Exercise: Discrete unknowns

(5/5 points)

Let Θ_1 and Θ_2 be some unobserved Bernoulli random variables and let X be an observation. Conditional on $X = x$, the posterior joint PMF of Θ_1 and Θ_2 is given by

$$p_{\Theta_1,\Theta_2|X}(\theta_1,\theta_2 \mid x) = \begin{cases} 0.26, & \text{if } \theta_1 = 0, \theta_2 = 0, \\ 0.26, & \text{if } \theta_1 = 0, \theta_2 = 1, \\ 0.21, & \text{if } \theta_1 = 1, \theta_2 = 0, \\ 0.27, & \text{if } \theta_1 = 1, \theta_2 = 1, \\ 0, & \text{otherwise.} \end{cases}$$

We can view this as a hypothesis testing problem where we choose between four alternative hypotheses: the four possible values of (Θ_1, Θ_2).

a) What is the estimate of (Θ_1, Θ_2) provided by the MAP rule?

(1,1) ▾ **Answer:** (1,1)

b) Once you calculate the estimate $(\hat{\theta}_1, \hat{\theta}_2)$ of (Θ_1, Θ_2), you may report the first component, $\hat{\theta}_1$, as your estimate of Θ_1. With this procedure, your estimate of Θ_1 will be

1 ▾ **Answer:** 1

c) What is the probability that Θ_1 is estimated incorrectly (the probability of error) when you use the procedure in part (b)?

0.52 **Answer:** 0.52

d) What is the MAP estimate of Θ_1 based on X, that is, the one that maximizes $p_{\Theta_1|X}(\theta_1 \mid x)$?

0 ▾ **Answer:** 0

e) The moral of this example is that an estimate of Θ_1 obtained by identifying the maximum of the joint PMF of all unknown random variables is

can be different from ▾ ✔ **Answer:** can be different from

the MAP estimate of Θ_1.

Answer:
a) The posterior is largest when $(\theta_1, \theta_2) = (1, 1)$.

b) The corresponding estimate of Θ_1 is the first component of $(1, 1)$, which is 1.

c) The probability of error is the posterior probability that $\Theta_1 = 0$, which is $0.26 + 0.26 = 0.52$.

d) The posterior PMF of Θ_1 is the marginal (posterior) PMF obtained from the joint posterior PMF:

$$\begin{aligned} p_{\Theta_1|X}(0 \mid x) &= 0.26 + 0.26 = 0.52, \\ p_{\Theta_1|X}(1 \mid x) &= 0.21 + 0.27 = 0.48. \end{aligned}$$

Hence, the MAP estimate is $\hat{\theta}_1 = 0$.

e) These can be different, as illustrated by parts (b) and (d).

You have used 1 of 1 submissions