MATHEMATICS

Second order derivative of log of vector

Asked 4 years ago Active 3 years, 11 months ago Viewed 1k times

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I have a vector of size n x 1 named α. Let $f(\alpha) = u \cdot \mathbf{1}^{\top} ln(\alpha)$ where u is scalar.

What is the $f'(\alpha)$ and $f''(\alpha)$ and equivalent **Matlab** code?

According to me the first derivative is

$$f'(\alpha) = u/\alpha$$

and equivalent MATLAB code is --

```
f_a_1 = u ./ a
```

and for the second derivative

$$f''(\alpha) = u \cdot (Diag(\alpha) * Diag(\alpha))^{-1}$$

Equivalent MATLAB code is

```
f_a_2 = u*inv(diag(a)*diag(a))
```

Is my inference correct?

calculus matrices matlab matrix-calculus

edited Nov 30 '15 at 11:08
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asked Nov 11 '15 at 9:52
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33 6

1 Answer

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Let $\alpha = (\alpha_1, \alpha_2, \dots, \alpha_n)$ You probably (see the note below to understand the doubts) defined function $f : \mathbb{R}^n \to \mathbb{R}$

$$f(\alpha) = f(\alpha_1, \alpha_2, \dots, \alpha_n) = u \sum_{i=1}^{n} \ln \alpha_i$$

therefore

If you need a vector gradient of f (a vector of partial derivatives), then you denote it as

$$\nabla f = (f_{\alpha_1}, \dots, f_{\alpha_n}) = \left(\frac{u}{\alpha_1}, \dots, \frac{u}{\alpha_n}\right)$$

and compute it in matlab as

> u./a

if you looking for a total derivative of f it is defined as $\nabla f \cdot \alpha$ and in your case is equal to un and another differentiation will be 0.

You can do

> -u./(a.^2)

in matlab (without converting it to diagonal matrices etc), but this is not a second derivative of your function.

I would say you have to really clarify your question.

Note, the ln of a matrix is defined for $n \times n$ matrices, so the notatoins of ln of vector are incorrect and misleading.

The truth is that

$$\exp \begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{n1} & \cdots & a_{nn} \end{bmatrix} \neq \begin{bmatrix} e^{a_{11}} & \cdots & e^{a_{1n}} \\ \vdots & \ddots & \vdots \\ e^{a_{n1}} & \cdots & e^{a_{nn}} \end{bmatrix}$$

neigher

$$\ln \begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{n1} & \cdots & a_{nn} \end{bmatrix} \neq \begin{bmatrix} \ln a_{11} & \cdots & \ln a_{1n} \\ \vdots & \ddots & \vdots \\ \ln a_{n1} & \cdots & \ln a_{nn} \end{bmatrix}$$

They are acutally defined trough power series of ln and exponential. See link and link

However in matlab the regular

> exp

and

> log

do an elementwise evaluation of matrix and vector entries, e,g.

> exp([a,b,c])

will return the value of

> [exp(a),exp(b),exp(c)].

In matlab, the true matrix ln and exponential implemented via

> logm

and

> expm

edited Nov 12 '15 at 9:51

answered Nov 11 '15 at 11:14

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