2.6.1 Summary Quiz: Effect of Fishing on Predator and Prey Fish

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In the next two problems, we confirm Ethan's statement that the average value of the sardine population over a cycle, $\overline{S}$, is equal to $\frac{\mathbf{c}}{\mathbf{d}}$. We can follow the same type of process we used to calculate $\overline{M}$.

Question 1

1/1 point (graded)

Solve for $S(t)$, but do not integrate:

$$\frac{dM}{dt} = -\mathbf{c}M + \mathbf{d}SM.$$

The solution is:

● $S(t) = \frac{1}{M \cdot d}\left(\frac{dM}{dt} + \mathbf{c}M\right)$ ✓

○ $S(t) = \frac{1}{M \cdot d}\left(\frac{dM}{dt} - \mathbf{c}M\right)$

○ $S(t) = \left(\frac{1}{dt} + \frac{\mathbf{c}}{\mathbf{d}}\right)$

○ $S(t) = \left(\frac{1}{dt} - \frac{\mathbf{c}}{\mathbf{d}}\right)$

○ $S(t) = \frac{1}{dt}$

Explanation

Solution: Add $\mathbf{c}M$ to both sides of the equation, then divide by $M \cdot d$. (We are writing it this way because $d \cdot M$ may be confused with the dM in $\frac{dM}{dt}$.)

$$\frac{dM}{dt} = -\mathbf{c}M + \mathbf{d}SM$$
$$S(t) = \frac{1}{M\cdot d}\left(\frac{dM}{dt} + cM\right)$$

Submit You have used 1 of 3 attempts

Answers are displayed within the problem

Question 2

1/1 point (graded)

The average value of the sardine population is defined as $\overline{S} = \frac{1}{L}\int_0^L S(t)dt$. Using your answer for $S(t)$ from the previous problem, compute the antiderivative $\int S(t)dt$ and select the correct answer below. Here k is any constant.

(From this answer you can figure out the average value of S, as Ethan mentioned.)

○ $\int S(t)dt = \frac{\mathbf{c}}{\mathbf{d}} + k$

● $\int S(t)dt = \frac{1}{d}\ln|M| + \frac{ct}{d} + k$ ✓

○ $\int S(t)dt = \frac{1}{d}\ln(t) + \frac{ct}{d} + k$

○ The function $S(t)$ is not integrable with respect to t. The value of $\overline{S}$ can be approximated but cannot be computed exactly.

Explanation

Solution: Since $S(t) = \frac{1}{M\cdot d}\left(\frac{dM}{dt} + cM\right) = \frac{1}{d}M\frac{dM}{dt} + \frac{\mathbf{c}}{\mathbf{d}}$, if we integrate both sides with respect to t we get:

$$\int S(t)dt = \int \frac{1}{d}M\frac{dM}{dt} + \frac{\mathbf{c}}{\mathbf{d}}\,dt = \frac{1}{d}\ln|M| + \frac{\mathbf{c}}{\mathbf{d}}\cdot t + k,$$

where k is any constant. This uses a substitution $u = M(t)$ to find the antiderivative $\int M\frac{dM}{dt}\,dt$. To find the average value of S, we evaluate this integral at L and 0 and divide the difference by L:

$$\int_0^L S(t)dt = \int_{M(0)}^{M(L)} \frac{dM}{M\cdot d} + \int_0^L \frac{\mathbf{c}}{\mathbf{d}}dt$$

$$\overline{S} = \frac{1}{L}\int_0^L S(t)dt = \frac{1}{d\cdot L}(\ln|M|)\Big|_{M=M(0)}^{M=M(L)} + \frac{1}{L}\left(\frac{\mathbf{c}}{\mathbf{d}}\cdot t\right)\Big|_0^L$$

$$\Rightarrow \overline{S} = \frac{\mathbf{c}}{\mathbf{d}}.$$

Recall that $M(0) = M(L)$, since L is the duration of one closed cycle.

Submit You have used 1 of 2 attempts

Answers are displayed within the problem

The next two problems explore what happens when the level of fishing is reduced, as happened during World War I. Here the effect of fishing in is represented by the term $-eS$ in the model below. This means that the “rate out" of sardine is proportional to the amount of sardine present, and e is the constant of proportionality. The similar situation occurs for marlin which are also caught by nets.

$$\frac{dS}{dt} = (a-e)S - \mathbf{b}SM$$

$$\frac{dM}{dt} = -(c+e)M + \mathbf{d}SM.$$

Recall that S represents the population of sardines (in hundreds of thousands), M the population of marlin (in hundreds), and t is time.

Question 3

2/2 points (graded)

When fishing is reduced, what happens to the average value of the populations of prey fish (S) and predator fish (M) over the length of a cycle?

The average value of S decreases ▼ ✓ **Answer:** decreases and the average value of M increases ▼ ✓ **Answer:** increases

Explanation

The average value of S decreases and the average value of M increases.

