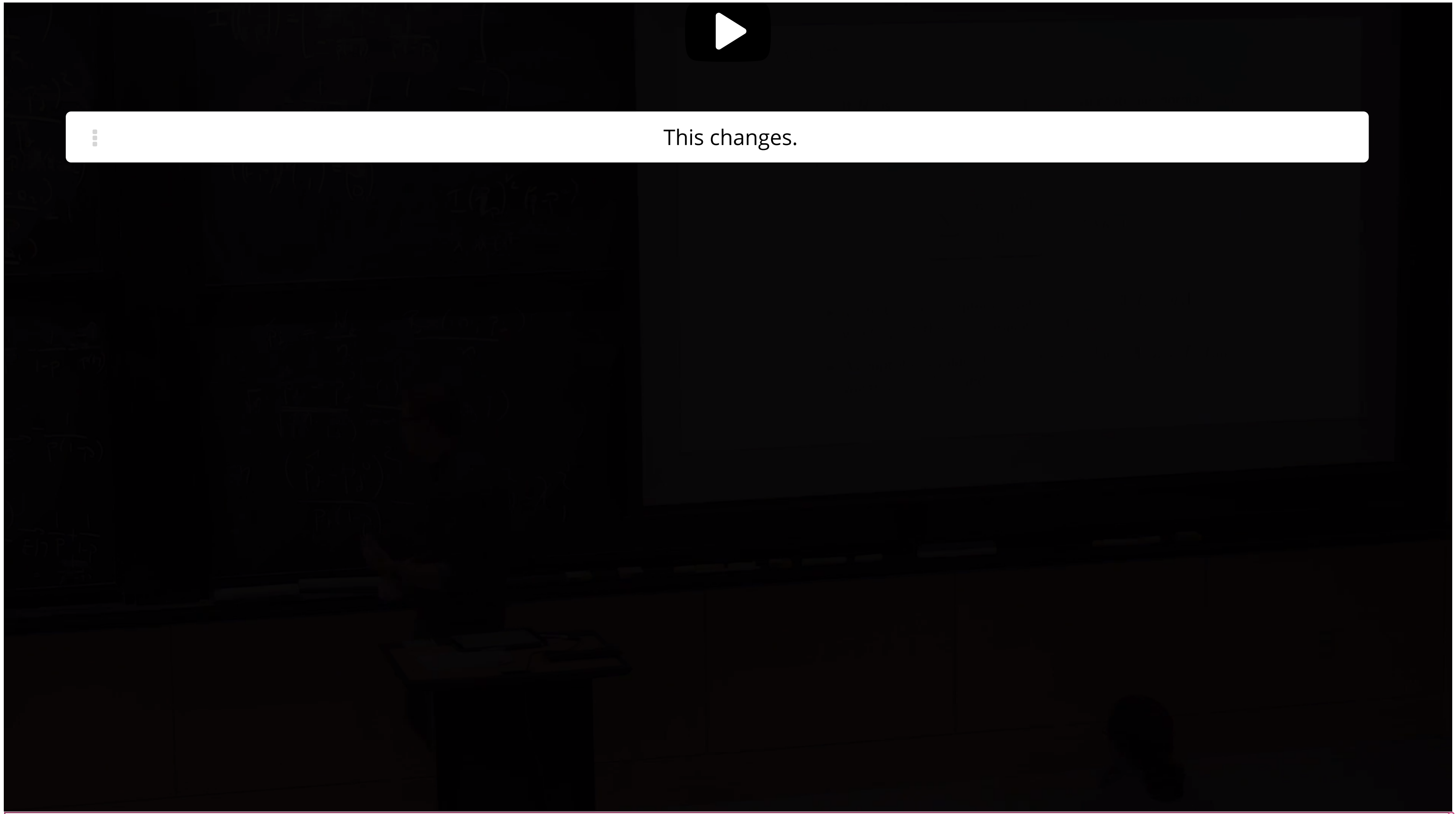
8. The Chi-Squared Test - Main Ideas

Informal Idea behind the Test Statistic and its Convergence Properties



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Topic: Unit 4 Hypothesis testing:Lecture 15: Goodness of Fit Test for Discrete Distributions / 8. The Chi-Squared Test - Main Ideas

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09:41 "If the physicists can't follow this, who will?"

discussion posted 6 days ago by SuhailWali

Richard Feynman must be having hiccups in his grave!!

