6. Maximum Likelihood Estimation for a Multivariate Standard Normal

Let $\mathbf{X}_1, \ldots, \mathbf{X}_n \overset{i.i.d.}{\sim} \mathcal{N}(\mu, \mathbf{1})$, where $\mu \in \mathbb{R}^d$ and $\mathbf{1}$ is the $d \times d$ identity matrix. (The $\mathbf{X}_i$ are random vectors.)

Recall the pdf defining the distribution $\mathcal{N}(\mu, \mathbf{1})$ is

$$f(\mathbf{x}) = \frac{1}{(2\pi)^{d/2}} \exp\left(-\frac{1}{2}(\mathbf{x}-\mu)^T \mathbf{1}(\mathbf{x}-\mu)\right)$$

(a)

1/1 point (graded)

What is the likelihood function $L(\mathbf{X}_1, \ldots, \mathbf{X}_n, \mu)$ for μ?

(Enter **(Sigma_i(norm(x_i-mu)^2))** for $\sum_{i=1}^{n} \|\mathbf{x}_i - \mu\|^2$.)

$L(\mathbf{X}_1, \ldots, \mathbf{X}_n, \mu) =$ exp(-(Sigma_i(norm(x_i-mu)^2))/2)/(2*pi)^((n*d)/2)

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Correct (1/1 point)

(b)

1/1 point (graded)
Compute the maximum likelihood estimator $\hat{\mu}_{MLE}$ for μ.

(Enter **barX_n** for the sample average.)

$\hat{\mu}_{MLE} =$ barX_n

Prove to yourself that the result you obtained above indeed maximizes the likelihood function. Is this step necessary?

STANDARD NOTATION

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Correct (1/1 point)

(c)

1/1 point (graded)
What is the distribution of $\hat{\mu}_{MLE}$?

- (selected) $\hat{\mu}_{MLE} \sim \mathcal{N}\left(\mu, \frac{1}{n}\mathbf{1}\right)$
- $\hat{\mu}_{MLE} \sim \mathcal{N}\left(\mu, \mathbf{1}\right)$
- $\hat{\mu}_{MLE} \sim \mathcal{N}\left(0, \frac{1}{n}\mathbf{1}\right)$
- $\hat{\mu}_{MLE} \sim \mathcal{N}\left(\mu, \frac{1}{\sqrt{n}}\mathbf{1}\right)$

✔

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✔ Correct (1/1 point)

(d)

1/1 point (graded)

What is the asymptotic variance of $\mathbf{A}\hat{\mu}_{MLE}$? (here, A is a fixed $m \times d$ matrix)

(If applicable, enter **trans(A)** for the transpose of a matrix A.)

A*trans(A) ✔

STANDARD NOTATION

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✔ Correct (1/1 point)

(e)

1/1 point (graded)

What is the asymptotic variance of $\|\hat{\mu}_{MLE}\|^2$?

(If applicable, enter **norm(v)** for the norm $\|\mathbf{v}\|$ of a vector $\mathbf{v}$, and **trans(v)** for the transpose $\mathbf{v}^T$ of a vector $\mathbf{v}$.)

4*norm(mu)^2 ✔

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✔ Correct (1/1 point)

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A friendly tip for E

discussion posted 2 days ago by LuisMasuelli

While there is a method which involves using the `mu -> norm(mu)` mapping as an estimator that will do the trick...

...the answer can be explained in TWO apparently different ways, which may confuse but their relationship is stated in the homework 0. Just don't forget to review the linear algebra part of homework 0 for this entire exercise (6).

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