Exercises: Sea level example

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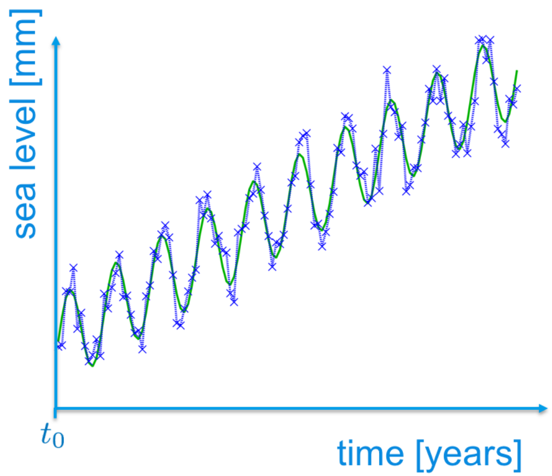
Sea level model: linear trend + annual signal

3/3 points (ungraded)



Assume we want to estimate the linear trend + annual signal (sine) as shown by the green graph in the figure (the blue crosses are the actual observations). What is the observation equation in this case (write down for yourself)?

Which are the unknown parameters? *Select all correct options.*

- [ ] the period T of the sine-function
- [x] the amplitute a of the sine-function
- [ ] the times of observation t_i
- [x] the rate of change of the sea level
- [x] the sea level l_0 at the first time of observation
- [ ] the sea levels at the times of observation

✔

Explanation
The observation equation becomes $E\{\underline{y}_i\} = l_0 + r \cdot \Delta t_i + a \sin(2\pi \Delta t_i)$, with Δt_i the known time differences with respect to t_0. The unknown parameters are l_0, r, and a.

We have monthly time gauge observations for a period of 2 years. What is the dimension of the A-matrix?

Number of rows:

24 ✔ **Answer:** 24

24

Number of columns:

3

✔ **Answer:** 3

3

Explanation

The dimension of the A-matrix is $m \times n$, with m the number of observations (24 in this case), and n the number of unknowns (3 in this case).

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✔ Correct (3/3 points)

In the following Matlab exercise you will be asked to construct the A-matrix for three different models:

1. $E\{\underline{y}_i\} = l_0 + r \cdot \Delta t_i$
2. $E\{\underline{y}_i\} = l_0 + a \sin(2\pi\Delta t_i)$
3. $E\{\underline{y}_i\} = l_0 + r \cdot \Delta t_i + a \sin(2\pi\Delta t_i)$

MATLAB EXERCISE SEA LEVEL RISE (EXTERNAL RESOURCE)

Sea level example

In this exercise you will set-up the A-matrix for the three models considered for the sea level observations (see Introduction text above this exercise).

The script will plot the model outcomes by solving the forward model for a given set of parameters $l_0 = 150$ mm, $r = 0.3$ mm/yr, $a = 0.5$ mm. This may help you to interpret the meaning of the different parameters.

The A-matrix for model 1 is already specified, fill in the correct expression for the other two models on lines 13 and 17.

Your Solution

Save Reset MATLAB Documentation (https://www.mathworks.com/help/)

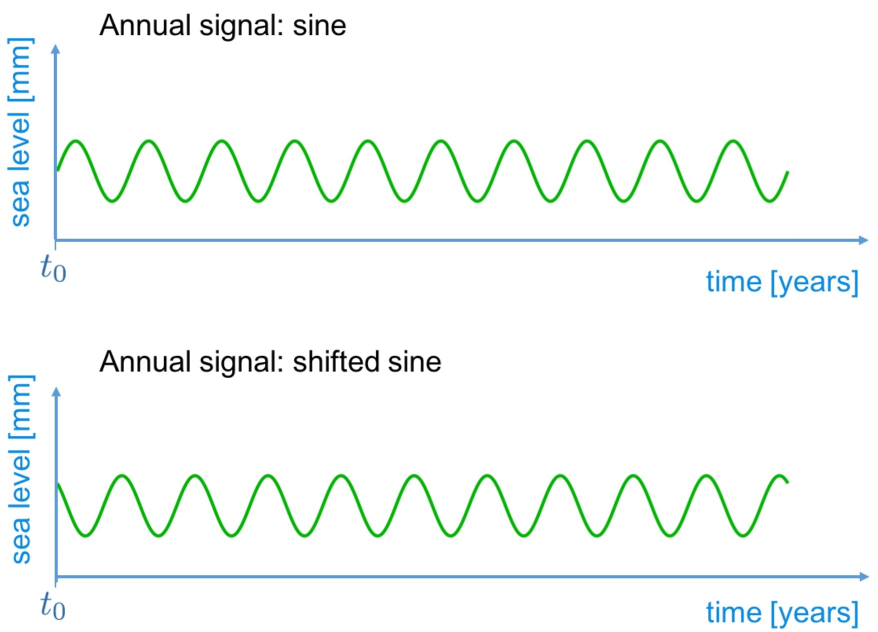
```
1  %% COLUMN VECTOR WITH TIMES OF OBSERVATION IN [YEARS]
2  t = (0:1/12:23/12)';
3
4  %% determine the number of observations
5  m = length(t);
6
7  %% MODEL 1: A-matrix for linear trend model, matrix is called A1
8  A1 = [ ones(m,1), t];
9
10 %% MODEL 2: A-matrix for annual signal model, matrix is called A2
11 %% NOTE: if x is a vector, y=sin(x) gives a vector with the sine
12 %% of the individual elements in x
13 A2 = [ ones(m,1), sin(2*pi*t)];  % REPLACE THE 0 WITH THE CORRECT EXPRESSION
14
15 %% MODEL 3: A-matrix for linear trend + annual signal model,
16 %% matrix is called A3
17 A3 = [ ones(m,1), t, sin(2*pi*t)];  % REPLACE THE 0 WITH THE CORRECT EXPRESSION
18
19 %% PLOT THE MODEL OUTCOMES for given values of the parameters
20 l0 = 150; % initial sea level at t0 in [mm]
```

```
l0 = 150; % initial sea level at t0 in [mm]
r  = 0.3; % rate of change in [mm/yr]
a  = 0.5; % amplitude of annual signal in [mm]
x1 = [l0 ; r];
x2 = [l0 ; a];
```

