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6.1.1 Summary Quiz Part I

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Question 1

1/1 point (graded)

Let $\theta_0 > 0$ and $b = \frac{\pi}{4}$. Which of the following are true about a pendulum whose solution is $\theta(t) = \theta_0 \cos\left(\sqrt{\frac{g}{l}}t + b\right)$?

(Hint: consider $\theta(0)$ and $\frac{d\theta}{dt}(0)$.)

- [ ] The maximum angle of the pendulum is θ_0 and the pendulum starts at θ_0.
- [ ] The maximum angle of the pendulum is $\pi/4$ and the pendulum starts at $\pi/4$.
- [x] The maximum angle of the pendulum is θ_0 but the pendulum starts at an angle less than θ_0. ✔
- [ ] The velocity at $t = 0$ is zero.
- [ ] The velocity at $t = 0$ is positive and the pendulum is moving toward the vertical.
- [x] The velocity at $t = 0$ is negative and the pendulum is moving toward the vertical. ✔
- [ ] None of the above.

Explanation

The maximum angle of the pendulum is θ_0 since the maximum value of the cosine function $\cos(x)$ is 1. However, $\theta(0) = \theta_0 \cos(\pi/4) = \theta_0 \frac{\sqrt{2}}{2} < \theta_0$. Thus the maximum angle of the pendulum is θ_0 but the pendulum starts at an angle less than θ_0. We can compute $\frac{d\theta}{dt}(0) = -\sqrt{\frac{g}{l}}\theta_0 \sin(\pi/4)$. Since $\theta_0 > 0$ the velocity is negative, and since $\theta(0) > 0$ this means the pendulum moves toward vertical.

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Answers are displayed within the problem

Question 2: Think About It...

1/1 point (graded)

Describe the trajectories, nullclines, and equilibria of the system

$$\frac{d\theta}{dt} = \alpha.$$

$$\frac{d\alpha}{dt} = -k\theta$$

where $k = \frac{g}{l}$ is a positive constant, and interpret each of these features in terms of a physical pendulum.

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t = sqrt(l/g)*(n*pi - b), for n an integer, is solution for
the first and t = sqrt(l/g)*(n*pi + pi/2 - b) is solution
for the second, so no equilibrium is there, except theta=0.
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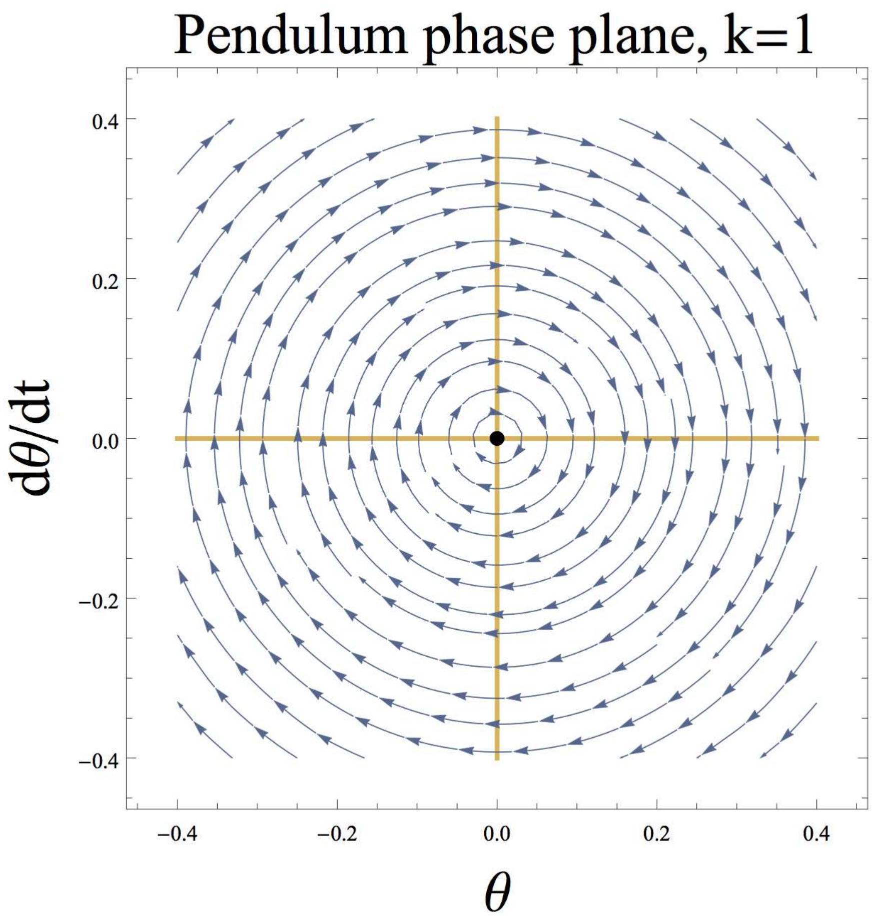
Thank you for your response.

