7.3.4 More Advanced (But Still Simple) Examples

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Calculator

Week 7 due Nov 20, 2023 01:42 IST Completed

7.3.4 More Advanced (But Still Simple) Examples

Video

Well, if we partition B into its columns and we then multiply that by A,

and we know that all we need to do is multiply the individual columns of B

by that matrix A. But we also know that we then

want to end up with the identity matrix.

And notice that the columns on the identity matrix are just the unit basis

vectors.

So now we can go and say, ah, typical column on the left

must equal a typical column on the right.

And therefore, A times the j-th column of the inverse of A

must equal to the j-th unit basis vector.

And since we know how to solve Ax equals to b.

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Reading Assignment

0 points possible (ungraded)

Read Unit 7.3.4 of the notes. [LINK]

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Calculator

Homework 7.3.4.1

1/1 point (graded)

$$\text{Find } \begin{pmatrix} -2 & 0 \\ 4 & 2 \end{pmatrix}^{-1} =$$

○ $\begin{pmatrix} -\frac{1}{2} & 1 \\ 0 & \frac{1}{2} \end{pmatrix}$

○ $\begin{pmatrix} \frac{1}{2} & 0 \\ 1 & \frac{1}{2} \end{pmatrix}$

● $\begin{pmatrix} -\frac{1}{2} & 0 \\ 1 & \frac{1}{2} \end{pmatrix}$

○ $\begin{pmatrix} -\frac{1}{2} & 0 \\ 1 & -\frac{1}{2} \end{pmatrix}$

✓

Homework 7.3.4.1 Compute $\begin{pmatrix} -2 & 0 \\ 4 & 2 \end{pmatrix}^{-1} = \begin{pmatrix} -\frac{1}{2} & 0 \\ 1 & \frac{1}{2} \end{pmatrix}$.

Answer: Here is how you can find the answer First, solve

$$\begin{pmatrix} -2 & 0 \\ 4 & 2 \end{pmatrix} \begin{pmatrix} \chi_0 \\ \chi_1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}.$$

If you do forward substitution, you see that the solution is $\begin{pmatrix} -\frac{1}{2} \\ 1 \end{pmatrix}$, which becomes the first column of A^{-1}.

Next, solve

$$\begin{pmatrix} -2 & 0 \\ 4 & 2 \end{pmatrix} \begin{pmatrix} \chi_0 \\ \chi_1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}.$$

If you do forward substitution, you see that the solution is $\begin{pmatrix} 0 \\ \frac{1}{2} \end{pmatrix}$, which becomes the second column of A^{-1}.

Check:

$$\begin{pmatrix} -2 & 0 \\ 4 & 2 \end{pmatrix} \begin{pmatrix} -\frac{1}{2} & 0 \\ 1 & \frac{1}{2} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

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Answers are displayed within the problem

Video

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Dr. Robert van de Geijn: OK, so the way to compute this inverse

is to note that the lower triangular matrix times its inverse partition

by columns must be equal to the identity partition by its columns.

That's what we talked about in the last video.

If you then say, well, I want to solve with

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Homework 7.3.4.2

1/1 point (graded)

$$\text{Find } \begin{pmatrix} 1 & -2 \\ 0 & 2 \end{pmatrix}^{-1} =$$

○ $\begin{pmatrix} 1 & -\frac{1}{2} \\ 0 & \frac{1}{2} \end{pmatrix}$

○ $\begin{pmatrix} 1 & -\frac{1}{2} \\ 0 & 1 \end{pmatrix}$

● $\begin{pmatrix} 1 & 1 \\ 0 & \frac{1}{2} \end{pmatrix}$

○ $\begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix}$

✓

Answer:

$$\begin{pmatrix} 1 & 1 \\ 0 & \frac{1}{2} \end{pmatrix}.$$

Calculator

Here is how you can find this matrix: First, solve

$$\begin{pmatrix} 1 & -2 \\ 0 & 2 \end{pmatrix} \begin{pmatrix} \chi_0 \\ \chi_1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}.$$

If you do back substitution, you see that the solution is $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$, which becomes the first column of A^{-1}.

