



MITx: 6.041x Introduction to Probability - The Science of Uncertainty



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Problem 1: Normal random variables

(5/5 points)

Let \mathbf{X} and \mathbf{Y} be normal random variables with means $\mathbf{0}$ and $\mathbf{2}$, respectively, and variances $\mathbf{1}$ and $\mathbf{9}$, respectively. Find the following, using the standard normal table . Express your answers to an accuracy of 4 decimal places.

1.

$$\mathbf{P}(\mathbf{X} > \mathbf{0.75}) =$$

✓ Answer: 0.2266

2.

$$\mathbf{P}(\mathbf{X} \leq \mathbf{-1.25}) =$$

✓ Answer: 0.1056

3. Let $\mathbf{Z} = (\mathbf{Y} - \mathbf{3})/\mathbf{4}$. Find the mean and the variance of \mathbf{Z} .

$$\mathbf{E}[\mathbf{Z}] =$$


✓ Answer: -0.25

$$\mathbf{var}(\mathbf{Z}) =$$


✓ Answer: 0.5625

Unit overview


Lec. 8: Probability density functions

Exercises 8 due Mar 18, 2016 at 23:59 UTC 

Lec. 9: Conditioning on an event; Multiple r.v.'s

Exercises 9 due Mar 18, 2016 at 23:59 UTC 


Lec. 10: Conditioning on a random variable; Independence; Bayes' rule

Exercises 10 due Mar 18, 2016 at 23:59 UTC 

Standard normal table

Solved problems

Problem Set 5

Problem Set 5 due Mar 18, 2016 at 23:59 UTC 

Unit summary

- ▶ Unit 6: Further topics on random variables
- ▶ Unit 7: Bayesian inference

4.

$$\mathbf{P}(-1 \leq Y \leq 2) =$$

0.3413447

✓ Answer: 0.3413

Answer:

1. \mathbf{X} is a standard normal, so by using the normal table, we have

$$\mathbf{P}(\mathbf{X} \leq 0.75) = \Phi(0.75) \approx 0.7734.$$

$$\text{Hence, } \mathbf{P}(\mathbf{X} > 0.75) = 1 - \mathbf{P}(\mathbf{X} \leq 0.75) \approx 1 - 0.7734 = 0.2266.$$

2. Since the distribution of \mathbf{X} is symmetric with respect to 0,

$$\mathbf{P}(\mathbf{X} \leq -1.25) = \mathbf{P}(\mathbf{X} \geq 1.25) = 1 - \Phi(1.25) \approx 1 - 0.8944 = 0.1056.$$

3. We have $\mathbf{E}[\mathbf{Z}] = \frac{1}{4}\mathbf{E}[\mathbf{Y}] - \frac{3}{4} = \frac{1}{4} \cdot 2 - \frac{3}{4} = -\frac{1}{4},$

$$\text{and } \mathbf{var}(\mathbf{Z}) = \frac{1}{4^2}\mathbf{var}(\mathbf{Y}) = \frac{1}{16} \cdot 9 = \frac{9}{16}.$$

4. By standardizing \mathbf{Y} and using the normal table, we have

$$\mathbf{P}(-1 \leq \mathbf{Y} \leq 2) = \mathbf{P}\left(\frac{-1-2}{3} \leq \frac{\mathbf{Y}-2}{3} \leq \frac{2-2}{3}\right)$$

$$= \mathbf{P}(-1 \leq \mathbf{Z} \leq 0)$$

$$= \mathbf{P}(0 \leq \mathbf{Z} \leq 1)$$

$$= \mathbf{P}(\mathbf{Z} \leq 1) - \mathbf{P}(\mathbf{Z} \leq 0)$$

$$= \Phi(1) - \Phi(0)$$

$$\approx 0.8413 - 0.5$$

$$= 0.3413,$$

- ▶ Exam 2
- ▶ Unit 8: Limit theorems and classical statistics
- ▶ Unit 9: Bernoulli and Poisson processes
- ▶ Unit 10: Markov chains
- ▶ Exit Survey
- ▶ Final Exam

where Z is a standard normal random variable.

You have used 1 of 3 submissions

Printable problem set available here .

DISCUSSION

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