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Course › Unit 3 Methods of Estimation › Lecture 8: Distance measures between distributions › 15. Likelihood of a Poisson Statistical Model

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15. Likelihood of a Poisson Statistical Model

Review: Statistical Model for a Poisson Distribution

2/2 points (graded)

Let $X_1, \ldots, X_n \overset{iid}{\sim} \text{Poiss}\,(\lambda^*)$ for some unknown $\lambda^* \in (0, \infty)$. Let $(E, \{\text{Poiss}\,(\lambda)\}_{\lambda \in \Theta})$ denote the corresponding statistical model. What is the smallest possible set that could be E?

- ○ $\mathbb{N} = \{1, 2, 3, \ldots\}$
- ● $\mathbb{N} \cup \{0\}$
- ○ $\mathbb{Z}$
- ○ $\mathbb{R}$

✓

The parameter space Θ can be written as an interval (a, ∞). What is the smallest value of a so that $\{\text{Poiss}\,(\lambda)\}_{\lambda \in (a, \infty)}$ represents all possible Poisson distributions?

$a =$ 0 ✓ **Answer:** 0.0

Solution:

A Poisson random variable takes values on all non-negative integers $\{0, 1, 2, \ldots\}$. Hence, the smallest possible sample space is $\mathbb{N} \cup \{0\}$.

A Poisson random variable is specified by its mean λ, which is allowed to be any positive real number. Hence, $a = 0$ is the correct choice.

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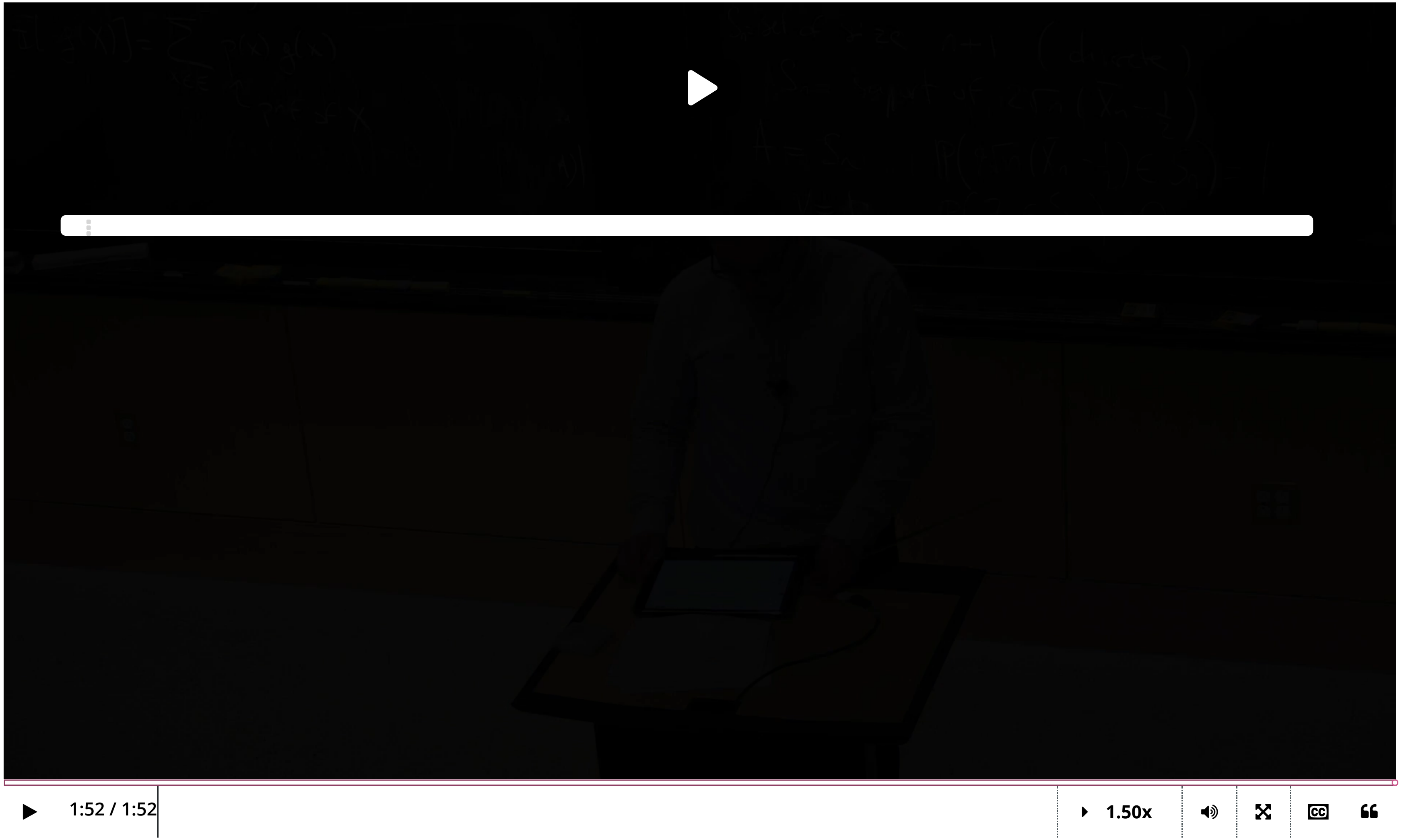
Answers are displayed within the problem

