2 3 5 7 11 13 17 19 23 29 31 37 41 43 47 53 59 61 67 71 73 79 83 89 97 101 103 107 109 113 127 131 137 139 149 151 157 163 167 173 179 181 191 193 197 199

More Fun with Prime Numbers

Week 5

Homework

Tetsushi Ito

Department of Mathematics
Kyoto University

Problem 1

The **elliptic curve**

$$Y^2 = X^3 - 4$$

has only two **integral points** (S,T) with T>0. One is (S,T)=(2,2). Find the second point.

- Finding rational or integral points on elliptic curves is an important problem.

Problem 1

$Y^2 = X^3 - 4$

$X \leq 1 \Rightarrow X^3 - 4 < 0$

$X=2 \Rightarrow X^3 - 4 = 4 \qquad Y=2 \qquad (2,2)$

$X=3 \Rightarrow X^3 - 4 = 23$

$X=4 \Rightarrow X^3 - 4 = 60$

$X=5 \Rightarrow X^3 - 4 = 121 \qquad Y=11 \qquad \mathbf{(5,11)}$

Answer $(S,T) = (5,11)$

Problem 1

- In 1929, Siegel proved every elliptic curve has only **finitely many** integral points. But finding all integral points is not easy.

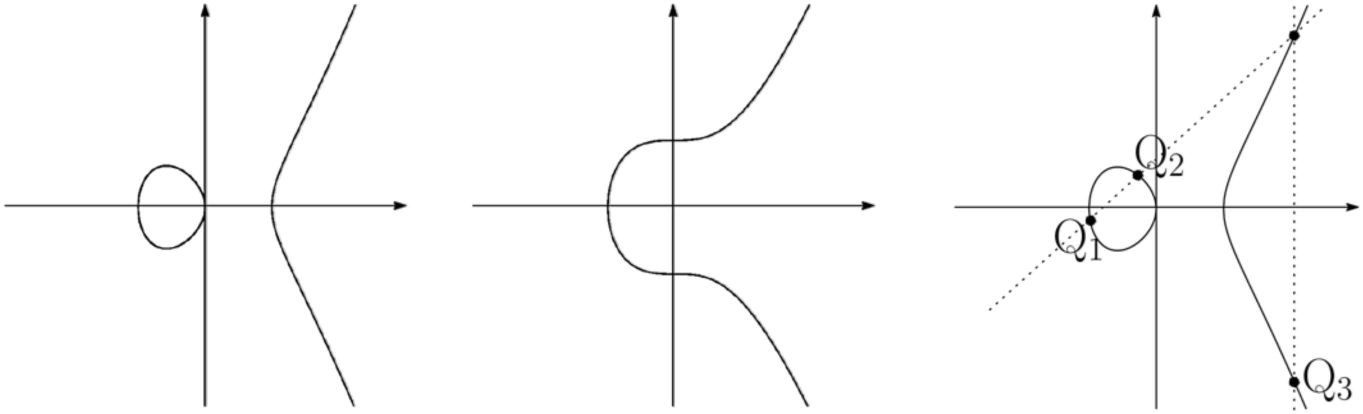
Example

$Y^2 = X^3 + 2X + 97$

(90086608, 855047718145) is an integral point.

Problem 1

- Finding rational points is much more difficult.
- An elliptic curve may have **finitely many** or **infinitely many** rational points.



$Y^2 = X^3 - X$ $\qquad$ $Y^2 = X^3 + 1$