Problem 2: Estimating the parameter of a geometric r.v.

(3/3 points)

We have k coins. The probability of Heads is the same for each coin and is the realized value q of a random variable Q that is uniformly distributed on $[0, 1]$. We assume that conditioned on $Q = q$, all coin tosses are independent. Let T_i be the number of tosses of the i^{th} coin until that coin results in Heads for the first time, for $i = 1, 2, \ldots, k$. (T_i includes the toss that results in the first Heads.)

You may find the following integral useful: For any non-negative integers k and m,

$$\int_0^1 q^k (1-q)^m dq = \frac{k!m!}{(k+m+1)!}.$$

1. Find the PMF of T_1. (Express your answer in terms of t using standard notation .)

   For $t = 1, 2, \ldots$, we have $p_{T_1}(t) =$ 1/(t*(t+1))

2. Find the least mean squares (LMS) estimate of Q based on the observed value, t, of T_1. (Express your answer in terms of t using standard notation .)

   $\mathbf{E}[Q \mid T_1 = t] =$ 2/(t+2)

3. We flip each of the k coins until they result in Heads for the first time. Compute the maximum a posteriori (MAP) estimate $\hat{q}$ of Q given the number of tosses needed, $T_1 = t_1, \ldots, T_k = t_k$, for each coin. Choose the correct expression for $\hat{q}$.

- $\hat{q} = \dfrac{k-1}{\sum_{i=1}^{k} t_i}$
- $\hat{q} = \dfrac{k}{\sum_{i=1}^{k} t_i}$ ✔
- $\hat{q} = \dfrac{k+1}{\sum_{i=1}^{k} t_i}$
- none of the above

You have used 1 of 2 submissions

DISCUSSION

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