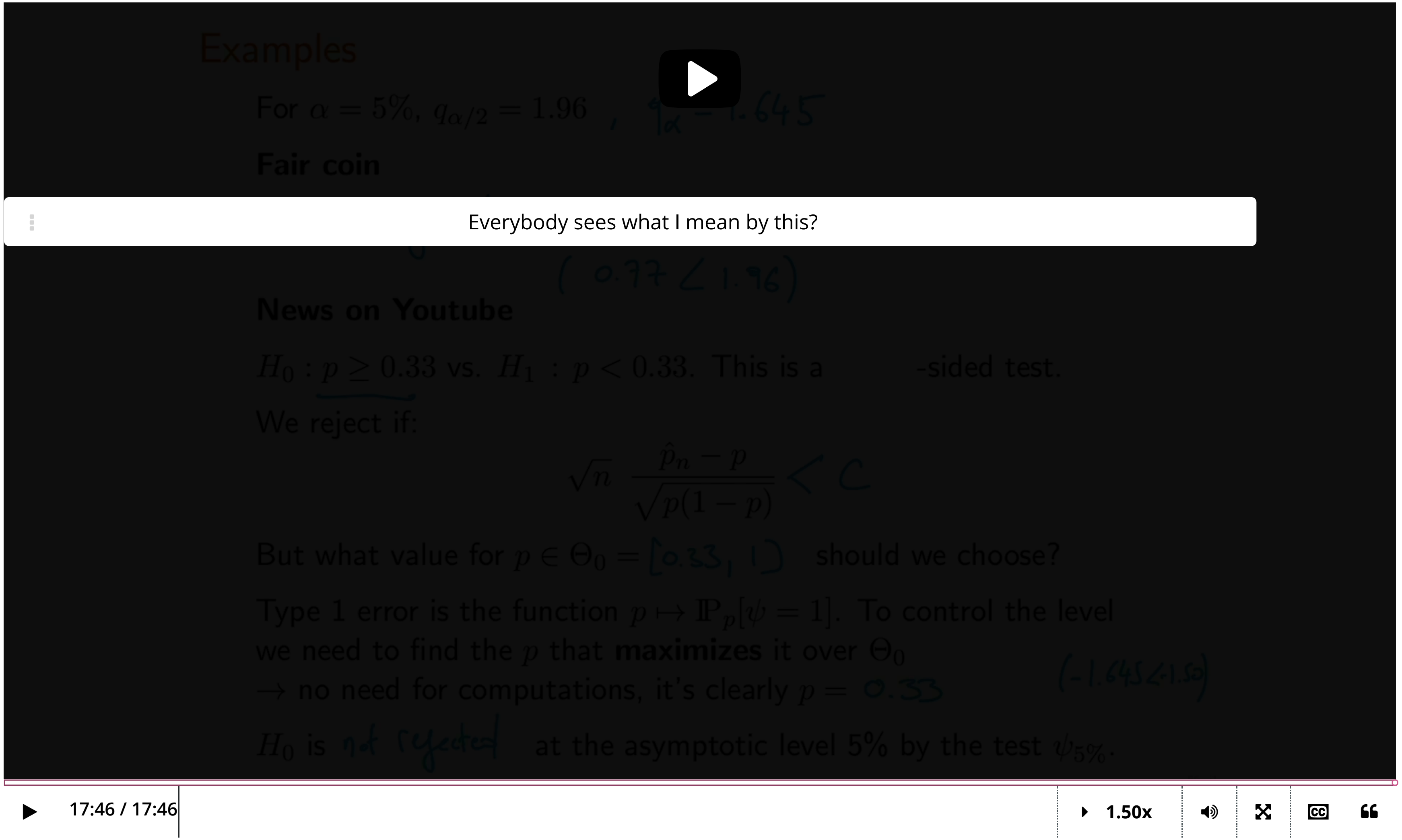
5. Worked Example: a One-Sided Test

Errors, Levels, and Conclusion of a One-Sided Test



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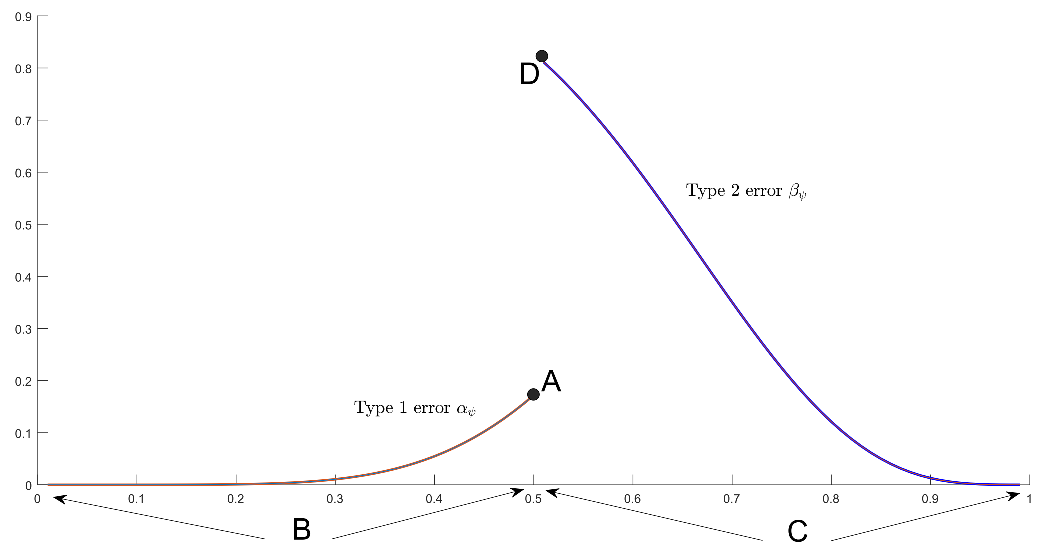
Visualizing Hypothesis Testing for a One-Sided Test

3/3 points (graded)

Let $X_1, \ldots, X_n \overset{iid}{\sim} \mathrm{Ber}\,(p^*)$ for some true parameter $p^* \in (0,1)$, and let $(\{0,1\}, \{P_p\}_{p\in(0,1)})$ denote the associated statistical model where $P_p = \mathrm{Ber}\,(p)$.

Suppose the null hypothesis is $H_0 : p^* \leq 1/2$ and the alternative hypothesis is $H_1 : p^* > 1/2$. Let ψ continue to denote the statistical test we will use. (Recall that a test takes value either 0 or 1. Usually it is of the form $\mathbf{1}\,(T_n > C)$ where C is a threshold to be specified and T_n is known as a **test statistic** . Be careful to not confuse **(tests** with **test statistics**.)

Consider the following graph of this hypothesis testing set-up.



- Continuous curve on the left: type 1 error, α_ψ, graphed as a function of θ.
- Continuous curve on the right: type 2 error, β_ψ, graphed as a function of θ.
- Horizontal axis: the parameter space $\Theta = (0,1)$.

Which letter indicates Θ_0, the region defined by the null hypothesis?

- ( ) A
- (•) B
- ( ) C
- ( ) D

✔

Which letter indicates Θ_1, the region defined by the alternative hypothesis?

- ( ) A
- ( ) B
- (•) C
- ( ) D

✔

Let $p \in (0,1)$ denote the point where the power is attained, i.e., the point where

$$\pi_\psi = \inf_{\Theta_1} \left(1 - \beta_\psi\left(p\right)\right).$$

Which letter indicates the ordered pair (p, π_ψ)?

- (•) A

- B
- C
- D

✔

Solution:

We consider the questions in order.

For the first question, since we are given that $H_0 : p \leq 1/2$, then the interval $(0, 1/2]$ defines Θ_0. Hence, letter B is the correct response.

For the second question, since we are given that $H_1 : p > 1/2$, then the interval $(1/2, 1)$ defines Θ_1. Hence, letter C is the correct response.

