12. Listening to Fourier series

Supplemental video

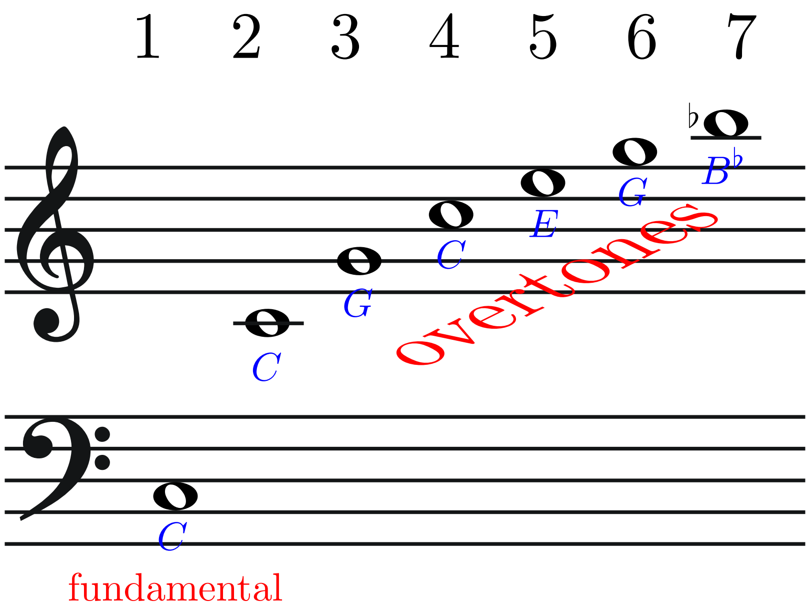
Check out this Vi Hart video called “What's up with Noises?" that goes into more detail about sounds and how we hear.

How we hear

Your ear is capable of decomposing a sound wave into its Fourier components of different frequencies. Each frequency corresponds to a certain pitch. Increasing the frequency produces a higher pitch. More precisely, multiplying the frequency by a number greater than 1 increases the pitch by what in music theory is called an **interval** . For example, multiplying the frequency by 2 raises the pitch by an octave, and multiplying by 3 raises the pitch an octave plus a perfect fifth.

When an instrument plays a note, it is producing a periodic sound wave in which typically many of the Fourier coefficients are nonzero. In a general Fourier series, the combination of the first two non-constant terms ($a_1 \cos t + b_1 \sin t$, if the period is 2π) is a sinusoid of some frequency ν, and the next combination (e.g., $a_2 \cos 2t + b_2 \sin 2t$) has frequency 2ν, and so on: the frequencies are the positive integer multiples of the lowest frequency ν. The note corresponding to the frequency ν is called the **fundamental** , and the notes corresponding to frequencies 2ν, 3ν, are called the **overtones** .

The musical staffs below show these for $\nu \approx 131\,\mathrm{Hz}$ (the C below middle C), with the integer multiplier shown at the top of the image.



Footnote

Most modern keyboard instruments divide the octave into 12 half-steps, each of which represents a frequency ratio of $2^{1/12}$. This means that intervals on such an instrument are only approximations to the pure intervals corresponding to rational number ratios. For instance,

- a fifth on a piano consists of 7 half-steps, for example C to G, hence a frequency ratio of $2^{7/12} \approx 1.4983$, whereas a pure fifth corresponds to a ratio of $3/2 = 1.5$.

- a major third on a piano consists of 4 half steps, for example C to E, hence a frequency ratio of $2^{4/12} \approx 1.2599$, whereas a pure third corresponds to a ratio of $5/4 = 1.25$.

- a major fourth on a piano consists of 5 half steps, for example G to C, hence a frequency ratio of $2^{5/12} \approx 1.3348$, whereas a pure fourth corresponds to a ratio of $4/3 = 1.333\ldots$.

(For reference, see the book *Temperament* by Stuart Isacoff.)

Question 12.1 Can you guess what note corresponds to 9ν?

Answer

As mentioned above, multiplying a frequency by 3 raises the pitch by an octave plus a fifth. Given that 3ν is the G above middle C, we go up an octave to the next G, and then up a fifth from there. This means that 9ν corresponds to the D a little over two octaves above middle C.

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