11.5.1 The Best Low Rank Approximation

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Week 11 due Dec 22, 2023 21:12 IST

11.5.1 The Best Low Rank Approximation

In the last slide of the below video you will find

$$
\begin{aligned}
x &= (A^TA)^{-1}A^Tb \\
&= ((U\Sigma V^T)^T(U\Sigma V^T))^{-1}(U\Sigma V^T)^Tb \\
&= (V\Sigma^TU^TU\Sigma V^T)^{-1}V\Sigma^TU^Tb \\
&= (V\Sigma\Sigma V^T)^{-1}V\Sigma U^Tb \\
&= ((V^T)^{-1}(\Sigma\Sigma)^{-1}V^{-1})V\Sigma U^Tb \\
&= V^T\Sigma^{-1}\Sigma^{-1}\Sigma U^Tb \\
&= V^T\Sigma^{-1}U^Tb
\end{aligned}
$$

The last two lines should be

$$
\begin{aligned}
&= V\Sigma^{-1}U^Tb \\
&= V\Sigma^{-1}\Sigma\Sigma^{-1}U^T
\end{aligned}
$$

Video

Start of transcript. Skip to the end.

Dr. Robert van de Geijn: Now we're going to touch very briefly on one

of the most important topics in linear algebra.

It's called the singular value decomposition, or SVD.

Unfortunately, this is an advanced topic that

0:00 / 0:00 2.0x

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Reading Assignment

0 points possible (ungraded)

Read Unit 11.5.1 of the notes. [LINK]

Done

Calculator

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Homework 11.5.1.2. - Probable typo on solution? 2

Homework 11.5.1.1

1/1 point (graded)

Let $B = U\Sigma V^T$ be the SVD of B, with $U \in \mathbb{R}^{m\times r}$, $\Sigma \in \mathbb{R}^{r\times r}$, and $V \in \mathbb{R}^{n\times r}$. Partition

$$U = \left(\begin{array}{c|c|c|c} u_0 & u_1 & \cdots & u_{r-1} \end{array} \right), \Sigma = \left(\begin{array}{c|c|c|c} \sigma_0 & 0 & \cdots & 0 \\ \hline 0 & \sigma_1 & \cdots & 0 \\ \hline \vdots & \vdots & \ddots & \vdots \\ \hline 0 & 0 & \cdots & \sigma_{r-1} \end{array} \right), V = \left(\begin{array}{c|c|c|c} v_0 & v_1 & \cdots & v_{r-1} \end{array} \right).$$

$$U\Sigma V^T = \sigma_0 u_0 v_0^T + \sigma_1 u_1 v_1^T + \cdots + \sigma_{r-1} u_{r-1} v_{r-1}^T.$$

Always ✔ Answer: Always

Answer: Always

$$\begin{aligned} B &= U\Sigma V^T \\ &= \underbrace{\underbrace{\left(\begin{array}{c|c|c|c} u_0 & u_1 & \cdots & u_{r-1} \end{array} \right) \left(\begin{array}{c|c|c|c} \sigma_0 & 0 & \cdots & 0 \\ \hline 0 & \sigma_1 & \cdots & 0 \\ \hline \vdots & \vdots & \ddots & \vdots \\ \hline 0 & 0 & \cdots & \sigma_{r-1} \end{array} \right)}_{\left(\begin{array}{c|c|c|c} \sigma_0 u_0 & \sigma_1 u_1 & \cdots & \sigma_{r-1} u_{r-1} \end{array} \right)} \underbrace{\left(\begin{array}{c|c|c|c} v_0 & v_1 & \cdots & v_{r-1} \end{array} \right)^T}_{\left(\begin{array}{c} v_0^T \\ \hline v_1^T \\ \hline \vdots \\ \hline v_{r-1}^T \end{array} \right)}}_{\sigma_0 u_0 v_0^T + \sigma_1 u_1 v_1^T + \cdots + \sigma_{r-1} u_{r-1} v_{r-1}^T.} \end{aligned}$$

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Answers are displayed within the problem

Homework 11.5.1.2

2/2 points (graded)

Let $B = U\Sigma V^T$ be the SVD of B with $U \in \mathbb{R}^{m\times r}$, $\Sigma \in \mathbb{R}^{r\times r}$, and $V \in \mathbb{R}^{n\times r}$.

- $\mathcal{C}(B) = \mathcal{C}(U)$

  Always ✔ Answer: Always

  Answer: Always

Calculator

Recall that if we can show that $\mathcal{C}(B) \subset \mathcal{C}(U)$ and $\mathcal{C}(U) \subset \mathcal{C}(B)$, then $\mathcal{C}(B) = \mathcal{C}(U)$.

$\mathcal{C}(B) \subset \mathcal{C}(U)$: Let $y \in \mathcal{C}(B)$. Then there exists a vector x such that $y = Bx$. But then $y = U \underbrace{\Sigma V^T x}_{z} = Uz$. Hence $y \in \mathcal{C}(U)$.

$\mathcal{C}(U) \subset \mathcal{C}(B)$: Let $y \in \mathcal{C}(U)$. Then there exists a vector x such that $y = Ux$. But

$$y = U \underbrace{\Sigma \overbrace{V^T V}^{I} \Sigma^{-1}}_{I} x = U\Sigma V^T \underbrace{V^T \Sigma^{-1} x}_{w} = Bw.$$ Hence $y \in \mathcal{C}(B)$.

- $\mathcal{R}(B) = \mathcal{C}(V)$

Always ✔ Answer: Always

Answer: Always
The proof is very similar, working with B^T since $\mathcal{R}(B) = \mathcal{C}(B^T)$.

Answer: Always
Recall that if we can show that $\mathcal{C}(B) \subset \mathcal{C}(U)$ and $\mathcal{C}(U) \subset \mathcal{C}(B)$, then $\mathcal{C}(B) = \mathcal{C}(U)$.

$\mathcal{C}(B) \subset \mathcal{C}(U)$: Let $y \in \mathcal{C}(B)$. Then there exists a vector x such that $y = Bx$. But then $y = U \underbrace{\Sigma V^T x}_{z} = Uz$. Hence $y \in \mathcal{C}(U)$.

$\mathcal{C}(U) \subset \mathcal{C}(B)$: Let $y \in \mathcal{C}(U)$. Then there exists a vector x such that $y = Ux$. But

$$y = U \underbrace{\Sigma \overbrace{V^T V}^{I} \Sigma^{-1}}_{I} x = U\Sigma V^T \underbrace{V^T \Sigma^{-1} x}_{w} = Bw.$$ Hence $y \in \mathcal{C}(B)$.

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