Homework

Homework due Jul 8, 2020 21:30 IST

The exercises below will count towards your grade. **You have only one chance to answer these questions.** Take your time, and think carefully before answering.

Problem 1

40/40 points (graded)

Imagine an island on which everyone is either a knight, who always tells the truth, or a knave, who always lies. Ten islanders, $S_0, S_1, \ldots, S_9$ are lined up. S_0 is in the back of the line; in front of her is S_1, in front of him is S_2, and so on. Every islander says:

"There is at least one person in front of me, and everyone in front of me is a knave."

Is S_0 a knight or a knave?

- ( ) S_0 is a knight.
- (•) S_0 is a knave.
- ( ) There is no coherent assignment of knight or knave status to S_0.
- ( ) It is consistent with the problem that S_0 be a knight, and consistent with the problem that S_0 be a knave.

Explanation

S_0 is a knave. We know that S_9 doesn't have anyone in front of her. So, S_9 is lying — so S_9 is a knave. So S_8 speaks truly when he says, "There is at least one person in front of me, and everyone in front of me is a knave." Since S_8 is telling the truth, S_8 must be a knight. But then we know that everyone else — S_7, S_6, ... and S_0 — are lying when they say "There is at least one person in front of me, and everyone in front of me is a knave," since in front of all of them is one person who isn't a knave. Since S_0 is lying, S_0 is a knave.

Next, imagine that there are a countable infinity of islanders $S_0, S_1, S_2, \ldots$ lined up (it's a big island — an infinitely big one). S_0 is at the back of the line; in front of her is S_1, in front of him is S_2, and so on. Every islander says:

"There is at least one person in front of me, and everyone in front of me is a knave."

Is S_0 a knight or a knave?

- ( ) S_0 is a knight.
- ( ) S_0 is a knave.
- (•) There is no coherent assignment of knight or knave status to S_0.
- ( ) It is consistent with the problem that S_0 be a knight, and consistent with the problem that S_0 be a knave.

Explanation

This problem is just Yablo's Paradox in disguise, with islanders instead of sentencess. To see that no coherent assignment of knight or knave status is possible for S_0, we will proceed by *reductio*. We will first consider the assumption that S_0 is a knight, and show that it leads to contradiction. We will then consider the assumption that S_0 is a knave, and show that it too leads to contradiction.

Suppose, first, that S_0 is a knight. Then what she says is true. So it is true that there is at least one person in front of S_0, and that everyone in front of her is a knave. That means, in particular, that S_1 is a knave. So everything S_1 says is false. So it is false that there is at least one person in front of S_1 and that everyone in front of S_1 is a knave. But since there are, in fact, people in front of S_1, the only way for this to be false is for there to be some knight in front of S_0, which contradicts the hypothesis that everything S_0 says is true.

Now suppose that S_0 is a knave. Then what she says is false. So it is false that there is at least one person in front of S_0 and that everyone in front of S_0 is a knave. Since there are, in fact, people in front of S_0, the only way for this to be false is for there to be some knight in front of S_0. Say that that knight is S_k. The same reasoning we used above to show that S_0 cannot be a knight can be used to show that S_k cannot be a knight.

Now, imagine the infinitely many islanders are lined up just as before. But this time every islander says:

"There is at least one person in front of me, and everyone in front of me is a knight."

Is S_0 a knight or a knave?

- S_0 is a knight.
- S_0 is a knave.
- There is no coherent assignment of knight or knave status to S_0.
- It is consistent with the problem that S_0 be a knight, and consistent with the problem that S_0 be a knave.

