MITx 18.02.1x
Multivariable Calculus 1: Vectors and Derivatives

Help sandipan_dey

Course Progress Dates Calendar Discussion Notes

Course/ Unit 1: Functions of two vari... / Lecture 3: Visualizing surfaces in three dime...

You are taking "Exam (Timed, No Correctness Feedback)" as a timed exam. Show more

End My Exam 44:33:51

Previous Next

8. Domain of definition

Bookmark this page

Calculator Hide Notes

Lecture due Aug 4, 2021 20:30 IST Completed

Explore

Recall that single variable functions are not necessarily defined for all input values. The input values for which the output values of a function exists are called its domain.

- The function $1/x$ is not defined for $x = 0$. The domain of $1/x$ is the set of all $x \neq 0$, that is all x except 0.
- The function $\ln(x)$ is not defined for $x \leq 0$. The domain of $\ln$ is all $x > 0$.

Multivariable functions also may not be defined for all pairs of inputs x and y.

Definition 8.1 The **domain** of a multivariable function $z = f(x, y)$ is the set of points (x, y) for which the function $f(x, y)$ exists (is finite and well-defined).

To determine the domain, you use algebra to determine where a function does not exist.

Examples 8.2

1. $z = x^2 + y^2$ is defined for all values of x and y in the xy-plane. It's domain is all points (x, y).
2. $z = \sqrt{y}$ is only defined for points in the xy-plane where $y \geq 0$. (This domain is sometimes called the upper half-plane.)
3. $z = \dfrac{1}{x+y}$ is only defined for points in the xy-plane where $x + y \neq 0$.
4. $z = \dfrac{1}{xy}$ exists as long as the denominator is nonzero. Thus the domain is the set of points (x, y) so that $xy \neq 0$. That is it is the set of points (x, y) so that $x \neq 0$ and $y \neq 0$. Geometrically, this is the points (x, y) that are not on either the x- or y-axes.

Domain concept check

1/1 point (graded)
(Choose all options below that correctly complete the following sentence.)

The domain of a multivariable function $z = f(x, y)$ is

- [ ] a subset of the real numbers
- [ ] a union of two subsets of the real numbers
- [x] a subset of the xy-plane
- [ ] the union of the set of all values x where $f(x, y)$ exists with the set of all values y where $f(x, y)$ exists
- [x] the set of all ordered pairs of points (x, y) so that $f(x, y)$ exists

Calculator Hide Notes

- [ ] None of the above

Solution:

The domain of definition is the set of all ordered pairs (x, y) such that the function $f(x, y)$ exists. In particular, this collection of points (x, y) is a subset of the the xy-plane. Let's go through each option to see why it is wrong or right.

- Note that a subset of the real numbers is a collection of points x, not a collection of ordered pairs (x, y). Thus is not correct.
- A union of two subsets of real numbers is not correct for the same reason as the above option.
- A subset of the xy-plane is some collection of points (x, y) which is what the domain is.
- The union of all x values such that $f(x, y)$ exists with the union of the set of all points y such that $f(x, y)$ exists is the union of two sets of real numbers, not a collection of points in the plane, thus is not correct.
- The set of all points (x, y) such that $f(x, y)$ exists is exactly the definition of the domain of definition!

Submit You have used 1 of 2 attempts

Answers are displayed within the problem

Find the domain 1

1/1 point (graded)
Find the domain of the function below.

$$z = \frac{1}{x^2 + y^2}$$

- ( ) The entire xy-plane
- (•) The set of points $(x, y) \neq (0, 0)$.
- ( ) The upper half plane $y \geq 0$.
- ( ) All points (x, y) so that $x \neq 0$.
- ( ) All points (x, y) so that $y \neq 0$.
- ( ) A unit disk in the xy-plane about the origin.
- ( ) The domain of this function is empty.

Solution:

This function is well-defined as long as the denominator is nonzero. That is as long as $x^2 + y^2 \neq 0$. But for all x and y, we have that

$$x^2 + y^2 \geq 0,$$

Calculator Hide Notes

with equality only happening if both $x = y = 0$.

