



MITx: 6.041x Introduction to Probability - The Science of Uncertainty



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Bookmark

Problem 4: Convolution calculations

(6/6 points)

1. Let the discrete random variable X be uniform on $\{0, 1, 2\}$ and let the discrete random variable Y be uniform on $\{3, 4\}$. Assume that X and Y are independent. Find the PMF of $X + Y$ using convolution. Determine the values of the constants a , b , c , and d that appear in the following specification of the PMF.

$$p_{X+Y}(z) = \begin{cases} a, & z = 3, \\ b, & z = 4, \\ c, & z = 5, \\ d, & z = 6, \\ 0, & \text{otherwise.} \end{cases}$$

 $a =$

1/6



Answer: 0.16667

 $b =$

1/3



Answer: 0.33333

 $c =$

1/3




Answer: 0.33333


▼ Unit 6: Further topics on random variables

Unit overview


Lec. 11: Derived distributions

Exercises 11 due Mar 30, 2016 at 23:59 UTC 

Lec. 12: Sums of independent r.v.'s; Covariance and correlation

Exercises 12 due Mar 30, 2016 at 23:59 UTC 


Lec. 13: Conditional expectation and variance revisited; Sum of a random number of independent r.v.'s

Exercises 13 due Mar 30, 2016 at 23:59 UTC 

Solved problems

Additional theoretical material

Problem Set 6

Problem Set 6 due Mar 30, 2016 at 23:59 UTC 

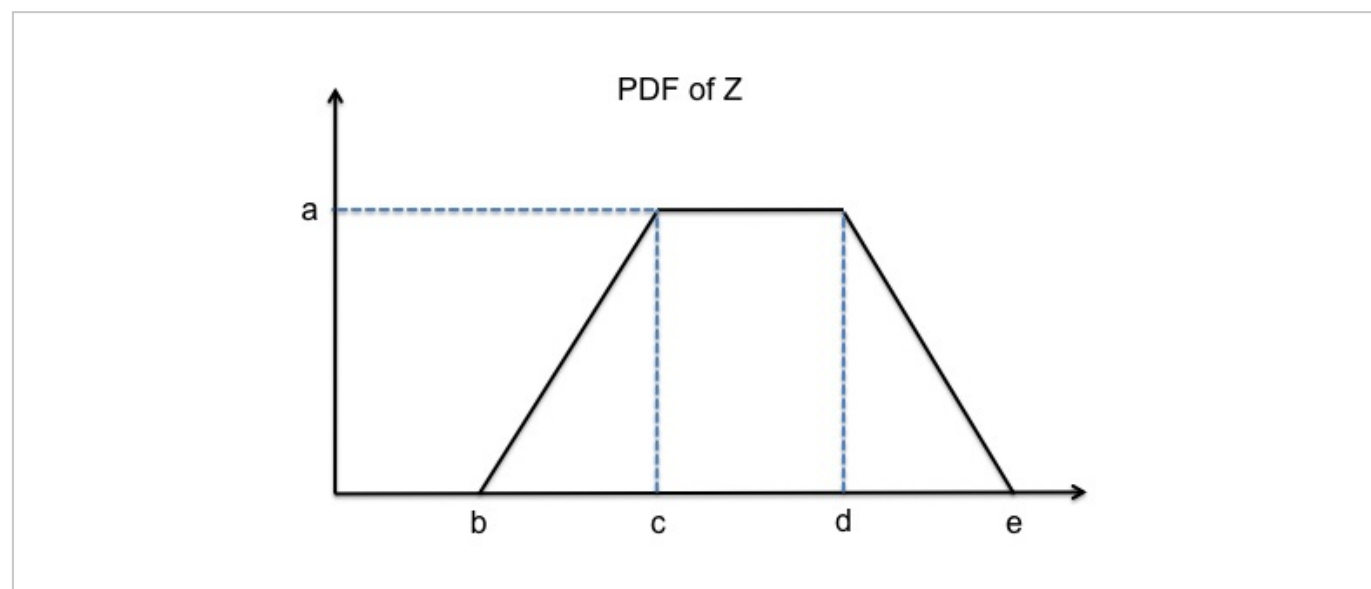
Unit summary

 $d =$



Answer: 0.16667

2. Let the random variable X be uniform on $[0, 2]$ and the random variable Y be uniform on $[3, 4]$. (Note that in this case, X and Y are continuous random variables.) Assume that X and Y are independent. Let $Z = X + Y$. Find the PDF of Z using convolution. The following figure shows a plot of this PDF. Determine the values of a , b , c , d , and e .


