Homework

Homework due Aug 19, 2020 21:30 IST

The exercises below will count towards your grade. **You have only one chance to answer these questions.** Take your time, and think carefully before answering.

Problem 1

20.0/20.0 points (graded)

Answer the following questions using the coding system described in "Numberint Turing Machines", above.

Which natural number codes the Turing Machine with program $0\ \_\ \_\ l\ 0$?

113190 ✔ **Answer:** 113190

113190

Explanation

In the relevant coding system, "_" is coded as 0, and "l" is coded as 2. So our Turing Machine corresponds to the sequence $\langle 0, 0, 0, 2, 0 \rangle$, which gets coded as

$$2^{0+1} \cdot 3^{0+1} \cdot 5^{0+1} \cdot 7^{2+1} \cdot 11^{0+1} = 113190$$

What does this Turing Machine do, on an empty input?

- ○ Goes rightward on the tape forever, leaving the tape unchanged.
- ○ Goes rightward on the tape forever, changing whatever is on the tape to a string of ones.
- ◉ Goes leftward on the tape forever, leaving the tape unchanged.

- ◯ Goes leftward on the tape forever, changing whatever is on the tape to a string of ones.

✔

Explanation
When this Turing Machine runs on an empty input, it goes leftward forever, leaving the tape unchanged.

Which natural number codes the Turing Machine with program $0\ 1\ \_\ r\ 0$?

6930 ✔ **Answer:** 6930

6930

Explanation
In the relevant coding system, "_" is coded as 0, and "r" is coded as 0. So our Turing Machine corresponds to the sequence $\langle 0, 1, 0, 0, 0 \rangle$, which gets coded as

$$2^{0+1} \cdot 3^{1+1} \cdot 5^{0+1} \cdot 7^{0+1} \cdot 11^{0+1} = 6930$$

What does this Turing Machine do, on an empty input?

- ◯ It will go rightward on the tape forever.
- ◯ It will go leftward on the tape forever.
- ◉ It will halt immediately.

✔

Explanation
It will halt immediately, as soon as it reaches a cell with a blank in it, because it has no instruction telling it what to do when reading a blank in state zero.

Submit You have used 1 of 1 attempt

ℹ Answers are displayed within the problem

Problem 2

10.0/10.0 points (graded)
Here is a Turing Machine:

$$0 \_ 1 R 1$$

$$1 \_ 1 L 2$$

$$2 1 \_ R 3$$

$$3 1 \_ L 0$$

What does this Turing Machine do, when run on an empty input?

- ○ It goes rightward forever, not changing anything.
- ◉ It goes back and forth forever, changing blanks to '1's and '1's to blanks.
- ○ It goes right and then left, back and forth forever, not changing anything.

✔

Explanation
This Turing Machine starts in state 0, enters a '1' in the blank cell, goes right, and enters state 1. It now again enters a '1' in the blank cell, but then goes left. So now it is back at the cell at which it began. It changes the '1' back to a blank, and moves right, where it enters state 3. It then changes the second '1' it wrote back to a blank, goes left, and starts the whole process over again. So it goes right changing blanks into '1's, left changing '1's into blanks, over and over.

Submit You have used 1 of 1 attempt

ℹ Answers are displayed within the problem

Problem 3

20.0/20.0 points (graded)
For each Turing Machine, say whether it halts on an empty input:

$$0 \_ \_ R 0$$

- ◉ Doesn't halt

◯ Halts

✔

Explanation
The machine just goes right indefinitely, so it doesn't halt.

$$0 \_ 1 \, R \, 1$$

◯ Doesn't halt

◉ Halts

✔

Explanation
This machine writes a '1' and then halts, since it goes to state 1 but there are no instructions for state 1.

$$0 \_ 1 \, R \, 1$$

$$1 \_ 1 \, R \, 2$$

$$2 \_ 1 \, R \, 3$$

$$3 \_ 1 \, R \, 4$$

◯ Doesn’t halt

◉ Halts

✔

Explanation
This machine writes four '1's and then halts, since it goes to state 4 but there are no instructions for state 4.

$$0 \_ 1 \, R \, 1$$

$$1 \_ 1 \, R \, 2$$

$$2 \_ 1 \, R \, 3$$

$$3 \_ 1 \, R \, 0$$

- (●) Doesn't halt
- ( ) Halts

✔

Explanation
This machine writes '1's indefinitely, since after writing four '1's it goes back to state 0 and starts again. (So it functions just as the machine "0_1R0".) So, it doesn't halt.

Submit You have used 1 of 1 attempt

ℹ Answers are displayed within the problem

Problem 4

20.0/20.0 points (graded)
For each of the following descriptions, determine whether there could be a Turing Machine satisfying that description.

A Turing Machine M such that given the code of a Turing Machine M' as input, M behaves as follows:

M outputs a 1 if M' halts when run on an empty input;

M outputs a 0 if M' doesn't halt when run on an empty input.

- ( ) Yes
- (●) No

✔

Explanation
No, since such a machine would allow us to compute the halting function, which we know not to be Turing computable.

A Turing Machine M such that given the code of a Turing Machine M' as input, M behaves as follows:

M halts if M' halts when run on an empty input;

M doesn't halt if M' doesn't halt when run on an empty input.

- (●) Yes
- ( ) No

✔

Explanation
Yes. All one needs is a Universal Turing Machine. (See the exercises here.)

Submit You have used 1 of 1 attempt

ℹ Answers are displayed within the problem

Problem 5

10/10 points (graded)
Could there be a function f such that, for some distinct numbers n and m, n and m both code Turing Machines that compute f?
Again, assume the coding system described in "Numbering Turing Machines", above.

- (●) Yes
- ( ) No

✔

Explanation
Yes. One way to see this is to note that if a Turing Machine M with two or more command lines computes f, then any Turing Machine that results from changing the order of M's command lines will also compute f, and is assigned a code different from the code assigned to M.

Submit You have used 1 of 1 attempt

ℹ Answers are displayed within the problem