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sandipan_dey >

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★ Course / Week 7: More Gaussian Elimination and ... / 7.2 When Gaussian Elimination ...

(1)

Next >

7.2.3 Permutations

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< Previous

Week 7 due Nov 20, 2023 01:42 IST Completed

7.2.3 Permutations





Start of transcript. Skip to the end.

Dr. Robert van de Geijn: As we work towards a solution for our problem, namely the problem that sometimes the equations need to be swapped, we're going to take a quick side tour into permutation matrices,

and then later we'll see how those fit into the picture.

Why don't you take a mamont and do

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Reading Assignments

0 points possible (ungraded)

Read Unit 7.2.3 of the notes. [LINK]

You REALLY need to read the text that goes with this unit!



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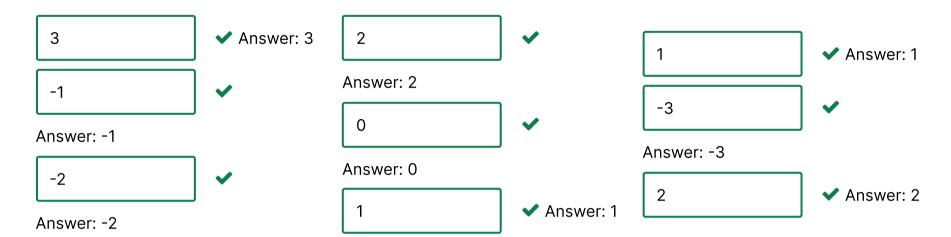
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Homework 7.2.3.1

9/9 points (graded) Compute

$$egin{pmatrix} 0 & 1 & 0 \ 0 & 0 & 1 \ 1 & 0 & 0 \end{pmatrix} egin{pmatrix} -2 & 1 & 2 \ 3 & 2 & 1 \ -1 & 0 & -3 \end{pmatrix} =$$



Answer:

$$\begin{pmatrix}
0 & 1 & 0 \\
\hline
0 & 0 & 1 \\
\hline
1 & 0 & 0
\end{pmatrix}
\begin{pmatrix}
-2 & 1 & 2 \\
\hline
3 & 2 & 1 \\
\hline
-1 & 0 & -3
\end{pmatrix} = \begin{pmatrix}
0 \times (-2 & 1 & 2) + 1 \times (3 & 2 & 1) + 0 \times (-1 & 0 & -3) \\
\hline
0 \times (-2 & 1 & 2) + 0 \times (3 & 2 & 1) + 1 \times (-1 & 0 & -3) \\
\hline
1 \times (-2 & 1 & 2) + 0 \times (3 & 2 & 1) + 0 \times (-1 & 0 & -3)
\end{pmatrix}$$

$$= \begin{pmatrix}
3 & 2 & 1 \\
\hline
-1 & 0 & -3 \\
\hline
-2 & 1 & 2
\end{pmatrix}.$$

Notice that multiplying the matrix by P from the left permuted the order of the rows in the matrix. Here is another way of looking at the same thing:

$$\left(\frac{e_1^T}{\frac{e_2^T}{e_0^T}}\right) \begin{pmatrix} -2 & 1 & 2 \\ 3 & 2 & 1 \\ -1 & 0 & -3 \end{pmatrix} = \begin{pmatrix} e_1^T \begin{pmatrix} -2 & 1 & 2 \\ 3 & 2 & 1 \\ -1 & 0 & -3 \end{pmatrix} \\ \hline e_2^T \begin{pmatrix} -2 & 1 & 2 \\ 3 & 2 & 1 \\ -1 & 0 & -3 \end{pmatrix} \\ \hline e_0^T \begin{pmatrix} -2 & 1 & 2 \\ 3 & 2 & 1 \\ -1 & 0 & -3 \end{pmatrix} \\ \hline e_0^T \begin{pmatrix} -2 & 1 & 2 \\ 3 & 2 & 1 \\ -1 & 0 & -3 \end{pmatrix} \end{pmatrix} = \begin{pmatrix} \frac{3 & 2 & 1}{-1 & 0 & -3} \\ \hline -2 & 1 & 2 \\ \hline -2 & 1 & 2 \end{pmatrix}.$$

Here we use the fact that $e_i^T A$ equals the *i*th row of A.

