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9. More practice with subspaces using nullspace and span Worked examples

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Identify the subspaces

1/1 point (graded)

Which of the following subsets of $\mathbb{R}^2$ are subspaces? Check all that apply.

- [ ] The set of all vectors $\begin{pmatrix} x \\ y \end{pmatrix}$ satisfying $x^2 + y^2 = 1$.
- [ ] The set of all vectors $\begin{pmatrix} x \\ y \end{pmatrix}$ satisfying $xy = 0$.
- [x] The set of all vectors $\begin{pmatrix} x \\ y \end{pmatrix}$ satisfying $2x + 3y = 0$. ✔
- [ ] None of these.

✔

Solution:

Only the set of vectors $\begin{pmatrix} x \\ y \end{pmatrix}$ satisfying $2x + 3y = 0$ is a subspace.

Algebraic explanation: Let S be the set. For S to be a vector space, it must satisfy all three conditions in the definition.

- The set of all vectors $\begin{pmatrix} x \\ y \end{pmatrix}$ satisfying $x^2 + y^2 = 1$ doesn't even satisfy the first condition, because the zero vector $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$ is not in S.

- The set of all vectors $\begin{pmatrix} x \\ y \end{pmatrix}$ satisfying $xy = 0$ satisfies the first condition: the zero vector is in S. It satisfies the second condition too: If $\begin{pmatrix} x \\ y \end{pmatrix}$ is one vector in S (so $xy = 0$) and c is any scalar, then the vector $c\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} cx \\ cy \end{pmatrix}$ satisfies

$$(cx)(cy) = c^2 xy = c^2(0) = 0,$$

so $c \begin{pmatrix} x \\ y \end{pmatrix}$ is in S.

However, it does not satisfy the third condition for **some** pairs of vectors in S: for example, $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 3 \end{pmatrix}$ are in S, but their sum $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$ is not in S.

- The set of all vectors $\begin{pmatrix} x \\ y \end{pmatrix}$ satisfying $2x + 3y = 0$ is a vector space, as we will now check. First, the zero vector is in S. Second, if $\begin{pmatrix} x \\ y \end{pmatrix}$ is any element of S (so $2x + 3y = 0$) and c is any scalar, then multiplying the equation by c gives

$$2(cx) + 3(cy) = 0,$$

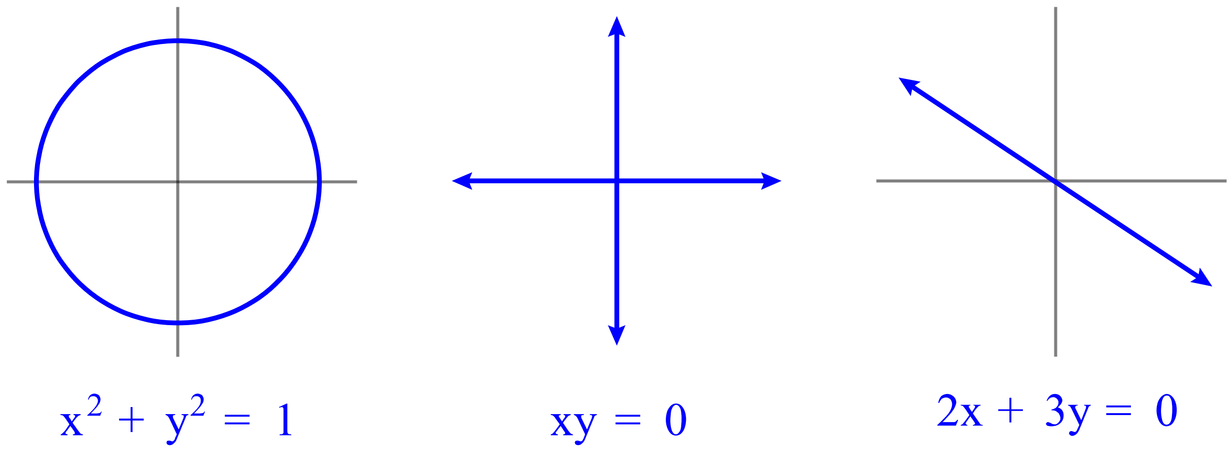
which shows that the vector $c \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} cx \\ cy \end{pmatrix}$ is in S. Third, if $\begin{pmatrix} x_1 \\ y_1 \end{pmatrix}$ and $\begin{pmatrix} x_2 \\ y_2 \end{pmatrix}$ are in S (so $2x_1 + 3y_1 = 0$ and $2x_2 + 3y_2 = 0$), then adding the equations shows that

