6. Determinants in 2×2

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Determinants in 2×2

For a 2×2 matrix $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, the number $\det(A) = ad - bc$ is called the **determinant** of A. We have already seen this expression show up in the formula for A^{-1}:

$$A^{-1} = \frac{1}{\det A}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix} \tag{5.69}$$

In order for the inverse matrix formula to work, we absolutely need the determinant to be non-zero. In fact, if the determinant is zero then it means there is a problem with the linear system, as we will see shortly.

Example (zero solutions)

Suppose we need to solve for $\vec{x}$ in $\begin{pmatrix} 1 & 3 \\ 2 & 6 \end{pmatrix}\vec{x} = \begin{pmatrix} 0 \\ 7 \end{pmatrix}$. If we try to take the inverse matrix, we find that the determinant equals zero ($6 \cdot 1 - 3 \cdot 2 = 0$). What is the problem?

In fact, there is no possible solution! The first equation says $x_1 + 3x_2 = 0$. The second equation says $2x_1 + 6x_2 = 7$. These equations cannot both be true, because multiplying the first equation by 2 gives a contradiction: it says that $2x_1 + 6x_2 = 0$ instead of 7.

Example (infinitely many solutions)

Suppose we need to solve for $\vec{x}$ in $\begin{pmatrix} 1 & 3 \\ 2 & 6 \end{pmatrix}\vec{x} = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$. Again the determinant is zero, but in this case there is a different problem.

There are infinitely many different solutions! The first equation says $x_1 + 3x_2 = 2$, and the second equation says $2x_1 + 6x_2 = 4$. But the second equation does not add any information, since it is logically equivalent to the first equation via multiplication by 2. Therefore, we are free to choose any value for x_1 and we can always find a value for x_2 that will make both equations true. An infinite family of solutions is given by $\begin{pmatrix} t \\ \frac{2-t}{3} \end{pmatrix}$.

Where does the determinant come from?

From the previous examples, you might have noticed that we will have a problem with a 2×2 linear system whenever the second equation is a multiple of the first equation. If the matrix $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, then this means we will have a problem if the vector $\begin{pmatrix} c \\ d \end{pmatrix}$ is a multiple of the vector $\begin{pmatrix} a \\ b \end{pmatrix}$. The only number t such that $at = c$ is $t = \frac{c}{a}$, so we will have a problem exactly when $b\frac{c}{a} = d$. By algebra, this is equivalent to $ad - bc = 0$.

Application to Area

The determinant also comes up in a completely separate problem. Given two points/v

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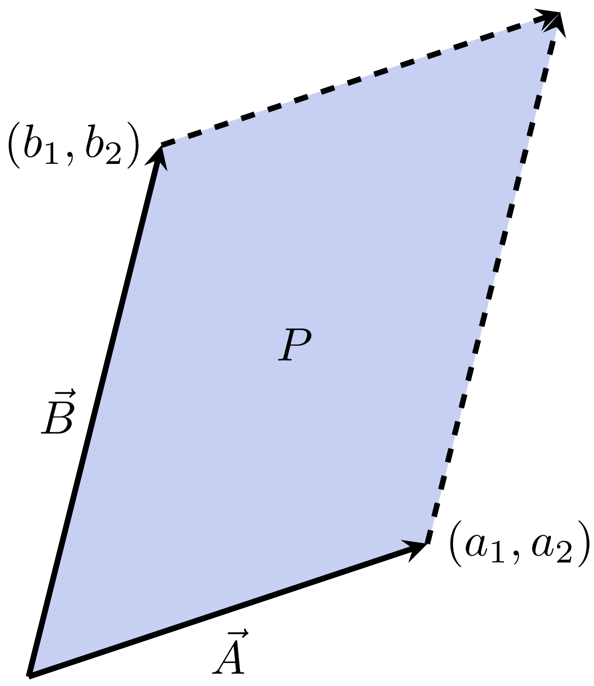
$\vec{B} = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$ in the plane, what is the area of the parallelogram P whose sides are described by $\vec{A}$ and $\vec{B}$?



It can be shown that

$$\mathrm{area}\,(P) = \left|\det \begin{pmatrix} a_1 & a_2 \\ b_1 & b_2 \end{pmatrix}\right| \tag{5.70}$$

For example, if $\vec{A} = \begin{pmatrix} 4 \\ 1 \end{pmatrix}$ and $\vec{B} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$ then $\mathrm{area}\,(P) = \det \begin{pmatrix} 4 & 1 \\ 1 & 3 \end{pmatrix} = 11.$

Note: It is also true that

$$\mathrm{area}\,(P) = \pm \det \begin{pmatrix} a_1 & b_1 \\ a_2 & b_2 \end{pmatrix} \tag{5.71}$$

In either case, the determinant is equal to $a_1 b_2 - a_2 b_1$. So you don't have to remember whether to use rows or columns for the points A and B.

Why does the determinant give an area?

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