8.2.5 Computing the Inverse of A via Gauss-Jordan Elimination, Alternative

Week 8 due Nov 26, 2023 15:12 IST

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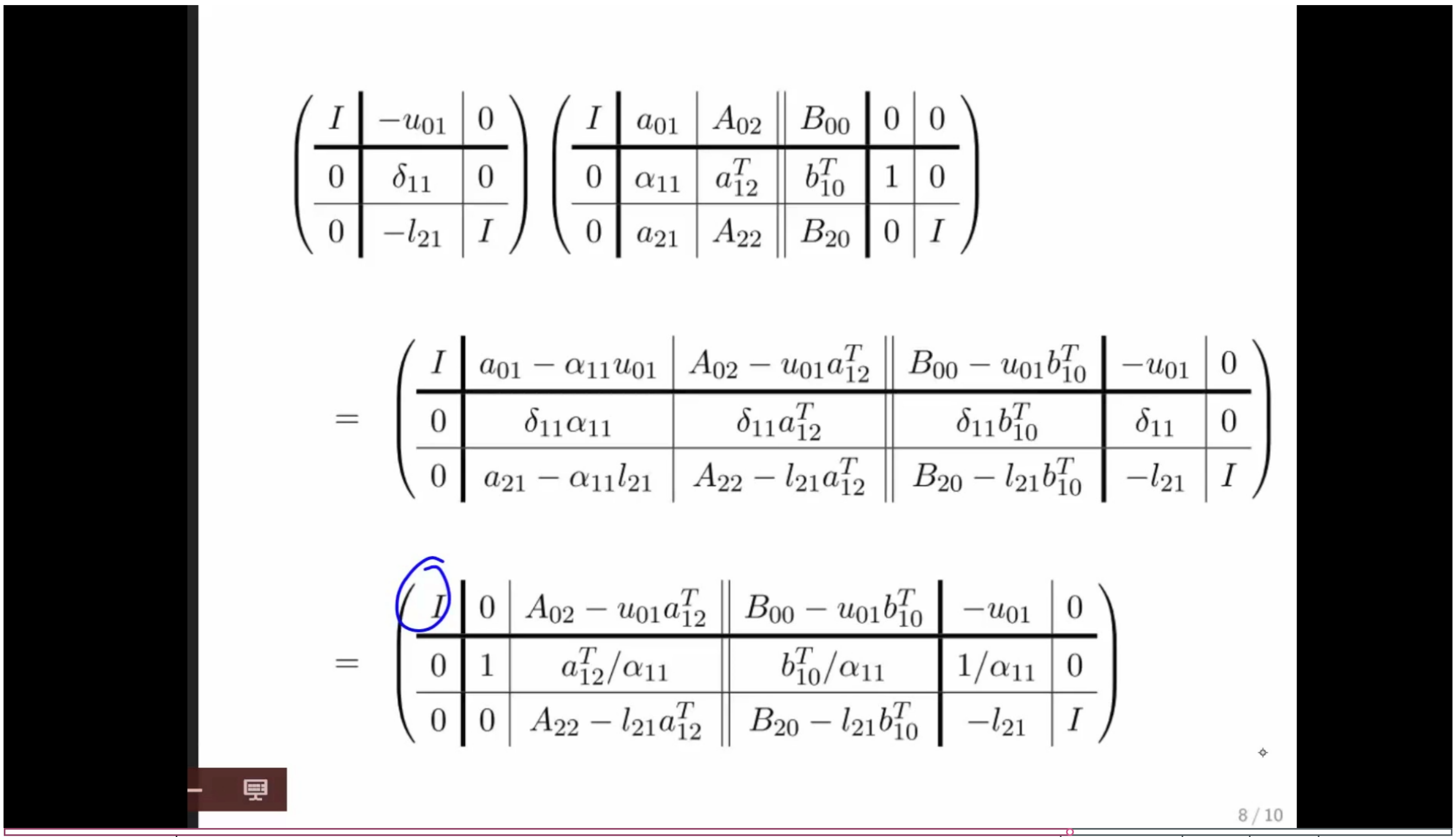

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Reading Assignment

0 points possible (ungraded)
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Homework 8.2.5.1

18/18 points (graded)

