



## MITx: 6.041x Introduction to Probability - The Science of Uncertainty



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## Exercise: CLT practice

(6/6 points)

The random variables  $X_i$  are i.i.d. with mean **2** and standard deviation equal to **3**. Assume that the  $X_i$  are nonnegative. Let  $S_n = X_1 + \cdots + X_n$ . Use the CLT to find good approximations to the following quantities. You may want to refer to the normal table . In parts (a) and (b), give answers with 4 decimal digits.



a)  $\mathbf{P}(S_{100} \leq 245) \approx$  Answer: 0.9332

b) We let  $N$  (a random variable) be the first value of  $n$  for which  $S_n$  exceeds **119**.




$\mathbf{P}(N > 49) \approx$  Answer: 0.8413

c) What is the largest possible value of  $n$  for which we have  $\mathbf{P}(S_n \leq 128) \approx 0.5$ ?


- ▶ Unit 6: Further topics on random variables
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#### Unit overview


##### Lec. 18: Inequalities, convergence, and the Weak Law of Large Numbers

Exercises 18 due Apr 27, 2016  
at 23:59 UTC 

##### Lec. 19: The Central Limit Theorem (CLT)

Exercises 19 due Apr 27, 2016  
at 23:59 UTC 

##### Lec. 20: An introduction to classical statistics

Exercises 20 due Apr 27, 2016  
at 23:59 UTC 

64



$n =$  Answer: 64

Answer:

We will use  $Z_n$  to refer to the standardized random variable  $(S_n - 2n)/(3\sqrt{n})$ .

a) We have

$$\mathbf{P}(S_{100} \leq 245) = \mathbf{P}\left(\frac{S_{100} - 2 \cdot 100}{3 \cdot \sqrt{100}} \leq \frac{245 - 2 \cdot 100}{3 \cdot \sqrt{100}}\right) = \mathbf{P}(Z_n \leq 1.5) \approx 0.9332.$$

b) The event  $N > 49$  is the same as the event  $S_{49} \leq 119$ . Its probability is

$$\mathbf{P}(S_{49} \leq 119) = \mathbf{P}\left(\frac{S_{49} - 2 \cdot 49}{3 \cdot \sqrt{49}} \leq \frac{119 - 2 \cdot 49}{3 \cdot \sqrt{49}}\right) = \mathbf{P}(Z_n \leq 1) \approx 0.8413.$$


c) We want  $n$  such that

$$0.5 \approx \mathbf{P}(S_n \leq 128) = \mathbf{P}\left(\frac{S_n - 2n}{3\sqrt{n}} \leq \frac{128 - 2n}{3\sqrt{n}}\right) = \Phi\left(\frac{128 - 2n}{3\sqrt{n}}\right).$$

## Solved problems

## Additional theoretical material

## Problem Set 8

Problem Set 8 due Apr 27, 2016  
at 23:59 UTC 

## Unit summary

But since  $0.5 = \Phi(0)$ , we must have  $(128 - 2n)/(3\sqrt{n}) = 0$ , so that  $n = 128/2 = 64$ .

A faster way to see the answer is to note that since the normal is symmetric around its mean, the relation  $\mathbf{P}(S_n \leq 128) \approx 0.50$  tells us that **128** should be equal to the mean,  **$2n$** , of  $S_n$ .

*You have used 1 of 3 submissions*

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