

## probability density of the maximum of samples from a uniform distribution

Asked 6 years, 6 months ago Active 2 months ago Viewed 31k times

Suppose

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 $X_1, X_2, \dots, X_n \sim Unif(0, \theta), iid$ 

and suppose

\* 8

 $\hat{ heta} = \max\{X_1, X_2, \dots, X_n\}$ 

How would I find the probability density of  $\hat{\theta}$ ?

Thank you!

probability probability-theory

edited Feb 24 '13 at 23:58

asked Feb 24 '13 at 22:05



nzo 17 1 5

are the Xs Independent? – Inquest Feb 24 '13 at 22:12 🖍

Here is a question for you: did you ask all your questions on MSE with no indication whatsoever on what you understood of the problem or what you tried before asking?
 Did Feb 24 '13 at 22:19

@Inquest, yes they are! Sorry that I forgot to point that out. - Enzo Feb 24 '13 at 23:06

1 @Did, no seriously I don't know what you are talking about. What do you mean by all the MSE questions? This is my first question that's related to MSE and I just told in in the comment what I did, so I am not sure what you are accusing me of. — Enzo Feb 25 '13 at 14:07

1 See. I thought you were talking about mean squared error. Well, I apologize if you didn't see what I have tried before asking, but I did indicate them somewhere in the questions, either in comments, or as a feedback of my own work. For example, on one of the proofs involving differential forms, I provided a copy of my own solution in the end. So I don't think you are in a right position to accuse me, and I don't feel comfortable about that. Thank you for helping me before though. — Enzo Feb 25 '13 at 16:35

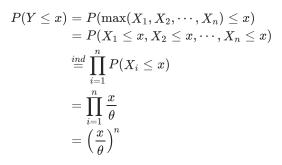
3 Answers



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edited Oct 29 '17 at 17:39

sligocki

answered Feb 24 '13 at 22:20

Inques

**5,130** 2 24

1 \_\_ In line 2, are the ns supposed to be xs or is this correct as is? – kram1032 Jul 26 '14 at 20:50



Let random variable W denote the maximum of the  $X_i$ . We will assume that the  $X_i$  are independent, else we can say very little about the distribution of W.



Note that the maximum of the  $X_i$  is  $\leq w$  if and only if **all** the  $X_i$  are  $\leq w$ . For w in the interval  $[0, \theta]$ , the probability that  $X_i \leq w$  is  $\frac{w}{\theta}$ . It follows by independence that the probability that  $W \leq w$  is  $\left(\frac{w}{\theta}\right)^n$ .

Thus, in our interval, the cumulative distribution function  $F_W(w)$  of W is given by

$$F_W(w) = \left(rac{w}{ heta}
ight)^n.$$

Differentiate to get the density function of W.

edited Jul 26 '14 at 21:02

answered Feb 24 '13 at 22:18



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k 37 445 8

There appears to be a TeX error. I can't edit it because it's just a single backslash that needs removal but math.SE asks me to change the post by at least 6 symbols. – kram1032 Jul 26 '14 at 20:51

↑ Thank you for telling me, yes I had written \w for w. − André Nicolas Jul 26 '14 at 21:02

https://math.stackexchange.com/questions/313390/probability-density-of-the-maximum-of-samples-from-a-uniform-distribution



The general formula for the probability density of the maximum of **any** iid sample set of the random variable  $x, M = max\{x_1, x_2, \dots, x_n\}$  is:



$$f_M(M=x) = n \ast F_x(x)^{n-1} \ast f_x(x)$$

where  $f_x(x)$  is the probability density of x, and  $F_x(x)$  is the cumulative distribution function of same.

In this case we have:  $f_x(x)=rac{1}{ heta}$  ,  $F_x(x)=rac{x}{ heta}$  , so we get:

$$f_M(M=x) = n*(rac{x}{ heta})^{n-1}*rac{1}{ heta} \ = rac{n*x^{n-1}}{ heta^n}$$

answered Jul 9 at 15:41

