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5. Scalar multiplication

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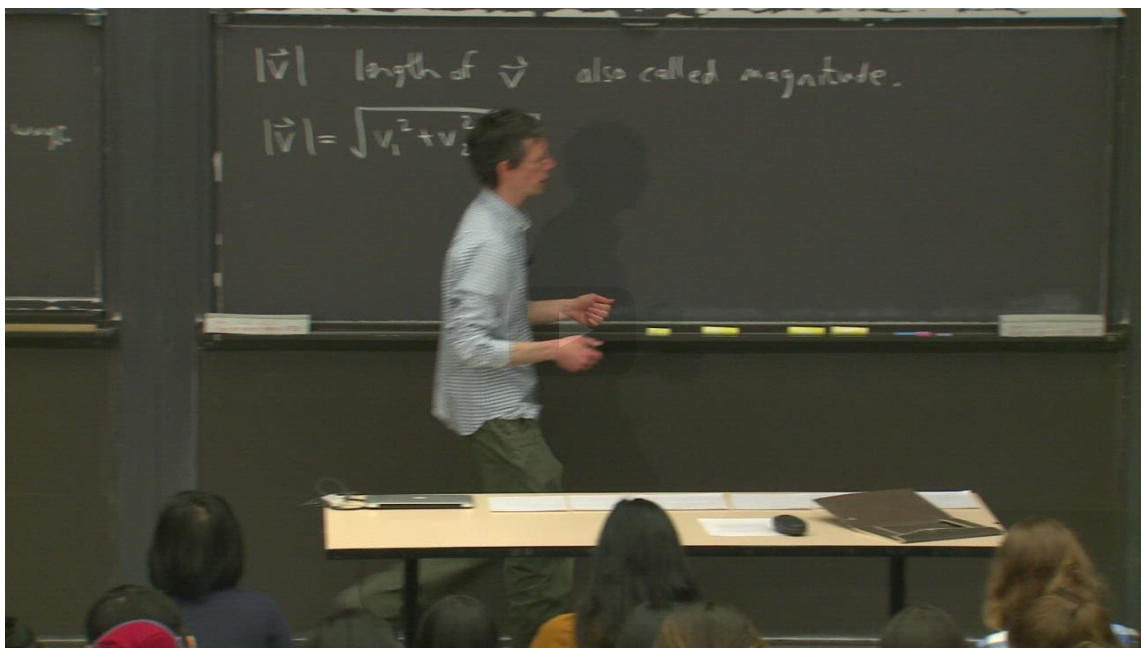
Lecture due Aug 18, 2021 20:30 IST Completed

Practice

Scalar multiplication with a vector



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PROFESSOR: Another thing that we can do with vectors

is we can scale them.

We can stretch them or contract them.

So if we write something like 2 times v, what it means

is it's a vector in the same direction as v,

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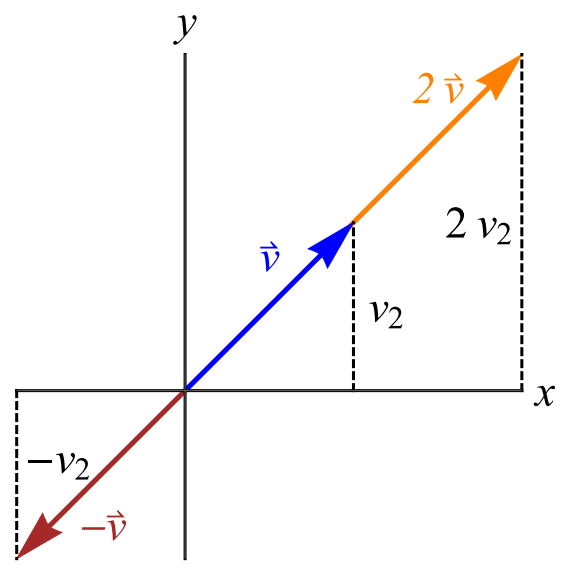
We can scale vectors. For example, $2\vec{v}$ is in the same direction as $\vec{v}$ but is twice as long.



