Exercise: Correlation coefficient

(1/1 point)

It is known that a for a standard normal random variable X, we have $\mathbf{E}[X^3] = 0$, $\mathbf{E}[X^4] = 3$, $\mathbf{E}[X^5] = 0$, $\mathbf{E}[X^6] = 15$. Find the correlation coefficient between X and X^3. Enter your answer as a number.

0.7745967 ✔ **Answer:** 0.77460

Answer:
Since $\mathbf{E}[X] = \mathbf{E}[X^3] = 0$, we have
$\mathbf{cov}(X, X^3) = \mathbf{E}[X \cdot X^3] = \mathbf{E}[X^4] = 3$. Furthermore, since $\mathbf{var}(X) = 1$ and $\mathbf{var}(X^3) = \mathbf{E}[X^6] = 15$, we obtain

$$\rho(X, X^3) = \frac{3}{\sqrt{1} \cdot \sqrt{15}} = \sqrt{3/5}.$$

Interestingly, even though the random variables are strongly dependent (the value of one determines the value of the other), the value of the correlation coefficient is moderate.

You have used 1 of 2 submissions

Lec. 12: Sums of independent r.v.'s; Covariance and correlation
Exercises 12 due Mar 30, 2016 at 23:59 UT

Lec. 13: Conditional expectation and variance revisited; Sum of a random number of independent r.v.'s
Exercises 13 due Mar 30, 2016 at 23:59 UT

Solved problems

Additional theoretical material

Problem Set 6
Problem Set 6 due Mar 30, 2016 at 23:59 UT

Unit summary