2. Volleyball

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Practice

Suppose a volleyball is served from a height of 2 meters with a velocity v and at angle θ. The highest point the ball's center will pass through, also called the apex, will be at position $X = \frac{v^2}{40}\sin(2\theta)$ and $Y = 2 + \frac{v^2}{40}\sin^2(\theta)$. Here X and Y are the distances (horizontal and vertical respectively) in meters from the server's position.

Derivation

The ball's trajectory is given by $x(t) = vt\cos(\theta)$ and $y(t) = 2 + vt\sin(\theta) - 10t^2$. Set $y'(t) = 0$ to get the time of the apex, $t^* = \frac{v}{20}\sin(\theta)$. Then evaluate $x(t^*)$ and $y(t^*)$ (we used the double-angle formula to make $x(t^*)$ simpler).

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For example, if $v = 14.42$ meters/second and $\theta = \pi/6$, then $X \approx 4.50$ meters and $Y \approx 3.30$ meters.

Volleyball 1

1/1 point (graded)

A volleyball server is standing 4.5 meters from a volleyball net whose highest point is 2.43 meters above the ground. A volleyball has diameter ≈ 0.2 meters. Suppose the server will serve in a direction perpendicular to the net. If the server serves with velocity 14.42 meters/second and at an angle of $\pi/6$ then:

Does the volleyball go over the net?

- (•) Yes, it goes over the net
- ( ) No, it hits the net
- ( ) Cannot be determined

✔

Solution:

Yes. The apex happens at $X = 4.50$, which is the position of the net. The ball's height is 3.30, which is much more than one ball radius above the net.

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Answers are displayed within the problem

Linearization

1/1 point (graded)

Compute the linearization of the relationship $v, \theta \implies x, y$ at the point $(v, \theta) = (14.42, \pi/6)$. Round your answers to two decimal places.

The equations for X and Y are reproduced below for convenience:

$$X = \frac{v^2}{40}\sin(2\theta)$$

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$$40$$

$$Y = 2 + \frac{v^2}{40}\sin^2(\theta).$$

(Enter a matrix using notation such as `[[a,b],[c,d]]`.)

[[0.6244043, 5.19841],[0.18025,4.501955]] ✔ **Answer:** [[0.62 , 5.20],[0.18 , 4.50]]

Solution:

We have

$$X = \frac{v^2}{40}\sin(2\theta) \quad (5.154)$$

$$Y = 2 + \frac{v^2}{40}\sin^2(\theta) \quad (5.155)$$

We compute the partial derivatives:

$$\begin{pmatrix} X_v & X_\theta \\ Y_v & Y_\theta \end{pmatrix} = \begin{pmatrix} \frac{v}{20}\sin(2\theta) & \frac{v^2}{20}\cos(2\theta) \\ \frac{v}{20}\sin(\theta)^2 & \frac{v^2}{20}\cos(\theta)\sin(\theta) \end{pmatrix} \quad (5.156)$$

We evaluate at the point $(v, \theta) = (14.42, \pi/6)$ for our final answer:

$$\begin{pmatrix} 0.62 & 5.20 \\ 0.18 & 4.50 \end{pmatrix} \quad (5.157)$$

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Volleyball 2

2/2 points (graded)

Suppose we would like the ball to come closer to the net. We will keep the x-coordinate of the apex at 4.50, but we would like to lower y by 0.15 meters. How should v and θ be adjusted? Use linearization to answer. Round your answers to two decimal places.

Increase v by 0.42071197 m/s ✔ **Answer:** 0.42

Increase θ by -0.05016181 radians ✔ **Answer:** -0.05

Solution:

Solve for $\vec{u}$ in

$$\begin{pmatrix} 0.62 & 5.20 \\ 0.18 & 4.50 \end{pmatrix}\vec{u} = \begin{pmatrix} 0 \\ -0.15 \end{pmatrix} \quad (5.158)$$

We obtain

$$\vec{u} = \begin{pmatrix} 0.42 \\ -0.05 \end{pmatrix} \quad (5.159)$$

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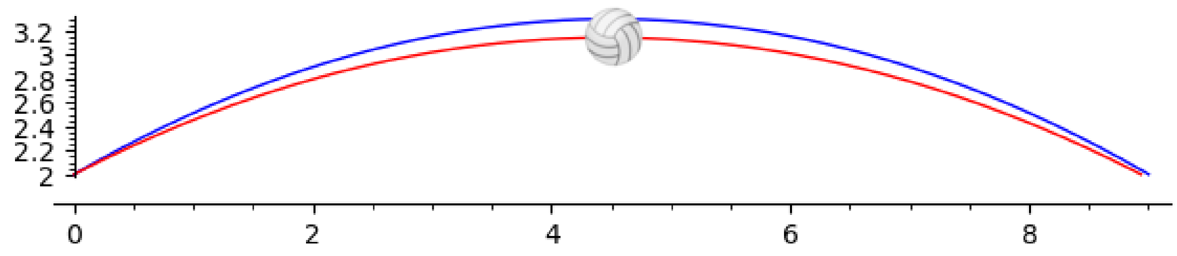
This means we should hit the ball a bit harder (increase its speed by 0.42 m/sec) and lower the angle by -0.05 radians $\approx 2.86°$.



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Volleyball 3

2/2 points (graded)
At the point $(v, \theta) = (14.42, \pi/6)$:

- Is the value of X more sensitive to changes in v or changes to θ (in absolute terms)?

  - ( ) v
  - (•) θ

  ✔

- Is the value of Y more sensitive to changes in v or changes to θ (in absolute terms)?

  - ( ) v
  - (•) θ

  ✔

Solution:

Since each row of the linearization has a larger number in the θ column, we see that X and Y are both more sensitive to θ.

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Answers are displayed within the problem

2. Volleyball

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[STAFF] Error in volleyball X and Y formulas 2

Great Volleyball problem, in reality a volleyball field is twice as long
Just a word of caution to fellow students who might know Volley Ball. Yes the problem assumes the field to ...

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