PurdueX: 416.2x Probability: Distribution Models & Continuous Random Variables

Hel

Unit 7: Quiz

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Unit 7: Quiz

The problems in the quiz will be automatically graded.

You are given only **ONE** attempt for each problem. You will be able to see the solution after you submit your answers.

If the answer is numeric and has more than 4 decimal places, just give your answers to **4 decimal places (0.0001 accuracy)** unless otherwise stated.

e.g. for 0.123456... you just need to round it up to 0.1235

Problem 1

3/3 points (graded)

1. Suppose that X is a random variable with density function

$$f_X(x) = \frac{2}{3}e^{-(2/3)x} \text{ for } x > 0$$

- Unit 9: Models of Continuous Random Variables
- Unit 10: Normal Distribution and Central Limit Theorem (CLT)
- Unit 11: Covariance, Conditional Expectation, Markov and Chebychev Inequalities
- Unit 12: Order Statistics, Moment Generating Functions, Transformation of RVs

and $f_X(x) = 0$ otherwise.

1a. Calculate $P(0.5 < X < 2.5)$.

0.5276557

1b. Calculate $P(X = 2.5)$. (Why do you get that value?)

0

1c. Find a formula for the CDF $F_X(x)$, calculate $F_X(1.5)$.

$F_X(1.5) =$ 0.6321206

Submit You have used 1 of 1 attempt

Problem 2

5/5 points (graded)

2. Suppose that X is a continuous random variable with a probability density function that is a positive constant on the interval $[8, 20]$, and is 0 otherwise.

2a. What is the positive constant mentioned above?

1/12

2b. Calculate $P(10 \leq X \leq 15)$.

5/12

2c. Find an expression for the CDF $F_X(x)$. Calculate the following values.

$F_X(7) =$ 0

$F_X(11) =$ 1/4

$F_X(30) =$ 1

Submit You have used 1 of 1 attempt

Problem 3

3/3 points (graded)

3. Suppose that X has CDF

$$F_X(x) = 1 - e^{-5x} \text{ for } x > 0$$

and $F_X(x) = 0$ otherwise.

3a. What is the 25th percentile of X? I.e., what is the value “a" such that $P(X \leq a) = 1/4$?

0.05753641 ✔

3b. What is the median (also called 50th percentile) of X, i.e., what is the value “a" such that $P(X \leq a) = 1/2$?

0.13862944 ✔

3c. What is the 75th percentile of X?

0.27725887 ✔

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Problem 4

6/6 points (graded)

4. Suppose that X has probability density function

$f_X(x) = x$ for $0 < x < 1$;

$= 2 - x$ for $1 < x < 2$,

$= 0$ otherwise.

4a. Find $P(X \le 3/4)$.

0.28125

4b. Find $P(X \le 5/4)$. (Hint: It is not necessary-but it could be easier-to first find the complementary probability.)

0.7187499

4c. Find a formula for the CDF $F_X(x)$. Calculate the following values.
(Hint: It is worthwhile to do this in a piecewise manner, since $f_X(x)$ is defined piecewise. I.e., it is helpful to find $F_X(x)$ for $0 < x < 1$ and then to find $F_X(x)$ for $1 < x < 2$.)

$F_X(-0.5) =$ 0

$F_X(0.5) =$ 0.125

$F_X(1.5) =$ 0.875

$F_X(2.5) =$ 1

4d. Do your answers to **a** and **b** each agree with your answer to **c**, in the specific cases $x = 3/4$ and $x = 5/4$?

Submit You have used 1 of 1 attempt

Problem 5

5/5 points (graded)

5. Suppose X and Y have a constant joint density on the square with vertices $(0,0),(4,0),(4,4),(0,4)$.

5a. For $0 < a < 4$, find $P(X+Y \leq a)$. Calculate the following value.

$P(X+Y \leq 2) =$ 1/8

5b. For $4 < a < 8$, find $P(X+Y \geq a)$. (Then the complement $P(X+Y \leq a)$ is easy.) Calculate the following value.

$P(X+Y \leq 6) =$ 7/8

5c. If you write $W = X+Y$, the work from **a** and **b** automatically yields an expression for the CDF $F_W(w) = P(W \leq w)$ of W. Differentiate this CDF $F_W(w)$ to find the density $f_W(w)$ of W. Calculate the following values.

