EXERCISE: INDEPENDENCE OF TWO EVENTS - III (2/2 points)

When is an event A independent of itself?

Choose one of the following possible answers:

- Always
- If and only if $\mathbf{P}(A) = 0$
- If and only if $\mathbf{P}(A) = 1$
- If and only if $\mathbf{P}(A)$ is either 0 or 1 ✔

Answer:
Using the definition, A is independent of itself if and only if

$$\mathbf{P}(A \cap A) = \mathbf{P}(A) \cdot \mathbf{P}(A).$$

Since $A \cap A = A$, we have $\mathbf{P}(A \cap A) = \mathbf{P}(A)$ and we obtain the equivalent condition

$$\mathbf{P}(A) = \mathbf{P}(A) \cdot \mathbf{P}(A),$$

or

$$\mathbf{P}(A) \cdot \big(1 - \mathbf{P}(A)\big) = 0,$$

and this happens if and only if $\mathbf{P}(A)$ is either 0 or 1.

You have used 1 of 2 submissions