CSE 473

Lecture 17

MDPs and Reinforcement Learning

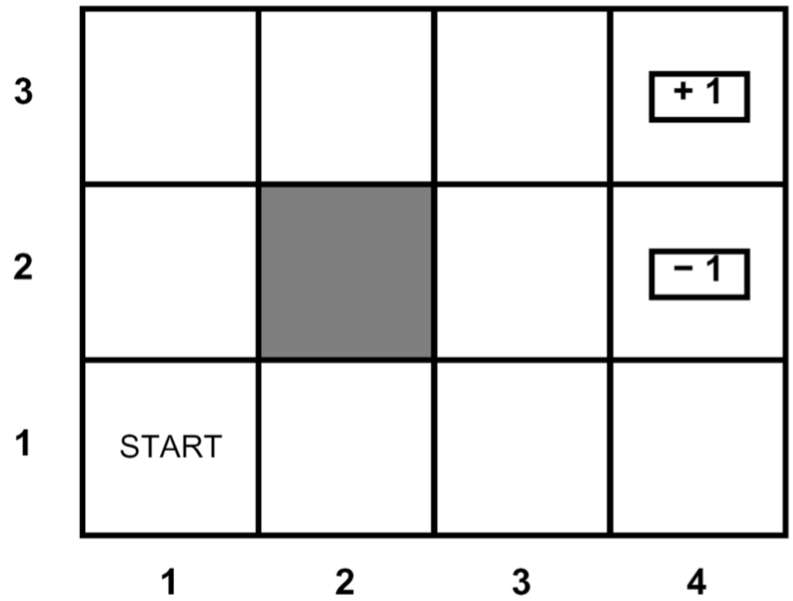
Unbeknownst to most students of psychology, Pavlov's first experiment was to ring a bell and cause his dog to attack Freud's cat.

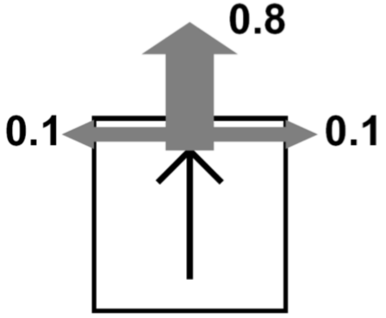
Today’s Outline

- MDPs
  - Policy iteration
  - Q-value iteration
- Reinforcement Learning
  - Q-learning

Recall: MDPs

- An MDP is defined by:
  - States $s \in S$
  - Actions $a \in A$
  - Transition function $T(s,a,s') = P(s' \mid s,a)$
  - Reward function $R(s, a, s')$
  - Start state



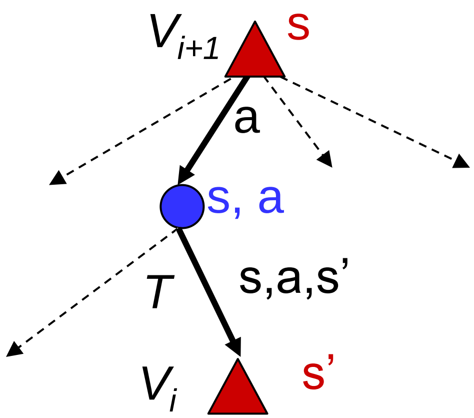


Recall: Value Iteration

- How do we compute *V*(s)* for all states *s*?
- Use iterative method called Value Iteration:
  - Start with $V_0^*(s) = 0$
  - Given V_i^*, calculate the values for all states for depth i+1:

$$V_{i+1}(s) \leftarrow \max_a \sum_{s'} T(s,a,s') \left[R(s,a,s') + \gamma V_i(s') \right]$$

  - Repeat until convergence



Is there a faster alternative to value iteration?

Yeah, crazy little thing called policy iteration!

