10.2.2 The Important Attributes of a Linear System

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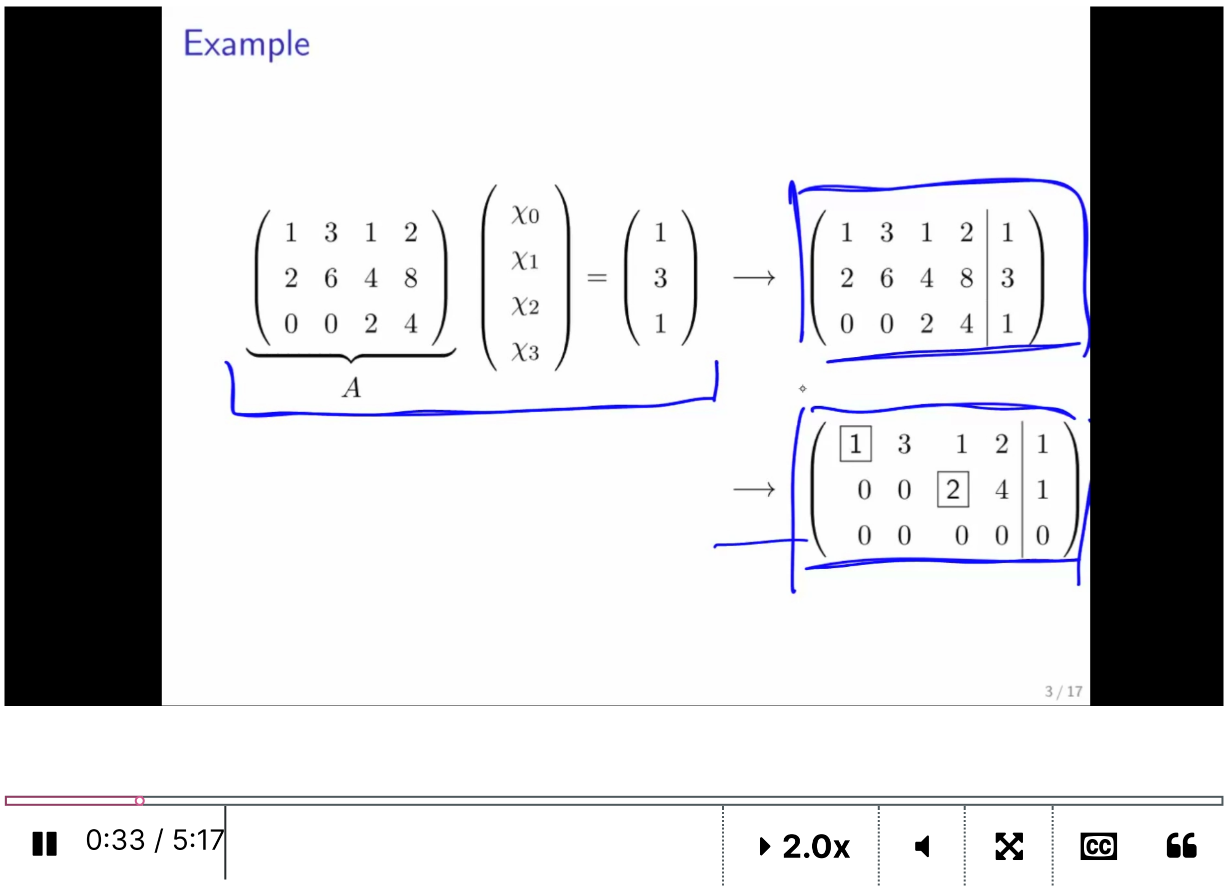
10.2.2 The Important Attributes of a Linear System

Video



Example

$$\underbrace{\begin{pmatrix} 1 & 3 & 1 & 2 \\ 2 & 6 & 4 & 8 \\ 0 & 0 & 2 & 4 \end{pmatrix}}_{A} \begin{pmatrix} \chi_0 \\ \chi_1 \\ \chi_2 \\ \chi_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \\ 1 \end{pmatrix} \longrightarrow \left(\begin{array}{cccc|c} 1 & 3 & 1 & 2 & 1 \\ 2 & 6 & 4 & 8 & 3 \\ 0 & 0 & 2 & 4 & 1 \end{array}\right)$$

$$\longrightarrow \left(\begin{array}{cccc|c} \boxed{1} & 3 & 1 & 2 & 1 \\ 0 & 0 & \boxed{2} & 4 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

So here is the list.