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1 response

markweitzman (Community TA)
6 days ago

When I was staff/student for MITx 8.06x Applications of Quantum Mechanics, several students asked me how that course compares with a Quantum Field Theory course (often a first year follow on graduate course for Phd. theoretical physics students). I answered that QFT is about an order of magnitude more difficult in terms of work and technical calculations. As an example below is my solution for a single problem (10.2) from a standard book An Introduction To Quantum Field Theory (Frontiers in Physics) 1st Edition by Michael E. Peskin (Author), Dan V. Schroeder (Author).

10.2 Renormalization of Yukawa theory. Consider the pseudoscalar Yukawa Lagrangian,

$$\mathcal{L} = \tfrac{1}{2}(\partial_\mu\phi)^2 - \tfrac{1}{2}m^2\phi^2 + \bar{\psi}(i\not{\partial} - M)\psi - ig\bar{\psi}\gamma^5\psi\phi,$$

where ϕ is a real scalar field and ψ is a Dirac fermion. Notice that this Lagrangian is invariant under the parity transformation $\psi(t,\mathbf{x}) \to \gamma^0\psi(t,-\mathbf{x})$, $\phi(t,\mathbf{x}) \to -\phi(t,-\mathbf{x})$, in which the field ϕ carries odd parity.

(a) Determine the superficially divergent amplitudes and work out the Feynman rules for renormalized perturbation theory for this Lagrangian. Include all necessary counterterm vertices. Show that the theory contains a superficially divergent 4ϕ amplitude. This means that the theory cannot be renormalized unless one includes a scalar self-interaction,

$$\delta\mathcal{L} = \frac{\lambda}{4!}\phi^4,$$

and a counterterm of the same form. It is of course possible to set the renormalized value of this coupling to zero, but that is not a natural choice, since the counterterm will still be nonzero. Are any further interactions required?

(b) Compute the divergent part (the pole as $d \to 4$) of each counterterm, to the one-loop order of perturbation theory, implementing a sufficient set of renormalization conditions. You need not worry about finite parts of the counterterms. Since the divergent parts must have a fixed dependence on the external momenta, you can simplify this calculation by choosing the momenta in the simplest possible way.

P. 344 10.2

$$\mathcal{L} = \tfrac{1}{2}(\partial_\mu\varphi)^2 - \tfrac{1}{2}m^2\varphi^2 + \psi(i\partial - M)\psi - ig\,\bar\psi\gamma^5\psi\varphi$$

P_e – electron propagators
P_φ – pseudoscalar propagators
N_e = external electrons
N_φ = external pseudoscalars

(a) $\text{Loops} = L = P_e + P_\varphi - V + 1$

$$3V = N_e + N_\varphi + 2P_\varphi + 2P_e$$

$$\begin{pmatrix} N_e + 2P_e = 2V \\ N_\varphi + 2P_\varphi = V \end{pmatrix}$$

D = Degree of Feynman Diagram (divergence) $= 4L - P_e - 2P_\varphi$

$$P_e = \frac{2V - N_e}{2} \qquad P_\varphi = \frac{V - N_\varphi}{2}$$

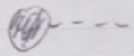
$$D = 2(2V - N_e) + 2(V - N_e) - 4V + 4 - \left(\frac{2V - N_e}{2}\right) - (V - N_\varphi)$$

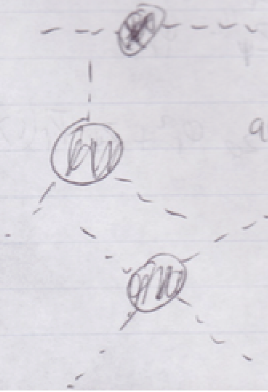
$= 4 - N_\gamma - \frac{3}{2} N_e$ (as in QED since vertex has ~~same~~ similar types of particles)

$D \geq 0$ Superficially divergent amplitudes

$$4 - N_\gamma - \frac{3}{2} N_e \geq 0$$

(a) $D=4$ irrelevant for scattering

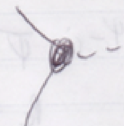
(b) $D=3$  vanishes due to invariance under parity.

