4. Solving the homogeneous tuned mass damper system

A tuned mass damper

2/2 points (graded)

Recall that the tuned mass damper system used to reduce the swaying in tall buildings can be modeled by the 4x4 inhomogeneous system.

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{y}_1 \\ \dot{y}_2 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \frac{-(k_1+k_2)}{m_1} & \frac{k_2}{m_1} & \frac{-(b_1+b_2)}{m_1} & \frac{b_2}{m_1} \\ \frac{k_2}{m_2} & \frac{-k_2}{m_2} & \frac{b_2}{m_2} & \frac{-b_2}{m_2} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ y_1 \\ y_2 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \frac{F}{m_1} \\ 0 \end{pmatrix}$$

In this problem, we will find the normal modes of the associated homogeneous system,

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{y}_1 \\ \dot{y}_2 \end{pmatrix} \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \frac{-(k_1+k_2)}{m_1} & \frac{k_2}{m_1} & \frac{-(b_1+b_2)}{m_1} & \frac{b_2}{m_1} \\ \frac{k_2}{m_2} & \frac{-k_2}{m_2} & \frac{b_2}{m_2} & \frac{-b_2}{m_2} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ y_1 \\ y_2 \end{pmatrix}.$$

In this problem, we will set
$m_1 = 1,\ k_1 = 1,\ b_1 = 0.001,\ m_2 = 0.05,\ k_2 = 1,\ b_2 = 0.01.$

The normal modes are real-valued functions that can be written in the form

$$\begin{aligned} n_1 &= e^{a_1 t}\cos(\omega_1 t + \phi_1)(c_1\mathbf{v}_1 + c_2\mathbf{v}_2) \\ n_2 &= e^{a_2 t}\cos(\omega_2 t + \phi_2)(c_3\mathbf{v}_3 + c_4\mathbf{v}_4). \end{aligned}$$

What are ω_1 and ω_2? (Let ω_1 be the term such that $a_1 > a_2$ in the expression above. Use the convention that frequencies are positive numbers.)

Use Matlab Online or other computer system to find the answer. Enter your answer to 4 decimal places in the answer boxes below.

$\omega_1 =$ 0.9747 ✔ **Answer:** 0.9747

$\omega_2 =$ 4.5868 ✔ **Answer:** 4.5868

Solution:

Using MATLAB (or your preferred math solver), you will find that the eigenvalues of the associated matrix for the given constants are

$$\begin{gathered} -0.1050 + 4.5868i \\ -0.1050 - 4.5868i \\ -0.0005 + 0.9747i \\ -0.0005 - 0.9747i \end{gathered}$$

The frequencies are the imaginary parts of these eigenvalues. The term e^{at} in the expression of the normal modes is determined by the eigenvalue where a is the real part of one of the eigenvalues. The eigenvalue with the largest real part is $-0.0005 + 0.9747i$. Thus

$$\omega_1 = \mathrm{Im}\left(-0.0005 + 0.9747i\right) = 0.9747,$$

and

$$\omega_2 = \mathrm{Im}\left(-0.1050 + 4.5868i\right) = 4.5868.$$

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Answers are displayed within the problem

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