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6. MLEs for general multinomial distribution

Maximum Likelihood Estimate

So pretty much, you can do exactly the same formula

by [INAUDIBLE] many of your documents

into a single document and then repeating the same story.

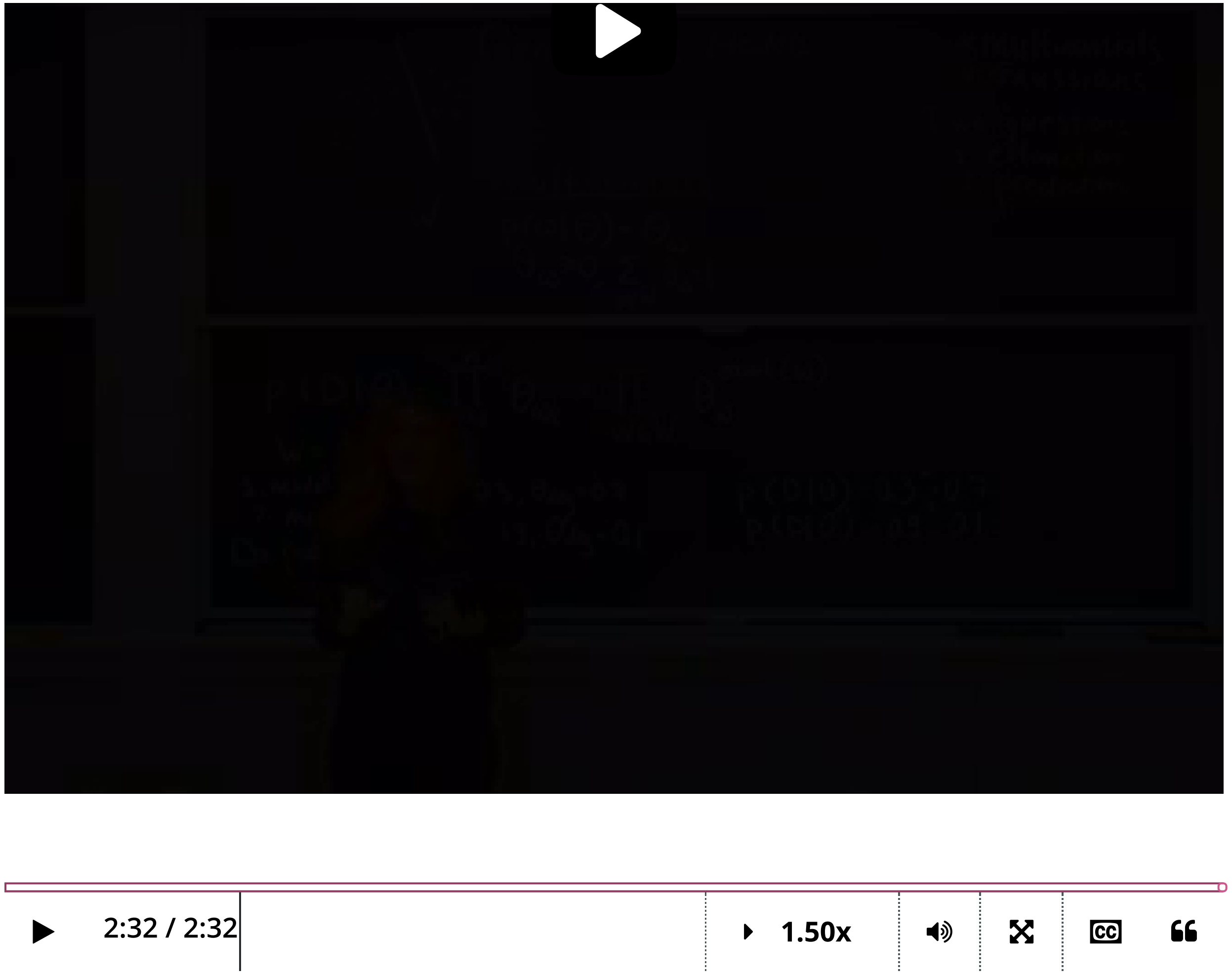
So now we're done with the discussion

of estimation for multinomial.

And with that, we are ready to start

talking about how can we use these multinomials to actually

do prediction.



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Deriving MLE estimates for a general multinomial distribution

1/1 point (graded)

In this problem, we will derive the maximum likelihood estimates for a multinomial distribution with more than 2 parameters. For this we employ a powerful optimization strategy called method of langrange multipliers.

First let $P(D|\theta)$ denote the probability of a multinomial model M with parameters $\theta = \{\theta_1, \theta_2 \ldots \theta_N\}$ generating a document D.

Which of the following option lists the correct expression for $P(D|\theta)$. Choose from options below.

