MITx: 6.041x Introduction to Probability - The Science of Uncertainty

Exercise: Theoretical properties

(1/2 points)

Let $\widehat{\Theta}$ be an estimator of a random variable Θ, and let $\widetilde{\Theta} = \widehat{\Theta} - \Theta$ be the estimation error.

a) In this part of the problem, let $\widehat{\Theta}$ be specifically the LMS estimator of Θ. We have seen that for the case of the LMS estimator, $\mathbf{E}[\widetilde{\Theta} \mid X = x] = 0$ for every x. Is it also true that $\mathbf{E}[\widetilde{\Theta} \mid \Theta = \theta] = 0$ for all θ? Equivalently, is it true that $\mathbf{E}[\widehat{\Theta} \mid \Theta = \theta] = \theta$ for all θ?

Yes ✖ **Answer:** No

b) In this part of the problem, $\widehat{\Theta}$ is no longer necessarily the LMS estimator of Θ. Is the property $\mathbf{var}(\Theta) = \mathbf{var}(\widehat{\Theta}) + \mathbf{var}(\widetilde{\Theta})$ true for every estimator $\widehat{\Theta}$?

No ✔ **Answer:** No

Answer:
a) There is no reason for this relation to be true. For an example, suppose that Θ is a Bernoulli random variable. With a noisy measurement, $\widehat{\Theta}$ will be somewhere in between 0 and 1, and therefore will never be equal to the true value of θ, which is either 0 or 1 exactly.

b) There is no reason for this to be the case. In fact, the variance of $\widehat{\Theta}$, for a poorly chosen estimator, can be larger than the variance of Θ. For an example, consider the usual model of an observation $X = \Theta + W$ and the estimator $\widehat{\Theta} = 100X$.

You have used 1 of 1 submissions