2.2.2 What is a linear transformation?

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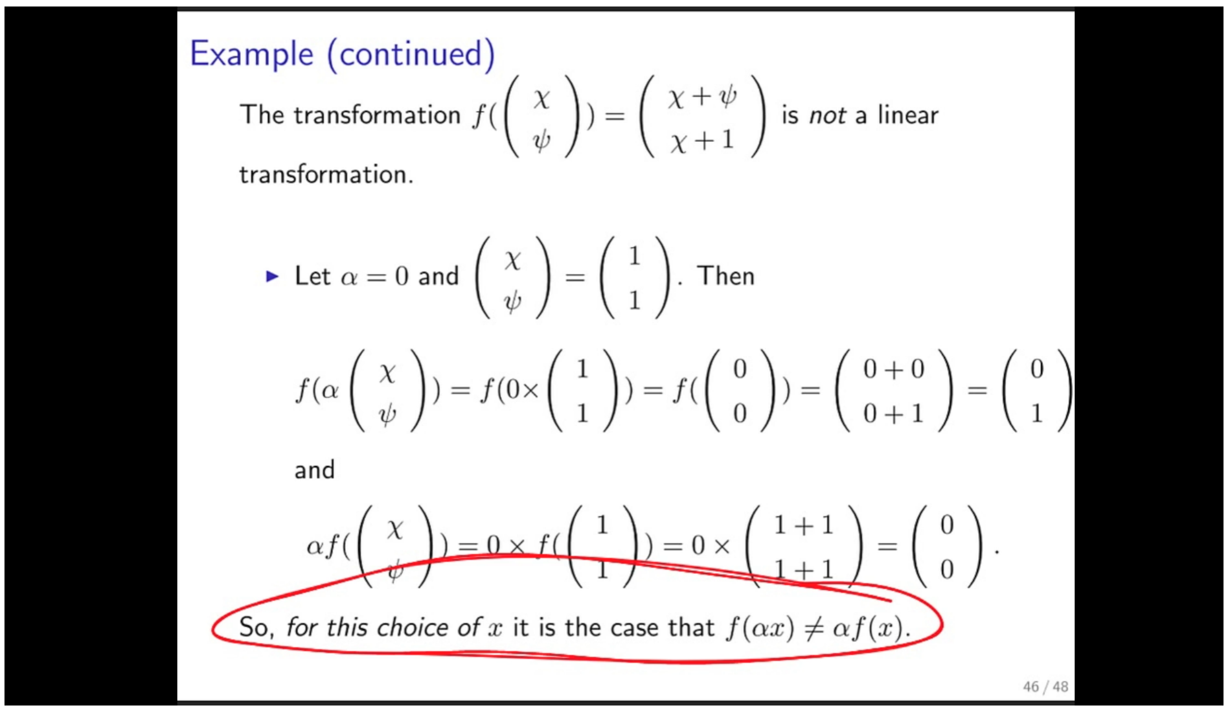
Week 2 due Oct 11, 2023 16:42 IST Completed

2.2.2 What is a linear transformation?



11:02 / 11:34 2.0x

You take that result vector and you multiply by 0 and you

get the zero vector.

And notice that these 2 results are not equal to each other.

Ah, so we have found one example for which scaling the vector first and

then transforming it is not the same as transforming it first and then

scaling it.

Therefore this cannot be a linear transformation.

We summarize that right here.

Linear transformations are special vector functions that have the

property that one could scale first and then transform or transform first

and then scale.

And one can add first and then transform.

Video

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Reading Assignment

0 points possible (ungraded)

Read Unit 2.2.2 of the notes. [LINK]

- [x] Done

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General proof for the 2nd example of the video for those who wants 1

Unique property of Linear Transformation

So I've just finished this lesson and hear me out, there is a unique property of Linear Transformation which can be detected from other not-lin...

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? Homework 2.2.2.7
Not sure how the answer in Homework 2.2.2.7 is proven. All options seem to be valid. Any insights? 11

Good explanation from Khan academy
If someone has a problem understanding linear transformations, I suggest waching: https://www.khanacademy.org/math/linear-algebra/matrix-t... 1

2.2.2.1 how do you know the transformation?
f(x0, x1) = (x0×1, x0) - just to confirm the operations of the vector function. 1. x0 = x0 * x1 , and 2. x1 = x1*x0 / x1 is that correct? I'm trying to un... 2

2.2.2.7
In my opinion the example with the norm is also an example where the scaling holds but the addition property does not hold. Justification : the t... 2

Proof of the homework 2.2.2.6
The transcript of the proof for Homework 2.2.2.6 is wrong. It's not for this video. 3

? Different Way To Prove Something Isn't a Linear Transformation?
The way outlined in the course to prove something *isn't* a linear transformation is by trying different values of alpha, chi, psi, etc. until we find ... 3

? 2.2.2 What is a linear transformation? 4

Homework 2.2.2.1

1/1 point (graded)

The vector function $f\left(\begin{pmatrix} \chi \\ \psi \end{pmatrix}\right) = \begin{pmatrix} \chi\psi \\ \chi \end{pmatrix}$ is a linear transformation.

