# 1.4.2 Summary Quiz: Survival or Extinction: The Effect of a Proportional Fishing Rate

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## Question 1: Think About It...

1 point possible (graded)

Wes described a bifurcation as ‘a dramatic change in the expected behavior of a system in response to a change in parameter.’ In the example of Wes’s research on cardiac rhythms, what is the bifurcation? In other words, what is the dramatic change in behavior of the system? And what is the parameter that changes to cause this bifurcation?

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## Question 2

1/1 point (graded)

Wes described a bifurcation as **"a dramatic change in the expected behavior of a system in response to a change in parameter."** In the fishing example the parameter is $\alpha$, the rate of fishing (or harvesting rate).

Which of the following explain why the critical value of $\alpha = 1000$ is considered a bifurcation? Choose the most complete answer.

You may find it helpful to think about the following questions first.

- What is the ‘expected behavior’ for values of $\alpha$ above and below that critical value?
- What is the ‘dramatic change in expected behavior’?

- ○ No matter the value of $\alpha$, if the starting population is low enough the population of fish will go extinct. This is the dramatic change.
- ○ When the fishing rate is less than the critical value of $\alpha = 1000$, the population never goes extinct but when the fishing rate is greater than the critical value of $\alpha = 1000$, the outcome will always be extinction. This is the dramatic change.
- ○ When the fishing rate is greater than the critical value $\alpha$If $\alpha = 1000$, the population never goes extinct but when the fishing rate is less than the critical value of $\alpha = 1000$, the outcome will always be extinction. This is the dramatic change.
- ○ When the fishing rate is greater than the critical value of $\alpha = 1000$, the population of fish will go extinct faster than when the fishing rate is below the critical value. This is the dramatic change.
- ◉ When the fishing rate is less than the critical value of $\alpha = 1000$, the population does not necessarily go extinct but when the fishing rate is greater than the critical value of $\alpha = 1000$, the outcome will always be extinction. This is the dramatic change. ✔
- ○ When the fishing rate is greater than the critical value of $\alpha = 1000$, the population does not necessarily go extinct but when the fishing rate is less than the critical value of $\alpha = 1000$, the outcome will always be extinction. This is the dramatic change.

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You have used 1 of 4 attempts

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## Question 3

1/1 point (graded)

The critical value of $\alpha$ corresponds to when the graph of $\frac{dP}{dt}$ versus $P$ intersects the horizontal axis exactly once. This means the graph of the parabola $y = \frac{1}{10}P(1 - \frac{P}{40000}) - \alpha$ intersects $y = 0$ only once. Which of the following computations will find the critical value $\alpha$ when this happens? There are two correct answers.

Hint: Translate this into a quadratic equation problem.

- ☐ Solve $\frac{dP}{dt} = \frac{1}{10}P(1 - \frac{P}{40000}) - \alpha = 0$ and solve for $\alpha$ in terms of $P$, and then set $P = 0$.
- ☐ Solve $\frac{dP}{dt} = \frac{1}{10}P(1 - \frac{P}{40000}) - \alpha = 0$ and solve for $P$ in terms of $\alpha$, and then set $\alpha = 0$.

- [x] Solve $\frac{dP}{dt} = \frac{1}{10}P(1 - \frac{P}{40000}) - \alpha = 0$, use the quadratic formula to solve for $P$ and see what $\alpha$ value means there is exactly one solution. ✔

- [ ] Solve $\frac{dP}{dt} = \frac{1}{10}P(1 - \frac{P}{40000}) - \alpha = 0$, use the quadratic formula to solve for $P$ and see what $\alpha$ value means there are no solutions.

- [x] The vertex of $y = \frac{1}{10}P(1 - \frac{P}{40000}) - \alpha$ always occurs at $P = 20000$ so the critical value happens when the vertex is exactly at $(20000, 0)$. We can solve $\frac{dP}{dt} = \frac{1}{10}20000(1 - \frac{20000}{40000}) - \alpha = 0$ for $\alpha$. ✔

✔

**Explanation**

The value of $\alpha$ is $1000$. We can compute this exactly by finding when the parabola $y = \frac{1}{10}P(1 - \frac{P}{40000}) - \alpha$ intersects the horizontal axis exactly once.

To see when the equation $\frac{1}{10}P(1 - \frac{P}{40000}) - \alpha = 0$ has exactly one solution, we can use the quadratic formula.

