2. Review bounded regions and constrained optimization

Bookmark this page

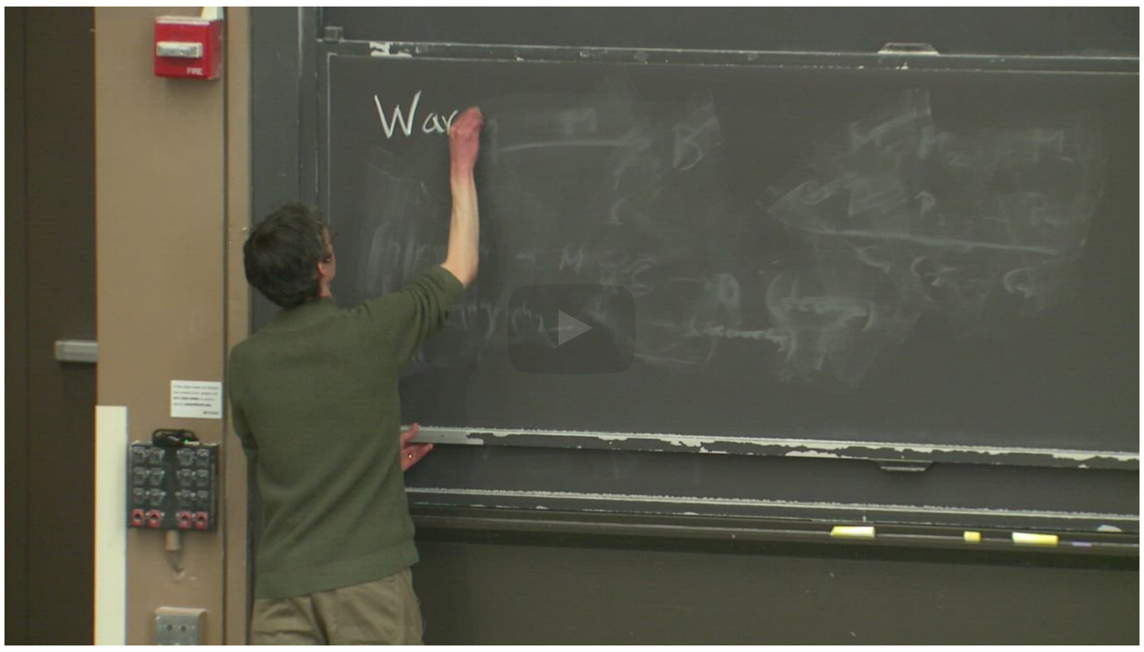
Review

In this lecture, we are going to learn how to solve constrained optimization problems when the constraint equation can be written as a level curve $g(x,y) = k$. In particular, we will learn the method of Lagrange multipliers, which specifies a relationship between the gradient of the function $f(x,y)$ we wish to maximize (or minimize) and the gradient of the constraint equation at the point that maximizes (or minimizes) $f(x,y)$ along the curve $g(x,y) = k$.

Before we get into the details, we are going to do some warm up problems to start thinking about how the gradient of a function along a constraint curve tells us whether that function increases or decreases along that curve. Pay particular attention to how this problem can be formulated as 'maximize $f(x,y)$ where (x,y) satisfies $g(x,y) = k$'.

We will start with a review of how to define closed and bounded regions.

Find the equation describing the region



0:00 / 0:00 2.0x

Start of transcript. Skip to the end.

PROFESSOR: Warm-up-- so here's our region R. It's a disk.

The center is at (2, comma 0), and the radius is 1.

And so the warm-up question is, find an equation

that describes this disk.

