



## MITx: 6.041x Introduction to Probability - The Science of Uncertainty



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**Exam 1**

Exam 1 due Mar 09, 2016 at 23:59 UTC



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## Problem 1: True or false

(4/4 points)

We are told that events  $A$  and  $B$  are conditionally independent, given a third event  $C$ , and that  $\mathbf{P}(B \mid C) > 0$ . For each one of the following statements, decide whether the statement is "Always true", or "Not always true."

1.  $A$  and  $B$  are conditionally independent, given the event  $C^c$ .

Not always true ▼



Answer: Not always true

2.  $A$  and  $B^c$  are conditionally independent, given the event  $C$ .

Always true ▼



Answer: Always true

3.  $\mathbf{P}(A \mid B \cap C) = \mathbf{P}(A \mid B)$

Not always true ▼



Answer: Not always true

- ▶ Unit 5: Continuous random variables
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4.  $\mathbf{P}(A \mid B \cap C) = \mathbf{P}(A \mid C)$

Always true ▼



Answer: Always true

Answer:

1. Not always true. Counterexample: Let  $\mathbf{X}, \mathbf{Y}$  be binary random variables. Consider a model with the following properties:  
 Conditioned on  $\mathbf{C}$ ,  $\mathbf{X}$  and  $\mathbf{Y}$  are independent.  
 Conditioned on  $\mathbf{C}^c$ ,  $\mathbf{X}$  and  $\mathbf{Y}$  are dependent.

Let  $\mathbf{A} = \{\mathbf{X} = 1\}$  and  $\mathbf{B} = \{\mathbf{Y} = 1\}$ . Then,  $\mathbf{A}$  and  $\mathbf{B}$  are conditionally independent given  $\mathbf{C}$ , but they will be generically dependent conditioned on  $\mathbf{C}^c$ .

2. Always true.

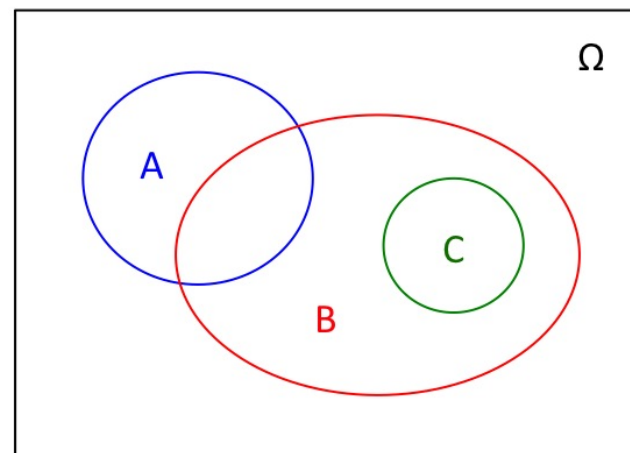
$$\mathbf{P}(\mathbf{A} \mid \mathbf{C}) = \mathbf{P}(\mathbf{A} \cap \mathbf{B} \mid \mathbf{C}) + \mathbf{P}(\mathbf{A} \cap \mathbf{B}^c \mid \mathbf{C})$$

$$\mathbf{P}(\mathbf{A} \mid \mathbf{C}) = \mathbf{P}(\mathbf{A} \mid \mathbf{C})\mathbf{P}(\mathbf{B} \mid \mathbf{C}) + \mathbf{P}(\mathbf{A} \cap \mathbf{B}^c \mid \mathbf{C})$$

$$\mathbf{P}(\mathbf{A} \mid \mathbf{C})(1 - \mathbf{P}(\mathbf{B} \mid \mathbf{C})) = \mathbf{P}(\mathbf{A} \cap \mathbf{B}^c \mid \mathbf{C})$$

$$\mathbf{P}(\mathbf{A} \mid \mathbf{C})\mathbf{P}(\mathbf{B}^c \mid \mathbf{C}) = \mathbf{P}(\mathbf{A} \cap \mathbf{B}^c \mid \mathbf{C}).$$

3. Not always true. Counterexample: Let  $\mathbf{P}(\mathbf{A}) > 0, \mathbf{P}(\mathbf{B}) > 0, \mathbf{P}(\mathbf{C}) > 0$ ,  $\mathbf{P}(\mathbf{A} \cap \mathbf{B}) > 0$  and  $\mathbf{P}(\mathbf{A} \cap \mathbf{C}) = 0$ . Furthermore, let  $\mathbf{C} \subset \mathbf{B}$ .



Show that  $A$  and  $B$  are conditionally independent given  $C$ :

$$\mathbf{P}(A \cap B \mid C) = 0 = (0)(1) = \mathbf{P}(A \mid C)\mathbf{P}(B \mid C)$$

Show that  $\mathbf{P}(A \mid B \cap C) \neq \mathbf{P}(A \mid B)$ :

$$\mathbf{P}(A \mid B \cap C) = 0 \neq \mathbf{P}(A \mid B) > 0$$

4. Always true. This is equivalent to the definition of independence of  $A$  and  $B$  in the conditional universe where  $C$  has occurred.

*You have used 1 of 1 submissions*



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