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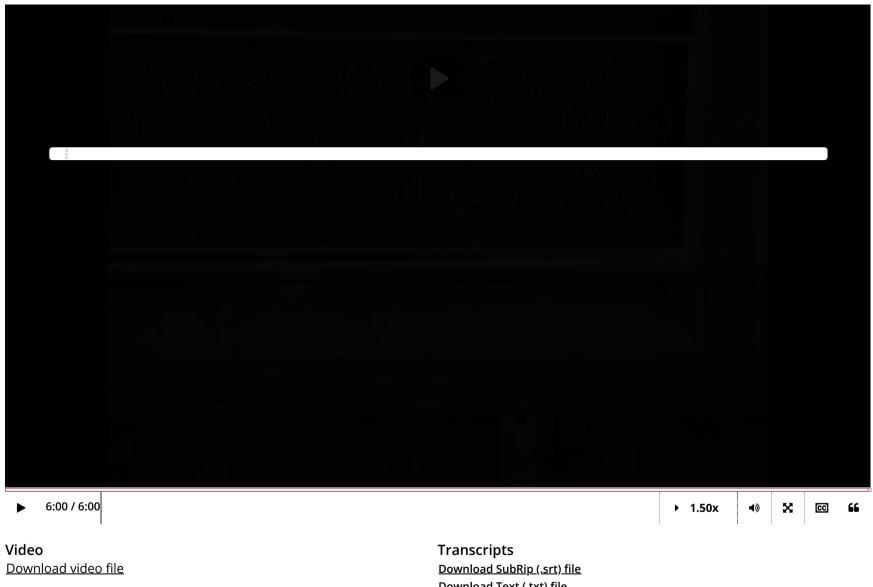
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13. Even and odd periodic functions Fourier series of even and odd periodic functions



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Even and odd symmetry

 $\bullet \ \ {\rm A \ function} \ f(t) \ {\rm is} \ {\bf even} \ {\rm if} \ f(-t) = f(t) \ {\rm for \ all} \ t.$

ullet A function f(t) is **odd** if f(-t)=-f(t) for all t.

lf

$$f\left(t
ight) \hspace{0.2in} = rac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nt + \sum_{n=1}^{\infty} b_n \sin nt,$$

then substituting -t for t gives

$$f\left(-t
ight) = rac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nt + \sum_{n=1}^{\infty} \left(-b_n
ight) \sin nt.$$

The right hand sides match if and only if $b_n=0$ for all n.

Conclusion: The Fourier series of an even function f has only cosine terms (including the constant term):

$$f\left(t
ight) =rac{a_{0}}{2}+\sum_{n=1}^{\infty }a_{n}\cos nt.$$

Similarly, the Fourier series of an odd function f has only sine terms:

$$f\left(t
ight) =\sum_{n=1}^{\infty }b_{n}\sin nt.$$

Fourier series of Square wave using oddness

The square wave,

$$\operatorname{Sq}\left(t
ight) = \left\{egin{array}{ll} 1 & 0 < t < \pi \ -1 & -\pi < t < 0 \end{array}
ight.,$$

is an odd function, so

$$\operatorname{Sq}\left(t
ight)=\sum_{n=1}^{\infty}b_{n}\sin nt$$

for some numbers b_n^{T} he Fourier coefficient formula says

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} \underbrace{\operatorname{Sq}(t) \sin nt}_{\text{even}} dt$$

$$= \frac{2}{\pi} \int_{0}^{\pi} \operatorname{Sq}(t) \sin nt \, dt \qquad \text{(the two halves of the integral are equal, by symmetry)}$$

$$= \frac{2}{\pi} \int_{0}^{\pi} \sin nt \, dt \qquad \text{(since Sq}(t) = 1 \text{ whenever } 0 < t < \pi)$$

$$= \frac{2(-\cos nt)}{\pi n} \Big|_{0}^{\pi}$$

$$= \frac{2}{\pi n} (-\cos n\pi + \cos 0)$$

$$= \begin{cases} \frac{4}{\pi n}, & \text{if } n \text{ is odd,} \\ 0, & \text{if } n \text{ is even.} \end{cases}$$

Thus

$$b_1=rac{arLambda}{\pi},\quad b_3=rac{arLambda}{2\pi},\quad b_5=rac{arLambda}{5\pi},\ldots$$

and all other Fourier coefficients are ()

Conclusion:

$$\operatorname{Sq}\left(t\right) = \frac{4}{\pi} \left(\sin t + \frac{\sin 3t}{3} + \frac{\sin 5t}{5} + \cdots \right).$$

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Concept check

1/1 point (graded)

The following functions are 2π -periodic. They are only defined on the interval from $-\pi$ to π . Identify all of the even periodic functions in the list below.

$$p\left(x
ight) = egin{cases} x & : x \in (0,\pi) \ \pi + x & : x \in (-\pi,0) \end{cases}$$

