Unit 4: Quiz

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Unit 4: Quiz

The problems in the quiz will be automatically graded.

You are given only **ONE** attempt for each problem. You will be able to see the solution after you submit your answers.

Problem 1

6/6 points (graded)

1a. Consider 5 fish in a bowl: 3 of them are red, and 1 is green, and 1 is blue. Select the fish one at a time, without replacement, until the bowl is empty. Let $X = 1$ if all of the red fish are selected, before the green fish is selected; and $X = 0$ otherwise. Find $\mathbb{E}(X)$. (Hint: You already found $p_X(1)$ on Monday, and $p_X(0)$ is just the complementary probability.)

0.25 ✔ **Answer:** 0.25

1b. Suppose that 60% of people in Chicago are fans of da Bears. Assume that the fans' preferences are independent. We interview 3 fans, and we let X denote the number of fans of da Bears. Find $\mathbb{E}(X)$.

L4.4: Expected Values of Functions of RVs

L4.5: Variance

L4.6: Practice

L4.7: Quiz

Unit 5: Models of Discrete Random Variables I

Unit 6: Models of Discrete Random Variables II

$p_X(0) =$ 0.064 ✔ **Answer:** 0.064

$p_X(1) =$ 0.288 ✔ **Answer:** 0.288

$p_X(2) =$ 0.432 ✔ **Answer:** 0.432

$p_X(3) =$ 0.216 ✔ **Answer:** 0.216

$p_X(x) = 0$ otherwise.

$\mathbb{E}(X) =$ 1.8 ✔ **Answer:** 1.8

Explanation

1a. We have $p_X(1) = 1/4$ and so $p_X(0) = 3/4$. Thus, we have
$\mathbb{E}(X) = (1)(1/4) + (0)(3/4) = 1/4$.
1b. The probability mass function of X is $p_X(0) = .4^3 = 0.064$;
$p_X(1) = (3)(.4^2)(.6) = 0.288$;
$p_X(2) = (3)(.4)(.6^2) = 0.432$;
$p_X(3) = .6^3 = 0.216$;
$p_X(x) = 0$ otherwise.
So we get $\mathbb{E}(X) = (0)(0.064) + (1)(0.288) + (2)(0.432) + (3)(0.216) = 1.8$.

Submit You have used 1 of 1 attempt

Problem 2

4/5 points (graded)

2. Suppose that a drawer contains 8 marbles: 2 are red, 2 are blue, 2 are green, and 2 are yellow. The marbles are rolling around in a drawer, so that all possibilities are equally likely when they are drawn. Alice chooses 2 marbles without replacement, and then Bob also chooses 2 marbles without replacement. Let Y denote the number of red marbles that Alice gets, and let X denote the number of red marbles that Bob gets.

2a. Find probability mass function for Y, i.e., for the number of red marbles that Alice gets, i.e., find $p_Y(y)$ for $y = 0, 1, 2$.

$p_Y(0) =$ 15/28 ✔ **Answer:** 0.5357

$p_Y(1) =$ 3/7 ✔ **Answer:** 0.4286

$p_Y(2) =$ 1/28 ✔ **Answer:** 0.0357

2b. Find $\mathbb{E}(Y)$.

0.5 ✔ **Answer:** 0.5

2c. Is $\mathbb{E}(X)$ the same as $\mathbb{E}(Y)$? I.e., is the expected number of marbles that Bob gets the same or different? Why or why not?

- ○ Yes
- ◉ No ✖

Explanation

2a. We compute the following: $p_Y(0) = \binom{2}{0}\binom{6}{2}/\binom{8}{2} = 15/28$, $p_Y(1) = \binom{2}{1}\binom{6}{1}/\binom{8}{2} = 3/7$, and $p_Y(2) = \binom{2}{2}\binom{6}{0}/\binom{8}{2} = 1/28$.
2b. We have $\mathbb{E}(Y) = (0)(15/28) + (1)(3/7) + (2)(1/28) = 1/2$.
2c. Yes, the probability mass functions of X and Y are the same, so their expected values are the same too.

Submit You have used 1 of 1 attempt

Problem 3

4/4 points (graded)
3. Consider two six sided dice. One die has 2 red, 2 green, and 2 blue sides. The other die has 3 red sides and 3 blue sides. Roll both dice, and let X denote the number of red sides that appear.

