Bipyramid

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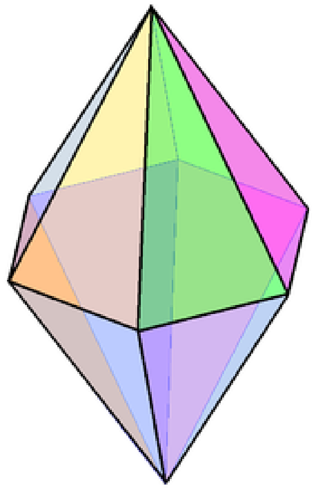
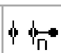
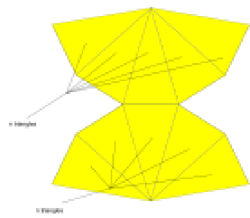
An *n*-gonal **bipyramid** or **dipyramid** is a polyhedron formed by joining an *n*-gonal pyramid and its mirror image base-to-base.

The referenced *n*-gon in the name of the bipyramids is not an external face but an internal one, existing on the primary symmetry plane which connects the two pyramid halves.

The face-transitive bipyramids are the dual polyhedra of the uniform prisms and will generally have isosceles triangle faces.

A bipyramid can be projected on a sphere or globe as *n* equally spaced lines of longitude going from pole to pole, and bisected by a line around the equator.

Bipyramid faces, projected as spherical triangles, represent the fundamental domains in the dihedral symmetry D_{nh}.

Set of bipyramids	
 (Example hexagonal form)	
Coxeter diagram	
Schläfli symbol	{ } + {n}
Faces	2*n* triangles
Edges	3*n*
Vertices	2 + *n*
Face configuration	V4.4.*n*
Symmetry group	D_{nh}, [*n*,2], (**n*22), order 4*n*
Rotation group	D_n, $[n,2]^+$, (*n*22), order 2*n*
Dual polyhedron	*n*-gonal prism
Properties	convex, face-transitive
Net	

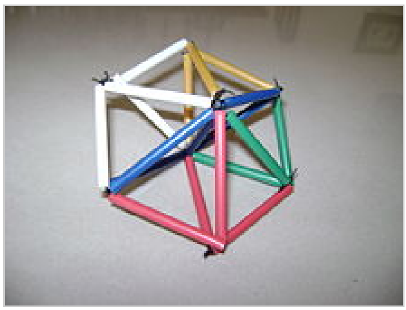
Contents

Volume

The volume of a bipyramid is $V=\frac{2}{3}Bh$ where *B* is the area of the base and *h* the height from the base to the apex. This works for any location of the apex, provided that *h* is measured as the perpendicular distance from the plane which contains the base.

The volume of a bipyramid whose base is a regular *n*-sided polygon with side length *s* and whose height is *h* is therefore:

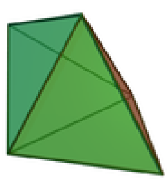
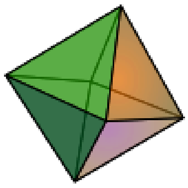
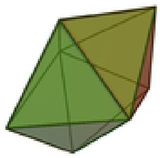
$$V = \frac{n}{6} h s^2 \cot \frac{\pi}{n}.$$



A bipyramid made with straws and elastics. An extra axial straw is added which doesn't exist in the simple polyhedron

Equilateral triangle bipyramids

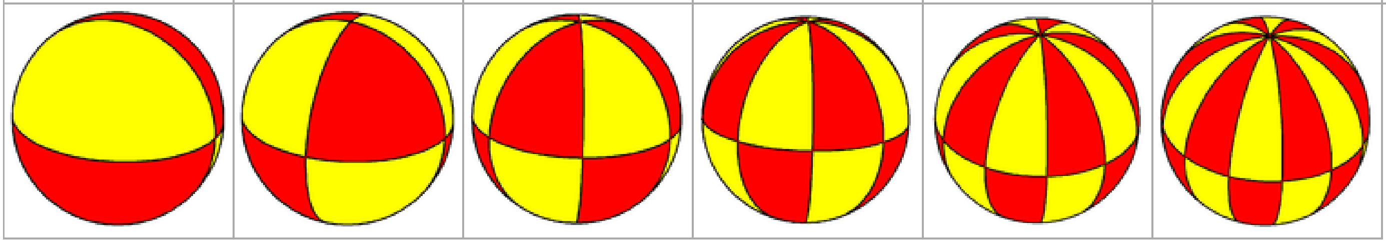
Only three kinds of bipyramids can have all edges of the same length (which implies that all faces are equilateral triangles, and thus the bipyramid is a deltahedron): the triangular, tetragonal, and pentagonal bipyramids. The tetragonal bipyramid with identical edges, or regular octahedron, counts among the Platonic solids, while the triangular and pentagonal bipyramids with identical edges count among the Johnson solids (J12 and J13).

