7. Examples of Parametric Models

Review: Sample Spaces of Distributions

4/4 points (graded)

Recall that a **sample space** of a random variable X is a set that contains all possible outcomes of X.

Note that the sample space of X is *not unique*. For example, if $X \sim \mathrm{Ber}(p)$, then both $\{0,1\}$ and $\mathbb{R}$ can serve as a sample space. However, in general, we associate a random variable with its smallest possible sample space (which would be $\{0,1\}$ if $X \sim \mathrm{Ber}(p)$).

Find the **smallest sample space** for each of the following random variables.

$X_1 \sim \text{Poiss}\,(\lambda)$, a **Poisson** random variable with parameter λ:

- $\{0,1\}$
- $\{x \in \mathbb{Z} : x \geq 0\}$ ✔
- $[0,\infty)$
- $(-\infty,\infty)$

$X_2 \sim \mathcal{N}\,(0,1)$, a **standard Gaussian (or normal)** random variable with mean 0 and variance 1:

- $\{0,1\}$
- $\{x \in \mathbb{Z} : x \geq 0\}$
- $[0,\infty)$
- $(-\infty,\infty)$ ✔

$X_3 \sim \exp(\lambda)$, an **exponential** random variable with parameter $\lambda > 0$:

- ○ $\{0,1\}$
- ○ $\{x \in \mathbb{Z} : x \geq 0\}$
- ◉ $[0,\infty)$ ✔
- ○ $(-\infty,\infty)$

$X_4 \sim \mathcal{I}(Y > 0)$ where Y is standard Gaussian and $\mathcal{I}$ is the **indicator function**.

Recall the definition of the indicator function is:

$$\mathcal{I}(Y > 0) = \begin{cases} 1 & \text{if } Y > 0 \\ 0 & \text{if } Y \leq 0. \end{cases}$$

- ◉ $\{0,1\}$ ✔
- ○ $\{x \in \mathbb{Z} : x \geq 0\}$

- $[0, \infty)$
- $(-\infty, \infty)$

Solution:

- A Poisson random variable is discrete and can take values on all non-negative integers.
- Gaussian random variables can take any real value.
- The Exponential distribution is continuous and is restricted to all non-negative real values.
- The final random variable is an indicator, so it must take values in $\{0, 1\}$. Note that X_4 is in fact Bernoulli.

Submit You have used 1 of 3 attempts

Answers are displayed within the problem

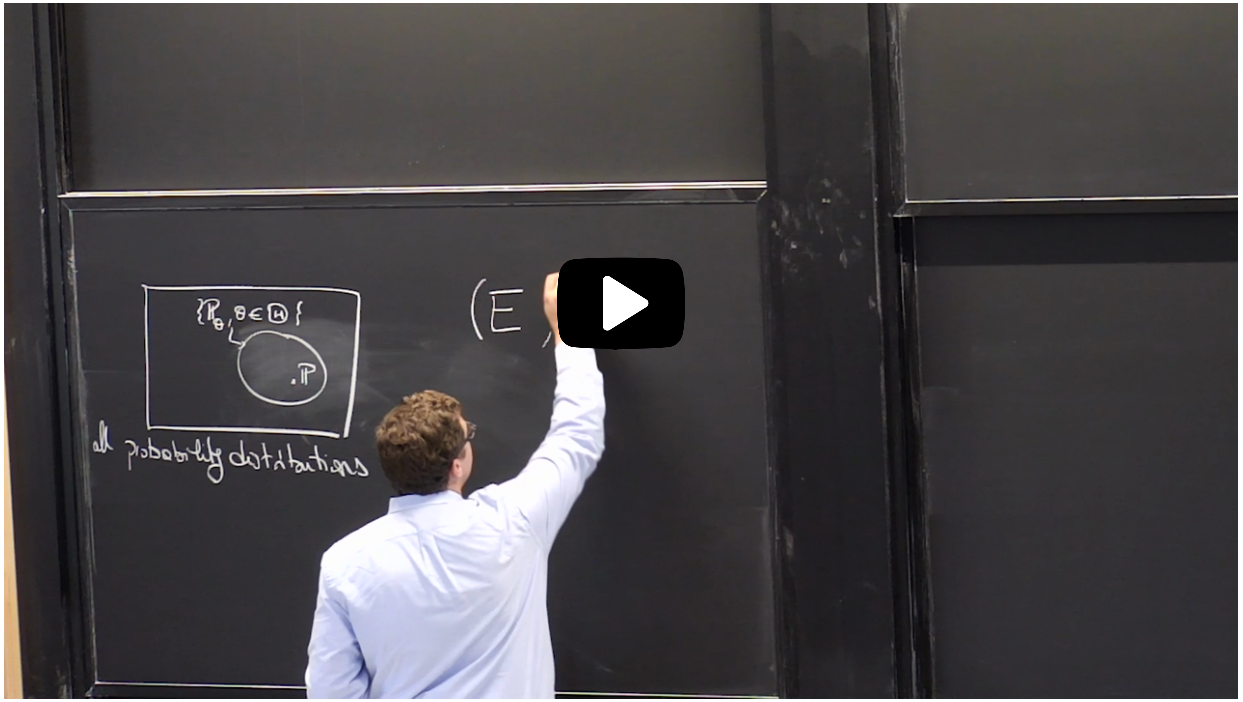
Examples of parametric and nonparametric models

