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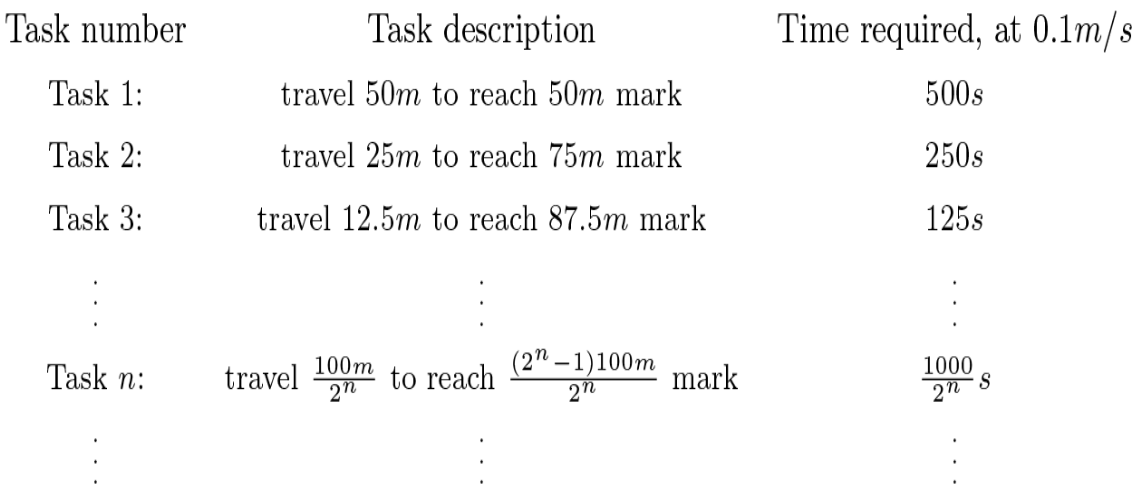
Assessment: Zeno's Paradox

Zeno’s Paradox

Paradox Grade: 2

It seems to me that from a contemporary perspective, Zeno’s paradox deserves a lowish paradoxicality grade. This is because nowadays the mathematical notion of a limit is commonplace. And limits can be used to show that—contrary to what is claimed above—it is possible to complete infinitely many tasks in a finite amount of time, if the time required to complete successive tasks deceases quickly enough.

Here is an example. Tortoises of the genus *Gopherus* have been clocked walking at speeds of about $0.1 m/s$. Suppose the distance from point A to point B is $100m$. This is how long it would take Marty the Tortoise to complete each of Zeno’s infinitely many tasks:



Task number	Task description	Time required, at $0.1m/s$
Task 1:	travel $50m$ to reach $50m$ mark	$500s$
Task 2:	travel $25m$ to reach $75m$ mark	$250s$
Task 3:	travel $12.5m$ to reach $87.5m$ mark	$125s$
$\vdots$	$\vdots$	$\vdots$
Task n:	travel $\frac{100m}{2^n}$ to reach $\frac{(2^n-1)100m}{2^n}$ mark	$\frac{1000}{2^n}s$
$\vdots$	$\vdots$	$\vdots$

But by using the mathematical notion of a limit, one can show that

$$500 + 250 + 125 + \ldots + \frac{1000}{2^n} + \ldots = 1000$$

So even though Marty must complete infinitely many tasks to get from point A to point B, he should be able to do so in $1000s$.

I certainly don’t want to suggest that Zeno was a fool—he was not. For someone in Zeno’s position, the paradox deserves a grade much higher than 2.

Video Review: Agustín Walks to the Door

So say that it takes me half a second

to reach the first one, and 1/4, and 1/8, and so forth.

