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3. Review tangent approximation

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Lecture due Sep 13, 2021 20:30 IST Completed

Review

Review the tangent plane

Start of transcript. Skip to the end.

PROFESSOR: Today, we are going to see

how to use what we saw last time about partial derivatives

To handle minimization or maximization problems involving

functions of several variables.

So we found that--

so remember last time we said that

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Given a function $z = f(x, y)$ of two variables, let's explore how the value of the function changes near a point (x_0, y_0) if we change x and y slightly,

$$x_0 \quad \rightsquigarrow \quad x_0 + \Delta x \tag{4.1}$$

$$y_0 \quad \rightsquigarrow \quad y_0 + \Delta y \tag{4.2}$$

We approximate how z has changed by the **approximation formula**

$$\boxed{\Delta z = f_x \Delta x + f_y \Delta y}$$

where $f_x(x_0, y_0)$ and $f_y(x_0, y_0)$ are the value of the partial derivatives at the original point.

This approximation formula comes from the **tangent plane approximation** .

Recall that

- $\dfrac{\partial f}{\partial x}(x_0, y_0) = a$ is the slope of the curve formed by intersecting the surface $z = f(x, y)$ with the plane $y = y_0$. This tangent line has the formula

$$z \quad = \quad z_0 + a(x - x_0) \tag{4.3}$$

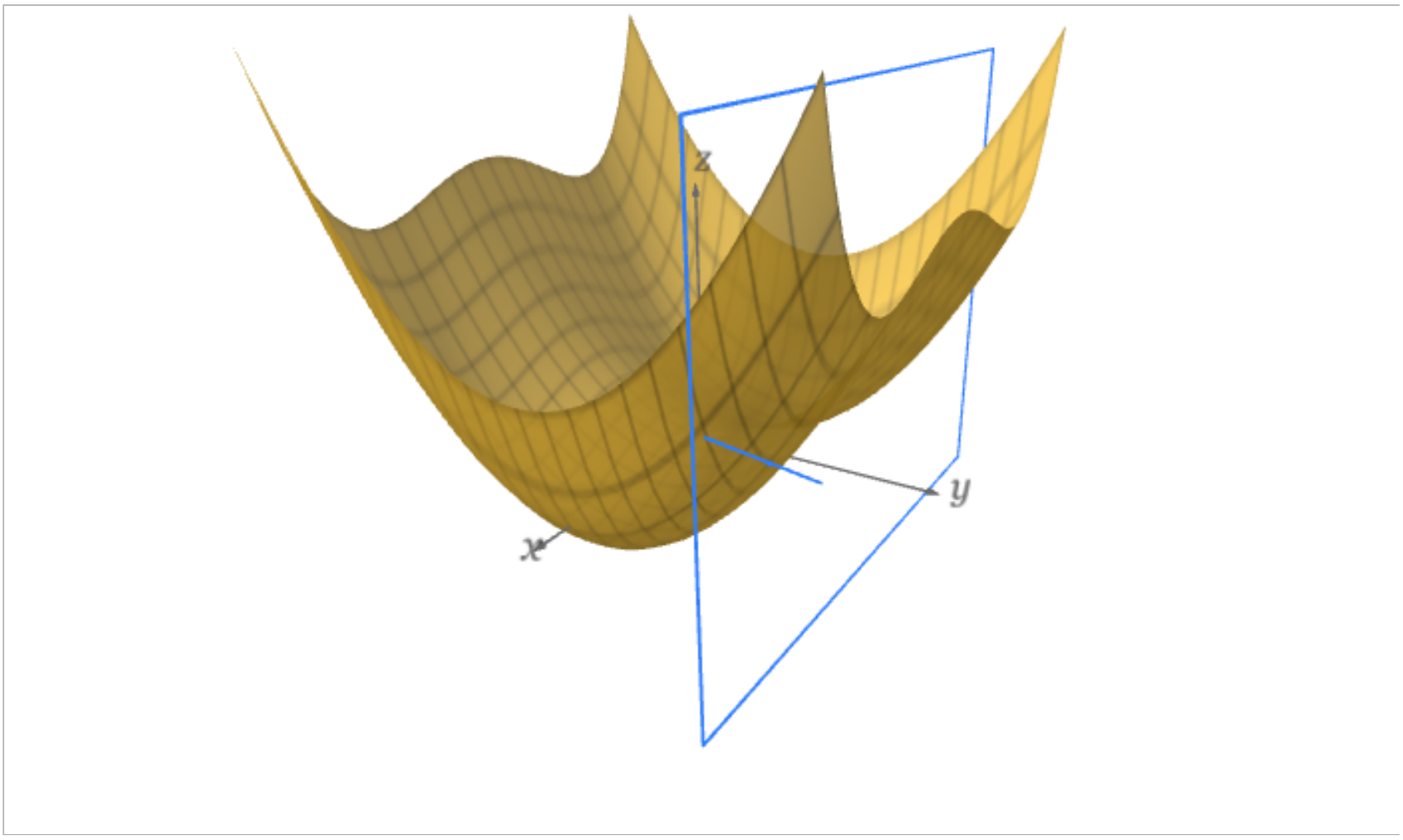
$$y \quad = \quad y_0 \tag{4.4}$$

where $z_0 = f(x_0, y_0)$.

▶ **Surface with tangent line in x-direction**

Calculator Hide Notes

▶ **Surface with tangent line in x-direction**



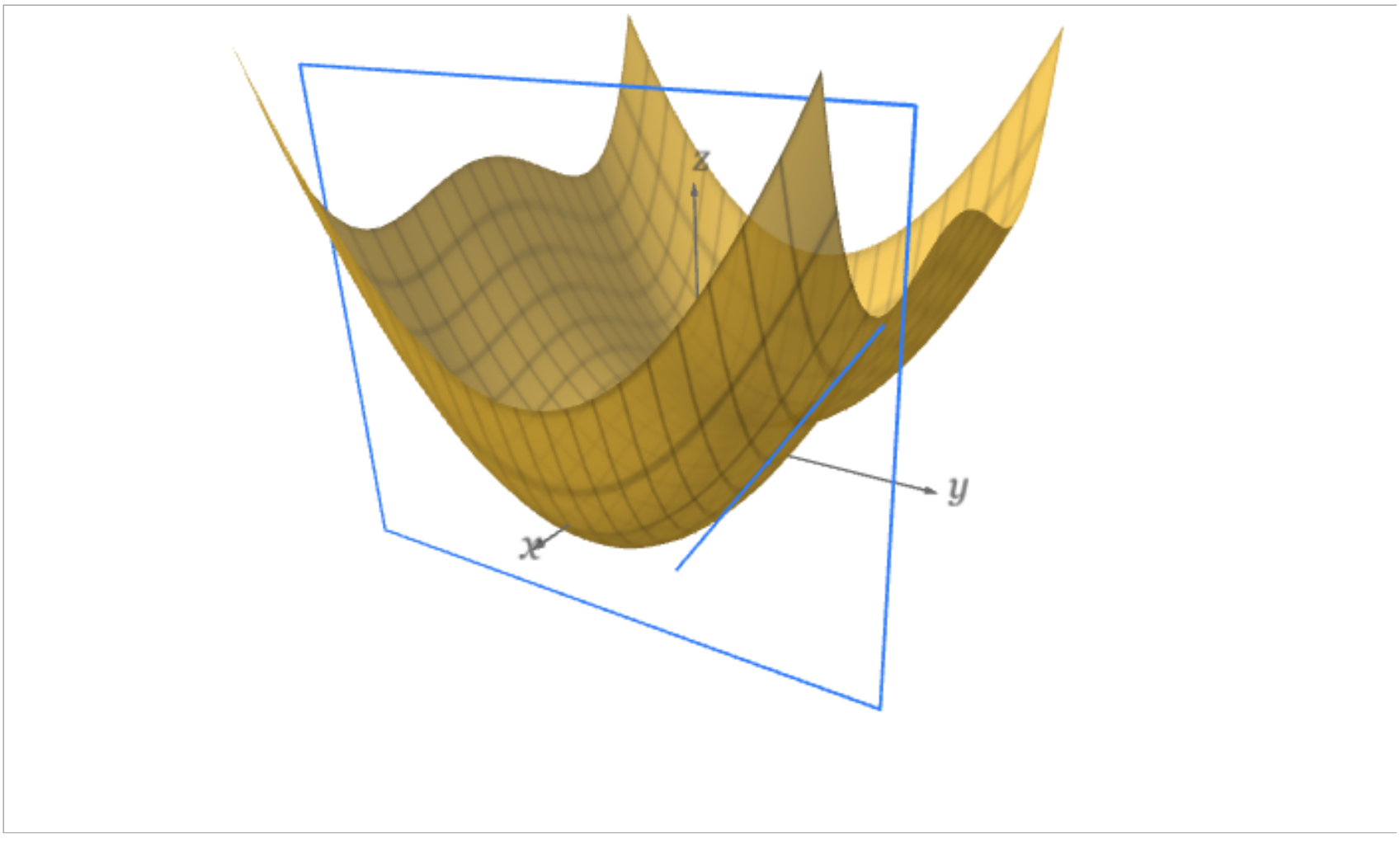
- $\dfrac{\partial f}{\partial y}(x_0, y_0) = b$ is the slope of the curve formed by intersecting the surface $z = f(x, y)$ with the plane $x = x_0$. This tangent line has the formula

