# 7.3.4 More Advanced (But Still Simple) Examples

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Week 7 due Nov 20, 2023 01:42 IST Completed

# 7.3.4 More Advanced (But Still Simple) Examples

## Video

Start of transcript. Skip to the end.

Dr. Robert van de Geijn: So now we're going

to look at slightly more complex examples.

And this is all in preparation for being able to discuss

the general case next week.

So here's the idea.

If we have a matrix A, can we multiply it

0:00 / 0:00 2.0x

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## Reading Assignment

0 points possible (ungraded)

Read Unit 7.3.4 of the notes. [LINK]

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## Homework 7.3.4.1

1/1 point (graded)

$$\text{Find} \begin{pmatrix} -2 & 0 \\ 4 & 2 \end{pmatrix}^{-1} =$$

- ( ) $\begin{pmatrix} -\frac{1}{2} & 1 \\ 0 & \frac{1}{2} \end{pmatrix}$
- ( ) $\begin{pmatrix} \frac{1}{2} & 0 \\ 1 & \frac{1}{2} \end{pmatrix}$
- (•) $\begin{pmatrix} -\frac{1}{2} & 0 \\ 1 & \frac{1}{2} \end{pmatrix}$
- ( ) $\begin{pmatrix} -\frac{1}{2} & 0 \\ 1 & -\frac{1}{2} \end{pmatrix}$

✓

**Homework 7.3.4.1** Compute $\begin{pmatrix} -2 & 0 \\ 4 & 2 \end{pmatrix}^{-1} = \begin{pmatrix} -\frac{1}{2} & 0 \\ 1 & \frac{1}{2} \end{pmatrix}$.

**Answer:** Here is how you can find the answer First, solve

$$\begin{pmatrix} -2 & 0 \\ 4 & 2 \end{pmatrix} \begin{pmatrix} \chi_0 \\ \chi_1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}.$$

If you do forward substitution, you see that the solution is $\begin{pmatrix} -\frac{1}{2} \\ 1 \end{pmatrix}$, which becomes the first column of $A^{-1}$.
Next, solve

$$\begin{pmatrix} -2 & 0 \\ 4 & 2 \end{pmatrix} \begin{pmatrix} \chi_0 \\ \chi_1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}.$$

If you do forward substitution, you see that the solution is $\begin{pmatrix} 0 \\ \frac{1}{2} \end{pmatrix}$, which becomes the second column of $A^{-1}$.
Check:

$$\begin{pmatrix} -2 & 0 \\ 4 & 2 \end{pmatrix} \begin{pmatrix} -\frac{1}{2} & 0 \\ 1 & \frac{1}{2} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

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Answers are displayed within the problem

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## Video

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Dr. Robert van de Geijn: OK, so the way to compute this inverse

is to note that the lower triangular matrix times its inverse partition

by columns must be equal to the identity partition by its columns.

That's what we talked about in the last video.

If you then say, well, I want to solve with the lower triangular matrix here

0:00 / 0:00 2.0x

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## Homework 7.3.4.2

1/1 point (graded)

$$\text{Find } \begin{pmatrix} 1 & -2 \\ 0 & 2 \end{pmatrix}^{-1} =$$

- [ ] $\begin{pmatrix} 1 & -\frac{1}{2} \\ 0 & \frac{1}{2} \end{pmatrix}$
- [ ] $\begin{pmatrix} 1 & -\frac{1}{2} \\ 0 & 1 \end{pmatrix}$
- [x] $\begin{pmatrix} 1 & 1 \\ 0 & \frac{1}{2} \end{pmatrix}$
- [ ] $\begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix}$

✔

**Answer:**

$$\begin{pmatrix} 1 & 1 \\ 0 & \frac{1}{2} \end{pmatrix}.$$

Here is how you can find this matrix: First, solve

Calculator

$$\begin{pmatrix} 1 & -2 \\ 0 & 2 \end{pmatrix} \begin{pmatrix} \chi_0 \\ \chi_1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}.$$

