MITx: 15.053x Optimization Methods in Business Analytics

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Lecture questions due Sep 20, 2016 at 19:30 IST

Recitation 2

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PART A

Consider the linear program:

$$\left.\begin{aligned} \max \quad & -2x+y \\ \text{s.t.:} \quad & \\ & -x+y \le 1 \text{ (Constraint 1)} \\ & x+y \ge 0 \text{ (Constraint 2)} \\ & x, y \ge 0 \text{ (Non-negativity)} \end{aligned}\right\}$$

How many extreme points does the feasible region have?

2 ✓ **Answer:** 2

2

EXPLANATION

Solution

1. There are two extreme points: (0, 0) and (0, 1).
2. There will be a picture of the feasible region and more detail after you have answered PART D

PART B

How many extreme rays does the feasible region have?

2 ✓ **Answer:** 2

2

EXPLANATION

Solution

1. There are two extreme rays. The first is the set of points $\{(\lambda, 0) : \lambda \geq 0\}$. The second is the set of points $\{(\lambda, 1 + \lambda) : \lambda \geq 0\}$.
2. There will be a picture of the feasible region and more detail after you have answered PART D

PART C

How many of the constraints are redundant?

1 ✓ **Answer:** 1

1

EXPLANATION

Solution

1. There is one redundant constraint. Constraint 2 is redundant because it is implied by the non-negativity of x, y.
2. There will be a picture of the feasible region and more detail after you have answered PART D

You have used 1 of 3 submissions

PART D

What is the optimal objective value?

