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1.2.2 Exploratory Quiz: Linear Demand Functions

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Here's a recap of some of the terminology we've just learned.

- **Demand for a good** is how much of that good people buy, and it depends of course on what the price of that good is. For example, demand for subway rides (how many rides are sold) depends on the price of a subway ride.
- A **demand function** expresses the demand for a good as function of its price. The independent variable is price, denoted by p, while the dependent variable is demand, denoted by q. In this section, we are looking at the **demand** for subway rides. Demand is measured as annual ridership of the subways (number of rides sold in one year), and this is a function of the fare, the **price** of a subway ride.

> **Side note:** Sometimes economists write a function with **price as a function of demand**. This is useful if you want to predict the price you should sell at to see a certain demand, but we'll stick with the perspective of demand as a function of price.

As Kiran commented, while price and demand are discrete quantities, economists do use continuous functions to model demand as a function of price because it allows them to use calculus tools (as we'll see) without being totally unrealistic. Let's see what we can do with continuous demand functions in the case of Boston and New York.

We'll pay careful attention to the units of the constants and variables in the demand functions because this will help us make sense of the model.

(This is in contrast with the Item Response Theory section, where we did not discuss units of θ, a, b, c and so forth, as they don't have a useful physical interpretation. This is the general rule of thumb for the course: we'll focus on **units** when it helps with the main learning goals, but avoid discussing them if they add unnecessary complexity.)

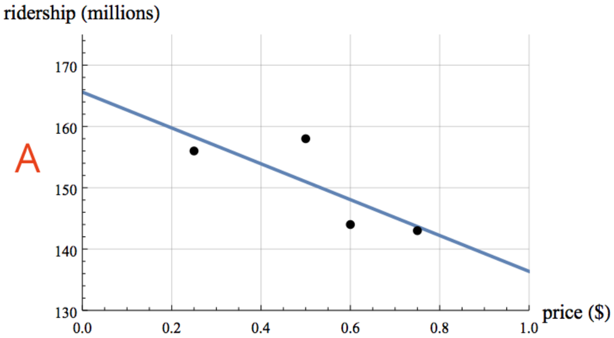
Question 1

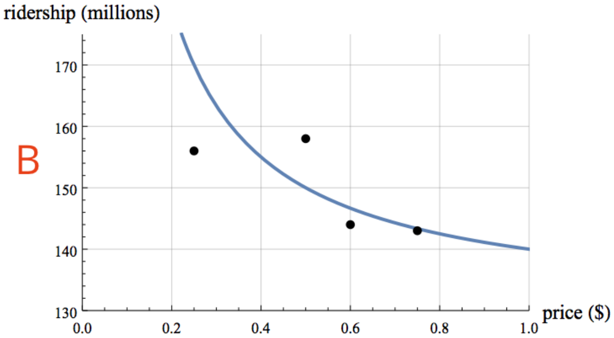
1/1 point (graded)

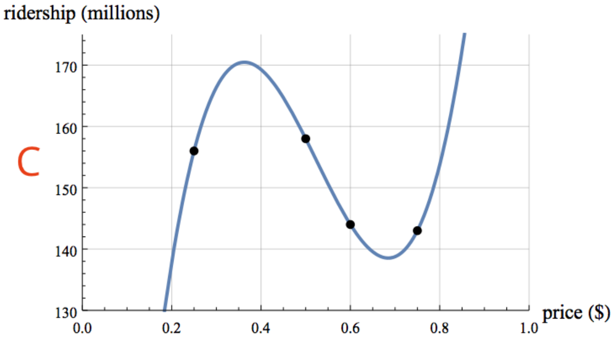
The simplest function is usually a **linear demand function** but other types of functions can be used to fit demand and price data. From the choices below, choose up to three that could be reasonable fits for the Boston data.

