3. Higher order partial derivatives

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Lecture due Sep 13, 2021 20:30 IST Completed

Explore

Consider a function $f(x, y)$. The second partial derivative with respect to x is computed by taking the partial derivative with respect to x twice. The notation for this is

$$\frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\right) = \frac{\partial^2 f}{\partial x^2} = (f_x)_x = f_{xx}. \tag{4.52}$$

Example 3.1 Let $f(x, y) = y^4 x^2$. Then the second partial with respect to x is given by

$$f_{xx} = \frac{\partial}{\partial x}\left(2y^4 x\right) = 2y^4. \tag{4.53}$$

We can also take second order derivatives in different variables. For example, if we first take the partial of f with respect to y and then take the partial with respect to x, we have

$$\frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right) = \frac{\partial^2 f}{\partial x \partial y} = (f_y)_x = f_{yx}. \tag{4.54}$$

Example 3.2 Let $f(x, y) = y^4 x^2$. Then

$$f_{yx} = \frac{\partial}{\partial x}\left(4y^3 x^2\right) = 8y^3 x. \tag{4.55}$$

Notice the two different notations (Leibniz and subscript). Using Leibniz notation, we first take the derivative of f with respect to the variable written closest to f. So

$$\frac{\partial^2 f}{\partial x \partial y} \tag{4.56}$$

means we first take the derivative with respect to y to obtain a new function $\partial f/\partial y$. We then take the derivative of that function with respect to x. Using subscript notation, we still first take the derivative of f with respect to the variable written closest to f, but in this case, the order we write them looks reversed because the variables are on the other side of f. For example,

$$f_{xy} = \frac{\partial^2 f}{\partial y \partial x} \tag{4.57}$$

as above.

Important Fact: If a function has continuous second partial derivatives, then $f_{xy} = f$ v
explore in this course satisfy this criteria, so we will not need to worry about the orde p

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derivatives.

Example 3.3 We saw that for $f(x, y) = y^4 x^2$, we have $f_{yx} = 8y^3 x$. Notice that we can also compute

$$f_{xy} = \frac{\partial}{\partial y}\left(2y^4 x\right) = 8y^3 x. \qquad (4.58)$$

Higher derivatives

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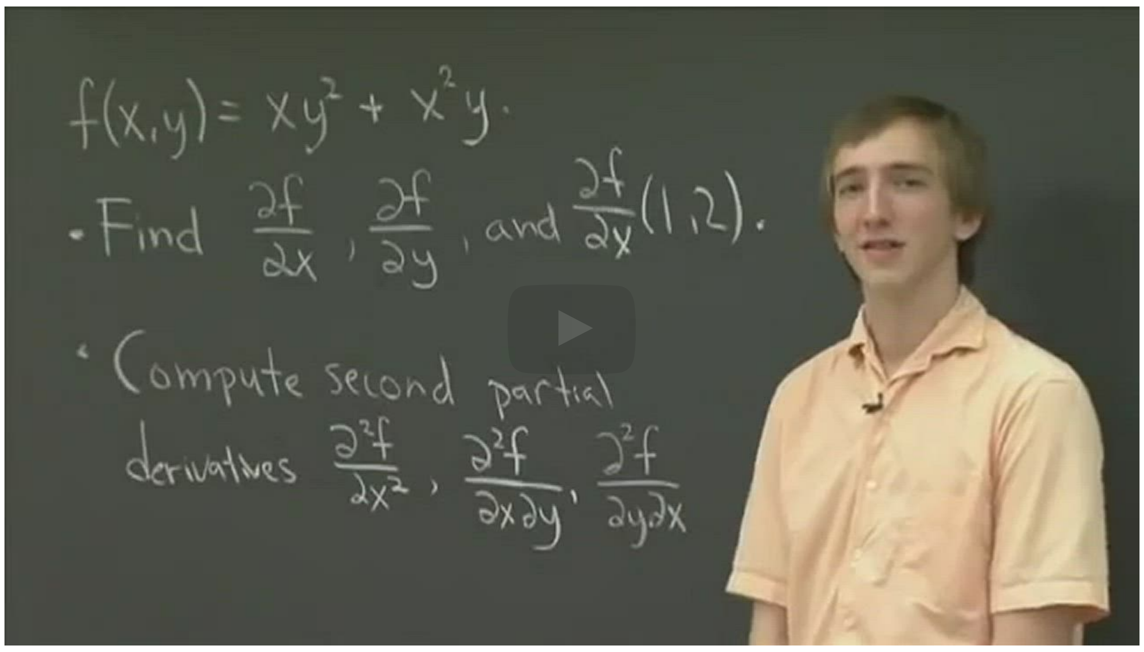

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PROFESSOR: Hello, and welcome back to recitation.

