

## <u>Lecture 14: Wald's Test, Likelihood</u> <u>Ratio Test, and Implicit Hypothesis</u>

Course > Unit 4 Hypothesis testing > Test

> 8. Review: Power of a Test

### **Audit Access Expires Dec 24, 2019**

You lose all access to this course, including your progress, on Dec 24, 2019.

# 8. Review: Power of a Test

Review: Power of a Test for Different Alternative Hypotheses

2/3 points (graded)

Recall that the power  $\pi_{\psi}$  of a test  $\psi$  for the hypotheses

$$H_0: heta^*\in\Theta_0$$

$$H_1: heta^*\in\Theta_1$$

is

$$\pi_{\psi} \; \equiv \; \inf_{ heta \in \Theta_{1}} \left( 1 - eta_{\psi} \left( heta 
ight) 
ight)$$

where  $eta_{\psi}\left( heta
ight)=\mathbf{P}_{ heta}\left(\psi=0
ight)$ , defined for  $heta\in\Theta_{1},\,$  is the **type 2 error rate** of  $\psi.$ 

Suppose  $X_1, \ldots, X_n$  are i.i.d. random variables (in 1 dimension). Assume the theorem of MLE applies so that  $\hat{\theta}^{\text{MLE}}$  is asymptotically normal. You use the test

$$\psi \; = \; \mathbf{1} \left( \sqrt{nI} \, \left| \hat{ heta}^{ ext{MLE}} - 0 
ight| > C_lpha 
ight),$$

which has level  $\alpha$  for some threshold  $C_{\alpha}$ , to test the hypotheses

$$H_0: heta^*=0$$

$$H_1: heta^* 
eq 0.$$

What is the *asymptotic* power  $\pi_{\psi}$  in terms of  $\alpha$ ?

$$\pi_{\psi} = egin{pmatrix} ext{1-alpha} & extbf{X} ext{ Answer: alpha} \ \hline & 1-lpha & ext{} \end{bmatrix}$$

Now, you use the same test  $\psi = \mathbf{1} \left( \sqrt{nI} \left| \hat{\theta}^{\mathrm{MLE}} - 0 \right| > C_{\alpha} \right)$  to test a different alternative hypothesis against the same null hypothesis:

$$H_0: heta^* = 0$$

$$H_1: heta^* = 1.$$

How do the (smallest) *asymptotic* level and the *asymptotic* power of  $\psi$  change with this change of the alternative hypothesis? (Choose one for each column.)

The (smallest) asymptotic level of  $\,\psi\,...\,$  the asymptotic power of  $\,\psi\,...\,$ 

increases	<ul><li>increases</li></ul>
decreases	decreases
stays the same	stays the same

(In general, how does the level and power of a test vary as  $\Theta_1$  shrinks?)

#### STANDARD NOTATION

#### Solution:

The power of  $\psi$  with  $H_1: heta^*
eq 0$  is

$$egin{array}{ll} \pi_{\psi} &=& \inf_{ heta 
eq 0} \left( 1 - eta_{\psi} \left( heta 
ight) 
ight) \ &=& \inf_{ heta 
eq 0} \mathbf{P}_{ heta} \left( \psi = 1 
ight) \, = \, \inf_{ heta 
eq 0} \mathbf{P}_{ heta} \left( \sqrt{nI} \left| \hat{ heta}^{ ext{MLE}} - 0 
ight| > C_{lpha} 
ight) \end{array}$$

Since  $\sqrt{nI}\left(\hat{\boldsymbol{\theta}}^{\mathrm{MLE}}-\boldsymbol{\theta}\right)\sim\mathcal{N}\left(0,1\right)$  (asymptotically if  $\boldsymbol{\theta}^{*}=\boldsymbol{\theta}$ ),  $\mathbf{P}_{\boldsymbol{\theta}}\left(\sqrt{nI}\left|\hat{\boldsymbol{\theta}}^{\mathrm{MLE}}-\boldsymbol{0}\right|>C_{\alpha}\right)$  decreases as  $\boldsymbol{\theta}\to0$  and approaches  $\mathbf{P}_{0}\left(\sqrt{nI}\left|\hat{\boldsymbol{\theta}}^{\mathrm{MLE}}-\boldsymbol{0}\right|>C_{\alpha}\right)=\alpha$  (sketch the probability as an area to see this). Hence  $\pi_{\psi}=\alpha$  in this case.

