PurdueX: 416.2x Probability: Distribution Models & Continuous Random Variables

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Unit 10: Quiz

The problems in the quiz will be automatically graded.

You are given only **ONE** attempt for each problem. You will be able to see the solution after you submit your answers.

If the answer is numeric and has more than 4 decimal places, just give your answers to **4 decimal places (0.0001 accuracy)** unless otherwise stated.

e.g. for 0.123456... you just need to round it up to 0.1235

Problem 1

3/3 points (graded)

1. Suppose that X is a Normal random variable with mean 1.2 and standard deviation 0.5.

1a. Find $P(1 < X < 2)$.

0.6006224

✓ **Answer:** 0.6006

L10.3: Central Limit Theorem

L10.4: Practice

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Quiz

- Unit 11: Covariance, Conditional Expectation, Markov and Chebychev Inequalities

- Unit 12: Order Statistics, Moment Generating Functions, Transformation of RVs

1b. Find $P(X > 1.4 \text{ or X} < 1)$.

0.6891565 ✓ **Answer:** 0.6892

1c. Find the probability that X is nonnegative.

0.9918025 ✓ **Answer:** 0.9918

Explanation

1a. Using Z to denote a standard normal random variable, we have

$$\begin{aligned} P(1 < X < 2) &= P(\tfrac{1-1.2}{0.5} < \tfrac{X-1.2}{0.5} < \tfrac{2-1.2}{0.5}) \\ &= P(-0.4 < Z < 1.6) = P(Z < 1.6) - P(Z \leq -0.4). \end{aligned}$$

Checking our table, we have $P(Z < 1.6) = 0.9452$ and

$$\begin{aligned} P(Z \leq -0.4) &= P(Z \geq 0.4) = 1 - P(Z < 0.4) \\ &= 1 - 0.6554 = 0.3446. \end{aligned}$$

Thus $P(1 < X < 2) = 0.9452 - 0.3446 = 0.6006$.

1b. We have

$$\begin{aligned} P(X > 1.4) &= P(\tfrac{X-1.2}{0.5} > \tfrac{1.4-1.2}{0.5}) \\ &= P(Z > 0.4) = 1 - P(Z \leq 0.4) \\ &= 1 - 0.6554 = 0.3446. \end{aligned}$$

Also, we have

$$\begin{aligned} P(X < 1) &= P(\tfrac{X-1.2}{0.5} < \tfrac{1-1.2}{0.5}) \\ &= P(Z < -0.4) = P(Z > 0.4) = 1 - P(Z \leq 0.4) \\ &= 1 - 0.6554 = 0.3446. \end{aligned}$$

So the desired probability is

$$P(X > 1.4 \text{ or X} < 1) = 0.3446 + 0.3446 = 0.6892.$$

1c. We compute

$$P(X \geq 0) = P(\frac{X-1.2}{0.5} \geq \frac{0-1.2}{0.5})$$
$$= P(Z \geq -2.40) = P(Z \leq 2.40) = 0.9918.$$

Submit You have used 1 of 1 attempt

Problem 2

3/3 points (graded)

2. Same setup as #1.

2a. Find a value a such that $P(X \leq a) = 0.10$.

0.5592242 ✓ **Answer:** 0.56

2b. Find a value b such that $P(X \geq b) = 0.10$.

1.840776 ✓ **Answer:** 1.84

2c. Find a value c such that $P(1.2 - c < X < 1.2 + c) = 0.30$.

0.1926602 ✓ **Answer:** 0.193

Explanation

2a. We compute

$0.10 = P(X \le a) = P(\frac{X-1.2}{0.5} \le \frac{a-1.2}{0.5}) = P(Z \le \frac{a-1.2}{0.5})$.

Thus,

$$0.90 = 1 - 0.10 = 1 - P(Z \le \frac{a-1.2}{0.5}) = P(Z > \frac{a-1.2}{0.5})$$
$$= P(Z < -\frac{a-1.2}{0.5}).$$

So we must have $-\frac{a-1.2}{0.5} = 1.28$. It follows that

$a = (-1.28)(0.5) + 1.2 = 0.56$.

