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2.4 Conditional probabilities are probabilities

Unit 2: Conditioning

Adapted from Blitzstein-Hwang Chapter 2.

When we condition on an event E , we update our beliefs to be consistent with this knowledge, effectively putting ourselves in a universe where we know that E occurred. Within our new universe, however, the laws of probability operate just as before. Conditional probability satisfies all the properties of probability! Therefore, any of the results we have derived about probability are still valid if we replace all unconditional probabilities with probabilities conditional on E .

For example, here are conditional forms of Bayes' rule and the law of total probability. These are obtained by taking the ordinary forms of Bayes' rule and LOTP and adding E to the right of the vertical bar everywhere.

THEOREM 2.4.1 (BAYES' RULE WITH EXTRA CONDITIONING).

Provided that $P(A \cap E) > 0$ and $P(B \cap E) > 0$, we have

$$P(A|B, E) = \frac{P(B|A, E)P(A|E)}{P(B|E)}.$$

THEOREM 2.4.2 (LOTP WITH EXTRA CONDITIONING).

Let A_1, \dots, A_n be a partition of S . Provided that $P(A_i \cap E) > 0$ for all i , we have

$$P(B|E) = \sum_{i=1}^n P(B|A_i, E)P(A_i|E).$$