9. Eigenvalues, Eigenvectors and Determinants(Optional)

Eigenvalues and Eigenvectors of a matrix (Optional)

0 points possible (ungraded)

Let $\mathbf{A} = \begin{pmatrix} 3 & 0 \\ \frac{1}{2} & 2 \end{pmatrix}$, $\mathbf{v} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ and $\mathbf{w} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$.

$\mathbf{A}\mathbf{v} = \lambda_1 \mathbf{v}$, where $\lambda_1 =$

3

✔ **Answer:** 3 .

$\mathbf{A}\mathbf{w} = \lambda_2 \mathbf{w}$, where $\lambda_2 =$

2

✔ **Answer:** 2 .

Therefore, $\mathbf{v}$ is an eigenvector of $\mathbf{A}$ with eigenvalue λ_1, and $\mathbf{w}$ is an eigenvector of $\mathbf{A}$ with eigenvalue λ_2.

Solution:

$$\mathbf{A}\mathbf{v} = \begin{pmatrix} 3 & 0 \\ \frac{1}{2} & 2 \end{pmatrix} \begin{pmatrix} 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 6 \\ 3 \end{pmatrix} \implies \lambda_1 = 3$$

$$\mathbf{A}\mathbf{w} = \begin{pmatrix} 3 & 0 \\ \frac{1}{2} & 2 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \end{pmatrix} \implies \lambda_2 = 2$$

Submit

You have used 1 of 3 attempts

Answers are displayed within the problem

Geometric Interpretation of Eigenvalues and Eigenvectors (Optional)

0 points possible (ungraded)

Let $\mathbf{A} = \begin{pmatrix} 3 & 0 \\ \frac{1}{2} & 2 \end{pmatrix}$, $\mathbf{v} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ and $\mathbf{w} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$. Recall from the previous exercise that $\mathbf{v}$ and $\mathbf{w}$ are eigenvectors of $\mathbf{A}$.

Suppose $\mathbf{x} = \mathbf{v} + 2\mathbf{w} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$. Then $\mathbf{Ax} = s\mathbf{v} + t\mathbf{w}$, where:

$s =$ 3 ✔ **Answer:** 3

and

$t =$ 4 ✔ **Answer:** 4 .

In particular, s describes the amount that $\mathbf{A}$ stretches $\mathbf{x}$ in the direction of $\mathbf{v}$, and $\frac{t}{2}$ (note the “2" in front of $\mathbf{w}$ in $\mathbf{x}$) describes the amount that $\mathbf{A}$ stretches $\mathbf{x}$ in the direction of $\mathbf{w}$.

Solution:

We have

$$\begin{aligned} \mathbf{Ax} &= \mathbf{A}(\mathbf{v} + 2\mathbf{w}) \\ &= \mathbf{Av} + 2\mathbf{Aw} \\ &= (3\mathbf{v}) + 2(2\mathbf{w}) \\ &= 3\mathbf{v} + 4\mathbf{w}. \end{aligned}$$

From this, we get $s = 3, t = 4$.

Submit You have used 1 of 3 attempts

Answers are displayed within the problem

Determinant and Eigenvalues (optional)

0 points possible (ungraded)

Recall that the **determinant** of a matrix indicates whether it is singular. For 2×2 matrices, it has the formula

$$\det \begin{pmatrix} a & b \\ c & d \end{pmatrix} = ad - bc$$

but for larger matrices, the formula is more complicated.

What is the determinant of the matrix $\mathbf{A} = \begin{pmatrix} 3 & 0 \\ \frac{1}{2} & 2 \end{pmatrix}$?

6

✔ **Answer:** 6

On the other hand, what is the product of the eigenvalues λ_1, λ_2 of $\mathbf{A}$? (We already computed this in the previous exercises.)

6

Answer: 6

Solution:

Plugging into the formula directly gives $3 \cdot 2 - 0 \cdot \frac{1}{2} = 6$. On the other hand, the eigenvalues are $\lambda_1 = 3$, $\lambda_2 = 2$, so the product is 6. This is not a coincidence; for general $n \times n$ matrices, the **product of the eigenvalues is always equal to the determinant**.

Submit

You have used 1 of 3 attempts

Answers are displayed within the problem

Trace and Eigenvalues

0 points possible (ungraded)

Recall that the **trace** of a matrix is the sum of the diagonal entries.

What is the trace of the matrix $\mathbf{A} = \begin{pmatrix} 3 & 0 \\ \frac{1}{2} & 2 \end{pmatrix}$?

5

Answer: 5

On the other hand, what is the sum of the eigenvalues λ_1, λ_2 of $\mathbf{A}$? (We already computed this in the previous exercises.)

5

✔ **Answer:** 5

Solution:

The diagonal sum is $3+2=5$. On the other hand, the eigenvalues are $\lambda_1 = 3$, $\lambda_2 = 2$, so the sum is 5. Just like the determinant, this is also not a coincidence. For general $n \times n$ matrices, the **sum of the eigenvalues is always equal to the trace of the matrix**.

Submit

You have used 1 of 3 attempts

Answers are displayed within the problem

Nullspace (Optional)

0 points possible (ungraded)

If a (nonzero) vector is in the nullspace of a square matrix $\mathbf{A}$, is it an eigenvector of $\mathbf{A}$?

yes

✔ **Answer:** yes

Which of the following are equivalent to the statement that 0 is an eigenvalue for a given square matrix $\mathbf{A}$? (Choose all that apply.)

- [x] There exists a nonzero solution to $\mathbf{A}\mathbf{v} = \mathbf{0}$. ✔

- [x] $\det(\mathbf{A}) = 0$ ✔
- [ ] $\det(\mathbf{A}) \neq 0$
- [ ] $\mathrm{NS}(\mathbf{A}) = \mathbf{0}$
- [x] $\mathrm{NS}(\mathbf{A}) \neq \mathbf{0}$ ✔

✔

Solution:

- If a vector $\mathbf{v}$ is in the nullspace of $\mathbf{A},$ then $\mathbf{A}\mathbf{v} = \mathbf{0} = (0)\,\mathbf{v}.$ So it is an eigenvector of $\mathbf{A}$ associated to the eigenvalue $0.$

- If 0 is an eigenvalue for a matrix $\mathbf{A},$ then by definition, there exists a nonzero solution to $\mathbf{A}\mathbf{v} = \mathbf{0};$ that is, $\mathrm{NS}(\mathbf{A}) \neq \mathbf{0},$ and this only happens if and only if $\det(\mathbf{A}) = 0.$

Submit

You have used 1 of 3 attempts

Answers are displayed within the problem

Discussion

Hide Discussion

Topic: Unit 0. Course Overview, Syllabus, Guidelines, and Homework on Prerequisites:Homework 0: Probability and Linear algebra Review / 9. Eigenvalues, Eigenvectors and Determinants(Optional)

Add a Post

Show all posts ▼ by recent activity ▼

[STAFF] Just curious
Is there any chance MITx will offer 18.06x? and/or 18.02x? Just wondering. I believe these subjects will really complement the MicroMasters Program. 7

Boosting grade
If I complete the ungraded, will this contribute to my score, if the graded answers are low? 3

? Request extension to deadline
For those of us who are finishing the last optional but important week of the probability course these deadlines are tough.Quiz 0 is substantial. Pers... 2

Sources
Could somebody recommend any good source to learn all this stuff? Preferably with detailed explanations of what is going on on each step of calcul... 7

NS(A) notation in Nullspace (Optional) 2

Learn About Verified Certificates