10. Variables and units

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Lecture due Aug 4, 2021 20:30 IST Completed

Reflect

In the first lecture, we mentioned that the world around us is multivariable, and that multivariable functions can be used to represent the world around us. So far, we've been thinking about x, y, and z as spatial variables, which we often expect to have units like meters, or inches. Let's think about what this means for how we represent functions first.

Suppose that x and y represent position in units of meters. And $z = f(x, y)$ is an altitude measured again in meters. Let's look at our first example function $z = x^2 + y^2$.

Both x^2 and y^2 have units of square meters. How can z have units of meters as we expect?

Secretly, there must be constant multiples that also have units embedded within this function that help all of the units to make sense. Really, what we have is

$$z = 1x^2 + 1y^2$$

where both of the constants 1 have units of 1/meters. In this way, every term in this function has the same units.

General rules for units of multivariable functions

1. All terms that are added together or equated in a multivariable function must have the same units. That is, we cannot add a term with units of meters to a term with units of square meters. It doesn't have a physical meaning that we can interpret.
2. Functions like $\sin$, $\cos$, $\ln$, $e^{f(x,y)}$ must have input arguments that are unit-less.
3. Taking derivatives changes units. (See exercise below.)

(Optional) Remark about unit-less input arguments

If you think about expressing a function like $\sin(f(x))$ in one variable as their Taylor series, it makes sense that an infinite sum of terms must have every term unit-less. Otherwise, we would break the first rule above. The same holds for multivariable functions with multivariable Taylor series!

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Example 10.1

Consider the ideal gas law

$$PV = nRT$$

This relationship involves 5 variables:

Variable	Quantity	Units
P	pressure	Newtons per square meter $(\mathrm{N}/\mathrm{m}^2)$
V	volume	cubic meters (m^3)

n	amount	number of moles (mol)
R	ideal gas constant	(Exercise below)
T	temperature	Kelvin (K)

Exercise

1.0/1 point (graded)
Find the units of the ideal gas constant R so that the equation makes physical sense.

(Type `N` for Newtons, type `m` for meters, type `mol` for moles, type `K` for Kelvins.)

N*m / K / mol

✔ **Answer:** N*m/(mol*K)

? INPUT HELP

Solution:

The units of both sides of the equation must be equal for this to make sense.

$$\underset{\mathrm{N/m^2}}{P}\,\underset{\mathrm{m^3}}{V} = \underset{\mathrm{mol}}{n}\,R\,\underset{\mathrm{K}}{T}$$

Therefore the units of R must be $\dfrac{\mathrm{N}\cdot\mathrm{m}}{\mathrm{mol}\cdot\mathrm{K}}$ for the units of both sides of this equation to balance.

Submit You have used 1 of 3 attempts

Answers are displayed within the problem

Units and partial derivatives

2.0/2 points (graded)
Suppose that a quantity $z = f(x,t)$ has units of temperature measured in Kelvins, the variable x has units of length measured in meters, and the variable t has units of time measured in seconds.

What are the units of the partial derivative f_x?

(Type `N` for Newtons, type `m` for meters, type `mol` for moles, type `K` for Kelvins, type `s` for seconds.)

K/m

✔ **Answer:** K/m

What are the units of the partial derivative f_t?

(Type `N` for Newtons, type `m` for meters, type `mol` for moles, type `K` for Kelvins, type `s` for seconds.)

K/s

✔ **Answer:** K/s

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Solution:

Calculator Hide Notes

Note that the units of the partial derivative f_x are the same as the units of its approxi per meter.

Similarly, the units of the partial derivative f_t are Kelvin per second.

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Answers are displayed within the problem

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Secret Constants
I would like to politely say I am having some difficulty with the propositions advanced in this section. One of the early examples of a ... 8

What is the function f that we have to find fx and fy derivatives?
What is the function f that we have to find fx and fy derivatives? 4

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