Problem 3: Oscar's lost dog in the forest

(5/6 points)

Oscar has lost his dog in either forest A (with probability 0.4) or in forest B (with probability 0.6).

If the dog is in forest A and Oscar spends a day searching for it in forest A, the conditional probability that he will find the dog that day is 0.25. Similarly, if the dog is in forest B and Oscar spends a day looking for it there, he will find the dog that day with probability 0.15.

The dog cannot go from one forest to the other. Oscar can search only in the daytime, and he can travel from one forest to the other only overnight.

The dog is alive during day 0, when Oscar loses it, and during day 1, when Oscar starts searching. It is alive during day 2 with probability $2/3$. In general, for $n \geq 1$, if the dog is alive during day $n-1$, then the probability it is alive during day n is $2/(n+1)$. The dog can only die overnight. Oscar stops searching as soon as he finds his dog, either alive or dead.

a) In which forest should Oscar look on the first day of the search to maximize the probability he finds his dog that day?

Forest A ▼ ✔ **Answer:** Forest A

b) Oscar looked in forest A on the first day but didn't find his dog. What is the probability that the dog is in forest A?

0.3333333 ✔ **Answer:** 0.33333

c) Oscar flips a fair coin to determine where to look on the first day and finds the dog on the first day. What is the probability that he looked in forest A?

0.5263158 ✔ **Answer:** 0.52632

d) Oscar decides to look in forest A for the first two days. What is the probability that he finds his dog alive for the first time on the second day?

0.05 ✔ **Answer:** 0.05

e) Oscar decides to look in forest A for the first two days. Given that he did not find his dog on the first day, find the probability that he does not find his dog dead on the second day.

0.9166667 ✖ **Answer:** 0.97222

f) Oscar finally finds his dog on the fourth day of the search. He looked in forest A for the first 3 days and in forest B on the fourth day. Given this information, what is the probability that he found his dog alive?

- Unit 10: Markov chains
- Exit Survey
- Final Exam

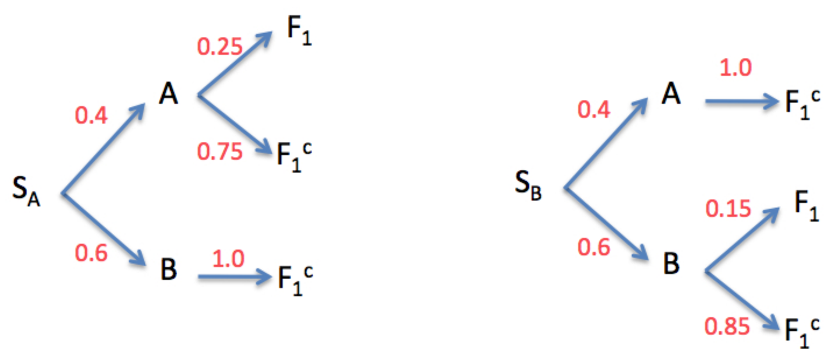
0.1333333

Answer: 0.13333

Answer:
We define the following events:

$$\begin{aligned} S_A &= \text{event that Oscar searches for his dog in forest A} \\ S_B &= \text{event that Oscar searches for his dog in forest B} \\ A &= \text{event that his dog is lost in forest A} \\ B &= \text{event that his dog is lost in forest B} \\ F_i &= \text{event that Oscar finds his dog on day } i \\ L_i &= \text{event that his dog is alive on day } i \end{aligned}$$

a) Oscar has two choices represented by the following tree diagrams:

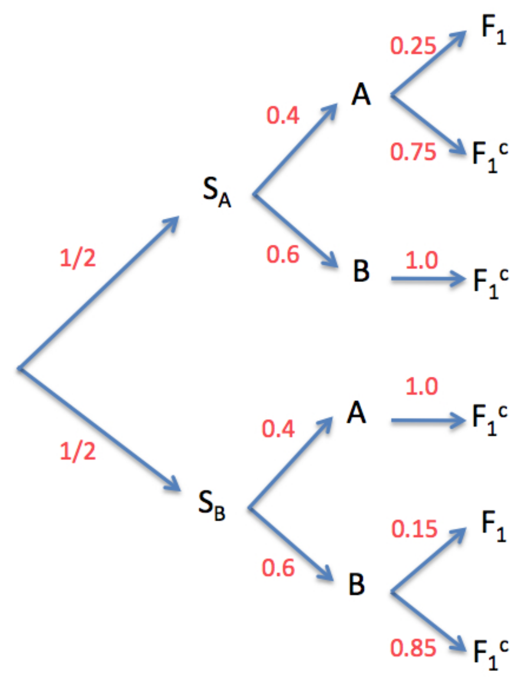


To make his choice, Oscar compares $\mathbf{P}(F_1 \mid S_A) = (0.4)(0.25) = 0.1$ with $\mathbf{P}(F_1 \mid S_B) = (0.6)(0.15) = 0.09$, and thus he should choose to search in forest A.