2b. We compute $0.10 = P(X \ge b) = P(\frac{X-1.2}{0.5} \ge \frac{b-1.2}{0.5}) = P(Z \ge \frac{b-1.2}{0.5})$. Thus $0.90 = 1 - 0.10 = 1 - P(Z \ge \frac{b-1.2}{0.5}) = P(Z < \frac{b-1.2}{0.5})$. So we must have $\frac{b-1.2}{0.5} = 1.28$. It follows that $b = (1.28)(0.5) + 1.2 = 1.84$.

2c. We compute that:

$$0.30 = P(1.2 - c < X < 1.2 + c) = P(\frac{-c}{0.5} < \frac{X-1.2}{0.5} < \frac{c}{0.5})$$
$$= P(\frac{-c}{0.5} < Z < \frac{c}{0.5}) = P(Z < \frac{c}{0.5}) - P(Z \le \frac{-c}{0.5}).$$

The second term of this last part is $P(Z \le \frac{-c}{0.5}) = P(Z \ge \frac{c}{0.5}) = 1 - P(Z < \frac{c}{0.5})$.

So altogether we get

$0.30 = 2P(Z < \frac{c}{0.5}) - 1$.

So $1.30 = 2P(Z < \frac{c}{0.5})$ and $0.65 = P(Z < \frac{c}{0.5})$.

Thus $\frac{c}{0.5} = 0.385$. So $c = (0.385)(0.5) = 0.193$.

Submit You have used 1 of 1 attempt

✔ Correct (3/3 points)

Problem 3

4/4 points (graded)

3. In a certain Chemistry class, the student scores are approximately normally distributed, with mean 72.5% and standard deviation 6.9%.

If the cutoffs on the exam are 90/80/70/60 for A, B, C, D, what percentage of students receive a score an A? B? C? D?
Give your answers to 2 decimal places.

$P(90 < X < 100) =$ 0.56 ✓ **Answer:** 0.55 $\%$

$P(80 < X < 90) =$ 13.29 ✓ **Answer:** 13.24 $\%$

$P(70 < X < 80) =$ 50.29 ✓ **Answer:** 50.27 $\%$

$P(60 < X < 70) =$ 32.35 ✓ **Answer:** 32.43 $\%$

Explanation

3. Let X denote a student's score.
The probability of an A grade is

$$\begin{aligned} P(90 < X < 100) &= P\left(\frac{90-72.5}{6.9} < \frac{X-72.5}{6.9} < \frac{100-72.5}{6.9}\right) \\ &= P(2.54 < Z < 3.98) = P(Z < 3.98) - P(Z \leq 2.54) \\ &= 1.0000 - 0.9945 = 0.0055. \end{aligned}$$

The probability of a B grade is

$$\begin{aligned} P(80 < X < 90) &= P\left(\frac{80-72.5}{6.9} < \frac{X-72.5}{6.9} < \frac{90-72.5}{6.9}\right) \\ &= P(1.09 < Z < 2.54) = P(Z < 2.54) - P(Z \leq 1.09) \\ &= 0.9945 - 0.8621 = 0.1324. \end{aligned}$$

The probability of a C grade is

$$P(70 < X < 80) = P(\frac{70-72.5}{6.9} < \frac{X-72.5}{6.9} < \frac{80-72.5}{6.9})$$
$$= P(-0.36 < Z < 1.09) = P(Z < 1.09) - P(Z \le -0.36).$$

We have $P(Z < 1.09) = 0.8621$ and

$$P(Z \le -0.36) = P(Z \ge 0.36) = 1 - P(Z < 0.36)$$
$$= 1 - 0.6406 = 0.3594.$$

So $P(70 < X < 80) = 0.8621 - 0.3594 = 0.5027$.

