# 3. Optimization

Part B due Oct 5, 2021 20:30 IST

**Practice**

## Minimize the distance to a sphere

3/3 points (graded)

Let $S$ be the unit sphere around the origin, defined by $x^2 + y^2 + z^2 = 1$. Find the point of $S$ which is closest to the point $(2, 2, 1)$.

Find the coordinates $(x, y, z)$.

$x =$ 2/3 ✔ **Answer:** 2/3

$y =$ 2/3 ✔ **Answer:** 2/3

$z =$ 1/3 ✔ **Answer:** 1/3

? INPUT HELP

**Solution:**

Assume that the point $(x, y, z)$ on the unit sphere makes the squared distance function

$$f(x, y, z) = (x-2)^2 + (y-2)^2 + (z-1)^2$$

achieve its minimal value. Then by the Lagrangian multiplier, we have

$$\nabla\left((x-2)^2 + (y-2)^2 + (z-1)^2\right) = \lambda \nabla\left(x^2 + y^2 + z^2\right).$$

This is equivalent to

$$x - 2 = \lambda x, \quad y - 2 = \lambda y, \quad z - 1 = \lambda z.$$

Rearranging this gives

$$(1-\lambda)\, x = 2, \qquad (6.294)$$

$$(1-\lambda)\, y = 2, \qquad (6.295)$$

$$(1-\lambda)\, z = 1. \qquad (6.296)$$

Squaring and summing together we find Hence

$$(1-\lambda)^2 \underbrace{(x^2 + y^2 + z^2)}_{=1} = 2^2 + 2^2 + 1^2 \qquad (6.297)$$

$$(1-\lambda)^2 = 9 \qquad (6.298)$$

which implies that $1 - \lambda = \pm 3$. Therefore the point we are looking for is either $\left(\frac{2}{3}, \frac{2}{3}, \frac{1}{3}\right)$ or $\left(-\frac{2}{3}, -\frac{2}{3}, -\frac{1}{3}\right)$.

According to the geometric condition, the fact that we are looking for the minimum ar

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$\boxed{\left(\frac{2}{3}, \frac{2}{3}, \frac{1}{3}\right)}$ is the answer.

Submit You have used 1 of 5 attempts

Answers are displayed within the problem

## 3. Optimization

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[Staff] Unit sphere vs ellipsoid 4
In "Minimize the distance to an ellipsoid," the title says ellipsoid, and the equation is an ellipsoid, but the text says "unit sphere."

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