

## MLE for a homogeneous Poisson process?

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If we have a data set consisting of event times  $\{t_1,t_2,\ldots,t_N\}$  and would like to model this as a Poisson process with intensity  $\lambda$ , how do we do it? Intuitively, I would expect that we can calculate the average waiting time  $w=\frac{1}{N-1}\sum_{i=2}^N(t_i-t_{i-1})$  and then set  $\hat{\lambda}=1/w$ ?



Is this correct? If so, how is it justified? Do we have to consider fitting a Poisson distribution to the number of events after doing some sort of binning?



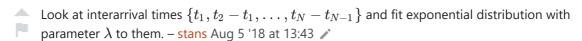
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poisson-distribution poisson-process

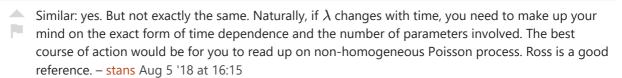
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edited Aug 5 '18 at 13:35

asked Aug 5 '18 at 13:25 theQman **507** 4 12







## 1 Answer





You can use Maximum Likelihood Estimation, either with synchronous data (time-binned data) or asynchronous data (time-stamped data). The likelihood function changes accordingly.



For time-binned (or synchronous) data, you can simply use the joint Poisson probability mass function for your observed counts as the likelihood function:



$$L = \prod_{i=1}^K rac{\lambda^{x_i}}{x_i!} ext{exp}(-\lambda)$$
,



where K is the number of bins,  $x_i$  the count of events in bin i, and  $\lambda$  the constant intensity that you want to estimate.

For asynchronous data, the likelihood is specified as follows:

$$L = \left[\prod_{i=1}^{N(T)} \, \lambda^*(t_i)
ight] \exp\!\left[-\int_0^T \lambda^*(s) ds
ight]$$
 ,

where N(T) is the number of points at end-of-sample time T, and  $\lambda^*(t)$  is the conditional intensity function, which is simply the constant  $\lambda^*(t)=\lambda$  for the homogeneous Poisson process.

In some cases including the homogeneous Poisson process, there are closed-form solutions for both cases (take logs, set derivative with respect to  $\lambda$  equal to zero, and solve for  $\lambda$ ). Otherwise the log-likelihood can be optimised numerically.

For more background on theory and estimation, these are good references:

- Lecture notes on temporal point processes by Rassmussen
- Daley, D. J.; Vere-Jones, D., An introduction to the theory of point processes. Vol. 1: **<u>Elementary theory and methods.</u>**, Probability and Its Applications. New York, NY: Springer. xxi, 469 p. (2003). ZBL1026.60061.

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