4. The Role of the Covariance Kernel

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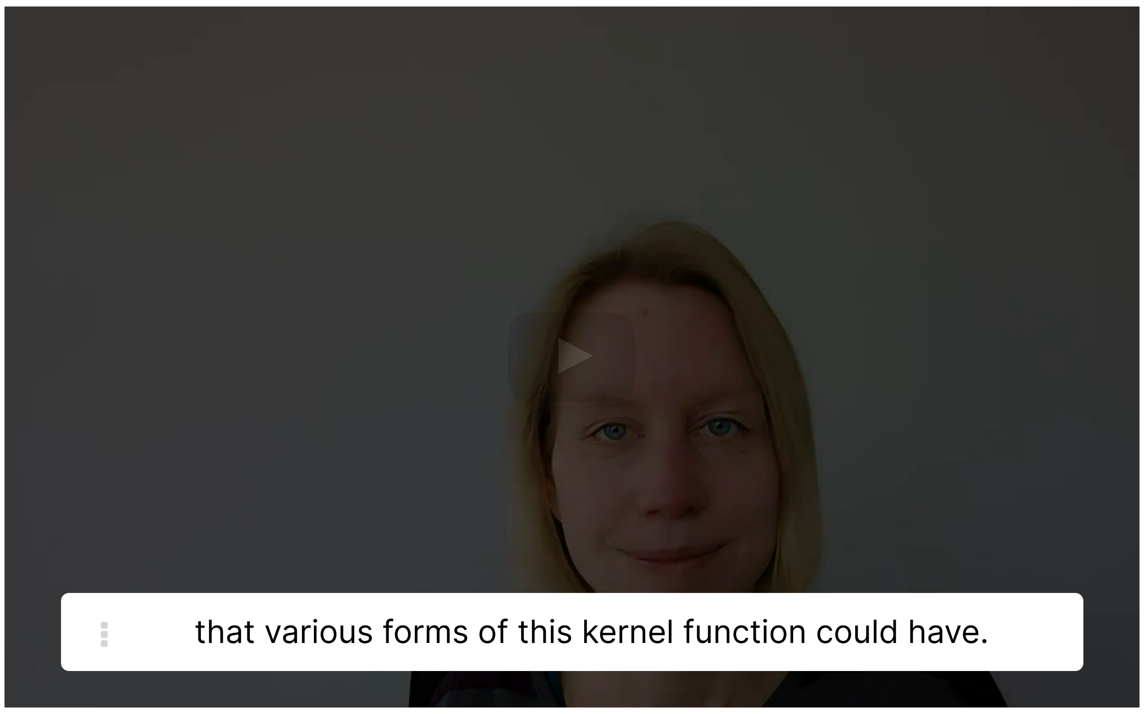
Exercises due Dec 1, 2021 17:29 IST

The Role of the Covariance Kernel



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distribution.

So you can essentially think of this as a Gaussian distribution

over infinitely many, but we only

ever look at finitely many at one point at a time.

And this is essentially the marginal distribution

of these finitely many.

So this is just-- you can think essentially

of the values, the regression values

as infinitely many Gaussian variables.

Now to specify it, again we need this covariance function

and we need to specify a mean for each location.

So hence this is actually also a function for every location.

If you have some prior knowledge

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Previously, we have presented an example of a covariance kernel defined as

$$k\left(Z_1, Z_2\right) = \exp\left(-\frac{\|Z_1 - Z_2\|^2}{2\ell^2}\right).$$

However, the obvious questions are

1. Why use such a kernel function?

2. Can I use any function as a kernel?

Why use such kernel function?

The use of this kernel function allows for a relatively easy and computationally efficient way to parametrize the correlations. Since the kernel function might be defined on the whole space, it allows for smooth computations over the support of the variables to be estimated. Such a parametrization allows for the introduction of other information like smoothness and dynamic behavior.

Can I use any function as a kernel?

The short answer is no. In general, any arbitrary function whose arguments are Z_1 and Z_2 will not be a valid covariance function.

Kernel Function

1/1 point (graded)

Definition 4.1 A function k of two arguments Z_1 and Z_2, mapping its inputs to $\mathbb{R}$ is called a *kernel*.

Can a kernel k such that $k\left(x, y\right) \neq k\left(y, x\right)$ be used to build a covariance function?

- ( ) Yes
- (•) No

✓

Solution:

The answer is No, because covariance matrix needs to be symmetric

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Answers are displayed within the problem

Moreover, covariance matrices need to be positive semi-definite. Thus, one needs to guarantee that the kernel function's output creates a positive definite matrix or a square of matrices.

Definition 4.2 If the covariance function is translation invariant, then it is called stationary.

This will happen if the covariance function depends on $Z_1 - Z_2$.

Definition 4.3 If the covariance function depends only on a norm $|Z_1 - Z_2|$, then it is called isotropic.

This means that the covariance function depends only on the distance between Z_1 and Z_2.

Definition 4.4 If the covariance function depends on $Z_1^\top Z_2$, then it is called dot product covariance.

Kernel Function

1/1 point (graded)

Is the the following kernel function isotropic?

$$k(Z_1, Z_2) = \exp\left(-\frac{\|Z_1 - Z_2\|^2}{2\ell^2}\right), \tag{7.6}$$

- (•) Yes
- ( ) No

✓

Solution:

The answer is yes, the kernel function is isotropic

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Is RBF isotropic?
To me it is very confusing according to the definition at scikit-learn: "RBF- It is parameterized by a length scale parameter , which ca... 1

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