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Homework 3: Introduction to

Course > Unit 2 Foundation of Inference > Hypothesis Testing

> 8. A Union-Intersection Test

Currently enrolled in Audit Track (expires December 25, 2019) Upgrade (\$300)

8. A Union-Intersection Test

Let X_1,\ldots,X_n be i.i.d. Bernoulli random variables with unknown parameter $p\in(0,1)$. Suppose we want to test

$$H_0: p \in [0.48, 0.51] \quad ext{vs} \quad H_1: p
otin [0.48, 0.51]$$

We want to construct an asymptotic test ψ for these hypotheses using \overline{X}_n . For this problem, we specifically consider the family of tests ψ_{c_1,c_2} where we reject the null hypothesis if either $\overline{X}_n < c_1 \le 0.48$ or $\overline{X}_n > c_2 \ge 0.51$ for some c_1 and c_2 that may depend on n, i.e.

$$\psi_{c_1,c_2} = \mathbf{1}\left((\overline{X}_n < c_1) \, \cup \, (\overline{X}_n > c_2)
ight) \qquad ext{where $c_1 < 0.48 < 0.51 < c_2$.}$$

Throughout this problem, we will discuss possible choices for constants c_1 and c_2 , and their impact to both the asymptotic and non-asymptotic level of the test.

(a)

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1/ 1 point (graded)

Which expression represents the (smallest asymptotic) level α of this test? Recall the (smallest asymptotic) level equals the maximum Type 1 error rate.

$$left{f O} \; lpha \; = \max_{p \in [0.48, 0.51]} \left({f P}_p \left(\overline{X}_n < c_1
ight) + {f P}_p \left(\overline{X}_n > c_2
ight)
ight)$$

$$igotimes lpha = \max_{p \in [0.48, 0.51]} \left(\max \left(\mathbf{P}_p \left(\overline{X}_n < c_1
ight), \mathbf{P}_p \left(\overline{X}_n > c_2
ight)
ight)
ight)$$

$$igcap lpha = \max_{p \in [0.48, 0.51]} \mathbf{P}_p \left(\overline{X}_n < c_1
ight)$$

$$igcap lpha = \max_{p \in [0.48, 0.51]} \mathbf{P}_p\left(\overline{X}_n > c_2
ight)$$

$$igcup lpha = \max_{p \in [0.48, 0.51]} \left(\mathbf{P}_p \left(\overline{X}_n < c_1
ight) \cdot \mathbf{P}_p \left(\overline{X}_n > c_2
ight)
ight)$$

~

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You have used 1 of 2 attempts

(b)

4.0/4 points (graded)

Use the central limit theorem and the approximation $\sqrt{p(1-p)} \approx \frac{1}{2}$ for $p \in [0.48, 0.51]$ to approximate $\mathbf{P}_p(\overline{X}_n < c_1)$ and $\mathbf{P}_p(\overline{X}_n > c_2)$ for large n. Express your answers as a formula in terms of c_1, c_2, n and p.

(Write **Phi** for the cdf of a Normal distribution, **c_1** for c_1 , and **c_2** for c_2 .)

$$\mathbf{P}_n\left(\overline{X}_n < c_1
ight)pprox \qquad \qquad \mathsf{Phi}(2 ext{*sqrt(n)*(c_1-p))}$$
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For what value of $\,p \in [0.48, 0.51]\,$ is the expression above for $\,{f P}_p\left(\overline{X}_n < c_1
ight)\,$ maximized?

$$\mathbf{P}_p\left(\overline{X}_n < c_1
ight)$$
 is max at $p=igg($ 0.48

$$\mathbf{P}_p\left(\overline{X}_n>c_2
ight)pprox \hspace{0.5cm} ext{ 1-Phi(2*sqrt(n)*(c_2-p))} \hspace{0.5cm} ullet$$

For what value of $\,p\in[0.48,0.51]\,$ is the expression above for $\,{f P}_p\left(\overline{X}_n>c_2
ight)\,$ maximized?

$$\mathbf{P}_p\left(\overline{X}_n>c_2
ight)$$
 is max at $\,p=igg[$ 0.51

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You have used 1 of 4 attempts

(c)

1.0/1 point (graded)

Next, we combine the results from parts (a) and (b).

Apply the inequality $\max_{x} (f(x) + g(x)) \leq \max_{x} f(x) + \max_{x} g(x)$ to the expression for the (asymptotic) level α obtained in part (a) and use the results from part (b) to give an upper bound on α .

Express your answer as a formula in terms of c_1 , c_2 , and n. (Write **Phi** for the cdf of a Normal distribution, **c** 1 for c_1 , and **c** 2 for c_2 .)

$$\alpha \le 1 + Phi(2*sqrt(n)*(c_1-0.48)) - Phi(2*sqrt(n)*(c_2-0.51))$$

(Food for thought: Is this upper bound tight? A bound is tight if equality may be achieved.)

