7. Estimating the Parameter for an Exponential Model

Estimating the Parameter for an Exponential Model

guy?

Well, the variance of T1, which as you can tell from here,

is one over lambda squared.

OK?

And that's two applications of integration by part.

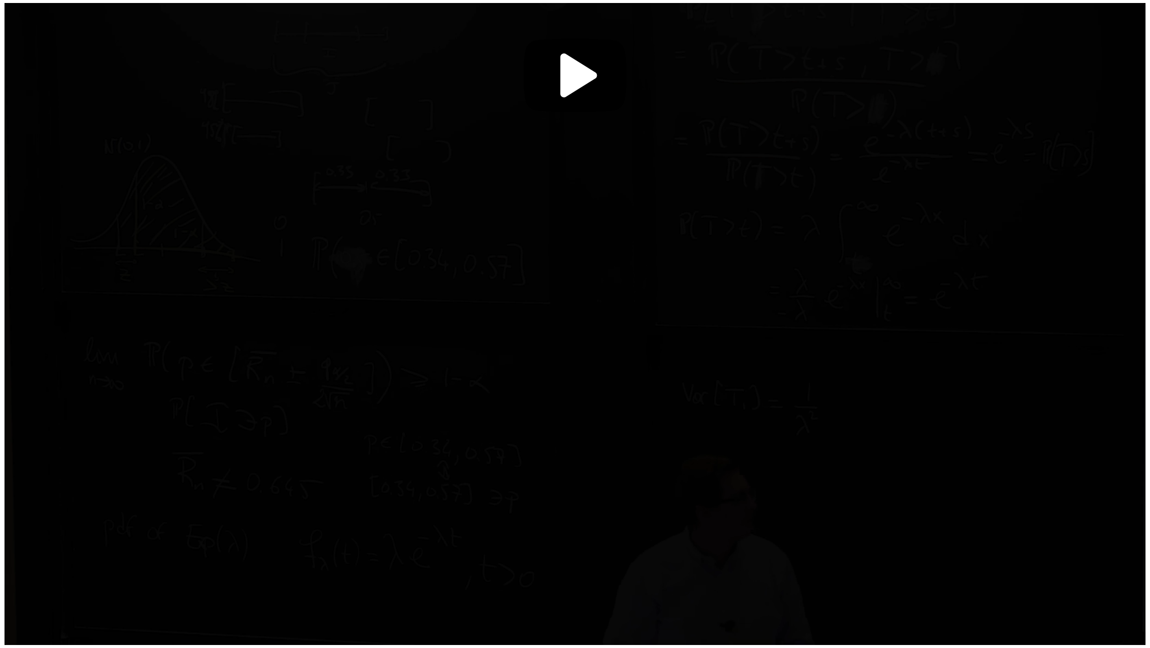
Right?

You have to integrate T squared against e

to the minus lambda T. OK?

And every time you're going to integrate,

you're going to have one lambda that goes to the denominator.



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Consistency and Biasedness

4/4 points (graded)

Let $X_1, X_2, \ldots, X_n \stackrel{iid}{\sim} \exp(\lambda)$. Let $\overline{X}_n := \frac{1}{n}\sum_{i=1}^{n} X_i$ denote the sample mean of the data set.

To which value does $\overline{X}_n$ converge (both a.s. and in probability) as $n \to \infty$?
(Choose all that apply)

- [x] $\mathbb{E}[X_i]$
- [ ] $\frac{1}{\mathbb{E}[X_i]}$
- [ ] $\mathbb{E}\left[\frac{1}{X_i}\right]$
- [ ] λ
- [x] $\frac{1}{\lambda}$

✔

To which value does $\frac{1}{\overline{X}_n}$ converge (both a.s. and in probability) as $n \to \infty$? (Choose all that apply)

- [ ] $\mathbb{E}[X_i]$
- [x] $\frac{1}{\mathbb{E}[X_i]}$
- [ ] $\mathbb{E}\left[\frac{1}{X_i}\right]$
- [x] λ
- [ ] $\frac{1}{\lambda}$

Which of the following is the bias of $\frac{1}{\overline{X}_n}$ as an estimator of λ? (Choose all that apply.)

