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5. Worked Example: a One-Sided Test

Errors, Levels, and Conclusion of a One-Sided Test

Examples

For $\alpha = 5\%$, $q_{\alpha/2} = 1.96$, $q_{\alpha} = 1.645$

Fair coin

Everybody sees what I mean by this?

News on Youtube

$H_0 : p \geq 0.33$ vs. $H_1 : p < 0.33$. This is a α -sided test.

We reject if:

$$\sqrt{n} \frac{\hat{p}_n - p}{\sqrt{p(1-p)}} < C$$

But what value for $p \in \Theta_0 = [0.33, 1)$ should we choose?

Type 1 error is the function $p \mapsto \mathbb{P}_p[\psi = 1]$. To control the level we need to find the p that **maximizes** it over Θ_0

\rightarrow no need for computations, it's clearly $p = 0.33$

H_0 is **not rejected** at the asymptotic level 5% by the test $\psi_{5\%}$.

17:46 / 17:46

1.50x

🔊 🔍 📄 🗣️

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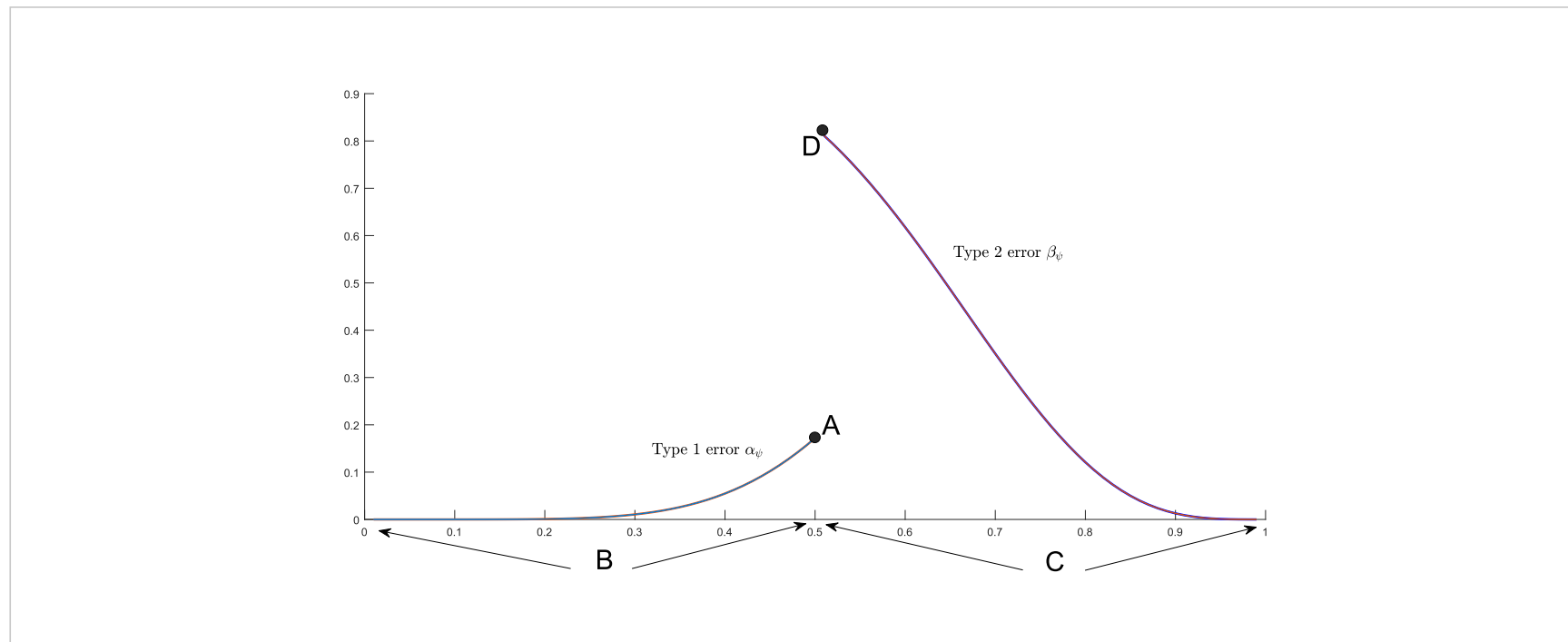
Visualizing Hypothesis Testing for a One-Sided Test

