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Exercise: Incorporating Observations in Graphical Models

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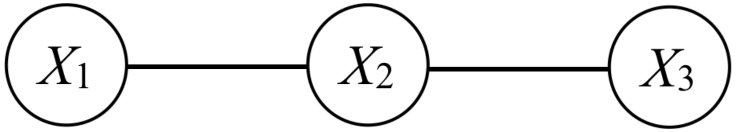
Exercise: Incorporating Observations in Graphical Models

7/7 points (graded)

Let's figure how incorporating observations works.

Recall the 3-node Markov chain we had earlier $X_1 \leftrightarrow X_2 \leftrightarrow X_3$. The graph was:



We have the factorization:

$$p_{X_1,X_2,X_3}(x_1,x_2,x_3) = \frac{1}{Z}\phi_1(x_1)\phi_2(x_2)\phi_3(x_3)\psi_{1,2}(x_1,x_2)\psi_{2,3}(x_2,x_3).$$

Suppose we condition on $X_2 = v$ for some fixed value v in the alphabet of X_2. We want to figure out the distribution $p_{X_1,X_3|X_2}(\cdot,\cdot \mid v)$. By the definition of conditional probability, Exercises due Oct 27, 2016 at 02:30 IST

Week 6: Special Case: Marginalization in Hidden Markov Models

Exercises due Oct 27, 2016 at 02:30 IST

Week 6: Homework 5

Homework due Oct 27, 2016 at 02:30 IST

Weeks 6 and 7: Mini-project on Robot Localization (to be posted)

$$\begin{aligned} p_{X_1,X_3|X_2}(x_1,x_3 \mid v) &= \frac{p_{X_1,X_2,X_3}(x_1,v,x_3)}{p_{X_2}(v)} \\ &= \frac{\frac{1}{Z}\phi_1(x_1)\phi_2(v)\phi_3(x_3)\psi_{1,2}(x_1,v)\psi_{2,3}(v,x_3)}{p_{X_2}(v)} \\ &= \frac{1}{Z\frac{p_{X_2}(v)}{\phi_2(v)}}\phi_1(x_1)\psi_{1,2}(x_1,v)\phi_3(x_3)\psi_{2,3}(v,x_3). \end{aligned}$$

Let's define the following:

$$\begin{aligned} Z' &\triangleq Z\frac{p_{X_2}(v)}{\phi_2(v)}, \\ \phi_1'(x_1) &\triangleq \phi_1(x_1)\psi_{1,2}(x_1,v), \\ \phi_3'(x_3) &\triangleq \phi_3(x_3)\psi_{2,3}(v,x_3). \end{aligned}$$

Notice that

$$p_{X_1,X_3|X_2}(x_1,x_3 \mid v) = \frac{1}{Z'}\phi_1'(x_1)\phi_3'(x_3)$$

corresponds to a new graphical model!

- In this new graphical model, how many nodes are there?

2

✓ **Answer:** 2

- In this new graphical model, how many edges are there? (Specify the minimum possible given the structure of the distribution.)

0 ✓ **Answer:** 0

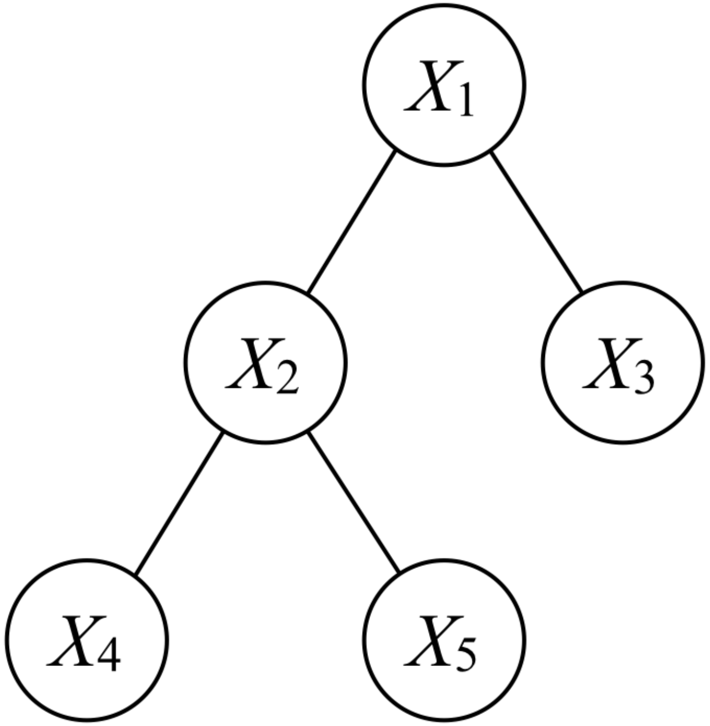
- In this new graphical model, is the graph the same as if you remove the node corresponding to X_2 in the original graph (and delete any edges that it participates in)?

