3. Kernels

In this question, we will practice kernel methods in classification.

3. (a)

1.0/1 point (graded)

Let $x, q \in \mathbb{R}^2$ be two feature vectors, and let $K(x, q) = (x^T q + 1)^2$. This is often known as a polynomial kernel. It's simple to compute: you just take the dot product between two feature vectors, add one, and then square the result. But what kind of feature mapping does this kernel implicitly use?

Assuming we can write $K(x, q) = \phi(x)^T \phi(q)$, derive an expression for $\phi(x)$.

Enter the solution as a vector $\phi(x) = [f_1(x_1, x_2), \cdots, f_N(x_1, x_2)]$.

$\phi(x) =$ [1, sqrt(2)*x_1, sqrt(2)*x_2, x_1^2, x_2^2, sqrt(2)*x_1*x_2] ✔

Answer: [x_1^2, x_2^2, sqrt(2)*x_1*x_2, sqrt(2)*x_1, sqrt(2)*x_2, 1]

Solution:

- We can rewrite the kernel as $K(x,q) = (x^T q + 1)^2 = \left(1 + \sum_{i=1}^{2} x_i q_i\right)^2 = (x_1 q_1 + x_2 q_2 + 1)^2$.
- Expanding and combining terms gives $x_1^2 q_1^2 + x_2^2 q_2^2 + 2x_1 x_2 q_1 q_2 + 2x_1 q_1 + 2x_2 q_2 + 1$.
- We can then rewrite this expression as $\phi(x)^T \phi(q)$ where $\phi(x) = [x_1^2, x_2^2, \sqrt{2}x_1 x_2, \sqrt{2}x_1, \sqrt{2}x_2, 1]$.

Submit You have used 1 of 3 attempts

Answers are displayed within the problem

3. (b)

1/1 point (graded)

As a simple example that uses this kernel, imagine that our feature vectors were bag of words vectors. In this example, give an intuitive interpretation of what the $\sqrt{2}x_1 x_2$ term in the expression for $\phi(x)$ you just wrote down means.

- consecutive co-appearance (bigram)

◉ co-appeareance in document ✔

Solution:

- Each token in the bag-of-word model only represents appearance in the document.
- Hence, $x_1 x_2$ represents co-appearance in a document.

Submit You have used 1 of 1 attempt

ℹ Answers are displayed within the problem

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Difficult to understand Kernels 5
Hello everyone, I am struggling to understand how to work with kernels, how to find what is the rigth _phi(x)_ and how to interpret. Lectures didn...

3a what to include? 3
Hi I am 2 out of 3 attempts on this problem and I have 1/2 credit so I just want to make sure before I try anything else. For the terms of phi(x), do ...

☑	Standard Notation How do I write a matrix in Standard Notation? Can't find it anywhere in the guide Thanks in advance	2
☑	3 (a) Invalid input issue I enter f_1(x_1,x_2) and get the error message that the first parenthesis is not closed while it is actually closed. How should I enter the functions s...	4
💬	Understanding Kernels To get a feel for kernels, I followed the following approach, and might be helpful for someone: 1. go through the lectures 2. Check out the followi...	1
💬	Test case I was stuck for 20 minutes after the grader marked my answer as wrong, while the test case was telling that everything is fine. If it's your case, th...	4
☑	3a I am entering the solution as : [[1], [sqrt(2)*x_1, sqrt(2)*x_2] , ... other terms are here... Getting this error : Vector and matrix expressions have be...	2
💬	[STAFF] Invalid Input: f_1 not permitted in answer as a function I am getting this error in (a). It is stated I need to put f_1(x_1^2), f2(x_1), ... but solver doesn't accept my answer. What is wrong?	6
☑	3a, error showing Invalid Input: x1 not permitted in answer as a variable. why? 3a, error showing Invalid Input: x1 not permitted in answer as a variable. why? how should I represent the answer? x also is invalid input. can any...	2
💬	3. (a)	8
☑	1 square bracket was opened without being closed (highlighted below) I keep getting this, not sure how to formulate my answer in correct format. any hint? My N is 3 and I used x1 and x2 in my answer.	5
☑	3a) phi(x) entry requirements	8
☑	[Staff] 3b - Clarification	10
💬	3b : How to interpret	