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2. Recap: Maximum Likelihood Estimators and Fisher information

Instructions:

For each of the following distributions, compute the maximum likelihood estimator based on n i.i.d. observations $X_1, \ldots, X_n$ and the Fisher information, if defined. If it is not, enter **DNE** in each applicable input box.

(a)

3/3 points (graded)

$$X_i \sim \text{Ber}(p), \quad p \in (0,1)$$

(Enter **barX_n** for the sample average $\overline{X}_n$

$$\overline{X}_n = \frac{1}{n}\sum_{i=1}^{n} X_i.$$

Maximum likelihood estimator $\hat{p} =$ barX_n ✔

Hint: Use the definition of Fisher information that leads to the shorter computation.

(If the Fisher information is not defined, enter **DNE**.)

Fisher information $I(p) =$ 1/p/(1-p) ✔

$\frac{1}{p\cdot(1-p)}$

Use Fisher Information to find the asymptotic variance $V(\hat{p})$ of the MLE $\hat{p}$.

$V(\hat{p}) =$ p*(1-p) ✔

$p \cdot (1-p)$

STANDARD NOTATION

Submit You have used 1 of 3 attempts

(b)

3/3 points (graded)

$$X_i \sim \textsf{Poiss}(\lambda), \quad \lambda > 0,$$

which means that each X_i has distribution

$$\mathbf{P}_\lambda(X = k) = e^{-\lambda}\frac{\lambda^k}{k!}, \quad k \in \mathbb{N}.$$

(Enter **barX_n** for the sample average $\overline{X}_n$.)

Maximum likelihood estimator $\hat{\lambda} =$ barX_n

(If the Fisher information is not defined, enter **DNE**.)

Fisher information $I(\lambda) =$ 1/lambda

$\frac{1}{\lambda}$

Use Fisher Information to find the asymptotic variance $V(\hat{\lambda})$ of the MLE $\hat{\lambda}$.

$V(\hat{\lambda}) =$ lambda

λ

STANDARD NOTATION

Submit You have used 1 of 3 attempts

(c)

3/3 points (graded)

$$X_i \sim \text{Exp}(\lambda), \quad \lambda > 0,$$

which means that each X_1 has density

$$f_\lambda(x) = \lambda e^{-\lambda x}, \quad x > 0.$$

(Enter **barX_n** for $\overline{X}_n$ the sample average.)

Maximum likelihood estimator $\hat{\lambda} =$ 1/barX_n ✔ (If the Fisher information is not defined, enter **DNE**.)

Fisher information $I(\lambda) =$ 1/lambda^2 ✔

$\frac{1}{\lambda^2}$

Use Fisher Information to find the asymptotic variance $V(\hat{\lambda})$ of the MLE $\hat{\lambda}$.

$V(\hat{\lambda}) =$ lambda^2 ✔

λ^2

STANDARD NOTATION

Submit You have used 1 of 3 attempts

(d)

7.0/7 points (graded)

$$X_i \sim \mathcal{N}\left(\mu, \sigma^2\right), \quad \mu \in \mathbb{R}, \sigma^2 > 0,$$

which means that each X_1 has density

$$f_{\mu,\sigma^2}(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right).$$

Hint: Keep in mind that we consider σ^2 as the parameter, not σ. You may want to write $\tau = \sigma^2$ in your computation.

(Enter **barX_n** for the sample average $\overline{X}_n$ and **bar(X_n^2)** for the sample average of second moments $\overline{X_n^2}$.)

Maximum likelihood estimator $\hat{\mu} =$ barX_n

(Enter **barX_n** for the sample average $\overline{X}_n$ and **bar(X_n^2)** for the sample average of second moments $\overline{X_n^2}$.)

Maximum likelihood estimator $\widehat{\sigma^2} =$ bar(X_n^2)-barX_n^2

Hint: One of the formulas for Fisher information will lead to a much shorter computation.

(If the Fisher information is not defined, enter **DNE** for all boxes below.)

$[I(\mu,\sigma^2)]_{1,1} =$ 1/sigma^2 $\frac{1}{\sigma^2}$, $[I(\mu,\sigma^2)]_{1,2} =$ 0 0

$[I(\mu,\sigma^2)]_{2,1} =$ 0 0 , $[I(\mu,\sigma^2)]_{2,2} =$ 1/2/sigma^4 $\frac{1}{2\cdot\sigma^4}$

Using the Fisher Information you obtain above, what is the asymptotic variance $V\left(\widehat{\sigma^2}\right)$ of the MLE $\widehat{\sigma^2}$?Compare this with your result from Homework 5 Problem 3.

$V\left(\widehat{\sigma^2}\right) =$ 2*sigma^4 ✔

$2 \cdot \sigma^4$

STANDARD NOTATION

Submit You have used 2 of 3 attempts

(e)

6/6 points (graded)

X_i follows a shifted exponential distribution with parameters $a \in \mathbb{R}$ and $\lambda > 0$. That means each X_i has density

$$f_{a,\lambda}(x) = \lambda e^{-\lambda(x-a)} \mathbf{1}\{x \geq a\}, \quad x \in \mathbb{R}.$$

(Enter **barX_n** for the sample average $\overline{X}_n$, and if applicable, use **min_i(X_i)** for $\min_{1 \leq i \leq n} X_i$).

Maximum likelihood estimator $\hat{a} =$ min_i(X_i) ✔

Maximum likelihood estimator $\hat{\lambda} =$ 1/(barX_n-min_i(X_i)) ✔

Hint: Think of the effect of the indicator function on the derivatives. (If the Fisher information is not defined, enter **DNE** in all boxes below.)

$[I(a,\lambda)]_{1,1} =$ DNE ✔ , $[I(a,\lambda)]_{1,2} =$ DNE ✔

DNE DNE

$[I(a,\lambda)]_{2,1} =$ DNE DNE , $[I(a,\lambda)]_{2,2} =$ DNE DNE

STANDARD NOTATION

Submit You have used 3 of 3 attempts

Correct (6/6 points)

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part e) Matrix Order

question posted about 16 hours ago by **nbourbon**

Is it ok to assume that the first value I_1_1 it corresponds to the second derivative of "a"? or of "lambda"?. I'm assuming that if I get one of the values "0" the grader expects "0" while "not defined" is a different thing right?

This post is visible to everyone.

Erocha (Community TA)
about 14 hours ago - marked as answer about 13 hours ago by **nbourbon**

Check slide 24. 0 is not the same as DNE. But pay attention to the hint.

thanks I got the green tick now... I was struggling with understanding that indicator effect and it's key.. was not intuitive at the beginning but now I got it

posted about 13 hours ago by **nbourbon**

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