Rewriting the quadratic in the form $y = aP^2 + bP + c$, we get $y = -\frac{1}{400000}P^2 + \frac{1}{10}P - \alpha$. The horizontal intercepts are $P$-values

$$P = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

We want to find the value of $\alpha$ so that there is only one solution, meaning the discriminant $b^2 - 4ac = 0$. Here $b^2 - 4ac = \frac{1}{100} - 4(-\frac{1}{400000}) \cdot (-\alpha)$. Setting this equal to zero and solving for $\alpha$ we get $\alpha = 1000$.

Another way to do this is to notice that the vertex of the parabola always occurs at $P = 20,000$ (you can find this by computing $P = -b/2a$). We want the $\alpha$ value such that when $P = 20,000$, the quadratic $y = \frac{1}{10}P(1 - \frac{P}{40000}) - \alpha$ is equal to zero. So we solve $\frac{1}{10}20000(1 - \frac{20000}{40000}) - \alpha = 0$ for $\alpha$ and get $\alpha = 1000$.

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You have used 3 of 4 attempts

Answers are displayed within the problem

Let's look at another possible situation where a change in parameter can cause a bifurcation.

If there are more fish, fishermen may be likely to catch more fish. Similarly, with less fish in the population, fisherman might have a harder time finding the fish and thus might catch less.

Thus it's reasonable to suppose that instead of harvesting at a constant rate, the fish are harvested at a rate **proportional to the population itself**. The constant of proportionality is $b$ and we have $0 \leq b \leq 1$. This will be the parameter of interest. (Note: fishing at a rate proportional to population was the assumption we made in the Population Dynamics section when we modeled fishing with nets.)

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## Question 4

1/1 point (graded)

Suppose the constant of proportionality is $b = 0.05$. This means the instantaneous rate of harvest is which percent of the population of fish per year?

- ○ 0.05%
- ○ 0.5%
- ◉ 5% ✔
- ○ 50%

**Explanation**

The value $b = 0.05$ equals $\frac{5}{100}$, so the rate of harvest is $5\%$. (Percent'' means per hundred''.)

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You have used 1 of 2 attempts

ℹ Answers are displayed within the problem

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## Question 5

1/1 point (graded)

Assume that *without fishing*, the population is modeled by $\frac{dP}{dt} = \frac{1}{10}P(1 - \frac{P}{40000})$. Which model incorporates that fish are harvested at a rate **proportional to the population itself**, with $b > 0$ the proportionality constant?

- ○ $\frac{dP}{dt} = \frac{1}{10}P(1 - \frac{P}{40000}) - b$
- ○ $\frac{dP}{dt} = \frac{1}{10}P(1 - \frac{P}{40000}) - bt$

- [x] $\frac{dP}{dt} = \frac{1}{10}P(1 - \frac{P}{40000}) - bP$ ✔
- [ ] $\frac{dP}{dt} = \frac{b}{10}P(1 - \frac{P}{40000})$
- [ ] $\frac{dP}{dt} = \frac{1}{10}P(1 - b\frac{P}{40000})$
- [ ] None of the above.

**Explanation**

*Answer: Assuming the number of fish harvested was a constant gave us the model* $\frac{dP}{dt} = \frac{1}{10}P\left(1 - \frac{P}{40000}\right) - \alpha$*. In our new model, fishing will remove* $bP$ *fish from the population rather than simply removing* $\alpha$ *fish. Therefore, we should replace* $\alpha$ *with* $bP$ *in our model to get:*

$$\frac{dP}{dt} = \frac{1}{10}P\left(1 - \frac{P}{40000}\right) - bP.$$

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You have used 1 of 3 attempts

