MITx: 6.008.1x Computational Probability and Inference

Hel

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Exercise: Big O Notation

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Exercise: Big O Notation

3/3 points (graded)

As a reminder, big O notation is given as follows:

Big O notation: We say that a function f that depends on a variable n is $\mathcal{O}(g(n))$ (read as "big O of g of n") if there exists some minimum n_0 such that for all $n \geq n_0$,

$$f(n) \leq c \cdot g(n)$$

for some constant $c > 0$.

For each of the following, decide whether the statement is true or false. Note that while we won't ask you to input a justification, you should be able to justify each of your answers mathematically.

- $3n^2 + 6n = \mathcal{O}(n^3)$.

● True ✓

○ False

Exercises due Oct 27, 2016 at 02:30 IST

Week 6: Special Case: Marginalization in Hidden Markov Models
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Week 6: Homework 5
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- $3n^2 + 6n = \mathcal{O}(n^2)$.

● True ✓

○ False

- $2^n = \mathcal{O}(n^2)$.

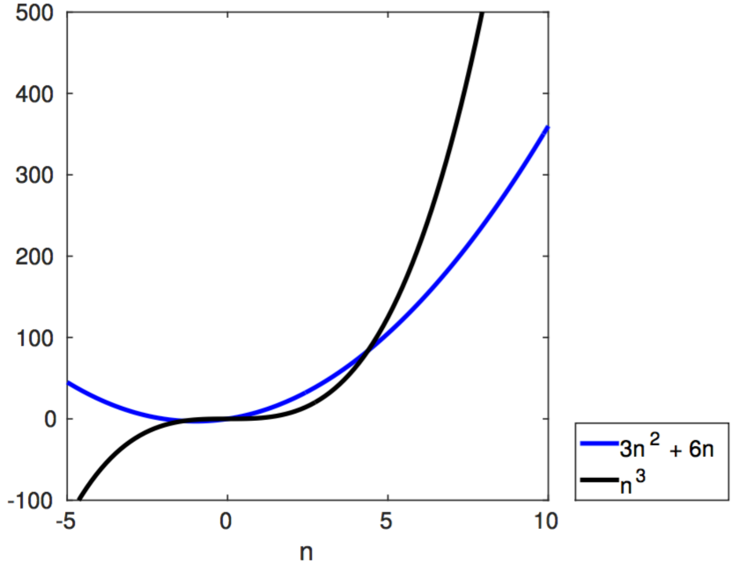
○ True

● False ✓

Solution:

- $3n^2 + 6n = \mathcal{O}(n^3)$.

  Solution: $\boxed{\text{True.}}$ Our intution is that this is true, since n^3 grows much faster than n^2. For example, we can choose $c = 1$ and plot the two functions:



We can see that indeed, $n^3 \geq 3n^2 + 6n$ for large n. Let's find the three intersection points of these two functions. Set:

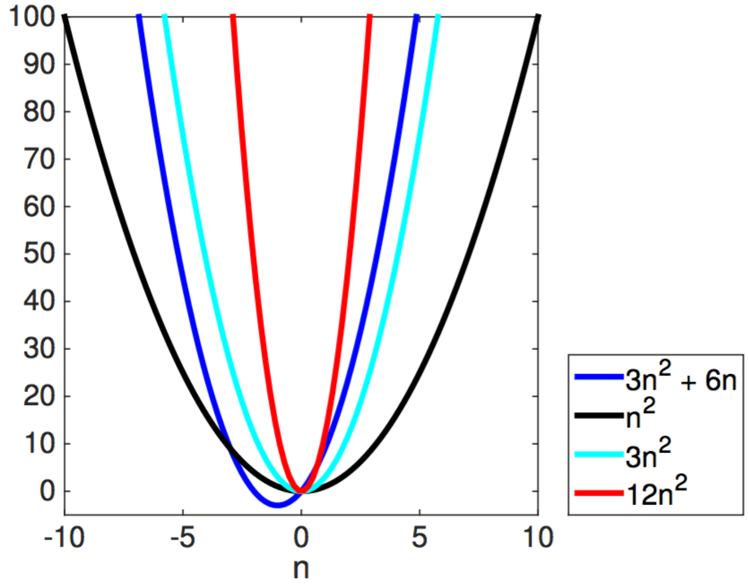
$$\begin{aligned} 3n^2 + 6n &= n^3 \\ 0 &= n^3 - 3n^2 - 6n \\ 0 &= n(n^2 - 3n - 6) \end{aligned}$$

There are three intersection points: $n = 0$, and (using the quadratic formula) $n = \frac{3 \pm \sqrt{33}}{2}$. We are interested in the largest of these, namely $n_0 = \frac{3 + \sqrt{33}}{2} \approx 4.37$.

Therefore, $3n^2 + 6n = \mathcal{O}(n^3)$ because for all $n \geq \frac{3 \pm \sqrt{33}}{2}$, $3n^2 + 6n \leq n^3$.

- $3n^2 + 6n = \mathcal{O}(n^2)$.

Solution: $\boxed{\text{True.}}$ Our intuition is that this is true because both $3n^2 + 6n$ and n^2 are polynomials of the same order. However, we cannot choose $c = 1$ this time because $3n^2 + 6n > n^2$ always. Let's plot the functions for increasing values of c:



We see that cn^2 dominates $3n^2 + 6n$ for large n as long as c is large enough. Let's choose $c = 12$. To find n_o, let's find the intersection points between $3n^2 + 6n$ and $12n^2$:

$$\begin{aligned} 3n^2 + 6n &= 12n^2 \\ 0 &= 9n^2 - 6n \\ 0 &= 3n(3n - 2) \end{aligned}$$

There are two intersection points: $n = 0$ and $n = \frac{2}{3}$. We see that for $c = 12$, we can choose $n_0 = \frac{2}{3}$. Therefore, $3n^2 + 6n = \mathcal{O}(n^2)$ because for all $n \geq \frac{2}{3}$, $3n^2 + 6n \leq 12n^2$.

- $2^n = \mathcal{O}(n^2)$.

  Solution: $\boxed{\text{False.}}$ Our intuition is that this is false: an exponential function like 2^n grows much more quickly than a polynomial function like n^2.

  To show that $2^n \neq \mathcal{O}(n^2)$, i.e., that there is no choice of n_0 and c such that for all $n \geq n_0$, $\frac{2^n}{n^2} \leq c$, suppose for contradiction that there were such a n_0 and constant c. Then this would imply that

  $$\lim_{n \to \infty} \frac{2^n}{n^2} \leq c,$$

  but this is a contradiction since

  $$\lim_{n \to \infty} \frac{2^n}{n^2} \overset{\text{l'Hopital's}}{=} \lim_{n \to \infty} \frac{(\ln 2) 2^n}{2n} \overset{\text{l'Hopital's}}{=} \lim_{n \to \infty} \frac{(\ln 2)^2 2^n}{2} = \infty.$$

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You have used 1 of 5 attempts

✔ Correct (3/3 points)