8. Graph Properties and Metrics - III

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Exercises due Oct 20, 2021 17:29 IST Completed

Graph Properties and Metrics - III

out

in the full network, well, if you

have different connected components,

so this is always going to be infinity.

Right?

Because there is no path between one connected component

and another connected components, if the distance

is going to be infinity.

So it's important, that when you start

studying diameter, et cetera, that you actually do this

in just the largest connected component or in one

connected component, but not over

0:00 / 0:00 2.0x

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Length of a Path, Diameter, and Average Distance

Let d_{ij} be the length of the shortest path (or the path with the smallest weight in the case of a weighted graph) between node i and j. The **diameter** of a graph is the largest distance between any two nodes:

$$\text{diameter} = \max_{i,j \in V} d_{i,j}.$$

We can also define the notion of **average shortest path length** :

$$\text{average path length} = \frac{1}{\binom{n}{2}} \sum_{i \le j} d_{ij}.$$

These notions are defined component-wise in case the graph is not connected. The definition of what constitutes a component, as we have seen before, varies between an undirected graph and a directed graph.

Line Graph

2/2 points (graded)
Consider a simple, unweighted, undirected, connected tree with n nodes.

1. What is an upper bound on the diameter of the graph?

n-1 ✔ **Answer:** n-1

2. Let the degree of every node of the graph be at most 2. Compute the average path length of the graph assuming that $n \ge 3$. *Hint*: First, try to understand what the structure of the graph looks like.

(n+1)/3 ✔ **Answer:** n - (2/3)*n + 1/3

Solution:

1. $n-1$. Since the graph is connected, the longest path length between any two nodes is at most $n-1$.

2. The graph is connected and every node has at most a degree of 2. This is the *line graph* – a graph where all nodes form a straight, connected line with the two nodes on the edges having a degree of 1 and the $n-2$ interior nodes having a degree of 2. Now that we know what the graph looks like, we can proceed to compute the average path length.

    1. The two nodes on the edges (a single pair of nodes) have a path length of $n-1$.

    2. There are two pairs of nodes that have a path length of $n-2$.

    3. There are three pairs of nodes that have a path length of $n-3$.

    4. So on...

    Therefore, the average path length is

$$\begin{aligned}\textbf{average path length} &= \frac{1}{\binom{n}{2}}\sum_{i\leq j} d_{ij}\\ &= \frac{1}{\binom{n}{2}}[(n-1)+2(n-2)+3(n-3)+\cdots+(n-1)(n-(n-1))]\\ &= \frac{1}{\binom{n}{2}}\left[n(1+2+\ldots(n-1))-(1+4+9+\cdots+(n-1)^2)\right]\\ &= \frac{1}{\binom{n}{2}}\left[\frac{n(n-1)n}{2}-\frac{n(n-1)(2(n-1)+1)}{6}\right]\\ &= n-\frac{2}{3}\cdot n+\frac{1}{3}\\ &= \frac{n+1}{3}\end{aligned}$$

    The above is valid when $n \geq 3$. When $n=2$, we only have one term overall and the average path length is equal to 1.

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Star Graph

1/1 point (graded)

Consider the sequence of star graphs: there is a central node and every other node has only one neighbor and it is the central node. Compute the limit of the average shortest path length of the sequence of star graphs as $n \to \infty$.

2 **Answer:** 2

Solution:

First, for a given n, there are $n-1$ path lengths of length 1 and $\binom{n-1}{2}$ path lengths of length 2 (these are all the path lengths and are also the shortest path lengths). Hence

$$\begin{aligned}\frac{1}{\binom{n}{2}}\sum_{i\leq j} d_{ij} &= \frac{2}{n(n-1)}\left[n-1+2\times\frac{(n-1)(n-2)}{2}\right]\\ &= \frac{2(n-1)}{n}.\end{aligned}$$

Taking the limit, we get the result that the average shortest path length converges to 2. This result should also

Taking the limit, we get the result that the average shortest path length converges to 2. This result should also intuitively make sense. Why?

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CAREFUL in "Star Graph": **every other node** means ALL other nodes, beside the central node

discussion posted 2 months ago by **TarekHamZawY**

.. and not one node yes and one node no .. do not be like your friend here :))

so if you have 9 nodes .. one is the central node .. all the other 8 nodes have neighbors (i.e. have edges with the central node) .. not only 4

if you are interested about the wrong problem I solved .. it gave zero .. still trying to reflect its meaning ..

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