Course › Unit 3 Methods of Estimation › Lecture 10: Consistency of MLE, Covariance Matrices, and Multivariate Statistics › 5. Review: Covariance

5. Review: Covariance

Note: The following exercises are a review of covariance, and will be discussed in lecture. We encourage that you attempt these exercises before watching the video.

Review: Covariance

2/2 points (graded)

If X and Y are random variables with respective means μ_X and μ_Y, then recall the **covariance** of X and Y (written Cov (X,Y)) is defined to be

$$\mathsf{Cov}\,(X,Y) = \mathbb{E}\left[(X-\mu_X)\,(Y-\mu_Y)\right].$$

Alternatively, one can show that this is equivalent to $\mathbb{E}\left[XY\right] - \mathbb{E}\left[X\right]\mathbb{E}\left[Y\right]$.

For each of the following statements, indicate whether it is true or false.

“Cov $(X, X) =$ Var (X)".

● True

○ False

✓

“Like the variance, the covariance between an arbitrary pair of RVs X and Y is always non-negative."

○ True

● False

✓

Solution:

- **True.** Cov $(X, X) = \mathbb{E}\left[(X - \mu_X)^2\right] =$ Var (X).
- **False.** Consider (X, Y) which is distributed uniformly over the set $\{(1, -1), (-1, 1)\}$. The marginal distributions of both X and Y are uniform over $\{\pm 1\}$, so $\mu_X = \mu_Y = 0$. On the other hand, $\mathbb{E}[XY] = -1$, so Cov $(X, Y) = -1$.

Submit You have used 1 of 1 attempt

ℹ Answers are displayed within the problem

Alternate Formula for Covariance

0/1 point (graded)

Let X and Y are random variables with respective means μ_X and μ_Y. Is it true that $\mathbb{E}\left[(X)\left(Y-\mu_Y\right)\right]=\textsf{Cov}\left(X,Y\right)$?

○ True ✓

● False

✗

Solution:

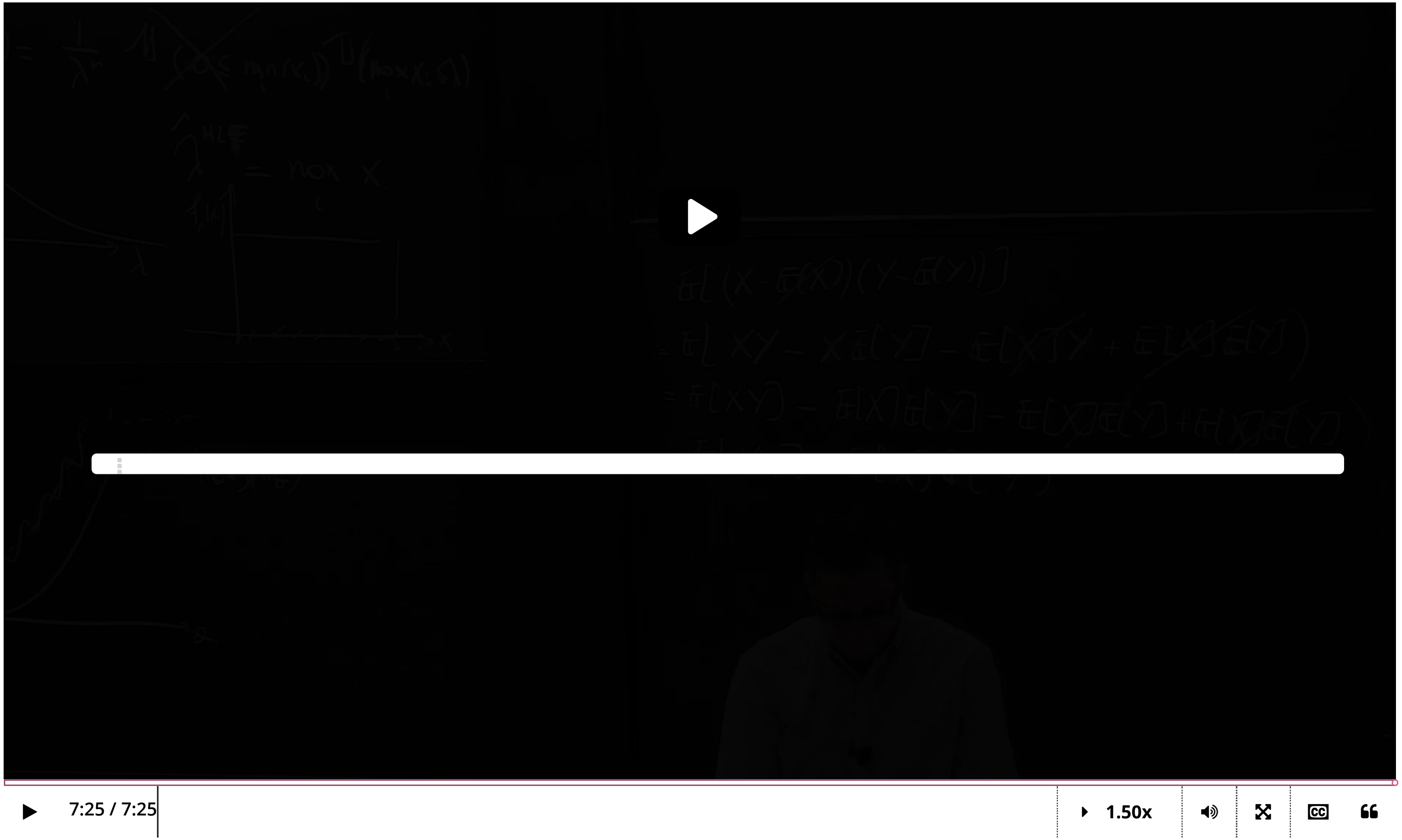
Indeed, $\mathbb{E}\left[(X)\left(Y-\mu_Y\right)\right]=\textsf{Cov}\left(X,Y\right)=\mathbb{E}\left[\left(X-\mu_X\right)(Y)\right]$. That is, it is sufficient to center one random variable around its mean when computing the covariance between two random variables. This can be seen from the following:

