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Hyperbolic functions - sinh, cosh, tanh, coth, sech, csch

DEFINITION OF HYPERBOLIC FUNCTIONS

Hyperbolic sine of x = sinh $x = (e^x - e^{-x})/2$

Hyperbolic cosine of x = cosh $x = (e^x + e^{-x})/2$

Hyperbolic tangent of x = tanh $x = (e^x - e^{-x})/(e^x + e^{-x})$

Hyperbolic cotangent of x = coth $x = (e^x + e^{-x})/(e^x - e^{-x})$

Hyperbolic secant of x = sech $x = 2/(e^x + e^{-x})$

⊟ **Indefinite Integrals**

⊞ **Integrals**

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⊟ **Spherical Triangle**

⊟ **Differential Equations**

⊟ **Beta Function**

⊟ **Mathematical Induction**

Hyperbolic cosecant of x = csch $x = 2/(e^x - e^{-x})$

RELATIONSHIPS AMONG HYPERBOLIC FUNCTIONS

tanh x = sinh x/cosh x

coth x = 1/tanh x = cosh x/sinh x

sech x = 1/cosh x

csch x = 1/sinh x

$\cosh^2 x - \sinh^2 x = 1$

$\text{sech}^2 x + \tanh^2 x = 1$

$\coth^2 x - \text{csch}^2 x = 1$

FUNCTIONS OF NEGATIVE ARGUMENTS

sinh(-x) = -sinh x

cosh(-x) = cosh x

tanh(-x) = -tanh x

csch(-x) = -csch x

sech(-x) = sech x

coth(-x) = -coth x

ADDITION FORMULAS

sinh *(x ± y)* = sinh *x* cosh *y* ± cosh *x* sinh *y*

cosh (x ± y) = cosh *x* cosh y ± sinh x sinh y

tanh(x ± y) = (tanh x ± tanh y)/(1 ± tanh x.tanh y)

coth(x ± y) = (coth x coth y ± l)/(coth *y* ± coth x)

DOUBLE ANGLE FORMULAS

sinh 2x = 2 sinh *x* cosh x

cosh 2x = $\cosh^2 x + \sinh^2 x = 2\cosh^2 x - 1 = 1 + 2\sinh^2 x$

tanh 2x = $(2\tanh x)/(1 + \tanh^2 x)$

HALF ANGLE FORMULAS

$$\sinh\frac{x}{2} = \pm\sqrt{\frac{\cosh x - 1}{2}}$$ [+ if x > 0, - if x < 0]

$$\cosh\frac{x}{2} = \sqrt{\frac{\cosh x + 1}{2}}$$

$$\tanh\frac{x}{2} = \pm\sqrt{\frac{\cosh x - 1}{\cosh x + 1}}$$ [+ if x > 0, - if x < 0]

$$= \frac{sinh(x)}{1 + cosh(x)} = \frac{cosh(x) - 1}{sinh(x)}$$

MULTIPLE ANGLE FORMULAS

sinh 3x = 3 sinh x + 4 $\sinh^3$ x

cosh 3x = 4 $\cosh^3$ x — 3 cosh x

tanh 3x = (3 tanh x + $\tanh^3$ x)/(1 + 3 $\tanh^2$x)

sinh 4x = 8 $\sinh^3$ x cosh x + 4 sinh x cosh x

cosh 4x = 8 $\cosh^4$ x — 8 $\cosh^2$ x + 1

tanh 4x = (4 tanh *x* + 4 $\tanh^3$ x)/(1 + 6 $\tanh^2$ x + $\tanh^4$ x)

