- Unit 0: Overview
- Entrance Survey
- Unit 1: Probability models and axioms
- Unit 2: Conditioning and independence
- Unit 3: Counting
- Unit 4: Discrete random variables
- Exam 1
- Unit 5: Continuous random variables

Problem 1: Independent normal random variables

(3/3 points)

Let U, V, and W be independent standard normal random variables (that is, independent normal random variables, each with mean 0 and variance 1), and let $X = 3U + 4V$ and $Y = U + W$. Give a numerical answer for each part below. You may want to refer to the standard normal table .

1. What is the probability that $X \geq 8$?

   $\mathbf{P}(X \geq 8) =$ 0.05479929 **Answer:** 0.0548

2. $\mathbf{E}[XY] =$ 3 **Answer:** 3

3. $\mathrm{var}(X+Y) =$ 33 **Answer:** 33

Answer:

- Unit 6: Further topics on random variables
- Unit 7: Bayesian inference
- Exam 2

Exam 2
Exam 2 due Apr 20, 2016 at 23:59 UTC

- Unit 8: Limit theorems and classical statistics
- Unit 9: Bernoulli and Poisson processes
- Unit 10: Markov chains
- Exit Survey
- Final Exam

1. Since X is a sum of independent normal random variables, X is also a normal random variable. Its mean and variance are

$$\begin{aligned} \mathbf{E}[X] &= \mathbf{E}[3U+4V] \\ &= 3\cdot\mathbf{E}[U] + 4\cdot\mathbf{E}[V] \\ &= 0, \\ \mathrm{var}(X) &= \mathrm{var}(3U+4V) \\ &= 9\cdot\mathrm{var}(U) + 16\cdot\mathrm{var}(V) \\ &= 25. \end{aligned}$$

Therefore,

$$\begin{aligned} \mathbf{P}(X \geq 8) &= \mathbf{P}\left(\frac{X-0}{5} \geq \frac{8-0}{5}\right) \\ &= 1-\Phi(1.6) \\ &\approx 1-0.9452 \\ &= 0.0548. \end{aligned}$$

2. Since U, V, and W are zero-mean and independent, we have

$$\begin{aligned} \mathbf{E}[XY] &= \mathbf{E}[(3U+4V)(U+W)] \\ &= \mathbf{E}[3U^2+3UW+4UV+4VW] \\ &= 3\cdot\mathbf{E}[U^2] + 3\cdot\mathbf{E}[U]\mathbf{E}[W] + 4\cdot\mathbf{E}[U]\mathbf{E}[V] + 4\cdot\mathbf{E}[V]\mathbf{E}[W] \\ &= 3\cdot\mathbf{E}[U^2] \end{aligned}$$

$$= 3 \cdot \left(\mathbf{var}(U) + (\mathbf{E}[U])^2\right)$$
$$= 3.$$

3. Since U, V, and W are independent with variance equal to 1, we have

$$\begin{aligned}\mathbf{var}(X+Y) &= \mathbf{var}(3U+4V+U+W)\\ &= \mathbf{var}(4U+4V+W)\\ &= \mathbf{var}(4U)+\mathbf{var}(4V)+\mathbf{var}(W)\\ &= 16\cdot 1+16\cdot 1+1\\ &= 33.\end{aligned}$$

You have used 2 of 2 submissions