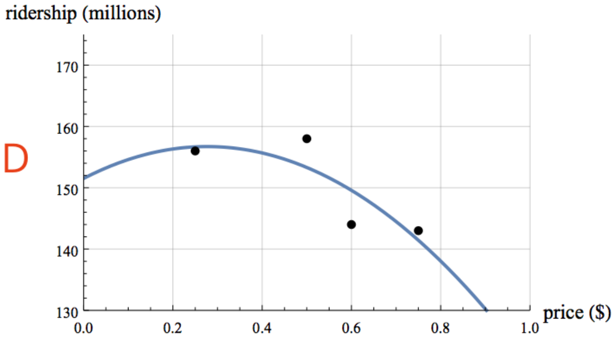
The function does not have to fit the data perfectly, but it should capture the general trend of the data or situation. In the previous video, we used a linear function to model the price and demand data. This function is called the line of best fit and is found using a technique called regression. Here "best" is defined as having the small- est distance between the data points and the line, roughly speaking. Finding this best fit is a multivariable calculus optimization problem.

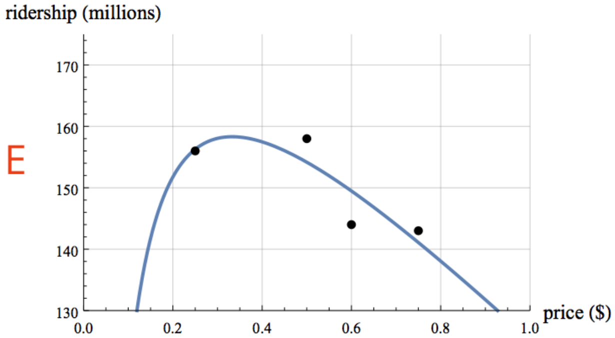
We won’t explore that in this course, but if you’re curious, you can check out Squared Error of Regression Line from Khan Academy.

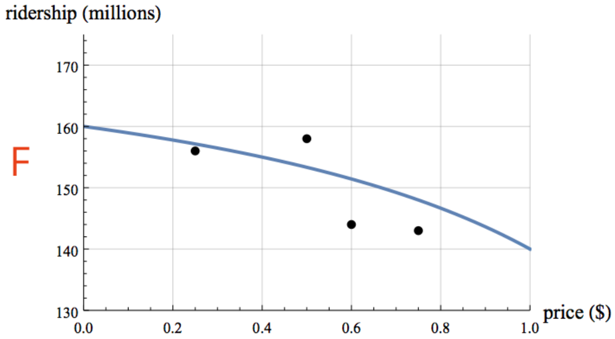












☑ Graph A ✓

☑ Graph B ✓

☐ Graph C

☐ Graph D

☐ Graph E

☑ Graph F ✓

✓

Explanation

People are more likely ride the subway if the price is low. Thus, we expect the graph of ridership versus price to decrease as price increases rather than alternating between increasing and decreasing ridership. Three graphs match this description. To choose between these graphs we would consider whether ridership could drop to zero, or whether there are some customers who will continue to ride the subway even at very high cost. We could also ask about limits on the number of customers who would ride the subway at low cost.
Note: While it's possible to find a curve that goes through each individual data point, like choice C, it's more important for a mathematical model to make sense with the context than to fit the data exactly.

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ℹ Answers are displayed within the problem

Questions 2-5

4/4 points (graded)

In the video, we used a linear model to describe demand in Boston,

$$q = -29p + 166$$

Let's explore this linear model more generally.

Suppose the **linear demand function**

$$q = mp + b$$

represents millions of rides demanded in a year in Boston (q) as a function of price in dollars (p) of the fare in Boston. Here q and p are variables and m and b are constants.

How can we interpret the constants b and m in this situation? The units of b are ...

○ dollars

● millions of rides in a year ✓

○ millions of rides in a year per dollar

○ dollars per millions of rides in a year

The units of the demand q are millions of rides per year, and b is the the demand when the price is zero. Thus the units on b must match those of q.

The sign of b is most likely ...

