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Odds

Odds are a numerical expression, usually expressed as a pair of numbers, used in both gambling and statistics. In statistics, the **odds for** or **odds of** some event reflect the likelihood that the event will take place, while **odds against** reflect the likelihood that it will not. In gambling, the odds are the ratio of payoff to stake, and do not necessarily reflect exactly the probabilities. Odds are expressed in several ways (see below), and sometimes the term is used incorrectly to mean simply the probability of an event.[1][2] Conventionally, gambling odds are expressed in the form "X to Y", where X and Y are numbers, and it is implied that the odds are odds against the event on which the gambler is considering wagering. In both gambling and statistics, the 'odds' are a numerical expression of the likelihood of some possible event.

In gambling, odds represent the ratio between the amounts staked by parties to a wager or bet.[3] Thus, odds of 6 to 1 mean the first party (normally a bookmaker) stakes six times the amount staked by the second party. In simplest terms, 6 to 1 odds means if you bet a dollar (the "1" in the expression), and you win you get paid six dollars (the "6" in the expression), or 6 x 1. If you bet two dollars you would be paid twelve dollars, or 6 x 2. If you bet three dollars and win, you would be paid eighteen dollars, or 6 x 3. If you bet one hundred dollars and win you would be paid six hundred dollars, or 6 x 100. If you lose any of those bets you would lose the dollar, or two dollars, or three dollars, or one hundred dollars.

In statistics, the odds for an event E are defined as a simple function of the probability of that possible event E. One drawback of expressing the uncertainty of this possible event as odds for is that to regain the probability requires a calculation. The natural way to interpret odds for (without calculating anything) is as the ratio of events to non-events in the long run. A simple example is that the (statistical) odds for rolling six with a fair die (one of a pair of dice) are 1 to 5. This is because, if one rolls the die many times, and keeps a tally of the results, one expects 1 six event for every 5 times the die does not show six. For example, if we roll the fair die 600 times, we would very much expect something in the neighborhood of 100 sixes, and 500 of the other five possible outcomes. That is a ratio of 100 to 500, or simply 1 to 5. To express the (statistical) odds against, the order of the pair is reversed. Hence the odds against rolling a six with a fair die are 5 to 1. The probability of rolling a six with a fair die is the single number 1/6, roughly 0.17.

The gambling and statistical uses of odds are closely interlinked. If a bet is a fair one, then the odds offered to the gamblers will perfectly reflect relative probabilities. A fair bet that a fair die will roll a six will pay the gambler $5 for a $1 wager (and return the bettor his or her wager) in the case of a six and nothing in any other case. The terms of the bet are fair, because on average, five rolls result in something other than a six, at a cost of $5, for every roll that results in a six and a net payout of $5. The profit and the expense exactly offset one another and so there is no advantage to gambling over the long run. If the odds being offered to the gamblers do not correspond to probability in this way then one of the parties to the bet has an advantage over the other. Casinos, for example, offer odds that place themselves at an advantage, which is how they guarantee themselves a profit and survive as businesses. The fairness of a particular gamble is more clear in a game involving relatively pure chance, such as the ping-pong ball method used in state lotteries in the United States. It is much harder to judge the fairness of the odds offered in a wager on a sporting event such as a football match.

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History

The language of odds such as "ten to one" for intuitively estimated risks is found in the sixteenth century, well before the development of mathematical probability.[4] Shakespeare wrote:

> Knew that we ventured on such dangerous seas
> That if we wrought out life 'twas ten to one
>
> — William Shakespeare, *Henry IV, Part II*, Act I, Scene 1 lines 181–2.

The sixteenth century polymath Cardano demonstrated the efficacy of defining odds as the ratio of favourable to unfavourable outcomes (which implies that the probability of an event is given by the ratio of favourable outcomes to the total number of possible outcomes[5]).

Terminology

Odds are expressed in the form X to Y, where X and Y are numbers. Usually, the word "to" is replaced by a symbol for ease of use. This is conventionally either a slash or hyphen, although a colon is sometimes seen. Thus, 6/1, 6-1 and 6:1 are all interchangeable.

