



Bookmarks

- ▶ Unit 0: Overview
- ▶ Entrance Survey
- ▶ Unit 1: Probability models and axioms
- ▶ Unit 2: Conditioning and independence
- ▶ Unit 3: Counting
- ▶ Unit 4: Discrete random variables
- ▶ Exam 1
- ▶ Unit 5: Continuous random variables
- ▶ Unit 6: Further topics on random variables
- ▼ Unit 7: Bayesian inference

Unit 7: Bayesian inference > Lec. 16: Least mean squares (LMS) estimation > Lec 16 Least mean squares LMS estimation vertical1



Bookmark

## Exercise: LMS estimation error

(3/3 points)

As in the previous exercise, let  $\Theta$  be the bias of a coin, i.e., the probability of Heads at each toss. We assume that  $\Theta$  is uniformly distributed on  $[0, 1]$ . Let  $K$  be the number of Heads in 9 independent tosses. We have seen that the LMS estimate of  $K$  is  $\mathbf{E}[K \mid \Theta = \theta] = n\theta$ .

a) Find the conditional mean squared error

$\mathbf{E}[(K - \mathbf{E}[K \mid \Theta = \theta])^2 \mid \Theta = \theta]$  if  $\theta = 1/3$ .



Answer: 2

b) Find the overall mean squared error of this estimation procedure.



Answer: 1.5

Answer:


a) This is the variance of the conditional distribution of  $K$ . Since the conditional distribution is binomial with parameters  $n = 9$  and  $\theta = 1/3$ , the conditional variance is  $9(1/3)(2/3) = 2$ .

b) This is the average of the conditional variance, averaged over all possible values of the observation  $\Theta$ , which has a uniform distribution:


$$\begin{aligned}
 \int_0^1 f_{\Theta}(\theta) \text{var}(K \mid \Theta = \theta) d\theta &= \int_0^1 9\theta(1 - \theta) d\theta \\
 &= \left( 9\frac{1}{2}\theta^2 - 9\frac{\theta^3}{3} \right) \Big|_0^1 \\
 &= 4.5 - 3 \\
 &= 1.5.
 \end{aligned}$$

You have used 1 of 2 submissions


**Unit overview****Lec. 14:  
Introduction to  
Bayesian inference**

Exercises 14 due Apr  
06, 2016 at 23:59 UTC 


**Lec. 15: Linear  
models with  
normal noise**

Exercises 15 due Apr  
06, 2016 at 23:59 UTC 


**Problem Set 7a**

Problem Set 7a due  
Apr 06, 2016 at 23:59  
UTC 


**Lec. 16: Least  
mean squares  
(LMS) estimation**

Exercises 16 due Apr  
13, 2016 at 23:59 UTC 

**Lec. 17: Linear  
least mean  
squares (LLMS)  
estimation**

Exercises 17 due Apr  
13, 2016 at 23:59 UTC 

**Problem Set 7b**

Problem Set 7b due  
Apr 13, 2016 at 23:59  
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**Solved problems****Additional  
theoretical  
material****Unit summary**

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