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**Exam 1**

Exam 1 due Mar 09, 2016 at  
23:59 UTC



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## Problem 3: A six-sided die

(3/3 points)

A fair, 6-sided die is rolled 6 times independently. Assume that the results of the different rolls are independent. Let  $(a_1, \dots, a_6)$  denote a typical outcome, where each  $a_i$  belongs to  $\{1, \dots, 6\}$ .

**Note:** Enter numerical answers; do not enter '!' or combinations. The following table for  $\binom{n}{k}$  up to  $n = 6$  has been provided for your convenience:

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		k						
		0	1	2	3	4	5	6
n	1	1	1					
	2	1	2	1				
	3	1	3	3	1			
	4	1	4	6	4	1		
	5	1	5	10	10	5	1	
	6	1	6	15	20	15	6	1

1. Find the probability that the results of the 6 rolls are all different. (Answer with at least 3 decimal digits.)



Answer: 0.01543

For any outcome  $\omega = (a_1, \dots, a_6)$ , let  $R(\omega)$  be the **set**  $\{a_1, \dots, a_6\}$ ; this is the set of numbers that showed up at least once in the different rolls. For example, if  $\omega = (2, 2, 5, 2, 3, 5)$ , then  $R(\omega) = \{2, 3, 5\}$ .

2. Find the probability that  $R(\omega)$  has exactly two elements. (Answer with at least 3 decimal digits.)

0.01993313

✓ Answer: 0.01993

3. Find the probability that  $R(\omega)$  has exactly three elements.

0.2314815

✓ Answer: 0.23148

Answer:

1. Let  $A$  be the event where the results of the 6 rolls are all the same. Since all outcomes are equally likely,  $P(A) = \frac{|A|}{|\Omega|}$ .

$$|A| = 6!$$

$$|\Omega| = 6^6$$

$$P(A) = \frac{6!}{6^6} = 5/324 \approx 0.01543$$

2. Let  $B$  be the event where  $R(\omega)$  has exactly two elements. We can find the number of elements in  $B$  by first choosing a pair of distinct numbers for  $R(\omega)$  – there are  $\binom{6}{2}$  choices. For each pair, we can then count the number of ways they can be assigned to a sequence of length 6 that consists of only those two numbers and has at least one of each. We see that there are  $\binom{6}{k}$  ways of constructing a sequence that consists of  $k$  repetitions of the first number, and  $6 - k$  repetitions of the second number, where  $1 \leq k \leq 5$ . Therefore,

$$|B| = \binom{6}{2} \sum_{k=1}^5 \binom{6}{k}$$

$$|\Omega| = 6^6$$

$$P(B) = \frac{|B|}{|\Omega|} = 155/7776 \approx 0.01993$$

3. Let  $C$  be the event where  $R(\omega)$  has exactly three elements. We can find the number of elements in  $C$  by first choosing a triple of distinct numbers for  $R(\omega)$  – there are  $\binom{6}{3}$  choices. For each triple, we can then count the number of ways they can be assigned to a sequence of length 6 that consists of only those three numbers and has at least one of each. We see that there are  $\binom{6}{k} \binom{6-k}{l}$  ways of constructing a sequence that consists of  $k$  repetitions of the first number,  $l$  repetitions of the second number, and  $6 - k - l$  repetitions of the third number, where  $1 \leq k \leq 4$  and  $1 \leq l \leq 5 - k$ . Therefore,

$$|C| = \binom{6}{3} \sum_{k=1}^4 \sum_{l=1}^{5-k} \binom{6}{k} \binom{6-k}{l}$$

$$|\Omega| = 6^6$$

By numerically evaluating the various entries in the formula for  $|C|$ , we find that:

$$P(C) = \frac{|C|}{|\Omega|} = 25/108 \approx 0.23148$$

*You have used 1 of 2 submissions*



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