14. Properties of determinants

Even for large matrices, we can easily compute determinants in special cases (such as for diagonal and upper triangular matrices). We record some properties of determinants that will help us compute determinants more easily.

The **diagonal** of a matrix consists of the entries a_{ij} with $i = j$.

A **diagonal matrix** is a square matrix that has zeros everywhere outside the diagonal:

$$\begin{pmatrix} a_{11} & 0 & 0 \\ 0 & a_{22} & 0 \\ 0 & 0 & a_{33} \end{pmatrix}$$

(it may have some zeros along the diagonal too).

An **upper triangular matrix** is a matrix whose entries strictly below the diagonal are all 0:

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ 0 & a_{22} & a_{23} \\ 0 & 0 & a_{33} \end{pmatrix}$$

(the entries on or above the diagonal may or may not be 0).

Example 14.1 Any square matrix in row-echelon form is upper triangular.

Theorem 14.2 The determinant of an upper triangular matrix equals the product of the diagonal entries.

For example,

$$\det \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ 0 & a_{22} & a_{23} \\ 0 & 0 & a_{33} \end{pmatrix} = a_{11}\, a_{22}\, a_{33}.$$

Proof. Why is the theorem true? The Laplace expansion along the first column shows that the determinant is a_{11} times a upper triangular minor with diagonal entries $a_{22}, \ldots, a_{nn}$.

□

Properties of determinants

Here are some properties of the determinant.

1. If $\mathbf{A}$ is a diagonal matrix, upper triangular matrix, or lower triangular matrix, then $\det \mathbf{A}$ is the product of the entries along the diagonal.
2. Multiplying an entire row by a scalar c multiplies $\det \mathbf{A}$ by c.
3. If one of the rows (or columns) is all $\mathbf{0}$, then $\det \mathbf{A} = 0$.
4. $\det(\mathbf{AB}) = \det(\mathbf{A})\det(\mathbf{B})$ (assuming that $\mathbf{A}$ and $\mathbf{B}$ are square matrices of the same size).

In particular, row operations multiply $\det \mathbf{A}$ by nonzero scalars, but do not change whether $\det \mathbf{A} = 0$.

Properties of determinants concept check I

1/1 point (graded)
Let $\mathbf{I}$ be the 100 by 100 identity matrix. What is $\det(\mathbf{I})$?

$\det(\mathbf{I}) =$ 1 ✔ **Answer:** 1

Solution:

The identity matrix is a diagonal matrix whose entries along the diagonal are all 1. Therefore the determinant is the product of these entries, which is 1.

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Properties of determinants concept check II

1/1 point (graded)

Let $\mathbf{A}$ be an invertible matrix. What is the determinant of $\mathbf{A}^{-1}$ in terms of the the determinant of $\mathbf{A}$? (Hint: consider the determinant of the product. $\mathbf{A}\mathbf{A}^{-1}$.)

- 0
- 1
- $\det(\mathbf{A})$
- $\dfrac{1}{\det(\mathbf{A})}$ ✔
- $1-\det(\mathbf{A})$
- None of these

Solution:

Appealing to the property of determinants of products as the hint suggests, we know that

$$\det\left(\mathbf{A}\right)\det\left(\mathbf{A}^{-1}\right)=\det(\mathbf{A}\mathbf{A}^{-1})=\det(\mathbf{I})=1.$$

Therefore

$$\det\left(\mathbf{A}^{-1}\right) = \frac{1}{\det\left(\mathbf{A}\right)}.$$

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