6. Moving average model

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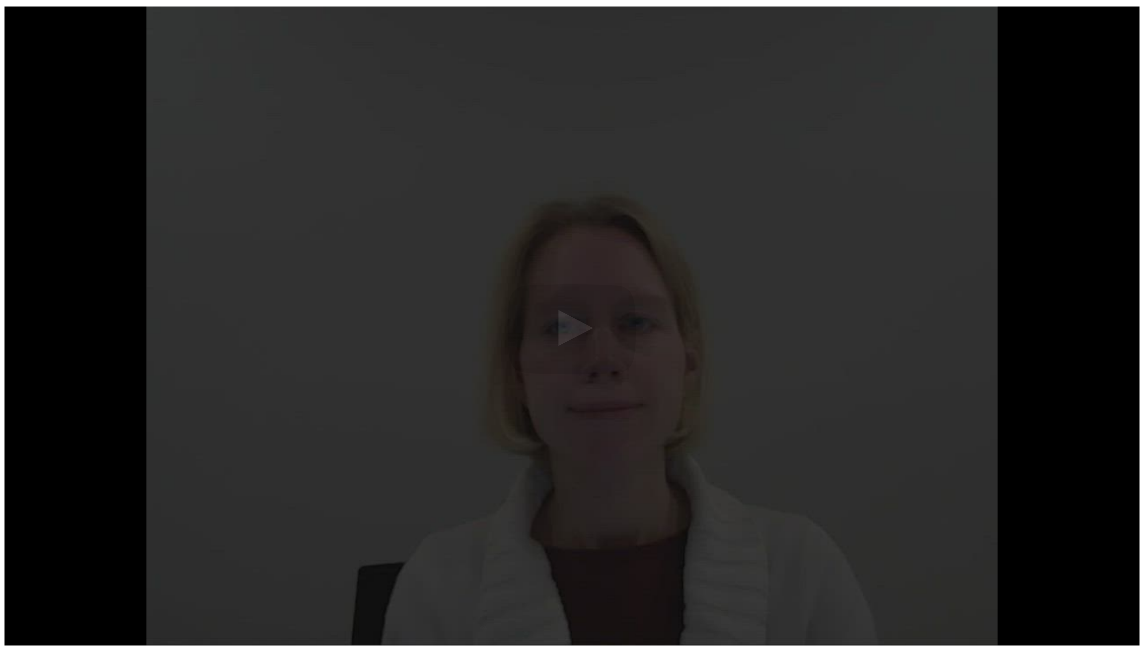
Moving average model



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Prof Jegelka: After exploring our first three statistical models,

that is white noise, autoregressive models,

and random walk models.

We'll now look at two other important models, and these

are moving average models, and

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A time series $\{X_t\}_t$ is a **moving average process** of order q, denoted by MA (q), if it can be represented as a weighted moving average

$$\begin{aligned} X_t &= W_t + \theta_1 W_{t-1} + \theta_2 W_{t-2} + \cdots + \theta_q W_{t-q} \\ &= \sum_{h=0}^{q} \theta_h W_{t-h} \end{aligned}$$

of a white noise series $\{W_t\}_t$.

Marginal mean of moving average model

1/1 point (graded)

What is the marginal mean μ_X of a moving average time series?

0 ✔ **Answer:** 0

Solution:

We have $\mu_X(t) = \mathbf{E}\left(\sum_{h=0}^{q} \theta_h W_{t-h}\right) = 0$.

