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> 6.2.1 Interactive: Bivariate Normal

## 6.2.1 Interactive: Bivariate Normal

### Bivariate Normal - Directions for Use

A graph of the Bivariate Normal joint PDF, where the components are marginally standard Normal and have correlation parameter  $\rho$ , is below. You can use the slider to change the correlation  $\rho$  in this graph. You can also drag the graph to rotate it and get a view from different angles.

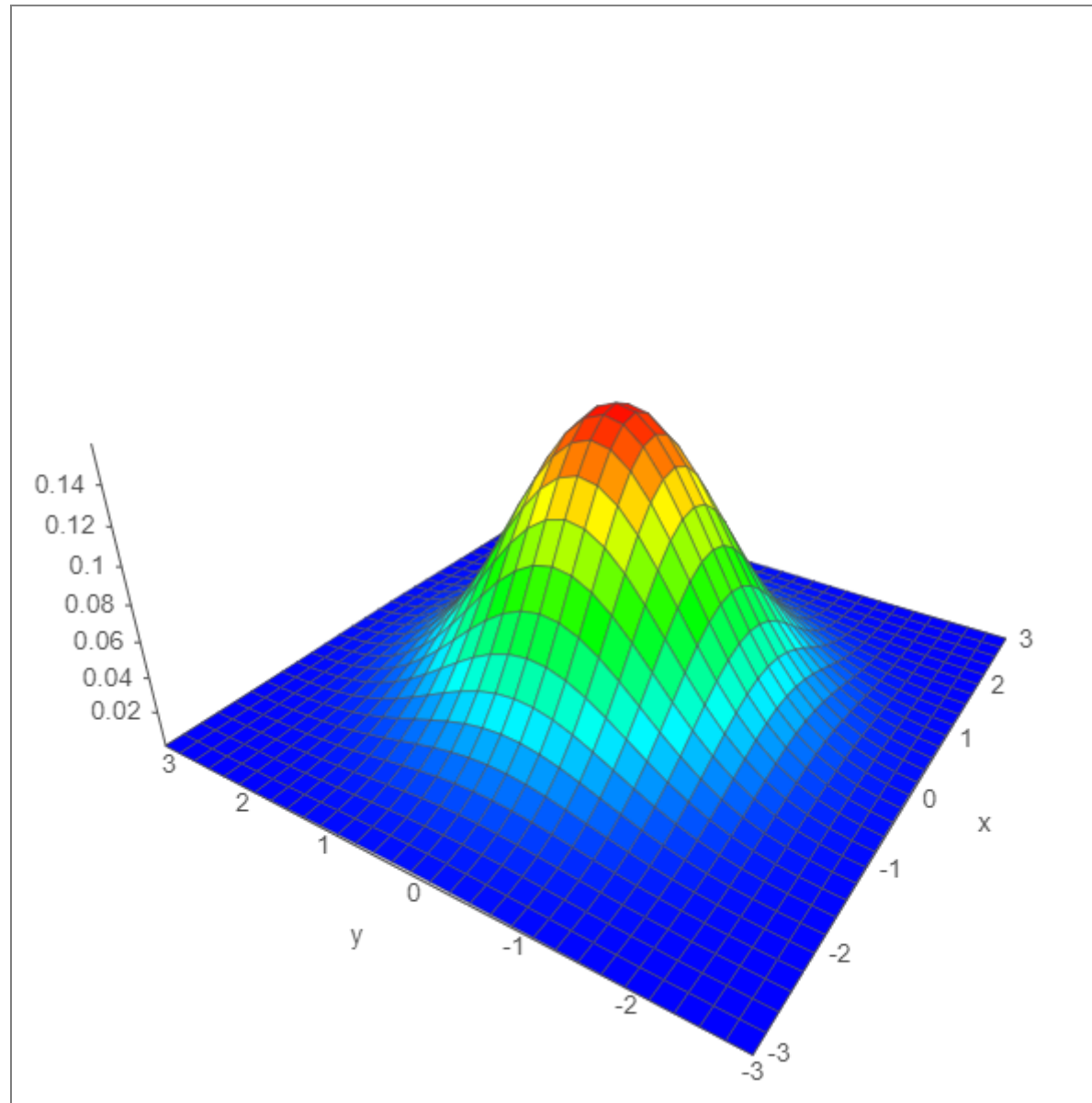
#### You SHOULD TRY:

Set the correlation at a few key values: 0, close to -1, and close to 1. What does the graph look like for values of  $\rho$  close to zero? What about when  $\rho$  is close to -1, and when it is close to 1?

## Controls

 $\rho$ : 0

Reset



## Bivariate Normal - Detailed Description



The Bivariate Normal is a 3-dimensional graph of the joint PDF of a Bivariate Normal random vector  $(X,Y)$ , where  $X$  and  $Y$  have correlation  $\rho$  and are marginally Normal with mean 0 and variance 1.

In our graph, the x-axis goes from -3 to 3, as does the y-axis. The graph looks like a mountain, with exactly one peak, which is when  $x = 0$  and  $y = 0$ .

Slicing the graph by intersecting it with a plane perpendicular to the x-axis or perpendicular to the y-axis yields a Univariate Normal curve. The contours of the graph, i.e., sets of  $(x,y)$  points where the density is a constant, are ellipses.

For  $\rho = 0$ ,  $X$  and  $Y$  are independent; for  $\rho$  not equal to 0,  $X$  and  $Y$  are correlated. For  $\rho = 0$ , the graph is rotationally symmetric, i.e., the contours are circles and the density at  $(x,y)$  depends only on how far  $(x,y)$  is from  $(0,0)$ , not on the angle that  $(x,y)$  is at. As  $\rho$  increases, the graph becomes more and more squashed and elongated, until when  $\rho$  is very close to 1, the graph is almost 2-dimensional, indicating an almost perfect linear relationship between  $X$  and  $Y$ , with this line having positive slope. The same is true as  $\rho$  decreases towards -1, except now the slope is negative.

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