1. Zipline

Introducing a new problem

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Write the first order system of differential equations

4 points possible (graded, results hidden)

The upshot is that setting $\dot{x} = v_x$, and $\dot{y} = v_y$, we can write the second order system of equations

$$\begin{aligned} \ddot{x} &= \frac{1}{m} F_x \\ \ddot{y} &= \frac{1}{m} F_y \end{aligned}$$

as a system of four first order (nonlinear) differential equations.

Find $\dot{x}$, $\dot{y}$, $\dot{v}_x$, and $\dot{v}_y$ in terms of v_x, v_y, F_x, F_y, m. Type **F_ x** for F_x; **F_ y** for F_y; **v_x** for v_x; **v_y** for v_y.

$\dot{x} =$ v_x

v_x

$\dot{y} =$ v_y

v_y

$\dot{v}_x =$ F_x/m

$\frac{F_x}{m}$

$\dot{v}_y =$ F_y/m

$\frac{F_y}{m}$

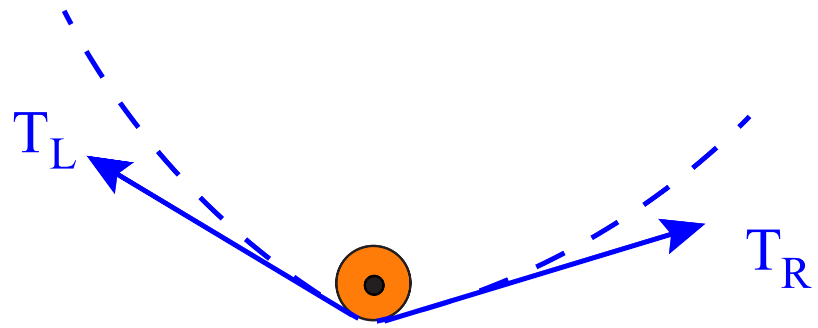
FORMULA INPUT HELP

Submit You have used 1 of 10 attempts

Answer submitted.

A simplification using torque balance

Note that the modeling of this system is much more complicated than any other system we have modeled thus far. One reason for this is the pulley which is what moves the rider along the cable. Because the mass of the pulley is negligible, the torques on the pulley must balance. What are the forces which may contribute to the torque?



There are two tension forces $\mathbf{T}_L$ and $\mathbf{T}_R$ tangent to the outside edge of the pulley pointing along the cable to the left and right of the pulley.

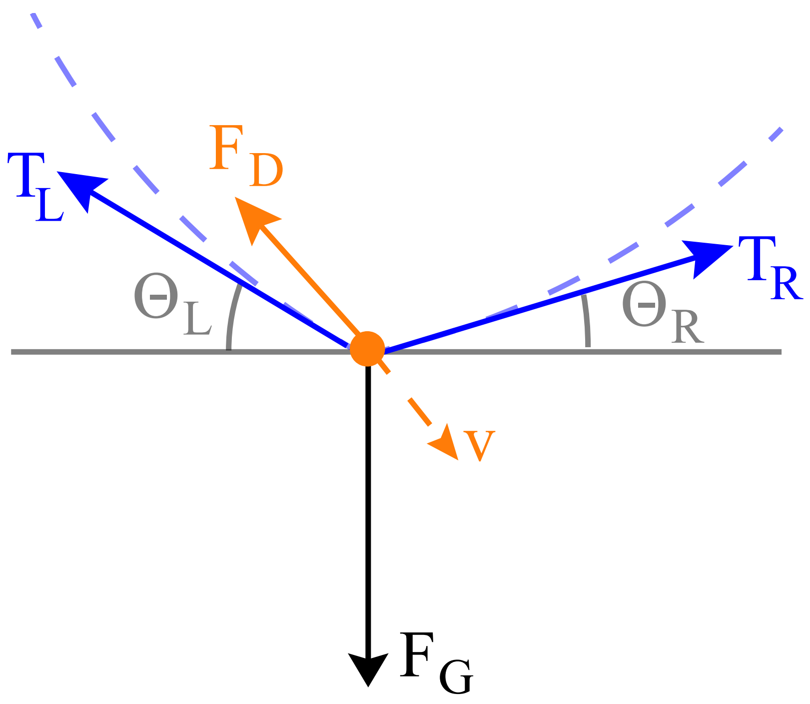
If we take the origin of the pulley to be the center of the pulley, then the forces exerted on the pulley by the rider are applied to the center and do not contribute to the torque. In this model, we will ignore friction in the pulley. Thus the only two forces contributing to the torque are the two tension forces. The tension forces have different directions at the same radial distance from the center of the pulley. Thus torque balance tells us that the magnitude of these two tension forces must be equal. We write the magnitude as T, setting

$$T = |\mathbf{T}_L| = |\mathbf{T}_R|.$$

Find horizontal and vertical components of force

2 points possible (graded, results hidden)

Modeling the rider as a point along the cable, there are four forces in total acting on the rider. The force of gravity $\mathbf{F}_G$ pointing down. Two tension forces $\mathbf{T}_L$ and $\mathbf{T}_R$ parallel to the zipline cable on the left and right of the rider. Based on the system you found in the previous problem, we need to find expressions for the horizontal and vertical components of the forces acting on the zipline rider.



Let the angle between the horizontal and the tension forces be denoted by θ_L and θ_R respectively. The final force is a damping force $\mathbf{F}_D$, which is parallel to the velocity vector of the rider but pointing in the opposite direction.

The magnitudes of these forces are

$$\begin{aligned} |\mathbf{F}_G| &= mg \\ |\mathbf{T}_L| &= T \\ |\mathbf{T}_R| &= T \\ |\mathbf{F}_D| &= c|\mathbf{v}|^2 \end{aligned}$$

The forces horizontal and vertical forces F_x and F_y are nonlinear functions that depend on the unknown variables T, θ_R, θ_L. These variables are determined by the shape of the cable. (To specify this system, we must next determine what these variables are.)