1 ✓ **Answer:** 1

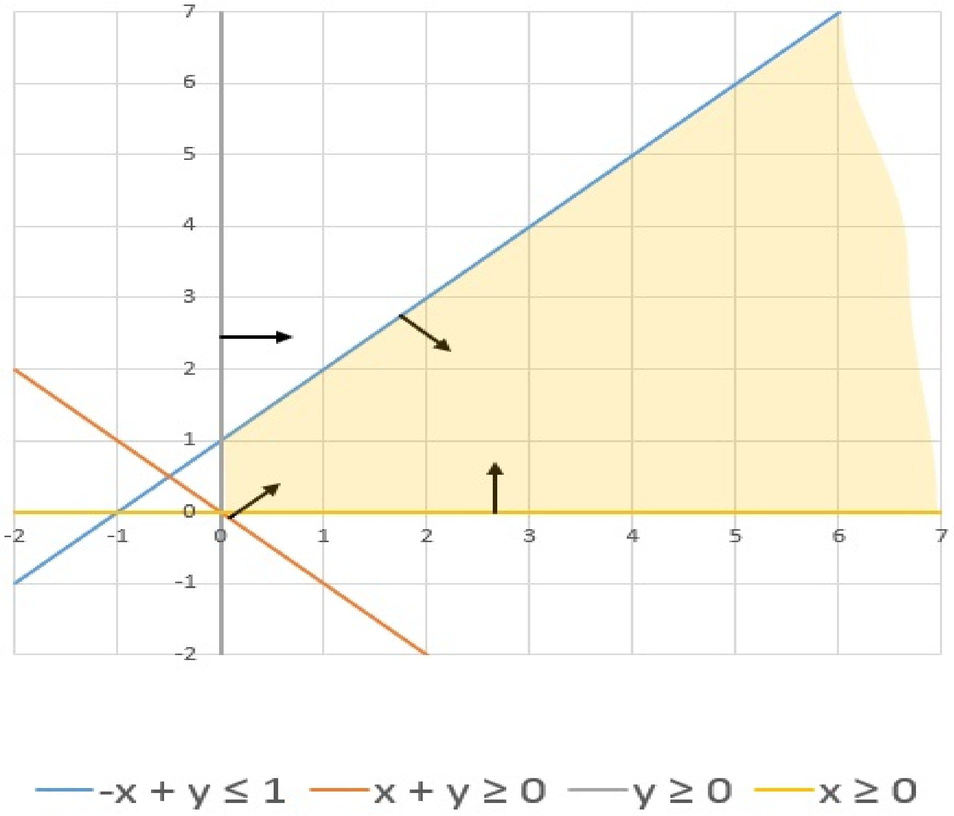
1

EXPLANATION

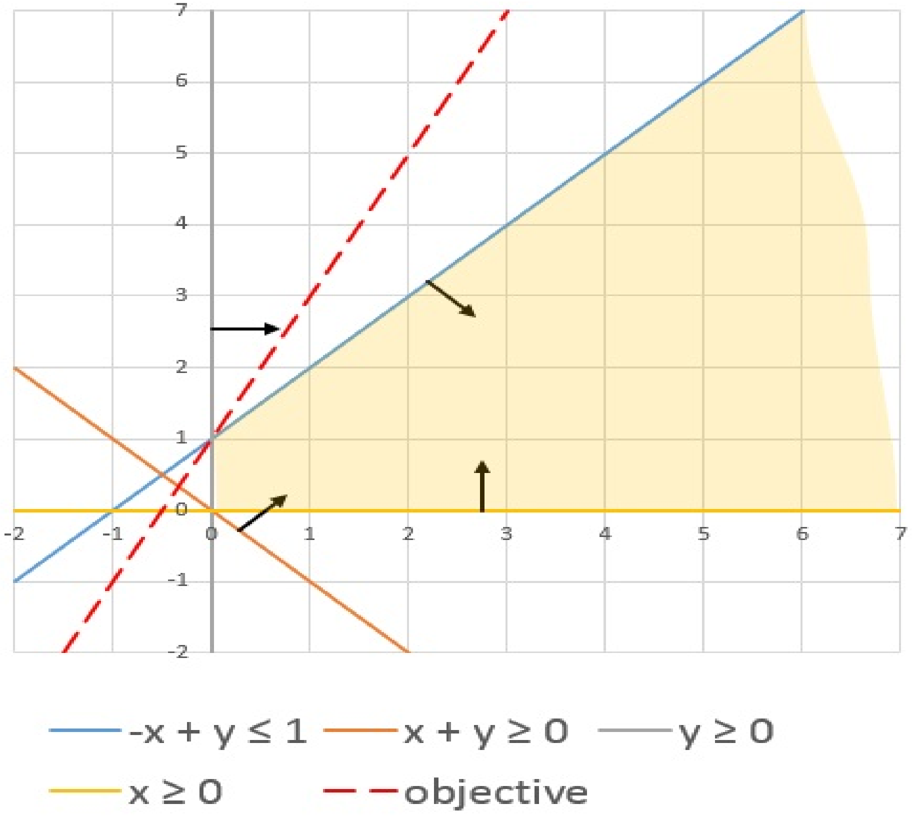
Solution

1. It is 1. Of the two extreme points $(0, 0), (0, 1)$, the extreme point (0, 1) gives the optimal solution, with objective value 1. There is no better solution.
2. There will be a picture of the feasible region and more detail after you have answered PART D

