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1.5 Summary Quiz: Outbreaks and Other Bifurcation Behavior

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Question 1

1/1 point (graded)

According to the model

$$\frac{dP}{dt} = \frac{1}{2}P\left(1 - \frac{P}{q}\right) - \frac{P^2}{1+P^2}$$

how can outbreak of budworms occur?

```
When q value goes above a threshold, the stable equilibrium
shifts to much higher value.
```

Thank you for your response.

Explanation

The model Wes presented took into account budworm reproduction rate (r), carrying capacity of the environment (q), and predation rate of warblers on budworms ($-\frac{P^2}{1+P^2}$). When q is sufficiently high, there are four equilibrium points for the budworm population, P: $P = 0$, a small stable equilibrium point, a mid-range unstable equilibrium, and a large stable equilibrium. The largest equilibrium is much larger than the smallest equilibrium. Thus, while the population is below the mid-range equilibrium point, it tends toward the smallest equilibrium.

Suppose P starts in this low-level range. Population fluctuations from other events not incorporated into the model in the budworms' environment can push P over the mid-range equilibrium value. Once P is greater than this mid-range value, the model predicts that P will increase until it reaches the largest equilibrium point. Thus, the budworm population "breaks out" from its normal lower level to attain a much higher stable population value.

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Answers are displayed within the problem

Question 2

1/1 point (graded)

A bifurcation is *"a dramatic change in the expected behavior of a system in response to a change in parameter."*

Here q is the parameter and the system is the budworm population over time. Recall that q_* is the carrying capacity at which the curve is tangent to the line.

When the carrying capacity is less than q_*, the population of budworms will always approach an equilibrium which is less than carrying capacity, regardless of starting population.

What is the dramatic change in the behavior of the budworm population when the carrying capacity is greater than q_*? (In other words, what is the bifurcation?)

- When the carrying capacity is less than q_*, the population of budworms will approach a non-zero equilibrium which is less than carrying capacity. But when the carrying capacity is greater than q_*, the population of budworms will approach an equilibrium equal to the carrying capacity, if the starting population is large enough. That is the dramatic change.
- When the carrying capacity is less than q_*, the population of budworms will approach a non-zero equilibrium which is less than carrying capacity. But when the carrying capacity is greater than q_*, the population of budworms will grow without bound for large enough starting populations. That is the dramatic change.
- When the carrying capacity is less than q_*, the population of budworms will approach a non-zero equilibrium which is less than carrying capacity. But when the carrying capacity is greater than q_*, the population of budworms will approach a new and larger equilibrium, if the starting population is large enough. That is the dramatic change. ✔
- When the carrying capacity is less than q_*, the population of budworms will approach a non-zero equilibrium which is less than carrying capacity. But when the carrying capacity is greater than q_*, the population of budworms will go extinct, if the starting population is small enough. That is the dramatic change.
- None of the above.

Explanation

When the parameter q is at the critical value q_*, the environment is in a boundary state between two dramatically different scenarios. If q is less than this critical value, the only long-term behavior for the budworm population is to tend toward a single small equilibrium value. In general, this value is much less than the carrying capacity, and the dominant force keeping the population below this value is predation. If q is larger than the critical value, the budworm population has an entirely different qualitative behavior: the budworms tend toward a similar small equilibrium point for small initial populations, but if the population ever increases beyond a certain mid-range value (given by the mid-range unstable equilibrium

point), then for fixed q, the population will no longer tend toward the small equilibrium value. Rather, the population will increase until it reaches another equilibrium value which is much larger than the small equilibrium point. (Note: this larger equilibrium is still less than the carrying capacity.)
This dramatic change in behavior occurs due to variation in the parameter q.

Submit You have used 1 of 3 attempts

Answers are displayed within the problem

Question 3

2/2 points (graded)
The bifurcation in this budworm example occurs when the curve $y = \frac{P}{1+P^2}$ is tangent to the line $y = \frac{1}{2}(1 - \frac{P}{q})$. What is the 'critical' value of q at which the bifurcation occurs?

Estimate the "critical" value of q when the curve is tangent to the line, using the dynamic Desmos graph. Give your answer as a range with each endpoint rounded to the nearest tenth.

Desmos Hint: By clicking on either equation on the left, you can see the intersection points highlighted (as well as max and min of the graph). Hovering the cursor over a point will show its' coordinates.

Enter numerical values for the lower and upper bound.

Lower Bound:

7.4 **Answer:** 7.4

7.4

Upper Bound:

7.5 **Answer:** 7.5

7.5

Explanation
$(7.4, 7.5)$ We adjust q until the two curves look almost tangent. To test when the two lines start to intersect, we click on one of the equations at the left. We find that $q \in (7.4, 7.5)$:
In the following plot, the two curves almost intersect with $q = 7.4$:

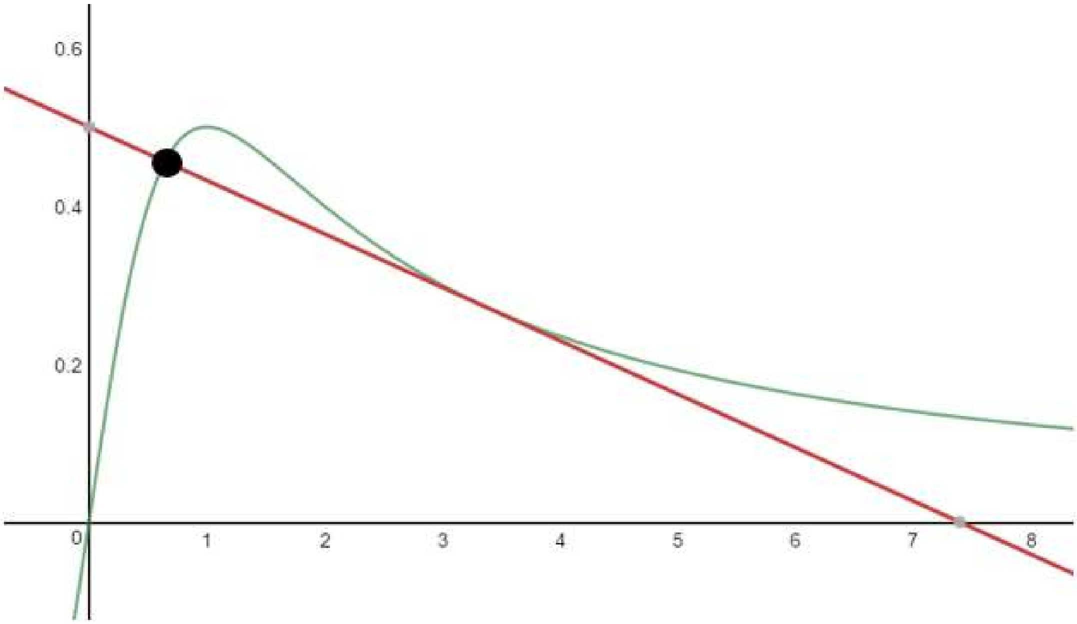

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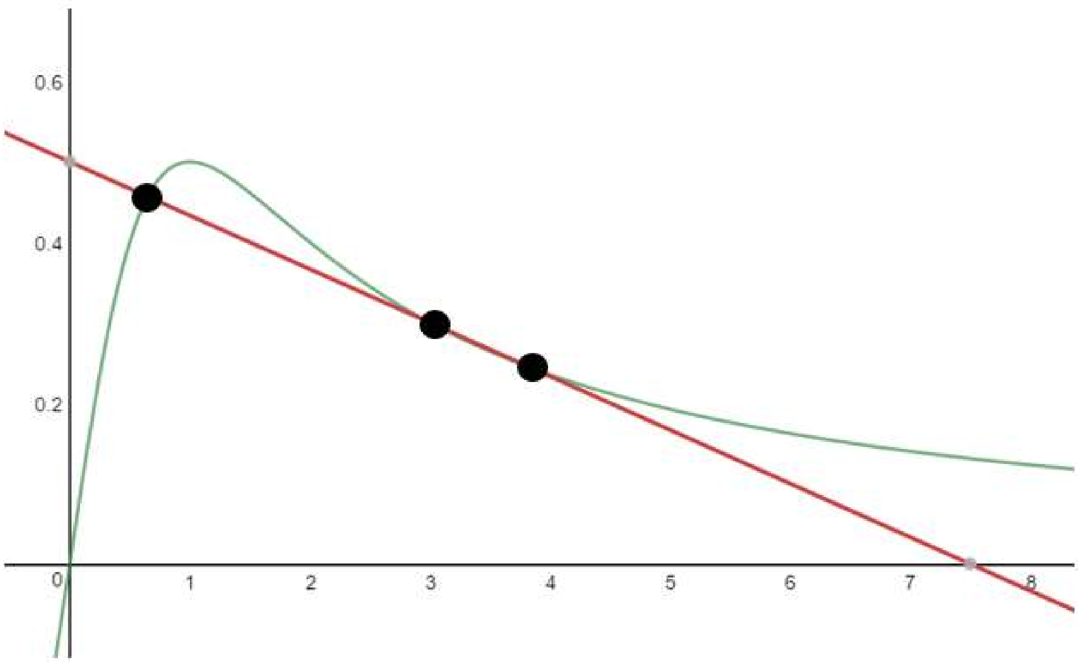
Image Description
When we increase q to 7.5, the two curves do barely intersect:



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Question 4

1/1 point (graded)

The bifurcation in this budworm example occurs when the curve $y = \frac{P}{1+P^2}$ is tangent to the line $y = \frac{1}{2}(1 - \frac{P}{q})$. Which of the following computations would allow you to compute this value exactly?

(Hint: We want to find the q-value where the line $y = \frac{1}{2}(1 - \frac{P}{q})$ is tangent to the curve $y = \frac{P}{1+P^2}$. Translate this to a calculus problem, by setting up the equation(s) you'd need to solve to find q. You do not need actually solve for q.)

- Take the derivative of $y = \frac{1}{2}(1 - \frac{P}{q})$ with respect to P, set it equal to zero and solve for q.
- Take the derivative of $y = \frac{1}{2}(1 - \frac{P}{q})$ and set it equal to the derivative of $y = \frac{P}{1+P^2}$. Solve for the value of q which makes this equation true.
- Solve a system of two equations in two unknowns. One equation is $\frac{P}{1+P^2} = \frac{1}{2}(1 - \frac{P}{q})$. For the other equation, take the derivative of $y = \frac{1}{2}(1 - \frac{P}{q})$ and set it equal to the derivative of $y = \frac{P}{1+P^2}$. Solve for the value of q and P which make these both true. ✔
- Solve a system of two equations in two unknowns. For the first equation equation, take the derivative of $y = \frac{1}{2}(1 - \frac{P}{q})$ and set it equal to zero. For the other equation, take the derivative of $y = \frac{P}{1+P^2}$ and set it equal to zero. Solve for the value of q and P which make these both true.
- None of the above.

Explanation

For two curves to be tangent at a point, two things must happen: the curves must be touching at that point and their slopes (i.e. derivatives) must be equal at that point.

This means we need to a system of two equations in two unknowns. One equation is $\frac{P}{1+P^2} = \frac{1}{2}(1 - \frac{P}{q})$.

For the other equation, we take the derivative of $y = \frac{1}{2}(1 - \frac{P}{q})$ and set it equal to the derivative of $y = \frac{P}{1+P^2}$.

Thus, we want to solve this set of equations to find the value of q and P which make both equations true.

We'll call the P-value where they intersect P_*. We call the critical q value q_*.

