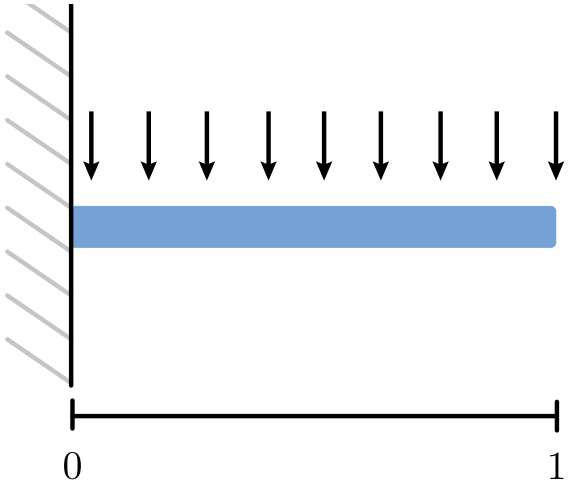
4. Horizontal beams and higher derivatives

In this next exercise, we will use linear algebra to discretize and solve the problem of a cantilever – a metal beam stuck into a wall at one end, and free at the other end.



we will solve the differential equation

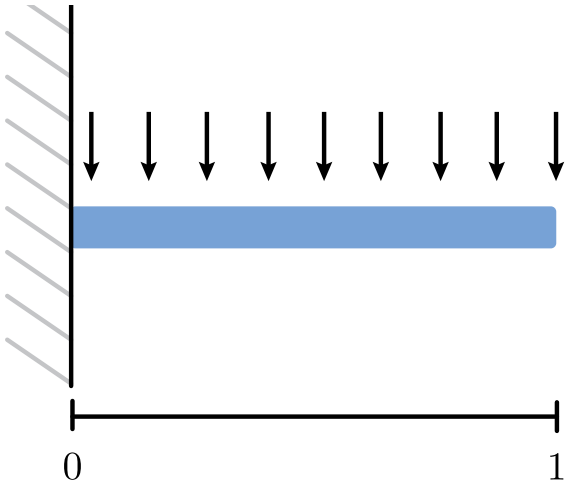
$$\frac{d^4v}{dx^4} = \frac{q(x)}{EI}$$

To solve this equation numerically, we must find a discretization of the fourth derivative. Additionally, we will have to find formulas for the boundary conditions. We will walk through finding these equations in the next few exercises.

The boundary conditions

2/2 points (graded)

In the physical setup shown here,



which of the following quantities are zero? (Each column is graded independently.)

- [x] $v(0)$
- [ ] $v(1)$
- [x] $v'(0)$
- [ ] $v'(1)$
- [ ] $v''(0)$
- [x] $v''(1)$
- [ ] $v'''(0)$
- [x] $v'''(1)$

✔ ✔

Submit You have used 1 of 3 attempts

Answers are displayed within the problem

Discrete formulas for the boundary conditions.

4/4 points (graded)

Find discrete formulas for the boundary conditions you identified in the previous problem in the special case where $N = 10$. That is we have discretized x in 10 evenly spaced points $x_1, \ldots, x_{10}$ and let $v_i = v(x_i)$.

(Reduce your formula to be linear relationships involving the v_i only. This is possible in this problem because the boundary conditions are homogeneous, so we can multiply through by a nonzero factor.)

(Type, for example, **v_{4}** for v_4. Note that the curly braces are required by the grader. A lack of curly braces will not be graded correctly.)

Boundary conditions at $x = 0$:

$0 =$ v_{1} ✔ **Answer:** v_1

$0 =$ (v_{1}-v_{2}) ✔ **Answer:** v_1-v_2

Boundary conditions at $x = 1$:

$0 =$ (v_{10}+v_{8}-2*v_{9}) ✔ **Answer:** -v_7+3*v_8-3*v_9+v_10

$0 =$ (v_{10}-3*v_{9}+3*v_{8}- ✔ **Answer:** v_8-2*v_9+v_10

Submit You have used 3 of 25 attempts

Answers are displayed within the problem

The fourth derivative

1/1 point (graded)

The first step is to discretize the beam. The variable x is a continuous variable from 0 to 1. Create a series of evenly spaced points $x_1, \ldots x_N$ with $x_1 = 0$ and $x_N = 1$. Then $\Delta x = x_{i+1} - x_i$.

Define $v_i = v(x_i)$.

Find a formula for the centered fourth derivative of v at x_i (assuming $i \geq 3$.)

