12. Consider the following algorithm for solving $Ux = b$, where U is upper triangular and x overwrites b.

Algorithm: $[b] :=$ Utrsv_nonunit_unb_var2(U, b)

Partition $U \rightarrow \left(\begin{array}{c|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right)$, $b \rightarrow \left(\begin{array}{c} b_T \\ \hline b_B \end{array}\right)$

where U_{BR} is 0×0, b_B has 0 rows

while $m(U_{BR}) < m(U)$ do

Repartition

$$\left(\begin{array}{c|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c|c|c} U_{00} & u_{01} & U_{02} \\ \hline u_{10}^T & \upsilon_{11} & u_{12}^T \\ \hline U_{20} & u_{21} & U_{22} \end{array}\right), \left(\begin{array}{c} b_T \\ \hline b_B \end{array}\right) \rightarrow \left(\begin{array}{c} b_0 \\ \hline \beta_1 \\ \hline b_2 \end{array}\right)$$

where υ_{11} is 1×1, β_1 has 1 row

$\beta_1 := \beta_1 / \upsilon_{11}$ 1 floating point operation

$b_0 := b_0 - \beta_1 u_{01}$ $\leftarrow$ $2(n-k-1)$ $n-k-1$

Continue with

$$\left(\begin{array}{c|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c|c|c} U_{00} & u_{01} & U_{02} \\ \hline u_{10}^T & \upsilon_{11} & u_{12}^T \\ \hline U_{20} & u_{21} & U_{22} \end{array}\right), \left(\begin{array}{c} b_T \\ \hline b_B \end{array}\right) \leftarrow \left(\begin{array}{c} b_0 \\ \hline \beta_1 \\ \hline b_2 \end{array}\right)$$

endwhile

Justify that this algorithm requires approximately n^2 floating point operations.

Total number will be $\sum_{k=0}^{n-1} 2(n-k-1) = \sum_{i=0}^{n-1} 2i = 2\sum_{i=0}^{n-1} i$

Let $i = n-k-1$ $\left(= 2\frac{(n-1)n}{2} \right)$

$\simeq n^2$