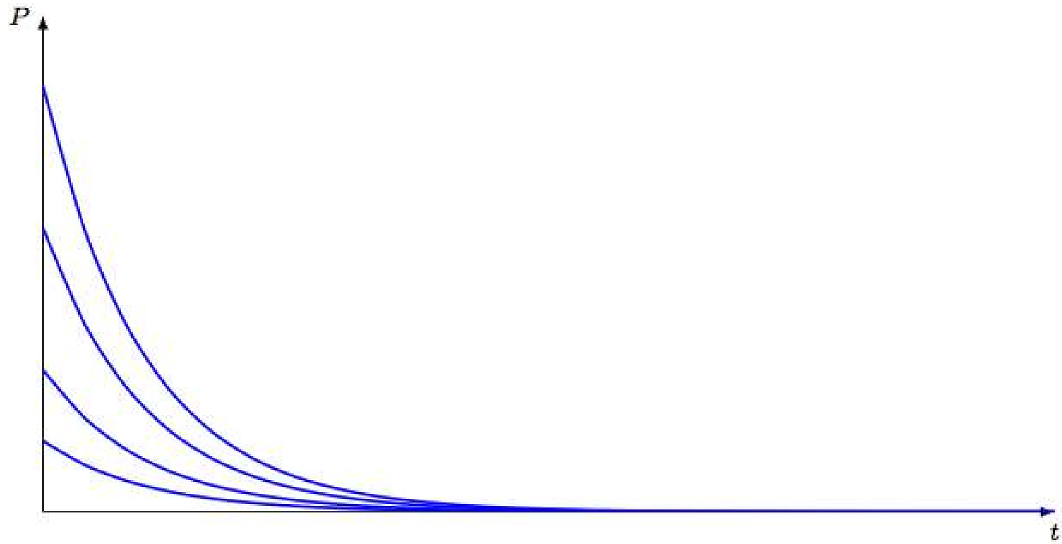
Answers are displayed within the problem

## Question 6

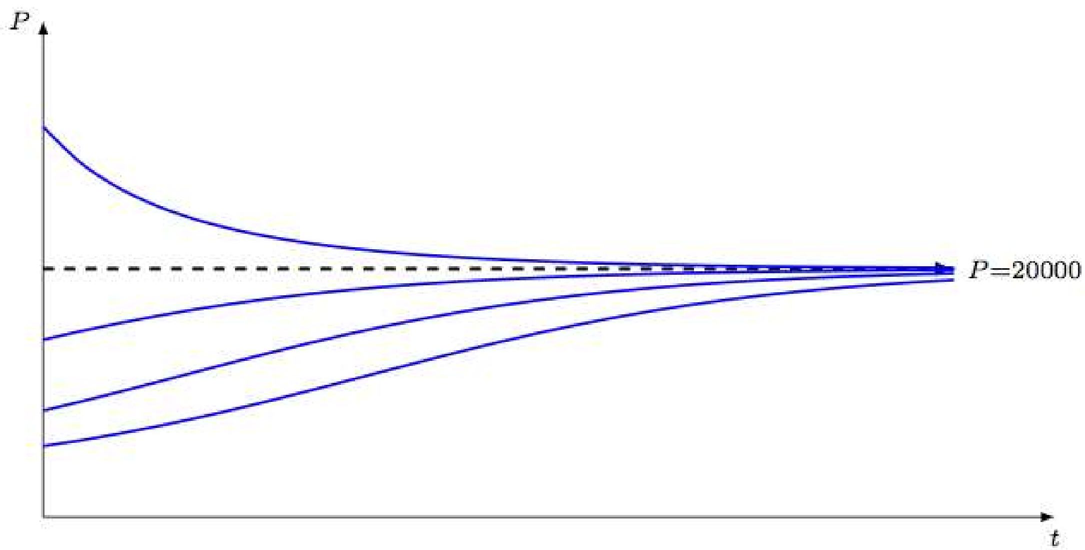
1/1 point (graded)

Which is the correct qualitative analysis of the differential equation in #2 in the case of $b = 0.05$? **Make sure you have the right model before going further!** See previous problem.

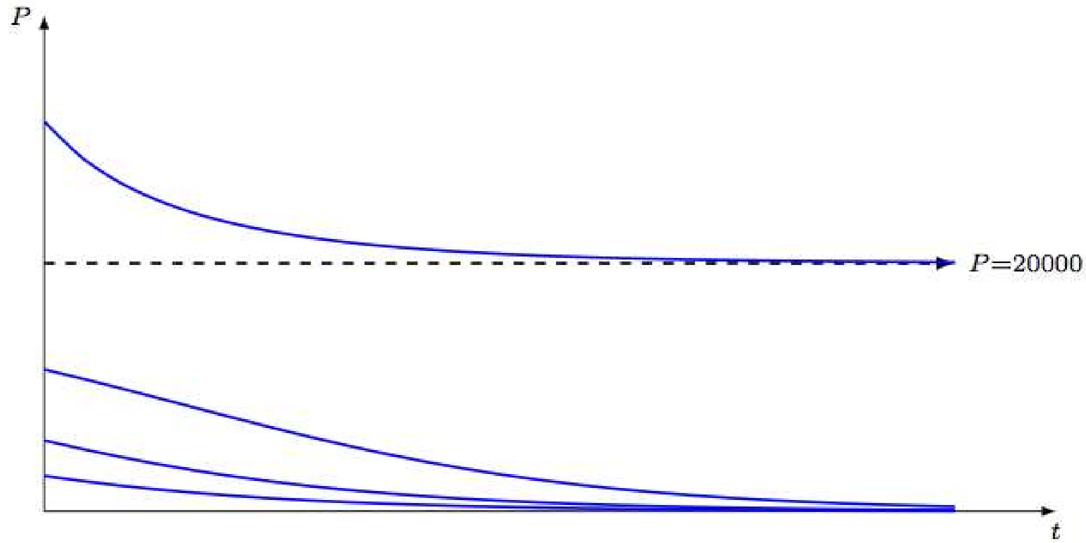
A.



