3. Application of Delta Method on Gamma Variables

The **Gamma distribution** $\mathrm{Gamma}\left(\alpha,\beta\right)$ with paramters $\alpha>0$, and $\beta>0$ is defined by the density

$$f_{\alpha,\beta}\left(x\right)=\frac{\beta^{\alpha}}{\Gamma\left(\alpha\right)}x^{\alpha-1}e^{-\beta x},\quad \text{for all} x\geq 0.$$

The Γ function is defined by

$$\Gamma\left(s\right)=\int_{0}^{\infty}x^{s-1}e^{-x}dx.$$

As usual, the constant $\frac{\beta^{\alpha}}{\Gamma(\alpha)}$ is a normalization constant that gives $\int_{0}^{\infty}f_{\alpha,\beta}\left(x\right)dx=1$.

In this problem, let $X_1,\ldots,X_n$ be i.i.d. Gamma variables with

$$\beta=\frac{1}{\alpha}\text{for some }\alpha>0.$$

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$,\ldots,X_n\sim\mathrm{Gamma}\left(\alpha,\frac{1}{\alpha}\right)$ random variables for some $\alpha>0$. The pdf for X_i is therefore

$$f_\alpha(x) = \frac{1}{\Gamma(\alpha)\,\alpha^\alpha} x^{\alpha-1} e^{-x/\alpha}, \quad \text{for all } x \geq 0.$$

(a)

1/1 point (graded)

What is the limit, in probability, of the sample average $\overline{X}_n$ of the sample in terms of α?

$\overline{X}_n \xrightarrow[n\to\infty]{\mathbf{P}}$ alpha^2 ✔

α^2

STANDARD NOTATION

Submit You have used 1 of 3 attempts

✔ Correct (1/1 point)

(b)

1/1 point (graded)

Use the result from the previous problem to give a consistent estimator $\hat{\alpha}$ of α in terms of $\overline{X}_n$.

(Enter barX_n for $\overline{X}_n$)

$\hat{\alpha} =$ sqrt(barX_n) ✔

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✔ Correct (1/1 point)

(c)

3/3 points (graded)

For the Delta method to apply, at what value of x does g need to be continuously differentiable? (Your answer should be in terms of α.)

$x =$ alpha^2 ✔

α^2

What distribution does $\sqrt{n}\,(\hat{\alpha} - \alpha)$ converge to as $n \to \infty$?

- ○ Gamma distribution
- ◉ Normal distribution
- ○ None of the above

✔

What is its asymptotic variance of $\hat{\alpha}$?

$\mathrm{Var}\,(\sqrt{n}\hat{\alpha}) = \mathrm{Var}\,(\sqrt{n}\,(\hat{\alpha} - \alpha)) =$ alpha/4 ✔

$\frac{\alpha}{4}$

STANDARD NOTATION

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✔ Correct (3/3 points)

(d)

4/4 points (graded)

Using the previous part, find confidence intervals for α with asymptotic level 90% using both the “solving" and the “plug-in" methods. Use $n = 25$, and $\overline{X}_n = 4.5$.

(Enter your answers accurate to 2 decimal places. Use the Gaussian estimate $q_{0.05} \approx 1.6448$ for best results.)

$\mathcal{I}_{\text{solve}} =$ [1.89 ✔ , 2.37 ✔]

$\mathcal{I}_{\text{plug-in}} =$ [1.88 ✔ , 2.36 ✔]

STANDARD NOTATION

Submit You have used 2 of 3 attempts

✔ Correct (4/4 points)

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[Staff] Can deadline for the next homework be extended? 2

As discussed by another student, I am also having the same problem. I have my midterms starting in my university from 25 Sept and would like to do some work ahead of tim...

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grading error? part (d) (I solve) 2

I think i got the correct quadratic equation and solved for roots but only the upper end got accepted, now i am thinking how on earth, is my only one root correct and another...

Part (d) - I think I have the correct solution for plug_in but the grader thinks otherwise — 1
Could there be problem with the round off? Thank you

[Staff] Is it possible to post content ahead a little more? — 3
I am not sure about the other people but one thing that could prevent me from finishing the program is something might come up someday that I could not work on the class...

I_solve in part d — 2

Unfamiliar with the gamma dist. — 2
I found this useful: [The gamma distribution - an introduction][1] [1]: https://www.youtube.com/watch?v=J0Yzmb_PY3Y