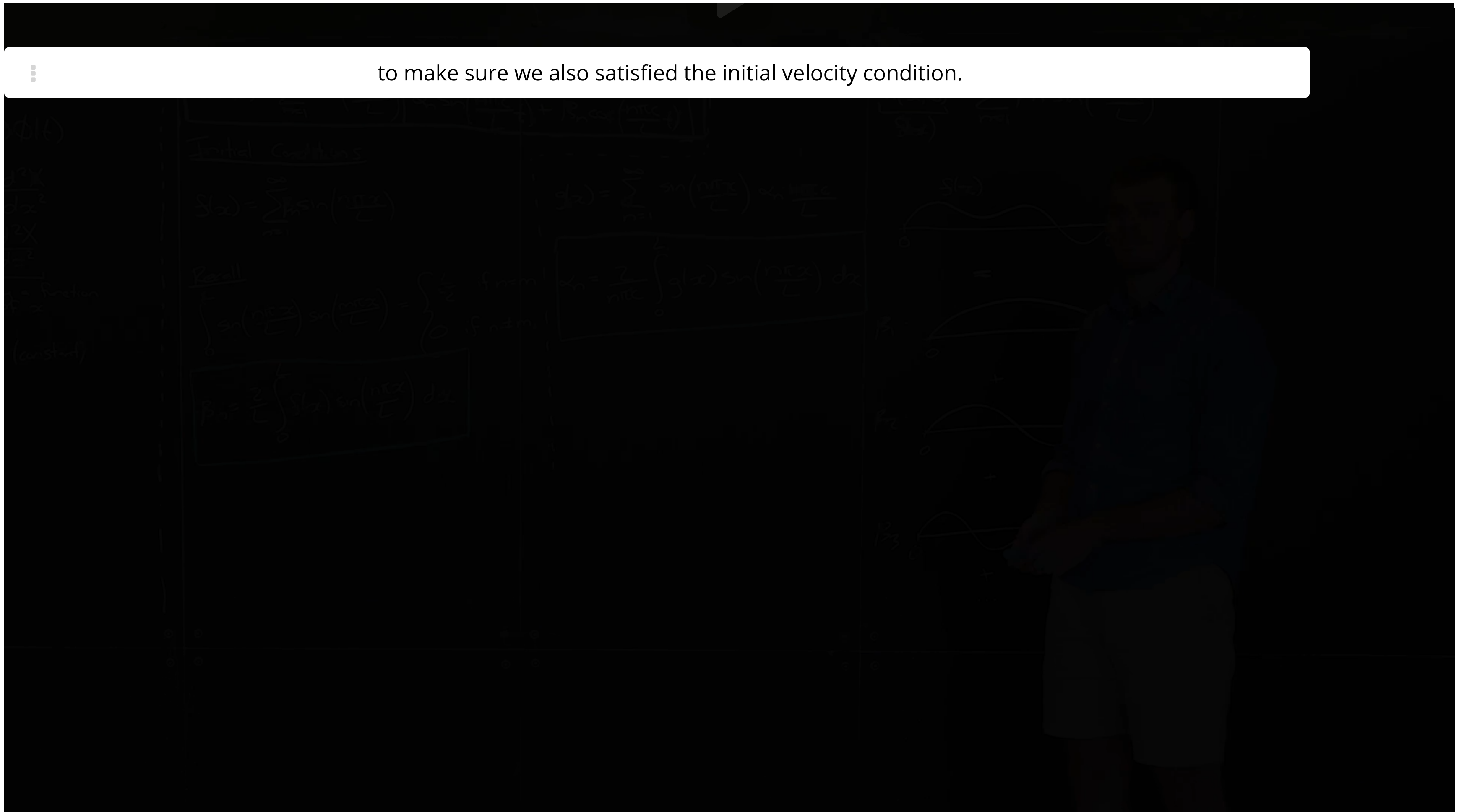
5. Initial conditions

Solving for initial conditions in the wave equation



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To specify a unique solution, we need two initial conditions: not only the initial position $u(x,0)$, but also the initial velocity $\frac{\partial u}{\partial t}(x,0)$, at each position of the string. (That **two** initial conditions are needed is related to the fact that the PDE is **second** -order in the t variable.)

