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## Unit overview

## Lec. 11: Derived distributions

Exercises 11 due Mar 30, 2016 at 23:59 UTC

## Lec. 12: Sums of independent r.v.'s; Covariance and correlation

Exercises 12 due Mar 30, 2016 at 23:59 UTC

## Lec. 13: Conditional expectation and variance revisited; Sum of a random number of independent r.v.'s

Exercises 13 due Mar 30, 2016 at 23:59 UTC

## Solved problems

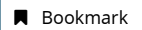
## Additional theoretical material

## Problem Set 6

Problem Set 6 due Mar 30, 2016 at 23:59 UTC

## Unit summary

Unit 6: Further topics on random variables &gt; Lec. 12: Sums of independent r.v.'s; Covariance and correlation &gt; Lec 12 Sums of independent r v s Covariance and correlation vertical4



## Exercise: Covariance properties

(3/3 points)

a) Is it true that  $\text{cov}(X, Y) = \text{cov}(Y, X)$ ?

True

✓ Answer: True

b) Find the value of  $a$  in the relation  $\text{cov}(2X, -3Y + 2) = a \cdot \text{cov}(X, Y)$ . $a =$ 

-6

✓ Answer: -6

c) Suppose that  $X$ ,  $Y$ , and  $Z$  are independent, with a common variance of 5. Then, $\text{cov}(2X + Y, 3X - 4Z) =$ 

30

✓ Answer: 30

Answer:

a) We have  $(X - \mathbf{E}[X])(Y - \mathbf{E}[Y]) = (Y - \mathbf{E}[Y])(X - \mathbf{E}[X])$ , and after taking expectations we obtain  $\text{cov}(X, Y) = \text{cov}(Y, X)$ .b) We have argued that  $\text{cov}(aX + b, Y) = a \cdot \text{cov}(X, Y)$ . Note that by symmetry, we also have  $\text{cov}(X, aY + b) = a \cdot \text{cov}(X, Y)$ . By using these relations,

$$\text{cov}(2X, -3Y + 2) = 2 \cdot \text{cov}(X, -3Y + 2) = 2 \cdot (-3) \cdot \text{cov}(X, Y) = -6 \text{cov}(X, Y).$$

c) Using linearity,

$$\begin{aligned} \text{cov}(2X + Y, 3X - 4Z) &= \text{cov}(2X + Y, 3X) + \text{cov}(2X + Y, -4Z) \\ &= \text{cov}(2X, 3X) + \text{cov}(Y, 3X) + \text{cov}(2X, -4Z) + \text{cov}(Y, -4Z) \\ &= 6 \text{var}(X) + 0 + 0 + 0 = 30, \end{aligned}$$

where the zeros are obtained because independent random variables have zero covariance.

You have used 1 of 2 submissions



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