12.2.4 Eigenvalues and Vectors of 3 × 3 Matrices

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Week 12 due Dec 29, 2023 10:42 IST Completed

12.2.4 Eigenvalues and Vectors of 3 x 3 Matrices

Video 12.2.4 Part 1

0:00 / 0:00 2.0x

Start of transcript. Skip to the end.

Dr. Robert van de Geijn: So let's move right along

and look at how to find the eigenvalues and vectors of a 3 by 3 matrix.

Let's review a little bit.

Here we have a 3 by 3 diagonal matrix.

How do you find its eigenvalues and eigenvectors?

Go and do the homework, and see me in

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Reading Assignment

0 points possible (ungraded)

Read Unit 12.2.4 of the notes. [LINK]

- [x] Done

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Answers are displayed within the problem

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? Question 5
Hi Professors! What does it mean when a matrix is defective?

Homework 12.2.4.1

Calculator

Homework 12.2.4.1

4/4 points (graded)

Let $A = \begin{pmatrix} 3 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 2 \end{pmatrix}$. Then which of the following are true:

- $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ is an eigenvector associated with eigenvalue 3.

  TRUE ✔ Answer: TRUE

  Just multiply it out.

- $\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ is an eigenvector associated with eigenvalue -1.

  TRUE ✔ Answer: TRUE

  Just multiply it out.

- $\begin{pmatrix} 0 \\ \chi_1 \\ 0 \end{pmatrix}$, where $\chi_1 \neq 0$ is a scalar, is an eigenvector associated with eigenvalue -1.

  TRUE ✔ Answer: TRUE

  Just multiply it out.

- $\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ is an eigenvector associated with eigenvalue 2.

  TRUE ✔ Answer: TRUE

  Just multiply it out.

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Answers are displayed within the problem

Homework 12.2.4.2

10.0/10.0 points (graded)

Let $A = \begin{pmatrix} \alpha_{0,0} & 0 & 0 \\ 0 & \alpha_{1,1} & 0 \\ 0 & 0 & \alpha_{2,2} \end{pmatrix}$. Then which of the following are true:

- $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ is an eigenvector associated with eigenvalue $\alpha_{0,0}$.

  TRUE ✔ Answer: TRUE

  Just multiply it out.

Calculator

- $\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ is an eigenvector associated with eigenvalue $\alpha_{1,1}$.

  TRUE ✔ Answer: TRUE

  Just multiply it out.

- $\begin{pmatrix} 0 \\ \chi_1 \\ 0 \end{pmatrix}$ where $\chi_1 \neq 0$ is an eigenvector associated with eigenvalue $\alpha_{1,1}$.

  TRUE ✔ Answer: TRUE

  Just multiply it out.

- $\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ is an eigenvector associated with eigenvalue $\alpha_{2,2}$.

  TRUE ✔ Answer: TRUE

  Just multiply it out.

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Video 12.2.4 Part 2

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Dr. Robert van de Geijn: So hopefully, you

remembered that the diagonal elements of a diagonal matrix are its eigenvalues,

and then the eigenvectors can be found as the unit basis vectors.

What about this one?

Here we have a 3 by 3 upper triangular matrix.

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Calculator

Homework 12.2.4.3

10.0/10.0 points (graded)

Let $A = \begin{pmatrix} 3 & 1 & -1 \\ 0 & -1 & 2 \\ 0 & 0 & 2 \end{pmatrix}$. Then which of the following are true:

- 3, -1, and 2 are eigenvalues of A.
- $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ is an eigenvector associated with eigenvalue 3.

  TRUE ✔ Answer: TRUE

  Just multiply it out.

- $\begin{pmatrix} -1/4 \\ 1 \\ 0 \end{pmatrix}$ is an eigenvector associated with eigenvalue -1.

  TRUE ✔ Answer: TRUE

  Just multiply it out.

- $\begin{pmatrix} -1/4\chi_1 \\ \chi_1 \\ 0 \end{pmatrix}$ where $\chi_1 \neq 0$ is an eigenvector associated with eigenvalue -1.

  TRUE ✔ Answer: TRUE

  Just multiply it out.

- $\begin{pmatrix} 1/3 \\ 2/3 \\ 1 \end{pmatrix}$ is an eigenvector associated with eigenvalue 2.

  TRUE ✔ Answer: TRUE

  Just multiply it out.

Submit

Answers are displayed within the problem

Video 12.2.4 Part 3

Start of transcript. Skip to the end.

Dr. Robert van de Geijn: So this

Calculator

should have remembered that again,

the diagonal elements of a triangular matrix are its eigenvalues.

And then once you subtract off the eigenvalue from the diagonal,

you should be able to reduce the system to row echelon form

and find the eigenvectors associated

0:00 / 0:00 2.0x

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Homework 12.2.4.4

1/1 point (graded)

Let $A = \begin{pmatrix} \alpha_{0,0} & \alpha_{0,1} & \alpha_{0,2} \\ 0 & \alpha_{1,1} & \alpha_{1,2} \\ 0 & 0 & \alpha_{2,2} \end{pmatrix}$. Then the eigenvalues of this matrix are $\alpha_{0,0}$, $\alpha_{1,1}$, and $\alpha_{2,2}$.

TRUE ✔ Answer: TRUE

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Answers are displayed within the problem

Homework 12.2.4.5

1/1 point (graded)

Consider $A = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$. Which of the following is true about this matrix:

(Mark all correct answers)

- [x] $(1, \begin{pmatrix} 1 \\ 3 \\ 1 \end{pmatrix})$ is an eigenpair.
- [x] $(0, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix})$ is an eigenpair.
- [x] $(0, \begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix})$ is an eigenpair.
- [x] This matrix is defective.

✔

Answer:

$$\det(\begin{pmatrix} 1-\lambda & 0 & 0 \\ 2 & -\lambda & 1 \\ 1 & 0 & -\lambda \end{pmatrix}) = [\underbrace{(1-\lambda)(-\lambda)^2}\ +0+0] - [\underbrace{0+0+0}].$$

Calculator

$$\underbrace{\underbrace{-(1-\lambda)\lambda^2}_{\lambda^3-\lambda^2} \quad 0}_{-(1-\lambda)\lambda^2}$$

So, $\lambda_0 = \lambda_1 = 0$ is a double root, while $\lambda_2 = 1$ is the third root.

$\lambda_2 = 1$:

$$A - \lambda_2 I = \begin{pmatrix} 0 & 0 & 0 \\ 2 & -1 & 1 \\ 1 & 0 & -1 \end{pmatrix}$$

We wish to find a nonzero vector in the null space:

$$\begin{pmatrix} 0 & 0 & 0 \\ 2 & -1 & 1 \\ 1 & 0 & -1 \end{pmatrix} \begin{pmatrix} \chi_0 \\ \chi_1 \\ \chi_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}.$$

By examination, I noticed that

$$\begin{pmatrix} 0 & 0 & 0 \\ 2 & -1 & 1 \\ 1 & 0 & -1 \end{pmatrix} \begin{pmatrix} 1 \\ 3 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}.$$

Eigenpair:

$$(1, \begin{pmatrix} 1 \\ 3 \\ 1 \end{pmatrix}).$$

$\lambda_0 = \lambda_1 = 0$:

$$A - 0I = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

Reducing this to row-echelon form gives us the matrix

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}.$$

Notice that there is only one free variable, and hence this matrix is defective! The sole linearly independent eigenvector associated with $\lambda = 0$ is

$$\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}.$$

Eigenpair:

$$(0, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix})$$

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