MITx: 14.310x Data Analysis for Social Scientists

Help

Bookmarks

Question 14 - 20

Bookmark this page

Question 14

1/1 point (graded)

Suppose that the PDF $f_X(x)$ of a random variable X is an even function. **Note:** $f_X(x)$ is an even function if $f_X(x) = f_X(-x)$.

Is it true that the random variables X and $-X$ are identically distributed?

- a. True ✔
- b. False

Explanation

This statement is true. The proof is the following $Y = -X$, and $g^{-1}(y) = -y$. Therefore, for every y:

$$f_Y(y) = f_X(g^{-1}(y)) = \left|\frac{d}{dy}g^{-1}(y)\right| = f_X(-y)|-1| = f_X(y)$$

Submit You have used 1 of 1 attempts

Correct (1/1 point)

Question 15

1/1 point (graded)

A couple decides to continue to have children until a daughter is born. What is the expected number of children this couple will have if the probability that a daughter is born is given by p?

- a. The expected number of children is given by $\frac{1}{p}$ ✔
- b. The expected number of children is given by $\frac{1-p}{p}$
- c. The expected number of children is given by $\frac{p}{p^3}$
- d. The expected number of children is given by $\frac{1}{p} - 1$

Explanation

If X is the number of children until the first daughter then $P(X = k) = (1-p)^{k-1}p$. Thus X is a geometric random variable and we have that as saw in the lecture:

$$\mathbb{E}\left[X\right] = \sum_{k=0}^{\infty} k(1-p)^{k-1}p = \frac{1}{p}$$

Submit You have used 1 of 2 attempts

Correct (1/1 point)

Question 16

1/1 point (graded)

Which of the following statements is correct? (Select all that apply)

- [x] a. If $f_X(x) = ax^{a-1}, 0 < x < 1, a > 0$ then $\mathbb{E}\left[X\right] = \frac{a}{a+1}$
- [ ] b. If $f_X(x) = \frac{1}{n}, x = 1, 2, \cdots, n, n > 0$ an integer then $\mathbb{E}\left[X\right] = \frac{n+2}{2}$
- [ ] c. If $f_X(x) = \frac{3}{2}(x-1)^2, 0 < x < 2$ then $\mathbb{E}\left[X\right] = 2$
- [ ] d. If $f_X(x) = ax^{a-1}, 0 < x < 1, a > 0$ then $\mathbb{E}\left[X\right] = 1 + \frac{1}{a}$
- [x] e. If $f_X(x) = \frac{3}{2}(x-1)^2, 0 < x < 2$ then $\mathbb{E}\left[X\right] = 1$

- [ ] If $f_X(x) = \frac{1}{n}, x = 1, 2, \cdots, n,$ for integer $n > 0$ then $\mathbb{E}[X] = \frac{n+1}{n}$

✔

Explanation

In this case we have that:

If $f_X(x) = ax^{a-1}, 0 < x < 1, a > 0$ then:

$$\mathbb{E}[X] = \int_0^1 ax^a dx = \frac{a}{a+1} x^{a+1} \Big|_0^1 = \frac{a}{a+1}$$

If $f_X(x) = \frac{1}{n}, x = 1, 2, \cdots, n,$ for integer $n > 0$ then:

$$\mathbb{E}[X] = \sum_{x=1}^n \frac{1}{n} = \frac{1}{n} \sum_{x=1}^n x = \frac{1}{n} \frac{(n+1)n}{2} = \frac{n+1}{2}$$

If $f_X(x) = \frac{3}{2}(x-1)^2, 0 < x < 2$ then:

$$\mathbb{E}[X] = \int_0^2 \frac{3}{2}(x-1)^2 = \frac{1}{2}(x-1)^3 \Big|_0^2 = \frac{1}{2} + \frac{1}{2} = 1$$

Submit You have used 2 of 2 attempts

Correct (1/1 point)

Question 17

1/1 point (graded)

Suppose that the random variable Y has a binomial distribution with n trials and success probability X, where n is a given constant and X is a uniform(0,1) random variable. What is $\mathbb{E}\left[Y\right]$?

