Problem 1: Two five-sided dice

(3/4 points)
You roll two five-sided dice. The sides of each die are numbered from 1 to 5. The dice are “fair" (all sides are equally likely), and the two die rolls are independent.

Part (a): Event A is “the total is 10" (i.e., the sum of the results of the two die rolls is 10).

1. Is event A independent of the event “at least one of the dice resulted in a 5"?

   No ✔ **Answer:** No

2. Is event A independent of the event “at least one of the dice resulted in a 1"?

   No ✔ **Answer:** No

Part (b): Event B is “the total is 8."

1. Is event B independent of getting “doubles" (i.e., both dice resulting in the same number)?

Problem Set 2 due Feb 17, 2016 at 23:59 UTC

- Unit 3: Counting
- Unit 4: Discrete random variables
- Exam 1
- Unit 5: Continuous random variables
- Unit 6: Further topics on random variables
- Unit 7: Bayesian inference
- Exam 2
- Unit 8: Limit theorems and classical statistics
- Unit 9: Bernoulli and Poisson processes

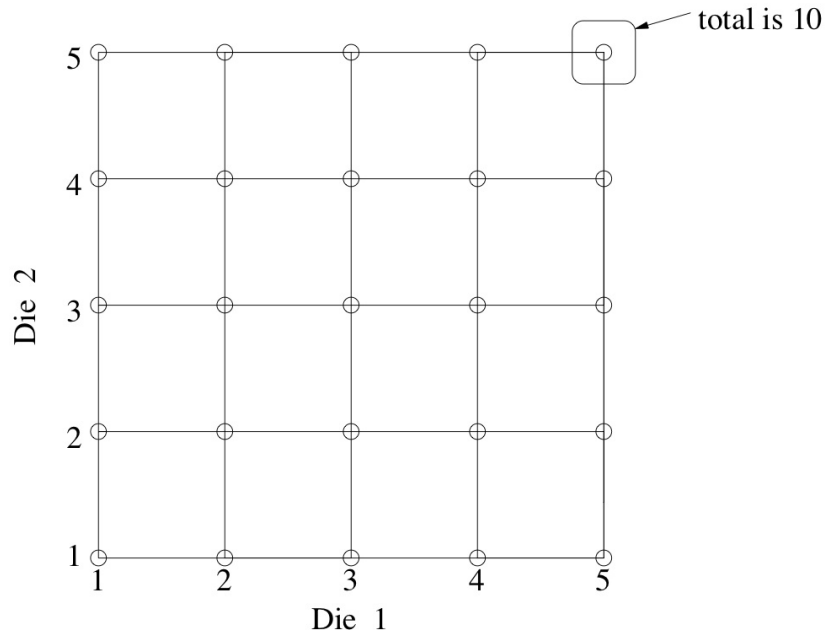
No ✔ **Answer:** No

2. Given that the total was 8, what is the probability that at least one of the dice resulted in a 3?

0.4 ✖ **Answer:** 0.66667

Answer:
Part (a):

1. No. A mathematical derivation is as follows:
   Let event A be “the total is 10," and event C be “at least one of the dice resulted in a 5".



Overall, there are 25 possible and equally likely outcomes. For a total of 10, we must get a 5 on both dice. Therefore, out of the 25 outcomes, only one of them will result in a total of 10. Therefore, $\mathbf{P}(A) = \frac{1}{25}$.

Next, for at least one die to result in a 5, we can have 5 on the first die, a 5 on the second die, or a 5 on both dice. This corresponds to 9 possible outcomes and so $\mathbf{P}(C) = \frac{9}{25}$.

We then notice that if we have a total of 10 (event A), then both dice must have resulted in a 5, and event C also occurs. Thus,

$$\mathbf{P}(A \cap C) = \mathbf{P}(A) = \frac{1}{25} \neq \mathbf{P}(A) \cdot \mathbf{P}(C) = \frac{1}{25} \cdot \frac{9}{25}.$$

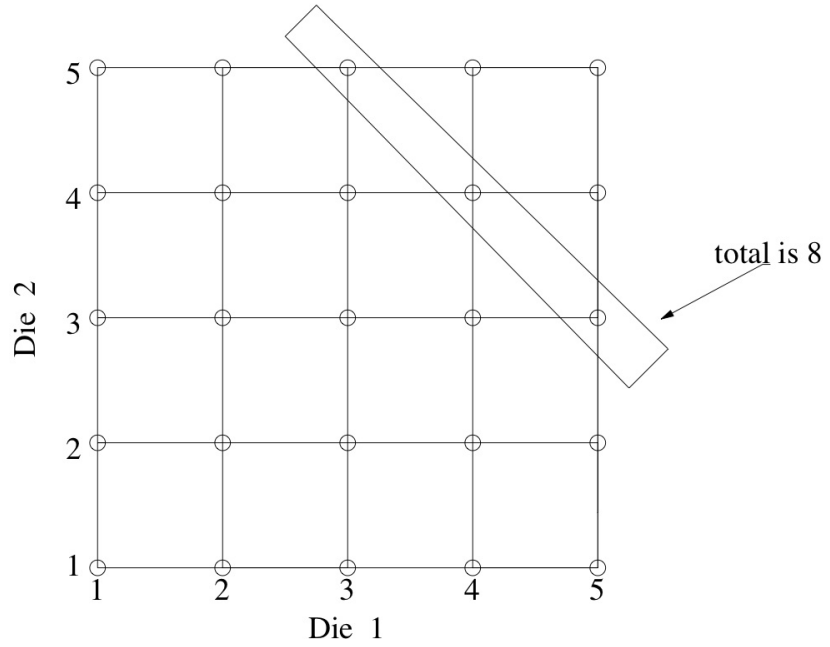
2. No. Let A be the event that “the total is 10", and let D be the event “at least one of the dice resulted in a 1". Similar to event C described in part 1, $\mathbf{P}(D) = \frac{9}{25}$.

   Next, let us consider $\mathbf{P}(A \cap D)$. We notice that if one of the dice resulted in a 1, it is impossible to get a total of 10. Therefore, $\mathbf{P}(A \cap D) = \mathbf{P}(\emptyset) = \mathbf{0}$, and

$$\mathbf{0} = \mathbf{P}(A \cap D) \neq \mathbf{P}(A) \cdot \mathbf{P}(D) > \mathbf{0}.$$

Part (b):

1. No. Let B be the event “the total is 8" and let E be the event that doubles are obtained.



Event B consists of the three outcomes $(3,5)$, $(4,4)$, and $(5,3)$. Therefore, $\mathbf{P}(B) = \frac{3}{25}$.

Event E occurs in 5 out of the 25 possible outcomes, and so $\mathbf{P}(E) = \frac{5}{25} = \frac{1}{5}$.

Therefore,

$$\mathbf{P}(B \cap E) = \mathbf{P}(\{(4,4)\}) = \frac{1}{25} \neq \mathbf{P}(B) \cdot \mathbf{P}(E) = \frac{3}{25} \cdot \frac{1}{5}.$$

2.

$$\begin{aligned}\mathbf{P}(\text{at least one 3} \mid \text{total is 8}) &= \frac{\mathbf{P}(\text{at least one 3 and total is 8})}{\mathbf{P}(\text{total is 8})} \\ &= \frac{\mathbf{P}(\{(3,5),(5,3)\})}{\mathbf{P}(B)} \\ &= \frac{2/25}{3/25} \\ &= \frac{2}{3}.\end{aligned}$$

You have used 1 of 1 submissions

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DISCUSSION

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