7. Maximum along the boundary continued: Lagrange multipliers

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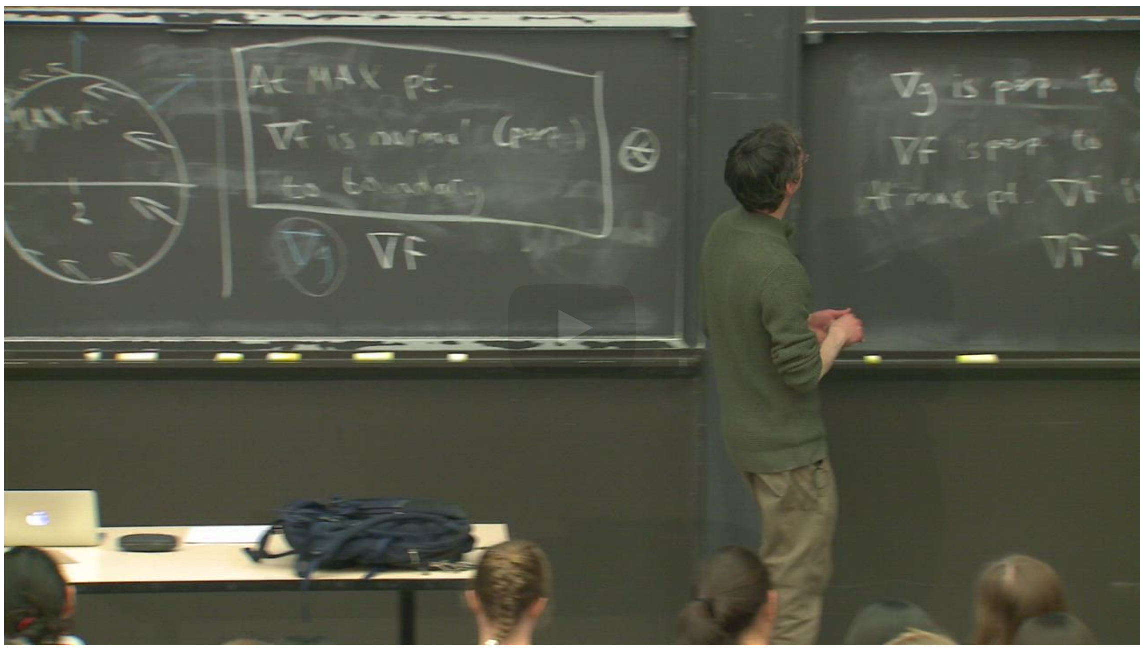

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INSTRUCTOR: So now let's find this maximum point.

So here's our function.

And we know that at the maximum point, this is true.

And so we're going to use this to compute

where the maximum point is.

So at the maximum point, now we

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Now that we know $\nabla f = \lambda \nabla g$ at the maximum point, let's find the maximum point.

Definition 7.1 The scalar λ is called the **Lagrange multiplier**.

We can use this relationship to find the maximum. We first write

$$\underbrace{\langle -2x, -2y+2\rangle}_{\nabla f} = \lambda \underbrace{\langle 2x-4, 2y\rangle}_{\nabla g} \qquad (4.164)$$

and set the components equal to each other to obtain

$$-2x = \lambda(2x-4) \qquad (4.165)$$

$$-2y+2 = \lambda(2y). \qquad (4.166)$$

We want to turn this system of two equations into a single equation about x and y. By solving for λ in 4.165, we obtain

$$\lambda = \frac{-2x}{2x-4}. \qquad (4.167)$$

Substituting this into 4.166, clearing denominators, and simplifying gives:

$$2y + x = 2.$$

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This is the equation of a line. The point at which this line intersects the boundary is where the maximum occurs. Solving 4.168 for x gives

$$x = 2 - 2y. \tag{4.169}$$

Recall that the equation for the boundary of R is $(x-2)^2 + y^2 = 1$. Substituting our equation for x into the equation for the boundary of R gives

