7. Wald's Test in 1 Dimension

Comparing Quantiles

1/1 point (graded)
Let $Z \sim \mathcal{N}(0,1)$. Then $Z^2 \sim \chi_1^2$.

The **quantile** $q_\alpha(\chi_1^2)$ of the χ_1^2-distibution is the number such that

$$\mathbf{P}\left(Z^2 > q_\alpha(\chi_1^2)\right) = \alpha.$$

Find the quantiles of the χ_1^2 distribution in terms of the quantiles of the normal distribution. That is, write $q_\alpha(\chi_1^2)$ in terms of $q_{\alpha'}(\mathcal{N}(0,1))$ where α' is a function of α.

(Enter **q(alpha)** for the quantile $q_\alpha(\mathcal{N}(0,1))$ of the normal distribution.)

$q_\alpha(\chi_1^2) =$ (q(alpha/2))^2 ✔ **Answer:** (q(alpha/2))^2

STANDARD NOTATION

Solution:

Since $Z^2 > q$ for any $q > 0$ if and only if $|Z| > \sqrt{q}$, we have

$$P\left(Z^2 > q_\alpha\left(\chi_1^2\right)\right) = P\left(|Z| > \sqrt{q_\alpha\left(\chi_1^2\right)}\right) = \alpha.$$

Since $Z \sim \mathcal{N}(0,1)$, $P\left(|Z| > \sqrt{q_\alpha\left(\chi_1^2\right)}\right) = \alpha$ if and only if

$$\sqrt{q_\alpha\left(\chi_1^2\right)} = q_{\alpha/2}\left(\mathcal{N}(0,1)\right)$$

Hence $q_\alpha\left(\chi_1^2\right) = q_{\alpha/2}\left(\mathcal{N}(0,1)\right)^2$.
For example, for $\alpha = 5\%$, using a table (e.g. https://people.richland.edu/james/lecture/m170/tbl-chi.html) or software (e.g. R), we have

$$\begin{aligned} q_{0.05}\left(\chi_1^2\right) &\approx 3.84. \\ \left(q_{0.025}\left(\mathcal{N}(0,1)\right)\right)^2 &\approx (1.96)^2 \approx 3.84. \end{aligned}$$