3a. Find $p_X(x)$ for $x = 0, 1, 2$.

$p_X(0) =$ 1/3 ✔ **Answer:** 0.3333

$p_X(1) =$ 1/2 ✔ **Answer:** 0.5

$p_X(2) =$ 1/6 ✔ **Answer:** 0.1667

3b. Find $\mathbb{E}(X)$.

0.8333333 ✔ **Answer:** 0.8333

Explanation

3a. We compute: $p_X(0) = (4/6)(3/6) = 1/3$,
$p_X(1) = (2/6)(3/6) + (4/6)(3/6) = 1/2$,
$p_X(2) = (2/6)(3/6) = 1/6$.
3b. The expected value of X is: $\mathbb{E}(X) = (0)(1/3) + (1)(1/2) + (2)(1/6) = 5/6$.

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Problem 4

5/5 points (graded)

4. Consider a collection of 9 bears. There is a family of red bears consisting of one father bear, one mother bear, and one baby bear. There is a similar green bear family, and a similar blue bear family. We draw 5 consecutive times from this collection *without replacement* (i.e., not returning the bear after each draw). We keep track (in order) of the kind of bears that we get. Let X denote the number of red bears selected.

4a. Find $p_X(x)$ for $x = 0, 1, 2, 3$.

$p_X(0) =$ 6/126 ✔ **Answer:** 0.04762

$p_X(1) =$ 45/126 ✔ **Answer:** 0.3571

$p_X(2) =$ 60/126 ✔ **Answer:** 0.4762

$p_X(3) =$ 15/126 ✔ **Answer:** 0.1190

4b. What is $\mathbb{E}(X)$?

1.6666667 ✔ **Answer:** 1.6667

Explanation

4a.
$p_X(0) = 1/21$, $p_X(1) = 5/14$,
$p_X(2) = 10/21$, $p_X(3) = 5/42$.
4b. We have $\mathbb{E}(X) = (0)(1/21) + (1)(5/14) + (2)(10/21) + (3)(5/42) = 5/3$.

Submit You have used 1 of 1 attempt

✔ Correct (5/5 points)

Problem 5

1/1 point (graded)

5. Suppose that 60% of people in Chicago are fans of da Bears. Assume that the fans' preferences are independent. We interview 3 fans, and we let X denote the number of fans of da Bears. For $i = 1, 2, 3$, let $X_i = 1$ if the ith person is a fan of da Bears, and let $X_i = 0$ otherwise. So we have $X = X_1 + X_2 + X_3$. Find $\mathbb{E}(X_i)$ for $i = 1, 2, 3$ first, and then find $\mathbb{E}(X)$.

$\mathbb{E}(X) =$ 1.8 ✔ **Answer:** 1.8

Explanation

5. Each X_i has $\mathbb{E}(X_i) = 0.6$, so
$\mathbb{E}(X) = \mathbb{E}(X_1 + X_2 + X_3)$
$= \mathbb{E}(X_1) + \mathbb{E}(X_2) + \mathbb{E}(X_3) = 0.6 + 0.6 + 0.6 = 1.8$.

Submit You have used 1 of 1 attempt

Problem 6

2/2 points (graded)

6. Suppose that a drawer contains 8 marbles: 2 are red, 2 are blue, 2 are green, and 2 are yellow. The marbles are rolling around in a drawer, so that all possibilities are equally likely when they are drawn. Alice chooses 2 marbles without replacement, and then Bob also chooses 2 marbles without replacement. Let Y denote the number of red marbles that Alice gets, and let X denote the number of red marbles that Bob gets.

6a. For $i = 1, 2$, let $Y_i = 1$ if the ith ball that Alice selects is red, and $Y_i = 0$ otherwise. So we have $Y = Y_1 + Y_2$. Find $\mathbb{E}(Y_i)$ for $i = 1, 2$, and then find $\mathbb{E}(Y)$.

$\mathbb{E}(Y) =$ 1/2 ✔ **Answer:** 0.5

6b. Temporarily view one of the red balls as having a #1 painted on it, and the other red ball as having a #2 painted on it. For $i = 1, 2$, let $Z_i = 1$ if the ith red ball is picked by Alice (at any time, i.e., on either of her roles), and $Z_i = 0$ otherwise. So we have $Y = Z_1 + Z_2$. Find $\mathbb{E}(Z_i)$ for $i = 1, 2$, and then find $\mathbb{E}(Y)$.

