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1.2.2 Quiz: Logistic Growth Model

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Here is Verhulst's logistic growth model:

$$\frac{dP}{dt} = rP\left(1 - \frac{P}{K}\right)$$

- P is the number of fish in the total population
- t is time in years
- r is a positive constant which is the constant of proportionality
- K is a positive constant representing the carrying capacity, the total population the environment can support.

We explore this model with a concrete example. Consider a population of fish living in a pond with a carrying capacity of 100 fish, $K = 100$, and a constant of proportionality, $r = 0.4$. The logistic growth model says:

$$\frac{dP}{dt} = 0.4P\left(1 - \frac{P}{100}\right)$$

What does this model predict will happen to the fish population over time? Certainly it depends on how many fish we have right now. Let's let $t = 0$ represent this moment, and assume the population of the fish at time $t = 0$ is some value P_0. (The value P_0 is calle an **initial condition** for the differential equation.)

Your first instinct may be to try to solve the differential equation for $P(t)$, the population at time t. However, we can determine a lot about what happens to the population over time by doing a **qualitative analysis** of this differential equation. In qualitative analysis, we don't solve the differential equation. We just determine the general shapes of the solution curves to the differential equation, by using the sign of the derivative $\frac{dP}{dt}$ to tell where the population is increasing, decreasing or constant.

For example, if $P = 50$, then we consider the sign of $\frac{dP}{dt}$.

$$\frac{dP}{dt} = 0.4(50)(1 - \frac{50}{100}) > 0.$$

Since $\frac{dP}{dt}$ is positive, then $P(t)$ is increasing at that instant.

What if $\frac{dP}{dt} = 0$ for some population value P_0? This means the population does not change; the population remains at the value P_0 for all time. We say that $P(t) = P_0$ is an **equilibrium solution** of the differential equation.

> **Side note:** In case you're curious, we can solve this differential equation this using a technique called *separation of variables*. For more on this, see Separable equations introduction from Khan Academy.

Question 1

1/1 point (graded)

Use the equation $\frac{dP}{dt} = 0.4P(1 - \frac{P}{100})$ to determine what values of P make $\frac{dP}{dt} = 0$.

(Remember, these values of P are called the **equilibrium solutions** of the differential equation. Can you interpret these in terms of fish populations?)

$P = 0$ ✔

- [ ] $P = 0.4$
- [ ] $P = 1$
- [x] $P = 100$ ✔
- [ ] None of these.

✔

Explanation

Equilibrium Solution 1: One equilibrium solution is $P_0 = 0$. In this case, the fish population is extinct – there are no fish.

Equilibrium Solution 2: The other equilibrium solution is $P_0 = 100$. In this case, the population is as large as the environment can support. The population can continue at this level but cannot grow.

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Answers are displayed within the problem

Question 2

1/1 point (graded)

If P is less than the carrying capacity of 100 but greater than 0, what is the sign of $\frac{dP}{dt}$? Use the equation $\frac{dP}{dt} = 0.4P(1 - \frac{P}{100})$ to answer this.

(Does this match your intuition? Is the population increasing, decreasing or neither?)

- (•) positive ✔
- ( ) negative
- ( ) zero ($\frac{dP}{dt} = 0$)

Explanation

Positive. The derivative $\frac{dP}{dt}$ is the product of $0.4P$ and $(1 - \frac{P}{100})$. The first factor is positive since P is. The second factor $(1 - \frac{P}{100})$ is also positive, since $0 < P < 100$ implies that $0 < \frac{P}{100} < 1$.

Because dP/dt is positive, P will increase. This matches our intuition that the population should increase as long as it is less than the carrying capacity, which is the population level that the environment can support.

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Answers are displayed within the problem

Question 3

1/1 point (graded)

If P is greater than the carrying capacity of 100, what is the sign of $\frac{dP}{dt}$? Use the equation $\frac{dP}{dt} = 0.4P(1 - \frac{P}{100})$ to answer this.

