



## MITx: 6.041x Introduction to Probability - The Science of Uncertainty



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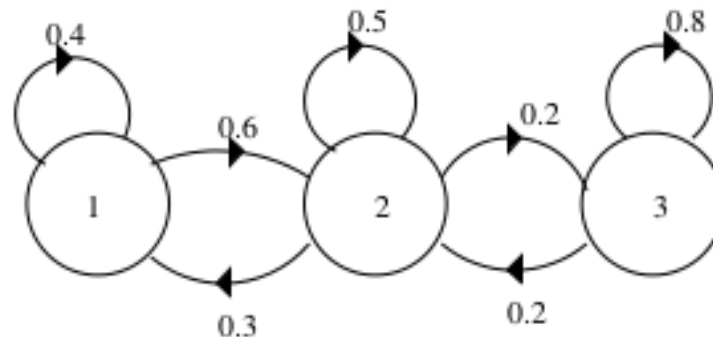
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## Exercise: Birth and death

(4/5 points)

Consider the Markov chain below. Let us refer to a transition that results in a state with a higher (respectively, lower) index as a birth (respectively, death). Calculate the following probabilities, assuming that when we start observing the chain, it is already in steady-state.



1. The steady-state probabilities for each state.

$$\pi_1 =$$

0.2



Answer: 0.2

► Unit 6: Further topics on random variables

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▼ **Unit 10: Markov chains**

Unit overview

**Lec. 24: Finite-state Markov chains**

Exercises 24 due May 18, 2016 at 23:59 UTC 

**Lec. 25: Steady-state behavior of Markov chains**

Exercises 25 due May 18, 2016 at 23:59 UTC 

$$\pi_2 =$$

✓ Answer: 0.4

$$\pi_3 =$$

✓ Answer: 0.4

2. The probability that the first transition we observe is a birth.

✓ Answer: 0.2

3. The probability that the first change of state we observe is a birth.

✗ Answer: 0.36


Answer:

1. The local balance equations take the form  $0.6\pi_1 = 0.3\pi_2$  and  $0.2\pi_2 = 0.2\pi_3$ . Together with the normalization equation, we get  $\pi_1 = 1/5$ ,  $\pi_2 = \pi_3 = 2/5$ .


2. We observe a birth if (i) we are in state 1 and the next transition is from 1 to 2, or (ii) we are in state 2 and the next transition is from 2 to 3. Hence, the desired probability is  $\pi_1 p_{12} + \pi_2 p_{23} = 1/5$ .

3. Note that a self-transition is not a change of state. If the state is 1, which happens with probability  $1/5$ , the first change of state is certain to be a birth. If the state is 2, which happens with probability  $2/5$ , the next change of state is to either 1 or 3. The probability that it is to 3 (i.e., a birth) is  $p_{23}/(p_{21} + p_{23}) = 0.2/(0.3 + 0.2) = 2/5$ .

**Lec. 26: Absorption probabilities and expected time to absorption**

Exercises 26 due May 18, 2016 at 23:59 UTC 

**Solved problems****Problem Set 10**

Problem Set 10 due May 18, 2016 at 23:59 UTC 

► Exit Survey

Finally, if the state is 3, the probability that the first change of state is a birth is equal to 0 since 3 is the highest state. Thus, the probability that the first change of state that we observe is a birth is equal to  $(1/5)(1) + (2/5)(2/5) = 9/25$ .

*You have used 2 of 2 submissions*

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