3. Asymptotics

LTI Consumer (External resource) (1.0 points possible)

Asymptotics for large time

Consider again three connected medicine storage tanks where tank 1 has a leak. For large time, the volume of medicine in each of the three tanks will decay to zero. This is because when we write our problem in matrix form $\dot{\mathbf{x}} = \mathbf{A}\mathbf{x}$ the matrix $\mathbf{A}$ has purely negative eigenvalues.

In the previous problem, we saw that initially solutions may still grow. This is known as the transient regime. Calculating what happens in the transient regime often requires knowing the exact solution of the problem, and the behavior may depend strongly on the initial conditions.

However, the long time solution of any problem of the form $\dot{\mathbf{x}} = \mathbf{A}\mathbf{x}$ will generically be dominated by the part of the solution corresponding to the eigenvalue with the largest real part. That is, if $\lambda_1 > \lambda_2 > \lambda_3$, with corresponding eigenvectors $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ then almost any long time solution will look like

$$\mathbf{x} \sim ce^{\lambda_1 t}\mathbf{v}_1$$

where c is a number that depends on the initial value and the eigenvector matrix.

Unfortunately, the output of MATLAB's "eig" function does not output the eigenvalues in any particular order. In this problem we will construct an algorithm to find the eigenvalue with largest real part. We will then compare the asymptotic solution with the exact solution to see how well it captures the decay of the solution.

Your Script

Save Reset MATLAB Documentation (https://www.mathworks.com/help/)

```
A = [ -2 1 0; 1 -2 1; 0 1 -1];
x0 = [0.5;0;0.5];
t = linspace(0,4,100);
% Store the eigenvectors and eigenvalues in matrices V and D respectively.
[V,D] = eig(A);

%Define I to be the index of the largest eigenvalue of A. That is the largest eig
% Hint try using sort() or max() and look into the MATLAB documentation for help.

[~, ind] = sort(diag(D),'descend');

I = ind;

% Defined lambda to be the maximum eigenvalue, and v to be the maximum eigenvecto


lambda = D(I(1),I(1));
v = V(:,I(1));
%
```

```
%
C = inv(V)*x0;
c = C(I(1),1);
asymp = c*exp(lambda*t)*v(1,1);
% use the matrix exponential to find the solution for the given initial condition
for m=1:length(t)
    x(:,m) = expm(A*t(m))*x0;
end
%

plot(t,x(1,:))
hold on
plot(t,asymp)
legend('x','asymp')
set(gca,'fontsize',18)
xlabel('Time')
ylabel('Volume')
```

▶ Run Script ? ()

Assessment: Correct

Submit ? ()

- V is correct
- D is correct
- I is correct
- lambda is correct
- v is correct
- Exponential matrix used
- x is correct

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