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10. The Fourier series of the square

<u>Course</u> > <u>Unit 1: Fourier Series</u> > <u>1. Introduction to Fourier Series</u> > wave

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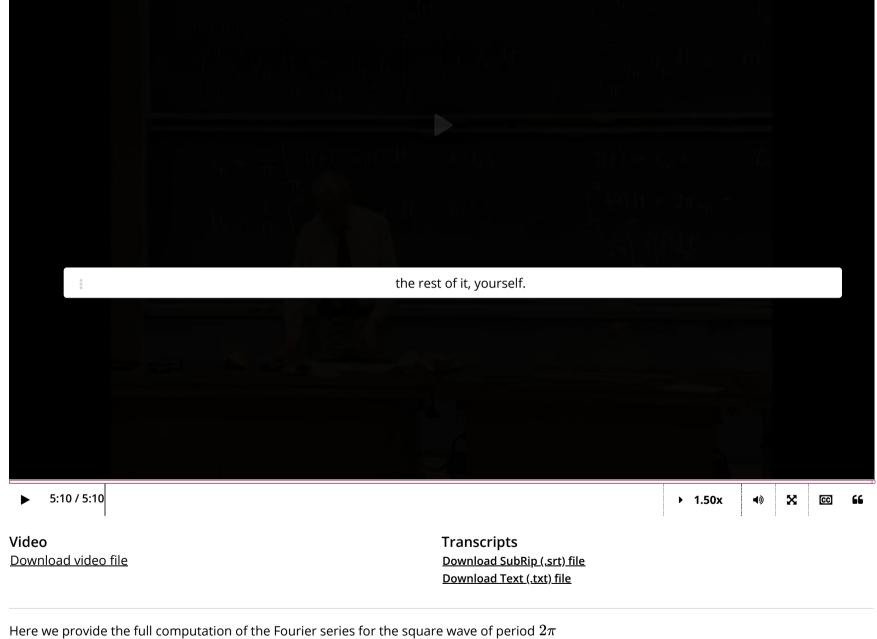
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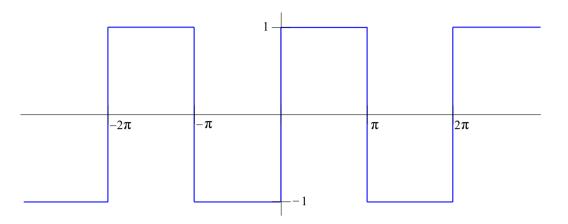
10. The Fourier series of the square wave

Video note: In the example video below, there is an error. Find the error. **Warning,** this is the most common error you will make in computing Fourier coefficients!

Square wave example



$$\mathrm{Sq}\left(t
ight) := \left\{egin{array}{ll} 1, & ext{if } 0 < t < \pi, \ -1 & ext{if } -\pi < t < 0. \end{array}
ight.$$



First we compute the constant term,

$$rac{a_0}{2} = rac{ ext{Signed area of one period}}{2\pi} = rac{\pi - \pi}{2\pi} = 0.$$

Next we compute the rest of the Fourier coefficients a_n , n>0.

$$egin{aligned} a_n &=& rac{1}{\pi}igg(\int_{-\pi}^{\pi}\operatorname{Sq}\left(t
ight)\cos\left(nt
ight)\,dtigg) \ &=& rac{1}{\pi}igg(-\int_{-\pi}^{0}\cos\left(nt
ight)\,dt+\int_{0}^{\pi}\cos\left(nt
ight)\,dtigg) \ &=& rac{1}{\pi}igg(-rac{\sin\left(nt
ight)}{n}igg|_{-\pi}^{0}+rac{\sin\left(nt
ight)}{n}igg|_{0}^{\pi}igg)=0 \end{aligned}$$

$$egin{array}{lll} b_n &=& rac{1}{\pi}igg(\int_{-\pi}^{\pi}\mathrm{Sq}\left(t
ight)\sin\left(nt
ight)\,dtigg) \ &=& rac{1}{\pi}igg(-\int_{-\pi}^{0}\sin\left(nt
ight)\,dt+\int_{0}^{\pi}\sin\left(nt
ight)\,dtigg) \ &=& rac{1}{\pi}igg(rac{\cos\left(nt
ight)}{n}igg|_{-\pi}^{0}-rac{\cos\left(nt
ight)}{n}igg|_{0}^{\pi}igg) \end{array}$$

Note that $\cos{(n\pi)} = \cos{(-n\pi)} = (-1)^n$, therefore

$$\frac{1}{\pi} \left(\frac{\cos(nt)}{n} \Big|_{-\pi}^{0} - \frac{\cos(nt)}{n} \Big|_{0}^{\pi} \right) = \frac{1}{\pi} \left[\left(\frac{1 - (-1)^{n}}{n} \right) - \left(\frac{(-1)^{n} - 1}{n} \right) \right]$$

$$= \frac{2}{\pi} \left(\frac{1 - (-1)^{n}}{n} \right)$$

$$= \begin{cases} \frac{4}{n\pi} & n \text{ odd,} \\ 0 & n \text{ even.} \end{cases}$$

Therefore the Fourier series for the square wave is

$$\operatorname{Sq}\left(t\right)\text{ "=" }\frac{4}{\pi}\bigg(\sin t+\frac{\sin 3t}{3}+\frac{\sin 5t}{5}+\cdots\bigg)=\frac{4}{\pi}\sum_{n\text{ odd}}^{\infty}\frac{\sin nt}{n}.$$

Video error

In the video above, the error is that the factor of $1/\pi$ is missing.

<u>Hide</u>

Remark 10.1 Note that while the square wave was undefined at the jump discontinuities at every integer multiple of π , the Fourier series is well defined and is 0 at each integer multiple of π (because $\sin(n\pi) = 0$ for all n)

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