MITx: 6.041x Introduction to Probability - The Science of Uncertainty

Bookmarks

- Unit 0: Overview
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Unit overview

Lec. 5: Probability mass functions and expectations
Exercises 5 due Mar 02, 2016 at 23:59 UTC

Bookmark

Problem 2: Three-sided dice

(9/9 points)

We have two fair three-sided dice, indexed by $i = 1, 2$. Each die has sides labelled 1, 2, and 3. We roll the two dice independently, one roll for each die. For $i = 1, 2$, let the random variable X_i represent the result of the ith die, so that X_i is uniformly distributed over the set $\{1, 2, 3\}$. Define $X = X_2 - X_1$.

1. Calculate the numerical values of following probabilities, as well as the expected value and variance of X:

$\mathbf{P}(X = 0) =$

1/3 ✔ **Answer:** 0.33333

$\mathbf{P}(X = 1) =$ 2/9 ✔ **Answer:** 0.22222

$\mathbf{P}(X = -2) =$ 1/9 ✔ **Answer:** 0.11111

$\mathbf{P}(X = 3) =$ 0 ✔ **Answer:** 0

Lec. 6: Variance; Conditioning on an event; Multiple r.v.'s
Exercises 6 due Mar 02, 2016 at 23:59 UTC

Lec. 7: Conditioning on a random variable; Independence of r.v.'s
Exercises 7 due Mar 02, 2016 at 23:59 UTC

Solved problems

Additional theoretical material

Problem Set 4
Problem Set 4 due Mar 02, 2016 at 23:59 UTC

Unit summary

- Exam 1
- Unit 5: Continuous random variables
- Unit 6: Further topics on random variables
- Unit 7: Bayesian

$\mathbf{E}[X] =$ 0 ✔ **Answer:** 0

$\mathrm{var}(X) =$ 4/3 ✔ **Answer:** 1.33333

2. Let $Y = X^2$. Calculate the following probabilities:

$\mathbf{P}(Y=0) =$

1/3 ✔ **Answer:** 0.33333

$\mathbf{P}(Y=1) =$ 4/9 ✔ **Answer:** 0.44444

$\mathbf{P}(Y=2) =$ 0 ✔ **Answer:** 0

Answer:

1. The sample space for the pair (X_1, X_2) has 9 equally likely outcomes. For each possible value x of X, we count the number of outcomes for which the difference $X_2 - X_1$ equals x, then multiply by $1/9$ to obtain $p_X(x)$.

$$p_X(x) = \begin{cases} 1/9, & x = -2 \text{ or } 2, \\ 2/9, & x = -1 \text{ or } 1, \\ 3/9, & x = 0, \\ 0, & \text{otherwise.} \end{cases}$$

$$\mathbf{E}[X] = \sum_{x=-2}^{2} x p_X(x) = (-2)\cdot\frac{1}{9} + (-1)\cdot\frac{2}{9} + (0)\cdot\frac{3}{9} + (1)\cdot\frac{2}{9} + (2)\cdot\frac{1}{9} = 0$$

We can also see that $\mathbf{E}[X]=0$ because the PMF is symmetric around 0, or because $\mathbf{E}[X_1]=\mathbf{E}[X_2]$, so that $\mathbf{E}[X]=\mathbf{E}[X_2-X_1]=\mathbf{E}[X_2]-\mathbf{E}[X_1]=0.$