$f_W(3) =$ 3/16

$f_W(5) =$ 3/16

$f_W(9) =$ 0

Submit You have used 1 of 1 attempt

Problem 6

3/3 points (graded)

6. Suppose X and Y have joint probability density function

$$f_{X,Y}(x,y) = 21e^{-3x-7y}$$

for $x > 0$ and $y > 0$; and $f_{X,Y}(x,y) = 0$ otherwise.

6a. Compute $P(Y \geq X)$.

3/10 **Answer:** 0.3

6b. Compute $P(Y \leq 3X)$.

7/8 **Answer:** 0.875

6c. Compute $P(Y \geq 1/10)$.

0.4965853 **Answer:** 0.4965853

Explanation

6a. We have $P(Y \geq X) = \int_0^\infty \int_x^\infty 21e^{-3x-7y}\, dy\, dx = \int_0^\infty 3e^{-10x}\, dx = 3/10$.

6b. We have $P(Y \leq 3X) = \int_0^\infty \int_{y/3}^\infty 21e^{-3x-7y}\, dx\, dy = \int_0^\infty 7e^{-8y}\, dy = 7/8$.

6c. We have $P(Y \geq 1/10) = \int_{1/10}^\infty \int_0^\infty 21e^{-3x-7y}\, dx\, dy = \int_{1/10}^\infty 7e^{-7y}\, dy = e^{-7/10}$.

Submit You have used 1 of 1 attempt

Correct (3/3 points)

Problem 7

4/4 points (graded)

7a. In the setup of question **6**, find the probability density function $f_X(x)$ of X. Then calculate the following values.

$f_X(\frac{1}{3}) =$ 1.103638 **Answer:** 1.103638

$f_X(-\frac{1}{3}) =$ 0 **Answer:** 0

7b. In the setup of question **6**, find the probability density function $f_Y(y)$ of Y.

$f_Y(0.1) =$ 3.476097 ✔ **Answer:** 3.476097

$f_Y(-0.1) =$ 0 ✔ **Answer:** 0

7c. Use your answer to **7b** to find $P(Y \geq 1/10)$. Does your answer agree with your answer to **6c**?

Explanation

7a. For $x > 0$, we have $f_X(x) = \int_0^\infty 21e^{-3x-7y}\,dy = 3e^{-3x}$, and for $x \leq 0$, we have $f_X(x) = 0$.
7b. For $y > 0$, we have $f_Y(y) = \int_0^\infty 21e^{-3x-7y}\,dx = 7e^{-7y}$, and for $y \leq 0$, we have $f_Y(y) = 0$.
7c. We have $P(Y \geq 1/10) = \int_{1/10}^\infty 7e^{-7y}\,dy = e^{-7/10}$, which agrees with **6c**.

Submit You have used 1 of 1 attempt

✔ Correct (4/4 points)

Problem 8

4/4 points (graded)

8. Consider a pair of random variables X and Y with joint probability density function $f_{X,Y}(x,y) = \frac{1}{8}xy$ for x, y in the triangle where $0 < x < 2$ and $0 < y < 2x$, and $f_{X,Y}(x,y) = 0$ otherwise.

8a. Are X and Y independent? Why or why not?

- Yes, X and Y are independent

No, X and Y are dependent ✔

8b. Find $P(X \leq 1)$ using the joint density $f_{X,Y}(x, y)$.

1/16 ✔ **Answer:** 0.0625

8c. Find the density $f_X(x)$. Calculate the following values.

$f_X(1) =$ 1/4 ✔ **Answer:** 0.25

$f_X(-1) =$ 0 ✔ **Answer:** 0

8d. Use the density $f_X(x)$ to find $P(X \leq 1)$. Does your answer agree with your answer to **b**?

Explanation

8a. Here X and Y are dependent. Perhaps the easiest way to see this is that their domain is not rectangular shaped (it is like a triangle shape).

8b. We have $P(X \leq 1) = \int_0^1 \int_0^{2x} \frac{1}{8} xy\,dy\,dx = \int_0^1 \frac{1}{4} x^3\,dx = 1/16$.

8c. The density of X is $f_X(x) = \int_0^{2x} \frac{1}{4} x^3\,dx = \frac{1}{4} x^3$ for $0 < x < 2$, and $f_X(x) = 0$ otherwise.

