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Unit overview

Lec. 8: Probability density functions

Exercises 8 due Mar 16, 2016 at 23:59 UTC

Lec. 9: Conditioning on an event; Multiple r.v.'s

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Bookmark

Exercise: Expected value rule and total expectation theorem

(5/8 points)

Let \mathbf{X} , \mathbf{Y} , and \mathbf{Z} be jointly continuous random variables. Assume that all conditional PDFs and expectations are well defined. E.g., when conditioning on $\mathbf{X} = \mathbf{x}$, assume that \mathbf{x} is such that $f_{\mathbf{X}}(\mathbf{x}) > 0$. For each one of the following formulas, state whether it is true for all choices of the function \mathbf{g} or false (i.e., not true for all choices of \mathbf{g}).

$$1. \mathbf{E}[g(\mathbf{Y}) | \mathbf{X} = \mathbf{x}] = \int g(\mathbf{y}) f_{\mathbf{Y}|\mathbf{X}}(\mathbf{y} | \mathbf{x}) d\mathbf{y}$$

True ▼



Answer: True

$$2. \mathbf{E}[g(\mathbf{y}) | \mathbf{X} = \mathbf{x}] = \int g(\mathbf{y}) f_{\mathbf{Y}|\mathbf{X}}(\mathbf{y} | \mathbf{x}) d\mathbf{z}$$

False ▼



Answer: False

$$3. \mathbf{E}[g(\mathbf{Y})] = \int \mathbf{E}[g(\mathbf{Y}) | \mathbf{Z} = \mathbf{z}] f_{\mathbf{Z}}(\mathbf{z}) d\mathbf{z}$$

True ▼



Answer: True

$$4. \mathbf{E}[g(\mathbf{Y}) | \mathbf{X} = \mathbf{x}, \mathbf{Z} = \mathbf{z}] = \int g(\mathbf{y}) f_{\mathbf{Y}|\mathbf{X},\mathbf{Z}}(\mathbf{y} | \mathbf{x}, \mathbf{z}) d\mathbf{y}$$

True ▼



Answer: True

$$5. \mathbf{E}[g(\mathbf{Y}) | \mathbf{X} = \mathbf{x}] = \int \mathbf{E}[g(\mathbf{Y}) | \mathbf{X} = \mathbf{x}, \mathbf{Z} = \mathbf{z}] f_{\mathbf{Z}|\mathbf{X}}(\mathbf{z} | \mathbf{x}) d\mathbf{z}$$

False ▼



Answer: True

Exercises 9 due Mar
16, 2016 at 23:59 UTC

Lec. 10:

Conditioning on a random variable; Independence; Bayes' rule

Exercises 10 due Mar
16, 2016 at 23:59 UTC

Standard normal table

Solved problems

Problem Set 5

Problem Set 5 due Mar
16, 2016 at 23:59 UTC

Unit summary

- ▶ Unit 6: Further
topics on
random
variables

6. $\mathbf{E}[g(X, Y) | Y = y] = \mathbf{E}[g(X, y) | Y = y]$

True ▾



Answer: True

7. $\mathbf{E}[g(X, Y) | Y = y] = \mathbf{E}[g(X, y)]$

True ▾



Answer: False

8. $\mathbf{E}[g(X, Z) | Y = y] = \int g(x, z) f_{X,Z|Y}(x, z | y) dy$

True ▾



Answer: False

Answer:

1. True. This is the usual expected value rule, applied to a conditional model where we are given that $\mathbf{X} = \mathbf{x}$.
2. False. Here the quantity inside the expectation, $g(\mathbf{y})$, is a number (not a random variable). The left-hand side is a function of \mathbf{y} , whereas on the right-hand side, \mathbf{y} , is a dummy variable that gets integrated away. So, the formula is wrong on a purely syntactical basis (the left-hand side depends on \mathbf{y} , while the right-hand side does not).
3. True. This is the total expectation theorem, where we condition on the events $\mathbf{Z} = \mathbf{z}$.
4. True. This is the usual expected value rule, applied to a conditional model where we are given that $\mathbf{X} = \mathbf{x}$ and $\mathbf{Z} = \mathbf{z}$.
5. True. This is the same total expectation theorem as in the third part, except that everything is calculated within a conditional model in which event $\mathbf{X} = \mathbf{x}$ is known to have occurred.
6. True. When we condition on $\mathbf{Y} = \mathbf{y}$, we know the value of \mathbf{Y} , and we can replace $g(\mathbf{X}, \mathbf{Y})$ by $g(\mathbf{X}, \mathbf{y})$.
7. False. Given that $\mathbf{Y} = \mathbf{y}$, we need to somehow take into account the conditional distribution of \mathbf{X} , whereas the right-hand side is determined by the unconditional PDF of \mathbf{X} .
8. False. The left-hand side is a function of \mathbf{y} , whereas the right-hand side (after \mathbf{y} is integrated out) is a function of \mathbf{x} and \mathbf{z} . The correct form (expected value rule, in a conditional model) is:

$$\mathbf{E}[g(X, Z) | Y = y] = \int \int g(x, z) f_{X, Z|Y}(x, z | y) dx dz.$$

You have used 1 of 1 submissions

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