# 4. System response

Now let's return to the original problem. Suppose that the input signal $f$ is an odd periodic function of period $2\pi$. Since $f$ is odd, the Fourier series of $f$ is a linear combination of sine functions

$$f(t) = b_1 \sin t + b_2 \sin 2t + b_3 \sin 3t + \cdots.$$

Let $f(t)$ be the input of the system

$$\ddot{x} + 50x = f(t).$$

By the superposition principle, the system response to $f(t)$ is

$$x(t) = b_1 \frac{1}{49} \sin t + b_2 \frac{1}{46} \sin 2t + b_3 \frac{1}{41} \sin 3t + \cdots.$$

Note that each Fourier component $\sin nt$ has a different gain: the gain depends on the frequency.

One could write a particular solution using sigma-notation:

$$x_p(t) = \sum_{n\geq 1} \frac{1}{50 - n^2} b_n \sin nt.$$

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### Forced Harmonic Oscillator

discussion posted 18 days ago by muondude

The equation $\frac{d^2x}{dt^2} + \omega x = f(t)$ is a forced harmonic oscillator. The solution has a resonance (large amplitude response) then the driving frequency of $f(t)$ equals $\omega$. In the example above this occurs when $n = \sqrt{50} \simeq 7.07$, so for integer $n$ this occurs when $n = 7$.

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**V 1**

16 days ago

In your explanation, shouldn't the driving frequency of $f(t)$ equal $\sqrt{w}$ ? (So that the input function's frequency matches that of the system's natural frequency)

We usually write frequencies with $\omega$, so he probably forgot the exponential in the equation rather than the square root. I think it was supposed to be $\frac{d^2x}{dt^2} + \omega^2 x = f(t)$, with $\omega$ as the natural frequency of the system.

posted 16 days ago by myyukiko

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