3. Application of Fourier series

Problem 3.1 Suppose that $f(t)$ is an odd periodic function of period 2π. Find the periodic function $x(t)$ of period 2π that is a solution to

$$\ddot{x} + 50x = f(t)$$

Think of $f(t)$ as the input signal, and the solution $x(t)$ as the system response (output signal).

Special case: What is the system response to the input signal $\sin(nt)$? In other words, what is a solution to

$$\ddot{x} + 50x = \sin(nt)$$

with the same (smallest) period as $\sin(nt)$?

Solution: First find the response to e^{int}, and then take the imaginary part. In other words, we first solve

$$\ddot{z} + 50z = e^{int}.$$

The characteristic polynomial is $P(r) = r^2 + 50$, so by ERF, the system response to e^{int} is

$$z = \frac{1}{P(in)} e^{int} = \frac{1}{50 - n^2} e^{int}.$$

The complex gain is $\frac{1}{50-n^2}$. Then

$$x = \operatorname{Im}\left(\frac{1}{50 - n^2} e^{int}\right) = \frac{1}{50 - n^2} \sin(nt)$$

is the system response to $\sin(nt)$. This explains all the rows of the table below except the last row.

Input Signal	**System Response**
e^{int}	$\frac{1}{50-n^2} e^{int}$
$\sin nt$	$\frac{1}{50-n^2} \sin nt$
$\sin t$	$\frac{1}{49} \sin t$
$\sin 2t$	$\frac{1}{46} \sin 2t$
$\sin 3t$	$\frac{1}{41} \sin 3t$
$\vdots$	$\vdots$
$\sum_{n \geq 1} b_n \sin nt$	$\sum_{n \geq 1} \frac{1}{50 - n^2} b_n \sin nt$

3. Application of Fourier series

Hide Discussion

Topic: Unit 1: Fourier Series / 3. Application of Fourier series

Add a Post

Show all posts | by recent activity

? Clarify meaning "same (smallest) period"

The last part of the problem statement says: "....with the same (smallest) period as sin(nt)?". I don't understand what this is asking.

3