3.2 Quiz: Phase Plane for the Simplified Pendulum Model

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We are now working with the simplified pendulum model:

$$\frac{d^2\theta}{dt^2} = -k\theta$$

which we expressed as a system by letting α be the angular velocity:

$$\frac{d\theta}{dt} = \alpha$$
$$\frac{d\alpha}{dt} = -k\theta$$

Question 1

1/1 point (graded)

Note: k is a positive constant in all cases.

At the point $(0.1, 0.1)$ in the $(\theta, \frac{d\theta}{dt})$ phase plane, the value of θ is:

You can add an optional tip or note related to the prompt like this.

- (selected) increasing ✔
- decreasing

- at a local maximum
- at a local minimum
- undefined

Explanation

At $(0.1, 0.1)$, the value of θ is increasing because $\frac{d\theta}{dt} = 0.1$ is positive.
Does this agree with the arrows drawn on your phase plane? If not, correct your drawing.

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Answers are displayed within the problem

Question 2

1/1 point (graded)

At the point $(0.1, 0.1)$ in the $(\theta, \frac{d\theta}{dt})$ phase plane, the value of $\frac{d\theta}{dt}$ is:

You can add an optional tip or note related to the prompt like this.

- increasing
- decreasing ✔
- at a local maximum
- at a local minimum
- undefined

Explanation

We know that $\frac{d^2\theta}{dt^2} = -k\theta$. Since $\theta = 0.1$, we have $\frac{d^2\theta}{dt^2} < 0$ and so $\frac{d\theta}{dt}$ is decreasing.
Does this agree with the arrows drawn on your phase plane? If not, correct your drawing and check your work in the other three quadrants of the plane.

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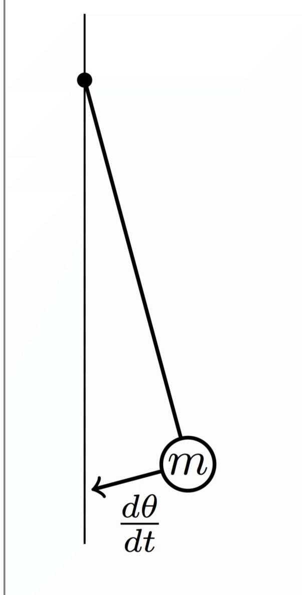
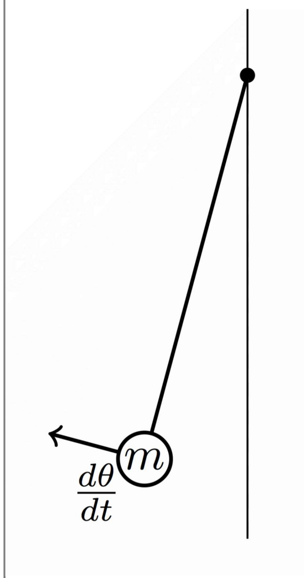
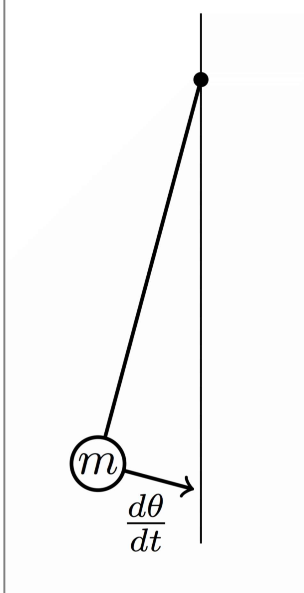
Answers are displayed within the problem

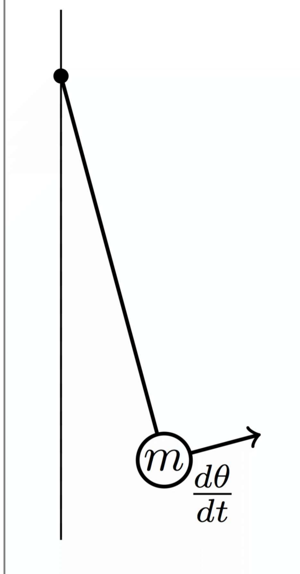
Question 3

4/4 points (graded)

Each image below shows the pendulum at different positions in its swing. Match each image of the pendulum with the quadrant of the phase plane corresponding to the sign of its angle and angular velocity.

