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## Unit 11: Quiz

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### Unit 11: Quiz

The problems in the quiz will be automatically graded.

You are given only **ONE** attempt for each problem. You will be able to see the solution after you submit your answers.

If the answer is numeric and has more than 4 decimal places, just give your answers to **4 decimal places (0.0001 accuracy)** unless otherwise stated.

e.g. for 0.123456... you just need to round it up to 0.1235

### Problem 1

1/1 point (graded)

**1.** Consider a collection of 9 bears. There is a family of red bears consisting of one father bear, one mother bear, and one baby bear. There is a similar green bear family, and a similar blue bear family. We draw 5 consecutive times from this collection *without replacement* (i.e., not returning the bear after each draw). Let  $X$  denote the number of red bears that are chosen, among these 5 selected bears. Find the variance of  $X$ .

## and Chebychev Inequalities

L11.1: Variance of Sums;  
Covariance; Correlation

L11.2: Conditional  
Expectation

L11.3: Conditional vs  
Independent

L11.4: Markov and  
Chebyshev Inequalities

L11.5: Practice

**L11.6: Quiz**  
Quiz



- ▶ Unit 12: Order Statistics, Moment Generating Functions, Transformation of RVs

0.5555556

✓ Answer: 0.5556

### Explanation

**1. Method #1.** We keep track (in order) of the kind of bears that we get. Let  $X$  denote the number of red bears selected. For  $i = 1, 2, 3, 4, 5$ , let  $X_i = 1$  if the  $i$ th bear selected is red, and  $X_i = 0$  otherwise.

So  $X = X_1 + X_2 + X_3 + X_4 + X_5$ .

Thus

$$\begin{aligned}\text{Var}(X) &= \text{Var}(X_1 + \cdots + X_5) \\ &= \sum_{i=1}^5 \text{Var}(X_i) + \sum_{i \neq j} \text{Cov}(X_i, X_j).\end{aligned}$$

We have

$$\text{Var}(X_i) = \mathbb{E}(X_i^2) - (\mathbb{E}(X_i))^2 = 1/3 - (1/3)^2 = 2/9.$$

Also,

$$\begin{aligned}\text{Cov}(X_i, X_j) &= \mathbb{E}(X_i X_j) - \mathbb{E}(X_i)\mathbb{E}(X_j) \\ &= (3/9)(2/8) - (1/3)(1/3) = -1/36.\end{aligned}$$

So altogether

$$\text{Var}(X) = (5)(2/9) + (20)(-1/36) = 5/9.$$

**Method #2.** Refer to the red father bear as red bear #1, and the red mother bear as red bear #2, and the red baby bear as red bear #3.

For  $i = 1, 2, 3$ , let  $Y_i = 1$  if the  $i$ th red bear is selected (at any time, i.e., on any of the five selections), and  $Y_i = 0$  otherwise.

So we have  $X = Y_1 + Y_2 + Y_3$ .

$$\text{Var}(X) = \text{Var}(Y_1 + Y_2 + Y_3) = \sum_{i=1}^3 \text{Var}(Y_i) + \sum_{i \neq j} \text{Cov}(Y_i, Y_j).$$

$$\text{We have } \text{Var}(Y_i) = \mathbb{E}(Y_i^2) - (\mathbb{E}(Y_i))^2 = 5/9 - (5/9)^2 = 20/81.$$

Also

$$\begin{aligned}\text{Cov}(Y_i, Y_j) &= \mathbb{E}(Y_i Y_j) - \mathbb{E}(Y_i)\mathbb{E}(Y_j) \\ &= (5/9)(4/8) - (5/9)(5/9) = -5/162.\end{aligned}$$

$$\text{So altogether } \text{Var}(X) = (3)(20/81) + (6)(-5/162) = 5/9.$$

You have used 1 of 1 attempt

**Problem 2**

2/2 points (graded)

**2.** Suppose that a drawer contains 8 marbles: 2 are red, 2 are blue, 2 are green, and 2 are yellow. The marbles are rolling around in a drawer, so that all possibilities are equally likely when they are drawn. Alice chooses 2 marbles without replacement, and then Bob also chooses 2 marbles without replacement. Let  $Y$  denote the number of red marbles that Alice gets, and let  $X$  denote the number of red marbles that Bob gets.