		
Triangular bipyramid	Square bipyramid (Octahedron)	Pentagonal bipyramid

Kalidescopic symmetry

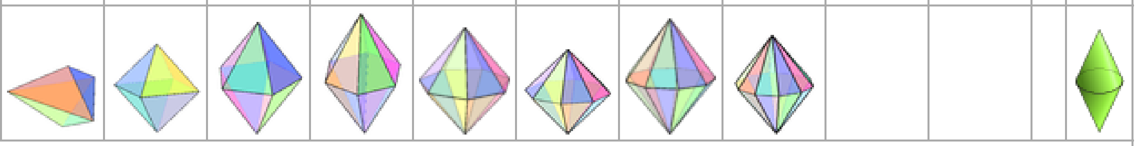
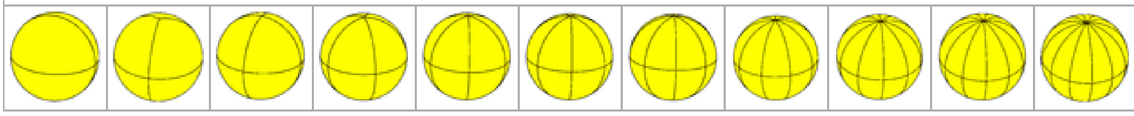
If the base is regular and the line through the apexes intersects the base at its center, the symmetry group of the *n*-agonal bipyramid has dihedral symmetry D_{nh} of order 4*n*, except in the case of a regular octahedron, which has the larger octahedral symmetry group O_h of order 48, which has three versions of D_{4h} as subgroups. The rotation group is D_n of order 2*n*, except in the case of a regular octahedron, which has the larger symmetry group O of order 24, which has three versions of D_4 as subgroups.

The digonal faces of a spherical 2*n*-bipyramid represents the fundamental domains of dihedral symmetry in three dimensions: D_{nh}, [n,2], (*n22), order *4n*. The reflection domains can be shown as alternately colored triangles as mirror images.

D_{1h}	D_{2h}	D_{3h}	D_{4h}	D_{5h}	D_{6h}	...
						

Forms

Family of bipyramids

2	3	4	5	6	7	8	9	10	11	12	...	∞
												
												
As spherical polyhedra												
												

Star bipyramids

Self-intersecting bipyramids exist with a star polygon central figure, defined by triangular faces connecting each polygon edge to these two points. A {p/q} bipyramid has Coxeter diagram .

5/2	7/2	7/3	8/3	9/2	9/4	10/3	11/2	11/3	11/4	11/5	12/5
											

Polychora with bipyramid cells

The dual of the rectification of each convex regular polychoron is a cell-transitive polychoron with bipyramidal cells. In the following, the apex vertex of the bipyramid is A and an equator vertex is E. The distance between adjacent vertices on the equator EE=1, the apex to equator edge is AE and the distance between the apices is AA. The bipyramid polychoron will have V_A vertices where the apices of N_A bipyramids meet. It will have V_E vertices where the type E vertices of N_E bipyramids meet. N_{AE} bipyramids meet along each type AE edge. N_{EE} bipyramids meet along each type EE edge. C_{AE} is the cosine of the dihedral angle along an AE edge. C_{EE} is the cosine of the dihedral angle along an EE edge. As cells must fit around an edge, $N_{AA} \cos^{-1}(C_{AA}) \leq 2\pi$, $N_{AE} \cos^{-1}(C_{AE}) \leq 2\pi$.