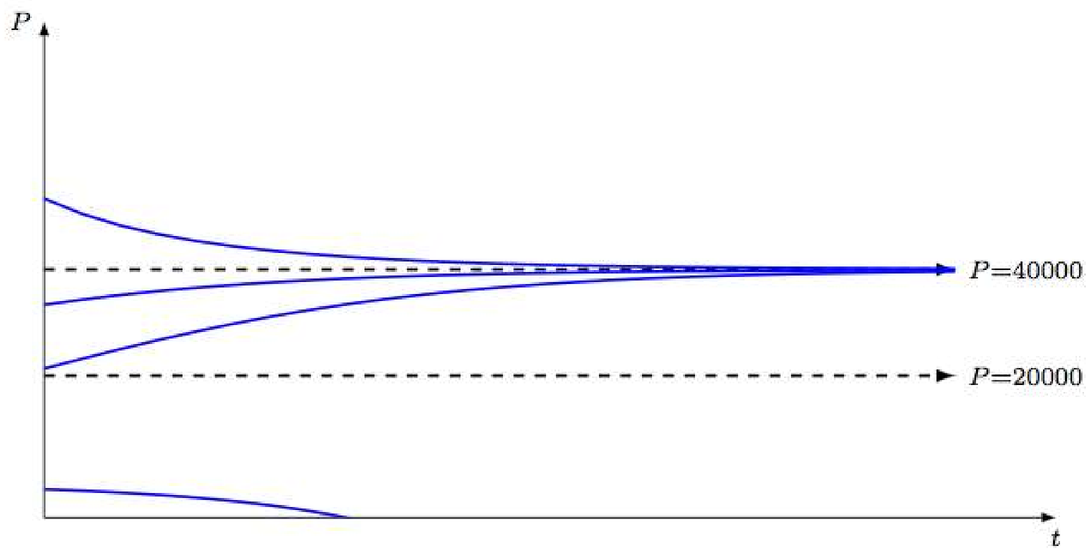

B.



C.




D.




**Explanation**

Graph B is the correct choice. When $b = 0.05$, there are two equilibrium solutions, $P = 0$ and $P = 20000$. For $0 < P < 20000$, the graph of $\frac{dP}{dt}$ versus $P$ is above the axis so the derivative is positive and thus $P$ is increasing. Thus for initial conditions less than $20000$, the solution curves will increase toward $P = 20000$.

For $P > 20000$, the graph of $\frac{dP}{dt}$ versus $P$ is below the axis so the derivative is negative and thus $P$ is decreasing. Thus for initial conditions greater than $20000$, the solution curves will decrease toward $P = 20000$.

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You have used 1 of 2 attempts

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## Question 7: Think About It...

1/1 point (graded)

The proportionality constant $b$ is a parameter of the differential equation. Let's consider the effect of the parameter $b$ on the behavior of solutions.

Intuitively, what do you expect to happen as $b$ gets large? In other words, as the proportionality constant which governs the rate of fishing increases, what will happen?