So let me draw a picture first, and then I'll give you

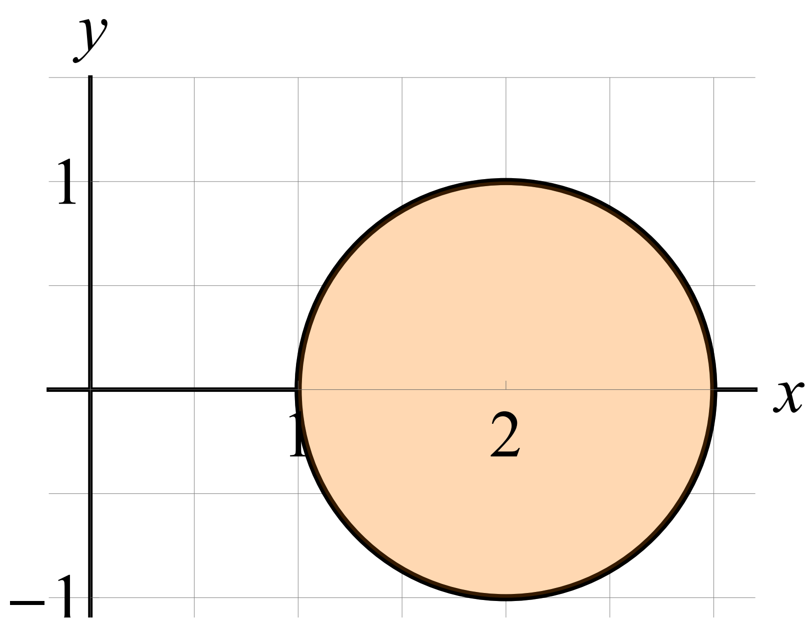
some choices for the equation.

So the disk looks like this.

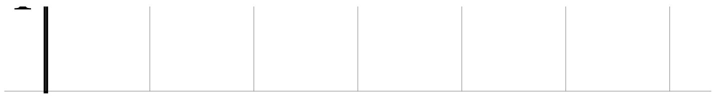
Video
Download video file

Transcripts
Download SubRip (.srt) file
Download Text (.txt) file

The region R is a disk with center $(2, 0)$ and radius 1.



Calculator Hide Notes



Find the equation that describes the disk of radius 1 centered at the point $(2, 0)$.

1. $(x-2)^2 + y^2 = 1$
2. $(x-4)^2 + y^2 = 1$
3. $(x-2)^2 + y^2 \leq 1$
4. $(x-4)^2 + y^2 \leq 1$

(Submit your answer in the poll below.)

POLL

Answer the question above. Select the number corresponding to the equation that describes the region.

RESULTS

1.	**14%**
2.	**1%**
3.	**84%**
4.	**1%**
I do not know how to think about this yet	**0%**

Submit

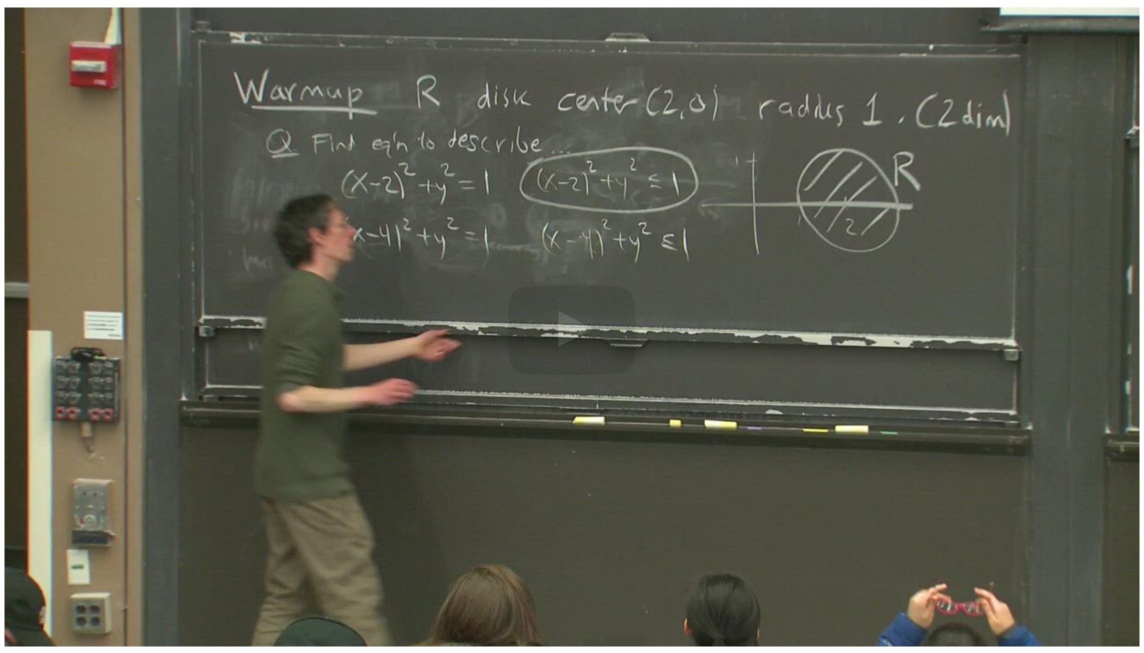
Results gathered from 472 respondents.

