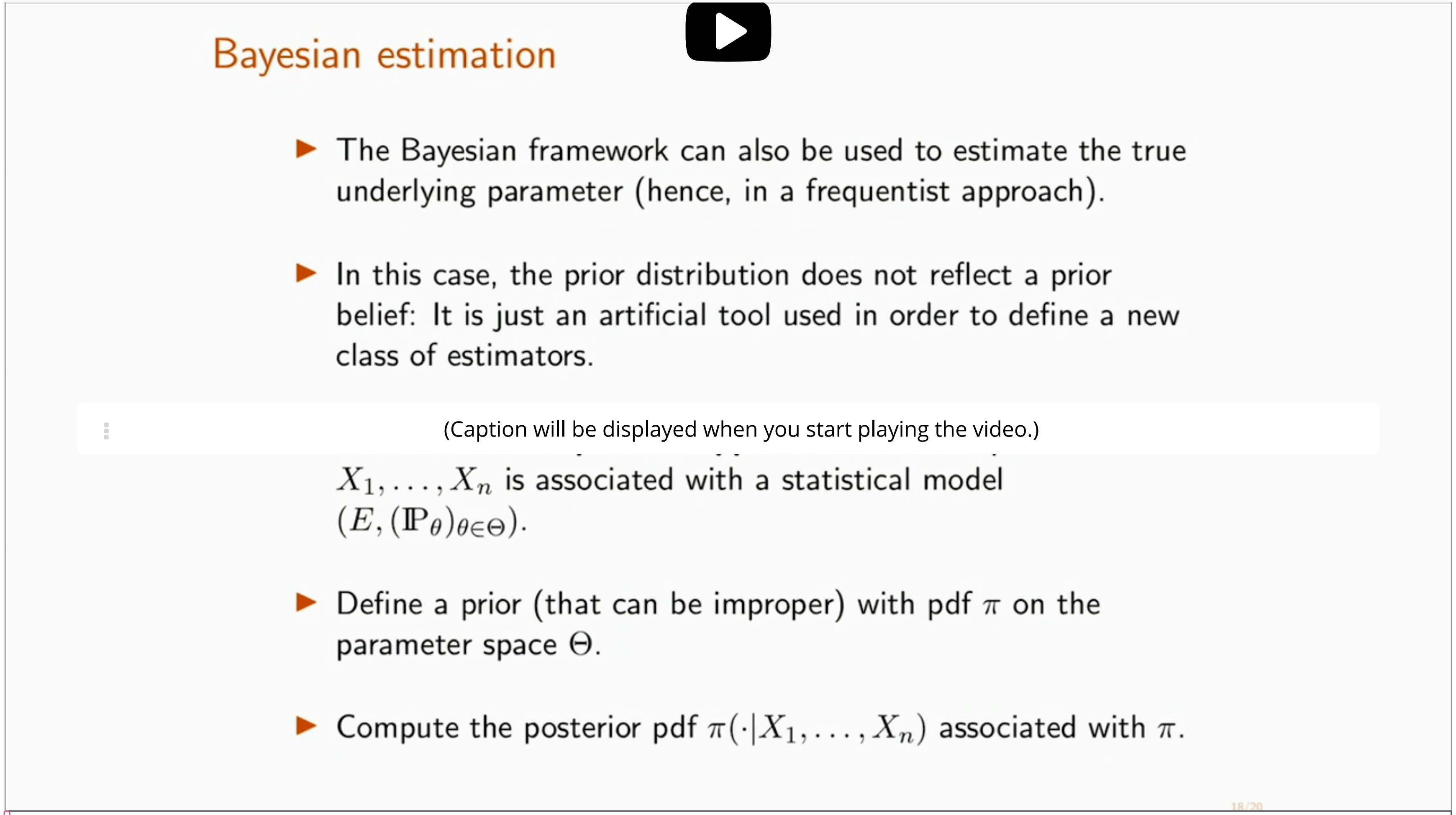
10. Bayesian Statistics for Estimation

Bayesian Estimation

Generating Speech Output



(Caption will be displayed when you start playing the video.)

0:00 / 0:00 1.50x

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(a)

4/4 points (graded)

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Consider the posterior distribution derived in the worked example from the previous lecture (here and here).

To recap, our parameter of interest is λ, prior distribution $\mathsf{Exp}\,(a)$, and likelihood $\mathsf{Poiss}\,(\lambda)$ for n observations $X_1, \ldots, X_n$. This is a Gamma distribution with parameters q_0 and λ_0 that you must get from the last two answerboxes in Worked Example Part II.

As before, recall the **Gamma distribution** , which is a probability distribution with parameters $q > 0$ and $\lambda > 0$, has support on $(0, \infty)$, and whose density is given by $f(x) = \frac{\lambda^q x^{q-1} e^{-\lambda x}}{\Gamma(q)}$. Here, Γ is the Euler Gamma function.