Why? The average value of S is $\frac{c+e}{d}$ and the average value of M is $\frac{a-e}{b}$.

Reduced fishing means the parameter e becomes smaller. This means that $\overline{S}$ becomes smaller (since the numerator is $c+e$ and e is smaller). This also means that $\overline{M}$ becomes larger (since the numerator is $a-e$ and e is smaller.).

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ℹ Answers are displayed within the problem

Question 4

1/1 point (graded)

Is this consistent with what D'Ancona saw during World War I --- an increase in the percent of predator fish in the net catches? Why or why not?

yes it is, since it explains the observed results.

✓

Thank you for your response.

Explanation

Solution:

We know the average values of the predator and prey fish in terms of the parameters in the problem: for a set of differential equations

$$\begin{aligned}\frac{dS}{dt} &= aS - bSM\\ \frac{dM}{dt} &= -\mathbf{c}M + dSM\end{aligned}$$

the average value of S is c/d and the average value of M is a/b, where the average is computed over one full closed cycle. Thus, if we change the coefficients (i.e., the parameters) in the equations such that $a \to a - e$ and $c \to c + e$, then we obtain the equations *with* fishing, but our rules for finding the average values in terms of equations' coefficients stay the same. Making these same substitutions into our formulas for average values, the new average value of S is $(c + e)/d$ and the new average value of M is $(a - e)/b$.

When fishing decreases, e decreases, decreasing the average population of sardines and increasing the average population of marlin. Thus, with less fishing during wartime, predator population increases and prey population decreases. Accordingly, the percent of predator fish per catch will, on average, increase during wartime, which is consistent with D'Ancona's observations.

Submit You have used 1 of 2 attempts

ℹ Answers are displayed within the problem

Question 5

1/1 point (graded)

Suppose you are a biologist working with the fishing industry. You've modeled the situation as:

$$\frac{dS}{dt} = (0.5 - e)S - 0.4SM$$

$$\frac{dM}{dt} = -(0.2 + e)M + 0.03SM$$

Here M and S represent the population of sardine and marlin, in hundreds and hundreds of thousands respectively. The variable e represents the proportion of fish caught. What would you recommend as the ideal level of fishing to **maximize the average value of the sardine** population over the length of a cycle and **not cause either population to go extinct**?

○ The best way to maximize the sardine population is to stop fishing: $e = 0$.

○ Fish at $e > 0.5$. The marlin population decreases as e increases, so fewer sardines are eaten by marlin leaving more sardine for fishing.

○ Fish at $e = 0.5$, removing half the sardine population via fishing (leaving the rest for marlin).

● Fish at a little less than $e = 0.5$, almost but not quite half the sardine population. ✓

○ Killing close to half the population by fishing is too much. Use $e = 0.4$.

Explanation

We know that for this system of differential equations, the average values (i.e. equilibrium values) for S and M are :

$$\overline{S} = \frac{c+e}{d} = \frac{0.2+e}{0.03}$$
$$\overline{M} = \frac{a-e}{b} = \frac{0.5-e}{0.4}$$

Contrary to what you might expect, increasing e increases $\overline{S}$. So we want to increase e, but by how much? If $e \geq 0.5$, then the average value of marlin over a cycle will, mathematically, be zero or negative. We cannot have a negative population of marlin, so we interpret this to mean that at some point during the cycle, the marlin population will reach zero. Once a population reaches zero, it is extinct and cannot be brought back. Thus, with $e = 0.5$ exactly, M will reach zero at some point during the first cycle, and at that point, $\frac{dM}{dt}$ will equal and remain at zero.

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Answers are displayed within the problem

Question 6

1/1 point (graded)

How would you convince the fishing industry that your recommendation makes sense?

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Since if we fish at higher rate then the marlin's will be
extinct that we don't want.
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✓

Thank you for your response.

Explanation

Use the mathematical details above.

The average values are here, and we see that increasing fishing e increases the average sardine population and decreases it for marlin.

$$\overline{S} = \frac{c+e}{d} = \frac{0.2+e}{0.03}$$
$$\overline{M} = \frac{a-e}{b} = \frac{0.5-e}{0.4}$$

To convince the fishing industry that just below 0.5 is the ideal fishing level, we could explain that fishing with e just below 0.5 maximizes sardines without putting marlin extinct.

If $e \geq 0.5$, the marlin will go extinct. Now it’s true that without marlin, the sardine population will grow without bound. This might sound great - but there may be unintended consequences of letting the marlin species go extinct.

Making e just less than 0.5 maximizes the sardine population over the duration of one full cycle, without putting marlin extinct.

Another way to appeal to it is to argue that $e > 0.5$ means we’re fishing at a rate greater than the sardine can reproduce so this would eventually lead to extinction of sardines.

Submit

Answers are displayed within the problem

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