Problem 4: Victor and his umbrella

(6/6 points)

Before leaving for work, Victor checks the weather report in order to decide whether to carry an umbrella. On any given day, with probability 0.2 the forecast is “rain" and with probability 0.8 the forecast is “no rain". If the forecast is “rain", the probability of actually having rain on that day is 0.8. On the other hand, if the forecast is “no rain", the probability of actually raining is 0.1.

1. One day, Victor missed the forecast and it rained. What is the probability that the forecast was “rain"?

   0.6666667 ✔ **Answer:** 0.66667

2. Victor misses the morning forecast with probability 0.2 on any day in the year. If he misses the forecast, Victor will flip a fair coin to decide whether to carry an umbrella. (We assume that the result of the coin flip is independent from the forecast and the weather.) On any day he sees the forecast, if it says “rain" he will always carry an umbrella, and if it says “no rain" he will not carry an umbrella. Let U be the event that “Victor is carrying an umbrella", and let N be the event that the forecast is “no rain". Are events U and N independent?

Problem Set 2 due Feb 17, 2016 at 23:59 UTC

- Unit 3: Counting
- Unit 4: Discrete random variables
- Exam 1
- Unit 5: Continuous random variables
- Unit 6: Further topics on random variables
- Unit 7: Bayesian inference
- Exam 2
- Unit 8: Limit theorems and classical statistics
- Unit 9: Bernoulli and Poisson processes

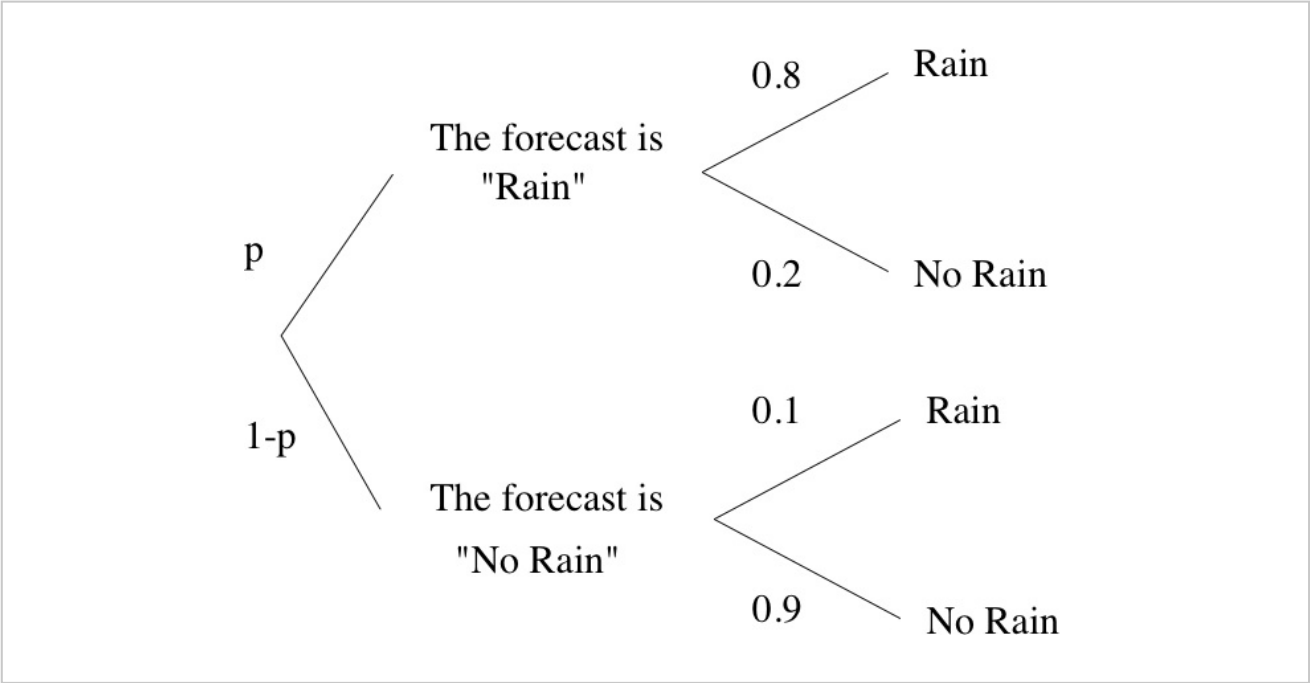
No ✔ **Answer:** No

3. Victor is carrying an umbrella and it is not raining. What is the probability that he saw the forecast?

0.2962963 ✔ **Answer:** 0.29630

Answer:

1. We can represent the scenario using the following tree diagram, where $p = 0.2$ is the probability that the forecast is “rain".

