1.8.1 Homework

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Week 1 due Oct 5, 2023 03:12 IST

1.8.1 Homework

Reading Assignment

0 points possible (ungraded)
Read Unit 1.8.1 of the notes. [LINK]

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Homework 1.8.1.1

1/1 point (graded)

Let $x = \begin{pmatrix} 2 \\ -1 \end{pmatrix}, y = \begin{pmatrix} \alpha \\ \beta - \alpha \end{pmatrix}, x = y$

Check all that hold true

- [x] $\alpha = 2$
- [x] $\beta = (\beta - \alpha) + \alpha = (-1) + 2 = 1$
- [x] $\beta - \alpha = -1$
- [x] $\beta - 2 = -1$
- [x] $x = 2e_0 - e_1$

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Homework 1.8.1.2

1/1 point (graded)

A displacement vector represents the length and direction of an imaginary, shortest, straight path between two locations. To illustrate this as well as to emphasize the difference between ordered pairs that represent positions and vectors, we ask you to map a trip we made.

Calculator

In 2012, we went on a journey to share our research in linear algebra. Below are some displacement vectors to d...

parts of this journey using longitude and latitude. For example, we began our trip in Austin, TX and landed in San Jose, CA. Austin has coordinates $30^\circ\ 15'\ \mathrm{N(orth)}, 97^\circ\ 45'\ \mathrm{W(est)}$ and San Jose's are $37^\circ\ 20'\ \mathrm{N}, 121^\circ\ 54'\ \mathrm{W}$. *(Notice that convention is to report first longitude and then latitude.)* If we think of using longitude and latitude as coordinates in a plane where the first coordinate is position E (positive) or W (negative) and the second coordinate is position N (positive) or S (negative), then Austin's location is $(-97^\circ\ 45', 30^\circ\ 15')$ and San Jose's are $(-121^\circ\ 54', 37^\circ\ 20')$. *(Here, notice the switch in the order in which the coordinates are given because we now want to think of E/W as the x coordinate and N/S as the y coordinate.)* For our displacement vector for this, our first component will correspond to the change in the x coordinate, and the second component will be the change in the second coordinate. For convenience, we extend the notion of vectors so that the components include units as well as real numbers. Notice that for convenience, we extend the notion of vectors so that the components include units as well as real numbers and 60 minutes ($'$)= 1 degree($^\circ$). Hence our displacement vector for Austin to San Jose is $\begin{pmatrix} -24^\circ\ 09' \\ 7^\circ\ 05' \end{pmatrix}$.

After visiting San Jose, we returned to Austin before embarking on a multi-legged excursion. That is, from Austin we flew to the first city and then from that city to the next, and so forth. In the end, we returned to Austin.

The following is a table of cities and their coordinates:

City	Coordinates	City	Coordinates
London	$00^\circ\ 08'$ W, $51^\circ\ 30'$ N	Austin	$-97^\circ\ 45'$ E, $30^\circ\ 15'$ N
Pisa	$10^\circ\ 21'$ E, $43^\circ\ 43'$ N	Brussels	$04^\circ\ 21'$ E, $50^\circ\ 51'$ N
Valencia	$00^\circ\ 23'$ E, $39^\circ\ 28'$ N	Darmstadt	$08^\circ\ 39'$ E, $49^\circ\ 52'$ N
Zürich	$08^\circ\ 33'$ E, $47^\circ\ 22'$ N	Krakow	$19^\circ\ 56'$ E, $50^\circ\ 4'$ N

Determine the order in which cities were visited, starting in Austin, given that the legs of the trip (given in order) had the following displacement vectors:

$$\begin{pmatrix} 102^\circ\ 06' \\ 20^\circ\ 36' \end{pmatrix} \to \begin{pmatrix} 04^\circ\ 18' \\ -00^\circ\ 59' \end{pmatrix} \to \begin{pmatrix} -00^\circ\ 06' \\ -02^\circ\ 30' \end{pmatrix} \to \begin{pmatrix} 01^\circ\ 48' \\ -03^\circ\ 39' \end{pmatrix} \to$$

$$\begin{pmatrix} 09^\circ\ 35' \\ 06^\circ\ 21' \end{pmatrix} \to \begin{pmatrix} -20^\circ\ 04' \\ 01^\circ\ 26' \end{pmatrix} \to \begin{pmatrix} 00^\circ\ 31' \\ -12^\circ\ 02' \end{pmatrix} \to \begin{pmatrix} -98^\circ\ 08' \\ -09^\circ\ 13' \end{pmatrix}$$

