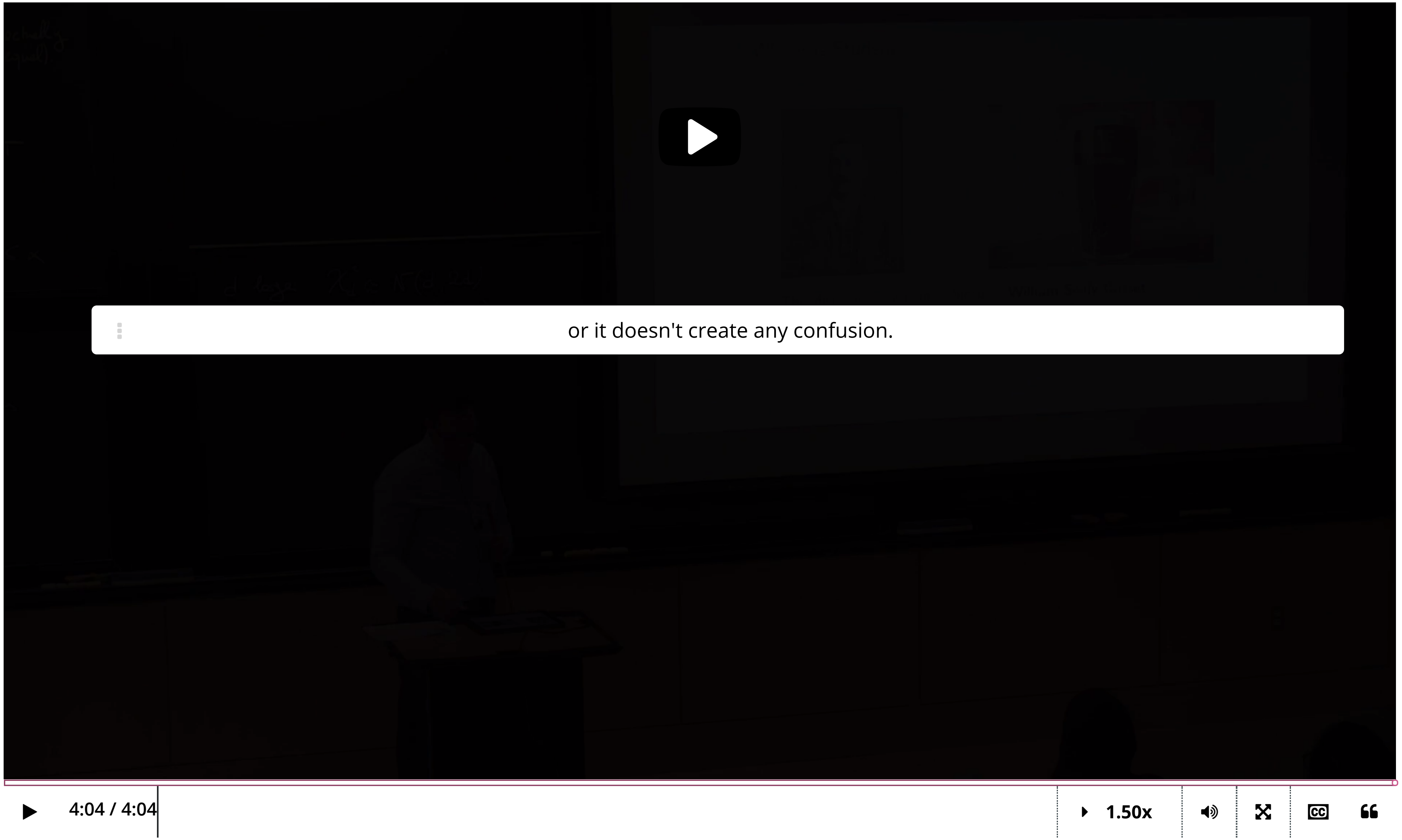
9. Student's T Distribution

Student's T Distribution: Definition



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Comparing Chi-Squared and Student's T Distribution

2/2 points (graded)

Consider the distribution χ_n^2 (χ-squared with n degrees of freedom). Let $f_n : \mathbb{R} \to \mathbb{R}$ denote the pdf of χ_n^2, and let A_n denote the maximizer of f_n (*i.e.*, the peak of the pdf of the distribution χ_n^2 is located at A_n).

What is $\lim_{n\to\infty} A_n$? (Answer heuristically, based on discussions in the lecture video about how the shape of the chi-squared distribution evolves with n.)

- 0
- 1
- ∞
- None of the above

✔

Consider the **Student's T Distribution** , which is defined to be the distribution of

$$T_n := \frac{Z}{\sqrt{V/n}}$$

where $Z \sim \mathcal{N}(0,1)$, $V \sim \chi_n^2$, and Z and V are independent. Let g_n denote the pdf of T_n, and let B_n denote the maximizer of g_n (*i.e.*, the peak of the pdf of the distribution T_n is located at B_n).

What is $\lim_{n\to\infty} B_n$? (*Hint:* What is the limit (in probability) of V/n?)

- 0
- 1
- ∞

None of the above

Solution:

The graph of the pdf of χ_n^2 in the slides shows that the peak of the distribution moves to the right as $n \to \infty$. Hence

$$\lim_{n\to\infty} A_n = \infty.$$

This is intuitive since we showed in a previous problem that $\mathbb{E}[X] = n$ if $X \sim \chi_n^2$.

As $n \to \infty$, the random variable V/n converges to 1 in probability. Hence, as $n \to \infty$,

$$T_n \xrightarrow[n\to\infty]{(d)} \mathcal{N}(0,1).$$

Since the distribution $\mathcal{N}(0,1)$ is peaked at the origin, this implies

$$\lim_{n\to\infty} B_n = 0.$$

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You have used 1 of 3 attempts

Answers are displayed within the problem

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