2. Review: Fundamental matrices

Fundamental matrix review 1

1/1 point (graded)

Assume that in each matrix, the columns are solutions to one system of ODEs.

Which of the following matrices could be fundamental matrices?

- [x] $\begin{pmatrix} \cos(t) & \sin(t) \\ -\sin(t) & \cos(t) \end{pmatrix}$ ✔
- [ ] $\begin{pmatrix} e^t & e^{2t} \\ -2e^t & -2e^{2t} \end{pmatrix}$
- [x] $\begin{pmatrix} e^{-5t} & e^t & e^t \\ 0 & e^t & 0 \\ 0 & 0 & e^t \end{pmatrix}$ ✔
- [x] $\begin{pmatrix} e^{it} & e^{-it} \\ -ie^{it} & ie^{-it} \end{pmatrix}$ ✔

✔

Solution:

We only need to check that $\mathbf{X}(0)$ is invertible! Let us first proceed in order of the choices:

- $\left.\begin{pmatrix} \cos(t) & \sin(t) \\ -\sin(t) & \cos(t) \end{pmatrix}\right|_{t=0} = I$ is invertible
- $\left.\begin{pmatrix} e^t & e^{2t} \\ -2e^t & -2e^{2t} \end{pmatrix}\right|_{t=0} = \begin{pmatrix} 1 & 1 \\ -2 & -2 \end{pmatrix}$ is singular.
- $\left.\begin{pmatrix} e^{-5t} & e^t & e^t \\ 0 & e^t & 0 \\ 0 & 0 & e^t \end{pmatrix}\right|_{t=0} = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ is invertible.
- $\left.\begin{pmatrix} e^{it} & e^{-it} \\ -ie^{it} & ie^{-it} \end{pmatrix}\right|_{t=0} = \begin{pmatrix} 1 & 1 \\ -i & i \end{pmatrix}$ is invertible.

Any of the invertible matrices above could be a fundamental matrix given an appropriate systems of ODEs.

Submit You have used 3 of 3 attempts

Answers are displayed within the problem

Fundamental matrix review 2

1/1 point (graded)

We also use the terms **fundamental matrix of a single higher order ODE** to mean a fundamental matrix of its companion system.

Determine which of the following second order ODEs or system of ODEs has the following fundamental matrix:

$$\begin{pmatrix} \cos(t) & \sin(t) \\ -\sin(t) & \cos(t) \end{pmatrix}$$

- [x] $\ddot{x} + x = 0$ ✔

- [ ] $\ddot{x} + \dot{x} + x = 0$
- [x] $\dot{\mathbf{x}} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \mathbf{x}$ ✔
- [ ] $\dot{\mathbf{x}} = \begin{pmatrix} 0 & 1 \\ -1 & 1 \end{pmatrix} \mathbf{x}$

✔

Solution:

Each system is the companion matrix of one of the second order ODEs, so we only need to check the systems. Inspection shows that

$$\frac{d}{dt}\begin{pmatrix} \cos(t) & \sin(t) \\ -\sin(t) & \cos(t) \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} \cos(t) & \sin(t) \\ -\sin(t) & \cos(t) \end{pmatrix}$$

Thus both $\ddot{x} + x = 0$ and its companion matrix $\dot{\mathbf{x}} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \mathbf{x}$ have a fundamental matrix $\begin{pmatrix} \cos(t) & \sin(t) \\ -\sin(t) & \cos(t) \end{pmatrix}$.

Submit You have used 1 of 3 attempts

Answers are displayed within the problem

2. Review: Fundamental matrices

Hide Discussion

Topic: Unit 3: Solving systems of first order ODEs using matrix methods / 2. Review: Fundamental matrices

Add a Post

< All Posts

If det(X(0)) != 0 then it will be != 0 for all t.

discussion posted 8 days ago by **PavelKrupets**

This is from part 7 Fundamental matrix. For 3rd matrix. Det[X(0)] != 0 but Det[X(1)] == 0. Unless I missed something (2*(t-1)*e^(-t)).

This post is visible to everyone.

Add a Response

6 responses

BlueFlow
8 days ago

you are correct pavel. i moved on by deciding that this question was about noticing the value of the determinant at t=0, which works for the grader.

the problem here is if you have a smart student they will get this one wrong by trying to construct the A for each given fundamental matrix.

- in this course the solutions are restricted to constant A, not the more typical A(t).
- but, it is easy to show by working backward that there is no A(t) leading to the third fundamental matrix.
- and pavel found a rather elegant way to show there is no such A(t). (the rightmost two colums are identical at t=1).

@BlueFlow:

Why particular $t = 0$? My approach was to check det. of a whole matrix as function of t.

posted 5 days ago by **sharov_am**

well, i am lazy and i just wanted a time where i could do the math by simply looking at the matrix and seeing the result.

the general reason you should be able to choose any time to evaluate the determinant is the theorem about the determinant remaining nonzero for all time. but that depends on the matrix being fundamental, which the third matrix initially was not.

posted 5 days ago by **BlueFlow**

Add a comment

JohnFMayne
6 days ago

Is the third matrix problematic? I have a red x on this one with one attempt left and I still can't figure out why my first attempt was wrong.
Visually, it is not linear, so how can it be linearly independent? Also, as noted, there is a t where the determinant = 0.

Add a comment

jfrench (Staff)
6 days ago

Sorry, we found this bug in beta testing, and I evidently failed to fix it! I'll fix and regrade in a way that doesn't hurt those who have gotten full points already.

Good luck... :)

posted 6 days ago by **yves-M** (Community TA)

? So some people have the green check while some other don't even though they submit the same answer?

posted 3 days ago by **subsole**

Add a comment

subsole
3 days ago

Is the grader okay now? I can't tell...

Yes it is.

posted about 23 hours ago by **yves-M** (Community TA)

then bad news.

posted about 20 hours ago by **subsole**

Add a comment

subsole
3 days ago

"the matrix $X(0)$ is nonsingular, namely $det(X(0)) \neq 0$". What about complex numbers?

@subsole: I don't understand your question.

posted about 23 hours ago by **yves-M** (Community TA)

Well... I mean... Is 0 + 3i $\neq 0$...? ...

posted about 20 hours ago by **subsole**

Yes, $3i \neq 0$; if $det(\mathbf{A}) = 3i$ then $\mathbf{A}$ is invertible. But I still don't see anything wrong with the grader.

posted about 19 hours ago by **yves-M** (Community TA)

Add a comment

subsole

about 20 hours ago

I still don't see why my answer is wrong. I want the matrices that have determinant not equal to zero. Why wrong?

You probably have done a mistake in your calculations. The solution is fine.

posted about 19 hours ago by **yves-M** (Community TA)

... I agree with the grader now. If I remember correctly it didn't perform this way in my first attempt.

posted about 16 hours ago by **subsole**

Add a comment

Showing all responses

Add a response:

Preview

Submit

Learn About Verified Certificates