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5.1 Quiz: How does gravity or pendulum weight and length affect the period?

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Why do clocks have pendulums? Does every pendulum swing at the same rate? Could a clock speed up or slow down if you changed the weight, length, or starting position of its pendulum? We need to think about the parameters of our model to answer this.

We will continue to work with the simplified pendulum model

$$\frac{d^2\theta}{dt^2} = -k\theta,$$

which assumes the angles the pendulum swings through are small. For the past few problems, we've also been assuming that $k = 1$.

For the past few problems, we've been assuming that $k = 1$.

What does this mean about length of the rod? Let's use standard international units, so gravity $g = -9.8m/s^2$. Since $k = \frac{g}{l}$, this means the pendulum rod is almost 10 meters long. The pendulums found in most clocks have $k > 1$, so we must abandon the assumption that $k = 1$. (Can you think why $k > 1$ makes more sense for a clock?) Here is the solution $\theta(t)$ we saw earlier with the general k, and the corresponding angular velocity $\frac{d\theta}{dt}(t)$.

$$\theta(t) = \theta_0 \cos(\sqrt{k}t)$$
$$\frac{d\theta}{dt}(t) = -\theta_0\sqrt{k}\sin(\sqrt{k}t).$$

How does changing the parameter k change the phase plane trajectories and the period of the pendulum?

Question 1

1/1 point (graded)

When $k = 1$, the solution trajectories are closed circles. How do the solution trajectories $(\theta_0 \cos(\sqrt{k}t), -\theta_0\sqrt{k}\sin(\sqrt{k}t))$ where $k > 1$ differ from the trajectories $(\theta_0 \cos(t), -\theta_0 \sin(t))$ where $k = 1$?

○ The trajectories will be dilated vertically, crossing the $\frac{d\theta}{dt}$ axis closer to zero but crossing the θ axis at the same locations.

● The trajectories will be dilated vertically, crossing the $\frac{d\theta}{dt}$ axis farther from zero but crossing the θ axis at the same locations.

✓

○ The trajectories will be dilated horizontally, crossing the θ axis closer to zero but crossing the $\frac{d\theta}{dt}$ axis at the same locations.

○ The trajectories will be dilated horizontally, crossing the θ axis farther from zero but crossing the $\frac{d\theta}{dt}$ axis at the same locations.

○ The trajectories will no longer be closed. They will be spirals toward the origin of the phase plane.

○ The shape of the trajectories will be unchanged.

○ None of the above.

Explanation

The factor of $\sqrt{k}$ in the input to the sine and cosine functions does not affect the shape of the curves, just the speed at which they are traversed. However, when $k > 1$, multiplying $\sin(\sqrt{k}t)$ by $\sqrt{k}$ increases the maximum value of $\frac{d\theta}{dt}$. The maximum value of θ is unchanged. Thus, the solution curves will dilate away from the origin along the $\frac{d\theta}{dt}$ axis but crossing the θ axis at the same locations.

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You have used 1 of 3 attempts

ℹ Answers are displayed within the problem

Question 2

1/1 point (graded)

Recall that the simplified pendulum model is $\frac{d^2\theta}{dt^2} = -k\sin(\theta)$ and we are looking at the family of solutions $\theta(t) = \theta_0 \cos(\sqrt{k}t)$.

We defined $k = \frac{g}{l}$ and are studying the family of solutions $\theta(t) = \theta_0 \cos(\sqrt{k}t)$. If we cut the length l of the pendulum's rod in half without changing any other parameters, how do we expect the pendulum's behavior to change?

(Hint: Use what you know about $k = 2$ versus $k = 1$, and remember how k and l are related.)

☐ The period will increase; the pendulum will take more time to traverse its arc.

☑ The period will decrease; the pendulum will take less time to traverse its arc. ✓

☐ The maximum angle of swing of the pendulum will increase.

☐ The maximum angle of swing of the pendulum will decrease.

☑ The maximum value of $\frac{d\theta}{dt}$ (angular velocity) will increase.