1st	2nd	3rd	4th
Brussels	Darmstadt	Zurich	Pisa
5th	**6th**	**7th**	**8th**
Krakow	London	Valencia	Austin

Calculator

Brussels → Darmstadt → Zurich → Pisa → Krakow → London → Valencia → Austin

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Homework 1.8.1.3

1/1 point (graded)

These days, high performance computers are called clusters and consist of many compute nodes, connected via a communication network. Each node of the cluster is basically equipped with a central processing unit (CPU), memory chips, a hard disk, and a network card. The nodes can be monitored for average power consumption (via power sensors) and application activity.

A system administrator monitors the power consumption of a node of such a cluster for an application that executes for two hours. This yields the following data:

Component	Average power (W)	Time in use (in hours)	Fraction of time in use
CPU	90	1.4	0.7
Memory	30	1.2	0.6
Disk	10	0.6	0.3
Network	15	0.2	0.1
Sensors	5	2.0	1.0

The energy, often measured in KWh, is equal to power times time. Notice that the total energy consumption can be found using the dot product of the vector of components' average power and the vector of corresponding time in use. What is the total energy consumed by this node in KWh? (The power is in Watts (W), so you will want to convert to Kilowatts (KW).)

0.181 ✔ Answer: 0.181

0.181

Let us walk you through this:

- The CPU consumes 90 Watts, is on 1.4 hours so that the energy used in two hours is 90×1.4 Watt-hours.
- If you analyze the energy used by every component and add them together, you get

$$(90 \times 1.4 + 30 \times 1.2 + 10 \times 0.6 + 15 \times 0.2 + 5 \times 2.0) \sim \mathrm{Wh} = 181 \sim \mathrm{Wh}.$$

- Convert to KWh by dividing by 1000, leaving us with the answer $.181 \sim \mathrm{KWh}$.

Now, let's set this up as two vectors, x and y. The first records the power consumption for each of the components and the other for the total time that each of the components is in use:

$$x = \begin{pmatrix} 90 \\ 30 \\ 10 \\ 15 \\ 5 \end{pmatrix} \quad \text{and} \quad y = 2 \begin{pmatrix} 0.7 \\ 0.6 \\ 0.3 \\ 0.1 \\ 1.0 \end{pmatrix}.$$

Calculator

Compute $x^T y$.

Verify that if you compute $x^T y$ you arrive at the same result as you did via the initial analysis where you added the energy consumed by the different components (before converting from Wh to KWh).

Think: How do the two ways of computing the answer relate?

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Homework 1.8.1.4

3/3 points (graded)

(Examples from statistics) Linear algebra shows up often when computing with data sets. In this homework, you find out how dot products can be used to define various sums of values that are often encountered in statistics.

Assume you observe a random variable and you let those sampled values be represented by $\chi_i, i = 0, 1, 2, 3, \cdots, n-1$. We can let x be the vector with components χ_i and $\vec{1}$ be a vector of size n with components all ones:

$$x = \begin{pmatrix} \chi_0 \\ \vdots \\ \chi_{n-1} \end{pmatrix}, \quad \text{and} \quad \vec{1} = \begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix}.$$

For any x, the sum of the values of x can be computed using the dot product operation as (check all that are true)

- [ ] $x^T x$
- [x] $\vec{1}^T x$
- [x] $x^T \vec{1}$

✔

The sample mean of a data set is the sum of the observed values (the random variable takes on) divided by the number of data points, n. In other words, if the values the random variable takes on are stored in vector x, then $\overline{x} = \frac{1}{n} \sum_{i=0}^{n-1} \chi_i$. Using a dot product operation, for all x this can be computed as (check all that are true)

- [ ] $\frac{1}{n} x^T x$
- [x] $\frac{1}{n} \vec{1}^T x$
- [x] $(\vec{1}^T \vec{1})^{-1} (x^T \vec{1})$

✔

Notice that $(\vec{1}^T \vec{1}) = n$ and hence $(\vec{1}^T \vec{1})^{-1} = 1/n$!!!

For any x, the sum of the squares of observations stored in (the elements of) a vector, x, can be computed using a dot product operation as (check all that are true)

- [x] $x^T x$
- [ ] $\vec{1}^T x$

Calculator

- [ ] $x^T\vec{1}$

✔

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Calculator