2.4.2 Practice with Matrix-Vector Multiplication

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Week 2 due Oct 11, 2023 16:42 IST Completed

2.4.2 Practice with Matrix-Vector Multiplication

Reading Assignment

0 points possible (ungraded)
Read Unit 2.4.2 of the notes. [LINK]

- [x] Done

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Matlab Error
Hi, I keep running into path problem. I tried PracticeGemv but something went wrong so I deleted all the folders and tried to upload the LAFF fol... 3

Homework 2.4.2.1

3/3 points (graded)

Compute $y = Ax$ when $A = \begin{pmatrix} -1 & 0 & 2 \\ -3 & 1 & -1 \\ -2 & -1 & 2 \end{pmatrix}$ and $x = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$

$y = \begin{pmatrix} \psi_0 \\ \psi_1 \\ \psi_2 \end{pmatrix}$

$\psi_0 =$ -1 Answer: -1 $\psi_1 =$ -3 Answer: -3

$\psi_2 =$ -2 Answer: -2

Explanation

Answer: $\begin{pmatrix} -1 \\ -3 \\ -2 \end{pmatrix}$, the first column of the matrix!

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Answers are displayed within the problem

Homework 2.4.2.2

Calculator

3/3 points (graded)

Compute $y = Ax$ when $A = \begin{pmatrix} -1 & 0 & 2 \\ -3 & 1 & -1 \\ -2 & -1 & 2 \end{pmatrix}$ and $x = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

$y = \begin{pmatrix} \psi_0 \\ \psi_1 \\ \psi_2 \end{pmatrix}$

$\psi_0 =$ 2 ✔ Answer: 2 $\psi_1 =$ -1 ✔ Answer: -1

$\psi_2 =$ 2 ✔ Answer: 2

Explanation

Answer: $\begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix}$, the third column of the matrix!

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Answers are displayed within the problem

Homework 2.4.2.3

1/1 point (graded)

If A is a matrix and e_j is a unit basis vector of appropriate length, then $Ae_j = a_j$ where a_j is the jth column of matrix A.

Always ✔ Answer: Always

Explanation

Answer: Always

If e_j is the j unit basis vector then

$$Ae_j = \left(a_0 \,\middle|\, a_1 \,\middle|\, \cdots \,\middle|\, a_j \,\middle|\, \cdots \,\middle|\, a_{n-1} \right) \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 1 \\ \vdots \\ 0 \end{pmatrix} = 0 \cdot a_0 + 0 \cdot a_1 + \cdots + 1 \cdot a_j + \cdots + 0 \cdot a_{n-1} = a_j.$$

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Homework 2.4.2.4

1/1 point (graded)

If x is a vector and e_i is a unit basis vector of appropriate length, then $e_i^T x$ equals the ith entry in

Calculator

Always ✔ Answer: Always

Explanation

Answer: Always (We saw this already in Week 1.)

$$e_i^T x = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix}^T \begin{pmatrix} \chi_0 \\ \chi_1 \\ \vdots \\ \chi_{i-1} \\ \chi_i \\ \chi_{i+1} \\ \vdots \\ \chi_{n-1} \end{pmatrix} = 0 \cdot \chi_0 + 0 \cdot \chi_1 + \cdots + 1 \cdot \chi_i + \cdots + 0 \cdot \chi_{n-1} = \chi_i.$$

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Answers are displayed within the problem

Homework 2.4.2.5

1/1 point (graded)

Compute

$$\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}^T \left(\begin{pmatrix} -1 & 0 & 2 \\ -3 & 1 & -1 \\ -2 & -1 & 2 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \right)$$

-2 ✔ Answer: -2

Explanation

Answer:

$$\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}^T \begin{pmatrix} -1 \\ -3 \\ -2 \end{pmatrix} = -2,$$

the $(2, 0)$ element of the matrix.

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Answers are displayed within the problem

Homework 2.4.2.6

1/1 point (graded)

Compute

$$\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}^T \left(\begin{pmatrix} -1 & 0 & 2 \\ -3 & 1 & -1 \\ -2 & -1 & 2 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \right)$$

-3 ✔ Answer: -3

Calculator

Explanation

Answer:

$$\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}^T \begin{pmatrix} -1 \\ -3 \\ -2 \end{pmatrix} = -3,$$

the $(1, 0)$ element of the matrix.

