8. Lines in 3D

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In previous lectures, we saw that a line in 3D could be represented as the intersection of planes. But this representation is cumbersome to work with, and it is easier to use parametric equations. The following video shows how to set up parametric lines in 3D.

Lines in 3D

I will switch to brighter colors.

So OK.

So apart from that, I claim now we

can find the position of this moving point.

Because-- well, this vector Q0Q1 we

can find from the coordinates of Q0 and Q1.

So we just subtract the coordinates of Q0

from those of Q1.

We'll get that vector Q0Q1 is (2, 1, minus 3).

OK.

So if I look at it--

well.

So let's call x of t, y of t, and z of t

the coordinates of the point that's moving on the line.

9:46 / 9:46 2.0x

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Suppose we have two points, Q_0 and Q_1, and we want to describe the motion of a particle that moves in a straight line from Q_0 to Q_1 in 3D. For example, suppose $Q_0 = (-1, 2, 2)$ and $Q_1 = (1, 3, -1)$. Let $Q(t)$ be the position of the moving point at time t. If we assume $Q(0) = Q_0$, $Q(1) = Q_1$, and that the point moves at constant speed from Q_0 to Q_1, then we can say that

$$\overrightarrow{Q_0 Q(t)} = t\overrightarrow{Q_0 Q_1}. \tag{6.80}$$

In words, the vector from Q_0 to $Q(t)$ is equal to t times the vector from Q_0 to Q_1.

It follows that, using the notation $Q(t) = (x(t), y(t), z(t))$,

$$x(t) = -1 + 2t \tag{6.81}$$

$$y(t) = 2 + t \tag{6.82}$$

$$z(t) = 2 - 3t \tag{6.83}$$

This can also be written as

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$$Q(t) = Q_0 + t\overrightarrow{Q_0Q_1} \tag{6.84}$$

In this form, the straight-line trajectory is more evident.

Now that we have it, what can we do with this parametric equation?

Intersect Line With Plane

Q1.

I'm going to draw them in completely random places.

Well, are Q0 and Q1 on the same side of a plane

or on different side--

on opposite sides of a plane?

Or could it be that maybe one of the points is in the plane?

So I think I'm going to let you vote on that.

Is that readable?

Is it too small?

OK.

So anyway, the question says, relative to the plane

x plus 2y plus 4z equals 7, these points, Q0 and Q1,

are they on the same side?

On opposite sides?

2:20 / 2:20 2.0x

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Having a parametric equation for the line gives us a better understanding of where the points are in 3D space. In particular, let's try to answer the question: where does the line through Q_0 and Q_1 intersect the plane $x + 2y + 4z = 7$?

Without a graph, it's hard to even visualize this question because we don't know where Q_0 and Q_1 are relative to the plane. Try to answer the next question without using a graph:

POLL

Relative to [mathjaxinline]x + 2y + 4z = 7[/mathjaxinline], the points [mathjaxinline]Q_0 = (-1,2,2)[/mathjaxinline] and [mathjaxinline]Q_1 = (1,3,-1)[/mathjaxinline] are on...

RESULTS

Same side	**9%**
Opposite sides	**82%**
One is in the plane	**3%**
Cannot decide	**6%**

Submit

Results gathered from 33 respondents.

FEEDBACK

Your response has been recorded

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Are the Points in the Plane?

These points are not in the plane,

but they're not in a plane in different ways, and one of them

somehow overshoots.

We get 11.

The other one, we only get 3.

That's less than 7.

If you think about how a plane splits space

into two half spaces on either side, well, one of them

is going to be the point where x plus 2y plus 4z is less than 7,

and the other one will be--

so that's somehow this side, and that's where Q1 is.

And the other side is where x plus 2y plus 4z is actually

bigger than 7.

And to go from one to the other, well,

3:29 / 3:29 2.0x

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We can substitute the x, y, and z values for Q_0 and Q_1 into the equation for the plane, and see if we get an equality or inequality.

The plane consists of all points (x, y, z) such that $x + 2y + 4z = 7$. Is Q_0 in this plane? Since $Q_0 = (-1, 2, 2)$, we can check the left-hand-side: $-2 + 2\,(2) + 4\,(2)$, which comes out to 10. Since $10 \neq 7$, we know Q_0 is not in the plane.

When we do the same calculation with Q_1, the left-hand-side becomes 3, which is also not 7.

This calculation tells us that Q_0 and Q_1 are on opposite sides of the plane. The point Q_0 is in the “half-space" described by $x + 2y + 4z > 7$, and Q_1 is in the opposite “half-space" described by $x + 2y + 4z < 7$.

