

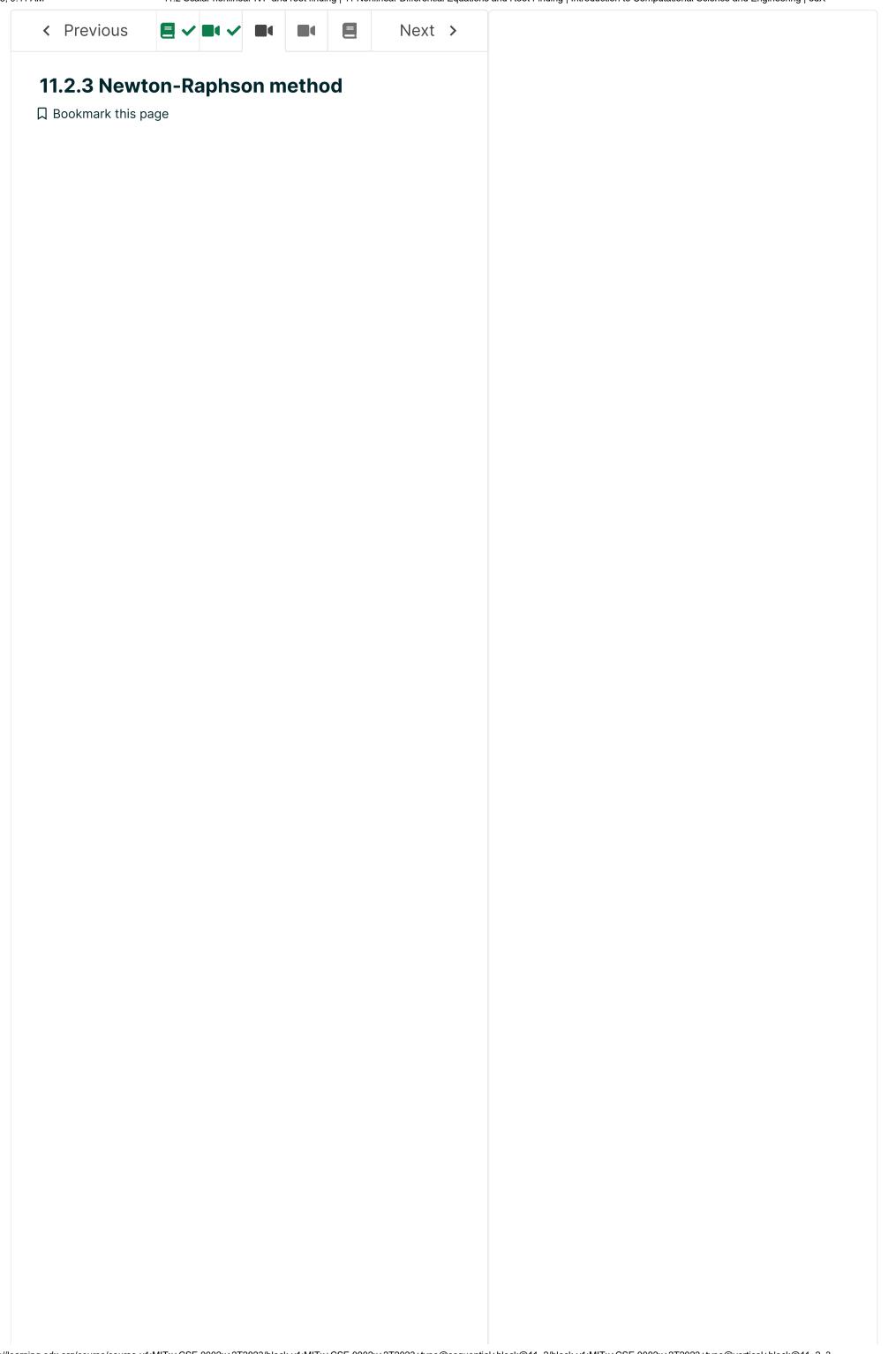
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sandipan_dey ~

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MO2.10

The Newton-Raphson method (often refered to just as Newton's method) fits a tangent line to the point $(x^k, r(x^k))$ on the graph of r, and defines x^{k+1} at the intersection of this tangent line with the x axis. The slope of this graph at the point x^k is $r'(x^k)$, and so the tangent line at $(x^k, r(x^k))$ satisfies the following equation,

$$r_{ an}\left(x
ight) = r\left(x^{k}
ight) + \left(x - x^{k}
ight)r'\left(x^{k}
ight),$$

Then, finding where this tangent intersects the x axis, defines the Newton iterate: $r_{
m tan}\left(x^{k+1}
ight)=0.$ Specifically,

$$x^{k+1} = x^k - rac{r\left(x^k
ight)}{r'\left(x^k
ight)}.$$
 (11.7)

An alternative (but equivalent) derivation is to perform a first-order Taylor series approximation of $r\left(x\right)$ about the current iterate x^{k} :

$$r\left(x^{k}+\Delta x
ight)pprox r\left(x^{k}
ight)+r'\left(x^{k}
ight)\Delta x$$

and then set this Taylor series approximation to zero to find Δx .

$$r\left(x^{k}
ight)+r'\left(x^{k}
ight)\Delta x=0\Rightarrow\Delta x=-rac{r\left(x^{k}
ight)}{r'\left(x^{k}
ight)}$$

And then update x^k by Δx :

$$x^{k+1} = x^k + \Delta x = x^k - rac{r\left(x^k
ight)}{r'\left(x^k
ight)}$$

which is the same result as Equation (11.7).

Convergence for Newton's method is very fast, when it occurs. Under some (reasonable) conditions, the error $\epsilon^k \equiv |x^*-x^k|$ obeys the following inequality:



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 $|\epsilon^{k+1}| \leq Cig(\epsilon^kig)^2,$



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Video on Newton's method and its application

Start of transcript. Skip to the end.