



Course > Unit 6 Linear Regression > Homework 10 Linear regression > 4. Heteroscedastic Regression

4. Heteroscedastic Regression

For the next three problems, consider the following setting.

We measure characteristics of n individuals, sampled randomly from a population. Let (X_i, y_i) be the observed data of the ith individual, where $y_i \in \mathbb{R}$ is the dependent variable and $X_i \in \mathbb{R}^p$ is the vector of p **deterministic** explanatory variables. Our goal is to estimate the coefficients of $\beta = (\beta_1, \dots, \beta_p)^T$ in the linear regression:

$$y_i = X_i^T eta + \epsilon_i, \qquad i = 1, \dots, n$$

We will consider the case where the model is potentially **heteroscedastic** (i.e. the error terms ϵ_i are **not** i.i.d.).

More specifically, assume that the vector $\boldsymbol{\epsilon} = (\epsilon_1, \dots, \epsilon_n)^T$ is a n-dimensional Gaussian with mean 0 and known **nonsingular** covariance matrix Σ . Denote by \mathbb{X} the matrix in $\mathbb{R}^{n \times p}$ whose rows are $\mathbf{X}_1^T, \dots, \mathbf{X}_n^T$ and by \mathbf{Y} the vector with coordinates y_1, \dots, y_n .

Instead of the usual Least Squares Estimator, instead consider the estimator $\hat{\beta}$ that minimizes, over all $\beta \in \mathbb{R}^p$,

$$(\mathbf{Y} - \mathbb{X}eta)^T \Sigma^{-1} \left(\mathbf{Y} - \mathbb{X}eta
ight).$$

(a) A Generalized Estimator

1/1 point (graded)

Let I_n be the $n \times n$ identity matrix. If $\Sigma = \sigma^2 I_n$ (i.e. homoscedastic ε) for some $\sigma^2 > 0$, then which one of the following statement about $\hat{\beta}$ must be true? Make no assumptions about the rank of \mathbb{X} .

- \bigcirc \hat{eta} has positive entries.
- ullet \hat{eta} is the least squares estimator.
- \hat{eta} is the unique minimizer of the specified loss.
- \bigcirc The components of \hat{eta} are independent.



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You have used 1 of 3 attempts

(b) The Maximum Likelihood Estimator

1/1 point (graded)

In this exercise, we will prove that $\hat{\beta}$ is equal to the Maximum Likelihood Estimator, even for general Σ . Recall the form of the n-dimensional Gaussian density from Lecture 10.

Let Σ be an arbitrary $n \times n$ matrix. The maximum likelihood estimator β_{MLE} is the value of β maximizes the log-likelihood function $\ell(\beta) = \ln L(\mathbb{X}, \mathbb{Y}; \beta)$.

Write down the function ℓ , in terms of \mathbb{X} , \mathbf{Y} , β , Σ , and n.

(Type **X** for \mathbb{X} , **Y** for \mathbb{Y} , **Sigma** for Σ . Type **trans(X)** for the transpose \mathbb{X}^T , **det(X)** for the determinant $\det \mathbb{X}$, and **X^(-1)** for the inverse \mathbb{X}^{-1} , of a matrix \mathbb{X} .)

$$\ell\left(eta
ight) = \ln L\left(\mathbb{X}, \mathbb{Y}; eta
ight) =$$

 $-(n/2)*ln(2*pi)-ln(det(Sigma))/2-1/2*(trans(Y-X*beta))*Sigma^(-1)*(Y-X*beta)$



STANDARD NOTATION

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✓ Correct (1/1 point)

(c)

2/2 points (graded)

Assume that the rank of $\mathbb X$ is p. (As a consequence, for any nonsingular n imes n matrix Q, the p imes p matrix product X^TQX is also nonsingular.)

Which of the following is a correct formula for \hat{eta} in terms of \mathbb{X} , \mathbb{Y} and Σ ?

$$\bullet \left(\mathbb{X}^T \Sigma^{-1} \mathbb{X} \right)^{-1} \mathbb{X}^T \Sigma^{-1} \mathbb{Y}$$

$$\bigcirc \left(\mathbb{X}^T\mathbb{X}\right)^{-1}\mathbb{X}^T\mathbb{Y}$$

$$igcup \left(\mathbb{X}^T \Sigma^{-1} \mathbb{X}
ight)^{-1} \mathbb{Y}^T \Sigma^{-1} \mathbb{X}$$

$$\bigcirc \ \mathbb{X}^{-1} \mathbb{Y}$$



Using the result from the above, which of the following correctly characterizes the distribution of $\hat{\beta}$?

