6. Optimization intuition: Worked example

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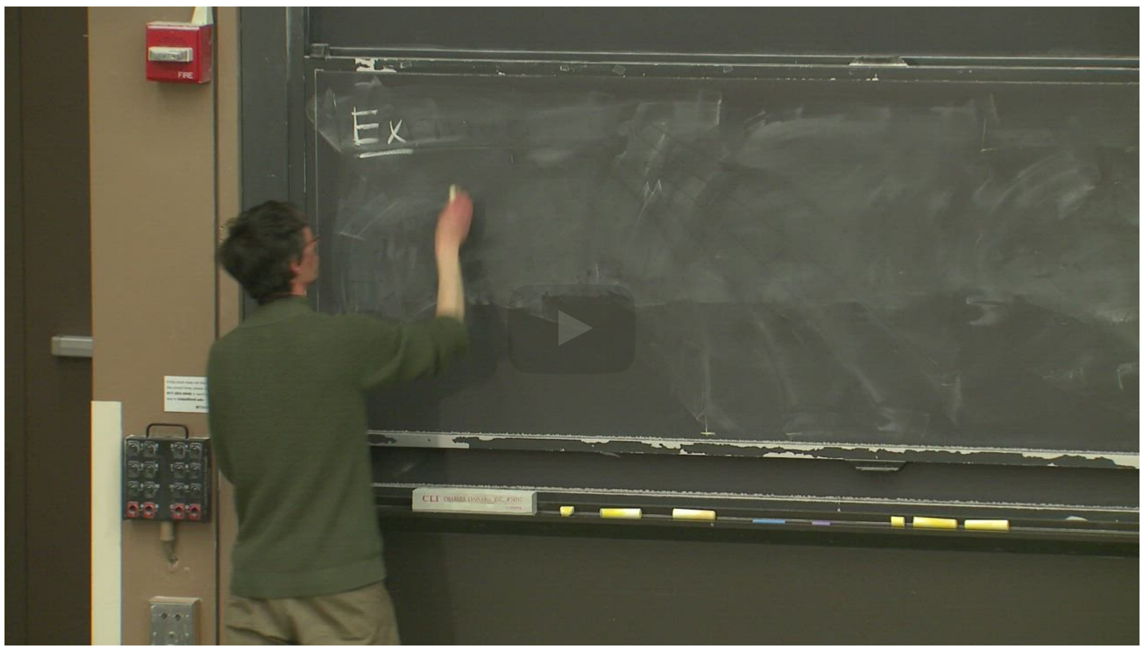
Reflect

Before we give you the methods for how to optimize functions of multiple variables, we are going to go through an example that will help us see how these methods come about.

Worked example

Start of transcript. Skip to the end.



0:00 / 0:00 2.0x

PROFESSOR: Here's an example.

So first of all, I'm going to draw for you the region of R,

which is a triangle.

So here's 2, 2, then this point (2, 2).

So that's our region, that triangle there.

And here is the function.

f of (x, y) is 2x minus x squared minus y.

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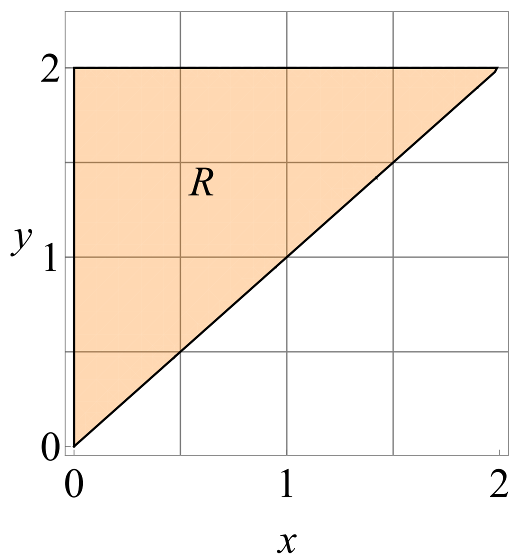
Example 6.1 Consider the function

$$f(x, y) = 2x - x^2 - y. \qquad (4.113)$$

We are going to find the maximum of f on the region R which is bounded by the y-axis, the line $y = x$, and the line $y = 2$. The region R is shown below.



