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Lecture 9: Introduction to Maximum

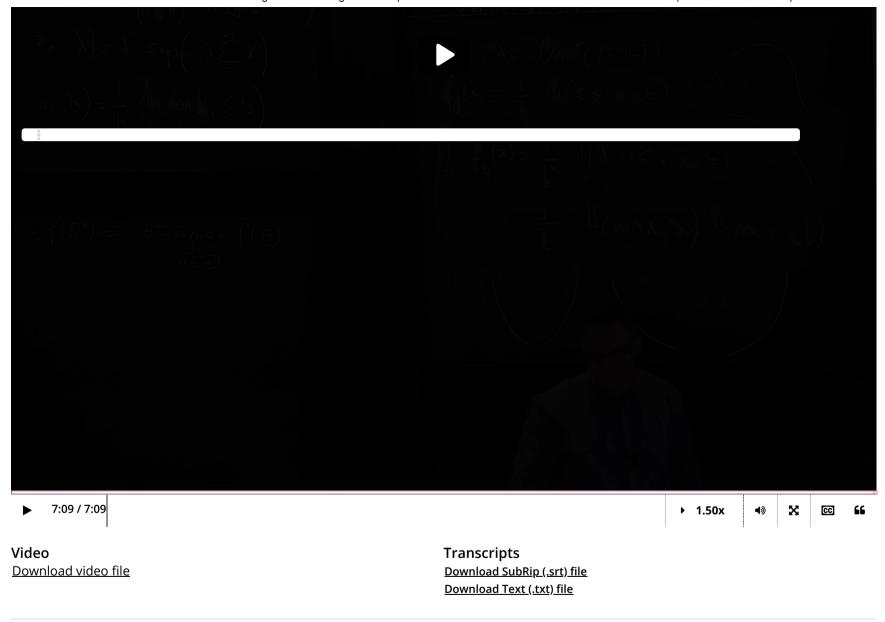
6. Interlude: Minimizing and

<u>Course</u> > <u>Unit 3 Methods of Estimation</u> > <u>Likelihood Estimation</u>

> Maximizing Functions

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6. Interlude: Minimizing and Maximizing Functions Concavity in 1 dimension

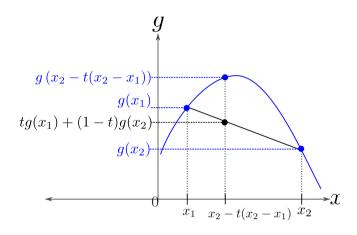


A function $g:I o\mathbb{R}$ is **concave** (or concave down), where I is an interval, if for all pairs of real numbers $x_1< x_2\in I$

$$g\left(tx_{1}+\left(1-t
ight)x_{2}
ight) \geq tg\left(x_{1}
ight)+\left(1-t
ight)g\left(x_{2}
ight) \qquad ext{ for all } 0 < t < 1.$$

Geometrically, this means that for $x_1 < x < x_2$, the graph of g is **above** the secant line connecting the two points $(x_1,g(x_1))$ and $(x_2,g(x_2))$

.



At $x=x_2-t$ $(x_2-x_1)=tx_1+(1-t)$ x_2 , the y-value of the graph of g is $g(x)=g(tx_1+(1-t)x_2)$, while the y-value of the secant line is $tg(x_1)+(1-t)$ $g(x_2)$.

If the inequality is strict, i.e. if

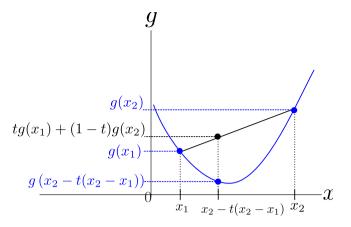
$$g(tx_1 + (1-t)x_2) > tg(x_1) + (1-t)g(x_2)$$
 for all $0 < t < 1$.

then g is **strictly concave** .

The definition for **(strictly) convex** is analogous. A function $g:I\to\mathbb{R}$ is **convex** (or concave up), where I is an interval, if for all pairs of real numbers $x_1< x_2\in I$

$$g\left(tx_1 + \left(1 - t\right)x_2\right) \le tg\left(x_1\right) + \left(1 - t\right)g\left(x_2\right)$$
 for all $0 < t < 1$.

Geometrically, this means that for $x_1 < x < x_2$, the graph of g is **below** the secant line connecting the two points $(x_1,g(x_1))$ and $(x_2,g(x_2))$



At $x=x_2-t$ $(x_2-x_1)=tx_1+(1-t)$ x_2 , the y-value of the graph of g is $g(x)=g(tx_1+(1-t)x_2)$, while the y-value of the secant line is $tg(x_1)+(1-t)$ $g(x_2)$.

If the inequality is strict, i.e. if

$$g\left(tx_1 + \left(1 - t\right)x_2\right) < tg\left(x_1\right) + \left(1 - t\right)g\left(x_2\right) \qquad ext{ for all } \ 0 < t < 1.$$

then g is **strictly convex** .

If in addition g is twice differentiable in the interval I, i.e. $g''\left(x\right)$ exists for all $x\in I$, then g is

- **concave** if and only if $g''(x) \leq 0$ for all $x \in I$;
- **strictly concave** if g''(x) < 0 for all $x \in I$;
- **convex** if and only if $q''(x) \ge 0$ for all $x \in I$;
- **strictly convex** if g''(x) > 0 for all $x \in I$;

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