

MITx: 15.053x Optimization Methods in Business Analytics

Heli

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Lecture

Lecture questions due Oct 18, 2016 at 19:30 IST

Recitation

Problem Set 6

Homework 6 due Oct 18, 2016 at 19:30 IST

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Problem 3

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PART A

2/2 points (graded)

In the facility location problem, the goal is to optimally place facilities so as to minimize transportation costs from the facilities to the customers. In practice, one rarely knows the demand of customers with a high degree of accuracy. In addition, the transportation costs themselves may vary with time and are subject to uncertainty. In this version of the facility location problem, we assume that all data is known.

In the first part of this problem, the goal is to locate one or more facilities out of five possible sites, which we designated as F_1, F_2, F_3, F_4 , and F_5 . The cost of selecting a facility is \$40. Coincidentally, there are also five customers that need to be serviced. We refer to them as C_1, C_2, C_3, C_4 and C_5 . The delivery cost from each possible facility site to each of the five customers is known. The cost of satisfying (all of) the demand of Customer C_j from site F_i is d_{ij} . (Note that the first index is for the site and the second is for the customer.) The delivery cost from all possible sites to all customers are given in Table 1 below.

Custo	mer C_1 Customer	C_2 Customer ($C_{f 3}$ Customer C	C_4 Customer C_5
Site F_1 30	15	59	78	27

Exit Survey

Site F_2 5	60	42	25	30	53
Site F_3 6	54	14	30	20	62
Site F_4 4	ļ6	19	66	48	11
Site F_5 1	9	40	60	31	27

Table 1: Delivery Cost from All Possible Sites to All Customers

The decision variables for this problem are as follows. We let $y_j=1$ if a facility is located at site F_j . Otherwise, $y_j=0$. We let $0\leq x_{ij}\leq 1$ represent the fraction of demand from C_j satisfied by facility F_i . The cost associated with customer C_1 would be

$$d_{11}x_{11} + d_{21}x_{21} + d_{31}x_{31} + d_{41}x_{41} + d_{51}x_{51}$$

One would also need to add constraints that ensures that $x_{ij}=0, \forall j=1,\ldots,5$ whenever $y_i=0$, which means we cannot use site F_i to serve any customer C_j if we don't locate a facility at site i; (2) all the customers' demand need to be satisfied.

Formulate this problem as a deterministic optimization problem in which the objective is to minimize the expected total cost which is the sum of the facility opening cost and the expected delivery cost.

Choose the correct objection function from below.

$$igcup ext{MIN } \sum_{i=1}^5 40y_i + \sum_{i=1}^5 \sum_{j=1}^5 x_{ij}$$

- $igcup MAX \sum_{i=1}^5 40y_i \sum_{i=1}^5 \sum_{j=1}^5 x_{ij}$
- $igcup MAX \sum_{i=1}^5 40y_i + \sum_{i=1}^5 \sum_{j=1}^5 d_{ij} x_{ij}$
- ullet MIN $\sum_{i=1}^{5} 40 y_i + \sum_{i=1}^{5} \sum_{j=1}^{5} d_{ij} x_{ij}$

Submit

You have used 1 of 2 attempts

PART B

2/2 points (graded)

Choose the correct constraint(s) from below:

Assume that a facility i is either used to serve a customer j or it is not, rather than a fractional amount.

- lacksquare $\sum_{i=1}^5 y_i = 1, orall j = 1, \ldots, 5$
- lacksquare $\sum_{i=1}^5 \sum_{j=1}^5 x_{ij} = 1, orall i = 1, \ldots, 5, orall j = 1, \ldots, 5$

$$extbf{Y} \quad x_{ij} \leq y_i, orall i=1,\ldots,5, orall j=1,\ldots,5$$

- $lacksquare x_{ij} \geq y_i, orall i=1,\ldots,5, orall j=1,\ldots,5$
- $extbf{\emptires} \; x_{ij} \in \{0,1\}, orall i=1,\ldots,5, orall j=1,\ldots,5$
- $lacksquare x_{ij} \geq 1, orall i=1,\ldots,5, orall j=1,\ldots,5$
- $extbf{ extit{y}} \ y_i \in \{0,1\}, orall i=1,\ldots,5$
- $lacksquare y_i \geq 0, orall i=1,\ldots,5$



Submit

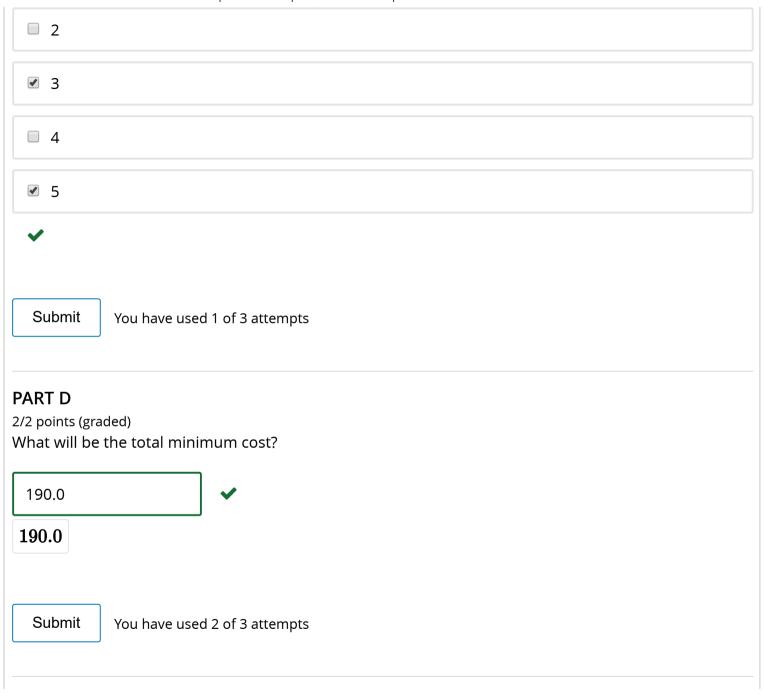
You have used 1 of 2 attempts

PART C

2/2 points (graded)

Solve the deterministic facility location you formulated above using Julia/JuMP. Which facilities are open? Select both of them.

