edX

Course > Section… > 1.1 A M… > 1.1.3 - …

1.1.3 - Exploratory Quiz: Species in Competition and Phase Plane Analysis

Bookmark this page

Question 1

1/1 point (graded)

The parameter value $\beta = 0$ corresponds to **no competition** between species $X$and Y. In this case, the system looks as follows:

$$\frac{dx}{dt} = x(1-x)$$

$$\frac{dy}{dt} = y(1-y)$$

Assuming we start with some members of both species $X$and Y, what are the possible long-term behaviors of the populations?

There are two ways to answer this.

- You can think of the system as a whole and do a phase plane analysis of the system.
- Alternately, since the species do not interact (the equation for y does not involve x and vice-versa), we can consider the two equations separately. Analyze the long-term behavior of the two species separately and then combine this information.

Recall that $x(t) = 1$ means species $X$is at its carrying capacity (saturation) and similarly for Y.

Assuming we start with some amount of species $X$and Y, which of the following are possible long-term behaviors of the populations?

- [ ] Species Xgoes extinct, Species Y approaches its carrying capacity
- [ ] Species Y goes extinct, Species Xapproaches its carrying capacity
- [ ] Species $X$and Y both go extinct
- [x] Species $X$and Y both approach their carrying capacity ✔
- [ ] None of the above.

✔

Explanation

Since the two species do not interact at all, we consider the long-term behavior of the two species separately and combine this information. The density of $x(t)$ is described by a logistic differential equation with solutions approaching the equilibrium solution $x(t) = 1$ over time. This means Xapproaches its carrying capacity. Since $y(t)$ has the same logistic differential equation, species Y will approach carrying capacity in the long-run too.

Note: if you do the phase plane analysis, we see there are four nullclines: $x = 0, x = 1, y = 0, y = 1$. The intersection of x and y nullclines create four equilibria: (0,0), (1,0), (0,1), (1,1). Orienting the null clines and regions created reveals that the equilibrium (0,0) is unstable (all trajectories head away from (0,0)). The two equilibrium on the axes are unstable (trajectories on the axis head toward them but all others head away toward (1,1) which is stable. This means if we start with an initial population where (x, y) is not on either axis, then $(x(t), y(t))$ approaches (1,1) and so both species will tend toward their carrying capacity.

Submit

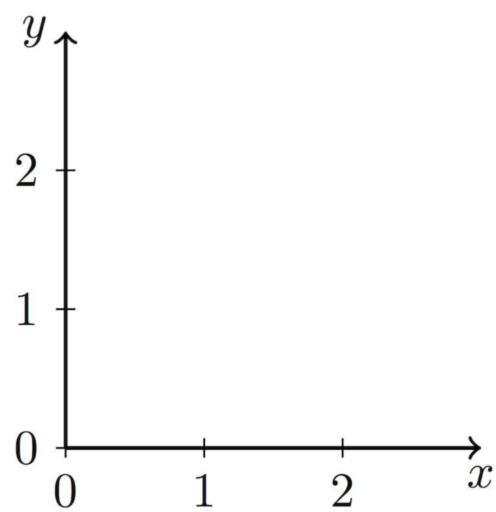
Answers are displayed within the problem

Now we will consider the case where there is some competition. Suppose that $\beta = \frac{1}{2}$. We'll call this "weak" competition.

$$\frac{dx}{dt} = x(1-x) - \frac{1}{2}xy$$

$$\frac{dy}{dt} = y(1-y) - \frac{1}{2}xy$$

Because there is interaction, we need to analyze the equations together. We can do this with phase plane analysis. The problems that follow will lead you through this process. Begin by drawing axes as follows:



View Larger Image
Image Description

Question 2

1/1 point (graded)

Suppose that we start at the point $\left(\frac{1}{4}, \frac{1}{4}\right)$. This means both species are at 25% of their individual carrying capacity.

What is the direction of the solution trajectory $(x(t), y(t))$ at this point?

Hint: consider the sign of $\frac{dx}{dt}$ and $\frac{dy}{dt}$ at this point.

- The trajectory is headed up and to the left.

