Observation Theory

Script V41 B – Estimation vs. Estimator

Welcome back!

Now that we learned about the concept of "Estimators"…

Let's look at some of their properties and see, in particular, what are the properties of the weighted least squares estimator.

To start, I want to point out the difference between two different kinds of estimators: linear vs. Nonlinear estimators.

We saw that, in the general case, an estimator is the function of the random variable y.

If this function is linear, then we can write it as a multiplication of a matrix, let's say matrix L, to the observable vector y.

In this case the estimator is a linear estimator.

For example, let's look at the weighted least squares estimator.

In this case, the weighted least squares estimator xhat is the multiplication of this matrix, I denote it by L, to the vector y.

So weighted least squairs is a LINEAR estimator.

Now, what can we say about the statistical properties of the linear estimators?

In our mathematical model with the functional model as the expectation of y equals Ax , and the stochastic model as the dispersion of y equals Qyy… if we know that observables are normally distributed, what can we say about the distribution of the least squares estimator xhat.

From statistics, we know an important property of linear function….

Which is a linear function of normally distributed random variables is again normally distributed.

Therefore, as weighted least squares is a linear estimator, we can say that if observables are normally distributed, the least squares xhat should be also normally distributed.

OK.

Now, what can we say about other statistical properties of xhat.

For example what is the mean or expectation of xhat.

If xhat is a linear estimator, for example as xhat equals Ly, then its expectation can be written as L multiply by expectation of y.

In the case of weighted least squares for example, the expectation of the estimator can be written as the multiplication of this matrix to the expectation of y.

From the functional model, we have the expectation of y equals Ax.

Inserting this equation here , we can see that these two terms cancels out each other and finally the expectation of xhat is equal to x or equal to the "true value" the unknown true value.

Although the true value of x always remains unknown, however this expectation-equals-x is an important conclusion.

This means on average, if have large realisations of xhat, we expect the mean value of the realizations to converge to the true value of x.

We can also reformulate it, and say that the expectation of xhat minus x is zero.

It means that the mean of the estimation error is zero, which indeed is a desirable property.

This property of expectation of xhat equals the true value… is called “unbiased property”.

That is our estimator is unbiased, and statistically its mean should be equal to the true value.

Well….

So far so good.

In this lecture, we learned that in our mathematical model with normally distributed observables, the weighted least squares estimator is a linear unbiased estimator, which itself has a normal distribution with the mean equals to the true value.

We can also ask now, what would be the dispersion of our estimator?

Or the covariance matrix of xhat?

Note that the Qxaht describes the precision of the estimator.

The detail derivation and interpretation of Qxhat and the quality of the estimators is the topic of the next week.

But here I just want to conclude this lecture by saying that, in general, if estimators are random variable, we are ideally interested in an estimator with, possibly, the lowest dispersion in Qxhat, or in other words, in the most precise estimator.

Let's see, in the next section of this week, how we can approach such an estimator.