UTAustinX UT.5.05x

Linear Algebra - Foundations to Frontiers

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E2.3.5 Question 9

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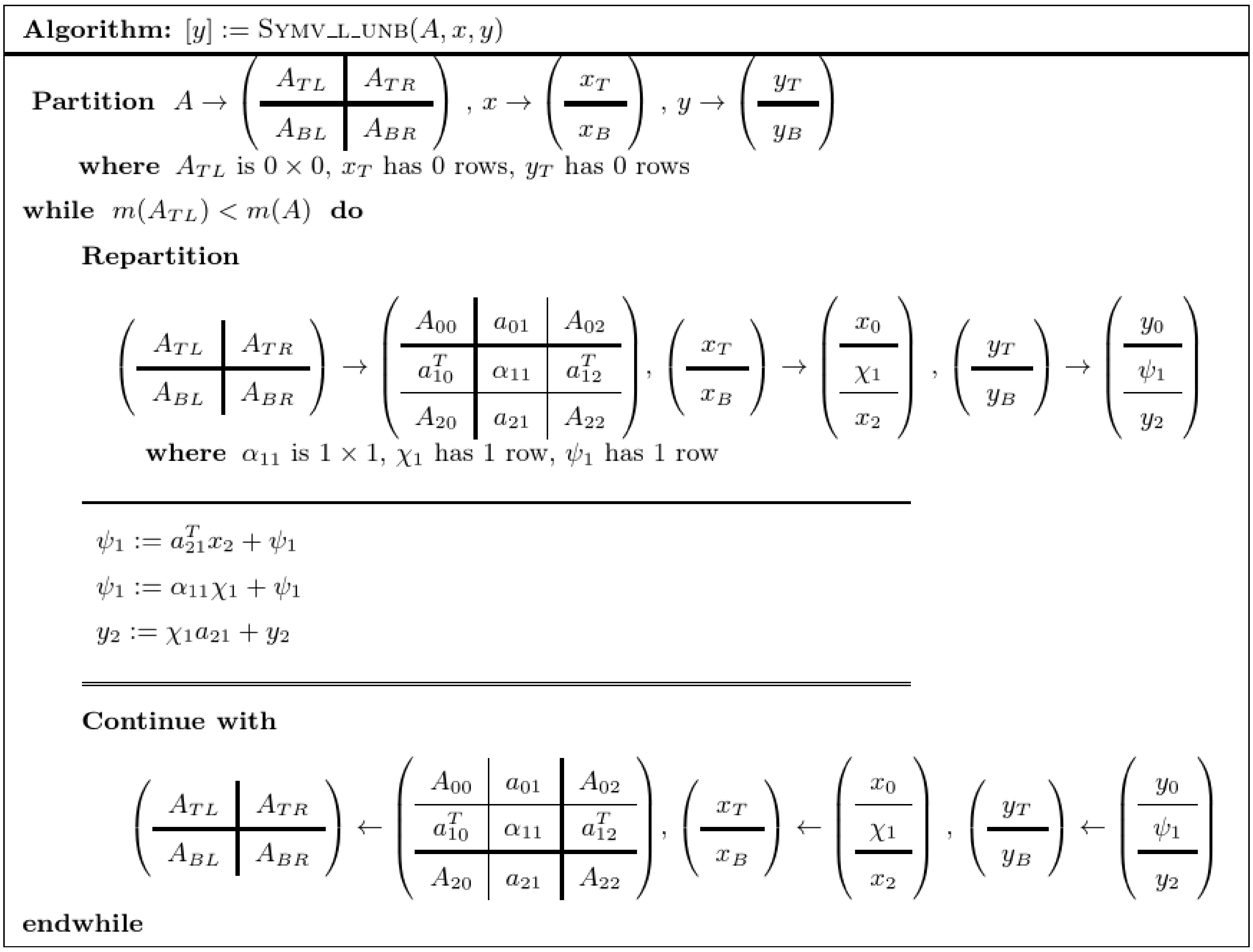
Exam 2 due Dec 3, 2023 04:42 IST Completed

E2.3.5 Question 9

Question 9

10.0/10.0 points (graded)

Consider the following algorithm for computing $y := Ax + y$ where A is an $n \times n$ symmetric matrix that is stored in the lower triangular part of the matrix.



