Determine a matrix knowing its eigenvalues and eigenvectors

I read through similar questions, but I couldn't find an answer to this:

How do you determine the symmetric matrix A if you know:

$\lambda_1 = 1,\ eigenvector_1 = \begin{pmatrix} 1 & 0 & -1 \end{pmatrix}^T;$

$\lambda_2 = -2,\ eigenvector_2 = \begin{pmatrix} 1 & 1 & 1 \end{pmatrix}^T;$

$\lambda_3 = 2,\ eigenvector_3 = \begin{pmatrix} -1 & 2 & -1 \end{pmatrix}^T;$

I tried to solve it as an equation system for each line, but it didn't work somehow.

I tried to find the inverse of the eigenvectors, but it brought a wrong matrix.

Do you know how to solve it?

Thanks!

(matrices) (eigenvalues-eigenvectors)

edited Jan 26 '15 at 0:09
abel
25.4k 1 16 45

asked Jan 25 '15 at 23:57
user3435407
406 1 4 12

math.stackexchange.com/questions/54818/… – Amzoti Jan 26 '15 at 0:04

3 Answers

Writing the matrix down in the basis defined by the eigenvectors is trivial. It's just

$$M = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 2 \end{pmatrix}.$$

Now, all we need is the change of basis matrix to change to the standard coordinate basis, namely:

$$S = \begin{pmatrix} 1 & 1 & -1 \\ 0 & 1 & 2 \\ -1 & 1 & -1 \end{pmatrix}.$$

This is just the matrix whose columns are the eigenvectors. We can change to the standard coordinate bases by computing SMS^{-1}. We get

$$SMS^{-1} = \frac{1}{6}\begin{pmatrix} 1 & -8 & -5 \\ -8 & 4 & -8 \\ -5 & -8 & 1 \end{pmatrix}.$$

You can check that this matrix has the desired eigensystem. For example,

$$\frac{1}{6}\begin{pmatrix} 1 & -8 & -5 \\ -8 & 4 & -8 \\ -5 & -8 & 1 \end{pmatrix}\begin{pmatrix} -1 \\ 2 \\ -1 \end{pmatrix} = \begin{pmatrix} -2 \\ 4 \\ -2 \end{pmatrix}.$$

edited Jan 26 '15 at 1:28

answered Jan 26 '15 at 0:18

Mark McClure
19k 1 32 56

sorry, I don't get it. Shouldn't be the result Ax = λx for all the three pairs of values and vectors? Like for example for the first pair: result-matrix * u1 = 1 * u1? – user3435407 Jan 26 '15 at 0:54

@user3435407 Yep - I guess I don't get what you don't get. I added one check. – Mark McClure Jan 26 '15 at 1:28

2 People: If you were searching for the answer of this question too, now I found a good live demonstration of the above mentioned solution here: youtube.com/watch?v=HWnCv4iHCDc – user3435407 Jan 26 '15 at 3:40

call the eigenvectors u_1, u_2 and u_3 the eigenvectors corresponding to the eigenvalues $1, -2,$ and $2.$ then

$$A = 1\frac{u_1u_1^T}{u_1^Tu_1} - 2\frac{u_2u_2^T}{u_2^Tu_2} + 2\frac{u_3u_3^T}{u_3^Tu_3}$$

you can verify this by computing $Au_1, \cdots$. this expression for A is called the spectral decomposition of a symmetric matrix.

answered Jan 26 '15 at 0:08

abel
25.4k 1 16 45

ok, thanks, I need to count it through first, I'll let you know when I managed it! – user3435407 Jan 26 '15 at 0:11

An $n \times n$ matrix with n independent eigenvectors can be expressed as $A = PDP^{-1}$, where D is the diagonal matrix $\mathrm{diag}(\lambda_1\ \lambda_2\ \cdots \lambda_n)$ and P is the matrix $(\vec{v}_1 \mid \vec{v}_2 \mid \cdots \mid \vec{v}_n)$ where v_i is the corresponding eigenvector to λ_i.

$$D = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

$$P = \begin{pmatrix} 1 & 0 & -1 \\ 1 & 1 & 1 \\ -1 & 2 & -1 \end{pmatrix}$$

edited Jun 18 '16 at 3:28

Michael Hardy
166k 19 150 364

answered Jan 26 '15 at 0:19

George
361 1 10