abc conjecture

From Wikipedia, the free encyclopedia

The ***abc* conjecture** (also known as the **Oesterlé–Masser conjecture**) is a conjecture in number theory, first proposed by Joseph Oesterlé (1988) and David Masser (1985). It is stated in terms of three positive integers, a, b and c (hence the name) that are relatively prime and satisfy $a + b = c$. If d denotes the product of the distinct prime factors of abc, the conjecture essentially states that d is usually not much smaller than c. In other words: if a and b are composed from large powers of primes, then c is usually not divisible by large powers of primes. The precise statement is given below.

The *abc* conjecture has already become well known for the number of interesting consequences it entails. Many famous conjectures and theorems in number theory would follow immediately from the *abc* conjecture. Goldfeld (1996) described the *abc* conjecture as "the most important unsolved problem in Diophantine analysis".

Several solutions have been proposed to the *abc* conjecture, the most recent of which is still being evaluated by the mathematical community.

Contents

Formulations

The abc conjecture can be expressed as follows: For every $\varepsilon > 0$, there are only finitely many triples of coprime positive integers $a + b = c$ such that $c > d^{1+\varepsilon}$, where d denotes the product of the distinct prime factors of abc.

To illustrate the terms used, if

> $a = 16 = 2^4$,
> $b = 17$, and
> $c = 16 + 17 = 33 = 3\cdot 11$,

then $d = 2\cdot 17\cdot 3\cdot 11 = 1122$, which is greater than c. Therefore, for all $\varepsilon > 0$, c is not greater than $d^{1+\varepsilon}$. According to the conjecture, most coprime triples where $a + b = c$ are like the ones used in this example, and for only a few exceptions is $c > d^{1+\varepsilon}$.

To add more terminology: For a positive integer n, the radical of n, denoted rad(n), is the product of the distinct prime factors of n. For example

- $\text{rad}(16) = \text{rad}(2^4) = 2$,
- $\text{rad}(17) = 17$,
- $\text{rad}(18) = \text{rad}(2\cdot 3^2) = 2\cdot 3 = 6$.

If a, b, and c are coprime[1] positive integers such that $a + b = c$, it turns out that "usually" $c < \text{rad}(abc)$. The *abc conjecture* deals with the exceptions. Specifically, it states that:

> **ABC Conjecture.** For every $\varepsilon > 0$, there exist only finitely many triples (a, b, c) of positive coprime integers, with $a + b = c$, such that
>
> $$c > \operatorname{rad}(abc)^{1+\varepsilon}.$$

An equivalent formulation states that:

> **ABC Conjecture II.** For every $\varepsilon > 0$, there exists a constant K_ε such that for all triples (a, b, c) of coprime positive integers, with $a + b = c$, the inequality
>
> $$c < K_\varepsilon \cdot \operatorname{rad}(abc)^{1+\varepsilon}$$
>
> holds.

A third equivalent formulation of the conjecture involves the *quality* $q(a, b, c)$ of the triple (a, b, c), defined as

$$q(a,b,c) = \frac{\log(c)}{\log(\operatorname{rad}(abc))}.$$

For example,

- $q(4, 127, 131) = \log(131) / \log(\text{rad}(4\cdot 127\cdot 131)) = \log(131) / \log(2\cdot 127\cdot 131) = 0.46820...$
- $q(3, 125, 128) = \log(128) / \log(\text{rad}(3\cdot 125\cdot 128)) = \log(128) / \log(30) = 1.426565...$

A typical triple (a, b, c) of coprime positive integers with $a + b = c$ will have $c < \mathrm{rad}(abc)$, i.e. $q(a, b, c) < 1$. Triples with $q > 1$ such as in the second example are rather special, they consist of numbers divisible by high powers of small prime numbers.

> **ABC Conjecture III.** For every $\varepsilon > 0$, there exist only finitely many triples (a, b, c) of coprime positive integers with $a + b = c$ such that $q(a, b, c) > 1 + \varepsilon$.

Whereas it is known that there are infinitely many triples (a, b, c) of coprime positive integers with $a + b = c$ such that $q(a, b, c) > 1$, the conjecture predicts that only finitely many of those have $q > 1.01$ or $q > 1.001$ or even $q > 1.0001$, etc. In particular, if the conjecture is true then there must exist a triple (a, b, c) which achieves the maximal possible quality $q(a, b, c)$.

