5. Censored data

In a given population, n individuals are sampled randomly, with replacement, and each sampled individual is asked whether his/her salary is greater than some fixed threshold z. Assume that the salary of a randomly chosen individual has the exponential distribution with unknown parameter λ. Asking whether the salary overcomes a given threshold rathen than directly asking for the salary increases the number people that are willing to answer and decreases the number of mistakes in the collected answers.

Denote by $X_1, \ldots, X_n$ the binary responses of the n sampled individuals, so that $X_i \in \{0, 1\}$. We call the X_i **censored data**.

(a)

2/2 points (graded)
What kind of distribution do the X_i s follow?

- Exponential distribution with parameter $\mu\,(\lambda)$

- (●) Bernoulli with parameter $\mu(\lambda)$
- ( ) Poisson with parameter $\mu(\lambda)$

✔

Give the parameter of this distribution in terms of λ and z:

Parameter $\mu(\lambda) =$ e^(-lambda*z) ✔

$e^{-\lambda \cdot z}$

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✔ Correct (2/2 points)

(b)

1/1 point (graded)

Let $\overline{X}_n$ be the proportion of sampled individuals whose response was 1 (corresponding to *Yes*). Convince yourself that $\overline{X}_n$ is asymptotically normal.

What is its asymptotic variance?

$V(\overline{X}_n) =$ e^(-lambda*z)*(1-e^(-lan ✔

$e^{-\lambda \cdot z} \cdot \left(1 - e^{-\lambda \cdot z}\right)$

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✔ Correct (1/1 point)

(c)

1/1 point (graded)

Find a function f such that $f(\overline{X}_n)$ is a consistent estimator of λ.

Write **barX_n** for the sample average $\overline{X}_n$.

$f(\overline{X}_n) =$ -ln(barX_n)/z ✔

$-\frac{\ln(barX_n)}{z}$

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✔ Correct (1/1 point)

(d)

1/1 point (graded)

Convince yourself that $f(\overline{X}_n)$ is asymptotically normal and compute its asymptotic variance.

$V(f(\overline{X}_n)) =$ (e^(lambda*z)-1)/z^2 ✔

$\frac{e^{\lambda \cdot z}-1}{z^2}$

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✔ Correct (1/1 point)

(e)

1/1 point (graded)

What equation must z satisfy in order to minimize the asymptotic variance computed in part (d)? Write this equation in the form $g_\lambda(z) = z$, where g_λ is a function that depends on the unknown parameter λ.

$g_\lambda(z) =$ 2*(1-e^(-lambda*z))/lam ✔

$\frac{2\cdot(1-e^{-\lambda\cdot z})}{\lambda}$

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✔ Correct (1/1 point)

(f)

1/1 point (graded)

Let $Y_1, \ldots, Y_n$ be the salaries of the n sampled people. If one could actually observe $Y_1, \ldots, Y_n$, what would be the Fisher information of Y, $I_Y(\lambda)$, depending on λ?

$I_Y(\lambda) =$ 1/lambda^2 ✔

$\frac{1}{\lambda^2}$

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✔ Correct (1/1 point)

(g)

1/1 point (graded)

In the model where only the X_i 's are observed (with fixed threshold z), what is the Fisher information? Denote it by $I_X(\lambda)$.

$I_X(\lambda)$ z^2/(e^(lambda*z)-1) ✔

$\frac{z^2}{e^{\lambda \cdot z}-1}$

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✔ Correct (1/1 point)

(h)

2/2 points (graded)

Compare $I_Y(\lambda)$ and $I_X(\lambda)$:

- (●) $I_Y(\lambda) \geq I_X(\lambda)$ for all λ
- ( ) $I_Y(\lambda) \leq I_X(\lambda)$ for all λ
- ( ) $I_Y(\lambda) \geq I_X(\lambda)$ for some λ, $I_Y(\lambda) < I_X(\lambda)$ for others.

✔

How do you interpret this in this model?

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It depends on the parameter λ whether the censored data or the actual data provides a better estimate.

The actual data always provides a better estimate

The censored data always provides a better estimate.

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Correct (2/2 points)

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? Parts e,g
For part e, not getting a neat answer from a solver. Maybe am way off in the method. For part g, compared to similar version of same problem with the parameter from this o... 4

e) trouble with Lambert W function
Hey, i got a green mark on d). So e) seemed quite trivial. But my function involves the Lambert W function (that surprised my) or to be precise an aproximation of the Output v... 1

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