9. Fitting autoregressive model

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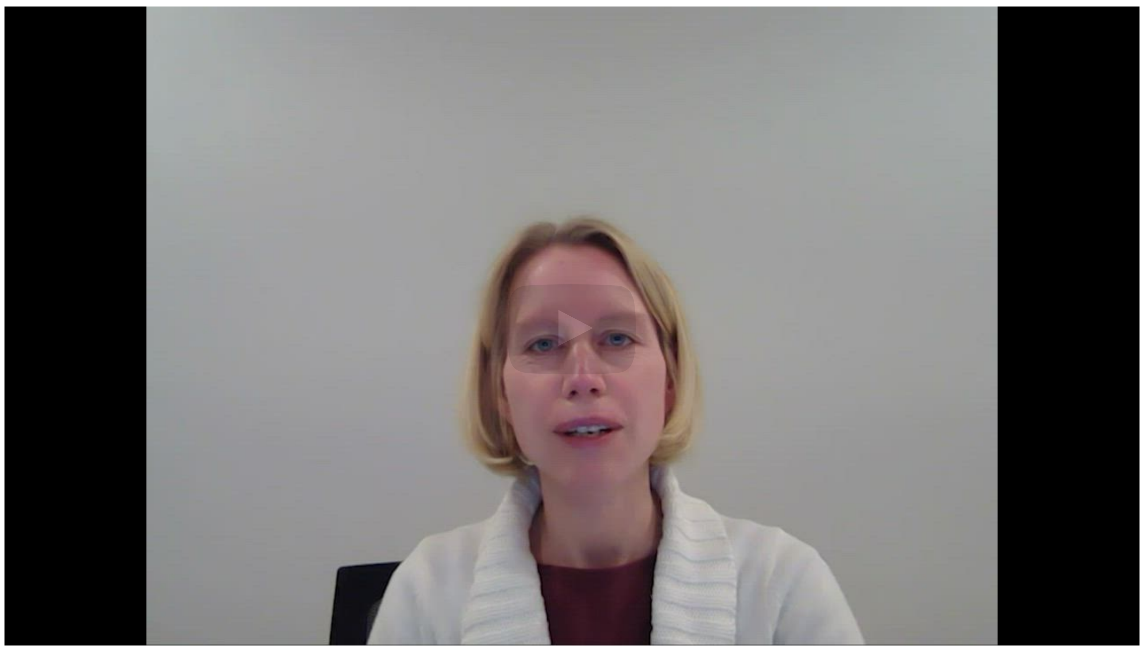
Fitting autoregressive model



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Prof Jegelka: We were talking about fitting statistical models,

and now we can make things very concrete by really

looking how we could fit an autoregressive model.

So this will actually be somewhat related

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AR (p) Parameter Estimation

For a given autoregressive order p, the AR (p) model has $p+1$ parameters that need to be estimated from data:

$$\phi_1, \phi_2, \ldots, \phi_p, \sigma_W^2.$$

We can estimate these parameters using the method of moments approach, which tells us to find $p+1$ moments that can be estimated from data, and $p+1$ equations that relate these estimable moments and the unknown parameters. Assuming that the time series $\{X_t\}$ is stationary, we can estimate the autocovariance function $\{\gamma_X(h)\}_{h=0}^{p}$ to get the required $p+1$ moments of the series. So the first step in estimation is to compute the autocovariances:

$$\hat{\gamma}_X(0), \hat{\gamma}_X(1), \hat{\gamma}_X(2), \ldots, \hat{\gamma}_X(p),$$

which we discussed before.

The second step is to find $p+1$ equations that relate these moments to the unknown parameters above:

$$\left(\gamma_X(0), \gamma_X(1), \gamma_X(2), \ldots, \gamma_X(p)\right) = \Gamma\left(\phi_1, \phi_2, \ldots, \phi_p, \sigma_W^2\right).$$

We will then have $p+1$ equations with $p+1$ unknowns, which will in general have a unique solution, so we obtain a plug-in estimator:

$$\left(\hat{\phi}_1, \hat{\phi}_2, \ldots, \hat{\phi}_p, \hat{\sigma}_W^2\right) = \Gamma^{-1}\left(\hat{\gamma}_X(0), \hat{\gamma}_X(1), \hat{\gamma}_X(2), \ldots, \hat{\gamma}_X(p)\right).$$

The consistency and asymptotic normality of the resulting estimator $(\hat{\phi}_1, \hat{\phi}_2, \ldots, \hat{\phi}_p, \hat{\sigma}_W^2)$ follows by the continuous mapping theorem and the Delta method from the corresponding properties of the estimator of the autocovariance function.

