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## Exercise: A criterion for independence

(1/1 point)

Suppose that the conditional PMF of $X$, given $Y=y$, is the same for every $y$ for which $p_Y(y)>0$. Is this enough to guarantee independence?

Yes ▾ ✔ **Answer:** Yes

Answer:
The condition given means that when I tell you the value of $Y$, the conditional PMF of $X$ will be the same. Thus, the value of $Y$ makes no difference, and, intuitively, we have independence.

For a formal argument, let $c(x)=p_{X|Y}(x\mid y)$; we can define $c(x)$ this way (without a dependence on $y$) since we are assuming that $p_{X|Y}(x\mid y)$ is the same for all $y$. Now,

$$p_{X,Y}(x,y)=p_Y(y)p_{X|Y}(x\mid y)=p_Y(y)c(x).$$

Summing over all $y$, we obtain

$$p_X(x)=\sum_y p_{X,Y}(x,y)=\sum_y p_Y(y)c(x)=c(x).$$

Therefore, $c(x)=p_X(x)$. It follows that $p_{X,Y}(x,y)=p_{X|Y}(x\mid y)p_Y(y)=c(x)p_Y(y)=p_X(x)p_Y(y)$, which establishes independence.

*You have used 1 of 1 submissions*