1. Economics and sustainability

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Problem Set B due Sep 13, 2021 20:30 IST Completed

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Problem 1 (a)

1.0/1 point (graded)

You run a manufacturing company that creates a specific part: a circuit board used in the next new iPhone.

You have designed two different machines that both create the same part. One machine is slow in production of parts, but is energy efficient. The other is fast in part production, but energy consumptive.

- Machine A creates 500 parts per hour, and draws 35 kWatt hours per hour in the process.
- Machine B creates 2500 parts per hour, and draws 200 kWatt hours per hour in the process.

(Note that a kWatt hour is a unit of Energy. That is, it has the same dimension as a Joule.)

Your company is located in a region that charges $ 12 per kWatt hour.

For each part that is created, you earn $ 1.

Let a be the number of hours per day that you run machine A. Let b be the number of hours per day that you run machine B. Each machine can run 0 hours, but no more than 24 hours in a day. The machines are independent and can be run simultaneously.

Find a profit function $p(a,b)$ in dollars per day that describes the total profit earned per day in terms of the number of hours per day you run each machine.

(Hint: Consider the revenue earned in producing a part in a hours on machine A minus the cost for energy, and similarly for machine B.)

$p(a,b) =$ 80*a+100*b dollars per day ✔ **Answer:** 80*a + 100*b

? INPUT HELP

Solution:

The total profit created by machine A is

$$1\,\frac{\text{dollars}}{\text{part}}\cdot 500\,\frac{\text{parts}}{\text{hr}}\cdot a\,\frac{\text{hrs}}{\text{day}} - 12\,\frac{\text{dollars}}{\text{kWatt hr}}\cdot 35\,\frac{\text{kWatt hr}}{\text{hr}}\cdot a\,\frac{\text{hrs}}{\text{day}}$$

The total profit created by machine B is

$$1\,\frac{\text{dollars}}{\text{part}}\cdot 2500\,\frac{\text{parts}}{\text{hr}}\cdot b\,\frac{\text{hrs}}{\text{day}} - 12\,\frac{\text{dollars}}{\text{kWatt hr}}\cdot 200\,\frac{\text{kWatt hr}}{\text{hr}}\cdot b\,\frac{\text{hrs}}{\text{day}}$$

Thus the total profit function is

$$p(a,b) = 80a + 100b \text{ dollars per day}$$

Submit You have used 1 of 5 attempts

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Answers are displayed within the problem

Problem 1(b)

3/3 points (graded)
Find the number of hours per day you should run each machine to maximize your profit.

$a =$ 24 hours per day ✔ **Answer:** 24 $b =$ 24 hours per day ✔ **Answer:** 24

What is the maximum profit?

4320 dollars per day ✔ **Answer:** 4320

Solution:

The function $p(a, b)$ is a linear function that increases with both variables a and b. Therefore the maximum will occur on the boundary where a and b are as large as possible, that is $a = b = 24$ hours per day.

The total profit earned by running both machines all day everyday is $ 4320 per day.

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Answers are displayed within the problem

Problem 1(c)

4.0/4 points (graded)
A new law just passed which penalizes the use of energy inefficient machines. Your energy company charges more the greater number of hours that machine B has been running.

- Machine A creates 500 parts per hour, and draws 35 kWatt hours per hour in the process.
- Machine B creates 2500 parts per hour, and draws 200 kWatt hours per hour in the process.

(Note that a kWatt hour is a unit of Energy. That is, it has the same dimension as a Joule.)

Your company is located in a region that charges $ 12 per kWatt hour. If you run machine B for b hours then you are charged an additional fee of $5b(b-1)$ dollars (in addition to the standard energy fee).

For each part created, you earn $ 1.

Let a be the number of hours per day that you run machine A. Let b be the number of hours per day that you run machine B. Each machine can run 0 hours, but no more than 24 hours in a day.

Find a profit function $p(a, b)$ in dollars per day that describes the total profit earned per day in terms of the number of hours per day you run each machine.

Find your new profit function.

$p(a, b) =$ 80*a+105*b-5*b^2 dollars per day ✔ **Answer:** 80*a + 105*b - 5*b^2

Maximize your profit given the new rules about energy usage for machine B.

$a =$ 24 hours per day ✔ **Answer:** 24 $b =$

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10.5 hours per day ✔ **Answer:** 10.5

What is the maximum profit?

