

5. Near resonance

Problem 5.1 If the system response of $\sin nt$ is $\frac{\sin nt}{50 - n^2}$, for which input signal $\sin nt$ is the gain the largest?

Solution: The gain is $\left| \frac{1}{50 - n^2} \right|$, which is largest when $|50 - n^2|$ is smallest. This happens for $n = 7$.

The gain for $\sin 7t$ is 1, and the next largest gain, occurring for $\sin 6t$ and $\sin 8t$, is $\frac{1}{14}$. Thus the system approximately filters out all the Fourier components of $f(t)$ except for the $\sin 7t$ term.

Problem 5.2 Let $x(t)$ be the periodic solution to

$$\ddot{x} + 50x = \frac{\pi}{4} \text{Sq}(t).$$

Which Fourier coefficient of $x(t)$ is largest? Which is second largest?

Solution: The input signal

$$\frac{\pi}{4} \text{Sq}(t) = \sum_{n \geq 1, \text{ odd}} \frac{\sin nt}{n}$$

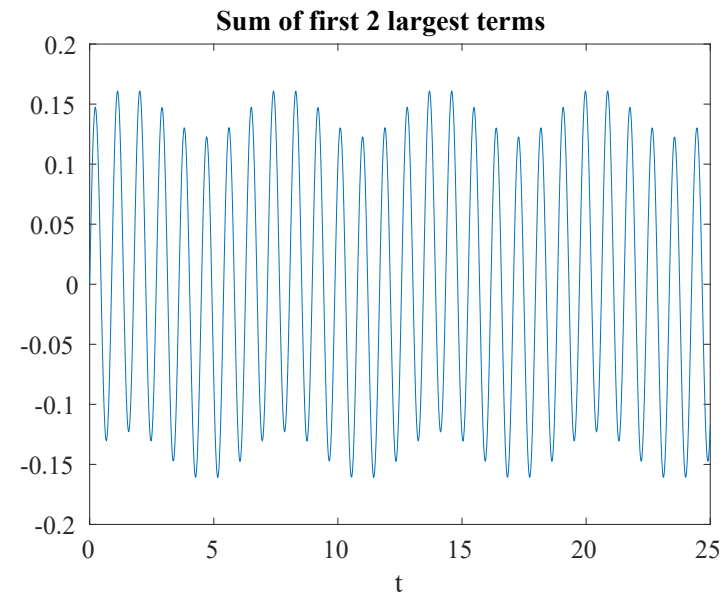
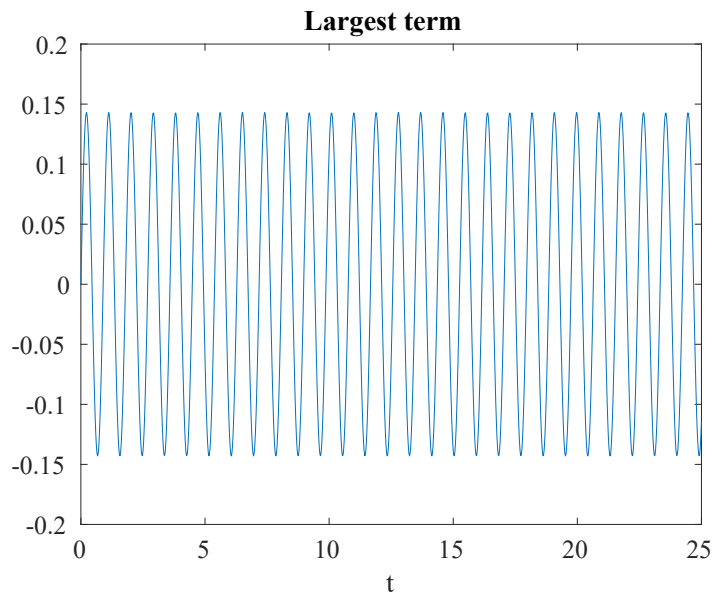


