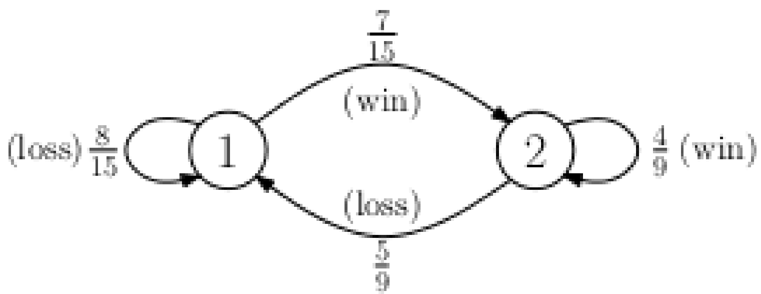
MITx: 6.041x Introduction to Probability - The Science of Uncertainty

Exercise: Frequency interpretations

(2/2 points)

Jack is a gambler who pays for his MIT tuition by spending weekends in Las Vegas. His latest game of choice is blackjack, which he plays using a fixed strategy. However, at this special blackjack table, the dealer uses one of 2 decks of cards for each hand. Using his fixed strategy, Jack wins with probability $7/15$ when deck #1 is used and with probability $4/9$ when deck #2 is used. Whenever deck #1 is used, if Jack wins, the dealer switches to deck #2 for the next hand, and if Jack loses, the dealer keeps using deck #1 for the next hand. Whenever deck #2 is used, if Jack wins, the dealer keeps using deck #2 for the next hand, and if Jack loses, the dealer switches to deck #1 for the next hand.

Jack's wins and losses can be modeled as the transitions of the following Markov chain, whose states correspond to the particular deck being used.



What is Jack's long-term probability of winning?

0.45652174 ✔ **Answer:** 0.45652

Answer:
The long-term frequency of winning can be found as the sum of the long-term frequency of transitions from state 1 to state 2 and from state 2 back to itself. These can be found from the steady-state probabilities π_1 and π_2, which are known to exist as the chain is aperiodic and recurrent. The local balance and normalization equations are

$$\begin{aligned} \frac{7}{15}\pi_1 &= \frac{5}{9}\pi_2 \\ \pi_1 + \pi_2 &= 1, \end{aligned}$$

which lead to the solution $\pi_1 = 25/46$ and $\pi_2 = 21/46$.

The probability of winning can now be found as

$$\pi_1 p_{12} + \pi_2 p_{22} = \frac{25}{46} \cdot \frac{7}{15} + \frac{21}{46} \cdot \frac{4}{9} = \frac{21}{46} = \pi_2,$$

and so for this particular game of blackjack, the long-term probability of winning also happens to equal the steady-state probability of being in state 2.

You have used 1 of 2 submissions