5.2.4 Matrix-Matrix Multiplication with Special Matrices

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Week 5 due Nov 6, 2023 22:42 IST

5.2.4 Matrix-Matrix Multiplication with Special Matrices

No introductory video

Reading Assignment

0 points possible (ungraded)

Read Unit 5.2.4 of the notes. [LINK]

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? triangular and diagonal matrix
Hi Maggie: Regarding Homework 5.2.4.12, could you please help to provide an example matrix which is a triangular but is not diagonal matrix? T... 2

Homework 5.2.4.1

21/21 points (graded)

Compute

- $\begin{pmatrix} 1 & -2 & -1 \\ 2 & 0 & 2 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} =$ 1 Answer: 1; 2 Answer: 2

- $\begin{pmatrix} 1 & -2 & -1 \\ 2 & 0 & 2 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} =$ -2 Answer: -2; 0 Answer: 0

- $\begin{pmatrix} 1 & -2 & -1 \\ 2 & 0 & 2 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} =$ -1 Answer: -1; 2 Answer: 2

- $\begin{pmatrix} 1 & -2 & -1 \\ 2 & 0 & 2 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} =$

1 Answer: 1	-2 Answer: -2	-1 Answer: -1
2 Answer: 2	0 Answer: 0	2 Answer: 2

Calculator

- $\begin{pmatrix} 1 & -2 & -1 \\ 2 & 0 & 2 \\ -1 & 3 & -1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} =$

1 ✔ Answer: 1	-2 ✔ Answer: -2	-1 ✔ Answer: -1
2 ✔ Answer: 2	0 ✔ Answer: 0	2 ✔ Answer: 2
-1 ✔ Answer: -1	3 ✔ Answer: 3	-1 ✔ Answer: -1

Explanation

- $\begin{pmatrix} 1 & -2 & -1 \\ 2 & 0 & 2 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$
- $\begin{pmatrix} 1 & -2 & -1 \\ 2 & 0 & 2 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} -2 \\ 0 \end{pmatrix}$
- $\begin{pmatrix} 1 & -2 & -1 \\ 2 & 0 & 2 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$
- $\begin{pmatrix} 1 & -2 & -1 \\ 2 & 0 & 2 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & -2 & -1 \\ 2 & 0 & 2 \end{pmatrix}$
- $\begin{pmatrix} 1 & -2 & -1 \\ 2 & 0 & 2 \\ -1 & 3 & -1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & -2 & -1 \\ 2 & 0 & 2 \\ -1 & 3 & -1 \end{pmatrix}$

Answer: There are at least two things to notice:

1. The first three results provide the columns for the fourth result. The fourth result provides the first two rows of the fifth result.
2. Multiplying the matrix from the right with the identity matrix does not change the matrix.

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Answers are displayed within the problem

Homework 5.2.4.2

18/18 points (graded)

Compute

- $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} =$

1 ✔ Answer: 1

2 ✔ Answer: 2

-1 ✔ Answer: -1

- $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} -2 \\ 0 \\ 3 \end{pmatrix} =$

-2 ✔ Answer: -2

0 ✔ Answer: 0

3 ✔ Answer: 3

Calculator

- $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} -1 \\ 2 \\ -1 \end{pmatrix} =$

-1 Answer: -1

2 Answer: 2

-1 Answer: -1

- $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & -2 & -1 \\ 2 & 0 & 2 \\ -1 & 3 & -1 \end{pmatrix} =$

1 Answer: 1	-2 Answer: -2	-1 Answer: -1
2 Answer: 2	0 Answer: 0	2 Answer: 2
-1 Answer: -1	3 Answer: 3	-1 Answer: -1

Explanation

- $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$
- $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} -2 \\ 0 \\ 3 \end{pmatrix} = \begin{pmatrix} -2 \\ 0 \\ 3 \end{pmatrix}$
- $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} -1 \\ 2 \\ -1 \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \\ -1 \end{pmatrix}$
- $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & -2 & -1 \\ 2 & 0 & 2 \\ -1 & 3 & -1 \end{pmatrix} = \begin{pmatrix} 1 & -2 & -1 \\ 2 & 0 & 2 \\ -1 & 3 & -1 \end{pmatrix}$

Answer: There are at least three things to notice:

1. The first three results provide the columns for the fourth result.
2. Multiplying the matrix from the left with the identity matrix does not change the matrix.
3. This homework and the last homework yield the same result.

