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1.5.1 Summary Quiz: Species in Competition and Bifurcation Values

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Question 1

1/1 point (graded)

If $\beta = \frac{1}{2}$ and we start with species X at 25% density and species Y at 30% density, what is the short term outcome for populations?

- (selected) Since $\frac{dx}{dt} > 0$ and $\frac{dy}{dt} > 0$ at this point, both populations increase in size in the short term. ✔
- Since $\frac{dx}{dt} < 0$ and $\frac{dy}{dt} < 0$ at this point, both populations decrease in size in the short term.
- Since $\frac{dx}{dt} > 0$ and $\frac{dy}{dt} < 0$ at this point, species X population increases and species Y population decreases in the short term.
- Since $\frac{dx}{dt} < 0$ and $\frac{dy}{dt} > 0$ at this point, species X population decreases and species Y population increases in the short term.
- None of the above.

Explanation

We can use the phase plane to see that $\frac{dx}{dt} > 0$ and $\frac{dy}{dt} > 0$ (or test the value of the derivatives at $x = 0.25, y = 0.3$). They are both positive, so both populations increase in the short term.

Submit You have used 2 of 3 attempts

Answers are displayed within the problem

Question 2

1/1 point (graded)

If $\beta = \frac{1}{2}$ and we start with species X at 25% density and species Y at 30% density, what is the long term outcome for the populations?

- Species X and Y each approach a stable population level at their respective carrying capacity.
- Species X and Y each approach a stable population level at about 67% of their respective carrying capacity. ✔
- Species X and Y each approach a stable population level at about 33% of their respective carrying capacity.
- Species X goes extinct, while species Y approaches its carrying capacity.
- Species Y goes extinct, while species X approaches its carrying capacity.
- Species X and Y go extinct.
- None of the above.

Explanation

This is addressed in the video in Section 1.2 and phase plane analysis in the quiz in Section 1.1. The equilibrium point $(2/3, 2/3)$ is stable so the trajectory $(x(t), y(t))$ starting with $(0.25, 0.3)$ approaches this point. This means both X and Y approach 2/3 of their carrying capacity or about 67%.

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You have used 1 of 3 attempts

Answers are displayed within the problem

Question 3

1/1 point (graded)

If $\beta = 2$ and we start with species X at 25% density and species Y at 30% density, what is the short term outcome for populations?

- Since $\frac{dx}{dt} > 0$ and $\frac{dy}{dt} > 0$ at this point, both populations increase in size in the short term. ✔
- Since $\frac{dx}{dt} < 0$ and $\frac{dy}{dt} < 0$ at this point, both populations decrease in size in the short term.

- Since $\frac{dx}{dt} > 0$ and $\frac{dy}{dt} < 0$ at this point, species X population increases and species Y population decreases in the short term.
- Since $\frac{dx}{dt} < 0$ and $\frac{dy}{dt} > 0$ at this point, species X population decreases and species Y population increases in the short term.
- None of the above.

Explanation

We can use the phase plane to see that $\frac{dx}{dt} > 0$ and $\frac{dy}{dt} > 0$ (or test the value of the derivatives at $x = 0.25, y = 0.3$). They are both positive, so both populations increase in the short term.

Submit You have used 1 of 3 attempts

Answers are displayed within the problem

Question 4

1/1 point (graded)

If $\beta = 2$ and we start with species X at 25% density and species Y at 30% density, what is the short term outcome for populations long term outcome for the populations?

- Species X and Y each approach a stable population level at their respective carrying capacity.
- Species X and Y each approach a stable population level at about 67% of their respective carrying capacity.
- Species X and Y each approach a stable population level at about 33% of their respective carrying capacity.
- Species X goes extinct, while species Y approaches its carrying capacity. ✔
- Species Y goes extinct, while species X approaches its carrying capacity.
- Species X and Y go extinct.
- None of the above.

