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Exercise: Correlation properties

(6/6 points)

As in the preceding example, let Z, V, and W be independent random variables with mean 0 and variance 1, and let $X = Z + V$ and $Y = Z + W$. We have found that $\rho(X, Y) = 1/2$.

a) It follows that:

$\rho(X, -Y) =$ -1/2 ✔ **Answer:** -0.5

$\rho(-X, -Y) =$ 1/2 ✔ **Answer:** 0.5

b) Suppose that X and Y are measured in dollars. Let X' and Y' be the same random variables, but measured in cents, so that $X' = 100X$ and $Y' = 100Y$. Then,

$\rho(X', Y') =$ 1/2 ✔ **Answer:** 0.5

c) Suppose now that $\tilde{X} = 3Z + 3V + 3$ and $\tilde{Y} = -2Z - 2W$. Then

$\rho(\tilde{X}, \tilde{Y}) =$ -1/2 ✔ **Answer:** -0.5

d) Suppose now that the variance of Z is replaced by a very large number. Then

$\rho(X, Y)$ is close to 1 ✔ **Answer:** 1

e) Alternatively, suppose that the variance of Z is close to zero. Then

$\rho(X, Y)$ is close to 0 ✔ **Answer:** 0

Answer:

We saw that a linear transformation $x \mapsto ax + b$ of a random variable does not change the value of the correlation coefficient, except for a possible sign change if the coefficient a is negative. Note that in the case of $\rho(-X, -Y)$, we have two sign changes, hence no sign change.

For the last two parts, if Z has a very large variance, then the terms V and W become insignificant, and $\rho(X, Y) \approx \rho(Z, Z) = 1$. And if Z has very small variance, then X and Y are approximately independent, so that $\rho(-X, -Y) \approx 0$. (These conclusions can also be justified by an exact calculation.)

You have used 1 of 2 submissions