2. Bounce

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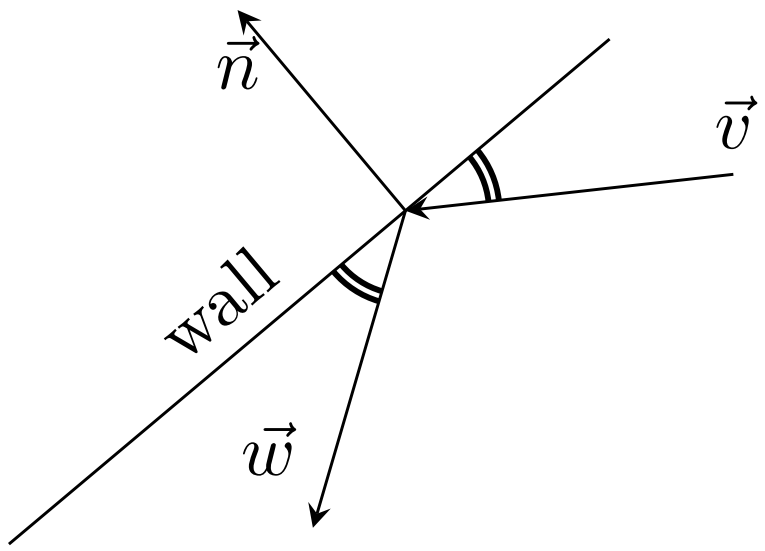
Problem Set B due Sep 15, 2021 20:30 IST

Practice

Setup

In Recitation 5, you practiced finding an outgoing velocity vector of a particle that bounces off a wall. In this problem, you will generalize this calculation and express the answer using matrices.

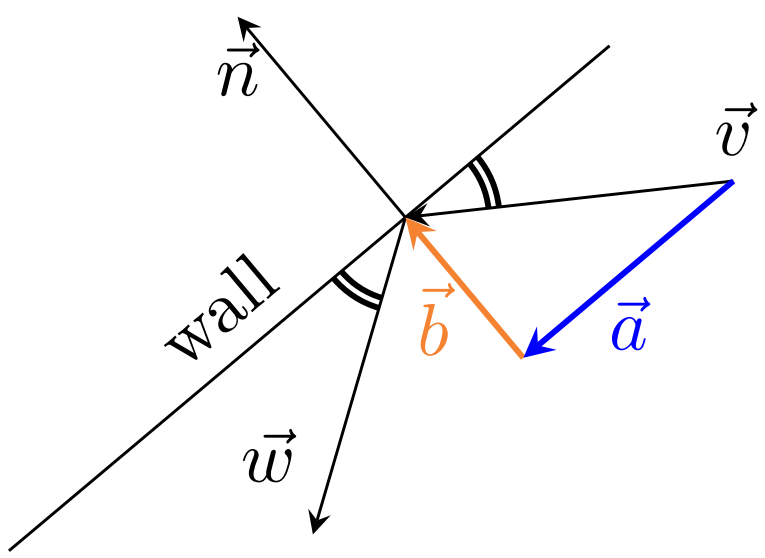
The following image depicts the setup:



In the picture, $\vec{v}$ is the velocity of the projectile before it hits the wall, and $\vec{w}$ is the velocity of the projectile after it hits the wall. The vector $\vec{n}$ is a normal vector to the wall. Assume $|\vec{n}| = 1$.

Physics tells us that the angle of incidence equals the angle of reflection – the two marked angles in the picture are equal. It also tells us that if there is no friction, then $|\vec{v}| = |\vec{w}|$. Given $\vec{v}$ and $\vec{n}$, our goal is to find $\vec{w}$.

We will break this big problem into several steps. We begin by breaking $\vec{v}$ into a piece parallel to $\vec{n}$ and a piece perpendicular to $\vec{n}$, as in the following picture.



Find w from a and b

1/1 point (graded)

Find a formula for $\vec{w}$ in terms of $\vec{a}$ and $\vec{b}$. Write `veca` and `vecb` for $\vec{a}$ and $\vec{b}$.

$\vec{w} =$ veca - vecb ✔ **Answer:** veca - vecb

Solution:

Bouncing off the wall reverses $\vec{b}$ and leaves $\vec{a}$ the same. Therefore, $\vec{w} = \vec{a} - \vec{b}$.

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Find length of b from v and n

1/1 point (graded)

What is the length of the vector $\vec{b}$? Enter a formula in terms of $\vec{v}$ and $\vec{n}$.

Write `vecv` for $\vec{v}$ and `vecn` for $\vec{n}$. You may write a dot product such as $\vec{u} \cdot \vec{w}$ using `vecu * vecw`.

