MITx: 6.041x Introduction to Probability - The Science of Uncertainty

Exercise: Covariance properties

(3/3 points)

a) Is it true that $\textbf{cov}(X,Y)=\textbf{cov}(Y,X)$?

True ✔ **Answer:** True

b) Find the value of a in the relation $\textbf{cov}(2X,-3Y+2)=a\cdot\textbf{cov}(X,Y)$.

$a=$ -6 ✔ **Answer:** -6

c) Suppose that X, Y, and Z are independent, with a common variance of 5. Then,

$\textbf{cov}(2X+Y,3X-4Z)=$ 30 ✔ **Answer:** 30

Answer:

a) We have $\big(X-\mathbf{E}[X]\big)\big(Y-\mathbf{E}[Y]\big)=\big(Y-\mathbf{E}[Y]\big)\big(X-\mathbf{E}[X]\big)$, and after taking expectations we obtain $\textbf{cov}(X,Y)=\textbf{cov}(Y,X)$.

b) We have argued that $\textbf{cov}(aX+b,Y)=a\cdot\textbf{cov}(X,Y)$. Note that by symmetry, we also have $\textbf{cov}(X,aY+b)=a\cdot\textbf{cov}(X,Y)$. By using these relations,

$$\textbf{cov}(2X,-3Y+2)=2\cdot\textbf{cov}(X,-3Y+2)=2\cdot(-3)\cdot\textbf{cov}(X,Y)=-6\,\textbf{cov}(X,Y).$$

c) Using linearity,

$$\begin{aligned}\textbf{cov}(2X+Y,3X-4Z)&=\textbf{cov}(2X+Y,3X)+\textbf{cov}(2X+Y,-4Z)\\&=\textbf{cov}(2X,3X)+\textbf{cov}(Y,3X)+\textbf{cov}(2X,-4Z)+\textbf{cov}(Y,-4\\&=6\,\textbf{var}(X)+0+0+0=30,\end{aligned}$$

where the zeros are obtained because independent random variables have zero covariance.

You have used 1 of 2 submissions