12.3.2 Eigenvalues of n x n Matrices

Week 12 due Dec 29, 2023 10:42 IST Completed

12.3.2 Eigenvalues of n x n Matrices

Video

Start of transcript. Skip to the end.

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Dr. Robert van de Geijn: So now that we have reviewed

a few simple matrices that are n x n but have special structure,

and we've seen that for those matrices it is relatively easy to find

the eigenvalues and eigenvectors, we're now

ready to discuss the general case of an n x n matrix.

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Homework 12.3.2.1

Calculator

Homework 12.3.2.1

10.0/10.0 points (graded)
If $A \in \mathbb{R}^{n \times n}$, then $\Lambda(A)$ has n distinct elements.

FALSE ✔ Answer: FALSE

The characteristic polynomial of A may have roots that have multiplicity greater than one. If $\Lambda(A) = \{\lambda_0, \lambda_1, \ldots, \lambda_{k-1}\}$, where $\lambda_i \neq \lambda_j$ if $i \neq j$, then

$$p_k(\lambda) = (\lambda - \lambda_0)^{n_0} (\lambda - \lambda_1)^{n_1} \cdots (\lambda - \lambda_{k-1})^{n_{k-1}}$$

with $n_0 + n_1 + \cdots + n_{k-1} = n$. Here n_j is the multiplicity of root λ_j.

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Answers are displayed within the problem

Homework 12.3.2.2

1/1 point (graded)
Let $A \in \mathbb{R}^{n \times n}$ and $\lambda \in \Lambda(A)$. Let S be the set of all vectors that satisfy $Ax = \lambda x$. (Notice that S is the set of all eigenvectors corresponding to λ *plus* the zero vector.) Then S is a subspace.

TRUE ✔ Answer: TRUE

The easiest argument is to note that $Ax = \lambda x$ is the same as $(A - \lambda I)x = 0$ so that S is the null space of $(A - \lambda I)$. But the null space is a subspace, so S is a subspace.

Alternative proof: Let $x, y \in S$ and $\alpha \in \mathbb{R}$. Then

- $x + y \in S$: Since $x, y \in S$ we know that$Ax = \lambda x$ and $Ay = \lambda y$. But then

$$A(x+y) = Ax + Ay = \lambda x + \lambda y = \lambda(x+y).$$

  Hence $x + y \in S$.

- $\alpha x \in S$: Since $x \in S$ we know that$Ax = \lambda x$. But then

$$A(\alpha x) = A(\alpha x) = \alpha A x = \alpha \lambda x = \lambda(\alpha x).$$

  Hence $\alpha x \in S$.

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Calculator