The strategy: preliminaries

Here is a strategy that $P_1, P_2, \ldots$ could use to guarantee that at most finitely many people get shot.

Let us represent an assignment of hats to individuals as an ω-sequence of zeroes and ones. A zero in the kth position means that P_k's hat is red, and a one in the kth position means that P_k's hat is blue. (So, for instance, the sequence $\langle 0, 1, 1, 1, 0, \ldots \rangle$ represents a scenario in which P_1's hat is red, P_2's hat is blue, and so forth.)

Let S be the set of all ω-sequences of zeroes and ones.

We'll start by **partitioning** S. In other words, we'll divide S into a family of non-overlapping "cells", whose union is S.

Cells are defined in accordance with the following principle:

> Sequences s and s' in S are members of the same cell if and only if there are at most finitely many numbers k such that the s and s' differ in the kth position.

For instance, $\langle 0, 0, 0, 0 \ldots \rangle$ and $\langle 1, 0, 0, 0 \ldots \rangle$ are in the same cell because they differ only in the first position. But the $\langle 0, 0, 0, 0 \ldots \rangle$ and $\langle 1, 0, 1, 0 \ldots \rangle$ are in different cells because they differ in infinitely many positions.

Problem 1

3/3 points (ungraded)

For $a, b, c \in S$, determine whether or not each of the following are true:

a is in the same cell as itself.

● True

○ False

✓

Explanation

a is in the same cell as itself because there are no numbers k such that a differs from itself in the kth position.

If a is in the same cell as b, then b is in the same cell as a.

● True

○ False

✓

Explanation

Suppose a is in the same cell as b. Then, for some finite n, there are n numbers k such that a differs from b in the kth position. But difference is symmetric. So it follows that there are n numbers k such that b differs from a in the kth position, which means that b is in the same cell as a.

If a is in the same cell as b and b is in the same cell as c, then a is in the same cell as c.

● True

○ False

✓

Explanation

Suppose a is in the same cell as b and b is in the same cell as c. Then there are finite numbers n and m such that there are n numbers k such that a differs from b in the kth position and m numbers k such that b differs from c in the kth position. Since $n + m$ is finite if n and m are, this means that there are at most $n + m$ numbers k such that a differs from c in the kth position. So a is in the same cell as c.

Submit

Answers are displayed within the problem

Problem 2

1/1 point (ungraded)

How many sequences does a given cell contain?

● $|\mathbb{N}|$

○ $|\mathbb{R}|$

○ 2

○ None of the above

✓

Explanation

A cell C contains as many sequences as there are natural numbers. Here is one way to see this. Let $\langle a_1, a_2, a_3, \ldots \rangle$ be an arbitrary sequence in C (with each a_i in $\{0, 1\}$). Since, by the definition of "cell", the sequences in C differ only finitely, we can use $\langle a_1, a_2, a_3, \ldots \rangle$ to list the elements of C:

- The first element of our list is the sequence that nowhere differs from $\langle a_1, a_2, a_3, \ldots \rangle$. In other words: it is just $\langle a_1, a_2, a_3, \ldots \rangle$ itself.

- The next element of our list is the sequence that differs from $\langle a_1, a_2, a_3, \ldots \rangle$ in at most the first position (and which has not already been counted). In other words: it is the sequence $\langle (1 - a_1), a_2, a_3, \ldots \rangle$.

- The next two elements of our list are the two sequences that differ from $\langle a_1, a_2, a_3, \ldots \rangle$ in at most the first two positions (and which have not already been counted). In other words: they are the sequences $\langle (1 - a_1), (1 - a_2), a_3, \ldots \rangle$ and

$\langle a_1, (1 - a_2), a_3, \ldots \rangle$.

- The next four elements of our list are the four sequences that differ from $\langle a_1, a_2, a_3, \ldots \rangle$ in at most the first three positions (and which have not already been counted). In other words: they are the sequences $\langle (1 - a_1), (1 - a_2), (1 - a_3), a_4, \ldots \rangle$, $\langle a_1, (1 - a_2), (1 - a_3), a_4, \ldots \rangle$, $\langle (1 - a_1), a_2, (1 - a_3), a_4, \ldots \rangle$ and $\langle a_1, a_2, (1 - a_3), a_4, \ldots \rangle$.

- And so forth

We can then use this enumeration of C to define a bijection between C and the natural numbers.

Submit

Answers are displayed within the problem

Problem 3

1/1 point (ungraded)

How many cells are there?

○ $|\mathbb{N}|$

● $|\mathbb{R}|$

○ 2

○ None of the above

✓

Explanation

There are more cells than there are natural numbers. (In fact, there are just as many cells as there are real numbers.)

To see this, suppose otherwise: suppose that there are no more cells than there are natural numbers. Then there can be no more sequences than there are natural numbers. For we have just seen that each cell is countable, and we saw in Lecture 1 that the union of countably many countable sets must be countable. But we also showed in Lecture 1 that the set of sequences of zeroes and ones has as many elements as the set of real numbers. So there must be more cells than there are natural numbers.

Submit

ℹ Answers are displayed within the problem

Discussion

Hide Discussion

Topic: Week 3 / The strategy: preliminaries

Add a Post

Show all posts ⌄ by recent activity ⌄

There are no posts in this topic yet.

✖