4. Maximum likelihood estimators

Instructions:

Let $X_1, \ldots, X_n$ be n i.i.d. random variables with pdf f_θ, where θ is an unknown parameter.

For each of the following questions, compute the likelihood function on paper and then find the maximum likelihood estimator for θ.

To encourage you to do the computations carefully rather than eliminate choices, you will only be given **1 or 2 attempts per question** .

(a)

1/1 point (graded)
Compute the likelihood function and the maximum likelihood estimator for θ for

$$f_\theta(x) = \tau\theta^\tau x^{-(\tau+1)}\mathbf{1}(x \geq \theta), \quad \theta > 0,$$

where $\tau > 0$ is a known constant.

- ○ $\max X_i$
- ◉ $\min X_i$
- ○ $\frac{1}{n}\sum X_i$
- ○ $n\tau$
- ○ ∞

✔

Solution:

The likelihood function is

$$L = \tau^n \theta^{n\tau} \prod_i X_i^{-(\tau+1)} \mathbf{1}\{\min_i X_i \geq \theta\}$$

For $\theta \leq \min_i X_i$, the log-likelihood function is

$$l = n\ln\tau + n\tau\ln\theta - (\tau+1)\sum_{i=1} \ln X_i$$

Take the derivative with respect to θ:

$$\frac{\partial l}{\partial \theta} = \frac{n\tau}{\theta} > 0.$$

Thus, L is an increasing function on $(0, \min_i X_i]$, and is 0 for $\theta > \min_i X_i$. Therefore,

$$\hat{\theta} = \min_i X_i$$

Submit You have used 1 of 2 attempts

Answers are displayed within the problem

(b)

3/3 points (graded)

Compute the likelihood function and the maximum likelihood estimator for θ for

$$f_\theta(x) = \sqrt{\theta} x^{\sqrt{\theta}-1} \mathbf{1}(0 < x < 1), \quad \theta > 0.$$

You will find that the maximum likelihood estimator for θ is of the form

$$\hat{\theta}^{\text{MLE}} = c_1 n^{c_2} \left(\sum_{i=1}^{n} \ln X_i \right)^{c_3}.$$

Enter the numbers c_1, c_2, c_3 below.

$c_1 =$ 1 **Answer:** 1

$c_2 =$ 2 ✔ **Answer:** 2

$c_3 =$ -2 ✔ **Answer:** -2

STANDARD NOTATION

Solution:

The likelihood function is

$$L = \theta^{n/2} \prod_i X_i^{\sqrt{\theta}-1} \mathbf{1}\{0 \leq X_i \leq 1\}.$$

The log-likelihood function is

$$l = \frac{n}{2} \ln \theta + (\sqrt{\theta} - 1) \sum_i \ln X_i.$$

Take the derivative with respect to θ and set it to 0:

$$\frac{\partial l}{\partial \theta} = \frac{n}{2\theta} + \frac{1}{2\theta^{1/2}} \sum_i \ln X_i = 0.$$

Then we get

$$\hat{\theta} = \frac{n^2}{\left(\sum \ln X_i\right)^2}.$$

Submit You have used 2 of 3 attempts

Answers are displayed within the problem

(c)

4/4 points (graded)

Compute the likelihood function and the maximum likelihood estimator for θ

$$f_\theta(x) = \theta\tau x^{\tau-1}\exp\{-\theta x^\tau\}\mathbf{1}(x \geq 0), \quad \theta > 0,$$

where $\tau > 0$ is a known constant.

You will find that the maximum likelihood estimator for θ is of the form

$$\hat{\theta}^{\text{MLE}} = c_1 n^{c_2}\left(\sum_{i=1}^{n} X_i^{c_3}\right)^{c_4}.$$

Enter the c_1, c_2, c_3, c_4 in terms of τ if applicable.

(Enter **tau** for τ.)

$c_1 =$ 1 **Answer:** 1

1

$c_2 =$ 1 **Answer:** 1

1

$c_3 =$ tau

τ

✔ **Answer:** tau

$c_4 =$ -1

-1

✔ **Answer:** -1

STANDARD NOTATION

Solution:

The likelihood function is

$$L = \theta^n \tau^n \prod_i X_i^{\tau-1} \exp\{-\theta \sum_i X_i^\tau\} \mathbf{1}\{X_i \geq 0\}.$$

The log-likelihood function is

$$l = n \ln \theta + n \ln \tau + (\tau - 1) \sum_i \ln X_i - \theta \sum_i X_i^\tau.$$

Take the derivative with respect to θ and set it to 0

$$\frac{\partial l}{\partial \theta} = \frac{n}{\theta} - \sum_i X_i^\tau = 0,$$

we get

$$\hat{\theta} = \frac{n}{\sum_i X_i^\tau}.$$

Submit You have used 1 of 3 attempts

Answers are displayed within the problem

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Hint: you can't simply sum up the X_i's

discussion posted 7 days ago by yiuming_h

I totally missed that we were working with random variables when doing the likelihood calculations, and kept getting the wrong answers. Thank you for this exercise, I think I understand likelihood a lot better now as a result

This post is visible to everyone.

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