A

Tables of Fourier Series and Transform Properties

Table A.1 Properties of the continuous-time Fourier series

$$x(t) = \sum_{k=-\infty}^{\infty} C_k \mathrm{e}^{jk\Omega t} \qquad C_k = \frac{1}{T} \int_{-T/2}^{T/2} x(t) \mathrm{e}^{-jk\Omega t} \mathrm{d}t$$

Property	Periodic function $x(t)$ with period $T = 2\pi/\Omega$	Fourier series C_k
Time shifting	$x(t \pm t_0)$	$C_k \mathrm{e}^{\pm jk\Omega t_0}$
Time scaling	$x(\alpha t)$, $\alpha > 0$	C_k with period $\frac{T}{\alpha}$
Differentiation	$\frac{\mathrm{d}}{\mathrm{d}t} x(t)$	$jk\Omega C_k$
Integration	$\int_{-\infty}^{t} x(t) dt < \infty$	$\frac{1}{jk\Omega} C_k$
Linearity	$\sum_i \alpha_i x_i(t)$	$\sum_i \alpha_i C_{ik}$
Conjugation	$x^*(t)$	C^*_{-k}
Time reversal	$x(-t)$	C_{-k}
Modulation	$x(t)\mathrm{e}^{jK\Omega t}$	C_{k-K}
Multiplication	$x(t)y(t)$	$\sum_{i=-\infty}^{\infty} C_{xi} C_{y(k-i)}$
Periodic convolution	$\int_T x(\theta) y(t-\theta) \mathrm{d}\theta$	$T C_{xk} C_{yk}$
Symmetry	$x(t) = x^*(t)$ real	$\begin{cases} C_k = C^*_{-k}\,, \lvert C_k \rvert = \lvert C_{-k} \rvert\,, \\ \mathrm{Re}\, C_k = \mathrm{Re}\, C_{-k}\,, \\ \mathrm{Im}\, C_k = -\mathrm{Im}\, C_{-k}\,, \\ \arg C_k = -\arg C_{-k} \end{cases}$
	$x(t) = x^*(t) = x(-t)$ real and even	$\begin{cases} C_k = C_{-k}\,, C_k = C^*_k\,, \\ \text{real and even} \end{cases}$
	$x(t) = x^*(t) = -x(-t)$ real and odd	$\begin{cases} C_k = -C_{-k}\,, C_k = -C^*_k\,, \\ \text{imaginary and odd} \end{cases}$
Parseval's theorem	$\frac{1}{T} \int_{-T/2}^{T/2} \lvert x(t) \rvert^2 \mathrm{d}t = \sum_{k=-\infty}^{\infty} \lvert C_k \rvert^2$	

Table A.2 Properties of the continuous-time Fourier transform

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)e^{j\omega t}d\omega \qquad X(j\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt$$

Property	Nonperiodic function $x(t)$	Fourier transform $X(j\omega)$
Time shifting	$x(t \pm t_0)$	$e^{\pm\omega t_0} X(j\omega)$
Time scaling	$x(\alpha t)$	$\frac{1}{\lvert\alpha\rvert} X\left(\frac{j\omega}{\alpha}\right)$
Differentiation	$\frac{d}{dt}x(t)$	$j\omega X(j\omega)$
Integration	$\int_{-\infty}^{t} x(t)dt$	$\frac{1}{j\omega}X(j\omega) + \pi X(j0)\delta(\omega)$
	$\int_{-\infty}^{\infty} x(t)dt$	$X(j0)$
Frequency integration	$2\pi x(0)$	$\int_{-\infty}^{\infty} X(j\omega)d\omega$
Linearity	$\sum_i \alpha_i x_i(t)$	$\sum_i \alpha_i X_i(j\omega)$
Conjugation	$x^*(t)$	$X^*(-j\omega)$
Time reversal	$x(-t)$	$X(-j\omega)$
Modulation	$x(t)e^{j\omega_0 t}$	$X(j\omega - j\omega_0)$
Multiplication	$x(t)y(t)$	$\frac{1}{2\pi}X(j\omega) \times Y(j\omega)$
Convolution	$x(t) * y(t)$	$X(j\omega)Y(j\omega)$
Symmetry	$x(t) = x^*(t)$ real	$\begin{cases} X(j\omega) = X^*(-j\omega)\,, \\ \lvert X(j\omega)\rvert = \lvert X(-j\omega)\rvert\,, \\ \operatorname{Re} X(j\omega) = \operatorname{Re} X(-j\omega)\,, \\ \operatorname{Im} X(j\omega) = -\operatorname{Im} X(-j\omega)\,, \\ \arg X(j\omega) = -\arg X(-j\omega) \end{cases}$
	$x(t) = x^*(t) = x(-t)$ real and even	$\begin{cases} X(j\omega) = X(-j\omega)\,, \\ X(j\omega) = X^*(j\omega)\,, \\ \text{real and even} \end{cases}$
	$x(t) = x^*(t) = -x(-t)$ real and odd	$\begin{cases} X(j\omega) = -X(-j\omega)\,, \\ X(j\omega) = -X^*(j\omega)\,, \\ \text{imaginary and odd} \end{cases}$
Rayleigh's theorem	$E_x = \int_{-\infty}^{\infty} \lvert x(t)\rvert^2 dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} \lvert X(j\omega)\rvert^2 d\omega$	

