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Problem 1: Independent uniform random variables

(3/3 points)

Let \mathbf{X} and \mathbf{Y} be independent random variables, each uniformly distributed on the interval $[0, 2]$.

1. Find the mean and variance of \mathbf{XY} .

$$\mathbf{E}[\mathbf{XY}] =$$



Answer: 1

$$\mathbf{var}[\mathbf{XY}] =$$



Answer: 0.77778

2. Find the probability that $\mathbf{XY} \geq 1$. Enter a numerical answer.

$$\mathbf{P}(\mathbf{XY} \geq 1) =$$



Answer: 0.40343

Answer:

1. \mathbf{X} and \mathbf{Y} are independent, so

$$\mathbf{E}[XY] = \mathbf{E}[X]\mathbf{E}[Y] = 1.$$

The variance is found using

$$\begin{aligned}\text{var}(XY) &= \mathbf{E}[(XY)^2] - (\mathbf{E}[XY])^2 \\ &= \mathbf{E}[X^2]\mathbf{E}[Y^2] - (\mathbf{E}[X]\mathbf{E}[Y])^2 \\ &= (\text{var}(X) + \mathbf{E}[X]^2)(\text{var}(Y) + \mathbf{E}[Y]^2) - 1 \\ &= (2^2/12 + 1)(2^2/12 + 1) - 1 \\ &= 7/9.\end{aligned}$$

2. The joint PDF of \mathbf{X} and \mathbf{Y} is given by


$$f_{X,Y}(x,y) = \begin{cases} 1/4, & \text{if } 0 \leq x, y \leq 2, \\ 0, & \text{otherwise.} \end{cases}$$

To calculate the probability of interest, we find the area over which we should integrate this joint PDF.

Consider the curve $\mathbf{XY} = 1$. Since $0 \leq \mathbf{X}, \mathbf{Y} \leq 2$, this curve is defined only for $1/2 \leq \mathbf{X} \leq 2$ and $\mathbf{Y} = 1/\mathbf{X}$. Since the event of interest is $\mathbf{XY} \geq 1$, we are looking for the area above this curve. Hence, we should integrate the joint PDF of \mathbf{X} and \mathbf{Y} over the range $x \in [1/2, 2]$ and $y \in [1/x, 2]$:

- ▶ Unit 6: Further topics on random variables
- ▶ Unit 7: Bayesian inference
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Final Exam

Final Exam due May 24, 2016 at 23:59 UTC 

$$\begin{aligned}\mathbf{P}(XY \geq 1) &= \int_{1/2}^2 \int_{1/x}^2 f_{X,Y}(x, y) \, dy \, dx \\ &= \int_{1/2}^2 \int_{1/x}^2 \frac{1}{4} \, dy \, dx \\ &= \frac{1}{4} \int_{1/2}^2 (2 - 1/x) \, dx \\ &= \frac{1}{4} (2x - \ln(x)) \Big|_{1/2}^2 \\ &= \frac{1}{4} (3 - (\ln(2) - \ln(1/2))) \\ &= \frac{1}{4} (3 - \ln(4)) \\ &\approx 0.40343.\end{aligned}$$

You have used 1 of 2 submissions

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