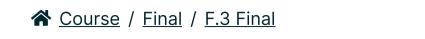


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sandipan_dey 🗸

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F.3.3 Final Questions 5-6

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Question 5

10.0/10.0 points (graded)

Consider the matrix
$$m{A}=egin{pmatrix} 0 & 1 \ 1 & 0 \ 2 & 1 \end{pmatrix}$$
 and vector $m{b}=egin{pmatrix} 1 \ 1 \ 1 \end{pmatrix}$.

1. Compute the linear least-squares solution, \hat{x} , to Ax = b.

$$\hat{x} = \begin{bmatrix} 1/3 & & \\ & &$$

$$ullet A^TA = egin{pmatrix} 0 & 1 \ 1 & 0 \ 2 & 1 \end{pmatrix}^T egin{pmatrix} 0 & 1 \ 1 & 0 \ 2 & 1 \end{pmatrix} = egin{pmatrix} 5 & 2 \ 2 & 2 \end{pmatrix}$$

$$ullet \left(A^TA
ight)^{-1} = egin{pmatrix} 5 & 2 \ 2 & 2 \end{pmatrix}^{-1} = rac{1}{(5)(2)-(2)(2)} igg(egin{pmatrix} 2 & -2 \ -2 & 5 \end{pmatrix} = rac{1}{6} igg(egin{pmatrix} 2 & -2 \ -2 & 5 \end{pmatrix} = igg(egin{pmatrix} 1/3 & -1/3 \ -1/3 & 5/6 \end{pmatrix}$$

$$ullet A^Tb = egin{pmatrix} 0 & 1 \ 1 & 0 \ 2 & 1 \end{pmatrix}^T egin{pmatrix} 1 \ 1 \ 1 \end{pmatrix} = egin{pmatrix} 3 \ 2 \end{pmatrix}$$

$$ullet (A^TA)^{-1}A^Tb = egin{pmatrix} 1/3 & -1/3 \ -1/3 & 5/6 \end{pmatrix} egin{pmatrix} 3 \ 2 \end{pmatrix} = egin{pmatrix} 1-2/3 \ -1+5/3 \end{pmatrix} = egin{pmatrix} 1/3 \ 2/3 \end{pmatrix}$$

2. Compute the projection of b onto the column space of A. Let us denote this with b.

$$\hat{b} = \begin{bmatrix} 2/3 & \checkmark & Answer: 2/3 \\ 1/3 & \checkmark & Answer: 1/3 \\ 4/3 & \checkmark & Answer: 4/3 \end{bmatrix}$$

$$A{(A^TA)}^{-1}A^Tb = egin{pmatrix} 0 & 1 \ 1 & 0 \ 2 & 1 \end{pmatrix} egin{pmatrix} 1/3 \ 2/3 \end{pmatrix} = egin{pmatrix} 0 + 2/3 \ 1/3 + 0 \ 2/3 + 2/3 \end{pmatrix} = egin{pmatrix} 2/3 \ 1/3 \ 4/3 \end{pmatrix}$$

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1 Answers are displayed within the problem