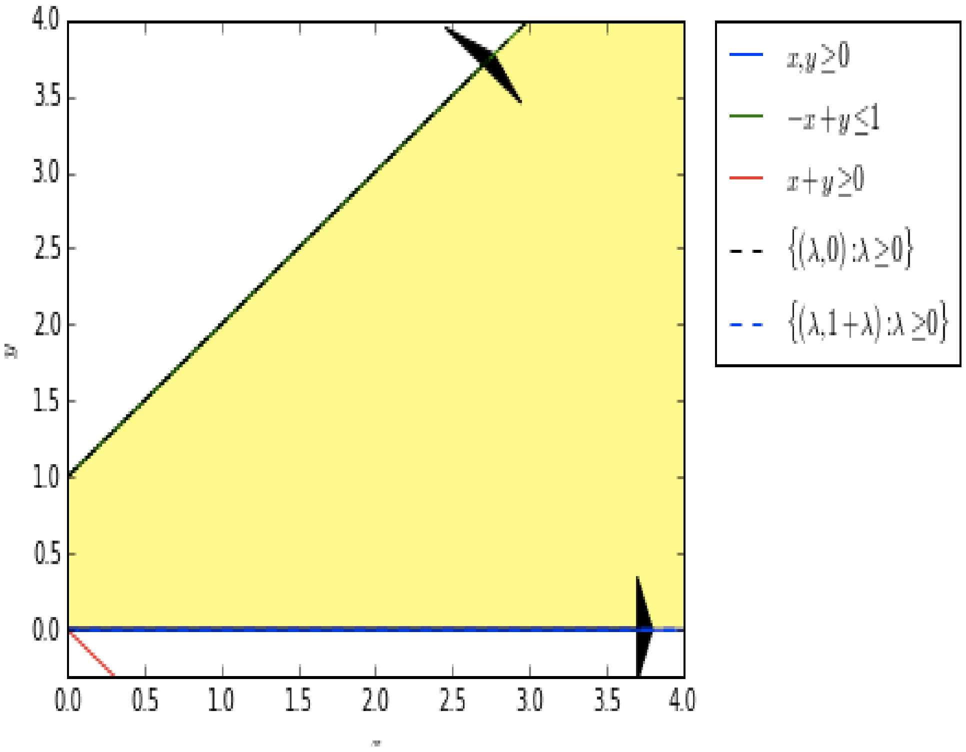
Constraints and feasible region



Optimal solution at $(0, 1)$



Extreme rays and constraints



PART E

Suppose that the objective function is changed to $-x + y$. Which of the following is true:

● There are an infinite number of optimal solutions. ✓

○ There are exactly two optimal solutions.

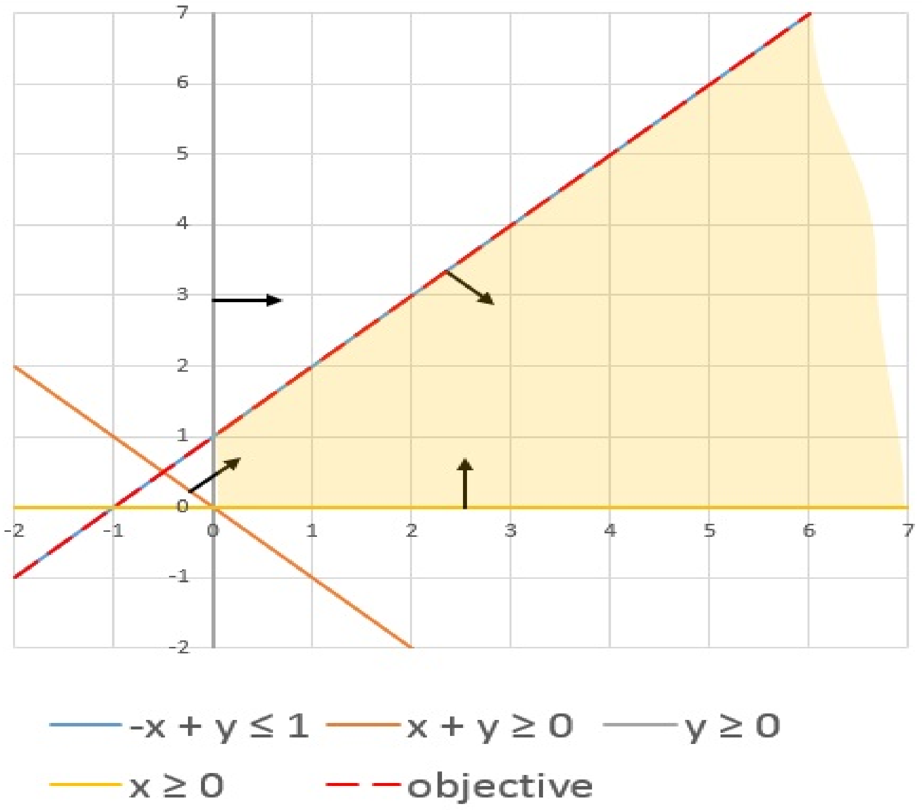
○ There is exactly one optimal solution

EXPLANATION

Solution

The set optimal solutions have an objective of 1.

The set of optimal solutions is $\{(\lambda, 1+\lambda) : \lambda \geq 0\}$, one of the two extreme rays.



PART F

Suppose the objective value is changed again. Which of the following objective functions lead to a problem for which the objective value is unbounded from above? In each case, the goal is to maximize.

