Problem 3: Hypothesis test with a continuous observation

(5/5 points)

Let Θ be a Bernoulli random variable that indicates which one of two hypotheses is true, and let $\mathbf{P}(\Theta = 1) = p$. Under the hypothesis $\Theta = 0$, the random variable X is uniformly distributed over the interval $[0, 1]$. Under the alternative hypothesis $\Theta = 1$, the PDF of X is given by

$$f_{X|\Theta}(x \mid 1) = \begin{cases} 2x, & \text{if } 0 \le x \le 1, \\ 0, & \text{otherwise.} \end{cases}$$

Consider the MAP rule for deciding between the two hypotheses, given that $X = x$.

1. Suppose for this part of the problem that $p = 3/5$. The MAP rule can choose in favor of the hypothesis $\Theta = 1$ if and only if $x \ge c_1$. Find the value of c_1.

   $c_1 =$ 1/3

2. Assume now that p is general such that $0 \le p \le 1$. It turns out that there exists a constant c such that the MAP rule always decides in favor of the hypothesis $\Theta = 0$ if and only if $p < c$. Find c.

   $c =$ 1/3

3. For this part of the problem, assume again that $p = 3/5$. Find the conditional probability of error for the MAP decision rule given that the hypothesis $\Theta = 0$ is true.

   $\mathbf{P}(\text{error} \mid \Theta = 0) =$ 2/3

4. Find the probability of error associated with the MAP rule as a function of p. Express your answer in terms of p using standard notation .

When $p \leq 1/3$, $\mathbf{P}(\text{error}) =$ p

When $p \geq 1/3$, $\mathbf{P}(\text{error}) =$ 3/2-1/(4*p)-(5*p)/4

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