Odds against

When the probability that the event will not happen is greater than the probability that it will, then the odds are "against" that event happening. Odds of 6 to 1, for example, are therefore sometimes said to be "6 to 1 *against*". To a gambler, "odds against" means that the amount he or she will win is greater than the amount staked.

Odds on

"Odds on" is the opposite of "odds against". It means that the event is more likely to happen than not. This is sometimes expressed with the smaller number first (1 to 2) but more often using the word "on" ("2 to 1 *on*") meaning that the event is twice as likely to happen as not. Note that the gambler who bets at "odds on" and wins will still be in profit, as his stake will be returned. For example, on a $2 bet, the gambler will be given $1 plus the returned stake of $2, yielding a $1 profit.

Even odds

"Even odds" occur when the probability of an event happening is exactly the same as it not happening. In common parlance, this is a "50-50 chance". Guessing heads or tails on a coin toss is the classic example of an event that has even odds. In gambling, it is commonly referred to as "even money" or simply "evens" (1 to 1, or 2 for 1). "Evens" implies that the payout will be one unit per unit wagered plus the original stake, that is, "double-your-money".

Better than/worse than evens

The term "better than evens" (or "worse than evens") varies in meaning depending on context. Looked at from the perspective of a gambler rather than a statistician, "better than evens" means "odds against". If the odds are evens (1:1), and one bets 10 units, one would be returned 20 units, making a profit of 10 units. If the gamble was paying 4:1 and the event occurred, one would make 50 units, or a profit of 40 units. So, it is "better than evens" from the gambler's perspective because it pays out more than one-for-one. If an event is more likely to occur than an even chance, then the odds will be "worse than evens", and the bookmaker will pay out less than one-for-one.

However, in popular parlance surrounding uncertain events, the expression "better than evens" usually implies a greater than 50% chance of the event occurring, which is exactly the opposite of the meaning of the expression when used in a gaming context.

Statistical usage

In statistics, odds are an expression of relative probabilities, generally quoted as the odds *in favor*. The odds (in favor) of an event or a proposition is the ratio of the probability that the event will happen to the probability that the event will not happen. Mathematically, this is a Bernoulli trial, as it has exactly two outcomes. In case of a finite sample space of equally likely outcomes, this is the ratio of the number of outcomes where the event occurs to the number of outcomes where the event does not occur; these can be represented as W and L (for Wins and Losses) or S and F (for Success and Failure). For example, the odds that a randomly chosen day of the week is a weekend are two to five (2:5), as days of the week form a sample space of seven outcomes, and the event occurs for two of the outcomes (Saturday and Sunday), and not for the other five.[6][7] Conversely, given odds as a ratio of integers, this can be represented by a probability space of a finite number of equally likely outcomes. These definitions are equivalent, since dividing both terms in the ratio by the number of outcomes yields the probabilities: $2:5=(2/7):(5/7).$ Conversely, the odds against is the opposite ratio. For example, the odds against a random day of the week being a weekend are 5:2.

Odds and probability can be expressed in prose via the prepositions *to* and *in:* "odds of so many *to* so many on (or against) [some event]" refers to *odds* – the ratio of numbers of (equally likely) outcomes in favor and against (or vice versa); "chances of so many [outcomes], *in* so many [outcomes]" refers to *probability* – the number of (equally like) outcomes in favour relative to the number for and against combined. For example, "odds of a weekend are 2 *to* 5", while "chances of a weekend are 2 *in* 7". In casual use, the words *odds* and *chances* (or *chance*) are often used interchangeably to vaguely indicate some measure of odds or probability, though the intended meaning can be deduced by noting whether the preposition between the two numbers is *to* or *in*.[8][9][10]

Mathematical relations

Odds can be expressed as a ratio of two numbers, in which case it is not unique – scaling both terms by the same factor does not change the proportions: 1:1 odds and 100:100 odds are the same (even odds). Odds can also be expressed as a number, by dividing the terms in the ratio – in this case it is unique (different fractions can represent the same rational number). Odds as a ratio, odds as a number, and probability (also a number) are related by simple formulas, and similarly odds in favor and odds against, and probability of success and probability of failure have simple relations. Odds range from 0 to infinity, while probabilities range from 0 to 1, and hence are often represented as a percentage between 0% and 100%: reversing the ratio switches odds for with odds against, and similarly probability of success with probability of failure.