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Dr. Robert van de Geijn: OK, so we're back,

and hopefully you went ahead and did that problem.

So let's walk through the solution.

The way I would do this, is I would use slicing and dicing.

And I would take the matrix A, and I

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Homework 7.2.3.2

3/3 points (graded)

Example: If

$$p = egin{pmatrix} 0 \ 1 \ 2 \ 3 \end{pmatrix} ext{ then } P\left(p
ight) = egin{pmatrix} 1 & 0 & 0 & 0 \ 0 & 1 & 0 & 0 \ 0 & 0 & 1 & 0 \ 0 & 0 & 0 & 1 \end{pmatrix}$$

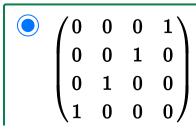
(Throughout, P(p) is the permutation matrix that orders the components of the vector to which it is applied according to the permutation vector p.)

lf

$$p=egin{pmatrix} 3\ 2\ 1\ 0 \end{pmatrix} ext{ then } P\left(p
ight)=$$

$$\begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{pmatrix}$$

$$\begin{pmatrix}
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
1 & 0 & 0 & 0
\end{pmatrix}$$



$\begin{pmatrix} 0 \\ 1 \end{pmatrix}$	1	0	0 \							
1	0	0	0							
$\begin{bmatrix} 0 \\ 0 \end{bmatrix}$	0	1	0							
0 /	0	0	1/							

~

lf

$$p=egin{pmatrix} 1\ 0\ 2\ 3 \end{pmatrix} ext{ then } P\left(p
ight)=$$

$$\begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{pmatrix}$$

$$\begin{pmatrix}
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
1 & 0 & 0 & 0
\end{pmatrix}$$

$$\begin{pmatrix}
0 & 0 & 0 & 1 \\
0 & 0 & 1 & 0 \\
0 & 1 & 0 & 0 \\
1 & 0 & 0 & 0
\end{pmatrix}$$

$$\begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

~

lf

$$p=egin{pmatrix}1\2\3\0\end{pmatrix} ext{ then }P\left(p
ight) =% egin{pmatrix}1\2\3\0\end{pmatrix}$$

$$\begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0; \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{pmatrix}$$

$$\begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

 $\bigcirc \left(\begin{smallmatrix} 0 & 0 & 0 & 1 \\ 2 & 2 & 1 & 2 \end{smallmatrix} \right)$

$$\left(\begin{array}{ccccc} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{array}\right)$$

$$\begin{pmatrix}
0 & 1 & 0 & 0 \\
1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{pmatrix}$$

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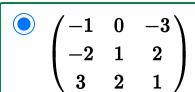
Homework 7.2.3.3

2/2 points (graded)

Let
$$p = egin{pmatrix} 2 \ 0 \ 1 \end{pmatrix}$$
 . Compute $P\left(p
ight) egin{pmatrix} -2 \ 3 \ -1 \end{pmatrix} =$

Let
$$p=egin{pmatrix}2\\0\\1\end{pmatrix}$$
 . Compute $P\left(p
ight)egin{pmatrix}-2&1&2\\3&2&1\\-1&0&-3\end{pmatrix}=$

- $\begin{pmatrix}
 2 & -2 & 1 \\
 1 & 3 & 2 \\
 -3 & -1 & 0
 \end{pmatrix}$



$$egin{pmatrix} -1 & 0 & 2 \ 1 & 1 & 2 \ -1 & 2 & 3 \end{pmatrix}$$



Answer:

$$P(p)\begin{pmatrix} -2\\3\\-1 \end{pmatrix} = \begin{pmatrix} -1\\-2\\3 \end{pmatrix} \quad \text{and} \quad P(p)\begin{pmatrix} -2 & 1 & 2\\3 & 2 & 1\\-1 & 0 & -3 \end{pmatrix} = \begin{pmatrix} -1 & 0 & -3\\-2 & 1 & 2\\3 & 2 & 1 \end{pmatrix}.$$

Hint: it is not necessary to write out P(p): the vector p indicates the order in which the elements and rows need to appear.

