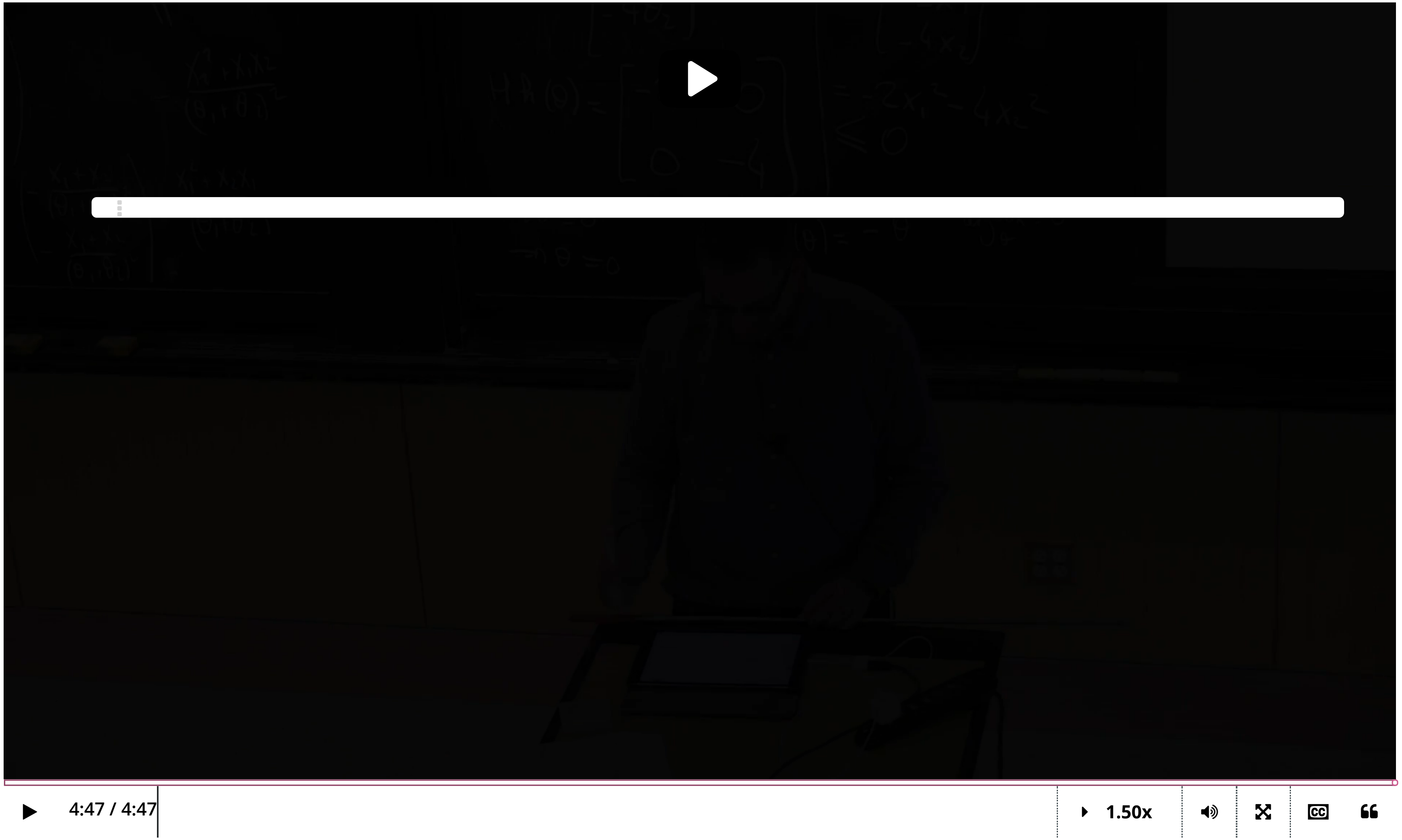
11. Strictly Concave Functions and Unique Maximizer

Strictly Concave Functions and Unique Maximizer



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Concavity and Convexity in Higher Dimensions II

2/2 points (graded)

As in the problem on the previous page, $f(x,y) = -2x^2 + \sqrt{2}xy - \frac{5}{2}y^2$. Based on your answer to the question, which of the following is true?

