Unit 2: Quiz

Bookmark this page

Unit 2: Quiz

The problems in the quiz will be automatically graded.

You are given only **ONE** attempt for each problem. You will be able to see the solution after you submit your answers.

Problem 1

2/2 points (graded)

1. It is estimated that, of the people viewing a certain movie over the weekend, 45 percent were adult women, 42 percent were adult men, and 13 percent were children.

If we randomly interview people at a theatre about the movie, each interview takes a few minutes, so we will not interview people in the same group or family. So we may assume that their gender and age classification is independent from person to person.

1a. What is the probability that the first five people we interview are all adults?

0.4984209

✔ **Answer:** 0.4984

1b. What is the probability that the first adult we interview is a female?

0.5172414 ✔ **Answer:** 0.5172

Explanation

1a. The probability is $(.45+.42)^5=(.87)^5=0.4984$.

1b. Let each person sampled be a trial. Treat the selection of a female as a good result, selection of a male as a bad result, and selection of a child as neutral. Then the probability that the first adult we interview is a female is $\frac{.45}{.45+.42}=0.5172$.

Submit You have used 1 of 1 attempt

Problem 2

3/3 points (graded)

2. Suppose that we roll a pair of (6 sided) dice until the first value appears that is 7 or less, and then we stop afterwards.

2a. What is the probability that exactly three (pairs of) rolls are required?

0.1012731 ✔ **Answer:** 0.1013

2b. What is the probability that at least three (pairs of) rolls are needed?

0.1736111 ✔ **Answer:** 0.1736

2c. What is the probability that, on the last rolled pair, we get a result of exactly 7?

2/7

✔ **Answer:** 0.2857

Explanation

2a. On a given roll, the probability that a value is 7 or less is $21/36$. So the probability that we get two results of 8 or higher, followed by a result of 7 or less, is $(15/36)^2(21/36) = 0.1013$.

2b. The probability that 1 or 2 rolls is sufficient is $\frac{21}{36} + (\frac{15}{36})(\frac{21}{36}) = 119/144 = 0.8264$. So the probability of the complementary event, i.e., the probability that 3 or more rolls are needed, is $1 - 119/144 = 25/144 = 0.1736$.

2c. We let the sum of the dice be a trial. Then a good trial is exactly a 7, a bad trial is a value (strictly) less than 7, and a neutral trial is (strictly) more than 7. (Notice that we stop when a good or bad trial occurs, i.e., when a roll of 7 or less occurs.) Then the probability of a good trial is $6/36$ and the probability of a bad trial is $15/36$. So the desired probability is $\frac{6/36}{6/36+15/36} = 6/21 = 2/7 = 0.2857$.

Submit You have used 1 of 1 attempt

Problem 3

1/1 point (graded)

3. Suppose that 11% of albums sold are country music; 15% are pop; 17% are R&B; and 29% are rock. There are several other kinds of genres not listed here. Suppose that we talk to people about their music choices. (Assume that the people selected have independent music preferences.) If we continue talking to people until we find someone whose top music choice is one of the four genres above, what is the probability that this person prefers rock?

0.4027778

✔ **Answer:** 0.4028

Explanation

3. We consider the chosen genre as a trial. A good trial is rock. A bad trial is country, pop, or R&B. A neutral trial is any other genre. Hence, we stop when we get a good or bad trial. So the probability that the person prefers rock is $\frac{.29}{.29+.11+.15+.17} = 0.4028$.

Submit You have used 1 of 1 attempt

Problem 4

2/2 points (graded)

4. Consider a red 4-sided die (numbered 1, 2, 3, 4), a green 4-sided die (also 1 to 4), and a blue 6-sided die (1 to 6). Roll the three dice (simultaneously) until the *sum* of the three dice equals 5, and then stop afterwards.

4a. On this final role of the dice, what is the probability that the red and green dice have the same values?

1/3 ✔ **Answer:** 0.3333

4b. Is the solution the same if the dice do not have colors? I.e., suppose that we roll two 4-sided white dice and one 6-sided white die, until the *sum* of the three dice equals 5, and then stop afterwards. On this final role of the dice, what is the probability that the 4-sided dice have the same values?