We will first find the critical points of the function. To do this, we compute the partial

$$f_x(x,y) = 2 - 2x \quad (4.114)$$

$$f_y(x,y) = -1. \quad (4.115)$$

The critical points occur when $\nabla f(x,y) = \vec{0}$, or in other words, when $f_x(x,y) = f_y(x,y) = 0$. Setting the above equations equal to 0 gives

$$f_x(x,y) = 2 - 2x = 0 \implies x = 1 \quad (4.116)$$

$$f_y(x,y) = -1 = 0 \implies ? \quad (4.117)$$

What does it mean if there are no critical points?

Because the second equation $f_y(x,y) = -1 = 0$ is never true, $f(x,y)$ has no critical points.

Since the region has no critical points, the maximum must occur on the boundary of the region R! Note that the boundary is not just the three corners of the triangle. The boundary consists of each edge of the triangle.

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Example continued

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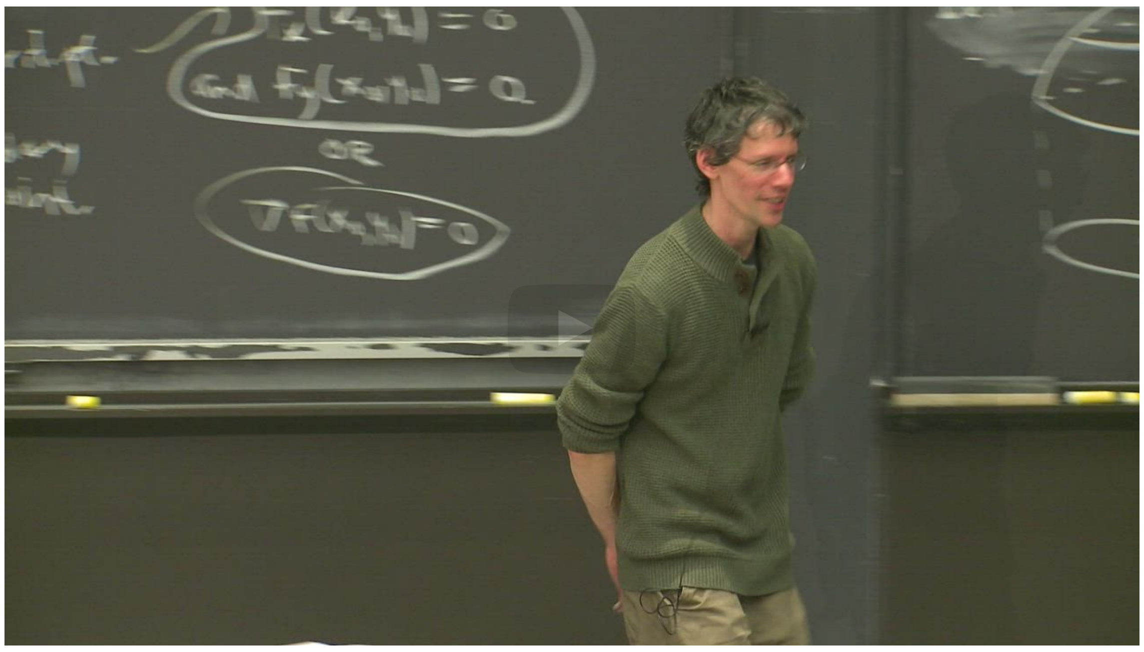

PROFESSOR: OK, good-- so now let's talk about it together.

I saw a few people in the audience who did like this.

Right, so this over here is an example.

There are no critical points for this graph.

We've seen lots of functions have no critical points

0:00 / 0:00 2.0x

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Checking the top edge

We will start by considering the top edge, where the computation is simpler. The top edge of the triangle is defined by $y = 2$, for $0 \le x \le 2$.

Substituting $y = 2$ into $f(x,y)$ gives

$$f(x,2) = 2x - x^2 - 2. \quad (4.118)$$

This is a single variable function, so to optimize we differentiate to look for critical points.