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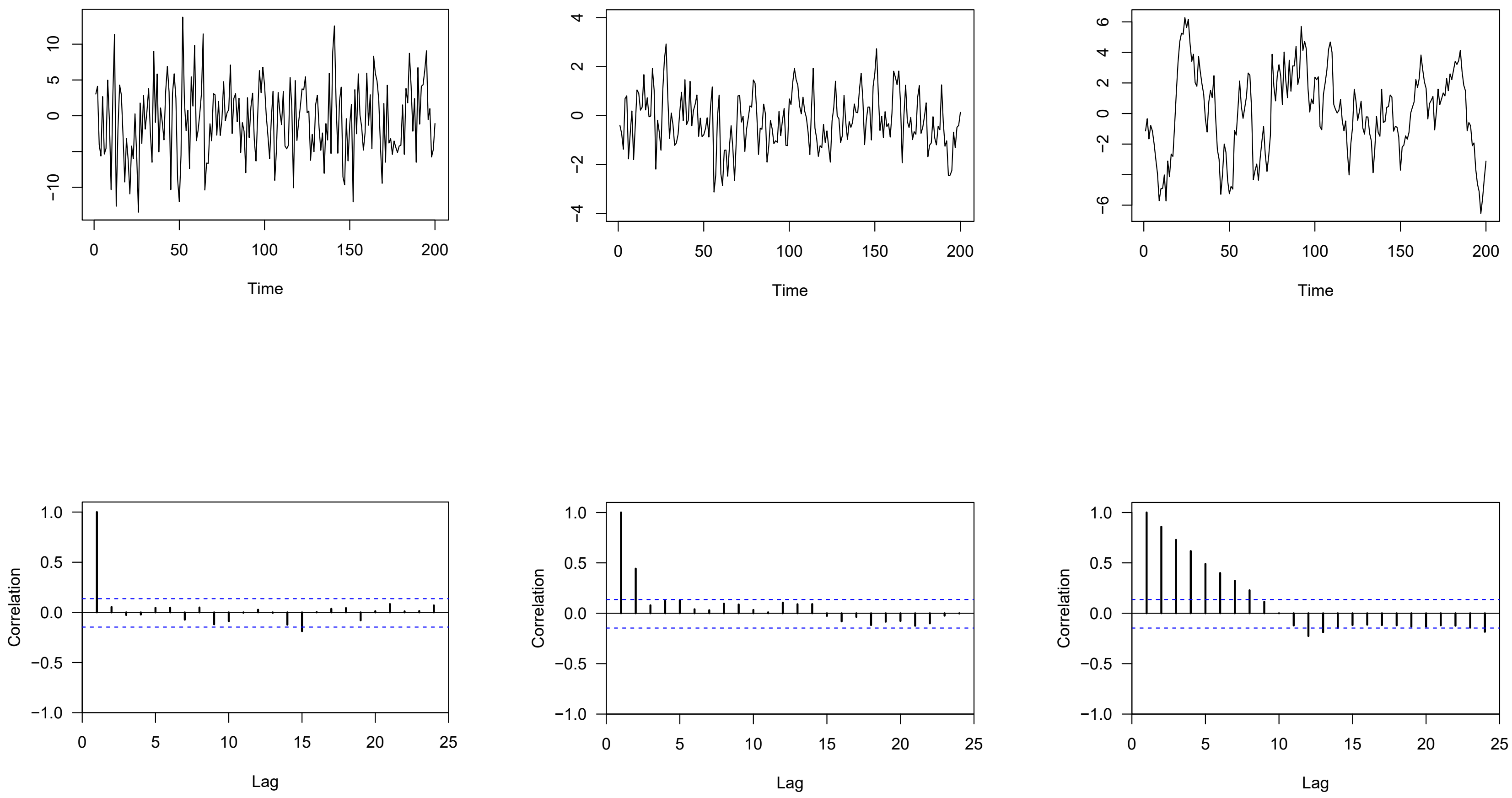
ACF of moving average model



WN MA (1) MA (10)

From the definition of a moving average time series, we find that the autocovariance function is

$$\begin{aligned}\gamma_X(h) &= \text{Cov}\Big(\sum_{j=0}^{q}\theta_j W_{t-j}, \sum_{k=0}^{q}\theta_k W_{t+h-k}\Big)\\ &= \sum_{j=0}^{q-h}\theta_j\theta_{j+h}\sigma_W^2 \quad \text{for} \quad 0 \le h \le q\end{aligned}$$

because W_j is uncorrelated with W_k for $j \neq k$.

ACF of moving average model

5/5 points (graded)

Let $X_t = W_t + \frac{1}{2}W_{t-1} + \frac{1}{3}W_{t-2}$ where $W_t \sim$ WN $(\sigma^2 = 1)$.

Compute the variance $\gamma_X(0)$ and autocovariance $\gamma_X(h)$ for the following gaps $h = 1, 2, 3, 4$.

(Enter an answer accurate to at least 2 decimal places).

Variance $\gamma_X(0) =$

49/36 ✔ **Answer:** 1.361

$\gamma_X(1)$

2/3 ✔ **Answer:** 0.667

$\gamma_X(2)$

1/3 ✔ **Answer:** 0.333

$\gamma_X(3)$

0 ✔ **Answer:** 0

$\gamma_X(4)$

0

Answer: 0

Solution:

$$\begin{aligned}\gamma_X(0) &= \mathrm{Var}\left(W_t + \frac{1}{2}W_{t-1} + \frac{1}{3}W_{t-2}\right)\\ &= \mathrm{Var}(W_t) + \frac{1}{4}\mathrm{Var}(W_{t-1}) + \frac{1}{9}\mathrm{Var}(W_{t-2})\\ &= 1 + \frac{1}{4} + \frac{1}{9} = \frac{49}{36}\end{aligned}$$

$$\begin{aligned}\gamma_X(1) &= \mathrm{Cov}\left(W_t + \frac{1}{2}W_{t-1} + \frac{1}{3}W_{t-2},\ W_{t-1} + \frac{1}{2}W_{t-2} + \frac{1}{3}W_{t-3}\right)\\ &= \frac{1}{2}\mathrm{Cov}(W_{t-1}, W_{t-1}) + \frac{1}{3}\cdot\frac{1}{2}\mathrm{Cov}(W_{t-2}, W_{t-2})\\ &= \frac{1}{2} + \frac{1}{3}\cdot\frac{1}{2} = \frac{4}{6}\end{aligned}$$

$$\begin{aligned}\gamma_X(2) &= \mathrm{Cov}\left(W_t + \frac{1}{2}W_{t-1} + \frac{1}{3}W_{t-2},\ W_{t-2} + \frac{1}{2}W_{t-3} + \frac{1}{3}W_{t-4}\right)\\ &= \frac{1}{3}\mathrm{Cov}(W_{t-2},\ W_{t-2}) = \frac{1}{3}\end{aligned}$$

The autocovariance $\gamma_X(h) = 0$ for all $h > 2$. **Remark:** In general, $\gamma_X(h) = 0$ for all $h > q$ because there are no common white noise terms in such terms of an $\mathrm{MA}(q)$ time series.

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Answers are displayed within the problem

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Lecture's aren't very good.
I know the last question can be easily solved because it was covered in the lecture. I even referred to the video but after looking at t... 16

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