E1.3.1 Exam Question 1

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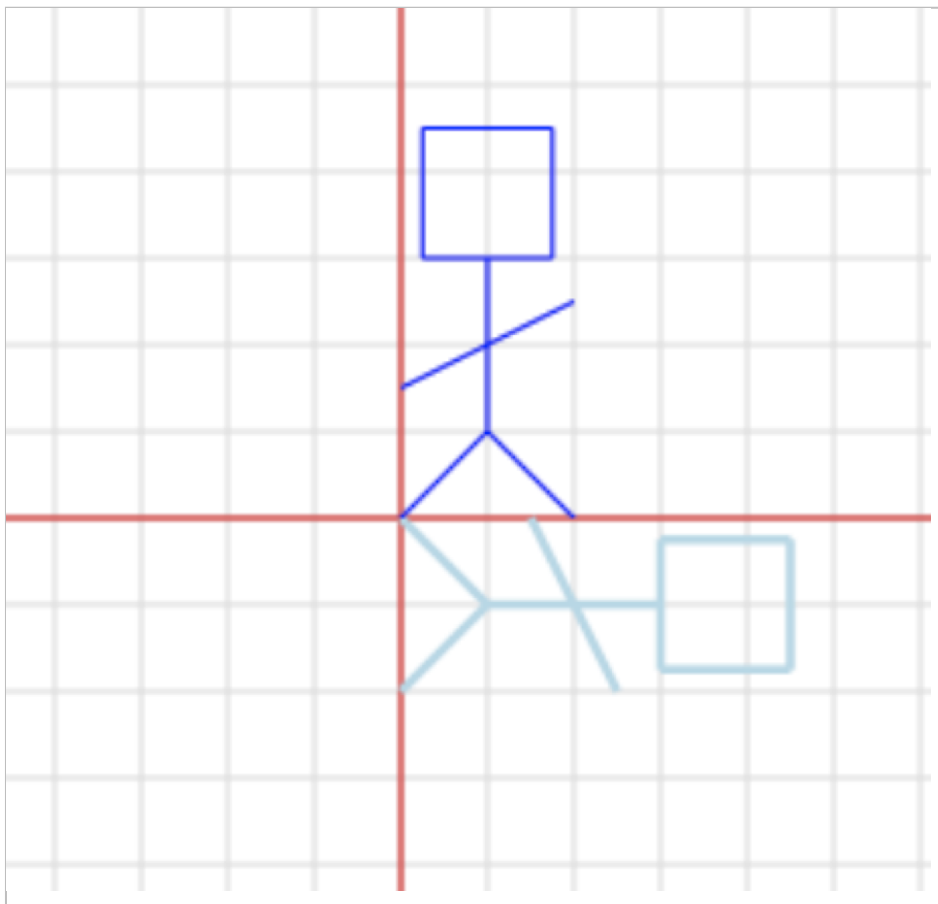
Exam 1 due Oct 31, 2023 09:12 IST Completed

Question 1

12/12 points (graded)

Consider the following picture of “Timmy":



1. What matrix, A, transforms Timmy (the blue figure drawn with thin lines that is in the top-right quadrant) into the target (the light blue figure drawn with thicker lines that is in the bottom-right quadrant)?

$A =$ [0] ✔ Answer: 0 [1] ✔ Answer: 1

[-1] ✔ Answer: -1 [0] ✔ Answer: 0

Answer:

Notice that $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ transforms into $\begin{pmatrix} 0 \\ -1 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ transforms into $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$. Thus, the matrix for this transformation is given by

$$A = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}.$$

2. What matrix, B, transforms the target Timmy (the light blue figure drawn with thicker lines that is in the bottom-right quadrant) into the original Timmy (the blue figure drawn with thin lines that is in the top-right quadrant)?

(Hint: You may want to look at how the vector $\begin{pmatrix} 0 \\ -1 \end{pmatrix}$ is transformed. How does this relate to how $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ is transformed?)

$B =$ [0] ✔ Answer: 0 [-1] ✔ Answer: -1

[1] ✔ Answer: 1 [0] ✔ Answer: 0

Answer:

Calculator

Notice that $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ transforms into $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$. Now, it is a little hard to see what part of Timmy $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ points to. But we know that $B\begin{pmatrix} 0 \\ 1 \end{pmatrix} = -B\begin{pmatrix} 0 \\ -1 \end{pmatrix}$ Now, $\begin{pmatrix} 0 \\ -1 \end{pmatrix}$ is a bit easier to identify in the picture, and it transforms into $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$. We notice that $\begin{pmatrix} 0 \\ -1 \end{pmatrix}$ transforms into $\begin{pmatrix} -1 \\ 0 \end{pmatrix}$. Thus, the matrix for this transformation is given by

$$B = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}.$$

3. With the resulting matrices A and B that you computed, evaluate BA.

   (Before you compute it by brute force, conjecture what the answer should be by thinking through how multiplication is defined. Then check what you think is the answer by computing it with the matrices.)

$BA =$ | 1 | ✔ Answer: 1 | 0 | ✔ Answer: 0
| 0 | ✔ Answer: 0 | 1 | ✔ Answer: 1

Answer:

If you think about it: applying A first and then B takes Timmy to the target and then back to Timmy. Thus, you can predict that the linear transformation that has the "action" of AB is the identity matrix:

$$BA = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

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