$$z = z_0 + b(y - y_0) \tag{4.5}$$

$$x = x_0 \tag{4.6}$$

where $z_0 = f(x_0, y_0)$.

▶ **Surface with tangent line in y-direction**



The tangent plane is the plane that is tangent to the surface $z = f(x, y)$ at the point (x_0, y_0) and contains both tangent lines.

$$z \approx z_0 + a(x - x_0) + b(y - y_0)$$

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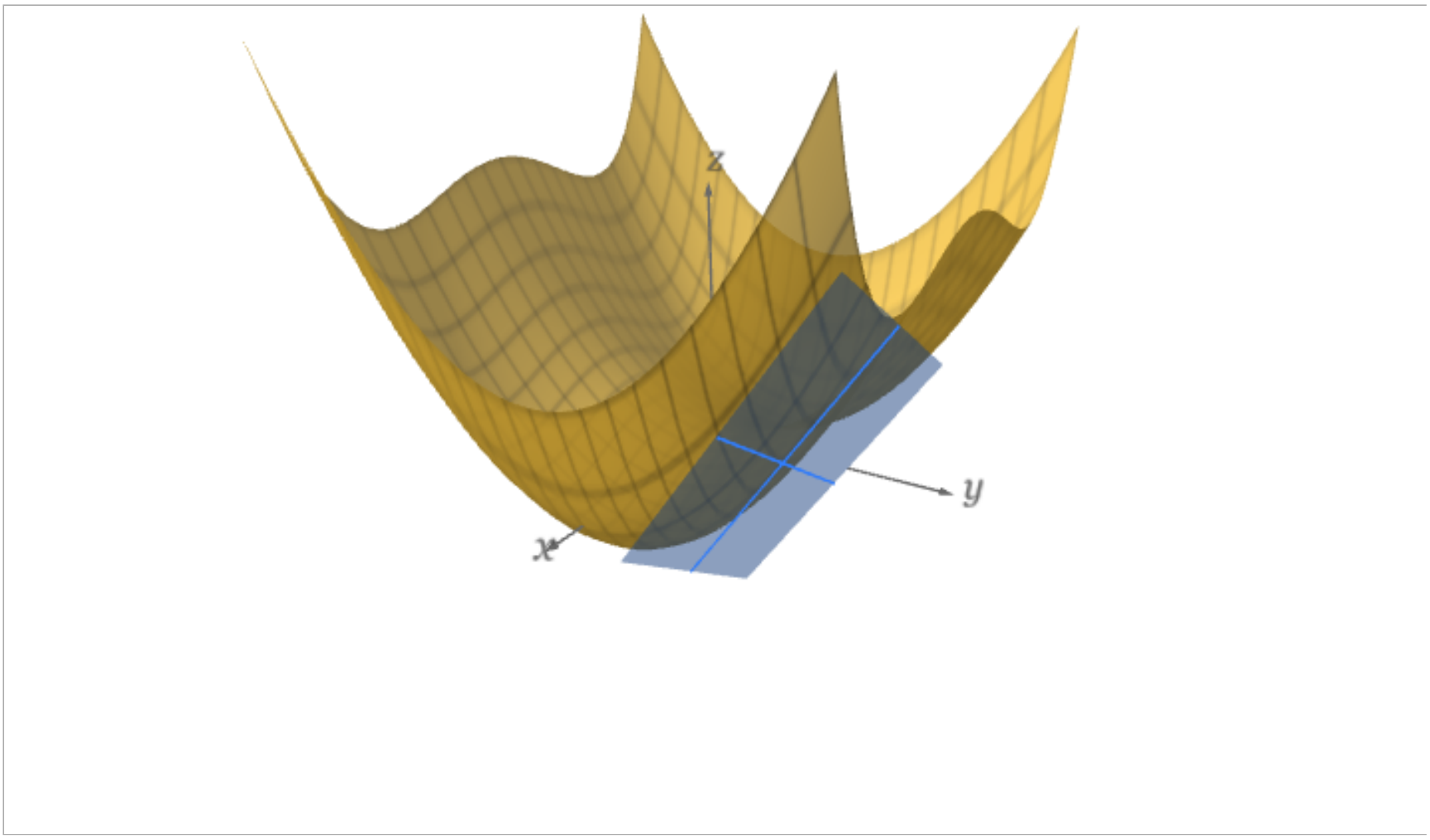
▶ **Surface with tangent plane**