B

Tables of Fourier Series and Transform of Basis Signals

Table B.1 The Fourier transform and series of basic signals

Signal $x(t)$	Transform $X(j\omega)$	Series C_k
1	$2\pi\delta(\omega)$	$C_0 = 1\ ,\ C_{k\neq 0} = 0$
$\delta(t)$	1	$C_k = \frac{1}{T}$
$u(t)$	$\frac{1}{j\omega} + \pi\delta(\omega)$	—
$u(-t)$	$-\frac{1}{j\omega} + \pi\delta(\omega)$	—
$e^{j\Omega t}$	$2\pi\delta(\omega - \Omega)$	$C_1 = 1\ ,\ C_{k\neq 1} = 0$
$\sum_{k=-\infty}^{\infty} C_k e^{jk\Omega t}$	$2\pi \sum_{k=-\infty}^{\infty} C_k \delta(\omega - k\Omega)$	C_k
$\cos\Omega t$	$\pi[\delta(\omega - \Omega) + \delta(\omega + \Omega)]$	$C_1 = C_{-1} = \frac{1}{2},\ C_{k\neq\pm 1} = 0$
$\sin\Omega t$	$\frac{\pi}{j}[\delta(\omega - \Omega) - \delta(\omega + \Omega)]$	$C_1 = -C_{-1} = \frac{1}{2j},\ C_{k\neq\pm 1} = 0$
$\frac{1}{\alpha^2+t^2}$	$e^{-\alpha\|\omega\|}$	$\frac{1}{T} e^{-\frac{2\pi\alpha\|k\|}{T}}$
Rectangular (Fig. 2.16c)	$A\tau \frac{\sin(\omega\tau/2)}{\omega\tau/2}$	$\frac{A}{q}\frac{\sin(k\pi/q)}{k\pi/q}$
Triangular (Fig. 2.21a)	$\frac{A\tau}{2}\frac{\sin^2(\omega\tau/4)}{(\omega\tau/4)^2}$	$\frac{A}{2q}\frac{\sin^2(k\pi/2q)}{(k\pi/2q)^2}$
Trapezoidal (Fig. 2.30)	$A\tau \frac{\sin(\omega\tau/2)}{\omega\tau/2}\frac{\sin(\omega\tau_s/2)}{\omega\tau_s/2}$	$\frac{A}{q}\frac{\sin(k\pi/q)}{k\pi/q}\frac{\sin(k\pi/q_s)}{k\pi/q_s}$
Ramp (Fig. 2.34b)	$\frac{A}{j\omega}\left[\frac{\sin(\omega\tau/2)}{\omega\tau/2} e^{j\frac{\omega\tau}{2}} - 1\right]$	$\frac{A}{j2\pi k}\left[\frac{\sin(k\pi/q)}{k\pi/q} e^{j\frac{k\pi}{q}} - 1\right]$
Ramp (Fig. 2.34c)	$\frac{A}{j\omega}\left[1 - \frac{\sin(\omega\tau/2)}{\omega\tau/2} e^{-j\frac{\omega\tau}{2}}\right]$	$\frac{A}{j2\pi k}\left[1 - \frac{\sin(k\pi/q)}{k\pi/q} e^{-j\frac{k\pi}{q}}\right]$
$\frac{\sin\alpha t}{\alpha t}$	$\begin{cases} \frac{\pi}{\alpha}, & \|\omega\| < \alpha \\ 0, & \|\omega\| > \alpha \end{cases}$	$\begin{cases} \frac{\pi}{\alpha T}, & \|k\| < \frac{\alpha T}{2\pi} \\ 0, & \|k\| > \frac{\alpha T}{2\pi} \end{cases}$
$e^{-\alpha t}u(t)$, $\mathrm{Re}\,\alpha > 0$	$\frac{1}{\alpha+j\omega}$	$\frac{1}{\alpha T+j2\pi k}$
$te^{-\alpha t}u(t)$, $\mathrm{Re}\,\alpha > 0$	$\frac{1}{(\alpha+j\omega)^2}$	$\frac{T}{(\alpha T+j2\pi k)^2}$

Table B.1 The Fourier transform and series of basic signals (*Contd.*)

