5.2.2 Properties

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5.2.2 Properties

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Reading Assignment

0 points possible (ungraded)
Read Unit 5.2.2 of the notes. [LINK]

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Correct

Homework 5.2.2.1

4/4 points (graded)

Let $A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $B = \begin{pmatrix} 0 & 2 & -1 \\ 1 & 1 & 0 \end{pmatrix}$, and $C = \begin{pmatrix} 0 & 1 \\ 1 & 2 \\ 1 & -1 \end{pmatrix}$. Compute

AB =

- ( ) $\begin{pmatrix} 1 & 3 \\ 1 & 5 \end{pmatrix}$
- (•) $\begin{pmatrix} 1 & 1 & 0 \\ 0 & 2 & -1 \end{pmatrix}$
- ( ) $\begin{pmatrix} 1 & 5 \\ 1 & 3 \end{pmatrix}$

(AB)C =

- (•) $\begin{pmatrix} 1 & 3 \\ \end{pmatrix}$

Calculator

$\begin{pmatrix} 1 & 5 \end{pmatrix}$

- [ ] $\begin{pmatrix} 1 & 1 & 0 \\ 0 & 2 & -1 \end{pmatrix}$
- [ ] $\begin{pmatrix} 1 & 5 \\ 1 & 3 \end{pmatrix}$

✓

BC =

- [ ] $\begin{pmatrix} 1 & 3 \\ 1 & 5 \end{pmatrix}$
- [ ] $\begin{pmatrix} 1 & 1 & 0 \\ 0 & 2 & -1 \end{pmatrix}$
- [x] $\begin{pmatrix} 1 & 5 \\ 1 & 3 \end{pmatrix}$

✓

A(BC) =

- [x] $\begin{pmatrix} 1 & 3 \\ 1 & 5 \end{pmatrix}$
- [ ] $\begin{pmatrix} 1 & 1 & 0 \\ 0 & 2 & -1 \end{pmatrix}$
- [ ] $\begin{pmatrix} 1 & 5 \\ 1 & 3 \end{pmatrix}$

✓

Explanation

- $AB = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 2 & -1 \end{pmatrix}$
- $(AB)C = \begin{pmatrix} 1 & 3 \\ 1 & 5 \end{pmatrix}$
- $BC = \begin{pmatrix} 1 & 5 \\ 1 & 3 \end{pmatrix}$
- $A(BC) = \begin{pmatrix} 1 & 3 \\ 1 & 5 \end{pmatrix}$

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Answers are displayed within the problem

Homework 5.2.2.2

1/1 point (graded)

Let $A \in \mathbb{R}^{mxn}$, $B \in \mathbb{R}^{nxk}$, and $C \in \mathbb{R}^{kxl}$, then $(AB)C = A(BC)$

Calculator

Let $A \in \mathbb{R}$, $B \in \mathbb{R}$, and $C \in \mathbb{R}$, then $(AB)C = A(BC)$.

Always

✔ Answer: Always

Explanation

Answer:

Proof 1:

Two matrices are equal if corresponding columns are equal. We will show that $(AB)C = A(BC)$ by showing that, for arbitrary j, the jth column of $(AB)C$ equals the jth column of $A(BC)$. In other words, that $((AB)C)e_j = (A(BC))e_j$.

$$
\begin{array}{l}
((AB)C)e_j \\
\quad = \quad < \text{Definition of matrix-matrix multiplication} > \\
(AB)Ce_j \\
\quad = \quad < \text{Definition of matrix-matrix multiplication} > \\
A(B(Ce_j)) \\
\quad = \quad < \text{Definition of matrix-matrix multiplication} > \\
A((BC)e_j) \\
\quad = \quad < \text{Definition of matrix-matrix multiplication} > \\
(A(BC))e_j.
\end{array}
$$

Proof 2 (using partitioned matrix-matrix multiplication):

$$
\begin{array}{l}
(AB)C \\
\quad = \quad < \text{Partition by columns} > \\
(AB)\left(\begin{array}{c|c|c|c} c_0 & c_1 & \cdots & c_{n-1} \end{array} \right) \\
\quad = \quad < \text{Partitioned matrix-matrix multiplication} > \\
\left(\begin{array}{c|c|c|c} (AB)c_0 & (AB)c_1 & \cdots & (AB)c_{n-1} \end{array} \right) \\
\quad = \quad < \text{Definition of matrix-matrix multiplication} > \\
\left(\begin{array}{c|c|c|c} A(Bc_0) & A(Bc_1) & \cdots & A(Bc_{n-1}) \end{array} \right) \\
\quad = \quad < \text{Partitioned matrix-matrix multiplication} > \\
A\left(\begin{array}{c|c|c|c} Bc_0 & Bc_1 & \cdots & Bc_{n-1} \end{array} \right) \\
\quad = \quad < \text{Partitioned matrix-matrix multiplication} > \\
A(B\left(\begin{array}{c|c|c|c} c_0 & c_1 & \cdots & c_{n-1} \end{array} \right)) \\
\quad = \quad < \text{Partition by columns} > \\
A(BC)
\end{array}
$$

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Homework 5.2.2.3

16/16 points (graded)

Let $A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $B = \begin{pmatrix} 2 & -1 \\ 1 & 0 \end{pmatrix}$, and $C = \begin{pmatrix} -1 & 1 \\ 0 & 1 \end{pmatrix}$