Policy Iteration: Motivation

- Problem with value iteration:

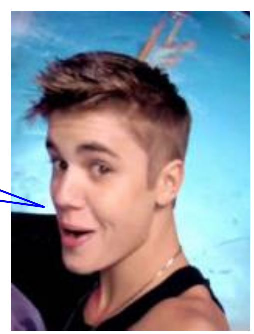
$$V_{i+1}(s) = \max_a \sum_{s'} T(s,a,s') \left[R(s,a,s') + \gamma V_i(s') \right]$$

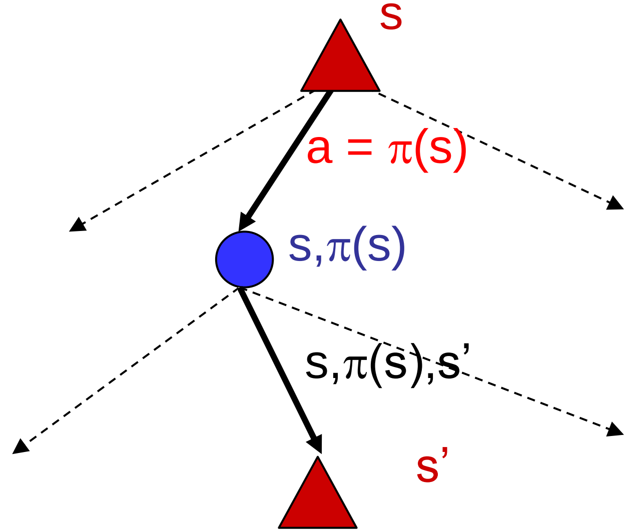
  - Considering *all actions* makes each iteration slow

- What if we compute values for some *fixed policy* $\pi(s)$?

$$V^{\pi}(s) = \sum_{s'} T(s,\pi(s),s')[R(s,\pi(s),s') + \gamma V^{\pi}(s')]$$

Look, no max,
so fast!





Policy Iteration

- Start with an arbitrary policy π_0
- Repeat until policy converges:

1. **Policy evaluation (fast)**: With fixed current policy π_k, iterate values until convergence:

$$V_0^{\pi_k}(s) = 0$$

$$V_{i+1}^{\pi_k}(s) \leftarrow \sum_{s'} T(s, \pi_k(s), s') \left[R(s, \pi_k(s), s') + \gamma V_i^{\pi_k}(s')\right]$$

2. **Policy improvement (slow but infrequent)**: Based on converged values in (2), update policy by choosing best action using one-step look-ahead:

$$\pi_{k+1}(s) = \arg\max_a \sum_{s'} T(s, a, s') \left[R(s, a, s') + \gamma V^{\pi_k}(s')\right]$$

Policy Iteration Complexity

- Problem size:
  - $|A|$ actions and $|S|$ states

- Each Iteration
  - Time: $O(|S|^3 + |A| \cdot |S|^2)$
  - Space: $O(|S|)$

- Num of iterations
  - Unknown, but can be fast in practice
  - Convergence is guaranteed

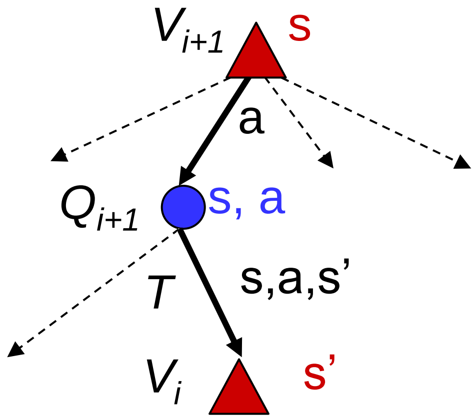
One last variation: Q-Value Iteration

- Value iteration updates values for states:

$$V_{i+1}(s) \leftarrow \max_a \sum_{s'} T(s,a,s') \left[R(s,a,s') + \gamma V_i(s')\right]$$

Equivalent to:

$$V_{i+1}(s) \leftarrow \max_a \; Q_{i+1}(s,a)$$



Why not update *Q*-values instead of *V*?!

