Exercise: Marginalization

(5 points possible)

Consider the following two joint probability tables.

I

W	1	0
sunny	1/2	0
rainy	0	1/6
snowy	0	1/3

Y

X	1	0
sunny	1/4	1/4
rainy	1/12	1/12
snowy	1/6	1/6

- Express the probability table for random variable X as a Python dictionary (the keys should be the Python strings `'sunny'`, `'rainy'`, and `'snowy'`). (Your answer should be the Python dictionary itself, and *not* the dictionary assigned to a variable, so please do not include, for instance, "`prob_table =`" before specifying your answer. You can use fractions. If you use decimals instead, please be accurate and use at least 5 decimal places.)

{1: 1/2, 0: 1/2}

? **Answer:** {'sunny': 1/2, 'rainy': 1/6, 'snowy': 1/3}

- Express the probability table for random variable Y as a Python dictionary (the keys should be the Python integers `0` and `1`). (Your answer should be the Python dictionary itself, and *not* the dictionary assigned to a variable, so please do not include, for instance, "`prob_table =`" before specifying your answer. You can use fractions. If you use decimals instead, please be accurate and use at least 5 decimal places.)

{1: 1/2, 0: 1/2} ? **Answer:** {1: 1/2, 0: 1/2}

- For two random variables U and V that take on values in the same alphabet, we say that U and V have the same distribution if $p_U(a) = p_V(a)$ for all a. For the above tables:

Do W and X have the same distribution?

- (•) Yes ✔
- ( ) No

?

Do I and Y have the same distribution?

- ( ) Yes ✔
- (•) No

?

Towards Infinity in Modeling Uncertainty (Week 4)
Exercises due Oct 13, 2016 at 02:30 IST

Homework 3 (Week 4)
Homework due Oct 13, 2016 at 02:30 IST

- For a pair of random variables (S,T), and another pair (U,V), we say that the pair (S,T) and the pair (U,V) have the same joint distribution if $p_{S,T}(a,b) = p_{U,V}(a,b)$ for all a, b.

  True or false: Consider two random variables (S,T) and (U,V), where S and U have the same distribution, and T and V have the same distribution. Then (S,T) and (U,V) have the same joint distribution.

  - ○ True
  - ◉ False ✔

  ?

Solution:

- Express the probability table for random variable X as a Python dictionary (the keys should be the Python strings `'sunny'`, `'rainy'`, and `'snowy'`).

  Solution: To get the probability table p_X, we sum across the columns in the table shown for $p_{X,Y}$:

$$\begin{aligned}\mathbb{P}(\text{sunny}) &= 1/4 + 1/4 = 1/2,\\ \mathbb{P}(\text{rainy}) &= 1/12 + 1/12 = 1/6,\\ \mathbb{P}(\text{snowy}) &= 1/6 + 1/6 = 1/3.\end{aligned}$$

  As a Python dictionary: **{'sunny': 1/2, 'rainy': 1/6, 'snowy': 1/3}"**

- Express the probability table for random variable Y as a Python dictionary (the keys should be the Python integers `0` and `1`).

  Solution: To get the probability table p_Y, we sum across the rows in the table shown for $p_{X,Y}$:

  $$\mathbb{P}(1) = 1/4 + 1/12 + 1/6 = 1/2,$$
  $$\mathbb{P}(0) = 1/4 + 1/12 + 1/6 = 1/2.$$

  As a Python dictionary: **{1: 1/2, 0: 1/2}**

- For two random variables U and V that take on values in the same alphabet, we say that U and V have the same distribution if $p_U(a) = p_V(a)$ for all a. For the above tables:

  Do W and X have the same distribution?

  Solution: To get the probability table p_W, we sum across the columns in the table shown for $p_{W,I}$:

  $$\mathbb{P}(\textbf{sunny}) = 1/2 + 0 = 1/2,$$
  $$\mathbb{P}(\textbf{rainy}) = 0 + 1/6 = 1/6,$$
  $$\mathbb{P}(\textbf{snowy}) = 0 + 1/3 = 1/3.$$

  Yes, this is the same distribution as p_X.

  Do I and Y have the same distribution?

  Solution: To get the probability table p_I, we sum across the rows in the table shown for $p_{W,I}$:

$$\mathbb{P}(1) = 1/2 + 0 + 0 = 1/2,$$
$$\mathbb{P}(0) = 0 + 1/6 + 1/3 = 1/2.$$

Yes, this is the same distribution as p_Y.

- For a pair of random variables (S,T), and another pair (U,V), we say that the pair (S,T) and the pair (U,V) have the same joint distribution if $p_{S,T}(a,b) = p_{U,V}(a,b)$ for all a,b.

  True or false: Consider two random variables (S,T) and (U,V), where S and U have the same distribution, and T and V have the same distribution. Then (S,T) and (U,V) have the same joint distribution.

  Solution:False. We just saw a counter-example! Consider when the pair (S,T) is equal to the pair (W,I) above, and when (U,V) is equal to the pair (X,Y) above. W and X have the same distribution. I and Y have the same distribution. However, the joint distribution for W and I is different from the joint distribution for X and Y, as we can see from their two tables above.

You have used 0 of 5 submissions