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6.1 Summing Up

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In this section we consider situations where the starting angular velocity of the pendulum is not $\mathbf{0}$, as well as review the predictions of our model, and reflect on why pendulums are useful for timekeeping.

General Solution to the Simplified Pendulum Model

For a small angles (when $heta pprox \sin(heta)$), we used the differential equation

$$rac{d^2 heta}{dt^2} = -\sqrt{rac{g}{l}} heta.$$

We found one family of solutions to this equation:

$$heta(t) = heta_0 \cos\!\left(\sqrt{rac{g}{l}}t
ight)$$

for any value of $heta_0$. Note that at t=0, the starting angle is $heta_0$ and $frac{d heta}{dt}=0$, implying that this angle is the maximum or minimum angle of the swing (depending on if $heta_0$ is positive or negative).

Therefore, this solution only describes situations where the pendulum starts from a maximum angle θ_0 with an angular velocity of 0.

What's the complete story?

• The complete solutions to this differential equation are.

$$heta(t) = heta_0 \cos\!\left(\sqrt{rac{g}{l}}t + b
ight)$$

where $heta_0$ and heta are any constants. We can check by differentiating that $heta(t)= heta_0\cos\Bigl(\sqrt{rac{g}{l}}t+b\Bigr)$ is a solution to $rac{d^2 heta}{dt^2}=-\sqrt{rac{g}{l}} heta$. Uniqueness theorems tell us that these must be all the solutions to the differential equation.

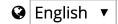
(Note that the solution $heta(t)=\sin(\sqrt{k}t)$ we found earlier corresponds to $heta_0=1$ and $b = -\pi/2$.)

These solutions are still represented in the phase plane. Recall that for the corresponding system of differential equations for θ and angular velocity $\frac{d\theta}{dt}$, the solution trajectories in the phase plane are closed loops and the length of a loop measured in time is the period of the pendulum. These solutions correspond to starting at different points on a loop (versus always starting on the horizontal axis as we did when b=0).

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