3. Closed and bounded regions

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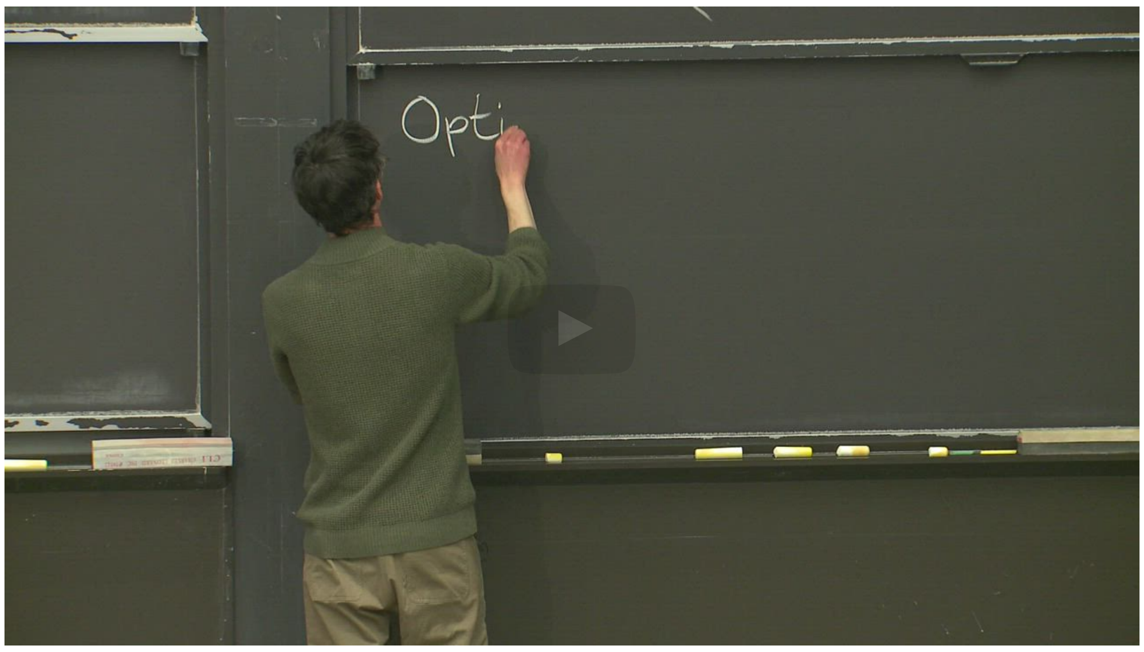
Local vs absolute extrema

Using the gradient to find local maxima and minima works well, but sometimes we want to find the absolute maximum or minimum value that a function has. For those values, we have to work harder. In fact, it is possible that a function has no absolute maximum or minimum. In this lecture, we will focus on optimizing a function inside of a closed and bounded region, in which case, we can be sure that there is an absolute max and min.

Goal: Find the absolute maximum or minimum of a function $f(x, y)$ on a region R.

Closed and bounded regions



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PROFESSOR: So optimization means, given a function

f of (x, y) and a region R, find the maximum

of the function on the region.

Find the maximum or the minimum of the function on the region.

So let's quickly remember how that worked in one-variable

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Recall the Extreme Value Theorem from single variable calculus.

The Extreme Value Theorem (Single variable)

If f is continuous on a closed interval $a \leq x \leq b$, then there are points at which f attains its maximum and its minimum in the interval $a \leq x \leq b$.

Recall that extrema can only occur

- at points where $f'(x) = 0$ (critical points),
- at points where $f'(x)$ is undefined, or
- at the endpoints of the closed interval: $x = a$ or $x = b$.

Some illustrations of these scenarios are shown below.