3/3 points (graded)

Let $X_1, \dots, X_n \stackrel{iid}{\sim} \text{Ber}(p^*)$ for some true parameter $p^* \in (0, 1)$, and let $(\{0, 1\}, \{P_p\}_{p \in (0, 1)})$ denote the associated statistical model where $P_p = \text{Ber}(p)$.

Suppose the null hypothesis is $H_0 : p^* \leq 1/2$ and the alternative hypothesis is $H_1 : p^* > 1/2$. Let ψ continue to denote the statistical test we will use. (Recall that a test takes value either 0 or 1. Usually it is of the form $\mathbf{1}(T_n > C)$ where C is a threshold to be specified and T_n is known as a **test statistic**. Be careful to not confuse **(tests with test statistics.)**

Consider the following graph of this hypothesis testing set-up.



- Continuous curve on the left: type 1 error, α_ψ , graphed as a function of θ .
- Continuous curve on the right: type 2 error, β_ψ , graphed as a function of θ .
- Horizontal axis: the parameter space $\Theta = (0, 1)$.

Which letter indicates Θ_0 , the region defined by the null hypothesis?

☐ A☒ B☐ C☐ D

Which letter indicates Θ_1 , the region defined by the alternative hypothesis?

☐ A☐ B☒ C☐ D

Let $p \in (0, 1)$ denote the point where the power is attained, i.e., the point where

$$\pi_\psi = \inf_{\Theta_1} (1 - \beta_\psi(p)).$$

Which letter indicates the ordered pair (p, π_ψ) ?

☒ A

☐ B

☐ C

☐ D
**Solution:**

We consider the questions in order.

For the first question, since we are given that $H_0 : p \leq 1/2$, then the interval $(0, 1/2]$ defines Θ_0 . Hence, letter B is the correct response.

For the second question, since we are given that $H_1 : p > 1/2$, then the interval $(1/2, 1)$ defines Θ_1 . Hence, letter C is the correct response.

The the third question, recall that the power of a test is given by

$$\pi_\psi = \inf_{p \in (0,1)} (1 - \beta_\psi(p)).$$

The continuous curve on the right, which graphs β_ψ , attains its maximum at $p = 1/2$, and this maximum is given by $\beta_\psi(1/2) = 0.8$. Therefore,

$$\pi_\psi = \inf_{p \in (0,1)} (1 - \beta_\psi(p)) = 1 - 0.8 = 0.2,$$

which implies that A is the correct response.

