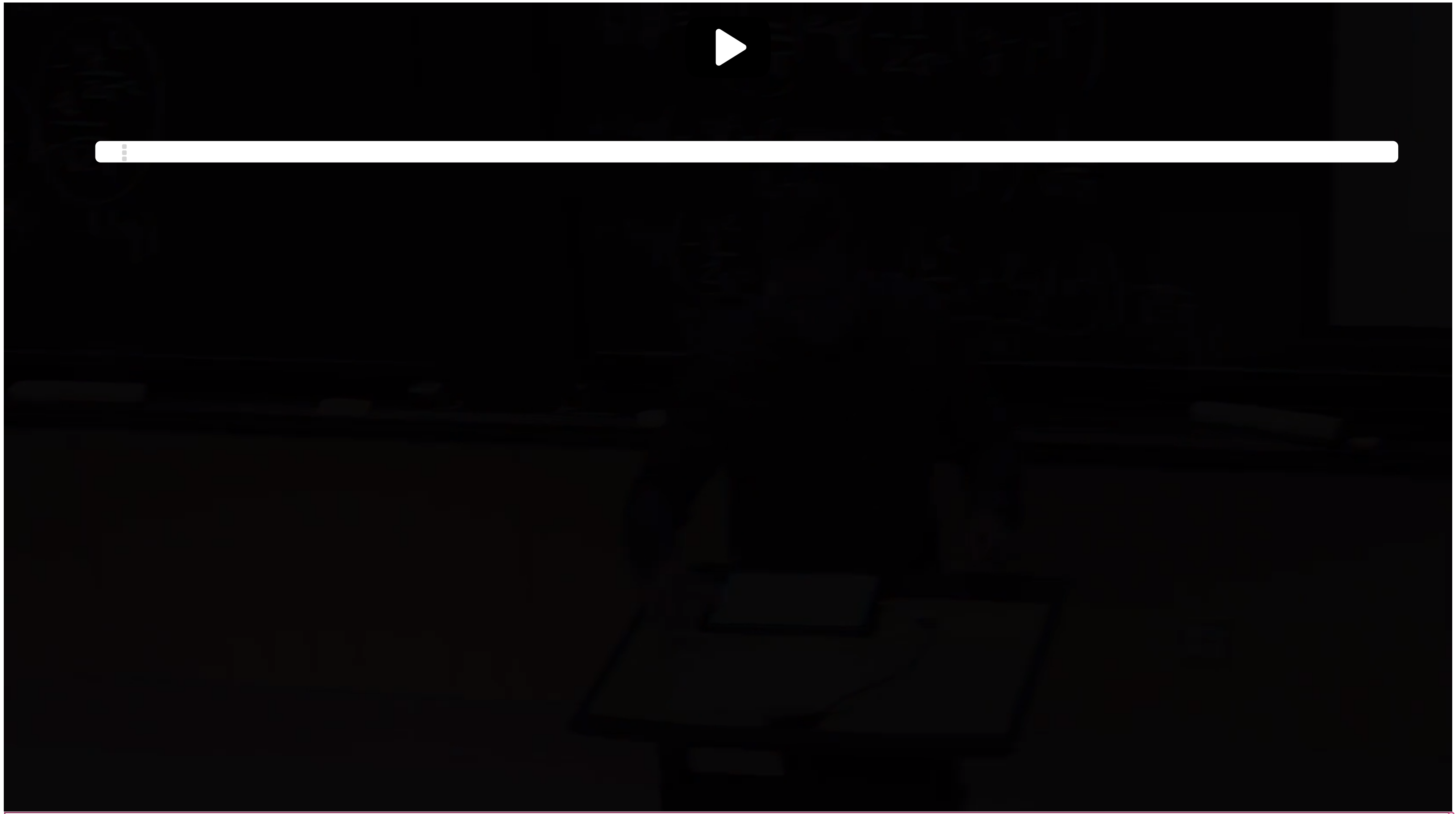
7. Exponential Family: Continuous Examples

Example: Gaussian Distribution



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Exponential Distribution as Exponential Families

4/4 points (graded)

Recall that the exponential distribution with parameter λ is given by the pdf by

$$f_\lambda(y) = \lambda e^{-\lambda y}.$$

Let $\theta = \lambda$. Rewrite $f_\lambda(y)$ in the form

$$f_\theta(y) = h(y)\exp(\eta(\theta)T(y) - B(\theta)),$$

and enter $\eta(\theta), T(y), B(\theta)$ below.

These functions are not unique. To get unique answers, let $h(y) = 1,$ and let the coefficient of y in $T(y)$ be $+1$.

$T(y) =$ y ✔ **Answer:** y

y

$\eta(\theta) =$ -theta ✔ **Answer:** -theta

$-\theta$

$B(\theta) =$ -ln(theta) ✔ **Answer:** -ln(theta)

$-\ln(\theta)$

If instead of $h(y) = 1,$ we had used $\tilde{h}(y) = C$ for some constant C, then what is $\widetilde{B}(\theta)$ in terms of $B(\theta)$ and C? That is, find $\widetilde{B}(\theta)$ such that the pdf $f_\theta(y)$ of $Y \sim$ **Exp** (θ) is

$$f_\theta(y) \ = \ \tilde{h}(y)\exp\left(\eta(\theta)T(y) - \widetilde{B}(\theta)\right).$$

(*Enter* ***B*** *for* $B(\theta)$ *and* ***C*** *for* C. *Your answer should be in terms of only* C *and* $B(\theta)$. *Enter "ln" for the natural logarithm.*)

$\widetilde{B}(\theta) =$ ln(C)+B ✔ **Answer:** B+ln(C)

STANDARD NOTATION

Solution:

$$f_\theta(y) = \theta e^{-\theta y} = e^{-(\theta)(y)-(-ln(\theta)}$$

Hence $\eta(\theta) = -\theta$, $T(y) = y$, $B(\theta) = -\ln(\theta)$. If instead $\tilde{h}(y) = C$ is used, then

$$f_\theta(y) = \theta e^{-\theta y} = Ce^{-(\theta)(y)-(-ln(\theta)+\ln(C))}$$

Hence $\widetilde{B}(\theta) = B(\theta) + \ln(C)$.

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Answers are displayed within the problem

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