12. Boundary conditions

Flux boundary condition

If we are prescribing the flow rate of CO_2 at the boundary, this is just the flux times the cross section

$$qA = -A\alpha \frac{\partial u}{\partial x}. \tag{3.61}$$

For a pipe of length L with boundaries at $x = 0, L$, after multiplying the constant factors all together, we get the conditions for $t > 0$ as

$$\frac{\partial u}{\partial x}(0,t) \quad = a \tag{3.62}$$

$$\frac{\partial u}{\partial x}(L,t) \quad = b \tag{3.63}$$

where a and b are constants. Note we could have a and b vary with time, but the techniques for solving such partial differential equations are beyond the level of this class. In general, boundary conditions that set the derivative at a boundary are known as **Neumann** boundary conditions.

Concentration boundary condition

The concentration boundary condition is similar to the above, with the difference being that we prescribe u itself instead of its derivative. We get an analogous set of equations to before, for $t > 0$

$$\begin{aligned} u\left(0,t\right) &= a, \qquad (3.64) \\ u\left(L,t\right) &= b. \qquad (3.65) \end{aligned}$$

We note that in mathematical terms these are called **Dirichlet** boundary conditions.

Other boundary conditions

Besides the two simple boundary conditions we described above, there are a few others that can be useful. One other boundary condition, not used as often but still important, is what is know as the **Robin** boundary condition. This condition has the form on the boundary

$$u + a\frac{\partial u}{\partial x} = b \qquad (3.66)$$

where a and b are constants. Such a condition is usually used to represent some sort of convective transport occurring at the boundaries. Imagine a glass of beer or soda with the top open to the atmosphere, and a wind is blowing over it. CO_2 naturally diffuses into the air above the beverage, and the wind will tend to carry it away. The above boundary condition deals with this case.

Make the analogy

2/2 points (graded)

In the analogous heat equation, the case of an insulated bar with ends held at $0^{\circ}\mathrm{C}$ corresponds to

● Dirichlet boundary conditions

○ Neumann boundary conditions

○ Robin boundary conditions

○ None of the above

✓

In the analogous heat equation, the case of an insulated bar with insulated ends corresponds to the

○ Dirichlet boundary conditions

● Neumann boundary conditions

○ Robin boundary conditions

○ None of the above

✓

Solution:

The ends of a thin (insulated) metal bar held at fixed temperatures corresponds to boundary conditions

$$\theta\left(0,t\right)=0 \qquad \theta\left(L,t\right)=0,$$

which is an example of Dirichlet boundary conditions.

The ends of a thin (insulated) metal bar with insulated ends corresponds to boundary conditions

$$\frac{\partial}{\partial x}\theta\left(0,t\right)=0 \qquad \frac{\partial}{\partial x}\theta\left(L,t\right)=0,$$

which is an example of Neumann boundary conditions.

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