3.3.2 Adding matrices

Week 3 due Oct 18, 2023 06:12 IST

3.3.2 Adding matrices

4:35 / 4:35 2.0x

Video

Download video file

Transcripts

Download SubRip (.srt) file

Download Text (.txt) file

Reading Assignment

0 points possible (ungraded)

Read Unit 3.3.2 of the notes. [LINK]

- [x] Done

Submit

Correct

Discussion

Hide Discussion

Topic: Week 3 / 3.1.2

Add a Post

Show all posts by recen

Calculator

Matrix operations for matrices that don't represent linear transformations
If I understand everything correctly, we found the definition of matrix addition by proving that if La and Lb are both linear transformations, then t... 7

Homework 3.3.2.7 Alternative Proof ? 2

homework 3.3.2.12
I am confused about items 1 and 4 on homework 3.3.2.12. If A and B are lower triangle matrices that have a corresponding element of the same ... 5

? I don't understand Homework 3.3.2.10
Can you clarify why this is always true? I looked back at the past similar proofs and still don't understand. 2

Finding an error in the pdf materials
Hi: I find a typo in the page 105. The 3rd question of Homewoek 3.3.2.12 should be "If A and B are unit lower triangular matrices then A-B is unit l... 2

Homework 3.3.2.1

1/1 point (graded)

The sum of two linear transformations is a linear transformation. More formally: Let $L_A : \mathbb{R}^n \to \mathbb{R}^m$ and $L_B : \mathbb{R}^n \to \mathbb{R}^m$ both be linear transformations and , for all $x \in \mathbb{R}^n$, define the function $L_C : \mathbb{R}^n \to \mathbb{R}^m$ by $L_C(x) = L_A(x) + L_B(x)$. $L_C(x)$ is a linear transformation.

Always ✔ Answer: Always

Explanation

Let $x, y \in \mathbb{R}^n$ and $\alpha \in \mathbb{R}$ and $L_C = L_A + L_B$. Then

- $$L_C(\alpha x) = \alpha L_C(x):$$
$$L_C(\alpha x) = L_A(\alpha x) + L_B(\alpha x) = \alpha L_A(x) + \alpha L_B(x)$$
$$= \alpha (L_A(x) + L_B(x)) = \alpha L_C(x).$$
- $$L_C(x+y) = L_C(x) + L_C(y):$$
$$L_C(x+y) = L_A(x+y) + L_B(x+y)$$
$$= L_A(x) + L_A(y) + L_B(x) + L_B(y) = L_A(x) + L_B(x) + L_A(y) + L_B(y)$$
$$= L_C(x) + L_C(y).$$

Submit

Answers are displayed within the problem

Homework 3.3.2.2

1/1 point (graded)

Algorithm: $[A] := \text{ADD\_MATRICES\_ALTERNATIVE}(A, B)$

Partition $A \to \left(\frac{A_T}{A_B}\right)$, $B \to \left(\frac{B_T}{B_B}\right)$

where A_T has 0 rows, B_T has 0 rows

while $m(A_T) < m(A)$ **do**

Repartition

$$\left(\frac{A_T}{A_B}\right) \to \left(\begin{array}{c} A_0 \\ \hline a_1^T \\ \hline A_2 \end{array}\right), \left(\frac{B_T}{B_B}\right) \to \left(\begin{array}{c} B_0 \\ \hline b_1^T \\ \hline B_2 \end{array}\right)$$

where a_1 has 1 row, b_1 has 1 row

Calculator

Continue with

$$\left(\frac{A_T}{A_B}\right) \leftarrow \left(\frac{A_0}{\frac{a_1^T}{A_2}}\right), \left(\frac{B_T}{B_B}\right) \leftarrow \left(\frac{B_0}{\frac{b_1^T}{B_2}}\right)$$

endwhile

What update will add B to A one row at a time, overwriting A with the result?

- [ ] $a_1 := 0$
- [ ] $a_1 := a_1 + b_1$
- [ ] $b_1 := a_1 + b_1$
- [x] $a_1^T := a_1^T + b_1^T$
- [ ] $b_1^T := a_1^T + b_1^T$

[explanation]

Answer: $a_1^T := a_1^T + b_1^T$

Submit

Answers are displayed within the problem

Homework 3.3.2.3

1/1 point (graded)

Let $A, B \in \mathbb{R}^{m \times n}$. $A + B = B + A$.

Always ✔ Answer: Always

Explanation

Transcripted in final section of this week

Calculator

Scanned solution from video

Robert's explanation

Proof 1: Let $C = A + B$ and $D = B + A$. We need to show that $C = D$. But

$$\gamma_{i,j} = \alpha_{i,j} + \beta_{i,j} = \beta_{i,j} + \alpha_{i,j} = \delta_{i,j}.$$

Hence $C = D$.

