4. Practice

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Lecture due Oct 5, 2021 20:30 IST

Practice

Practice

3/3 points (graded)

Suppose a particle has position at time t given by $(t^3, (t-1)(t+2))$. Find the speed and velocity at $t = 2$.

To enter your answer, let the velocity vector $\vec{v} = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$.

$v_1 =$ 12 ✔ **Answer:** 12

$v_2 =$ 5 ✔ **Answer:** 5

Speed: 13 ✔ **Answer:** 13

? INPUT HELP

Solution:

Differentiating, we have $x'(t) = 3t^2$ and $y'(t) = 2t + 1$, so the velocity vector is given by evaluating these at $t = 2$, giving $\begin{pmatrix} 12 \\ 5 \end{pmatrix}$.

The speed is given by $|\vec{v}| = \sqrt{12^2 + 5^2} = \sqrt{169} = 13$.

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Answers are displayed within the problem

Find the y

2/2 points (graded)

Suppose a particle has $x(t) = 6t^2 + 1$. Let $y(t) = at^2 + bt$ for constants a and b. Find the values of a and b such that the particle's speed at all times $t > 0$ is given by $20t$.

$a =$ 8 ✔ **Answer:** 8

$b =$ 0 ✔ **Answer:** 0

Solution:

We can compute $\vec{v}$ as follows:

$$\vec{v} = \sqrt{x'(t)^2 + y'(t)^2} \tag{6.45}$$

$$= \sqrt{144t^2 + 4a^2t^2 + 4abt + b^2} \tag{6.46}$$

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Since we need $v = 20t$, squaring both sides, we need

$$144t^2 + 4a^2t^2 + 4abt + b^2 = 400t^2 \tag{6.47}$$

for all $t > 0$. Since there is no constant term on the right side, we obtain $b = 0$. This leaves us to solve for a in

$$144t^2 + 4a^2t^2 = 400t^2 \tag{6.48}$$

By algebra, we obtain $a = \pm 8$.

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Conceptual 1

1/1 point (graded)
Suppose a particle has constant velocity. Does it follow that the particle has constant speed?

Yes, it follows ✔ **Answer:** Yes, it follows

Solution:

Since speed is the magnitude of the velocity vector, whenever the velocity is the same, the speed will also be the same.

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Answers are displayed within the problem

Conceptual 2

1/1 point (graded)
Suppose a particle has constant speed. Does it follow that the particle has constant velocity?

No, not always ✔ **Answer:** No, not always

Solution:

Constant speed does not imply constant velocity. A particle could have changing velocity, which points in different directions over time, but as long as the magnitude of the velocity stays the same, the speed would be constant. An example is given by the circle trajectory: $(x(t), y(t)) = (\cos t, \sin t)$.

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Answers are displayed within the problem

Estimate Velocity

1/1 point (graded)
Suppose a particle has $\vec{r} = \begin{pmatrix} \cos t \\ \sin t \end{pmatrix}$. For what value of t between 0 and 2π will the velocity vector $\vec{v}$ be a $\begin{pmatrix} 1 \\$

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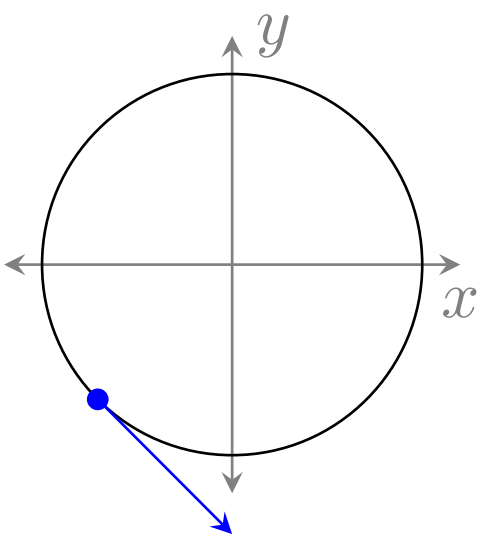
positive multiple of $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$?

5*pi/4 ✔ **Answer:** 5*pi/4

? INPUT HELP

Solution:

One may find the answer using inverse trigonometric functions. Alternatively, recall that the velocity vector will point in the direction tangent to the circle. We need this direction to point along the vector $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$. By visualizing the trajectory of a circle, one might guess that this will happen at the multiple of $\pi/4$ that lands in the third quadrant, and indeed it does.



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