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Homework 6 Maximum Likelihood

4. Maximum Likelihood Estimation,

Course > Unit 3 Methods of Estimation > Estimation and Method of Moments > Tests, and Confidence Intervals

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## 4. Maximum Likelihood Estimation, Tests, and Confidence Intervals Setup:

Let  $X_1, \ldots, X_n \overset{iid}{\sim} X$  be distributed i.i.d. with probability density function

$$f_{ heta}\left(x
ight) = (x/ heta^{2}) \exp\left(-x^{2}/2 heta^{2}
ight) \, \mathbf{1}\left(x \geq 0
ight), heta > 0.$$

(a)

3/3 points (graded)

Let  $l(\theta) = \ln L(X_1, \dots, X_n, \theta)$  denote the log likelihood. Find the critical point of  $l(\theta)$ . (The critical point is unique because KL divergence is definite.)

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(If applicable, enter  $\operatorname{\bf barX_n}$  for  $\overline{X}_n$  and  $\operatorname{\bf bar(X_n^2)}$  for  $\overline{X_n^2}$  .)

Critical point of  $l\left( heta 
ight)$  is at  $heta = \int \left| \operatorname{sqrt(bar(X_n^2)/2)} \right|$ 

Find the second derivative  $l''=rac{d^2l}{d heta^2}$  of  $l\left( heta
ight)$  . Your answer should be a function of heta and the data  $X_1,\,\ldots,\,X_n$  .

(Do **not** evaluate l'' at the critical point at this stage.)

(If applicable, enter  ${f Sigma_i(X_i)}$  for  $\sum_{i=1}^n X_i$  and and  ${f Sigma_i(X_i^2)}$  for  $\sum_{i=1}^n X_i^2$  .)

$$l''=rac{d^2l}{d heta^2}= oxed{2*n/theta^2-3* Sigma_i(X_i^2)/theta^4}$$

Using the second derivative test, is the critical point you obtain above a global maximum, a global minimum, or neither of  $l\left(\theta\right)$  in the domain  $\theta>0$ ?

global maximum

global minimum

neither

~

What can you conclude about the maximum likelihood estimator  $\hat{\theta}$  for  $\theta$ ? (There is no answer box for this question.)

STANDARD NOTATION

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You have used 1 of 3 attempts

✓ Correct (3/3 points)

(b)

1/1 point (graded)

What is the Fisher information  $I(\theta)$  of the random variables  $X_i$ ?

$$I(\theta) = 4/\text{theta^2}$$

STANDARD NOTATION

Submit

You have used 1 of 3 attempts

✓ Correct (1/1 point)

(c)

2/2 points (graded)

Use the theorem for the MLE to write down the asymptotic distribution of the MLE  $\hat{ heta}$  .

Give an asymptotic 95% confidence interval  $\mathcal{I}_{ ext{plug-in}}$  for  $\theta$  using the plug-in method. (You may use I in the answer box below to denote  $I(\hat{\theta})$ , the Fisher Information, which you found in the previous part, evaluated at  $\hat{\theta}$ .)

(If applicable, enter I for  $I(\hat{\theta})$ , **hattheta** for  $\hat{\theta}$ , and **q(alpha)** for  $q_{\alpha}$  for any numerical value  $\alpha$ . Recall  $q_{\alpha}$  denotes the value such that  $\mathbf{P}(Z \geq q_{\alpha}) = \alpha$  for  $Z \sim \mathcal{N}(0,1)$ .)

(Do not worry if the parser does not render properly; the graders will work independently. To render properly, add parentheses around Generating Speech Output (q(alpha)).)

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 $\mathcal{I}_{ ext{plug-in}} = [A,B]$  where

A = hattheta-(q(0.025))/sqrt(n\*l)

 $B = \frac{1}{100} \left( \frac{1}{100} \right) \left( \frac{1}{100} \right$ 

STANDARD NOTATION

Submit

You have used 2 of 3 attempts

✓ Correct (2/2 points)

(d)

1/1 point (graded)

Use the results from the previous parts to give a test with asymptotic level  $\alpha$  for testing

$$H_0: heta=1 \quad ext{ v.s.} \quad H_1: heta 
eq 1.$$

Suppose n=100 and the data gives  $\overline{X}_n=1.5$  and  $\overline{X}_n^2=4.0$ . Find the p-value associated to this data for this hypothesis test.

(If applicable, enter **Phi(z)** for the cdf  $\Phi(z)$  of a normal variable Z, **q(alpha)** for  $q_{\alpha}$  for any numerical value  $\alpha$ .)

p-value: 2-2\*Phi(8.28)

**Correction Note:** An earlier version gave the data  $\bar{X}_n=2.5$  instead, which led to the variance being negative, i.e. impossible data! The grader has no issue.

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