1. Curves

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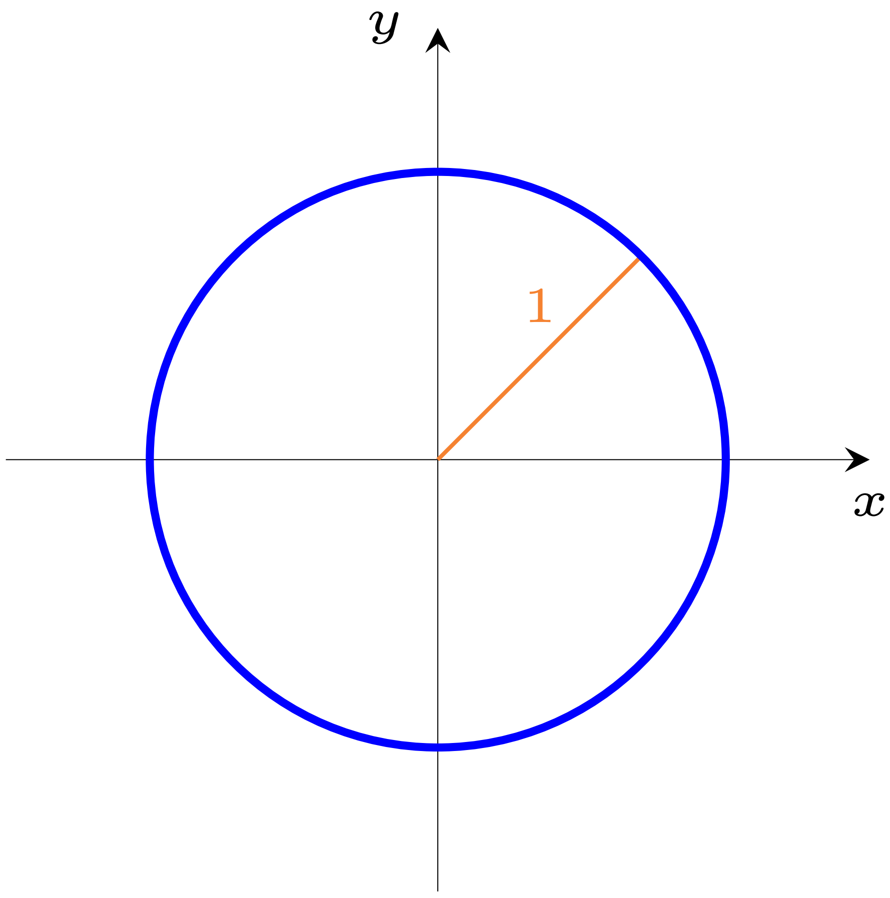
Lecture due Oct 5, 2021 20:30 IST

Synthesize

Descriptions of Curves

We now have several ways of describing a curve. Consider the unit circle:



The image above may be called a “graphical description". But for doing calculus, we need descriptions that use equations. We have two methods available to us:

As a level curve

We can describe this curve as the solution set to $x^2 + y^2 = 1$. In fact this is a level curve of the function $g(x, y) = x^2 + y^2$.

As a parametric trajectory

We can also describe this curve as a parametric trajectory. In this case, we have $\vec{r}(t) = \begin{pmatrix} \cos t \\ \sin t \end{pmatrix}$. Describing a curve as a parametric trajectory is sometimes called “parameterizing the curve."

Comparison

Both methods are useful descriptions of the curve. But they don't contain exactly the same information. The “level curve" description only describes a subset of the plane, whereas the “parametric trajectory" description additionally describes the motion of a moving point (which has speed, velocity, etc.). Both descriptions give completely different methods for finding tangent vectors.

Ask Yourself

Question 1: How do you find the tangent vector to a curve in the plane when it is given as a parametric equation?