If you do back substitution, you see that the solution is $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$, which becomes the first column of $A^{-1}$.
Next, solve

$$\begin{pmatrix} 1 & -2 \\ 0 & 2 \end{pmatrix} \begin{pmatrix} \chi_0 \\ \chi_1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}.$$

If you do back substitution, you see that the solution is $\begin{pmatrix} 1 \\ \frac{1}{2} \end{pmatrix}$, which becomes the second column of $A^{-1}$.
Check:

$$\begin{pmatrix} 1 & -2 \\ 0 & 2 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 0 & \frac{1}{2} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

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## Homework 7.3.4.3

1/1 point (graded)

**Let $\alpha_{0,0} \neq 0$ and $\alpha_{1,1} \neq 0$. Then** $\begin{pmatrix} \alpha_{0,0} & 0 \\ \alpha_{1,0} & \alpha_{1,1} \end{pmatrix}^{-1} = \begin{pmatrix} \frac{1}{\alpha_{0,0}} & 0 \\ -\frac{\alpha_{1,0}}{\alpha_{0,0}\alpha_{1,1}} & \frac{1}{\alpha_{1,1}} \end{pmatrix}$

True ✔ Answer: True

**Answer:** True
Here is how you can find the matrix: First, solve

$$\begin{pmatrix} \alpha_{0,0} & 0 \\ \alpha_{1,0} & \alpha_{1,1} \end{pmatrix} \begin{pmatrix} \chi_0 \\ \chi_1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}.$$

If you solve the lower triangular system, you see that the solution is $\begin{pmatrix} \frac{1}{\alpha_{0,0}} \\ -\frac{\alpha_{1,0}}{\alpha_{0,0}\alpha_{1,1}} \end{pmatrix}$, which becomes the first column of $A^{-1}$.
Next, solve

$$\begin{pmatrix} \alpha_{0,0} & 0 \\ \alpha_{1,0} & \alpha_{1,1} \end{pmatrix} \begin{pmatrix} \chi_0 \\ \chi_1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}.$$

If you solve this system, you find the solution $\begin{pmatrix} 0 \\ \frac{1}{\alpha_{1,1}} \end{pmatrix}$, which becomes the second column of $A^{-1}$.
Check:

$$\begin{pmatrix} \alpha_{0,0} & 0 \\ \alpha_{1,0} & \alpha_{1,1} \end{pmatrix} \begin{pmatrix} \frac{1}{\alpha_{0,0}} & 0 \\ -\frac{\alpha_{1,0}}{\alpha_{0,0}\alpha_{1,1}} & \frac{1}{\alpha_{1,1}} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

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## Homework 7.3.4.4

1/1 point (graded)

Partition lower triangular matrix $L$ as

$$L = \left( \begin{array}{c|c} L_{00} & 0 \\ \hline l_{10}^T & \lambda_{11} \end{array} \right)$$

Assume that $L$ has no zeroes on its diagonal. Then

$$L^{-1} = \left( \begin{array}{c|c} L_{00}^{-1} & 0 \\ \hline -\frac{l_{10}^T L_{00}^{-1}}{\lambda_{11}} & \frac{1}{\lambda_{11}} \end{array} \right)$$

True

Answer: True

**Answer:** True Stictly speaking, one needs to show that $L_{00}$ has an inverse... This would require a proof by induction. We'll skip that part. Instead, we'll just multiply out:

$$\begin{aligned} \left( \begin{array}{c|c} L_{00} & 0 \\ \hline l_{10}^T & \lambda_{11} \end{array} \right) \left( \begin{array}{c|c} L_{00}^{-1} & 0 \\ \hline -\frac{l_{10}^T L_{00}^{-1}}{\lambda_{11}} & \frac{1}{\lambda_{11}} \end{array} \right) &= \left( \begin{array}{c|c} L_{00} L_{00}^{-1} & L_{00} 0 + 0 \frac{1}{\lambda_{11}} \\ \hline l_{10}^T L_{00}^{-1} - \lambda_{11} \frac{l_{10}^T L_{00}^{-1}}{\lambda_{11}} & l_{10}^T \times 0 + \lambda_{11} \frac{1}{\lambda_{11}} \end{array} \right) \\ &= \left( \begin{array}{c|c} I & 0 \\ \hline 0 & 1 \end{array} \right). \end{aligned}$$

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Answers are displayed within the problem

### Video

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Dr. Robert van de Geijn: So now we get to a more general case.

