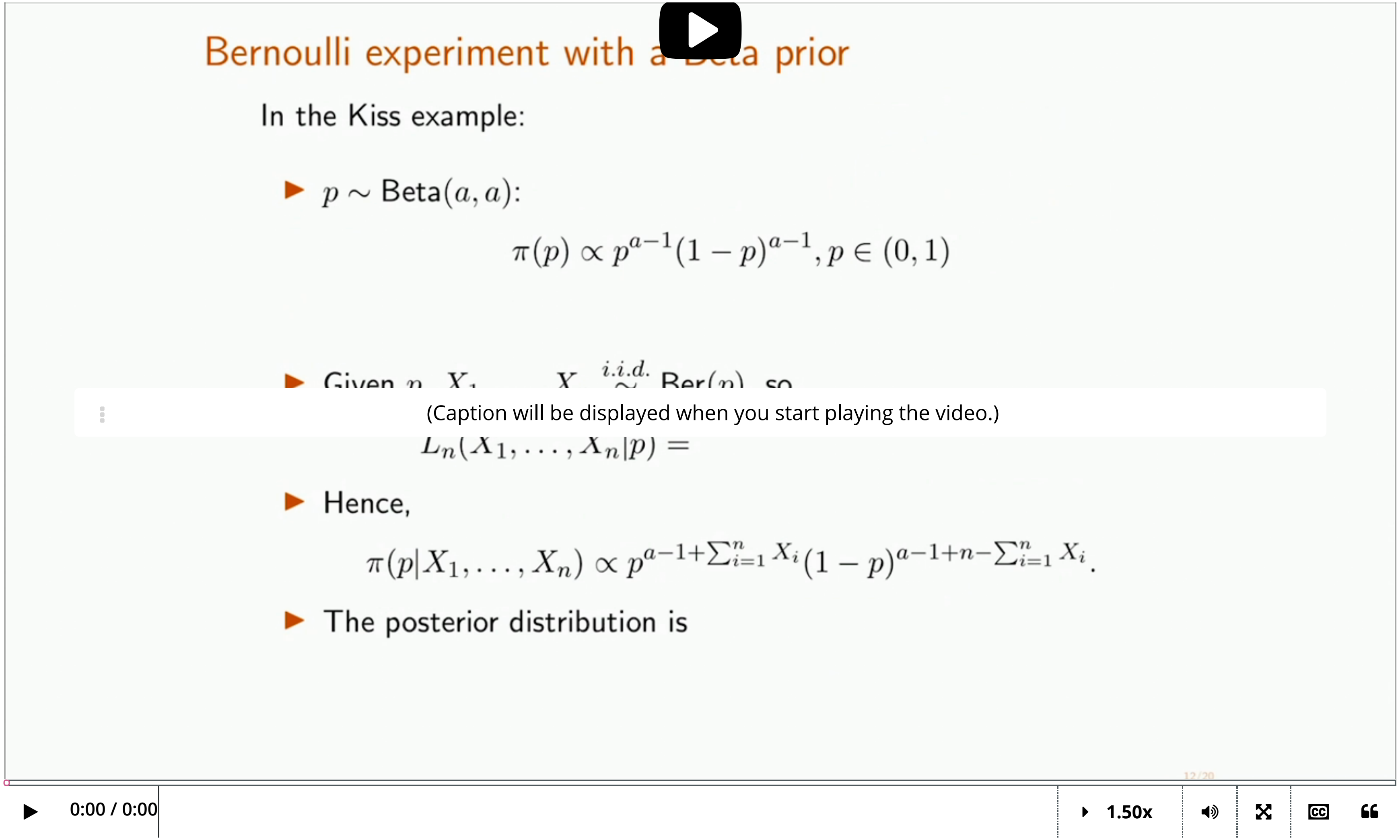
# 9. Bayes' Formula with the Beta Distribution

## Application: Bernoulli Experiment with the Beta Prior

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## Posterior Update : A Concrete Example

2/2 points (graded)

You are playing a computer game, where at each step you either succeed with probability $\theta$ or fail with probability $1-\theta$. Assume that the outcomes of the games are independent. Based on previous knowledge, you select a prior $\pi(\theta) \sim \mathrm{Beta}(a,b)$.

Assume that you play this game for $12$ rounds, with $7$ successes and $5$ failures. Find the posterior distribution $\pi(\theta|X_1,\ldots,X_{12})$.

It is known that the posterior is a Beta distribution, so you just need to state the parameters $a'$ and $b'$ below (the primes simply indicate that these parameters are the updated versions of aforementioned $a, b$).

$a' =$

a+7

✔ **Answer:** a+7

$a+7$

$b' =$ b+5

✔ **Answer:** b+5

$b+5$

STANDARD NOTATION

**Solution:**

- Note that if we interpret $X_i = 1$ for a success and $X_i = 0$ for a failure, the experiment simply is an instance of i.i.d. Bernoulli trials. $L_n(X_1,\ldots,X_n|\theta)$ therefore computes as

$$p_n(X_1,\ldots,X_n|\theta) = \theta^{\sum_{i=1}^n X_i}(1-\theta)^{n-\sum_{i=1}^n X_i}.$$

- Using the update rule for the Beta prior discussed in lecture,

$$\begin{aligned}\pi(\theta|X_1,\ldots,X_n) &\propto p_n(X_1,\ldots,X_n|\theta)\,\pi(\theta)\\ &\propto \theta^{a-1}(1-\theta)^{b-1}\theta^{\sum_{i=1}^n X_i}(1-\theta)^{n-\sum_{i=1}^n X_i}\\ &\propto \theta^{a+\sum_{i=1}^n X_i-1}(1-\theta)^{b+n-\sum_{i=1}^n X_i-1}.\end{aligned}$$

Therefore, the updated versions are

$$a' = a + \sum_{i=1}^{n} X_i = a + 7,$$

and

$$b' = b + n - \sum_{i=1}^{n} X_i = b + 5.$$

Thus,

$$\pi\left(\theta | X_1, \ldots, X_{12}\right) \sim \mathrm{Beta}\left(a + 7, b + 5\right).$$

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You have used 1 of 3 attempts

Answers are displayed within the problem

## Advantages of Bayesian View

1/1 point (graded)

Which one of the following statements below illustrates the advantages of Bayesian view over the frequentist approach?

- (selected) The Bayesian approach gives statisticians some freedom to reflect prior belief.
- The Bayesian approach is computationally more tractable.

An estimator that takes the maximum of the posterior distribution obtained via Bayes rule is strictly closer to the actual parameter than the maximum likelihood estimator.

**Solution:**

The first choice is the correct answer.

- The main power of Bayesian approach comes from the fact that, designer can reflect the prior information in terms of a cleverly-engineered prior distribution.
- The second statement is false. Bayesian approach is actually computationally more expensive: normalizing the posterior distribution (via Bayes' rule) involves computing an integral in the denominator which might not have a simple solution.
- The third item is also false. A counterexample is that the maximum a-posteriori and maximum likelihood are actually the same if one has a uniform prior.

Submit You have used 1 of 1 attempt

Answers are displayed within the problem

## Discussion

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**Topic:** Unit 5 Bayesian statistics:Lecture 17: Introduction to Bayesian Statistics / 9. Bayes' Formula with the Beta Distribution

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? [Staff] Posterior Update answer
I gave a correct answer and grader response was "'a" is not permitted" and when checked it was permitted?
4

Beta prior + Experiment => Beta posterior. Everytime or only with Bernoulli experiments ?
I can't get these two sentences together: > So here's our conjugate thing. You can see that no matter what you > observe, if you start with a beta prior, you're going to get a be...
2