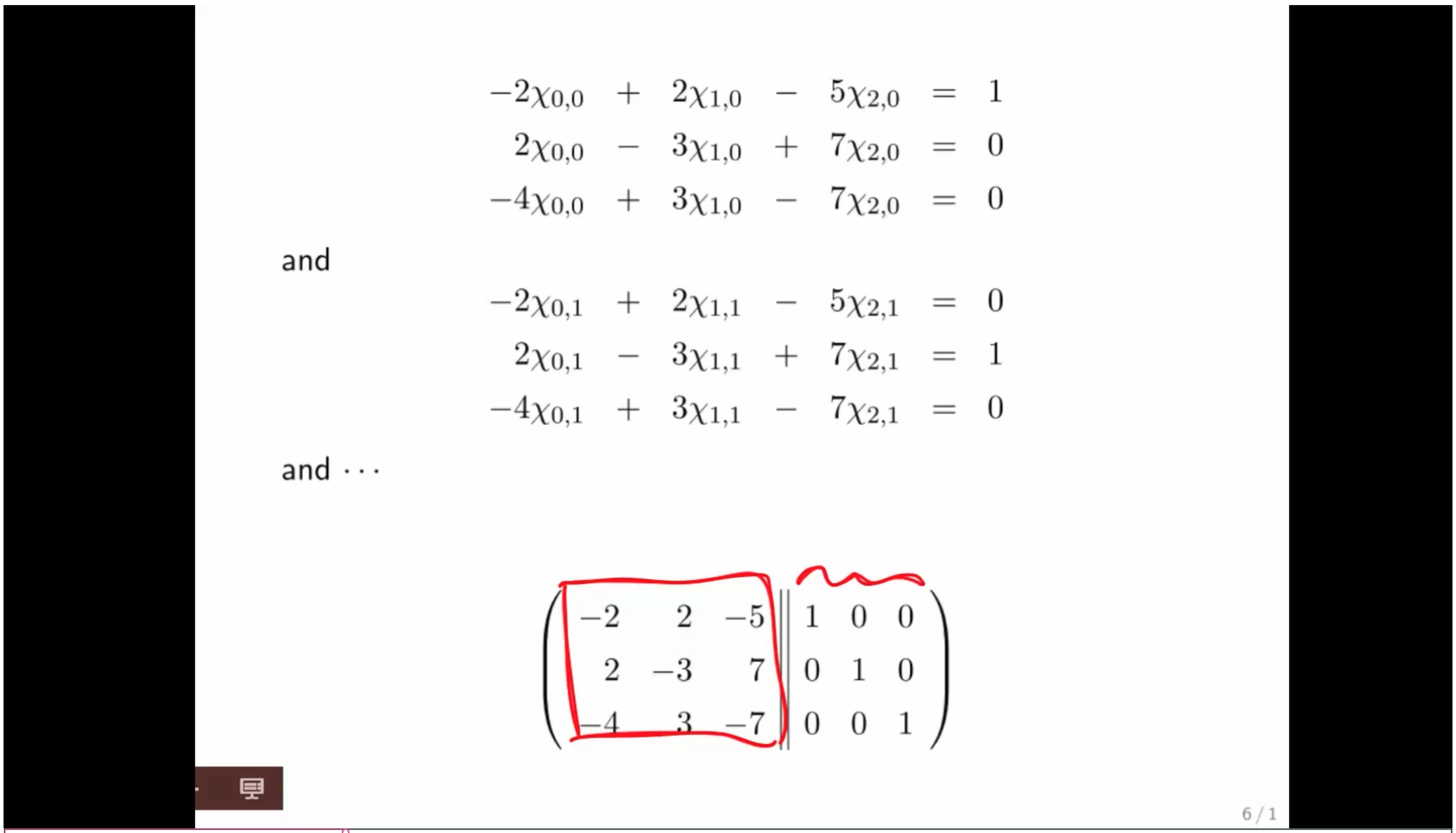
8.2.4 Computing the Inverse of A via Gauss-Jordan Elimination

Bookmark this page

Week 8 due Nov 26, 2023 15:12 IST

8.2.4 Computing the Inverse of A via Gauss-Jordan Elimination



1:16 / 5:18 2.0x

Video

Download video file

Transcripts

Download SubRip (.srt) file

Download Text (.txt) file

Reading Assignment

0 points possible (ungraded)

Read Unit 8.2.4 of the notes. [LINK]

Done

Submit

Answers are displayed within the problem

Discussion

Topic: Week 8 / 8.2.4

Hide Discussion

Calculator

Show all posts by recent activity

There are no posts in this topic yet.

