8.2.2 Solving Ax = b via Gauss-Jordan Elimination, Gauss Transforms

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Week 8 due Nov 26, 2023 15:12 IST

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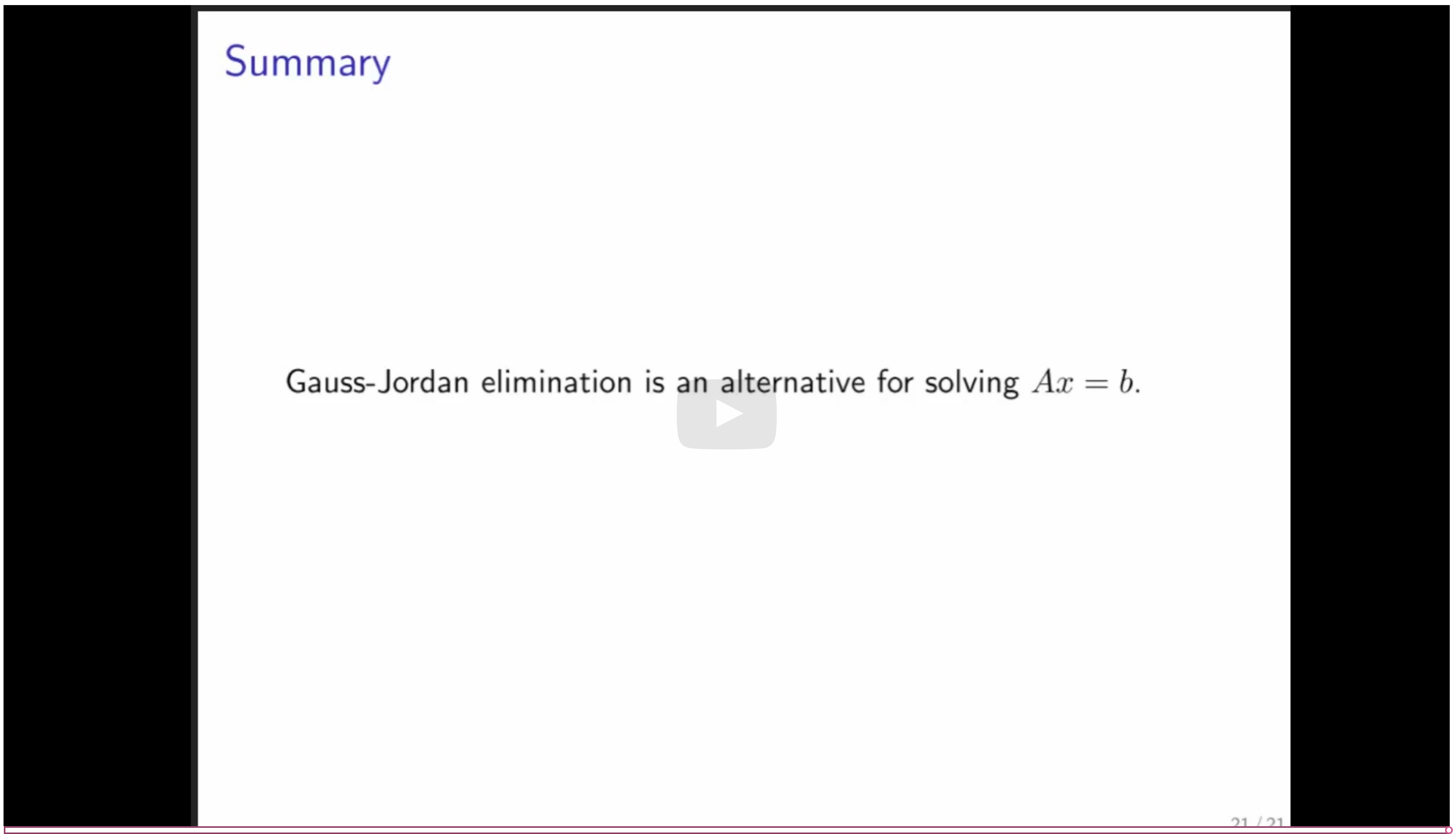

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Reading Assignment

0 points possible (ungraded)

Read Unit 8.2.2 of the notes. [LINK]

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Calculator

8.2.2. Typo in algorithm?

? Signs switched in HW? 2
It looks like the homework problems below have signs switched from the algorithm discussed in the lecture. Put another way, everywhere we w...

