Exercises: Best Linear Unbiased Estimation (1)

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BLUE of the travel time

1/1 point (ungraded)

Two cars drive for an equal period of time over a long and straight road, with different but constant velocities, $v_1 = 10$ m/s and $v_2 = 20$ m/s. After this time, the traveled distance is measured for each car. You want to know the time that the cars were driving.

The distance measurements are uncorrelated, with an equal standard deviation $\sigma = 2$ m.

If the cars are driving 10 m/s and 20 m/s, and the measurements are 548 m and 1098 m, respectively, compute the best estimate $\hat{t}$ of the travel time t.

Tip: first set up the system of observation equations and define the covariance matrix of the observations. Then work out the equations as discussed in the video and reader.

54.88

✔ **Answer:** 54.88

54.88

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✔ Correct (1/1 point)

BLUE of the travel time (cont'd)

3/3 points (ungraded)

In the previous question, does the value of σ have any effect on your estimate?

- [ ] Yes
- [x] No

✔

This time assume that the first measurement is more precise with $\sigma_1 = 1$m, and the precision of the second observation is the same as before ($\sigma_2 = 2$m).

What is the BLU estimate of the travel time?

54.85 ✔ **Answer:** 54.85

54.85

In this case, do the values of σ_1 and σ_2 have any effect on your estimate?

- [x] Yes
- [ ] No

✔

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✔ Correct (3/3 points)

Melting glacier

1/1 point (ungraded)

The height of a point on a melting glacier is observed in 6 consecutive years.

The first 2 observations have a precision of 1.2 m, the remaining observations have a precision of 0.3 m.

The 6 observations are given by:

$$y = [101.726,\ \ 99.300,\ \ 95.774,\ \ 94.411,\ \ 91.486,\ \ 89.969]^T$$

We will apply BLUE and ordinary (i.e., unweighted) least squares estimation using Matlab. But first try to answer the following question.

When applying BLUE, which of the following statements is true?

- (•) The residuals corresponding to the first 2 observations are likely to be larger. ✔
- ( ) BLUE will give identical results as WLSE with weights equal to the variances.
- ( ) Both statements are not true.

Explanation
The first 2 observations have poorer precision and will get a lower weight, and therefore it is likely that the associated residuals are larger.
BLUE and WLSE will be identical if the weights are set equal to the inverse variances.

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✔ Correct (1/1 point)

MATLAB EXERCISE: MELTING GLACIER (EXTERNAL RESOURCE)

the covariance matrix of the observations Q_{yy}, and completes the code to calculate the estimates.

You are also asked to calculate the residuals. Compare the residuals obtained with BLUE and LS, from which you should see that indeed the first 2 residuals are larger in case of BLUE due to the lower precision (and thus weight). This can also be seen in the figure that you will plot.

Your Solution

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```
%% times of observation [years]
t    = [0 1 2 3 4 5]';
%% number of observations
m  = length(t);
%% observed heights [m]
y    = [101.726   99.300   95.774   94.411   91.486   89.969]';

% Covariance matrix: Qyy
Qyy = [1.2^2, zeros(1,5);0,1.2^2, zeros(1,4);
       zeros(1,2),0.3^2, zeros(1,3);zeros(1,3),0.3^2, zeros(1,2);
       zeros(1,4),0.3^2, 0; zeros(1,5),0.3^2];

% Design matrix
A = [ones(m,1), t];

% What is the least squares estimate (xhat_LS) and
% the best linear unbiased estimate (xhat_BLU)
xhat_LS  = inv(A'*A)*(A'*y);
xhat_BLU = inv(A'*inv(Qyy)*A)*(A'*inv(Qyy)*y);

% Calculate the corresponding residuals
ehat_LS  = y - A*xhat_LS;
ehat_BLU = y - A*xhat_BLU;

% plot of the results (no editing required for this part)
ehat_LS
ehat_BLU
figure
```

```
29 plot(t, y, '.b','markersize',20)
30 hold on;grid on
31 plot(t, A*xhat_LS, 'r','linewidth',2)
32 plot(t, A*xhat_BLU, 'color',[0 0.6 0],'linewidth',2)
```