

## Find the number of ways the commander can be chosen

Asked 4 days ago Active 4 days ago Viewed 106 times



Here is the question I'm trying to solve:



n soldiers standing in a line are divided into several non-empty units and then a commander is chosen for each unit. Count the number of ways this can be done.



My approach: Let C(x) be the required generating function. Considering the type A structures on the non empty intervals as  $a_k=1$ , the generating function is given as  $A(x)=\sum_{k\geq 1} 1\cdot x^k$ which gives,  $A(x) = \frac{x}{1-x}$ .



Next the type B structures are given by  $b_k = \binom{k}{1}$  giving the generating function  $B(x) = \sum_{k \geq 1} {k \choose 1} \cdot x^k$  which gives,  $B(x) = \frac{x}{(1-x)^2}$ .

Now 
$$C(x) = B(A(x))$$

Solving for C(x) I get

$$C(x)=rac{x\cdot (1-x)}{\left(1-2x
ight)^2}$$

Is my approach correct? How do I find the coefficient of  $x^n$  in C(x)?

Edit: Any hints on where I'm going wrong here? Because the expected answer doesn't match the answer I have calculated. Any help is appreciated.

combinatorics generating-functions

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edited Sep 6 at 13:07

asked Sep 6 at 10:34



Is the commander one of the soldiers in the unit? - Henry Sep 6 at 10:38



Yes the commander is one of the soldiers in the unit. - lettuce Sep 6 at 10:44



It looks as if the coefficient from your generating function may be  $a_n = (n+1)2^{n-2}$  at least for  $n \geq 1$ . You can derive this from your generating function since its denominator suggests  $a_n = 4a_{n-1} - 4a_{n-2}$ . Manual calculation from your original question confirms this starts  $1, 3, 8, \ldots$ Henry Sep 6 at 10:49



Thanks Also is my approach correct? Because the expected answer is different. - lettuce Sep 6 at

The expression I calculated doesn't hold for n = 4, for example. The expected value is 21 but my expression gives 20. However, it matches the expected values till n = 3. – lettuce Sep 6 at 13:19

2 Answers





I think your generating function may be wrong. In particular when n=4, I think it gives 20 when I can count 21 cases





I think if  $b_{n,k}$  is the number of choices when you have n soldiers and the first group has k individuals then you can consider adding an additional soldier at the beginning so you can say



$$b_{n+1,k+1} = rac{k+1}{k} b_{n,k}$$



for  $k \geq 1$  while

$$b_{n+1,1} = \sum_{k=1}^n b_{n,k}$$

which leads to results like

- $b_{n,k} = k \, b_{n+1-k,1}$
- $b_{n+1,1} = b_{n,1} + \sum_{m=1}^{n} b_{m,1}$
- $b_{n+1,1} = 3b_{n,1} b_{n-1,1}$
- sine the number you want  $a_n = b_{n+1,1}$ :

$$a_{n,1} = 3a_{n-1,1} - a_{n-2,1}$$

Since the numbers are  $1, 3, 8, 21, \ldots$  when  $n = 1, 2, 3, 4, \ldots$ , this leads to a generating function of the form  $\frac{x}{1 - 3x + x^2}$  if you think the answer is 0 when n = 0, or of the form

$$\frac{1-2x+x^2}{1-3x+x^2}$$
 if you think the answer is 1 when  $n=0$ .

This is a second order recurrence and can be solved related to  $\frac{3\pm\sqrt{5}}{2}$  but can also be written by saying the coefficient of  $x^n$  is Fib(2n).

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edited Sep 6 at 15:13

answered Sep 6 at 13:40

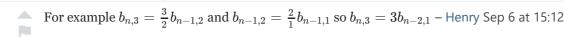


Thanks for your help! I understand it now. - lettuce Sep 6 at 14:38



Could you please explain how you reached the listed results? - lettuce Sep 6 at 15:02

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- Please correct me if i am wrong, you said that "when i count 21 cases". So, i decided to count so as to reach same value but i could not. My procedure was if n=4, then there are 4 ways to divide it into non-empty sets such that  $\{1-1-1-1\}$ ,  $\{1-3\}$ ,  $\{2-2\}$ ,  $\{2-1-1\}$ , we must also select a commander in each unit . I thought that we can calculate them such that 1 ways for  $\{1-1-1-1\}$ ,  $4 \times C(3,1) = 12$  for  $\{1-3\}$ ,  $(6 \times 2 \times 2)/2 = 12$  for  $\{2-2\}$ ,  $(6 \times 2 \times 2)/2 = 12$  for  $\{2-1-1\}$  , so 1+12+12+12=37 . Wht i missing ? – Bulbasaur Sep 6 at 15:22 🧪
- @ Your  $\{1-1-1-1\}, \{1-3\}, \{2-2\}, \{2-1-1\}$  generate  $1+3\times 2+2^2+2\times 3=17$  but you are missing  $\{4\}$  with 4 more possibilities making 21 – Henry Sep 6 at 15:32



We use

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$$\frac{1}{(1-2x)^2} = 1 + 2(2x) + 3(2x)^2 + \dots$$



Hence



$$C(x) = (x - x^2)(1 + 2(2x) + 3(2x)^2...)$$

So, coefficient of  $x^n$  is  $n cdot 2^{n-1} - (n-1)2^{n-2}$ .

Hence the answer is  $(n+1)2^{n-2}$ 

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answered Sep 6 at 10:59

