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Problem Set B due Aug 18, 2021 20:30 IST Completed



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5(a)

2.0/2 points (graded)

You are swimming along the surface of a large natural hot water spring. The temperature is hottest near the geothermal heat sources, and cools off inversely proportional to the distance from the heat source as you move away. The hot spring you have found has two heat sources. One is located below $m{x}=m{0}$, $m{y}=m{0}$. The other is located at x=-10, y=20.

You do a quick approximation of the temperature of this hot spring to be

$$T\left({x,y} \right) = rac{{450}}{{\sqrt {{x^2} + {y^2} + 1} }} + rac{{420}}{{\sqrt {{{\left({x + 10}
ight)^2} + {{\left({y - 20}
ight)^2} + 1}} }}$$

where temperature $m{T}$ is measured in Celsius and $m{x}$ and $m{y}$ along the surface of the hot water spring are measured in meters.

You enter the pool on the edge (20,20) where it is 30 degrees Celsius.

What is the gradient of the temperature at that point?

(Enter answer as a vector quantity with 4 decimal places of accuracy in each component surrounded by square brackets: [a,b].)

[-0.8628929, -0.397003] **Answer:** [-0.8629, -0.3970]

At the given point, what rate does the temperature rise per unit distance in that direction?

0.9498397

degrees Celsius per meter **Answer:** 0.9498

Solution:

To figure out the direction the temperature increases fastest at the point (20, 20) we compute the gradient.

$$T_x(x,y) = \frac{-450x}{\sqrt{x^2 + y^2 + 1}^3} + \frac{-420(x+10)}{\sqrt{(x+10)^2 + (y-20)^2 + 1}}$$
 (3.159)

$$T_x\left(20,20
ight) \ = \ rac{-450\left(20
ight)}{\sqrt{400+400+1}^3} + rac{-420\left(20+10
ight)}{\sqrt{\left(30
ight)^2+1}} pprox -0.8629$$
 (3.160)

$$T_y(x,y) = \frac{-450y}{\sqrt{x^2 + y^2 + 1}^3} + \frac{-420(y - 20)}{\sqrt{(x + 10)^2 + (y - 20)^2 + 1}}$$
 (3.161)

$$T_y\left(20,20\right) = \frac{-450\left(20\right)}{\sqrt{400+400+1}^3} + \frac{-420\left(0\right)}{\sqrt{\left(30\right)^2+1}^3} \approx -0.3970$$
 (3.162)

Thus the temperature increases most quickly in the direction $\langle -0.8629, -0.3970 \rangle$.

The magnitude of the gradient here is the "slope" or how quickly the temperature is ri 🖬 Calculator 🦼 🏕 Hide Notes

travelled. $|\langle -0.8629, -0.3970 \rangle| \approx 0.9498$

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You have used 0 of 5 attempts

1 Answers are displayed within the problem

5(b)

3.0/3 points (graded)

You swim in this direction and continue in a straight line until the temperature reaches a balmy 40 degrees Celsius. *Approximately* how far must you swim?

10.52809

meters **Answer:** 10.5285

What is the position you swim to?

(Enter the point as an ordered pair, separated by commas, and surrounded by round parentheses: (a,b). Each component should be entered to 2 decimal places.)

(10.44, 15.60)

Answer: (10.46,15.61)

What is the actual temperature there? (Answer to the nearest degree.)

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Solution:

You want to travel in the direction $\langle -0.8629, -0.3970 \rangle = \nabla T$ until the temperature has increased by 10 degrees celsius. To get the units right, we first need to convert this direction to a unit vector: $\langle -0.9085, -0.4180 \rangle = \frac{\nabla T}{|\nabla T|}$.

Now we can use linear approximation to find the distance we need to travel to increase the temperature by 10.

$$10 = \Delta T pprox T_x \left(-0.9085d
ight) + T_y \left(-0.4180d
ight) =
abla T \cdot rac{
abla T}{|
abla T|} d = |
abla T| d$$

Therefore $d=10/|\nabla T|pprox 10.5$ meters.

Your position after swimming is $(20 - 0.9085(10.5), 20 - 0.4180(10.5)) \approx (10.46, 15.61)$.

