4. Optimization and linear approximation

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Optimization along the boundary

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PROFESSOR: Let's see what people are thinking.

So we know that the maximum is on the boundary,

so I'm just going to move the cursor--

move the little-- the cursor along the boundary.

And you guys will tell me, keep going, or stop there

0:00 / 0:00 2.0x

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We can start to think about optimization by first zooming in on the function $f(x,y)$ and using the linear approximation. Say we start at (x,y) and move to a point $(x+\Delta x, y+\Delta y)$. We want to know if doing this causes the change in f to be positive, negative, or zero. The linear approximation is given by

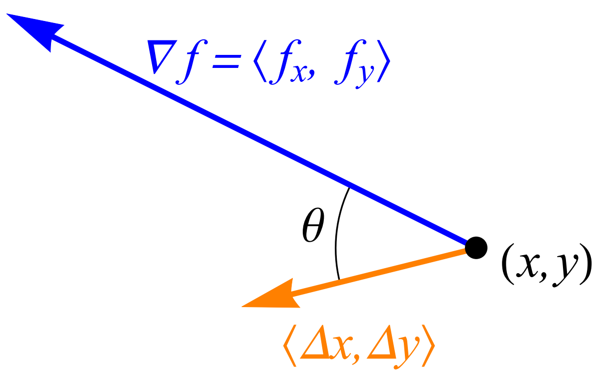
$$f(x+\Delta x, y+\Delta y) \approx f(x,y) + \underbrace{f_x(x,y)\,\Delta x + f_y(x,y)\,\Delta y}_{\text{change in } f} \qquad (4.155)$$

where we have highlighted the change in f as the part of the linear approximation that deviates from $f(x,y)$.

We can write the change in f as the following dot product

$$\text{change in } f = f_x(x,y)\,\Delta x + f_y(x,y)\,\Delta y = \langle f_x, f_y\rangle \cdot \langle \Delta x, \Delta y\rangle = |\langle f_x, f_y\rangle||\langle \Delta x, \Delta y\rangle| \cos\theta \qquad (4.156)$$

where θ is the angle between the gradient $\langle f_x, f_y\rangle$ and $\langle \Delta x, \Delta y\rangle$. An illustration of this is shown below.



From the equation for the change in f, we can conclude the following:

- If $\theta < \frac{\pi}{2}$, then $\cos\theta > 0$ and therefore the change in f is positive.

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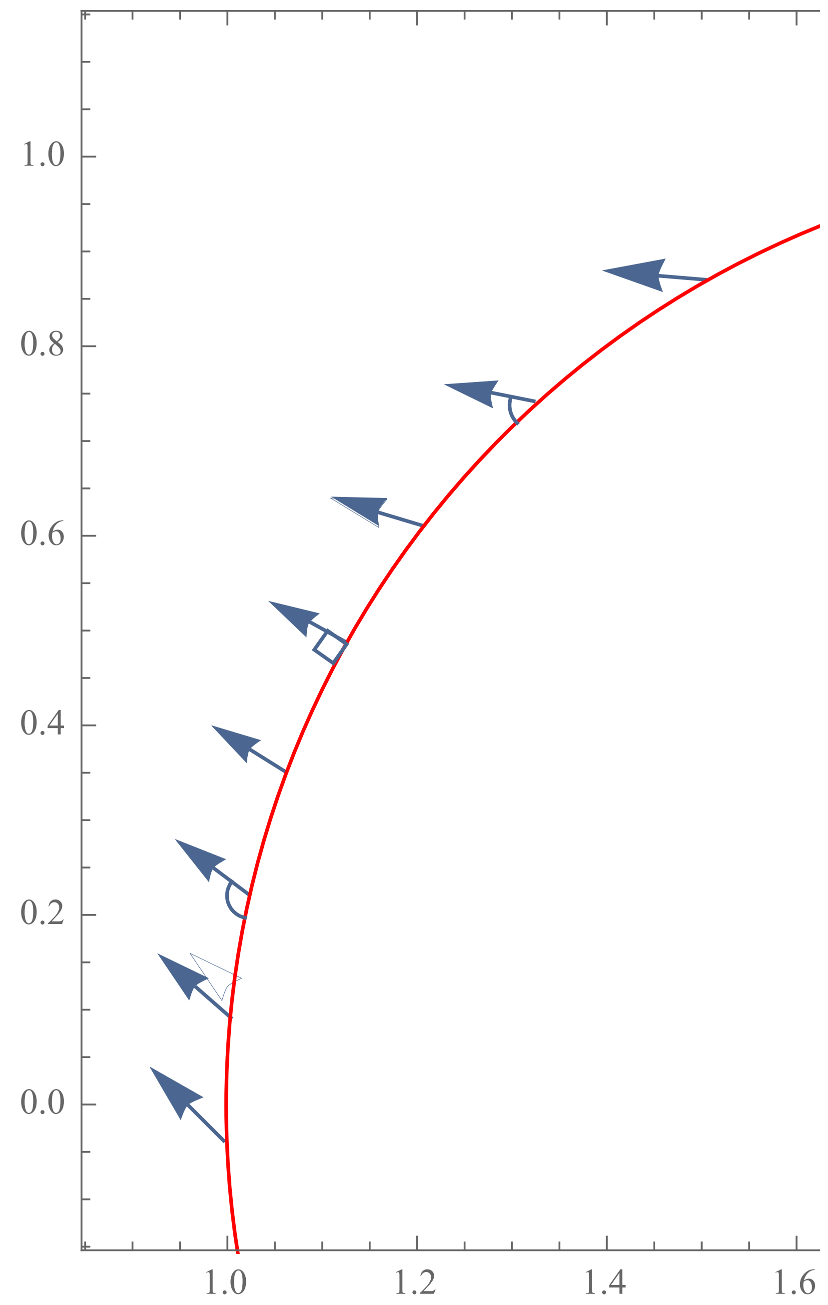
- If $\theta > \frac{\pi}{2}$, then $\cos\theta < 0$ and therefore the change in f is negative.
- If $\theta = \frac{\pi}{2}$, then $\cos\theta = 0$ and therefore the change in f is zero.

What does this mean when we are looking at the change of a function $f(x, y)$ as we move along some curve?



- If you move in a direction along a curve that makes an angle less than $\pi/2$ with the gradient of a f, the function f is increasing as you move along the curve in that direction.
- If you move in a direction along a curve that makes an angle greater than $\pi/2$ with the gradient of f, the function f decreasing as you move along the curve in that direction.
- If you move in a direction along a curve that makes an angle equal to $\pi/2$ with the gradient of f, the function f is neither increasing or decreasing.

Therefore the function f is as its largest when we reach a point along the curve such that the curve and the gradient are perpendicular to each other. At this point, the function does not increase as you move in either direction. In all other cases, there is a direction you can move that will increase f.

Key point : The maximum of f along a boundary curve occurs when the gradient ∇f is normal (perpendicular) to the boundary.

The argument we gave was setup for a general function $f(x, y)$ moving along a direction specified by a curve C. None of this argument was specific to our example, so this holds more generally.

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? lecture 9th last line
The 9th last line says 'it would get bigger' - my understanding is he is speaking of the value of the function which will become smalle... 2

? [staff] typo? 3

Another method of checking if the direction of travel = increasing function
Using vector projection of the gradient onto <Δx,Δy> and see if it's the same direction of the gradient. 2

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