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Course > Section 3: From X-rays to CT scans: Mathematics and Medical Imaging >
1.7 Summary Quiz: From X-rays to CT-scans > 1.7.2 Summary Quiz Part 2

1.7.2 Summary Quiz Part 2

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Question 1

1/1 point (graded)

As Margo discussed, the resulting intensity of an x-ray passing through an object depends on the object's material and the thickness.

If the object is uniform with attenuation coefficient $\mu \geq 0$ and thickness Δx, the Lambert-Beer model describes precisely the resulting intensity. (The object we're considering in this case is one-dimensional, like a line.)

Where does this model come from? Consider a fixed attenuation μ. Let $I(t)$ be the resulting intensity after the light travels t units through the object.

The Lambert-Beer model is based on the assumption that the change of intensity I with respect to distance traveled t through the object is proportional to the intensity at that point. Furthermore, the constant of proportionality comes from the attenuation coefficient μ of the material.

Translate this statement into a differential equation.

- $\frac{dI}{dt} = \mu t$
- $\frac{dI}{dt} = \mu I$
- $\frac{dI}{dt} = \mu I_0$
- $\frac{dI}{dt} = -\mu t$
- $\frac{dI}{dt} = -\mu I$ ✔
- $\frac{dI}{dt} = -\mu I_0$

Explanation

A: $\frac{dI}{dt} = -\mu I$ is the correct differential equation. $\frac{dI}{dt}$ is the change of intensity I with respect to distance traveled t through the object and is proportional to I, the intensity at that point. The constant $-\mu$ is the constant of proportionality, and it is negative. This is because intensity I decreases as the x-ray passes through an object, so $\frac{dI}{dt}$ should be negative.

We can also check that $I = I_0 e^{-\mu t}$ satisfies this differential equation and none of the others.

For more detail, see : https://en.wikipedia.org/wiki/Beer%E2%80%93Lambert_law

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Answers are displayed within the problem

Question 2

1/1 point (graded)

Imagine a single x-ray of initial intensity I_0 traveling through a uniform object (one made of material with a constant attenuation coefficient, μ). Furthermore, assume the x-ray travels through a portion which is L cm thick. What is the formula for the output intensity of the x-ray?

- The output intensity is $I_0 - \mu L$.
- The output intensity is $I_0 - e^{-\mu L}$.
- The output intensity is $I_0 e^{-\mu L}$. ✔
- The output intensity is $I_0 e^{-\mu} L$.
- None of the above

Explanation

If the object is made of uniform material, then $\mu(x) = \mu$, a constant. Recall that the output intensity $I = I_0 e^{-\int_0^L \mu\, dx} = I_0 e^{-\mu L}$.

We could also use the Lambert-Beer model Margo introduced for uniform objects. We know the length of the object is L and the attenuation is μ, so the output intensity when the x-ray passes through the object is $I_0 e^{-\mu L}$.

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Question 3

1/1 point (graded)

This next problem is NOT about x-rays or CT-scans but gives another example of integration in a model from medicine. (Source: This problem is based on a model found in *MEASURING CARDIAC OUTPUT, Brindell Horelidc and Sinan Koont, UMAP, Education Development Center, Inc.*.)

One method for measuring blood flow from the heart, called **cardiac output**, is the Dye Dilution Method.

Here's how it works:

- The rate of flow F is the volume of blood leaving the heart per unit time. We assume F is a constant, and the goal is to estimate F . (We'll assume its units are milliliters per second).
- An amount A of dye is injected into the bloodstream. (We'll use milligrams as its units of the dye.)
- A special instrument measures the concentration of dye at time t in the blood leaving the heart, $c(t)$, until the dye has cleared T seconds later. (The units are milligrams/milliliter.) Here time is measure from the moment of injection.
- Concentrations are measured at n equally spaced time intervals, $t_1, t_2, .., t_n = T$, each length $\Delta t = T/n$.

Based on these concentration readings, we can estimate the cardiac output. How?

- We estimate the amount of dye leaving the heart at each time interval. For example, in the first time interval, $[t_0, t_1]$, the volume of blood leaving is the flow rate, F (ml/second) multiplied by the length of time Δt (seconds). The amount of dye leaving in that interval is thus approximately $c(t_1) \cdot F\Delta t$, the concentration of dye in the blood multiplied by the volume of blood.
  Note: this is an approximation because we're using the concentration measurement at t_1 as the concentration for that whole time interval.

- We do this for each time interval, $[t_i, t_{i+1}]$, to estimate the amount of dye leaving in each time interval. The sum of these amounts is approximately equal to the total dye, A . We can then solve for an estimate of F , the flow rate.

Following the process above, write an approximation for the total amount of dye leaving the blood. Your expression will involve quantities such as A, F, T or Δt. Solve for F to find an expression that expressions estimates the flow rate F of blood from the heart.

- ○ $F \approx \frac{A}{T}$
- ○ $F \approx \sum_{i=1}^{n} Ac(t_i)$
- ○ $F \approx \sum_{i=1}^{n} Ac(t_i)\Delta t$
- ◉ $F \approx \frac{A}{\sum_{i=1}^{n} c(t_i)\Delta t}$ ✔
- ○ $F \approx \frac{1}{A}\sum_{i=1}^{n} c(t_i)\Delta t$
- ○ None of the above.

Explanation

Following the process, we get:

$$A \approx c(t_1) \cdot F\Delta t + c(t_2) \cdot F\Delta t + \ldots + c(t_n) \cdot F\Delta t = \sum_{i=1}^{n} c(t_i) F\Delta t.$$

Since F is a constant, we can factor it out to get:

$$A \approx F \sum_{i=1}^{n} c(t_i)\Delta t.$$

Solving for F we get:

$$F \approx \frac{A}{\sum_{i=1}^{n} c(t_i)\Delta t}.$$

Notice that if we take the limit as $n \to \infty$, we get an integral expression for flow rate $F = \frac{A}{\int_0^T c(t)\, dt}$.

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Answers are displayed within the problem

Question 4

1/1 point (graded)

If we take the approximation for F above and take the limit as $n \to \infty$, we get an integral expression which gives the flow rate F exactly. What is the physical interpretation of letting $n \to \infty$?

- ○ Adding dye continuously into the bloodstream
- ● Measuring the concentration at smaller and smaller time intervals ✔
- ○ T, the time it takes to clear the bloodstream gets longer and longer
- ○ None of the above

Explanation

Letting $n \to \infty$ means $\Delta t = T/n$ is getting closer to 0. In other words, we measure the concentration at smaller and smaller time intervals.

Since $F \approx \frac{A}{\sum_{i=1}^{n} c(t_i)\Delta t}$, taking $n \to \infty$, we get the integral expression for flow rate $F = \frac{A}{\int_0^T c(t)\, dt}$.

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Answers are displayed within the problem

Question 5

1/1 point (graded)

What are benefits of a CT-scan versus an x-ray? Can you think of any drawbacks?

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Better detailed view in CT scan with less superimposing.
Exposure to x-ray.
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Thank you for your response.

Explanation

The benefits of a CT-scan are that we can more clearly distinguish the different features in a slice of the object, because we are not looking at superimposed slices like in an x-ray.

Some drawbacks of a CT are the amount of radiation and the cost of such an image (x-rays are usually in the range of 100-200, while CT-scans can be closer to 1000 or more. (CT-machines cost more and are expensive to maintain). (See How much does an x-ray cost? and How much does a CT Scan Cost? from Honor Health.)

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Answers are displayed within the problem