13.2.5 Types of Optimization Problems

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So far we have talked about *unconstrained* optimization formulations, where the question is to minimize $J\left(a_0, \ldots, a_{N-1}\right)$ without any further condition on $a_0, \ldots, a_{N-1}$. In practice one often encounters *constrained* optimization formulations, of the form

$$\min_{a_0, \ldots, a_{N-1}} J\left(a_0, \ldots, a_{N-1}\right), \tag{13.21}$$

$$\text{s.t.} \quad \left(a_0, \ldots, a_{N-1}\right) \in S, \tag{13.22}$$

meaning the point $\left(a_0, \ldots, a_{N-1}\right)$ belongs to some set S in N dimensions. The acronym $s.t.$ means “subject to". One might encounter equality constraints (like $2a_0 - 3a_1 = 5$), or inequality constraints (like $2a_0 - 3a_1 \geq 5$).

Here is a rundown of some important types of optimization problems (also called “optimization programs") that we encounter in CSE.

1. *Linear programming* is when the objective is linear, and the constraints are linear as well (either equality or inequality). For instance, the optimal transportation problem for a commodity between supply nodes S_i and demand nodes D_j, with edge costs c_{ij} between any pair of supply and demand nodes, where x_{ij} is the amount of commodity transported from node i to j, reads

$$\min_{x_{ij}} \sum_{ij} c_{ij} x_{ij}, \tag{13.23}$$

$$\text{s.t.} \quad \sum_j x_{ij} \leq S_i \tag{13.24}$$

$$\sum_i x_{ij} \leq D_j. \tag{13.25}$$

2. *Quadratic programming* involves quadratic objectives (like $3a_0^2 + 4a_1^2 - a_0 a_1 + 5a_0 - 2a_1$), but still linear constraints. For instance, the equilibrium conditions for a system of interconnected springs and masses obeys a linear system of equations $Ku = f$. But, it can equivalently be formulated via the minimization of the energy function

$$E(u) = \frac{1}{2} \sum_{ij} u_i K_{ij} u_j - \sum_j u_j f_j. \tag{13.26}$$

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$$\text{s.t.} \quad Ax = b \tag{13.28}$$

3. *Convex programming* involves convex objectives