13.1.2 Example: Minimizing mass of a metal can

MO2.11

Suppose a cylindrical metal can is being designed to hold a volume V. Let the radius of the can be r and the height of the can be h. Then, the volume is

$$V = \pi r^2 h \tag{13.1}$$

Since V will be given, then we can re-arrange this equation to give h as a function of V and r,

$$h = \frac{V}{\pi r^2} \tag{13.2}$$

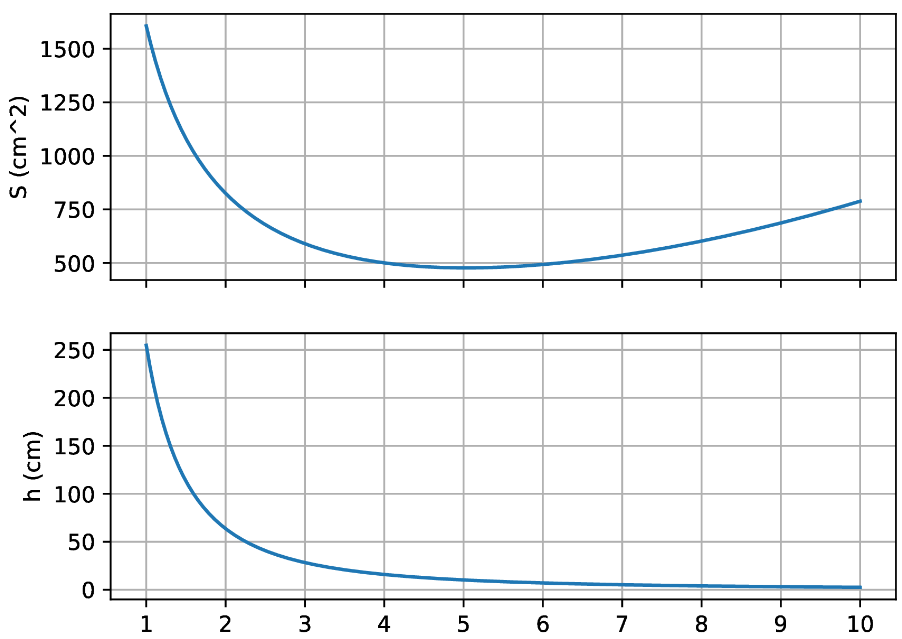
Then, assume that the metal is well approximated as having uniform density and thickness, and so the mass of the metal used in the can will be directly proportional to the surface area of the can, i.e.

$$m = \rho_m t_m S \tag{13.3}$$

where ρ_m and t_m are the density and thickness of the metal, and S is the surface area of the can. For a given ρ_m and t_m, minimizing the mass of the metal used is equivalent to minimizing S. The surface area of the can is:

$$S = 2\pi r^2 + 2\pi r h = 2\pi r^2 + \frac{2V}{r} \tag{13.4}$$

Let's consider the specific case of a can to hold $V = 800\,\mathrm{cm}^3$. Figure 13.1 shows the variation of S and h with respect to r. From the plot, the minimum $S \approx 500\,\mathrm{cm}^2$ occurs for $r \approx 5.0$ cm and $h \approx 10.0$ cm.



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r (cm)

Figure 13.1: Surface area S and height h of a cylindrical can with 800 cm^3 volume as a function of the can radius r

Beyond plotting $S(r)$ to find the minimum S and corresponding r (and h) values, let's consider what the mathematical conditions are at the minimum of S, specifically in terms of the first and second derivatives of S with respect to r:

- First derivative: At the minimum, the first derivative of S must be zero, i.e. $\mathrm{d}S/\mathrm{d}r\left(r_{\text{min}}\right) = 0$. Suppose that instead, $\mathrm{d}S/\mathrm{d}r < 0$ at $r = r_{\text{min}}$. Then, by increasing r a small amount $r = r_{\text{min}} + \epsilon$, the value of S would decrease (where ϵ is a small perturbation). Similarly, suppose that instead, $\mathrm{d}S/\mathrm{d}r > 0$ at $r = r_{\text{min}}$. Then a small decrease in $r = r_{\text{min}} - \epsilon$ would result in a decrease in S. Thus, $\mathrm{d}S/\mathrm{d}r\left(r_{\text{min}}\right) = 0$ at the minimum.

- Second derivative: Consider a Taylor series of S with respect to r at the minimum:

$$S\left(r_{\text{min}} + \epsilon\right) \approx S\left(r_{\text{min}}\right) + \epsilon\frac{\mathrm{d}S}{\mathrm{d}r}\left(r_{\text{min}}\right) + \frac{1}{2}\epsilon^2\frac{\mathrm{d}^2 S}{\mathrm{d}r^2}\left(r_{\text{min}}\right) + \frac{1}{6}\epsilon^6\frac{\mathrm{d}^3 S}{\mathrm{d}r^3}\left(r_{\text{min}}\right) + \cdots \tag{13.5}$$

  Since we have established the first derivative is zero this minimum, then this becomes:

$$S\left(r_{\text{min}} + \epsilon\right) \approx S\left(r_{\text{min}}\right) + \frac{1}{2}\epsilon^2\frac{\mathrm{d}^2 S}{\mathrm{d}r^2}\left(r_{\text{min}}\right) + \frac{1}{6}\epsilon^3\frac{\mathrm{d}^3 S}{\mathrm{d}r^3}\left(r_{\text{min}}\right) + \cdots \tag{13.6}$$

  If the second derivative of S is positive, then S will

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  second derivative is negative, then S would decrease and then r_{min} would actually not be a

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$$\left(V \right)^{1/3} \quad (13.10)$$