7. Linear Regression - Basic Setup

Linear Regression: The Function for Conditional Expectation of Y Given a value x

(Caption will be displayed when you start playing the video.)

0:00 / 0:00 1.50x

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In **Linear Regression** , *we will work with the assumption that the regression function* $\nu(x) := \mathbb{E}[Y|X=x]$ *is linear*, so that

$$\nu(x) = a + bx$$

for some pair (a, b).

In this unit, we will be studying the **Least Squares Estimator** . It is an estimator $(\hat{a}, \hat{b})$ so that $\hat{Y} = \hat{a} + \hat{b}X$ is "close" (in some distance metric) to the actual Y as often as possible.

A Minimization Problem

1/1 point (graded)

Let X be an arbitrary random variable, with mean μ and variance σ^2 . In terms of μ and σ^2 , which scalar k is the unique minimizer of the function $f(k) = \mathbb{E}[(X-k)^2]$?

Hint: Write $f(k)$ as a quadratic in k.

mu

μ

STANDARD NOTATION

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Correct (1/1 point)

An Estimator

1/1 point (graded)

Let (X, Y) be a pair of random variables for which the regression function $\nu(x) = \mathbb{E}[Y|X=x]$ takes the form

$$\nu(x) = a + bx$$

for some pair of real numbers (a, b).

What is a random variable $\hat{Y}$ that is a function of X that minimizes

$$\mathbb{E}\left[(Y-\hat{Y})^2|X=x\right]$$

over all possible choices of $\hat{Y}$ and for all x? Enter your answer in terms of a, b and the random variable X (capital letter “X").

(*Remark: for a clean, quick solution, it may be helpful to review the law of iterated expectations:* $\mathbb{E}_{X,Y}[\cdot] = \mathbb{E}_X[\mathbb{E}_Y[\cdot \mid X]]$, *where* $\mathbb{E}_Y[\cdot|X]$ *denotes the conditional expectation, which is a random variable. Use the insight from the previous exercise.*)

a+b*X

✔ **Answer:** a + b*X

$a + b \cdot X$

STANDARD NOTATION

Solution:

For each realization x of X, the previous exercise tells us that $\mathbb{E}_Y[(Y-\hat{y})^2|X=x]$ is minimized by $\hat{y} = \nu(x)$. Since $\nu(x) = a + bx$ is a minimizer for each choice of x, $\nu(X)$ is a minimizer over all choices of $\hat{Y}$.

These two exercises verify that the Least Squares Estimator is consistent in the following sense: **using the actual distribution on (X, Y), the true pair (a, b) itself is a least squares estimator.** It may or may not be unique; we will address this in the following sections.

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You have used 1 of 3 attempts

Answers are displayed within the problem

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