sum of bernoulli distributed random variables

I have two random variables X_1 and X_2 with $P(X_1 = x_1) = (1-p)^{1-x_1}p^{x_1}$ and for X_2 the same and $x_1 \in \{0,1\}$. I want to know how $Y = X_1 + X_2$ is distributed. This is what i did: $P(Y=y) = P(X_1 + X_2 = y) = \sum_{x=0}^{1} P(X_1 = x, X_2 = y - x) =$

$$\sum_{x=0}^{1} P(X_1 = x)P(X_2 = y - x) =$$

$$\sum_{x=0}^{1}(1-p)^{1-x}p^x(1-p)^{1-(y-x)}p^{y-x} = \sum_{x=0}^{1}(1-p)^{2-y}p^y = 2p^y(1-p)^{2-y}$$

I think this is wrong because I think the answer should be $\binom{2}{y}p^y(1-p)^{2-y}$ So what did I do wrong? X_1 and X_2 are independent.

(probability)

edited Feb 1 '13 at 10:46

asked Feb 1 '13 at 10:34

Badshah

956 7 21

There's not enough information to find the distribution of Y; you need to know the joint distribution of X_1 and X_2. Perhaps you intended them to be independent? Also, what's k doing there? It never occurs again after you introduce it. – joriki Feb 1 '13 at 10:42

@joriki I meant x_1 and yes, they are independent. – Badshah Feb 1 '13 at 10:46

Please add that missing information to the question itself; people shouldn't have to read the comments in

order to make sense of the question. – joriki Feb 1 '13 at 10:48

2 Answers

This looks far too complicated.

You have two random variables, each of which takes - the value 1 with probability p and - the value 0 with probability $1-p$.

If they are independent then their sum takes

- the value 2 with probability p^2
- the value 1 with probability $2p(1-p)$
- the value 0 with probability $(1-p)^2$

The sum is a binomial random variable.

A particular problem with your expressions is that you have not restricted the values X_2 can take. For example when $y=0$ the expression $\sum_{x=0}^{1} P(X_1=x)P(X_2=y-x)$ means $P(X_1=0)P(X_2=0-0)+P(X_1=1)P(X_2=0-1)$ but you should not have the second term in that sum since $P(X_2=-1)=0$ rather than the positive value you give it

answered Feb 1 '13 at 10:52

Henry
67.1k 3 38 103

so which restriction should I give to X_2 ? – Badshah Feb 1 '13 at 10:58

@Badshah: It has to be 0 or 1 so you need

$$\sum_{x=\max(0,y-1)}^{\min(1,y)} P(X_1=x)P(X_2=y-x)$$

– Henry Feb 1 '13 at 11:08

What you wrote is not correct (first step) because according to that $P(Y=0)=\sum_{x=0}^{1}P(X_1=x,X_2=-x)$ and X_2 can't be negative. Moreover, if $X_1=x$ and $X_2=y-x$ they can't be independent.

Sorry guys but I don't see where I can put this as a comment :S :S

answered Feb 1 '13 at 10:52

dann
11 1

why cant they be independent? – Badshah Feb 1 '13 at 11:03

Because as you write them, one depends on the result of the other. – dann Feb 1 '13 at 11:08

I would use either a reasoning based on "observing", draw a Venn's diagram and you'll see easily what is the distribution of Y. Or mathematically, the sum of two indep. r.v. is the convolution. – dann Feb 1 '13 at 11:12

Mmm sorry you're right, I see it now. They are independent since X_1 and X_2 are ssumed to be ind. – dann Feb 1 '13 at 11:14