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Exercise: The hat problem

(2/2 points)

Consider the hat problem, with $n = 10$. What is the expected value of $X_3X_6X_7$?

$\mathbf{E}[X_3X_6X_7] =$ 1/720 ✔ **Answer:** 0.00139

Answer:
By symmetry, this is the same as $\mathbf{E}[X_1X_2X_3]$. Since the product $X_1X_2X_3$ is either zero or one, this is the same as

$$\mathbf{P}(X_1X_2X_3 = 1) = \mathbf{P}(X_1 = 1) \cdot \mathbf{P}(X_2 = 1 \mid X_1 = 1) \cdot \mathbf{P}(X_3 = 1 \mid X_1 = X_2 = 1$$

By thinking in terms of the sequential description of the process, we have seen that $\mathbf{P}(X_1 = 1) = 1/10$ and $\mathbf{P}(X_2 = 1 \mid X_1 = 1) = 1/9$. By a similar argument, given that the first two people obtained their own hats, the third person is faced with 8 hats, one of which is his/her own, and has probability $\mathbf{P}(X_3 = 1 \mid X_1 = X_2 = 1) = 1/8$ of picking it. Thus, the final answer is $(1/10) \cdot (1/9) \cdot (1/8)$.

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