



## MITx: 6.041x Introduction to Probability - The Science of Uncertainty



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## Problem 2: Calculation with PDFs

(3/3 points)

Let  $\mathbf{X}$  be a random variable that takes non-zero values in  $[1, \infty)$ , with a PDF of the form

$$f_X(x) = \begin{cases} \frac{c}{x^3}, & \text{if } x \geq 1, \\ 0, & \text{otherwise.} \end{cases}$$

Let  $\mathbf{U}$  be a uniform random variable on  $[0, 2]$ . Assume that  $\mathbf{X}$  and  $\mathbf{U}$  are independent.

1. What is the value of the constant  $c$ ?

 $c =$  Answer: 2

2.

► Unit 6: Further topics on random variables

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▼ Exam 2

### Exam 2

Exam 2 due Apr 20, 2016 at 23:59 UTC



► Unit 8: Limit theorems and classical statistics

1/4



$\mathbf{P}(X \leq U) =$  Answer: 0.25

3. Find the PDF of  $D = 1/X$ . Express your answer in terms of  $d$  using standard notation .

2\*d



For  $0 \leq d \leq 1$ ,  $f_D(d) =$  Answer: 2\*d

Answer:

1. The distribution must integrate to 1. Since

$$\int_1^{\infty} \frac{c}{x^3} dx = -\frac{c}{2x^2} \Big|_1^{\infty} = \frac{c}{2},$$

we have  $c = 2$ .

2. Since  $X \geq 1$  and  $U \leq 2$ , the event of interest occurs only if  $1 \leq X \leq U \leq 2$ . Using the law of total probability and the independence of  $X$  and  $U$ , we have

$$\mathbf{P}(X \leq U) = \int_1^2 \mathbf{P}(X \leq u) f_U(u) du$$

$$\begin{aligned}
&= \int_1^2 \left( \int_1^u f_X(x) dx \right) f_U(u) du \\
&= \int_1^2 \left( \int_1^u \frac{2}{x^3} dx \right) \frac{1}{2} du \\
&= \int_1^2 \left( 1 - \frac{1}{u^2} \right) \cdot \frac{1}{2} du \\
&= \frac{1}{4}.
\end{aligned}$$

3. Since  $\mathbf{X}$  takes values in  $[1, \infty)$ ,  $\mathbf{D}$  takes values in  $[0, 1]$ . We use the method of derived distributions to find the CDF of  $\mathbf{D}$ . For  $0 \leq d \leq 1$ ,

$$\begin{aligned}
F_D(d) &= \mathbf{P}(D \leq d) \\
&= \mathbf{P}(X \geq 1/d) \\
&= \int_{1/d}^{\infty} \frac{2}{x^3} dx \\
&= d^2.
\end{aligned}$$

The complete CDF of  $\mathbf{D}$  is

$$F_D(d) = \begin{cases} 0, & \text{if } d < 0, \\ d^2, & \text{if } 0 \leq d \leq 1, \\ 1, & \text{if } d > 1. \end{cases}$$

Differentiating the CDF gives the PDF of  $D$ :

$$f_D(d) = \begin{cases} 2d, & \text{if } 0 \leq d \leq 1, \\ 0, & \text{otherwise.} \end{cases}$$

Alternatively, the same result can be achieved without explicitly computing the CDF of  $D$ . We express the CDF of  $D$  in terms of the CDF of  $X$  and use the chain rule of differentiation. For  $0 \leq d \leq 1$ ,

$$\begin{aligned} F_D(d) &= \mathbf{P}(D \leq d) \\ &= \mathbf{P}(X \geq 1/d) \\ &= 1 - F_X(1/d), \\ f_D(d) &= -f_X(1/d) \cdot \frac{-1}{d^2} \\ &= -2d^3 \cdot \frac{-1}{d^2} \\ &= 2d. \end{aligned}$$

Hence, we obtain the same PDF:

$$f_D(d) = \begin{cases} 2d, & \text{if } 0 \leq d \leq 1, \\ 0, & \text{otherwise.} \end{cases}$$

*You have used 2 of 2 submissions*

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