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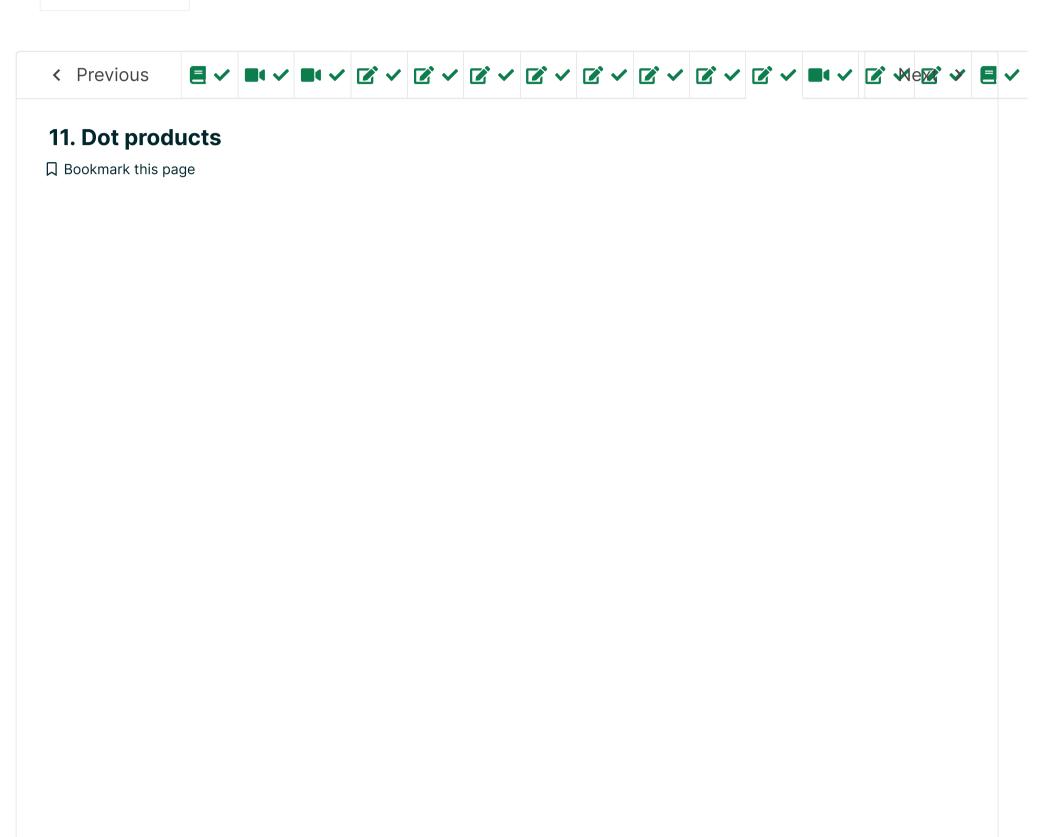


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Lecture due Aug 18, 2021 20:30 IST Completed



**Explore** 

Another operation we can do with vectors is called the **dot product**.

#### **Definition 11.1**

The **dot product** of two vectors  $ec{v}=\langle v_1,v_2
angle$  and  $ec{w}=\langle w_1,w_2
angle$  is defined as

$$\vec{v} \cdot \vec{w} = \langle v_1, v_2 \rangle \cdot \langle w_1, w_2 \rangle = v_1 w_1 + v_2 w_2. \tag{3.20}$$

### **→** Spoiler: Dot products in higher dimensions

Consider two vectors of length n given by  $ec v=\langle v_1,v_2,\ldots,v_n
angle$  and  $ec w=\langle w_1,w_2,\ldots,w_n
angle$ . The dot product of these vectors is

$$ec{v}\cdotec{w}=\langle v_1,v_2,\ldots,v_n
angle\cdot\langle w_1,w_2,\ldots,w_n
angle=v_1w_1+v_2w_2+\cdots+v_nw_n.$$

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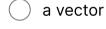
## Dot product concept check

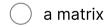
1/1 point (graded)

From the definition of the dot product, what kind of quantity is  $\vec{v} \cdot \vec{w}$ ?



a scalar







### Solution:

The dot product is a scalar. To see this, let's consider the dot product of vectors  $ec{v}=\langle 2,1 
angle$  and  $ec{w}=\langle 2,3 
angle$ . This gives

$$ec{v}\cdotec{w}=\langle 2,1
angle\cdot\langle 2,3
angle=2\,(2)+1\,(3)=7,$$

which is a number (also called a scalar).

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You have used 1 of 1 attempt

## Dot product practice

3.0/3 points (graded)

Compute the following.

Let 
$$ec{v}=\langle 1,3 
angle$$
 and  $ec{w}=\langle 0,-2 
angle$ . Then  $ec{v}\cdot ec{w}=$  -6

Let 
$$ec{s}=\langle 11,-2
angle$$
 and  $ec{r}=\langle 1,1
angle$ . Then  $ec{s}\cdotec{r}=$  9

Let  $ec{u}=\langle 2a,3b
angle$  and  $ec{q}=\langle b,-4a
angle$  where a and b are constants.

