PurdueX: 416.1x Probability: Basic Concepts & Discrete Random Variables

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Unit 3: Quiz

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The problems in the quiz will be automatically graded.

You are given only **ONE** attempt for each problem. You will be able to see the solution after you submit your answers.

Problem 1

6/6 points (graded)

1. Roll three (6-sided) dice. Let X denote the maximum of the values that appear.

1a. Find $P(X = 1)$.

1/216

✔ **Answer:** 0.00462963

1b. Find $P(X = 2)$.

7/216

✔ **Answer:** 0.0324074

1c. Find $P(X = 3)$.

19/216

Answer: 0.087963

1d. Find $P(X = 4)$.

37/216

Answer: 0.171296

1e. Find $P(X = 5)$.

61/216

Answer: 0.282407

1f. Find $P(X = 6)$.

91/216

Answer: 0.421296

[Hint: It might be helpful to first find the values of $P(X \le x)$.]

Explanation

1. We have $X \le x$ if and only if all of the values on the three dice are less than or equal to x. Thus, $P(X \le x) = x^3/216$. So we get:

$P(X = 1) = P(X \le 1) = 1/216$
$P(X = 2) = P(X \le 2) - P(X \le 1) = 8/216 - 1/216 = 7/216$
$P(X = 3) = P(X \le 3) - P(X \le 2) = 27/216 - 8/216 = 19/216$
$P(X = 4) = P(X \le 4) - P(X \le 3) = 64/216 - 27/216 = 37/216$
$P(X = 5) = P(X \le 5) - P(X \le 4) = 125/216 - 64/216 = 61/216$

$P(X = 6) = P(X \le 6) - P(X \le 5) = 216/216 - 125/216 = 91/216$

By the way, these probabilities (of course) sum to 1.

Submit You have used 1 of 1 attempt

Problem 2

4/4 points (graded)

2. Consider a collection of 9 bears. There is a family of red bears consisting of one father bear, one mother bear, and one baby bear. There is a similar green bear family, and a similar blue bear family. We draw 3 consecutive times from this collection *without replacement* (i.e., not returning the bear after each draw). Let X denote the number of red bears that are chosen.

2a. Find $P(X = 0)$.

5/21 ✔ **Answer:** 0.238095

2b. Find $P(X = 1)$.

15/28 ✔ **Answer:** 0.535714

2c. Find $P(X = 2)$.

3/14 ✔ **Answer:** 0.214286

2d. Find $P(X = 3)$.

1/84

✔ **Answer:** 0.0119048

Explanation

2. The probabilities are:

$$P(X=0) = \frac{\binom{3}{0}\binom{6}{3}}{\binom{9}{3}} = \frac{5}{21}; \qquad P(X=1) = \frac{\binom{3}{1}\binom{6}{2}}{\binom{9}{3}} = \frac{15}{28};$$

$$P(X=2) = \frac{\binom{3}{2}\binom{6}{1}}{\binom{9}{3}} = \frac{3}{14}; \qquad P(X=3) = \frac{\binom{3}{3}\binom{6}{0}}{\binom{9}{3}} = \frac{1}{84}.$$

The general formula is $P(X = x) = \binom{3}{x}\binom{6}{3-x}/\binom{9}{3}$. Again, the probabilities sum to 1.

Submit You have used 1 of 1 attempt

Problem 3

2/2 points (graded)

3. Roll a 6-sided die until the first value of “3" that appears, and then stop afterwards. Let X denote the number of rolls that are needed.

3a. Give a formula for $P(X > x)$, where x is a nonnegative integer.

(5/6)^x

✔ **Answer:** (5/6)^x

You entered:

$$\frac{5}{6}^{x}$$

3b. Give a formula for $P(X = x)$, where x is a positive integer.

(5/6)^(x-1)*(1/6)

✔ **Answer:** (1/6)*(5/6)^(x-1)

You entered:

$$\frac{1}{6}\frac{5}{6}^{x-1}$$

3c. Verify that the probabilities in **(3b)** have a sum of 1.

Explanation

3a. We have $X > x$ if the first x rolls have no 3's. Thus, we have $P(X > x) = (5/6)^x$.

