edX

4. One-sided Test vs Wald's Test

In the problems on this page, $X_1, \ldots, X_n \overset{iid}{\sim} \mathrm{Exp}\,(\lambda)$, where $\lambda > 0$ is an unknown parameter. In this series of problems, we will compare two tests for the following null and alternative hypotheses:

$$H_0 \quad : \lambda \leq 1$$

$$H_1 \quad : \lambda > 1.$$

MLE and Fisher Information for an Exponential Statistical Model

2/2 points (graded)
What is the MLE $\hat{\lambda}$ for an exponential statistical model?

(Enter **barX_n** for $\overline{X}_n$.)

$\hat{\lambda}$ = 1/barX_n   ✔

What is the Fisher information $I(\lambda)$ for an exponential statistical model?

$$I(\lambda) = \boxed{\text{1/lambda^2}} \quad \checkmark$$

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✔ Correct (2/2 points)

Test Statistic Based on the MLE for an Exponential Statistical Model

1/1 point (graded)

Assume that the technical conditions hold so that the MLE $\hat{\lambda}_n^{MLE}$ of an exponential statistical model is asymptotically normal. Then it follows that

$$\frac{\sqrt{n}\,(\hat{\lambda}_n^{MLE} - \lambda)}{g(\hat{\lambda}_n^{MLE})} \xrightarrow[n\to\infty]{(d)} N(0,1)$$

where $g(\hat{\lambda}_n^{MLE})$ is an expression that depends on $\hat{\lambda}_n^{MLE}$.

What is $g(\hat{\lambda}_n^{MLE})$?

(Enter **hatlambda** for $\hat{\lambda}_n^{MLE}$.)

$$g(\hat{\lambda}_n^{MLE}) = \boxed{\text{hatlambda}} \quad \checkmark$$

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✔ Correct (1/1 point)

Evaluating the Test Based on the MLE

1/1 point (graded)
Let us define the test statistic

$$T_n = \frac{\sqrt{n}\left(\hat{\lambda}_n^{MLE} - 1\right)}{g\left(\hat{\lambda}_n^{MLE}\right)}$$

where $g\left(\hat{\lambda}_n^{MLE}\right)$ is the expression from the previous problem.

We define the test $\psi = \mathbf{1}\left(T_n > \tau\right)$, where τ is a chosen so that ψ is a test at asymptotic level $\alpha = 0.05$. Suppose we observe $\overline{X}_n = 0.83$.

Does the test ψ **reject** or **fail to reject** H_0 on this data set? **Use $n = 100$.**

○ Fail to reject

● Reject

✔

✔ Correct (1/1 point)

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Wald's Test

2/2 points (graded)

Recall the test-statistic T_n from the previous problem, and let T_n^{Wald} denote the test-statistic associated to Wald's test for the hypotheses H_0 and H_1.

Express T_n^{Wald} in terms of T_n.

(Enter **T_n** for T_n.)

$T_n^{Wald} = $

T_n ^2   ✔

T_n^2

Which of the following is true about T_n^{Wald} if we assume that $\lambda = 1$?

○ T_n^{Wald} is distributed as $\mathcal{N}(0,1)$.

○ T_n^{Wald} is asymptotically distributed as χ_2^2.

○ T_n^{Wald} is distributed as χ_1^2.

● T_n^{Wald} is asymptotically distributed as χ_1^2.

✔

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Evaluating Wald's Test on a Sample Data Set

1/1 point (graded)

Consider the test $\psi^{Wald} = \mathbf{1}\left(T_n^{Wald} > \tau\right)$ where τ is set so that the test ψ^{Wald} has asymptotic level 0.05. Suppose you observe $\overline{X}_n = 0.83$.

Does the test ψ^{Wald} **reject** or **fail to reject** on the given data set? **Use $n = 100$.**

- ⦿ Fail to reject

- ○ Reject

✔

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✔ Correct (1/1 point)

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