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Measure

The easiest way to get clear about the notion of measure is to start by thinking about the notion of *length*.

Let us start by considering the notion of length as it applies to line segments $[a, b]$, where a and b are real numbers such that $a \leq b$. Although it is natural to think of of line segments in spatial terms, here we will model them as sets of real numbers. More specifically, we will think of the line segment $[a, b]$ as the set of real numbers x such that $a \leq x \leq b$. So, for instance, we take $[\frac{1}{4}, \frac{1}{2}]$ to be the set $\{x : \frac{1}{4} \leq x \leq \frac{1}{2}\}$. (We will treat $[a, a] = \{a\}$ as a line segment, and therefore regard a point as a special case of a line segment.)

A nice feature of line segments is that there is a simple recipe for characterizing their length:

$$\text{Length}([a, b]) = b - a.$$

Accordingly, $[\frac{1}{2}, \frac{3}{4}]$ has length $\frac{1}{4}$, because $\frac{3}{4} - \frac{1}{2} = \frac{1}{4}$; and $[2, 2]$ has length 0, because $2 - 2 = 0$. Now suppose that we wish to generalize the notion of length by applying it to sets of real numbers other than line segments. Suppose, for example, that we wish to assign a "length" to the set:

$$\left[0, \frac{1}{4}\right] \cup \left[\frac{1}{2}, 1\right]$$

It is natural to say the following:

$$\text{Length}\left(\left[0, \frac{1}{4}\right] \cup \left[\frac{1}{2}, 1\right]\right) = \text{Length}\left([0, \frac{1}{4}]\right) + \text{Length}\left([\frac{1}{2}, 1]\right) = \frac{1}{4} + \frac{1}{2} = \frac{3}{4}$$

And as it turns out, there is a systematic way of extending this general idea to an rich and interesting family of sets of real numbers. They are known as the *Borel Sets*, in honor of the great French mathematician Émile Borel.

I'll tell you more about them in the next subsection.

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