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2. Find the blast radius and energy

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Problem Set B due Sep 13, 2021 20:30 IST Completed

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Problem 2(a)

2/2 points (graded)

When the atomic bomb was developed, there was significant photographic data collected from test blasts. There is a story often told when teaching dimensional analysis about G. I. Taylor who was able to compute the energy of test blasts based on (then) declassified photographs. We will walk you through the thought process for how this was accomplished and practice some methods learned in this unit.

The photographic data consisted of time stamps and a visual of the blast at each time, which could be used to determine the radius of the blast.

Below we give you a table of such data. (Note all data is fabricated, but has the correct orders of magnitude.)

time t (milliseconds)	**radius** R (meters)
36.2	156
49.8	175
64.6	214
98.3	228

We are interested in describing the radius R in meters as a function of time t in **seconds**.

Use least squares approximation to fit the data to a line: $R = mt + b$. (Answer should involve no more than three significant figures. Entry to the nearest integer will be graded correctly.)

$m =$ 1168.4869 ✔

$b =$ 120.540905 ✔

Submit You have used 2 of 5 attempts

Problem 2(b)

4.0/4 points (graded)

There are four physical variables at play in this experiment. The radius of the blast, R, the time t, the energy released E, and the air density ρ.

Determine the dimensions of each variable in terms of length L, time T, and mass M,

Dimension of R: L ✔ **Answer:** L

Dimension of t: T ✔ **Answer:** T

Dimension of E: M*L^2*T^(-2) ✔ **Answer:** M*L^2/T^2

Dimension of ρ: M*L^(-3) ✔ **Answer:** M/L^3

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Solution:

The radius R has dimension of length L.

Time t has dimension of time T.

Energy E has dimension of ML^2/T^2, (Note that we've computed this before.)

Air density is a measure of mass per volume, which has dimension M/L^3.

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Answers are displayed within the problem

Problem 2(c)

3/3 points (graded)
Use dimensional analysis to find a relationship for the radius R in terms of ρ, E, and t by balancing dimension.

$$R \propto E^a \rho^b t^c$$

(Note that $\propto$ means that R is proportional to this quantity. There is constant proportionality that cannot be determined from dimensional analysis alone.)

$a =$ 1/5 ✔ **Answer:** 1/5

$b =$ -1/5 ✔ **Answer:** -1/5

$c =$ 2/5 ✔ **Answer:** 2/5

Solution:

We want to multiply E, ρ and t in such a way that the dimension has length.

Writing out the dimensions, this means we are looking for numbers a, b, and c so that

$$[L] = \left[\frac{ML^2}{T^2}\right]^a \left[\frac{M}{L^3}\right]^b [T]^c$$

We know that we must cancel all mass dimensions. This tells us that $b = -a$. Similarly we must cancel all dimensions of time. This tells us that $c = 2a$.

Rewriting in terms of a, this tells us

$$[L] = \left[\frac{ML^2}{T^2}\right]^a \left[\frac{M}{L^3}\right]^{-a} [T]^{2a} = \left[L^{5a}\right]$$

This tells us that $a = 1/5$, $b = -1/5$, and $c = 2/5$.

Submit You have used 2 of 3 attempts

Answers are displayed within the problem

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Problem 2(d)

3/3 points (graded)

Based on the equation you found above, the linear fit obtained in part (a) is likely not a very good model. Instead, transform the data to obtain a linear fit that determines a power law.

$$R \approx wt^p$$

Find the power and constant multiple. (Answer to three significant figures.)

Here is the data again.

time t (milliseconds)	**radius** R (meters)
36.2	156
49.8	175
64.6	214
98.3	228

(Enter both as decimals (or integers) with 3 significant figures of accuracy only. However, keep 4 decimal digits of data accuracy in your transformed data before transforming back.)

(Enter accurate to 3 decimal places.)

Power $p =$ 0.4005213 ✔ **Answer:** 0.401

(Enter the nearest integer.)

Constant multiple $w =$ 597 ✔ **Answer:** 597

Write the equation for R determined by your data.

