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[Lecture 10: Consistency of MLE,  
Covariance Matrices, and](#)

10. Multivariate Central Limit  
> Theorem

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## 10. Multivariate Central Limit Theorem

**Note:** The following exercise will be presented in the video that follows. We encourage you to attempt it before watching the video.

### Vector Version of the Central Limit Theorem

1/1 point (graded)

Let  $\mathbf{X}$  be a random vector of dimension  $d \times 1$  and let  $\mu$  and  $\Sigma$  be its mean and covariance. Let  $\mathbf{X}_1, \dots, \mathbf{X}_n$  be i.i.d. copies of  $\mathbf{X}$ . Let  $\bar{\mathbf{X}}_n \triangleq \frac{1}{n} \sum_{i=1}^n \mathbf{X}_i$ .

Based on your knowledge of the central limit theorem for a single random variable, select from the following the correct shift and scale factor for  $\bar{\mathbf{X}}_n$  so that  $\bar{\mathbf{X}}_n$  could potentially converge to the Gaussian random vector  $\mathcal{N}(0, I_{d \times d})$ .

☐  $\sqrt{d} \cdot \Sigma^{-\frac{1}{2}} (\bar{\mathbf{X}}_n - \mu)$

☐  $\sqrt{d} \cdot \Sigma^{-1} (\bar{\mathbf{X}}_n - \mu)$

☐  $\sqrt{n} \cdot \Sigma^{-1} (\bar{\mathbf{X}}_n - \mu)$

☒  $\sqrt{n} \cdot \Sigma^{-\frac{1}{2}} (\bar{\mathbf{X}}_n - \mu)$

☐ None of the above**Solution:**

The shift of  $\mathbf{X}$  by  $\mu$  is the correct shift that needs to be applied in order to center the random vector.

The scaling factor should be  $\sqrt{n}\Sigma^{-\frac{1}{2}}$  because it mimics the single variable CLT case most closely. In particular, the division by  $\sqrt{\sigma^2}$  in the single variable CLT case is being taken care of by the inverse of the square root of  $\Sigma$ .

**Note:** Of course, this is only a heuristic discussion that is meant to test how you can potentially generalize the single variable CLT. This is not a proof and the solution is also written as guesswork.

You have used 1 of 3 attempts

 Answers are displayed within the problem

## Multivariate Central Limit Theorem

The multivariate CLT

The CLT may be generalized to averages or random vectors (also vectors of averages).  
 Let  $X_1, \dots, X_n \in \mathbb{R}^d$  be independent copies of a random vector  $X$  such that  $\mathbb{E}[X] = \mu$ ,  $\text{Cov}(X) = \Sigma$ .

$$\sqrt{n}(\bar{X}_n - \mu) \xrightarrow{d} N(0, \Sigma)$$

Equivalently

$$\frac{\bar{X}_n - \mu}{\sqrt{n}} \xrightarrow{d} N_d(0, I_d)$$

(Caption will be displayed when you start playing the video.)



## Video

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(Optional) Multivariate Convergence in Distribution and Proof of Multivariate CLT

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### A covariance matrix is a positive definite matrix and vice-versa. Why?

discussion posted 3 days ago by [mbh038](#)

Wow.

'Any covariance matrix is a positive definite matrix, any positive definite matrix is a covariance matrix'.

Anyone care to explain why this is so?

This post is visible to everyone.

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1 response

[markweitzman](#) (Community TA)

3 days ago

See: [Positive definite Real Symmetric Matrix and its Eigenvalues](#)

If I understood correctly, we also assume here and in the rest of the course that variances (diagonal elements) are non-zeros. Since they already were greater or equal to zero (e.g., because they are variances) now we know that they are positive.

We can diagonalize  $\Sigma$ . When a matrix is in a diagonal form all its eigenvalues are on the diagonal. And in our case they all are positive, so the matrix is, obviously, positive definite.

Reverse (if a matrix  $A$  is positive definite then it is a covariance matrix) I believe can be proved in the same way. Lets diagonalize  $A$  and get some matrix  $\Sigma_Y$ . Now  $\Sigma_Y$  is a covariance matrix of a certain random vector  $Y_1, \dots, Y_n$ , so (since the transformation is done by using orthogonal, i.e. invertible, matrices) now we can trace back the origin of our  $Y_1, \dots, Y_n$  and find the original  $X_1, \dots, X_n$  and their covariance matrix  $\Sigma_X$ .

posted a day ago by [Alexander Andrianov](#)

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