Explanation

This is addressed in the video in Section 1.3 and phase plane picture which follows it. The equilibrium point $(1/3, 1/3)$ is not stable. The trajectory starts in the southwest region at $(0.25, 0.3)$ heads up and to the right but eventually crosses the red-nullcline into the northwest region where the trajectory approaches (0,1) since $\frac{dx}{dt} < 0$ and $\frac{dy}{dt} > 0$.This means Y approaches its carrying capacity while X goes extinct.

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Question 5

1/1 point (graded)

Wes described a bifurcation as "a dramatic change in the expected behavior of a system in response to a change in parameter." Why does the competition value $\beta = 1$ correspond to a bifurcation? Assuming that both species X and Y are present at the starting time, what is the dramatic change in expected behavior of these populations when we change from $\beta < 1$ to $\beta > 1$?

Choose the most complete answer.

- When the competition parameter $\beta < 1$, the populations will always approach their full carrying capacity. When the competition parameter $\beta > 1$, the populations will approach coexistence equilibrium but lower than their carrying capacity. That is the dramatic change.
- When the competition parameter $\beta < 1$, the populations will always approach a coexistence equilibrium (both species are present). When the competition parameter $\beta > 1$, both populations will go extinct, for most starting conditions. That is the dramatic change.
- When the competition parameter $\beta < 1$, the populations will always approach a coexistence equilibrium (both species are present). When the competition parameter $\beta > 1$, one population will reach its carrying capacity and the other will go extinct, for most starting conditions. That is the dramatic change. ✔
- When the competition parameter $\beta < 1$, one population will reach its carrying capacity and the other will go extinct, for most starting conditions. When the competition parameter $\beta > 1$, the populations will always approach a coexistence equilibrium (both species are present). That is the dramatic change.
- None of the above.

Explanation

There are two qualitatively different situations, and the change happens when $\beta = 1$. For weak competition $\beta < 1$, the populations will approach coexistence equilibrium (where each reach a level lower than their carrying capacity).

However, for strong competition $\beta > 1$, one population will reach its carrying capacity and the other will go extinct (unless they start with the exact same amount).

This is the dramatic change: we go from coexistence to competitive exclusion.

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You have used 1 of 3 attempts

Answers are displayed within the problem

Question 6

1/1 point (graded)

In the case of $\beta = 1$, the two diagonal null clines for $\frac{dx}{dt}$ and $\frac{dy}{dt}$ are identical, creating infinitely many equilibrium points along the line $y = 1 - x$.

Furthermore, looking at the sign of $\frac{dx}{dt}$ and $\frac{dy}{dt}$, we see they are both negative for points above the line and both positive for points below the line. This means solution trajectories head toward that line over time.

But where exactly do they head and how? In straight lines? In exponential or parabolic curves? In other words, what's the relation of $y(t)$ and $x(t)$?

It turns out in the case of $\beta = 1$, we can determine exactly the shape of the trajectories by considering $\frac{dy}{dx}$:

By the chain rule,

$$\frac{dy}{dt} = \frac{dy}{dx}\frac{dx}{dt}.$$

Solving for $\frac{dy}{dx}$ we get

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{y(1-y) - xy}{x(1-x) - xy}$$

Simplify the right hand expression and use **separation of variables** to solve for the relationship between $x(t)$ and $y(t)$. (If you are not familiar with this technique of solving differential equations, check out the video Separable equations introduction from Khan Academy.)

- The population densities of X and Y are exponentially related as follows: $y(t) = e^{x(t)} + C$ for some constant C

- ○ The population densities of X and Y are exponentially related: $y(t) = Ce^{x(t)}$ for some constant C.
- ◉ The population densities of X and Y are proportional: $y(t) = Cx(t)$ for some constant C. ✔
- ○ The population densities of X and Y are linearly related: $y(t) = x(t) + C$ for some constant C.
- ○ None of the above.

