Problem 3: Weight fluctuation

(2/2 points)

Uncle Henry has been having trouble keeping his weight constant. In fact, during each week, his weight changes from the beginning of the week to the end of the week by a random amount, uniformly distributed between -0.5 and 0.5 pounds. Assuming that his weight change during any given week is independent of his weight change during any other week, approximate the probability that at the end of 50 weeks Uncle Henry will have had a net change in weight of at least +3 pounds. You may want to refer to the standard normal table .

0.07082235 ✔ **Answer:** 0.0708

Answer:

Let W_i be Uncle Henry's weight change during week i, where the W_i's are i.i.d. uniform random variables distributed on $[-0.5, 0.5]$. Let $W = \sum_{i=1}^{50} W_i$ be his net weight change over the course of 50 weeks. We are looking for $\mathbf{P}(W \geq 3)$.

Since we have a sum of a large number of i.i.d. random variables, we can apply the central limit theorem to approximate this probability. Note that

$$\begin{aligned}\mathbf{E}[W] &= 0\\ \mathbf{var}(W) &= 50 \cdot \mathbf{var}(W_1) = 50 \cdot (1^2/12) = 25/6.\end{aligned}$$

Hence, by the central limit theorem,

$$\begin{aligned}\mathbf{P}(W \geq 3) &= \mathbf{P}\left(\frac{W-0}{\sqrt{25/6}} \geq \frac{3-0}{\sqrt{25/6}}\right)\\ &\approx 1 - \Phi\left(\frac{3}{\sqrt{25/6}}\right)\\ &\approx 0.0708.\end{aligned}$$

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