 $a =$



Answer: 0.5

 $b =$



Answer: 3

 $c =$



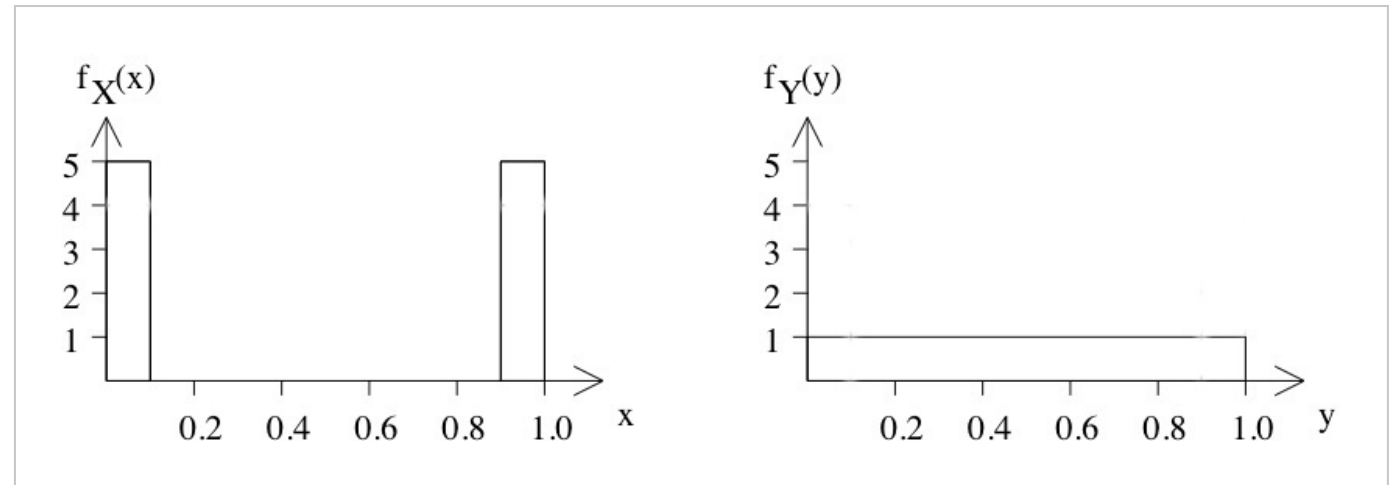
Answer: 4

- ▶ Unit 7: Bayesian inference
- ▶ Exam 2
- ▶ Unit 8: Limit theorems and classical statistics
- ▶ Unit 9: Bernoulli and Poisson processes
- ▶ Unit 10: Markov chains
- ▶ Exit Survey
- ▶ Final Exam

$d =$ ✓ Answer: 5

$e =$ ✓ Answer: 6

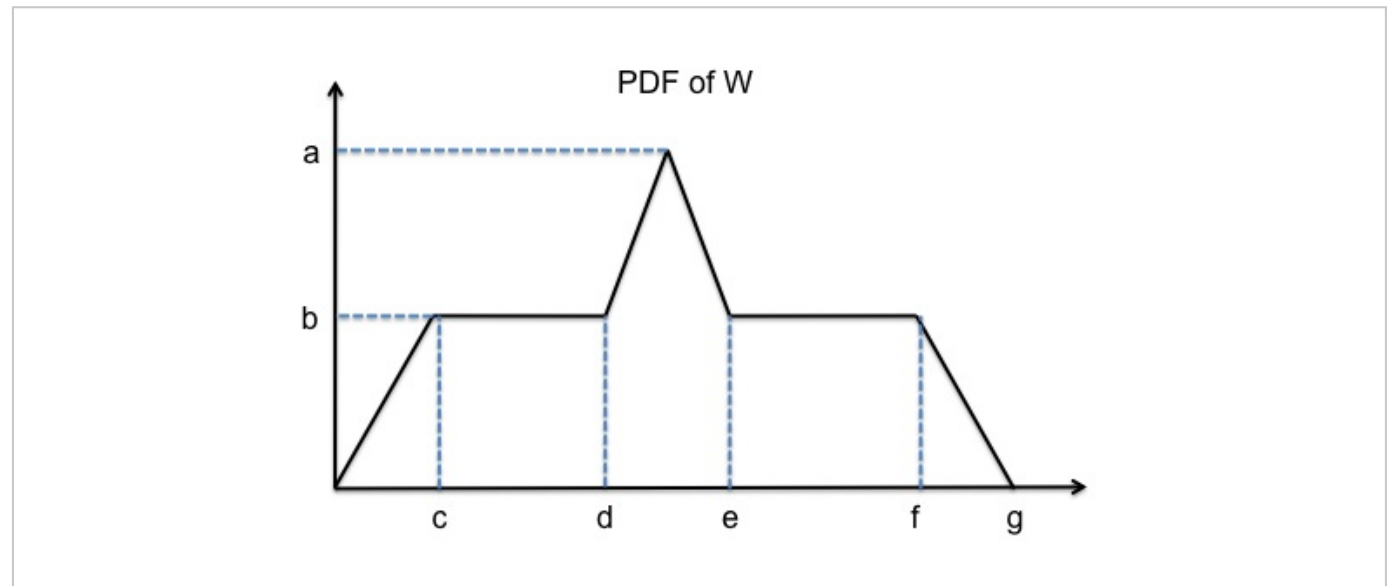
3. Let \mathbf{X} and \mathbf{Y} be two independent random variables with the PDFs shown below. below.