(Does this match your intuition? Is the population increasing, decreasing or neither?)

- positive
- negative ✔
- zero ($\frac{dP}{dt} = 0$)

Explanation

Negative. $\frac{dP}{dt}$ is the product of $0.4P$ and $(1 - \frac{P}{100})$. The first factor is positive since P is. The second factor $(1 - \frac{P}{100})$ is negative, since $P > 100$ implies that $\frac{P}{100} > 1$, so $1 - \frac{P}{100} < 0$.

Because dP/dt is negative, P will decrease. This matches our intuition that the population should decrease if it has exceeded carrying capacity, which is the population level that the environment can support.

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Question 4

1/1 point (graded)

Suppose that the starting population P_0 (the initial condition) is large enough to reproduce but much much less than 100, so that $\frac{P_0}{100}$ is almost 0. What behavior would you expect from $P(t)$ in this case?

Use the equation $\frac{dP}{dt} = 0.4P(1 - \frac{P}{100})$ to help you answer this.

- no growth
- increase linearly
- decrease linearly
- increase exponentially ✔
- increase quadratically
- other

Explanation

In this case, $\frac{dP}{dt} = .04P\left(1 - \frac{P}{100}\right) \approx 0.4P$, since $\frac{P}{100} \approx 0$. At this point, the rate of growth is almost proportional to the size of the population, indicating exponential growth.

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Question 5

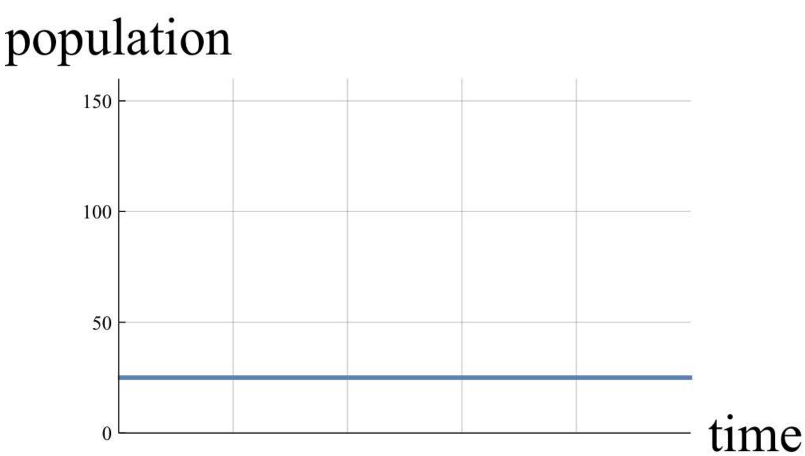
1/1 point (graded)

Suppose that we start with a population of 25 fish. This is the initial condition $P_0 = 25$. From the graphs below, choose the most plausible graph of $P(t)$, the population of fish, as a function of time t. (Hint: Use what you've learned from the differential equation for P. We're not expecting you to solve the differential equation to answer this question!)

Note: there are no values on the time axis, as we're looking for a qualitative sense of the graph of $P(t)$.

Image Description for all answers A-E

A.



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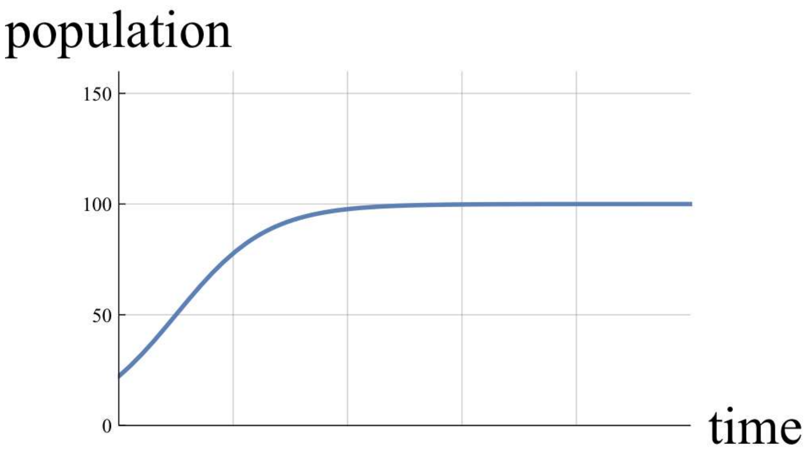

