Life with Countable Additivity

In this section I'll try to give you a sense of why the issue is not entirely straightforward. I'll start by mentioning an awkward consequence of *accepting* Countable Additivity. Then I'll point to an awkward consequence of *not accepting* Countable Additivity.

An awkward consequence of accepting Countable Additivity

Imagine that God has selected a positive integer, and that you have no idea which. For n a positive integer, what credence should you assign to the proposition, G_n, that God selected n?

Countable Additivity entails that your credences should remain undefined, unless you're prepared to give different answers for different choices of n.

To see this, suppose otherwise. Suppose that for some real number r between 0 and 1, you assign credence r to each proposition G_n.

What real number could r be? It must either be equal to zero or greater than zero.

First, suppose that r is equal to zero. Then Countable Additivity entails:

$$c_S\left(G_1 \text{ or } G_2 \text{ or } G_3 \text{ or } \ldots\right) = c_s\left(G_1\right) + c_s\left(G_2\right) + c_s\left(G_3\right) + \ldots = \underbrace{0+0+0+0\ldots}_{\text{once for each integer}} = 0$$

But G_1 or G_2 or G_3 or $\ldots$ is just the proposition that God selects some positive integer. So we would end up with the unacceptable conclusion that you're certain that God won't select a positive integer after all.

Now suppose that r is greater than zero. Then Countable Additivity entails:

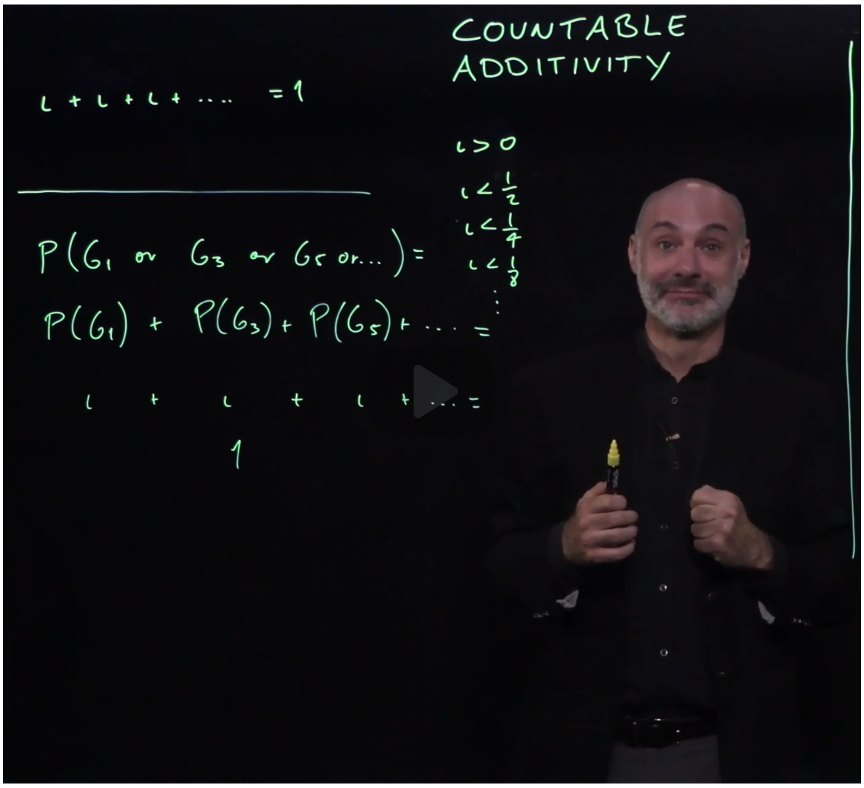
$$c_S\left(G_1 \text{ or } G_2 \text{ or } G_3 \text{ or } \ldots\right) = c_s\left(G_1\right) + c_s\left(G_2\right) + c_s\left(G_3\right) + \ldots = \underbrace{r+r+r+r\ldots}_{\text{once for each integer}} = \infty$$

But since probabilities are always real numbers between 0 and 1, this contradicts the assumption that c_S is a probability function.

The moral is that when Countable Additivity is in place there is no way of assigning probabilities to the G_n, unless one is prepared to assign different probabilities to different G_n.

More generally: Countable Additivity entails that there is no way of distributing probability *uniformly* across a countably infinite set of (mutually exclusive and jointly exhaustive) propositions.

Video Review: Countable Additivity



appeal

to infinitesimal values, and hope that that's going to solve

our problem.

The hard truth, I think, is that if you have countable

additivity, then you simply cannot have a uniform

probability distribution over a countably infinite set.

12:31 / 12:31 1.50x

End of transcript. Skip to the start.

Video

Download video file

Transcripts

Download SubRip (.srt) file

Download Text (.txt) file

Problem 1

1/1 point (ungraded)

As before, God has selected a number. But this time your credences are as follows:

> Your credence that God selected the number 1 is $1/2$, your credence that God selected the number 2 is $1/4$, your credence that God selected the number 3 is $1/8$, and so forth. (In general, your credence that God selected positive natural number n is $1/2^n$.)

Assuming your credence function satisfies Countable Additivity, what is your credence that God selected a natural number?

1

✔ **Answer:** 1

1

Explanation

As before, let G_n be the proposition that God selected the number n. Then the proposition that God selected some positive integer or other can be expressed as

$$G_1 \text{ or } G_2 \text{ or } G_3 \text{ or } \ldots$$

But, by Countable Additivity,

$$c_s\left(G_1 \text{ or } G_2 \text{ or } \ldots\right) = c_S\left(G_1\right) + c_S\left(G_2\right) + \ldots = 1/2 + 1/4 + \ldots + 1/2^n + \ldots = 1$$

(If you'd like to know more about why $1/2 + 1/4 + \ldots + 1/2^n + \ldots = 1$ is true, have a look at the Wikipedia entry on convergent series.)

Submit

Answers are displayed within the problem

Discussion

Hide Discussion

Topic: Week 6 / Life with Countable Additivity

Add a Post

Show all posts by recent activity

The probability is well defined in the limit
The probability that God picks the number r from among the first k integers = 1/k The probability tha... 3

Different view

what if we use density in infinite addition for example: iota + iota + iota + iota + iota + iota + ... = 1 bu... 1