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3.

### Two sample t-statistic

1/1 point (graded)

Let

- $X_1, \dots, X_n \stackrel{\text{iid}}{\sim} N(\mu_0, \sigma_0^2)$
- $Y_1, \dots, Y_m \stackrel{\text{iid}}{\sim} N(\mu_1, \sigma_1^2)$
- the  $X_i$ 's are independent of the  $Y_i$ 's,

where  $\mu_0, \mu_1, \sigma_0^2, \sigma_1^2$  are **unknown** parameters.

What is the two-sample  $t$ -statistic  $T_{n,m}$ ?

☐  $(\bar{X}_n - \bar{Y}_m) \sqrt{(\hat{\sigma}_1^2 + \hat{\sigma}_0^2) / (m + n)}$

☒  $\frac{\bar{X}_n - \bar{Y}_m}{\sqrt{\hat{\sigma}_1^2/m + \hat{\sigma}_0^2/n}}$

☐ 
$$\frac{\bar{X}_n - \bar{Y}_m}{\sqrt{\sigma_1^2/m + \sigma_0^2/n}}$$

☐ 
$$\frac{\bar{X}_n - \bar{Y}_m}{\sqrt{\sigma_1^2 + \sigma_0^2/(m+n)}}$$

☐ The  $t$ -statistic cannot be defined for two samples.

You have used 1 of 3 attempts

**i** Answers are displayed within the problem

## Distribution of $t$ -statistic with Small Sample Size

2/2 points (graded)

Continuing with the setup as above, suppose the data you collected are summarized as follows:

- Sample sizes:  $n = m = 5$ ;
- Sample means:  $\bar{X}_n = 1.2, \bar{Y}_m = 1.0$ ;
- Sample variances:  $\widehat{\sigma}_1^2 = \widehat{\sigma}_2^2 = 0.5$ .

Which of the following distributions **best approximates** the distribution of the  $t$ -statistic  $T_{n=5,m=5}$  under the null hypothesis  $H_0 : \mu_0 = \mu_1$ ?

☐ Normal distribution with mean  $\mu \neq 0$

☐ Standard normal distribution

☒ t-distribution

☐ F-distribution



If applicable, specify the degrees of freedom of the distribution above (i.e. the distribution that **best approximates** the distribution of the  $t$ -statistic). If not applicable, enter  $-1$ .

Degrees of freedom:   Answer: 8

### Solution:

Since the sample sizes are small, the two-sample  $T$ -statistic  $T_{5,5} = \frac{\bar{X}_n - \bar{Y}_m}{\sqrt{\widehat{\sigma}_1^2/m + \widehat{\sigma}_0^2/n}}$  is approximated by a  $t$ -distribution with degree of freedom given by the Welch-Satterthwaite formula. A sanity check is that the degree of freedom should be at least  $\min m, n = 5$ .

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I chose the last option for this question. Because I thought at this stage there is no hypothesis yet. And the test shall always be based/related to the hypotheses. So at this step th...

1

<p>💬 <a href="#">[Res] Degrees of freedom</a></p> <p><a href="#">[note: unverified result, could be wrong] To "test" my answer of 8, I implemented the WelchSatterthwaite Formula (not really needed in this case, as n is equal to m): WelchSat...</a></p>	4
<p>💬 <a href="#">[Polling] What are your answers for this page?</a></p> <p><a href="#">1) 2nd choice 2) t-dist 3) 8</a></p>	9
<p>💬 <a href="#">[NOTE FROM STAFF] Last problem on this page small typo</a></p> <p><a href="#">👤 Staff</a></p>	1
<p>💬 <a href="#">[Staff] Typo in Deleted by MW-CTA</a></p> <p><a href="#">Deleted by MW-CTA</a></p>	3
<p>? <a href="#">[Staff]: you have an error in your sigma squared subscripts in the last question</a></p> <p><a href="#">Hi, There is an error in your variance subscripts, these should be 0 and 1; there is no subscript 2. Not that it matters for the solutions since you made them equal; but would b...</a></p>	2
<p>💬 <a href="#">[staff]issue with the problem</a></p> <p><a href="#">[...]</a></p>	4
<p>💬 <a href="#">[Edited by staff to remove exam content]</a></p> <p><a href="#">[Edited by staff to remove exam content].</a></p>	3

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