MITx: 6.041x Introduction to Probability - The Science of Uncertainty

Problem 1: Convergence in probability

(6/6 points)

For each of the following sequences, determine the value to which it converges in probability.

(a) Let $X_1, X_2, \ldots$ be independent continuous random variables, each uniformly distributed between -1 and 1.

1. Let $U_i = \dfrac{X_1 + X_2 + \cdots + X_i}{i}, \quad i = 1, 2, \ldots$.
   What value does the sequence U_i converge to in probability?

   0

2. Let $W_i = \max(X_1, X_2, \ldots, X_i), \quad i = 1, 2, \ldots$.
   What value does the sequence W_i converge to in probability?

1

3. Let $V_i = X_1 \cdot X_2 \cdot \cdots \cdot X_i, \quad i = 1, 2, \ldots$.
What value does the sequence V_i converge to in probability?

0

(b) Let $X_1, X_2, \ldots$ be independent identically distributed random variables with $\mathbf{E}[X_i] = 2$ and $\mathbf{var}(X_i) = 9$, and let $Y_i = X_i/2^i$.

1. What value does the sequence Y_i converge to in probability?

0

2. Let $A_n = \dfrac{1}{n}\sum_{i=1}^{n} Y_i$. What value does the sequence A_n converge to in probability?

0

3.

Solved problems

Additional theoretical material

Problem Set 8
Problem Set 8 due Apr 27, 2016 at 23:59 UTC

Unit summary

Unit 9: Bernoulli and Poisson processes

Let $Z_i = \frac{1}{3}X_i + \frac{2}{3}X_{i+1}$ for $i = 1, 2, \ldots$, and let $M_n = \frac{1}{n}\sum_{i=1}^{n} Z_i$ for $n = 1, 2, \ldots$.

What value does the sequence M_n converge to in probability?

2 ✔

You have used 1 of 2 submissions

Printable problem set available here .

DISCUSSION

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