$\frac{t^{n-1}}{(n-1)!}\mathrm{e}^{-\alpha t}u(t)$, $\mathrm{Re}\,\alpha > 0$	$\frac{1}{(\alpha+j\omega)^n}$	$\frac{T^{n-1}}{(\alpha T+j2\pi k)^n}$
$\mathrm{e}^{-\alpha\|t\|}$, $\alpha > 0$	$\frac{2\alpha}{\alpha^2+\omega^2}$	$\frac{2\alpha T}{\alpha^2 T^2+4\pi^2 k^2}$
$\mathrm{e}^{-\alpha^2 t^2}$	$\frac{\sqrt{\pi}}{\alpha}\mathrm{e}^{-\frac{\omega^2}{4\alpha^2}}$	$\frac{\sqrt{\pi}}{\alpha T}\mathrm{e}^{-\frac{\pi^2 k^2}{\alpha^2 T^2}}$

C_k corresponds to $x(t)$ repeated with period T, τ and τ_s are durations, $q = \frac{T}{\tau}$, and $q_\mathrm{s} = \frac{T}{\tau_\mathrm{s}}$.

Table B.2 The Fourier transform and series of complex signals

Signal $y(t)$	Transform $Y(j\omega)$	Series C_k
Burst of N pulses with known $X(j\omega)$	$X(j\omega)\frac{\sin(\omega NT/2)}{\sin(\omega T/2)}$	$\frac{1}{T_1}X\left(j\frac{2k\pi}{T_1}\right)\frac{\sin(k\pi/q_2)}{\sin(k\pi/Nq_2)}$
Rectangular pulse-burst (Fig. 2.47)	$A\tau\frac{\sin(\omega\tau/2)}{\omega\tau/2}\frac{\sin(\omega NT/2)}{\sin(\omega T/2)}$	$\frac{A}{q_1}\frac{\sin(k\pi/q_1)}{k\pi/q_1}\frac{\sin(k\pi/q_2)}{\sin(k\pi/Nq_2)}$
Triangular pulse-burst	$\frac{A\tau}{2}\frac{\sin^2(\omega\tau/4)}{(\omega\tau/4)^2}\frac{\sin(\omega NT/2)}{\sin(\omega T/2)}$	$\frac{A}{2q_1}\frac{\sin^2(k\pi/2q_1)}{(k\pi/2q_1)^2}\frac{\sin(k\pi/q_2)}{\sin(k\pi/Nq_2)}$
Sinc-shaped pulse-burst	$\begin{cases}\frac{A\pi}{\alpha}\frac{\sin(\omega NT/2)}{\sin(\omega T/2)}, & \|\omega\| < \alpha\\ 0, & \|\omega\| > \alpha\end{cases}$	$\begin{cases}\frac{A\pi}{\alpha T_1}\frac{\sin(k\pi/q_2)}{\sin(k\pi/Nq_2)}, & \|k\| < \frac{\alpha T_1}{2\pi}\\ 0, & \|k\| > \frac{\alpha T_1}{2\pi}\end{cases}$

C_k corresponds to $y(t)$ repeated with period T_1, τ is pulse duration, T is period of pulse in the burst, T_1 is period of pulse-bursts in the train, $q_1 = \frac{T_1}{\tau}$, and $q_2 = \frac{T_1}{NT}$.

C

Tables of Hilbert Transform and Properties

Table C.1 Properties of the Hilbert transform

$$y(t) = \frac{1}{\pi}\int_{-\infty}^{\infty}\frac{\hat{y}(\theta)}{\theta - t}\mathrm{d}\theta \qquad \hat{y}(t) = \frac{1}{\pi}\int_{-\infty}^{\infty}\frac{y(\theta)}{t-\theta}\mathrm{d}\theta$$

Property	Function $y(t)$	Transform $\hat{y}(t)$
Filtering	$y(t)$ is constant	$\hat{y}(t)$ is zero
	is real	is real
	is even (odd)	is odd (even)
Causality	If $y(t)$ is causal with known transform $Y(j\omega) = Y_{\mathrm{r}}(\omega) + jY_{\mathrm{i}}(\omega)$, then:	
	$Y_{\mathrm{r}}(\omega)$	$Y_{\mathrm{i}}(\omega)$
	$Y_{\mathrm{i}}(\omega)$	$-Y_r(\omega)$
Linearity	$\sum_i \alpha_i y_i(t)$	$\sum_i \alpha_i \hat{y}_i(t)$
Time shifting	$y(t \pm \theta)$	$\hat{y}(t \pm \theta)$
Time reversal	$y(-t)$	$-\hat{y}(-t)$
Scaling	$y(at)$	$\hat{y}(at)$
	$y(-at)$	$-\hat{y}(-at)$
Multiple transform	$\hat{y}(t)$	$-y(t)$
		$\mathcal{H}^3 y(t) = \mathcal{H}^{-1} y(t)$
		$\mathcal{H}^4 y(t) = y(t)$
		$\mathcal{H}^n y(t) \overset{\mathcal{F}}{\Leftrightarrow} [-j\,\mathrm{sgn}(\omega)]^n Y(j\omega)$
Differentiation	$\frac{\mathrm{d}}{\mathrm{d}t} y(t)$	$\frac{\mathrm{d}}{\mathrm{d}t}\hat{y}(t)$
Integration	$\int_a^b y(t)\mathrm{d}t$, a and b are constants	0
Convolution	$y_1(t) * y_2(t)$	$y_1(t) * \hat{y}_2(t)$
Autocorrelation	$y(t) * y(t)$	$\widehat{y(t) * \hat{y}(t)}$
Multiplication	$ty(t)$	$t\hat{y}(t) + \frac{1}{\pi}\int_{-\infty}^{\infty} y(t)\mathrm{d}t$