$$Q_{i+1}(s,a) \leftarrow \sum_{s'} T(s,a,s') \left[R(s,a,s') + \gamma V_i(s') \right]$$ i.e.,

$$Q_{i+1}(s,a) \leftarrow \sum_{s'} T(s,a,s') \left[R(s,a,s') + \gamma \max_{a'} Q_i(s',a')\right]$$

Q-Value Iteration

Initialize each Q-state: $Q_0(s,a) = 0$

Repeat

For all Q-states s,a

Compute $Q_{i+1}(s,a)$ from Q_i by Bellman update:

$$Q_{i+1}(s,a) \leftarrow \sum_{s'} T(s,a,s') \left[R(s,a,s') + \gamma \max_{a'} Q_i(s',a') \right]$$

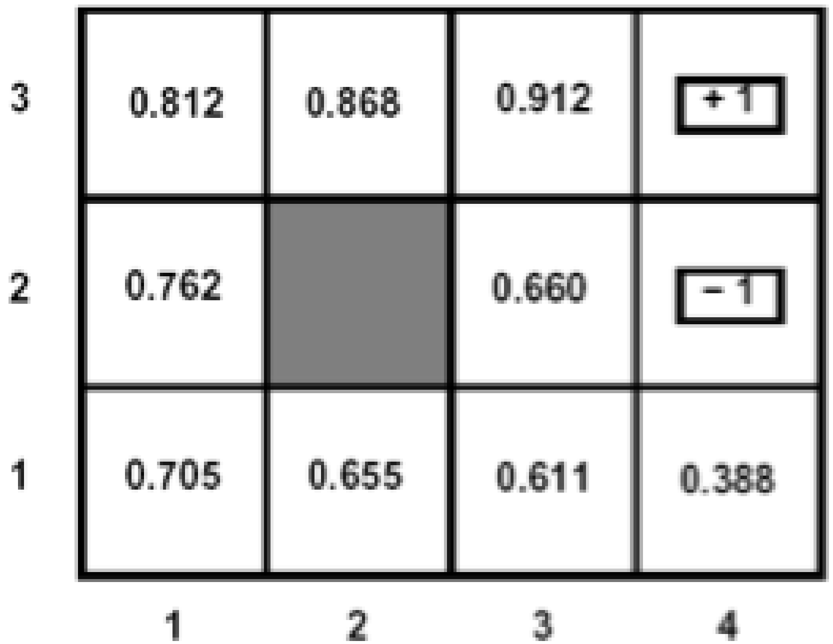
Until $\max_{s,a} |Q_{i+1}(s,a) - Q_i(s,a)| < \boldsymbol{\varepsilon}$

(i.e., until convergence of all Q values;

$\boldsymbol{\varepsilon}$ is a small positive value)

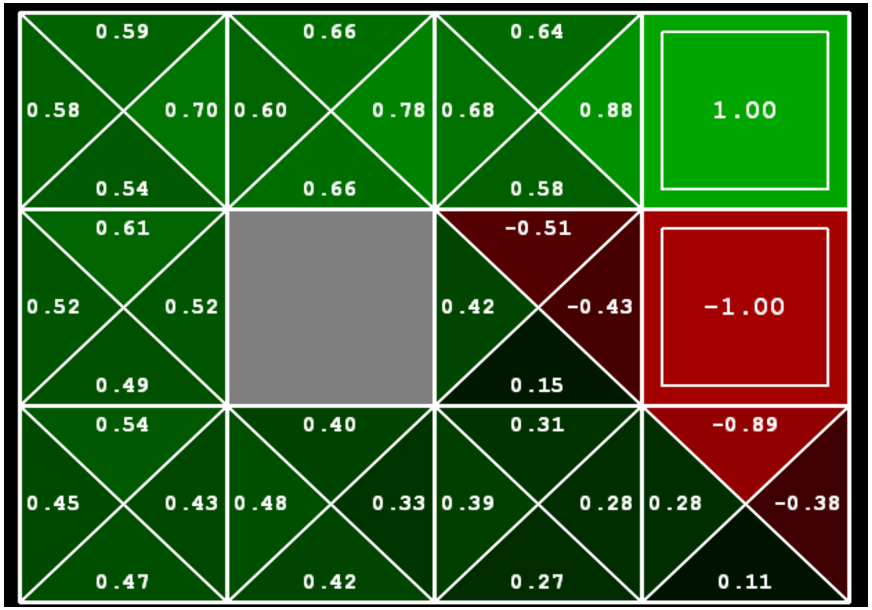
Example: Q-Value Iteration

Value Iteration



Numbers show V(s)