$(v_{i+2} - 3v_{i+1} + 3v_i - v_{i-1}) / (\Delta x)^3$

- ○ $(v_{i+1} - 3v_i + 3v_{i-1} + v_{i-2}) / (\Delta x)^3$
- ◉ $(v_{i+2} - 4v_{i+1} + 6v_i - 4 * v_{i-1} + v_{i-2}) / (\Delta x)^4$
- ○ $(v_{i+2} + 4v_{i+1} + 6v_i + 4 * v_{i-1} + v_{i-2}) / (\Delta x)^4$

✔

Submit You have used 1 of 3 attempts

Answers are displayed within the problem

Putting it all together

Putting what you discovered in the previous exercises, we can write the differential equation

$$\frac{d^4v}{dx^4} = \frac{q(x)}{EI}$$

as a linear system as follows:

$$\begin{pmatrix} * & * & 0 & 0 & & 0 \cdots & 0 \\ * & * & 0 & 0 & & 0 \cdots & 0 \\ \circ & \circ & \circ & \circ & 0 & \cdots & 0 \\ 0 & \circ & \circ & \circ & \circ & \cdots & 0 \\ & \ddots & \ddots & \ddots & \ddots & & \\ 0 & \cdots & \circ & \circ & \circ & \circ & 0 \\ 0 & \cdots & 0 & 0 & * & * & * \\ 0 & \cdots & 0 & 0 & * & * & * \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \\ v_3 \\ \vdots \\ v_{N-2} \\ v_{N-1} \\ v_N \end{pmatrix} = \frac{\Delta x^4}{EI} \begin{pmatrix} 0 \\ 0 \\ q\left(x_3\right) \\ \vdots \\ q\left(x_{N-2}\right) \\ 0 \\ 0 \end{pmatrix}$$

where the rows of $*$ correspond to the boundary conditions, and the rows of $\circ$ correspond to the fourth derivative.

Fourth order system with boundary conditions (External resource)

4. Horizontal beams and higher derivatives

Topic: Unit 2: Boundary value problems and PDEs / 4. Horizontal beams and higher derivatives

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?	MatrixA Which function should be used to callout for matrixA? I used A=run(matrixA) but it is giving error.	2
☑	[Staff] Discrete formulas for the boundary conditions. Should not the question specify whether to use the forwards, centered or backwards derivatives?	4
💬	Way of finding formulas for n-th derivative Can you tell me what is the formula for finding any n-th derivative. I understand formula for 1st but 2nd,3rd and 4th is not in my power (exponential decreasing power :))?	2
💬	Boundary conditions versus initial conditions and ode45	1
💬	[staff] wrong answers shown The answers shown for the b.c. at x=1 are interchanged.	3
☑	deltax Hello, What's the best way to write delta x?	2
💬	Any short trick to create matrix A in above script? My MATLAB code for creating matrix A is: A=gallery('tridiag',10,*,*,*); w=diag(ones(8)); B= diag(w,-2); C= diag(w,2); A=A+B+C; A(1,1)=*;A(1,2)=*;A(1,3)=*; A(2,1)=*;A(2,2)=*;A(2,3)=...	2
💬	Putting it all together - MATLAB Hello, In the end when we summarize everything I have a problem with b. The first two values of b (b(1) and b(2)) can be directly calculated from the BC, but for the last ones, ...	3

Cantilever setup

Put together the equations you found above into a matrix equation and solve to find the displacement v as a function of horizontal position x.

- We have created the matrix $\mathbf{A}$ for you in an attached file called matrixA.m; it is initialized by calling matrixA at the begining of the script. You can refer to the matrix $\mathbf{A}$ in your own work.
- Create a column vector $\mathbf{b}$ with 10 entries that is -0.0000001 everywhere except for the positions that specify boundary conditions.
- Solve $\mathbf{Av} = \mathbf{b}$ for the displacement $\mathbf{v}$.
- Create a column vector $\mathbf{x}$ of 10 equally spaced points between 0 and 1.
- Plot $\mathbf{x}$ on the horizontal axis and $\mathbf{v}$ on the vertical axis.

Script

Save Reset MATLAB Documentation (https://www.mathworks.com/help/)

```
% Call the preloaded matlab file that defines the matrix A
matrixA;

%You can now use the matrix A, it is defined.

% Create a vector b that is zero for boundary conditions and -0.0000001 in every other entry.
b = ones(10,1)*(-1e-7)

% Create a vector v that solves Av = b.
v = A\b;

%Create a column vector x of 10 evenly spaced points between 0 and 1 (for plotting)
x = linspace(0,1,10)';
%Plot v on the vertical axis, and x on the horizontal axis.
plot(x, v);
```

Run Script