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Lec. 4: Counting

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Solved problems**Problem Set 3**

Problem Set 3 due Feb 24, 2016 at 23:59 UTC

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Problem 5: Hats in a box

(5/5 points)

Each one of n persons, indexed by $1, 2, \dots, n$ has a clean hat and throws it into a box. The persons then pick hats from the box, at random. Every assignment of the hats to the persons is equally likely. In an equivalent model, each person picks a hat, one at a time, in the order of their index, with each one of the remaining hats being equally likely to be picked. Find the probability of the following events.

1. Every person gets his or her own hat back.

☒ $\frac{1}{n!}$ ✓

☐ $\frac{1}{(n+1)!}$

☐ $\frac{1}{n}$

☐ $\frac{1}{n+1}$

2. Each one of persons $1, \dots, m$ gets his or her own hat back, where $1 \leq m \leq n$.

☐ $\frac{(n+m)!}{n!}$

☒ $\frac{(n-m)!}{n!}$ ✓

☐ $\frac{n!}{(n+m)!}$

☐ $\frac{m!}{n!}$

3. Each one of persons $1, \dots, m$ gets back a hat belonging to one of the last m persons (persons $n-m+1, \dots, n$), where $1 \leq m \leq n$.

☒ $\frac{1}{\binom{n}{m}}$ ✓

☐ $\frac{m}{\binom{n}{m}}$

☐ $\frac{n-m}{\binom{n}{m}}$

☐ $\frac{n}{\binom{n}{m}}$

Now assume, in addition, that every hat thrown into the box has probability p of getting dirty (independently of what happens to the other hats or who has dropped or picked it up). Find the probability that:

4. Persons $1, \dots, m$ will pick up clean hats.

☐ $(1-p)^{n-m}$

☐ $m(1-p)^m$

☒ $(1 - p)^m$ ✓

☐ $m(1 - p)^{n-m}$

5. Exactly m persons will pick up clean hats.

☐ $\frac{\binom{n}{m}}{n!} (1 - p)^m p^{n-m}$

☐ $(1 - p)^m p^{n-m}$

☐ $\binom{n}{m} (1 - p)^{n-m} p^m$

☒ $\binom{n}{m} (1 - p)^m p^{n-m}$ ✓

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DISCUSSION

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