Zeta Function of Elliptic Curves (7)

- The **zeta function** (**L-function**) of an elliptic curve is defined by

$$L(\mathrm{E}, s) = \left(\prod_{\mathrm{P}} \frac{1}{1 - (\mathrm{P} + 1 - \mathrm{N_P})\,\mathrm{P}^{-s} + \mathrm{P}^{-2s}} \right) \times (\text{Bad Factors})$$

Modularity Theorem (2nd form)
For an elliptic curve E, there is a **modular form f** satisfying

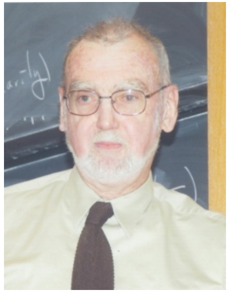
L(E,s) = L(f,s).

Zeta Function of Elliptic Curves (8)

- Modularity is only the beginning of the whole story.
- We have more general **symmetric power L-functions** $\mathbf{L(Sym^N\ E, s)}$ for $N \geq 1$.
- General Reciprocity Laws proposed by Langlands (**Langlands's program**) predict they are related to **automorphic forms**.

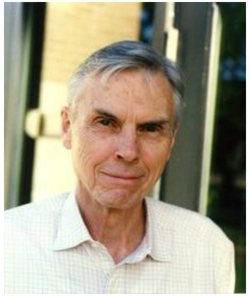
Zeta Function of Elliptic Curves (9)

- Partial solutions of Langlands's Conj implies striking applications, e.g., the **Sato-Tate Conj** (solved in 2011).


Robert Phelan Langlands (1936-)


Andrew John Wiles (1953-)

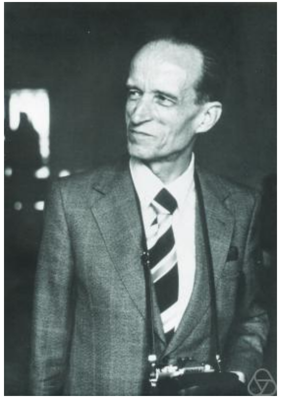

Mikio Sato (1928-)


John Torrence Tate, Jr. (1925-)

https://en.wikipedia.org/wiki/Robert_Langlands
Notices of the AMS, vol 54, Num 2 (2007), p.210
https://en.wikipedia.org/wiki/John_Tate

Interlude: Five Fundamental Operations

"There are **five fundamental operations** in mathematics: **addition**, **subtraction**, **multiplication**, **division**, and **modular forms**." (Martin Eichler, 1912-1992)



https://en.wikipedia.org/wiki/Martin_Eichler