

MITx: 6.041x Introduction to Probability - The Science of Uncertainty

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Unit overview

Lec. 5: Probability mass functions and expectations

Exercises 5 due Mar 02, 2016 at 23:59 UTC

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Problem 3: PMF, expectation, and variance

(6/6 points)

The random variables $oldsymbol{X}$ and $oldsymbol{Y}$ have the joint PMF

$$p_{X,Y}(x,y)=\left\{egin{aligned} c\cdot(x+y)^2,& ext{if }x\in\{1,2,4\} ext{ and }y\in\{1,3\},\ 0,& ext{otherwise}. \end{aligned}
ight.$$

All answers in this problem should be numerical.

1. Find the value of the constant c.

2. Find $\mathbf{P}(Y < X)$.

Lec. 6: Variance; Conditioning on an event; Multiple r.v.'s

Exercises 6 due Mar 02, 2016 at 23:59 UTC

Lec. 7: Conditioning on a random variable; Independence of r.v.'s

Exercises 7 due Mar 02, 2016 at 23:59 UTC

Solved problems

Additional theoretical material

Problem Set 4

Problem Set 4 due Mar 02, 2016 at 23:59 UTC

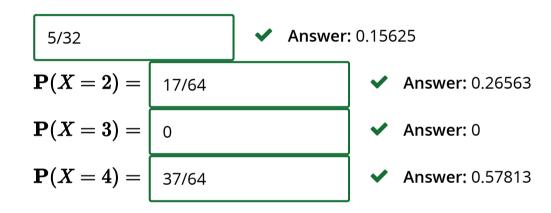
Unit summary

- Exam 1
- Unit 5: Continuous random variables
- Unit 6: Further topics on random variables
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3. Find $\mathbf{P}(Y=X)$.

4. Find the following probabilities.

$$P(X = 1) =$$



5. Find the expectations $\mathbf{E}[X]$ and $\mathbf{E}[XY]$.

6. Find the variance of X.

inference

- ▶ Exam 2
- Unit 8: Limit theorems and classical statistics
- Unit 9: Bernoulli and Poisson processes
- Unit 10: Markov chains
- Exit Survey
- Final Fxam

$$var(X) =$$

1.46875

Answer: 1.46875

Answer:

1. From the joint PMF, there are six (x, y) pairs with nonzero probability mass. These pairs are (1,1),(1,3),(2,1),(2,3),(4,1),(4,3). Because the probability of the entire sample space must equal 1, we have:

$$c(1+1)^2 + c(1+3)^2 + c(2+1)^2 + c(2+3)^2 + c(4+1)^2 + c(4+3)^2 = 1.$$

Solving for c, we get $c=\frac{1}{128}$.

2. There are three possible outcomes for which y < x: (2,1), (4,1), (4,3).

$$\mathbf{P}(Y < X) = p_{X,Y}(2,1) + p_{X,Y}(4,1) + p_{X,Y}(4,3) = rac{9}{128} + rac{25}{128} + rac{49}{128} = rac{83}{128}.$$

3. There is only one possible outcome for which y=x: (1,1).

$$\mathbf{P}(Y=X)=p_{X,Y}(1,1)=rac{4}{128}.$$

4. We use the formula $p_X(x) = \sum_y p_{X,Y}(x,y)$.

For example, $p_X(2)=p_{X,Y}(2,1)+p_{X,Y}(2,3)=rac{34}{128}$. More generally, we find that

$$p_X(x) = egin{cases} 20/128, & ext{if } x=1, \ 34/128, & ext{if } x=2, \ 74/128, & ext{if } x=4, \ 0, & ext{otherwise.} \end{cases}$$

5. We have

$$\mathbf{E}[X] = \sum_x x p_X(x) = 1 \cdot rac{20}{128} + 2 \cdot rac{34}{128} + 4 \cdot rac{74}{128} = 3.$$

Using the expected value rule,

$$\mathbf{E}[XY] \; = \sum_x \sum_y xy p_{X,Y}(x,y)$$

$$= 1 \cdot \frac{4}{128} + 2 \cdot \frac{9}{128} + 4 \cdot \frac{25}{128} + 3 \cdot \frac{16}{128} + 6 \cdot \frac{25}{128} + 12 \cdot \frac{49}{128}$$
$$= \frac{227}{32}.$$

6. The variance of a random variable X can be computed as $\mathbf{E}[X^2] - (\mathbf{E}[X])^2$ or as $\mathbf{E}[(X - \mathbf{E}[X])^2]$. We use the second approach here. We have

$$\mathrm{var}(X) = (1-3)^2 rac{20}{128} + (2-3)^2 rac{34}{128} + (4-3)^2 rac{74}{128} = rac{47}{32}.$$

You have used 1 of 4 submissions

DISCUSSION

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