11.3.2 Orthonormal Vectors (Continued)

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Week 11 due Dec 22, 2023 21:12 IST Completed

11.3.2 Orthonormal Vectors (Continued)

Homework 11.3.2.4

10.0/10.0 points (graded)

1. $$\begin{pmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{pmatrix}^T \begin{pmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{pmatrix} =$$

1	0
Answer: 1	Answer: 0
0	1
Answer: 0	Answer: 1

2. $$\begin{pmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{pmatrix}^T \begin{pmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{pmatrix} =$$

1	0
Answer: 1	Answer: 0
0	1
Answer: 0	Answer: 1

3. The vectors $\begin{pmatrix} -\sin(\theta) \\ \cos(\theta) \end{pmatrix}, \begin{pmatrix} \cos(\theta) \\ \sin(\theta) \end{pmatrix}$ are mutually orthonormal.

TRUE Answer: TRUE

4. The vectors $\begin{pmatrix} \sin(\theta) \\ \cos(\theta) \end{pmatrix}, \begin{pmatrix} \cos(\theta) \\ -\sin(\theta) \end{pmatrix}$ are mutually orthonormal. True/False

TRUE Answer: TRUE

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Answers are displayed within the problem

Video 11.3.2 Part 5

Start of transcript. Skip to the end.

Dr. Robert van de Geijn: OK.

So what happens when you mul

these out?

Calculator

Well, from the insights from an earlier homework,

we know that this is just equal to the identity.

And similarly, notice that this is actually

0:00 / 0:00 2.0x

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Homework 11.3.2.5

1/1 point (graded)

Let $q \in \mathbb{R}^m$ be a unit vector (which means it has length one). Then the matrix that projects vectors onto $\mathrm{Span}\left(\{q\}\right)$ is given by qq^T.

TRUE ✔ Answer: TRUE

The matrix that projects onto $\mathrm{Span}\left(\{q\}\right)$ is given by $q(q^Tq)^{-1}q^T$. But $q^Tq = 1$ since q is of length one. Thus $q\underbrace{\left(q^Tq\right)^{-1}}_{1}q^T = qq^T$.

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Answers are displayed within the problem

Video 11.3.2 Part 6

Start of transcript. Skip to the end.

Dr. Robert van de Geijn: So the answer is that this is true as well.

And why is that?

Well, the matrix that projects onto the span of q is given by that.

That's because the matrix that projects onto the span of vector a

was given by a, a transpose a inverse, a

0:00 / 0:00 2.0x

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Homework 11.3.2.5

1/1 point (graded)

Let $q \in \mathbb{R}^m$ be a unit vector (which means it has length one). Let $x \in \mathbb{R}^m$. Then the component of x in the direction of q (in $\mathrm{Span}\left(\{q\}\right)$) is given by $q^T x q$.

TRUE ✔ Answer: TRUE

In the last exercise we saw that the matrix that projects onto $\mathrm{Span}\left(\{q\}\right)$ is given by qq^T. Thus, the component of x in the direction of q is given by $qq^T x = q\left(q^T x\right) = q^T x q$ (since $q^T x$ is a scalar).

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Answers are displayed within the problem

Video 11.3.2 Part 7

Start of transcript. Skip to the end.

Dr. Robert van de Geijn: Well, in the last exercise,

we saw that multiplying by matrix Q Q transpose projects onto the span of Q.

So if you multiply that times x, we get this.

But then we recognize that the result of the product is just a scalar.

0:00 / 0:00 2.0x

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Homework 11.3.2.6

10.0/10.0 points (graded)

Let $Q \in \mathbb{R}^{m \times n}$ have orthonormal columns (which means $Q^T Q = I$). Then the matrix that projects vectors onto the column space of Q, $\mathcal{C}\left(Q\right)$, is given by QQ^T.

TRUE ✔ Answer: TRUE

Calculator

The matrix that projects onto $\mathcal{C}(Q)$ is given by $Q(Q^TQ)^{-1}Q^T$. But then $Q\underbrace{(Q^TQ)^{-1}}_{I^{-1}=I}Q^T = QQ^T$.

Submit

Answers are displayed within the problem

Video 11.3.2 Part 8

Start of transcript. Skip to the end.

Dr. Robert van de Geijn: And this turns out to be true as well.

And why is that?

Well, we know that for general matrix A, the formula

for projecting onto the column space of A is given by this matrix right here.

If we substitute Q in for A, we get this

0:00 / 0:00 2.0x

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Homework 11.3.2.7

10.0/10.0 points (graded)

Let $Q \in \mathbb{R}^{m\times n}$ have orthonormal columns (which means $Q^TQ = I$). Then the matrix that projects vectors onto the space orthogonal to the columns of Q, $\mathcal{C}(Q)^\perp$, is given by $I - QQ^T$.

TRUE ✔ Answer: TRUE

In the last problem we saw that the matrix that projects onto $\mathcal{C}(Q)$ is given by QQ^T. Hence, the matrix that projects onto the space orthogonal to $\mathcal{C}(Q)$ is given by $I - QQ^T$.

Submit

Answers are displayed within the problem

Calculator

Video 11.3.2 Part 9

Video 11.3.2 Part 3

Start of transcript. Skip to the end.

Dr. Robert van de Geijn: So this is true as well.

And that's because we know that the formula for general matrix A

was this right here.

If we then substitute in Q, we get this.

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