Exercises: Overall Model Test (Matlab exercise)

Three sets of observed heights are given, collected in 6 consecutive years.

```
%% 3 sets of observed heights [m]

y1  = [ 101.726  99.300  95.774  94.411  91.486  89.969]';

y2  = [ 101.726  99.300  95.774  101.411  91.486  89.969]';

y3  = [ 101.726  99.300  95.774  101.411  98.486  96.969]';
```

Our default model is given by:

$$E\{\underline{y}\} = Ax = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 2 \\ 1 & 3 \\ 1 & 4 \\ 1 & 5 \end{bmatrix}; D\{\underline{y}\} = Q_{yy} = \mathrm{diag}([1.44 \;\; 1.44 \;\; 0.09 \;\; 0.09 \;\; 0.09 \;\; 0.09])$$

Hence the covariance matrix is a diagonal matrix with the given values on its diagonal.

We will apply the overall model test for 5 cases:

- Case 1: **unbiased case**; default model with $y1$ as realization of $\underline{y}$.
- Case 2: **outlier (4th observation)**; default model with $y2$ as realization of $\underline{y}$.
- Case 3: **systematic bias (last 3 observations)**; default model with $y3$ as realization of $\underline{y}$.
- Case 4: **wrong stochastic model**; same as case 1, but with $D\{\underline{y}\} = Q'_{yy}$ (see below).
- Case 5: **wrong A-matrix**; same as case 1 but with $E\{\underline{y}\} = A'x$ (see below).

where

$$A' = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 4 \\ 1 & 9 \\ 1 & 16 \\ 1 & 25 \end{bmatrix};$$

$$Q'_{yy} = 0.09 I_6$$

DIFFERENT ERRORS (MATLAB) (EXTERNAL RESOURCE)

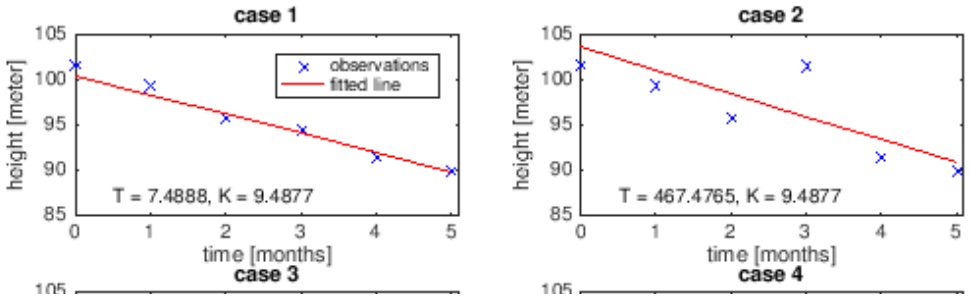
```
82 ylabel('height [meter]')
83 title('case 4')
84 text(0.5,87,['T = ' num2str(T4) ', K = ' num2str(K)])
85
86 subplot(325)
87 plot(t,y1,'xb')
88 hold on
89 plot(t,A*xhat5,'r')
90 set(gca,'xlim',[0 5.1],'ylim',[85 105])
91 xlabel('time [months]')
92 ylabel('height [meter]')
93 title('case 5')
94 text(0.5,87,['T = ' num2str(T5) ', K = ' num2str(K)])
95
```

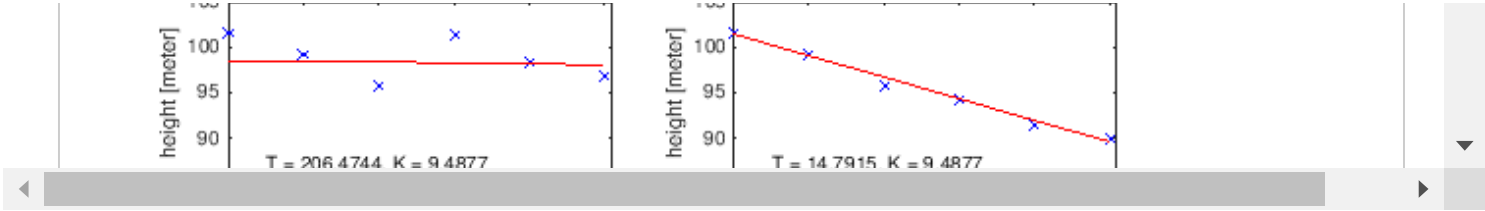
Run Submit for Assessment

Assessment Tests: Correct

✔ Code ran without errors

Output





Different errors (cont'd)

1/1 point (ungraded)

Which of the following statements is true (at least one is correct):

- [ ] an outlier does not affect the residuals of the other observations.
- [ ] a systematic bias cannot be detected with the overall model test.
- [x] if you apply a stochastic model with too small variance, the test statistic will be too large. ✔

✔

Explanation

- An outlier will generally affect the other residuals as well.
- In case of a constant systematic bias in all observations it is not possible to detect it. If for instance a subset of the observations is affected, the bias may be detected.
- In the Matlab example the overall model test is rejected due to the wrong choice of the covariance matrix (case 4). Especially the residuals of the first 2 observations are considered to be larger than expected based on the assumed standard deviation of 0.3 whereas the true standard deviation was 1.2.

Submit

✔ Correct (1/1 point)