MITx: 6.041x Introduction to Probability - The Science of Uncertainty

Exercise: Non-Poisson random incidence

(2/2 points)

The consecutive interarrival times of a certain arrival process are i.i.d. random variables that are equally likely to be 5, 10, or 15 minutes. Find the expected value of the length of the interarrival time seen by an observer who arrives at some particular time, unrelated to the history of the process.

35/3

Answer: 11.66667

Answer:
Following the same argument as in the preceding video, out of every 30 minutes, there will be (in an average sense) 5 minutes (a fraction of 1/6 of the total) covered by intervals of length 5, 10 minutes (a fraction of 2/6) covered by intervals of length 10, and 15 minutes (a fraction of 3/6 of the total) covered by intervals of length 15. Thus, the observer has probability 1/6, 2/6, and 3/6, of seeing an interval of length 5, 10, and 15, respectively. The expected value is

$$\frac{1}{6}\cdot 5+\frac{2}{6}\cdot 10+\frac{3}{6}\cdot 15=\frac{70}{6}.$$

Note that this is larger than the average interarrival time, which is

$$\frac{1}{3}\cdot(5+10+15)=10.$$

In case you are curious, if a typical interarrival interval T has probability p_k of having length k, then the probability that the observer sees an interval S of length k is proportional to kp_k. Since probabilites need to sum to 1,

$$\mathbf{P}(S=k)=\frac{kp_k}{\sum_k kp_k}=\frac{kp_k}{\mathbf{E}[T]}.$$

It follows that

$$\mathbf{E}[S]=\sum_k k\frac{kp_k}{\sum_k kp_k}=\frac{\sum_k k^2p_k}{\mathbf{E}[T]}=\frac{\mathbf{E}[T^2]}{\mathbf{E}[T]}.$$

It can be shown that the expression $\mathbf{E}[S] = \mathbf{E}[T^2]/\mathbf{E}[T]$ is the correct one also for the continuous time case. As an illustration, suppose that interarrival times are exponential with rate λ, so that we are dealing with a Poisson process. In that case, $\mathbf{E}[T] = 1/\lambda$, $\mathbf{E}[T^2] = 2/\lambda^2$, so that $\mathbf{E}[S] = 2/\lambda$, which agrees with our earlier analysis of random incidence in the Poisson process.

You have used 1 of 2 submissions