9. Quadratic Risk and Variance

Quadratic Risk

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The Quadratic Risk

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We would like to analyze the *typical error in* $\hat{\boldsymbol{\beta}}$ *compared to the true parameter,* $\boldsymbol{\beta}$, which we may define as $\mathbb{E}\left[\|\hat{\boldsymbol{\beta}}-\boldsymbol{\beta}\|_2^2\right]$. On the other hand, we might also consider the *typical error between the predictions* $\mathbb{X}\hat{\boldsymbol{\beta}}$ *and the observations* $\mathbf{Y}$, which we may define as $\mathbb{E}\left[\|\mathbf{Y}-\mathbb{X}\hat{\boldsymbol{\beta}}\|_2^2\right]$.

These are respectively called the **quadratic risk of** $\hat{\boldsymbol{\beta}}$ and the **prediction error** . (The prediction error will be discussed in the next video.)

What happens to these errors as σ^2 increases?

- The error in $\hat{\boldsymbol{\beta}}$ increases, but the prediction error decreases.
- The error in $\hat{\boldsymbol{\beta}}$ decreases, but the prediction error increases.
- Both errors decrease.
- Both errors increase.

Solution:

σ^2 is the variance of each coordinate of ϵ. As the σ^2 increases, the data becomes more noisy. In particular, the task of estimating $\hat{\boldsymbol{\beta}}$ ought to become harder, and it is intuitive that $\mathbf{Y}$ becomes further from the prediction $\mathbb{X}\hat{\boldsymbol{\beta}}$. To make this concrete, recall the following formulas, which hold in the homoscedastic Gaussian case:

$$\mathbb{E}[\|\hat{\boldsymbol{\beta}}-\boldsymbol{\beta}\|_2^2 = \sigma^2 \mathbf{tr}\left((\mathbb{X}^T\mathbb{X})^{-1}\right)$$

$$\mathbb{E}\left[\|\mathbf{Y}-\mathbb{X}\hat{\boldsymbol{\beta}}\|_2^2\right] = \sigma^2\,(n-p)$$

(In our scenario, $n = 1000$ and $p = 2$.)

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? Prediction error
Does it increase with more data? And decreases if there are more parameters to estimate, given the same data?
2