Explanation

$$\frac{dy}{dx} = \frac{y(1-y) - xy}{x(1-x) - xy} = \frac{y(1-y-x)}{x(1-x-y)} = \frac{y}{x}.$$

Then using separation of variables we have

$$\int \frac{1}{y}\,dy = \int \frac{1}{x}\,dx$$

This leads to

$$\ln(y) = \ln(x) + K$$

for any constant K. Thus

$$y = e^{\ln(x)+K} = xe^K = Cx$$

for a non-zero constant C.
There are other ways to rule out the other choices using just the symmetry of the system.

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You have used 1 of 3 attempts

Answers are displayed within the problem

Let's look at another example of species in equal competition. This time instead of varying the strength of competition between the two species, we will vary the *carrying capacity* K *of one species*.

$$\frac{dx}{dt} = 0.1x(1 - \frac{x}{K}) - 0.02xy$$

$$\frac{dy}{dt} = 0.1y(1 - \frac{y}{20}) - 0.02xy$$

Here $x(t)$ and $y(t)$ are actual population size, not densities.

Question 7: Think About It...

1/1 point (graded)

Intuitively, what do you expect to happen as K gets large? Take your best guess.

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When K is "large", it is also possible for species X to
reach carrying capacity and Y to go extinct.
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Thank you for your response.

This is an answer based on one possible intuition. If K is small the environment can't support much of species X, so it is likely to be overrun by competition from Y.

As K gets large, X has a larger carrying capacity and we may expect that it can grow larger and compete more with Y and thus survive and maybe even outcompete Y.

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You have used 1 of 2 attempts

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Question 8

1/1 point (graded)

Find the equations for the nullclines $\frac{dx}{dt} = 0$. (These may depend on the carrying capacity K of species X .)

- [x] $x = 0$ ✔
- [ ] $y = 0$
- [ ] $y = 5 - 5x$
- [x] $y = 5 - \frac{5}{K}x$ ✔
- [ ] $y = 20 - 4x$

- [ ] $y = 20 - \frac{1}{4}x$
- [ ] $y = 20 - \frac{4}{K}x$
- [ ] None of these.

✔

Explanation

We look at where $\frac{dy}{dt} = 0.1x(1 - \frac{x}{K}) - 0.02xy$ is equal to zero. After factoring the right hand side, we get

$$\frac{dx}{dt} = 0.1x(1 - \frac{x}{K} - .2y)$$

which means $x = 0$ and $1 - \frac{x}{K} - 0.2y = 0$ are nullclines. The latter line is $y = 5 - \frac{5}{K}x$ in slope-intercept form.

Submit You have used 1 of 5 attempts

Answers are displayed within the problem

Question 9

1/1 point (graded)

Find the equations for the nullclines $\frac{dy}{dt} = 0$. (These may depend on the carrying capacity K of species X.

- [ ] $x = 0$
- [x] $y = 0$ ✔
- [ ] $y = 5 - 5x$
- [ ] $y = 5 - \frac{5}{K}x$
- [x] $y = 20 - 4x$ ✔
- [ ] $y = 20 - \frac{1}{4}x$

- [ ] $y = 20 - \frac{4}{K}x$

- [ ] None of these.

✔

Explanation

We look at where $\frac{dy}{dt} = 0.1y(1 - \frac{y}{20}) - 0.02xy$ is equal to zero. After factoring the right hand side, we get

$$\frac{dy}{dt} = 0.1y(1 - \frac{y}{20}) - 0.02xy = 0.1y(1 - \frac{y}{20} - 0.2x)$$

which means $y = 0$ and $1 - \frac{y}{20} - 0.2x = 0$ are nullclines. The latter line is $y = 20 - 4x$ in slope-intercept form.

Submit You have used 1 of 5 attempts

Answers are displayed within the problem

Question 10

1/1 point (graded)

Use a graphing tool like Desmos to explore what happens to the nullclines when you vary K. (In Desmos, if you type in something like $y = 2x + K$, it will give you the option to 'add a slider' for K. This way you can adjust the value of K.)

This system has a single bifurcation. Estimate the value of K which corresponds to this bifurcation, that is, to a major qualitative change in the phase plane.

5 ✔ **Answer:** 5

5

Explanation

$K = 5$.