$\mathbb{E}(Y) =$ 1/2 ✔ **Answer:** 0.5

Explanation

6a. Each Y_i has $\mathbb{E}(Y_i) = 1/4$, so $\mathbb{E}(Y) = \mathbb{E}(Y_1 + Y_2) = \mathbb{E}(Y_1) + \mathbb{E}(Y_2) = 1/4 + 1/4 = 1/2$.
6b. Each Z_i has $\mathbb{E}(Z_i) = 1/4$, so $\mathbb{E}(Y) = \mathbb{E}(Z_1 + Z_2) = \mathbb{E}(Z_1) + \mathbb{E}(Z_2) = 1/4 + 1/4 = 1/2$.

Submit You have used 1 of 1 attempt

✔ Correct (2/2 points)

Problem 7

1/1 point (graded)

7. Consider two six sided dice. One die has 2 red, 2 green, and 2 blue sides. The other die has 3 red sides and 3 blue sides. Roll both dice, and let X denote the number of red sides that appear. Treat the red/green/blue die as die #1, and the red/blue die as die #2. Let $X_i = 1$ if the ith die is red, or $X_i = 0$ otherwise. So we have $X = X_1 + X_2$. Find $\mathbb{E}(X_i)$ for $i = 1, 2$, and then find $\mathbb{E}(X)$.

$\mathbb{E}(X) =$ 0.8333333 ✔ **Answer:** 0.8333

Explanation

7. We have $\mathbb{E}(X_1) = 1/3$ and $\mathbb{E}(X_2) = 1/2$, so
$\mathbb{E}(X) = \mathbb{E}(X_1 + X_2) = \mathbb{E}(X_1) + \mathbb{E}(X_2) = 1/3 + 1/2 = 5/6$.

Submit You have used 1 of 1 attempt

Problem 8

2/2 points (graded)

8. Consider a collection of 9 bears. There is a family of red bears consisting of one father bear, one mother bear, and one baby bear. There is a similar green bear family, and a similar blue bear family. We draw 5 consecutive times from this collection *without replacement* (i.e., not returning the bear after each draw). We keep track (in order) of the kind of bears that we get. Let X denote the number of red bears selected.

8a. For $i = 1, 2, 3, 4, 5$, let $X_i = 1$ if the ith bear selected is red, and $X_i = 0$ otherwise. So we have $X = X_1 + X_2 + X_3 + X_4 + X_5$. Find $\mathbb{E}(X_i)$ for $i = 1, 2, 3, 4, 5$, and then find $\mathbb{E}(X)$.

$\mathbb{E}(X) =$ 1.6666667 ✔ **Answer:** 1.6667

8b. Refer to the red father bear as red bear #1, and the red mother bear as red bear #2, and the red baby bear as red bear #3. For $i = 1, 2, 3$, let $Y_i = 1$ if the ith red bear is selected (at any time, i.e., on any of the five selections), and $Y_i = 0$ otherwise. So we have $X = Y_1 + Y_2 + Y_3$. Find $\mathbb{E}(Y_i)$ for

$i = 1, 2, 3$, and then find $\mathbb{E}(X)$.

$\mathbb{E}(X) =$ 1.6666667 ✔ **Answer:** 1.6667

Explanation

8a. We have $\mathbb{E}(X_i) = 1/3$ for each i, so
$\mathbb{E}(X) = \mathbb{E}(X_1 + \cdots + X_5) = \mathbb{E}(X_1) + \cdots + \mathbb{E}(X_5)$
$= 1/3 + 1/3 + 1/3 + 1/3 + 1/3 = 5/3$.
8b. We have $\mathbb{E}(Y_i) = 5/9$ for each i, so
$\mathbb{E}(X) = \mathbb{E}(Y_1 + Y_2 + Y_3) = \mathbb{E}(Y_1) + \mathbb{E}(Y_2) + \mathbb{E}(Y_3)$
$= 5/9 + 5/9 + 5/9 = 5/3$.

