MITx: 6.041x Introduction to Probability - The Science of Uncertainty

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Problem 1: Fred promotes dog food

(5/6 points)

Fred is giving away samples of dog food. He makes visits door to door, but he gives a sample away (one can of dog food) only on those visits for which the door is answered *and* a dog is in residence. On any visit, the probability of the door being answered is $3/4$, and the probability that any given household has a dog is $2/3$. Assume that the events “Door answered" and “A dog lives here" are independent and also that events related to different households are independent.

1. What is the probability that Fred gives away his first sample on his third visit?

   1/8 ✔ **Answer:** 0.125

2. Given that he has given away exactly four samples on his first eight visits, what is the conditional probability that Fred will give away his fifth sample on his eleventh visit?

   1/8 ✔ **Answer:** 0.125

3. What is the probability that he gives away his second sample on his fifth visit?

1/8 ✔ **Answer:** 0.125

4. Given that he did not give away his second sample on his second visit, what is the conditional probability that Fred will give away his second sample on his fifth visit?

1/6 ✔ **Answer:** 0.16667

5. We will say that Fred “needs a new supply" immediately *after* the visit on which he gives away his last sample. If he starts out with two samples, what is the probability that he completes at least five visits before he needs a new supply?

5/16 ✔ **Answer:** 0.3125

6. If he starts out with exactly 10 samples, what is the expected value of the number of homes with dogs where Fred visits but leaves no samples (because the door was not answered) before he needs a new supply?

5 ✖ **Answer:** 3.33333

Answer:
A successful (i.e., the door is answered and a dog is in residence) visit (“trial") occurs with probability $p = \frac{3}{4} \cdot \frac{2}{3} = \frac{1}{2}$.

1.

Solved problems

Additional theoretical material

Problem Set 9
Problem Set 9 due May 11, 2016 at 23:59 UTC

Unit summary

- Unit 10: Markov chains
- Exit Survey

Fred will give away his first sample on the third visit if and only if the first two visits are failures and the third is a success. Since the trials are independent, the probability of this sequence of events is simply

$$(1-p)(1-p)p = \frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2} = \frac{1}{8}.$$

2. Given the conditioning event, the event of interest occurs if and only if there are failures on the ninth and tenth trials and a success on the eleventh trial. For a Bernoulli process, the outcomes of all trials are independent, and again our answer is

$$(1-p)(1-p)p = \frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2} = \frac{1}{8}.$$

3. We desire the probability that Y_2, the time to the second arrival, is equal to five. We know that $p_{Y_2}(y)$ is a Pascal PMF of order 2, and we have

$$p_{Y_2}(5) = \binom{5-1}{2-1}p^2(1-p)^{5-2} = 4\cdot\left(\frac{1}{2}\right)^5 = \frac{1}{8}.$$

4. Here we require the conditional probability that Y_2 is equal to 5, given that it is greater than 2.

$$\mathbf{P}(Y_2 = 5 | Y_2 > 2) \;=\; \frac{p_{Y_2}(5)}{\mathbf{P}(Y_2 > 2)} = \frac{p_{Y_2}(5)}{1 - p_{Y_2}(2)}$$

$$= \frac{\binom{5-1}{2-1}p^2(1-p)^{5-2}}{1-\binom{2-1}{2-1}p^2(1-p)^0} = \frac{4\cdot\left(\frac{1}{2}\right)^5}{1-\left(\frac{1}{2}\right)^2} = \frac{1}{6}.$$

5. The probability that Fred will complete at least five visits before he needs a new supply is equal to the probability that Y_2 is greater than or equal to 5.

$$\begin{aligned}\mathbf{P}(Y_2 \geq 5) &= 1-\mathbf{P}(Y_2 \leq 4) = 1-\sum_{\ell=2}^{4}\binom{\ell-1}{2-1}p^2(1-p)^{\ell-2} \\ &= 1-\left(\frac{1}{2}\right)^2 - \binom{2}{1}\left(\frac{1}{2}\right)^3 - \binom{3}{1}\left(\frac{1}{2}\right)^4 = \frac{5}{16}.\end{aligned}$$

6. For this part of the problem, we are only concerned with visits to houses with dogs. At each such visit, either (i) the door is answered (“success"), which happens with probability 3/4, and a sample is given, or (ii) the door is not answered (“failure"), which happens with probability 1/4. We wish to determine the expected number of failures until the 10th success. The expected number of trials (visits to houses with dogs) up to and including the 10th success is $10/(3/4) = 40/3$. The number of failures is the number of such visits minus the number of successes. Therefore, the expected number of failures is $(40/3) - 10 = 10/3$.

You have used 2 of 2 submissions

Printable problem set available here .

DISCUSSION

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