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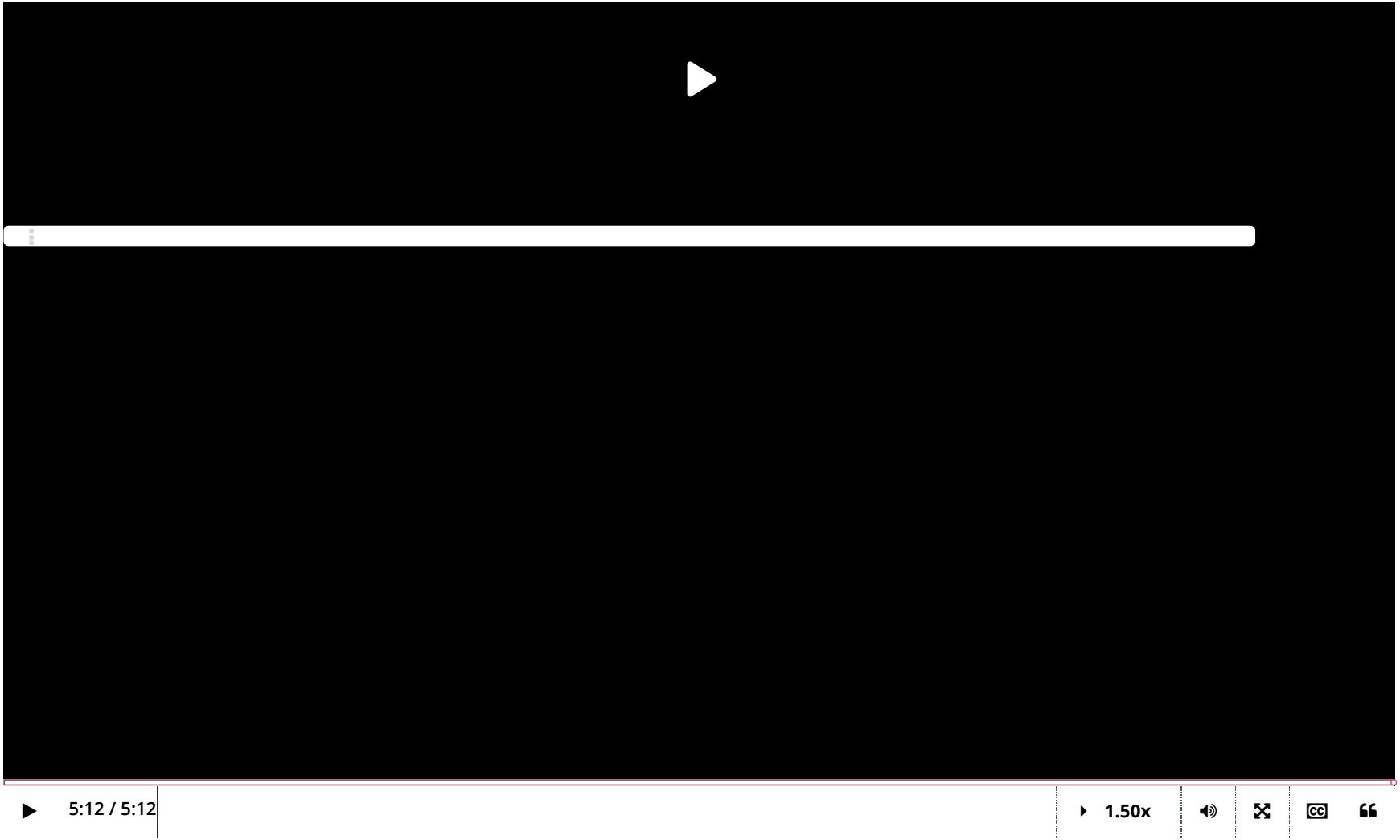
9. Worked Example: Concavity and Composition of Functions

Worked Example: Hessian and Concavity



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Combination of Convex functions

3/3 points (graded)

Let f_1, f_2 be convex functions on $\mathbb{R}$.

Determine if the following functions are necessarily convex or concave.

Hint: Recall a function $g : I \to \mathbb{R}$ is convex in the interval I is an interval, if for all pairs of real numbers $x_1 < x_2 \in I$

$$g\left(tx_1 + (1-t)\,x_2\right) \leq tg\left(x_1\right) + (1-t)\,g\left(x_2\right) \qquad \text{for all } 0 \leq t \leq 1.$$

- $3f_1 + 2f_2$:

  - (●) Convex
  - ( ) Concave
  - ( ) Cannot be determined without more information

  ✔

- $-10f_1$:

  - ( ) Convex
  - (●) Concave
  - ( ) Cannot be determined without more information

  ✔

- $f_2 f_1$:

  - ( ) Convex

Concave

Cannot be determined without more information ✔

Solution:

Given f_1, f_2 are convex, we have

$$f_1(tx_1 + (1-t)x_2) \leq tf_1(x_1) + (1-t)f_1(x_2) \qquad \text{for all } 0 \leq t \leq 1$$

and the same holds for f_2.

- The same inequality holds for $g = 3f_1 + 2f_2$:

$$\begin{aligned} g(tx_1 + (1-t)x_2) &= 3f_1(tx_1 + (1-t)x_2) + 2f_2(tx_1 + (1-t)x_2) \\ &\leq 3(tf_1(x_1) + (1-t)f_1(x_2)) + 2(tf_2(x_1) + (1-t)f_2(x_2)) \\ &= tg(x_1) + (1-t)g(x_2). \end{aligned}$$

  Hence $3f_1 + 2f_2$ is also convex.

  Remark: In general, any function $c_1f_1 + c_2f_2$ where $c_1, c_2 > 0$ is convex of $f_1,\ f_2$ are.

- $-10f_1$ is concave, because it is negative of a convex function.

- f_1f_2 is not necessary convex For example, is $f_1(x) = x$, and $f_2 = x^2$, then $(f_1f_2)(x) = x^3$ which is neither convex nor concave. Other examples of f_1 and f_2, e.g. $f_1 = f_2 = x^2$ will lead to f_1f_2 being convex.

Submit You have used 2 of 2 attempts

Answers are displayed within the problem

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? Both concave and convex (non-strictly)? 1

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