

EdX and its Members use cookies and other tracking technologies for performance, analytics, and marketing purposes. By using this website, you accept this use. Learn more about these technologies in the [Privacy Policy](#).



[Lecture 10: Consistency of MLE,  
Covariance Matrices, and](#)

2. Maximum Likelihood Estimator of  
> Uniform Statistical Model

[Course](#) > [Unit 3 Methods of Estimation](#) > [Multivariate Statistics](#)

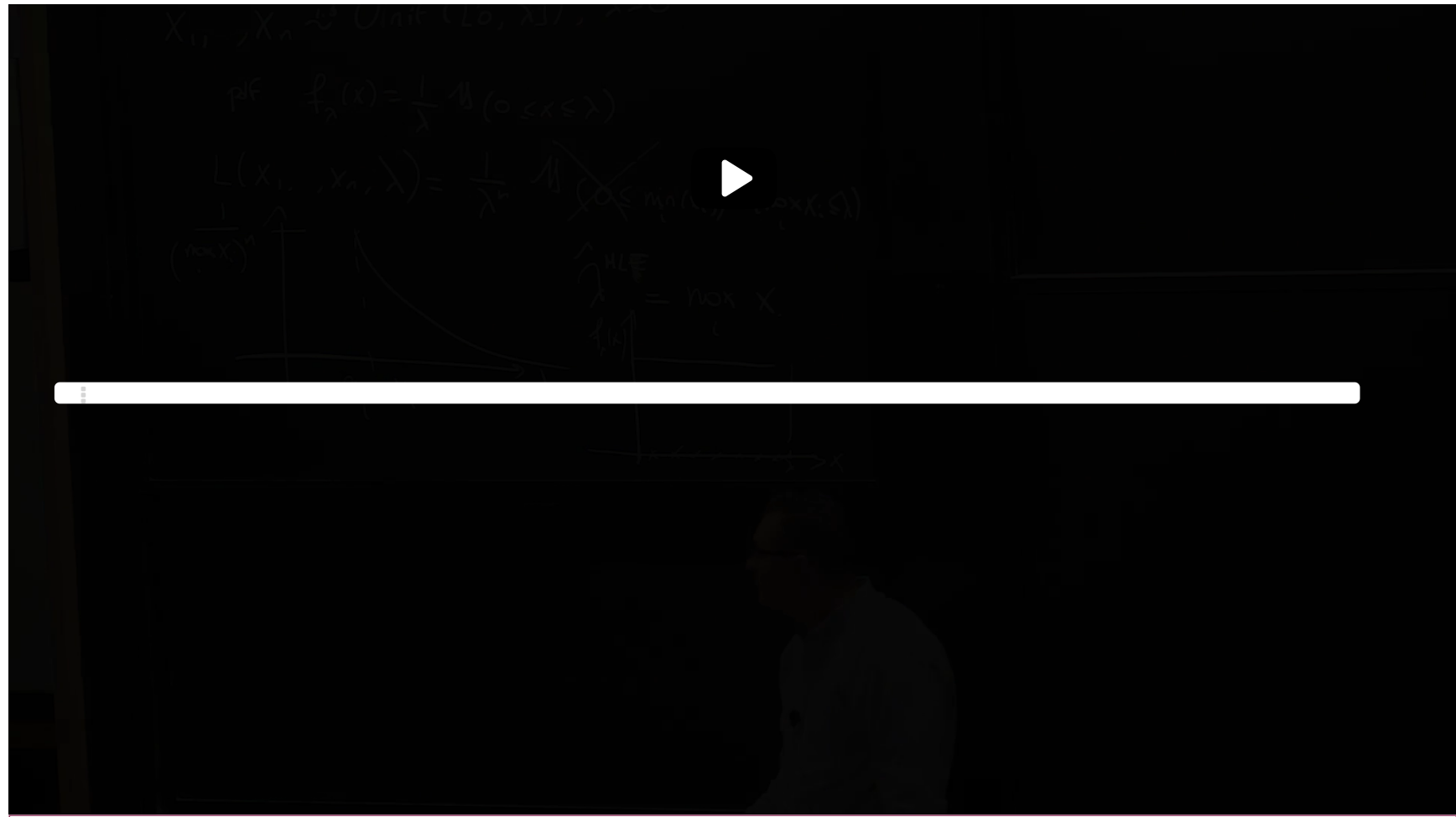
**Audit Access Expires Dec 24, 2019**

You lose all access to this course, including your progress, on Dec 24, 2019.

