1.8 Quiz: Summary Quiz

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Question 1

1/1 point (graded)

The histogram below shows our small sample of 100 household incomes using a bin width of $20,000/year. If we define households to be "middle income" if their annual income is between $40,000/year and $120,000/year, approximately what percentage of the households in our subsample are middle income?

Round your answer to the nearest percent.

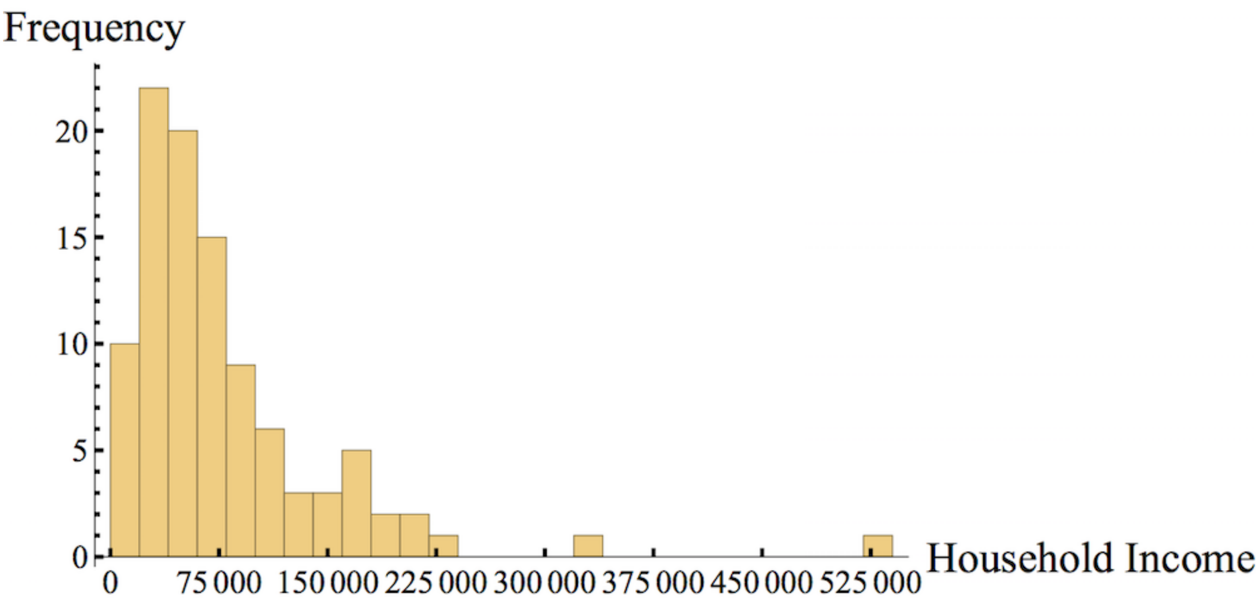

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Image Description

50 % **Answer:** 50

50

Explanation

We know that there are 100 households in the sample. From the histogram we can see that there are 20 households in the sample with income between $40,000/year and $60,000/year, 15 with income between $60,000/year and $80,000/year, and so on up to

\$120,000/year. Our final calculation looks like:

$$(20 + 15 + 9 + 6)/100 = 50/100 = 0.5.$$

This is 50%.

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Answers are displayed within the problem

Question 2

1/1 point (graded)

Let's think more about the continuous model of our data.

Recall that the mean of the continuous model was \$79,450.

Using a computer, we can compute that $F(79450) \approx 0.62$, where $F(x)$ is the cumulative distribution function for the probability density function $f(x)$. What can we say, if anything, based on this value?

(NOTE: In all these choices, we are using 'US household' to mean a US household earning some income.)

- ○ The model predicts that the chance of a US household earning the mean income is 62%.
- ○ The model predicts that more than half of US households made above the mean income.
- ◉ The model predicts that more than half of US households made below the mean income. ✔
- ○ The model predicts that exactly half of US households made below the mean income and half made above the mean income.
- ○ We don't have enough information to say any of these things.

Explanation

$F(79450) \approx 0.62$ means that the probability a random household makes less than or equal to \$79450/year is 62%. From this, we can predict that 62% of US households earning some income made below the mean income.

