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Homework 5: Maximum Likelihood

4. Maximum Likelihood Estimator

<u>Course</u> > <u>Unit 3 Methods of Estimation</u> > <u>Estimation</u>

> for Curved Gaussian

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4. Maximum Likelihood Estimator for Curved Gaussian

(a)

1.0/1 point (graded)

Note: To avoid too much double jeopardy, the solution to part (a) will be available once you have either answered it correctly or reached the maximum number of attempts.

Let X_1,\ldots,X_n be n i.i.d. random variables with distribution $\mathcal{N}\left(\theta,\theta\right)$ for some unknown $\theta>0$.

Compute the maximum likelihood estimator $\hat{\theta}$ for θ in terms of the sample averages of the linear and quadratic means, i.e. \overline{X}_n and \overline{X}_n^2

(Enter **barX_n** for \overline{X}_n and **bar(X_n^2**) for $\overline{X_n^2}$. Note that **barX_n^2** represents $(\overline{X_n})^2$, and is **not** equal to **bar(X_n^2)** with the brackets.

$$\hat{\theta} = \boxed{ (-1+sqrt(1+4*bar(X_n^2)) }$$

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STANDARD NOTATION

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You have used 3 of 3 attempts

(b)

4/4 points (graded)

We want to compute the asymptotic variance of $\hat{ heta}$ via two methods.

In this problem, we apply the Central Limit Theorem and the 1-dimensional Delta Method. We will compare this with the approach using the Fisher information next week.

First, compute the limit and asymptotic variance of $\overline{X_n^2}$.

The limit to which $\overline{X_n^2}$ converges in probability, also known as its ${f P}$ -limit , is

$$\overline{X_n^2} \xrightarrow[n o \infty]{\mathbf{P}}$$
 theta^2+theta $m{ heta}^2 + heta$

The asymptotic variance $\,V\,(\overline{X_n^2})\,$ of $\,\overline{X_n^2}$, which is equal to $\,$ $\,$ $\,$ $\,$ $\,$ $\,$ $\,$ is

$$V\left(\overline{X_n^2}
ight) = \mathsf{Var}\left(X_1^2
ight) = egin{bmatrix} 4 imes \mathsf{theta} imes 2 imes 4 \cdot heta^3 + 2 \cdot heta^2 \end{bmatrix}$$

Now, write $\hat{\theta}$ as the function of $\overline{X_n^2}$ you found in part (a),

$$\hat{ heta} = g(\overline{X}_n^2)$$

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and give its first derivative, g'(x),

What can you conclude about the asymptotic variance $V\left(\hat{ heta}
ight)$ of $\hat{ heta}$?

$$V(\hat{\theta}) =$$

$$2*\text{theta}^2/(1+2*\text{theta})$$

STANDARD NOTATION

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You have used 2 of 3 attempts

✓ Correct (4/4 points)

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Part (b) asymptotic variance

+

discussion posted 22 minutes ago by sandipan dey

*

Leadby-liked the problem very much and also interested to know about the thought-process to come up with a function g(.) s.t. $g(\theta^2 + \theta) = \theta$ Generating Speech Output structing the problem.

It's nice to notice that the quadratic $heta^2+ heta=1$ has the +ve root $rac{-1+\sqrt{5}}{2}=g\left(1
ight)$.

Question: Why asymptotic variance of $\bar{X_n^2}$ i.e. $Var\left(\bar{X_n^2}\right) = Var\left(\bar{X_1^2}\right)$, whereas $Var\left(\bar{X_n^2}\right) = Var\left(\frac{1}{n}.\sum_{i=1}^n X_i^2\right) = \frac{1}{n}Var\left(X_i^2\right)$? Is it

because when $n o\infty$ the (sampling) distribution of X_n^2 tends to be same as the (population) distribution of X_i^2 ?

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