Likelihood of a Poisson Statistical Model



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Practice: Compute Likelihood of a Poisson Statistical Model

3/3 points (graded)

Let $X_1, \ldots, X_n \overset{iid}{\sim} \text{Poiss}(\lambda^*)$ for some unknown $\lambda^* \in (0, \infty)$. You construct the associated statistical model $(E, \{\text{Poiss}(\lambda)\}_{\lambda \in \Theta})$ where E and Θ are defined as in the answers to the previous question.

Suppose you observe two samples $X_1 = 1, X_2 = 2$. What is $L_2(1, 2, \lambda)$? Express your answer in terms of λ.

$L_2(1, 2, \lambda) =$ e^(-2*lambda)*lambda^3/2 ✓ **Answer:** e^(-2*lambda)*lambda^3/2

$\frac{e^{-2 \cdot \lambda} \cdot \lambda^3}{2}$

Next, you observe a third sample $X_3 = 3$ that follows $X_1 = 1$ and $X_2 = 2$. What is $L_3(1, 2, 3, \lambda)$?

$L_3(1, 2, 3, \lambda) =$ e^(-3*lambda)*lambda^6/12 ✓ **Answer:** e^(-3*lambda)*lambda^6/12

$\frac{e^{-3 \cdot \lambda} \cdot \lambda^6}{12}$

Suppose your data arrives in a different order: $X_1 = 2, X_2 = 3, X_3 = 1$. What is $L_3(2, 3, 1, \lambda)$?

$L_3(2, 3, 1, \lambda) =$ e^(-3*lambda)*lambda^6/12 ✓ **Answer:** e^(-3*lambda)*lambda^6/12

$\frac{e^{-3 \cdot \lambda} \cdot \lambda^6}{12}$

STANDARD NOTATION

Solution:

The probability mass function of $\text{Poiss}(\lambda)$ is $x \mapsto e^{-\lambda} \frac{\lambda^x}{x!}$ where $x \in \mathbb{N} \cup \{0\}$. Hence by definition

$$L_n(x_1, \ldots, x_n, \lambda) = \prod_{i=1}^{n} e^{-\lambda} \frac{\lambda^{x_i}}{x_i!} = e^{-n\lambda} \frac{\lambda^{\sum_{i=1}^{n} x_i}}{x_1! \cdots x_n!}.$$

Hence, first we plug in $n = 2$, $x_1 = 1,$ and $x_2 = 2$:

$$L_2\left(1,2,\lambda\right) = e^{-2\lambda}\frac{\lambda^{1+2}}{2!1!} = e^{-2\lambda}\frac{\lambda^3}{2}.$$

When the next sample arrives, we can simply evaluate the density of a Poisson at the observation:

$$P\left(X_3 = 3\right) = e^{-\lambda}\frac{\lambda^3}{3!}, \quad X \sim \mathrm{Poiss}\left(\lambda\right)$$

and multiply this by the previous response:

$$L_3\left(1,2,3,\lambda\right) = e^{-\lambda}\frac{\lambda^3}{3!}L_2\left(1,2,\lambda\right) = e^{-3\lambda}\frac{\lambda^6}{12}.$$

Remark 1: Observe that we can compute the likelihood sequentially as the data arrives, updating it in the previous fashion after each new observation.

