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## Exercise: The posterior of a coin's bias

(3/3 points)

Let  $\Theta$  be a continuous random variable that represents the unknown bias (i.e., the probability of Heads) of a coin.

a) The prior PDF  $f_{\Theta}$  for the bias of a coin is of the form

$$f_{\Theta}(\theta) = a\theta^9(1 - \theta), \quad \text{for } \theta \in [0, 1],$$

where  $a$  is a normalizing constant. This indicates a prior belief that the bias  $\Theta$  of the coin is

High ▼



Answer: High

b) We flip the coin 10 times independently and observe 1 Heads and 9 Tails. The posterior PDF of  $\Theta$  will be of the form  $c\theta^m(1 - \theta)^n$ , where  $c$  is a normalizing constant and where

$m =$

10



Answer: 10

$n =$

10



Answer: 10


Answer:

a) Because of the high exponent, the term  $\theta^9$  is very small when  $\theta$  is small. This prior, as can also be seen by plotting it, is concentrated on high values of  $\theta$  and indicates a prior belief in favor of large values.


b) As we saw in the last video, the power to which  $\theta$  (respectively,  $1 - \theta$ ) is raised needs to be incremented by the number of Heads (respectively, Tails) observed, leading to  $m = 9 + 1 = 10$  and  $n = 1 + 9 = 10$ . Notice that the resulting posterior is symmetric around 0.5.

This exercise indicates that the strength of the "evidence" incorporated in a prior with  $\alpha = 9$  and  $\beta = 1$  is exactly counterbalanced by observing 1 Heads and 9 Tails. Differently said, a


**Unit overview****Lec. 14:  
Introduction to  
Bayesian  
inference**

Exercises 14 due Apr  
06, 2016 at 23:59 UTC 


**Lec. 15: Linear  
models with  
normal noise**

Exercises 15 due Apr  
06, 2016 at 23:59 UTC 


**Problem Set 7a**

Problem Set 7a due  
Apr 06, 2016 at 23:59  
UTC 


**Lec. 16: Least  
mean squares  
(LMS) estimation**

Exercises 16 due Apr  
13, 2016 at 23:59 UTC 

**Lec. 17: Linear  
least mean  
squares (LLMS)  
estimation**

Exercises 17 due Apr  
13, 2016 at 23:59 UTC 

**Problem Set 7b**

Problem Set 7b due  
Apr 13, 2016 at 23:59  
UTC 

**Solved problems****Additional  
theoretical  
material****Unit summary**

prior with  $\alpha = 9$  and  $\beta = 1$  can be thought of as equivalent to prior "evidence" based on 9 Heads and 1 Tails.

*You have used 1 of 1 submissions*

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