And let's just go through this list using the example from the last unit.

Notice that in the last unit, we started with a linear systems

with three equations and four unknowns.

You then wrote that as a appended system.

And you then transformed that into row echelon form.

And, in this particular case, the last row

ends up simply becoming zero is equal to zero.

And therefore we really, for all practical purposes,

end up with two equations and four unknowns.

Now let's go through all of these attributes one

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Reading Assignment

0 points possible (ungraded)

Read Unit 10.2.2 of the notes. [LINK]

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? Homework 10.2.2.5

On Homework 10.2.2.5 I don't understand how you can know for sure that the subsequent rows will all be reduced to 0 without knowing what

Calculator

? Homework 10.2.2.1 row space 2
For the question "Which of the following are a basis for the row space of A?" , the notes say "Thus, all you need to do is list the rows in the matri...

Important attributes of a linear system and/or matrix

Consider $Ax = b$. The following are important attributes:

- The row-echelon form of the system.
- The pivots.
- The free variables.
- The dependent variables.
- A specific solution. **Often called a particular solution.**
- A basis for the null space.
  Something we should have mentioned before: The null space is often called the *kernel* of the matrix.
- A general solution.
  Often called a complete solution.
- A basis for the column space.
  Something we should have mentioned before: The column space is often called the *range* of the matrix.
- A basis for the row space.
  The row space is the subspace of all vectors that can be created by taking linear combinations of the rows of a matrix. In other words, the row space of A equals $\mathcal{C}(A^T)$ (the column space of A^T).
- The dimension of the row and column space.
- The rank of the matrix.
- The dimension of the null space.

Homework 10.2.2.1

22/22 points (graded)

Consider

$$\begin{pmatrix} 1 & 2 & 2 \\ 2 & 4 & 5 \end{pmatrix} \begin{pmatrix} \chi_0 \\ \chi_1 \\ \chi_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 4 \end{pmatrix}$$

- Reduce the system to row echelon form (but not reduced row echelon form).

$$\begin{matrix} \alpha_{0,0} & \alpha_{0,1} & \alpha_{0,2} \\ \alpha_{0,0} & \alpha_{0,1} & \alpha_{0,2} \end{matrix} =$$

1 ✓ Answer: 1
2 ✓ Answer: 2
2 ✓ Answer: 2
0 ✓ Answer: 0
0 ✓ Answer: 0
1 ✓ Answer: 1

$$\begin{matrix} \gamma_0 \\ \gamma_1 \end{matrix} =$$

1 ✓ Answer: 1
2 ✓ Answer: 2

$$\left(\begin{array}{ccc|c} \boxed{1} & 2 & 2 & 1 \\ 0 & 0 & \boxed{1} & 2 \end{array}\right)$$

Calculator

Identify the free variables (mark all):

☐ χ_0

☑ χ_1

☐ χ_2

☐ $none$

✓

Identify the dependent variables (mark all):

☑ χ_0

☐ χ_1

☑ χ_2

☐ $none$

✓

What is the dimension of the column space?

2 ✓ Answer: 2

What is the dimension of the row space?

2 ✓ Answer: 2

What is the dimension of the null space?