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Answers are displayed within the problem

Homework 2.4.2.7

1/1 point (graded)

Let A be a m x n matrix and $\alpha_{i,j}$ its (i, j) element. Then $\alpha_{i,j} = e_i^T\,(Ae_j)$.

Always ✔ Answer: Always

Explanation

Answer: Always

From a previous exercise we know that $Ae_j = a_j$, the jth column of A. From another exercise we know that $e_i^T a_j = \alpha_{i,j}$, the ith component of the jth column of A.

Later, we will see that $e_i^T A$ equals the ith row of matrix A and that $\alpha_{i,j} = e_i^T(Ae_j) = e_i^T Ae_j = (e_i^T A)e_j$ (this kind of multiplication is associative).

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Answers are displayed within the problem

Homework 2.4.2.8

12/12 points (graded)

Compute

$$\begin{pmatrix} \chi_0 \\ \chi_1 \\ \chi_2 \end{pmatrix} = \begin{pmatrix} 2 & -1 \\ 1 & 0 \\ -2 & 3 \end{pmatrix} \left((-2) \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right)$$

$\chi_0 =$ 2 ✔ Answer: 2 $\chi_1 =$ 0 ✔ Answer: 0 $\chi_2 =$ -6 ✔ Answer: -6

$$\begin{pmatrix} \chi_0 \\ \chi_1 \\ \chi_2 \end{pmatrix} = (-2) \begin{pmatrix} 2 & -1 \\ 1 & 0 \\ -2 & 3 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$\chi_0 =$ 2 ✔ Answer: 2 $\chi_1 =$ 0 ✔ Answer: 0 $\chi_2 =$ -6 ✔ Answer: -6

$$\begin{pmatrix} \chi_0 \\ \chi_1 \\ \chi_2 \end{pmatrix} = \begin{pmatrix} 2 & -1 \\ 1 & 0 \\ -2 & 3 \end{pmatrix} \left(\begin{pmatrix} 0 \\ 1 \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \end{pmatrix} \right)$$

Calculator

$\chi_0 =$ 1 Answer: 1 $\chi_1 =$ 1 Answer: 1 $\chi_2 =$

1 Answer: 1

$$\begin{pmatrix} \chi_0 \\ \chi_1 \\ \chi_2 \end{pmatrix} = \begin{pmatrix} 2 & -1 \\ 1 & 0 \\ -2 & 3 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} + \begin{pmatrix} 2 & -1 \\ 1 & 0 \\ -2 & 3 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$\chi_0 =$ 1 Answer: 1 $\chi_1 =$ 1 Answer: 1 $\chi_2 =$

1 Answer: 1

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Answers are displayed within the problem

Homework 2.4.2.9

1/1 point (graded)

Let $A \in \mathbb{R}^{mxn}$; $x, y \in \mathbb{R}^n$; and $\alpha \in \mathbb{R}$. Then

$A(\alpha x) = \alpha A x$.

$A(x + y) = Ax + Ay$

In other words, matrix-vector multiplication is a linear transformation.