First, we compute the derivatives:

$$\begin{aligned}
y_1 &= \frac{1}{2}\left(1 - \frac{P}{q}\right) \\
\frac{dy_1}{dP} &= -\frac{1}{2q} \\
y_2 &= \frac{P}{1+P^2} \\
\frac{dy_2}{dP} &= P\left(-\frac{2P}{(1+P^2)^2}\right) + \frac{1}{1+P^2} \quad \text{(Using the product or quotient rule)} \\
&= \frac{1-P^2}{(1+P^2)^2}
\end{aligned}$$

We need to solve for P_* and q_* such that the following two conditions hold:

$$\begin{aligned}
y_1(P_*) &= y_2(P_*) \\
\frac{dy_1}{dP}(P = P_*, q = q_*) &= \frac{dy_2}{dP}(P = P_*, q = q_*)
\end{aligned}$$

The first conditions ensures that the lines are touching at $P = P_*$, and the second condition dictates that the two curves have the same slope at that point. In terms of our equations, these two conditions give:

$$\begin{aligned}
\frac{1}{2}\left(1 - \frac{P_*}{q_*}\right) &= \frac{P_*}{1+P_*^2} \\
-\frac{1}{2q_*} &= \frac{1-P_*^2}{(1+P_*^2)^2}
\end{aligned}$$

This system has two independent equations for two unknowns, P_* and q_*, so we can use these equations to solve for both unknowns, determining q_* exactly.
Although you are not asked to solve for q exactly, these two solutions can be solved analytically as follows:
Starting with the second equation above:

$$\begin{aligned}
(1+P_*^2)^2 &= 2q_*(P_*^2 - 1) \\
1 + 2P_*^2 + P_*^4 &= 2q_*P_*^2 - 2q_* \\
0 &= P_*^4 + 2(1-q_*)P_*^2 + (1-2q_*) \\
\text{(Using the quadratic formula) } P_*^2 &= \frac{-2(1-q) \pm \sqrt{4(1-q_*)^2 - 4(1-2q_*)}}{2} \\
&= \frac{-2 + 2q_* \pm \sqrt{4 - 8q_* + 4q_*^2 - 4 + 8q_*}}{2} \\
&= -1 + 2q_*^2 \text{ or } -1
\end{aligned}$$

P_*^2 cannot be equal to -1 since P_* is a real number, so $P_* = \sqrt{2q_* - 1}$. We now substitute this into the first of our original two equations and solve for q_*:

$$\begin{aligned}\frac{1}{2}\left(1-\frac{\sqrt{2q_*-1}}{q_*}\right) &= \frac{\sqrt{2q_*-1}}{1+2q_*-1}\\ 1-\frac{\sqrt{2q_*-1}}{q_*} &= \frac{\sqrt{2q_*-1}}{q_*}\\ q_* &= 2\sqrt{2q_*-1}\\ q_*^2-8q_*+4 &= 0\\ q_* &= 4\pm 2\sqrt{3}=\{7.46, 0.54\}\end{aligned}$$

We recognize $q_* = 7.46$ as the critical value corresponding to the bifurcation point, which we estimated in part (a) (note that the simulation device is not exact). The other solution for q_* is not physical. If we go back to our expression for P_* in terms of q_*, we see that $q_* = 0.54$ gives an imaginary value for P_*, so we discard this solution and are left with the solution we expect.

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Answers are displayed within the problem

The remaining questions are about the following model of fish population with harvesting:

$$\frac{dP}{dt} = 0.1P\left(1-\frac{P}{20}\right) - \frac{aP}{10+P}$$

Here P is the population of fish at time t (in years) and a is a positive parameter. The effect of fishing here is represented by the term $\frac{aP}{10+P}$ which has units fish/year. (Note: this is different than the fishing models we considered in the previous section of the course.)

We'll analyze the meaning of the fishing term and then use the strategy of graphical analysis to identify any bifurcations in the system.

Question 5

1/1 point (graded)

Sketch a graph of the fishing rate $y = \frac{aP}{10+P}$. Using your graph, choose all correct descriptions of the relationship between the fishing rate and the size of the population.

- [ ] For small values of P, the fishing rate is larger than it is for large values of P.

- [x] For small values of P, the fishing rate is smaller than it is for large values of P.
- [x] The fishing rate increases as the population size P increases, but there is a limit on how large the rate can be.
- [ ] The fishing rate increases as the population size P increases, and there is no limit on how large the rate can be.
- [ ] None of the above.

Explanation

Below is the plot of $y = \frac{aP}{10+P}$, with the function plotted in blue. The red line is $y = a$, to which the function asymptotes as $P \to \infty$. Note that the vertical axis is in units of a.

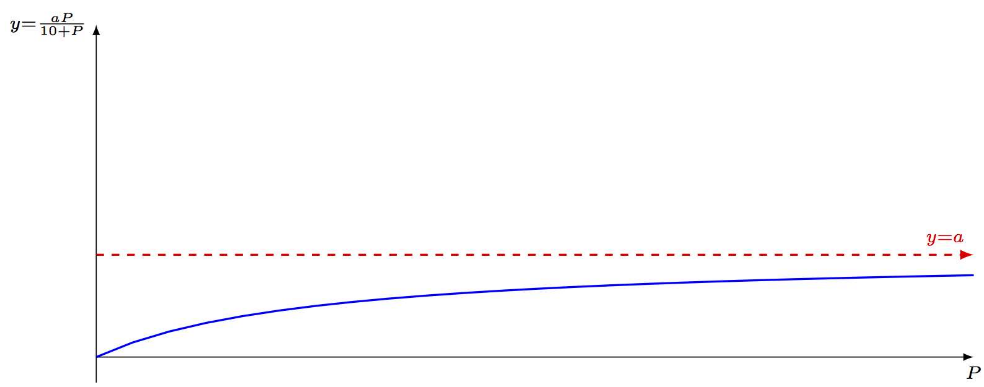

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Image Description
This fishing rate as a function of P is an increasing function, so for small values of P, the fishing rate is smaller than it is for large values of P. However, as P gets larger, the fishing rate approaches the constant value a.
This is a plausible fishing model for two reasons: in reality, since it is harder to catch fish when the population is small, the fishing rate should be smaller for small values of P. On the other hand, when the fish population is large, the fishing rate is limited by total number of fishermen there are and the maximum number of fish each fisherman can catch per unit time. Thus, in the case of large P, all fishermen will be fishing to their maximum capacities, and the total fishing rate will approach a constant maximum value of a.

Submit You have used 2 of 3 attempts

Answers are displayed within the problem

Question 6

1/1 point (graded)

To analyze the equilibria of this differential equation, we'll use the graphical strategy that Wes used for the budworm model.

