MITx: 6.041x Introduction to Probability - The Science of Uncertainty

Exercise: Steady-state behavior

(1/2 points)

In the previous video, we have seen that for a homogeneous discrete-time Markov chain with m states and one aperiodic recurrent class, the steady-state probabilities $\pi_1, \ldots, \pi_m$ can be found as the unique solution to the balance equations

$$\pi_j = \sum_k \pi_k p_{kj} \qquad j = 1, \ldots, m,$$

together with the normalization equation $\sum_{j=1}^{m} \pi_j = 1.$

In order to derive this system of equations, we have used one type of recursion for $r_{ij}(n)$. Inspired by the fact that there are many ways to write such recursions, let us see if similar other balance equations can be obtained. For each of the following systems of equations, decide whether, when combined with the normalization equation, it also has the steady-state probabilities as the unique solution.

1. $\pi_j = \sum_k p_{ik}\pi_j \qquad j = 1, \ldots, m$

Yes ✖ **Answer:** No

2. $\pi_j = \sum_k \pi_k r_{kj}(2) \qquad j = 1, \ldots, m$

Yes ✔ **Answer:** Yes

Answer:

1. No. Since $\sum_k p_{ik} = 1$ for all i, each of these m equations simply say that $\pi_j = \pi_j$, which does not even have a unique solution, not to mention a unique solution that also gives the correct steady-state probabilities.

2. Yes. We obtain the given system of equations by taking the limit as n goes to infinity on both sides of $r_{ij}(n) = \sum_k r_{ik}(n-2)r_{kj}(2)$.

You have used 1 of 1 submissions