# 7.2.3 Permutations

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Week 7 due Nov 20, 2023 01:42 IST Completed

# 7.2.3 Permutations

### Video

Start of transcript. Skip to the end.

Dr. Robert van de Geijn: As we work towards a solution for our problem,

namely the problem that sometimes the equations need to be swapped,

we're going to take a quick side tour into permutation matrices,

and then later we'll see how those fit into the picture.

0:00 / 0:00 2.0x

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## Reading Assignments

0 points possible (ungraded)
Read Unit 7.2.3 of the notes. [LINK]

**You REALLY need to read the text that goes with this unit!**

- [x] Done

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Calculator

## Homework 7.2.3.1

9/9 points (graded)

Compute

$$\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} -2 & 1 & 2 \\ 3 & 2 & 1 \\ -1 & 0 & -3 \end{pmatrix} =$$

3 ✔ Answer: 3

-1 ✔ Answer: -1

-2 ✔ Answer: -2

2 ✔ Answer: 2

0 ✔ Answer: 0

1 ✔ Answer: 1

1 ✔ Answer: 1

-3 ✔ Answer: -3

2 ✔ Answer: 2

**Answer:**

$$\left(\begin{array}{c|c|c} 0 & 1 & 0 \\ \hline 0 & 0 & 1 \\ \hline 1 & 0 & 0 \end{array}\right) \left(\begin{array}{ccc} -2 & 1 & 2 \\ \hline 3 & 2 & 1 \\ \hline -1 & 0 & -3 \end{array}\right) = \left(\begin{array}{c} 0 \times \begin{pmatrix} -2 & 1 & 2 \end{pmatrix} + 1 \times \begin{pmatrix} 3 & 2 & 1 \end{pmatrix} + 0 \times \begin{pmatrix} -1 & 0 & -3 \end{pmatrix} \\ \hline 0 \times \begin{pmatrix} -2 & 1 & 2 \end{pmatrix} + 0 \times \begin{pmatrix} 3 & 2 & 1 \end{pmatrix} + 1 \times \begin{pmatrix} -1 & 0 & -3 \end{pmatrix} \\ \hline 1 \times \begin{pmatrix} -2 & 1 & 2 \end{pmatrix} + 0 \times \begin{pmatrix} 3 & 2 & 1 \end{pmatrix} + 0 \times \begin{pmatrix} -1 & 0 & -3 \end{pmatrix} \end{array}\right)$$

$$= \left(\begin{array}{ccc} 3 & 2 & 1 \\ \hline -1 & 0 & -3 \\ \hline -2 & 1 & 2 \end{array}\right).$$

Notice that multiplying the matrix by $P$ from the left permuted the order of the rows in the matrix. Here is another way of looking at the same thing:

$$\left(\begin{array}{c} e_1^T \\ \hline e_2^T \\ \hline e_0^T \end{array}\right) \begin{pmatrix} -2 & 1 & 2 \\ 3 & 2 & 1 \\ -1 & 0 & -3 \end{pmatrix} = \left(\begin{array}{c} e_1^T \begin{pmatrix} -2 & 1 & 2 \\ 3 & 2 & 1 \\ -1 & 0 & -3 \end{pmatrix} \\ \hline e_2^T \begin{pmatrix} -2 & 1 & 2 \\ 3 & 2 & 1 \\ -1 & 0 & -3 \end{pmatrix} \\ \hline e_0^T \begin{pmatrix} -2 & 1 & 2 \\ 3 & 2 & 1 \\ -1 & 0 & -3 \end{pmatrix} \end{array}\right) = \left(\begin{array}{ccc} 3 & 2 & 1 \\ \hline -1 & 0 & -3 \\ \hline -2 & 1 & 2 \end{array}\right).$$

Here we use the fact that $e_i^T A$ equals the $i$th row of $A$.

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Answers are displayed within the problem

### Video

Calculator

Start of transcript. Skip to the end.

Dr. Robert van de Geijn: OK, so we're back,

and hopefully you went ahead and did that problem.

So let's walk through the solution.

The way I would do this, is I would use slicing and dicing.

And I would take the matrix A, and I would partition it into rows

0:00 / 0:00 2.0x

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## Homework 7.2.3.2

3/3 points (graded)

Example: If

$$p = \begin{pmatrix} 0 \\ 1 \\ 2 \\ 3 \end{pmatrix} \text{ then } P(p) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

(Throughout, $P(p)$ is the permutation matrix that orders the components of the vector to which it is applied according to the permutation vector $p$.)

