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☆ Course / Unit 3: Optimization / Lecture 12: Least squares approximation



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Summarize

Big Picture

Given a set of data points, (x_i,y_i) the simplest possible relationship we can hope to find between the data is a linear one. That is, we want to find a line y=ax+b that best fits these data. Our "variables" are now the a and $m{b}$, the slope and $m{y}$ -intercept that determine the linear relationship.

In order to find the best possible a and b, we minimize an error function. The error function we defined squares the difference between the y_i to the predicted value ax_i+b :

$$D\left(a,b
ight) = \sum_{i=1}^{n} \left(y_i - \left(ax_i + b
ight)
ight)^2.$$

Thus finding the best fit requires minimizing this function of two variables a and b, which is an (unconstrained) optimization problem!

Mechanics

To fit a collection of data (x_i,y_i) to a straight line that minimizes the squares of the differences between the predicted $oldsymbol{y}$ values and the actual $oldsymbol{y_i}$, solve the system:

$$\left(\sum_{i=1}^{n} x_i^2\right) a + \left(\sum_{i=1}^{n} x_i\right) b = \sum_{i=1}^{n} x_i y_i \tag{4.218}$$

$$\left(\sum_{i=1}^{n} x_i\right) a + nb = \sum_{i=1}^{n} y_i. \tag{4.219}$$

where n is the number of data points.

- ullet To fit your data (x_i,y_i) to a power law $y=cx^p$, first transform your data $(X_i,Y_i)=(\ln x_i,\ln y_i)$, then use least squares approximation. The power p will be the slope.
- ullet To fit your data (x_i,y_i) to an exponential law $y=ce^{kx}$, first transform your data $(X_i,Y_i)=(x_i,\ln y_i)$. Then use least squares approximation. The exponential multiple $m{k}$ is the slope of the least squares fitting to the transformed data.

Ask Yourself

In order to define "best fit" we have to make a decision about how to quantify the error. A common choice is to minimize the square of the difference between predicted and observed values. Since this method involves minimizing squares, the method is called "least-squares".

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→ What is the point of drawing a straight line through data?

Fitting a straight line to data isn't just for making pretty pictures! Once we have a line that fits our data well, we can "extrapolate", which means we can make a prediction for what would have happened for values we weren't able to test. For example, if you find the line of best-fit between $m{x}=$ heights of marathon runners and y = marathon time, then you can use the line to predict the time for a new marathon runner, even if all you know is their height. Of course the prediction won't be perfect, but it's a good start. If you study statistics, you can learn how to create a "confidence interval" around the predicted

Lecture 12: Least squares approximation | Unit 3: Optimization | Multivariable Calculus 1: Vectors and Derivatives | edX In this sense, least squares approximation is an example of *machine learning*, the process by which a computer can be "trained" on some data (here it would be the (x_i,y_i) pairs that we measure) and the computer "learns" to predict what would happen for other data (new values of x_i outside of the training set). <u>Hide</u> 8. Summary **Hide Discussion** Topic: Unit 3: Optimization / 8. Summary **Add a Post** by recent activity > Show all posts ~ Something fascinating: transforming proxy into equality! 5 **≜** Community TA <u>Typos - [STAFF]</u> 2 Next Up: Recitation 12: Fitting a parabola to data Previous 11 min + 4 activities

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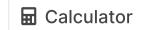
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