$R \approx$ 597*t^(2/5) ✔ **Answer:** 597*t^(0.401)

$597 \cdot t^{\frac{2}{5}}$

Solution:

To transform the data, we take the natural log of both sides. Then we perform the linear regression. The transformed data is

$\ln(t)$	$\ln(R)$
-3.3187	5.0499
-2.9997	5.1648
-2.7395	5.3660
-2.3197	5.4293

The linear fit we find is $\ln R \approx 0.4010 \ln t + 6.3917$. Taking the exponential of both sides, to get back to a power law for our original expression we get

$$R \approx e^{0.4010 \ln t + 6.3917} = e^{6.39174} e^{0.4010 \ln t} = 597 t^{0.401}$$

Note that we were looking for $p = 2/5 = 0.4$, and we computed $p = 0.401$ from our data, which is quite close.

Submit You have used 7 of 7 attempts

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Answers are displayed within the problem

Problem 2(e)

2/2 points (graded)

Based on dimensional analysis we found a formula for $R = kE^a\rho^b t^c$, where k is a constant determined by physical properties. If you knew k, you could find the energy E. (In the folklore, G. I. Taylor picked up some sand from the blast, watched it drift in the wind and intuited the value of k. In reality, he used his intuition from dimensional analysis and physical insight to carry out significant mathematical modeling.)

For us, we will assume that $k = 1$. At sea level air has density 1.225 kg/m^3.

Determine the energy E of the blast in the test data from your computed constant above.

$E \approx$ 92818190000000 Joules ✔ **Answer:** (10^(13), 10^(14))

In the US, the average home uses about $400,000,000$ Joules per year.

How long (in years) could the energy produced by the blast power a single home?

232045.5 years ✔ **Answer:** (25000, 250000)

Solution:

Dimensional analysis tells us that

$$R \approx k\left(\frac{E}{\rho}\right)^{1/5} t^{2/5} = 1 \cdot \left(\frac{E}{1.225}\right)^{1/5} t^{2/5}.$$

Our data fitting told us that $R \approx 597t^{2/5}$. Therefore $E \approx 597^5\,(1.225) = 9.29 \times 10^{13}$.

Dividing the total energy by the energy used by a home per year gives us

$$\frac{9.29 \times 10^{13} \text{ Joules}}{4 \times 10^8 \text{ Joules per year}} = 232,245 \text{ years}$$

To three significant figures this is 232,000 years!

Submit You have used 1 of 10 attempts

Answers are displayed within the problem

For more information about the problem above, you may be interested in the following reference. This reference is behind a paywall, but you may be able to access it at a local public or university library.

Michael A.B. Deakin (2011) G.I. Taylor and the Trinity test, International Journal of Mathematical Education in Science and Technology, 42:8, 1069-1079, DOI: 10.1080/0020739X.2011.562324

2. Find the blast radius and energy

Topic: Unit 3: Optimization / 2. Find the blast radius and energy

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Least Squares in Python PS3B 2(a) & 2(d)		3
Great section!	This is really cool, one of the best sections in the courses i took! Great!!! I like to get more of this kind.	2
Problem 2(e): Density dimension?	Probably I am wrong, but I think we have to adapt the dimension of density \rho from kg/dm^3 to kg/m^3.	4
Question 2(b) - What is meant by "dimensions" here?	We may have covered this in the previous lectures but I don't think I recall going over "dimensions". What is meant by "dimension...	3
Problem 2(e)	Since we do not have the time, is there something we are supposed to assume to get the energy? I am not sure I understand ho...	3
2(a) and 2(d)	Hello, On 2(a), I get b right but m wrong On 2(d), I get p right but w wrong Using the same model, assuming I did not make a mist...	6
[Staff] 2(e) solution typo		2
[STAFF] Problem 2b is not shown	I see this error message instead of Problem 2b: "Could not format HTML for problem. Contact course staff in the discussion foru...	3
2b	What can I do to see (and answer) 2b?	3
[STAFF] 2(d) grader problem	I got 2(d) p and w correct but R isn't, could staad please check on this? I got only one attempt left	6
[STAFF] Question 2(b) HTML won't show	When I wrote the answers for question b, I can't submit. but when I refreshed the page it says "Could not format HTML for proble...	2
2C proportionality constant	Are we to assume that the proportionality constant is unitless for this problem?	2
[Staff] 2(d) milliseconds to seconds	Maybe it is worth mentioning again in 2(d) that milliseconds need to be converted to seconds. I think many people (including me...	3
[Staff] 2(a) and 2(d): Is the grader correct?		

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