MITx: 6.041x Introduction to Probability - The Science of Uncertainty

Bookmarks

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Bookmark

Problem 1: The PDF of exp(X)

(6/6 points)

Let X be a random variable with PDF f_X. Find the PDF of the random variable $Y = e^X$ for each of the following cases:

1. For general f_X, when $y > 0$, $f_Y(y) =$

- $f_X\left(\frac{e^y}{y}\right)$
- $f_X\left(\frac{\ln y}{y}\right)$
- $\frac{f_X(\ln y)}{y}$ ✔
- none of the above

- Unit 6: Further topics on random variables
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2. When $f_X(x) = \begin{cases} 1/3, & \text{if } -2 < x \leq 1, \\ 0, & \text{otherwise,} \end{cases}$

we have $f_Y(y) = \begin{cases} g(y), & \text{if } a < y \leq b, \\ 0, & \text{otherwise.} \end{cases}$

Give a formula for $g(y)$ and the values of a and b using standard notation . (In your answers, you may use the symbol 'e' to denote the base of the natural logarithm.)

$g(y) =$ 1/(3*y) ✔ **Answer:** 1/(3*y)

$a =$ e^(-2) ✔ **Answer:** 0.13534

$b =$ e ✔ **Answer:** 2.71828

3. When $f_X(x) = \begin{cases} 2e^{-2x}, & \text{if } x > 0, \\ 0, & \text{otherwise,} \end{cases}$

we have $f_Y(y) = \begin{cases} g(y), & \text{if } a < y, \\ 0, & \text{otherwise.} \end{cases}$

Give a formula for $g(y)$ and the value of a using the standard notation .

$g(y) =$ 2/y^3 ✔ **Answer:** 2/(y^3)

- Unit 7: Bayesian inference
- Exam 2
- Unit 8: Limit theorems and classical statistics
- Unit 9: Bernoulli and Poisson processes
- Unit 10: Markov chains
- Exit Survey
- Final Exam

$a =$ 1 Answer: 1

4. When X is a standard normal random variable, we have, for $y > 0$, $f_Y(y) =$

- $\frac{1}{\sqrt{2\pi}} e^{-\frac{(\ln y)^2}{2}}$
- $\frac{1}{\sqrt{2\pi}} \frac{e^{-\frac{(\ln y)^2}{2}}}{y}$ ✔
- $\frac{1}{\sqrt{2\pi}} e^{-\frac{(\ln y)^2}{2y}}$
- $\frac{1}{\sqrt{2\pi}} \frac{e^{-\frac{\ln y}{2}}}{y}$
- none of the above

Answer:

1.

Since $Y = e^X$ is a strictly monotonic function of the continuous random variable X, we can apply the corresponding formula for derived distributions to obtain $f_Y(y) = \frac{f_X(\ln y)}{|y|}$. Note that $Y = e^X > 0$ and so the PDF of Y is nonzero only for $y > 0$. Thus, we can remove the absolute value in the denominator to arrive at the simpler expression $f_Y(y) = \frac{f_X(\ln y)}{y}$ for $y > 0$.

2. By applying the derived distribution formula, we have
$$f_Y(y) \;=\; \begin{cases} 1/(3y), & \text{if } e^{-2} < y \le e, \\ 0, & \text{otherwise.} \end{cases}$$

3. By applying the derived distribution formula, we have
$$f_Y(y) \;=\; \begin{cases} 2/(y^3), & \text{if } 1 < y, \\ 0, & \text{otherwise.} \end{cases}$$

4. We apply the result found in part (1), with the specific PDF $f_X(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$, to obtain

$$f_Y(y) = \frac{1}{\sqrt{2\pi}} \frac{e^{-\frac{\ln^2 y}{2}}}{y}, \text{ for } y > 0.$$

You have used 2 of 2 submissions

Printable problem set available here .

DISCUSSION

Click “Show Discussion" below to see discussions on this problem.