2.3 Unit 2 Practice Problems

Unit 2: Conditioning

Adapted from Blitzstein-Hwang Chapter 2.

Problem 1

1/1 point (graded)

A spam filter is designed by looking at commonly occurring phrases in spam. Suppose that 80% of email is spam. In 10% of the spam emails, the phrase "free money" is used, whereas this phrase is only used in 1% of non-spam emails. A new email has just arrived, which does mention "free money". What is the probability that it is spam?

40/41

$\frac{40}{41}$

Submit You have used 1 of 99 attempts

Problem 2

1/1 point (graded)

The screens used for a certain type of cell phone are manufactured by 3 companies, A, B, and C. The proportions of screens supplied by A, B, and C are 0.5, 0.3, and 0.2, respectively, and their screens are defective with probabilities 0.01, 0.02, and 0.03, respectively.

Given that the screen on such a phone is defective, what is the probability that Company A manufactured it?

5/17

$\frac{5}{17}$

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For Problem 3

A family has 3 children, creatively named A, B, and C.

Problem 3a

1/1 point (graded)

(a) Discuss intuitively whether the event "A is older than B" is independent of the event "A is older than C".

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Not independent since the event P(A > B | A > C) > P(A > B)
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Thank you for your response.

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Problem 3b

1/1 point (graded)

(b) Find the probability that A is older than B, given that A is older than C.

2/3

$\frac{2}{3}$

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FOR PROBLEM 4

Consider the Monty Hall problem, except that Monty enjoys opening door 2 more than he enjoys opening door 3, and if he has a choice between opening these two doors, he opens door 2 with probability $p = \frac{3}{4}$.

To recap: there are three doors, behind one of which there is a car (which you want), and behind the other two of which there are goats (which you don't want). Initially, all possibilities are equally likely for where the car is. You choose a door, which for concreteness we assume is door 1. Monty Hall then opens a door to reveal a goat, and offers you the option of switching. Assume that Monty Hall knows which door has the car, will always open a goat door and offer the option of switching, and as above assume that if Monty Hall has a choice between opening door 2 and door 3, he chooses door 2 with probability $p = \frac{3}{4}$.

Problem 4a

1/1 point (graded)

(a) Find the unconditional probability that the strategy of always switching succeeds (unconditional in the sense that we do not condition on which of doors 2 or 3 Monty opens).

2/3 ✔ **Answer:** 2/3

$\frac{2}{3}$

Solution

Let C_j be the event that the car is hidden behind door j and let W be the event that we win using the switching strategy. Using the law of total probability, we can find the unconditional probability of winning:

$$\begin{aligned} P(W) &= P(W|C_1)P(C_1) + P(W|C_2)P(C_2) + P(W|C_3)P(C_3) \\ &= 0 \cdot 1/3 + 1 \cdot 1/3 + 1 \cdot 1/3 \\ &= 2/3 \end{aligned}$$

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Answers are displayed within the problem

Problem 4b

1/1 point (graded)

(b) Find the probability that the strategy of always switching succeeds, given that Monty opens door 2.

4/7 ✔ **Answer:** 4/7

$\frac{4}{7}$

Solution

A tree method works well here (delete the paths which are no longer relevant after the conditioning, and reweight the remaining values by dividing by their sum), or we can use Bayes' rule and the law of total probability (as below).

Let D_i be the event that Monty opens Door i. Note that we are looking for $P(W|D_2)$, which is the same as $P(C_3|D_2)$ as we first choose Door 1 and then switch to Door 3. By Bayes' rule and the law of total probability,

$$\begin{aligned}P(C_3|D_2) &= \frac{P(D_2|C_3)P(C_3)}{P(D_2)}\\ &= \frac{P(D_2|C_3)P(C_3)}{P(D_2|C_1)P(C_1)+P(D_2|C_2)P(C_2)+P(D_2|C_3)P(C_3)}\\ &= \frac{1\cdot 1/3}{p\cdot 1/3+0\cdot 1/3+1\cdot 1/3}\\ &= \frac{1}{1+p}=\frac{4}{7}.\end{aligned}$$

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Answers are displayed within the problem

Problem 4c

1/1 point (graded)

(c) Find the probability that the strategy of always switching succeeds, given that Monty opens door 3.

4/5 ✔ **Answer:** 4/5

$\frac{4}{5}$

Solution

The structure of the problem is the same as Part (b) (except for the condition that $p \geq 1/2$, which was not needed above). Imagine repainting doors 2 and 3, reversing which is called which. By Part (b) with $1-p$ in place of p, $P(C_2|D_3)=\frac{1}{1+(1-p)}=\frac{1}{2-p}=\frac{4}{5}$.

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Answers are displayed within the problem

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