PART A

(1/1 point)

The local post office requires full-time employees to meet demands that vary from day to day. The number of full-time employees required on each day is given in Table 1. Each full-time employee must work five consecutive days and then receive two days off. For example, an employee who works Monday to Friday must be off on Saturday and Sunday. This schedule for employees repeats every week. We want to formulate an integer program that the post office can use to minimize the number of full-time employees who must be hired.

Table 1: Requirements for the local post office

	Number of full time employees required
1=Monday	17
2=Tuesday	13
3=Wednesday	15
4=Thursday	19

5=Friday	14
6=Saturday	16
7=Sunday	11

We define the decision variables as follows:

- Let x_1 be the number of full-time employees who work Monday to Friday
- Let x_2 be the number of full-time employees who work Tuesday to Saturday
- $\ldots$
- Let x_7 be the number of full-time employees who work Sunday to Thursday

Then the integer program to minimize the number of total employees is given as follows:

$$\left.\begin{array}{ll} \min & z = x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 \\ \text{s.t.:} & \\ & x_1 + x_4 + x_5 + x_6 + x_7 \geq 17 \quad (1) \\ & x_1 + x_2 + x_5 + x_6 + x_7 \geq 13 \quad (2) \\ & x_1 + x_2 + x_3 + x_6 + x_7 \geq 15 \quad (3) \\ & x_1 + x_2 + x_3 + x_4 + x_7 \geq 19 \quad (4) \\ & x_1 + x_2 + x_3 + x_4 + x_5 \geq 14 \quad (5) \\ & x_2 + x_3 + x_4 + x_5 + x_6 \geq 16 \quad (6) \\ & x_3 + x_4 + x_5 + x_6 + x_7 \geq 11 \quad (7) \\ & x_1, x_2, x_3, x_4, x_5, x_6, x_7 \in Z^+ \end{array}\right\}$$

Constraints (1) – (7) represents that the number of employees who work on Monday to Sunday respectively should satisfy the requirement stated in Table 1. The objective function is the total number of employees required.

The following parts are INDEPENDENT from each other. They are all based on the formulation above.

Use the spreadsheet pset3_p4.xlsx to solve the above integer program. What is the minimum workers that the post office needs?

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You have used 1 of 3 submissions

PART B

(1/1 point)

Let $s_2, s_5, \text{ and } s_7$ (assume integers) denote the surplus number of workers on Tuesday, Friday, and Sunday. For example, if there are 18 workers on Tuesday, then the $s_2 = 5$. Suppose that the current objective is to minimize $z = max\{s_2, s_5, s_7\}$. Reformulate that as an IP. Include all constraints that contain $s_2, s_5 \text{ or } s_7$, other than the non-negativity and integrality constraints, as well as any other changes from the original formulation. Which of the following are necessary changes to the original integer program? Select 6 of the constraints.

- [ ] $z \le s_2$ (new constraint)
- [x] $z \ge s_2$ (new constraint)
- [ ] $z \le s_5$ (new constraint)
- [x] $z \ge s_5$ (new constraint)
- [ ] $z \le s_7$ (new constraint)
- [x] $z \ge s_7$ (new constraint)
- [ ] $x_1 + x_2 + x_5 + x_6 + x_7 + s_2 = 13$ (replace constraint 2)
- [x] $x_1 + x_2 + x_5 + x_6 + x_7 - s_2 = 13$ (replace constraint 2)

- [ ] $x_1 + x_2 + x_3 + x_4 + x_5 + s_5 = 14$ (replace constraint 5)
- [x] $x_1 + x_2 + x_3 + x_4 + x_5 - s_5 = 14$ (replace constraint 5)
- [ ] $x_3 + x_4 + x_5 + x_6 + x_7 + s_7 = 11$ (replace constraint 7)
- [x] $x_3 + x_4 + x_5 + x_6 + x_7 - s_7 = 11$ (replace constraint 7)

You have used 1 of 3 submissions

PART C

(1/1 point)