Here is a image of the input that will create these images in Desmos.

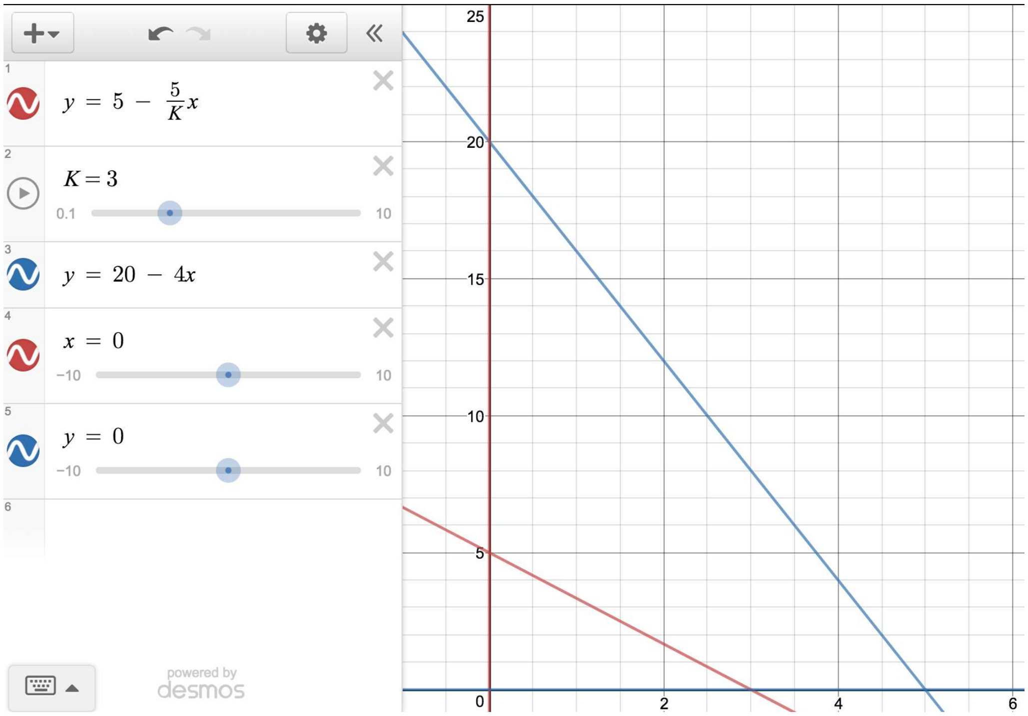

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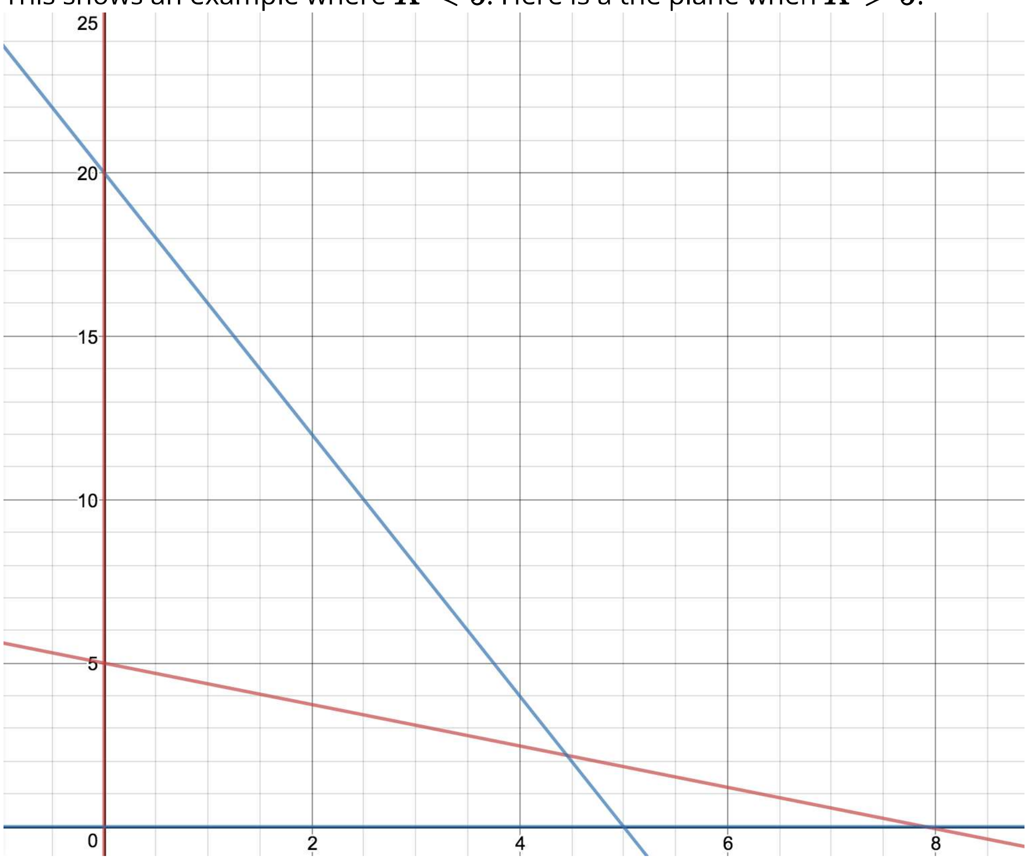
Image Description

