Exercise: Moments of the Beta distribution

(2/2 points)

Suppose that Θ takes values in $[0,1]$ and its PDF is of the form

$$f_\Theta(\theta) = a\theta(1-\theta)^2, \quad \text{for } \theta \in [0,1],$$

where a is a normalizing constant.

Use the formula

$$\int_0^1 \theta^\alpha (1-\theta)^\beta \, d\theta = \frac{\alpha!\,\beta!}{(\alpha+\beta+1)!}$$

to find the following:

a) $a =$ 12 Answer: 12

b) $\mathbf{E}[\Theta^2] =$ 1/5 Answer: 0.2

Answer:
a) Let $\mathrm{I}(\alpha,\beta)$ be the integral in the formula given in the problem statement. The normalizing constant must be equal to $1/\mathrm{I}(1,2)$: this is needed for the PDF to integrate to 1. We have $\mathrm{I}(1,2) = 2!/4! = 1/12$, so that $a = 12$.

b)

$$\mathbf{E}[\Theta^2] = \int_0^1 \theta^2 f_\Theta(\theta)\,d\theta = \int_0^1 a\theta^3(1-\theta)^2\,d\theta = a\cdot\mathrm{I}(3,2) = 12\cdot\frac{3!\,2!}{6!} = \frac{1}{5}$$

You have used 1 of 2 submissions