5. Adjacency Matrix

Exercises due Oct 20, 2021 17:29 IST Completed

Discussions: Representation of Networks and Adjacency Matrix

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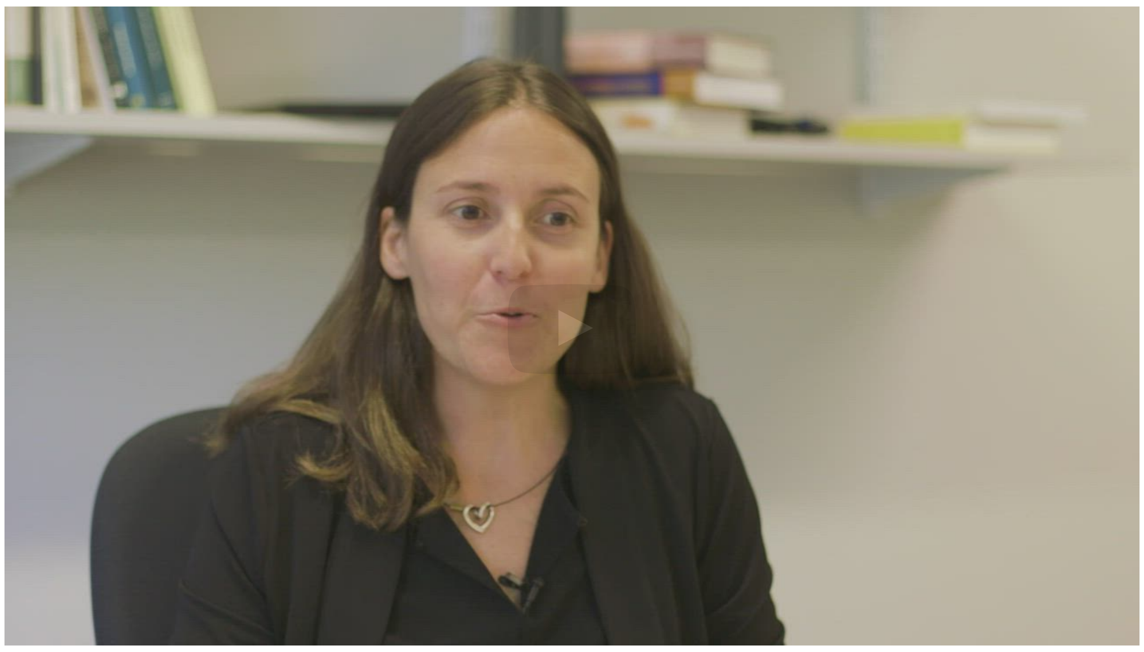

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Prof Uhler: OK, so welcome back.

So now that we have introduced or looked at different network

examples and seen, also discussed,

different motivations for studying networks, today

and in this video, we'll discuss different ways

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Definition of the adjacency matrix

In this course module, the adjacency matrix, A_{ij} will be defined as

$$\begin{aligned} A_{ij} &= 1 \quad \text{if there exists an edge from } i \text{ to } j \\ A_{ij} &= 0 \quad \text{otherwise} \end{aligned}$$

For an undirected graph, $A_{ij} = 1$ implies that $A_{ji} = 1$.

For a directed graph, A_{ij} and A_{ji} are independent.

Note that another convention for the adjacency matrix exists, where $A_{ij} = 1$ implies an edge from j to i. That convention will be not be used in this course, instead the above mentioned convention will be used.

Adjacency Matrix

1/2 points (graded)

Let A be the adjacency matrix of an unweighted graph. State whether the following are **True** or **False**.

1. A non-zero A_{ii} represents a loop from node i to itself.

- (selected) True
- False

✔

2. $A = A^T$ if and only if the graph is undirected.

- (selected) True

False ✔

✖

Solution:

1. **True.** Follows from the definition.

2. **False.** While an undirected graph has the property $A = A^T$, it is not necessary that the graph be undirected for this property to hold. If this property is true for a directed graph then it means that there is a directed edge from node i to j whenever there is a directed edge from node j to i.

