Problem 6: Random incidence under Erlang interarrivals

(2 points possible)

A single dot is placed on a very long length of yarn at the textile mill. The yarn is then cut into pieces. The lengths of the different pieces are independent, and the length of each piece is distributed according to the same PDF $f_X(x)$. Let R be the length of the piece that includes the dot. Determine the expected value of R in each of the following cases.

In each part below, express your answer in terms of λ using standard notation . Enter 'lambda' for λ.

1. Suppose that $f_X(x) = \begin{cases} \lambda e^{-\lambda x}, & x \geq 0, \\ 0, & x < 0. \end{cases}$

$\mathbf{E}[R] =$ 1/lambda ✖ **Answer:** 2/lambda

2. Suppose that $f_X(x) = \begin{cases} \frac{\lambda^3 x^2 e^{-\lambda x}}{2}, & x \geq 0, \\ 0, & x < 0. \end{cases}$

$\mathbf{E}[R] =$ 3/lambda ✖ **Answer:** 4/lambda

Answer:

1. Here, the lengths of the pieces of yarn are independent and exponentially distributed with parameter λ. As explained on pages 322-324 of the text, due to the memorylessness of the exponential, the distribution of the length of the piece of yarn containing the dot is a second order Erlang. Thus, $\mathbf{E}[R] = 2\mathbf{E}[X] = 2/\lambda$.

2. Here, X is an Erlang of order 3. Think of sections on the yarn, each exponentially distributed with parameter λ. We can then interpret each piece of yarn as *three* consecutive sections of exponentially distributed lengths. The piece of yarn with the dot will have the dot in one of these three sections. By the standard random incidence analysis, the expected length of that section will be $2/\lambda$. However, the piece of yarn containing the dot also consists of two other sections, each with an expected length of $1/\lambda$. Thus, the total expected length of the piece of yarn containing the dot is $4/\lambda$.

You have used 2 of 2 submissions

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