Annual signal with shifted sine

3/3 points (ungraded)

In reality we may need to consider a shifted version of the sine-function in case we are modelling an annual signal. See the example in the figure, where the top panel shows the default sine-model, the bottom panel shows the shifted sine-model.



Note that we do not consider a linear trend here.

What would be the corresponding observation equation for the shifted sine-function?

- ○ $E\{\underline{y}_i\} = l_0 + a\sin(2\pi\Delta t_i)$
- ◉ $E\{\underline{y}_i\} = l_0 + a\sin(2\pi(\Delta t_i + t_s))$ ✔
- ○ $E\{\underline{y}_i\} = l_0 + s + a\sin(2\pi\Delta t_i)$
- ○ $E\{\underline{y}_i\} = l_0 + a\cos(2\pi\Delta t_i)$

Explanation

The sine-function is shifted by t_s along the horizontal axis. You could also say that the initial phase at t_0 was equal to $\phi_0 = 2\pi t_s$. The fourth option would have been correct if the time shift was exactly equal to 6 months (half of the period of the annual signal).

What would be the number of unknowns for this model?

3 ✔ **Answer:** 3

3

Explanation

l_0, a, t_s

Which of the following statements is true for the shifted sine-model:

- [ ] l_0 is the initial sea level at t_0
- [x] l_0 is the mean sea level in one year
- [ ] The observation equations are linear in all unknown parameters.

Explanation

l_0 is the sea level when the sine-term is equal to zero, this is not the case at t_0. In fact, l_0 is equal to the mean sea level during a one year period (since we assume an annual signal). This is also true for the unshifted sine-model, in that case both the first and second statement would have been true. The observation equation is still linear in l_0 and a, but not in t_s. See the following note.

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Correct (3/3 points)

Sometimes it is possible to apply a re-parameterization such that the observation equations do become a linear function of the new parameters. Re-parameterization means that we define new parameters, which are a function of the original ones.

We will show how this can be done for the model based on the annual signal with shifted sine.

The observation equation is given by

$$E\{\underline{y}_i\} = l_0 + a\sin(2\pi\Delta t_i + \phi_0)$$

with $\phi_0 = 2\pi t_s$.

The observation equation is non-linear in ϕ_0. We will now apply some basic laws from trigonometry to do a re-parameterization.

From trigonometry we know that

$$\sin(u+v) = \sin u\cos v + \cos u\sin v$$

Applying this to the last term of our observation equation we get:

$$\begin{aligned} a\sin(2\pi\Delta t_i + \phi_0) &= a(\sin 2\pi\Delta t_i\cos\phi_0 + \cos 2\pi\Delta t_i\sin\phi_0) \\ &= a_s\sin 2\pi\Delta t_i + a_c\cos 2\pi\Delta t_i \end{aligned}$$

with

$$a_s = a\cos\phi_0 \quad \text{and} \quad a_c = a\sin\phi_0$$

Hence, we have defined two new parameters as a function of the original parameters a and ϕ_0.

Note that we can also find the inverse relations:

$$a = \sqrt{a_c^2 + a_s^2} = \sqrt{a^2\cos^2\phi_0 + a^2\sin^2\phi_0}$$

since: $\cos^2 \phi_0 + \sin^2 \phi_0 = 1$.

And

$$\phi_0 = \arctan \frac{a_c}{a_s}$$

since: $\tan \phi_0 = \frac{\sin \phi_0}{\cos \phi_0}$.

The linear observation equation becomes thus:

$$E\{\underline{y}_i\} = l_0 + a_s \sin(2\pi\Delta t_i) + a_c \cos(2\pi\Delta t_i)$$