

Problem 3

Find the maximum length of **arithmetic progressions** consisting of **prime numbers only** whose **initial term is 5**.

Theorem (Green-Tao, 2004)

For any given N , there exists an **arithmetic progression** of length N consisting of **prime numbers only**.



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[https://en.wikipedia.org/wiki/Ben_Green_\(mathematician\)](https://en.wikipedia.org/wiki/Ben_Green_(mathematician))

https://en.wikipedia.org/wiki/Terence_Tao

Problem 3

Arithmetic Progression (A=initial term)

$$A + KB \quad (K = 0, 1, 2, 3, \dots)$$

Example (A=5, B=6)

- ◆ (K=0) $A = 5$
- ◆ (K=1) $A = 5+6 = 11$
- ◆ (K=2) $A = 5+2 \times 6 = 17$
- ◆ (K=3) $A = 5+3 \times 6 = 23$
- ◆ (K=4) $A = 5+4 \times 6 = 29$
- ◆ (K=5) $A = 5+5 \times 6 = \mathbf{35}$ (**Not prime**)

Problem 3

5 11 17 23 29 (length = 5)

In fact, 5 is the **maximum length** because

5 + KB ($K = 5$) is always divisible by 5

\Rightarrow it is **not** a prime number.

Answer 5

Remark If the initial term is A ,
the maximum length is $\leq A$.