FEEDBACK

Your response has been recorded

Is the maximum inside or on the boundary?

Start of transcript. Skip to the end.



0:00 / 0:00 2.0x

PROFESSOR: This is the right answer.

And so let's talk for a second about why it's less

than or equal to versus equal.

So if I said x minus 2 squared plus y squared is equal to 1,

then that would be--

that would say that the distance from the point (2, 0)

Video

Download video file

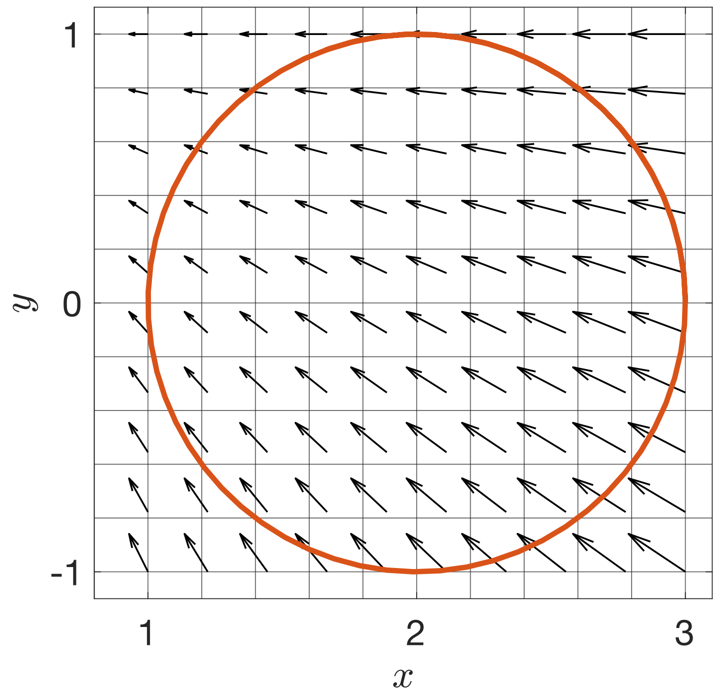
Transcripts

Download SubRip (.srt) file

Download Text (.txt) file

Calculator Hide Notes

Consider the gradient of the differentiable function $f(x, y)$ inside the disk shown below.



Since $f(x, y)$ is continuous, there is a point (a, b) where the function is largest on this disk. Is this point (a, b) on the inside of the disk or on the boundary circle?

POLL

Does the function f(x,y) attain its maximum on the inside of the disk or on the boundary circle?

RESULTS

Inside the disk	**2%**
On the boundary	**97%**
I do not know how to think about this yet	**1%**

Submit

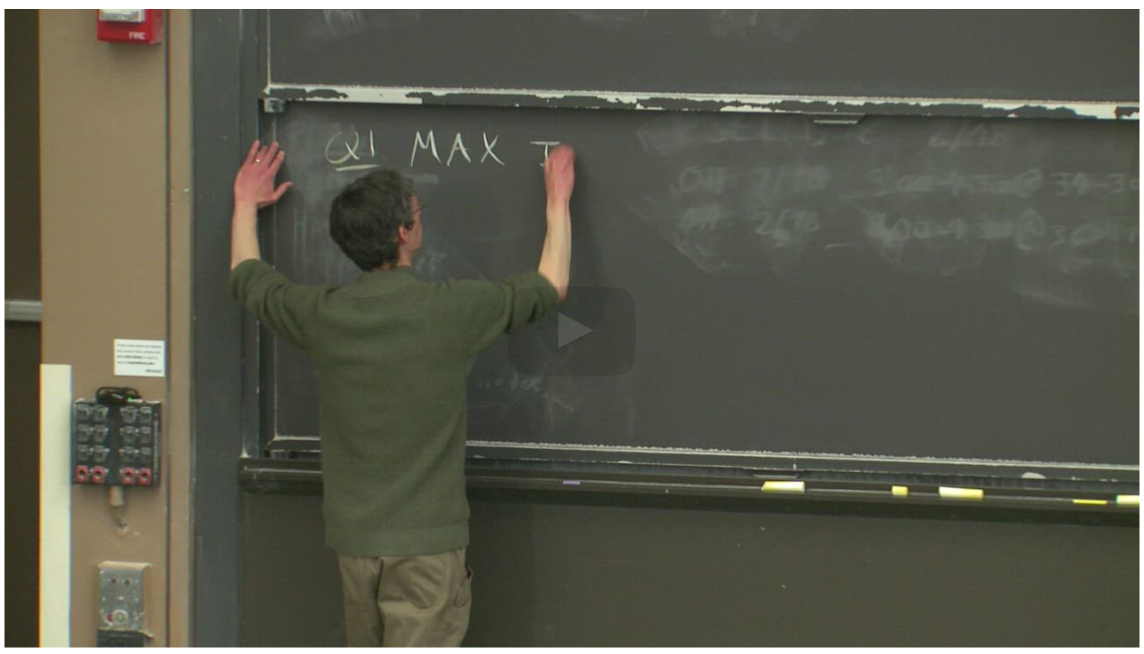
Results gathered from 473 respondents.

