Observation Theory

Script V11A

Welcome to this first week of the course “Observation Theory… estimating the unknown”

To get started right away, let’s have a closer look at the title of the course.

Three key words are important in the title.

These are observation, (so we are observing/ or measuring something) estimation, which I will address in a minute, and the unknown, which refers to some kind of parameter that we are interested in.

The relation between these two fields (that is the red arrow) needs to be established: it is what we call an ‘estimation problem’.

So what is an estimation problem?

In this course, estimation has a specific meaning, which is different from the ordinary use of the word estimation

In ordinary life we could easily ‘estimate’ the number of walnuts in this image.

If we have enough time to count them, everybody would probably come up with the same answer.

So, there would be practically no error.

However in the right image, if I would ask you to ‘estimate’ the number of walnuts in the bowl, you would probably need to start ‘guessing’.

Different people would arrive at different answers, and the likelihood to get the ‘correct’ or ‘right’ answer, would be very low.

In fact, only by looking at this picture, there is no way to really know whether we found the right answer.

So, here the measurement/observation is our visual perception of the picture with the walnuts, and the unknown parameter is the true (but unknown) number of walnuts.

The relation between the observations and the unknowns can actually be viewed in two directions.

If we would already know the correct answer for the parameters, and we know how they are related to the observations, we call this the forward problem.

In most cases this is easy: if we know what the answer is, we can find a set of observations that matches perfectly.

Our problem occurs in the other direction.

We call this the inverse problem.

How do we “determine” the unknown parameters, given a particular set of observations.

Since the inverse problem is notoriously difficult, we often cannot determine, or calculate, the right answer.

We have to engage in a process to get as close to the unknown parameters as possible.

This is what we refer to as estimation.

Now let's get back to our video of the canal measurement

The four student groups were challenged to measure the width of the canal.

Thus, the unknown parameter was the same for every one.

Ideally, the outcome should be a single number.

However, each team came up with a different method to estimate this parameter:

Two teams, the upper ones, attempted to measure it directly:

Swim to the other side and measure it with a rope, or use a laser distance measurement to measure it electronically

The other two teams used indirect measurements:

They measured a different distance and aligned a triangle in a specific way, or they measured angles using a fancy instrument, in combination with another distance.

Thus, using a different set of measurements, they tried to estimate the same unknown parameter.

Looking at these four methods, which one would you prefer?

Please take a couple of moments to think about it.

You can press the pause button to interrupt the video temporarily.

Welcome back.

And?

Which choice did you make?

What is the best way to measure the width of the canal?

Perhaps you first wanted to get some more information.

What would be good questions to ask?

First of all, we need a criterion to know what we mean with the best way to measure the width of the canal.

Best could refer to the cheapest way to find it, or the fastest way, or the safest way…

In this course we will define best usually as the method in which the precision of the estimated parameter is optimal.

But on the other hand, perhaps we don't even need an answer with 10 decimal digits…

Another question to ask is whether the students performed their measurement only once, or whether they repeated it several times?

Obviously, measuring it several times is preferable, since it reduces the risk of having an outlier.

Moreover, we would be able to apply some kind of averaging to improve the quality of our estimation.

It would also give us a way to say something about the reliability of the result.

This is a concept we will discuss later.

If we take a good look at these stills from the video, there is one more thing we should consider.

Are the students really estimating the same unknown parameter?

What is the width of the canal: where is it defined?

The sides may not be parallel, and they may not be straight lines.

And how easy is it to determine where the side of the canal is?

Determining whether we are really estimating the same parameter is a problem by itself.

If both teams here would send you their results, they may be both very precise, but still very different.

In this course, we want to introduce a very systematic and conceptual approach in dealing with estimation problems.

It involves a physical world, and a mathematical world.

Conceptually, when we deal with a real-life problem, we can always distinguish three stages.

In the first stage our problem is defined in the real world, but we need to translate it to the mathematical world.

In this mathematical world we can apply all our tricks and methods to get optimal numbers as a result.

We call this 'linking the mathematical model to the real world'.

In the last stage, we need to go in the other direction: the results from the procedures in the mathematical world need to be transferred back to reality, where decisions need to be made.

We call this the 'unlinking' or the disengagement of the mathematical model.

In the canal video, we need to compare the numerical result with our knowledge of the real world.

If the width of the canal would be in the order of kilometres, something went wrong.

The stage in between is the actual parameter estimation procedure, which takes place entirely within the abstraction of the mathematical model.

This conceptual framework is very important to keep in mind at all times, as both worlds cannot exist without each other.

In this video, we introduced a number of important concepts: observations (or measurements), unknowns (or parameters) and the relation between the two, for which we engage in the linking and unlinking of a mathematical model.

With estimation theory, we try to get an estimate, which is as close as possible to the unknown parameter, where the optimality criterion is defined in terms of the precision of the estimated parameters.