- Determine $\delta_{0,0}$, $\lambda_{1,0}$, $\lambda_{2,0}$ so that

$$\left(\begin{array}{c|cc} \delta_{0,0} & 0 & 0 \\ \lambda_{1,0} & 1 & 0 \\ \lambda_{2,0} & 0 & 1 \end{array}\right)\left(\begin{array}{c|cc|ccc} -1 & -4 & -2 & 1 & 0 & 0 \\ \hline 2 & 6 & 2 & 0 & 1 & 0 \\ -1 & 0 & 3 & 0 & 0 & 1 \end{array}\right) = \left(\begin{array}{c|cc|ccc} 1 & 4 & 2 & -1 & 0 & 0 \\ \hline 0 & -2 & -2 & 2 & 1 & 0 \\ 0 & 4 & 5 & -1 & 0 & 1 \end{array}\right)$$

$\delta_{0,0} =$ -1 ✔ Answer: -1

$\lambda_{1,0} =$ 2 ✔ Answer: 2

$\lambda_{2,0} =$ -1 ✔ Answer: -1

- Determine $\upsilon_{0,1}$, $\delta_{1,1}$, and $\lambda_{2,1}$ so that

$$\left(\begin{array}{c|c|c} 1 & \upsilon_{0,1} & 0 \\ \hline 0 & \delta_{1,1} & 0 \\ \hline 0 & \lambda_{2,1} & 1 \end{array}\right)\left(\begin{array}{c|cc|ccc} 1 & 4 & 2 & -1 & 0 & 0 \\ \hline 0 & -2 & -2 & 2 & 1 & 0 \\ \hline 0 & 4 & 5 & -1 & 0 & 1 \end{array}\right) = \left(\begin{array}{cc|c|cc|c} 1 & 0 & -2 & 3 & 2 & 0 \\ \hline 0 & 1 & 1 & -1 & -\frac{1}{2} & 0 \\ \hline 0 & 0 & 1 & 3 & 2 & 1 \end{array}\right)$$

$\upsilon_{0,1} =$ 2 ✔ Answer: 2

$\delta_{1,1} =$ -1/2 ✔ Answer: -1/2

$\lambda_{2,1} =$ 2 ✔ Answer: 2

- Determine $\upsilon_{0,2}$, $\upsilon_{0,2}$, and $\delta_{2,2}$ so that

$$\left(\begin{array}{cc|c} 1 & 0 & \upsilon_{0,2} \\ 0 & 1 & \upsilon_{1,2} \\ \hline 0 & 0 & \delta_{2,2} \end{array}\right)\left(\begin{array}{cc|c|cc|c} 1 & 0 & -2 & 3 & 2 & 0 \\ 0 & 1 & 1 & -1 & -\frac{1}{2} & 0 \\ \hline 0 & 0 & 1 & 3 & 2 & 1 \end{array}\right) = \left(\begin{array}{cc|c|cc|c} 1 & 0 & 0 & 9 & 6 & 2 \\ 0 & 1 & 0 & -4 & -\frac{5}{2} & -1 \\ \hline 0 & 0 & 1 & 3 & 2 & 1 \end{array}\right)$$

$\upsilon_{0,2} =$ 2 ✔ Answer: 2

$\upsilon_{1,2} =$ -1 ✔ Answer: -1

$\delta_{2,2} =$ 1 ✔ Answer: 1

1	0	0
✔ Answer: 1	✔ Answer: 0	✔ Answer: 0

Calculator

$$\begin{pmatrix} -1 & -4 & -2 \\ 2 & 6 & 2 \\ -1 & 0 & 3 \end{pmatrix} \begin{pmatrix} 9 & 6 & 2 \\ -4 & -\frac{5}{2} & -1 \\ 3 & 2 & 1 \end{pmatrix} =$$

0 ✔ Answer: 0	1 ✔ Answer: 1	0 ✔ Answer: 0
0 ✔ Answer: 0	0 ✔ Answer: 0	1 ✔ Answer: 1

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Homework 8.2.5.2

1/1 point (graded)

Assume below that all matrices and vectors are partitioned “conformally" so that the operations make sense.

