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Lecture 6: Introduction to

Hypothesis Testing, and Type 1 and

15. Type 2 Error and Power of a

<u>Course</u> > <u>Unit 2 Foundation of Inference</u> > <u>Type 2 Errors</u>

> Statistical Test

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15. Type 2 Error and Power of a Statistical Test Type 2 Error and Power of a Statistical Test



(Caption will be displayed when you start playing the video.)

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Testing the Support of a Uniform Variable: Type 2 Error of a Test

1/1 point (graded)

As on the previous page, let $X_1,\ldots,X_n\stackrel{iid}{\sim} \mathrm{Unif}\,[0, heta]$ for an unknown parameter heta and we designed the statistical test

$$\psi_n = \mathbf{1}(\max_{1 \leq i \leq n} X_i > 1/2)$$

to decide between the null and alternative hypotheses

$$H_0: heta \ \le 1/2$$

$$H_1: heta > 1/2.$$

Recall from lecture that the **type 2 error (rate)** of a test ψ_n is the **function**

$$egin{aligned} eta_{\psi_n} : \Theta_1 &
ightarrow \mathbb{R} \ & heta & \mapsto \mathbf{P}_{ heta} \left(\psi_n = 0
ight) \end{aligned}$$

where \mathbf{P}_{θ} ($\psi_n=0$) is the probability of the event $\psi_n=0$ under the probability distribution \mathbf{P}_{θ} when $\theta\in\Theta_1$, i.e. the probability of not rejecting H_0 when H_1 is true. In this example, the region Θ_1 defining the alternative hypothesis is $(1/2,\infty)$, and $\mathbf{P}_{\theta}=\mathrm{Unif}[0,\theta]$.

Evaluate $\mathbf{P}_{ heta}\left(\psi_n=0
ight)=\mathbf{P}_{ heta}\left(\max_{1\leq i\leq n}X_i\leq 1/2
ight)$ at heta=1/2, the boundary between Θ_0 and Θ_1 .

$$\mathbf{P}_{ heta=1/2}\left(\max_{1\leq i\leq n}X_i\leq 1/2
ight)=$$
 1

Solution:

$$egin{aligned} eta_{\psi_n}\left(1/2
ight) &= \mathbf{P}_{1/2}(\max_{1 \leq i \leq n} X_i < 1/2) \ &= \mathbf{P}_{1/2}\left(X_1 < 1/2
ight) \ldots \mathbf{P}_{1/2}\left(X_n < 1/2
ight) \ &= 1 imes 1 \ldots imes 1 = 1 \end{aligned}$$

where we applied independence of the X_i 's in the second line.

Generating Speech Output

You have used 1 of 3 attempts

1 Answers are displayed within the problem

Testing the Support of a Uniform Variable: Type 2 Error of a Test Continued

3/3 points (graded)

As above, let $X_1,\dots,X_n\stackrel{iid}{\sim}\mathrm{Unif}[0, heta]$ for an unknown parameter heta and we designed the statistical test

$$\psi_n = \mathbf{1}(\max_{1 \leq i \leq n} X_i > 1/2)$$

to decide between the null and alternative hypotheses

$$H_0: \theta \leq 1/2$$

$$H_1: \theta > 1/2.$$

Recall from lecture that the **type 2 error** of a test ψ_n is the **function**

$$egin{aligned} eta_{\psi_n}:\Theta_1 &
ightarrow [0,1] \ & heta &
ightarrow \mathbf{P}_{ heta}\left(\psi_n=0
ight) \end{aligned}$$

where \mathbf{P}_{θ} ($\psi_n=0$) is the probability of the event $\psi_n=0$ under the probability distribution \mathbf{P}_{θ} when $\theta\in\Theta_1$, i.e. the probability of not rejecting H_0 when H_1 is true.

In this example, $\Theta_1=(1/2,\infty)$, and $\mathbf{P}_{\theta}=\mathrm{Unif}[0,\theta]$.

What is $\beta_{\psi_n}(\theta)$?

$$eta_{\psi_n}\left(heta
ight)=egin{array}{c} (1/2/\mathrm{theta})^n \ \hline \left(rac{1}{2\cdot heta}
ight)^n \end{array}$$

Find
$$\lim_{ heta o 1/2} eta_{\psi_n} \left(heta
ight)$$
.

$$\lim_{ heta o 1/2} eta_{\psi_n} \left(heta
ight) =$$
 1 $lacksquare$ Answer: 1

Find
$$\lim_{ heta o \infty} eta_{\psi_n} \left(heta
ight)$$
.

$$\lim_{ heta o\infty}eta_{\psi_n}\left(heta
ight)=egin{bmatrix} 0 \ 0 \end{bmatrix}$$
 Answer: 0

STANDARD NOTATION

Solution:

For any $\theta \in \Theta_1 = [1/2, \infty)$,

$$egin{aligned} eta_{\psi_n}\left(heta
ight) &= \left.\mathbf{P}_{ heta}\left(\psi_n=0
ight) \right. = \left.\mathbf{P}_{ heta}\left(\max_{1\leq i\leq n}X_i < 1/2
ight) \\ &= \left.\mathbf{P}_{ heta}\left(X_1 < 1/2
ight) \ldots \mathbf{P}_{ heta}\left(X_n < 1/2
ight) = \left(rac{1/2}{ heta}
ight)^n. \end{aligned}$$

As heta o 1/2,

$$eta_{\psi_n}\left(heta
ight)
ightarrow \left(rac{1/2}{1/2}
ight)^n = 1.$$

As
$$heta o \infty$$
 ,

$$eta_{\psi_n}\left(heta
ight)=\left(rac{1/2}{ heta}
ight)^n
ightarrow 0.$$

Remark: This test is rather extreme example in that it minimizes type-1 error while maximizing the type-2 error. In general, we want to design tests so that the type-1 and type-2 error are both controlled. These types of trade-offs are crucial to consider in the context of hypothesis testing.

