1.6.3 Elasticity and Maximizing Revenue

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In our model, we found that revenue is maximized when $p = 2.86$, close to three dollars! Now this would be a high price for a subway ticket in 1980. Clearly it would not have been reasonable for the Boston MBTA to set prices there, and this is true for many reasons as we discussed earlier.

But leaving this aside for a moment, let's look at how maximizing revenue relates to elasticity. Previously, we saw that elasticity was not constant for the linear demand curve of Boston. Elasticity at different points was different, or in other words, riders react differently at different prices. What's the sweet spot for elasticity to maximize revenue?

For example, elasticity at 0.5 was about $-.1$ while at 1 dollar it was more negative, $-.2$. When we look at elasticity at the price point where revenue **is at its maximum**, at $p = 2.86$, we notice that it equals -1. So at the point of maximum revenue, elasticity is -1, meaning a percent increase in price causes the percent decrease in demand of the exact same size.

Informally, this makes sense.

- When demand is inelastic, we predict that raising prices increases revenue, since percent change in demand is less than percent change in price.
- When demand is elastic, we predict that lowering prices increases revenue, since percent change in demand is greater than percent change in price.
- So to maximize revenue we either raise the price until demand stops being inelastic or lower it until demand stops being elastic. This boundary is $E = -1$.

But is this always true? Does the calculus support our intuition? The answer is yes. If there is a maximum revenue, it has to occur where the absolute value of the elasticity is 1.

This is the sweet spot, the boundary between elastic and inelastic demand, where we can balance the benefit of an increase in price with the drawback of a decrease in demand.

To show this, let's consider a good whose demand function $q(p)$ we don't know explicitly, but which we assume is differentiable and non-zero. The revenue function for this good looks like price times demand:

$$R(p) = p \cdot q(p)$$

Let's do a thought experiment. If $R(p)$ has a maximum, it would have to be at a critical point, and we want to show that elasticity is -1 at that critical point.

- Since our function is differentiable, the only critical points are when the derivative equals zero. We take the derivative using the chain rule and get

$$R'(p) = q(p) + p \cdot q'(p)$$

- When we set $R'(p)$ equal to zero,

$$R'(p) = q(p) + p \cdot q'(p) = 0$$

  we can't solve for the price p explicitly because we don't know the demand function $q(p)$ or its derivative $q'(p)$ explicitly. Instead we'll try to connect $R'(p)$ to the point price elasticity formula, $E = q'(p)\frac{p}{q}$.

- We rearrange the equation by subtracting $q(p)$ from both sides and then dividing by $q(p)$:

$$q(p) + p \cdot q'(p) = 0$$

$$p \cdot q'(p) = -q(p)$$

$$\frac{p}{q(p)} q'(p) = -1$$

$$E(p) = -1.$$

  So we get that if $R(p)$ has a critical point, this will happen when when the point price elasticity is -1. The converse is true as well: if elasticity is -1, then $R(p)$ has a critical point.

  What can we conclude? If $R(p)$ has a maximum at some point, then point price elasticity is −1 at that point as well: a percent increase in price causes the exact same percent decrease in demand at that point. Now, as we saw already, it may not be realistic or even desirable for a transit authority to set fare prices to where elasticity is -1 in order to maximize revenue. However, knowing the effect of elasticity on revenue can help inform changes in price in certain situations, like monopolies.

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