Problem 2: Hypothesis test between two coins

(5/6 points)

Alice has two coins. The probability of Heads for the first coin is $1/3$, and the probability of Heads for the second is $2/3$. Other than this difference, the coins are indistinguishable. Alice chooses one of the coins at random and sends it to Bob. The random selection used by Alice to pick the coin to send to Bob is such that the first coin has a probability p of being selected. Assume that $0 < p < 1$ and that Bob knows the value of p. Bob tries to guess which of the two coins he received by tossing it 3 times in a row and observing the outcome. Assume that for any particular coin, all tosses of that coin are independent.

1. Given that Bob observed k Heads out of the 3 tosses (where $k = 0, 1, 2, 3$), what is the conditional probability that he received the first coin?

- $\frac{1}{2^{3-k}}$
- $\frac{2^{3-k}p}{2^{3-k}p + 2^k(1-p)}$ ✔
- $\frac{p}{2^{3-k}}$
- $\frac{1}{1 + \frac{p}{1-p}2^{3-2k}}$

2. We define an error to have occurred if Bob decides that he received one coin from Alice, but he actually received the other coin. He decides that he received the first coin when the number of Heads, k, that he observes on the 3 tosses satisfies a certain condition. When one of the following conditions is used, Bob will minimize the probability of error. Choose the correct threshold condition.

- ○ $k \geq \frac{3}{2} + \frac{1}{2}\log_2 \frac{p}{1-p}$
- ◉ $k \leq \frac{3}{2} + \frac{1}{2}\log_2 \frac{p}{1-p}$ ✔
- ○ $k \leq \frac{1}{2}\log_2 \frac{p}{1-p}$
- ○ $k \geq \frac{1}{2}\log_2 \frac{p}{1-p}$
- ○ none of the above

3. For this part, assume that $p = 2/3$.

(a) What is the probability that Bob will guess the coin correctly using the decision rule from part 2?

0.6714597 ✖

(b) Suppose instead that Bob tries to guess which coin he received without tossing it. He still guesses the coin in order to minimize the probability of error. What is the probability that Bob will guess the coin correctly under this scenario?

2/3 ✔

4. Suppose that we increase p. Then does the number of different values of k for which Bob decides that he received the first coin increase, decrease, or stay the same?

It increases or stays the same. ▾ ✔

5. Find the values of p for which Bob will never decide that he received the first coin, regardless of the outcome of the 3 tosses.

p is less than 1/9 ✔

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