15. Lecture 1 Summary

Summarize

Big Picture

1. Multivariable functions are harder to visualize. One way to visualize such a function is to look at slices of the function at different heights. The plot looks like a hiking map and can be used to understand the graph in space more easily.

2. There is more than one notion of a derivative when we consider multivariable functions. Here we look at slicing the function with vertical planes in the x and y direction to obtain the notion of a partial derivative.

Mechanics

Definition 15.1 The **level curves** of a function $f(x,y)$ are given by $f(x,y) = k$ where k is a constant.

Definition 15.2 The **partial derivative of $f(x,y)$ with respect to** x is defined by

$$f_x(x,y) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x, y) - f(x,y)}{\Delta x}. \tag{2.33}$$

The Leibniz notation for this is $\dfrac{\partial f}{\partial x}$.

Definition 15.3 The **partial derivative of $f(x,y)$ with respect to** y is defined by

$$f_y(x,y) = \lim_{\Delta y \to 0} \frac{f(x, y+\Delta y) - f(x,y)}{\Delta y}. \tag{2.34}$$

The Leibniz notation for this is $\dfrac{\partial f}{\partial y}$.

Spoiler: Partial derivatives in higher dimensions

Definition 15.4 For a function in n dimensions $f(x_1, x_2, \ldots, x_n)$, the partial derivative with respect to the variable x_k is defined by

$$f_{x_k} = \lim_{\Delta x_k \to 0} \frac{f(x_1, \ldots, x_k + \Delta x_k, \ldots, x_n) - f(x_1, \ldots, x_k, \ldots, x_n)}{\Delta x_k} \tag{2.35}$$

for $1 \leq k \leq n$. The Leibniz notation for this is $\dfrac{\partial f}{\partial x_k}$.

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Ask yourself

Is there a product rule for partial derivatives?

Yes, it is the same product rule from single-variable-calculus. For example, you can use the product rule to find $\frac{\partial f}{\partial x}(1-x)(2x+3y) = (-1)(2x+3y) + (1-x)(2)$. We use the product rule because we have the product of two functions that both depend on x.

You can skip the product rule if one of the terms in the product is constant with respect to x. For example, $\frac{\partial}{\partial x}(1-x)e^y = -e^y$.

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Is there a quotient rule or chain rule for partial derivatives?

Yes, they are the same rules as in single variable calculus. You need to use the quotient rule when you have $\frac{\partial}{\partial x}\frac{A}{B}$ and A, B each depend on x. You need to use the chain rule when you have something like $\frac{\partial}{\partial x}\sin(A)$ and A depends on x.

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Is a function of two variables two-dimensional or three-dimensional?

The phrasing is ambiguous: it's best to say the function $f(x,y)$ depends on **two real variables**. The confusion arises because the **graph of $f(x,y)$ is three-dimensional**, being made up of the points $(x, y, f(x,y))$. However, the **contour plot of $f(x,y)$ is two-dimensional** since it consists of the points (x,y) such that $f(x,y) = k$ for fixed values of k.

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Surfaces and curves on planes, spaces,
The last question about the dimension of a two-variable function makes me think that we need a 3D space to represent it, since the ... 2

[STAFF] typo 2

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