5. Constrained optimization: Big picture

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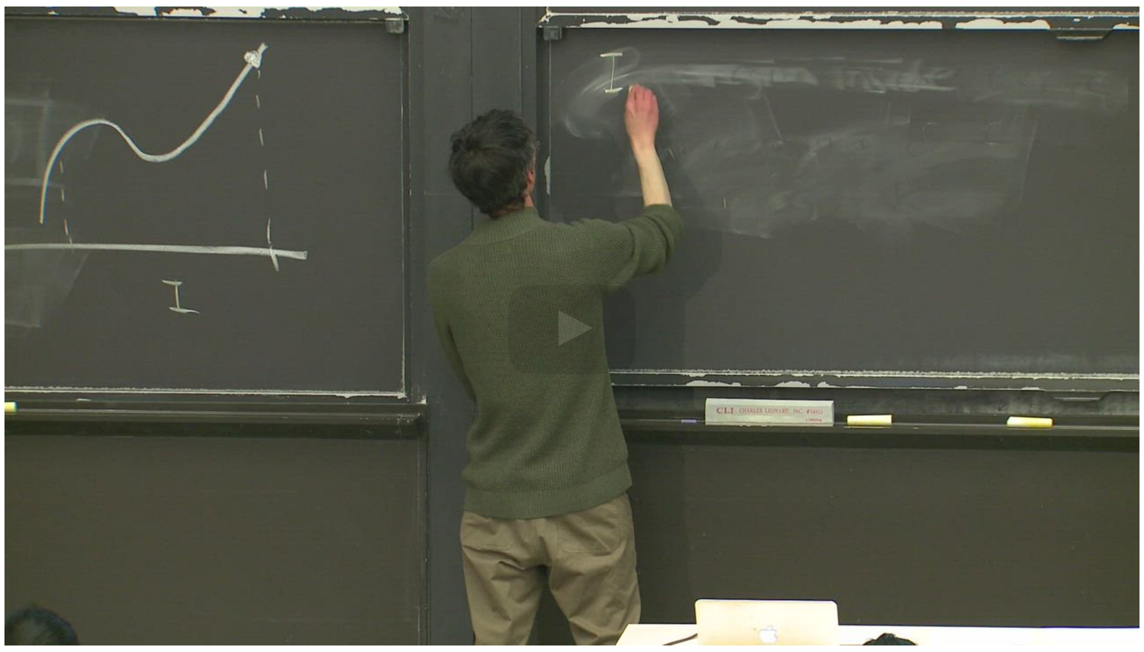
Definition 5.1 Finding the absolute maximum or minimum of a function restricted to a region R is referred to as **constrained optimization**. The boundary curve of R is often referred to as the **constraint equation**.

Constrained optimization



OK, so one thing that could happen

is that the maximum occurs at a critical point.

That happens in the example we just looked at.

So this is where the maximum was,

and it's at a point where the gradient vanishes--

which, of course, just means that the x derivative and the y

derivative vanish.

So that's one thing that could happen.

Another thing that could happen is

it could occur at a boundary point.

OK, so I'm going to try to draw two pictures that

0:00 / 0:00 2.0x

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Theorem

If a function $f\left(x,y\right)$ of two variables is differentiable on a closed bounded region R, then $f\left(x,y\right)$ attains its absolute maximum (or absolute minimum) on R. Furthermore, either

- the absolute maximum (or absolute minimum) occurs at a critical point, or
- the absolute maximum (or absolute minimum) occurs on the boundary of R.

Some examples are shown below. In the next sections, you will learn how to find extrema for functions of multiple variables defined on specific regions.

Example 5.3 The function $f\left(x,y\right) = -x^2 - y^2$ over the region $x^2 + y^2 \leq 1$ has an absolute maximum at the point $(0,0)$. Notice that this point is located at a critical point since $f_x = -2x$ and $f_y = -2y$.

