## 11.2.2 Bisection method

MO2.10

One of the most common methods for root finding in a scalar equation is the bisection method. The bisection methods starts from two points $a^0$ and $b^0$ such that either

$$r\left(a^0\right) > 0, \qquad \text{and} \qquad r\left(b^0\right) < 0.$$

or

$$r\left(a^0\right) < 0, \qquad \text{and} \qquad r\left(b^0\right) > 0.$$

When $r$ is continous, there must exist a root in the interval $\left[a^0, b^0\right]$. At stage $k$, assume that we have obtained an interval $\left[a^k, b^k\right]$ such that the same sign properties hold, i.e. $r\left(a^k\right)$ and $r\left(b^k\right)$ have opposite signs. The bisection method consists in subdividing the interval $\left[a^k, b^k\right]$ in two and keeping the half in which there must be a root. Let $m^k = \left(a^k + b^k\right)/2$. The key concept to deciding which half to keep is to determine whether $r\left(m^k\right)$ has the opposite sign as $r\left(a^k\right)$ or $r\left(b^k\right)$. This can be done by multiplying $r\left(a^k\right) r\left(m^k\right)$ and $r\left(b^k\right) r\left(m^k\right)$ and checking the signs of these multiplications. When the sign is negative, then the corresponding $r$ values have the opposite sign and there must still be a root between the corresponding $x$ values.

Here's the bisection algorithm based upon this concept:

- If $r\left(a^k\right) r\left(m^k\right) > 0$, then it is the interval $\left[m^k, b^k\right]$ which is of interest. We put $a^{k+1} = m^k$ and $b^{k+1} = b^k$.
- If $r\left(a^k\right) r\left(m^k\right) < 0$, then it is the interval $\left[a^k, m^k\right]$ which is of interest. We put $a^{k+1} = a^k$ and $b^{k+1} = m^k$.
- If $r\left(a^k\right) r\left(m^k\right) = 0$, then $m^k$ is a root and we stop the algorithm.

In practice, this iteration is stopped once $r\left(m^k\right)$ gets small enough. Let $x^*$ be the unknown root. The error

$$\left|x^* - m^k\right| \le \frac{1}{2}\left|b^k - a^k\right| = 2^{-k-1}\left|b^0 - a^0\right|$$

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