11.2.2 An Application: Rank-1 Approximation

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Week 11 due Dec 22, 2023 21:12 IST Completed

11.2.2 An Application: Rank-1 Approximation

Video

Start of transcript. Skip to the end.

Dr. Robert van de Geijn: So let's look at Rank-1 Approximation.

Here's a picture.

Think of this picture as a matrix.

Think of this red line as representing the jth column so that the matrix can

be thought of as consisting of n columns.

0:00 / 0:00 2.0x

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Reading Assignment

0 points possible (ungraded)

Read Unit 11.2.2 of the notes. [LINK]

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Homework 11.2.2.1: How is $s_j \in \mathbf{T}$? 4

? rank-1 approximation
In practice, I guess rank-1 (amd rank 2) are of any use only in pictures where columns share a lot of similarity? Or are there any theoretical pur

Calculator

Homework 11.2.2.1

1/1 point (graded)
Let $\mathbf{S}$ and $\mathbf{T}$ be subspaces of $\mathbb{R}^m$ and $\mathbf{S} \subset \mathbf{T}$.

$\mathbf{dim}(\mathbf{S}) \leq \mathbf{dim}(\mathbf{T})$.

Always ✔ Answer: Always

Proof by contradiction:

Let $\mathbf{dim}(\mathbf{S}) = k$ and $\mathbf{dim}(\mathbf{T}) = n$, where $k > n$. Then we can find a set of k vectors $\{s_0, \ldots, s_{k-1}\}$ that form a basis for $\mathbf{S}$ and a set of n vectors $\{t_0, \ldots, t_{n-1}\}$ that form a basis for $\mathbf{T}$.

Let

$$S = \left(\begin{array}{c|c|c|c} s_0 & s_1 & \cdots & s_{k-1} \end{array} \right) \quad \text{and} \quad T = \left(\begin{array}{c|c|c|c} t_0 & t_1 & \cdots & t_{n-1} \end{array} \right).$$

$s_j \in \mathbf{T}$ and hence can be written as $s_j = T x_j$. Thus

$$S = T \underbrace{\left(\begin{array}{c|c|c|c} x_0 & x_1 & \cdots & x_{k-1} \end{array} \right)}_{X} = TX$$

But X is $n \times k$ which has more columns that it has rows. Hence, there must exist vector $z \neq 0$ such that $Xz = 0$.

But then

$$Sz = TXz = T0 = 0$$

and hence S does not have linearly independent columns. But, we assumed that the colums of S formed a basis, and hence this is a contradiction. We conclude that $\mathbf{dim}(\mathbf{S}) \leq \mathbf{dim}(\mathbf{T})$.

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Answers are displayed within the problem

Homework 11.2.2.2

10.0/10.0 points (graded)
Let $u \in \mathbb{R}^m$ and $v \in \mathbb{R}^n$. Then the $m \times n$ matrix uv^T has a rank of at most one.

TRUE ✔ Answer: TRUE

Let $y \in \mathcal{C}(uv^T)$. We will show that then $y \in \text{Span}(\{u\})$ and hence $\mathcal{C}(uv^T) \subset \text{Span}(\{u\})$.

$$\begin{array}{l} y \in \mathcal{C}(uv^T) \\ \quad \Rightarrow \quad < \text{there exists a } x \in \mathbb{R}^n \text{ such that } y = uv^T x > \\ y = uv^T x \\ \quad \Rightarrow \quad < u\alpha = \alpha u \text{ when } \alpha \in \mathbb{R} > \\ y = (v^T x)\, u \\ \quad \Rightarrow \quad < \text{Definition of span and } v^T x \text{ is a scalar} > \end{array}$$

Calculator

$$y \in \text{Span}(\{u\})$$

Hence $\dim(\mathcal{C}(uv^T)) \leq \dim(\text{Span}(\{u\})) \leq 1$. Since $\text{rank}(uv^T) = \dim(\mathcal{C}(uv^T))$ we conclude that $\text{rank}(uv^T) \leq 1$.

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Homework 11.2.2.3

1/1 point (graded)

Let $u \in \mathbb{R}^m$ and $v \in \mathbb{R}^n$. Then uv^T has rank equal to zero if
(Mark all correct answers.)

- [x] $u = 0$ (the zero vector in $\mathbb{R}^m$).
- [x] $v = 0$ (the zero vector in $\mathbb{R}^n$).
- [ ] Never.
- [ ] Always.

✓

$u = 0$ (the zero vector in $\mathbb{R}^m$).

$v = 0$ (the zero vector in $\mathbb{R}^n$).

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