8.2.3 Solving A x = b via Gauss-Jordan Elimination: Multiple Right-Hand Sides

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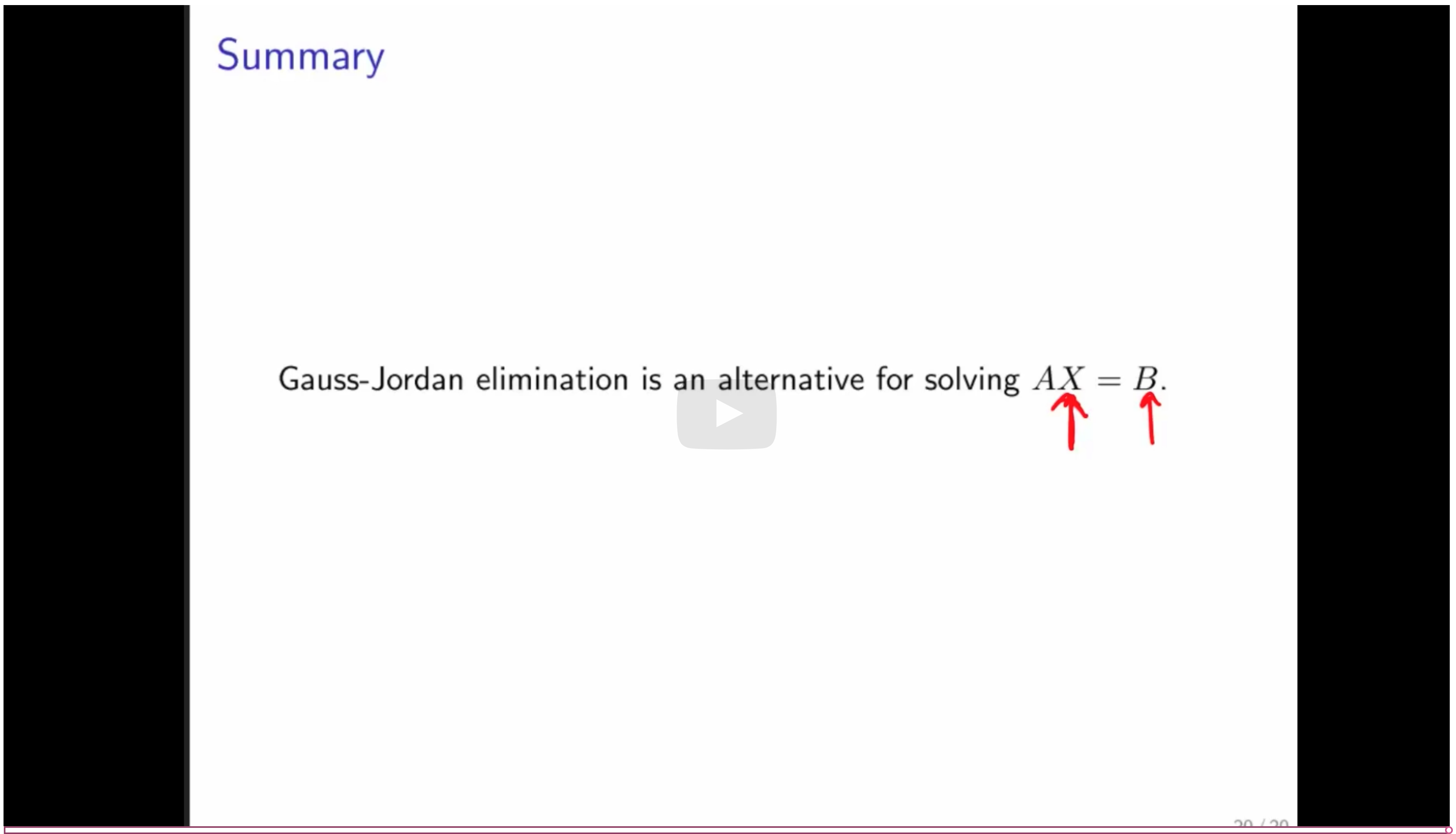

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Reading Assignment

0 points possible (ungraded)

Read Unit 8.2.3 of the notes. [LINK]

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In the below problem(s) we use two lines to separate the matrix from the appended right-hand sides. Unfortunately, this does not typeset nicely. If this bothers or confuses you, you may want to look at the exercises in the notes instead.

