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Lecture
Lecture questions due Oct 04, 2016 at 19:30 IST

Recitation

Problem Set 4
Homework 4 due Oct 04, 2016 at 19:30 IST

PART A

(1/1 point)

We now analyze the problem of a firm trying to decide on the opening of several lockboxes. In this problem we have to take into account the binary decisions of opening/not opening each potential lockbox, as well as an assignment problem to decide which regions should send their money to each lockbox. There are fixed costs for opening a lockbox, and there are opportunity costs for not opening a lockbox. This problem is based on an example from: G. Cornuéjols and R. Tütüncü, *Optimization Methods in Finance.*

Consider a national firm that receives checks from all over the United States. Due to the vagaries of the U.S. Postal Service, as well as the banking system, there is a variable delay from when the check is postmarked (and hence the customer has met her obligation) and when the check clears (and when the firm can use the money). For instance, a check mailed in Boston sent to a Boston address might clear in just 3 days. A similar check sent to Los Angeles might take 7 days to clear. It is in the firm's interest to have the check clear as quickly as possible since then the firm can use the money. In order to speed up this clearing process, firms open offices (called lockboxes) in different cities to handle the checks.

For example, suppose we receive payments from 3 regions (West, East, and South). The average daily value from each region is as follows: $625,000 from the West, $850,000 from the East, and $350,000 from the South. We are considering opening lockboxes in Los Angeles, Chicago, and/or Boston. Operating a lockbox costs $150,000 per year. Currently, all checks are mailed to Seattle, where the firm is based. We can assume that handling checks in Seattle does not cost extra money. The average days from mailing to clearing is given in Table 1. Which lockboxes should we open?

Table 1: Average clearing times for checks mailed from one region to L.A., Chicago, Boston, and Seattle.

	L.A.	Chicago	Boston	Seattle
West	2	4	6	2
East	7	5	3	6
South	4	5	6	5

First we must calculate the lost interest for each possible assignment. For example, if the West sends its checks to a lockbox in Boston, then on average there will be \$3,750,000 (= 6 x \$625,000) in process on any given day. Assuming a fixed investment rate of 6%, this corresponds to a yearly loss of \$225,000. We can calculate the losses for the other combinations in a similar fashion; we obtain Table 2.

Table 2: Yearly opportunity cost in thousands of US dollars for checks sent from West, East, South to L.A., Chicago, Boston, and Seattle.

	L.A.	Chicago	Boston	Seattle

West	75	150	225	75
East	357	255	153	306
South	84	105	126	105

We can open as many lockboxes as we need. Our goal is to determine the decision that minimizes total costs for the firm: how many lockboxes should be opened, and where. Note that this problem is an integer linear program. We will use the following variables. Let y_j be a binary variable that is 1 if lockbox j is opened and 0 if it is not, $j = 1, \ldots, 3$ (L.A., Chicago, Boston). Note that checks can always be handled in Seattle: there is no cost associated to opening a lockbox in Seattle. Let x_{ij} be 1 if region i sends its checks to lockbox $j, i = 1, \ldots, 3$ (West, East, South) and $j = 1, \ldots, 4$ ($j = 4$ corresponding to Seattle).

Formulate the objective function of the problem, taking into account the cost of opening lockboxes and opportunity costs due to lost interest. As mentioned in the problem's description, we can assume that checks can be handed in Seattle without having to pay extra for opening a lockbox there.

- MIN $150(y_1 + y_2 + y_3) + 75x_{11} + 150x_{12} + 225x_{13} + 75x_{14} + 357x_{21} + 255x_{22} + 153x_{23} + 306x_{24} + 84x_{31} + 105x_{32} + 126x_{33} + 105x_{34}$ ✔
- MAX $150(y_1 + y_2 + y_3) + 75x_{11} + 150x_{21} + 225x_{31} + 75x_{41} + 357x_{12} + 255x_{22} + 153x_{32} + 306x_{42} + 84x_{13} + 105x_{23} + 126x_{33} + 105x_{43}$
- MAX $150(y_1 + y_2 + y_3)$

- MIN $75x_{11} + 150x_{12} + 225x_{13} + 75x_{14} + 357x_{21} + 255x_{22} + 153x_{23} + 306x_{24} + 84x_{31} + 105x_{32} + 126x_{33} + 105x_{34}$

You have used 1 of 3 submissions

PART B

(1/1 point)

Formulate the constraints that a region cannot send checks to a closed lockbox. *Hint: For each region i and for each lockbox j, add a constraint that is equivalent to "if $x_{ij} = 1$, then $y_j = 1$".* There are two correct ways of formulating these constraints. Select both of them below.

- [x] $$\left.\begin{aligned} x_{11} + x_{21} + x_{31} &\leq 3y_1 \\ x_{12} + x_{22} + x_{32} &\leq 3y_2 \\ x_{13} + x_{23} + x_{33} &\leq 3y_3 \end{aligned}\right\}$$

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$$\left.\begin{aligned} x_{11} + x_{21} + x_{31} &\geq 3y_1 \\ x_{12} + x_{22} + x_{32} &\geq 3y_2 \\ x_{13} + x_{23} + x_{33} &\geq 3y_3 \end{aligned}\right\}$$

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$$\left.\begin{aligned} x_{11} \leq y_1 \quad x_{21} \leq y_1 \quad x_{31} &\leq y_1 \\ x_{12} \leq y_1 \quad x_{22} \leq y_1 \quad x_{32} &\leq y_2 \\ x_{13} \leq y_3 \quad x_{23} \leq y_1 \quad x_{33} &\leq y_3 \end{aligned}\right\}$$

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$$\left.\begin{aligned} x_{11} \geq y_1 \quad x_{21} \geq y_1 \quad x_{31} &\geq y_1 \\ x_{12} \geq y_1 \quad x_{22} \geq y_1 \quad x_{32} &\geq y_2 \\ x_{13} \geq y_3 \quad x_{23} \geq y_1 \quad x_{33} &\geq y_3 \end{aligned}\right\}$$

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You have used 1 of 3 submissions

PART C

(1/1 point)

There are constraints that ensure that the checks from each region are mailed to a lock box.

Which of the constraints below is the "West-assigned region constraint"?

- $x_{11} + x_{21} + x_{31} + x_{41} \leq 1$
- $x_{11} + x_{12} + x_{13} + x_{14} \leq 1$
- $x_{11} + x_{21} + x_{31} + x_{41} = 1$
- $x_{11} + x_{12} + x_{13} + x_{14} = 1$ ✔

You have used 1 of 3 submissions

PART D

(3/3 points)

Formulate this instance of the Lockbox Problem in Excel, and solve it using Solver or OpenSolver. What is the optimal cost? Error checking hint. It is between 480 and 495.

483 ✔

You have used 1 of 3 submissions

PART E

(1/1 point)

In the optimal solution, is a lockbox opened at Chicago?

- (●) No ✔
- ( ) Yes

You have used 1 of 1 submissions


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