8.2.1 Models of time-dependent phenomena

MO2.4

The mathematical models used to describe time-dependent phenomena in Nature and social systems are frequently **differential equations**, i.e., relationships between functions *and their derivatives*.

Where do models involving derivatives come from? The most important example is perhaps $\vec{F} = m\vec{a}$, Newton's second law, that we have already encountered. Here the basic quantity is the position $\vec{x}\,(t)$ of a particle or an object as a function of time t. In general, $\vec{x}\,(t) = (x_1\,(t)\,, x_2\,(t)\,, x_3\,(t))$ is a vector in 3 dimensions for each time t, but there are also cases where we only consider two, or one component of $\vec{x}\,(t)$ at a time (and we might still denote those by $x\,(t)$ when convenient). Then the velocity vector $\vec{v}\,(t)$ is the time derivative of $\vec{x}\,(t)$, and the acceleration vector $\vec{a}\,(t)$ is the time derivative of $\vec{v}\,(t)$:

$$\vec{v}\,(t) = \frac{\mathrm{d}\vec{x}}{\mathrm{d}t}(t)\,, \qquad \vec{a}\,(t) = \frac{\mathrm{d}\vec{v}}{\mathrm{d}t}(t) = \frac{\mathrm{d}^2\vec{x}}{\mathrm{d}t^2}(t)\,. \tag{8.4}$$

The force vector $\vec{F}$ acting on the object at $\vec{x}\,(t)$ is either given as a function of time, or is itself a function of $\vec{x}$ and/or $\vec{v}$. Either way, “$F = ma$" becomes a differential equation.

More generally, mathematical models involving derivatives range from trustworthy elegant theories (like Newton's laws of gravity; or Euler's equation of fluid

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