



## MITx: 6.041x Introduction to Probability - The Science of Uncertainty



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## Exercise: Non-Poisson random incidence

(2/2 points)

The consecutive interarrival times of a certain arrival process are i.i.d. random variables that are equally likely to be 5, 10, or 15 minutes. Find the expected value of the length of the interarrival time seen by an observer who arrives at some particular time, unrelated to the history of the process.

35/3


**Answer:** 11.66667**Answer:**

Following the same argument as in the preceding video, out of every 30 minutes, there will be (in an average sense) 5 minutes (a fraction of  $1/6$  of the total) covered by intervals of length 5, 10 minutes (a fraction of  $2/6$ ) covered by intervals of length 10, and 15 minutes (a fraction of  $3/6$  of the total) covered by intervals of length 15. Thus, the observer has probability  $1/6$ ,  $2/6$ , and  $3/6$ , of seeing an interval of length 5, 10, and 15, respectively. The expected value is


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#### Unit overview

##### Lec. 21: The Bernoulli process

Exercises 21 due May 11, 2016 at 23:59 UTC 

##### Lec. 22: The Poisson process

Exercises 22 due May 11, 2016 at 23:59 UTC 

##### Lec. 23: More on the Poisson process

$$\frac{1}{6} \cdot 5 + \frac{2}{6} \cdot 10 + \frac{3}{6} \cdot 15 = \frac{70}{6}.$$

Note that this is larger than the average interarrival time, which is


$$\frac{1}{3} \cdot (5 + 10 + 15) = 10.$$

In case you are curious, if a typical interarrival interval  $T$  has probability  $p_k$  of having length  $k$ , then the probability that the observer sees an interval  $S$  of length  $k$  is proportional to  $kp_k$ . Since probabilities need to sum to 1,

$$\mathbf{P}(S = k) = \frac{kp_k}{\sum_k kp_k} = \frac{kp_k}{\mathbf{E}[T]}.$$

It follows that


$$\mathbf{E}[S] = \sum_k k \frac{kp_k}{\sum_k kp_k} = \frac{\sum_k k^2 p_k}{\mathbf{E}[T]} = \frac{\mathbf{E}[T^2]}{\mathbf{E}[T]}.$$

Exercises 23 due May 11, 2016  
at 23:59 UTC 

Solved problems

Additional theoretical  
material

Problem Set 9

Problem Set 9 due May 11,  
2016 at 23:59 UTC 

Unit summary

► Unit 10: Markov  
chains

It can be shown that the expression  $\mathbf{E}[S] = \mathbf{E}[T^2]/\mathbf{E}[T]$  is the correct one also for the continuous time case. As an illustration, suppose that interarrival times are exponential with rate  $\lambda$ , so that we are dealing with a Poisson process. In that case,  $\mathbf{E}[T] = 1/\lambda$ ,  $\mathbf{E}[T^2] = 2/\lambda^2$ , so that  $\mathbf{E}[S] = 2/\lambda$ , which agrees with our earlier analysis of random incidence in the Poisson process.

*You have used 1 of 2 submissions*

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