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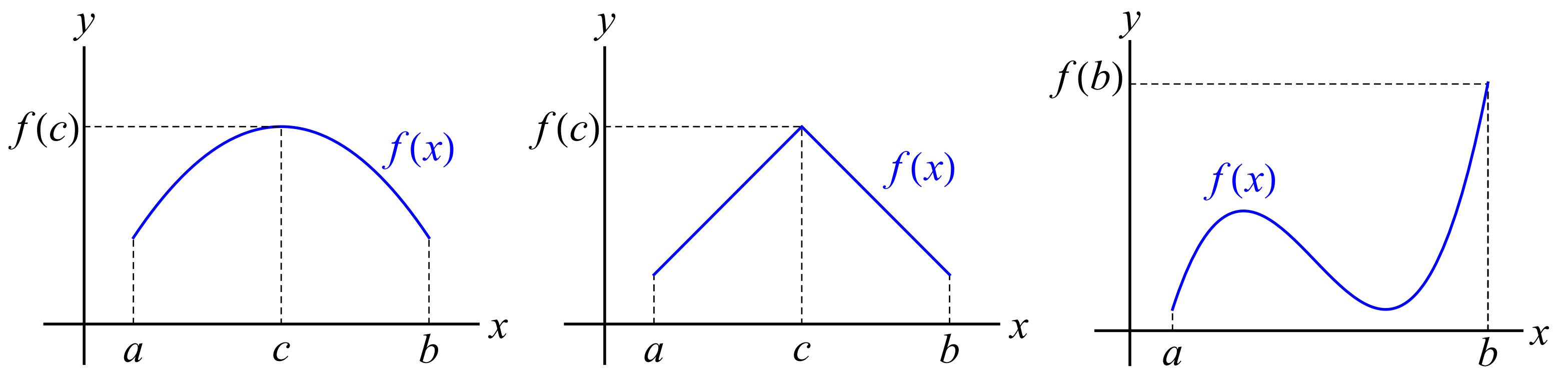


A similar idea can be applied to optimizing functions of two variables. To explain this, we need some terminology.

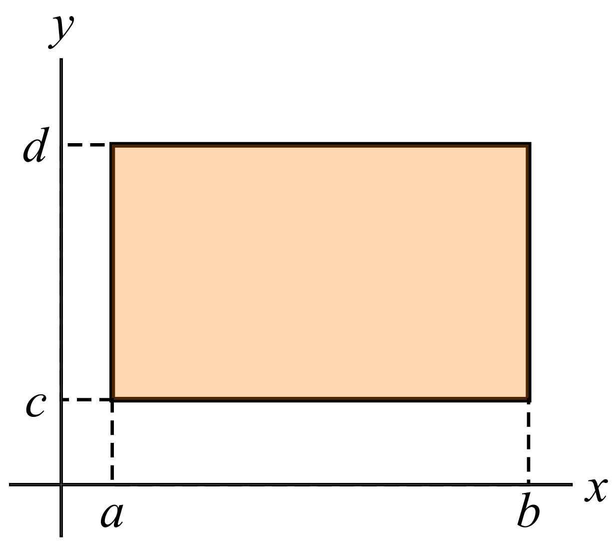
We have already seen that the multivariable version of a critical point is a point (x, y) at which $\nabla f(x, y) = \vec{0}$. Now we need a multivariable version of a closed interval.

Closed intervals in 1 dimension

In 1D, a closed interval is given by all the values of x such that $a \leq x \leq b$. Notice that a and b are included in the interval. A visualization of this on the number line is shown in the figure below.

Closed intervals generalized to 2 dimensions

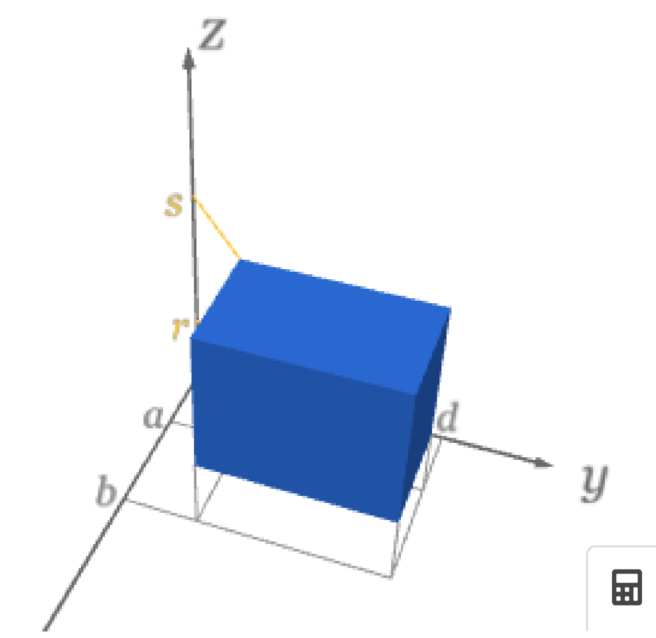
The simplest analog to this in two dimensions is a rectangle of finite width and height given by the ordered pairs (x, y) such that $a \leq x \leq b$ and $c \leq y \leq d$.



