MITx: 6.041x Introduction to Probability - The Science of Uncertainty

Problem 7: Bayes' rule

(2/2 points)

Let K be a discrete random variable with PMF

$$p_K(k) = \begin{cases} 1/3, & \text{if } k = 1, \\ 2/3, & \text{if } k = 2, \\ 0 & \text{otherwise.} \end{cases}$$

Conditional on $K = 1$ or 2, random variable Y is exponentially distributed with parameter 1 or $1/2$, respectively.

Using Bayes' rule, find the conditional PMF $p_{K|Y}(k \mid y)$. Which of the following is the correct expression for $p_{K|Y}(2 \mid y)$ when $y \geq 0$?

- $$\frac{\frac{1}{3}e^{-y/2}}{\frac{1}{3}e^{-y}+\frac{2}{3}e^{-y/2}}$$
- $$\frac{e^{-y/2}}{e^{-y}+e^{-y/2}}$$ ✔
- $$\frac{\frac{1}{3}e^{-y}}{\frac{1}{3}e^{-y}+\frac{2}{3}e^{-y/2}}$$
- $$\frac{e^{-y}}{e^{-y}+e^{-y/2}}$$

Answer:
Applying Bayes' rule, we have

$$p_{K|Y}(k \mid y) = \frac{p_K(k) f_{Y|K}(y \mid k)}{f_Y(y)}.$$

By the total probability theorem,

$$\begin{aligned} f_Y(y) &= \sum_k p_K(k) f_{Y|K}(y \mid k) \\ &= p_K(1) f_{Y|K}(y \mid 1) + p_K(2) f_{Y|K}(y \mid 2) \\ &= \frac{1}{3} e^{-y} + \frac{2}{3} \cdot \frac{1}{2} e^{-y/2} \\ &= \frac{1}{3} e^{-y} + \frac{1}{3} e^{-y/2}. \end{aligned}$$

Hence, for $k = 2$, we have

$$\begin{aligned} p_{K|Y}(2 \mid y) &= \frac{p_K(2) f_{Y|K}(y \mid 2)}{f_Y(y)} \\ &= \frac{\frac{1}{3} e^{-y/2}}{\frac{1}{3} e^{-y} + \frac{1}{3} e^{-y/2}} \\ &= \frac{e^{-y/2}}{e^{-y} + e^{-y/2}}. \end{aligned}$$

You have used 1 of 2 submissions

DISCUSSION

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