10. Vector decomposition

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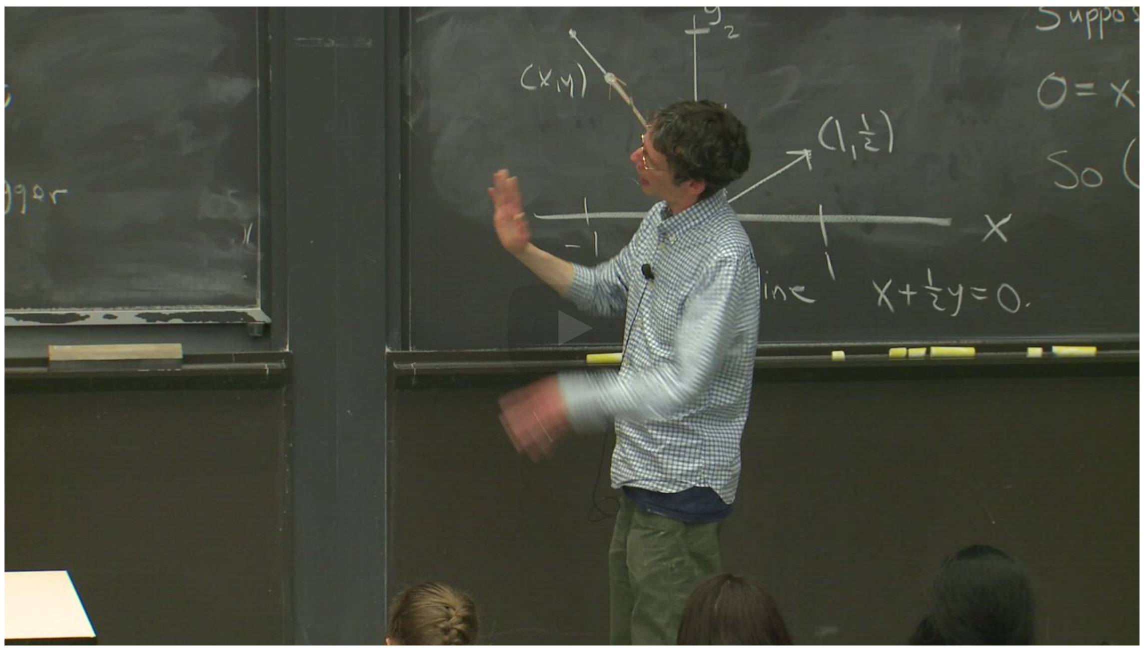
Note about video: The class 8.01 mentioned in the following video is a physics class (classical mechanics) at MIT.

Motivation from physics



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PROFESSOR: So this is more than just an exercise

in trying to figure out how big dot products are

by looking at a picture.

It's actually used a lot in problems about vectors.

And let me give you a motivating example, so similar to things

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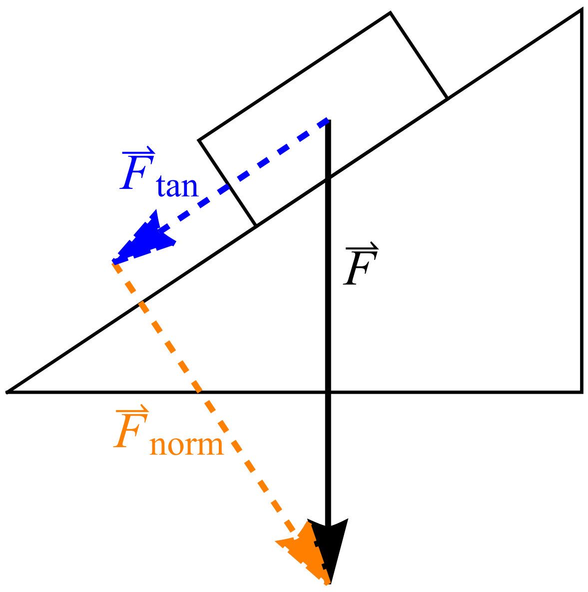
Consider a mass on an inclined plane where $\vec{g}$ is the force of gravity.



The vector that forms the tangent component of $\vec{g}$ is labeled $\vec{g}_{tan}$. This vector is tangent to the inclined plane. The vector that forms the normal component of $\vec{g}$ is labeled $\vec{g}_{norm}$ and is normal to the inclined plane. We have

$$\vec{g} = \vec{g}_{tan} + \vec{g}_{norm}.$$

The general math problem here is to find the tangential and normal components of the vector. Consider the two vectors $\vec{v}$ and $\vec{u}$ below.

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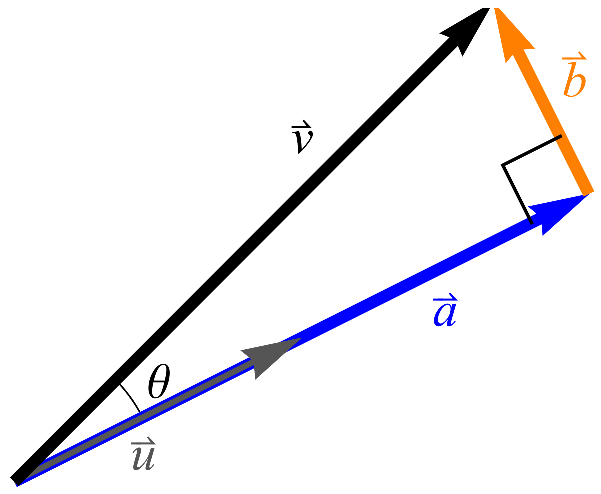

We want to decompose $\vec{v}$ into a piece in the $\vec{u}$ direction and a piece that is normal to $\vec{u}$ so that

$$\vec{v} = \vec{a} + \vec{b}$$

where

- $\vec{a}$ is the component of $\vec{v}$ in the $\vec{u}$ direction, and
- $\vec{b}$ is the component of $\vec{v}$ perpendicular to the $\vec{u}$ direction.