● Yes ✓

○ No

We can always view conditioning (and thus incorporating observations) as fixing the value(s) of whichever random variable(s) we observe, which always has the effect that you just saw: the pairwise potentials involving the observed random variables become part of new node potentials involving the unobserved (also called *hidden* or *latent*) random variables. Since these pairwise potentials corresponded to edges that were present, and now they have become part of node potentials instead, the effect on the graph is that we deleted the nodes that we made observations for.

Let's consider a graphical model where the graph is the graph we've seen before:



If we condition on X_2, we get a new graph. In this new graph:

- Is there a path from the node for X_1 to the node for X_4?

○ Yes

● No ✓

- Is there a path from the node for X_1 to the node for X_3?

● Yes ✓

○ No

- Conditioned on X_2, what can you say about X_1 and X_4?

● X_1 and X_4 are conditionally independent given X_2 ✓

○ X_1 and X_4 are not conditionally independent given X_2

○ We cannot conclude whether or not X_1 and X_4 are conditionally independent given X_2.

- Conditioned on X_2, what can you say about X_1 and X_3?

○ X_1 and X_3 are conditionally independent given X_2

○ X_1 and X_3 are not conditionally independent given X_2

● We cannot conclude whether or not X_1 and X_3 are conditionally independent given X_2. ✓

The graph for a graphical model enables us to easily read off conditional independence statements (which are also called *conditional independencies*)!

Solution:

Notice that

$$p_{X_1,X_3|X_2}(x_1,x_3 \mid v) = \frac{1}{Z'}\phi'_1(x_1)\phi'_3(x_3)$$

corresponds to a new graphical model!

- In this new graphical model, how many nodes are there?

  Solution: The distribution is over 2 random variables X_1 and X_3 so there are $\boxed{2}$ nodes.

- In this new graphical model, how many edges are there? (Specify the minimum possible given the structure of the distribution.)

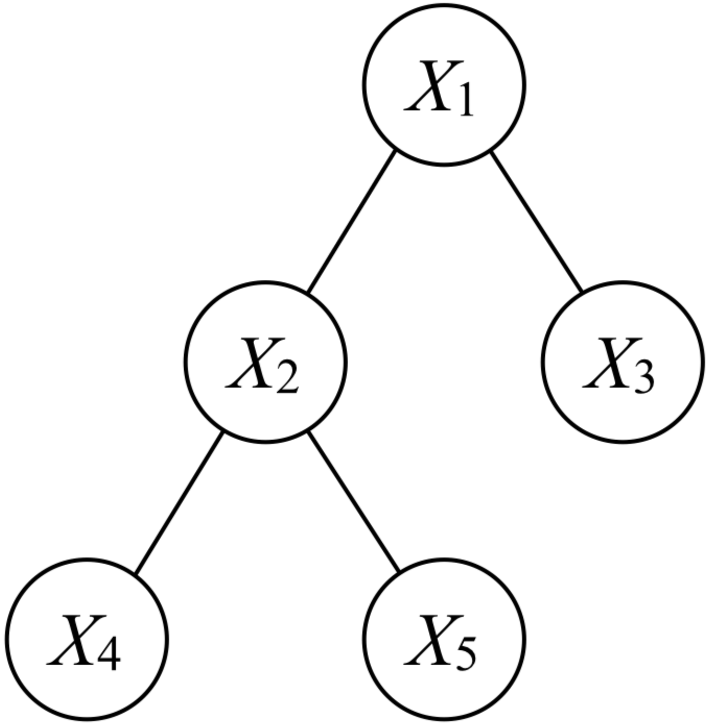
  Solution: There are no pairwise factors so we can get away with $\boxed{0}$ edges.

