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Question 14 - 20

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Question 14

1/1 point (graded)

Suppose that the PDF $f_X(x)$ of a random variable X is an even function. **Note:** $f_X(x)$ is an even function if $f_X(x) = f_X(-x)$.

Is it true that the random variables X and $-X$ are identically distributed?

☒ a. True ✓

☐ b. False


Explanation

This statement is true. The proof is the following $Y = -X$, and $g^{-1}(y) = -y$. Therefore, for every y :


$$f_Y(y) = f_X(g^{-1}(y)) = \left| \frac{d}{dy} g^{-1}(y) \right| = f_X(-y) | -1 | = f_X(y)$$

▼ **Module 5: Moments of a Random Variable, Applications to Auctions, & Intro to Regression**

Moments of a Distribution and Auctions

Finger Exercises due Oct 31, 2016 at 05:00 IST 

Expectation, Variance, and an Introduction to Regression

Finger Exercises due Oct 31, 2016 at 05:00 IST 

Module 5: Homework

Homework due Oct 24, 2016 at 05:00 IST 

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You have used 1 of 1 attempts

✓ Correct (1/1 point)

Question 15

1/1 point (graded)

A couple decides to continue to have children until a daughter is born. What is the expected number of children this couple will have if the probability that a daughter is born is given by p ?

- ☒ a. The expected number of children is given by $\frac{1}{p}$ ✓
- ☐ b. The expected number of children is given by $\frac{1-p}{p}$
- ☐ c. The expected number of children is given by $\frac{p}{p^3}$
- ☐ d. The expected number of children is given by $\frac{1}{p} - 1$

Explanation

If X is the number of children until the first daughter then $P(X = k) = (1 - p)^{k-1}p$. Thus X is a geometric random variable and we have that as saw in the lecture:

$$\mathbb{E}[X] = \sum_{k=0}^{\infty} k(1-p)^{k-1}p = \frac{1}{p}$$

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Question 16

1/1 point (graded)

Which of the following statements is correct? (Select all that apply)

☒ a. If $f_X(x) = ax^{a-1}, 0 < x < 1, a > 0$ then $\mathbb{E}[X] = \frac{a}{a+1}$ ✓

☐ b. If $f_X(x) = \frac{1}{n}, x = 1, 2, \dots, n, n > 0$ an integer then $\mathbb{E}[X] = \frac{n+2}{2}$

☐ c. If $f_X(x) = \frac{3}{2}(x-1)^2, 0 < x < 2$ then $\mathbb{E}[X] = 2$

☐ d. If $f_X(x) = ax^{a-1}, 0 < x < 1, a > 0$ then $\mathbb{E}[X] = 1 + \frac{1}{a}$

☒ e. If $f_X(x) = \frac{3}{2}(x-1)^2, 0 < x < 2$ then $\mathbb{E}[X] = 1$ ✓

☐ If $f_X(x) = \frac{1}{n}, x = 1, 2, \dots, n$, for integer $n > 0$ then $\mathbb{E}[X] = \frac{n+1}{n}$



Explanation

In this case we have that:

If $f_X(x) = ax^{a-1}, 0 < x < 1, a > 0$ then:

$$\mathbb{E}[X] = \int_0^1 ax^a dx = \frac{a}{a+1} x^{a+1} \Big|_0^1 = \frac{a}{a+1}$$

If $f_X(x) = \frac{1}{n}, x = 1, 2, \dots, n$, for integer $n > 0$ then:

$$\mathbb{E}[X] = \sum_{x=1}^n \frac{1}{n} = \frac{1}{n} \sum_{x=1}^n x = \frac{1}{n} \frac{(n+1)n}{2} = \frac{n+1}{2}$$

If $f_X(x) = \frac{3}{2}(x-1)^2, 0 < x < 2$ then:

$$\mathbb{E}[X] = \int_0^2 \frac{3}{2}(x-1)^2 = \frac{1}{2}(x-1)^3 \Big|_0^2 = \frac{1}{2} + \frac{1}{2} = 1$$

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✓ Correct (1/1 point)

Question 17

1/1 point (graded)

Suppose that the random variable Y has a binomial distribution with n trials and success probability X , where n is a given constant and X is a uniform(0,1) random variable. What is $\mathbb{E}[Y]$?

