MITx: 6.041x Introduction to Probability - The Science of Uncertainty

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Problem 2: Functions of a standard normal

(3/3 points)

The random variable X has a standard normal distribution. Find the PDF of the random variable Y, where:

1. $Y = 3X - 1$.

- $f_Y(y) = \frac{1}{3} f_X(3(y+1))$
- $f_Y(y) = 3 f_X(3(y+1))$
- $f_Y(y) = \frac{1}{3} f_X(\frac{y+1}{3})$ ✔
- $f_Y(y) = 3 f_X(\frac{y+1}{3})$

2. $Y=3X^2-1$. For $y\geq -1$,

- $f_Y(y)=\frac{1}{6}\cdot\sqrt{\frac{3}{y+1}}f_X\left(\sqrt{\frac{y+1}{3}}\right)$
- $f_Y(y)=\frac{1}{3}\cdot\sqrt{\frac{y+1}{3}}f_X\left(\sqrt{\frac{y+1}{3}}\right)$
- $f_Y(y)=\frac{1}{3}\cdot\sqrt{\frac{3}{y+1}}f_X\left(\sqrt{\frac{y+1}{3}}\right)$ ✔
- $f_Y(y)=\frac{1}{3}\cdot\frac{y+1}{3}f_X\left(\sqrt{\frac{y+1}{3}}\right)$
- $f_Y(y)=\frac{1}{3}\cdot\frac{3}{y+1}f_X\left(\sqrt{\frac{y+1}{3}}\right)$

Answer:

1. $Y=3X-1$. We know that when $Y=aX+b$, with $a\neq 0$, we have

$$f_Y(y) = \frac{1}{|a|} f_X\left(\frac{y-b}{a}\right).$$

Therefore,

$$f_Y(y) = \frac{1}{3} f_X\left(\frac{y+1}{3}\right), \text{ for all } y.$$

Note that the fact that X is a standard normal random variable did not matter.

2. $Y = 3X^2 - 1$. We will find the CDF of Y and then differentiate to find the PDF. For $y \geq -1$, we have

$$\begin{aligned} F_Y(y) &= \mathbf{P}(Y \leq y) \\ &= \mathbf{P}\left(X^2 \leq \frac{y+1}{3}\right) \\ &= \mathbf{P}\left(-\sqrt{\frac{y+1}{3}} \leq X \leq \sqrt{\frac{y+1}{3}}\right) \\ &= F_X\left(\sqrt{\frac{y+1}{3}}\right) - F_X\left(-\sqrt{\frac{y+1}{3}}\right), \end{aligned}$$

and therefore, using the chain rule and also the fact that the standard normal PDF is symmetric about zero,

$$f_Y(y) = \frac{1}{3} \cdot \sqrt{\frac{3}{y+1}} f_X\left(\sqrt{\frac{y+1}{3}}\right), \text{ for } y \geq -1.$$

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DISCUSSION

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