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Homework 7.2.3.4

1/1 point (graded)

Let
$$p=egin{pmatrix}2\\0\\1\end{pmatrix}$$
 be a permutation vector and $P=P\left(p
ight)$ be a permutation matrix. Compute $egin{pmatrix}-2&1&2\\3&2&1\\-1&0&-3\end{pmatrix}P^T=$

$$egin{pmatrix} igoldsymbol{2} & -2 & 1 \ 1 & 3 & 2 \ -3 & -1 & 0 \ \end{pmatrix}$$

$$egin{pmatrix} igg(egin{pmatrix} 1 & 2 & -2 \ 2 & 1 & 3 \ 0 & -3 & -1 \end{pmatrix}$$

$$egin{pmatrix} -1 & 0 & -3 \ -2 & 1 & 2 \ 3 & 2 & 1 \end{pmatrix}$$

$$egin{pmatrix} -1 & 0 & 2 \ 1 & 1 & 2 \ -1 & 2 & 3 \end{pmatrix}$$



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$$\begin{pmatrix} -2 & 1 & 2 \\ 3 & 2 & 1 \\ -1 & 0 & -3 \end{pmatrix} \begin{pmatrix} 0 & 0 & 1 \\ \hline 1 & 0 & 0 \\ \hline 0 & 1 & 0 \end{pmatrix}^{T} = \begin{pmatrix} -2 & 1 & 2 \\ 3 & 2 & 1 \\ -1 & 0 & -3 \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 \\ \hline 0 & 0 & 1 \\ \hline 1 & 0 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} (0) \begin{pmatrix} -2 \\ 3 \\ -1 \end{pmatrix} + (0) \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} + (1) \begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix} | (1) \begin{pmatrix} -2 \\ 3 \\ -1 \end{pmatrix} + (0) \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} + (0) \begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix} | (0) \begin{pmatrix} -2 \\ 3 \\ -1 \end{pmatrix} + (1) \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} + (0) \begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix})$$

$$= \begin{pmatrix} 2 & | -2 & | 1 \\ 1 & 3 & | 2 \\ -3 & | -1 & | 0 \end{pmatrix}$$

Alternatively:

$$\begin{pmatrix} -2 & 1 & 2 \ 3 & 2 & 1 \ -1 & 0 & -3 \end{pmatrix} \begin{pmatrix} e_2^T \\ \hline e_0^T \\ \hline e_1 \end{pmatrix}^T = \begin{pmatrix} -2 & 1 & 2 \ 3 & 2 & 1 \ -1 & 0 & -3 \end{pmatrix} \begin{pmatrix} e_2 & | e_0 & | e_1 \end{pmatrix}$$

$$= \begin{pmatrix} \begin{pmatrix} -2 & 1 & 2 \ 3 & 2 & 1 \ -1 & 0 & -3 \end{pmatrix} e_2 & \begin{pmatrix} -2 & 1 & 2 \ 3 & 2 & 1 \ -1 & 0 & -3 \end{pmatrix} e_0 & \begin{pmatrix} -2 & 1 & 2 \ 3 & 2 & 1 \ -1 & 0 & -3 \end{pmatrix} e_1$$

$$= \begin{pmatrix} \begin{pmatrix} 2 \ 1 \ -3 \end{pmatrix} & \begin{pmatrix} -2 \ 3 \ -1 \end{pmatrix} & \begin{pmatrix} 1 \ 2 \ 0 \end{pmatrix} & = \begin{pmatrix} 2 & | -2 & | 1 \ 1 & 3 & 2 \ -3 & -1 & | 0 \end{pmatrix}$$

Hint: it is not necessary to write out P(p): the vector p indicates the order in which the columns need to appear. In this case, you can go directly to the answer

$$\left(\begin{array}{c|c|c} 2 & -2 & 1 \\ 1 & 3 & 2 \\ -3 & -1 & 0 \end{array}\right).$$

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Homework 7.2.3.5

1/1 point (graded)

Let
$$p=\left(k_0,\cdots,k_{n-1}
ight)^T$$
 be a permutation vector. Consider $oldsymbol{x}=egin{pmatrix} \dfrac{\chi_0}{\chi_1} \\ \vdots \\ \overline{\chi_{n-1}} \end{pmatrix}$.

