2 3 5 7 11 13 17 19 23 29 31 37 41 43 47 53 59 61 67 71 73 79 83 89 97 101 103 107 109 113 127 131 137 139 149 151 157 163 167 173 179 181 191 193 197 199

More Fun with Prime Numbers

Week 3

Homework

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Problem 1

Fill a positive integer in each blank.

$$7 = \square^2 + 3\times\square^2$$
$$31 = \square^2 + 3\times\square^2$$
$$127 = \square^2 + 3\times\square^2$$

- A **prime number** P is written as

$$P = X^2 + 3\times Y^2$$

if and only if

$$P = 3 \text{ or } P \equiv 1 \pmod 3.$$

Pierre de Fermat
(1607?-1665)

https://en.wikipedia.org/wiki/Pierre_de_Fermat

Problem 1

$7 = 2\times3 + 1 \equiv 1 \pmod 3$

$31 = 10\times3 + 1 \equiv 1$

$127 = 42\times3 + 1 \equiv 1$

- By **Fermat's thm**, P is written as

$$\mathbf{P = X^2 + 3\times Y^2}$$

  for some X, Y.

- Find X and Y.

Pierre de Fermat
(1607?-1665)

Problem 1

Answer

$$7 = \mathbf{2}^2 + 3\times\mathbf{1}^2$$
$$31 = \mathbf{2}^2 + 3\times\mathbf{3}^2$$
$$127 = \mathbf{10}^2 + 3\times\mathbf{3}^2$$

Pierre de Fermat
(1607?-1665)