8. Worked Examples

Note: The following exercises will be presented in lecture, but we encourage you to attempt these yourselves first.

Concept Check: Upper Bound on TV

1/1 point (graded)

Give the smallest number M such that $\mathrm{TV}(\mathbf{P},\mathbf{Q}) \leq M$ for **any** probability measures $\mathbf{P},\mathbf{Q}$.

$M =$ 1 ✔ **Answer:** 1

(Find a pair of distributions $\mathbf{P},\mathbf{Q}$ such that $\mathrm{TV}(\mathbf{P},\mathbf{Q}) = M$.)

Solution:

Using the definition of total variation distance,

$$\mathrm{TV}(\mathbf{P},\mathbf{Q}) = \max_{A\subset E} |\mathbf{P}(A) - \mathbf{Q}(A)|,$$

we can say that if the maximum is obtained using a set A_1 such that $\mathbf{P}(A_1) \geq \mathbf{Q}(A_1)$, then

$$\mathrm{TV}\left(\mathbf{P},\mathbf{Q}\right)=\left|\mathbf{P}\left(A_1\right)-\mathbf{Q}\left(A_1\right)\right|\leq\mathbf{P}\left(A_1\right)\leq 1.$$

A similar argument can be made for the case when the the maximum is obtained using a set A_2 such that $\mathbf{Q}\left(A_2\right)>\mathbf{P}\left(A_2\right)$.

An example pair $\mathbf{P},\mathbf{Q}$ where the bound is met with equality: $E=\{1,2\},\mathbf{P}\left(1\right)=1,\mathbf{Q}\left(2\right)=1$.
Remark:. In general, when the support of $\mathbf{P}$ does not intersect with the support $\mathbf{Q}$, then $\mathrm{TV}\left(\mathbf{P},\mathbf{Q}\right)=1$.

Submit You have used 1 of 3 attempts

Answers are displayed within the problem

Computing Total Variation III

1/1 point (graded)
Compute $\mathrm{TV}\left(\mathsf{Exp}\left(1\right),\mathsf{Unif}\left[0,1\right]\right)$.

Hint: Use the formula $\frac{1}{2}\int_0^\infty\left|f\left(x\right)-g\left(x\right)\right|\mathrm{d}x$ where f and g are the probability density functions of $\mathsf{Exp}\left(1\right)$, and $\mathsf{Unif}\left[0,1\right]$ respectively.

$\mathrm{TV}\left(\mathsf{Exp}\left(1\right),\mathsf{Unif}\left[0,1\right]\right)=$ e^(-1) **Answer:** e^(-1)

e^{-1}

Solution:

Let f and g represent the density functions of $\mathsf{Exp}\left(1\right)$ and $\mathsf{Unif}\left[0,1\right]$, respectively.

$$\frac{1}{2}\int_{-\infty}^{\infty}\left|f\left(x\right)-g\left(x\right)\right|\mathrm{d}x=\frac{1}{2}\left(\int_0^1\left|1-e^{-x}\right|\mathrm{d}x+\int_1^\infty e^{-x}\mathrm{d}x\right)$$

$$= \frac{1}{2}\left(\left(1 - 1 + \frac{1}{e}\right) + \frac{1}{e}\right) = \frac{1}{e}.$$

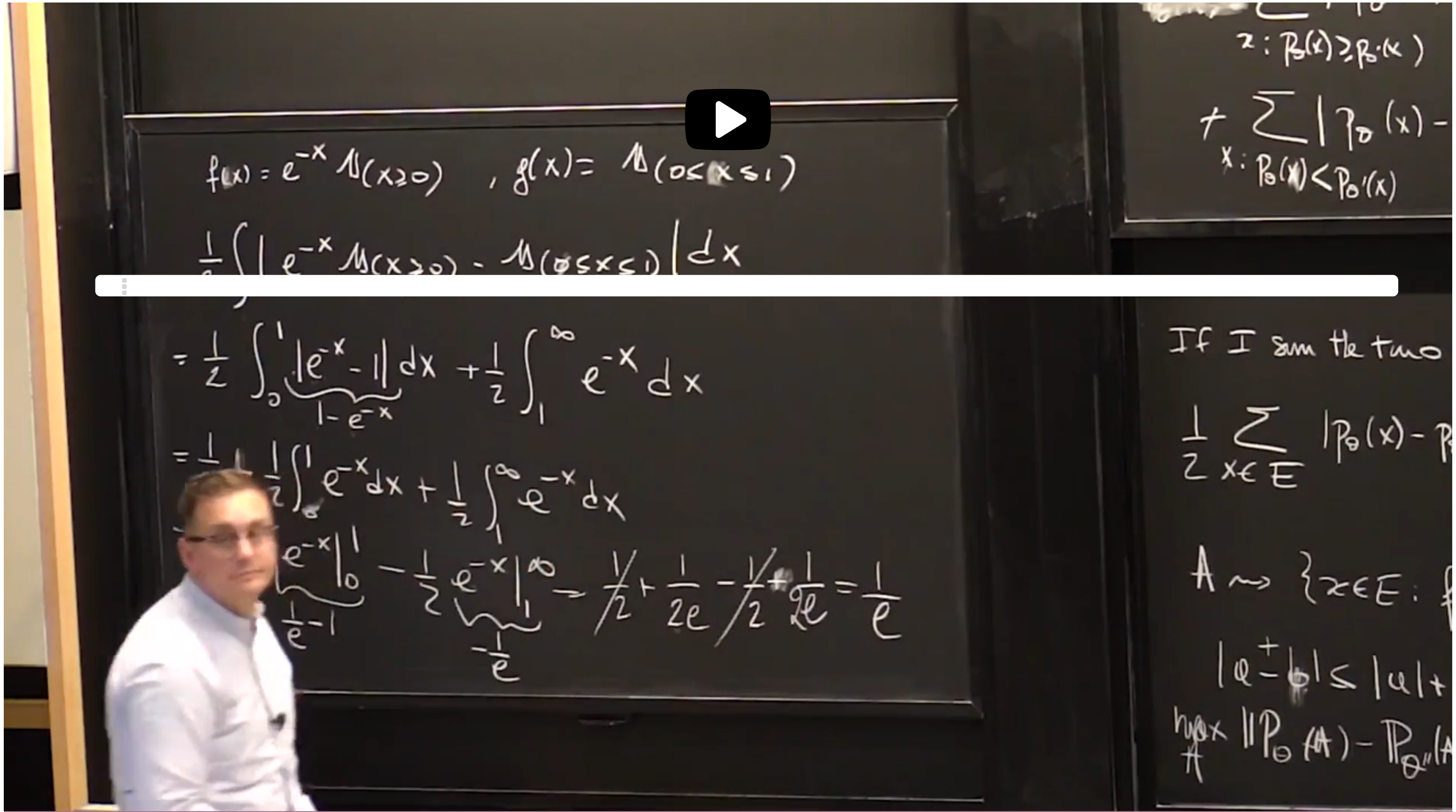
Remark: Even though the two distributions have different sample spaces, we can take the union of the two as the sample space for both, and integrate over it.

In general, the total variation distance between two distributions with probability density functions f, g is $\frac{1}{2}\int_{-\infty}^{\infty} |f(x) - g(x)| \mathrm{d}x$. Also, note that both densities in this case are equal to 0 when $x < 0$.

Submit You have used 3 of 3 attempts

Answers are displayed within the problem

Worked Examples on Total Variation Distance



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