2. An introduction to partial differential equations for traveling waves

Bookmark this page

Problem Set B due Aug 18, 2021 20:30 IST Completed

Explore

Problem 2(a)

3.0/3 points (graded)

Consider the function

$$f(x,t) = \sin(x - vt)$$

where $v > 0$.

Find the gradient.

$f_x(x,t) =$ cos(x-v*t) **Answer:** cos(x-v*t) $f_t(x,t) =$ -v*cos(x-v*t) **Answer:** -v*cos(x-v*t)

Where is the gradient equal to 0? (Describe as a relationship of x in terms of t, v, and an integer n.)

$x =$ v*t + (2*n+1)*pi/2 for $n = 0, \pm 1, \pm 2, \ldots$ **Answer:** v*t+(2*n+1)*pi/2

? INPUT HELP

Solution:

First we compute the gradient.

$$f_x(x,t) \;=\; \sin'(x - vt)\,\frac{d(x-vt)}{dx} \;=\; \cos(x-vt)\; f_t(x,t) \;=\; \sin'(x-vt)\,\frac{d(x-vt)}{dt} \;=\; -v\cos(x-$$

The gradient is given by the vector $\langle f_x, f_t \rangle = \langle \cos(x-vt), -v\cos(x-vt) \rangle$.

Next we solve for where the gradient is zero. The gradient is zero exactly where $\cos(x - vt) = 0$. And this happens whenever

