

<u>Help</u> Ţ sandipan_dey ~

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★ Course / Week 7: More Gaussian Elimination and ... / 7.2 When Gaussian Elimination ...

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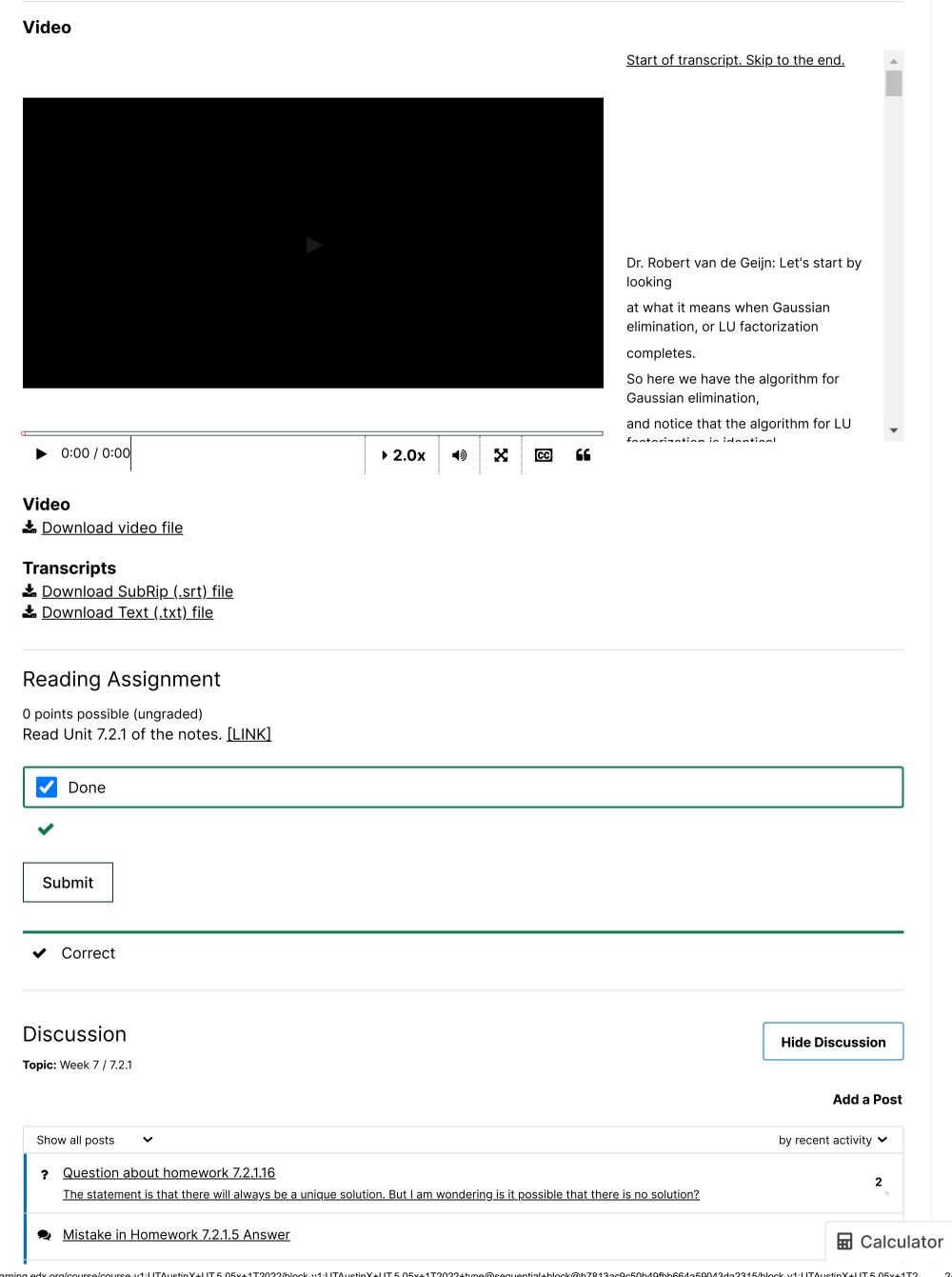
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7.2.1 When Gaussian Elimination Works

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Week 7 due Nov 20, 2023 01:42 IST

7.2.1 When Gaussian Elimination Works



- Question on "only 0 that may or may not have been encountered is the very, very last entry, in the upper triangular matrix" Greetings, Could you mind explaining a bit on the following contents mentioned in your video? "And notice again, we may end up dividing by 0. B...
- 7.2.1.7 PictureFLAME (Spoiler answer given). It is (kind of) possible to get it working in PictureFLAME. For some reason, beta1 can't be used to store intermediate values, but lambda11 can. In...
- Variant 2 of Lower Triangular Solve I don't understand how the second variant of the lower triangular solve algorithm works. Are you proceeding from bottom right to top left? How ...

