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3. Formalizing the problem

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Lecture due Sep 13, 2021 20:30 IST Completed

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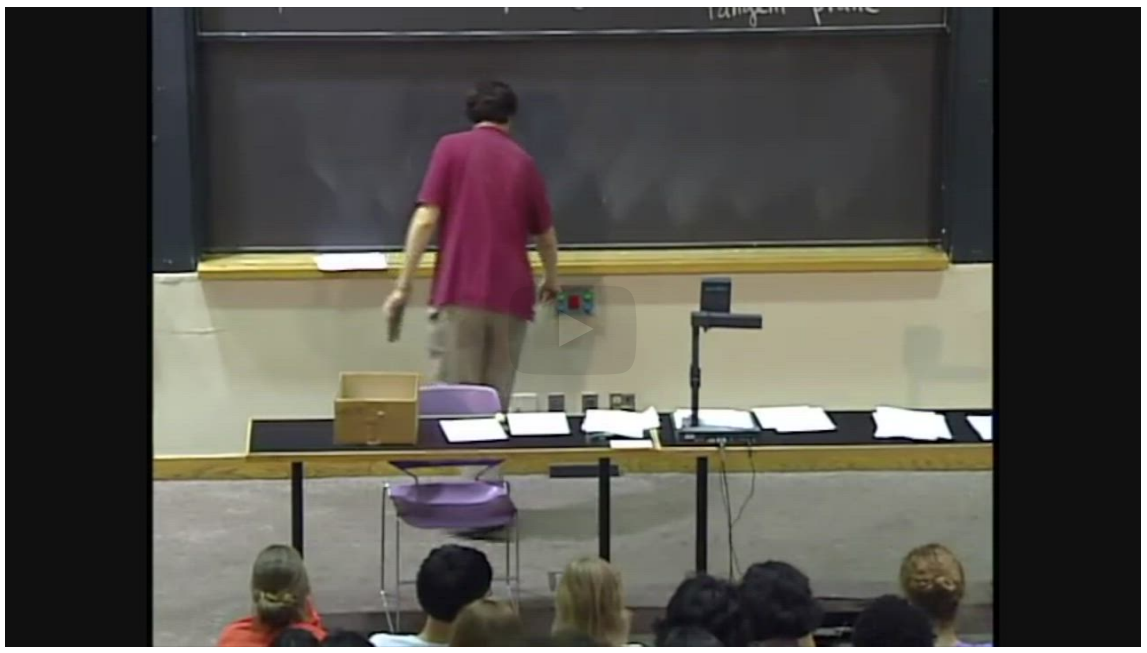
Identifying the unknowns



Start of transcript. Skip to the end.

PROFESSOR: One tricky thing here that I

want to draw your attention to is what are the unknowns here?

So the natural answer would be to say, well,

the unknowns are x and y.

That's not actually the case, OK?

0:00 / 0:00 2.0x

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Notice that the points (x_i, y_i) are known, because these are the discrete values in our given experimental data. The unknowns are the values a and b that allow us to describe the line that best fits the data.

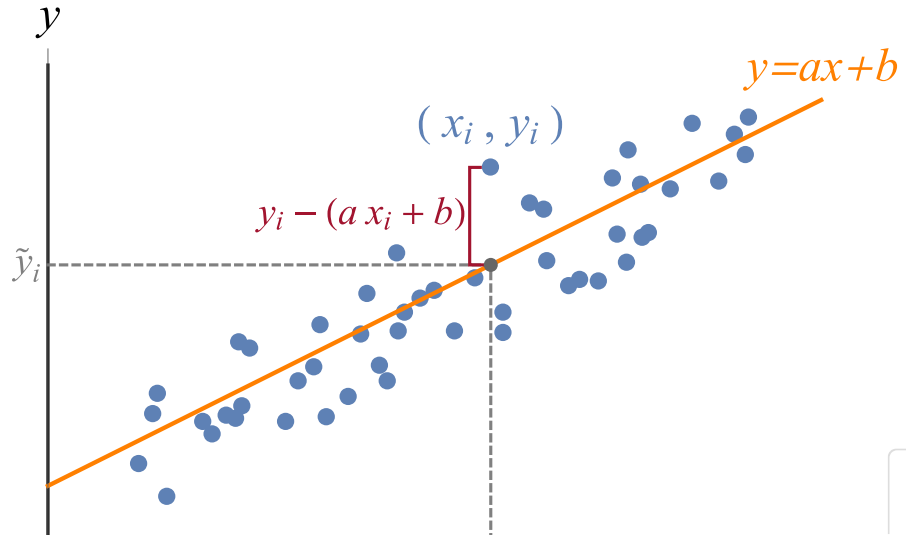
We also have to decide what we mean by the "best" line. We want to minimize a measure of the deviation between the data points and the line $y = ax + b$. The height of the data point at x_i is given by y_i. The height of the line $y = ax + b$ at the point x_i is given by

$$\tilde{y}_i = ax_i + b.$$

So the deviation between the height y_i and the corresponding value on the line $y = ax + b$ is given by

$$y_i - \tilde{y}_i = y_i - (ax_i + b)\,. \tag{4.196}$$

An illustration of this is shown in the figure below.



