4. Column space

Find y

1/1 point (graded)

Find the value of y such that the columns of $\mathbf{A} = \begin{pmatrix} 1 & 1 & 2 \\ 3 & y & 4 \\ 5 & 1 & 6 \end{pmatrix}$ linearly dependent?

$y =$ 1 ✔ **Answer:** 1

Solution:

There are many ways to solve this problem.

One way to find the value of y such that the columns are the matrix of $\mathbf{A}$ are linearly dependent is to determine what value of y allows you to express the second column is a linear combination of the first and third column. Observe that

$$\begin{pmatrix} 2 \\ 4 \\ 6 \end{pmatrix} - \begin{pmatrix} 1 \\ 3 \\ 5 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}.$$

Therefore the value of y that makes the matrix singular is $y = 1$.

Another solution is to compute the determinant in terms of y, and find the value of y that makes the determinant 0:

$$\begin{aligned} \det(\mathbf{A}) &= 1(6y-4) - 1(18-20) + 2(3-5y) \\ &= -4y+4. \end{aligned}$$

Therefore the determinant equals zero when $y = 1$.

Submit You have used 1 of 5 attempts

Answers are displayed within the problem

A vector not in the column space

1/1 point (graded)

Let $\mathbf{A} = \begin{pmatrix} 1 & 1 & 2 \\ 3 & y & 4 \\ 5 & 1 & 6 \end{pmatrix}$ be the matrix where y is the value you found in the previous problem.

Which of the vectors are **not** in the column space of $\mathbf{A}$? (Choose all that are not in the column space.)

- [ ] $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$
- [ ] $\begin{pmatrix} 1 \\ 3 \\ 5 \end{pmatrix}$
- [x] $\begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$ ✔
- [x] $\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$ ✔

☑ $\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ ✔

✔

Solution:

We will go through the columns one by one.

- $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ is the second column of $\mathbf{A}$, therefore it is in the column space of $\mathbf{A}$.
- $\begin{pmatrix} 1 \\ 3 \\ 5 \end{pmatrix}$ is the first column of $\mathbf{A}$, therefore it is in the column space of $\mathbf{A}$.
- $\begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$ is perpendicular to both of the columns of $\mathbf{A}$. If we dot this vector with the two column vectors we get zero. Therefore this vector is orthogonal to every vector in the span of the columns, it is not in the column space. In fact, we can use this vector to give a defining equation for the column space. The column space of $\mathbf{A}$ is the same as the nullspace of $\begin{pmatrix} 1 & -2 & 1 \end{pmatrix}$, in other words, all solutions to the equation $x - 2y + z = 0$.
- $\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$ is not in the column space because it does not satisfy the defining equation.
- $\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ is not in the column space because it does not satisfy the defining equation.

Submit You have used 1 of 4 attempts

Answers are displayed within the problem

MATLAB exploration

Continue to set $\mathbf{A} = \begin{pmatrix} 1 & 1 & 2 \\ 3 & y & 4 \\ 5 & 1 & 6 \end{pmatrix}$, where the value of y is such that the columns are linearly dependent as found in the first problem.

Let the vector $\mathbf{c}$ be a vector not in the column space of $\mathbf{A}$. (See the previous problem.)

Then $\mathbf{Ax} = \mathbf{c}$ has no solution. What solution do you find if you try to solve this equation using MATLAB?

Let the vector $\mathbf{b}$ be the first column of $\mathbf{A}$. What solution do you find if you type `Ab` into MATLAB?

Note that you do get an error message before the answer is displayed, but an answer will be displayed for you in both cases.

4. Column space

Hide Discussion

Topic: Unit 2: Linear Algebra, Part 2 / 4. Column space

Add a Post

Show all posts ▼ by recent activity ▼

inv(A)*b
Think it should be inv(A)*b. 1

Learn About Verified Certificates