5. Critical point type for a quadratic function

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Lecture due Sep 13, 2021 20:30 IST Completed

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Warm up

2.0/2 points (graded)

Assume $b^2 - 4ac \neq 0$. Find the critical point of a generic quadratic function $w(x, y) = ax^2 + bxy + cy^2$. Then find the formula for the tangent plane at that critical point.

(Enter the critical point as an ordered pair surrounded by round parentheses: e.g. `(a,b)` for (a, b).)

Critical point: (0,0) ✔ **Answer:** (0,0)

Tangent plane at critical point: $w \approx$ 0 ✔ **Answer:** 0

Solution:

Computing the partial derivatives we find:

$$\begin{aligned} w_x &= 2ax + by \\ w_y &= bx + 2cy \end{aligned}$$

Setting them both equal to zero, we can solve for x and y. We multiply the top equation by b and the bottom equation by $-2a$ and add together to get

$$\begin{aligned} 0 &= 2abx + b^2 y \\ 0 &= -2abx + -4acy \\ 0 &= 0 + (b^2 - 4ac)\, y \end{aligned}$$

This tells us that if $b^2 - 4ac \neq 0$ then $y = 0$. Similarly we can multiply the top equation by $2c$ and the bottom equation by $-b$ and add them together to get

$$\begin{aligned} 0 &= 4acx + 2bcy \\ 0 &= -b^2 x - 2bcy \\ 0 &= (4ac - b^2)\, x + 0 \end{aligned}$$

This tells us that if $4ac - b^2 \neq$ then $x = 0$. Thus in general, the critical point is $(0, 0)$. However, in the case that $b^2 - 4ac = 0$, our two equations are multiples of each other and we get a line of critical points. (This is a degenerate case that we will not consider.)

The tangent plane is then $w(x, y) \approx w(0, 0) + 0\Delta x. + 0\Delta y = 0$ near the critical point at the origin.

Submit You have used 1 of 3 attempts

Answers are displayed within the problem

Quadratic function exploration

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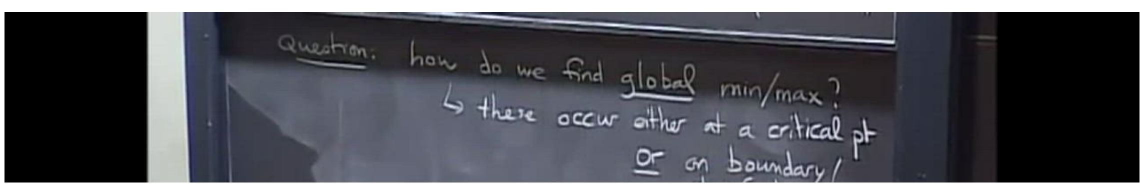

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0:00 / 0:00 2.0x

PROFESSOR: So we'll use something

that's known as the second derivative test.

And, in principle, the idea is kind of

similar to what you do with a function of one valuable.

Namely, a function of one valuable,

if the derivative is 0, then that you

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Recall that we learned about first derivatives using the linear approximation and tangent planes. We will soon see how the second derivative test relates to quadratic approximations. So to explain why the second derivative test works, we will first consider a quadratic function of two variables

$$w(x,y) = ax^2 + bxy + cy^2 \tag{4.59}$$

for $a \neq 0$.

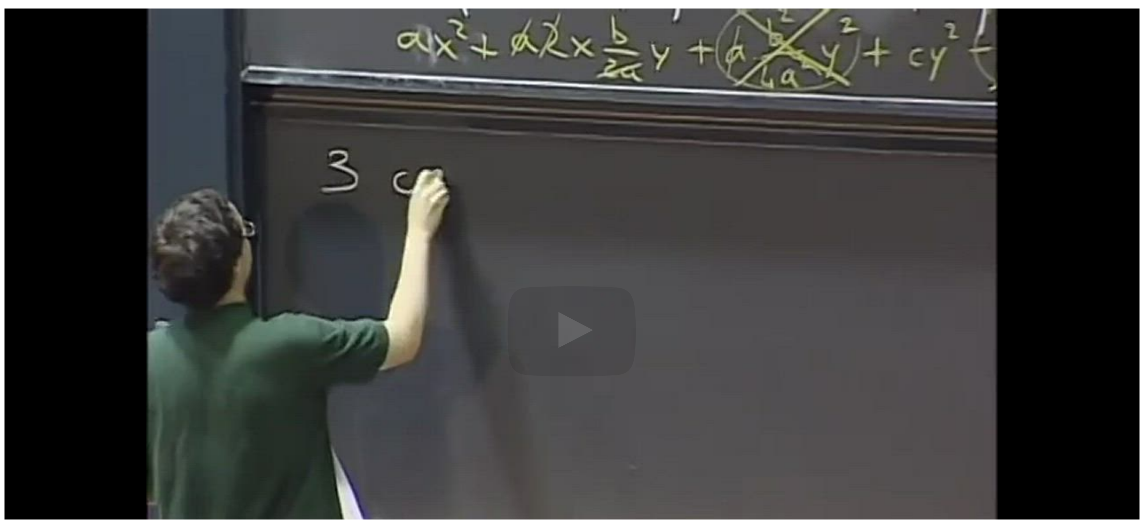
Note: We know that a critical point of $w(x,y)$ is the origin $(0,0)$. The following steps are to help us classify what type of critical point the origin is based on the values of a, b, and c.

We can rewrite $w(x,y)$ by completing the square.

$$\begin{aligned} w(x,y) &= a\left(x^2 + \frac{b}{a}xy\right) + cy^2 && (4.60)\\ &= a\left(x + \frac{b}{2a}y\right)^2 + \left(c - \frac{b^2}{4a}\right)y^2 && (4.61)\\ &= \frac{1}{4a}\left[4a^2\left(x + \frac{b}{2a}y\right)^2 + \left(4ac - b^2\right)y^2\right]. && (4.62)\end{aligned}$$

We do not expect you to have seen how to rewrite this equation on your own, but we invite you to expand equation 4.62 to see that it is equal to our original equation 4.59. Our next goal is to determine the behavior of the terms in 4.62 based on the values of a, b and c.

Cases for a quadratic function



will be three cases,

and that's good news for us.

Because, after all, we want to distinguish

between three possibilities.

So, let's first, do away with the most complicated one.

What if 4ac minus b squared is negative?

Well if it's negative, then it means,

what I have between the brackets is--

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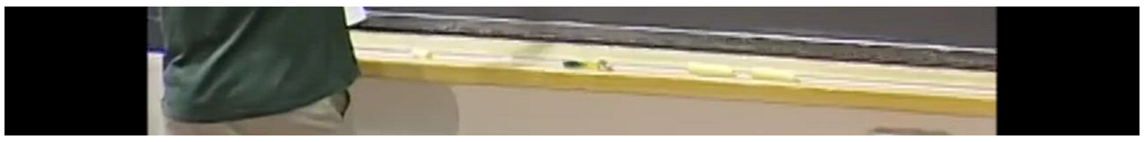

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so the first guy is obviously a positive quantity

while the second one will be something negative times y

squared.

So it will be a negative quantity.

OK

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[Staff] Meaning of “Saddle”	4
What does the professor mean when he says "all of these guys are smaller than x and y"? In the first video at 1:33, he says that "all of these guys are smaller than x and y," so the tangent plane is just w=0. What is this supp...	2
Lol Did that calculation for a 0? lol 😄	2
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minor typo in warm up solution. Hello. "This tells us that if 4ac−b2≠ then" needs a zero. Best wishes.	1

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