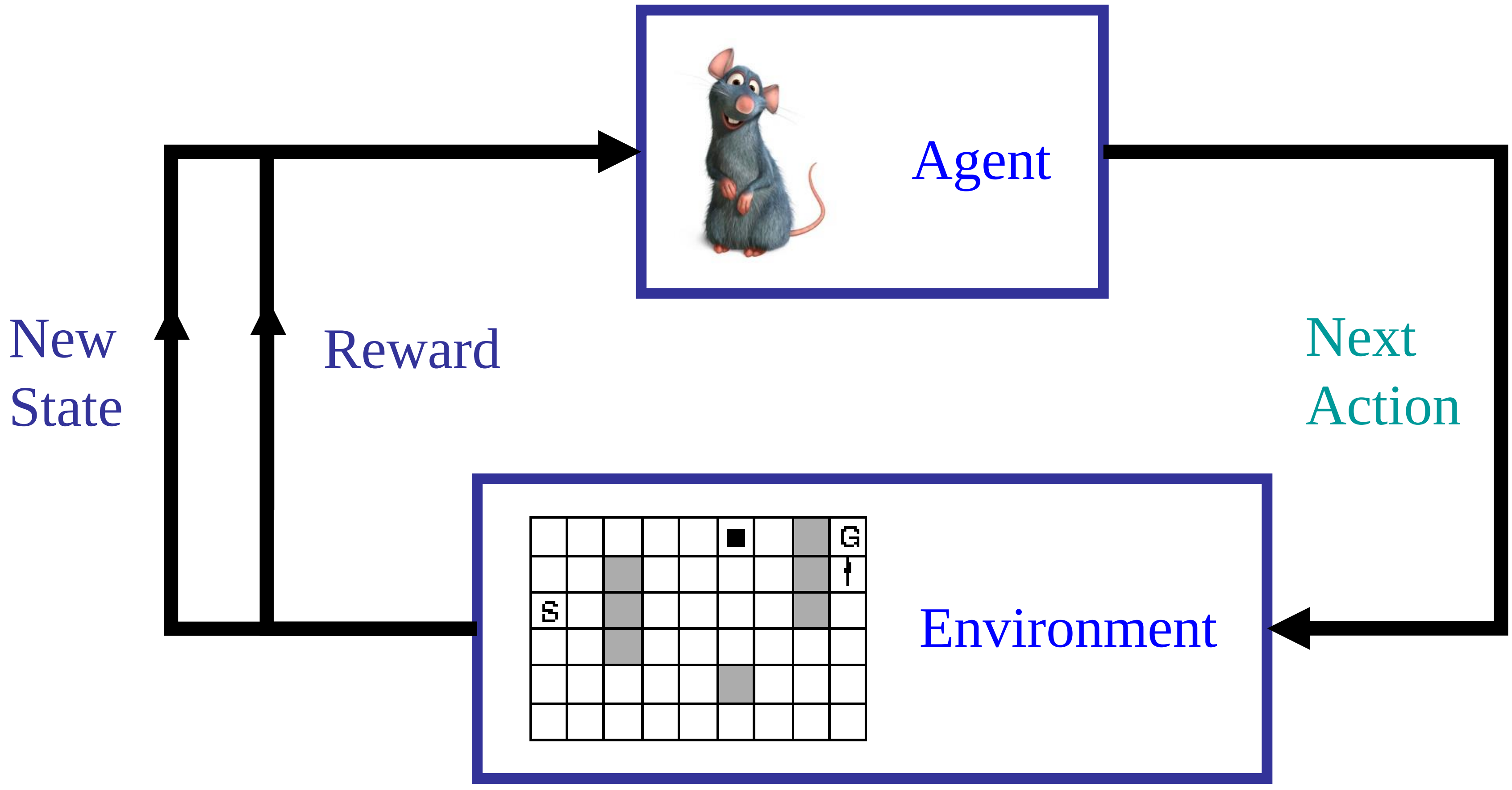
Q-Value Iteration



Numbers show Q(s,a)

What if we don’t know the transition model T(s,a,s’) and reward model R(s,a)?!