$\bigcirc \mathcal{N}\left(0,\Sigma ight)$	
$\mathcal{N}\left(0,\left(\mathbb{X}^{T}\Sigma^{-1}\mathbb{X} ight)^{-1} ight)$	
$\mathcal{N}\left(0,\left(\mathbb{X}^{T}\mathbb{X} ight)^{-1} ight)$	

$$left{igotimes \mathcal{N} \left(eta, \left(\mathbb{X}^T \Sigma^{-1} \mathbb{X}
ight)^{-1}
ight)}$$

$$\bigcirc \, \mathcal{N} \, (eta, (\mathbb{X}^T \mathbb{X})^{-1})$$



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You have used 2 of 3 attempts

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(b) The Maximum Likelihood Estimator incomplete log-likelihood?



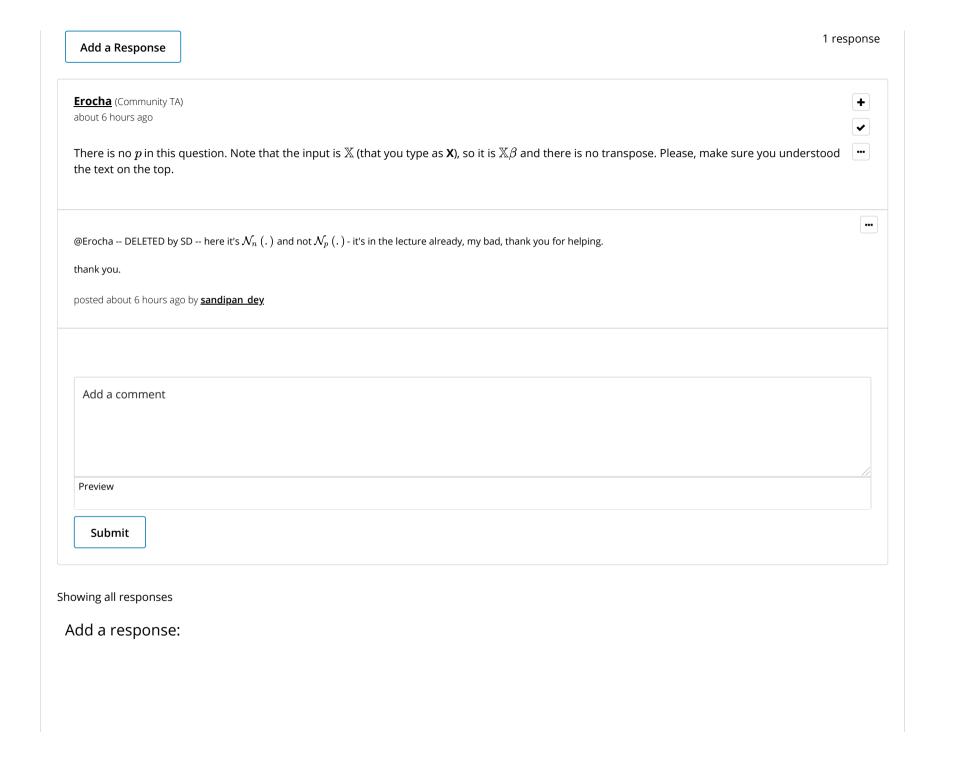
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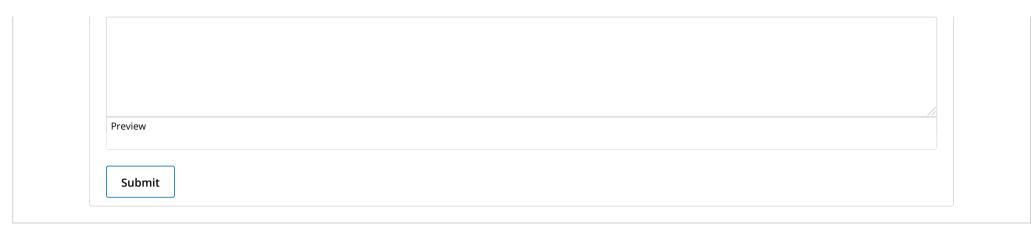
*

For MLE, we have log-likelihood function of the form $l(X,Y;\beta,p)=f_1(p)+f_2(\Sigma)+f_3(X,Y,\beta,\Sigma)$. But grader does not accept $f_1(p)$ and when I entered $f_2(\Sigma)+f_3(X,Y,\beta,\Sigma)$ it got rejected. Should we get rid of the $f_2(\Sigma)$ part too? Also, as per the initial definition we have $Y=X^T\beta+\epsilon$, for MLE we are asked to minimize $(Y-X\beta)^T\Sigma^{-1}(Y-X\beta)$ (notice it's $X\beta$ and NOT $X^T\beta$ here), should not it be $X^T\beta$ instead in the objective function?



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