6. Data fitting and power laws

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Not all models are linear, so we can't expect to fit all data with lines accurately.

More generally, when we have a relationship of the form $y = cx^p$ for some constant c and power p, we call this relationship a power law. Power laws occur all over the natural and manmade world.

Example 6.1

The force of gravity between masses is given by the formula

$$F = \frac{Gm_1m_2}{r^2} = (Gm_1m_2)\, r^{-2}$$

where m_1 and m_2 are the masses of the objects, and r is the distance between them. This force and planet separation satisfy a power law, where the power is -2.

It turns out that you can transform your data to obtain new data that will allow you to use a procedure of least squares approximation to do a power law fitting of your data! We will work through an example together to explain the procedure.

One of Kepler's Laws says that there is a power law relationship between the mean distance of a planet and the period of its revolution about the sun. Letting x represent the mean distance (measured in Astronomical Units, AU) and y represent the period of revolution about the sun in its orbit (measured in days). This means we expect a relationship $y = cx^p$ to hold, for some unknown values of c and p. Let's use some real-world data to find the values of c and p.

Here are the data for the mean distances and periods of revolution of the planets in our solar system.

Planet	**Mean Distance (AU)**	**Period (days)**
Mercury	0.389	87.77
Venus	0.724	224.7
Earth	1	365.25
Mars	1.524	686.95
Jupiter	5.2	4332.62
Saturn	9.51	10759.2

We are going to use these data to find this power law using least squares approximation on a transformed data set. Here are a couple of problems to get started.

Take the natural log

1/1 point (graded)
Take the natural logarithm of both sides of the equation $y = cx^p$.

Let $Y = \ln(y)$, let $X = \ln(x)$, and $C = \ln(c)$.

Find an equation for Y in terms fo X, C, and p. (Note the answer box is case sensitiv Calculator Hide Notes

$Y =$ C+p*X ✔ **Answer:** C+p*X

$C + p \cdot X$

? INPUT HELP

Solution:

Taking the log of both sides of the power law $y = cx^p$ we get

$$\ln y = \ln\left(cx^p\right) = \ln c + p \ln x \tag{4.214}$$

Substituting $Y = \ln(y)$ and $X = \ln(x)$, we get a linear relationship

Therefore $Y = C + pX$.

Submit You have used 1 of 3 attempts

Answers are displayed within the problem

Finding the power

1/1 point (graded)

Given data x_i and y_i satisfying $y_i = cx_i^p$, you find transformed data $Y_i = \ln y_i$ and $X_i = \ln x_i$. Which quantity best describes the power p?

- (selected) The slope, m, of the best fit line $Y_i = mX_i + b$
- The exponential of the slope, e^m, of the best fit line $Y_i = mX_i + b$
- The Y-intercept, b, of the best fit line $Y_i = mX_i + b$
- The exponential of the Y-intercept, e^b, of the best fit line $Y_i = mX_i + b$
- None of the above

✔

Solution:

The best fit line is $Y_i = \ln c + pX_i$. The best fit line to these transformed data will have a slope p. Thus the slope of the best fit line is what gives us the power in the power law.

Note that to get the constant multiple, we need to take the exponential of the Y-intercept of this best fit line, which gives us $e^{\ln c} = c$.

Submit You have used 1 of 2 attempts

Answers are displayed within the problem

The procedure for data fitting to a power law

1. Take the logarithm of all data. (This is equivalent to plotting your data on log-log sc

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2. Perform least square approximation to the transformed data.

3. The slope is the power in the power law. The intercept in the vertical axis is the natural log of the multiplicative constant.

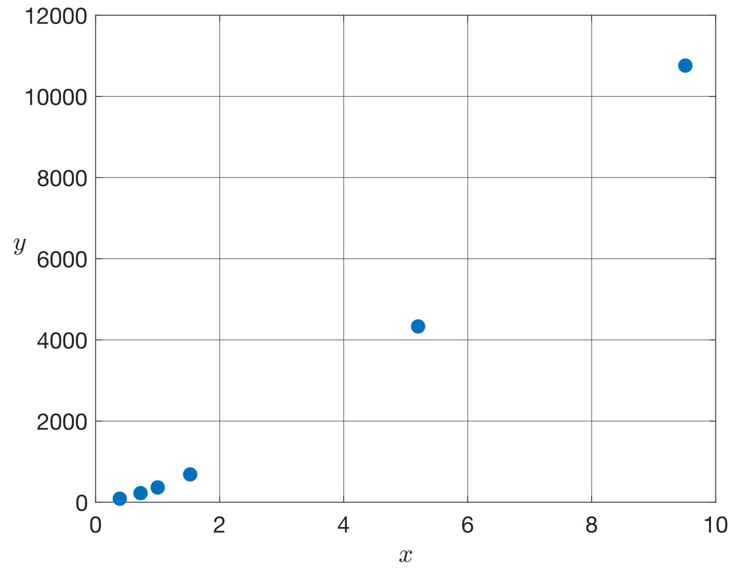
4. (Optional) Take the exponential of the least square fit line to obtain the equation of the power law.

The reason the last step is optional is that the linear fit in log-log gives us all the information we need!