This should make some sense, given the fact that very high incomes affect the mean and make it higher than the median. (Recall: by definition of median, the median value such that such that a randomly chosen X has a 50% chance of being higher than that value and a 50% chance of being lower. So the model automatically predicts that exactly half of households made less than the median income.)

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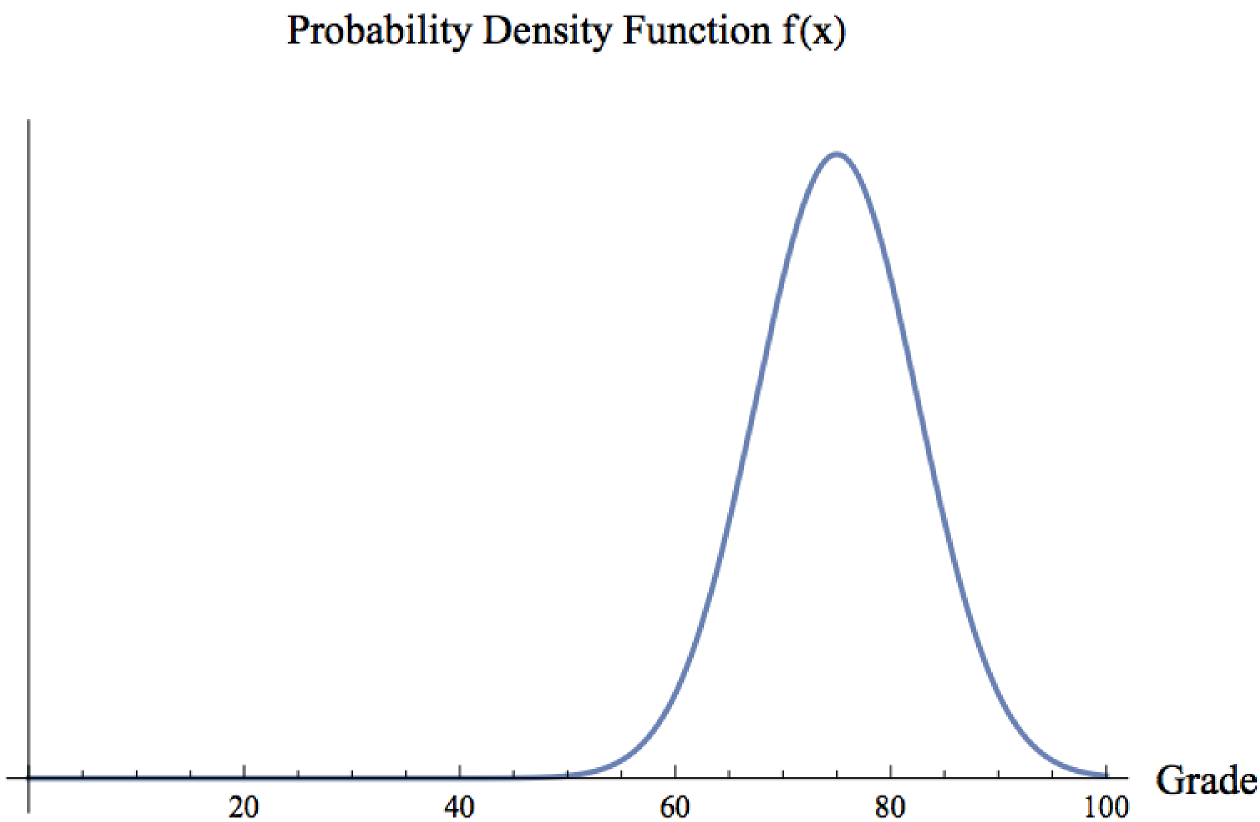
Answers are displayed within the problem

Question 3

4/4 points (graded)

You've probably heard of the *normal distribution* or 'bell curve.' These are often used to model the distribution of test scores, heights, and so on in a general population. For example, consider the following normal distribution. Suppose that the continuous random variable X represents a score on a quiz and that the function $f(x)$ is the probability density function for X.



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This means $P(a \leq X \leq b) = \int_a^b f(x)\, dx$, or in other words, the probability of getting a quiz score between a and b is equal to the area under the curve $f(x)$ between a and b.