3b. From **(3a)**, we compute

$$P(X = x) = P(X > x - 1) - P(X > x) = (5/6)^{x-1} - (5/6)^x$$

$$= (1 - 5/6)(5/6)^{x-1} = (1/6)(5/6)^{x-1}.$$

3c. We can verify

$$\sum_{x=1}^{\infty}(1/6)(5/6)^{x-1} = (1/6)\sum_{x=1}^{\infty}(5/6)^{x-1}$$

$$= (1/6)(1 + 5/6 + (5/6)^2 + (5/6)^3 + \cdots) = (1/6)\frac{1}{1 - 5/6} = 1.$$

Submit You have used 1 of 3 attempts

Problem 4

7/7 points (graded)

4. Suppose that a drawer contains 8 marbles: 2 are red, 2 are blue, 2 are green, and 2 are yellow. The marbles are rolling around in a drawer, so that all possibilities are equally likely when they are drawn. Alice selects marbles (without replacement) until she gets a red marble, and then she stops afterwards.

Let X denote the number of draws that are needed until the first red appears.

4a. Find $P(X = 1)$.

1/4

✔ **Answer:** 0.25

4b. Find $P(X = 2)$.

3/14

✔ **Answer:** 0.214286

4c. Find $P(X = 3)$.

5/28

✔ **Answer:** 0.178571

4d. Find $P(X = 4)$.

1/7

✔ **Answer:** 0.142857

4e. Find $P(X = 5)$.

3/28

✔ **Answer:** 0.107143

4f. Find $P(X = 6)$.

1/14

✔ **Answer:** 0.0714286

4g. Find $P(X = 7)$.

1/28

✔ **Answer:** 0.0357143

Explanation

4. We see that $X = x$ if the xth marble is red and any other afterwards (from the $(x + 1)$st marble to the 8th marble) is red too. There are $\binom{8}{2} = 28$ ways to choose which two marbles are red. So the desired probability is $P(X = x) = (8 - x)/28$.

If you did not notice the fact above, you can also go case by case, to compute:

$P(X = 1) = 2/8 = 1/4 = 7/28$

$P(X = 2) = (6/8)(2/7) = 3/14 = 6/28$

$P(X = 3) = (6/8)(5/7)(2/6) = 5/28$

$P(X = 4) = (6/8)(5/7)(4/6)(2/5) = 1/7 = 4/28$

$P(X = 5) = (6/8)(5/7)(4/6)(3/5)(2/4) = 3/28$

$P(X = 6) = (6/8)(5/7)(4/6)(3/5)(2/4)(2/3) = 1/14 = 2/28$

$P(X = 7) = (6/8)(5/7)(4/6)(3/5)(2/4)(1/3)(2/2) = 1/28$

Again, the probabilities do sum to 1.

Submit You have used 1 of 1 attempt

Problem 5

1/1 point (graded)

5. Suppose that we choose cards from a standard 52-card deck, with replacement and shuffling in between cards, until the first card with value 6, 7, 8, 9, or 10 appears, and then we stop. Let X be the number of flips needed.

Find $F_X(x)$, the CDF of X, for integers $x \geq 1$. Then calculate $F_X(2)$. Give your answer to 4 decimal places.

0.6213

✔ **Answer:** 0.6213

Explanation

5. The mass of X is $p_X(x) = (32/52)^{x-1}(20/52)$, for integers $x \geq 1$. So the CDF of X, for an integer $x \geq 1$, is $F_X(x) = \sum_{j=1}^{x}(32/52)^{j-1}(20/52) = (20/52)\frac{1-(32/52)^x}{1-32/52} = 1-(32/52)^x$.

Submit

You have used 1 of 3 attempts

Problem 6

2/2 points (graded)

6a. Roll a die until the first 5 appears. Let X denote the number of rolls needed. Find the probability that X is even.

5/11

✔ **Answer:** 0.454545

6b. Suppose that $P(Y = y) = pq^{y-1}$ for integers $y \geq 1$, where $q = 1 - p$. Find the probability that Y is even. (If you believe your answer is correct but still got wrong, please try a different form of your expression. The automatic grading is not yet perfect. Sorry for the inconvenience. Hint: both p and q appear in the numerator whereas only q shows up in the denominator.)