Examples of triples with small radical

The condition that $\varepsilon > 0$ is necessary for the truth of the conjecture, as there exist infinitely many triples a, b, c with $\mathrm{rad}(abc) < c$. For instance, such a triple may be taken as

$a = 1$,
$b = 2^{6n} - 1$,
$c = 2^{6n}$.

As a and c together contribute only a factor of two to the radical, while b is divisible by 9, $\mathrm{rad}(abc) < 2c/3$ for these examples, if n > 1. This is because $\mathrm{rad}(abc) = \mathrm{rad}(a)\mathrm{rad}(b)\mathrm{rad}(c) = 2\mathrm{rad}(b)$. $b = 64^n - 1^n = (64 - 1)(...) = 3^2 \times 7 \times (...)$. So $b = 3^2r$ for some r. So $\mathrm{rad}(b) = \mathrm{rad}(3^2r) \leq 3r = b/3$. So $\mathrm{rad}(abc) = 2\mathrm{rad}(b) \leq 2b/3 < 2c/3$.

By replacing the exponent $6n$ by other exponents forcing b to have larger square factors, the ratio between the radical and c may be made arbitrarily small. Specifically, replacing $6n$ by $p(p - 1)n$ for an arbitrary prime $p > 2$ will make b divisible by p^2, because $2^{p(p-1)} \equiv 1 \pmod{p^2}$ and $2^{p(p-1)} - 1$ will be a factor of b.

A list of the highest-quality triples (triples with a particularly small radical relative to c) is given below; the highest quality, 1.6299, was found by Eric Reyssat (Lando & Zvonkin 2004, p. 137) for

$a = 2$,
$b = 3^{10}\cdot 109 = 6{,}436{,}341$,
$c = 23^5 = 6{,}436{,}343$,
$\mathrm{rad}(abc) = 15042$.

Some consequences

The *abc* conjecture has a large number of consequences. These include both known results (some of which have been proven separately since the conjecture has been stated) and conjectures for which it gives a conditional proof. While an earlier proof of the conjecture would have been more significant in terms of consequences, the *abc* conjecture itself remains of interest for the other conjectures it would prove, together with its numerous links with deep questions in number theory.

- Thue–Siegel–Roth theorem on diophantine approximation of algebraic numbers (Bombieri 1994)
- Fermat's Last Theorem for all sufficiently large exponents (already proven in general by Andrew

Wiles) (Granville & Tucker 2002)
- The Mordell conjecture (already proven in general by Gerd Faltings) (Elkies 1991)
- It is equivalent to Vojta's conjecture. (Van Frankenhuijsen 2002)
- The Erdős–Woods conjecture except for a finite number of counterexamples (Langevin 1993)
- The existence of infinitely many non-Wieferich primes in every base $b > 1$ (Silverman 1988)
- The weak form of Marshall Hall's conjecture on the separation between squares and cubes of integers (Nitaj 1996)
- The Fermat–Catalan conjecture, a generalization of Fermat's last theorem concerning powers that are sums of powers (Pomerance 2008)
- The L function $L(s,\chi_d)$ formed with the Legendre symbol, has no Siegel zero (this consequence actually requires a uniform version of the *abc* conjecture in number fields, not only the *abc* conjecture as formulated above for rational integers) (Granville & Stark 2000)
- $P(x)$ has only finitely many perfect powers for integral x for P a polynomial with at least three simple zeros.[2]
- A generalization of Tijdeman's theorem concerning the number of solutions of $y^m = x^n + k$ (Tijdeman's theorem answers the case $k = 1$), and Pillai's conjecture (1931) concerning the number of solutions of $Ay^m = Bx^n + k$.
- It is equivalent to the Granville–Langevin conjecture, that if f is a square-free binary form of degree $n > 2$, then for every real $\beta > 2$ there is a constant $C(f, \beta)$ such that for all coprime integers x,y, the radical of $f(x, y)$ exceeds $C\cdot\max\{|x|,|y|\}^{n-\beta}$.[3][4]
- It is equivalent to the modified Szpiro conjecture, which would yield a bound of $\mathrm{rad}(abc)^{1.2+\varepsilon}$ (Oesterlé 1988).
- Dąbrowski (1996) has shown that the *abc* conjecture implies that the Diophantine equation $n! + A = k^2$ has only finitely many solutions for any given integer A.
- There are $\sim c_f N$ positive integers $n \leq N$ for which $f(n)/B'$ is square-free, with $c_f > 0$ a positive constant defined as $c_f = \prod_{\text{prime } p} x_i \left(1 - \frac{\omega_f(p)}{p^{2+q_p}}\right)$. (Granville 1998)