Homework 8.2.2.1

51/51 points (graded)

$$\left(\begin{array}{c|cc} 1 & 0 & 0 \\ \hline 1 & 1 & 0 \\ -2 & 0 & 1 \end{array}\right)\left(\begin{array}{c|cc|c} -2 & 2 & -5 & -7 \\ \hline 2 & -3 & 7 & 11 \\ -4 & 3 & -7 & -9 \end{array}\right) =$$

-2	2	-5	-7
✔ Answer: -2	✔ Answer: 2	✔ Answer: -5	✔ Answer: -7
0	-1	2	4
✔ Answer: 0	✔ Answer: -1	✔ Answer: 2	✔ Answer: 4
0	-1	3	5
✔ Answer: 0	✔ Answer: -1	✔ Answer: 3	✔ Answer: 5

$$\left(\begin{array}{c|c|c} 1 & 2 & 0 \\ \hline 0 & 1 & 0 \\ \hline 0 & -1 & 1 \end{array}\right)\left(\begin{array}{c|c|c|c} -2 & 2 & -5 & -7 \\ \hline 0 & -1 & 2 & 4 \\ \hline 0 & -1 & 3 & 5 \end{array}\right) =$$

-2	0	-1	1
✔ Answer: -2	✔ Answer: 0	✔ Answer: -1	✔ Answer: 1
0	-1	2	4
✔ Answer: 0	✔ Answer: -1	✔ Answer: 2	✔ Answer: 4
0	0	1	1
✔ Answer: 0	✔ Answer: 0	✔ Answer: 1	✔ Answer: 1

$$\left(\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & -2 \\ \hline 0 & 0 & 1 \end{array}\right)\left(\begin{array}{cc|c|c} -2 & 0 & -1 & 1 \\ 0 & -1 & 2 & 4 \\ \hline 0 & 0 & 1 & 1 \end{array}\right) =$$

-2	0	0	2
✔ Answer: -2	✔ Answer: 0	✔ Answer: 0	✔ Answer: 2
0	-1	0	2
✔ Answer: 0	✔ Answer: -1	✔ Answer: 0	✔ Answer: 2
0	0	1	1
✔ Answer: 0	✔ Answer: 0	✔ Answer: 1	✔ Answer: 1

$$\begin{pmatrix} -1/2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\left(\begin{array}{ccc|c} -2 & 0 & 0 & 2 \\ 0 & -1 & 0 & 2 \\ 0 & 0 & 1 & 1 \end{array}\right) =$$

Calculator

1	0	0	-1
✔ Answer: 1	✔ Answer: 0	✔ Answer: 0	✔ Answer: -1
0	1	0	-2
✔ Answer: 0	✔ Answer: 1	✔ Answer: 0	✔ Answer: -2
0	0	1	1
✔ Answer: 0	✔ Answer: 0	✔ Answer: 1	✔ Answer: 1

Use the above exersise to compute $x = \begin{pmatrix} \chi_0 \\ \chi_1 \\ \chi_2 \end{pmatrix}$ that solves $\begin{array}{rrrr} -2\chi_0 + & 2\chi_1 - & 5\chi_2 = & -7 \\ 2\chi_0 - & 3\chi_1 + & 7\chi_2 = & 11 \\ -4\chi_0 + & 3\chi_1 - & 7\chi_2 = & -9 \end{array}$

$x =$

-1 ✔ Answer: -1

-2 ✔ Answer: -2

1 ✔ Answer: 1

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Answers are displayed within the problem

