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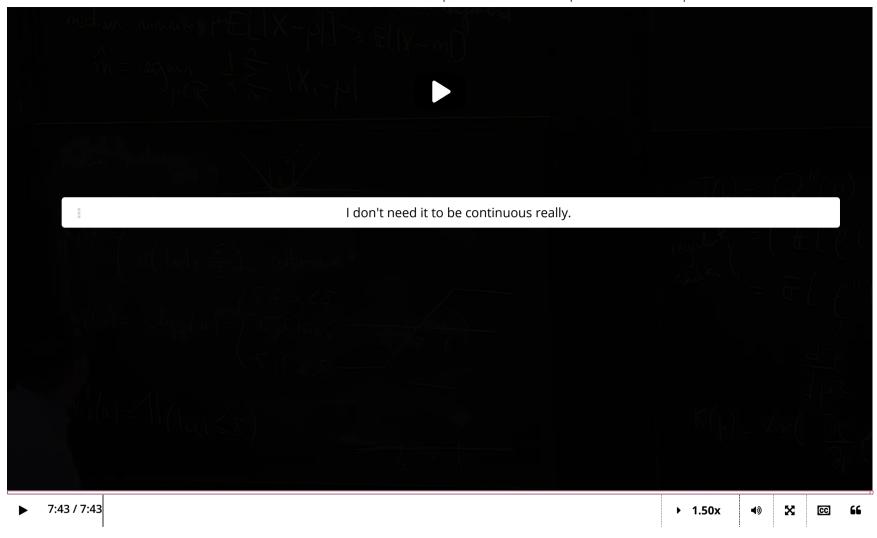
Course > Unit 3 Methods of Estimation > Lecture 12: M-Estimation > 7. Robust Statistics and Huber's Loss

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7. Robust Statistics and Huber's Loss Motivation and Introduction to Huber's Loss



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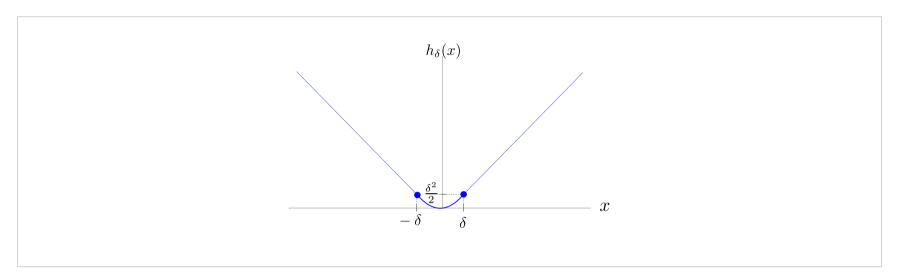
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Huber's Loss

3/3 points (graded)

Huber's loss is defined to be

$$h_{\delta}\left(x
ight)=\left\{ egin{array}{ll} rac{x^{2}}{2} & ext{if} \; \left|x
ight| \leq \delta \ \delta\left(\left|x
ight| - \delta/2
ight) & ext{if} \; \left|x
ight| > \delta \end{array}
ight.$$



Let k denote the smallest integer such that the $rac{d^k}{dx^k}h_\delta\left(x
ight)$ is **not** a continuous function.

What is k?

The function $rac{d^k}{dx^k}h_\delta\left(x
ight)$ is discontinuous at two points $x_1,x_2\in\mathbb{R}$ where $x_1< x_2.$

What are x_1 and x_2 in terms of δ ?

$$x_1 = egin{bmatrix} - ext{delta} & lacksquare & lacks$$

STANDARD NOTATION

Solution:

Observe that

$$rac{\partial h_\delta}{\partial x}(x) = egin{cases} x & ext{if } |x| < \delta \ \delta & ext{if } x > \delta \ -\delta & ext{if } x < -\delta, \end{cases}$$

which is a continuous function. However, the next derivative

$$rac{\partial^2 h_\delta}{\partial^2 x}(x) = egin{cases} 1 & ext{if } |x| < \delta \ 0 & ext{if } |x| > \delta \end{cases}$$

has discontinuities at $x=\pm\delta$. In particular, $\frac{\partial^2 h_\delta}{\partial^2 x}(\pm\delta)$ is not defined. Therefore, for the first question, we conclude that k=2. For the second question, $x_1=-\delta$ and $x_2=\delta$.

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You have used 1 of 3 attempts

• Answers are displayed within the problem

Comparing Huber's Loss and the absolute value function

1/1 point (graded)

Recall Huber's loss $h_{\delta}(x)$ as defined in the previous problem. The absolute value function is defined to be |x|.

Which of the following statements are true? (Choose all that apply.)

Both Huber's loss and the absolute value are differentiable everywhere.

For x>0 sufficiently large, both Huber's loss and the absolute value are both linear functions.

In the intervals where $h_{\delta}\left(x\right)$ is a linear function, both Huber's loss and the absolute value function have the same slope.

Both Huber's loss and the absolute value function are convex.



Solution:

We examine the choices in order.

- "Both Huber's loss and the absolute value are differentiable everywhere." is incorrect. It is true that Huber's loss is differentiable everywhere. However, |x| is not differentiable at x=0.
- "For x>0 sufficiently large, both Huber's loss and the absolute value are both linear functions." is correct. This is certainly true for the absolute function, as |x|=x if x>0. Moreover, if $x>\delta$, then we have $h_{\delta}\left(x\right)=\delta\left(x-\delta/2\right)$ which is also a linear function.
- "In the intervals where $h_{\delta}\left(x\right)$ is a linear function, both Huber's loss and the absolute value function have the same slope." is incorrect. For example, if $x>\delta$, then |x| has slope +1. However, $h_{\delta}\left(x\right)$ has slope δ , which is not necessarily equal to 1.
- "Both Huber's loss and the absolute value function are convex." is correct. This is evident from the graphs of |x| and $h_{\delta}(x)$.

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