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#### Homework 4: TV distance, KL-

Course > Unit 3 Methods of Estimation > Divergence, and Introduction to MLE > 2. Compute Total Variation Distance

Currently enrolled in Audit Track (expires December 25, 2019) Upgrade (\$300)

# 2. Compute Total Variation Distance

(a)

3/3 points (graded)

Compute the total variation distance between

$$\mathbf{P} = X$$
 and  $\mathbf{Q} = X + c$ , where  $X \sim \mathsf{Ber}(p)$ ,  $p \in (0,1)$ , and  $c \in \mathbb{R}$ .

(If applicable, enter **abs(x)** for |x|. Simplify your answer to have the minimum number of absolute signs possible.)

For  $c \notin \{-1, 0, 1\}$ :

For c=0:

$$\mathsf{TV}(\mathbf{P},\mathbf{Q}) = \boxed{0}$$

For c=1 or c=-1:

$$\mathsf{TV}(\mathbf{P}, \mathbf{Q}) = \boxed{\frac{1 + \mathsf{abs}(1 - 2 \cdot p)}{2}}$$

STANDARD NOTATION

Submit

You have used 1 of 2 attempts

✓ Correct (3/3 points)

(b)

2/2 points (graded)

Compute the total variation distance between

$$\mathbf{P} = \mathsf{Ber}\left(p
ight) \quad ext{and} \quad \mathbf{Q} = \mathsf{Ber}\left(q
ight), \quad ext{where } p,q \in \left[0,1
ight].$$

(If applicable, enter **abs(x)** for |x|.)

$$\mathsf{TV}\left(\mathbf{P},\mathbf{Q}
ight) = egin{bmatrix} \mathsf{abs}\left(p ext{-}q
ight) \ & \mathsf{abs}\left(p-q
ight) \end{bmatrix}$$

Let  $X_1,\ldots,X_n$  be n i.i.d. Bernoulli random variables with some parameter  $p\in[0,1]$ , and  $\bar{X}_n$  be their empirical average. Consider the total variation distance  $\mathsf{TV}(\mathsf{Ber}(\bar{X}_n),\mathsf{Ber}(p))$  between  $\mathsf{Ber}(\bar{X}_n)$  and  $\mathsf{Ber}(p)$  as a function of the random variable  $\bar{X}_n$ , and hence a random variable itself. Does  $\mathsf{TV}(\mathsf{Ber}(\bar{X}_n),\mathsf{Ber}(p))$  necessarily converge in probability to a constant? If yes, enter the constant below; if not; enter DNE.

$$\mathsf{TV}\left(\mathsf{Ber}\left(ar{X}_n
ight),\mathsf{Ber}\left(p
ight)
ight) \xrightarrow[n o \infty]{} \mathbb{Q}$$

STANDARD NOTATION

Submit

You have used 1 of 2 attempts

✓ Correct (2/2 points)

(c)

1/1 point (graded)

Compute the total variation distance between

$$P = \mathsf{Unif}([0, s]) \quad \text{and} \quad Q = \mathsf{Unif}([0, t]), \quad \text{where } 0 < s < t.$$

STANDARD NOTATION

Submit

You have used 1 of 2 attempts

✓ Correct (1/1 point)

(d)

1/1 point (graded)

Let  $X \sim N(\mu, \sigma^2)$  and  $Y \sim \text{Ber}(p)$ . Compute the total variation distance between the distributions of  $\operatorname{sign}(X)$  and Y-1. Note that  $\operatorname{sign}(X)$  is a function of the random variable with

$${
m sign}\,(X) \,=\, egin{cases} 1 & {
m if}\,\,X > 0 \ 0 & {
m if}\,\,X = 0 \ -1 & {
m if}\,\,X < 0. \end{cases}$$

(If applicable, enter **abs(x)** for |x|, **Phi(x)** for  $\Phi\left(x\right)=\mathbf{P}\left(Z\leq x\right)$  where  $Z\sim\mathcal{N}\left(0,1\right),$  and **q(alpha)** for  $q_{\alpha},$  the  $1-\alpha$  -quantile of a standard normal distribution, e.g. enter **q(0.01)** for  $q_{0.01}$ .)

$$\mathsf{TV}\left(\operatorname{sign}\left(X\right),Y-1\right) = \boxed{ (\mathsf{p+1-Phi}(-\mathsf{mu/sigma}))/2 + \mathsf{abs}(\mathsf{p-1+Phi}(-\mathsf{mu/sigma}))/2 }$$

STANDARD NOTATION

Submit

You have used 2 of 3 attempts

✓ Correct (1/1 point)

(e)

1/1 point (graded)

Compute the total variation distance between

$$\mathbf{P} = \mathsf{Ber}\left(p
ight) \quad ext{and} \quad \mathbf{Q} = \mathsf{Poiss}\left(p
ight), \quad ext{where } p \in \left(0,1
ight).$$



**STANDARD NOTATION** 

Submit

You have used 1 of 2 attempts

✓ Correct (1/1 point)

### Discussion

**Topic:** Unit 3 Methods of Estimation:Homework 4: TV distance, KL-Divergence, and Introduction to MLE / 2. Compute Total Variation Distance

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## Not getting why I got Part (d) not accepted

discussion posted 21 minutes ago by sandipan\_dey

I got my answer as (1/4+p/2)+|1/4-p/2| for Part (d), but it's not accepted by the grader, any hint in terms of whether I am in the right direction? thank you very much in advance. It can also be written as max(1/2,p) but I think it will again get rejected by the grader.

[EDIT] my bad, i wrongly assumed  $X \sim N(0,1)$  . Now it got accepted.

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