Homework 8.2.3.1

18/18 points (graded)

$$\left(\begin{array}{r|rr} 1 & 0 & 0 \\ \hline 1 & 1 & 0 \\ -2 & 0 & 1 \end{array}\right)\left(\begin{array}{r|rr||rr} -2 & 2 & -5 & -7 & 8 \\ \hline 2 & -3 & 7 & 11 & -13 \\ -4 & 3 & -7 & -9 & 9 \end{array}\right) =$$

$$\left(\begin{array}{r|rr||rr} -2 & 2 & -5 & -7 & \beta_{0,1} \\ \hline 0 & -1 & 2 & 4 & \beta_{1,1} \\ 0 & -1 & 3 & 5 & \beta_{2,1} \end{array}\right)$$

$\beta_{0,1} =$ 8 ✔ Answer: 8

$\beta_{1,1} =$ -5 ✔ Answer: -5

$\beta_{2,1} =$ -7 ✔ Answer: -7

$$\left(\begin{array}{r|r|r} 1 & 2 & 0 \\ \hline 0 & 1 & 0 \\ \hline 0 & -1 & 1 \end{array}\right)\left(\begin{array}{r|r|r||rr} -2 & 2 & -5 & -7 & 8 \\ \hline 0 & -1 & 2 & 4 & -5 \\ \hline 0 & -1 & 3 & 5 & -7 \end{array}\right) =$$

$$\left(\begin{array}{r|r|r||rr} -2 & 0 & -1 & 1 & \beta_{0,1} \\ \hline 0 & -1 & 2 & 4 & \beta_{1,1} \\ \hline 0 & 0 & 1 & 1 & \beta_{2,1} \end{array}\right)$$

$\beta_{0,1} =$ -2 ✔ Answer: -2

$\beta_{1,1} =$ -5 ✔ Answer: -5

$\beta_{2,1} =$ -2 ✔ Answer: -2

$$\left(\begin{array}{rr|r} 1 & 0 & 1 \\ 0 & 1 & -2 \\ \hline 0 & 0 & 1 \end{array}\right)\left(\begin{array}{rr|r||rr} -2 & 0 & -1 & 1 & -2 \\ 0 & -1 & 2 & 4 & -5 \\ \hline 0 & 0 & 1 & 1 & -2 \end{array}\right) =$$

$$\left(\begin{array}{rr|r||rr} -2 & 0 & 0 & 2 & \beta_{0,1} \\ 0 & -1 & 0 & 2 & \beta_{1,1} \\ \hline 0 & 0 & 1 & 1 & \beta_{2,1} \end{array}\right)$$

$\beta_{0,1} =$ -4 ✔ Answer: -4

Calculator

$\beta_{1,1} =$ -1 ✔ Answer: -1

$\beta_{2,1} =$ -2 ✔ Answer: -2

$$\begin{pmatrix} -1/2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \left(\begin{array}{ccc|c|cc} -2 & 0 & 0 & & 2 & -4 \\ 0 & -1 & 0 & & 2 & -1 \\ 0 & 0 & 1 & & 1 & -2 \end{array}\right) =$$

$$\left(\begin{array}{ccc|c|cc} 1 & 0 & 0 & & -1 & \beta_{0,1} \\ 0 & 1 & 0 & & -2 & \beta_{1,1} \\ 0 & 0 & 1 & & 1 & \beta_{2,1} \end{array}\right)$$