The probability of a D grade is

$$P(60 < X < 70) = P(\frac{60-72.5}{6.9} < \frac{X-72.5}{6.9} < \frac{70-72.5}{6.9})$$
$$= P(-1.81 < Z < -0.36) = P(Z < -0.36) - P(Z \le -1.81).$$

We have $P(Z < -0.36) = 0.3594$ (as in the previous part) and

$$P(Z \le -1.81) = P(Z \ge 1.81)$$
$$= 1 - P(Z < 1.81) = 1 - 0.9649 = 0.0351.$$

So $P(60 < X < 70) = 0.3594 - 0.0351 = 0.3243$.

Submit You have used 1 of 2 attempts

✓ Correct (4/4 points)

Problem 4

2/2 points (graded)

4. Suppose that the heights of blades of grass are Normally distributed, with each height having expected value 4 inches and standard deviation 0.75 inches.

4a. What is the chance that a random blade of grass is 9 cm or less? (Just FYI: There are 2.54 cm per inch.)

0.2712874

✓ **Answer:** 0.2709

4b. A seed company wants to know the value a such that 90% of the blades of grass are between height $4 - a$ inches and $4 + a$ inches. What is the right value of a?

1.23364

✓ **Answer:** 1.24

Explanation

4a. If X is the length of the blade of grass in inches, we have

$$\begin{aligned} P(X \le \tfrac{9}{2.54}) &= P(X \le 3.54) = P(\tfrac{X-4}{0.75} \le \tfrac{3.54-4}{0.75}) \\ &= P(Z \le -0.61) = P(Z \ge 0.61) = 1 - P(Z < 0.61) \\ &= 1 - 0.7291 = 0.2709. \end{aligned}$$

4b. We have

$$\begin{aligned} 0.90 &= P(4 - a < X < 4 + a) = P(\tfrac{-a}{0.75} < \tfrac{X-4}{0.75} < \tfrac{a}{0.75}) \\ &= P(\tfrac{-a}{0.75} < Z < \tfrac{a}{0.75}) = P(Z < \tfrac{a}{0.75}) - P(Z \le \tfrac{-a}{0.75}). \end{aligned}$$

The second term is $P(Z \le \frac{-a}{0.75}) = P(Z \ge \frac{a}{0.75}) = 1 - P(Z < \frac{a}{0.75})$.

Thus $0.90 = 2P(Z < \frac{a}{0.75}) - 1$.

So $1.90 = 2P(Z < \frac{a}{0.75})$, and thus $0.95 = P(Z < \frac{a}{0.75})$.

So we get $\frac{a}{0.75} = 1.65$.

So the desired a is $(0.75)(1.65) = 1.24$.

Submit You have used 1 of 1 attempt

✓ Correct (2/2 points)

Problem 5

2/2 points (graded)

5. Consider 40 students in a Chemistry class that each have random amounts of liquid in their beakers. Suppose that the amount of liquid in each beaker is Normally distributed with mean 0.20 liters and standard deviation 0.05 liters.

5a. Find the probability that the class has between 7.9 and 8.1 liters of liquid altogether.

0.2481704 ✓ **Answer:** 0.2510

5b. Find the value of b such that there is a 95 percent chance that total quantity of liquid is between $8-b$ and $8+b$ liters altogether.

0.619795 ✓ **Answer:** 0.6272

Explanation

5a. Use X_1, ..., X_{40} to denote the 40 liquid amounts, so

$$P(7.9 \leq X_1 + \cdots + X_{40} \leq 8.1)$$

$$= P\left(\frac{7.9-40(0.20)}{\sqrt{40(0.05)^2}} \leq \frac{X_1+\cdots+X_{40}-40(0.20)}{\sqrt{40(0.05)^2}} \leq \frac{8.1-40(0.20)}{\sqrt{40(0.05)^2}}\right)$$