Use the Desmos graph here to explore what happens to the graph of $\frac{dP}{dt}$ versus $P$ when you change $b$.

```
At b<=0.1 there is exactly one equilibrium where the fish
population reduces to zero,
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Thank you for your response.

This is explored in the following problems.

Submit You have used 1 of 2 attempts

Answers are displayed within the problem

## Question 8

1/1 point (graded)

There are **two** qualitatively different situations. Estimate the value of $b$ for which the situation changes, to the nearest tenth. Enter a numerical value for your answer.

0.1 **Answer:** 0.1

$0.1$

**Explanation**

*Answer:* $b = 0.1$ *(accept a range of* $\pm 0.05$.*)*

As before, for each different value of our parameter the graph of $\frac{dP}{dt}$ vs. $P$ is a parabola. When $b$ is small, the parabola intercepts the $P$-axis twice, at $P = 0$ and at a positive $P$-value. For larger $b$, the parabola intercepts the $P$-axis at $P = 0$ and a negative $P$ value. This means that for large enough values of $b$, the derivative $\frac{dP}{dt}$ will be negative for all positive population values $P$. The change in behavior occurs at approximately $b = 0.1$.

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Answers are displayed within the problem

## Question 9

7/7 points (graded)

We've identified one critical value of b. For each situation, find all statements and graphs below that correspond to it.

- "large" $b$ values above critical value
- "small" $b$ values below critical value

A. There is one semi-stable equilibrium value of P .

neither ▼ **Answer:** neither

B. There is one stable equilibrium point. Another equilibrium point is at P = 0.

small b ▼ **Answer:** small b

C. There is only one equilibrium point: P = 0, representing extinction

large b ▼ **Answer:** large b

D. When P is near 0, the population tends to increase

small b ▼ **Answer:** small b

E. When P is near 0, the population tends toward extinction

large b ▼ **Answer:** large b

F. For all non-zero values of P , the size of the population tends toward a non-zero equilibrium point

small b ▼ **Answer:** small b

G. No matter how large P is, the system tends toward extinction (P = 0).

large b ▼ **Answer:** large b

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You have used 2 of 5 attempts

Answers are displayed within the problem

## Question 10

1/1 point (graded)

This system has one bifurcation. Compute exactly the value of b for which it occurs. Enter a numerical value for your answer.

