# 3. Matrices

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Problem Set B due Sep 15, 2021 20:30 IST

**Practice**

## Rotation determinant

1/1 point (graded)

Let $R_\theta$ be the matrix that rotates a two-dimensional vector by $\theta$ counter-clockwise (that is, the vector $R_\theta \vec{v}$ is obtained by rotating $\vec{v}$ counter-clockwise by the angle $\theta$). Find the determinant of $R_\theta$.

$\det R_\theta =$ 1 ✔ **Answer:** 1

**Solution:**

We have

$$R_\theta = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \tag{5.222}$$

Therefore the determinant is given by $\cos^2\theta + \sin^2\theta$, which equals $1$.

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## Inverse Rotation

1.0/1 point (graded)

Find the inverse of the rotation matrix $R_\theta$. Type `theta` for $\theta$.

(Enter a matrix using notation such as `[[a,b],[c,d]]`.)

[[cos(theta),sin(theta)],[-sin(theta),cos(theta)]] ✔

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## Matrix Transpose

1.0/1 point (graded)

Given a matrix $A$, we write $A^T$ for the matrix whose entry in row $i$ and column $j$ is given by the entry in row $j$ and column $i$ of $A$. This $A^T$ is called the *transpose* of $A$. For example,

$$A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \iff A^T = \begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix} \tag{5.223}$$

Visually, one obtains the transpose by "reflecting" the matrix entries across the top-left to bottom-right diagonal.

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Let $M = \begin{pmatrix} 0 & 1 \\ 2 & -1 \end{pmatrix}$. Find $M^T$.

(Enter a matrix using notation such as `[[a,b],[c,d]]`.)

$M^T =$ [[0,2],[1,-1]] ✔ **Answer:** [[0, 2],[1,-1]]

**Solution:**

For a $2 \times 2$ matrix, we interchange the lower-left and top-right entries to obtain the transpose $\begin{pmatrix} 0 & 2 \\ 1 & -1 \end{pmatrix}$.

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## Orthogonal Matrices

1/1 point (graded)

A square matrix $A$ is called *orthogonal* if $A^T = A^{-1}$. Which of the following matrices are orthogonal? Choose all that apply.

- [ ] $\begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$
- [x] $R_\theta$ (rotation matrix)
- [x] $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$
- [x] $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

✔

**Solution:**

The answer is found by computing $A^T$ and $A^{-1}$ for each matrix $A$ and seeing if they are equal.

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## Find Orthogonal Matrices

4/4 points (graded)

Find four distinct orthogonal $2 \times 2$ matrices, each of which has top-left entry equal to $\frac{-1}{\sqrt{2}}$.

(Enter a matrix using notation such as `[[a,b],[c,d]]`.)

[[-1/sqrt(2),-1/sqrt(2)],[1/sqrt( ✔ **Answer:** [ [-1/sqrt(2) , -1/sqrt(2) ],[ 1/sqrt(2) , -1/sqrt(2)] ]

[[-1/sqrt(2),1/sqrt(2)],[-1/sqrt( ✔ **Answer:** [ [-1/sqrt(2) , 1/sqrt(2) ],[ -1/sqrt(2) , -1/sqrt(2)] ]

[[-1/sqrt(2),1/sqrt(2)],[1/sqrt(2 ✔ **Answer:** [ [-1/sqrt(2) , 1/sqrt(2) ],[ 1/sqrt(2) , 1/

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[[-1/sqrt(2),1/sqrt(2)],[1/sqrt(2 ✔ **Answer:** [ [-1/sqrt(2) , 1/sqrt(2) ],[ 1/sqrt(2) , 1/sqrt(2)] ]

[[-1/sqrt(2),-1/sqrt(2)],[-1/sqrt ✔ **Answer:** [ [-1/sqrt(2) , -1/sqrt(2) ],[ -1/sqrt(2) , 1/sqrt(2)] ]

**Solution:**

For a $2 \times 2$ matrix to be orthogonal, we need $A^{-1} = A^T$, in symbols:

$$\begin{pmatrix} a & c \\ b & d \end{pmatrix} = \frac{1}{ad-bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix} \tag{5.224}$$

By matching the top-left or bottom-right entries, we see that $ad - bc$ equals either $1$ (case 1) or $-1$ (case 2).

In case 1, we have $d = a$ and $c = -b$, so the $ad - bc = 1$ constraint becomes $a^2 + b^2 = 1$. With $a = \frac{-1}{\sqrt{2}}$, the solutions are $b = \pm\frac{1}{\sqrt{2}}$.

$$M_0 = \begin{pmatrix} \frac{-1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \end{pmatrix} \quad \text{or} \quad M_1 = \begin{pmatrix} \frac{-1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \end{pmatrix} \tag{5.225}$$

In fact, these are the rotation matrices $R^{3\pi/4}$ and $R^{-3\pi/4}$.

In case 2, we have $d = -a$ and $c = b$, so the $ad - bc = -1$ constraint becomes $-a^2 - b^2 = -1$. With $a = \frac{-1}{\sqrt{2}}$, again we get the solutions $b = \pm\frac{1}{\sqrt{2}}$. The corresponding matrices are:

$$M_2 = \begin{pmatrix} \frac{-1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \quad \text{or} \quad M_3 = \begin{pmatrix} \frac{-1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \tag{5.226}$$

It follows that these are the only four orthogonal $2 \times 2$ matrices with $a = \frac{-1}{\sqrt{2}}$.

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## 3. Matrices

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