I'm sorry.

It has to be an integer.

It has to be an integer.

But for a Poisson, this is more precise.



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It's always going to be the case.

So just give-- always, thank you,

always give the most possible precise answer.

I could actually write z here.

Don't do it.

I could write z.

All integers.

But it's certainly a superset of what I'm looking for.

But I should not, because I'm never going

to get any negative numbers.

So if I know ahead of time I'm only

going to get positive integers, let

me just write a non-negative integers, natural integer.

And here I write the same thing.

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Statistical Model Definition Concept check

1/1 point (graded)

Which of the following is a statistical model?

- $\left(\{1\}, (\mathsf{Ber}\,(p))_{p\in(0,1)}\right)$
- $\left(\{0,1\}, (\mathsf{Ber}\,(p))_{p\in(0.2,0.4)}\right)$ ✔
- Both of the above
- None of the above

Solution:

Solution in video below.

The set $\{1\}$ is not the sample space of the distribution $\mathrm{Ber}\,(p)$, so the first choice $\left(\{1\}, (\mathsf{Ber}\,(p))_{p\in(0,1)}\right)$ is not a statistical model. On the other hand, $\left(\{0,1\}, (\mathsf{Ber}\,(p))_{p\in(0.2,0.4)}\right)$ is a valid statistical model.

Remark: In the model $\left(\{0,1\}, (\mathsf{Ber}\,(p))_{p\in(0.2,0.4)}\right)$, the parameter p is restricted to be in the interval $(0.2, 0.4)$. Such a restriction is perfectly valid, and can be useful for performing modeling tasks.

Submit You have used 1 of 2 attempts

Answers are displayed within the problem

A Non-Example of a Statistical Model

0 points possible (ungraded)
(This problem is strictly pedagogical and is ungraded.)

Let $\mathcal{U}([0,a])$ denote the uniform distribution on the interval $[0,a]$. Let $X_1,\ldots,X_n \overset{iid}{\sim} \mathcal{U}([0,a])$ for some unknown $a>0$. Which one of the following is *not* a statistical model associated with this statistical experiment?

- $\left([0,a],\left(\mathcal{U}([0,a])\right)_{a>0}\right)$ ✔
- $\left(\mathbb{R}_+,\left(\mathcal{U}([0,a])\right)_{a>0}\right)$
- Neither choice above is a statistical model.

