



## MITx: 6.041x Introduction to Probability - The Science of Uncertainty



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## Problem 3: The sample mean

(5/5 points)

Let  $\mathbf{X}$  be a continuous random variable. We know that it takes values between  $\mathbf{0}$  and  $\mathbf{3}$ , but we do not know its distribution or its mean and variance. We are interested in estimating the mean of  $\mathbf{X}$ , which we denote by  $\mathbf{h}$ . We will use  $\mathbf{1.5}$  as a conservative value (upper bound) for the standard deviation of  $\mathbf{X}$ . To estimate  $\mathbf{h}$ , we take  $\mathbf{n}$  i.i.d. samples  $\mathbf{X}_1, \mathbf{X}_2, \dots, \mathbf{X}_n$ , which all have the same distribution as  $\mathbf{X}$ , and compute the sample mean

$$H = \frac{1}{n} \sum_{i=1}^n X_i.$$

1. Express your answers for this part in terms of  $\mathbf{h}$  and  $\mathbf{n}$  using standard notation .

 $\mathbf{E}[H] =$ 

h



Given the available information, the smallest upper bound for  $\mathbf{var}(H)$  is:

► Unit 6: Further topics on random variables


► Unit 7: Bayesian inference

► Exam 2


▼ Unit 8: Limit theorems and classical statistics

#### Unit overview


#### Lec. 18: Inequalities, convergence, and the Weak Law of Large Numbers

Exercises 18 due Apr 27, 2016 at 23:59 UTC 

#### Lec. 19: The Central Limit Theorem (CLT)

Exercises 19 due Apr 27, 2016 at 23:59 UTC 

#### Lec. 20: An introduction to classical statistics

Exercises 20 due Apr 27, 2016 at 23:59 UTC 

2.25/n



2. Calculate the smallest possible positive value of  $n$  such that the standard deviation of  $H$  is guaranteed to be at most **0.01**.

22500



This minimum value of  $n$  is:

3. We would like to be at least **99%** sure that our estimate is within **0.05** of the true mean  $h$ . Using the Chebyshev inequality, calculate the minimum value of  $n$  that will achieve this.

90000



This minimum value of  $n$  is:


4. Assume that  $X$  is uniformly distributed on  $[0, 3]$ . Using the Central Limit Theorem, identify the most appropriate expression for a **95%** confidence interval for  $h$ .

●  $\left[ H - \frac{1.96}{\sqrt{n}}, H + \frac{1.96}{\sqrt{n}} \right]$

## Solved problems

## Additional theoretical material

## Problem Set 8

Problem Set 8 due Apr 27, 2016  
at 23:59 UTC 

## Unit summary

- Unit 9: Bernoulli and Poisson processes

☐  $\left[ H - \frac{\sqrt{1.96 \cdot 3}}{\sqrt{4n}}, H + \frac{\sqrt{1.96 \cdot 3}}{\sqrt{4n}} \right]$

☒  $\left[ H - \frac{1.96\sqrt{3}}{\sqrt{4n}}, H + \frac{1.96\sqrt{3}}{\sqrt{4n}} \right]$  ✓

☐  $\left[ H - \frac{1.96 \cdot 3}{\sqrt{4n}}, H + \frac{1.96 \cdot 3}{\sqrt{4n}} \right]$

*You have used 1 of 2 submissions*

## DISCUSSION

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