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[Lecture 6: Introduction to Hypothesis Testing, and Type 1 and](#)

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> 12. Statistical Tests

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12. Statistical Tests

Statistical Tests

Asymmetry in the hypotheses

▶ H_0 and H_1 do not play a symmetric role: the data is only used to try to disprove H_0

▶ In particular lack of evidence, does not mean that H_0 is true

▶ how to choose the threshold c .

▶ A *test* is a statistic $\psi \in \{0, 1\}$ such that:

- ▶ If $\psi = 0$, H_0 is not rejected;
- ▶ If $\psi = 1$, H_0 is rejected. $\Leftrightarrow H_1$

▶ Coin example: $H_0: p = 1/2$ vs. $H_1: p \neq 1/2$.

▶ $\psi = \mathbb{I}\left\{\frac{\sqrt{n}|\bar{X}_n - \frac{1}{2}|}{\sqrt{0.5(1.0.5)}} > C\right\}$, for some $C > 0$.

▶ How to choose the *threshold* C ?

$\psi(x) = 1 \Leftrightarrow \psi(x) = 0$

▶ 6:19 / 6:19

▶ 1.50x ▶ 🔊 🔍 📄 🗣️

Video[Download video file](#)**Transcripts**[Download SubRip \(.srt\) file](#)[Download Text \(.txt\) file](#)**Which Statistics are Tests?**

1/1 point (graded)

Recall that a **statistic** is, intuitively speaking, a function that can be computed from the data.

A **(statistical) test** is an **statistic** whose output is **always** either 0 or 1, and like an estimator, does not depend explicitly on the value of true unknown parameter.

Let $X_1, \dots, X_n \stackrel{iid}{\sim} \text{Ber}(\theta)$ for some unknown parameter $\theta \in (0, 1)$. Which of the following statistics are also tests?

(Recall that $\mathbf{1}(A)$ is the indicator defined as follows: $\mathbf{1}(A) = \begin{cases} 1 & \text{if } A \text{ occurs} \\ 0 & \text{otherwise} \end{cases}$.)

(Choose all that apply.)

☐ \bar{X}_n

☒ $\mathbf{1}(\bar{X}_n > 0.5)$

☒ $\mathbf{1}(|\bar{X}_n - 0.5| > 0.01)$

☐ $\mathbf{1}(|\bar{X}_n - \theta| > 0.5)$

☒ $\mathbf{1}(\bar{X}_n \text{ is a rational number})$



Solution:

We examine the choices in order.

- $\bar{X}_n := \frac{1}{n} \sum_{i=1}^n X_i$ is **not** a statistical test, because the sample average is not **always** either 0 or 1.
- $\mathbf{1}(\bar{X}_n > 0.5)$ is a statistical test. Its expression only depends on the sample (and not the true parameter), and since it is an indicator, it takes values only in $\{0, 1\}$.

- $\mathbf{1}(|\bar{X}_n - 0.5| > 0.01)$ is a statistical test. Its expression only depends on the sample (and not the true parameter), and since it is an indicator, it takes values only in $\{0, 1\}$.
- $\mathbf{1}(|\bar{X}_n - \theta| > 0.5)$ is **not** a statistical test because its output depends on the unknown parameter θ .
- $\mathbf{1}(\bar{X}_n \text{ is a rational number})$ is a statistical test. Its expression only depends on the sample (and not the true parameter), and since it is an indicator, it takes values only in $\{0, 1\}$. This is a rather bizarre test, but it does satisfy all required properties.

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Applying a Statistical Test on a Data Set

1/1 point (graded)

Let $X_1, \dots, X_n \stackrel{iid}{\sim} N(\mu, 1)$ where μ is an unknown parameter. You are interested in answering the **question of interest**: "Does $\mu = 0$?". To do so you construct the **null hypothesis** $H_0 : \mu = 0$ and the **alternative hypothesis** $H_1 : \mu \neq 0$.

You design the test

$$\psi = \mathbf{1}(\sqrt{n} |\bar{X}_n| > 0.25).$$

If $\psi = 1$, you will **reject** the null hypothesis, and if $\psi = 0$, you will **fail to reject**. For simplicity, we will set the sample size to be $n = 7$.

On which of the following data sets would you reject the null hypothesis?
(Choose all that apply. Feel free to use computational tools.)

☒ $-1.0, -0.8, -2.9, 1.4, 0.3, -0.8, 1.4$

☒ $-1.7, -0.1, -0.2, 0.3, 0.3, -0.9, -0.03$

☐ $-0.2, 0.6, 1.1, -0.9, 0.1, -1.2, 1.1$


Solution:

We examine the choices in order.

- The first choice is correct. For this data set, we compute $\sqrt{7} \bar{X}_7 \approx -0.9072$. Since $|-0.9072| > 0.25$, we reject.
- The second choice is correct. For this data set, we compute $\sqrt{7} \bar{X}_7 \approx -0.8768$. Since $|-0.8768| > 0.25$, we reject.
- The third choice is incorrect. For this data set, we compute $\sqrt{7} \bar{X}_7 \approx -0.2267$. Since $|0.2267| \leq 0.25$, we fail to reject.

Remark 1: It is useful to keep in mind the following mnemonic,

$$\psi = 0 \Rightarrow H_0$$

$$\psi = 1 \Rightarrow H_1.$$

Of course, the implications above are informal and should not be taken literally. To be precise, we say that if $\psi = 0$, we fail to reject H_0 , and if $\psi = 1$, then we reject H_0 in favor of H_1 .

Remark 2: If we assume the null hypothesis $H_0 : \mu = 0$, then since the variance is known to be 1, the CLT guarantees that

$$\sqrt{n} \bar{X}_n \xrightarrow[n \rightarrow \infty]{(d)} \mathcal{N}(0, 1).$$

The quantiles of $\mathcal{N}(0, 1)$ can be understood using tables or computational software, so if n is very large, then we can approximate the probability of our test ψ **rejecting** or **failing to reject** under the null hypothesis. This concept will be further explored in the next page where we explore the "type 1" and "type 2 error" of a test.

You have used 1 of 2 attempts

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