(c) $D=2$  needs counterterm

(d) $D=1$ also vanishes by parity

(e) $D=0$ will need counterterm (vanishes in QED due to gauge invariance see Prob 10.2(b))

(f)  $D=1$ needs counterterm

(g)  $D=0$ needs counterterm

With addition of $\delta\mathcal{L} = \frac{-\lambda}{4!}\varphi^4$ (note sign change from book's λ)

Check divergence criterion

Additional Interaction vertex V_λ (V= electron vertex)

$V + 4V_\lambda = N_\varphi + 2P_\varphi$

$L = P_e + P_\varphi - V - V_\lambda + 1$

$2V = N_e + 2P_e$

$P_\varphi = \frac{(V+4V_\lambda) - N_\varphi}{2}$

$P_e = \frac{2V - N_e}{2}$

$$D = 4(P_e + P_\varphi - V - V_\lambda + 1) - 2P_\varphi - P_e$$

$$= 3P_e + 2P_\varphi - 4V - 4V_\lambda + 4$$

$$= 3\left(\frac{2V - N_e}{2}\right) + \cancel{2}\left(\frac{V + 4V_\lambda - N_\varphi}{\cancel{2}}\right) - 4V - 4V_\lambda + 4$$

$$= -\frac{3}{2}N_e - N_\varphi + 4$$

Thus no change (~~as~~ expected since $\lambda\varphi^4$ is a dimension 4 operator)

Rescale fields $\varphi = Z_\varphi^{1/2}\varphi_r$
$\psi = Z_\psi^{1/2}\psi_r$

$$\mathcal{L} = \frac{1}{2}Z_\varphi(\partial_\mu\varphi_r)^2 - \frac{1}{2}m^2 Z_\varphi \varphi_r^2 + Z_\psi\bar{\psi}_r(i\not\partial - M)\psi_r - ig Z_\varphi^{1/2} Z_\psi \bar{\psi}_r\gamma^5\psi_r\varphi_r - \frac{\lambda}{4!}Z_\varphi^2\varphi_r^4$$

Rewriting $\mathcal{L}$

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \varphi_r)^2 - \frac{1}{2} m_p^2 \varphi_r^2 + \bar{\Psi}_r (i\not\partial - M_p)\Psi_r - i g_p \bar{\Psi}_r \gamma^5 \Psi_r \varphi_r - \frac{\lambda_p}{4!}\varphi_r^4$$

$$+ \frac{1}{2}(Z_\varphi - 1)(\partial_\mu \varphi_r)^2 - \frac{1}{2}(m^2 Z_\varphi - m_p^2)\varphi_r^2 + \bar{\Psi}_r \left[i\not\partial (Z_\Psi - 1) - (M Z_\Psi - M_p) \right] \Psi_r$$

$$- i g_p \left(g \frac{Z_\varphi^{1/2} Z_\Psi}{g_p} - 1 \right) \bar{\Psi}_r \gamma^5 \Psi_r \varphi_r - \frac{(\lambda Z_\varphi^2 - \lambda_p)}{4!} \varphi_r^4$$

Defining:

$\delta Z_\varphi = Z_\varphi - 1$

$\delta Z_\Psi = Z_\Psi - 1$

$\delta m_\varphi = m^2 Z_\varphi - m_\varphi^2$

$\delta M_e = M Z_\Psi - M_e$

$Z_\varphi^{1/2} Z_\Psi g = g_p Z_1 \qquad \delta Z_1 = Z_1 - 1$

$\delta_\lambda = \lambda Z_\varphi^2 - \lambda_p$

m_p = physical mass of pseudoscalar particle

M_p = physical mass of electron

g_p = Physical Coupling Constant of electron pseudoscalar interaction

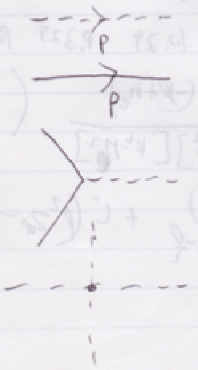
λ_p = Physical Coupling Constant of φ^4 interaction

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \varphi_r)^2 - \frac{1}{2} m_\varphi^2 \varphi_r^2 + \bar{\Psi}_r (i\not\partial - M_e)\Psi_r - i g_p \bar{\Psi}_r \gamma^5 \Psi_r \varphi_r - \frac{\lambda_p}{4!}\varphi_r^4$$

$$+ \frac{1}{2}\delta Z_\varphi (\partial_\mu \varphi_r)^2 - \frac{1}{2}\delta m_\varphi \varphi_r^2 + \bar{\Psi}_r (i\not\partial \, \delta Z_\Psi - \delta M_e)\Psi_r - i \delta Z_1 g_p \bar{\Psi} \gamma^5 \Psi \varphi$$

$$- \frac{\delta_\lambda \varphi_r^4}{4!}$$

Feynman rules:



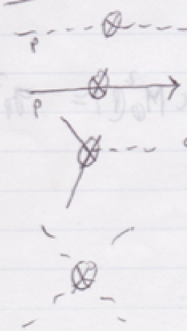
$\dfrac{i}{p^2 - m_\varphi^2 + i\epsilon}$

$\dfrac{i}{\not p - M_e + i\epsilon}$

$g_p \gamma^5$

$-i\lambda_p$

Counterterms



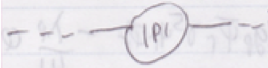
$i(p^2 \delta Z_\varphi - \delta m_\varphi)$

$i(\not p \, \delta Z_\Psi - \delta M_e)$

$g_p \, \delta Z_1 \gamma^5$

$-i\delta_\lambda$

2 point pseudoscalar point function:


$$= -i M_\varphi^2(p^2)$$


$$= \frac{i}{p^2 - M_\varphi^2 - M_\varphi^2(p^2)}$$

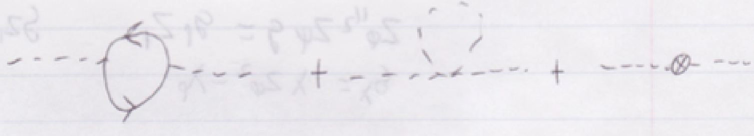
As on P.328 P+S

Renormalization prescription:

$$M_\varphi^2(p^2)\Big|_{p^2 = M_\varphi^2} = 0$$

$$\frac{\partial}{\partial p^2} M_\varphi^2(p^2)\Big|_{p^2 = M_\varphi^2} = 0$$

$$-i M_\varphi^2(p^2) \overset{(1\text{-loop})}{=}$$



$$-i M_\varphi^2(p^2) = \underset{\substack{\uparrow \\ \text{fermion loop}}}{(-1)}\, g_p^2 \int \frac{d^d k}{(2\pi)^d} \operatorname{tr} \frac{i(\not k + \not p + M_e)\gamma^5\, i(\not k + M_e)\gamma_5}{[(k+p)^2 - M_e^2][k^2 - M_e^2]}$$

$$+ \frac{-i\lambda}{2} \frac{1}{(4\pi)^{d/2}} \frac{\Gamma(1-\frac{d}{2})}{(M_\varphi^2)^{1-\frac{d}{2}}} + i\left(p^2 Z_\varphi - \delta M_\varphi\right)$$

$\uparrow$ 10.29 P.328 P+S

$$-i M_\varphi^2(p^2) = g_p^2 \int \frac{d^d k}{(2\pi)^d} \frac{\operatorname{tr}(\not k + \not p + M_e)(-\not k + M_e)}{[(k+p)^2 - M_e^2][k^2 - M_e^2]}$$

(? γ^5 in d dimensions)

$$- \frac{i\lambda}{2} \frac{1}{(4\pi)^{d/2}} \frac{\Gamma(1-\frac{d}{2})}{(M_\varphi^2)^{1-\frac{d}{2}}} + i\left(p^2 Z_\varphi - \delta M_\varphi\right)$$

$$-i M_\phi^2(p^2) = 4g_p^2 \int \frac{d^dk}{(2\pi)^d} \frac{-k\cdot(k+p)+m_e^2}{[(p+k)^2-m_e^2][k^2-m_e^2]} - \frac{i\lambda}{2}\frac{1}{(4\pi)^{d/2}}\frac{\Gamma(1-\frac{d}{2})}{(M_\phi^2)^{1-\frac{d}{2}}} + i(p^2 Z_\phi - \delta m_\phi)$$

$$l = k + xp$$

$$-i M_\phi^2(p^2) = 4g_p^2 \int_0^1 dx \int \frac{d^dl}{(2\pi)^d} \frac{-[l^2 - x(1-x)p^2]+m_e^2}{[l^2+x(1-x)p^2-m_e^2]} + \ldots$$

$$= 4g_p^2\, i \int_0^1 dx \int \frac{d^dl_E}{(2\pi)^d} \frac{l_E^2 + x(1-x)p^2 + m_e^2}{(l_E^2+\Delta)^2} + \ldots \qquad \Delta = m_e^2 - x(1-x)p^2$$

$$-i M_\phi^2(p^2) = \frac{4ig_p^2}{(4\pi)^{d/2}} \int_0^1 dx\, \frac{d}{2}\Gamma\left(1-\frac{d}{2}\right)\left(\frac{1}{\Delta}\right)^{1-\frac{d}{2}} + \left[x(1-x)p^2+m_e^2\right]\Gamma\left(2-\frac{d}{2}\right)\left(\frac{1}{\Delta}\right)^{2-\frac{d}{2}} + \ldots$$