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Answers are displayed within the problem

Homework 5.2.4.3

1/1 point (graded)
Let $A \in \mathbb{R}^{m \times n}$ and let I denote the identity matrix of appropriate size.

$AI = IA = A$

Always Answer: Always

Explanation
Transcripted in final section of this week

Answer: Always
Partition A and I by columns:

Calculator

$$A = \left(a_0 \,\middle|\, a_1 \,\middle|\, \cdots \,\middle|\, a_{n-1} \right) \text{ and } I = \left(e_0 \,\middle|\, e_1 \,\middle|\, \cdots \,\middle|\, e_{n-1} \right)$$

and recall that e_j equals the jth unit basis vector.

$AI = A$:

$$\begin{aligned}
& AI \\
& \quad = \quad < \text{Partition } I \text{ by columns } > \\
& A\left(e_0 \,\middle|\, e_1 \,\middle|\, \cdots \,\middle|\, e_{n-1} \right) \\
& \quad = \quad < \text{Partitioned matrix-matrix multiplication } > \\
& \left(Ae_0 \,\middle|\, Ae_1 \,\middle|\, \cdots \,\middle|\, Ae_{n-1} \right) \\
& \quad = \quad < a_j = Ae_j > \\
& \left(a_0 \,\middle|\, a_1 \,\middle|\, \cdots \,\middle|\, a_{n-1} \right) \\
& \quad = \quad < \text{Partition } A \text{ by columns } > \\
& A
\end{aligned}$$

$IA = A$:

$$\begin{aligned}
& IA \\
& \quad = \quad < \text{Partition } A \text{ by columns } > \\
& I\left(a_0 \,\middle|\, a_1 \,\middle|\, \cdots \,\middle|\, a_{n-1} \right) \\
& \quad = \quad < \text{Partitioned matrix-matrix multiplication } > \\
& \left(Ia_0 \,\middle|\, Ia_1 \,\middle|\, \cdots \,\middle|\, Ia_{n-1} \right) \\
& \quad = \quad < Ix = x > \\
& \left(a_0 \,\middle|\, a_1 \,\middle|\, \cdots \,\middle|\, a_{n-1} \right) \\
& \quad = \quad < \text{Partition } A \text{ by columns } > \\
& A
\end{aligned}$$

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Answers are displayed within the problem

Homework 5.2.4.4

12/12 points (graded)

Compute

- $\begin{pmatrix} 1 & -2 & -1 \\ 2 & 0 & 2 \end{pmatrix} \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix} =$ 2 ✔ Answer: 2; 4 ✔ Answer: 4
- $\begin{pmatrix} 1 & -2 & -1 \\ 2 & 0 & 2 \end{pmatrix} \begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix} =$ 2 ✔ Answer: 2; 0 ✔ Answer: 0
- $\begin{pmatrix} 1 & -2 & -1 \\ 2 & 0 & 2 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ -3 \end{pmatrix} =$ 3 ✔ Answer: 3; 6 ✔ Answer: 6

Calculator

$\begin{pmatrix} -3 \end{pmatrix}$ -6 Answer: -6

- $\begin{pmatrix} 1 & -2 & -1 \\ 2 & 0 & 2 \end{pmatrix}\begin{pmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -3 \end{pmatrix} =$

2 Answer: 2 | 2 Answer: 2 | 3 Answer: 3

4 Answer: 4 | 0 Answer: 0 | -6 Answer: -6

Explanation

- $\begin{pmatrix} 1 & -2 & -1 \\ 2 & 0 & 2 \end{pmatrix}\begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$
- $\begin{pmatrix} 1 & -2 & -1 \\ 2 & 0 & 2 \end{pmatrix}\begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \end{pmatrix}$
- $\begin{pmatrix} 1 & -2 & -1 \\ 2 & 0 & 2 \end{pmatrix}\begin{pmatrix} 0 \\ 0 \\ -3 \end{pmatrix} = \begin{pmatrix} 3 \\ -6 \end{pmatrix}$
- $\begin{pmatrix} 1 & -2 & -1 \\ 2 & 0 & 2 \end{pmatrix}\begin{pmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -3 \end{pmatrix} = \begin{pmatrix} 2 & 2 & 3 \\ 4 & 0 & -6 \end{pmatrix}$

Answer: Notice the relation between the above problems.