Applying permutation matrix
$$P=P\left(p
ight)$$
 to x yields $Px=\left(egin{array}{c} \dfrac{\chi_{k_0}}{\chi_{k_1}} \\ \vdots \\ \chi_{k_{n-1}} \end{array}
ight)$

Always ~

✓ Answer: Always

Answer: Always

$$Px = P(p)x = \left(\frac{\frac{e_{k_0}^T}{e_{k_1}^T}}{\vdots}\right) x = \left(\frac{\frac{e_{k_0}^T x}{e_{k_1}^T x}}{\vdots}\right) = \left(\frac{\chi_{k_0}}{\chi_{k_1}}\right).$$

(Recall that $e_i^T x = \chi_i$.)

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Homework 7.2.3.6

1/1 point (graded)

Let
$$p=\left(k_0,\cdots,k_{n-1}
ight)^T$$
 be a permutation vector . Consider $A=egin{pmatrix} rac{ ilde{a}_0^T}{ ilde{a}_1^T} \\ \hline dots \\ \hline ilde{a}_{n-1}^T \end{pmatrix}$.

Applying permutation matrix $P=P\left(p
ight)$ to A yields $PA=egin{pmatrix} rac{oldsymbol{a}_{k_0}^T}{oldsymbol{ ilde{a}}_{k_{n-1}}^T} \end{pmatrix}$.

Always

✓ Answer: Always

Answer: Always

$$PA = P(p)A = \begin{pmatrix} \frac{e_{k_0}^T}{e_{k_1}^T} \\ \vdots \\ \hline e_{k_{n-1}}^T \end{pmatrix} A = \begin{pmatrix} \frac{e_{k_0}^T A}{e_{k_1}^T A} \\ \vdots \\ \hline e_{k_{n-1}}^T A \end{pmatrix} = \begin{pmatrix} \frac{\widetilde{a}_{k_0}^T}{\widetilde{a}_{k_1}^T} \\ \vdots \\ \hline \widetilde{a}_{k_{n-1}}^T A \end{pmatrix}.$$

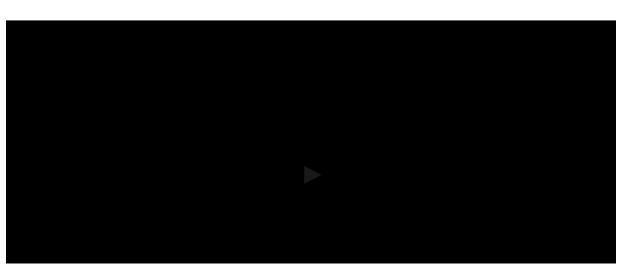
(Recall that $e_i^T A$ equals the *i*th row of A.)

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Dr. Robert van de Geijn: OK, so you
were asked to look at a general
permutation

Calculator

matrix and to look and see what that does to a vector x.

Now, if we apply the permutation matrix to x, where the permutation matrix is defined by this integers vector right

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Homework 7.2.3.7

1/1 point (graded)

Let $p=\left(\,k_0,\cdots,k_{n-1}\,
ight)^T$ be a permutation vector, $P=P\left(p
ight)$ be the associated permutation matrix, and $A = \left(egin{array}{c|c} a_0 & a_1 & \dots & a_{n-1} \end{array}
ight).$

$$AP^T = \left(egin{array}{c|c} a_{k_0} & a_{k_1} & \dots & a_{k_{n-1}} \end{array}
ight).$$

Always

Answer: Always

Answer: Always

Recall that unit basis vectors have the property that $Ae_k = a_k$.

