6.2.1 Reducing a System of Linear Equations to an Upper Triangular System

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Week 6 due Nov 13, 2023 12:12 IST Completed

6.2.1 Reducing a System of Linear Equations to an Upper Triangular System

Start of transcript. Skip to the end.

Dr. Robert van de Geijn: So we're going to look

at how to reduce a system of linear equations

to an upper triangular system, and this is probably a method

that you've seen before for solving simultaneous equations.

0:00 / 0:00 2.0x

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Reading Assignment

0 points possible (ungraded)
Read Unit 6.2.1 of the notes. [LINK]

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HW 6.2.1.1 3

6.2.1.3

For problem 6.2.1.3 ... in the solution to the problem, the work below where it says "We compute the value of p2(n) for n = 0,1,2:" where is the ...

Calculator

Video for Homework 6.2.1.1

Start of transcript. Skip to the end.

Dr. Robert van de Geijn: So what we've done

is we've created a web page where you can practice

different ways of performing Gaussian elimination.

Notice that it's organized by Unit, 6.2.1, 2.2, 2.3, et cetera.

0:00 / 0:00 2.0x

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Homework 6.2.1.1

3/3 points (graded)

Practice reducing a system of linear equations to an upper triangular system of linear equations by visiting the "Practice with Gaussian Elimination" webpage we created for you. For now, only work with the top part of that webpage.

Problem 1 in that webpage starts with the system of linear equations

$$\begin{array}{rcrcrcrcrcrcr} 1 & \chi_0 & + & 1 & \chi_1 & + & 2 & \chi_2 & = & -1 \\ 3 & \chi_0 & + & 1 & \chi_1 & + & 7 & \chi_2 & = & -7 \\ 1 & \chi_0 & + & 7 & \chi_1 & + & 1 & \chi_2 & = & 7 \end{array}$$

and yields the upper triangular system

$$\begin{array}{rcrcrcrcrcr} 1 & \chi_0 & + & 1 & \chi_1 & + & 2 & \chi_2 & = & -1 \\ & & & \alpha_{1,1} & \chi_1 & + & 1 & \chi_2 & = & \beta_1 \\ & & & & & & \alpha_{2,2} & \chi_2 & = & -4 \end{array}$$

Enter the values for $\alpha_{1,1}, \alpha_{2,2}$, and β_1 below:

$\alpha_{1,1} =$

-2

Answer: -2

$\alpha_{2,2} =$

2

Answer: 2

$\beta_1 =$

Calculator

-4 Answer: -4

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Answers are displayed within the problem

Homework 6.2.1.2

3/3 points (graded)

$$\begin{array}{rcrcrcr} -2\chi_0 & + & \chi_1 & + & 2\chi_2 & = & 0 \\ 4\chi_0 & - & \chi_1 & - & 5\chi_2 & = & 4 \\ 2\chi_0 & - & 3\chi_1 & - & \chi_2 & = & -6 \end{array}$$

$$\begin{pmatrix} \chi_0 \\ \chi_1 \\ \chi_2 \end{pmatrix} =$$

-1 Answer: -1

2 Answer: 2

-2 Answer: -2

Answer:

$$\begin{array}{rcrcrcr} -2\chi_0 & + & \chi_1 & + & 2\chi_2 & = & 0 \\ 4\chi_0 & - & \chi_1 & - & 5\chi_2 & = & 4 \\ 2\chi_0 & - & 3\chi_1 & - & \chi_2 & = & -6 \end{array} \longrightarrow \begin{array}{rcrcrcr} -2\chi_0 & + & \chi_1 & + & 2\chi_2 & = & 0 \\ & & \chi_1 & - & \chi_2 & = & 4 \\ & & -2\chi_1 & + & \chi_2 & = & -6 \end{array} \longrightarrow$$

$$\begin{array}{rcrcrcr} -2\chi_0 & + & \chi_1 & + & 2\chi_2 & = & 0 \\ & & \chi_1 & - & \chi_2 & = & 4 \\ & & & & -\chi_2 & = & 2 \end{array} \longrightarrow \left\{ \begin{array}{rcl} -\chi_2 = 2 & \Rightarrow & \chi_2 = -2 \\ \chi_1 - (-2) = 4 & \Rightarrow & \chi_1 = 2 \\ -2\chi_0 + (2) + 2(-2) = 0 & \Rightarrow & \chi_0 = -1 \end{array} \right.$$

