MATHEMATICS

probability density of the maximum of samples from a uniform distribution

Asked 6 years, 6 months ago Active 2 months ago Viewed 31k times

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Suppose

$$X_1, X_2, \ldots, X_n \sim Unif(0, \theta), iid$$

and suppose

$$\hat{\theta} = \max\{X_1, X_2, \ldots, X_n\}$$

How would I find the probability density of $\hat{\theta}$?

Thank you!

probability probability-theory

edited Feb 24 '13 at 23:58

asked Feb 24 '13 at 22:05

Enzo
407 1 5 12

are the Xs Independent? – Inquest Feb 24 '13 at 22:12

2 Here is a question for you: did you ask all your questions on MSE with no indication whatsoever on what you understood of the problem or what you tried before asking? – Did Feb 24 '13 at 22:19

@Inquest, yes they are! Sorry that I forgot to point that out. – Enzo Feb 24 '13 at 23:06

1 @Did, no seriously I don't know what you are talking about. What do you mean by all the MSE questions? This is my first question that's related to MSE and I just told in in the comment what I did, so I am not sure what you are accusing me of. – Enzo Feb 25 '13 at 14:07

1 I see. I thought you were talking about mean squared error. Well, I apologize if you didn't see what I have tried before asking, but I did indicate them somewhere in the questions, either in comments, or as a feedback of my own work. For example, on one of the proofs involving differential forms, I provided a copy of my own solution in the end. So I don't think you are in a right position to accuse me, and I don't feel comfortable about that. Thank you for helping me before though. – Enzo Feb 25 '13 at 16:35

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3 Answers

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$$\begin{aligned} P(Y \leq x) &= P(\max(X_1, X_2, \cdots, X_n) \leq x) \\ &= P(X_1 \leq x, X_2 \leq x, \cdots, X_n \leq x) \\ &\overset{ind}{=} \prod_{i=1}^{n} P(X_i \leq x) \\ &= \prod_{i=1}^{n} \frac{x}{\theta} \\ &= \left(\frac{x}{\theta}\right)^n \end{aligned}$$

edited Oct 29 '17 at 17:39
sligocki
223 1 4

answered Feb 24 '13 at 22:20
Inquest
5,130 2 24 50

1 In line 2, are the ns supposed to be xs or is this correct as is? – kram1032 Jul 26 '14 at 20:50

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Let random variable W denote the maximum of the X_i. We will assume that the X_i are independent, else we can say very little about the distribution of W.

Note that the maximum of the X_i is $\leq w$ if and only if **all** the X_i are $\leq w$. For w in the interval $[0, \theta]$, the probability that $X_i \leq w$ is $\frac{w}{\theta}$. It follows by independence that the probability that $W \leq w$ is $\left(\frac{w}{\theta}\right)^n$.

Thus, in our interval, the cumulative distribution function $F_W(w)$ of W is given by

$$F_W(w) = \left(\frac{w}{\theta}\right)^n.$$

Differentiate to get the density function of W.

edited Jul 26 '14 at 21:02

answered Feb 24 '13 at 22:18
André Nicolas
460k 37 445 840

There appears to be a TeX error. I can't edit it because it's just a single backslash that needs removal but math.SE asks me to change the post by at least 6 symbols. – kram1032 Jul 26 '14 at 20:51

Thank you for telling me, yes I had written \w for w. – André Nicolas Jul 26 '14 at 21:02

1

The general formula for the probability density of the maximum of **any** iid sample set of the random variable x, $M = max\{x_1, x_2, \ldots, x_n\}$ is:

$$f_M(M = x) = n * F_x(x)^{n-1} * f_x(x)$$

where $f_x(x)$ is the probability density of x, and $F_x(x)$ is the cumulative distribution function of same.

In this case we have: $f_x(x) = \frac{1}{\theta}$, $F_x(x) = \frac{x}{\theta}$, so we get:

$$f_M(M = x) = n * (\frac{x}{\theta})^{n-1} * \frac{1}{\theta}$$

$$= \frac{n * x^{n-1}}{\theta^n}$$

answered Jul 9 at 15:41

אלימלך שרייבר
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