A way out?

My favorite answer to the Two-Envelope Paradox is due to Broome, and to New York University philosopher David Chalmers. One starts by asking a simple question: in a Two-Envelope setup such as Broome's, what is the expected value of a given envelope? As I'll ask you to verify below, the answer is that each envelope has *infinite* expected value.

As it turns out, this is no accident: one can prove that a two-envelope setup can only lead to trouble when the expected value of the envelopes is infinite. But, as Broome and Chalmers point out, we have independent reasons for thinking that decisions involving options of infinite expected value are problematic.

Consider, for example, the **St. Petersburg Paradox**. A coin is tossed until it lands Heads. If it lands Heads on the nth toss you are given $\$2^n$. How much should you pay for the privilege of playing such a game? The expected value of playing is infinite:

$$EV\,(\text{play}) = \frac{1}{2^1}\cdot 2^1 + \frac{1}{2^2}\cdot 2^2 + \cdots = 1 + 1 + \ldots = \infty$$

So (assuming you value only money) the Principle of Value Maximization entails that you should be willing to pay *any finite amount of money* for the privilege of playing. And this is surely wrong: it is surely reasonable to balk at the prospect of paying, e.g. a million dollars for a chance to play.

I'm inclined to think that Broome and Chalmers are right to think that decisions involving options of infinite expected value are problematic, and that this helps answer the Two-Envelope Paradox. I must confess, however, that I do not feel ready to let the matter rest. Like the St. Petersburg Paradox, the Two-Envelope Paradox suggests that our decision theoretic tools fall short when we try to reason with infinite expected value. But I'm not sure I have a satisfactory way of theorizing about such cases. If I were to assign the Two-Envelope Paradox a "paradoxicality grade" of the kind we used in Lecture 3, I would choose a number somewhere between 6 and 8. It is clear that the paradox brings out some important limitations of decision-theory, but I do not feel able to gauge just how important they are.

Problem 1

1/1 point (ungraded)

Suppose that an envelope is filled using Broom's method: a die is tossed until it lands One or Two. If the die first lands One or Two on the kth toss, the envelope is filled with 2^{k-1} dollars. What is the expected value of the envelope?

- ( ) $2^{k-1} \cdot k$
- ( ) $2^{k-1} + k^2$
- (•) ∞

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Explanation

We know from the previous exercises that the probability that the die first lands One or Two on the kth toss (and that the envelope is therefore filled with 2^{k-1}) is $2^{k-1}/3^k$. So the expected value of the envelope is:

$$\begin{aligned} EV\,(\text{envelope}) &= \frac{2^0}{3^1}\cdot 2^0 + \frac{2^1}{3^2}\cdot 2^1 + \frac{2^2}{3^3}\cdot 2^2 + \ldots \\ &= \frac{(2^0)^2}{3^1} + \frac{(2^1)^2}{3^2} + \frac{(2^2)^2}{3^3} + \ldots \\ &= \frac{(2^2)^0}{3^1} + \frac{(2^2)^1}{3^2} + \frac{(2^2)^2}{3^3} + \ldots \\ &= \frac{4^0}{3^1} + \frac{4^1}{3^2} + \frac{4^2}{3^3} + \ldots \\ &= \infty \end{aligned}$$

To verify that that the final identity holds, note that $\frac{4^k}{3^{k+1}} < \frac{4^{k+1}}{3^{k+2}}$ for any k, since $\frac{4^{k+1}}{3^{k+2}} = \frac{4^k}{3^{k+1}} \cdot \frac{4}{3}$

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