14. Likelihood of a Discrete Distribution

Preparation: Equivalent Expressions for the pmf of a Bernoulli Distribution

1/1 point (graded)

Which of the following function $f(x)$, when restricted to the domain $x \in \{0,1\}$, is equal to the pmf f of the probability distribution $\mathrm{Ber}(p)$? Assume that $p \in (0,1)$. (Choose all that apply.) (Recall that if $X \sim \mathrm{Ber}(p)$, then $p = P(X=1)$.)

- [x] $f(x) = \begin{cases} p & \text{if } x=1 \\ 1-p & \text{if } x=0 \end{cases}$
- [x] $f(x) = p^x (1-p)^{1-x}$
- [x] $f(x) = xp + (1-x)(1-p)$
- [ ] $f(x) = \begin{cases} 1 & \text{if } x=1 \\ 0 & \text{if } x=0 \end{cases}$

✔

Solution:

We will explain in the order of the choices.

- $f(x) = \begin{cases} p & \text{if } x = 1 \\ 1-p & \text{if } x = 0 \end{cases}$ is correct. A random variable $X \sim \mathrm{Ber}(p)$, by definition, has sample space $\{0, 1\}$ and satisfies $P(X = 1) = p$ and $P(X = 0) = 1 - p$. The given function is just a restatement of that definition.
- $f(x) = p^x (1-p)^{1-x}$ is correct. Note that $f(1) = p$ and $f(0) = 1 - p$, so this is the same as the function considered in the first choice.
- $f(x) = xp + (1-x)(1-p)$ is correct. It also satisfies $f(1) = p$ and $f(0) = 1 - p$, so f is the same as the function considered in the first choice.
- $f(x) = \begin{cases} 1 & \text{if } x = 1 \\ 0 & \text{if } x = 0 \end{cases}$ is incorrect. This is actually the probability mass function of $\mathrm{Ber}(1)$, but we have assumed $p \in (0, 1)$.

Submit You have used 2 of 2 attempts

Answers are displayed within the problem

Review: Statistical Model for a Bernoulli Distribution

3/3 points (graded)

Let $X_1, \ldots, X_n \overset{iid}{\sim} \mathrm{Ber}(p^*)$ for some unknown $p^* \in (0, 1)$. Let $(E, \{\mathrm{Ber}(p)\}_{p \in \Theta})$ denote the corresponding statistical model. What is the smallest possible set that could be E?

- ( ) $\{0\}$
- ( ) $\{-1, 1\}$
- (•) $\{0, 1\}$

- $\mathbb{R}$

The parameter space Θ can be written as an interval $[a, b]$. What is the smallest possible interval so that $\{\text{Ber}(p)\}_{p\in\Theta}$ represents all possible Bernoulli distributions?

$a =$ 0 **Answer:** 0.0

$b =$ 1 **Answer:** 1.0

Solution:

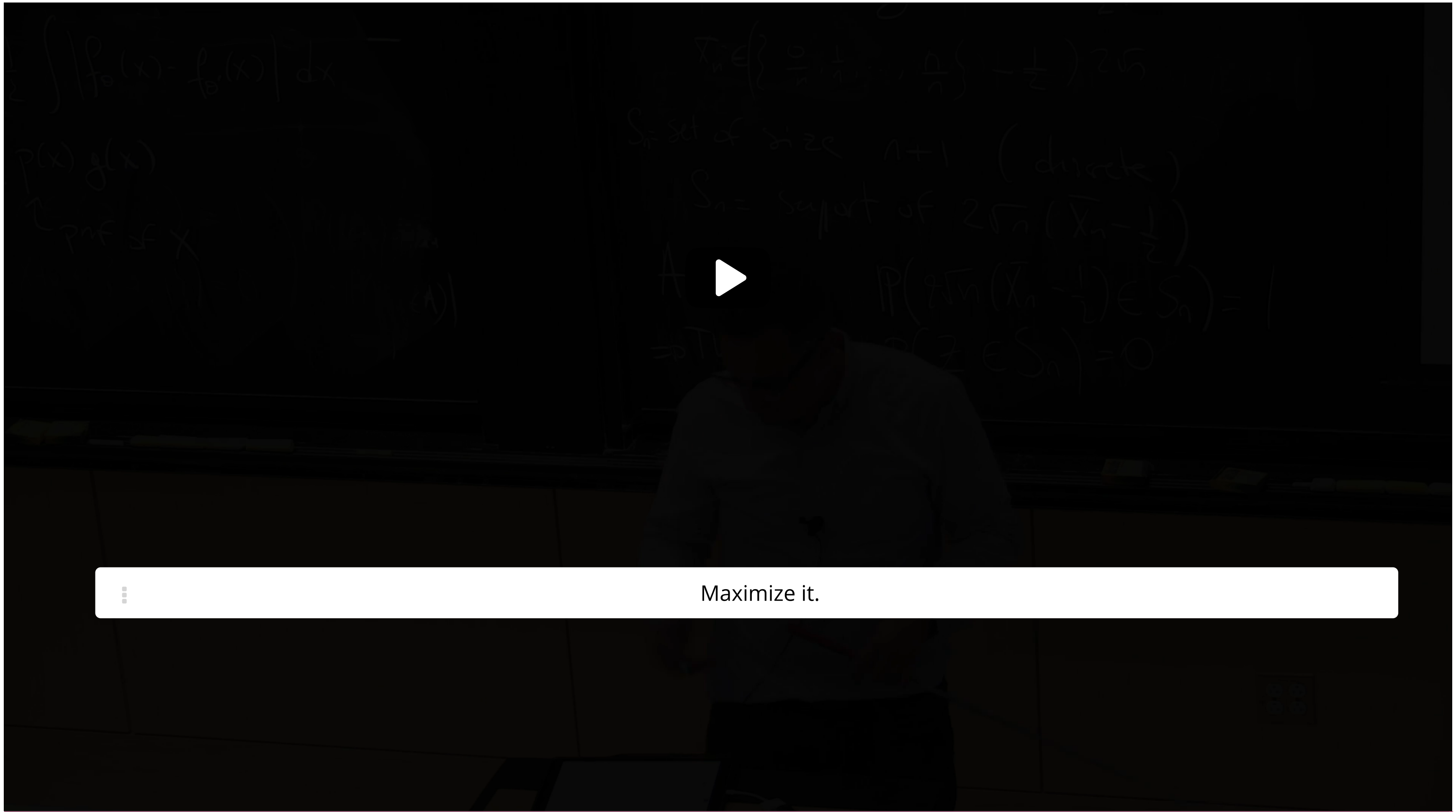
Since a Bernoulli random variable is either 0 or 1, the smallest possible sample space is $\{0, 1\}$.

If $\Theta = [0, 1]$, then $\{\text{Ber}(p)\}_{p\in[0,1]}$ is the set of all possible Bernoulli distributions, as desired.

Submit You have used 1 of 2 attempts

Answers are displayed within the problem

Likelihood of a Discrete Distribution



6:45 / 6:45 1.50x

Video

[Download video file](#)

Transcripts

[Download SubRip (.srt) file](#)

[Download Text (.txt) file](#)

Concept Check: Interpreting the Likelihood

1/1 point (graded)

Let $(E, \{P_\theta\}_{\theta\in\Theta})$ denote a discrete statistical model. Let p_θ denote the pmf of P_θ. Let $X_1, \ldots, X_n \stackrel{iid}{\sim} P_{\theta^*}$ where the parameter θ^* is unknown. Then the **likelihood** is the function

$$\begin{aligned} L_n : E^n \times \Theta \ &\to \mathbb{R} \\ (x_1, \ldots, x_n, \theta) \ &\mapsto \prod_{i=1}^{n} p_\theta(x_i). \end{aligned}$$

For our purposes, we think of $x_1, \ldots, x_n$ as observations of the random variables $X_1, \ldots, X_n$.

Which of the following are true about the likelihood L_n? (Choose all that apply.)

- [x] It is the joint pmf of n iid samples from the distribution P_θ.
- [x] It is a function of the sample $X_1 = x_1, \ldots, X_n = x_n$.
- [x] It is a function of the parameter θ, where θ ranges over all possible values of in the parameter space Θ.
- [ ] It is the joint pmf of n iid samples from the true distribution P_{θ^*}.

✔

Solution:

We examine the choices in order.

- "It is the joint pmf of n iid samples from the distribution P_θ." is correct. If $Y_1, \ldots, Y_n \stackrel{iid}{\sim} P_\theta$, then by independence, the joint pmf of these variables is given by a product:

$$P(Y_1 = x_1, \ldots, Y_n = x_n) = \prod_{i=1}^{n} p_\theta(x_i).$$

Remark 1: We use Y_i to denote these variables to differentiate from the samples X_i that come from the true distribution P_{θ^*}.