Table C.2 Useful relations between $y(t)$ and its Hilbert transform $\hat{y}(t)$

Property	Relation
Orthogonality	$\int\limits_{-\infty}^{\infty} y(t)\hat{y}(t)\mathrm{d}t = 0$
Integration	$\int\limits_{-\infty}^{\infty} y_1(t)\hat{y}_2(t)\mathrm{d}t = \int\limits_{-\infty}^{\infty} \hat{y}_1(t)y_2(t)\mathrm{d}t$
	$\int\limits_{-\infty}^{\infty} y_1(t)y_2(t)\mathrm{d}t = \int\limits_{-\infty}^{\infty} \hat{y}_1(t)\hat{y}_2(t)\mathrm{d}t$
	$\int\limits_{-\infty}^{\infty} y^2(t)\mathrm{d}t = \int\limits_{-\infty}^{\infty} \hat{y}^2(t)\mathrm{d}t$
Energy	$\int\limits_{-\infty}^{\infty} \|y(t)\|^2(t)\mathrm{d}t = \int\limits_{-\infty}^{\infty} \|\hat{y}(t)\|^2\mathrm{d}t$
Autocorrelation	$\int\limits_{-\infty}^{\infty} y(t)y(t-\theta)\mathrm{d}t = \int\limits_{-\infty}^{\infty} \hat{y}(t)\hat{y}(t-\theta)\mathrm{d}t$

Table C.3 The Hilbert transform of analytic signals

Property	Signal	Transform
Analytic signal	$y_\mathrm{a}(t) = y(t) + j\hat{y}(t)$	$-jy_\mathrm{a}(t) = \hat{y}(t) - jy(t)$
Multiplication	$y_{\mathrm{a}1}(t)y_{\mathrm{a}2}(t)$	$-jy_{\mathrm{a}1}(t)y_{\mathrm{a}2}(t)$
		$= \hat{y}_{\mathrm{a}1}(t)y_{\mathrm{a}2}(t)$
		$= y_{\mathrm{a}1}(t)\hat{y}_{\mathrm{a}2}(t)$
Power n	$y_\mathrm{a}^n(t)$	$-jy_\mathrm{a}^n(t)$
Real product	$\mathrm{Re}[y_{\mathrm{a}1}(t)y_{\mathrm{a}2}(t)]$	$\mathrm{Im}[y_{\mathrm{a}1}(t)y_{\mathrm{a}2}(t)]$
Imaginary product	$\mathrm{Im}[y_{\mathrm{a}1}(t)y_{\mathrm{a}2}(t)]$	$-\mathrm{Re}[y_{\mathrm{a}1}(t)y_{\mathrm{a}2}(t)]$

Table C.4 Products of the analytic signals

Product	Relation
$y_{\mathrm{a}1}(t)y_{\mathrm{a}2}(t)$	$= j\hat{y}_{\mathrm{a}1}(t)y_{\mathrm{a}2}(t) = jy_{\mathrm{a}1}(t)\hat{y}_{\mathrm{a}2}(t)$
$\hat{y}_{\mathrm{a}1}(t)y_{\mathrm{a}2}(t)$	$= -jy_{\mathrm{a}1}(t)y_{\mathrm{a}2}(t) = y_{\mathrm{a}1}(t)\hat{y}_{\mathrm{a}2}(t)$

