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9. Practice Exam

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A hiker is hiking in a mountainous region. The height of the landscape at (x, y) is $h(x, y) = xy + xy^2$. The hiker is at $(1, 1)$ when she reaches a fork. One path goes in a straight line towards $(2, 2)$ and the other path goes in a straight line towards $(3, 1)$. Which path starts out more steeply?

- (●) The path that goes in a straight line towards $(2, 2)$ is steeper.
- ( ) The path that goes in a straight line towards $(3, 1)$ is steeper.
- ( ) They are equally as steep.

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Solution:

For convenience, we first compute the gradient at $(1, 1)$.

$$\begin{aligned} \nabla h(x, y) &= (h_x, h_y) = (y + y^2, x + 2xy) \\ \nabla h(1, 1) &= (2, 3). \end{aligned}$$

Now we compare both paths.

1. Path to $(2, 2)$

   Here, the hiker is walking toward the direction of $\langle 2, 2\rangle - \langle 1, 1\rangle = \langle 1, 1\rangle$. A unit normal vector in this direction is $\frac{1}{\sqrt{2}}\langle 1, 1\rangle$. Therefore the instantaneous steepness at $(1, 1)$ in the direction toward $(2, 2)$ is

$$D_{\vec{u}} h(1, 1) = \nabla h(1, 1) \cdot \vec{u} = \langle 2, 3\rangle \cdot \frac{1}{\sqrt{2}}\langle 1, 1\rangle = \frac{5}{\sqrt{2}}$$

2. Path to $(3, 1)$

   Here, the hiker is walking toward the direction of $\langle 3, 1\rangle - \langle 1, 1\rangle = \langle 2, 0\rangle$. A unit normal vector in this direction is $\frac{1}{2}\langle 2, 0\rangle = \langle 1, 0\rangle$. Therefore the instantaneous steepness at $(1, 1)$ in the direction toward $(3, 1)$ is

$$D_{\vec{u}} h(1, 1) = \nabla h(1, 1) \cdot \vec{u} = \langle 2, 3\rangle \cdot \langle 1, 0\rangle = 2.$$

Since

$$\frac{5}{\sqrt{2}} > 2$$

we see that the path toward $(2, 2)$ starts out more steeply.

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