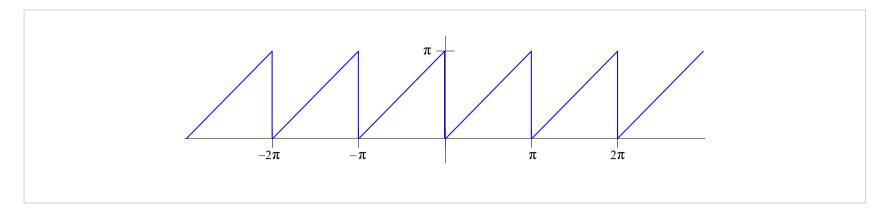
$$oldsymbol{Q}\left(x
ight) = egin{cases} x & : x \in (0,\pi) \ -x & : x \in (-\pi,0) \end{cases}$$

$$ho r\left(x
ight) =x \quad :x\in \left(-\pi ,\pi
ight)$$

$$s\left(x
ight) = \left\{egin{array}{ll} x^2 & :x\in(0,\pi) \ -x^2 & :x\in(-\pi,0) \end{array}
ight.$$

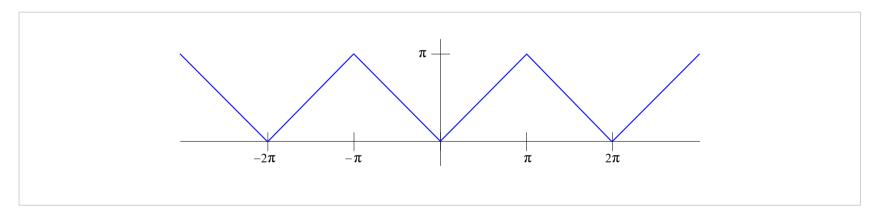
$$lacksquare t\left(x
ight) =\,x^{2}\quad :x\in \left(-\pi ,\pi
ight)$$

• The graph of p(x) is below.



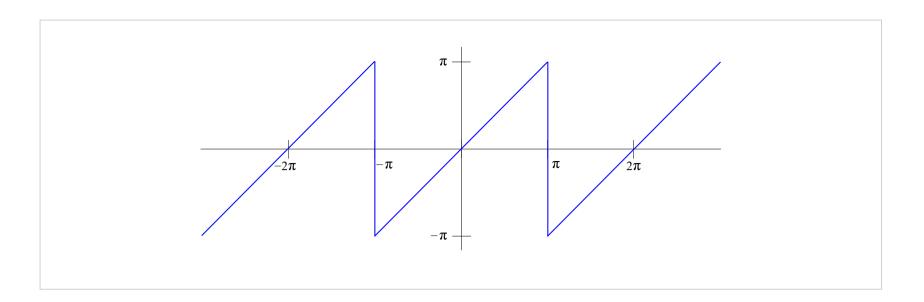
Consider values of x so that $0 < x < \pi$. Note that $p(-x) = \pi - x$, and p(x) = x. Since $\pi - x \neq x$ this function is neither even nor odd.

• The graph of q(x) is below.



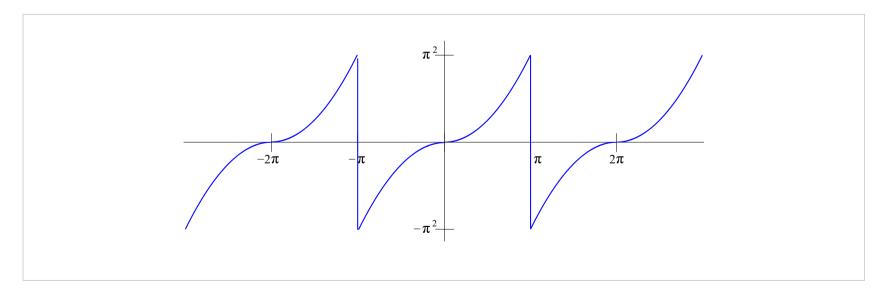
Consider values of x so that $0 < x < \pi$. Note that q(-x) = x, and q(x) = x. Since q(-x) = q(x) this function is even.

• The graph of $r\left(x\right)$ is below.

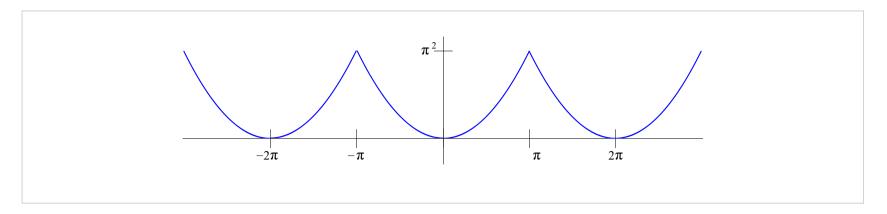


Consider values of x so that $0 < x < \pi$. Note that r(-x) = -x, and r(x) = x. Since r(-x) = -r(x) this function is odd.

• The graph of $s\left(x\right)$ is below.



• The graph of $t\left(x\right)$ is below.



Consider values of x so that $0 < x < \pi$. Note that $t(-x) = x^2$, and $t(x) = x^2$. Since t(-x) = t(x) this function is even.

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• Answers are displayed within the problem

13. Even and odd periodic functions

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by n, n even, terms are zero for square fn

For the square function, since it is odd, its F.S. consists of sines, and we found the even n terms vanish. O.K. All these vanishing terms have a base period pi, while the square f...

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