Proof 2:

$$\begin{aligned}
A + B &= \begin{pmatrix} \alpha_{0,0} & \alpha_{0,1} & \cdots & \alpha_{0,n-1} \\ \alpha_{1,0} & \alpha_{1,1} & \cdots & \alpha_{1,n-1} \\ \vdots & \vdots & & \vdots \\ \alpha_{m-1,0} & \alpha_{m-1,1} & \cdots & \alpha_{m-1,n-1} \end{pmatrix} + \begin{pmatrix} \beta_{0,0} & \beta_{0,1} & \cdots & \beta_{0,n-1} \\ \beta_{1,0} & \beta_{1,1} & \cdots & \beta_{1,n-1} \\ \vdots & \vdots & & \vdots \\ \beta_{m-1,0} & \beta_{m-1,1} & \cdots & \beta_{m-1,n-1} \end{pmatrix} \\
&= \begin{pmatrix} \alpha_{0,0} + \beta_{0,0} & \alpha_{0,1} + \beta_{0,1} & \cdots & \alpha_{0,n-1} + \beta_{0,n-1} \\ \alpha_{1,0} + \beta_{1,0} & \alpha_{1,1} + \beta_{1,1} & \cdots & \alpha_{1,n-1} + \beta_{1,n-1} \\ \vdots & \vdots & & \vdots \\ \alpha_{m-1,0} + \beta_{m-1,0} & \alpha_{m-1,1} + \beta_{m-1,1} & \cdots & \alpha_{m-1,n-1} + \beta_{m-1,n-1} \end{pmatrix} \\
&= \begin{pmatrix} \beta_{0,0} + \alpha_{0,0} & \beta_{0,1} + \alpha_{0,1} & \cdots & \beta_{0,n-1} + \alpha_{0,n-1} \\ \beta_{1,0} + \alpha_{1,0} & \beta_{1,1} + \alpha_{1,1} & \cdots & \beta_{1,n-1} + \alpha_{1,n-1} \\ \vdots & \vdots & & \vdots \\ \beta_{m-1,0} + \alpha_{m-1,0} & \beta_{m-1,1} + \alpha_{m-1,1} & \cdots & \beta_{m-1,n-1} + \alpha_{m-1,n-1} \end{pmatrix} \\
&= \begin{pmatrix} \beta_{0,0} & \beta_{0,1} & \cdots & \beta_{0,n-1} \\ \beta_{1,0} & \beta_{1,1} & \cdots & \beta_{1,n-1} \\ \vdots & \vdots & & \vdots \\ \beta_{m-1,0} & \beta_{m-1,1} & \cdots & \beta_{m-1,n-1} \end{pmatrix} + \begin{pmatrix} \alpha_{0,0} & \alpha_{0,1} & \cdots & \alpha_{0,n-1} \\ \alpha_{1,0} & \alpha_{1,1} & \cdots & \alpha_{1,n-1} \\ \vdots & \vdots & & \vdots \\ \alpha_{m-1,0} & \alpha_{m-1,1} & \cdots & \alpha_{m-1,n-1} \end{pmatrix} \\
&= B + A
\end{aligned}$$

Submit

Answers are displayed within the problem

Homework 3.3.2.4

1/1 point (graded)

Let $A, B, C \in \mathbb{R}^{m \times n}$. $(A + B) + C = A + (B + C)$.

Always ✔ Answer: Always

Explanation

Answer: Always

Let's introduce the notation $(A)_{i,j}$ for the i, j element of A. Then

$$\begin{aligned}
((A + B) + C)_{i,j} &= (A + B)_{i,j} + (C)_{i,j} = ((A)_{i,j} + (B)_{i,j}) + (C)_{i,j} = (A)_{i,j} + ((B)_{i,j} + (C)_{i,j}) \\
&= (A)_{i,j} + (B + C)_{i,j} = (A + (B + C))_{i,j}.
\end{aligned}$$

Hence $(A + B) + C = A + (B + C)$.

Submit

Calculator

Submit

Answers are displayed within the problem

Homework 3.3.2.5

1/1 point (graded)
Let $A, B \in \mathbb{R}^{m \times n}$ and $\gamma \in \mathbb{R}$. $\gamma (A + B) = \gamma A + \gamma B.$

Always ✔ Answer: Always

Explanation

Answer: Always

(Using the notation from the last proof.)

$$(\gamma(A+B))_{i,j} \quad = \quad \gamma(A+B)_{i,j} = \gamma((A)_{i,j} + (B)_{i,j}) = \gamma(A)_{i,j} + \gamma(B)_{i,j} = (\gamma A + \gamma B)_{i,j}$$

Hence, the i, j element of $\gamma(A+B)$ equals the i, j element of $\gamma A + \gamma B$, establishing the desired result.

