Homework Problem: Alice's Coins

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10.0 points possible (graded)

Alice has five coins in a bag: two coins are normal, two are double-headed, and the last one is double-tailed. She reaches into the bag and randomly pulls out a coin. Without looking at the coin she drew, she tosses it.

- **(a)** What is the probability that once the coin lands, the side of the coin that is face-down is heads? (Please be precise with at least 3 decimal places, unless of course the answer doesn't need that many decimal places. You could also put a fraction.)

Answer: 3/5

- **(b)** The coin lands and shows heads face-up. What is the probability that the face-down side is heads? (Please be precise with at least 3 decimal places, unless of course the answer doesn't need that many decimal places. You could also put a fraction.)

Answer: 2/3

Alice discards the first coin **(the one from part (b) that landed and showed heads face-up)**, reaches again into the bag and draws out a random coin. Again, without looking at it, she tosses it.

- **(c)** What is the probability that the coin shows heads face-up? (Please be precise with at least 3 decimal places, unless of course the answer doesn't need that many decimal places. You could also put a fraction.)

Answer: 13/24

Solution:

- **(a)** What is the probability that once the coin lands, the side of the coin that is face-down is heads?

  Solution: The parts of this problem become much easier to answer when we can condition on which type of coin she drew. Therefore, we will rely on total probability. Using total probability requires us to determine the conditional probability of the event (the face-down side of the coin is heads) and the prior probability of pulling out each kind of coin.

  Let C_F, C_H, and C_T be the events that she pulled a fair coin, double-headed, and double-tailed coin, respectively. These events are collectively exhaustive. Let D_H be the event that the face-down side is heads. We apply total probability:

$$\begin{aligned}\mathbb{P}(D_H) &= \mathbb{P}(D_H|C_F)\mathbb{P}(C_F) + \mathbb{P}(D_H|C_H)\mathbb{P}(C_H) + \mathbb{P}(D_H|C_T)\mathbb{P}(C_T) \\ &= \frac{1}{2}\cdot\frac{2}{5} + 1\cdot\frac{2}{5} + 0\cdot\frac{1}{5} \\ &= \boxed{\frac{3}{5}}.\end{aligned}$$

- **(b)** The coin lands and shows heads face-up. What is the probability that the face-down side is heads?

Solution: Let U_H be the event that the face-up side is heads. Then we are interested in $\mathbb{P}(D_H|U_H)$. Using the definition of conditional probability, we can write $\mathbb{P}(D_H|U_H) = \mathbb{P}(D_H \cap U_H)/\mathbb{P}(U_H)$. By symmetry of the coins, the marginal (unconditional) probabilities $\mathbb{P}(U_H)$ and $\mathbb{P}(D_H)$ are equal. We can find $\mathbb{P}(D_H \cap U_H)$ using total probability:

$$\begin{aligned}\mathbb{P}(U_H \cap D_H) &= \mathbb{P}(U_H \cap D_H|C_F)\mathbb{P}(C_F) + \mathbb{P}(U_H \cap D_H|C_H)\mathbb{P}(C_H) + \\ &\quad \mathbb{P}(U_H \cap D_H|C_T)\mathbb{P}(C_T) \\ &= 0 \cdot \frac{2}{5} + 1 \cdot \frac{2}{5} + 0 \cdot \frac{1}{5} \\ &= \frac{2}{5}.\end{aligned}$$

Our final answer is obtained by $\mathbb{P}(D_H \cap U_H)/\mathbb{P}(U_H) = (2/5)/(3/5) = \boxed{\frac{2}{3}}$.

Alice discards the first coin, reaches again into the bag and draws out a random coin. Again, without looking at it, she tosses it.

- **(c)** What is the probability that the coin shows heads face-up?

  Solution: Let $U_{H,2}$ be the event that the face-up side of the second coin is heads. Since reasoning about the second coin would be easier if we knew what the first coin was, we will again use total probability:

$$\begin{aligned}\mathbb{P}(U_{H,2}|U_H) &= \mathbb{P}(U_{H,2}|U_H, C_F)\mathbb{P}(C_F|U_H) + \mathbb{P}(U_{H,2}|U_H, C_H)\mathbb{P}(C_H|U_H) + \\ &\quad \mathbb{P}(U_{H,2}|U_H, C_T)\mathbb{P}(C_T|U_H)\end{aligned}$$

  Since $\mathbb{P}(C_T|U_H) = 0$, the last term is 0. Here, U_H and $U_{H,2}$ are *conditionally independent* given C_F: if we know that the first coin was fair, then its outcome won't affect the outcome of the second, and similarly for C_H. Computing $\mathbb{P}(U_{H,2}|C_F)$ proceeds like the calculation we did in part

(a), but with one fewer fair coin in the bag. Similarly, $\mathbb{P}(U_{H,2}|C_H)$ is similar to part (a) with one fewer double-headed coin in the bag:

$$\begin{aligned}\mathbb{P}(U_{H,2}|C_F) &= \frac{1}{2}\cdot\frac{1}{4}+1\cdot\frac{2}{4}+0\cdot\frac{1}{4}\\ &= \frac{5}{8}\\ \mathbb{P}(U_{H,2}|C_H) &= \frac{1}{2}\cdot\frac{2}{4}+1\cdot\frac{1}{4}+0\cdot\frac{1}{4}\\ &= \frac{1}{2}\end{aligned}$$