Fermat's Last theorem

Fermat's Last Theorem was proven by Andrew Wiles, and the proof is famous for its difficulty. But if a strong effective form of the *abc* conjecture is correct, the proof of Fermat's Last theorem becomes much shorter and easier as follows:[5]
If *abc* conjecture is correct when $K = 1$ and $\varepsilon = 1$, and when the co-prime natural numbers A, B, C satisfy an equation $A + B = C$, we have $C < (\mathrm{rad}(ABC))^2$.

We assume the co-prime natural numbers a^n, b^n, c^n satisfy $a^n + b^n = c^n$, replacing A to a^n, B to b^n, C to c^n. This equation $a^n + b^n = c^n$ is the Fermat's Last theorem. Then we get:

$$c^n < (\mathrm{rad}(a^n b^n c^n))^2 = (\mathrm{rad}(abc))^2 \leq (abc)^2 < (c^3)^2 = c^6.$$

(Because $\mathrm{rad}(x^n) = \mathrm{rad}(x), \mathrm{rad}(x) \leq x, a\cdot b\cdot c < c\cdot c\cdot c$.)

Now we get:

$$c^n < c^6.$$

That is why n must be smaller than 6. But for exponents $n = 3, 4, 5$, we already have proofs, which were proved before (Fermat, Euler, Dirichlet or Legendre), so no three positive integers a, b, and c can satisfy the equation $a^n + b^n = c^n$ for any integer value of $n > 2$.[6]

In this argument one can allow ε to be smaller and *K* to be larger, at the cost of requiring explicit checks that there are no *small* solutions to the Fermat equation. These checks are possible for reasonable values of ε and *K*, but it is possible (even perhaps likely) that a proof of the *abc* conjecture will give only ineffective bounds on *K* in terms of ε. In this case the deduction yields only the weaker statement that there are finitely many solutions to the Fermat equation. This is stronger than Faltings' theorem, which states that there are finitely many solutions to the Fermat equation for each *n*.

Theoretical results

The abc conjecture implies that *c* can be bounded above by a near-linear function of the radical of *abc*. However, exponential bounds are known. Specifically, the following bounds have been proven:

$c < \exp\left(K_1 \operatorname{rad}(abc)^{15}\right)$ (Stewart & Tijdeman 1986),

$c < \exp\left(K_2 \operatorname{rad}(abc)^{\frac{2}{3}+\varepsilon}\right)$ (Stewart & Yu 1991), and

$c < \exp\left(K_3 \operatorname{rad}(abc)^{\frac{1}{3}+\varepsilon}\right)$ (Stewart & Yu 2001).

In these bounds, K_1 is a constant that does not depend on *a*, *b*, or *c*, and K_2 and K_3 are constants that depend on ε (in an effectively computable way) but not on *a*, *b*, or *c*. The bounds apply to any triple for which $c > 2$.

Computational results

In 2006, the Mathematics Department of Leiden University in the Netherlands, together with the Dutch Kennislink science institute, launched the ABC@Home project, a grid computing system, which aims to discover additional triples *a*, *b*, *c* with rad(*abc*) < *c*. Although no finite set of examples or counterexamples can resolve the *abc* conjecture, it is hoped that patterns in the triples discovered by this project will lead to insights about the conjecture and about number theory more generally.

Distribution of triples with $q > 1$[7]

	$q > 1$	$q > 1.05$	$q > 1.1$	$q > 1.2$	$q > 1.3$	$q > 1.4$
$c < 10^2$	6	4	4	2	0	0
$c < 10^3$	31	17	14	8	3	1
$c < 10^4$	120	74	50	22	8	3
$c < 10^5$	418	240	152	51	13	6
$c < 10^6$	1,268	667	379	102	29	11
$c < 10^7$	3,499	1,669	856	210	60	17
$c < 10^8$	8,987	3,869	1,801	384	98	25
$c < 10^9$	22,316	8,742	3,693	706	144	34
$c < 10^{10}$	51,677	18,233	7,035	1,159	218	51
$c < 10^{11}$	116,978	37,612	13,266	1,947	327	64
$c < 10^{12}$	252,856	73,714	23,773	3,028	455	74
$c < 10^{13}$	528,275	139,762	41,438	4,519	599	84
$c < 10^{14}$	1,075,319	258,168	70,047	6,665	769	98
$c < 10^{15}$	2,131,671	463,446	115,041	9,497	998	112
$c < 10^{16}$	4,119,410	812,499	184,727	13,118	1,232	126
$c < 10^{17}$	7,801,334	1,396,909	290,965	17,890	1,530	143
$c < 10^{18}$	14,482,065	2,352,105	449,194	24,013	1,843	160