2471.25 dollars per day ✔ **Answer:** 2471.25

? INPUT HELP

Solution:

The new profit function is

$$p\left(a,b\right) = 500a - 12\left(35\right)a + 2500b - 12\left(200\right)b - \left(5\left(b-1\right)\right)b = 80a + 105b - 5b^2$$

The constraint is $0 \leq a \leq 24, 0 \leq b \leq 24$.

Because this function increases in a, it has no critical points and we can assume that the maximum will occur where $a = 24$. Restricting to that edge we maximize the function

$$q\left(b\right) = p\left(24,b\right) = 1920 + 105b - 5b^2$$

This function has a critical point where $q'\left(b\right) = 105 - 10b = 0$, which is where $b = 10.5$ hours.

The value of the function at this point is $p\left(24, 10.5\right) = 1920 + 105\left(10.5\right) - 5(10.5)^2 = 2471.25$ dollars per day. This is the maximum. We verify by check the end points.

When $b = 0$ the profit is 1920 dollars per day. When $b = 24$ the profit is 1560 dollars per day.

Thus the maximum is running machine A for $a = 24$ hours per day and running machine B for $b = 10.5$ hours per day.

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Answers are displayed within the problem

Problem 1(d)

3/3 points (graded)
Suppose the government imposes a cap on the total amount of energy you can use per day at 2940 kWatt hrs. (Note this is the same amount of energy used in the previous problem.)

- Machine A creates 500 parts per hour, and draws 35 kWatt hours per hour in the process.
- Machine B creates 2500 parts per hour, and draws 200 kWatt hours per hour in the process.

(Note that a kWatt hour is a unit of Energy. That is, it has the same dimension as a Joule.)

Your company is located in a region that charges $ 12 per kWatt hour.

For each part, created, you earn $ 1.

Let a be the number of hours per day that you run machine A. Let b be the number of hours per day that you run machine B. Each machine can run 0 hours, but no more than 24 hours in a day.

The profit function $p\left(a,b\right)$ is the same as the profit function you found in part (a).

Find the maximum number of hours you should run each machine.

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$a =$

24 hours per day ✔ **Answer:** 24

$b =$

10.5 hours per day ✔ **Answer:** 10.5

What is the maximum profit?

2970 dollars per day ✔ **Answer:** 2970

Solution:

The total profit function is

$$p(a, b) = 80a + 100b \text{ dollars per day}$$

We want to maximize this function subject to the constraint $35a + 200b \leq 2940$. Note that because our profit function does not have critical points, we can restrict our attention to the boundary of the region.

Now we solve the Lagrange multiplier problem subject to the constraint $g(a, b) = 35a + 200b = 2940$.

