9. Applying the Delta Method

Applying the Delta Method

Start of transcript. Skip to the end.

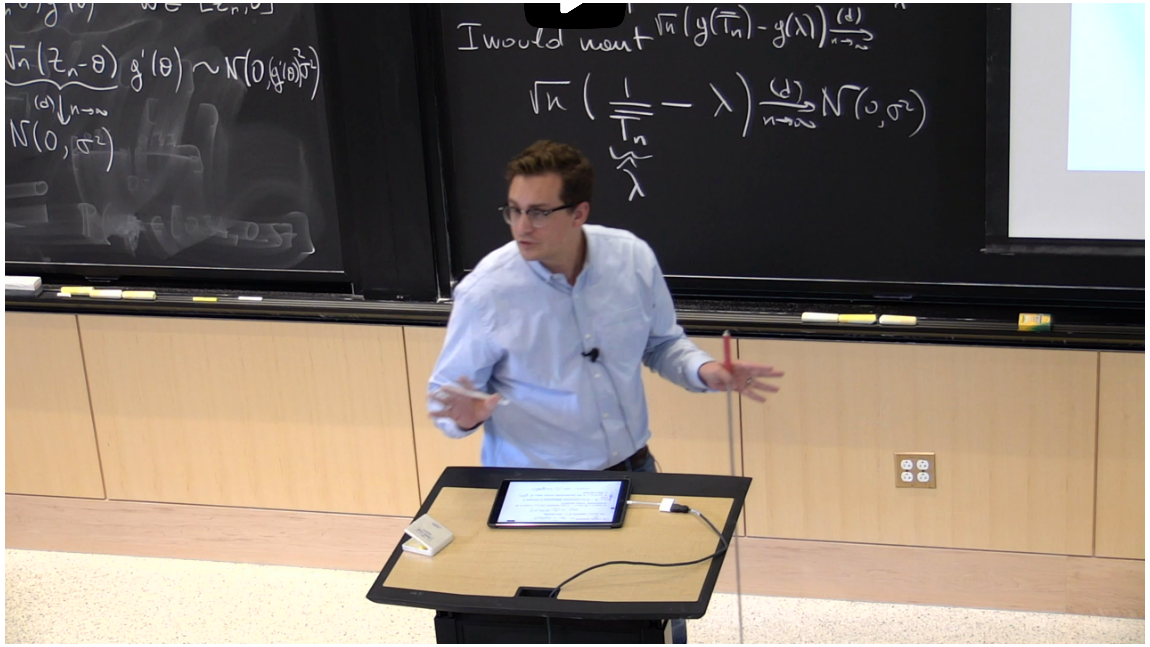
Everybody knows where this is coming from?

I didn't drop it on you like unprepared?

So this is where the correction comes from.

When you apply the delta method, be very careful.

In our example, in the T example, what was theta?



Was it lambda?

Was it 1/lambda?

0:00 / 0:00 1.50x

Video

Download video file

Transcripts

Download SubRip (.srt) file

Download Text (.txt) file

An Estimator for the Mean of an Exponential Random Variable

1/1 point (graded)

In the next two problems, we will repeat the computation in lecture.

Let $X_1, \ldots, X_n \sim \exp(\lambda)$ where $\lambda > 0$.

Since $\mathbb{E}[X] = \dfrac{1}{\lambda}$, by the central limit theorem,

$$\sqrt{n}\left(\frac{1}{n}\sum_{i=1}^{n} X_i - \frac{1}{\lambda}\right) \xrightarrow[n\to\infty]{(d)} N\left(0, \sigma^2\right).$$

What is σ^2 in terms of λ?

$\sigma^2 =$ 1/lambda^2

$\frac{1}{\lambda^2}$

STANDARD NOTATION

Submit You have used 1 of 3 attempts

Applying the Delta Method to an Exponential Random Variable

1/1 point (graded)

As above, let $X_1, \ldots, X_n \sim \exp(\lambda)$ where $\lambda > 0$. Let $\overline{X}_n = \frac{1}{n}\sum_{i=1}^{n} X_i$ denote the sample mean. By the CLT, we know that

$$\sqrt{n}\left(\overline{X}_n - \frac{1}{\lambda}\right) \xrightarrow[n\to\infty]{(d)} N\left(0, \sigma^2\right)$$

for some value of σ^2 that depends on λ, which you computed in the problem above.

If we set g to be

$$\begin{aligned} g:\ & \mathbb{R} \to \mathbb{R} \\ & x \mapsto 1/x, \end{aligned}$$

then by the Delta method,

$$\sqrt{n}\left(g\left(\overline{X}_n\right)-g\left(\frac{1}{\lambda}\right)\right)\xrightarrow[n\to\infty]{(d)} N\left(0,\tau^2\right).$$

where τ^2 is the asymptotic variance and can be expressed in terms of λ.

