8.6.1 Linear scalar IVP and exponential solution

MO2.4 **MO2.5**

The behavior of a linear scalar ($M = 1$) IVP provides useful insight into the behavior of much more complex systems of equations. Further, many phenomena can be described by a linear scalar IVP. The meaning of linear is the $\underline{f}$ is at most a linear function of $\underline{u}$. For the scalar $M = 1$ case, consider the following linear differential model equation,

$$\frac{\mathrm{d}u}{\mathrm{d}t} = \lambda u \tag{8.48}$$

where λ is a constant (i.e. independent of time). Thus, for this linear scalar IVP, $f = \lambda u$.

Recall that the derivative of the exponential function $\exp(\lambda t)$ is $\lambda \exp(\lambda t)$, i.e.

$$\frac{\mathrm{d}}{\mathrm{d}t}[\exp(\lambda t)] = \lambda \exp(\lambda t) \tag{8.49}$$

Comparing this to Equation (8.48), we see that the solution to that equation is,

$$u(t) = u_I \exp(\lambda t) \tag{8.50}$$

For $\lambda < 0$, then as t increases, the state $u(t)$ decreases in magnitude. This is known as an exponentially decaying solution. And vice-versa, for $\lambda > 0$, the state $u(t)$ increases in magnitude as t increases. This is known as an exponentially growing solution. An example of these solutions is shown for $\lambda = -1$ and $\lambda = 1$ with $u(0) = 1$ in Figure 8.11.

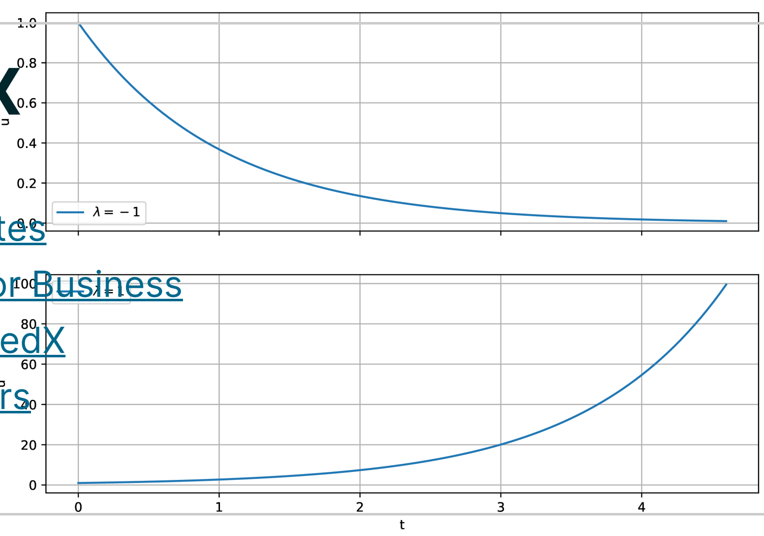

Figure 8.11: Example of exponentially decaying ($\lambda = -1$) and growing ($\lambda = 1$) solutions with $u(0) = 1$

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2.0x

one of the
simplest,
probably the