● positive ✓

○ negative

○ b = 0

○ positive or negative depending on price

When price (p) equals zero, our model predicts that ridership will be b (in millions). We expect the value of b to be positive. Of course, the price is unlikely to be zero, but we might estimate this using the population of the city – how people live or visit in the city, and how many of those people do we expect to use the subway if it were free?

The units of m are ...

○ dollars

○ millions of rides in a year

● millions of rides in a year per dollar ✓

○ dollars per millions of rides in a year

Since m is the slope of the function $q = mx + b$, the units of m are units of q per units of p. This is millions of rides in a year per dollar. This slope represents how ridership will change given a fare change, in terms of millions of rides each year per dollar change in fare. A slope of m, for example, means ridership will change by m million rides per year for each dollar increase in fare. Another way to reason about units is that the product of m and p (price) is some portion of q (millions of riders in a year). To help us think about this question, we might say "m times dollars equals millions of riders". This suggests that when we multiply by m we're "dividing by dollars" and "multiplying by millions of riders"; the units would be millions annual rides per dollar.

The sign of m is most likely ...

○ positive

● negative ✓

○ m = 0

○ positive or negative depending on price

Again, m is the slope of the function $q = mx + b$. We expect that if the price increases, then the number of rides in a year will decrease, so the sign of the slope m should be negative.

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Answers are displayed within the problem

Questions 6-7

2/2 points (graded)

We can model the situation in New York City with a linear function as well. Let

$$Q = MP + B$$

represent millions of rides in a year in New York City (Q) as a function of price in dollars(P) of the fare.

(We use capital letters for New York and smaller for Boston since New York is the bigger city!)

We expect the value of B to be _______ the value of b.

larger in absolute value than ▼ ✓**Answer:** larger in absolute value than

B and b are the ridership levels in New York and Boston, respectively, when the price is zero. Because New York City is much larger than Boston, we expect more people to ride the subway there. In other words, B should be much larger than b (and we expect both to be positive).

We expect the value of M to be ________ the value of m.

larger in absolute value than ▼ ✓**Answer:** larger in absolute value than

Explanation
M and m are the annual ridership per dollar in New York and Boston, respectively.
For the same change in fare price, the decrease in rides demanded will likely be greater in New York City because there are more people (more riders) affected by the fare change compared to Boston.
Thus we expect M to be larger in absolute value than m because New York is a much larger city than Boston.

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Answers are displayed within the problem

Question 8

1/1 point (graded)
Here are models of the demand in each city (created using line of best fit, as mentioned in the first problem):

$$Boston : q = -29p + 166$$

$$New\ York : Q = -267P + 1195$$

Can we use the values of $m = -29$ and $M = -267$ to decide which city's riders will react more dramatically to a price change of a dollar?

For a price increase of 1 dollar, do we expect more people to stop riding the subway in Boston or New York City? (Why?)

○ Boston

● New York City ✓

○ the amounts for both cities will be roughly equal

○ not possible to say

Explanation

Answer:

The decrease in ridership in New York City for a dollar increase is 267 since $M = -267$. This is larger in absolute value than $m = -29$, which represents the decrease in ridership in Boston for a dollar increase. We expect $|M| > |m|$ simply because New York City is a larger city. In other words, for a price increase of 1 dollar we expect more people to stop riding in New York City.

For this reason, using the slopes to quantify “a more dramatic reaction” is difficult.

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ℹ Answers are displayed within the problem

Question 9

1/1 point (graded)

For a price increase of 1 dollar, do we expect the percent change in ridership to be greater in New York City or Boston?

○ Boston

○ New York City

○ the amounts for both cities will be roughly equal

● not possible to say ✓

Now it’s harder to say. The values of M and m by themselves aren’t enough to determine the percent change in ridership. It’s difficult to say whether the reaction by Boston riders was really more dramatic than that of New York City riders. We’ll see how we can address this issue in the next section.

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ℹ Answers are displayed within the problem

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