Homework 8.2.2.2

1/1 point (graded)

Homework 8.2.2.2 This exercise shows you how to use MATLAB to do the heavy lifting for Homework 8.2.2.1. Again solve

$$\begin{array}{rcrcrcr} -2\chi_0 & + & 2\chi_1 & - & 5\chi_2 & = & -7 \\ 2\chi_0 & - & 3\chi_1 & + & 7\chi_2 & = & 11 \\ -4\chi_0 & + & 3\chi_1 & - & 7\chi_2 & = & -9 \end{array}$$

via Gauss-Jordan elimination. This time we set this up as an appended matrix:

$$\left(\begin{array}{rrr|r} -2 & 2 & -5 & -7 \\ 2 & -3 & 7 & 11 \\ -4 & 3 & -7 & -9 \end{array}\right).$$

We can enter this into MATLAB as

```
A = [
-2  2 -5  ??
 2 -3  7  ??
-4  3 -7  ??
]
```

(You enter `??`.) Create the Gauss transform, G_0, that zeroes the entries in the first column below the diagonal:

```
G0 = [
 1  0  0
??  1  0
??  0  1
]
```

(You fill in the `??`). Now apply the Gauss transform to the appended system:

```
A0 = G0 * A
```

Calculator

Similarly create G_1,

```
G1 = [
 1 ??  0
 0  1  0
 0 ??  1
]
```

A_1, G_2, and A_2, where A_2 equals the appended system that has been transformed into a diagonal system. Finally, let D equal to a diagonal matrix so that $A_3 = D * A2$ has the identity for the first three columns.
You can then find the solution to the linear system in the last column.

☑ Done/Skipped

Homework_8_2_2_2_Answer.m

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Homework 8.2.2.3

1/1 point (graded)
Assume below that all matrices and vectors are partitioned "conformally" so that the operations make sense.

$$\left(\begin{array}{c|c|c} I & -u_{01} & 0 \\ \hline 0 & 1 & 0 \\ \hline 0 & -l_{21} & I \end{array}\right)\left(\begin{array}{c|c|c||c} D_{00} & a_{01} & A_{02} & b_0 \\ \hline 0 & \alpha_{11} & a_{12}^T & \beta_1 \\ \hline 0 & a_{21} & A_{22} & b_2 \end{array}\right) =$$

$$\left(\begin{array}{c|c|c||c} D_{00} & a_{01} - \alpha_{11} u_{01} & A_{02} - u_{01} a_{12}^T & b_0 - \beta_1 u_{01} \\ \hline 0 & \alpha_{11} & a_{12}^T & \beta_1 \\ \hline 0 & a_{21} - \alpha_{11} l_{21} & A_{22} - l_{21} a_{12}^T & b_2 - \beta_1 l_{21} \end{array}\right)$$

Always ✔ Answer: Always

Just multiply it out. (Partitioned matrix-matrix multiplication)

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Answers are displayed within the problem

Homework 8.2.2.4

1/1 point (graded)
Assume below that all matrices and vectors are partitioned "conformally" so that the operations make sense and that $\alpha_{11} \neq 0$. Choose

- $u_{01} := a_{01} / \alpha_{11}$ and
- $l_{21} := a_{21} / \alpha_{11}$

$$\left(\begin{array}{c|c|c} I & -u_{01} & 0 \\ \hline 0 & 1 & 0 \\ \hline 0 & -l_{21} & I \end{array}\right)\left(\begin{array}{c|c|c||c} D_{00} & a_{01} & A_{02} & b_0 \\ \hline 0 & \alpha_{11} & a_{12}^T & \beta_1 \\ \hline 0 & a_{21} & A_{22} & b_2 \end{array}\right) =$$

Calculator

$$\left(\begin{array}{c|c|c|c}
D_{00} & 0 & A_{02} - u_{01} a_{12}^T & b_0 - \beta_1 u_{01} \\ \hline
0 & \alpha_{11} & a_{12}^T & \beta_1 \\ \hline
0 & 0 & A_{22} - l_{21} a_{12}^T & b_2 - \beta_1 l_{21}
\end{array}\right)$$

Always ✔ Answer: Always

Just multiply it out. (Partitioned matrix-matrix multiplication)

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Answers are displayed within the problem

The discussion in this unit motivates the algorithm `GaussJordan_Part1`, which transforms A to a diagonal matrix and updates the right-hand side accordingly, and `GaussJordan_Part2`, which transforms the diagonal matrix A an identity matrix and updates the right-hand side accordingly.

