3. Boundary conditions for the hanging cable

On the previous page, we found a differential equation for the shape of a hanging cable to be

$$y'' = \frac{1}{A}\sqrt{1+(y')^2},$$

where A is determined by the mass per unit length of the cable, the horizontal tension in the zipline cable, and gravity.

Squaring both sides, this is

$$(y'')^2 = \frac{1}{A^2}(1+(y')^2).$$

Differentiating implicitly this becomes

$$2y''y''' = \frac{1}{A^2}(2y'y'').$$

This equation is true if

$$y'' = 0 \qquad \text{or} \qquad y''' = \frac{1}{A^2}y'.$$

The differential equation $y'' = 0$ is solved by the straight line $y = ax + b$. This is the solution in the case that the length of cable is exactly equal to the distance between the end points. In all other cases, we must solve the differential equation $y''' = \frac{1}{A^2}y'$.

This is a third order differential equation, but we may solve it by setting $u = y'$ and solving the associated second order differential equation. Check for yourself that the differential equation

$$u'' = \frac{1}{A^2}u$$

is solved by a hyperbolic sinusoid of the form

$$u = \sinh\left(\frac{x - c_1}{A}\right).$$

Let us rename A to be the constant c_2, to make it clear that this function has two free parameters:

$$u = \sinh\left(\frac{x - c_1}{c_2}\right).$$

Therefore if $y' = u$, then y is a generalized hyperbolic cosine

$$y = c_3 + c_2 \cosh\left(\frac{x - c_1}{c_2}\right),$$

where c_1, c_2, and c_3 must be determined by initial conditions (or boundary conditions).

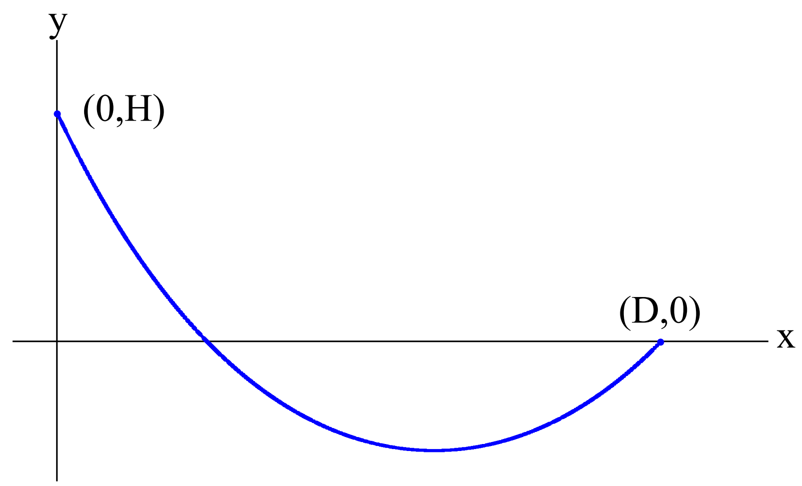
Boundary conditions

In order to solve for the unknown constants c_3, c_2, and c_1, there are three conditions that must be satisfied:

1. The catenary must pass through the point (0,H).

2. The catenary must pass through the point (D,0).

3. The arc length of the catenary is L.



These conditions are called boundary conditions. Unlike initial conditions, these conditions do not uniquely specify a catenary. In this case, these boundary conditions specify two catenaries, one concave up and one concave down. Thus we can specify the catenary by forcing ourselves to choose the catenary that is concave up.

The boundary conditions give us a system of equations in terms of the vector of parameters $\mathbf{c} = \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix}$:

$$y(\mathbf{c}, 0) = H, \quad y(\mathbf{c}, D) = 0, \quad \mathbf{length}(y(\mathbf{c})) = L.$$

In order for MATLAB to solve for $\mathbf{c}$ using **`fsolve`** (see documentation here), we should rewrite these three equations in the form

$$F_1(\mathbf{c}) = 0, \quad F_2(\mathbf{c}) = 0, \quad F_3(\mathbf{c}) = 0.$$

We ask you to specify the first two equations in the MATLAB code below.

Solving for the boundary conditions (External resource) (1.0 points possible)

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? end points in scripts do not correspond with text 2

Convergence of fsolve 3
The title of this post was just to catch your attention. The code converges to the correct boundary conditions yet when submitted it indicates "cOut Incorrect". I played with th...

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