Given odds (in favor) as the ratio W:L (Wins:Losses), the odds in favor (as a number) o_f and odds against (as a number) o_a can be computed by simply dividing, and are multiplicative inverses:

$$\begin{aligned} o_f &= W/L = 1/o_a \\ o_a &= L/W = 1/o_f \\ o_f \cdot o_a &= 1 \end{aligned}$$

Analogously, given odds as a ratio, the probability of success or failure can be computed by dividing, and the probability of success and probability of failure sum to unity (one), as they are the only possible outcomes. In case of a finite number of equally likely outcomes, this can be interpreted as the number of outcomes where the event occurs divided by the total number of events:

$$\begin{aligned} p &= W/(W+L) = 1-q \\ q &= L/(W+L) = 1-p \\ p+q &= 1 \end{aligned}$$

Given a probability *p*, the odds as a ratio is $p : q$ (probability of success to probability of failure), and the odds as numbers can be computed by dividing:

$$\begin{aligned} o_f &= p/q = p/(1-p) = (1-q)/q \\ o_a &= q/p = (1-p)/p = q/(1-q) \end{aligned}$$

Conversely, given the odds as a number o_f, this can be represented as the ratio $o_f : 1$, or conversely $1 : (1/o_f) = 1 : o_a$, from which the probability of success or failure can be computed:

$$\begin{aligned} p &= o_f/(o_f+1) = 1/(o_a+1) \\ q &= o_a/(o_a+1) = 1/(o_f+1) \end{aligned}$$

Thus if expressed as a fraction with a numerator of 1, probability and odds differ by exactly 1 in the denominator: a probability of 1 *in* 100 (1/100 = 1%) is the same as odds of 1 *to* 99 (1/99 = 0.0101... = $0.\overline{01}$), while odds of 1 *to* 100 (1/100 = 0.01) is the same as a probability of 1 *in* 101 (1/101 = 0.00990099... = $0.\overline{0099}$). This is a minor difference if the probability is small (close to zero, or "long odds"), but is a major difference if the probability is large (close to one).

These are worked out for some simple odds:

odds (ratio)	o_f	o_a	p	q
1:1	1	1	50%	50%
0:1	0	∞	0%	100%
1:0	∞	0	100%	0%
2:1	2	0.5	67%	33%
1:2	0.5	2	33%	67%
4:1	4	0.25	80%	20%
1:4	0.25	4	20%	80%
9:1	9	$0.\overline{1}$	90%	10%
10:1	10	0.1	$90.\overline{90}$%	$9.\overline{09}$%
99:1	99	$0.\overline{01}$	99%	1%
100:1	100	0.01	$99.\overline{0099}$%	$0.\overline{90}$%

These transforms have certain special geometric properties: the conversions between odds for and odds against (resp. probability of success with probability of failure) and between odds and probability are all Möbius transformations (fractional linear transformations). They are thus specified by three points (sharply 3-transitive). Swapping odds for and odds against swaps 0 and infinity, fixing 1, while swapping probability of success with probability of failure swaps 0 and 1, fixing .5; these are both order 2, hence circular transforms. Converting odds to probability fixes 0, sends infinity to 1, and sends 1 to .5 (even odds are 50% likely), and conversely; this is a parabolic transform.

Applications

In probability theory and Bayesian statistics, odds may sometimes be more natural or more convenient than probabilities. This is often the case in problems of sequential decision making as for instance in problems of how to stop (online) on a **last specific event** which is solved by the odds algorithm. Similar ratios are used elsewhere in Bayesian statistics, such as the Bayes factor.

The odds are a ratio of probabilities; an odds ratio is a ratio of odds, that is, a ratio of ratios of probabilities. Odds-ratios are often used in analysis of clinical trials. While they have useful mathematical properties, they can produce counter-intuitive results: an event with an 80% probability of occurring is four times *more likely* to happen than an event with a 20% probability, but the *odds* are 16 times higher on the less likely event (4–1 *against*, or 4) than on the more likely one (1–4, or 4–1 *on*, or 0.25).

