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4. Estimation of an exponential parameter

(a)

1/1 point (graded)
Let $X_1, \ldots, X_n$ be i.i.d. Exp(λ) random variables, where λ is unknown.

What is the distribution of $\min_i (X_i)$? Enter the pdf $f_{\min}(x)$ of $\min_i (X_i)$ in terms of x.

$f_{\min}(x)$ n*lambda*exp(-n*lamb ✓

$n \cdot \lambda \cdot \exp\left(-n \cdot \lambda \cdot x\right)$

STANDARD NOTATION

You have used 2 of 3 attempts

Submit

(b)

1/1 point (graded)

Use the previous question to give an **unbiased** estimator $\hat{\theta}$ for $1/\lambda$.
(Enter `min`, with no subscripts, for the expression $\min_i (X_i)$.

$\hat{\theta} =$ n*min ✓

$n \cdot min$

STANDARD NOTATION

Submit You have used 1 of 3 attempts

(c)

2/2 points (graded)

What is the variance and quadratic risk of the unbiased estimator $\hat{\theta}$ in the previous part?

Var $\left(\hat{\theta}\right) =$ 1/lambda^2 ✓

$\frac{1}{\lambda^2}$

Quadratic risk of $\hat{\theta}$: 1/lambda^2 ✓

$\frac{1}{\lambda^2}$

STANDARD NOTATION

Submit You have used 1 of 3 attempts

(d)

2/3 points (graded)

Compute $\mathbf{P}\left(\frac{1}{\lambda} \geq \frac{n \min_i X_i}{\ln(5)}\right)$.

$\mathbf{P}\left(\frac{1}{\lambda} \geq \frac{n \min_i X_i}{\ln(5)}\right) =$ 4/5 ✓

This computation allows us to compute a confidence interval. The interpretation is as follows:

Let α be a value such that $1-\alpha = \mathbf{P}\left(\frac{1}{\lambda} \leq \frac{n \min_i (X_i)}{\ln(5)}\right)$. (This value depends on the answer you just computed.)

Based on this setup, the corresponding, non-asymptotic, one-sided confidence interval at level $1-\alpha$ for $1/\lambda$ is:
(Type `min` for $\min(X_i)$.)
(Note the confidence interval is finite.)
Note: The value of α is unusually large ($\alpha > 0.5$) in this problem. Please do not worry and proceed with the question as written.

[0 ✓ , min/ln(5) ✗]

0 $\frac{min}{\ln(5)}$

STANDARD NOTATION

Submit You have used 3 of 3 attempts

✱ Partially correct (2/3 points)

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Not sure what to do get the non-asymptotic interval in the last step

question posted 4 days ago by **RitterSebastian**

Got all the other questions, but confused about what to with the last part of (d).

We are trying to get the interval that $P_\theta\left(|n*min_i\left(X_i\right)-\frac{1}{\lambda}|>x\right)=\alpha$ right?

(also, is the inequality $1-\alpha=\mathbf{P}\left(\frac{1}{\lambda}\leq\frac{n\min_i\left(X_i\right)}{\ln\left(5\right)}\right)$ reversed on purpose from the first part - Seems like a very low $1-\alpha$, no?)

Seeing as this is the non-asymptotic C.I I assume we can't use CLT here?

I think I have the lower bound given 1/λ, as I'm guessing this is trivial?

This post is visible to everyone.

Cool7

4 days ago - marked as answer a day ago by **karenechu** (Staff)

1. We are trying to get interval $P\left(\text{lower bound}<=\frac{1}{\lambda}<=\text{higher bound}\right)=1-\alpha$
2. Yes, $1-\alpha$ is not a big number, e.g. <0.5
3. We can't use CLT here as we don't have many r.v. add together. Check question(a) as a hint.

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2 other responses

cmhcheung

3 days ago

Can our answer depend on f_min? If not, how to express the lower bound as lambda is not allowed.

This is one-sided confidence interval as stated in question. Lower bound doesn't need lambda to express.

posted 3 days ago by **Cool7**

You are calling the

'non-asymptotic, one-sided confidence interval'

for $1/\lambda$. That and considering what the the distribution of $min\left(X_i\right)$ is from the pdf f_{min} you found in the first part should be enough to arrive at what the lower bound should be.

posted 3 days ago by **lyanhminh11**

Got it right, thanks

posted 2 days ago by **cmhcheung**

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karenechu (Staff)
a day ago

Sorry, we will add a note about α being unusually large. (We should have done so from the last term.)

For the upper bound I got an expression like

$P\left(\frac{1}{\lambda} - \hat{\theta} \leq x\right) = P\left(\frac{1}{\lambda} \leq \hat{\theta} + x\right) = 1 - \alpha$, where $x = (ln\,(1/5) - 1)\,.\,min_i(X_i)$

but rejected by the grader and the quantity $ln\,(1/5) - 1 = -0.37866506544038814$ is also negative (I think it should always be positive).

The value of α and the lower bound is accepted by the grader. Any hint where I am doing wrong? thanks in advance.

posted about 9 hours ago by **sandipan_dey**

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