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1. Lecture 5

The following can be done after Lecture 5.

5-1

5.0/5.0 points (graded)

Consider the system

$$\begin{aligned} \dot{x_1} &= 6x_1 - x_2 - 4x_3 + x_4 \\ \dot{x_2} &= 4x_1 - 6x_2 + 5x_3 - x_4 \\ \dot{x_3} &= -6x_1 + x_2 - 5x_3 \\ \dot{x_4} &= 4x_1 - 5x_2 + x_4. \end{aligned}$$

If

$$\begin{pmatrix} \dot{x_1} \\ \dot{x_2} \\ \dot{x_3} \\ \dot{x_4} \end{pmatrix} = A \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix}$$

is the vector equation for the given system, what is the fourth entry in the third row of the matrix $\mathbf{A}$?

0

Answer: 0

0

Solution:

Look carefully at the vector equation and the system. From the vector on the left-hand side, we see that the third row of $\mathbf{A}$ has to be related to $\dot{x}_3$. From the ordering of the variables in the vector on the left-hand side, we see that the fourth entry has to be the coefficient in front of x_4 in the equation for $\dot{x}_3$. From the system we see that this coefficient is equal to 0.

Submit You have used 1 of 5 attempts

Answers are displayed within the problem

5-2

5.0/5.0 points (graded)

Consider the system

$$\begin{aligned} \dot{x} &= x + 3y - z, \\ \dot{y} &= -2y + 4z, \\ \dot{z} &= 3z. \end{aligned}$$

Which of the following is the general solution of the system?
(Check all that apply.)

- [ ] $c_1 e^{t} + c_2 e^{-2t} + c_3 e^{3t}$
- [ ] $c_1 \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + c_2 \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} + c_3 \begin{pmatrix} 7 \\ 8 \\ 10 \end{pmatrix}$
- [x] $c_1 e^{t} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + c_2 e^{-2t} \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} + c_3 e^{3t} \begin{pmatrix} 7 \\ 8 \\ 10 \end{pmatrix}$ ✓

✓

Solution:

The system is $\dot{\mathbf{x}} = \mathbf{A}\mathbf{x}$ with

$$\mathbf{A} = \begin{pmatrix} 1 & 3 & -1 \\ 0 & -2 & 4 \\ 0 & 0 & 3 \end{pmatrix}.$$

Since the matrix is upper triangular, the eigenvalues are its diagonal entries $1,\ -2$ and $3.$ Computing $\mathrm{NS}(A - I)$ using back-substitution (the matrix is already in row-echelon form) shows that the eigenspace of 1 has the vector $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ as a basis. Similarly, the eigenspace of -2 has $\begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$ as a basis, and the eigenspace of 3 has $\begin{pmatrix} 7 \\ 8 \\ 10 \end{pmatrix}$ as a basis.

The general solution is

$$\mathbf{x} = c_1 e^{t} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + c_2 e^{-2t} \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} + c_3 e^{3t} \begin{pmatrix} 7 \\ 8 \\ 10 \end{pmatrix}.$$

Submit

You have used 1 of 2 attempts

Answers are displayed within the problem

5-3

8.0/8.0 points (graded)

For the system

$$\begin{aligned} \ddot{x} &= 5x + 7\dot{y} \\ \ddot{y} &= 2\dot{x} + 3y, \end{aligned}$$

which of the following is true?

(Suggestion: Convert it to a **first-order** system.)

- The set of solutions is a 1-dimensional vector space.
- The set of solutions is a 2-dimensional vector space.
- The set of solutions is a 4-dimensional vector space. ✔
- The set of solutions is not a vector space.

Solution:

The set of solutions is a 4-dimensional vector space.

To convert this to a first-order system, introduce new function variables $u := \dot{x}$ and $v := \dot{y}$. Then the original system is equivalent to the first-order system

$$\begin{aligned} \dot{x} &= u \\ \dot{y} &= v \\ \dot{u} &= 5x + 7v \\ \dot{v} &= 2u + 3y. \end{aligned}$$

This is a first-order homogeneous linear system of 4 ODEs in 4 unknown functions, so the set of solutions is a 4-dimensional vector space.

Submit You have used 1 of 2 attempts

Answers are displayed within the problem

5-4

10/10 points (graded)

Suppose that $\mathbf{X} = \begin{pmatrix} 2e^{3t} & 3e^{-2t} \\ 5e^{3t} & 7e^{-2t} \end{pmatrix}$ is a fundamental matrix for a system $\dot{\mathbf{x}} = \mathbf{A}\mathbf{x}$. Let $\mathbf{x}(t) = (x(t), y(t))$ be the solution satisfying the initial condition $\mathbf{x}(0) = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$. What is $y(t)$?

65*exp(3*t)-63*exp(-2*t)

Answer: 65*e^(3*t)-63*e^(-2*t)

$65 \cdot \exp(3 \cdot t) - 63 \cdot \exp(-2 \cdot t)$

Solution:

The answer is $y(t) = 65e^{3t} - 63e^{-2t}$.

The general solution to the system has the form

$$\mathbf{x} = \mathbf{X}(t) \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = c_1 \begin{pmatrix} 2e^{3t} \\ 5e^{3t} \end{pmatrix} + c_2 \begin{pmatrix} 3e^{-2t} \\ 7e^{-2t} \end{pmatrix}.$$

Setting $t = 0$ and using the initial condition leads to

$$\begin{pmatrix} -1 \\ 2 \end{pmatrix} = c_1 \begin{pmatrix} 2 \\ 5 \end{pmatrix} + c_2 \begin{pmatrix} 3 \\ 7 \end{pmatrix},$$

which is the system

$$\begin{aligned} 2c_1 + 3c_2 &= -1 \\ 5c_1 + 7c_2 &= 2, \end{aligned}$$

whose solution is $c_1 = 13$, $c_2 = -9$. Thus the particular solution is

$$\mathbf{x} = 13 \begin{pmatrix} 2e^{3t} \\ 5e^{3t} \end{pmatrix} - 9 \begin{pmatrix} 3e^{-2t} \\ 7e^{-2t} \end{pmatrix}.$$

Taking the second coordinate function gives $y(t) = 65e^{3t} - 63e^{-2t}$.

Submit You have used 3 of 10 attempts

Answers are displayed within the problem

5-5

5.0/5.0 points (graded)

Let $\mathbf{S} = \begin{pmatrix} 2 & 3 \\ 5 & 7 \end{pmatrix}$. Let $\mathbf{D} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$. Let $\mathbf{A} = \mathbf{SDS}^{-1}$. What is the sum of the entries of $\mathbf{A}^{1000}$?

2

Answer: 2

2

Solution:

The sum of the entries is $\mathbf{2}$.

We have $\mathbf{A}^{1000} = \mathbf{SD}^{1000}\mathbf{S}^{-1}$. But $\mathbf{D}^2 = \mathbf{I}$, so $\mathbf{D}^{1000} = \mathbf{I}$. Thus

$$\mathbf{A}^{1000} = \mathbf{SD}^{1000}\mathbf{S}^{-1} = \mathbf{SIS}^{-1} = \mathbf{SS}^{-1} = \mathbf{I}.$$

The sum of its entries is $1 + 0 + 0 + 1 = 2$.

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Answers are displayed within the problem

1. Lecture 5

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