Homework Problem: Blue Green Tree

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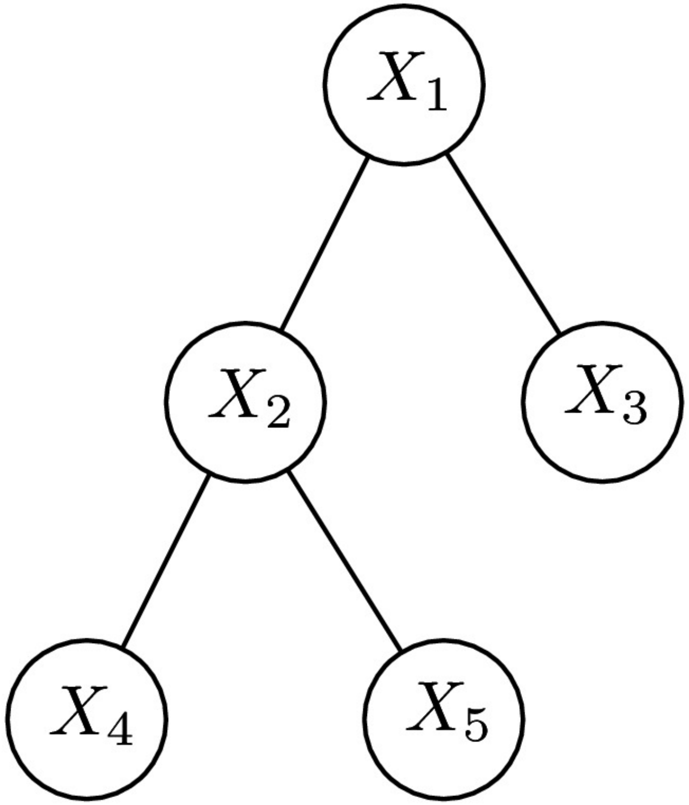
5/5 points (graded)

What this problem is getting you to see (October 13, 2016): *Many times, people set up a probability distribution by directly specifying what the potential tables are in a graphical model. In particular, unlike the previous homework problem, this problem is best approached by directly setting up the potential tables of the graphical model, which of course implies what the joint probability table is for the random variables of interest.*

In this problem, we'll look at a tree structure where each node has a color – either blue or green. A blue node can only have green nodes as its neighbors and vice-versa. In other words, there is a color change at every edge. We'll consider a probabilistic version of a blue-green tree, where each node is blue or green with some probability p. The color change at each edge implies a flip in the probability of being blue or green. For example, in the simplest graph with two nodes, if it is given that node 1 is blue, then its neighbor, node 2, must be green.

Consider the graph shown below:



The joint distribution of the graph can be expressed in the form

$$p_X(x) = \frac{1}{Z}\prod_{i\in V}\phi_i(x_i)\prod_{(i,j)\in E}\psi_{i,j}(x_i, x_j).$$

- **(a)** Define a suitable pairwise potential table $\psi_{i,j}$. (Note that for this problem, the pairwise potential tables associated with every edge is actually the same table, which only needs to be specified once.)

  Hint: Your answer should try to force a flip in the probability of being blue at every edge.

  Note that a pairwise potential table can be specified in a way where you can always scale all the entries by an arbitrary positive constant. However, to make this problem autogradable, please provide your answers below so that the sum of the pairwise potential table's entries is **2**.

  $\psi(\mathbf{blue}, \mathbf{blue}) =$ 0

  $\psi(\mathbf{blue}, \mathbf{green}) =$ 1

  $\psi(\mathbf{green}, \mathbf{blue}) =$ 1

  $\psi(\mathbf{green}, \mathbf{green}) =$ 0

- **(b)** Suppose that for nodes 4 and 5, we favor each of them being blue with weight 0.8 and green with weight 0.2 (note that these in general don't have to sum to 1). Also, suppose that we favor node 3 being blue with weight 0.6 and green with weight 0.4. We have no preference for nodes 1 and 2, equally favoring blue and green for them. Define suitable node potential tables ϕ_i.

  Again, in general we can always scale the entries of each node potential table by a positive constant. To make this part autogradable, please provide each node potential table so that the entries add up to **1**. As a result this means that to specify ϕ_1 for instance, you only need to specify $\phi_1(\mathbf{blue})$ since $\phi_1(\mathbf{green}) = 1 - \phi_1(\mathbf{blue})$.

$\phi_1(\text{blue}) =$ 0.5 ✔

$\phi_2(\text{blue}) =$ 0.5 ✔

$\phi_3(\text{blue}) =$ 0.6 ✔

$\phi_4(\text{blue}) =$ 0.8 ✔

$\phi_5(\text{blue}) =$ 0.8 ✔

Submit You have used 1 of 5 attempts

✔ Correct (5/5 points)