✖

Homework 8.2.4.1

45/45 points (graded)

Evaluate

- $$\left(\begin{array}{r|rr} 1 & 0 & 0 \\ \hline 1 & 1 & 0 \\ -2 & 0 & 1 \end{array}\right)\left(\begin{array}{r|rr|r|rr} -2 & 2 & -5 & 1 & 0 & 0 \\ \hline 2 & -3 & 7 & 0 & 1 & 0 \\ -4 & 3 & -7 & 0 & 0 & 1 \end{array}\right) = \left(\begin{array}{r|rr|r|rr} -2 & 2 & -5 & \beta_{0,0} & \beta_{0,1} & \beta_{0,2} \\ \hline 0 & -1 & 2 & \beta_{1,0} & \beta_{1,1} & \beta_{1,2} \\ 0 & -1 & 3 & \beta_{2,0} & \beta_{2,1} & \beta_{2,2} \end{array}\right)$$

$$\left(\begin{array}{ccc} \beta_{0,0} & \beta_{0,1} & \beta_{0,2} \\ \hline \beta_{1,0} & \beta_{1,1} & \beta_{1,2} \\ \beta_{2,0} & \beta_{2,1} & \beta_{2,2} \end{array}\right) =$$

1	0	0
✔ Answer: 1	✔ Answer: 0	✔ Answer: 0
1	1	0
✔ Answer: 1	✔ Answer: 1	✔ Answer: 0
-2	0	1
✔ Answer: -2	✔ Answer: 0	✔ Answer: 1

- $$\left(\begin{array}{rr|r} 1 & 2 & 0 \\ \hline 0 & 1 & 0 \\ \hline 0 & -1 & 1 \end{array}\right)\left(\begin{array}{rr|r|rr|r} -2 & 2 & -5 & 1 & 0 & 0 \\ \hline 0 & -1 & 2 & 1 & 1 & 0 \\ \hline 0 & -1 & 3 & -2 & 0 & 1 \end{array}\right) = \left(\begin{array}{rr|r|rr|r} -2 & 0 & -1 & \beta_{0,0} & \beta_{0,1} & \beta_{0,2} \\ \hline 0 & -1 & 2 & \beta_{1,0} & \beta_{1,1} & \beta_{1,2} \\ \hline 0 & 0 & 1 & \beta_{2,0} & \beta_{2,1} & \beta_{2,2} \end{array}\right)$$

$$\left(\begin{array}{ccc} \beta_{0,0} & \beta_{0,1} & \beta_{0,2} \\ \hline \beta_{1,0} & \beta_{1,1} & \beta_{1,2} \\ \hline \beta_{2,0} & \beta_{2,1} & \beta_{2,2} \end{array}\right) =$$

3	2	0
✔ Answer: 3	✔ Answer: 2	✔ Answer: 0
1	1	0
✔ Answer: 1	✔ Answer: 1	✔ Answer: 0
-3	-1	1
✔ Answer: -3	✔ Answer: -1	✔ Answer: 1

- $$\left(\begin{array}{rrr|} 1 & 0 & 1 \\ 0 & 1 & -2 \\ \hline 0 & 0 & 1 \end{array}\right)\left(\begin{array}{rrr|rrr|} -2 & 0 & -1 & 3 & 2 & 0 \\ 0 & -1 & 2 & 1 & 1 & 0 \\ \hline 0 & 0 & 1 & -3 & -1 & 1 \end{array}\right) = \left(\begin{array}{rrr|rrr|} -2 & 0 & 0 & \beta_{0,0} & \beta_{0,1} & \beta_{0,2} \\ 0 & -1 & 0 & \beta_{1,0} & \beta_{1,1} & \beta_{1,2} \\ \hline 0 & 0 & 1 & \beta_{2,0} & \beta_{2,1} & \beta_{2,2} \end{array}\right)$$

$$\left(\begin{array}{ccc} \beta_{0,0} & \beta_{0,1} & \beta_{0,2} \\ \beta_{1,0} & \beta_{1,1} & \beta_{1,2} \\ \hline \beta_{2,0} & \beta_{2,1} & \beta_{2,2} \end{array}\right) =$$