POWERS OF HYPERBOLIC FUNCTIONS

$\sinh^2$ x = ½cosh 2x — ½

$\cosh^2$ x = ½cosh 2x + ½

$\sinh^3$ x = ¼sinh 3x — ¾sinh x

$\cosh^3$ x = ¼cosh 3x + ¾cosh x

$\sinh^4 x = 3/8 - ½\cosh 2x + 1/8\cosh 4x$

$\cosh^4 x = 3/8 + ½\cosh 2x + 1/8\cosh 4x$

SUM, DIFFERENCE AND PRODUCT OF HYPERBOLIC FUNCTIONS

sinh x + sinh *y* = 2 sinh ½(x + *y)* cosh ½(x - y)

sinh x - sinh y = 2 cosh ½(x + y) sinh ½(x - y)

cosh x + cosh *y* = 2 cosh ½(x + y) cosh ½(x - y)

cosh x - cosh *y* = 2 sinh ½(x + y) sinh ½(x — y)

sinh x sinh *y* = ½(cosh *(x + y)* - cosh (x - y))

cosh x cosh y = ½(cosh *(x + y)* + cosh (x — y))

sinh x cosh y = ½(sinh *(x + y)* + sinh (x - y))

EXPRESSION OF HYPERBOLIC FUNCTIONS IN TERMS OF OTHERS

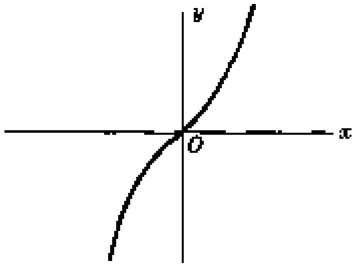
In the following we assume x > 0. If *x* < 0 use the appropriate sign as indicated by formulas in the section "Functions of Negative Arguments"

~	$sinhx = u$	$coshx = u$	$tanhx = u$	$cothx = u$	$sechx = u$	$cschx = u$
$sinhx$	u	$\sqrt{u^2-1}$	$\frac{u}{\sqrt{1-u^2}}$	$\frac{1}{\sqrt{u^2-1}}$	$\frac{\sqrt{1-u^2}}{u}$	$\frac{1}{u}$
$coshx$	$\sqrt{1+u^2}$	u	$\frac{1}{\sqrt{1-u^2}}$	$\frac{u}{\sqrt{u^2-1}}$	$\frac{1}{u}$	$\frac{\sqrt{1+u^2}}{u}$
$tanhx$	$\frac{u}{\sqrt{1+u^2}}$	$\frac{\sqrt{u^2-1}}{u}$	u	$\frac{1}{u}$	$\sqrt{1-u^2}$	$\frac{1}{\sqrt{1+u^2}}$

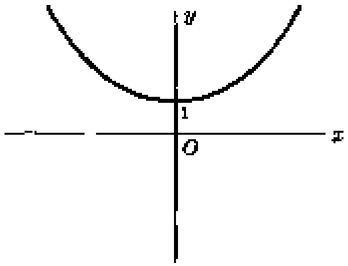
$coth x$	$\frac{\sqrt{1+u^2}}{u}$	$\frac{u}{\sqrt{u^2-1}}$	$\frac{1}{u}$	u	$\frac{1}{\sqrt{1-u^2}}$	$\sqrt{1+u^2}$
$sech x$	$\frac{1}{\sqrt{1+u^2}}$	$\frac{1}{u}$	$\sqrt{1-u^2}$	$\frac{\sqrt{u^2-1}}{u}$	u	$\frac{u}{\sqrt{1+u^2}}$
$csch x$	$\frac{1}{u}$	$\frac{1}{\sqrt{u^2-1}}$	$\frac{\sqrt{1-u^2}}{u}$	$\sqrt{u^2-1}$	$\frac{u}{\sqrt{1-u^2}}$	u

