Data Analysis: Statistical Modeling and Computation in Applications

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5. Introduction to Clustering

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Exercises due Oct 20, 2021 17:29 IST Completed

Clustering Measures

1/1 point (graded)

Which of the following could be valid measures of clustering? Pick the ones that will make sense more often than not. That is, choose only the ones that do not just apply under a few contrived settings. For this problem, think of clustering as a notion that captures how close a set of nodes are with respect to other nodes in the graph.

| Nodes with the same degree. |
|--|
| Nodes with node identification values (node index) close to each other. |
| Nodes with edges connecting each other more often than their edges connecting outside of them. |
| Nodes that are "key" to the connectivity of the graph in the following sense: in the absence of them (along with their edges), the graph is no longer connected. |
| |

Solution:

- 1. Nodes with the same degree No, two nodes with the same degree need not be close to each other.
- 2. Nodes with node identification values (node index) close to each other No, this is not a good measure of clustering. It is dependent on the index we give to every node.
- 3. Nodes with edges connecting each other more often than their edges connecting outside of them Yes, this captures some measure of clustering.
- 4. Nodes that are "key" to the connectivity of the graph in the following sense: in the absence of them (along with their edges), the graph is no longer connected – No, this notion captures nodes that are critical to the graph. Such nodes need not be close to each other in the graphical sense (such as in 3.)

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You have used 2 of 2 attempts

1 Answers are displayed within the problem

Graph Laplacian

We now introduce the **graph Laplacian matrix** of a graph. Throughout the remainder of this lecture, we assume only that the graph is simple and undirected. Everything discussed here also works for undirected, weighted, simple graphs.

The **graph Laplacian matrix** $m{L}$ is defined as

$$L=D-A$$
,

where $m{A}$ is the adjacency matrix and $m{D}$ is the degree matrix. The **degree matrix** for an undirected, unweighted graph is a matrix whose off-diagonal elements are equal to $oldsymbol{0}$ and whose diagonal elements are given by

$$D_{ii} = \sum_j A_{ij}.$$

In other words, the degree matrix of an undirected, unweighted, simple graph is simply a diagonal matrix whose diagonal entries are the degrees of the nodes. In the case of a weighted, undirected, simple graph the definition is the same but the interpretation no longer concerns the degree of the nodes, but rather the sum weight of the

Graph Laplacian – Example

4/4 points (graded)

Consider the adjacency matrix

$$A = egin{pmatrix} 0 & 1 & 0 \ 1 & 0 & 0 \ 0 & 0 & 0 \end{pmatrix}.$$

1. How many components does the graph have?

2. What are the diagonal values of the Laplacian matrix of the graph?

First diagonal element: 1

Second: 1

Third: 0

Answer: 1

Answer: 0

Solution:

- 1. The first and second node are connected to each other, while the third node is not connected at all. Thus there are two components: $\{\{1,2\},\{3\}\}$.
- 2. The diagonal elements of $m{A}$ are zero (as it is simple), so the diagonal elements of $m{L}$ come from $m{D}$. The first diagonal element is the degree of the first node, etc.

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You have used 1 of 2 attempts

• Answers are displayed within the problem

Symmetry

1/1 point (graded)

Let L be the Laplacian of an undirected simple graph (weighted or unweighted). **True** or **False**? The Laplacian is symmetric.





Solution:

The adjacency matrix and the degree matrix are both symmetric – hence the Laplacian is also symmetric.

