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## Outer Measure of the complement of a Vitali Set in [0,1] equal to 1

Asked 7 years, 9 months ago Active 1 month ago Viewed 2k times



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I am trying to prove the first part of exercise 33, ch. 1 in Stein and Shakarchi (*Real Analysis*). I am running into some difficulties following the hint though. Here is the problem (note, N is a Vitali set constructed in [0,1]):



Show that the set [0,1]-N has outer measure  $m_*(N^c)=1$ . [Hint: argue by contradiction, and pick a measurable set such that  $N^c\subset U\subset [0,1]$  and  $m_*(U)\leq 1-\epsilon$ .



A)

I know that both N and its complement are not measurable, so neither are countable. I know that measurable subsets of non-measurable sets have measure o. I am not sure how to proceed given the proof though. A point to note: the book does not work with the inner

measure at all, and even though I have used the inner measure in a previous course, I do not think I am allowed to for this proof.

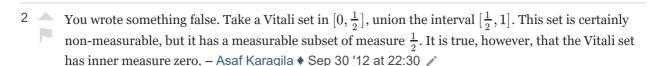
real-analysis

measure-theory

edited Sep 30 '12 at 23:08 user940

asked Sep 30 '12 at 22:16





One way to see that Vitali set have inner measure zero is Lemma 2 in <a href="here">here</a> – leo Oct 1 '12 at 22:56

## 2 Answers

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Recall the key property of the Vitali set N is that the sets  $N \oplus q_i$ , where  $\oplus$  is addition mod 1 and  $\mathbb{Q} = \{q_1, q_2, \ldots\}$ , are disjoint. (Not sure if this is the same notation that S&S use.)



Now, consider the sets  $U^c \oplus q_i$ . Show that they are disjoint measurable sets and each has the same measure as  $U^c$ , which is greater than  $\epsilon$ . Therefore, what can you say about the measure of  $V = \bigcup_i U^c \oplus q_i$ ? This should give you a contradiction.



answered Jan 8 at 6:18

Nate Eldredge

79.7k 11 100 201



**Hint:** By Theorem 3.2,  $m_*(I) = m_*(U) + m_*(U^c)$ .

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