11. Checking independence using the nullspace
Independence and the nullspace of a matrix

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One way to check independence using the nullspace

Given vectors $\mathbf{v}_1$, $\mathbf{v}_2$, ..., $\mathbf{v}_n$, here is how to test if they are linearly independent.

1. Create a matrix $\mathbf{A}$ whose columns are the vectors $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_n$.

2. Find the nullspace of $\mathbf{A}$.

3. If the nullspace contains only the zero vector, then the vectors $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_n$ are linearly independent. If there are any nonzero vectors in the nullspace, then the vectors $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_n$ are linearly dependent.

Summary: Linear dependence is a property of a list of **vectors** . But to test linear dependence, put those vectors into a **matrix** and compute its nullspace.

Checking independence using the nullspace

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Are the vectors $\begin{pmatrix} 2 \\ 3 \\ -1 \\ 4 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ -3 \\ 6 \end{pmatrix}, \begin{pmatrix} 9 \\ 11 \\ 8 \\ -4 \end{pmatrix}$ linearly dependent?

- linearly dependent
- linearly independent ✔

Solution:

We place the vectors into the columns of a matrix and perform row operations to put the matrix into row echelon form. If the matrix has free variables, then the nullspace is nonzero and the vectors are linearly dependent. Otherwise, they are independent.

$$\begin{pmatrix} 2 & 1 & 9 \\ 3 & 2 & 11 \\ -1 & -3 & 8 \\ 4 & 6 & -4 \end{pmatrix} \rightarrow \begin{pmatrix} 2 & 1 & 9 \\ 0 & 1/2 & -5/2 \\ 0 & -5/2 & 25/2 \\ 0 & 4 & -22 \end{pmatrix} \rightarrow \begin{pmatrix} 2 & 1 & 9 \\ 0 & 1/2 & -5/2 \\ 0 & 0 & 0 \\ 0 & 0 & -2 \end{pmatrix} \rightarrow \begin{pmatrix} 2 & 1 & 9 \\ 0 & 1/2 & -5/2 \\ 0 & 0 & -2 \\ 0 & 0 & 0 \end{pmatrix}$$

The row echelon form has pivots in every column, so the nullspace of this matrix is $\{\mathbf{0}\}$. This tells us that these vectors are linearly independent.

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Answers are displayed within the problem

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