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2. Maximum Likelihood Estimation

Extension Note: Homework 5 due date has been extended by 1 day to **August 17 23:59UTC** .

Consider a general multinomial distribution with parameters θ . Recall that the likelihood of a dataset \mathcal{D} is given by:

$$P(\mathcal{D}; \theta) = \prod_{i=1}^{|\theta|} \theta_i^{c_i}$$

where c_i is the occurrence count of the i -th event.

The MLE of θ is the setting of θ that maximizes $P(\mathcal{D}; \theta)$. In lecture we derived this to be

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Unigram Model

4/4 points (graded)

Consider the sequence:

A B A B B C A B A A B C A C

A unigram model considers just one character at a time and calculates $p(w)$ for $w \in \{A, B, C\}$.

What is the MLE estimate of θ ? Give your result to three decimal places.

θ_A^*  **Answer:** 0.4285714286

θ_B^*  **Answer:** 0.3571428571

θ_C^*  **Answer:** 0.2142857143

Using the MLE estimate of θ on \mathcal{D} , which of the following sequences is most likely?

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[End My Exam](#)46:28:56 ☒ ABB ✓☐ AAC**Solution:**

We calculate the MLE as $\frac{\text{count}(w)}{N}$ where $N = 14$ and the counts are 6, 5, and 3.

For comparing probabilities in part two, we simply multiply. We only need to compare the numerators: $6 \times 5 \times 3$, 5^3 , 6×5^2 , and $6^2 \times 3$.

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You have used 1 of 3 attempts

 Answers are displayed within the problem

Bigram Model 1

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$$p(\mathcal{D}, \theta) = \prod_{w_1, w_2 \in \mathcal{D}} p(w_2 | w_1)$$

where w_2 is a word that follows w_1 in the corpus.

This is also a multinomial model. Assume the vocab size is N . How many parameters are there?

Grading note: The formula above contains an error: the probability $p(\mathcal{D}; \theta)$ in a bigram model is generally:

$$p(\mathcal{D}; \theta) = p(w_0) \prod_{w_1, w_2 \in \mathcal{D}} p(w_2 | w_1)$$

where w_0 is the first word, and (w_1, w_2) is a pair of consecutive words in the document. In this case, the number of parameters is $(N - 1) + (N^2 - N) = N^2 - 1$. However, with the model as written above, there are only parameters $N^2 - N$.

The grader is now fixed to accept both as correct and regrading is happening.

✓ Answer: $N^2 - 1$

STANDARD NOTATION

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$$p(\mathcal{D}; \theta) = p(w_0) \prod_{w_1, w_2 \in \mathcal{D}} p(w_2 | w_1)$$

where w_0 is the first word, and (w_1, w_2) is a pair of consecutive words in the document. Denote the set of all N words by V . The set of parameters is

$$\{p(w_0) : w_0 \in V\} \cup \{p(w_1 | w_2) : w_1 \in V, w_2 \in V\}$$

and the only constraints on these parameters are

$$\begin{aligned} \sum_{w_0 \in V} p(w_0) &= 1 \\ \sum_{w_1 \in V} p(w_1 | w_2) &= 1 \quad \text{for all } w_2 \in V. \end{aligned}$$

Hence, the number of parameters is $(N - 1) + (N^2 - N) = N^2 - 1$. (Note that this is also the number of parameters $p(w_1, w_2)$ where $w_1 \in V, w_2 \in V$, which determine the joint distribution.

Solution to the problem as written:

The likelihood of D in bigram model was given as

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without taking into account the likelihood $p(w_0)$ of the first word. In this case, the parameters are

$$\{p(w_1|w_2) : w_1 \in V, w_2 \in V\}$$

where $\sum_{w_1 \in V} p(w_1|w_2) = 1$ for all $w_2 \in V$. Hence, the number of parameters is $N^2 - N$.

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You have used 2 of 3 attempts

i Answers are displayed within the problem

Bigram Model 2

1/1 point (graded)

Which of the following represents the MLE for the **conditional probability** $p(w_2 | w_1)$?

☐
$$\frac{\text{count}(w_1, w_2)}{\sum_{w'_1, w'_2 \in \mathcal{D}} \text{count}(w'_1, w'_2)}$$

☐
$$\frac{\text{count}(w_1, w_2)}{\sum_{w'_1, w_2 \in \mathcal{D}} \text{count}(w'_1, w_2)}$$

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$$\frac{p(w_1, w_2)}{\sum_{w_1, w'_2 \in \mathcal{D}} \text{count}(w_1, w'_2)}$$

Solution:

This is a simple application of Bayes Rule:

$$p(w_2 | w_1) = \frac{p(w_1, w_2)}{p(w_1)}$$

To compute $p(w_1)$, we marginalize out w_2 .

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i Answers are displayed within the problem

Bigram Model 3

1/1 point (graded)

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If you estimate θ on this, what probability will be assigned to the following test sequence? Assume the starting probabilities of all characters $p(w|\text{null})$ is uniform. Give your answer to three decimal places.

A A B C B A B

0



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You have used 1 of 3 attempts

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? [Bigram Model 1](#)

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<input checked="" type="checkbox"/> Bigram Model 2. Bayes rule or conditional probability?	2
Hi. Is it a Bayes Rule or simple application of cond. probability? Thanks in advance.	
<input type="checkbox"/> Bigram Model I - Clarification for the students	13
<input checked="" type="checkbox"/> [Staff] Bigram Model 1	3
<input type="checkbox"/> [STAFF] I disagree with Q1 answer	4
<input type="checkbox"/> [Staff] Bigram Model 1: there maybe a Solution smaller than standard	1
p(w_i w_j)=p(w_i,w_j)/p(w_j)
p(D;theta)=prod(p(w_2 w_1))=prod(p(w_1,w_2)/p(w_1))
p(w_i)=sum_j(p(w_i,w_j))
So we o...	
<input type="checkbox"/> [Staff] Please check my answer for Bigram	1
Please check my answer to bigram 3. It says to give answer to 3 decimal places but the grader mark me wrong. thanks	
<input type="checkbox"/> Bigram Model 1 - Hints for those trying	6
I will try to give some hints on the Bigram Model 1 question since many seem to have been confused with this problem. In case I am saying too ...	
<input checked="" type="checkbox"/> [STAFF] Bigram model 1 - not clear at all	20
<input type="checkbox"/> Bigram - what is the assumption about p(null w)	5
Dear Staff, I see we are considering p(w null). How about p(null w)? I do not see it mentioned any where in the bigram questions. Should we add...	
<input type="checkbox"/> Yes it is a Markov chain...also, that's not the issue for the number of parameters	1
Community TA	

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