# 9. Further examples

# Linear regression model and Cox proportional Hazard model

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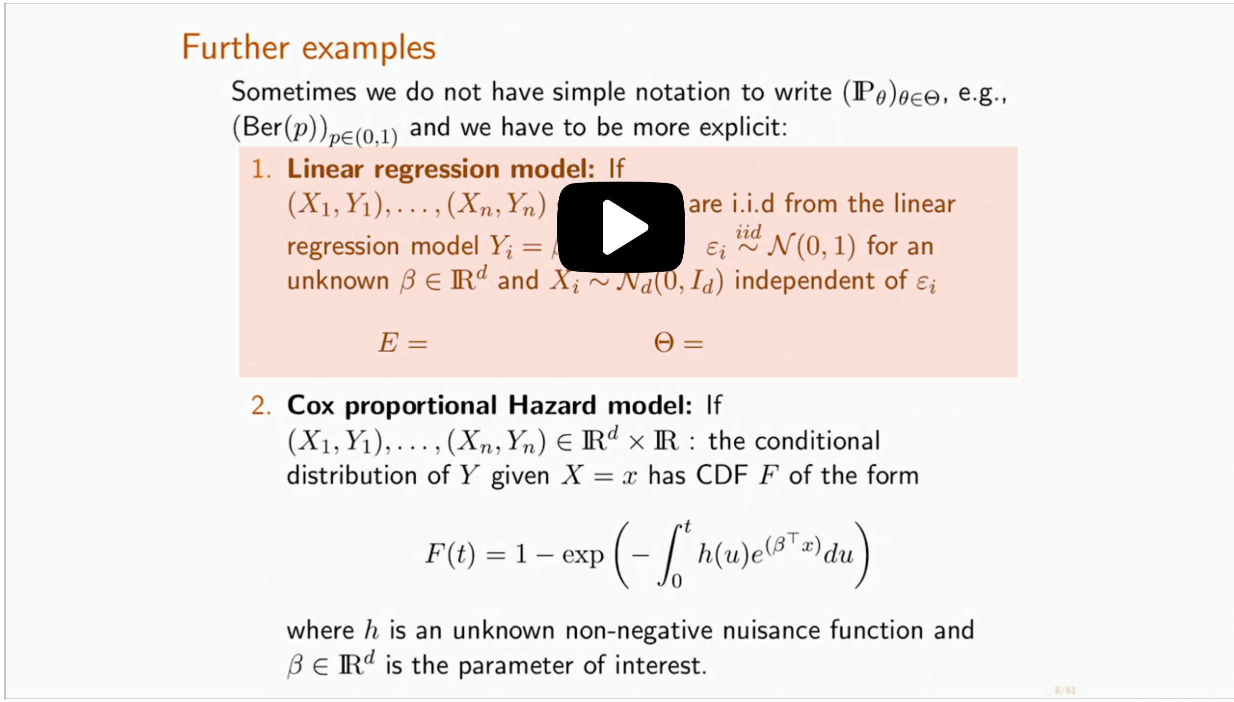


There's a couple more examples that I wanted you to look at.

One is linear regression.

So this is something we will come back to.

So you can see also that it's not

entirely trivial to write what the distribution is.

I'll write it on the board for you.

But it's not very easy.

So this is a linear regression model.

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# Linear regression as a statistical model I

1/2 points (graded)

Consider the linear regression model introduced in the slides and lecture, restated below:

**Linear regression model** : $(X_1, Y_1), \ldots, (X_n, Y_n) \in \mathbb{R}^d \times \mathbb{R}$ are i.i.d from the linear regression model $Y_i = \beta^\top X_i + \varepsilon_i, \quad \varepsilon_i \overset{iid}{\sim} \mathcal{N}(0,1)$ for an unknown $\beta \in \mathbb{R}^d$ and $X_i \sim \mathcal{N}_d(0, I_d)$ independent of $\varepsilon_i$.

Suppose that $\beta = \mathbf{1} \in \mathbb{R}^d$, which denotes the $d$-dimensional vector with all entries equal to $1$.

What is the mean of $Y_1$?

