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10. MLEs for gaussian distribution

MLEs for gaussian distribution

But we are going to make it slightly more interesting

and, instead of telling you that we're just

going to always assume we have one mean and one variance,

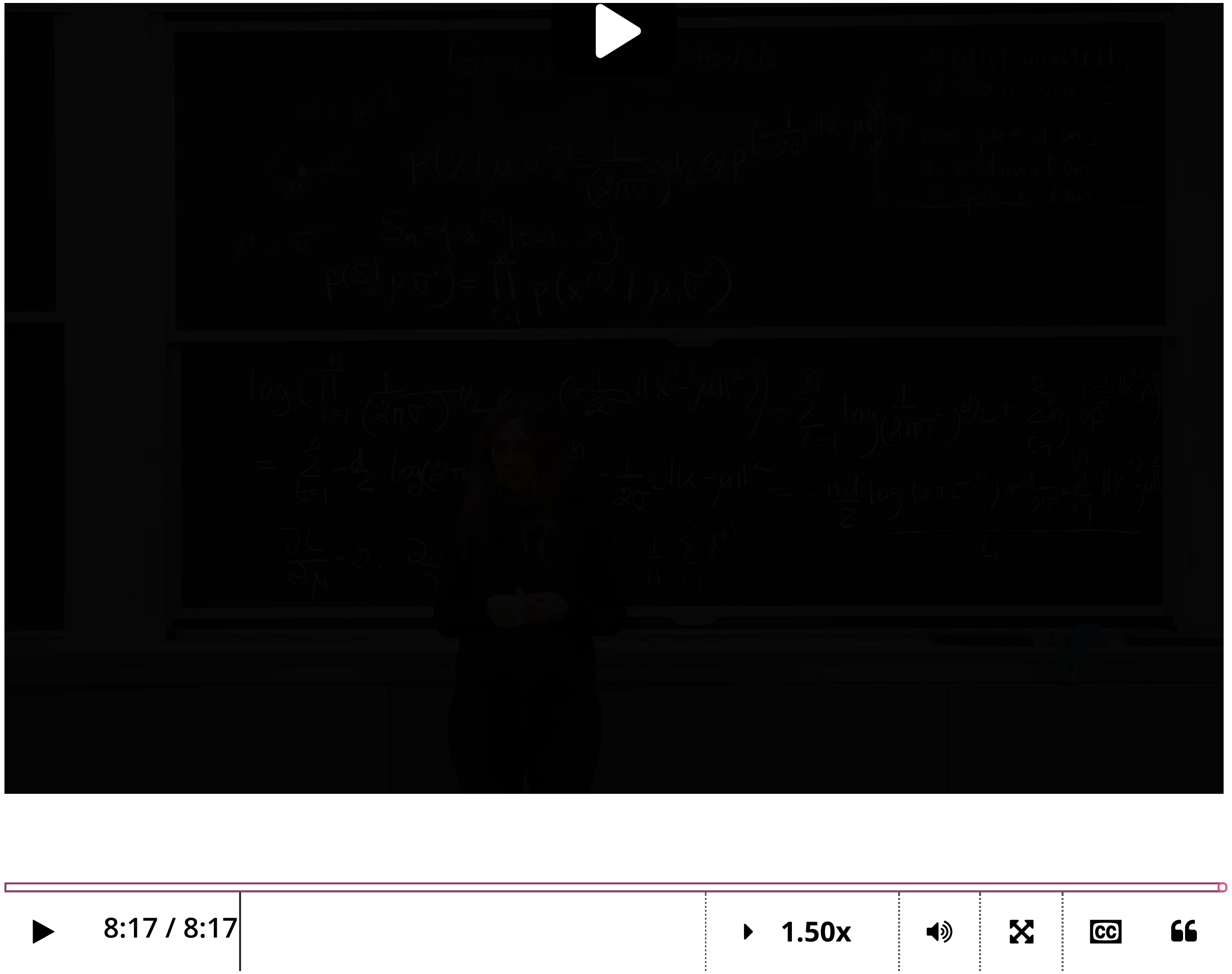
we would actually assume that there

can be many different means, and there can be many different

crowns.

We will see how this expansion is done

and how we can estimate it.



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MLE estimates for a gaussian distribution

1/1 point (graded)

In this problem, we will derive the maximum likelihood estimates for a gaussian model.

Let X be a gaussian random variable in d-dimensional real space (R^d) with mean μ and standard deviation σ.

Note that μ, σ are the parameters of a gaussian generative model.