Submit

Answers are displayed within the problem

Homework 5.2.4.5

18/18 points (graded)

- $\begin{pmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -3 \end{pmatrix}\begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} =$

2 Answer: 2

-2 Answer: -2

3 Answer: 3

- $\begin{pmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -3 \end{pmatrix}\begin{pmatrix} -2 \\ 0 \\ 3 \end{pmatrix} =$

-4 Answer: -4

0 Answer: 0

-9 Answer: -9

-2 Answer: -2

Calculator

2 ✔ Answer: 2

- $\begin{pmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -3 \end{pmatrix} \begin{pmatrix} -1 \\ 2 \\ -1 \end{pmatrix} =$

-2 ✔ Answer: -2

3 ✔ Answer: 3

- $\begin{pmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -3 \end{pmatrix} \begin{pmatrix} 1 & -2 & -1 \\ 2 & 0 & 2 \\ -1 & 3 & -1 \end{pmatrix} =$

2 ✔ Answer: 2	-4 ✔ Answer: -4	-2 ✔ Answer: -2
-2 ✔ Answer: -2	0 ✔ Answer: 0	-2 ✔ Answer: -2
3 ✔ Answer: 3	-9 ✔ Answer: -9	3 ✔ Answer: 3

Explanation

- $\begin{pmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -3 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} = \begin{pmatrix} 2 \\ -2 \\ 3 \end{pmatrix}$
- $\begin{pmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -3 \end{pmatrix} \begin{pmatrix} -2 \\ 0 \\ 3 \end{pmatrix} = \begin{pmatrix} -4 \\ 0 \\ -9 \end{pmatrix}$
- $\begin{pmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -3 \end{pmatrix} \begin{pmatrix} -1 \\ 2 \\ -1 \end{pmatrix} = \begin{pmatrix} -2 \\ -2 \\ 3 \end{pmatrix}$
- $\begin{pmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -3 \end{pmatrix} \begin{pmatrix} 1 & -2 & -1 \\ 2 & 0 & 2 \\ -1 & 3 & -1 \end{pmatrix} = \begin{pmatrix} 2 & -4 & -2 \\ -2 & 0 & -2 \\ 3 & -9 & 3 \end{pmatrix}$

Answer: Notice the relation between the above problems.

Submit

Answers are displayed within the problem

Homework 5.2.4.6

1/1 point (graded)

Let $A \in \mathbb{R}^{m \times n}$ and let D denote the diagonal matrix with diagonal elements $\delta_0, \delta_1, \cdots, \delta_{n-1}$. Partition A by columns :

$$A = \left(a_0 \mid a_1 \mid \ldots \mid a_{n-1} \right).$$

$$AD = \left(\delta_0 a_0 \mid \delta_1 a_1 \mid \ldots \mid \delta_{n-1} a_{n-1} \right).$$

Always ✔ Answer: Always

Explanation

Transcribed in final section of this week

Answer: Always

AD

Calculator

$$= \quad < \text{Partition } A \text{ by columns, } D \text{ by elements } >$$

$$\left(\begin{array}{c|c|c|c} a_0 & a_1 & \cdots & a_{n-1} \end{array} \right) \left(\begin{array}{c|c|c|c} \delta_0 & 0 & \cdots & 0 \\ \hline 0 & \delta_1 & \cdots & 0 \\ \hline \vdots & \vdots & \ddots & \vdots \\ \hline 0 & 0 & \vdots & \delta_{n-1} \end{array} \right)$$

$$= \quad < \text{Partitioned matrix-matrix multiplication} >$$

$$\left(\begin{array}{c|c|c|c} a_0\delta_0 & a_1\delta_1 & \cdots & a_{n-1}\delta_{n-1} \end{array} \right)$$

$$= \quad < x\beta = \beta x >$$

$$\left(\begin{array}{c|c|c|c} \delta_0 a_0 & \delta_1 a_1 & \cdots & \delta_{n-1} a_{n-1} \end{array} \right).$$

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Answers are displayed within the problem

