edX

2. Subsets and subspaces
Identify the subspaces

1/1 point (graded)

Choose all of the following subsets of $\mathbb{R}^4$ which are vector spaces.

☑ The set of vectors in $\mathbb{R}^4$ whose coordinates are all equal to each other. ✔

☐ The set consisting of the two vectors $\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$.

☑ The set of vectors in $\mathbb{R}^4$ whose coordinates sum to 0. ✔

☑ The set of vectors $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix}$ such that $x_1 = x_2$ and $x_1 + x_2 + x_3 = x_4$. ✔

☐ The set of vectors such that at least one coordinate is 0.

✔

Solution:

- The set of vectors in $\mathbb{R}^4$ whose coordinates are all equal to each other is the set of vectors of the form

$$\begin{pmatrix} c \\ c \\ c \\ c \end{pmatrix} = c \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}.$$

Therefore it is the span of a single vector, so it is a vector space.

- The set consisting of the two vectors $\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$ is not a vector space, because it does not contain the zero vector.

- The set of vectors in $\mathbb{R}^4$ whose coordinates sum to 0 is equivalent to the set of vectors $\mathbf{x}$ such that $\mathbf{Ax} = 0$ where $\mathbf{A} = \begin{pmatrix} 1 & 1 & 1 & 1 \end{pmatrix}$. Thus the set in question is the nullspace of the matrix $\mathbf{A}$, which is a vector space. In particular, this is a 3 dimensional vector subspace of $\mathbb{R}^4$.

- The set of vectors $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix}$ such that $x_1 = x_2$ and $x_1 + x_2 + x_3 = x_4$ is equal to the set of solutions to the homogeneous system

$$\begin{aligned} x_1 - x_2 &= 0 \\ x_1 + x_2 + x_3 - x_4 &= 0. \end{aligned}$$

Therefore this set is the nullspace of the matrix

$$\mathbf{A} = \begin{pmatrix} 1 & -1 & 0 & 0 \\ 1 & 1 & 1 & -1 \end{pmatrix},$$

which is a vector space. In particular this describes a two dimensional subspace of $\mathbb{R}^4$.

- The set of vectors such that at least one coordinate is 0 is not a vector space. For example, the vectors $\begin{pmatrix} 1 \\ 1 \\ 1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 1 \\ 1 \\ 1 \end{pmatrix}$ are in the set, but their sum $\begin{pmatrix} 1 \\ 2 \\ 2 \\ 1 \end{pmatrix}$ is not.

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ℹ   Answers are displayed within the problem

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?	**Need hint on Q2** I believe that all the selections meet the three criteria of vector space except for one, but I got it wrong. …	10
💬	**at least one coordinate is 0…** So for the last selection… does the vector in the set has a zero element in any arbitrary coordinates? or is…	2

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