**2a.** Find the covariance of  $X$  and  $Y$ .

✓ Answer: -0.1071429

**2b.** Find the correlation of  $X$  and  $Y$ .

✓ Answer: -0.3333333

**Explanation**

**2a.** First we note that  $0 \leq X \leq 2$  and  $0 \leq Y \leq 2$ , with the additional constraint that  $0 \leq X + Y \leq 2$ . So we have  $XY = 1$  if  $X = 1$  and  $Y = 1$ , or otherwise  $XY = 0$ . (You can try the various combinations of the  $X$  and  $Y$ , if you do not see this immediately.) Thus

$\mathbb{E}(XY) = (1)P(X = 1 \& Y = 1) + (0)(1 - P(X = 1 \& Y = 1))$ . So

$$\mathbb{E}(XY) = \frac{\binom{2}{1}\binom{6}{1}}{\binom{8}{2}} \frac{\binom{1}{1}\binom{5}{1}}{\binom{6}{2}} = 1/7. \text{ Thus}$$

$$\text{Cov}(X, Y) = \mathbb{E}(XY) - \mathbb{E}(X)\mathbb{E}(Y) = 1/7 - (1/2)(1/2) = -3/28.$$

**2b.** We have  $\text{Var}(X) = \mathbb{E}(X^2) - (\mathbb{E}(X))^2$ .

We calculate

$$\begin{aligned} \mathbb{E}(X^2) &= (0^2)(6/8)(5/7) + (1^2)((2/8)(6/7) + (6/8)(2/7)) + (2^2)(2/8)(1/7) \\ &= 4/7 \end{aligned}$$

and we know  $\mathbb{E}(X) = 1/2$ , so  $\text{Var}(X) = 4/7 - (1/2)^2 = 9/28$ .

Similarly,  $\text{Var}(Y) = 9/28$ . So the correlation of  $X$  and  $Y$  is

$$\rho(X, Y) = \frac{\text{Cov}(X, Y)}{\sqrt{\text{Var}(X) \text{Var}(Y)}} = \frac{-3/28}{\sqrt{(9/28)(9/28)}} = -1/3.$$

Submit

You have used 1 of 1 attempt

✓ Correct (2/2 points)

### Problem 3

1/1 point (graded)

**3.** A bag of candy contains 10 green M&M's and 10 red M&M's. Suppose that 10 students pick 2 candies each, without replacement. Let  $X$  denote the number of students who get one red and one green candy. Find  $\text{Var}(X)$ .

2.639726

✓ Answer: 2.6397

**Explanation**

3. We can write  $X = X_1 + \cdots + X_{10}$  where  $X_j = 1$  if the  $j$ th pair has 1 red and 1 green, or  $X_j = 0$  otherwise. Then  $\mathbb{E}(X_j) = 10/19$  for each  $j$ . Also,

$$\text{Var}(X) = \text{Var}(X_1 + \cdots + X_{10}) = \sum_{i=1}^{10} \text{Var}(X_i) + \sum_{i \neq j} \text{Cov}(X_i, X_j).$$

We have

$$\text{Var}(X_i) = \mathbb{E}(X_i^2) - (\mathbb{E}(X_i))^2 = 10/19 - (10/19)^2 = 90/361 \text{ for each } i.$$

$$\text{Also } \text{Cov}(X_i, X_j) = \mathbb{E}(X_i X_j) - \mathbb{E}(X_i)\mathbb{E}(X_j)$$

$$= (10/19)(9/17) - (10/19)^2 = 10/6137 \text{ for each } i \neq j.$$

So altogether we have

$$\text{Var}(X) = (10)(90/361) + (90)(10/6137) = 16200/6137 = 2.64.$$

Submit

You have used 1 of 1 attempt

✓ Correct (1/1 point)

**Problem 4**

0/1 point (graded)

4. Suppose  $X$  and  $Y$  have joint density  $f_{X,Y}(x,y) = e^{1-x}$  for  $x, y$  in the region where  $0 < x < y < 1$ , and  $f_{X,Y}(x,y) = 0$  otherwise. Find the covariance of  $X$  and  $Y$ .

