Confidence interval of multivariate gaussian distribution

I want to actually get the confidence interval of gaussian distribution. I want to know how I can use the covariance matrix and check if the obtained mui vector for the multivariate gaussian distribution actually satisfied the confidence interval. I have a mui vector and the actual values to be obtained. How can I use covariance matrix and the actual values plust mui vector to verify if it satisfied the confidence interval

normal-distribution

asked Jun 5 '12 at 23:44
user31820
426 1 8 22

This post is answering your question. stats.stackexchange.com/questions/7882/… – user4581 Jun 5 '12 at 23:48

I want to know how I can get the standard deviation such that I can check if the true value is within mui+-standard deviation. It's tricky when it comes to covariance matrix – user31820 Jun 6 '12 at 0:00

1 A confidence region for what? The mean vector? It would be an ellipsoid involving the inverse of the sample covariance matrix. – Michael Chernick Jun 6 '12 at 0:02

1 Yeah, so if I have a sample lets say x vector. How can I check if that sample lies within the 68 percent region. I mean I can get the standard deviation from the covariance matrix for each variable of the multivariate random vector. Then for each element of the x vector I can check if it lies within the +- standard deviation of the elements of the mui vector. Is this the way to go? – user31820 Jun 6 '12 at 0:38

no you construct the 68% confidence ellipse. Find the contour of constant density that contains 68% of the distribution for the sample mean vector within it. – Michael Chernick Jun 6 '12 at 2:54

1 Answer

The quantity $y = (x-\mu)^T\Sigma^{-1}(x-\mu)$ is distributed as χ^2 with k degrees of freedom (where k is the length of the x and μ vectors). Σ is the (known) covariance matrix of the multivariate

Gaussian.

When Σ is unknown, we can replace it by the sample covariance matrix $S = \frac{1}{n-1}\sum_i (x_i - \overline{x})(x_i - \overline{x})^T$, where $\{x_i\}$ are the n data vectors, and $\overline{x} = \frac{1}{n}\sum_i x_i$ is the sample mean. The quantity $t^2 = n(\overline{x} - \mu)^T S^{-1}(\overline{x} - \mu)$ is distributed as Hotelling's T^2 distribution with parameters k and $n-1$.

An ellipsoidal confidence set with coverage probability $1-\alpha$ consists of all μ vectors such that $n(\overline{x} - \mu)^T S^{-1}(\overline{x} - \mu) \leq T^2_{k,n-k}(1-\alpha)$. The critical values of T^2 can be computed from the F distribution. Specifically, $\frac{n-k}{k(n-1)}t^2$ is distributed as $F_{k,n-k}$.

Source: Wikipeda Hotelling's T-squared distribution

edited Jun 18 '13 at 4:42

answered Jun 18 '13 at 4:11

Tom Dietterich
181 3 6