The equations Γ are known as the **Yule-Walker equations** and can be obtained as follows. Start from the definition of the AR (p) model

$$X_t = \phi_1 X_{t-1} + \phi_2 X_{t-2} + \cdots + \phi_p X_{t-p} + W_t,$$

multiply both sides of the equation by the column vector

$$(X_t, X_{t-1}, \ldots, X_{t-p})^\top$$

and take expectation of both side. We obtain:

$$\begin{aligned}
\mathbf{E}[X_t X_t] &= \mathbf{E}[\phi_1 X_t X_{t-1} + \phi_2 X_t X_{t-2} + \cdots + \phi_p X_t X_{t-p} + X_t W_t] \\
\mathbf{E}[X_{t-1} X_t] &= \mathbf{E}[\phi_1 X_{t-1} X_{t-1} + \phi_2 X_{t-1} X_{t-2} + \cdots + \phi_p X_{t-1} X_{t-p} + X_{t-1} W_t] \\
&\vdots \\
\mathbf{E}[X_{t-p} X_t] &= \mathbf{E}[\phi_1 X_{t-p} X_{t-1} + \phi_2 X_{t-p} X_{t-2} + \cdots + \phi_p X_{t-p} X_{t-p} + X_{t-p} W_t]
\end{aligned}$$

In terms of the autocovariance function, this give us the following equations:

$$\begin{aligned}
\gamma_X(0) &= \phi_1 \gamma_X(1) + \phi_2 \gamma_X(2) + \cdots + \phi_p \gamma_X(p) + \sigma_W^2 \\
\gamma_X(1) &= \phi_1 \gamma_X(0) + \phi_2 \gamma_X(1) + \cdots + \phi_p \gamma_X(p-1) \\
&\vdots \\
\gamma_X(p) &= \phi_1 \gamma_X(p-1) + \phi_2 \gamma_X(p-2) + \cdots + \phi_p \gamma_X(0)
\end{aligned}$$

ACF and AR(p) model

1/1 point (graded)

Does the autocovariance function tell us the order p of the AR (p) model?

- [ ] True
- [x] False

✔

Solution:

False. Unfortunately, the ACF does not tell us the order of the AR (p) model, because the ACF is non-zero for all h and decays exponentially fast.

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ℹ Answers are displayed within the problem

Yule-Walker equations

0/1 point (graded)

Is the moment function $\Gamma(\cdot)$ in Yule-Walker equations continuous?

- True ✔
- ◉ False

✖

Solution:

True. The autocovariance terms are related to the parameters of the model via additions and multiplications, which are continuous operations. More formally, if we let

$$\mathbf{r} = \begin{pmatrix} \gamma_X(0) \\ \ldots \\ \gamma_X(p) \end{pmatrix} \qquad \mathbf{R} = \begin{pmatrix} \gamma(1) & \gamma(2) & \ldots & \gamma(p) & 1 \\ \gamma(0) & \gamma(1) & \ldots & \gamma(p-1) & 0 \\ \vdots & & & & \\ \gamma(p-1) & \gamma(p-2) & \ldots & \gamma(0) & 0 \end{pmatrix} \qquad \mathbf{\Phi} = \begin{pmatrix} \phi_1 \\ \ldots \\ \phi_p \\ \sigma_W^2 \end{pmatrix}$$

then the Yule-Walker equation can be written as

$$\mathbf{r} = \mathbf{R}\mathbf{\Phi}$$

so that the function Γ is multiplication by the matrix $\mathbf{R}$.

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[STAFF] Is the moment function continuous?

question posted 2 months ago by **vstone9**

I'm not quite convinced with the solution. is Γ a matrix which is a set of equations $\gamma(h)$?

This post is visible to everyone.

Syed_SB

2 months ago - marked as answer 2 months ago by **vstone9**

Γ is a function/transformation which maps parameters to the the covariances.

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