ABC@Home had found 23.8 million triples.[8]

Highest quality triples[9]

	q	a	b	c	Discovered by
1	1.6299	2	$3^{10}\cdot 109$	23^5	Eric Reyssat
2	1.6260	11^2	$3^2\cdot 5^6\cdot 7^3$	$2^{21}\cdot 23$	Benne de Weger
3	1.6235	$19\cdot 1307$	$7\cdot 29^2\cdot 31^8$	$2^8\cdot 3^{22}\cdot 5^4$	Jerzy Browkin, Juliusz Brzezinski
4	1.5808	283	$5^{11}\cdot 13^2$	$2^8\cdot 3^8\cdot 17^3$	Jerzy Browkin, Juliusz Brzezinski, Abderrahmane Nitaj
5	1.5679	1	$2\cdot 3^7$	$5^4\cdot 7$	Benne de Weger

Note: the *quality* $q(a, b, c)$ of the triple (a, b, c) is defined above.

Refined forms, generalizations and related statements

The *abc* conjecture is an integer analogue of the Mason–Stothers theorem for polynomials.

A strengthening, proposed by Baker (1998), states that in the *abc* conjecture one can replace rad(*abc*) by

$$\varepsilon^{-\omega}\,\mathrm{rad}(abc),$$

where ω is the total number of distinct primes dividing *a*, *b* and *c* (Bombieri & Gubler 2006, p. 404).

Andrew Granville noticed that the minimum of the function $\left(\varepsilon^{-\omega}\,\mathrm{rad}(abc)\right)^{1+\varepsilon}$ over $\varepsilon > 0$ occurs when $\varepsilon = \dfrac{\omega}{\log(\mathrm{rad}(abc))}$.

This incited Baker (2004) to propose a sharper form of the *abc* conjecture, namely:

$$c < \kappa\,\mathrm{rad}(abc)\frac{(\log(\mathrm{rad}(abc)))^{\omega}}{\omega!}$$

with κ an absolute constant. After some computational experiments in order to find a value for κ, he found that a value of $\frac{6}{5}$ was admissible.

This version is called "explicit *abc* conjecture".

From the previous inequality, Baker deduced a stronger form of the original *abc* conjecture: let *a*, *b*, *c* be coprime positive integers with $a + b = c$; then we have $c < \left(\mathrm{rad}(abc)\right)^{1+\frac{3}{4}}$.

Baker (1998) also describes related conjectures of Andrew Granville that would give upper bounds on *c* of the form

$$K^{\Omega(abc)}\,\mathrm{rad}(abc),$$

where Ω(*n*) is the total number of prime factors of *n* and

$$O(\mathrm{rad}(abc)\Theta(abc)),$$

where Θ(*n*) is the number of integers up to *n* divisible only by primes dividing *n*.

Robert, Stewart & Tenenbaum (2014) proposed more precise inequality based on Robert & Tenenbaum (2013). Let $k = \mathrm{rad}(abc)$. They conjectured there is a constant C_1 such that

$$c < k\exp\left(4\sqrt{\frac{3\log k}{\log\log k}}\left(1 + \frac{\log\log\log k}{2\log\log k} + \frac{C_1}{\log\log k}\right)\right)$$

holds whereas there is a constant C_2 such that

$$c > k\exp\left(4\sqrt{\frac{3\log k}{\log\log k}}\left(1 + \frac{\log\log\log k}{2\log\log k} + \frac{C_2}{\log\log k}\right)\right)$$

holds infinitely often.

Browkin & Brzeziński (1994) formulated the n conjecture—a version of the *abc* conjecture involving $n > 2$ integers.