$$x - vt = \frac{\pi}{2} + n\pi, \qquad n \text{ any integer.}$$

Therefore the gradient is zero along the lines given by the equations

$$x = vt + \frac{\pi}{2} + n\pi, \qquad n \text{ any integer.}$$

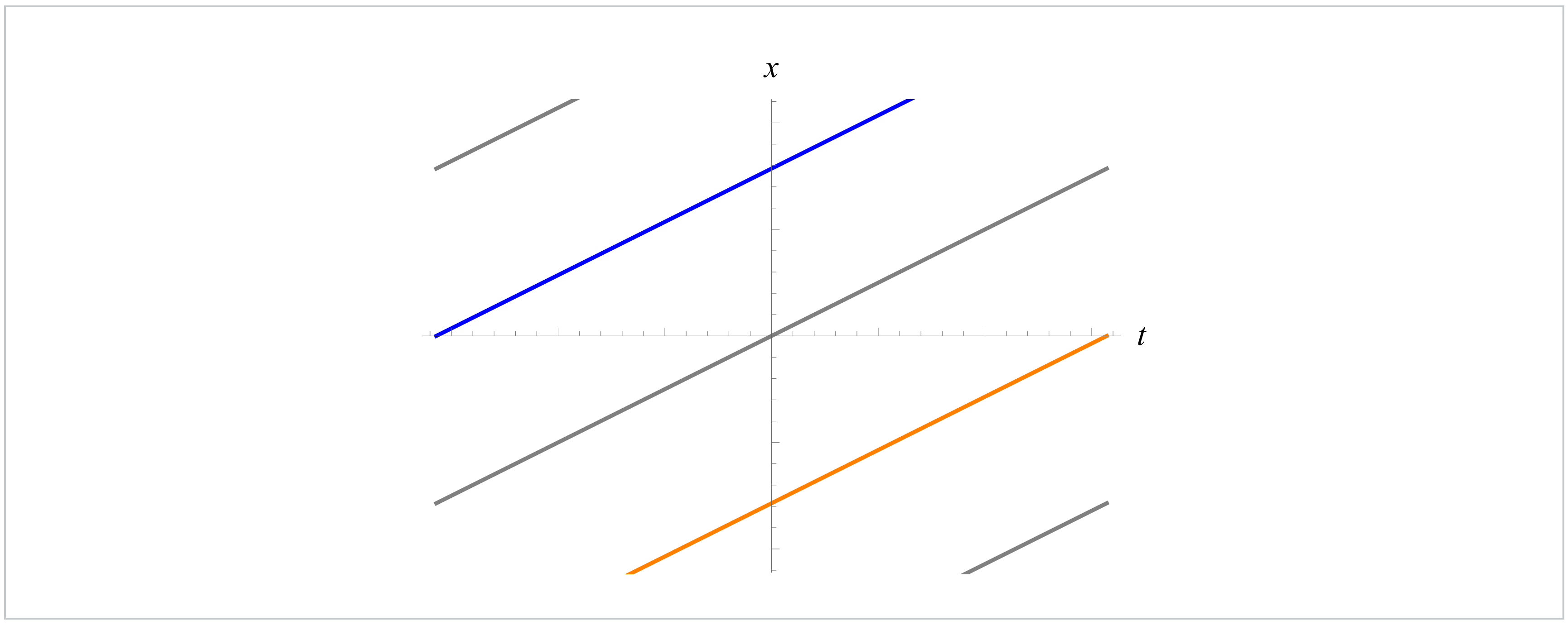
Submit You have used 1 of 5 attempts

Answers are displayed within the problem

Calculator Hide Notes

2(b)

5/5 points (graded)
The plot of the level curves of heights -1, 0, and 1 of the function $f(x,t) = \sin(x - vt)$ are below. The level curve of height 1 is shown in blue, the level curves of height 0 are shown in gray, and the level curve of height -1 is shown in orange.



What is the slope of the level curves?

v ✔ **Answer:** v

What is the speed of the traveling wave?

v ✔ **Answer:** v

What is the x-intercept of the level curve of height 1?

pi/2 ✔ **Answer:** pi/2

What is the x-intercept of the level curve of height -1?

-pi/2 ✔ **Answer:** -pi/2

Which of the following best describes the critical points of the function?

- [ ] Isolated points where the gradient is 0.
- [ ] The level curves of height 0
- [x] The level curves of height 1.
- [x] The level curves of height -1.
- [ ] None of the above.

✔

? INPUT HELP

Solution:

The level curves are the lines

Calculator Hide Notes

$$\sin(x - vt) = c.$$

In particular, the level curves of height 0 are where $\sin(x - vt) = 0$, which is where

$$x - vt = n\pi.$$

In other words, the lines

$$x = vt + n\pi$$

These level curves all have slope v.

The x-intercept of the level curve of height 1 is the smallest positive value of x where $\sin(x - v \cdot 0) = 1$. This is $x = \pi/2$.

The x-intercept of the level curve of height -1 is the largest negative value of x where $\sin(x - v \cdot 0) = -1$. This is $x = -\pi/2$.

Submit You have used 2 of 7 attempts

Answers are displayed within the problem

2(c)

1/1 point (graded)
What is the relationship between f_x and f_t?

$f_t(x,t) =$ -v $f_x(x,t)$ ✔ **Answer:** -v

? INPUT HELP

Solution:

The relationship is given by $f_t = -vf_x$. We can rewrite this as $f_t + vf_x = 0$. Such a relationship, described as an equation involving partial derivatives of a function, is called a **partial differential equation** . Partial differential equations are used to model most natural phenomena: traffic flow, heat, waves, Schroedinger's equation, and more.

Submit You have used 1 of 5 attempts

Answers are displayed within the problem

Problem 2(d)

1/1 point (graded)
Which of the following functions satisfy the relationship between f_t and f_x you discovered in the problem above?

(Choose all that apply.)

- [ ] $\sin(x + vt)$
- [x] $\cos(x - vt)$

Calculator Hide Notes

- [ ] $\cos(x + vt)$
- [x] $e^{-(x-vt)^2}$
- [x] $\tanh(x - vt)$
- [ ] $\sin(x)\cos(vt)$
- [x] $g(x - vt)$ for an arbitrary function g
- [ ] $g(x + vt)$ for an arbitrary function g

✔

Solution:

We can show that any function of the type $g(x - vt)$ satisfies $g_t + vg_x = 0$.

$$g_x(x - vt) = g'(x - vt) \quad (3.147)$$
$$g_t(x - vt) = -vg'(x - vt) \quad (3.148)$$
$$\longrightarrow \quad g_t(x,t) + vg_x(x,t) = 0 \quad (3.149)$$

Functions of the form $g(x + vt)$ do not satisfy this relationship however do satisfy the relationship $g_t - vg_x = 0$.

$$g_x(x + vt) = g'(x + vt) \quad (3.150)$$
$$g_t(x + vt) = vg'(x + vt) \quad (3.151)$$
$$\longrightarrow \quad g_t(x,t) - vg_x(x,t) = 0 \quad (3.152)$$

And the function $\sin(x)\cos(vt)$ satisfies the relationship $g_{tt} = v^2 g_{xx}$, but not the (partial differential) equation of interest.

