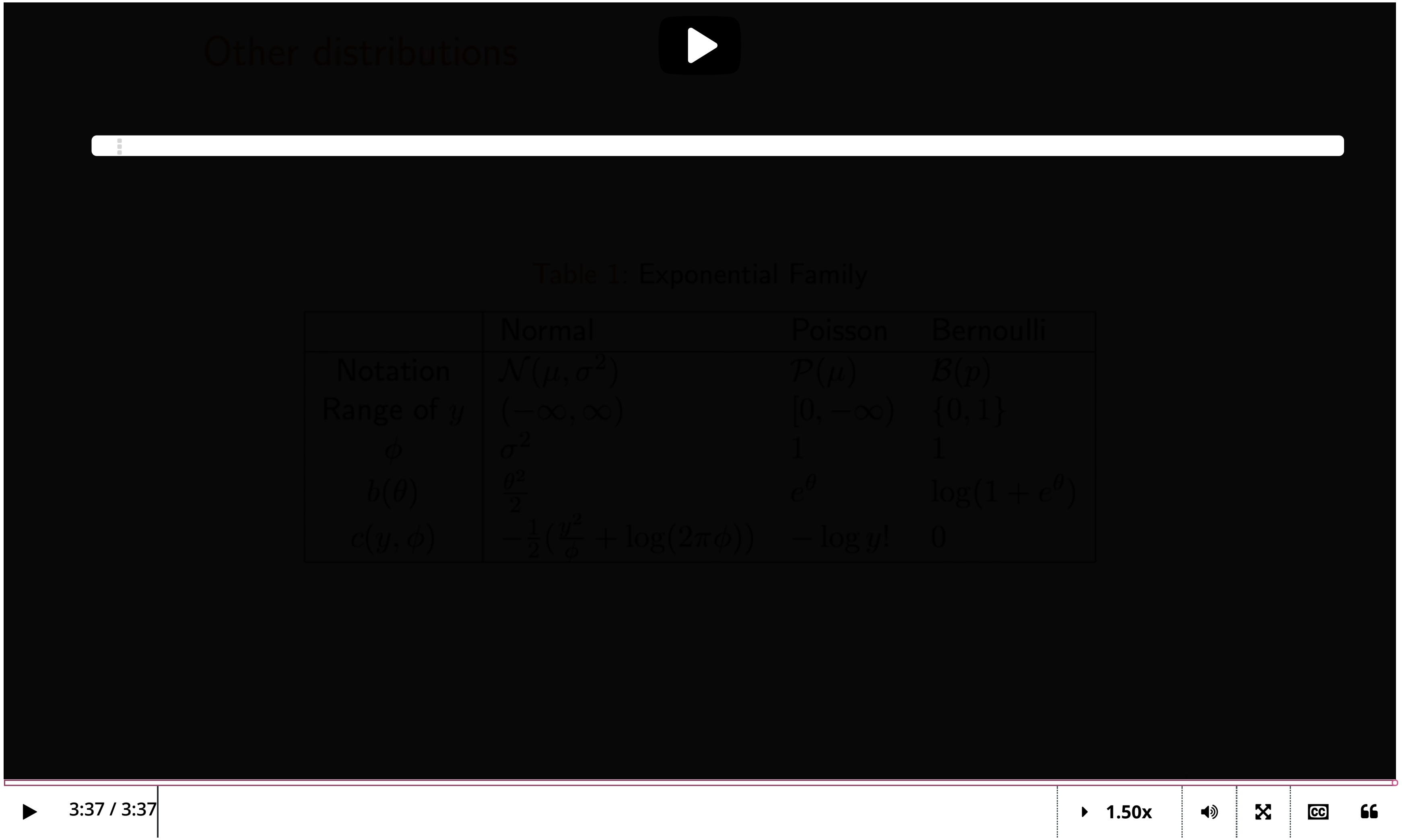
11. One-Parameter Canonical Exponential Families

Worked example: Find B for Bernoulli Distribution



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Practice: Binomial Distribution as a Canonical Exponential Family

3/3 points (graded)

Recall from a previous problem that the pmf of a Binomial distribution Binom (n, p) with known n can be written as

$$f_p(y) = \binom{n}{y} e^{y(\ln(p)-\ln(1-p))+n\ln(1-p)}.$$

Rewrite $f_p(y)$ as the pmf of a canonical exponential family:

$$f_\theta(y) = \exp\left(\frac{y\theta - b(\theta)}{\phi} + c(y,\phi)\right).$$

Enter the canonical parameter $\theta,$ in terms of p, the dispersion parameter $\phi,$ and the function $b(\theta)$ below.

$\theta =$ ln(p/(1-p)) ✔ **Answer:** ln(p)-ln(1-p)

$\ln\left(\frac{p}{1-p}\right)$

$b(\theta) =$ n*ln(1+e^theta) ✔ **Answer:** n*ln(1+e^(theta))

$n\cdot\ln\left(1+e^{\theta}\right)$

$\phi =$ 1 ✔ **Answer:** 1

1

STANDARD NOTATION

Solution:

Pattern matching, we have

$$f_p(y) = \exp\left(\underbrace{y(\ln(p)-\ln(1-p))}_{\theta} + \underbrace{n\ln(1-p)}_{-b(\theta)} + \underbrace{\ln\left(\binom{n}{y}\right)}_{c(y,\phi)}\right).$$

That is, the dispersion parameter is $\phi = 1,$ and the canonical parameter is $\theta = \ln(p) - \ln(1-p)\,.$
To find $b(\theta)\,,$ first invert $\theta(p)$:

$$\theta \;=\; \ln\left(\frac{p}{1-p}\right) \;\iff\; p \;=\; \frac{e^\theta}{1+e^\theta}$$

Plugging this into $n\ln(1-p)$ gives

$$b(\theta) \;=\; -n\ln(1-p) \;=\; n\ln(1+e^\theta)\,.$$

Finally, $c(y,\phi) = \ln\left(\binom{n}{y}\right)$ (remember n is known).

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Answers are displayed within the problem

Properties of Canonical Exponential Families

1/1 point (graded)
Which of the following are examples of one-parameter canonical exponential families for **nonzero** θ?
(Select all that apply.)

- [x] Normal$(\mu, 1)$
- [x] Poisson(λ)
- [ ] Uniform$([0, a])$
- [x] Bernoulli(p)

- [x] Binomial$(1000, p)$
- [x] Exponential(λ)

Solution:

Every choice here is an example of a canonical exponential family except the uniform distribution parametrized by a. There is no way to write it in the form f_θ described in the beginning of this section: we require $y\theta$ to show up in the exponent, yet the density does not depend on the value of y.

Every other distribution can be expressed as a canonical exponential family; just as before, apply the trick of writing some function $f(y)$ as $e^{\ln f(y)}$, and identify η (using the generalized exponential family notation) as θ.

For example, Bernoulli(p) has the distribution $p^y(1-p)^{1-y}$, with y taking one of two values 0 or 1. We can write it as

$$p^y(1-p)^{1-y} = e^{y\ln p + (1-y)\ln(1-p)} = e^{y\ln\frac{p}{1-p} + \ln(1-p)}$$

and take $\theta = \ln\frac{p}{1-p}$ and $b(\theta) = -\ln(1-p) = \ln(1+e^\theta)$. (Here, we also took $\phi = 1$ and $c(y,\phi) = 0$.)

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Answers are displayed within the problem

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