For each of the following probability questions, choose the expression (either a number or an integral) that gives its value. *You may use a choice more than once.*

A. $P(0 < X < 70)$

- 0
- 1
- $\int_0^{70} f(x)\,dx$ ✔
- $\int_{70}^{\infty} f(x)\,dx$
- $\int_{70}^{100} f(x)\,dx$
- $\int_{70}^{99} f(x)\,dx$
- None of the above.

B. $P(X = 70)$

- 0 ✔
- 1
- $\int_0^{70} f(x)\,dx$
- $\int_{70}^{\infty} f(x)\,dx$
- $\int_{70}^{100} f(x)\,dx$
- $\int_{70}^{99} f(x)\,dx$
- None of the above.

C. $P(70 < X < 100)$

- 0
- 1
- $\int_0^{70} f(x)\,dx$
- $\int_{70}^{\infty} f(x)\,dx$
- $\int_{70}^{100} f(x)\,dx$ ✔
- $\int_{70}^{99} f(x)\,dx$
- None of the above.

D. $P(70 < X \leq 100)$

- 0
- 1
- $\int_0^{70} f(x)\,dx$
- $\int_{70}^{\infty} f(x)\,dx$
- $\int_{70}^{100} f(x)\,dx$ ✔
- $\int_{70}^{99} f(x)\,dx$
- None of the above.

Recall that $P(a < X < b) = \int_a^b f(x)\,dx$ and that $P(X = a) = 0$ for a continuous random variable X and any value of a. Thus, the value of $P(0 < X < 70)$ is given by $\int_0^{70} f(x)\,dx$.

Also, $P(X = 70) = 0$ since the area under a curve on a point interval is 0 (the probability that $X = 70$ exactly is zero).

Both $P(70 < X < 100)$ and $P(70 < X \leq 100)$ are given by $\int_{70}^{100} f(x)\,dx$ by the definition of $P(a < X < b)$ and the fact that $P(X = b) = 0$.

While some graders might use only integer grades, there is no guarantee that there are no grades of 99.5 on this quiz and so we cannot assume that $\int_{70}^{100} f(x)\,dx$ and $\int_{70}^{99} f(x)\,dx$ describe the same value.

Submit You have used 1 of 8 attempts

Answers are displayed within the problem

Question 4

1/1 point (graded)

We're still thinking about the random variable X as quiz score from 0 to 100. Here's the graph of $f(x)$, the probability density function for X.

