5. Improper Prior: Example

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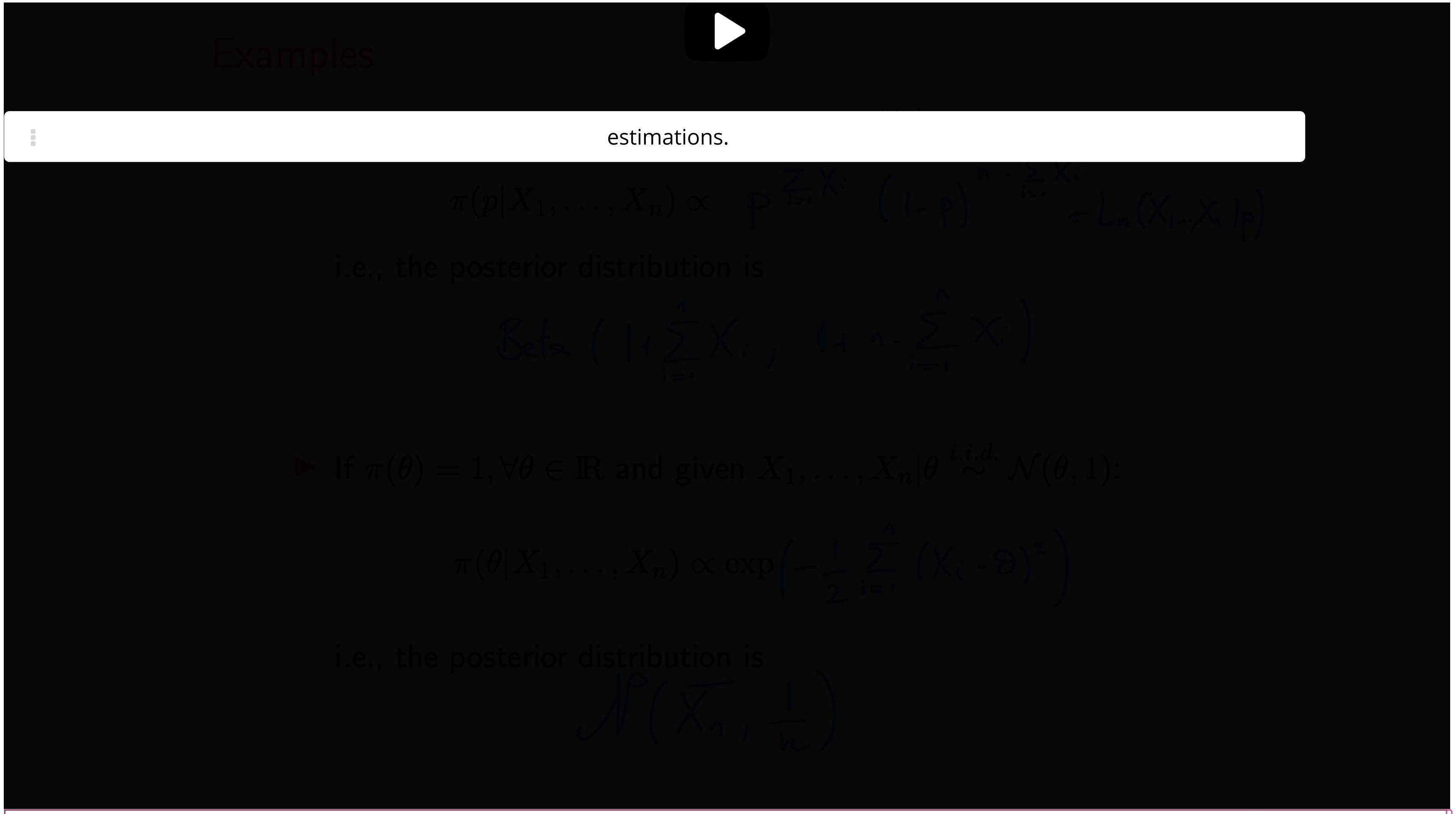

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Gaussian Prior on Gaussian Observations

2/2 points (graded)

Let $X_1, X_2, \ldots, X_n$ be random variables; which are i.i.d., conditional on θ, and such that,

$$p\left(X_i|\theta\right) = N\left(\theta, 1\right),$$

where $p\left(X_i|\theta\right)$ is the conditonal density of X_i given θ. Furthermore, we assume the prior $\pi\left(\theta\right) \sim N\left(\mu, 1\right)$. Let

$$\pi\left(\theta|X_1, \ldots, X_n\right) \sim N\left(\alpha, \beta^2\right).$$

Find, α and β^2.

- $\alpha =$

- [x] $\frac{1}{n+1}((\sum_{i=1}^{n} X_i) + \mu)$
- [ ] $\frac{1}{n}(\sum_{i=1}^{n} X_i + \mu)$
- [ ] $\frac{1}{n+1}(\sum_{i=1}^{n} X_i) + \mu$
- [ ] $\frac{1}{n}(\sum_{i=1}^{n} X_i)$

✔

- $\beta^2 =$

- [x] $\frac{1}{n+1}$
- [ ] $\frac{1}{n}$
- [ ] $\frac{\mu}{n}$

- $\frac{\mu}{n+1}$

Solution:

We begin by recalling that,

$$\begin{aligned}\pi\left(\theta|X_1,\ldots,X_n\right) &\propto p_n\left(X_1,\ldots,X_n|\theta\right)\pi\left(\theta\right)\\ &\propto \exp\left(-\frac{1}{2}\sum_{i=1}^{n}\left(X_i-\theta\right)^2\right)\exp\left(-\frac{1}{2}(\theta-\mu)^2\right).\end{aligned}$$

We now study the last quantity, keeping in mind that, Gaussian distribution is a conjugate prior of itself; hence, we expect the resulting distribution to be a Gaussian. For this, we need to arrive at a formula of the form,

$$\exp\left(-\frac{1}{2}\sum_{i=1}^{n}\left(X_i-\theta\right)^2\right)\exp\left(-\frac{1}{2}(\theta-\mu)^2\right)\propto\exp\left(-\frac{1}{2\beta^2}(\theta-\alpha)^2\right).$$

Now, we begin doing the algebra, after removing $\exp$'s from both sides.

$$\begin{aligned}&-\frac{1}{2}\sum_{i=1}^{n}\left(X_i-\theta\right)^2-\frac{1}{2}(\theta-\mu)^2\\ &=-\frac{1}{2}\left(\left(n+1\right)\theta^2-2\theta\left(\left[\sum_{i=1}^{n}X_i\right]+\mu\right)\right)+C\\ &=-\frac{n+1}{2}\left(\theta^2-2\theta\frac{\left[\sum_{i=1}^{n}X_i\right]+\mu}{n+1}\right)+C\\ &=-\frac{1}{2(n+1)^{-1}}(\theta-\alpha)^2+C',\end{aligned}$$

where C and C''s are constants; and,

$$\alpha = \frac{1}{n+1}\left(\left[\sum_{i=1}^{n} X_i\right] + \mu\right) \quad \text{and} \quad \beta^2 = \frac{1}{n+1}.$$

This is, essentially, the same formula as derived in the lecture, except that n is replaced with $n+1$ (since we have a prior now), and the summation also takes μ into account. In a sense, you may view this as, $n+1$ observations, where the first n are $X_1, \ldots, X_n$ and the last one is from $N(\mu, 1)$.

Submit You have used 1 of 3 attempts

Answers are displayed within the problem

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If you are stuck on this one, consider referencing Lecture 15 from 6.431x

It discusses recognizing normal PDFs and their mean and variance when they are represented in perhaps a less familiar fashion.

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