1. Robots

Problem Set B due Aug 18, 2021 20:30 IST Completed

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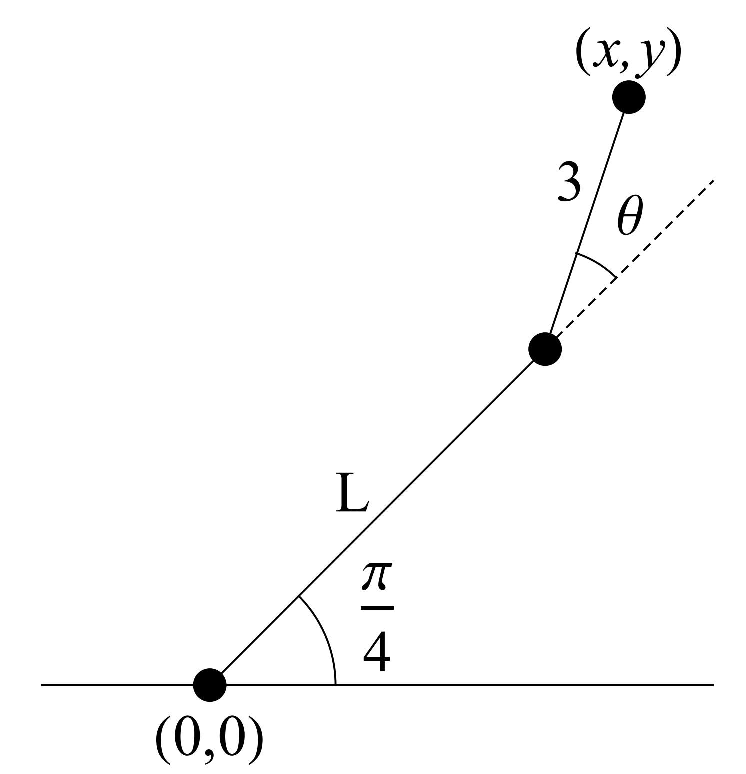
Problem 1 (a)

1.0/1 point (graded)

Consider the robot arm shown below.



There is a joint at the origin $(0, 0)$, and a bar extends from this joint. The bar is fixed at an angle of $\pi/4$ with the x-axis, but its length L is adjustable. At the end of the first bar is a second joint, and a bar of length 3 extends out of this second joint. Let θ be the angle between the first bar and the second bar as in the picture. We will refer to the end of the second bar as the tip of the robot's finger.

In terms of L and θ, find the position of the tip of the robot's finger represented by the point (x, y). (Enter your answer as an order pair in parentheses, e.g. `(x, y)`. You may type `theta` for θ, and `pi` for the mathematical constant π.)

$(x, y) =$

(L/sqrt(2)+3*cos(theta+pi/4),L/sqrt(2)+3*sin(theta+pi/4))

Answer: (L*sqrt(2)/2+3*cos(theta+pi/4),L*sqrt(2)/2+3*sin(theta+pi/4))

? INPUT HELP

Solution:

The tip of the finger is the sum of two vectors: (1) the one from the origin to the joint along the first bar, and (2) the one along the second bar from the joint to the tip. We calculate these individually. The first one is given by

$$\langle L\cos(\pi/4), L\sin(\pi/4)\rangle = \langle L/sqrt2, L/\sqrt{2}\rangle.$$

For the second vector, notice that if the second bar is at an angle θ as drawn, it the second bar forms an angle of $\frac{\pi}{4} + \theta$ with the x-axis.