0.1

**Answer:** .1

$0.1$

**Explanation**

Answer: The bifurcation occurs when the number of equilibrium points changes, so we start by finding the equilibrium points at which $\frac{dP}{dt} = 0$. First, combine like terms of the equation for $\frac{dP}{dt}$:

$$\begin{aligned}\frac{dP}{dt} &= \frac{1}{10}P\left(1 - \frac{P}{40,000}\right) - bP\\ &= \frac{1}{10}P - bP - \frac{P^2}{400,000}\\ &= -\frac{1}{400,000}P^2 + \left(\frac{1}{10} - b\right)P.\end{aligned}$$

Factor this, or apply the quadratic formula, to find that $\frac{dP}{dt} = 0$ when $P = 0$ and when $P = 400,000\left(\frac{1}{10} - b\right)$. This means that the system has two equilibrium points when $b \neq \frac{1}{10}$ and one when $b = \frac{1}{10}$.

Since the model only makes sense for $P \geq 0$, the system effectively has two equilibria for $b < \frac{1}{10}$ and one ($P = 0$, or extinction) for $b \geq \frac{1}{10}$.

This makes sense because the growth rate of the population is about $\frac{1}{10}$. If the rate of fishing exceeds the rate of reproduction the population will eventually become extinct.

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You have used 1 of 3 attempts

Answers are displayed within the problem

## Question 11

1/1 point (graded)

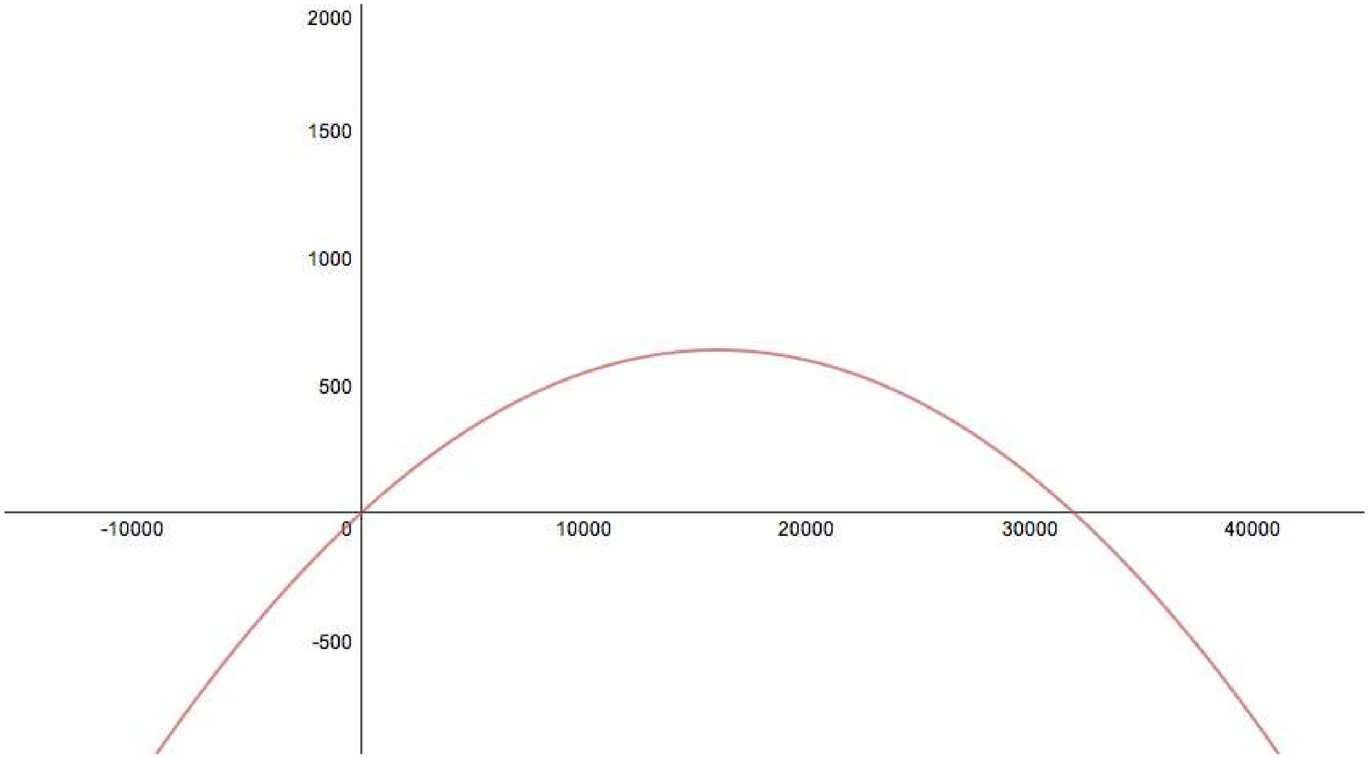
Wes described a bifurcation as ‘a dramatic change in the expected behavior of a system in response to a change in parameter.’ Here b is the parameter.

Which of the following explain why the ‘critical’ value of b is considered a bifurcation? Choose the most complete answer.

- No matter the value of b, the population of fish will go extinct. This is the dramatic change.
- When the harvesting rate is above the critical value of b, no matter what the starting population, the outcome will be extinction, whereas below the critical value the population does not necessarily go extinct.
- When the harvesting rate is below the critical value of b, no matter what the starting population, the outcome will be extinction, whereas above the critical value the population does not necessarily go extinct.
- When the harvesting rate is above the critical value of b, no matter what the starting population, the outcome will be extinction, whereas below the critical value the population will approach a stable level. ✔
- When the harvesting rate is below the critical value of b, no matter what the starting population, the outcome will be extinction, whereas above the critical value the population will approach a stable level.
