2. Lecture 6

The following can be done after Lecture 6.

6-1

5.0/5.0 points (graded)

Let $\mathbf{D} := \begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix}$. What is the upper right entry of $e^{t\mathbf{D}}$?

0 ✔ **Answer:** 0

0

Solution:

0.

In general, if $\mathbf{D}$ is a diagonal matrix with diagonal entries $\lambda_1, \ldots, \lambda_n$, then $e^{t\mathbf{D}}$ is a diagonal matrix with diagonal entries $e^{\lambda_1 t}, \ldots, e^{\lambda_n t}$. The upper right entry of $e^{t\mathbf{D}}$ in the problem is not on the diagonal, so it is 0.

Submit You have used 1 of 10 attempts

Answers are displayed within the problem

6-2

5.0/5.0 points (graded)

Let $\mathbf{A} = \begin{pmatrix} 2 & 2 \\ -2 & -2 \end{pmatrix}$. What is the lower right entry of $e^{t\mathbf{A}}$? (Hint: What is $\mathbf{A}^2$?)

1-2*t

✔ **Answer:** 1-2*t

$1 - 2 \cdot t$

FORMULA INPUT HELP

Solution:

$1 - 2t$.

Since $\mathbf{A}^2 = 0$,

$$\begin{aligned} e^{t\mathbf{A}} &= \mathbf{I} + t\mathbf{A} + \frac{(t\mathbf{A})^2}{2!} + \frac{(t\mathbf{A})^3}{3!} + \cdots \\ &= \mathbf{I} + t\mathbf{A} \\ &= \begin{pmatrix} 1+2t & 2t \\ -2t & 1-2t \end{pmatrix}. \end{aligned}$$

Submit

You have used 1 of 10 attempts

Answers are displayed within the problem

6-3

10.0/10.0 points (graded)

Let $\mathbf{A} = \begin{pmatrix} 3 & 2 \\ 0 & 3 \end{pmatrix}$. What is the upper right entry of $e^{t\mathbf{A}}$?

2*t*exp(3*t)

✔ **Answer:** 2*t*e^(3*t)

$2 \cdot t \cdot \exp(3 \cdot t)$

Solution:

$2te^{3t}$.

Let $\mathbf{N} := \begin{pmatrix} 0 & 2 \\ 0 & 0 \end{pmatrix}$. Then $\mathbf{A} = 3\mathbf{I} + \mathbf{N}$. Scalar multiples of $\mathbf{I}$ commute with any matrix, and $\mathbf{N}^2 = 0$, so

$$\begin{aligned} e^{t\mathbf{A}} &= e^{t(3\mathbf{I})} e^{t\mathbf{N}} \\ &= e^{3t\mathbf{I}}(\mathbf{I} + t\mathbf{N} + 0 + 0 + \cdots) \\ &= \begin{pmatrix} e^{3t} & 0 \\ 0 & e^{3t} \end{pmatrix} \begin{pmatrix} 1 & 2t \\ 0 & 1 \end{pmatrix} \\ &= \begin{pmatrix} e^{3t} & 2te^{3t} \\ 0 & e^{3t} \end{pmatrix}. \end{aligned}$$

Submit You have used 2 of 15 attempts

Answers are displayed within the problem

6-4

10.0/10.0 points (graded)

Find the first coordinate of $\begin{pmatrix} 5 \\ 7 \end{pmatrix}$ with respect to the basis $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$, $\begin{pmatrix} 3 \\ 2 \end{pmatrix}$ of $\mathbb{R}^2$.

-11 **Answer:** -11

-11

Solution:

The first coordinate is -11.

We need to solve

$$\begin{pmatrix} 5 \\ 7 \end{pmatrix} = c_1 \begin{pmatrix} 2 \\ 1 \end{pmatrix} + c_2 \begin{pmatrix} 3 \\ 2 \end{pmatrix};$$

then the answer is c_1. This amounts to the system

$$\begin{aligned} 2c_1 + 3c_2 &= 5 \\ c_1 + 2c_2 &= 7. \end{aligned}$$

To find c_1, eliminate c_2 by taking 2 times the first equation minus 3 times the second equation:

$$c_1 = 2(5) - 3(7) = -11.$$

Submit

Answers are displayed within the problem

6-5

10/10 points (graded)

The function $10\cos(5t) + 8\sin(5t)$ lies in the complex vector space with basis e^{5it}, e^{-5it}. Find its second coordinate with respect to that basis.

