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Homework 6 Maximum Likelihood

<u>Course</u> > <u>Unit 3 Methods of Estimation</u> > <u>Estimation and Method of Moments</u> > 5. Censored data

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5. Censored data

In a given population, n individuals are sampled randomly, with replacement, and each sampled individual is asked whether his/her salary is greater than some fixed threshold z. Assume that the salary of a randomly chosen individual has the exponential distribution with unknown parameter λ . Asking whether the salary overcomes a given threshold rathen than directly asking for the salary increases the number people that are willing to answer and decreases the number of mistakes in the collected answers.

Denote by X_1,\ldots,X_n the binary responses of the n sampled individuals, so that $X_i\in\{0,1\}$. We call the X_i censored data .

(a)

2/2 points (graded)

What kind of distribution do the X_i s follow?

<u>Expo</u>nential distribution with parameter $\mu(\lambda)$

lacksquare Bernoulli with parameter $\,\mu\left(\lambda
ight)$

igcap Poisson with parameter $\,\mu\left(\lambda
ight)$



Give the parameter of this distribution in terms of λ and z:

Parameter
$$\mu\left(\lambda\right)=$$

$$\begin{array}{c} & \\ & \\ e^{\left(-\operatorname{lambda*z}\right)} \end{array}$$

Submit

You have used 1 of 2 attempts

✓ Correct (2/2 points)

(b)

1/1 point (graded)

Let \overline{X}_n be the proportion of sampled individuals whose response was 1 (corresponding to Yes). Convince yourself that \overline{X}_n is asymptotically normal.

What is its asymptotic variance?

$$V\left(\overline{X}_n
ight) = egin{bmatrix} \mathrm{e}^{-(-\mathrm{lambda*z})*(1-\mathrm{e}^{-(-\mathrm{lambda*z})}} \ & ullet \ e^{-\lambda \cdot z} \cdot \left(1 - e^{-\lambda \cdot z}
ight) \end{bmatrix}$$

Submit

You have used 1 of 3 attempts

✓ Correct (1/1 point)

(c)

1/1 point (graded)

Find a function f such that $f(\overline{X}_n)$ is a consistent estimator of λ .

Write ${f barX_n}$ for the sample average $\,\overline{X}_n$.

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You have used 1 of 3 attempts

✓ Correct (1/1 point)

(d)

1/1 point (graded)

Convince yourself that $f(\overline{X}_n)$ is asymptotically normal and compute its asymptotic variance.

Submit

You have used 1 of 3 attempts

✓ Correct (1/1 point)

(e)

1/1 point (graded)

What equation must z satisfy in order to minimize the asymptotic variance computed in part (d)? Write this equation in the form $g_{\lambda}(z)=z$, where g_{λ} is a function that depends on the unknown parameter λ .

$$g_{\lambda}\left(z
ight)=$$
 2*(1-e^(-lambda*z))/lam wo $rac{2\cdot\left(1-e^{-\lambda\cdot z}
ight)}{\lambda}$

Submit

You have used 2 of 3 attempts

✓ Correct (1/1 point)

(f)

1/1 point (graded)

Let Y_1, \ldots, Y_n be the salaries of the n sampled people. If one could actually observe Y_1, \ldots, Y_n , what would be the Fisher information of Y, $I_Y(\lambda)$, depending on λ ?

$$I_{Y}\left(\lambda
ight)= egin{array}{c} 1/ ext{lambda^2} \end{array}$$

Submit

You have used 1 of 3 attempts

✓ Correct (1/1 point)

(g)

1/1 point (graded)

In the model where only the X_i 's are observed (with fixed threshold z), what is the Fisher information? Denote it by $I_X(\lambda)$.

$$I_X(\lambda)$$
 $z^2/(e^{(lambda*z)-1)}$ z^2

Submit

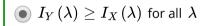
You have used 2 of 3 attempts

✓ Correct (1/1 point)

(h)

2/2 points (graded)

Compare $I_{Y}\left(\lambda\right)$ and $I_{X}\left(\lambda\right)$:



 $igcup I_{Y}\left(\lambda
ight)\leq I_{X}\left(\lambda
ight)$ for all λ

 $\bigcap I_{Y}\left(\lambda
ight) \geq I_{X}\left(\lambda
ight)$ for some λ , $I_{Y}\left(\lambda
ight) < I_{X}\left(\lambda
ight)$ for others.

~

Generating Speech Output interpret this in this model?

The ac	tual data always provides a better estim	nate		
The ce	ensored data always provides a better es	stimate.		
~				
Submit	You have used 1 of 2 attempts			
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