

sandipan\_dey >

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### 11.5.1 The Best Low Rank Approximation

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Week 11 due Dec 22, 2023 21:12 IST

# 11.5.1 The Best Low Rank Approximation

In the last slide of the below video you will find

$$x = (A^{T}A)^{-1}A^{T}b$$

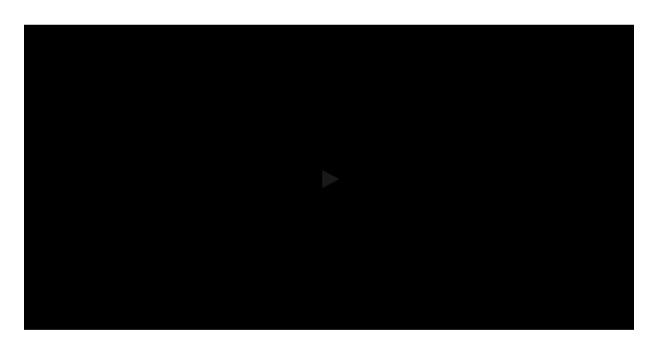
- $= \ ((U\Sigma V^T)^T(U\Sigma V^T))^{-1}(U\Sigma V^T)^Tb$
- $= \ (V\Sigma^T U^T U \Sigma V^T)^{-1} V \Sigma^T U^T b$
- $= \quad (V \Sigma \Sigma V^T)^{-1} V \Sigma U^T b$
- $= \quad ((V^T)^{-1}(\Sigma\Sigma)^{-1}V^{-1})V\Sigma U^Tb$
- $= \quad V^T \Sigma^{-1} \Sigma^{-1} \Sigma U^T b$
- $= V^T \Sigma^{-1} U_I^T b$

The last two lines should be

$$= V \Sigma^{-1} U^T b$$

$$= V \Sigma^{-1} \Sigma \Sigma^{-1} U^T$$

#### **Video**



Start of transcript. Skip to the end.

Dr. Robert van de Geijn: Now we're going to touch very briefly on one of the most important topics in linear algebra.

It's called the singular value decomposition, or SVD.

Unfortunately, this is an advanced topic that

acco bourned a tunical introductory

**o**:00 / 0:00

▶ 2.0x

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#### Video

**▲** Download video file

### **Transcripts**

### Reading Assignment

0 points possible (ungraded) Read Unit 11.5.1 of the notes. [<u>LINK</u>]



**⊞** Calculator

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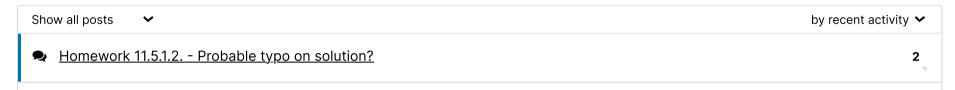
✓ Correct

Discussion

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**Topic:** Week 11 / 11.5.1

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#### Homework 11.5.1.1

1/1 point (graded)

Let  $B=U\Sigma V^T$  be the SVD of B, with  $U\in\mathbb{R}^{m imes r}$  ,  $\Sigma\in\mathbb{R}^{r imes r}$  , and  $V\in\mathbb{R}^{n imes r}$  . Partition

$$U = \left(egin{array}{c|c|c} u_0 & u_1 & \cdots & u_{r-1} \end{array}
ight), \Sigma = \left(egin{array}{c|c|c} \sigma_0 & 0 & \cdots & 0 \ \hline 0 & \sigma_1 & \cdots & 0 \ \hline \hline 0 & \sigma_1 & \cdots & 0 \ \hline dots & dots & \ddots & dots \ \hline 0 & 0 & \cdots & \sigma_{r-1} \end{array}
ight), V = \left(egin{array}{c|c|c|c} v_0 & v_1 & \cdots & v_{r-1} \end{array}
ight).$$
  $U \Sigma V^T = \sigma_0 u_0 v_1^T + \sigma_1 u_1 v_1^T + \cdots + \sigma_{r-1} u_{r-1} v_r^T \ldots$ 

Always ✓ Answer: Always

Answer: Always

$$= \underbrace{\left(\begin{array}{c|c|c} u_0 & u_1 & \cdots & 0 \\ \hline u_0 & u_1 & \cdots & 0 \\ \hline & \vdots & \vdots & \ddots & \vdots \\ \hline & 0 & 0 & \cdots & \sigma_{r-1} \end{array}\right)}_{\left(\begin{array}{c|c|c} \sigma_0 u_0 & \sigma_1 u_1 & \cdots & \sigma_{r-1} u_{r-1} \end{array}\right)} \underbrace{\left(\begin{array}{c|c|c} v_0 & v_1 & \cdots & v_{r-1} \end{array}\right)^T}_{\left(\begin{array}{c|c} v_0^T & \cdots & \sigma_{r-1} u_{r-1} \end{array}\right)}$$

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• Answers are displayed within the problem

#### Homework 11.5.1.2

2/2 points (graded)

Let  $B=U\Sigma V^T$  be the SVD of B with  $U\in\mathbb{R}^{m imes r}$  ,  $\Sigma\in\mathbb{R}^{r imes r}$  , and  $V\in\mathbb{R}^{n imes r}$  .

•  $\mathcal{C}(B) = \mathcal{C}(U)$ 

Always 

✓ Answer: Always

■ Calculator

Recall that if we can show that  $C(B) \subset C(U)$  and  $C(U) \subset C(B)$ , then C(B) = C(U).

 $C(B) \subset C(U)$ : Let  $y \in C(B)$ . Then there exists a vector x such that y = Bx. But then  $y = U \Sigma V^T x = Uz$ . Hence  $y \in C(U)$ .

z

 $C(U) \subset C(B)$ : Let  $y \in C(U)$ . Then there exists a vector x such that y = Ux. But

 $y = U \underbrace{\sum_{i=1}^{N} \sum_{j=1}^{N} x_{i}}_{I} = U \sum_{i=1}^{N} \underbrace{\sum_{j=1}^{N} x_{j}}_{W} = Bw. \text{ Hence } y \in C(B).$ 

•  $\mathcal{R}(B) = \mathcal{C}(V)$ 

Always ~

Answer: Always

Answer: Always

The proof is very similar, working with  $B^T$  since  $\mathcal{R}(B) = \mathcal{C}(B^T)$ .

Answer: Always

Recall that if we can show that  $C(B) \subset C(U)$  and  $C(U) \subset C(B)$ , then C(B) = C(U).

 $C(B) \subset C(U)$ : Let  $y \in C(B)$ . Then there exists a vector x such that y = Bx. But then  $y = U \underbrace{\Sigma V^T x}_{Z} = Uz$ . Hence  $y \in C(U)$ .

 $\mathcal{C}(U) \subset \mathcal{C}(B)$ : Let  $y \in \mathcal{C}(U)$ . Then there exists a vector x such that y = Ux. But

$$y = U \sum_{I} \underbrace{\nabla^{T} V \Sigma^{-1}}_{I} x = U \Sigma V^{T} \underbrace{V^{T} \Sigma^{-1} x}_{W} = Bw. \text{ Hence } y \in C(B).$$

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