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5(c)

3.0/3 points (graded)

Your sense of direction isn't the best. Instead of swimming in the direction of the gradient, you swim in the direction $\langle -1,0\rangle$. How fast is the temperature changing the moment you begin swimming (i.e. at the point (20,20))? (Give at least 2 decimal places.)

0.8628929

degrees Celsius per meter **Answer:** 0.8629

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You swim 9 meters in the direction $\langle -1,0 \rangle$. What is the *actual* temperature where you end up? (Enter to the nearest degree.)

degrees Celsius **✓ Answer:** 39.6733

Which direction should you now start swimming so that the temperature does not change?

(Enter answer as a vector quantity to 4 decimal places in each component surrounded by square brackets: [a,b].)

[0.7546347, -1.3642] **Answer:** [-0.7546, 1.3642]

Solution:

The rate of change in the direction $\langle -1,0 \rangle$ is the directional derivative

$$D_{\langle -1,0\rangle}T(20,20) = \nabla T(20,20) \cdot (-1,0)$$
 (3.163)

$$= -T_x(20, 20) = 0.8629 \text{ degrees Celsius per meter}$$
 (3.164)

If you swim 9 meters in the negative x direction, you end up at (11,20). The temperature at this point is 39.6 degrees Celsius.

We compute the gradient at the point (11,20). Then we find a vector normal to this vector.

$$T_x(x,y) = \frac{-450x}{\sqrt{x^2 + y^2 + 1}^3} + \frac{-420(x+10)}{\sqrt{(x+10)^2 + (y-20)^2 + 1}}$$
 (3.165)

$$T_x\left(11,20
ight) = \frac{-450\left(11
ight)}{\sqrt{121+400+1}^3} + \frac{-420\left(21
ight)}{\sqrt{\left(21
ight)^2+1}^3} pprox -1.3642$$
 (3.166)

$$T_y(x,y) = \frac{-450y}{\sqrt{x^2 + y^2 + 1}^3} + \frac{-420(y - 20)}{\sqrt{(x + 10)^2 + (y - 20)^2 + 1}}$$
 (3.167)

$$T_y(11,20) = \frac{-450(20)}{\sqrt{121+400+1}^3} + 0 \approx -0.7546$$
 (3.168)

Therefore one answer is $\langle -0.7546, 1.3642 \rangle$. (Another answer is the opposite direction. Or any other scalar multiple of this direction.)

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You have used 0 of 5 attempts

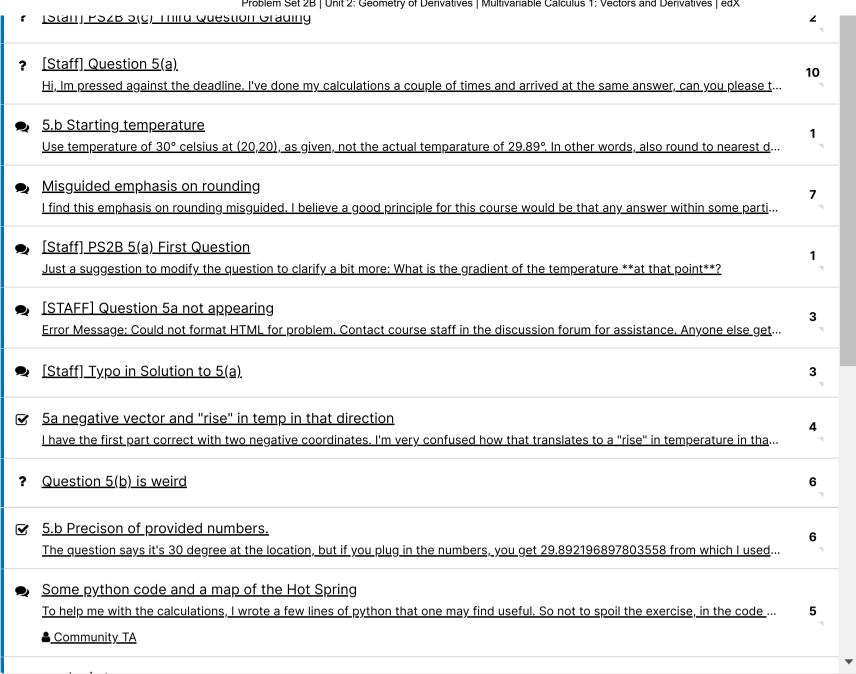
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5. Swimming in Iceland

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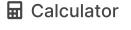
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