MATHEMATICS

Conditional expectation of a random walk given that it is positive

Asked 5 years, 7 months ago Active today Viewed 276 times

2

1

Let $\{\xi_k\}$ is a sequence of iid random variables with $E(\xi_1) = 0$ and $E(\xi_1)^2 = \sigma^2 < \infty$. Define the random walk $Y_n = \sum_{k=1}^n \xi_k$. Is it necessarily true that the conditional expectation $E(Y_n|Y_n > 0) = \mathcal{O}(\sqrt{n})$ as $n \to \infty$?

From the classical CLT, we know that $\frac{Y_n}{\sqrt{n}} \Rightarrow N(0, \sigma^2)$, in which the arrow denotes weak convergence as $n \to \infty$. This seems to heavily support the suggested limiting behavior, as do the results of some computations I've done with specific choices for $\{\xi_k\}$. However, I'm stuck on how to prove the bound concretely. Is there some twist on CLT that I'm just not seeing, or is there more to the picture? Maybe I need to assume more on the random walk in question.

Any assistance provided will be much appreciated.

probability probability-theory conditional-expectation random-walk

Share Cite Edit Follow Flag

asked Feb 11 '16 at 6:43
srnoren
537 1 3 14

I am not sure that this is the answer you are looking for. But Try proving that $(Y_n)^2 - n\sigma^2$ is a Martingale. This gives you that the mean of $(Y_n)^2$ is $n\sigma^2$ – Kore-N Feb 11 '16 at 7:33

1 Answer

Active Oldest Votes

2

Let $W_n = n^{-1/2} Y_n$. Then

$$n^{-1/2}\mathsf{E}[Y_n \mid Y_n > 0] = \mathsf{E}[W_n \mid W_n > 0] \leq \frac{\sqrt{\mathsf{E}|W_n|^2}}{\mathsf{P}(W_n > 0)} = \frac{\sigma}{\mathsf{P}(W_n > 0)},$$

and $\mathsf{P}(W_n > 0) \to 1/2$ (by the CLT), i.e., for any $\epsilon > 0$, $\mathsf{P}(W_n > 0) \geq \frac{1}{2} - \epsilon$ for n large enough.

Share Cite Edit Follow Flag edited 1 hour ago

answered Feb 11 '16 at 8:55
d.k.o.
22.4k 1 14 41

Ah, I was so close! I tried going down this route, but I didn't see the Jensen's Inequality. Thank you!

– srnoren Feb 11 '16 at 17:35