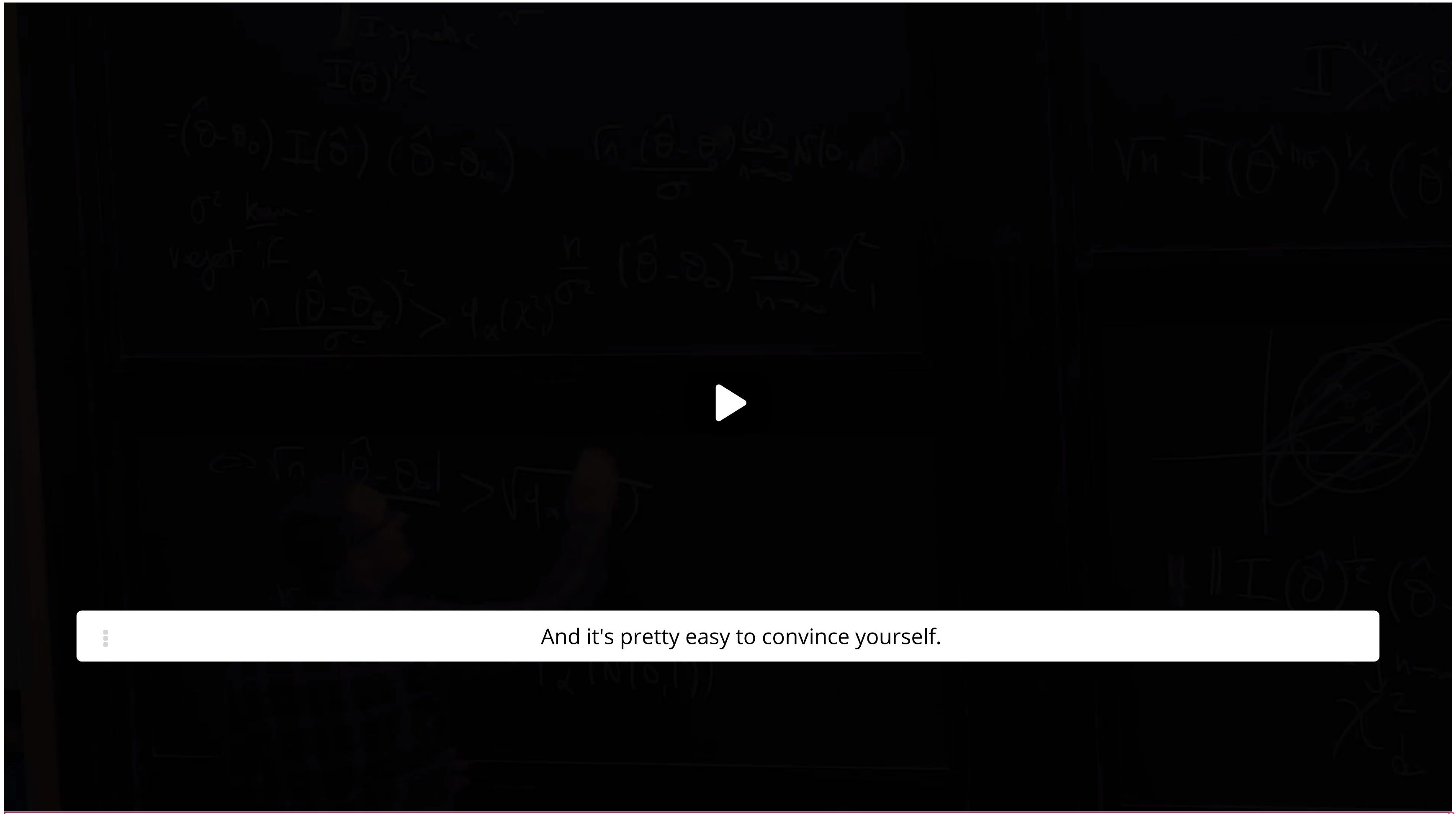
Submit You have used 3 of 3 attempts

Answers are displayed within the problem

Video note: In the video below at 5:27, Prof Rigollet misprinted on the board: the bottom inequality should read:

$$\sqrt{n}\frac{\left|\hat{\theta}-\theta\right|}{\sigma} > q_{\alpha/2}\left(\mathcal{N}(0,1)\right).$$

Wald's Test Continued



5:28 / 5:28 1.50x

Video

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Wald's Test in 1 dimension

In 1 dimension, Wald's Test coincides with the two-sided test based on on the asymptotic normality of the MLE.

Given the hypotheses

$$H_0 : \theta^* = \theta_0$$

$$H_1 : \theta^* \neq \theta_0,$$

a two-sided test of level α, based on the asymptotic normality of the MLE, is

$$\psi_\alpha = \mathbf{1}\left(\sqrt{nI(\theta_0)}\left|\hat{\theta}^{\text{MLE}} - \theta_0\right| > q_{\alpha/2}(\mathcal{N}(0,1))\right)$$

where the Fisher information $I(\theta_0)^{-1}$ is the asymptotic variance of $\hat{\theta}^{\text{MLE}}$ under the null hypothesis.

On the other hand, a Wald's test of level α is

$$\begin{aligned}\psi_\alpha^{\text{Wald}} &= \mathbf{1}\left(nI(\theta_0)\left(\hat{\theta}^{\text{MLE}} - \theta_0\right)^2 > q_\alpha(\chi_1^2)\right)\\ &= \mathbf{1}\left(\sqrt{nI(\theta_0)}\left|\hat{\theta}^{\text{MLE}} - \theta_0\right| > \sqrt{q_\alpha(\chi_1^2)}\right).\end{aligned}$$

Using the result from the problem above, we see that the two-sided test of level α is the same as Wald's test at level α.

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Q(alpha)

question posted 8 days ago by nbourbon

In the video, professor shows an equality where the SQRT(Q(alpha) and Q(Chi) are the same... but a simple derivation of Q(Chi) was rejected by the grader. What am I missing?. Thanks!

This post is visible to everyone.

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1 response

SuhailWali
8 days ago

Read the text followed by the grader carefully

got it, I missed the fact that one was two sided so that needed to be taken into consideration ... now I fixed it. thanks

posted 7 days ago by **nbourbon**

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