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Previous Next

15. Linear approximation, understanding tangent planes

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Recitation due Aug 4, 2021 20:30 IST Completed

Practice

We will study the function $f(x,y) = y^2 - x^3 + xy - x$ by using linear approximation. This function is complicated, and it would be challenging to draw its level curves without a computer. On the other hand, we will see that the linear approximation helps us to understand some things about the function and its level curves.

1. Compute the value at a point

1/1 point (graded)
Let $f(x,y) = y^2 - x^3 + xy - x$.

Compute $f(1,1)$.

$f(1,1) =$ 0 ✔ **Answer:** 0

Submit You have used 1 of 10 attempts

Answers are displayed within the problem

2. Compute partial derivatives, and evaluate

4.0/4 points (graded)
Let $f(x,y) = y^2 - x^3 + xy - x$ as above.

Compute $f_x(x,y)$ and $f_y(x,y)$.

$f_x(x,y) =$ -3*x^2+y-1 ✔ **Answer:** -3*x^2+y-1

$f_y(x,y) =$ 2*y+x ✔ **Answer:** 2*y+x

? INPUT HELP

Plug in to find $f_x(1,1)$ and $f_y(1,1)$.

$f_x(1,1) =$ -3 ✔ **Answer:** -3

$f_y(1,1) =$ 3 ✔ **Answer:** 3

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3. Compute linear approximation, evaluate

4.0/4 points (graded)
Let $f(x,y) = y^2 - x^3 + xy - x$ as above.

Calculator Hide Notes

Find a formula the linear approximation of f around the point $(1,1)$ at the point (x,y) or at the point $(1+\Delta x, 1+\Delta y)$.

$f(x,y) \approx$ 3*y-3*x ✔ **Answer:** -3*(x-1)+3*(y-1)

? INPUT HELP

Use it to approximate $f(1.01,1)$, $f(1,1.01)$ and $f(1.01,1.01)$.

$f(1.01,1) \approx$ -0.03 ✔ **Answer:** -0.03

$f(1,1.01) \approx$ 0.03 ✔ **Answer:** 0.03

$f(1.01,1.01) \approx$ 0 ✔ **Answer:** 0

Submit You have used 1 of 15 attempts

Answers are displayed within the problem

4. Sketch level curves

1.0/1 point (graded)
Let $f(x,y) = y^2 - x^3 + xy - x$ as above.

Using the **linear approximation** , try to sketch the level curves of f inside the box where $.5 \leq x \leq 1.5$ and $.5 \leq y \leq 1.5$. (This region is outlined with a black box.)

Sketch the level curves of f at heights -1.5, 0, and 1.5. Use the specified tool for each level curve.

Note that when drawing the function, you may choose between using the freeform or spline drawing tool using the dropdown menu in the menu of the sketch response tool. The freeform tool draws like a regular pencil using your mouse. The spline tool allows you to pick a series of discrete points, and it then connects them (once you have two or more points) with a curve.

Pro tip: To draw disconnected segments using the spline tool, complete one segment and then click to any other tool on the tool bar and then back to the spline tool.

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Answer: See solution.

✔

Good Job

Solution:

Here is a picture of some level curves of f in the box where $.5 \leq x \leq 1.5$ and $.5 \leq y \leq 1.5$.

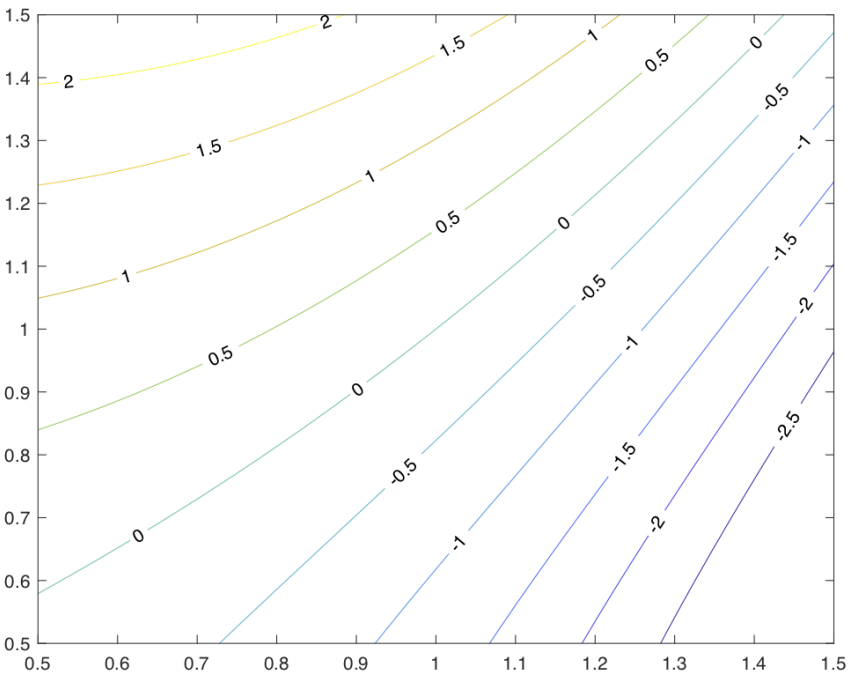

Figure 6: Level curves of f for $.5 \leq x \leq 1.5$

For comparison, here are the level curves that are predicted by the linear approximation.

