7. Completed worked example

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Reflect

On the page before last, Prof. Auroux found that the function

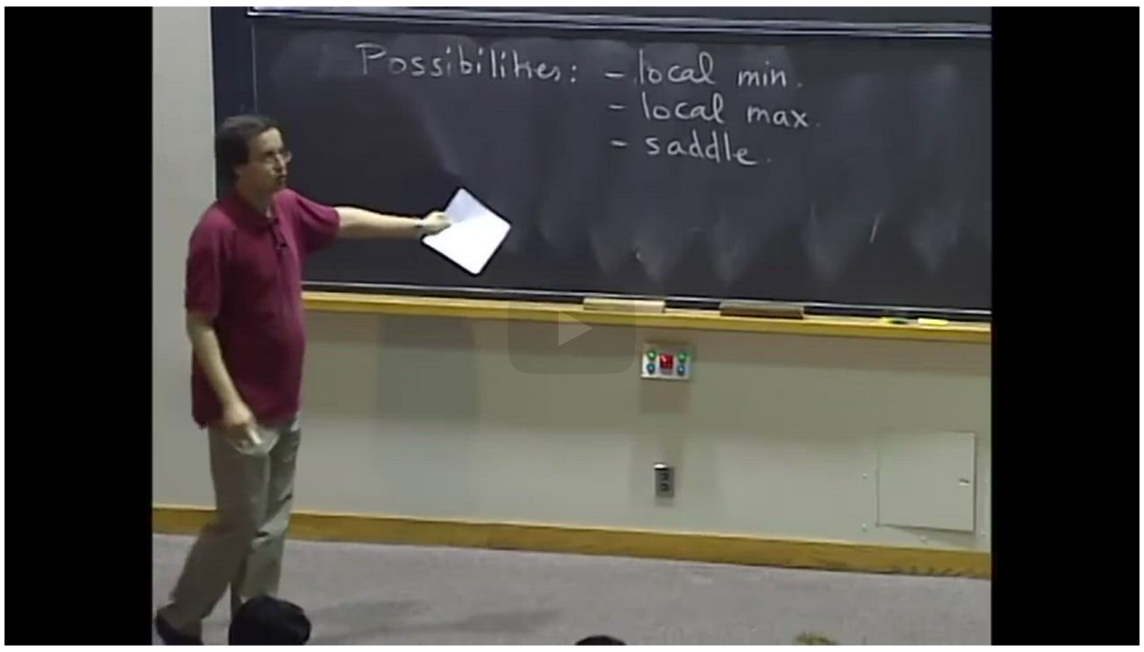
$$f(x,y) = x^2 - 2xy + 3y^2 + 2x - 2y$$

has one critical point at $(-1,0)$.

In the video that follows, he continues with this example to determine the type of critical point by hand.

Find the type of critical point

Start of transcript. Skip to the end.



0:00 / 0:00 2.0x

PROFESSOR: Tomorrow we'll see how

to decide which one it is in general, using

second derivatives.

For this time let's just try to do it by hand.

So I just want to observe--

in fact, I can try to--

these examples that I have here,

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The analysis for the function $f(x,y) = x^2 - 2xy + 3y^2 + 2x - 2y$ involves trying to write it as a sum of two squares.

Note that this technique is useful, but we do not expect you to know that this is possible immediately by inspection! In fact, Prof. Auroux knew he could write this as a sum of two squares because he started with a sum of two squares and expanded them to create this example for you. We work out the algebra for you here so that you can follow the steps more carefully at your own pace.

First we note that the first two terms $x^2 - 2xy$ in the expression look like part of the sum of two squares: $x^2 - 2xy = (x-y)^2 - y^2$ where we've subtracted off a y^2 term to cancel the one added by including the squared term.

Therefore we can write the entire function as

$$x^2 - 2xy + 3y^2 + 2x - 2y = (x-y)^2 - y^2 + 3y^2 + 2x - 2y \quad (4.33)$$

$$= (x-y)^2 + 2y^2 + 2x - 2y \quad (4.34)$$

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$$= \quad (x-y)^2 + 2y^2 + 2(x-y) \tag{4.35}$$

Next notice that $(x-y)^2 + 2x - 2y = (x-y)^2 + 2(x-y)$, which again looks like the sum of squares. We can write this as $(x-y)^2 + 2(x-y) = ((x-y)+1)^2 - 1$ where we've subtracted off a -1 that does exist on the left side.

Therefore we can write the entire function as

$$(x-y)^2 + 2y^2 + 2(x-y) \quad = \quad ((x-y)+1)^2 - 1 + 2y^2 \tag{4.36}$$

$$= \quad ((x-y)+1)^2 + 2y^2 - 1 \tag{4.37}$$

This allows us to write our original function as a sum of squares

$$f(x,y) = x^2 - 2xy + 3y^2 + 2x - 2y = \underbrace{((x-y)+1)^2}_{\geq 0} + \underbrace{2y^2}_{\geq 0} - 1 \geq -1.$$

Observe that the two square terms in this expression of the function are both zero exactly at the critical point.

$$f(-1,0) = \underbrace{((-1-0)+1)^2}_{=0} + \underbrace{2\cdot 0^2}_{=0} - 1 = -1$$

Therefore the critical point we found is a local minimum.

> **Remark 7.1** At this point, we can say that this is actually a global minimum since we know that this function must be greater than or equal to -1 which is the global minimum value. Since the function achieves this global minimum value at this critical point, it must be a global minimum.

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Conclusion that the function is a minimum 2

I may be missing something, but I do not understand how the function being greater or equal to -1 translates to it being a local and e...

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