0	1	1
✔ Answer: 0	✔ Answer: 1	✔ Answer: 1
7	3	-2
✔ Answer: 7	✔ Answer: 3	✔ Answer: -2

Calculator

-3	-1	1
Answer: -3	Answer: -1	Answer: 1

- $$\begin{pmatrix} -\frac{1}{2} & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \left(\begin{array}{ccc|ccc} -2 & 0 & 0 & 0 & 1 & 1 \\ 0 & -1 & 0 & 7 & 3 & -2 \\ 0 & 0 & 1 & -3 & -1 & 1 \end{array} \right) = \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & \beta_{0,0} & \beta_{0,1} & \beta_{0,2} \\ 0 & 1 & 0 & \beta_{1,0} & \beta_{1,1} & \beta_{1,2} \\ 0 & 0 & 1 & \beta_{2,0} & \beta_{2,1} & \beta_{2,2} \end{array} \right)$$

$$\begin{pmatrix} \beta_{0,0} & \beta_{0,1} & \beta_{0,2} \\ \beta_{1,0} & \beta_{1,1} & \beta_{1,2} \\ \beta_{2,0} & \beta_{2,1} & \beta_{2,2} \end{pmatrix} =$$

0	-1/2	-1/2
Answer: 0	Answer: -1/2	Answer: -1/2
-7	-3	2
Answer: -7	Answer: -3	Answer: 2
-3	-1	1
Answer: -3	Answer: -1	Answer: 1

- $$\begin{pmatrix} -2 & 2 & -5 \\ 2 & -3 & 7 \\ -4 & 3 & -7 \end{pmatrix} \begin{pmatrix} 0 & -\frac{1}{2} & -\frac{1}{2} \\ -7 & -3 & 2 \\ -3 & -1 & 1 \end{pmatrix} = \begin{pmatrix} \beta_{0,0} & \beta_{0,1} & \beta_{0,2} \\ \beta_{1,0} & \beta_{1,1} & \beta_{1,2} \\ \beta_{2,0} & \beta_{2,1} & \beta_{2,2} \end{pmatrix}$$

$$\begin{pmatrix} \beta_{0,0} & \beta_{0,1} & \beta_{0,2} \\ \beta_{1,0} & \beta_{1,1} & \beta_{1,2} \\ \beta_{2,0} & \beta_{2,1} & \beta_{2,2} \end{pmatrix} =$$

1	0	0
Answer: 1	Answer: 0	Answer: 0
0	1	0
Answer: 0	Answer: 1	Answer: 0
0	0	1
Answer: 0	Answer: 0	Answer: 1

Submit

Answers are displayed within the problem

Homework 8.2.4.2

1/1 point (graded)

In this exercise, you will use MATLAB to compute the inverse of a matrix using the techniques discussed in this unit.

Initialize	```A = [-2 2 -5 ; 2 -3 7 ; -4 3 -7]```

Calculator

Create an appended matrix by appending the identity	`A_appended = [ A    eye(size(A)) ]`

Create the first Gauss transform to introduce zeros in the first column (fill in the ?s).	`G0 = [`
	`1  0  0`
	`?  1  0`
	`?  0  1 ]`

Apply the Gauss transform to the appended system	`A0 = G0 * A_appended`

Create the second Gauss transform to introduce zeros in the second column	`G1 = [`
	`1  ?  0`
	`0  1  0`
	`0  ?  1 ]`

Apply the Gauss transform to the appended system	`A1 = G1 * A0`

Create the third Gauss transform to introduce zeros in the third column	`G2 = [`
	`1  0  ?`
	`0  1  ?`
	`0  0  1 ]`

Apply the Gauss transform to the appended system	`A2 = G2 * A1`

Create a diagonal matrix to set the diagonal elements to one	`D3 = [`
	`-1/2  0  0`
	`0  -1  0`
	`0   0  1 ]`

Apply the diagonal matrix to the appended system	`A3 = D3 * A2`
Extract the (updated) appended columns	`Ainv = A3( :, 4:6 )`
Check that the inverse was computed	`A * Ainv`

Calculator

The result should be a 3×3 identity matrix.