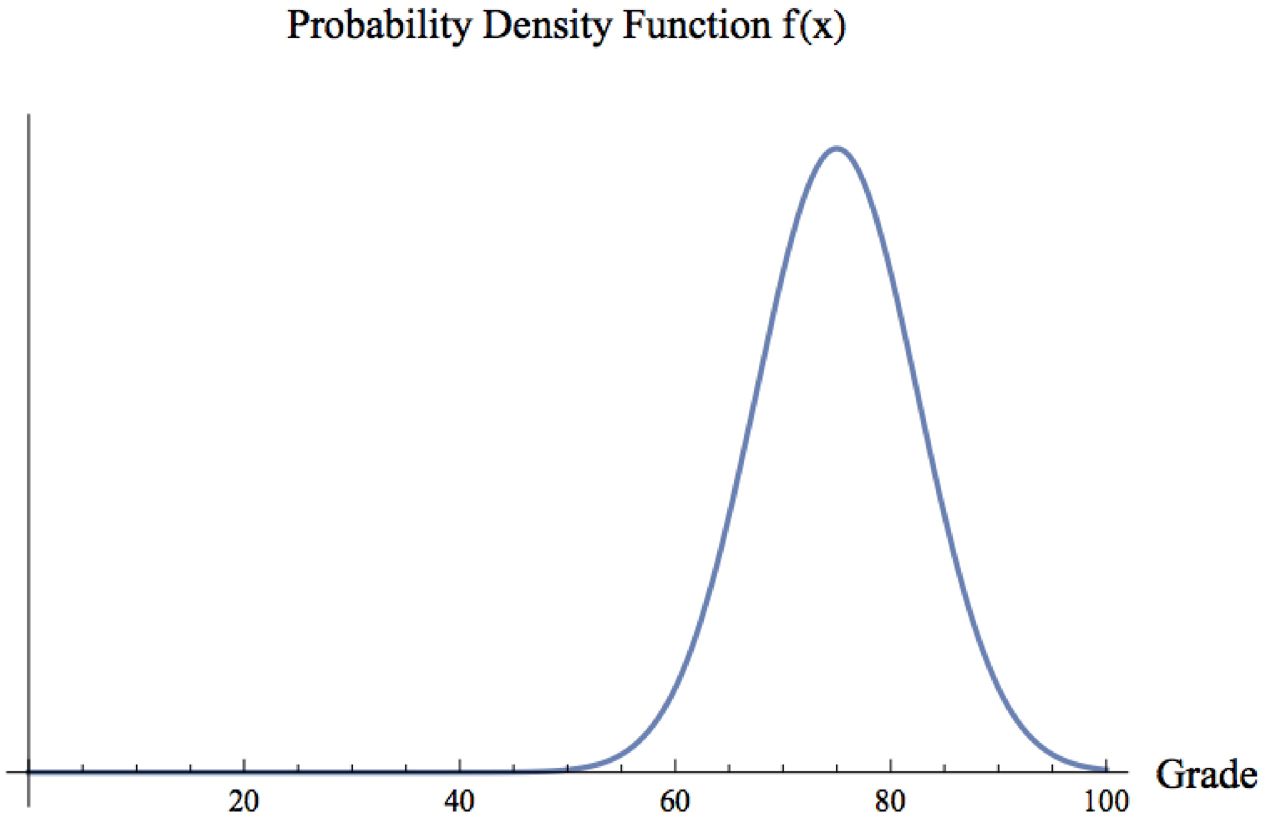

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Image Description

According to this model, is it more likely for someone to get a score above 70 or below 70 or are the two equally likely? In other words, compare the probability of getting a score above 70 to getting a score below 70.

- It is more likely to score above 70 than below. ✔
- It is more likely to score below 70 than above.
- Getting a score above or below 70 is equally likely.

Explanation

It is more likely to get a score above 70, or in other words, the probability of getting a score above 70 is greater than the probability of getting a score less than 70. This is because the area under the curve from 70 to 100 is greater than the area from 0 to 70.

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Answers are displayed within the problem

Question 5

1/1 point (graded)

Consider a random variable X defined on all values $(-\infty, \infty)$ and the associated probability density function $f(x)$ and cumulative distribution function $F(x)$.

Which of the following are true? (There are four correct answers.)

- [x] The cumulative distribution function is never decreasing. ✔
- [ ] The probability density function is never decreasing.
- [x] The cumulative distribution function has an asymptote at $y = 1$. ✔
- [ ] The probability density function has an asymptote at $y = 1$.
- [ ] The area under the cumulative distribution function on $(-\infty, a)$ is the value of the probability density function at $x = a$.
- [x] The area under the probability density function on $(-\infty, a)$ is the value of the cumulative distribution function at $x = a$. ✔
- [x] The derivative of the cumulative distribution function is the probability density function. ✔
- [ ] The derivative of the probability density function is the cumulative distribution function.

✔

Given a probability density function $f(x)$, we defined the cumulative distribution function to be $F(x) = \int_{-\infty}^{x} f(t)\,dt$. This means **choice 6 is true.** $F(a) = \int_{-\infty}^{a} f(t)\,dt$.

Also, because probabilities can never be negative, $f(x) \geq 0$ for all x and so the cumulative distribution function $F(x)$ is never decreasing (as x increases, either F is constant for a period (if $f(x) = 0$) or we accumulate more area, increasing the value of $F(x)$).

Choice 2 is wrong. As in the example of household incomes, the probability density function may increase or decrease.

Choice 3 is correct. Because probabilities can never be greater than 1 (100% certainty), $\int_{-\infty}^{\infty} f(t)\,dt = 1$. Therefore the cumulative distribution function has an asymptote at $y = 1$. As a goes to ∞, the value of $F(a) = \int_{-\infty}^{a} f(t)\,dt$ goes to 1.

