Exercise: Conditional PMFs

(7/7 points)

For each of the formulas below, state whether it is true or false.

a) $p_{X,Y,Z}(x,y,z) = p_Y(y)\, p_{Z|Y}(z \mid y)\, p_{X|Y,Z}(x \mid y,z)$

True ✔ **Answer:** True

b) $p_{X,Y|Z}(x,y \mid z) = p_X(x)\, p_{Y|Z}(y \mid z)$

False ✔ **Answer:** False

c) $p_{X,Y|Z}(x,y \mid z) = p_{X|Z}(x \mid z)\, p_{Y|X,Z}(y \mid x,z)$

True ✔ **Answer:** True

d) $\sum_x p_{X,Y|Z}(x,y \mid z) = 1$

False ✔ **Answer:** False

e) $\sum_x \sum_y p_{X,Y|Z}(x,y \mid z) = 1$

True ✔ **Answer:** True

f) $p_{X,Y|Z}(x,y \mid z) = \dfrac{p_{X,Y,Z}(x,y,z)}{p_Z(z)}$

True ✔ **Answer:** True

g) $p_{X|Y,Z}(x \mid y,z) = \dfrac{p_{X,Y,Z}(x,y,z)}{p_{Y,Z}(y,z)}$

True ✔ **Answer:** True

Answer:
a) True. This is the usual multiplication rule for the probability of three events occurring simultaneously.

b) False. This does not follow from any of the formulas we have developed.

c) True. This is the usual multiplication rule for the event $\{X = x \text{ and } Y = y\}$, in a conditional model in which it is given that the event $\{Z = z\}$ has occurred.

d) False. The left-hand side is a function of y, whereas the right-hand side is not.

e) True. This is the usual normalization property, in a conditional model in which it is given that the event $\{Z = z\}$ has occurred.

f) True. This is just the formula for the conditional probability $\mathbf{P}(X = x, Y = y \mid Z = z)$.

g) True. This is just the formula for the conditional probability $\mathbf{P}(X = x \mid Y = y, Z = z)$.

You have used 1 of 1 submissions