The problem I like to work with you

now is, simply, to compute some partial derivatives using

the definitions we learned today in lecture.

So first we're going to compute the partial derivative

in the x direction, of this function

xy squared plus x squared y.

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Practice 1

3/3 points (graded)
Let $u(x, t) = x^4 t^2 - x^3 t^5$.

Compute:

$u_{xx}(x, t) =$ 12*x^2*t^2-6*x*t^5 ✔ **Answer:** 12*x^2*t^2-6*x*t^5

$u_{tt}(x, t) =$ 2*x^4-20*x^3*t^3 ✔ **Answer:** 2*x^4-20*x^3*t^3

$u_{xt}(x, t) =$ 8*x^3*t-15*x^2*t^4 ✔ **Answer:** 8*x^3*t-15*x^2*t^4

Solution:

- $$\begin{aligned} u_{xx}(x, t) &= \frac{\partial}{\partial x}\left(4x^3t^2 - 3x^2t^5\right) \\ &= 12x^2t^2 - 6xt^5 \end{aligned}$$

$$u_{tt}(x, t) = \frac{\partial}{\partial t}\left(2x^4t - 5x^3t^4\right)$$

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- $$\begin{aligned} u_{tt}(x,t) &= \frac{\partial}{\partial t}\left(2x^4 t - 5x^3 t^4\right) \\ &= 2x^4 - 20x^3 t^3 \end{aligned}$$
- $$\begin{aligned} u_{xt}(x,t) &= \frac{\partial}{\partial t}\left(4x^3 t^2 - 3x^2 t^5\right) \\ &= 8x^3 t - 15x^2 t^4 \end{aligned}$$

Note that for the third computation, we could have computed

$$\begin{aligned} u_{tx}(x,t) &= \frac{\partial}{\partial x}\left(2x^4 t - 5x^3 t^4\right) \\ &= 8x^3 t - 15x^2 t^4. \end{aligned}$$

Submit You have used 2 of 5 attempts

Answers are displayed within the problem

Practice 2

3/3 points (graded)

Let $g(x,y) = e^{2y}\cos(3x)$.

Compute:

$g_{xx}(x,y) =$ -9*e^(2*y)*cos(3*x) **Answer:** -9*exp(2*y)*cos(3*x)

$g_{yy}(x,y) =$ 4*e^(2*y)*cos(3*x) **Answer:** 4*exp(2*y)*cos(3*x)

$g_{xy}(x,y) =$ -6*e^(2*y)*sin(3*x) **Answer:** -6*exp(2*y)*sin(3*x)

Solution:

- $$\begin{aligned} g_{xx}(x,y) &= \frac{\partial}{\partial x}\left(-3e^{2y}\sin(3x)\right) \\ &= -9e^{2y}\cos(3x) \end{aligned}$$
- $$\begin{aligned} g_{yy}(x,y) &= \frac{\partial}{\partial y}\left(2e^{2y}\cos(3x)\right) \\ &= 4e^{2y}\cos(3x) \end{aligned}$$
- $$\begin{aligned} g_{xy}(x,y) &= \frac{\partial}{\partial y}\left(-3e^{2y}\sin(3x)\right) \\ &= -6e^{2y}\sin(3x) \end{aligned}$$

Note that for the third computation, we could have computed

$$\begin{aligned} g_{yx}(x,y) &= \frac{\partial}{\partial x}\left(2e^{2y}\cos(3x)\right) \\ &= -6e^{2y}\sin(3x). \end{aligned}$$

Submit You have used 2 of 5 attempts

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What textbook did they use when they taught this class live at MIT ?
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Geometric significance of Fxy
Fxx can be thought of as a measure of how much the function bends in x direction. Is there any geometric significance like that for F...
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