(q*p)/(1-q^2)

✔ **Answer:** p*q/(1-q^2)

You entered:

$$\frac{pq}{-q^2 + 1}$$

Explanation

6a. The mass of X is $p_X(x) = (5/6)^{x-1}(1/6)$, for integers $x \geq 1$. The probability X is even is $(5/6)^1(1/6) + (5/6)^3(1/6) + (5/6)^5(1/6) + (5/6)^7(1/6) + \cdots$
$= (5/6)(1/6)(1 + (5/6)^2 + (5/6)^4 + (5/6)^6 + \cdots) = (5/6)(1/6)\frac{1}{1-(5/6)^2} = 5/11$.

6b. The probability Y is even is
$pq + pq^3 + pq^5 + pq^7 + \cdots = pq(1 + q^2 + q^4 + q^6 + \cdots) = \frac{pq}{1-q^2}$.

Submit

You have used 2 of 6 attempts

Problem 7

2/2 points (graded)

7a. Roll a die until the first 5 appears. Let X denote the number of rolls needed. Find the probability that X is a multiple of 3.

0.2747253

✔ **Answer:** 0.274725

7b. Suppose that $P(Y = y) = pq^{y-1}$ for integers $y \geq 1$, where $q = 1 - p$. Find the probability that Y is a multiple of 3.

(q^2*p)/(1-q^3)

✔ **Answer:** p*q^2/(1-q^3)

You entered:

$$\frac{pq^2}{-q^3 + 1}$$

Explanation

7a. The mass of X is $p_X(x) = (5/6)^{x-1}(1/6)$, for integers $x \geq 1$. The probability X is a multiple of three is

$$(5/6)^2(1/6) + (5/6)^5(1/6) + (5/6)^8(1/6) + (5/6)^{11}(1/6) + \cdots$$
$$= (5/6)^2(1/6)(1 + (5/6)^3 + (5/6)^6 + (5/6)^9 + \cdots)$$
$$= (5/6)^2(1/6)\frac{1}{1-(5/6)^3} = 25/91.$$

7b. The probability Y is a multiple of 3 is

$$pq^2 + pq^5 + pq^8 + pq^{11} + \cdots = pq^2(1 + q^3 + q^6 + q^9 + \cdots) = \frac{pq^2}{1-q^3}.$$

Submit You have used 1 of 1 attempt

Problem 8

1/1 point (graded)

8. Suppose Alice flips 4 coins and Bob flips 4 coins. Find the probability that Alice and Bob get the exact same number of heads.

0.2734375

✔ **Answer:** 0.273438

Explanation

8. The probability that they get the exact same number of heads is
$(1/16)(1/16) + (4/16)(4/16) + (6/16)(6/16) + (4/16)(4/16) + (1/16)(1/16) = 35/128$.

Submit You have used 1 of 1 attempt

✔ Correct (1/1 point)

Problem 9

0/1 point (graded)

9. Let Alice roll a 6-sided die and let X denote the result of her roll. Let Bob roll a pair of 4-sided dice and let Y denote the sum of the two values on his two dice. Find $P(X < Y)$.

1/4

✖ **Answer:** 0.65625

Explanation

9. The probability mass function of X is $p_X(x) = 1/6$ for integers $1 \le x \le 6$. The probability mass function of Y is $p_Y(2) = 1/16$, $p_Y(3) = 2/16$, $p_Y(4) = 3/16$, $p_Y(5) = 4/16$, $p_Y(6) = 3/16$, $p_Y(7) = 2/16$, $p_Y(8) = 1/16$. Also, X and Y are independent in this problem. So the desired probability is

$$P(X < 2, Y = 2) + P(X < 3, Y = 3) + P(X < 4, Y = 4)$$
$$+P(X < 5, Y = 5) + P(X < 6, Y = 6) + P(Y = 7) + P(Y = 8)$$

which simply turns out to be:

$$\left(\frac{1}{6}\right)\left(\frac{1}{16}\right) + \left(\frac{2}{6}\right)\left(\frac{2}{16}\right) + \left(\frac{3}{6}\right)\left(\frac{3}{16}\right) + \left(\frac{4}{6}\right)\left(\frac{4}{16}\right) + \left(\frac{5}{6}\right)\left(\frac{3}{16}\right) + \frac{2}{16} + \frac{1}{16}$$
$$= 63/96 = 21/32 = 0.65625.$$

Submit You have used 1 of 1 attempt

✖ Incorrect (0/1 point)

Problem 10

7/7 points (graded)

10. Consider 5 fish in a bowl: 3 of them are red, and 1 is green, and 1 is blue. Select the fish one at a time, without replacement, until the bowl is empty.

