

Independent chances of 3 events $\frac{a}{a+x}$, $\frac{b}{b+x}$, $\frac{c}{c+x}$

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Here's a problem from my probability textbook:







X 1 Of three independent events the chance that the first *only* should happen is a; the chance of the second *only* is b; the chance of the third *only* is c. Show that the independent chances of the three events are respectively

$$\frac{a}{a+x}$$
, $\frac{b}{b+x}$, $\frac{c}{c+x}$,

where x is a root of the equation

$$(a+x)(b+x)(c+x) = x^2.$$

Here's what I did. We have

$$a=p_1(1-p_2)(1-p_3), \quad b=(1-p_1)p_2(1-p_3), \quad c=(1-p_1)(1-p_2)p_3.$$

Without loss of generality let's consider a. We have

$$p_1 = \frac{a}{(1 - p_2)(1 - p_3)}.$$

If we assume the result we want to show, then this equals

$$p_1=rac{a}{\left(1-rac{b}{b+x}
ight)\left(1-rac{c}{c+x}
ight)}=rac{a}{rac{x^2}{(b+x)(c+x)}}=rac{a}{a+x}.$$

However, we assumed in part what we wanted to show, which possibly makes this circular.

Another observation I noticed is that

$$p_1p_2p_3 + p_1p_2(1-p_3) + p_1(1-p_2)p_3 + (1-p_1)p_2p_3 + (1-p_1)(1-p_2)(1-p_3) + a + c = 1.$$

However, despite what I've tried, I'm stuck and do not know how to proceed further. Could anybody help me? Is there a way to turn my circular approach into a noncircular one?

probability combinatorics algebra-precalculus polynomials

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edited 50 mins ago

asked 1 hour ago





2 — Hint: think about what x should be as an expression of p_1, p_2, p_3 . – user3257842 39 mins ago

3 Answers

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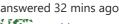
Note that you want $p_1=rac{a}{a+x}$. Since $a=p_1(1-p_2)(1-p_3)$, then $a + x = (1 - p_2)(1 - p_3)$. You can further simplify $x = (1 - p_1)(1 - p_2)(1 - p_3)$. This satisfies the equation in the question.



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sankhya 11 2

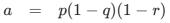


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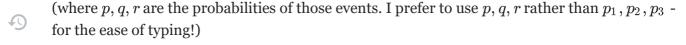
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$$b = (1-p)q(1-r)$$

$$c = (1-p)(1-q)r$$



Now, let x = (1-p)(1-q)(1-r) - the probability that *none* of the events happened. Then, obviously:

$$\frac{a}{a+x} = \frac{p(1-q)(1-r)}{p(1-q)(1-r)+(1-p)(1-q)(1-r)}
= \frac{p(1-q)(1-r)}{(1-q)(1-r)}
= p$$

and similarly for q and r. Moreover:

$$(a+x)(b+x)(c+x) = (1-q)(1-r) \cdot (1-p)(1-r) \cdot (1-p)(1-q)$$

= $(1-p)^2 (1-q)^2 (1-r)^2$
= x^2

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answered 35 mins ago



Stinking Bishop **12.7k** 14 29



To get corner cases out of the way, if $p_1 = 1$ then b = c = 0, and x = 1 - a will satisfy all the wanted properties. If $p_1 = 0$, then a = 0 and the problem reduces to a similar one with only

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1 two events. The same goes for p_2 and p_3 , so from here on assume $0 < p_1 < 1, 0 < p_2 < 1$, and $0 < p_3 < 1$.



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From your observation

$$a=p_1(1-p_2)(1-p_3), \quad b=(1-p_1)p_2(1-p_3), \quad c=(1-p_1)(1-p_2)p_3$$

we can get

$$(1-p_1)(1-p_2)(1-p_3)=rac{1-p_1}{p_1}a=rac{1-p_2}{p_2}b=rac{1-p_3}{p_3}c$$

If we call this value $x = (1 - p_1)(1 - p_2)(1 - p_3)$, then x > 0 and

$$x = \frac{1 - p_1}{p_1}a$$

$$\frac{x}{a} = \frac{1}{p_1} - 1$$

$$\frac{1}{p_1} = \frac{a + x}{a}$$

$$p_1 = \frac{a}{a + x}$$

Solving for p_2 and p_3 works just the same.

To show that $(a+x)(b+x)(c+x)=x^2$, it will be easier to look at the left side divided by x^3 :

$$\frac{(a+x)(b+x)(c+x)}{x^3} = \left(1 + \frac{a}{x}\right)\left(1 + \frac{b}{x}\right)\left(1 + \frac{c}{x}\right)$$

$$= \left(1 + \frac{p_1}{1-p_1}\right)\left(1 + \frac{p_2}{1-p_2}\right)\left(1 + \frac{p_3}{1-p_3}\right)$$

$$= \frac{1}{(1-p_1)(1-p_2)(1-p_3)} = \frac{1}{x}$$

Now just multiply both sides by x^3 to get the polynomial equation in the question.

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