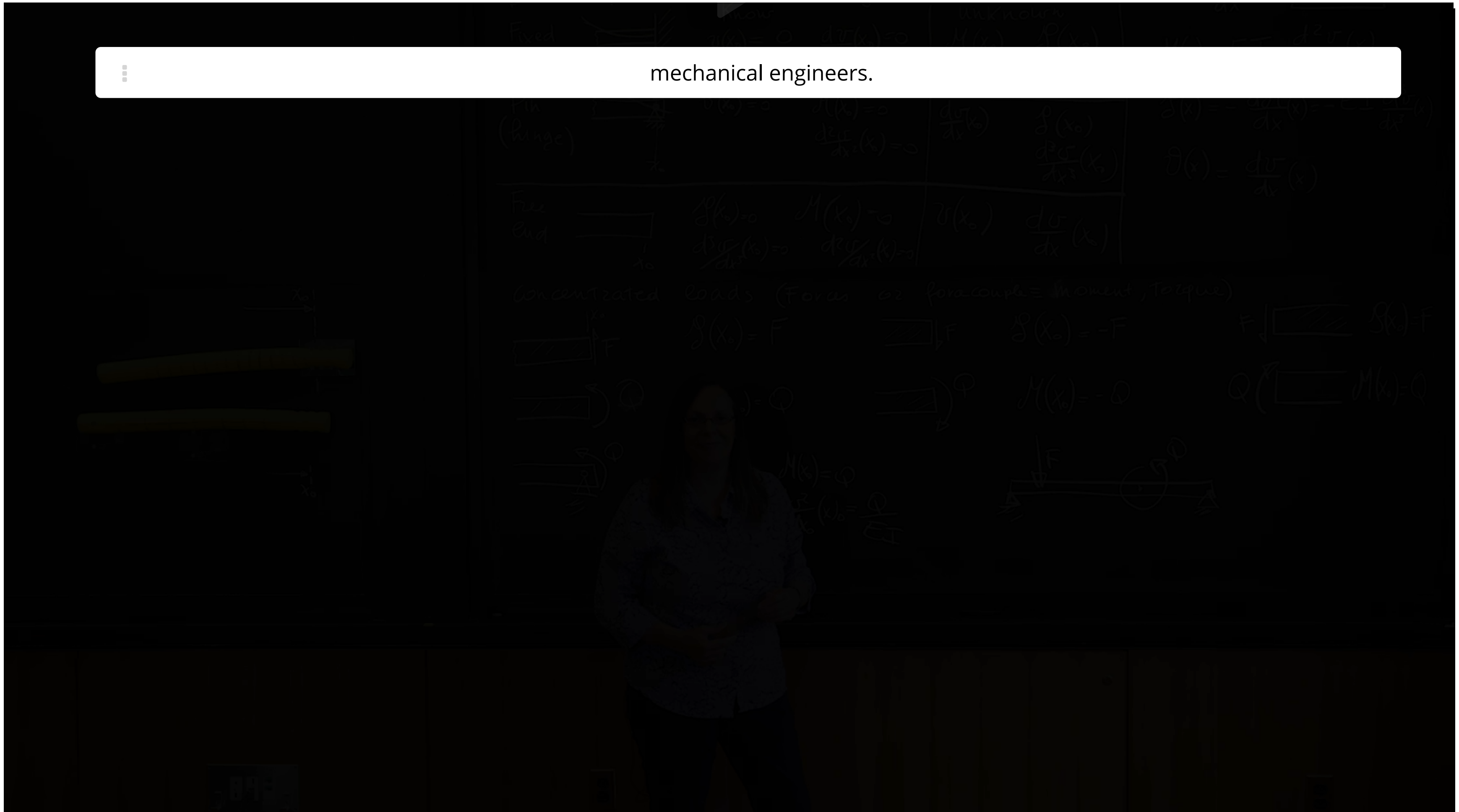
9. Horizontal beams and boundary conditions

Boundary conditions



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1.50x

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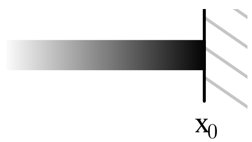
Recall that the equation governing the static deflection of a slender horizontal beam under a load $q_y(x)$ is given by

