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11. Dot products

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Lecture due Aug 18, 2021 20:30 IST Completed

Explore

Another operation we can do with vectors is called the **dot product**.

Definition 11.1

The **dot product** of two vectors $\vec{v} = \langle v_1, v_2 \rangle$ and $\vec{w} = \langle w_1, w_2 \rangle$ is defined as

$$\vec{v} \cdot \vec{w} = \langle v_1, v_2 \rangle \cdot \langle w_1, w_2 \rangle = v_1 w_1 + v_2 w_2. \tag{3.20}$$

Spoiler: Dot products in higher dimensions

Consider two vectors of length n given by $\vec{v} = \langle v_1, v_2, \ldots, v_n \rangle$ and $\vec{w} = \langle w_1, w_2, \ldots, w_n \rangle$. The dot product of these vectors is

$$\vec{v} \cdot \vec{w} = \langle v_1, v_2, \ldots, v_n \rangle \cdot \langle w_1, w_2, \ldots, w_n \rangle = v_1 w_1 + v_2 w_2 + \cdots + v_n w_n.$$

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Dot product concept check

1/1 point (graded)
From the definition of the dot product, what kind of quantity is $\vec{v} \cdot \vec{w}$?

● a scalar

○ a vector

○ a matrix

✓

Solution:

The dot product is a scalar. To see this, let's consider the dot product of vectors $\vec{v} = \langle 2, 1 \rangle$ and $\vec{w} = \langle 2, 3 \rangle$. This gives

$$\vec{v} \cdot \vec{w} = \langle 2, 1 \rangle \cdot \langle 2, 3 \rangle = 2\,(2) + 1\,(3) = 7,$$

which is a number (also called a scalar).

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Dot product practice

3.0/3 points (graded)
Compute the following.

Let $\vec{v} = \langle 1, 3\rangle$ and $\vec{w} = \langle 0, -2\rangle$. Then $\vec{v}\cdot\vec{w} =$ -6 ✓ **Answer:** -6

Let $\vec{s} = \langle 11, -2\rangle$ and $\vec{r} = \langle 1, 1\rangle$. Then $\vec{s}\cdot\vec{r} =$ 9 ✓ **Answer:** 9

Let $\vec{u} = \langle 2a, 3b\rangle$ and $\vec{q} = \langle b, -4a\rangle$ where a and b are constants.

Then $\vec{u}\cdot\vec{q} =$ -10*a*b ✓ **Answer:** -10*a*b

Solution:

$$\vec{v}\cdot\vec{w} = \langle 1,3\rangle\cdot\langle 0,-2\rangle = (1)(0) + (3)(-2) = 0 - 6 = -6 \qquad (3.21)$$
$$\vec{s}\cdot\vec{r} = \langle 11,-2\rangle\cdot\langle 1,1\rangle = (11)(1) + (-2)(1) = 11 - 2 = 9 \qquad (3.22)$$
$$\vec{u}\cdot\vec{q} = \langle 2a,3b\rangle\cdot\langle b,-4a\rangle = (2a)(b) + (3b)(-4a) = 2ab - 12ab = -10ab. \qquad (3.23)$$

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Vector properties

1/1 point (graded)
Let $\vec{u}$, $\vec{v}$, and $\vec{w}$ be vectors (with the same number of components) and let c be a scalar.

Which of the following properties of vectors are true? (Choose all that are true.)

☐ $\vec{v}\cdot\vec{v} = |\vec{v}|$

☑ $\vec{v}\cdot\vec{w} = \vec{w}\cdot\vec{v}$

☑ $c(\vec{v}+\vec{w}) = c\vec{v} + c\vec{w}$

☐ $\vec{u}\cdot(\vec{v}+\vec{w}) = |\vec{u}|(\vec{v}+\vec{w})$

✓

Solution:

We explain these properties that involve dot products in terms of the algebraic definition given on this page. We leave it as an exercise for you to verify that the geometric definition of the dot product introduced on the next page can be used to verify these properties as well!

1.
$$\begin{aligned}\vec{v}\cdot\vec{v} &= \langle v_1, v_2\rangle\cdot\langle v_1, v_2\rangle \\ &= v_1^2 + v_2^2 \\ &= |\vec{v}|^2 \neq |\vec{v}|\end{aligned}$$

2.
$$\begin{aligned}\vec{v}\cdot\vec{w} &= \langle v_1, v_2\rangle\cdot\langle w_1, w_2\rangle \\ &= v_1 w_1 + v_2 w_2\end{aligned}$$

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$$\begin{aligned} &= w_1 v_1 + w_2 v_2 \quad \text{(multiplication of numbers commutes)} \\ &= \vec{w} \cdot \vec{v} \quad \text{(rewrite as a dot product)}\checkmark \end{aligned}$$

3.

$$\begin{aligned} c\left(\vec{v}+\vec{w}\right) &= c\left(\langle v_1, v_2\rangle + \langle w_1, w_2\rangle\right) \\ &= c\langle v_1 + w_1, v_2 + w_2\rangle \\ &= \langle c\left(v_1 + w_1\right), c\left(v_2 + w_2\right)\rangle \quad \text{(distribute across)} \\ &= \langle cv_1 + cw_1, cv_2 + cw_2\rangle \quad \text{(rewrite as sum of vectors)} \\ &= \langle cv_1, cv_2\rangle + \langle cw_1, cw_2\rangle \\ &= c\vec{v} + c\vec{w}\checkmark \end{aligned}$$

4. $\vec{u} \cdot (\vec{v}+\vec{w}) = |\vec{u}|\left(\vec{v}+\vec{w}\right)$ cannot be true since the expression on the left hand side is a dot product of vectors, which is a scalar quality, and the expression on the right hand side is a scalar times a vector, which is a vector.

The property that is true is that the dot product distributes over vector sums. $\vec{u} \cdot (\vec{v}+\vec{w}) = \vec{u}\cdot\vec{v} + \vec{u}\cdot\vec{w}$ because

$$\begin{aligned} \vec{u} \cdot (\vec{v}+\vec{w}) &= \langle u_1, u_2\rangle \cdot \langle v_1 + w_1, v_2 + w_2\rangle \quad \text{(take dot product)} \\ &= u_1\left(v_1 + w_1\right) + u_2\left(v_2 + w_2\right) \quad \text{(multiply out)} \\ &= u_1 v_1 + u_1 w_1 + u_2 v_2 + u_2 w_2 \quad \text{(rearrange)} \\ &= \left(u_1 v_1 + u_2 v_2\right) + \left(u_1 w_1 + u_2 w_2\right) \quad \text{(find hidden dot products)} \\ &= \vec{u}\cdot\vec{v} + \vec{u}\cdot\vec{w} \end{aligned}$$

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Answers are displayed within the problem

11. Dot products

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Typo 6
For the last question (Vector Properties), the solution for the 4th box has a typo in the "multiply out" line., The second u_1 should be ...

[STAFF] Vector properties q2

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