Let $X = 1$ if all of the red fish are selected, before the green fish is selected; and $X = 0$ otherwise.

Let $Y = 1$ if all of the red fish are selected, before the blue fish is selected; and $Y = 0$ otherwise.

10a. Find the joint probability mass function of X and Y.

$p_{X,Y}(0,0) =$ 12/20 ✔ **Answer:** 0.6

$p_{X,Y}(0,1) =$ 3/20 ✔ **Answer:** 0.15

$p_{X,Y}(1,0) =$ 3/20 ✔ **Answer:** 0.15

$p_{X,Y}(1,1) =$ 2/20 ✔ **Answer:** 0.1

10b. Make sure that the four probabilities $p_{X,Y}(0,0)$, $p_{X,Y}(0,1)$, $p_{X,Y}(1,0)$, and $p_{X,Y}(1,1)$ from part 3a have a sum of 1.

10c. Find the probability $p_X(1)$. Find the probability $p_Y(1)$.

$p_X(1) =$ 1/4 ✔ **Answer:** 0.25 $p_Y(1) =$ 1/4 ✔

Answer: 0.25

10d. Are X and Y independent?

- ○ Yes
- ◉ No ✔

Explanation

10a. We have

- $p_{X,Y}(0,0) = 3/5$ ($X = Y = 0$ exactly when the last fish is red);
- $p_{X,Y}(0,1) = (1/5)(3/4) = 3/20$ ($X = 0$ and $Y = 1$ if the last is blue & the 4th is red);
- $p_{X,Y}(1,0) = (1/5)(3/4) = 3/20$ ($X = 1$ and $Y = 0$ if the last is green & the 4th is red);
- $p_{X,Y}(1,1) = (2/5)(1/4) = 1/10$ ($X = Y = 1$ if the last two fish are green and blue).

10b. We verify that $3/5 + 3/20 + 3/20 + 1/10 = 1$.
10c. We have $X = 1$ if the green fish is last, or if the green fish is 4th and the blue fish is last. So $p_X(1) = 1/5 + (1/5)(1/4) = 1/4$.
Another way to see this is that, when paying attention to only the 3 reds and the 1 green, we have $X = 1$ only if the green comes after all 3 reds, so $p_X(1) = 1/4$.
Similarly, we have $p_Y(1) = 1/4$.
10d. The random variables X and Y are dependent since $p_{X,Y}(1,1) \neq p_X(1)p_Y(1)$.

Submit You have used 1 of 1 attempt

✔ Correct (7/7 points)

Problem 11

1/1 point (graded)

11. Suppose that a person rolls a 6-sided die until the first occurrence of 4 appears, and then the person stops afterwards. Let Y denote the number of rolls that are needed. Let X denote the number of rolls (during this same experiment) on which a value of 3 appears. Find a formula for $p_{X|Y}(x \mid y)$.

- $\binom{y}{x}(1/5)^x(4/5)^{y-x}$
- $\binom{y-1}{x}(1/5)^x(4/5)^{y-1-x}$ ✔
- $\binom{y}{x-1}(1/5)^{x-1}(4/5)^{y-x+1}$
- $\binom{y-1}{x}(4/5)^x(1/5)^{y-1-x}$

Explanation

11. If we are given $Y = y$, then the first $y - 1$ rolls do not have any occurrences of 4, but the other 5 results are equally likely. So the probability that exactly x out of these $y - 1$ results are 3's is:
$p_{X|Y}(x \mid y) = \binom{y-1}{x}(1/5)^x(4/5)^{y-1-x}$.

Submit You have used 1 of 1 attempt

✔ Correct (1/1 point)