Homework 5.2.4.7

1/1 point (graded)

Let $A \in \mathbb{R}^{m \times n}$ and let D denote the diagonal matrix with diagonal elements $\delta_0, \delta_1, \cdots, \delta_{m-1}$. Partition A by rows :

$$A = \left(\begin{array}{c} \tilde{a}_0^T \\ \hline \tilde{a}_1^T \\ \hline \vdots \\ \hline \tilde{a}_{m-1}^T \end{array} \right).$$

$$DA = \left(\begin{array}{c} \delta_0 \tilde{a}_0^T \\ \hline \delta_1 \tilde{a}_1^T \\ \hline \vdots \\ \hline \delta_{m-1} \tilde{a}_{m-1}^T \end{array} \right).$$

Always ✔ Answer: Always

Explanation

Transcripted in final section of this week

Answer: Always

$$DA = \left(\begin{array}{c|c|c|c} \delta_0 & 0 & \cdots & 0 \\ \hline 0 & \delta_1 & \cdots & 0 \\ \hline \vdots & \vdots & \ddots & \vdots \\ \hline 0 & 0 & \vdots & \delta_{m-1} \end{array} \right) \left(\begin{array}{c} \tilde{a}_0^T \\ \hline \tilde{a}_1^T \\ \hline \vdots \\ \hline \tilde{a}_{m-1}^T \end{array} \right) = \left(\begin{array}{c} \delta_0 \tilde{a}_0^T \\ \hline \delta_1 \tilde{a}_1^T \\ \hline \vdots \\ \hline \delta_{m-1} \tilde{a}_{m-1}^T \end{array} \right)$$

by simple application of partitioned matrix-matrix multiplication.

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Answers are displayed within the problem

Calculator

Homework 5.2.4.8

9/9 points (graded)

Compute

$$\begin{pmatrix} 1 & -1 & -2 \\ 0 & 2 & 3 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} -2 & 1 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix} =$$

-2 ✔	0 ✔	-5 ✔
Answer: -2	Answer: 0	Answer: -5
0 ✔	2 ✔	7 ✔
Answer: 0	Answer: 2	Answer: 7
0 ✔	0 ✔	1 ✔
Answer: 0	Answer: 0	Answer: 1

$$\begin{pmatrix} -2 & 0 & -5 \\ 0 & 2 & 7 \\ 0 & 0 & 1 \end{pmatrix}$$

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Answers are displayed within the problem

Homework 5.2.4.9

9/9 points (graded)

Compute the following, using what you know about partitioned matrix-matrix multiplication:

$$\left(\begin{array}{cc|c} 1 & -1 & -2 \\ 0 & 2 & 3 \\ \hline 0 & 0 & 1 \end{array}\right) \left(\begin{array}{cc|c} -2 & 1 & -1 \\ 0 & 1 & 2 \\ \hline 0 & 0 & 1 \end{array}\right) =$$

-2	0	-5
✔ Answer: -2	✔ Answer: 0	✔ Answer: -5
0	2	7
✔ Answer: 0	✔ Answer: 2	✔ Answer: 7
0	0	1
✔ Answer: 0	✔ Answer: 0	✔ Answer: 1

Answer:

$$\left(\begin{array}{cc|c} 1 & -1 & -2 \\ 0 & 2 & 3 \\ \hline 0 & 0 & 1 \end{array}\right) \left(\begin{array}{cc|c} -2 & 1 & -1 \\ 0 & 1 & 2 \\ \hline 0 & 0 & 1 \end{array}\right)$$

$$= \left(\begin{array}{c|c} \begin{pmatrix} 1 & -1 \\ 0 & 2 \end{pmatrix}\begin{pmatrix} -2 & 1 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} -2 \\ 3 \end{pmatrix}\begin{pmatrix} 0 & 0 \end{pmatrix} & \begin{pmatrix} 1 & -1 \\ 0 & 2 \end{pmatrix}\begin{pmatrix} -1 \\ 2 \end{pmatrix} + \begin{pmatrix} -2 \\ 3 \end{pmatrix}(1) \\ \hline \begin{pmatrix} 0 & 0 \end{pmatrix}\begin{pmatrix} -2 & 1 \\ 0 & 1 \end{pmatrix} + (1)\begin{pmatrix} 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 \end{pmatrix}\begin{pmatrix} -1 \\ 2 \end{pmatrix} + (1)(1) \end{array}\right)$$

$$= \left(\begin{array}{c|c} \begin{pmatrix} -2 & 0 \\ 0 & 2 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} -3 \\ 4 \end{pmatrix} + \begin{pmatrix} -2 \\ 3 \end{pmatrix} \\ \hline \end{array}\right) = \left(\begin{array}{cc|c} -2 & 0 & -5 \\ 0 & 2 & 7 \\ \hline \end{array}\right)$$

