L9 PROBLEM 3 (4/4 points)

1. For questions 1 and 2, consider our previous problem (permutations of 3 students in a line).

   When represented as a tree, each node will have how many children?

   2

   2

   Answer: 2

   EXPLANATION:

   From each permutation, there are two possible swaps (Students 1st and 2nd in line, and Students 2nd and 3rd in line).

2. Given two permutations, what is the maximum number of swaps it will take to reach one from the other?

   3

   3

   Answer: 3

   EXPLANATION:

   We can label the permutations a through f and construct a graph as such:

   ```
   a <--> b <--> c <--> d <--> e <--> f <--> a
   ```

   Clearly, it takes a maximum of 3 movements along edges to get from any node to another.
   The maximum number of permutations happens when we go from ABC to CBA. We have to go ABC -> BAC -> BCA -> CBA.

3. For questions 3 and 4, consider the general case of our previous problem (permutations of *n* students in a line). Give your answer in terms of *n*.

   When represented as a tree, each node will have how many children?

   n-1

   $n-1$

   Answer: n - 1

Explanation:

In any given permutation, *n* students are lined up. Since one may only swap the positions of two adjacent students, there are exactly $n-1$ pairs we are able to swap. Each of these swaps will create a distinct ordering, so there are exactly $n-1$ childern of each node.

4. Given two permutations, what is the maximum number of swaps it will take to reach one from the other?

n*(n-1)/2

$$\frac{n \cdot (n-1)}{2}$$

Answer: n * (n - 1)/2

EXPLANATION:

Answer: $n \cdot \frac{n-1}{2}$

Consider the case where the two permutations whose exchange would take the maximum number of swaps. Clearly these are two whose orders are opposite. It takes $n-1$ swaps to move the last person in line to the first position. This leaves the rest of the line's old order intact.

Next it takes $n-2$ swaps to move the last person in line to the second position. We continue until only one more swap is needed (switching the last two people in line). This takes $(n-1)+(n-2)+\ldots+2+1 = n \cdot \frac{n-1}{2}$ swaps.

Reminder: You do not lose points for trying a problem multiple times, nor do you lose points if you hit "Show Answer". If this problem has you stumped after you've tried it a few times, feel free to reveal the solution.

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