Then 
$$\vec{u} \cdot \vec{q} = \boxed{-10*a*b}$$

#### **Solution:**

$$\vec{v} \cdot \vec{w} = \langle 1, 3 \rangle \cdot \langle 0, -2 \rangle = (1)(0) + (3)(-2) = 0 - 6 = -6$$
 (3.21)

$$\vec{s} \cdot \vec{r} = \langle 11, -2 \rangle \cdot \langle 1, 1 \rangle = (11)(1) + (-2)(1) = 11 - 2 = 9$$
 (3.22)

$$\vec{u} \cdot \vec{q} = \langle 2a, 3b \rangle \cdot \langle b, -4a \rangle = (2a)(b) + (3b)(-4a) = 2ab - 12ab = -10ab.$$
 (3.23)

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You have used 1 of 5 attempts

**1** Answers are displayed within the problem

## Vector properties

1/1 point (graded)

Let  $\vec{u}_i$ ,  $\vec{v}_i$ , and  $\vec{w}$  be vectors (with the same number of components) and let c be a scalar.

Which of the following properties of vectors are true? (Choose all that are true.)

$$ec{v}\cdotec{v}=ertec{v}ert$$

$$\vec{v} \cdot \vec{w} = \vec{w} \cdot \vec{v}$$

$$c\left( ec{v}+ec{w}
ight) =cec{v}+cec{w}$$

$$ec{oldsymbol{u}}\cdot ec{oldsymbol{u}}\cdot (ec{oldsymbol{v}} + ec{oldsymbol{w}}) = |ec{oldsymbol{u}}|\, (ec{oldsymbol{v}} + ec{oldsymbol{w}})$$

### Solution:

We explain these properties that involve dot products in terms of the algebraic definition given on this page. We leave it as an exercise for you to verify that the geometric definition of the dot product introduced on the next page can be used to verify these properties as well!

1.

$$egin{array}{lll} ec{v} \cdot ec{v} &=& \langle v_1, v_2 
angle \cdot \langle v_1, v_2 
angle \ &=& v_1^2 + v_2^2 \ &=& \left| ec{v} 
ight|^2 
eq \left| ec{v} 
ight| \end{array}$$

2.

$$ec{v}\cdotec{w} \ = \ \langle v_1,v_2
angle\cdot\langle w_1,w_2
angle \ = \ v_1w_1+v_2w_2$$

$$= w_1v_1 + w_2v_2$$
 (multiplication of numbers commutes)  
 $= \vec{w} \cdot \vec{v}$  (rewrite as a dot product) $\checkmark$ 

3.

$$egin{array}{lll} c\left(ec{v}+ec{w}
ight) &=& c\left(\left\langle v_{1},v_{2}
ight
angle +\left\langle w_{1},w_{2}
ight
angle 
ight) \ &=& c\left\langle v_{1}+w_{1},v_{2}+w_{2}
ight
angle \ &=& \left\langle c\left(v_{1}+w_{1}
ight),c\left(v_{2}+w_{2}
ight)
ight
angle & ext{ (distribute across)} \ &=& \left\langle cv_{1}+cw_{1},cv_{2}+cw_{2}
ight
angle & ext{ (rewrite as sum of vectors)} \ &=& \left\langle cv_{1},cv_{2}
ight
angle +\left\langle cw_{1},cw_{2}
ight
angle \ &=& cec{v}+cec{w}\checkmark \end{array}$$

4.  $\vec{u} \cdot (\vec{v} + \vec{w}) = |\vec{u}| (\vec{v} + \vec{w})$  cannot be true since the expression on the left hand side is a dot product of vectors, which is a scalar quality, and the expression on the right hand side is a scalar times a vector, which is a vector.

The property that is true is that the dot product distributes over vector sums.  $\vec{u} \cdot (\vec{v} + \vec{w}) = \vec{u} \cdot \vec{v} + \vec{u} \cdot \vec{w}$  because

$$egin{aligned} ec{u}\cdot(ec{v}+ec{w}) &= \langle u_1,u_2
angle\cdot\langle v_1+w_1,v_2+w_2
angle & ext{(take dot product)} \ &= u_1\left(v_1+w_1
ight)+u_2\left(v_2+w_2
ight) & ext{(multiply out)} \ &= u_1v_1+u_1w_1+u_2v_2+u_2w_2 & ext{(rearrange)} \ &= \left(u_1v_1+u_2v_2
ight)+\left(u_1w_1+u_2w_2
ight) & ext{(find hidden dot products)} \ &= ec{u}\cdotec{v}+ec{u}\cdotec{w} \end{aligned}$$

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You have used 1 of 5 attempts

**1** Answers are displayed within the problem

### 11. Dot products

**Hide Discussion** 

**Topic:** Unit 2: Geometry of Derivatives / 11. Dot products

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Typo For the last question (Vector Properties), the solution for the 4th box has a typo in the "multiply out" line., The secon	6 d u_1 should be
STAFF] Vector properties q2	2

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