- "It is a function of the sample $X_1 = x_1, \ldots, X_n = x_n$." is correct. To construct the likelihood, we observe samples $X_1 = x_1, \ldots, X_n = x_n$ and then compute $L_n(x_1, \ldots, x_n, \theta)$.

- "It is a function of the parameter θ, where θ ranges over all possible values of in the parameter space Θ" is correct. As θ varies over Θ, the likelihood $L_n(x_1, \ldots, x_n, \theta)$ takes on different values. This is evident from the dependence on θ in the definition of the likelihood.

  Remark 2: Later on we will maximize L_n (as a function of θ) to define the **maximum likelihood estimator** . Hence, it is a crucial property that the likelihood is a function of the parameter.

- "It is the joint pmf of n iid samples from the distribution P_{θ^*}." is incorrect. The likelihood takes as input all possible θ, not just the true parameter θ^*. Note how the likelihood is defined for general θ, not just the true parameter θ^*.

Submit You have used 1 of 2 attempts

Answers are displayed within the problem

Likelihood of a Bernoulli Statistical Model

1/1 point (graded)

Let $X_1, \ldots, X_n \overset{iid}{\sim} \text{Ber}(p^*)$ for some unknown $p^* \in (0,1)$. Let $(E, \{\text{Ber}(p)\}_{p\in\Theta})$ denote the corresponding statistical model constructed in the previous question.

What is the likelihood L_n of this statistical model? (Choose all that apply.)

Hint: Use the pmf's in the second and third choices from the first problem on this page: "Preparation Equivalent Expressions for the pmf of a Bernoulli Distribution".

- [x] $L_n(x_1, \ldots, x_n, p) = p^{\sum_{i=1}^n x_i}(1-p)^{n-\sum_{i=1}^n x_i}$.

- [ ] $L_n(x_1,\ldots,x_n,p) = p^{n-\sum_{i=1}^n x_i}(1-p)^{\sum_{i=1}^n x_i}$.
- [ ] $L_n(x_1,\ldots,x_n,p) = p^{\sum_{i=1}^n x_i}(1-p)^{\sum_{i=1}^n x_i}$.
- [x] $L_n(x_1,\ldots,x_n,p) = \prod_{i=1}^n (x_i p + (1-x_i)(1-p))$

✔

Solution:

We examine the choices in order.

- As shown in the previous problem, we can write the pmf of a Bernoulli as $x \mapsto p^x(1-p)^{1-x}$. Hence,

$$L_n(x_1,\ldots,x_n,p) = \prod_{i=1}^n p^{x_i}(1-p)^{1-x_i} = p^{\sum_{i=1}^n x_i}(1-p)^{n-\sum_{i=1}^n x_i}.$$

$$\begin{aligned} L(x_1,\ldots,x_n,p) &= \prod_{i=1}^n p^{x_i}(1-p)^{1-x_i} \\ &= p^{\sum_{i=1}^n x_i}(1-p)^{n-\sum_{i=1}^n x_i}. \end{aligned}$$

  Hence the first answer choice is correct.

- The second and third choices $L_n(x_1,\ldots,x_n,p) = p^{n-\sum_{i=1}^n x_i}(1-p)^{\sum_{i=1}^n x_i}$ and $L_n(x_1,\ldots,x_n,p) = p^{\sum_{i=1}^n x_i}(1-p)^{\sum_{i=1}^n x_i}$ are incorrect. Note that they are slight algebraic modifications of the first choice, so these formulas cannot be correct.

- If we use the expression $f(x) = xp + (1-x)(1-p)$ for the pmf of $\mathrm{Ber}(p)$, then

$$L(x_1, \ldots, x_n, p) = \prod_{i=1}^{n} (x_i p + (1 - x_i)(1 - p))$$

is, by definition, the likelihood. Hence, the last answer choice is also correct.

Remark: Although the last answer choice is formally correct, the formula is much more difficult to work with. It is often more convenient to use $p^{\sum_{i=1}^{n} x_i}(1-p)^{n-\sum_{i=1}^{n} x_i}$ for the likelihood of a Bernoulli statistical model.

Submit You have used 1 of 2 attempts

Answers are displayed within the problem

Discussion

Hide Discussion

Topic: Unit 3 Methods of Estimation:Lecture 8: Distance measures between distributions / 14. Likelihood of a Discrete Distribution

Add a Post

Show all posts ▼ by recent activity ▼

There are no posts in this topic yet.

✖

Learn About Verified Certificates