Attempts at solution

Lucien Szpiro attempted a solution in 2007, but it was found to be incorrect.[10]

In August 2012, Shinichi Mochizuki released a series of four preprints containing a claim to a proof of the *abc* conjecture.[11] Mochizuki calls the theory on which this proof is based "inter-universal Teichmüller theory (IUT)", and it has other applications, including a proof of Szpiro's conjecture and Vojta's conjecture.[12] Experts were expected to take months to check Mochizuki's new mathematical machinery, which was developed over decades in 500 pages of preprints and several of his prior papers.[13]

When an error in one of the articles was pointed out by Vesselin Dimitrov and Akshay Venkatesh in October 2012, Mochizuki posted a comment on his website acknowledging the mistake, stating that it would not affect the result, and promising a corrected version in the near future.[14] He revised all of his papers on "inter-universal Teichmüller theory", the latest of which is dated September 2015.[11] Mochizuki has refused all requests for media interviews, but released progress reports in December 2013[15] and December 2014.[16] According to Mochizuki, verification of the core proof is "for all practical purposes, complete." However, he also stated that an official declaration shouldn't happen until some time later in the 2010s, due to the importance of the results and new techniques. In addition, he predicts that there are no proofs of the abc conjecture that use significantly different techniques than those used in his papers.[16] There was a workshop on IUT at Kyoto University in March 2015, another one was held at Clay Mathematics Institute in December 2015 [17] and a third one will be held at the Research Institute for Mathematical Sciences in Kyoto in July 2016.[18]

> Fesenko has estimated that it would take an expert in arithmetic geometry some 500 hours to understand his work. So far, only four mathematicians say that they have been able to read the entire proof.[19]

See also

- List of unsolved problems in mathematics

Notes

1. When $a + b = c$, coprimeness of a, b, c implies pairwise coprimeness of a, b, c. So in this case, it does not matter which concept we use.
2. http://www.math.uu.nl/people/beukers/ABCpresentation.pdf
3. Mollin (2009)
4. Mollin (2010) p. 297
5. Granville, Andrew; Tucker, Thomas (2002). "It’s As Easy As abc" (http://www.ams.org/notices/200210/fea-granville.pdf). Notices of the AMS 49 (10): 1224–1231.
6. Yamazaki, Takao"Fermat's Last Theorem and ABC conjecture (http://www.math.tohoku.ac.jp/~ytakao/papers/abc.pdf)"(Japanese), Sugaku seminar Oct 2010.
7. "Synthese resultaten", *RekenMeeMetABC.nl* (in Dutch), retrieved October 3, 2012.
8. "Data collected sofar", *ABC@Home*, retrieved April 30, 2014
9. "100 unbeaten triples". *Reken mee met ABC*. 2010-11-07.
10. "Finiteness Theorems for Dynamical Systems", Lucien Szpiro, talk at Conference on L-functions and Automorphic Forms (on the occasion of Dorian Goldfeld's 60th Birthday), Columbia University, May 2007. See Woit, Peter (May 26, 2007), "Proof of the abc Conjecture?", *Not Even Wrong*.
11. Mochizuki, Shinichi (May 2015). *Inter-universal Teichmuller Theory I: Construction of Hodge Theaters*, *Inter-universal Teichmuller Theory II: Hodge-Arakelov-theoretic Evaluation*, *Inter-universal Teichmuller Theory III: Canonical Splittings of the Log-theta-lattice.*, *Inter-universal Teichmuller Theory IV: Log-volume Computations and Set-theoretic Foundations*, available at http://www.kurims.kyoto-u.ac.jp/~motizuki/papers-

english.html
12. Ball, Phillip (10 September 2012), "Proof claimed for deep connection between primes", *Nature*, doi:10.1038/nature.2012.11378.
13. Cipra, Barry (September 12, 2012), "ABC Proof Could Be Mathematical Jackpot", News, *Science*.
14. Kevin Hartnett (3 November 2012). "An ABC proof too tough even for mathematicians". *Boston Globe*.
15. "On the Verification of Inter-Universal Teichmüller Theory: A Progress Report (as of December 2013)" by Shinichi Mochizuki (http://www.kurims.kyoto-u.ac.jp/~motizuki/IUTeich%20Verification%20Report%202013-12.pdf)
16. "On the Verification of Inter-Universal Teichmüller Theory: A Progress Report (as of December 2014)" by Shinichi Mochizuki (http://www.kurims.kyoto-u.ac.jp/~motizuki/IUTeich%20Verification%20Report%202014-12.pdf)
17. http://www.claymath.org/events/iut-theory-shinichi-mochizuki
18. https://www.maths.nottingham.ac.uk/personal/ibf/files/kyoto.iut.html
19. Castelvecchi, Davide (7 October 2015). "The biggest mystery in mathematics: Shinichi Mochizuki and the impenetrable proof". *Nature* **526**. doi:10.1038/526178a.

