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6.1 Summing Up

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In this section we consider situations where the starting angular velocity of the pendulum is not 0, as well as review the predictions of our model, and reflect on why pendulums are useful for timekeeping.

General Solution to the Simplified Pendulum Model

For a small angles (when $\theta \approx \sin(\theta)$), we used the differential equation

$$\frac{d^2\theta}{dt^2} = -\sqrt{\frac{g}{l}}\theta.$$

We found one family of solutions to this equation:

$$\theta(t) = \theta_0 \cos\left(\sqrt{\frac{g}{l}}t\right)$$

for any value of θ_0. Note that at $t = 0$, the starting angle is θ_0 and $\frac{d\theta}{dt} = 0$, implying that this angle is the maximum or minimum angle of the swing (depending on if θ_0 is positive or negative).

Therefore, this solution only describes situations where the pendulum starts from a maximum angle θ_0 with an angular velocity of 0.

What's the complete story?

- The complete solutions to this differential equation are:

$$\theta(t) = \theta_0 \cos\left(\sqrt{\frac{g}{l}}t + b\right)$$

where θ_0 and b are any constants. We can check by differentiating that $\theta(t) = \theta_0 \cos\left(\sqrt{\frac{g}{l}}t + b\right)$ is a solution to $\frac{d^2\theta}{dt^2} = -\sqrt{\frac{g}{l}}\theta$. Uniqueness theorems tell us that these must be all the solutions to the differential equation.

(Note that the solution $\theta(t) = \sin(\sqrt{k}t)$ we found earlier corresponds to $\theta_0 = 1$ and $b = -\pi/2$.)

- These solutions are still represented in the phase plane. Recall that for the corresponding system of differential equations for θ and angular velocity $\frac{d\theta}{dt}$, the solution trajectories in the phase plane are closed loops and the length of a loop measured in time is the period of the pendulum. These solutions correspond to starting at different points on a loop (versus always starting on the horizontal axis as we did when $b = 0$).

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