FALSE ✔ Answer: FALSE

After you answer, try to prove your response. Be sure to check the solution, since part of what we want you to learn is often in the solution to a problem. (This is the last time we repeat this.)

Explanation

Transcripted in final section of this week

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Answer: FALSE The first check should be whether $f(0) = 0$. The answer in this case is *yes*. However,

$$f(2\begin{pmatrix} 1 \\ 1 \end{pmatrix}) = f(\begin{pmatrix} 2 \\ 2 \end{pmatrix}) = \begin{pmatrix} 2 \times 2 \\ 2 \end{pmatrix} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$$

and

$$2f(\begin{pmatrix} 1 \\ 1 \end{pmatrix}) = 2\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \end{pmatrix}.$$

Hence, there is a vector $x \in \mathbb{R}^2$ and $\alpha \in \mathbb{R}$ such that $f(\alpha x) \neq \alpha f(x)$. We conclude that this function is *not* a linear transformation.
(Obviously, you may come up with other examples that show the function is not a linear transformation.)

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Answers are displayed within the problem

Homework 2.2.2.2

1/1 point (graded)

$f\left(\begin{pmatrix} \chi_0 \\ \chi_1 \\ \chi_2 \end{pmatrix}\right) = \begin{pmatrix} \chi_0 + 1 \\ \chi_1 + 2 \\ \chi_2 + 3 \end{pmatrix}$ is a linear transformation.

FALSE ✔ Answer: FALSE

Explanation

Answer: FALSE

In Homework 1.4.6.1 you saw a number of examples where $f(\alpha x) \neq \alpha f(x)$.

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Homework 2.2.2.3

1/1 point (graded)

$f\left(\begin{pmatrix} \chi_0 \\ \chi_1 \\ \chi_2 \end{pmatrix}\right) = \begin{pmatrix} \chi_0 \\ \chi_0 + \chi_1 \\ \chi_0 + \chi_1 + \chi_2 \end{pmatrix}$ is a linear transformation.

TRUE ✔ Answer: TRUE

Explanation

Answer: TRUE

Pick arbitrary $\alpha \in \mathbb{R}$, $x = \begin{pmatrix} \chi_0 \\ \chi_1 \\ \chi_2 \end{pmatrix}$, and $y = \begin{pmatrix} \psi_0 \\ \psi_1 \\ \psi_2 \end{pmatrix}$. Then

- Show $f(\alpha x) = \alpha f(x)$:

$$f(\alpha x) = f(\alpha \begin{pmatrix} \chi_0 \\ \chi_1 \\ \chi_2 \end{pmatrix}) = f(\begin{pmatrix} \alpha\chi_0 \\ \alpha\chi_1 \\ \alpha\chi_2 \end{pmatrix}) = \begin{pmatrix} \alpha\chi_0 \\ \alpha\chi_0 + \alpha\chi_1 \\ \alpha\chi_0 + \alpha\chi_1 + \alpha\chi_2 \end{pmatrix}$$

  and

$$\alpha f(x) = \alpha f(\begin{pmatrix} \chi_0 \\ \chi_1 \\ \chi_2 \end{pmatrix}) = \alpha \begin{pmatrix} \chi_0 \\ \chi_0 + \chi_1 \\ \chi_0 + \chi_1 + \chi_2 \end{pmatrix} = \begin{pmatrix} \alpha\chi_0 \\ \alpha(\chi_0 + \chi_1) \\ \alpha(\chi_0 + \chi_1 + \chi_2) \end{pmatrix} = \begin{pmatrix} \alpha\chi_0 \\ \alpha\chi_0 + \alpha\chi_1 \\ \alpha\chi_0 + \alpha\chi_1 + \alpha\chi_2 \end{pmatrix}.$$

  Thus, $f(\alpha x) = \alpha f(x)$.