Phase Plane Quadrant	Pendulum
The first quadrant: $\theta > 0$ and $\frac{d\theta}{dt} > 0$. Pendulum D ▼ ✔ **Answer:** Pendulum D	Pendulum A  View Larger Image Image Description

The second quadrant: $\theta < 0$ and $\frac{d\theta}{dt} > 0$. Pendulum C ▼ ✔ **Answer:** Pendulum C	Pendulum B  View Larger Image Image Description
The third quadrant: $\theta < 0$ and $\frac{d\theta}{dt} < 0$. Pendulum B ▼ ✔ **Answer:** Pendulum B	Pendulum C  View Larger Image Image Description

<table>
<tr>
<td>The fourth quadrant: $\theta > 0$ and $\frac{d\theta}{dt} < 0$.
Pendulum A ▼ ✔ **Answer:** Pendulum A</td>
<td>Pendulum D


View Larger Image
Image Description</td>
</tr>
</table>

Explanation

When the pendulum is on the right ($\theta > 0$) and moving to the right ($\frac{d\theta}{dt} > 0$), as in image D, this corresponds to the first quadrant.
When the pendulum is on the left ($\theta < 0$) and moving to the right ($\frac{d\theta}{dt} > 0$), as in image C, this corresponds to the second quadrant.
When the pendulum is on the left ($\theta < 0$) and moving to the left ($\frac{d\theta}{dt} < 0$), as in image B, this corresponds to the third quadrant.
When the pendulum is on the right ($\theta > 0$) and moving to the left ($\frac{d\theta}{dt} < 0$), as in image A, this corresponds to the fourth quadrant.

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You have used 1 of 3 attempts

Answers are displayed within the problem

Question 4

4/4 points (graded)

At each point $(\theta, \frac{d\theta}{dt})$, we can determine the direction of the solution trajectory as time t progresses using the system of differential equations. Match each quadrant to the corresponding direction of solution trajectories $(\theta(t), \frac{d\theta}{dt}(t))$.

Note: $\frac{d\theta}{dt}$ is positive as the pendulum swings to the right and negative as it swings left. A negative velocity is increasing' when it is approaching 0.

Quadrant	Angle
The point is in the first quadrant: $\theta > 0$ and $\frac{d\theta}{dt} > 0$.	The angle θ is decreasing and the velocity $\frac{d\theta}{dt}$ is increasing. (↖) Quadrant III ▼ ✔ **Answer:** Quadrant III
The point is in the second quadrant: $\theta < 0$ and $\frac{d\theta}{dt} > 0$.	The angle θ is decreasing and the velocity $\frac{d\theta}{dt}$ is decreasing. (↙) Quadrant IV ▼ ✔ **Answer:** Quadrant IV
The point is in the third quadrant: $\theta < 0$ and $\frac{d\theta}{dt} < 0$.	The angle θ is increasing and the velocity $\frac{d\theta}{dt}$ is decreasing. (↘) Quadrant I ▼ ✔ **Answer:** Quadrant I
The point is in the fourth quadrant: $\theta > 0$ and $\frac{d\theta}{dt} < 0$.	The angle θ is increasing and the velocity $\frac{d\theta}{dt}$ is increasing. (↗) Quadrant II ▼ ✔ **Answer:** Quadrant II

Explanation

In the first quadrant, the pendulum is on the right ($\theta > 0$) and moving to the right ($\frac{d\theta}{dt} > 0$). The value of $\frac{d^2\theta}{dt^2} = -k\theta$ is negative, so $\frac{d\theta}{dt}$ is decreasing. (↘)

In the second quadrant, the pendulum is on the left ($\theta < 0$) and moving to the right ($\frac{d\theta}{dt} > 0$). The value of $\frac{d^2\theta}{dt^2} = -k\theta$ is positive, so $\frac{d\theta}{dt}$ is increasing. (↗)

In the third quadrant, the pendulum is on the left ($\theta < 0$) and moving to the left ($\frac{d\theta}{dt} < 0$). The value of $\frac{d^2\theta}{dt^2} = -k\theta$ is positive, so $\frac{d\theta}{dt}$ is increasing. (↖)

In the fourth quadrant, the pendulum is on the right ($\theta > 0$) and moving to the left. The value of $\frac{d\theta}{dt}$ is negative and decreasing. (↙)

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You have used 2 of 3 attempts

Answers are displayed within the problem

Question 5

1/1 point (graded)

Imagine starting with a pendulum at a small angle to the right of the vertical and angular velocity zero. This means the point is on the positive horizontal axis. Which quadrant will the corresponding solution trajectory move into next?