b)The desired probability is

$$\mathbf{P}(A \mid S_A \cap F_1^c) = \frac{\mathbf{P}(A \cap S_A \cap F_1^c)}{\mathbf{P}(S_A \cap F_1^c)} = \frac{(0.4)(0.75)}{(0.4)(0.75) + (0.6)(1)} = \frac{1}{3}.$$

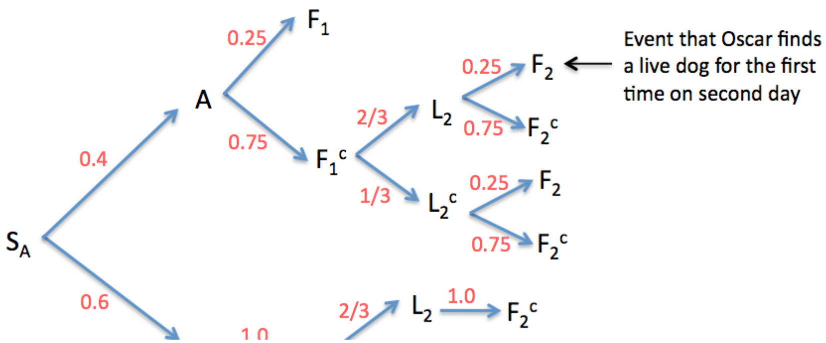
c) We can combine the two diagrams in part (a) to get the following diagram:



The desired probability is

$$\mathbf{P}(S_A \mid F_1) = \frac{\mathbf{P}(S_A \cap F_1)}{\mathbf{P}(F_1)} = \frac{(0.5)(0.4)(0.25)}{(0.5)(0.4)(0.25) + (0.5)(0.6)(0.15)} = \frac{10}{19}.$$

d) The following tree diagram illustrates the sequence of possible events:



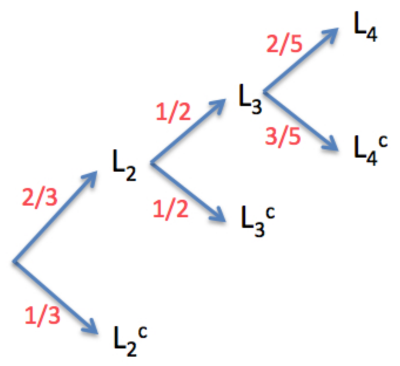
The desired probability is

$$\mathbf{P}(A \cap F_1^c \cap L_2 \cap F_2 \mid S_A) = (0.4)(0.75)(2/3)(0.25) = 0.05.$$

e) We can use the same diagram as in part (d).

$$\begin{aligned}
&\mathbf{P}(\text{Oscar does not find dead dog on day 2} \mid F_1^c \cap S_A) \\
&= 1 - \mathbf{P}(\text{Oscar does find dead dog on day 2} \mid F_1^c \cap S_A) \\
&= 1 - \frac{\mathbf{P}(S_A \cap A \cap F_1^c \cap L_2^c \cap F_2)}{\mathbf{P}(F_1^c \cap S_A)} \\
&= 1 - \frac{(0.4)(0.75)(1/3)(0.25)}{(0.4)(0.75) + (0.6)(1.0)} \\
&= \frac{35}{36}.
\end{aligned}$$

f) We know that Oscar found his dog and we know it took 4 days. It doesn't matter, then, where he searched. We just want the probability the dog survived to day 4.



This probability is

$$\mathbf{P}(L_4) = \left(\frac{2}{2+1}\right)\left(\frac{2}{3+1}\right)\left(\frac{2}{4+1}\right) = \frac{2}{15}.$$

You have used 2 of 2 submissions

DISCUSSION

Click “Show Discussion" below to see discussions on this problem.