2. Differential of y = f(x)

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Lecture due Oct 5, 2021 20:30 IST

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Differential of $y = f(x)$

Recall from single-variable calculus that if $y = x^2$ then $\frac{dy}{dx} = 2x$. It is common to think of $\frac{dy}{dx}$ as just the Leibniz notation for the derivative. But we sometimes also write $dy = 2x\,dx$, obtained by "multiplying by dx". Is this an abuse of notation, or is there some deeper meaning?

In fact, there is a deeper meaning. The symbols "dy" and "dx" represent the "infinitesimal versions" of Δy and Δx. In contrast to "dy" and "dx", the symbols Δy and Δx can be given numerical values, and they mean that if you change x by Δx, it causes y to change by Δy. But when we replace the Δ's by d's, we express the fact that we are imagining these quantities tending towards zero.

$$\frac{dy}{dx} = 2x \;\text{ means }\; \frac{\Delta y}{\Delta x} \approx 2x \text{ for small values of } \Delta x \tag{6.108}$$

In the same way,

$$dy = 2x\,dx \;\text{ means }\; \Delta y \approx 2x\,\Delta x \text{ for small values of } \Delta x \tag{6.109}$$

Using the dy and dx symbols, we can state linear approximation as an equality ($=$ instead of $\approx$). This equality becomes approximate once we replace the dx and dy with Δx and Δy.

Example 2.1 Thinking in terms of differentials can give you more facility in estimating numbers like $(9.1)^2$.

The relevant function is $y = x^2$. By "differentiation", if $y = x^2$ then $dy = 2x\,dx$. Since we know $y = 81$ when $x = 9$, increasing x by 0.1 will cause y to change a little from 81. The differentials tell us that this change in y is approximately $2x\Delta x$, so y should change by about $(18)\cdot 0.1 = 1.8$. Thus $(9.1)^2 \approx 82.8$.

Difference between dy, dx and $\Delta y, \Delta x$

Based on the above example, it might seem unnecessary to use dy, dx when we could have written the solution using the more familiar Δy and Δx. Why the need for a new notation if it expresses the same thing?

The answer is that writing $dy = 2x\,dx$ expresses a more precise statement than writing $\Delta y \approx 2x\,\Delta x$.

The $\approx$ symbol is flawed because its meaning is highly context-dependent. In some contexts $\approx$ means "equal up to the first decimal point", but in other contexts, it could mean "equal up to the sixth decimal point". Neither of these is what we mean in this context. In particular, we mean that the accuracy of the approximation gets better and better as Δx gets closer and closer to 0.

In other words, writing $dy = 2x\,dx$ expresses something about a limit, whereas $\Delta y \approx 2x\,\Delta x$ does not.

Using differentials is particularly useful in multivariable calculus, because we sometimes need to keep track of multiple linear approximations at one time. The symbols dx and dy work as placeholders to tell us where the numerical values of Δy and Δx will go in the end.

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Check your understanding

1/1 point (graded)

Suppose $y = \sqrt{x}$. Which of the following is the correct expression for dy?

○ $dy = \dfrac{dx}{\sqrt{x}}$

○ $dy = \dfrac{\sqrt{x}}{2}\,dx$

○ $dy = \sqrt{x}\,dx$

● $dy = \dfrac{dx}{2\sqrt{x}}$

○ $dy = \dfrac{2\,dx}{\sqrt{x}}$

✓

Solution:

We have $\frac{dy}{dx} = \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{2}}$. Therefore, $dy = \frac{dx}{2\sqrt{2}}$.

Submit You have used 1 of 3 attempts

Answers are displayed within the problem

Practice Estimation

1/1 point (graded)

Use the answer to the previous question to answer: which of the following is closest to $\sqrt{42}$?

○ 6.1

○ 6.2

○ 6.3

○ 6.4

● 6.5

✓

Solution:

If $y = \sqrt{x}$, then at $x = 36$ we have $y = 6$. If x increases by 6, then from $dy = \frac{dx}{2\sqrt{x}}$ we expect y to increase by about $\frac{6}{2\sqrt{36}} = \frac{1}{2}$. Therefore 6.5 should be closest.

Another way to obtain the same answer is to start from $x = 49, y = 7$ and decrease x by 7, which will cause y to decrease by about $\frac{1}{2}$.

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Practice Differentials

2/2 points (graded)

Suppose $y = x^3$. What is dy?

Type `dx` for dx. Use `*` to denote multiplication, such as `dy = 2*x*dx`.

$dy =$ 3*x^2*dx ✓ **Answer:** 3*x^2*dx

? INPUT HELP

Estimate the value of $(5.1)^3$ without a calculator. Try to think about this problem using differentials. Typing the exact value of $(5.1)^3$ will be marked incorrect.

$(5.1)^3 \approx$ 132.5 ✓ **Answer:** 132.5

Solution:

By "differentiation", we have:

$$dy = 3x^2\,dx. \tag{6.110}$$

This tells us that if we push x from 5 to 5.1, then the change in y will be approximately $3(5)^2\,(0.1) = 7.5$. Since $y = 125$ for $x = 5$, we expect $y \approx 132.5$ for $x = 5.1$.

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