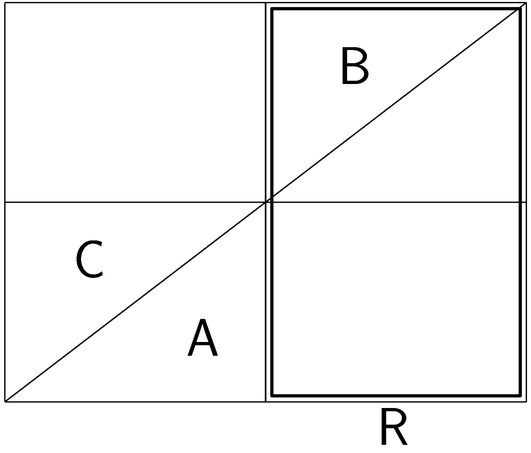
The Quadratic Reciprocity Law (8)

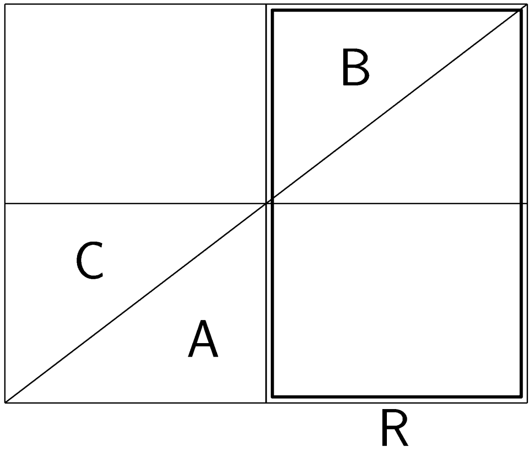
- $(X,Y) \leftrightarrow (P-X, Q-Y)$ is one-to-one between
  (a) lattice points in the interior of △A
  with **odd** x-coord
  (b) lattice points in the interior of △B
  with **even** x-coord
- # of lattice points in
  the interior of the rectangle
  R is a multiple of Q−1 (**even**)



The Quadratic Reciprocity Law (9)

- N = # of lattice points in the interior of △A
  ⇒ **M ≡ N (mod 2)**.
  $$(-1)^N = (-1)^M = \left(\frac{Q}{P}\right)$$

- N′ = # of lattice points in the interior of △C
  $$(-1)^{N'} = \left(\frac{P}{Q}\right)$$



The Quadratic Reciprocity Law (10)

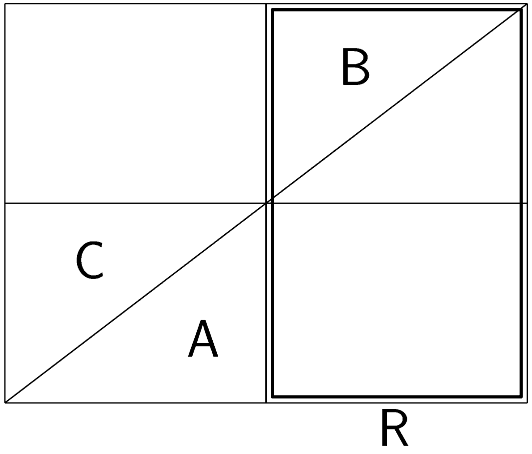
End of the Proof of QRL

$N + N' =$ # of lattice points in the interior of $\triangle A + \triangle C$

$$= \frac{P-1}{2}\frac{Q-1}{2}$$

$$(-1)^{N} \times (-1)^{N'} = \left(\frac{Q}{P}\right) \times \left(\frac{P}{Q}\right)$$

$$= (-1)^{N+N'} = (-1)^{\frac{P-1}{2}\frac{Q-1}{2}}$$



Interlude: Three Epoch-making Mathematicians (1)

Eisenstein proved many foundational results on prime numbers. He proved the **Cubic Reciprocity Law** and the **Quartic Reciprocity Law**.

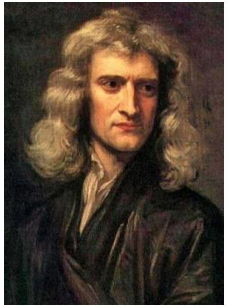
Ferdinand Gotthold Max Eisenstein
(1823-1852)

Interlude: Three Epoch-making Mathematicians (2)

"There had been only three epoch-making mathematicians: Archimedes, Newton, and Eisenstein." (Carl Friedrich Gauss)

Carl Friedrich Gauss (1777-1855)

Archimedes of Syracuse (c.287-c.212 BC)



Isaac Newton (1642-1726/27)

Ferdinand Gotthold Max Eisenstein (1823-1852)

https://en.wikipedia.org/wiki/Archimedes
https://en.wikipedia.org/wiki/Isaac_Newton