11. Physical boundary conditions

The Diffusion Equation governs the evolution of the $\mathrm{CO_2}$ concentration profile inside of the length of the pipe. Our pipe is a finite length L. How do we handle what happens at either end of the pipe?

We need to specify the initial condition, i.e. the initial concentration distribution of $\mathrm{CO_2}$,

$$u_0 = u\left(x, 0\right),$$

and constraints that tell us what the boundary condtions are at all positive times.

Question 11.1 What are the boundary conditions? What can we specify?

The most natural thing to prescribe in a **physical** sense would be the flow rate of $\mathrm{CO_2}$ into/out of the ends of the pipe: $-\alpha\frac{\partial u}{\partial x}(0, t)$ and $-\alpha\frac{\partial u}{\partial x}(L, t)$. Another, less physically intuitive example in this case, but which will be mathematically simpler, is the case where the $\mathrm{CO_2}$ concentration is prescribed at each end of the pipe: $u\left(0, t\right) = c_1$ and $u\left(L, t\right) = c_2$. We can imagine this is due to the pipe connecting into a large reservoir of constant concentration; even as $\mathrm{CO_2}$ flows into and out of the pipe, the reservoir is so large that its net concentration remains constant. This could be represented as a large tank in a brewery with small pipes attached, for example.

Side Note: Fluid vs. Cell Flow

It is important to note that for the model created above, we are only considering the diffusion of $\mathrm{CO_2}$ in a non-flowing liquid, and **not** the transport of $\mathrm{CO_2}$ by the flow of the liquid itself. Such a process is called advection, and is modeled by a different set of partial differential equations. Also, one can combine both of these methods of transport, and this results in a partial differential equation called the Advection

Diffusion Equation. For now we will assume that the fluid is near-stationary so that all the transport is dominated by diffusion, so we will be fine using the Diffusion Equation for our models.

Hide

11. Physical boundary conditions

Hide Discussion

Topic: Unit 2: Boundary value problems and PDEs / 11. Physical boundary conditions

Add a Post

Show all posts by recent activity

There are no posts in this topic yet.

✖