$$f_X(x) = \begin{cases} 5, & \text{if } 0 \leq x \leq 0.1 \text{ or } 0.9 \leq x \leq 1, \\ 0, & \text{otherwise.} \end{cases}$$

$$f_Y(y) = \begin{cases} 1, & \text{if } 0 \leq y \leq 1, \\ 0, & \text{otherwise.} \end{cases}$$

Let $W = X + Y$. The following figure shows a plot of the PDF of W . Determine the values of a, b, c, d, e, f , and g .

 $a =$

✓ Answer: 1

 $b =$

✓ Answer: 0.5

 $c =$

✓ Answer: 0.1

 $d =$

✓ Answer: 0.9

 $e =$

✓ Answer: 1.1

 $f =$

✓ Answer: 1.9

$g =$

2



Answer: 2

Answer:

$$1. \quad p_{X+Y}(z) = \begin{cases} 1/6, & z \in \{3, 6\} \\ 1/3, & z \in \{4, 5\} \\ 0, & \text{otherwise.} \end{cases}$$

2. If $3 \leq z \leq 6$, we have

$$\begin{aligned} f_{X+Y}(z) &= \int_{-\infty}^{\infty} f_X(x) f_Y(z-x) dx \\ &= \int_{\max(0, z-4)}^{\min(2, z-3)} \frac{1}{2} dx \\ &= (\min(2, z-3) - \max(0, z-4))/2. \end{aligned}$$

The PDF of $X + Y$ is then

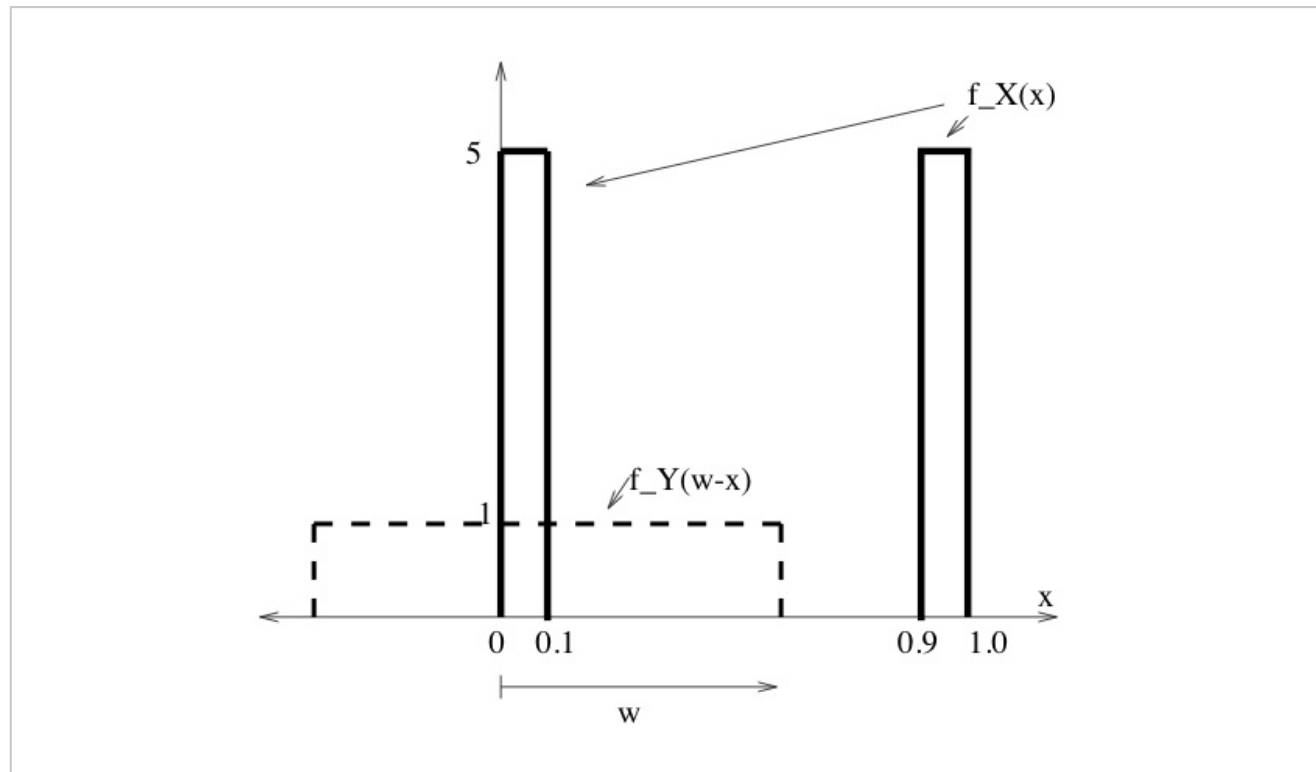
$$f_{X+Y}(z) = \begin{cases} \frac{z-3}{2}, & 3 \leq z < 4, \\ \frac{1}{2}, & 4 \leq z < 5, \\ \frac{6-z}{2}, & 5 \leq z \leq 6, \\ 0, & \text{otherwise.} \end{cases}$$

