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<u>Unit 0. Course Overview, Syllabus,</u> <u>Guidelines, and Homework on</u>

Homework 0: Probability and Linear

<u>Course</u> > <u>Prerequisites</u> > <u>algebra Review</u>

> 6. Probability tables

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6. Probability tables

Gaussian probabilities

4/4 points (graded)

Let $X\sim N\left(1,2.25\right)$. As a reminder, the 2.25 here represents the value of σ^2 . Using the normal probability table below, compute the following probabilities:

Normal probability table

<u>Show</u>

$${f P}\left(|X-2|\leq 1
ight)= oxed{0.4087887802741321}$$
 $lacktriangledow{Answer: 0.4082}$

$$\mathbf{P}(X^2 > 4) = \boxed{0.2752426694951021}$$
 \checkmark Answer: 0.2752427

$$\mathbf{P}(X^2 - 2X - 1 > 0) = \begin{bmatrix} 0.3457785861511602 \end{bmatrix}$$
 Answer: 0.3457786

STANDARD NOTATION

Solution:

First, note that for $\,Z \sim \mathcal{N}\left(0,1
ight)$, $\,x > 0$, we have

$$\mathbf{P}\left(Z\leq -x
ight)=\mathbf{P}\left(Z\geq x
ight)=1-\mathbf{P}\left(Z\leq x
ight),$$

and

$$\mathbf{P}\left(Z\geq x
ight)=1-\mathbf{P}\left(Z\leq x
ight).$$

Moreover, if $X \sim \mathcal{N}\left(1,2.25\right)$, we can write it as X=1.5Z+1 , where $Z \sim \mathcal{N}\left(0,1\right)$. This allows us to reduce all probabilities to the ones that are listed in the table.

In particular,

$$\begin{array}{lll} \mathbf{P}(X>1) = & \mathbf{P}(1.5Z+1>1) = \mathbf{P}(1.5Z>0) \\ = & \mathbf{P}(Z\geq0) = 1 - \mathbf{P}(Z\leq0) = 1 - 0.5000 = 0.5000, \\ \mathbf{P}(|X-2|\leq1) = & \mathbf{P}(-1\leq(X-2)\leq1) = \mathbf{P}(-1\leq(1.5Z+1-2)\leq1) \\ = & \mathbf{P}(0\leq1.5Z\leq2) \\ \simeq & \mathbf{P}(0\leq Z\leq1.33) \\ = & \mathbf{P}(Z\leq1.33) - \mathbf{P}(Z\leq0) \simeq 0.9082 - 0.5000 = 0.4082 \\ \\ \mathbf{P}(X^2>4) = & \mathbf{P}(|X|>2) = \mathbf{P}(|1.5Z+1|>2) \\ = & \mathbf{P}(1.5Z+1\leq-2) + \mathbf{P}(1.5Z+1\geq2) \\ = & \mathbf{P}(Z\leq-2) + \mathbf{P}\left(Z\geq\frac23\right) \\ = & 1 - \mathbf{P}(Z\leq2) + 1 - \mathbf{P}\left(Z\leq\frac23\right) \\ \simeq & 2 - 0.9772 - 0.7486 = 0.2742 \\ \\ \mathbf{P}(X^2-2X-1>0) = & \mathbf{P}(|X-1|^2-2>0) = \mathbf{P}(|X-1|>\sqrt2) \\ = & \mathbf{P}(|1.5Z|>\sqrt2) \end{array}$$

$$= \mathbf{P}(Z > \frac{\sqrt{2}}{1.5}) + \mathbf{P}(Z < -\frac{\sqrt{2}}{1.5})$$

$$= 2 - 2\mathbf{P}(Z < \frac{\sqrt{2}}{1.5})$$

$$\simeq 2 - 2(0.8264)$$

$$= 0.3472.$$

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You have used 2 of 3 attempts

1 Answers are displayed within the problem

Approximation of Binomial variables

1/1 point (graded)

Using the normal probability table, evaluate approximately ${f P}\,(X>400)$, where $\,X\,$ is a binomial random variable with parameters $\,1000\,$ and $\,.3\,$.

Normal probability table

Show

$$\mathbf{P}\left(X>400
ight)\simeq igg|$$
 0

✓ Answer: 0.0002

STANDARD NOTATION

Solution:

A binomial distribution with parameters (n,p) has expectation np and variance np(1-p). Hence, by the Central Limit Theorem, we have

$$rac{1}{\sqrt{np\left(1-p
ight)}}(X-np) \stackrel{ ext{(D)}}{\longrightarrow} Z \sim \mathcal{N}\left(0,1
ight).$$

The probability in question can therefore be approximated by

$$\mathbf{P}(X > 400) = \mathbf{P}\left(\frac{1}{\sqrt{1000 \times 0.3 \times 0.7}}(X - 300) > \frac{100}{\sqrt{1000 \times 0.3 \times 0.7}}\right)$$

$$\simeq 1 - \mathbf{P}\left(Z \le \frac{100}{\sqrt{1000 \times 0.3 \times 0.7}}\right)$$

$$\simeq 1 - \mathbf{P}\left(Z \le 6.90\right)$$

$$< 1 - 0.9998 = 0.0002.$$

Note: This is only an estimate, because the probability table ends here. In fact, the probability is approximately $\,7 imes10^{-12}$.

