# The Easy Hat-Problem

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## Paradox Grade: 0

You may have heard of the following brain teaser:

> *Setup:* Ten prisoners, $P_1, P_2, \ldots P_{10}$, are standing in line. $P_1$ is at the end of the line, in front of her is $P_2$, in front of him is $P_3$, and so forth, with $P_{10}$ standing at the front of the line. Each of them is wearing a hat. The color of each prisoner's hat determined by the toss of a fair coin: people whose coin lands Heads get a blue hat, people whose coin lands Tails get a red hat. Nobody knows the color of her own hat, but each prisoner can see the colors of the hats of everyone standing in front of her. (For instance, $P_6$ can see the colors of the hats worn by $P_7, P_8, P_9$, and $P_{10}$.) Now assume that a guard will start at the end of the line, and ask each prisoner to guess the color of her own hat. Prisoners answer in turn, with everyone being in a position to hear the answers of everyone else. Prisoners who correctly call out the color of their own hats will be spared. Everyone else will be shot.
>
> *Problem:* if the prisoners are allowed to confer with each other beforehand, is there a strategy they can agree upon in advance to improve their chances of surviving?

The answer is "yes".

In fact, the prisoners can guarantee that $P_2, \ldots, P_{10}$ survive, and that $P_1$ gets a 50% chance of survival. (I'll ask you to verify this in the exercise below.)

That's a fun puzzle. It's a bit tricky, but it's certainly not paradoxical. Here we will be concerned with a much harder version of the hat-problem, one which arguably *is* paradoxical. Read on …

## Problem 1

1/1 point (ungraded)

Identify a strategy that guarantees that $P_2, \ldots, P_{10}$ survive, and gives $P_1$ a 50% chance of survival.

- ◯ Each person, $P_1, \ldots, P_{10}$, looks at the color of the hat immediately in front of them. If the person sees a red hat, they say "Blue!". If they see a blue hat, they say "Red!".
- ◯ Let $P_1$ announce "Red!" if they see more red hats than blue hats. Announce "Blue!" otherwise.
- ◉ Let $P_1$ announce "Red!" if they see an odd number of red hats and announce "Blue!" otherwise.
- ◯ Person $P_1$ announces the color of the hat of person $P_2$. Everyone else announces *the opposite* of the color that was announced previously.
- ◯ Let everyone announce "red!" if they see an even number of blue hats and announce "Blue!" otherwise.

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**Explanation**

Let $P_1$ say "Red!" if she sees an odd number of red hats in front of her, and "Blue!" otherwise. Since the outcomes fair coin tosses are independent of one another, this gives $P_1$ a 50% chance of survival.

Now consider things from $P_2$'s perspective. If $P_1$ said "Red!", $P_2$ knows that the number of red hats amongst $P_2, \ldots, P_{10}$ is odd. But $P_2$ can see the colors of the hats on $P_3, \ldots, P_{10}$, so she has all the information she needs to figure out whether her own hat is red. (The case in which $P_1$ says "Blue!" is similar.)

$P_2$ now says the color of her hat. So $P_3$ knows (i) that the number of red hats amongst $P_2, \ldots, P_{10}$ is odd, (ii) the color of $P_2$'s hat, and (iii) the colors of the hats on $P_4, \ldots, P_{10}$. So she has all the information she needs to figure out the color of her own hat.

By iterating this process, $P_2, \ldots, P_{10}$ can each figure out the color of their own hats.

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Answers are displayed within the problem

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