2. Student's T Test

Deriving the Student's T Test from Likelihood Ratio

2/2 points (graded)

Let $X_1, \ldots, X_n \stackrel{iid}{\sim} X \sim \mathcal{N}(\mu_1, \sigma_1^2)$. Consider the null and alternative hypotheses

$$\begin{aligned} H_0 &: \mu_1 = 5 \\ H_1 &: \mu_1 \neq 5. \end{aligned}$$

Assume that μ_1 is not known, but σ_1^2 is known. The test statistic T_n' for the likelihood ratio test associated to the above hypothesis can be expressed in terms of n, $\overline{X}_n$, and σ_1^2.

What is T_n'?

(Enter **barX_n** for $\overline{X}_n$, and **sigma_1^2** for σ_1^2.)

Generating Speech Output

n*(barX_n-5)^2/(sigma_1^2) ✔

STANDARD NOTATION

If σ_1^2 were unknown and we used the estimator $\widetilde{\sigma_1^2} = \frac{1}{n-1}\sum_i \left(X_i - \overline{X}_n\right)^2$ in **both log-likelihoods**, what would be the distribution of $\sqrt{T_n'}$?

- t_{n-1}
- t_n
- $|t_{n-1}|$
- None of the above.

STANDARD NOTATION

Submit You have used 3 of 4 attempts

Correct (2/2 points)

Introducing Another Sample

1/1 point (graded)

Let $Y_1, \ldots, Y_m \overset{iid}{\sim} Y \overset{iid}{\sim} N\left(\mu_2, \sigma_2^2\right)$ denote another sample, and assume that X's are independent of the Y's.

What is the distribution of $\overline{X}_n - \overline{Y}_m$?

- $N\left(\mu_1 - \mu_2, \frac{\sigma_1^2}{n} + \frac{\sigma_2^2}{m}\right)$

Generating Speech Output

$N\left(\mu_1-\mu_2, \frac{\sigma_1^2}{n}-\frac{\sigma_2^2}{m}\right)$

$N\left(\mu_1+\mu_2, \sigma_1^2+\sigma_2^2\right)$

None of the above.

Submit

You have used 1 of 3 attempts

Correct (1/1 point)

Test Statistic for a Two-Sample Test

1/1 point (graded)

Recall that $X_1, \ldots, X_n \stackrel{iid}{\sim} N\left(\mu_1, \sigma_1^2\right), Y_1, \ldots, Y_m \stackrel{iid}{\sim} N\left(\mu_2, \sigma_2^2\right)$, and the two samples are independent of one another. Consider the null and alternative hypotheses

$$\begin{aligned} H_0 &: \mu_1 \leq \mu_2 \\ H_1 &: \mu_1 > \mu_2. \end{aligned}$$

What is the test statistic T_n for the two-sample student's T test associated to H_0 and H_1?Express your answer in terms of $n, m, \hat{\sigma_1}^2, \hat{\sigma_2}^2, \overline{X}_n$, and $\overline{Y}_m$.

(Enter **barX_n** for $\overline{X}_n$, **barY_m** for $\overline{Y}_m$, **hat(sigma_1^2)** for $\widehat{\sigma_1^2}$, and **hat(sigma_2^2)** for $\widehat{\sigma_2^2}$.)

$T_n =$ (barX_n-barY_m)/sqrt(hat(sigma_1^2) /n+hat(sigma_2^2)/m)

STANDARD NOTATION

Generating Speech Output

Submit You have used 1 of 4 attempts

✔ Correct (1/1 point)

Applying the Welch-Satterthwaite Formula

2/2 points (graded)

Suppose we observe $\overline{X}_n = 6.2, \overline{Y}_m = 6, \hat{\sigma_1}^2 = 0.1$, and $\hat{\sigma_2}^2 = 0.2$ with $n = 50$ and $m = 50$.

Using the Welch-Satterthwaite formula, what is the approximate number of degrees of freedom for the test statistic T_n?

89 ✔

What is the p-value for this test?

(You may consult a table of values or use software for the student's T distribution.)

0.005728596679356102 ✔

Submit You have used 1 of 3 attempts

✔ Correct (2/2 points)

Discussion

Hide Discussion

Topic: Unit 4 Hypothesis testing:Homework 7 / 2. Student's T Test

Add a Post

‹ All Posts

Generating Speech Output

Likelihood ratio test statistic (first question)

question posted about 8 hours ago by **corderfj**

I have 2 $\sum$ terms in my test statistic formula coming from the log likelihood function of a gaussian. Based on the description of the solution, the grader only accepts n, $\overline{X}_n$ and σ_1^2. Any tips on how to either get rid of these $\sum$ terms or enter them in the grader?

This post is visible to everyone.

Add a Response

1 response

Gaylyn

about an hour ago

Think about what $\hat{\mu}^{MLE}$ is equal to.

Add a comment

Showing all responses

Add a response:

Preview

Submit

Generating Speech Output

Learn About Verified Certificates