

### MITx: 6.041x Introduction to Probability - The Science of Uncertainty

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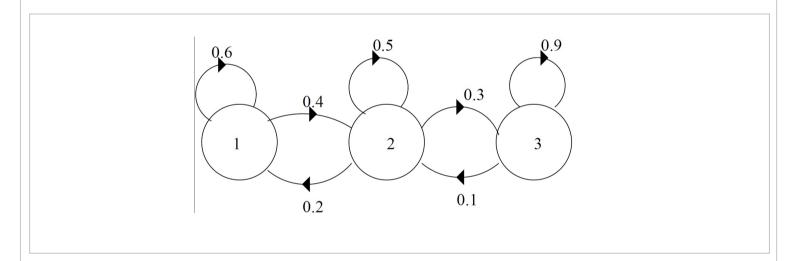
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# Problem 4: A simple Markov chain

(10/10 points)

Consider a Markov chain  $\{X_0, X_1, \ldots\}$ , specified by the following transition probability graph.



1.  $\mathbf{P}(X_2 = 2 \mid X_0 = 1) = \boxed{0.44}$ 

2. Find the steady-state probabilities  $\pi_1$ ,  $\pi_2$ , and  $\pi_3$  associated with states 1, 2, and 3, respectively.

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Unit overview

Lec. 24: Finite-state Markov chains

Exercises 24 due May 18, 2016 at 23:59 UTC

Lec. 25: Steady-state behavior of Markov chains Exercises 25 due May 18, 2016 at 23:59 UTC



 $\pi_2 = \boxed{ 2/9 }$ 

$$\pi_3 = \boxed{ 2/3 }$$

3. For  $n=1,2,\ldots$ , let  $Y_n=X_n-X_{n-1}$ . Thus,  $Y_n=1$  indicates that the nth transition was to the right,  $Y_n=0$  indicates that it was a self-transition, and  $Y_n=-1$  indicates that it was a transition to the left.

$$\lim_{n\to\infty}\mathbf{P}(Y_n=1)=\boxed{1/9}$$

4. Is the sequence  $Y_1, Y_2, \ldots$  a Markov chain?



5. Given that the nth transition was a transition to the right ( $Y_n=1$ ), find (approximately) the probability that the state at time n-1 was state 1 (i.e.,  $X_{n-1}=1$ ). Assume that n is large.

## Lec. 26: Absorption probabilities and expected time to absorption

Exercises 26 due May 18, 2016 at 23:59 UTC

#### Solved problems

#### **Problem Set 10**

Problem Set 10 due May 18, 2016 at 23:59 UTC



Exit Survey



6. Suppose that  $X_0=1$ . Let T be the first *positive* time index n at which the state is equal to **1**.

$$\mathbf{E}[T] = \boxed{9}$$

7. Does the sequence  $X_1, X_2, X_3, \ldots$  converge in probability to a constant?



8. Let  $Z_n = \max\{X_1,\ldots,X_n\}$ . Does the sequence  $Z_1,Z_2,Z_3,\ldots$  converge in probability to a constant?





You have used 2 of 2 submissions

## **DISCUSSION**

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