8. A Union-Intersection Test

Let $X_1, \ldots, X_n$ be i.i.d. Bernoulli random variables with unknown parameter $p \in (0,1)$. Suppose we want to test

$$H_0 : p \in [0.48, 0.51] \quad \text{vs} \quad H_1 : p \notin [0.48, 0.51]$$

We want to construct an asymptotic test ψ for these hypotheses using $\overline{X}_n$. For this problem, we specifically consider the family of tests ψ_{c_1,c_2} where we reject the null hypothesis if either $\overline{X}_n < c_1 \leq 0.48$ or $\overline{X}_n > c_2 \geq 0.51$ for some c_1 and c_2 that may depend on n, i.e.

$$\psi_{c_1,c_2} = \mathbf{1}\left((\overline{X}_n < c_1) \cup (\overline{X}_n > c_2)\right) \qquad \text{where } c_1 < 0.48 < 0.51 < c_2.$$

Throughout this problem, we will discuss possible choices for constants c_1 and c_2, and their impact to both the asymptotic and non-asymptotic level of the test.

(a)

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1/1 point (graded)

Which expression represents the (smallest asymptotic) level α of this test? Recall the (smallest asymptotic) level equals the maximum Type 1 error rate.

- (selected) $\alpha = \max_{p\in[0.48,0.51]} \left(\mathbf{P}_p\left(\overline{X}_n < c_1\right) + \mathbf{P}_p\left(\overline{X}_n > c_2\right)\right)$
- $\alpha = \max_{p\in[0.48,0.51]} \left(\max\left(\mathbf{P}_p\left(\overline{X}_n < c_1\right), \mathbf{P}_p\left(\overline{X}_n > c_2\right)\right)\right)$
- $\alpha = \max_{p\in[0.48,0.51]} \mathbf{P}_p\left(\overline{X}_n < c_1\right)$
- $\alpha = \max_{p\in[0.48,0.51]} \mathbf{P}_p\left(\overline{X}_n > c_2\right)$
- $\alpha = \max_{p\in[0.48,0.51]} \left(\mathbf{P}_p\left(\overline{X}_n < c_1\right) \cdot \mathbf{P}_p\left(\overline{X}_n > c_2\right)\right)$

✔

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(b)

4.0/4 points (graded)

Use the central limit theorem and the approximation $\sqrt{p(1-p)} \approx \frac{1}{2}$ for $p \in [0.48, 0.51]$ to approximate $\mathbf{P}_p\left(\overline{X}_n < c_1\right)$ and $\mathbf{P}_p\left(\overline{X}_n > c_2\right)$ for large n. Express your answers as a formula in terms of c_1, c_2, n and p.

(Write **Phi** for the cdf of a Normal distribution, **c_1** for c_1, and **c_2** for c_2.)

$\mathbf{P}_p\left(\overline{X}_n < c_1\right) \approx$ Phi(2*sqrt(n)*(c_1-p)) ✔

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For what value of $p \in [0.48, 0.51]$ is the expression above for $\mathbf{P}_p\left(\overline{X}_n < c_1\right)$ maximized?

$\mathbf{P}_p\left(\overline{X}_n < c_1\right)$ is max at $p =$ 0.48

$\mathbf{P}_p\left(\overline{X}_n > c_2\right) \approx$ 1-Phi(2*sqrt(n)*(c_2-p))

For what value of $p \in [0.48, 0.51]$ is the expression above for $\mathbf{P}_p\left(\overline{X}_n > c_2\right)$ maximized?

$\mathbf{P}_p\left(\overline{X}_n > c_2\right)$ is max at $p =$ 0.51

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(c)

1.0/1 point (graded)

Next, we combine the results from parts (a) and (b).

Apply the inequality $\max_x (f(x) + g(x)) \leq \max_x f(x) + \max_x g(x)$ to the expression for the (asymptotic) level α obtained in part (a) and use the results from part (b) to give an upper bound on α.

Express your answer as a formula in terms of c_1, c_2, and n.
(Write **Phi** for the cdf of a Normal distribution, **c_1** for c_1, and **c_2** for c_2.)

$\alpha \leq$ 1+Phi(2*sqrt(n)*(c_1-0.48))-Phi(2*sqrt(n)*(c_2-0.51))

(*Food for thought:* Is this upper bound tight? A bound is tight if equality may be achieved.)