Enter…Reinforcement Learning (RL)



Agent doesn’t know the effects of actions

Agent doesn’t know which states are good

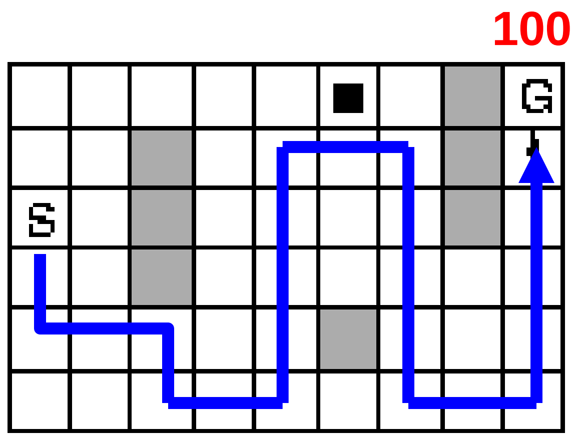
Try different actions and learn policy by trial-and-error!

Example: Animal Learning

- RL studied experimentally for more than 80 years in psychology and brain science
  - Rewards: food, pain, hunger, drugs, etc.
  - Evidence for RL in the brain via a chemical called dopamine
- Example: foraging
  - Bees can learn near-optimal foraging policy in field of artificial flowers with controlled nectar supplies

RL solves the “Credit Assignment” Problem

I’m in state 43,	reward = 0,	action = 2
“ “ “ 39,	“ = 0,	“ = 4
“ “ “ 22,	“ = 0,	“ = 1
“ “ “ 21,	“ = 0,	“ = 1
“ “ “ 21,	“ = 0,	“ = 1
“ “ “ 13,	“ = 0,	“ = 2
“ “ “ 54,	“ = 0,	“ = 2
“ “ “ 26,	“ = **100**,	



Yippee! I got to a state with a big reward!

But which of the actions along the way actually helped you get there??

RL solves this Credit Assignment problem

The Reinforcement Learning (RL) Problem

- Given: Set of states **S** and actions **A**
  - Do not know transition probabilities T
  - Do not know reward function R

- Interact with environment at each time step *t* :
  - Environment gives new state $\mathbf{s}_t$ and reward r_t
  - Choose next action $\mathbf{a}_t$

- **Goal**: Learn policy π that maximizes expected discounted sum of rewards

Two main approaches to RL

- Model-based approaches:
  - Explore environment & learn model T=P(**s**'|**s**,**a**) and R(**s**,**a**,**s'**)
  - Use model to compute policy MDP-style
  - Works well when state-space is small
- Model-free approach:
  - Don't learn a model
  - Learn value function (Q value) or policy *directly*
  - Works better when state space is large

Comparison of approaches

- Model-based approaches:

Learn T + R

$|S|^2|A| + |S||A|$ parameters (E.g., $200^2*10+200*10 = 402{,}000$)

- Model-free approach:

Learn Q

$|S||A|$ parameters (E.g., $200*10 = 2{,}000$)

We will focus on Q-learning (model-free approach)

- Adapt Q-value iteration idea to get “Q-learning”

