5. Maximum Likelihood Estimator

Review: Maximizing composite functions

0/1 point (graded)

The **arguments of the minima** (*resp.* **arguments of the maxima**) of a function $f(x)$, denoted by $\mathrm{argmin} f(x)$ (*resp.* $\mathrm{argmax}\, f(x)$), is the value(s) of x at which $f(x)$ is minimum (*resp.* maximum). We can also restrict to a subset S of the domain of f, and denote by $\mathrm{argmin}_{x\in S} f(x)$ (*resp.* $\mathrm{argmax}_{x\in S}\, f(x)$) the value(s) of $x \in S$ at which $f(x)$ is minimum (*resp.* maximum) over S.

Let $f(x) > 0$ be continuous **positive** function with $\max_x f(x) = 1.$ (Note that $\max_x f(x)$ is the maximum value of the function, which is different from $\mathrm{argmax} f(x)$, the value of the argument x at which the function is maximum.)

Which of the following functions of $f(x)$ has the same argmax as $f(x)$? In other words, which of the following attain their maxima at the same x-value(s) as $f(x)$?
(Choose all that apply.)

- [x] $f(x)^2$ ✔
- [x] $\sqrt{f(x)}$ ✔

- [x] $\ln(f(x))$ ✔
- [x] $-\ln\left(\frac{1}{f(x)}\right)$ ✔
- [ ] $\cos(f(x))$
- [ ] $-\cos(2f(x))$ ✔

✖

Solution:

We go through the choices in order.

- Since y^2, $\sqrt{y}$, $\ln(y) = -\ln\left(\frac{1}{y}\right)$ are all **strictly increasing** functions, their value increases as y increases. Hence, the functions $f(x)^2$, $\sqrt{f(x)}$, $\ln(f(x))$, $-\ln\left(\frac{1}{f(x)}\right)$ attain their maxima when $f(x)$ attain its maximum, which is at $x = \operatorname{argmax} f(x)$.
- The cosine function is strictly decreasing in $(0, \pi)$. Given $\max_x f(x) = 1 < \pi$, $\cos(f(x))$ is in fact minimum when $f(x)$ is maximum.
- On the other hand, $-\cos(2y)$ is strictly increasing for $0 < 2y < \pi$. Since $\max_x 2f(x) = 2 < \pi$, we conclude that $-\cos(2f(x))$ is maximum again when $f(x)$ is maximum, at $x = \operatorname{argmax} f(x)$.

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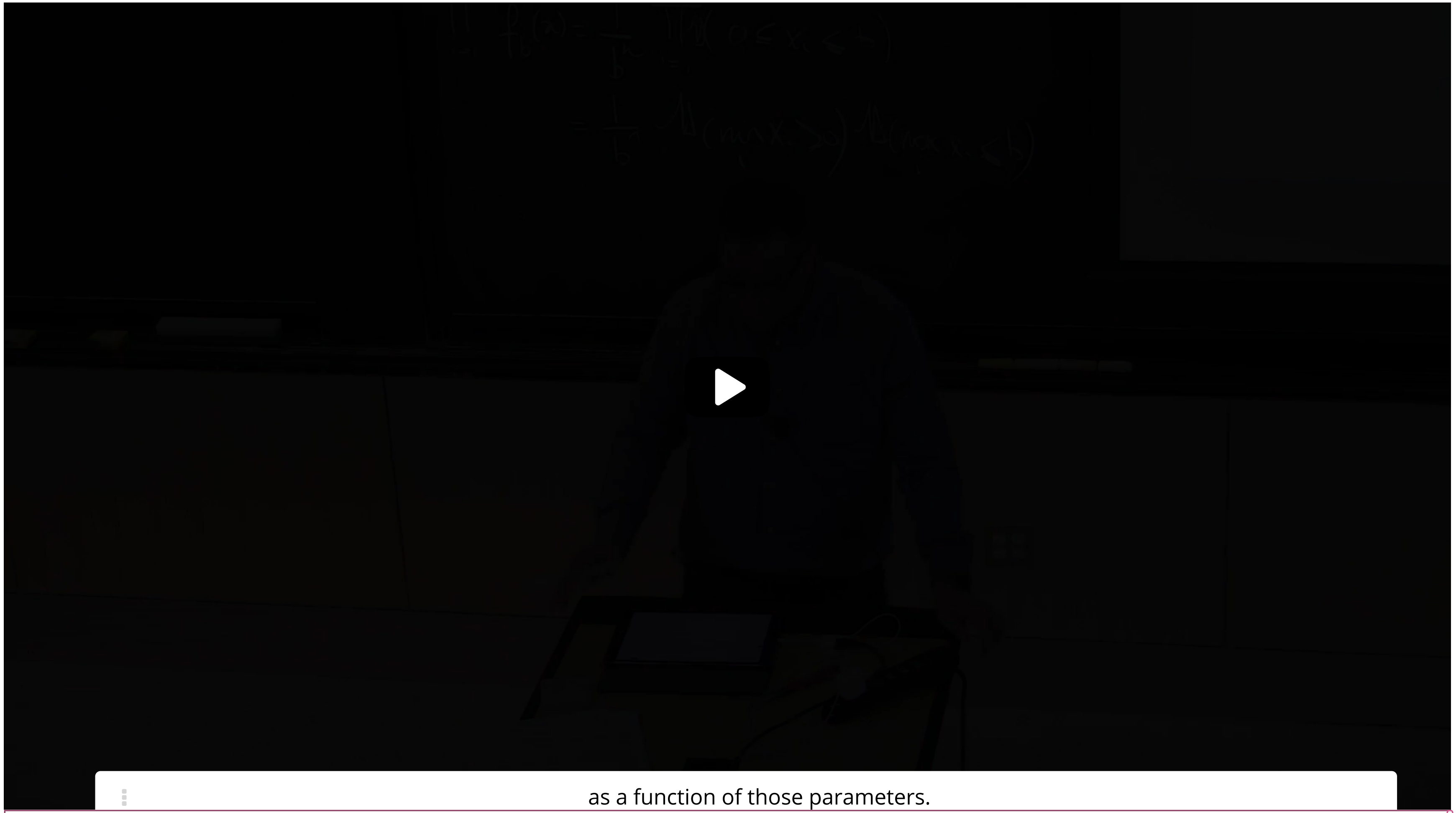
Definition of Maximum Likelihood Estimator and Log Likelihood



