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Exercise: Discrete unknown and continuous observation

(2/2 points)

Similar to the last example, suppose that $X = \Theta + W$, where Θ is equally likely to take the values -1 and 1, and where W is standard normal noise, independent of Θ. We use the estimator $\widehat{\Theta}$, with $\widehat{\Theta} = 1$ if $X > 0$ and $\widehat{\Theta} = -1$ otherwise. (This is actually the MAP estimator for this problem.)

a) Let us assume that the true value of Θ is 1. In this case, our estimator makes an error if and only if W is "small". The conditional probability of error given the true value of Θ is 1, that is, $\mathbf{P}(\widehat{\Theta} \neq 1 \mid \Theta = 1)$, is equal to

- (●) $\Phi(-1)$ ✔
- ( ) $\Phi(0)$
- ( ) $\Phi(1)$

where Φ is the standard normal CDF.

b) For this problem, the overall probability of error is easiest found using the formula

- ( ) $\mathbf{P}(\widehat{\Theta} \neq \Theta) = \int \mathbf{P}(\widehat{\Theta} \neq \Theta \mid X = x) f_X(x)\, dx$
- (●) $\mathbf{P}(\widehat{\Theta} \neq \Theta) = \sum_\theta \mathbf{P}(\widehat{\Theta} \neq \theta \mid \Theta = \theta)\, p_\Theta(\theta)$ ✔

Answer:
a) We have

$$\begin{aligned}\mathbf{P}(\widehat{\Theta} \neq 1 \mid \Theta = 1) &= \mathbf{P}(\Theta + W \leq 0 \mid \Theta = 1) = \mathbf{P}(1 + W \leq 0 \mid \Theta = 1) \\ &= \mathbf{P}(1 + W \leq 0) = \mathbf{P}(W \leq -1) = \Phi(-1).\end{aligned}$$

b) Similar to part (a), $\mathbf{P}(\widehat{\Theta} \neq \theta \mid \Theta = \theta)$ is easy to calculate for either choice of $\theta = -1$ or $\theta = 1$. For this reason, the second formula is easy to implement.

Exercises 16 due Apr 13, 2016 at 23:59 UTC

Lec. 17: Linear least mean squares (LLMS) estimation
Exercises 17 due Apr 13, 2016 at 23:59 UTC

Problem Set 7b
Problem Set 7b due Apr 13, 2016 at 23:59 UTC

Solved problems

Additional theoretical material

Unit summary

You have used 1 of 1 submissions