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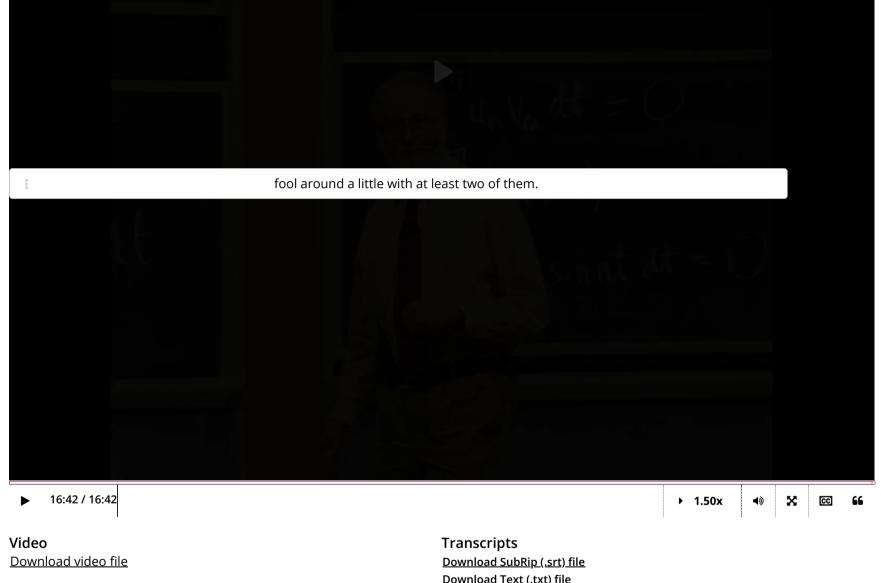
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7. (Optional) proof of Orthogonality Proof of orthogonality



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Proof of orthogonality

Here we prove that if $m \neq n$, then $\sin{(nt)}$ and $\cos{(nt)}$ are orthogonal to $\sin{(mt)}$ and $\cos{(mt)}$. To do this we use that fact that both $\sin{(nt)}$ and $\cos{(nt)}$ solve the differential equation $\ddot{x} + n^2x = 0$, and both $\sin{(mt)}$ and $\cos{(mt)}$ solve the differential equation $\ddot{x} + m^2x = 0$.

Supose that $u\left(t\right)$ solves the differential equation $\ddot{x}+n^2x=0$. And suppose $v\left(t\right)$ solves the differential equation $\ddot{x}+m^2x=0$ and $n\neq m$.

Then in particular

$$\int_{-\pi}^{\pi}\ddot{u}\left(t
ight)v\left(t
ight)\,dt \quad = \quad \dot{u}\left(t
ight)v\left(t
ight)ig|_{-\pi}^{\pi} - \int_{-\pi}^{\pi}\dot{u}\left(t
ight)\dot{v}\left(t
ight)\,dt.$$

Note that $\dot{u}\left(t\right)v\left(t\right)|_{-\pi}^{\pi}=0$. Both $u\left(t\right)$ and $v\left(t\right)$ are both periodic on $\left(-\pi,\pi\right]$, therefore so is $\dot{u}\left(t\right)v\left(t\right)$. Therefore the difference $\dot{u}\left(\pi\right)v\left(\pi\right)-\dot{u}\left(-\pi\right)v\left(-\pi\right)=0$.

Thus this reduces to the expression

$$\int_{-\pi}^{\pi}\ddot{u}\left(t
ight)v\left(t
ight)\;dt=-\int_{-\pi}^{\pi}\dot{u}\left(t
ight)\dot{v}\left(t
ight)\;dt.$$

A similar argument shows that

$$\int_{-\pi}^{\pi}u\left(t
ight)\ddot{v}\left(t
ight)\,dt=-\int_{-\pi}^{\pi}\dot{u}\left(t
ight)\dot{v}\left(t
ight)\,dt.$$

However, by using the substitution from the differential equation, we have that $\ddot{u}\left(t\right)=-n^{2}u\left(t\right)$ and $\ddot{v}\left(t\right)=-m^{2}v\left(t\right)$. Therefore it follows that

$$n^{2}\int_{-\pi}^{\pi}u\left(t
ight) v\left(t
ight) \,dt=m^{2}\int_{-\pi}^{\pi}u\left(t
ight) v\left(t
ight) \,dt.$$

Note that since we have assumed that m
eq n, the only way for this to hold is for the integral $\int_{-\pi}^{\pi} u\left(t
ight)v\left(t
ight)\,dt = 0$.

Remark 7.1 In disguise, this is the same as showing that two eigenvectors corresponding to distinct eigenvalues of a symmetric matrix are orthogonal.

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7. (Optional) proof of Orthogonality

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| Q | Generalization? Can this proof be generalized with another set of basis functions that satisfy a different ODE? In other words if {u,v} satisfy some linear ODE and form a basis set for some vec | 6 |
| € | Do we have resources for the other two ways to prove it? Can you please give insights or provide external resources on the proof of orthogonality using trig identities and complex exponential? I think it could be interesting to compa | 3 |
| Q | eigenvectors corresponding to distinct eigenvalues of a symmetric matrix | 2 |
| € | What's the intuition behind considering the given integral in the 'Using ODE' method? In the video, Prof. Mattuck says that this is the general method for discovering orthogonal bases, but he doesn't explain the reason for selecting the integral as the starting po | 6 |

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