PART A

Three different items are to be routed through three machines. Each item must be processed first on machine 1, then on machine 2, and finally on machine 3. The sequence of items may differ for each machine. Assume that the times t_{ij} required to perform the work on item i by machine j are known and are integers. Our objective is to minimize the total time necessary to process all the items. In PARTS A to E, we will formulate the problem as an integer program. (All of the parts are put together in PART F.)

Let x_{ij} denote the time at which job i starts being processed on machine j. The objective is to minimize z, which is the first time at which all of the jobs are completed. What additional constraints are needed to ensure that z is the earliest time at which all jobs are completed? When you are ready to have the answer shown, click below.

◉ I am ready to have the answer shown ✔

SOLUTION

Job i starts being processed on machine j at time x_{ij}. Hence, job i is finished being processed at time $x_{ij} + t_{ij}$ on machine j. Then we need to make sure that z is at least as much as the time when all processing is completed on machine 3. Hence,

$z \geq \max\{x_{13} + t_{13}, x_{23} + t_{23}, x_{33} + t_{33}\}$. We express it using linear constraints as follows.

$$z \geq x_{13} + t_{13}$$
$$z \geq x_{23} + t_{23}$$
$$z \geq x_{33} + t_{33}$$

PART B

Write the constraints that ensure that jobs 1 and 2 do not overlap when scheduled on Machine 1. When you are ready to have the answer shown, click below.

HINT: you will need to use the big M method.

◉ I am ready to have the answer shown ✔

SOLUTION

We need that either jobs 2 starts after job 1 is finished or job 1 starts after job 2 is finished (on machine 1). Hence, we need that either $x_{21} \geq x_{11} + t_{11}$ or $x_{11} \geq x_{21} + t_{21}$. We add a new binary variable w and the constraints $x_{21} \geq x_{11} + t_{11} - Mw; x_{11} \geq x_{21} + t_{21} - M(1 - w)$. If $M \geq t_{11} + t_{12} + t_{13}$, then M is sufficiently large.

PART C

Write the constraints that ensure that no two of the jobs overlap when scheduled on any of the three machines. When you are ready to have the answer shown, click below.

◉ I am ready to have the answer shown ✔

SOLUTION

We write the constraints that ensure that jobs i and k do not overlap on machine j, for all $i, j, k \in \{1, 2, 3\}$. For each constraint, we introduce a new binary variable.

$$x_{kj} \geq x_{ij} + t_{ij} - M w_{ijk}, \ \forall i, j, k \in \{1, 2, 3\}$$
$$x_{ij} \geq x_{kj} + t_{kj} - M(1 - w_{ijk}), \ \forall i, j, k \in \{1, 2, 3\}$$
$$w_{ijk} \in \{0, 1\}, \ \forall i, j, k \in \{1, 2, 3\}$$

PART D

Write the constraint that ensures that the time that job 1 runs on Machine 2 is at least equal to the time that it completes on Machine 1.

◉ I am ready to have the answer shown ✔

SOLUTION

$x_{12} \geq x_{11} + t_{11}$

PART E

Write the constraints that ensure that the time that job i runs on Machine j is at least equal to the time that it completes on Machine $j-1$ for all $i = 1$ to 3 and $j = 2$ to 3.

I am ready to have the answer shown ✔

SOLUTION

$x_{ij} \geq x_{i(j-1)} + t_{i(j-1)} \quad \textbf{for } i = 1 \textbf{ to } 3 \textbf{ and } j = 2 \textbf{ to } 3.$

PART F

Are you ready to see the entire formulation?

I am ready to have the answer shown ✔

SOLUTION

$$
\begin{aligned}
\textbf{min} \quad & z \\
\textbf{s.t.:} \quad & \\
& z \geq x_{13} + t_{13} \\
& z \geq x_{23} + t_{23} \\
& z \geq x_{33} + t_{33} \\
& x_{kj} \geq x_{ij} + t_{ij} - M w_{ijk} \\
& x_{ij} \geq x_{kj} + t_{kj} - M(1 - w_{ijk}) \quad \forall i, j, k \in \{1, 2, 3\} \\
& w_{ijk} \in \{0, 1\} \quad \forall i, j, k \in \{1, 2, 3\} \\
& \\
& x_{ij} \geq x_{i(j-1)} + t_{i(j-1)} \quad \forall i \in \{1, 2, 3\}, j \in \{2, 3\}
\end{aligned}
$$