2. Solve the boundary value problem

Find eigenvalues, eigenfunctions, and normal modes

3/3 points (graded)

Consider a cylinder with two open ends of length L. Longitudinal air waves/ pressure waves along the midline of the cylinder satisfy the PDE

$$\frac{\partial^2 p}{\partial t^2} = c^2 \frac{\partial^2 p}{\partial x^2}, \qquad 0 < x < L,\ t > 0.$$

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}, \qquad 0 < x < L,\ t > 0.$$

where p is the pressure, u is the horizontal displacement of air molecules, and c is the speed of sound in the ambient air.

Let us find solutions for u.

Separating variables and looking for solutions of the form $u(x,t) = v(x)\,w(t)$ leads to solving the ODEs

$$\begin{aligned} \frac{d^2 v}{dx^2} &= \lambda v, \qquad 0 < x < L \\ \frac{d^2 w}{dt^2} &= \lambda c^2 w, \qquad t > 0 \end{aligned}$$

Solving leads to a family of solutions $u_n(x,t) = v_n(x)\, w_n(t)$ for different values λ_n, which all satisfy the boundary conditions.

Find λ_n, v_n, and w_n for $n = 1, 2, 3 \ldots$.

$\lambda_n =$ -n^2*pi^2/L^2 ✔ **Answer:** -n^2*pi^2/L^2

$$-\frac{n^2 \cdot \pi^2}{L^2}$$

$v_n(x) =$ cos(n*pi/L*x) ✔ **Answer:** cos(n*pi*x/L)

$$\cos\left(\frac{n \cdot \pi}{L} \cdot x\right)$$

(Let the unknown constant in front of the cosine term be a, and the unknown term in front of the sine term be b.)

$w_n(t) =$ a*cos(n*pi*c/L*t)+b*sin ✔ **Answer:** a*cos(n*pi*t*c/L)+b*sin(n*pi*c*t/L)

$$a \cdot \cos\left(\frac{n \cdot \pi \cdot c}{L} \cdot t\right) + b \cdot \sin\left(\frac{n \cdot \pi \cdot c}{L} \cdot t\right)$$

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Solve the initial value problem

5/5 points (graded)

Suppose that the initial displacement takes the form

$$u(x,0) = \frac{2x}{L} - 1, \qquad 0 < x < L.$$

Let the initial velocity of the displacement be zero (for convenience).

The general solution takes the form

$$u(x,t) = a_0/2 + \sum_{n=1}^{\infty} \left(a_n \cos(\omega_n t) + b_n \sin(\omega_n t)\right) \cos(\alpha_n x)$$

Determine what type of periodic extension is needed of this initial condition to solve for the coefficients in the Fourier series. Then use the initial condition to find the coefficients.

Hint: The triangle wave $T(z)$ of period 2π has Fourier series $T(z) = \frac{\pi}{2} - \frac{4}{\pi} \sum_{n \text{ odd}} \frac{\cos(nz)}{n^2}$.

The sawtooth wave $W(z)$ of period 2π has Fourier series $W(z) = 2\sum_{n=1}^{\infty} \frac{(-1)^{n+1} \sin(nz)}{n}$.

$a_0/2 =$ 0 ✔ **Answer:** 0

0

n odd, $a_n =$ -8/pi^2/n^2 ✔ **Answer:** -8/(pi^2*n^2)

$-\frac{8}{\pi^2 \cdot n^2}$

n even, $a_n =$ 0 ✔ **Answer:** 0

0

n odd, $b_n =$ 0 ✔ **Answer:** 0

0

n even, $b_n =$ 0 ✔ **Answer:** 0

0

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Correct solution for $v_n(x)$? 5