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10. Summary

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Summarize

Big Picture

The partial derivatives of a function together form a vector quantity. This vector quantity gives us information about the shape and slope of the graph of the function.

Mechanics

Gradients

Definition 10.1

The vector $\langle f_x, f_y \rangle$ is called the **gradient** of f.

The abbreviation for the gradient of f is ∇f.

Vector fields

Definition 10.2

A **vector field** on the plane is a function that attaches a vector to each point (x, y) in the plane.

Equivalent definitions:

A vector field is a function $\mathbf{F}$ that maps points in the plane to vectors:

$$\mathbf{F}(x, y) = \langle F_1(x, y), F_2(x, y) \rangle.$$

A vector field is sometimes called a vector-valued function.

The magnitude and direction of the gradient

Theorem

1. At any point (x_0, y_0), the vector $\langle f_x(x_0, y_0), f_y(x_0, y_0) \rangle$ is perpendicular to the level curve of f through (x_0, y_0).
2. ∇f points in the direction of steepest increase.
3. $|\nabla f|$ is the slope of that increase.

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Ask Yourself

Why is the gradient useful?

Computing the gradient is relatively straight forward, and it gives us a lot of information about a potentially complicated function. It tells us in which direction the function is increasing most quickly and the slope of the tangent plane in that direction.

We can also use the gradient to write the approximate change in the function value near any point as a hidden dot product:

$$\Delta f \approx \nabla f \cdot \langle \Delta x, \Delta y \rangle.$$

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What is the direction of steepest decrease?

The gradient points in the direction of steepest increase, so the negative gradient points in the direction of steepest decrease!

The gradient is the key in machine learning algorithms, which make guesses and refine their models based on the direction that minimizes the error function of the model fastest.

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? Negative gradient
This is from the last question. If the gradient is a vector, what does it mean "negative gradient"? 3

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