→ Taking only infinite part

$$-i M_\phi^2(p^2) = \frac{4ig_p^2}{(4\pi)^{d/2}} \int_0^1 dx\; 2\left(\frac{-2}{\epsilon}\right)\left[m_e^2 - x(1-x)p^2\right] + \left(\frac{2}{\epsilon}\right)\left[x(1-x)p^2+m_e^2\right] + -\frac{i\lambda}{2}\frac{1}{(4\pi)^2}\left(\frac{-2}{\epsilon}\right)M_\phi^2 + i(p^2 Z_\phi - \delta m_\phi)$$

$$-M_\phi^2(p^2) = \frac{4g_p^2}{(4\pi)^2}\left(\frac{1}{\epsilon}\right)(-2m_e^2) + \frac{4g_p^2}{(4\pi)^2\epsilon}\left(p^2\right) + p^2 Z_\phi - \delta m_\phi - \frac{\lambda}{2}\frac{1}{(4\pi)^2}\left(\frac{-2}{\epsilon}\right)M_\phi^2$$

$$0 = \frac{-\partial M_\phi^2(p^2)}{\partial p^2}\bigg|_{p^2=m_\phi^2} = \frac{+4g_p^2}{16\pi^2}\frac{1}{\epsilon} + Z_\phi \rightarrow \boxed{Z_\phi = \frac{-g_p^2}{4\pi^2}\frac{1}{\epsilon}}$$

$$0 = -M_\phi^2(p^2)\big|_{p^2=m_\phi^2} = \frac{-8g_p^2 m_e^2}{16\pi^2}\frac{1}{\epsilon} - \delta m_\phi + \frac{\lambda}{\epsilon}\frac{m_\phi^2}{16\pi^2} \rightarrow \boxed{\delta m_\phi = \frac{1}{\epsilon}\left(\frac{\lambda m_\phi^2}{16\pi^2} - \frac{g_p^2 m_e^2}{2\pi^2}\right)}$$

$$-i M_\phi^2(p^2) = 4g_p^2 \int \frac{d^d k}{(2\pi)^d} \frac{-k\cdot(k+p) + m_e^2}{[(p+k)^2 - m_e^2][k^2 - m_e^2]} - \frac{i\lambda}{2}\frac{1}{(4\pi)^{d/2}}\frac{\Gamma(1-\frac{d}{2})}{(M_\phi^2)^{1-\frac{d}{2}}}$$

$$+ i(p^2 z_\phi - \delta m_\phi)$$

$$l = k + xp$$

$$-i M_\phi^2(p^2) = 4g_p^2 \int_0^1 dx \int \frac{d^d l}{(2\pi)^d} \frac{-[l^2 - x(1-x)p^2] + m_e^2}{[l^2 + x(1-x)p^2 - m_e^2]} + \dots$$

$$= 4g_p^2 i \int_0^1 dx \int \frac{d^d l_E}{(2\pi)^d} \frac{l_E^2 + x(1-x)p^2 + m_e^2}{(l_E^2 + \Delta)^2} \quad \Delta = m_e^2 - x(1-x)p^2$$

$$+ \dots$$

$$-i M_\phi^2(p^2) = \frac{4 i g_p^2}{(4\pi)^{d/2}} \int_0^1 dx \, \frac{d}{2}\Gamma(1-\tfrac{d}{2})\left(\frac{1}{\Delta}\right)^{1-\frac{d}{2}} + [x(1-x)p^2 + m_e^2]\,\Gamma(2-\tfrac{d}{2})\left(\frac{1}{\Delta}\right)^{2-\frac{d}{2}} + \dots$$