Of the four sample statistics (mean, median, mode, variance), the Gamma distribution has a simple closed form for three of them. Look up statistics for the Gamma distribution, then for the three that have a simple closed form, calculate them and express your answer in terms of a, n, and $\sum_{i=1}^{n} X_i$ (use **SumXi**), otherwise enter -1.

Note that depending on your source, the format of the Gamma distribution may be different, so you must make sure that you have the correct corresponding parameters.

mean:

(SumXi+1)/(a+n)

$\frac{SumXi+1}{a+n}$

median:

-1

-1

mode:

(SumXi)/(a+n)

$\frac{SumXi}{a+n}$

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(SumXi+1)/(a+n)^2

$\frac{SumXi+1}{(a+n)^2}$

STANDARD NOTATION

Submit You have used 1 of 3 attempts

(b)

4/4 points (graded)

Suppose we have the improper prior $\pi(\lambda) \propto e^{-a\lambda}$, $\lambda \in \mathbb{R}$ (and $a \geq 0$). Conditional on λ, we have observations $X_1, X_2, \cdots, X_n \stackrel{\text{i.i.d}}{\sim} \mathsf{N}(\lambda, 1)$. Compute the posterior distribution $\pi(\lambda | X_1, X_2, \ldots, X_n)$, then provide the following statistics on the posterior distribution.

Use **SumXi** for $\sum_{i=1}^{n} X_i$.

mean:

(SumXi -a)/n

$\frac{SumXi-a}{n}$

variance:

1/n

$\frac{1}{n}$

$q_{0.025}$ (cutoff for highest 2.5%):

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(SumXi -a)/n+1.96/sqrt(n ✔

$\frac{SumXi - a}{n} + \frac{1.96}{\sqrt{n}}$

STANDARD NOTATION

True or False: The variance of this distribution models our uncertainty about the value of the parameter λ.

- (•) True
- ( ) False

✔

Submit You have used 2 of 3 attempts

(c)

1.0/2 points (graded)

Now, suppose that we instead have the proper prior $\pi(\lambda) \sim \mathsf{Exp}(a)$ $(a > 0)$. Again, just as in part (b): conditional on λ, we have observations $X_1, X_2, \cdots, X_n \overset{\text{i.i.d}}{\sim} \mathsf{N}(\lambda, 1)$. You may assume that $a < \sum_{i=1}^{n} X_i$. Compute the posterior distribution $\pi(\lambda | X_1, X_2, \ldots, X_n)$, then provide the following statistics on the posterior distribution. Write `Phi` for the CDF function $\Phi()$ and `PhiInv` for its inverse.

Use **SumXi** for $\sum_{i=1}^{n} X_i$.

median:

(SumXi-a)/n+(PhiInv(0.75 ✖

mode:

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(SumXi-a)/n

$\frac{SumXi-a}{n}$

STANDARD NOTATION

Submit You have used 3 of 3 attempts

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Topic: Unit 5 Bayesian statistics:Lecture 18: Jeffreys Prior and Bayesian Confidence Interval / 10. Bayesian Statistics for Estimation

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Disagreement with Wikipedia page?

question posted 2 days ago by kirill94

I believe the last part (c) disagrees with wikipedia article on trunkated normal: https://en.wikipedia.org/wiki/Truncated_normal_distribution

Substituting α and β I've got another answer to (c)...

This post is visible to everyone.

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3 responses

aajagnow

2 days ago

I double checked that page (thanks for the link, by the way). The stats there worked for me, but I had to be very, very careful.

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Add a comment

markweitzman (Community TA)
2 days ago

I did the calculation and then compared with the wikipedia page, and they agree and get green check marks.

Add a comment

sandipan dey
about 2 hours ago

I thought the median will be $\mu + \Phi^{-1}\left(\frac{\Phi(0)+\Phi(\infty)}{2}\right)\sigma = \mu + \Phi^{-1}\left(\frac{0.5+1}{2}\right)\sigma = \mu + \Phi^{-1}\left(0.75\right)\sigma$, where μ and σ^2 are same as computed in the earlier problems, since posterior is $\propto \mathcal{N}\left(\mu, \sigma^2\right)$ with the left part with the corresponding standardized value ≤ 0 being truncated. But I got it marked wrong.