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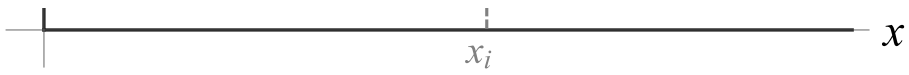

Figure 18: The deviation in y-value between the data point (x_i, y_i) is the difference between the value y_i and the value $\tilde{y}_i$. The value $\tilde{y}_i$ is the height along the line $y = ax + b$ at the point x_i.

As we saw in the video, we will minimize the total square deviation, which is given by the quantity

$$D(a,b) = \sum_{i=1}^{n} [y_i - (ax_i + b)]^2. \tag{4.197}$$

In order to minimize this quantity, which is a function of a and b, we need to find its critical points. To do this, we must first compute the partial derivatives of D with respect to a and b.

Understanding the formula for the deviation

0/1 point (graded)

Find the deviation function as a function of a and b for the following data points.

$(0,0)$, $(1,0)$, $(2,2)$.

$D(a,b) =$ 5*a^2+3*b^2-6*a*b-8*a+4*b ✖ **Answer:** b^2 + (a+b)^2 + (2-2*a-b)^2

$5 \cdot a^2 + 3 \cdot b^2 - 6 \cdot a \cdot b - 8 \cdot a + 4 \cdot b + 4$

Solution:

Plug in the data points into the formula for the deviation we get

$$\begin{aligned} D(a,b) &= \sum_{i=1}^{3} (y_i - ax_i - b)^2 && (4.198)\\ &= (0-0-b)^2 + (0-a-b)^2 + (2-2a-b)^2 && (4.199)\\ &= b^2 + (a+b)^2 + (2-2a-b)^2 && (4.200) \end{aligned}$$

It helps to plug in points to see that every term in this function is the square of a number. And that as a function, it really is a function of a and b, not x_i and y_i.

Submit You have used 3 of 7 attempts

Answers are displayed within the problem

Understanding the deviation function

1/1 point (graded)

The function $D(a,b)$ is a sum of squares. That means that $D(a,b) \geq 0$.

What must be true for this function $D(a,b) = 0$?

- ○ The data are on a horizontal line.
- ○ The data lie along a line. ✔
- ◉ This can never happen.

✔

Solution:

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The only way for the deviation to be zero, is if every square term is zero. That is you can find numbers a and b so that $y = ax + b$ and the data all lie along that line.

Note that this will never happen with real experimental data as real data has measurement error and linear models are always an approximation. In practice, this means that the function $D(a, b) > 0$.

Submit You have used 1 of 2 attempts

Answers are displayed within the problem

Understanding the deviation function

Recall that the function $D(a, b) \geq 0$ is a sum of squares.

Suppose that (a, b) is a critical point of the deviation function $D(a, b)$ for a set of data.

What type of critical point must (a, b) be? (Select your answer in the poll below.)

POLL
What type of critical point is it?

RESULTS

Local minimum	**82%**
Local maximum	**8%**
Saddle	**5%**
I do not know how to think about this yet	**5%**

Submit

Results gathered from 450 respondents.

FEEDBACK
Your response has been recorded

How to verify the type of critical point.

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3. Formalizing the problem

Topic: Unit 3: Optimization / 3. Formalizing the problem

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[staff] found correct answer but was marked wrong
for the first question I found the correct answer but I was marked wrong because I expanded the whole squares and now my three tr... 4

Does anyone have any idea of how to prove using the Cauchy-Shwarz inequality?
"Try proving that this is in fact a minimum using the second derivative test and the Cauchy–Schwarz inequality." I was wondering if a... 1

[Staff]Typo in the text
In the first line below the video, *because these are the discrete values **are** our given experimental dat... 2

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? [STAFF] How to verify the type of the critical point - typo
Shouldn't the B vector consist of lowercase b components in the explanation? 2

[Staff] typo in formule 4.196
Community TA 2

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