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► Introduction

▼ 1. Probability and Inference

Introduction to Probability (Week 1)

Exercises due Sep 22, 2016 at 02:30 IST

**Probability Spaces and Events (Week 1)**

Exercises due Sep 22, 2016 at 02:30 IST

**Random Variables (Week 1)**

Exercises due Sep 22, 2016 at 02:30 IST

**Jointly Distributed Random Variables (Week 2)**

Exercises due Sep 29, 2016 at 02:30 IST

**Conditioning on Events (Week 2)**

Exercises due Sep 29, 2016 at 02:30 IST



1. Probability and Inference > Inference with Bayes' Theorem for Random Variables (Week 3) > Exercise: Bayes' Theorem for Random Variables - Medical Diagnosis, Continued

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Exercise: Bayes' Theorem for Random Variables - Medical Diagnosis, Continued

(4/4 points)

Recall the medical diagnosis setup from before, summarized in these tables:

		Prob.
X	healthy	0.999
	infected	0.001

Homework 1 (Week 2)

Homework due Sep 29, 2016 at 02:30 IST

**Inference with Bayes' Theorem for Random Variables (Week 3)**

Exercises due Oct 06, 2016 at 02:30 IST

**Independence Structure (Week 3)**

Exercises due Oct 06, 2016 at 02:30 IST

**Homework 2 (Week 3)**

Homework due Oct 06, 2016 at 02:30 IST

**Notation Summary (Up Through Week 3)****Mini-project 1: Movie Recommendations (Week 3)**

Mini-projects due Oct 13, 2016 at 02:30 IST



		X	
		healthy	infected
Y	positive	0.01	0.99
	negative	0.99	0.01

Recall that Bayes' theorem is given by

$$p_{X|Y}(x | y) = \frac{p_X(x)p_{Y|X}(y | x)}{\sum_{x'} p_X(x')p_{Y|X}(y | x')}$$

for all values x that random variable X can take on.

Use Bayes' theorem to compute the following probabilities: (Please be precise with at least 3 decimal places, unless of course the answer doesn't need that many decimal places. You could also put a fraction.)

- $p_{X|Y}(\text{healthy} | \text{positive}) =$ Answer: 0.9098360656

- $p_{X|Y}(\text{healthy} \mid \text{negative}) =$

✓ Answer: 0.9999898889

What is the MAP estimate for X given $Y = \text{positive}$?

☒ healthy ✓

☐ infected

What is the MAP estimate for X given $Y = \text{negative}$?

☒ healthy ✓

☐ infected

Solution:

Use Bayes' theorem to compute the following probabilities:

$$\begin{aligned}
 & p_{X|Y}(\text{healthy} \mid \text{positive}) \\
 &= \frac{p_{X|Y}(\text{positive} \mid \text{healthy})p_X(\text{healthy})}{p_{X|Y}(\text{positive} \mid \text{healthy})p_X(\text{healthy}) + p_{X|Y}(\text{positive} \mid \text{infected})p_X(\text{infected})} \\
 \bullet &= \frac{0.01 \times 0.999}{0.01 \times 0.999 + 0.99 \times 0.001} \\
 &\approx \boxed{0.9098360656}.
 \end{aligned}$$

$$\begin{aligned}
 & p_{X|Y}(\text{healthy} \mid \text{negative}) \\
 &= \frac{p_{X|Y}(\text{negative} \mid \text{healthy})p_X(\text{healthy})}{p_{X|Y}(\text{negative} \mid \text{healthy})p_X(\text{healthy}) + p_{X|Y}(\text{negative} \mid \text{infected})p_X(\text{infected})} \\
 \bullet &= \frac{0.99 \times 0.999}{0.99 \times 0.999 + 0.01 \times 0.001} \\
 &\approx \boxed{0.999989889}.
 \end{aligned}$$

Note that if $Y = \text{positive}$, then the probability that $X = \text{infected}$ is just 1 minus the first probability computed. In this case, it is clear that **healthy** is the MAP estimate of X given $Y = \text{positive}$ because

$$\underbrace{p_{X|Y}(\text{healthy} \mid \text{positive})}_{\approx 0.9098360656} > \underbrace{p_{X|Y}(\text{infected} \mid \text{positive})}_{\approx 1 - 0.9098360656 = 0.0901639344}.$$

Similarly, if $Y = \text{negative}$, the MAP estimate for X is still **healthy** since

$$\underbrace{p_{X|Y}(\text{healthy} \mid \text{negative})}_{\approx 0.999989889} > \underbrace{p_{X|Y}(\text{infected} \mid \text{negative})}_{\approx 1 - 0.999989889 = 0.000010111}.$$

You have used 1 of 5 submissions

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