16. Applications of diagonalization

Powers of a matrix

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Suppose that $\mathbf{A} = \mathbf{SDS}^{-1}$. Then

$$\mathbf{A}^3 = \mathbf{SD}\underbrace{\mathbf{S}^{-1}\mathbf{S}}_{\text{cancels}}\mathbf{D}\underbrace{\mathbf{S}^{-1}\mathbf{S}}_{\text{cancels}}\mathbf{DS}^{-1} = \mathbf{SD}^3\mathbf{S}^{-1}.$$

More generally, for any integer $n \geq 0$,

$$\mathbf{A}^n = \mathbf{SD}^n\mathbf{S}^{-1}.$$

Problem 16.1 Compute $\mathbf{A}^{10}$ for $\mathbf{A} = \begin{pmatrix} -2 & 1 & 1 \\ 1 & -2 & 1 \\ 1 & 1 & -2 \end{pmatrix}$.

Solution : We have previously diagonalized this matrix:

$$\begin{aligned} \mathbf{A} \quad &= \quad \mathbf{SDS}^{-1} \qquad \text{where} \quad \mathbf{S} = \begin{pmatrix} 1 & -1 & -1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} \\ & \qquad\qquad\qquad\qquad\qquad \mathbf{D} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & -3 \end{pmatrix}. \end{aligned}$$

This gives

$$\mathbf{A}^{10} \quad = \quad \left(\mathbf{SDS}^{-1}\right)$$

$$= \mathbf{S}\mathbf{D}^{10}\mathbf{S}^{-1}$$

$$= \begin{pmatrix} 1 & -1 & -1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 \\ 0 & (-3)^{10} & 0 \\ 0 & 0 & (-3)^{10} \end{pmatrix} \begin{pmatrix} 1 & -1 & -1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}^{-1}$$

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