Calculator

$$\left(\begin{array}{c|c} \begin{pmatrix} 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 \end{pmatrix} & (0+0)+(1) \end{array} \right) \quad \left(\begin{array}{cc|c} 0 & 0 & 1 \end{array} \right)$$

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Answers are displayed within the problem

Homework 5.2.4.10

1/1 point (graded)

Let $U, R \in \mathbb{R}^{n \times n}$ be uppertriangular matrices.

The product UR is an upper triangular matrix.

Always ✔ Answer: Always

Explanation

Answer: Always We will prove this by induction on n, the size of the square matrices.

Base case: $n = 1$. If $U, R \in \mathbb{R}^{1\times 1}$ then they are scalars. (Scalars are inherently upper triangular since they have no elements below the diagonal!). But then UR is also a scalar, which is an upper triangular matrix. Thus the result is true for $n = 1$.

Inductive Step: Induction Hypothesis (I.H.): Assume the result is true for $n = N$, where $N \geq 1$.

We will show the result is true for $n = N + 1$.

Let U and R be $n \times n$ upper triangular matrices with $n = N + 1$. We can partition

$$U = \left(\begin{array}{c|c} U_{00} & u_{01} \\ \hline 0 & \upsilon_{11} \end{array} \right) \quad \text{and} \quad R = \left(\begin{array}{c|c} R_{00} & r_{01} \\ \hline 0 & \rho_{11} \end{array} \right),$$

where U_{00} and R_{00} are $N \times N$ matrices and are upper triangular themselves. Now,

$$\begin{aligned} UR &= \left(\begin{array}{c|c} U_{00} & u_{01} \\ \hline 0 & \upsilon_{11} \end{array} \right) \left(\begin{array}{c|c} R_{00} & r_{01} \\ \hline 0 & \rho_{11} \end{array} \right) \\ &= \left(\begin{array}{c|c} U_{00}R_{00} + u_{01}0 & U_{00}r_{01} + u_{01}\rho_{11} \\ \hline 0R_{00} + \upsilon_{11}0 & 0r_{01} + \upsilon_{11}\rho_{11} \end{array} \right) = \left(\begin{array}{c|c} U_{00}R_{00} & U_{00}r_{01} + u_{01}\rho_{11} \\ \hline 0 & \upsilon_{11}\rho_{11} \end{array} \right). \end{aligned}$$

By the I.H., $U_{00}R_{00}$ is upper triangular. Hence,

$$UR = \left(\begin{array}{c|c} U_{00}R_{00} & U_{00}r_{01} + u_{01}\rho_{11} \\ \hline 0 & \upsilon_{11}\rho_{11} \end{array} \right)$$

is upper triangular.

By the Principle of Mathematical Induction (PMI), the result holds for all n.

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Answers are displayed within the problem

Calculator

Homework 5.2.4.11

1/1 point (graded)

The product of an n x n lower triangular matrix times an n x n lower triangular matrix is a lower triangular matrix.

Always ✔ Answer: Always

Explanation

Always!

We prove this by induction on n, the size of the square matrices.

Let A and B be lower triangular matrices.

Base case: $n = 1$. If $A, B \in \mathbb{R}^{1\times 1}$ then they are scalars. (Scalars are inherently lower triangular since they have no elements below the diagonal!). But then AB is also a scalar, which is an lower triangular matrix. Thus the result is true for $n = 1$.

Inductive Step: Induction Hypothesis (I.H.): Assume the result is true for $n = N$, where $N \geq 1$.

We will show the result is true for $n = N + 1$.