Taking only infinite part

$$-i M_\phi^2(p^2) = \frac{4 i g_p^2}{(4\pi)^{d/2}} \int_0^1 dx \; 2\left(\frac{-2}{\epsilon}\right)[m_e^2 - x(1-x)p^2] + \left(\frac{2}{\epsilon}\right)[x(1-x)p^2 + m_e^2] +$$

$$-\frac{i\lambda}{2}\frac{1}{(4\pi)^2}\left(\frac{-2}{\epsilon}\right) M_\phi^2$$

$$+ i(p^2 z_\phi - \delta m_\phi)$$

$$-M_\phi^2(p^2) = \frac{4g_p^2}{(4\pi)^2}\left(\frac{1}{\epsilon}\right)(-2m_e^2) + \frac{4g_p^2}{(4\pi)^2\epsilon}\left(\frac{p^2}{1}\right) + p^2 z_\phi - \delta m_\phi - \frac{\lambda}{2}\frac{1}{(4\pi)^2}\left(\frac{-2}{\epsilon}\right) m_\phi^2$$

$$0 = \frac{-\partial M_\phi^2(p^2)}{\partial p^2}\bigg|_{p^2 = m_\phi^2} = \frac{+4g_p^2}{16\pi^2}\frac{1}{\epsilon} + z_\phi \rightarrow \boxed{z_\phi = \frac{-g_p^2}{4\pi^2}\frac{1}{\epsilon}}$$

$$0 = -M_\phi^2(p^2)\big|_{p^2 = m_\phi^2} = \frac{-8g_p^2 m_e^2}{16\pi^2}\frac{1}{\epsilon} - \delta m_\phi + \frac{\lambda}{\epsilon}\frac{m_\phi^2}{16\pi^2} \rightarrow \boxed{\delta m_\phi = \frac{1}{\epsilon}\left(\frac{\lambda m_\phi^2}{16\pi^2} - \frac{g_p^2 m_e^2}{2\pi^2}\right)}$$

infinite part

$$-i\Sigma(p) = \frac{i g_p^2}{16\pi^2}\int_0^1 dx\, (x\not{p} - m_e)\frac{2}{\epsilon} + i(\not{p}\, \delta z_\psi - \delta m_e)$$

$$0 = \frac{\partial\, -i\Sigma(p)}{\partial p}\Big|_{p=m_e} = \frac{i g_p^2}{16\pi^2}\frac{1}{\epsilon} + i\,\delta Z_\psi$$

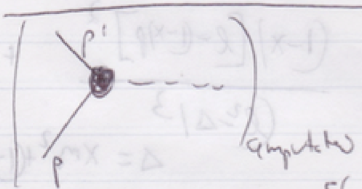
$$\boxed{\delta Z_\psi = \frac{-g_p^2}{16\pi^2}\left(\frac{1}{\epsilon}\right)}$$

$$0 = -i\Sigma(p)\big|_{p=m_e} = \frac{i g_p^2}{16\pi^2}\,\frac{2}{\epsilon}\left(\frac{1}{2}m_e - m_e\right) + i m_e \frac{-g_p^2}{16\pi^2}\frac{1}{\epsilon} - i\,\delta m_e$$

$$0 = \frac{m_e}{\epsilon}\left[\frac{-g_p^2}{16\pi^2} - \frac{g_p^2}{16\pi^2}\right] - \delta m_e$$

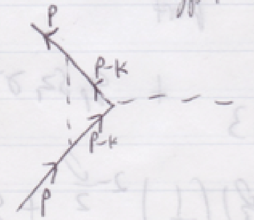
$$\boxed{\delta m_e = \frac{-2m_e}{\epsilon}\frac{g_p^2}{16\pi^2}}$$

Electron - pseudoscalar vertex



$$= g_p \Gamma^5(p', p)$$

$$g_p \Gamma^5(0) = g_p \gamma^5$$



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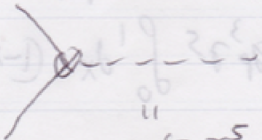


$$\| \qquad g_p\,\delta Z_1\,\gamma^5$$

$$g_p^3 \int \frac{d^4k}{\ldots}\ \gamma_5\ i(\not p - \not k + m_e)\,\gamma_5\ i\,(\not p - \not k + m_e)\,\gamma_5\ i \ldots + g_p\,\delta Z_1\,\gamma^5$$

$$\left[(p-k)^2-m_e^2\right]\left[(p+k)^2-m_e^2\right]\left[k^2-m_\varphi^2\right]$$

$$= i g_p^3 \gamma^5 \int \frac{d^d k}{(2\pi)^d} \frac{(\not p-\not k+m_e)(\not p-\not k-m_e)}{\left[(p-k)^2-m_e^2\right]^2\left[k^2-m_\varphi^2\right]} + g_p \delta z_1 \gamma^5$$

using 646 p. 190

$$= i g_p^3 \gamma^5 \int_0^1 dx\, dy\, \delta(x+y-1) \int \frac{d^d k}{(2\pi)^d} \frac{(p-k)^2-m_e^2}{\left[x(k^2-m_\varphi^2)+y[(p+k)^2-m_e^2]\right]^3}$$

