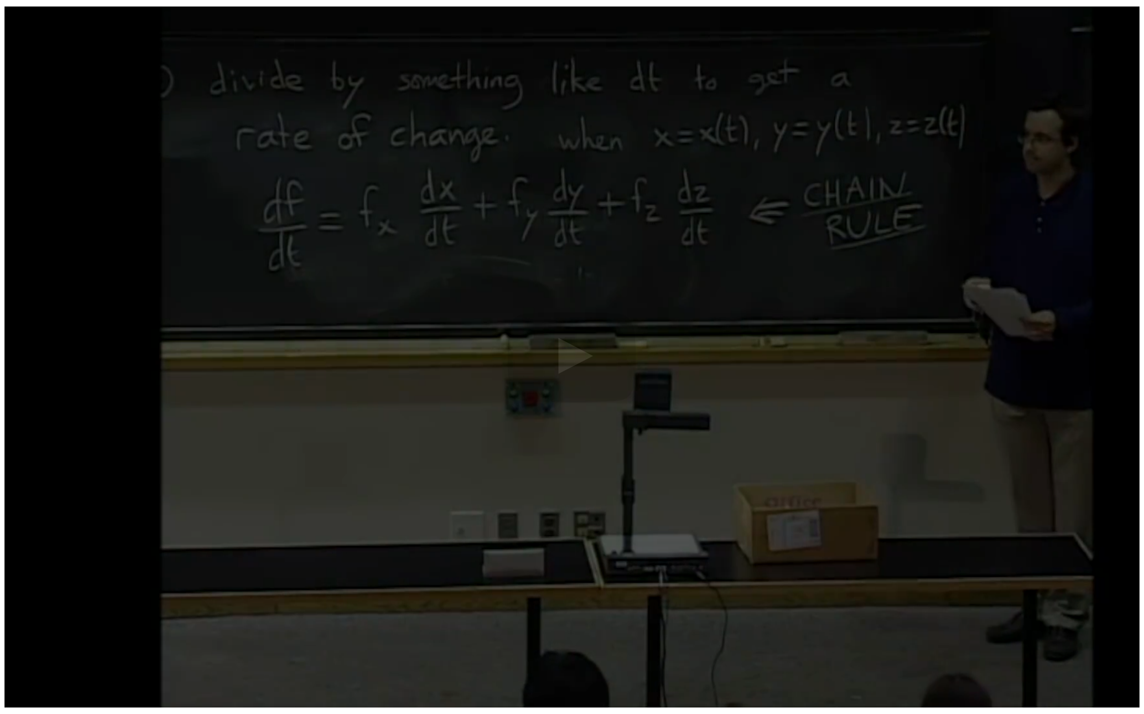
4. Multivariable Chain Rule

Explore

Dividing Differentials



2:40 / 2:40 2.0x

And that corresponds to the situation

where x is a function of t, y is a function of t,

and z is a function of t.

Then, that means you can plug in these values into f to get--

well, the value of f will depend on t

And then you can find the rate of change

with t of a value of f.

So these are the basic rules.

And this is known as the chain rule.

It's one instance of a chain rule, which tells you

when you have a function that depends on something

and that something in turn depends on something else

how to find the rate of change of a function

Video
Download video file

Transcripts
Download SubRip (.srt) file
Download Text (.txt) file

Multivariable Chain Rule

The **multivariable chain rule** is needed when we need to differentiate a function whose inputs are controlled by another variable. Imagine a function that depends on x, y and z such as $f = f(x, y, z)$. Now imagine that we cannot control x, y, and z directly and instead they each depend on a parameter t. This means changing the variable t will cause the function f to change, and we would like to know the corresponding rate of change $\frac{df}{dt}$.

