12.2.1 Explicit and Implicit Methods

MO2.7

The stability issue observed with the Forward Euler method turns out to be a problem for the other methods we have also learned about (i.e. Runge-Kutta methods and leap frog). The common feature all of these methods is that they all extrapolate from the current state $\underline{v}^n$ and the current forcing $\underline{f}\left(\underline{v}^n, t^n\right)$ to estimate $\underline{v}^{n+1}$. This is clear for Forward Euler,

$$\underline{v}^{n+1} = \underline{v}^n + \Delta t \underline{f}\left(\underline{v}^n, t^n\right) \qquad (12.21)$$

where $\underline{v}^{n+1}$ is explicitly calculated from $\underline{v}^n$ and $\underline{f}\left(\underline{v}^n, t^n\right)$ and hence Forward Euler (and our other methods so far) are known as *explicit* methods.

A different approach is to use an *implicit* method, the simplest implicit method the Backward Euler method given by:

$$\underline{v}^{n+1} = \underline{v}^n + \Delta t \underline{f}\left(\underline{v}^{n+1}, t^{n+1}\right) \qquad (12.22)$$

The meaning of the term *implicit* arises because the value of $\underline{v}^{n+1}$ depends implicitly on itself through the appearance of $\underline{f}\left(\underline{v}^{n+1}, t^{n+1}\right)$. Thus, in order to find $\underline{v}^{n+1}$ we will need to solve the implicit relationship appearing in Equation (12.22). Except for some simple Initial Value Problems (like this oscillating combustion example), solving these implicit relationships will be much more expensive (per iteration) then the explicit methods we have been using. However, the benefit of implicit methods is that they will generally allow much large Δt to be taken before the method becomes unstable. For example, the Backward Euler method is stable for any Δt (i.e. there is no limit for this method).

However, while Backward Euler is more stable than Forward Euler, the order of accuracy is the same as Forward Euler. Specifically, Backward Euler is first order ($p = 1$) accurate just like Forward Euler. Thus, while in principle any Δt will produce a stable result using Backward Euler, the timestep may still need to be quite small in order to achieve acceptable accuracy. Implicit methods do exist which have $p > 1$ order of accuracy, e.g. the trapezoidal method is an implicit, second-order accurate method. The trapezoidal method is given by,

(12.23)

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