1.3 Naive definition of probability

Unit 1: Probability and Counting

Adapted from Blitzstein-Hwang Chapter 1.

Historically, the earliest definition of the probability of an event was to count the number of ways the event could happen and divide by the total number of possible outcomes for the experiment. We call this the *naive definition* since it is restrictive and relies on strong assumptions; nevertheless, it is important to understand, and useful when not misused.

> DEFINITION 1.3.1 (NAIVE DEFINITION OF PROBABILITY).
>
> Let A be an event for an experiment with a finite sample space S. The *naive probability* of A is
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> $$P_{\text{naive}}(A) = \frac{|A|}{|S|} = \frac{\text{number of outcomes favorable to } A}{\text{total number of outcomes in } S},$$
>
> where $|A|$ is the size (cardinality) of set A.

The naive definition is very restrictive in that it requires S to be finite, with equal mass for each pebble. It has often been misapplied by people who assume equally likely outcomes without justification and make arguments to the effect of "either it will happen or it won't, and we don't know which, so it's 50-50". For example, if we don't know whether or not there is life on Saturn, should we conclude that it is 50-50? What about *intelligent* life on Saturn, which seems like it should be strictly less likely than there being any form of life on Saturn? But there are several important types of problems where the naive definition *is* applicable, such as when there is *symmetry* in the problem that makes the outcomes equally likely.