5. Temperature and humidity

Problem Set B due Oct 5, 2021 20:30 IST

Apply

In hot weather, we regulate our body temperatures by sweating. When sweat evaporates from our skin, evaporative cooling allows us to feel "apparent temperatures" that are significantly lower than the actual air temperature. The apparent temperature we feel can increase because of increases in the actual temperature and because of increases in humidity, the latter because sweat evaporates less quickly when there is more moisture in the air around us.

One simple measure of "apparent temperature" is the **wet-bulb temperature** , or the temperature that air reaches when water is evaporated into it until it reaches 100% relative humidity. **Humidity** is expressed as either mass of water vapor per volume of moist air, or as mass of water vapor per mass of dry air (usually in grams per kilogram). **Relative humidity** is defined as the ratio of the absolute humidity to the maximum humidity possible given the temperature. By definition, the relative humidity is a number between 0 and 1.

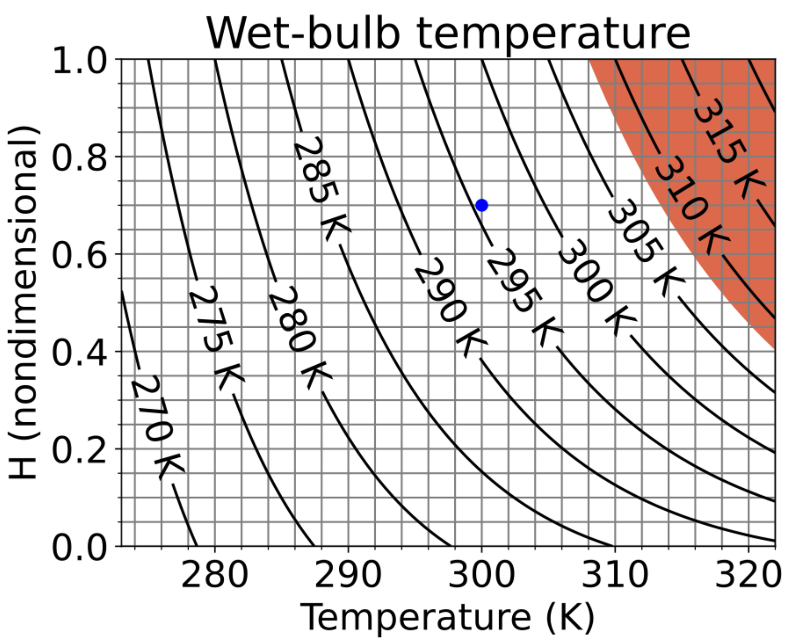
Wet-bulb temperature W is a function of both temperature T and relative humidity H (i.e., $W = W(T, H)$) and is always lower than the actual temperature, because evaporating water into air will cool the air while increasing humidity, and the wet-bulb temperature increases with increasing actual temperature and increasing humidity. It is also a useful measure of heat stress: humans cannot survive exposure to wet-bulb temperatures above 95°F for more than a few hours. As global climate changes, it is important to understand how the heat stress of a region can change to make different regions more or less habitable.

Wet-bulb temperature is related to the actual temperature and the relative humidity by the following implicitly defined function:

$$T - W = a\left(e^{-b/W} - He^{-b/T}\right) \qquad (6.320)$$

The equation above is a statement about energy balance: it says that the energy consumed to evaporate water (right-hand side) is equal to the energy provided by cooling the air (left-hand side). The independent variables are T (temperature) and H (relative humidity). The dependent variable is $W = W(T, H)$, and $a = 4.2 \times 10^9$ Kelvins and $b = 5400$ Kelvins are constants.

The equation above cannot be solved for W expressed in terms of elementary functions. Instead, the wet-bulb temperature is typically calculated numerically as a function of T and H. The figure below shows level curves of W for T between 273 K and 322 K (32°F to 120°F) and H between 0 and 1 (0% to 100%). The region where the wet-bulb temperature is above 95°F (308 K), or above the limit humans can tolerate, is shaded in red. A warm summertime day in Boston, with relative humidity around 70% (H = 0.7), temperature around 85 °F (T = 300 K), and a wet-bulb temperature of 295 K (71° F) is marked with a blue dot.



