Stochastic Model

This week is about the mathematical model, describing the relation between our observations and unknowns. In the first part, the functional model was introduced.

In this part we will introduce the second part of the mathematical model, namely the stochastic model.

First a very brief review of the functional model in which the unknown parameters x are written as a function of the observables y.

Their relationship can be either linear with design matrix A as shown here or non-linear, but we mainly will focus on the first case.

The random errors in our observations are accounted for by explicitly adding them on the right-hand side of the equations. But since we know the random errors have an expectation of zero, the models can also be expressed like this where we see that the expectation of our observables is assumed to have the specific functional relation with the unknowns x.

So let's now introduce the so-called stochastic model, which we need to describe the uncertainty in our observations due to the random errors.

Recall from the first week that the spread in the outcomes of the random errors can be described by the standard deviation or covariance matrix in case of a random vector. Therefore, the stochastic model is in fact provided by the covariance matrix of the random errors.

The structure of the covariance matrix is shown in the bottom right, with variance on the diagonal and covariances as off-diagonal elements.

We saw in the past that the variance of a random error equals the variance of the corresponding observable. The same holds for the covariance matrices.

This may seem logical if we look at the functional model, where the Ax part is deterministic, meaning that there is no uncertainty involved in that part.

The complete mathematical model is given by the combination of the functional model and the stochastic model.

In summary, the functional model is given by the first moment of the observable vector, being the expectation. The stochastic model is the second central moment, which is the dispersion.

In the remainder of this lecture we will look at the sea level rise example again.

We will start with the same example as before, where we want to estimate the rate at which the sea level has been rising in the past ten years in IJmuiden. We will use monthly sea level observations collected at the tide gauge station over there. And in this case we are only interested in the initial sea level at the first time of observation, and the rate of change. The functional model was shown to be this one.

If all observations are assumed to have the same precision, given by standard deviation sigma, the stochastic model takes the simple form of a scaled identity matrix.

But let's now assume that at a certain time k the tide gauge was replaced by a newer and better version, such that the precision became better as well.

In that case, the standard deviations after time k will be smaller, and the stochastic model would be given by this covariance matrix.

Summarizing, we have formulated the complete mathematical model comprising the functional and stochastic model, which are given by the expectation and the dispersion of the observables. Furthermore, we looked at an example with independent and therefore uncorrelated tide gauge observations.

In the next video we will look at an example with observations which are correlated in time.