9. Example Polar Coordinates

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Lecture due Oct 5, 2021 20:30 IST

Synthesize

Chain Rule Polar Coordinates

depends on x and y, then, in fact,

you can plug these and get the function of r and theta.

And then you can ask yourself, well,

what is partial f over partial r?

And that's going to be-- well, you

want to take partial f, partial x, partial x,

partial r plus partial f, partial y, partial y,

partial r.

And so that will end up being, actually,

f sub x times cosine theta plus f sub y times sine theta.

And you can do the same thing to find partial f, partial theta.

So you can express derivatives either in terms of x and y

or in terms of r and theta with simple relations between them.

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A useful application of the chain rule would be when we need to switch between rectangular and polar coordinates.

Suppose a quantity f varies in the plane with x and y. Perhaps we already know $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$, but what we really want to know are the partial derivatives of f with respect to the polar coordinates r and θ. The chain rule gives us a way to find these partial derivatives without writing f explicitly in terms of r and θ.

In particular, the chain rule tells us that

$$\frac{\partial f}{\partial r} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial r} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial r}. \qquad (6.179)$$

This equation follows from the "more variables" version of the chain rule.

Next, we have

$$x = r\cos\theta \qquad (6.180)$$

$$y = r\sin\theta \qquad (6.181)$$

Therefore we have

$$\frac{\partial x}{\partial r} = \cos\theta \qquad (6.182)$$

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$$\frac{\partial y}{\partial r} = \sin\theta \tag{6.183}$$

So if we know $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ we can write $\frac{\partial f}{\partial r}$ as:

$$\frac{\partial f}{\partial r} = \underbrace{\frac{\partial f}{\partial x}}_{f_x}\cos\theta + \underbrace{\frac{\partial f}{\partial y}}_{f_y}\sin\theta \tag{6.184}$$

In a similar way it is possible to write $\frac{\partial f}{\partial \theta}$ in terms of $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$.

Example 9.1 Let $f(x, y) = xy$. Suppose we would like to know $\frac{\partial f}{\partial r}$, that is, the derivative of f with respect to r. It is straightforward to compute the partial derivatives with respect to x and y:

$$\frac{\partial f}{\partial x} = y \tag{6.185}$$

$$\frac{\partial f}{\partial y} = x \tag{6.186}$$

Then we will use the formula

$$\frac{\partial f}{\partial r} = \underbrace{\frac{\partial f}{\partial x}}_{f_x}\cos\theta + \underbrace{\frac{\partial f}{\partial y}}_{f_y}\sin\theta \tag{6.187}$$

All that remains is to rewrite $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ in terms of r and θ:

$$\frac{\partial f}{\partial x} = r\sin\theta \tag{6.188}$$

$$\frac{\partial f}{\partial y} = r\cos\theta \tag{6.189}$$

Then we have

$$\frac{\partial f}{\partial r} = r\sin\theta\cos\theta + r\cos\theta\sin\theta \tag{6.190}$$

This answer can be further simplified to

$$\frac{\partial f}{\partial r} = r\sin(2\theta) \tag{6.191}$$

This simplified expression can lend us some insight about the function $f(x, y)$. For example, notice that $\sin(2\theta)$ is positive for $0 < \theta < \pi/2$ and negative for $\pi/2 < \theta < \pi$. It follows that if r is increased within quadrant 1 then f will increase. But if r is increased within quadrant 2, then f will decrease.

We can also see from $\frac{\partial f}{\partial r} = r\sin(2\theta)$ that, as r increases, the quantity f will increase most rapidly when $\theta = \pi/4$. In other words, moving out from the origin, f increases most rapidly along the line $y = x$.

Polar coordinates chain rule

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Suppose a quantity f varies in the plane with x and y. Which of the following is the correct expression for $\frac{\partial f}{\partial \theta}$ in terms of $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$?

- ○ $\frac{\partial f}{\partial \theta} = \frac{\partial f}{\partial x}\tan\theta + \frac{\partial f}{\partial y}\tan\theta$
- ○ $\frac{\partial f}{\partial \theta} = \frac{\partial f}{\partial x} r\cos\theta + \frac{\partial f}{\partial y} r\sin\theta$
- ● $\frac{\partial f}{\partial \theta} = -\frac{\partial f}{\partial x} r\sin\theta + \frac{\partial f}{\partial y} r\cos\theta$
- ○ $\frac{\partial f}{\partial \theta} = \frac{\partial f}{\partial x}\cos\theta + \frac{\partial f}{\partial y}\sin\theta$

✔

Solution:

By the chain rule, we have

$$\frac{\partial f}{\partial \theta} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial \theta} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial \theta} \tag{6.192}$$

From

$$x = r\cos\theta \tag{6.193}$$

$$y = r\sin\theta \tag{6.194}$$

we can find

$$\frac{\partial x}{\partial \theta} = -r\sin\theta \tag{6.195}$$

$$\frac{\partial y}{\partial \theta} = r\cos\theta \tag{6.196}$$

By substitution, we obtain the answer:

$$\frac{\partial f}{\partial \theta} = -\frac{\partial f}{\partial x} r\sin\theta + \frac{\partial f}{\partial y} r\cos\theta \tag{6.197}$$

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Answers are displayed within the problem

Polar coordinates chain rule 2

1/1 point (graded)

Let $f(x, y) = xy$. Using the answer to the previous problem, what is $\frac{\partial f}{\partial \theta}$?

Answer in terms of r and θ. Type `theta` for θ.

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$\frac{\partial f}{\partial \theta} =$ r^2*cos(2*theta) ✔ **Answer:** r^2*cos(2*theta)

? INPUT HELP

Solution:

The partial derivatives of f are

$$\frac{\partial f}{\partial x} = y \tag{6.198}$$

$$\frac{\partial f}{\partial y} = x \tag{6.199}$$

Writing these in terms of r and θ we have

$$\frac{\partial f}{\partial x} = r\sin\theta \tag{6.200}$$

$$\frac{\partial f}{\partial y} = r\cos\theta \tag{6.201}$$

From the previous problem, we have

$$\frac{\partial f}{\partial \theta} = -\frac{\partial f}{\partial x} r\sin\theta + \frac{\partial f}{\partial y} r\cos\theta \tag{6.202}$$

Therefore, by substitution we get the answer:

$$\frac{\partial f}{\partial \theta} = -r^2\sin^2\theta + r^2\cos^2\theta \tag{6.203}$$

This answer can be further simplified to

$$\frac{\partial f}{\partial \theta} = r^2\cos(2\theta)\,. \tag{6.204}$$

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