This shows an example where $K < 5$. Here is a the plane when $K > 5$.



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Question 11

1/1 point (graded)

For each qualitatively different phase plane, give a representative sketch, labeled according to the value of K, for example "small" K or "large" K. Indicate all equilibrium, orient the null-clines and include arrows to indicate the direction of the flow in the various regions.

```
When K, the carrying capacity of species X, is "small",
there is only one option: species Y will reach its carrying
capacity while species X will go extinct. When K is
"large", it is also possible for species X to reach
carrying capacity and Y to go extinct.
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✔

Thank you for your response.

The phase planes are discussed in later questions.

Submit You have used 1 of 2 attempts

Answers are displayed within the problem

Question 12

1/1 point (graded)

Consider the case where K, the carrying capacity of species X, is 'small'. Assuming we start with some members of both species X and Y, which are possible long-term behaviors of the system?

(Note: choose all which are theoretically possible, even if they do not seem biologically so.)

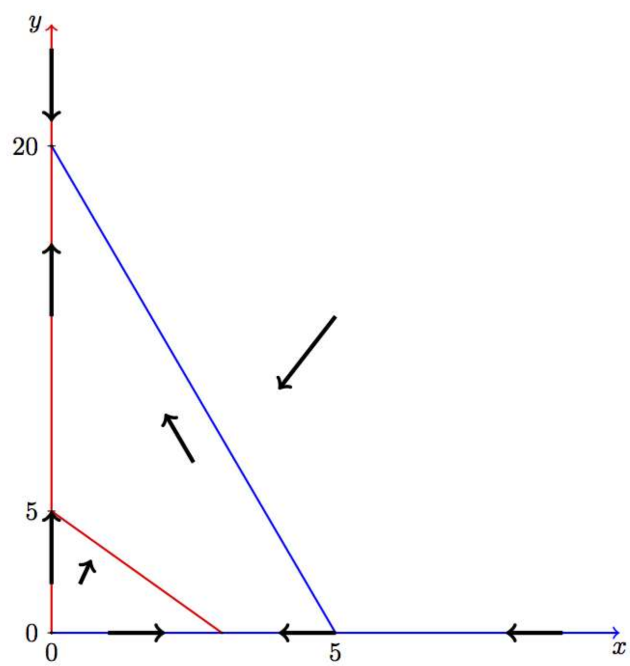
- [x] Species X goes extinct, Species Y approaches its carrying capacity ✔
- [ ] Species Y goes extinct, Species X approaches its carrying capacity
- [ ] Species X and Y both go extinct
- [ ] Species X and Y both approach their carrying capacities

- [ ] Species X and Y both approach a population level below their carrying capacity
- [ ] None of the above.

Explanation

As we can see from the phase plane, there are three equilibrium points: (0,0), (0,20) and $(K,0)$. The null clines create three regions. From the direction of arrows we see that if we start in the first quadrant, all trajectories eventually enter the middle region where they head toward (0,20), which represents that X goes extinct and Y reaches carrying capacity.

A second plot showing rough trajectories is also provided.



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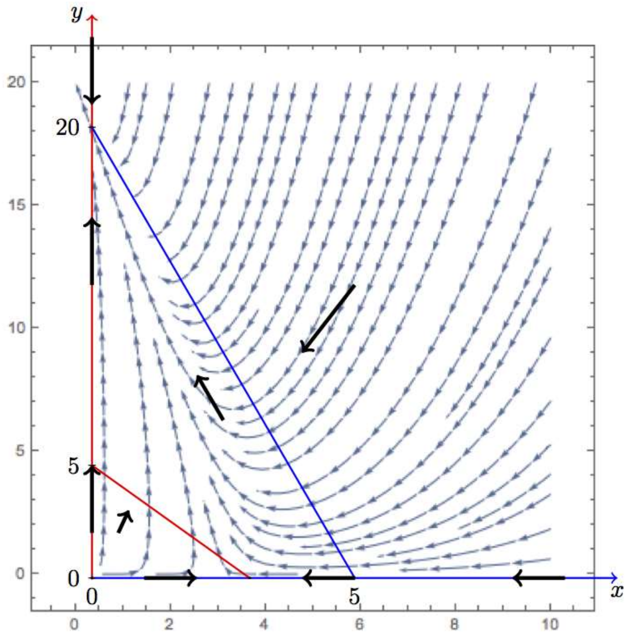