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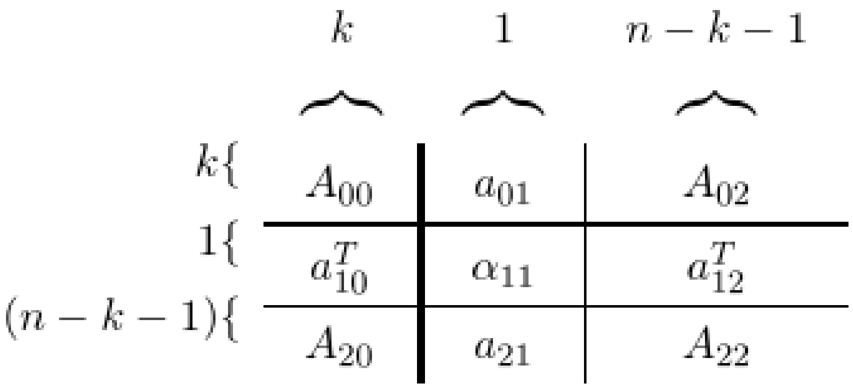
Algorithm: $[y] := \text{SYMV\_L\_UNB}(A, x, y)$
Partition $A \rightarrow \left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right)$, $x \rightarrow \left(\begin{array}{c} x_T \\ \hline x_B \end{array}\right)$, $y \rightarrow \left(\begin{array}{c} y_T \\ \hline y_B \end{array}\right)$
    where $A_{TL}$ is $0 \times 0$, $x_T$ has 0 rows, $y_T$ has 0 rows
while $m(A_{TL}) < m(A)$ do
    Repartition
        $\left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c|c|c} A_{00} & a_{01} & A_{02} \\ \hline a_{10}^T & \alpha_{11} & a_{12}^T \\ \hline A_{20} & a_{21} & A_{22} \end{array}\right)$, $\left(\begin{array}{c} x_T \\ \hline x_B \end{array}\right) \rightarrow \left(\begin{array}{c} x_0 \\ \hline \chi_1 \\ \hline x_2 \end{array}\right)$, $\left(\begin{array}{c} y_T \\ \hline y_B \end{array}\right) \rightarrow \left(\begin{array}{c} y_0 \\ \hline \psi_1 \\ \hline y_2 \end{array}\right)$
        where $\alpha_{11}$ is $1 \times 1$, $\chi_1$ has 1 row, $\psi_1$ has 1 row

    $\psi_1 := a_{21}^T x_2 + \psi_1$
    $\psi_1 := \alpha_{11} \chi_1 + \psi_1$
    $y_2 := \chi_1 a_{21} + y_2$

