2. Review of Parametric Hypothesis Testing

Review: Goal of Hypothesis Testing

1/1 point (graded)

You have i.i.d. data $X_1, \ldots, X_n$ generated by a distribution $\mathbf{P}_\theta$ for some unknown parameter $\theta \in \mathbb{R}$. You would like to test some null hypothesis H_0 against an alternative hypothesis H_1.

What is the purpose of hypothesis testing?

- ○ To solve exactly for the true parameter θ.
- ○ To provide a consistent estimator for the true parameter θ.
- ○ To develop an estimator that is close to the true parameter θ.
- ● To decide with a quantified probability of error whether or not θ lies in a certain region of the parameter set.

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Solution:

We provide the correct response and then discuss the incorrect ones.

The goal of hypothesis testing is "To decide with a quantified probability of error whether or not θ lies in a certain region of the parameter set." The null and the alternative hypotheses describe complementary subsets of the parameter set. A statistical test is a data dependent rule that decides whether or not to reject the statement (hypothesis) that the unknown true parameter θ belongs to the subset described by H_0 or fail to reject it. In designing a statistical test, we must quantify how likely it is that the observed sample is generated by a probability distribution $\mathbf{P}_\theta$ for θ in H_0.

- "To solve exactly for the true parameter θ." is incorrect. In general in statistics, since we only have samples from the distribution, it will not be possible to solve for θ exactly.

- The second and third choices "To provide a consistent estimator for the true parameter θ." and "To develop an estimator that is close to the true parameter θ." are incorrect. These are some of the goals of parameter estimation.

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You have used 1 of 2 attempts

Answers are displayed within the problem

Review: Rejection Region

1/1 point (graded)

Setup:

You have samples $X_1, \ldots, X_n \stackrel{iid}{\sim} \mathrm{Ber}\,(p^*)$ for some true parameter $p^* \in (0,1)$. Let $\left(\{0,1\}, \{\mathbf{P}_p\}_{p\in(0,1)}\right)$ denote the associated statistical model, where $\mathbf{P}_p = \mathrm{Ber}\,(p)$.

You conduct a hypothesis test between

- a null hypothesis $H_0 : p^* \in \Theta_0$ and

- an alternative hypothesis $H_1 : p^* \in \Theta_1$,

where $\Theta_0, \Theta_1 \subset (0,1)$ and Θ_0 and Θ_1 are disjoint.

You construct a statistical **test**

$$\psi : \{0,1\}^n \to \{0,1\}$$

which takes as input the sample $(X_1, \ldots, X_n)$. If $\psi(X_1, \ldots, X_n) = 1$, you will **reject** the null H_0 in favor of the alternative H_1, and otherwise you will **fail to reject** the null.

Recall that the **rejection region** R_ψ describes which outcomes $(x_1, \ldots, x_n)$ will result in $\psi(x_1, \ldots, x_n) = 1$ and, hence, rejection of the null.

Questions:
The rejection region is a subset of ...
(Choose all that apply.)

- [ ] $(\Theta_0)^n$ where Θ_0 defines the null hypothesis H_0 in the parameter space Θ
- [ ] $(\Theta_1)^n$ where Θ_1 defines the alternative hypothesis H_1 in the parameter space Θ
- [ ] $(\Theta)^n$ where Θ is the parameter space
- [x] E^n where E is the sample space of X_i

✔

Solution:

The rejection region is by definition the set of all observed outcomes for which H_0 will be rejected by the test $\psi = \mathbf{1}((X_1, \ldots, X_n) \in R_\psi)$. It is a subset of E^n, where E is the sample space of X_i.

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Review : What Type of Objects are These?

5/5 points (graded)

Setup as above:

You have samples $X_1, \ldots, X_n \stackrel{iid}{\sim} \mathrm{Ber}(p^*)$ for some true parameter $p^* \in (0,1)$. Let $(\{0,1\}, \{\mathbf{P}_p\}_{p\in(0,1)})$ denote the associated statistical model, where $\mathbf{P}_p = \mathrm{Ber}(p)$.

You conduct a hypothesis test between

- a null hypothesis $H_0 : p^* \in \Theta_0$ and
- an alternative hypothesis $H_1 : p^* \in \Theta_1$,

where $\Theta_0, \Theta_1 \subset (0,1)$ and Θ_0 and Θ_1 are disjoint.

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Recall that the **rejection region** R_ψ describes which samples $(X_1, \ldots, X_n)$ will result in $\psi(X_1, \ldots, X_n) = 1$ and, hence, rejection of the null.

Let α_ψ and β_ψ denote the **type 1 error** and **type 2 error**, respectively.

Questions:

Determine which type of mathematical object each of the following is.(You are encouraged to review the definitions from the slides of the last lecture.)

(Choose one for each column.)

