1. Linear approximation of a piston

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Part A due Oct 5, 2021 20:30 IST

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5A-1

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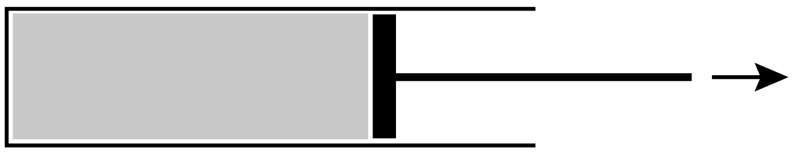
Suppose that we have a gas in a piston which can expand and contract – see the picture.



We write V for the volume of the gas in the piston, T for its temperature, and P for its pressure. The ideal gas law is an important equation that relates these quantities:

$$PV = nRT.$$

Here n is the number of particles in the gas in the piston, and R is a universal constant. As we expand the piston and increase V, n doesn't change because no gas goes into or out of the piston, and R doesn't change. In a real world situation, the value of nR would probably be messy, but for this problem, suppose that $nR = 6$, so we have

$$PV = 6T. \qquad (6.252)$$

Suppose that initially, $V = 2$, $P = 3$, and $T = 1$. We slightly expand the piston so that V increases by a small quantity ΔV. Suppose that as we expand the piston, PV^2 remains constant. So $PV^2 = 12$ throughout the process. (This is actually physically reasonable.)

a.) We can think of T as a function of two variables $T(P, V)$, and we can think about the relationship $PV^2 = 12$ as a relationship that expresses the fact that $P = P(V)$ depends on V. Find the derivative of T with respect to V, $\frac{dT}{dV}$, using the chain rule at $V = 2$, $P = 3$ and $T = 1$.

$\frac{dT}{dV} =$ -1/2 ✔ **Answer:** -1/2

b.) When V increases from 2 to $2 + \Delta V$, what is the approximate change in the pressure, ΔP? Give your answer in terms of ΔV.

(Type `DeltaV` for ΔV.)

$\Delta P \approx$ -3*DeltaV ✔ **Answer:** -3*DeltaV

c.) When V increases from 2 to $2 + \Delta V$, what is the approximate change in the temperature, ΔT? Again, give your answer in terms of ΔV.

(Type `DeltaV` for ΔV.)

$\Delta T \approx$ -1/2*DeltaV ✔ **Answer:** -0.5*DeltaV

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Solution:

(a.) To find $\frac{dT}{dV}$ we have to use the chain rule! Write $T = f(P, V) = \frac{1}{6}PV$.

$$\frac{dT}{dV} = f_P \frac{dP}{dV} + f_V \tag{6.253}$$

$$= \frac{V}{6}\frac{dP}{dV} + \frac{P}{6} \tag{6.254}$$

Taking the total differential of $PV^2 = 12$ we find

$$(dP)\,V^2 + 2PV\,(dV) = 0.$$

We use this differential to find $\frac{dP}{dV}$. In particular, we can write this as

$$\frac{dP}{dV} = \frac{-2PV}{V^2} = \frac{-2P}{V} \tag{6.255}$$

Thus

$$\frac{dT}{dV} = \frac{1}{6}\left(V\frac{\partial P}{\partial V} + P\right) \tag{6.256}$$

$$= \frac{1}{6}\left(\frac{-2P}{V}V + P\right) = \frac{-P}{6} \tag{6.257}$$

When $V = 2$, $P = 3$ and $T = 1$, the value of this total derivative is $-1/2$.

(b.) The gradient vector of the function $f(P, V) = PV^2$ at the point $(3, 2)$ is

$$\nabla(f)(3,2) = \langle V^2, 2PV\rangle|_{(3,2)} = \langle 4, 12\rangle.$$

By linear approximation, we obtain that

$$0 = f(3 + \Delta P, 2 + \Delta V) - f(3,2) = \nabla(f)(3,2) \cdot \langle \Delta P, \Delta V\rangle,$$

which gives

$$4\Delta P + 12\Delta V = 0.$$

Hence ΔP = -3 ΔV .

(c.) The gradient vector of the function $T(P, V) = (1/6)\,PV$ at the point $(3, 2)$ is

$$\nabla(T)(3,2) = \langle (1/6)\,V, (1/6)\,P\rangle|_{(3,2)} = \left(\frac{1}{3}, \frac{1}{2}\right).$$

Passing from the total differential to the linear approximation we find

$$dT = \frac{1}{3}dP + \frac{1}{2}dV$$

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$$\Delta T \approx \frac{1}{3}\Delta P + \frac{1}{2}\Delta V \tag{6.259}$$

Plugging in our expression for ΔP from (b) we get

$$\Delta T \approx \frac{1}{3}(-3\Delta V) + \frac{1}{2}\Delta V = -\Delta V + \frac{1}{2}\Delta V = \frac{-1}{2}\Delta V. \tag{6.260}$$

Hence

$$\boxed{\Delta T \approx -\frac{1}{2}\Delta V.}$$

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1. Linear approximation of a piston

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[STAFF] Possible grader error dT/dV

discussion posted 2 days ago by **valleymd**

I believe that there is an error in the grader value for dT/dV. I presume a numerical anser was desired. My answer for dT/dV gave me the correct answer for ΔT.

This post is visible to everyone.

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1 response

jfrench (Staff)
a day ago

Hmm. A numerical answer is being asked for!

Ah there is a bug in the grader! Your answer is right! (Also is the answer in the solution.) Will fix and regrade!

posted a day ago by **jfrench** (Staff)

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