



## MITx: 6.041x Introduction to Probability - The Science of Uncertainty



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Bookmark

## Exercise: Sample mean bounds

(2/2 points)

By the argument in the last video, if the  $X_i$  are i.i.d. with mean  $\mu$  and variance  $\sigma^2$ , and if  $M_n = (X_1 + \dots + X_n)/n$ , then we have an inequality of the form

$$\mathbf{P}(|M_n - \mu| \geq \epsilon) \leq \frac{a\sigma^2}{n},$$

for a suitable value of  $a$ .

a) If  $\epsilon = 0.1$ , then the value of  $a$  is:



Answer: 100

b) If we change  $\epsilon = 0.1$  to  $\epsilon = 0.1/k$ , for  $k \geq 1$  (i.e., if we are interested in  $k$  times higher accuracy), how should we change  $n$  so that the value of the upper bound does not change from the value calculated in part (a)?

$n$  should



stay the same



increase by a factor of  $k$



increase by a factor of  $k^2$



decrease by a factor of  $k$



none of the above

Answer:

## ▶ Exam 2

## ▼ Unit 8: Limit theorems and classical statistics

## Unit overview

**Lec. 18: Inequalities, convergence, and the Weak Law of Large Numbers**

Exercises 18 due Apr 27, 2016 at 23:59 UTC

**Lec. 19: The Central Limit Theorem (CLT)**

Exercises 19 due Apr 27, 2016 at 23:59 UTC

**Lec. 20: An introduction to classical statistics**

Exercises 20 due Apr 27, 2016 at 23:59 UTC

## Solved problems

## Additional theoretical material

**Problem Set 8**

Problem Set 8 due Apr 27, 2016 at 23:59 UTC

## Unit summary

a) Chebyshev's inequality yields

$$\mathbf{P}(|M_n - \mu| \geq \epsilon) \leq \frac{\sigma^2}{n\epsilon^2},$$

so that  $a = 1/\epsilon^2 = 1/0.1^2 = 100$ .

b) In order to keep the same upper bound, the term  $n\epsilon^2$  in the denominator needs to stay constant. If we reduce  $\epsilon$  by a factor of  $k$ , then  $\epsilon^2$  gets reduced by a factor of  $k^2$ . Thus,  $n$  will have to be increased by a factor of  $k^2$ .

*You have used 1 of 2 submissions*

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