14. Review Exercises

Transformations of Random Variables

2/2 points (graded)

Consider a random variable Y with distribution $p_\theta(y)$ for some θ, coming from a canonical exponential family.

Let $Z = Y + a$, where a is a constant. Denote by $q_\theta(z)$ the density of Z, which is parametrized by θ.

Is q_θ also a member of some canonical exponential family?

- (selected) Yes
- No

✔

Now instead suppose $Z = \lambda Y$, where $\lambda \neq 0$ is constant. This again determines some density $\tilde{q}_\theta(z)$ of Z.

Is $\tilde{q}_\theta$ also a member of some canonical exponential family?

- (selected) Yes

No

Solution:

For the first part: we have $q_\theta(z) = p_\theta(z-a)$. In particular,

$$q_\theta(z) = \exp\left(\frac{(z-a)\theta - b(\theta)}{\phi} + c(z-a,\phi)\right) = \exp\left(\frac{z\theta - (b(\theta) + a\theta)}{\phi} + c(z-a,\phi)\right)$$

Let $\tilde{b}(\theta) = b(\theta) + a\theta$ and $\tilde{c}(z,\phi) = c(z-a,\phi)$ which demonstrates that this is indeed contained in a canonical exponential family.

A similar argument (exercise) shows the same answer for the second part, where we instead use $\tilde{q}_\theta(z) = p_\theta(z/\lambda)$.

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Answers are displayed within the problem

(Ungraded) Re-parametrization

0 points possible (ungraded)

Ungrading note: The third part of this problem is unclear and need to be reworked. For now, we have ungraded this problem.

Let $\mathbf{x} = (X_1, X_2)$ where X_1, X_2 are positive random variables, and suppose $\mu(x_1, x_2) = \mathbb{E}[Y|X = (x_1, x_2)]$ is given by

$$\mathbb{E}[Y|X = (x_1, x_2)] = 1000\exp\left(x_1^2 - x_2^2\right).$$

Answer the following questions.

- True or False: $\ln \mu(\mathbf{x})$ is linear in $\mathbf{x}$.

True

False

- True or False: There is an invertible reparametrization $\widetilde{\mathbf{x}}$ of $\mathbf{x}$ for which $Y|\widetilde{\mathbf{x}}$ is a generalized linear model.

True

False

- If there *were* a reparametrization $\widetilde{\mathbf{x}}$, would Jeffreys prior change? That is, would Jeffreys prior be computed using a different formula?

Yes

No

Solution:

- No. Note that $\ln \mu(\mathbf{x}) = \ln \delta + \alpha x_1^2 - \beta x_2^2$. In particular, it is quadratic in $\mathbf{x}$.
- Yes. Since x_1, x_2 are positive, so we can equivalently use a reparametrization, $\widetilde{\mathbf{x}} = (x_1^2, x_2^2)$. From here, $\ln \mu(\mathbf{x})$ is linear.
- No. This is a consequence of the fact that Jeffreys prior is parametrization-invariant.

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