The the third question, recall that the power of a test is given by

$$\pi_\psi = \inf_{p \in (0,1)} \left(1 - \beta_\psi (p)\right).$$

The continuous curve on the right, which graphs β_ψ, attains its maximum at $p = 1/2$, and this maximum is given by $\beta_\psi (1/2) = 0.8$. Therefore,

$$\pi_\psi = \inf_{p \in (0,1)} \left(1 - \beta_\psi (p)\right) = 1 - 0.8 = 0.2,$$

which implies that A is the correct response.

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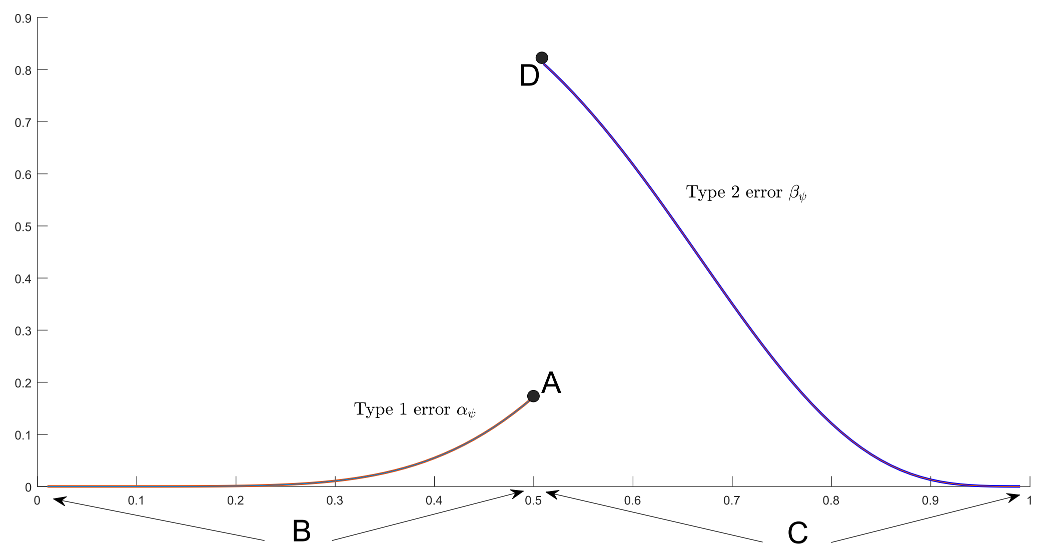
Answers are displayed within the problem

Level of a statistical test

1/1 point (graded)

As in the previous question, let $X_1, \ldots, X_n \stackrel{iid}{\sim} \mathrm{Ber}(p^*)$ for some true parameter $p^* \in (0,1)$, and let $(\{0,1\}, \{P_p\}_{p\in(0,1)})$ denote the associated statistical model where $P_p = \mathrm{Ber}(p)$.

Suppose the null hypothesis is $H_0 : p^* \leq 1/2$ and the alternative hypothesis is $H_1 : p^* > 1/2$. Let ψ continue to denote the statistical test we will use. Consider the graphic below from the previous problem.



- Continuous curve on the left: type 1 error, α_ψ, graphed as a function of θ.
- Continuous curve on the right: type 2 error, β_ψ, graphed as a function of θ.

- Horizontal axis: the parameter space $\Theta = (0, 1)$.

Which of the following are **levels** of ψ? (Choose all that apply.)

- [ ] 5 %
- [ ] 10 %
- [x] 20 %

✔

Solution:

The level of ψ is given by any real $\alpha \in \mathbb{R}$ such that

$$\alpha_\psi(p) \leq \alpha, \quad \text{for all } p \in \Theta_0 = (0, 1/2]$$

That is, the type 1 error is uniformly bounded above by α. According to the graph, the continuous curve on the left curve stays below 0.2, but not below 0.05 and 0.1. Thus $0.2 = 20\%$ is the correct response.

Remark: In general, we will describe the level of a test by the *smallest* possible level α, but this is not strictly necessary.

Submit You have used 1 of 2 attempts

Answers are displayed within the problem

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[STAFF] Level of a statistical test
The curve drawn does not do justice to the explanation provided in the solution.
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