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6. Speed of traveling waves

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Problem Set B due Aug 4, 2021 20:30 IST Completed

The dimensions of parameters

2/2 points (graded)

Consider the traveling wave function defined by the equation $w\left(x,t\right)=\sin\left(ax-bt\right)$. The expression $ax-bt$ is dimensionless, that is, it has no units. If x has a dimension of length, and t has a dimension of time, what are the dimensions of the parameters a and b respectively?

(Type `L` for a dimension of length. Type `T` for the dimension time. Your dimensions should be mathematical expressions of `L` and `T`.)

a has dimension L^(-1) ✓ **Answer:** 1/L

b has dimension T^(-1) ✓ **Answer:** 1/T

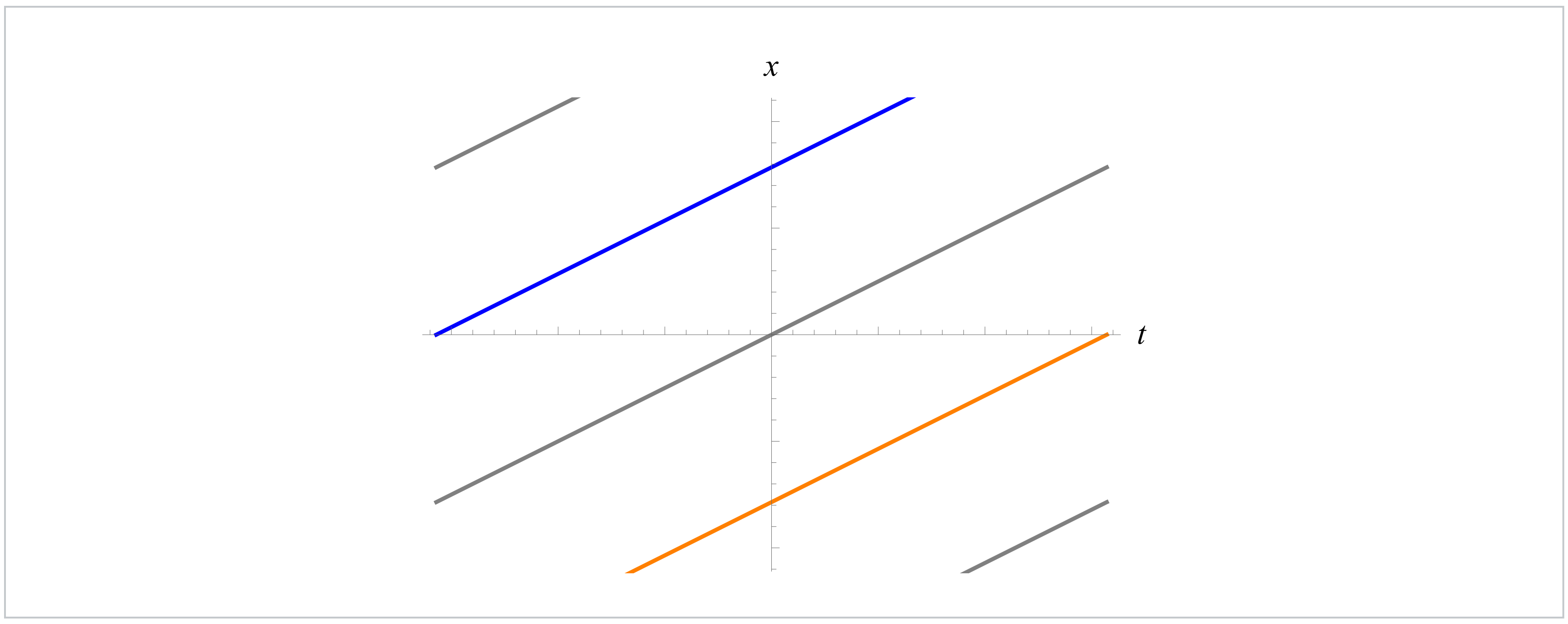
Submit You have used 1 of 3 attempts

ℹ Answers are displayed within the problem

The slopes of level curves from level curves

2/2 points (graded)

We've learned that functions of the form $w\left(x,t\right)=\sin\left(ax-bt\right)$ are called traveling waves. The level curves of height 1, 0, and -1 of such a traveling wave are shown in the graph below. The level curves of height 0 are shown in gray. The level curve of height 1 is blue, and the level curve of height -1 is orange.



What are the slopes of the level curves of $w\left(x,t\right)=\sin\left(ax-bt\right)$ in terms of a and b?

b/a ✓ **Answer:** b/a

What is the dimension of this quantity?

(Type `L` for a dimension of length. Type `T` for the dimension time. Your dimensions should be mathematical expressions of `L` and `T`.)

L/T ✓ **Answer:** L/T

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Finding the velocity of the traveling wave

1/1 point (graded)
We continue to explore the function $w(x,t) = \sin(ax - bt)$.

Use the level curves to find the speed of the traveling wave. To do this, we choose a feature of our curve, say the maximum, then we measure its position at two points in time, and use distance = rate $*$ time to find the rate of change. The maximum is represented as one level curve of height 1. To see how the x-coordinate changes as the t-coordinate increases, we measure how far (Δx) a point on that level curves moves over a fixed time interval (Δt).