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Answers are displayed within the problem

Question 13

1/1 point (graded)
Consider the case where K, the carrying capacity of species X, is "large". Assuming we start with some members of both species X and Y, which are possible long-term behaviors of the system?

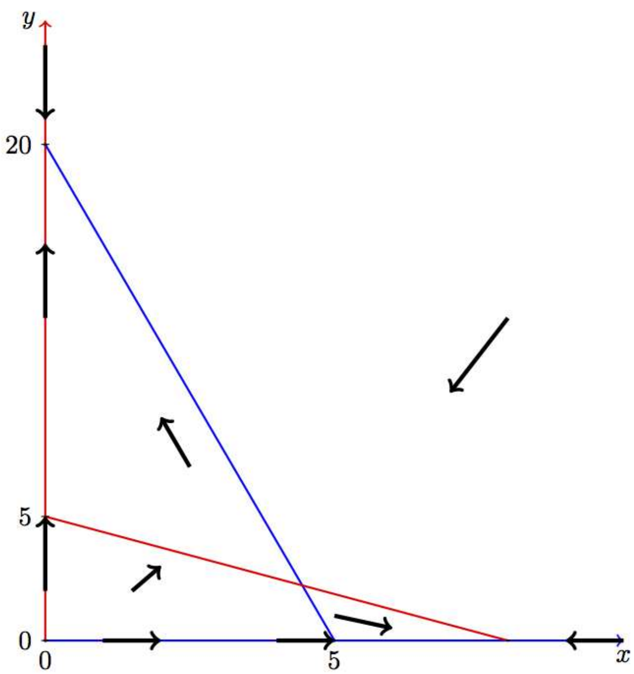
(Note: choose all which are theoretically possible, even if they do not seem biologically so.)

- [x] Species X goes extinct, Species Y approaches its carrying capacity ✔
- [x] Species Y goes extinct, Species X approaches its carrying capacity ✔
- [ ] Species X and Y both go extinct
- [ ] Species X and Y both approach their carrying capacities
- [x] Species X and Y both approach a population level below their carrying capacity ✔

- [ ] None of the above.

✔

Explanation
As we can see from the phase plane, there are four equilibrium points: (0,0), (0,20) and $(K, 0)$ and a coexistence equilibrium. The null clines create four regions. From the direction of arrows we see that trajectories do one of three things: (1) enter the top left triangular region where they head toward (0,20), which represents that X goes extinct and Y reaches carrying capacity, (2) enter the bottom right triangular region where they head toward $(K, 0)$, which represents that Y goes extinct and X reaches carrying capacity, or (3) approach an unstable coexistence equilibrium. (This is theoretically possible but not biologically so, because of random population fluctuations.)
A second plot showing rough trajectories is also provided.



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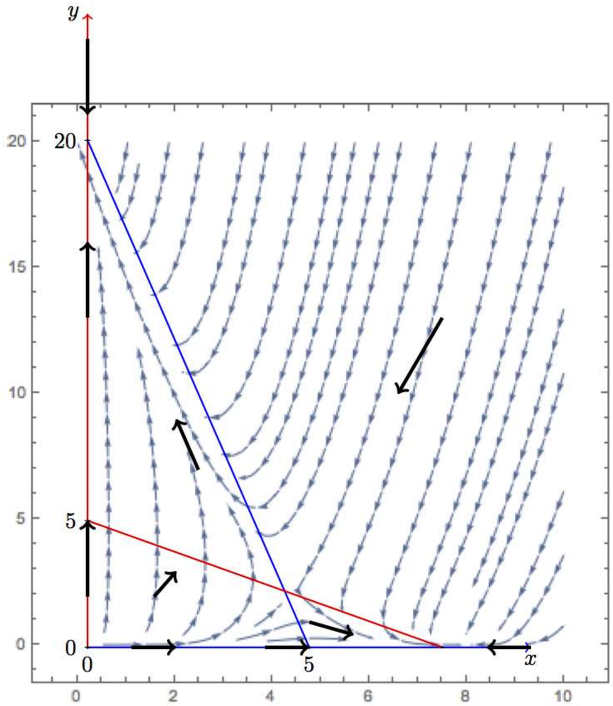

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Question 14

1/1 point (graded)

This system has a single bifurcation when K, the carrying capacity of X, changes from "small" to "large". Which of the following best describe the "dramatic change in the expected behavior" of the system, assuming we start with members of both species?

- [ ] When K, the carrying capacity of species X, is "small", the populations will approach a coexistence equilibrium. When K is "large", species X will reach its carrying capacity while species Y will go extinct.
