11. Concept checks

The normal modes of the wave equation

$$\frac{\partial^2}{\partial t^2}u = c^2\frac{\partial^2}{\partial x^2}u, \quad u(0,t) = u(\pi/2,t) = 0, t > 0$$

on $0 \leq x \leq \pi/2, t \geq 0$ are given by

$$u_k(x,t) \quad = \quad \sin(2kx)\left(A\cos(2ckt) + B\sin(2ckt)\right).$$

Interpreting frequency

2/2 points (graded)

If the equation above represents a vibrating string, then the main note heard is the lowest frequency standing wave. What is the main note heard? What is the frequency and angular frequency you hear when the string is plucked?

Angular frequency: 2*c ✔ **Answer:** 2*c

$2\cdot c$

Frequency: c/pi ✔ **Answer:** c/pi

$\frac{c}{\pi}$

Solution:

The main note, from the mode u_1, has angular frequency $2c$. The frequncy is $\frac{2c}{2\pi} = \frac{c}{\pi}$. You will also hear the higher harmonics at the frequencies $\frac{ck}{\pi}$, $k = 2, 3, \ldots$. (The sound waves induced by the vibrating string depend on the frequencies in t of the modes.)

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Changing the length

1/1 point (graded)

If the length of the string is longer what happens to the sound?

The note is

- ◯ higher
- ◉ lower
- ◯ unchanged

✔

Solution:

Longer strings have lower frequencies, lower notes. Shorter strings have higher frequencies, higher notes.
If the length of the string is L, then the equations $v''(x) = \lambda v(x)$, $v(0) = v(L) = 0$ lead to solutions $v_k(x) = \sin(k\pi x/L)$. The associated angular frequencies in the t variable are $\frac{kc\pi}{L}$, so the larger the L, the smaller the $\frac{kc\pi}{L}$ and the lower the note.

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Understanding the Wave Equation

1/1 point (graded)
When you tighten the string of a musical instrument such as a guitar, piano, or cello, the note gets higher. How does the parameter change in the differential equation $u_{tt} = c^2 u_{xx}$?

- ( ) c decreases
- (•) c increases
- ( ) c stays the same; the constant of the integral curve increases

✔

Solution:

When you tighten the string, you are increasing the tension T. Since the wave speed c is proportional to $\sqrt{T}$, the wave speed also increases. The frequencies heard are proportional to c, so the only way to increase the frequency $\frac{kc\pi}{L}$ keeping the length L fixed and the string desnity μ fixed is to increase the constant c. (Tightening the string increases the tension in the string and increases the spring constant, which corresponds to c.)

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