- Quadrant I
- Quadrant II
- Quadrant III
- (selected) Quadrant IV ✔
- None of the above. The solution trajectory is a single point (an equilibrium).

Explanation

The pendulum starts at some point $(\theta_0, 0)$ in the $(\theta, \frac{d\theta}{dt})$ phase plane. This point is on the positive horizontal axis.

At that instant $\frac{d\theta}{dt} = 0$, so the arrow indicating the direction of motion in the phase plane will be vertical. To see if it is up or down, we look at $\frac{d^2\theta}{dt^2} = -k\theta_0 < 0$. Thus the arrow is down and the trajectory is moving into the fourth quadrant.

Physical intuition agrees with this – the pendulum will move left toward the center, so from a starting velocity of zero, the velocity will decrease. Thus the acceleration of the pendulum is negative.

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Answers are displayed within the problem

Question 6

1/1 point (graded)

What might the solution curves look like on the phase plane diagram? Choose all options below which are consistent our findings up to this point.

We are asking you to select which trajectories could be possible given what we know about the phase plane so far in this section.

(**Note:** Only one of these is actually the correct trajectory, but knowing this requires more information. If you have found such additional information or have previous knowledge that allows you to know the correct trajectory, please answer without using this information.)

(There at least two valid choices.)

- [ ] The solution curves will be horizontal line segments. The point $(\theta, \frac{d\theta}{dt})$ will move straight back and forth over time.
- [ ] The solution curves will be vertical line segments. The point $(\theta, \frac{d\theta}{dt})$ will move straight up and down over time.
- [x] The point $(\theta, \frac{d\theta}{dt})$ will trace out clockwise loops around the origin. It will move right, down, left, then up over time. ✔
- [ ] The point $(\theta, \frac{d\theta}{dt})$ will trace out counter-clockwise loops around the origin. It will move left, down, right, then up over time.
- [x] The point $(\theta, \frac{d\theta}{dt})$ will follow a clockwise spiral in toward the origin. ✔
- [ ] The point $(\theta, \frac{d\theta}{dt})$ will follow a counter-clockwise spiral in toward the origin.
- [x] The point $(\theta, \frac{d\theta}{dt})$ will follow a clockwise spiral away from the origin. ✔
- [ ] The point $(\theta, \frac{d\theta}{dt})$ will follow a counter-clockwise spiral away from the origin.
- [ ] We cannot determine any information about the solution curves without a formula describing $\frac{d\theta}{dt}$ in terms of t.

✔

Explanation

Based on our analysis up to this point, the solution curves will be clockwise loops or spirals. When the pendulum is at its lowest point and moving to the right, $\theta = 0$ and $\frac{d\theta}{dt}$ is at a (positive) maxiumum. From there, θ increases and $\frac{d\theta}{dt}$ decreases until θ reaches its maximum value and $\frac{d\theta}{dt} = 0$. Then the pendulum swings back to the left and the point $(\theta, \frac{d\theta}{dt})$ sweeps back under $(0, 0)$ until θ reaches its minimum (negative) value and $\frac{d\theta}{dt}$ is again 0.

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Answers are displayed within the problem

Question 7

1/1 point (graded)

Of the choices you selected above, which is most consistent with your intuition?

```
The point (θ,dθdt) will trace out counter-clockwise loops
around the origin. It will move left, down, right, then up
over time.
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✔

Thank you for your response.

Explanation

Real pendulums swing back and forth through smaller and smaller arcs until they eventually come to rest. This is consistent with a clockwise spiral in toward the origin.

In our model we are assuming that there is no friction or air resistance, so this may mean we that the solution curves are clockwise loops around the origin. We'll see in the next section.

Note that a clockwise spiral out would imply a larger and larger arc of the pendulum which is physically impossible without some added energy to the system.

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Answers are displayed within the problem

Question 8

1/1 point (graded)

Which of the following, if any, are solutions to the differential equation below?

$$\frac{d^2\theta}{dt^2} = -k\theta$$

- [ ] $\theta(t) = \sin(kt)$
- [x] $\theta(t) = \sin(\sqrt{k}t)$
- [ ] $\theta(t) = \cos(kt)$
- [x] $\theta(t) = \cos(\sqrt{k}t)$
- [ ] None of the above.

✔

Submit You have used 1 of 4 attempts

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