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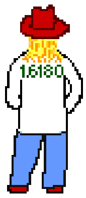
The Monty Hall Problem



For a review of basic concepts, see Introduction to Probability and Permutations and Combinations.

Let's Make a Deal!

Imagine that the set of Monty Hall's game show *Let's Make a Deal* has three closed doors. Behind one of these doors is a car; behind the other two are goats. The contestant does not know where the car is, but Monty Hall does.

The contestant picks a door and Monty opens one of the remaining doors, one he knows *doesn't* hide the car. If the contestant has already chosen the correct door, Monty is equally likely to open either of the two remaining doors.

After Monty has shown a goat behind the door that he opens, the contestant is always given the option to switch doors. **What is the probability of winning the car if she stays with her first choice? What if she decides to switch?**

One way to think about this problem is to consider the sample space, which Monty alters by opening one of the doors that has a goat behind it. In doing so, he effectively removes one of the two losing doors from the sample space.

We will assume that there is a winning door and that the two remaining doors, A and B, both have goats behind them. There are *three options:*

1. The contestant first chooses the door with the car behind it. She is then shown either door A or door B, which reveals a goat. If she changes her choice of doors,

she loses. If she stays with her original choice, she wins.

2. The contestant first chooses door A. She is then shown door B, which has a goat behind it. If she switches to the remaining door, she wins the car. Otherwise, she loses.

3. The contestant first chooses door B. She is then is shown door A, which has a goat behind it. If she switches to the remaining door, she wins the car. Otherwise, she loses.

Each of the above three options has a 1/3 probability of occurring, because the contestant is equally likely to begin by choosing any one of the three doors. In two of the above options, the contestant wins the car if she switches doors; in only one of the options does she win if she does not switch doors. When she switches, she wins the car twice (the number of favorable outcomes) out of three possible options (the sample space). Thus the probability of winning the car is 2/3 if she switches doors, which means that she should *always* switch doors - unless she wants to become a goatherd.

This result of 2/3 may seem counterintuitive to many of us because we may believe that the probability of winning the car should be 1/2 once Monty has shown that the car is not behind door A or door B. Many people reason that since there are two doors left, one of which must conceal the car, the probability of winning must be 1/2. This would mean that switching doors would not make a difference. As we've shown above through the three different options, however, this is not the case.

One way to convince yourself that 2/3 is the correct probability is to do a simulation with a friend. Have your friend impersonate Monty Hall and you be the contestant. Keep track of how often you win the car by switching doors and by not switching doors. Computer simulations are also available:

- The Monty Hall Page - Prof. Francois Bergeron
- The Let's Make a Deal Applet - R. Webster West (Java-capable browser needed)
- Monty Hall Dilemma - A. Bogomolny (Java-capable browser needed)
- Monte Hall's Paradox - Francois Bergeron (applet - Java-capable browser needed)
- The Monty Hall Problem - Steven R. Costenoble (Java-capable browser needed)
- A Mathematica Notebook file to download

If you're still not convinced that 2/3 is the correct probability, here are two more ways to think about the problem.

1. It seems to make sense that you have a 1/3 chance of picking the correct door. This means, however, that since the probabilities must add up to one - and the car has to be *somewhere* - you also have a 2/3 chance of *not* picking the correct door. In other words, you are more likely *not* to win the car than to win it.

   Imagine that Monty opens a door and shows that there's only a goat behind it. Consider that the car is more likely to be behind a door other than the one you choose. Monty has just shown that one of those two doors - which *together* have the greater probability of concealing the car - actually conceals a goat. This means that you should definitely switch doors, because the remaining door now has a 2/3 chance of concealing the car. Why? Well, your first choice still has a 1/3 probability of being the correct door, so the additional 2/3 probability must be somewhere else. Since you know that one of the two doors that previously shared the 2/3 probability does *not* hide the car, you should switch to the other door, which still has a 2/3 chance of concealing the car.

2. What if there were 1,000 doors? You would have a 1/1,000 chance of picking the correct door. If Monty opens 998 doors, all of them with goats behind them, the door that you chose first will still have a 1/1,000 chance of being the one that conceals the car, but the other remaining door will have a 999/1,000 probability of being the door that is concealing the car. Here switching sounds like a pretty good idea.

   For a a discussion of this puzzle translated into Spanish, see the SNARK math puzzles list, in particular Rodolfo Valeiras' messages of 21 July and 25 July 1997; also search for the keywords "tres puertas."

From the Dr. Math archives:

- Probability: Let's Make a Deal
- Let's Make a Deal Strikes Again
- Which Box Contains the Ring?
- Card Game Analogous to Monty Hall?

On the Web:

Let's Make a Deal - Francois Bergeron
Marilyn is tricked by a game show host - Herb Weiner

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