- $A(B+C)$

Calculator

1	✔ Answer: 1	1	✔ Answer: 1
1	✔ Answer: 1	0	✔ Answer: 0

- $AB + AC$

1	✔ Answer: 1	1	✔ Answer: 1
1	✔ Answer: 1	0	✔ Answer: 0

- $(A + B)\,C$

-2	✔ Answer: -2	2	✔ Answer: 2
-2	✔ Answer: -2	2	✔ Answer: 2

- $AC + BC$

-2	✔ Answer: -2	2	✔ Answer: 2
-2	✔ Answer: -2	2	✔ Answer: 2

- $A(B+C) = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}.$
- $AB + AC = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}.$
- $(A+B)C = \begin{pmatrix} -2 & 2 \\ -2 & 2 \end{pmatrix}.$
- $AC + BC = \begin{pmatrix} -2 & 2 \\ -2 & 2 \end{pmatrix}.$

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Answers are displayed within the problem

Homework 5.2.2.4

1/1 point (graded)

Let $A \in \mathbb{R}^{m\times k}$, $B \in \mathbb{R}^{k\times n}$, and $C \in \mathbb{R}^{k\times n}$ then $A\,(B + C) = AB + AC$.

Always ✔ Answer: Always

Explanation

Answer: Always

$A(B+C)$

Calculator

$$
\begin{array}{l}
\quad = \quad < \text{ Partition } B \text{ and } C \text{ by columns } > \\
A\left(\left(\begin{array}{c|c|c|c} b_0 & b_1 & \cdots & b_{n-1} \end{array} \right) + \left(\begin{array}{c|c|c|c} c_0 & c_1 & \cdots & c_{n-1} \end{array} \right)\right) \\
\quad = \quad < \text{ Definition of matrix addition } > \\
A\left(\begin{array}{c|c|c|c} b_0 + c_0 & b_1 + c_1 & \cdots & b_{n-1} + c_{n-1} \end{array} \right) \\
\quad = \quad < \text{ Partitioned matrix-matrix multiplication } > \\
\left(\begin{array}{c|c|c|c} A(b_0 + c_0) & A(b_1 + c_1) & \cdots & A(b_{n-1} + c_{n-1}) \end{array} \right) \\
\quad = \quad < \text{ Matrix-vector multiplication distributes } > \\
\left(\begin{array}{c|c|c|c} Ab_0 + Ac_0 & Ab_1 + Ac_1 & \cdots & Ab_{n-1} + Ac_{n-1} \end{array} \right) \\
\quad = \quad < \text{ Defintion of matrix addition } > \\
\left(\begin{array}{c|c|c|c} Ab_0 & Ab_1 & \cdots & Ab_{n-1} \end{array} \right) + \left(\begin{array}{c|c|c|c} Ac_0 & Ac_1 & \cdots & Ac_{n-1} \end{array} \right) \\
\quad = \quad < \text{ Partitioned matrix-matrix multiplication } > \\
A\left(\begin{array}{c|c|c|c} b_0 & b_1 & \cdots & b_{n-1} \end{array} \right) + \left(\begin{array}{c|c|c|c} Ac_0 & Ac_1 & \cdots & Ac_{n-1} \end{array} \right) \\
\quad = \quad < \text{ Partition by columns } > \\
AB + AC.
\end{array}
$$

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Answers are displayed within the problem

Homework 5.2.2.5

1/1 point (graded)

If $A \in \mathbb{R}^{m \times k}, B \in \mathbb{R}^{m \times k}$, **and** $C \in \mathbb{R}^{k \times n}$, **then** $(A + B)\,C = AC + BC$

True ✔ Answer: True

Explanation

Answer: True

$$
\begin{array}{l}
(A + B)C \\
\quad = \quad < \text{Partition } C \text{ by columns.} > \\
(A + B)\left(\begin{array}{c|c|c|c} c_0 & c_1 & \cdots & c_{n-1} \end{array} \right) \\
\quad = \quad < DE \text{ means } D \text{ multiplies each of the columns of } E > \\
\left(\begin{array}{c|c|c|c} (A + B)c_0 & (A + B)c_1 & \cdots & (A + B)c_{n-1} \end{array} \right) \\
\quad = \quad < \text{Definition of matrix addition} > \\
\left(\begin{array}{c|c|c|c} Ac_0 + Bc_0 & Ac_1 + Bc_1 & \cdots & Ac_{n-1} + Bc_{n-1} \end{array} \right) \\
\quad = \quad < D + E \text{ means adding corresponding columns} > \\
\left(\begin{array}{c|c|c|c} Ac_0 & Ac_1 & \cdots & Ac_{n-1} \end{array} \right) + \left(\begin{array}{c|c|c|c} Bc_0 & Bc_1 & \cdots & Bc_{n-1} \end{array} \right) \\
\quad = \quad < DE \text{ means } D \text{ multiplies each of the columns of } E > \\
A\left(\begin{array}{c|c|c|c} c_0 & c_1 & \cdots & c_{n-1} \end{array} \right) + B\left(\begin{array}{c|c|c|c} c_0 & c_1 & \cdots & c_{n-1} \end{array} \right) \\
\quad = \quad < \text{Partition } C \text{ by columns} > \\
AC + BC.
\end{array}
$$

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Calculator

Answers are displayed within the problem

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