Exercises: Uniqueness of solutions

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Underdetermined matrices

1/1 point (ungraded)

Let the system of equations by given by $y = A_i x$, with $i = 1, 2, 3, 4$. Which of the following A_i results in a underdetermined system?

- [ ] $A_1 = \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}$
- [x] $A_2 = \begin{bmatrix} 3 & 6 \\ 1 & 2 \\ 1 & 2 \end{bmatrix}$
- [x] $A_3 = \begin{bmatrix} 3 & 1 & 2 \\ 1 & 0 & \frac{2}{3} \\ 6 & 1 & 4 \end{bmatrix}$

- 4. Best Linear Unbiased Estimation (BLUE)
- Pre-knowledge Mathematics
- MATLAB Learning Content

$$A_4 = \begin{bmatrix} 3 & 1 & 2 \\ 1 & 0 & 2 \\ 6 & 1 & 4 \end{bmatrix}$$

✔

Explanation
The rank of A_2 is 1, whereas $n = 2$; the second column is simply 2 times the first column.
The rank of A_3 is 2, whereas $n = 3$; the third column is 2/3 times the first column.
In these cases, the columns are not independent.
You may also compute the rank of a matrix with Matlab, using e.g. rank(A2) with A2 the matrix of interest.

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✔ Correct (1/1 point)

Is there a solution?

5/5 points (ungraded)
Assume we have a system of equations $y = Ax$.

For each A-matrix, select one of the options:

$$A = \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}$$

- (•) system will have one solution ✔
- ( ) system may have a solution
- ( ) system does not have a solution

Answer
Correct: $\mathbf{rank}(A) = 2 = m = n$

$$A = \begin{bmatrix} 3 & 6 \\ 1 & 2 \\ 1 & 2 \end{bmatrix}$$

- ( ) system will have one solution
- (•) system may have a solution ✔
- ( ) system does not have a solution

Answer
Correct: $\mathbf{rank}(A) = 1$, and $m = 3$. The rank is smaller than m so there might be a solution.

$$A = \begin{bmatrix} 1 & 6 \\ 1 & 2 \\ 1 & 3 \end{bmatrix}$$

- system will have one solution
- system may have a solution ✔
- system does not have a solution

Answer
Correct:
$\mathbf{rank}(A) = 2$, and $m = 3$. The rank is smaller than m so there might be a solution. If there is a solution, it will be unique since $n = 2 = \mathbf{rank}(A)$.

$$A = \begin{bmatrix} 3 & 1 & 2 \\ 1 & 0 & \frac{2}{3} \\ 6 & 1 & 4 \end{bmatrix}$$

- system will have one solution
- system may have a solution ✔
- system does not have a solution

Answer
Correct:
$\mathbf{rank}(A) = 2$, and $m = 3$, so it may have a solution, which will not be unique, since $n = 3 > \mathbf{rank}(A)$.

$$A = \begin{bmatrix} 3 & 1 & 2 \\ 1 & 0 & 2 \\ 6 & 1 & 4 \end{bmatrix}$$

- (•) system will have one solution ✔
- ( ) system may have a solution
- ( ) system does not have a solution

Answer
Correct: $\mathbf{rank}(A) = 3 = m = n$

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✔ Correct (5/5 points)

True or False

2/2 points (ungraded)

Underdetermined systems do not have a solution.

false ▼ ✔ **Answer:** false

Explanation

If an underdetermined system is consistent, it will have an infinite number of solutions.

Underdetermined systems are always inconsistent.

false ▼ ✔ **Answer:** false

Explanation

A system is inconsistent if it does not have a solution.

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✔ Correct (2/2 points)