5. Justification

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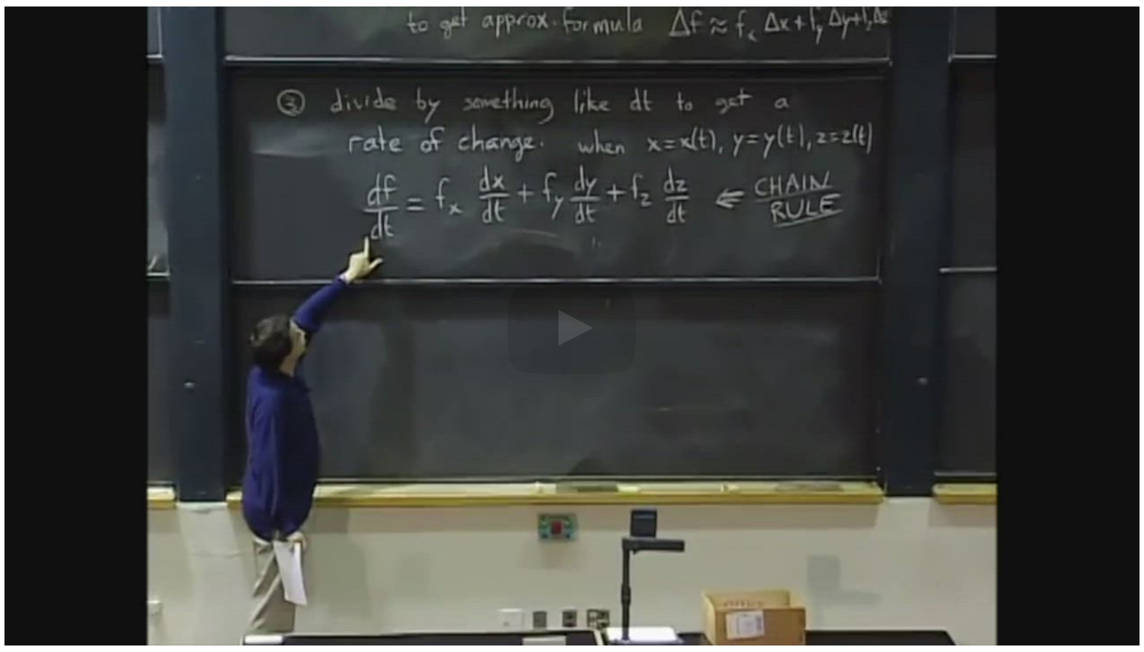
Justify the Chain Rule



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PROFESSOR: I mean, you probably have a right

to expect some reason for why this works.

Why is this valid?

After all, I first told you we have

this new mysterious object.

And then I'm telling you, well, we can do that.

But, you know, I kind of have pulled it

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From now on, we will sometimes just use the phrase "chain rule" instead of "multivariable chain rule."

Why is the chain rule true?

On the previous page, we claimed that you could “divide everything by dt" to obtain the chain rule. To justify this requires clearly interpreting differentials. However, the justifications given here are not entirely rigorous, and are just meant to give you an idea of what is going on when we write equations with differentials.

1st attempt

One justification is that, if x, y, and z are all functions of t, then we can compute dx, dy, and dz directly using single-variable calculus.

$$dx = x'(t)\,dt, \quad dy = y'(t)\,dt, \quad dz = z'(t)\,dt. \tag{6.128}$$

By substitution, we obtain the "chain rule" statement made on the previous page.

$$df = f_x x'(t)\,dt + f_y y'(t)\,dt + f_z z'(t)\,dt \tag{6.129}$$

$$= (f_x x'(t) + f_y y'(t) + f_z z'(t))\,dt \tag{6.130}$$

Now we have an equation for df in terms of only t and dt. The coefficient on dt must be the derivative of f with respect to t.

2nd attempt

Another way to convince ourselves of the chain rule is to replace the d's by Δ's. This removes any ambiguity about the meaning.