FEEDBACK

Your response has been recorded

Increasing or decreasing as you move clockwise?

Start of transcript. Skip to the end.



PROFESSOR: Maximum inside or on the boundary.

Answer?

It's on the boundary.

OK.

So now we want to think about where on the boundary.

So suppose that we start at (2, comma, minus 1).

0:00 / 0:00 2.0x

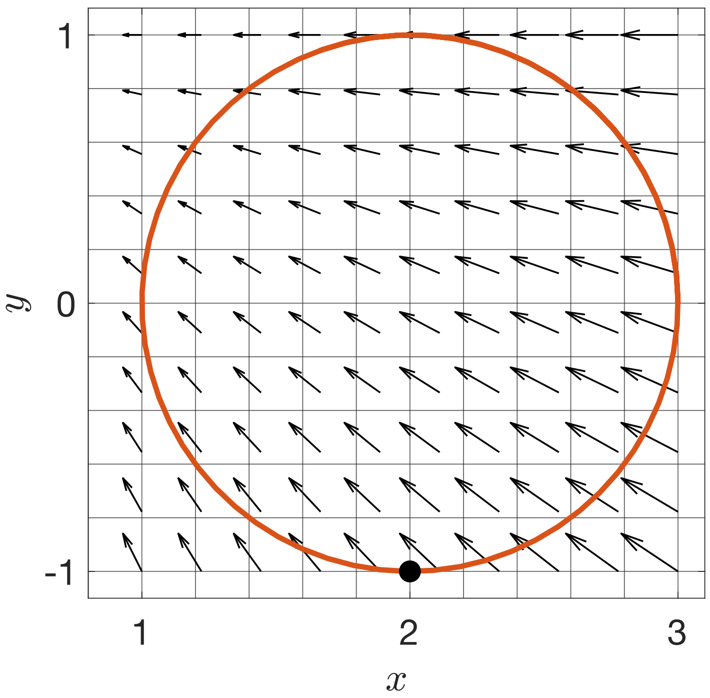
Calculator Hide Notes

Video
Download video file

Transcripts
Download SubRip (.srt) file
Download Text (.txt) file

Recall that the maximum of f over a closed and bounded region will occur either at a critical point or along the boundary. For there to be a critical point, we would need $\nabla f = 0$. This means that $|\nabla f| = 0$, or in other words, that the arrows of the gradient field would have a magnitude of 0. This does not occur in the figure shown and so the maximum must occur along the boundary.

Consider the gradient of the differentiable function $f(x, y)$ inside the disk shown below.



If you begin at the point $(2, -1)$ on the boundary of the circle, and move along the boundary clockwise (to the left), does the function $f(x, y)$ increase or decrease?

