Module 5 Assessment - Part 1

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Confidence interval interpretation

1.0/1.0 point (graded)

Assuming normally distributed observables, and a nonlinear estimator $\underline{\hat{x}} = G(\underline{y})$ with variance $\sigma^2_{\hat{x}}$, is the following statement true or false?

For all kinds of function G, the 95 percent confidence interval is $[\hat{x} - 1.96\sigma_{\hat{x}}, \hat{x} + 1.96\sigma_{\hat{x}}]$

- True
- False ✔

Answer

Correct:

If G is nonlinear, the estimator is not normally distributed, and so the statement is not correct.

Submit You have used 1 of 1 attempt

Graded Assignment due Feb 8, 2017 17:30 IST

Q&A Forum

Feedback

- 6. Does the estimate make sense?
- Pre-knowledge Mathematics
- MATLAB Learning Content

Precision of average

1.0/1.0 point (graded)

An unknown parameter $\boldsymbol{x}$ is estimated based by taking the average of $\boldsymbol{m}$ observations: $\underline{\hat{x}} = \frac{1}{m}\sum_i^m \underline{y}_i$ where $\underline{y}_i$ are independent and $\underline{y}_i \sim \mathrm{N}(0, \sigma_y^2)$.

What will be the standard deviation of the estimator $\underline{\hat{x}}$?

- ( ) $\frac{1}{m}\sigma_y^2$
- (•) $\frac{1}{\sqrt{m}}\sigma_y$ ✔
- ( ) $\frac{1}{m}\sigma_y$

Feedback

$$\underline{\hat{x}} = \begin{bmatrix}\frac{1}{m} & \frac{1}{m} & \dots & \frac{1}{m}\end{bmatrix}\begin{bmatrix}\underline{y}_1 \\ \underline{y}_2 \\ \vdots \\ \underline{y}_m\end{bmatrix}, \quad \sigma_{\hat{x}}^2 = \begin{bmatrix}\frac{1}{m} & \frac{1}{m} & \dots & \frac{1}{m}\end{bmatrix}(\sigma_y^2 I_m)\begin{bmatrix}\frac{1}{m} \\ \frac{1}{m} \\ \vdots \\ \frac{1}{m}\end{bmatrix} = \frac{\sigma_y^2}{m}, \quad \sigma_{\hat{x}} = \frac{\sigma_y}{\sqrt{m}}$$

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precesion for room-length estimate

0/3 points (graded)

The width of a lecture room has been measured with steps. The result of each measurement l_i is about 9 steps. The mean distance of a step $\underline{x}$ is 1 m, and the standard deviation $\sigma_x = 0.2$ m. The observations are assumed to be normally distributed. We calculate the final estmate by averaging all the measurments of the room width.

How many times should the measurement be done to reach a precision (standard deviation) of one micrometre (10^{-6} m)

- ( ) $3.6\ .\ 10^{11}$ times ✔
- (•) $3.6\ .\ 10^{12}$ times ✖
- ( ) $3.6\ .\ 10^{13}$ times

Feedback

Each measuemnet of the room-wdith is the sum of the 9 steps ($\underline{l}_i = \sum_1^9 \underline{x}_i$), and so, based on error propagation, we get $\sigma_{l_i} = \sqrt{9}\sigma_x$=0.6 m. The estimator of width is obtained by taking the everage over all N measurments l_i. Hence, the standard deviation of the room-width estimator $\underline{\hat{l}}$ is: $\sigma_{\hat{l}} = \frac{\sigma_{l_i}}{\sqrt{N}}$ m. We want $\sigma_{\hat{l}}$ to be 10^{-6} m, so $10^{-6} = \frac{0.6}{\sqrt{N}}$, and so $N = (0.6/10^{-6})^2 = 3.6\ .\ 10^{11}$.

Submit You have used 1 of 1 attempt

✖ Incorrect (0/3 points)

BLUE confidence interval

1/1 point (graded)

An object is moving along a straight line with constant but unknown speed v. It started at the origin $y = 0$ at $t = 0$. Uncorrelated observations y_i of the object's distance from the origin $y = 0$ have been made at corresponding time instants $t_i = i$ seconds ($i = 1, 2, \ldots, m$). The precision of the observations is given by $\sigma_{y_i} = i\sigma_0$.

What is the precision of BLUE of v ?

- $\sigma_{\hat{v}} = \frac{\sigma_0^2}{m}$
- $\sigma_{\hat{v}} = \frac{\sigma_0}{m^2}$
- $\sigma_{\hat{v}} = \frac{\sigma_0}{\sqrt{m}} \sum_i^m \frac{y_i}{i^2}$
- (selected) $\sigma_{\hat{v}} = \frac{\sigma_0}{\sqrt{m}}$ ✔

Feedback

$$\sigma_{\hat{v}}^2 = (A^T Q_{yy}^{-1} A)^{-1} = \sigma_0^2 \left([1\ 2 \ldots\ m]^T \begin{bmatrix} 1 & & & \\ & 4 & & \\ & & \ddots & \\ & & & m^2 \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 2 \\ \vdots \\ m \end{bmatrix} \right)^{-1} = \sigma_0^2 \left(\sum_i^m 1 \right)^{-1} = \frac{\sigma_0^2}{m}.$$

So $\sigma_{\hat{v}} = \frac{\sigma_0}{\sqrt{m}}$

Submit You have used 1 of 1 attempt

Correct (1/1 point)

BLUE confidence interval (cont.)

