1.6.2 Summary Quiz: Part 2: Spectral Radiance of Electromagnetic Radiation

Bookmark this page

As we've seen, simplification of a theory via Taylor approximation can help us connect a more complicated theory to a simpler theory, one that is well-studied and is well understood. The last problems in this quiz are another example of this. (Taylor approximation is also useful in studying systems of differential equations, such as motion of a pendulum.)

Let's think about the **spectral radiance of electromagnetic radiation** at all wavelengths from a black body at a given temperature. An example of an object that is treated as a black body is the sun. Spectral radiance is how much energy per time (per unit area per unit solid angle) the object emits.

Planck's law, empirically derived by Max Planck in 1900, gives the spectral radiance of electromagnetic radiation of such an object. This law was instrumental in developing quantum mechanics.

Here is one formulation of Planck's law:

$$R = \frac{8\pi\nu^3 h}{c^3\left(e^{h\nu/kT} - 1\right)}$$

where

- T is temperature of the object, measured in Kelvin,
- ν is frequency of the radiated light,
- c is the speed of light, $\approx 3 \cdot 10^8$ m/s,
- k is Boltzmann's constant, $1.38 \cdot 10^{-23}$ m$^2\cdot$ kg/(s$^2 \cdot K$)
- h is Planck's constant $6.626 \cdot 10^{-34}$ m$^2\cdot$kg/s

Note: Frequency has an inverse relationship to wavelength, and is equal to the speed of the wave divided by the wavelength. In this case our "waves" are light with speed c. So we could also formulate the law in terms of wavelength instead.

Question 5

1/1 point (graded)

Find the first-order (linear) Taylor approximation of the exponential e^x around the center $x = 0$. Then let $x = h\nu/kT$ to make a first-order approximation of Planck's law.

Which is the first-order approximation of Planck's law?

- $\frac{8\pi h\nu^3}{c^3}$
- $\frac{8\pi\nu^2 kT}{c^3}$ ✔
- $\frac{8\pi h\nu^3 kT}{c^3(2+h\nu/(kT))}$
- $\frac{16\pi\nu k^2T^2}{c^3 h}$
- None of the above.

Explanation

The first order Taylor approximation (linear approximation) of $f(x) = e^x$ around $x = 0$ is $f(0) + f'(0)x = 1 + x$. We then let $x = h\nu/kT$ to get $e^{h\nu/kT} \approx 1 + h\nu/kT$. Finally, we replace the denominator of Planck's law $e^{h\nu/kT} - 1$ with this approximation. So we now have $\frac{8\pi\nu^3 h}{c^3(e^{h\nu/kT}-1)} \approx \frac{8\pi\nu^3 h}{c^3(h\nu/kT)} = \frac{8\pi\nu^2 kT}{c^3}$.

This is known as Rayleigh Jean's law and was discovered by classical physics arguments around the same time as Planck's law. See the Wikipedia entry for more information.

Submit

You have used 1 of 3 attempts

Answers are displayed within the problem

Question 6

1/1 point (graded)

The first-order approximation of Planck's law from the previous problem will be a good approximation for spectral radiance in which situations?

- $h\nu/kT$ is approximately one
- $h\nu$ is large compared to kT
- $h\nu$ is small compared to kT ✔
- None of the above.

Explanation
When we used the Taylor approximation of the exponential, we used the Taylor expansion for e^x around $x = 0$. Since we let $x = h\nu/kT$, this will only be a good approximation if $h\nu/kT \approx 0$, meaning $h\nu$ is small compared to kT.

Submit You have used 1 of 2 attempts

Answers are displayed within the problem

Question 7

1/1 point (graded)
What does Planck's law predict for the spectral radiance as $\nu \to \infty$? (Note: frequency ν and wavelength are inversely related, so larger frequency corresponds to shorter wavelengths.)

- $8\pi\nu^2 kT/c^3$
- 0 ✔
- ∞
- $8\pi\nu^2/c^3$

Explanation

To find the limit as $\nu \to \infty$, we need to take the limit of the full law as a function of ν:

$$\lim_{\nu\to\infty} R(\nu) = \lim_{\nu\to\infty} \frac{8\pi\nu^3 h}{c^3\left(e^{h\nu/kT} - 1\right)} = \lim_{\nu\to\infty} \frac{8\pi h}{c^3} \frac{\nu^3}{e^{h\nu/kT} - 1} = 0.$$

since the first factor is a constant, and the denominator $e^{h\nu/kT} - 1$ goes to ∞ faster than $\nu^3 \to \infty$.

Submit You have used 1 of 2 attempts

Answers are displayed within the problem

Problem 8

1/1 point (graded)

The first-order Talyor approximation of Planck's law disagrees greatly with Planck's law as frequency gets large (or wavelengths get small). What does the first-order Taylor approximation predict for spectral radiance as $\nu \to \infty$ (small wavelengths)?

- $8\pi\nu^2 kT/c^3$
- 0
- ∞ ✔
- $8\pi\nu^2/c^3$

Explanation

The approximation is $\frac{8\pi\nu^2 kT}{c^3}$. This is proportional to ν^2, so as $\nu \to \infty$, the approximation goes to ∞. Infinite radiance is very different from what Planck's predicts, which is 0 radiance.

Submit You have used 1 of 2 attempts

Answers are displayed within the problem

The disagreement between Planck's and the low-frequency approximation happens when frequencies get large. As frequency gets large, this means that wavelengths get small. Smaller wavelengths just outside the visible spectrum correspond to ultraviolet light, hence this disagreement was named the "ultraviolet catastrophe."

Learn About Verified Certificates

English ▼

POWERED BY OPENedX

Download on the App Store

ANDROID APP ON Google play