2.2.1 Computing Average Value

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We saw in the previous section that the solution trajectory (the curve traced out by $(S(t), M(t))$ over time) is a closed curve and the sardine and marlin populations oscillate between maximum and minimum values. Given this, how do we get an overall sense of the size of the sardine and marlin populations over time?

Why do we care? Knowing the size of a fluctuating population is important to environmental scientists and the fishing industry as they think about how to responsibly manage resources.

As Ethan suggested, one way to get a sense of population size to take the average value of the populations over the cycle. Let L denote the length of one solution cycle - this is the amount of time it takes for the fish populations to return to their starting values. (We saw an example of a specific cycle length in the previous part. However, in general we don't know the length of any given cycle, so that's why we call it L.)

We now take the **average value** of the functions describing the populations of sardine and marlin, respectively, over the cycle length L. The **average value of a function over an interval** is defined using an integral. We integrate the function over the interval and then divide by the length of the interval.

> For an explanation of why this definition makes sense, review the video on Average value of a function over a closed interval from Khan Academy.

We have a cycle of length L. For simplicity, we'll work with the interval $[0, L]$. (We can do this because our differential equations system do not explicitly depend on time.) The average value of the function $M(t)$ describing marlin population over the cycle is:

$$\overline{M} = \frac{1}{L}\int_0^L M(t)\,dt.$$

You first instinct might be to try to compute this integral directly. This is not possible because we don't have an explicit formula for $M(t)$. (Remember: we didn't solve the system of differential equations for $M(t)$ or $S(t)$ explicitly.)

However, it turns out knowing the the system of differential equations

$$\frac{dS}{dt} = \mathbf{a}S - \mathbf{b}SM$$

$$\frac{dM}{dt} = -\mathbf{c}M + \mathbf{d}SM.$$

is enough to find the average value of the sardine and marlin populations over a cycle! That's exciting because our goal is to have a sensible way of discussing the size of a fluctuating population.

The following quizzes and videos will guide you through this. As we go through this, remember that

- t is the independent variable;
- $S(t)$ and $M(t)$ are functions which depend on time (so when we write S we really mean $S(t)$ and $M(t)$);
- $\mathbf{a}, \mathbf{b}, \mathbf{c},$ and $\mathbf{d}$ are *parameters* of this model representing rates of growth, death, predation, synthesis. They are constant within a particular context (for a particular predator and prey relationship) but may be different for different species.

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