Next, solve

$$\begin{pmatrix} 1 & -2 \\ 0 & 2 \end{pmatrix} \begin{pmatrix} \chi_0 \\ \chi_1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}.$$

If you do back substitution, you see that the solution is $\begin{pmatrix} 1 \\ \frac{1}{2} \end{pmatrix}$, which becomes the second column of A^{-1}.

Check:

$$\begin{pmatrix} 1 & -2 \\ 0 & 2 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 0 & \frac{1}{2} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

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Homework 7.3.4.3

1/1 point (graded)

Let $\alpha_{0,0} \neq 0$ and $\alpha_{1,1} \neq 0$. Then $\begin{pmatrix} \alpha_{0,0} & 0 \\ \alpha_{1,0} & \alpha_{1,1} \end{pmatrix}^{-1} = \begin{pmatrix} \frac{1}{\alpha_{0,0}} & 0 \\ -\frac{\alpha_{1,0}}{\alpha_{0,0}\alpha_{1,1}} & \frac{1}{\alpha_{1,1}} \end{pmatrix}$

True ✓ Answer: True

Answer: True

Here is how you can find the matrix: First, solve

$$\begin{pmatrix} \alpha_{0,0} & 0 \\ \alpha_{1,0} & \alpha_{1,1} \end{pmatrix} \begin{pmatrix} \chi_0 \\ \chi_1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}.$$

If you solve the lower triangular system, you see that the solution is $\begin{pmatrix} \frac{1}{\alpha_{0,0}} \\ -\frac{\alpha_{1,0}}{\alpha_{0,0}\alpha_{1,1}} \end{pmatrix}$, which becomes the first column of A^{-1}.

Next, solve

$$\begin{pmatrix} \alpha_{0,0} & 0 \\ \alpha_{1,0} & \alpha_{1,1} \end{pmatrix} \begin{pmatrix} \chi_0 \\ \chi_1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}.$$

If you solve this system, you find the solution $\begin{pmatrix} 0 \\ \frac{1}{\alpha_{1,1}} \end{pmatrix}$, which becomes the second column of A^{-1}.

Check:

$$\begin{pmatrix} \alpha_{0,0} & 0 \\ \alpha_{1,0} & \alpha_{1,1} \end{pmatrix} \begin{pmatrix} \frac{1}{\alpha_{0,0}} & 0 \\ -\frac{\alpha_{1,0}}{\alpha_{0,0}\alpha_{1,1}} & \frac{1}{\alpha_{1,1}} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

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Homework 7.3.4.4

1/1 point (graded)

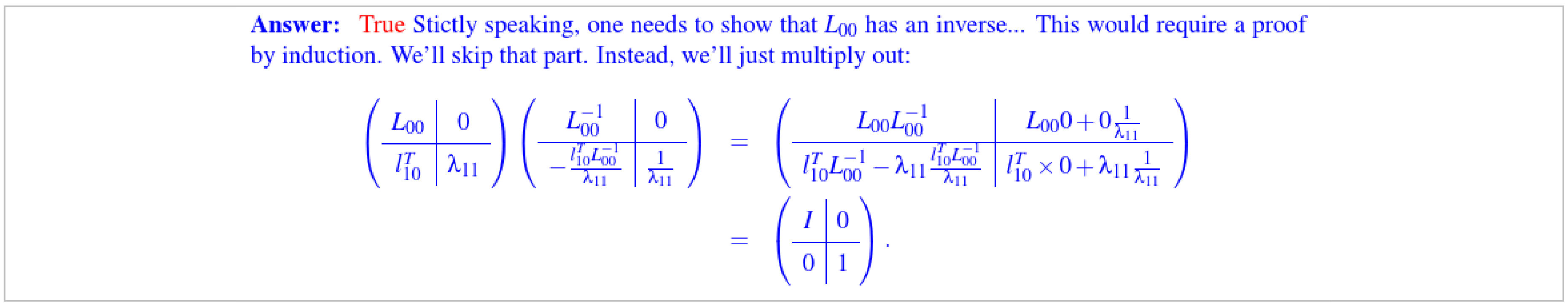
Partition lower triangular matrix L as

$$L = \left(\begin{array}{c|c} L_{00} & 0 \\ \hline l_{10}^T & \lambda_{11} \end{array} \right)$$