Homework 7.2.1.1

1/1 point (graded)

Let $L \in \mathbb{R}^{1 imes 1}$ be a unit lower triangular matrix.

Lx = b, where x is the unknown and b is given, has a unique solution.

Always

Answer: Always

Explanation

Answer: Always

Since L is 1×1 , it is a scalar:

$$(1)(\chi_0) = (\beta_0).$$

From basic algebra we know that then $\chi_0 = \beta_0$ is the unique solution.

Submit

Answers are displayed within the problem

Homework 7.2.1.2

2/2 points (graded)

Give the solution of $egin{pmatrix} 1 & 0 \ 2 & 1 \end{pmatrix} egin{pmatrix} \chi_0 \ \chi_1 \end{pmatrix} = egin{pmatrix} 1 \ 2 \end{pmatrix}$

1

Answer: 1

0

Answer: 0

Answer: The above translates to the system of linear equations

$$\chi_0 = 1$$

$$2\chi_0 + \chi_1 = 2$$

which has the solution

$$\begin{pmatrix} \chi_0 \\ \chi_1 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 - (2)(1) \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}.$$

Submit

Answers are displayed within the problem

3

2

3

3/3 points (graded)

Give the solution of
$$egin{pmatrix} 1&0&0\\2&1&0\\-1&2&1 \end{pmatrix} egin{pmatrix} \chi_0\\\chi_1\\\chi_2 \end{pmatrix} = egin{pmatrix} 1\\2\\3 \end{pmatrix}$$

1

✓ Answer: 1

0

✓ Answer: 0

4

✓ Answer: 4

Answer: A clever way of solving the above is to slice and dice:

$$\underbrace{\begin{pmatrix}
1 & 0 & 0 \\
2 & 1 & 0 \\
\hline
-1 & 2 & 1
\end{pmatrix}}_{1} \begin{pmatrix} \chi_{0} \\ \chi_{1} \\ \hline \chi_{2} \end{pmatrix} = \underbrace{\begin{pmatrix}
1 \\ 2 \\ 3
\end{pmatrix}}_{3}$$

$$\underbrace{\begin{pmatrix}
1 & 0 \\ 2 & 1
\end{pmatrix}}_{1} \begin{pmatrix} \chi_{0} \\ \chi_{1} \end{pmatrix}}_{1} \begin{pmatrix} \chi_{0} \\ \chi_{1} \end{pmatrix} + \begin{pmatrix} \chi_{2} \\ \chi_{1} \end{pmatrix} + \begin{pmatrix} \chi_{2} \\ \chi_{1} \end{pmatrix} + \begin{pmatrix} \chi_{2} \\ \chi_{1} \end{pmatrix} = \underbrace{\begin{pmatrix}
1 \\ 2 \\ 3
\end{pmatrix}}_{3}$$

Hence, from the last exercise, we conclude that

$$\left(\begin{array}{c} \chi_0 \\ \chi_1 \end{array}\right) = \left(\begin{array}{c} 1 \\ 0 \end{array}\right).$$

We can then compute χ_2 by substituting in:

$$\left(\begin{array}{cc} -1 & 2 \end{array}\right) \left(\begin{array}{c} \chi_0 \\ \chi_1 \end{array}\right) + \left(\begin{array}{c} \chi_2 \end{array}\right) = 3$$

So that

$$\chi_2 = 3 - \begin{pmatrix} -1 & 2 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = 3 - (-1) = 4.$$

Thus, the solution is the vector

$$\begin{pmatrix} \chi_0 \\ \chi_1 \\ \chi_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 4 \end{pmatrix}.$$

Submit

1 Answers are displayed within the problem

Homework 7.2.1.4

1/1 point (graded)

Let $L \in \mathbb{R}^{2 imes 2}$ be a unit lower triangular matrix.

 $\boldsymbol{L}\boldsymbol{x}=\boldsymbol{b}$, where \boldsymbol{x} is the unknown and \boldsymbol{b} is given, has a unique solution.



