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## Exercise: Markov property

(2/3 points)

For each of the following definitions of the state  $X_n$  at time  $n$  ( $n = 1, 2, \dots$ ), determine whether the Markov property is satisfied.

1.  $X_n$  is a sequence of independent discrete random variables.

Yes ▼



Answer: Yes


2. You have  $m$  distinct boxes, numbered **1** through  $m$ , each containing some tokens. On each token is written an integer from **1** to  $m$ . Each box contains at least one token, but different boxes may contain different numbers of tokens. A box may also contain multiple tokens with the same number. Assume that you know the distribution of tokens in each box.

At time 0, you pick one box at random, say box  $i$ . You pick one of the tokens in box  $i$  randomly (each token in the box is equally likely to be chosen), read the corresponding number (say  $j$ ), and put the token back in box  $i$ . At the next time slot, you pick one of

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Unit overview

### Lec. 24: Finite-state Markov chains

Exercises 24 due May 18, 2016 at 23:59 UTC 

### Lec. 25: Steady-state behavior of Markov chains

the tokens in box  $j$  randomly (each token in the box is equally likely to be chosen) and repeat this process forever. At time  $n$ , you will be choosing tokens from some box. Let  $X_n$  be the number of this box.

No ▼

✗ Answer: Yes

3. Alice and Bob take turns tossing a fair coin. Assume that tosses are independent. Whenever the result is Heads, Alice gives **1** dollar to Bob, and whenever it is Tails, Bob gives **1** dollar to Alice. Alice starts with  **$A$**  dollars and Bob starts with  **$B$**  dollars, for some positive integers  **$A$**  and  **$B$** . They keep playing until one player goes broke. Let  $X_n$  be the amount of money that Alice has after the  $n$ th toss.

Yes ▼

✓ Answer: Yes


Answer:

1. Yes. Trivially, the independence assumption ensures that we have


$$\mathbf{P}(X_{n+1} = j \mid X_n = i, X_{n-1} = i_{n-1}, \dots, X_0 = i_0) = \mathbf{P}(X_{n+1} = j).$$

In this case, not only does the past not matter, but the present ( $X_n$ ) also has no influence.

- 2.


Exercises 25 due May 18, 2016  
at 23:59 UTC 

**Lec. 26: Absorption  
probabilities and  
expected time to  
absorption**

Exercises 26 due May 18, 2016  
at 23:59 UTC 

**Solved problems**

**Problem Set 10**

Problem Set 10 due May 18,  
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Yes. The set of states is  $\{1, 2, \dots, m\}$ . Given that  $X_n = i$ , the probabilities of what the next box will be at time  $n + 1$  depend only on the tokens in box  $i$  and not on the past values of  $X_0, \dots, X_{n-1}$  (i.e., the past boxes), and so the Markov property is satisfied.

3. Yes. The set of states is  $\{0, 1, 2, 3, \dots, A + B\}$ . If  $X_n = 0$  or  $X_n = A + B$ , the game stops (i.e., we stay in state  $0$  or  $A + B$ , respectively). If  $1 \leq X_n \leq A + B - 1$ , then the value of  $X_n$  is the only knowledge needed in order to describe the probabilities of what  $X_{n+1}$  will be next: it will be  $X_n + 1$  with probability  $1/2$  and  $X_n - 1$  with probability  $1/2$ . Hence, the Markov property is satisfied.

*You have used 1 of 1 submissions*

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