$$AP^{T} = A \begin{pmatrix} e_{k_{0}}^{T} \\ e_{k_{1}}^{T} \\ \vdots \\ e_{k_{n-1}}^{T} \end{pmatrix}^{T} = A \left(e_{k_{0}} \mid e_{k_{1}} \mid \cdots \mid e_{k_{n-1}} \right)$$

$$= \left(Ae_{k_{0}} \mid Ae_{k_{1}} \mid \cdots \mid Ae_{k_{n-1}} \right) = \left(a_{k_{0}} \mid a_{k_{1}} \mid \cdots \mid a_{k_{n-1}} \right).$$

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Dr. Robert van de Geijn: OK, we're back. Now notice that this matrix P is a really the matrix--

the permutation matrix P associated with the permutation vector littl **⊞** Calculator

simply has the unit basis vectors ordered in the order indicated

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Homework 7.2.3.8

1/1 point (graded)

If $m{P}$ is a permutation matrix, then so is $m{P}^{m{T}}$.

TRUE

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Homework 7.2.3.9

12/12 points (graded)

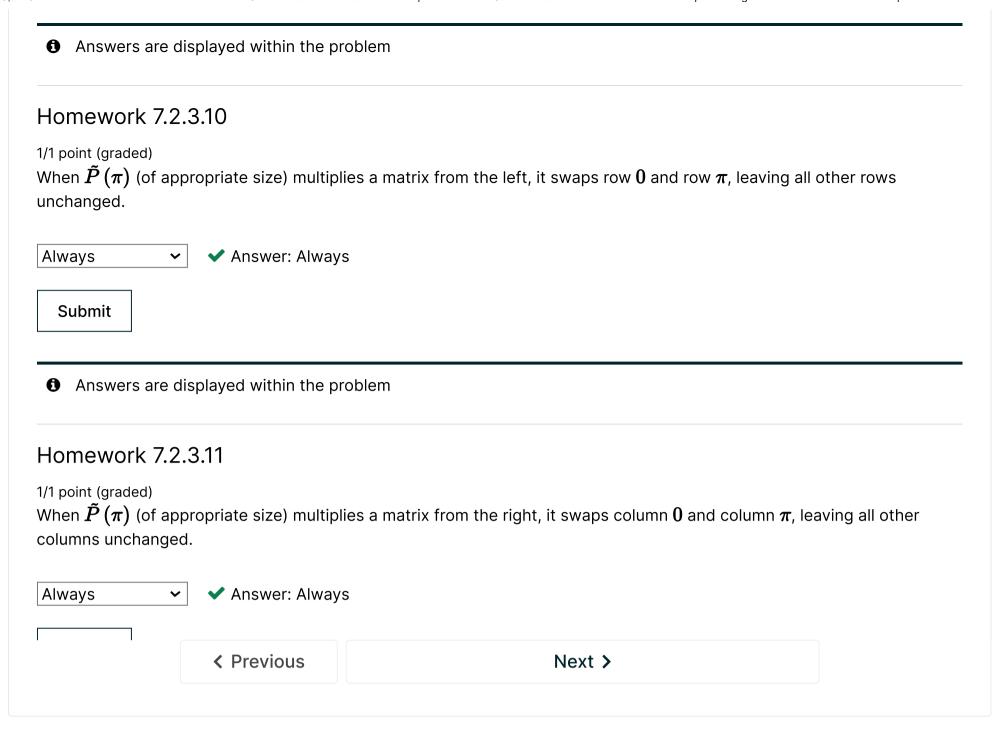
Recall:

$$\widetilde{P}(\pi) = \begin{pmatrix} e_{\pi}^{T} \\ e_{1}^{T} \\ \vdots \\ e_{\pi-1}^{T} \\ e_{0}^{T} \\ e_{\pi+1}^{T} \\ \vdots \\ e_{n-1}^{T} \end{pmatrix} = \begin{pmatrix} 0 & 0 & \cdots & 0 & 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 & 0 & 0 & \cdots & 0 \\ 1 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 1 \end{pmatrix}$$

Compute

$$\tilde{P}(1)\begin{pmatrix} -2\\3\\-1 \end{pmatrix} = \begin{bmatrix} -2\\-1 \end{bmatrix}$$
 Answer: -2

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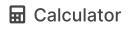
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