GRAPHS OF HYPERBOLIC FUNCTIONS

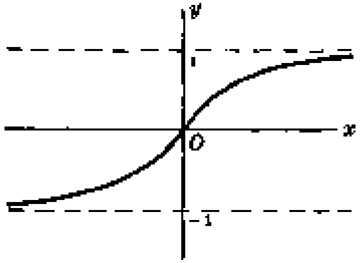
y = sinh x



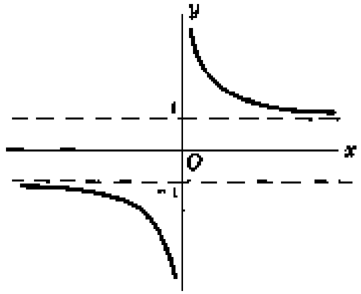
y = cosh x



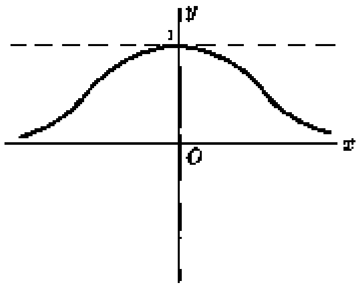
y = tanh x



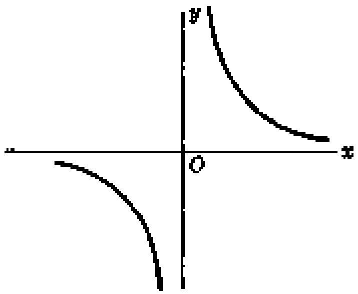
y = coth x



y = sech x



y = csch x



INVERSE HYPERBOLIC FUNCTIONS

If *x* = sinh *y*, then *y* = $\sinh^{-1}$ a is called the *inverse hyperbolic sine* of *x.* Similarly we define the other inverse hyperbolic functions. The inverse hyperbolic functions are multiple-valued and as in the case of inverse trigonometric functions we restrict ourselves to principal values for which they can be considered as single-valued.

The following list shows the principal values [unless otherwise indicated] of the inverse hyperbolic functions expressed in terms of logarithmic functions which are taken as real valued.

$$\sinh^{-1} x = \ln(x + \sqrt{x^2 + 1}) \quad -\infty < x < \infty$$

$$\cosh^{-1} x = \ln(x + \sqrt{x^2 - 1}) \quad x \geq l \ [\cosh^{-1} x > 0 \text{ is principal value}]$$

$$\tanh^{-1} x = \frac{1}{2} \ln \frac{(1+x)}{(1-x)} \quad -1 < x < 1$$

$$\coth^{-1} x = \frac{1}{2} \ln \frac{(x+1)}{(x-1)} \quad x > 1 \text{ or } x < -1$$

$$\operatorname{sech}^{-1} x = \ln(\frac{1}{x} + \sqrt{\frac{1}{x^2} - 1}) \quad 0 < x \leq l \ [\operatorname{sech}^{-1} x > 0 \text{ is principal value}]$$

$$\operatorname{csch}^{-1} x = \ln(\frac{1}{x} + \sqrt{\frac{1}{x^2} + 1}) \quad x \neq 0$$

RELATIONS BETWEEN INVERSE HYPERBOLIC FUNCTIONS

csch^{-1} x = $\sinh^{-1}$ (1/x)

sech^{-1} x = $\cosh^{-1}$ (1/x)

$\coth^{-1}$ x = $\tanh^{-1}$ (1/x)

$\sinh^{-1}(-x) = -\sinh^{-1}x$

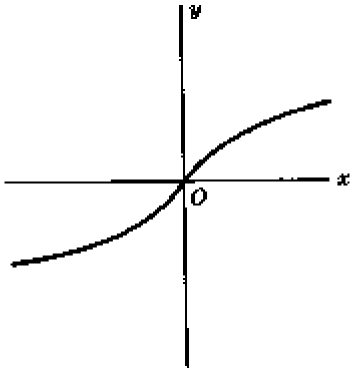
$\tanh^{-1}(-x) = -\tanh^{-1}x$

$\coth^{-1}(-x) = -\coth^{-1}x$

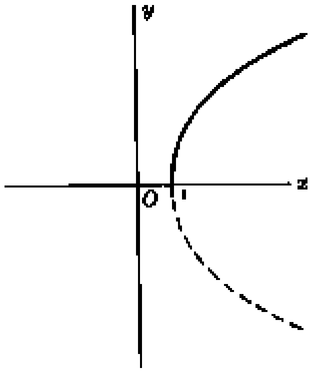
$\operatorname{csch}^{-1}(-x) = -\operatorname{csch}^{-1}x$