8d. Yes! We have $P(X \leq 1) = \int_0^1 \frac{1}{4} x^3\,dx = 1/16$.

Submit You have used 1 of 1 attempt

✔ Correct (4/4 points)

Problem 9

3/3 points (graded)

9. Suppose X and Y have joint density $f_{X,Y}(x,y) = 10e^{-3x-2y}$ for x, y in the region where $0 < x < y$, and $f_{X,Y}(x,y) = 0$ otherwise.

9a. Find $P(Y > 2X)$. (Just a side comment, not a hint: We already know $P(Y > X) = 1$.)

5/7 ✔ **Answer:** 0.7142857

9b. Find the density $f_X(x)$ of X. Calculate the following values.

$f_X(1) =$ 0.03368973 ✔ **Answer:** 0.03368973

$f_X(-1) =$ 0 ✔ **Answer:** 0

Explanation

9a. We have $\int_0^\infty \int_{2x}^\infty 10e^{-3x-2y}\,dy\,dx = \int_0^\infty 5e^{-7x}\,dx = 5/7$.

9b. We have $f_X(x) = \int_x^\infty 10e^{-3x-2y}\,dy = 5e^{-5x}$ for $x > 0$, and $f_X(x) = 0$ otherwise.

Submit You have used 1 of 1 attempt

✔ Correct (3/3 points)

Problem 10

4/5 points (graded)

10. Suppose X, Y has joint density

$f_{X,Y}(x,y) = \frac{1}{225}(5-x)(6-y)$ if $0 \leq x \leq 5$ and $0 \leq y \leq 6$,

$= 0$, otherwise.

10a. Are X and Y independent? Why or why not?

- (selected) Yes, X and Y are independent ✔
- No, X and Y are dependent

10b. Find the density $f_X(x)$ of X. Calculate the following values.

$f_X(2.5) =$ 1/45 ✘ **Answer:** 0.2

$f_X(10) =$ 0 ✔ **Answer:** 0

10c. Find the density $f_Y(y)$ of Y.

$f_Y(5) =$ 1/18 **Answer:** 0.05555556

$f_Y(10) =$ 0 **Answer:** 0

Explanation

10a. Yes, X and Y are independent. Their density is defined in a rectangular region, and it can be factored into x and y parts.

10b. We have $f_X(x) = \int_0^6 \frac{1}{225}(5-x)(6-y)\,dy = \frac{2}{25}(5-x)$, for $0 \le x \le 5$, and $f_X(x) = 0$ otherwise.

10c. We have $f_Y(y) = \int_0^5 \frac{1}{225}(5-x)(6-y)\,dx = \frac{1}{18}(6-y)$, for $0 \le y \le 6$, and $f_Y(y) = 0$ otherwise.

Submit You have used 1 of 1 attempt

✱ Partially correct (4/5 points)

Problem 11

2/2 points (graded)

11. Suppose X is a continuous random variable with density $f_X(x) = 3e^{-3x}$ for $x > 0$, and $f_X(x) = 0$ otherwise. Suppose Y is a continuous random variable with density $f_Y(y) = 5e^{-5y}$ for $y > 0$, and $f_Y(y) = 0$ otherwise. Finally, suppose that X and Y are independent. Define Z as the minimum of X and Y, i.e., $Z = \min(X, Y)$.

11a. Find the density $f_Z(z)$ of Z. Calculate the following value.

$f_Z(0.5) =$ 0.1465251 **Answer:** 0.1465251

11b. Find $P(Z > 1/10)$.

0.449329 **Answer:** 0.4493

Explanation

11a. For $z > 0$, we have

$$P(Z \geq z) = P(X \geq z \,\&\, Y \geq z) = P(X \geq z)P(Y \geq z)$$

$= (\int_z^\infty 3e^{-3x}\,dx)(\int_z^\infty 5e^{-5y}\,dy)$

$= e^{-3z}e^{-5z} = e^{-8z}$.

Thus $F_Z(z) = P(Z \leq z) = 1 - e^{-8z}$ for $z > 0$. So $f_Z(z) = 8e^{-8z}$ for $z > 0$, and $f_Z(z) = 0$ otherwise.

11b. We have $P(Z > 1/10) = \int_{1/10}^\infty 8e^{-8z}\,dz = e^{-4/5} = 0.4493$.

Submit You have used 1 of 1 attempt

Correct (2/2 points)