






< Previous





Next >

# 11.3.1 Newton's method for systems

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MO2.10

We can also use Newton's method for problems with systems of equations (i.e.  $M > 1$ ). A system of nonlinear equations is of the form

$$r_i(x_0, \dots, x_{M-1}) = 0, \quad i = 0, \dots, M - 1.$$

We take the same number of equations and unknowns, so that we may be in a situation where there is one solution (rather than a continuum of solutions or no solution at all.) Whether the system has zero, one or several solutions is still a question that needs to be addressed separately. The shorthand notation for the system is  $\underline{r}(\underline{x}) = 0$

By analogy with the 1D case we perform a Taylor expansion about  $\underline{x}^k$ :

$$\underline{r}(\underline{x}^k + \Delta \underline{x}) \approx \underline{r}(\underline{x}^k) + \nabla \underline{r}(\underline{x}^k) \Delta \underline{x}$$

The gradient  $\nabla \underline{r}$  is known as the Jacobian. It is an  $M \times M$  matrix that we will refer to symbolically as  $J$ . Note that the  $(i, j)$  element of  $J$  is:

$$J_{i,j} = \frac{\partial r_i}{\partial x_j} \tag{11.8}$$

Continuing with the derivation of Newton's method for a system, we set the Taylor series approximation to zero to find  $\Delta \underline{x}$ :

$$\underline{r}(\underline{x}^k) + J(\underline{x}^k) \Delta \underline{x} = 0 \Rightarrow J(\underline{x}^k) \Delta \underline{x} = -\underline{r}(\underline{x}^k)$$

which is a system of linear equations that must be solved for each iteration. And finally, we update  $\underline{x}^k$  by

< Previous

Next >

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2/3



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