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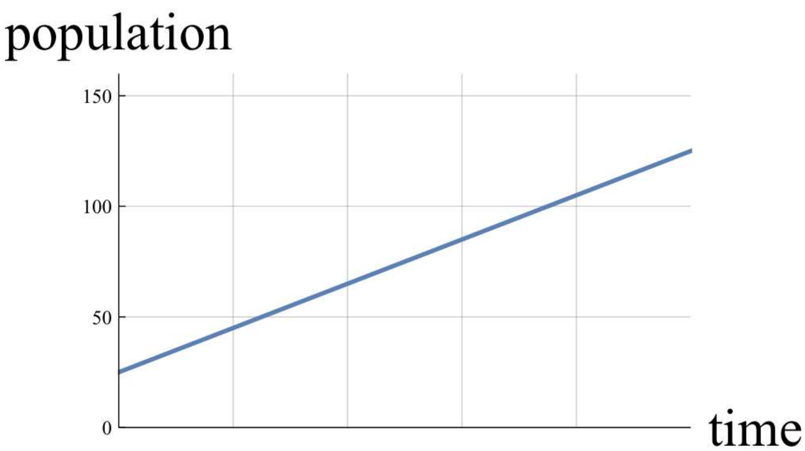
C.



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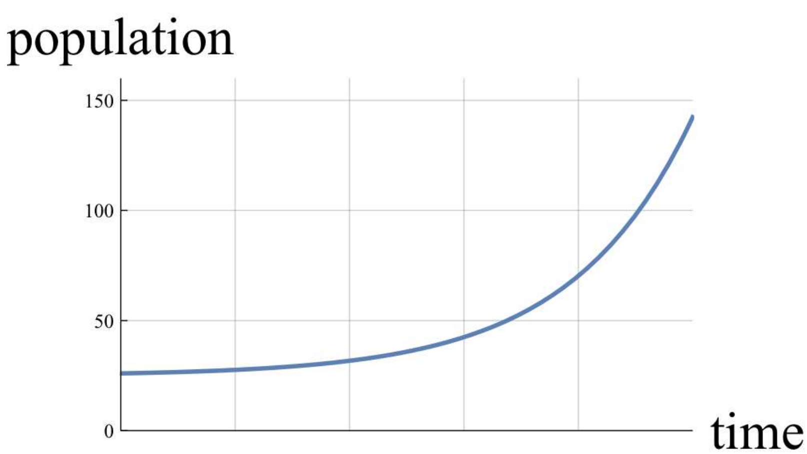
D.



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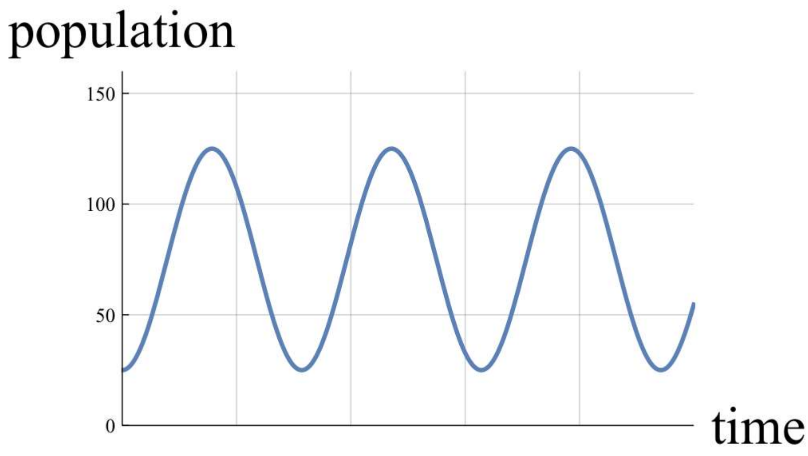
E.