- ○ $P(D|\theta) = \sum_{w \in W} \theta_w^{count(w)}$
- ● $P(D|\theta) = \Pi_{w \in W} \theta_w^{count(w)}$ ✓
- ○ $P(D|\theta) = \Pi_{w \in W} count(w)_w^{\theta}$
- ○ $P(D|\theta) = \Pi_{w \in W} \theta_w + count(w)$

Solution:

Recall from the lecture that

$$P(D|\theta) = \Pi_{i=1}^{n} \theta_{w_i} = \Pi_{w \in W} \theta_w^{count(w)}$$

Submit You have used 1 of 2 attempts

Answers are displayed within the problem

Constraints

1/1 point (graded)

Which of the following options lists the right set of constraints on the parameters θ_w of the multinomial model.

● $\theta_w \geq 0, \sum_{w \in W} \theta_w = 1$ ✓

○ $\theta_w \geq 0, \sum_{w \in W} \theta_w < 1$

○ $\theta_w < 0, \sum_{w \in W} \theta_w > -1$

○ $\theta_w \geq 0, \sum_{w \in W} \theta_w \geq 1$

Solution:

Note that θ_w denotes the probability of model M choosing the word w. Since it's a probability, its value must lie between 0 and 1. Therefore, $0 \leq \theta_w \leq 1$.

Further, all the above probability values must also sum up to 1. That is, $\sum_{w \in W} \theta_w = 1$.

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Stationary points for lagrange function

2/2 points (graded)

Let's recall the function that we're trying to optimize:

$$max_\theta P\left(D|\theta\right) = \Pi_{w \in W} \theta_w^{count(w)}$$

Let us take natural logarithm on both sides of the equation in order to bring down the exponent

$$max_\theta logP\left(D|\theta\right) = \sum_{w \in W} count\left(w\right) log\theta_w$$

subject to the following constraints,

$$\sum w \in W\theta_w = 1$$

Let's define a function L called the lagrange function for the sake of the above defined constrained optimization problem:

$$L = logP\left(D|\theta\right) + \lambda \sum_{w\in W}\left(\theta_w - 1\right)$$

where λ is a constant.

Consider finding stationary points for L by solving for equation obtained by setting its derivative to zero. That is,

$$\frac{\partial L}{\partial \theta_w}\left(logP\left(D|\theta\right) + \sum_{w\in W}\lambda\left(\theta_w - 1\right)\right) = 0$$

Solve for θ_w from the above equation. Choose the right answer for θ_w from options below.

○ $\theta_w = \frac{-\lambda}{count(w)}$

○ $\theta_w = \lambda \times count\left(w\right)$

○ $\theta_w = -\lambda \times count\left(w\right)$

● $\theta_w = \frac{-count(w)}{\lambda}$ ✓

Now, apply the constraint that $\sum_{w\in W}\theta_w = 1$, we get that λ is:

● $\lambda = -\sum_{w\in W} count\,(w)$ ✓

○ $\lambda = \sum_{w\in W} count\,(w)$

○ $\lambda = -\sum_{w\in W} count\,(w) \times \theta_w$

○ $\lambda = \sum_{w\in W} count\,(w) \times \theta_w$

Solution:

$$\frac{\partial L}{\partial \theta_w}(logP\,(D|\theta) + \sum_{w\in W} \lambda\,(\theta_w - 1)) = 0$$

$$= \frac{\partial logP\,(D|\theta_w)}{\partial \theta_w} + \lambda = 0$$

$$= \frac{\partial log\Pi_{w\in W}\theta_w^{count(w)}}{\partial \theta_w} + \lambda = 0$$

$$= \frac{\partial \sum_{w \in W} log\theta_w \times count\left(w\right)}{\partial \theta_w} + \lambda = 0$$

$$= \frac{count\left(w\right)}{\theta_w} + \lambda = 0$$

$$\theta_w = -\frac{count\left(w\right)}{\lambda}$$

If we apply the constraint that $\sum_{w \in W} \theta_w = 1$ we get

$$\sum_{w \in W} \theta_w = 1$$

$$\sum_{w \in W} -\frac{count\left(w\right)}{\lambda} = 1$$

$$\sum_{w \in W} count\left(w\right) = -\lambda$$

$$\lambda = -\sum_{w \in W} count\left(w\right)$$

Substituting this expression for λ back into our previous expression for θ_w we get

$$\theta_w = -\frac{count\left(w\right)}{\lambda}$$

$$\theta_w = \frac{count\left(w\right)}{\sum_{w \in W} count\left(w\right)}$$

Note that $\theta_w > 0$ and $\sum_{w \in W} \theta_w = 1$ satisfying all the constraints. These set of θ_w parameters are the maximum likelihood estimates for this multinomial generative distribution.

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ℹ Answers are displayed within the problem

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