Counting Prime Numbers (8)

Prime numbers in Arithmetic Progressions

A, B ≥ 1 **relatively prime** (GCD(A,B)=1)

$\pi_{A,B}(N)$ = the number of prime numbers P of the form **P = A + K B** with **P ≤ N**

Theorem (de la Vallée-Poussin, 1896)

$$\lim_{N\to\infty} \frac{\pi_{A,B}(N)}{N/\log(N)} = \frac{1}{\phi(B)}$$

Counting Prime Numbers (9)

Moreover, de la Vallée-Poussin's Thm implies:

Dirichlet's Thm on Arithmetic Progression (1837):
there are **infinitely many** prime numbers of the form
P = A + K B.

Peter Gustav Lejeune Dirichlet (1805-1859)

https://en.wikipedia.org/wiki/Peter_Gustav_Lejeune_Dirichlet

Interlude: the largest known prime

- Euclid's Thm and PNT do **not** give concrete prime numbers.
- Currently, the **largest known prime number** is $\mathbf{2^{74,207,281} - 1}$, a number with 22,338,618 digits. (found in Jan 2016)
- Prime numbers of the form $\mathbf{P = 2^n - 1}$ are **Mersenne primes**.

Marin
Mersenne
(1588-1648)

https://en.wikipedia.org/wiki/Marin_Mersenne