4. Configuration model

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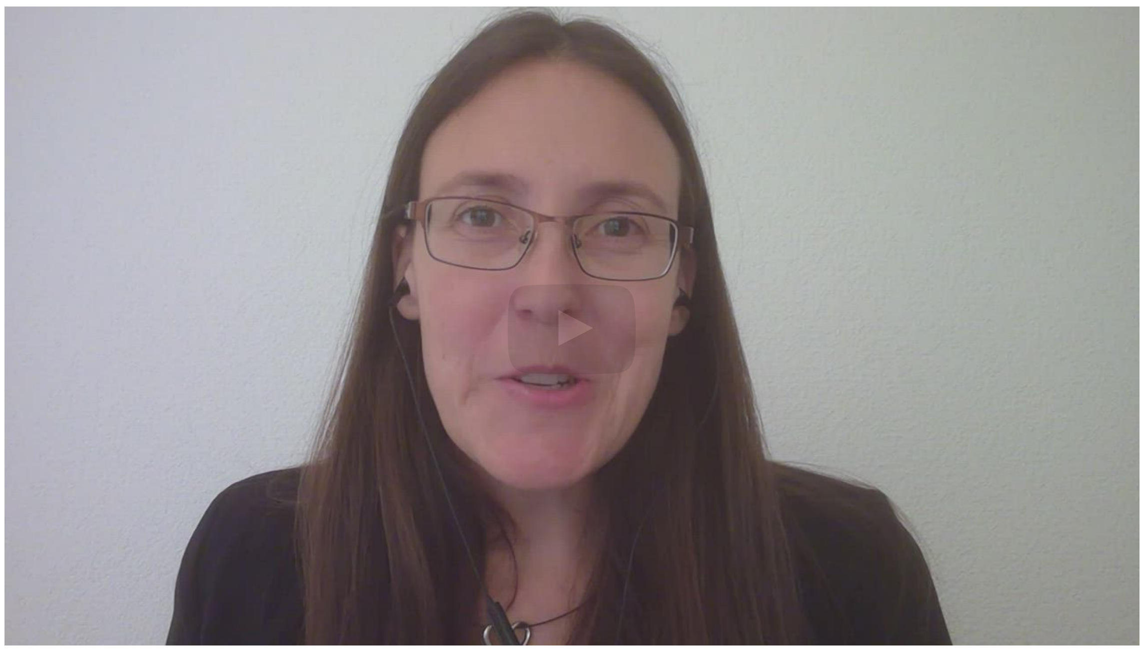
Exercises due Oct 27, 2021 17:29 IST Completed

The Configuration, Price and Small-World models



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Prof Uhler: Welcome back to this third video

and this third lecture on the network module,

where we're talking about different kinds of network

models.

We've already discussed Erdos-

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Configuration model

1/1 point (graded)

Unfortunately the Erdos-Renyi model does not produce a power-law degree distribution. Power-law degree distributions are commonly observed in natural networks, so we desire a graphical model that will produce such a distribution.

In fact, we can go one step further and define a graphical model that can produce any desired degree distribution. This is known as the Configuration model.

The configuration model starts with a list of the desired degree distributions $\{k_1, k_2, \dots, k_n\}$, with each k_i denoting the desired degree of node i. We then assign to each node a number of “stubs", s_i that is initially equal to the desired degree for that node: $s_i = k_i$. Each stub can be thought of as “half of an edge" as $\sum_i k_i = 2m$ where m is the number of edges for the graph.

Then, two stubs are selected uniformly at random. This means that a node, i, is selected with probability $\frac{s_i}{\sum_l s_l}$, and a node, j, is selected with probability $\frac{s_j}{\sum_l s_l}$. Note that this implies i can be the same as j.

Then, these stubs are connected together, removing them, and forming an edge. This means that an edge, $\{i, j\}$ is inserted into the edge list for the graph. Then, the number of stubs for node i and j are both reduced by one: $s_i \leftarrow s_i - 1$, $s_j \leftarrow s_j - 1$.

This process repeats until $\sum_l s_l = 0$.

We can generate the initial degree list by sampling values of k from our desired degree distribution: for example, a power-law distribution. Suppose we use a power-law distribution, can any list of samples drawn from this distribution be used to construct the Configuration model?