What is the asymptotic variance τ^2 in terms of λ?
(Choose all that apply.)

- [ ] $g'\left(\lambda\right)\text{Var}X$
- [ ] $g'\left(\lambda\right)\frac{1}{\lambda^2}$
- [x] $g'(E\left[X\right])^2\text{Var}X$
- [x] $g'\left(\frac{1}{\lambda}\right)^2\frac{1}{\lambda^2}$
- [ ] $\dfrac{1}{\lambda^2}$
- [x] λ^2

Submit

You have used 1 of 3 attempts

When does the delta method apply?

1/1 point (graded)

Let $X_1, X_2, \ldots \overset{\text{i.i.d.}}{\sim} X$. The distribution of X depends on a **positive** parameter θ, which is a function of the mean μ, i.e $\theta = g(\mu)$. You estimate θ by the estimator $\hat{\theta} = g(\overline{X}_n)$.

For which function g can the delta method be applied? Remember that $\theta > 0$.
(Choose all that apply.)

- [x] $g(x) = x^3$
- [x] $g(x) = \sqrt{x}$
- [x] $g(x) = \ln(x)$
- [ ] $g(x) = \begin{cases} x & \text{if } x \leq 1 \\ 2x-1 & \text{if } x > 1 \end{cases}$
- [x] $g(x) = \frac{1}{x-1}$

✔

Solution:

For the Delta method to apply, g' exists and is continuous at $\mathbb{E}[X] = g^{-1}(\theta)$. Since θ and $\mu = \mathbb{E}[X]$ are unknown, for the Delta method to apply, we need to make sure g is continuously differentiable at all possible values of $\mathbb{E}[X]$ given that $\theta > 0$. Let us first go through the correct choices:

1. $g(x) = x^3$ is continuously differentiable everywhere.

2. $g(x) = \sqrt{x}$ is continously differentiable for all $x > 0$. Given any $\theta > 0$, $\mu = g^{-1}(\theta) = \theta^2 > 0$. So for all possible values of $\mathbb{E}[X]$, g satisfies the requirement; hence Delta method applies.

3. Similarly, $g(x) = \ln x$ is continously differentiable for all $x > 0$. Given any $\theta > 0$, $\mu = g^{-1}(\theta) = e^{\theta} > 0$. Again, Delta method applies.

4. $g(x) = \frac{1}{x-1}$ is continously differentiable everywhere except at $x = 1$. However, inverting $\theta = g(\mu) = \frac{1}{\mu-1}$ gives $\mu = \frac{1}{\theta} + 1$, so $\mu \neq 1$ for all $\theta > 0$. Hence the Delta method applies.

Here is the incorrect choice: $g(x) = \begin{cases} x & \text{if } x \leq 1 \\ 2x-1 & \text{if } x \geq 1 \end{cases}$" is a 1-to-1 piecewise linear function and is continuously differentiable everywhere except at $x = 1$. Observe that $g(1) = 1$, hence when $\theta = 1$, $\mu = 1$. There is a possible value of μ when $g'(\mu)$ does not exist, so the Delta method does not apply.

Submit You have used 2 of 2 attempts

Answers are displayed within the problem

Discussion

Hide Discussion

Topic: Unit 2 Foundation of Inference:Lecture 5: Delta Method and Confidence Intervals / 9. Applying the Delta Method

Add a Post

All Posts

When does delta method apply

discussion posted 2 days ago by cmhcheung

Considering that the mean of X can be negative, it seems all five functions can result in a negative g or error. Does it mean no function can apply?

This post is visible to everyone.

Add a Response

3 responses

nbourbon

2 days ago

one condition that is key to evaluate is that the function needs to be differentiable. You will notice that there is/are cases where the function won't meet that condition for all points

Add a comment

RitterSebastian
a day ago

That was my first thought too

Add a comment

Cool7
about 4 hours ago

We need $g'\left(x\right)$ exists around μ, and $\theta = g\left(\mu\right) > 0$. That means $g\left(x\right)$ needs to be differentiable for the range $g\left(x\right) > 0$. $g\left(x\right) <= 0$ won't happen in this setup, thus differentiable or not doesn't matter.

Add a comment

Showing all responses

Add a response:

Preview

Submit

Audit Access Expires Dec 24, 2019

You lose all access to this course, including your progress, on Dec 24, 2019.

Upgrade by Nov 4, 2019 to get unlimited access to the course as long as it exists on the site. **Upgrade now**

Learn About Verified Certificates