Problem 5: Covariance for the multinomial

(5/5 points)

Consider n independent rolls of a k-sided fair die with $k \geq 2$: the sides of the die are labelled $1, 2, \ldots, k$ and each side has probability $1/k$ of facing up after a roll. Let the random variable X_i denote the number of rolls that result in side i facing up. Thus, the random vector $(X_1, \ldots, X_k)$ has a multinomial distribution.

1. Which of the following statements is correct? Try to answer without doing any calculations.

- X_1 and X_2 are uncorrelated.
- X_1 and X_2 are positively correlated.
- X_1 and X_2 are negatively correlated. ✔

2. Find the covariance, $\mathbf{cov}(X_1, X_2)$, of X_1 and X_2. Express your answer as a function of n and k using standard notation . *Hint:* Use indicator variables to encode the result of each roll.

$\mathbf{cov}(X_1, X_2) =$ -n/k^2 ✔

3. Suppose now that the die is biased, with a probability $p_i \neq 0$ that the result of any given die roll is i, for $i = 1, 2, \ldots, k$. We still consider n independent tosses of this biased die and define X_i to be the number of rolls that result in side i facing up.

Generalize your answer to part 2: Find $\mathbf{cov}(X_1, X_2)$ for this case of a biased die. Express your answer as a function of n, k, p_1, p_2 using standard notation . Write p_1 and p_2 as `p_1` and `p_2`, respectively, and wrap them in parentheses in your answer; i.e., enter `(p_1)` and `(p_2)`.

$\mathbf{cov}(X_1, X_2) =$ -n*p_1*p_2 ✔

You have used 1 of 2 submissions

DISCUSSION

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