just computing divergent part:

$$i g_p^3 \gamma^5 \int \frac{d^d k}{(2\pi)^d} \int_0^1 dx \frac{2(1-x)k^2}{\left[k^2-2(1-x)p\cdot k+(1-x)p^2-x m_\varphi^2-(1-x)m_e^2\right]^3}$$

$$+ g_p \delta z_1 \gamma^5$$

let $\ell = k+(1-x)p$

$$= 2 i g_p^3 \gamma^5 \int_0^1 dx \int \frac{d^d \ell}{(2\pi)^d} \frac{(1-x)\left[\ell-(1-x)p\right]^2}{(\ell^2-\Delta)^3} + g_p \delta z_1 \gamma^5$$

$$\Delta = x m_\varphi^2+(1-x)m_e^2-p^2 x(1-x)$$

Again just keeping divergent part

$$-2 g_p^3 \gamma^5 \int_0^1 dx\, (1-x) \frac{\ell_E^2}{(\ell_E^2+\Delta)^3} + g_p \delta z_1 \gamma^5$$

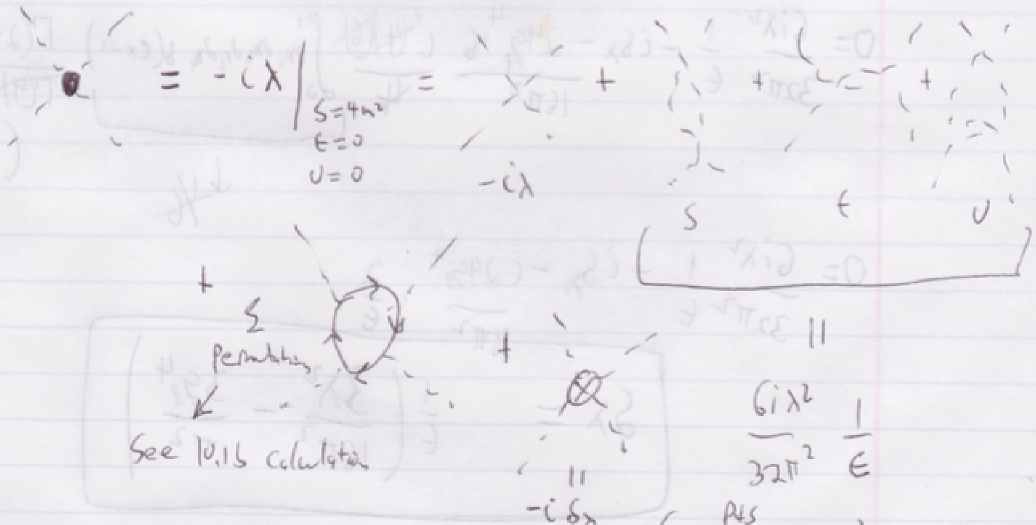
$$= -2 g_p^3 \gamma^5 \int_0^1 dx\, (1-x) \frac{d}{2} \frac{\Gamma(2-\frac{d}{2})}{2}\left(\frac{1}{\Delta}\right)^{2-\frac{d}{2}} + g_p \delta z_1 \gamma^5$$

$$= \frac{-2g_p^3\gamma^5}{16\pi^2}\int_0^1 dx\,(1-x)\frac{2}{\epsilon} + g_p\delta z_1\gamma^5$$

$$= \frac{-2g_p^3\gamma^5}{16\pi^2}\left(\frac{1}{\epsilon}\right) + g_p\delta z_1\gamma^5$$

so $\boxed{\delta z_1 = \frac{2g_p^2}{16\pi^2}\left(\frac{1}{\epsilon}\right)}$

4 point pseudo-scalar function



$= -i\lambda\Big|_{\substack{s=4m^2\\t=0\\u=0}} =$

$-i\lambda$ + s + t + u

||

$\frac{6i\lambda^2}{32\pi^2}\frac{1}{\epsilon}$

(see P&S p. 327)