References

- Baker, Alan (1998). "Logarithmic forms and the *abc*-conjecture". In Győry, Kálmán. *Number theory. Diophantine, computational and algebraic aspects. Proceedings of the international conference, Eger, Hungary, July 29-August 2, 1996*. Berlin: de Gruyter. pp. 37–44. ISBN 3-11-015364-5. Zbl 0973.11047.
- Baker, Alan (2004). "Experiments on the *abc*-conjecture". *Publ. Math. Debrecen* **65**: 253–260.
- Bombieri, Enrico (1994). "Roth's theorem and the abc-conjecture". *preprint* (ETH Zürich).
- Bombieri, Enrico; Gubler, Walter (2006). *Heights in Diophantine Geometry*. New Mathematical Monographs **4**. Cambridge University Press. doi:10.2277/0521846153. ISBN 978-0-521-71229-3. Zbl 1130.11034.
- Browkin, Jerzy; Brzeziński, Juliusz (1994). "Some remarks on the *abc*-conjecture". *Math. Comp.* **62** (206): 931–939. doi:10.2307/2153551. JSTOR 2153551.
- Browkin, Jerzy (2000). "The *abc*-conjecture". In Bambah, R. P.; Dumir, V. C.; Hans-Gill, R. J. *Number Theory*. Trends in Mathematics. Basel: Birkhäuser. pp. 75–106. ISBN 3-7643-6259-6.
- Dąbrowski, Andrzej (1996). "On the diophantine equation $x! + A = y^2$". *Nieuw Archief voor Wiskunde, IV.* **14**: 321–324. Zbl 0876.11015.
- Elkies, N. D. (1991). "ABC implies Mordell". *Intern. Math. Research Notices* **7** (7): 99–109. doi:10.1155/S1073792891000144.
- Goldfeld, Dorian (1996). "Beyond the last theorem". *Math Horizons* (September): 26–34. JSTOR 25678079.
- Goldfeld, Dorian (2002). "Modular forms, elliptic curves and the abc-conjecture". In Wüstholz, Gisbert. *A panorama in number theory or The view from Baker's garden. Based on a conference in honor of Alan Baker's 60th birthday, Zürich, Switzerland, 1999*. Cambridge: Cambridge University Press. pp. 128–147. ISBN 0-521-80799-9. Zbl 1046.11035.
- Gowers, Timothy; Barrow-Green, June; Leader, Imre, eds. (2008). *The Princeton Companion to Mathematics*. Princeton: Princeton University Press. pp. 361–362, 681. ISBN 978-0-691-11880-2.
- Granville, A. (1998). "ABC Allows Us to Count Squarefrees" (PDF). *International Mathematics Research Notices* **1998**: 991–1009. doi:10.1155/S1073792898000592.
- Granville, Andrew; Stark, H. (2000). "ABC implies no "Siegel zeros" for L-functions of characters with negative exponent" (PDF). *Inventiones Mathematicae* **139**: 509–523. doi:10.1007/s002229900036.
- Granville, Andrew; Tucker, Thomas (2002). "It's As Easy As abc" (PDF). *Notices of the AMS* **49** (10): 1224–1231.
- Guy, Richard K. (2004). *Unsolved Problems in Number Theory*. Berlin: Springer-Verlag. ISBN 0-387-20860-7.
- Lando, Sergei K.; Zvonkin, Alexander K. (2004). *Graphs on Surfaces and Their Applications*. *Encyclopaedia of Mathematical Sciences: Lower-Dimensional Topology II* **141** (Springer-Verlag). ISBN 3-540-00203-0.
- Langevin, M. (1993). "Cas d'égalité pour le théorème de Mason et applications de la conjecture *abc*". *Comptes rendus de l'Académie des sciences* (in French) **317** (5): 441–444.
- Masser, D. W. (1985). "Open problems". In Chen, W. W. L. *Proceedings of the Symposium on Analytic Number Theory*. London: Imperial College.
- Mollin, R.A. (2009). "A note on the ABC-conjecture" (PDF). *Far East J. Math. Sci.* **33** (3): 267–275. ISSN 0972-0871. Zbl 1241.11034.