☑ $x - 10y$ ✓

☐ $-2x + y$

☐ $-2x + 2y$

☑ $-2x + 3y$ ✓

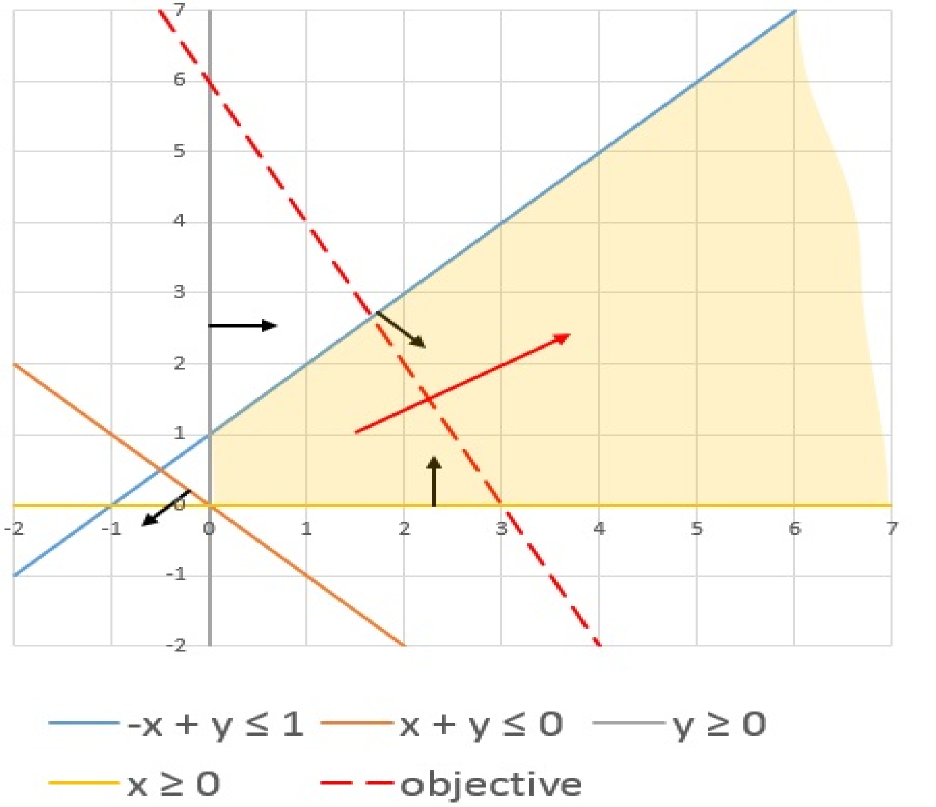
✓

EXPLANATION

Solution

- $x - 10y$
- $-2x + 3y$

In both of these cases, we can move along an extreme ray.



PART G

Suppose the objective is changed to minimize $(x-1)^2+(y-2)^2$. What is the optimal objective value?

0

✓ **Answer:** 0

0

EXPLANATION

Solution

The optimum solution is $(1, 2)$ with objective value 0. Because each term of the objective function is guaranteed to be nonnegative, there can be no better solution. Note that $(1, 2)$ is not an extreme point of the feasible region. When optimizing a nonlinear function, the optimum is not guaranteed to occur at an extreme point.

PART H

Suppose the objective is changed to minimize $x^2 + (y - 5)^2$. If (x^*, y^*) denotes the optimal solution, what is x^*? (Note: this problem goes a bit beyond what was taught in lecture videos.)

2

✓ **Answer:** 2

2

EXPLANATION

Solution

The optimal solution will be the point in the feasible region that is closest (in Euclidean distance) to the point [mathjaxinline](0, 5)[/mathjaxinline]. There are several ways of obtaining the answer, including good visual intuition or the use of Solver. If you look at the problem graphically, you will notice that the optimum solution must satisfy the constraint [mathjaxinline]-x + y = 1[/mathjaxinline]. Thus, [mathjaxinline]y = x + 1[/mathjaxinline]. We can then rewrite the objective as "Minimize [mathjaxinline]x^2 + (x-4)^2[/mathjaxinline], which is equal to [mathjaxinline]2x^2 - 8x + 16 = 2(x-2)^2 + 8[/mathjaxinline]. This is optimized at [mathjaxinline]x = 2[/mathjaxinline], with objective value 8.