$|\vec{b}| =$ vecv*vecn ✔ **Answer:** vecv*vecn

Solution:

By trigonometry we have $|\vec{b}| = |v| \cos\theta$, where θ is the angle between $\vec{v}$ and $\vec{b}$. Since $\vec{b}$ is parallel to $\vec{n}$, this θ is also the angle between $\vec{v}$ and $\vec{n}$. Since $|\vec{n}| = 1$, we have $\cos\theta = \frac{v \cdot n}{|\vec{v}|}$. Substituting this $\cos\theta$ in to the above formula for $|\vec{b}|$ we obtain $|\vec{b}| = \vec{v} \cdot \vec{n}$.

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Find b from v and n

1/1 point (graded)

Find a formula for $\vec{b}$ in terms of $\vec{v}$ and $\vec{n}$.

Write `vecv` for $\vec{v}$ and `vecn` for $\vec{n}$. You may also write a dot product between two vectors using `*`. To write a scaled vector such as $2\vec{v}$, enter `2*vecv`.

$\vec{b} =$ (vecv*vecn)*vecn ✔ **Answer:** (vecv*vecn)*vecn

Solution:

Since $\vec{b}$ is parallel to $\vec{n}$, there exists k such that $\vec{b} = k\vec{n}$. Since $|\vec{n}| = 1$, we have $k = |\vec{b}|$, which is $\vec{v} \cdot \vec{n}$. Thus $\vec{b} = (\vec{v} \cdot \vec{n}) \cdot \vec{n}$.

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Find a from v and n

1/1 point (graded)

Find a formula for $\vec{a}$ from $\vec{v}$ and $\vec{n}$.

Write `vecv` for $\vec{v}$ and `vecn` for $\vec{n}$. You may also write a dot product between two vectors using `*`. To write a scaled vector such as $2\vec{v}$, enter `2*vecv`.

$\vec{a} =$ vecv - (vecv*vecn)*vecn ✔ **Answer:** vecv-(vecv*vecn)*vecn

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Solution:

Since $\vec{v} = \vec{a} + \vec{b}$, we have $\vec{a} = \vec{v} - \vec{b}$. Substituting the value for $\vec{b}$ found in the previous problem, we obtain $\vec{a} = \vec{v} - (v \cdot \vec{n})\,\vec{n}$.

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Find w from v and n

1/1 point (graded)
Find a formula for $\vec{w}$ from $\vec{v}$ and $\vec{n}$.

Write `vecv` for $\vec{v}$ and `vecn` for $\vec{n}$. You may also write a dot product between two vectors using `*`. To write a scaled vector such as $2\vec{v}$, enter `2*vecv`.

$\vec{w} =$ vecv - 2*(vecv*vecn)*vecn ✔ **Answer:** vecv-2*(vecv*vecn)*vecn

Solution:

Since $\vec{w} = \vec{a} - \vec{b}$, and $\vec{v} = a + b$, we have $\vec{w} = \vec{v} - 2\vec{b}$. Substituting the value for $\vec{b}$ found in the previous problem, we obtain $\vec{w} = \vec{v} - 2\,(v \cdot \vec{n}) \cdot \vec{n}$.

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Find the matrix

1/1 point (graded)
Write $\vec{v} = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$ and $\vec{n} = \begin{pmatrix} n_1 \\ n_2 \end{pmatrix}$. There is a matrix M such that $\vec{w} = M\vec{v}$. What is M?

Enter `n_1` for n_1 and `n_2` for n_2.

(Enter a matrix using notation such as `[[a,b],[c,d]]`.)

$M =$
[[1-2*n_1^2,-2*n_1*n_2],[-2*n_1*n_2,1-2*n_2^2]] ✔

Answer: [[1 - 2*n_1^2 , -2*n_1*n_2],[-2*n_1*n_2 , 1 - 2*n_2^2]]

Solution:

We found previously that $\vec{w} = \vec{v} - 2\,(\vec{v} \cdot \vec{n})\,\vec{n}$. Letting $c = \vec{v} \cdot \vec{n}$, we have:

$$\begin{pmatrix} w_1 \\ w_2 \end{pmatrix} = \begin{pmatrix} v_1 - 2cn_1 \\ v_2 - 2cn_2 \end{pmatrix} \tag{5.217}$$

Substituting the value of c, we have

$$\begin{pmatrix} w_1 \\ w_2 \end{pmatrix} = \begin{pmatrix} v_1 - 2v_1 n_1^2 - 2v_2 n_2 n_1 \\ v_2 - 2v_1 n_1 n_2 - 2v_2 n_2^2 \end{pmatrix}$$

(5.218)

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To find the matrix, we need to recognize the entries of $\vec{w}$ as "a number times v_1" plus "a number times v_2". We can factor out v_1 and v_2 to obtain:

$$\begin{pmatrix} w_1 \\ w_2 \end{pmatrix} = \begin{pmatrix} (1-2n_1^2)\,v_1 + (-2n_1n_2)\,v_2 \\ (-2n_1n_2)\,v_1 + (1-2n_2^2)\,v_2 \end{pmatrix} \tag{5.219}$$

Therefore, the matrix M is given by

$$1-2n_1^2 \quad -2n_1n_2 \tag{5.220}$$

$$-2n_1n_2 \quad 1-2n_2^2 \tag{5.221}$$

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