MITx: 6.041x Introduction to Probability - The Science of Uncertainty




- Unit 0: Overview
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- Unit 1: Probability models and axioms
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# Exercise: A variation on merging

(1/2 points)

We start with two independent Bernoulli processes, $X_n$ and $Y_n$, with parameters $p$ and $q$, respectively. We form a new process $Z_n$ by recording an arrival in a given time slot if and only if **both** of the original processes record an arrival in that same time slot. Mathematically, $Z_n = X_n Y_n$.

The new process $Z_n$ is also Bernoulli with parameter

p*q

**Answer:** p*q

(Enter an algebraic function of $p$ and $q$ using standard notation .)

Suppose that the two Bernoulli processes $X_n$ and $Y_n$ are dependent. We still assume, however, that the pairs $(X_n, Y_n)$ are independent. E.g., $(X_1, Y_1)$ is independent from $(X_2, Y_2)$, etc. Is the process $Z_n$ guaranteed to be Bernoulli?

Yes ▾ **Answer:** No

- Unit 6: Further topics on random variables
- Unit 7: Bayesian inference
- Exam 2
- Unit 8: Limit theorems and classical statistics
- **Unit 9: Bernoulli and Poisson processes**

**Unit overview**

**Lec. 21: The Bernoulli process**
Exercises 21 due May 11, 2016 at 23:59 UTC

**Lec. 22: The Poisson process**
Exercises 22 due May 11, 2016 at 23:59 UTC

**Lec. 23: More on the Poisson process**


Answer:
The merged process records an arrival if and only if both of the original processes record an arrival, which happens with probability $pq$.

In the second case, since the pairs $(X_n, Y_n)$ are independent, the random variables $Z_n$ are also independent. However, there is nothing in the statement that would ensure that the $Z_n$ are identically distributed. Thus, $Z_n$ is not guaranteed to be a Bernoulli process. For example, consider the special case of $p = q$ and suppose that $Y_1 = X_1$ but $Y_n$ is independent of $X_n$ for $n > 1$. Then $\mathbf{P}(Z_1 = 1) = p$ while $\mathbf{P}(Z_n = 1) = p^2$ for $n > 1$, violating the time-homogeneity property of Bernoulli processes.

*You have used 1 of 1 submissions*

Exercises 23 due May 11, 2016 at 23:59 UTC

**Solved problems**

**Additional theoretical material**

**Problem Set 9**
Problem Set 9 due May 11, 2016 at 23:59 UTC

**Unit summary**

Unit 10: Markov chains