1/3 ✔ **Answer:** 0.3333

Explanation

4a. We let each of the simultaneous (triples of) rolls of the three dice count as a trial. A good trial has a sum of 5 and the green and blue dice have the same values. A bad trial has a sum of 5 but the green and blue dice do not have the same values. A neutral trial does not have a sum of 5. So a good trial has probability $P(\{(1,1,3),(2,2,1)\}) = 2/96$, and a bad trial has probability $P(\{(1,3,1),(3,1,1),(2,1,2),(1,2,2)\}) = 4/96$. So the desired probability is $\frac{2/96}{2/96+4/96} = 2/6 = 1/3$.

4b. The solution is exactly the same, even if the dice do not have colors. If you are worried about distinguishing the two 4-sided dice, just put one into your left hand and one into your right hand, and everything proceeds as above.

Submit You have used 1 of 1 attempt

Problem 5

2/2 points (graded)

5a. Consider two six sided dice. One die has 2 red, 2 green, and 2 blue sides. The other die has 3 red sides and 3 blue sides. One die is selected at random and rolled. The result is red. What is the conditional probability that the first die (i.e., the die with 3 colors) was chosen?

2/5 ✔ **Answer:** 0.4

5b. Consider two cards. One is black on both sides. The other has one white side and one black side. If a card is chosen at random and the side facing up is black, what is the conditional probability that the other side is black too?

2/3 ✔ **Answer:** 0.6667

Explanation

5a. If A is the event that the first die was used, and B is the event that red appears, then we have $P(A\cap B) = (1/2)(2/6) = 2/12$ and $P(B) = (1/2)(2/6) + (1/2)(3/6) = 5/12$, so $P(A \mid B) = \frac{P(A\cap B)}{P(B)} = \frac{2/12}{5/12} = 2/5$.

Another method of solution is to note that there are 5 ways that red can appear, and all five of these ways are *equally likely*, but only 2 of the ways are on the first die, so the desired conditional probability is $2/5$.

5b. If A is the event that the card with black on both sides was chosen, and B is the event that black appears facing up, then we have $P(A\cap B) = (1/2)(1) = 1/2$ and $P(B) = (1/2)(1) + (1/2)(1/2) = 3/4$, so $P(A \mid B) = \frac{P(A\cap B)}{P(B)} = \frac{1/2}{3/4} = 2/3$.

Another method of solution is that there are 3 black sides that can appear, and all three of these ways are *equally likely*. Since 2 of these 3 are on the card with black on both sides, then the desired conditional probability is $2/3$.

Submit You have used 1 of 1 attempt

Problem 6

6/6 points (graded)

6. Consider a standard deck of 52 cards. Shuffle the cards, and then deal them onto the table, one at a time, without replacement.

6a. Find the probability that the first card dealt is a queen.

1/13 ✔ **Answer:** 0.0769

6b. Find the probability that the last card dealt is a queen.

1/13

Answer: 0.0769

6c. Find the probability that the third card dealt is a queen.

1/13

Answer: 0.0769

6d. Find the probability that the jth card dealt is a queen, for $1 \le j \le 52$.

1/13

Answer: 0.0769

6e. Find the conditional probability that the 19th card dealt is a queen, given that the first card dealt is a queen.

1/17

Answer: 0.0588

6f. Find the conditional probability that the 19th card dealt is a queen, given that the first and seventh cards dealt are queens.

2/50

Answer: 0.04

Explanation

6abcd. In each of these problems, any of the 52 cards could appear in the position under discussion, and all 52 of these cards are *equally likely* to appear in that position, but only 4 of these 52 are queens, so the desired conditional probability is $4/52$.

6e. If A is the event that the 19th card is a queen, and B is the event that the 1st card is a queen, then we have $P(A \cap B) = (4/52)(3/51)$ and $P(B) = (4/52)$, so $P(A \mid B) = \frac{P(A \cap B)}{P(B)} = \frac{(4/52)(3/51)}{4/52} = 3/51$
.
Another method is to note that, once we have seen that the 1st card is a queen, there are 51 other cards that could appear at the 19th position, and they are all *equally likely*, and 3 of them are queens, so the desired conditional probability is $3/51$.
6f. If A is the event that the 19th card is a queen, and B is the event that the 1st and 7th cards are queens, then we have $P(A \cap B) = (4/52)(3/51)(2/50)$ and $P(B) = (4/52)(3/51)$, so
$P(A \mid B) = \frac{P(A \cap B)}{P(B)} = \frac{(4/52)(3/51)(2/50)}{(4/52)(3/15)} = 2/50$.
Another method is to note that, once we have seen that the 1st and 7th cards are queens, there are 50 other cards that could appear at the 19th position, and they are all *equally likely*, and 2 of them are queens, so the desired conditional probability is $2/50$.