And the exercise is to show that the inverse of this lower triangular matrix

partitioned like that is given by this.

The way to verify that that is indeed the inverse

is a matter of just multiplying the two matrices together.

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## Homework 7.3.4.5

1/1 point (graded)

The inverse of a lower triangular matrix with no zeroes on its diagonal is a lower triangular matrix.

TRUE ✔ Answer: TRUE

**Answer:** True

Proof by induction on $n$, the size of the square matrix.

Let $L$ be the lower triangular matrix.

**Base case:** $n = 1$. Then $L = (\lambda_{11})$, with $\lambda_{11} \neq 0$. Clearly, $L^{-1} = (1/\lambda_{11})$.

**Inductive step:** Inductive Hypothesis: Assume that the inverse of any $n \times n$ lower triangular matrix with no zeroes on its diagonal is a lower triangular matrix.

We need to show that the inverse of any $(n+1) \times (n+1)$ lower triangular matrix, $L$, with no zeroes on its diagonal is a lower triangular matrix.

Partition

$$L = \left( \begin{array}{c|c} L_{00} & 0 \\ \hline l_{10}^T & \lambda_{11} \end{array} \right)$$

We know then that $L_{00}$ has no zeroes on its diagonal and $\lambda_{11} \neq 0$. We also saw that then

$$L^{-1} = \left( \begin{array}{c|c} L_{00}^{-1} & 0 \\ \hline -\frac{1}{\lambda_{11}} l_{10}^T L_{00}^{-1} & \frac{1}{\lambda_{11}} \end{array} \right)$$

Hence, the matrix has an inverse, and it is lower triangular.

By the **Principle of Mathematical Induction**, the result holds.

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Answers are displayed within the problem

The answer to the last exercise suggests an algorithm for inverting a lower triangular matrix. See if you can implement it!

## Homework 7.3.4.7

1/1 point (graded)

$$\text{Find} \begin{pmatrix} 1 & 2 \\ 1 & 1 \end{pmatrix}^{-1} =$$

- ○ $\begin{pmatrix} 1 & 2 \\ 1 & 1 \end{pmatrix}$
- ◉ $\begin{pmatrix} -1 & 2 \\ 1 & -1 \end{pmatrix}$
- ○ $\begin{pmatrix} -1 & \frac{1}{2} \\ 1 & -1 \end{pmatrix}$

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○ $\begin{pmatrix} 1 & \frac{1}{2} \\ 1 & 1 \end{pmatrix}$

✔

**Answer:**

$$\begin{pmatrix} -1 & 2 \\ 1 & -1 \end{pmatrix}.$$

Here is how you can find this matrix: First, you compute the LU factorization. Since there is only one step, this is easy:

$$\begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 0 & -1 \end{pmatrix}.$$

Thus,

$$\begin{pmatrix} 1 & 2 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 0 & -1 \end{pmatrix}.$$

Next, you solve

$$\begin{pmatrix} 1 & 2 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} \chi_0 \\ \chi_1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

by solving $\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} \zeta_0 \\ \zeta_1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ followed by $\begin{pmatrix} 1 & 2 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} \chi_0 \\ \chi_1 \end{pmatrix} = \begin{pmatrix} \zeta_0 \\ \zeta_1 \end{pmatrix}$. If you do this right, you get $\begin{pmatrix} -1 \\ 1 \end{pmatrix}$, which becomes the first column of the inverse.

You solve

$$\begin{pmatrix} 1 & 2 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} \chi_0 \\ \chi_1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

in a similar manner, yielding the second column of the inverse, $\begin{pmatrix} 2 \\ -1 \end{pmatrix}$.