If

$$p = \begin{pmatrix} 3 \\ 2 \\ 1 \\ 0 \end{pmatrix} \text{ then } P(p) =$$

- ( ) $\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

- ( ) $\begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{pmatrix}$

- (x) $\begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$

Calculator

$$\begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

If

$$p = \begin{pmatrix} 1 \\ 0 \\ 2 \\ 3 \end{pmatrix} \text{ then } P(p) =$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

If

$$p = \begin{pmatrix} 1 \\ 2 \\ 3 \\ 0 \end{pmatrix} \text{ then } P(p) =$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0; \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 0 & 1 \end{pmatrix}$$

Calculator

$$\begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

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## Homework 7.2.3.3

2/2 points (graded)

$$\text{Let } p = \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}. \text{ Compute } P(p) \begin{pmatrix} -2 \\ 3 \\ -1 \end{pmatrix} =$$

$$\begin{pmatrix} -2 \\ 3 \\ -1 \end{pmatrix}$$

$$\begin{pmatrix} -1 \\ 3 \\ -2 \end{pmatrix}$$

$$\begin{pmatrix} -1 \\ -2 \\ 3 \end{pmatrix}$$

$$\begin{pmatrix} -2 \\ -1 \\ 3 \end{pmatrix}$$

$$\text{Let } p = \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}. \text{ Compute } P(p) \begin{pmatrix} -2 & 1 & 2 \\ 3 & 2 & 1 \\ -1 & 0 & -3 \end{pmatrix} =$$

$$\begin{pmatrix} -1 & 0 & -3 \\ -2 & 0 & 2 \\ 0 & 3 & 4 \end{pmatrix}$$

$$\begin{pmatrix} 2 & -2 & 1 \\ 1 & 3 & 2 \\ -3 & -1 & 0 \end{pmatrix}$$

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- [x] $\begin{pmatrix} -1 & 0 & -3 \\ -2 & 1 & 2 \\ 3 & 2 & 1 \end{pmatrix}$
- [ ] $\begin{pmatrix} -1 & 0 & 2 \\ 1 & 1 & 2 \\ -1 & 2 & 3 \end{pmatrix}$

✔

**Answer:**

$$P(p)\begin{pmatrix} -2 \\ 3 \\ -1 \end{pmatrix} = \begin{pmatrix} -1 \\ -2 \\ 3 \end{pmatrix} \quad \text{and} \quad P(p)\begin{pmatrix} -2 & 1 & 2 \\ 3 & 2 & 1 \\ -1 & 0 & -3 \end{pmatrix} = \begin{pmatrix} -1 & 0 & -3 \\ -2 & 1 & 2 \\ 3 & 2 & 1 \end{pmatrix}.$$

Hint: it is not necessary to write out $P(p)$: the vector $p$ indicates the order in which the elements and rows need to appear.

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## Homework 7.2.3.4

1/1 point (graded)

Let $p = \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}$ be a permutation vector and $P = P(p)$ be a permutation matrix. Compute $\begin{pmatrix} -2 & 1 & 2 \\ 3 & 2 & 1 \\ -1 & 0 & -3 \end{pmatrix} P^T =$

- [x] $\begin{pmatrix} 2 & -2 & 1 \\ 1 & 3 & 2 \\ -3 & -1 & 0 \end{pmatrix}$
- [ ] $\begin{pmatrix} 1 & 2 & -2 \\ 2 & 1 & 3 \\ 0 & -3 & -1 \end{pmatrix}$
- [ ] $\begin{pmatrix} -1 & 0 & -3 \\ -2 & 1 & 2 \\ 3 & 2 & 1 \end{pmatrix}$
- [ ] $\begin{pmatrix} -1 & 0 & 2 \\ 1 & 1 & 2 \\ -1 & 2 & 3 \end{pmatrix}$

✔

Calculator

**Answer:**

$$\left(\begin{array}{r|r|r} -2 & 1 & 2 \\ 3 & 2 & 1 \\ -1 & 0 & -3 \end{array}\right) \left(\begin{array}{c|c|c} 0 & 0 & 1 \\ \hline 1 & 0 & 0 \\ \hline 0 & 1 & 0 \end{array}\right)^T = \left(\begin{array}{r|r|r} -2 & 1 & 2 \\ 3 & 2 & 1 \\ -1 & 0 & -3 \end{array}\right) \left(\begin{array}{c|c|c} 0 & 1 & 0 \\ \hline 0 & 0 & 1 \\ \hline 1 & 0 & 0 \end{array}\right)$$

