5. Review constrained optimization intuition

Reflect

Visualization of constrained optimization

the white curve

and the blue curve are tangent here.

If the blue curve was going across the white curve,

like over here, that wouldn't be the maximum.

Because we could go to bigger x.

But at the maximum, they should be tangent to each other.

And because they're tangent to each other,

they have the same tangent direction.

And they have the same normal direction.

So this exercise, it was partly about keeping f and g straight

and visualizing the gradient of a function

and partly about thinking why this thing is true.

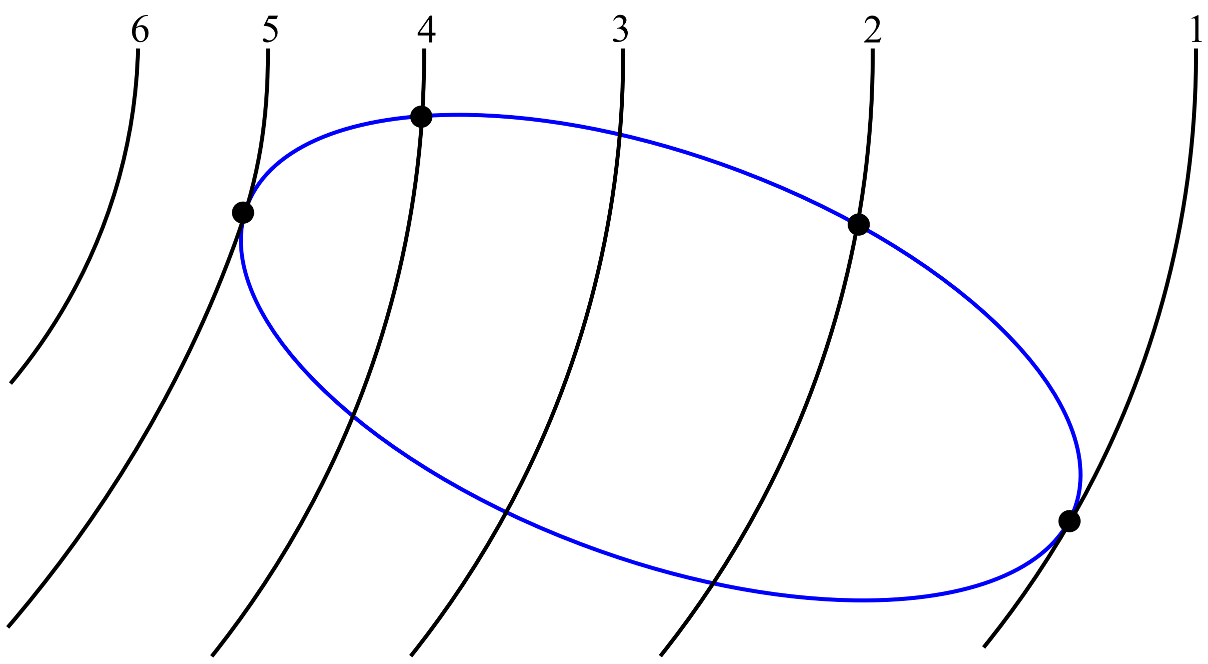
2:01 / 2:01 2.0x

Video
Download video file

Transcripts
Download SubRip (.srt) file
Download Text (.txt) file

Let C be an ellipse. Suppose that we can describe C as the level curve of height 2 of some function $g(x,y)$. Inside C, we know that $g(x,y) < 2$.

The level curves of a function $f(x,y)$ of heights 6, 5, 4, 3, 2, and 1 are depicted in the image below.



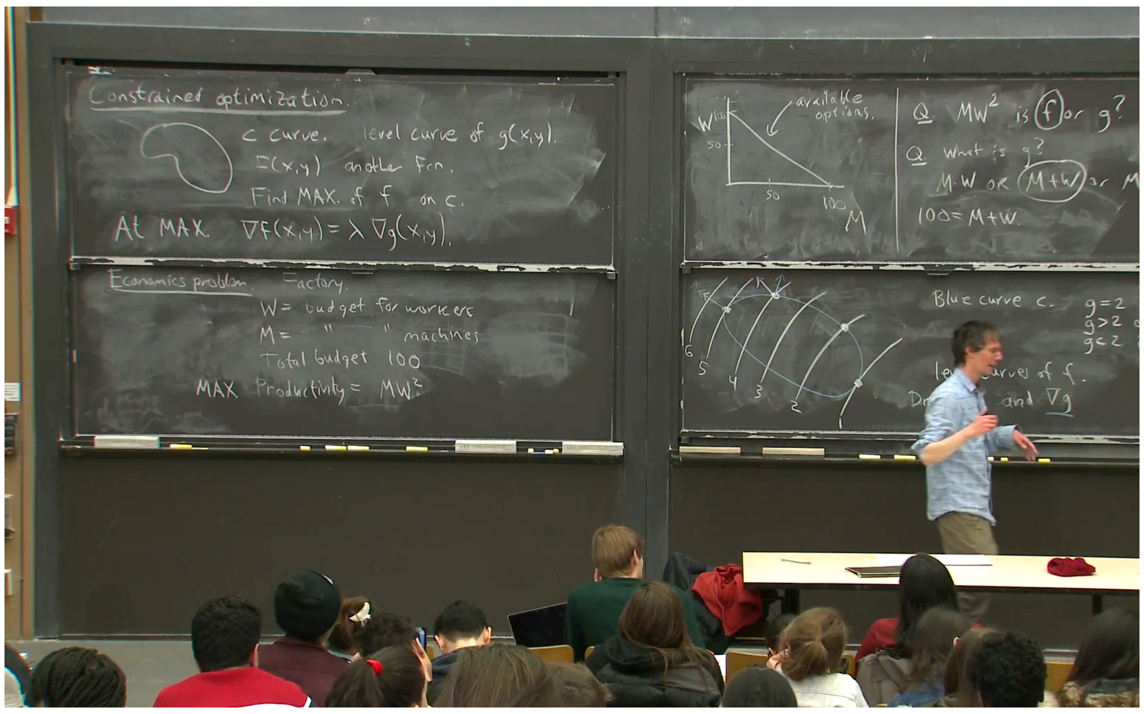
On your own paper, make a sketch of the level curves of f and the curve g. Sketch the gradient of f in black and the gradient of g in blue. Then compare your answer with Prof. Guth's solution, which is presented in the next video.