Polychoron Properties									Bipyramid Properties					
Dual of	Coxeter diagram	Cells	V_A	V_E	N_A	N_E	N_{AE}	N_{EE}	Cell	Coxeter diagram	AA	AE**	C_{AE}	C_{EE}
Rectified 5-cell		10	5	5	4	6	3	3	Triangular bipyramid		$\frac{2}{3}$	0.667	$-\frac{1}{7}$	$-\frac{1}{7}$
Rectified tesseract		32	16	8	4	12	3	4	Triangular bipyramid		$\frac{\sqrt{2}}{3}$	0.624	$-\frac{2}{5}$	$\frac{1}{5}$
Rectified 24-cell		96	24	24	8	12	4	3	Triangular bipyramid		$\frac{2\sqrt{2}}{3}$	0.745	$\frac{1}{11}$	$-\frac{5}{11}$
Rectified 120-cell		1200	600	120	4	30	3	5	Triangular bipyramid		$\frac{\sqrt{5}-1}{3}$	0.613	$-\frac{10+9\sqrt{5}}{61}$	$\frac{12\sqrt{5}-7}{61}$
Rectified 16-cell		24*	8	16	6	6	3	3	Square bipyramid		$\sqrt{2}$	1	$-\frac{1}{3}$	$-\frac{1}{3}$
Rectified cubic honeycomb		∞	∞	∞	6	12	3	4	Square bipyramid		1	0.866	$-\frac{1}{2}$	0
Rectified 600-cell		720	120	600	12	6	3	3	Pentagonal bipyramid		$\frac{5+3\sqrt{5}}{5}$	1.447	$-\frac{11+4\sqrt{5}}{41}$	$-\frac{11+4\sqrt{5}}{41}$

*The rectified 16-cell is the regular 24-cell and vertices are all equivalent – octahedra are regular bipyramids. **Given numerically due to more complex form.

Higher dimensions

In general, a *bipyramid* can be seen as an *n*-polytope constructed with a (*n*−1)-polytope in a hyperplane with two points in opposite directions, equal distance perpendicular from the hyperplane. If the (*n*−1)-polytope is a regular polytope, it will have identical pyramids facets. An example is the 16-cell, which is an octahedral bipyramid, and more generally an n-orthoplex is an (n-1)-orthoplex bypyramid.

See also

- Trapezohedron

References

- Anthony Pugh (1976). *Polyhedra: A visual approach*. California: University of California Press Berkeley. ISBN 0-520-03056-7. Chapter 4: Duals of the Archimedean polyhedra, prisma and antiprisms

External links

- Weisstein, Eric W., "Dipyramid" (http://mathworld.wolfram.com/Dipyramid.html), *MathWorld*.
- Olshevsky, George, *Bipyramid*

Wikimedia Commons has media related to ***Bipyramids***.

(http://web.archive.org/web/20070204075028/members.aol.com/Polycell/glossary.html#Bipyramid) at *Glossary for Hyperspace*.

- The Uniform Polyhedra (http://www.mathconsult.ch/showroom/unipoly/)
- Virtual Reality Polyhedra (http://www.georgehart.com/virtual-polyhedra/vp.html) The Encyclopedia of Polyhedra
  - VRML models (George Hart) (http://www.georgehart.com/virtual-polyhedra/alphabetic-list.html) <3> (http://www.georgehart.com/virtual-polyhedra/vrml/triangular_dipyramid.wrl) <4> (http://www.georgehart.com/virtual-polyhedra/vrml/octahedron.wrl) <5> (http://www.georgehart.com/virtual-polyhedra/vrml/pentagonal_dipyramid.wrl) <6> (http://www.georgehart.com/virtual-polyhedra/vrml/hexagonal_dipyramid.wrl) <7> (http://www.georgehart.com/virtual-polyhedra/vrml/heptagonal_dipyramid.wrl) <8> (http://www.georgehart.com/virtual-polyhedra/vrml/octagonal_dipyramid.wrl) <9> (http://www.georgehart.com/virtual-polyhedra/vrml/enneagonal_dipyramid.wrl) <10> (http://www.georgehart.com/virtual-polyhedra/vrml/decagonal_dipyramid.wrl)
    - Conway Notation for Polyhedra (http://www.georgehart.com/virtual-polyhedra/conway_notation.html) Try: "dP**n**", where **n** = 3, 4, 5, 6, ... example "dP4" is an octahedron.

Categories: Polyhedra | Pyramids and bipyramids
