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10. Tangent plane

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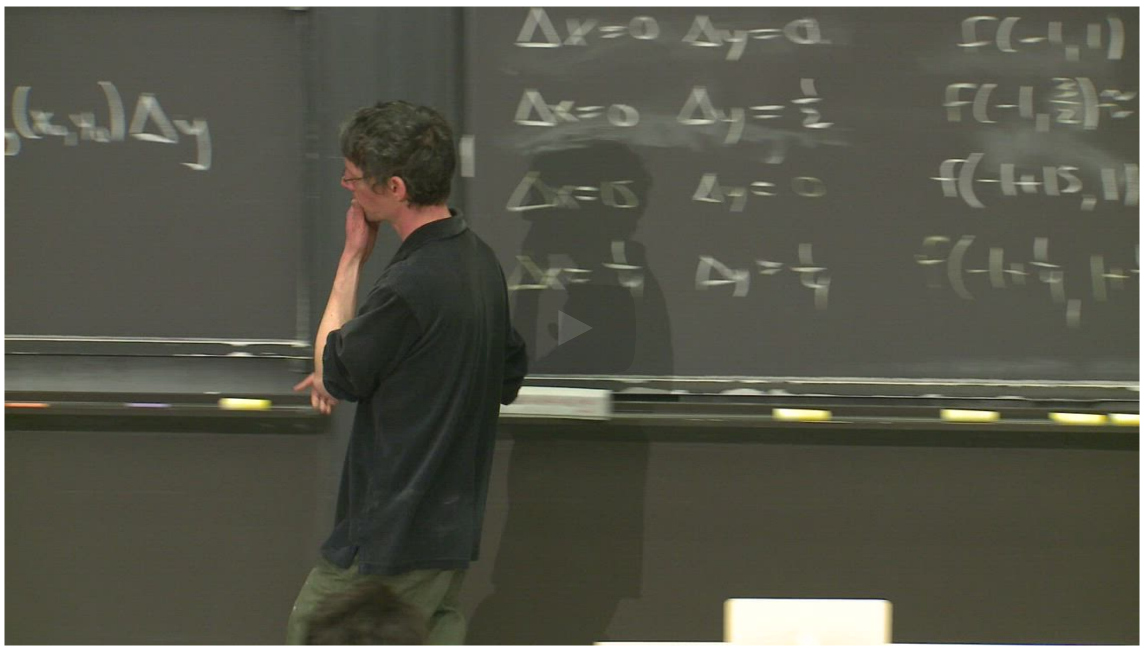
Lecture due Aug 4, 2021 20:30 IST Completed

Synthesize

Tangent plane



0:00 / 0:00 2.0x

Start of transcript. Skip to the end.

PROFESSOR: Yeah, so that was a great lead-in.

In the multivariable case, for f of (x,y),

the linear approximation is going to be a plane.

And then the step I want to make next is like, what would

the equation be for the plane?

So starting from here, how do we get

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We can express a linear approximation in terms of Δx and Δy, or x and y.

If (x,y) is near $(-1,1)$, we write

$$f(x,y) \approx f(-1,1) + f_x(-1,1)\underbrace{(x-(-1))}_{\Delta x} + f_y(-1,1)\underbrace{(y-1)}_{\Delta y} \qquad (2.37)$$

$$= 2 - 2(x-(-1)) + 2(y-1) = -2 - 2x + 2y. \qquad (2.38)$$

Note that the graph of $-2-2x+2y$ is a plane. It is the tangent plane to the graph of f at $(-1,1,2)$.

Definition 10.1

- The linear approximation of a function $f(x,y)$ near a point (a,b) is the equation for a **tangent plane**.
- The equation for the **tangent plane** is

