



Bookmarks

- ▶ Unit 0: Overview
- ▶ Entrance Survey
- ▶ Unit 1: Probability models and axioms
- ▶ Unit 2: Conditioning and independence
- ▶ Unit 3: Counting
- ▶ Unit 4: Discrete random variables
- ▶ Exam 1
- ▶ Unit 5: Continuous random variables
- ▼ Unit 6: Further topics on random variables

Unit overview

Lec. 11: Derived distributions

Exercises 11 due Mar 30, 2016 at 23:59 UTC

Unit 6: Further topics on random variables > Problem Set 6 > Problem 5 Vertical: Covariance for the multinomial

Bookmark

Problem 5: Covariance for the multinomial

(5/5 points)

Consider n independent rolls of a k -sided fair die with $k \geq 2$: the sides of the die are labelled $1, 2, \dots, k$ and each side has probability $1/k$ of facing up after a roll. Let the random variable X_i denote the number of rolls that result in side i facing up. Thus, the random vector (X_1, \dots, X_k) has a multinomial distribution.

1. Which of the following statements is correct? Try to answer without doing any calculations.

- ☐ X_1 and X_2 are uncorrelated.
- ☐ X_1 and X_2 are positively correlated.
- ☒ X_1 and X_2 are negatively correlated. ✓

2. Find the covariance, $\text{cov}(X_1, X_2)$, of X_1 and X_2 . Express your answer as a function of n and k using standard notation. *Hint:* Use indicator variables to encode the result of each roll.

$$\text{cov}(X_1, X_2) = \boxed{-n/k^2} \quad \checkmark$$

3. Suppose now that the die is biased, with a probability $p_i \neq 0$ that the result of any given die roll is i , for $i = 1, 2, \dots, k$. We still consider n independent tosses of this biased die and define X_i to be the number of rolls that result in side i facing up.

Generalize your answer to part 2: Find $\text{cov}(X_1, X_2)$ for this case of a biased die. Express your answer as a function of n, k, p_1, p_2 using standard notation. Write p_1 and p_2 as p_1 and p_2 , respectively, and wrap them in parentheses in your answer; i.e., enter (p_1) and (p_2) .

$$\text{cov}(X_1, X_2) = \boxed{-n \cdot p_1 \cdot p_2} \quad \checkmark$$

Lec. 12: Sums of independent r.v.'s; Covariance and correlation

Exercises 12 due Mar 30, 2016 at 23:59 UTC

Lec. 13: Conditional expectation and variance revisited; Sum of a random number of independent r.v.'s

Exercises 13 due Mar 30, 2016 at 23:59 UTC

Solved problems

Additional theoretical material

Problem Set 6

Problem Set 6 due Mar 30, 2016 at 23:59 UTC

Unit summary

You have used 1 of 2 submissions

DISCUSSION

Click "Show Discussion" below to see discussions on this problem.

© All Rights Reserved



© edX Inc. All rights reserved except where noted. EdX, Open edX and the edX and Open EdX logos are registered trademarks or trademarks of edX Inc.

POWERED BY
OPENedX