Let A and B be $n \times n$ lower triangular matrices with $n = N + 1$. We can partition

$$A = \left(\begin{array}{c|c} A_{00} & 0 \\ \hline a_{10}^T & \alpha_{11} \end{array}\right) \quad \text{and} \quad B = \left(\begin{array}{c|c} B_{00} & 0 \\ \hline b_{10}^T & \beta_{11} \end{array}\right),$$

where A_{00} and B_{00} are $N \times N$ matrices and are lower triangular themselves. Now,

$$\begin{aligned} AB &= \left(\begin{array}{c|c} A_{00} & 0 \\ \hline a_{10}^T & \alpha_{11} \end{array}\right)\left(\begin{array}{c|c} B_{00} & 0 \\ \hline b_{10}^T & \beta_{11} \end{array}\right) \\ &= \left(\begin{array}{c|c} A_{00}B_{00} + 0b_{10}^T & A_{00}0 + 0\beta_{11} \\ \hline a_{10}^T B_{00} + \alpha_{11} b_{10}^T & a_{10}^T 0 + \alpha_{11}\beta_{11} \end{array}\right) = \left(\begin{array}{c|c} A_{00}B_{00} & 0 \\ \hline a_{10}^T B_{00} + \alpha_{11} b_{10}^T & \alpha_{11}\beta_{11} \end{array}\right). \end{aligned}$$

By the I.H., $A_{00}B_{00}$ is lower triangular. Hence,

$$AB = \left(\begin{array}{c|c} A_{00}B_{00} & 0 \\ \hline a_{10}^T B_{00} + \alpha_{11} b_{10}^T & \alpha_{11}\beta_{11} \end{array}\right).$$

is lower triangular.

By the Principle of Mathematical Induction (PMI), the result holds for all n.

Submit

Answers are displayed within the problem

Homework 5.2.4.12

1/1 point (graded)

The product of an n x n lower triangular matrix times an n x n upper triangular matrix is a diagonal matrix.

Sometimes ✔ Answer: Sometimes

Explanation

Diagonal matrices are both upper and lower triangular. Multiply them together, and you get a diagonal matrix.

But take any lower triangular matrix that is not diagonal and multiply it by an upper triangular matrix (diagonal or not), and you don't get a diagonal matrix.

Submit

Answers are displayed within the problem

Calculator

Homework 5.2.4.13

1/1 point (graded)

Let $A \in \mathbb{R}^{m \times n}$.

$A^T A$ is symmetric.

Always ✔ Answer: Always

Explanation

Answer: Always

$$(A^T A)^T = A^T (A^T)^T = A^T A.$$

Hence, $A^T A$ is symmetric.

Submit

Answers are displayed within the problem

Homework 5.2.4.14

45/45 points (graded)

Compute

- $\begin{pmatrix} -1 \\ 1 \\ 2 \end{pmatrix} \begin{pmatrix} -1 & 1 & 2 \end{pmatrix} =$

1 ✔ Answer: 1 | -1 ✔ Answer: -1 | -2 ✔ Answer: -2
-1 ✔ Answer: -1 | 1 ✔ Answer: 1 | 2 ✔ Answer: 2
-2 ✔ Answer: -2 | 2 ✔ Answer: 2 | 4 ✔ Answer: 4

- $\begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix} \begin{pmatrix} 2 & 0 & -1 \end{pmatrix} =$

4 ✔ Answer: 4 | 0 ✔ Answer: 0 | -2 ✔ Answer: -2
0 ✔ Answer: 0 | 0 ✔ Answer: 0 | 0 ✔ Answer: 0
-2 ✔ Answer: -2 | 0 ✔ Answer: 0 | 1 ✔ Answer: 1

- $\left(\begin{array}{c|c} -1 & 2 \\ 1 & 0 \\ 2 & -1 \end{array}\right) \left(\begin{array}{ccc} -1 & 1 & 2 \\ \hline 2 & 0 & -1 \end{array}\right) =$

5 ✔ Answer: 5 | -1 ✔ Answer: -1 | -4 ✔ Answer: -4
-1 ✔ Answer: -1 | 1 ✔ Answer: 1 | 2 ✔ Answer: 2
-4 ✔ Answer: -4 | 2 ✔ Answer: 2 | 5 ✔ Answer: 5

