11.2.5 An Application: Rank-k Approximation

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Week 11 due Dec 22, 2023 21:12 IST

11.2.5 An Application: Rank-k Approximation

Video

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Dr. Robert van de Geijn: The way we discussed

the theory behind the rank two approximation was completely general.

We didn't really put a restriction on the number of columns in our matrix A.

What that means is that we can now look at what

happens as we choose more and more

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Reading Assignment

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Read Unit 11.2.5 of the notes. [LINK]

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? Homework 11.2.5.2 Cholesky
Using the Cholesky algorithm in the function is producing an image of a black box for me. Do you know what the problem may be? Using the LU ... 3

Video

Calculator

Start of transcript. Skip to the end.

PROFESSOR: So the answer is that it's true.

Well, it'd better be true because otherwise

why have we been calling this a rank k approximation?

And this is the key insight.

If you have two subspaces of Rn, and S is a subspace of T,

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Homework 11.2.5.1

1/1 point (graded)

Let $U \in \mathbb{R}^{m \times k}$ and $V \in \mathbb{R}^{n \times k}$. Then the $m \times n$ matrix UV^T has a rank of at most k.

TRUE ✔ Answer: TRUE

Answer: True

Again, we build on the insight that if $S, T \subset \mathbb{R}^m$ are subspaces and $S \subset T$, then $\dim(S) \le \dim(T)$. Here $T = \mathcal{C}(\{U\})$ and $S = \mathcal{C}(UV^T)$.

Now, clearly $\mathrm{rank}(U) = \dim(\mathcal{C}(U)) \le k$ since U is a $m \times k$ matrix. Let $y \in \mathcal{C}(UV^T)$. We will show that then $y \in \mathcal{C}(U)$.

$$\begin{aligned}
& y \in \mathcal{C}(UV^T) \\
& \quad \Rightarrow \quad < \text{there exists a } x \in \mathbb{R}^n \text{ such that } y = UV^T x > \\
& y = UV^T x \\
& \quad \Rightarrow \quad < z = V^T x > \\
& y = Uz \\
& \quad \Rightarrow \quad < \text{Definition of column space} > \\
& y \in \mathcal{C}(U).
\end{aligned}$$

Hence $\mathrm{rank}(UV^T = \dim(\mathcal{C}(UV^T)) \le \dim(\mathcal{C}(U)) = \mathrm{rank}(U) \le k$.

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Answers are displayed within the problem

Homework 11.2.5.2

1 point possible (graded)

We discussed in this section that the projection of B onto the column space of A is given by $A(A^TA)^{-1}A^TB$.

Calculator

compute $V = (A^T A)^{-1} A^T B$, then AV is an approximation to B that requires only $m \times k$ matrix A and $k \times n$ matrix V.

To compute V, we can perform the following steps:

- Form $C = A^T A$.
- Compute the LU factorization of C, overwriting C with the resulting L and U.
- Compute $V = A^T B$.
- Solve $LX = V$, overwriting V with the solution matrix X.
- Solve $UX = V$, overwriting V with the solution matrix X.
- Compute the approximation of B as $A \cdot V$ (A times V). In practice, you would not compute this approximation, but store A and V instead, which typically means less data is stored.

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