

Unit 4 Unsupervised Learning (2

4. Computation Complexity of K-

Course > weeks)

> <u>Lecture 14. Clustering 2</u> > Means and K-Medoids

4. Computation Complexity of K-Means and K-Medoids Computation Complexity of K-Means and K-Medoids

the new representative.

And what we know, this is the place where all the action will happen, because in K-means in this step, we were violating both assumptions.

First of all, we were selecting the point

which may not necessarily be the member of the original set.

And the second point is the reason our representative look

like a centroid was because we did this derivation

and we directly built on the assumption that our distance metric happened to be square Euclidean distance.



So here we would expect that the change will happen.

And let's just intuitively see what we would

like to get in this step 2b.

Let's say your cluster looks like something like this.



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Computation Complexity of K-Means

1/1 point (graded)

Remember that the K-Means algorithm is given by

- 1. Randomly select z_1, \ldots, z_k
- 2. Iterate
 - 1. Given $z_1, \ldots z_k$, assign each $x^{(i)}$ to the closest z_i . i.e., assign each $x^{(i)}$ such that

$$\operatorname{Cost}\left(z_{1}, \ldots z_{k}
ight) = \sum_{j=1}^{n} \min_{j=1, ..., k} \left\|x^{(i)} - z_{j}
ight\|^{2}$$

2. Given C_1, \ldots, C_k find the best representatives z_1, \ldots, z_k such that

$$z_j = rac{\sum_{i \in C_j} x^{(i)}}{|C_j|}$$

Assuming that there are n data points $\{x_1, \ldots, x_n\}$, k clusters and representatives, and each $x_i \in \{x_1, \ldots, x_n\}$ is a vector of dimension d, what is the computational complexity for one complete iteration of the k-means algorithm?

- \circ $\mathcal{O}(n)$
- $\mathcal{O}(nk)$

- \circ $\mathcal{O}\left(nk^2
 ight)$

Solution:

In line 2.1, we go through each of the n x_i , and iterate through each of the k z_j 's for each x_i (to find the closest z_j). This iteration is $\mathcal{O}(nk)$. And because each x_i has length d, the total iteration is $\mathcal{O}(ndk)$.

Line 2.2 is similar.

Note that because 2.1 and 2.2 both take $\mathcal{O}(ndk)$, one complete iteration takes $\mathcal{O}(ndk)$.

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You have used 1 of 2 attempts

1 Answers are displayed within the problem

Computation Complexity of K-Medoids

2/2 points (graded)

Remember that the K-Medoids algorithm is given by

1. Randomly select z_1,\ldots,z_k

- 2. Iterate
 - 1. Given $z_1, \ldots z_k$, assign each $x^{(i)}$ to the closest z_j . i.e., assign each $x^{(i)}$ such that

$$\operatorname{Cost}\left(z_{1}, \ldots z_{k}
ight) = \sum_{i=1}^{n} \min_{j=1, \ldots, k} \left\|x^{(i)} - z_{j}
ight\|^{2}$$

2. Given $C_j \in \left\{C_1,\ldots,C_k
ight\}$ find the best representative $z_j \in \left\{x_1,\ldots,x_n
ight\}$ such that

$$\sum_{x^{(i)} \in C_j} \mathrm{dist}\,(x^{(i)},z_j)$$

is minimal.

What is the complexity of step 2a?

- $\mathcal{O}(n)$
- \circ $\mathcal{O}(nk)$
- $^{\circ}~\mathcal{O}\left(nk^{2}
 ight)$

O (ndk) ✓

Now what is the complexity of step 2b?

- \circ $\mathcal{O}\left(ndk
 ight)$
- $\mathcal{O}\left(nk^2\right)$
- $\bigcirc \mathcal{O}(nk^2d)$

Solution:

Note that step 2.1 of the K-Medoids is the same as that of K-Means, so the time complexity is $\mathcal{O}(ndk)$. Note that step 2.2 of K-Medoids has an additional loop of iterating through the n points $z_j \in \{x_1, \dots, x_n\}$ which takes $\mathcal{O}(n)$. Thus step 2.2 takes $\mathcal{O}(n^2dk)$.

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You have used 1 of 3 attempts

