Homework Problem: Alternate-Reality Potter Family Tree

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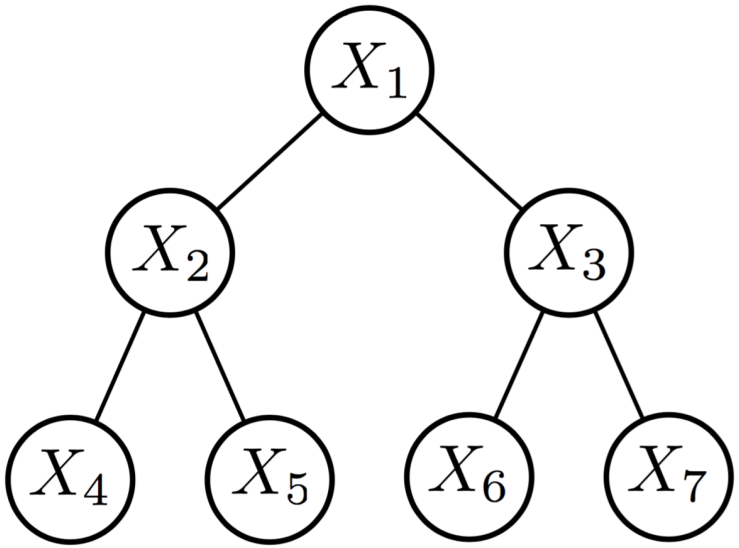
10/10 points (graded)

Consider the family tree below, which shows three generations of the Potter family. In this alternate reality, everyone only has one parent. X_1 is the parent of X_2 and X_3, X_2 is the parent of X_4 and X_5, and X_3 is the parent of X_6 and X_7.

Each person can be either a wizard, who can perform magic, or a muggle, who cannot. If a member of the family is a wizard, his or her children have a 50% chance of inheriting his magical abilities, and they are muggles otherwise. A muggle has 25% chance of having wizard children. Suppose you are the person at node X_4.



- **(a)** Let each variable X_i take one of two states: “wizard" or “muggle." You know you are a wizard, but you don't know anything about the rest of your relatives in the tree. We define ϕ_{wiz} and ϕ_0 as follows:

		$\phi_{\text{wiz}}(x)$	$\phi_0(x)$
x	wizard	1	1
	muggle	0	1

The joint distribution can be represented by this graph with the node potentials defined as $\phi_i(x_i) = \phi_0(x_i)$ for $i \in \{1, 2, 3, 5, 6, 7\}$, and $\phi_4(x_4) = \phi_{\text{wiz}}(x_4)$.

All the edge potentials can be chosen so that they are the same function $\psi(\cdot,\cdot)$, where the first argument is always the parent value and the second argument is always the child value. Determine such a function ψ.

Please provide the **exact** answer for these four quantities.

$\psi(\textbf{wizard}, \textbf{wizard}) =$ 0.5 ✔

$\psi(\textbf{wizard}, \textbf{muggle}) =$ 0.5 ✔

$\psi(\textbf{muggle}, \textbf{wizard}) =$ 0.25 ✔

$\psi(\textbf{muggle}, \textbf{muggle}) =$ 0.75 ✔

- **(b)** Evaluate the messages $m_{2\to 1}(\cdot)$ and $m_{3\to 1}(\cdot)$ according to the sum-product algorithm, expressing your answers in the form of tables.

  For each message table, please normalize the entries so that they add to 1.

  (Please be precise with at least 3 decimal places, unless of course the answer doesn't need that many decimal places. You could also put a fraction.)

  $m_{2\to 1}(\textbf{wizard}) =$ 0.5454545454545455 ✔

  $m_{3\to 1}(\textbf{wizard}) =$ 0.5 ✔

- **(c)** Determine the probability that your grandparent, X_1, was a wizard.

  (Please be precise with at least 3 decimal places, unless of course the answer doesn't need that many decimal places. You could also put a fraction.)

  0.545454545455 ✔

- **(d)** We now extend the tree from 3 generations to n generations. Every parent continues to have exactly two children, so there are 2^{k-1} individuals in the kth generation. In this larger tree, you continue to be in the last (i.e., nth) generation.

  Before knowing whether or not you are a wizard, you use the sum-product algorithm to find the probability that the earliest ancestor (the root) was a wizard.

  Then, you discover that you are a wizard, but know nothing about the rest of your relatives. In order to determine the probability that your earliest ancestor (the root) is a wizard given this information, how many messages must be recomputed?

  Provide your answer in terms of n.

  In this part, please provide your answer as a mathematical formula (and not as Python code). Use ^ for exponentiation, e.g., x^2 denotes x^2. Explicitly include multiplication using *, e.g. x*y is xy.

  n-1 ✔

  $n-1$

Submit You have used 3 of 5 attempts

✔ Correct (10/10 points)

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What's the trick in Potter b

discussion posted 2 days ago by **RobertOstas**

Hello,

I got right answers in part a and right answer for $m_{3\to 1}(W)$ in part b. However my answer for $m_{2\to 1}(W)$ in part b is not correct.

I...

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in part (d) where does n start to count?

question posted 4 days ago by **myjy**

if n starts from 0 the answer will change.

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2 responses

myjy

4 days ago

it starts from 1.

Add a comment

sandipan_dey

about 4 hours ago

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nice question

discussion posted 3 days ago by **SamS2**

A nice question. The language used here:

"A muggle has 25% chance of having wizard children."

could be tightened up.

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Potter, Part b

discussion posted 2 days ago by **smax13**

Hi,

> All the edge potentials can be chosen so that they are the same function ψ(·,·), where the first argument is always the parent value ...

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Potter Family Tree Part D

question posted 4 days ago by **JoonhoPark**

I would like some clarification in this part. I am pretty sure I have a formula needed to come up with number of message computations needed...

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What is one message?

question posted a day ago by **PavelKrupets**

When asked how many messages needed to be recalculated I entered number of messages for each RV value. Like: m21(w), m21(m).

I assume that is wrong and one message is m21(.)?

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1 response

smax13

a day ago

2 messages for an edge: one forward and one backward. But we use only one direction.

I modeled it in my head for a 3 node network then 4 and 5 .... at which point I had an answer to test. And it worked.

posted a day ago by **kiwitrader** **Community TA**

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Harry Potter part (a)

discussion posted 4 days ago by **mrBB**

A minor issue with/question about part (a) of the question: shouldn't it specify a normalization requirement for ψ? When I normalized to...

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Harry Potter part b

discussion posted 4 days ago by **MikeRead68**

I'm a bit confused. The part b question states "expressing your answers in the form of tables". Does that mean in the form on a Python dictionary,...

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