[Note: It might look strange to have a joint probability density function of  $\mathbf{X}$  and  $\mathbf{Y}$  with no  $\mathbf{y}$ 's in it, but this is OK. This function is constant with regard to  $\mathbf{y}$ , i.e., it does not change as  $\mathbf{y}$  changes. You can check, for instance, that  $f_{X,Y}(x, y)$  is a valid probability density function because it is nonnegative and because  $\int_0^1 \int_x^1 e^{1-x} dy dx = 1$ .]

[Hint: Just to save you having to do so many integration by parts, for your convenience, we have:  
 $\int_0^1 e^{-x} dx = 1 - e^{-1}$  and  $\int_0^1 x e^{-x} dx = 1 - 2e^{-1}$  and  $\int_0^1 x^2 e^{-x} dx = 2 - 5e^{-1}$  and  
 $\int_0^1 x^3 e^{-x} dx = 6 - 16e^{-1}$ .]

Cov( $X, Y$ ) =  ✖ Answer: 0.0238

#### Explanation

4. We have  $\text{Cov}(X, Y) = \mathbb{E}(XY) - \mathbb{E}(X)\mathbb{E}(Y)$ .

We compute

$$\begin{aligned}\mathbb{E}(XY) &= \int_0^1 \int_x^1 x y e^{1-x} dy dx = \int_0^1 x e^{1-x} \int_x^1 y dy dx \\ &= \int_0^1 x e^{1-x} (1 - x^2)/2 dx = \int_0^1 x e^{1-x} (1 - x^2)/2 dx \\ &= \frac{e}{2} \int_0^1 e^{-x} (x - x^3) dx = 7 - (5/2)(e) = 0.2043.\end{aligned}$$

Also we compute

$$\begin{aligned}\mathbb{E}(X) &= \int_0^1 \int_x^1 x e^{1-x} dy dx = \int_0^1 x e^{1-x} \int_x^1 1 dy dx \\ &= \int_0^1 x e^{1-x} (1 - x) dx = e \int_0^1 e^{-x} (x - x^2) dx \\ &= 3 - e = 0.2817\end{aligned}$$

and

$$\begin{aligned}\mathbb{E}(Y) &= \int_0^1 \int_x^1 y e^{1-x} dy dx = \int_0^1 e^{1-x} \int_x^1 y dy dx \\ &= \int_0^1 e^{1-x} (1 - x^2)/2 dx = \frac{e}{2} \int_0^1 e^{-x} (1 - x^2) dx \\ &= 2 - e/2 = 0.6409.\end{aligned}$$

So we conclude that

$$\begin{aligned}\text{Cov}(X, Y) &= \mathbb{E}(XY) - \mathbb{E}(X)\mathbb{E}(Y) \\ &= 0.2043 - (0.2817)(0.6409) = 0.0238.\end{aligned}$$

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You have used 1 of 1 attempt

### Problem 5

4/5 points (graded)

5. Suppose  $X$  and  $Y$  have joint probability density function

$$f_{X,Y}(x, y) = 70e^{-3x-7y}$$

for  $0 < x < y$ , and  $f_{X,Y}(x, y) = 0$  otherwise.