- When the harvesting rate is above the critical value of b, the population of fish will go extinct faster than when the harvesting rate is below the critical value.

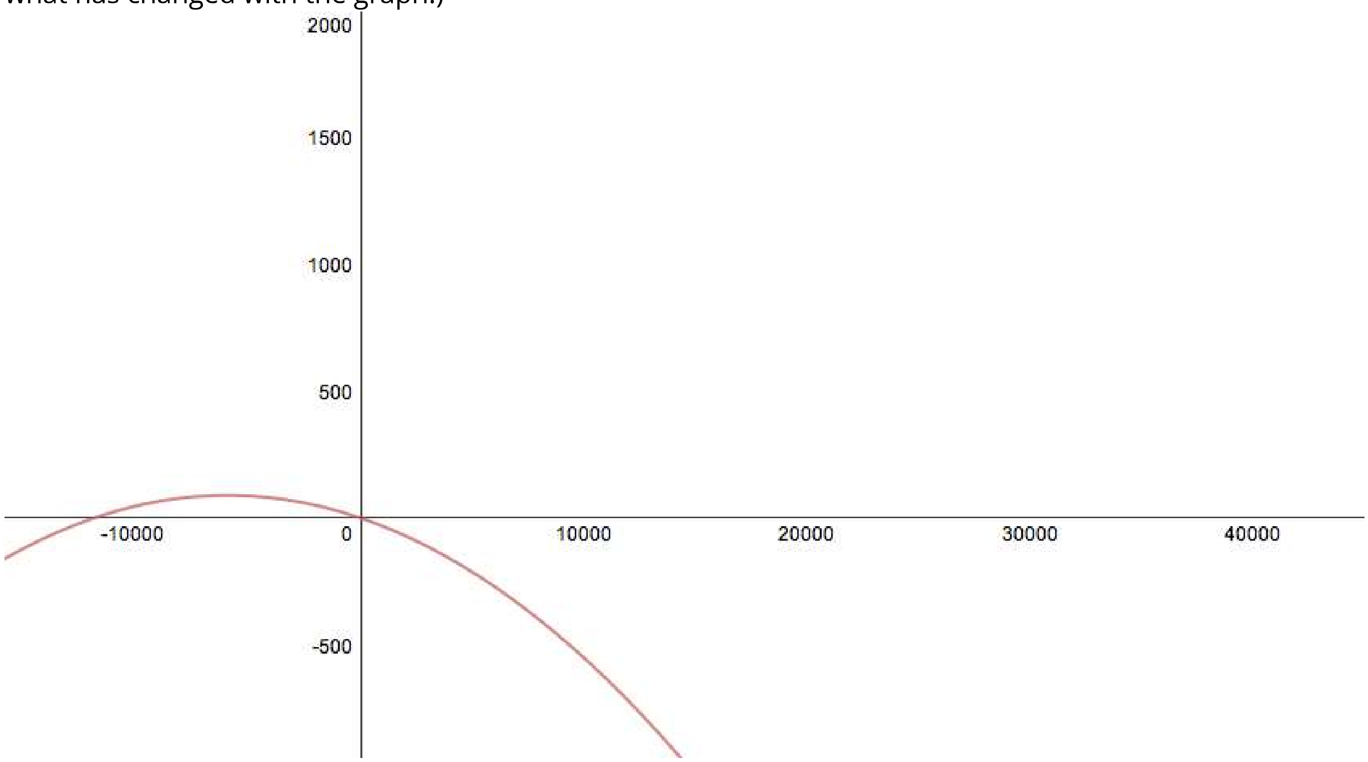
**Explanation**

There are two qualitatively different situations, and the change happens when $b = \frac{1}{10}$. For ‘small’ $b$ ( $b < 1/10$), there are two equilibria. For ‘large’ $b$ ($b > 1/10$), there is only one equilibrium. Here’s $b = 0.02$. The two equilibria are $P = 0$ and $P = 32,000$. For $0 < P < 32,000$, the graph is above the axis so the derivative is positive and thus $P$ is increasing. We would indicate that with an arrow on the horizontal axis pointing right. For $P > 32,000$, the graph is below the axis so the derivative is negative and thus $P$ is decreasing. We would indicate that with an arrow on the horizontal axis pointing left.




The only possible outcome is for the population to approach the stable equilibrium which is less than the carrying capacity.

Here's $b = 0.13$. The only equilibrium is $P = 0$. For $P > 0$, the graph is below the axis so the derivative is negative and thus $P$ is decreasing. We would indicate that with an arrow on the horizontal axis pointing left. (Note that the values of $P < 0$ make no sense in this context. But we include them so we can see what has changed with the graph.)




The only possible outcome is for the population to approach the zero equilibrium and thus go extinct.

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Answers are displayed within the problem

## Question 12: Think About It...

1/1 point (graded)

Can you see intuitively why the value of $b$ you get makes sense? (Hint: consider the reproduction rate of the fish, as reflected in the coefficient $\frac{1}{10}$ of the $P$ term in the model without fishing.)

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Since at b=1/10, the differential equation becomes P^2 = 0,
leading to exactly one zero solution.
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Thank you for your response.

*A: Without fishing, the fish population grows at a rate proportional to itself with constant* $\frac{1}{10}$ *but up to a limit – represented by the* $P^2$ *term.*

When we include fishing at a rate proportional to the fish population, if that constant of proportionality $b$ exceeds $\frac{1}{10}$, the rate of growth, the population cannot possibly sustain itself and will go extinct.

In other words, the fish won't reproduce fast enough to keep up with the fishing.

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