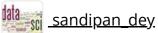


<u>lp</u>



<u>Unit 5: Averages, Law of Large</u>

5.5 Law of the unconscious

> 5.1 Reading > statistician (LOTUS)

Numbers, and Central Limit

Course > Theorem

5.5 Law of the unconscious statistician (LOTUS) Unit 5: Averages

Adapted from Blitzstein-Hwang Chapters 4, 5, and 10.

As we saw in the St. Petersburg paradox, E(g(X)) does *not* equal g(E(X)) in general if g is not linear. So how do we correctly calculate E(g(X))? Since g(X) is an $\underline{r.v.}$, one way is to first find the distribution of g(X) and then use the definition of expectation. Perhaps surprisingly, it turns out that it is possible to find E(g(X)) directly using the distribution of X, without first having to find the distribution of g(X). This is done using the *law of the unconscious statistician* (LOTUS).

THEOREM 5.5.1 (LOTUS).

If X is a discrete r.v. and g is a function from $\mathbb R$ to $\mathbb R$, then

$$E(g(X)) = \sum_x g(x) P(X=x),$$

where the sum is taken over all possible values of \boldsymbol{X} .

This means that we can get the expected value of g(X) knowing only P(X=x), the <u>PMF</u> of X; we don't need to know the PMF of g(X). The name comes from the fact that in going from E(X) to E(g(X)) it is tempting just to change x to g(x) in the definition, which can be done very easily and mechanically, perhaps in a state of unconsciousness. On second thought, it may sound too good to be true that finding the distribution of g(X) is not needed for this calculation, but LOTUS says it is true. We will omit a general proof of LOTUS, but let's see why it is true in some special cases. Let X have support $0,1,2,\ldots$ with probabilities p_0,p_1,p_2,\ldots , so the PMF is $P(X=n)=p_n$. Then X^3 has support $0^3,1^3,2^3,\ldots$ with probabilities p_0,p_1,p_2,\ldots , so

$$E(X) = \sum_{n=0}^{\infty} n p_n,$$

$$\vec{E}(\vec{X}^2) = \sum_{n=0}^{\infty} n^2 p_n.$$

As claimed by LOTUS, to edit the expression for E(X) into an expression for $E(X^3)$, we can just change the n in front of the p_n to an n^3 ; the p_n is unchanged, and we can still use the PMF of $\stackrel{.}{X}$.

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