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Answers are displayed within the problem

Homework 6.2.1.3

3/3 points (graded)

Compute the coefficients $\gamma_0, \gamma_1,$ and γ_2 so that

$$\sum_{i=0}^{n-1} i = \gamma_0 + \gamma_1 n + \gamma_2 n^2$$

Hint: let $p_2(n) = \gamma_0 + \gamma_1 n + \gamma_2 n^2$. Evaluate $p_2(0), p_2(1),$ and $p_2(2)$ by plugging n into the expression. Then also evaluate $\sum_{i=0}^{n-1} i$. This then gives you three equations in three unknowns (the coefficients). Then you solve!

In other words: if $n = 0$ then

$$\gamma_0 + \gamma_1 \times 0 + \gamma_2 \times 0^2 = \sum_{i=0}^{0-1} i$$

or

$$\gamma_0 + 0 \times \gamma_1 + 0 \times \gamma_2 = 0$$

since $\sum_{i=0}^{-1} i = 0$ (because the sum over an "empty range" is defined to equal zero). That is your first equation. Similarly, create the second equation and third equation by setting $n = 1$ and $n = 2,$ respectively. Then solve your system of linear equations with three equations in three unknowns.

Calculator

$\gamma_0 =$

γ_0

0

Answer: 0

$\gamma_1 =$

-1/2

Answer: -.5

$\gamma_2 =$

1/2

Answer: .5

Answer: Earlier in this course, as an example when discussing proof by induction and then again later when discussing the cost of a matrix-vector multiplication with a triangular matrix and the solution of a triangular system of equations, we encountered

$$\sum_{i=0}^{n-1} i.$$

Now, you may remember that the equalled some quadratic (second degree) polynomial in n, but not what the coefficients of that polynomial were:

$$\sum_{i=0}^{n-1} i = \gamma_0 + \gamma_1 n + \gamma_2 n^2,$$

for some constant scalars γ_0, γ_1, and γ_2. What if you wanted to determine what these coefficients are? Well, you now know how to solve linear systems, and we now see that determining the coefficients is a matter of solving a linear system.
Starting with

$$p_2(n) = \sum_{i=0}^{n-1} i = \gamma_0 + \gamma_1 n + \gamma_2 n^2,$$

we compute the value of $p_2(n)$ for $n = 0, 1, 2$:

$$\begin{array}{lccccccccc}
p_2(0) = \sum_{i=0}^{(0)-1} i & = & \gamma_0(0)^0 & + & \gamma_1(0) & + & \gamma_2(0)^2 & = & 0 & = & 0 \\
p_2(1) = \sum_{i=0}^{(1)-1} i & = & \gamma_0(1)^0 & + & \gamma_1(1) & + & \gamma_2(1)^2 & = & 0 & = & 0 \\
\sum_{i=0}^{(2)-1} i & = & \gamma_0(2)^0 & + & \gamma_1(2) & + & \gamma_2(2)^2 & = & 0+1 & = & 1
\end{array}$$

or, in matrix notation,

$$\begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & 2 & 4 \end{pmatrix} \begin{pmatrix} \gamma_0 \\ \gamma_1 \\ \gamma_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

One can then solve this system to find that

$$\begin{pmatrix} \gamma_0 \\ \gamma_1 \\ \gamma_2 \end{pmatrix} = \begin{pmatrix} 0 \\ -\frac{1}{2} \\ \frac{1}{2} \end{pmatrix}$$

so that

$$\sum_{i=0}^{n-1} i = \frac{1}{2} n^2 - \frac{1}{2} n$$

Previous Next

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