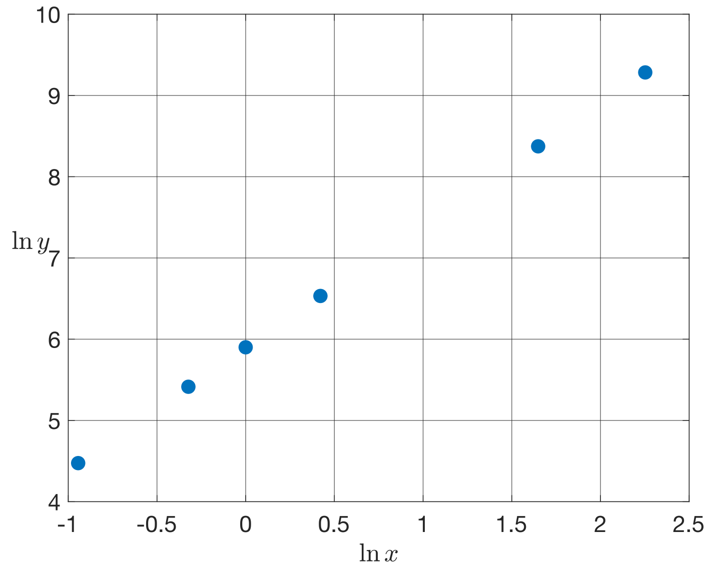
Example 6.2

Let's find the power law of our data from Kepler's third law example.

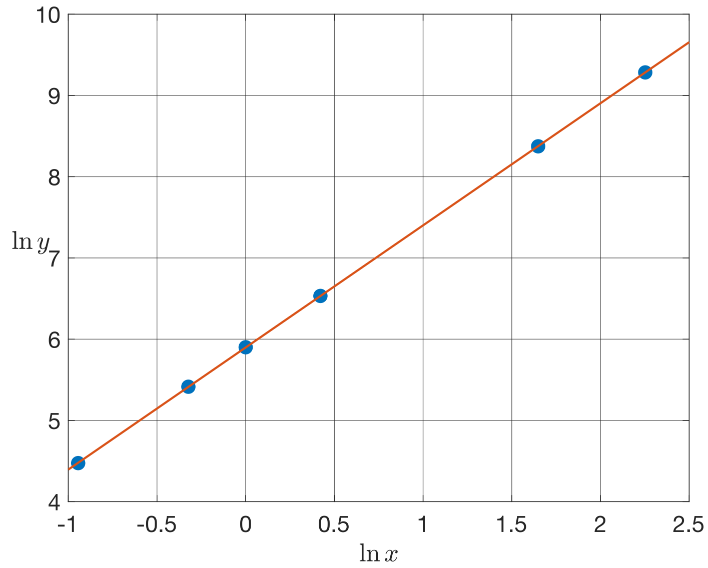
Here, our variables are x, the mean distance of the planet from the sun, and y, the days it takes the planet to orbit the sun. Here is a plot of the data from the table above.



Here is a plot of the data of the transformed data $\ln x$ and $\ln y$.



Running least squares approximation on the transformed data, we obtain the line pictured in orange.



The slope (obtained here using MATLAB) was found to be 1.5031. In fact, Kepler's third law states that p = 3/2. We can conclude that our data matched this law very well.

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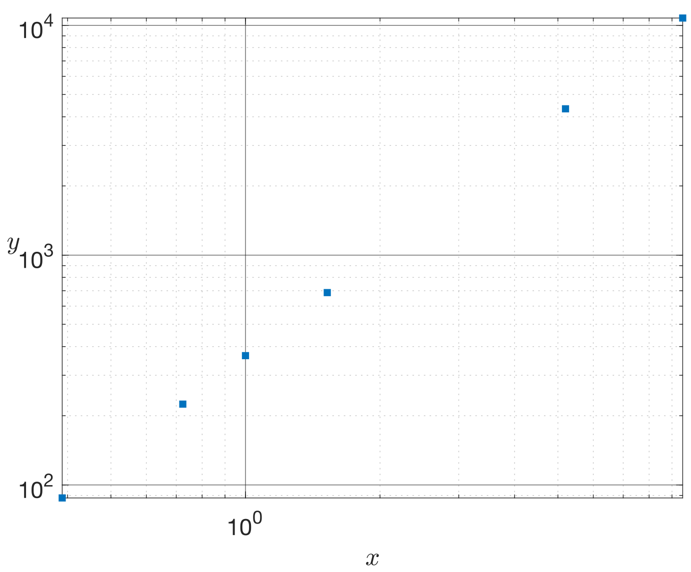
(Kepler's third law states that the squares of the time it takes a planet to orbit its sun is directly proportional to the cube of the mean between the planet and its sun.)

Remark 6.3

Note that you can obtain the same plot as the transformed data by transforming your axes rather than transforming the data. You can plot on log scale rather than the usual scale, and you get the same picture! Such a plot is called a log-log plot.



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Remark 6.3 (log log plot?)
Is it really called the log log plot, or is it a typo and should be corrected to log plot? 5

Calculating without Taking Logarithm
If I calculate the least squares directly by D=(y_i-c*x_i^p)^2 and finding the minimum of D(if I can solve it), will the answer be differen... 4

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