4. Modeling Assumptions in Regression

Review: Joint, Conditional, and Marginal Distributions

2/2 points (graded)

Let (X, Y) be a pair of random variables with **joint** density $h(x, y) = x + y$ over the space $[0, 1]^2$.

What is the **marginal density** of X? We denote this by writing $h(x)$.

$h(x) =$ x+1/2 ✔ **Answer:** 0.5+x

$x + \frac{1}{2}$

What is the **conditional density** of Y given $X = x$? We denote this by writing $h(y|x)$.

$h(y|x) =$ (x+y)/(x+1/2) ✔ **Answer:** (x+y)/(0.5+x)

$\frac{x+y}{x+\frac{1}{2}}$

STANDARD NOTATION

Solution:

The marginal density $h(x)$ is computed by integrating over y:

$$\begin{aligned} h(x) &= \int_0^1 h(x,y)\,dy \\ &= \left[xy + \frac{y^2}{2}\right]_0^1 \\ &= x + \frac{1}{2} \end{aligned}$$

The conditional density is computed as the ratio:

$$\begin{aligned} h(y|x) &= \frac{h(x,y)}{h(x)} \\ &= \frac{x+y}{x+\frac{1}{2}} \end{aligned}$$

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Conditional Variance Given x

1/1 point (graded)

Consider the joint density setup as in the previous problem. What is the variance of Y given $X = x$?

Var $(Y|X=x) =$ (x^2/12+x/12+1/72)/(x+1 **Answer:** (x/3+1/4)/(0.5+x) - ((x/2+1/3)/(0.5+x))^2 + y*0

$$\frac{\frac{x^2}{12}+\frac{x}{12}+\frac{1}{72}}{\left(x+\frac{1}{2}\right)^2}$$

STANDARD NOTATION

Solution:

The conditional density $h(y|x)$ is

$$h(y|x) = \frac{x+y}{0.5+x}.$$

We need to compute the expectations $\mathbb{E}[Y|X=x]$ and $\mathbb{E}[Y^2|X=x]$ in order to compute the conditional variance of Y given $X=x$.

$$\begin{aligned}\mathbb{E}[Y|X=x] &= \int_{y=0}^{y=1} \frac{y(x+y)}{0.5+x} \mathrm{d}y \\ &= \frac{1}{0.5+x} \int_0^1 yx + y^2 \mathrm{d}y \\ &= \frac{1}{0.5+x} \left[\frac{x}{2} + \frac{1}{3}\right] \\ &= \frac{\frac{x}{2}+\frac{1}{3}}{0.5+x}\end{aligned}$$

Similarly,

$$\begin{aligned}\mathbb{E}[Y^2|X=x] &= \int_0^1 \frac{y^2(x+y)}{0.5+x} \mathrm{d}y \\ &= \frac{1}{0.5+x} \int_0^1 y^2 x + y^3 \mathrm{d}y \\ &= \frac{1}{0.5+x} \left[\frac{x}{3} + \frac{1}{4}\right] \\ &= \frac{\frac{x}{3}+\frac{1}{4}}{0.5+x}\end{aligned}$$

Therefore, the conditional variance given $X=x$ is

$$\text{Var}(Y \mid X = x) = \mathbb{E}[Y^2|X = x] - \mathbb{E}[Y|X = x]^2 = \frac{\frac{x}{3} + \frac{1}{4}}{0.5 + x} - \left(\frac{\frac{x}{2} + \frac{1}{3}}{0.5 + x}\right)^2.$$

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Review: Joint, Conditional, and Marginal Distributions: Discrete Example

4/4 points (graded)

Let X be a discrete Poisson random variable with parameter λ. Given $X = x$, let Y be the binomial random variable **Binom** (x, p), where p is the binomial parameter.

Given $X = x$, what are the values that Y can take?

Lower limit of Y given $X = x$: 0 ✔ **Answer:** 0

Upper limit of Y given $X = x$: x ✔ **Answer:** x

x

Is $\mathbb{E}[Y \mid X = x]$ a linear function of x?

- (•) Yes
- ( ) No

✔

What is $\mathbb{E}[Y]$?

Hint: Use the tower property of expectation (law of iterated expectation).

lambda*p

✔ **Answer:** p*lambda

$\lambda \cdot p$

STANDARD NOTATION

Solution:

Given $X = x$, it is clear that Y can take values in the set $\{0, 1, \ldots, x\}$. The expectation of Y given $X = x$ is xp as $Y|X = x$ is a binomial random variable with parameters x and p. Therefore, this expectation is a linear function of x. Using the law of iterated expectation,

$$\begin{aligned} \mathbb{E}\left[Y\right] &= \mathbb{E}_X\left[\mathbb{E}\left[Y|X\right]\right] \\ &= \mathbb{E}_X\left[Xp\right] = p \cdot \lambda. \end{aligned}$$

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You have used 1 of 3 attempts

Answers are displayed within the problem

Modeling Assumptions

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