One way of finding $\frac{df}{dt}$ is to use differentials. We know:

$$df = f_x\, dx + f_y\, dy + f_z\, dz. \tag{6.122}$$

Now “divide everything by dt" to obtain a formula for $\frac{df}{dt}$:

$$\frac{df}{dt} = f_x \frac{dx}{dt} + f_y \frac{dy}{dt} + f_z \frac{dz}{dt}. \tag{6.123}$$

There you have it: this is the multivariable chain rule (at least, one manifestation of the multivariable chain rule). It gives us a recipe for finding $\frac{df}{dt}$ in terms of the intervening rates of change.

Example 4.1 An example might make the idea more clear. Imagine a box of height y with a square base of width x. Then the volume is given by $V = x^2 y$. Now suppose that we

Calculator Hide Notes

values of x and y directly, but they each depend on a parameter t. In this example, let's imagine $x = (1+t)^2$ and $y = 3t$. Now changing the variable t will cause the volume V to change, and we would like to know the corresponding rate of change, that is, the value of $\frac{dV}{dt}$.

We can use differentials. We know:

$$dV = V_x\,dx + V_y\,dy \qquad (6.124)$$

Now "divide everything by dt" to obtain a formula for $\frac{dV}{dt}$:

$$\frac{dV}{dt} = V_x \frac{dx}{dt} + V_y \frac{dy}{dt} \qquad (6.125)$$

.

We find V_x and V_y from $V = x^2 y$. We find $\frac{dx}{dt}$ from $x = (1+t)^2$ and $\frac{dy}{dt}$ from $y = 3t$.

$$\frac{dV}{dt} = \underbrace{(2xy)}_{V_x}\underbrace{(2\,(1+t))}_{\frac{dx}{dt}} + \underbrace{(x^2)}_{V_y}\,\underbrace{(3)}_{\frac{dy}{dt}} \qquad (6.126)$$

Now we write everything in terms of t to obtain:

$$\frac{dV}{dt} = (2)\,(1+t)^2\,(3t)\,(2\,(1+t)) + (1+t)^4\,(3)\,, \qquad (6.127)$$

which simplifies to $\frac{dV}{dt} = 12t(1+t)^3 + 3(1+t)^4$.

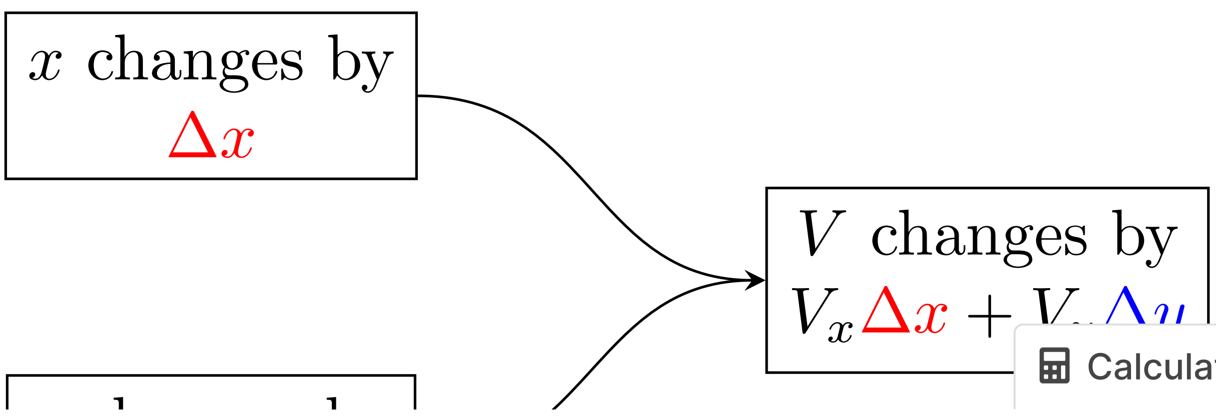
Remark 4.2 In this example, one could also proceed by finding the explicit formula for V in terms of t. We would plug in $x = (1+t)^2$ and $y = 3t$ to $V = x^2 y$ to get $V = (1+t)^4\,(3t)$. Now one can take a single-variable derivative to obtain $\frac{dV}{dt}$, obtaining the same answer in the end.

