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3. Constrained to an ellipse

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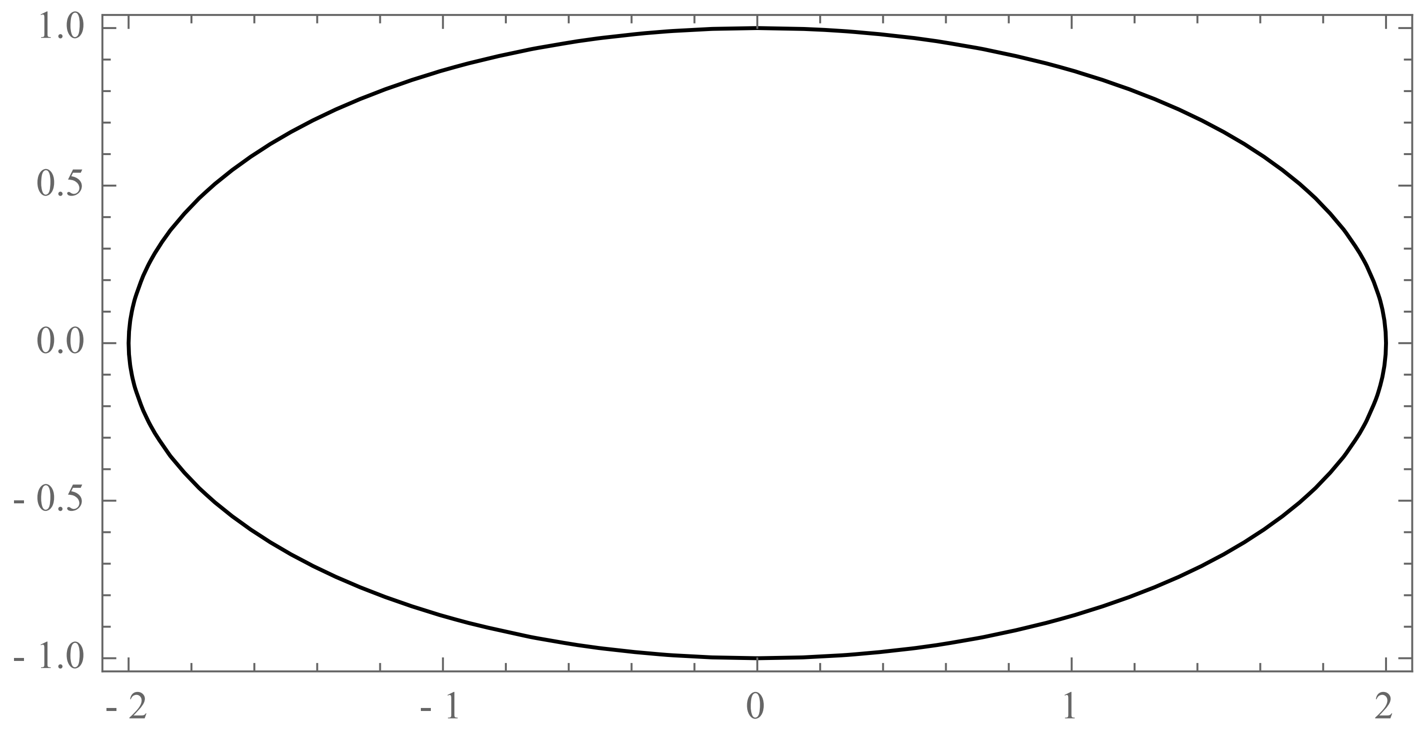
Problem Set A due Sep 13, 2021 20:30 IST Completed

Practice

3(a)

1.0/1 point (graded)

Let c be the curve $(1/4)\,x^2 + y^2 = 1$. This curve is an ellipse. Here is a picture of it.



Find all the points on c where the vector $\langle 1, 1\rangle$ is perpendicular to c. (In other words, find all the points of c where the normal vector is parallel to $\langle 1, 1\rangle$.)

(Enter points between round parentheses. Separate critical points by semicolons. For example, `(0,0);(1,1)`.)

(4/sqrt(5),1/sqrt(5));(-4/sqrt(5),-1/sqrt(5))

? INPUT HELP

Submit You have used 2 of 3 attempts

3(b)

1.0/1 point (graded)

Let c be the curve $(1/4)\,x^2 + y^2 = 1$.

Let $f\,(x, y) = x + y$. Find all the points of c where ∇f is perpendicular to c. (Hint: you won't have to compute much after the previous problem.)

(4/sqrt(5),1/sqrt(5));(-4/sqrt(5),-1/sqrt(5))

Answer: (4/sqrt(5),1/sqrt(5));(-4/sqrt(5),-1/sqrt(5))

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Solution:

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Note that $\nabla f = \langle 1, 1 \rangle$, so this is exactly what we computed in the previous problem!

