Exercise: Boy or Girl Paradox

(3 points possible)

Alice has two children. Let's look at the probability that both children are girls, given different observations. We'll assume that the underlying finite probability space is as follows for Alice's children:

Outcome	Probability
1st child: girl, 2nd child: girl	1/4
1st child: girl, 2nd child: boy	1/4
1st child: boy, 2nd child: girl	1/4
1st child: boy, 2nd child: boy	1/4

- What is the probability that both children are girls? (This is an unconditional probability in that we aren't given any observations.) (Please be precise with at least 3 decimal places, unless of course the answer doesn't need that many decimal places. You could also put a fraction.)

1/4 ? **Answer:** 1/4

- What is the probability that both children are girls given that the younger child is a girl? (Please be precise with at least 3 decimal places, unless of course the answer doesn't need that many decimal places. You could also put a fraction.)

1/2 ? **Answer:** 1/2

- What is the probability that both children are girls given that at least one child is a girl? (Please be precise with at least 3 decimal places, unless of course the answer doesn't need that many decimal places. You could also put a fraction.)

1/3 ? **Answer:** 1/3

Solution:

Let's use G to denote that the child is a girl and B to denote that the child is a boy. Thus, the ordering GB would mean that the first child is a girl and the second child is a boy. Using this encoding, the finite sample space for this problem becomes $\Omega = \{GG, GB, BG, BB\}$.

- Since we are not given any observations and the gender of each child is independent of the other, the probability of having two girls, the probability of having two boys, the probability of the older one being a boy and the younger one being a girl and the probability of the older one being a girl and the younger one being a boy are the same. Thus, the probability of having two girls is $\frac{1}{4}$.

- We now observe that the second child is a girl. This reduces the sample space to $\Omega = \{BG, GG\}$. The first child can be either a boy or a girl with equal probability. Thus, the probability that both children are girls given that the younger child is a girl becomes $\frac{1}{2}$.

Towards Infinity in Modeling Uncertainty (Week 4)
Exercises due Oct 13, 2016 at 02:30 IST

Homework 3 (Week 4)
Homework due Oct 13, 2016 at 02:30 IST

- Now we observe that at least one of the two children is a girl, but we don't necessarily know if it's the older or younger child. Thus, our sample space is reduced to $\Omega = \{BG, GG, GB\}$. Each outcome is equally likely, thus the probability that both children are girls given that at least one of them is a girl becomes $\frac{1}{3}$.

You have used 0 of 5 submissions