3. Centrality Measures – Introduction

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Exercises due Oct 20, 2021 17:29 IST Completed

Centrality Measures

0:00 / 0:00 1.50x

Start of transcript. Skip to the end.

Prof Uhler: OK, welcome back to the second lecture

in this networks module.

So we already discussed a little bit this module,

got some intuition on actually how to find

important nodes in a network.

Video

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Video note: At 3:57, the convention for the definitions of k_i^{in} and k_i^{out} are different the one stated below.

Video note: At 7:00, the definition of betweenness centrality uses a normalization factor of n^{-2}, in the exercises below, the more common definition with no normalization factor will be used (shown in the last unit).

The **degree centrality** measures the importance of nodes in terms of the degree of a node. For a directed graph, define the in-degree k_i^{in} and the out-degree k_i^{out} of a node to be the sum over the ith column or the row of the adjacency matrix respectively:

$$k_i^{\text{in}} = \sum_j A_{ji}, \quad k_i^{\text{out}} = \sum_j A_{ij}.$$

Note that (unlike in the slides) we use the convention where $A_{ij} = 1$ indicates an edge going **from node i to node j**. The convention used can differ in different fields applications.

The degree centrality only captures importance up to one-hop neighbors of a node. Depending upon the application, this may not be representative of the importance of a node in the overall graph.

High Degrees

2/2 points (graded)

Let an undirected graph have n nodes. Let the edges be selected according to the following random model: every possible edge (including self loop) is present with probability p independent of every other edge.

There is an inequality, known as the *Markov inequality*, that gives an upper bound on the tail probability of a non-negative random variable:

Let X be a nonnegative random variable and $\epsilon > 0$, then

$$P(X \geq \epsilon) \leq \frac{\mathbb{E}[X]}{\epsilon}.$$

(You may recall this from the course *Probablity–The Science of Uncertainty and Data*.)

1. Using Markov inequality, obtain an upper bound on the probability that, for any given node, there are at least $n-1$ edges connected to this node in this graph.

n*p/(n-1) ✔ **Answer:** n*p/(n-1)

2. Now, find the exact probability that, for any given node, there are at least $n-1$ edges connected to this node in this graph.

n*p^(n-1)-(n-1)*p^n ✔ **Answer:** n*p^(n-1)*(1-p) + p^n

Solution:

1. Let X represent the random variable that is the number of edges connected to a node in this graph generation model. X is a binomial random variable with parameters n, p. The expected value of a binomial random variable is np. Therefore,

$$P(X \geq n-1) \leq \frac{np}{n-1}.$$

2. This probability is equal to

$$\begin{aligned} P(X \geq n-1) &= P_{\text{Binomial}(n,p)}(n-1) + P_{\text{Binomial}(n,p)}(n) \\ &= n \cdot p^{n-1}(1-p) + p^n. \end{aligned}$$

Submit You have used 1 of 3 attempts

Answers are displayed within the problem

A Matrix Equation – Preparation for Eigenvector Centrality

2/2 points (graded)

Let A be an adjacency matrix of size $n \times n$. Assume that the graph is an unweighted graph. Use the convention that $A_{ij} = 1$ indicates an edge going **from node i to node j**.

Let $\mathbf{x}$ be an all-ones vector of size $n \times 1$. What does entry i of the vector $A\mathbf{x}$ represent?

- (selected) Entry i of vector $A\mathbf{x}$ is equal to the out-degree of node i.
- Entry i of vector $A\mathbf{x}$ is equal to the in-degree of node i.

✔

Let $\mathbf{x}^T$ be an all-ones vector of size $1 \times n$. What does entry i of the vector $\mathbf{x}^T A$ represent?

- Entry i of vector $\mathbf{x}^T A$ is equal to the out-degree of node i.
- (selected) Entry i of vector $\mathbf{x}^T A$ is equal to the in-degree of node i.

✔

Solution:

This follows from the definition of A for an unweighted graph.

Submit You have used 1 of 1 attempt

Answers are displayed within the problem

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exact probability for any given node to have n-1 edges

question posted 2 months ago by **bhbenam**

The question seems simple, but the grader refused my answer. Does anyone have a problem with that?

This post is visible to everyone.

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3 responses

Romario-Garcia
2 months ago

Hi!

I don't know if you are considering that the question is asking for the probability of **at least** *n-1* edges. In my approach I end up with a somehow complicate sum of *n-1* terms, which I don't know how to solve. Could someone provide a tip on how to approach this question?

I don't suppose the answer is too complicated since the probability of having an edge is p and they are all independent.

posted 2 months ago by **bhbenam**

I have an idea of just looking at the self loop, but I only have one attempt remaining and the last ;-)

posted 2 months ago by **bhbenam**

I think you are right, and you can solve it by using the binomial distribution.

posted 2 months ago by **abc497662892**

Yes, binomial distribution is the way to go. You may want to think about how many ways to choose (n-1) edges from n edges and so forth.

posted 2 months ago by **lam_trinh** (Community TA)

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Dmitrii_Ivanov (Community TA)
2 months ago

Hi!

The question asks not for probability a node has exactly $(n-1)$ edges, but **at least** $(n-1)$ edges. Hope, this helps!

Hi, for a simple graph with n nodes, the highest degree is n-1 for a given node in the graph, right? Unless the graph allows multi edges between a pair of node and self loop in the graph, then the probability of a node with n edges is zero. Is my understanding correct?

posted 2 months ago by **apcshark**

you forgot self edge, which in this case is allowed

posted 2 months ago by **SunPenguin**

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yz2001zzx
2 months ago

After a long time (stuck in here for about 1hr).....I realized that there are just two meaningful PMF terms for P(x>=n-1), you just decompose P(x>=n-1) (remember the binomial is a discrete distribution) into two terms and sum them up and you will get the answer.

How do you handle "for any given node" in these two questions? Once you have an edge selected for one node, that means it's at the same time also selected for another node, right?

posted 2 months ago by **graftedlife**

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