2. Bounce

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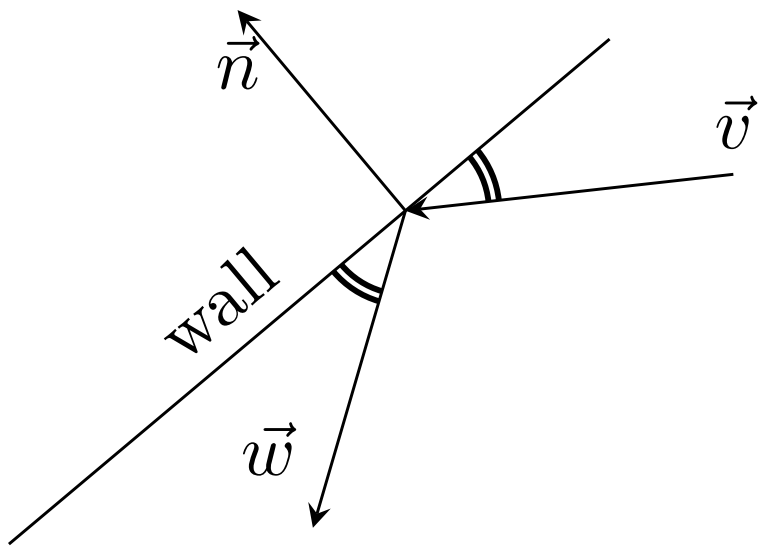
Problem Set B due Sep 15, 2021 20:30 IST

Practice

Setup

In Recitation 5, you practiced finding an outgoing velocity vector of a particle that bounces off a wall. In this problem, you will generalize this calculation and express the answer using matrices.

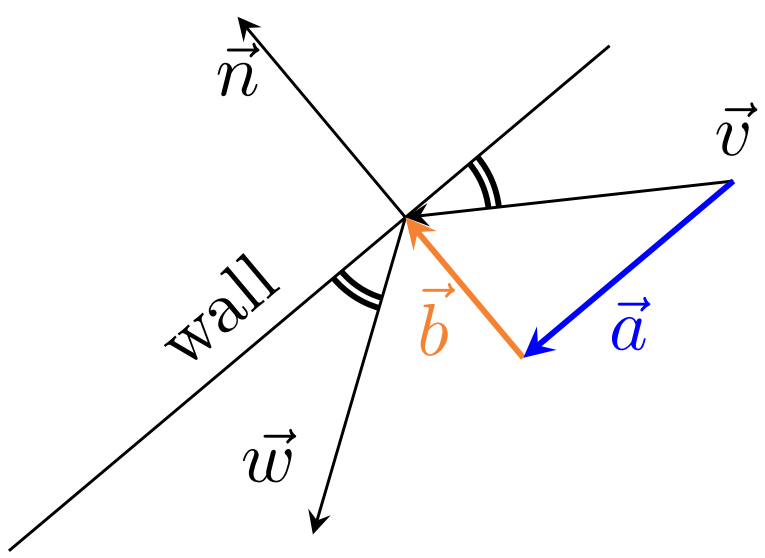
The following image depicts the setup:



In the picture, $\vec{v}$ is the velocity of the projectile before it hits the wall, and $\vec{w}$ is the velocity of the projectile after it hits the wall. The vector $\vec{n}$ is a normal vector to the wall. Assume $|\vec{n}| = 1$.

Physics tells us that the angle of incidence equals the angle of reflection – the two marked angles in the picture are equal. It also tells us that if there is no friction, then $|\vec{v}| = |\vec{w}|$. Given $\vec{v}$ and $\vec{n}$, our goal is to find $\vec{w}$.

We will break this big problem into several steps. We begin by breaking $\vec{v}$ into a piece parallel to $\vec{n}$ and a piece perpendicular to $\vec{n}$, as in the following picture.



Find w from a and b

1/1 point (graded)

Find a formula for $\vec{w}$ in terms of $\vec{a}$ and $\vec{b}$. Write `veca` and `vecb` for $\vec{a}$ and $\vec{b}$.

$\vec{w} =$ veca - vecb ✔

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Find length of b from v and n

1/1 point (graded)

What is the length of the vector $\vec{b}$? Enter a formula in terms of $\vec{v}$ and $\vec{n}$.

Write `vecv` for $\vec{v}$ and `vecn` for $\vec{n}$. You may write a dot product such as $\vec{u} \cdot \vec{w}$ using `vecu * vecw`.

$|\vec{b}| =$ vecv*vecn ✔

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Find a from v and n

1/1 point (graded)

Find a formula for $\vec{a}$ from $\vec{v}$ and $\vec{n}$.

Write `vecv` for $\vec{v}$ and `vecn` for $\vec{n}$. You may also write a dot product between two vectors using `*`. To write a scaled vector such as $2\vec{v}$, enter `2*vecv`.

$\vec{a} =$ vecv - (vecv*vecn)*vecn ✔

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Find w from v and n

1/1 point (graded)

Find a formula for $\vec{w}$ from $\vec{v}$ and $\vec{n}$.

Write `vecv` for $\vec{v}$ and `vecn` for $\vec{n}$. You may also write a dot product between two vectors using `*`. To write a scaled vector such as $2\vec{v}$, enter `2*vecv`.

$\vec{w} =$ vecv - 2*(vecv*vecn)*vecn ✔

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2. Bounce

Topic: Unit 4: Matrices and Linearization / 2. Bounce

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Alternative method: Eigen vectors and Eigen values of M

discussion posted 7 days ago by **yves-M** (Community TA)

I find it fascinating that this problem can be modeled with a Matrix.

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- Intuitively M must have -1 and 1 for Eigen values with respective Eigen vectors $\vec{n}$ and $\vec{n}^{\perp}$ (a vector perpendicular to $\vec{n}$, for example $\vec{n}^{\perp} = \langle -n_2, n_1 \rangle$). [Since an incoming particule in the direction of $\vec{n}$ should bounce back, while a particule coming along $\vec{n}^{\perp}$ would slide along the wall].
- An indeed, we can check that it is the case of the solution found in (*Find the matrix*).

With the above intuition, one could have found M by multiplying three matrices. From right to left:

- (a) the change-of-basis matrix (also called transition matrix) $R_{-\theta}$ to change basis from $\vec{i}, \vec{j}$ to $\vec{n}, \vec{n}^{\perp}$
- then (b) D a diagonal matrix with -1 and 1 on its diagonal
- then (c) R_{θ} to return from $\vec{n}, \vec{n}^{\perp}$ basis to $\vec{i}, \vec{j}$ basis. Note that $R_{-\theta} = {R_{\theta}}^{T}$ (if $|\vec{n}| = |\vec{n}^{\perp}| = 1$) and R_{θ} is easily expressed in terms of n_1 and n_2

Physics becomes easy when Algebra comes at the rescue :)

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