5a. Find the probability density function  $f_X(x)$  of  $X$ . Then compute  $f_X(0.1)$

$$f_X(0.1) = 3.678794$$

✓ Answer: 3.678794

5b. Use the formula of  $f_X(x)$  you found in 5a to find  $f_{Y|X}(y | x) = \frac{f_{X,Y}(x,y)}{f_X(x)}$  for fixed  $x > 0$ . Then compute  $f_{Y|X}(0.1 | 0.2)$  and  $f_{Y|X}(0.2 | 0.1)$

$$f_{Y|X}(0.1 | 0.2) =$$

14.09627

✗ Answer: 0

$$f_{Y|X}(0.2 | 0.1) =$$

3.476097

✓ Answer: 3.476097

**5c.** Use the formula of  $f_{Y|X}(y | x)$  you found in **5b** to find  $\mathbb{E}(Y | X = x) = \int_x^\infty y f_{Y|X}(y | x) dy$ , for a fixed  $x > 0$ . Then compute  $\mathbb{E}(Y | X = \frac{6}{7})$

$$\mathbb{E}(Y | X = \frac{6}{7}) =$$

1

✓ Answer: 1

**5d.** Use the formula of  $\mathbb{E}(Y | X = x)$  you found in **5c** to find  $\mathbb{E}(Y) = \int_0^\infty \mathbb{E}(Y | X = x) f_X(x) dx$ .

17/70

✓ Answer: 0.2429

### Explanation

**5a.** For  $x > 0$ , we have  $f_X(x) = \int_x^\infty 70e^{-3x-7y} dy = 10e^{-10x}$ ; for  $x < 0$ , we have  $f_X(x) = 0$ .

**5b.** For  $x > 0$ , we have  $f_{Y|X}(y | x) = \frac{f_{X,Y}(x,y)}{f_X(x)} = \frac{70e^{-3x-7y}}{10e^{-10x}} = 7e^{7x-7y}$  for  $y > x$ ; and  $f_{Y|X}(y | x) = 0$  for  $y \leq x$ .

**5c.** For  $x > 0$ , we have  $\mathbb{E}(Y | X = x) = \int_x^\infty (y)(7e^{7x-7y}) dy = x + 1/7$ .

**5d.** We compute  $\mathbb{E}(Y) = \int_0^\infty (x + 1/7)(10e^{-10x}) dx = 1/10 + 1/7 = 17/70 = 0.2429$ .

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You have used 1 of 1 attempt



\* Partially correct (4/5 points)

### Problem 6

2/2 points (graded)

6. Roll two 4-sided dice. Let  $X$  denote the maximum value, and let  $Y$  denote the minimum value.

6a. Find  $\mathbb{E}(X \mid Y = 3)$ .

✓ Answer: 3.6667

6b. Find  $\mathbb{E}(Y \mid X = 3)$ .

✓ Answer: 1.8

### Explanation

6a. The conditional mass of  $X$ , given  $Y = 3$ , is  $f_{X|Y}(3 \mid 3) = 1/3$ ;  $f_{X|Y}(4 \mid 3) = 2/3$ ; and  $f_{X|Y}(x \mid 3) = 0$  otherwise. So  $\mathbb{E}(X \mid Y = 3) = (3)(1/3) + (4)(2/3) = 11/3$ .

6b. The conditional mass of  $Y$ , given  $X = 3$ , is  $f_{Y|X}(1 \mid 3) = 2/5$ ;  $f_{Y|X}(2 \mid 3) = 2/5$ ;  $f_{Y|X}(3 \mid 3) = 1/5$ ; and  $f_{Y|X}(y \mid 3) = 0$  otherwise. So  $\mathbb{E}(Y \mid X = 3) = (1)(2/5) + (2)(2/5) + (3)(1/5) = 9/5$ .

Submit

You have used 1 of 1 attempt

**Problem 7**

2/2 points (graded)

**7.** Consider a pair of continuous random variables  $X, Y$  with constant joint density on the triangle with vertices at  $(0, 0)$ ,  $(2, 0)$ , and  $(0, 8)$ .

