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8. The One-Dimensional Delta  
> Method

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## 8. The One-Dimensional Delta Method

### Applying Linear Functions to a Random Sequence

3/3 points (graded)

Let  $(Z_n)_{n \geq 1}$  be a sequence of random variables such that

$$\sqrt{n}(Z_n - \theta) \xrightarrow[n \rightarrow \infty]{(d)} Z$$

for some  $\theta \in \mathbb{R}$  and some random variable  $Z$ .

Let  $g(x) = 5x$  and define another sequence by  $Y_n = g(Z_n)$ .

The sequence  $\sqrt{n}(Y_n - g(\theta))$  converges. In terms of  $Z$ , what random variable does it converge to?

$$\sqrt{n}(Y_n - g(\theta)) \xrightarrow[n \rightarrow \infty]{(d)} Y.$$

(Answer in terms of  $Z$ )

$Y =$

5\*Z

✓ Answer: 5\*Z

5 · Z

What theorem did we invoke to compute  $Y$ ?  
(There can be more than 1 acceptable answers.)

☐ Laws of large number

☐ Central Limit theorem

☒ Slutsky theorem ✓

☒ Continuous mapping theorem

✓

If  $\text{Var}(Z) = \sigma^2$ , what is  $\text{Var}(Y)$ ? This is the asymptotic variance of  $(Y_n)_{n \geq 1}$ .  
(Answer in terms of  $\sigma^2$ .)

$\text{Var}(Y) =$

25\*sigma^2

✓ Answer: 25\*sigma^2

25 ·  $\sigma^2$

STANDARD NOTATION

**Solution:**

1.

$$\begin{aligned}\sqrt{n}(Y_n - g(\theta)) &= \sqrt{n}(g(Z_n) - g(\theta)) = \sqrt{n}(5Z_n - 5\theta) \\ &= 5(\sqrt{n}(Z_n - \theta)) \xrightarrow[n \rightarrow \infty]{(d)} 5Z\end{aligned}$$

by the continuous mapping theorem because  $5(\sqrt{n}(Z_n - \theta))$  is a linear and hence continuous function of  $Z_n$  in the last step.

Alternatively, since we were given that  $\sqrt{n}(Z_n - \theta) \xrightarrow[n \rightarrow \infty]{(d)} Z$ , and 5 converges trivially in probability to itself, we can also use Slutsky theorem to conclude.

2. Since  $Y = 5Z$ ,  $\text{Var}(Y) = 25\text{Var}(Z) = 25\sigma^2$ .

Submit

You have used 1 of 2 attempts

**i** Answers are displayed within the problem

**Video note:** In the video below, there is an important misprint at roughly 1:26, which will be corrected in the video on the next page. The Central limit theorem applied to  $\bar{T}_n$  should read

$$\sqrt{n}\left(\bar{T}_n - \frac{1}{\lambda}\right) \xrightarrow[n \rightarrow \infty]{(d)} \mathcal{N}\left(0, \frac{1}{\lambda^2}\right).$$

## the Delta Method

The problem is that this is not something of the form estimator of  $\lambda$  minus  $\lambda$ .

The video shows a lecture on blackboards. The left blackboard contains a normal distribution curve with a shaded area under the curve, and a diagram showing a confidence interval for a sample mean. The right blackboard shows the derivation of the probability of a sum of exponential variables exceeding a threshold, leading to a normal distribution approximation.

Left blackboard content:

$$\lim_{n \rightarrow \infty} P(\bar{R}_n \in [\bar{R}_n \pm \frac{1}{\sqrt{n}}]) \geq 1 - \alpha$$

$$P[\bar{R}_n \in [0.34, 0.57]] \geq 1 - \alpha$$

$$\bar{R}_n \approx 0.45$$

$$\text{pdf of } E_p(\lambda) \quad f_\lambda(t) = \lambda e^{-\lambda t}, t > 0$$

Right blackboard content:

$$P(T > t+s | T > t) = P(T > t+s, T > t) / P(T > t)$$

$$= \frac{P(T > t+s)}{P(T > t)} = \frac{e^{-\lambda(t+s)}}{e^{-\lambda t}} = e^{-\lambda s} = P(T > s)$$

$$P(T > t) = \lambda \int_t^\infty e^{-\lambda x} dx$$

$$= \frac{\lambda}{-\lambda} e^{-\lambda x} \Big|_t^\infty = e^{-\lambda t}$$

$$\text{Var}[T_i] = \frac{1}{\lambda^2}$$

Right?

What I would want to see is something that looks like square root of n one over T n bar, which is actually my lambda hat, minus lambda converges to some Gaussian as n goes to infinity in distribution.

Maybe zero and some sigma squared here.

Right?

That's what I want to see.

Because once I know how to do this, then I can start unpacking my confidence intervals



## Video

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## Transcripts

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## (Optional) Proof of the Delta Method

[Show](#)

## Discussion


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 [Delta method](#)

2

[Hello, Does anyone know as to why this is called the delta method? Thanks](#) [Error in video at 1:31](#)

2

 [There is a bug in question 2](#)

2

[The question 2 is a single-select question, but may accepts many answers as valid.](#)[Learn About Verified Certificates](#)

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