Observation Theory

Script V4PA – Non-linear Least Squares estimation

Welcome to this unit on non-linear least-squares estimation.

This course mainly focuses on linear estimation problems, by which we mean that the unknown parameters x have a linear relationship with the observables y, and this linear relationship is described by the functional model.

Together with the stochastic model this allows to obtain the best linear unbiased estimator of x.

But what now if the model is non-linear?

In this unit we will look at this particular situation.

We will start with some examples of non-linear models and have a look at their observation equations.

Then we will present the principle of non-linear least squares estimation and associated properties.

For the first example we will look at a two-dimensional positioning problem.

So in this example we are only interested in the horizontal position of a target u.

Hence, the unknowns in this case are the coordinates of point u.

Let's furthermore assume that we know the coordinates of another point r .

And finally, the observation that we will take is the distance between the two points r and u.

With this we have identified all the elements of the observation equation.

But what is the functional relationship between all these variables?

For that we need Pythagoras' law.

See the triangle in the figure..

Pythagoras tells us that the square of the observed distance is equal Delta-x squared plus Delta-y squared.

If we take the square root, this gives the observed distance on the left-hand side of the equation.

Note that Delta-x is the difference between the x-coordinates of r and u, and Delta-y the difference between the y-coordinates, so we plugged that into the equation and highlighted the unknown coordinates in red.

Having one observation and 2 unknowns does not allow to get a solution for the position of course.

Therefore we need to observe the distance to more points with known coordinates.

For instance here we added a second point s with known coordinates, and we can similarly as before, set-up the observation equation for this new distance observation.

Now we have 2 observations, 4 known coordinates, and 2 unknown coordinates.

Note that this example is in fact based on the same principle as positioning with GPS, where r and s would represent GPS satellites with known coordinates.

A big difference though, is that with GPS we are working in a 3-dimensional space, so the z-coordinate enters into the problem.

Time to look at another example, for which I will take you back to the measurements of the canal width.

So far, you haven't been looking at the measurements of one group, the Cool Guys…

They were using a so-called Total Station to measure angles.

The principle is to point the instrument at the targets of interest, and you measure the corresponding directions – this will then give you the angle between the two directions.

Let's now see what our estimation problem looks like.

The approach is to indirectly determine the canal width, by first determining the coordinates of points P and Q.

Like in the previous example we know that the width w can be calculated from those coordinates with Pythagoras' law.

The coordinates of P and Q will be determined in a local coordinate system as shown here.

The origin is one of the points, we call it R, where the cool guys set up the instrument.

The x-axis goes through the point S, which is the other point where they set up the instrument.

Let's now zoom in on one of the angle measurements that they took.

It is the angle between the directions to points P and S as measured from point R.

The angle is called alpha PRS.

How is this angle related to the unknown coordinates of point P?

In order to see that, look at the highlighted triangle here.

Then you may see that the tangent of the angle is equal to Delta-y divided by Delta-x.

These are the coordinate differences between points P and R And since R is in the origin, its coordinates are zero and we get this result.

The observation equation follows finally by taking the arctangent.

This is the non-linear observation equation for this angle measurement.

In total the cool guys took 4 angle measurements, as shown here, plus one distance measurement between points R and S.

In the exercises you will be asked to set up the observation equations for the remaining observations yourself.

Having seen these examples of non-linear observation equations, forming a non-linear functional model, we can introduce the principle non-linear least-squares estimation in the next video.