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Review of Week 1

Laws of Prime Numbers

Week 2 Problems due Feb 03, 2016 at 23:30 UTC

Homework 2

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Completion Checklist 2 due Feb 03, 2016 at 23:30 UTC

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## HOMEWORK 2-1 (1/1 point)

Choose all pairs of positive integers, which are relatively prime.

☒ (4, 7)☐ (10, 18)☒ (35, 99)☒ (122, 177)☐ (12,345, 111,111)

## HOMEWORK 2-2 (3/3 points)

Confirm Fermat's theorem on sums of two squares for the following prime numbers. Write two squares in ascending order.

(1)  $233 = A^2 + B^2$

(2)  $349 = C^2 + D^2$

(3)  $821 = E^2 + F^2$

A

8



8|

B

13



13|

C

5



5|

D

18



18|

E

14



14|

F

25



25|

## HOMEWORK 2-3 (2/2 points)

The following stronger statement holds: a prime number  $P$  whose remainder is 1 when we divide it by 4 is written as the sum of two squares in a unique way. In other words, for such a prime number  $P$ , there exists a unique pair  $(X, Y)$  of positive integers satisfying

$$P = X^2 + Y^2 \quad (X < Y).$$

This statement is no longer true for composite numbers. For a concrete example, consider 221, which is a composite number. It is written as the sum of two squares in two different ways:

$$221 = A^2 + B^2 = C^2 + D^2 \quad (A < B, C < D, A < C)$$

Find positive integers A,B,C,D satisfying the above condition.

A	B	C	D
<input type="text" value="5"/>	<input type="text" value="14"/>	<input type="text" value="10"/>	<input type="text" value="11"/>
✓	✓	✓	✓
<input type="text" value="5 "/>	<input type="text" value="14 "/>	<input type="text" value="10 "/>	<input type="text" value="11 "/>

#### HOMWORK 2-4 (4/4 points)

Calculate integers modulo 11. Write the integers between 0 and 10 in the blanks.

(1)  $2 - (4 + 5) \equiv A \pmod{11}$

(2)  $5 \times 9 \times 8 \equiv B \pmod{11}$

(3)  $3^{123} \equiv C \pmod{11}$

(4)  $10! \times 10! \times 10! \times 10! \times 10! \equiv D \pmod{11}$  (Hint: Use Wilson's theorem.)

A	B	C	D
<input type="text" value="4"/>	<input type="text" value="8"/>	<input type="text" value="5"/>	<input type="text" value="10"/>
✓	✓	✓	✓
<input type="text" value="4 "/>	<input type="text" value="8 "/>	<input type="text" value="5 "/>	<input type="text" value="10 "/>

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