$$((2-2y)-2)^2 + y^2 = 1 \tag{4.170}$$

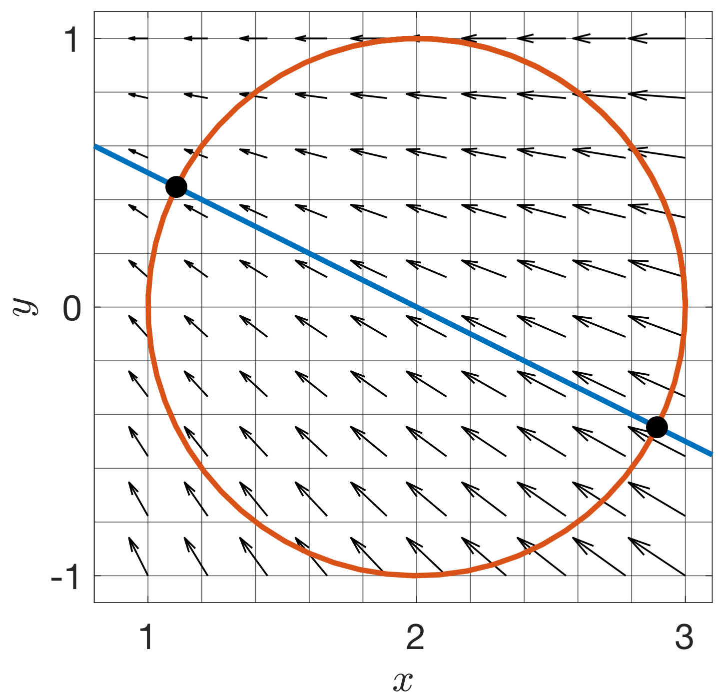
$$(-2y)^2 + y^2 = 1 \tag{4.171}$$

$$y = \pm\frac{1}{\sqrt{5}}. \tag{4.172}$$

Substituting these values of y into $x = 2 - 2y$ gives the ordered pairs

$$\left(2 - \frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}}\right) \text{ and } \left(2 + \frac{2}{\sqrt{5}}, -\frac{1}{\sqrt{5}}\right) \tag{4.173}$$

Notice that we obtained two possible answers. The figure below shows an illustration of $\nabla f(x,y)$, the boundary of R, and the line $2y + x = 2$. The points at which $2y + x = 2$ intersects the boundary of R are indicated by black circular markers.



To determine which point of intersection is the maximum, plug these values into $f(x,y)$. We see that

$$f\left(2 - \frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}}\right) = 2\sqrt{5} - 1 \approx 3.4721 \tag{4.174}$$

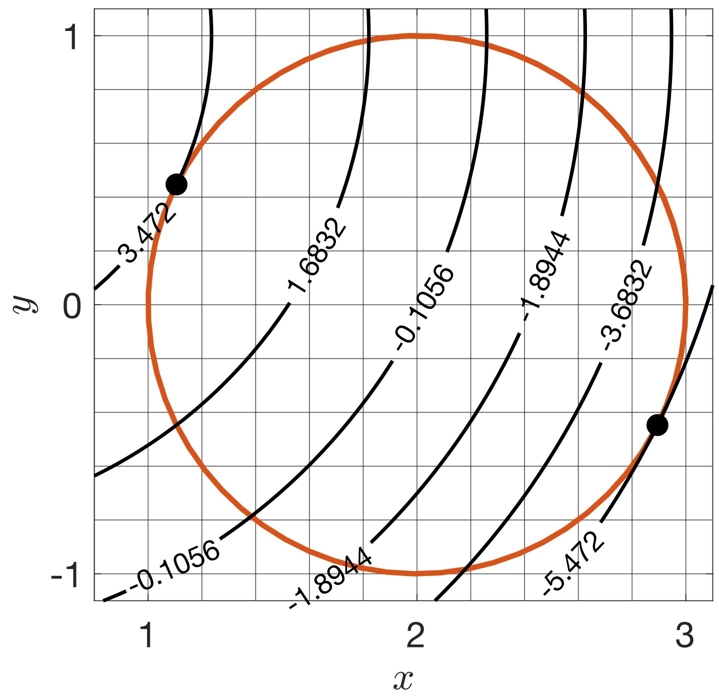
$$f\left(2 + \frac{2}{\sqrt{5}}, -\frac{1}{\sqrt{5}}\right) = -1 - 2\sqrt{5} \approx -5.4721. \tag{4.175}$$

So the maximum of $f(x,y)$ over the region R occurs at

$$(x,y) = \left(2 - \frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}}\right). \tag{4.176}$$

But what does the other point of intersection represent? It turns out that the other point is precisely where the minimum of $f(x,y)$ occurs over the boundary R. At this point, $\nabla f(x,y)$ points in th Calculator or Hide Notes
$\nabla g(x,y)$

The figure below shows the level curves of $f(x, y)$ along with the boundary of the region R. The two points we found using the Lagrange multiplier method are indicated by black circles. Notice that they both occur at points where the level curves of f are tangent to the boundary of R.



Remark 7.2 This process works for any function $f(x, y)$ along any curve C.

1. Describe the curve C as a level curve $g(x, y) = k$.
2. Then the maximum of $f(x, y)$ along C occurs where $\nabla f = \lambda \nabla g$.

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? Can we gat a saddle/inflection point? 3
Is is stated that we can use this method to get min\max on a boundary. Could this method also yield saddle (or inflection) point on t...

Division by 0? 1

min max directions 5

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