Always Answer: Always

Explanation

Answer: Always

$$\begin{aligned}
A(\alpha x) &= \begin{pmatrix} \alpha_{0,0} & \alpha_{0,1} & \cdots & \alpha_{0,n-1} \\ \alpha_{1,0} & \alpha_{1,1} & \cdots & \alpha_{1,n-1} \\ \vdots & \vdots & & \vdots \\ \alpha_{m-1,0} & \alpha_{m-1,1} & \cdots & \alpha_{m-1,n-1} \end{pmatrix} \begin{pmatrix} \alpha\chi_0 \\ \alpha\chi_1 \\ \vdots \\ \alpha\chi_{n-1} \end{pmatrix} \\
&= \begin{pmatrix} \alpha_{0,0}(\alpha\chi_0) + \alpha_{0,1}(\alpha\chi_1) + \cdots + \alpha_{0,n-1}(\alpha\chi_{n-1}) \\ \alpha_{1,0}(\alpha\chi_0) + \alpha_{1,1}(\alpha\chi_1) + \cdots + \alpha_{1,n-1}(\alpha\chi_{n-1}) \\ \vdots \\ \alpha_{m-1,0}(\alpha\chi_0) + \alpha_{m-1,1}(\alpha\chi_1) + \cdots + \alpha_{m-1,n-1}(\alpha\chi_{n-1}) \end{pmatrix} \\
&= \begin{pmatrix} \alpha\alpha_{0,0}\chi_0 + \alpha\alpha_{0,1}\chi_1 + \cdots + \alpha\alpha_{0,n-1}\chi_{n-1} \\ \alpha\alpha_{1,0}\chi_0 + \alpha\alpha_{1,1}\chi_1 + \cdots + \alpha\alpha_{1,n-1}\chi_{n-1} \\ \vdots \\ \alpha\alpha_{m-1,0}\chi_0 + \alpha\alpha_{m-1,1}\chi_1 + \cdots + \alpha\alpha_{m-1,n-1}\chi_{n-1} \end{pmatrix} \\
&= \begin{pmatrix} \alpha(\alpha_{0,0}\chi_0 + \alpha_{0,1}\chi_1 + \cdots + \alpha_{0,n-1}\chi_{n-1}) \\ \alpha(\alpha_{1,0}\chi_0 + \alpha_{1,1}\chi_1 + \cdots + \alpha_{1,n-1}\chi_{n-1}) \\ \vdots \\ \alpha(\alpha_{m-1,0}\chi_0 + \alpha_{m-1,1}\chi_1 + \cdots + \alpha_{m-1,n-1}\chi_{n-1}) \end{pmatrix} \\
&= \begin{pmatrix} \alpha_{0,0}\chi_0 + \alpha_{0,1}\chi_1 + \cdots + \alpha_{0,n-1}\chi_{n-1} \\ \alpha_{1,0}\chi_0 + \alpha_{1,1}\chi_1 + \cdots + \alpha_{1,n-1}\chi_{n-1} \end{pmatrix}
\end{aligned}$$

Calculator

$$= \alpha \begin{pmatrix} \vdots \\ \alpha_{m-1,0}\chi_0 + \alpha_{m-1,1}\chi_1 + \cdots + \alpha_{m-1,n-1}\chi_{n-1} \end{pmatrix}$$

$$= \alpha \begin{pmatrix} \alpha_{0,0} & \alpha_{0,1} & \cdots & \alpha_{0,n-1} \\ \alpha_{1,0} & \alpha_{1,1} & \cdots & \alpha_{1,n-1} \\ \vdots & \vdots & & \vdots \\ \alpha_{m-1,0} & \alpha_{m-1,1} & \cdots & \alpha_{m-1,n-1} \end{pmatrix} \begin{pmatrix} \chi_0 \\ \chi_1 \\ \vdots \\ \chi_{n-1} \end{pmatrix} = \alpha A x$$

$$A(x+y) = \begin{pmatrix} \alpha_{0,0} & \alpha_{0,1} & \cdots & \alpha_{0,n-1} \\ \alpha_{1,0} & \alpha_{1,1} & \cdots & \alpha_{1,n-1} \\ \vdots & \vdots & & \vdots \\ \alpha_{m-1,0} & \alpha_{m-1,1} & \cdots & \alpha_{m-1,n-1} \end{pmatrix} \left(\begin{pmatrix} \chi_0 \\ \chi_1 \\ \vdots \\ \chi_{n-1} \end{pmatrix} + \begin{pmatrix} \psi_0 \\ \psi_1 \\ \vdots \\ \psi_{n-1} \end{pmatrix} \right)$$

$$= \begin{pmatrix} \alpha_{0,0} & \alpha_{0,1} & \cdots & \alpha_{0,n-1} \\ \alpha_{1,0} & \alpha_{1,1} & \cdots & \alpha_{1,n-1} \\ \vdots & \vdots & & \vdots \\ \alpha_{m-1,0} & \alpha_{m-1,1} & \cdots & \alpha_{m-1,n-1} \end{pmatrix} \begin{pmatrix} \chi_0 + \psi_0 \\ \chi_1 + \psi_1 \\ \vdots \\ \chi_{n-1} + \psi_{n-1} \end{pmatrix}$$

$$= \begin{pmatrix} \alpha_{0,0}(\chi_0 + \psi_0) + \alpha_{0,1}(\chi_1 + \psi_1) + \cdots + \alpha_{0,n-1}(\chi_{n-1} + \psi_{n-1}) \\ \alpha_{1,0}(\chi_0 + \psi_0) + \alpha_{1,1}(\chi_1 + \psi_1) + \cdots + \alpha_{1,n-1}(\chi_{n-1} + \psi_{n-1}) \\ \vdots \\ \alpha_{m-1,0}(\chi_0 + \psi_0) + \alpha_{m-1,1}(\chi_1 + \psi_1) + \cdots + \alpha_{m-1,n-1}(\chi_{n-1} + \psi_{n-1}) \end{pmatrix}$$

