15.053x, Optimization Methods in Business Analytics

Fall, 2016

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A glossary of notation and terms used in 15.053x

NOTATION AND TERMINOLOGY

The purpose of this document is to provide a glossary of notation and terminology relevant to 15.053x. We will add notation and terminology throughout the semester, and we will update this document once a week. If you would like terms or notation added, contact the TA, Khizar Qureshi.

For a comprehensive (and mathematically advanced) glossary of mathematical terms used in optimization, see the Math Programming Glossary, which was developed by Harvey Greenberg.

MATHEMATICAL NOTATION

- $\sum_{i \in S} x_i$ = The sum of x_i where the sum is over all indices i in the set S. We refer to this type of notation as *summation notation*.
- $|x|$ = the absolute value of x. (This assumes that x is a single variable.)
- $\lfloor x \rfloor$ = the *floor* of x. That is, x rounded down to the nearest integer. For example, $\lfloor 2.3 \rfloor = 2$; $\lfloor -1.1 \rfloor = -2$; $\lfloor x \rfloor = x$ if x is an integer.
- $\lceil x \rceil$ = the *ceiling* of x. That is, x rounded up to the nearest integer. For example, $\lceil 2.3 \rceil = 3$; $\lceil -1.1 \rceil = -1$; $\lceil x \rceil = x$ if x is an integer.
- $x^+ = \max\{0, x\}$. This is often referred to as the *positive part* of x.
- $x^- = \min\{0, x\}$. This is often referred to as the *negative part* of x.

TYPES OF OPTIMIZATION MODELS.

By an *optimization model* (or *optimization problem*) we mean a problem in which there is a single objective function (max or min) subject to constraints. An alternative term that is commonly used is *mathematical program*. We also refer to them as *maximization problems* or *minimization problems*.

- *Linear Program*: an optimization model in which the objective is linear and the constraints are linear.
- *Mixed Integer Linear Program*: an optimization model in which the objective is linear and the constraints are linear, and some (or all) of the variables are constrained to be integer valued. It is called a *Pure Integer Program* if every variable is required to take on an integer value. It is called a *Binary Integer Program* (or a 0-1 Integer Program) if every variable is required to be 0 or 1.
- *Nonlinear Program*. This is the common name that refers to any possible optimization model. Remember that nonlinear programs include linear programs as a special case.

OTHER TERMINOLOGY

- *Bounded feasible region*. We say that a feasible region is *bounded* if there is some positive number *M* such that every decision variable is guaranteed to be between -*M* and *M*. If a feasible region is not bounded, we say that it is *unbounded*.
- *Constraints*: Inequalities (or equalities) to impose limitations on the decision variables.
- *CBC.* The solution algorithm that is freely available and is commonly used in conjunction with OpenSolver to solve linear programs and integer programs.
- *Decision variables*. The variables that represent the decisions or choices to be made. If you are using spreadsheet optimization, these variables are the values in *Changing Cells* or *Changing Variable Cells*.
- *Excel Solver*. The optimization software that is included with Microsoft Excel. (With Google Sheets, the free software is called *Solver*.) It can be used to solve linear programs (simplex method) or integer programs (simplex method) or nonlinear programs (GRG Nonlinear).
- *Feasible*. A point is said to be *feasible* if it satisfies all of the constraints of the optimization model. (A point represents the assignment of values to each of the decision variables.) The *feasible region* is the set of all feasible points.
- *Free*. A decision variable *x* is called *free* if it can be either positive or negative. If a variable is free, we also say that it is *unconstrained in sign*.

- *Infeasible*. A point is said to be *infeasible* if it violates one or more constraints of the optimization model. An optimization model is said to be *infeasible* if there are no feasible points (equivalently, there are no solutions).
- *Integrality constraint*. A constraint stipulating that one or more variables of a model are required to be integer valued.
- *Non-negativity constraints*. The constraints that constrain variables to be greater than or equal to 0.
- *Objective Function*. In an optimization model, the goal is to either minimize or maximize the objective function.
- *OpenSolver.* Spreadsheet modeling software that can be used to set up an optimization problem and call an algorithm to solve it. OpenSolver is freely available on the web at www.OpenSolver.org. OpenSolver can, in principle, be used to model and solve optimization problems with any number of variables. (Excel Solver is limited to 200 variables.) In reality, extremely large problems may take up more memory than is available in your computer, and they may require too much time to solve. In 15.053x, we typically use CBC to solve linear and integer programs. In addition, OpenSolver works with other optimization software such as CPLEX and Gurobi.
- *Optimal solution*. A *solution* refers to a feasible point. Suppose that one is trying to solve a maximization problem, and that the objective function is $f(\ )$. A solution x^* is called *optimal* (or *maximal*) if for any other feasible solution x', $f(x^*) \geq f(x')$. If it were a minimization problem, then x^* would be called an *optimal* (or *minimal*) *solution* if for any other solution x', $f(x^*) \leq f(x')$.
- Simplex Algorithm. The most commonly used method for solving linear programs. It was developed by George Dantzig in 1947.
- Solution. Typically a *solution* refers to a feasible point of an optimization model. The term "infeasible solution" sounds like a paradox. But, the term *infeasible solution is widely used to* refer to a point that is infeasible. That is, it is not a solution.
- *Unbounded*. We say that a feasible region is *unbounded* if it is not bounded. That is, for any positive number *M*, there is some feasible solution x' such that some variable of x' has absolute value larger than *M*. We say that the optimal objective value of a maximization problem is *unbounded from above* if there is a sequence of feasible solutions whose objective values goes off to (converges to) ∞. Similarly, we say that the optimal objective value of a minimization problem is unbounded from below if there is a sequence of feasible solutions whose objective values converge to -∞.