Choice 7 is correct. The fundamental theorem of calculus tells us that the derivative of the cumulative distribution function is the probability density function. In other words, the derivative of the cumulative distribution function is the probability density function because differentiation is the opposite of integration.

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Answers are displayed within the problem

Question 7

1/1 point (graded)

Consider a random variable X defined on all values $(-\infty, \infty)$, and the associated probability density function $f(x)$ and cumulative distribution function $F(x)$.

Which of the following represent the median value of this continuous model?

- [ ] $F(0.5)$
- [ ] $f(0.5)$
- [ ] $F'(0.5)$
- [x] The value m such that $\int_{-\infty}^{m} f(x)\,dx = 0.5$ ✔
- [ ] $\int_0^{0.5} f(x)\,dx$
- [x] $F^{-1}(0.5)$ ✔

✔

Explanation

The median value m has the property that $\int_{-\infty}^{m} f(x)\,dx = 0.5$. Since $F(m) = \int_{-\infty}^{m} f(x)\,dx = 0.5$, this means we can also find the median by computing $m = F^{-1}(0.5)$.

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Answers are displayed within the problem

Question 8

2/2 points (graded)

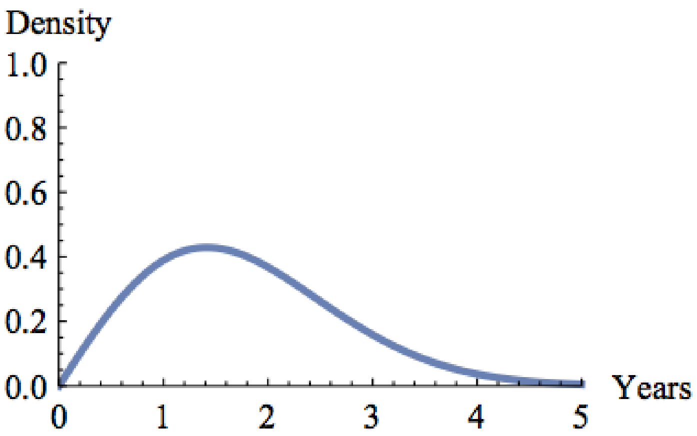
Probability density functions are used a lot in modeling lifetimes of technology (like light bulbs and cell phones, for example) and of people (in medical treatment trials, for example).

Let's consider the lifetime of three different cell phone brands. Suppose the following are graphs of the probability density functions for the lifetime variable for each brand. (Hint: translate each into a question about the probability of the cell phone having a certain lifespan.)

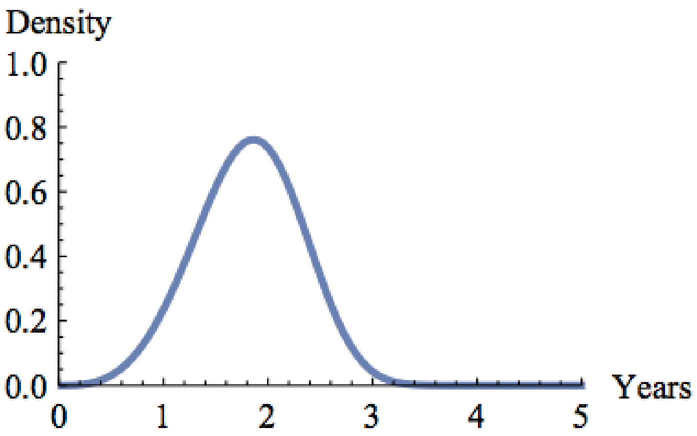
View Larger Image (Graph A) | View Larger Image (Graph B) | View Larger Image (Graph C) | Image Description (All Graphs)

Which cell phone has the greatest chance (the greatest probability) of failing within the first year after purchase?

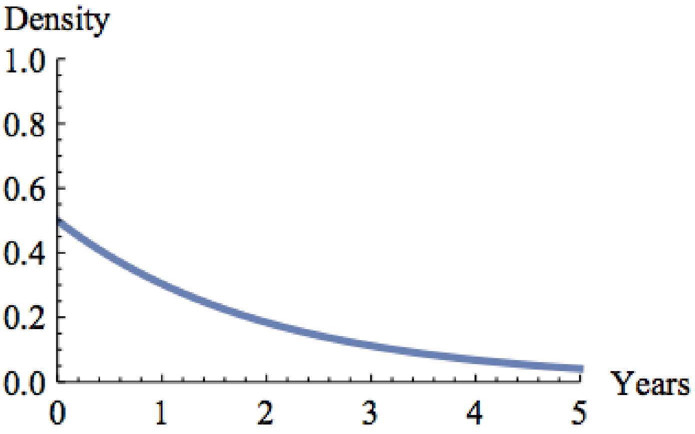
a.



b.

