1. Parametric curves

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Problem Set B due Oct 5, 2021 20:30 IST Completed

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Find the trajectory of the oar

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A single scull (row boat) advances along the y-axis in the xy-plane at a constant speed U. It is propelled by a pair of blades (oars) of radius R that sweep out a total angle $2\pi/3$ in the xy-plane. The angular position of the blade relative to the perpendicular to the boat is given by $\theta = \pi/3 \sin(at)$ where a is a constant. At time $t = 0$, the center of the boat is at the origin $(0, 0)$, and the tip of its right blade (the point P) is at $(R, 0)$.

Find the parametric equations for the trajectory of the blade tip P.

$x(t) =$ R*cos(pi/3*sin(a*t)) ✔ **Answer:** R*cos(pi/3*sin(a*t))

$y(t) =$ U*t+R*sin(pi/3*sin(a*t)) ✔ **Answer:** U*t + R*sin(pi/3*sin(a*t))

? INPUT HELP

Solution:

To determine the position of the oar tip as the boat moves, we parameterize the boat motion and the oar motion relative to the boat motion separately, and then add them together.

The boat moves along the y-axis with constant velocity U. Thus the parametric equations for the boat are

$$\begin{pmatrix} b_1(t) \\ b_2(t) \end{pmatrix} = \begin{pmatrix} 0 \\ Ut \end{pmatrix} \tag{6.278}$$

The motion of the oar relative to the boat is given by the arc

$$\begin{pmatrix} P_1(t) \\ P_2(t) \end{pmatrix} = \begin{pmatrix} R\cos(\theta(t)) \\ R\sin(\theta(t)) \end{pmatrix} = \begin{pmatrix} R\cos\left(\frac{\pi}{3}\sin(at)\right) \\ R\sin\left(\frac{\pi}{3}\sin(at)\right) \end{pmatrix} \tag{6.279}$$

Adding these together, we get a parametric equation for the position of the tip of the blade of the oar.

$$\begin{pmatrix} x(t) \\ y(t) \end{pmatrix} = \begin{pmatrix} b_1(t) \\ b_2(t) \end{pmatrix} + \begin{pmatrix} P_1(t) \\ P_2(t) \end{pmatrix} \tag{6.280}$$

$$= \begin{pmatrix} R\cos\left(\frac{\pi}{3}\sin(at)\right) \\ Ut + R\sin\left(\frac{\pi}{3}\sin(at)\right) \end{pmatrix} \tag{6.281}$$

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Answers are displayed within the problem

Find the velocity of the oar

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A single scull (row boat) advances along the y-axis in the xy-plane at a constant speed U. It is propelled by a pair of blades (oars) of radius R that sweep out a total angle $2\pi/3$ in the xy-plane. The angular position of the blade relative to the perpendicular to the boat is given by $\theta = \pi/3 \sin(at)$ where a is a constant. At time $t = 0$, the center of the boat is at the origin $(0, 0)$, and the tip of its right blade (the point P) is at $(R, 0)$.

Find the velocity $\vec{v} = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$ of the blade tip.

$v_1(t) =$ -pi/3*a*R*sin(pi/3*sin(a*t))*c ✔ **Answer:** -R*pi*a/3*sin(pi/3*sin(a*t))*cos(a*t)

$v_2(t) =$ U+pi/3*a*R*cos(pi/3*sin(a*t)) ✔ **Answer:** U + R*pi*a/3*cos(pi/3*sin(a*t))*cos(a*t)

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Solution:

The velocity is obtained by taking the time derivative of $x(t)$ and $y(t)$.

$$\begin{pmatrix} x'(t) \\ y'(t) \end{pmatrix} = \begin{pmatrix} -R\frac{a\pi}{3}\cos(at)\sin\left(\frac{\pi}{3}\sin(at)\right) \\ U + R\frac{a\pi}{3}\cos(at)\cos\left(\frac{\pi}{3}\sin(at)\right) \end{pmatrix} \quad (6.282)$$

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Where is speed maximized and in which direction?

