MITx: 15.053x Optimization Methods in Business Analytics

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PART A

Suppose that you are interested in choosing a set of investments, labeled for simplicity $\{1, \ldots, 7\}$. Let x_i be a binary variable representing whether or not we choose the i^{th} investment.

Model the following constraints using integer and linear constraints. (Other nonlinear constraints are not permitted.) In each part of this problem, we will have four choices, and two of the answers will be (alternative) correct answers. Select both of the correct answers.

You cannot invest in all of them.

For every "practice problem" in 15.053x, you may have an unlimited number of attempts and checks. You have the option of revealing the answer any time after the first guess.

- [x] $x_1 + 2x_2 + 2x_3 + 2x_4 + 2x_5 + 2x_6 + 2x_7 \leq 12$ ✔
- [ ] $x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 \leq 7$
- [ ] $x_1 + x_2 + x_3 + x_4 + x_5 + x_6 \leq 6$
- [x] $x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 \leq 6$ ✔

✔

EXPLANATION

Solution

- $x_1 + 2x_2 + 2x_3 + 2x_4 + 2x_5 + 2x_6 + 2x_7 \leq 12$
- $x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 \leq 6$

  The point of Problem 1 is that there are multiple ways of representing integer programs. Some are better in practice because they help the algorithm to find the optimum faster. But that topic is too vast to be included here. For this instance, the constraint $x_1 + 2x_2 + 2x_3 + 2x_4 + 2x_5 + 2x_6 + 2x_7 \leq 12$ is valid because the only integral solution that is forbidden by the constraint is the "selecting all investments".

PART B

Investment 1 cannot be chosen if investment 3 is chosen. Select two correct answers from the following four choices.

- ☐ $x_1 - x_3 \leq 0$
- ☑ $x_1 + x_3 \leq 1$ ✔

- [x] $3x_1 + 4x_3 \leq 6$ ✔

- [ ] $2x_1 + x_3 \leq 3$

✔

EXPLANATION

Solution

- $x_1 + x_3 \leq 1$
- $3x_1 + 4x_3 \leq 6$

  This is similar to Part A. The only solution that is forbidden is the one in which /(x_1 = x_3 = 1 \). Constraints (2) and (3) only eliminate this one solution.

PART C

Investment 4 can be chosen only if investment 2 is also chosen. Select two correct answers from the following four choices.

- [x] $-x_2 + x_4 \leq 0$ ✔

- [ ] $x_2 + x_4 \leq 1$
- [ ] $x_2 + x_4 \geq 1$
- [x] $x_2 - x_4 \geq 0$ ✔

✔

EXPLANATION

Solution

- $-x_2 + x_4 \leq 0$
- $x_2 - x_4 \geq 0$

  In this case, constraint (4) is constraint (1) multiplied by -1. They are equivalent linear inequalities.

PART D

You must choose either at least one of the investments 1, 2, 3, or at least two investments from 4, 5, 6, 7 or both. Select two correct answers from the following four choices.

- [ ] $x_1 + x_2 + x_3 + 2x_4 + 2x_5 + 2x_6 + 2x_7 \geq 2$
- [x] $2x_1 + 2x_2 + 2x_3 + x_4 + x_5 + x_6 + x_7 \geq 2$ ✔
- [ ] $2x_1 + 2x_2 + 2x_3 + 3x_4 + 3x_5 + 3x_6 + 3x_7 \geq 3$
- [x] $3x_1 + 3x_2 + 3x_3 + 2x_4 + 2x_5 + 2x_6 + 2x_7 \geq 3$ ✔

✔

EXPLANATION

Solution

- $2x_1 + 2x_2 + 2x_3 + x_4 + x_5 + x_6 + x_7 \geq 2$
- $3x_1 + 3x_2 + 3x_3 + 2x_4 + 2x_5 + 2x_6 + 2x_7 \geq 3$

The second constraint will be satisfied if any of $x_1, x_2, \text{ and } x_3$ are 1. It would also be satisfied if two of $x_4, x_5, x_6, \text{ and } x_7$ are 1. The same is true of the fourth constraint.

PART E

You must choose both of investments 1 and 2, or choose investment 4, or choose all three of these investments. Select two correct answers from the following four choices.

- [x] $x_1 + x_2 + 2x_4 \geq 2$ ✔
- [ ] $x_1 x_2 + x_4 \geq 1$
- [ ] $x_1 + x_2 + x_4 \geq 2$
- [x] $3x_1 + 4x_2 + 7x_4 \geq 7$ ✔

✔

EXPLANATION

Solution

- $x_1 + x_2 + 2x_4 \geq 2$
- $3x_1 + 4x_2 + 7x_4 \geq 7$

This problem is similar to PART D, and a similar explanation to the one in PART D is valid.