Ask yourself

- Which direction does ∇f point, and how do you know it isn't the opposite direction?

- Which direction does ∇g point, and how do you know it isn't the opposite direction?

Solution



1:59 / 2:37 2.0x

If the blue curve was going across the white curve,

like over here, that wouldn't be the maximum.

Because we could go to bigger x.

But at the maximum, they should be tangent to each other.

And because they're tangent to each other,

they have the same tangent direction.

And they have the same normal direction.

So this exercise, it was partly about keeping f and g straight

and visualizing the gradient of a function

and partly about thinking why this thing is true.

I want to say I would really love

for you both to be able to solve optimization problems by using

Video

Download video file

Transcripts

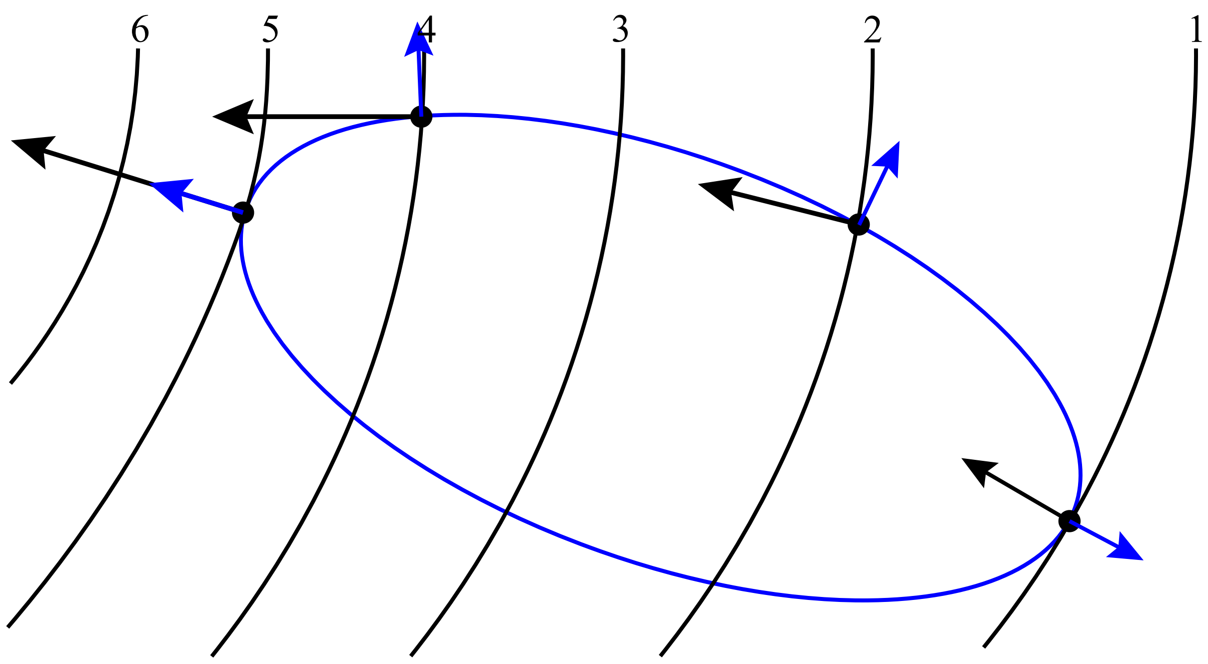
Download SubRip (.srt) file

Download Text (.txt) file

Solution

The gradient is normal to level curves, and points in the direction the function is increasing.

Therefore the gradient vectors of $\nabla f(x, y)$ are pointing normal to each level curve and to the left, in the direction that the heights are increasing. The height of the level curve defining the ellipse is 2, and $g(x, y) > 2$ outside of the ellipse. Therefore the gradient vectors $\nabla g(x, y)$ are always normal to the ellipse, and pointing outwards.



Hide

5. Review constrained optimization intuition

Hide Discussion

Topic: Review / 5. Review constrained optimization intuition

Calculator Hide Notes

Show all posts by recent activity

There are no posts in this topic yet.

✖

Previous Next

edX

About
Affiliates
edX for Business
Open edX
Careers
News

Legal

Terms of Service & Honor Code
Privacy Policy
Accessibility Policy
Trademark Policy
Sitemap

Connect

Blog
Contact Us
Help Center
Media Kit
Donate

GET IT ON Google play
Download on the App Store

Calculator Hide Notes