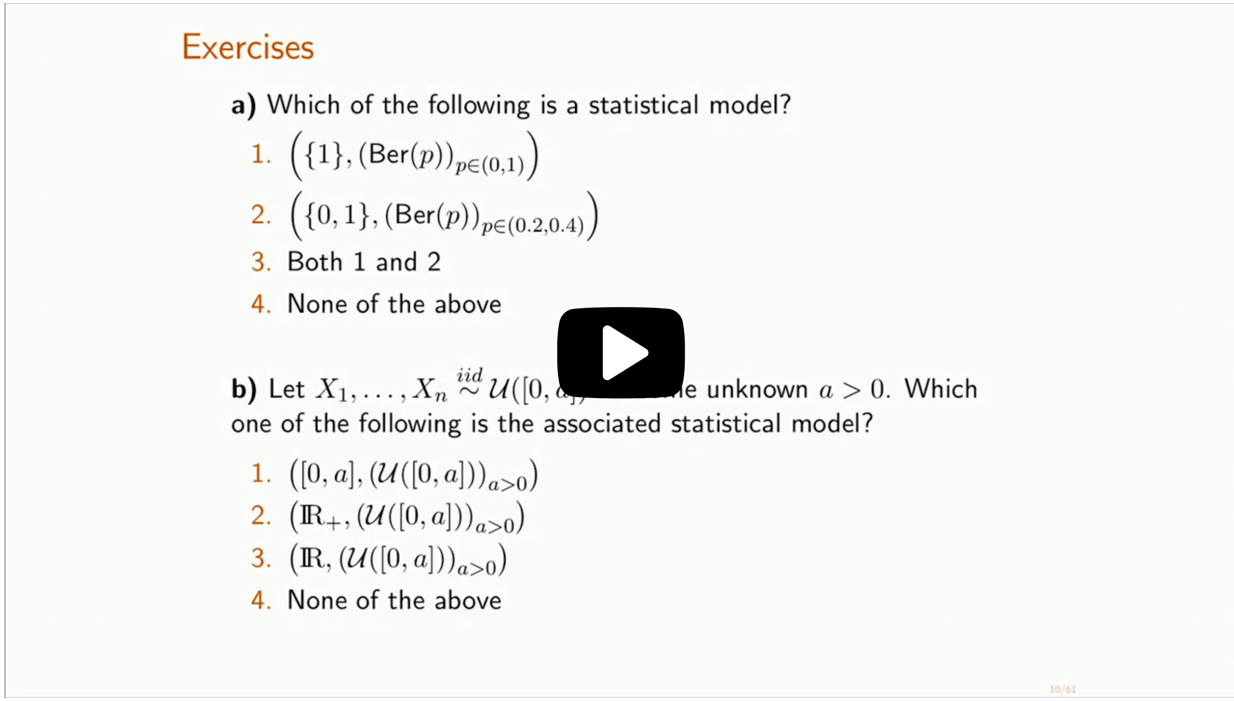
Solution:

See video below.
The first choice $\left([0,a],\left(\mathcal{U}([0,a])\right)_{a>0}\right)$ is not a statistical model because the sample space, as written, depends on an unknown parameter a.
The second choice $\left(\mathbb{R}_+,\left(\mathcal{U}([0,a])\right)_{a>0}\right)$ is a statistical model because for any value of a, the random variables $X_1,\ldots,X_n$ will have sample space contained in the interval $[0,\infty)=\mathbb{R}_+$.

Submit You have used 2 of 2 attempts

Answers are displayed within the problem

Worked example: Definition of Statistical model



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Start of transcript. Skip to the end.

OK, so which one is a statistical model?

OK, so either-- well there's clearly only one answer

that's valid here.

So which one do we have?

Who says one?

Actually, let's just go for it.

Is one a valid statistical model?

Who says yes?

Who says no?

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? Need some more details on sample space
Hi all, in the second video prof. Philippe explains why a sample space cannot be defined in terms of an unknown paramater. But I cannot manage to ... 2

[STAFF] Grader is wrong on "A Non-Example of a Statistical Model"
Not a big deal, as the question is ungraded, but the grader accepts a wrong answer and rejects the correct one.
Note that the explanation in "Sh... 3

[Staff] Typo in the Lecture Notes 5

Is 'sample space' the domain or subset of the range of a random variable? 9

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