Circular permutations with repetitions

n distinct objects have $n!$ (linear) permutations and thus $(n-1)!$ circular permutations.

Now consider m objects, some identical, r_1 of the first kind, r_2 of the second kind, ..., r_k of the kth kind. These n objects have $\frac{m!}{r_1!r_2!...r_k!}$ (linear) permutations.

Can we likewise reason that these m objects have $\frac{(m-1)!}{r_1!r_2!...r_k!}$ circular permutations? I think the answer is no, but can someone explain the intuition why the reasoning that worked earlier doesn't work here? Also, what is the correct number of circular permutations for these $m!$ objects?

(I am hoping for an answer that's suitable for high school students. Thanks.)

(combinatorics)

edited Jul 17 at 2:17 asked Jul 16 at 11:51

Kenny LJ
1,151 7 23

The correct calculation will allow for the fact that under certain circimstances, some arrangements of the circle will be indistinct from each other because for TWO reasons: Rotational symmetry of the circle, and rotational symmetry of the pattern $m_1, m_2, m_4 ...$ due to indistinct elements. Therefore the prior method will have subtracted these twice so you will need to add them back in, by the inclusion-exclusion principle. – Robert Frost Jul 16 at 12:07

Hint: try some small examples. The case $k = 1$, $m > 1$ should give 1 permutation, but your formula gives $1/m$ (not even a whole number). A better guess based on the argument for the case with no identifications would be $(m-1)!/((r_1-1)! \ldots r_k!)$, but that will be an overestimate unless $r_1 = 1$: try the case $k = 2$, $r_1 = 2$, $r_2 = 3$ to see why. – Rob Arthan Jul 16 at 12:13

1 This can be done using Burnside's Lemma, as you are counting equivalence classes of permutations of elements in a circle under rotational equivalence. See en.m.wikipedia.org/wiki/Burnside%27s_lemma – Tad Jul 16 at 18:47

This problem recently appeared at this MSE link. – Marko Riedel Jul 16 at 19:38