

## MITx: 6.041x Introduction to Probability - The Science of Uncertainty

Bookmarks

Unit 0: Overview

- Survey ▶ Unit 1:

Entrance

- Probability models and axioms
- ▶ Unit 2: Conditioning and independence
- ▶ Unit 3: Counting
- Unit 4: Discrete random variables
- Exam 1
- ▶ Unit 5: Continuous random variables
- ▼ Unit 6: Further topics on random variables

Unit overview

Lec. 11: Derived distributions Exercises 11 due Mar 30, 2016 at 23:59 UT 🗗 Unit 6: Further topics on random variables > Lec. 13: Conditional expectation and variance revisited; Sum of a random number of independent r.v.'s > Lec 13 Conditional expectation and variance revisited Sum of a random number of independent r v s vertical4

■ Bookmark

# Exercise: Conditional variance definition

(4/5 points)

For each one of the following statements, indicate whether it is true or false.

(a) If X = Y (i.e., the two random variables always take the same values), then  $\operatorname{var}(X \mid Y) = 0$ .

True 

Answer: True

(b) If X = Y (the two random variables always take the same values), then  $\operatorname{var}(X \mid Y) = \operatorname{var}(X)$ .

False ▼

Answer: False

(c) If Y takes on the value y, then the random variable var(X | Y) takes the value

$$\mathbf{E}[(X - \mathbf{E}[X | Y = y])^2 | Y = y].$$

True ▼

✓ Answer: True

(d) If Y takes on the value y, then the random variable  $\operatorname{var}(X \mid Y)$  takes the value

$$\mathbf{E}[(X - \mathbf{E}[X | Y])^2 | Y = y.]$$

False ▼

**Answer:** True

Lec. 12: Sums of independent r.v.'s; Covariance and correlation

Exercises 12 due Mar 30, 2016 at 23:59 UT (3)

Lec. 13: Conditional expectation and variance revisited; Sum of a random number of independent r.v.'s

Exercises 13 due Mar 30, 2016 at 23:59 UT 🗗

### Solved problems

Additional theoretical material

Problem Set 6

Problem Set 6 due Mar 30, 2016 at 23:59 UT (3)

## **Unit summary**

▶ Unit 7: Bayesian inference (e) If Y takes on the value y, then the random variable  $\operatorname{var}(X \mid Y)$  takes the value

$$\mathbf{E}\big[(X-\mathbf{E}[X])^2\,|\,Y=y.\,\big]$$

False ▼

**Answer:** False

#### Answer:

- (a) Conditioned on Y, X is deterministic, and var(X | Y = y) = 0. This implies that the random variable  $\operatorname{var}(X \mid Y)$  is identically equal to zero. Thus, the statement is true.
- (b) False, because the previous statement is true.
- (c) This statement is just the definition of the numerical value of the conditional variance. We are in a universe where the event  $oldsymbol{Y}=oldsymbol{y}$  is known to have occurred, and every expectation is replaced by the corresponding conditional expectation.
- (d) The outer expectation places us in a universe where Y=y. Given this information, the value of the random variable  $\mathbf{E}[X \mid Y]$  becomes a known quantity, equal to  $\mathbf{E}[X \mid Y = y]$ . Thus, this statement is equivalent to the preceding one and is true.
- (e) This is false, because all expectations should be conditional on the universe (Y = y) within which we are working. For a concrete counterexample, suppose that X is zero-mean and that Y = X. Then, as in part (a),  $\operatorname{var}(X \,|\, Y=y)=0$ . On the other hand, since  $\mathbf{E}[X] = \mathbf{0}$ , we have

$$\mathbf{E}ig[(X - \mathbf{E}[X])^2 \,|\, Y = yig] = \mathbf{E}[X^2 \,|\, Y = y] = \mathbf{E}[Y^2 \,|\, Y = y] = y^2.$$

You have used 1 of 1 submissions

© All Rights Reserved



© edX Inc. All rights reserved except where noted. EdX, Open edX and the edX and Open EdX logos are registered trademarks or trademarks of edX Inc.

