$$= \left( (0)\begin{pmatrix} -2 \\ 3 \\ -1 \end{pmatrix} + (0)\begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} + (1)\begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix} \middle| (1)\begin{pmatrix} -2 \\ 3 \\ -1 \end{pmatrix} + (0)\begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} + (0)\begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix} \middle| (0)\begin{pmatrix} -2 \\ 3 \\ -1 \end{pmatrix} + (1)\begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} + (0)\begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix} \right)$$

$$= \left(\begin{array}{r|r|r} 2 & -2 & 1 \\ 1 & 3 & 2 \\ -3 & -1 & 0 \end{array}\right)$$

Alternatively:

$$\begin{pmatrix} -2 & 1 & 2 \\ 3 & 2 & 1 \\ -1 & 0 & -3 \end{pmatrix} \left(\begin{array}{c} e_2^T \\ \hline e_0^T \\ \hline e_1 \end{array}\right)^T = \begin{pmatrix} -2 & 1 & 2 \\ 3 & 2 & 1 \\ -1 & 0 & -3 \end{pmatrix} \left( e_2 \,\middle|\, e_0 \,\middle|\, e_1 \right)$$

$$= \left( \begin{pmatrix} -2 & 1 & 2 \\ 3 & 2 & 1 \\ -1 & 0 & -3 \end{pmatrix} e_2 \middle| \begin{pmatrix} -2 & 1 & 2 \\ 3 & 2 & 1 \\ -1 & 0 & -3 \end{pmatrix} e_0 \middle| \begin{pmatrix} -2 & 1 & 2 \\ 3 & 2 & 1 \\ -1 & 0 & -3 \end{pmatrix} e_1 \right)$$

$$= \left( \begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix} \middle| \begin{pmatrix} -2 \\ 3 \\ -1 \end{pmatrix} \middle| \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} \right) = \left(\begin{array}{r|r|r} 2 & -2 & 1 \\ 1 & 3 & 2 \\ -3 & -1 & 0 \end{array}\right)$$

Hint: it is not necessary to write out $P(p)$: the vector $p$ indicates the order in which the columns need to appear. In this case, you can go directly to the answer

$$\left(\begin{array}{r|r|r} 2 & -2 & 1 \\ 1 & 3 & 2 \\ -3 & -1 & 0 \end{array}\right).$$

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Answers are displayed within the problem

## Homework 7.2.3.5

1/1 point (graded)

Let $p = (k_0, \cdots, k_{n-1})^T$ be a permutation vector. Consider $x = \left(\begin{array}{c} \chi_0 \\ \hline \chi_1 \\ \hline \vdots \\ \hline \chi_{n-1} \end{array}\right)$.

Applying permutation matrix $P = P(p)$ to $x$ yields $Px = \left(\begin{array}{c} \chi_{k_0} \\ \hline \chi_{k_1} \\ \hline \vdots \\ \hline \chi_{k_{n-1}} \end{array}\right)$.

Always ✔ Answer: Always

**Answer:** Always

$$Px = P(p)x = \left(\begin{array}{c} e_{k_0}^T \\ \hline e_{k_1}^T \\ \hline \vdots \\ \hline e_{k_{n-1}}^T \end{array}\right) x = \left(\begin{array}{c} e_{k_0}^T x \\ \hline e_{k_1}^T x \\ \hline \vdots \\ \hline e_{k_{n-1}}^T x \end{array}\right) = \left(\begin{array}{c} \chi_{k_0} \\ \hline \chi_{k_1} \\ \hline \vdots \\ \hline \chi_{k_{n-1}} \end{array}\right).$$

Calculator

(Recall that $e_i^T x = \chi_i$.)

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## Homework 7.2.3.6

1/1 point (graded)

Let $p = (\, k_0, \cdots, k_{n-1}\,)^T$ be a permutation vector . Consider $A = \left( \begin{array}{c} \tilde{a}_0^T \\ \hline \tilde{a}_1^T \\ \hline \vdots \\ \hline \tilde{a}_{n-1}^T \end{array} \right)$.

Applying permutation matrix $P = P(p)$ to $A$ yields $PA = \left( \begin{array}{c} \tilde{a}_{k_0}^T \\ \hline \tilde{a}_{k_1}^T \\ \hline \vdots \\ \hline \tilde{a}_{k_{n-1}}^T \end{array} \right)$.