Table C.5 The Hilbert transform of basic signals

Signal	Transform
$\delta(x)$	$\frac{1}{\pi x}$
$\delta'(x)$	$-\frac{1}{\pi x^2}$
$\cos x$	$\sin x$
$\sin x$	$-\cos x$
$\cos(ax)J_n(bx) \quad [0 < b < a]$	$-\sin(ax)J_n(bx)$
$\frac{\sin x}{x}$	$\frac{\sin^2 x/2}{x/2}$
e^{jx}	$j\mathrm{e}^{jx}$
$\frac{a}{x^2+a^2} \quad [a > 0]$	$\frac{x}{x^2+a^2}$
$\frac{ab-x^2}{(ab-x^2)^2+x^2(a+b)^2}$	$\frac{x(a+b)}{(ab-x^2)^2+x^2(a+b)^2}$
$\delta(x-a)-\delta(x+a)$	$\frac{2}{\pi}\frac{a}{x^2-a^2}$
$\delta(x-a)+\delta(x+a)$	$\frac{2}{\pi}\frac{x}{x^2-a^2}$
$\mathrm{e}^{-x} \quad [x > 0]$	$\frac{1}{\pi}\mathrm{e}^{-x}\mathrm{Ei}(x)$
sign $x\mathrm{e}^{-\|x\|}$	$\frac{1}{\pi}[\mathrm{e}^{-x}\ \mathrm{Ei}\ (x)+\mathrm{e}^{x}\mathrm{Ei}(-x)]$
$\mathrm{e}^{-\|x\|}$	$\frac{1}{\pi}[\mathrm{e}^{-x}\mathrm{Ei}(x)-\mathrm{e}^{x}\mathrm{Ei}(-x)]$
$\|x\|^{v-1} \quad [0 < \mathrm{Re}v < 1]$	$\cot\left(\frac{\pi v}{2}\right)\|x\|^{v-1}$ sign x
$\|x\|^{v-1}$ sign $x \quad [0 < \mathrm{Re}v < 1]$	$-\tan\left(\frac{\pi v}{2}\right)\|x\|^{v-1}$
sign x	$-\infty$
Even rectangular pulse $u(x+\tau)-u(x-\tau)$	$\frac{1}{\pi}\ln\left\|\frac{x+\tau}{x-\tau}\right\|$
Odd rectangular pulse $-u(x+\tau)+2u(x)-u(x-\tau)$	$-\frac{1}{\pi}\left[\ln\left\|\frac{x+\tau}{x}\right\|+\ln\left\|\frac{x-\tau}{x}\right\|\right]$
Ramp pulse $\frac{1}{\tau}(\tau-x)[u(x)-u(x-\tau)]$	$\frac{1}{\pi}\left[\frac{x-\tau}{\tau}\ln\left\|\frac{x-\tau}{x}\right\|+1\right]$
Triangular pulse $\frac{1}{\tau}(\tau-\|x\|)[u(x+\tau)-u(x-\tau)]$	$\frac{1}{\pi}\left[\frac{x+\tau}{\tau}\ln\left\|\frac{x+\tau}{x}\right\|+\frac{x-\tau}{\tau}\ln\left\|\frac{x-\tau}{x}\right\|\right]$
Sawtooth pulse $\frac{1}{\tau}(\tau-\|x\|)[-u(x+\tau)+2u(x)-u(x-\tau)]$	$\frac{1}{\pi}\left[\frac{x-\tau}{\tau}\ln\left\|\frac{x-\tau}{x}\right\|-\frac{x+\tau}{\tau}\ln\left\|\frac{x+\tau}{x}\right\|+2\right]$

D

Mathematical Formulas

Limits:

- $\lim\limits_{x\to a} \frac{f(x)}{g(x)} = \lim\limits_{x\to a} \frac{\partial f(x)/\partial x}{\partial g(x)/\partial x}$ (L'Hospital's rule)
- $\lim\limits_{x\to 0} \frac{\sin x}{x} = 1$
- $\lim\limits_{x\to 0} \frac{\sin Nx}{\sin x} = N$
- $\int\limits_0^\infty \sin bx \,\mathrm{d}x = \lim\limits_{\alpha\to 1} \int\limits_0^\infty x^{\alpha-1} \sin bx \,\mathrm{d}x = \frac{\Gamma(\alpha)}{b^\alpha} \sin \frac{\alpha\pi}{2}\Big|_{\alpha=1} = \frac{1}{b}$

Trigonometric identities:

- $\mathrm{e}^{jx} = \cos x + j \sin x$ (Euler's formula)
- $\mathrm{e}^{(\alpha+jx)} = \mathrm{e}^{\alpha}(\cos x + j \sin x)$
- $\cos x = \frac{\mathrm{e}^{jx}+\mathrm{e}^{-jx}}{2}$
- $\sin x = \frac{\mathrm{e}^{jx}-\mathrm{e}^{-jx}}{2j}$
- $\cos^2 x + \sin^2 x = 1$
- $\cos^2 x - \sin^2 x = \cos 2x$
- $2 \cos x \sin x = \sin 2x$
- $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$
- $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$
- $\cos x \cos y = \frac{1}{2}[\cos (x + y) + \cos (x - y)]$
- $\sin x \sin y = \frac{1}{2}[\cos (x - y) - \cos (x + y)]$