Although solving explicitly for wet-bulb temperature requires numerical calculations, l

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useful tool for quantifying the sensitivity of wet-bulb temperature to changes in temperature and humidity. In the following problems, we will use linear approximation to examine how heat stress during a warm summertime day in Boston changes when the weather becomes hotter and more humid.

Remark 5.1

Relative humidity itself is determined from physical measurements of the wet-bulb and apparent temperatures. That is, $H = H(T, W)$. However, in the series of questions that follows, we consider H and T to be independently measured quantities.

Identify the constants and variables

2/2 points (graded)

We are interested in understanding the wet-bulb temperature.

Identify the independent variables in the equation for the wet-bulb temperature.

- [x] T
- [x] H
- [ ] a
- [ ] b
- [ ] None of the above

✓

Identify the constants determined by material and physical properties.

- [ ] T
- [ ] H
- [x] a
- [x] b
- [ ] None of the above

✓

Solution:

The independent variables are T and H. Note we are also told that H depends on T, although the relationship was not explicit!

The a and b are constants.

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Answers are displayed within the problem

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Take the differential

1/1 point (graded)
Using the implicit function

$$T - W = a\left(e^{-b/W} - He^{-b/T}\right), \tag{6.321}$$

find an expression for the differential dW in terms of the variables H, T and W, the differentials dT and dH and the constants a and b.

To take the differential of an implicitly defined function, we take the differential of each side independently.

(You answer will involve `a`, `b`, the variables `T`, `W`, and `H`, and the differentials `dH`, and `dT`.)

$dW =$

((1+(a*b*H*e^(-b/T))/T^2)*dT+a*e^(-b/T)*dH)/(1+(a*b*e^(-b/W))/W^2)

Answer: (a*e^(-b/T)*dH + (1+a*b*H/T^2*e^(-b/T))*dT)/(1+a*b/W^2*e^(-b/W))

Solution:

To implicitly differentiation, we will rearrange and then take the total differential.

$$W + ae^{-b/W} = T + aHe^{-b/T} \tag{6.322}$$

$$dW + a\left(b/W^2\right)e^{-b/W}\,dW = dT + ae^{-b/T}\,dH + aH\left(b/T^2\right)e^{-b/T^2}\,dT \tag{6.323}$$

$$dW\left(1 + \frac{ab}{W^2}e^{-b/W}\right) = ae^{-b/T}\,dH + (1 + \frac{abH}{T^2}e^{-b/T})\;dT \tag{6.324}$$

$$dW = \frac{ae^{-b/T}dH + (1 + \frac{abH}{T^2}e^{-b/T})\;dT}{1 + \frac{ab}{W^2}e^{-b/W}} \tag{6.325}$$

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Sensitivity to temperature and humidity

2/2 points (graded)
A warm summertime day in Boston has relative humidity around 70% (H = 0.7), temperature around 85 °F (T = 300 K), and a wet-bulb temperature of W = 295 K (71° F). We are interested in determining how sensitive is the wet-bulb temperature to increases in the actual temperature and to increases in relative humidity. Recall that $a = 4.2 \times 10^9$ Kelvins and $b = 5400$ Kelvins.

(Enter as an expression to 2 decimal places.)

$W_T =$ 0.9391309 **Answer:** 0.94

(Enter as an expression to 1 decimal place.)

$W_H =$ 16.29493 **Answer:** 16.3

Solution:

Use the expression for dW to find $\dfrac{\partial W}{\partial T}$ and $\dfrac{\partial W}{\partial H}$.

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$$dW = \frac{ae^{-b/T}\,dH + \left(1 + \frac{abH}{T^2}e^{-b/T}\right)\,dT}{1 + \frac{ab}{W^2}e^{-b/W}} \quad (6.326)$$

$$\frac{\partial W}{\partial T} = \frac{\left(1 + \frac{abH}{T^2}e^{-b/T}\right)}{1 + \frac{ab}{W^2}e^{-b/W}} \quad (6.327)$$

$$\frac{\partial W}{\partial H} = \frac{ae^{-b/T}}{1 + \frac{ab}{W^2}e^{-b/W}} \quad (6.328)$$

Plugging in the appropriate values for W, T, H, a and b, we get

$$W_T = 0.94 \quad (6.329)$$

$$W_H = 16 \quad (6.330)$$

Submit

You have used 1 of 5 attempts

Answers are displayed within the problem

Which creates a larger heat stress?