$$= P(-0.32 \leq Z \leq 0.32) = 0.6255 - (1 - 0.6255)$$
$$= 0.2510.$$

5b. We have

$$\begin{aligned} 0.95 &= P(8 - b \leq X_1 + \cdots + X_{40} \leq 8 + b) \\ &= P\left(\frac{8-b-40(0.20)}{\sqrt{40(0.05)^2}} \leq \frac{X_1+\cdots+X_{40}-40(0.20)}{\sqrt{40(0.05)^2}} \leq \frac{8+b-40(0.20)}{\sqrt{40(0.05)^2}}\right) \\ &= P\left(-\frac{b}{0.32} \leq Z \leq \frac{b}{0.32}\right) \\ &= P\left(Z \leq \frac{b}{0.32}\right) - \left(1 - P\left(Z \leq \frac{b}{0.32}\right)\right) \\ &= 2P\left(Z \leq \frac{b}{0.32}\right) - 1. \end{aligned}$$

So $P\left(Z \leq \frac{b}{0.32}\right) = 0.975$. Thus $\frac{b}{0.32} = 1.96$.

So we get $b = (0.32)(1.96) = 0.6272$.

Submit You have used 1 of 2 attempts

✓ Correct (2/2 points)

Problem 6

2/2 points (graded)

6. Consider 5000 stones whose weights are Normally distributed, each weight having expected value 70 grams, and standard deviation of 8 grams. Let μ denote the expected weight of the entire collection of stones, and let σ^2 denote the variance of the entire collection of stones.

6a. Find the probability that the total weight of the stones exceeds 349000 grams.

0.9614501

✓ **Answer:** 0.9616

6b. Find the probability that the total weight of the stones is within 500 grams of the expected value μ, i.e., if X_1, ..., X_{5000} are the individual weights, then calculate the probability $P(|X_1+\cdots+X_{5000}-\mu|\le 500)$, i.e., $P(\mu-500\le X_1+\cdots+X_{5000}\le\mu+500)$.

0.6232409

✓ **Answer:** 0.6212

Explanation

6a. Use X_1, ..., X_{5000} to denote the weights of the stones, so

$$\begin{aligned}
&P(349000\le X_1+\cdots+X_{5000})\\
&\qquad=P\Big(\frac{349000-5000(70)}{\sqrt{5000(8)^2}}\le\frac{X_1+\cdots+X_{40}-5000(70)}{\sqrt{5000(8)^2}}\Big)\\
&\qquad=P(-1.77\le Z)=P(1.77\ge Z)=0.9616.
\end{aligned}$$

6b. We have

$$\begin{aligned}
&P(\mu-500\le X_1+\cdots+X_{5000}\le\mu+500)\\
&\qquad=P\Big(\frac{\mu-500-5000(70)}{\sqrt{5000(8)^2}}\le\frac{X_1+\cdots+X_{5000}-5000(70)}{\sqrt{5000(8)^2}}\le\frac{\mu+500-5000(70)}{\sqrt{5000(8)^2}}\Big)\\
&\qquad=P\Big(-\frac{500}{565.69}\le Z\le\frac{500}{565.69}\Big)=P(-0.88\le Z\le 0.88)\\
&\qquad=P(Z\le 0.88)-(1-P(Z\le 0.88))\\
&\qquad=0.8106-(1-0.8106)=0.6212.
\end{aligned}$$

Submit

You have used 1 of 1 attempt

✓ Correct (2/2 points)

Problem 7

2/2 points (graded)

7. Suppose that the heights of blades of grass are Normally distributed, with each height having expected value 4 inches and standard deviation 0.75 inches. Also suppose that the heights are independent.

7a. When ten blades of grass are randomly selected, find the probability that the average height of these blades of grass is between 3.5 and 4.5 inches, i.e., $P(3.5 \leq \frac{X_1+\cdots+X_{10}}{10} \leq 4.5)$.

0.964985

✓ **Answer:** 0.9652

7b. When ten blades of grass are randomly selected, find the value of "a" such that the probability is 0.90 that the average height of these blades of grass is between $4-a$ and $4+a$ inches, i.e., $P(4-a \leq \frac{X_1+\cdots+X_{10}}{10} \leq 4+a) = 0.90$.

