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Russell's Paradox

Mathematicians sometimes talk about the *set* of prime numbers, or the *set* of functions from real numbers to real numbers.

Suppose we wanted to formalize this talk of sets. What could we use as axioms?

When the great German mathematician Gottlob Frege started thinking about sets, he proposed a version of the following axiom. (It seemed obviously true at the time.)

> **Frege's Axiom**
> Select any objects you like (the prime numbers, for example, or the functions from real numbers to real numbers). There is a set whose members are all and only the objects you selected.

In fact, Frege's Axiom is inconsistent. This was discovered by the British philosopher Bertrand Russell. Russell's argument is devastatingly simple, and is sometimes known as **Russell's Paradox**:

> Consider the objects that are not members of themselves. (The empty set has no members, for example, so it is an object that is not a member of itself.) An immediate consequence of Frege's Axiom is that there is a set that has all and only the non-self-membered sets as members. Let us honor Russell by calling this set R.
>
> Now consider the following question: is R a member of R? The definition of R tells us that R's members are exactly the objects that have a certain property: the property of not being members of oneself. So we know that R is a member of R if and only if it has that property. In other words: R is a member of itself if and only if it is not a member of itself.
>
> That's a contradiction So R can't really exist. So Frege's Axiom must be false.

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