For a plucked string, it is reasonable to assume that the initial velocity is 0, so one initial condition is $\frac{\partial u}{\partial t}(x,0) = 0$. What condition does this impose on the a_n and b_n? Well, for the general solution above,

$$\begin{aligned} \frac{\partial u}{\partial t} &= \sum_{n\geq 1} -na_n \sin(nt)\sin(nx) + \sum_{n\geq 1} nb_n \cos(nt)\sin(nx) \\ \frac{\partial u}{\partial t}(x,0) &= \sum_{n\geq 1} nb_n \sin(nx), \end{aligned}$$

so the initial condition says that $b_n = 0$ for every n; in other words,

$$u(x,t) = \sum_{n\geq 1} a_n \cos(nt)\sin(nx).$$

If we also knew the initial position $u(x,0)$, we could solve for the a_n by extending to an odd, period 2π function of x and using the Fourier coefficient formula.

Wave equation mathlet

The first few coefficients are the amplitudes of the “harmonics" in the Wave Equation Mathlet.

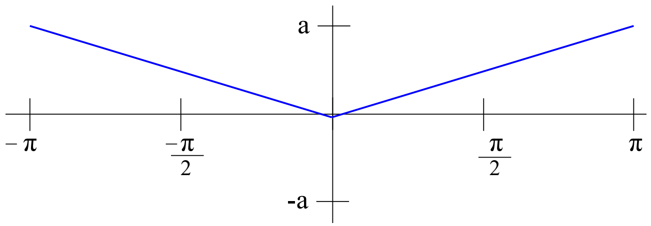
$$u(x,0) = \begin{cases} \frac{2a}{\pi}x & 0 < x < \pi/2 \\ \frac{2a}{\pi}(\pi - x) & \pi/2 < x < \pi \end{cases},$$

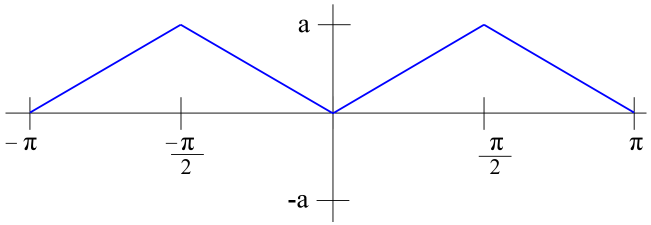
where a is a small positive number.

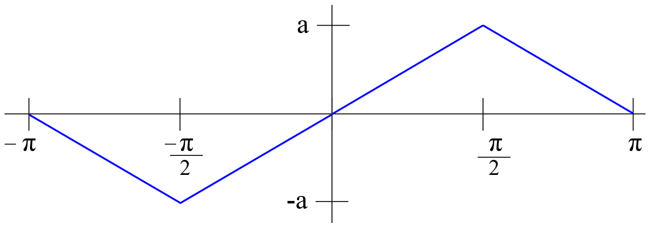
The general solution takes the form

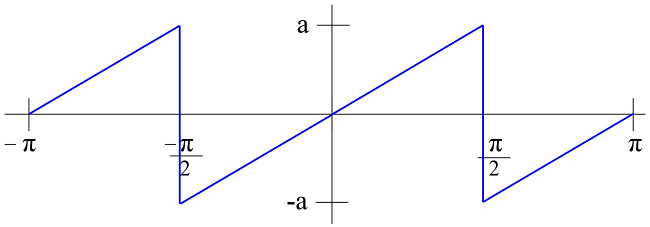
$$u(x,t) = \sum_{n \geq 1} a_n \cos(nt) \sin(nx).$$

To solve for the Fourier coefficients a_n, which of the following periodic extensions of $u(x,0)$ must you use?