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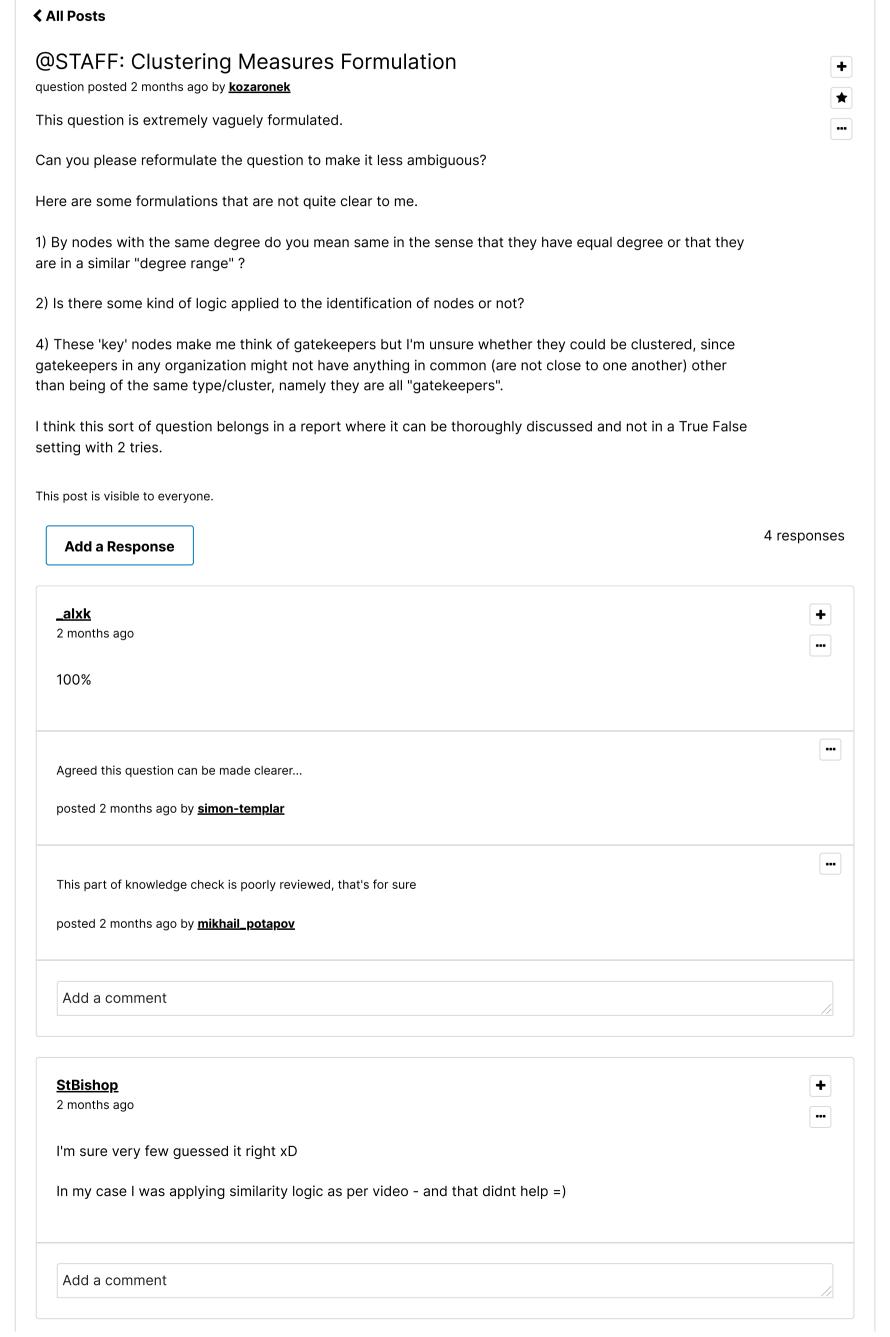
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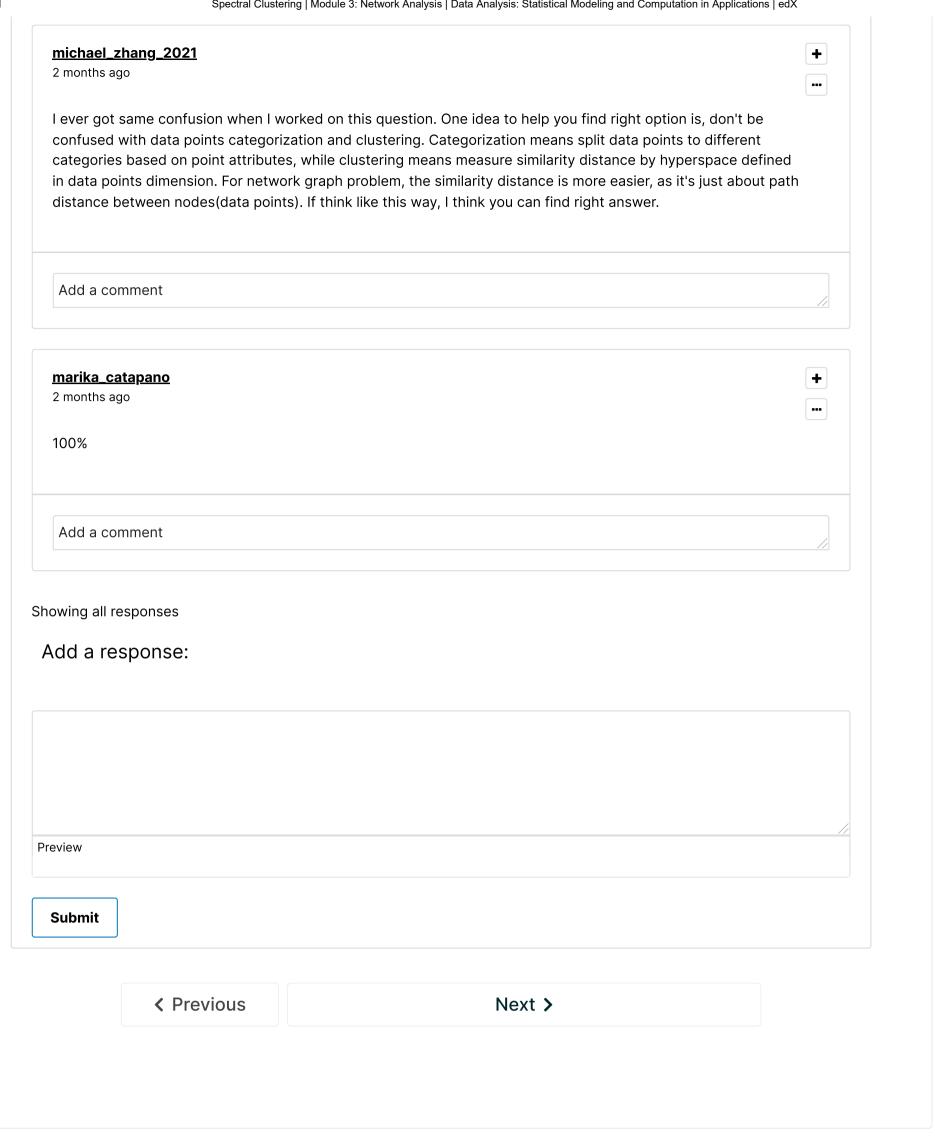
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