Finding the Fisher's Information in a normal distribution with known μ and unknown σ^2

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I have a point statistic $x_i, \ldots, x_n X \in N(\mu, \sigma^2)$, μ is known.

I have to apply the Rao-cramer theorem but calculating the Fisher's information I stumbled upon this problem:

$$I(\sigma) = -E(\frac{n}{\gamma} + 3\sum(\frac{x_i-\mu)^2}{\sigma^2}) = \frac{n}{\gamma} + 3\frac{E(\sum(x_i-\mu)^2)}{E\sigma^4} = \frac{n}{\gamma} + \frac{3}{\sigma^4}E(\sum(x_i-\mu)^2)$$

$$E(\sum(x_i-\mu)^2) = ?$$

$\mathrm{E}[X] = \int_{\mathbb{R}} x f(x)\, dx$. But what is f(x) here ? Could it possibly be the function itself $\mathrm{E}[X] = \int_{\mathbb{R}} x \sum(x_i - \mu)2\, dx$.

From wiki, we know that Fisher's information is:

$$\begin{pmatrix} \frac{1}{\sigma^2} & 0 \\ 0 & \frac{1}{2\sigma^4} \end{pmatrix}$$

But I need a number, what is that matrix supposed to mean?

What is $I(\sigma^2)$ for a normal distribution with μ - known and σ^2 - unknown?

probability statistics probability-distributions normal-distribution

asked Apr 14 '18 at 16:48
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2 Answers

Let $\sigma^2 = \theta$, thus $X \sim N(\mu, \theta)$, hence

$$l(\theta) = -\tfrac{1}{2}\ln\theta - \frac{(x-\mu)^2}{2\theta} + \text{constant}$$

$$l'(\theta) = -\frac{1}{2\theta} + \frac{(x-\mu)^2}{2\theta^2}$$

$$-\mathbb{E}l''(\theta) = -\mathbb{E}[\frac{1}{2\theta^2} - \frac{(x-\mu)^2}{\theta^3}] = -\frac{1}{2\theta^2} + \frac{1}{\theta^2} = \frac{1}{2\theta^2}.$$

Use the additive property of Fisher's information to get the Info. for sample of size n, i.e.,

$$I_{X_1,\ldots,X_n}(\theta) = \frac{n}{2\theta^2} = \frac{n}{2\sigma^4},$$

for the observed information replace σ^2 with

$$S^2 = \frac{\sum_{i=1}^{n}(X_i-\mu)^2}{n}.$$

(And note that $\operatorname{var}(X) = \mathbb{E}(X-\mu)^2 = \sigma^2$).

edited May 28 '18 at 20:23
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answered Apr 14 '18 at 17:17
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Fisher information is only a matrix when we have 2 or more unknown parameters. Since μ is known, we will get a single number.

For a random sample $X_1,\ldots,X_n$, the Fisher information *for the sample* can be defined as

$$I_X(\theta) = -n\,\mathrm{E}\left[\frac{\partial^2 \ln f(X|\theta)}{\partial\theta^2}\right]$$

where here $\theta = \sigma^2$. So we don't need to consider the individual X_i.

answered Apr 14 '18 at 17:19
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