6. Pure resonance

What happens if instead of considering the differential equation

$$\ddot{x} + 50x = \frac{\pi}{4}\mathrm{Sq}\,(t)\,,$$

we change 50 to 49

$$\ddot{x} + 49x = \frac{\pi}{4}\mathrm{Sq}\,(t)?$$

Pure resonance concept check

1/1 point (graded)

Which of the following is true of the ODE

$$\ddot{x} + 49x = \frac{\pi}{4}\mathrm{Sq}\,(t)?$$

- There are no solutions.

- ( ) There is exactly one solution. It is not periodic.
- ( ) There is exactly one solution. It is periodic.
- (•) There are infinitely many solutions. None are periodic.
- ( ) There are infinitely many solutions. Only one is periodic.
- ( ) There are infinitely many solutions. All are periodic.

✔

Solution:

There are infinitely many solutions, but none of them are periodic. Here is why: For $n \neq 7$, we can solve $\ddot{x} + 49x = \sin nt$ using complex replacement and ERF since in is not a root of $r^2 + 49$. For $n = 7$, we can still solve $\ddot{x} + 49x = \sin 7t$ (the existence and uniqueness theorem guarantees this), but the solution requires generalized ERF, and involves t, and hence is not periodic: it turns out that one solution is $-\frac{t}{14}\cos 7t$.

For the input signal $\mathrm{Sq}(t)$, we can find a solution x_p by superposition: most of the terms will be periodic, but one of them will be $\frac{1}{7}\left(-\frac{t}{14}\cos 7t\right)$, and this makes the whole solution x_p non-periodic.

There are infinitely many other solutions, which differ by adding the homogeneous solution, namely $x_p + c_1 \cos 7t + c_2 \sin 7t$ for any c_1 and c_2. These solutions still include the $\frac{1}{7}\left(-\frac{t}{14}\cos 7t\right)$ term and hence are not periodic.

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You have used 1 of 3 attempts

Answers are displayed within the problem

Remark 6.1 If the ODE had been

$$\ddot{x} + 36x = \frac{\pi}{4}\mathrm{Sq}(t)$$

then all solutions would have been periodic, because $\frac{\pi}{4}\mathrm{Sq}(t)$ has no $\sin 6t$ term in its Fourier series.

In general, for a periodic function f, the ODE $P(D)\,x = f(t)$ has a periodic solution if and only if for each term $\cos \omega t$ or $\sin \omega t$ appearing with a nonzero coefficient in the Fourier series of f, the number $i\omega$ is not a root of $P(r)$.

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- I don't understand why that answer is correct. — 2
  I don't want to spoil the right answer, but I don't understand the explanation in the answer. I was lucky to complete this task half intuitively/half rendomly. I doubt about corre...
- "Meaning of most of the terms will be periodic" — 5
- Recommendation — 3
  a poorly treated topic but highly valued questions in the questionnaires as a recommendation to improve in future courses
- general case expanation — 4
  Would you like to explain general case more detailed? I need it for my homework, :) Thanks.
- [staff] Use of the word 'solution' — 3
  It seems to me that there is only one steady state solution composed of many terms in a series. Are we to interpret the word 'solutions' above to mean the terms of the series?
- Resonance understanding — 4