Submit You have used 1 of 1 attempt

✔ Correct (6/6 points)

Problem 7

1/1 point (graded)
7. Suppose two 6-sided dice are rolled, and the sum is 8 or larger. What is the conditional probability that at least one value of 4 appears on the dice?

1/3 ✔ **Answer:** 0.3333

Explanation

7. If A is the event that at least one value of 4 appears on the dice, and B is the event that the sum is 8 or larger, then we have $P(A \cap B) = 5/36$ and $P(B) = 15/36$, so
$P(A \mid B) = \frac{P(A \cap B)}{P(B)} = \frac{5/36}{15/36} = 5/15 = 1/3$.
Another method of solution is to note that there are 15 ways that the sum can be 8 or larger, and all 15 of these ways are *equally likely*, but only 5 of the ways will have at least one value of 4 on the dice, so the desired conditional probability is $5/15 = 1/3$.

Submit You have used 1 of 1 attempt

Problem 8

3/3 points (graded)

8. Consider a red 4-sided die (numbered 1, 2, 3, 4), a green 4-sided die (also 1 to 4), and a blue 6-sided die (1 to 6). Roll the three dice (simultaneously).

Let B denote the event that the sum of the three dice is 5. Let A_j (for $j = 0, 1, 2$) denote the event that exactly j of the 4-sided dice have value 2.

Find the values of $P(A_j \mid B)$ for each j. Make sure that these three conditional probabilities sum to 1, i.e., that $P(A_0 \mid B) + P(A_1 \mid B) + P(A_2 \mid B) = 1$.

$P(A_0 \mid B) =$ 1/2 ✔ **Answer:** 0.5

$P(A_1 \mid B) =$ 1/3 ✔ **Answer:** 0.3333

$P(A_2 \mid B) =$ 1/6 ✔ **Answer:** 0.1667

Explanation

8. We have $P(A_0 \mid B) = \frac{P(A_0 \cap B)}{P(B)} = \frac{3/96}{6/96} = 3/6 = 1/2$, and $P(A_1 \mid B) = \frac{P(A_1 \cap B)}{P(B)} = \frac{2/96}{6/96} = 2/6 = 1/3$, and $P(A_2 \mid B) = \frac{P(A_2 \cap B)}{P(B)} = \frac{1/96}{6/96} = 1/6$. Indeed, these conditional probabilities do sum to 1.

Submit You have used 1 of 1 attempt

Problem 9

3/3 points (graded)

9. According to cars.com, the percentages of cars sold are given in the second row of the following table, and a risk score for each color is given in the third row, using data from a 2007 report at Monash University by S. Newstead and A. D'Elia.

color:	white	black	silver	gray	red	blue	brown	yellow	green	other
% of cars:	23%	18%	16%	13%	10%	9%	5%	3%	2%	1%
risk rating	1	1.06	1.10	1.10	1.08	1.05	1.03	0.99	1	1%

If a car is picked at random according to the percentages above, and its risk rating is found to be higher than 1.07, what is the conditional probability that:

9a. it is a silver car?

16/39

✔ **Answer:** 0.4103

9b. it is a gray car?

13/39

✔ **Answer:** 0.3333

9c. it is a red car?

10/39

✔ **Answer:** 0.2564

Explanation

9a. Let S, G, R denote (respectively) the probability that the car is silver, gray, or red. So we have $P(S \mid S \cup G \cup R) = \frac{P(S \cap (S \cup G \cup R))}{P(S \cup G \cup R)}$. In the numerator, we have $S \cap (S \cup G \cup R)) = S$, because a car is only in S and in $S \cup G \cup R$ if it is (indeed) in S! So we get $P(S \mid S \cup G \cup R) = \frac{P(S)}{P(S \cup G \cup R)}$. The events S, G, and R are disjoint, so this yields $P(S \mid S \cup G \cup R) = \frac{P(S)}{P(S)+P(G)+P(R)} = \frac{.16}{.16+.13+.10} = 0.4103$.

9b. Similar to part 1a, we have $P(G \mid S \cup G \cup R) = \frac{P(G)}{P(S)+P(G)+P(R)} = \frac{.13}{.16+.13+.10} = 0.3333$.

9c. Similar to part 1a, we have $P(R \mid S \cup G \cup R) = \frac{P(R)}{P(S)+P(G)+P(R)} = \frac{.10}{.16+.13+.10} = 0.2564$.