1 ✓ Answer: 1

Which of the following are a basis for the column space. (Mark all)

☐ $\begin{pmatrix}1\\0\end{pmatrix},\begin{pmatrix}2\\0\end{pmatrix}$

☑ $\begin{pmatrix}1\\2\end{pmatrix},\begin{pmatrix}2\\5\end{pmatrix}$

☐ $\begin{pmatrix}1\\2\end{pmatrix},\begin{pmatrix}2\\4\end{pmatrix}$

☐ $\begin{pmatrix}1\\2\end{pmatrix},\begin{pmatrix}2\\4\end{pmatrix},\begin{pmatrix}2\\5\end{pmatrix}$

✓

Calculator

Which of the following are a basis for the row space of A?

☐ $\begin{pmatrix}1\\0\end{pmatrix},\begin{pmatrix}2\\1\end{pmatrix}$

☐ $\begin{pmatrix}1&2&2\end{pmatrix},\begin{pmatrix}0&0&1\end{pmatrix},$

☑ $\begin{pmatrix}1\\2\\2\end{pmatrix},\begin{pmatrix}0\\0\\1\end{pmatrix},$

☑ $\begin{pmatrix}1\\2\\2\end{pmatrix},\begin{pmatrix}2\\4\\5\end{pmatrix}$

✓

What is the rank of the matrix?

2 ✓ Answer: 2

Give the general solution (please use the consistent approach explained in the video so we all end up with the same solution...)

-3 ✓ Answer: -3

-2 ✓ Answer: -2

0 ✓ Answer: 0

$+\alpha$

1 ✓ Answer: 1

2 ✓ Answer: 2

0 ✓ Answer: 0

$$\begin{pmatrix}\boxed{-3}\\0\\\boxed{2}\end{pmatrix}+\alpha\begin{pmatrix}\boxed{-2}\\1\\\boxed{0}\end{pmatrix}.$$

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Answers are displayed within the problem

Homework 10.2.2.2

10/10 points (graded)

Consider $Ax=b$ where

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$\begin{pmatrix}0&1&0&3\end{pmatrix}$ $\begin{pmatrix}\beta_0\end{pmatrix}$

$$A = \begin{pmatrix} 0 & 2 & 0 & 6 \end{pmatrix} \quad \text{and} \quad b = \begin{pmatrix} \beta_1 \end{pmatrix}.$$

Identify the row echelon form of this equation:

$$\left(\begin{array}{cccc|c} \alpha_{0,0} & \alpha_{0,1} & \alpha_{0,2} & \alpha_{0,3} & \gamma_0 \\ \alpha_{1,0} & \alpha_{1,1} & \alpha_{1,2} & \alpha_{1,3} & \gamma_1 \end{array}\right)$$

$\begin{matrix} \alpha_{0,0} & \alpha_{0,1} & \alpha_{0,2} & \alpha_{0,3} \\ \alpha_{1,0} & \alpha_{1,1} & \alpha_{1,2} & \alpha_{1,3} \end{matrix} =$

0	1	0	3
✓ Answer: 0	✓ Answer: 1	✓ Answer: 0	✓ Answer: 3
0	0	0	0
✓ Answer: 0	✓ Answer: 0	✓ Answer: 0	✓ Answer: 0

$\begin{matrix} \gamma_0 \\ \gamma_1 \end{matrix} =$

○ $\begin{matrix} \beta_0 \\ \beta_1 \end{matrix}$

○ $\begin{matrix} \beta_0 \\ 0 \end{matrix}$

● $\begin{matrix} \beta_0 \\ \beta_1 - 2\beta_0 \end{matrix}$

○ $\begin{matrix} \beta_0 \\ \beta_1 + 2\beta_0 \end{matrix}$

✓

$$\left(\begin{array}{cccc|c} 0 & 1 & 0 & 3 & \beta_0 \\ 0 & 0 & 0 & 0 & \beta_1 - 2\beta_0 \end{array}\right)$$

When is the equation guaranteed to have a solution? (Mark all)

☐ When $\beta_1 = 0$.

☑ When $\beta_1 - 2\beta_0 = 0$.

☑ When $\beta_1 = 2\beta_0$.

☑ When $\gamma_1 = 0$.

