3. Review the exponential response formula

Recall that the exponential response formula gives us a quick method for finding the particular solution to any linear, constant coefficient, differential equations whose input can be expressed in terms of an exponential function.

The exponential response formula (ERF): Let P be a polynomial with real, constant coefficients, $D = \frac{d}{dt}$ a differential operator, and r a (real or complex) number. If $P(r) \neq 0$, then a particular solution to the inhomogeneous differential equation

$$P(D)\,y = e^{rt} \qquad \text{is given by} \qquad y_p = \frac{e^{rt}}{P(r)}.$$

Caveat

Caveat: If $P(r) = P'(r) = P''(r) = \ldots = P^{(k-1)}(r) = 0$, but $P^{(k)}(r) \neq 0$, then a particular solution to $P(D)\,y = e^{rt}$ is given by

$$y_p = \frac{t^k e^{rt}}{P^{(k)}(r)}.$$

Hide

Sinusoidal input:

$P(D)\,x = \cos(\omega t)$	is the real part of	$P(D)\,z = e^{i\omega t}$;
$P(D)\,x = \sin(\omega t)$	is the imaginary part of	$P(D)\,z = e^{i\omega t}$.

Therefore

- a particular solution to $P(D)\,x = \cos(\omega t)$ is given by $x_p = \mathrm{Re}\left[\frac{e^{i\omega t}}{P(i\omega)}\right]$;
- a particular solution to $P(D)\,x = \sin(\omega t)$ is given by $x_p = \mathrm{Im}\left[\frac{e^{i\omega t}}{P(i\omega)}\right]$.

Review ERF

1/1 point (graded)
Use ERF to find a particular solution to the differential equation

$$(D^3 + D + 3)\,x = \cos 2t.$$

$x_p =$ (cos(2*t)-2*sin(2*t))/15 ✔ **Answer:** cos(2*t)/15-2*sin(2*t)/15

$\frac{\cos(2\cdot t) - 2\cdot\sin(2\cdot t)}{15}$

FORMULA INPUT HELP

Solution:

A particular solution is given by finding the real part of

$$z_p = \frac{e^{i2t}}{(i2)^3 + (i2) + 3} = \frac{e^{i2t}}{3 - i6}.$$

Therefore

$$x_p = \mathrm{Re}\left[\frac{e^{i2t}}{3-i6}\right] = \mathrm{Re}\left[\frac{(\cos(2t) + i\sin(2t))(3+i6)}{45}\right] = \frac{\cos 2t}{15} - \frac{2\sin 2t}{15}.$$

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Answers are displayed within the problem

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{Staff} exponential response formula 6

? Linear independence 1

What would we do if Particular solution become our general solution ?

I am unable to enter my solution into the question.

I am unable to enter my solution into the question. I've tried multiple forms for the answer, but in every case, I get the message: "Sorry, couldn't parse formula." I had no prob... 6

[Staff] The caveat has a caveat... 3

Will it accept both radians and degrees? Also, amplitude-phase form vs other form?
Radians and degrees both okay to use? Also, does it care whether we express answer in the form that involves only one sinusoidal function (the amplitude-phase form) or in t... 3

How do you type Real or Imaginary in the assignment box
As title 5

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