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(d)

2.0/2 points (graded)

Suppose that we wish to have a level $\alpha=0.05$. What c_1 and c_2 will achieve $\alpha=0.05$? Choose c_1 and c_2 by setting the expressions you obtained above for $\max_{p\in[0.48,0.51]}\mathbf{P}_p\left(\overline{X}_n< c_1\right)$ and $\max_{p\in[0.48,0.51]}\mathbf{P}_p\left(\overline{X}_n> c_2\right)$ to both be 0.025.

(If applicable, enter **q(alpha)** for q_{α} , the $1-\alpha$ -quantile of a standard normal distribution, e.g. enter **q(0.01)** for $q_{0.01}$.)

$$c_1 = \boxed{ (q(0.975))/(2*sqrt(n))+0.4 }$$

$$c_2 = \left| \begin{array}{c} (\mathsf{q}(0.025))/(2 \text{*sqrt(n)}) + 0.5 \end{array} \right|$$

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You have used 2 of 3 attempts

(e)

2/2 points (graded)

We will now show that the values we just derived for c_1 and c_2 are in fact too conservative.

Recall the expression from part (b) for $\mathbf{P}_p\left(\overline{X}_n < c_1\right)$ for large n. For p>0.48 (note the strict inequality), find $\lim_{n \to \infty} \mathbf{P}_p\left(\overline{X}_n < c_1\right)$.

$$\lim_{n o\infty}\mathbf{P}_{p>0.48}\left(\overline{X}_n< c_1
ight)=egin{array}{c} \mathtt{0} \end{array}$$

Similarly, for p<0.51 (note the strict inequality), find $\lim_{n\to\infty}\mathbf{P}_p\left(\overline{X}_n>c_2\right)$. Use the expression you found in part (b) for $\mathbf{P}_p\left(\overline{X}_n>c_2\right)$.

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$$\lim_{n o\infty}\mathbf{P}_{p<0.51}\left(\overline{X}_n>c_2
ight)=igg[$$
 0

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You have used 2 of 3 attempts

(f)

2/2 points (graded)

Note: This part of the problem will contain multiple steps but you would only enter answers to the final step. Also **refer to the recitation 4 last video** *Composite Test* for related ideas.

Next, we analyze the asymptotic test given different possible values of p, in order to choose suitable and sufficiently-tight c_1 and c_2 . Looking more closely at part (d), we may note that the asymptotic behavior of the expressions for the errors are different depending on whether p=0.48, 0.48 , or <math>p=0.51.

Based on your answers and work from the previous part, evaluate the asymptotic Type 1 error

$$\mathbf{P}\left(\overline{X}_{n} < c_{1}
ight) + \mathbf{P}\left(\overline{X}_{n} > c_{2}
ight).$$

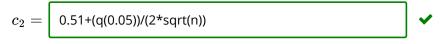
on each of the three cases for the value of p in terms of c_1 , c_2 , and n, and determine in each case which component(s) of the Type 1 error will converge to zero.

This would allow you to come up with a new set of conditions for c_1 and c_2 in terms of n, given the desired level of 5%. Enter these values (in terms of n) below.

(If applicable, enter **q(alpha)** for q_{α} , the $1-\alpha$ -quantile of a standard normal distribution, e.g. enter **q(0.01)** for $q_{0.01}$. Do not worry about the parser not rendering **q(alpha)** properly; the grader will work nonetheless. You could also enclose **q(alpha)** by brackets for the rendering to show properly.)

$$c_1 = \boxed{ 0.48-(q(0.05))/(2*sqrt(n)) }$$

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STANDARD NOTATION

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You have used 3 of 3 attempts

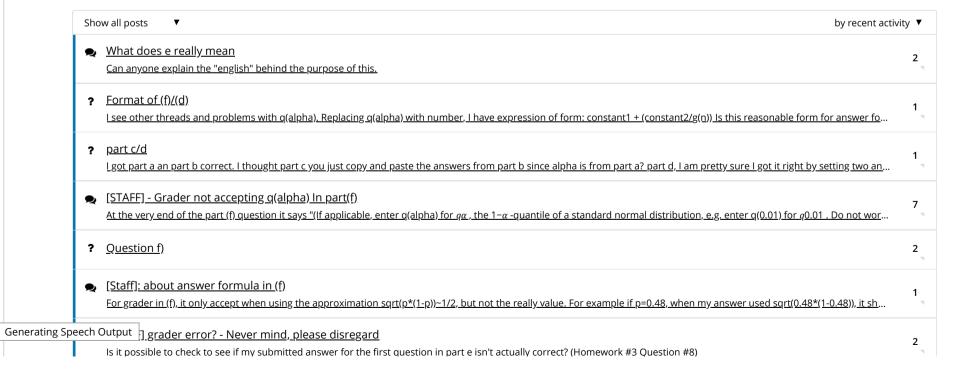
✓ Correct (2/2 points)

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✓ Part b, q(alpha)

Lbelieve we will need to put the expression of the CDF evaluated at C1, C2 but the equivalent normalized values. When I'm trying to normalize c2, I found that within the Phi st...

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3

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