You have used 1 of 3 attempts

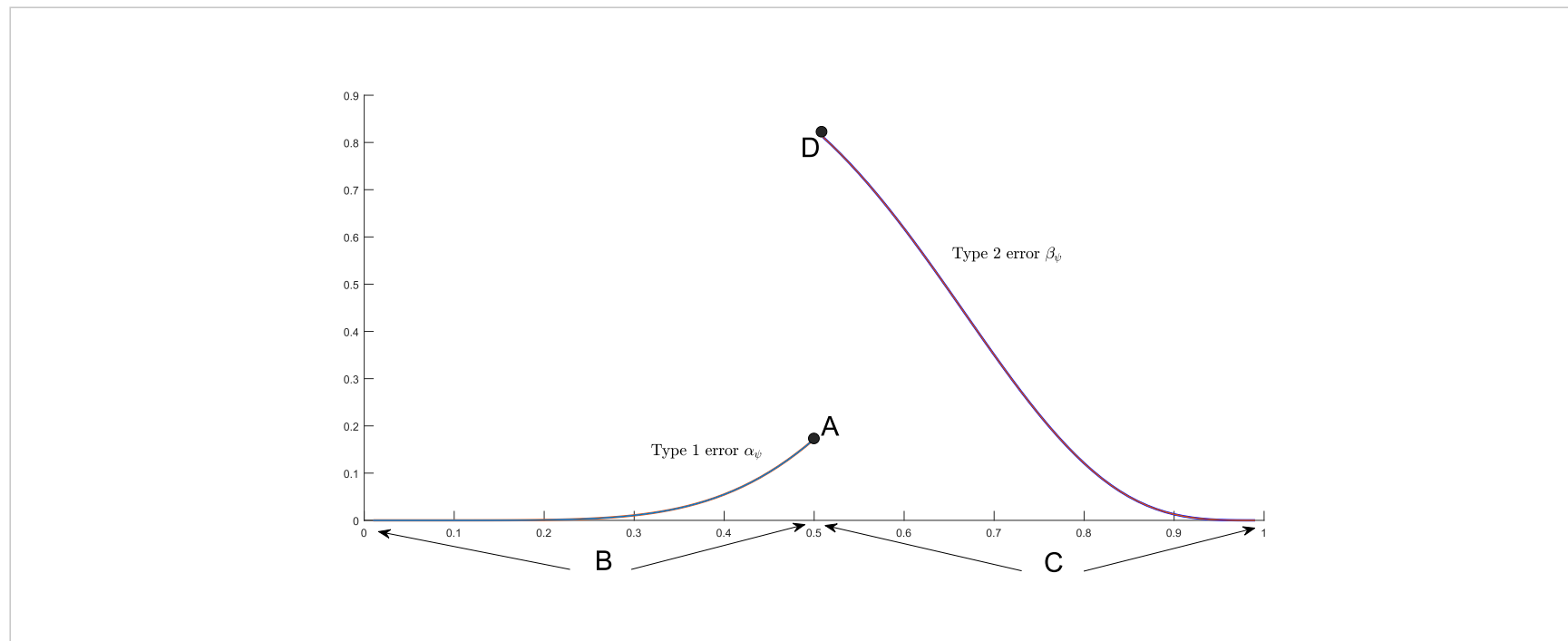
i Answers are displayed within the problem

Level of a statistical test

1/1 point (graded)

As in the previous question, let $X_1, \dots, X_n \stackrel{iid}{\sim} \text{Ber}(p^*)$ for some true parameter $p^* \in (0, 1)$, and let $(\{0, 1\}, \{P_p\}_{p \in (0, 1)})$ denote the associated statistical model where $P_p = \text{Ber}(p)$.

Suppose the null hypothesis is $H_0 : p^* \leq 1/2$ and the alternative hypothesis is $H_1 : p^* > 1/2$. Let ψ continue to denote the statistical test we will use. Consider the graphic below from the previous problem.



- Continuous curve on the left: type 1 error, α_ψ , graphed as a function of θ .
- Continuous curve on the right: type 2 error, β_ψ , graphed as a function of θ .

- Horizontal axis: the parameter space $\Theta = (0, 1)$.

Which of the following are **levels** of ψ ? (Choose all that apply.)

☐ 5 %

☐ 10 %

☒ 20 %



Solution:

The level of ψ is given by any real $\alpha \in \mathbb{R}$ such that


$$\alpha_{\psi}(p) \leq \alpha, \quad \text{for all } p \in \Theta_0 = (0, 1/2]$$

That is, the type 1 error is uniformly bounded above by α . According to the graph, the continuous curve on the left curve stays below 0.2, but not below 0.05 and 0.1. Thus $0.2 = 20\%$ is the correct response.

Remark: In general, we will describe the level of a test by the *smallest* possible level α , but this is not strictly necessary.

Submit

You have used 1 of 2 attempts

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**[STAFF] [Level of a statistical test](#)**[The curve drawn does not do justice to the explanation provided in the solution.](#)

5

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