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11.3.1 Newton's method for systems

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MO2.10

We can also use Newton's method for problems with systems of equations (i.e. $M > 1$). A system of nonlinear equations is of the form

$$r_i\left(x_0, \ldots, x_{M-1}\right) = 0, \qquad i = 0, \ldots, M-1.$$

We take the same number of equations and unknowns, so that we may be in a situation where there is one solution (rather than a continuum of solutions or no solution at all.) Whether the system has zero, one or several solutions is still a question that needs to be addressed separately. The shorthand notation for the system is $\underline{r}\left(\underline{x}\right) = 0$

By analogy with the 1D case we perform a Taylor expansion about $\underline{x}^k$:

$$\underline{r}\left(\underline{x}^k + \Delta\underline{x}\right) \approx \underline{r}\left(\underline{x}^k\right) + \nabla\underline{r}\left(\underline{x}^k\right)\Delta\underline{x}$$

The gradient $\nabla\underline{r}$ is known as the Jacobian. It is an $M \times M$ matrix that we will refer to symbolically as J. Note that the (i, j) element of J is:

$$J_{i,j} = \frac{\partial r_i}{\partial x_j} \tag{11.8}$$

Continuing with the derivation of Newton's method for a system, we set the Taylor series approximation to zero to find $\Delta\underline{x}$:

$$\underline{r}\left(\underline{x}^k\right) + J\left(\underline{x}^k\right)\Delta\underline{x} = 0 \Rightarrow J\left(\underline{x}^k\right)\Delta\underline{x} = -\underline{r}\left(\underline{x}^k\right)$$

which is a system of linear equations that must be solved for each iteration. And finally, we update x^k by