1.3.3 The Meaning of Critical Values and Bifurcations

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The following questions are to help you make sense of the graphs Wes presented corresponding to the "small", "critical" and "large" values of alpha.

Recall that:

$$\frac{dP}{dt} = \frac{1}{10} P \left(1 - \frac{P}{40000}\right) - \alpha$$

As α increases, the graph of $\frac{dP}{dt}$ versus P takes on each of the three forms shown below. (You should review your earlier analysis of these graphs. Did you correctly indicate whether the population was increasing or decreasing by drawing arrows on the P axis? If not, do so now.)

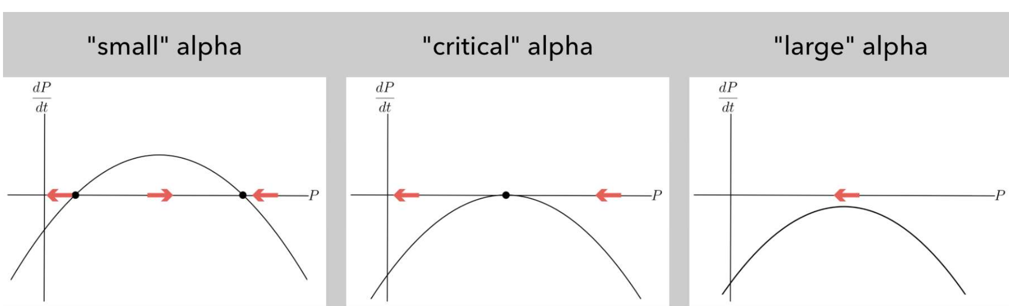

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Image Description

What is the significance of the 'critical' value of α? The 'critical value' is the one for which the graph of $\frac{dP}{dt}$ versus P intersects the horizontal axis exactly once. We can show that this value is $\alpha = 1000$, as you'll do in the summary quiz.

> A **critical value** of a parameter is a value where a bifurcation happens; in other words, there is a sudden qualitative change in the behavior of solutions. We can see this from the graphs, since the graph of $\frac{dP}{dt}$ goes from having two horizontal intercepts when $\alpha < 1000$ to zero horizontal intercepts when $\alpha > 1000$.

In this next quiz, we'll determine what this means about the long-term behavior of solutions to the system.

> **Note:** If you'd like another perspective on the graph of $\frac{dP}{dt}$ versus P, check out the optional section that follows. There we will show how to use this graph to do a qualitative analysis of solutions, like those in the section on Population Dynamics.