Derivation of decomposition

Given $\vec{u}$ and $\vec{v}$, find $\vec{a}$ and $\vec{b}$.

What do we know about $\vec{a}$? Well, we know $\vec{a}$ is in the same direction as $\vec{u}$, which means

$$\vec{a} = \lambda\vec{u}$$

for some number $\lambda > 0$. Now, we need to find λ.

We saw in the previous question that

$$\vec{u}\cdot\vec{v} = \vec{u}\cdot\vec{a} = \vec{u}\cdot(\lambda\vec{u}) = \lambda\left(\vec{u}\cdot\vec{u}\right).$$

So λ is given by

$$\lambda = \frac{\vec{u}\cdot\vec{v}}{\vec{u}\cdot\vec{u}}.$$

This means

$$\boxed{\vec{a} = \left(\frac{\vec{u}\cdot\vec{v}}{\vec{u}\cdot\vec{u}}\right)\vec{u}}.$$

Now, how do we find $\vec{b}$? We already found $\vec{a}$ and we know that $\vec{v} = \vec{a} + \vec{b}$. So

$$\boxed{\vec{b} = \vec{v} - \vec{a}}.$$

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Example 10.1

Decompose the vector $\vec{v} = \langle 1, 2\rangle$ into components that point in the direction of $\vec{u} = \langle 1, 1\rangle$ and normal to $\vec{u}$.

Using the formula derived above, we want to write $\vec{v} = \vec{a} + \vec{b}$, where

$$\vec{a} = \frac{\langle 1,1\rangle \cdot \langle 1,2\rangle}{\langle 1,1\rangle \cdot \langle 1,1\rangle}\langle 1,1\rangle = \frac{3}{2}\langle 1,1\rangle = \langle 3/2, 3/2\rangle.$$

Then we use vector subtraction to find the component normal to $\vec{a}$:

$$\vec{b} = \langle 1,2\rangle - \langle 3/2, 3/2\rangle = \langle -1/2, 1/2\rangle$$

Sanity check: Verify that $\vec{a}$ and $\vec{b}$ are normal by taking their dot product:

$$\langle 3/2, 3/2\rangle \cdot \langle -1/2, 1/2\rangle = -3/4 + 3/4 = 0✓$$

Does vector length and angle matter?

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PROFESSOR: Yeah

STUDENT: So it doesn't matter if v or u have different size,

if u is bigger than v or the other way around?

PROFESSOR: OK, yeah, so the question is, so in the picture,

v is longer than u, and the question is, does that matter.

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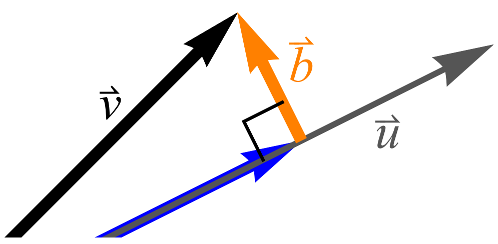
Does vector length matter?

What if $|\vec{v}| < |\vec{u}|$? Does it matter which vector is longer?

Let's draw a picture.



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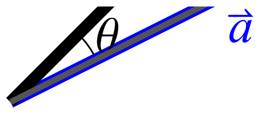
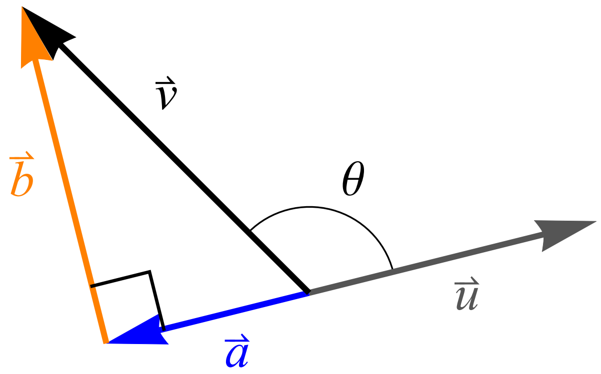


The same formula will work.

What if the angle is obtuse?

What if $\theta > \pi/2$? Then the picture would look like the following.



In this case, $\lambda < 0$ and $\vec{a}$ is in the opposite direction of $\vec{u}$.

Remark 10.2

- If we replace $\vec{u}$ by $2\vec{u}$ or $\frac{1}{3}\vec{u}$, the $\vec{a}$ and $\vec{b}$ components don't change.
- If $|\vec{u}| = 1$, then

$$\vec{u} \cdot \vec{u} = u_1^2 + u_2^2 = |\vec{u}|^2 = 1$$

and the formula becomes

$$\vec{a} = (\vec{v} \cdot \vec{u})\,\vec{u}.$$

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	[staff] decomposing of other vector I am just wondering , can we decompose u vector into two small pieces at same time and find out relationship between components ...	2
?	[STAFF] What happened to the course 8.01x on EDX? Hello, staff. It's an offtopic quesiton, but maybe you know. Parallel to 18.02x, I have registered for mechanics course, 8.01x for Septe...	5
	very simple derivation Thats the simplest derivation of the projection formula ive seen. Very easy to understand.	3
?	Why vec(u) * vec(a) = vec(v) * vec(u)? I don't understand the reason why vec(u) * vec(a) = vec(v) * vec(u) ? Can anyone explain this?	7
	Physics conventions Just a note on the labeling of the "g" vector. It would be more conventional (and consistent with physics notation) to refer to g" as th...	5

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