Problem 4

For simplicity, suppose our rating matrix is a $2x2$ matrix and we are looking for a rank-1 solution UV^T so that user and movie features U and V are both $2x1$ matrices. The observed rating matrix has only a single entry:

$$Y = \begin{bmatrix} ? & 1 \\ ? & ? \end{bmatrix} \tag{6.4}$$

In order to learn user/movie features, we minimize

$$J(U,V) = \left(\frac{1}{2} \sum_{(a,i)\in D} \left(Y_{ai} - [UV^T]_{ai}\right)^2 \right) + \lambda \left(U_1^2 + V_1^2\right) \tag{6.5}$$

where U_1 and V_1 are the first components of the vectors U and V respectively (if $U = [u_1, u_2]$, then $U_1 = u_1$), the set D is just the observed entries of the matrix Y , in this case just $(1, 2)$.

Note that the regularization we use applies only to the first coordinate of user/movie features . We will see how things get a bit tricky with this type of partial regularization.

Correction Note (July 30 21:00UTC): An earlier version does not include the clarifcation “where U_1 and V_1 are the first compoments of the vectors U and V respectively."

Correction Note (Aug 4 03:00UTC): Added an example of what U_1 means: if $U = [u_1, u_2]$, then $U_1 = u_1$).

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4. (1)

1.0/1 point (graded)

If we initialize $U = [u \quad 1]^T$, for some $u > 0$, what is the solution to the vector $V = [v_1 \quad v_2]^T$ as a function of λ and u?

(Enter V as a vector, enclosed in square brackets, and components separated by commas, e.g. type `[u,lambda+1]` if $V = [u \quad \lambda + 1]^T$.)

$V =$ [0,1/u] ✔ **Answer:** [0,1/u]

STANDARD NOTATION

Solution:

Notice that J only regularizes on the first coordinate. Thus, we only want to minimize $J(v_1, v_2) = \frac{1}{2}(1 - uv_2)^2 + \lambda(v_1^2 + u^2)$ given that $V = [v_1, v_2]^T$. We can see that J is minimized when $v_1 = 0, v_2 = \frac{1}{u}$. Therefore,

$$V = \left[0, \frac{1}{u}\right]^T. \tag{6.6}$$

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You have used 1 of 3 attempts

Answers are displayed within the problem

4. (2)

0/1 point (graded)

What is the resulting value of $J(U, V)$ as a function of λ and u?

(Type `lambda` for λ).

0

Answer: (lambda*u^2)

0

STANDARD NOTATION

Solution:

Notice that J only regularizes on the first coordinate. Therefore,

$$\begin{aligned} J(U,V) &= \frac{1}{2}\sum_{(a,i)\in D}(Y_{ai} - [UV^T]_{ai})^2 + \lambda(U_1^2 + V_1^2) \\ &= \frac{1}{2}(1 - u_1 v_2)^2 + \lambda(u_1^2 + v_1^2) \\ &= \frac{1}{2}\left(1 - u \cdot \frac{1}{u}\right)^2 + \lambda(u^2 + 0^2) \\ &= \lambda u^2 \end{aligned}$$

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Answers are displayed within the problem

4. (3)

1/1 point (graded)

If we continue to iteratively solve for U and V, what would U and V converge to?

- (selected, correct) U goes to $[0,1]$, V goes to $[0,\infty]$
- U goes to $[0,0]$, V goes to $[0,0]$

- U goes to $[0, 1]$, V goes to $[0, 0]$
- U goes to $[0, \infty]$, V goes to $[1, 0]$

Correction Note (July 29): In an earlier version, V was missing in all choices.

Solution:

The regularization error is minimized when u_1 and v_1 are 0. Over many iterations, u_1 will eventually converge to zero. The squared error term is $\frac{1}{2}(1 - u_1 v_2)^2$ is minized when $u_1 v_2 = 1$. Since u_1 converges to 0, v_2 diverges to ∞.

Submit You have used 1 of 3 attempts

Answers are displayed within the problem

4. (4)

3/3 points (graded)

Not all rating matrices Y can be reproduced by UV^T when we restrict the dimensions of U and V to be 2×1.

For each matrix below, answer “Yes" or “No" according to whether it can be reproduced by such U and V of size 2×1.

$$Y = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

- (selected) yes ✔
- no

$$Y = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

- yes
- (selected) no ✔

$$Y = \begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix}$$

- (selected) yes ✔
- no

Solution:

In order for matrix Y to be reproduced by UV^T we must have $[u_1, u_2]^T \times [v_1, v_2] = Y$. For the second matrix, this would require $u_1 \times v_1 = 1$, $u_1 \times v_2 = 0$, $u_2 \times v_1 = 0$, $u_2 \times v_2 = 1$, which has no solution.
The first matrix can be represented as $[1-1]^T \times [1-1]$ and the third can be represented as $[1-1]^T \times [11]$.

Submit You have used 1 of 3 attempts

Answers are displayed within the problem

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? [staff] 4.(2) 4
I think (you may check), that when i submitted my exam the clarification, that the 1/2 only affected the summation was not in place. My answer w...

? For 4(1) - (2) one iteration 2

[Staff] 4.1 typo in solution 2

? [STAFF] Question on 4.3 7

Midterm deadline passed, some of my answers "not submitted"?
I'm a bit worried because some of the answers I submitted seem to have not been recorded. I submitted all my answers and then clicked "Submi...
1

[STAFF]: the set D is just the observed entries of the matrix Y , in this case just (1,2): is this correct?
Since the matrix has only 1 observed entry 1.
9

Clarification needed about 4.1
--- edited ---
4

Clarification needed: --- edited ---
--- edited ---
2

Typo ("compoment") in clarification
[Edited to remove exam content]
2

[Edited to remove exam content]
[Edited to remove exam content] Please email 686exam@mit.edu for further questions.
3

Just wondering when this type of regularization is useful if it leads to a bad solution
Just wondering when this type of regularization is useful (if at all) , if it leads to a bad solution, any intuition (or it's just there for theoretical prope...
1
Following

[STAFF] J Expression
[Edited to remove exam content]A
3

[staff] clarification of notation
[Edited to remove exam content]
3