$$2(x_1 + x_2) + 3(y_1 + y_2) = 0,$$

which says that the vector

$$\begin{pmatrix} x_1 \\ y_1 \end{pmatrix} + \begin{pmatrix} x_2 \\ y_2 \end{pmatrix} = \begin{pmatrix} x_1 + x_2 \\ y_1 + y_2 \end{pmatrix}$$

is in S. Thus S is a vector space.



Alternate solution: We could have also solved this example using the fact that the only subspaces of $\mathbb{R}^2$ are zero, lines through the origin, and all of $\mathbb{R}^2$.

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Answers are displayed within the problem

Identify the subspaces II

1/1 point (graded)

Which of the following are subspaces of $\mathbb{R}^2$? Check all that apply.

- [x] $y = x$ ✔
- [x] $\{\mathbf{0}\}$ ✔
- [ ] $y = 2x + 3$
- [ ] $y = 2$
- [x] $x = 0$ ✔

- [ ] $y = 1/x$
- [x] $y = -4x$ ✔
- [ ] $y = -x^2 + 1$
- [ ] $y = x^2$

✔

Solution:

Recall that the full list of subspaces of $\mathbb{R}^2$ is:

1. the point $\mathbf{0}$;
2. any line passing through the origin.
3. the entire plane $\mathbb{R}^2$.

The curves $y = 1/x$, $y = -x^2 + 1$, and $y = x^2$ are not subspaces of $\mathbb{R}^2$: both $y = 1/x$ and $y = -x^2 + 1$ fail to contain the origin $\mathbf{0}$, and $y = x^2$ fails to be closed under addition and scalar multiplication.

The lines $y = 2$ and $y = 2x + 3$ do not contain the zero vector, so they cannot be subspaces of $\mathbb{R}^2$.

The set $\{\mathbf{0}\}$ is a subspace of $\mathbb{R}^2$ by itself.

The lines $y = x$, $x = 0$ and $y = -4x$ are all subspaces because they all pass through the origin, and are closed under addition and scalar multiplication:

- The line $y = x$ is the set of vectors $\left\{\begin{pmatrix} a \\ a \end{pmatrix} \text{ where } a \text{ is real}\right\}$. Note that $\mathbf{0}$ is in this set, $c\begin{pmatrix} a \\ a \end{pmatrix} = \begin{pmatrix} ca \\ ca \end{pmatrix}$ remains in this set, and $\begin{pmatrix} a \\ a \end{pmatrix} + \begin{pmatrix} b \\ b \end{pmatrix} = \begin{pmatrix} a+b \\ a+b \end{pmatrix}$ also remains in the set.

- The line $x = 0$ is the set of vectors $\{\begin{pmatrix} 0 \\ a \end{pmatrix} \textbf{ where } a \textbf{ is real}\}$. Note that $\mathbf{0}$ is in this set, $c\begin{pmatrix} 0 \\ a \end{pmatrix} = \begin{pmatrix} 0 \\ ca \end{pmatrix}$ remains in this set, and $\begin{pmatrix} 0 \\ a \end{pmatrix} + \begin{pmatrix} 0 \\ b \end{pmatrix} = \begin{pmatrix} 0 \\ a+b \end{pmatrix}$ also remains in the set.

- The line $y = -4x$ is the set of vectors $\{\begin{pmatrix} a \\ -4a \end{pmatrix} \textbf{ where } a \textbf{ is real}\}$. Note that $\mathbf{0}$ is in this set, $c\begin{pmatrix} a \\ -4a \end{pmatrix} = \begin{pmatrix} ca \\ -4ca \end{pmatrix}$ remains in this set, and $\begin{pmatrix} a \\ -4a \end{pmatrix} + \begin{pmatrix} b \\ -4b \end{pmatrix} = \begin{pmatrix} a+b \\ -4(a+b) \end{pmatrix}$ also remains in the set.

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Answers are displayed within the problem

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