- Factor out P from the right hand side of

$$\frac{dP}{dt} = 0.1P\left(1 - \frac{P}{20}\right) - \frac{aP}{10+P}.$$

  You should have something now in the form

$$\frac{dP}{dt} = P[\textbf{Term 1} - \textbf{Term 2}].$$

  This reveals at least one equilibrium solution, namely $P = 0$.

- Use Desmos to plot graphs of $y =$ Term 1 and $y =$ Term 2. (In Desmos, if you type in something like $y = 2x + a$, it will give you the option to "add a slider" for a. This way you can adjust the value of a.)

- Explore what happens to these graphs as you change the value of the parameter a. How many qualitatively different situations are there? How many equilibrium points does each have? (Remember: the intersection points of $y =$ Term 1 and $y =$ Term 2 correspond to equilibrium points since they correspond to where the non-P factor of $\frac{dP}{dt} = 0$.)

```
Changing the value of a, we  have different number of
equilibria depending on whether a is

"small" (one non-zero equilibrium)
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Thank you for your response.

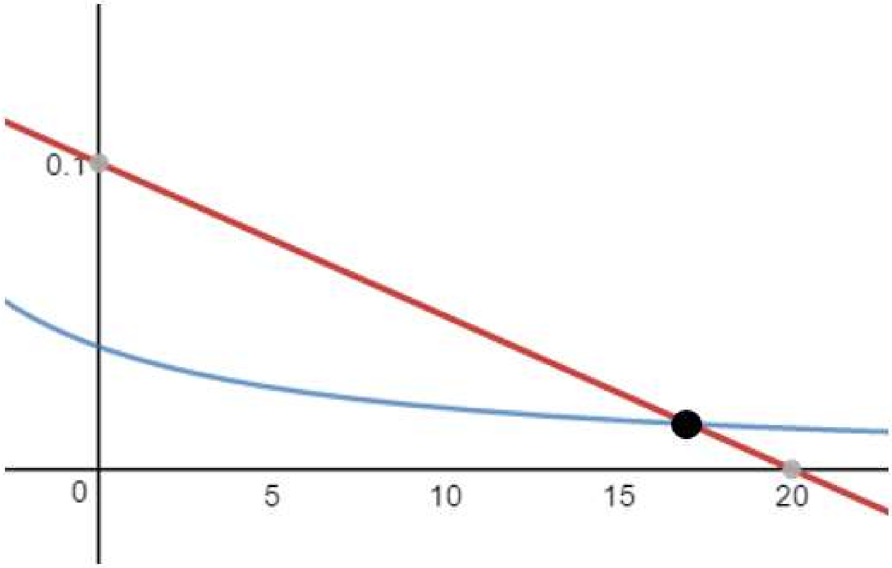
Explanation

We factor out P from the right-hand side of the differential equation to obtain:

$$\frac{dP}{dt} = P\left(0.1\left(1 - \frac{P}{20}\right) - \frac{a}{10+P}\right)$$

Equilibria occur at $\frac{dP}{dt} = 0$, so this factorization shows there is (at least) an equilibrium at $P = 0$. We know there should also be equilibria at the points where $0.1(1 - \frac{P}{20}) - \frac{a}{10+P} = 0$.

We now have this in the form $\frac{dP}{dt} = P[\textbf{Term 1} - \textbf{Term 2}]$, where $\textbf{Term 1} = \mathbf{0.1}(\mathbf{1} - \frac{P}{\mathbf{20}})$ and $\textbf{Term 2} = \frac{a}{10+P}$. When we plot these two terms, we find that adjusting a allows for 4 qualitatively different situations:



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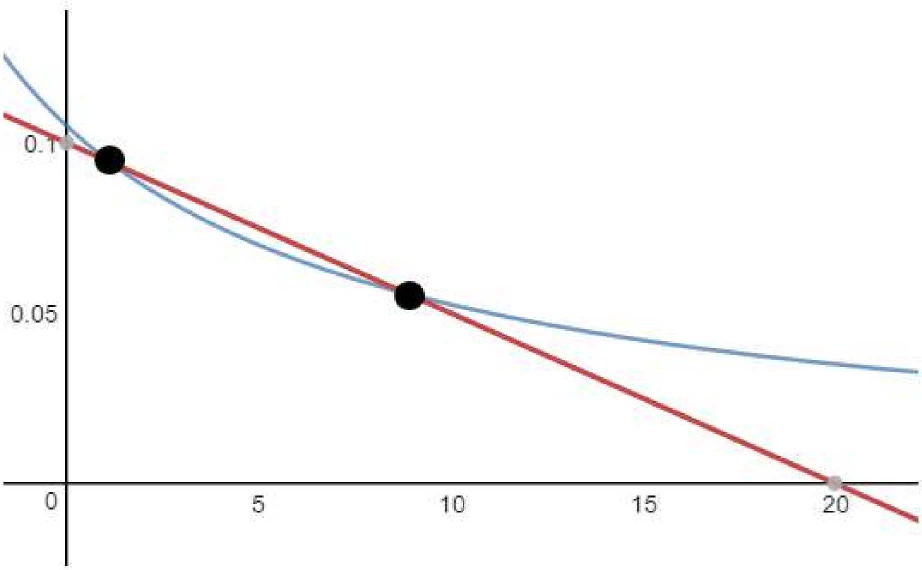

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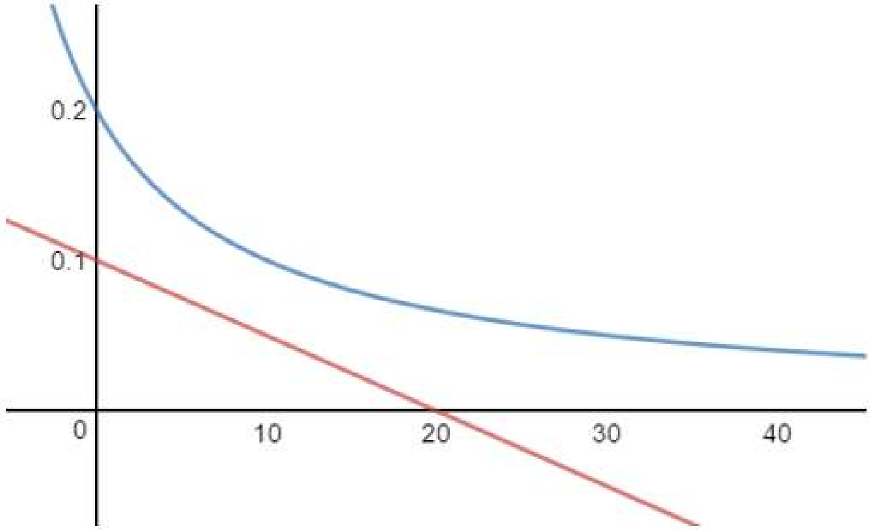

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Image Description

In each plot, the blue curve represents $y = \frac{a}{10+P}$, and the red curve represents $y = 0.1(1 - \frac{P}{20})$. The top plot shows the situation with very small a ($a = 0.4$), the middle plot shows medium-small a ($a \approx 1.05$), the lower-left plot shows medium-big a ($a \approx 1.13$), and the bottom plot shows big a ($a = 2$). We see that for small a, there is one non-zero equilibrium point; for medium-small a, there are two non-zero equilibrium points; for medium-big a, there is again one non-zero equilibrium point; and for big a, there are no non-zero equilibrium points. Note that $P = 0$ is an equilibrium point for all of these situations.

Additionally, note that we must be careful only to consider intersection points for which $P \geq 0$. In particular, the "small a" and "medium-small a" scenarios both have two intersection points, but with small a, one of those intersection points occurs with $P < 0$, so it is not physical (in the above plot, we show only the physical intersection points).

Equilibrium points are those values of P such that $\frac{dP}{dt} = 0$. Since Term 1 and Term 2 have opposite signs in the right-hand side of our differential equation, $\frac{dP}{dt} = 0$ when $\text{Term 1} = \text{Term 2}$. This occurs when the two curves intersect in our plots above.

Submit

You have used 1 of 2 attempts

Answers are displayed within the problem

Question 7

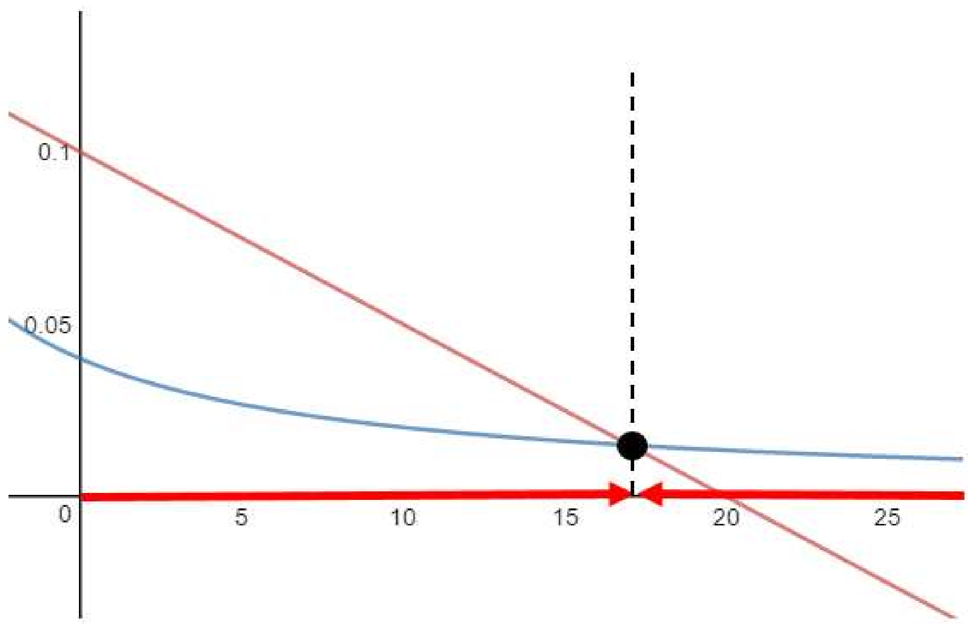
1/1 point (graded)

