3. Parametrized curves and linear approximation

Part A due Oct 5, 2021 20:30 IST

Practice

5A-6(a)

10/10 points (graded)

Let c be the curve parametrized by $(x(t), y(t)) = (\sin 2t, \sin t)$ as t goes from 0 to π.

Compute the values of $(x(t), y(t))$ at $t = 0, \pi/4, \pi/2, 3\pi/4, \pi$.

t	$x(t)$	$y(t)$
0	0 ✔ **Answer:** 0	0 ✔ **Answer:** 0
$\pi/4$	1 ✔ **Answer:** 1	1/sqrt(2) ✔ **Answer:** 1/sqrt(2)
$\pi/2$	0 ✔ **Answer:** 0	1 ✔ **Answer:** 1
$3\pi/4$	-1 ✔ **Answer:** -1	1/sqrt(2) ✔ **Answer:** 1/sqrt(2)
π	0 ✔ **Answer:** 0	0 ✔ **Answer:** 0

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Solution:

$(x(0), y(0)) = \boxed{(0,0)}$,

$(x(\pi/4), y(\pi/4)) = \boxed{(1, \frac{1}{\sqrt{2}})}$,

$(x(\pi/2), y(\pi/2)) = \boxed{(0,1)}$,

$(x(3\pi/4), y(3\pi/4)) = \boxed{(-1, \frac{1}{\sqrt{2}})}$,

$(x(\pi), y(\pi)) = \boxed{(0,0)}$.

Submit You have used 1 of 5 attempts

Answers are displayed within the problem

5A-6(b)

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10/10 points (graded)
Let c be the curve parametrized by $(x(t), y(t)) = (\sin 2t, \sin t)$ as t goes from 0 to π.

Compute the velocity $\langle x'(t), y'(t)\rangle$ at $t = 0, \pi/4, \pi/2, 3\pi/4, \pi$.

t	$x'(t)$		$y'(t)$	
0	2	✔ Answer: 2	1	✔ Answer: 1
$\pi/4$	0	✔ Answer: 0	1/sqrt(2)	✔ Answer: 1/sqrt(2)
$\pi/2$	-2	✔ Answer: -2	0	✔ Answer: 0
$3\pi/4$	0	✔ Answer: 0	-1/sqrt(2)	✔ Answer: -1/sqrt(2)
π	2	✔ Answer: 2	-1	✔ Answer: -1

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Solution:

We have $\langle x'(t), y'(t)\rangle = \langle 2\cos 2t, \cos t\rangle$.

So $\langle x'(0), y'(0)\rangle = \boxed{\langle 2, 1\rangle}$,

$\langle x'(\pi/4), y'(\pi/4)\rangle = \boxed{\langle 0, \frac{1}{\sqrt{2}}\rangle}$,

$\langle x'(\pi/2), y'(\pi/2)\rangle = \boxed{\langle -2, 0\rangle}$,

$\langle x'(3\pi/4), y'(3\pi/4)\rangle = \boxed{\langle 0, -\frac{1}{\sqrt{2}}\rangle}$,

$\langle x'(\pi), y'(\pi)\rangle = \boxed{\langle 2, -1\rangle}$.

Submit You have used 1 of 5 attempts

Answers are displayed within the problem

5A-6(c)

1/1 point (graded)
Using the information from the first two parts, sketch the parametrized curve. The sketch doesn't have to be beautiful, but it should incorporate the information from both (a) and (b) above.

- Label the points on the curve at $t = 0, \pi/4, \pi/2,$ and $3\pi/4$ using the **Positions** point tool.
- Draw the velocity vectors for times $t = 0, \pi/4, \pi/2, 3\pi/4$ and π starting from appropriate position using the **Velocities** arrow tool. (The relative magnitude need not be correct, but the directi