Always

Answer: Always

Explanation

Answer: Always

Since L is 2×2 , the linear system has the form

$$\begin{pmatrix} 1 & 0 \\ \lambda_{1,0} & 1 \end{pmatrix} \begin{pmatrix} \chi_0 \\ \chi_1 \end{pmatrix} = \begin{pmatrix} \beta_0 \\ \beta_1 \end{pmatrix}.$$

But that translates to the system of linear equations

$$\chi_0 = \beta_0$$
$$\lambda_{1,0}\chi_0 + \chi_1 = \beta_1$$

which has the unique solution

$$\left(\begin{array}{c} \chi_0 \\ \chi_1 \end{array}\right) = \left(\begin{array}{c} \beta_0 \\ \beta_1 - \lambda_{1,0} \chi_0 \end{array}\right).$$

Submit

Answers are displayed within the problem

Homework 7.2.1.5

1/1 point (graded)

Let $L \in \mathbb{R}^{3 imes 3}$ be a unit lower triangular matrix.

 $\boldsymbol{L}\boldsymbol{x}=\boldsymbol{b}$, where \boldsymbol{x} is the unknown and \boldsymbol{b} is given, has a unique solution.

Always ✓ Answer: Always

Explanation

Answer: Always

Notice

$$\underbrace{\begin{pmatrix}
1 & 0 & 0 \\
\lambda_{1,0} & 1 & 0 \\
\hline
\lambda_{2,0} & \lambda_{2,1} & 1
\end{pmatrix}}_{\lambda_{2,0} & \lambda_{2,1} & 1} \underbrace{\begin{pmatrix}
\chi_{0} \\
\chi_{1} \\
\chi_{2}
\end{pmatrix}}_{\lambda_{2,0} & \lambda_{2,1} & 1
\end{pmatrix}}_{= \underbrace{\begin{pmatrix}
\beta_{0} \\
\beta_{1} \\
\beta_{2}
\end{pmatrix}}_{\lambda_{2,0} & \lambda_{2,1} & 1
\end{pmatrix}}_{= \underbrace{\begin{pmatrix}
\beta_{0} \\
\beta_{1} \\
\beta_{2}
\end{pmatrix}}_{\beta_{2}}$$

Hence, from the last exercise, we conclude that the unique solutions for χ_0 and χ_1 are

$$\begin{pmatrix} \chi_0 \\ \chi_1 \end{pmatrix} = \begin{pmatrix} \beta_0 \\ \beta_1 - \lambda_{1,0} \beta_0 \end{pmatrix}.$$

We can then compute χ_2 by substituting in:

(vo) . . .

$$\left(\begin{array}{cc} \lambda_{2,0} & \lambda_{2,1} \end{array}\right) \left(\begin{array}{c} \lambda^{0} \\ \chi_{1} \end{array}\right) + \left(\begin{array}{c} \chi_{2} \end{array}\right) = \beta_{2}$$

So that

$$\chi_2 = \beta_2 - \left(\begin{array}{cc} \lambda_{2,0} & \lambda_{2,1} \end{array}\right) \left(\begin{array}{c} \beta_0 \\ \beta_1 - \lambda_{1,0}\beta_0 \end{array}\right)$$

Since there is no ambiguity about what χ_2 must equal, the solution is unique:

$$\begin{pmatrix} \chi_0 \\ \chi_1 \\ \chi_2 \end{pmatrix} = \begin{pmatrix} \beta_0 \\ \beta_1 - \lambda_{1,0}\beta_0 \\ \beta_2 - \begin{pmatrix} \lambda_{2,0} & \lambda_{2,1} \end{pmatrix} \begin{pmatrix} \beta_0 \\ \beta_1 - \lambda_{1,0}\beta_0 \end{pmatrix}.$$

Submit

• Answers are displayed within the problem

Homework 7.2.1.6

1/1 point (graded)

Let $L \in \mathbb{R}^{n \times n}$ be a unit lower triangular matrix.

 ${m L}{m x}={m b}$, where ${m x}$ is the unknown and ${m b}$ is given, has a unique solution.

Always ✓ ✓ Answer: Always

Explanation

Always

The last exercises were meant to make you notice that this can be proved with a proof by induction on the size, n, of L.

Base case: n = 1. In this case L = (1), $x = (\chi_1)$ and $b = (\beta_1)$. The result follows from the fact that $(1)(\chi_1) = (\beta_1)$ has the unique solution $\chi_1 = \beta_1$.