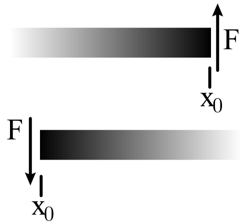
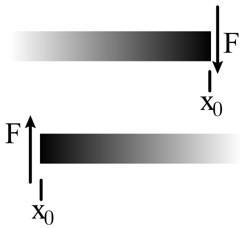
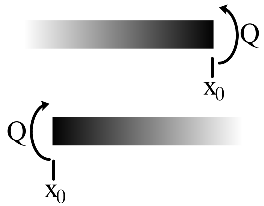
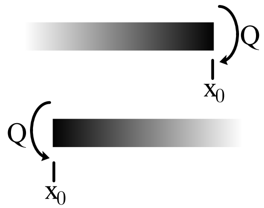
$$EI\frac{d^4v(x)}{dx^4} = q_y(x),$$

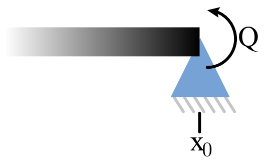
where

- The angle of deflection is $\theta(x) = \frac{dv}{dx}(x)$
- The bending moment is $M(x) = EI\frac{d^2v}{dx^2}(x)$
- The shear force is $S(x) = -EI\frac{d^3v}{dx^3}(x)$

Table of constraints on a horizontal beam with corresponding boundary conditions

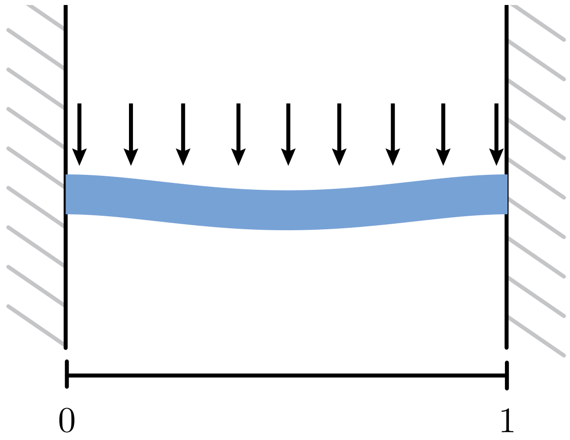
Constraints	**Drawing**	**Boundary conditions**	**Unknowns**
Fixed (in wall)		$v(x_0) = 0$, and $\frac{dv}{dx}(x_0) = 0$	$\frac{d^2v}{dx^2}(x_0)$ and $\frac{d^3v}{dx^3}(x_0)$
Pinned (on hinge)		$v(x_0) = 0$, and $\frac{d^2v}{dx^2}(x_0) = 0$	$\frac{dv}{dx}(x_0)$ and $\frac{d^3v}{dx^3}(x_0)$
Free		$\frac{d^2v}{dx^2}(x_0) = 0$, and $\frac{d^3v}{dx^3}(x_0) = 0$	$v(x_0)$ and $\frac{dv}{dx}(x_0)$

Free with applied shear force		$\frac{d^2v}{dx^2}(x_0) = 0$, and $\frac{d^3v}{dx^3}(x_0) = -F/EI$	$v(x_0)$ and $\frac{dv}{dx}(x_0)$
		$\frac{d^2v}{dx^2}(x_0) = 0$, and $\frac{d^3v}{dx^3}(x_0) = F/EI$	$v(x_0)$ and $\frac{dv}{dx}(x_0)$
Free with applied torque		$\frac{d^2v}{dx^2}(x_0) = Q/EI$, and $\frac{d^3v}{dx^3}(x_0) = 0$	$v(x_0)$ and $\frac{dv}{dx}(x_0)$
		$\frac{d^2v}{dx^2}(x_0) = -Q/EI$, and $\frac{d^3v}{dx^3}(x_0) = 0$	$v(x_0)$ and $\frac{dv}{dx}(x_0)$

Pinned with applied torque		$v\left(x_0\right)=0$, and $\dfrac{d^2v}{dx^2}\left(x_0\right)=Q/EI$	$\dfrac{dv}{dx}\left(x_0\right)$ and $\dfrac{d^3v}{dx^3}\left(x_0\right)$

Examples of boundary conditions for a horizontal beam

Example 1: Both ends fixed

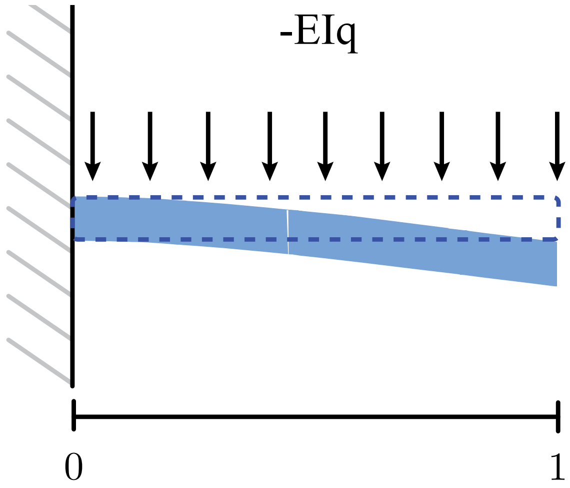


In this case, both the right and left sides are clamped to the wall, so $v\left(0\right)=0$ and $v\left(1\right)=0$. Because the bar is perpendicular to the wall, it is also the case that $\dfrac{dv}{dx}\left(0\right)=0$ and $\dfrac{dv}{dx}\left(1\right)=0$, giving a full set of boundary conditions:

$$v\left(0\right)=\frac{dv}{dx}\left(0\right)\quad=0 \tag{3.24}$$

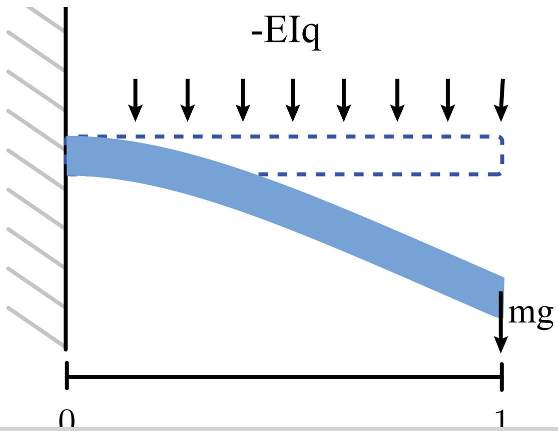
$$v\left(1\right)=\frac{dv}{dx}\left(1\right)\quad=0 \tag{3.25}$$

Example 2: One end fixed, one end free



In this case, we still find that $v(0) = \frac{dv}{dx}(0) = 0$ and the left endpoint is fixed to the wall. Where the bar hangs free on the right side, the displacement must satisfy the boundary conditions $\frac{d^2v}{dx^2}(1) = 0$ and $\frac{d^3v}{dx^3}(1) = 0$. The second and third derivative terms are proportional to the **bending moment** and **shear force** .

Example 3: One end fixed, one end has a hanging mass



In this case, we still find that $v(0) = \frac{dv}{dx}(0) = 0$ and the left endpoint is fixed to the wall. Where the bar hangs free on the right side, the fact that there is a point force says that $\frac{d^2v}{dx^2}(1) = 0$ and $\frac{d^3v}{dx^3}(1) = \frac{mg}{EI}$, where mg is the magnitude of the point force, E is the material constant of elasticity relating stress and strain, and I is the moment of inertia. (To learn more, take 2.001!)

Boundary conditions, 1

8/8 points (graded)

Find the boundary conditions describing a beam that is pinned at the left (on a hinge) end and fixed at the right end (in a wall).

Enter the boundary condition, or enter UNK for an unknown constraint.

$v(0) =$ 0 ✔ **Answer:** 0 $v(L) =$ 0 ✔ **Answer:** 0

$\frac{dv}{dx}(0) =$ UNK ✔ **Answer:** UNK $\frac{dv}{dx}(L) =$ 0 ✔ **Answer:** 0

$\frac{d^2v}{dx^2}(0) =$ 0 ✔ **Answer:** 0 $\frac{d^2v}{dx^2}(L) =$ UNK ✔ **Answer:** UNK

$\frac{d^3v}{dx^3}(0) =$ UNK ✔ **Answer:** UNK $\frac{d^3v}{dx^3}(L) =$ UNK ✔ **Answer:** UNK

FORMULA INPUT HELP

Solution:

The end at $x = 0$ is pinned, which tells us that $v(0) = 0$. The pin is frictionless and thus it allows rotation but provides no moment, hence $\frac{d^2v}{dx^2}(0) = 0$. The first and third derivatives of the deflection at $x = 0$ are unknown.

The end at $x = L$ is fixed, which tells us that $v(L) = 0$ and $\frac{dv}{dx}(L) = 0$. The second and third derivatives at $x = L$ are unknown.

Submit You have used 1 of 4 attempts

Answers are displayed within the problem

Boundary conditions, 2

8/8 points (graded)

Find the boundary conditions for a beam fixed in a wall on the right with a free left end. There is a shear force pointing upwards of magnitude F on the left end.

Enter the boundary condition, or enter UNK for an unknown constraint.