- Show $f(x + y) = f(x) + f(y)$:

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$$f(x+y) = f\left(\begin{pmatrix} \chi_0 \\ \chi_1 \\ \chi_2 \end{pmatrix} + \begin{pmatrix} \psi_0 \\ \psi_1 \\ \psi_2 \end{pmatrix}\right) = f\left(\begin{pmatrix} \chi_0+\psi_0 \\ \chi_1+\psi_1 \\ \chi_2+\psi_2 \end{pmatrix}\right) = \begin{pmatrix} \chi_0+\psi_0 \\ (\chi_0+\psi_0)+(\chi_1+\psi_1) \\ (\chi_0+\psi_0)+(\chi_1+\psi_1)+(\chi_2+\psi_2) \end{pmatrix}$$

and

$$\begin{aligned} f(x)+f(y) &= f\left(\begin{pmatrix} \chi_0 \\ \chi_1 \\ \chi_2 \end{pmatrix}\right) + f\left(\begin{pmatrix} \psi_0 \\ \psi_1 \\ \psi_2 \end{pmatrix}\right) = \begin{pmatrix} \chi_0 \\ \chi_0+\chi_1 \\ \chi_0+\chi_1+\chi_2 \end{pmatrix} + \begin{pmatrix} \psi_0 \\ \psi_0+\psi_1 \\ \psi_0+\psi_1+\psi_2 \end{pmatrix} \\ &= \begin{pmatrix} \chi_0+\psi_0 \\ (\chi_0+\chi_1)+(\psi_0+\psi_1) \\ (\chi_0+\chi_1+\chi_2)+(\psi_0+\psi_1+\psi_2) \end{pmatrix} = \begin{pmatrix} \chi_0+\psi_0 \\ (\chi_0+\psi_0)+(\chi_1+\psi_1) \\ (\chi_0+\psi_0)+(\chi_1+\psi_1)+(\chi_2+\psi_2) \end{pmatrix} \end{aligned}$$

Hence $f(x+y) = f(x)+f(y)$.

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Homework 2.2.2.4

1/1 point (graded)

If $L : \mathbb{R}^n \to \mathbb{R}^m$ is a linear transformation, then $L(0) = 0$. (Recall that here 0 represents vectors of appropriate size whose components are all 0.)

Always ✔ Answer: Always

Explanation

Transcripted in final section of this week

Answer: Always. We know that for all scalars α and vector $x \in \mathbb{R}^n$ it is the case that $L(\alpha x) = \alpha L(x)$. Now, pick $\alpha = 0$. We know that for this choice of α it has to be the case that $L(\alpha x) = \alpha L(x)$. We conclude that $L(0x) = 0L(x)$. But $0x = 0$. (Here the first 0 is the scalar 0 and the second is the vector with n components all equal to zero.) Similarly, regardless of what vector $L(x)$ equals, multiplying it by the scalar zero yields the vector 0 (with m zero components). So, $L(0x) = 0L(x)$ implies that $L(0) = 0$.

A typical mathematician would be much more terse, writing down merely: Pick $\alpha = 0$. Then

$$L(0) = L(0x) = L(\alpha x) = \alpha L(x) = 0L(x) = 0.$$

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There are actually many ways of proving this:

$$L(0) = L(x - x) = L(x + (-x)) = L(x) + L(-x) = L(x) + (-L(x)) = L(x) - L(x) = 0.$$

Alternatively, $L(x) = L(x + 0) = L(x) + L(0)$, hence $L(0) = L(x) - L(x) = 0$.

Typically, it is really easy to evaluate $f(0)$. Therefore, if you think a given vector function f is *not* a linear transformation, then you may want to first evaluate $f(0)$. If it does not evaluate to the zero vector, then you know it is not a linear transformation.

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Homework 2.2.2.5

1/1 point (graded)

Let $f : \mathbb{R}^n \to \mathbb{R}^m$ and $f(0) \neq 0$. Then f is not a linear transformation.

TRUE ✔ Answer: TRUE

Explanation

Transcripted in final section of this week

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Homework 2.2.2.6

1/1 point (graded)

If $f : \mathbb{R}^n \to \mathbb{R}^m$ and $f(0) = 0$.

Then f is a linear transformation.