Submit You have used 1 of 7 attempts

ℹ Answers are displayed within the problem

Problem 2(e)

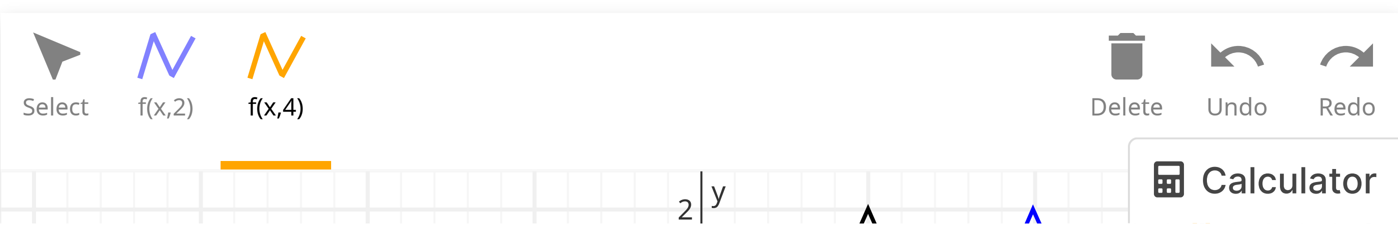
1.0/1 point (graded)

Suppose you are given a piecewise continuous function $g(u)$.

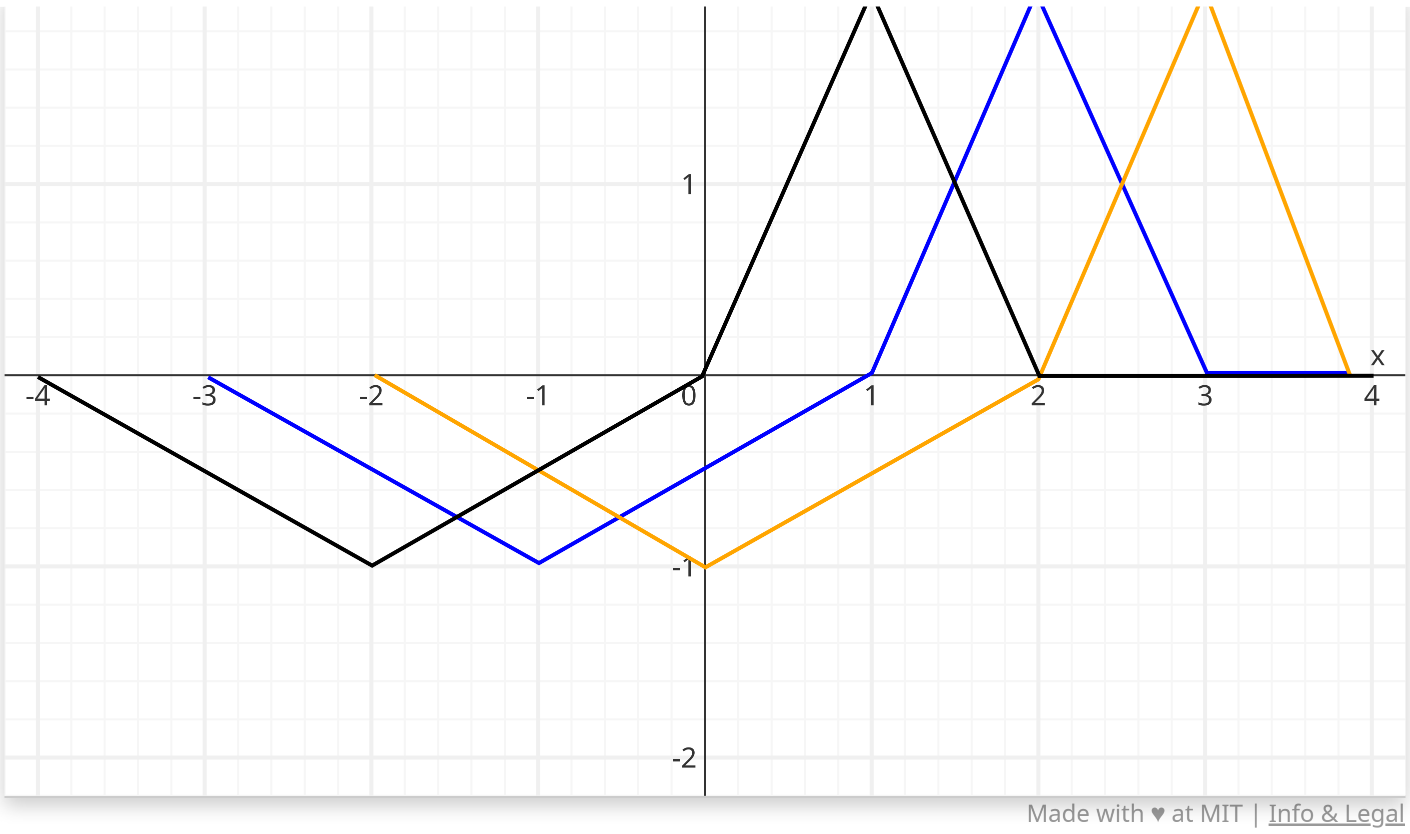
We create a new multivariable function that satisfies the equation $f_t + \frac{1}{2} f_x = 0$ where the derivative exists by defining $f(x,t) = g(x - (1/2)t)$.

