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4. Maximum likelihood estimators

Instructions:

Let X_1, \dots, X_n be n i.i.d. random variables with pdf f_θ , where θ is an unknown parameter.

For each of the following questions, compute the likelihood function on paper and then find the maximum likelihood estimator for θ .

To encourage you to do the computations carefully rather than eliminate choices, you will only be given **1 or 2 attempts per question**.

(a)

1/1 point (graded)

Compute the likelihood function and the maximum likelihood estimator for θ for

$$f_{\theta}(x) = \tau \theta^{\tau} x^{-(\tau+1)} \mathbf{1}(x \geq \theta), \quad \theta > 0,$$

where $\tau > 0$ is a known constant.

☐ $\max X_i$
☒ $\min X_i$
☐ $\frac{1}{n} \sum X_i$
☐ $n\tau$
☐ ∞


Solution:

The likelihood function is

$$L = \tau^n \theta^{n\tau} \prod_i X_i^{-(\tau+1)} \mathbf{1}\{\min_i X_i \geq \theta\}$$

For $\theta \leq \min_i X_i$, the log-likelihood function is

$$l = n \ln \tau + n\tau \ln \theta - (\tau + 1) \sum_{i=1}^n \ln X_i$$

Take the derivative with respect to θ :

$$\frac{\partial l}{\partial \theta} = \frac{n\tau}{\theta} > 0.$$

Thus, L is an increasing function on $(0, \min_i X_i]$, and is 0 for $\theta > \min_i X_i$. Therefore,

$$\hat{\theta} = \min_i X_i$$

You have used 1 of 2 attempts

i Answers are displayed within the problem

(b)

3/3 points (graded)

Compute the likelihood function and the maximum likelihood estimator for θ for

$$f_{\theta}(x) = \sqrt{\theta} x^{\sqrt{\theta}-1} \mathbf{1}(0 < x < 1), \quad \theta > 0.$$

You will find that the maximum likelihood estimator for θ is of the form

$$\hat{\theta}^{\text{MLE}} = c_1 n^{c_2} \left(\sum_{i=1}^n \ln X_i \right)^{c_3}.$$

Enter the numbers c_1 , c_2 , c_3 below.

$c_1 =$

✓ Answer: 1

$c_2 =$

2

✓ Answer: 2

 $c_3 =$

-2

✓ Answer: -2

STANDARD NOTATION

Solution:

The likelihood function is

$$L = \theta^{n/2} \prod_i X_i^{\sqrt{\theta}-1} \mathbf{1}\{0 \leq X_i \leq 1\}.$$

The log-likelihood function is

$$l = \frac{n}{2} \ln \theta + (\sqrt{\theta} - 1) \sum_i \ln X_i.$$

Take the derivative with respect to θ and set it to 0:

$$\frac{\partial l}{\partial \theta} = \frac{n}{2\theta} + \frac{1}{2\theta^{1/2}} \sum_i \ln X_i = 0.$$

Then we get

$$\hat{\theta} = \frac{n^2}{(\sum \ln X_i)^2}.$$

Submit

You have used 2 of 3 attempts

i Answers are displayed within the problem

(c)

4/4 points (graded)

Compute the likelihood function and the maximum likelihood estimator for θ

$$f_{\theta}(x) = \theta \tau x^{\tau-1} \exp\{-\theta x^{\tau}\} \mathbf{1}(x \geq 0), \quad \theta > 0,$$

where $\tau > 0$ is a known constant.You will find that the maximum likelihood estimator for θ is of the form

$$\hat{\theta}^{\text{MLE}} = c_1 n^{c_2} \left(\sum_{i=1}^n X_i^{c_3} \right)^{c_4}.$$

Enter the c_1 , c_2 , c_3 , c_4 in terms of τ if applicable.(Enter **tau** for τ .)

$c_1 =$ ✓ Answer: 1

$c_2 =$ ✓ Answer: 1

$c_3 =$

tau

✓ Answer: tau

 τ $c_4 =$

-1

✓ Answer: -1

-1

STANDARD NOTATION

Solution:

The likelihood function is

$$L = \theta^n \tau^n \prod_i X_i^{\tau-1} \exp\{-\theta \sum_i X_i^\tau\} \mathbf{1}\{X_i \geq 0\}.$$

The log-likelihood function is

$$l = n \ln \theta + n \ln \tau + (\tau - 1) \sum_i \ln X_i - \theta \sum_i X_i^\tau.$$

Take the derivative with respect to θ and set it to 0

$$\frac{\partial l}{\partial \theta} = \frac{n}{\theta} - \sum_i X_i^\tau = 0,$$

we get

$$\hat{\theta} = \frac{n}{\sum_i X_i^\tau}.$$

Submit

You have used 1 of 3 attempts

i Answers are displayed within the problem

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Maximum likelihood estimators

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Hint: you can't simply sum up the X_i's

discussion posted 7 days ago by [yiuming_h](#)

I totally missed that we were working with random variables when doing the likelihood calculations, and kept getting the wrong answers. Thank you for this exercise, I think I understand likelihood a lot better now as a result

This post is visible to everyone.



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