5. Chain rule

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Part A due Oct 5, 2021 20:30 IST

Practice

5A-8

1/1 point (graded)

Let $f(x,y) = x^3 + y^2 + xy$. Suppose that a point is moving through the plane. At time t, the point is at $(x(t), y(t)) = (t^2, e^{t-1})$. Use linear approximation to estimate the change in f as t goes from 1 to 1.01. In other words, approximate

$$f(x(1.01), y(1.01)) - f(x(1), y(1)).$$

Hint: Use the chain rule.

0.11 ✔ **Answer:** 0.11

Solution:

We want to compute df where $f(x,y) = x^3 + y^2 + xy$ and $(x(t), y(t)) = (t^2, e^{t-1})$.

We compute the differential of f using the chain rule:

$$df = \left(\frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial y}\frac{dy}{dt}\right) dt. \tag{6.271}$$

We compute the partial derivatives and derivatives we need to evaluate this.

$$\frac{\partial f}{\partial x} = 3x^2 + y \tag{6.272}$$

$$\frac{\partial f}{\partial y} = 2y + x \tag{6.273}$$

$$x'(t) = 2t \tag{6.274}$$

$$y'(t) = e^{t-1} \tag{6.275}$$

To evaluate these partial derivatives at time $t = 1$, we need to know that $x(1) = 1$, and $y(1) = 1$.

Thus

$$\Delta f = \frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial y}\frac{dy}{dt}dt \tag{6.276}$$

$$\Delta f \approx \left((3(1)^2 + 1)2 + (2(1) + 1)(1)\right)\Delta t = (8+3)\Delta t = 11\Delta t. \tag{6.277}$$

This tells us that

$$f(x(1.01), y(1.01)) - f(x(1), y(1)) \approx 11(0.01) = 0.11$$

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