Submit

Answers are displayed within the problem

Homework 3.3.2.6

1/1 point (graded)
Let $A, B \in \mathbb{R}^{m \times n}$ and $\beta, \gamma \in \mathbb{R}$. $(\beta + \gamma) A = \beta A + \gamma A.$

Always ✔ Answer: Always

Explanation

Answer: Always

(Using the notation from the last proof.)

$$((\beta+\gamma)A)_{i,j} \quad = \quad (\beta+\gamma)(A)_{i,j} = \beta(A)_{i,j} + \gamma(A)_{i,j} = (\beta A)_{i,j} + (\gamma A)_{i,j}.$$

Hence, the i, j element of $(\beta + \gamma)A$ equals the i, j element of $\beta A + \gamma A$, establishing the desired result.

Submit

Answers are displayed within the problem

Homework 3.3.2.7

1/1 point (graded)
Let $A, B \in \mathbb{R}^{m \times n}$. $(A + B)^T = A^T + B^T.$

Always ✔ Answer: Always

Explanation

Answer: Always

$$\left(\left(\begin{array}{cccc} \alpha_{0,0} & \alpha_{0,1} & \cdots & \alpha_{0,n-1} \end{array} \right. \right. \quad \left. \begin{array}{cccc} \beta_{0,0} & \beta_{0,1} & \cdots & \beta_{0,n-1} \end{array} \right.$$

Calculator

$$
\begin{aligned}
(A+B)^T &= \left(\begin{pmatrix} \alpha_{1,0} & \alpha_{1,1} & \cdots & \alpha_{1,n-1} \\ \vdots & \vdots & & \vdots \\ \alpha_{m-1,0} & \alpha_{m-1,1} & \cdots & \alpha_{m-1,n-1} \end{pmatrix} + \begin{pmatrix} \beta_{1,0} & \beta_{1,1} & \cdots & \beta_{1,n-1} \\ \vdots & \vdots & & \vdots \\ \beta_{m-1,0} & \beta_{m-1,1} & \cdots & \beta_{m-1,n-1} \end{pmatrix} \right) \\
&= \begin{pmatrix} \alpha_{0,0}+\beta_{0,0} & \alpha_{0,1}+\beta_{0,1} & \cdots & \alpha_{0,n-1}+\beta_{0,n-1} \\ \alpha_{1,0}+\beta_{1,0} & \alpha_{1,1}+\beta_{1,1} & \cdots & \alpha_{1,n-1}+\beta_{1,n-1} \\ \vdots & \vdots & & \vdots \\ \alpha_{m-1,0}+\beta_{m-1,0} & \alpha_{m-1,1}+\beta_{m-1,1} & \cdots & \alpha_{m-1,n-1}+\beta_{m-1,n-1} \end{pmatrix}^T \\
&= \begin{pmatrix} \alpha_{0,0}+\beta_{0,0} & \alpha_{1,0}+\beta_{1,0} & \cdots & \alpha_{m-1,0}+\beta_{m-1,0} \\ \alpha_{0,1}+\beta_{0,1} & \alpha_{1,1}+\beta_{1,1} & \cdots & \alpha_{m-1,1}+\beta_{m-1,1} \\ \vdots & \vdots & & \vdots \\ \alpha_{0,n-1}+\beta_{0,n-1} & \alpha_{1,n-1}+\beta_{1,n-1} & \cdots & \alpha_{m-1,n-1}+\beta_{m-1,n-1} \end{pmatrix} \\
&= \begin{pmatrix} \alpha_{0,0} & \alpha_{1,0} & \cdots & \alpha_{m-1,0} \\ \alpha_{0,1} & \alpha_{1,1} & \cdots & \alpha_{m-1,1} \\ \vdots & \vdots & & \vdots \\ \alpha_{0,n-1} & \alpha_{1,n-1} & \cdots & \alpha_{m-1,n-1} \end{pmatrix} + \begin{pmatrix} \beta_{0,0} & \beta_{1,0} & \cdots & \beta_{m-1,0} \\ \beta_{0,1} & \beta_{1,1} & \cdots & \beta_{m-1,1} \\ \vdots & \vdots & & \vdots \\ \beta_{0,n-1} & \beta_{1,n-1} & \cdots & \beta_{m-1,n-1} \end{pmatrix} \\
&= \begin{pmatrix} \alpha_{0,0} & \alpha_{0,1} & \cdots & \alpha_{0,n-1} \\ \alpha_{1,0} & \alpha_{1,1} & \cdots & \alpha_{1,n-1} \\ \vdots & \vdots & & \vdots \\ \alpha_{m-1,0} & \alpha_{m-1,1} & \cdots & \alpha_{m-1,n-1} \end{pmatrix}^T + \begin{pmatrix} \beta_{0,0} & \beta_{0,1} & \cdots & \beta_{0,n-1} \\ \beta_{1,0} & \beta_{1,1} & \cdots & \beta_{1,n-1} \\ \vdots & \vdots & & \vdots \\ \beta_{m-1,0} & \beta_{m-1,1} & \cdots & \beta_{m-1,n-1} \end{pmatrix}^T \\
&= A^T + B^T
\end{aligned}
$$

