6. Types of critical points

Explore

Types of critical points

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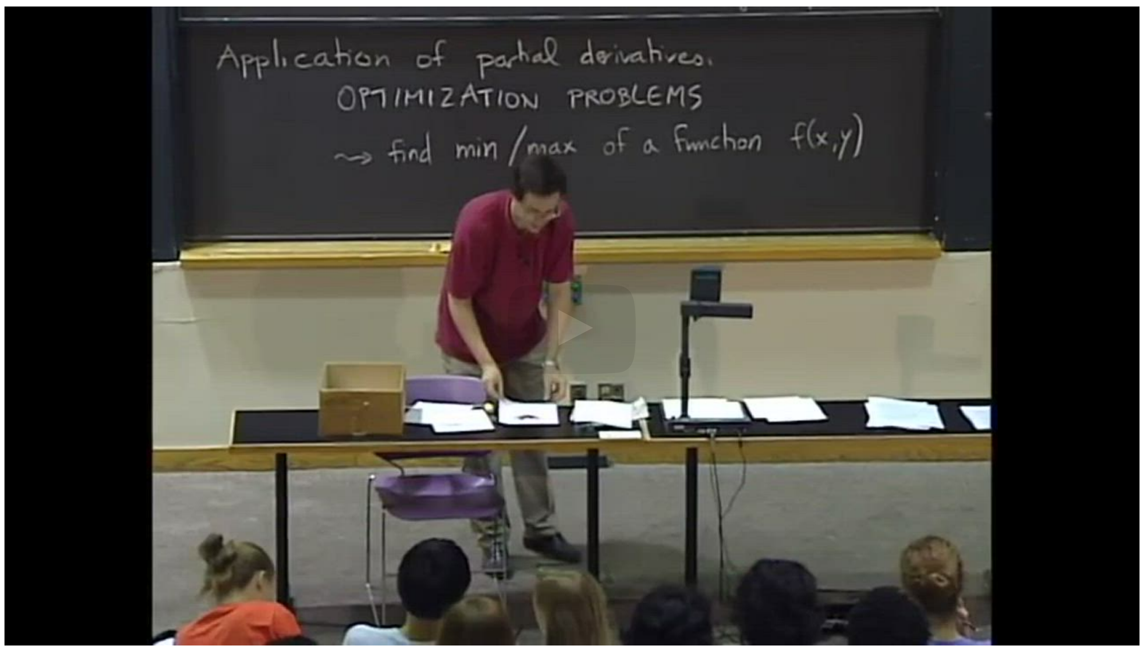

0:00 / 0:00 2.0x

[PROFESSOR]:: Let me point out to you immediately that there's

more than maximum and minimum.

So, remember, we saw the example of x squared plus y

squared that has a critical point,

but the critical point is obviously a minimum.

And of course it could be a local minimum

because it could be that, if you have

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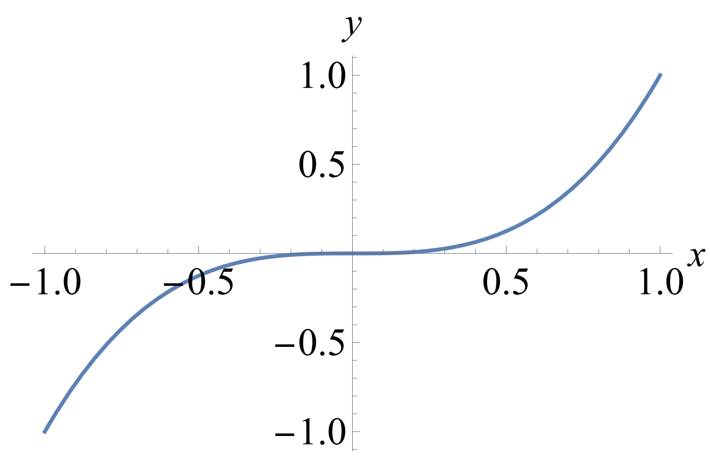
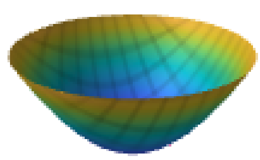
Just as we saw in single-variable calculus, the derivative equaling zero is a necessary but not sufficient condition for finding local extrema. This means that a critical point (where $f'(x) = 0$) might be a local max or min but it might not. For example, $f(x) = x^3$ has a critical point at $x = 0$. But the function is monotonically increasing everywhere and therefore $x = 0$ is not a local max or min.



