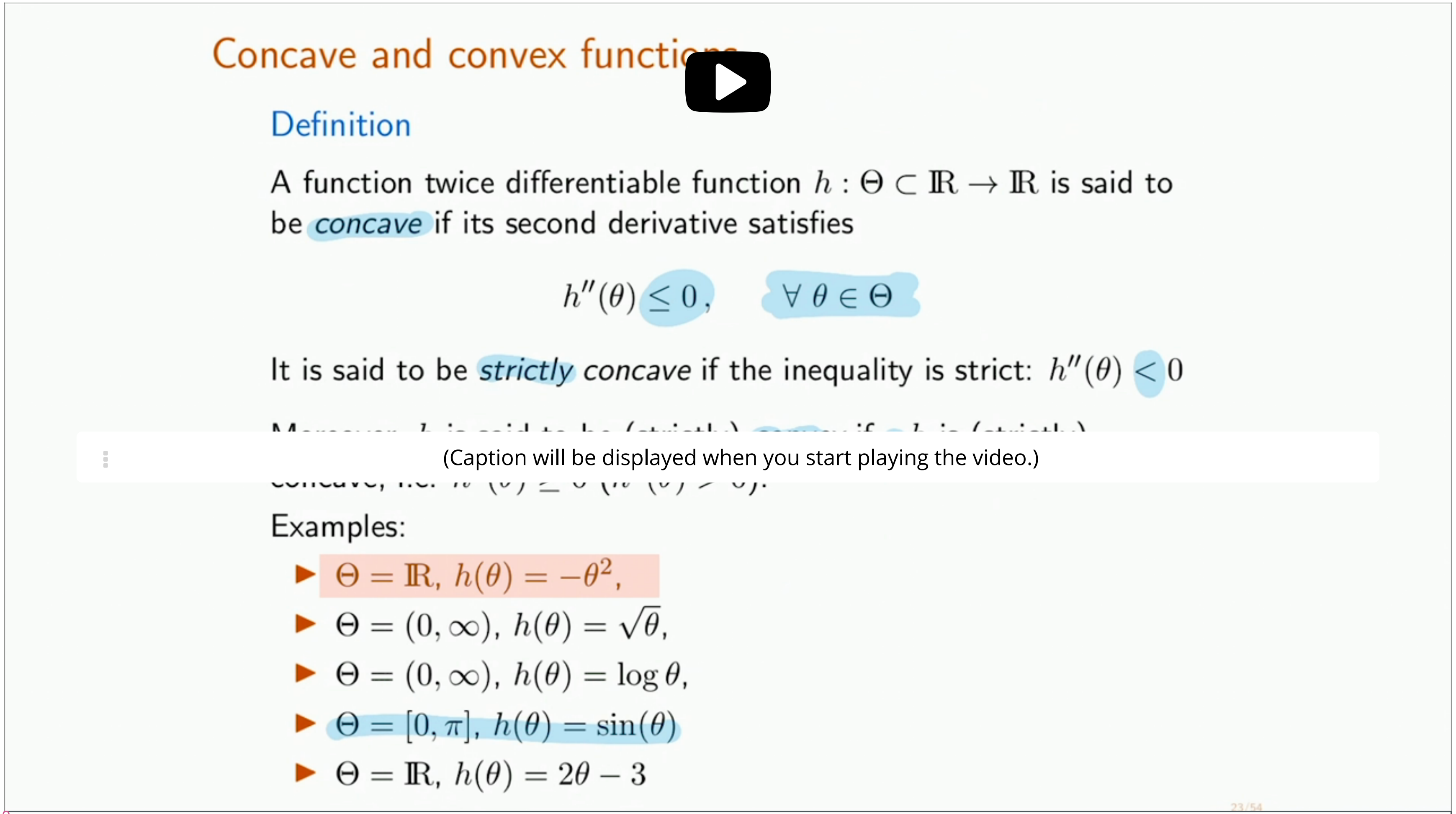
7. Worked examples: Concavity in 1 dimension

Worked Examples: Concavity in 1 dimensions



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Review: 1D Optimization via Calculus

4/4 points (graded)

(For this problem, you are welcome to use any computational tools that would be helpful.)