- Mollin, Richard A. (2010). *Advanced number theory with applications*. Boca Raton, FL: CRC Press. ISBN 978-1-4200-8328-6. Zbl 1200.11002.
- Nitaj, Abderrahmane (1996). "La conjecture *abc*". *Enseign. Math.* (in French) **42** (1–2): 3–24.
- Oesterlé, Joseph (1988), "Nouvelles approches du "théorème" de Fermat", *Astérisque*, Séminaire Bourbaki exp 694 (161): 165–186, ISSN 0303-1179, MR 992208
- Pomerance, Carl (2008). "Computational Number Theory". *The Princeton Companion to Mathematics*. Princeton University Press. pp. 361–362.
- Silverman, Joseph H. (1988). "Wieferich's criterion and the *abc*-conjecture". *Journal of Number Theory* **30** (2): 226–237. doi:10.1016/0022-314X(88)90019-4. Zbl 0654.10019.
- Robert, Olivier; Stewart, Cameron L.; Tenenbaum, Gérald (2014). "A refinement of the abc conjecture". *Bull. London Math. Soc.* **46** (6): 1156–1166. doi:10.1112/blms/bdu069.
- Robert, Olivier; Tenenbaum, Gérald (2013). "Sur la répartition du noyau d'un entier". *Indag. Math.* **24**: 802–914. doi:10.1016/j.indag.2013.07.007.
- Stewart, C. L.; Tijdeman, R. (1986). "On the Oesterlé-Masser conjecture". *Monatshefte für Mathematik* **102** (3): 251–257. doi:10.1007/BF01294603.
- Stewart, C. L.; Yu, Kunrui (1991). "On the *abc* conjecture". *Mathematische Annalen* **291** (1): 225–230. doi:10.1007/BF01445201.
- Stewart, C. L.; Yu, Kunrui (2001). "On the *abc* conjecture, II". *Duke Mathematical Journal* **108** (1): 169–181. doi:10.1215/S0012-7094-01-10815-6.
- Van Frankenhuijsen, Machiel (2002). "The ABC conjecture implies Vojta's height inequality for curves". *J. Number Theory* **95** (2): 289–302. doi:10.1006/jnth.2001.2769. MR 1924103.

External links

- ABC@home (http://abcathome.com/) Distributed Computing project called ABC@Home.
- Easy as ABC (http://bit-player.org/2007/easy-as-abc): Easy to follow, detailed explanation by Brian Hayes.
- Weisstein, Eric W., "abc Conjecture" (http://mathworld.wolfram.com/abcConjecture.html), *MathWorld*.
- Abderrahmane Nitaj's ABC conjecture home page (http://www.math.unicaen.fr/~nitaj/abc.html)
- Bart de Smit's ABC Triples webpage (http://www.math.leidenuniv.nl/~desmit/abc/)
- http://www.math.columbia.edu/~goldfeld/ABC-Conjecture.pdf
- The amazing ABC conjecture (http://www.sciencenews.org/sn_arc97/12_6_97/mathland.htm)
- The ABC's of Number Theory (http://dash.harvard.edu/bitstream/handle/1/2793857/Elkies%20-%20ABCs%20of%20Number%20Theory.pdf?sequence=2) by Noam D. Elkies
- Questions about Number (http://www.math.harvard.edu/~mazur/papers/scanQuest.pdf) by Barry Mazur
- Philosophy behind Mochizuki’s work on the ABC conjecture (http://mathoverflow.net/questions/106560/philosophy-behind-mochizukis-work-on-the-abc-conjecture) on MathOverflow
- ABC Conjecture (http://michaelnielsen.org/polymath1/index.php?title=ABC_conjecture) Polymath project wiki page linking to various sources of commentary on Mochizuki's papers.
- abc Conjecture (http://www.youtube.com/watch?v=RkBl7WKzzRw) Numberphile video
- News about IUT by Mochizuki (http://www.kurims.kyoto-u.ac.jp/~motizuki/news-english.html)

Categories: Conjectures | Number theory | Unsolved problems in mathematics

---