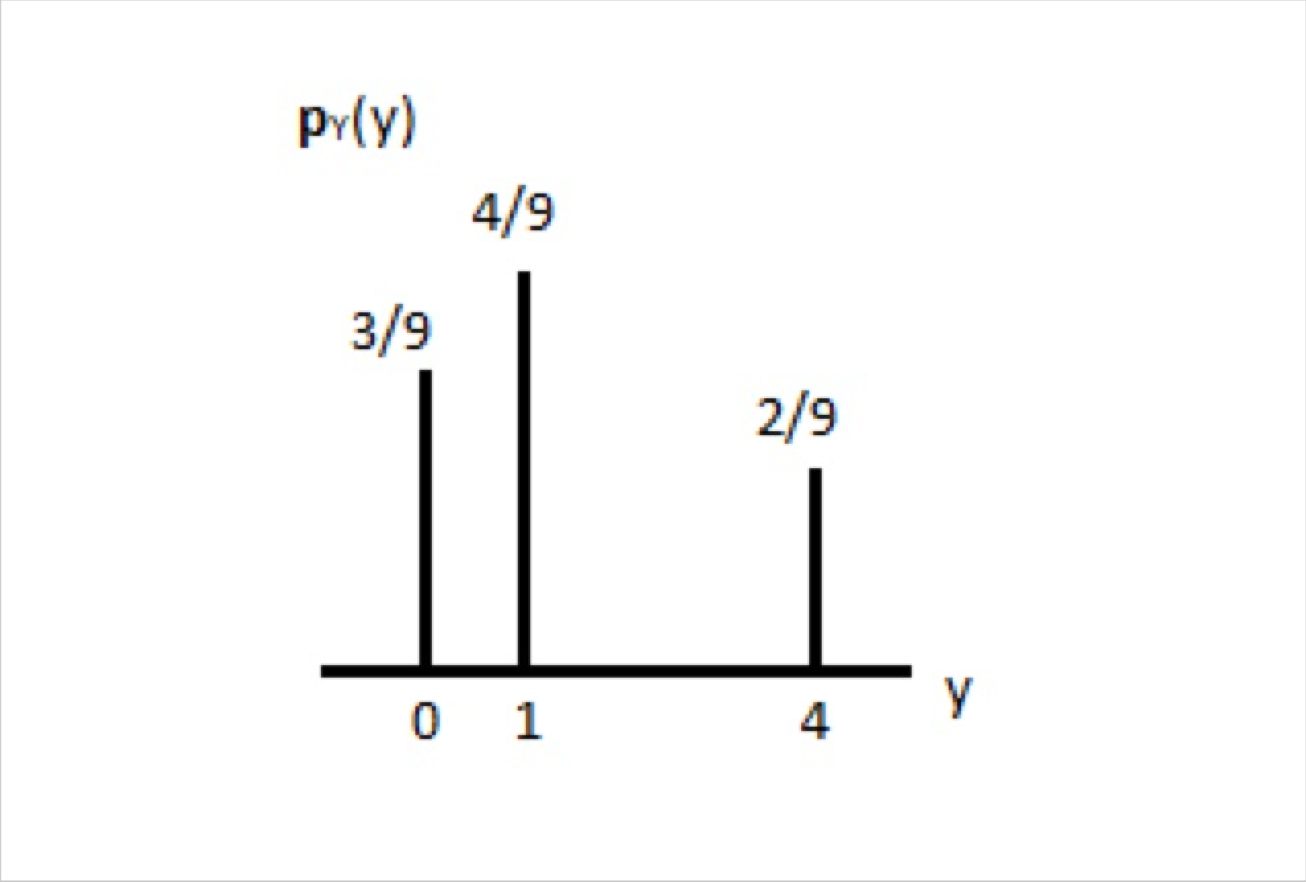
To find the variance of X, we note that $\mathbf{var}(X)=\mathbf{E}[(X-\mathbf{E}[X])^2]=\mathbf{E}[X^2]$, and so

$$\mathbf{E}[X^2] = \sum_{x=-2}^{2} x^2 p_X(x) = 4\cdot\frac{1}{9} + 1\cdot\frac{2}{9} + 0\cdot\frac{3}{9} + 1\cdot\frac{2}{9} + 4\cdot\frac{1}{9} = \frac{4}{3}.$$

2. Let $Y=X^2$. By matching the possible values of X and their probabilities to the possible values of Y, we obtain

$$p_Y(y) = \begin{cases} 2/9, & y=4, \\ 4/9, & y=1, \\ 3/9, & y=0, \\ 0, & \text{otherwise.} \end{cases}$$

A plot of the PMF of Y is shown below:



You have used 1 of 2 submissions

DISCUSSION

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