Setting derivative with respect to x equal to zero gives:

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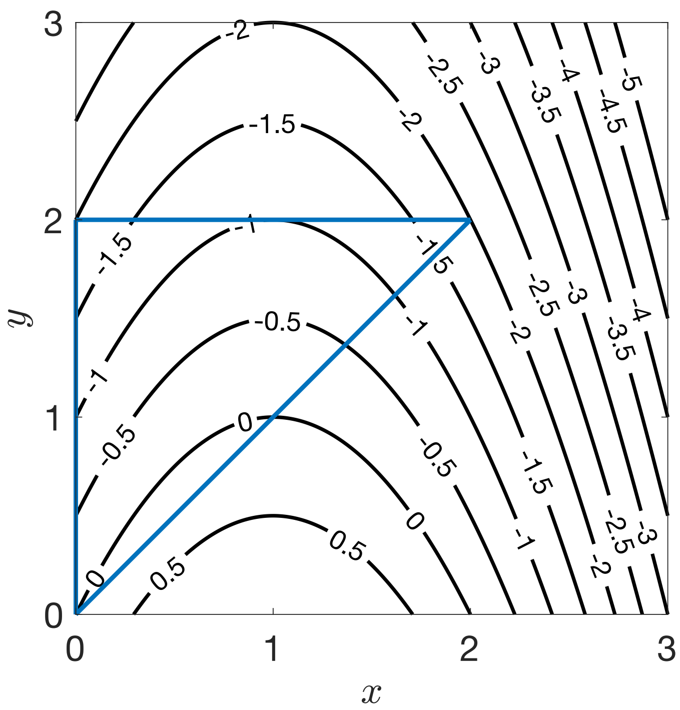
$$2 - 2x = 0 \tag{4.119}$$

$$x = 1 \tag{4.120}$$

Plugging into our formula for $f(x,y)$ we get $f(1,2) = 2 - 1 - 2 = -1$.

Graphical intuition

We will first visualize how this picture relates to the level curves below.



Notice that all the values along the vertical edge and horizontal edge are negative. Along the diagonal edge, the function starts at a height of 0, becomes positive, decreases back down to 0, and then becomes negative. Thus graphically, we can tell that the maximum will have to occur along the diagonal edge.

Checking the diagonal edge

So we are only going to consider the diagonal edge.

Along the diagonal edge, we have $y = x$ and $0 \leq x \leq 2$. Substituting $y = x$ into $f(x,y)$ gives

$$f(x, y = x) = 2x - x^2 - x = x - x^2, \;\; 0 \leq x \leq 2. \tag{4.121}$$

Let's call this new function $g(x)$. So we have

$$g(x) = x - x^2, \;\; 0 \leq x \leq 2. \tag{4.122}$$

We want to choose x to maximize the function. Notice that this is now a single-variable problem. Following the procedure from single-variable calculus, we test the critical points and end points. The critical points are given by

$$g'(x) = 1 - 2x = 0 \implies x = \frac{1}{2} \tag{4.123}$$

and the end points are $x = 0$ and $x = 2$. Substituting these values in for $g(x)$ give

$$g(0) \quad = 0 - 0^2 = 0 \tag{4.124}$$

$$g(1/2) \quad = \frac{1}{2} - \left(\frac{1}{2}\right)^2 = \frac{1}{4} \tag{4.125}$$

$$g(2) \quad = 2 - 2^2 = -2.$$

So the maximum occurs at $x = 1/2$. Going back to our original function $f(x, y)$, this means the maximum occurs when $x = 1/2$ and $y = x = 1/2$ and the maximum value is

$$f(1/2, 1/2) = \frac{1}{4}. \quad (4.127)$$

Conclusion

The maximum of f is the maximum value among the largest values found along each edge. If you followed the same procedure along the side edge, you would see that the maximum indeed occurs along the diagonal edge.

This was a very conceptual introduction to finding the maximum of a function over a closed and bounded region. We hope it gives you a sense for how these types of problems relate to level curves. We will formalize an approach to solving these types of problems in the following lecture.

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- [staff] typo in example 6.1 — 3
- typo: example 6.1 — 2
  "the region R which is bounded by the y-axis, the x-axis, the line y = x, and the line y = 2." R isn't bounded by the x-axis.
- ? left edge — 2
  The video passes on finding the maximum for the left edge analytically. But I thought I'd give it a go because it's a vertical line segm...
- Cases of extrema on vertices of boundary. — 4
- Matlab can solve the problem — 2
  define function f = @(x,y) 2.*x - x.^2 - y; define mesh [xx, yy] = meshgrid(x,y); evaluate function g = f(xx,yy); add boundary g(xx>yy)...
- [typo] need a word — 2
  Community TA
- [STAFF] error in eq 4.118 — 2
  f(x,1) → f(x,2)
- [STAFF] Error in Eq (4.127) — 2
  Eq (4.127) should be f(1/2,1/2) = 1/4

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