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2. Matrix exponential

LTI Consumer (External resource) (1.0 / 1.0 points)

Using matrix exponential (expm)

Recall that for a linear system

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x}$$

with initial conditions

$$\mathbf{x}(0) = \mathbf{x}_0$$

we can write the solution in terms of the matrix exponential

$$\mathbf{x}(t) = e^{\mathbf{A}t}\mathbf{x}_0$$

This is often impossible to write down by hand, but Matlab has its function which can calculate matrix exponentials for us. If we have an $N \times N$ matrix $\mathbf{A}$, we can define $\mathbf{B} = e^{\mathbf{A}}$ in MATLAB by

```
B = expm(A);
```

Let's return to the example of three connected tanks, where tank 1 has a leak, from recitation 5. Again we suppose that tank 1 contains some volume of one fluid, and tank 3 contains the same volume of another fluid. These two tanks are connected to the mixing tank (tank 2) by pipes of equal width. Tank 2 is initially empty. You found that all of the eigenvalues are negative. This means that h_1, h_2 and h_3 always decay to zero.

In the next problem, you will show that you can create any mixture in the second tank, but you stop the apparatus from running at the right time to make sure you maximize the yield.

1. Use the template below to plot the solution for the system

$$\dot{\mathbf{x}} = \begin{bmatrix} -(a+\mu) & a & 0 \\ a & -2a & a \\ 0 & a & -a \end{bmatrix} \mathbf{x}$$

with $a = 1$, $\mu = 0.5$ and the initial conditions:

$$\mathbf{x}_0 = \begin{bmatrix} 2/3 \\ 0 \\ 1/3 \end{bmatrix}$$

You should find the analytic solution by calculating matrix exponentials.

2. Estimate the time at which we should stop the apparatus running so as to maximize the yield in the second tank. You should search in the range

$t \in [0.5, 1.5]$.

Your Script

Save Reset MATLAB Documentation (https://www.mathworks.com/help/)

```
%We've set the values of a and mu as
%specified in the problem for you.
a = 1.0;
mu = 0.5;
%Enter the matrix A as defined above,
% the initial value vector x0, and create
% a row vector t of 400 equally spaced
%time values as t ranges from 0.5 to 1.5;
A = [-a-mu,a,0;a,-2*a,a;0,a,-a];
x0 = [2/3;0;1/3];
t = linspace(0.5,1.5,400);
%At each time t, define x in terms of the exponenetial matrix expm(A), t, and the
for m=1:length(t)
    x(:,m) = expm(A*t(m))*x0;
end
%Let h1 be the height of fluid in tank 1.
%Let h2 be the height of fluid in tank 2.
%Let h3 be the height of fluid in tank 3.
%Define h1, h2, and h3 in terms of x.
h1 = x(1,:);
h2 = x(2,:);
h3 = x(3,:);
%Plot h1, h2 and h3 (do not edit code for plotting)
plot(t,h1,t,h2,t,h3,'LineWidth',2)
set(gca,'fontsize',18)
legend('h1','h2','h3')
xlabel('Time')
ylabel('Volume')
title('Time series')
%Determine the time tmax when the height in tank 2 is a maximum.
%Hint: look up the function max() in the matlab documentation.

[vmax,tmaxi] = max(h2);
tmax = t(tmaxi);
```

Run Script ()

Assessment: Correct

Submit ()

Check if A is correctly defined

Check if x0 is correctly defined

Check if t is correctly defined

Check if h1 is correct

Check if h2 is correct

Check if h3 is correct

Time when h2 is maximized found

Output

2. Matrix exponential

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tmax
I did this in a bit of a convoluted way but its rejecting my tmax while everything else is correct and ey... 9

Time array to use not specified
Just wanted to note that the question does not explicitly state any format to use for time discretisatio... 2

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