Exercise: The Product Rule for Random Variables - Medical Diagnosis Revisited

(4/4 points)

Let's revisit the medical diagnosis problem we saw earlier. We now use random variables to construct a joint probability table.

Let random variable X represent the patient's condition — whether “healthy" or “infected", with the following distribution for X:

X	Prob.
healthy	0.999
infected	0.001

Meanwhile, the test outcome Y for whether the patient is infected is either “positive" (for the disease) or “negative". As before, the test is 99% accurate, which means that the conditional probability table for Y given X is as follows (note that we also show how to write things out as a single table):

$Y \mid X = \text{healthy}$	Prob.
positive	0.01
negative	0.99

$Y \mid X = \text{infected}$	Prob.
positive	0.99
negative	0.01

$p_{Y \mid X}$

Y \ X	healthy	infected
positive	0.01	0.99
negative	0.99	0.01

Alternative way to write out the tables for $p_{Y \mid X}(\cdot \mid \text{healthy})$ and $p_{Y \mid X}(\cdot \mid \text{infected})$ in 1 table

Using the product rule for random variables, what are the four entries for the joint probability table? **Please provide the exact answer for these four quantities.**

$p_{X,Y}(\text{healthy}, \text{positive}) =$ 0.00999 ✔ **Answer:** 0.00999

$p_{X,Y}(\text{healthy}, \text{negative}) =$ 0.98901 ✔ **Answer:** 0.98901

$p_{X,Y}(\text{infected}, \text{positive}) =$ 0.00099 ✔ **Answer:** 0.00099

$p_{X,Y}(\text{infected}, \text{negative}) =$ 1e-05 ✔ **Answer:** 0.00001

Solution:

$$\begin{aligned} p_{X,Y}(\text{healthy}, \text{positive}) &= p_X(\text{healthy}) p_{Y|X}(\text{positive} \mid \text{healthy}) \\ &= 0.999 \times 0.01 \\ &= \boxed{0.00999} \end{aligned}$$

$$\begin{aligned} p_{X,Y}(\text{healthy}, \text{negative}) &= p_X(\text{healthy}) p_{Y|X}(\text{negative} \mid \text{healthy}) \\ &= 0.999 \times 0.99 \\ &= \boxed{0.98901} \end{aligned}$$

$$\begin{aligned} p_{X,Y}(\text{infected}, \text{positive}) &= p_X(\text{infected}) p_{Y|X}(\text{positive} \mid \text{infected}) \\ &= 0.001 \times 0.99 \\ &= \boxed{0.00099} \end{aligned}$$

$$\begin{aligned} p_{X,Y}(\text{infected}, \text{negative}) &= p_X(\text{infected}) p_{Y|X}(\text{negative} \mid \text{infected}) \\ &= 0.001 \times 0.01 \\ &= \boxed{0.00001} \end{aligned}$$

You have used 1 of 5 submissions