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[Lecture 8: Distance measures](#)

7. Properties of Total Variation

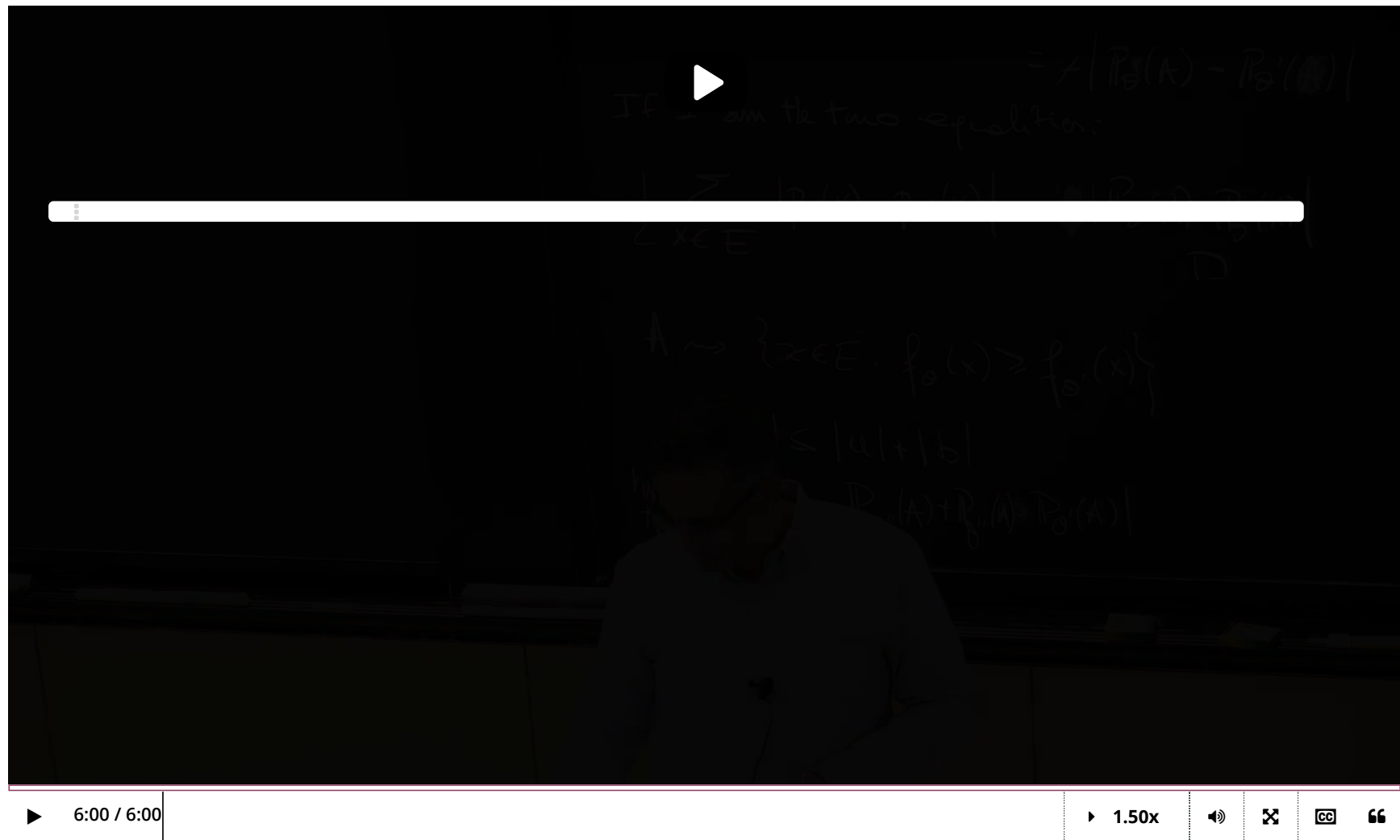
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> Distance

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7. Properties of Total Variation Distance

Properties of Total Variation Distance



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Let d be a function that takes two probability measures \mathbf{P} and \mathbf{Q} and maps them to a real number $d(\mathbf{P}, \mathbf{Q})$. Then d is a **distance** on probability measures if the following four axioms hold. (Here, \mathbf{P} , \mathbf{Q} , and \mathbf{V} are all probability measures.)

