6. Linear algebra and Fourier analogy

Here we provide a table that outlines the analogy between linear algebra and Fourier techniques. On the left hand side, we present a general linear algebra example, on the right hand side, we illustrate the analogy using the specific example we worked through on the previous two pages. We wanted to introduce this analogy early, but you may want to come back and review this analogy once you have had more practice solving the heat equation.

System of ODEs	**The Heat Equation**
vector $\mathbf{v}$	function $v\left(x\right)$
matrix A	linear operator $\frac{d^2}{dx^2}$
$A\mathbf{v} = \mathbf{f}$	$\frac{d^2}{dx^2}v\left(x\right) = f\left(x\right)$ on $0 < x < \pi; v\left(0\right) = v\left(\pi\right) = 0$
eigenvalue-eigenvector problem	eigenvalue-eigenfunction problem
$A\mathbf{v} = \lambda\mathbf{v}$	$\frac{d^2}{dx^2}v = \lambda v$, $v\left(0\right) = 0$, $v\left(\pi\right) = 0$
eigenvalues $\lambda_1, \lambda_2, \ldots \lambda_N$	eigenvalues $\lambda = -1, -4, -9, \ldots, -n^2, \ldots$, for $n = 1, 2, 3, \ldots$.
eigenvectors $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_n$	eigenfunctions $v\left(x\right) = \sin nx$ for $n = 1, 2, 3, \ldots$.
linear system of ODEs	Heat Equation with boundary conditions
$\dot{\mathbf{x}} = A\mathbf{x}$	$\dot{\theta} = \frac{\partial^2}{\partial x^2}\theta$ on $0 < x < \pi, \quad \theta\left(0, t\right) = 0, \quad \theta\left(\pi, t\right) = 0$

normal modes: $e^{\lambda_n t}\mathbf{v}_n$ for $n = 1, \ldots, N$	normal modes: $e^{\lambda t} v(x) = e^{-n^2 t}\sin nx$ for $n = 1, 2, 3 \ldots.$
General solution: $\mathbf{u}(t) = \sum c_n e^{\lambda_n t}\mathbf{v}_n$	General solution: $\theta(x,t) = \sum b_n e^{-n^2 t}\sin nx$
Solve $\mathbf{u}(0) = \sum c_n \mathbf{v}_n$ to get the c_n	Solve $\theta(x,0) = \sum b_n \sin nx$ to get the b_n

Warning: This analogy does carry through in other examples and different homogeneous boundary conditions. We used the specific example for illustration purposes only.

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