Exercise: Conditional PDFs

(2/2 points)

The random variables X and Y are jointly continuous, with a joint PDF of the form

$$f_{X,Y}(x,y) = \begin{cases} cxy, & \text{if } 0 \le x \le y \le 1, \\ 0, & \text{otherwise,} \end{cases}$$

where c is a normalizing constant.

For $x \in [0, 0.5]$, the conditional PDF $f_{X|Y}(x \mid 0.5)$ is of the form ax^b. Find a and b. Your answers should be numbers.

$a =$ 8 **Answer:** 8

$b =$ 1 **Answer:** 1

Answer:

We have $f_{X|Y}(x \mid 0.5) = \dfrac{f_{X,Y}(x, 0.5)}{f_Y(0.5)}$.

Having fixed $y = 0.5$, the conditional PDF is to be viewed as a function of x. For those values of x that are possible (i.e., $x \in [0, 0.5]$), the conditional PDF will be proportional to the joint PDF, hence of the form ax, for some constant a. This implies that $b = 1$. To find the normalizing constant, we use the normalization equation

$$1 = \int_0^{0.5} f_{X|Y}(x \mid 0.5)\, dx = \int_0^{0.5} ax\, dx = a \cdot \frac{x^2}{2}\Big|_0^{0.5} = \frac{a}{8},$$

which yields $a = 8$.

You have used 2 of 2 submissions