Let $f(x) = \frac{1}{3}x^3 - x^2 - 3x + 10$ defined on the interval $[-4, 4]$.

Let x_1 and x_2 be the critical points of f, and let's impose that $x_1 < x_2$. Fill in the next two boxes with the values of x_1 and x_2, respectively:
(Recall that the **critical points** of f are those $x \in \mathbb{R}$ such that $f'(x) = 0$.)

$x_1 =$ -1 ✔ **Answer:** -1

$x_2 =$ 3 ✔ **Answer:** 3

Fill in the next two boxes with the values of $f''(x_1)$ and $f''(x_2)$, respectively:

$f''(x_1) =$ -4 ✔ **Answer:** -4

$f''(x_2) =$ 4 ✔ **Answer:** 4

Solution:

Observe that

$$f'(x) = x^2 - 2x - 3 = (x-3)(x+1).$$

Hence the **critical points** are $x_1 = -1$ and $x_2 = 3$. The **second derivative** is

$$f''(x) = 2x - 2$$

so that

$$f''(x_1) = -4, \quad f''(x_2) = -4.$$

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Answers are displayed within the problem

Review: 1D Optimization via Calculus (Continued)

4/4 points (graded)
(For this problem, you are welcome to use any computational tools that would be helpful.)

Recall that x_1 and x_2 are the critical points of the function $f(x) = \frac{1}{3}x^3 - x^2 - 3x + 10.$

According to the second derivative test, x_1 is a ...

- (selected) Local Maximum
- Local Minimum
- None of the above

✔

and x_2 is a

- Local Maximum
- (selected) Local Minimum

None of the above

At what value of x is the (global) minimum value of $f(x)$ attained on the interval $[-4, 4]$?

-4

Answer: -4

At what value of x the (global) maximum value of $f(x)$ attained on the interval $[-4, 4]$?

-1

Answer: -1

Solution:

The previous problem implies that f is concave at x_1 and convex at x_2, so x_1 is a **local maximum** and x_2 is a **local minimum**. To figure out the *global* extrema, we need to test the critical points as well as the endpoints: -4 and 4. We compute that

$$f(x_1) = \frac{35}{3} \approx 11.6666, \quad f(x_2) = 1$$

$$f(-4) = -\frac{46}{3} \approx -15.33333, \quad f(4) = 10/3 \approx 3.3333$$

Hence the **maximum value** of f on $[-4, 4]$ is $\frac{35}{3} \approx 11.6666$ and the **minimum value** is $-\frac{46}{3} \approx -15.33333$.

Remark: It is very important to remember to test the endpoints when doing optimization.

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You have used 1 of 2 attempts

Answers are displayed within the problem

Strict Concavity

1/1 point (graded)
Which of the following functions are strictly concave? (Choose all that apply.) (Recall that a twice-differentiable function $f : I \to \mathbb{R}$, where I is a subset of $\mathbb{R}$, is **strictly concave** if $f''(x) < 0$ for all $x \in I$.)

- [ ] $f_1(x) = x$ on $\mathbb{R}$
- [x] $f_2(x) = -e^{-x}$ on $\mathbb{R}$
- [x] $f_3(x) = x^{0.99}$ on the interval $(0, \infty)$
- [ ] $f_4(x) = x^2$ on $\mathbb{R}$

Solution:

- $f_1(x) = x$ is **not** strictly concave because $f_1''(x) = 0$.
- $f_2(x) = -e^{-x}$ is strictly concave because $f_2''(x) = -e^{-x} < 0$ for all $x \in \mathbb{R}$.
- $f_3(x) = x^{0.99}$ is strictly concave because $f_3''(x) = (0.99)(-.01)\,x^{-1.01} < 0$ for all $x \in (0, \infty)$.
- $f_4(x) = x^2$ is **not** strictly concave because $f_4''(x) = 2 > 0$. In fact, this function is strictly *convex*.

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Answers are displayed within the problem

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last question

Be careful with the sentence Strictly Concave... as I unfortunately made the mistake twice of not realizing about it and put one that is concave but not strictly concave which g... 3