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Exam 1
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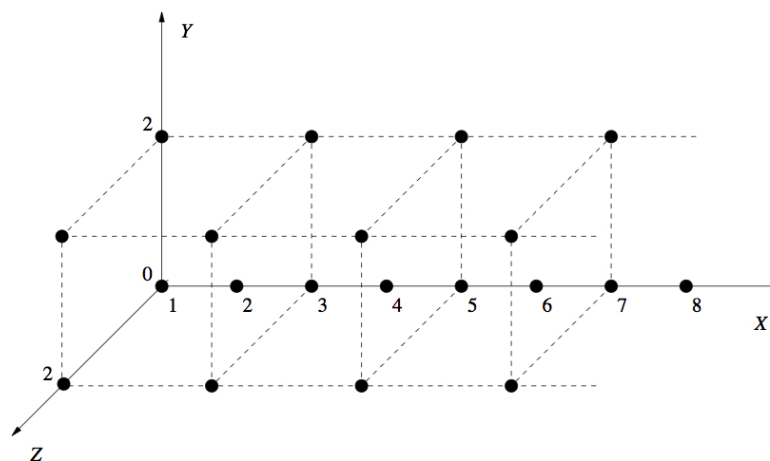
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Problem 5: Joint PMF calculations - Part 2

(5/5 points)

Note: The problem statement from part 1 has been repeated here for your convenience.

Consider three random variables X , Y , and Z , associated with the same experiment. The random variable X is geometric with parameter $p \in (0, 1)$. If X is even, then Y and Z are equal to zero. If X is odd, (Y, Z) is uniformly distributed on the set $S = \{(0, 0), (0, 2), (2, 0), (2, 2)\}$. The figure below shows all the possible values for the triple (X, Y, Z) that have $X \leq 8$. (Note that the X axis starts at 1 and that a complete figure would extend indefinitely to the right.)



random variables

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1. Find the joint PMF $p_{X,Y,Z}(x, y, z)$. Express your answers in terms of x and p using standard notation .

If x is odd and $(y, z) \in \{(0, 0), (0, 2), (2, 0), (2, 2)\}$,

$$p_{X,Y,Z}(x, y, z) = \boxed{(1-p)^{(x-1)}p/4} \quad \checkmark \text{ Answer: } (1/4)*p*(1-p)^{(x-1)}$$

If x is even and $(y, z) = (0, 0)$,

$$p_{X,Y,Z}(x, y, z) = \boxed{(1-p)^{(x-1)}p} \quad \checkmark \text{ Answer: } p*(1-p)^{(x-1)}$$

2. Find $p_{X,Y}(x, 2)$, for when x is odd. Express your answer in terms of x and p using standard notation .

If x is odd,

$$p_{X,Y}(x, 2) = \boxed{(1-p)^{(x-1)}p/2} \quad \checkmark \text{ Answer: } (1/2)*p*(1-p)^{(x-1)}$$

3. Find $p_Y(2)$. Express your answer in terms of p using standard notation .

$$p_Y(2) = \boxed{1/(4-2*p)} \quad \checkmark \text{ Answer: } 1/(2*(2-p))$$

4. Find $\text{var}(Y + Z \mid X = 5)$.

$$\boxed{2}$$

\checkmark Answer: 2

Answer:

1. An easy way to derive $p_{X,Y,Z}(x, y, z)$ uses the multiplication rule: $p_X(x) \cdot p_{Y,Z|X}(y, z|x)$. Note that X is geometric with parameter p . Conditioned on X even, $(Y, Z) = (0, 0)$ with probability 1. Conditioned on X odd, $p_{Y,Z|X}(y, z) = \frac{1}{4}$ for $(y, z) \in \{(0, 0), (0, 2), (2, 0), (2, 2)\}$.

$$p_{X,Y,Z}(x, y, z) = \begin{cases} \frac{1}{4}p(1-p)^{x-1}, & \text{if } x \text{ is odd and } (y, z) \in \{(0, 0), (0, 2), (2, 0), (2, 2)\} \\ p(1-p)^{x-1}, & \text{if } x \text{ is even and } (y, z) = (0, 0) \\ 0, & \text{otherwise.} \end{cases}$$

2. $p_{X,Y}(x, 2) = \sum_z p_{X,Y,Z}(x, 2, z)$. From part 1, we know that when x is odd and $(y, z) \in \{(0, 0), (0, 2), (2, 0), (2, 2)\}$, $p_{X,Y,Z}(x, y, z) = \frac{1}{4}p(1-p)^{x-1}$ and so:

$$\begin{aligned} p_{X,Y}(x, 2) &= p_{X,Y,Z}(x, 2, 0) + p_{X,Y,Z}(x, 2, 2) \\ &= \frac{1}{2}p(1-p)^{x-1}, \end{aligned}$$

when x is odd.

3. $p_Y(2) = \sum_x p_{X,Y}(x, 2)$. Since $p_{X,Y}(x, 2)$ is non-zero only when x is odd, we can use the result from the previous question to find:

$$p_Y(2) = \sum_{x \text{ is odd}} p_{X,Y}(x, 2)$$

$$\begin{aligned}
&= \frac{1}{2} \sum_{x \text{ is odd}} p(1-p)^{x-1} \\
&= \frac{1}{2} (p(1-p)^0 + p(1-p)^2 + p(1-p)^4 + p(1-p)^6 + \dots) \\
&= \frac{p}{2} ((1-p)^0 + (1-p)^2 + (1-p)^4 + (1-p)^6 + \dots) \\
&= \frac{p}{2} \left(\frac{1}{1 - (1-p)^2} \right) \\
&= \frac{1}{2(2-p)}
\end{aligned}$$

4. If $\mathbf{X} = 5$, then \mathbf{Y} and \mathbf{Z} are uniformly distributed on the set \mathbf{S} specified in the problem statement, so $\mathbf{Y} + \mathbf{Z}$ takes the values 0 and 4 with probability $\frac{1}{4}$, and takes the value 2 with probability $\frac{1}{2}$. This PMF is symmetric about 2, so the mean value of $\mathbf{Y} + \mathbf{Z}$ is evidently 2. Hence the variance is

$$(0-2)^2 \frac{1}{4} + (4-2)^2 \frac{1}{4} = 2.$$

You have used 1 of 2 submissions



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