Algorithm: $[A, b] := \text{GAUSSJORDAN\_PART1}(A, b)$

Partition $A \rightarrow \left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right)$, $b \rightarrow \left(\begin{array}{c} b_T \\ \hline b_B \end{array}\right)$

where A_{TL} is 0×0, b_T has 0 rows

while $m(A_{TL}) < m(A)$ **do**

Repartition

$$\left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c|c|c} A_{00} & a_{01} & A_{02} \\ \hline a_{10}^T & \alpha_{11} & a_{12}^T \\ \hline A_{20} & a_{21} & A_{22} \end{array}\right), \left(\begin{array}{c} b_T \\ \hline b_B \end{array}\right) \rightarrow \left(\begin{array}{c} b_0 \\ \hline \beta_1 \\ \hline b_2 \end{array}\right)$$

$a_{01} := a_{01}/\alpha_{11}$ $(= u_{01})$

$a_{21} := a_{21}/\alpha_{11}$ $(= l_{21})$

$A_{02} := A_{02} - a_{01} a_{12}^T$ $(= A_{02} - u_{01} a_{12}^T)$

$A_{22} := A_{22} - a_{21} a_{12}^T$ $(= A_{22} - l_{21} a_{12}^T)$

$b_0 := b_0 - \beta_1 a_{01}$ $(= b_2 - \beta_1 u_{01})$

$b_2 := b_2 - \beta_1 a_{21}$ $(= b_2 - \beta_1 l_{21})$

$a_{01} := 0$ (zero vector)

$a_{21} := 0$ (zero vector)

Continue with

$$\left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c|c|c} A_{00} & a_{01} & A_{02} \\ \hline a_{10}^T & \alpha_{11} & a_{12}^T \\ \hline A_{20} & a_{21} & A_{22} \end{array}\right), \left(\begin{array}{c} b_T \\ \hline b_B \end{array}\right) \leftarrow \left(\begin{array}{c} b_0 \\ \hline \beta_1 \\ \hline b_2 \end{array}\right)$$

endwhile

Algorithm: $[A, b] := \text{GAUSSJORDAN\_PART2}(A, b)$

Partition $A \rightarrow \left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right)$, $b \rightarrow \left(\begin{array}{c} b_T \\ \hline b_B \end{array}\right)$

where A_{TL} is 0×0, b_T has 0 rows

while $m(A_{TL}) < m(A)$ **do**

Calculator

Repartition

$$\left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c|c|c} A_{00} & a_{01} & A_{02} \\ \hline a_{10}^T & \alpha_{11} & a_{12}^T \\ \hline A_{20} & a_{21} & A_{22} \end{array}\right), \left(\begin{array}{c} b_T \\ \hline b_B \end{array}\right) \rightarrow \left(\begin{array}{c} b_0 \\ \hline \beta_1 \\ \hline b_2 \end{array}\right)$$

$\beta_1 := \beta_1 / \alpha_{11}$

$\alpha_{11} := 1$

Continue with

$$\left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c|c|c} A_{00} & a_{01} & A_{02} \\ \hline a_{10}^T & \alpha_{11} & a_{12}^T \\ \hline A_{20} & a_{21} & A_{22} \end{array}\right), \left(\begin{array}{c} b_T \\ \hline b_B \end{array}\right) \leftarrow \left(\begin{array}{c} b_0 \\ \hline \beta_1 \\ \hline b_2 \end{array}\right)$$

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