Logical Symbols

The arithmetical symbols we've discussed so far allow us to name numbers.

But we do not yet have the resources to express any *claims* about numbers: we are not yet able to express any thoughts capable of being true or false. One cannot, for example, express the thought that two times one equals one times two.

Once the identity symbol, "=", is in place, however, we can express the thought that two times one equals one times two as "$2 \times 1 = 1 \times 2$". We can also express false thoughts; for example, "$2 = 3$".

The logical operators "$\neg$" and "&" allow us to build complex claims out of simpler ones:

- "$\neg$" is read "It is not the case that", and can be used to formulate sentences like:

  $\neg(1 = 2)$
  (*Read:* "It is not the case that one equals two")

- "&" is read "and", and can be used to formulate sentences like:

  $(1 + 2 = 3)\ \&\ (2 + 1 = 3)$
  (*Read:* "One plus two equals three, and two plus one equals three").

None of the symbols we have discussed so far enables us to express generality. For example, we are in a position to express commutativity claims about particular numbers (e.g. "$2 \times 1 = 1 \times 2$"), but we are unable to state the the fact that multiplication is commutative *in general*.

We cannot say:

For *any* numbers n and m, the product of n and m equals the product of m and n.

To express general claims of this kind in L we need the quantifier-symbol "$\forall$" and the variables "x_0", "x_1", "x_2", ... The quantifier symbol "$\forall$" expresses universal generalization, and is read "every number is such that". Each variable "x_i" works like a name that is yet to be assigned a particular referent, and is read "it". When quantifiers and variables are combined, they allow us to say all sorts of interesting things.

For instance:

$\forall x_0\,(x_0 = x_0)$
(*Read:* "every number is such that it is identical to it")

It is important to have variables with different indices to avoid ambiguity. Consider, for example, the sentence:

$\forall x_1 \forall x_2\,(x_1 \times x_2 = x_2 \times x_1)$

If the different variables weren't indexed, we'd be forced to read this sentence as:

every number is such that every number is such that it times it equals it times it

which can be read in different ways, depending on what one takes the various "it"s to refer to. With indices in place, however, we avoid ambiguity by saying:

every number (call it x_1) is such that every number (call it x_2) is such that it (i.e. x_1) times it (i.e. x_2) equals it (i.e. x_2) times it (i.e. x_1)

or more succinctly:

every number x_1 and every number x_2 are such that x_1 times x_2 equals x_2 times x_1

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I'm going to write a list of symbols on the board.

And our language is going to be restricted

to strings of those symbols.

So anything that our language says

has to be built up from things on this list.

OK, here's my list.

So let me go through the list and tell you

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? Existential quantifier

In the list of symbols, shouldn't there be an existential quantifier (backwards capital "E") after the...

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