The Philosophical Significance of Godel's Theorem

From a mathematical point of view, Gödel's Theorem is an incredibly interesting result.

But it also has far-reaching philosophical consequences.

I'll tell you about one of them in this section.

Certainty

Sometimes our best theories of the natural world turn out to be mistaken. The geocentric theory of the universe turned out to be mistaken. So did Newtonian physics. We now have different theories of the natural world. But how can we be sure that they won't turn out to be mistaken too? Certainty would appear elusive.

Many years ago, when I was young and reckless, I used to think that even if certainty remained forever elusive in our theories of the natural world, mathematical theories were different. I used to think that when it comes to mathematics, we really can aspire to absolute certainty. Alas, my youthful self was mistaken. It is a consequence of Gödel's Theorem that absolute certainty is no more possible in our mathematical theories than it is in our theories of the natural world.

There is an important disanalogy between physical theories and their mathematical counterparts. When Copernicus proposed the heliocentric theory of the universe, he defended his hypothesis by arguing that it was simpler than rival theories. But, of course, the fact that a theory is simple does not guarantee that the theory is true. Copernicus's theory is false as he proposed it, since we now know that the planetary orbits are not circular, as he claimed, but rather elliptical. In contrast, when Euclid proposed that there are infinitely many prime numbers, he justified his hypothesis with a *proof* from basic principles. Since Euclid's proof is valid, it *guarantees* that if his basic principles are correct, then his conclusion is correct as well. So it is natural to suppose—and so I assumed in my youth—that Euclid, unlike Copernicus, established his result *conclusively*.

Unfortunately, there is a catch. A mathematical proof is always a *conditional* result: it shows that its conclusion is true *provided* that the basic principles on which the proof is based are true.

This means that in order to show conclusively that a mathematical sentence is true, it is not enough to give a proof from basic principles.

We must also show that our basic principles—our axioms—are true, and that our basic rules of inference are valid.

How could one show that an axiom is true, or that an axiom is valid? It is tempting to answer that rules and axioms are principles so basic that they are absolutely obvious: their correctness is immediately apparent to us. There is, for example, an arithmetical axiom that says that different natural numbers must have different successors. What could possibly be more obvious?

Sadly, the fact that an axiom seems obvious is not a guarantee of its truth. Some mathematical axioms that have seemed obviously true to great mathematicians have turned out to be false. Let me give you an example.

Discussion

Hide Discussion

Topic: Week 10 / The Philosophical Significance of Godel's Theorem

Add a Post

Show all posts by recent activity

? Proving the Inconsistency of an Axiomatic System 1
Gödel's Second Theorem (quoted from above) *If an axiomatic system is at least as strong a...