1 ✔ | -2 ✔ | 2

Calculator

Answer: 1 Answer: -2 Answer: 2

- $\begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix} \begin{pmatrix} 1 & -2 & 2 \end{pmatrix} =$

-2 ✔ Answer: -2	4 ✔ Answer: 4	-4 ✔ Answer: -4
2 ✔ Answer: 2	-4 ✔ Answer: -4	4 ✔ Answer: 4

- $\left(\begin{array}{cc|c} -1 & 2 & 1 \\ 1 & 0 & -2 \\ 2 & -1 & 2 \end{array}\right) \left(\begin{array}{ccc} -1 & 1 & 2 \\ 2 & 0 & -1 \\ \hline 1 & -2 & 2 \end{array}\right) =$

6 ✔ Answer: 6	-3 ✔ Answer: -3	-2 ✔ Answer: -2
-3 ✔ Answer: -3	5 ✔ Answer: 5	-2 ✔ Answer: -2
-2 ✔ Answer: -2	-2 ✔ Answer: -2	9 ✔ Answer: 9

- $$\begin{pmatrix} -1 \\ 1 \\ 2 \end{pmatrix} \begin{pmatrix} -1 & 1 & 2 \end{pmatrix} = \begin{pmatrix} 1 & -1 & -2 \\ -1 & 1 & 2 \\ -2 & 2 & 4 \end{pmatrix}.$$

- $$\begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix} \begin{pmatrix} 2 & 0 & -1 \end{pmatrix} = \begin{pmatrix} 4 & 0 & -2 \\ 0 & 0 & 0 \\ -2 & 0 & 1 \end{pmatrix}.$$

- $$\left(\begin{array}{c|c} -1 & 2 \\ 1 & 0 \\ 2 & -1 \end{array}\right) \left(\begin{array}{ccc} -1 & 1 & 2 \\ \hline 2 & 0 & -1 \end{array}\right) = \begin{pmatrix} 1 & -1 & -2 \\ -1 & 1 & 2 \\ -2 & 2 & 4 \end{pmatrix} + \begin{pmatrix} 4 & 0 & -2 \\ 0 & 0 & 0 \\ -2 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 5 & -1 & -4 \\ -1 & 1 & 2 \\ -4 & 2 & 5 \end{pmatrix}.$$

- $$\begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix} \begin{pmatrix} 1 & -2 & 2 \end{pmatrix} = \begin{pmatrix} 1 & -2 & 2 \\ -2 & 4 & -4 \\ 2 & -4 & 4 \end{pmatrix}.$$

- $$\left(\begin{array}{cc|c} -1 & 2 & 1 \\ 1 & 0 & -2 \\ 2 & -1 & 2 \end{array}\right) \left(\begin{array}{ccc} -1 & 1 & 2 \\ 2 & 0 & -1 \\ \hline 1 & -2 & 2 \end{array}\right) =$$

Answer:

$$\begin{pmatrix} -1 & 2 \\ 1 & 0 \\ 2 & -1 \end{pmatrix} \begin{pmatrix} -1 & 1 & 2 \\ 2 & 0 & -1 \end{pmatrix} + \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix} \begin{pmatrix} 1 & -2 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} 5 & -1 & -4 \\ -1 & 1 & 2 \\ -4 & 2 & 5 \end{pmatrix} + \begin{pmatrix} 1 & -2 & 2 \\ -2 & 4 & -4 \\ 2 & -4 & 4 \end{pmatrix} = \begin{pmatrix} 6 & -3 & -2 \\ -3 & 5 & -2 \\ -2 & -2 & 9 \end{pmatrix}.$$

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Calculator

Answers are displayed within the problem

Homework 5.2.4.15

1/1 point (graded)
Let $x \in \mathbb{R}^n$.

xx^T is symmetric.

Always ✔ Answer: Always

Explanation

Answer: Always
Proof 1: Since $A^T A$ is symmetric for any matrix $A \in \mathbb{R}^{m \times n}$ and vector $A = x^T \in \mathbb{R}^n$ is just the special case where the matrix is a vector.

