5. Inverse Matrix

Bookmark this page

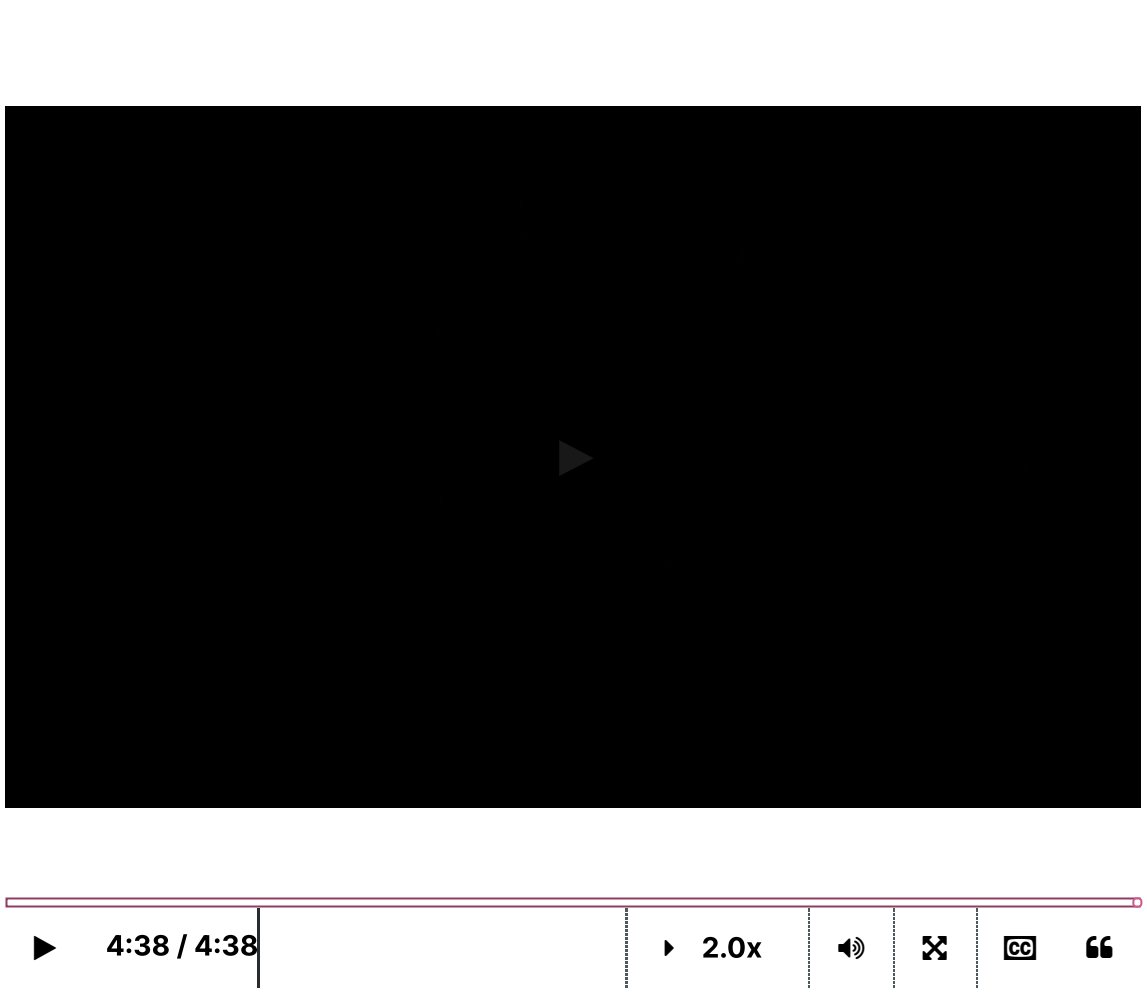
Lecture due Sep 15, 2021 20:30 IST

Explore

Inverse Matrix

There is another method that allows us to solve for $\vec{x}$ in $A\vec{x} = \vec{b}$ without having to do elimination. One may use the **inverse matrix**.

