6. Graph Properties and Metrics - I

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Exercises due Oct 20, 2021 17:29 IST Completed

In the video below, we discuss exploring a social network (a criminal network) by finding the degree distribution. In the first recitation in this module, we will plot and estimate the degree distribution of a social network.

Discussions: Summary Statistics of Networks

0:00 / 0:00 2.0x

Start of transcript. Skip to the end.

Prof Uhler: OK, so welcome back.

So in this video, we will analyze

how to get a first impression of a network.

So say I hand you a large adjacency matrix,

and I just want to find a few summary

statistics of getting a first impression

Video

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Some basic summary statistics for networks

0:00 / 0:00 1.50x

Start of transcript. Skip to the end.

Prof Uhler: OK, so that's about how to represent networks.

And then what we also started discussing already last time

are how do we get a first impression of the network.

Now, you get a network with, say, thousands of nodes,

let's get a first impression of it.

Video

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Video note: At 9:30, the definition for sparsity should be stated as “The number of edges does not grow proportionally with the square of the number of nodes."

Connected Components

Given an **undirected graph** a **connected component** is a subset of nodes $V' \subset V$ such that the induced graph on V' has the following properties: There exists a walk from v_i to v_j whenever $v_i, v_j \in V'$ and there is no walk from v_i to v_j whenever $v_i \in V'$ and $v_j \in V \setminus V'$.

The notion of a connected component as defined for an undirected graph does not translate directly to the case of a directed graph where walks have directions. In a **directed graph** a related notion we can define is **strong connectivity** . A set of nodes $V' \subset V$ is said to be **strongly connected** if every vertex in V' is reachable from every other vertex in V' and there exists some vertex in V' and some vertex in $V \setminus V'$ such that there is no directed path between such vertices in at least one direction.

Adjacency Matrix, Connected Components

8/8 points (graded)
Consider the following adjacency matrix. You may use any computational tool to answer the questions that follow.

$$A = \begin{pmatrix} 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \end{pmatrix}$$

Raw matrix

Python:

```
[[0, 1, 0, 1, 0, 1, 0, 0, 0, 0],
[1, 0, 1, 1, 1, 0, 0, 0, 0, 0],
[0, 1, 0, 0, 0, 0, 0, 0, 0, 0],
[1, 1, 0, 0, 0, 1, 0, 1, 1, 0],
[0, 1, 0, 0, 0, 0, 0, 0, 1, 1],
[1, 0, 0, 1, 0, 0, 0, 0, 0, 0],
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1],
[0, 0, 0, 1, 0, 0, 0, 0, 0, 0],
[0, 0, 0, 1, 1, 0, 0, 0, 0, 1],
[0, 0, 0, 0, 1, 0, 1, 0, 1, 0]]
```

Mathematica:

```
{{0, 1, 0, 1, 0, 1, 0, 0, 0, 0},
{1, 0, 1, 1, 1, 0, 0, 0, 0, 0},
{0, 1, 0, 0, 0, 0, 0, 0, 0, 0},
{1, 1, 0, 0, 0, 1, 0, 1, 1, 0},
{0, 1, 0, 0, 0, 0, 0, 0, 1, 1},
{1, 0, 0, 1, 0, 0, 0, 0, 0, 0},
{0, 0, 0, 0, 0, 0, 0, 0, 0, 1},
{0, 0, 0, 1, 0, 0, 0, 0, 0, 0},
{0, 0, 0, 1, 1, 0, 0, 0, 0, 1},
{0, 0, 0, 0, 1, 0, 1, 0, 1, 0}}
```

Hide

1. Does the adjacency matrix represent a simple graph?

- [x] Yes

- ( ) No

✓

2. Can the adjacency matrix potentially represent an undirected graph?

- (•) Yes
- ( ) No

✓

3. Is the graph connected?

- (•) Yes
- ( ) No

✓

4. What is the minimum ℓ such that A^{ℓ} contains no entry equal to 0? If such an ℓ does not exist, then enter -1.

4 ✓ **Answer:** 4

5. How many connected components does the graph have?

1 ✓ **Answer:** 1

6. What is the maximum degree of a node in the graph?

5 ✓ **Answer:** 5

7. How many walks of length 5 are there from node 0 (represented by first row/column) to itself?

46 ✓ **Answer:** 46

8. Is the following statement **True** or **False**? “For an undirected graph, that is **not** weighted or a multigraph, the diagonal entries of A^2 are equal to the degree of the nodes."

