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Bookmark

Exercise: Normal unknown and additive noise

(3/4 points)

As in the last video, let $\mathbf{X} = \boldsymbol{\Theta} + \mathbf{W}$, where $\boldsymbol{\Theta}$ and \mathbf{W} are independent normal random variables and \mathbf{W} has mean zero.

a) Assume that \mathbf{W} has positive variance. Are \mathbf{X} and \mathbf{W} independent?

No ▼



Answer: No

b) Find the MAP estimator of $\boldsymbol{\Theta}$ based on \mathbf{X} if $\boldsymbol{\Theta} \sim N(1, 1)$ and $\mathbf{W} \sim N(0, 1)$, and evaluate the corresponding estimate if $\mathbf{X} = 2$.

 $\hat{\theta} =$

3/2



Answer: 1.5

c) Find the MAP estimator of $\boldsymbol{\Theta}$ based on \mathbf{X} if $\boldsymbol{\Theta} \sim N(0, 1)$ and $\mathbf{W} \sim N(0, 4)$, and evaluate the corresponding estimate if $\mathbf{X} = 2$.

 $\hat{\theta} =$

2/5



Answer: 0.4

d) For this part of the problem, suppose instead that $\mathbf{X} = 2\boldsymbol{\Theta} + 3\mathbf{W}$, where $\boldsymbol{\Theta}$ and \mathbf{W} are standard normal random variables. Find the MAP estimator of $\boldsymbol{\Theta}$ based on \mathbf{X} under this model and evaluate the corresponding estimate if $\mathbf{X} = 2$.

 $\hat{\theta} =$

16/25



Answer: 0.30769

Answer:

a) They are not independent. This is intuitively clear because \mathbf{W} has an effect on \mathbf{X} . Another way to see it is that we have (by independence of $\boldsymbol{\Theta}$ and \mathbf{W}) that $\mathbf{E}[\boldsymbol{\Theta}\mathbf{W}] = \mathbf{E}[\boldsymbol{\Theta}] \mathbf{E}[\mathbf{W}] = 0$, which leads to

$$\mathbf{E}[\mathbf{X}\mathbf{W}] = \mathbf{E}[(\boldsymbol{\Theta} + \mathbf{W})\mathbf{W}] = \mathbf{E}[\mathbf{W}^2] \neq 0 = \mathbf{E}[\mathbf{X}] \mathbf{E}[\mathbf{W}],$$

Unit overview

Lec. 14:
Introduction to
Bayesian inference

 Exercises 14 due Apr
 06, 2016 at 23:59 UTC

Lec. 15: Linear
models with
normal noise

 Exercises 15 due Apr
 06, 2016 at 23:59 UTC

Problem Set 7a

 Problem Set 7a due
 Apr 06, 2016 at 23:59
 UTC

Lec. 16: Least
mean squares
(LMS) estimation

 Exercises 16 due Apr
 13, 2016 at 23:59 UTC

Lec. 17: Linear
least mean
squares (LLMS)
estimation

 Exercises 17 due Apr
 13, 2016 at 23:59 UTC

Problem Set 7b

 Problem Set 7b due
 Apr 13, 2016 at 23:59
 UTC

Solved problems

 Additional
 theoretical
 material

Unit summary

which in turn implies that \mathbf{X} and \mathbf{W} are not independent.

b) If we focus on the terms that involve θ , the posterior is of the form

$$c(x)e^{-(\theta-1)^2/2}e^{-(x-\theta)^2/2}.$$

To find the MAP estimate, we set the derivative with respect to θ of the exponent to zero, so that $(\hat{\theta} - 1) + (\hat{\theta} - x) = 0$, or $\hat{\theta} = (1 + x)/2$, which, when $x = 2$, evaluates to $3/2$.

c) If we focus on the terms that involve θ , the posterior is of the form

$$c(x)e^{-\theta^2/2}e^{-(x-\theta)^2/(2 \cdot 4)}.$$

To find the MAP estimate, we set the derivative with respect to θ of the exponent to zero, so that $\hat{\theta} + (\hat{\theta} - x)/4 = 0$, or $\hat{\theta} = x/5$, which, when $x = 2$, evaluates to $2/5$.

d) Note that conditional on $\Theta = \theta$, the random variable \mathbf{X} is normal with mean 2θ and variance 9 . If we focus on the terms that involve θ , the posterior is of the form

$$c(x)e^{-\theta^2/2}e^{-(x-2\theta)^2/(2 \cdot 9)}.$$

To find the MAP estimate, we set the derivative with respect to θ of the exponent to zero, so that $\hat{\theta} + 2(2\hat{\theta} - x)/9 = 0$, or $\hat{\theta} = 2x/13$, which, when $x = 2$, evaluates to $4/13$.

You have used 3 of 3 submissions

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