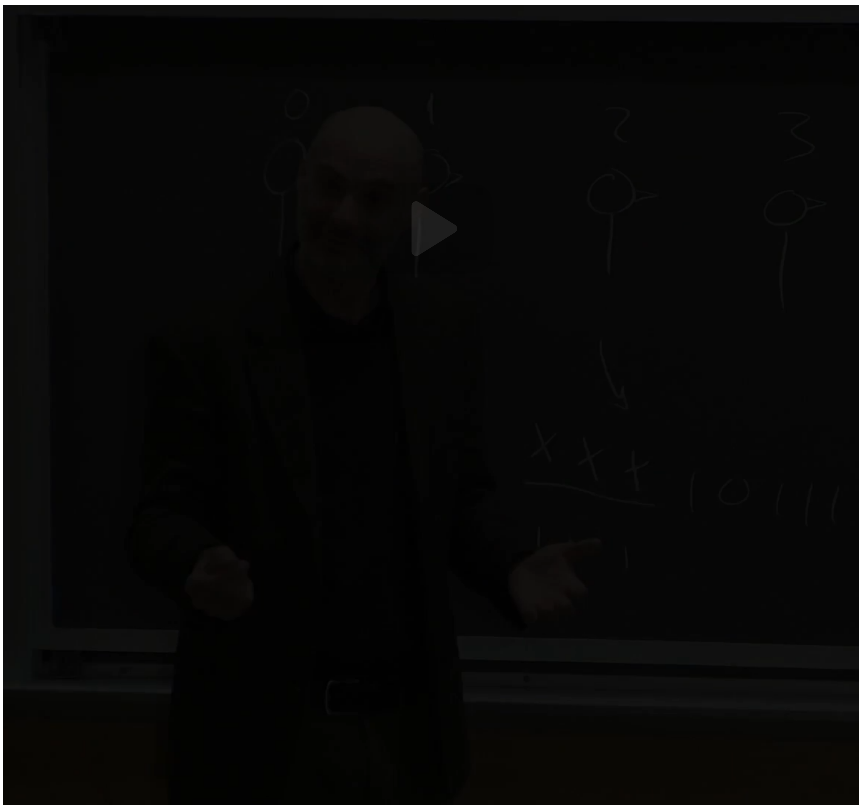
So if everyone chooses in accordance

with the representative, only finitely many people

are going to get it wrong, because there are only

finitely many spots where the two sequences disagree.

So at most, finitely many



people are going to get shot.

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I must be wrong about something... (resolved)

question posted 5 days ago by LHP22

So: Hypothetically, they still don't know what their hat is so isn't it hypothetically POSSIBLE for all of them to get it wrong?

Edit: Because they are able to identify the cell it is in, and because the cell's members are only finitely different it is impossible for infinitely many people to die because there are only a finite number of differences. So even though it doesn't seem like anyone has any more information, they collectively have reduced the number of possibilities.

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Cosmo_Grant (Staff)
4 days ago - marked as answer 4 days ago by **Cosmo_Grant** (Staff)

Question asked, question answered!

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