1/1 point (graded)

A warm summertime day in Boston has relative humidity around 70% (H = 0.7), temperature around 85 °F (T = 300 K), and a wet-bulb temperature of W = 295 K (71° F).

Use linear approximation to determine which would produce a larger increase in heat stress as measured by wet-bulb temperature:

- ○ a 5° F (2.8 K) increase in actual temperature
- ◉ a 0.2 increase (20%) in relative humidity
- ○ they increase heat stress by the same amount

✓

Solution:

Using the differential as a placeholder for the linear approximation, we get

$$dW = \frac{\partial W}{\partial T}dT + \frac{\partial W}{\partial H}dH \quad (6.331)$$

$$\Delta W \approx \frac{\partial W}{\partial T}\Delta T + \frac{\partial W}{\partial H}\Delta H \quad (6.332)$$

We compare ΔW for the two cases under consideration.

- $\Delta T = 2.8$ and $\Delta H = 0$, and
- $\Delta T = 0$ and $\Delta H = 0.20$,

In the first case,

$$\Delta W \approx 0.94\Delta T = 0.94\,(2.8) = 2.6 \quad (6.333)$$

In the second case,

$$\Delta W \approx 16\Delta H = 16.3\,(0.2) = 3.2$$

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Thus we see that increasing humidity by 20% will have a greater affect on the wet-bulb temperature.

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Increasing actual temperature

2/2 points (graded)

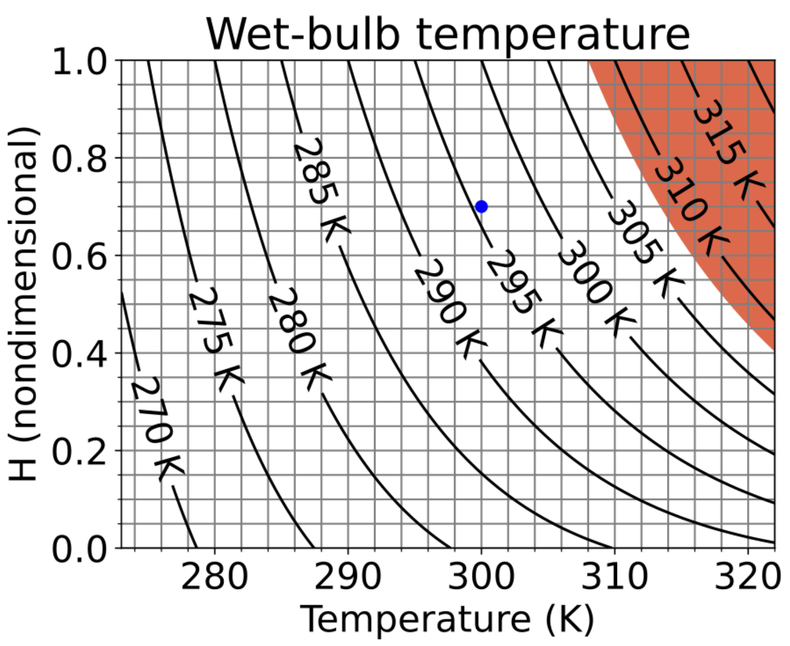
A warm summertime day in Boston has relative humidity around 70% (H = 0.7), temperature around 85 °F (T = 300 K), and a wet-bulb temperature of W = 295 K (71° F).

Approximate how large of an increase in actual temperature would be required to exceed the maximum wet-bulb temperature (308 K) that humans can tolerate.

(Round to one decimal place.)

13.84259 K ✔ **Answer:** 13.8

Compare the approximation to the temperature increase seen in the level curves image. According to the image, how large of an increase in actual temperature would be required to exceed the maximum wet-bulb temperature (308 K) that humans can tolerate? (Enter to the nearest degree.)



(Enter to one decimal place.)

14 K ✔ **Answer:** 13.5

Solution:

Increasing the wet-bulb temperature to the human limit of 308 K requires a 13 K increase from the 295 K wet-bulb temperature typical of a warm summer day in Boston. This requires an actual temperature increase of about

$$dT = dW \frac{1}{\frac{\partial W}{\partial T}} \tag{6.335}$$

$$= \frac{13}{0.94} \approx 13.98 \text{ K} \tag{6.336}$$

So to the nearest degree Kelvin, the change in temperature is 13.8 K, which is about 24° F.