Explanation

The trajectories are clockwise curves. When $k = 1$, the trajectories are circles centered on the origin. More generally, they are ellipses. Because of our simplifying assumptions (no friction, air resistance, or stretching) the trajectories are closed. Our model predicts that each swing of the pendulum will be exactly the same as the last. The θ-nullcline is where $\alpha = 0$. This means the angle is not changing where $\alpha = 0$. This reflects the fact that the pendulum stops at each end of its swing ($\frac{d\theta}{dt} = 0$) before changing direction. The α-nullcline is where $\theta = 0$. This reflects the fact that the velocity is greatest when the pendulum is hanging straight down ($\theta = 0$).

The system has one equilibrium point at $(0, 0)$. This means there is an initial angle of 0 and an initial velocity of 0. The equilibrium represents the solution in which the pendulum hangs straight down without moving.



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Question 3

1/1 point (graded)

What are the possible long term behaviors of a pendulum obeying the equation $\frac{d^2\theta}{dt^2} = -k\theta$? Choose the best answer below.

(Again, we continue to assume that the angle θ is small throughout the swing.)

- ◯ The graph of θ vs. time for a swinging pendulum has a horizontal asymptote at $\theta = 0$.
- ◉ A swinging pendulum will continue to swing with the same period indefinitely. ✔
- ◯ A swinging pendulum will continue to swing, but its period will increase in duration and its angle of swing will decrease in size until it is nearly stationary.
- ◯ A swinging pendulum will continue to swing, but its period will gradually decrease in duration and its angle of swing will increase in size until it stops.
- ◯ None of the above.

Explanation

This mathematical model predicts that the pendulum will continue to swing indefinitely. This comes from the periodic solutions to the differential equation.

Our model does not include details like air resistance and friction, which eventually slow a pendulum to a stop. However, real pendulums can swing for a very long time. If the angle they swing through is small, their period changes little over time but the width of their arc (represented by θ_0 in our model) will eventually decrease to 0.

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Answers are displayed within the problem

Question 4

1/1 point (graded)

Find the period of the function $\theta(t) = \theta_0 \cos\left(\sqrt{\frac{g}{l}}t + b\right)$. Which of the following affect the period of the pendulum?

Choose all that apply.

- [ ] Time, t.
- [x] Acceleration due to gravity.
- [x] Length of rod l.
- [ ] Starting angle θ_0.
- [ ] Starting velocity.
- [ ] Mass of bob, m.
- [ ] None of the above – all pendulums have the same period.

Explanation

In our model, the position of the pendulum is described by $\theta = \theta_0 \cos\left(\sqrt{\frac{g}{l}}t + b\right)$, where θ_0 and b are determined by the initial angle and velocity of the pendulum.

This function has a period of $\frac{2\pi}{\sqrt{k}} = 2\pi\sqrt{\frac{l}{g}}$. The period is the amount of time t it takes the pendulum to return to its initial condition. It depends on the acceleration due to gravity and length of the rod.

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Answers are displayed within the problem

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