2. Statistics, Estimators, Consistency, and Asymptotic Normality

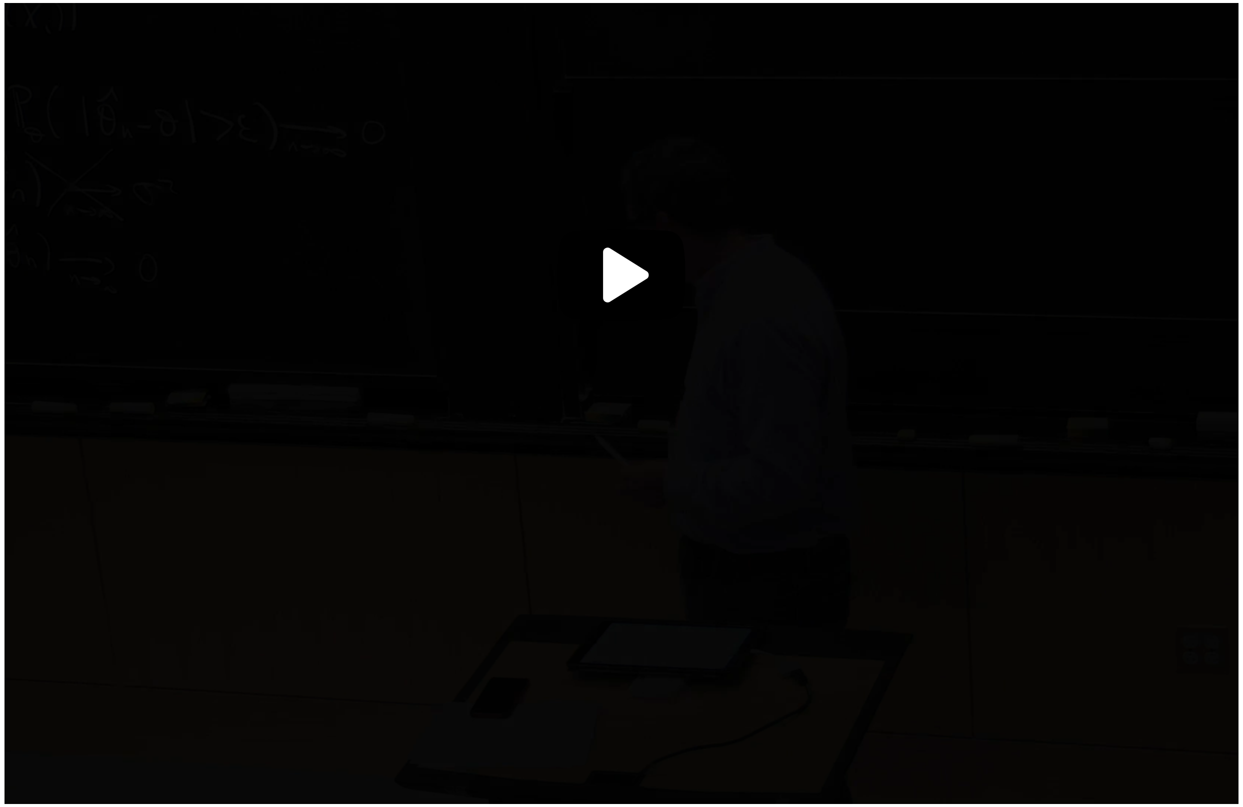
Statistics, Estimators, Consistency, and Asymptotic Normality

which is 0 almost all the time.

So I decide to use the word "asymptotic variance"

to denote the variance of theta hat n,

but once rescaled by the square root of n,



so that I have something that's meaningful

and not always equal to 0.

Is that clear?

End of transcript. Skip to the start.

10:33 / 10:33 1.50x

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Which Statistics are Estimators?

1/1 point (graded)

Let $X_1, \ldots, X_n \stackrel{iid}{\sim} P_\theta$ where the distribution P_θ depends on an unknown parameter $\theta \in \mathbb{R}$. Which of the following statistics are considered **estimators** ?
(Choose all that apply.)

- [ ] θ
- [x] 4.2
- [x] $\sum_{i=1}^{n} i^2 X_i^i$
- [x] $\frac{1}{n}\sum_{i=1}^{n} X_i$
- [ ] $\frac{1}{n}\sum_{i=1}^{n} X_i - \theta$

✔

Solution:

Recall that heuristically, a statistic is a function of the data that can be easily computable, and an estimator is a statistic whose expression does not depend on the unknown parameter θ.

Note that the first and last choices, θ and $\frac{1}{n}\sum_{i=1}^{n} X_i - \theta$ both have some explicit dependence of θ, so they cannot be estimators.