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Answers are displayed within the problem

3(c)

2/2 points (graded)
Let c be the curve $(1/4)\, x^2 + y^2 = 1$. Let $f\,(x, y) = x + y$.

Find the maximum of f on c and the minimum of f on c.

Maximal value on c: sqrt(5) ✔ **Answer:** sqrt(5)

Minimal value on c: -sqrt(5) ✔ **Answer:** -sqrt(5)

? INPUT HELP

Solution:

The maximum and minimum values of f occur at the points along c where the gradient of f is parallel to the gradient of c. These are the points we found above. We plug into the formula for f to find the maximum and minimum values.

$$f\,(4/\sqrt{(5)}\,, 1/\sqrt{(5)}) \;=\; 4/\sqrt{(5)} + 1/\sqrt{(5)} = 5/\sqrt{5} = \sqrt{5} \qquad (4.235)$$

$$f\,(-4/\sqrt{(5)}\,, -1/\sqrt{(5)}) \;=\; -4/\sqrt{(5)} - 1/\sqrt{(5)} = -5/\sqrt{5} = -\sqrt{5} \qquad (4.236)$$

Thus the maximum value is $\sqrt{5}$ and the minimum is $-\sqrt{5}$.

Submit You have used 1 of 3 attempts

Answers are displayed within the problem

3(d)

1/1 point (graded)
Let c be the same curve $(1/4)\, x^2 + y^2 = 1$. Consider a new function $h\,(x, y) = xy$. Find the maximal value of h on the curve c.

Maximal value on c: 1 ✔ **Answer:** 1

Solution:

We solve the Lagrange multiplier problem. $\nabla h\,(x, y) = \langle y, x \rangle$. The normal vector to the curve is the same, it is $\langle x/2, 2y \rangle$. Solve the system

$$y \;=\; \lambda x/2 \qquad (4.237)$$

$$x \;=\; \lambda 2y \qquad (4.238)$$

Use the first equation to solve for λ and plug into the second equation:

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$$2y/x = \lambda \quad (4.239)$$

$$x = (2y/x)\,2y = 4y^2/x \quad (4.240)$$

$$x^2 = 4y^2 \quad (4.241)$$

Plug in the formula for x^2 into the equation for c to get

$$(1/4)\left(4y^2\right) + y^2 = 1 \quad (4.242)$$

$$y^2 + y^2 = 1 \quad (4.243)$$

$$y^2 = 1/2 \quad (4.244)$$

$$y = \pm 1/\sqrt{2} \quad (4.245)$$

We have four candidates, $(2/\sqrt{2}, 1/\sqrt{2})$, $(-2/\sqrt{2}, 1/\sqrt{2})$, $(2/\sqrt{2}, -1/\sqrt{2})$, and $(-2/\sqrt{2}, -1/\sqrt{2})$. Note that when they have the same sign the value is maximal, and is equal to 1. (When the have opposite signs it is the minimum, which is -1.)