For small values of a, there is only one intersection of the two terms. This corresponds to one non-zero equilibrium solution. What can you say about this equilibrium from the graph?

- The non-zero equilibrium is unstable. This means the population will either grow larger and larger or will go extinct, depending if the population starts above or below the equilibrium.
- The non-zero equilibrium is stable. This means no matter the starting population, the population will approach the equilibrium number of fish. ✔
- The non-zero equilibrium is semi-stable. If the population starts above the equilibrium, it will approach the equilibrium number of fish. For populations less than the equilibrium, they will go extinct.
- The non-zero equilibrium is semi-stable. If the population starts above the equilibrium, it will grow larger and larger. For populations less than the equilibrium, they will approach the equilibrium number of fish.
- There is not enough information to determine its stability.

Explanation

Here is an example of the graph for a small a:



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We can determine the sign of $\frac{dP}{dt}$ for a given value of P by looking at which graph is above the other. Recall that $\frac{dP}{dt}$ is proportional to the difference of two terms: $\frac{dP}{dt} \propto \text{Term 1} - \text{Term 2}$, where the proportionality factor is P. Since P is always positive, this means that $\frac{dP}{dt}$ will be positive if $\text{Term 1} > \text{Term 2}$ and negative if $\text{Term 1} < \text{Term 2}$. In terms of the graphs, the red graph represents Term 1, and the blue graph represents Term 2. So $\frac{dP}{dt}$ is positive when the red graph is above the blue graph, and negative when the blue graph is above the red graph.
We indicate the sign of $\frac{dP}{dt}$ with arrows on the horizontal axis, with a left arrow meaning $\frac{dP}{dt} > 0$ and a right arrow meaning $\frac{dP}{dt} < 0$.
We see that for small a, the equilibrium point is stable because for initial population values below or above this equilibrium value, the population will increase or decrease toward the equilibrium.

Submit You have used 1 of 3 attempts

Answers are displayed within the problem

Question 8

2/2 points (graded)
As you experimented with changing the value of a, you probably noticed you can have different number of equilibria depending on whether a is

- "small" (one non-zero equilibrium)
- "medium" (two non-zero equilibria)
- "large" (no non-zero equilibria).

Enter a numerical value.

At approximately what value of the parameter a does the situation change from "small" to "medium"?

1.01 ✔ **Answer:** 1

1.01

At approximately what value of the parameter a does the situation change from "medium" to "large"?

1.14 ✔ **Answer:** 1.13

1.14

Submit You have used 1 of 4 attempts

Answers are displayed within the problem

Question 9

1/1 point (graded)

For each qualitatively different situation ('small', 'medium', 'large'), make a representative sketch of the graph, labeled according to the value of a (for example 'small' a, 'medium' a, 'large' a).

Indicate the intersection points and include arrows along the horizontal axis to indicate where P will be increasing or decreasing. (Remember: you can determine the sign of $\frac{dP}{dt}$ by thinking about which graph is above the other.)

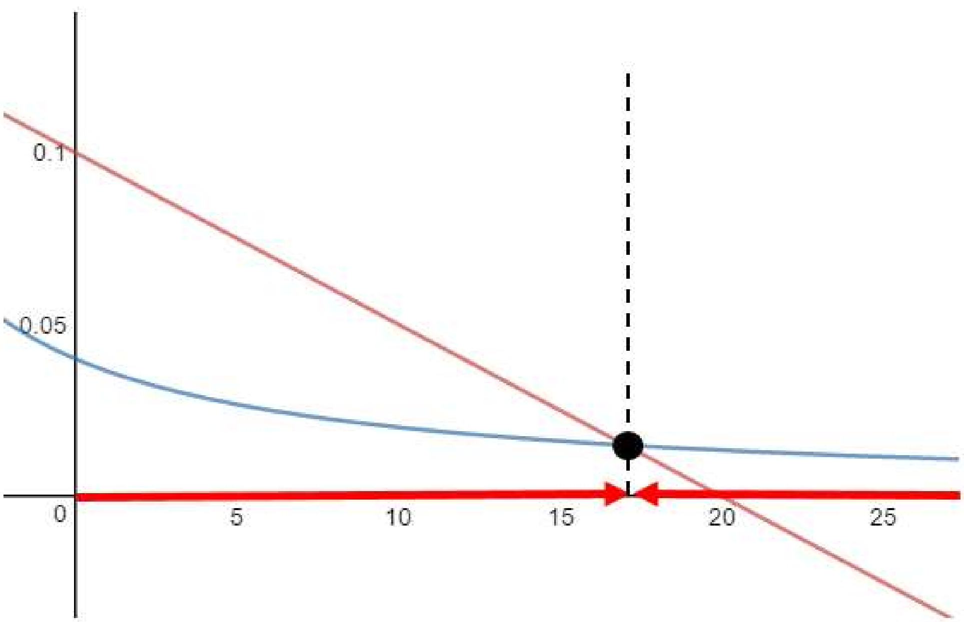
Then use this to determine the stability of the non-zero equilibrium solution(s). (We've already done this in the case of 'small' a.)