Recall from the lecture that, the probability density function for a gaussian random variable is given as follows:

$$f_X\left(x|\mu,\sigma^2\right) = \frac{1}{\left(2\pi\sigma^2\right)^{d/2}} e^{-\|x-\mu\|^2/2\sigma^2}$$

Let $S_n = \{X^{(1)}, X^{(2)}, \dots X^{(t)}\}$ be identical idependent random variables following a gaussian distribution with mean μ and variance σ^2 .

Then their joint probability density function is given by

$$\Pi_{t=1}^{n} P\left(x^{(t)}|\mu,\sigma^2\right) = \Pi_{t=1}^{n} \frac{1}{\left(2\pi\sigma^2\right)^{d/2}} e^{-\left\|x^{(t)}-\mu\right\|^2/2\sigma^2}$$

Taking logarithm of the above function, we get

$$log\{\Pi_{t=1}^{n} \frac{1}{\left(2\pi\sigma^2\right)^{d/2}} e^{-\left\|x^{(t)}-\mu\right\|^2/2\sigma^2}\}$$

$$= \sum_{t=1}^{n} log \frac{1}{\left(2\pi\sigma^2\right)^{d/2}} + \sum_{t=1}^{n} log e^{-\left\|x^{(t)}-\mu\right\|^2/2\sigma^2}$$

$$= \sum_{t=1}^{n} -\frac{d}{2} log\left(2\pi\sigma^2\right) + \sum_{t=1}^{n} log e^{-\left\|x^{(t)}-\mu\right\|^2/2\sigma^2}$$

$$logP\left(S_n|\mu,\sigma^2\right) = -\frac{nd}{2} log\left(2\pi\sigma^2\right) - \frac{1}{2\sigma^2}\sum_{t=1}^{n} \left\|x^{(t)}-\mu\right\|^2$$

Compute the partial derivative $\frac{\partial logP(S_n|\mu,\sigma^2)}{\partial\mu}$ using the above derived expression for $P\left(S_n|\mu,\sigma^2\right)$.

Choose the correct expression from options below.

○ $\frac{\partial logP(S_n|\mu,\sigma^2)}{\partial\mu} = -\frac{1}{\sigma^2}\sum_{t=1}^{n} \left\|x^{(t)}-\mu\right\|$

● $\frac{\partial log P(S_n|\mu,\sigma^2)}{\partial \mu} = \frac{1}{\sigma^2}\sum_{t=1}^{n}||x^{(t)}-\mu||$ ✓

○ $\frac{\partial log P(S_n|\mu,\sigma^2)}{\partial \mu} = \frac{1}{\mu^2}\sum_{t=1}^{n}||x^{(t)}-\mu||$

○ $\frac{\partial log P(S_n|\mu,\sigma^2)}{\partial \mu} = -\frac{1}{\mu^2}\sum_{t=1}^{n}||x^{(t)}-\mu||$

Solution:

$$\frac{\partial}{\partial \mu} log P\left(S_n|\mu,\sigma^2\right)$$

$$= -\frac{1}{2\sigma^2}\sum_{t=1}^{n} -2||x^{(t)}-\mu||$$

$$= \frac{1}{\sigma^2}\sum_{t=1}^{n}||x^{(t)}-\mu||$$

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ℹ Answers are displayed within the problem

MLE for the mean

1/1 point (graded)

Use the answer from the previous problem in order to solve the following equation

$$\frac{\partial log P\left(S_n|\mu, \sigma^2\right)}{\partial \mu} = 0$$

Compute expression for $\hat{\mu}$ that is a solution for the above equation.

Choose the correct expression from options below

○ $\hat{\mu} = \Pi_{t=1}^{n} x^{(t)}$

○ $\hat{\mu} = \frac{\Pi_{t=1}^{n} x^{(t)}}{n}$

○ $\hat{\mu} = \sum_{t=1}^{n} x^{(t)}$

● $\hat{\mu} = \frac{\sum_{t=1}^{n} x^{(t)}}{n}$ ✓

Solution:

Recall from the previous solution that

$$\frac{\partial logP\left(S_n|\mu,\sigma^2\right)}{\partial\mu} = \frac{1}{\sigma^2}\sum_{t=1}^{n}\|x^{(t)} - \mu\|$$