$\mathbb{E}[Y_1] =$ 0 ✔ **Answer:** 0

What is the variance of $Y_1$? (Express your answer in terms of $d$.)

$\text{Var}(Y_1) =$ 1 ✖ **Answer:** d+1

$1$

STANDARD NOTATION

**Solution:**

By definition of the model and setting $\beta = \mathbf{1}$, we have

$$Y_1 = \beta^T X_1 + \varepsilon_1 = \mathbf{1}^T X_1 + \varepsilon_1 = \varepsilon_1 + \sum_{j=1}^{d} X_{1,j}.$$

where $X_{i,j}$ denotes the $j$'th coordinate of $X_i \sim \mathcal{N}(0, I_d)$. By linearity of expectation,

$$\mathbb{E}[Y_1] = \mathbb{E}[\varepsilon_1] + \sum_{j=1}^{d} \mathbb{E}[X_{1,j}] = 0$$

Next we compute the variance. Since $X_{1,1}, \ldots, X_{1,d}, \varepsilon_i$ are mutually independent, the variance is additive:

$$\mathrm{Var}[Y_1] = \mathrm{Var}[\varepsilon_1] + \sum_{j=1}^{d} \mathrm{Var}[X_{1,j}] = d + 1$$

because $X_{1,1}, \ldots, X_{1,d}, \varepsilon_1 \overset{iid}{\sim} N(0,1)$.

Submit You have used 2 of 2 attempts

Answers are displayed within the problem

## Linear regression as a statistical model II

2/2 points (graded)

Recall the linear regression model as introduced above in the previous question. This model is parametric, although it is not written in the standard notation previously introduced for parametric statistical models. In this problem, you will explicitly write the linear regression model as a parametric statistical model.

We will represent the linear regression model as an ordered pair $(E, \{P_\beta\}_{\beta \in \Theta})$. Here $E$ denotes the sample space associated to the distribution $P_\beta$, where $P_\beta$ is defined as follows for $\beta \in \mathbb{R}^d$:

*The random ordered pair* $(X, Y) \subset \mathbb{R}^d \times \mathbb{R}$ *is distributed as* $P_\beta$ *if:*

- $X \sim N(0, I_d)$,
- $Y \sim \beta^T X + \varepsilon$, *where* $\varepsilon \sim N(0,1)$ *and* $\varepsilon$ *is independent of* $X$.

The set $\Theta$ in the ordered pair $(E, \{P_\beta\}_{\beta \in \Theta})$ denotes the parameter space for this model.

The sample space for the linear regression model can be written $E = \mathbb{R}^k$ for some integer $k$. What is $k$? (Express your answer in terms of $d$.)

*Hint:* You should use the fact that $\mathbb{R}^{m+n} = \mathbb{R}^m \times \mathbb{R}^n$ for all integers $m, n \geq 0$.

$k =$ d+1 ✔ **Answer:** d+1

$d + 1$

The parameter space for the model can be written as $\Theta = \mathbb{R}^j$ for some integer $j$. What is $j$? (Express your answer in terms of $d$.)

$j =$ d ✔ **Answer:** d

$d$

STANDARD NOTATION

**Solution:**

The statistical experiment is given by the iid sample $(X_1, Y_1), \ldots, (X_n, Y_n)$. Where $X_i \sim N(0, I_d)$ and $Y_i = \beta^T X_i + \varepsilon_i$ for $\varepsilon_i \sim N(0,1)$ and some true parameter $\beta \in \mathbb{R}^d$. In particular, $X_i \in \mathbb{R}^d$ and $Y_i \in \mathbb{R}$. Therefore, $(X_i, Y_i) \in \mathbb{R}^{d+1}$, so indeed $E = R^{d+1}$ is the sample space for this model. We conclude that $k = d + 1$.

This model is parametrized by the vector $\beta \in \mathbb{R}^d$. That is, specifying the value of $\beta$ uniquely determines the distribution of $(X_1, Y_1), \ldots, (X_n, Y_n)$. Hence, the parameter is $\beta$, and the parameter space is $\Theta = \mathbb{R}^d$. We conclude that $j = d$.

Submit You have used 1 of 2 attempts

Answers are displayed within the problem

## Discussion

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