Homework

Homework due Jul 29, 2020 21:30 IST

The exercises below will count towards your grade. **You have only one chance to answer these questions**. Take your time, and think carefully before answering.

Problem 1

20.0/20.0 points (graded)

Suppose a fair die is tossed ten consecutive times. Consider these two possible outcomes:

> **Outcome 1:** The ten tosses result in the sequence $\langle 6, 6, 6, 6, 6, 6, 6, 6, 6, 6 \rangle$ (in that order).
>
> **Outcome 2:** The ten tosses result in the sequence $\langle 6, 1, 1, 6, 1, 6, 1, 6, 1, 1 \rangle$ (in that order).

Which outcome is more probable?

- **Outcome 1** is more probable than **Outcome 2**.
- **Outcome 2** is more probable than **Outcome 1**.
- (selected) **Outcome 1** and **Outcome 2** are equally probable.

✔

Explanation

They are equally probable. This is so because the outcome of each toss is independent of the outcome of the preceding tosses. So, the probability of **Outcome 1** is $(1/6)^{10}$, as is the probability of **Outcome 2**.

Next, suppose you are dealt a hand of ten cards from a standard deck of 52 cards, half of which are red (R) and half of which are black (B). Consider these two possible outcomes:

Outcome X: You are dealt a sequence of the form $\langle B, B, B, B, B, B, B, B, B, B \rangle$ (in that order).

Outcome Y: You are dealt a sequence of the form $\langle B, R, R, B, R, B, R, B, R, R \rangle$ (in that order).

Which outcome is more probable?

- ○ **Outcome X** is more probable than **Outcome Y**.
- ◉ **Outcome Y** is more probable than **Outcome X**.
- ○ **Outcome X** and **Outcome Y** are equally probable.

✔

Explanation

Outcome Y is more probable than **Outcome X** because as more and more black cards are dealt, it becomes less and less probable that additional black cards will be dealt. Here's the same point, put a little more precisely: of the ordered 10-tuples of cards that can be built from a standard pack of cards,

$$26 \times 25 \times 24 \times 23 \times 22 \times 21 \times 20 \times 19 \times 18 \times 17$$

of them correspond to **Outcome X**, and

$$26 \times 26 \times 25 \times 25 \times 24 \times 24 \times 23 \times 23 \times 22 \times 21$$

of them correspond to **Outcome Y**. Since the latter number is larger, **Outcome Y** is more probable.

Submit   You have used 1 of 1 attempt

Answers are displayed within the problem

Problem 2

20.0/20.0 points (graded)

Suppose you are about to be dealt a two-card hand from a standard deck of cards (which consists of 52 cards, four of which are aces and thirteen of which are diamonds). The cards are picked totally at random.

What credence should you assign to the proposition that your two-card hand will contain at least one ace? (Represent your answer as a fraction of whole numbers.)

(If you have not attempted a problem like this before, you might find online resources such as this one helpful.)

33/221

✔ **Answer:** 396/2652

$\frac{33}{221}$

Explanation

The number of ordered pairs of cards that can be built from a standard pack of cards is $52 \times 51 = 2652$. Since we have no more reason to think that one of these outcomes will occur than we have for thinking that any other will occur, your credence that any one of them will is $1/2652$. Of our ordered pairs, 4×3 pairs contain two aces, 4×48 have an ace only in their first position, and 48×4 have an ace only in their second position. So the total number of pairs with at least one ace is:

$$(4 \times 3) + (4 \times 48) + (48 \times 4) = 396.$$

Since each of them has a probability of $1/2652$ of occurring, the probability of being dealt a pair with at least one ace is $396/2652 = 33/221$. And so *that* should be your credence that you will be dealt a pair with an ace.

Suppose again that you are dealt a two-card hand from a standard deck of cards (13 out of 52 of which are diamonds). And again, the cards are picked totally at random. This time, you are offered a bet. If your hand contains at least one diamond, you get $\$100$. If it doesn't, you lose $\$10$.