Extension to higher dimension: rectangular prism

In three dimensions, the analogue of an interval is a rectangular prism consisting of all ordered triples (x, y, z) such that $a \leq x \leq b$, $c \leq y \leq d$, and $r \leq z \leq s$. A visualization of this in 3-D space is shown in the figure below.

▶ **Rectangular prism**



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Generalizing to n-dimensions, the analogue of an interval consists of ordered n-tuples $(x_1, x_2, \ldots, x_n)$ such that $a_1 \leq x_1 \leq b_1$, $a_2 \leq x_2 \leq b_2$, ..., $a_n \leq x_n \leq b_n$. For $n > 3$, this is much harder to visualize!

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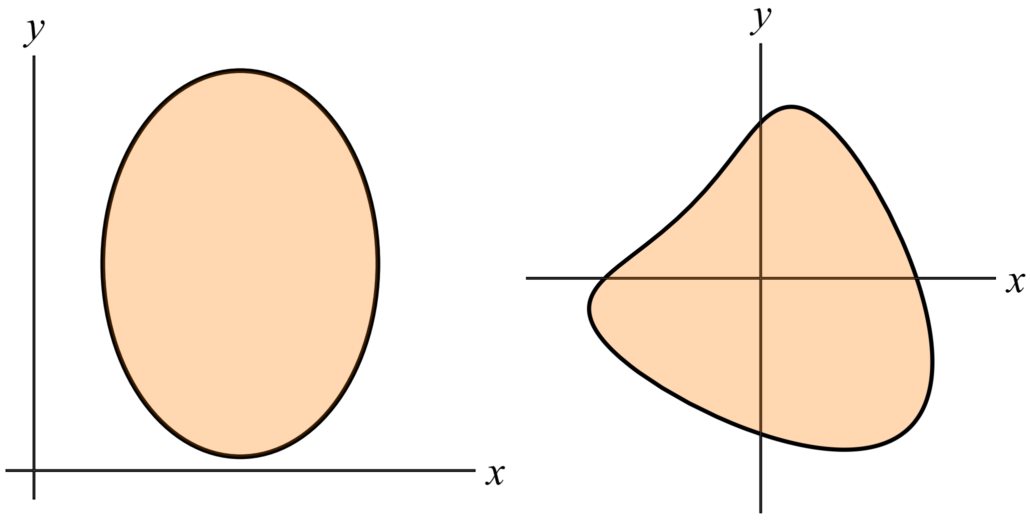
General closed and bounded regions

Notice that the boundaries are included in these spaces because we are constructing them from closed intervals. These regions are **closed and bounded**. The definition below is not a formal definition, but will suffice for this discussion.

Definition 3.1 A **bounded region** in the plane is a region that fits inside of a rectangular region of finite width and finite height. A **closed region** is a region that includes its boundary.

Examples 3.2 Some other examples of closed and bounded regions are shown below.



Optional: Footnote on Cartesian products

A rectangular region defined by points in the plane (x, y) such that $a \leq x \leq b$ and $c \leq y \leq d$ can be described as a **Cartesian product** . The notation for this is $I_1 \times I_2$ where $I_1 = \{x \text{ such that } a \leq x \leq b\}$ and $I_2 = \{y \text{ such that } c \leq y \leq d\}$. This type of set is called a Cartesian product and is a way of describing a set of ordered pairs in which the first variable (in our case, x) lies in the set I_1 and the second variable (in our case, y) lies in the interval I_2. Notice that the Cartesian product notation looks like a product of closed intervals.

Thus another way to think about the generalization of a closed interval in 2 dimensions is as a cartesian product of 2 closed intervals. Similarly, the generalization of a closed interval in n dimensions is the Cartesian product of n-closed intervals: $I_1 \times I_2 \times \cdots \times I_n$.

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"We have already seen that the multivariable version of a critical point is a point (x,y) at which grad(f)=0" 3
"We have already seen that the multivariable version of a critical point is a point (x,y) at which grad(f)=0" Shouldn't grad(f) be a zero ...

Missing adjective in Local vs absolute extrema paragraph
I think the adjective "closed" should be added to the introductive sentence: "In this lecture, we will focus on optimizing a function ins... 2

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