4. Linearization

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Problem Set A due Sep 15, 2021 20:30 IST

Practice

Linearize

1/1 point (graded)

Suppose A and B are obtained from x, y by

$$A = xy^2 + x^2 - 6xy - y^2 + 7x + 7y - 11 \tag{5.191}$$

$$B = xy - 4x - y + 4 \tag{5.192}$$

Compute the linearization of the relationship $x, y \implies A, B$ at the point $(1, 3)$.

(Enter a matrix using notation such as `[[a,b],[c,d]]`.)

$M =$ [[0,1],[-1,0]] ✔ **Answer:** [[0, 1],[-1,0]]

Solution:

We compute the partial derivatives:

$$\frac{\partial A}{\partial x} = y^2 + 2x - 6y + 7 \qquad \frac{\partial A}{\partial y} = 2xy - 7x - 2y + 7$$

$$\frac{\partial B}{\partial x} = y - 4 \qquad \frac{\partial B}{\partial y} = x - 1$$

Evaluating each at the point $(x, y) = (1, 3)$ we obtain

$$\frac{\partial A}{\partial x} = 0 \qquad \frac{\partial A}{\partial y} = 1$$

$$\frac{\partial B}{\partial x} = -1 \qquad \frac{\partial B}{\partial y} = 0$$

Therefore, the linearization is given by the matrix of partial derivatives:

$$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \tag{5.193}$$

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Answers are displayed within the problem

Recognize

1/1 point (graded)

Which of the following describes the matrix you found in the previous problem?

- The identity matrix
- Rotation by $\pi/2$ counter-clockwise

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- (●) Rotation by $\pi/2$ clockwise
- ( ) Reflection across the line $y = x$
- ( ) Reflection across the line $y = -x$

✔

Solution:

The matrix $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ represents rotation by $\pi/2$ clockwise, that is, $R_{-\pi/2}$. One can quickly verify this by checking that the vector $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ is sent to $\begin{pmatrix} 0 \\ -1 \end{pmatrix}$, and the vector $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ is sent to $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$.

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Answers are displayed within the problem

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