Comparing to the level curves image, it looks like the actual temperature increase required is closer to 13.5 K.

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Increasing relative humidity

2/2 points (graded)

A warm summertime day in Boston has relative humidity around 70% (H = 0.7), temperature around 85 °F (T = 300 K), and a wet-bulb temperature of W = 295 K (71° F).

Approximate the maximum wet-bulb temperature experienced by increasing the relative humidity to 1. (Enter to the nearest tenth, one decimal place.)

299.9 K **Answer:** 299.9

Note that when the relative humidity is equal to 1, we expect the wet-bulb and actual temperatures to agree. What is the error in your approximation above?

(Enter as a percentage. Do not enter the percent symbol. It is provided for you. Three decimal places is enough.)

0.033 % **Answer:** 0.033

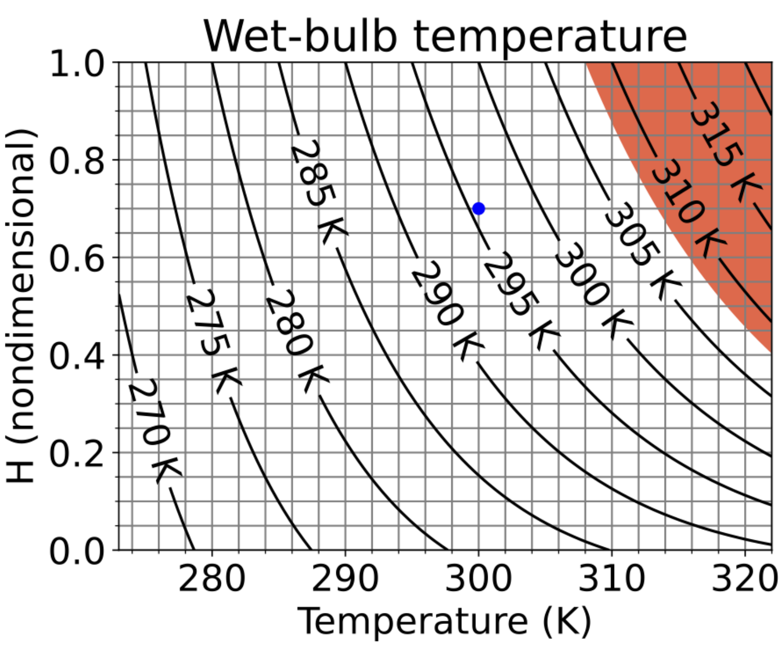
Solution:

The maximum possible ΔH on a day with $H = 0.7$ is $\Delta H = 0.3$ (because H takes on values between 0 and 1). Therefore

$$\begin{aligned} dW &= \frac{\partial W}{\partial H} dH && (6.337) \\ \Delta W &\approx \frac{\partial W}{\partial H} \Delta H && (6.338) \\ \Delta W &\approx 16.3 \Delta H && (6.339) \\ &\approx 16\,(0.3) = 4.89 && (6.340) \end{aligned}$$

Therefore $W + \Delta W \approx 299.9$. Which is close to the 300 we would expect. The error is on the order of 0.00033, or 0.033%.

So, for typical summertime conditions in Boston, the wet-bulb temperature cannot increase above the human tolerance limit through increases in relative humidity alone. This is consistent with the level curves of wet-bulb temperature.



If you move upward from the blue dot, increasing relative humidity while keeping temp

100% relative humidity before intersecting with the red shaded area.

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5. Temperature and humidity

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- Increasing relative humidity one chance only! 1
- [STAFF] What % error definition is being used in Increasing Relative Humidity 5
  Are you using the formula 100*(approximate value-actual value)/(actual value)? Do you expect a percentage error with the appropria...
- [STAFF] Problem with answer input for Take the differential 6
  Even though I have the correct equation, as revealed by subsequent ansers in this exercise, the grader keeps giving me the messag...
- [Staff] PS5B not showing up in "Progress" tab 1 new_ 3
  My "Progress" tab is empty for PS5B, even though I've answered most of the questions.
- [Staff] Increasing Actual Temperature - nearest degree vs one decimal place 5
  The instructions say to read off the level curves diagram to one decimal place, but I don't think we can read that accurately, did you ...

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