Exercise: Uniform PDF

(3/3 points)

Let X be uniform on the interval $[1, 3]$. Suppose that $1 < a < b < 3$. Then,

(a) $\mathbf{P}(a \leq X \leq b) =$ (b-a)/2 ✔ **Answer:** (b-a)/2

(Your answer to part (a) should be an algebraic expression involving a and b.)

(b) $\mathbf{E}[X] =$ 2 ✔ **Answer:** 2

(c) $\mathbf{E}[X^3] =$ 10 ✔ **Answer:** 10

Answer:

(a) The value of the PDF on the interval $[1, 3]$ must be equal to 1/2, so that it integrates to 1. Thus, $\mathbf{P}(a \leq X \leq b) = \int_a^b \frac{1}{2}\, dx = \frac{b-a}{2}$.

(b) The expected value of a uniform is the midpoint of its range: $\mathbf{E}[X] = (1+3)/2 = 2$.

(c) Using the expected value rule,

$$\mathbf{E}[X^3] = \int_1^3 x^3 \cdot \frac{1}{2}\, dx = \frac{1}{2} \cdot \frac{1}{4} x^4 \Big|_1^3 = \frac{1}{2} \cdot \frac{1}{4} \cdot (81 - 1) = 10.$$

You have used 1 of 2 submissions