9.5.2 Linear Independence

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Week 9 due Dec 9, 2023 18:12 IST Completed

9.5.2 Linear Independence

Video

Start of transcript. Skip to the end.

Dr. Robert van de Geijn: We're after describing subspaces

that have an infinite number of vectors in them

with a finite number of vectors.

And we would like to have as few vectors as possible

0:00 / 0:00 2.0x

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Reading Assignment

0 points possible (ungraded)

Read Unit 9.5.2 of the notes. [LINK]

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? The column space of A and linearly independency of A's columns 2
Hi, In the last of this unit Dr. Robert van de Geijn said these two are equivalent: 1- The column space of A is R^n 2- A has linearly independent co...

? Question on 9.5.2 Linear Independence
Greetings! I do not quite understand that a matrix, which has more columns than rows, indicates we have vectors in the null space. It is mentic

Calculator

Homework 9.5.2.1

1/1 point (graded)

$$\text{Span}\left(\begin{pmatrix}1\\0\\1\end{pmatrix},\begin{pmatrix}0\\0\\1\end{pmatrix}\right)=\text{Span}\left(\begin{pmatrix}1\\0\\1\end{pmatrix},\begin{pmatrix}0\\0\\1\end{pmatrix},\begin{pmatrix}1\\0\\3\end{pmatrix}\right)$$

TRUE ✔ Answer: TRUE

- $S \subset T$: Let $x \in \text{Span}\left(\begin{pmatrix}1\\0\\1\end{pmatrix},\begin{pmatrix}0\\0\\1\end{pmatrix}\right)$ Then there exist α_0 and α_1 such that $x=\alpha_0\begin{pmatrix}1\\0\\1\end{pmatrix}+\alpha_1\begin{pmatrix}0\\0\\1\end{pmatrix}$. This in turn means that
$$x=\alpha_0\begin{pmatrix}1\\0\\1\end{pmatrix}+\alpha_1\begin{pmatrix}0\\0\\1\end{pmatrix}+(0)\begin{pmatrix}1\\0\\3\end{pmatrix}.$$
Hence $x \in \text{Span}\left(\begin{pmatrix}1\\0\\1\end{pmatrix},\begin{pmatrix}0\\0\\1\end{pmatrix},\begin{pmatrix}1\\0\\3\end{pmatrix}\right)$.
- $T \subset S$: Let $x \in \text{Span}\left(\begin{pmatrix}1\\0\\1\end{pmatrix},\begin{pmatrix}0\\0\\1\end{pmatrix},\begin{pmatrix}1\\0\\3\end{pmatrix}\right)$ Then there exist α_0, α_1, and α_2 such that $x=\alpha_0\begin{pmatrix}1\\0\\1\end{pmatrix}+\alpha_1\begin{pmatrix}0\\0\\1\end{pmatrix}+\alpha_2\begin{pmatrix}1\\0\\3\end{pmatrix}$. But $\begin{pmatrix}1\\0\\3\end{pmatrix}=\begin{pmatrix}1\\0\\1\end{pmatrix}+2\begin{pmatrix}0\\0\\1\end{pmatrix}$. Hence
$$x=\alpha_0\begin{pmatrix}1\\0\\1\end{pmatrix}+\alpha_1\begin{pmatrix}0\\0\\1\end{pmatrix}+\alpha_2\left(\begin{pmatrix}1\\0\\1\end{pmatrix}+2\begin{pmatrix}0\\0\\1\end{pmatrix}\right)=(\alpha_0+\alpha_2)\begin{pmatrix}1\\0\\1\end{pmatrix}+(\alpha_1+2\alpha_2)\begin{pmatrix}0\\0\\1\end{pmatrix}.$$
Therefore $x \in \text{Span}\left(\begin{pmatrix}1\\0\\1\end{pmatrix},\begin{pmatrix}0\\0\\1\end{pmatrix}\right)$.

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Homework 9.5.2.2

1/1 point (graded)

Let the set of vectors $\{v_0, v_1, \ldots, v_{n-1}\} \subset \mathbb{R}^m$ be linearly dependent. Then at least one of these vectors can be written as a linear combination of the others.

TRUE ✔ Answer: TRUE

Since the vectors are linearly dependent, then there must exist $\chi_0, \chi_1, \ldots, \chi_{n-1} \in \mathbb{R}$ such that $\chi_0 a_0 + \chi_1 a_1 + \cdots + \chi_{n-1} a_{n-1} = 0$ and for at least one i, $0 \le i < n$, $\chi_i \neq 0$. But then

Calculator

$\chi_0 a_0 + \chi_1 a_1 + \cdots + \chi_{n-1} a_{n-1} = 0$ and for at least one j, $0 \leq j < n$, $\chi_j \neq 0$. But then

$$\chi_j a_j = -\chi_0 a_0 + -\chi_1 a_1 - \cdots - \chi_{j-1} a_{j-1} - \chi_{j+1} a_{j+1} - \cdots - \chi_{n-1} a_{n-1}$$

and therefore

$$a_j = -\frac{\chi_0}{\chi_j} a_0 + -\frac{\chi_1}{\chi_j} a_1 - \cdots - \frac{\chi_{j-1}}{\chi_j} a_{j-1} - \frac{\chi_{j+1}}{\chi_j} a_{j+1} - \cdots - \frac{\chi_{n-1}}{\chi_j} a_{n-1}.$$

In other words, a_j can be written as a linear combination of the other $n-1$ vectors.

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Homework 9.5.2.3

1/1 point (graded)

Let $U \in \mathbb{R}^{n \times n}$ be an upper triangular matrix with nonzeroes on its diagonal. Then its columns are linearly independent.

TRUE ✔ Answer: TRUE

We saw in a previous week that $Ux = b$ has a unique solution if U is upper triangular with nonzeroes on its diagonal. Hence $Ux = 0$ has the unique solution $x = 0$ (the zero vector). This implies that U has linearly independent columns.

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Homework 9.5.2.4

1/1 point (graded)

Let $L \in \mathbb{R}^{n \times n}$ be a lower triangular matrix with nonzeroes on its diagonal. Then its rows are linearly independent. (Hint: How do the rows of L relate to the columns of L^T?)

ALWAYS ✔ Answer: ALWAYS

We saw in a previous week that $Lx = b$ has a unique solution if L is lower triangular with nonzeroes on its diagonal. Hence $Lx = 0$ has the unique solution $x = 0$ (the zero vector). This implies that L has linearly independent columns.

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