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Exercise: The discrete Bayes rule

(1/1 point)

The bias of a coin (i.e., the probability of Heads) can take three possible values, $1/4$, $1/2$, or $3/4$, and is modeled as a discrete random variable Q with PMF

$$p_Q(q) = \begin{cases} 1/6, & \text{if } q = 1/4, \\ 2/6, & \text{if } q = 2/4, \\ 3/6, & \text{if } q = 3/4, \\ 0, & \text{otherwise.} \end{cases}$$

Let K be the total number of Heads in two independent tosses of the coin. Find $p_{Q|K}(3/4 \mid 2)$.

3/4 ✔ **Answer:** 0.75

Answer:
The Bayes rule for discrete random variables gives

$$p_{Q|K}(3/4 \mid 2) = \frac{p_Q(3/4) p_{K|Q}(2 \mid 3/4)}{p_K(2)} = \frac{(3/6)\cdot(3/4)^2}{p_K(2)} = \frac{(3/6)\cdot(3/4)^2}{3/8} = \frac{3}{4}.$$

To find $p_K(2)$, we used the total probability theorem:

$$p_K(2) = \sum_q p_Q(q) p_{K|Q}(2 \mid q) = (1/6)\cdot(1/4)^2 + (2/6)\cdot(2/4)^2 + (3/6)\cdot(3/4)^2 = 3$$

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