8. Let A and B both be $n \times n$ matrices and both be invertible.

$C = AB$ is invertible

Always/Sometimes/Never

Justify (prove) your answer.

Let $D = C^{-1}$

Claim $D = B^{-1} A^{-1}$

then

$CD = I$

and $DC = I$

$$CD = (AB)(B^{-1} A^{-1})$$
$$= A (BB^{-1}) A^{-1}$$
$$= (A \ I) A^{-1}$$
$$= A A^{-1}$$
$$= I$$

D is C^{-1}

$$DC = (B^{-1} A^{-})(AB)$$
$$= B^{-1} (A^{-1} A) B$$
$$= B^{-1} I B$$
$$= B^{-1} B$$
$$= I$$

8