3. Bias of Estimators; Jensen's Inequality

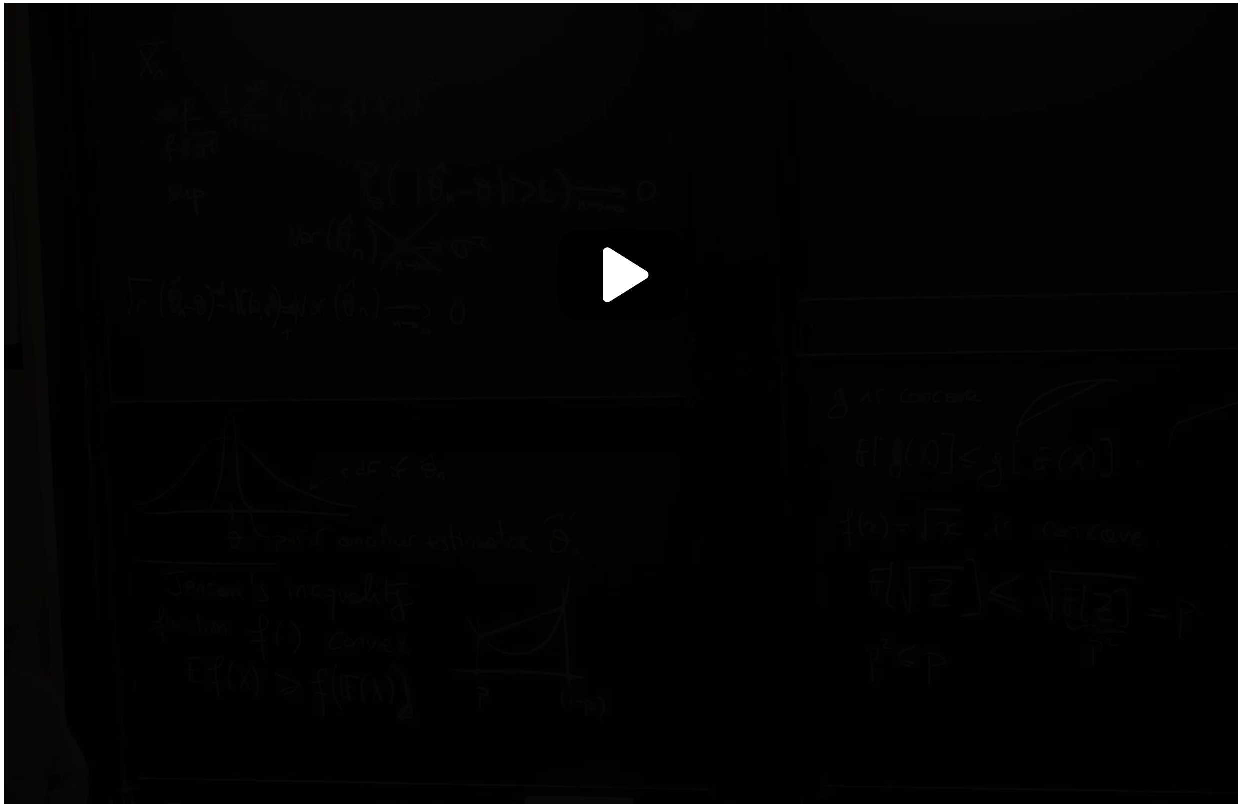
Bias Estimators and an application of Jensen's Inequality

itself, which is precisely the definition of a convex

function.

The way you want to remember Jensen, and the way it goes,

is by saying the chord represents the expectation,



and of course, the function represents the function

of the expectation.

End of transcript. Skip to the start.

12:22 / 12:22

1.50x

Video

Download video file

Transcripts

Download SubRip (.srt) file

Download Text (.txt) file

The Expectation of the Average

1/1 point (graded)

Let $X_1, \ldots, X_n \stackrel{iid}{\sim} \mathcal{U}([a, a+1])$ where a is an unknown parameter. Let $\overline{X}_n = \frac{1}{n}\sum_{i=1}^{n} X_i$ denote the sample mean. In terms of a, what is $\mathbb{E}[\overline{X}_n]$?

$\mathbb{E}\left[\overline{X}_n\right] =$ a+1/2 ✔ **Answer:** a+1/2

$a + \frac{1}{2}$

Solution:

Note that since the X_i's are identically distributed, by linearity of expectation,

$$\mathbb{E}[\overline{X}_n] = \frac{1}{n}\sum_{i=1}^{n}\mathbb{E}[X_i] = \mathbb{E}[X_1] = a + \frac{1}{2}.$$

Submit You have used 1 of 3 attempts

Answers are displayed within the problem

Computing Bias

1/1 point (graded)

Recall: Let $\hat{\theta}_n$ denote an estimator for a true parameter θ. Here n specifies the sample size. The **bias** of $\hat{\theta}_n$ is defined to be

$$\mathbb{E}\left[\hat{\theta}_n\right] - \theta.$$

Let $X_1, \ldots, X_n$ be defined as in the previous question. Compute the bias of the estimator $\overline{X}_n$ with respect to the parameter a.

1/2

✔ **Answer:** .5

Solution:

The bias is given by $\mathbb{E}\left[\overline{X}_n\right] - a = 1/2$, where we applied the previous part. Note that this implies that $\overline{X}_n - \frac{1}{2}$ is an unbiased estimator.

Submit

You have used 1 of 3 attempts

Answers are displayed within the problem

(Optional) Jensen's Inequality

Show

(Optional) Expectation of nonlinear functions and Jensen's Inequality

0 points possible (ungraded)

Let X be a positive random variable with expectation λ. How does $\mu = \mathbb{E}\left[\frac{1}{X}\right]$ compare to $\frac{1}{\lambda}$?

- ○ In general, μ and λ are not comparable
- ◉ $\mu \geq \frac{1}{\lambda}$
- ○ $\mu \leq \frac{1}{\lambda}$

✔

Solution:

Note that the function $x \mapsto \frac{1}{x}$ is a convex function on $(0, \infty)$, hence we can use Jensen's inequality that implies

$$\mathbb{E}\left[f(X)\right] \geq f\left(\mathbb{E}\left[X\right]\right)$$

for all convex functions f to conclude

$$\mu = \mathbb{E}\left[\frac{1}{X}\right] \geq \frac{1}{\mathbb{E}[X]} = \frac{1}{\lambda}.$$

Submit You have used 1 of 1 attempt

Answers are displayed within the problem

Discussion

Hide Discussion

Topic: Unit 2 Foundation of Inference:Lecture 4: Parametric Estimation and Confidence Intervals / 3. Bias of Estimators; Jensen's Inequality

Add a Post

Show all posts ▼ by recent activity ▼

computing bias

I'm assuming `theta` is `mu` of a Uniform Random Variable?. If yes, then that estimator is unbiased and the answer wouldn't include "a" but since i... 6

Learn About Verified Certificates