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5. Critical points practice

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Lecture due Sep 13, 2021 20:30 IST Completed

Practice

Critical points concept check

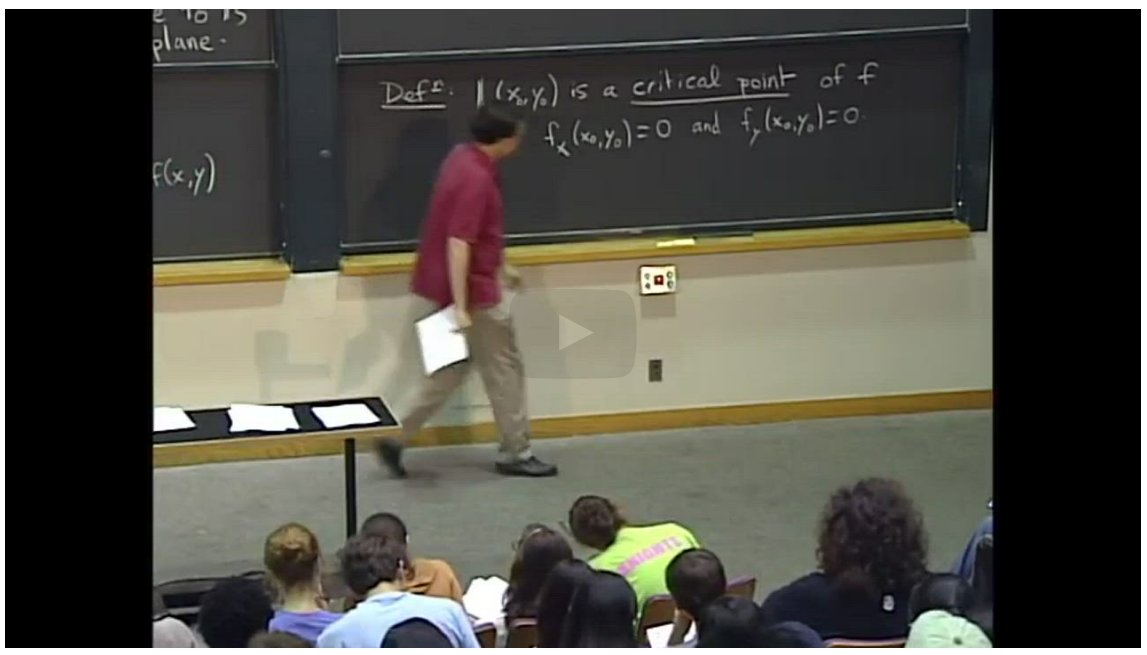
1/1 point (graded)

A point (x_0, y_0) is a critical point of $f(x, y)$ if

- $f(x_0, y_0) = 0$
- $f_x(x_0, y_0) = 0$
- $f_y(x_0, y_0) = 0$
- $f_x(x_0, y_0) = 0$ or $f_y(x_0, y_0) = 0$
- ✔ $f_x(x_0, y_0) = 0$ and $f_y(x_0, y_0) = 0$

Submit You have used 1 of 2 attempts

Worked example



0:00 / 0:00 2.0x

Start of transcript. Skip to the end.

PROFESSOR: So let's see an example.

So let's say, I give you the function f of (x, y)

equals x squared minus 2xy plus 3 y squared plus 2x minus 2 y.

OK.

And let's try to figure out whether we

can minimize or maximize this.

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To find the critical points of a function

$$f(x, y) = x^2 - 2xy + 3y^2 + 2x - 2y$$

We compute the partial derivatives and set them both equal to zero

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We compute the partial derivatives and set them both equal to zero

$$f_x(x,y) = 2x - 2y + 2 = 0 \tag{4.17}$$

$$f_y(x,y) = -2x + 6y - 2 = 0 \tag{4.18}$$

To solve both of these equations simultaneously, we first take their sum, which tells us

$$4y = 0 \quad \Longrightarrow \quad y = 0$$

Plugging in $y = 0$ into the first equation gives us

$$2x + 2 = 0 \quad \Longrightarrow \quad x = -1.$$

Therefore there is one critical point, when $x = -1$ and $y = 0$, the point $(-1, 0)$.