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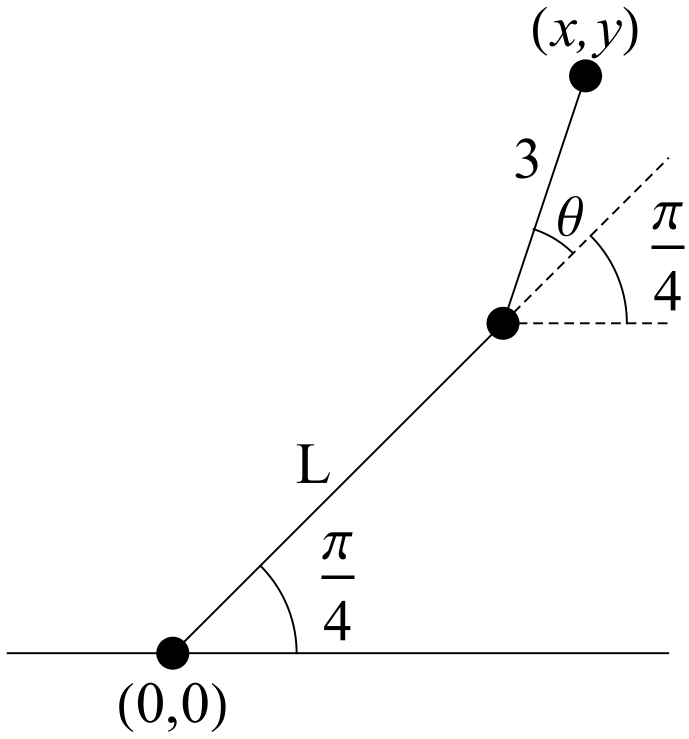

So the vector from the joint to the tip is given by

$$\langle 3\cos(\theta+\pi/4), 3\sin(\theta+\pi/4)\rangle.$$

Adding these two vectors gives the position of the tip of the figure

$$\left(\frac{L}{\sqrt{2}}+3\cos\left(\frac{\pi}{4}+\theta\right), \frac{L}{\sqrt{2}}+3\sin\left(\frac{\pi}{4}+\theta\right)\right)$$

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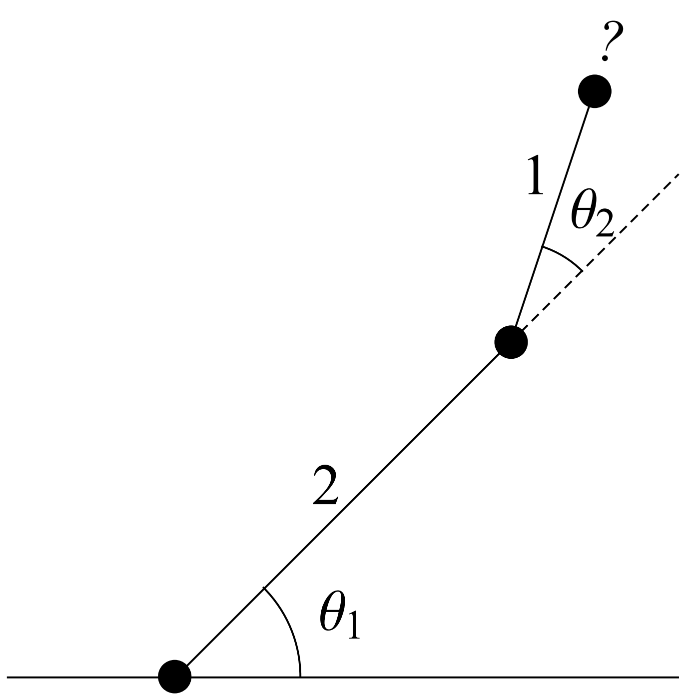
Answers are displayed within the problem

Problem 1(b)

2/2 points (graded)

In Lecture 4, we began studying a simple robotic arm with two joints. Here is a picture of the robot.



The robot arm has a base along the x-axis. There is a joint at $(0,0)$, and a bar of length 2 that comes out of this joint. Let θ_1 be the angle from the positive x axis to the first bar, as in the picture. At the end of the first bar is a second joint, and a second bar of length 1 comes out of the second joint. Let θ_2 be th
bar and the second bar, as in the picture. We will refer to the end of the second bar a

Let the position of the tip of the finger by $(x(\theta_1,\theta_2), y(\theta_1,\theta_2))$.

