



MITx: 6.041x Introduction to Probability - The Science of Uncertainty



Bookmarks

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Bookmark

Problem 2: Functions of a standard normal

(3/3 points)

The random variable X has a standard normal distribution. Find the PDF of the random variable Y , where:

1. $Y = 3X - 1$.

☐ $f_Y(y) = \frac{1}{3} f_X(3(y + 1))$

☐ $f_Y(y) = 3 f_X(3(y + 1))$


☒ $f_Y(y) = \frac{1}{3} f_X\left(\frac{y+1}{3}\right)$ ✓

☐ $f_Y(y) = 3 f_X\left(\frac{y+1}{3}\right)$


▼ Unit 6: Further topics on random variables

Unit overview


Lec. 11: Derived distributions

Exercises 11 due Mar 30, 2016 at 23:59 UTC 

Lec. 12: Sums of independent r.v.'s; Covariance and correlation

Exercises 12 due Mar 30, 2016 at 23:59 UTC 


Lec. 13: Conditional expectation and variance revisited; Sum of a random number of independent r.v.'s

Exercises 13 due Mar 30, 2016 at 23:59 UTC 

Solved problems

Additional theoretical material

Problem Set 6

Problem Set 6 due Mar 30, 2016 at 23:59 UTC 

Unit summary

2. $Y = 3X^2 - 1$. For $y \geq -1$,

☐ $f_Y(y) = \frac{1}{6} \cdot \sqrt{\frac{3}{y+1}} f_X\left(\sqrt{\frac{y+1}{3}}\right)$

☐ $f_Y(y) = \frac{1}{3} \cdot \sqrt{\frac{y+1}{3}} f_X\left(\sqrt{\frac{y+1}{3}}\right)$

☒ $f_Y(y) = \frac{1}{3} \cdot \sqrt{\frac{3}{y+1}} f_X\left(\sqrt{\frac{y+1}{3}}\right)$ ✓

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Answer:

1. $Y = 3X - 1$. We know that when $Y = aX + b$, with $a \neq 0$, we have

- ▶ Unit 7: Bayesian inference
- ▶ Exam 2
- ▶ Unit 8: Limit theorems and classical statistics
- ▶ Unit 9: Bernoulli and Poisson processes
- ▶ Unit 10: Markov chains
- ▶ Exit Survey
- ▶ Final Exam

$$f_Y(y) = \frac{1}{|a|} f_X\left(\frac{y-b}{a}\right).$$

Therefore,

$$f_Y(y) = \frac{1}{3} f_X\left(\frac{y+1}{3}\right), \text{ for all } y.$$

Note that the fact that \mathbf{X} is a standard normal random variable did not matter.

2. $\mathbf{Y} = 3\mathbf{X}^2 - 1$. We will find the CDF of \mathbf{Y} and then differentiate to find the PDF. For $y \geq -1$, we have

$$\begin{aligned} F_Y(y) &= \mathbf{P}(Y \leq y) \\ &= \mathbf{P}\left(X^2 \leq \frac{y+1}{3}\right) \\ &= \mathbf{P}\left(-\sqrt{\frac{y+1}{3}} \leq X \leq \sqrt{\frac{y+1}{3}}\right) \\ &= F_X\left(\sqrt{\frac{y+1}{3}}\right) - F_X\left(-\sqrt{\frac{y+1}{3}}\right), \end{aligned}$$

and therefore, using the chain rule and also the fact that the standard normal PDF is symmetric about zero,

$$f_Y(y) = \frac{1}{3} \cdot \sqrt{\frac{3}{y+1}} f_X\left(\sqrt{\frac{y+1}{3}}\right), \text{ for } y \geq -1.$$

You have used 2 of 2 submissions

DISCUSSION

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