DelftX: OT.1x Observation theory: Estimating the Unknown

Help

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Graded Assignment due Feb 8, 2017 17:30 IST

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Exercises: geometry of least squares

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Projection onto range space

1/1 point (ungraded)

Which of the following expressions is the orthogonal projector P_A, projecting orthogonally on the range space of A?

● $P_A = A(A^TA)^{-1}A^T$ ✓

○ $P_A = (A^TA)^{-1}A^T$

○ $P_A = (A^TA)^{-1}A^TA$

○ $P_A = A^T(A^TA)^{-1}A$

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Mid-survey

Feedback

- 4. Best Linear Unbiased Estimation (BLUE)
- 5. How precise is the estimate?
- Pre-knowledge Mathematics
- MATLAB Learning Content

✓ Correct (1/1 point)

Some matrix algebra

1/1 point (ungraded)

Use the correct expression for P_A from the previous expression to check the following.

Which of the following expressions is/are correct?

☑ $P_A^T = P_A$

☐ $P_A + P_A = P_A$

☑ $P_A P_A = P_A$

✓

Answer

Correct:

$P_A P_A = A(A^T A)^{-1} A^T \cdot A(A^T A)^{-1} A^T = A \cdot I \cdot (A^T A)^{-1} A^T = P_A$;

$P_A^T = (A(A^T A)^{-1} A^T)^T = (A^T)^T)((A^T A)^{-1})^T (A)^T = A(A^T A)^{-1} A^T = P_A$;

$P_A + P_A = 2P_A$ The property of $P_A P_A = P_A$ is the required condition for a matrix P to be a projector. It indicates that if P applies twice on any vector, it gives the same results as it applies once.

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✓ Correct (1/1 point)

True or False questions

2/2 points (ungraded)

Try to determine if the following statements are TRUE or FALSE.

The orthogonal projection of y on $\mathrm{R}(A)$ guarantees that the elements of $\hat{e}$ are minimized.

○ TRUE

● FALSE ✓

Answer

Correct:

The length (i.e. the norm) of $\hat{e}$ is minimized, being equal to the sum of the squared elements of $\hat{e}$.

For a consistent system of observation equations, y would be in the range space $\mathrm{R}(A)$

● TRUE ✓

○ FALSE

Answer

Correct:
True, but unfortunately in reality the system of observation equations generally isn't consistent due to measurement errors.

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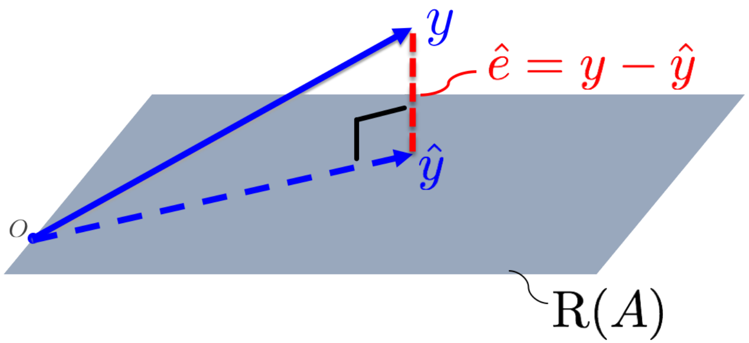
✓ Correct (2/2 points)

Decomposition

1/1 point (ungraded)

It can be shown that $\hat{y}$ is equal to the orthogonal projection of y on the range space of A:

$$\hat{y} = A \cdot \hat{x} = A \cdot (A^T A)^{-1} A^T y = P_A y$$



Now check for yourself which of the following expressions is correct for $\hat{e}$. *Select all correct answers*

☑ $\hat{e} = (I_m - P_A)y$

☐ $\hat{e} = P_A^T y$

☐ $\hat{e} = P_A \hat{y}$

☑ $\|\hat{e}\|^2 = \|y\|^2 - \|\hat{y}\|^2$

✓

Explanation

$\hat{e} = y - \hat{y} = y - P_A y.$

The last expression follows from Pythagoras's theorem, see the figure with the decomposition. Vector y is the hypothenuse, and its squared length $\|y\|^2$ is equal to the squared lengths of the other sides of the triangle (recall: $\hat{y} \perp \hat{e}$).

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✓ Correct (1/1 point)