```
small: the single equilibrium is stable
medium: the first equilibrium is unstable and the second
one is stable
```

Thank you for your response.

Explanation

Small a:



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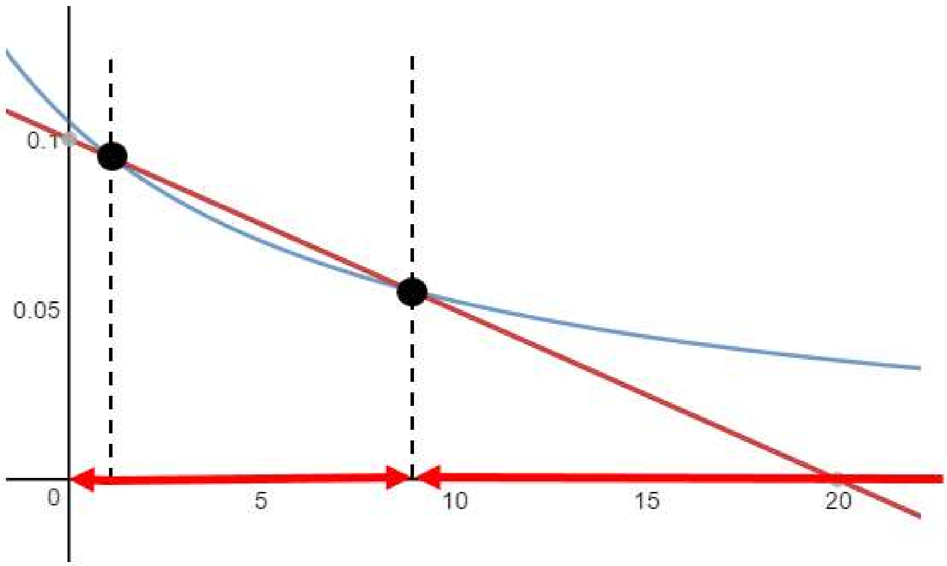
Medium-small a:



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Large a:

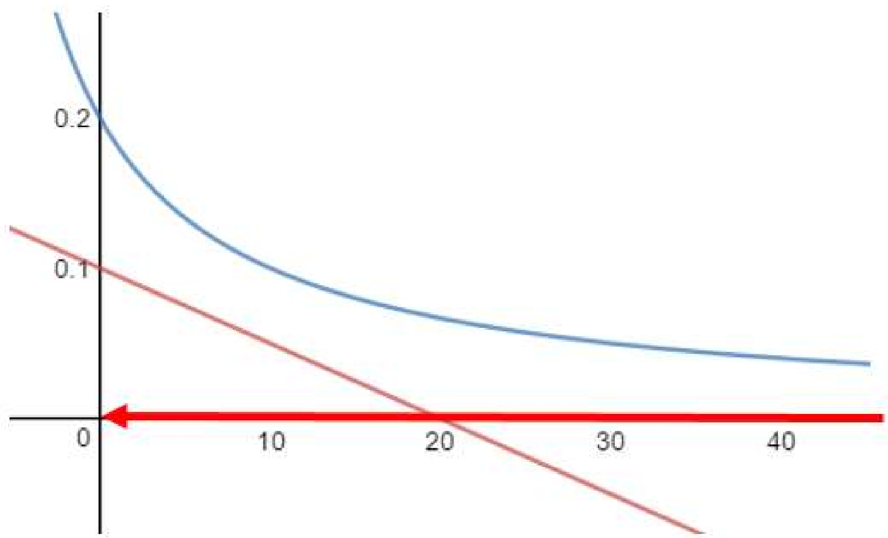

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Image Description
In each situation, we can determine the sign of $\frac{dP}{dt}$ for a given value of P by looking at which graph is above the other. Recall that $\frac{dP}{dt}$ is proportional to the difference of two terms: $\frac{dP}{dt} \propto \text{Term 1} - \text{Term 2}$, where the proportionality factor is P. Since P is always positive, this means that $\frac{dP}{dt}$ will be positive if $\text{Term 1} > \text{Term 2}$ and negative if $\text{Term 1} < \text{Term 2}$. In terms of the graphs, the red graph represents Term 1, and the blue graph represents Term 2. So $\frac{dP}{dt}$ is positive when the red graph is above the blue graph, and negative when the blue graph is above the red graph.

For small a, there is only one stable equilibrium point. For initial population values below or above this equilibrium value, the population will tend toward the equilibrium.

For medium a, there is one stable nonzero equilibrium solution and one unstable nonzero equilibrium solution; the unstable equilibrium solution occurs at a smaller P value than the stable one. For large a, there is no non-zero equilibrium solution.

Submit You have used 1 of 2 attempts

Answers are displayed within the problem

Question 10

1/1 point (graded)

Which of the following are possible long-term behaviors of the population in the situation that there are two non-zero equilibrium solutions ("medium" a)?

- [x] The population can decrease toward zero (go extinct). ✔
- [ ] The population can approach the smaller equilibrium solution in the long term.
- [x] The population can approach the larger equilibrium solution in the long term. ✔
- [ ] The population can grow larger and larger without bound.
- [ ] Other.

✔

Explanation

For medium a, there is one stable nonzero equilibrium solution and one unstable nonzero equilibrium solution; the unstable equilibrium solution occurs at a smaller P value than the stable one. For initial populations above the unstable equilibrium point, the population value will tend toward the stable equilibrium point, increasing or decreasing as needed. However, for initial populations below the unstable equilibrium point, the population will decrease to $P = 0$(extinction).

Note: this is qualitatively similar to the one of the situations in the logistic model with fishing at a constant rate from the previous section.

$$\frac{dP}{dt} = \frac{1}{10}P\left(1 - \frac{P}{40000}\right) - \alpha.$$

