The Principle of Countable Additivity

Some philosophers and mathematicians think that the (finite) Additivity principle we considered in Section 6.2.1 needs to be strengthened to the following:

> **Countable Additivity**
> Let $A_1, A_2, A_3, \ldots$ be a countable list of propositions, and suppose that A_i and A_j are incompatible whenever $i \neq j$. Then:
>
> $$p\left(A_1 \text{ or } A_2 \text{ or } A_3 \text{ or } \ldots\right) = p\left(A_1\right) + p\left(A_2\right) + p\left(A_3\right) + \ldots$$

There are two main reasons to hope for the adoption of Countable Additivity.

The first is that it is needed to prove a number of important results in probability theory, including certain versions of the Law of Large Numbers.

The second is that probability functions that fail to be countably additive can have mathematically awkward properties. We'll consider one of these properties below. We'll also talk about the fact that adopting Countable Additivity is not without costs.

My own view—and that of many others—is that *on balance* the advantages outweigh the costs. But it is important to be clear that the issue is not entirely straightforward.

(Incidentally, why stop with *countable* Additivity? Why not go all the way, and require probability distributions to be additive with respect to *any* infinite set of mutually exclusive propositions, regardless of cardinality? The answer is that the resulting mathematics is not nice. Consider, for example, a dart on a random trajectory to the unit interval, $[0, 1]$. A principle of uncountable additivity would entail that, for some number $r \in [0, 1]$, the probability that the dart lands on r is x for some $x > 0$, which would be an awkward result, since r is smaller than a subinterval of $[0, 1]$ of size x. We'll return to this sort of issue in Lecture 7.)

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