MITx: 6.041x Introduction to Probability - The Science of Uncertainty

Exercise: People in the park

(2/2 points)
Busy people arrive at the park according to a Poisson process with rate $\lambda_1 = 3$/hour and stay in the park for exactly $1/6$ of an hour. Relaxed people arrive at the park according to a Poisson process with rate $\lambda_2 = 2$/hour and stay in the park for exactly half an hour. The arrivals of busy and relaxed people are independent processes. An observer visits the park at a specific time and sees B busy and R relaxed people at the park at that moment.

For both parts below, use standard notation . If your answer involves the exponential function, use notation such as `e^(3)`.

a) Find that probability that $B = 0$. *Hint:* Think about what must have happened in the immediate past. Recall also the formula for the Poisson PMF with parameter λ:

$$\frac{\lambda^k e^{-\lambda}}{k!}, \quad \text{for } k = 0, 1, 2, \ldots.$$

$\mathbf{P}(B=0) =$ e^(-1/2)

Answer: e^(-0.5)

b) Find the probability that $B+R=1$.

$\mathbf{P}(B+R=1) =$ (3/2)*e^(-3/2)

Answer: 1.5*e^(-1.5)

Answer:
a) The busy people that the observer sees are exactly those busy people who arrived during the last (1/6)th of an hour. It is therefore a Poisson random variable with parameter $3 \cdot (1/6) = 1/2$. The desired probability is $e^{-1/2}$.

b) By the same argument, R is an independent Poisson random variable with parameter $2 \cdot (1/2) = 1$. Thus, $B+R$ is a Poisson random variable with parameter 1.5. Using the formula for the Poisson PMF,

$$\mathbf{P}(B+R=1) = 1.5e^{-1.5}.$$

You have used 2 of 2 submissions

Exercises 23 due May 11, 2016 at 23:59 UTC

Solved problems

Additional theoretical material

Problem Set 9
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Unit summary

Unit 10: Markov chains