12. Worked examples

Example 12.1 Solve the linear system $4x + 3y + z = 5$.

Solution

This linear system describes a plane in $\mathbb{R}^3$. We can represent this equation as the matrix equation:

$$\begin{pmatrix} 4 & 3 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = 5.$$

To find the general solution we notice that the matrix $\begin{pmatrix} 4 & 3 & 1 \end{pmatrix}$ is already is row echelon form. It has one pivot, and 2 free columns. The pivot variable or dependent variable is x. The free variables or independent variables are y and z.

Set the free variables equal to parameters since they cannot be determined from the equation:

$$\begin{aligned} y &= c_1, \\ z &= c_2. \end{aligned}$$

Then solve for x using back substitution:

$$\begin{aligned} 4x + 3y + z &= 5 \\ 4x + 3c_1 + c_2 &= 5 \end{aligned}$$

$$x = \frac{5-3c_1-c_2}{4}$$

Putting everything together we get

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \frac{5-3c_1-c_2}{4} \\ c_1 \\ c_2 \end{pmatrix} = \begin{pmatrix} 5/4 \\ 0 \\ 0 \end{pmatrix} + c_1 \begin{pmatrix} -3/4 \\ 1 \\ 0 \end{pmatrix} + c_2 \begin{pmatrix} -1/4 \\ 0 \\ 1 \end{pmatrix}.$$

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Example 12.2 Solve the linear system

$$\begin{aligned} x + \ y + \ z + \ w &= 4 \\ x + 2y + 3z + 4w &= 7 \\ y + 2z + 3w &= 3 \end{aligned}$$

Solution

First, put the system of linear equations into matrix form:

$$\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 3 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} = \begin{pmatrix} 4 \\ 7 \\ 3 \end{pmatrix}.$$

Next we form the augmented matrix and put it into row echelon form:

$$\left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 4 \\ 1 & 2 & 3 & 4 & 7 \\ 0 & 1 & 2 & 3 & 3 \end{array}\right) \longrightarrow \left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 4 \\ 0 & 1 & 2 & 3 & 3 \\ 0 & 1 & 2 & 3 & 3 \end{array}\right) \longrightarrow \left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 4 \\ 0 & 1 & 2 & 3 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right).$$

The row echelon form of the augmented matrix has two pivots in orange. Therefore it has two free columns (not counting the augmented column).

The **dependent** or **pivot variables** are x and y. The **independent** or **free variables** are z and w.

Set the free variables equal to new parameters,

$$\begin{aligned} z &= c_1, \\ w &= c_2. \end{aligned}$$

The second to last row in the augmented matrix is equivalent to the equation

$$y + 2z + 3w = 3.$$

Use back substitution to solve for y in terms of the parameters.

$$\begin{aligned} y + 2c_1 + 3c_2 &= 3 \\ y &= 3 - 2c_1 - 3c_2 \end{aligned}$$

The first row in the augmented matrix is equivalent to the equation

$$x + y + z + w = 4.$$

Use back substitution to solve for x:

$$\begin{aligned} x + (3 - 2c_1 - 3c_2) + c_1 + c_2 &= 4 \\ x + 3 - c_1 - 2c_2 &= 4 \\ x &= 1 + c_1 + 2c_2. \end{aligned}$$

Writing the solution in vector form we find that the general solution is

$$\begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} = \begin{pmatrix} 1+c_1+2c_2 \\ 3-2c_1-3c_2 \\ c_1 \\ c_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \\ 0 \\ 0 \end{pmatrix} + c_1 \begin{pmatrix} 1 \\ -2 \\ 1 \\ 0 \end{pmatrix} + c_2 \begin{pmatrix} 2 \\ -3 \\ 0 \\ 1 \end{pmatrix}.$$

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setting y and z as independent parameters

So, When I solved this matrix, I chose y and z as my independent parameters. I get a different answer... 5

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