Let s_1 be the number of excess workers on Monday. Suppose that there is a penalty for having too many workers on Monday. The linear penalty due to s_1 is 0.1. That is, there is no penalty for having 17 workers; there is a penalty of 0.1 for having 18 workers; there is a penalty of 0.2 for having 19 workers (0.1 for both the 18th and 19th workers). The penalty increases if there are more than 19 workers. Here the linear penalty is .25 per worker. For example, if there are 20 workers, the total penalty is .45, which includes a penalty of .25 for the 20th worker. The objective function is to minimize the penalty cost for extra workers on Monday. Let $s_1^{'}$ be the surplus for the first 2 extra workers and $s_1^{''}$ be the surplus for extra workers more than two. Thus $s_1 = s_1^{'} + s_1{''}$.

What would the new objective function be?

- [x] $\text{MIN } 0.1s_1{'} + 0.25s_1^{''}$

- [ ] $\text{MIN } 0.25s_1{}' + 0.1s_1^{''}$
- [ ] $\text{MIN } 0.1s_1{}' + 0.1s_1^{''}$

✔

You have used 1 of 2 submissions

PART D

(1/1 point)
In order to incorporate the requirement in the previous part, what modifications should be made to the Monday constraint (constraint 1)? Select three of the following.

- [x] $x_1 + x_4 + x_5 + x_6 + x_7 - s_1^{'} - s_1^{''} = 17$
- [ ] $x_1 + x_4 + x_5 + x_6 + x_7 - s_1^{'} = 17$
- [ ] $x_1 + x_4 + x_5 + x_6 + x_7 - s_1^{''} = 17$
- [ ] $x_1 + x_4 + x_5 + x_6 + x_7 + s_1^{'} + s_1^{''} = 17$
- [ ] $x_1 + x_4 + x_5 + x_6 + x_7 + s_1^{'} = 17$

- [ ] $x_1 + x_4 + x_5 + x_6 + x_7 + s_1^{''} = 17$
- [x] $0 \le s_1^{'} \le 2$
- [x] $s_1^{''} \ge 0$
- [ ] $s_1^{''} \ge 2$

You have used 1 of 3 submissions

PART E

(1/1 point)

Suppose we want to minimize the absolute difference between the number of workers who start on Saturday (x_6) and the number of workers who start on Tuesday (x_2). Let z be the new objective. Which of the following are necessary changes to the original integer program?

- [x]

$$\left.\begin{array}{lr} \textbf{min} & z \text{ (new objective)} \\ \textbf{s.t.:} & \\ & z \ge x_2 - x_6 \text{ (new constraint)} \\ & z \ge x_6 - x_2 \text{ (new constraint)} \end{array}\right\}$$

$$\left.\begin{array}{lr} \max & z \text{ (new objective)} \\ \text{s.t.:} & \\ & z \leq x_2 - x_6 \text{ (new constraint)} \\ & z \leq x_6 - x_2 \text{ (new constraint)} \end{array}\right\}$$

$$\left.\begin{array}{lr} \min & z \text{ (new objective)} \\ \text{s.t.:} & \\ & z \leq x_2 - x_6 \text{ (new constraint)} \\ & z \leq x_6 - x_2 \text{ (new constraint)} \end{array}\right\}$$

$$\left.\begin{array}{lr} \max & z \text{ (new objective)} \\ \text{s.t.:} & \\ & z \geq x_2 - x_6 \text{ (new constraint)} \\ & z \geq x_6 - x_2 \text{ (new constraint)} \end{array}\right\}$$

You have used 1 of 3 submissions

PART F

(1/1 point)
Suppose that we added the constraint that $x_i \leq 19$ for each $i = 1, \ldots, 7$.

Would this change the feasible region?

Hint: You'll need to check the original formulation.

- No
- Yes ✔

You have used 1 of 1 submissions

PART G

(1/1 point)
Would this change the optimal solution?

- ◉ No ✔
- ○ Yes

You have used 1 of 1 submissions