- $\sin x \cos y = \frac{1}{2}[\sin(x+y) + \sin(x-y)]$
- $\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$
- $\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$
- $\cos x + \cos y = 2\cos\frac{x+y}{2}\cos\frac{x-y}{2}$
- $\cos x - \cos y = -2\sin\frac{x+y}{2}\sin\frac{x-y}{2}$
- $\sin x \pm \sin y = 2\sin\frac{x\pm y}{2}\cos\frac{x\mp y}{2}$
- $a\cos x + b\sin x = r\sin(x+\varphi) = r\cos(x-\psi)$,
  $r = \sqrt{a^2+b^2}$, $\sin\varphi = \frac{a}{r}$, $\cos\varphi = \frac{b}{r}$, $\sin\psi = \frac{b}{r}$, $\cos\psi = \frac{a}{r}$
- $\frac{d}{dx}\arcsin x = \frac{1}{\sqrt{1-x^2}}$
- $\frac{d}{dx}\arccos x = -\frac{1}{\sqrt{1-x^2}}$
- $\frac{d}{dx}\arctan x = \frac{1}{1+x^2}$
- $\frac{d}{dx}\mathrm{arccot}\, x = -\frac{1}{1+x^2}$

Hyperbolic identities:

- $\sinh x = -\sinh(-x) = \pm\sqrt{\cosh^2 x - 1} = \pm\sqrt{\frac{1}{2}(\cosh 2x - 1)} = \frac{\mathrm{e}^x - \mathrm{e}^{-x}}{2}$
- $\cosh x = \cosh(-x) = \sqrt{\sinh^2 x + 1} = \sqrt{\frac{1}{2}(\cosh 2x + 1)} = 2\cosh^2\frac{x}{2} - 1 = \frac{\mathrm{e}^x + \mathrm{e}^{-x}}{2}$
- $\tanh x = \frac{\sinh x}{\cosh x} = \frac{\mathrm{e}^x - \mathrm{e}^{-x}}{\mathrm{e}^x + \mathrm{e}^{-x}}$
- $\coth x = \frac{\cosh x}{\sinh x} = \frac{\mathrm{e}^x + \mathrm{e}^{-x}}{\mathrm{e}^x - \mathrm{e}^{-x}}$
- $\cosh^2 x - \sinh^2 x = 1$
- $\cosh x + \sinh x = \mathrm{e}^x$
- $\cosh x - \sinh x = \mathrm{e}^{-x}$

Exponents:

- $\mathrm{e}^{\ln x} = x$
- $\frac{\mathrm{e}^x}{\mathrm{e}^y} = \mathrm{e}^{x-y}$
- $\mathrm{e}^x\mathrm{e}^y = \mathrm{e}^{x+y}$
- $(\mathrm{e}^x)^\alpha = \mathrm{e}^{\alpha x}$

Logarithms:

- ▷ $\ln \mathrm{e}^x = x$
- ▷ $\ln \frac{x}{y} = \ln x - \ln y$
- ▷ $\ln \alpha x = \ln \alpha + \ln x$
- ▷ $\ln x^\alpha = \alpha \ln x$

Extension to series:

- ▷ $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \ldots + (-1)^n \frac{x^{2n+1}}{(2n+1)!} + \ldots$
- ▷ $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots + (-1)^n \frac{x^{2n}}{(2n)!} + \ldots$
- ▷ $\mathrm{e}^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots + \frac{x^n}{n!} + \ldots$
- ▷ $\mathrm{e}^{jz\cos\phi} = \sum_{n=-\infty}^{\infty} j^n J_n(z) \mathrm{e}^{jn\phi}$

Series:

- ▷ $\sum_{n=0}^{N-1} x^n = \frac{1-x^N}{1-x}$, $x \neq 1$ (by geometric progression)
- ▷ $\sum_{n=0}^{N-1} \mathrm{e}^{\alpha n} = \frac{1-\mathrm{e}^{\alpha N}}{1-\mathrm{e}^{\alpha}}$
- ▷ $\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}$, $|x| < 1$
- ▷ $\sum_{k=1}^{\infty} \frac{\sin^2(k\pi/q)}{k^2} = \frac{\pi^2(q-1)}{2q^2}$

Indefinite integrals:

- ▷ $\int f'(x)g(x)\mathrm{d}x = f(x)g(x) - \int f(x)g'(x)\mathrm{d}x$ (integration by parts)
- ▷ $\int f(x)\mathrm{d}x = \int f[g(y)]g'(y)\mathrm{d}y$ $[x = g(y)]$ (change of variable)
- ▷ $\int \frac{dx}{x} = \ln |x|$
- ▷ $\int \mathrm{e}^x \mathrm{d}x = \mathrm{e}^x$
- ▷ $\int a^x \mathrm{d}x = \frac{a^x}{\ln a}$
- ▷ $\int \sin x \, \mathrm{d}x = -\cos x$
- ▷ $\int \cos x \, \mathrm{d}x = \sin x$

▷ $\int x \begin{Bmatrix} \sin x \\ \cos x \end{Bmatrix} \mathrm{d}x = \begin{Bmatrix} \sin x \\ \cos x \end{Bmatrix} \mp x \begin{Bmatrix} \cos x \\ \sin x \end{Bmatrix}$