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When $P = 25$, $\frac{dP}{dt}$ is positive and so P will increase. We know that the graph of $P(t)$ has positive slope whenever the value of P is less than 100. There are three graphs with this property. However, graphs C and D continue to rise as P passes 100, which does not fit with the differential equation for P. We know for populations above the carrying capacity of 100, dP/dt is negative and the population will decrease.

So the correct choice is B. Notice this graph becomes less and less steep as P approaches 100, becoming almost horizontal. This is consistent with the fact that as P approaches 100, the rate of change gets closer to zero.

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You have used 1 of 3 attempts

Answers are displayed within the problem

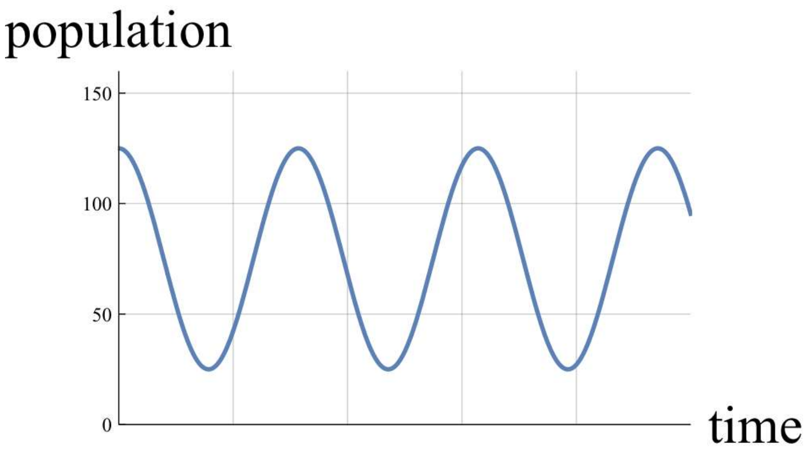
Question 6

1/1 point (graded)

Now suppose we start with a population of 125 fish, $P_0 = 125$. Of the graphs below, choose the most plausible graph for $P(t)$, the population of fish as a function of time t.

Image Description for all answers A-E

A.



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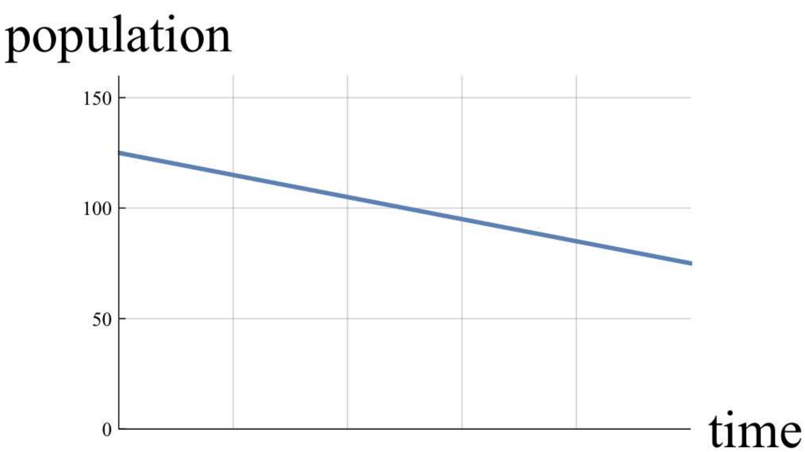
B.



