MITx: 6.041x Introduction to Probability - The Science of Uncertainty

- Unit 0: Overview
- Entrance Survey
- Unit 1: Probability models and axioms
- Unit 2: Conditioning and independence
- Unit 3: Counting
- Unit 4: Discrete random variables
- Exam 1
- Unit 5: Continuous random variables

Exercise: The time of the kth arrival

(2 points possible)

Let Y_k be the time of the kth arrival in a Poisson process with parameter $\lambda = 1$. In particular, $\mathbf{E}[Y_k] = k$.

Is it true that $\mathbf{P}(Y_k \geq k) = 1/2$ for any finite k?

Yes ✖ **Answer:** No

Is it true that $\lim_{k\to\infty} \mathbf{P}(Y_k \geq k) = 1/2$?

No ✖ **Answer:** Yes

Answer:
Consider the special case of $k = 1$. Then, $\mathbf{P}(Y_1 \geq 1) = e^{-1} \neq 1/2$.

- Unit 6: Further topics on random variables
- Unit 7: Bayesian inference
- Exam 2
- Unit 8: Limit theorems and classical statistics
- **Unit 9: Bernoulli and Poisson processes**

Unit overview

Lec. 21: The Bernoulli process
Exercises 21 due May 11, 2016 at 23:59 UTC

Lec. 22: The Poisson process
Exercises 22 due May 11, 2016 at 23:59 UTC

Lec. 23: More on the Poisson process

When k is large, the central limit theorem applies because Y_k is the sum of k i.i.d. (exponential) random variables. Its (standardized) distribution is approximately normal, hence approximately symmetric around its mean. More formally, using the fact that the variance of an exponential with parameter 1 is 1, we have

$$\lim_{k\to\infty} \mathbf{P}(Y_k \geq k) = \lim_{k\to\infty} \mathbf{P}\left(\frac{Y_k - k}{\sqrt{k}} \geq 0\right) = \Phi(0) = \frac{1}{2},$$

where Φ is is the standard normal CDF.

You have used 1 of 1 submissions

Exercises 23 due May 11, 2016 at 23:59 UTC

Solved problems

Additional theoretical material

Problem Set 9
Problem Set 9 due May 11, 2016 at 23:59 UTC

Unit summary

Unit 10: Markov chains