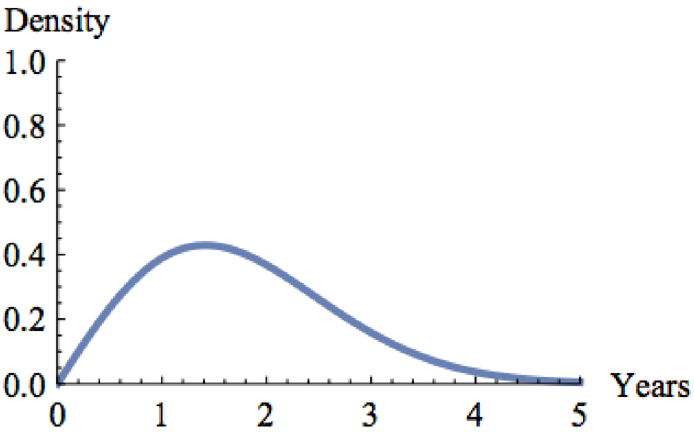


c.

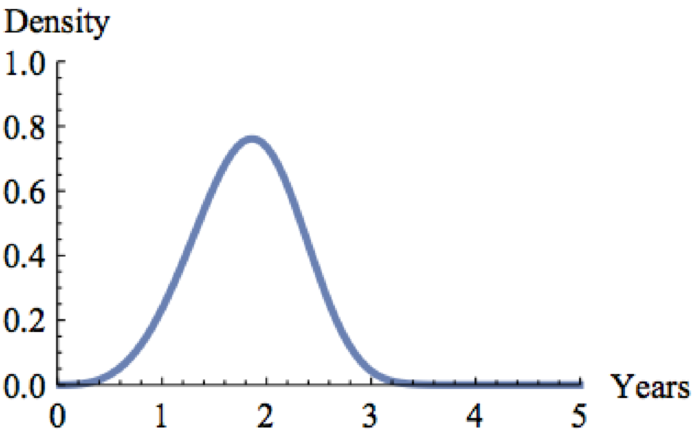


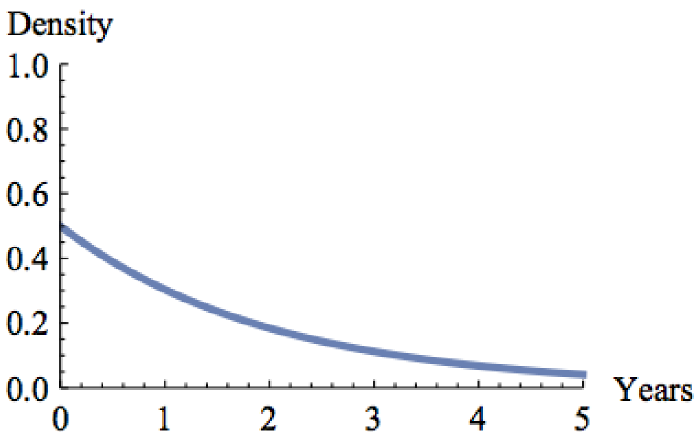
Which has the least chance (probability) of failing withing the first year?

a.



b.



c. 

We can rank these by comparing the area under the probability density function graph on $[0, 1]$ for all three brands.

Cell phone brand C has the greatest probability of failure in the first year, as the area under the curve on $[0, 1]$ looks to be greatest (more than 0.3, or 30%). Cell phone brand A has the next highest probability of failure at about 0.2 (20%). Cell phone brand B appears to have a probability of failure less than 0.1 or 10% in the first year. These values were obtained by estimating the area between the curves shown and the interval $[0, 1]$ on the horizontal axis.

