



## MITx: 6.041x Introduction to Probability - The Science of Uncertainty



Bookmarks

- ▶ Unit 0: Overview
- ▶ Entrance Survey
- ▶ Unit 1: Probability models and axioms
- ▶ Unit 2: Conditioning and independence
- ▶ Unit 3: Counting
- ▶ Unit 4: Discrete random variables
- ▶ Exam 1
- ▶ Unit 5: Continuous random variables

Unit 8: Limit theorems and classical statistics &gt; Problem Set 8 &gt; Problem 2 Vertical: Find the limits

Bookmark

## Problem 2: Find the limits

(3/3 points)

Let  $S_n$  be the number of successes in  $n$  independent Bernoulli trials, where the probability of success for each trial is  $1/2$ . Provide a numerical value, to a precision of 3 decimal places, for each of the following limits. You may want to refer to the standard normal table .

1.

$$\lim_{n \rightarrow \infty} \mathbf{P} \left( \frac{n}{2} - 20 \leq S_n \leq \frac{n}{2} + 20 \right) = \boxed{0} \quad \checkmark \quad \text{Answer: 0}$$

2.

$$\lim_{n \rightarrow \infty} \mathbf{P} \left( \frac{n}{2} - \frac{n}{3} \leq S_n \leq \frac{n}{2} + \frac{n}{3} \right) = \boxed{1} \quad \checkmark \quad \text{Answer: 1}$$

3.

$$\lim_{n \rightarrow \infty} \mathbf{P} \left( \frac{n}{2} - \frac{\sqrt{n}}{4} \leq S_n \leq \frac{n}{2} + \frac{\sqrt{n}}{4} \right) = \boxed{0.3829249} \quad \checkmark$$


Answer: 0.383

Answer:


- ▶ Unit 6: Further topics on random variables
- ▶ Unit 7: Bayesian inference
- ▶ Exam 2
- ▼ Unit 8: Limit theorems and classical statistics

#### Unit overview


##### Lec. 18: Inequalities, convergence, and the Weak Law of Large Numbers

Exercises 18 due Apr 27, 2016 at 23:59 UTC 

##### Lec. 19: The Central Limit Theorem (CLT)

Exercises 19 due Apr 27, 2016 at 23:59 UTC 

##### Lec. 20: An introduction to classical statistics

Exercises 20 due Apr 27, 2016 at 23:59 UTC 

Note first that  $S_n = X_1 + \cdots + X_n$ , where the  $X_i$  are independent Bernoulli random variables with parameter  $1/2$ . Hence,  $\mathbf{E}[S_n] = n/2$  and  $\mathbf{var}(S_n) = n/4$ .

1. Let us fix some  $\epsilon > 0$ . No matter how small  $\epsilon$  is, as long as  $n$  is large enough, we will have  $\epsilon\sqrt{n} > 20$ . Therefore,

$$\begin{aligned} \mathbf{P}\left(\frac{n}{2} - 20 \leq S_n \leq \frac{n}{2} + 20\right) &\leq \mathbf{P}\left(\frac{n}{2} - \epsilon\sqrt{n} \leq S_n \leq \frac{n}{2} + \epsilon\sqrt{n}\right) \\ &= \mathbf{P}\left(-\frac{\epsilon\sqrt{n}}{\sqrt{n/4}} \leq \frac{S_n - n/2}{\sqrt{n/4}} \leq \frac{\epsilon\sqrt{n}}{\sqrt{n/4}}\right) \\ &= \mathbf{P}\left(-2\epsilon \leq \frac{S_n - n/2}{\sqrt{n/4}} \leq 2\epsilon\right). \end{aligned}$$

By the Central Limit Theorem,

$$\lim_{n \rightarrow \infty} \mathbf{P}\left(-2\epsilon \leq \frac{S_n - n/2}{\sqrt{n/4}} \leq 2\epsilon\right) = \Phi(2\epsilon) - \Phi(-2\epsilon).$$


Since this is true for any  $\epsilon > 0$ , it is also true in the limit as  $\epsilon \rightarrow 0^+$ . The final answer then follows from the fact that

$$\lim_{\epsilon \rightarrow 0^+} \Phi(2\epsilon) - \Phi(-2\epsilon) = \Phi(0) - \Phi(0) = 0.$$

## Solved problems

## Additional theoretical material

## Problem Set 8

Problem Set 8 due Apr 27, 2016  
at 23:59 UTC 

## Unit summary

- ▶ Unit 9: Bernoulli and Poisson processes
- ▶ Unit 10: Markov chains
- ▶ Exit Survey
- ▶ Final Exam

2. The event  $\frac{n}{2} - \frac{\sqrt{n}}{4} \leq S_n \leq \frac{n}{2} + \frac{\sqrt{n}}{4}$  is the same as the event  $|(S_n/n) - (1/2)| \leq 1/3$ . By the weak law of large numbers, the probability of this event converges to 1 as  $n \rightarrow \infty$ .

3. By the Central Limit Theorem,

$$\begin{aligned} \lim_{n \rightarrow \infty} \mathbf{P} \left( \frac{n}{2} - \frac{\sqrt{n}}{4} \leq S_n \leq \frac{n}{2} + \frac{\sqrt{n}}{4} \right) &= \lim_{n \rightarrow \infty} \mathbf{P} \left( \left| \frac{S_n - n/2}{\sqrt{n/4}} \right| \leq \frac{\sqrt{n}/4}{\sqrt{n/4}} \right) \\ &= \Phi(1/2) - \Phi(-1/2) \\ &\approx 0.383. \end{aligned}$$

*You have used 1 of 2 submissions*

## DISCUSSION

Click "Show Discussion" below to see discussions on this problem.

© All Rights Reserved



© edX Inc. All rights reserved except where noted. EdX, Open edX and the edX and Open EdX logos are registered trademarks or trademarks of edX Inc.