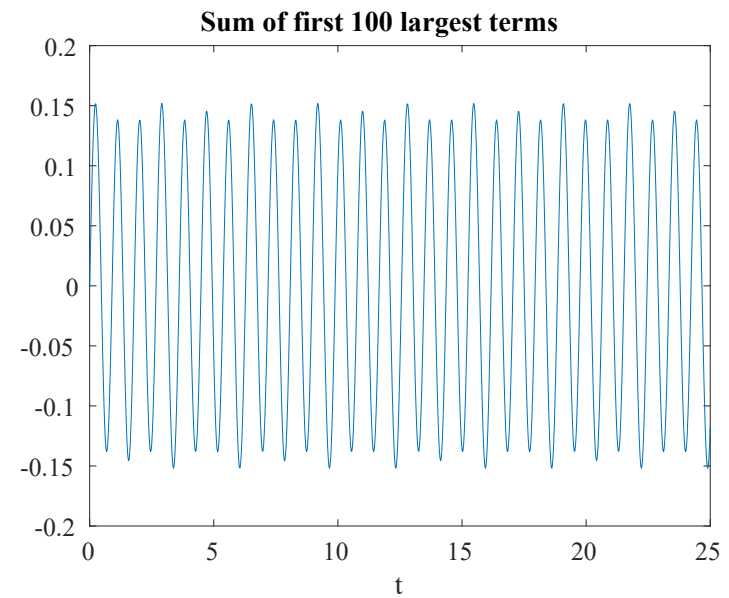
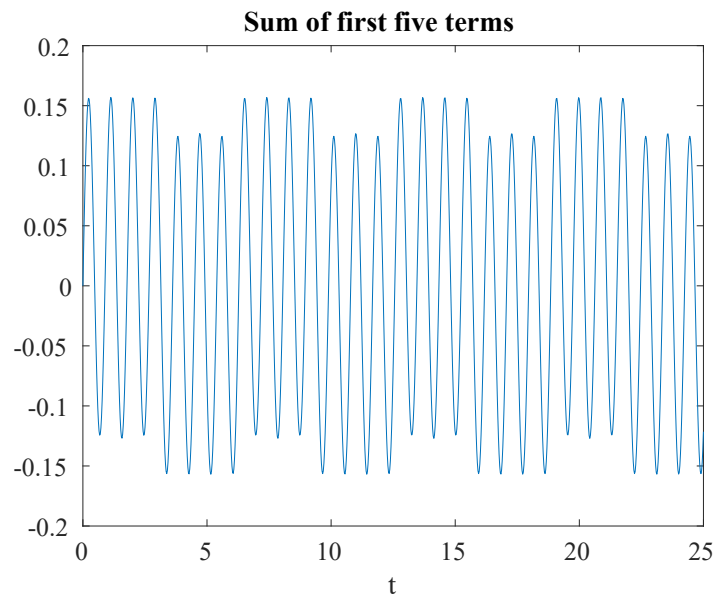
elicits the system response

$$x(t) = \sum_{n \geq 1, \text{ odd}} \left(\frac{1}{50 - n^2} \right) \frac{\sin nt}{n}$$
$$\approx 0.020 \sin t + 0.008 \sin 3t + 0.008 \sin 5t + 0.143 \sin 7t - 0.004 \sin 9t - (\text{even smaller terms})$$

so the coefficient of $\sin 7t$ is largest, and the coefficient of $\sin t$ is second largest. (This makes sense since the Fourier coefficient $\frac{1}{(50-n^2)n}$ is large only when one of n or $50 - n^2$ is small.)

Remark 5.3 Even though the system response is a complicated Fourier series, with infinitely many terms, only one or two are significant, and the rest are negligible.





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What does this sentence mean "Thus the system approximately filters out all the Fourier components of $f(t)$ except for the $\sin 7t$ term"

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