The function $y = f(x,0)$ is shown below in black. Plot the function $f(x,t)$ on the x axis at the time values $t = 2$ and $t = 4$ as specified in the sketching tool. That is, plot $y = f(x,2)$ in blue, and plot $y = f(x,4)$ in orange.

Note that the functions are piecewise linear, so the drawing tools given will draw piecewise linear function connecting points you select on the canvas below.



Calculator Hide Notes



Answer: See solution.

Well done

Solution:

The graph of $y = f(x, 2)$ is the graph of $y = f(x, 0)$ but shifted to the right 1 unit. Similarly, the graph of $y = f(x, 4)$ is the same as the graph of $y = f(x, 0)$ but shifted to the right by 2 units.

Submit You have used 1 of 25 attempts

Answers are displayed within the problem

2. An introduction to partial differential equations for traveling waves

Hide Discussion

Topic: Unit 2: Geometry of Derivatives / 2. An introduction to partial differential equations for traveling waves

Add a Post

Show all posts by recent activity

- 2(b) Critical Points - answer details 8
  Hi, I don't see any detailed answer for this - would anyone help with some details? I got to answer by selecting the places which ...
- Warning: in 2(c) answer box.... 2
  Warning: in 2(c) answer box.... If you are thinking of entering "foo*", instead enter "foo". I don't think this gives away the answer.
- ? [staff] velocity of the traveling wave? 2
  When asked for a velocity I now usually expect the answer should be a vector, and not a scalar. In this problem it seems the read...
- Relationship of partial derivatives and proofs 2
  Could someone explain me the proofs? I don't understand why v is multiplied by f_xx instead of f_tt given that the partial derivati...
- ? Question Problem (2b) 2
  The values for the slope calculation should be eyeballed from the graph ? And should there be a relation between a unit on the g...
- ? 2(e) 3
  Any hints for problem 2(e)? I cannot figure out how to start.
- Difficulty comprehending the question 7
  Hello fellow students, I am wondering if I am the only one here who is having trouble comprehending what the questions really w...
- [tip 2(e)] Check your blue line
  Community TA

Calculator Hide Notes

2b: the slope 3

2b - heights 1 and -1 3
Clearly, sin(pi/2) = 1, then the answer for the x-intercept (vertical line in this case) should be in terms of pi, however every time I ...

? 2(b) 4
I'm kind of confused with this and sure I'm missing something really simple connecting these questions with the graph. For exam...

[Staff] Swapped Components in Problem 2(a) 5

? Grader on 2(a) Part 3 is too strict. 3
If n is allowed to be any positive or negative integer then $+n$ is equivalent to $-n$, but the grader will mark $-k*n$ as incorrect a...

< Previous Next >

edX

About
Affiliates
edX for Business
Open edX
Careers
News

Legal

Terms of Service & Honor Code
Privacy Policy
Accessibility Policy
Trademark Policy
Sitemap

Calculator Hide Notes

Connect

Blog
Contact Us
Help Center
Media Kit
Donate

Calculator Hide Notes