Inductive step: Inductive Hypothesis (I.H.): Assume that Lx = b has a unique solution for all $L \in \mathbb{R}^{n \times n}$ and right-hand side vectors b.

We now want to show that then Lx = b has a unique solution for all $L \in \mathbb{R}^{(n+1)\times(n+1)}$ and right-hand side vectors b.

Partition

$$L \to \left(egin{array}{c|c} L_{00} & 0 \\ \hline l_{10}^T & \lambda_{11} \end{array}
ight), \quad x \to \left(egin{array}{c|c} x_0 \\ \hline \chi_1 \end{array}
ight) \quad ext{and} \quad b \to \left(egin{array}{c|c} b_0 \\ \hline eta_1 \end{array}
ight),$$

where, importantly, $L_{00} \in \mathbb{R}^{n \times n}$. Then Lx = b becomes

$$\underbrace{\begin{pmatrix} L_{00} & 0 \\ l_{10}^T & \lambda_{11} \end{pmatrix} \begin{pmatrix} x_0 \\ \chi_1 \end{pmatrix}}_{\begin{pmatrix} L_{00}x_0 \\ \hline l_{10}^T x_0 + \lambda_{11}\chi_1 \end{pmatrix}} = \begin{pmatrix} b_0 \\ \hline \beta_1 \end{pmatrix}$$

or

$$l_{10}^T x_0 + \lambda_{11} \chi_1 = \beta_1$$

By the Inductive Hypothesis, we know that $L_{00}x_0 = b_0$ has a unique solution. But once x_0 is set, $\lambda_{11}\chi_1 = \beta_1 - l_{10}^T x_0$ uniquely determines χ_1 .

By the **Principle of Mathematical Induction**, the result holds.

Submit

Answers are displayed within the problem

Homework 7.2.1.7

1/1 point (graded)

The proof for the last exercise suggests an alternative algorithm (Variant 2) for solving Lx=b when L is unit lower triangular. Use the below partial algorithm to state this alternative algorithm.

$$\textbf{Algorithm:} \ [b] := \texttt{LTRSV_UNB_VAR2}(L,b)$$

Partition
$$L \rightarrow \left(\begin{array}{c|c} L_{TL} & 0 \\ \hline L_{BL} & L_{BR} \end{array}\right)$$
 , $b \rightarrow \left(\begin{array}{c|c} b_T \\ \hline b_B \end{array}\right)$

where L_{TL} is 0×0 , b_T has 0 rows

while $m(L_{TL}) < m(L)$ do

Repartition

$$\left(\begin{array}{c|c|c} L_{TL} & 0 \\ \hline L_{BL} & L_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c|c|c} L_{00} & 0 & 0 \\ \hline l_{10}^T & \lambda_{11} & 0 \\ \hline L_{20} & l_{21} & L_{22} \end{array}\right), \left(\begin{array}{c} b_T \\ \hline b_B \end{array}\right) \rightarrow \left(\begin{array}{c} b_0 \\ \hline \beta_1 \\ \hline b_2 \end{array}\right)$$

where λ_{11} is 1×1 , β_1 has 1 row

Continue with

$$\left(\begin{array}{c|c|c} L_{TL} & 0 \\ \hline L_{BL} & L_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c|c|c} L_{00} & 0 & 0 \\ \hline l_{10}^T & \lambda_{11} & 0 \\ \hline L_{20} & l_{21} & L_{22} \end{array}\right), \left(\begin{array}{c} b_T \\ \hline b_B \end{array}\right) \leftarrow \left(\begin{array}{c} b_0 \\ \hline \beta_1 \\ \hline b_2 \end{array}\right)$$

endwhile

Next, implement it, yielding

• [b_out] = Ltrsv_unb_var2(L, b)

You can check that they compute the right answers with the script in

• test_Ltrsv_unb_var2.m

Unfortunately, PictureFLAME does not work for this problem.



done/skip



Algorithm:

Algorithm: $[b] := LTRSV_UNB_VAR2(L,b)$

Partition
$$L \to \left(\begin{array}{c|c} L_{TL} & 0 \\ \hline L_{BL} & L_{BR} \end{array}\right)$$
, $b \to \left(\begin{array}{c|c} b_T \\ \hline b_B \end{array}\right)$
where L_{TL} is 0×0 , b_T has 0 rows

while $m(L_{TL}) < m(L)$ do

Repartition

$$\left(\begin{array}{c|c|c} L_{TL} & 0 \\ \hline L_{BL} & L_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c|c|c} L_{00} & 0 & 0 \\ \hline l_{10}^T & \lambda_{11} & 0 \\ \hline L_{20} & l_{21} & L_{22} \end{array}\right), \left(\begin{array}{c} b_T \\ \hline b_B \end{array}\right) \rightarrow \left(\begin{array}{c} b_0 \\ \hline \beta_1 \\ \hline b_2 \end{array}\right)$$

where λ_{11} is 1×1 , β_1 has 1 row

$$\beta_1 := \beta_1 - l_{10}^T b_0$$

Continue with

$$\left(\begin{array}{c|c|c} L_{TL} & 0 \\ \hline L_{BL} & L_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c|c|c} L_{00} & 0 & 0 \\ \hline l_{10}^T & \lambda_{11} & 0 \\ \hline L_{20} & l_{21} & L_{22} \end{array}\right), \left(\begin{array}{c} b_T \\ \hline b_B \end{array}\right) \leftarrow \left(\begin{array}{c} b_0 \\ \hline \beta_1 \\ \hline b_2 \end{array}\right)$$

endwhile

Implementation: Ltrsv_unb_var2.m

Submit

1 Answers are displayed within the problem

Homework 7.2.1.8

1/1 point (graded)

Let $L \in \mathbb{R}^{n \times n}$ be a unit lower triangular matrix.

Lx=0, where 0 is the zero vector of size n, has the unique solution x=0.

Always ~

Answer: Always

Explanation

Always/Sometimes/Never Answer: Always

Obviously x = 0 is a solution. But a previous exercise showed that when L is a unit lower triangular matrix, Lx = b has a unique solution for all b. Hence, it has a unique solution for b = 0.

Submit

Answers are displayed within the problem

Homework 7.2.1.9

1/1 point (graded)

Let $U \in \mathbb{R}^{1 imes 1}$ be an upper triangular matrix with no zeroes on its diagonal.

Ux=b, where $oldsymbol{x}$ is the unknown and $oldsymbol{b}$ is given, has a unique solution.

Answer: Aiways

Explanation

Answer: Always

Since U is 1×1 , it is a nonzero scalar

$$(v_{0,0})(\chi_0) = (\beta_0).$$

From basic algebra we know that then $\chi_0 = \beta_0/\nu_{0,0}$ is the unique solution.

Submit

Answers are displayed within the problem

Homework 7.2.1.10

2/2 points (graded)

Give the solution of
$$egin{pmatrix} -1 & 1 \ 0 & 2 \end{pmatrix} egin{pmatrix} \chi_0 \ \chi_1 \end{pmatrix} = egin{pmatrix} 1 \ 2 \end{pmatrix}$$

0

Answer: 0

1

Answer: 1

Answer: The above translates to the system of linear equations

$$-1\chi_0 + \chi_1 = 1$$

 $2\chi_1 = 2$

which has the solution

$$\begin{pmatrix} \chi_0 \\ \chi_1 \end{pmatrix} = \begin{pmatrix} (1 - \chi_1)/(-1) \\ 2/2 \end{pmatrix} = \begin{pmatrix} (1 - (1))/(-1) \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}.$$

Submit

Answers are displayed within the problem

Homework 7.2.1.11

3/3 points (graded)

Give the solution of
$$\begin{pmatrix} -2 & 1 & -2 \\ 0 & -1 & 1 \\ 0 & 0 & 2 \end{pmatrix} \begin{pmatrix} \chi_0 \\ \chi_1 \\ \chi_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}$$

-1

✓ Answer: -1

0

Answer: 0

1

Answer: 1

9/29/23, 2:11 PM

Answer: A clever way of solving the above is to slice and dice:

$$\underbrace{\begin{pmatrix}
-2 & 1 & -2 \\
0 & -1 & 1 \\
0 & 0 & 2
\end{pmatrix}}_{0} \underbrace{\begin{pmatrix}
\chi_0 \\
\chi_1 \\
\chi_2
\end{pmatrix}}_{2} = \underbrace{\begin{pmatrix}
0 \\
1 \\
2
\end{pmatrix}}_{2}$$

$$\underbrace{\begin{pmatrix}
-2\chi_0 + \begin{pmatrix}
1 & -2
\end{pmatrix}}_{0} \begin{pmatrix}
\chi_1 \\
\chi_2
\end{pmatrix}}_{2} = \underbrace{\begin{pmatrix}
0 \\
1 \\
2
\end{pmatrix}}_{2}$$