$$= \begin{pmatrix} \alpha_{0,0}\chi_0 + \alpha_{0,0}\psi_0 + \alpha_{0,1}\chi_1 + \alpha_{0,1}\psi_1 + \cdots + \alpha_{0,n-1}\chi_{n-1} + \alpha_{0,n-1}\psi_{n-1} \\ \alpha_{1,0}\chi_0 + \alpha_{1,0}\psi_0 + \alpha_{1,1}\chi_1 + \alpha_{1,1}\psi_1 + \cdots + \alpha_{1,n-1}\chi_{n-1} + \alpha_{1,n-1}\psi_{n-1} \\ \vdots \\ \alpha_{m-1,0}\chi_0 + \alpha_{m-1,0}\psi_0 + \alpha_{m-1,1}\chi_1 + \alpha_{m-1,1}\psi_1 + \cdots + \alpha_{m-1,n-1}\chi_{n-1} + \alpha_{m-1,n-1}\psi_{n-1} \end{pmatrix}$$

$$= \begin{pmatrix} \alpha_{0,0}\chi_0 + \alpha_{0,1}\chi_1 + + \cdots + \alpha_{0,n-1}\chi_{n-1} + \alpha_{0,0}\psi_0 + \alpha_{0,1}\psi_1 + \cdots + \alpha_{0,n-1}\psi_{n-1} \\ \alpha_{1,0}\chi_0 + \alpha_{1,1}\chi_1 + \cdots + \alpha_{1,n-1}\chi_{n-1} + \alpha_{1,0}\psi_0 + \alpha_{1,1}\psi_1 + \cdots + \alpha_{1,n-1}\psi_{n-1} \\ \vdots \\ \alpha_{m-1,0}\chi_0 + \alpha_{m-1,1}\chi_1 + \cdots + \alpha_{m-1,n-1}\chi_{n-1} + \alpha_{m-1,0}\psi_0 + \alpha_{m-1,1}\psi_1 + \cdots + \alpha_{m-1,n-1}\psi_{n-1} \end{pmatrix}$$

$$= \begin{pmatrix} \alpha_{0,0}\chi_0 + \alpha_{0,1}\chi_1 + + \cdots + \alpha_{0,n-1}\chi_{n-1} \\ \alpha_{1,0}\chi_0 + \alpha_{1,1}\chi_1 + \cdots + \alpha_{1,n-1}\chi_{n-1} + \\ \vdots \\ \alpha_{m-1,0}\chi_0 + \alpha_{m-1,1}\chi_1 + \cdots + \alpha_{m-1,n-1}\chi_{n-1} \end{pmatrix} + \begin{pmatrix} \alpha_{0,0}\psi_0 + \alpha_{0,1}\psi_1 + \cdots + \alpha_{0,n-1}\psi_{n-1} \\ \alpha_{1,1}\psi_1 + \cdots + \alpha_{1,n-1}\psi_{n-1} \\ \vdots \\ \alpha_{m-1,0}\psi_0 + \alpha_{m-1,1}\psi_1 + \cdots + \alpha_{m-1,n-1}\psi_{n-1} \end{pmatrix}$$

$$= \begin{pmatrix} \alpha_{0,0} & \alpha_{0,1} & \cdots & \alpha_{0,n-1} \\ \alpha_{1,0} & \alpha_{1,1} & \cdots & \alpha_{1,n-1} \\ \vdots & \vdots & & \vdots \\ \alpha_{m-1,0} & \alpha_{m-1,1} & \cdots & \alpha_{m-1,n-1} \end{pmatrix} \begin{pmatrix} \chi_0 \\ \chi_1 \\ \vdots \\ \chi_{n-1} \end{pmatrix} + \begin{pmatrix} \alpha_{0,0} & \alpha_{0,1} & \cdots & \alpha_{0,n-1} \\ \alpha_{1,0} & \alpha_{1,1} & \cdots & \alpha_{1,n-1} \\ \vdots & \vdots & & \vdots \\ \alpha_{m-1,0} & \alpha_{m-1,1} & \cdots & \alpha_{m-1,n-1} \end{pmatrix} \begin{pmatrix} \psi_0 \\ \psi_1 \\ \vdots \\ \psi_{n-1} \end{pmatrix}$$

$$= Ax + Ay$$

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Answers are displayed within the problem

