2. Combining topics

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Part B due Oct 5, 2021 20:30 IST

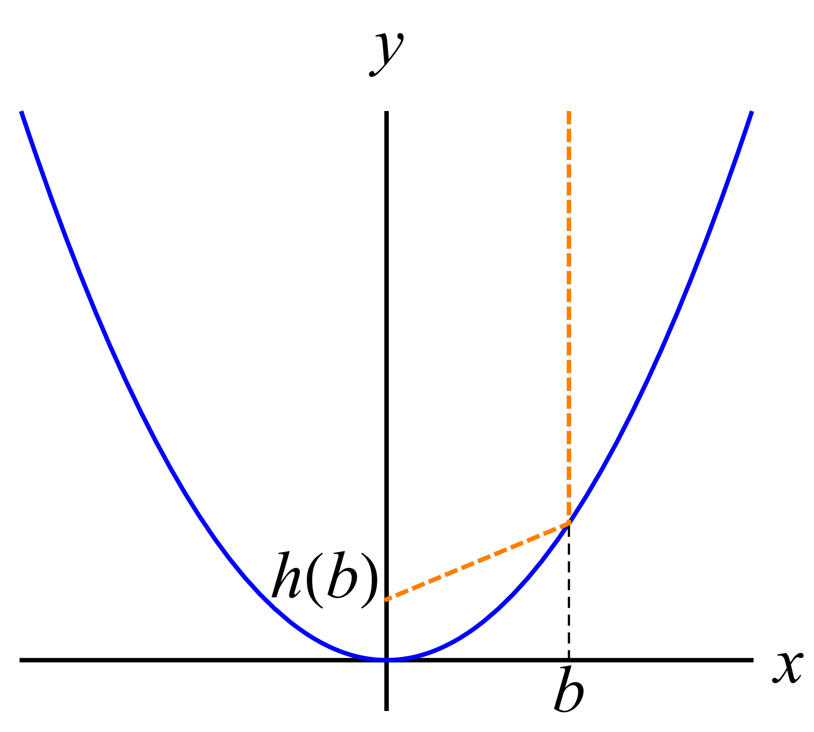
Synthesize

In this problem set, we will review topics from earlier in the course as well as explore concepts from Unit 5.

Parabolic mirror

2/2 points (graded)
Here is a problem that puts together several of the ingredients we've been studying.

Let c denote the parabola $y = x^2$. Suppose that a projectile is traveling vertically down towards the point $(b, 0)$ with velocity $\langle 0, -1 \rangle$ – see the picture. It bounces off the parabola and hits the y-axis. Let $h(b)$ denote the height where it hits the y-axis.



When the projectile bounces off the parabola, the angle of incidence is equal to the angle of reflection (with respect to the tangent line to the parabola). On the last problem set, you figured out that if a projectile with velocity $\vec{v}$ bounces off of a wall with normal vector $\vec{n}$, then the outgoing velocity $\vec{w}$ is given by

$$\vec{w} = \vec{v} - 2\frac{\vec{v} \cdot \vec{n}}{\vec{n} \cdot \vec{n}}\vec{n}. \qquad (6.293)$$

You also know how to find the normal vector to the parabola at any given point.

Combining all these ideas, find the outgoing velocity $\vec{w}$ of the projectile after it bounces off the parabola.

(Your answer will depend on b. Enter vectors as coordinates surrounded by square brackets: e.g. `[-1,0]`.)

[(-4*b)/(1+4*b^2),(1-4*b^2)/(

✔ **Answer:** [-4*b/(4*b^2+1), (1-4*b^2)/(4*b^2+1)]

Find the height $h(b)$ where the projectile traveling vertically from the point $(b, 0)$ hits the y-axis.

1/4

✔ **Answer:** 1/4

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Solution:

We know that $\vec{v} = \langle 0, -1 \rangle$. Assume $A = (b, b^2)$ is the point where the projectile hits the parabola. Then we can calculate the normal vector

$$\vec{n} = \nabla\left(x^2 - y\right) = \langle 2x, -1 \rangle.$$

At the point (b, b^2) the normal vector is $\langle 2b, -1 \rangle$. We apply the formula to find the reflected vector

$$\vec{w} = \langle 0, -1 \rangle - 2\frac{1}{4b^2+1}\langle 2b, -1 \rangle = \boxed{\langle -\frac{4b}{4b^2+1}, \frac{1-4b^2}{4b^2+1} \rangle}.$$

Recall that $A = (b, b^2)$. Let $B = (0, h(b))$ is the point where the projectile hits the y-axis. We know that the vector $\overrightarrow{AB}$ is parallel to $\vec{w}$. Thus we may assume it is $\lambda\vec{w}$. Hence the vector from the origin to the point B can be expressed as

$$(b, b^2) + \lambda\vec{w} = \left(b - \lambda\frac{4b}{4b^2+1}, b^2 + \lambda\frac{1-4b^2}{1+4b^2}\right).$$

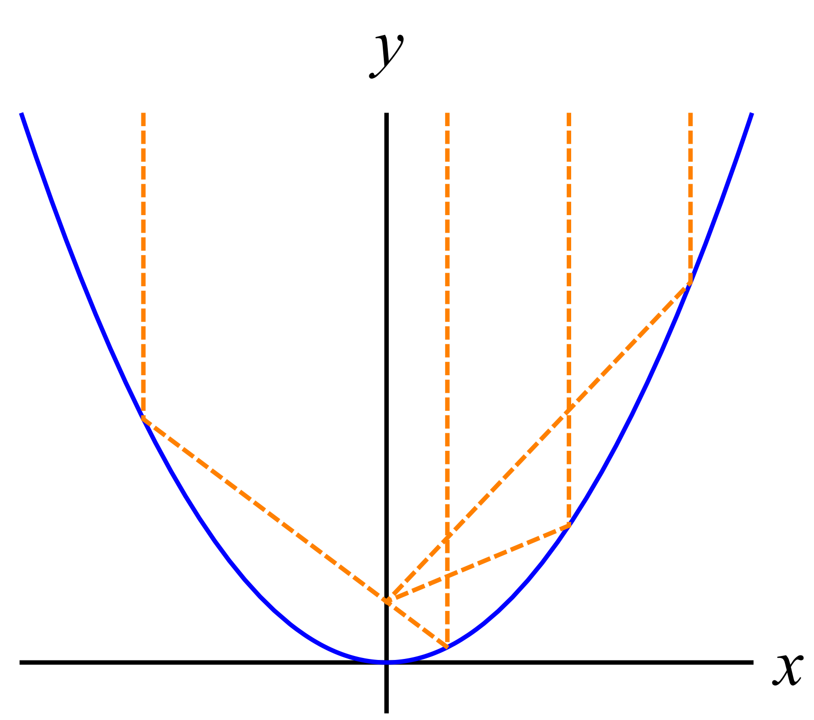
Since B is of the form $(0, h(b))$, we solve λ by the equation

$$0 = b - \lambda\frac{4b}{4b^2+1}.$$

In particular, $\lambda = \frac{4b^2+1}{4}$. We finally get

$$h(b) = b^2 + \lambda\frac{1-4b^2}{1+4b^2} = b^2 + \frac{4b^2+1}{4}\cdot\frac{1-4b^2}{1+4b^2} = \boxed{\frac{1}{4}}.$$

(We see that the height $h(b)$ does not depend on the value of b!)



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Answers are displayed within the problem

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