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(d)

2.0/2 points (graded)

Suppose that we wish to have a level $\alpha = 0.05$. What c_1 and c_2 will achieve $\alpha = 0.05$? Choose c_1 and c_2 by setting the expressions you obtained above for $\max_{p\in[0.48,0.51]} \mathbf{P}_p\left(\overline{X}_n < c_1\right)$ and $\max_{p\in[0.48,0.51]} \mathbf{P}_p\left(\overline{X}_n > c_2\right)$ to both be 0.025.

(If applicable, enter **q(alpha)** for q_α, the $1-\alpha$ -quantile of a standard normal distribution, e.g. enter **q(0.01)** for $q_{0.01}$.)

$c_1 =$ (q(0.975))/(2*sqrt(n))+0.4

$c_2 =$ (q(0.025))/(2*sqrt(n))+0.5

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(e)

2/2 points (graded)

We will now show that the values we just derived for c_1 and c_2 are in fact too conservative.

Recall the expression from part (b) for $\mathbf{P}_p\left(\overline{X}_n < c_1\right)$ for large n. For $p > 0.48$ (note the strict inequality), find $\lim_{n\to\infty} \mathbf{P}_p\left(\overline{X}_n < c_1\right)$.

$\lim_{n\to\infty} \mathbf{P}_{p>0.48}\left(\overline{X}_n < c_1\right) =$ 0

Similarly, for $p < 0.51$ (note the strict inequality), find $\lim_{n\to\infty} \mathbf{P}_p\left(\overline{X}_n > c_2\right)$. Use the expression you found in part (b) for $\mathbf{P}_p\left(\overline{X}_n > c_2\right)$.

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$\lim_{n\to\infty} \mathbf{P}_{p<0.51}\left(\overline{X}_n > c_2\right) =$ 0

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(f)

2/2 points (graded)

Note: This part of the problem will contain multiple steps but you would only enter answers to the final step. Also **refer to the recitation 4 last video** *Composite Test* for related ideas.

Next, we analyze the asymptotic test given different possible values of p, in order to choose suitable and sufficiently-tight c_1 and c_2. Looking more closely at part (d), we may note that the asymptotic behavior of the expressions for the errors are different depending on whether $p = 0.48, 0.48 < p < 0.51$, or $p = 0.51$.

Based on your answers and work from the previous part, evaluate the asymptotic Type 1 error

$$\mathbf{P}\left(\overline{X}_n < c_1\right) + \mathbf{P}\left(\overline{X}_n > c_2\right).$$

on each of the three cases for the value of p in terms of c_1, c_2, and n, and determine in each case which component(s) of the Type 1 error will converge to zero.

This would allow you to come up with a new set of conditions for c_1 and c_2 in terms of n, given the desired level of 5%. Enter these values (in terms of n) below.

(If applicable, enter **q(alpha)** for q_α, the $1-\alpha$ -quantile of a standard normal distribution, e.g. enter **q(0.01)** for $q_{0.01}$. Do not worry about the parser not rendering **q(alpha)** properly; the grader will work nonetheless. You could also enclose **q(alpha)** by brackets for the rendering to show properly.)

$c_1 =$ 0.48-(q(0.05))/(2*sqrt(n))

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$c_2 =$ 0.51+(q(0.05))/(2*sqrt(n))

STANDARD NOTATION

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Correct (2/2 points)

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What does e really mean
Can anyone explain the "english" behind the purpose of this. 2

Format of (f)/(d)
I see other threads and problems with q(alpha). Replacing q(alpha) with number, I have expression of form: constant1 + (constant2/g(n)) Is this reasonable form for answer fo... 1

part c/d
I got part a an part b correct. I thought part c you just copy and paste the answers from part b since alpha is from part a? part d, I am pretty sure I got it right by setting two an... 1

[STAFF] - Grader not accepting q(alpha) In part(f)
At the very end of the part (f) question it says "(If applicable, enter q(alpha) for q_α , the $1-\alpha$ -quantile of a standard normal distribution, e.g. enter q(0.01) for $q_{0.01}$. Do not wor... 7

Question f) 2

[Staff]: about answer formula in (f)
For grader in (f), it only accept when using the approximation sqrt(p*(1-p))~1/2, but not the really value. For example if p=0.48, when my answer used sqrt(0.48*(1-0.48)), it sh... 1

Generating Speech Output] grader error? - Never mind, please disregard
Is it possible to check to see if my submitted answer for the first question in part e isn't actually correct? (Homework #3 Question #8) 2

Part b, q(alpha)

I believe we will need to put the expression of the CDF evaluated at C1, C2 but the equivalent normalized values. When I'm trying to normalize c2, I found that within the Phi st... 3

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