Using Chebyshev's inequality to obtain lower bounds

I need help with a question I found in Master Stats. I'm unaware of Chebyshev's inequality hence I can't do this question, can anyone help.

Q) A company produces planks whose length is a random variable of mean 2.5m and standard deviation 0.1m. Use Chebyshev's inequality to obtain a lower bound on the probability that the length of planks does not differ more than 0.5m from the mean length.

Thanks in advance

(probability)

asked May 10 '12 at 9:37

Ricky Rozay

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4 Have you looked up Chebyshev's inequality? en.wikipedia.org/wiki/Chebyshev's_inequality – Daan Michiels May 10 '12 at 10:34

1 Answer

Let X be a random variable with mean μ and standard deviation $\sigma > 0$. Then the Chebyshev Inequality says that if $k > 0$, then

$$P(|X-\mu| \geq k\sigma) \leq \frac{1}{k^2}.$$

In our case, X is the length of a plank chosen at random from the company's production. Then $\mu = 2.5$ and $\sigma = 0.1$. We want to find k such that $k\sigma = 0.5$. Thus $k = \frac{0.5}{\sigma} = \frac{0.5}{0.1} = 5$. We conclude that

$$P(|X-2.5| \geq 0.5) \leq \frac{1}{5^2}.$$

It follows that

$$P(|X-2.5| < 0.5) \geq 1 - \frac{1}{5^2}. \qquad (*)$$

This is not *quite* what we want, since we want to find a number p such that $P(|X-2.5| \leq 0.5) \geq p$. One can argue that the probability that the difference is *absolutely exactly* 0.5 is 0, so that $(*)$ gives us the inequality we want. That gives a lower bound of $1 - \frac{1}{5^2} = 0.96$. There is a probability of at least 0.96 that the plank does not differ by more than 0.5 from the mean 2.5.

Typically, the Chebyshev Inequality gives very *conservative* estimates. In our case, though Chebyshev says that $P(|X-2.5| \geq 0.5) \leq \frac{1}{5^2}$, the actual probability is likely to be substantially smaller than $\frac{1}{5^2}$. Thus the lower bound of 0.96 is likely conservative. Informally, more than 96% of the production will be "within spec."

edited May 10 '12 at 13:49

answered May 10 '12 at 13:41

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but isn't the formula for Chebyshev Inequality P(|X−μ|≥k)≤σ^2/k^2 or is the formula you used just a rearrangement? – methuselah May 15 '12 at 13:06

I used the equivalent version $P(|X-\mu| \geq k\sigma) \leq \frac{1}{k^2}$. So my k is the number of "standard deviation units." You can see that (in this case, as always) we get the same thing. Your k is 0.5, and with your k we have $\sigma^2/k^2 = (0.1)^2/(0.5)^2 = 1/5^2$. – André Nicolas May 15 '12 at 13:14