Submit You have used 1 of 1 attempt

✔ Correct (2/2 points)

Problem 9

2/2 points (graded)

9. Suppose that 60% of people in Chicago are fans of da Bears. Assume that the fans' preferences are independent. We interview 3 fans, and we let X denote the number of fans of da Bears.

9a. Find $\mathbb{E}(X^2)$ using the probability mass function of X.

$\mathbb{E}(X^2) =$ 3.96 Answer: 3.96

9b. Find $\mathbb{E}(X^2)$ in a different way, namely, using the fact that $X = X_1 + X_2 + X_3$, where the X_j's are independent indicators. Expand $(X_1 + X_2 + X_3)^2$ into 9 terms, where 6 of them will behave one way, and the other 3 will behave another way. You are expected to obtain the same answer as 9a.

9c. Find $\mathrm{Var}(X)$ using your answer to $\mathbb{E}(X^2)$ and your answer to $\mathbb{E}(X)$ from Problem 1b or Problem 5.

$\mathrm{Var}(X) =$ 0.72 Answer: 0.72

9d. Find $\mathrm{Var}(X)$ in a different manner, namely, by writing $X = X_1 + X_2 + X_3$ for some independent indicators: First find $\mathrm{Var}(X_j)$ for each j, and then use the fact that the variance of the sum of independent random variables equals the sum of the variances. You are expected to obtain the same answer as 9c.

Explanation

9a. The mass of X is $P(X = j) = \binom{3}{j}(.60)^j(.40)^{3-j}$ for $0 \leq j \leq 3$,

so we get

$\mathbb{E}(X^2) = 0^2 P(X = 0) + 1^2 P(X = 1) + 2^2 P(X = 2) + 3^2 P(X = 3)$

$= 99/25 = 3.96$.

9b. Let X_1, X_2, X_3 denote (respectively) whether the 1st, 2nd, 3rd person is a fan of da Bears. Then $X = X_1 + X_2 + X_3$, so $\mathbb{E}(X^2) = \mathbb{E}((X_1 + X_2 + X_3)^2)$, which has 6 terms of the form $\mathbb{E}(X_i X_j)$ (for $i \neq j$) and 3 terms of the form $\mathbb{E}(X_j^2)$. Since X_i and X_j are independent for $i \neq j$, then $\mathbb{E}(X_i X_j) = \mathbb{E}(X_i)\mathbb{E}(X_j) = (.6)(.6) = .36$. Also, since indicators only take on values 0 or 1, then $\mathbb{E}(X_j^2) = \mathbb{E}(X_j) = .6$. Thus $\mathbb{E}(X^2) = (6)(.36) + (3)(.6) = 99/25 = 3.96$.

9c. We have $\mathrm{Var}(X) = \mathbb{E}(X^2) - (\mathbb{E}(X))^2 = 3.96 - (1.8)^2 = 0.72$.
9d. Since the X_j's are independent,
$\mathrm{Var}(X) = \mathrm{Var}(X_1 + X_2 + X_3) = \mathrm{Var}(X_1) + \mathrm{Var}(X_2) + \mathrm{Var}(X_3)$. We have
$\mathrm{Var}(X_j) = \mathbb{E}(X_j^2) - (\mathbb{E}(X_j))^2 = .6 - (.6)^2 = 0.24$, so $\mathrm{Var}(X) = 3(0.24) = 0.72$.

Submit You have used 1 of 1 attempt

Problem 10

2/2 points (graded)

10. Suppose that a drawer contains 8 marbles: 2 are red, 2 are blue, 2 are green, and 2 are yellow. The marbles are rolling around in a drawer, so that all possibilities are equally likely when they are drawn. Alice chooses 2 marbles without replacement, and then Bob also chooses 2 marbles without replacement. Let Y denote the number of red marbles that Alice gets.

10a. Find $\mathbb{E}(Y^2)$ using the probability mass function of Y.

0.5714286 ✔ **Answer:** 0.5714

10b. Find $\mathbb{E}(Y^2)$ in a different way, namely, using the fact that $Y = Y_1 + Y_2$, where the Y_j's are **dependent** indicators. Expand $(Y_1 + Y_2)^2$ into 4 terms, where 2 of them will behave one way, and the other 2 will behave another way. You are expected to obtain the same answer as 10a.

10c. Find $\mathrm{Var}(Y)$ using your answer to $\mathbb{E}(Y^2)$ and your answer to $\mathbb{E}(Y)$ from Problem 2 or Problem 6.

