



MITx: 6.041x Introduction to Probability - The Science of Uncertainty



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Bookmark

Problem 3: A Joint PDF given by a simple formula

(4/4 points)

Random variables X and Y are distributed according to the joint PDF

$$f_{X,Y}(x,y) = \begin{cases} ax, & \text{if } 1 \leq x \leq 2 \text{ and } 0 \leq y \leq x, \\ 0, & \text{otherwise.} \end{cases}$$


1. Find the constant a . $a =$ 

Answer: 0.42857


2. Determine the marginal PDF $f_Y(y)$. (Your answer can be either numerical or algebraic functions of y).For $0 \leq y \leq 1$,

Unit overview


Lec. 8: Probability density functions

Exercises 8 due Mar 18, 2016 at 23:59 UTC 

Lec. 9: Conditioning on an event; Multiple r.v.'s

Exercises 9 due Mar 18, 2016 at 23:59 UTC 


Lec. 10: Conditioning on a random variable; Independence; Bayes' rule

Exercises 10 due Mar 18, 2016 at 23:59 UTC 

Standard normal table

Solved problems

Problem Set 5

Problem Set 5 due Mar 18, 2016 at 23:59 UTC 

Unit summary

- ▶ Unit 6: Further topics on random variables
- ▶ Unit 7: Bayesian inference

$$f_Y(y) = \boxed{9/14}$$

✓ Answer: 0.64286

For $1 < y \leq 2$,

$$f_Y(y) = \boxed{(3/14)*(4-y^2)}$$

✓ Answer: (3/14)*(4-y^2)

3. Determine the conditional expectation of $1/X$ given that $Y = 3/2$.

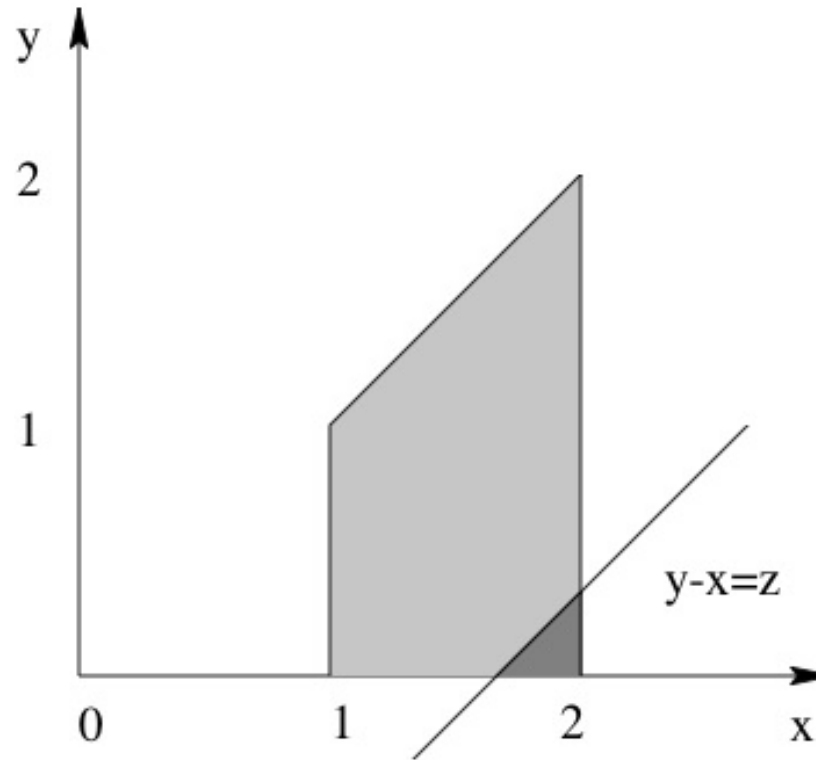
$$\mathbf{E}[1/X \mid Y = 3/2] = \boxed{4/7}$$

✓ Answer: 0.57143

Answer:

Let us draw the region where $f_{X,Y}(x,y)$ is nonzero:

- ▶ Exam 2
- ▶ Unit 8: Limit theorems and classical statistics
- ▶ Unit 9: Bernoulli and Poisson processes
- ▶ Unit 10: Markov chains
- ▶ Exit Survey
- ▶ Final Exam



1. The joint PDF has to integrate to 1. From

$$\int_1^2 \int_0^x ax \, dy \, dx = \frac{7}{3}a = 1,$$

we get $a = \frac{3}{7}$.

2. To find the marginal PDF of Y , we integrate the joint PDF over x :

$$\begin{aligned}
 f_Y(y) &= \int_{-\infty}^{+\infty} f_{X,Y}(x,y) dx \\
 &= \begin{cases} \int_1^2 \frac{3}{7}x dx, & \text{if } 0 \leq y \leq 1, \\ \int_y^2 \frac{3}{7}x dx, & \text{if } 1 < y \leq 2, \\ 0, & \text{otherwise,} \end{cases} \\
 &= \begin{cases} \frac{9}{14}, & \text{if } 0 \leq y \leq 1, \\ \frac{3}{14}(4 - y^2), & \text{if } 1 < y \leq 2, \\ 0, & \text{otherwise.} \end{cases}
 \end{aligned}$$

3. We first find the conditional PDF of X given $Y = 3/2$.

$$f_{X|Y}\left(x \middle| \frac{3}{2}\right) = \frac{f_{X,Y}(x, \frac{3}{2})}{f_Y(\frac{3}{2})} = \frac{\frac{3}{7}x}{\frac{3}{14}\left(4 - \left(\frac{3}{2}\right)^2\right)} = \frac{8}{7}x, \text{ for } \frac{3}{2} \leq x \leq 2,$$

and equals 0 otherwise.

Then,

$$\mathbf{E} \left[\frac{1}{X} \mid Y = \frac{3}{2} \right] = \int_{-\infty}^{\infty} \frac{1}{x} \cdot f_{X|Y} \left(x \mid \frac{3}{2} \right) dx = \int_{3/2}^2 \frac{1}{x} \cdot \frac{8}{7} x dx = \frac{4}{7}.$$

You have used 2 of 3 submissions

DISCUSSION

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