1.3.2 Exploratory Quiz: What is the bifurcation for this system?

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Question 1: Think About It...

1/1 point (graded)

Is Coexistence Possible with Strong Competition?

Wes says in the "strong" competition case, like when $\beta = 2$, whichever species has a "head start" will competitively exclude the other. But what happens if neither species has a head start?

Assume we start with $x = \frac{1}{2}$ and $y = \frac{1}{2}$, so that the two species X and Y start with exactly the same proportion of their respective carrying capacity. What do you think happens to species X and Y in the long run?

- Do you think coexistence is theoretically possible with strong competition? Try to answer this intuitively, knowing you have two species with perfectly balanced competition. Then consider what the differential equations imply (hint: look at the phase plane.)
- Do you think coexistence is possible with strong competition in nature (a real-world biological system)? Why or why not?

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If both of them have equal numbers to start with it will
result in a co-existence.
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Thank you for your response.

Intuitively, if the species are evenly matched and they start with equal population densities then their densities will stay equal (since the competition affects each species equally).

Looking at the equations, we see that if we start with the same size population ($x(0) = y(0)$), then the rates of change of the two populations will be equal. Thus x and y will change in exactly the same way thus making $x(t) = y(t)$ for all time t. Thus the two species will end up at the same size, which is represented by the equilibrium at $(1/3, 1/3)$.

You might wonder: is it ever possible to observe this type of equilibrium in nature? The answer is no. Accounting for real world random fluctuations of the two species, they will not reach the coexistence equilibrium.

This is because the differential equation is intended to model the mean value of the species population density (the expected value), not an actual population density. In reality, population density is a discrete not continuous quantity, and random events can have small impacts on population. These random fluctuations in population would push one species slightly ahead of the other at some point in time. The species which is pushed ahead will then drive the other species to extinction, and the remaining species will reach the equilibrium density of 1.

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Question 2

1/1 point (graded)

Use the dynamic graph in Desmos to estimate the value of β that corresponds to the bifurcation where the system changes from coexistence to competitive exclusion.

1 ✔ **Answer:** 1

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Explanation

The value is $\beta = 1$.

The two red and blue diagonal nullclines are now the same line, meaning there is a whole line segment of equilibrium points. This range of equilibrium points corresponds to the segment of the line $y = -x + 1$ which lies in quadrant I of the coordinate plane.

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Question 3

1/1 point (graded)

Consider the system when β is equal to the critical value where the system changes from coexistence to competitive exclusion. What is the phase plane like? How many non-zero equilibrium points are there?

(**Note:** The equilibrium point (0,0) corresponding to zero population for both X and Y is always an equilibrium point for the system.)

A **stable** equilibrium point is one where for all nearby initial conditions, the solution trajectories approach that point. We use **unstable** to mean an equilibrium point which is not stable.

Choose the best answer.

- There are two non-zero equilibrium points (1,0) and (0,1) corresponding to saturation of one species and extinction of the others.
- There are three non-zero equilibrium points (1,0) and (0,1) corresponding to saturation of one species and extinction of the others, and a third point corresponding to stable coexistence.
- There are three non-zero equilibrium points (1,0) and (0,1) corresponding to saturation of one species and extinction of the others, and a third point corresponding to unstable coexistence.
- Infinitely many. There is a line segment of equilibrium points from (1,0) to (0,1) and solution trajectories head toward that line over time. ✔
- Infinitely many. There is a line segment of unstable equilibrium points from (1,0) to (0,1) and solution trajectories head away from that line over time.

Explanation

Infinitely many. The two diagonal nullclines are now the same line, $y = 1 - x$. This means there is a whole line segment of equilibrium points.

Looking at the sign of $\frac{dx}{dt}$ and $\frac{dy}{dt}$, we see they are both negative for points above the line and both positive for points below the line. This means solution trajectories head toward that line over time.

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