2. Maximum Likelihood Estimator of Uniform Statistical Model

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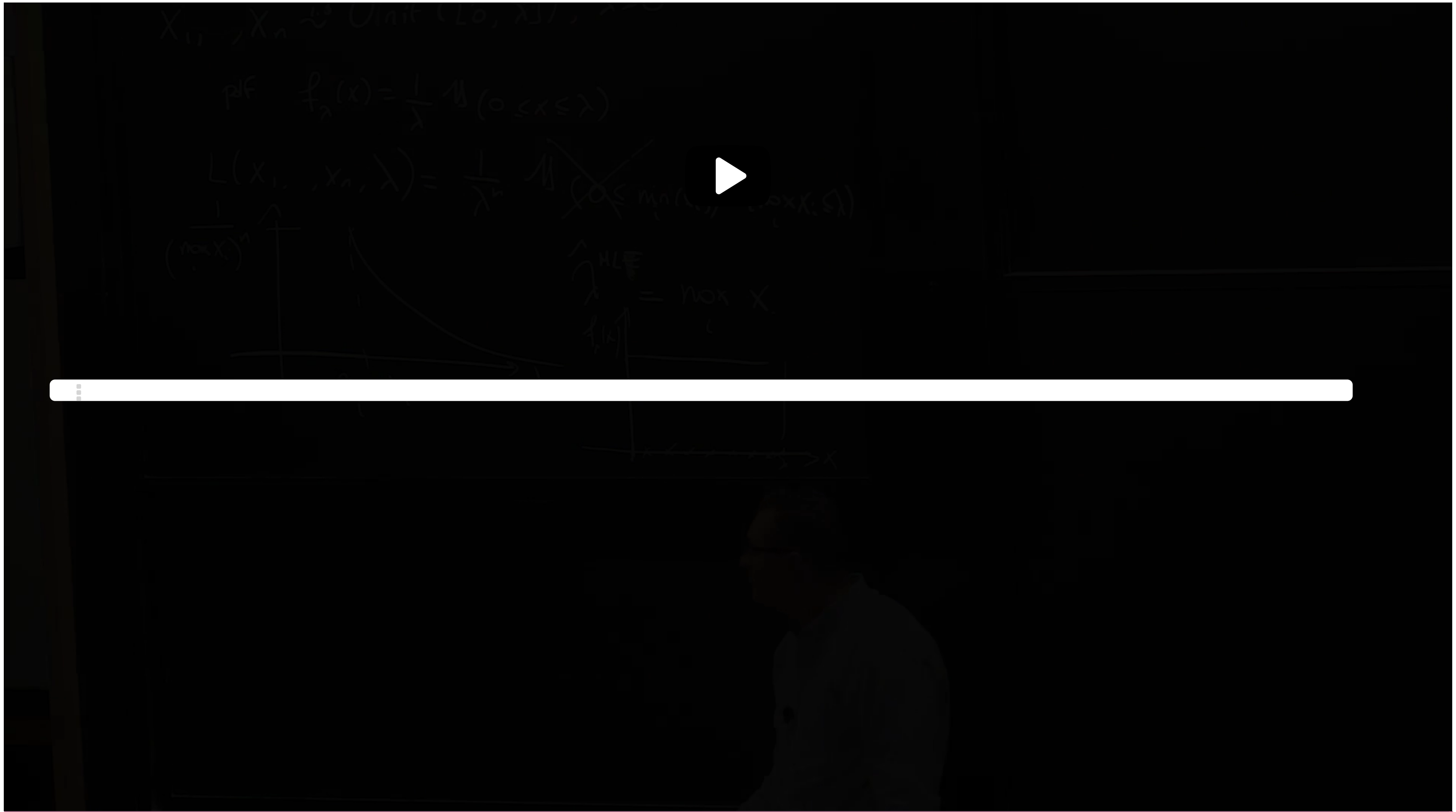

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Concept Check: Maximum Likelihood Estimator for a Uniform Statistical Model

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Let $X_1, \ldots, X_n \overset{iid}{\sim} \mathrm{Unif}\,[0, \theta^*]$ where θ^* is an unknown parameter. We constructed the associated statistical model $\left(\mathbb{R}_{\geq 0}, \{\mathrm{Unif}\,[0, \theta]\}_{\theta > 0}\right)$ (where $\mathbb{R}_{\geq 0}$ denotes the nonnegative reals).

For any $\theta > 0$, the density of $\mathrm{Unif}\,[0, \theta]$ is given by $f(x) = \frac{1}{\theta}\mathbf{1}\,(x \in [0, \theta])$. Recall that

$$\mathbf{1}\,(x \in [0, \theta]) = \begin{cases} 1 & \text{if } x \in [0, \theta] \\ 0 & \text{otherwise.} \end{cases}$$

Hence we can use the product formula and compute the likelihood to be

$$L_n(x_1, \ldots, x_n, \theta) = \prod_{i=1}^{n} \left(\frac{1}{\theta}\mathbf{1}\,(x_i \in [0, \theta])\right) = \frac{1}{\theta^n}\mathbf{1}\,(x_i \in [0, \theta] \;\forall\, 1 \leq i \leq n)\,.$$

For the fixed values $(1, 3, 2, 2.5, 5, 0.1)$ (think of these as observations of random variables $X_1, \ldots, X_6$), what value of θ maximizes $L_6(1, 3, 2, 2.5, 5, 0.1, \theta)$?

5

✔ **Answer:** 5

Solution:

Observe that

$$L_6(1, 3, 2, 2.5, 5, 0.1, \theta) = \frac{1}{\theta^6}\mathbf{1}\,(\{1, 3, 2, 2.5, 5, 0.1\} \subset [0, \theta])\,.$$

If $\theta < \max\{1, 3, 2, 2.5, 5, 0.1\}$, then we have $\{1, 3, 2, 2.5, 5, 0.1\} \not\subset [0, \theta]$. By the definition of the indicator function, this means $L_6(1, 3, 2, 2.5, 5, 0.1, \theta) = 0$ for $\theta < \max\{1, 3, 2, 2.5, 5, 0.1\} = 5$. Hence, when maximizing $L_6(1, 3, 2, 2.5, 5, 0.1, \theta)$, we need to consider $\theta \in [5, \infty)$. Restricted to this interval, we observe that

$$L_6\left(1, 3, 2, 2.5, 5, 0.1, \theta\right) = \frac{1}{\theta^n}.$$

The above is a decreasing function on $[5, \infty)$, so the maximum is attained when $\theta = \max\{1, 3, 2, 2.5, 5, 0.1\} = 5.$

Remark: In general, the maximum likelihood estimator for θ^* in this uniform statistical model is

$$\widehat{\theta_n}^{MLE} = \max_{1 \leq i \leq n} X_i.$$

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