6. Deterministic Design with Gaussian Noise

Review of Multi-Dimensional Gaussians

1/1 point (graded)

The n-dimensional Gaussian $\mathcal{N}_n(\mu, \Sigma)$ with mean μ and covariance matrix Σ has density

$$f(\mathbf{x}) = \frac{\exp\left(-\frac{1}{2}(\mathbf{x}-\mu)^T \Sigma^{-1}(\mathbf{x}-\mu)\right)}{\sqrt{(2\pi)^n \det\Sigma}}$$

for all $\mathbf{x} \in \mathbb{R}^n$.

Let $\mathbf{X} \sim \mathcal{N}_n(0, \Sigma)$, so that it is centered at the origin. If we have $\mathbf{Y} = M\mathbf{X}$ for some matrix M, it turns out that $\mathbf{Y}$ is also an n-dimensional Gaussian, $\mathcal{N}_n(0, \Sigma_{\mathbf{Y}})$. Which of the following provides a correct formula for the Covariance $\Sigma_{\mathbf{Y}}$ of $\mathbf{Y}$?

(*Hint: Recall the formula* $\Sigma_{\mathbf{Y}} = \mathbb{E}\left[(\mathbf{Y}-\mathbb{E}[\mathbf{Y}])(\mathbf{Y}-\mathbb{E}[\mathbf{Y}])^T\right]$.)

- $M\Sigma M^{-1}$
- $M^{-1}\Sigma M$

- [x] $M\Sigma M^T$
- [ ] $M^T\Sigma M$

Solution:

This can be directly computed as hinted.

$$\begin{aligned}\Sigma_{\mathbf{Y}} &= \mathbb{E}\left[\left(\mathbf{Y}-\mathbb{E}\left[\mathbf{Y}\right]\right)\left(\mathbf{Y}-\mathbb{E}\left[\mathbf{Y}\right]\right)^T\right]\\ &= \mathbb{E}\left[\left(M\mathbf{X}-\mathbb{E}\left[M\mathbf{X}\right]\right)\left(M\mathbf{X}-\mathbb{E}\left[M\mathbf{X}\right]\right)^T\right]\\ &= \mathbb{E}\left[M\left(\mathbf{X}-\mathbb{E}\left[\mathbf{X}\right]\right)\left(\mathbf{X}-\mathbb{E}\left[\mathbf{X}\right]\right)^T M^T\right]\\ &= M\mathbb{E}\left[\left(\mathbf{X}-\mathbb{E}\left[\mathbf{X}\right]\right)\left(\mathbf{X}-\mathbb{E}\left[\mathbf{X}\right]\right)^T\right]M^T\\ &= M\Sigma M^T.\end{aligned}$$

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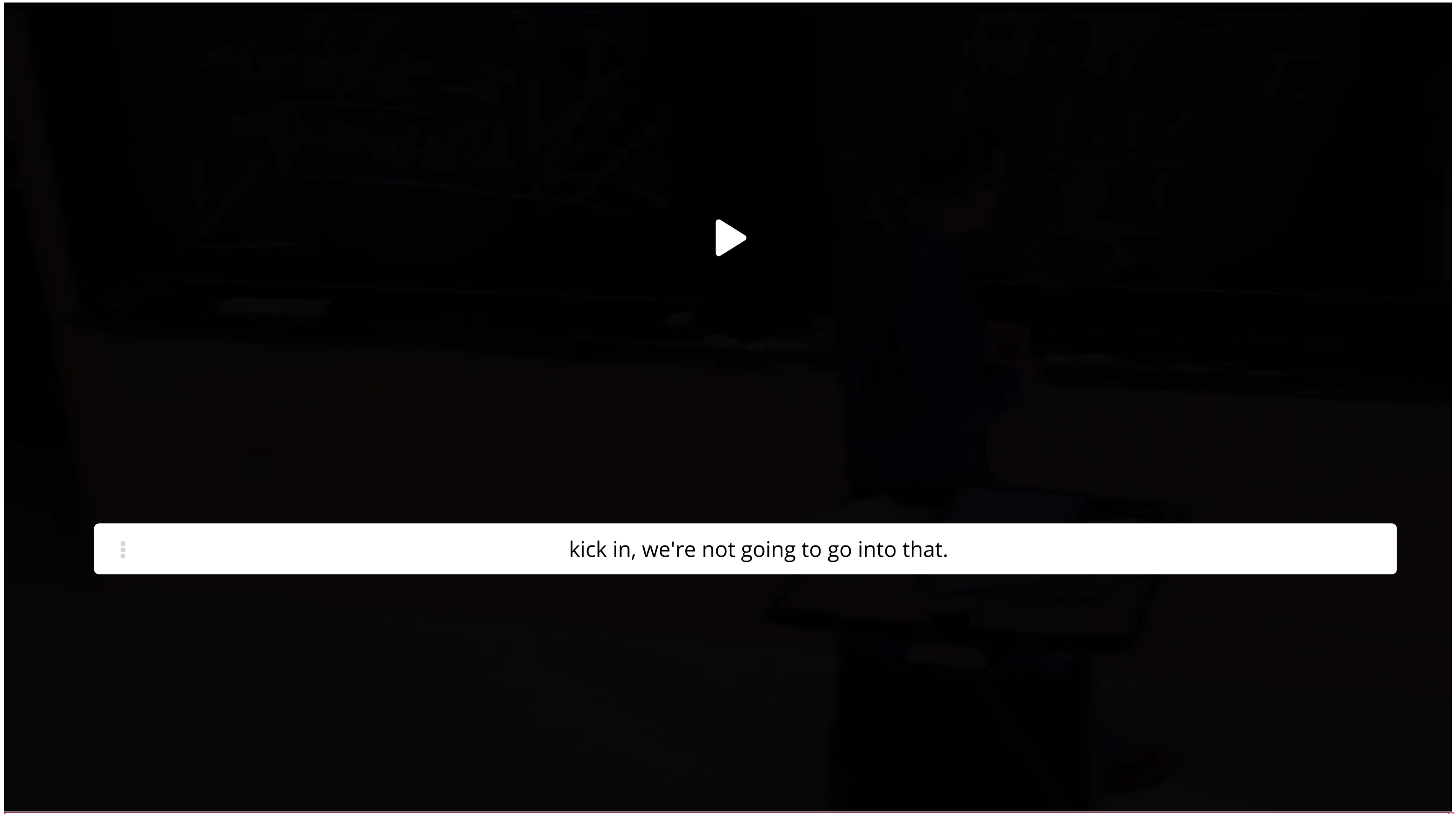
The Least Square Estimator is the MLE in Deterministic Design



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could be an inside joke...

question posted 2 days ago by rickytyagi

but want to be sure I'm not missing relevant info, what does the prof. say at around 0:30 "The first one says that the LSE is the MLE because I like TLA.."??

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1 response

Erocha (Community TA)
2 days ago

Two Letter Acronym - like MIT

Is two instead of three also an inside joke?

posted 2 days ago by DriftingWoods

reminds me of this acronym-rich clip:
https://www.youtube.com/watch?v=wXlvy3sTTBk

posted a day ago by mbh038

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