4. Graphing in three dimensions, an example

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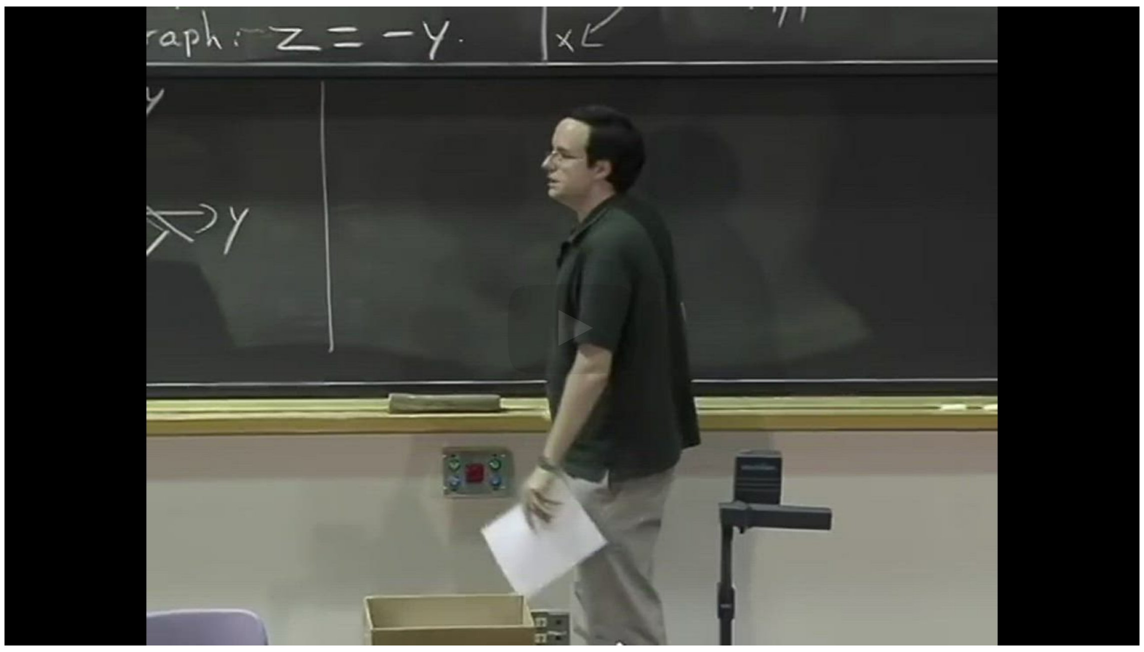

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PROFESSOR: So let's say that we have a slightly more

complicated function, how do we see it?

So let's do another example.

Let's say I give you f of (x, y) equals 1

minus x squared minus y squared.

So we try to picture what the surface

z equals 1 minus x squared minus y squared looks like.

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In this example, we want to understand the graph of the function $z = 1 - x^2 - y^2$.

We will use a process where we **slice** through our surface with different planes and looking at the curves we get to gain a better understanding of the full surface. What do we mean by **slice** ?

> **Definition 4.1** A **slice** of a surface $z = f(x, y)$ by a plane is the **intersection** of the plane and the surface. That is, it is the collection of points that lie on both the surface and the plane simultaneously.

First we look at the slice of the surface with the yz-plane. This plane is defined by the equation $x = 0$. The way we find the intersection is to find all points on the equation for the surface that also satisfy $x = 0$. Mathematically, we perform this operation by substituting the equation for the plane $x = 0$ into the equation for the surface.

$$z = 1 - 0^2 - y^2 = 1 - y^2$$

This is an upside down parabola with vertex at $z = 1$. In three dimensions, it looks like this.