Therefore all points (x, y) in the xy-plane except for the origin $(0, 0)$ are in the domain of this function.

Submit You have used 1 of 2 attempts

Answers are displayed within the problem

Find the domain 2

1/1 point (graded)
Find the domain of the function below.

$$z = \frac{1}{x^2 + y^2 + 1}$$

- (selected) All points (x, y) in the xy-plane.
- The set of points $(x, y) \neq (0, 0)$.
- The upper half plane $y \geq 0$.
- All points (x, y) so that $x \neq 0$.
- All points (x, y) so that $y \neq 0$.
- A unit disk in the xy-plane about the origin.
- The unit square defined by (x, y) such that $0 \leq x, y \leq 1$.
- The domain of this function is empty.

✔

Solution:

This function is well defined as long as the denominator is nonzero. That is as long as $x^2 + y^2 + 1 \neq 0$. But for all x and y, we have that

$$x^2 + y^2 + 1 \geq 1 \neq 0.$$

Therefore all points (x, y) in the xy-plane are in the domain of this function.

Submit You have used 1 of 2 attempts

Answers are displayed within the problem

Find the domain 3

1/1 point (graded)

Calculator Hide Notes

1/1 point (graded)
Find the domain of the function below.

$$z = \sqrt{1 - (x^2 + y^2)}$$

- All points (x, y) in the xy-plane.
- The set of points $(x, y) \neq (0, 0)$.
- The upper half plane $y \geq 0$.
- All points (x, y) so that $x \neq 0$.
- All points (x, y) so that $y \neq 0$.
- A unit disk in the xy-plane about the origin.
- The unit square defined by (x, y) such that $0 \leq x, y \leq 1$.
- The domain of this function is empty.

Solution:

The function is well defined as long as the term in the square root is nonnegative. That is if

$$1 - (x^2 + y^2) \geq 0 \qquad \longrightarrow \qquad x^2 + y^2 \leq 1.$$

This inequality defines a unit disk in the xy-plane centered about the origin.

Submit You have used 2 of 2 attempts

Answers are displayed within the problem

Find the domain 4

1/1 point (graded)
Find the domain of the function below.

$$z = \sqrt{-1 - (x^2 + y^2)}$$

- All points (x, y) in the xy-plane.
- The set of points $(x, y) \neq (0, 0)$.
- The upper half plane $y \geq 0$.
- All points (x, y) so that $x \neq 0$.

Calculator Hide Notes

- All points (x, y) so that $y \neq 0$.
- A unit circle in the xy-plane about the origin.
- The unit square defined by (x, y) such that $0 \leq x, y \leq 1$.
- The domain of this function is empty.

Solution:

We only concern ourselves with functions of real variables in this course. Since for any value of x and y, the expression $-1-(x^2+y^2) \leq -1$, this function is complex valued, hence the domain is empty. It is not well defined for any points in the xy-plane.

Submit You have used 1 of 2 attempts

Answers are displayed within the problem

8. Domain of definition

Hide Discussion

Topic: Unit 1: Functions of two variables / 8. Domain of definition

Add a Post

Show all posts by recent activity

- ? y=sqrt(negative real number) — When y=sqrt(-3) then we plot the function 3i on a complex plane. So why it is said that negative real square root function doesn't exi... 2
- ? [Edit] Title should be Definition of Domain? — Hello, I don't know if this is potentially a reference or anything, but think that the title might make more sense if it was "Definition of ... 3
- [staff] Domain concept check — The answer is missing an explanation of the correct answer(s). Also even after completing the question it isn't clear to me which is c... 3
- Find the domain 3 — Hi! Should we have a strict inequality? Because, the function may be undefined, if we have a 0. Thanks! 3
- ? Domain Concept Check — Is "the set of all pairs of points (x,y) so that f(x,y) exists" intentionally worded as such rather than as the "set of all ordered pairs of p... 2

< Previous Next >

Calculator Hide Notes