Final Exam

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Final Exam Problem 1

1.0/1.0 point (graded)

Choose the incorrect statement.

- The precise birth year of Pierre de Fermat is not known. According to a theory, he was born in 1607, and 1607 is a prime number.
- Leonhard Euler was born in 1707, and 1707 is a prime number. ✔
- Johann Carl Friedrich Gauss was born in 1777, and 1777 is a prime number.
- Gotthold Eisenstein was born in 1823, and 1823 is a prime number.

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Final Exam Problem 2

1.0/1.0 point (graded)

Today, advanced Reciprocity Laws on prime numbers are often described using modular forms and automorphic forms. Eichler is one of the pioneers of the modern theory of modular forms and elliptic curves in the middle of the 20th century. It is claimed that he once said "there are five fundamental operations in mathematics: addition, subtraction, multiplication, division, and ___________."

Choose the correct answer to fill in the blank.

- Prime Numbers
- Reciprocity Laws
- Elliptic Curves
- Modular Forms ✔

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Final Exam Problem 3

4.0/4.0 points (graded)

In 1734, Euler proved the following equality (Basel Problem):

$$1+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}+\frac{1}{5^2}+\frac{1}{6^2}+\ldots=\frac{\pi^2}{6}$$

The sum of the inverses of the squares divided by π^2 is equal to $\frac{1}{6}$. On the other hand, it is known that the alternating sum of the sum of the inverses of the squares divided by π^2 is also a rational number. In other words, the equality

$$1-\frac{1}{2^2}+\frac{1}{3^2}-\frac{1}{4^2}+\frac{1}{5^2}-\frac{1}{6^2}+\ldots=\frac{A\pi^2}{B}$$

is true for some positive relatively prime integers A, B. Calculate A and B.

$A=$

1 ✔

1

$B =$

12

12

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Final Exam Problem 4

Assume that integers $A, B, C,$ and D satisfy the following:

$$2^{560} \equiv A \pmod{561} \qquad 0 \leq A \leq 560$$

$$3^{560} \equiv B \pmod{561} \qquad 0 \leq B \leq 560$$

$$5^{560} \equiv C \pmod{561} \qquad 0 \leq C \leq 560$$

$$7^{560} \equiv D \pmod{561} \qquad 0 \leq D \leq 560$$

Final Exam Problem 4-1

1.0/1.0 point (graded)

Find A.

$A =$

1

1

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Final Exam Problem 4-2

1.0/1.0 point (graded)

Find B.

$B =$

375

$\mathbf{375}$

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Final Exam Problem 4-3

1.0/1.0 point (graded)

Find C.

$C =$

1

$\mathbf{1}$

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Final Exam Problem 4-4

1.0/1.0 point (graded)

Find D.

$D =$

1

$\mathbf{1}$

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Final Exam Problem 5

3.0/3.0 points (graded)

You are working in an intelligence agency. You found a ciphertext $Y = 46$. It is known that it is encrypted by the RSA cryptosystem with parameter $N = 91$ and (public) encryption key $E = 29$.

What is the plaintext X?

$X =$

37

37

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Final Exam Problem 6

3.0/3.0 points (graded)

The elliptic curve

$$Y^2 = X^3 + 2X + 1$$

has only three integral points (S, T) with $T > 0$. Two of them are $(S, T) = (0, 1), (1, 2)$.

Find the third point.

$S =$

8

8

$T =$

23

$\mathbf{23}$

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Final Exam Problem 7

4.0/4.0 points (graded)

Let’s play a game with prime numbers using a pencil and paper! The prime magic square consists of 3×3 cells as shown in the following figure

A	B	C
D	E	F
G	H	I

satisfying the following rules:

a) Each cell should contain an integer from 1 to 9.
b) Different cells should contain different integers.
c) In each row, the sum of the three cells should be a prime number. In other words, $A+B+C, D+E+F$, and $G+H+I$ should be prime numbers.
d) Similarly, in each column, the sum of the three cells should be a prime number. In other words, $A+D+G, B+E+H$, and $C+F+I$ should be prime numbers.
e) In the diagonal from the top-left cell to the bottom-right cell, the sum of the three cells should be a prime number. In other words, $A+E+I$ should be a prime number.
f) However, the sum $C+E+G$ need not be a prime number.

Here is an example of a prime magic square:

2	4	5
8	6	9
7	1	3

Your task is to make a prime magic square. The cells B, G, H, I are already fixed as $B = 7, G = 3, H = 2, I = 6$:

A	7	C
D	E	F
3	2	6

Fill integers in the remaining cells so that the 3×3 cells become a prime magic square.

Then calculate the 5-digit integer $A \times 10000 + C \times 1000 + D \times 100 + E \times 10 + F$.

$A \times 10000 + C \times 1000 + D \times 100 + E \times 10 + F =$

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