4. Review constrained optimization

Review

Poll solution, and constrained optimization example

want to spend on machines.

And let's say that the productivity

is M times W squared.

I don't claim that that's actually very realistic,

but just to give a sense of a real world-ish problem.

OK, and of course our goal is to figure out

how to allocate our budget so that we have the most

productivity, so we want to maximize this.

So let me make a little picture.

So let's say this is M and this is W, and maybe over here

is 100, and over here is 100.

There a total budget of 100, so our available options

look like that, it's supposed to be a straight line

6:26 / 6:26 2.0x

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If C is a curve we can describe as the level curve of a function $g(x, y)$, and $f(x, y)$ is another function that we want to maximize along C, the the maximum occurs where the gradient of f is a multiple of the gradient of g:

$$\nabla f = \lambda \nabla g.$$

Something that can be confusing is understanding which function plays what role. Let's look at an example.

Economics problem

0 points possible (ungraded)
Suppose we have a factory.

$$W = \text{budget for workers} \quad (7.52)$$
$$M = \text{budget for machines} \quad (7.53)$$

We have a total budget of 100. And both $M > 0$ and $W > 0$.

Our goal is to maximize the **productivity** of our factory, which is measured as MW^2.

In our framework, is MW^2 the function f or the function g?

- (selected) MW^2 is the function f
- MW^2 is the function g

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MW is the function g

Solution:

The productivity is the function that we want to maximize, thus it plays the role of the function f in our Lagrange multiplier method.

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Answers are displayed within the problem

Question solution

And we're trying to maximize it.

And we're only allowed to choose points on this curve.

Follow-up question, what should be g in this problem?

Is it M times W, or M plus W, or just M?

Thumbs up for M times W, M plus W, that's right.

So what is our constraint?

Our constraint is that our total budget is M plus W.

So this is our curve, and it's a level curve at M plus W, cool.

So hopefully, thinking about a situation like that

will help think of the role of f, and the role of g,

and not get confused between them.

1:24 / 1:24 2.0x

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Find the solution

0 points possible (ungraded)
Our constraint is $M + W = 100$. So the function $g(x, y) = M + W$.

Find the budget for workers and machines that optimizes the productivity.

$M =$ 100/3 **Answer:** 33.33333

$W =$ 200/3 **Answer:** 66.6666

Solution:

So let's solve this problem. At the optimal point (M_0, W_0), we will have

$$\nabla f = \lambda \nabla g \tag{7.54}$$

$$f_M = \lambda g_M \tag{7.55}$$

$$f_W = \lambda g_W$$

Calculator Hide Notes

We get the following system:

$$W_0^2 = \lambda \quad (7.57)$$
$$2M_0W_0 = \lambda \quad (7.58)$$

Thus we get $W_0^2 = 2M_0W_0$. This tells us that $W_0 = 0$ or $W_0 = 2M_0$. Let's compare the value of the productivity function in each case.

- If $W_0 = 0$, then $M_0 = 100$, and $W_0M_0^2 = 0$.
- If $W_0 = 2M_0$, then $W_0 + M_0 = 3M_0 = 100$, so $M_0 = 331/3$ and $W_0 = 662/3$. Thus the productivity is $M_0W_0^2 = (33\,1/3)\,(67\,2/3)^2 > 0$, thus this is the maximum.
- We should also check the boundary $M = 0$ and $W = 0$, but the productivity is equal to zero there, thus the productivity will not achieve the maximum along the boundary.

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Answers are displayed within the problem

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? [Staff] Typo in solution to finding W and M
Solution states that M=331/3 and M=662/3, but it is wrong
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