$$\nabla p(a, b) = \langle 80, 100 \rangle \quad (4.281)$$

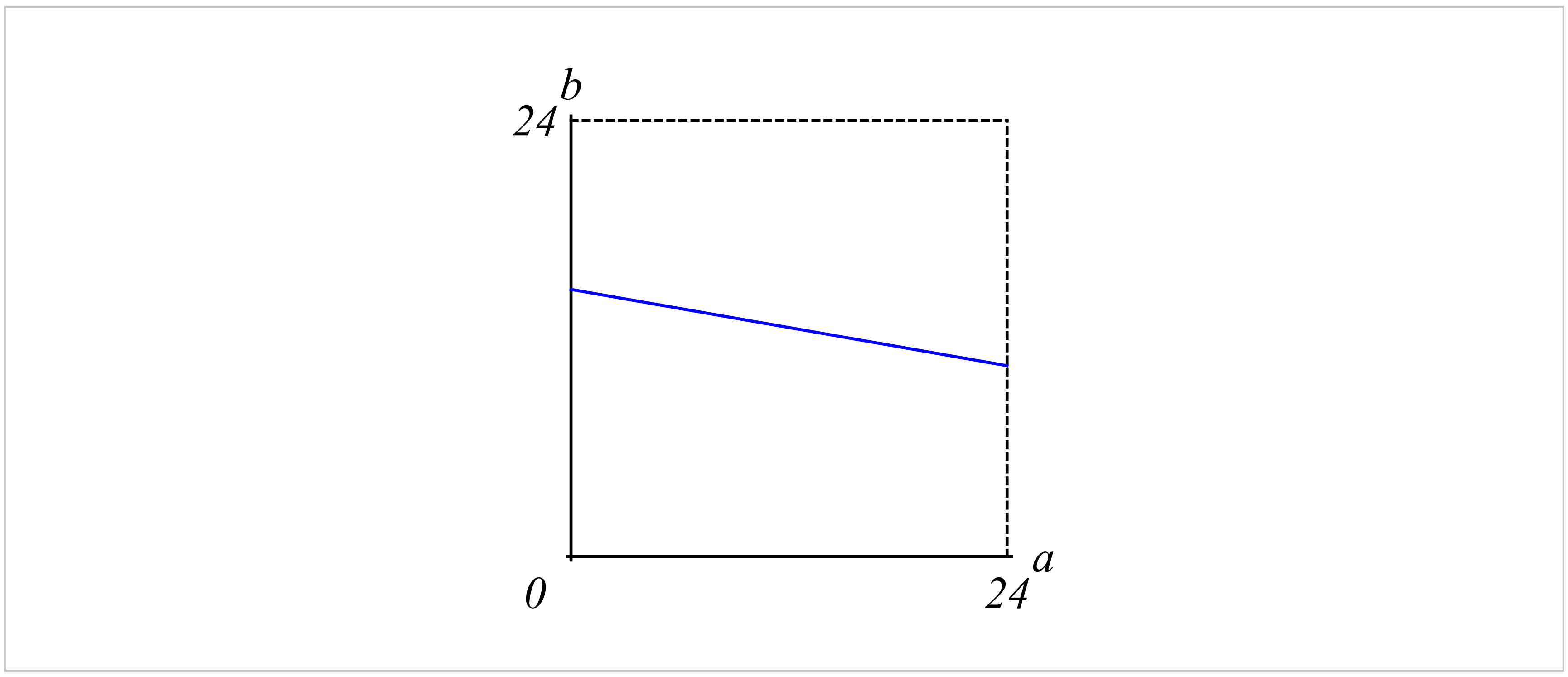
$$\nabla g(a, b) = \langle 35, 200 \rangle \quad (4.282)$$

$$80 = \lambda 35 \quad (4.283)$$

$$100 = \lambda 200 \quad (4.284)$$

There is no solution!

We need to check the boundary of our region of valid (a, b) points. What does the region we are maximizing look like? It is the region below the constraint $35a + 200b = 2940$ and within the square $0 \leq a, b \leq 24$.



We consider along the constraint equation, but also must check along the other boundaries of the region of valid (a, b) points. There are two points we need to consider. The point $a = 0$, $b = 2940/200 = 14.7$, and $a = 24$, and $b = (2940 - 35(24))/200 = 10.5$. The profit at each end point is below.

$$p(24, 10.5) = 2970 \text{ dollars per day} \quad (4.285)$$

$$p(0, 14.7) = 1470 \text{ dollars per day} \quad (4.286)$$

Therefore the maximum occurs when $a = 24$ and $b = 10.5$, with maximum profit 2970 dollars per day.

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You have used 1 of 7 attempts

Answers are displayed within the problem

Reflection

Observe that we added constraints into our problem in two different ways. The first way introduced a change in our profit. This changed the function we were maximizing. In the second case we added an energy constraint that was more naturally observed as changing the region of definition. The total energy used in the maximum case was the same in both instances.

Try to think about the following questions based on the work you did in this problem. You can also start a discussion in the forums!

Question 1 : How do you know how new constraints should be added into your model? (Hint: Think about units.)

Question 2 : Which do you think is more effective strategy for curbing energy usage? Adding monetary penalties for energy usage, or capping energy usage? (Hint: This may be a trick question.)

1. Economics and sustainability

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- Problem 1(d) is simpler than that 4
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  Find the maximum number of hours you should run each *part*. That should be "machine". 2
- Economics and sustainability:Reflection
  > Question 1 : How do you know how new constraints should be added into your model? (Hint: Think about units.) There are two con... 1
- ? [STAFF] Could not format HTML for problem.
  @ staff : the text of the problem could not be formatted. 8
- Real life answer to this problem :)
  Community TA 7
- [Staff] Typo in the solution to 1(d)
  I think there is a typo in the solution to 1(d), just before the graph ("a" is missing): "the region below the constraint 35[a] + " 2
- One approach to questions one and two 1
- stuck in problem (d) max profit
  Using the constraints I got the max number of hours (a and b), they are marked correct. Then, I use the objective function (the profit... 5
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