- Unit 0: Overview
- Entrance Survey
- Unit 1: Probability models and axioms
- Unit 2: Conditioning and independence
- Unit 3: Counting
- Unit 4: Discrete random variables
- Exam 1
- Unit 5: Continuous random variables
- Unit 6: Further topics on random variables
- **Unit 7: Bayesian inference**

Problem 3: Radiation from a remote star

(3/3 points)

Caleb builds a particle detector and uses it to measure radiation from a remote star. On any given day, the number of particles, Y, that hit the detector is distributed according to a Poisson distribution with parameter x. The parameter x is unknown and is modeled as the value of a random variable X that is exponentially distributed with parameter $\mu > 0$:

$$f_X(x) = \begin{cases} \mu e^{-\mu x}, & \text{if } x \geq 0, \\ 0, & \text{otherwise.} \end{cases}$$

The conditional PMF of the number of particles hitting the detector is

$$p_{Y|X}(y \mid x) = \begin{cases} \frac{e^{-x} x^y}{y!}, & \text{if } y = 0, 1, 2, \ldots, \\ 0, & \text{otherwise.} \end{cases}$$

(a) Find the MAP estimate of X based on the observed value y of Y. Express your answer in terms of y and μ. Use 'mu' to denote μ.

$\hat{x}_{\text{MAP}}(y) =$ y/(mu+1) ✔

(b) Our goal is to find the LMS estimate for X based on the observed particle count y.

1. We can show that the conditional PDF of X given Y is of the form

$$f_{X|Y}(x \mid y) = \frac{\lambda^{y+1}}{y!} x^y e^{-\lambda x}, \quad x > 0, y \geq 0.$$

Express λ as a function of μ. You may find the following equality useful:

Unit overview

Lec. 14: Introduction to Bayesian inference
Exercises 14 due Apr 06, 2016 at 23:59 UTC

Lec. 15: Linear models with normal noise
Exercises 15 due Apr 06, 2016 at 23:59 UTC

Problem Set 7a
Problem Set 7a due Apr 06, 2016 at 23:59 UTC

Lec. 16: Least mean squares (LMS) estimation
Exercises 16 due Apr 13, 2016 at 23:59 UTC

Lec. 17: Linear least mean squares (LLMS) estimation
Exercises 17 due Apr 13, 2016 at 23:59 UTC

Problem Set 7b
Problem Set 7b due Apr 13, 2016 at 23:59 UTC

Solved problems

Additional theoretical material

Unit summary

$$\int_0^\infty a^{y+1} x^y e^{-ax}\,dx = y!, \quad \text{for any } a > 0.$$

$\lambda =$ mu+1

2. Find the LMS estimate of X based on the observed particle count y. Express your answer in terms of y and μ. *Hint:* You may want to express $x f_{X|Y}(x \mid y)$ in terms of $f_{X|Y}(x \mid y+1)$.

$\hat{x}_{LMS}(y) =$ (y+1)/(mu+1)

You have used 1 of 2 submissions

DISCUSSION

Click “Show Discussion" below to see discussions on this problem.