    Continue with
        $\left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c|c|c} A_{00} & a_{01} & A_{02} \\ \hline a_{10}^T & \alpha_{11} & a_{12}^T \\ \hline A_{20} & a_{21} & A_{22} \end{array}\right)$, $\left(\begin{array}{c} x_T \\ \hline x_B \end{array}\right) \leftarrow \left(\begin{array}{c} x_0 \\ \hline \chi_1 \\ \hline x_2 \end{array}\right)$, $\left(\begin{array}{c} y_T \\ \hline y_B \end{array}\right) \leftarrow \left(\begin{array}{c} y_0 \\ \hline \psi_1 \\ \hline y_2 \end{array}\right)$
endwhile
```

During the kth iteration this captures the sizes of the submatrices:



$$\begin{array}{r|c|c|c} & \overbrace{\qquad}^{k} & \overbrace{\qquad}^{1} & \overbrace{\qquad}^{n-k-1} \\ k\{ & A_{00} & a_{01} & A_{02} \\ \hline 1\{ & a_{10}^T & \alpha_{11} & a_{12}^T \\ \hline (n-k-1)\{ & A_{20} & a_{21} & A_{22} \end{array}$$

(a) $\psi_1 := a_{21}^T x_2 + \psi_1$ is an example of a __________ operation.

dot ✓ Answer: dot

During the kth iteration it requires approximately __________ floating point operations.

○ $2k$

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○ $2\,(n-1)$

● $2\,(n-k-1)$

✓

(b) $y_2 := \chi_1 a_{21} + y_2$ is an example of a __________ operation.

axpy ✓ Answer: axpy

During the kth iteration it requires approximately __________ floating point operations.

○ $2k$

○ $2\,(n-1)$

● $2\,(n-k-1)$

✓

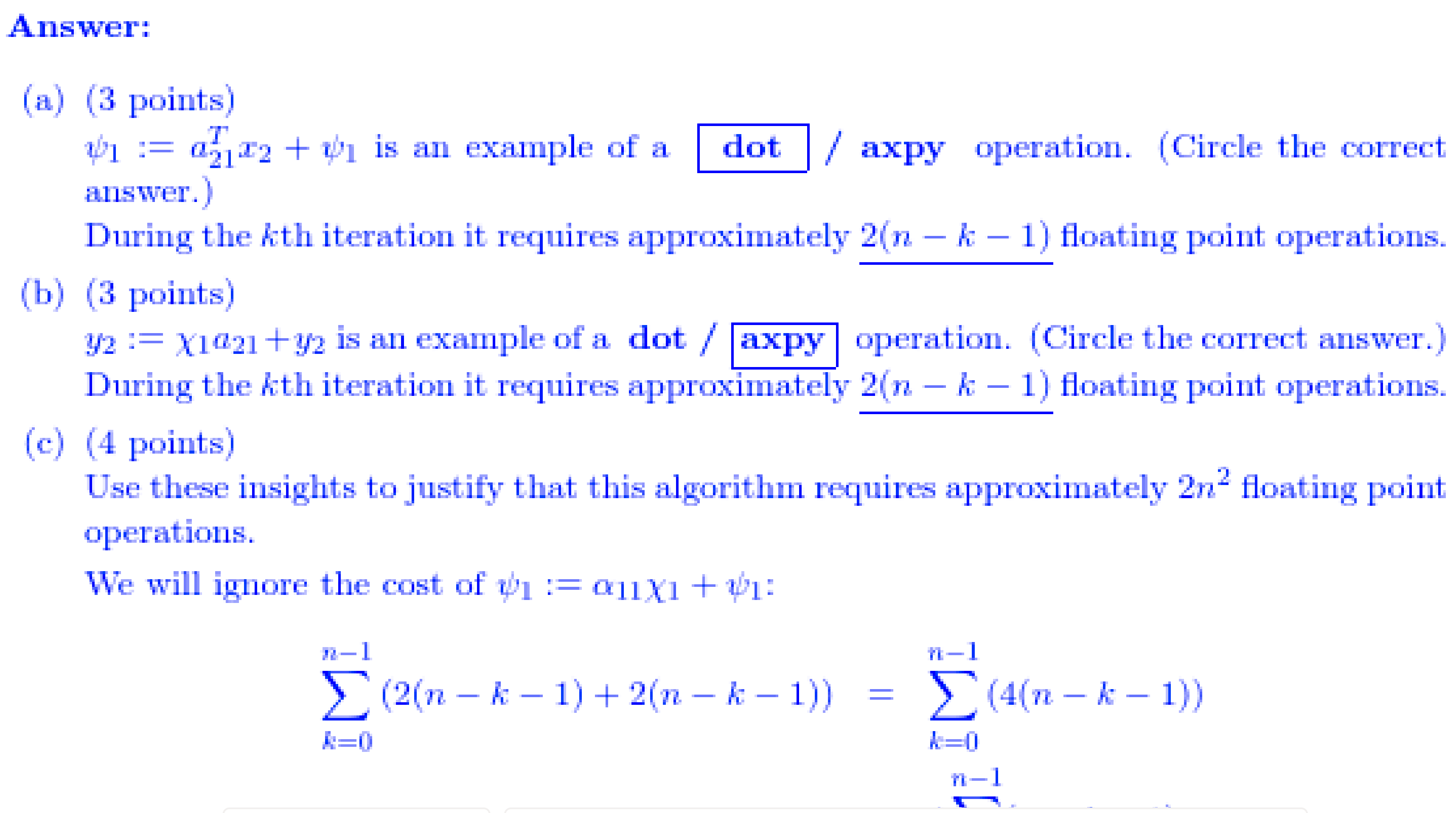
(c) What is the approximate cost of the above algorithm (in floating point operations) when executed with an $n \times n$ matrix.

○ $2n^3$

● $2n^2$

○ $2(n-k-1)^2$

✓



Answer:

(a) (3 points)
$\psi_1 := a_{21}^T x_2 + \psi_1$ is an example of a $\boxed{\textbf{dot}}$ / **axpy** operation. (Circle the correct answer.)
During the kth iteration it requires approximately $\underline{2(n-k-1)}$ floating point operations.

(b) (3 points)
$y_2 := \chi_1 a_{21} + y_2$ is an example of a **dot** / $\boxed{\textbf{axpy}}$ operation. (Circle the correct answer.)
During the kth iteration it requires approximately $\underline{2(n-k-1)}$ floating point operations.

(c) (4 points)
Use these insights to justify that this algorithm requires approximately $2n^2$ floating point operations.
We will ignore the cost of $\psi_1 := \alpha_{11}\chi_1 + \psi_1$:

$$\sum_{k=0}^{n-1} (2(n-k-1) + 2(n-k-1)) = \sum_{k=0}^{n-1} (4(n-k-1))$$

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