Exercises: Weighted Least Squares Estimation

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True or False

7/7 points (ungraded)

If measurement 1 is considered to be more important than measurement 2, we want to give measurement 2 a higher weight.

- ○ True
- ◉ False ✔

If we would copy a particular measurement two times, so we have the same measurement three times in our system of observation equations, it would get three times the weight, compared to the situation where we only have it one time in our system of observation equations.

- ◉ True ✔
- ○ False

If we want to give particular measurements a different weight, we need to introduce a weight matrix.

- (•) True ✔
- ( ) False

If we have four observations, the weight matrix has the size 4×1.

- ( ) True
- (•) False ✔

In the normal equations, the weight matrix needs to be used twice.

- (•) True ✔
- ( ) False

For a system of equations $y = Ax$, we obtain the normal equations by pre-multiplying both sides with $A^T W$, where W is the weight matrix.

- (•) True ✔
- ( ) False

The weighted least-squares method is a special case of the unweighted least squares method.

- ( ) True
- (•) False ✔

Answer
Correct: Ordinary least squares is a special case of weighted least squares

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✔ Correct (7/7 points)

Canal width - ordinary least squares

6/6 points (ungraded)

Suppose we have 4 distance measurements of the canal width (in meters):
$y = [10.1 \quad 9.8 \quad 10.2 \quad 10.3]^T$.

Our goal is to estimate the canal width $\hat{x}$.

Compute the value of the normal matrix $A^T A$ [give exact answer]

4

✔ **Answer:** 4

4

What is the ordinary least squares estimate of the canal width $\hat{x}$ (in meters)?

10.1

✔ **Answer:** 10.1

10.1

What are the values in the vector of residuals $\hat{e}$? Make sure to enter the values in the right order (corresponding to the observation vector y)

0.0

✔ **Answer:** 0

0.0

-0.3

✔ **Answer:** -0.3

-0.3

0.1

Answer: 0.1

0.1

0.2

Answer: 0.2

0.2

Explanation

Note that the least squares estimate here is simply the mean of the observations.

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Correct (6/6 points)

Canal width - weighted least squares

6/6 points (ungraded)

For the same problem as in the previous exercise, we know that the last two observations were taken by a very experienced surveyor, while the first two measurements were taken by an amateur surveyor. Therefore we decide to give more weight to the last two observations.

It is the decided that the weights for observations 1 to 4 are as follows: $w_1 = 1$, $w_2 = 1$, $w_3 = 2$, $w_4 = 2$ (i.e., the weigth matrix is a diagonal matrix with the aforementioned weights on the diagonal).

Compute the value of the normal matrix $A^T WA$ [give the exact answer]

6

✔ **Answer:** 6

6

What is the weighted least squares estimate of the canal width $\hat{x}$ (in meters) [answer to 2 decimal places]?

10.15

✔ **Answer:** 10.15

10.15

What are the values in the vector of residuals $\hat{e}$? Make sure to enter the values in the right order (corresponding to the observation vector y). [answer to 2 decimal places]

-0.05

✔ **Answer:** -0.05

-0.05

-0.35

✔ **Answer:** -0.35

-0.35

0.05

Answer: 0.05

0.05

0.15

Answer: 0.15

0.15

Explanation

Note that the normal matrix returns the sum of the weights $\sum w_i$. Also note that the last two residuals are smaller since their corresponding observations had a larger weight. In other words: by assigning a larger weight to certain observations, we demand the least squares solution to be closer to those observations.

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Correct (6/6 points)

Canal width - ordinary vs. weighted least squares

1/1 point (ungraded)

You and your fellow students are asked to measure the canal width. You all decide to take a different approach (just like the teams in the canal width video). Alice is asked to come up with an estimate of the canal width using all m observations. Based on her own 'ranking', Alice decides to give different weights to the different observations.

Which of the following is the equation that Alice will use to estimate the canal width? The individual weights are called w_i ($i = 1, \ldots, m$)

- $\hat{x} = \frac{1}{m}\sum_{i=1}^{m} y_i$
- $\hat{x} = \frac{1}{\sum_{i=1}^{m} w_i}\sum_{i=1}^{m}(w_i y_i)$ ✔
- $\hat{x} = \frac{1}{\sum_{i=1}^{m} w_i}\sum_{i=1}^{m} y_i$
- $\hat{x} = \sum_{i=1}^{m}\frac{y_i}{w_i}$

Explanation

$$\hat{x} = \left(\begin{bmatrix}1 & 1 & \ldots & 1\end{bmatrix}\begin{bmatrix}w_1 & & & \\ & w_2 & & \\ & & \ddots & \\ & & & w_m\end{bmatrix}\begin{bmatrix}1\\1\\\vdots\\1\end{bmatrix}\right)^{-1}\begin{bmatrix}1 & 1 & \ldots & 1\end{bmatrix}\begin{bmatrix}w_1 & & & \\ & w_2 & & \\ & & \ddots & \\ & & & w_m\end{bmatrix}\begin{bmatrix}y_1\\y_2\\\vdots\\y_m\end{bmatrix}$$

Note: this is the equation for the weighted average! Hence, the weighted least squares is equal to the weighted average in case we have m observations with $E\{\underline{y}_i\} = x$ ($i = 1, \ldots, m$).
Check yourself what happens if all weights are identical.

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✔ Correct (1/1 point)