Problem 1: Independent normal random variables

(3/3 points)

Let U, V, and W be independent standard normal random variables (that is, independent normal random variables, each with mean 0 and variance 1), and let $X = 3U + 4V$ and $Y = U + W$. Give a numerical answer for each part below. You may want to refer to the standard normal table .

1. What is the probability that $X \geq 8$?

$\mathbf{P}(X \geq 8) =$ 0.05479929 ✔

Answer: 0.0548

2.

$\mathbf{E}[XY] =$ 3 ✔

Answer: 3

3.

$\mathbf{var}(X+Y) =$ 33 ✔ **Answer:** 33

Answer:

1. Since X is a sum of independent normal random variables, X is also a normal random variable. Its mean and variance are

$$\begin{aligned}\mathbf{E}[X] &= \mathbf{E}[3U+4V] \\ &= 3\cdot\mathbf{E}[U]+4\cdot\mathbf{E}[V] \\ &= 0, \\ \mathbf{var}(X) &= \mathbf{var}(3U+4V) \\ &= 9\cdot\mathbf{var}(U)+16\cdot\mathbf{var}(V) \\ &= 25.\end{aligned}$$

Therefore,

$$\begin{aligned}\mathbf{P}(X\geq 8) &= \mathbf{P}\left(\frac{X-0}{5}\geq\frac{8-0}{5}\right) \\ &= 1-\Phi(1.6) \\ &\approx 1-0.9452 \\ &= 0.0548.\end{aligned}$$

2. Since U, V, and W are zero-mean and independent, we have

$$\begin{aligned}\mathbf{E}[XY] &= \mathbf{E}[(3U+4V)(U+W)] \\ &= \mathbf{E}[3U^2+3UW+4UV+4VW] \\ &= 3\cdot\mathbf{E}[U^2]+3\cdot\mathbf{E}[U]\mathbf{E}[W]+4\cdot\mathbf{E}[U]\mathbf{E}[V]+4\cdot\mathbf{E}[V]\mathbf{E}[W] \\ &= 3\cdot\mathbf{E}[U^2] \\ &= 3\cdot\left(\mathbf{var}(U)+(\mathbf{E}[U])^2\right) \\ &= 3.\end{aligned}$$

3. Since U, V, and W are independent with variance equal to 1, we have

$$\begin{aligned}\mathbf{var}(X+Y) &= \mathbf{var}(3U+4V+U+W) \\ &= \mathbf{var}(4U+4V+W) \\ &= \mathbf{var}(4U)+\mathbf{var}(4V)+\mathbf{var}(W) \\ &= 16\cdot 1+16\cdot 1+1 \\ &= 33.\end{aligned}$$

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