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Answers are displayed within the problem

In the next three problems, we'll look more at the Gamma function for the case of a = 1:

$$f(x) = Ce^{-\frac{1}{b}x}$$

(Here we define $f(x) = 0$ for $x < 0$.

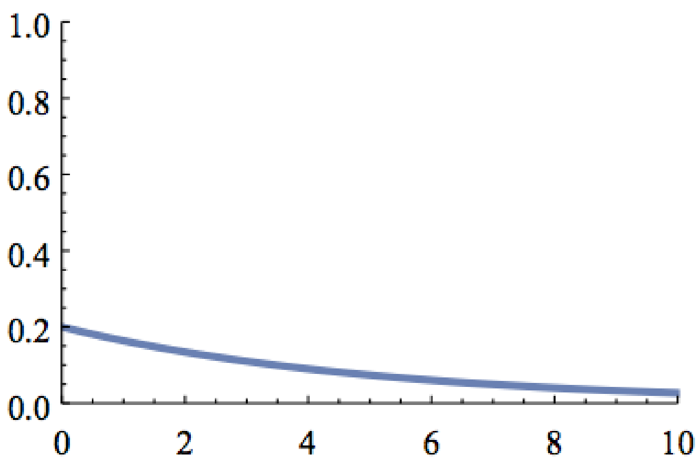
This is called the **exponential distribution**, since $f(x)$ is just an exponential function.

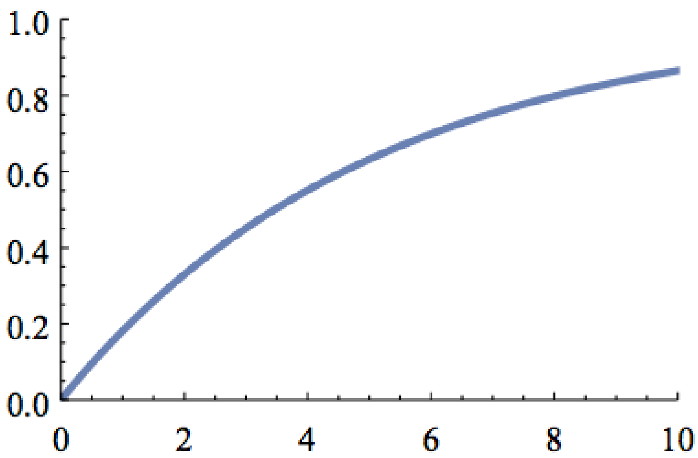
Question 9

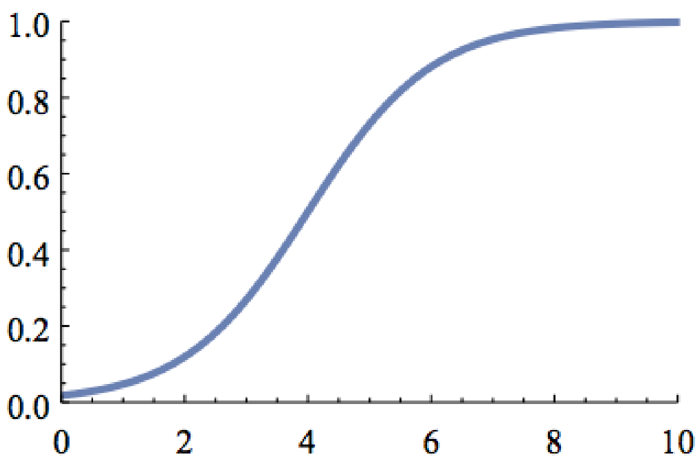
1/1 point (graded)

Let's consider the case when $b = 5$. Which of the following could be the graph of the cumulative distribution function for $f(x)$?

View Larger Image (Graph A) | View Larger Image (Graph B) | View Larger Image (Graph C) | Image Description (All Graphs)

a. 

b. 

c. 

None of the above

Explanation

Cumulative distribution functions are always increasing, and their output goes to 1 as their input goes to infinity. Two of the graphs shown have this property, graph b. and c.

