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Lecture due Aug 18, 2021 20:30 IST Completed



Explore

We can also find the rate of change of a function f(x,y) in the direction of a vector \vec{v} that is not a unit vector. We do this by creating a unit vector \hat{u} from \vec{v} by dividing \vec{v} by its magnitude. Doing this gives the vector

$$\hat{\boldsymbol{u}} = \frac{1}{|\vec{\boldsymbol{v}}|}\vec{\boldsymbol{v}} \tag{3.109}$$

and apply the formula for the directional derivative $\nabla f \cdot \hat{u}$. We will work through the calculations step-by-step in the following exercise.

Directional derivative exercise

Our goal will be to find the directional derivative of $f(x,y)=x-y^2-2y$ at the point (0,1/2) in the direction $ec v=\langle 1,1\rangle$.

Step 1: Find the unit vector

1.0/1 point (graded)

Given $\vec{v} = \langle 1, 1 \rangle$, find the unit vector \hat{u} that points in the direction of \vec{v} .

(Enter the vector in the form [a,b]. That is surround your vector by square brackets, and separate entries by a comma. Note that the entries of your vector must be numbers.)

$$\hat{\boldsymbol{u}} = \begin{bmatrix} 1/\text{sqrt}(2),1/\text{sqrt}(2) \end{bmatrix}$$
 \checkmark Answer: $[1/\text{sqrt}(2),1/\text{sqrt}(2)]$

Solution:

Notice that $|\vec{v}|=\sqrt{2}$, so we need to construct our unit vector that is in the same direction as \vec{v} . Dividing \vec{v} by its magnitude gives

$$\hat{u}=\langle rac{1}{\sqrt{2}},rac{1}{\sqrt{2}}
angle.$$

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You have used 1 of 5 attempts

1 Answers are displayed within the problem

Step 2: Find the gradient

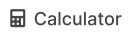
1.0/1 point (graded)

Given $f(x,y)=x-y^2-2y$, find the gradient of f at the point (0,1/2).

(Enter the vector in the form <code>[a,b]</code>. That is surround your vector by square brackets, and separate entries by a comma. Note that the entries of your vector must be numbers.)

$$abla f\left(0,1/2
ight) = egin{bmatrix} egin{bmatr$$





Solution:

We first compute

$$abla f(x,y) = \langle 1, -2y - 2 \rangle.$$

Then we evaluate at (0,1/2) to obtain

$$\nabla f(0,1/2) = \langle 1, -3 \rangle.$$

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Step 3: Find the directional derivative

1/1 point (graded)

Use the information from Steps 1 and 2 to find the directional derivative of $f(x,y)=x-y^2-2y$ at the point (0,1/2) in the direction of the vector $\vec{v}=\langle 1,1\rangle$.

Solution:

We have

$$\hat{\pmb{u}} = \langle rac{1}{\sqrt{2}}, rac{1}{\sqrt{2}}
angle$$

and

$$\nabla f(0,1/2) = \langle 1, -3 \rangle.$$

So we compute

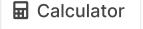
$$D_{\hat{u}}f\left(0,1/2
ight)=
abla f\left(0,1/2
ight)\cdot\langlerac{1}{\sqrt{2}},rac{1}{\sqrt{2}}
angle=rac{1}{\sqrt{2}}-rac{3}{\sqrt{2}}.$$

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1 Answers are displayed within the problem

Remark 7.1 If \vec{v} is a vector with magnitude not equal to 1, a compact way of writing the directional derivative of f in the direction of a vector \vec{v} is



$$D_{ec{v}}f\left(x,y
ight) =rac{ec{v}}{\leftert ec{v}
ightert }.$$

Practice

1/1 point (graded)

Let $g\left(x,t\right)=e^{2t}\cos\left(3x\right)$. Find the directional derivative of g at the point $\left(x,t\right)=\left(\pi,1\right)$ in the direction of the vector $ec{v}=\langle 3,-4
angle$.

Solution:

We compute the gradient

$$abla g\left(x,t
ight) = \left\langle -3\sin\left(3x
ight)e^{2t}, 2e^{2t}\cos\left(3x
ight)
ight
angle$$

and so

$$abla g\left(\pi,1
ight) =\langle 0,-2e^{2}
angle .$$

This gives

$$D_{ec{v}}g\left(\pi,1
ight) =
abla g\left(\pi,1
ight) \cdot rac{\left\langle 3,-4
ight
angle}{\left|\left\langle 3,-4
ight
angle}
ight| = \left\langle 0,-2e^2
ight
angle \cdot rac{1}{5}\langle 3,-4
ight
angle = rac{8}{5}e^2.$$

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7. Directional derivatives with non-unit vectors

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