Hence, from the last exercise, we conclude that

$$\left(\begin{array}{c} \chi_1 \\ \chi_2 \end{array}\right) = \left(\begin{array}{c} 0 \\ 1 \end{array}\right).$$

We can then compute χ_0 by substituting in:

$$-2\chi_0 + \left(\begin{array}{cc} 1 & -2 \end{array}\right) \left(\begin{array}{c} \chi_1 \\ \chi_2 \end{array}\right) = 0$$

So that

$$-2\chi_2 = 0 - \begin{pmatrix} 1 & -2 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = 0 - (-2) = 2.$$

Thus, the solution is the vector

$$\begin{pmatrix} \chi_0 \\ \chi_1 \\ \chi_2 \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}.$$

Submit

Answers are displayed within the problem

Homework 7.2.1.12

1/1 point (graded)

Let $U \in \mathbb{R}^{2 imes 2}$ be an upper triangular matrix with no zeroes on its diagonal.

 $Uoldsymbol{x}=oldsymbol{b}$, where $oldsymbol{x}$ is the unknown and $oldsymbol{b}$ is given, has a unique solution.

Always **✓**

Answer: Always

Explanation

Answer: Always

Since U is 2×2 , the linear system has the form

$$\left(\begin{array}{cc} \upsilon_{0,0} & \upsilon_{0,1} \\ 0 & \upsilon_{1,1} \end{array}\right) \left(\begin{array}{c} \chi_0 \\ \chi_1 \end{array}\right) = \left(\begin{array}{c} \beta_0 \\ \beta_1 \end{array}\right).$$

But that translates rto the system of linear equations

$$v_{0,0}\chi_0 + v_{0,1}\chi_1 = \beta_0$$

 $v_{1,1}\chi_1 = \beta_1$

which has the unique solution

$$\left(\begin{array}{c}\chi_0\\\chi_1\end{array}\right)=\left(\begin{array}{c}(\beta_0-\upsilon_{0,1}\chi_1)/\upsilon_{0,0}\\\\\beta_1/\upsilon_{1,1}\end{array}\right).$$

Submit

1 Answers are displayed within the problem

Homework 7.2.1.13

1/1 point (graded)

Let $U \in \mathbb{R}^{3 imes 3}$ be an upper triangular matrix with no zeroes on its diagonal.

Ux = b, where x is the unknown and b is given, has a unique solution.

