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Exercise: Continuous unknown and observation

(4/4 points)

Let Θ and X be jointly continuous nonnegative random variables. A particular value x of X is observed and it turns out that $f_{\Theta|X}(\theta | x) = 2e^{-2\theta}$, for $\theta \geq 0$.

The following facts may be useful: for an exponential random variable Y with parameter λ , we have $\mathbf{E}[Y] = 1/\lambda$ and $\mathbf{var}(Y) = 1/\lambda^2$.

a) The LMS estimate (conditional expectation) of Θ is



Answer: 0.5

b) The conditional mean squared error $\mathbf{E}[(\Theta - \hat{\Theta}_{\text{LMS}})^2 | X = x]$ is



Answer: 0.25

c) The MAP estimate of Θ is



Answer: 0

d) The conditional mean squared error $\mathbf{E}[(\Theta - \hat{\Theta}_{\text{MAP}})^2 | X = x]$ is



Answer: 0.5







Answer:

a) The posterior PDF is exponential with parameter 2. The LMS estimate is the mean of this distribution, which is 1/2.

b) Since $\hat{\Theta}_{\text{LMS}}$ is the conditional mean, the mean squared error is the conditional variance, that is, the variance of an exponential random variable with parameter 2, and is equal to 1/4.

c) The posterior PDF, which is exponential, is largest at zero.

Unit overview

**Lec. 14:
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inference**Exercises 14 due Apr
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models with
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06, 2016 at 23:59 UTC **Problem Set 7a**Problem Set 7a due
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UTC **Solved problems****Additional
theoretical
material****Unit summary**

d) Since $\hat{\Theta} = 0$, the conditional mean squared error is the second moment of the exponential distribution (that is, of the form $\mathbf{E}[Y^2]$, where Y is exponential with parameter 2). Using the formula $\mathbf{E}[Y^2] = \text{var}(Y) + (\mathbf{E}[Y])^2$, we obtain

$$\mathbf{E}[Y^2] = \frac{1}{4} + \left(\frac{1}{2}\right)^2 = \frac{1}{2}.$$

Note that the LMS estimator results in a smaller mean squared error.

You have used 1 of 2 submissions

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