8.3.1 Definition

MO2.4

In the previous section, you've seen several examples of time-dependent phenomena and their model equations. Next, we will introduce the idea of an Initial Value Problem (IVP), which is the general mathematical form of these types of phenomenon and model equations.

Definition 1 (Initial Value Problem (IVP)).

Let $\underline{u}(t)$ *be a time-dependent vector of* M *states,*

$$\underline{u}(t) = \begin{bmatrix} u_0(t) \\ u_1(t) \\ \vdots \\ u_{M-2}(t) \\ u_{M-1}(t) \end{bmatrix} \tag{8.35}$$

with an initial condition, $\underline{u}(t_I) = \underline{u}_I$*. The evolution of the state from the initial condition at* $t = t_I$ *until the final time* $t = t_F$ *is governed by the system of differential equations,*

$$\frac{\mathrm{d}\underline{u}}{\mathrm{d}t} = \underline{f}(\underline{u}, t) \quad \text{for} \quad t_I < t < t_F \tag{8.36}$$

Remark 10.

When $M = 1$*, we refer to the problem as a scalar IVP. For convenience and clarity, we drop the underline notation and write the scalar IVP as,*

$$\frac{\mathrm{d}u}{\mathrm{d}t} = f(u, t) \quad \text{for} \quad t_I < t < t_F \tag{8.37}$$

with $u(t_I) = u_I$*.*

Remark 11.

While the IVP description above is for a range of time $t_I < t < t_F$*, often we may not know a precise final time. Rather, we are interested in solving the problem until the time when some event occurs. For example, in the coffee cooling, perhaps we want to know how long we have until the temperature drops to* $T_c = 70°C$*. Or, for the water tank problem, we may want to run the simulation until the tank is completely empty. In either case, we will not know ahead of time*

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