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You have used 1 of 3 attempts

• Answers are displayed within the problem

Testing the Support of a Uniform Variable: : Power of a Test

1/1 point (graded)

The **power** of the test ψ_n is defined to be

$$\pi_{\psi_{n}}=\inf_{ heta\in\Theta_{1}}\left(1-eta_{\psi_{n}}\left(heta
ight)
ight).$$

Continuing from the problem above, what is the power π_{ψ_n} ?

$$\pi_{\psi_n} = egin{bmatrix} \mathtt{0} & \hspace{0.5cm} \checkmark \hspace{0.5cm} \mathsf{Answer:} \hspace{0.5cm} \mathtt{0} \end{array}$$

Solution:

A priori we have that

$$\pi_{\psi_n} = \inf_{ heta \in [1/2,\infty)} \left(1 - P_{ heta}\left(\psi_n = 0
ight)
ight) = \inf_{ heta \in [1/2,\infty)} P_{ heta}\left(\psi_n = 1
ight) \geq 0].$$

Moreover, we computed above that $eta_{\psi_n}\left(1/2
ight)=P_{0.5}\left[\psi_n=0
ight]=1$. Thus,

$$\pi_{\psi_n}=0.$$

Remark: The power of a test is the largest lower bound on the probability that if H_1 is true, that indeed H_0 is rejected in favor of H_1 . In this example, as $\theta \in \Theta_1$ approaches the boundary 1/2, the probability of rejecting H_0 decreases and approaches 0.

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You have used 1 of 3 attempts

• Answers are displayed within the problem

Testing the Support of a Uniform Variable: Graphing the errors

1/1 point (graded)

As above, let $X_1,\ldots,X_n\stackrel{iid}{\sim}\mathrm{Unif}[0, heta]$ for an unknown parameter heta and we designed the statistical test

$$\psi_n = \mathbf{1}(\max_{1 \leq i \leq n} X_i > 1/2)$$

to decide between the null and alternative hypotheses

$$H_0: heta \leq 1/2$$

$$H_1: \theta > 1/2.$$

Let $\alpha_{\psi_n}(\theta)$ and $\beta_{\psi_n}(\theta)$ denote the type 1 and type 2 errors respectively.

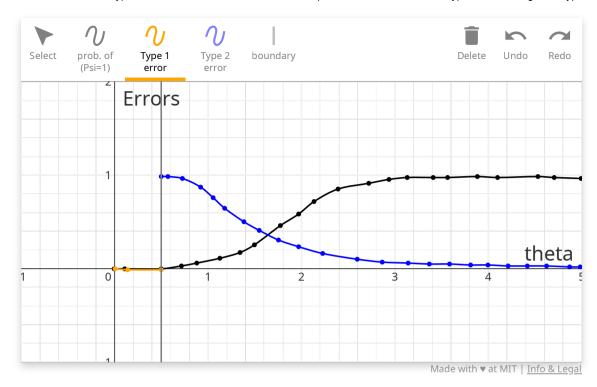
On the graph below, do the following:

• Place a vertical line at the boundary of Θ_0 and Θ_1 using the **boundary tool**.

- Sketch the graph of \mathbf{P}_{θ} ($\psi_n=1$) as a function of θ using the **probabilty of rejecting null** tool.
- Sketch the graph of the type 1 error $\alpha_{\psi_n}\left(\theta\right)$ on the **correct domain** using the **type 1 error** tool.
- Sketch the graph of the type 2 error $\beta_{\psi_n}\left(\theta\right)$ on the **correct domain** using the **type 2 error** tool.

Note: To use the spline tool for sketching the graphs, click on point on the graph, and the tool will connect these points with a smooth curve.

For each curve, you will be graded on its domain, its limiting values, its value on the boundary between Θ_0 and Θ_1 , and its shape and continuity.

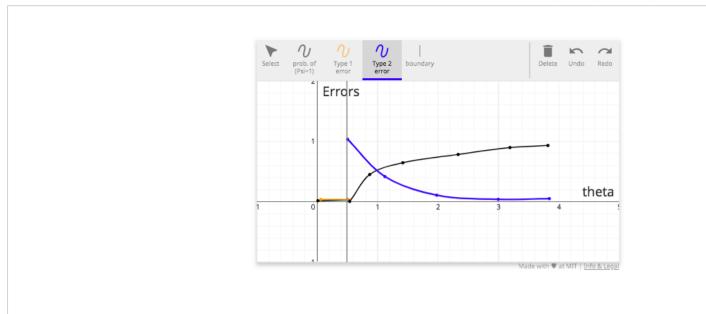


Answer: See solution.



Good job on the graph of the probability of rejecting the null! Good job on the graph of the type 1 error! Good job on the graph of the type 2 error!

Solution:



Submit

You have used 1 of 10 attempts

• Answers are displayed within the problem

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Testing the Support of a Uniform Variable: Type 2 Error of a Test Continued

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