Assume that L has no zeroes on its diagonal. Then

$$L^{-1} = \left(\begin{array}{c|c} L_{00}^{-1} & 0 \\ \hline -\frac{l_{10}^T L_{00}^{-1}}{\lambda_{11}} & \frac{1}{\lambda_{11}} \end{array} \right)$$

True ✓ Answer: True



Answer: True Stictly speaking, one needs to show that L_{00} has an inverse... This would require a proof by induction. We'll skip that part. Instead, we'll just multiply out:

$$\left(\begin{array}{c|c} L_{00} & 0 \\ \hline l_{10}^T & \lambda_{11} \end{array} \right) \left(\begin{array}{c|c} L_{00}^{-1} & 0 \\ \hline -\frac{l_{10}^T L_{00}^{-1}}{\lambda_{11}} & \frac{1}{\lambda_{11}} \end{array} \right) = \left(\begin{array}{c|c} L_{00}L_{00}^{-1} & L_{00}0 + 0\frac{1}{\lambda_{11}} \\ \hline l_{10}^T L_{00}^{-1} - \lambda_{11}\frac{l_{10}^T L_{00}^{-1}}{\lambda_{11}} & l_{10}^T \times 0 + \lambda_{11}\frac{1}{\lambda_{11}} \end{array} \right)$$
$$= \left(\begin{array}{c|c} I & 0 \\ \hline 0 & 1 \end{array} \right).$$

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Video

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Dr. Robert van de Geijn: So now we get to a more general case.

And the exercise is to show that the inverse of this lower triangular matrix

partitioned like that is given by this.

The way to verify that that is indeed the inverse

is a matter of just multiplying the two matrices together.

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Homework 7.3.4.5

1/1 point (graded)

The inverse of a lower triangular matrix with no zeroes on its diagonal is a lower triangular matrix.

TRUE ⌄ ✓ Answer: TRUE

Answer: True

Proof by induction on n, the size of the square matrix.

Let L be the lower triangular matrix.

Base case: $n = 1$. Then $L = (\lambda_{11})$, with $\lambda_{11} \neq 0$. Clearly, $L^{-1} = (1/\lambda_{11})$.

Inductive step: Inductive Hypothesis: Assume that the inverse of any $n \times n$ lower triangular matrix with no zeroes on its diagonal is a lower triangular matrix.

We need to show that the inverse of any $(n+1) \times (n+1)$ lower triangular matrix, L, with no zeroes on its diagonal is a lower triangular matrix.

Partition

$$L = \left(\begin{array}{c|c} L_{00} & 0 \\ \hline l_{10}^T & \lambda_{11} \end{array}\right)$$

We know then that L_{00} has no zeroes on its diagonal and $\lambda_{11} \neq 0$. We also saw that then

$$L^{-1} = \left(\begin{array}{c|c} L_{00}^{-1} & 0 \\ \hline -\frac{1}{\lambda_{11}} l_{10}^T L_{00}^{-1} & \frac{1}{\lambda_{11}} \end{array}\right)$$

Hence, the matrix has an inverse, and it is lower triangular.

By the **Principle of Mathematical Induction**, the result holds.

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The answer to the last exercise suggests an algorithm for inverting a lower triangular matrix. See if you can implement it!

Homework 7.3.4.7

1/1 point (graded)

$$\text{Find } \begin{pmatrix} 1 & 2 \\ 1 & 1 \end{pmatrix}^{-1} =$$

○ $\begin{pmatrix} 1 & 2 \\ 1 & 1 \end{pmatrix}$

● $\begin{pmatrix} -1 & 2 \\ 1 & -1 \end{pmatrix}$

Calculator

○ $\begin{pmatrix} -1 & \frac{}{2} \\ 1 & -1 \end{pmatrix}$

○ $\begin{pmatrix} 1 & \frac{1}{2} \\ 1 & 1 \end{pmatrix}$

✓

Answer:

$$\begin{pmatrix} -1 & 2 \\ 1 & -1 \end{pmatrix}.$$

Here is how you can find this matrix: First, you compute the LU factorization. Since there is only one step, this is easy:

$$\begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 0 & -1 \end{pmatrix}.$$

Thus,

$$\begin{pmatrix} 1 & 2 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 0 & -1 \end{pmatrix}.$$

Next, you solve

$$\begin{pmatrix} 1 & 2 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} \chi_0 \\ \chi_1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

by solving $\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} \zeta_0 \\ \zeta_1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ followed by $\begin{pmatrix} 1 & 2 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} \chi_0 \\ \chi_1 \end{pmatrix} = \begin{pmatrix} \zeta_0 \\ \zeta_1 \end{pmatrix}$. If you do this right, you get $\begin{pmatrix} -1 \\ 1 \end{pmatrix}$, which becomes the first column of the inverse.

