The Quadratic Reciprocity Law (1)

The Quadratic Reciprocity Law

$P \neq Q$ **odd** prime numbers

(1) If $P \equiv 1 \pmod 4$ or $Q \equiv 1 \pmod 4$,

P is QR (mod Q) $\Leftrightarrow$ **Q is QR** (mod P)

(2) If $P \equiv Q \equiv 3 \pmod 4$,

P is QR (mod Q) $\Leftrightarrow$ **Q is not QR** (mod P).

$$\left(\frac{Q}{P}\right) = (-1)^{\frac{P-1}{2}\frac{Q-1}{2}} \left(\frac{P}{Q}\right)$$

The Quadratic Reciprocity Law (2)

Supplements to QRL

P **odd** prime number

(3) **−1 is QR** (mod P) ⇔ **P ≡ 1 (mod 4)**.

(4) **2 is QR** (mod P) ⇔ **P ≡ 1 or 7 (mod 8)**.

$$\left(\frac{-1}{P}\right) = (-1)^{(P-1)/2} \qquad \left(\frac{2}{P}\right) = (-1)^{(P^2-1)/8}$$

The Quadratic Reciprocity Law (3)

- Currently, more than two hundred proofs of QRL are known.
- We shall present Eisenstein's beautiful proof.
  It is an improvement of Gauss's third proof.

Ferdinand Gotthold Max Eisenstein (1823-1852)

https://en.wikipedia.org/wiki/Gotthold_Eisenstein