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7. Worked example

Problem 7.1 Find all the eigenvalues, eigenvectors, and eigenspaces of

$$\mathbf{A} = \begin{pmatrix} -2 & 1 & 1 \\ 1 & -2 & 1 \\ 1 & 1 & -2 \end{pmatrix}.$$

Solution:

We found previously that the eigenvalues are 0, -3, where -3 is of multiplicity 2.

Eigenspace of 0:

This is NS(A), that is, the set of all solutions to

$$\mathbf{A}\mathbf{v} = \mathbf{0}$$

$$\begin{pmatrix} -2 & 1 & 1 \\ 1 & -2 & 1 \\ 1 & 1 & -2 \end{pmatrix} \mathbf{v} = \mathbf{0}.$$

We reduce the matrix to reduced row-echelon form:

$$egin{pmatrix} -2 & 1 & 1 \ 1 & -2 & 1 \ 1 & 1 & -2 \end{pmatrix} \quad o \quad egin{pmatrix} -2 & 1 & 1 \ 0 & -3/2 & 3/2 \ 0 & 3/2 & -3/2 \end{pmatrix} \quad o \quad egin{pmatrix} -2 & 1 & 1 \ 0 & -3/2 & 3/2 \ 0 & 0 & 0 \end{pmatrix}$$

Hence, z is a free parameter, and the eigenspace is given by

Eigenspace of
$$0 = NS(\mathbf{A}) = Span \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$
.

Eigenspace of -3:

We use the fact that NS(-3I - A) = NS(A + 3I) to avoid negative sign errors. The set of all solutions to

$$(\mathbf{A} + 3\mathbf{I})\mathbf{v} = \mathbf{0}$$

$$\begin{pmatrix} -2+3 & 1 & 1 \\ 1 & -2+3 & 1 \\ 1 & 1 & -2+3 \end{pmatrix} \mathbf{v} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \mathbf{v} = \mathbf{0}.$$

The matrix can be reduced to:

$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

Hence, both $m{y}$ and $m{z}$ can be free parameters and $m{x} = -m{y} - m{z}$. The eigenspace is given by

$$\text{Eigenspace of } -3 \, = \, \text{NS} \left(\mathbf{A} + 3 \mathbf{I} \right) \quad = \quad \text{Span} \left(\begin{array}{c} -1 \\ 1 \\ 0 \end{array} \right), \left(\begin{array}{c} -1 \\ 0 \\ 1 \end{array} \right).$$

This is a 2-dimensional vector space spanned by two independent vectors

$$egin{pmatrix} -1 \ 1 \ 0 \end{pmatrix}, egin{pmatrix} -1 \ 0 \ 1 \end{pmatrix}$$
 . Geometrically, this consists of all vectors on the plane $x=-y-z$.

Conclusion: The eigenvalues and corresponding eigenspaces of the matrix

$$\begin{pmatrix} -2 & 1 & 1 \\ 1 & -2 & 1 \\ 1 & 1 & -2 \end{pmatrix}$$
 are:

Eigenvalue Corresponding eigenspace

$$\lambda=0 \quad ; \quad \mathrm{Span} egin{pmatrix} 1 \ 1 \ 1 \end{pmatrix}$$

$$\lambda = -3 \quad ; \quad \mathrm{Span}\left(egin{pmatrix} -1 \ 1 \ 0 \end{pmatrix}, egin{pmatrix} -1 \ 0 \ 1 \end{pmatrix}
ight)$$

(The eigenvectors for each eigenvalue are all vectors in the corresponding eigenspace.) Notice that the eigenspace of $\bf -3$ is $\bf 2$ -dimensional.

Eigenvectors concept check

1/1 point (graded)

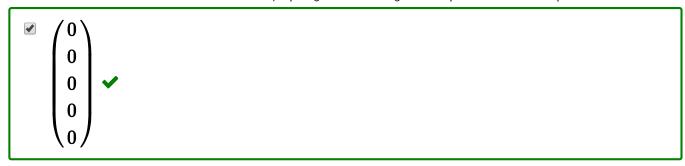
A 5×5 matrix **A** has eigenvalues 5, 1, 0.5.

If $\begin{pmatrix} 2 \\ 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}$ are two eigenvectors associated to the eigenvalue ${\bf 1}.$ Which of the

following must also be eigenvectors with eigenvalue ${f 1}$?

(Choose all that apply.)

 $\begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$





Solution:

Any linear combination of two eigenvectors corresponding to the same eigenvalue λ is again an eigenvector corresponding to λ (even if it is the $\bf 0$ vector).

• The vector $\begin{pmatrix} 0\\1\\0\\0\\0 \end{pmatrix}$ cannot be a linear combination of $\begin{pmatrix} 2\\0\\0\\1\\0 \end{pmatrix}$ and $\begin{pmatrix} 0\\0\\0\\1\\0 \end{pmatrix}$, because any

linear combinations of these given eigenvectors must have ${f 0}$ in the second component.

• All other choices of vector are linear combinations of the given eigenvectors and so are eigenvectors themselves.

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You have used 3 of 3 attempts

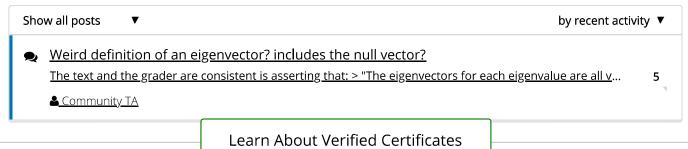
1 Answers are displayed within the problem

7. Worked example

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