Similar behavior can occur for functions of multiple variables. There are three types of critical points for a function of 2 variables.

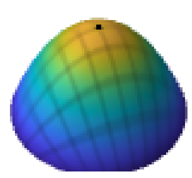
Local minimum

A point whose value is smaller than any nearby point.



Local maximum

A point whose value is larger than any nearby point.



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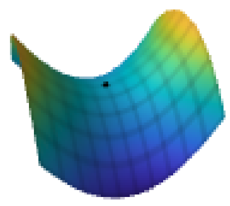
Saddle point

A point that is neither a minimum or a maximum, but can look like either depending on the direction you look at it from.



Example 6.1 Consider the function

$$f(x,y) = x^2 - y^2. \tag{4.31}$$

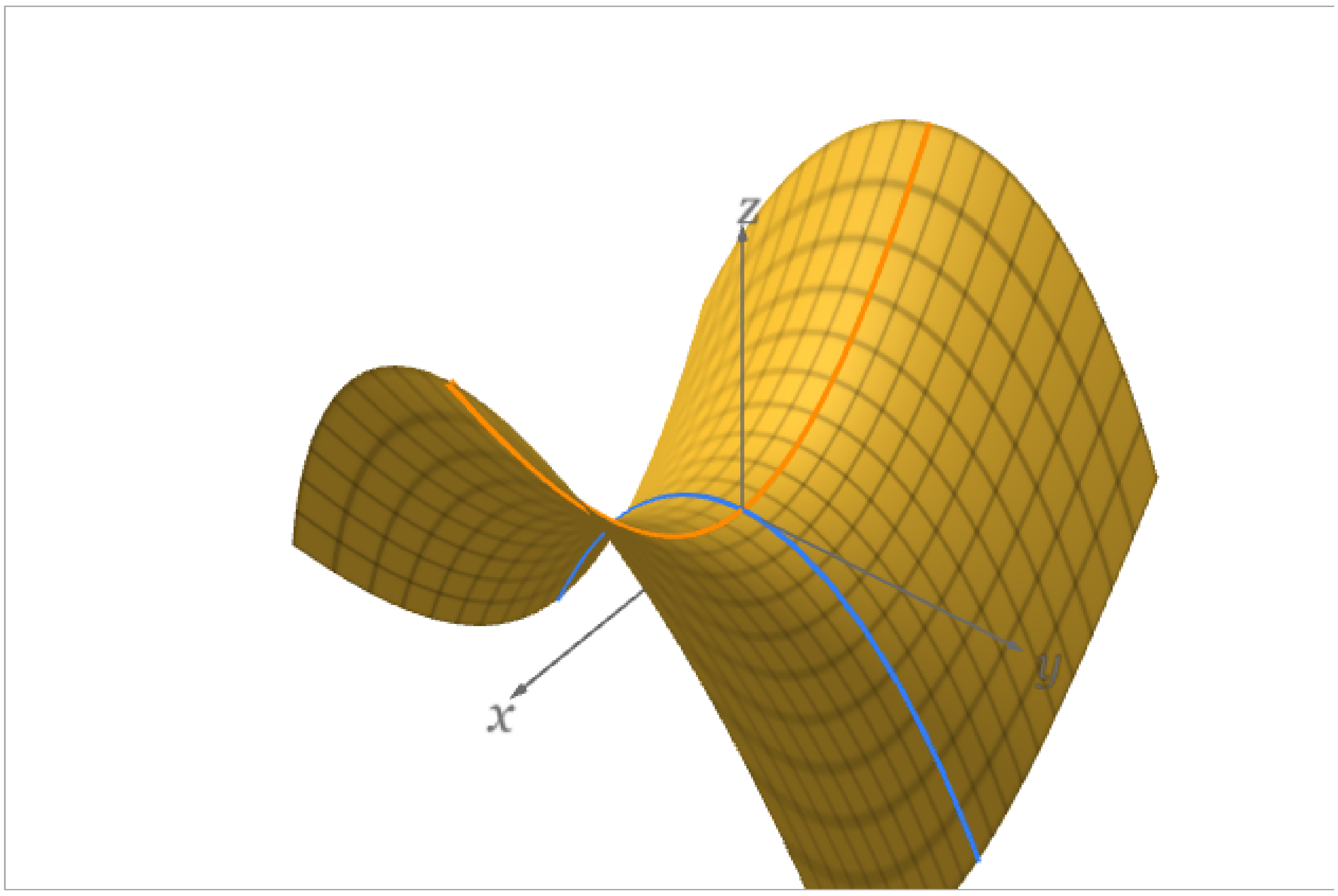
The partial derivatives are

$$f_x(x,y) = 2x \text{ and } f_y(x,y) = -2y \tag{4.32}$$

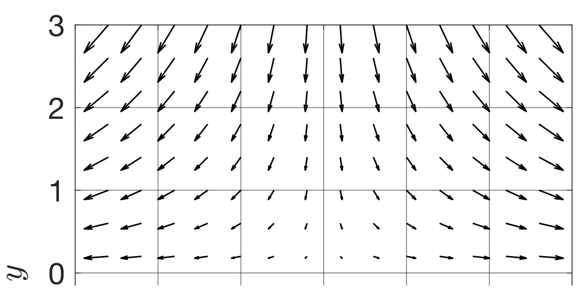
which means $(x,y) = (0,0)$ is a critical point of $f(x,y)$. However, if we inspect the graph of f (below), we see that $(0,0)$ is a **saddle point**.

