



MITx: 6.041x Introduction to Probability - The Science of Uncertainty



Bookmarks



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Unit overview

Lec. 8: Probability density functions

Exercises 8 due Mar 16, 2016 at 23:59 UTC

Lec. 9: Conditioning on an event; Multiple r.v.'s

Exercises 9 due Mar 16, 2016 at 23:59 UTC

Lec. 10: Conditioning on a random variable; Independence; Bayes' rule

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Standard normal table

Solved problems

Problem Set 5

Problem Set 5 due Mar 16, 2016 at 23:59 UTC

Unit summary

Unit 5: Continuous random variables > Lec. 10: Conditioning on a random variable; Independence; Bayes' rule > Lec 10 Conditioning on a random variable Independence Bayes rule vertical4

Exercise: Independence and expectations II

(3/3 points)

Let \mathbf{X} , \mathbf{Y} , and \mathbf{Z} be independent jointly continuous random variables, and let \mathbf{g} , \mathbf{h} , \mathbf{r} be some functions. For each one of the following formulas, state whether it is true for all choices of the functions \mathbf{g} , \mathbf{h} , and \mathbf{r} , or false (i.e., not true for all choices of these functions). Do not attempt formal derivations; use an intuitive argument.

$$1. \mathbf{E}[g(\mathbf{X}, \mathbf{Y})h(\mathbf{Z})] = \mathbf{E}[g(\mathbf{X}, \mathbf{Y})] \cdot \mathbf{E}[h(\mathbf{Z})]$$

True ▼



Answer: True

$$2. \mathbf{E}[g(\mathbf{X}, \mathbf{Y})h(\mathbf{Y}, \mathbf{Z})] = \mathbf{E}[g(\mathbf{X}, \mathbf{Y})] \cdot \mathbf{E}[h(\mathbf{Y}, \mathbf{Z})]$$

False ▼



Answer: False

$$3. \mathbf{E}[g(\mathbf{X})r(\mathbf{Y})h(\mathbf{Z})] = \mathbf{E}[g(\mathbf{X})] \cdot \mathbf{E}[r(\mathbf{Y})] \cdot \mathbf{E}[h(\mathbf{Z})]$$

True ▼



Answer: True

Answer:

1. True. Using our intuitive understanding of independence, the pair of random variables (\mathbf{X}, \mathbf{Y}) does not provide any information on \mathbf{Z} . Therefore, (\mathbf{X}, \mathbf{Y}) and \mathbf{Z} are independent. It follows that $\mathbf{g}(\mathbf{X}, \mathbf{Y})$ and $\mathbf{h}(\mathbf{Z})$ are independent, from which the formula follows.
2. False. The random variable \mathbf{Y} appears in both functions \mathbf{g} and \mathbf{h} , so that $\mathbf{g}(\mathbf{X}, \mathbf{Y})$ and $\mathbf{h}(\mathbf{Y}, \mathbf{Z})$ will be, in general, dependent. For an example, suppose that $\mathbf{g}(\mathbf{X}, \mathbf{Y}) = \mathbf{h}(\mathbf{Y}, \mathbf{Z}) = \mathbf{Y}$, in which case the statement becomes $\mathbf{E}[\mathbf{Y}^2] = (\mathbf{E}[\mathbf{Y}])^2$, which we know to be false in general.
3. True. Using the first part, and then again the independence of \mathbf{X} with \mathbf{Y} , we have

$$\mathbf{E}[g(\mathbf{X})r(\mathbf{Y})h(\mathbf{Z})] = \mathbf{E}[g(\mathbf{X})r(\mathbf{Y})] \cdot \mathbf{E}[h(\mathbf{Z})] = \mathbf{E}[g(\mathbf{X})] \cdot \mathbf{E}[r(\mathbf{Y})] \cdot \mathbf{E}[h(\mathbf{Z})]$$

You have used 1 of 1 submissions

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