Similarly, we see that

$$L_3\left(2,3,1,\lambda\right) = e^{-3\lambda}\frac{\lambda^6}{12}.$$

Remark 2: Observe that the likelihood of observations $X_1 = x_1, \ldots, X_n = x_n$ is independent of the *order* in which these observations arrive.

You have used 1 of 3 attempts

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Properties of the Likelihood

1/1 point (graded)

Let $(E, \{P_\theta\}_{\theta\in\Theta})$ denote a discrete statistical model. Let p_θ denote the pmf of P_θ. Let $X_1, \ldots, X_n \overset{iid}{\sim} P_{\theta^*}$ where the parameter θ^* is unknown. Then the **likelihood** is the function

$$\begin{aligned} L_n : E^n \times \Theta &\to \mathbb{R} \\ (x_1, \ldots, x_n, \theta) &\mapsto \prod_{i=1}^{n} p_\theta(x_i). \end{aligned}$$

For our purposes, we think of $x_1, \ldots, x_n$ as observations of the random variables $X_1, \ldots, X_n$.

Which of the following are properties of the likelihood L_n? (Choose all that apply.)

Hint: It may be useful to consider your responses from the previous question.

☐ The likelihood does not change with the parameter θ.

☑ The likelihood can be updated sequentially as new samples are observed. For example, $L_3(x_1, x_2, x_3, \theta) = L_1(x_3, \theta) L_2(x_1, x_2, \theta)$

☑ The likelihood is symmetric: it doesn't matter the order in which we plug in the observations. For example, $L_4(x_1, x_2, x_3, x_4, \theta) = L_4(x_2, x_3, x_1, x_4, \theta)$ and this is true for any rearrangement of x_1, x_2, x_3, x_4.

☐ If we eliminate a single observation, then the likelihood remains unchanged. For example, $L_3(x_1, x_2, x_3, \theta) = L_2(x_1, x_2, \theta)$

✓

Solution:

We examine the choices in order.

- "The likelihood does not change with the parameter θ." is incorrect. Rather, it is crucial that we interpret the likelihood L_n as a function of θ. That is, L_n varies as θ ranges over the parameter space Θ. This is evident in the likelihoods for the Bernoulli and Poisson models in the previous problems.

- "The likelihood can be updated sequentially as new samples are observed. For example, $L_3\left(x_1, x_2, x_3, \theta\right) = L_1\left(x_3, \theta\right) L_2\left(x_1, x_2, \theta\right)$." is also correct. In the previous problem, we saw that to compute the likelihood after observing $X_3 = 3$, we simply took the old likelihood $L_2\left(1, 2, \lambda\right)$ and multiplied it by $L_1\left(3, \lambda\right)$. Note that $L_1\left(x_3, \theta\right) = p_\theta\left(x_3\right)$, the density of P_θ evaluated at the new observation. Inspection of the defining formula

  $$L_n\left(x_1, \ldots, x_n, \theta\right) = \prod_{i=1}^{n} p_\theta\left(x_i\right)$$

  implies that the likelihood can be updated sequentially in this fashion.

- "The likelihood is symmetric..." is correct. We observed in the previous problem that observing the samples in a different order does not affect the likelihood. This is also evident from the definition of the likelihood: we can take the product

  $$\prod_{i=1}^{n} p_\theta\left(x_i\right)$$

  in any order, and the result will still be the same.

- "If we eliminate a single observation, then the likelihood remains unchanged..." is incorrect. In the previous question, we saw that for a Poisson statistical model, $L_2\left(1, 2, \lambda\right)$ and $L_3\left(1, 2, 3, \lambda\right)$ do not have the same formula. Hence, deleting an observation from the sample will change the likelihood.

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ℹ Answers are displayed within the problem

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