You are taking "Exam (Timed, No Correctness Feedback)" as a timed exam. Show more

End My Exam 22:46:42

10. Summary

Bookmark this page

Summarize

Big Picture

A differentiable function $f(x, y)$ of two variables on a closed bounded region R attains an absolute maximum (and absolute minimum) on R.

- The absolute maximum (or minimum) occurs at a critical point, or
- the absolute maximum (or minimum) occurs on the boundary of R.

Key point 1 : Along the boundary, the maximum occurs when the gradient ∇f is normal (perpendicular) to the boundary.

Key point 2 : If the boundary of the region R is described as the level curve $g(x, y) = k$. Then the maximum occurs where ∇f and ∇g point in the same (or opposite) direction:

$$\boxed{\nabla f = \lambda \nabla g}.$$

Mechanics

The method of **Lagrange multipliers** is used to optimize a function $f(x, y)$ (find the max or min) along a curve C described as a level curve $g(x, y) = k$ for some function $g(x, y)$. The curve C is called the **constraint**. A summary of the steps is given below.

1. Solve the following system of equations

$$f_x(x, y) = \lambda g_x(x, y) \qquad (4.193)$$

$$f_y(x, y) = \lambda g_y(x, y) \qquad (4.194)$$

$$g(x, y) = k \qquad (4.195)$$

   for x and y. (The scalar λ is called the **Lagrange multiplier**.)

2. Compute the value of $f(x, y)$ at each point found in Step 1.
3. Identify which points give the maxima and minima of $f(x, y)$.

Ask Yourself

How do you determine which function plays which role?

The function f is the function whose maximum and minimum we want to find. The function g describes the constraint.

Hide

If a function is only defined along a curve and has no meaning otherwise, do you still check critical points?

No! If you only care about a function along a curve C, the restricted domain is this curve, and not the interior. In this case, it is enough to find the maximum and minimum along the boundary. Note that any critical points along the boundary will be found by the method of Lagrange Multipliers already!

Hide

Calculator Hide Notes

10. Summary

Topic: Unit 3: Optimization / 10. Summary

Hide Discussion

Add a Post

Show all posts by recent activity

< Previous

Next Up: Recitation 11: Practice with Lagrange Multiplier Problems
11 min + 4 activities >