Inverse Matrix



Video
Download video file

Transcripts
Download SubRip (.srt) file
Download Text (.txt) file

The Inverse Matrix (2×2)

If $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ then the inverse of A is

$$A^{-1} = \frac{1}{ad - bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix} \tag{5.63}$$

The most important property of A^{-1} is that

$$\textbf{(Inverse Matrix Property)}\ AA^{-1} = A^{-1}A = I. \tag{5.64}$$

(Recall that I is the identity matrix). This property can be used to define the inverse of larger matrices. To see why this property is important, imagine we want to solve for $\vec{x}$ in the equation $A\vec{x} = \vec{b}$. We can multiply both sides by A^{-1} and simplify:

$$A^{-1}A\vec{x} = \vec{b}$$

Calculator Hide Notes

$$\vec{x} = A^{-1}\vec{b} \tag{5.66}$$

This gives us a recipe for finding $\vec{x}$: we can just multiply out $A^{-1}\vec{b}$. This method is also useful if we need to solve for $\vec{x}$ several times for different values of $\vec{b}$.

Example

Let $A = \begin{pmatrix} -1 & 1 \\ 1 & 3 \end{pmatrix}$. Then A^{-1} is given by

$$A^{-1} = \frac{1}{-4}\begin{pmatrix} 3 & -1 \\ -1 & -1 \end{pmatrix} \tag{5.67}$$

Suppose we want to solve for $\vec{x}$ in $A\vec{x} = \begin{pmatrix} 1 \\ 7 \end{pmatrix}$. The solution is given by

$$\vec{x} = A^{-1}\begin{pmatrix} 1 \\ 7 \end{pmatrix} = \frac{1}{-4}\begin{pmatrix} 3 & -1 \\ -1 & -1 \end{pmatrix}\begin{pmatrix} 1 \\ 7 \end{pmatrix} = \frac{1}{-4}\begin{pmatrix} -4 \\ 8 \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \end{pmatrix} \tag{5.68}$$

This agrees with the solution we found by elimination.

Find an Inverse

1/1 point (graded)

Let $A = \begin{pmatrix} 1 & 2 \\ 1 & -1 \end{pmatrix}$.

(Enter a matrix using notation such as `[[a,b],[c,d]]`.)

$A^{-1} =$ [[1/3,2/3],[1/3,-1/3]] ✔ **Answer:** [[1/3, 2/3],[1/3, -1/3]]

? INPUT HELP

Solution:

Using the formula $A^{-1} = \frac{1}{ad-bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$, we have $A^{-1} = \frac{1}{-3}\begin{pmatrix} -1 & -2 \\ -1 & 1 \end{pmatrix}$.

Submit You have used 2 of 3 attempts

Answers are displayed within the problem

Solve a System

2/2 points (graded)

Let $A = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}$. Find A^{-1} and solve for $\vec{x}$ in $A\vec{x} = \begin{pmatrix} 5 \\ 0 \end{pmatrix}$.

(Enter a matrix using notation such as `[[a,b],[c,d]]`.)

$A^{-1} =$ [[1,-1],[-1,2]] ✔ **Answer:** [[1,-1],[-1,2]]

(Enter a vector using notation such as `[a,b]`.)

Calculator Hide Notes

$\vec{x} =$ [5,-5] ✔ **Answer:** [5,-5]

? INPUT HELP

Solution:

$A^{-1} = \begin{pmatrix} 1 & -1 \\ -1 & 2 \end{pmatrix}$. So $\vec{x} = A^{-1} \begin{pmatrix} 5 \\ 0 \end{pmatrix} = \begin{pmatrix} 5 \\ -5 \end{pmatrix}$.

Submit You have used 1 of 3 attempts

Answers are displayed within the problem

Caution

The inverse matrix method will not always work because the denominator $ad - bc$ could equal zero. This is explored in more detail on the next page.

5. Inverse Matrix

Topic: Unit 4: Matrices and Linearization / 5. Inverse Matrix

Hide Discussion

Add a Post

Show all posts by recent activity

[STAFF] Grader issue Question 1 8

Community TA

Previous Next

Calculator Hide Notes