- [ ] When K, the carrying capacity of species X, is "small", the populations will approach a coexistence equilibrium. When K is "large", species Y will reach its carrying capacity while species X will go extinct.

- [ ] When K, the carrying capacity of species X, is "small", there is only one option: species X will reach its carrying capacity while species Y will go extinct. When K is "large", X and Y will reach a coexistence equilibrium.

- [ ] When K, the carrying capacity of species X, is "small", there is only one option: species Y will reach its carrying capacity while species X will go extinct. When K is "large", X and Y will reach a coexistence equilibrium.

- [x] When K, the carrying capacity of species X, is "small", there is only one option: species Y will reach its carrying capacity while species X will go extinct. When K is "large", it is also possible for species X to reach carrying capacity and Y to go extinct. ✔

- [ ] When K, the carrying capacity of species X, is "small", there is only one option: species X will reach its carrying capacity while species Y will go extinct. When K is "large", it is also possible for species Y to reach carrying capacity and X to go extinct.

- [ ] None of the above.

✔

Explanation
If K is small, then species Y will reach its carrying capacity while species X will go extinct. If K is sufficiently large, then it is possible for the reverse to happen: species X will reach its carrying capacity and Y will go extinct.
The dramatic change is that we go from Y always competitively excluding X, to a situation where X may survive and competitively exclude Y as well. Theoretically, for large K, it is also possible for X and Y to reach a coexistence equilibrium. This is not biologically possible, however, because this is an unstable equilibrium. This means random population fluctuations will push the species level towards one of the other trajectories headed toward competitive exclusion.

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Answers are displayed within the problem

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