**7a.** Fix  $y$  with  $0 < y < 8$ . Find  $\mathbb{E}(X \mid Y = y)$ , then compute  $\mathbb{E}(X \mid Y = 4)$ .

✓ Answer: 0.5

**7b.** Fix  $x$  with  $0 < x < 2$ . Find  $\mathbb{E}(Y \mid X = x)$ , then compute  $\mathbb{E}(Y \mid X = 1)$

✓ Answer: 2

**Explanation**

**7a.** For  $0 < y < 8$ , we have  $f_Y(y) = \int_0^{(8-y)/4} 1/8 dx = (1/8)(8-y)/4 = (8-y)/32$ . So we get

$$f_{X|Y}(x \mid y) = \frac{f_{X,Y}(x,y)}{f_Y(y)} = \frac{1/8}{(8-y)/32} = 4/(8-y). \text{ So we conclude}$$

$$\mathbb{E}(X \mid Y = y) = \int_0^{(8-y)/4} (x)(4/(8-y)) dx = (8-y)/8.$$

**7b.** For  $0 < x < 2$ , we have  $f_X(x) = \int_0^{8-4x} 1/8 dy = (2-x)/2$ . So we get

$$f_{Y|X}(y \mid x) = \frac{f_{X,Y}(x,y)}{f_X(x)} = \frac{1/8}{(2-x)/2} = 1/(4(2-x)). \text{ So we conclude}$$

$$\mathbb{E}(Y \mid X = x) = \int_0^{8-4x} (y)1/(4(2-x)) dy = 4 - 2x.$$

You have used 1 of 1 attempt

✓ Correct (2/2 points)

### Problem 8

3/3 points (graded)

**8.** Suppose that a drawer contains 8 marbles: 2 are red, 2 are blue, 2 are green, and 2 are yellow. The marbles are rolling around in a drawer, so that all possibilities are equally likely when they are drawn. Alice chooses 2 marbles without replacement, and then Bob also chooses 2 marbles without replacement. Let  $Y$  denote the number of red marbles that Alice gets, and let  $X$  denote the number of red marbles that Bob gets.

**8a.** Find  $\mathbb{E}(X \mid Y = 0)$ .

0.666667

✓ Answer: 0.6667

**8b.** Find  $\mathbb{E}(X \mid Y = 1)$ .

0.333333

✓ Answer: 0.3333

**8c.** Find  $\mathbb{E}(X \mid Y = 2)$ .

0

✓ Answer: 0

### Explanation

**8a.** Intuitively, if Alice got no reds, then Bob is drawing 2 marbles from a collection of 2 reds and 4 non-reds, so  $\mathbb{E}(X \mid Y = 0) = 2/6 + 2/6 = 2/3$ .

**8b.** Intuitively, if Alice got 1 red, then Bob is drawing 2 marbles from a collection of 1 red and 5 non-reds, so  $\mathbb{E}(X | Y = 1) = 1/6 + 1/6 = 1/3$ .

**8c.** Intuitively, if Alice got 2 reds, then Bob is drawing 2 marbles from a collection of 0 reds and 6 non-reds, so  $\mathbb{E}(X | Y = 2) = 0/6 + 0/6 = 0$ .

Here is a more formal way to solve question 8. We have

$$p_{X,Y}(2, 0) = \frac{\binom{6}{0} \binom{2}{2}}{\binom{8}{2}} \frac{\binom{6}{2} \binom{0}{0}}{\binom{6}{2}} = (1/28)(1) = 1/28$$

$$p_{X,Y}(0, 2) = \frac{\binom{6}{2} \binom{2}{0}}{\binom{8}{2}} \frac{\binom{4}{0} \binom{2}{2}}{\binom{6}{2}} = (15/28)(1/15) = 1/28$$

$$p_{X,Y}(1, 1) = \frac{\binom{6}{1} \binom{2}{1}}{\binom{8}{2}} \frac{\binom{5}{1} \binom{1}{1}}{\binom{6}{2}} = (3/7)(1/3) = 1/7.$$

