6. Theoretical Application

Lecture due Oct 5, 2021 20:30 IST

Synthesize

Product Rule From Chain Rule

function of two variables

u and v. But u and v themselves are actually

going to be functions of t.

Then, well, dg dt is going to be partial g, partial u.

How much is that?

How much is partial g partial u?

1 over v times du dt plus--

well, next, we need to have partial g over partial v.

What's the derivative of this with respect to v?

Well, here, we need to know how to differentiate the inverse.

It's minus u over v squared times dv dt.

And that's actually the usual quotient rule just written

in a slightly different way.

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You may have noticed that when we did the example on the previous page the “old way" we had to use a product rule. When we did it the “new way" we somehow avoided using a product rule. This gives a hint that, in fact, this new chain rule can be used to prove the product rule.

Justify the product rule

Let's see how it is done. For functions u and v that depend on t, we want to prove:

$$\textbf{(Want to prove)} \quad \frac{d\,(uv)}{dt} = v\frac{du}{dt} + u\frac{dv}{dt} \tag{6.143}$$

We can give a name to the function described by the product uv, let's call it $f = uv$. The chain rule gives us a way of computing the derivative of f with respect to t via the dependence of f on u and v.

$$\textbf{(Differentiate with chain rule)} \quad \frac{df}{dt} = \frac{d\,(uv)}{dt} = f_u\frac{du}{dt} + f_v\frac{dv}{dt} \tag{6.144}$$

We compute f_u and f_v from $f = uv$. Substituting, we get:

$$\textbf{(Simplify)} \quad \frac{d\,(uv)}{dt} = v\frac{du}{dt} + u\frac{dv}{dt} \tag{6.145}$$

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This is indeed the familiar product rule from single-variable calculus.

Justify the quotient rule

A similar calculation can be done to justify the quotient rule. If we instead let $g = \frac{u}{v}$, then we can say

$$\frac{dg}{dt} = \frac{d\left(u/v\right)}{dt} = g_u \frac{du}{dt} + g_v \frac{dv}{dt} \tag{6.146}$$

We compute g_u and g_v from $g = \frac{u}{v}$. Therefore,

$$\frac{d\left(u/v\right)}{dt} = \frac{1}{v}\frac{du}{dt} + \frac{-u}{v^2}\frac{dv}{dt} = \frac{u'v - v'u}{v^2} \tag{6.147}$$

Indeed we obtain the familiar quotient rule.

Check your understanding

1/1 point (graded)
Through the following problems, you will derive the "triple product rule."

Suppose $f = abc$. What is df? Type `da` and `db` and `dc` for da and db and dc. Don't forget `*` for multiplication.

$df =$ b*c*da+c*a*db+a*b*dc ✔ **Answer:** b*c*da + a*c*db + a*b*dc

? INPUT HELP

Solution:

The partial derivatives of f are

$$f_a = bc \tag{6.148}$$
$$f_b = ac \tag{6.149}$$
$$f_c = ab \tag{6.150}$$

The answer is obtained by plugging these in to the total differential $df = f_a\,da + f_b\,db + f_c\,dc$.

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Answers are displayed within the problem

Triple Product Rule

1/1 point (graded)
Now suppose a, b, and c are all functions of t. Then $f = abc$ becomes a function of t. What is $\frac{df}{dt}$?

Type `a'`, `b'` and `c'` for the derivatives of a, b, and c with respect to t.

$\frac{df}{dt} =$ b*c*a'+c*a*b'+a*b*c' ✔ **Answer:** b*c*a' + a*c*b' + a*b*c'

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Solution:

Since $df = (bc)\ da + (ac)\ db + (ab)\ dc$, we divide by dt to obtain

$$\frac{df}{dt} = (bc)\frac{da}{dt} + (ac)\frac{db}{dt} + (ab)\frac{dc}{dt} \tag{6.151}$$

This is the "triple product rule."

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Triple Product Rule Practice

1/1 point (graded)

What is the derivative of $f(t) = (1+t^2)(\sin t)(e^t)$? Use the triple product rule you found above.

(1+t^2)*sin(t)*e^t + (1+t^2)*cos(t)*e^t + 2*t*sin(t)*e^t

Answer: (sin(t))*(e^t)*(2*t) + (1+t^2)*(e^t)*(cos(t)) + (1+t^2)*(sin(t))*(e^t)

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Solution:

The triple product rule says that for a triple product $f = abc$ the derivative of f is:

$$\frac{df}{dt} = (bc)\frac{da}{dt} + (ac)\frac{db}{dt} + (ab)\frac{dc}{dt} \tag{6.152}$$

In our case we have $a = 1+t^2$, $b = \sin t$ and $c = e^t$. Plugging in, we obtain:

$$f'(t) = (\sin t)(e^t)(2t) + (1+t^2)(e^t)(\cos t) + (1+t^2)(\sin t)(e^t) \tag{6.153}$$

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Answers are displayed within the problem

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? a prime?
i tried type a' as input, but it's not accepted.... neither do aprime, pls help... 2

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