3.

Setup:

We continue with the problem on the previous page.

Recall the lifetime (in months) of a battery is modeled by a random variable X that has pdf

$$f_\theta(x) = K\theta^x \mathbf{1}(x > 0) \qquad \text{where } K = \ln(1/\theta)$$

for an unknown parameter $\theta \in (0,1)$.

Assume instead that we cannot actually observe the lifetime of the batteries. Instead, we only observe if the battery is still working after τ months for some known τ to be chosen later (this is called **censored data**).

Let $Y_1, \ldots, Y_n$ be our observations where $Y_i = \mathbf{1}(X_i > \tau)$ indicates that the ith battery is still working after τ months. Our goal is to estimate $\theta \in (0,1)$ (the parameter for the pdf of X) based on this new data.

Distribution of Y

2/2 points (graded)
What kind of distribution does $Y = \mathbf{1}(X > \tau)$ follow?

- Standard Gaussian
- Gaussian with mean $\mu \neq 0$
- Bernoulli
- Cannot be determined

✔

What is $\mathbb{E}[Y_i]$?

$\mathbb{E}[Y_i] =$ theta^tau ✔ **Answer:** theta^tau

θ^τ

STANDARD NOTATION

Submit You have used 1 of 3 attempts

Answers are displayed within the problem

Statistical Model

1/1 point (graded)
What is the statistical model $(E, \mathbf{P}_{\theta \in \Theta})$ associated to this experiment?

- $\left(\{0,1\}, \text{Ber}(\theta^\tau)_{\tau\in(0,1)}\right)$
- $\left(\{0,1\}, \text{Ber}(\theta^\tau)_{\theta\in(0,1)}\right)$
- $\left((-\infty,\infty), \mathcal{N}(0,\theta^\tau)_{\theta\in(0,1)}\right)$
- $\left((-\infty,\infty), \mathcal{N}(\mu,\theta^\tau)_{\tau\in(0,1)}\right)$

✔

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MLE

2.0/2.0 points (graded)

Find the maximum likelihood estimator $\tilde{\theta}$ of θ **based on the censored data**.

(Enter **barY_n** for $\overline{Y_n}$ and **tau** for τ.)

$\tilde{\theta} =$

(barY_n)^(1/tau) ✔ **Answer:** barY_n^(1/tau)

STANDARD NOTATION

Solution:

the maximum likelihood estimator $\tilde{\theta}$ of θ **based on the censored data** is given by $\hat{\theta} = \left(\overline{Y_n}\right)^{\frac{1}{\tau}}$.

Submit You have used 1 of 3 attempts

Answers are displayed within the problem

Consistency

1/1 point (graded)

Is $\tilde{\theta}$ a consistent estimator? If yes, enter the limit in probability; that is, enter the constant C such that $\tilde{\theta} \xrightarrow[n\to\infty]{\mathbf{P}} C$ (where the convergence is in probability). If no, enter **-999**.

$C =$ theta ✔ **Answer:** theta

θ

Submit You have used 1 of 3 attempts

Answers are displayed within the problem

Asymptotic Variance

3.0/3.0 points (graded)

The quantity $\sqrt{n}\,(\tilde{\theta} - \theta)$ converges in distribution to $\mathcal{N}\,(0, \tilde{\sigma^2})$. Find the asymptotic variance $\tilde{\sigma^2}$.

$\tilde{\sigma^2} =$ (theta^(2-tau)-theta^2)/tau^2 ✔ **Answer:** 1/tau^2*theta^(-tau+2)*(1-theta^tau)

$\frac{\theta^{2-\tau}-\theta^2}{\tau^2}$

Submit You have used 1 of 3 attempts

Answers are displayed within the problem

Different Tau's

1/1 point (graded)
Previous studies indicate that $\theta > 1/3$. Would you recommend using $\tau = 1$ or $\tau = 2$? Select the correct reasoning.

(Choose all that apply.)

- [ ] $\tau = 1$ is recommended.
- [x] $\tau = 2$ is recommended.
- [x] The recommended τ value should give a smaller asymptotic variance.
- [ ] The recommended τ value should give a larger asymptotic variance.

Submit You have used 1 of 3 attempts

Answers are displayed within the problem

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	Is the answer for the Asymptotic variance correct? Per MLE question, theta is the function of Yn bar, or (barY_n)^(1/tau). If we take the derivative and square it, it should be (barY_n)^(2/tau - 2) / tau^2. Then the asy. var. sho...	31
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