To choose between these two, we can use the fact that $f(x)$ is the derivative of $F(x)$, the cumulative distribution function since by definition $F(x) = \int_0^x Ce^{-\frac{1}{5}t}\,dt$. Since $f(x) = Ce^{-\frac{1}{5}x}$ with $C > 0$ is always decreasing, this means $F(x)$ is always concave down.
Thus qualitatively, the graph in **b.** could be the cumulative distribution function, while the graph in **c.** could not (because there is a change in concavity). Thus the correct answer is choice **b.**

Submit You have used 1 of 2 attempts

Answers are displayed within the problem

Question 10

1/1 point (graded)
Nina said that one property of a probability density function $f(x)$ is that the total area under the graph of /(f(x)/ must equal 1:

$$\int_{-\infty}^{\infty} f(x)\,dx = 1.$$

Suppose the probability density function is the exponential distribution $f(x) = Ce^{-\frac{1}{b}x}$ (with $f(x) = 0$ for $x < 0$).

(Note: This is not something you can see 'at a glance.' It requires computing the integral so make sure you have a pencil and paper handy. You may also want to refresh how to compute improper integrals, integrals with an infinite bound.)

What must the value of C be to satisfy the property that total area equals 1?

- b
- $1/b$ ✔
- e^b

e^{-b}

None of the above

We know that $f(x) = 0$ for $x < 0$, so we have $\int_0^\infty f(x)\,dx = 1$. Thus:

$$\begin{aligned}\int_0^\infty f(x)\,dx &= \int_0^\infty Ce^{-\frac{1}{b}x}\,dx \\ &= \lim_{t\to\infty}\int_0^t Ce^{-\frac{1}{b}x}\,dx \\ &= \lim_{t\to\infty} C\left(-\frac{b}{1}\right)e^{-\frac{1}{b}x}\Big|_0^t \\ &= \lim_{t\to\infty} C\left(-\frac{b}{1}\right)\left(e^{-\frac{1}{b}t} - e^{-\frac{1}{b}0}\right) \\ &= C(-b)(0-1) \\ &= bC.\end{aligned}$$

Since the integral must equal 1, this means $C = \frac{1}{b}$.

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Answers are displayed within the problem

Question 11

1/1 point (graded)

Nina mentioned a cool property of the Gamma distribution. She said the mean of the continuous model with

$$f(x) = Cx^{a-1}e^{-x/b}$$

is the product of the parameters a and b.

Let's investigate this in the special case of the **exponential distribution** which means $a = 1$.

Get out your pencil and paper and compute the mean

$$E[X] = \int_0^\infty x\,f(x)\,dx$$

for $f(x) = \frac{1}{b}e^{-x/b}$.

Remember you can check if you get the right answer: it should be the product of $a = 1$ and b.

Choose all correct answers below.

- [ ] We can use substitution with $u = -x/b$ and the fact that $\frac{d}{dx}e^x = e^x$ to find the antiderivative $\int x\,f(x)\,dx$.
- [x] We can use integration by parts with $u = x$ and $dv = e^{-x/b}dx$ to find the antiderivative $\int x\,f(x)\,dx$. ✔
- [ ] We can use integration by parts with $u = e^{-x/b}$ and $dv = xdx$ to find the antiderivative $\int x\,f(x)\,dx$.
- [ ] The antiderivative of $\frac{1}{b}e^{-x/b}$ is $e^{-x/b}$ and the antiderivative of x is $\frac{1}{2}x^2$, so the antiderivative $\int x\,f(x)\,dx = \frac{1}{2}x^2e^{-x/b}$
- [ ] None of these.

✔

Explanation

We need to compute

$$\begin{aligned} E[X] = \int_0^\infty x\,f(x)\,dx &= \int_0^\infty xCe^{-\frac{1}{b}x}\,dx \\ &= C\lim_{t\to\infty}\int_0^t xe^{-\frac{1}{b}x}\,dx \end{aligned}$$

The antiderivative $\int xCe^{-\frac{1}{b}x}\,dx$ can be computed by integration by parts, using $u = x$ and $dv = e^{-x/b}$.

In doing so, you can show that $E[X] = b$, which is the product of $a = 1$ and b.

Note: The last choice is not correct: the antiderivative of a product of functions is not the product of the antiderivatives of each factor.

The first and third choice are not mathematically incorrect but they do not lead to a simpler form of the antiderivative problem (they are "dead ends".)

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