MATHEMATICS

Expectation time - General birth and death process

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I study continuous time Markov chain and more specifically birth and death processes. I am trying to understand how to calculate the expectation time it takes to start from a state i to a state $i+1$. I am using the book "Introduction to probability models" by Sheldon Ross but I have trouble understanding it and I did not find alternative notes on my problem so I am asking here.

We consider a general birth and death process with birth rate $\{\lambda_n\}$ and death rates $\{\mu_n\}$, where $\mu_0 = 0$ and we denote T_i as the time it takes starting from state i to enter state $i+1$. Since the times of death and births are exponential, we already know that $E[T_0] = \frac{1}{\lambda_0}$.

Then, the book follows this logic: For $i > 0$, we condition whether the first transition takes the process into state $i-1$ or $i+1$, i.e we let:

$$I_i = \begin{cases} 0, \text{ if the first transition from } i \text{ is } i+1 \\ 1, \text{ if the first transition from } i \text{ is } i-1 \end{cases}$$

And therefore we note that:

$E[T_i \mid I_i = 1] = \frac{1}{\lambda_i+\mu_i}$,

$E[T_i \mid I_i = 0] = \frac{1}{\lambda_i+\mu_i} + E[T_{i-1}] + E[T_i]$

(Because independently of whether the first transition is a birth or a death, the time until it occurs is the minimum and thus ~Exp$(\lambda_i + \mu_i)$.

And here is my problem:

To compute $E[T_i]$ I simply apply the following formula:

$$E[T_i] = E[T_i \mid I_i = 1] \cdot P(I_i = 1) + E[T_i \mid I_i = 0] \cdot P(I_i = 0)$$

and after working it out, I find:

$$E[T_i] = \frac{1}{\lambda_i+\mu_i} + \frac{\mu_i}{\lambda_i(\lambda_i+\mu_i)} + \frac{\mu_i}{\lambda_i}E[T_{i-1}]$$

But the book gives:

"Since the probability that the first transition is a birth is $\frac{\lambda_i}{(\lambda_i+\mu_i)}$ we have :"

$E[T_i] = \frac{1}{\lambda_i+\mu_i} + \frac{\mu_i}{(\lambda_i+\mu_i)}(E[T_{i-1}] + E[T_i])$

$$\implies E[T_i] = \frac{1}{\lambda_i} + \frac{\mu_i}{\lambda_i} E[T_{i-1}]$$

Obviously the use of my formula is not appropriate here and I added useless terms, but I don't see which ones and the reasoning behind it. Understanding these kind of steps would help me a lot to better grasp this topic, so any help or idea is really appreciated, thanks!

stochastic-processes expectation markov-process birth-death-process

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$$\frac{1}{\lambda_i + \mu_i} + \frac{\mu_i}{\lambda_i(\lambda_i + \mu_i)} = \frac{1}{\lambda_i}$$

hence the two recursions are equivalent. – Did Jul 16 '17 at 10:48

I feel a bit foolish I did not see this mini step.. Thanks for your help! – user386721 Jul 16 '17 at 12:53