11.3.7 The QR Factorization (Again)

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Week 11 due Dec 22, 2023 21:12 IST

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Video error slide 2 (0:30)

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discussion posted about 17 hours ago by **encipher**

In the video on slide 2 (starts around the 30 second mark), on the last line, on the right most column $q_0 r_{12}^T$ should be $q_1 r_{12}^T$.

Also the same error is on the next slide which starts around 2:30.

I did not see this error in the Week11.pdf notes.

Best regards.

11.3.7 The QR Factorization (Again)

Video

Start of transcript. Skip to the end.

0:00 / 0:00 2.0x

Dr. Robert van de Geijn: What we have now

is a systematic way of computing the QR factorization

by recognizing it to be the same as the Gram-Schmidt process.

But if I asked you to now give me an algorithm using the FLAME notation,

Video

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Reading Assignment

0 points possible (ungraded)

Read Unit 11.3.7 of the notes. [LINK]

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- $\|a_1^\perp\|_2 = q_1^Ta_1$? 2
- The link between $a_1^\perp$ and Pythagoras 1
- Why not overwrite A with Q? 3
- Bugs in laff_norm2.m
  1. Using boolean && instead of bit-wise & on line 15 will be faster. 2. Missing semicolon on line 20, causing an undesired $x\ \mathbf{max} = 0.0$ print ev... 2

Homework 11.3.7.1

1/1 point (graded)

Algorithm: $[Q, R] := \mathrm{QR}(A, Q, R)$

Partition $A \rightarrow \left(\begin{array}{c|c} A_L & A_R \end{array}\right)$, $Q \rightarrow \left(\begin{array}{c|c} Q_L & Q_R \end{array}\right)$, $R \rightarrow \left(\begin{array}{c|c} R_{TL} & R_{TR} \\ \hline R_{BL} & R_{BR} \end{array}\right)$

where A_L and Q_L have 0 columns, R_{TL} is 0×0

while $n(A_L) < n(A)$ **do**

Repartition

$$\left(\begin{array}{c|c} A_L & A_R \end{array}\right) \rightarrow \left(\begin{array}{c|c|c} A_0 & a_1 & A_2 \end{array}\right), \left(\begin{array}{c|c} Q_L & Q_R \end{array}\right) \rightarrow \left(\begin{array}{c|c|c} Q_0 & q_1 & Q_2 \end{array}\right),$$

$$\left(\begin{array}{c|c} R_{TL} & R_{TR} \\ \hline R_{BL} & R_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c|c|c} R_{00} & r_{01} & R_{02} \\ \hline r_{10}^T & \rho_{11} & r_{12}^T \\ \hline R_{20} & r_{21} & R_{22} \end{array}\right)$$

$r_{01} := Q_0^T a_1$

$a_1^\perp := a_1 - Q_0 r_{01}$

$\rho_{11} := \|a_1^\perp\|_2$

$q_1 = a_1^\perp / \rho_{11}$

Continue with

$$\left(\begin{array}{c|c} A_L & A_R \end{array}\right) \leftarrow \left(\begin{array}{c|c|c} A_0 & a_1 & A_2 \end{array}\right), \left(\begin{array}{c|c} Q_L & Q_R \end{array}\right) \leftarrow \left(\begin{array}{c|c|c} Q_0 & q_1 & Q_2 \end{array}\right),$$

$$\left(\begin{array}{c|c} R_{TL} & R_{TR} \\ \hline R_{BL} & R_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c|c|c} R_{00} & r_{01} & R_{02} \\ \hline r_{10}^T & \rho_{11} & r_{12}^T \\ \hline R_{20} & r_{21} & R_{22} \end{array}\right)$$

endwhile

Implement the above algorithm for computing the QR factorization of a matrix:

```
[ Q_out, R_out ] = QR_unb( A, Q, R )
```

where `A` and `Q` are $m \times n$ matrices and `R` is an $n \times n$ matrix. You will want to use the routines `laff_gemv`, `laff_norm`, and `laff_invscal`. (Alternatively, use native operations.) Store the routine in

```
LAFFSpring2015 -> Programming -> Week11 -> QR_unb.m
```

Test the routine with

```
A = [  1 -1  2
       2  1 -3
      -1  3  2
       0 -2 -1 ];

Q = zeros( 4, 3 );
R = zeros( 3, 3 );
```

Calculator

```
[ Q_out, R_out ] = QR_unb( A, Q, R );
```

Next, see if $A = QR$:

```
A - Q_out * R_out
```

This should equal, approximately, the zero matrix. Check if Q has mutually orthogonal columns:

```
Q_out' * Q_out
```

This should equal, approximately, the identity matrix.

Finally, repeat the above, but with matrix

```
epsilon = 1e-8

A = [     1     1      1
       epsilon  0      0
          0  epsilon   0
          0     0   epsilon ]

Q = zeros( 4, 3 );
R = zeros( 3, 3 );
[ Q_out, R_out ] = QR_unb( A, Q, R );
```

Again, check if $A = QR$ and if Q has mutually orthogonal columns. To understand what went wrong, you may want to read Robert's notes for his graduate class. For details, see the enrichment for this week.

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✔

See file `Week11/QR_unb_Answer.m` in the `Week11.zip` file you downloaded.

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