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Exercise: Expected value rule and total expectation theorem

(5/8 points)

Let X, Y, and Z be jointly continuous random variables. Assume that all conditional PDFs and expectations are well defined. E.g., when conditioning on $X = x$, assume that x is such that $f_X(x) > 0$. For each one of the following formulas, state whether it is true for all choices of the function g or false (i.e., not true for all choices of g).

1. $$\mathbf{E}\big[g(Y)\,|\,X=x\big] = \int g(y) f_{Y|X}(y\,|\,x)\,dy$$

True ✔ **Answer:** True

2. $$\mathbf{E}\big[g(y)\,|\,X=x\big] = \int g(y) f_{Y|X}(y\,|\,x)\,dy$$

False ✔ **Answer:** False

3. $$\mathbf{E}\big[g(Y)\big] = \int \mathbf{E}\big[g(Y)\,|\,Z=z\big]\, f_Z(z)\,dz$$

True ✔ **Answer:** True

4. $$\mathbf{E}\big[g(Y)\,|\,X=x, Z=z\big] = \int g(y) f_{Y|X,Z}(y\,|\,x,z)\,dy$$

True ✔ **Answer:** True

5. $$\mathbf{E}\big[g(Y)\,|\,X=x\big] = \int \mathbf{E}\big[g(Y)\,|\,X=x, Z=z\big]\, f_{Z|X}(z\,|\,x)\,dz$$

False ✘ **Answer:** True

6. $\mathbf{E}\big[g(X,Y)\,|\,Y=y\big] = \mathbf{E}\big[g(X,y)\,|\,Y=y\big]$

True ✔ **Answer:** True

7. $\mathbf{E}\big[g(X,Y)\,|\,Y=y\big] = \mathbf{E}\big[g(X,y)\big]$

True ✖ **Answer:** False

8. $\mathbf{E}\big[g(X,Z)\,|\,Y=y\big] = \int g(x,z) f_{X,Z|Y}(x,z\,|\,y)\,dy$

True ✖ **Answer:** False

Answer:

1. True. This is the usual expected value rule, applied to a conditional model where we are given that $X=x$.

2. False. Here the quantity inside the expectation, $g(y)$, is a number (not a random variable). The left-hand side is a function of y, whereas on the right-hand side, y, is a dummy variable that gets integrated away. So, the formula is wrong on a purely syntactical basis (the left-hand side depends on y, while the right-hand side does not).

3. True. This is the total expectation theorem, where we condition on the events $Z=z$.

4. True. This is the usual expected value rule, applied to a conditional model where we are given that $X=x$ and $Z=z$.

5. True. This is the same total expectation theorem as in the third part, except that everything is calculated within a conditional model in which event $X=x$ is known to have occurred.

6. True. When we condition on $Y=y$, we know the value of Y, and we can replace $g(X,Y)$ by $g(X,y)$.

7. False. Given that $Y=y$, we need to somehow take into account the conditional distribution of X, whereas the right-hand side is determined by the unconditional PDF of X.

8. False. The left-hand side is a function of y, whereas the right-hand side (after y is integrated out) is a function of x and z. The correct form (expected value rule, in a conditional model) is:

$$\mathbf{E}[g(X,Z)\,|\,Y=y] = \int\int g(x,z) f_{X,Z|Y}(x,z\,|\,y)\,dx\,dz.$$

You have used 1 of 1 submissions