$$f(x,y) \approx f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b),$$

or in terms of Δx and Δy, it can be written as

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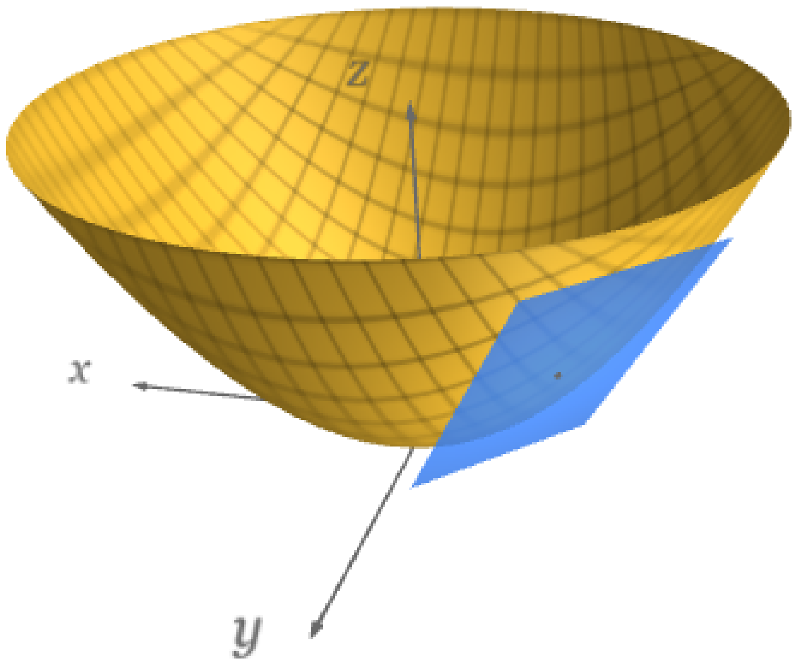
$$f(a+\Delta x, b+\Delta y) \approx f(a,b) + f_x(a,b)\,\Delta x + f_y(a,b)\,\Delta y.$$

Geometrically, the **tangent plane** of a two variable function $f(x,y)$ at a point (a,b) is a plane that contains the slope of every line tangent to the surface $z = f(x,y)$ at the point $(a,b,f(a,b))$.

▶ **Tangent plane**



The point in 3D

1/1 point (graded)

Where did the point $(-1,1,2)$ come from?

- ○ 2 is the value found by taking the sum of the absolute values of x and y
- ◉ 2 is the value of z found by plugging in $(-1,1)$ into $z = f(x,y)$
- ○ This point does not make sense, there is nothing 3-dimensional here

✓

Solution:

To think about the three dimensional picture, we first need to imagine our function $z = f(x,y)$ as designating a height z above each point (x,y) in the plane.

The point $(x,y,z) = (-1,1,2)$ exactly corresponds to plugging in $(-1,1)$ into our function $f(x,y)$ to determine the height, or value z at that point on the function.

The tangent plane is a plane that passes through the point $(-1,1,2)$. Further if you intersect the tangent plane with the plane $y = 1$, you get a line that is tangent to the function $f(x,1)$ with slope $f_x(-1,1)$. If you intersect the tangent plane with the plane $x = -1$, you get a line that is tangent to the function $f(-1,y)$ with slope $f_y(-1,1)$.

Submit You have used 1 of 2 attempts

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Answers are displayed within the problem

Answers are displayed within the problem

Compare the function and its tangent plane

1/1 point (graded)

We think of the tangent plane of a function $f(x,y)$ at the point $(x,y)=(a,b)$ as a linear approximation to the function near that point.

The tangent plane to the function $f(x,y)$ at the point $(x,y)=(a,b)$ has the equation:

$$T(x,y)=f(a,b)+f_x(a,b)(x-a)+f_y(a,b)(y-b)$$

Compare the function $f(x,y)$ and this equation of the tangent plane $T(x,y)$ that is tangent to the function at the point (a,b). Which of the following are necessarily true?

- [x] $f(a,b)=T(a,b)$
- [ ] $f(x,y)=T(x,y)$
- [x] $f_x(a,b)=T_x(a,b)$
- [ ] $f_x(x,y)=T_x(x,y)$
- [x] $f_y(a,b)=T_y(a,b)$
- [ ] $f_y(x,y)=T_y(x,y)$

✔

Solution:

The tangent plane has the same value as the function at the point (a,b), and it has the same partial derivatives at (a,b).

At nearby points, we can assume that the value of the function and the tangent plane are close, but we certainly shouldn't expect them to be exactly equal.

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10. Tangent plane

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? [STAFF] Typo in 2.38
Shouldn't it be -2-2x+2y?? 9

[Staff] Wording Suggestion
> *At nearby points, we can assume that the value of the function and the tangent plane are close, but they are certainly not exactly... 3

Compare the fuunction and its tangent plane
Great exercise. It is the 3-D equivalent of the skate park example in calculus 1a. 2

When would be tangent to 3d shape is just line instead of plane ?

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For curve of the form X^2 - Y , would tangent be a line ? 3

[STAFF] 4 attempts on a question with 3 choices

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