▷ $\int \frac{\mathrm{d}x}{x(ax^r+b)} = \frac{1}{rb} \ln \left| \frac{x^r}{ax^r+b} \right|$

▷ $\int \sin^2 x \, \mathrm{d}x = -\frac{1}{4} \sin 2x + \frac{x}{2}$

▷ $\int \cos^2 x \, \mathrm{d}x = \frac{1}{4} \sin 2x + \frac{x}{2}$

▷ $\int x \mathrm{e}^{\alpha x} \mathrm{d}x = \mathrm{e}^{\alpha x} \left(\frac{x}{\alpha} - \frac{1}{\alpha^2} \right)$

▷ $\int x^2 \mathrm{e}^{\alpha x} \mathrm{d}x = \mathrm{e}^{\alpha x} \left(\frac{x^2}{\alpha} - \frac{2x}{\alpha^2} + \frac{2}{\alpha^3} \right)$

▷ $\int x^\lambda \mathrm{e}^{\alpha x} \mathrm{d}x = \frac{1}{\alpha} x^\lambda \mathrm{e}^{\alpha x} - \frac{\lambda}{\alpha} \int x^{\lambda-1} \mathrm{e}^{\alpha x} \mathrm{d}x$

▷ $\int \mathrm{e}^{-(ax^2+bx+c)} \mathrm{d}x = \frac{1}{2} \sqrt{\frac{\pi}{a}} \mathrm{e}^{\frac{b^2-4ac}{4a}} \operatorname{erf} \left(x\sqrt{a} + \frac{b}{2\sqrt{a}} \right)$

▷ $\int \frac{1}{x} \mathrm{e}^{\alpha x} \mathrm{d}x = \mathrm{Ei}(\alpha x) \quad [\alpha \neq 0]$

▷ $\int \frac{1}{\sqrt{x}} \mathrm{e}^{-\alpha x} \mathrm{d}x = \sqrt{\frac{\pi}{\alpha}} \operatorname{erf} \left(\sqrt{\alpha x} \right) \quad [\alpha > 0]$

▷ $\int \frac{\mathrm{e}^x}{x^2+\alpha^2} \mathrm{d}x = \frac{1}{\alpha} \operatorname{Im}[\mathrm{e}^{j\alpha} \operatorname{Ei}(x - j\alpha)]$

Definite integrals:

▷ $\int\limits_{-\infty}^{\infty} \frac{\sin \alpha x}{x} \mathrm{d}x = \pi$

▷ $\int\limits_{-\infty}^{\infty} \mathrm{e}^{-\alpha x^2} \mathrm{d}x = \sqrt{\frac{\pi}{\alpha}}$

▷ $\int\limits_{-\infty}^{\infty} x^2 \mathrm{e}^{-\alpha x^2} \mathrm{d}x = \sqrt{\pi} \alpha^{-3/2}$

▷ $\int\limits_{0}^{\infty} \frac{\sin \alpha x}{x} \mathrm{d}x = \frac{\pi}{2} \operatorname{sgn} \alpha$

▷ $\int\limits_{0}^{\infty} \frac{\sin^2 \alpha x}{x^2} \mathrm{d}x = \frac{\pi\alpha}{2}$

▷ $\int\limits_{-\infty}^{\infty} \frac{\sin^4 \alpha x}{x^2} \mathrm{d}x = \frac{\pi\alpha}{2}$

▷ $\int\limits_{0}^{\infty} \frac{\mathrm{d}x}{\alpha^2+x^2} = \frac{\pi}{2\alpha}$

▷ $\int\limits_{0}^{\infty} x^{\alpha-1} \sin bx \, \mathrm{d}x = \frac{\Gamma(\alpha)}{b^\alpha} \sin \frac{\alpha\pi}{2}$

▷ $\int\limits_0^\infty \sin bx \, \mathrm{d}x = \frac{1}{b}$

▷ $\int\limits_{-\infty}^{\infty} \frac{\sin x}{x(x-\alpha)} \mathrm{d}x = \frac{\pi}{\alpha}(\cos\alpha - 1)$, α is real

▷ $\int\limits_{-\infty}^{\infty} \frac{\cos(ax)}{b^2-x^2} \mathrm{d}x = \frac{\pi}{2b} \sin(ab)$, $a, b > 0$

Special functions:

▷ $\mathrm{erf}\,(x) = \frac{2}{\sqrt{\pi}} \int\limits_0^x \mathrm{e}^{-t^2} dt$ (Error function)

▷ $\mathrm{Ei}\,(x) = -\int\limits_{-x}^{\infty} \frac{\mathrm{e}^{-t}}{t} \mathrm{d}t = \int\limits_{-\infty}^{x} \frac{\mathrm{e}^{t}}{t} \mathrm{d}t$ $[x < 0]$ (Exponential-integral function)