3D Line Strike Plane

you see that 7 is actually right in between--

it's the average of these two numbers.

So it would make sense that it's halfway in between Q 0 and Q 1,

that we will get 7.

And then at that time Q at time 1/2--

well, let's plug the values.

So minus 1 plus 2t will be 0.

2 plus t will be 2 and 1/2 or 5/2.

And 2 minus 3/2 will be 1/2.

So this is where the line intersects the plane.

So you see that's actually a pretty easy way

of finding where a line and a plane intersect

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3:30 / 3:30 2.0x

If we can find a parametric equation of the line

and an equation of a plane, then we

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We can carry out the same comparison for the general point $Q(t)$. When the value strikes 7, we know that $Q(t)$ will be precisely inside of the plane.

When does $Q(t)$ enter the plane?

To answer this question, we look for the value of t such that $x(t)+2y(t)+4z(t)=7$. Simplifying:

$$x(t)+2y(t)+4z(t) = (-1+2t)+2(2+t)+4(2-3t) \tag{6.85}$$

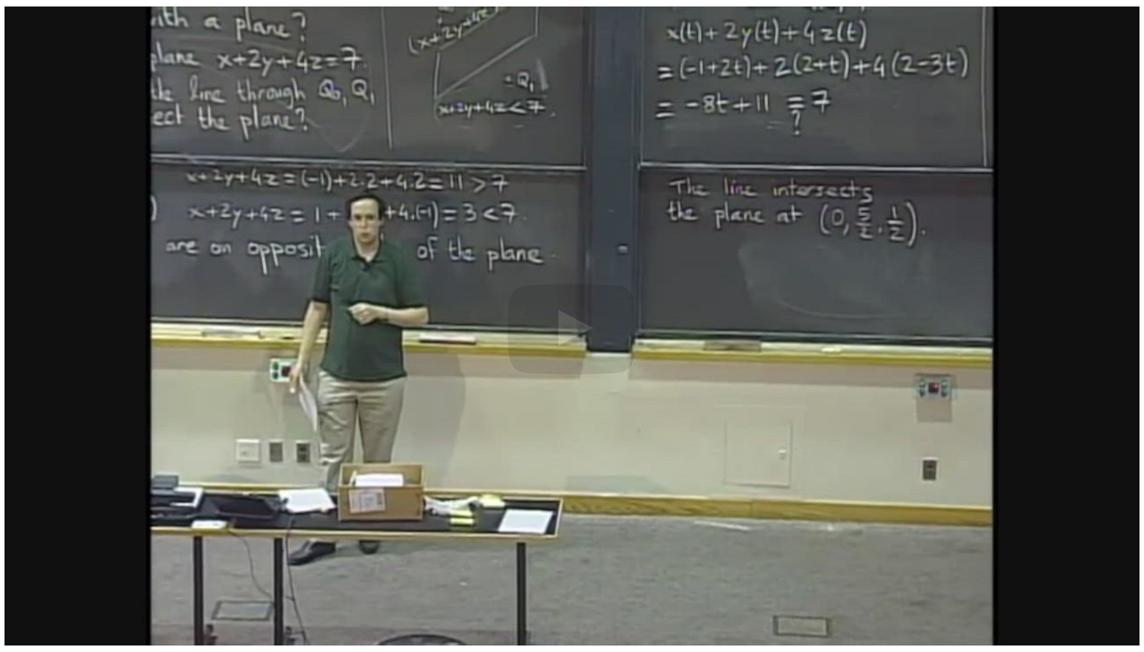
$$= -8t+11 \tag{6.86}$$

When does this equal 7? By algebra, this happens when $t=1/2$.

Thus we have learned that at time $t=1/2$ the moving point strikes the plane.

Parametric Line Questions

Start of transcript. Skip to the end.



PROFESSOR: OK.

Are there questions about this?

Yes.

STUDENT: [INAUDIBLE].

PROFESSOR: Sorry.

Can you say that again?

STUDENT: [INAUDIBLE].

0:00 / 0:00 2.0x

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Q and A

What if we had found that the equation for t had no solution?

This would mean that the line described by $Q(t)$ would never intersect the plane. This could happen if the line is parallel to the plane.

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What if we had found multiple solutions for t?

This would mean that the line is completely contained inside of the plane.

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Takeaway: The parametric equation gives us a lot of insight into what's going on, even when we don't have access to a graph.

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