# 1.4.5 Quiz: Creating a Phase Plane for Marlin and Sardine

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**Before we get started, here are a few ground rules:**

- We will use the horizontal axis to represent values of $S$, the sardine population, and the vertical axis to represent the marlin population $M$. *(This is just a choice - we could make the other choice and proceed similarly. It would not affect the end conclusions from the phase plane analysis.)*
- Since populations can't be negative, we'll focus on the first quadrant where $S$ and $M$ are non-negative. *(For a system of differential equations modeling things like position and velocity of a pendulum we would consider all four quadrants.)*

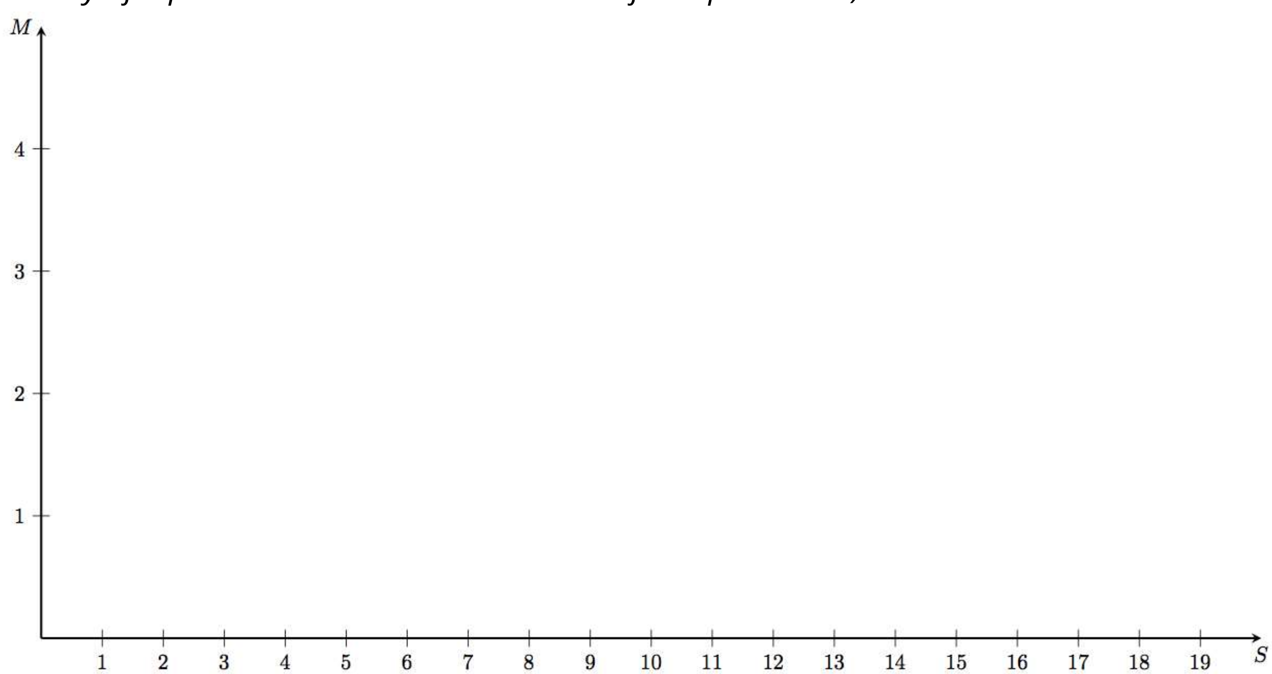


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Image Description

**Sketch these axes on a sheet of paper.** (Feel free to use this printable PDF version of the axes above.)

We'll start by considering the point $(S, M) = (3, 1)$. This represents a fish population with 300,000 sardines and 100 marlin. Locate this point on your graph, then answer the questions below.

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## Question 1

1/1 point (graded)

According to the system of equations:

$$\frac{dS}{dt} = 0.5S - 0.4SM$$

$$\frac{dM}{dt} = -0.2M + 0.03SM,$$

when $S = 3$ and $M = 1$, is the value of $S$ increasing, decreasing, or constant?

- (•) increasing ✔
- ( ) decreasing
- ( ) constant

When $S = 3$ and $M = 1$,

$$\frac{dS}{dt} = 0.5S - 0.4SM = 1.5 - 1.2 > 0.$$

So the sardine population is increasing.

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You have used 1 of 2 attempts

Answers are displayed within the problem

## Multiple Choice

1/1 point (graded)

When $S = 3$ and $M = 1$, is the value of $M$ increasing, decreasing, or constant?

- increasing
- decreasing ✔
- constant

Answer: When $S = 3$ and $M = 1$,

$$\frac{dM}{dt} = -0.2M + 0.03SM = -0.2 + 0.09 < 0.$$

So $M$ is decreasing. 300,000 sardines are not enough to feed 100 marlin.

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Answers are displayed within the problem

Now we will include the information from the previous questions in the phase-plane.

- Using your previous answers, **add an arrow to your graph** indicating the direction the solution curve $(S(t), M(t))$ is moving when $S = 3$ and $M = 1$. *(Don't worry about the exact length and direction of the arrow – this is just a rough sketch.)*

- Choose a few more points in your phase plane, **calculate the values** of $\frac{dS}{dt}$ and $\frac{dM}{dt}$ at those points, and **sketch arrows** indicating how the fish populations are changing under those conditions.

Now, we can't possibly check every point in the entire quadrant. Instead, we'll use general properties of the equations to see what's happening more generally (and save us some effort). If the sardine population, $S$, is decreasing at a point $(S, M)$ then it will be decreasing at nearby points in the phase plane, where $\frac{dS}{dt}$ is also negative. In order for $\frac{dS}{dt}$ to become positive (for the sardine population to increase), we must pass through a point where $\frac{dS}{dt} = 0$. We'll summarize this in a key fact:

> **Key Fact:** Because the system of differential equations is continuous, $\frac{dS}{dt}$ can only change from negative to negative or positive to negative if $\frac{dS}{dt} = 0$. The same is true for $\frac{dM}{dt}$.

This means it can be very helpful to determine where the derivative $\frac{dS}{dt}$ equals zero. (This should remind you of what we did when analyzing the logistic equation for population $P$ in a previous subsection: we found where $\frac{dP}{dt} = 0$.)

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## Question 3

2/2 points (graded)

Enter your answers as numerical values.

If

$$\frac{dS}{dt} = 0.5S - 0.4SM$$

$$\frac{dM}{dt} = -0.2M + 0.03SM,$$

find values of $M$ and $S$ for which $\frac{dS}{dt} = 0$. (It helps to factor the equation for $\frac{dS}{dt}$ first.)

S=

0

✔ **Answer:** 0

$0$

M=

1.25

✔ **Answer:** 5/4

$1.25$

**Explanation**

Since $\frac{dS}{dt} = 0.5S - 0.4SM = S(0.5 - 0.4M)$,
$\frac{dS}{dt} = 0$ when $S = 0$ or $M = 5/4$.

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You have used 1 of 4 attempts

Answers are displayed within the problem

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