Computing functions on a Turing Machine

Let f be a function from natural numbers to natural numbers. An ordinary computer (running an ordinary computer program) computes f if and only if the following holds for each natural number n:

> If you give the ordinary computer n as input, you'll get $f(n)$ as output.

Turing Machines can be used to implement a version of this idea.

For we can think of a Turing Machine as taking number n ($n \geq 0$) as **input** if it starts out with a tape that contains only a sequence of n ones (with the reader positioned at the left-most one, if $n > 0$).

And we can think of the Turing Machine as delivering number $f(n)$ as **output** if it halts with a tape that contains only a sequence of $f(n)$ ones (with the reader positioned at the left-most one, if $f(n) > 0$).

Finally, we can say that a Turing Machine **computes** a function $f(x)$ if and only if it it delivers $f(n)$ as output whenever it is given n as input. (For a function to be **Turing-computable** is for it to be computed by some Turing Machine.)

Notice, incidentally, that a similar definition could be used to define computability for functions from n-tuples of natural numbers to natural numbers. For we can think of a Turing Machine as taking a sequence of natural numbers $\langle n_1, \ldots, n_k \rangle$ as **input** if it starts out with a tape that contains only a sequence composed of the following: a sequence of n_1 ones (or a blank, if $n_1 = 0$), followed by a blank, followed by a sequence of n_2 ones (or a blank, if $n_2 = 0$), followed by a blank, followed by …, followed by a sequence of n_k ones (or a blank, if $n_k = 0$), with the reader positioned at the left-most one of the left-most sequence of ones (unless $n_1 = 0$, in which case the reader is positioned at the blank corresponding to n_1).

Video Review: Computing A Function

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Let me first say what it is for a function to be

computable by a Turing Machine.

So say you have a function f from the natural numbers

to the natural numbers.

So in other words, for each natural number,

the function outputs a natural number.

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