6.2.2 Interactive: Patterns in Sequences

In Example 6.5.8 of this unit, we encountered the following surprising result: flipping a fair coin repeatedly until the pattern HT appears requires 4 flips on average, but for the pattern HH we need 6 flips on average! This interactive provides a simulation for this problem, and allows you to explore what happens with much more general patterns in sequences with two possible symbols (taken here to be x and o rather than H and T).

Patterns in Sequences - Directions for Use

1. Enter your desired pattern of x's and o's (with length between 1 and 15), using the "Add 'x'" and "Add 'o'" buttons.
2. Press "Generate Sequence" to generate a random sequence of x's and o's, where each term is x with probability 1/2 and o with probability 1/2, independently. Appearances of your pattern in the sequence are highlighted. **If your pattern has not yet appeared for the first time in the sequence shown on screen, the sequence is lengthened (without being shown on screen) until your pattern appears.**
3. Press "Generate Sequence 100 Times" to generate 100 such sequences (you can also press it repeatedly to accumulate a lot more data).
4. The Results panel gives summary statistics for how long it took for your pattern to appear. **The counts shown do not include the sequence itself.** For example, the theoretical expected value for the time until xo is 2 (rather than 4) and for the time until xx it is 4 (rather than 6).

> **You Should Try:**
>
> - Check that the mean and standard deviation make sense when the pattern has length 1 (just x or o).
> - Check that the means make sense when the pattern has length 2 (in particular, comparing xo to xx).
> - Which sequences of length 3 have the *smallest* possible average waiting time? Which sequences of length 3 have the *largest* possible average waiting time? Explore this via simulation.

Controls

Show screen reader enhancements: ☐

Add 'x' | Add 'o' | Reset

Pattern:

Generate Sequence | Generate Sequence 100 Times

Results

Current Sequence

Overall