1. Asymptotic Variance of MLE for Curved Gaussian

(a)

3/3 points (graded)

Let $X_1, \ldots, X_n$ be n i.i.d. random variables with distribution $\mathcal{N}(\theta, \theta)$ for some unknown $\theta > 0$.

In the last homework, you have computed the maximum likelihood estimator $\hat{\theta}$ for θ in terms of the sample averages of the linear and quadratic means, i.e. $\overline{X}_n$ and $\overline{X_n^2}$, and applied the CLT and delta method to find its asymptotic variance.

In this problem, you will compute the asymptotic variance of $\hat{\theta}$ via the Fisher Information.

Denoting the log likelihood for one sample by $\ell(\theta, x)$, compute the second derivative $\frac{d^2}{d\theta^2}\ell(\theta, x)$.

$\frac{d^2}{d\theta^2}\ell\left(\theta,x\right) =$ 1/2/theta^2-x^2/theta^3 ✔

$\frac{1}{2\cdot\theta^2}-\frac{x^2}{\theta^3}$

Then, compute the Fisher information $I\left(\theta\right)$.

as

$$I\left(\theta\right) = -\mathbb{E}\left[\frac{d^2}{d\theta^2}\ell\left(\theta,X\right)\right].$$

$I\left(\theta\right) =$ 1/2/theta^2+1/theta ✔

$\frac{1}{2\cdot\theta^2}+\frac{1}{\theta}$

Finally, what does this tell us about the asymptotic variance of $\hat{\theta}$?

$V\left(\hat{\theta}\right) =$ 2*theta^2/(1+2*theta) ✔

$\frac{2\cdot\theta^2}{1+2\cdot\theta}$

STANDARD NOTATION

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barX_n^2

question posted about 7 hours ago by **nbourbon**

is it expected that I continue to have a bar(X_n)^2 after the second derivative? .. since the grader is not translating that then I believer it's not expecting it... so I'm wondering how can that term go away?. Unless I need to replace that with it's expectation at that point?

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1 response

DriftingWoods
about 4 hours ago

"Denoting the log likelihood for one sample by ..."

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