8.7.1 Discretization

MO2.4

The basic philosophy of the numerical methods we will study for solving IVPs is to start from a known initial state, $\underline{u}(t_I) = \underline{u}_I$, and somehow approximate the solution a small time forward, $\underline{u}(t_I + \Delta t)$ where Δt is a small time increment. Then, we repeat this process and move forward to the next time to find an approximation to $\underline{u}(t_I + 2\Delta t)$, and so on. This is known as discretizing the solution, as we have moved from representing infinitely many times t, i.e. all t from t_I to t_F to a representation at a discrete (i.e. finite) set of time points. This discrete representation is shown in Figure 8.12. In the limit as $\Delta t \rightarrow 0$, the discrete solution representation approaches the exact solution.

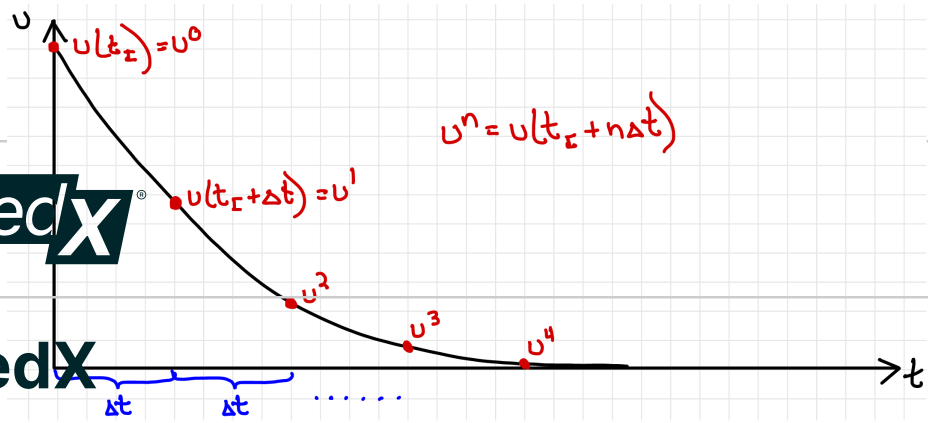

Figure 8.12: Discrete representation of the exact solution in which $u(t)$ is sampled at $t^n = t_I + n\Delta t$ giving $u^n = u(t^n)$

We will consider the situation in which Δt is fixed for the entire integration from $t = t_I$ to t_F. However, the best methods for solving IVPs tend to be adaptive methods in which Δt is adjusted depending on the current approximation.

Let's first put some notation in place. Superscripts will be used to indicate a particular iteration, that is t^n denotes the time at iteration n. Thus, assuming constant Δt,

$$t^n = t_I + n\Delta t. \tag{8.54}$$

The approximation from the numerical method will be defined as $\underline{v}$. Thus, using the superscript notation,

$$\underline{v}^n = \text{the approximation of } \underline{u}(t^n). \tag{8.55}$$

Video introducing discretization

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