

Expected value of conditional Poisson process

Asked 2 days ago Active today Viewed 63 times



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I have been trying to solve this issue for quite a while. So, lets say that we have a Poisson process, $N=(N_t,t\geq 0)$ and the $\lambda=3$. Lets say that $Y=(N_2|N_6=3)$. Find Ee^Y . However, I am stuck on understanding the notation of Y and how should I process it to continue the calculation of expected value. Should I find joint density function?







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Distribution of Y is the conditional distribution of N_2 given $N_6=3$. Can you figure out this conditional distribution? – StubbornAtom 2 days ago

3 Answers





1

Let us assume we have a Poisson process with an arrival rate of λ . After some time t, N_t unobserved arrivals have occurred. After some more time, say τ , we observe that $N_{t+\tau}$ arrivals have occurred. What is the distribution of N_t given the observed value of $N_{t+\tau}$?

As it happens, the memoryless property of the Poisson process implies that the time of any



given arrival is uniformly distributed over, in this case, $[0,t+\tau]$. This implies that the probability p that any given arrival in $[0,t+\tau]$ actually shows up in the interval [0,t] is just $p=t/(t+\tau)$, the fraction of the total time that occurred before t, and is independent of the time of any other arrival. If we have $N_{t+\tau}$ arrivals overall, the number that arrive in [0,t] is therefore distributed Binomial $(N_{t+\tau},\frac{t}{(t+\tau)})$



In this case, we have t=2, $t+\tau=6$, and $N_{t+\tau}=3$. Substituting gives us the probability distribution of $Y=(N_2|N_6=3)$, which is a Binomial (3,1/3) distribution. Getting from this to $\mathbb{E}e^Y$ is a straightforward calculation.

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I understand that it seem straightforward calculation for most of you, but it is quite opposite for me, unfortunately. I suppose that this calculation should be done like this:

$$Ee^Y = \sum_{y=0}^3 e^y {3 \choose y} rac{1}{3}^y rac{2}{3}^{3-y}$$

? Am I right on this? – Gvidas Pranauskas yesterday 🖍



Yes, you're right. You can save yourself some effort if you have some familiarity with moment generating functions (as per @StubbornAtom 's excellent answer.) – jbowman yesterday



Thank you for thoroughly explained solution to this problem. - Gvidas Pranauskas 14 hours ago



The notation ' $Y=(N_2\mid N_6=3)$ ' implies that the distribution of the random variable Y is the conditional distribution of N_2 given $N_6=3$.





Now if $(N_t)_{t\geq 0}$ is a Poisson process with intensity parameter $\lambda(>0)$, then the following holds:



- $N_t \sim \text{Poisson}(\lambda t)$.
- ullet $N_{t+s}-N_s\sim \mathrm{Poisson}(\lambda t)$ is independent of N_s .

Using this information, one can find the conditional distribution of N_s given N_t for 0 < s < t. This turns out to be a standard distribution, and you are required to find/recall the moment generating function of this distribution.

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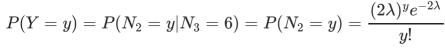


Thanks for reminding the moment generating function:) - Gvidas Pranauskas 14 hours ago



Due to memoryless / independent properties of Poisson process, we have







$$\therefore E[e^Y] = \sum_{y=0}^{\infty} P(Y=y). e^y = \sum_{y=0}^{\infty} \frac{(2\lambda)^y e^{-2\lambda}}{y!}. e^y = e^{-2\lambda} \sum_{y=0}^{\infty} \frac{(2\lambda e)^y}{y!} = e^{-2\lambda}. e^{2\lambda e}.$$

$$= e^{2\lambda(e-1)}$$

Now, if we want to compute expectation the conditional Poisson Process for the same interval, let's say, if in a given interval of length 6λ we have 3 arrivals, what's the expected number of arrivals in a subinterval of length 2λ of that interval, then we can proceed as follows.

 $N_6=3$ restricts $N_2\leq 3$ (since there is 3 arrivals in interval of length 6λ , then in length 2λ sub-interval of it, we must have less or equal arrivals).

Hence, $P(Y=y)=P(N_2=y)=rac{e^{-2\lambda y}(2\lambda)^y}{y!}$, where $y\in\{0,1,2,3\}$ (considering memoryless and independence properties of the Poisson process).

$$\therefore E[e^Y] = \sum_{y=0}^3 P(Y=y).\,e^y = \sum_{y=0}^3 \frac{e^{-2\lambda y}(2\lambda)^y}{y!}.\,e^y = \sum_{y=0}^3 \frac{e^{(1-2\lambda)y}.(2\lambda)^y}{y!}\text{, where we have }\lambda = 3$$

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edited 20 hours ago

answered 2 days ago



