



## MITx: 6.041x Introduction to Probability - The Science of Uncertainty



Bookmarks



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**Lec. 4: Counting**

Exercises 4 due Feb 24, 2016 at 23:59 UTC

**Solved problems****Problem Set 3**

Problem Set 3 due Feb 24, 2016 at 23:59 UTC

## Unit 3: Counting &gt; Lec. 4: Counting &gt; Lec 4 Counting vertical2

**Exercise: Counting committees**

(2/2 points)

We start with a pool of  $n$  people. A chaired committee consists of  $k \geq 1$  members, out of whom one member is designated as the chairperson. The expression  $k \binom{n}{k}$  can be interpreted as the number of possible chaired committees with  $k$  members. This is because we have  $\binom{n}{k}$  choices for the  $k$  members, and once the members are chosen, there are then  $k$  choices for the chairperson. Thus,

$$c = \sum_{k=1}^n k \binom{n}{k}$$

is the total number of possible chaired committees of any size.

Find the value of  $c$  (as a function of  $n$ ) by thinking about a different way of forming a chaired committee: first choose the chairperson, then choose the other members of the committee. The answer is of the form

$$c = (\alpha + n^\beta) 2^{\gamma n + \delta}.$$

What are the values of  $\alpha$ ,  $\beta$ ,  $\gamma$ , and  $\delta$ ?

$\alpha =$	<input type="text" value="0"/>	✓ Answer: 0
$\beta =$	<input type="text" value="1"/>	✓ Answer: 1
$\gamma =$	<input type="text" value="1"/>	✓ Answer: 1
$\delta =$	<input type="text" value="-1"/>	✓ Answer: -1

Answer:

We first choose the chairperson, for which there are  $n$  choices, and then choose an arbitrary subset of the remaining  $n - 1$  people, who will be the remaining committee members. For example, this arbitrary subset could be the empty set, which would mean that the committee is of size 1: only the chairperson. There are  $2^{n-1}$  possible subsets of a set with  $n - 1$  elements, and so there are  $2^{n-1}$  ways of choosing the

remaining committee members. Thus, an alternative expression for the number of possible chaired committees of any size is  $n2^{n-1}$ , from which we can extract the values of  $\alpha$ ,  $\beta$ ,  $\gamma$ , and  $\delta$ .

*You have used 1 of 2 submissions*

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