



Bookmarks

- ▶ Unit 0: Overview
- ▶ Entrance Survey
- ▶ Unit 1: Probability models and axioms
- ▶ Unit 2: Conditioning and independence
- ▶ Unit 3: Counting
- ▶ Unit 4: Discrete random variables
- ▶ Exam 1
- ▶ Unit 5: Continuous random variables

Exam 2 > Exam 2 > Exam 2 vertical 1

Bookmark

Problem 2: Calculation with PDFs

(3/3 points)

Let X be a random variable that takes non-zero values in $[1, \infty)$, with a PDF of the form

$$f_X(x) = \begin{cases} \frac{c}{x^3}, & \text{if } x \geq 1, \\ 0, & \text{otherwise.} \end{cases}$$

Let U be a uniform random variable on $[0, 2]$. Assume that X and U are independent.

1. What is the value of the constant c ?

 $c =$ 

Answer: 2

2.

 $\mathbf{P}(X \leq U) =$ 

Answer: 0.25

▶ Unit 6: Further topics on random variables

▶ Unit 7: Bayesian inference

▼ Exam 2

Exam 2

Exam 2 due Apr 20, 2016 at 23:59 UTC



▶ Unit 8: Limit theorems and classical statistics

▶ Unit 9: Bernoulli and Poisson processes

▶ Unit 10: Markov chains

▶ Exit Survey

▶ Final Exam

3. Find the PDF of $D = 1/X$. Express your answer in terms of d using standard notation.

For $0 \leq d \leq 1$, $f_D(d) =$

2*d

✓ Answer: 2*d

Answer:

1. The distribution must integrate to 1. Since

$$\int_1^{\infty} \frac{c}{x^3} dx = -\frac{c}{2x^2} \Big|_1^{\infty} = \frac{c}{2},$$

we have $c = 2$.

2. Since $X \geq 1$ and $U \leq 2$, the event of interest occurs only if $1 \leq X \leq U \leq 2$. Using the law of total probability and the independence of X and U , we have

$$\begin{aligned} \mathbf{P}(X \leq U) &= \int_1^2 \mathbf{P}(X \leq u) f_U(u) du \\ &= \int_1^2 \left(\int_1^u f_X(x) dx \right) f_U(u) du \\ &= \int_1^2 \left(\int_1^u \frac{2}{x^3} dx \right) \frac{1}{2} du \end{aligned}$$

$$\begin{aligned}
 &= \int_1^2 \left(1 - \frac{1}{u^2}\right) \cdot \frac{1}{2} du \\
 &= \frac{1}{4}.
 \end{aligned}$$

3. Since \mathbf{X} takes values in $[1, \infty)$, \mathbf{D} takes values in $[0, 1]$. We use the method of derived distributions to find the CDF of \mathbf{D} . For $0 \leq d \leq 1$,

$$\begin{aligned}
 F_D(d) &= \mathbf{P}(D \leq d) \\
 &= \mathbf{P}(X \geq 1/d) \\
 &= \int_{1/d}^{\infty} \frac{2}{x^3} dx \\
 &= d^2.
 \end{aligned}$$

The complete CDF of \mathbf{D} is

$$F_D(d) = \begin{cases} 0, & \text{if } d < 0, \\ d^2, & \text{if } 0 \leq d \leq 1, \\ 1, & \text{if } d > 1. \end{cases}$$

Differentiating the CDF gives the PDF of \mathbf{D} :

$$f_D(d) = \begin{cases} 2d, & \text{if } 0 \leq d \leq 1, \\ 0, & \text{otherwise.} \end{cases}$$

Alternatively, the same result can be achieved without explicitly computing the CDF of D . We express the CDF of D in terms of the CDF of X and use the chain rule of differentiation. For $0 \leq d \leq 1$,

$$\begin{aligned} F_D(d) &= \mathbf{P}(D \leq d) \\ &= \mathbf{P}(X \geq 1/d) \\ &= 1 - F_X(1/d), \\ f_D(d) &= -f_X(1/d) \cdot \frac{-1}{d^2} \\ &= -2d^3 \cdot \frac{-1}{d^2} \\ &= 2d. \end{aligned}$$

Hence, we obtain the same PDF:

$$f_D(d) = \begin{cases} 2d, & \text{if } 0 \leq d \leq 1, \\ 0, & \text{otherwise.} \end{cases}$$

You have used 2 of 2 submissions

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