0.3901113

✓ **Answer:** 0.3899

Explanation

7a. We have

$$P\left(3.5 \leq \frac{X_1+\cdots+X_{10}}{10} \leq 4.5\right)$$
$$= P((3.5)(10) \leq X_1+\cdots+X_{10} \leq (4.5)(10))$$

$$
\begin{aligned}
&= P\big(\tfrac{(3.5)(10)-10(4)}{\sqrt{10(0.75)^2}} \le \tfrac{X_1+\cdots+X_{10}-10(4)}{\sqrt{10(0.75)^2}} \le \tfrac{(4.5)(10)-10(4)}{\sqrt{10(0.75)^2}}\big)\\
&= P(-2.11 \le Z \le 2.11) = 0.9826 - (1 - 0.9826)\\
&= 0.9652.
\end{aligned}
$$

7b. We have

$$
\begin{aligned}
0.90 &= P\big(4 - a \le \tfrac{X_1+\cdots+X_{10}}{10} \le 4 + a\big)\\
&= P((4-a)(10) \le X_1 + \cdots + X_{10} \le (4+a)(10))\\
&= P\big(\tfrac{(4-a)(10)-10(4)}{\sqrt{10(0.75)^2}} \le \tfrac{X_1+\cdots+X_{10}-10(4)}{\sqrt{10(0.75)^2}} \le \tfrac{(4+a)(10)-10(4)}{\sqrt{10(0.75)^2}}\big)\\
&= P\big(-\tfrac{10a}{2.37} \le Z \le \tfrac{10a}{2.37}\big)\\
&= P\big(Z \le \tfrac{10a}{2.37}\big) - (1 - P(Z \le \tfrac{10a}{2.37}))\\
&= 2P\big(Z \le \tfrac{10a}{2.37}\big) - 1.
\end{aligned}
$$

So $P\big(Z \le \frac{10a}{2.37}\big) = 0.95$. Thus $\frac{10a}{2.37} = 1.645$. So we get $a = (2.37)(1.645)/10 = 0.3899$.

Submit You have used 1 of 1 attempt

✓ Correct (2/2 points)

Problem 8

2/2 points (graded)

8. Consider the weights of 5 encyclopedia books and 20 novels. The weight of each encyclopedia book is Normally distributed with mean 6 pounds and standard deviation 0.8 pounds. The weight of each novel is Normally distributed with mean 1.4 pounds and standard deviation 0.3 pounds. All of the weights are assumed to be independent.

8a. Find the probability that the total weight of the books does not exceed 60 pounds.

0.8144533 ✓ **Answer:** 0.8133

8b. Find the probability that the total weight of the books is between 58 and 62 pounds. (Note: The average weight is not 60 pounds, so this problem is not symmetric around 60.)

0.4631809 ✓ **Answer:** 0.4633

Explanation

8a. Use X_1, ..., X_5 to denote the weights of the encyclopedias and Y_1, ..., Y_{20} to denote the weights of the novels, so we compute

$$P(X_1 + \cdots + X_5 + Y_1 + \cdots + Y_{20} \leq 60)$$
$$= P\left(\frac{X_1+\cdots+X_5+Y_1+\cdots+Y_{20}-(5)(6)-(20)(1.4)}{\sqrt{(5)(0.8)^2+(20)(0.3)^2}} \leq \frac{60-(5)(6)-(20)(1.4)}{\sqrt{(5)(0.8)^2+(20)(0.3)^2}}\right)$$
$$= P(Z \leq 0.89) = 0.8133.$$