Find the horizontal and vertical components of the total force acting on the rider.

Enter your answer in terms of $T, \theta_L, \theta_R, c, v_x$ and v_y (where $v_x = \dot{x}$ and $v_y = \dot{y}$), m, and g.

Type **theta_ R** for θ_R; **theta_ L** for θ_L; **v_x** for v_x; **v_y** for v_y.

$F_x =$ T*cos(theta_R) - T*cos(theta_L)-c*v_x*sqrt(v_x^2+v_y^2)

$$T \cdot \cos(\theta_R) - T \cdot \cos(\theta_L) - c \cdot v_x \cdot \sqrt{v_x^2 + v_y^2}$$

$F_y =$ T*sin(theta_R) + T*sin(theta_L)-m*g-c*v_y*sqrt(v_x^2+v_y^2)

$$T \cdot \sin(\theta_R) + T \cdot \sin(\theta_L) - m \cdot g - c \cdot v_y \cdot \sqrt{v_x^2 + v_y^2}$$

FORMULA INPUT HELP

Submit You have used 8 of 25 attempts

Answer submitted.

1. Zipline

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Zipline horizontal and vertical components of force

question posted 2 days ago by PatentGuy

Just confirming that positive is to the right for x and upward for y. Also, since mg is not allowed in the answer, I assume we're only finding the positive (upward) vertical force components.

This post is visible to everyone.

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1 response

jfrench (Staff)
2 days ago

yes! Also m*g is allowed in the answer.

And does c take care of the directional component according to the diagram, so that cv_x^2 points in the $-x$ direction and cv_y^2 points in the $+y$ direction? Or do we have to change the sign?

posted 2 days ago by **PatentGuy**

Correct!

posted 2 days ago by **jfrench** (Staff)

Heh heh. I'm assuming you mean it's correct that we *don't* have to change the sign, i.e., c takes care of the direction so we can simply add the appropriate cv^2 component to each entry.

posted 2 days ago by **PatentGuy**

Same doubt here.

posted 2 days ago by **OscarSG**

If mg is allowed in the answer, you should say. Enter your answer in terms of $T, \theta_L, \theta_R, c, v_x, v_y, m, g$

posted 2 days ago by **OscarSG**

Sure, Oscar I'll add m and g into the description.

And now I think I understand the original question.

You can look at the formula $|\mathbf{F}_d| = -c|\mathbf{v}^2|$ to determine that the c does not include the negative sign natively.

posted 2 days ago by **jfrench** (Staff)

the horizontal and vertical components of $\mathbf{v}$ are v_x and v_y right?

posted 2 days ago by **OscarSG**

Do we need to write the absolute value symbol in our response?

posted 2 days ago by **OscarSG**

@PatentGuy c is just a (positive) constant. v_x^2 also is always positive, so $F_x = cv_x^2$ would always point in the positive x-direction, irrespective of the sign of v_x, and therefore can't be right. Hint: v_x and v_y can be either positive or negative, depending on their direction, so you'll need those.

@OscarSG Yes, v_x and v_y are the components of $\mathbf{v}$. I don't think you need absolute value function/symbols in your response, although the grader kan can handle `abs(.)`.

posted a day ago by **mrBB** (Community TA)

Thanks, that makes sense, and I appreciate the helpful points you make.

posted a day ago by **PatentGuy**

@mrBB I'm stuck with this problem. We need to decompose 4 forces. Since one force is pointing directly downwards. One component of F will have only 3 terms and the other will have 4 terms. Also taking into consideration the assumptions about the sign of v_x and v_y we discussed before? Are my assumptions correct?

posted a day ago by **OscarSG**

Yes, that all seems to make sense. :-)

posted about 23 hours ago by **mrBB** (Community TA)

@mrBB you posted before Hint: v_x and v_y can be either positive or negative, depending on their direction, so you'll need those. what do you mean by that?

posted about 22 hours ago by **OscarSG**

If the rider is moving to the right, $v_x > 0$, if he is moving to the left $v_x < 0$.
If the rider is getting closer to the earth's center, $v_y < 0$, if he is moving upward $v_y > 0$.

posted about 19 hours ago by **yves-M** (Community TA)

@mrBB @yves-M I did all possible things that i could think of (including using abs(.), to ensure that damping force vector's direction / magnitude is correct), to consider the sign of the velocity vector's hor/vert components, but my answer I think is still not correct, any other hint :)? I am using $F_D = -c|v|^2\hat{v}$, where $\hat{v}$ is the unit vector represented using v only, for both hor/vert force components.

posted about 18 hours ago by **sandipan_dey** well...

- small hint: $\vec{\mathbf{v}} = v_x.\vec{\mathbf{e}}_x + v_y.\vec{\mathbf{e}}_y$ And $\vec{\mathbf{F}}_{Damping} = -c.\|\vec{v}\|^2.\frac{\vec{\mathbf{v}}}{\|\vec{v}\|}$. This should help you decompose $\vec{\mathbf{F}}_{Damping} = Fd_x.\vec{\mathbf{e}}_x + Fd_y.\vec{\mathbf{e}}_y$
- big hint: watch closely the video a few pages from here.

posted about 15 hours ago by **yves-M** (Community TA)

+x points to the right. +y points up.

Vector addition head to tail so shouldn't it be: $\overrightarrow{F_y} - \overrightarrow{F_x} = \overrightarrow{F_D}$?

posted about 14 hours ago by **subsole**

@yves-M the small hint is very useful, for magnitude we should consider the velocity, not its components, that's something i was missing. Thanks you very much.

posted less than a minute ago by **sandipan_dey**

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