6. Modeling Inter-arrival Times of a Subway System

Review: Exponential Random Variables

3/3 points (graded)

Let $X \sim \exp(\lambda)$ for some $\lambda > 0$. Which of the following is the (smallest possible) sample space for X?

- $\mathbb{N}$
- $\mathbb{Z}$
- $[0, \infty)$ (selected)
- $(-\infty, \infty)$

✔

Which of the following is the probability density function (pdf) for X? (Assume that $x > 0$).

- (selected) $\lambda e^{-\lambda x}$
- $\frac{1}{\lambda} e^{-\lambda x}$
- $\lambda e^{\lambda x}$
- $\lambda e^{-\lambda x^2}$

✔

What is $\mathbb{E}[X]$?
(By now, you may simply memorize this and not rederive it everytime.)

$\mathbb{E}[X] =$ 1/lambda ✔ **Answer:** 1/lambda

$\frac{1}{\lambda}$

STANDARD NOTATION

Solution:

- An exponential random variable takes values on all positive real numbers. Therefore, the smallest possible sample space for X is given by $[0, \infty)$.
- By definition, the density of an exponential random variable is given by the function $x \mapsto \lambda e^{-\lambda x}$.
- For completeness, we use the formula for the density to compute the mean of an exponential random variable. By definition and integration by parts,

$$\begin{aligned}\mathbb{E}[X] &= \int_0^\infty x\lambda e^{-\lambda x}\,dx \\ &= -xe^{-\lambda x}\Big|_\infty^0 + \int_0^\infty e^{-\lambda x}\,dx \\ &= 0 - \frac{1}{\lambda}e^{-\lambda x}\Big|_\infty^0 \\ &= \frac{1}{\lambda}.\end{aligned}$$

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Answers are displayed within the problem

Video note: In the video below, the “T" refers to the subway train in the public transportation system in Boston.

Modeling Inter-arrival Times of a Subway System

Start of transcript. Skip to the end.

So let's move on to another example.

Some of you might be familiar with this.

This is the T. In particular, this is the red line.

I don't know where it is actually.

And so I do take the T sometimes,

and I would like to model the arrival times of the T.

So if you take the T, you know it's definitely

a random process.

And even though they tell you ahead

0:00 / 0:00

1.50x

Video

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Memoryless Property of Exponential Random Variables

2/2 points (graded)

Let $X \sim \exp(1)$. What is $\mathbf{P}(X > 3)$?

$\mathbf{P}(X > 3) =$ e^(-3) ✔ **Answer:** exp(-3)

Let $t > 0$. What is $\mathbf{P}(X > t + 3 | X > t)$?

$\mathbf{P}\left(X > t+3 | X > t\right) =$ e^(-3) ✔ **Answer:** exp(-3)

Solution:

The density of $\exp\left(1\right)$ is given by e^{-x}. Therefore,

$$\mathbf{P}\left(X > 3\right) = \int_3^\infty e^{-x}\,dx = -e^{-x}\Big|_3^\infty = e^{-3}.$$

Next, by the memoryless property of the exponential distribution, for any $s, t > 0$, it holds that

$$\mathbf{P}\left(X > s+t | X > t\right) = \mathbf{P}\left(X > s\right).$$

Apply the above equality with $s = 3$ shows that

$$\mathbf{P}\left(X > 3+t | X > t\right) = \mathbf{P}\left(X > 3\right) = \exp\left(-3\right).$$

Submit You have used 1 of 3 attempts

Answers are displayed within the problem

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