Exercise: Sample mean bounds

(2/2 points)

By the argument in the last video, if the X_i are i.i.d. with mean μ and variance σ^2, and if $M_n = (X_1 + \cdots + X_n)/n$, then we have an inequality of the form

$$\mathbf{P}\big(|M_n - \mu| \geq \epsilon\big) \leq \frac{a\sigma^2}{n},$$

for a suitable value of a.

a) If $\epsilon = 0.1$, then the value of a is:

100 ✔

Answer: 100

b) If we change $\epsilon = 0.1$ to $\epsilon = 0.1/k$, for $k \geq 1$ (i.e., if we are interested in k times higher accuracy), how should we change n so that the value of the upper bound does not change from the value calculated in part (a)?

n should

- ○ stay the same
- ○ increase by a factor of k
- ● increase by a factor of k^2 ✔
- ○ decrease by a factor of k
- ○ none of the above

Answer:

a) Chebyshev's inequality yields

$$\mathbf{P}\big(|M_n - \mu| \geq \epsilon\big) \leq \frac{\sigma^2}{n\epsilon^2},$$

so that $a = 1/\epsilon^2 = 1/0.1^2 = 100$.

b) In order to keep the same upper bound, the term $n\epsilon^2$ in the denominator needs to stay constant. If we reduce ϵ by a factor of k, then ϵ^2 gets reduced by a factor of k^2. Thus, n will have to be increased by a factor of k^2.

You have used 1 of 2 submissions