Problem 5: Indicator variables

(6/6 points)

Consider a sequence of independent tosses of a biased coin at times $k = 0, 1, 2, \ldots, n$. On each toss, the probability of Heads is p, and the probability of Tails is $1 - p$.

A reward of one unit is given at time k, for $k \in \{1, 2, \ldots, n\}$, if the toss at time k resulted in Tails and the toss at time $k - 1$ resulted in Heads. Otherwise, no reward is given at time k.

Let R be the sum of the rewards collected at times $1, 2, \ldots, n$.

We will find $\mathbf{E}[R]$ and $\mathbf{var}(R)$ by carrying out a sequence of steps. Express your answers below in terms of p and/or n using standard notation . Remember to write '*' for all multiplications and to include parentheses where necessary.

We first work towards finding $\mathbf{E}[R]$.

1. Let I_k denote the reward (possibly 0) given at time k, for $k \in \{1, 2, \ldots, n\}$. Find $\mathbf{E}[I_k]$.

$\mathbf{E}[I_k] =$ (1-p)*p

2. Using the answer to part 1, find $\mathbf{E}[R]$.

$\mathbf{E}[R] =$ n*(1-p)*p

The variance calculation is more involved because the random variables $I_1, I_2, \ldots, I_n$ are not independent. We begin by computing the following values.

3. If $k \in \{1, 2, \ldots, n\}$, then

$\mathbf{E}[I_k^2] =$ (1-p)*p

4. If $k \in \{1, 2, \ldots, n - 1\}$, then

Exercises 7 due Mar 02, 2016 at 23:59 UTC

Solved problems

Additional theoretical material

Problem Set 4
Problem Set 4 due Mar 02, 2016 at 23:59 UTC

Unit summary

Unit 5: Continuous random variables

$\mathbf{E}[I_k I_{k+1}] =$ 0

5. If $k \geq 1$, $\ell \geq 2$, and $k + \ell \leq n$, then

$\mathbf{E}[I_k I_{k+\ell}] =$ (1-p)^2*p^2

6. Using the results above, calculate the numerical value of $\mathbf{var}(R)$ assuming that $p = 3/4$, $n = 10$.

$\mathbf{var}(R) =$ 0.890625

You have used 1 of 3 submissions

Your answers have been saved but not graded. Click 'Check' to grade them.

DISCUSSION

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