



[Course](#) > [Unit 1: Fourier Series](#) > [3. Solving ODEs with Fourier Series](#) > [2. Review the exponential response formula](#)

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2. Review the exponential response formula

Recall that the exponential response formula gives us a quick method for finding the particular solution to any linear, constant coefficient, differential equations whose input can be expressed in terms of an exponential function.

The exponential response formula (ERF): Let P be a polynomial with real, constant coefficients, $D = \frac{d}{dt}$ a differential operator, and r a (real or complex) number. If $P(r) \neq 0$, then a particular solution to the inhomogeneous differential equation

$$P(D)y = e^{rt} \quad \text{is given by} \quad y_p = \frac{e^{rt}}{P(r)}.$$

Caveat

Caveat: If $P(r) = P'(r) = P''(r) = \dots = P^{(k-1)}(r) = 0$, but $P^{(k)}(r) \neq 0$, then a particular solution to $P(D)y = e^{rt}$ is given by



$$y_p = \frac{t^k e^{rt}}{P^{(k)}(r)}.$$

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Sinusoidal input:

$$P(D)x = \cos(\omega t)$$

is the real part of

$$P(D)z = e^{i\omega t};$$

$$P(D)x = \sin(\omega t)$$

is the imaginary part of

$$P(D)z = e^{i\omega t}.$$

Therefore

- a particular solution to $P(D)x = \cos(\omega t)$ is given by $x_p = \operatorname{Re} \left[\frac{e^{i\omega t}}{P(i\omega)} \right];$
- a particular solution to $P(D)x = \sin(\omega t)$ is given by $x_p = \operatorname{Im} \left[\frac{e^{i\omega t}}{P(i\omega)} \right].$

2. Review the exponential response formula

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[A particular solution](#)

3

A particular solution, So I guess we could add a solution of the homogeneous equation $P(D)y = 0$ to get other solutions using superposition.

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