- $d(\mathbf{P}, \mathbf{Q}) = d(\mathbf{Q}, \mathbf{P})$ (symmetric)
- $d(\mathbf{P}, \mathbf{Q}) \geq 0$ (nonnegative)
- $d(\mathbf{P}, \mathbf{Q}) = 0 \iff \mathbf{P} = \mathbf{Q}$ (definite)
- $d(\mathbf{P}, \mathbf{V}) \leq d(\mathbf{P}, \mathbf{Q}) + d(\mathbf{Q}, \mathbf{V})$ (triangle inequality)

In the above, $\mathbf{P} = \mathbf{Q}$ means $\mathbf{P}(A) = \mathbf{Q}(A)$ for $A \subset E$, where E is the common sample space of \mathbf{P} and \mathbf{Q} .

The total variation distance (TV) is a distance on probability measures.

Symmetry and Definiteness of Total Variation Distance

1/1 point (graded)

Let \mathbf{P} be a probability measure. Which of the following is (are) true?

☐ One can find a measure $\mathbf{Q} \neq \mathbf{P}$ such that $\text{TV}(\mathbf{P}, \mathbf{Q}) = 0$.

☒ $\text{TV}(\mathbf{P}, \mathbf{Q}) = \text{TV}(\mathbf{Q}, \mathbf{P})$.



Solution:

Choice 1 is not true because of the following: By definition, $\mathbf{Q} \neq \mathbf{P}$ means that there is some set A of non-zero measure over which the measures \mathbf{Q} and \mathbf{P} are not the same. Therefore, over this set A , $|\mathbf{P}(A) - \mathbf{Q}(A)| > 0$, which implies that $\text{TV}(\mathbf{P}, \mathbf{Q}) \neq 0$.

Choice 2 (symmetry) is true because for any set A , $|\mathbf{P}(A) - \mathbf{Q}(A)| = |\mathbf{Q}(A) - \mathbf{P}(A)|$.

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i Answers are displayed within the problem

Triangle Inequality

1/1 point (graded)

Which of the following quantities is greater than or equal to $\text{TV}(\text{Ber}(.5), \text{Ber}(0.3))$?

(Choose all that apply.)

☒ $\text{TV}(\text{Ber}(0.5), \text{Ber}(0.1)) + \text{TV}(\text{Ber}(0.1), \text{Ber}(0.3))$

☒ $\text{TV}(\text{Ber}(0.5), \text{Poiss}(5)) + \text{TV}(\text{Ber}(0.3), \text{Poiss}(5))$

☒ $\text{TV}(\text{Bin}(7, 0.4), \text{Ber}(0.5)) + \text{TV}(\text{Ber}(0.3), \text{Bin}(7, 0.4))$



Solution:

Recall the triangle inequality states that for distributions \mathbf{P} , \mathbf{Q} , and \mathbf{V} :

$$\text{TV}(\mathbf{P}, \mathbf{V}) \leq \text{TV}(\mathbf{P}, \mathbf{Q}) + \text{TV}(\mathbf{Q}, \mathbf{V}).$$

- If we set $\mathbf{P} = \text{Ber}(0.5)$, $\mathbf{V} = \text{Ber}(0.3)$, and $\mathbf{Q} = \text{Ber}(0.1)$, then applying the triangle inequality above gives the first upper bound.
- In the second choice, set $\mathbf{P} = \text{Ber}(0.5)$, $\mathbf{V} = \text{Ber}(0.3)$, and $\mathbf{Q} = \text{Poiss}(5)$ and apply the triangle inequality.
- In the third choice, set $\mathbf{P} = \text{Ber}(0.5)$, $\mathbf{V} = \text{Ber}(0.3)$, and $\mathbf{Q} = \text{Bin}(7, 0.4)$ and apply the triangle inequality.

Remark: Implicitly we are also using the symmetry property of total variation: $\text{TV}(\mathbf{P}, \mathbf{Q}) = \text{TV}(\mathbf{Q}, \mathbf{P})$.

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? Symmetry and Definiteness of Total Variation Distance

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I guess the question should be: Let P and Q be probability measures... - To avoid confusion.

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