GRAPHS OF INVERSE HYPERBOLIC FUNCTIONS

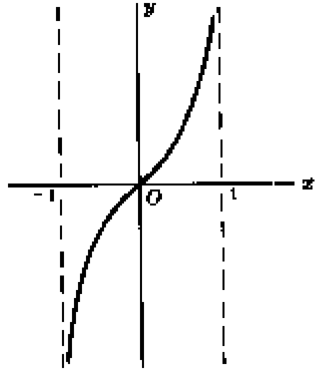
$y = \sinh^{-1}x$



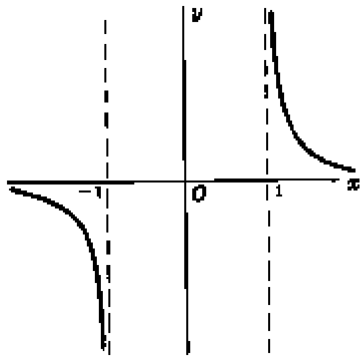
$y = \cosh^{-1}x$



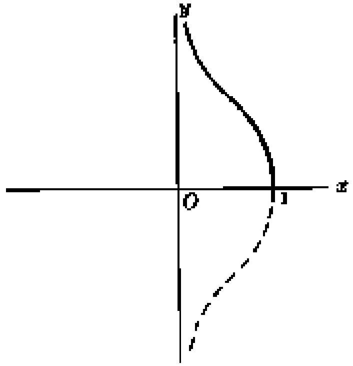
$y = \tanh^{-1}x$



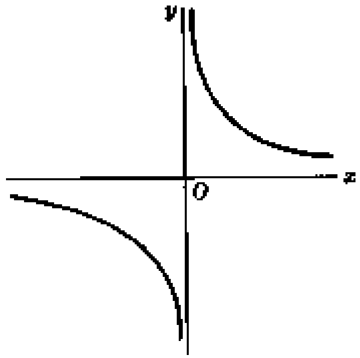
$y = \coth^{-1}x$



$y = \operatorname{sech}^{-1} x$



$y = \operatorname{csch}^{-1} x$



RELATIONSHIP BETWEEN HYPERBOLIC AND TRIGONOMETRIC FUNCTIONS

sin(ix) = i sinh x	cos(ix) = cosh x	tan(ix) = i tanh x
csc(ix) = -i csch x	sec(ix) = sech x	cot(ix) = -i coth x
sinh(ix) = i sin x	cosh(ix) = cos x	tanh(ix) = i tan x
csch(ix) = -i csc x	sech(ix) = sec x	coth(ix) = -i cot x

PERIODICITY OF HYPERBOLIC FUNCTIONS

In the following *k* is any integer.

sinh (x + 2kπi) = sinh x   csch (x + 2kπi) = csch x

cosh (x + 2kπi) = cosh x   sech (x + *2kπi)* = sech x

tanh (x + kπi) = tanh *x*   coth (x + *kπi)* = coth x

RELATIONSHIP BETWEEN INVERSE HYPERBOLIC AND INVERSE TRIGONOMETRIC FUNCTIONS

$\sin^{-1}(ix) = i\sinh^{-1}x$ $\quad$ $\sinh^{-1}(ix) = i\sin^{-1}x$

$\cos^{-1}x = \pm i\cosh^{-1}x$ $\quad$ $\cosh^{-1}x = \pm i\cos^{-1}x$

$\tan^{-1}(ix) = i\tanh^{-1}x$ $\quad$ $\tanh^{-1}(ix) = i\tan^{-1}x$

$\cot^{-1}(ix) = -i\coth^{-1}x$ $\quad$ $\coth^{-1}(ix) = -i\cot^{-1}x$

$\sec^{-1}x = \pm i\,\mathrm{sech}^{-1}x$ $\quad$ $\mathrm{sech}^{-1}x = \pm i\sec^{-1}x$

$\csc^{-1}(ix) = -i\,\mathrm{csch}^{-1}x$ $\quad$ $\mathrm{csch}^{-1}(ix) = -i\csc^{-1}x$

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