If we use the same test  $\psi$  for the alternative hypothesis  $H_1: heta^* = 1$ , then

$$egin{array}{ll} \pi_{\psi} &=& \mathbf{P}_{ heta=1} \left( \sqrt{nI} \left| \hat{ heta}^{ ext{MLE}} - 0 
ight| > C_{lpha} 
ight) \end{array}$$

which is greater than  $\left.\mathbf{P}_{ heta=0}\left(\sqrt{nI}\left|\hat{ heta}^{ ext{MLE}}-0
ight|>C_{lpha}
ight)=lpha$ . (Again, sketch the probability as an area to see this.)

On the other hand, the alternative hypothesis has no effect on the level of the test once the test has been fixed.

Submit

You have used 2 of 2 attempts

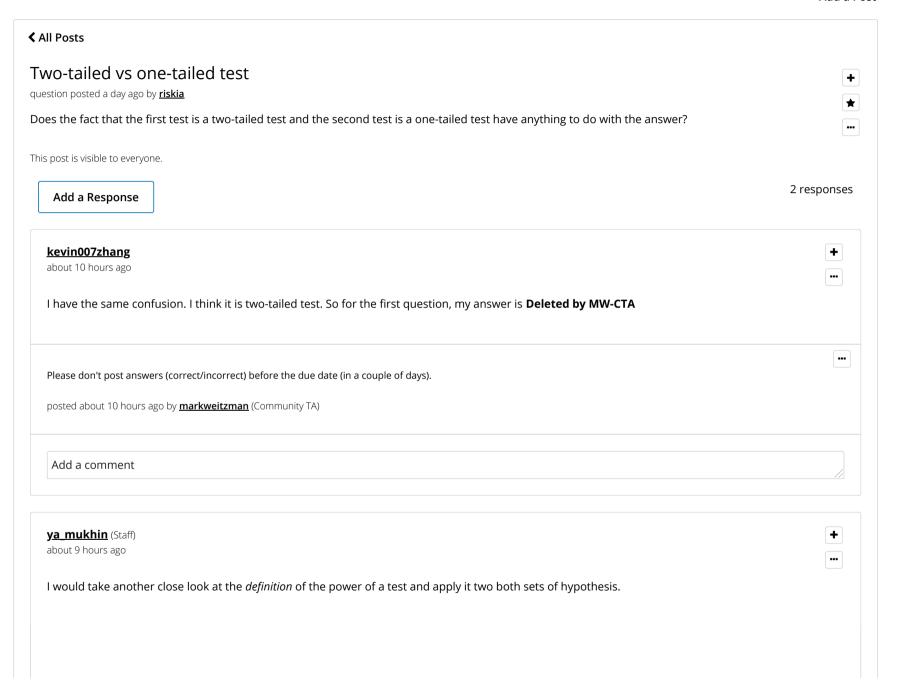
• Answers are displayed within the problem

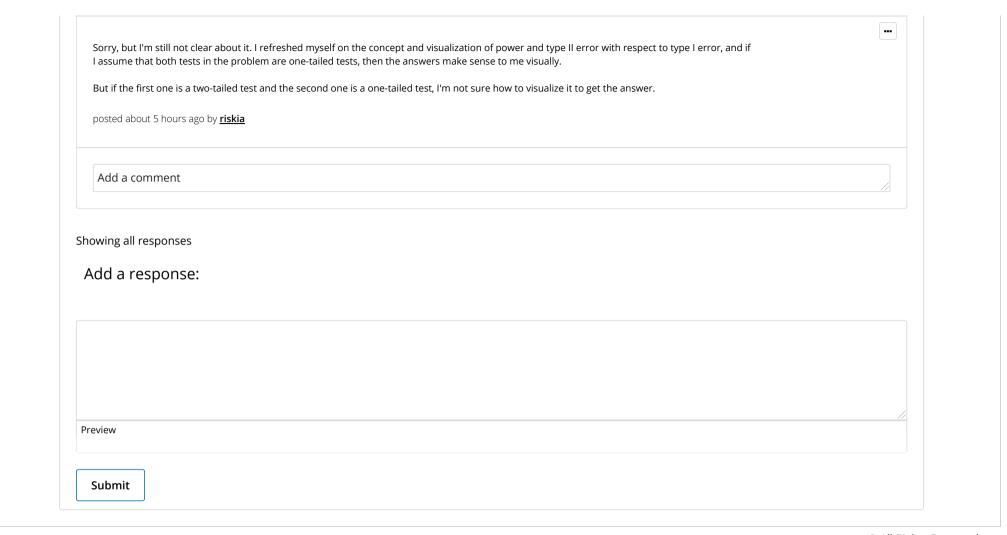
# Discussion

**Hide Discussion** 

**Topic:** Unit 4 Hypothesis testing:Lecture 14: Wald's Test, Likelihood Ratio Test, and Implicit Hypothesis Test / 8. Review: Power of a Test

#### Add a Post





© All Rights Reserved