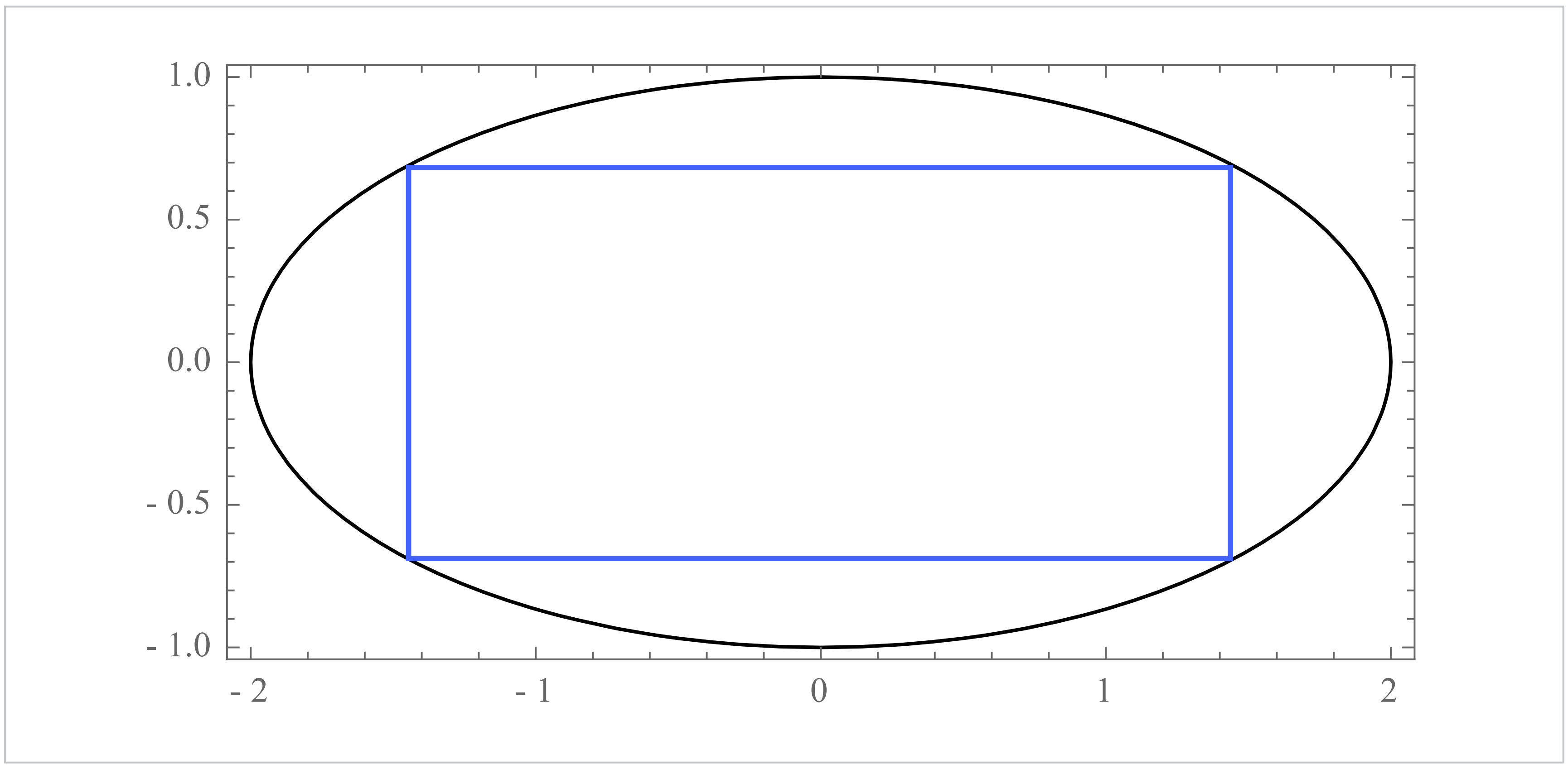
Submit You have used 2 of 10 attempts

Answers are displayed within the problem

4.

2/2 points (graded)

Let c be the ellipse $x^2 + 4y^2 = 4$. Given a point (x, y) on the ellipse, imagine cutting a rectangular box out of the ellipse with corners at (x, y), $(x, -y)$, $(-x, y)$, and $(-x, -y)$. (See the picture.)



How can we choose the point (x, y) (with $x > 0$ and $y > 0$) to make the area of the rectangle as large as possible?

$x =$ sqrt(2) **Answer:** sqrt(2)

$y =$ 1/sqrt(2) **Answer:** sqrt(2)/2

? INPUT HELP

Solution:

The area of the rectangle with corners at (x, y), $(x, -y)$, $(-x, y)$, and $(-x, -y)$ is $h(x, y) = 4xy$. We want to maximize this function subject to the constraint that $x^2 + 4y^2 = 4$, which says th Calculator e Hide Notes

First, observe that $h(x,y) = 4f(x,y)$, where $f(x,y) = xy$ as in the previous problem. Similarly, $x^2 + 4y^2 = 4$ is exactly the same as the curve $(1/4)x^2 + y^2 = 1$. Therefore the locations where the gradient of h is parallel to the normal to the ellipse will be the exact same points (x,y) found in the previous problem.

Thus we choose the point that lies in the first quadrant, $(x,y) = (\sqrt{2}, 1/\sqrt{2})$. (Note the other corners are the other points where the gradient of the function is parallel to the normal vector to the ellipse.)

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Answers are displayed within the problem

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4 solutions in 3(a) and 3(b)
For 3(a) and 3(b), I arrived at four solutions. I ultimately got the two correct solutions by looking at the geometry (the second pair of ... 5

"subject to the constraint"
The solution to 3(a) includes the phrase "subject to the constraint" in the first sentence of the second paragraph. Is that constraint n... 2

Stumped on #4
I thought I was on the right track until I entered my answers and they came back wrong. Is the Area formula 2y*sqrt(4x^2+4y^2) ? I ... 2

Question 4
question 4 is the same as the worked example in recitation 11. ive worked the question out and checked the recitation but my answe... 7

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[STAFF] Error in solution of 4 (last paragraph)
(x,y) = (the answer for the x co-ordinate is incorrect, y co-ordinate correct) 3

[Staff] Missing Backslashes 2

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