✓

Echelon form:

$$\left(\begin{array}{cccc|c} 0 & 1 & 0 & 3 & \beta_0 \\ 0 & 0 & 0 & 0 & \beta_1 - 2\beta_0 \end{array}\right)$$

This is consistent only if $\beta_1 - 2\beta_0 = 0$. In other words, when $\beta_1 = 2\beta_0$. General solution: Note that χ_0, χ_2, ar free variables. Thus, a general solution has the form

Calculator

$$\begin{pmatrix}0\\ \square\\ 0\\ 0\end{pmatrix}+\alpha\begin{pmatrix}1\\ \square\\ 0\\ 0\end{pmatrix}+\beta\begin{pmatrix}0\\ \square\\ 1\\ 0\end{pmatrix}+\gamma\begin{pmatrix}0\\ \square\\ 0\\ 1\end{pmatrix}.$$

Solving for the boxes yields the special solution (the first vector) and the vectors in the null spaces (the other three vectors):

$$\begin{pmatrix}0\\ \boxed{\beta_0}\\ 0\\ 0\end{pmatrix}+\alpha\begin{pmatrix}1\\ \boxed{0}\\ 0\\ 0\end{pmatrix}+\beta\begin{pmatrix}0\\ \boxed{0}\\ 1\\ 0\end{pmatrix}+\gamma\begin{pmatrix}0\\ \boxed{\text{-3}}\\ 0\\ 1\end{pmatrix}.$$

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Answers are displayed within the problem

Homework 10.2.2.3

4/4 points (graded)

Which of these statements is a correct definition of the rank of a given matrix $A \in \mathbb{R}^{m \times n}$?

1. The number of nonzero rows in the row reduced form of A.

   TRUE ✓ Answer: TRUE

2. The number of columns minus the number of rows, $n - m$.

   FALSE ✓ Answer: FALSE

3. The number of columns minus the number of free columns in the row reduced form of A. (Note: a free column is a column that does not contain a pivot.)

   TRUE ✓ Answer: TRUE

4. The number of 1s in the row reduced form of A.

   FALSE ✓ Answer: FALSE

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Answers are displayed within the problem

Homework 10.2.2.4

19/19 points (graded)

Compute

$$\begin{pmatrix}-1\\ 2\end{pmatrix}\begin{pmatrix}3 & 1 & 2\end{pmatrix}$$

-3	✓	1	✓	-2	✓
6	✓	-2	✓	4	

Calculator

9 ✓	-3 ✓	6 ✓

Reduce the matrix to row echelon form.

-3 ✓	1 ✓	-2 ✓
0 ✓	0 ✓	0 ✓
0 ✓	0 ✓	0 ✓

What is the rank of this matrix?

1 ✓

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Homework 10.2.2.5

1/1 point (graded)

Let $u \in \mathbb{R}^m$ and $v \in \mathbb{R}^n$ so that uv^T is an $m \times n$ matrix. Let $k = \text{rank}\,(uv^T)$. Then

○ $k = 0$

○ $k = 1$

● $k = 0$ or $k = 1$

○ $k = n$

○ $k = m$

○ not enough information

✓

$k = 0$ or $k = 1$

Partition u into elements. Then

$$uv^T = \begin{pmatrix} v_0 \\ v_1 \\ \vdots \\ v_{m-1} \end{pmatrix} v^T = \begin{pmatrix} v_0 v^T \\ v_1 v^T \\ \vdots \\ v_{m-1} v^T \end{pmatrix}$$

and for simplicity assume that $v_0 \neq 0$.

Now, imagine reducing this matrix to row echelon form. The resulting matrix equals

$$\begin{pmatrix} v_0 v^T \\ 0 \\ \vdots \\ 0 \end{pmatrix}$$

Calculator

where the 0s indicates rows of zeroes.

Notice that there is at most one pivot in the matrix and hence $k \leq 1$.

Notice that this exercise explains the name “rank-1 update" when computing $A := \alpha u v^T + A$.

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