



[Unit 4 Unsupervised Learning \(2 weeks\)](#)

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## 5. Maximum Likelihood Estimate

### Maximum Likelihood Estimate

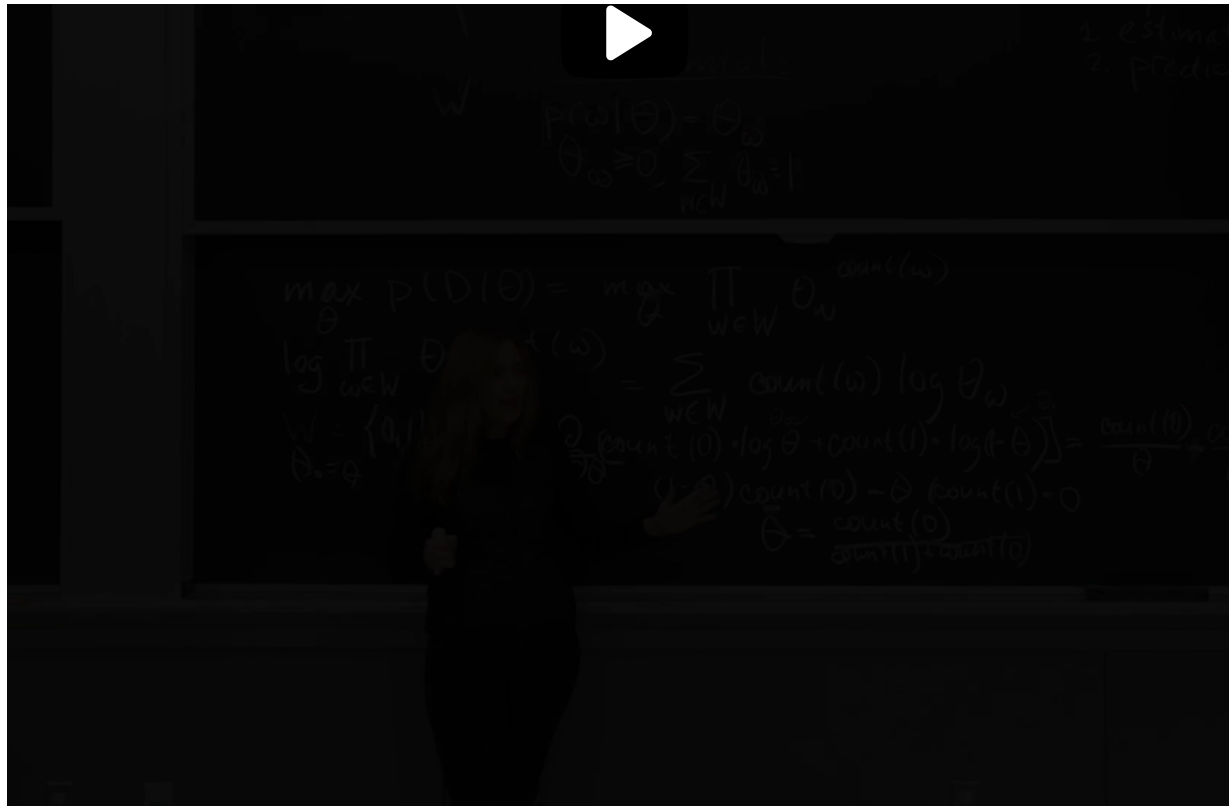
Exactly what this formula says.

But what we've demonstrated here is that, by properly defining our maximum likelihood criteria

and making the estimation, we're actually getting the parameters that we are expected to get.

And in some cases, for some model, you can kind of intuitively say what they should look like.

**In other cases not, but this mechanism always works.**



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Number of parameters

1/1 point (graded)

For the following set of questions, let us consider generating documents that are English letter sequences (assume no spaces or punctuation), i.e. the vocabulary  $W = \{a, b, c, \dots, z\}$  is made up of all the letters in the English alphabet.

We would like to generate documents using this vocabulary using a multinomial model  $M$ . As described in the lecture, what is the minimal number of parameters that the model  $M$  should have? Enter your answer below.

✓ Answer: 25

### Solution:

Recall from the lecture that for multinomial generative models we have a parameter  $\theta_w$  for each word  $w \in W$ . However, since the parameters should sum up to one, we can express one of the parameters as 1 minus the sum of all others. Since the vocabulary size for this example is 26, our model  $M$  can have 25 parameters to express the probability of each letter.

You have used 1 of 2 attempts

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**i** Answers are displayed within the problem

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## Maximum Likelihood estimate

1/1 point (graded)

Let  $\theta^* = \theta_a^*, \theta_b^*, \dots, \theta_z^*$  be the parameters of the multinomial model  $M^*$  that maximize the likelihood of generating a document  $D$ .

Further, it is known that the letter 'e' is twice as more likely to occur than the letter 'z' in document  $D$ .

Which of the following options is a correct expression relating  $\theta_e^*$  and  $\theta_z^*$ ?

☐  $\theta_z^* = 2 * \theta_e^*$

☒  $\theta_e^* = 2 * \theta_z^*$  ✓

☐  $\theta_z^* = \theta_e^*$

☐  $\theta_z^* + \theta_e^* = 2$

**Solution:**

Recall from the lecture that for any  $w \in W$  we have,

$$\theta_w^* = \frac{\text{count}(w)}{\sum_{w' \in W} \text{count}(w')}$$

Since  $\text{count}(e) = 2 * \text{count}(z)$ , we can conclude that  $\theta_e^* = 2 * \theta_z^*$

Submit

You have used 1 of 1 attempt

**i** Answers are displayed within the problem

## Maximum Likelihood Estimate for Poisson Distribution

2/2 points (graded)

Maximum Likelihood Estimate (MLE) is a very general method that can be applied to both continuous and discrete distributions. In this problem, we assume we have a training data  $x_1, x_2, \dots, x_n$  that are drawn from a Poisson distribution:

$$P(X = x) = \frac{\lambda^x e^{-\lambda}}{x!}$$

and we want to use MLE to fit the parameter  $\lambda$  with the training data. We can do so by first computing the log likelihood of our training data, which is:

☐  $\log \lambda \sum_i x_i + n\lambda + \sum_i \log(x_i!)$

☒  $\log \lambda \sum_i x_i - n\lambda - \sum_i \log(x_i!) \checkmark$

☐  $\log \lambda \prod_i x_i - n\lambda - \prod_i \log(x_i!)$

☐  $\log \lambda \prod_i x_i + n\lambda + \prod_i \log(x_i!)$

In the next step, we maximize this log likelihood function by taking the derivative. What is the resulting estimator for  $\lambda$ ?

☒  $\frac{1}{n} \sum_i x_i$  ✓

☐  $\frac{1}{n} \prod_i x_i$

☐  $\sum_i x_i$

☐  $\prod_i x_i$

Is it in accordance with the definition of  $\lambda$  in Poisson distribution?

**Solution:**

The loglikelihood of the data is:

$$\begin{aligned} \log \prod_i P(X = x_i) &= \log \prod_i \frac{\lambda^{x_i} e^{-\lambda}}{x_i!} \\ &= \sum_i \log(\lambda^{x_i}) + \log(e^{-\lambda}) - \log(x_i!) \\ &= \log \lambda \sum_i x_i - n\lambda - \sum_i \log(x_i!) \end{aligned}$$

Take the derivative to  $\lambda$ , we have

$$\frac{1}{\lambda} \sum_i x_i - n = 0$$
$$\lambda = \frac{1}{n} \sum_i x_i$$

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You have used 1 of 2 attempts

 Answers are displayed within the problem

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