Do row operations change the column space of a matrix?

I know that

(i) row operations do not change the row space

(ii) column operations do not change the column space

and (iii) row rank = column rank (but this is sort of unrelated, I think).

But, is it true that row operations do not change *both* the row space and the column space of a matrix?

Thanks,

EDIT: I am guessing that it's most likely true, since in Guassian elimination, solving Ax=b involves *only* row operations -- there's something about column operations that makes the algorithm not work, I think (according to the book by Friedberg, Insel and Spence.)

(linear-algebra) (matrices) (systems-of-equations) (matrix-equations)

edited Jul 28 '15 at 1:44

asked Jul 28 '15 at 1:41
User001
1

5 Answers

Row operations in general do change the column space. Consider the following matrix

$$\begin{pmatrix} 1 & 0 & 2 \\ 2 & 1 & 1 \\ 1 & 1 & -1 \end{pmatrix}.$$

Row reducing, we get

$$\begin{pmatrix} 1 & 0 & 2 \\ 2 & 1 & 1 \\ 0 & 0 & 0 \end{pmatrix}.$$

The span of these columns is the set $\left\{ \begin{pmatrix} a \\ b \\ 0 \end{pmatrix} : a, b \in \mathbb{R} \right\}$ but this is not the span of the original columns.

answered Jul 28 '15 at 1:46
Cameron Williams
22k 4 34 77

Hi @CameronWilliams: take your 3x3 matrix, for example, and add a parameter to it, to a few entries, say, adding +α to entries a_{11}, a_{13} and a_{32}. Call this matrix A. I want to find for which values of α will Ax=b have a unique solution. This is easy; I row-reduce to eliminate as many of the entries with a α term attached to it, but I only use the operation that is adding a multiple of one row to another row, which doesn't change the determinant. – User001 Jul 28 '15 at 2:35

When my matrix is simplified enough (not necessarily in row-echelon form), I then use the definition of determinant, and it is easy to see which values of α make the determinant = zero. So not choosing these values makes the determinant nonzero, and I get that the linear system always has a unique solution for suitably chosen α. But the reason for my posting this question is: can I use this much simpler matrix to now find for which parameters β, which is attached to the *image* vector, b, will the system have infinitely many solutions or no solutions. – User001 Jul 28 '15 at 2:36

I want to use the simplified matrix, use Guassian elimination, row-reduce the augmented matrix, and when I get a zero row (which I will surely get, since det (A) = 0), then I can see which values of β I must pick to keep the linear system *consistent*, which will then give infinitely many solutions to Ax=b. – User001 Jul 28 '15 at 2:36

I used this simplified matrix, and it seems to work, e.g., I am finding parameters β such that the system is consistent and gives infinitely many solutions -- meaning, the values I got match the solutions, which aren't always correct. Is this just a coincidence, and that I should *not* be using the simplified matrix from case I as my starting point for cases II and III (infinitely many solutions or no solutions)? Thanks, – User001 Jul 28 '15 at 2:36

1 @LebronJames You're an amazing basketball player so I don't want to offend your sentiments but you should make a new post for this question since it's unrelated to this one seemingly. – Cameron Williams Jul 28 '15 at

make a new post for this question since it's unrelated to this one seemingly. – Cameron Williams Jul 28 '15 at 2:42

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The answer is no. I'll give an example that can be generalized to any matrix size and any field.

Consider $A = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$. Applying row operation to annihilate the second row, we get $B = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$. The column space of A is the subspace of all multiples of $[1\ 1]^T$. The column space of B is the subspace of all multiples of $[1\ 0]^T$. Clearly, we can generalize this to any size by appending columns and rows of zeros to A and B.

edited Jul 29 '15 at 7:47 answered Jul 28 '15 at 4:24
chhro
861 2 8

Hi @chhro, thanks so much for this simple and helpful example :-) – User001 Jul 28 '15 at 6:55

Note: Row operations do leave column dependencies unchanged. That is, if some linear combination of the columns of the original matrix is the zero-vector, then so is the same linear combinations of changed columns under row operations. However, as noted in other answers, the column space does not (in general) stay the same.

answered Jul 28 '15 at 1:49
paw88789
27.4k 1 20 44

No it is not true.

Row operations leaves the row space and null space unchanged, but can change the column space.

That is, row operations do not affect the linear dependence relations among the columns, but can change the linear dependence relations among the rows.

edited Jul 28 '15 at 1:56 answered Jul 28 '15 at 1:46
Lucas
1,190 1 6 24

Suppose that $C_1, \ldots, C_n$ are the columns of a matrix. If an ERO is applied through a matrix E, then the new columns are $EC_1, \ldots, EC_n$. If $C_{j1}, \ldots, C_{jr}$ are linearly independent then so are $EC_{j1}, \ldots, EC_{jr}$ and vice versa due to invertibility of E.

edited Feb 16 '17 at 18:40 answered Feb 16 '17 at 18:26
John Vaibhaw Kumar
923 4 14 1