

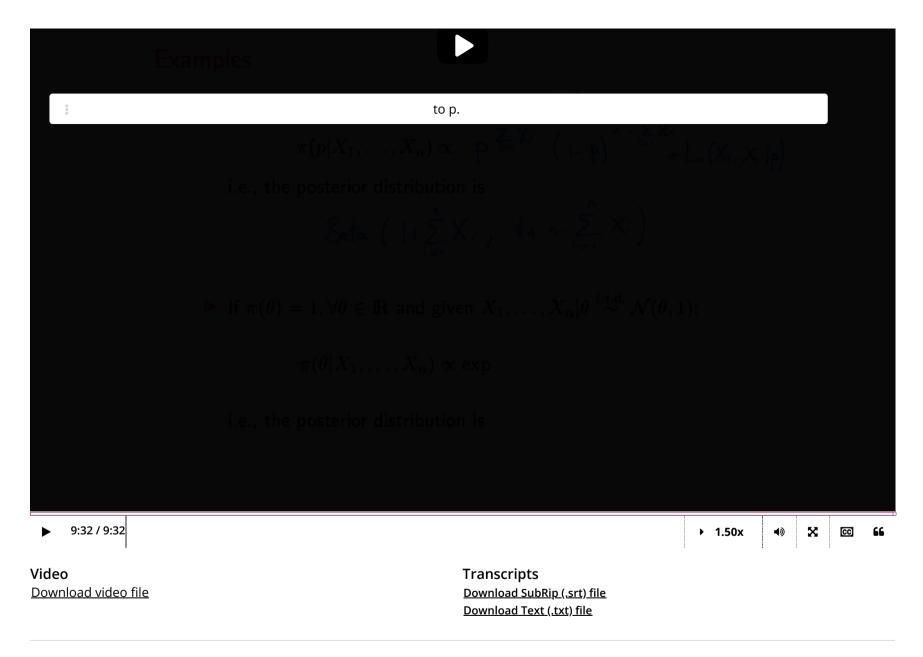


Lecture 18: Jeffreys Prior and

<u>Course</u> > <u>Unit 5 Bayesian statistics</u> > <u>Bayesian Confidence Interval</u>

> 3. Choosing a Prior

# 3. Choosing a Prior Choosing a Prior



### Uniform Priors: True or False

1/1 point (graded)

Select from the following statements the **true** ones for uniform priors. (In this question, we also allow *improper* priors.)

- They can be defined only on parameter sets  $\Theta$  with a finite number of possible values.
- They should integrate to 1 (or if the distribution is discrete; should sum to 1)
- ✔ They reflect an equal belief in each possible hypothesis.
- The maximum a-posteriori and maximum likelihood estimators when using such a prior would always be the same.



#### Solution:

- The first choice is false. As discussed in the lecture, they can be defined on infinite sets or even non-discrete distributions with an uncountably infinite number of possible parameter values.
- The second choice is also false. If  $\pi(\cdot)$  is improper, then it will definitely not integrate to 1 by definition.
- The third choice is correct. A uniform prior reflects an "equal" belief in each of the possible hypothesis.
- The last choice is also correct. Recall that the maximum-a-posterior estimator maximizes  $\pi(\theta|X_1,\ldots,X_n)$  while the maximum likelihood estimator maximizes  $L_n(X_1,\ldots,X_n|\theta)$ , both taken as functions of  $\theta$ . By Bayes' rule, we have that

$$\pi(\theta|X_1,\ldots,X_n) \propto L_n(X_1,\ldots,X_n|\theta) \pi(\theta) \propto L_n(X_1,\ldots,X_n|\theta),$$

where the first proportionality is due to Bayes' rule and the second proportionality is due to  $\pi(\cdot)$  being uniform. The two statistics, when taken as a function of  $\theta$ , are therefore identical up to a constant of proportionality. Hence, while the maximum values might be different, the value of  $\theta$  attaining the maximum for both quantities are the same.

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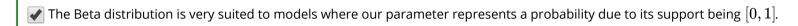
You have used 2 of 3 attempts

**1** Answers are displayed within the problem

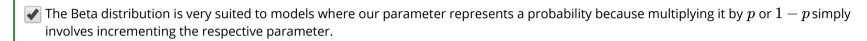


1/1 point (graded)

One specific prior discussed in the previous lecture is the Beta distribution, which was then demonstrated in a scenario with a Bernoulli statistical model. Which of the follwing statements is/are true about the Beta distribution, written as Beta  $(\alpha, \beta) \propto p^{\alpha-1} (1-p)^{\beta-1}$ ?



The Beta distribution is very suited to models where our parameter represents a probability due to its maximum always being close to  $\frac{1}{2}$ .





#### **Solution:**

The first and third statements are correct.

- The first statement is correct. The Beta distribution indeed has support on the interval [0,1], which is also the range of possible probabilities. Thus, using the Beta distribution to model possible parameters p would allow us to exactly cover the feasible range.
- The second statement is incorrect. The Beta family of distributions is very flexible and does not constrain us to symmetric shapes (which happens if we instead use a Gaussian prior). Indeed, if you recall the calculation of modes from the previous lecture, the mode can be at 0 or 1 for certain special cases. In the general case  $\alpha>1$ ,  $\beta>1$ , the mode is at  $\frac{\alpha-1}{\alpha+\beta-2}$ , which could range anywhere in (0,1) depending on  $\alpha$  and  $\beta$ .
- The third statement is correct. Mathematically, it is easy to see that multiplying the PDF of a Beta distribution by p or 1-p increments the  $\alpha$  or  $\beta$  coefficient, respectively, by 1. In practical terms, it is very common in statistical applications, as you've seen in the previous lecture, to multiply the likelihood function by either p or 1-p depending on the outcome of a binary trial.

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You have used 1 of 2 attempts

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## Discussion

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