$v(0) =$ UNK **Answer:** UNK $v(L) =$ 0 **Answer:** 0

$\frac{dv}{dx}(0) =$ UNK ✔ **Answer:** UNK

$\frac{dv}{dx}(L) =$ 0 ✔ **Answer:** 0

$\frac{d^2v}{dx^2}(0) =$ 0 ✔ **Answer:** 0

$\frac{d^2v}{dx^2}(L) =$ UNK ✔ **Answer:** UNK

$\frac{d^3v}{dx^3}(0) =$ F/E/I ✔ **Answer:** F/(E*I)

$\frac{d^3v}{dx^3}(L) =$ UNK ✔ **Answer:** UNK

FORMULA INPUT HELP

Solution:

The beam is free at the left but with an applied shear force pointing up. On the left side, an upward pointing shear force is negative, thus $S(0) = -F$, so $\frac{d^3v}{dx^3}(0) = F/EI$. We know that the bending moment is zero, so $\frac{d^2v}{dx^2}(0) = 0$. The position and first derivative at $x = 0$ are unknown.

The right end is fixed into the wall, thus $v(L) = \dfrac{dv}{dx}(L) = 0$, as in the previous problem.

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Boundary conditions, 3

8/8 points (graded)

Find the boundary conditions for a beam pinned on the right with a free left end. There is a shear force pointing downwards of magnitude F on the left end, and a bending moment Q curving down on the right.

Enter the boundary condition, or enter UNK for an unknown constraint.

$v(0) =$	UNK	✔ **Answer:** UNK	$v(L) =$	0	✔ **Answer:** 0
$\frac{dv}{dx}(0) =$	UNK	✔ **Answer:** UNK	$\frac{dv}{dx}(L) =$	UNK	✔ **Answer:** UNK
$\frac{d^2v}{dx^2}(0) =$	0	✔ **Answer:** 0	$\frac{d^2v}{dx^2}(L) =$	-Q/E/I	✔ **Answer:** -Q/(E*I)
$\frac{d^3v}{dx^3}(0) =$	-F/E/I	✔ **Answer:** -F/(E*I)	$\frac{d^3v}{dx^3}(L) =$	UNK	✔ **Answer:** UNK

FORMULA INPUT HELP

Solution:

As in the previous problem, the beam is free at the left but with an applied shear force pointing down. So $\frac{d^3v}{dx^3}(0) = -F/EI$ and $\frac{d^2v}{dx^2}(0) = 0$. The position and first derivative at $x = 0$ are unknown.

The right end is pinned with an applied moment at $x = L$. Thus $v(L) = 0$ and $M(L) = -Q$, because the moment makes the beam frown. Since $M(L) = EI\frac{d^2v}{dx^2}(L)$, it follows that $\frac{d^2v}{dx^2}(L) = -Q/EI$.

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You have used 1 of 4 attempts

Answers are displayed within the problem

Boundary conditions, 4

8/8 points (graded)

Find the boundary conditions for a beam fixed in a wall on the right with a free left end. There is a bending moment Q curving up on the left.

Enter the boundary condition, or enter UNK for an unknown constraint.

$v(0) =$ UNK ✔ **Answer:** UNK

$v(L) =$ 0 ✔ **Answer:** 0

$\frac{dv}{dx}(0) =$ UNK ✔ **Answer:** UNK

$\frac{dv}{dx}(L) =$ 0 ✔ **Answer:** 0

$\frac{d^2v}{dx^2}(0) =$ Q/E/I ✔ **Answer:** Q/(E*I)

$\frac{d^2v}{dx^2}(L) =$ UNK ✔ **Answer:** UNK

$\frac{d^3v}{dx^3}(0) =$ 0 ✔ **Answer:** 0

$\frac{d^3v}{dx^3}(L) =$ UNK ✔ **Answer:** UNK

FORMULA INPUT HELP

Solution:

The beam is fixed on the right at $x = L$, thus $v(L) = 0$ and $v'(L) = 0$. The second and third derivative of the deflection at $x = L$ are unknown.

The left end is free with an applied upward moment $M(0) = Q$ (positive because it makes the beam smile). Thus $\frac{d^2 v}{dx^2}(0) = Q/EI$ and $\frac{d^3 v}{dx^3}(0) = 0$ due to force balance.

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9. Horizontal beams and boundary conditions

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? [Staff] Boundary conditions, 2

I put the correct answer at the first time but it was not accepted. Then I changed the sign in my answer and the answer again was not accepted (because it was not correct). C...

2

? Force's sign in "Free with applied shear force"

I fell a little confused about force's sign when applying a force to the free end; can you help me understand when (and why) it should be positive and negative? Thanks :)

4