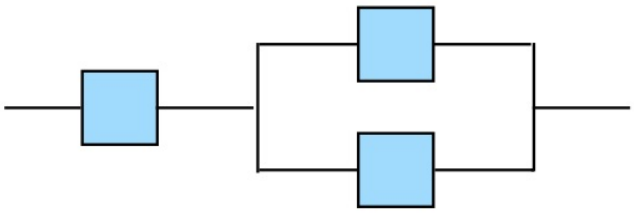
EXERCISE: RELIABILITY (4/4 points)

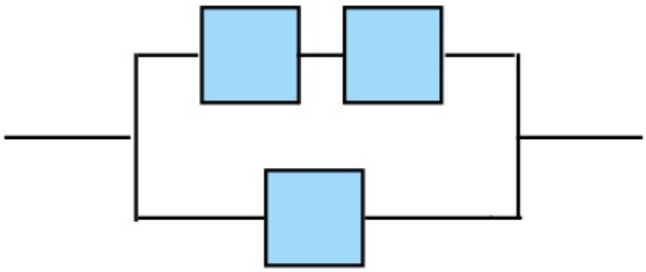
Suppose that each unit of a system is up with probability $2/3$ and down with probability $1/3$. Different units are independent. For each one of the systems shown below, calculate the probability that the whole system is up (that is, that there exists a path from the left end to the right end, consisting entirely of units that are up).

1. What is the probability that the following system is up?



0.5925926 ✔ **Answer:** 0.59259

2. What is the probability that the following system is up?



0.8148148 ✔ **Answer:** 0.81481

Answer:

1. In the first diagram, the parallel connection of the two units (on the right) is down when both units fail, which happens with probability $(1/3) \cdot (1/3) = 1/9$. Therefore the parallel connection is up with probability $1 - 1/9 = 8/9$. The overall system is up if the first unit is up (probability $2/3$) and the parallel connection is also up (probability $8/9$), which happens with probability $(8/9) \cdot (2/3) = 16/27$.

2. In the second diagram, the top path is up when both of its units are up – this happens with probability $(2/3) \cdot (2/3) = 4/9$. Thus it fails with probability $1 - 4/9 = 5/9$. The overall system fails when the top path fails (probability $5/9$) and the bottom path also fails (probability $1/3$). Thus the probability of failure is $(5/9) \cdot (1/3) = 5/27$. It follows that the probability that the system is up (does not fail) is $1 - 5/27 = 22/27$.

You have used 1 of 2 submissions