2. Rotating a vector

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Lecture due Sep 15, 2021 20:30 IST

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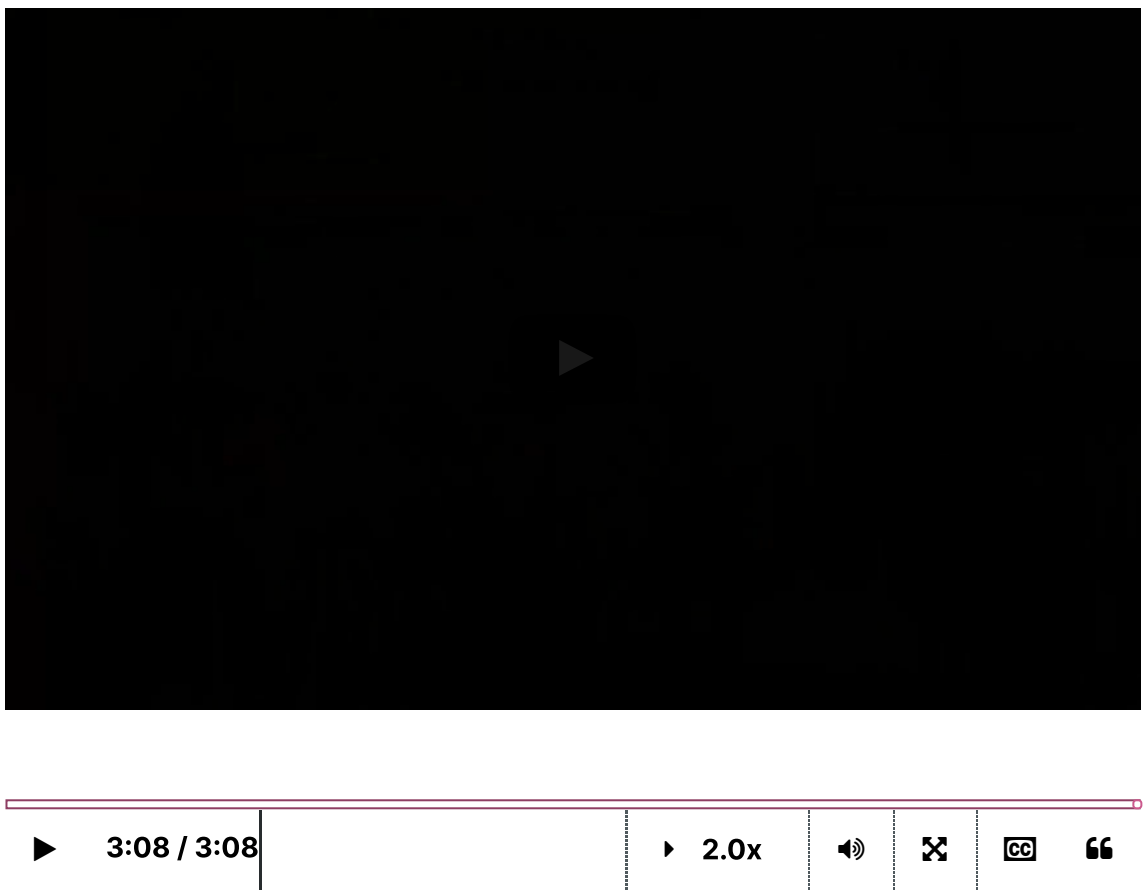
Rotate a Vector



instead of being in the x direction,

let's think about the case where v is in the y direction.

So v is 0 comma v2.

So I'll draw that one in red.

So there is v. And then we rotate it by an angle theta

and get a new vector.

We'll call that one w.

And I'd like you to find w.

So take a minute and talk with your neighbor,

try to figure out what is w.

And I'll put up some choices.

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Rotating a vector introduction

Recall: a vector is a pair of numbers, which we can visualize by drawing an arrow in the plane. We are going to introduce matrices and linear functions by solving a concrete problem: how does rotating a vector change its coordinates?

Notation: In previous sections, we would typically write a vector as $< v_1, v_2 >$. In this section, we will be using the notation $\begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$ instead, because it is better suited to working with matrices.

Consider a vector $\vec{v} = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$. Suppose we want to rotate $\vec{v}$ counterclockwise by an angle θ to get a new vector $\vec{w}$.

First, consider the case $v_2 = 0$, so $\vec{v} = \begin{pmatrix} v_1 \\ 0 \end{pmatrix}$. Let's find $\vec{w}$ where $\vec{w}$ is the vector created by rotating this $\vec{v}$ counterclockwise by an angle θ.

Since $\vec{v} = \begin{pmatrix} v_1 \\ 0 \end{pmatrix}$, we know that

$$|\vec{w}| = |\vec{v}| = v_1. \tag{5.1}$$

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Then, using the components of the right-triangle, we see that

$$\vec{w} = \begin{pmatrix} |\vec{v}| \cos\theta \\ |\vec{v}| \sin\theta \end{pmatrix} = \begin{pmatrix} v_1 \cos\theta \\ v_1 \sin\theta \end{pmatrix}. \tag{5.2}$$

So

$$\boxed{\vec{w} = \begin{pmatrix} v_1 \cos\theta \\ v_1 \sin\theta \end{pmatrix}}. \tag{5.3}$$

Vertical Example

1/1 point (graded)
Vertical Example

Now consider the case when $\vec{v} = \begin{pmatrix} 0 \\ v_2 \end{pmatrix}$. Find $\vec{w}$ where $\vec{w}$ is the vector created by rotating $\vec{v}$ counterclockwise by an angle θ.

- $\begin{pmatrix} -v_2 \cos\theta \\ v_2 \sin\theta \end{pmatrix}$
- ✔ $\begin{pmatrix} -v_2 \sin\theta \\ v_2 \cos\theta \end{pmatrix}$
- $\begin{pmatrix} v_2 \sin\theta \\ v_2 \cos\theta \end{pmatrix}$

✔

Solution:

The correct answer is $\begin{pmatrix} -v_2 \sin\theta \\ v_2 \cos\theta \end{pmatrix}$.

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