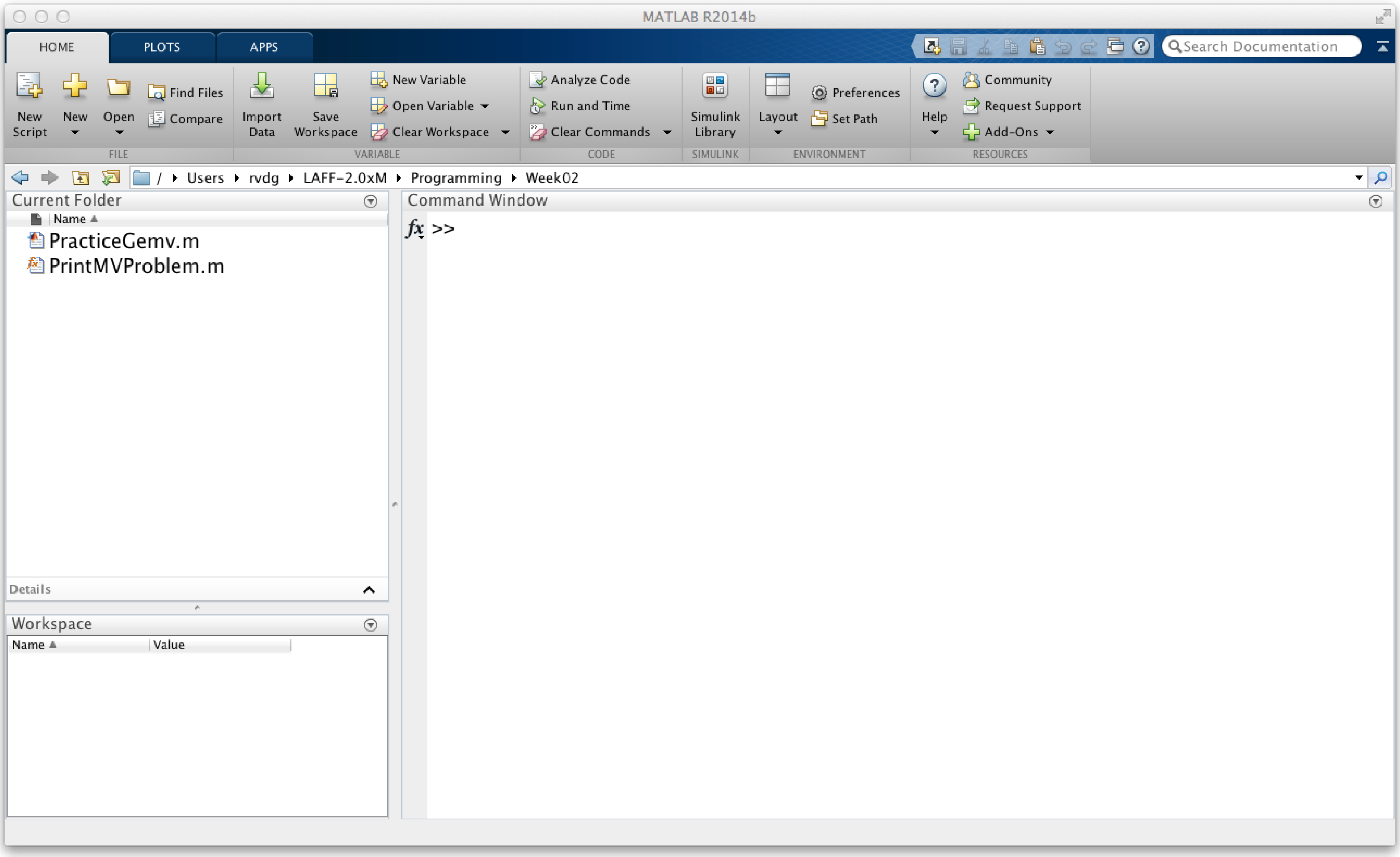
Calculator

Homework 2.4.2.10

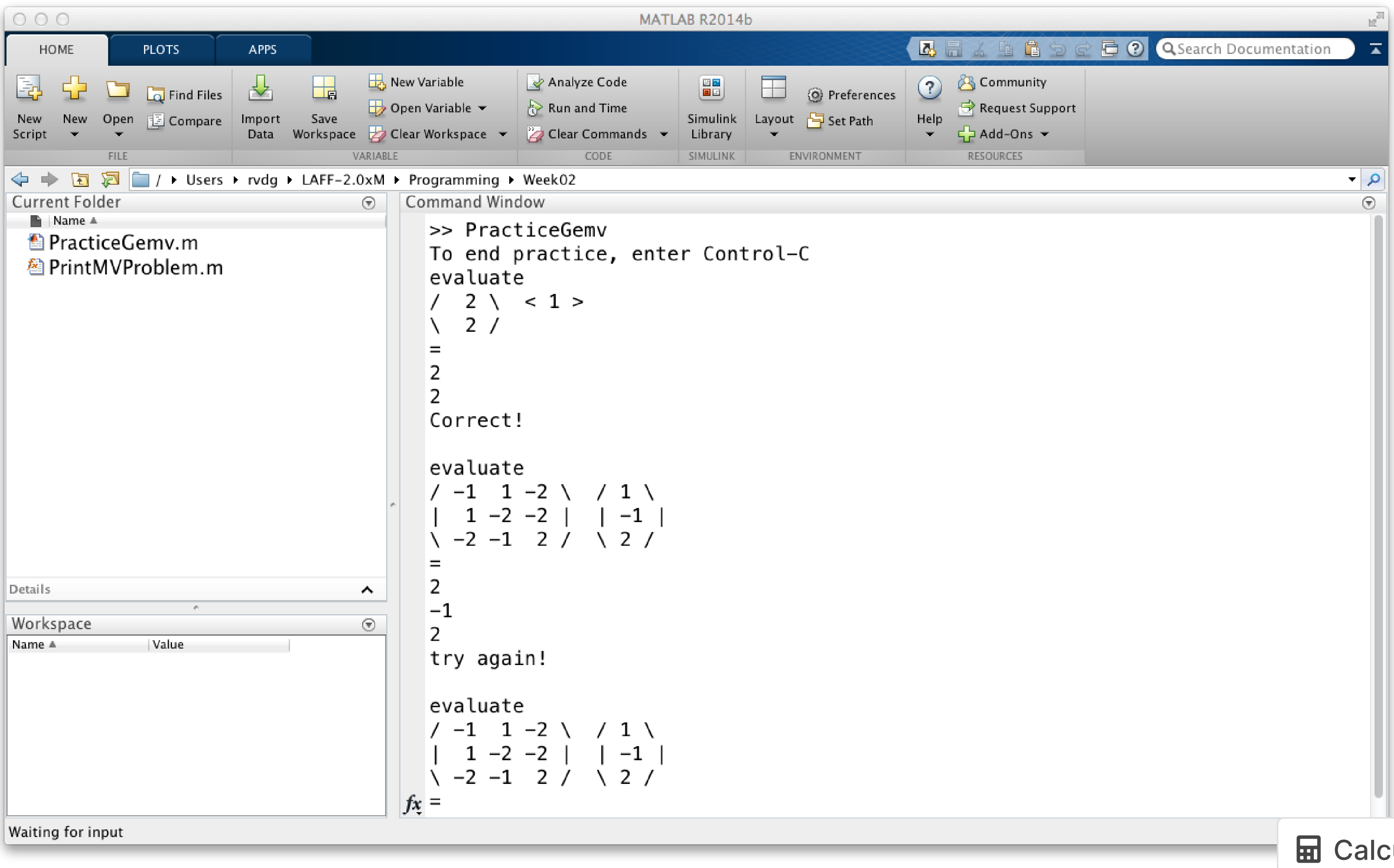
You can practice as little or as much as you want!

The following illustrations are for the desktop version of Matlab, but it should be pretty easy to figure out what to do instead with Matlab Online.

Log on to Matlab Online and change the current directory to the directory where these files exist so that your window looks something like



Then type `PracticeGemv` in the command window and you get to practice all the matrix-vector multiplications you want! For example, after a bit of practice my window looks like



THIS IS NOT A GRADED HOMEWORK. NO BOX TO CHECK!

What you notice is that the result vector is entered as a column of numbers, rather than a vector as MATLAB would normally expect.

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Calculator