On the other hand, the remaining expressions 4.2, $\sum_{i=1}^{n} i^2 X_i^i$, and $\frac{1}{n}\sum_{i=1}^{n} X_i$ only depend on $X_1, \ldots, X_n$ (and not θ), so they are indeed estimators.

Remark: The second estimator, 4.2, is potentially a very poor choice as it does not depend on the data set. But according to the definition, it is still considered an estimator.

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Answers are displayed within the problem

Consistency of an Estimator

1/1 point (graded)

An estimator $\hat{\theta}_n$ is **weakly consistent** if $\lim_{n \to \infty} \hat{\theta}_n = \theta$, where the convergence is in probability.

Suppose that in the previous problem the unknown parameter θ is the common mean of $X_1, \ldots, X_n$. Assume that $\theta \neq 4.2$. Which of the following is a weakly consistent estimator for θ? (Choose all that apply.)

- [ ] θ

- [ ] 4.2
- [ ] $\sum_{i=1}^{n} i^2 X_i^i$
- [x] $\frac{1}{n}\sum_{i=1}^{n} X_i$
- [ ] $\frac{1}{n}\sum_{i=1}^{n} X_i - \theta$

Solution:

By the weak law of large numbers, $\displaystyle \frac{1}{n}\sum_{i=1}^{n} X_i \to \mathbb{E}[X_1] = \theta$ in probability, so this is the correct choice.

From the previous question, the first and last choice, θ and $\frac{1}{n}\sum_{i=1}^{n} X_i - \theta$, are not even estimators, so these options are incorrect.

Since $\theta \neq 4.2$, this estimator cannot be consistent. Finally, there is no guarantee that $\sum_{i=1}^{n} i^2 X_i^i$ converges to θ. In fact, for many choices of distribution, this statistic will diverge to ∞.

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Answers are displayed within the problem

Quantifying Consistency

1/1 point (graded)

Note: The problem statement has been changed to asking about convergence in probability instead of almost surely. Attempts will be reset.

Let $X_1, \ldots, X_n \overset{iid}{\sim} \mathrm{Ber(p)}$. Let $\overline{X}_n$ be the estimator given by $\dfrac{1}{n}\sum_{i=1}^{n} X_i$.

What is the smallest constant c such that

$$n^c\left(\overline{X}_n - p\right) = n^c\left(\frac{1}{n}\sum_{i=1}^{n} X_i - p\right)$$

does **not** converge to 0 in probability as $n \to \infty$?

1/2

Answer: .5

Solution:

Let $\sigma = \sqrt{p\,(1-p)}$ denote the common standard deviation of $X_1, \ldots, X_n$. By the central limit theorem,

$$\frac{\sqrt{n}}{\sigma}\left(\overline{X}_n - p\right) = \frac{\sqrt{n}}{\sigma}\left(\frac{1}{n}\sum_{i=1}^{n} X_i - p\right) \to N(0,1)$$

where the convergence is in distribution. As a result, we see that for $c < 1/2$,

$$n^c\left(\overline{X}_n - p\right) = \frac{\sigma}{n^{1/2-c}}\frac{\sqrt{n}}{\sigma}\left(\overline{X}_n - p\right) \approx \frac{\sigma}{n^{1/2-c}} N(0,1) \to 0$$

in probability as $n \to \infty$. Hence, $c = 1/2$ is the smallest possible value of c such that

$$n^c\left(\overline{X}_n - p\right) = n^c\left(\frac{1}{n}\sum_{i=1}^{n} X_i - p\right)$$

does **not** converge to 0 in probability as $n \to \infty$.

Remark: As defined in the third video in this section, this implies that the estimator $\overline{X}_n$ is $\sqrt{n}$-consistent. This means that the estimator $\overline{X}_n$ converges to the true parameter at a relatively fast rate, so this gives us something stronger than just consistency.

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You have used 2 of 3 attempts

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Problem#3: Quantifying Consistency

question posted 5 days ago by rickytyagi

I'm not following the logic in the solution, someone kindly clarify.

This post is visible to everyone.

Jeevesh1

3 days ago - marked as answer 3 days ago by **rickytyagi**

Let X denote the expression in question without the n^c outside.

1.) Whenever the sample mean converges a.s. to p, X will converge a.s. to 0 .

2.) Sample mean converges to p a.s (By strong Law of Large Numbers)

3.)If you multiply the expression X by $\sqrt{n}$, you will get a Standard Normal Gaussian.(Z). Now we see what would have happed , if we would have used a different power of n ??

4.) If you now divide Z by n^c, $0 < c < 0.5$ your gaussian will go to 0 at every point as $n \to \infty$. This is equivalent to multiplying X by some power of n less than 0.5 .

5.) If you would use some power of n, higher than 0.5, it'd be equivalent to multiplying the Gaussian Z , by some positive power of n. Which would cause the Gaussian to explode at all points as $n \to \infty$.

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