8. Review: Power of a Test

Review: Power of a Test for Different Alternative Hypotheses

2/3 points (graded)

Recall that the power π_ψ of a test ψ for the hypotheses

$$H_0 : \theta^* \in \Theta_0$$
$$H_1 : \theta^* \in \Theta_1$$

is

$$\pi_\psi = \inf_{\theta \in \Theta_1} (1 - \beta_\psi(\theta))$$

where $\beta_\psi(\theta) = \mathbf{P}_\theta(\psi = 0)$, defined for $\theta \in \Theta_1,$ is the **type 2 error rate** of ψ.

Suppose $X_1, \ldots, X_n$ are i.i.d. random variables (in 1 dimension). Assume the theorem of MLE applies so that $\hat{\theta}^{\mathrm{MLE}}$ is asymptotically normal. You use the test

$$\psi = \mathbf{1}\left(\sqrt{nI}\left|\hat{\theta}^{\text{MLE}} - 0\right| > C_\alpha\right),$$

which has level α for some threshold C_α, to test the hypotheses

$$H_0 : \theta^* = 0$$
$$H_1 : \theta^* \neq 0.$$

What is the *asymptotic* power π_ψ in terms of α?

$\pi_\psi =$ 1-alpha ✖ **Answer:** alpha

$1 - \alpha$

Now, you use the same test $\psi = \mathbf{1}\left(\sqrt{nI}\left|\hat{\theta}^{\text{MLE}} - 0\right| > C_\alpha\right)$ to test a different alternative hypothesis against the same null hypothesis:

$$H_0 : \theta^* = 0$$
$$H_1 : \theta^* = 1.$$

How do the (smallest) *asymptotic* level and the *asymptotic* power of ψ change with this change of the alternative hypothesis? (Choose one for each column.)

The (smallest) asymptotic level of ψ ...	the asymptotic power of ψ ...
○ increases	● increases
○ decreases	○ decreases
● stays the same	○ stays the same
✔	✔

(In general, how does the level and power of a test vary as Θ_1 shrinks?)

STANDARD NOTATION

Solution:

The power of ψ with $H_1 : \theta^* \neq 0$ is

$$\begin{aligned}\pi_\psi &= \inf_{\theta\neq 0}\left(1-\beta_\psi(\theta)\right)\\ &= \inf_{\theta\neq 0}\mathbf{P}_\theta(\psi=1) = \inf_{\theta\neq 0}\mathbf{P}_\theta\left(\sqrt{nI}\left|\hat{\theta}^{\text{MLE}}-0\right| > C_\alpha\right)\end{aligned}$$

Since $\sqrt{nI}\left(\hat{\theta}^{\text{MLE}}-\theta\right)\sim\mathcal{N}(0,1)$ (asymptotically if $\theta^*=\theta$), $\mathbf{P}_\theta\left(\sqrt{nI}\left|\hat{\theta}^{\text{MLE}}-0\right| > C_\alpha\right)$ decreases as $\theta\to 0$ and approaches $\mathbf{P}_0\left(\sqrt{nI}\left|\hat{\theta}^{\text{MLE}}-0\right| > C_\alpha\right)=\alpha$ (sketch the probability as an area to see this). Hence $\pi_\psi=\alpha$ in this case.

If we use the same test ψ for the alternative hypothesis $H_1 : \theta^* = 1,$ then

$$\pi_\psi = \mathbf{P}_{\theta=1}\left(\sqrt{nI}\left|\hat{\theta}^{\text{MLE}}-0\right| > C_\alpha\right)$$

which is greater than $\mathbf{P}_{\theta=0}\left(\sqrt{nI}\left|\hat{\theta}^{\text{MLE}}-0\right| > C_\alpha\right)=\alpha.$ (Again, sketch the probability as an area to see this.)

On the other hand, the alternative hypothesis has no effect on the level of the test once the test has been fixed.

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Answers are displayed within the problem

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Two-tailed vs one-tailed test

question posted a day ago by riskia

Does the fact that the first test is a two-tailed test and the second test is a one-tailed test have anything to do with the answer?

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2 responses

kevin007zhang
about 10 hours ago

I have the same confusion. I think it is two-tailed test. So for the first question, my answer is **Deleted by MW-CTA**

Please don't post answers (correct/incorrect) before the due date (in a couple of days).

posted about 10 hours ago by markweitzman (Community TA)

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ya_mukhin (Staff)
about 9 hours ago

I would take another close look at the *definition* of the power of a test and apply it two both sets of hypothesis.

Sorry, but I'm still not clear about it. I refreshed myself on the concept and visualization of power and type II error with respect to type I error, and if I assume that both tests in the problem are one-tailed tests, then the answers make sense to me visually.

But if the first one is a two-tailed test and the second one is a one-tailed test, I'm not sure how to visualize it to get the answer.

posted about 5 hours ago by **riskia**

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