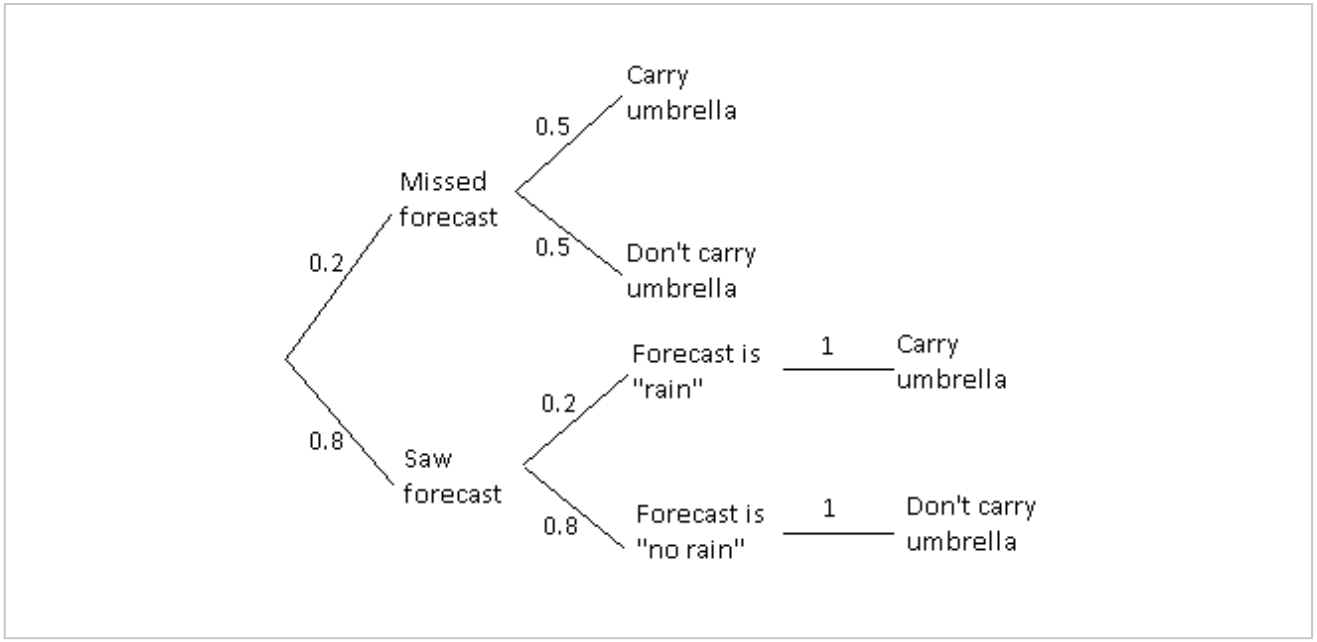


- Unit 10: Markov chains
- Exit Survey
- Final Exam

Let A be the event that the forecast was “rain", and let B be the event that it rained. Then,

$$\mathbf{P}(A \mid B) = \frac{\mathbf{P}(A)\mathbf{P}(B \mid A)}{\mathbf{P}(B)} = \frac{(0.2)(0.8)}{(0.2)(0.8)+(0.8)(0.1)} = \frac{2}{3}.$$

2. The tree diagram in this case is the following:



$$\begin{aligned}
\mathbf{P}(U) &= (0.2)(0.5)+(0.8)(1)(0.2) = 0.26 \\
\mathbf{P}(N) &= 0.8 \\
\mathbf{P}(U \cap N) &= \mathbf{P}(U \cap N \mid \text{Missed forecast})\mathbf{P}(\text{Missed forecast})
\end{aligned}$$

$$\begin{aligned} &+\mathbf{P}(U \cap N \mid \textbf{Saw forecast})\mathbf{P}(\textbf{Saw forecast}) \\ = \ &(0.5 \cdot 0.8)(0.2) + (0)(0.8) = 0.08 \end{aligned}$$

Since $\mathbf{P}(U \cap N) \neq \mathbf{P}(U)\mathbf{P}(N)$, the two events are not independent.

