Moment Generating Function for a Continuous Uniform Random Variable U distributed on the interval $[a, b]$.

$$M_u(t) = E[e^{tu}] = \int_a^b (e^{tu}) \frac{1}{b-a} \, du$$

$$E[U] = \frac{\partial}{\partial t} M_u(t) \Big|_{t=0} = M'_u(0) = \frac{a+b}{2} \checkmark$$

$$E[U^2] = \frac{\partial^2}{\partial t^2} M_u(t) \Big|_{t=0} = M''_u(0) = \frac{a^2 + ab + b^2}{3}$$

$$\text{So } \operatorname{Var}(U) = E(U^2) - (E(U))^2 = \frac{a^2 + ab + b^2}{3} - \left(\frac{a+b}{2}\right)^2$$

$$= \frac{(b-a)^2}{12} \cdot \checkmark$$