Exercises: Non-linear Least Squares (2)

Bookmark this page

Point on a circle

2/2 points (ungraded)

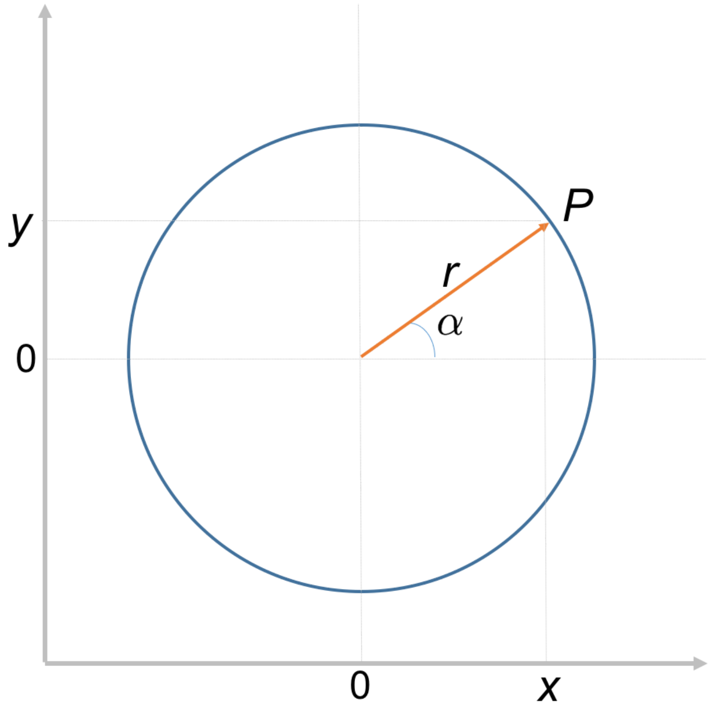
It is known that point P is on a circle with radius $r = 2$ meters. We are interested in the unknown angle α. Based on an observation of the y-coordinate, we would like to get an estimate of this angle. Our observation is $y = 1.02$.

Q&A Forum

4.ⓔ Non-linear Least Squares (optional topic)

Feedback

- 5. How precise is the estimate?
- 6. Does the estimate make sense?
- Pre-knowledge Mathematics
- MATLAB Learning Content



What is the corresponding non-linear observation equation?

- ○ $E\{\underline{y}\} = r\cos\alpha$
- ◉ $E\{\underline{y}\} = r\sin\alpha$ ✔
- ○ $E\{\underline{y}\} = r\tan\alpha$

Explanation

According to the figure we have $\sin\alpha = \frac{y}{r}$

We will now apply the iteration procedure to estimate α based on linearizing the observation equation. Our initial guess for α is: $\alpha_{[0]} = 30^o$.

What is the value for $\Delta y_{[0]} = y - a(x_{[0]})$? *Give your answer in degrees to 2 decimal places*

0.02

✔ **Answer:** 0.02

0.02

Explanation

$\Delta y_{[0]} = 1.02 - 2\sin(30)$

Submit

Point on a circle (cont'd)

4/4 points (ungraded)

Continuing with previous result, we will use the linear approximation of our non-linear function to get a new estimate for the angle α. Of course this is not really necessary in order to calculate α, since you can simply use the inverse sine-function ($\arcsin$) to calculate it. However, we do this is in order to illustrate the principle of non-linear least squares.

The first step is to find the linearized observation equation in the form:

$$\Delta y_{[0]} = \partial_\alpha a(\alpha_{[0]})\Delta\alpha_{[0]}$$

where $a(\alpha)$ is equal to the found $E\{\underline{y}\}$ from the previous question.

Note: our unknown parameter is now called α and not x.

The derivative $\partial_\alpha a(\alpha)$ is equal to:

- ( ) $-r\cos\alpha$
- ( ) $r\sin\alpha$
- (•) $r\cos\alpha$ ✔
- ( ) $-r\sin\alpha$

Note: if you use Matlab, note that the $\sin$ and $\cos$ functions work with radians and not degrees. Either you should convert the angles to radians, or use sind and cosd in Matlab.

Calculate $\Delta\alpha_{[0]}$. *Give your answer in degree and upto 2 decimal places.*

$\Delta\alpha_{[0]} =$

0.66 ✔ **Answer:** 0.66

0.66

What is then the new estimate of α. *Give your answer in degree and upto 2 decimal places.*

$\alpha_{[1]} =$

30.66 ✔ **Answer:** 30.66

30.66

Now use this estimate as your new guess for the angle, and repeat the procedure. What will be the new estimate $\alpha_{[2]}$? *Give your answer in degree and upto 2 decimal places.*

30.66 ✔ **Answer:** 30.66

30.66

Explanation

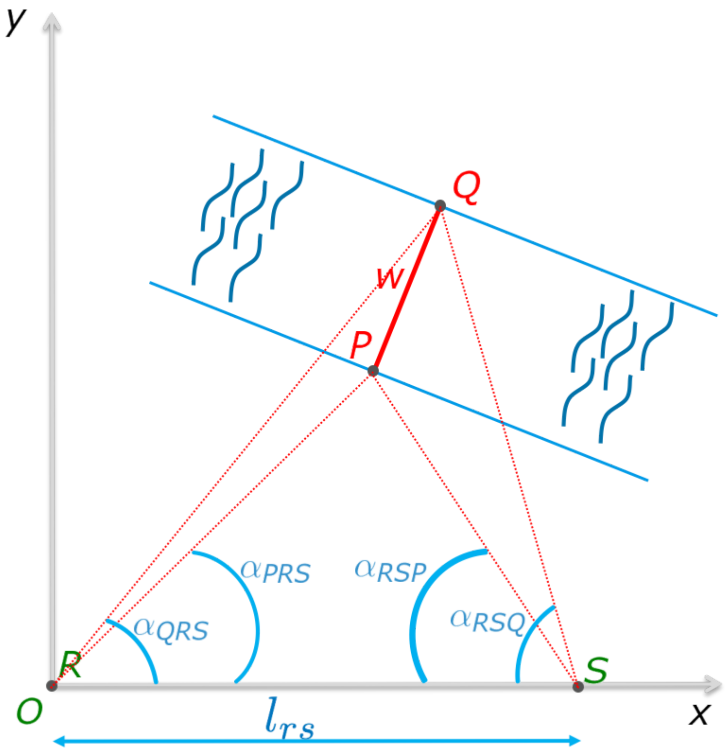
$$\Delta\alpha_{[0]} = \frac{\Delta y_{[0]}}{r \cos \alpha_{[0]}} = \frac{0.02}{2 \cos 30}$$

