10. Back-substitution

Key point of row echelon form : Matrices in row echelon form correspond to systems that are ready to be solved immediately by **back-substitution.** To perform back-substitution, you solve for each variable in reverse order (from bottom row to top row). (On the next page, we will see that you must introduce a parameter for each variable that can not be directly expressed in terms of later variables, and substitute values into earlier equations once they are known.)

Example problem The augmented matrix in row echelon form

$$\left(\begin{array}{ccc|c} 1 & -1 & 4 & 1 \\ 0 & 8 & -4 & 0 \\ 0 & 0 & 3 & 1 \end{array}\right)$$

describes a system of equations

$$\begin{aligned} x - y + 4z &= 1 \\ 8y - 4z &= 0 \\ 3z &= 1 \end{aligned}$$

Find the general solution to the system.

Solution: Solve the last equation first to get.

$$z = 1/3.$$

Substitute this into the second to last equation to get :

$$\begin{aligned} 8y - 4(1/3) &= 0 \\ y &= 1/6. \end{aligned}$$

Now substitute both values into the first equation to get

$$\begin{aligned} x - 1/6 + 4/3 &= 1 \\ x &= 1 + 1/6 - 4/3 = -1/6. \end{aligned}$$

Conclusion: The general solution written as a column vector is

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -1/6 \\ 1/6 \\ 1/3 \end{pmatrix}.$$

Worked example: elimination and back substitution

Start of transcript. Skip to the end.

Hi, welcome to recitation.

My name is Martina and I'll be your recitation instructor

for some of these linear algebra videos.

Today's problem is a straightforward

solve the following linear system

with four equations and four unknowns

(Caption will be displayed when you start playing the video.)

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Back-substitution practice

3/3 points (graded)

The augmented matrix in row echelon form

$$\left(\begin{array}{ccc|c} 2 & 3 & 5 & 7 \\ 0 & 1 & 4 & 6 \\ 0 & 0 & 3 & 9 \end{array}\right)$$

represents the system

$$\begin{pmatrix} 2 & 3 & 5 \\ 0 & 1 & 4 \\ 0 & 0 & 3 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 7 \\ 6 \\ 9 \end{pmatrix}.$$

Use back-substitution to find the general solution.

$x =$ 5 ✔ **Answer:** 5

$y =$ -6 ✔ **Answer:** -6

$z =$ 3 ✔ **Answer:** 3

Solution:

The last equation says

$$3z = 9,$$

i.e., $z = 3$. Plugging this value into the second equation we get

$$y + 4(3) = 6,$$

which gives $y = -6$. Plugging the value of y and z into the first equation we get

$$2x + 3(-6) + 5(3) = 7,$$

which gives $x = 5$.

Submit You have used 1 of 4 attempts

Answers are displayed within the problem

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Typo in the solution to "Back-substitution practice" 2

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