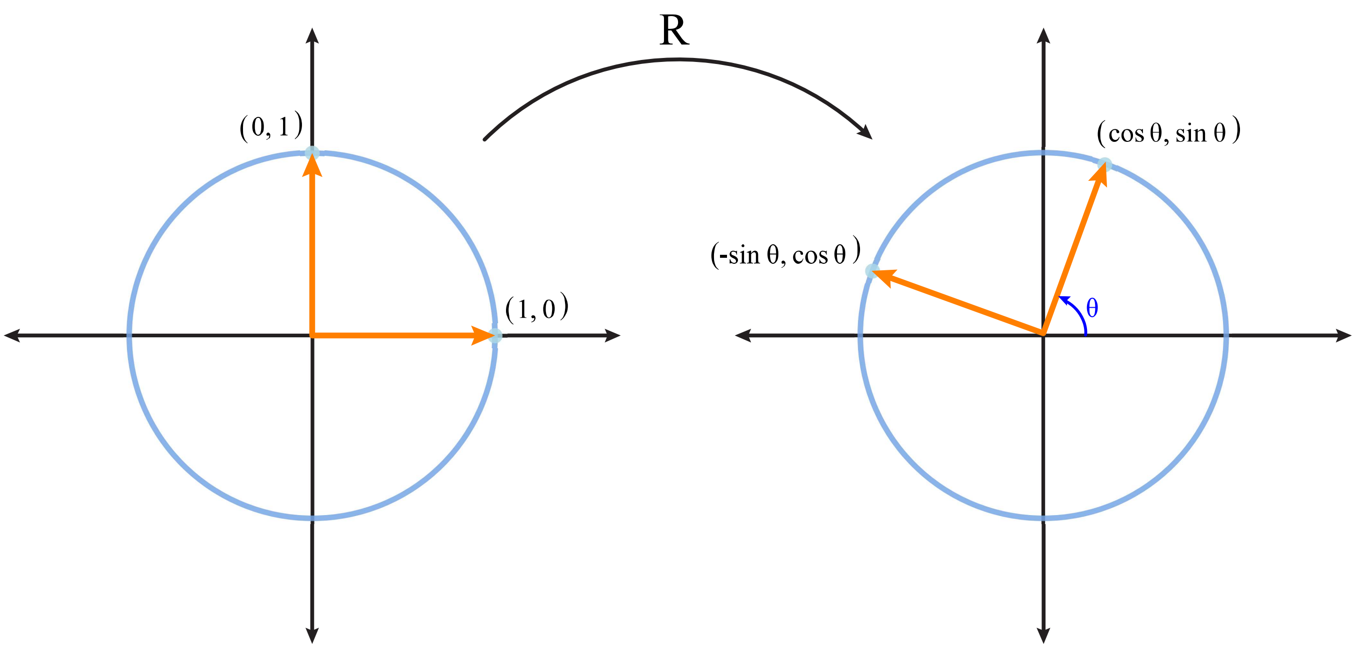
7. Representing functions with matrices

It turns out that if we have a function from $\mathbb{R}^n$ to $\mathbb{R}^m$ that we know comes from multiplication on the left with a matrix $\mathbf{A}$, we can find this matrix by looking at where this matrix sends the standard basis vectors!

Problem 7.1 Given θ, there is a 2×2 matrix R whose associated function rotates each vector in $\mathbb{R}^2$ counterclockwise by the angle θ. What is it?



Solution: The rotation maps $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ to $\begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ to $\begin{pmatrix} -\sin\theta \\ \cos\theta \end{pmatrix}$. The matrix with columns $\mathbf{R}\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\mathbf{R}\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ is the matrix we want, and that this approach for finding a matrix representing a function always works. Thus

$$\begin{aligned}(\text{first column of }\mathbf{R}) &= \mathbf{R}\begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}\cos\theta\\ \sin\theta\end{pmatrix}\\ (\text{second column of }\mathbf{R}) &= \mathbf{R}\begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}-\sin\theta\\ \cos\theta\end{pmatrix},\end{aligned}$$

