Exercise: Iterated expectations

(7/8 points)

In this exercise, do not attempt formal mathematical derivations, which would actually involve some subtle issues when we go beyond discrete random variables. Rather, use your understanding of the concepts involved. For each one of the statements below, indicate whether it is true or false.

(a) The law of iterated expectations tells us that $\mathbf{E}\big[\mathbf{E}[X \mid Y]\big] = \mathbf{E}[X]$. Suppose that we want apply this law in a conditional universe, given another random variable Z, in order to evaluate $\mathbf{E}[X \mid Z]$. Then:

$$\mathbf{E}\big[\mathbf{E}[X \mid Y, Z] \mid Z\big] = \mathbf{E}[X \mid Z]$$

True ✔ **Answer:** True

$$\mathbf{E}\big[\mathbf{E}[X \mid Y] \mid Z\big] = \mathbf{E}[X \mid Z]$$

True ✖ **Answer:** False

$$\mathbf{E}\big[\mathbf{E}[X \mid Y, Z]\big] = \mathbf{E}[X \mid Z]$$

False ✔ **Answer:** False

(b) Determine whether each of the following statements about the quantity $\mathbf{E}\big[g(X,Y) \mid Y, Z\big]$ is true or false.

The quantity $\mathbf{E}\big[g(X,Y) \mid Y, Z\big]$ is:

- a random variable

  True ✔ **Answer:** True

- a number

  False ✔ **Answer:** False

- a function of (X, Y)

  False ✔ **Answer:** False

- a function of (Y, Z)

  True ✔ **Answer:** True

- a function of Z only

  False ✔ **Answer:** False

Answer:
(a) The first statement is correct: it is just the law of iterated expectations where all the expectations now involve the additional conditioning on Z.

The second statement is incorrect because the inner conditional expectation should be evaluated in a conditional universe where Z is given. For a concrete counterexample, suppose that X and Y are independent and zero mean, and that $X = Z$. Because of independence, $\mathbf{E}[X \mid Y] = \mathbf{E}[X] = 0$, and the left-jand side evaluates to zero. On the other hand, the right-hand side is equal to Z .

For the third statement, note that the left-hand side is a number (the ordinary expectation of the random variable $\mathbf{E}[X \mid Y, Z]$), whereas the right-hand side is a random variable (a function of Z). Hence the statement is incorrect.

(b) A conditional expectation is generally a random variable, a function of the random variables on which we are conditioning, and so a function of (Y, Z) in this case.

You have used 1 of 1 submissions