0.3214286 **Answer:** 0.3214

Explanation

10a. The mass of Y is $P(Y=j)=\binom{2}{j}\binom{6}{2-j}/\binom{8}{2}$ for $0 \le j \le 2$,
so we get
$\mathbb{E}(Y^2)=0^2P(Y=0)+1^2P(Y=1)+2^2P(Y=2)$
$=(0)(15/28)+(1)(3/7)+(4)(1/28)=4/7=0.5714.$
10b. We have $\mathbb{E}(Y^2)=\mathbb{E}((Y_1+Y_2)^2)=\mathbb{E}(Y_1^2)+2\mathbb{E}(Y_1Y_2)+\mathbb{E}(Y_2^2)$. We note $\mathbb{E}(Y_1^2)=\mathbb{E}(Y_1)=1/4$, and $\mathbb{E}(Y_2^2)=\mathbb{E}(Y_2)=1/4$, and $\mathbb{E}(Y_1Y_2)=(2/8)(1/7)=1/28$. Thus $\mathbb{E}(Y^2)=1/4+(2)(1/28)+1/4=4/7$.
10c. We have $\mathrm{Var}(Y)=\mathbb{E}(Y^2)-(\mathbb{E}(Y))^2=4/7-(1/2)^2=9/28=0.3214$.

Submit You have used 1 of 1 attempt

Problem 11

2/2 points (graded)

11. Consider two six sided dice. One die has 2 red, 2 green, and 2 blue sides. The other die has 3 red sides and 3 blue sides. Roll both dice, and let X denote the number of red sides that appear.

11a. Find $\mathbb{E}(X^2)$ using the probability mass function of X.

1.166667 **Answer:** 1.1667

11b. Find $\mathbb{E}(X^2)$ in a different way, namely, using the fact that $X = X_1 + X_2$, where the X_j's are independent indicators. Expand $(X_1 + X_2)^2$ into 4 terms, where 2 of them will behave one way, and the other 2 will behave another way. You are expected to obtain the same answer as 11a.

11c. Find $\mathrm{Var}(X)$ using your answer to $\mathbb{E}(X^2)$ and your answer to $\mathbb{E}(X)$ from Problem 3 or Problem 7.

0.4722222 ✔ **Answer:** 0.4722

11d. Find $\mathrm{Var}(X)$ in a different manner, namely, by writing $X = X_1 + X_2$ for some independent indicators: First find $\mathrm{Var}(X_j)$ for each j, and then use the fact that the variance of the sum of independent random variables equals the sum of the variances. You are expected to obtain the same answer as 11c.

Explanation

11a. The mass of X is $P(X = 0) = 1/3$; $P(X = 1) = 1/2$; $P(X = 2) = 1/6$.
Thus, we get
$\mathbb{E}(X^2) = 0^2 P(X = 0) + 1^2 P(X = 1) + 2^2 P(X = 2)$
$= (0)(1/3) + (1)(1/2) + (4)(1/6) = 7/6 = 1.1667$.
11b. We have $\mathbb{E}(X^2) = \mathbb{E}((X_1 + X_2)^2) = \mathbb{E}(X_1^2) + 2\mathbb{E}(X_1X_2) + \mathbb{E}(X_2^2)$. We note $\mathbb{E}(X_1^2) = \mathbb{E}(X_1) = 1/3$ and $\mathbb{E}(X_2^2) = \mathbb{E}(X_2) = 1/2$, and $\mathbb{E}(X_1X_2) = (1/3)(1/2) = 1/6$. Thus $\mathbb{E}(X^2) = 1/3 + (2)(1/3)(1/2) + 1/2 = 7/6 = 1.1667$.
11c. We have $\mathrm{Var}(X) = \mathbb{E}(X^2) - (\mathbb{E}(X))^2 = 7/6 - (5/6)^2 = 17/36 = 0.4722$.
11d. Since the X_j's are independent, $\mathrm{Var}(X) = \mathrm{Var}(X_1 + X_2) = \mathrm{Var}(X_1) + \mathrm{Var}(X_2)$. We have
$\mathrm{Var}(X_1) = \mathbb{E}(X_1^2) - (\mathbb{E}(X_1))^2 = 1/3 - (1/3)^2 = 2/9$, and
$\mathrm{Var}(X_2) = \mathbb{E}(X_2^2) - (\mathbb{E}(X_2))^2 = 1/2 - (1/2)^2 = 1/4$, so
$\mathrm{Var}(X) = 2/9 + 1/4 = 17/36 = 0.4722$.