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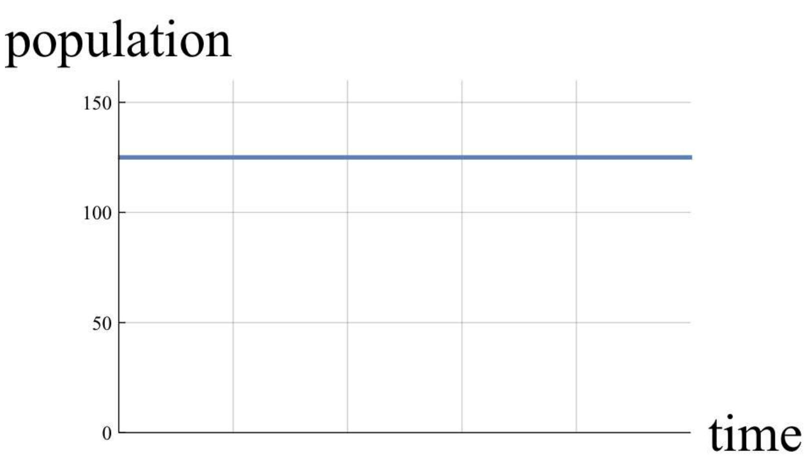
C.



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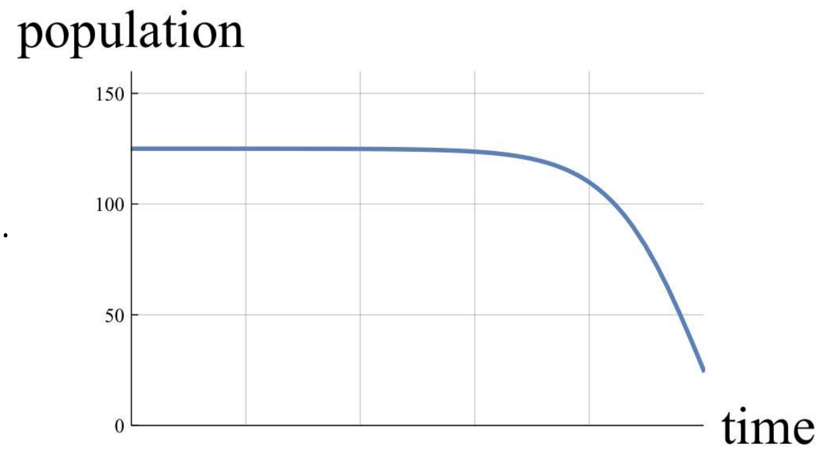
D.



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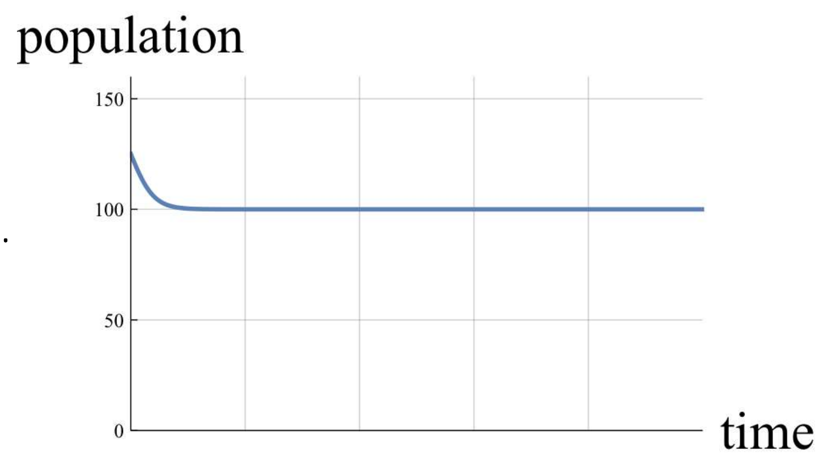
E.



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When $P = 125$, dP/dt is negative and so P will decrease. Since P will decrease for any value of P greater than 100, P will continue to decrease over time as long as P is greater than 100. There are three graphs which do this. However, graphs B and D continue to decrease as P passes 100, and we know from the previous problem that P is increasing for populations below 100 because dP/dt is positive.

So the correct choice is E. Notice that this graph becomes less and less steep as P approaches 100, becoming almost horizontal. This is consistent with the fact that as P approaches 100, the rate of change dP/dt gets closer to zero.

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