Answer

Suppose C is the trajectory of $\vec{r}(t) = \begin{pmatrix} x(t) \\ \end{pmatrix}$. To find the tangent vector at a po

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such that $\vec{r}(t_0) = (x_0, y_0)$. Then compute $\vec{v}(t) = \frac{d\vec{r}}{dt} = \begin{pmatrix} x'(t) \\ y'(t) \end{pmatrix}$, and the vector $\vec{v}(t_0)$ will point in a direction tangent to C at the point (x_0, y_0).

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Question 2: How do you find the tangent vector to a curve in the plane when it is given as a level curve?

Answer

Suppose C is a curve described by the equation $g(x, y) = k$. To find the tangent vector at a point (x_0, y_0), first compute the gradient $\nabla g(x, y)$. Then the vector $\nabla g(x_0, y_0)$ will be normal to the curve C at the point (x_0, y_0). Rotating this normal vector by $\pm\pi/2$ gives us the desired tangent vector.

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Note that, in three dimensions, the equation $g(x, y, z) = k$ describes a level surface, rather than a curve. Therefore, when working in three dimensions, we (almost) always exclusively use parametric equations to describe curves.

Describe an ellipse

1/1 point (graded)

The curve described by $\left(\frac{x}{2}\right)^2 + y^2 = 1$ is known as an ellipse. Which of the following parameterizations has this ellipse as its trajectory for $t > 0$? Choose all that apply.

- [ ] $\begin{pmatrix} \cos 2t \\ \sin t \end{pmatrix}$
- [x] $\begin{pmatrix} 2\cos t \\ \sin t \end{pmatrix}$
- [ ] $\begin{pmatrix} 2\cos t \\ 2\sin t \end{pmatrix}$
- [ ] $\begin{pmatrix} \cos^2 t \\ \sin t \end{pmatrix}$
- [ ] $\begin{pmatrix} \cos t \\ \sin^2 t \end{pmatrix}$
- [x] $\begin{pmatrix} 2\sin t \\ \cos t \end{pmatrix}$

✔

Solution:

One way to find out is to substitute the parametric equations in to the equation $\left(\frac{x}{2}\right)^2 + y^2$ and see if the result simplifies to 1. This equation holds only for the second and fifth choices.

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Answers are displayed within the problem

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Describe a trajectory

1/1 point (graded)

Let $\vec{r}\,(t) = \begin{pmatrix} 2+2t^2 \\ 4-t^2 \end{pmatrix}$. Find a function $g\,(x,y)$ such that the level curve $g\,(x,y) = 0$ contains the trajectory of $\vec{r}\,(t)$.

$g\,(x,y) =$ x+2*y-10 ✔ **Answer:** x+2*y-10

? INPUT HELP

Solution:

This $\vec{r}\,(t)$ has a straight-line trajectory, since

$$\vec{r}\,(t) = \begin{pmatrix} 2 \\ 4 \end{pmatrix} + t^2 \begin{pmatrix} 2 \\ -1 \end{pmatrix}. \tag{6.94}$$

Since the trajectory is along the vector $\begin{pmatrix} 2 \\ -1 \end{pmatrix}$, it has a slope of $-1/2$. Furthermore, the trajectory goes through the point $(2,4)$. Therefore, it is described by the equation $y-4 = \frac{-1}{2}(x-2)$. Multiplying both sides by 2, moving everything to one side, and simplifying gives $x+2y-10$.

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Insight for "Describe a trajectory"

discussion posted about an hour ago by **stanlcod15**

I spent a ridiculous amount of time on this problem. It's a bit weird to wrap your head around, but not all that difficult in hindsight. I'm sharing some of my thoughts to perhaps prevent others from getting snagged like I did.

Since $\vec{r}\,(t) =< x\,(t)\,, y\,(t) >$ can be rewritten in the form $\vec{r}\,(0) + f\,(t) \cdot \vec{w}$ we know the trajectory travels along a line in the x, y plane. The key insight is you can deduce the slope of the trajectory in cartesian coordinates via the velocity vector $\vec{v}\,(t)$. With that slope and the starting point $\vec{r}\,(0)$ you can write an equation for the line the trajectory travels along.

I don't want to give too much away, but if you run into the same problem I did that should help get you going.

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