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Expectation of the min of two independent random variables?

How do you compute the minimum of two independent random variables in the general case ?

In the particular case there would be two uniforms variables with difference support, how should one proceed ?

EDIT: specified that they were independent and that the the uniform variables do not have obligatory the same support range.

(probability) (probability-distributions) (random-variables)

edited Feb 21 '13 at 18:35

asked Feb 19 '13 at 17:16
BlueTrin
232 1 2 10

Do you want the *uncorrelated* case (as in the title), the *general* case (as in the body), or the *independent* case (as in the solution below)? – Byron Schmuland Feb 19 '13 at 17:34

1 Knowing their distributions and that they're independent would enable you find find the distribution of the minimum, as in the answer below, but knowing only that they're uncorrelated does not. But I'm not sure whether it would allow you to find the expected value of the minimum. – Michael Hardy Feb 19 '13 at 17:45

3 Answers

$F_{X,Y}(x,y)$ be the joint cumulative distribution function. Then, for $Z = \min(X,Y)$

$$\begin{aligned} 1 - F_Z(z) &= \mathbb{P}\left(\min(X,Y) > z\right) = \mathbb{P}\left(X > z, Y > z\right) \\ &= 1 - \mathbb{P}\left(X \leqslant z\right) - \mathbb{P}\left(Y \leqslant z\right) + \mathbb{P}\left(X \leqslant z, Y \leqslant z\right) \end{aligned}$$

where the inclusion exclusion principle was applied to get the last equality. Thus

$$F_Z(z) = \mathbb{P}\left(X \leqslant z\right) + \mathbb{P}\left(Y \leqslant z\right) - \mathbb{P}\left(X \leqslant z, Y \leqslant z\right) = F_X(z) + F_Y(z) - F_{X,Y}(z,z)$$

Notice that we have not used the information about the correlation of X and Y.

Let's consider an example. Let $F_{X,Y}(x,y) = F_X(x)F_Y(y)\left(1 + \alpha(1-F_X(x))(1-F_Y(y))\right)$, known as Farlie-Gumbel-Morgenstern copula, and let $F_X(x)$ and $F_Y(y)$ be cdfs of uniform random variables on the unit interval. Then, for $0 < z < 1$

$$F_Z(z) = 2z - z^2\left(1 + \alpha(1-z)^2\right)$$

leading to

$$\mathbb{E}\left(Z\right) = \int_0^1 z F_Z^\prime(z) \mathrm{d}z = \frac{1}{3}\left(1 + \frac{\alpha}{10}\right)$$

edited Jun 2 '15 at 20:06

answered Feb 19 '13 at 17:49
Sasha
53.7k 4 76 147

You need some information about the relationship between the random variables. I will assume here that they are independent, otherwise, please modify your question wit this info.

Let $M = \min\{A, B\}$. Then

$\mathbb{P}[M > x] = \mathbb{P}[A > x, B > x] = \mathbb{P}[A > x]\mathbb{P}[B > x]$, where the last step is by independence. Hence,

$$\begin{aligned} F_M(x) &= \mathbb{P}[M \leq x] = 1 - \mathbb{P}[M > x] \\ &= 1 - (1 - F_A(x))(1 - F_B(x)) \\ &= F_A(x) + F_B(x) - F_A(x)F_B(x). \end{aligned}$$

If you need pdf instead of CDF, take derivatives to find

$f_M(x) = (1 - f_A(x))(1 - F_B(x)) + (1 - F_A(x))(1 - f_B(x))$.

In case of standard uniform random variables, $F_U(x) = x$ for all $x \in (0, 1)$, 0 to the left and 1 to the right. So $F_M(x) = 1 - (1 - x)^2$ for all x in $(0, 1)$ and 0 to the left and 1 to the right, and you can take derivatives to find that $f_M(x) = 2(1 - x)$ for $x \in (0, 1)$ and 0 otherwise.

edited Feb 19 '13 at 17:34 answered Feb 19 '13 at 17:22
gt6989b
14.4k 2 13 30

2 But the question said "uncorrelated" rather than "independent". "Uncorrelated" is weaker than "independent". – Michael Hardy Feb 19 '13 at 17:43

Let: $U = \min(X, Y)$, where $\min(X, Y) \leq z$.

$Pr(\min(X, Y) > z) = Pr((X > z) \cap (Y > z))$.

$Pr(U > z) = Pr(X > z) * Pr(Y > z)$.

$Pr(U \geq z) = (1 - Fx(z)) * (1 - Fy(z))$.

$Fu(z) = 1 - (1 - Fx(z)) * (1 - Fy(z))$.

Thus,

$F_{\min}(x, y) = Fx(z) + Fy(z) - Fx(z) * Fy(z)$.

edited Oct 3 '13 at 13:11 answered Sep 24 '13 at 16:14
rschwieb sky-light
63.9k 10 58 146 231 1 3 14

Yes, I mean *Pr(min(X,Y)>z)=Pr(X>z∩Y>z)* as I know in probability theory this is correct, and its basics of probability theory. – sky-light Oct 3 '13 at 13:07

OK, I made that edit... but you should feel free to correct such things if you can. – rschwieb Oct 3 '13 at 13:11

You can pick up a lot by just looking at the edit history, but I think there is also a help page here on using TeX. Let me know if you need help finding it. LaTeX is very useful. – rschwieb Oct 4 '13 at 11:32