Remaining question: Is it a maximum or a minimum, and how do we know?

Before we can determine what type of critical point it is, we should understand the different types of critical points. But first, it is your turn to practice finding critical points.

Critical points practice 1

1.0/1 point (graded)
Find the critical points of

$$h(x,y) = \frac{8}{3}x^3 + \frac{1}{3}y^3 - 8xy. \tag{4.19}$$

(Enter ordered pairs in parentheses, e.g. (x, y). If there is more than one point, separate with semicolons, e.g. (a, b); (c, d). You may type e for Euler's number, and pi for the mathematical constant π.)

Critical points of $h(x,y)$: (0,0);(2,4) ✔ **Answer:** (0,0);(2,4)

Solution:

We have

$$h_x(x,y) = 8x^2 - 8y \tag{4.20}$$

$$h_y(x,y) = y^2 - 8x. \tag{4.21}$$

Setting equation 4.20 to 0 gives

$$y = x^2. \tag{4.22}$$

Substituting 4.22 into 4.21 and setting it equal to zero gives

$$0 = \left(x^2\right)^2 - 8x = x^4 - 8x = x\left(x^3 - 8\right). \tag{4.23}$$

This implies $x = 0$ or $x = 2$. When $x = 0$, the equation $y = x^2$ gives $y = 0$. When $x = 2$, the equation $y = x^2$ gives $y = 4$. So the critical points of $h(x,y)$ are $(0,0)$ and $(2,4)$.

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Submit You have used 1 of 3 attempts

Answers are displayed within the problem

Critical points practice 2

1.0/1 point (graded)
Find the critical points of

$$g(x,y) = \frac{1}{2}x^6 + \frac{1}{2}y^6 - 3xy + 3. \tag{4.24}$$

(Enter ordered pairs in parentheses, e.g. (x, y). If there is more than one point, separate with semicolons, e.g. (a, b); (c, d). You may type e for Euler's number, and pi for the mathematical constant π.)

Critical points of $g(x,y)$: (0,0);(1,1);(-1,-1) ✔ **Answer:** (0,0);(1,1);(-1,-1)

Solution:

We have

$$g_x(x,y) = 3x^5 - 3y \tag{4.25}$$
$$g_y(x,y) = 3y^5 - 3x. \tag{4.26}$$

Setting these equations to zero gives the following two equations

$$y = x^5 \ (\text{from } g_x = 0) \tag{4.27}$$
$$x = y^5 \ (\text{from } g_y = 0). \tag{4.28}$$

Substituting 4.27 into 4.26 and setting it equal to zero gives

$$0 = 3y^5 - 3x = 3(x^5)^5 - 3x = 3x\left(x^{24} - 1\right). \tag{4.29}$$

Therefore, the critical points occur when $x = 0$, $x = 1$, and $x = -1$. Substituting these values for x back into the equation $y = x^5$ gives the ordered pairs

$$(0,0),\ (1,1),\ (-1,-1). \tag{4.30}$$

Submit You have used 1 of 3 attempts

Answers are displayed within the problem

5. Critical points practice

Topic: Unit 3: Optimization / 5. Critical points practice

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[Staff] Could not format HTML for problem. Contact course staff in the discussion forum for assistance. 5
I keep getting this problem; it was working fine until yesterday.

First problem score- critical points practice 1

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The first and second time I attempted it, I got a partial score 0.5/1. The third time it was marked wrong but I still got one of the critic...

	minor typo in solution to concept check Hello. " ∇f(x0,y0)=0." needs a vector symbol over the zero. Best wishes. P.S. Bonus. Typo in the solution to problem one: "Subsitutin...	2
?	Might "Critical Points Practice 1" and "C.P.P 2" Have Additional Critical Points? Practice 1 Equation (4.23) in the solution given could be considered to have more solutions than recognized by the the rest of the so...	9
	question 1 grader Hi, I gave three critical points for this answer, two of which were the given answer and 1 that is valid but missing from the set of ans...	2
	[Staff] grades not showing for this lecture Hi Staff- It doesn't look like I'm getting credit for the exercises in this lecture when I check the 'Progress' tab despite my answers bei...	5

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