$\beta_{0,1} =$ 2 ✔ Answer: 2

$\beta_{1,1} =$ 1 ✔ Answer: 1

$\beta_{2,1} =$ -2 ✔ Answer: -2

Use the above exercises to compute $x_0 = \begin{pmatrix} \chi_{0,0} \\ \chi_{1,0} \\ \chi_{2,0} \end{pmatrix}$ and $x_1 = \begin{pmatrix} \chi_{0,1} \\ \chi_{1,1} \\ \chi_{2,1} \end{pmatrix}$ that solves

$$\begin{array}{rrrr} -2\chi_{0,0}+ & 2\chi_{1,0}- & 5\chi_{2,0} = & -7 \\ 2\chi_{0,0}- & 3\chi_{1,0}+ & 7\chi_{2,0} = & 11 \\ -4\chi_{0,0}+ & 3\chi_{1,0}- & 7\chi_{2,0} = & -9 \end{array}$$

and

$$\begin{array}{rrrr} -2\chi_{0,1}+ & 2\chi_{1,1}- & 5\chi_{2,1} = & 8 \\ 2\chi_{0,1}- & 3\chi_{1,1}+ & 7\chi_{2,1} = & -13 \\ -4\chi_{0,1}+ & 3\chi_{1,1}- & 7\chi_{2,1} = & 9 \end{array}$$

$$\begin{pmatrix} \chi_{0,0} & \chi_{0,1} \\ \chi_{1,0} & \chi_{1,1} \\ \chi_{2,0} & \chi_{2,1} \end{pmatrix} =$$

-1 ✔ Answer: -1 | 2 ✔ Answer: 2

-2 ✔ Answer: -2 | 1 ✔ Answer: 1

1 ✔ Answer: 1 | -2 ✔ Answer: -2

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Answers are displayed within the problem

Homework 8.2.3.2

1/1 point (graded)

Homework 8.2.3.2 This exercise shows you how to use MATLAB to do the heavy lifting for Homework 8.2.3.1. Start with the appended system:

$$\left(\begin{array}{c|cc||cc} -2 & 2 & -5 & -7 & 8 \\ \hline 2 & -3 & 7 & 11 & -13 \\ -4 & 3 & -7 & -9 & 9 \end{array}\right)$$

Enter this into MATLAB as

Calculator

```
A = [
-2  2 -5  ?? ??
 2 -3  7  ?? ??
-4  3 -7  ?? ??
]
```

(You enter `??`.) Create the Gauss transform, G_0, that zeroes the entries in the first column below the diagonal:

```
G0 = [
 1  0  0
??  1  0
??  0  1
]
```

(You fill in the `??`). Now apply the Gauss transform to the appended system:

```
A0 = G0 * A
```

Similarly create G_1,

```
G1 = [
 1 ??  0
 0  1  0
 0 ??  1
]
```

A_1, G_2, and A_2, where A_2 equals the appended system that has been transformed into a diagonal system. Finally, let D equal to a diagonal matrix so that $A_3 = D * A2$ has the identity for the first three columns.
You can then find the solutions to the linear systems in the last column.

☑ Done/Skipped

✔

Homework_8_2_3_2_Answer.m

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Homework 8.2.3.3

39/39 points (graded)
Evaluate

- $$\left(\begin{array}{c|cc} 1 & 0 & 0 \\ \hline \lambda_{1,0} & 1 & 0 \\ \lambda_{2,0} & 0 & 1 \end{array}\right)\left(\begin{array}{c|cc|cc} 3 & 2 & 10 & -7 & 16 \\ \hline -3 & -3 & -14 & 9 & -25 \\ 3 & 1 & 4 & -5 & 3 \end{array}\right) = \left(\begin{array}{c|cc|cc} 3 & 2 & 10 & \beta_{0,0} & \beta_{0,1} \\ \hline 0 & -1 & -4 & \beta_{1,0} & \beta_{1,1} \\ 0 & -1 & -6 & \beta_{2,0} & \beta_{2,1} \end{array}\right)$$

$\lambda_{1,0} =$ 1 ✔ Answer: 1

$\lambda_{2,0} =$ -1 ✔ Answer: -1

-7 ✔ Answer: -7	16 ✔ Answer: 16

$$\begin{pmatrix} \beta_{0,0} & \beta_{0,1} \\ \beta_{1,0} & \beta_{1,1} \\ \beta_{2,0} & \beta_{2,1} \end{pmatrix} =$$

