# 3. Speed and Velocity

Lecture due Oct 5, 2021 20:30 IST

## Explore

### Trajectory versus Motion

It is important to remember that the particle's *trajectory* is not the same as the particle's *motion*. Looking only at the plot of the trajectory, we would not be able to tell the speed of the particle or the direction of motion.

Can we figure out the speed and direction of motion just by looking at the formulas for $x\left(t\right)$ and $y\left(t\right)$? The answer is yes, but first, we need to clarify the difference between **speed** and **velocity**.

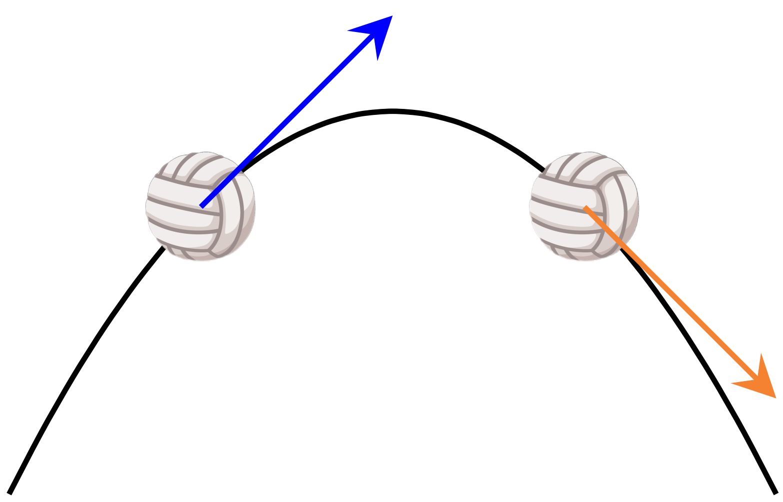
### Speed versus Velocity

**Speed** is a nonnegative real number that measures how fast the particle is moving. If you were to toss a tennis ball in the air, it would start with a high speed, which decreases down to zero at the peak, and then the speed would increase again as the ball accelerates towards the ground.

On the other hand, **velocity** has a *directional component*. For a tossed tennis ball, it starts with an upwards velocity, but after reaching its peak, the velocity begins to point downwards.

### Same Speed Different Velocity

To take another example, imagine serving a volleyball. The following figure shows its trajectory:



Noto Emoji from Github by User: Behdad (Apache License 2.0)

The two arrows represent the velocity vectors of the volleyball at two moments in time. Comparing these two points in time, the volleyball has the same speed but different velocity.

### Find the velocity vector

**Definition 3.1** (Velocity in 2D) If a particle follows a parametric equation $\left(x\left(t\right), y\left(t\right)\right)$ then its **velocity vector** at time $t$ is the vector $\begin{pmatrix} x'\left(t\right) \\ y'\left(t\right) \end{pmatrix}$.

It is also common to write this vector using the differential notation:

$$\frac{d\vec{r}}{dt} = \begin{pmatrix} x'\left(t\right) \\ y'\left(t\right) \end{pmatrix}$$

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Again we are using the standard notation $\vec{r}$ for $\begin{pmatrix} x(t) \\ y(t) \end{pmatrix}$. The differential notation should be understood in the same way as in single-variable calculus: it is the limit of the ratio $\frac{\Delta\vec{r}}{\Delta t}$ as $\Delta t$ goes to zero (here $\Delta\vec{r}$ is a vector and $\Delta t$ is a real number).

**Example 3.2** Our first example was $(x(t), y(t)) = (t^2, 1+t^2)$. The velocity vector is therefore $\begin{pmatrix} 2t \\ 2t \end{pmatrix}$. We see that the velocity vector is parallel to $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$, the trajectory of the particle.

## Find the speed

1/1 point (graded)

Speed is the magnitude of the velocity vector. Which of these expressions is the correct notation for speed?

- ○ $\frac{d|\vec{r}|}{dt}$
- ○ $\frac{d\vec{r}}{dt}$
- ● $\left|\frac{d\vec{r}}{dt}\right|$

✔

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✔ Correct (1/1 point)

**Example 3.3** Again let's consider $(x(t), y(t)) = (t^2, 1+t^2)$. The velocity vector is $\begin{pmatrix} 2t \\ 2t \end{pmatrix}$. The speed is given by $\sqrt{8t^2}$, which shows that the particle is accelerating, that is, its speed is increasing over time.

Note that the velocity vector $\vec{v}$ depends on the time $t$, since at different times the particle will be moving in different directions. This $\vec{v}(t)$ is another example of a vector-valued function.

## 3. Speed and Velocity

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