8. Augmented matrix and row operations

Example 8.1

Consider the following system.

$$\begin{aligned} 8y - 4z &= 0 \\ x - y + 4z &= 1 \\ -x - 5y + 2z &= 0, \end{aligned}$$

We can encode all the information of this system in this **augmented matrix,** which is a new matrix with one extra column formed by putting the vector $\mathbf{b}$ on the right side of the matrix $\mathbf{A}$:

$$\left(\begin{array}{c|c} \mathbf{A} & \mathbf{b} \end{array} \right) = \left(\begin{array}{ccc|c} 0 & 8 & -4 & 0 \\ 1 & -1 & 4 & 1 \\ -1 & -5 & 2 & 0 \end{array} \right).$$

Equation Operations

One way to solve a linear system is by performing the following operations repeatedly, in some order:

- Multiply an equation by a nonzero number.
- Interchange two equations.

- Add a multiple of one equation to another equation.

Each of these operations are valid because they don't change the solution set.

Row operations

The equation operations correspond to operations on the augmented matrix, called **elementary row operations** :

- Multiply a row by a nonzero number.
- Interchange two rows.
- Add a multiple of one row to another row (while leaving the first row as it was).

We can do the following pairs of corresponding equation and row operations on the system and the augmented matrix:

1. Multiply an equation by a nonzero number.

$$\begin{aligned} (3)(8)y + (3)(-4)z &= (3)(0) \\ x - y + 4z &= 1 \\ -x - 5y + 2z &= 0, \end{aligned}$$

1. Multiply a row by a nonzero number.

$$\left(\begin{array}{ccc|c} (3)(0) & (3)(8) & (3)(-4) & (3)(0) \\ 1 & -1 & 4 & 1 \\ -1 & -5 & 2 & 0 \end{array}\right).$$

2. Interchange two equations.

$$\begin{aligned} x - y + 4z &= 1 \\ 8y - 4z &= 0 \\ -x - 5y + 2z &= 0, \end{aligned}$$

2. Interchange two rows.

$$\left(\begin{array}{ccc|c} 1 & -1 & 4 & 1 \\ 0 & 8 & -4 & 0 \\ -1 & -5 & 2 & 0 \end{array}\right).$$

3. Add a multiple of one equation to another equation.

Adding 2 times the second equation to the third one we get:

$$\begin{aligned} 8y - 4z &= 0 \\ x - y + 4z &= 1 \\ x - 7y + 10z &= 2 \end{aligned}$$

3. Add a multiple of one row to another row (while leaving the first row as it was).

Adding 2 times the second row to the third one we get:

$$\left(\begin{array}{ccc|c} 0 & 8 & -4 & 0 \\ 1 & -1 & 4 & 1 \\ 1 & -7 & 10 & 2 \end{array}\right).$$

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