



#### Lecture 17: Introduction to Bayesian

<u>Course</u> > <u>Unit 5 Bayesian statistics</u> > <u>Statistics</u>

> 11. Worked Example Part II

# 11. Worked Example Part II

**Note:** The problems in this vertical depend on the final answer from Worked Example Part I. **You must have the answer to the final answerbox in order to answer the questions here.** 

We now consider the **Gamma distribution** , which is a probability distribution with parameters q>0 and  $\lambda>0$  , has support on  $(0,\infty)$  , and whose density is given by

$$f\left( x
ight) =rac{\lambda ^{q}x^{q-1}e^{-\lambda x}}{\Gamma \left( q
ight) }.$$

Here,  $\, \Gamma \,$  is the Euler Gamma function.

### Simplifying the Gamma Distribution

1/1 point (graded)

We will use proportionality notation in order to simplify the Gamma Distribution. But first, we perform a cosmetic change of variables to avoid repetitive notation with our answer in Part I: we write our parameters instead as  $\lambda_0$  and  $q_0$ .

From the expression for the Gamma distribution given above, remove outermost multipliers to simplify it in such a way that our expression for f(1) is  $e^{-\lambda_0}$  regardless of the value of  $q_0$ .

Use **q\_0** for  $q_0$  and **lambda\_0** for  $\lambda_0$ .

#### Solution:

Note that we want a function of x, so we are able to pull out factors that do not depend on the variable x. (i.e. are purely constants or a factor whose value only depends on variables other than x). From  $f(x) = \frac{\lambda^q x^{q-1} e^{-\lambda x}}{\Gamma(q)}$ , we can notice that both  $\lambda^q$  in the numerator and  $\Gamma(q)$  in the denominator are independent of x, so removing those reduces our expression to  $x^{q-1}e^{-\lambda x}$ .

Making a slight tweak of variables so that we use  $\lambda_0$  and  $q_0$  instead, as specified, gives  $f(x) \propto x^{q_0-1}e^{-\lambda_0 x}$ , and it can be seen (as an exercise) that this expression for f(x) satisfies  $f(1) = e^{-\lambda_0}$ .

Submit

You have used 1 of 3 attempts

• Answers are displayed within the problem

## Interpreting the Posterior Distribution

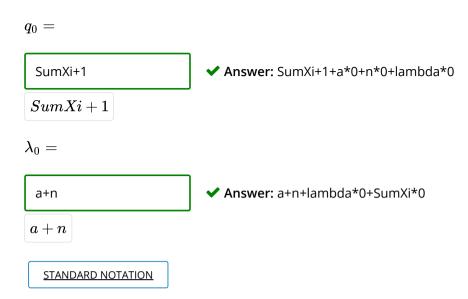
3/3 points (graded)

Compare this with the posterior distribution you computed from Part I, which you should see is a Gamma distribution. What is the corresponding variable, and what are its parameters?

Use **SumXi** for  $\sum_{i=1}^n X_i$ .

$$x =$$

lambda ✓ Answer: lambda+a\*0+n\*0+SumXi\*0



### **Solution:**

In Part I, we derived the posterior distribution (as a function of  $\lambda$ ) to be

$$e^{-(a+n)\lambda}\lambda^{\sum_{i=1}^n X_i}.$$

Here, it is the variable  $\lambda$  that is supposed to be distributed according to a Gamma distribution, hence we must write  $x=\lambda$ .

From here, we need to match the remaining variables. The exponent of x (vis.  $\lambda$ ) in the general Gamma distribution is  $q_0-1$  and in our posterior distribution is  $\sum_{i=1}^n X_i$ , so we could write  $q_0=(\sum_{i=1}^n X_i)+1$ . Similarly,  $\lambda_0$  is what multiplies x in the exponent of e, which we see is a+n in our posterior distribution, so a+n.

Submit

You have used 1 of 3 attempts

• Answers are displayed within the problem

### Discussion

**Hide Discussion** 

Topic: Unit 5 Bayesian statistics:Lecture 17: Introduction to Bayesian Statistics / 11. Worked Example Part II

#### Add a Post



© All Rights Reserved