- In this new graphical model, is the graph the same as if you remove the node corresponding to X_2 in the original graph (and delete any edges that it participates in)?

Solution: $\boxed{\text{Yes}}$. The new graph is just X_1 and X_3 as two isolated circles.

We can always view conditioning (and thus incorporating observations) as fixing the value(s) of whichever random variable(s) we observe, which always has the effect that you just saw: the pairwise potentials involving the observed random variables become part of new node potentials involving the unobserved (also called *hidden* or *latent*) random variables. Since these pairwise potentials corresponded to edges that were present, and now they have become part of node potentials instead, the effect on the graph is that we deleted the nodes that we made observations for.

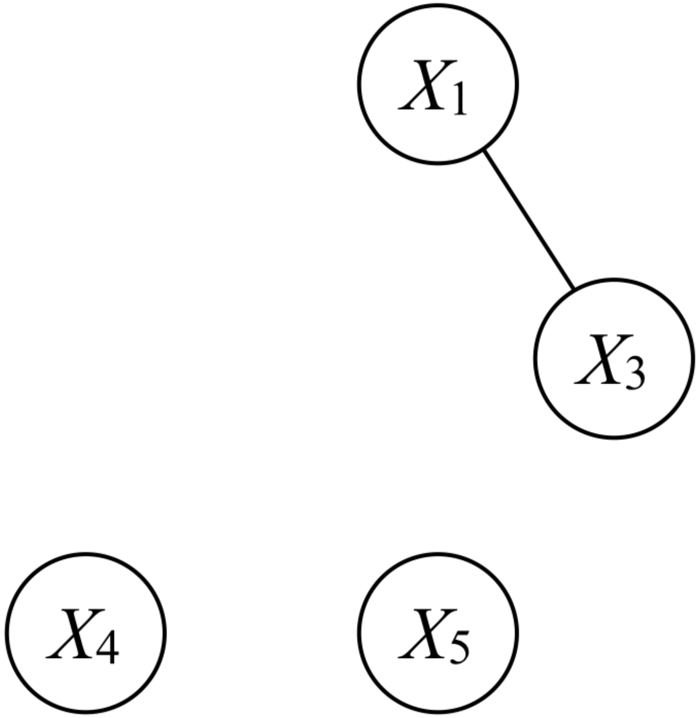
Let's consider a graphical model where the graph is the graph we've seen before:



If we condition on X_2, we get a new graph. In this new graph:

- Is there a path from the node for X_1 to the node for X_4?

  Solution: First off this new graph looks like:



$\boxed{\text{No}}$, there is no path from X_1 to X_4.

- Is there a path from the node for X_1 to the node for X_3?

  Solution: $\boxed{\text{Yes}}$, there is no path from X_1 to X_3.

- Conditioned on X_2, what can you say about X_1 and X_4?

  Solution: Conditioned on X_2, there is no path from X_1 to X_4 in the new graph so $\boxed{X_1 \perp X_4 \mid X_2}$.

- Conditioned on X_2, what can you say about X_1 and X_3?

  Solution: Conditioned on X_2, there is a path from X_1 to X_3 in the new graph. Without additional assumptions, we do not know whether X_1 and X_3 are conditionally independent or not given X_2. For example, it could have been that X_1 and X_3 were independent to begin with in which case they would remain independent.

Submit

You have used 2 of 5 attempts

✓ Correct (7/7 points)

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