$$\begin{aligned}\mathbb{E}\left[(X)\left(Y-\mu_Y\right)\right] &= \mathbb{E}\left[XY\right]-\mathbb{E}\left[X\mu_Y\right]\\ &= \mathbb{E}\left[XY\right]-\mathbb{E}\left[X\right]\mathbb{E}\left[Y\right].\end{aligned}$$

Submit You have used 1 of 1 attempt

Answers are displayed within the problem

Covariance: Definition and Formula



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Bilinearity of Covariance

1/1 point (graded)

Let $X,\ Y,\ Z$ be random variables and $a,\ b$ be constants. Indicate whether the following statement is true or false.

"Covariance is bilinear, i.e. $\textsf{Cov}\left(aX+bY,Z\right)=a\textsf{Cov}\left(X,Z\right)+b\textsf{Cov}\left(Y,Z\right)$."

Hint: Use the result from the problem immediately above.

● True

○ False

✓

Submit You have used 1 of 1 attempt

✓ Correct (1/1 point)

Example of Covariance I

1/1 point (graded)

Let $A=X+Y$ and $B=X-Y$. Let $\mu_X=\mathbb{E}\left[X\right]$, $\mu_Y=\mathbb{E}\left[Y\right]$, $\tau_X=\textsf{Var}\left(X\right)$, $\tau_Y=\textsf{Var}\left(Y\right)$ and $c=\textsf{Cov}\left(X,Y\right)$. In terms of $\mu_X,\ \mu_Y,\ \tau_X,\ \tau_Y,$ and $c,$ what is $\textsf{Cov}\left(A,B\right)$?

(Enter **mu_X** for μ_X, **tau_X** for τ_X.)

tau_X-tau_Y ✓ **Answer:** tau_X-tau_Y

$\tau_X-\tau_Y$

STANDARD NOTATION

Solution:

Expand out the definition of covariance using bi-linearity (see the solution to the previous question):

$$\begin{aligned}\textsf{Cov}\,(A,B) &= \textsf{Cov}\,(X+Y,X-Y)\\ &= \textsf{Cov}\,(X+Y,X) - \textsf{Cov}\,(X+Y,Y)\\ &= \textsf{Cov}\,(X,X) + \textsf{Cov}\,(Y,X) - \textsf{Cov}\,(X,Y) - \textsf{Cov}\,(Y,Y)\\ &= \textsf{Var}\,(X) - \textsf{Var}\,(Y)\\ &= \tau_X - \tau_Y.\end{aligned}$$

Submit

You have used 1 of 4 attempts

Answers are displayed within the problem

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☑ All terms required in 2nd question? 2

? possible error in first question? 5
If my browser is correct, what I see is that the question is Cov(x,x) = Var(x)^n. I see an "n". and based on that I answered ... although the grader is marking me a red although d...

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