Observation Theory

Script V22C – Properties of the functional model (part2)

Now let’s focus on the full column rank systems.

In the systems with the rank of A equals to number of unknowns or ’n’, two scenarios can happen:

1- either the rank of the 'm by n' matrix A is equal to both m and n.

2- or the rank of A is equal to n but smaller than m.

In the first case, we are dealing with a square A matrix, where the number of observations or equations is the same as the number of unknowns.

This is the case of, for example when we have two equations and two unknowns, or 10 equations and 10 unknowns.

These systems are consistent as the rank of A is equal to m, and a unique solution for them exists.

In these systems, the solution can be simply computed by multiplication of inverse of matrix A by the observation vector y.

These systems are called ‘determined systems’.

The second scenario, however, is more complicated.

We have more observations than unknowns, and the consistency is not guaranteed, as the rank is smaller than the number of observations.

Here I want to introduce a new and important quantity.

The number of observations ‘m’ minus the rank of A results is a quantity which is called “redundancy” of the system.

In our case where we have a full column rank matrix (that is the rank is equal to n), the redundancy is equal to the number of observations minus the number of unknowns.

These kinds of systems with the number of equations larger than the number of unknowns or with a positive redundancy are called ‘overdetermined systems.'

Whether or not a solution exists for an overdetermined system depends on the actual entries of the observation vector y.

If the observation vector is in the range space of A, the system is consistent.

But since, in practice, perfect observations do not exist, and measurements are always contaminated by errors, we more often deal with inconsistent systems with no solutions.

In such a scenario the observation vector is outside the range space of the matrix A.

So perhaps it is better, instead of writing the system of equations as Y equals Ax, to write y approximately equals Ax, in order to emphasize the potential inconsistency in the equations.

The question now is that what to do when the system is inconsistent.

In fact, this is the central question of the rest of this course.

In summary, in this section we learned that for the functional models in the form of system of observation equations, that is y equals Ax:

1) Assuming consistency, the solution is unique if the rank of A is equal to number of unknowns

2) For cases with rank of A smaller than 'n'

The system is 'underdetermined' and there are more than one solution.

Underdetermined systems are not discussed further in this course.

3) For the full column rank systems, with rank(A) equals 'n' and equals to 'm' (or when we have the same number of observations and unknowns) … the system is determined and the unique solution can simply computed as: xhat is equal to the inverse of A multiply by y.

And the final conclusion is:

4) The full column rank systems, with the number of unknowns smaller than the number of observations, are overdetermined.

These systems may be consistent or not.

However, due to measurement errors, we often, in practice, deal with inconsistent systems, for which, theoretically, there is no exact solution.

In the rest of this course, we mainly focus on how to handle the inconsistent and overdetermined systems.

It is now a good time, in the next unit, to look at some examples of these different kinds of the functional model ..

And get a better sense of the concepts of redundancy and over or under-determined systems.