Problem 1: Determining the type of a lightbulb

(3/3 points)

The lifetime of a type-A bulb is exponentially distributed with parameter λ. The lifetime of a type-B bulb is exponentially distributed with parameter μ, where $\mu > \lambda > 0$. You have a box full of lightbulbs of the same type, and you would like to know whether they are of type A or B. Assume an *a priori* probability of $1/3$ that the box contains type-B lightbulbs.

1. You observe the value t_1 of the lifetime, T_1, of a lightbulb. A MAP decision rule implies that the lightbulb is of type A if and only if $t_1 \geq \alpha$.

   Assuming that $\mu \geq 2\lambda$, find α. Express your answer in terms of μ and λ. Use 'mu', 'lambda' and 'ln' to denote μ, λ, and the natural logarithm function, respectively. For example, $\ln\frac{2\mu}{\lambda}$ should be entered as 'ln((2*mu)/lambda)'.

   $\alpha =$ ln(mu/(2*lambda))/(mu ✔

2. Assuming that $\mu \geq 2\lambda$, what is the probability of error of the MAP estimator?

   - ● $\frac{1}{3}e^{-\mu\alpha} + \frac{2}{3}(1 - e^{-\lambda\alpha})$ ✔
   - ○ $\frac{2}{3}e^{-\mu\alpha} + \frac{1}{3}(1 - e^{-\lambda\alpha})$
   - ○ $\frac{1}{3}(1 - e^{-\mu\alpha}) + \frac{2}{3}e^{-\lambda\alpha}$
   - ○ $\frac{2}{3}(1 - e^{-\mu\alpha}) + \frac{1}{3}e^{-\lambda\alpha}$

3.

Unit overview

Lec. 14: Introduction to Bayesian inference
Exercises 14 due Apr 06, 2016 at 23:59 UTC

Lec. 15: Linear models with normal noise
Exercises 15 due Apr 06, 2016 at 23:59 UTC

Problem Set 7a
Problem Set 7a due Apr 06, 2016 at 23:59 UTC

Lec. 16: Least mean squares (LMS) estimation
Exercises 16 due Apr 13, 2016 at 23:59 UTC

Lec. 17: Linear least mean squares (LLMS) estimation
Exercises 17 due Apr 13, 2016 at 23:59 UTC

Problem Set 7b
Problem Set 7b due Apr 13, 2016 at 23:59 UTC

Solved problems

Additional theoretical material

Unit summary

- Unit 8: Limit theorems and classical statistics

Assume that $\lambda = 2$ and $\mu = 3$. Find the LMS estimate of T_2, the lifetime of another lightbulb from the same box, based on observing $T_1 = 2$. Assume that conditioned on the bulb type, bulb lifetimes are independent. (For this part, you will need a calculator. Provide an answer with an accuracy of two decimal places.)

LMS estimate of $T_2 =$ 0.48 ✔

You have used 2 of 2 submissions

Printable problem set available here .

DISCUSSION

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