

MITx: 6.041x Introduction to Probability - The Science of Uncertainty



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Exam 1

Exam 1 due Mar 09, 2016 at 23:59 UTC

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■ Bookmark

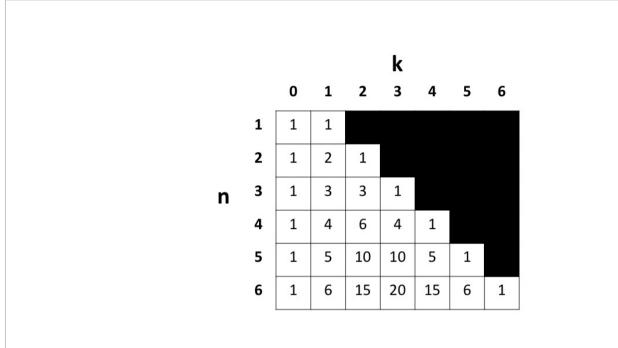
Problem 3: A six-sided die

(3/3 points)

A fair, 6-sided die is rolled 6 times independently. Assume that the results of the different rolls are independent. Let (a_1,\ldots,a_6) denote a typical outcome, where each a_i belongs to $\{1,\ldots,6\}$.

Note: Enter numerical answers; do not enter '!' or combinations. The following table for $\binom{n}{k}$ up to n=6 has been provided for your convenience:

- Unit 5: Continuous random variables
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1. Find the probability that the results of the 6 rolls are all different. (Answer with at least 3 decimal digits.)

0.0154321

V

Answer: 0.01543

For any outcome $\omega=(a_1,\ldots,a_6)$, let $R(\omega)$ be the **set** $\{a_1,\ldots,a_6\}$; this is the set of numbers that showed up at least once in the different rolls. For example, if $\omega=(2,2,5,2,3,5)$, then $R(\omega)=\{2,3,5\}$.

2.

Find the probability that $R(\omega)$ has exactly two elements. (Answer with at least 3 decimal digits.)

0.01993313

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Answer: 0.01993

3. Find the probability that $R(\omega)$ has exactly three elements.

0.2314815

V

Answer: 0.23148

Answer:

1. Let A be the event where the results of the 6 rolls are all the same. Since all outcomes are equally likely, $\mathbf{P}(A) = \frac{|A|}{|\Omega|}$.

$$|A|=6! \ |\Omega|=6^6 \ P(A)=rac{6!}{6^6}=5/324pprox 0.01543$$

2. Let B be the event where $R(\omega)$ has exactly two elements. We can find the number of elements in B by first choosing a pair of distinct numbers for $R(\omega)$ – there are $\binom{6}{2}$ choices. For each pair, we can then count the number of ways they can be assigned to a sequence of length 6 that consists of only those two numbers and has

at least one of each. We see that there are $\binom{6}{k}$ ways of constructing a sequence that consists of k repetitions of the first number, and 6-k repetitions of the second number, where $1 \leq k \leq 5$. Therefore,

$$egin{aligned} |B| &= inom{6}{2} \sum_{k=1}^5 inom{6}{k} \ |\Omega| &= 6^6 \ P(B) &= rac{|B|}{|\Omega|} = 155/7776 pprox 0.01993 \end{aligned}$$

3. Let C be the event where $R(\omega)$ has exactly three elements. We can find the number of elements in C by first choosing a triple of distinct numbers for $R(\omega)$ – there are $\binom{6}{3}$ choices. For each triple, we can then count the number of ways they can be assigned to a sequence of length 6 that consists of only those three numbers and has at least one of each. We see that there are $\binom{6}{k}\binom{6-k}{l}$ ways of constructing a sequence that consists of k repetitions of the first number, k repetitions of the second number, and k0 repetitions of the third number, where k1 k2 and k3 and k4 k5 k5 k6. Therefore,

$$egin{array}{l} |C| = inom{6}{3} \sum_{k=1}^4 \sum_{l=1}^{5-k} inom{6}{k} inom{6-k}{l} \ |\Omega| = 6^6 \end{array}$$

By numerically evaluating the various entries in the formula for $|{m C}|$, we find that:

$$P(C) = rac{|C|}{|\Omega|} = 25/108 pprox 0.23148$$

You have used 1 of 2 submissions

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