For small enough α, there is a smaller unstable equilibrium solution and a larger stable solution, meaning that the population is will likely approach that larger equilibrium unless it starts below that smaller equilibrium.

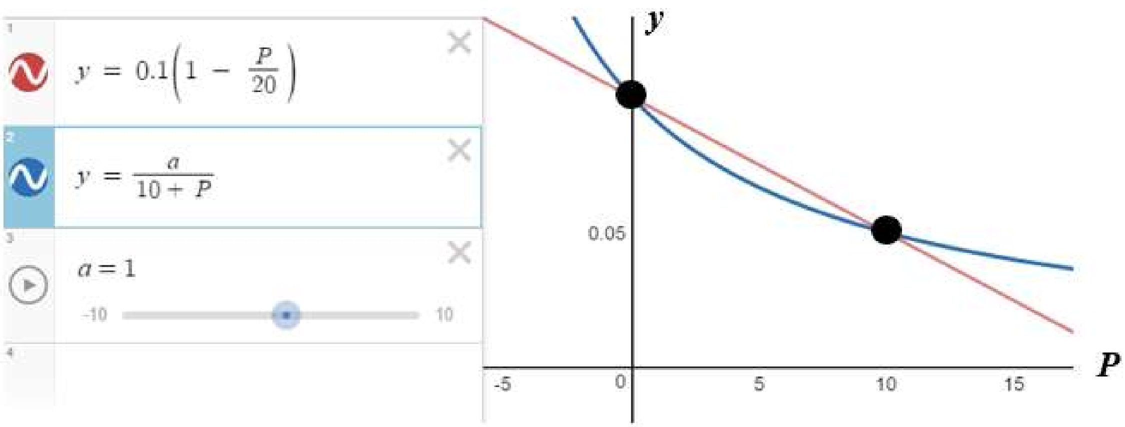
Submit You have used 1 of 3 attempts

Answers are displayed within the problem

There are two bifurcations in this system: the boundary between 'small' a and 'medium' a behavior, and the boundary between 'medium' a and 'large' a behavior.

- The first boundary value, which we will call a_{sm}, is the situation in which the two curves start to intersect twice for non-negative P values. Here the smaller equilibrium point occurs exactly at $P = 0$.

  Notice that this boundary case is qualitatively the same as in the small a case. We can see this by noticing that in this boundary case, there are still two equilibrium points: one non-zero and one at $P = 0$, and the stability of these two equilibria is the same as for the nonzero and $P = 0$ equilibria in the small a case.

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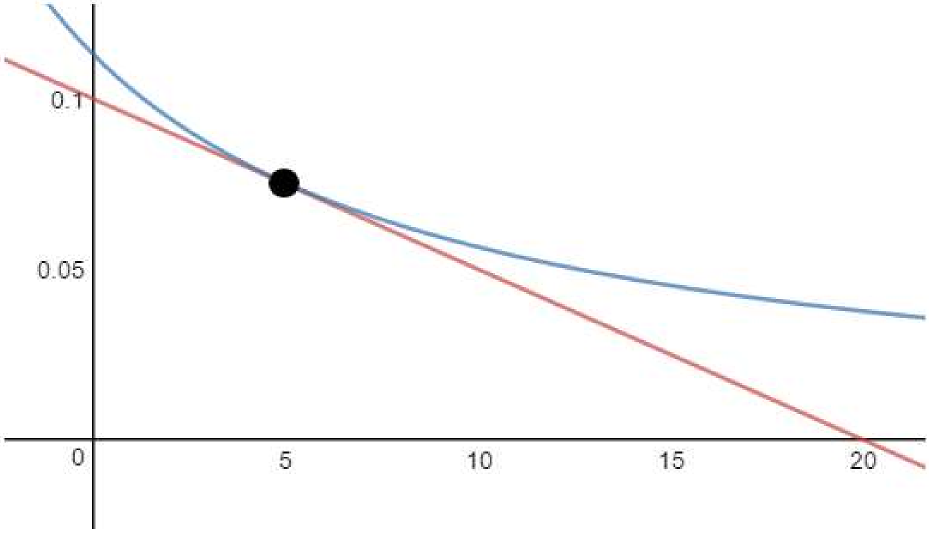
- The second boundary value, which we call a_{ml}, is when the curve is exactly tangent to the line at one positive value of P. This is qualitatively different from the medium a and large a situations as we'll now explore.



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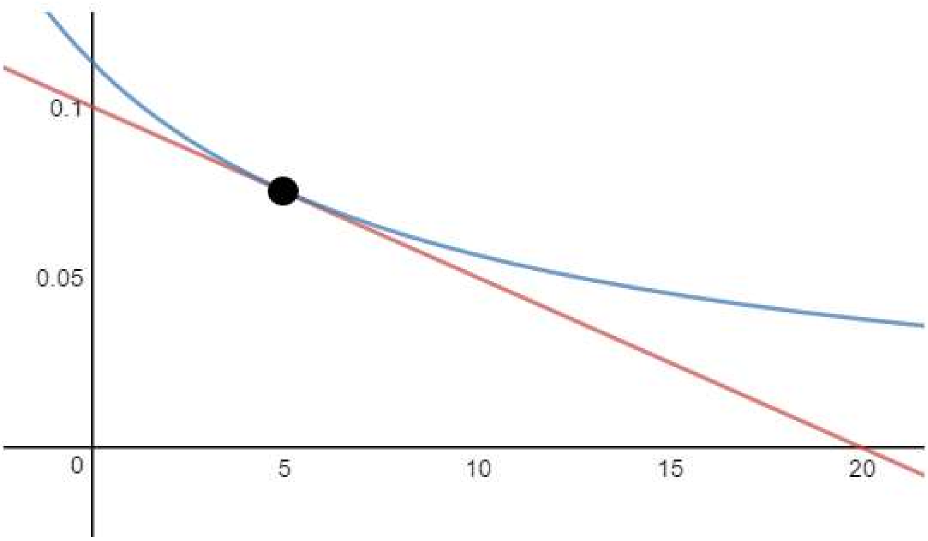
Question 11

1/1 point (graded)
Consider the boundary case $a = a_{ml}$, when the curve is exactly tangent to the line at one positive value of P. This is qualitatively different from the medium a and large a situations, since the system transitions from two non-zero equilibria to no non-zero equilibria.



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In this boundary case, the one non-zero equilibrium is semi-stable. This means, according to the model, for populations less than the equilibrium, they will go extinct. But if the population starts above the semi-stable equilibrium, it will approach that equilibrium number of fish.

In reality, why might we expect that the fish population will go extinct, no matter the starting population of fish?

- In reality, fishing will always cause extinction in the long-run since the fish can never reproduce fast enough to replace the fish that are harvested.
- In reality, there are random fluctuations in the population. If the population starts above the equilibrium population, it will decrease toward that semi-stable equilibrium, and eventually fluctuate below it which it will decrease toward P = 0. Initial populations below the semi-stable equilibrium point also decrease toward P = 0. ✔
- In reality, the effect of fishing combined with predators will always drive a fish population to extinction.
- None of the above.

Explanation