We know from linear approximation that

$$\Delta f \approx f_x\,\Delta x + f_y\,\Delta y + f_z\,\Delta z \tag{6.131}$$

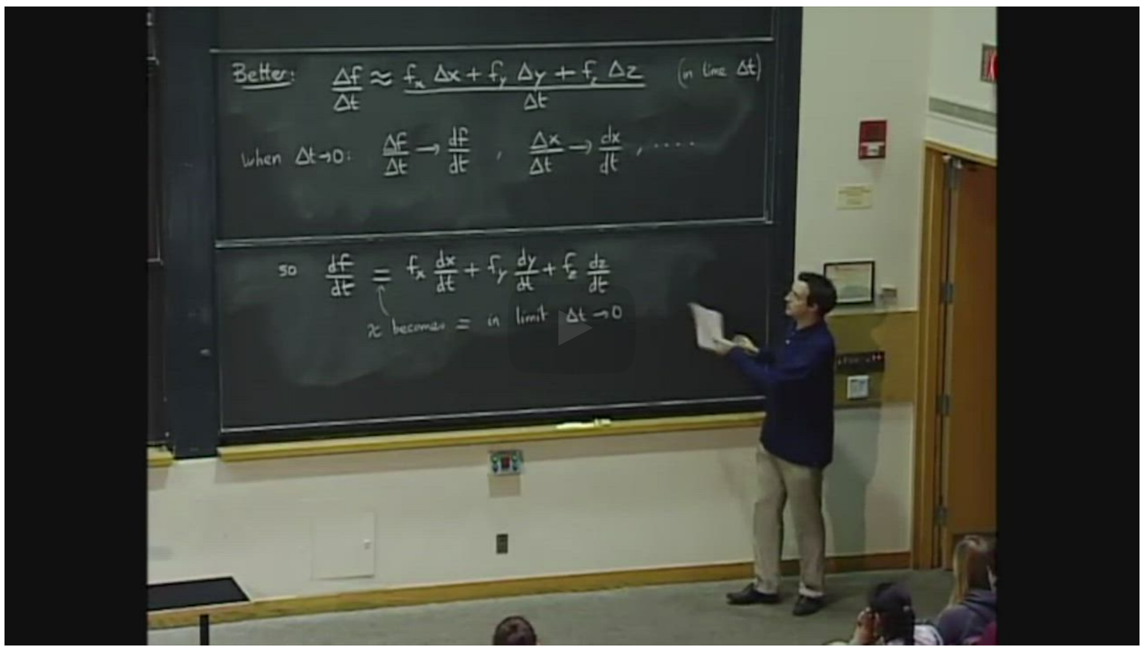
Dividing by Δt gives

$$\frac{\Delta f}{\Delta t} \approx \frac{f_x\,\Delta x + f_y\,\Delta y + f_z\,\Delta z}{\Delta t} \tag{6.132}$$

$$\frac{\Delta f}{\Delta t} \approx f_x\frac{\Delta x}{\Delta t} + f_y\frac{\Delta y}{\Delta t} + f_z\frac{\Delta z}{\Delta t} \tag{6.133}$$

Now if we imagine Δt moving towards zero, we can replace all Δ's with d's to get a correct statement.

Example of the Chain Rule



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PROFESSOR: OK, so let's check maybe things in an example.

Let's say that we really don't have any faith in these things.

And so let's try to do it.

So let's say I give you a function

that's x squared y plus z.

And let's say that maybe x will be t, y will be e to the t,

and z will be sine t.

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Example 5.1 Let's look at an example of the chain rule in practice. Suppose we have a quantity w that depends on x, y, and z as

$$w = x^2 y + z \tag{6.134}$$

Now suppose we can't control x, y, and z directly, but they each depend on the parameter t:

$$x(t) = t \tag{6.135}$$

$$y(t) = e^t \tag{6.136}$$

$$z(t) = \sin t \tag{6.137}$$

Changing the variable t will cause w to change, and we would like to know the corresponding rate of change $\frac{dw}{dt}$.

The chain rule tells us that

$$\frac{dw}{dt} = \underbrace{(2xy)}_{w_x}\frac{dx}{dt} + \underbrace{x^2}_{w_y}\frac{dy}{dt} + \underbrace{1}_{w_z}\frac{dz}{dt} \qquad (6.138)$$

Now substituting in the formulas for x, y, and z gives a final answer:

$$\frac{dw}{dt} = 2te^t + t^2e^t + \cos t \qquad (6.139)$$

Checking the answer with the old way

Since the above method used a new technique (the chain rule) let's make sure we get the same answer using our old methods. Namely, in this case, we can write down the full formula for w as a function of t and take a single-variable derivative.

$$w = x^2y + z \qquad (6.140)$$

$$w(t) = t^2e^t + \sin t \qquad (6.141)$$

$$w'(t) = 2te^t + t^2e^t + \cos t \qquad (6.142)$$

Indeed, the answers match.

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