1.6.2 Quiz: Maximizing Revenue in the Boston Example

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Let's come back to the question of maximizing revenue. Revenue is computed as the price of a good p times demand function for that good, $q(p)$ (the number of units sold).

$$R(p) = p \cdot q(p)$$

In the case of the Boston model, if we use the linear model for demand $q(p) = -29p + 166$, the annual revenue function looks like

$$R(p) = p \cdot q(p) = p(-29p + 166)$$

The units are millions of dollars, because p is in dollars per ride and $q(p)$ is in millions of rides per year. Intuitively, to maximize revenue, you might think just we should raise the price we charge. After all, for each ride taken, we'd be bringing in more money. But as we've seen, increase in price usually causes a decrease in demand, so this may not in fact increase revenue. The question is whether there is some sweet spot where we can balance the benefit of an increase in price with the drawback of a decrease in demand. Since $R(p)$ is a differentiable function, we can use calculus to find this the price to charge in order to maximize revenue. You'll do that now, and think about how it relates to elasticity.

Question 1

1/1 point (graded)

Compute $R'(p)$ and evaluate $R'(0.5)$. Interpret your answer below

- ○ $R'(0.5) < 0$ so we should decrease the price to increase revenue.
- ○ $R'(0.5) < 0$ so we should increase the price to increase revenue.
- ○ $R'(0.5) = 0$ so we should keep the price the same, since we are already at the maximum revenue.
- ○ $R'(0.5) > 0$ so we should decrease the price to increase revenue.
- ● $R'(0.5) > 0$ so we should increase the price to increase revenue. ✔

None of the above.

Explanation

The derivative of $R(p) = -29p^2 + 166p$ is $R'(p) = -58p + 166$.
$R'(0.5) = -58 + 166 = 137$. Since $R'(0.5) > 0$, the function R is increasing at $p = 0.5$. Thus increasing price at that instant will increase revenue.

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Question 2

1/1 point (graded)

Use techniques of calculus to find the value p at which the function $R(p)$ has a maximum. Round your answer to the nearest hundredth (cent).

2.86 **Answer:** 2.86

2.86

Explanation

The derivative of $R(p) = -29p^2 + 166p$ is $R'(p) = -58p + 166$.
We solve $R'(p) = 0$ to find critical points. There is only one critical point $p = 2.86$, rounded to the nearest cent. Since the second derivative $R''(p) = -58$ is negative, the graph of R is concave down at$p = 2.86$ and thus this is indeed a maximum.
Note: in this example, since $R(p)$ is a quadratic, we could also use facts about quadratics to find the location of the maximum value of R.
$p = 2.86$, rounded to the nearest cent.

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Question 3

1/1 point (graded)

Previously, you computed a formula for the elasticity at any price point:

$$E(p) = \frac{-29 \cdot p}{-29p + 166}$$

Compute the elasticity at the value p at which the function $R(p)$ has a maximum, and round to the nearest tenth.

Using this value, choose the best answer.

- [ ] At this price point, an increase of 1% in price will cause a decrease in demand greater than 1%.
- [x] At this price point, an increase of 1% in price will cause a decrease in demand of 1%. ✔
- [ ] At this price point, an increase of 1% in price will cause a decrease in demand less than 1%.
- [ ] There is not enough information to determine the effect on demand of a change of 1% in price.

Explanation

$E(p) = \frac{-82.94}{83.06} = -1$, rounded to the nearest tenth.This means an increase of 1% in price will cause a decrease in demand of 1%, since their ratio must be −1.
In the next section, we'll learn why the maximum revenue occurs when the elasticity is exactly −1. Why might this make sense, given what elasticity tells us about how a percent change in price affects a change in percent demand?

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