☐ a. This is given by n ☒ b. This is given by $\frac{n}{2}$ ✓☐ c. This is given by $\frac{n}{3}$ ☐ d. This is given by $\frac{X}{n}$ **Explanation**

In general, we have that since Y is binomial with probability success X then $\mathbb{E}[Y|X] = nX$. Using this, and the law of iterated expectations, we have that:

$$\mathbb{E}[Y] = \mathbb{E}[\mathbb{E}[Y|X]] = \mathbb{E}[nX].$$

Since \mathbf{X} is a uniform variable between $\mathbf{0}$ and $\mathbf{1}$, then we know that $\mathbb{E}[\mathbf{nX}] = \frac{n}{2}$.

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You have used 1 of 2 attempts

✓ Correct (1/1 point)

Question 18

1/1 point (graded)

Suppose that the random variable \mathbf{Y} has a binomial distribution with \mathbf{n} trials and success probability \mathbf{X} , where \mathbf{n} is a given constant and \mathbf{X} is a uniform(0,1) random variables. What is $\mathbf{Var}(\mathbf{Y})$?

- ☐ a. This is given by $\frac{n^2}{6} + \frac{n}{12}$
- ☐ b. This is given by $\frac{n^2}{18} + \frac{n}{6}$
- ☒ c. This is given by $\frac{n^2}{12} + \frac{n}{6}$ ✓
- ☐ d. This is given by $\frac{n^2}{18} + \frac{n}{12}$

Explanation

Here we use the law of total probability. We have that:

$$\text{Var}(Y) = \text{Var}(\mathbb{E}[Y|X]) + \mathbb{E}[\text{Var}(Y|X)] = \text{Var}(nX) + \mathbb{E}[nX(1-X)] = \frac{n^2}{12} + \frac{n}{6}$$

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✓ Correct (1/1 point)

Question 19

1/1 point (graded)

Assume that $Y = \alpha + \beta X + U$, where $\beta = \frac{\rho_{XY}\sigma_Y}{\sigma_X}$ and $\alpha = \mu_Y - \beta\mu_X$. What is the expected value of U ?

- ☐ a. The expected value of U is $\alpha + \beta\mu_X$.
- ☐ b. The expected value of U is μ_Y .
- ☒ c. The expected value of U is 0 . ✓
- ☐ d. The expected value of U is α .

Explanation

In the case we have that:

$$\begin{aligned}\mathbb{E}[U] &= \mathbb{E}[Y - \alpha - \beta X] \\ &= \mu_Y - \alpha - \beta \mu_X \\ &= \mu_Y - \mu_Y + \beta \mu_X - \beta \mu_X = 0\end{aligned}$$

You have used 1 of 2 attempts

✓ Correct (1/1 point)

Question 20

1/1 point (graded)

Assume that $Y = \alpha + \beta X + U$, where $\beta = \frac{\rho_{XY}\sigma_Y}{\sigma_X}$ and $\alpha = \mu_Y - \beta\mu_X$. What is $\text{cov}(X, U)$? (Select all that apply).

☐ a. We have that $\text{cov}(X, U) = \text{var}(X)$

☐ b. We have that $\text{cov}(X, U) = \sigma_X \sigma_U$

☒ c. We have that $\text{cov}(X, U) = 0$ ✓

☐ d. We have that $\text{cov}(X, U) = \sigma_X \sigma_Y$

☒ e. We have that $\text{cov}(X, U) = \rho_{XU} \sigma_X \sigma_U$ ✓



Explanation

By definition we know that $\rho_{XU} = \frac{\text{cov}(X, U)}{\sigma_X \sigma_U}$. In this case we also have that:

$$\begin{aligned}
 \text{cov}(X, U) &= \text{cov}(X, Y - \alpha - \beta X) \\
 &= \text{cov}(X, Y) - \beta \text{var}(X) \\
 &= \text{cov}(X, Y) - \left(\frac{\rho_{XY} \sigma_Y}{\sigma_X} \right) \text{var}(X) \\
 &= \text{cov}(X, Y) - \left(\frac{\frac{\text{cov}(X, Y)}{\sigma_Y \sigma_X} \sigma_Y}{\sigma_X} \right) \text{var}(X) \\
 &= \text{cov}(X, Y) - \left(\frac{\frac{\text{cov}(X, Y)}{\sigma_X}}{\sigma_X} \right) \text{var}(X) \\
 &= \text{cov}(X, Y) - \text{cov}(X, Y) = 0
 \end{aligned}$$

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You have used 1 of 2 attempts

✓ Correct (1/1 point)

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