9.2.1 Accuracy and Convergence

MO2.4 MO2.6

As the timestep is decreased, i.e. $\Delta t \to 0$, we would like our numerical method to better approximate the exact solution to the model equations. Let's define the error in our approximation at timestep n to be:

$$\underline{e}^n \equiv \underline{v}^n - \underline{u}\left(t^n\right) \tag{9.1}$$

Since this is $\underline{u}\left(t^n\right)$ is in general an M-dimensional vector of states, then so are $\underline{v}^n$ and $\underline{e}^n$. We can define the components of this error then,

$$e_m^n = v_m^n - u_m\left(t^n\right) \tag{9.2}$$

for each $m = 0$ to $M - 1$. Then, to measure the error for the entire simulation (i.e. over all timesteps), we'll take the maximum magnitude,

$$\left(e_m\right)_{\max} = \max_n \left|e_m^n\right| \tag{9.3}$$

Then, in terms of these error definitions, we desire that the maximum error in each state goes to zero as $\Delta t \to 0$. This concept is known as convergence and is stated mathematically as follows:

Definition 2 (Convergence).

A numerical method for an Initial Value Problem is convergent from $t = t_I$ *to* t_F *if for all* m,

$$\left(e_m\right)_{\max} \to 0 \qquad \text{as} \qquad \Delta t \to 0.$$

While convergence is a clear requirement for a good numerical method, the rate at which the method converges is also important. This rate is known as the order of accuracy.

Definition 3 (Order of Accuracy).

A method has an order of accuracy of p *if, for all* m,

$$\left(e_m\right)_{\max} = O\left(\Delta t^p\right) \qquad \text{as} \qquad \Delta t \to 0,$$

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$$(e_m)_{\max} = C\Delta t^p \qquad \text{as} \qquad \Delta t \to 0.$$