2. Review the exponential response formula

Recall that the exponential response formula gives us a quick method for finding the particular solution to any linear, constant coefficient, differential equations whose input can be expressed in terms of an exponential function.

The exponential response formula (ERF): Let P be a polynomial with real, constant coefficients, $D = \frac{d}{dt}$ a differential operator, and r a (real or complex) number. If $P(r) \neq 0$, then a particular solution to the inhomogeneous differential equation

$$P(D)\,y = e^{rt} \qquad \text{is given by} \qquad y_p = \frac{e^{rt}}{P(r)}.$$

Caveat

Caveat: If $P(r) = P'(r) = P''(r) = \ldots = P^{(k-1)}(r) = 0$, but $P^{(k)}(r) \neq 0$, then a particular solution to $P(D)\,y = e^{rt}$ is given by

$$y_p = \frac{t^k e^{rt}}{P^{(k)}(r)}.$$

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Sinusoidal input:

$P(D)\,x = \cos(\omega t)$	is the real part of	$P(D)\,z = e^{i\omega t}$;
$P(D)\,x = \sin(\omega t)$	is the imaginary part of	$P(D)\,z = e^{i\omega t}$.

Therefore

- a particular solution to $P(D)\,x = \cos(\omega t)$ is given by $x_p = \mathrm{Re}\left[\dfrac{e^{i\omega t}}{P(i\omega)}\right]$;
- a particular solution to $P(D)\,x = \sin(\omega t)$ is given by $x_p = \mathrm{Im}\left[\dfrac{e^{i\omega t}}{P(i\omega)}\right]$.

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A particular solution

A particular solution. So I guess we could add a solution of the homogeneous equation P(D) y = 0 to get other solutions using superposition. 3

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