5. Katz Centrality

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Exercises due Oct 20, 2021 17:29 IST Completed

Katz Centrality

Start of transcript. Skip to the end.

Prof Uhler: OK, so now, there can actually be some problems.

And it's important to talk through this.

So this is actually not really meaningful for any directed network.

So in particular, let's think about this.

So say like, now, the importance

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Video note: In these slides, the out-edges Katz centrality is used. For these exercises we will use the more common in-edges Katz Centrality, which is defined below.

Katz Centrality

There is an issue with eigenvector centrality. Consider the adjacency matrix of a **directed, acyclic graph (DAG)** . A DAG is a directed graph with no cycles (no self loops as well). The issue with eigenvector centrality in the case of DAGs is that the adjacency matrix A of a DAG has the property that A^ℓ contains all entries equal to 0 for some value ℓ (and hence for all values greater than ℓ). This leads to an issue in the application of the Perron-Frobenius theorem that we alluded to in the definition of eigenvector centrality – there is no convergence to a non-zero vector in the series of updates starting with an initial centrality vector. In particular, $\left(\mathbf{y}^k\right)^T \to \mathbf{0}^T$ as $k \to \infty$.

This leads **Katz centrality** , a centrality measure that corrects this issue with eigenvector centrality in the case of DAGs. The update equation is modified to be

$$\left(\mathbf{y}^{k+1}\right)^T = \alpha\left(\mathbf{y}^k\right)^T A + \beta \mathbf{1}^T,$$

where α is chosen in the interval $(0, 1/\lambda_{\max}(A))$. We can show that with this update equation

$$\text{as } k \to \infty, \quad \left(\mathbf{y}^k\right)^T \to \mathbf{v}^T,$$

where $\mathbf{v}^T = \beta \mathbf{1}^T (\mathbf{I} - \alpha A)^{-1}$. For the case of DAGs, $\lambda_{\max} = 0$ and hence we can simply choose any value of α ; for example, $\alpha = 1$.

Directed Acyclic Graphs (DAGs)

4/4 points (graded)
Let A be the adjacency matrix of a DAG.

1. What is the sum of the diagonal values of A?

0 ✓ **Answer:** 0

2. A is **nilpotent** . That is, A^ℓ has all entries equal to 0 for some value of ℓ. What is the determinant of A?

0 ✓ **Answer:** 0

3. Let λ be an eigenvalue of a matrix $\boldsymbol{M}$ (not necessarily the adjacency matrix of a DAG). Let $\mathbf{v}$ be an eigenvector of $\boldsymbol{M}$. What is $\boldsymbol{M}^\ell\mathbf{v}$ equal to in terms of the corresponding eigenvalue λ and $\mathbf{v}$? Use `v` for $\mathbf{v}$, `lambda` for λ, `l` for ℓ.

lambda^l*v ✓ **Answer:** lambda^l * v

4. Do the fact that A^ℓ is a zero matrix and 3. above together imply that all the eigenvalues of the adjacency matrix of a DAG are equal to 0?

● Yes

○ No

✓

Solution:

1. **0.** A DAG has no self loops.
2. **Also equal to 0** since $0 = \det\left(A^\ell\right) = \left(\det\left(A\right)\right)^\ell$.
3. $\lambda^\ell\mathbf{v}$. This can be seen by induction or by writing out what $A^\ell\mathbf{v}$ is explicitly.
4. **Yes.** If there is a non-zero eigenvalue, then we can obtain a contradiction in 3. for the case of the adjacency matrix of a DAG.

Submit You have used 1 of 3 attempts

Answers are displayed within the problem

Katz Centrality

5/5 points (graded)

Consider the adjacency matrix

$$A = \begin{pmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}.$$

Raw matrix

Python:

```
    [[0,1,1,1],
[0,0,0,0],
[0,0,0,0],
[0,0,0,0]]
```

Mathematica:

```
        {{0, 1, 1, 1},
    {0, 0, 0, 0},
    {0, 0, 0, 0},
    {0, 0, 0, 0}}
```

Hide

1. Does it make sense to talk of the eigenvector centrality?

○ Yes

● No

✓

2. Compute the Katz centrality values using a computational tool. In **networkx**, use the default values of $\alpha = 0.1$ and $\beta = 1$ for the `networkx.katz_centrality` function. Make sure your resulting centrality vector is normalized to $\sqrt{\sum_i v_i^2} = 1$ if necessary. Provide an answer accurate to at least three **significant figures** (graded to 1% tolerance).

Note: In this problem, use the in-edges Katz centrality, as defined in the notes above.

Node 0: 0.46473941 ✓ **Answer:** 0.4647

Node 1: 0.51121335 ✓ **Answer:** 0.5112

Node 2: 0.51121335 ✓ **Answer:** 0.5112

Node 3: 0.51121335 ✓ **Answer:** 0.5112

Solution:

1. **No**. The graph is a DAG.
2. Because the graph is a DAG, we can set the value of α to be any value larger than zero, so the default value of $\alpha = 0.1$ in **networkx** is appropriate. Using these default values for α and β, we obtain that the centrality values are equal to **0.4647, 0.5112, 0.5112, 0.5112.**.

   Python:

   ```
   graph = networkx.from_numpy_matrix(np.array(A), create_using=networkx.DiGraph)
   networkx.katz_centrality(graph, alpha=0.1, beta=1)
   ```

   Mathematica:

   ```
   KatzCentrality[AdjacencyGraph[A], 0.1, 1]
   ```

   Note that Mathematica does not normalize the centrality vector, so make sure you do so after computation.

Submit You have used 1 of 2 attempts

Answers are displayed within the problem

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Katz Centrality

discussion posted 2 months ago by **estuardozapeta**

Hi,

I don't know if I did something wrong, use networkx.katz_centrality as follows:

G = nx.convert_matrix.from_numpy_matrix (A)

centrality = nx.katz_centrality (G, normalized = True)

It seems that it is not correct. Any hint?. It could be for the decimals?

This post is visible to everyone.

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2 responses

vilgalys (Staff)
2 months ago

That looks like the right procedure - make sure A is the right type? Should be a numpy array.

Other than that, just make sure you're entering to 3 significant figures and this looks sound to me.

Add a comment

marika_catapano
2 months ago

I had the same problem. It was wrong only for node 0, the rest was correct. Might be dofferent versions of the package?

Hope this is useful!

https://networkx.org/documentation/stable/reference/algorithms/generated/networkx.algorithms.centrality.katz_centrality.html

posted 2 months ago by **asalimw**

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