The approximation formula is saying that the change in f can be approximated by looking in the change in the value of the tangent plane.

$$\Delta z = z - z_0 \approx a\,(x - x_0) + b\,(y - y_0)$$

Tangent plane approximation

3/3 points (graded)

Consider the function $z = x^2 + 2xy - y^3$.

Use the approximation formula above to find Δz near the point $(1, 1)$ for the Δx and Δy given below.

1. $\Delta x = 0.1$ and $\Delta y = 0$, then $\Delta z =$ 0.4 ✔ **Answer:** 0.4

2. $\Delta x = 0$ and $\Delta y = 0.1$, then $\Delta z =$ -0.1 ✔ **Answer:** -0.1

3. $\Delta x = 0.1$ and $\Delta y = 0.1$, then $\Delta z =$ 0.3 ✔ **Answer:** 0.3

Solution:

First we compute the partial derivatives and evaluate at our given point.

$$f_x\,(x, y) = 2x + 2y \qquad f_x\,(1, 1) = 4 \tag{4.7}$$
$$f_y\,(x, y) = 2x - 3y^2 \qquad f_y\,(1, 1) = -1 \tag{4.8}$$

Next we approximate the change in the output of our function Δz using our tangent plane approximation formula.

1. $\Delta x = 0.1$ and $\Delta y = 0$, then

$$\Delta z = f_x\,(1, 1)\,\Delta x + f_y\,(1, 1)\,\Delta y = 4\,(0.1) - 1\,(0) = 0.4$$

Calculator Hide Notes

2. $\Delta x = 0$ and $\Delta y = 0.1$, then

$$\Delta z = f_x(1,1)\,\Delta x + f_y(1,1)\,\Delta y = 4(0) - 1(0.1) = -0.1$$

3. $\Delta x = 0.1$ and $\Delta y = 0.1$, then

$$\Delta z = f_x(1,1)\,\Delta x + f_y(1,1)\,\Delta y = 4(0.1) - 1(0.1) = 0.3$$

Submit You have used 1 of 3 attempts

Answers are displayed within the problem

3. Review tangent approximation

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[Staff] Proof-reading 1
(Just an FYI in case it helps refine this course for future runs) It seems to me the formula immediately above the *Surface with tange...

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In some of the earlier courses you had PDF files available for the summary sections. It would be great if you could put links to the PD

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