# Problem 3: The sample mean

(5/5 points)

Let $X$ be a continuous random variable. We know that it takes values between $0$ and $3$, but we do not know its distribution or its mean and variance. We are interested in estimating the mean of $X$, which we denote by $h$. We will use $1.5$ as a conservative value (upper bound) for the standard deviation of $X$. To estimate $h$, we take $n$ i.i.d. samples $X_1, X_2, \ldots, X_n$, which all have the same distribution as $X$, and compute the sample mean

$$H = \frac{1}{n}\sum_{i=1}^{n} X_i.$$

1. Express your answers for this part in terms of $h$ and $n$ using standard notation .

$\mathbf{E}[H] =$ h

Given the available information, the smallest upper bound for $\mathbf{var}(H)$ is:

- Unit 6: Further topics on random variables
- Unit 7: Bayesian inference
- Exam 2
- **Unit 8: Limit theorems and classical statistics**

**Unit overview**

**Lec. 18: Inequalities, convergence, and the Weak Law of Large Numbers**
Exercises 18 due Apr 27, 2016 at 23:59 UTC

**Lec. 19: The Central Limit Theorem (CLT)**
Exercises 19 due Apr 27, 2016 at 23:59 UTC

**Lec. 20: An introduction to classical statistics**
Exercises 20 due Apr 27, 2016 at 23:59 UTC


2.25/n ✔

2. Calculate the smallest possible positive value of $n$ such that the standard deviation of $H$ is guaranteed to be at most $0.01$.

   22500 ✔

   This minimum value of $n$ is:

3. We would like to be at least $99\%$ sure that our estimate is within $0.05$ of the true mean $h$. Using the Chebyshev inequality, calculate the minimum value of $n$ that will achieve this.

   90000 ✔

   This minimum value of $n$ is:

4. Assume that $X$ is uniformly distributed on $[0,3]$. Using the Central Limit Theorem, identify the most appropriate expression for a $95\%$ confidence interval for $h$.

   - ○ $\left[H-\frac{1.96}{\sqrt{n}}, H+\frac{1.96}{\sqrt{n}}\right]$

Solved problems

Additional theoretical material

**Problem Set 8**
Problem Set 8 due Apr 27, 2016 at 23:59 UTC

Unit summary

Unit 9: Bernoulli and Poisson processes

$$\left[H-\frac{\sqrt{1.96\cdot 3}}{\sqrt{4n}}, H+\frac{\sqrt{1.96\cdot 3}}{\sqrt{4n}}\right]$$

$$\left[H-\frac{1.96\sqrt{3}}{\sqrt{4n}}, H+\frac{1.96\sqrt{3}}{\sqrt{4n}}\right]$$

$$\left[H-\frac{1.96\cdot 3}{\sqrt{4n}}, H+\frac{1.96\cdot 3}{\sqrt{4n}}\right]$$

*You have used 1 of 2 submissions*

## DISCUSSION

Click “Show Discussion" below to see discussions on this problem.