- a. This is given by n
- b. This is given by $\frac{n}{2}$ (selected, correct)
- c. This is given by $\frac{n}{3}$
- d. This is given by $\frac{X}{n}$

Explanation

In general, we have that since Y is binomial with probability success X then $\mathbb{E}\left[Y|X\right] = nX$. Using this, and the law of iterated expectations, we have that:

$$\mathbb{E}\left[Y\right] = \mathbb{E}\left[\mathbb{E}\left[Y|X\right]\right] = \mathbb{E}\left[nX\right].$$

Since X is a uniform variable between 0 and 1, then we know that $\mathbb{E}\left[nX\right] = \frac{n}{2}$.

Submit You have used 1 of 2 attempts

✔ Correct (1/1 point)

Question 18

1/1 point (graded)

Suppose that the random variable Y has a binomial distribution with n trials and success probability X, where n is a given constant and X is a uniform(0,1) random variables. What is $Var(Y)$?

- a. This is given by $\frac{n^2}{6} + \frac{n}{12}$
- b. This is given by $\frac{n^2}{18} + \frac{n}{6}$
- c. This is given by $\frac{n^2}{12} + \frac{n}{6}$ ✔
- d. This is given by $\frac{n^2}{18} + \frac{n}{12}$

Explanation

Here we use the law of total probability. We have that:

$$Var(Y) = Var\left(\mathbb{E}\left[Y|X\right]\right) + \mathbb{E}\left[Var(Y|X)\right] = Var(nX) + \mathbb{E}\left[nX(1-X)\right] = \frac{n^2}{12} + \frac{n}{6}$$

Submit You have used 1 of 2 attempts

Correct (1/1 point)

Question 19

1/1 point (graded)

Assume that $Y = \alpha + \beta X + U$, where $\beta = \frac{\rho_{XY}\sigma_Y}{\sigma_X}$ and $\alpha = \mu_Y - \beta\mu_X$. What is the expected value of U?

a. The expected value of U is $\alpha + \beta\mu_X$.

b. The expected value of U is μ_Y

c. The expected value of U is 0.

d. The expected value of U is α

Explanation

In the case we have that:

$$\mathbb{E}[U] = \mathbb{E}[Y - \alpha - \beta X]$$

$$= \mu_Y - \alpha - \beta\mu_X$$

$$= \mu_Y - \mu_Y + \beta\mu_X - \beta\mu_X = 0$$

Submit You have used 1 of 2 attempts

Correct (1/1 point)

Question 20

1/1 point (graded)

Assume that $Y = \alpha + \beta X + U$, where $\beta = \frac{\rho_{XY}\sigma_Y}{\sigma_X}$ and $\alpha = \mu_Y - \beta\mu_X$. What is $cov(X, U)$? (Select all that apply).

- [ ] a. We have that $cov(X, U) = var(X)$
- [ ] b. We have that $cov(X, U) = \sigma_X \sigma_U$
- [x] c. We have that $cov(X, U) = 0$
- [ ] d. We have that $cov(X, U) = \sigma_X \sigma_Y$

e. We have that $cov(X,U) = \rho_{XU}\sigma_X\sigma_U$

Explanation

By definition we know that $\rho_{XU} = \frac{cov(X,U)}{\sigma_X\sigma_U}$. In this case we also have that:

$$cov(X,U) = cov(X, Y - \alpha - \beta X)$$

$$= cov(X,Y) - \beta var(X)$$

$$= cov(X,Y) - \left(\frac{\rho_{XY}\sigma_Y}{\sigma_X}\right) var(X)$$

$$= cov(X,Y) - \left(\frac{\frac{cov(X,Y)}{\sigma_Y\sigma_X}\sigma_Y}{\sigma_X}\right) var(X)$$

$$= cov(X,Y) - \left(\frac{\frac{cov(X,Y)}{\sigma_X}}{\sigma_X}\right) var(X)$$

$$= cov(X,Y) - cov(X,Y) = 0$$

Submit You have used 1 of 2 attempts

Correct (1/1 point)