



## MITx: 6.041x Introduction to Probability - The Science of Uncertainty



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## Unit overview

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v s vertical5

Bookmark

## Exercise: Independence and variances

(3/3 points)

The pair of random variables  $(X, Y)$  is equally likely to take any of the four pairs of values  $(0, 1)$ ,  $(1, 0)$ ,  $(-1, 0)$ ,  $(0, -1)$ . Note that  $X$  and  $Y$  each have zero mean.

a) Find  $\mathbf{E}[XY]$ . $\mathbf{E}[XY] =$ 

0



Answer: 0

b) For this pair of random variables  $(X, Y)$ , is it true that  $\text{var}(X + Y) = \text{var}(X) + \text{var}(Y)$ ?

Yes ▼



Answer: Yes

c) We know that if  $X$  and  $Y$  are independent, then  $\text{var}(X + Y) = \text{var}(X) + \text{var}(Y)$ . Is the converse true? That is, does the condition  $\text{var}(X + Y) = \text{var}(X) + \text{var}(Y)$  imply independence?

No ▼



Answer: No

Answer:

a) At each possible outcome, we have  $XY = 0$ , and therefore  $\mathbf{E}[XY] = 0$ .

b) Since the random variables have zero mean,  $\mathbf{E}[X + Y] = 0$ ,  $\text{var}(X) = \mathbf{E}[X^2]$ , and  $\text{var}(Y) = \mathbf{E}[Y^2]$ . Combining this with the result from part (a), we conclude that

$$\begin{aligned}
 \text{var}(X + Y) &= \mathbf{E}[(X + Y)^2] - (\mathbf{E}[X + Y])^2 \\
 &= \mathbf{E}[(X + Y)^2] \\
 &= \mathbf{E}[X^2] + 2\mathbf{E}[XY] + \mathbf{E}[Y^2] \\
 &= \mathbf{E}[X^2] + \mathbf{E}[Y^2] \\
 &= \text{var}(X) + \text{var}(Y).
 \end{aligned}$$

Exercises 7 due Mar  
02, 2016 at 23:59 UTC

Solved problems

Additional  
theoretical  
material

Problem Set 4

Problem Set 4 due Mar  
02, 2016 at 23:59 UTC

Unit summary

c) We have here an example of two random variables that satisfy the condition  $\text{var}(X + Y) = \text{var}(X) + \text{var}(Y)$ . But these random variables are not independent. For example, the information that  $X = 1$  tells us that the value of  $Y$  must be zero.

*You have used 1 of 1 submissions*

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