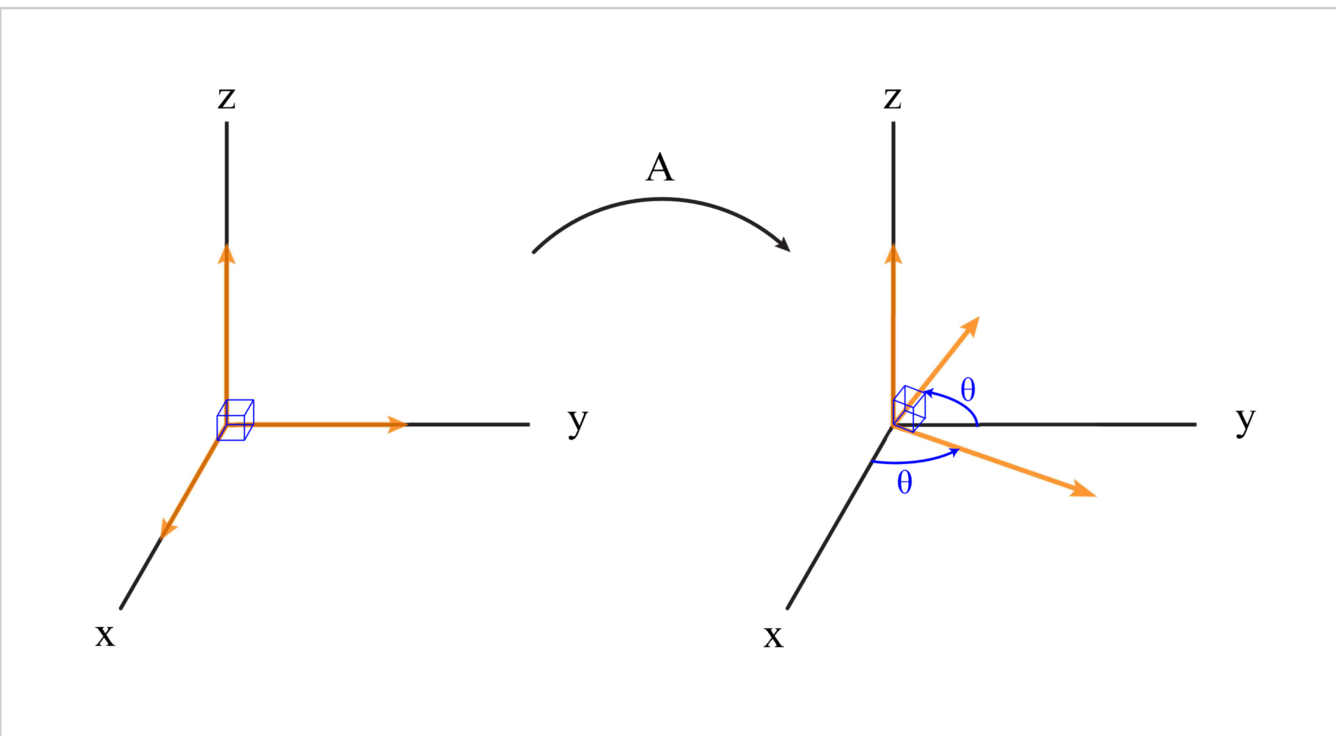
so

$$\mathbf{R} = \begin{pmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{pmatrix}.$$

Rotation in three dimensions

1/1 point (graded)

What is the matrix $\mathbf{A}$ which rotates the xy-plane in $\mathbb{R}^3$ counterclockwise by an angle θ about the z-axis?



- [ ] $\begin{pmatrix} \cos\theta & 1 & 0 \\ \sin\theta & 0 & -\sin\theta \\ 0 & 0 & \cos\theta \end{pmatrix}.$

- [ ] $\begin{pmatrix} \cos\theta & -\sin\theta & \cos\theta \\ \sin\theta & \cos\theta & \sin\theta \\ \cos\theta & -\sin\theta & \cos\theta \end{pmatrix}.$

- [x] $\begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}.$ ✔

- [ ] $\begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{pmatrix}.$

Solution:

The z-axis is fixed by this rotation, so

$$\mathbf{A}\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}.$$

On the other hand, the x and y basis vectors rotate by θ in the plane, but still don't have a z-coordinate after the rotation; only their x and y components change. So, similar to the example above,

$$\mathbf{A}\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} \cos\theta \\ \sin\theta \\ 0 \end{pmatrix},$$

and

$$\mathbf{A}\begin{pmatrix}0\\1\\0\end{pmatrix}=\begin{pmatrix}-\sin\theta\\ \cos\theta\\ 0\end{pmatrix},$$

so

$$\mathbf{A}=\begin{pmatrix}\cos\theta & -\sin\theta & 0\\ \sin\theta & \cos\theta & 0\\ 0 & 0 & 1\end{pmatrix}.$$

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Answers are displayed within the problem

Depicting functions as matrices

1/1 point (graded)

The function associated to a matrix $\mathbf{A}$ sends the vector $\begin{pmatrix}1\\0\end{pmatrix}$ to the vector $\begin{pmatrix}2\\-2\\3\end{pmatrix}$, and sends the vector $\begin{pmatrix}0\\1\end{pmatrix}$ to the vector $\begin{pmatrix}1\\-2\\0\end{pmatrix}$.

What is the first column of the matrix $\mathbf{A}$?

- $\begin{pmatrix}2\\-2\\0\end{pmatrix}$
- $\begin{pmatrix}4\\-1\\-1\end{pmatrix}$

- ○ $\begin{pmatrix} 1 \\ -2 \\ 0 \end{pmatrix}$
- ◉ $\begin{pmatrix} 2 \\ -2 \\ 3 \end{pmatrix}$ ✔
- ○ It cannot be determined without more information.

Solution:

The vector resulting vector from $\mathbf{A}\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ is actually the first column of $\mathbf{A},$ so we get that the first column of $\mathbf{A}$ is $\mathbf{f}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \\ -2 \\ 3 \end{pmatrix}.$

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Geometric examples in $\mathbb{R}^3$: Here are some functions from $\mathbb{R}^3$ to $\mathbb{R}^3$ that can be represented as a matrix: reflections across a plane through the origin, rotations about a line through the origin, and projections onto a plane or line through the origin.

Nonexample: A function from $\mathbb{R}^3$ to $\mathbb{R}^3$ which translates all points by a fixed amount cannot be represented by multiplication by a matrix.

(Optional) When can a function be represented by a matrix?

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