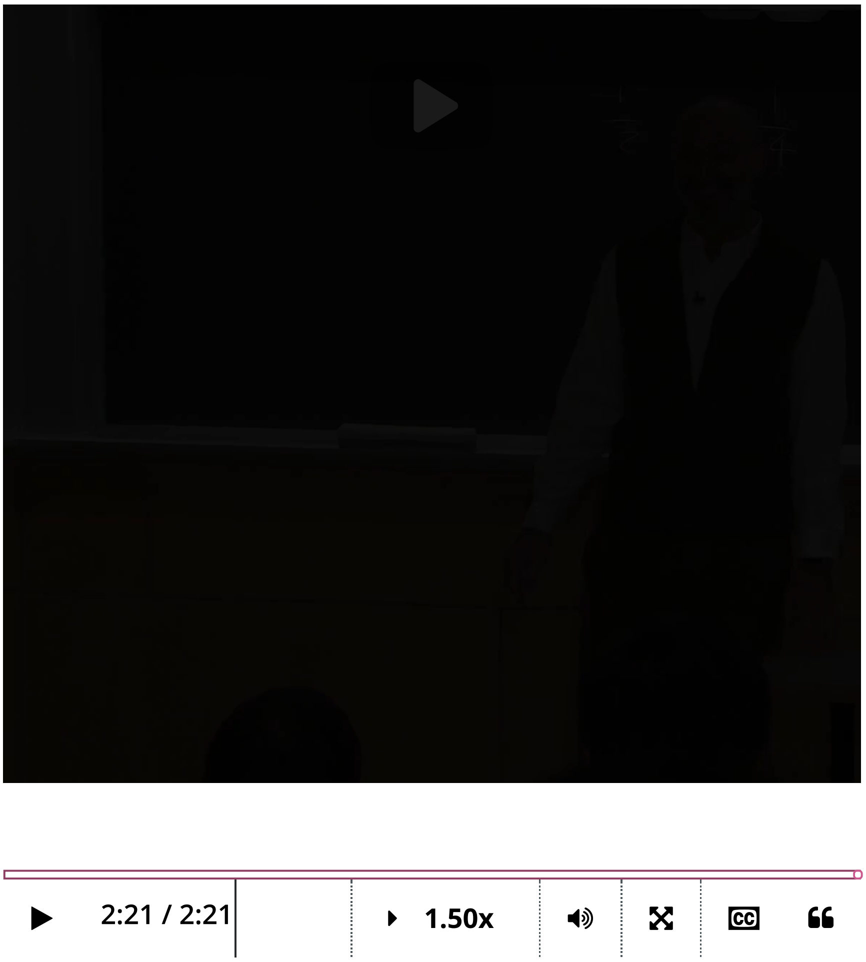
Since this series adds up to one,

that means that I'll make it to the door in one second.

And that's fine.

OK.

So far, so good.



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Problem 1

1/1 point (ungraded)

Is the following true or false?

$$\lim_{n\to\infty}\left(500+250+125+\ldots+\frac{1000}{2^n}\right)=1000$$

● True

○ False

✓

Explanation

Let $f(n) = \frac{1000}{2^1} + \frac{1000}{2^2} + \cdots + \frac{1000}{2^n}$. Then it is easy to verify that for each n, $f(n) = 1000 - \frac{1000}{2^n}$. (*Proof*: It is obvious that $f(1) = \frac{1000}{2^1}$. For $n > 1$, we may assume that $n = m + 1$, and that $f(m) = 1000 - \frac{1000}{2^m}$. It follows that $f(n) = 1000 - \frac{1000}{2^m} + \frac{1000}{2^n}$. But $\frac{1000}{2^m} = 2\frac{1000}{2^n}$. So $f(n) = 1000 - 2\frac{1000}{2^n} + \frac{1000}{2^n} = 1000 - \frac{1000}{2^n}$.)

Since $f(n) = 1000 - \frac{1000}{2^n}$ for each n, $\lim_{n\to\infty} f(n)$ must be 1000. To see this, note that for any $\epsilon > 0$ we can find a δ such that for any $k > \delta$, $|1000 - f(k)| < \epsilon$: simply let δ be such that $\frac{1000}{2^\delta} < \epsilon$.

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ℹ Answers are displayed within the problem

Problem 2

1/1 point (ungraded)

Suppose Marty slows down as he approaches the 100m mark:

Task number	Task description	Speed
Task 1:	travel $50m$ to reach $50m$ mark	$\frac{1}{10\cdot 2^1} m/s$
Task 2:	travel $25m$ to reach $75m$ mark	$\frac{1}{10\cdot 2^2} m/s$
Task 3:	travel $12.5m$ to reach $87.5m$ mark	$\frac{1}{10\cdot 2^3} m/s$
$\vdots$	$\vdots$	$\vdots$
Task n:	travel $\frac{100m}{2^n}$ to reach $\frac{(2^n-1)100m}{2^n}$ mark	$\frac{1}{10\cdot 2^n} m/s$
$\vdots$	$\vdots$	$\vdots$

On these revised assumptions, is it possible for Marty to make it to the 100m mark in a finite amount of time?

○ Yes

● No

✓

Explanation

No. Under our new assumptions, Marty would require an infinite amount of time to make it to the $1000m$ mark.

To see this, note that Marty will have to complete each of the following tasks before he can make it to the 1000m mark:

Task number	Task description	Speed	Time required
Task 1:	travel $50m$ to reach $50m$ mark	$\frac{1}{10\cdot 2^1}m/s$	$1000s$
Task 2:	travel $25m$ to reach $75m$ mark	$\frac{1}{10\cdot 2^2}m/s$	$1000s$
Task 3:	travel $12.5m$ to reach $87.5m$ mark	$\frac{1}{10\cdot 2^3}m/s$	$1000s$
$\vdots$	$\vdots$	$\vdots$	$\vdots$
Task n:	travel $\frac{100m}{2^n}$ to reach $\frac{(2^n-1)100m}{2^n}$ mark	$\frac{1}{10\cdot 2^n}m/s$	$1000s$
$\vdots$	$\vdots$	$\vdots$	$\vdots$

So the number of seconds that Marty would need to complete her infinitely many tasks is $1000 + 1000 + 1000 + \ldots = \infty$.

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So how did the old greek phylosophers resolve this paradox?
If in the ancient greeks they didn't know about the concept of limit I guess they probably came with ...
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