Although we get the same answer either way, differentiating $V = (1+t)^4\,(3t)$ requires using a product rule. Using differentials, we only had to use the power rule.

Later, we will see some situations in which the only way to proceed is by using the multivariable chain rule.

Chain Rule in Pictures

The following diagram shows how changing x and y causes V to change.



Calculator Hide Notes

If x and y depend on t, then the change in t indirectly causes a change in V. The following diagram shows how changing t causes V to change. Notice the “chain" of variables, giving rise to the “chain rule".

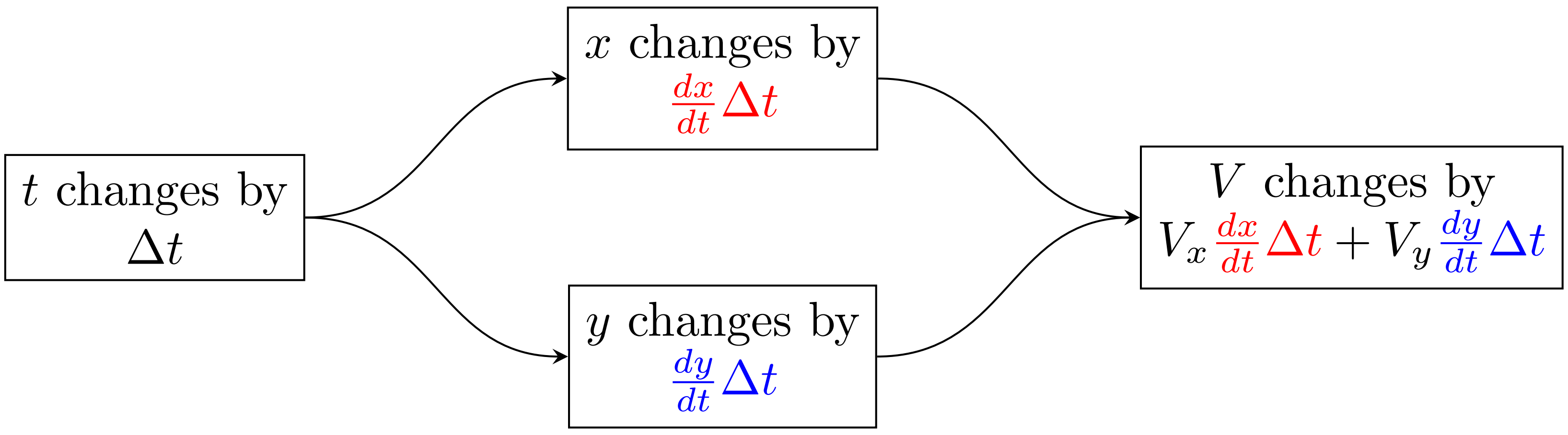


Photo by Joey Kyber from Pexels

What is the statement of the chain rule?

The multivariable chain rule differs from the single-variable chain rule because, rather than a single formula, it represents a general principle. In words, the theorem behind the chain rule says **any partial derivative can be computed by looking at the chain of transformations that take the input to the output, and forming the product of the partial derivatives for each link in the chain** (as shown in the diagrams above). It is possible to package this statement into formulas, which are given at the end of this lecture.

4. Multivariable Chain Rule

Hide Discussion

Topic: Unit 5: Curves and Surfaces / 4. Multivariable Chain Rule

Add a Post

Show all posts by recent activity

There are no posts in this topic yet.

✖

< Previous Next >

Calculator Hide Notes

edX

About
Affiliates
edX for Business
Open edX
Careers
News

Legal

Terms of Service & Honor Code
Privacy Policy
Accessibility Policy
Trademark Policy
Sitemap

Connect

Blog
Contact Us
Help Center
Media Kit
Donate

GET IT ON Google play
Download on the App Store

© 2021 edX Inc. All rights reserved.
深圳市恒宇博科技有限公司 粤ICP备17044299号-2

Calculator Hide Notes