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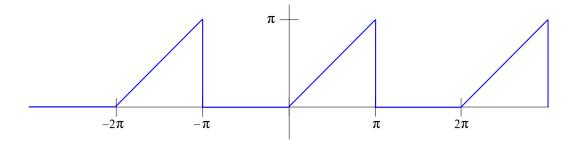
11. A worked example

Find the Fourier series of the 2π -periodic function

$$f\left(t
ight) = \left\{ egin{array}{ll} t & 0 < t < \pi \ 0 & -\pi < t < 0 \end{array}
ight..$$

Solution: In this example, we will work out the terms a_n and leave it to you to find the coefficients b_n .

First, draw a picture.



Next, find the constant term:

$$rac{a_0}{2} = rac{1}{2\pi} \int_{-\pi}^{\pi} f\left(t
ight) \, dt = rac{ ext{area under one period of curve}}{ ext{length of period}} = rac{\pi^2/2}{2\pi} = rac{\pi}{4}.$$

Now for the other terms a_n :

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(t) \cos(nt) dt$$

$$= \frac{1}{\pi} \int_{-\pi}^{0} 0 \cos(nt) dt + \frac{1}{\pi} \int_{0}^{\pi} t \cos(nt) dt$$

$$= \frac{1}{\pi} \int_{0}^{\pi} t \cos(nt) dt$$

Evaluating this integral, we find

$$rac{1}{\pi} \int_0^\pi t \cos{(nt)} \; dt = rac{1}{\pi} rac{\cos{(n\pi)} - 1}{n^2} = rac{1}{\pi} rac{(-1)^n - 1}{n^2} = egin{cases} rac{-2}{\pi n^2} & n ext{ odd} \ 0 & n ext{ even} \end{cases}.$$

Therefore the Fourier series is given by

$$f\left(t
ight) = rac{\pi}{4} + \sum_{n \, ext{odd}} rac{-2}{\pi n^2} ext{cos}\left(nt
ight) + \sum_n b_n \, ext{sin}\left(nt
ight).$$

We leave the computation of the b_n as an exercise for you.

Finish the example

1/1 point (graded)

Find the coefficients b_n of the Fourier series for the 2π -periodic function

$$f(t) = egin{cases} t & 0 < t < \pi \ 0 & -\pi < t < 0 \end{cases}.$$

(Find a formula that holds for both n even and n odd.)

Solution:

We can use computer assistance or integration by parts to see that

$$b_n = rac{1}{\pi} \int_0^{\pi} t \sin{(nt)} \; dt = rac{-\cos{(n\pi)}}{n} = rac{(-1)^{n+1}}{n}.$$

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