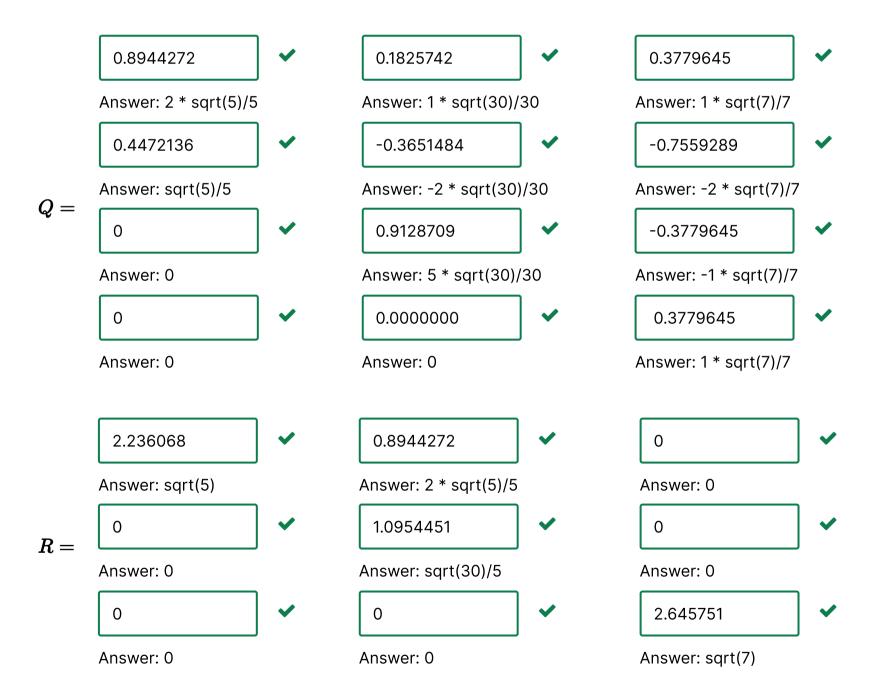
Question 6

10.0/10.0 points (graded)

Consider
$$A=egin{pmatrix}2&1&1\1&0&-2\0&1&-1\0&0&1\end{pmatrix}$$
 .

1. Compute the QR factorization of \boldsymbol{A} .

(Yes, you may use a IPython Notebook to perform the calculations... I did it by hand, and got it wrong the first time!)



$$Q = \left(egin{array}{c|c} rac{\sqrt{5}}{5} egin{array}{c} 2 \ 1 \ 0 \ 0 \end{array} & rac{\sqrt{30}}{30} egin{array}{c} 1 \ -2 \ 5 \ 0 \end{array} & rac{\sqrt{282}}{282} egin{array}{c} 5 \ -10 \ -11 \ 6 \end{array} \end{array}
ight)$$

$$R = \left(egin{array}{ccc} \sqrt{5} & 2\sqrt{(5)}/5 & 0 \ 0 & \sqrt{30}/5 & \sqrt{30}/6 \end{array}
ight)$$

■ Calculator

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$$\begin{pmatrix} 0 & 0 & \sqrt{282}/6 \end{pmatrix}$$

$$Q = \begin{pmatrix} \frac{\sqrt{5}}{5} \begin{pmatrix} 2\\1\\0\\0 \end{pmatrix} & \frac{\sqrt{30}}{30} \begin{pmatrix} 1\\-2\\5\\0 \end{pmatrix} & \frac{\sqrt{7}}{7} \begin{pmatrix} 1\\-2\\-1\\1 \end{pmatrix} \end{pmatrix}$$

$$R = \begin{pmatrix} \sqrt{5} & 2\sqrt{(5)/5} & 0\\ 0 & \sqrt{30/5} & 0\\ 0 & 0 & \sqrt{7} \end{pmatrix}$$

•
$$\rho_{0,0} = ||a_0||_2 = \sqrt{4+1} = \sqrt{5}$$

$$q_0 = a_0/\rho_{0,0} = \frac{\sqrt{5}}{5} \begin{pmatrix} 2\\1\\0\\0 \end{pmatrix}$$

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•
$$\rho_{0,1} = q_0^T a_1 = \begin{bmatrix} \frac{\sqrt{5}}{5} \begin{pmatrix} 2\\1\\0\\0 \end{pmatrix} \end{bmatrix}^T \begin{pmatrix} 1\\0\\1\\0 \end{pmatrix} = \frac{2\sqrt{5}}{5}$$

$$a_1^{\perp} = a_1 - \rho_{0,1} q_0 = \begin{pmatrix} 1\\0\\1\\0 \end{pmatrix} - \frac{2\sqrt{5}}{5} \begin{bmatrix} \frac{\sqrt{5}}{5} \begin{pmatrix} 2\\1\\0\\0 \end{pmatrix} \end{bmatrix}$$

$$= \begin{pmatrix} 1\\0\\1\\0 \end{pmatrix} - \frac{2}{5} \begin{pmatrix} 2\\1\\0\\0 \end{pmatrix}$$

$$= \begin{pmatrix} 1\\0\\1\\0\\0 \end{pmatrix} - \frac{2}{5} \begin{pmatrix} 2\\1\\0\\0\\0 \end{pmatrix}$$

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