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Concept Check: Interpreting the Maximum Likelihood Estimator

1/1 point (graded)

Let $X_1, \ldots, X_n \stackrel{iid}{\sim} \mathbf{P}_{\theta^*}$ be discrete random variables. We construct a statistical model $(E, \{\mathbf{P}_\theta\}_{\theta \in \mathbb{R}})$ where $\mathbf{P}_\theta$ has pmf p_θ. We observe our sample to be $X_1 = x_1, X_2 = x_2, \ldots, X_n = x_n$. The **maximum likelihood estimator** for θ^* is defined to be

$$\hat{\theta}_n^{MLE} = \text{argmax}_{\theta \in \mathbb{R}} \left(\prod_{i=1}^{n} p_\theta(X_i) \right).$$

Which of the following is a correct interpretation of the maximum likelihood estimator (MLE) when applied to the sample $X_1 = x_1, X_2 = x_2, \ldots, X_n = x_n$?
(Choose all that apply.)

- [x] The value of θ that maximizes the probability that $\mathbf{P}_\theta$ generates the data set $(x_1, \ldots, x_n)$.
- [x] The value of θ that minimizes an estimator of the KL divergence between $\mathbf{P}_\theta$ and the true distribution $\mathbf{P}_{\theta^*}$.
- [ ] It is the true parameter θ^*

✔

Solution:

- "The value of θ that maximizes the probability that $\mathbf{P}_\theta$ generates the data set $(x_1, \ldots, x_n)$." is correct. Since the likelihood is the joint density of n iid samples from $\mathbf{P}_\theta$,

$$\mathbf{P}_\theta[X_1 = x_1, \ldots, X_n = x_n] = L_n(x_1, \ldots, x_n, \theta).$$

Hence, the MLE finds $\hat{\theta}_n$ that maximizes the probability that $x_1, \ldots, x_n$ were sampled from $P_{\hat{\theta}_n}$.

- "The value of θ that minimizes the KL divergence between $\mathbf{P}_\theta$ and the true distribution $\mathbf{P}_{\theta^*}$." is correct. In fact, this is how the MLE was derived from KL divergence. See the third section "Parameter Estimation via KL Divergence" of this lecture to review this fact.
- "It is the true parameter θ^*" is incorrect. The MLE is an estimator– it is constructed from the finite amount of data $x_1, \ldots, x_n$ that we are given– so we can't hope for it to exactly recover the true parameter.

Remark: Under some technical conditions the MLE is a **weakly consistent estimator** for θ^*, meaning that the MLE will converge to θ^* in probability under these conditions. However, there are examples of statistical models where the maximum likelihood estimator will **not** converge to the true parameter.

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