- (•) True
- ( ) False

✓

Solution:

1. **Yes.** There are no multiple edges between any two nodes and there are no self loops.
2. **Yes.** This is because $A = A^T$.
3. **Yes.** This is because for $\ell \geq 4$, A^{ℓ} has no element equal to 0.
4. **4.** One can see this by computing the powers of A using a computational tool.
5. **1.** This is because the graph is connected (as we concluded in a previous part). That is, there is a path

between any two nodes in the graph.

6. **5.** This can be seen by computing the degree of each node.

7. **46.** This can be seen by computing A^5.

8. **True.** This follows from the definition of degree of a node.

Submit You have used 1 of 5 attempts

Answers are displayed within the problem

Family Tree

5/5 points (graded)

Consider the **family tree** of a family of people. Assume that the edges are directed and an edge represents a biological $(\text{parent}, \text{child})$ relationship. Also, assume that the graph has a finite number of nodes and edges (so that one or both of the parents of some nodes are not represented in this tree).

1. What is the minimum in-degree of a node?

0 ✔ **Answer:** 0

2. What is the maximum in-degree of a node?

2 ✔ **Answer:** 2

3. What is the minimum out-degree of a node?

0 ✔ **Answer:** 0

4. If there are n people in this family tree, what is the maximum possible out-degree?

n-1 ✔ **Answer:** n-1

5. Are self loops possible in this tree?

- ○ Yes
- ● No

✔

Solution:

1. **0.** This is because we have a finite number of nodes in the graph. There must exist at least one person in the tree whose parent(s) are not represented in the family tree.

2. **2.** A child in the family tree can have up to 2 parents represented in the tree.

3. **0.** Not every node in the tree is required to have a child node.

4. $n-1$. The maximum out-degree occurs when one node in the tree is the parent of all the other nodes represented in the tree.

5. **No.** We guess this one is obvious.

Submit You have used 2 of 3 attempts

Answers are displayed within the problem

Storing a Graph

1/1 point (graded)
Is storing the entire adjacency matrix of a graph memory efficient for **any** graph?

- Yes, any other representation of the graph would consume the same amount of storage space.
- No, there are cases when other representations are more memory efficient.

Solution:

An *edge list*, which is simply the list of edges of the graph, consumes far less memory if, for example, an undirected or a directed graph is *very sparse* (many more 0's in the adjacency matrix than 1's).

Submit You have used 1 of 1 attempt

Answers are displayed within the problem

Degree Sum

1/1 point (graded)
Let $k_1, \ldots, k_n$ be the degrees of an n-node undirected, simple graph. Let e be the number of edges in the graph. What is $\sum_{i=1}^n k_i$ equal to?

2*e **Answer:** 2*e

Solution:

The degree of a node in a simple, undirected graph is the number of edges emanating from the node. The sum of the degrees is an expression that counts each edge $\{v_i, v_j\}$ twice; once from the perspective of v_i and the other time from the perspective of v_j. Therefore, the summation is equal to $2e$, where e is the number of edges in the graph.

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Answers are displayed within the problem

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Family tree 4: in the family of n people max out-degree

question posted 2 months ago by **Kathi_007**

Any hints of this question? I thought I got it right: If there are only 2 generations: parents and kids then the max out-degree is n-2. But its the wrong answer.

This post is visible to everyone.

Kathi_007
2 months ago - marked as answer 2 months ago by **Iam_trinh** (Community TA)

Solved it. Hint: one or both of the parents of some nodes are not represented in this tree.

I still don't get it. I can't come up with an answer that works in weird cases like when n = 1 or n = 2. It seems for n>=3, the answer should be 2*(n-2)?

posted 2 months ago by **SunPenguin**

Unless we are working with a case where we can have out and in edges without nodes on both sides? wouldn't that not be an edge?

posted 2 months ago by **SunPenguin**

Hi @SunPenguin, it's not 2*(n-2).

Think about the maximum number of child nodes that a node can have and that should give you the maximum out-degree. And keep in mind that *the graph has a finite number of nodes and edges (so that one or both of the parents of some nodes are not represented in this tree).*

posted 2 months ago by **Iam_trinh** (Community TA)

Got it! I had confused and thought the question meant the maximum total of out-degrees in the entire graph. The question is asking what is maximum possible out-degree *for a single node*

posted 2 months ago by **SunPenguin**

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