This answer can also be found by calculating the convolution graphically.

3. We have

$$f_W(w) = \int_{-\infty}^{\infty} f_X(x) f_Y(w - x) dx.$$

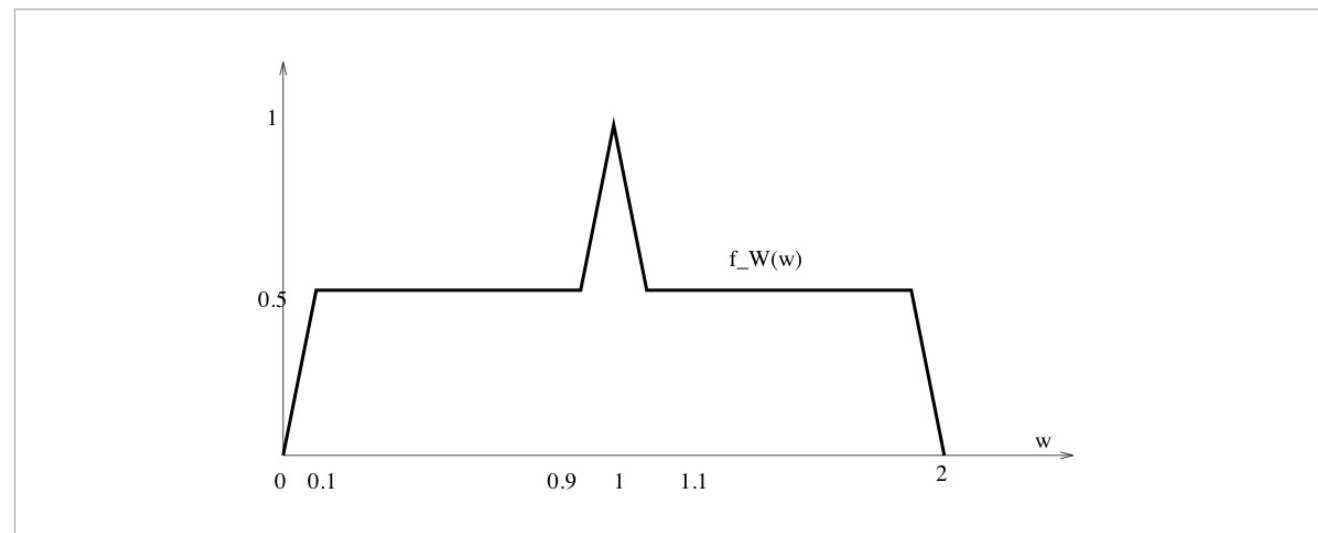
Graphically, $f_Y(w - x)$, as a function of x , is obtained by “flipping” $f_Y(x)$ about the $x = 0$ axis, then shifting the plot to the right by w . The PDF $f_X(x)$ is then sketched on the same plot.



From this graph we compute the integral of the product of the curves for any given w . By visualizing the graph as w is varied, we obtain

$$f_W(w) = \begin{cases} 5w, & 0 \leq w \leq 0.1, \\ 0.5, & 0.1 \leq w \leq 0.9, \\ 5(0.1 + (w - 0.9)), & 0.9 \leq w \leq 1.0, \\ 5(0.1 + (1.1 - w)), & 1.0 \leq w \leq 1.1, \\ 0.5, & 1.1 \leq w \leq 1.9, \\ 5(2.0 - w), & 1.9 \leq w \leq 2.0, \\ 0, & \text{otherwise.} \end{cases}$$

Pictorially,



You have used 1 of 2 submissions

DISCUSSION

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