$$p_{X,Y}(1, 0) = \frac{\binom{6}{1} \binom{2}{1}}{\binom{8}{2}} \frac{\binom{5}{2} \binom{1}{0}}{\binom{6}{2}} = (3/7)(2/3) = 2/7.$$

$$p_{X,Y}(0, 1) = \frac{\binom{6}{2} \binom{2}{0}}{\binom{8}{2}} \frac{\binom{4}{1} \binom{2}{1}}{\binom{6}{2}} = (15/28)(8/15) = 2/7.$$

$$p_{X,Y}(0, 0) = \frac{\binom{6}{2} \binom{2}{0}}{\binom{8}{2}} \frac{\binom{4}{2} \binom{2}{0}}{\binom{6}{2}} = (15/28)(2/5) = 3/14.$$

So  $p_Y(0) = 1/28 + 2/7 + 3/14 = 15/28$  and  $p_Y(1) = 1/7 + 2/7 = 3/7$  and  $p_Y(2) = 1/28$ .

Thus:

**8a.** We have  $p_{X|Y}(x | 0) = \frac{p_{X,Y}(x,0)}{p_Y(0)}$ . Thus  $p_{X|Y}(0 | 0) = \frac{3/14}{15/28} = 2/5$  and

$p_{X|Y}(1 | 0) = \frac{2/7}{15/28} = 8/15$  and  $p_{X|Y}(2 | 0) = \frac{1/28}{15/28} = 1/15$ , so

$\mathbb{E}(X | Y = 0) = (0)(2/5) + (1)(8/15) + (2)(1/15) = 2/3$ .

**8b.** We have  $p_{X|Y}(x|1) = \frac{p_{X,Y}(x,1)}{p_Y(1)}$ . Thus  $p_{X|Y}(0|1) = \frac{2/7}{3/7} = 2/3$  and  $p_{X|Y}(1|1) = \frac{1/7}{3/7} = 1/3$ , so  $\mathbb{E}(X|Y=1) = (0)(2/3) + (1)(1/3) = 1/3$ .

**8c.** We have  $p_{X|Y}(x|2) = \frac{p_{X,Y}(x,2)}{p_Y(2)}$ . Thus  $p_{X|Y}(0|2) = \frac{1/28}{1/28} = 1$ , so  $\mathbb{E}(X|Y=2) = (0)(1) = 0$ .

You have used 1 of 1 attempt

✓ Correct (3/3 points)

### Problem 9

2/2 points (graded)

**9a.** Suppose that, in a certain course, the expected value of a student's grade is 0.80. Even without knowing anything else about the distribution of the grade, find an upper bound on the probability that a student earns 0.95 or higher in the course.

✓ Answer: 0.84

**9b.** In addition to knowing that the expected value of a student's grade is 0.80, suppose that you also know that the standard deviation of a student's grade is 0.05. Find a bound on the probability that the student's grade is in the range between 0.73 and 0.87.

✓ Answer: 0.4898

**Explanation**

**9a.** If  $X$  denotes the grade, then  $P(X \geq 0.95) \leq \frac{\mathbb{E}(X)}{0.95} = \frac{0.80}{0.95} = 0.84$ .

**9b.** We have  $P(0.73 \leq X \leq 0.87) = P(|X - 0.80| \leq 0.07) = P(|X - 0.80| \leq k\sigma_X)$  where  $\sigma_X = 0.05$  is the standard deviation, and  $k = 0.07/0.05$ . So we have

$$P(0.73 \leq X \leq 0.87) \geq 1 - \frac{1}{(0.07/0.05)^2} = 0.49.$$

Submit

You have used 2 of 2 attempts

✓ Correct (2/2 points)

**Problem 10**

3/3 points (graded)

**10.** A box of cereal contains, on average, 22oz of cereal inside (and, therefore, this is the amount claimed on the box), with standard deviation of **0.3oz**. Use  $X$  to denote the amount of cereal in such a box.