- [x] The trajectory is headed up and to the right. ✔
- [ ] The trajectory is headed down and to the left.
- [ ] The trajectory is headed down and to the right.
- [ ] None of these.

Explanation

At this point, $\frac{dx}{dt} > 0$ and $\frac{dy}{dt} > 0$, thus $x(t)$ and $y(t)$ are both increasing. This means the trajectory traced out by $(x(t), y(t))$ is headed up and to the right.

Submit

Answers are displayed within the problem

Question 3

1/1 point (graded)

One of the nullclines where $\frac{dx}{dt} = 0$ is $x = 0$. What is the other nullcline?

- [ ] $x = 1$
- [ ] $y = 0$
- [ ] $y = 1$
- [ ] $y = 1 - x$
- [x] $y = 2 - 2x$ ✔

- $y = \frac{1}{2} - \frac{1}{2}x$
- None of the above

Explanation

To find nullclines where $\frac{dx}{dt} = 0$, we look for factors of the right hand side of the equation for $\frac{dx}{dt}$. By factoring out x from $\frac{dx}{dt} = x(1-x) - \frac{1}{2}xy = x(1 - x - \frac{1}{2}y)$, we see a second factor. This is zero when $1 - x = \frac{1}{2}y$ which is the same as $y = 2 - 2x$.

Submit

Answers are displayed within the problem

Question 4

1/1 point (graded)

One of the nullclines where $\frac{dy}{dt} = 0$ is $y = 0$. What is the other nullcline?

- $x = 0$
- $x = 1$
- $y = 1$
- $y = 1 - x$
- $y = 2 - 2x$
- $y = \frac{1}{2} - \frac{1}{2}x$

- [x] None of the above ✔

Explanation

To find nullclines where $\frac{dy}{dt} = 0$, we look for factors of the right hand side of the equation for $\frac{dy}{dt}$. By factoring out y from $\frac{dy}{dt} = y(1-y) - \frac{1}{2}xy = y(1 - y - \frac{1}{2}x)$, we see a second factor. This is zero when $1 - \frac{1}{2}x = y$.

Submit

Answers are displayed within the problem

Question 5

1/1 point (graded)

The point $(0, 0)$ is one equilibrium of the system (corresponding to no members of either species X or Y). There are three other equilibrium points of the system. What are they?

Hint: Use your work with the nullclines.

- [x] $(1, 0)$ ✔
- [x] $(0, 1)$ ✔
- [ ] $(\frac{1}{2}, \frac{1}{2})$
- [x] $(\frac{2}{3}, \frac{2}{3})$ ✔
- [ ] $(1, 1)$
- [ ] Other equilibrium point not listed here.

✔

Explanation

Equilibrium points occur when both $\frac{dx}{dt} = 0$ and $\frac{dy}{dt} = 0$, so we look at intersections of the corresponding nullclines. The nullclines $x = 0$ and $y = 0$ intersect at (0,0). The nullclines $x = 0$ and $y = 1 - \frac{1}{2}x$ intersect at $(0, 1)$. The nullclines $y = 2 - 2x$ and $y = 0$ intersect at $(1, 0)$. The nullclines $y = 2 - 2x$ and $y = 1 - \frac{1}{2}x$ intersect at $(2/3, 2/3)$.

Submit

Answers are displayed within the problem

Question 6

1/1 point (graded)

Now that you have identified the nullclines and equilibrium points, sketch arrows to indicate the direction of solution trajectories along the nullclines and in the regions they create. Use this to answer the following question.

Assuming we start with some members of both species X and Y, which of the following are possible long-term behaviors of the system?

- [ ] Species X goes extinct, Species Y approaches its carrying capacity
- [ ] Species Y goes extinct, Species X approaches its carrying capacity
- [ ] Species X and Y both go extinct
- [ ] Species X and Y both approach their carrying capacities
- [x] Species X and Y both approach a population level below their carrying capacity
- [ ] None of the above.

Explanation

Species X and Y both approach a stable population level below their carrying capacity. See the next video for the phase plane analysis and explanation.

Submit

Answers are displayed within the problem