Proof 2:

$$xx^T = \begin{pmatrix} \chi_0 \\ \chi_1 \\ \vdots \\ \chi_{n-1} \end{pmatrix} \begin{pmatrix} \chi_0 \\ \chi_1 \\ \vdots \\ \chi_{n-1} \end{pmatrix}^T = \begin{pmatrix} \chi_0 \\ \chi_1 \\ \vdots \\ \chi_{n-1} \end{pmatrix} \begin{pmatrix} \chi_0 & \chi_1 & \cdots & \chi_{n-1} \end{pmatrix}$$
$$= \begin{pmatrix} \chi_0\chi_0 & \chi_0\chi_1 & \cdots & \chi_0\chi_{n-1} \\ \chi_1\chi_0 & \chi_1\chi_1 & \cdots & \chi_1\chi_{n-1} \\ \vdots & \vdots & & \vdots \\ \chi_{n-1}\chi_0 & \chi_{n-1}\chi_1 & \cdots & \chi_{n-1}\chi_{n-1} \end{pmatrix}.$$

Since $\chi_i\chi_j = \chi_j\chi_i$, the (i,j) element of xx^T equals the (j,i) element of xx^T. This means xx^T is symmetric.

Proof 3: $(xx^T)^T = (x^T)^T x^T = xx^T$.

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Homework 5.2.4.16

1/1 point (graded)
Let $A \in \mathbb{R}^{n \times n}$ be symmetric and $x \in \mathbb{R}^n$.

$A + xx^T$ is symmetric.

Always ✔ Answer: Always

Explanation

If matrices $A, B \in \mathbb{R}^{nxn}$ are symmetric, then $A + B$ is symmetric since $(A+B)^T = A^T + B^T = A + B$. In this case, $B = xx^T$.

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Homework 5.2.4.17

1/1 point (graded)
Let $A \in \mathbb{R}^{m \times n}$.

Calculator

AA^T is symmetric.

Always ✔ Answer: Always

Explanation

Answer: Always

Proof 1: $(AA^T)^T = (A^T)^T A^T = AA^T$.

Proof 2: We know that $A^T A$ is symmetric. Take $B = A^T$. Then $AA^T = B^T B$ and hence AA^T is symmetric.

Proof 3:

$$\begin{aligned} AA^T &= \left(a_0 \mid a_1 \mid \cdots \mid a_{n-1} \right) \left(a_0 \mid a_1 \mid \cdots \mid a_{n-1} \right)^T \\ &= \left(a_0 \mid a_1 \mid \cdots \mid a_{n-1} \right) \begin{pmatrix} a_0^T \\ \hline a_1^T \\ \hline \vdots \\ \hline a_{n-1}^T \end{pmatrix} \\ &= a_0 a_0^T + a_1 a_1^T + \cdots + a_{n-1} a_{n-1}^T. \end{aligned}$$

But each $a_j a_j^T$ is symmetric (by a previous exercise) and adding symmetric matrices yields a symmetric matrix. Hence, AA^T is symmetric.

Proof 4:
Proof by induction on n.

Base case: $A = \left(a_0 \right)$, where a_0 is a vector. Then $AA^T = a_0 a^T$. But we saw in an earlier homework that if x is a vector, then xx^T is symmetric.

Induction Step: Assume that AA^T is symmetric for matrices with $n = N$ columns, where $N \geq 1$. We will show that AA^T is symmetric for matrices with $n = N + 1$ columns.
Let A have $N + 1$ columns.

$$\begin{aligned} & AA^T \\ = & \quad < \text{Partition } A > \\ & \left(A_0 \mid a_1 \right) \left(A_0 \mid a_1 \right)^T \\ = & \quad < \text{Transpose partitioned matrix} > \\ & \left(A_0 \mid a_1 \right) \begin{pmatrix} A_0^T \\ \hline a_1^T \end{pmatrix} \\ = & \quad < \text{Partitioned matrix-matrix multiplication} > \\ & A_0 A_0^T + a_1 a_1^T \end{aligned}$$

Now, by the I.H. $A_0 A_0^T$ is symmetric. From a previous exercise we know that xx^T is symmetric and hence $a_1 a_1^T$ is. From another exercise we know that adding symmetric matrices yields a symmetric matrix.

By the Principle of Mathematical Induction (PMI), the result holds for all n.

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Homework 5.2.4.18

1/1 point (graded)
Let $A, B \in \mathbb{R}^{n \times n}$ be symmetric matrices.

AB is symmetric.

Sometimes ✔

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