In lecture, we computed a formula for the position as a function of θ_1 and θ_2.

$$(x(\theta_1,\theta_2), y(\theta_1,\theta_2)) = \left(2\cos\theta_1 + \cos(\theta_1+\theta_2), 2\sin\theta_1 + \sin(\theta_1+\theta_2)\right).$$

Suppose that the robot starts with $\theta_1 = \pi/6$ and $\theta_2 = \pi/3$. Using the formula above, we can compute that the tip of the robot's finger is at $(\sqrt{3}, 2)$. We want the robot to move the tip of its finger a small distance straight up - to the point $(\sqrt{3}, 2.01)$. How we should adjust θ_1 and θ_2 to get that to happen?

This problem is too complicated to solve exactly! We would have to solve the equations

$$\sqrt{3} = x(\theta_1,\theta_2) = 2\cos\theta_1 + \cos(\theta_1+\theta_2),$$

and

$$2.01 = y(\theta_1,\theta_2) = 2\sin\theta_1 + \sin(\theta_1+\theta_2).$$

These equations are sufficiently complicated that Larry (the instructor of the course) has no idea how to solve them. But you can use the linear approximation to get a good approximate answer. If you take the linear approximation of $x(\theta_1,\theta_2)$ and the linear approximation of $y(\theta_1,\theta_2)$, then you will have much simpler functions to work with, and these simpler functions still give a good approximation of the real behavior of the robot.

Find $\Delta\theta_1$ and $\Delta\theta_2$.

$\Delta\theta_1 \approx$ 0.01/sqrt(3) ✔ **Answer:** 0.01/sqrt(3)

$\Delta\theta_2 \approx$ -0.02/sqrt(3) ✔ **Answer:** -0.02/sqrt(3)

? INPUT HELP

Solution:

Let $\theta_1 = \frac{\pi}{6} + \Delta\theta_1$ and $\theta_2 = \frac{\pi}{3} + \Delta\theta_2$. Then

$$\begin{aligned}
\Delta x = x(\theta_1,\theta_2) - x(\pi/6,\pi/3) &\approx \frac{\partial x}{\partial\theta_1}\Delta\theta_1 + \frac{\partial x}{\partial\theta_2}\Delta\theta_2 && (3.140)\\
\sqrt{3}-\sqrt{3} &\approx \left(-2\sin\left(\frac{\pi}{6}\right) - \sin\left(\frac{\pi}{6}+\frac{\pi}{3}\right)\right)\Delta\theta_1 - \sin\left(\frac{\pi}{6}+\frac{\pi}{3}\right)\Delta\theta_2 && (3.141)\\
0 &\approx -2\Delta\theta_1 - \Delta\theta_2 && (3.142)
\end{aligned}$$

Similarly,

$$\begin{aligned}
\Delta y = y(\theta_1,\theta_2) - y(\pi/6,\pi/3) &\approx \frac{\partial y}{\partial\theta_1}\Delta\theta_1 + \frac{\partial y}{\partial\theta_2}\Delta\theta_2 && (3.143)\\
2.01 - 2 &\approx \left(2\cos\left(\frac{\pi}{6}\right) + \cos\left(\frac{\pi}{6}+\frac{\pi}{3}\right)\right)\Delta\theta_1 + \cos\left(\frac{\pi}{6}+\frac{\pi}{3}\right)\Delta\theta_2 && (3.144)\\
0.01 &\approx \sqrt{3}\Delta\theta_1 \qquad (\text{since } \cos(\pi/2) = 0) && (3.145)
\end{aligned}$$

Therefore $\Delta\theta_1 = 0.01/\sqrt{3}$ and $\Delta\theta_2 = -0.02/\sqrt{3}$.

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Answers are displayed within the problem

1. Robots

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- Robots: Solving for theta1 and theta2
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- ? The solution got cropped
  Am I the only one who experience this? Actually, we can still see it by shrinking the notations (and make the blue arrows appear), th... — 5
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  Could someone please give me a hint in part 1(b). Thank you :3 — 2
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