5+4*i

Answer: 4*i + 5

$5 + 4 \cdot i$

Solution:

The answer is $5 + 4i$,

because

$$\begin{aligned} 10\cos(5t) + 8\sin(5t) &= 10\left(\frac{e^{5it} + e^{-5it}}{2}\right) + 8\left(\frac{e^{5it} - e^{-5it}}{2i}\right) \\ &= 5(e^{5it} + e^{-5it}) - 4i(e^{5it} - e^{-5it}) \\ &= (5-4i)e^{5it} + (5+4i)e^{-5it}. \end{aligned}$$

Submit

Answers are displayed within the problem

6-6

5.0/5.0 points (graded)

The vectors $\mathbf{v}_1 := \begin{pmatrix} 4/5 \\ 3/5 \end{pmatrix}$ and $\mathbf{v}_2 := \begin{pmatrix} -3/5 \\ 4/5 \end{pmatrix}$ form an orthonormal basis of $\mathbb{R}^2$. Find the first coordinate of $\begin{pmatrix} 10 \\ 15 \end{pmatrix}$ with respect to this basis.

17

Answer: 17

17

Solution:

The first coordinate is

$$\begin{aligned} \begin{pmatrix} 10 \\ 15 \end{pmatrix} \cdot \mathbf{v}_1 &= \begin{pmatrix} 10 & 15 \end{pmatrix} \cdot \begin{pmatrix} 4/5 \\ 3/5 \end{pmatrix} \\ &= 8 + 9 = 17. \end{aligned}$$

Submit

Answers are displayed within the problem

6-7

10.0/10.0 points (graded)

Vectors $\mathbf{w_1}, \mathbf{w_2}, \mathbf{w_3}$ form an orthogonal basis for $\mathbb{R}^3$. Given that $\mathbf{w_1} = \begin{pmatrix} 2 \\ 3 \\ 5 \end{pmatrix}$, what is the first coordinate of the vector $\mathbf{v} := \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}$ with respect to this basis?

(Enter as a fraction or decimal to three places.)

0.34210526

✔ **Answer:** 13/38

0.34210526

Solution:

The first coordinate is $13/38$.

If $\mathbf{v} = c_1\mathbf{w_1} + c_2\mathbf{w_2} + c_3\mathbf{w_3}$, then $\mathbf{v} \cdot \mathbf{w_1} = c_1\mathbf{w_1} \cdot \mathbf{w_1} + 0 + 0$ (since $\mathbf{w_1}$ is orthogonal to $\mathbf{w_2}$ and $\mathbf{w_3}$), so the first coordinate is

$$c_1 = \frac{\mathbf{v} \cdot \mathbf{w_1}}{\mathbf{w_1} \cdot \mathbf{w_1}} = \frac{\begin{pmatrix} 0 & 1 & 2 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 3 \\ 5 \end{pmatrix}}{\begin{pmatrix} 2 & 3 & 5 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 3 \\ 5 \end{pmatrix}} = \frac{13}{38}.$$

Submit

Answers are displayed within the problem

2. Lecture 6

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Topic: Unit 3: Solving systems of first order ODEs using matrix methods
/ 2. Lecture 6

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[Staff] Problems 6-4 ~ 6-7 have unlimited attempts

discussion posted about 22 hours ago by Wan-TehChang

Just FYI: Problems 6-4 ~ 6-7 have unlimited attempts.

Wan-Teh

This post is visible to everyone.

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1 response

Wan-TehChang
about 20 hours ago

I just remembered that a downside of unlimited attempts is that there is no way to see the solution before the due date unless the student already has the correct answer. So it would be good to set a finite number of attempts.

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