



Bookmarks

- ▶ Unit 0: Overview
- ▶ Entrance Survey
- ▶ Unit 1: Probability models and axioms
- ▶ Unit 2: Conditioning and independence
- ▶ Unit 3: Counting
- ▶ Unit 4: Discrete random variables
- ▶ Exam 1
- ▼ **Unit 5: Continuous random variables**

Unit 5: Continuous random variables &gt; Problem Set 5 &gt; Problem 7 Vertical: Bayes' rule

Bookmark

## Problem 7: Bayes' rule

(2/2 points)

Let  $K$  be a discrete random variable with PMF


$$p_K(k) = \begin{cases} 1/3, & \text{if } k = 1, \\ 2/3, & \text{if } k = 2, \\ 0 & \text{otherwise.} \end{cases}$$

Conditional on  $K = 1$  or  $2$ , random variable  $Y$  is exponentially distributed with parameter  $1$  or  $1/2$ , respectively.


Using Bayes' rule, find the conditional PMF  $p_{K|Y}(k | y)$ . Which of the following is the correct expression for  $p_{K|Y}(2 | y)$  when  $y \geq 0$ ?

## Unit overview


### Lec. 8: Probability density functions

Exercises 8 due Mar 18, 2016 at 23:59 UTC 

### Lec. 9: Conditioning on an event; Multiple r.v.'s

Exercises 9 due Mar 18, 2016 at 23:59 UTC 


### Lec. 10: Conditioning on a random variable; Independence; Bayes' rule

Exercises 10 due Mar 18, 2016 at 23:59 UTC 

## Standard normal table

## Solved problems

### Problem Set 5

Problem Set 5 due Mar 18, 2016 at 23:59 UTC 

## Unit summary

- ▶ Unit 6: Further topics on random variables
- ▶ Unit 7: Bayesian inference

☐ 
$$\frac{\frac{1}{3}e^{-y/2}}{\frac{1}{3}e^{-y} + \frac{2}{3}e^{-y/2}}$$

☒ 
$$\frac{e^{-y/2}}{e^{-y} + e^{-y/2}} \quad \checkmark$$

☐ 
$$\frac{\frac{1}{3}e^{-y}}{\frac{1}{3}e^{-y} + \frac{2}{3}e^{-y/2}}$$

☐ 
$$\frac{e^{-y}}{e^{-y} + e^{-y/2}}$$

Answer:  
Applying Bayes' rule, we have

- ▶ Exam 2
- ▶ Unit 8: Limit theorems and classical statistics
- ▶ Unit 9: Bernoulli and Poisson processes
- ▶ Unit 10: Markov chains
- ▶ Exit Survey
- ▶ Final Exam

$$p_{K|Y}(k | y) = \frac{p_K(k)f_{Y|K}(y | k)}{f_Y(y)}.$$

By the total probability theorem,

$$\begin{aligned} f_Y(y) &= \sum_k p_K(k)f_{Y|K}(y | k) \\ &= p_K(1)f_{Y|K}(y | 1) + p_K(2)f_{Y|K}(y | 2) \\ &= \frac{1}{3}e^{-y} + \frac{2}{3} \cdot \frac{1}{2}e^{-y/2} \\ &= \frac{1}{3}e^{-y} + \frac{1}{3}e^{-y/2}. \end{aligned}$$

Hence, for  $k = 2$ , we have

$$\begin{aligned} p_{K|Y}(2 | y) &= \frac{p_K(2)f_{Y|K}(y | 2)}{f_Y(y)} \\ &= \frac{\frac{1}{3}e^{-y/2}}{\frac{1}{3}e^{-y} + \frac{1}{3}e^{-y/2}} \\ &= \frac{e^{-y/2}}{e^{-y} + e^{-y/2}}. \end{aligned}$$

*You have used 1 of 2 submissions*

## DISCUSSION

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