▶ **Maximum occurs at critical point**

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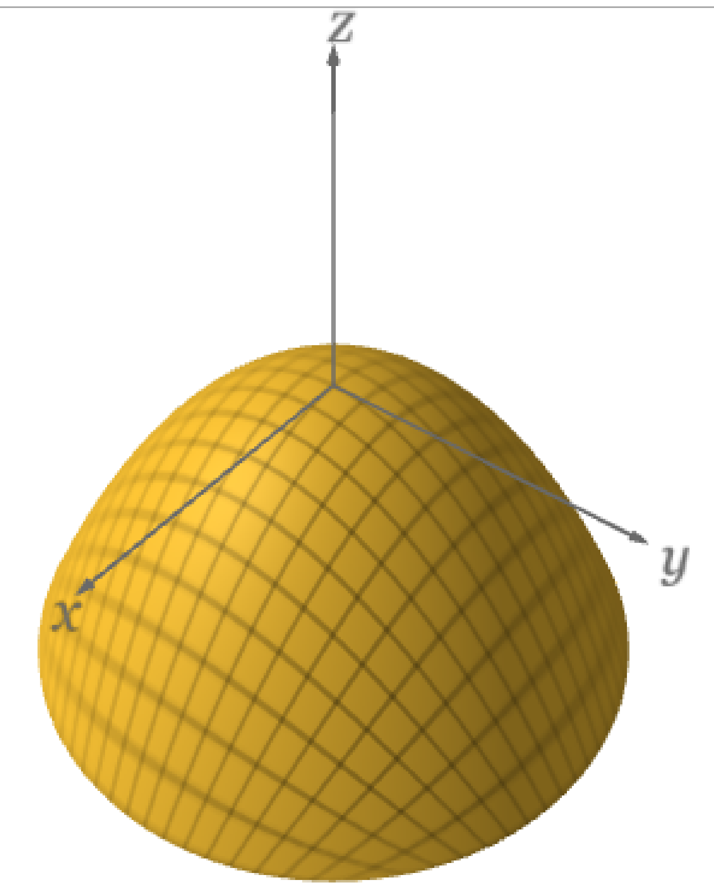
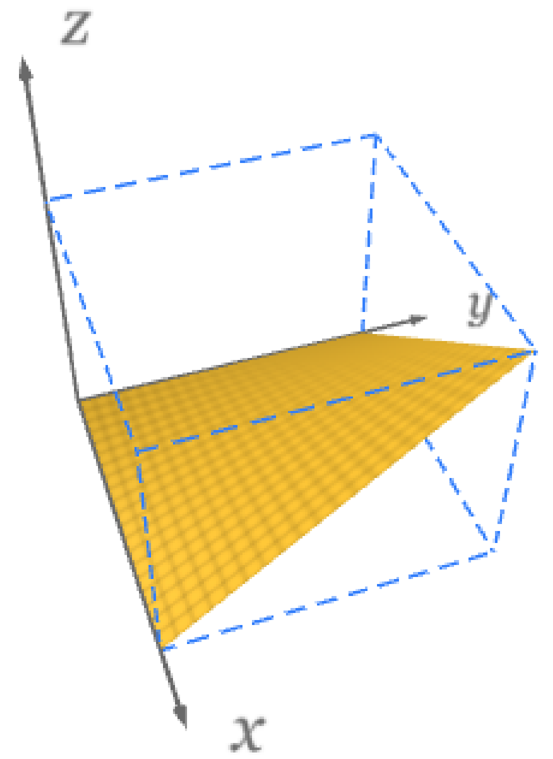


Example 5.4 The function $f(x,y) = xy$ over the region $0 \le x \le 1, 0 \le y \le 1$ has an absolute maximum at the point $(1,1)$. Notice that this point is **not** located at a critical point but is on the boundary of the region.

▶ **MAXIMUM OCCURS ON BOUNDARY**



Example 5.5 Sometimes the absolute maximum (or absolute minimum) of a function occurs at more than one point. For example, the function $f(x,y) = x$ over the region $0 \le x \le 1, 0 \le y \le 1$ has a maximum value of 1, which occurs along the line $x = 1, 0 \le y \le 1$. Notice that these points are **not** located at critical points but are on the boundary of the region.

▶ **MAXIMUM OCCURS ALONG BOUNDARY**

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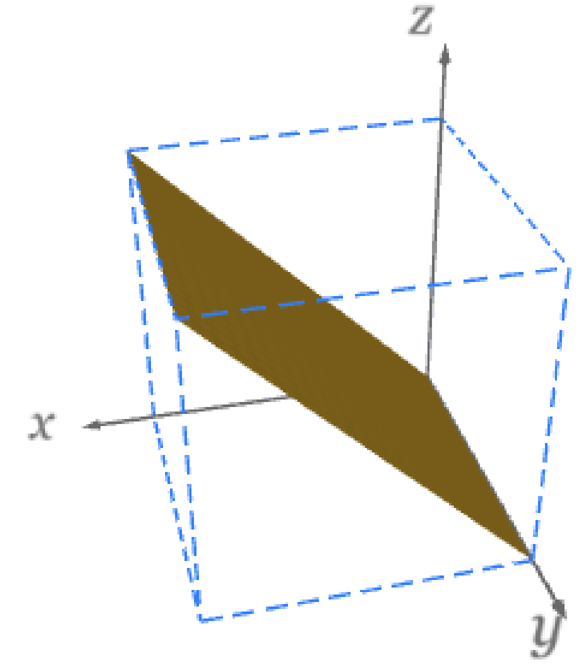

Remark 5.6

In 18.01, the process involved finding critical points, and then comparing values of the function at critical points to the values at the boundary, which is two points.

In 18.02 the process starts the same. We start by finding critical points. But now the boundary is a curve. So there are infinitely many points we have to consider. And so now we are going to start thinking about how we can identify maxima (or minima) along a boundary.

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