2 ✔ Answer: 2	-9 ✔ Answer: -9

Calculator

2	-13
Answer: 2	Answer: -13

- $$\left(\begin{array}{c|c|c} 1 & \upsilon_{0,1} & 0 \\ \hline 0 & 1 & 0 \\ \hline 0 & \lambda_{2,1} & 1 \end{array}\right)\left(\begin{array}{c|c|c|c} 3 & 2 & 10 & -7 \quad 16 \\ \hline 0 & -1 & -4 & 2 \quad -9 \\ \hline 0 & -1 & -6 & 2 \quad -13 \end{array}\right) = \left(\begin{array}{c|c|c|c} 3 & 0 & 2 & \beta_{0,0} \quad \beta_{0,1} \\ \hline 0 & -1 & -4 & \beta_{1,0} \quad \beta_{1,1} \\ \hline 0 & 0 & -2 & \beta_{2,0} \quad \beta_{2,1} \end{array}\right)$$

$\upsilon_{0,1} =$ 2 Answer: 2

$\lambda_{2,1} =$ -1 Answer: -1

-3	-2
Answer: -3	Answer: -2

$$\begin{pmatrix} \beta_{0,0} & \beta_{0,1} \\ \beta_{1,0} & \beta_{1,1} \\ \beta_{2,0} & \beta_{2,1} \end{pmatrix} =$$

2	-9
Answer: 2	Answer: -9
0	-4
Answer: 0	Answer: -4

- $$\left(\begin{array}{cc|c} 1 & 0 & \upsilon_{0,2} \\ 0 & 1 & \upsilon_{1,2} \\ \hline 0 & 0 & 1 \end{array}\right)\left(\begin{array}{cc|c|c} 3 & 0 & 2 & -3 \quad -2 \\ 0 & -1 & -4 & 2 \quad -9 \\ \hline 0 & 0 & -2 & 0 \quad -4 \end{array}\right) = \left(\begin{array}{cc|c|c} 3 & 0 & 0 & \beta_{0,0} \quad \beta_{0,1} \\ 0 & -1 & 0 & \beta_{1,0} \quad \beta_{1,1} \\ \hline 0 & 0 & -2 & \beta_{2,0} \quad \beta_{2,1} \end{array}\right)$$

$\upsilon_{0,2} =$ 1 Answer: 1

$\upsilon_{1,2} =$ -2 Answer: -2

-3	-6
Answer: -3	Answer: -6

$$\begin{pmatrix} \beta_{0,0} & \beta_{0,1} \\ \beta_{1,0} & \beta_{1,1} \\ \beta_{2,0} & \beta_{2,1} \end{pmatrix} =$$

2	-1
Answer: 2	Answer: -1
0	-4
Answer: 0	Answer: -4

Calculator

- $$\begin{pmatrix} \delta_{0,0} & 0 & 0 \\ 0 & \delta_{1,1} & 0 \\ 0 & 0 & \delta_{2,2} \end{pmatrix} \left(\begin{array}{ccc|cc} 3 & 0 & 0 & -3 & -6 \\ 0 & -1 & 0 & 2 & -1 \\ 0 & 0 & -2 & 0 & -4 \end{array}\right) = \left(\begin{array}{ccc|cc} 1 & 0 & 0 & \beta_{0,0} & \beta_{0,1} \\ 0 & 1 & 0 & \beta_{1,0} & \beta_{1,1} \\ 0 & 0 & 1 & \beta_{2,0} & \beta_{2,1} \end{array}\right)$$

$\delta_{0,0} =$ 1/3 ✔ Answer: 1/3

$\delta_{1,1} =$ -1 ✔ Answer: -1

$\delta_{2,2} =$ -1/2 ✔ Answer: -1/2

$$\begin{pmatrix} \beta_{0,0} & \beta_{0,1} \\ \beta_{1,0} & \beta_{1,1} \\ \beta_{2,0} & \beta_{2,1} \end{pmatrix} =$$