- ○ Yes
- ◉ No

✔

Solution:

The sum of the node degrees must be even for any graph. When we draw samples from a power-law distribution, we may end up with $\sum_i k_i$ being odd. This cannot be used in the Configuration model, as it does not represent a valid graph.

We can address this using rejection sampling. If we draw a degree list for which the sum is odd, we can simple discard it (rejection) and draw a new degree list. We can repeat this until we draw a degree list for which the sum is even.

Submit You have used 1 of 1 attempt

Answers are displayed within the problem

Number of stub pairs

2/2 points (graded)

Suppose that node i has degree k_i and node j has degree k_j in a Configuration model with n nodes and total number of edges m.

What is the total number of possible pairings of stubs between these two nodes? (Use `k_i` for k_i and `k_j` for k_j.)

k_i*k_j **Answer:** k_i * k_j

Suppose that we have selected a stub from node i, what is the probability of it being connected to any other stub in the model? (Use `m` for m.)

$p_{\text{pair}} =$

1/(2*m-1) **Answer:** 1/(2*m-1)

Solution:

As there are k_i stubs for node i and k_j stubs for node j, and it is possible all stubs for node i to be connected to any stub from node j, we have that the total number of pairs is $k_i k_j$.

Once we have selected a stub, there are $2m - 1$ other stubs that it could possibly be connected to. So the probability of this stub being connected to any particular other stub in the model is $1/(2m-1)$. We can also see that there are $2m - 1$ stub pairs that involve any given stub, and as this given stub must be connected somewhere in the graph, the probability of any particular pair in again $1/(2m-1)$.

Submit You have used 2 of 2 attempts

Answers are displayed within the problem

Edge count

1/1 point (graded)

Use the correct solution to the previous question to answer this problem.

Let I_l be an indicator variable for stub pair l being selected to form an edge in the graph. We know that $\mathbb{E}[I_l] = p_{\text{pair}}$.

In addition, the number of edges, $h_{i,j}$, between nodes i and j is equal to the sum of these indicator variables for all stub pairs between i and j:

$$h_{i,j} = \sum_{l \in \{\text{stub pairs between } i \text{ and } j\}} I_l$$

What is the expected value of $h_{i,j}$?

$\mathbb{E}[h_{i,j}] =$

(k_i*k_j)/(2*m-1) ✔ **Answer:** k_i*k_j/(2*m-1)

Solution:

We have

$$\begin{aligned} \mathbb{E}[h_{i,j}] &= \sum_{l \in \{\text{stub pairs between } i \text{ and } j\}} \mathbb{E}[I_l] \\ &= \sum_{l \in \{\text{stub pairs between } i \text{ and } j\}} p_{\text{pair}} \\ &= k_i k_j p_{\text{pair}} \\ &= \frac{k_i k_j}{2m-1} \end{aligned}$$

Submit You have used 2 of 2 attempts

Answers are displayed within the problem

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Number of stub pairs - input validation

question posted 3 months ago by **paulbartusch**

Hi, although I don't know, if my answer is correct, I get a validation error:

Invalid Input: k_j not permitted in answer as a variable

for answers on the question on stub pairs. Is there something wrong with the grader?

This post is visible to everyone.

gurdeep213
3 months ago - marked as answer 2 months ago by **lam_trinh** (Community TA)

Ain't nothing wrong. Please use "only" the variables mentioned in the specific question part.

Add a comment

kozaronek
2 months ago - marked as answer 2 months ago by **lam_trinh** (Community TA)

k_j should be accepted by the grader for the first answer box. You probably put k_j into the second answer box as well. Pay attention to what is being asked. The second question asks for the general probability of a stub connecting to any other stub in the network.

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1 other response

abc497662892
2 months ago

Actually, I think the second question is really confusing! I think a better description is "Suppose that we have selected a stub from node i, what is the probability of it being connected to ***a particular stub (or another given stub)*** in the model?"

Thank you. your comment helped me to get the answer. It is indeed confusing with the sentence phrasing I believe.

posted 2 months ago by **ashokmeruva**

I agree, there is ambiguity in the question (I thought of the probability of picking any of the other 2m -1 instead of a particular one)

posted 2 months ago by **ediazr**

+1 Thank you!

posted 2 months ago by **andsann**

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