▷ $\mathrm{Ei}\,(x) = \mathrm{e}^x \left[\frac{1}{x} + \int\limits_0^\infty \frac{\mathrm{e}^{-t}}{(x-t)^2} \mathrm{d}t \right]$ $[x > 0]$ (Exponential-integral function)

▷ $S(x) = \frac{2}{\sqrt{2\pi}} \int\limits_0^x \sin t^2 \mathrm{d}t = \int\limits_0^x \sin \frac{\pi t^2}{2} \mathrm{d}t$ (Fresnel integral)

▷ $C(x) = \frac{2}{\sqrt{2\pi}} \int\limits_0^x \cos t^2 \mathrm{d}t = \int\limits_0^x \cos \frac{\pi t^2}{2} \mathrm{d}t$ (Fresnel integral)

▷ $J_n(z) = \frac{1}{2\pi} \int\limits_{-\pi}^{\pi} \mathrm{e}^{-j(n\theta - z\sin\theta)} \mathrm{d}\theta$ (Bessel function of the first kind)

References

1. Anderson, J. B., Aulin, T., and Sundberg, C.-E. *Digital Phase Modulation,* New York, Plenum Press, 1986.
2. Baskakov, S. I. *Radio Circuits and Signals,* 2nd edn., Moscow, Vysshaya Shkola, 1988 (in Russian).
3. Bracewell, R. N. *The Fourier Transform and its Applications,* 3rd edn., New York, McGraw-Hill, 1999.
4. Chen, C.-T. *Signals and Systems,* 3rd edn., New York, Oxford University Press, 2004.
5. Cracknell, A. and Hayes, L. *Introduction to Remote Sensing,* London, Taylor & Francis, 1990.
6. Gonorovsky, I. S. *Radio Circuits and Signals,* 2nd edn., Moscow, Radio i sviaz, 1986 (in Russian).
7. Gradshteyn, I. S. and Ryzhik, I. M. *Tables of Integrals, Series, and Products,* San Diego, CA, Academic Press, 1980.
8. Elali, T. and Karim, M. A. *Continuous Signals and Systems with MATLAB,* Boca Raton, FL, CRC Press, 2001.
9. Frerking, M. *Digital Signal Processing in Communication Systems,* Boston, MA, Kluwer, 1994.
10. Haykin, S. and Van Veen, B. *Signals and Systems,* 2nd edn., New York, Wiley, 2002.
11. Hsu, H. P. *Signals and Systems,* New York, McGraw-Hill, 1995.
12. Komarov, I. V. and Smolskiy, S. M. *Fundamentals of Short-Range FM Radars,* Boston, MA, Artech, 2003.
13. Lathi, B. P. *Linear Systems and Signals,* New York, Oxford University Press, 2002.
14. Le Chevalier, F. *Principles of Radar and Sonar Signal Processing,* Norwood, MA, Artech, 2002.
15. Logston, T. *Understanding the Navstar GPS, GIS, and IVHS,* 2nd edn., New York, Chapman & Hall, 1995.
16. Nathanson, F. E., Reilly, J. P., and Cohen, M. N. *Radar Design Principles,* 2nd edn., Mendham, NJ, SciTech Publishing, 1999.
17. Oppenheim, A. V., Willsky, A. S., and Hamid Nawab, S., *Signals and Systems,* 2nd edn., New York, Prentice-Hall, 1994.

18. Poularikas, A. D. *The Handbook of Formulas and Tables for Signal Processing,* Boca Raton, FL, CRC Press, 1999.
19. Roberts, M. J. *Signals and Systems Analysis Using Transform Methods and MATLAB,* New York, McGraw-Hill, 2004.
20. Schwartz, M. *Mobile Wireless Communications,* Cambridge University Press, 2005.
21. Soliman, S. S. and Srinath, M. D. *Continuous and Discrete Signals and Systems,* New York, Prentice-Hall, 1990.
22. Tetley, L. and Calcutt, D. *Electronic Navigation Systems,* 3rd edn., Oxford, Elsevier, 2001.
23. Taylor, F. J. *Principles of Signals and Systems,* New-York, McGraw Hill, 1994.
24. Tikhonov, V. I. and Mironov, M. A, *Markov Processes,* 2nd edn., Moscow, Sovetskoe Radio, 1977 (in Russian).
25. Trakhtman, A. M. and Trakhtman, V. A. Tables of Hilbert transform, *Radiotekhnika*, 25, 3, 1970.
26. Vig, J. *Introduction to Quartz Frequency Standards,* Fort Monmouth, US Army SLCET-TR-92-1 (Rev. 1), 1992.
27. Shannon, C. E. A mathematical theory of communication, *The Bell System Technical Journal*, 25, 379–423, July 1948.

Index