▶ **The slice of the function in the yz-plane**

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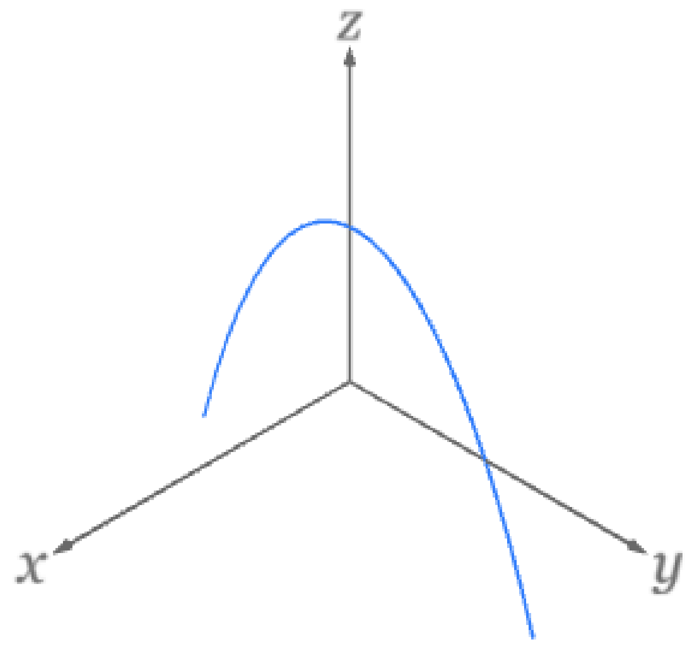


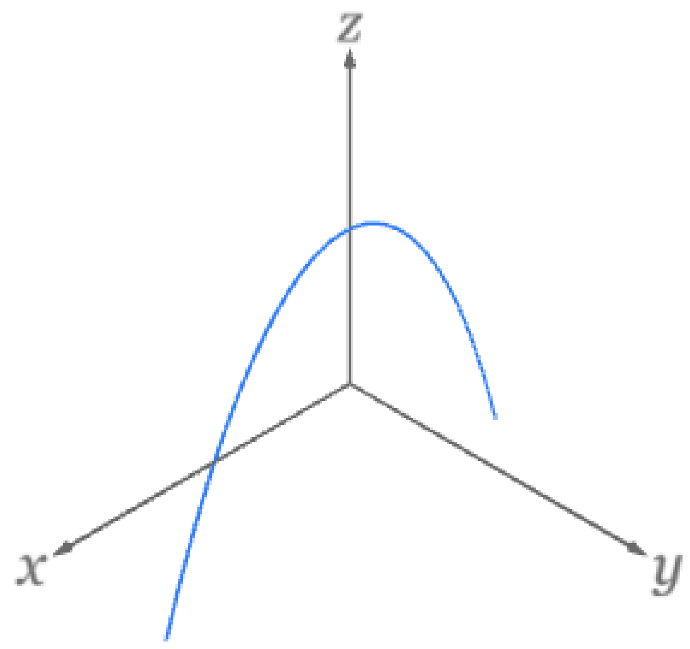
Next, we find the slice of the surface by the xz-plane. This plane is defined by the equation $y = 0$. Thus to find the intersection of the surface with this plane, we substitute $y = 0$ into the equation for the surface.

$$z = 1 - x^2 - 0^2 = 1 - x^2$$

This is an upside down parabola with vertex at $z = 1$.

We can add this curve to three dimensional graph. It looks like this.

▶ **The slice of the function in the xz-plane**



Finally, we might want to see what the intersection with the xy-plane is. This plane is defined by the equation $z = 0$. Plugging this into the equation for our graph we get

$$\begin{aligned} 0 &= 1 - x^2 - y^2 \\ x^2 + y^2 &= 1 \end{aligned}$$

which is the the equation for the unit circle in the xy-plane.