Assuming you value $\$n$ to degree n, what is the expected value of taking the bet? (Represent your answer as a fraction of whole numbers.)

1310/34

✔ **Answer:** 102180/2652

$\frac{1310}{34}$

Explanation

We first need to calculate the probability that you'll be dealt a hand with at least one diamond. The probability that you'll be dealt any given two-card hand (i.e., ordered pair) is $1/2652$. Of our ordered pairs, 13×12 pairs contain two diamonds, 13×39 have a diamond only in their first position, and 39×13 have a diamond only in their second position. So the total number of pairs with at least one diamond is

$$(13 \times 12) + (13 \times 39) + (39 \times 13) = 1170$$

Since each of them has a probability of $1/2652$ of occurring, the probability of being dealt a pair with at least one diamond is $1170/2652$. We can now perform the expected value calulation

$$EV\,(\text{Bet}) = (100 \times 1170/2652) + (-10 \times 1482/2652)$$
$$EV\,(\text{Bet}) = 117000/2652 - 14820/2652$$
$$EV\,(\text{Bet}) = 102180/2652 = 8515/221$$

Submit You have used 1 of 1 attempt

Answers are displayed within the problem

Problem 3

20/20 points (graded)

Recall the following connection between objective and subjective probability:

> *The Objective-Subjective Connection*: The objective probability of A at time t is the subjective probability that a perfectly rational agent would assign to A, if she had perfect information about events before t and no information about events after t.

And recall that a *deterministic world* is such that given a full specification of the world at any given time, the laws can be used to determine a full specification of the world at any future time.

Now, suppose that we live in a deterministic world, and that I am about to toss an ordinary coin. Assuming that the Objective-Subjective Connection holds, what is the *objective* probability that the toss will land heads?

- ○ 0
- ○ $.5$
- ○ 1
- ● Either 0 or 1

✔

Explanation

It is either 0 or 1. For the coin will either land heads or it won't. If it will, then, since we live in a deterministic world, a perfectly rational agent with access to full information about the initial conditions, should be able to figure out that the coin will land heads. So she would assign a subjective probability of 1 to the proposition that the coin lands heads. By the Objective-Subjective connection, the current objective probability that the coin will land heads is 1. If the coin won't land heads, a similar reasoning shows that the current objective probability that the coin will land heads is 0.

What *subjective* probability should you assign to the the proposition that the toss will land heads? (Assume you're in an ordinary situation and don't know the exact details of the forces acting on the coin toss.)

- ( ) 0
- (•) $.5$
- ( ) 1
- ( ) Either 0 or 1

✔

Explanation

Orindary coins land heads about half the time, when tossed. This is all the information you have to go on, in the above scenario, when deciding your subjective probability that the coin will land heads. So you should assign it subjective probability $.5$.

Submit You have used 1 of 1 attempt

Answers are displayed within the problem

Problem 4

10/10 points (graded)

You are offered a chance to play a game that works as follows. A coin will be tossed ten times. If all ten tosses are Tails, you get $\$100,000$. If not, you get $\$2^n$ if the coin first lands Heads on the nth toss.

Assuming you value $\$n$ to degree n, what is the expected value of playing this game? (Represent your answer as a fraction of whole numbers.)

3445/32

Answer: 110240/1024

$$\frac{3445}{32}$$

Explanation

The expected value calculation goes like this:

$$EV\left(Bet\right) = (\sum_{n=1}^{10} (1/2)^n \times \$2^n) + ((1/2)^{10} \times \$100,000)$$

$$EV\left(Bet\right) = 10 + \frac{100,000}{1024}$$

$$EV\left(Bet\right) = \frac{10,240 + 100,000}{1024}$$

$$EV\left(Bet\right) = \frac{110,240}{1024}$$

Submit You have used 1 of 1 attempt

Answers are displayed within the problem