$$\left(\begin{array}{c|c|c} I & -u_{01} & 0 \\ \hline 0 & \delta_{11} & 0 \\ \hline 0 & -l_{21} & I \end{array}\right) \left(\begin{array}{c|c|c||c|c|c} I & a_{01} & A_{02} & B_{00} & 0 & 0 \\ \hline 0 & \alpha_{11} & a_{12}^T & b_{10}^T & 1 & 0 \\ \hline 0 & a_{21} & A_{22} & B_{20} & 0 & I \end{array}\right)$$

$$= \left(\begin{array}{c|c|c||c|c|c} I & a_{01}-\alpha_{11}u_{01} & A_{02}-u_{01}a_{12}^T & B_{00}-u_{01}b_{10}^T & -u_{01} & 0 \\ \hline 0 & \delta_{11}\alpha_{11} & \delta_{11}a_{12}^T & \delta_{11}b_{10}^T & \delta_{11} & 0 \\ \hline 0 & a_{21}-\alpha_{11}l_{21} & A_{22}-l_{21}a_{12}^T & B_{20}-l_{21}b_{10}^T & -l_{21} & I \end{array}\right)$$

Always ✔ Answer: Always

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Homework 8.2.5.3

1/1 point (graded)

Assume below that all matrices and vectors are partitioned “conformally" so that the operations make sense and that $\alpha_{11} \neq 0$.

Choose

- $u_{01} := a_{01}/\alpha_{11}$; and
- $\delta_{11} := 1/\alpha_{11}$; and
- $l_{21} := a_{21}/\alpha_{11}$.

Consider the following expression:

$$\left(\begin{array}{c|c|c} I & -u_{01} & 0 \\ \hline 0 & \delta_{11} & 0 \\ \hline 0 & -l_{21} & I \end{array}\right) \left(\begin{array}{c|c|c||c|c|c} I & a_{01} & A_{02} & B_{00} & 0 & 0 \\ \hline 0 & \alpha_{11} & a_{12}^T & b_{10}^T & 1 & 0 \\ \hline 0 & a_{21} & A_{22} & B_{20} & 0 & I \end{array}\right)$$

Calculator

$$
= \left(\begin{array}{c|c|c||c|c|c}
I & 0 & A_{02} - u_{01} a_{12}^T & B_{00} - u_{01} b_{10}^T & -u_{01} & 0 \\ \hline
0 & 1 & a_{12}^T / \alpha_{11} & b_{10}^T / \alpha_{11} & 1/\alpha_{11} & 0 \\ \hline
0 & 0 & A_{22} - l_{21} a_{12}^T & B_{20} - l_{21} b_{10}^T & -l_{21} & I
\end{array}\right)
$$

Always ✔ Answer: Always

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The above observations justify the following alternative "One Sweep" algorithm for Gauss-Jordan elimination" for inverting a matrix.

Algorithm: $[B] := \text{GJ\_INVERSE\_ALT}(A, B)$

Partition $A \rightarrow \left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right)$, $B \rightarrow \left(\begin{array}{c|c} B_{TL} & B_{TR} \\ \hline B_{BL} & B_{BR} \end{array}\right)$

where A_{TL} is 0×0, B_{TL} is 0×0

while $m(A_{TL}) < m(A)$ **do**

Repartition

$$
\left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c|c|c} A_{00} & a_{01} & A_{02} \\ \hline a_{10}^T & \alpha_{11} & a_{12}^T \\ \hline A_{20} & a_{21} & A_{22} \end{array}\right), \left(\begin{array}{c|c} B_{TL} & B_{TR} \\ \hline B_{BL} & B_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c|c|c} B_{00} & b_{01} & B_{02} \\ \hline b_{10}^T & \beta_{11} & b_{12}^T \\ \hline B_{20} & b_{21} & B_{22} \end{array}\right)
$$

where α_{11} is 1×1, β_{11} is 1×1

$$
\begin{array}{c|c|c}
 & a_{01} := a_{01}/\alpha_{11} & A_{02} := A_{02} - a_{01} a_{12}^T \\ \hline
 & & \\ \hline
 & a_{21} := a_{21}/\alpha_{11} & A_{22} := A_{22} - a_{21} a_{12}^T
\end{array}
\qquad
\begin{array}{c|c|c}
B_{00} := B_{00} - a_{01} b_{10}^T & b_{01} := -a_{01} & \\ \hline
 & & \\ \hline
B_{20} := B_{20} - a_{21} b_{10}^T & b_{21} := -a_{21} &
\end{array}
$$

(Note: above a_{01} and a_{21} must be updated before the operations to their right.)

$$
\begin{array}{c|c|c}
 & a_{01} := 0 & \\ \hline
 & \alpha_{11} := 1 & a_{12}^T := a_{12}^T/\alpha_{11} \\ \hline
 & a_{21} := 0 &
\end{array}
\qquad
\begin{array}{c|c|c}
 & & \\ \hline
b_{10}^T := b_{10}^T/\alpha_{11} & \beta_{11} = 1/\alpha_{11} & \\ \hline
 & &
\end{array}
$$

(Note: above α_{11} must be updated last.)