-1 ✔ Answer: -1	-2 ✔ Answer: -2
-2 ✔ Answer: -2	1 ✔ Answer: 1
0 ✔ Answer: 0	2 ✔ Answer: 2

- Use the above exercises to compute $x_0 = \begin{pmatrix} \chi_{00} \\ \chi_{10} \\ \chi_{20} \end{pmatrix}$ and $x_1 = \begin{pmatrix} \chi_{01} \\ \chi_{11} \\ \chi_{21} \end{pmatrix}$ that solve

$$\begin{array}{rcrcrcr} 3\chi_{00} & + & 2\chi_{10} & + & 10\chi_{20} & = & -7 \\ -3\chi_{00} & - & 3\chi_{10} & - & 14\chi_{20} & = & 9 \\ 3\chi_{00} & + & 1\chi_{10} & + & 4\chi_{20} & = & -5 \end{array} \quad \text{and} \quad \begin{array}{rcrcrcr} 3\chi_{00} & + & 2\chi_{10} & + & 10\chi_{20} & = & 16 \\ -3\chi_{00} & - & 3\chi_{10} & - & 14\chi_{20} & = & -25 \\ 3\chi_{00} & + & 1\chi_{10} & + & 4\chi_{20} & = & 3 \end{array}$$

$$\begin{pmatrix} \chi_{0,0} & \chi_{0,1} \\ \chi_{1,0} & \chi_{1,1} \\ \chi_{2,0} & \chi_{2,1} \end{pmatrix} =$$

-1 ✔ Answer: -1	-2 ✔ Answer: -2
-2 ✔ Answer: -2	1 ✔ Answer: 1
0 ✔ Answer: 0	2 ✔ Answer: 2

(You may want to use MATLAB like in the last homework to do the computation for you.)

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Answers are displayed within the problem

Calculator

Homework 8.2.3.4

1/1 point (graded)

Assume below that all matrices and vectors are partitioned "conformally" so that the operations make sense.

$$\left(\begin{array}{c|c|c} I & -u_{01} & 0 \\ \hline 0 & 1 & 0 \\ \hline 0 & -l_{21} & I \end{array}\right)\left(\begin{array}{c|c|c||c} D_{00} & a_{01} & A_{02} & B_0 \\ \hline 0 & \alpha_{11} & a_{12}^T & b_1^T \\ \hline 0 & a_{21} & A_{22} & B_2 \end{array}\right) =$$
$$\left(\begin{array}{c|c|c||c} D_{00} & a_{01}-\alpha_{11}u_{01} & A_{02}-u_{01}a_{12}^T & B_0-u_{01}b_1^T \\ \hline 0 & \alpha_{11} & a_{12}^T & b_1^T \\ \hline 0 & a_{21}-\alpha_{11}l_{21} & A_{22}-l_{21}a_{12}^T & B_2-l_{21}b_1^T \end{array}\right)$$

Always ✔ Answer: Always

Just multiply it out. (Partitioned matrix-matrix multiplication)

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Homework 8.2.3.5

1/1 point (graded)

Assume below that all matrices and vectors are partitioned "conformally" so that the operations make sense and that $\alpha_{11} \neq 0$. Choose

- $u_{01} := a_{01}/\alpha_{11}$ and
- $l_{21} := a_{21}/\alpha_{11}$

$$\left(\begin{array}{c|c|c} I & -u_{01} & 0 \\ \hline 0 & 1 & 0 \\ \hline 0 & -l_{21} & I \end{array}\right)\left(\begin{array}{c|c|c||c} D_{00} & a_{01} & A_{02} & B_0 \\ \hline 0 & \alpha_{11} & a_{12}^T & b_1^T \\ \hline 0 & a_{21} & A_{22} & B_2 \end{array}\right) = \left(\begin{array}{c|c|c||c} D_{00} & 0 & A_{02}-u_{01}a_{12}^T & B_0-u_{01}b_1^T \\ \hline 0 & \alpha_{11} & a_{12}^T & b_1^T \\ \hline 0 & 0 & A_{22}-l_{21}a_{12}^T & B_2-l_{21}b_1^T \end{array}\right)$$

Always ✔ Answer: Always

Just multiply it out. (Partitioned matrix-matrix multiplication)

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Answers are displayed within the problem

The discussion in this unit motivates the algorithm `GaussJordan_MRHS_Part1`, which transforms A to a diagonal matrix and updates the right-hand side accordingly, and `GaussJordan_MRHS_Part2`, which transforms the diagonal matrix A an identity matrix and updates multiple right-hand sides accordingly.

