4. Application to geometry

Recitation due Aug 18, 2021 20:30 IST Completed

Apply

Shortest distance

1.0/1.0 point (graded)

Food for thought.

Let L be the line $2x - y = 1$, just like in previous problems. Find the point of L which is closest to $(1, 0)$.

Hint: Sketch L, and draw the shortest line segment going from $(1, 0)$ to L. This line segment is perpendicular to L.

(Enter a point as two numbers surrounded by round parentheses: e.g. type `(1,0)` for $(1, 0)$.)

(3/5,1/5) ✔ **Answer:** (3/5,1/5)

Solution:

We know that the distance will be perpendicular to the graph of $2x - y = 1$. Therefore, this distance will be parallel to the vector $\langle 2, -1 \rangle$. So to reach the desired point, we need to add some some unknown multiple of $\langle 2, -1 \rangle$ to the point $(1, 0)$ until reaching the line. So we need to answer the question: for what value of λ is the point $P = \langle 1, 0 \rangle + \lambda \langle 2, -1 \rangle$ in the line $2x - y = 1$?

The point P is given by:

$$P = (1 + 2\lambda, -\lambda) \tag{3.79}$$

Plugging these values in for x and y in the equation for the line leads to the equation:

$$2\,(1 + 2\lambda) - (-\lambda) = 1 \tag{3.80}$$

The solution is $\lambda = -1/5$. Therefore, the desired point P is given by $(1 - 2/5, 1/5) = (3/5, 1/5)$.

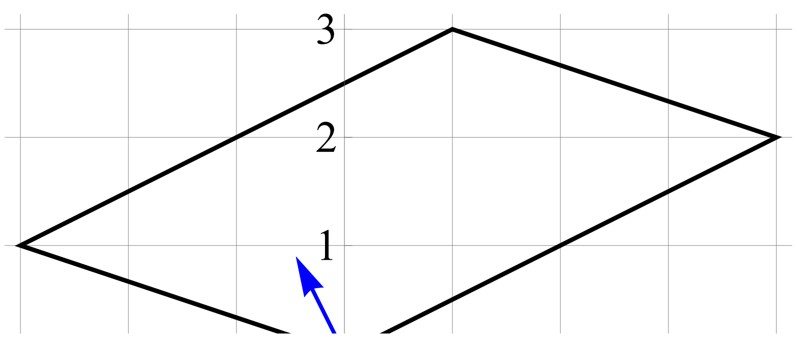
Submit You have used 2 of 10 attempts

Answers are displayed within the problem

Area of a parallelogram

1.0/1.0 point (graded)

Consider the parallelogram with vertices at $(0, 0)$, $(4, 2)$, $(1, 3)$, and $(-3, 1)$.



Let's call the side from $(0, 0)$ to $(4, 2)$ the base of the parallelogram. The unit normal vector to the base is $\langle -2/\sqrt{20}, 4/\sqrt{20}\rangle$, which is drawn in the picture. The length of the base is $\sqrt{20}$. Find the area of the parallelogram.

Hint: The area is the base times the height. You can find the height by using dot products.

10 ✔ **Answer:** 10

Solution:

The length of the base is $\sqrt{20}$ by the Pythagorean Theorem. What about the length of the height?

Let the drawn normal vector be $\vec{n}$, and let $\vec{v}$ be the vector pointing from $(0, 0)$ to $(-3, 1)$. By trigonometry, the length of the height is given by $|\vec{v}| \cos\theta$ where θ is the angle between $\vec{v}$ and $\vec{n}$. The only unknown is $\cos\theta$, which we can find using dot products:

$$\cos\theta = \frac{\vec{n} \cdot \vec{v}}{|\vec{n}||\vec{v}|} \tag{3.81}$$

Therefore, we have

$$\begin{aligned} \text{area} &= \sqrt{20} \cdot (\text{height}) && (3.82) \\ &= \sqrt{20} \cdot \left(|\vec{v}| \cdot \frac{\vec{n} \cdot \vec{v}}{|\vec{n}||\vec{v}|}\right) && (3.83) \\ &= \sqrt{20} \cdot (\vec{n} \cdot \vec{v}) && (3.84) \end{aligned}$$

The last step results from cancelling the common $|\vec{v}|$ and using $|\vec{n}| = 1$.

Since $\vec{v} = \langle -3, 1\rangle$, we have $\vec{n} \cdot \vec{v} = 6/\sqrt{20} + 4/\sqrt{20} = 10/\sqrt{20}$. Thus,

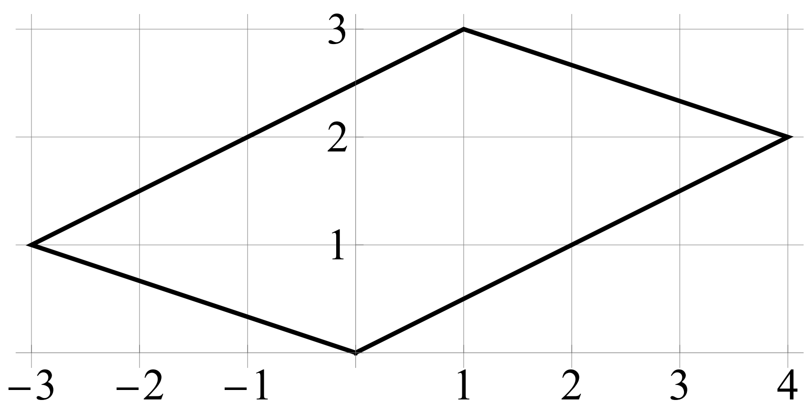
$$\text{area} = \sqrt{20} \cdot \frac{10}{\sqrt{20}} = 10 \tag{3.85}$$

Submit You have used 1 of 10 attempts

Answers are displayed within the problem

Discuss

Food for thought. Consider the parallelogram with vertices at $(0, 0)$, $(4, 2)$, $(1, 3)$, and $(-3, 1)$. The parallelogram is shown in the picture below.



How can we use vectors and dot products to check that these points are in fact the corners of a perfect parallelogram? (Hint: How is a parallelogram defined?)

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4. Application to geometry

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Alternative Method for Shortest Distance I am afraid that my algebra knowledge is keeping me from learning/applying the vector ideas covered thus far in this class. I found th...	2
I typed an answer for question 1 and it says partially corrsect. its the right answer i only got half credit	2
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Alternative Method for Area of a Parallelogram: Scalar Multiplication The unit normal vector given is in the same line as the point (-1,2), therefore the vector (-1,2) = λ*(-2/sqrt(20),4/sqrt(20)). The dista...	1
? [staff] Shortest distance Two different approaches to this question have been used and both gave the same answer. However, this answer has been marked ...	7
What is a perfect parallelogram? Is it the one whose opposite sides are equal ore something different?	6
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Is it coincidence? I used dot product between two sides without using normal vector to find the area which turned out to be correct. Is it mere coinced...	4
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[Staff] Typo in Area of parallelogram solution In the *Area of parallelogram* solution, line (3.85), the equal sign is placed incorrectly.	2

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