Always

Answer: Always

Explanation

Answer: Always Notice

$$\underbrace{\begin{pmatrix} \frac{\upsilon_{0,0} & \upsilon_{0,1} & \upsilon_{0,2} \\ 0 & \upsilon_{1,1} & \upsilon_{1,2} \\ 0 & 0 & \upsilon_{2,2} \end{pmatrix} \begin{pmatrix} \frac{\chi_0}{\chi_1} \\ \chi_2 \end{pmatrix}}_{= \underbrace{\begin{pmatrix} \frac{\beta_0}{\beta_1} \\ \beta_2 \end{pmatrix}}_{= \underbrace{\begin{pmatrix} \frac{\upsilon_{0,0}}{\beta_1} \\ \frac{\upsilon_{1,1}}{\zeta_2} \\ 0 & \upsilon_{2,2} \end{pmatrix} \begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix}}_{= \underbrace{\begin{pmatrix} \frac{\upsilon_{0,0}}{\beta_1} \\ \frac{\upsilon_{1,1}}{\zeta_2} \\ 0 & \upsilon_{2,2} \end{pmatrix} \begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix}}_{= \underbrace{\begin{pmatrix} \frac{\upsilon_{0,0}}{\beta_1} \\ \frac{\upsilon_{0,0}}{\zeta_2} \\ \frac{\upsilon_{0,0}}{\zeta_2} \end{pmatrix} \begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix}}_{= \underbrace{\begin{pmatrix} \frac{\upsilon_{0,0}}{\beta_1} \\ \frac{\upsilon_{0,0}}{\zeta_2} \\ \frac{\upsilon_{0,0}}{\zeta_2} \end{pmatrix} \begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix}}_{= \underbrace{\begin{pmatrix} \frac{\upsilon_{0,0}}{\beta_1} \\ \frac{\upsilon_{0,0}}{\zeta_2} \end{pmatrix} \begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix}}_{= \underbrace{\begin{pmatrix} \frac{\upsilon_{0,0}}{\beta_1} \\ \frac{\upsilon_{0,0}}{\zeta_2} \end{pmatrix} \begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix} \end{pmatrix}}_{= \underbrace{\begin{pmatrix} \frac{\upsilon_{0,0}}{\beta_1} \\ \frac{\upsilon_{0,0}}{\zeta_2} \end{pmatrix} \begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix} \begin{pmatrix} \frac{\upsilon_{0,0}}{\zeta_2} \end{pmatrix} \begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix} \begin{pmatrix} \frac{\upsilon_{0,0}}{\zeta_2} \end{pmatrix} \begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix} \end{pmatrix}}_{= \underbrace{\begin{pmatrix} \frac{\upsilon_{0,0}}{\zeta_2} \\ \frac{\upsilon_{0,0}}{\zeta_2} \end{pmatrix} \begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix} \begin{pmatrix} \frac{\upsilon_{0,0}}{\zeta_2} \end{pmatrix} 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\end{pmatrix} \end{pmatrix}}_{= \underbrace{\begin{pmatrix} \frac{\upsilon_{0,0}}{\zeta_2} \\ \frac{\upsilon_{0,0}}{\zeta_2} \end{pmatrix} \begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix} \end{pmatrix}}_{= \underbrace{\begin{pmatrix} \frac{\upsilon_{0,0}}{\zeta_2} \\ \frac{\upsilon_{0,0}}{\zeta_2} \end{pmatrix} \begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix} \end{pmatrix}}_{= \underbrace{\begin{pmatrix} \frac{\upsilon_{0,0}}{\zeta_2} \\ \frac{\upsilon_{0,0}}{\zeta_2} \end{pmatrix} \begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix} \end{pmatrix}}_{= \underbrace{\begin{pmatrix} \frac{\upsilon_{0,0}}{\zeta_2} \\ \frac{\upsilon_{0,0}}{\zeta_2} \end{pmatrix} \begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix} \end{pmatrix}}_{= \underbrace{\begin{pmatrix} \frac{\upsilon_{0,0}}{\zeta_2} \\ \frac{\upsilon_{0,0}}{\zeta_2} \end{pmatrix} \begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix} \end{pmatrix}}_{= \underbrace{\begin{pmatrix} \frac{\upsilon_{0,0}}{\zeta_2} \\ \frac{\upsilon_{0,0}}{\zeta_2} \end{pmatrix} \begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix} \end{pmatrix}}_{= \underbrace{\begin{pmatrix} \frac{\upsilon_{0,0}}{\zeta_2} \\ \frac{\upsilon_{0,0}}{\zeta_2} \end{pmatrix} \begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix} \end{pmatrix}}_{= \underbrace{\begin{pmatrix} \frac{\upsilon_{0,0}}{\zeta_2} \\ \frac{\upsilon_{0,0}}{\zeta_2} \end{pmatrix} \begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix} \end{pmatrix}}_{= \underbrace{\begin{pmatrix} \frac{\upsilon_{0,0}}{\zeta_2} \\ \frac{\upsilon_{0,0}}{\zeta_2} \end{pmatrix} \begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix} \end{pmatrix}}_{= \underbrace{\begin{pmatrix} \frac{\upsilon_{0,0}}{\zeta_2} \\ \frac{\upsilon_{0,0}}{\zeta_2} \end{pmatrix} \begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix} \end{pmatrix}}_{= \underbrace{\begin{pmatrix} \frac{\upsilon_{0,0}}{\zeta_2} \\ \frac{\upsilon_{0,0}}{\zeta_2} \end{pmatrix} \begin{pmatrix} \chi_1 \\ \chi_1 \\ \chi_2 \end{pmatrix} \end{pmatrix}}_{= \underbrace{\begin{pmatrix} \frac{\upsilon_{0,0}}{\zeta_2} \\ \frac{\upsilon_{0,0}}{\zeta_2} \end{pmatrix} \begin{pmatrix} \chi_1 \\ \chi_1 \\ \chi_2 \end{pmatrix} \end{pmatrix}}_{= \underbrace{\begin{pmatrix} \frac{\upsilon_{0,0}}{\zeta_2} \\ \frac{\upsilon_{0,0}}{\zeta_2} \end{pmatrix} \begin{pmatrix} \chi_1 \\ \chi_1 \\ \chi_2 \end{pmatrix} \end{pmatrix}}_{= \underbrace{\begin{pmatrix} \frac{\upsilon_{0,0}}{$$

