Exercises: Deriving the equations

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Weight matrix

2/2 points (ungraded)

We have a set of 3 observations, which are correlated, where the weight of the third observation is four times higher than the weight of the second observation. Which weight matrix would match with this description?

- $$\begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 8 \end{bmatrix}$$
- $$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$
- $$\begin{bmatrix} 2 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 2 \end{bmatrix}$$

- [x] $\begin{bmatrix} 3 & 2 & 1 \\ 2 & 2 & 1 \\ 1 & 1 & 8 \end{bmatrix}$ ✔

The normal equation leads to the expressions of $\hat{x}$, $\hat{y}$, and $\hat{e}$. Indicate which expressions below are correct:

- [ ] $\hat{x} = (A^T W A)^{-1} A^T y$
- [ ] $\hat{y} = (A^T W A)^{-1} y$
- [ ] $\hat{y} = A(A^T W A)^{-1} y$
- [ ] $\hat{e} = \hat{y} - A\hat{x}$
- [x] None of the above

✔

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✔ Correct (2/2 points)

Moving object

3/3 points (ungraded)

We collected 3 distance measurements to an object moving along a line at constant speed at $t_i = i$, $i = 0, 1, 2$. The unknown initial position on the line is x_0, and the unknown speed is v.

The observations are given by $y = [1.10 \ \ 3.20 \ \ 6.03]^T$.

We apply weighted least squares. The first 2 observations are given equal weight, the last observation is given twice the weight of the other observations.

Which of the following expressions is correct?

- $\hat{y}_i = 0.98 + 2.46t_i$
- $\hat{y}_i = 0.97 + 2.5t_i$ ✔
- $\hat{y}_i = 0.95 + 2.5t_i$
- $\hat{y}_i = 1.0 + 2.4t_i$

Explanation

$$\hat{x} = \begin{bmatrix} \hat{x}_0 \\ \hat{v} \end{bmatrix} = \left(\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 2 \end{bmatrix} \right)^{-1} \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} 1.10 \\ 3.20 \\ 6.03 \end{bmatrix} = \begin{bmatrix} 0.97 \\ 2.5 \end{bmatrix}$$

What is the weighted sum of squared residuals $\hat{e}^T W \hat{e}$? [give your answer to one decimal place]

0.1 ✔ **Answer:** 0.1

0.1

Explanation

Use $\hat{e} = y - \hat{y}$.

Will it be possible to find a solution for x_0 and v resulting in a smaller value for the weighted sum of squared residuals?

- ○ No
- ○ Yes, this is definitely possible.
- ◉ Yes, but only with another weight matrix W. ✔

Explanation

Note that for this particular weight matrix, it is not possible to find a solution for x_0 and v resulting in a smaller value for the weighted sum of squared residuals (according to the weighted least squares principle!).

However, the weighted sum of squared residuals might be smaller with another weight matrix. Try for example $W = I$. It does not mean that the corresponding solution is better if it was realistic to give the third observation a larger weight.

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✔ Correct (3/3 points)