For this critical value of a, there is one semi-stable nonzero equilibrium solution. For initial populations above this equilibrium solution, the population will tend to decrease until it comes close to the equilibrium value. In an ideal world these populations will then remain at the semi-stable equilibrium. However, in reality, random fluctuations in population will eventually push the population below the semi-stable equilibrium point, after which they will tend to $P = 0$. Initial populations below the semi-stable equilibrium solution will similarly tend to $P = 0$.

Choice A is not correct. It is possible to fish at a rate slower than reproduction and thus maintain a fish population over time. This is the goal of sustainable fishing. We saw models of this in the first section on bifurcations.

Choice C is not correct. It is possible for fish to survive even with predators and fishing, as we saw in the Lotka-Volterra example.

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Answers are displayed within the problem

Question 12

1/1 point (graded)

This system has two bifurcations, one at the critical value between 'small' and 'medium' a, which we call a_{sm}, and one between 'medium' and 'large' a, which we call a_{ml}. Which best describes the 'major change in the expected behavior' of the population that occurs for each bifurcation?

- [ ] For a_{sm}, the major change is that extinction is not possible for $a < a_{sm}$ and survival is not possible for all values $a > a_{sm}$.
- [x] For a_{sm}, the major change is that extinction is not possible for $a < a_{sm}$ but becomes a possible outcome for values $a > a_{sm}$. ✔
- [ ] For a_{sm}, the major change is that extinction is possible for $a < a_{sm}$ but is no longer a possible outcome once $a > a_{sm}$.
- [ ] For a_{lm}, the major change is that extinction is possible for $a < a_{lm}$ but extinction is no longer possible for $a > a_{lm}$.
- [x] For a_{lm}, the major change is that survival of the population is possible for $a < a_{lm}$ but extinction is the only possibility for $a > a_{lm}$. ✔

✔

Explanation

In the small a case, increasing to a stable equilibrium value is the only possible long-term behavior for the population. When the system transitions from small a to medium a at $a = a_{sm}$ a second non-zero equilibrium value appears, allowing the population now to have two long-term behaviors instead of one. This new equilibrium is smaller than the first and unstable. This means extinction becomes one of two possible outcomes. When will a population go extinct? That depends on the relative value of the initial population and the small unstable equilibrium.

When a reaches the value a_{ml}, the two equilibria merge into a a non-zero semi-stable equilibrium. Above this a value, there are no nonzero equilibrium solutions. and the only long-term behavior is extinction. This is a major change since below the critical a_{ml} value, there are two long-term behaviors, one of which is survival, tending toward a nonzero population value.

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