



MITx: 6.041x Introduction to Probability - The Science of Uncertainty



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Problem 6: True or False II

(3/3 points)

Determine whether each of the following statement is true (i.e., always true) or false (i.e., not always true).

1. Let X be a random variable that takes values between 0 and c only, for some $c \geq 0$, so that $\mathbf{P}(0 \leq X \leq c) = 1$. Then, $\text{var}(X) \leq c^2/4$.

True ▼



Answer: True

2. X and Y are continuous random variables. If $X \sim N(\mu, \sigma^2)$ (i.e., normal with mean μ and variance σ^2), $Y = aX + b$, and $a > 0$, then $Y \sim N(a\mu + b, a\sigma^2)$.

False ▼



Answer: False

3. The expected value of a non-negative continuous random variable X , which is defined by $\mathbf{E}[X] = \int_0^\infty x f_X(x) dx$, also satisfies $\mathbf{E}[X] = \int_0^\infty \mathbf{P}(X > t) dt$.


True ▼




Answer: True

Unit overview


Lec. 8: Probability density functions

Exercises 8 due Mar 18, 2016 at 23:59 UTC 

Lec. 9: Conditioning on an event; Multiple r.v.'s

Exercises 9 due Mar 18, 2016 at 23:59 UTC 


Lec. 10: Conditioning on a random variable; Independence; Bayes' rule

Exercises 10 due Mar 18, 2016 at 23:59 UTC 

Standard normal table

Solved problems

Problem Set 5

Problem Set 5 due Mar 18, 2016 at 23:59 UTC 

Unit summary

- ▶ Unit 6: Further topics on random variables
- ▶ Unit 7: Bayesian inference

Answer:

1. The statement is true. Since $0 \leq X \leq c$,

$$\begin{aligned}\mathbf{E}[X^2] &= \mathbf{E}[XX] \\ &\leq \mathbf{E}[cX] \\ &= c\mathbf{E}[X].\end{aligned}$$

Therefore,

$$\begin{aligned}\text{var}(X) &= \mathbf{E}[X^2] - (\mathbf{E}[X])^2 \\ &\leq c\mathbf{E}[X] - (\mathbf{E}[X])^2 \\ &= c^2 \left(\frac{\mathbf{E}[X]}{c} \right) - c^2 \left(\frac{\mathbf{E}[X]}{c} \right)^2 \\ &= c^2 \left(\frac{\mathbf{E}[X]}{c} \left(1 - \frac{\mathbf{E}[X]}{c} \right) \right) \\ &= c^2 [\alpha(1 - \alpha)] \\ &\leq c^2/4,\end{aligned}$$

where $\alpha = \mathbf{E}[X]/c$. The last inequality is obtained by noticing that the function $\alpha(1 - \alpha)$ is largest at $\alpha = 1/2$, where it takes a value of $1/4$.

2. The statement is false. The correct statement is: $Y \sim N(a\mu + b, a^2\sigma^2)$.

- ▶ Exam 2
- ▶ Unit 8: Limit theorems and classical statistics
- ▶ Unit 9: Bernoulli and Poisson processes
- ▶ Unit 10: Markov chains
- ▶ Exit Survey
- ▶ Final Exam

3. The statement is true. By performing an interchange of the order of integration, we obtain

$$\begin{aligned}
 \int_0^\infty \mathbf{P}(X > t) dt &= \int_0^\infty \int_t^\infty f_X(x) dx dt \\
 &= \int_0^\infty \int_0^x f_X(x) dt dx \\
 &= \int_0^\infty x f_X(x) dx \\
 &= \mathbf{E}[X].
 \end{aligned}$$

This result is analogous to the result for discrete random variables that was shown in the Unit 4 solved problem .

You have used 1 of 1 submissions

DISCUSSION

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