Definition 5.1

A **scalar** is a (real or complex) number. In this class, we will almost always consider real scalars.

To multiply a vector $\vec{v} = \langle v_1, v_2 \rangle$ by a scalar c, we multiply each component by c as follows:

$$c\vec{v} = c\langle v_1, v_2 \rangle = \langle cv_1, cv_2 \rangle. \tag{3.6}$$

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- If $\vec{v}$ is in the **same direction** as $\vec{w}$, then $\vec{v} = \lambda\vec{w}$ for some **positive number** $\lambda > 0$.
- If $\vec{v}$ is in the **opposite direction** as $\vec{w}$, then $\vec{v} = \lambda\vec{w}$ for some **negative number** $\lambda < 0$.

Question: Is $(2, 3)$ in the same direction as the vector $(4, 7)$?

This is the same as asking is $(4, 7) = \lambda\,(2, 3)$ for some positive λ? For this to be true, we would need

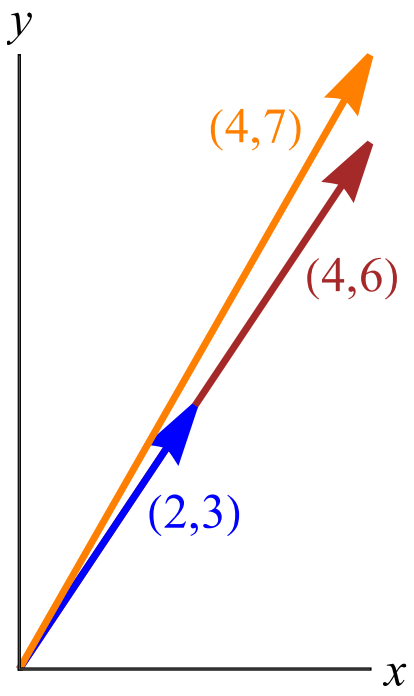
$$(4, 7) = \lambda\,(2, 3) = (2\lambda, 3\lambda) \implies 4 = 2\lambda \text{ and } 7 = 3\lambda.$$

But this would mean $\lambda = 2$ and $\lambda = 7/3$, which is not possible. Since there is no λ that works, the vectors are not in the same direction.

On the other hand,

$$(4, 6) = 2\,(2, 3)$$

and so $(4, 6)$ is in the same direction as $(2, 3)$.



Spoiler: Scalar multiplication in higher dimension.

Consider a scalar c and a vector with n components given by $\vec{v} = \langle v_1, v_2, \ldots, v_n \rangle$. Then $c\vec{v}$ is given by

$$c\vec{v} = c\langle v_1, v_2, \cdots, v_n \rangle = \langle cv_1, cv_2, \cdots, cv_n \rangle. \tag{3.7}$$

Hide

Scalar multiplication and graders

0 points possible (ungraded)
This problem is to help familiarize yourself with entering vectors or their scalar multiples into an automatically graded problem. It is ungraded.

Given a vector $\vec{v}$ and a scalar c, enter $c\vec{v}$ for the following values.

(Enter your answer as a vector with two components inside square brackets, e.g., [1,1]. Note you can type 2*[1,1] for the vector $\langle 2, 2 \rangle$.)

$\vec{v} = \langle 3, 4 \rangle$, $c = 3$, $c\vec{v} =$ [9,12] ✔ **Answer:** [9,12]

$c\vec{w} =$

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$\vec{w} = \langle 2, -8 \rangle, c = -7,$ [-14,56] ✔ **Answer:** [-14,56]

$\vec{u} = \langle -32, 24 \rangle, c = -1/4,$ $c\vec{u} =$ [8,-6] ✔ **Answer:** [8,-6]

Solution:

We have:

$$3\langle 3,4\rangle = \langle (3)(3), (3)(4)\rangle = \langle 9, 12\rangle \tag{3.8}$$

$$-7\langle 2,-8\rangle = \langle (-7)(2), (-7)(-8)\rangle = \langle -14, 56\rangle \tag{3.9}$$

$$-\frac{1}{4}\langle -32, 24\rangle = \langle (-1/4)(-32), (-1/4)(24)\rangle = \langle 8, -6\rangle \tag{3.10}$$

Submit You have used 1 of 3 attempts

Answers are displayed within the problem

Why are they called scalars?