You solve

$$\begin{pmatrix} 1 & 2 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} \chi_0 \\ \chi_1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

in a similar manner, yielding the second column of the inverse, $\begin{pmatrix} 2 \\ -1 \end{pmatrix}$.

Check:

$$\begin{pmatrix} 1 & 2 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} -1 & 2 \\ 1 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

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Homework 7.3.4.8

1/1 point (graded)

Calculator

$$\text{If } \alpha_{0,0}\alpha_{1,1} - \alpha_{1,0}\alpha_{0,1} \neq 0 \text{ then}$$

$$\begin{pmatrix} \alpha_{0,0} & \alpha_{0,1} \\ \alpha_{1,0} & \alpha_{1,1} \end{pmatrix}^{-1} = \frac{1}{\alpha_{0,0}\alpha_{1,1} - \alpha_{1,0}\alpha_{0,1}} \begin{pmatrix} \alpha_{1,1} & -\alpha_{0,1} \\ -\alpha_{1,0} & \alpha_{0,0} \end{pmatrix}$$

True ✓ Answer: True

Answer: True

Check:

$$\begin{aligned} & \begin{pmatrix} \alpha_{0,0} & \alpha_{0,1} \\ \alpha_{1,0} & \alpha_{1,1} \end{pmatrix} \frac{1}{\alpha_{0,0}\alpha_{1,1} - \alpha_{1,0}\alpha_{0,1}} \begin{pmatrix} \alpha_{1,1} & -\alpha_{0,1} \\ -\alpha_{1,0} & \alpha_{0,0} \end{pmatrix} \\ = & \frac{1}{\alpha_{0,0}\alpha_{1,1} - \alpha_{1,0}\alpha_{0,1}} \begin{pmatrix} \alpha_{0,0} & \alpha_{0,1} \\ \alpha_{1,0} & \alpha_{1,1} \end{pmatrix} \begin{pmatrix} \alpha_{1,1} & -\alpha_{0,1} \\ -\alpha_{1,0} & \alpha_{0,0} \end{pmatrix} \\ = & \frac{1}{\alpha_{0,0}\alpha_{1,1} - \alpha_{1,0}\alpha_{0,1}} \begin{pmatrix} \alpha_{0,0}\alpha_{1,1} - \alpha_{0,1}\alpha_{1,0} & -\alpha_{0,0}\alpha_{0,1} + \alpha_{0,1}\alpha_{0,0} \\ \alpha_{1,0}\alpha_{1,1} - \alpha_{1,1}\alpha_{1,0} & -\alpha_{1,0}\alpha_{0,1} + \alpha_{1,1}\alpha_{0,0} \end{pmatrix} \\ = & \frac{1}{\alpha_{0,0}\alpha_{1,1} - \alpha_{1,0}\alpha_{0,1}} \begin{pmatrix} \alpha_{0,0}\alpha_{1,1} - \alpha_{0,1}\alpha_{1,0} & 0 \\ 0 & \alpha_{1,1}\alpha_{0,0} - \alpha_{1,0}\alpha_{0,1} \end{pmatrix} \\ = & \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \end{aligned}$$

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Homework 7.3.4.9

1/1 point (graded)

The 2×2 matrix $A = \begin{pmatrix} \alpha_{0,0} & \alpha_{0,1} \\ \alpha_{1,0} & \alpha_{1,1} \end{pmatrix}$ has an inverse if and only if $\alpha_{0,0}\alpha_{1,1} - \alpha_{1,0}\alpha_{0,1} \neq 0.$

True ✓

Submit

In the below video, towards the end, the instructor misspeaks and says that the inverse exists if and only if the determinant is zero. The correct statement is that the inverse exists if and only if the determinant is **not** zero.

Video

Start of transcript. Skip to the end.

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