FINAL EXAM PROBLEM 1 (3/3 points)

Do you know periodical cicadas? The life cycle of 13-year cicadas is 13 years. They emerge in huge numbers from the ground every 13 years. Sometimes, more than one million cicadas per acre appear at the same time. Similarly, the life cycle of 17-year cicadas is 17 years. They emerge in huge numbers from the ground every 17 years. Imagine that both of the 13-year cicadas and the 17-year cicadas inhabit in the same city. Using the fact that 13 and 17 are prime numbers, calculate the period of simultaneous emergence of both 13-year and 17-year cicadas in this city. (Note that this is just a mathematical question. You should not consider any biological, climatic, or environmental matters to solve this problem.)

221 ✔

221

FINAL EXAM PROBLEM 2 (4/4 points)

Assume that you performed the Sieve of Eratosthenes for integers less than 100,000. Under the process, all integers which are not prime numbers are removed from the list. What is the last integer removed from the list?

99221 ✔

99221

FINAL EXAM PROBLEM 3 (3/3 points)

By Dirichlet's theorem on arithmetic progressions, there are infinitely many prime numbers which are congruent to 7 modulo 8. The first 10 such prime numbers are

7 23 31 47 (A) (B) (C) 127 (D) 167.

Write a prime number in each of the blanks.

A

71 ✔

71

B

79 ✔

79

C

103 ✔

103

D

151 ✔

151

FINAL EXAM PROBLEM 4 (3/3 points)

It is known that 239,693 is written as the sum of two squares in two different ways. In other words, there exist positive integers A,B,C,D satisfying

$$239{,}693 = A^2 + B^2 = C^2 + D^2 \quad (A < B,\ C < D,\ A < C).$$

Find such integers A,B,C,D.

A

173 ✔

173

B

458 ✔

458

C

278 ✔

278

D

403 ✔

403

FINAL EXAM PROBLEM 5 (4/4 points)

There are infinitely many right triangles with area 5 whose sides are rational numbers. The simplest example is a right triangle with **legs 3/2, 20/3, and hypotenuse 41/6**. The second simplest example is a right

triangle with **legs 1519/492, A/B, and hypotenuse C/D**. Fill an integer in each of the blanks. Write fractions in lowest terms.

A

4920 ✔

4920

B

1519 ✔

1519

C

3344161 ✔

3344161

D

747348 ✔

747348

FINAL EXAM PROBLEM 6 (3/3 points)

Let's consider Tunnell's theorem when P=59.

(1) Calculate the number of triples (X,Y,Z) of integers satisfying $59=2X^2+Y^2+8Z^2$. Do not forget to count triples whose entries are not necessarily positive integers.

36 ✔

36

(2) Calculate the number of triples (A,B,C) of integers satisfying $59=2A^2+B^2+32C^2$. Do not forget to count triples whose entries are not necessarily positive integers.

20 ✔

20

(3) What can we conclude using the results of (1), (2), and Tunnell's theorem?

- We can conclude that 59 is a congruent number.

- (●) We can conclude that 59 is not a congruent number. ✔
- ( ) We can not conclude anything because the Birch and Swinnerton-Dyer Conjecture is still unsolved.