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Unit overview

Lec. 5: Probability mass functions and expectations
Exercises 5 due Mar 02, 2016 at 23:59 UTC

Lec. 6: Variance; Conditioning on an event; Multiple r.v.'s
Exercises 6 due Mar 02, 2016 at 23:59 UTC

Lec. 7: Conditioning on a random variable; Independence of r.v.'s
Exercises 7 due Mar 02, 2016 at 23:59 UTC

Solved problems

Additional theoretical material

Problem Set 4
Problem Set 4 due Mar 02, 2016 at 23:59 UTC

Unit summary

- ▶ Unit 5: Continuous random variables

Unit 4: Discrete random variables > Lec. 7: Conditioning on a random variable; Independence of r.v.'s > Lec 7
Conditioning on a random variable Independence of r v s vertical2



Exercise: Independence

(5/5 points)

Let \mathbf{X} , \mathbf{Y} , and \mathbf{Z} be discrete random variables.

a) Suppose that \mathbf{Z} is identically equal to 3, i.e., $\mathbf{P}(\mathbf{Z} = 3) = 1$. Is \mathbf{X} guaranteed to be independent of \mathbf{Z} ?

Yes ▾

✓ Answer: Yes

b) Would either of the following be an appropriate definition of independence of the pair (\mathbf{X}, \mathbf{Y}) from \mathbf{Z} ?

- $p_{\mathbf{X}, \mathbf{Y}, \mathbf{Z}}(\mathbf{x}, \mathbf{y}, \mathbf{z}) = p_{\mathbf{X}}(\mathbf{x}) p_{\mathbf{Y}}(\mathbf{y}) p_{\mathbf{Z}}(\mathbf{z})$, for all $\mathbf{x}, \mathbf{y}, \mathbf{z}$

No ▾

✓ Answer: No

- $p_{\mathbf{X}, \mathbf{Y}, \mathbf{Z}}(\mathbf{x}, \mathbf{y}, \mathbf{z}) = p_{\mathbf{X}, \mathbf{Y}}(\mathbf{x}, \mathbf{y}) p_{\mathbf{Z}}(\mathbf{z})$, for all $\mathbf{x}, \mathbf{y}, \mathbf{z}$

Yes ▾

✓ Answer: Yes

c) Suppose that $\mathbf{X}, \mathbf{Y}, \mathbf{Z}$ are independent. Is it true that \mathbf{X} and \mathbf{Y} are independent?

Yes ▾

✓ Answer: Yes

d) Suppose that $\mathbf{X}, \mathbf{Y}, \mathbf{Z}$ are independent. Is it true that (\mathbf{X}, \mathbf{Y}) is independent from \mathbf{Z} ?

Yes ▾

✓ Answer: Yes

Answer:

a) Since \mathbf{Z} is deterministic, the value of \mathbf{Z} does not provide any information, and so, intuitively, we have independence. For a formal argument, suppose that $\mathbf{z} \neq 3$. Then, $p_{\mathbf{X}, \mathbf{Z}}(\mathbf{x}, \mathbf{z}) = 0 = p_{\mathbf{X}}(\mathbf{x}) p_{\mathbf{Z}}(\mathbf{z})$. And for $\mathbf{z} = 3$, $p_{\mathbf{X}, \mathbf{Z}}(\mathbf{x}, 3) = \mathbf{P}(\mathbf{X} = \mathbf{x}, \mathbf{Z} = 3) = \mathbf{P}(\mathbf{X} = \mathbf{x}) = \mathbf{P}(\mathbf{X} = \mathbf{x}) \cdot 1 = p_{\mathbf{X}}(\mathbf{x}) p_{\mathbf{Z}}(3)$, so that the definition of independence is satisfied.

b) The second definition is correct, because it says that events of the form $\{\mathbf{X} = \mathbf{x} \text{ and } \mathbf{Y} = \mathbf{y}\}$ are independent from events of the form $\{\mathbf{Z} = \mathbf{z}\}$. On the other hand, the first imposes the stronger requirement that \mathbf{X} is also independent of \mathbf{Y} .

c) Intuitively, since $\mathbf{X}, \mathbf{Y}, \mathbf{Z}$ are independent, none of the random variables provides information about the others. For a formal argument,

$$p_{\mathbf{X}, \mathbf{Y}}(\mathbf{x}, \mathbf{y}) = \sum_{\mathbf{z}} p_{\mathbf{X}, \mathbf{Y}, \mathbf{Z}}(\mathbf{x}, \mathbf{y}, \mathbf{z}) = \sum_{\mathbf{z}} p_{\mathbf{X}}(\mathbf{x}) p_{\mathbf{Y}}(\mathbf{y}) p_{\mathbf{Z}}(\mathbf{z}) = p_{\mathbf{X}}(\mathbf{x}) p_{\mathbf{Y}}(\mathbf{y}) \sum_{\mathbf{z}} p_{\mathbf{Z}}(\mathbf{z}) = p_{\mathbf{X}}(\mathbf{x}) p_{\mathbf{Y}}(\mathbf{y})$$

d) Intuitively, the value of the pair (\mathbf{X}, \mathbf{Y}) provides no information about the random variable \mathbf{Z} . We will verify that the appropriate definition of independence of (\mathbf{X}, \mathbf{Y}) from \mathbf{Z} from part (b) is satisfied. We first use independence of $\mathbf{X}, \mathbf{Y}, \mathbf{Z}$, and then the fact, from part (c), that $p_{\mathbf{X}}(\mathbf{x}) p_{\mathbf{Y}}(\mathbf{y}) = p_{\mathbf{X}, \mathbf{Y}}(\mathbf{x}, \mathbf{y})$, to obtain

$$p_{X,Y,Z}(x,y,z) = p_X(x)p_Y(y)p_Z(z) = p_{X,Y}(x,y)p_Z(z),$$

as desired.

You have used 1 of 1 submissions

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