Problem 2

$E : Y^2 = X^3 + AX + B$ elliptic curve

$f(q) = q + C_2q^2 + C_3q^3 + C_4q^4 + C_5q^5 + \cdots$

modular form associated with E

N_P = # of **mod P points** on E.

If $4A^3+27B^2$ is not divisible by 13, and $C_{13}=5$, calculate N_{13}.

- **Reciprocity Law** for elliptic curves (**modularity**)

Problem 2

Modularity

- $f(q) = q + \mathbf{C_2}q^2 + \mathbf{C_3}q^3 + C_4q^4 + \mathbf{C_5}q^5 + \cdots$
- $\mathbf{N_P}$ = # of mod P points

➢ For every $P \geq 5$ not dividing $4A^3+27B^2$,

$$\mathbf{C_P = P + 1 - N_P}.$$

➢ If $C_{13}=5$,

$$\mathbf{5 = C_{13} = 13 + 1 - N_{13}}.$$

Answer $N_{13} = 9$