POLL
Starting from (2,-1) and moving to the left along the boundary, the function f(x,y)

RESULTS

Increases	**83%**
Decreases	**16%**
I do not know how to think about this yet	**1%**

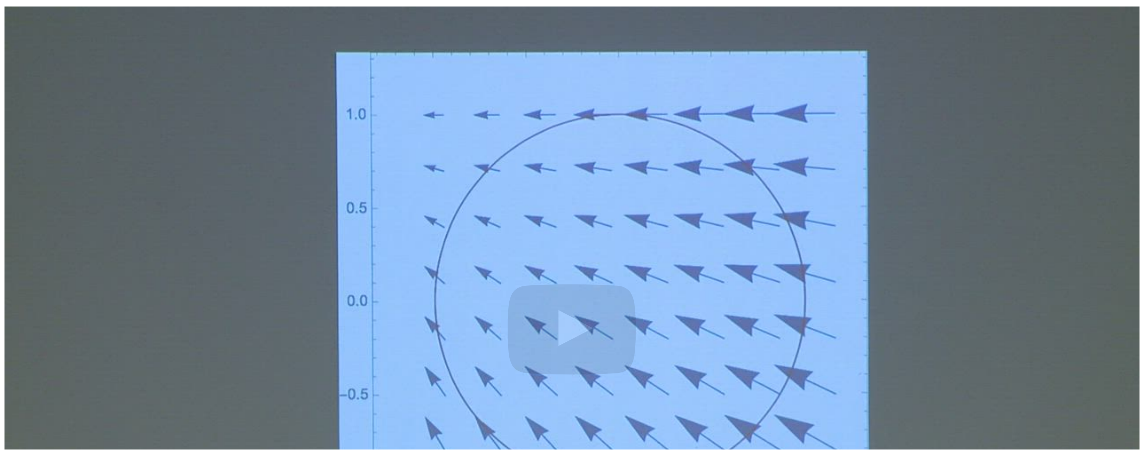
Submit

Results gathered from 466 respondents.

FEEDBACK
Your response has been recorded

Where is the maximum?

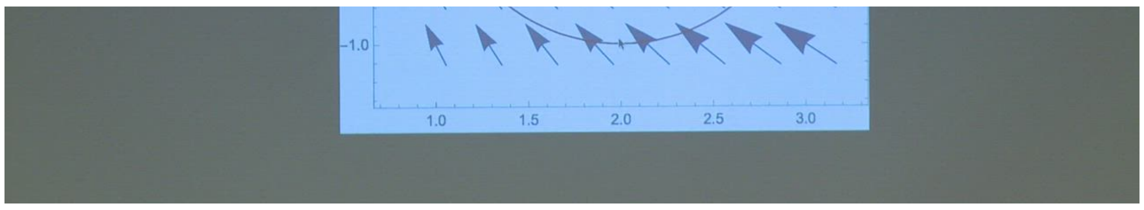
Start of transcript. Skip to the end.



PROFES... think.

Calculator

Hide Notes



0:00 / 0:00 2.0x

Thumbs up for getting bigger.
Thumbs up for getting smaller.
Good.
OK.
That's right.
The function is getting bigger.
At office hours yesterday, I came up
with a little, I don't know, slightly

Video
Download video file

Transcripts
Download SubRip (.srt) file
Download Text (.txt) file

Let's summarize the video above.

We've seen that level curves of a function $f(x, y)$ are perpendicular to the gradient of that function. The gradient points in the direction of steepest increase of that function.

The linear approximation of a function at a point is its tangent plane. That tangent plane is defined by a dot product equation specifying that the gradient of f at that point is normal to the level curves of that tangent plane. The direction of the plane's steepest increase is in the direction of the gradient of f at the given point.

Thus if you imagine such a tangent plane along the curve at a particular gradient vector, and image continuing along a curve, you are just asking if you move uphill or not. And in this case, we saw that we are going uphill. We are not moving uphill as steeply as possible, but it is still overall in the uphill direction.

---

2. Review bounded regions and constrained optimization

Hide Discussion

Topic: Unit 3: Optimization / 2. Review bounded regions and constrained optimization

Add a Post

Show all posts | by recent activity

[STAFF] Tiny Emendation — 2
"whether that function increases and decreases along that curve." Here 'and' should probably be replaced by 'or'.

minor typo, first paragraph — 2
"...which specifies a relationship between the gradient **of the** function f(x,y)..."

< Previous | Next >

Calculator | Hide Notes

edX

About
Affiliates
edX for Business
Open edX
Careers
News

Legal

Terms of Service & Honor Code
Privacy Policy
Accessibility Policy
Trademark Policy
Sitemap

Connect

Blog
Contact Us
Help Center
Media Kit
Donate

Calculator Hide Notes