✔

Explanation

It is consistent with the description of the problem that everyone is a knight, and therefore that S_0 is a knight. To see this, suppose that everyone is, indeed, a knight. Then for, any S_k, it is true that there is at least one person in front of S_k, and that everyone in front of S_k is a knight. So what S_k says is true. So S_k is indeed a knight. This means that our hypothesis that everyone is a knight is consistent with the description of the problem But it is also consistent with the problem that everyone is a knave, and therefore that S_0 is a knave. To see this, suppose that everyone is, indeed, a knave. Then for, any S_k, it is false that everyone in front of S_k is a knight (and therefore false that there is at least one person in front of S_k, *and* that everyone in front of S_k is a knight). So what S_k says is false. So S_k is indeed a knave. This means that our hypothesis that everyone is a knave is consistent with the description of the problem

Submit You have used 1 of 1 attempt

Answers are displayed within the problem

Problem 2

20/20 points (graded)

Lazy wants to run from A to B, but he likes to take one-second breaks. He first stops halfway between A and B and takes a one-second break. He then stops halfway between *that* point and B and takes a one-second break, and so on. More generally, for each $k \geq 1$, Lazy takes a break at a distance of $(B - A)/2^k$ from B. Assume that the traveling itself takes Lazy no time at all.

Is there a positive integer n such that after n seconds Lazy has reached point B?

- Yes
- No (selected) ✔

If so, what is it? (If you answered 'No' above, enter '0')

0 ✔ **Answer:** 0

0

Explanation

No, there is no such positive integer n. For suppose otherwise. Then there is some positive integer n such that Lazy will have reached B after n seconds. But that means she must have taken no more than n breaks. But after n breaks she will be at a distance from B of $(1/2^n)$ the distance between B and A. So if Lazy takes no more than n breaks, then she cannot be any closer to B than $(1/2^n)$ the distance between B and A.

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Answers are displayed within the problem

Problem 3

15/30 points (graded)

Fool has infinitely many dollar bills, and has labeled each of them with a different natural number (its 'serial number'). One minute before midnight, Fool gives you a dollar bill. Half a minute later, he gives you two dollars. Fifteen seconds later, he gives you four dollars. And so forth. (For each $i \geq 0$, Fool gives you 2^i dollars 2^{-i} minutes before midnight) There is, however, a catch. Each time you receive money from Fool, you are required to put together all your dollar bills and burn the one with the lowest serial number.

Assume that, at midnight, you will have every dollar bill that you received from Fool and did not burn. How much money will you have at midnight?

0

✔ **Answer:** 0

0

Explanation

You will have zero dollars at midnight. To see this, suppose otherwise. Then you are left with one or more bills at midnight. Let k be the serial number of one of them. After the time at which you received bill k infinitely many burnings took place, but there are at most $k-1$ bills you could have burned before burning bill k. So you must have burnt bill k after all.

Next, imagine that you give Fool one dollar at one minute before midnight. Half a minute later, he gives you two dollars. Fifteen seconds later, you give Fool one dollar. Seven and a half seconds later, Fool gives you four dollars. And so forth. (In general, for each $i \geq 0$, you give Fool one dollar at 2^{-2i} minutes before midnight and Fool gives you 2^{i+1} dollars at $2^{-(2i+1)}$ minutes before midnight.)

Assume that, at midnight, you have every dollar bill that you received from Fool and did not return. How much money will you have at midnight?

- (●) Zero dollars
- ( ) Infinitely many dollars
- ( ) A finite amount of dollars
- ( ) It depends ✔

✖

Explanation

It depends on which of your bills you give Fool. If you always give Fool the bill with the smallest serial number, then you will end up with zero dollars (as shown by the answer to the previous problem). On the other hand, if you always give back one of the bills you were just given, then you will end up with infinitely many dollars. (And you can easily arrange to end up with k many bills for any natural number k.)

Submit

You have used 1 of 1 attempt

Answers are displayed within the problem

Problem 4

0/10 points (graded)

You and I each toss a fair coin, and are asked to guess how the other person's coin landed. (Neither of us has any information about how the other's coin landed.) If at least one of us guesses correctly we both get a prize, otherwise we get nothing.

Is there a strategy that we could agree upon ahead of time which would guarantee that we win the prize?

- ( ) Yes ✔
- (•) No
- ( ) It is not determined by the description of the case

✖

Explanation

Here is one strategy: you guess that my coin landed on the same side as yours, and I guess that your coin landed on the opposite side of mine. If the coins landed alike, you guess correctly and we win; if the coins landed differently, I guess correctly and we win.

Submit You have used 1 of 1 attempt

Answers are displayed within the problem

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