$$\alpha_{[1]} = \alpha_{[0]} + \Delta\alpha_{[0]} = 30 + 0.01.$$

Submit

Canal width by the cool team

1/1 point (ungraded)



In the canal width estimation problem using the cool team's measurements, we do have the following observations, knowns and unknowns:

Knowns: $x_R = 0$, $y_R = 0$, $y_S = 0$

Unknowns: x_P, y_P, x_Q, y_Q, x_S

Observables : $\underline{\alpha}_{PRS}, \underline{\alpha}_{QRS}, \underline{\alpha}_{RSP}, \underline{\alpha}_{RSQ}, \underline{l}_{RS}$

We start with showing how to linearize the observation equation of angle α_{PRS}

Recall from the video that if we have a single non-linear observation equation $y = a(x)$ with x an $n \times 1$ vector, then the linearized observation equation, using an initial guess $x_{[0]}$ for x, reads:

$$\Delta y_{[0]} = y - a(x_{[0]}) = J_{[0]} \Delta x_{[0]}$$

with gradient vector

$$J_{[0]} = \begin{bmatrix} \frac{\partial a(x_{[0]})}{\partial x_1} & \frac{\partial a(x_{[0]})}{\partial x_2} & \ldots & \frac{\partial a(x_{[0]})}{\partial x_n} \end{bmatrix}$$

containing the partial derivatives with respect to all unknown parameters.

So let's look at observation α_{PRS} and its non-linear observation equation:

$$E\{\underline{\alpha}_{PRS}\} = a(x) = \arctan(\frac{y_P}{x_P})$$

In order to find the partial derivatives, we first need to review the chain rule of differentiation: if f is a function of g, and g on its turn is a function of x then

$$\frac{\partial f}{\partial x} = \frac{\partial f}{\partial g} \cdot \frac{\partial g}{\partial x}$$

In our case we have $f = \arctan(g)$, and $g = \frac{y_P}{x_P}$:

$$\frac{\partial a(x)}{\partial x_P} = \frac{\partial f}{\partial g} \cdot \frac{\partial g}{\partial x_P}$$

$$= \frac{1}{1+g^2} \cdot \left(-\frac{y_P}{x_p^2}\right)$$

$$= \frac{1}{1+(\frac{y_P}{x_P})^2} \cdot \left(-\frac{y_P}{x_P^2}\right)$$

$$= \frac{x_P^2}{x_P^2+y_P^2} \cdot \left(-\frac{y_P}{x_P^2}\right)$$

$$= \frac{-y_P}{x_P^2+y_P^2}$$

Try to take the partial derivatives with respect to the other unknown parameters yourself and then answer the following question.

Which of the following is the correct gradient vector for the linearized observation equation of $E\{\underline{\alpha}_{PRS}\} = \arctan(\frac{y_P}{x_P})$? *For notational convenience, we omitted the iteration index here.*

- $J = \begin{bmatrix} \frac{-y_P}{x_P^2+y_P^2} & \frac{x_P}{x_P^2+y_P^2} \end{bmatrix}$
- $J = \begin{bmatrix} \frac{-y_P}{x_P^2+y_P^2} & \frac{-x_P}{x_P^2+y_P^2} \end{bmatrix}$
- $J = \begin{bmatrix} \frac{-y_P}{x_P^2+y_P^2} & \frac{x_P}{x_P^2+y_P^2} & 0 & 0 & 0 \end{bmatrix}$ ✔

- $J = \left[\frac{-y_P}{x_P^2+y_P^2} \;\; \frac{-x_P}{x_P^2+y_P^2} \;\; 0 \;\; 0 \;\; 0 \right]$
- $J = \left[\frac{-y_P}{x_P^2+y_P^2} \;\; \frac{x_P}{x_P^2+y_P^2} \;\; \frac{-y_Q}{x_Q^2+y_Q^2} \;\; \frac{x_Q}{x_Q^2+y_Q^2} \;\; 0 \right]$
- $J = \left[\frac{-y_P}{x_P^2+y_P^2} \;\; \frac{x_P}{x_P^2+y_P^2} \;\; \frac{-y_Q}{x_Q^2+y_Q^2} \;\; \frac{-x_Q}{x_Q^2+y_Q^2} \;\; 0 \right]$

Explanation

The gradient vector must have length $n = 5$. Similarly as before we can find:

$$\frac{\partial a(x)}{\partial y_P} = \frac{\partial f}{\partial g} \cdot \frac{\partial g}{\partial y_P} = \frac{x_P^2}{x_P^2 + y_P^2} \cdot (\frac{1}{x_P})$$

In this case $a(x)$ is not a function of the other unknown parameters, so the corresponding partial derivatives are equal to 0.

Submit

✔ Correct (1/1 point)