- ( ) f has a unique (global) minimizer.
- (•) f has a unique (global) maximizer.
- ( ) f has more than one (global) minimizer
- ( ) f has more than one (global) maximizer

✔

Where is the critical point of f? (If there is more than one critical point, just enter one of them.)

(Enter your answer as a vector, e.g., type **[3,2]** for the vector $\begin{pmatrix} 3 \\ 2 \end{pmatrix}$, which corresponds to the point $(3,2)$ on the (x,y)-plane).

Critical point of f: [0,0] ✔ **Answer:** [0,0]

STANDARD NOTATION

Solution:

Since f is twice-differentiable and strictly concave, we know there will be a unique global maximum.

The critical points of f are those points where $\nabla f = (\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}) = 0$. We compute that

$$\nabla f = (-4x + \sqrt{2}y, -5y + \sqrt{2}x),$$

which is 0 if and only if $x = y = 0$.

Submit You have used 1 of 2 attempts

Answers are displayed within the problem

Concavity Concept Check

0/1 point (graded)

Let $f : \mathbb{R}^2 \to \mathbb{R}$ be a twice-differentiable function such that the Hessian matrix $\mathbf{H}f(0,0)_{1,1} > 0$. Is f concave?

- Yes
- No ✔
- Not possible to determine from given information

✖

Solution:

Recall that f is concave at $(0,0)$ if for all $(x,y) \in \mathbb{R}^2$,

$$(x,y)\,\mathbf{H}f\,(0,0)\begin{pmatrix} x \\ y \end{pmatrix} < 0.$$

By expanding and using the definition of the Hessian, we see that

$$(x,y)\,\mathbf{H}f(0,0)\begin{pmatrix} x \\ y \end{pmatrix} = x^2\frac{\partial^2}{\partial x^2}f(0,0) + xy\left(\frac{\partial^2}{\partial x\partial y}f(0,0) + \frac{\partial^2}{\partial y\partial x}f(0,0)\right) + y^2\frac{\partial^2}{\partial y^2}f(0,0).$$

By assumption, we know that $\frac{\partial^2}{\partial x^2}f(0,0) > 0$. Hence,

$$(1,0)^T\mathbf{H}f(0,0)\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \frac{\partial^2}{\partial x^2}f(0,0) > 0.$$

This violates the definition of concavity, so the correct response is "No."

Submit You have used 1 of 1 attempt

Answers are displayed within the problem

Note: The following problem will be presented in lecture, but we encourage you to attempt it first.

Intuition for Optimizing Concave Functions

1/1 point (graded)

Let $f : \mathbb{R} \to \mathbb{R}$ be a twice-differentiable function that has a critical point and is strictly concave. Recall that the critical point is a **unique maximizer** of f.

You choose an initial guess for the maximizer $x_0 = 0$ (which may be very far from the true maximizer). You compute the derivative and observe that $f'(x_0) < 0$. Where is the maximizer of f?

- [x] To the left of $x_0 = 0$

- To the right of $x_0 = 0$
- Very far from x_0
- Very close to x_0

✔

Solution:

Graphically, a strictly concave function that has a critical point looks like a hill. If you are to the right of the peak (*i.e.*, the maximum), then the hill is sloping downward. If you are to the left of the peak, then the hill is sloping upward.

More formally, a differentiable function that is strictly concave has a strictly decreasing derivative. The first derivative is zero at the critical point, so it must be positive to the left of the maximum (which implies the function is increasing) and negative to the right of the maximum (which implies the function is decreasing).

Thus the first choice, "To the left of $x_0 = 0$", is correct.

Submit You have used 1 of 1 attempt

Answers are displayed within the problem

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