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Problem 12

2/2 points (graded)

12. Consider a collection of 9 bears. There is a family of red bears consisting of one father bear, one mother bear, and one baby bear. There is a similar green bear family, and a similar blue bear family. We draw 5 consecutive times from this collection *without replacement* (i.e., not returning the bear after each draw). We keep track (in order) of the kind of bears that we get. Let X denote the number of red bears selected. For $i = 1, 2, 3, 4, 5$, let $X_i = 1$ if the ith bear selected is red, and $X_i = 0$ otherwise. Refer to the red father bear as red bear #1, and the red mother bear as red bear #2, and the red baby bear as red bear #3. For $i = 1, 2, 3$, let $Y_i = 1$ if the ith red bear is selected (at any time, i.e., on any of the five selections), and $Y_i = 0$ otherwise.

12a. Find $\mathbb{E}(X^2)$ using the probability mass function of X.

3.333333 ✔ **Answer:** 3.3333

12b. Find $\mathbb{E}(X^2)$ in a different way, namely, using the fact that $X = X_1 + \cdots + X_5$ (the X_j's are **dependent** indicators). Expand $(X_1 + \cdots + X_5)^2$ into 25 terms, where 20 of them will behave one way, and the other 5 will behave another way. You are expected to obtain the same answer as 12a.

12c. Find $\mathbb{E}(X^2)$ in a different way, namely, using the fact that $X = Y_1 + Y_2 + Y_3$ (the Y_j's are **dependent** indicators). Expand $(Y_1 + Y_2 + Y_3)^2$ into 9 terms, where 6 of them will behave one way, and the other 3 will behave another way. You are expected to obtain the same answer as 12a.

12d. Find $\mathrm{Var}(X)$ using your answer to $\mathbb{E}(X^2)$ and your answer to $\mathbb{E}(X)$ from Problem 4 or Problem 8.

0.5555556 ✔ **Answer:** 0.5556

Explanation

12a. The mass of X is $P(X=j)=\binom{3}{j}\binom{6}{5-j}/\binom{9}{5}$ for $0\le j\le 3$,
so we get
$\mathbb{E}(X^2)=0^2P(X=0)+1^2P(X=1)+2^2P(X=2)+3^2P(X=3)$
$=(0)(1/21)+(1)(5/14)+(4)(10/21)+(9)(5/42)=10/3=3.3333$.
12b. We have $\mathbb{E}(X^2)=\mathbb{E}((X_1+\cdots+X_5)^2)$, which has 20 terms of the form $\mathbb{E}(X_iX_j)$ (for $i\ne j$) and 5 terms of the form $\mathbb{E}(X_j^2)$. We have $\mathbb{E}(X_iX_j)=(3/9)(2/8)=1/12=0.0833$. Also, since indicators only take on values 0 or 1, then $\mathbb{E}(X_j^2)=\mathbb{E}(X_j)=3/9=1/3=0.3333$. Thus
$\mathbb{E}(X^2)=(20)(1/12)+(5)(1/3)=(20)(0.0833)+(5)(0.3333)=10/3=3.3333$.
12c. We have $\mathbb{E}(X^2)=\mathbb{E}((Y_1+Y_2+Y_3)^2)=3\mathbb{E}(Y_1^2)+6\mathbb{E}(Y_1Y_2)$. We note
$\mathbb{E}(Y_1^2)=\mathbb{E}(Y_1)=5/9=0.5556$, and $\mathbb{E}(Y_1Y_2)=(5/9)(4/8)=5/18=0.2778$. Thus
$\mathbb{E}(X^2)=(3)(5/9)+(6)(5/18)=(3)(0.5556)+(6)(0.2778)=10/3=3.3333$.
12d. We have $\mathrm{Var}(X)=\mathbb{E}(X^2)-(\mathbb{E}(X))^2=10/3-(5/3)^2=5/9=0.5556$.

Submit You have used 1 of 1 attempt

✔ Correct (2/2 points)