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5. Variation of parameters Variation of parameters



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Long ago, we used **variation of parameters** to solve single first order inhomogeneous linear ODEs

$$\dot{x} + p(t)x = r(t).$$

We first find a solution x_h to the associated homogeneous equation, seek a particular solution of the form $x_p(t) = u(t)x_h(t)$, and use the original inhomogeneous ODE to solve for the unknown function u(t).

Now, we are going to use the same idea to solve an inhomogeneous linear $n \times n$ system of ODEs:

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{r},$$

where ${\bf r}$ is a vector-valued function of ${\bf t}$.

First, find a basis of solutions to the corresponding homogeneous system

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x}$$
.

Call the basis solutions $\mathbf{x}_1(t)$, $\mathbf{x}_2(t)$, \cdots , $\mathbf{x}_n(t)$. The general homogeneous solution is any linear combination of these:

Notice that in 1 dimension, $x_h(t) = cx_1(t) = x_1(t)c$, but in higher dimensions, $\mathbf{x}_h(t) = \mathbf{X}\mathbf{c}$ where \mathbf{c} is a column vector and must be placed to the right of the fundamental matrix \mathbf{X} .

To find a particular solution, we let the coefficients c_i vary with time. In other words, replace the constant vector ${f c}$ by the vector **function**

$$\mathbf{v}(t) = egin{pmatrix} v_1(t) \ v_2(t) \ dots \ v_n(t) \end{pmatrix}.$$

Now, substitute $\mathbf{x} = \mathbf{X}\mathbf{v}(t)$ in the original system:

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{r}$$
 $\dot{\mathbf{X}}\mathbf{v} + \mathbf{X}\dot{\mathbf{v}} = \mathbf{A}\mathbf{X}\mathbf{v} + \mathbf{r}$ (product rule of differentiation)
 $\mathbf{A}\mathbf{X}\mathbf{v} + \mathbf{X}\dot{\mathbf{v}} = \mathbf{A}\mathbf{X}\mathbf{v} + \mathbf{r}$ ($\dot{\mathbf{X}} = \mathbf{A}\mathbf{X}$)
 $\mathbf{X}\dot{\mathbf{v}} = \mathbf{r}$
 $\dot{\mathbf{v}} = \mathbf{X}^{-1}\mathbf{r}$ (\mathbf{X} invertible).

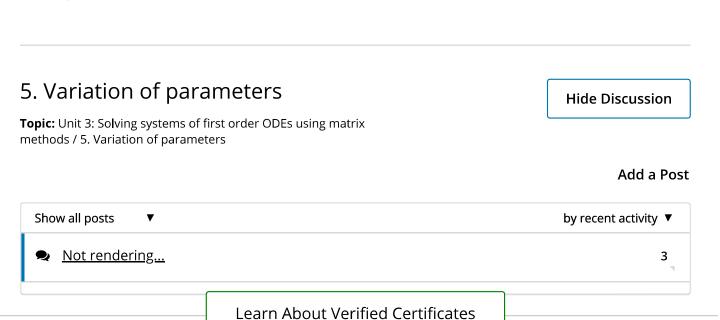
This means

$$\mathbf{v}(t) = \int \mathbf{X}^{-1} \mathbf{r} \, dt.$$

and the general solution to the inhomogeneous system is

for any fundamental matrix $\, {f X} \,$ of the associated homogeneous system.

This is a family of solutions because the indefinite integral on the right hand side will result in a constant of integration. Note that the constant of integration is a column vector.



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