Name: ____________________

LAFF Spring 15
Sample Exam 2

1. Compute the following:

(a) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & -2 & 0 \\ 0 & 1 & 0 \\ 0 & 4 & 1 \end{pmatrix} \begin{pmatrix} 1 & -2 & 0 \\ 0 & 2 & 0 \\ 0 & 4 & 1 \end{pmatrix}$

Answer: (The diagonal matrix scales only the second row, by 2.)

(b) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix}^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

Answer: (Inverting a diagonal matrix means inverting the entries on the diagonal.)

(c) $\begin{pmatrix} 1 & -2 & 0 \\ 0 & 1 & 0 \\ 0 & 4 & 1 \end{pmatrix}^{-1} = \begin{pmatrix} 1 & +2 & 0 \\ 0 & 1 & 0 \\ 0 & -4 & 1 \end{pmatrix}$

Answer: (Inverting a Gauss transform is a matter of negating the off-diagonal elements.)

(d) $\begin{pmatrix} 1 & -2 & 0 \\ 0 & 2 & 0 \\ 0 & 4 & 1 \end{pmatrix}^{-1} =$

Answer: Here there is a trick. Use the other answers and the fact that $(AB)^{-1} = B^{-1}A^{-1}$.

$$
\begin{aligned}
\begin{pmatrix} 1 & -2 & 0 \\ 0 & 2 & 0 \\ 0 & 4 & 1 \end{pmatrix}^{-1} &= \left[\begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & -2 & 0 \\ 0 & 1 & 0 \\ 0 & 4 & 1 \end{pmatrix}\right]^{-1} \\
&= \begin{pmatrix} 1 & -2 & 0 \\ 0 & 1 & 0 \\ 0 & 4 & 1 \end{pmatrix}^{-1}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix}^{-1} \\
&= \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 0 & -4 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1 \end{pmatrix} \\
&= \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1/2 & 0 \\ 0 & -2 & 1 \end{pmatrix}
\end{aligned}
$$

Alternatively, you can work with the appended system to compute the inverse:

$$
\begin{aligned}
\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\left(\begin{array}{ccc|ccc} 1 & -2 & 0 & 1 & 0 & 0 \\ 0 & 2 & 0 & 0 & 1 & 0 \\ 0 & 4 & 1 & 0 & 0 & 1 \end{array}\right) &\longrightarrow \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{pmatrix}\left(\begin{array}{ccc|ccc} 1 & -2 & 0 & 1 & 0 & 0 \\ 0 & 2 & 0 & 0 & 1 & 0 \\ 0 & 4 & 1 & 0 & 0 & 1 \end{array}\right) \\
&\longrightarrow \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1 \end{pmatrix}\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 2 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & -2 & 1 \end{array}\right) \\
&\longrightarrow \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 2 & 0 & 0 & 1/2 & 0 \\ 0 & 0 & 1 & 0 & -2 & 1 \end{array}\right)
\end{aligned}
$$

so that the inverse is

$$\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1/2 & 0 \\ 0 & -2 & 1 \end{pmatrix}.$$