Previous Next

9.3.1 2nd order Runge-Kutta Method (RK2)

MO2.4 MO2.7

A popular second-order ($p = 2$) Runge-Kutta (RK2) method is known as the modified Euler method. The basic idea is to attempt to estimate the slope $\mathrm{d}\underline{u}/\mathrm{d}t$ at the middle of a timestep (i.e. at $t^n + \Delta t/2$) and use that estimated slope in the Forward Euler method. Specifically, the modified Euler RK2 method is given by,

$$\underline{a} = \Delta t \underline{f}\left(\underline{v}^n, t^n\right) \tag{9.7}$$

$$\underline{b} = \Delta t \underline{f}\left(\underline{v}^n + \underline{a}/2, t^n + \Delta t/2\right) \tag{9.8}$$

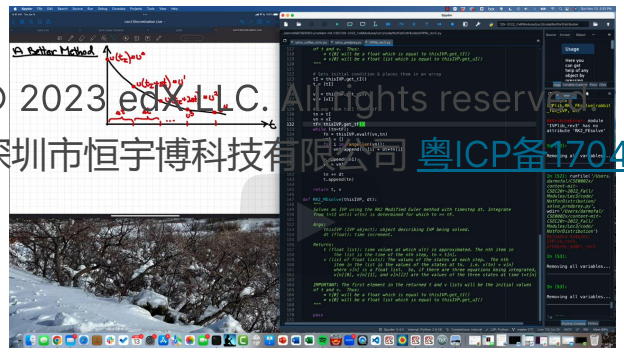
$$\underline{v}^{n+1} = \underline{v}^n + \underline{b} \tag{9.9}$$

The final update involves extrapolating the full timestep from $\underline{v}^n$ using $\underline{b}$. And, $\underline{b}$ is evaluated using the estimated slope at $t^n + \Delta t/2$.

Note that $\underline{f}$ is evaluated twice to calculate the new value of $\underline{v}^{n+1}$: once to determine $\underline{a}$ and once to determine $\underline{b}$. So, per iteration, it is essentially twice as expensive as Forward Euler (since the cost of evaluating $\underline{f}$ usually is the dominant cost of either Forward Euler or modified Euler).

In the following video, we discuss the modified Euler method and implement it in our `IVPlib` code. Then we show the performance of the modified Euler method on the coffee and rabbit-fox problem. The Python codes discussed in this video are available in the following zip file.

Video on modified Euler RK2 method and its implementation



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PROFESSOR: We're going to look at a better

Previous Next

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