Submit You have used 1 of 1 attempt

Problem 10

1/1 point (graded)

10. The genres of the songs on a student's iPod are: 15% country music, 21% pop, 24% R&B, and 40% rock. Suppose that 90% of the country music songs have a fiddle, 18% of the pop songs have a fiddle, none of the R&B songs have a fiddle, and 10% of the rock songs have a fiddle. If a song is randomly selected from the iPod, and it happens to have fiddle music in the song, what is the conditional probability that it is a country song?

0.6343985 ✔ **Answer:** 0.6344

Explanation

10. Let C be the event that it is a country song, and let F be the event that the selected song has a fiddle. So $P(C \mid F) = \frac{P(C \cap F)}{P(F)} = \frac{(.15)(.90)}{(.15)(.90)+(.21)(.18)+(.24)(0)+(.40)(.10)} = \mathbf{0.6344}$.

Submit You have used 1 of 1 attempt

Problem 11

1/1 point (graded)

11. Roll a 4-sided die and a 6-sided die. Given that the 4-sided die has a result of 1, 2, or 3 (but not 4), find the conditional probability that the sum of the two dice is 5 or larger.

2/3 ✔ **Answer:** 0.6667

Explanation

11. Let B_1, B_2, B_3 denote the events that the 4-sided die has a result of 1, 2, or 3 (resp.). Let A be the event that the sum of the dice is 5 or larger. Then $P(A \mid B_1 \cup B_2 \cup B_3) = \frac{P(A \cap (B_1 \cup B_2 \cup B_3))}{P(B_1 \cup B_2 \cup B_3)} = \frac{P(A \cap B_1) + P(A \cap B_2) + P(A \cap B_3)}{P(B_1) + P(B_2) + P(B_3)}$, where the last equality is true since the B_j's are disjoint. Also $P(A \cap B_j) = P(B_j)P(A \mid B_j)$, so we get $P(A \mid B_1 \cup B_2 \cup B_3) = \frac{P(B_1)P(A \mid B_1) + P(B_2)P(A \mid B_2) + P(B_3)P(A \mid B_3)}{P(B_1) + P(B_2) + P(B_3)} = \frac{(1/4)(3/6) + (1/4)(4/6) + (1/4)(5/6)}{1/4 + 1/4 + 1/4} = 2$

.

An alternative method is to recognize that there are 18 equally likely outcomes in which the 4-sided die has a result of 1, 2, or 3, and exactly 12 of these 18 outcomes has a sum of 5 or larger on the dice, so the desired probability is $12/18 = 2/3$.

Submit You have used 1 of 1 attempt

Problem 12

2/2 points (graded)

12. Suppose that a drawer contains 8 marbles: 2 are red, 2 are blue, 2 are green, and 2 are yellow. The marbles are rolling around in a drawer, so that all possibilities are equally likely when they are drawn. Alice chooses 2 marbles without replacement, and then Bob chooses 2 marbles. Let A denote the event that Alice's 2 marbles have a matching color. Let B denote the event that Bob's 2 marbles have a matching color.

12a. Find $P(A \mid B^c)$, i.e., given that Bob's marbles *did not have* a matching color, find the probability that Alice's marbles had a matching color.

2/15 ✔ **Answer:** 0.1333

12b. Find $P(A^c \mid B^c)$, i.e., given that Bob's marbles *did not have* have a matching color, find the probability that Alice's marbles did not have a matching color.

13/15

✔ **Answer:** 0.8667

Explanation

12a. We have

$$P(A \mid B^c) = \frac{P(A \cap B^c)}{P(B^c)} = \frac{P(A)P(B^c \mid A)}{P(A)P(B^c \mid A) + P(A^c)P(B^c \mid A^c)}$$

$$= \frac{(1/7)(4/5)}{(1/7)(4/5) + (6/7)((2/6)(1) + (4/6)(4/5))} = 2/15.$$

12b. We have

$$P(A^c \mid B^c) = \frac{P(A^c \cap B^c)}{P(B^c)} = \frac{P(A^c)P(B^c \mid A^c)}{P(A)P(B^c \mid A) + P(A^c)P(B^c \mid A^c)}$$

$$= \frac{(6/7)((2/6)(1) + (4/6)(4/5))}{(1/7)(4/5) + (6/7)((2/6)(1) + (4/6)(4/5))} = 13/15.$$

Submit You have used 1 of 1 attempt