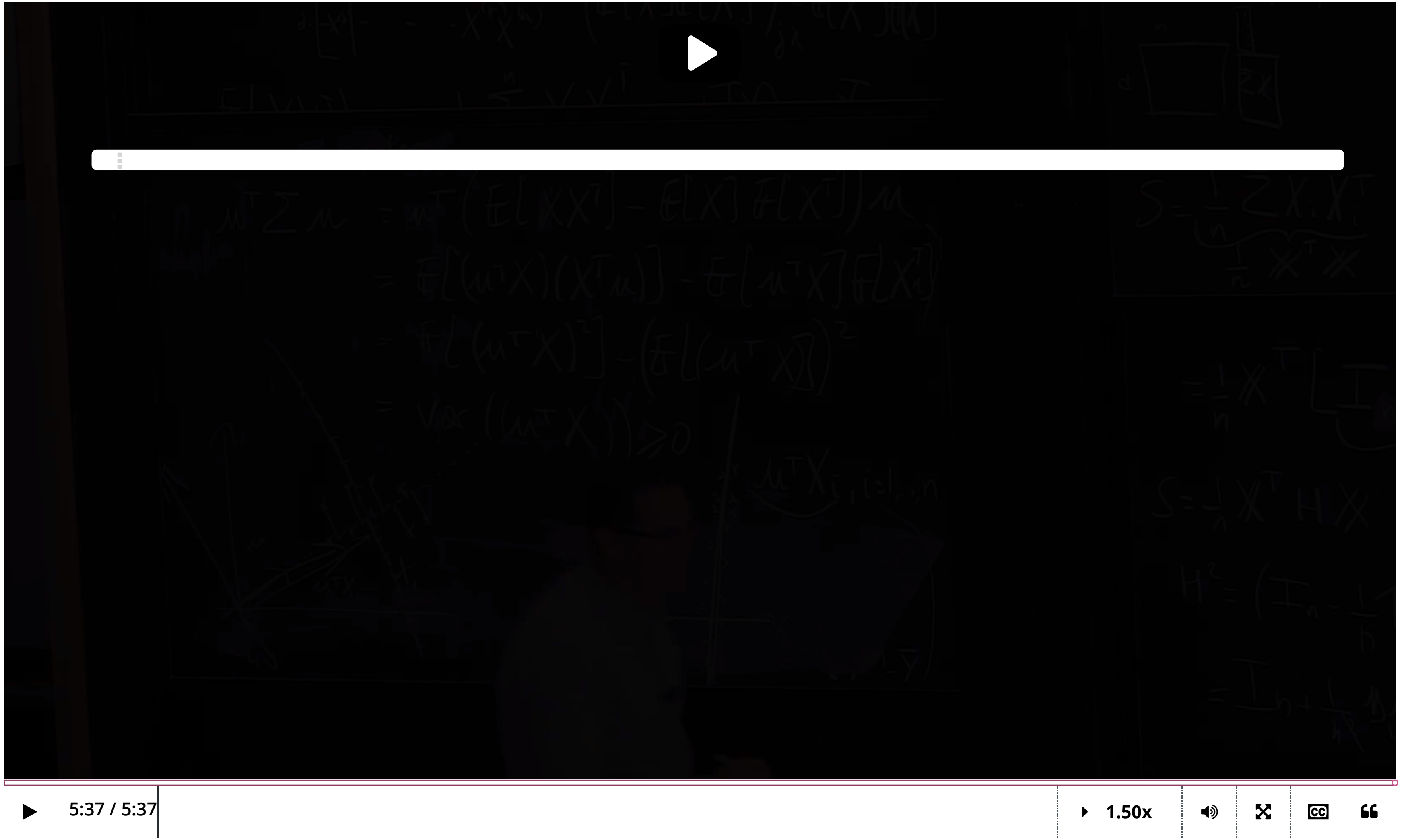
5. Review of Linear Algebra Required for this Lecture

Review of an Important Concept in Linear Algebra - Spectral Decomposition of Positive Semi-Definite Matrices



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Concept Check: Eigenvalues in Spectral Decomposition

3/3 points (ungraded)

Assume that Σ is **positive semi-definite**. Let $\Sigma = PDP^T$ be the spectral decomposition of Σ. Answer whether the following statements are **True** or **False**.

"Eigenvalues of Σ" are always non-negative (greater than or equal to zero)."

● True

○ False

✓

"If $\mathbf{x}$ is an eigenvector of Σ corresponding to eigenvalue λ, then $\mathbf{x}^T \Sigma \mathbf{x} \leq \lambda_{\max} \|\mathbf{x}\|^2$, where $\lambda_{\max}$ is the largest eigenvalue of Σ."

● True

○ False

✓

"The sum of the diagonal elements of Σ, also known as the trace of Σ, is equal to the sum of the eigenvalues of Σ."

● True

○ False

✓

Solution:

All the statements are **true**. Let us examine the statements in order:

- Since we are given that Σ is positive semi-definite, its eigenvalues are non-negative.
- If $\mathbf{x}$ is an eigenvector corresponding to an eigenvalue λ, then

$$\begin{aligned} \mathbf{x}^T\Sigma\mathbf{x} &= \mathbf{x}^T(\lambda\mathbf{x}) \\ &= \lambda\|\mathbf{x}\|^2 \\ &\leq \lambda_{\max}\|\mathbf{x}\|^2, \end{aligned}$$

since $\|\mathbf{x}\|^2$ is non-negative.

- The property that the sum of diagonal elements of the eigenvalues is equal to the trace of Σ is true because of the following. Let $\mathbf{p}_i$ be the columns of P

$$\begin{aligned} \text{trace}\,(\Sigma) &= \text{sum of diagonal elements of } \Sigma \\ &= \text{sum of diagonal elements of } (PDP^T) \\ &= \sum_i \lambda_i \|\mathbf{p}_i\|^2 = \sum_i \lambda_i, \end{aligned}$$

since $\mathbf{p}_i$'s have a norm of 1.

Submit You have used 1 of 2 attempts

Answers are displayed within the problem

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