Solution:

The general solution is a sine series in x, therefore it must be odd. That eliminates the first two choices.

The next point is that it must satisfy the initial condition on $0 < x < \pi$, which is given by a triangle of height a that peaks at $\pi/2$. Therefore the odd extension of this function is a 2π periodic function that is given by the graph in the third option.

The fourth option is the odd periodic sawtooth wave of period π, which does not match the initial condition given.

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Answers are displayed within the problem

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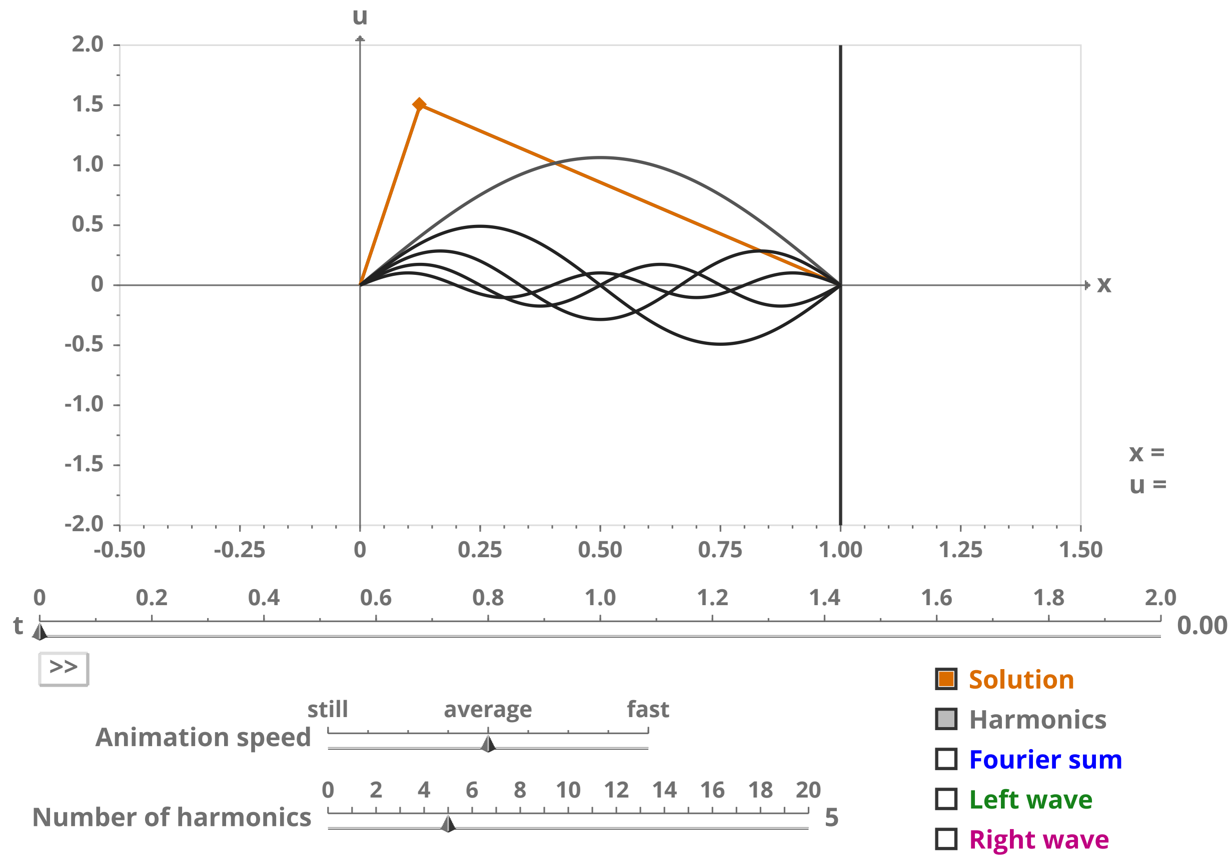

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Solve for the initial condition

1/1 point (graded)

Suppose a guitar string of length π is plucked. The initial velocity is zero, and the initial position is given by the function