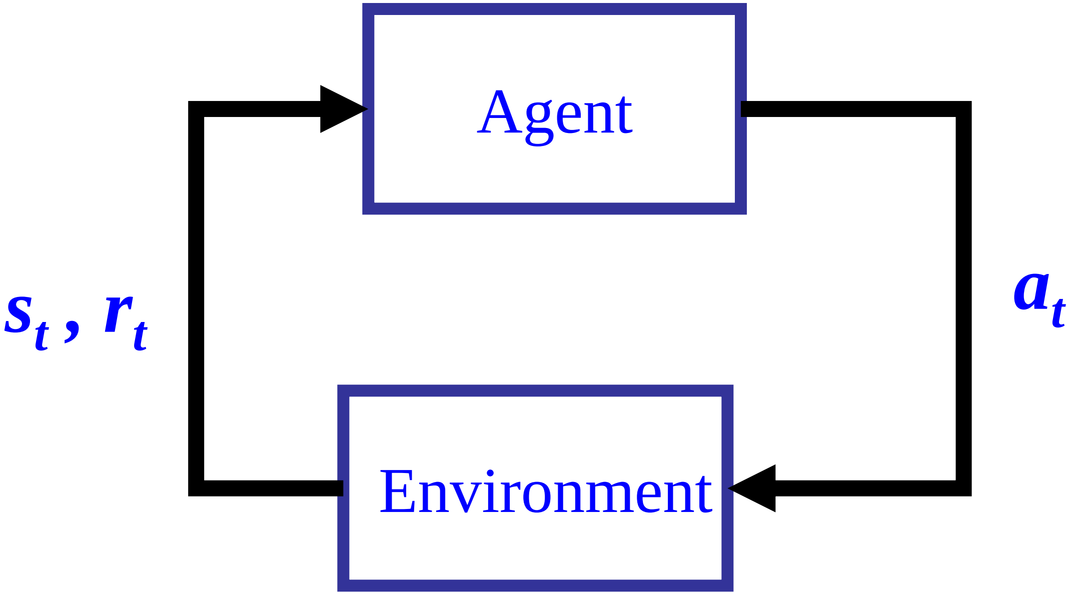
Recall: Q-value iteration

$$Q_{i+1}(s,a) \leftarrow \sum_{s'} T(s,a,s') \left[R(s,a,s') + \gamma \max_{a'} Q_i(s',a') \right]$$

In RL, we don't have this!

But we get a sample at each time step *t*:

$$(s_t, a_t, r_t, s_{t+1})$$



Q-learning Idea

Instead of expectation under *T*:

$$Q_{i+1}(s,a) \leftarrow \sum_{s'} T(s,a,s') \left[R(s,a,s') + \gamma \max_{a'} Q_i(s',a')\right]$$

what if we compute a running average of *Q* from all samples received thus far?

$$Q(s,a) \leftarrow \frac{1}{t} \sum_{t\,\text{samples}} \left(r + \gamma \max_{a'} Q(s',a')\right)$$

Why does this compute the correct expectation? Because environment produces samples at the right frequencies!

Running Average

- Running average of t samples of a quantity x:

$$\bar{x}_t = \frac{x_1 + x_2 + \ldots x_{t-1} + x_t}{t}$$

$$= \frac{x_1 + x_2 + \ldots x_{t-1}}{t} \cdot \frac{(t-1)}{(t-1)} + \frac{x_t}{t}$$

$$= \frac{(t-1)}{t}\bar{x}_{t-1} + \frac{1}{t} x_t$$

$$= (1-\alpha)\bar{x}_{t-1} + \alpha\, x_t \quad \text{where } \alpha = 1/t$$

- Running average of Q:

$$Q(s,a) \leftarrow (1-\alpha)Q(s,a) + \alpha(r + \gamma \max_{a'} Q(s',a'))$$

Q-Learning

- Q-Learning = Online ***sample-based*** Q-value iteration. At each time step:
  - Execute action and get new sample (s,a,s',r)
  - Incorporate new sample into **running average of *Q***:

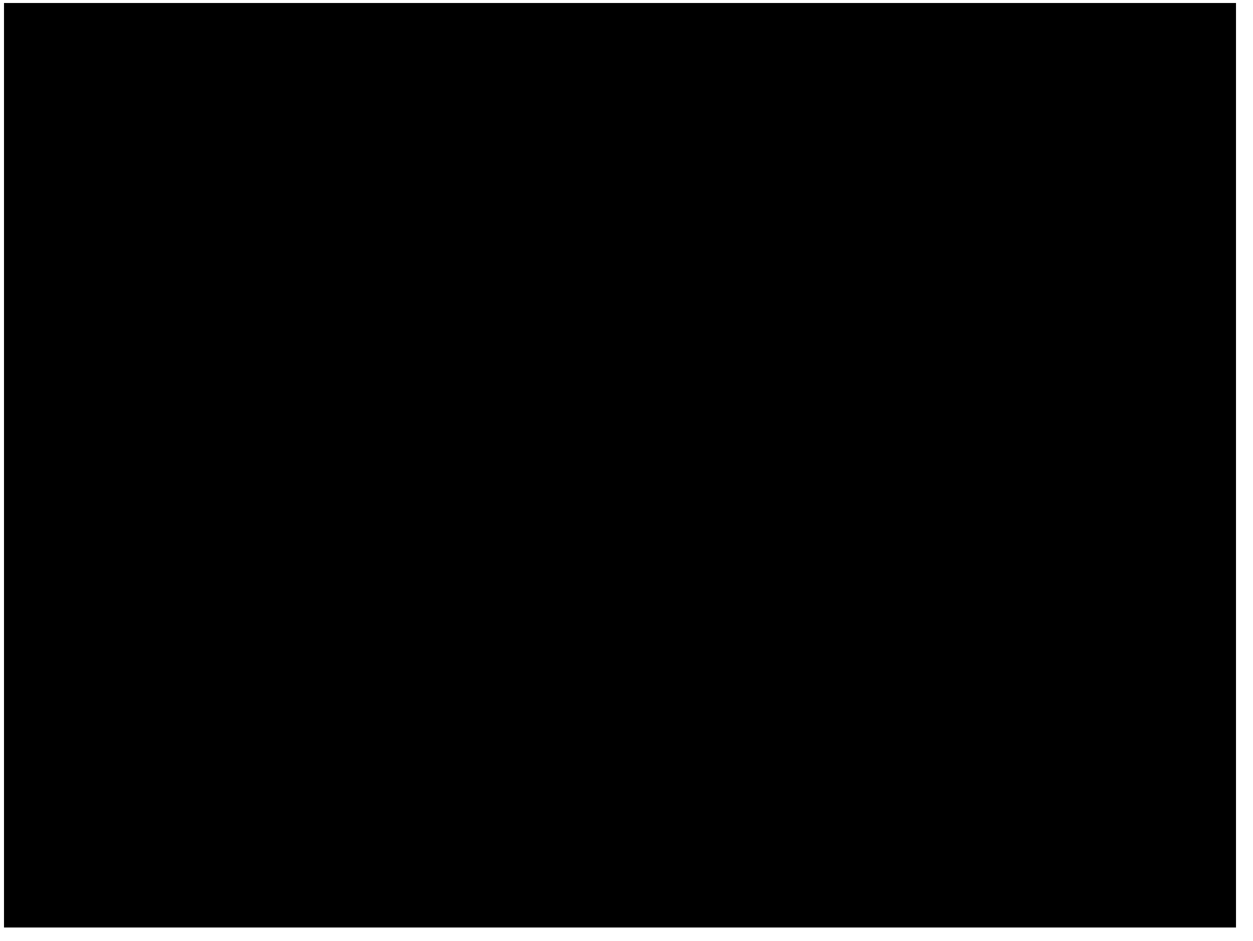
$$Q(s,a) \leftarrow (1-\alpha)Q(s,a) + \alpha(r + \gamma \max_{a'} Q(s',a'))$$

where α is the learning rate $(0 < \alpha < 1)$.

  - Update policy:

$$\pi(s) = \arg\max_{a} Q(s,a))$$

Q-learning example
(with manual control)



Next Time

- More on Model-Free Reinforcement learning
- To Do
  - Finish Chapter 17
  - Read Chapter 21
  - Start Project #3