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Foundations of Mathematics > Set Theory > Partial Orders >

Totally Ordered Set

A total order (or "totally ordered set," or "linearly ordered set") is a [set](#) plus a relation on the set (called a [total order](#)) that satisfies the conditions for a [partial order](#) plus an additional condition known as the comparability condition. A [relation](#) \leq is a total order on a [set](#) S (" \leq totally orders S ") if the following properties hold.

1. Reflexivity: $a \leq a$ for all $a \in S$.
2. Antisymmetry: $a \leq b$ and $b \leq a$ implies $a = b$.
3. Transitivity: $a \leq b$ and $b \leq c$ implies $a \leq c$.
4. Comparability ([trichotomy law](#)): For any $a, b \in S$, either $a \leq b$ or $b \leq a$.

The first three are the axioms of a [partial order](#), while addition of the [trichotomy law](#) defines a total order.

Every finite totally ordered set is [well ordered](#). Any two totally ordered sets with k elements (for k a nonnegative integer) are [order isomorphic](#), and therefore have the same [order type](#) (which is also an [ordinal number](#)).

SEE ALSO:
[Order Isomorphic](#), [Order Type](#), [Partial Order](#), [Relation](#), [Total Order](#), [Trichotomy Law](#), [Well Ordered Set](#)

REFERENCES:
Séroul, R. *Programming for Mathematicians*. Berlin: Springer-Verlag, p. 23, 2000.

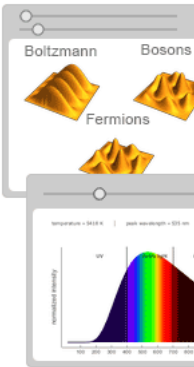
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(A union B) intersect C

THINGS TO TRY:
= (A union B) intersect C
= Cantor's Paradox
= local maximum calculator

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