# PART A

Consider the problem:

$$
\left.
\begin{array}{lr}
\max & x_1 + x_2 \\
\text{s.t.:} & \\
 & x_1 + x_2 \leq 8 \\
 & -x_1 + x_2 \leq 2 \\
 & x_1 - x_2 \leq 4 \\
 & x_2 \geq 0 \\
 & x_1 \in \{0, 1, 4, 6\}
\end{array}
\right\}
$$

When you formulate this problem as an IP, how many binary variables need to be added (Assume that no non-binary variables are added)?

3

**Answer:** [3, 4]

$3$

**SOLUTION**

There are three resource constraints, and two additional logical constraints.

## PART B

Formulate the problem as an equivalent integer program

I am ready to have the answer shown

**SOLUTION**

Everything except the condition that $x_1$ can only be one of four values is already linear. We can use the standard way of doing this by introducing 4 binary variables $w_1, \ldots, w_4$ and imposing $x_1 = 0$
.

$$w_1 + w_2 + 4w_3 + 6w_4$$
$$w_1 + w_2 + w_3 + w_4 \leq 1$$

However, it is easy to see that we need only three binary variables as follows

$$x_1 = w_1 + 4w_2 + 6w_3$$
$$w_1 + w_2 + w_3 \leq 1$$

# PART C

How would your answer to the previous part change if the objective function were changed to:

$$\text{MAX } x_1^2 + 2x_2$$

*NOTE: It is possible to modify the problem in many different ways. See if you can find a way that adds a single decision variable $y$ (which will be equal to $(x_1)^2$) and only one additional constraint.*

I am ready to have the answer shown

**SOLUTION**

Let $y = x_1^2$, then note that $y$ is either $0, 1, 16$ or 36, depending on whether $x_1$ is $0, 1, 4$ or 6 respectively. Hence we add the constraint $y = w_1 + 16w_2 + 36w_3$ where $w_1, w_2, w_3$ are as described above (in the solution with 3 binary variables). The objective and all constraints are now linear.