1.1.2 A System of Differential Equations for Species in Equal Competition

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Wes introduced the following model of two species X and Y which are in equal competition for the same resources.

$$\frac{dx}{dt} = x(1-x) - \beta xy$$

$$\frac{dy}{dt} = y(1-y) - \beta xy$$

Let's review what this model is describing.

- Here $x(t)$ is the **density** of the population of species X at time t and $y(t)$ is the density of the population of species Y at time t.

  What does **density** of a species mean? It means we consider the size of the population of species as a fraction of the carrying capacity of that species. For example, $x(t) = 0.3$ means that the size of the population of species X is 30% of the carrying capacity for X. When $x(t) = 1$, this means the population of species X is at its carrying capacity (saturation.)

  Note that the carrying capacity of species X is the number of those rabbits that the environment can support, **when** the competing rabbit species Y is not present. Similarly, the carrying capacity of species Y is the number of those rabbits that the environment can support, **when** the competing rabbit species X is not present.

When both species are present, this affects the carrying capacity of X and Y and we call the result the **effective carrying capacity**. (This is discussed more later.)

- Here β is a non-negative parameter which represents the strength of the competition for some common resource (for example, food in the garden.)

  The fact that the term $-\beta xy$ appears in the differential equations for both X and Y represents that the populations are in equal competition. (Neither population is "better" than the other at obtaining the common resource.)

Qualitative Analysis via Phase Planes: What Happens in the Long Term?

The solution to the system of differential equations is a pair of functions $x(t)$ and $y(t)$. The focus in this section will be to understand the qualitative behavior of these solutions. In other words, we want to understand what will happen to the populations in the long term. To do this, we will use **phase plane analysis** which was introduced in the section Population Dynamics I.

- A **phase plane** is a way to visualize the relationship between $x(t)$ and $y(t)$, by thinking about how x is changing with time and how y is changing with time.

- We are interested in the general direction of a **solution trajectory**, a curve representing the path traced out by the points $(x(t), y(t))$ for all time t.

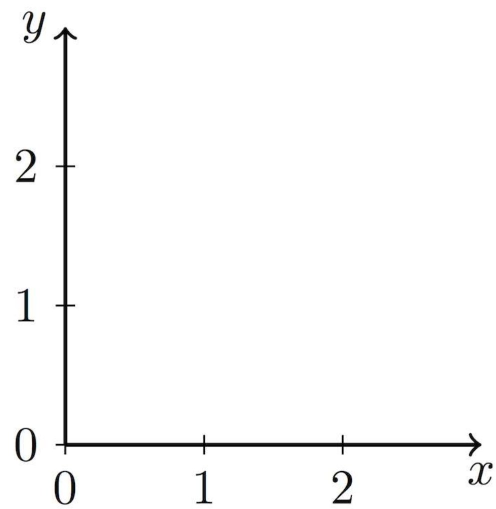
- We use the horizontal axis to represent values of x, the density of species X, and the vertical axis to represent the density of species Y. *(This is just a choice - we could make the other choice and proceed similarly. It would not affect the end conclusions from the phase plane analysis.)*

- Since populations can't be negative, we focus on the first quadrant where x and y are non-negative.

- We identify the **nullclines** (where $\frac{dx}{dt} = 0$ and where $\frac{dy}{dt} = 0$). and the **equilibrium point(s)** where these nullclines intersect (neither x nor y is changing).

- Then we sketch arrows to indicate the direction of solution trajectories along the nullclines and in the regions they create. From this we try to determine what can happen over time to the point $(x(t), y(t))$.

- A **stable** equilibrium point is one where for all nearby initial conditions, the solution trajectories approach that point. We use **unstable** to mean an equilibrium point which is not stable.



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