- [x] Done/Skip

Homework_8_2_4_2_Answer.m.

Submit

Answers are displayed within the problem

Homework 8.2.4.3

18/18 points (graded)
Compute

- $$\begin{pmatrix} 3 & 2 & 9 \\ -3 & -3 & -14 \\ 3 & 1 & 3 \end{pmatrix}^{-1} =$$

5/3	1	-1/3
Answer: 5/3	Answer: 1	Answer: -1/3
-11	-6	5
Answer: -11	Answer: -6	Answer: 5
2	1	-1
Answer: 2	Answer: 1	Answer: -1

- $$\begin{pmatrix} 2 & -3 & 4 \\ 2 & -2 & 3 \\ 6 & -7 & 9 \end{pmatrix}^{-1} =$$

-3/2	1/2	1/2
Answer: -3/2	Answer: 1/2	Answer: 1/2
0	3	-1
Answer: 0	Answer: 3	Answer: -1
1	2	-1
Answer: 1	Answer: 2	Answer: -1

Submit

Answers are displayed within the problem

Homework 8.2.4.4

1/1 point (graded)
Assume below that all matrices and vectors are partitioned “conformally" so that the operations make sense.

$$\left(\begin{array}{c|c|c} I & -u_{01} & 0 \\ \hline 0 & 1 & 0 \end{array}\right)\left(\begin{array}{c|c|c||c|c|c} D_{00} & a_{01} & A_{02} & B_{00} & 0 & 0 \\ \hline 0 & \alpha_{11} & a^T & b^T & 1 & 0 \end{array}\right)$$

Calculator

$$
\left(\begin{array}{c|c|c} 0 & 1 & 0 \\ \hline 0 & -l_{21} & I \end{array}\right)
\left(\begin{array}{c|c|c||c|c|c} 0 & \alpha_{11} & a_{12}^T & b_{10}^T & 1 & 0 \\ \hline 0 & a_{21} & A_{22} & B_{20} & 0 & I \end{array}\right)
$$

$$
= \left(\begin{array}{c|c|c||c|c|c} D_{00} & a_{01} - \alpha_{11} u_{01} & A_{02} - u_{01} a_{12}^T & B_{00} - u_{01} b_{10}^T & -u_{01} & 0 \\ \hline 0 & \alpha_{11} & a_{12}^T & b_{10}^T & 1 & 0 \\ \hline 0 & a_{21} - \alpha_{11} l_{21} & A_{22} - l_{21} a_{12}^T & B_{20} - l_{21} b_{10}^T & -l_{21} & I \end{array}\right)
$$

Always ✔ Answer: Always

Submit

ℹ Answers are displayed within the problem

Homework 8.2.4.5

1/1 point (graded)

Assume below that all matrices and vectors are partitioned “conformally" so that the operations make sense and that $\alpha_{11} \neq 0$.

Choose

- $u_{01} := a_{01}/\alpha_{11}$; and
- $l_{21} := a_{21}/\alpha_{11}$.

Consider the following expression:

$$
\left(\begin{array}{c|c|c} I & -u_{01} & 0 \\ \hline 0 & 1 & 0 \\ \hline 0 & -l_{21} & I \end{array}\right)
\left(\begin{array}{c|c|c||c|c|c} D_{00} & a_{01} & A_{02} & B_{00} & 0 & 0 \\ \hline 0 & \alpha_{11} & a_{12}^T & b_{10}^T & 1 & 0 \\ \hline 0 & a_{21} & A_{22} & B_{20} & 0 & I \end{array}\right)
$$

$$
= \left(\begin{array}{c|c|c||c|c|c} D_{00} & 0 & A_{02} - u_{01} a_{12}^T & B_{00} - u_{01} b_{10}^T & -u_{01} & 0 \\ \hline 0 & \alpha_{11} & a_{12}^T & b_{10}^T & 1 & 0 \\ \hline 0 & 0 & A_{22} - l_{21} a_{12}^T & B_{20} - l_{21} b_{10}^T & -l_{21} & I \end{array}\right)
$$

Always ✔ Answer: Always

Submit

ℹ Answers are displayed within the problem

The above observations justify the following two algorithms for Gauss-Jordan elimination" for inverting a matrix.

