Fun with Prime Numbers (4)

Invitation to the Mysterious World of Mathematics

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The ABC conjecture

- We have seen prime numbers

  2 3 5 7 11 13 17 19 23 29 31 37 41 43⋯

  have many interesting properties.
- Prime factorization: any $N \geq 1$ can be written as a product of prime numbers.

  $$N = P_1 \times \cdots \times P_M$$

  Prime factors $P_1, \cdots, P_M$ are unique up to permutation.

The ABC conjecture (2)

- Difficult to understand how prime numbers behave in addition or subtraction.
- Twin Prime Conjecture: are there infinitely many prime numbers P,Q with Q−P=2?
- Goldbach Conjecture: is every even N≥4 written as N=P+Q for prime numbers P,Q?

The ABC conjecture (3)

- The ABC conjecture is yet another example of conjectures concerning additive properties of integers.
- Assume integers A,B,C satisfy

$$A + B = C,$$

  and A,B are relatively prime.
- The ABC conjecture concerns the size of prime factors of ABC.

The ABC conjecture (4)

- In 1980's, Oesterlé and Masser formulated a simply-looking conjecture concerning A+B=C. It is called the ABC conjecture.

Joseph Oesterlé (1954-)

Reference: https://en.wikipedia.org/wiki/Joseph_Oesterl%C3%A9

David Masser (1948-)

Reference: https://en.wikipedia.org/wiki/David_Masser

The ABC conjecture (5)

ABC Conjecture (rough form)

Let A,B,C be positive integers satisfying

- A + B = C
- A, B are relatively prime.

Then, there are 'large prime numbers' appearing in the prime factorization of ABC.

The ABC conjecture (6)

Definition

- A triple of positive integers (A,B,C) is an ABC triple if A+B=C, and A,B are relatively prime.
- Let N be the product of all distinct prime factors of ABC. It is the conductor of (A,B,C).

The ABC conjecture (7)

ABC Conjecture (precise form)
For every $\varepsilon > 0$, there exist only finitely many ABC triples (A,B,C) satisfying

$$C > N^{1+\varepsilon}.$$

- Despite its simple form, it is a very strong conjecture.
- If the ABC conjecture is true, we can prove many number theoretic problems.

The ABC conjecture (8)

- In 2012, Mochizuki released a proof of the ABC conjecture. His papers consist of several hundred pages long. Experts are checking his proof.

Shinichi Mochizuki
(1969-)

Reference: http://www.kurims.kyoto-u.ac.jp/~motizuki/top-english.html