✓

☐ The maximum value of $\frac{d\theta}{dt}$ (angular velocity) will decrease.

☐ None of the above.

✓

Explanation

Neither choice (c) or (d) are correct. The maximum angle of swing is determined by θ_0, which doesn't depend on l.

The parameters k and l are inversely proportional. Cutting the rod length in half doubles the k-value. We'll look at a specific example.

When $k = 1$, our model predicts that $\theta(t) = \theta_0 \cos(t)$ with period 2π. Cutting the length of the rod in half gives us $k = 2$, so $\theta(t) = \theta_0 \cos(\sqrt{2}t)$ and the pendulum has period $2\pi/\sqrt{2}$ which is less than 2π. The shorter pendulum takes less time to traverse its arc. (This makes physical sense - the longer the pendulum, the longer the arc.)

From the previous problem, we saw that the trajectories are stretched vertically when $k > 1$ compared to $k = 1$, which means the maximum angular velocity is greater. We can also directly compute this: $\frac{d\theta}{dt} = -\theta_0\sqrt{k}\,\text{sin}(\sqrt{k}t)$. The amplitude of this sine curve is $\theta_0\sqrt{k}$. Decreasing l increases k and increases the maximum value of $\frac{d\theta}{dt}$. This is consistent with the observation that the pendulum takes less time to traverse its arc.

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You have used 1 of 4 attempts

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Question 3

1/1 point (graded)

Recall that the simplified pendulum model is $\frac{d^2\theta}{dt^2} = -k\theta$ where $k = \frac{g}{l}$. We are studying the family of solutions $\theta(t) = \theta_0 \cos(\sqrt{k}t)$. If we cut the mass m of the pendulum's bob in half, how do we expect its behavior to change? Mark all that apply.

☐ The period will decrease; the pendulum will take less time to traverse its arc.

☐ The period will increase; the pendulum will take more time to traverse its arc.

☐ The maximum angle of swing of the pendulum will increase.

☐ The maximum angle of swing of the pendulum will decrease.

☐ The maximum value of $\frac{d\theta}{dt}$ (angular velocity) will increase.

☐ The maximum value of $\frac{d\theta}{dt}$ (angular velocity) will decrease.

☑ None of the above.

✓

✓

Explanation

Explanation

The mass parameter m does not appear in our differential equation for $\theta(t)$. According to our model, the value of m does not affect the behavior of the pendulum.

In practice, friction and air resistance have a more noticeable effect if the bob is not massive enough.

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Question 4

1/1 point (graded)

You are moving to the moon and take your favorite pendulum clock with you. The gravity on the moon is less than on earth. Assuming its mechanisms are still working as before, which of the following are true?

☑ The period will increase; the pendulum will take more time to traverse its arc. ✓

☐ The period will decrease; the pendulum will take less time to traverse its arc.

☐ The maximum angle of swing of the pendulum will increase.

☐ The maximum angle of swing of the pendulum will decrease.

☐ The maximum value of $\frac{d\theta}{dt}$ (angular velocity) will increase.

☑ The maximum value of $\frac{d\theta}{dt}$ (angular velocity) will decrease. ✓

☐ None of the above.

✓

Explanation

We can reason about this similarly to the length question. The maximum angle of swing is determined by the θ_0, which doesn't depend on g.

Decreasing gravity decreases the value of k since they are directly proportional. Decreasing k increases the period, since the period is $\frac{2\pi}{\sqrt{k}}$. Thus the pendulum will take more time to traverse its arc. (This makes physical sense — there is less gravity pulling it back down to vertical.)

When we look at $\frac{d\theta}{dt} = -\theta_0\sqrt{k}\sin(\sqrt{k}t)$, we see the amplitude of this sine curve is $\theta_0\sqrt{k}$. Decreasing k thus decreases the maximum value of $\frac{d\theta}{dt}$. This is consistent with the observation that the pendulum takes more time to traverse its arc.

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