Algorithm: $[A,B] := \text{GAUSSJORDAN\_MRHS\_PART1}(A,B)$

Partition $A \rightarrow \left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right)$, $B \rightarrow \left(\begin{array}{c} B_T \\ \hline B_B \end{array}\right)$

where A_{TL} is 0×0, B_T has 0 rows

while $m(A_{TL}) < m(A)$ **do**

Calculator

Repartition

$$\left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c|c|c} A_{00} & a_{01} & A_{02} \\ \hline a_{10}^T & \alpha_{11} & a_{12}^T \\ \hline A_{20} & a_{21} & A_{22} \end{array}\right), \left(\begin{array}{c} B_T \\ \hline B_B \end{array}\right) \rightarrow \left(\begin{array}{c} B_0 \\ \hline b_1^T \\ \hline B_2 \end{array}\right)$$

$$\begin{array}{ll} a_{01} := a_{01}/\alpha_{11} & (= u_{01}) \\ a_{21} := a_{21}/\alpha_{11} & (= l_{21}) \\ A_{02} := A_{02} - a_{01}a_{12}^T & (= A_{02} - u_{01}a_{12}^T) \\ A_{22} := A_{22} - a_{21}a_{12}^T & (= A_{22} - l_{21}a_{12}^T) \\ B_0 := B_0 - a_{01}b_1^T & (= B_0 - u_{01}b_1^T) \\ B_2 := B_2 - a_{21}b_1^T & (= B_2 - l_{21}b_1^T) \\ a_{01} := 0 \quad \text{(zero vector)} & \\ a_{21} := 0 \quad \text{(zero vector)} & \end{array}$$

Continue with

$$\left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c|c|c} A_{00} & a_{01} & A_{02} \\ \hline a_{10}^T & \alpha_{11} & a_{12}^T \\ \hline A_{20} & a_{21} & A_{22} \end{array}\right), \left(\begin{array}{c} B_T \\ \hline B_B \end{array}\right) \leftarrow \left(\begin{array}{c} B_0 \\ \hline b_1^T \\ \hline B_2 \end{array}\right)$$

endwhile

Algorithm: $[A, B] := \text{GAUSSJORDAN\_MRHS\_PART2}(A, B)$

Partition $A \rightarrow \left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right), B \rightarrow \left(\begin{array}{c} B_T \\ \hline B_B \end{array}\right)$

where A_{TL} is 0×0, B_T has 0 rows

while $m(A_{TL}) < m(A)$ **do**

Repartition

$$\left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c|c|c} A_{00} & a_{01} & A_{02} \\ \hline a_{10}^T & \alpha_{11} & a_{12}^T \\ \hline A_{20} & a_{21} & A_{22} \end{array}\right), \left(\begin{array}{c} B_T \\ \hline B_B \end{array}\right) \rightarrow \left(\begin{array}{c} B_0 \\ \hline b_1^T \\ \hline B_2 \end{array}\right)$$

$$\begin{array}{l} b_1^T := (1/\alpha_{11})b_1^T \\ \alpha_{11} := 1 \end{array}$$

Continue with

$$\left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c|c|c} A_{00} & a_{01} & A_{02} \\ \hline a_{10}^T & \alpha_{11} & a_{12}^T \\ \hline A_{20} & a_{21} & A_{22} \end{array}\right), \left(\begin{array}{c} B_T \\ \hline B_B \end{array}\right) \leftarrow \left(\begin{array}{c} B_0 \\ \hline b_1^T \\ \hline B_2 \end{array}\right)$$

endwhile

Calculator

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