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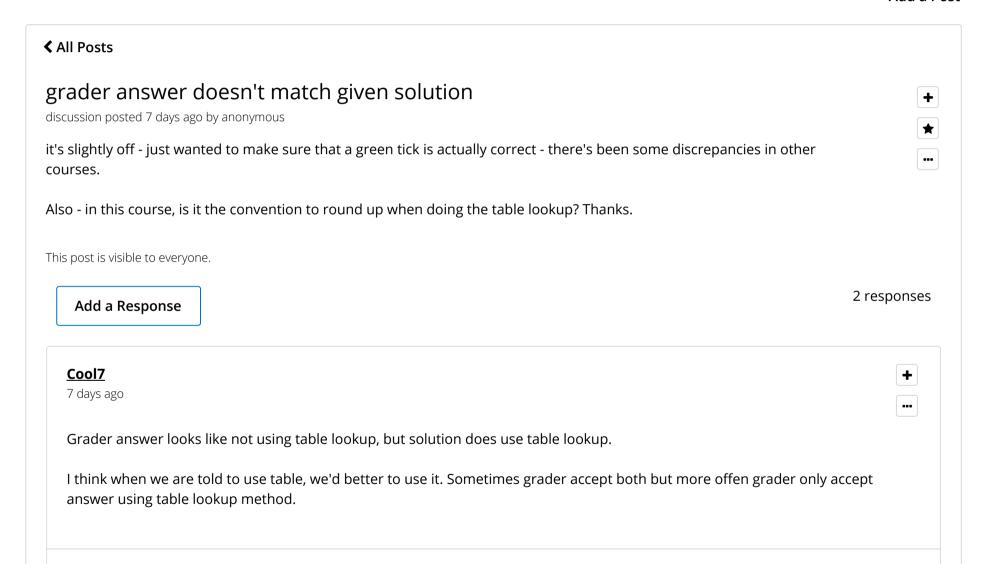
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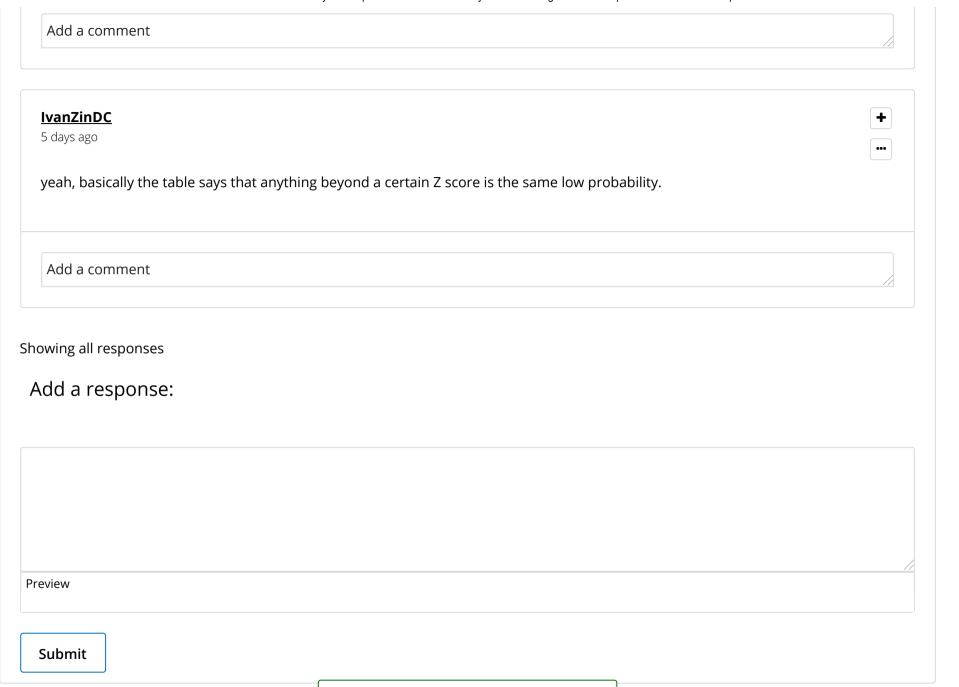
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