Submit

Answers are displayed within the problem

Homework 3.3.2.8

1/1 point (graded)
Let $A, B \in \mathbb{R}^{n\times n}$ be symmetric matrices. $A + B$ is symmetric.

Always ✔ Answer: Always

Explanation

Answer: Always

Let $C = A + B$. We need to show that $\gamma_{j,i} = \gamma_{i,j}$.

$$
\begin{aligned}
& \gamma_{j,i} \\
= & \quad < \text{Definition of matrix addition} > \\
& \alpha_{j,i} + \beta_{j,i} \\
= & \quad < A \text{ and } B \text{ are symmetric} > \\
& \alpha_{i,j} + \beta_{i,j} \\
= & \quad < \text{Definition of matrix addition} > \\
& \gamma_{i,j}
\end{aligned}
$$

Calculator

Submit

Answers are displayed within the problem

Homework 3.3.2.9

1/1 point (graded)

Let $A, B \in \mathbb{R}^{n\times n}$ be symmetric matrices. $A - B$ is symmetric.

Always ✔ Answer: Always

Explanation

Answer: Always

Let $C = A - B$. We need to show that $\gamma_{j,i} = \gamma_{i,j}$.

$$\begin{aligned}
& \gamma_{j,i} \\
= & \quad < \text{Definition of matrix addition} > \\
& \alpha_{j,i} - \beta_{j,i} \\
= & \quad < A \text{ and } B \text{ are symmetric} > \\
& \alpha_{i,j} - \beta_{i,j} \\
= & \quad < \text{Definition of matrix addition} > \\
& \gamma_{i,j}
\end{aligned}$$

Submit

Answers are displayed within the problem

Homework 3.3.2.10

1/1 point (graded)

Let $A, B \in \mathbb{R}^{n\times n}$ be symmetric matrices and $\beta, \gamma \in \mathbb{R}$. $\beta A + \gamma B$ is symmetric.

Always ✔ Answer: Always

Explanation

Answer: Always

Let $C = \alpha A + \beta B$. We need to show that $\gamma_{j,i} = \gamma_{i,j}$. The proof is similar to many proofs we have seen.

Submit

Answers are displayed within the problem

Homework 3.3.2.11

6/6 points (graded)

If A and B are lower triangular matrices then $A + B$ is lower triangular.

TRUE ✔ Answer: TRUE

If A and B are strictly lower triangular matrices then $A + B$ is strictly lower triangular.

Calculator

TRUE ✔ Answer: TRUE

If A and B are unit lower triangular matrices then $A + B$ is unit lower triangular.

FALSE ✔ Answer: FALSE

If A and B are upper triangular matrices then $A + B$ is upper triangular.

TRUE ✔ Answer: TRUE

If A and B are strictly upper triangular matrices then $A + B$ is strictly upper triangular.

TRUE ✔ Answer: TRUE

If A and B are unit upper triangular matrices then $A + B$ is unit upper triangular.

FALSE ✔ Answer: FALSE

Submit

Answers are displayed within the problem

Homework 3.3.2.12

6/6 points (graded)

If A and B are lower triangular matrices then $A - B$ is lower triangular.

TRUE ✔ Answer: TRUE

If A and B are strictly lower triangular matrices then $A - B$ is strictly lower triangular.

TRUE ✔ Answer: TRUE

If A and B are unit lower triangular matrices then $A - B$ is unit lower triangular.

FALSE ✔ Answer: FALSE

If A and B are upper triangular matrices then $A - B$ is upper triangular.

TRUE ✔ Answer: TRUE

If A and B are strictly upper triangular matrices then $A - B$ is strictly upper triangular.

TRUE ✔ Answer: TRUE

If A and B are unit upper triangular matrices then $A - B$ is unit upper triangular.

FALSE ✔ Answer: FALSE

Submit

Answers are displayed within the problem

Calculator

< Previous Next >

edX

About
Affiliates
edX for Business
Open edX
Careers
News

Legal

Terms of Service & Honor Code
Privacy Policy
Accessibility Policy
Trademark Policy
Sitemap
Cookie Policy
Your Privacy Choices

Connect

Idea Hub
Contact Us
Help Center
Security
Media Kit

GET IT ON Google play
Download on the App Store

Calculator