In some cases the log-odds are used, which is the logit of the probability. Most simply, odds are frequently multiplied or divided, and log converts multiplication to addition and division to subtractions.

Example #1

> There are 5 pink marbles, 2 blue marbles, and 8 purple marbles. What are the odds in favor of picking a blue marble?

Answer: The odds in favour of a blue marble are 2:13. One can equivalently say, that the odds are 13:2 *against*. There are 2 out of 15 chances in favour of blue, 13 out of 15 against blue.

In probability theory and statistics, where the variable p is the probability in favor of a binary event, and the probability against the event is therefore 1-p, "the odds" of the event are the quotient of the two, or $\frac{p}{1-p}$. That value may be regarded as the relative probability the event will happen, expressed as a fraction (if it is less than 1), or a multiple (if it is equal to or greater than one) of the likelihood that the event will not happen.

In the very first example at top, saying the odds of a Sunday are "one to six" or, less commonly, "one-sixth" means the probability of picking a Sunday randomly is one-sixth the probability of not picking a Sunday. While the mathematical probability of an event has a value in the range from zero to one, "the odds" in favor of that same event lie between zero and infinity. The odds against the event with probability given as p are $\frac{1-p}{p}$. The odds against Sunday are 6:1 or 6/1 = 6. It is 6 times as likely that a random day is not a Sunday.

Example #2

> There are 5 red marbles, 2 green marbles, and 8 yellow marbles. What are the odds against picking a yellow marble?

Answer: 7:8

Gambling usage

The use of odds in gambling facilitates betting on events where the relative probabilities of outcomes varied. For example, on a coin toss or a match race between two evenly matched horses, it is reasonable for two people to wager level stakes. However, in more variable situations, such as a multi-runner horse race or a football match between two unequally matched sides, betting "at odds" provides a perspective on the relative likelihoods of the possible outcomes.

In the modern era, most fixed odds betting takes place between a betting organisation, such as a bookmaker, and an individual, rather than between individuals. Different traditions have grown up in how to express odds to customers, older eras came with betting odds between people, today which is illegal in most countries, it was referred as "odding", an underground slang word with origins based in the Bronx.

Fractional odds

Favoured by bookmakers in the United Kingdom and Ireland, and also common in horse racing, fractional odds quote the net total that will be paid out to the bettor, should he or she win, relative to the stake.[11] Odds of 4/1 would imply that the bettor stands to make a £400 profit on a £100 stake. If the odds are 1/4, the bettor will make £25 on a £100 stake. In either case, having won, the bettor always receives the original stake back; so if the odds are 4/1 the bettor receives a total of £500 (£400 plus the original £100). Odds of 1/1 are known as *evens* or *even money*.

The numerator and denominator of fractional odds are always integers, thus if the bookmaker's payout was to be £1.25 for every £1 stake, this would be equivalent to £5 for every £4 staked, and the odds would therefore be expressed as 5/4. However, not all fractional odds are traditionally read using the lowest common denominator. For example, given that there is a pattern of odds of 5/4, 7/4, 9/4 and so on, odds which are mathematically 3/2 are more easily compared if expressed in the equivalent form 6/4.

Fractional odds are also known as *British odds, UK odds,*[12] or, in that country, *traditional odds*. They are typically represented with a "/" but can also be represented with a "-", e.g. 4/1 or 4-1. Odds with a denominator of 1 are often presented in listings as the numerator only.

A variation of fractional odds is known as *Hong Kong* odds. Fractional and Hong Kong odds are actually exchangeable. The only difference is that the UK odds are presented as a fractional notation (e.g. 6/5) whilst the Hong Kong odds are decimal (e.g. 1.2). Both exhibit the net return.

