3. Extra Dependencies

Recitation due Oct 5, 2021 20:30 IST Completed

Derivative of Composition

2/2 points (graded)

Suppose $g(x, y)$ has

$$\frac{\partial g}{\partial x} = 2x, \qquad \text{and} \qquad \frac{\partial g}{\partial y} = e^{-y^2}. \tag{6.234}$$

Let

$$f(x, y, z) = x^2 - \ln(y) + 2z, \tag{6.235}$$

and let

$$w = f(x, y, g(x, y)). \tag{6.236}$$

Compute the partial derivatives of w.

$\frac{\partial w}{\partial x} =$ 6*x ✓

$\frac{\partial w}{\partial y} =$ -1/y+2*e^(-y^2) ✓

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Surface

2/2 points (graded)

Let S be the surface described by the equation $\frac{1}{x} + \arctan(y + 2z) = 1$. Compute $\frac{\partial x}{\partial z}$ and $\frac{\partial y}{\partial z}$ along S at the point (x, y, z).

Note: In this context, $\frac{\partial x}{\partial z}$ is the instantaneous rate of change of x with respect to z, holding y constant. In other words, if z were to change slightly and y were held constant, then x would have to change in order to maintain the equation $\frac{1}{x} + \arctan(y + 2z) = 1$. Then $\frac{\partial x}{\partial z}$ is the associated instantaneous rate of change of x.

$\frac{\partial x}{\partial z} =$ (2*x^2)/(1+(y+2*z)^2) ✓ **Answer:** 2*x^2/(1+(y+2*z)^2)

$\frac{\partial y}{\partial z} =$ -2 ✓ **Answer:** -2

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Solution:

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Approach 1: Differentials

We define $w(x,y,z) = \frac{1}{x} + \arctan(y+2z)$. Then we compute the total differential of w:

$$dw = \frac{-1}{x^2}\,dx + \frac{1}{1+(y+2z)^2}\,dy + \frac{2}{1+(y+2z)^2}\,dz \tag{6.243}$$

On the surface S, we have $dw = 0$. Solving for dx in the resulting equation, we have:

$$dx = \frac{x^2}{1+(y+2z)^2}\,dy + \frac{2x^2}{1+(y+2z)^2}\,dz \tag{6.244}$$

Now we can obtain $\frac{\partial x}{\partial z}$ by reading off the coefficient on dz. Thus, $\frac{\partial x}{\partial z} = \frac{2x^2}{(1+(y+2z)^2}$.

In a similar way, we can solve for dy instead:

$$dy = \left(\frac{1+(y+2z)^2}{x^2}\right)\,dx + (-2)\;dz \tag{6.245}$$

It follows that $\frac{\partial y}{\partial z} = -2$.

Approach 2: Implicit Differentiation

We may also use regular single-variable implicit-differentiation. We have the equation:

$$1 = \frac{1}{x} + \arctan(y+2z) \tag{6.246}$$

Now take $\frac{\partial}{\partial z}$ of both sides (treating y as a constant):

$$\frac{\partial}{\partial z}(1) \;= \frac{\partial}{\partial z}\left(\frac{1}{x} + \arctan(y+2z)\right) \tag{6.247}$$

$$0 \;= \frac{-1}{x^2}\frac{\partial x}{\partial z} + \frac{2}{1+(y+2z)^2} \tag{6.248}$$

Solving for $\frac{\partial x}{\partial z}$ gives

$$\frac{\partial x}{\partial z} = \frac{2x^2}{1+(y+2z)^2} \tag{6.249}$$

Using an analogous procedure to find $\frac{\partial y}{\partial z}$, we have:

$$\frac{\partial}{\partial z}(1) \;= \frac{\partial}{\partial z}\left(\frac{1}{x} + \arctan(y+2z)\right) \tag{6.250}$$

$$0 \;= 0 + \frac{1}{1+(y+2z)^2}\left(\frac{\partial y}{\partial z} + 2\right)$$

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Solving for $\frac{\partial y}{\partial z}$ again produces $\frac{\partial y}{\partial z} = -2.$

Approach 3: Chain Rule

One can also use the chain rule to obtain the same answers. This approach is essentially the same as Approach 1.

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Answers are displayed within the problem

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[STAFF] Is the grader working correctly for Surface?

discussion posted a day ago by **valleymd**

Is the grader working properly on this problem (Surface) or am I missing something?

This post is visible to everyone.

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4 responses

zhanybeksuleimenov
a day ago

My answers were marked wrong despite they are the same as in the solution. Problem with the grader.

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DuncanLL (Staff)
about 19 hours ago

Apologies valleymd and zhanybeksuleimenov! There was indeed a mistake in the grader. I've fixed it, reset attempts, and rescored previous submissions. Thank you for pointing this out.

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jfrench (Staff)
about 19 hours ago

@valleymd the grader is fixed! If you resubmit it should work correctly now.

Thanks

posted about 9 hours ago by **valleymd**

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sandipan_dey
about 11 hours ago

For $\partial x/\partial z$ I tried to express the surface equation as z=f(x) and then compute the partial derivative but grader did not accept, although grader accepted $\partial y/\partial z$ when solved using similar approach. I have no clue where I am doing wrong, any help will be appreciated, thanks in advance.

Did you remember to invert the partial derivative?

An easier way to solve it might be to take differentials of the original equation, set $\partial y = 0$, then solve for $\frac{\partial x}{\partial z}$.

posted about 11 hours ago by **martincmartin**

Yeah I inverted, it did not work, grader does not seem to use the identity $sec^2 x - tan^2 x = 1$ / does not like the answer involving the variable x only, anyway it worked by direct differentiation, thank you very much

posted less than a minute ago by **sandipan_dey**

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