With the following code in python, I tried for a few different μ, σ and the median returned by python's *truncnorm.median()* function and my computed value were exactly same, as shown below (also wikipedia's formula $\mu + \Phi^{-1}\left(\frac{\Phi(-\mu/\sigma)+\Phi(\infty)}{2}\right)\sigma$ with the standardized values do not seem to work, as shown below):

```
from scipy.stats import truncnorm
mu = 10
sigma = 5
print(truncnorm.median(a=0, b=np.Inf, loc=mu, scale=sigma))   # scipy's implementation
# 13.372448750980409
print(mu+norm.ppf(0.75)*(sigma))                                                  # my formula
# 13.372448750980409
print(mu+norm.ppf((1+norm.cdf(-mu/sigma))/2)*(sigma))          # wikipedia's formula
# 10.142584632954586
```

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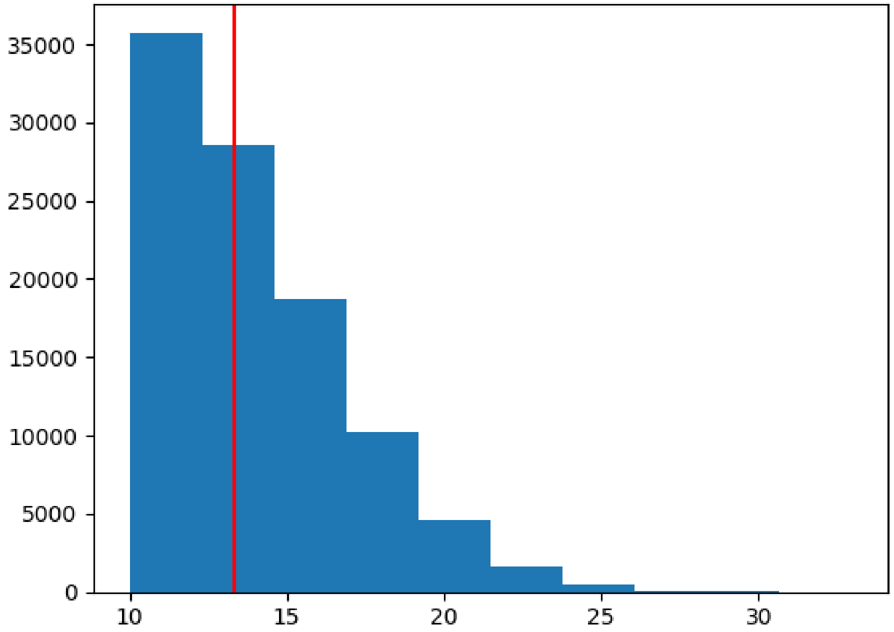

Did not get what's going wrong here, can anyone help? thanks in advance.

Also, I was thinking of writing some MCMC / variational inference code to approximate the posterior distribution, but never done it before (may be some examples in **computational Bayesian** will be useful, since I am a beginner in probabilistic programming).

[UPDATE] I think I got the issue, a parameter value passed to the scipy function was wrong (it needs to be standardized), wiki's formula is correct.

The correct code should be:

```
print(truncnorm.median(a=-mu/sigma, b=np.Inf, loc=mu, scale=sigma))   # scipy's implementation
# 10.142584632954586
```

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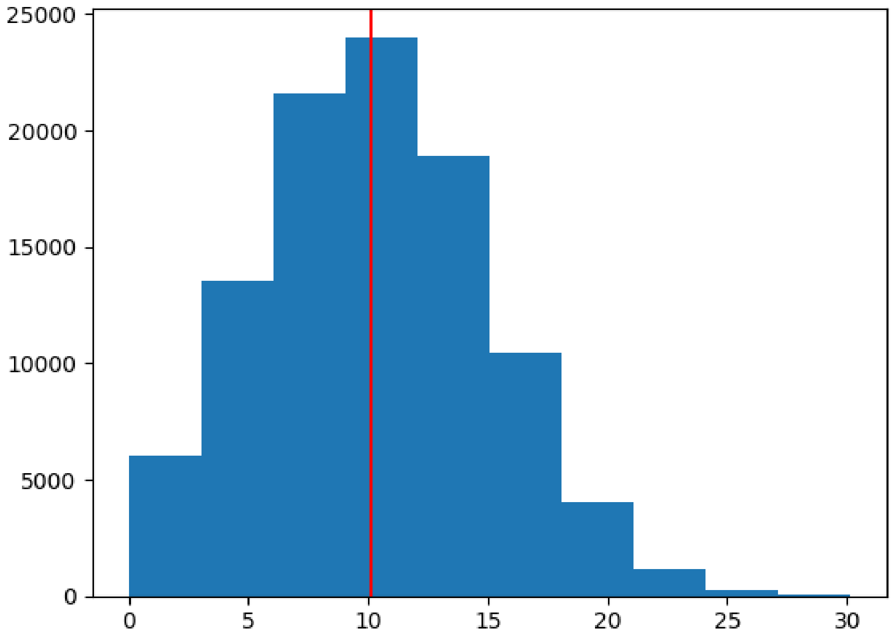

You seem determined to post answers (often incorrect but still basically complete answers) before the deadline. This is not a proper way to get help and is against the spirit of the honor code. Also the answer is available by hitting show answer.

posted about an hour ago by **markweitzman** (Community TA)

Okay, deleting, anyway I did not reveal much information than the ones already available in wikipedia and open source codes. Also, I ended up with a wrong answer even with a last available attempt.

posted less than a minute ago by **sandipan_dey**

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Showing all responses

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