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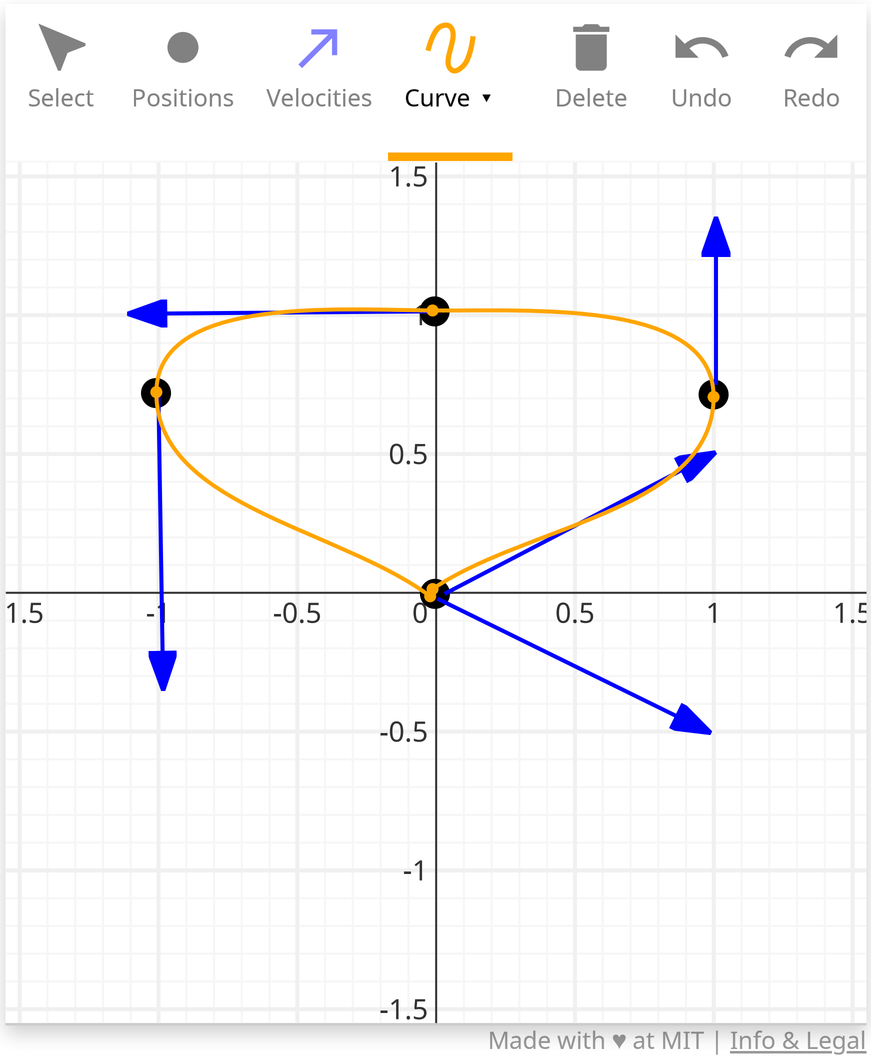
- Sketch the trajectory of the parametric curve using the **Curve** spline or freeform tool.



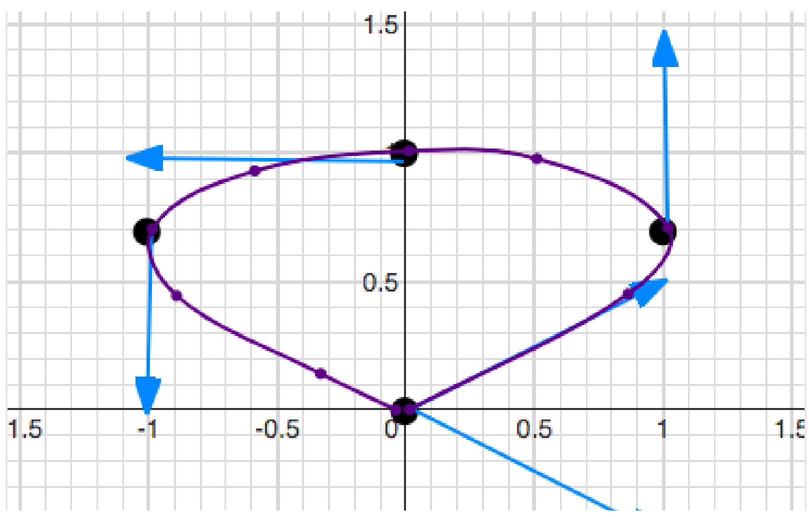
Answer: See solution.

Well done

Solution:

See the figure below. We also briefly indicate some important features of this sketch (this wasn't necessary to turn in, but might be helpful):

- All points are on the upper half plane.
- The lowest points on the curve are $(x(0), y(0))$, $(x(\pi), y(\pi))$, which coincide (they're both at the origin). However, their corresponding tangent vectors $(x'(0), y'(0))$, $(x'(\pi), y'(\pi))$ don't coincide. This means that there is a sort of “corner" where the curve is supposed to close up.
- The highest point is $(x(\pi/2), y(\pi/2))$. Its tangent vector is (necessarily) horizontal and points left.
- The rightmost and leftmost points are $(x(\pi/2), y(\pi/2))$, $(x(3\pi/4), y(3\pi/4))$. They are symmetric about the y axis. Their tangent vectors are (necessarily) vertical and point up and down, respectively.



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