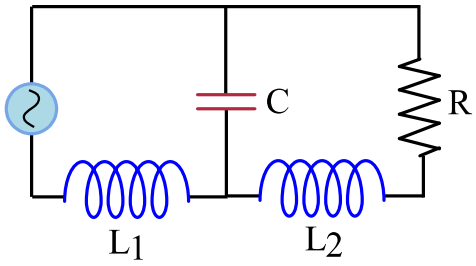
1. RLC circuit

Consider the circuit



In general, the voltage and current in any circuit with standard components can be modeled by a first-order system of ODE's. In the diagram above, suppose that the input (power supply) is given as $V_{in}(t) = \sin \omega t$. Let V_1 be the voltage drop across the inductor with inductance L_1, and let I_1 be the current through that inductor. Similarly, let V_2 and I_2 be the voltage drop across and the current through the inductor with inductance L_2. Let V_C be the voltage drop across the capacitor with capacitance C and let I_C be the current through the capacitor. Let V_R be the voltage drop across the resistor of resistance R, and let I_R be the current through the resistor.

There are eight dependent variables (I's and V's), and the eight equations governing the system are

1. $I_R = I_2$

2. $I_1 = I_C + I_2$

3. $V_C = V_R + V_2$

4. $V_C + V_1 = \sin \omega t$

5. $V_R = RI_R$

6. $\dot{V}_C = \frac{I_C}{C}$

7. $V_1 = L_1 \dot{I}_1$

8. $V_2 = L_2 \dot{I}_2$

RLC (a) (External resource) (1.0 / 1.0 points)

Find the matrix system

Let $\mathbf{x} = \begin{bmatrix} V_C \\ I_1 \\ I_2 \end{bmatrix}$. The equations above can be written as a 3×3 system

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{b}(t).$$

Find $\mathbf{A}$ and $\mathbf{b}$.

Enter your answer in terms of the symbolic entries listed here and defined in the template below.

- R for R
- C for C
- L1 for L_1
- L2 for L_2
- w for ω
- t for t

Your Script

Save Reset MATLAB Documentation (https://www.mathworks.com/help/)

```
% Create symbols used in defining A and b
R = sym('R');
C = sym('C');
L1 = sym('L1');
L2 = sym('L2');
w = sym('w');
t = sym('t');

%Define the matrix A and the vector b below.
%Note that b should be a column vector for the differential equation to make sense.

A = [0,1/C,-1/C;-1/L1,0,0;1/L2,0,-R/L2];
b = [0;sin(w*t)/L1;0];
```

▶ Run Script ()

Assessment: Correct

Submit ()

Check the matrix A

Check the vector b

RLC (b)

1/1 point (graded)

Let $L_1 = L_2 = C = 1$. Find the characteristic polynomial $\det(\lambda \mathbf{I} - \mathbf{A})$ (in terms of R).

(Enter as a polynomial is **lambda** with leading coefficient $+1$.)

lambda^3+R*lambda^2+2*lambda+R

Answer: lambda^3+R*lambda^2+2*lambda+R

$\lambda^3 + R \cdot \lambda^2 + 2 \cdot \lambda + R$

FORMULA INPUT HELP

Solution:

The characteristic polynomial is $\det(\lambda \mathbf{I} - \mathbf{A})$, which is

$$\begin{vmatrix} \lambda & -1 & 1 \\ 1 & \lambda & 0 \\ -1 & 0 & \lambda + R \end{vmatrix} = \lambda^2(\lambda + R) + \lambda + (\lambda + R) = \lambda^3 + R\lambda^2 + 2\lambda + R.$$

Submit You have used 2 of 3 attempts

Answers are displayed within the problem

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