Exercise: Describing events

(4/4 points)

Events related to the Poisson process can be often described in two equivalent ways: in terms of numbers of arrivals during certain intervals or in terms of arrival times. The first description involves discrete random variables, the second continuous random variables.

Let $N(t)$ be the number of arrivals during the time interval $[0,t]$ in a Poisson process. Let Y_k be the time of the kth arrival.

a) The event $\{N(5) > 1\}$ is equivalent to the event $\{Y_k \leq b\}$, for suitable b and k. Find b and k.

$b =$ 5 ✔ **Answer:** 5

$k =$ 2 ✔ **Answer:** 2

b) The event $\{2 < Y_3 \leq Y_4 \leq 5\}$ is equivalent to the event $\{N(2) \leq a \text{ and } N(5) \geq b\}$. Find a and b.

$a =$ 2 ✔ **Answer:** 2

$b =$ 4 ✔ **Answer:** 4

Answer:
a) We have $N(5) > 1$ if and only if we have had two or more arrivals by time 5, i.e., $T_2 \leq 5$. Thus, $b = 5$ and $k = 2$.

b) We have $2 < Y_3 \leq Y_4 \leq 5$ if and only if by time 2 we have not yet had 3 arrivals (i.e., $N(2) \leq 2$) and by time 5 we have had at least 4 arrivals (i.e., $N(5) \geq 4$). Thus, $a = 2$ and $b = 4$.

You have used 2 of 2 submissions

Exercises 23 due May 11, 2016 at 23:59 UTC

Solved problems

Additional theoretical material

Problem Set 9
Problem Set 9 due May 11, 2016 at 23:59 UTC

Unit summary