Hence, from the last exercise, we conclude that the unique solutions for χ_0 and χ_1 are

$$\begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix} = \begin{pmatrix} (\beta_1 - \upsilon_{1,2}\chi_2)/\upsilon_{1,1} \\ \beta_2/\upsilon_{2,2} \end{pmatrix}.$$

We can then compute χ_0 by substituting in:

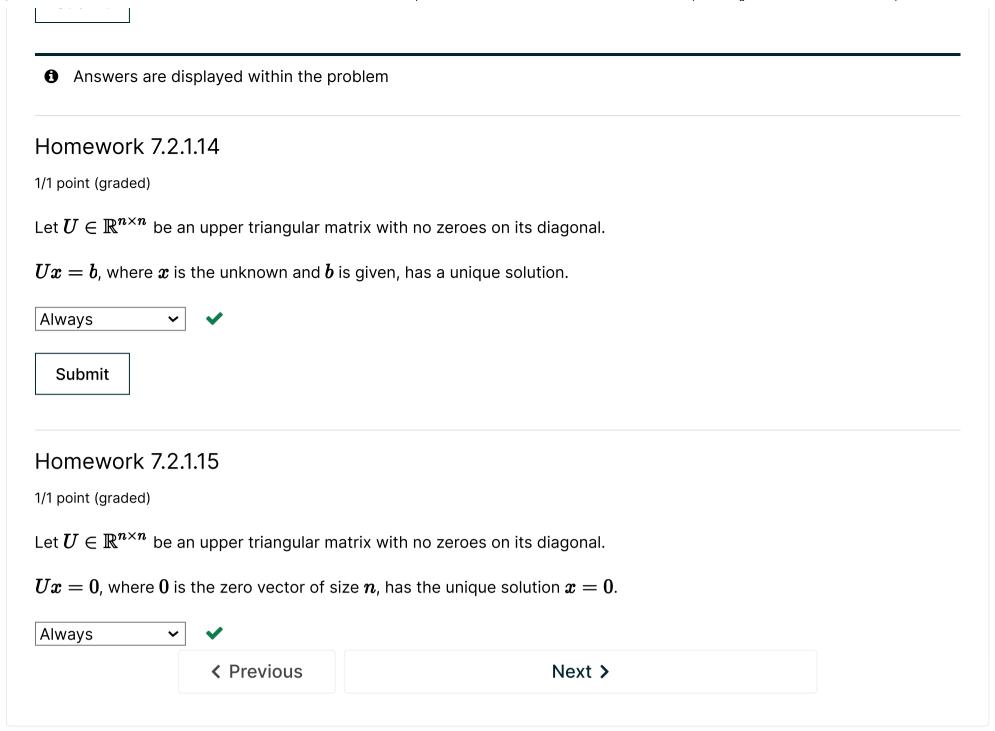
$$\mathbf{v}_{0,0}\mathbf{\chi}_0 + \begin{pmatrix} \mathbf{v}_{0,1} & \mathbf{v}_{0,2} \end{pmatrix} \begin{pmatrix} \mathbf{\chi}_1 \\ \mathbf{\chi}_2 \end{pmatrix} = \mathbf{\beta}_0$$

So that

$$\chi_0 = \left(\beta_0 - \left(\begin{array}{cc} \upsilon_{0,1} & \upsilon_{0,2} \end{array}\right) \left(\begin{array}{c} \chi_1 \\ \chi_2 \end{array}\right)\right) / \upsilon_{0,0}$$

Since there is no ambiguity about what χ_2 must equal, the solution is unique:

$$\begin{pmatrix} \chi_0 \\ \chi_1 \\ \chi_2 \end{pmatrix} = \begin{pmatrix} \begin{pmatrix} \beta_0 - \begin{pmatrix} \upsilon_{0,1} & \upsilon_{0,2} \end{pmatrix} \begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix} \end{pmatrix} / \upsilon_{0,0} \\ (\beta_1 - \upsilon_{1,2}\chi_2) / \upsilon_{1,1} \\ \beta_2 / \upsilon_{2,2} \end{pmatrix}.$$



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