▶ **The slice of the function in the xy-plane**

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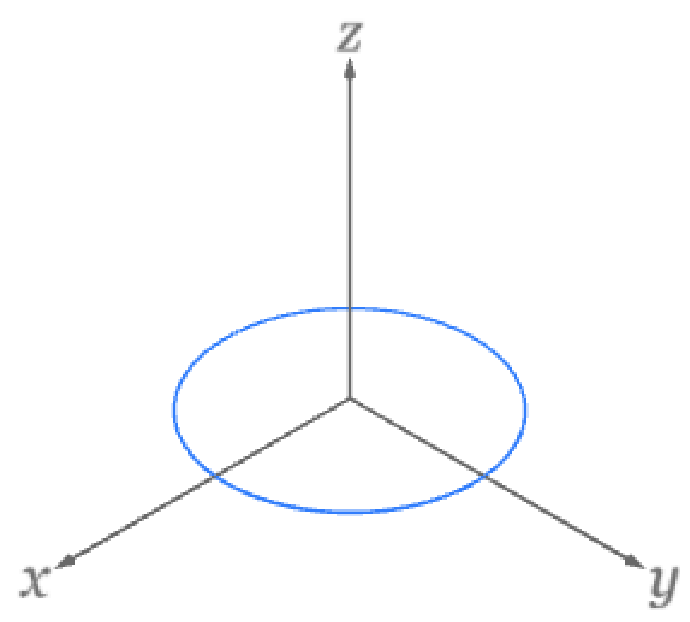
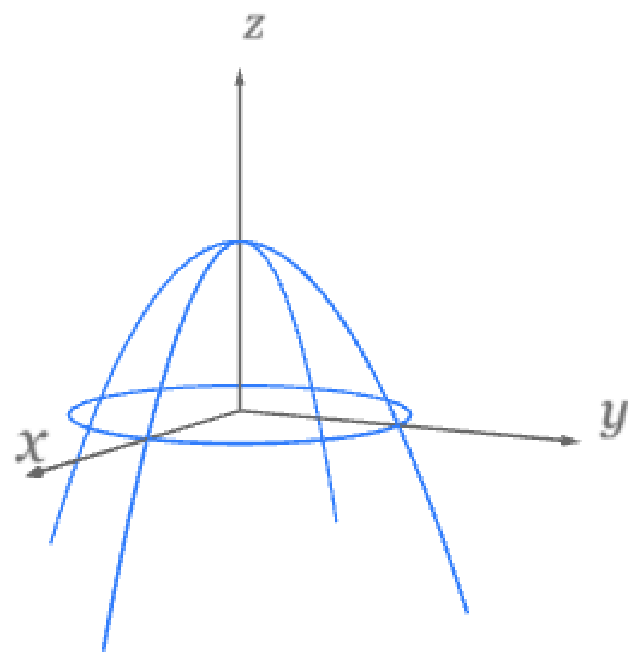


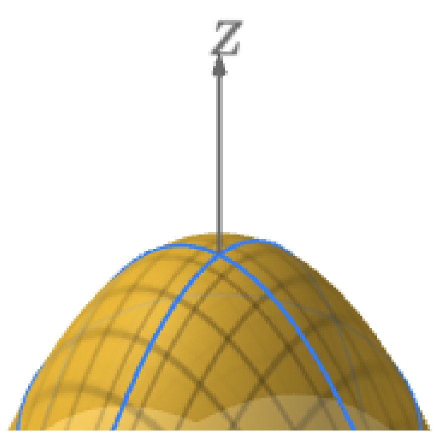
Adding this curve into our graph in three dimensions we get the following.

▶ All three slices together



We can see that what we have is a surface of revolution, and upside down paraboloid with maximum on the z-axis at $z = 1$.

▶ The graph of the surface with intersection curves



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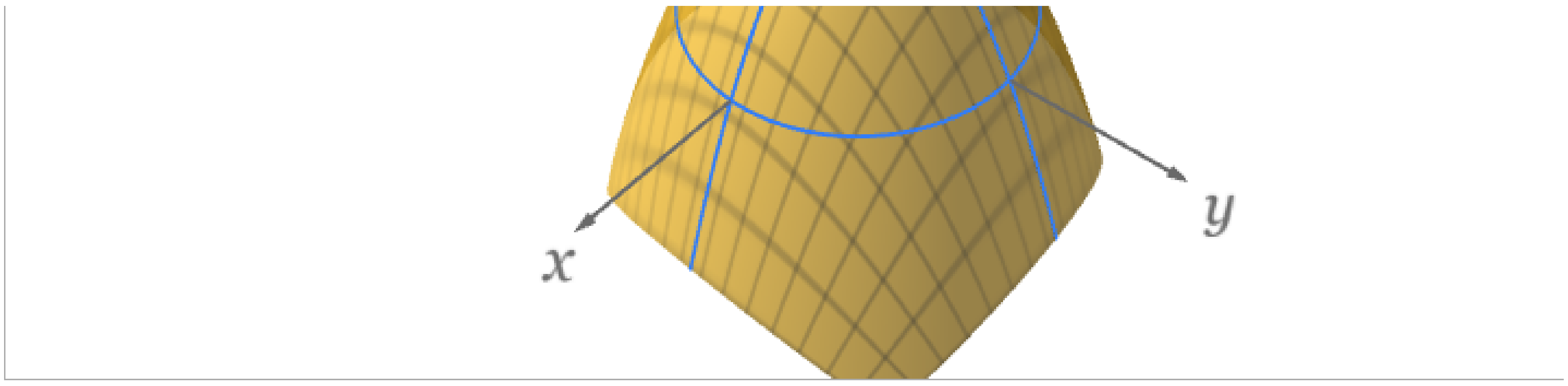

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