   If the statement "$A = A^T$ if and only if the graph is undirected" were correct, then this would imply that any directed graph with $A = A^T$ is also equivalent to an undirected graph with the same adjacency matrix. However, we can't have this. Consider a simple directed graph with two nodes and adjacency matrix

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}.$$

   We have $A = A^T$, and we can also write this graph as $G = (V = \{1,2\}, E = \{(1,2), (2,1)\})$. Note that E has two elements, and that there is a cycle in this graph.

   On the other hand, an undirected graph with the same adjacency matrix would be written as $G' = (V = \{1,2\}, E = \{\{1,2\}\})$. Note that E has one element, and while there is a path of length 2 from node 1 to itself, this graph has no cycle (as the definition of a cycle does not allow repeated edges). Therefore, these two graphs cannot be considered equivalent, as G has two edges, while G' has one edge; and, G has a cycle, while G' has no cycles.

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Answers are displayed within the problem

Powers of the Adjacency Matrix

Theorem: Let A be the adjacency matrix of an unweighted graph (could be directed/undirected, simple/multigraph). For any $\ell \geq 1$, A^ℓ contains the following elements: A^ℓ_{ij}, which is the element in row i and column j of A^ℓ, is the number of walks of length ℓ from node i to node j.

Proof: The proof of this statement follows from induction with the following base cases: For $\ell = 1$, the statement is true by the definition of an adjacency matrix. For $\ell = 2$, let r_i be row i of A and c_j be column j of A. By definition, the entries of r_i are the number of walks of length 1 from node i to any other node. Similarly, the entries of c_j are the number of walks of length 1 from any other node to node j. Since entry A^2_{ij} is the vector inner product of r_i and c_j it is then also equal to the number of walks of length 2 from node i to j via all possible intermediate nodes.

Inductively, we assume that A^ℓ_{kj} is equal to the number of walks with length ℓ from node k to node j. Then

$$\begin{aligned} A^{\ell+1}_{ij} &= [AA^\ell]_{ij} \\ &= \sum_k A_{ik} A^\ell_{kj}. \end{aligned}$$

We can see now that $A_{ik}A^\ell_{kj}$ will be zero if there is no walk from i to k, and it will be equal to A^ℓ_{kj} if there is. Thus $A^{\ell+1}_{ij}$ is equal to the number of walks of length $\ell + 1$ from node i to j and the proof is completed.

Adjacency Matrix - Walks of Length 2 and 3

2/2 points (graded)
Consider the following adjacency matrix:

$$A = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

Let the nodes be numbered 1, 2, and 3, with A_{ij} representing the number of directed edges from node i to node j.

1. How many walks of length 2 are there from node 1 to 2?

1

Answer: 1

2. How many walks of length 3 are there from node 1 to 2?

1

Answer: 1

Solution:

1. A^2 has entry A^2_{12} equal to 1. Therefore, there is exactly 1 walk of length 2 from node 1 to 2.
2. A^3 has entry A^3_{12} equal to 1. Therefore, there is exactly 1 walk of length 3 from node 1 to 2.

Identify the walks by sketching the graph.

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Answers are displayed within the problem

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Why the inner product inc walk length from 1 to 2 AND connects node i to node j?

question posted 2 months ago by anonymous

I am baffled that the matrices could be used like that but unfortunately, I do not see how did the magic happened :// I know the math of it and how to answer and so on but I would not guess ever that the power is the walk-length!!

Could someone either refer me to a material where I could see an explanation or perhaps demystify it for me here? Thanks!

This post is visible to everyone.

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1 response

lam_trinh (Community TA)
2 months ago

Hello

Hello,

I shared this resource in another post, it'll help you with the intuition to answer your question:
https://quickmathintuitions.org/finding-paths-length-n-graph/

The link to the post where there're more resources from myself and others are here:
https://courses.edx.org/courses/course-v1:MITx+6.419x+3T2021/discussion/forum/6419x_3T2021_discussion_networks_lec1_basics-tab6_discussion/threads/61581474b9dd920473a133e8

Thanks this was interesting.

posted 2 months ago by **jtourkis**

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