Sometimes ✔

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Homework 2.2.2.7

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1/1 point (graded)
For which of the following is $f(\alpha x) = \alpha f(x)$ for all α and all x but there are examples for x and y such that $f(x+y) \neq f(x) + f(y)$

- (selected) $f\begin{pmatrix} \chi_0 \\ \chi_1 \end{pmatrix} = \begin{cases} \chi_0 & if\ \chi_0 = \chi_1 \\ 0 & otherwise \end{cases}$
- $f(x) = \|x\|_2$
- $f(x) = x$
- $f(x) = 3x$
- None of the above

✔

Explanation

Answer: $f(\begin{pmatrix} 0 \\ 1 \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \end{pmatrix}) = f(\begin{pmatrix} 1 \\ 1 \end{pmatrix}) = 1$ but $f(\begin{pmatrix} 0 \\ 1 \end{pmatrix}) + f(\begin{pmatrix} 1 \\ 0 \end{pmatrix}) = 0 + 0 = 0.$

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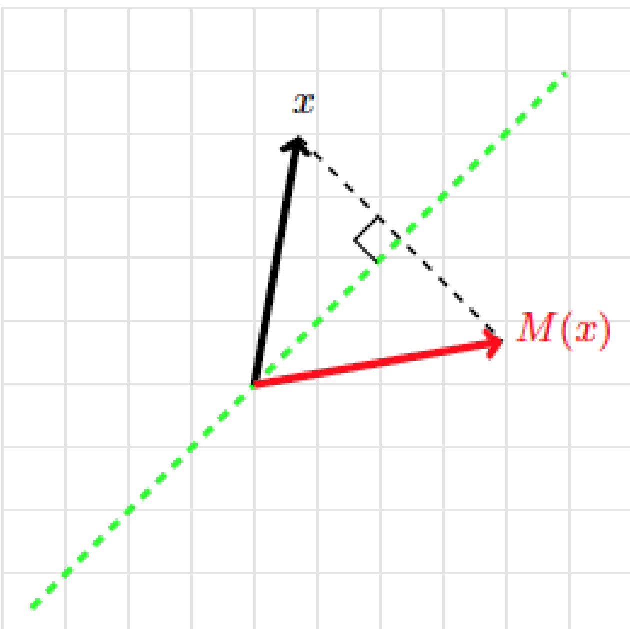
Homework 2.2.2.8

1/1 point (graded)
$f(\begin{pmatrix} \chi_0 \\ \chi_1 \end{pmatrix}) = \begin{pmatrix} \chi_1 \\ \chi_0 \end{pmatrix}$ is a linear transformation.

TRUE ✔ Answer: TRUE

Explanation

Answer: TRUE
This is actually the reflection with respect to 45 degrees line that we talked about earlier:



Pick arbitrary $\alpha \in \mathbb{R}$, $x = \begin{pmatrix} \chi_0 \\ \chi_1 \end{pmatrix}$, and $y = \begin{pmatrix} \psi_0 \\ \psi_1 \end{pmatrix}$. Then

- Show $f(\alpha x) = \alpha f(x)$:

$$f(\alpha x) = f(\alpha \begin{pmatrix} \chi_0 \\ \chi_1 \end{pmatrix}) = f(\begin{pmatrix} \alpha\chi_0 \\ \alpha\chi_1 \end{pmatrix}) = \begin{pmatrix} \alpha\chi_1 \\ \alpha\chi_0 \end{pmatrix} = \alpha \begin{pmatrix} \chi_1 \\ \chi_0 \end{pmatrix} = \alpha f(\begin{pmatrix} \chi_0 \\ \chi_1 \end{pmatrix}).$$

- Show $f(x+y) = f(x) + f(y)$:

$$f(x+y) = f(\begin{pmatrix} \chi_0 \\ \chi_1 \end{pmatrix} + \begin{pmatrix} \psi_0 \\ \psi_1 \end{pmatrix}) = f(\begin{pmatrix} \chi_0 + \psi_0 \\ \chi_1 + \psi_1 \end{pmatrix}) = \begin{pmatrix} \chi_1 + \psi_1 \\ \chi_0 + \psi_0 \end{pmatrix}$$

and

$$\begin{aligned} f(x) + f(y) &= f(\begin{pmatrix} \chi_0 \\ \chi_1 \end{pmatrix}) + f(\begin{pmatrix} \psi_0 \\ \psi_1 \end{pmatrix}) = \begin{pmatrix} \chi_1 \\ \chi_0 \end{pmatrix} + \begin{pmatrix} \psi_1 \\ \psi_0 \end{pmatrix} \\ &= \begin{pmatrix} \chi_1 + \psi_1 \\ \chi_0 + \psi_0 \end{pmatrix}. \end{aligned}$$

Hence $f(x+y) = f(x) + f(y)$.

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