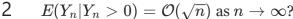


Conditional expectation of a random walk given that it is positive

Asked 5 years, 7 months ago Active today Viewed 276 times



Let $\{\xi_k\}$ is a sequence of iid random variables with $E(\xi_1)=0$ and $E(\xi_1)^2=\sigma^2<\infty$. Define the random walk $Y_n=\sum_{k=1}^n \xi_k$. Is it necessarily true that the conditional expectation





X

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From the classical CLT, we know that $\frac{Y_n}{\sqrt{n}} \Rightarrow N(0, \sigma^2)$, in which the arrow denotes weak convergence as $n \to \infty$. This seems to heavily support the suggested limiting behavior, as do the results of some computations I've done with specific choices for $\{\xi_k\}$. However, I'm stuck on how to prove the bound concretely. Is there some twist on CLT that I'm just not seeing, or

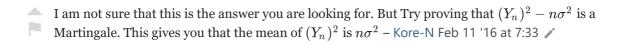


Any assistance provided will be much appreciated.

probability probability-theory conditional-expectation random-walk

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asked Feb 11 '16 at 6:43
srnoren
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1 Answer





Let $W_n=n^{-1/2}Y_n.$ Then



$$n^{-1/2} \mathsf{E}[Y_n \mid Y_n > 0] = \mathsf{E}[W_n \mid W_n > 0] \leq rac{\sqrt{\mathsf{E}|W_n|^2}}{\mathsf{P}(W_n > 0)} = rac{\sigma}{\mathsf{P}(W_n > 0)},$$



and $\mathsf{P}(W_n>0)\to 1/2$ (by the CLT), i.e., for any $\epsilon>0,\,\mathsf{P}(W_n>0)\ge \frac{1}{2}-\epsilon$ for n large enough.

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edited 1 hour ago

answered Feb 11 '16 at 8:55



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- srnoren Feb 11 '16 at 17:35