10. Differentiating Fourier series

Key Insight : if a function is differentiable, you can simply differentiate its Fourier series term by term to obtain the Fourier series for the derivative.

Note:

There are many subtle, but important, questions in Fourier series that we will not cover here (but which courses such as 18.100 (Real Analysis) do, at least partially). For example: If I write some arbitrary Fourier series, how do I know if it comes from a differentiable function? If I differentiate term by term the Fourier series for a function that is not differentiable (like the square wave function), is the result the Fourier series for something?

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Example 10.1 The Fourier series for the period 2π triangle wave is:

$$g(t) = \frac{\pi}{2} - \frac{4}{\pi}\left(\cos t + \frac{\cos 3t}{3^2} + \frac{\cos 5t}{5^2} + \dots\right)$$

What is $g'(t)$?

Solution

Differentiating the Fourier series $g(t)$ term-by-term gives

$$g'(t) = \frac{4}{\pi}\left(\sin t + \frac{\sin 3t}{3} + \frac{\sin 5t}{5} + \dots\right),$$

which is the Fourier series of the 2π-periodic square wave!

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This page is perhaps a little too simplistic. Maybe it only applies to functions that are continuous and piecewise differentiable, but not functions with step discontinuities (like ...

typo: Is the power 2 miss? 3

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