Rejection Region: Type 1 Error: Level: Type 2 Error: Power:

A number.	A number.	A number.	A number.	A number.
A set.	A set.	A set.	A set.	A set.
A function.	A function.	A function.	A function.	A function.

Solution:

We recall the definitions of each object in the context of the statistical model $\left(\{0,1\}, \{\mathbf{P}_p\}_{p\in(0,1)}\right)$.

The rejection region is defined to be

$$R_\psi := \{\mathbf{x} \in \{0,1\}^n : \psi(\mathbf{x}) = 1\},$$

so this is a **set**.

The type 1 error is defined to be

$$\begin{aligned} \alpha_\psi : \Theta_0 &\to [0,1] \\ p &\mapsto P_p(\psi = 1), \end{aligned}$$

so this is a **function** of p.

A level of a test is defined to be a **number** α such that

$$\alpha \geq \alpha_\psi(p) \qquad \text{for all } p \in \Theta_0 \text{. or equivalently} \quad \alpha \geq \sup_{p\in\Theta_0} \alpha_\psi(p)$$

The type 2 error is defined to be

$$\begin{aligned} \beta_\psi : \Theta_1 &\to [0,1] \\ p &\mapsto P_p(\psi = 0), \end{aligned}$$

so this is a **function** of p.

The power π_ψ is defined as

$$\pi_\psi := \inf_{p \in \Theta_1} (1 - \beta_\psi(p)).$$

This greatest lower bound of $(1 - \beta_\psi(p))$ over a set of p values is a **number**.

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Answers are displayed within the problem

Review: Type 1 vs. Type 2 Error

2/2 points (graded)

Setup as above:

let $X_1, \ldots, X_n \stackrel{iid}{\sim} \mathrm{Ber}(p^*)$ for some true parameter $p^* \in (0,1)$, and let $(\{0,1\}, \{P_p\}_{p \in (0,1)})$ denote the associated statistical model where $P_p = \mathrm{Ber}(p)$.

Hypotheses for this problem:

You would like to hypothesis test between two simple hypotheses:

$$H_0 : \; p^* \in \Theta_0 = \{1/2\}$$

$$H_1 : \; p^* \in \Theta_1 = \{3/4\}.$$

That is, each of the regions defined by the null and alternative hypotheses consists of a single point in the parameter space $\Theta = [0,1]$.

You constructed a statistical test ψ, and let α_ψ and β_ψ denote the **type 1 error** and **type 2 error**, respectively, associated to this test.

Questions:
What does $\alpha_\psi\,(1/2)$ represent?

- (selected) The probability that we **reject** $p^* = 1/2$ in favor of $p^* = 3/4$ even though **in fact** $p^* = 1/2$
- The probability that we **fail to reject** $p^* = 1/2$ in favor of $p^* = 3/4$ given **in fact** $p^* = 1/2$
- The probability that we **reject** $p^* = 1/2$ in favor of $p^* = 3/4$ given **in fact** $p^* = 3/4$
- The probability that we **fail to reject** $p^* = 1/2$ in favor of $p^* = 3/4$ even though **in fact** $p^* = 3/4$

✔

What does $\beta_\psi\,(3/4)$ represent?

- The probability that we **reject** $p^* = 1/2$ in favor of $p^* = 3/4$ even though **in fact** that $p^* = 1/2$
- The probability that we **fail to reject** $p^* = 1/2$ in favor of $p^* = 3/4$ given that **in fact** $p^* = 1/2$
- The probability that we **reject** $p^* = 1/2$ in favor of $p^* = 3/4$ given that **in fact** $p^* = 3/4$
- (selected) The probability that we **fail to reject** $p^* = 1/2$ in favor of $p^* = 3/4$ even though **in fact** $p^* = 3/4$

✔

Solution:

Let's consider the first question. If $\psi = 1$, then we would reject the null-hypothesis $p^* \in \Theta_0 = \{1/2\}$. Therefore $\alpha_\psi(1/2) = \mathbf{P}_{1/2}(\psi = 1)$ is the probability of **rejecting** $p^* \in \Theta_0 = \{1/2\}$ in favor of $p^* \in \Theta_1 = \{3/4\}$ even when in fact $p^* \in \Theta_0 = \{1/2\}$.

Now let's consider the second question. If $\psi = 0$, then we would fail to reject the null hypothesis $p^* \in \Theta_0 = \{1/2\}$ in favor of the alternative hypothesis $p^* \in \Theta_1 = \{3/4\}$. Therefore $\beta_\psi(3/4) = \mathbf{P}_{3/4}(\psi = 0)$ is the probability of not rejecting $H_0 : p^* \in \Theta_0 = \{1/2\}$ even when $p^* \in \Theta_1 = \{3/4\}$.

The other two choices are probabilities when the correct conclusions are made, not errors.

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