**10a.** Find a bound on the probability that the stated weight is wrong by **0.5oz** or more. I.e., find a bound on  $P(|X - 22| \geq 0.5)$ .

9/25

✓ Answer: 0.36

**10b.** Can you find a bound on the probability that the box of cereal has at least 24oz of cereal?

11/12

✓ Answer: 0.9166

**10c.** Without knowing more about the problem, should we use Markov's inequality to give a bound on  $P(X > 21)$ ? Why or why not?

☐ Yes☒ No ✓

#### Explanation

**10a.** We have  $P(|X - 22| \geq 0.5) = P(|X - 22| \geq k\sigma_X)$  where  $\sigma_X = 0.3$  and  $k = 0.5/0.3$ . So we get  $P(|X - 22| \geq 0.5) \leq \frac{1}{(0.5/0.3)^2} = 0.36$ .

**10b.** We have  $P(X \geq 24) \leq \frac{22}{24} = 0.92$ , by the Markov inequality.

**10c.** We have  $P(X \geq 21) \leq \frac{22}{21} = 1.05$ , but of course we automatically have an even better bound (without using the Markov inequality), namely,  $P(X \geq 21) \leq 1$ . So the Markov inequality does not give us any additional information in this case.

You have used 1 of 1 attempt

✓ Correct (3/3 points)

**Problem 11**

2/2 points (graded)

**11.** An agricultural consultant has determined that the number of bees that should appear at noon in a certain flowerbed, on a randomly chosen day in the summertime, has an expected value of 15 bees, with standard deviation of 3 bees.

**11a.** Find a bound on the probability that 20 or more bees are present on such a day.

✓ Answer: 0.75

**11b.** Find a bound on the probability there are between 10 to 20 bees (inclusive) on such a day.

✓ Answer: 0.64

**Explanation**

**11a.** We use  $X$  for the number of bees. Then we get  $P(X \geq 20) \leq \frac{15}{20} = 0.75$ .

**11b.** We have  $P(10 \leq X \leq 20) = P(|X - 10| \leq 5) = P(|X - 10| \leq k\sigma_X)$  where  $\sigma_X = 3$  and  $k = 5/3$ . So we get  $P(10 \leq X \leq 20) \geq 1 - \frac{1}{(5/3)^2} = 0.64$ .

You have used 1 of 1 attempt

✓ Correct (2/2 points)



**Problem 12**

4/4 points (graded)

**12.** The number of customers in a large sandwich restaurant is randomly distributed. Over time, the manager has estimated that the average number of customers at lunchtime is 30, with standard deviation of 5.

**12a.** Find a bound on the probability that there are between 20 to 40 customers (inclusive) at lunchtime.

✓ Answer: 0.75

**12b.** Find a bound on the probability that there are at least 40 customers at lunchtime.

✓ Answer: 0.75

**12c.** Find a bound on the probability that there are at least 50 customers at lunchtime.

✓ Answer: 0.60

**12d.** Find a bound on the probability that there are at least 60 customers at lunchtime.

✓ Answer: 0.50

**Explanation**

**12a.** We use  $X$  to denote the number of customers. Then  $P(20 \leq X \leq 40) = P(|X - 30| \leq 10) = P(|X - 30| \leq k\sigma_X)$  where  $\sigma_X = 5$  and  $k = 10/5 = 2$ . So we conclude  $P(20 \leq X \leq 40) \geq 1 - \frac{1}{2^2} = 3/4$ .

**12b.** We have  $P(X \geq 40) \leq \frac{30}{40} = 3/4 = 0.75$ .

**12c.** We have  $P(X \geq 50) \leq \frac{30}{50} = 3/5 = 0.60$ .

**12d.** We have  $P(X \geq 60) \leq \frac{30}{60} = 1/2 = 0.50$ .

You have used 1 of 1 attempt

✓ Correct (4/4 points)

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