5. Leaky tank

Leaky tank (External resource) (0.0 / 1.0 points)

Leaky tank

We now consider a slightly different system. Suppose that tank 1 contains some volume of one fluid, and tank 3 contains the same volume of another fluid. These two tanks are connected to the mixing tank (tank 2) by pipes of equal width. Tank 2 is initially empty. We set $a_{12} = a_{23} = 1$ and $a_{13} = 0$. As we showed in the previous section, if we let the system run, then after a while we will end up with a mixture of the two fluids in tank 2, and the volume of the mixture will be one third of the total of the initial volumes of tank 1 and 3.

Unfortunately, tank 1 has a leak in it! This means we must adjust the equation for h_1, to include a leaking rate $Q_{leak} = -\mu h_1$ with $\mu > 0$, so that the system becomes:

$$\frac{dh_1}{dt} = (h_2 - h_1) - \mu h_1,$$
$$\frac{dh_2}{dt} = (h_3 - h_2) + (h_1 - h_2),$$
$$\frac{dh_3}{dt} = (h_2 - h_3).$$

We can then recast this in matrix form as

$$\dot{\mathbf{x}} = \begin{bmatrix} -1-\mu & 1 & 0 \\ 1 & -2 & 1 \\ 0 & 1 & -1 \end{bmatrix} \mathbf{x}$$

with $\mathbf{x}$ defined as before. Let the initial conditions be

$$\mathbf{x}(0) = \begin{bmatrix} 1/2 \\ 0 \\ 1/2 \end{bmatrix}.$$

We know that the solution can be written in the form

$$\mathbf{x}(t) = c_1 e^{\lambda_1 t}\mathbf{v}_1 + c_2 e^{\lambda_2 t}\mathbf{v}_2 + c_3 e^{\lambda_3 t}\mathbf{v}_3.$$

Calculate the $\lambda_i, c_i, \mathbf{v}_i$ using MATLAB, and plot the solutions. Take $\mu = 1$.

Your Script

Save Reset MATLAB Documentation (https://www.mathworks.com/help/)

```
% Input the 3x3 matrix describing your linear system as a variable A
```

```
% Input the 3x3 matrix describing your linear system as a variable A
A = [-2,1,0;1,-2,1;0,1,-1];
% Input your initial conditions as a column vector x0
x0 =[1/2,0,1/2];
% Use eig(A) to find the eigenvalues and eigenvectors of A
% Define the eigenvectors as column vectors v1, v2 and v3,
% and the eigenvalues as lambda1, lambda2, lambda3
% so that A*v1 = lambda1*v1, etc.
[V,D] = eig(A);
v1 = V(:,1);
v2 = V(:,2);
v3 = V(:,3);
lambda1 = D(1,1);
lambda2 = D(2,2);
lambda3 = D(3,3);
% Calculate the column vector c = [c1;c2;c3] from the initial conditions using in
c = inv(V)*x0';
c1 = c(1,1); c2 = c(2,1); c3 = c(3,1);
%%% Define a row vector t with 100 equally spaced entries,
%%% beginning with 0 and ending at 4.
t = linspace(0,4);
%%% Define three row vectors h1, h2 and h3, with entries corresponding
%%% to h1(t), h2(t) and h3(t) evaluated at each time in t.
h = c1*v1*exp(lambda1*t)+c2*v2*exp(lambda2*t)+c3*v3*exp(lambda3*t);
h1 = h(1,:);
h2 = h(2,:);
h3 = h(3,:);
%%% Now use plot to plot the three vectors against time on the same figure
plot(t, h1);
hold on
plot(t, h2);
plot(t, h3);
set(gca,'fontsize',18)
xlabel('Time')
ylabel('Volume')
title('Time series')
```

Run Script ()

Previous Assessment: Correct

Submit ()

Value of c

Value of h1

Value of h2

5. Leaky tank

Hide Discussion

Topic: Unit 3: Solving systems of first order ODEs using matrix methods / 5. Leaky tank

Add a Post

Show all posts ▼ by recent activity ▼

There are no posts in this topic yet.

✖

Learn About Verified Certificates