8b. We compute

$$P(58 \leq X_1 + \cdots + X_5 + Y_1 + \cdots + Y_{20} \leq 62)$$
$$= P\left(\frac{58-(5)(6)-(20)(1.4)}{\sqrt{(5)(0.8)^2+(20)(0.3)^2}} \leq \frac{X_1+\cdots+X_5+Y_1+\cdots+Y_{20}-(5)(6)-(20)(1.4)}{\sqrt{(5)(0.8)^2+(20)(0.3)^2}} \leq \frac{62-(5)(6)-(20)(1.4)}{\sqrt{(5)(0.8)^2+(20)(0.3)^2}}\right)$$
$$= P(0 \leq Z \leq 1.79) = 0.9633 - 0.5000 = 0.4633.$$

Submit You have used 1 of 1 attempt

✓ Correct (2/2 points)

Problem 9

3/3 points (graded)

9. Suppose that the time that a customer waits at a receiving center in the Union is Exponentially distributed, with an average wait of 2.5 minutes. Also, suppose that the students' waiting times are independent. Suppose that we survey 500 students, and we add up their waiting times. Let X denote the total waiting time of the 500 students.

9a. What kind of distribution does X have?

○ Uniform

○ Exponential

● Gamma ✓

○ Beta

9b. Write an integral expression for the probability that the total waiting time is less than 20 hours, i.e., less than 1200 minutes. In other words, write an integral for $P(X \leq 1200)$. You do not need to evaluate the integral.

○ $\int_{\infty}^{1200} \frac{(1/2.5)^{500}}{500!} x^{500} e^{-x/2.5}\, dx$

○ $\int_{0}^{1200} \frac{(1/2.5)^{500}}{500!} x^{500} e^{-x/2.5}\, dx$

○ $\int_{\infty}^{1200} \frac{(1/2.5)^{500}}{499!} x^{499} e^{-x/2.5}\, dx$

● $\int_{0}^{1200} \frac{(1/2.5)^{500}}{499!} x^{499} e^{-x/2.5}\, dx$ ✓

9c. Approximate the probability in **9b**.

0.186282 ✓ **Answer:** 0.1867

Explanation

9a. The random variable X is a Gamma random variable with $r = 500$ and $\lambda = \frac{1}{2.5}$.

9b. We have $P(X \leq 1200) = \int_{0}^{1200} \frac{(1/2.5)^{500}}{499!} x^{499} e^{-x/2.5}\, dx$.

9c. We compute

$$P(X \le 1200) = P\left(\frac{X-500(2.5)}{\sqrt{500(1/(2.5)^2)}} \le \frac{1200-500(2.5)}{\sqrt{500(1/(2.5)^2)}}\right)$$
$$\approx P(Z \le -0.89) = P(Z \ge 0.89)$$
$$= 1 - P(Z < 0.89) = 1 - 0.8133 = 0.1867.$$

Submit You have used 1 of 1 attempt

✓ Correct (3/3 points)

Problem 10

3/3 points (graded)

10. Consider 5000 stones whose weights are Normally distributed, each weight having expected value 70 grams, and standard deviation of 8 grams. A stone is considered “big" if it weighs 80 grams or more. (Assume that the weights are independent.) Let X denote the number of big stones found in the collection.

10a. What kind of distribution does X have?

○ Bernoulli

● Binomial ✓

○ Geometric

○ Poisson

10b. Write a sum for the probability that there are 500 or fewer “big" stones in the collection. You do not need to evaluate the sum.

● $\sum_{x=0}^{500} \binom{5000}{x} (0.1056)^x (1-0.1056)^{5000-x}$ ✓

○ $\sum_{x=0}^{5000} \binom{500}{x} (0.1056)^x (1-0.1056)^{500-x}$

○ $\sum_{x=0}^{500} \binom{5000}{x} (0.1056)^{5000-x} (1-0.1056)^x$

○ $\sum_{x=0}^{5000} \binom{500}{x} (0.1056)^{500-x} (1-0.1056)^x$

10c. Approximate the probability in **10b**.

