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EXERCISE: UNIFORM PROBABILITIES ON THE INTEGERS (1/1 point)

Let the sample space be the set of positive integers. Is it possible to have a "uniform" probability law, that is, a probability law that assigns the same probability c to each positive integer?

[No ▾]   ✔ **Answer:** No

Answer:
Suppose that $c = 0$. Then, by countable additivity,

$$1 = \mathbf{P}(\Omega) = \mathbf{P}\big(\{1\} \cup \{2\} \cup \{3\} \cdots \big) = \mathbf{P}(\{1\}) + \mathbf{P}(\{2\}) + \mathbf{P}(\{3\}) + \cdots = 0 + 0 + 0 + \cdots$$

which is a contradiction.

Suppose that $c > 0$. Then, there exists an integer k such that $kc > 1$. By additivity,

$$\mathbf{P}\big(\{1, 2, \ldots, k\}\big) = kc > 1,$$

which contradicts the normalization axiom.

You have used 1 of 1 submissions