Observation Theory Script V52A – Precision

We now know how errors in the observations will propagate into the errors of the estimators.

When we talk about the precision of the estimator, what do we actually mean?

And how should we interpret this precision?

As a small recap: we looked at the linear system of observation equations, and describe the first and the second moment of the observations with the Expectation and the Dispersion operator.

Using Best Linear Unbiased Estimation, we find an expression for the best estimator of the unknown parameters,: which is x-hat And then we find, using the error propagation laws, the Expectation and the Dispersion of that value, x-hat.

For the Expectation part, this expression states that the estimator for x-hat is unbiased.

This follows from the fact that the expectation of x-hat is equal to x.

For the Dispersion part, we see that the dispersion for x-hat is described by its covariance matrix.

We now would like to have a closer look at the covariance matrix If we have a one-dimensional variable, the covariance matrix is a variance.

If the variable is y, and it has a certain expectation value E{y}, we can interpret the square-root of the variance to be representative of the spread (or dispersion) of the actual values around that expectation value.

We often use the concept of CLICK an 'error-bar' , to indicate this precision.

The error-bar can indicate the 'one-sigma' variation range, or the two- or three-sigma variation range.

This error-bar can also be regarded as a 'confidence interval' In case of a normal distribution, we would represent this spread with the well known bell-curve, or Gaussian.

The one-sigma error bar indicates 68.3% of the distribution.

The two and three sigma intervals correspond with 95 and 99.7% We call this the 68-95-99.7 rule Note that the variance is defined irrespective of the shape of the probability distribution.

If we have two variables, the covariance matrix is a 2 by 2 matrix.

On the diagonals, we have the variance of the first and the second variable, and on the off-diagonal terms we have the covariance of the two variables.

Let's look at an example to interpret this For example, when the two variables represent the east and the north coordinate of a certain point.

In this case, the covariance matrix gives us information on the distribution of points, or the likelihood that a point is within a certain range.

From the distribution in this figure, we see that the range of variation is larger in the north-south direction than in the east-west direction.

So the lower right term of the covariance matrix will be larger than the upper left term.

From the fact that the point distribution is not oriented along the coordinate axes We deduce that the off-diagonal terms of the matrix are not zero.

If we have more than two variables, the covariance matrix will have corresponding dimensions.

These dimensions may be very large.

This implies that it will be more difficult (or even impossible) to visualize the corresponding distribution in a two-dimensional figure, but in essence this does not matter.

The matrix just expresses the variances of each variable, as well as the covariances between different variables.

Now we can more or less ‘interpret’, or read, a covariance matrix when we have obtained one.

In the next video, we will look at some examples.