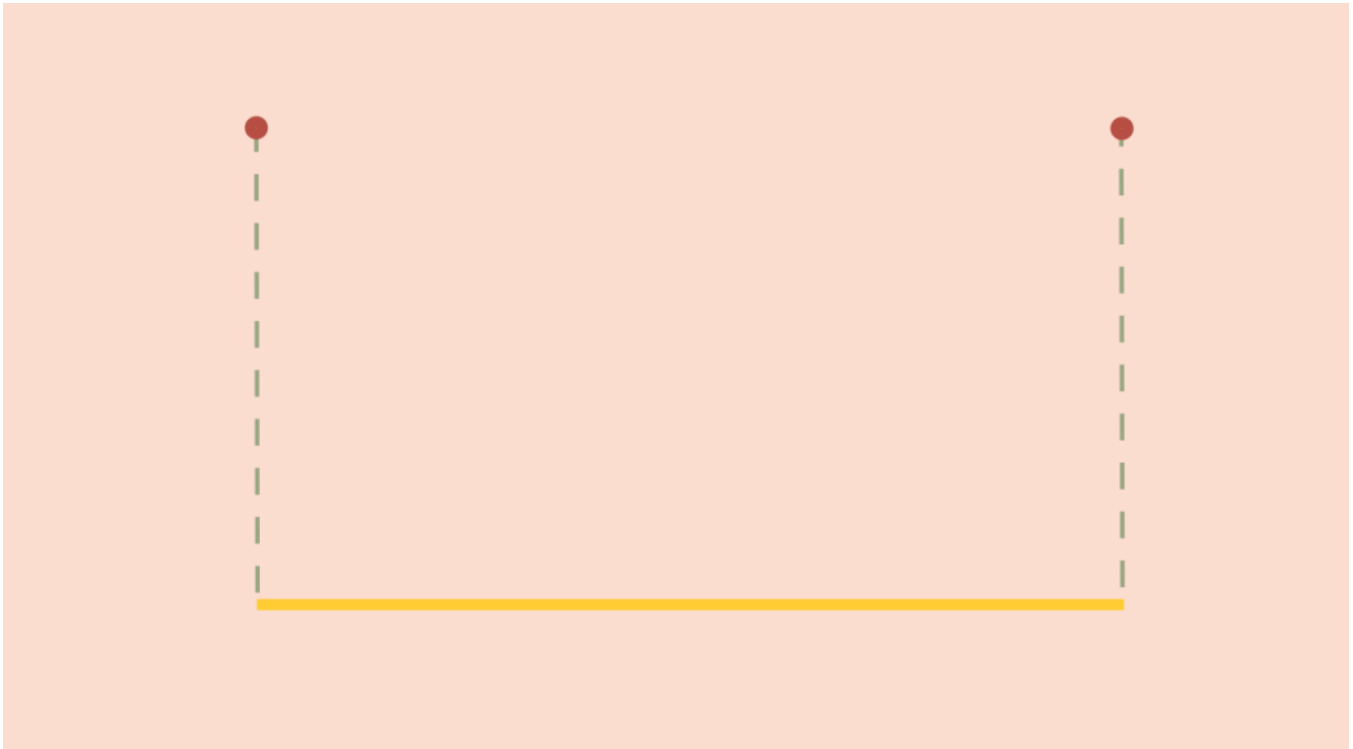


Math Counterexamples



ANALYSIS

A DISCONTINUOUS REAL CONVEX FUNCTION

SEPTEMBER 13, 2015 | JEAN-PIERRE MERX | [LEAVE A COMMENT](#)

Consider a function f defined on a real interval $I \subset \mathbb{R}$. f is called **convex** if:

$$\forall x, y \in I \forall \lambda \in [0, 1] : f((1 - \lambda)x + \lambda y) \leq (1 - \lambda)f(x) + \lambda f(y)$$

Suppose that I is a closed interval: $I = [a, b]$ with $a < b$. For $a < s < t < u < b$ one can prove that:

$$\frac{f(t) - f(s)}{t - s} \leq \frac{f(u) - f(s)}{u - s} \leq \frac{f(u) - f(t)}{u - t}.$$

It follows from those relations that f has left-hand and right-hand derivatives at each point of the interior of I . And therefore that f is continuous at each point of the interior of I .

Is a convex function defined on an interval I continuous at all points of the interval? That might not be the case and a simple example is the function:

$$f : \left\{ \begin{array}{ll} [0, 1] & \longrightarrow \mathbb{R} \\ x & \longmapsto 0 \text{ for } x \in (0, 1) \\ x & \longmapsto 1 \text{ else} \end{array} \right.$$

It can be easily verified that f is convex. However, f is not continuous at 0 and 1.

◀ ANALYSIS ◀ CONTINUITY ◀ CONVEXITY ◀ MAPS ◀ REAL-ANALYSIS