The European odds also represent the potential winnings (net returns), but in addition they factor in the stake (e.g. 6/5 or 1.2 plus 1 = 2.2).[13]

Decimal odds

Favoured in continental Europe, Australia, New Zealand and Canada, decimal odds quote the ratio of the payout amount, *including* the original stake, to the stake itself.[14] Decimal odds are generally considered the easiest to deal with as they reflect the inverse of the probability of an outcome.[15] Therefore, the decimal odds of an outcome are equivalent to the decimal value of the fractional odds plus one.[16] Thus even odds 1/1 are quoted in decimal odds as 2.00. The 4/1 fractional odds discussed above are quoted as 5.00, while the 1/4 odds are quoted as 1.25. This is considered to be ideal for parlay betting, because the odds to be paid out are simply the product of the odds for each outcome wagered on. Decimal odds are also favoured by betting exchanges because they are the easiest to work with for trading.

Decimal odds are also known as *European odds*, *digital odds* or *continental odds*.[12]

Moneyline odds

Moneyline odds are favoured by American bookmakers. The figure quoted is either positive or negative.

- When moneyline odds are positive, the figure indicates how much money will be won on a $100 wager (this is done for an outcome that is considered less likely to happen than not). For example, a net payout of 4/1 would be quoted as +400.
- When moneyline odds are negative, the figure indicates how much money must be wagered to win $100 (this is done for an outcome is considered more likely to happen than not). For example, a net payout of 1/4 would be quoted as -400.

Moneyline odds are often referred to as *American odds*. A "Moneyline" wager refers to odds on the straight-up outcome of a game with no consideration to a point spread. In most cases, the favorite will have negative moneyline odds (less payoff for a safer bet) and the underdog will have positive moneyline odds (more payoff for a risky bet). However, if the teams are evenly matched, *both* teams can have a negative line at the same time (e.g. -110 -110 or -105 -115), due to house take.

Gambling odds versus probabilities

In gambling, the odds on display do not represent the true chances (as imagined by the bookmaker) that the event will or will not occur, but are the amount that the bookmaker will pay out on a winning bet, together with the required stake. In formulating the odds to display the bookmaker will have included a profit margin which effectively means that the payout to a successful bettor is less than that represented by the true chance of the event occurring. This profit is known as the 'over-round' on the 'book' (the 'book' refers to the old-fashioned ledger in which wagers were recorded, and is the derivation of the term 'bookmaker') and relates to the sum of the 'odds' in the following way:

In a 3-horse race, for example, the true probabilities of each of the horses winning based on their relative abilities may be 50%, 40% and 10%. The total of these three percentages is 100%, thus representing a fair 'book'. The true odds against winning for each of the three horses are 1-1, 3-2 and 9-1 respectively.

In order to generate a profit on the wagers accepted, the bookmaker may decide to increase the values to 60%, 50% and 20% for the three horses, respectively. This represents the odds against each, which are 4-6, 1-1 and 4-1, in order. These values now total 130%, meaning that the book has an overround of 30 (130−100). This value of 30 represents the amount of profit for the bookmaker if he gets bets in good proportions on each of the horses. For example, if he takes £60, £50, and £20 of stakes respectively for the three horses, he receives £130 in wagers but only pays £100 back (including stakes), whichever horse wins. And the expected value of his profit is positive even if everybody bets on the same horse. The art of bookmaking is in setting the odds low enough so as to have a positive expected value of profit while keeping the odds high enough to attract customers, and at the same time attracting enough bets for each outcome to reduce his risk exposure.

A study on soccer betting found that the probability for the home team to win was generally about 3.4% less than the value calculated from the odds (for example, 46.6% for even odds). It was about 3.7% less for wins by the visitors, and 5.7% less for draws.[17]

Making a profit in gambling involves predicting the relationship of the true probabilities to the payout odds. Sports information services are often used by professional and semi-professional sports bettors to help achieve this goal.

The odds or amounts the bookmaker will pay are determined by the total amount that has been bet on all of the possible events. They reflect the balance of wagers on either side of the event, and include the deduction of a bookmaker's brokerage fee ("vig" or vigorish).

Also, depending on how the betting is affected by jurisdiction, taxes may be involved for the bookmaker and/or the winning player. This may be taken into account when offering the odds and/or may reduce the amount won by a player.

See also

- Galton box
- Gaming mathematics
- Formal mathematical specification of logistic regression
- Optimal stopping
- Statistical association football predictions

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