- [x] $\mathbb{E}\left[\frac{1}{\overline{X}_n}\right] - \lambda$
- [x] $\mathbb{E}\left[\frac{1}{\overline{X}_n}\right] - \frac{1}{\mathbb{E}[X_i]}$
- [x] $\mathbb{E}\left[\frac{1}{\overline{X}_n}\right] - \frac{1}{\mathbb{E}[\overline{X}_n]}$
- [ ] $\frac{1}{\mathbb{E}[X_i]} - \lambda$
- [ ] $\frac{1}{\mathbb{E}[X_i]} - \frac{1}{\mathbb{E}[\overline{X}_n]}$

Which of the following are properties of $\frac{1}{\overline{X}_n}$ as an estimator of λ? (Choose all that apply.)

- [x] consistent
- [ ] unbiased

✔

Solution:

- By the (strong/weak) law of large numbers

$$\overline{X}_n = \frac{\sum_{i=1}^n X_i}{n} \xrightarrow[n\to\infty]{a.s/\mathbf{P}} \mathbb{E}[X_i] = \frac{1}{\lambda}.$$

- On the other hand, by the continuous mapping theorem

$$\frac{1}{\overline{X}_n} \xrightarrow[n\to\infty]{a.s/\mathbf{P}} \frac{1}{\mathbb{E}[X_i]} = \lambda.$$

- Hence, we can answer the last part immediately: $\frac{1}{\overline{X}_n}$ is a consistent estimator of λ.

- However,

$$\mathbb{E}\left[\frac{1}{\overline{X}_n}\right] \neq \frac{1}{\mathbb{E}\left[\overline{X}_n\right]} = \lambda.$$

So the bias of $\frac{1}{\overline{X}_n}$ as an estimator of $\lambda = \frac{1}{\mathbb{E}[X_i]} = \frac{1}{\mathbb{E}\left[\overline{X}_n\right]}$ is

$$\text{Bias} = \mathbb{E}\left[\frac{1}{\overline{X}_n}\right] - \frac{1}{\mathbb{E}\left[\overline{X}_n\right]}.$$

Remark: Since the function $\frac{1}{x}$ is convex (by the shape of its graph or by $\left(\frac{1}{x}\right)'' = \frac{2}{x^3} > 0$), Jensen's inequality gives $\mathbb{E}\left[\frac{1}{\overline{X}_n}\right] > \frac{1}{\mathbb{E}\left[\overline{X}_n\right]}$ and hence the bias is greater than zero.

Submit You have used 1 of 2 attempts

Answers are displayed within the problem

Review: Central Limit Theorem

1/1 point (graded)

The **Central Limit Theorem** states that if $X_1, \ldots, X_n$ are i.i.d. and

$$\mathbb{E}[X_1] = \mu < \infty \quad ; \qquad \mathsf{Var}(X_1) = \sigma^2 < \infty,$$

then

$$\sqrt{n}\left[\left(\frac{1}{n}\sum_{i=1}^{n} X_i\right) - \mu\right] \xrightarrow[n\to\infty]{(d)} Z \qquad \text{where } Z \sim \mathcal{N}(0, ?)\,.$$

What is $\mathsf{Var}(Z)$? (Express your answer in terms of n, μ and σ).

$\mathsf{Var}(Z) =$ sigma^2 **Answer:** sigma^2

σ^2

STANDARD NOTATION

Solution:

For any n,

$$\mathsf{Var}\sqrt{n}\left(\overline{X}_n - \mu\right) = n\mathsf{Var}\left(\overline{X}_n\right) = \mathsf{Var}\left(X_i\right) = \sigma^2.$$

The central limit theorem states as $n \to \infty$, the distribution of $\sqrt{n}\left(\overline{X}_n - \mu\right)$ becomes Gaussian with the variance above (and mean 0); that is,

$$\sqrt{n}\left(\overline{X}_n - \mu\right) \xrightarrow[n\to\infty]{(d)} \mathcal{N}\left(0, \sigma^2\right).$$

Note: The variance of Z is called the **asymptotic variance** of $\overline{X}_n$, even though it equals the variance of $\sqrt{n}\overline{X}_n$.

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Answers are displayed within the problem

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? Review: Central Limit Theorem
Could someone please elaborate on the **note** on **asymptotic variance** in the review problem's solution? I don't understand its significance. 2

Poor Notation
Since Z is almost always used to represent a standard normal variable/distribution, I think another letter should be used here say W. 1

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