G1 (1/1 point)

8 people are lining up to purchase tickets, 4 people are holding 10 Yuan, 4 people are holding 20 Yuan, the ticket price is 10 Yuan. The ticket booth does not have money at the first place, find out how many different possible ways of the arrangement of 8 people that they can successfully purchase the tickets?

8064

8064

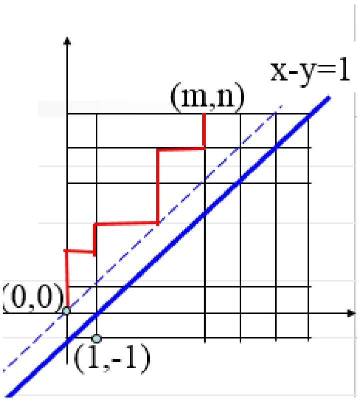
Answer: 8064

EXPLANATION

Let n=4, assume of using 2n dimensional 0,1 vector to represent a type of queueing condition, make this vector as $(a_1 a_2 a_3 \ldots a_{2n}), i = 1, 2, 3, \ldots, n. a_i = 1$represents the i^th of customer is holding 10 Yuan, $a_i = 0$ represents the i^th of customer is holding 20 Yuan.

If so, there are vector of n number of '0' element, n number of '1' element, total C(2n,n).

Each vector may have a one-to-one correspondence from (0,0) point to (n,n) point, which is departed from (0,0) point， means move an unit along x axis, means move an unit along y axis.To ensure the customer may purchase a ticket successfully, it may not appear the condition of unable to change 10 Yuan, the path must satisfy $x \le y$ condition.



The problem is equaled to find out the path from (0,0) to (n,n) , which does not cross $y = x$ line, thus $y >= x$, the path that we are looking for is the paths which can touch line $x = y$, but can not pass through line $x = y$. Therefore, we move line $x = y$ to right by one unit to get another line $x - y = 1$. Then the constraint that the path could touch $x = y$ but not go through it has translated into the constraint that the path should not have any point of intersection with line $x - y = 1$.

It is not that easy to directly compute the number of paths which do not have any point of intersection with line $x - y = 1$. Then we change to compute how many paths HAVE intersections with $x - y = 1$. For each path from (0,0) to (n,n), if it has touched the line $x - y = 1$ at the first time at point P, we can correspondingly draw a mirrored path from (1,-1) to P. (1,-1) is the symmetry point of (0,0) to line $x - y = 1$. Therefore, every path from (1,-1) to (n,n) corresponds to a path from (0,0) to (n,n) which have intersection point with $x - y = 1$, which is also corresponding to a path from(0,0) to (n,n) which pass through line x=y. The number of those paths are C(2n,n-1).

To obtain the paths which do not pass through the line x=y, we need to use subtraction rule as following:

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$$C(2n,n)-C(2n,n-1)=C(8,4)-C(8,3)=14$$

And, the person which is lining up to purchase ticket are of different person, so the queueing solution number is 14x4!x4!=8064

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