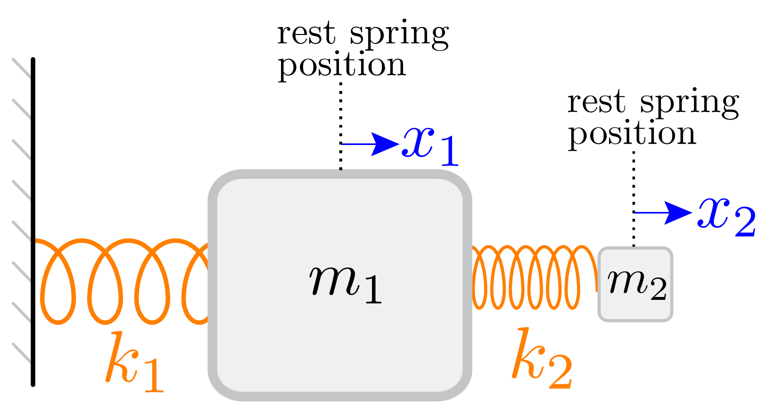
17. Unforced tuned mass damper

Angular frequencies for the unforced tuned mass damper

1.0/1 point (graded)

Recall from a previous video a swaying building with a tuned mass damper installed can be modelled as a forced coupled oscillator.



The companion system, in the case when $F = 0,$ and when no dashpots are present ($b_1 = b_2 = 0$), can be written as

$$\frac{d}{dt}\begin{pmatrix} x_1 \\ x_2 \\ y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{k_1+k_2}{m_1} & \frac{k_2}{m_1} & 0 & 0 \\ \frac{k_2}{m_2} & -\frac{k_2}{m_2} & 0 & 0 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ y_1 \\ y_2 \end{pmatrix}.$$

This can also be obtained from the previous coupled oscillator by setting $k_3 = 0$.

Set the parameters to $m_1 = 100,\ m_2 = 1$ and $k_1 = 100,\ k_2 = 1,$ to model the facts that the building is much heavier than the mass in the tuned mass damper, and the building is much stiffer as a spring that the one in the tuned mass damper. We will now follow the procedure as in the text above to find the pure sinusoidal modes of this system.

First, rewrite the above system in a more compact form:

$$\frac{d}{dt}\begin{pmatrix} \mathbf{x} \\ \mathbf{y} \end{pmatrix} = \begin{pmatrix} \mathbf{0} & \mathbf{I} \\ \mathbf{B} & \mathbf{0} \end{pmatrix}\begin{pmatrix} \mathbf{x} \\ \mathbf{y} \end{pmatrix} \qquad \text{where} \quad \mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \text{ and } \mathbf{y} = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}.$$

Write down the matrix $\mathbf{B}$.

(Enter **[a,b;c,d]** for the matrix $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$. That is, use commas to separate entries within the same row and **semicolon** to separate rows, and **square bracket** the entire matrix. Similarly for larger matrices.)

$\mathbf{B} =$ [-101/100, 1/100;1,-1] ✔

Submit You have used 1 of 3 attempts

Find the eigenvalues and angular frequencies

4/4 points (graded)

Continuing from the problem above, find the eigenvalues of $\mathbf{B}$.

$\lambda_1 =$ -1.1051249 ✔ **Answer:** -201/200+sqrt(401)/200

-1.1051249

$\lambda_2 =$ -0.9048751 ✔ **Answer:** -201/200-sqrt(401)/200

-0.9048751

What are the angular frequencies ω_1, ω_2 of the two pure sinusoidal modes of unforced tuned mass damper?

$\omega_1 =$ 1.051249 ✔ **Answer:** (sqrt(401)-1)/20

1.051249

$\omega_2 =$ 0.9512492 ✔ **Answer:** (sqrt(401)+1)/20

0.9512492

Solution:

The eigenvalues of $\mathbf{B}$ are the roots of the equation:

$$\left(\lambda+\frac{101}{100}\right)(\lambda+1)-\frac{1}{100}=\lambda^2+\left(2+\frac{1}{100}\right)\lambda+1=0 \qquad \Longrightarrow \qquad \lambda=-\frac{201}{200}\pm\frac{\sqrt{401}}{200}.$$

Note that both of these eigenvalues are negative.

The eigenvalues of $\begin{pmatrix}\mathbf{0} & \mathbf{I}\\ \mathbf{B} & \mathbf{0}\end{pmatrix}$ are the square roots of the eigenvalues of $\mathbf{B}$ and so are purely imaginary:

$$\pm i\frac{1}{10}\sqrt{\frac{201-\sqrt{401}}{2}}=i\frac{\sqrt{401}-1}{20}$$

$$\pm i\frac{1}{10}\sqrt{\frac{201+\sqrt{401}}{2}}=i\frac{\sqrt{401}+1}{20}$$

Therefore, the angular frequencies of the pure sinusoidal modes are $\dfrac{\sqrt{401}-1}{20}$ and $\dfrac{\sqrt{401}+1}{20}$.

Note: In the homework, you will find the normal modes of coupled oscillators with dashpots.

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You have used 1 of 3 attempts

Answers are displayed within the problem

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angular frequencies 24

Physics Approach 8
The material of this lecture is treated more generally in physics courses (mechanics, waves, math methods, normal modes etc.) ...

angular frequency? 2
I have no idea of the relationship between the angular frequency of the mass damper with the parameters of the equations or ...

Angular velocity 2
Hi, My angular velocities are correct yet they have been marked wrong? Can someone please assist? Thanks.

Unit 3: 17. Unforced tuned mass damper 7
I do not understand why angular frequencies are being asked will the eigenvalues are not complex.

B Matrix
Hi, My B matrix has shown that it is partially correct, is that because it requires multiplying by w^2 (omega squared)? Thanks.
2

Angular frequencies
Is there something wrong with the grader on this part?
8

? [Staff] Find the eigenvalues and angular frequencies.
Community TA
2