Setting the above expression to zero, we get:

$$\frac{1}{\sigma^2}\sum_{t=1}^{n}\|x^{(t)} - \hat{\mu}\| = 0$$

$$\sum_{t=1}^{n}\|x^{(t)} - \hat{\mu}\| = 0$$

$$\sum_{t=1}^{n} (x^{(t)}) - n\hat{\mu} = 0$$

Resulting in the final expression for $\hat{\mu}$ as follows:

$$\hat{\mu} = \frac{\sum_{t=1}^{n} x^{(t)}}{n}$$

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MLE for the variance

1/1 point (graded)

Compute the partial derivative $\frac{\partial log P(S_n|\mu,\sigma^2)}{\partial\sigma^2}$ using the above derived expression for $P\left(S_n|\mu,\sigma^2\right)$ which is restated below as well:

$$log P\left(S_n|\mu,\sigma^2\right) = -\frac{nd}{2} log\left(2\pi\sigma^2\right) - \frac{1}{2\sigma^2}\sum_{t=1}^{n} \left\|x^{(t)} - \mu\right\|^2$$

Choose the correct expression from options below.

○ $\frac{\partial logP(S_n|\mu,\sigma^2)}{\partial\sigma^2} = \frac{nd}{2\sigma^2} + \frac{\sum_{t=1}^{n} \|x^{(t)}-\mu\|^2}{2(\sigma^2)^2}$

● $\frac{\partial logP(S_n|\mu,\sigma^2)}{\partial\sigma^2} = -\frac{nd}{2\sigma^2} + \frac{\sum_{t=1}^{n} \|x^{(t)}-\mu\|^2}{2(\sigma^2)^2}$ ✓

○ $\frac{\partial logP(S_n|\mu,\sigma^2)}{\partial\sigma^2} = \frac{nd}{2\sigma^2} - \frac{\sum_{t=1}^{n} \|x^{(t)}-\mu\|^2}{2(\sigma^2)^2}$

○ $\frac{\partial logP(S_n|\mu,\sigma^2)}{\partial\sigma^2} = \frac{\sum_{t=1}^{n} \|x^{(t)}-\mu\|^2}{2(\sigma^2)^2}$

Solution:

$$\frac{\partial logP\left(S_n|\mu,\sigma^2\right)}{\partial\sigma^2} = \frac{\partial}{\partial\sigma^2}\{-\frac{nd}{2}log\left(2\pi\sigma^2\right)\} - \frac{\partial}{\sigma^2}\{\frac{1}{2\sigma^2}\sum_{t=1}^{n}\|x^{(t)}-\mu\|^2\}$$

$$\frac{\partial logP\left(S_n|\mu,\sigma^2\right)}{\partial\sigma^2} = -\frac{nd}{2\sigma^2} + \frac{\sum_{t=1}^{n}\|x^{(t)}-\mu\|^2}{2(\sigma^2)^2}$$

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MLE for the variance

1/1 point (graded)

Using the answer from the previous problem in order to solve the equation

$$\frac{\partial logP\left(S_n|\mu,\sigma^2\right)}{\partial\sigma^2}=0$$

Compute expression for $\hat{\sigma}^2$ that is a solution for the above equation.

Choose the correct expression from options below

● $\hat{\sigma}^2 = \frac{\sum_{t=1}^{n}\left(x^{(t)}-\mu\right)^2}{nd}$ ✓

○ $\hat{\sigma}^2 = -\frac{\sum_{t=1}^{n}\left(x^{(t)}-\mu\right)^2}{nd}$

○ $\hat{\sigma}^2 = -\frac{\sum_{t=1}^{n}\left(x^{(t)}-\mu\right)^2}{n}$

○ $\hat{\sigma}^2 = -\frac{\Pi_{t=1}^{n}(x^{(t)}-\mu)^2}{nd}$

Solution:

Recall from the previous solution that

$$\frac{\partial logP\left(S_n|\mu,\sigma^2\right)}{\partial\sigma^2} = -\frac{nd}{2\sigma^2} + \frac{\sum_{t=1}^{n}\left\|x^{(t)}-\mu\right\|^2}{2(\sigma^2)^2}$$

Setting the above expression to zero, we get:

$$-\frac{nd}{2\sigma^2} + \frac{\sum_{t=1}^{n}\left\|x^{(t)}-\mu\right\|^2}{2(\sigma^2)^2} = 0$$

$$nd = \frac{\sum_{t=1}^{n}\left\|x^{(t)}-\mu\right\|^2}{\sigma^2}$$

The above equation leads us to our final expression for $\hat{\sigma}^2$:

$$\hat{\sigma}^2 = \frac{\sum_{t=1}^{n}\left(x^{(t)}-\mu\right)^2}{nd}$$

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[Staff] Question 1 2

LaTeX notation \log 1