

G1 (1/1 point)

$x_1 + x_2 + x_3 + x_4 = 20, 1 \leq x_1 \leq 6, 0 \leq x_2 \leq 7, 4 \leq x_3 \leq 8, 2 \leq x_4 \leq 6$, calculate the number of integral solutions.

96

96

Answer: 96**EXPLANATION**

Let $y_1 = x_1 - 1, y_2 = x_2, y_3 = x_3 - 4, y_4 = x_4 - 2$ we have

$$y_1 + y_2 + y_3 + y_4 = 13, 0 \leq y_1 \leq 5, 0 \leq y_2 \leq 7, 0 \leq y_3 \leq 4, 0 \leq y_4 \leq 4$$

the number of integral solutions of $y_1 + y_2 + y_3 + y_4 = 13, y_i \geq 0, i = 1, 2, 3, 4$ is $C(13 + 4 - 1, 13) = 560$

We represent the solution set of $y_1 + y_2 + y_3 + y_4 = 13, y_1 \geq 6, y_2 \geq 0, y_3 \geq 0, y_4 \geq 0$ by A_1

We represent the solution set of $y_1 + y_2 + y_3 + y_4 = 13, y_1 \geq 0, y_2 \geq 8, y_3 \geq 0, y_4 \geq 0$ by A_2

We represent the solution set of $y_1 + y_2 + y_3 + y_4 = 13, y_1 \geq 0, y_2 \geq 0, y_3 \geq 5, y_4 \geq 0$ by A_3

We represent the solution set of $y_1 + y_2 + y_3 + y_4 = 13, y_1 \geq 0, y_2 \geq 0, y_3 \geq 0, y_4 \geq 5$ by A_4

Solve $|A_1|$:

Let $z_1 = y_1 - 6, z_i = y_i, i = 2, 3, 4$

$$z_1 + z_2 + z_3 + z_4 = 7, z_i \geq 0, i = 1, 2, 3, 4, |A_1| = C(7 + 3, 7) = C(10, 3) = 120$$

Similarly, we have

$$\begin{aligned} |A_2| &= 56, |A_3| = 165, |A_4| = 165, |A_1 \cap A_2| = 0 \\ |A_1 \cap A_3| &= |A_1 \cap A_4| = 10, |A_2 \cap A_3| = |A_2 \cap A_4| = 1 \\ |A_3 \cap A_4| &= 20 \\ |A_1 \cap A_2 \cap A_3| &= |A_1 \cap A_2 \cap A_4| = |A_1 \cap A_3 \cap A_4| = |A_2 \cap A_3 \cap A_4| = 0 \\ |A_1 \cap A_2 \cap A_3 \cap A_4| &= 0 \\ |\overline{A_1} \cap \overline{A_2} \cap \overline{A_3} \cap \overline{A_4}| &= 560 - (120 + 56 + 165 + 165) + (10 + 10 + 1 + 1 + 20) \\ &= 96 \end{aligned}$$

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