L9 PROBLEM 5 (5/5 points)

Challenge Problem! This problem is difficult and may stump you, but we include it because it is very interesting, especially for those who are more mathematically inclined.

Don't worry if you can't get all the math behind it, and don't get discouraged. Remember that you do not lose points for trying a problem multiple times, nor do you lose points if you hit "Show Answer". If this problem has you stumped after you've tried it, feel free to reveal the solution and read our explanations.

In the following examples, assume all graphs are undirected. That is, an edge from A to B is the same as an edge from B to A and counts as exactly one edge.

A **clique** is an unweighted graph where each node connects to all other nodes. We denote the clique with n nodes as **KN**. Answer the following questions in terms of n.

1. How many edges are in **KN**?

   n*(n-1)/2

   $$\frac{n \cdot (n-1)}{2}$$

   Answer: n * (n - 1)/2

   EXPLANATION:

   Answer: $n \cdot \frac{(n-1)}{2}$

   In a directed graph, each node would connect to all other nodes, yielding $n \cdot (n-1)$ edges. In our undirected graph, an edge from A to B and from B to A are the same edge, so there are, in fact, half as many.

   Alternatively - if you are familiar with the binomial coefficient - see that for each edge, you must choose two nodes to connect. Thus there are

   $$\binom{n}{2} = n \cdot \frac{(n-1)}{2}$$

   edges.

2. Consider the new version of DFS. This traverses paths until all non-circular paths from the source to the destination have been found, and returns the shortest one.

   Let A be the source node, and B be the destination in **KN**. How many paths of length 2 exist from A to B?

   n-2

   $$n-2$$

Answer: n - 2

EXPLANATION:

Answer: $n-2$

We have a source A and a destination B. Paths of length 2 contain exactly three three nodes. We must select one more node to place in the middle of our path. As we cannot select the A or B, we are left with N - 2 choices to construct a path.

3. How many paths of length 3 exist from A to B?

(n-2)*(n-3)

$(n-2)\cdot(n-3)$

Answer: (n - 2) * (n - 3)

EXPLANATION:

Answer: $(n-2)\cdot(n-3)$

Use the same reasoning as used for the previous problem. After knowing our source and destination, we must travel through 2 additional nodes without touching any node twice. For the first node, we have $n-2$ choices, and for the second, we have $n-3$ choices.

Note that this is equivalent to $\dfrac{(n-2)!}{(n-4)!}$

4. Continuing the logic used above, calculate the number of paths of length m from A to B, where $1 \leq m \leq (n-1)$, and write this number as a ratio of factorials.

To indicate a factorial, please enter `fact(n)` to mean $n!$; `fact(n+2)` to mean $(n+2)!$, etc.

The 'logic above' from the last part of the problem

Click to see the solution for the previous problem, if you want some guidence on how to think about this problem part

Answer: $(n-2)\cdot(n-3)$

Use the same reasoning as used for the previous problem. After knowing our source and destination, we must travel through 2 additional nodes without touching any node twice. For the first node, we have $n-2$ choices, and for the second, we have $n-3$ choices.

Note that this is equivalent to $\dfrac{(n-2)!}{(n-4)!}$

fact(n-2)/fact(n-m-1)

$\dfrac{\text{fact}(n-2)}{\text{fact}(n-m-1)}$

Answer: fact(n - 2)/fact(n - m - 1)

EXPLANATION:

Answer: $\dfrac{(n-2)!}{(n-m-1)!}$

Following the previous problems, it is clear that in choosing our first node between A and B, we have $(n-2)$ choices. Similarly, in choosing the second, we have $(n-3)$ choices.

In fact, in choosing the *j*th node, we have $(n-j-1)$ choices. Taking the product from *j* = 1 to *m* - 1 (since *m - 1* nodes exist between A and B in a path of length *m*), we get $\dfrac{(n-2)!}{(n-m-1)!}$

5. Using the fact that for any *n*, $\frac{1}{0!}+\frac{1}{1!}+\frac{1}{2!}+\ldots+\frac{1}{n!}\leq e \text{ for all } n$ where e is some constant, determine the asymptotic bound on the number of paths explored by DFS. For simplicity, write $O(n)$ as just n, $O\left(n^2\right)$ as n^2, etc.

fact(n-2)

$\text{fact}(n-2)$

Answer: fact(n - 2)

EXPLANATION:

Answer: $O((n-2)!)$

Note that DFS will traverse every path from A to B. To calculate the number of paths, we must sum the paths of every length (from 1 to $n-1$). This sum can be written as:

$$\frac{(n-2)!}{(n-2)!}+\frac{(n-2)!}{(n-3)!}+\frac{(n-2)!}{(n-4)!}+\ldots+\frac{(n-2)!}{0!}$$

This is equal to $(n-2)!\cdot\left(\frac{1}{0!}+\frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+\ldots+\frac{1}{(n-2)!}\right)$.

Since $\left(\frac{1}{0!}+\frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+\ldots+\frac{1}{(n-2)!}\right)\leq e$ which is a constant, the number of paths is $O((n-2)!)$.

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