2.0/2 points (graded)

When we do math with vectors, we call real numbers scalars because when we multiply vectors by scalars, it has the effect of scaling the length of the vector.

Find the length of the vector $\langle 3, 4\rangle$.

Find the length of the vector $c\langle 3, 4\rangle = \langle 3c, 4c\rangle$, where c is a scalar.

(Hint: c could be positive, negative, or zero. For help in entering formulas, use the input help button.)

$|\langle 3, 4\rangle| =$ 5 ✔ **Answer:** 5

$|\langle 3c, 4c\rangle| =$ 5*abs(c) ✔ **Answer:** 5*abs(c)

? INPUT HELP

Solution:

$$|\langle 3, 4\rangle| = \sqrt{3^2 + 4^2} = 5$$

$$|\langle 3c, 4c\rangle| = \sqrt{3^2c^2 + 4^2c^2} = |c|\sqrt{3^2 + 4^2} = 5|c|$$

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Answers are displayed within the problem

Sketch the vectors

1.0/1 point (graded)

Let $\vec{v} = \langle -1, 2\rangle$ and $c = -1/2$. Sketch the original vector $\vec{v}$ in blue, and sketch the vector after scalar multiplication $c\vec{v}$ in orange.

You can move a drawn arrow around the page using the select button an clicking on the arrow to drag

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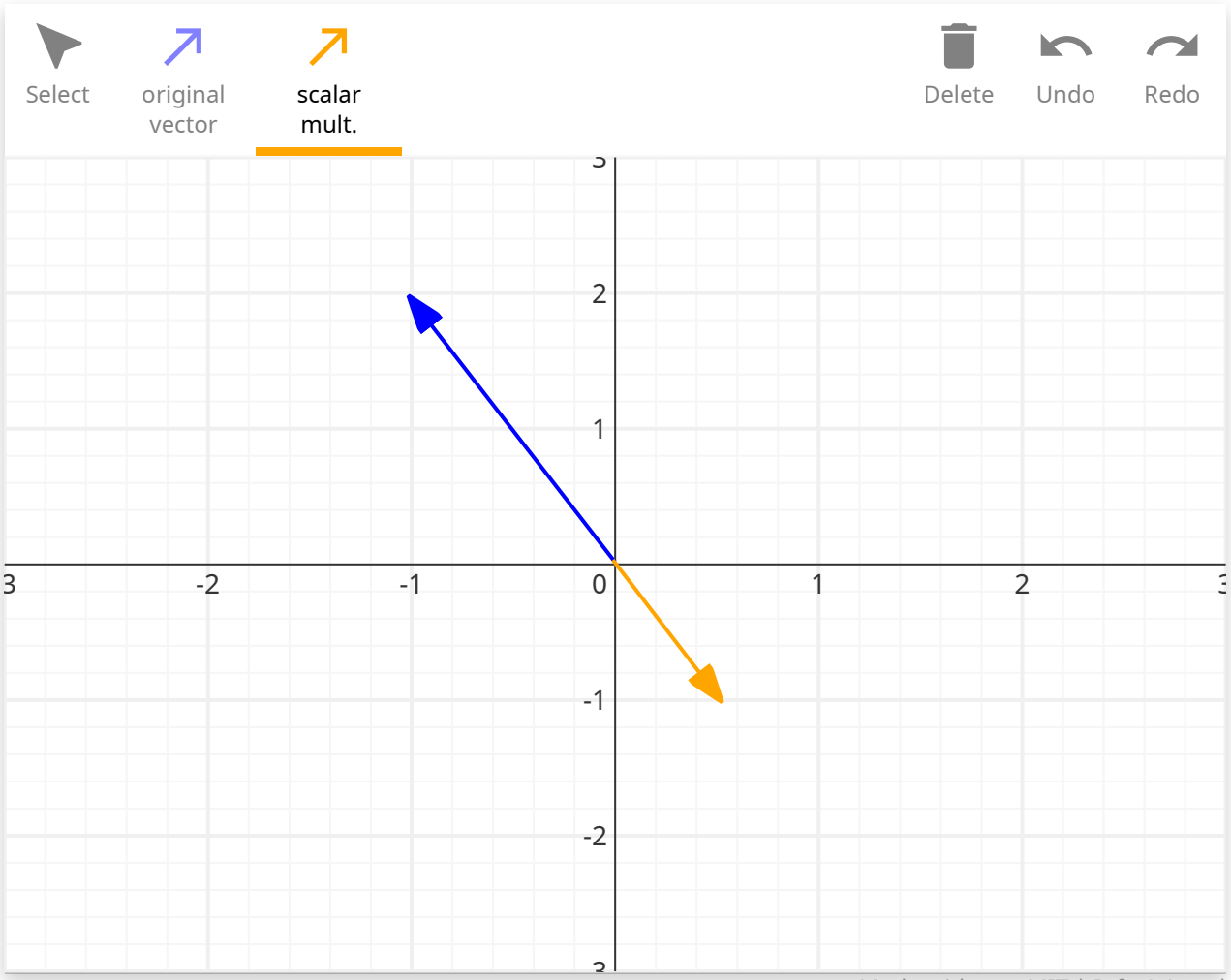
drawn arrow, click select and then click and drag either the start or end point of the vector.