+ $\sum$ permutations

See 10.1b calculation

+ $= -i\delta_\lambda$

$$-i\lambda = -i\lambda + \frac{6i\lambda^2}{32\pi^2}\frac{1}{\epsilon} - i\delta_\lambda - \sum_{\text{perms}} g_p^4\int\frac{d^4k}{(2\pi)^4}\frac{\mathrm{Tr}\left[\gamma^5 (i\not k)\gamma^5 i(\not p_1+\not k_1)\gamma^5 i(\not p_1+\not p_2+\not k)\gamma^5 i(\not p_1+\not p_2+\not p_3+\not k)\right]}{(k^2-m_e^2)\left[(p_1+k)^2-m_e^2\right]\left[(p_1+p_2+k)^2-m_e^2\right]\left[(p_1+p_2+p_3+k)^2-m_e^2\right]}$$

fermion loop

Taking divergent parts only:

$$0 = \frac{6i\lambda^2}{32\pi^2}\frac{1}{\epsilon} - i\delta_\lambda - \sum_{\text{perms}} g_p^4 \int \frac{d^4k}{(2\pi)^4} \frac{\mathrm{Tr}\, \not{k}\not{k}\not{k}\not{k}}{(k^2-m_e^2)\left[(k+p_1)^2-m_e^2\right]\left[(k+p_1+p_2)^2-m_e^2\right]\left[(k+p_1+p_2+p_3)^2-m_e^2\right]}$$

each permutation gives same result for divergent part and there are 6 of them.

$$0 = \frac{6i\lambda^2}{32\pi^2}\frac{1}{\epsilon} - i\delta_\lambda - \frac{24 g_p^4}{(2\pi)^4} 3! \int_0^1 dx_1 dx_2 dx_3 dx_4\, \delta(x_1+x_2+x_3+x_4-1) \int \frac{d^4k\,(k^2)^2}{(k^2-\Delta)^4}$$

Δ = some combination of p_i's, m_e, after completing square and keeping only divergent parts

$$0 = \frac{6i\lambda^2}{32\pi^2}\frac{1}{\epsilon} - i\delta_\lambda - \frac{24 g_p^4 \cdot 6}{16\pi^2}\frac{i(4)(6)}{4} \underbrace{\int_0^1 dx_1 dx_2 dx_3 dx_4\, \delta(\Sigma x - 1)}_{\downarrow\, 1/6} \frac{\Gamma(2-\frac{d}{2})}{\Gamma(4)}$$

(see A.47 P+S)

$$0 = \frac{6i\lambda^2}{32\pi^2}\frac{1}{\epsilon} - i\delta_\lambda - \frac{i\,24 g_p^4}{16\pi^2}\cdot\frac{2}{\epsilon}$$

$$\boxed{\delta\lambda = \frac{1}{\epsilon}\left(\frac{3\lambda^2}{16\pi^2} - \frac{3g_p^4}{\pi^2}\right)}$$

@markweitzman I agree.My remark was sarcastic. I think there is/has always been a banter between mathematicians and Physicists and Professor was just making a comment on those lines. I don't think anyone really believes physics is easy as pie and a Physicist need not prove what he is doing is hard. :)

posted 6 days ago by **SuhailWali**

I understand what you meant, what I meant to show is how much more work/calculation and mathematics is required for theoretical physics, than for probably any other discipline (including mathematics). There is a top theoretical physicist and author (and Nobel prize winner) Steven Weinberg, who in his books always makes remarks/jokes like in kindergarten we learned all the representations of the spinor groups in 4 dimensions, now will do it in n dimensions etc. So for theoretical physicists chi squared distributions etc. are like nursery school. :-)

posted 6 days ago by **markweitzman** (Community TA)

Some MIT students say that as they progress through their first 4 years, that the academic rigor becomes noticeably easier to them. Does theoretical physics, like say the above example problem, become significantly easier for students after a few years?

posted about 18 hours ago by **lygie**

Definitely not at least for theoretical physics - it only gets harder and harder.

posted about 17 hours ago by **markweitzman** (Community TA)

@markweitzman never took a QFT course (also never knew beyond the basics of the Schrodinger equation / finite potential wall in quantum mechanics at any point in time :( ), the solution looks very fascinating (wish i could understand a bit of the math and also the intuitive meaning), but it seems that at few places in your solution you needed to consider some special cases (e.g., for deg. of freedom D) and rewriting the Lagrangian and setting the partial derivatives to 0 to find critical points and applying few rules (e.g., Feynman's) whenever required (so are you basically solving a hard constrained optimization problem?).

I am just curious to know: given the theory (rules etc.) does it require one to be much more creative / imaginative to come up with a solution like yours in QFT, or it's difficult because it's lengthy / involves many symbols / the concept is hard (but once understood, if applied carefully in steps one can arrive at the solution without much difficulty), when compared to a mathematical or statistical proof.

posted 3 minutes ago by **sandipan_dey**

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