7. How to process signals: High, Low, and Mid pass Filters

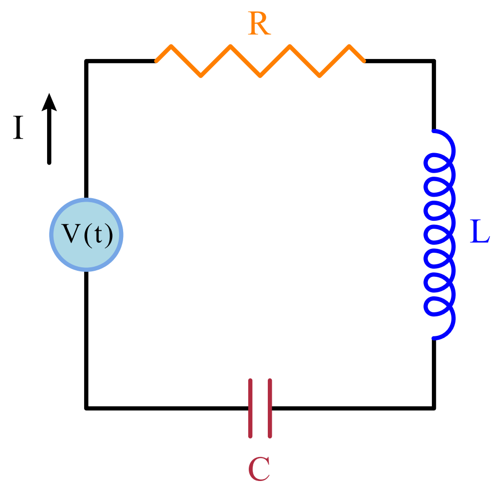
You've seen a little bit about how our ears actually hear. In the next two MATLAB exercises, we are going to start learning how to process audio files.

Before we get into the practice of processing signals, let us think a bit about the electronics involved in creating filters for real world signals. In this simplified example, our system is an RLC series circuit. We would like to process our sound signals by passing them through some circuit which will modify the signal in some way.

To get started, let's first review what we learned about RLC circuits in the course Introduction to Differential Equations, the lecture on Resonance and RLC circuits.

Review of series RLC circuits

Here is an image of a series RLC circuit.



The diagram shows symbols for four standard electronic components:

R	resistance of the resistor (ohms)
L	inductance of the inductor $(\mathrm{henries})$
C	capacitance of the capacitor (farads)
V	voltage source (volts)

The quantities $R,\ L,\ C$ are constants while the voltage source V is a function of time.

We say that current $I\,(t)$ flows clockwise. Since it's a series circuit, the current through any of the wires is the same. The power source produces a voltage **increase** of $V\,(t)$ volts at time t. This voltage increase may vary with time, and may be negative as well as positive.

The voltage drop across each component is determined by its relationship to the current flowing through it.

$$\text{Voltage drop across the inductor:} \quad V_L\,(t) = L\dot{I}\,(t)\,. \tag{2.1}$$

$$\text{Voltage drop across the resistor:} \quad V_R\,(t) = RI\,(t)\,. \tag{2.2}$$

$$\text{Voltage drop across the capacitor:} \quad C\dot{V}_C(t) = I(t)\,. \tag{2.3}$$

In what follows, we will typically think of $V(t)$ as being determined by a sound signal. Depending on how we want to process the signal, we will consider the system response to be any one of the voltage drops across the various components in our circuit.

One of the interesting features of this system is that the differential equation describing the series circuit is the same for all three possible system responses, but the way the input signal shows up is different for all three.

$$\begin{aligned}
L\ddot{V}_R + R\dot{V}_R + \frac{1}{C}V_R &= R\dot{V} \\
L\ddot{V}_L + R\dot{V}_{L+}\frac{1}{C}V_L &= L\ddot{V} \\
L\ddot{V}_C + R\dot{V}_{C+}\frac{1}{C}V_C &= \frac{1}{C}V
\end{aligned}$$

For now, imagine that our input signal $V(t)$ is a single sinusoid of angular frequency ω. (More general (sound) signals are a superposition of such functions, so this is an OK assumption.)

The system response is determined by the complex gain of the system. If $V(t) = \cos(\omega t)$, then

$$\begin{aligned}
V_R(t) &= \mathrm{Re}\left(\frac{iR\omega}{(\frac{1}{C} - L\omega^2) + iR\omega}e^{i\omega t}\right) \\
V_L(t) &= \mathrm{Re}\left(\frac{-L\omega^2}{(\frac{1}{C} - L\omega^2) + iR\omega}e^{i\omega t}\right) \\
V_C(t) &= \mathrm{Re}\left(\frac{\frac{1}{C}}{(\frac{1}{C} - L\omega^2) + iR\omega}e^{i\omega t}\right)
\end{aligned}$$

High pass, low pass, and midpass filters

0 points possible (ungraded)

When we input a signal which is a superposition of periodic terms into an RLC circuit system, we think of what happens to the signal in creating the system response as filtering that signal in some way.

- A **low pass filter** is a system that takes in a signal, and damps out high frequency signals, allowing the low frequency signals to pass through.
- A **high pass filter** is a system that takes in a signal, and damps out low frequency signals, allowing the high frequency signals to pass through.
- A **mid pass filter** is a system that takes in a signal, and damps out both the low and high frequency signals, allowing mid-range frequency signals to pass through.

In considering a series RLC circuit, identify which system response corresponds to which type of filtering process.

The system with response V_R is a	The system with response V_L is a	The system with response V_C is a
○ high pass filter.	● high pass filter.	○ high pass filter.
○ low pass filter.	○ low pass filter.	● low pass filter.
● mid pass filter.	○ mid pass filter.	○ mid pass filter.
✔	✔	✔

Solution:

To understand how the system responds to signals of different frequencies, we can look at the magnitude of the complex gain for each system response.

- $\left|G_R(\omega)\, e^{i\omega t}\right| = \left|\dfrac{iR\omega}{(\frac{1}{C} - L\omega^2) + iR\omega}\right|$. Observe that for ω near zero, the complex gain has magnitude near zero as well. For ω large, the complex gain is zero as well. You can check that there is a finite value ω_r somewhere in between where this positive function (the absolute value of the complex gain) must be positive and have a local maximum. Thus if we consider a series RLC circuit where the response is the voltage drop across the resistor, this has the effect of damping out low and high frequency inputs, and letting midrange signals through. Thus it is a mid pass filter.

- $\left|G_L(\omega)\, e^{i\omega t}\right| = \left|\dfrac{-L\omega^2}{\left(\frac{1}{C} - L\omega^2\right) + iR\omega}\right|$. Observe that for ω near zero, the magnitude of the system response is also near zero, but as ω approaches infinity, the magnitude of the response is near 1. Therefore if we consider a series RLC circuit whose response is the voltage drop across the inductor, this has the effect of damping out low frequencies and letting high frequencies pass through. It is a high pass filter.

- $\left|G_C(\omega)\, e^{i\omega t}\right| = \left|\dfrac{\frac{1}{C}}{\left(\frac{1}{C} - L\omega^2\right) + iR\omega}\right|$. Observe that for ω near zero, the magnitude of the system response is 1, but as ω approaches infinity, the magnitude of the response is near 0. Therefore if we consider a series RLC circuit whose response is the voltage drop across the capacitor, this has the effect of damping out high frequencies and letting low frequency signals pass through. It is a low pass filter.

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Answers are displayed within the problem

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