Continue with

$$
\left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c|c|c} A_{00} & a_{01} & A_{02} \\ \hline a_{10}^T & \alpha_{11} & a_{12}^T \\ \hline A_{20} & a_{21} & A_{22} \end{array}\right), \left(\begin{array}{c|c} B_{TL} & B_{TR} \\ \hline B_{BL} & B_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c|c|c} B_{00} & b_{01} & B_{02} \\ \hline b_{10}^T & \beta_{11} & b_{12}^T \\ \hline B_{20} & b_{21} & B_{22} \end{array}\right)
$$

endwhile

Homework 8.2.5.4

1/1 point (graded)

Implement the above algorithm yielding the function

- `[ B_out ] = GJ_Inverse_alt_unb( A, B )`. Assume that it is called as

```
Ainv = GJ_Inverse_alt_unb( A, B )
```

Matrices A and B must be square and of the same size.

Check that it computes correctly with the script

Calculator

- `test_GJ_Inverse_alt_unb.m` (In `LAFF-2.0xM/Programming/Week08/`).

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✔

Our implementation: `GJ_Inverse_alt_unb.m`.

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Challenge 8.2.5.5

1/1 point (graded)
If you are very careful, you can overwrite matrix A with its inverse without requiring the matrix B.

Modify the algorithm in the above figure so that it overwrites A with its inverse without the use of matrix B yielding the function

- `[ A_out ] = GJ_Inverse_inplace_unb( A ).`

Check that it computes correctly with the script

- `test_GJ_Inverse_inplace_unb.m` (In `LAFF-2.0xM/Programming/Week06/`).

☑ Done/Skip

✔

The modified algorithm:

Algorithm: $[A] := \mathrm{GJ\_INVERSE\_INPLACE}(A)$

Partition $A \rightarrow \left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right)$

where A_{TL} is 0×0

while $m(A_{TL}) < m(A)$ **do**

Repartition

$$\left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c|c|c} A_{00} & a_{01} & A_{02} \\ \hline a_{10}^T & \alpha_{11} & a_{12}^T \\ \hline A_{20} & a_{21} & A_{22} \end{array}\right)$$

where α_{11} is 1×1

$$\left(\begin{array}{c|c|c} & a_{01} := a_{01}/\alpha_{11} & A_{02} := A_{02} - a_{01}a_{12}^T \\ \hline & & \\ \hline & a_{21} := a_{21}/\alpha_{11} & A_{22} := A_{22} - a_{21}a_{12}^T \end{array}\right) \qquad \left(\begin{array}{c|c|c} A_{00} := A_{00} - a_{01}a_{10}^T & a_{01} := -a_{01} & \\ \hline & & \\ \hline A_{20} := A_{20} - a_{21}a_{10}^T & a_{21} := -a_{21} & \end{array}\right)$$

(Note: above a_{01} and a_{21} must be updated before the operations to their right.)

$$\left(\begin{array}{c|c|c} & & \\ \hline & & a_{12}^T := a_{12}^T/\alpha_{11} \\ \hline & & \end{array}\right) \qquad \left(\begin{array}{c|c|c} & & \\ \hline a_{10}^T := a_{10}^T/\alpha_{11} & \alpha_{11} = 1/\alpha_{11} & \\ \hline & & \end{array}\right)$$

(Note: above α_{11} must be updated last.)

Continue with

$$\left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c|c|c} A_{00} & a_{01} & A_{02} \\ \hline a_{10}^T & \alpha_{11} & a_{12}^T \\ \hline A_{20} & a_{21} & A_{22} \end{array}\right)$$

endwhile

Calculator

Our implementation: `GJ_Inverse_inplace_unb.m`.

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