Algorithm: $[A, B] := \text{GJ\_INVERSE\_PART1}(A, B)$

Partition $A \rightarrow \left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right)$, $B \rightarrow \left(\begin{array}{c|c} B_{TL} & B_{TR} \\ \hline B_{BL} & B_{BR} \end{array}\right)$

where A_{TL} is 0×0, B_{TL} is 0×0

while $m(A_{TL}) < m(A)$ **do**

Repartition

$$
\left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c|c|c} A_{00} & a_{01} & A_{02} \\ \hline a_{10}^T & \alpha_{11} & a_{12}^T \\ \hline A_{20} & a_{21} & A_{22} \end{array}\right), \left(\begin{array}{c|c} B_{TL} & B_{TR} \\ \hline B_{BL} & B_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c|c|c} B_{00} & b_{01} & B_{02} \\ \hline b_{10}^T & \beta_{11} & b_{12}^T \\ \hline B_{20} & b_{21} & B_{22} \end{array}\right)
$$

where α_{11} is 1×1, β_{11} is 1×1

Calculator

$$
\left(\begin{array}{c|c|c} & a_{01} := a_{01}/\alpha_{11} & A_{02} := A_{02} - a_{01}a_{12}^T \\ \hline & & \\ \hline & a_{21} := a_{21}/\alpha_{11} & A_{22} := A_{22} - a_{21}a_{12}^T \end{array}\right)
\quad
\left(\begin{array}{c|c|c} B_{00} := B_{00} - a_{01}b_{10}^T & b_{01} := -a_{01} & \\ \hline & & \\ \hline B_{20} := B_{20} - a_{21}b_{10}^T & b_{21} := -a_{21} & \end{array}\right)
$$

(Note: a_{01} and a_{21} on the left need to be updated first.)

$a_{01} := 0$ (zero vector)

$a_{21} := 0$ (zero vector)

Continue with

$$\left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c|c|c} A_{00} & a_{01} & A_{02} \\ \hline a_{10}^T & \alpha_{11} & a_{12}^T \\ \hline A_{20} & a_{21} & A_{22} \end{array}\right), \left(\begin{array}{c|c} B_{TL} & B_{TR} \\ \hline B_{BL} & B_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c|c|c} B_{00} & b_{01} & B_{02} \\ \hline b_{10}^T & \beta_{11} & b_{12}^T \\ \hline B_{20} & b_{21} & B_{22} \end{array}\right)$$

endwhile

Algorithm: $[A, B] := \text{GJ\_INVERSE\_PART2}(A, B)$

Partition $A \rightarrow \left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right), B \rightarrow \left(\begin{array}{c|c} B_{TL} & B_{TR} \\ \hline B_{BL} & B_{BR} \end{array}\right)$

where A_{TL} is 0×0, B_{TL} is 0×0

while $m(A_{TL}) < m(A)$ **do**

Repartition

$$\left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c|c|c} A_{00} & a_{01} & A_{02} \\ \hline a_{10}^T & \alpha_{11} & a_{12}^T \\ \hline A_{20} & a_{21} & A_{22} \end{array}\right), \left(\begin{array}{c|c} B_{TL} & B_{TR} \\ \hline B_{BL} & B_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c|c|c} B_{00} & b_{01} & B_{02} \\ \hline b_{10}^T & \beta_{11} & b_{12}^T \\ \hline B_{20} & b_{21} & B_{22} \end{array}\right)$$

where α_{11} is 1×1, β_{11} is 1×1

$b_{10}^T := b_{10}^T/\alpha_{11}$

$\beta_{11} := \beta_{11}/\alpha_{11}$

$b_{12}^T := b_{12}^T/\alpha_{11}$

$\alpha_{11} := 1$

Continue with

$$\left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c|c|c} A_{00} & a_{01} & A_{02} \\ \hline a_{10}^T & \alpha_{11} & a_{12}^T \\ \hline A_{20} & a_{21} & A_{22} \end{array}\right), \left(\begin{array}{c|c} B_{TL} & B_{TR} \\ \hline B_{BL} & B_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c|c|c} B_{00} & b_{01} & B_{02} \\ \hline b_{10}^T & \beta_{11} & b_{12}^T \\ \hline B_{20} & b_{21} & B_{22} \end{array}\right)$$

endwhile

< Previous Next >

Calculator