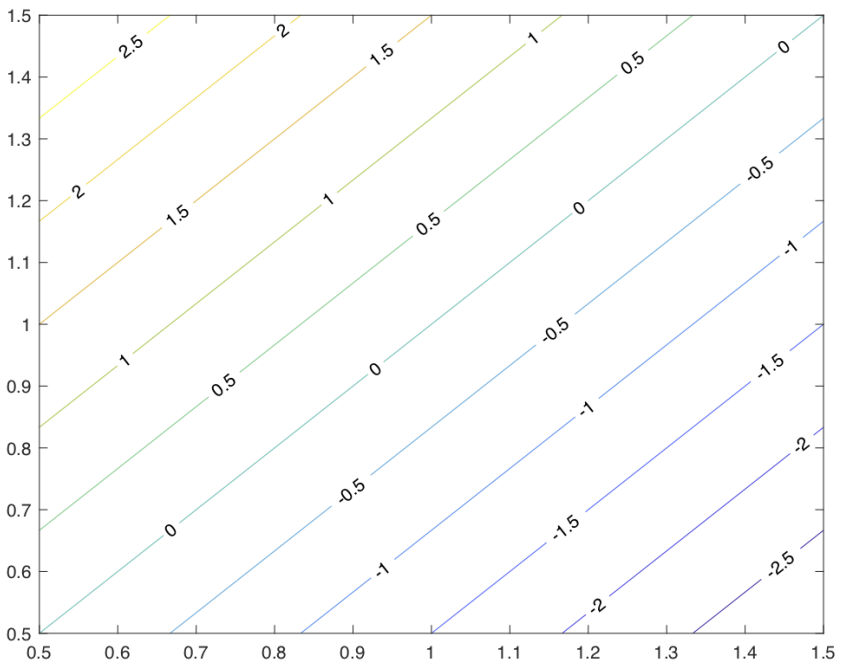

Figure 7: Level curves of the linear approximation of f for $.5 \leq x \leq 1.5$

Submit You have used 1 of 25 attempts

Answers are displayed within the problem

The problems so far helped us understand what the linear approximation is. Now, let's keep exploring how to use it.

5. Find a zero

1/1 point (graded)

Let $f(x, y) = y^2 - x^3 + xy - x$ as above.

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I am looking for a value of x close to 1 where $f(x, 0.99) = 0$. Approximate this value of x.

$x =$ 0.99 ✔ **Answer:** 0.99

Pro tip: Check your work! Evaluate $f(x, 0.99)$ and compute the error. Compare the value you found to the image of the level curves and see if it looks right.

Solution:

We are looking for $x = 1 + \Delta x$ so that

$$\begin{aligned} 0 &= f(1,1) + f_x(1,1)\,\Delta x + f_y(1,1)\,(-0.01) && (2.39)\\ &= 0 - 3\Delta x - 0.03 && (2.40)\end{aligned}$$

This implies that $\Delta x = -0.01$, so $x = 1 + \Delta x = 0.99$.

Submit You have used 1 of 10 attempts

Answers are displayed within the problem

6. Find a property with constraint

1.0/1 point (graded)
Let $f(x, y) = y^2 - x^3 + xy - x$ as above.

I am looking for a point (x, y) close to $(1, 1)$ where $f(x, y) = 0.01$ and $x + y = 2$. Approximate this point (x, y).

(Enter coordinates correctly up to 5 decimal places. Type as an ordered pair surrounded by round parentheses: e.g. `(1,1)`.)

$(x, y) =$ (0.9983333, 1.001667) ✔ **Answer:** (0.99833,1.00167)

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Solution:

We are looking for $x = 1 + \Delta x$, and $y = 1 + \Delta y$ so that

$$2 = x + y = (1 + \Delta x) + (1 + \Delta y) = 2 + \Delta x + \Delta y.$$

Thus we are looking for $\Delta x = -\Delta y$.

$$\begin{aligned} 0.01 &= f(1,1) + f_x(1,1)\,\Delta x + f_y(1,1)\,\Delta y && (2.41)\\ &= 0 - 3\Delta x + 3\Delta y && (2.42)\\ &= 0 - 3\Delta x - 3\Delta x && (2.43)\\ \Delta x &= -0.01/6 && (2.44)\end{aligned}$$

This implies that $x = 1 - (0.01/6)$ and $y = 1 + (0.01/6)$.

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Answers are displayed within the problem

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15. Linear approximation, understanding tangent planes

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Problem 5		2
[Staff] An approach querry. Hello there, This post is regarding question number 5. I solved it using L.A at (1,1). The equation for the L.A is 3dY-3dX, which basical...		2
[Rec 1-2] 4. Sketch level curves Community TA	4 new	15
Error in 4. Sketch level curves. I used the linear approxiamation and I received this error *Check the x coordinate of your level curve of height -1.5 where y=0.5* I a...		4
Rec 2, problem 15 parts 5 & 6 I obtained the solutions by the simple approach that we have found the linear approximation equation and are given the z-value we ...		1
[Staff] Sketch level curves - very few attempts There are only 3 attempts for *Sketch level curves*. Perhaps more were intended, since SketchInput questions usually have 10 to 25...		5
[Staff] Please check solution to Problem 6. Hi Staff, Please check the equations 2.41 - 2.44.		3
[STAFF] Error in Problem 6 In the last problem I get the following error: $extrakeys\neg allowed \circ data\,[' \to \leq rance']\,.\,Got 0.0001$ Please take a look.		10

Previous Next

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