Always ✔ Answer: Always

**Answer:** Always

$$PA = P(p)A = \left( \begin{array}{c} e_{k_0}^T \\ \hline e_{k_1}^T \\ \hline \vdots \\ \hline e_{k_{n-1}}^T \end{array} \right) A = \left( \begin{array}{c} e_{k_0}^T A \\ \hline e_{k_1}^T A \\ \hline \vdots \\ \hline e_{k_{n-1}}^T A \end{array} \right) = \left( \begin{array}{c} \tilde{a}_{k_0}^T \\ \hline \tilde{a}_{k_1}^T \\ \hline \vdots \\ \hline \tilde{a}_{k_{n-1}}^T \end{array} \right).$$

(Recall that $e_i^T A$ equals the $i$th row of $A$.)

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### Video

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Dr. Robert van de Geijn: OK, so you were asked to look at a general permutation

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0:00 / 0:00 2.0x

matrix and to look and see what that does to a vector x.

Now, if we apply the permutation matrix to x, where the permutation matrix is

defined by this integers vector right

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## Homework 7.2.3.7

1/1 point (graded)

Let $p = (k_0, \cdots, k_{n-1})^T$ be a permutation vector, $P = P(p)$ be the associated permutation matrix, and $A = \left( a_0 \mid a_1 \mid \ldots \mid a_{n-1} \right)$.

$AP^T = \left( a_{k_0} \mid a_{k_1} \mid \ldots \mid a_{k_{n-1}} \right)$.

Always ✔ Answer: Always

**Answer:** Always

Recall that unit basis vectors have the property that $Ae_k = a_k$.

$$\begin{aligned} AP^T &= A \begin{pmatrix} e_{k_0}^T \\ e_{k_1}^T \\ \vdots \\ e_{k_{n-1}}^T \end{pmatrix}^T = A \left( e_{k_0} \mid e_{k_1} \mid \cdots \mid e_{k_{n-1}} \right) \\ &= \left( Ae_{k_0} \mid Ae_{k_1} \mid \cdots \mid Ae_{k_{n-1}} \right) = \left( a_{k_0} \mid a_{k_1} \mid \cdots \mid a_{k_{n-1}} \right). \end{aligned}$$

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**Video**

Start of transcript. Skip to the end.

Dr. Robert van de Geijn: OK, we're back.

Now notice that this matrix P is a really the matrix--

the permutation matrix P associated with the permutation vector littl

We know that matrix P, itself, as

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simply has the unit basis vectors ordered in the order indicated

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## Homework 7.2.3.8

1/1 point (graded)

If $P$ is a permutation matrix, then so is $P^T$.

TRUE

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## Homework 7.2.3.9

12/12 points (graded)

Recall:

$$\widetilde{P}(\pi) = \begin{pmatrix} \boxed{e_\pi^T} \\ e_1^T \\ \vdots \\ e_{\pi-1}^T \\ \boxed{e_0^T} \\ e_{\pi+1}^T \\ \vdots \\ e_{n-1}^T \end{pmatrix} = \left(\begin{array}{c|ccc|c|ccc} 0 & 0 & \cdots & 0 & 1 & 0 & \cdots & 0 \\ \hline 0 & 1 & \cdots & 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 & 0 & 0 & \cdots & 0 \\ \hline 1 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\ \hline 0 & 0 & \cdots & 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 1 \end{array}\right)$$

Compute

$$\tilde{P}(1)\begin{pmatrix} -2 \\ 3 \\ -1 \end{pmatrix} =$$

3 Answer: 3

-2 Answer: -2

-1 Answer: -1

$$\tilde{P}(1)\begin{pmatrix} -2 & 1 & 2 \\ 3 & 2 & 1 \\ -1 & 0 & -3 \end{pmatrix} =$$

| | | |
|---|---|---|
| 3 Answer: 3 | 2 Answer: 2 | 1 Answer: 1 |
| -2 Answer: -2 | 1 Answer: 1 | 2 Answer: 2 |
| -1 Answer: -1 | 0 Answer: 0 | -3 Answer: -3 |

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## Homework 7.2.3.10

1/1 point (graded)

When $\tilde{P}(\pi)$ (of appropriate size) multiplies a matrix from the left, it swaps row $0$ and row $\pi$, leaving all other rows unchanged.

Always ✔ Answer: Always

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## Homework 7.2.3.11

1/1 point (graded)

When $\tilde{P}(\pi)$ (of appropriate size) multiplies a matrix from the right, it swaps column $0$ and column $\pi$, leaving all other columns unchanged.

Always ✔ Answer: Always

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