

## MITx: 6.041x Introduction to Probability - The Science of Uncertainty

Bookmarks

Unit 4: Discrete random variables > Lec. 7: Conditioning on a random variable; Independence of r.v.'s > Lec 7 Conditioning on a random variable Independence of r v s vertical6

- Unit 0: Overview
- **Entrance Survey**
- Unit 1: Probability models and axioms
- Unit 2: Conditioning and independence
- Unit 3: Counting
- ▼ Unit 4: Discrete random variables

Unit overview

Lec. 5: Probability mass functions and expectations

Exercises 5 due Mar 02, 2016 at 23:59 UTC

Lec. 6: Variance; Conditioning on an event; Multiple r.v.'s Exercises 6 due Mar 02, 2016

Lec. 7: Conditioning on a random variable; Independence of r.v.'s Exercises 7 due Mar 02, 2016 at 23:59 UTC

Solved problems

at 23:59 UTC

Additional theoretical material

Problem Set 4 Problem Set 4 due Mar 02, 2016 at 23:59 UTC

**Unit summary** 

Unit 5: Continuous random variables

Exercise: The hat problem

(2/2 points)

Consider the hat problem, with n=10. What is the expected value of  $X_3X_6X_7$ ?

$$\mathbf{E}[X_3 X_6 X_7] = \begin{bmatrix} 1/720 \\ \checkmark \text{ Answer: 0.00139} \end{bmatrix}$$

Answer:

By symmetry, this is the same as  $\mathbf{E}[X_1X_2X_3]$ . Since the product  $X_1X_2X_3$  is either zero or one, this is the same as

$$\mathbf{P}(X_1X_2X_3=1) = \mathbf{P}(X_1=1) \cdot \mathbf{P}(X_2=1 \mid X_1=1) \cdot \mathbf{P}(X_3=1 \mid X_1=X_2=1)$$

By thinking in terms of the sequential description of the process, we have seen that  $\mathbf{P}(X_1=1)=1/10$  and  $\mathbf{P}(X_2=1\mid X_1=1)=1/9$ . By a similar argument, given that the first two people obtained their own hats, the third person is faced with 8 hats, one of which is his/her own, and has probability  $\mathbf{P}(X_3=1\mid X_1=X_2=1)=1/8$  of picking it. Thus, the final answer is  $(1/10) \cdot (1/9) \cdot (1/8)$ .

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