



MITx: 6.041x Introduction to Probability - The Science of Uncertainty



Bookmarks

- ▶ Unit 0: Overview
- ▶ Entrance Survey
- ▶ Unit 1: Probability models and axioms
- ▶ Unit 2: Conditioning and independence
- ▶ Unit 3: Counting
- ▶ Unit 4: Discrete random variables
- ▼ Exam 1

Exam 1

Exam 1 due Mar 09, 2016 at
23:59 UTC



Exam 1 > Exam 1 > Exam 1 vertical

Bookmark

Problem 1: True or false

(4/4 points)

We are told that events A and B are conditionally independent, given a third event C , and that $\mathbf{P}(B \mid C) > 0$. For each one of the following statements, decide whether the statement is "Always true", or "Not always true."

1. A and B are conditionally independent, given the event C^c .

Not always true ▼



Answer: Not always true

2. A and B^c are conditionally independent, given the event C .

Always true ▼



Answer: Always true

3. $\mathbf{P}(A \mid B \cap C) = \mathbf{P}(A \mid B)$

Not always true ▼



Answer: Not always true

4. $\mathbf{P}(A \mid B \cap C) = \mathbf{P}(A \mid C)$

Always true ▾



Answer: Always true

- ▶ Unit 5: Continuous random variables
- ▶ Unit 6: Further topics on random variables
- ▶ Unit 7: Bayesian inference
- ▶ Exam 2
- ▶ Unit 8: Limit theorems and classical statistics
- ▶ Unit 9: Bernoulli and Poisson processes
- ▶ Unit 10: Markov chains
- ▶ Exit Survey
- ▶ Final Exam

Answer:

1. Not always true. Counterexample: Let \mathbf{X}, \mathbf{Y} be binary random variables. Consider a model with the following properties:

Conditioned on \mathbf{C} , \mathbf{X} and \mathbf{Y} are independent.

Conditioned on \mathbf{C}^c , \mathbf{X} and \mathbf{Y} are dependent.

Let $\mathbf{A} = \{\mathbf{X} = 1\}$ and $\mathbf{B} = \{\mathbf{Y} = 1\}$. Then, \mathbf{A} and \mathbf{B} are conditionally independent given \mathbf{C} , but they will be generically dependent conditioned on \mathbf{C}^c .

2. Always true.

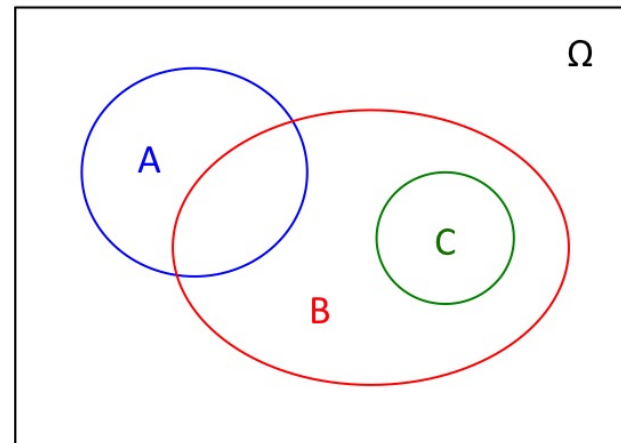
$$\mathbf{P}(\mathbf{A} \mid \mathbf{C}) = \mathbf{P}(\mathbf{A} \cap \mathbf{B} \mid \mathbf{C}) + \mathbf{P}(\mathbf{A} \cap \mathbf{B}^c \mid \mathbf{C})$$

$$\mathbf{P}(\mathbf{A} \mid \mathbf{C}) = \mathbf{P}(\mathbf{A} \mid \mathbf{C})\mathbf{P}(\mathbf{B} \mid \mathbf{C}) + \mathbf{P}(\mathbf{A} \cap \mathbf{B}^c \mid \mathbf{C})$$

$$\mathbf{P}(\mathbf{A} \mid \mathbf{C})(1 - \mathbf{P}(\mathbf{B} \mid \mathbf{C})) = \mathbf{P}(\mathbf{A} \cap \mathbf{B}^c \mid \mathbf{C})$$

$$\mathbf{P}(\mathbf{A} \mid \mathbf{C})\mathbf{P}(\mathbf{B}^c \mid \mathbf{C}) = \mathbf{P}(\mathbf{A} \cap \mathbf{B}^c \mid \mathbf{C}).$$

3. Not always true. Counterexample: Let $\mathbf{P}(\mathbf{A}) > 0$, $\mathbf{P}(\mathbf{B}) > 0$, $\mathbf{P}(\mathbf{C}) > 0$, $\mathbf{P}(\mathbf{A} \cap \mathbf{B}) > 0$ and $\mathbf{P}(\mathbf{A} \cap \mathbf{C}) = 0$. Furthermore, let $\mathbf{C} \subset \mathbf{B}$.



Show that A and B are conditionally independent given C :
 $\mathbf{P}(A \cap B \mid C) = 0 = (0)(1) = \mathbf{P}(A \mid C)\mathbf{P}(B \mid C)$

Show that $\mathbf{P}(A \mid B \cap C) \neq \mathbf{P}(A \mid B)$:
 $\mathbf{P}(A \mid B \cap C) = 0 \neq \mathbf{P}(A \mid B) > 0$

4. Always true. This is equivalent to the definition of independence of A and B in the conditional universe where C has occurred.

You have used 1 of 1 submissions



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