You are taking "Exam (Timed, No Correctness Feedback)" as a timed exam. Show more

End My Exam 43:49:17

Previous Next

7. Directional derivatives with non-unit vectors

Bookmark this page

Calculator Hide Notes

Lecture due Aug 18, 2021 20:30 IST Completed

Explore

We can also find the rate of change of a function $f(x,y)$ in the direction of a vector $\vec{v}$ that is not a unit vector. We do this by creating a unit vector $\hat{u}$ from $\vec{v}$ by dividing $\vec{v}$ by its magnitude. Doing this gives the vector

$$\hat{u} = \frac{1}{|\vec{v}|}\vec{v} \tag{3.109}$$

and apply the formula for the directional derivative $\nabla f \cdot \hat{u}$. We will work through the calculations step-by-step in the following exercise.

Directional derivative exercise

Our goal will be to find the directional derivative of $f(x,y) = x - y^2 - 2y$ at the point $(0, 1/2)$ in the direction $\vec{v} = \langle 1, 1 \rangle$.

Step 1: Find the unit vector

1.0/1 point (graded)
Given $\vec{v} = \langle 1, 1 \rangle$, find the unit vector $\hat{u}$ that points in the direction of $\vec{v}$.

(Enter the vector in the form `[a,b]`. That is surround your vector by square brackets, and separate entries by a comma. Note that the entries of your vector must be numbers.)

$\hat{u} =$ [1/sqrt(2),1/sqrt(2)] ✔ **Answer:** [1/sqrt(2),1/sqrt(2)]

Solution:

Notice that $|\vec{v}| = \sqrt{2}$, so we need to construct our unit vector that is in the same direction as $\vec{v}$. Dividing $\vec{v}$ by its magnitude gives

$$\hat{u} = \langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \rangle.$$

Submit You have used 1 of 5 attempts

Answers are displayed within the problem

Step 2: Find the gradient

1.0/1 point (graded)
Given $f(x,y) = x - y^2 - 2y$, find the gradient of f at the point $(0, 1/2)$.

(Enter the vector in the form `[a,b]`. That is surround your vector by square brackets, and separate entries by a comma. Note that the entries of your vector must be numbers.)

$\nabla f(0, 1/2) =$ [1,-3] ✔ **Answer:** [1,-3]

Calculator Hide Notes

Solution:

We first compute

$$\nabla f\left(x,y\right)=\langle 1,-2y-2\rangle.$$

Then we evaluate at $(0,1/2)$ to obtain

$$\nabla f\left(0,1/2\right)=\langle 1,-3\rangle.$$

Submit You have used 1 of 5 attempts

Answers are displayed within the problem

Step 3: Find the directional derivative

1/1 point (graded)

Use the information from Steps 1 and 2 to find the directional derivative of $f\left(x,y\right)=x-y^2-2y$ at the point $(0,1/2)$ in the direction of the vector $\vec{v}=\langle 1,1\rangle$.

$D_{\vec{v}}f\left(0,1/2\right)=$ -sqrt(2) ✔ **Answer:** 1/sqrt(2)-3/sqrt(2)

Solution:

We have

$$\hat{u}=\langle\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}\rangle$$

and

$$\nabla f\left(0,1/2\right)=\langle 1,-3\rangle.$$

So we compute

$$D_{\hat{u}}f\left(0,1/2\right)=\nabla f\left(0,1/2\right)\cdot\langle\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}\rangle=\frac{1}{\sqrt{2}}-\frac{3}{\sqrt{2}}.$$

Submit You have used 1 of 5 attempts

Answers are displayed within the problem

Remark 7.1 If $\vec{v}$ is a vector with magnitude not equal to 1, a compact way of writing the directional derivative of f in the direction of a vector $\vec{v}$ is

$$\nabla f\cdot\vec{n}$$

Calculator Hide Notes

$$D_{\vec{v}} f(x,y) = \frac{\quad}{|\vec{v}|}.$$

Practice

1/1 point (graded)

Let $g(x,t) = e^{2t}\cos(3x)$. Find the directional derivative of g at the point $(x,t) = (\pi, 1)$ in the direction of the vector $\vec{v} = \langle 3, -4 \rangle$.

$D_{\vec{v}} g(\pi, 1) =$ 8*e^2/5 ✔ **Answer:** 8*e^2/5

Solution:

We compute the gradient

$$\nabla g(x,t) = \langle -3\sin(3x)\, e^{2t}, 2e^{2t}\cos(3x) \rangle$$

and so

$$\nabla g(\pi, 1) = \langle 0, -2e^2 \rangle.$$

This gives

$$D_{\vec{v}} g(\pi,1) = \nabla g(\pi,1) \cdot \frac{\langle 3,-4\rangle}{|\langle 3,-4\rangle|} = \langle 0, -2e^2\rangle \cdot \frac{1}{5}\langle 3,-4\rangle = \frac{8}{5}e^2.$$

Submit You have used 2 of 5 attempts

Answers are displayed within the problem

7. Directional derivatives with non-unit vectors

Hide Discussion

Topic: Unit 2: Geometry of Derivatives / 7. Directional derivatives with non-unit vectors

Add a Post

Show all posts by recent activity

There are no posts in this topic yet.

✖

Previous Next ›

Calculator Hide Notes

edX

Calculator Hide Notes