MATHEMATICS

Intuitive definition for curvature [duplicate]

Asked 4 years, 5 months ago Active 4 years, 5 months ago Viewed 1k times

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Understanding the formula for curvature (4 answers)
Closed last month.

I was reading about curvature of a curve but I didn't understand it.

I'm looking for an easy definition and also I want to know is there any general formula for finding curvature of the all curves ?

Note : I looked at Wolfram Math Wolrd: *Curvature* but It was really hard to me

differential-geometry curves curvature

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edited Mar 25 '17 at 22:13 Rodrigo de Azevedo 18k 4 31 87

asked Mar 25 '17 at 15:23 S.H.W 4,159 4 15 42

Are you asking about the curvature of curves $\gamma : [0,1] \to \mathbb{R}^3$ or specifically about the curvature of the graph of a function? – Olivier Mar 25 '17 at 16:14

@Olivier My mean is curvature of the graph of a function. – S.H.W Mar 25 '17 at 16:22

4 Answers

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In the plane curvature is rate of change of slope ϕ w.r.t. arc s, $\tan\phi = \dfrac{dy}{dx}$, where $ds^2 = dx^2 + dy^2$

This all that is important by way of definition. Physically it is rate at which a curve turns, the rest is just manipulation:

Curvature

$$= \frac{d\phi}{ds} == \frac{d\tan^{-1}\frac{dy}{dx}}{ds} = \frac{d\tan^{-1} y'}{\frac{dx\, ds}{dx}} = \frac{y''/(1+y'^2)}{\sec\phi}$$

But

$$\sec\phi = \sqrt{1+y'^2}$$

Plug in, curvature general formula /differential relation in rectangular coordinates becomes

$$\frac{y''}{(1+y'^2)^{\frac{3}{2}}} \tag{1}$$

It is constant for a circle. $k=1$ for circle, else for ellipse $k=b/a$

$$y = k\sqrt{a^2-x^2}$$

$$y^{'} = k\frac{-x}{\sqrt{a^2-x^2}}$$

$$y^{''} = k\frac{-a^2}{(a^2-x^2)^{\frac{3}{2}}}$$

Can you now use formula (1)?

Or if you choose $x=0$ it simplifies to $\dfrac{b}{a^2}$.

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edited Mar 26 '17 at 4:38

answered Mar 25 '17 at 16:11

Narasimham

34.6k 7 32 79

We are answering two somewhat different questions, and I'm not sure which one OP was asking about... – Olivier Mar 25 '17 at 16:13

I also could not figure out what exactly he wants.. concept or derivation of formula in the plane or what? – Narasimham Mar 25 '17 at 16:21

@Narasimham Can you compute curvature for a circle and an ellipse ? – S.H.W Mar 25 '17 at 16:29

As above, you should get $1/a$ answer. – Narasimham Mar 25 '17 at 16:38

@Narasimham Okay , but I did it for an ellipse but I got a complicated expression and didn't get $\frac{b^2}{a}$ or $\frac{a^2}{b}$ – S.H.W Mar 25 '17 at 21:03

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> I was reading about curvature of a curve but I didn't understand it.

Imagine your in a car, travelling a road in the mountains at constant speed. When entering curves, you feel pushed sideways. The more the road is curved, the more you are feeling the effect. This is because, even though you are travelling at constant speed, there is an effect. This is because, even though you are travelling at constant speed, there is an acceleration when you change directions. This acceleration is directed towards the *center of curvature* of the curve. For instance, if you are travelling on a circular road, then you are constantly accelerating towards the center of the circle.

The curvature κ of a curve at a point is the magnitude of this acceleration when you are travelling at *unit speed*.

In other words, given a planar curve $\gamma : [0,1] \to \mathbb{R}^2$ such that $\|\gamma'(t)\| = 1$ (constant unit speed), the curvature of γ at t is defined as

$$\kappa(t) = \|\gamma''(t)\|.$$

In general, if you are travelling at a constant positive speed $v = \|\gamma'(t)\| > 0$, then the curve $\alpha(t) = \gamma(t/v)$ has unit speed, so that

$$\kappa(t) = \|\alpha''(t)\| = \frac{\|\gamma''(t/v)\|}{v^2}.$$

> is there any general formula for finding curvature of the all curves?

Yes. In general, if $\gamma : [0,1] \to \mathbb{R}^3$ is a curve that never stops ($\|\gamma'(t)\| > 0$, for all t), then the formula of the curvature of γ is

$$\kappa(t) = \frac{\|\gamma'(t) \times \gamma''(t)\|}{\|\gamma'(t)\|^3},$$

where $\times$ denotes the cross product of $\mathbb{R}^3$. The physical interpretation of curvature does not change in this general case.