We can compute the probabilities $\mathbb{P}(C_F|U_H)$ and $\mathbb{P}(C_H|U_H)$ using Bayes' rule as $1/3$ and $2/3$ respectively:

$$\begin{aligned}\mathbb{P}(C_F|U_H) &= \frac{\mathbb{P}(U_H|C_F)\mathbb{P}(C_F)}{\mathbb{P}(U_H)}\\ &= \frac{(1/2)(2/5)}{(1/2)(2/5)+1(2/5)}\\ &= \frac{1}{3},\\ \mathbb{P}(C_H|U_H) &= \frac{\mathbb{P}(U_H|C_H)\mathbb{P}(C_H)}{\mathbb{P}(U_H)}\\ &= \frac{1(2/5)}{(1/2)(2/5)+1(2/5)}\\ &= \frac{2}{3}.\end{aligned}$$

Putting the pieces together,

$$\begin{aligned}\mathbb{P}(U_{H,2}|U_H) &= \mathbb{P}(U_{H,2}|U_H, C_F)\mathbb{P}(C_F|U_H) + \mathbb{P}(U_{H,2}|U_H, C_H)\mathbb{P}(C_H|U_H) \\ &= \frac{5}{8}\cdot\frac{1}{3} + \frac{1}{2}\cdot\frac{2}{3} \\ &= \boxed{\frac{13}{24}}\end{aligned}$$

This is lower than our answer from part (a): if we've removed a coin that produces heads, our chance of obtaining heads from the bag decreases.

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Discussion

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Part B

discussion posted 13 days ago by **Mickeygabz**

I disagree with answer in Part B. The answer should be 2/5

Probability that the both upper head and lower head = probability that the coin tossed...

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Understanding the given solution

question posted 2 months ago by **JanakRajChadha**

$$\begin{eqnarray} \mathbb{P} (U_{H,2} | U_H) &=& \mathbb{P}(U_{H,2} | U_H, C_F) \mathbb{P}(C_F | U_H) + \mathbb{P}(U_{H,2}...$$

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How to do this computationally?

discussion posted 2 months ago by **DG84**

I solved this with just pen and paper. Can someone point me to a python implementation after the deadline? / could the instructors do so?

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Event vs Outcome vs Random Variable

discussion posted 2 months ago by **lokeshhh**

I am having some confusion with the notation. I wanted to write down the given quantities in this problem. So I did the following:

> X: coin...

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simultaneity not the same as intersection ?

discussion posted 2 months ago by **dnogues**

why when we discuss the simultaneity of a condition and an outcome we annotate them as A,B and not A^B ?

This is in relation to question (c)...

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why isn't C=9/16? OH... oops time's up

question posted 2 months ago by **hedges333**

The initial set is {HT, HT, HH, HH, TT}. 6 out of 10 are heads, so the answer to (a) is 6/10.

The first coin toss shows face up heads. After...

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How i solved part C correctly

discussion posted 2 months ago by **abomostafa**

There will be four scenarios in the first draw so that the coin could show head face-up in the second draw:-

- Scenario A: the first draw was...

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For those having trouble with Part C

discussion posted 2 months ago by **Robox**

I was looking at the hints and I felt that I wanted to provide a different perspective.

So here's the question:

> Alice discards the first...

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Solution discussion

discussion posted 2 months ago by **JanakRajChadha**

\begin{eqnarray} \mathbb{P} (U_{H,2} | U_H) &=& \mathbb{P}(U_{H,2} | U_H, C_F) \mathbb{P}(C_F | U_H) + \mathbb{P}(U_{H,2}...

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Part c approach

discussion posted 2 months ago by **JanakRajChadha**

Although I've used up my submissions for part c, I think I have finally solved it correctly. Just wanted to share my approach with the online...

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A hint for (c)

discussion posted 2 months ago by **Bio_Geek**

After some struggle I found an easy way to solve it:

Consider that the first split in a decision tree is whether 'HH' or 'HT' coin was discarded...

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Part B in Alice's coins

question posted 2 months ago by **ashikyan**

Is this correct way of thinking about part B?

Alice tosses the first coin with probability of [same answer as A] of it landing with heads face...

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avoid distracting details

discussion posted 2 months ago by **SamS2**

Part (c) took me longer to solve than I care to admit. I ran it as a simulation first to have a number to check against. In the end the solution...

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Coalescing events

question posted 2 months ago by **Jean-Victor**

I saw the problem as one of selecting heads or tails from a bucket, coalescing the events of picking a coin and tossing it into one event. Initially,...

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Request for solutions

discussion posted 2 months ago by **bicepjai**

Please provide solution for previous home work after dead line. Going nuts not knowing the solution.

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B seems so simple. yet i got it wrong

discussion posted 2 months ago by **Traumfabrik**

When i see heads up, it cannot be the tail only coin. It can be only 1 of 4 coins, and 2 of them have a heads face down as well. so my answer...

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Solution put online after due date?

discussion posted 2 months ago by **GeorgeLogos**

Are you going to put a example solution online after the due date?

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Part (b) subtlety?

discussion posted 2 months ago by **vladislavsd**

I am re-reading the problem statement again and again but this escapes me. Here is how I see it:

A random coin is picked (1/5) and is flipped:...

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Part C : What am I doing wrong?

discussion posted 2 months ago by **ripande**

I am not getting correct answer for part C. Would appreciate if anyone can point out where I am going wrong.

1. I am assuming that the coin...

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Part C - fault in reasoning or just wrong?

discussion posted 2 months ago by **AngelosToytziaridis**

I'm confused about part C. Either there is a small fault in my reasoning or I'm just wrong... I reason as follows. First of all I define the...

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