



MITx: 6.041x Introduction to Probability - The Science of Uncertainty



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Problem 6: Random incidence under Erlang interarrivals

(2 points possible)

A single dot is placed on a very long length of yarn at the textile mill. The yarn is then cut into pieces. The lengths of the different pieces are independent, and the length of each piece is distributed according to the same PDF $f_X(x)$. Let R be the length of the piece that includes the dot. Determine the expected value of R in each of the following cases.

In each part below, express your answer in terms of λ using standard notation. Enter 'lambda' for λ .

1. Suppose that $f_X(x) = \begin{cases} \lambda e^{-\lambda x}, & x \geq 0, \\ 0, & x < 0. \end{cases}$

$$\mathbf{E}[R] =$$

1/lambda


✗ Answer: 2/lambda

2. Suppose that $f_X(x) = \begin{cases} \frac{\lambda^3 x^2 e^{-\lambda x}}{2}, & x \geq 0, \\ 0, & x < 0. \end{cases}$


- ▶ Unit 6: Further topics on random variables
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- ▼ **Unit 9: Bernoulli and Poisson processes**

Unit overview


Lec. 21: The Bernoulli process

Exercises 21 due May 11, 2016 at 23:59 UTC 

Lec. 22: The Poisson process

Exercises 22 due May 11, 2016 at 23:59 UTC 

Lec. 23: More on the Poisson process

Exercises 23 due May 11, 2016 at 23:59 UTC 

$$\mathbf{E}[R] = 3/\lambda$$

✗ Answer: $4/\lambda$

Answer:


1. Here, the lengths of the pieces of yarn are independent and exponentially distributed with parameter λ . As explained on pages 322-324 of the text, due to the memorylessness of the exponential, the distribution of the length of the piece of yarn containing the dot is a second order Erlang. Thus, $\mathbf{E}[R] = 2\mathbf{E}[X] = 2/\lambda$.
2. Here, X is an Erlang of order 3. Think of sections on the yarn, each exponentially distributed with parameter λ . We can then interpret each piece of yarn as *three* consecutive sections of exponentially distributed lengths. The piece of yarn with the dot will have the dot in one of these three sections. By the standard random incidence analysis, the expected length of that section will be $2/\lambda$. However, the piece of yarn containing the dot also consists of two other sections, each with an expected length of $1/\lambda$. Thus, the total expected length of the piece of yarn containing the dot is $4/\lambda$.

You have used 2 of 2 submissions

DISCUSSION

Click "Show Discussion" below to see discussions on this problem.

Solved problems**Additional theoretical material****Problem Set 9**

Problem Set 9 due May 11, 2016
at 23:59 UTC 

Unit summary

- ▶ Unit 10: Markov chains
- ▶ Exit Survey

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