Upgrade by Nov 4, 2019 to get unlimited access to the course as long as it exists on the site. [Upgrade now](#)

## 2. Maximum Likelihood Estimator of Uniform Statistical Model

### Maximum Likelihood Estimator of Uniform Statistical Model



▶ 9:26 / 9:26

▶ 1.50x



## Video

[Download video file](#)

## Transcripts

[Download SubRip \(.srt\) file](#)[Download Text \(.txt\) file](#)

## Concept Check: Maximum Likelihood Estimator for a Uniform Statistical Model

1/1 point (graded)

Let  $X_1, \dots, X_n \stackrel{iid}{\sim} \text{Unif}[0, \theta^*]$  where  $\theta^*$  is an unknown parameter. We constructed the associated statistical model  $(\mathbb{R}_{\geq 0}, \{\text{Unif}[0, \theta]\}_{\theta > 0})$  (where  $\mathbb{R}_{\geq 0}$  denotes the nonnegative reals).

For any  $\theta > 0$ , the density of  $\text{Unif}[0, \theta]$  is given by  $f(x) = \frac{1}{\theta} \mathbf{1}(x \in [0, \theta])$ . Recall that

$$\mathbf{1}(x \in [0, \theta]) = \begin{cases} 1 & \text{if } x \in [0, \theta] \\ 0 & \text{otherwise.} \end{cases}$$

Hence we can use the product formula and compute the likelihood to be

$$L_n(x_1, \dots, x_n, \theta) = \prod_{i=1}^n \left( \frac{1}{\theta} \mathbf{1}(x_i \in [0, \theta]) \right) = \frac{1}{\theta^n} \mathbf{1}(x_i \in [0, \theta] \ \forall 1 \leq i \leq n).$$

For the fixed values  $(1, 3, 2, 2.5, 5, 0.1)$  (think of these as observations of random variables  $X_1, \dots, X_6$ ), what value of  $\theta$  maximizes  $L_6(1, 3, 2, 2.5, 5, 0.1, \theta)$ ?

✓ Answer: 5

**Solution:**

Observe that

$$L_6(1, 3, 2, 2.5, 5, 0.1, \theta) = \frac{1}{\theta^6} \mathbf{1}(\{1, 3, 2, 2.5, 5, 0.1\} \subset [0, \theta]).$$

If  $\theta < \max\{1, 3, 2, 2.5, 5, 0.1\}$ , then we have  $\{1, 3, 2, 2.5, 5, 0.1\} \not\subset [0, \theta]$ . By the definition of the indicator function, this means  $L_6(1, 3, 2, 2.5, 5, 0.1, \theta) = 0$  for  $\theta < \max\{1, 3, 2, 2.5, 5, 0.1\} = 5$ . Hence, when maximizing  $L_6(1, 3, 2, 2.5, 5, 0.1, \theta)$ , we need to consider  $\theta \in [5, \infty)$ . Restricted to this interval, we observe that

$$L_6(1, 3, 2, 2.5, 5, 0.1, \theta) = \frac{1}{\theta^n}.$$

The above is a decreasing function on  $[5, \infty)$ , so the maximum is attained when  $\theta = \max\{1, 3, 2, 2.5, 5, 0.1\} = 5$ .

**Remark:** In general, the maximum likelihood estimator for  $\theta^*$  in this uniform statistical model is

$$\widehat{\theta}_n^{MLE} = \max_{1 \leq i \leq n} X_i.$$

[Submit](#)

You have used 1 of 2 attempts

**i** Answers are displayed within the problem

## Discussion

[Hide Discussion](#)

**Topic:** Unit 3 Methods of Estimation:Lecture 10: Consistency of MLE, Covariance Matrices, and Multivariate Statistics / 2. Maximum Likelihood Estimator of Uniform Statistical Model

[Add a Post](#)

Show all posts ▼

by recent activity ▼

There are no posts in this topic yet.

✕

[Learn About Verified Certificates](#)

© All Rights Reserved