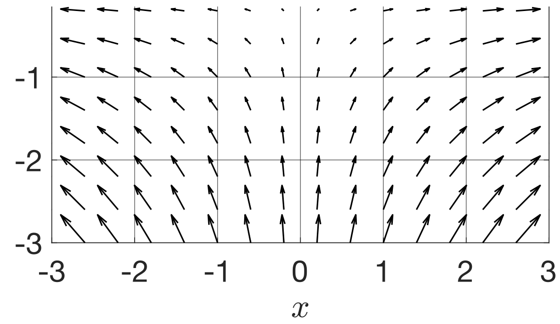
In the x-direction along the curve $f(x,0) = x^2$, the function has a local minimum. (See the orange curve in the interactive image below.) However, in the y-direction along the curve $f(0,y) = -y^2$, the curve has a local maximum. (See the blue curve in the interactive image below). Thus $(0,0)$ is neither a local max nor a local min.

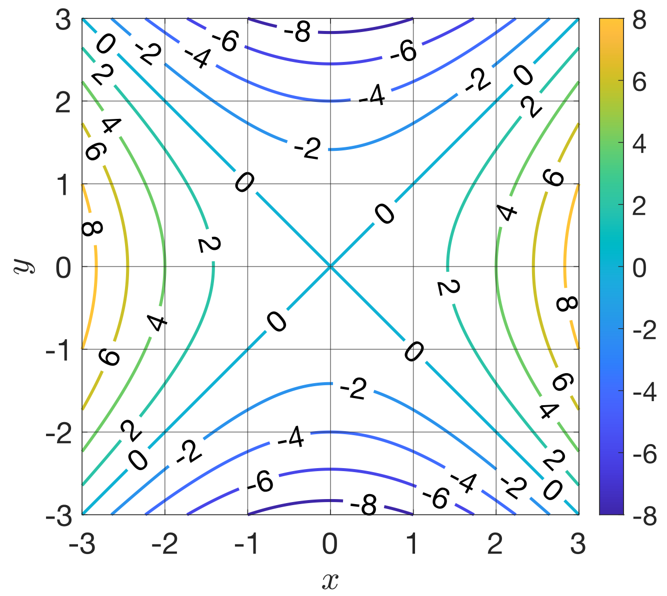
▶ **SADDLE POINT**



The gradient field for $f(x,y) = x^2 - y^2$ is shown below on the left, while the level curves are shown below on the right.







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[staff] Possible error
In the example 6.1, I think the orange curve is for f(x,0)=x^2 and the blue curve is for f(0,y)=-y^2, which is contrary to what the text i... 3

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