Share Cite Edit Follow Flag edited Mar 26 '17 at 2:27 answered Mar 25 '17 at 15:54

Olivier
3,778 16 29

So we can consider a physical interpretation for all curves and find the curvature ? – S.H.W Mar 25 '17 at 16:00

I'm not sure I understand correctly your question. The idea behind curvature (magnitude of acceleration when travelling at constant speed) is the same for planar and non-planar curves. However, in $\mathbb{R}^3$ and when the curve is not already parametrized with constant speed, the formula for curvature is a bit more complicated. – Olivier Mar 25 '17 at 16:09

Can you compute curvature for a circle and an ellipse ? – S.H.W Mar 25 '17 at 16:33

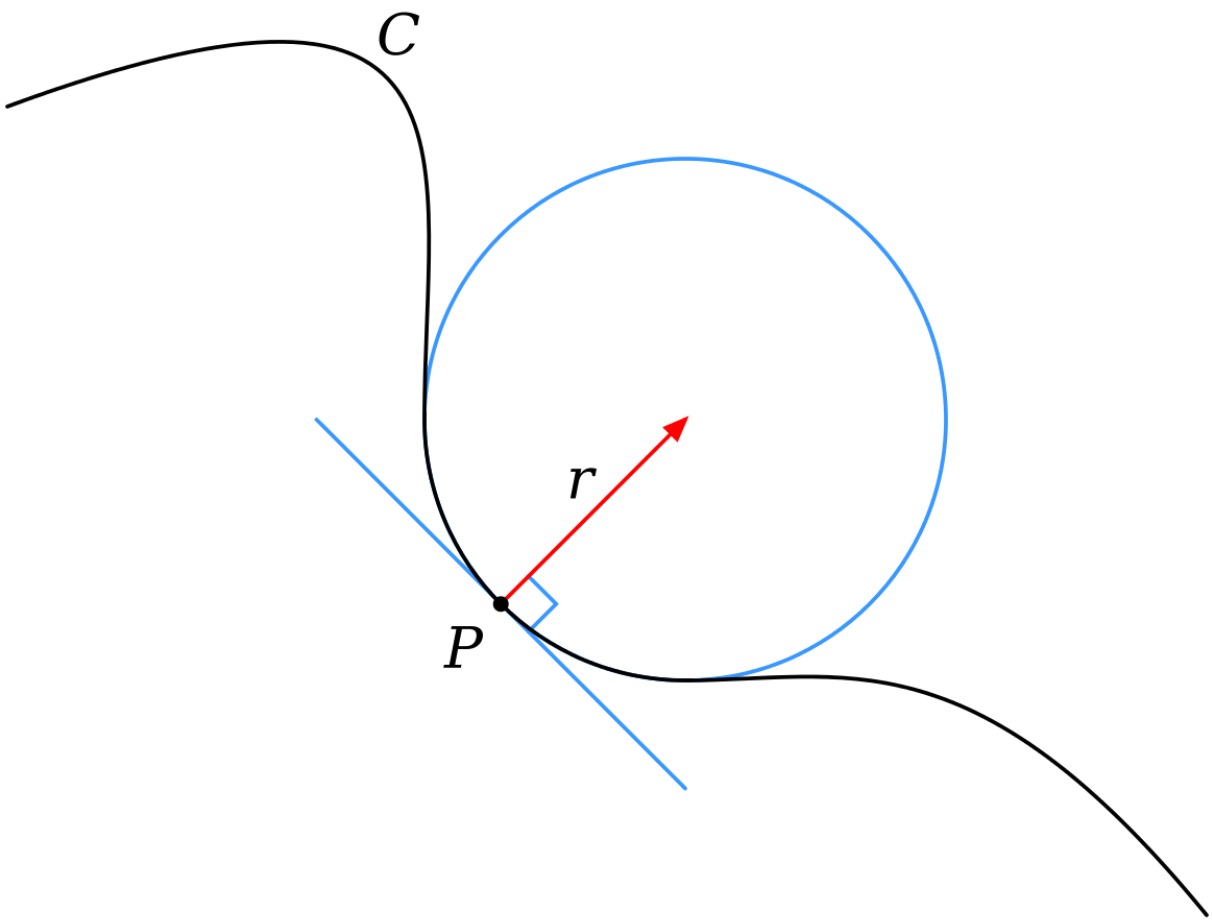
1 Yes... but no. Try doing the circle by yourself, starting from the unit speed parametrization $t \mapsto (R\cos(t/R), R\sin(t/R))$. For the ellipse, you can use the parametrization $t \mapsto (a\cos(t), b\sin(t), 0)$ and the general formula for the curvature in $\mathbb{R}^3$. – Olivier Mar 25 '17 at 16:37

(+1) how should we define the signed curvature according to your final formula? :) – Hosein Rahnama May 29 '17 at 6:12

|

3

The curvature of a *plane* curve C at a point P is the reciprocal of the radius of the osculating circle.



The higher the curvature at a point on the curve, the smaller the radius of the osculating circle. A circle has the same nonzero curvature at every point. A line has zero curvature at every point.

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answered Mar 25 '17 at 22:32
Rodrigo de Azevedo
18k 4 31 87

2

There are two notions of the curvature of a curve in an 2-dimensional Euclidean vector space. The first one is due to Euler (1775) who defines the curvature to be the limit of the quotient of the change of the tangent vector compared by the length of the observed interval.

The second one is a cinematic-dynamic measurement of the orientated curvature κ_c of curve c given by

$$\kappa_c = \frac{\text{orientated normal-component of the acceleration}}{\text{square of } c\text{'s velocity}}$$

Share Cite Edit Follow Flag edited Mar 25 '17 at 19:03 answered Mar 25 '17 at 17:57

Michael Hoppe
15.9k 3 26 45

Should n't that be normal *acceleration* in numerator? – Narasimham Mar 25 '17 at 18:48

@Narasimham Of course, corrected, thanks. – Michael Hoppe Mar 25 '17 at 19:04