Answer: See solution.

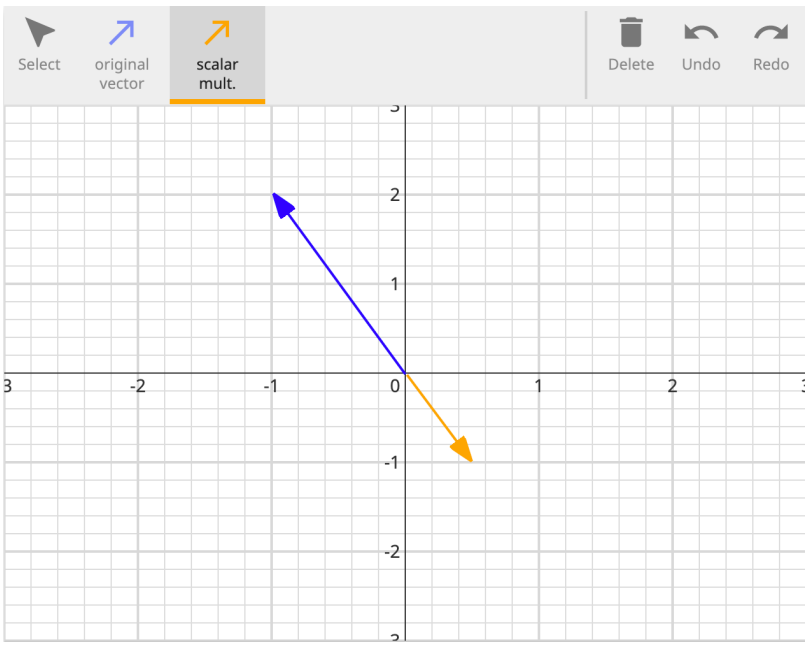
Great job!

Solution:

The vector $\vec{v} = \langle -1, 2 \rangle$ begins at the origin and extends to the coordinate $(-1, 2)$. The vector

$$c\vec{v} = \frac{-1}{2}\langle -1, 2 \rangle = \langle 1/2, -1 \rangle \tag{3.11}$$

starts at the origin and extends to the coordinate $(1/2, -1)$.



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5. Scalar multiplication

Topic: Unit 2: Geometry of Derivatives / 5. Scalar multiplication

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- [Staff] HTML Problem — 7
- Sketching the Vectors — 4
- assumption for scaling vectors — 3
  We are told in the lecture that if vector **v** is in the same direction as vector **w**, that means we can multiply vector **w** with ...
- [Edit] Relabeling vector from (3,7) to (4,7) — 2
  Hello, in the second diagram from the top of this page, the orange vector is labelled (3,7), but the vector discussed in text before it i...

Previous Next

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