8.2.5 Water emptying from a tank

MO2.4

Consider a tank holding water with density ρ. The upper surface of the water is exposed to the atmosphere (i.e. the tank is open on top). The cross-sectional area of the tank is A_t. The water is emitted from a small orifice at the bottom of the tank. As the water exits through the orifice, the water height $h\left(t\right)$ changes. Let the velocity of the water at this orifice be u_o and the cross-sectional area of the orifice be A_o (which we will assume is much smaller than A_t, i.e. $A_o/A_t << 1$). This exit velocity can be estimated assuming idealized flow conditions as

$$u_o = \sqrt{2gh} \tag{8.20}$$

Thus, u_0 decreases as h decreases.

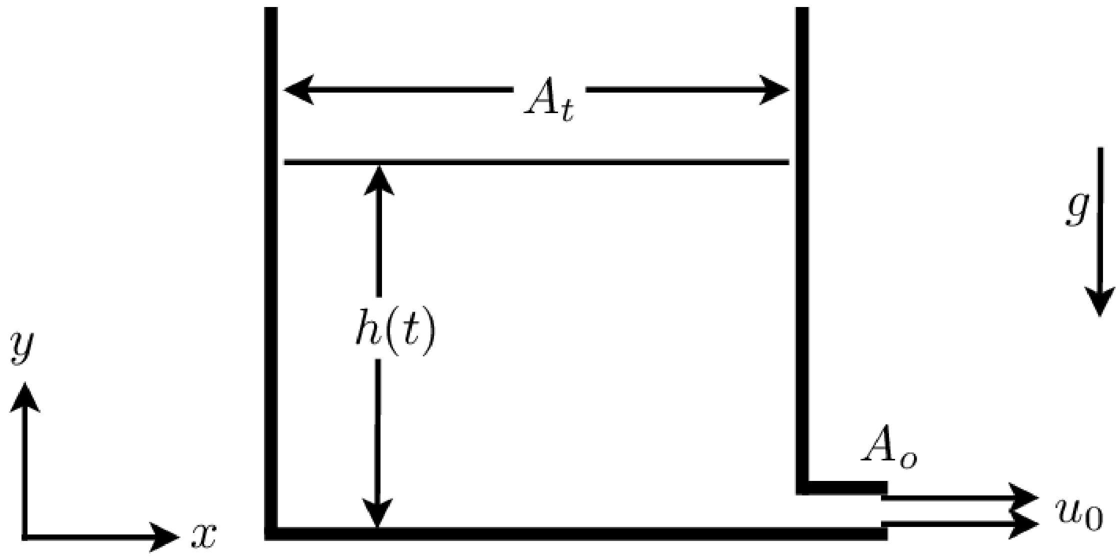

Figure 8.5: Water emptying from a tank

The key principle in deriving this model differential equation known as the conservation of mass and it states that the mass of water in the tank ($m_w\left(t\right)$) decreases by the rate at which the water (mass) is emitted at the orifice. Let's call the rate at which mass flows out of the orifice $\dot{m}_o$. Then conservation of mass gives,

$$\frac{\mathrm{d}m_w}{\mathrm{d}t} = -\dot{m}_o \tag{8.21}$$

At any instant, the mass of water in the tank, $m_w\left(t\right)$ is related to the volume of water $\mathrm{Vol}_w\left(t\right)$ and the density ρ through $m_w = \rho\mathrm{Vol}_w$. The water volume is simply $A_t h$. Thus,

$$m_w\left(t\right) = \rho\mathrm{Vol}_w\left(t\right) = \rho A_t h\left(t\right) \tag{8.22}$$

The mass flow rate out of the orifice can be shown to be $\dot{m}_o = \rho A_o u_o$ which upon substitution of Equation (8.20) gives,

$$\dot{m}_o = \rho A_o \sqrt{2gh} \tag{8.23}$$

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2

< Previous Next >

experimental results.

Remark 9.

The exact solution for the differential model equation (i.e.