Homework Problem: Blue Green Tree, Continued

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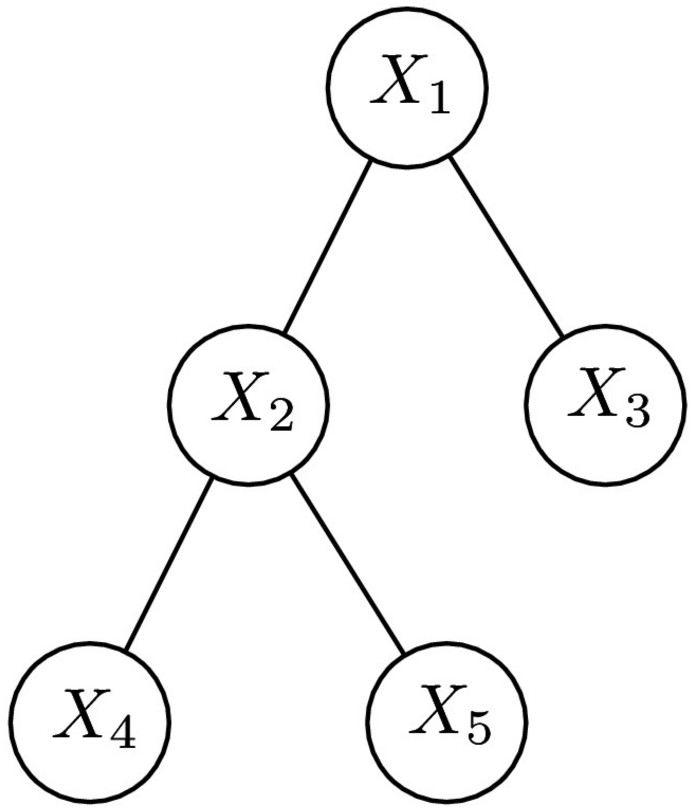
5/5 points (graded)

This problem continues off the previous homework's "Blue Green Tree" problem and requires that you've completed that problem (in particular, you will be working off of the potential functions determined in that problem). As a reminder the problem setup is below:

In this problem, we'll look at a tree structure where each node has a color – either blue or green. A blue node can only have green nodes as its neighbors and vice-versa. In other words, there is a color change at every edge. We'll consider a probabilistic version of a blue-green tree, where each node is blue or green with some probability p. The color change at each edge implies a flip in the probability of being blue or green. For example, in the simplest graph with two nodes, if it is given that node 1 is blue, then its neighbor, node 2, must be green.

Consider the graph shown below:



The joint distribution of the graph can be expressed in the form

$$p_X(x) = \frac{1}{Z} \prod_{i \in V} \phi_i(x_i) \prod_{(i,j) \in E} \psi_{i,j}(x_i, x_j).$$

- **(c)** Apply the sum-product algorithm on this tree and determine messages $m_{4\to2}$, $m_{5\to2}$, $m_{3\to1}$, $m_{1\to2}$. Find the marginal probability that node 2 is blue.

  For each message, please normalize its table so that the entries sum to 1.

  (Please be precise with at least 3 decimal places, unless of course the answer doesn't need that many decimal places. You could also put a fraction.)

  $m_{4\to2}(\text{blue}) =$ 0.2

  $m_{5\to2}(\text{blue}) =$ 0.2

  $m_{3\to1}(\text{blue}) =$ 0.4

  $m_{1\to2}(\text{blue}) =$ 0.6

  $p_{X_2}(\text{blue}) =$ 0.0857142857143

- **(d)** Now consider that instead of forcing a color change completely, we favor two adjacent nodes having different colors with weight 0.8 and having the same color with weight 0.2 (these in general don't have to sum to 1). Also, it is given that node 3 and 5 are deterministically green, and node 4 is deterministically blue. Define new edge and node potentials and repeat part (c).

  For each message, please normalize its table so that the entries sum to 1.

(Please be precise with at least 3 decimal places, unless of course the answer doesn't need that many decimal places. You could also put a fraction.)

$m_{4\to 2}(\text{blue}) =$ 0.2

$m_{5\to 2}(\text{blue}) =$ 0.8

$m_{3\to 1}(\text{blue}) =$ 0.8

$m_{1\to 2}(\text{blue}) =$ 0.32

$p_{X_2}(\text{blue}) =$ 0.32

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