0.1021746

✓ **Answer:** 0.1022

Explanation

10a. Let Y denote the weight of such a stone. The probability that such a stone is “big" is

$$P(Y \geq 80) = P\left(\frac{Y-70}{8} \geq \frac{80-70}{8}\right) = P(Z \geq 1.25)$$
$$= 1 - P(Z < 1.25) = 1 - 0.8944 = 0.1056.$$

Therefore, X is a Binomial random variable with $n = 5000$ and $p = 0.1056$.

10b. We have $P(X \leq 500) = \sum_{x=0}^{500} \binom{5000}{x} (0.1056)^x (1-0.1056)^{5000-x}$.

10c. We have

$$P(X \leq 500) = P(X \leq 500.5)$$
$$= P\left(\frac{X - 5000(0.1056)}{\sqrt{5000(0.1056)(0.8944)}} \leq \frac{500.5 - 5000(0.1056)}{\sqrt{5000(0.1056)(0.8944)}}\right)$$
$$\approx P(Z \leq -1.27) = P(Z \geq 1.27)$$
$$= 1 - P(Z < 1.27) = 1 - 0.8980 = 0.1020.$$

Submit You have used 1 of 1 attempt

✓ Correct (3/3 points)

Problem 11

2/2 points (graded)

11. Let X, Y be independent Poisson random variables, with $\mathbb{E}(X) = 5000$ and $\mathbb{E}(Y) = 4900$.

11a. Find a double sum for the probability that X is strictly less than Y. I.e., find a double sum for $P(X < Y)$. You do not need to evaluate the double sum.

● $\sum_{x=0}^{\infty} \frac{(e^{-5000})(5000^x)}{x!} \sum_{y=x+1}^{\infty} \frac{(e^{-4900})(4900^y)}{y!}$ ✓

○ $\sum_{x=0}^{\infty} \frac{(e^{-4900})(4900^x)}{x!} \sum_{y=x+1}^{\infty} \frac{(e^{-5000})(5000^y)}{y!}$

○ $\sum_{x=0}^{y} \frac{(e^{-5000})(5000^x)}{x!} \sum_{y=x+1}^{\infty} \frac{(e^{-4900})(4900^y)}{y!}$

○ $\sum_{x=0}^{y} \frac{(e^{-4900})(4900^x)}{x!} \sum_{y=x+1}^{\infty} \frac{(e^{-5000})(5000^y)}{y!}$

11b. Approximate the probability in **11a**.

0.156224 ✓ **Answer:** 0.1562

Explanation

11a. We have $P(X < Y) = \sum_{x=0}^{\infty} \frac{(e^{-5000})(5000^x)}{x!} \sum_{y=x+1}^{\infty} \frac{(e^{-4900})(4900^y)}{y!}$.

11b. We have

$$P(X < Y) = P(X - Y < 0) = P(X - Y < -0.5)$$
$$= P\left(\frac{X - Y - (5000 - 4900)}{\sqrt{5000 + 4900}} \le \frac{-0.5 - (5000 - 4900)}{\sqrt{5000 + 4900}}\right)$$
$$\approx P(Z \le -1.01) = P(Z \ge 1.01)$$
$$= 1 - P(Z < 1.01) = 1 - 0.8438 = 0.1562.$$

Submit You have used 1 of 1 attempt

✓ Correct (2/2 points)

Problem 12

1/1 point (graded)

12. Consider 300 Continuous Uniform random variables, each of which has constant density on the interval $(0, 10)$. Assume that these random variables are independent. Find the probability that their sum is greater than 1600.

0.02275013 ✓ **Answer:** 0.0228

Explanation

12. We write U_1, ..., U_{300} for these Continuous Uniform random variables. Then we have

$$
\begin{aligned}
P(U_1 + \cdots + U_{300} > 1600) &= P\left(\frac{U_1+\cdots+U_{300} - 300(5)}{\sqrt{300(25/3)}} > \frac{1600 - 300(5)}{\sqrt{300(25/3)}}\right) \\
&= P(Z > 2) = 1 - P(Z \le 2) \\
&= 1 - 0.9772 = 0.0228.
\end{aligned}
$$

Submit You have used 1 of 1 attempt