2/2 points (graded)

In the previous question, assume that $\sigma_0 = 0.1$ m, and again $\sigma_{y_i} = i\sigma_0$.

How many measurements m are required if we want the 95% confidence interval to be $[\hat{v} - 2, \quad \hat{v} + 2]$ cm/s ?

The answer should be an integer numbert.

96

Answer: 96

96

Feedback

$$1.96\frac{\sigma_0}{\sqrt{m}} = 1.96\frac{10\text{cm}}{\sqrt{m}} = 2, \quad m \approx 96$$

Submit You have used 2 of 2 attempts

✔ Correct (2/2 points)

Confidence interval for average

1/1 point (graded)

Assume we have 10 observations of an unknown distance x. To estimate the distance, we take the average of the 10 observations. The results is $\hat{x} = 2$m. If the observations are all from a normal distribution with precesion of 5cm, then what is the 95% confidence interval

The 95% confidence interval is 200 $\pm$? cm. Insert your answer in cm upto 2 decimal places.

3.10 ✔ **Answer:** 3.1

3.10

Submit You have used 2 of 2 attempts

✔ Correct (1/1 point)

Expectation BLUE residuals (mean propagation)

2/2 points (graded)

Assume a linear model of $\mathrm{E}\{\underline{y}\} = Ax$ and $\mathrm{D}\underline{y} = Q_{yy}$. If $\underline{\hat{x}}$ is the BLU estimator, and $\underline{\hat{e}} = \underline{y} - A\underline{\hat{x}}$ is the BLU estimator of residuals, then what will be the expectation of residuals vector $\mathrm{E}\{\underline{e}\}$?

- ( ) $\mathrm{E}\{\underline{e}\} = A(x - \hat{x})$
- (•) $\mathrm{E}\{\underline{e}\} = 0$ ✔
- ( ) $\mathrm{E}\{\underline{e}\} = x - \hat{x}$

Feedback

$$\mathrm{E}\{\underline{e}\} = \left((I - A(A^T Q_{yy}^{-1} A)^{-1} A^T Q_{yy}^{-1}\right)\mathrm{E}\{\underline{y}\} =$$
$$\left(I - A(A^T Q_{yy}^{-1} A)^{-1} A^T Q_{yy}^{-1}\right) Ax = Ax - Ax = 0$$

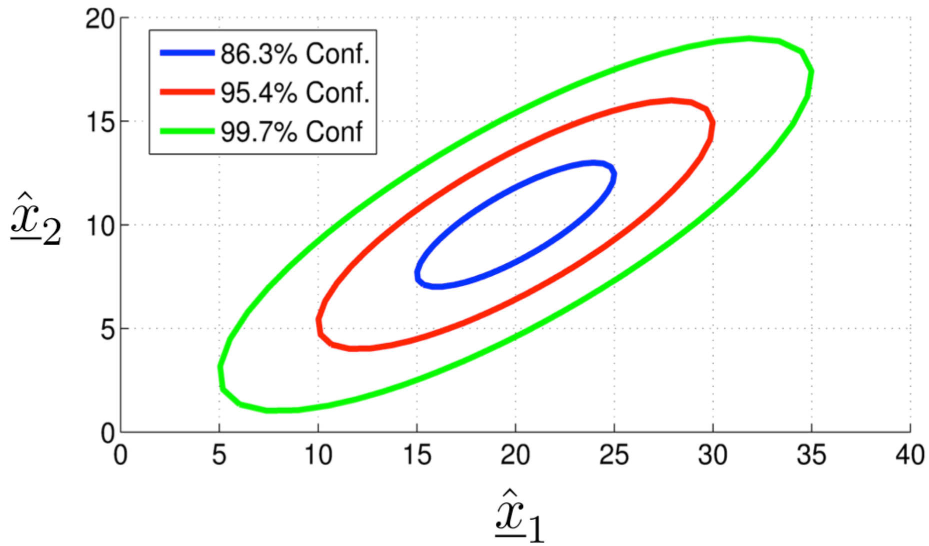
Submit   You have used 1 of 1 attempt

✔ Correct (2/2 points)

Confidence interval

1.0/1.0 point (graded)

Assume the estimator $\underline{\hat{x}}$ and its covarainec matrix $Q_{\hat{x}\hat{x}}$ of unknown parameters $x = [x_1 \;\; x_2]^{\mathrm{T}}$.



If $\hat{x} = [20 \quad 10]$, the above confidence-interval plot are corresponding to which covariance matrix?

- (selected) $\begin{bmatrix} 25 & 12 \\ 12 & 9 \end{bmatrix}$ ✔
- $\begin{bmatrix} 25 & 0 \\ 0 & 9 \end{bmatrix}$ ✔

$\begin{bmatrix} 25 & -12 \\ -12 & 9 \end{bmatrix}$ ✔

$\begin{bmatrix} 9 & 12 \\ 12 & 25 \end{bmatrix}$ ✔

Submit You have used 1 of 1 attempt