The velocity of the traveling wave is given by:

b/a ✓ **Answer:** b/a

? INPUT HELP

Solution:

The velocity of a traveling wave is a measure of how far a point at the maximum (say $x = \pi/2a$ and $t = 0$) has moved over a time interval Δt.

Let's explore how we do this measurement with a general level curve of height C.

First note that all level curves take the form $ax - bt = C$, where $C = \sin^{-1}(h)$ where h is the height of the level curve. To see how far a point on a level curve has moved over a time Δt, we look at a point on a level curve at two times t_1 and t_2 where $\Delta t = t_2 - t_1$.

$$\begin{aligned}
ax - bt &= C \\
x &= C/a + b/at \\
x_1 &= C/a + \frac{b}{a}t_1 \\
x_2 &= C/a + \frac{b}{a}t_2 \\
\Delta x = x_2 - x_1 &= \frac{b}{a}(t_2 - t_1) = \frac{b}{a}\Delta t \\
\frac{\Delta x}{\Delta t} = \frac{b}{a}
\end{aligned}$$

Observe that this computation is exactly a measurement of the slope of the level curve! Therefore the slope of any level curve is the velocity of the traveling wave.

Sanity check

Observe that the dimension of the expression we found does in fact have dimension L/T, which is the dimension of velocity.

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Answers are displayed within the problem

The velocity from partial derivatives, 1

1/1 point (graded)

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Let's answer the same question using partial derivatives. Which of the following is the velocity of the traveling wave?

○ w_x

○ w_t

● Neither

✓

Solution:

The function w without any additional information, or without a constant multiple with units is dimensionless. Thus w_x has dimension $1/L$ and w_t has dimension $1/T$, so neither can be the velocity of the traveling wave.

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The velocity from partial derivatives, 2

1/1 point (graded)

Let x_0 be the position where the wave has a maximum at time $t = 0$. If we let a small amount of time pass, say Δt, then the wave's maximum will be at a new position x_1.

Which of the following equations governs the relationship between x_1 and x_0?

○ $x_1 = x_0$

● $w\left(x_0, 0\right) = w\left(x_1, \Delta t\right)$

○ $w\left(x_0, \Delta t\right) = w\left(x_1, \Delta t\right)$

○ $x_1 = x_0 + \Delta t$

○ None of the above.

✓

Solution:

We consider a point x_0 at time $t = 0$. The value of the level curve is $w\left(x_0, 0\right) = h = 1$ since we assume it is a maximum.

If this maximum is at a point x_1 after a time Δt, this means that this point lies on the same level curve. That is $w\left(x_1, \Delta t\right) = h = 1$. Therefore the relationship between x_1 and x_0 is

$$w\left(x_0, 0\right) = w\left(x_1, \Delta t\right).$$

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The velocity from partial derivatives, 3

1/1 point (graded)
Using your answer to the above question, apply a linear approximation, and solve for the value of $\Delta x = x_1 - x_0$
.

Which of the following is Δx?

○ $\left(w_x/w_t\right)\Delta t$

○ $-\left(w_x/w_t\right)\Delta t$

○ $\left(w_t/w_x\right)\Delta t$

● $-\left(w_t/w_x\right)\Delta t$

○ None of the above

✓

Solution:

Take a tangent plane approximation to $w\left(x_1, \Delta t\right)$.

$$\begin{aligned} w\left(x_1, \Delta t\right) &= w\left(x_0 + \Delta x, \Delta t\right) \\ w\left(x_0 + \Delta x, \Delta t\right) &\approx w\left(x_0, 0\right) + w_x \Delta x + w_t \Delta t \end{aligned}$$

We know that at this time we are on the same level curve if

$$w\left(x_0 + \Delta x, \Delta t\right) = w\left(x_0, 0\right)$$

This is true if and only if

$$w_x \Delta x + w_t \Delta t \approx 0$$

This gives us an expression for Δx in terms of Δt and the partial derivatives of w:

$$\Delta x \approx \frac{-w_t}{w_x}\Delta t.$$

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The velocity from partial derivatives, 4

1/1 point (graded)
Which of the following is the velocity of the traveling wave?

○ w_x/w_t

○ $-w_x/w_t$

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○ w_t/w_x

● $-w_t/w_x$

○ None of the above

✓

Solution:

Because $\Delta x/\Delta t$ is a good measurement of velocity when Δx and Δt are small, and from the problem above we have that

$$\frac{\Delta x}{\Delta t} \approx \frac{(-w_t/w_x)\,\Delta t}{\Delta t} = \frac{-w_t}{w_x},$$

this ratio of partial derivatives is also a measure of this velocity. Note the negative sign!

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6. Speed of traveling waves

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?	Which part of my derivation of velocity is wrong?	5
	Extra materials Is there some extra materials on the velocity of traveling waves that you would point to? struggled with the intuition behind this and ...	2
	Finding the velocity of the traveling wave FWIW, I thought the answer was a numerical one given the instructions to measure the graph. Actually, the answer is trivial, and alge...	5
☑	The velocity from partial derivatives, 3	3
?	Explanation for 'The slopes of level curves from level curves' question Hi, would it please be possible to have an explanation for 'The slopes of level curves from level curves' question ? I understand why t...	2
	(Staff) perhaps introduce standard symbols for wavenumber and angular velocity	3
	Use of linear approximation I don't understand the justification for using linear approximation in this situation. You have Δx = x1 - x0, which is the distance betwe...	3

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