1



PART E

3/3 points (graded)

In the second part of the problem, we will consider the stochastic version. The delivery cost of satisfying customers from the facilities is uncertain. It depends on which future scenario occurs. There are five possible scenarios. Below is the probability of each scenario: p_1, p_2, \ldots, p_5 .

	Scenario 1	Scenario 2	Scenario 3	Scenario 4	Scenario 5
Probability	0.15	0.15	0.25	0.2	0.25

We will model this as a 2-stage stochastic optimization problem. In the first stage, the decision is which of the five facilities to select. Similarly, we let $y_j=1$ if a facility is located at site F_i . Otherwise, $y_i=0$. The cost of opening up a facility is \$40.

After the facilities are selected, the decision maker learns which of the five scenarios occurs. We let the scenarios be designated as S_1, S_2, S_3, S_4 and S_5 . We let d_{ij}^k denote the cost of satisfying demand of customer C_j using facility F_i under scenario S_k . Note that the first subscript corresponds to the facility and the second subscript corresponds to the customer. The data of d_{ij}^k is stored in pset6_p1_data.xlsx

We let $x_{ij}^k=1$ if customer C_j is served by facility F_i under scenario S_k . Otherwise, it is 0.

Model the problem of minimizing the following sum: the cost of opening facilities plus the expected cost of satisfying demand for customers from the open facilities

Choose the correct objection function from below.

- $igcup MIN \sum_{k=1}^5 \sum_{i=1}^5 \sum_{j=1}^5 d_{ij}^k x_{ij}^k$
- $igcap ext{MIN } \sum_{i=1}^5 40 y_i + \sum_{k=1}^5 \sum_{i=1}^5 \sum_{j=1}^5 d_{ij}^k x_{ij}^k$
- ullet MIN $\sum_{i=1}^5 40 y_i + \sum_{k=1}^5 p_k \sum_{i=1}^5 \sum_{j=1}^5 d_{ij}^k x_{ij}^k$

Submit

You have used 1 of 3 attempts

PART F

3/3 points (graded)

Choose all necessary constraint(s) from below.

$$extstyle \sum_{i=1}^5 x_{ij}^k = 1, orall j = 1, \ldots, 5, orall k = 1, \ldots, 5$$

$$lacksquare \sum_{j=1}^5 x_{ij}^k = 1, orall i = 1, \ldots, 5, orall k = 1, \ldots, 5$$

$$ot igwedge x_{ij}^k \leq y_i, orall i=1,\ldots,5, orall j=1,\ldots,5, orall k=1,\ldots,5$$

$$lacksquare x_{ij}^k \geq y_i, orall i=1,\ldots,5, orall j=1,\ldots,5, orall k=1,\ldots,5$$

$$extbf{Y} \quad x_{ij}^k \in \{0,1\}, orall i=1,\ldots,5, orall j=1,\ldots,5$$

$$lacksquare x_{ij}^k \geq 1, orall i=1,\ldots,5, orall j=1,\ldots,5$$

$$extbf{Y} y_j \in \{0,1\}, orall j=1,\ldots,5$$



Submit

You have used 1 of 3 attempts

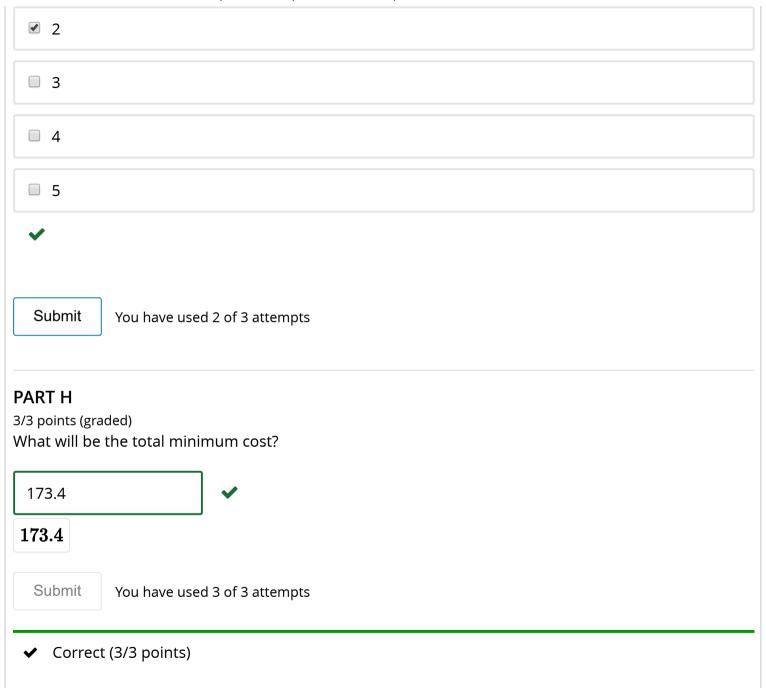
PART G

3/3 points (graded)

Solve the stochastic facility location problem you formulated above using Julia/JuMP. Complete PS6_p1_partfg.jl to solve the problem. We already provide data in it.

Which facilities will be opened under the optimal solution?

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