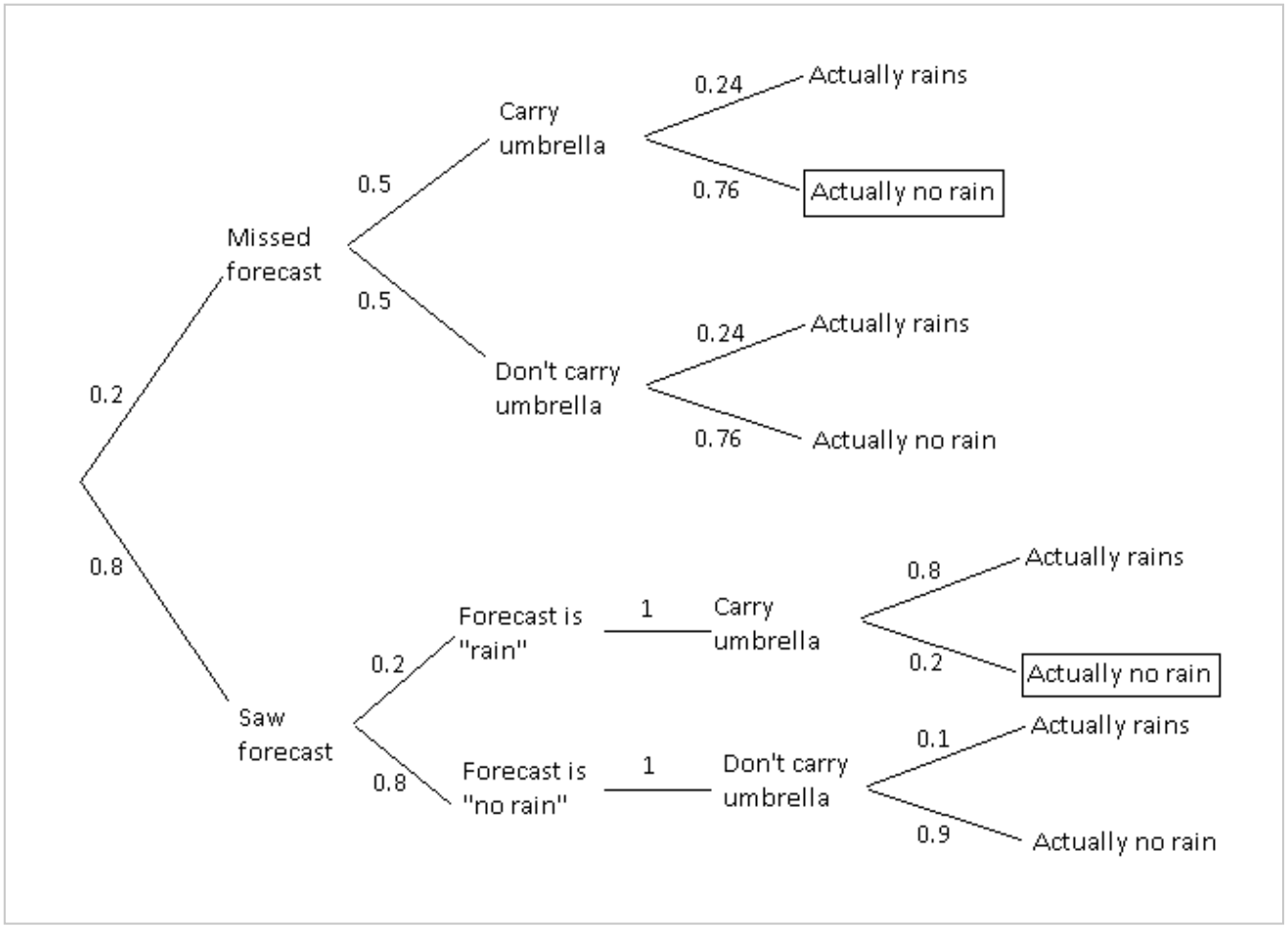
3. Let us first find the probability that it actually rains:

$$\mathbf{P}(\textbf{Actually rains}) = (0.8)(0.2) + (0.1)(0.8) = 0.24.$$

Thus,

$$\mathbf{P}(\textbf{Actually no rain}) = 1 - 0.24 = 0.76.$$

We can then extend the tree from part 2 as follows:



The event that Victor is carrying an umbrella and it is not raining corresponds to the two boxed cases in the figure above, of which the lower one is the one where Victor saw the forecast. Therefore, the desired probability is

$$\begin{aligned}&\mathbf{P}(\text{Saw forecast} \mid \text{Umbrella and actually no rain})\\ &= \frac{(0.8)(0.2)(1)(0.2)}{(0.8)(0.2)(1)(0.2)+(0.2)(0.5)(0.76)}\end{aligned}$$

$$= \frac{8}{27}.$$

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DISCUSSION

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