Check:

$$\begin{pmatrix} 1 & 2 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} -1 & 2 \\ 1 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

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Answers are displayed within the problem

## Homework 7.3.4.8

1/1 point (graded)

If $\alpha_{0,0}\alpha_{1,1} - \alpha_{1,0}\alpha_{0,1} \neq 0$ then

$$\begin{pmatrix} \alpha_{0,0} & \alpha_{0,1} \\ \end{pmatrix}^{-1} = \frac{1}{\quad} \begin{pmatrix} \alpha_{1,1} & -\alpha_{0,1} \\ \end{pmatrix}$$

Calculator

$$\begin{pmatrix} \alpha_{1,0} & \alpha_{1,1} \end{pmatrix} \quad \alpha_{0,0}\alpha_{1,1} - \alpha_{1,0}\alpha_{0,1} \begin{pmatrix} -\alpha_{1,0} & \alpha_{0,0} \end{pmatrix}$$

True ✔ Answer: True

**Answer:** True
Check:

$$\begin{aligned}
&\begin{pmatrix} \alpha_{0,0} & \alpha_{0,1} \\ \alpha_{1,0} & \alpha_{1,1} \end{pmatrix} \frac{1}{\alpha_{0,0}\alpha_{1,1} - \alpha_{1,0}\alpha_{0,1}} \begin{pmatrix} \alpha_{1,1} & -\alpha_{0,1} \\ -\alpha_{1,0} & \alpha_{0,0} \end{pmatrix} \\
&= \frac{1}{\alpha_{0,0}\alpha_{1,1} - \alpha_{1,0}\alpha_{0,1}} \begin{pmatrix} \alpha_{0,0} & \alpha_{0,1} \\ \alpha_{1,0} & \alpha_{1,1} \end{pmatrix} \begin{pmatrix} \alpha_{1,1} & -\alpha_{0,1} \\ -\alpha_{1,0} & \alpha_{0,0} \end{pmatrix} \\
&= \frac{1}{\alpha_{0,0}\alpha_{1,1} - \alpha_{1,0}\alpha_{0,1}} \begin{pmatrix} \alpha_{0,0}\alpha_{1,1} - \alpha_{0,1}\alpha_{1,0} & -\alpha_{0,0}\alpha_{0,1} + \alpha_{0,1}\alpha_{0,0} \\ \alpha_{1,0}\alpha_{1,1} - \alpha_{1,1}\alpha_{1,0} & -\alpha_{1,0}\alpha_{0,1} + \alpha_{1,1}\alpha_{0,0} \end{pmatrix} \\
&= \frac{1}{\alpha_{0,0}\alpha_{1,1} - \alpha_{1,0}\alpha_{0,1}} \begin{pmatrix} \alpha_{0,0}\alpha_{1,1} - \alpha_{0,1}\alpha_{1,0} & 0 \\ 0 & \alpha_{1,1}\alpha_{0,0} - \alpha_{1,0}\alpha_{0,1} \end{pmatrix} \\
&= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}
\end{aligned}$$

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## Homework 7.3.4.9

1/1 point (graded)

The $2 \times 2$ matrix $A = \begin{pmatrix} \alpha_{0,0} & \alpha_{0,1} \\ \alpha_{1,0} & \alpha_{1,1} \end{pmatrix}$ has an inverse if and only if $\alpha_{0,0}\alpha_{1,1} - \alpha_{1,0}\alpha_{0,1} \neq 0$.

True ✔ Answer: True

**Answer:** True
This is an immediate consequence of the last exercise.

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In the below video, towards the end, the instructor misspeaks and says that the inverse exists if and only if the determinant is zero. The correct statement is that the inverse exists if and only if the determinant is **not** zero.

**Video**

Start of transcript. Skip to the end.

Calculator

0:00 / 0:00 2.0x

Dr. Robert van de Geijn: So let's look at inverting a general two

by two matrix.

The conjecture is that a formula for its inverse

is given by what's on the right, right here.

And notice that that formula can only be computed if this right here

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