5. Price's model and small-world model

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Exercises due Oct 27, 2021 17:29 IST Completed

Price's model

Price introduced this directed network model in 1976, based on the concept of preferential attachment.

The model starts by assuming a graph that has an average node out-degree of c. During the generation of subsequent nodes, c will be kept constant. Each subsequent node is generated one at a time using the following process.

A new node is inserted into the graph. This node is connected to, on average, c already existing nodes (the exact mechanism for choosing the out-degree is not important as long as the average is maintained). The target node for each one of these connections is selected in proportion to the in-degree of that node. Because new nodes will have an in-degree of zero, this needs to be modified to “in proportion to the in-degree plus β" to ensure that new nodes can be connected to subsequent nodes. Therefore, if the in-degree of node i is k_i, then node i is selected as the target for each connection with probability

$$p_i = \frac{k_i + \beta}{\sum_i (k_i + \beta)}.$$

This is called a “preferential attachment" model, as new nodes are preferentially attached to already well connected nodes. The result is a kind of “rich get richer" process, and, as the number of new nodes overwhelms the existing nodes in the graph, the node in-degree distribution starts to follow a power-law:

$$p(k) \propto k^{2+\beta/c}$$

Circulant model

1/1 point (graded)

The previous models tend to have clustering coefficients that are smaller than what is typically observed in nature. We can attempt to address this issue by generating a graph that naturally has a large number of triplets (triangles).

To do this, we can create a circulant graph. In a circulant graph, the nodes are arranged into a circle and each node is connected to c of its closest neighbors.

Suppose we have a circulant graph with $n = 9$ and $c = 2$. What kind of graph is this?

● Cycle graph.

○ Complete graph.

○ Bipartite graph.

✓

Solution:

If $c = 2$, then each node is connected to the adjacent two nodes in the circle, and thus forms a single cycle. Therefore it is a cycle graph.

As $n = 9$, and each node is only connected to two other nodes, the graph cannot be complete. Also, as n is odd, the graph cannot be bipartite (as it has an odd cycle).

Submit You have used 1 of 2 attempts

Answers are displayed within the problem

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Small world model

When $c > 2$, the circulant graph will naturally start forming triplets. The number of triplets increases with c, which leads to a large clustering coefficient.

However, if c is fixed and n grows, then the diameter of the graph grows as $\mathcal{O}(n)$. This means that the graph does not represent a small world (where the diameter grows as $\mathcal{O}(\ln n)$).

This is an undesirable property, as most natural networks are also small world networks. To address this, we can add “short circuits" to the graph.

This can be done by rewiring a few randomly selected edges. Let one of these selected edges be written as $\{i, j\}$ so that it connects node i to node j. Then the edge is rewired to $\{i, k\}$ where k is randomly and uniformly selected from all nodes in the graph. In particular, k is chosen such that $k \neq i$ (no self-loops) and such that $\{i, k\}$ is not already an edge in the graph (avoids creating a multi-graph).

For large n, k is very likely to be a large distance from i, and so this rewiring procedure drastically reduces the diameter of the graph. Only a few edges need to be rewired this way to achieve a small world model, and so the clustering coefficient is largely preserved. Unfortunately, the degree distribution stays very close to c, and so it does not replicate a power-law distribution.

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power-law distribution sign

discussion posted 2 months ago by **Marc_Svensson**

Hi! I am confused about the sign of the power-law degree distribution formula:

$$p_k \propto k^{2+\beta/c}$$

My problem with this formula is the one that appears in the slides:

$$p_k \sim k^{-\alpha}$$

where

$$\alpha = 2 + \frac{\beta}{c}$$

Notice the minus.

This post is visible to everyone.

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1 response

trungaero

2 months ago

I think the one in the slide is correct. Power law distribution always have a negative slope.

I agree! I wanted to point out the difference, sorry if i implied the slides one was incorrect, i also think it is correct.

posted 2 months ago by **Marc_Svensson**

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