2/2 points (graded)

A single scull (row boat) advances along the y-axis in the xy-plane at a constant speed U. It is propelled by a pair of blades (oars) of radius R that sweep out a total angle $2\pi/3$ in the xy-plane. The angular position of the blade relative to the perpendicular to the boat is given by $\theta = \pi/3 \sin(at)$ where a is a constant. At time $t = 0$, the center of the boat is at the origin $(0, 0)$, and the tip of its right blade (the point P) is at $(R, 0)$.

What is the maximum speed of the blade tip, and what is the direction of the velocity at the corresponding time?

Max speed: U+R*a*pi/3 ✔ **Answer:** U+R*a*pi/3

$U + \frac{R \cdot a \cdot \pi}{3}$

Direction (unit vector): [0,1] ✔ **Answer:** [0,1]

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Solution:

In the previous problem, we computed the velocity. The speed is the magnitude of the velocity vector, which is

$$\left(\left(R\frac{a\pi}{3}\right)^2 \cos^2(at)\sin^2\left(\frac{\pi}{3}\sin(at)\right) + U^2\right. \quad (6.283)$$

$$\left. +2UR\frac{a\pi}{3}\cos(at)\cos\left(\frac{\pi}{3}\sin(at)\right) + \left(R\frac{a\pi}{3}\right)^2 \cos^2(at)\cos^2\left(\frac{\pi}{3}\sin(at)\right)\right)^{1/2} \quad (6.284)$$

$$= \sqrt{U^2 + 2UR\frac{a\pi}{3}\cos(at)\cos\left(\frac{\pi}{3}\sin(at)\right) + \left(R\frac{a\pi}{3}\right)^2 \cos^2(at).}$$

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Note that the velocity is as large as possible when $\cos(at) = 1$ and $\cos\left(\frac{\pi}{3}\sin(at)\right) = 1$. Note that this happens when

$$at = 2k\pi \quad k \text{ any integer} \tag{6.286}$$

In particular, this holds at $t = 0$, at which time the speed is

$$\sqrt{U^2 + 2UR\frac{a\pi}{3} + \left(R\frac{a\pi}{3}\right)^2} \tag{6.287}$$

$$= \sqrt{\left(U + R\frac{a\pi}{3}\right)^2} \tag{6.288}$$

$$= U + R\frac{a\pi}{3}. \tag{6.289}$$

When $t = \pi/a$ the speed is not maximized because we get a negative sign on the middle term

$$\sqrt{U^2 - 2UR\frac{a\pi}{3} + \left(R\frac{a\pi}{3}\right)^2} \tag{6.290}$$

To find the direction, we plug $t = 0$ into the velocity vector to get

$$\begin{pmatrix} x'(0) \\ y'(0) \end{pmatrix} = \begin{pmatrix} -R\frac{a\pi}{3}\cos(0)\sin(0) \\ U + R\frac{a\pi}{3}\cos(0)\cos(0) \end{pmatrix} \tag{6.291}$$

$$= \begin{pmatrix} 0 \\ U + R\frac{a\pi}{3} \end{pmatrix} \tag{6.292}$$

Thus the direction is in the positive y-direction.

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Answers are displayed within the problem

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Getting a wrong y(t) any clue will be appreciated

discussion posted about 10 hours ago by **sandipan_dey**

I am getting x(t) accepted by the grader but not y(t), i thought it would be pretty straight-forward (although I am not using the fact that the total angle $2\pi/3$ is swept by the oars). Any clue will be appreciated, thanks in advance.

This post is visible to everyone.

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1 response

jfrench (Staff)
about 8 hours ago

Remember that the boat is also moving with constant velocity in the y direction. :) You have

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respect to the boat correct!

...

Thank you very much for the hint, I forgot to take care of it

posted 40 minutes ago by **sandipan_dey**

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