# 14.1.1 Introduction to probabilistic thinking

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MO2.8 MO2.13

We now turn our attention to situations in which uncertainty and/or randomness have an important impact. While often these concepts are introduced by considering coin flips, we will take a different approach which is a more common scenario in computational science and engineering. Specifically, we will consider how uncertainty impacts the behavior of a modeled system.

Let's consider the Martian lander problem which was used in our first problem set. A key design concern is at what altitude the lander will reach a slow enough velocity to deploy the parachute. If that altitude is too low, then the parachute may not be able to sufficiently slow the lander down before reaching the Martian surface. Recall the simple model we used for the entry and descent of the lander through the Martian atmosphere as shown in Figure 14.1.

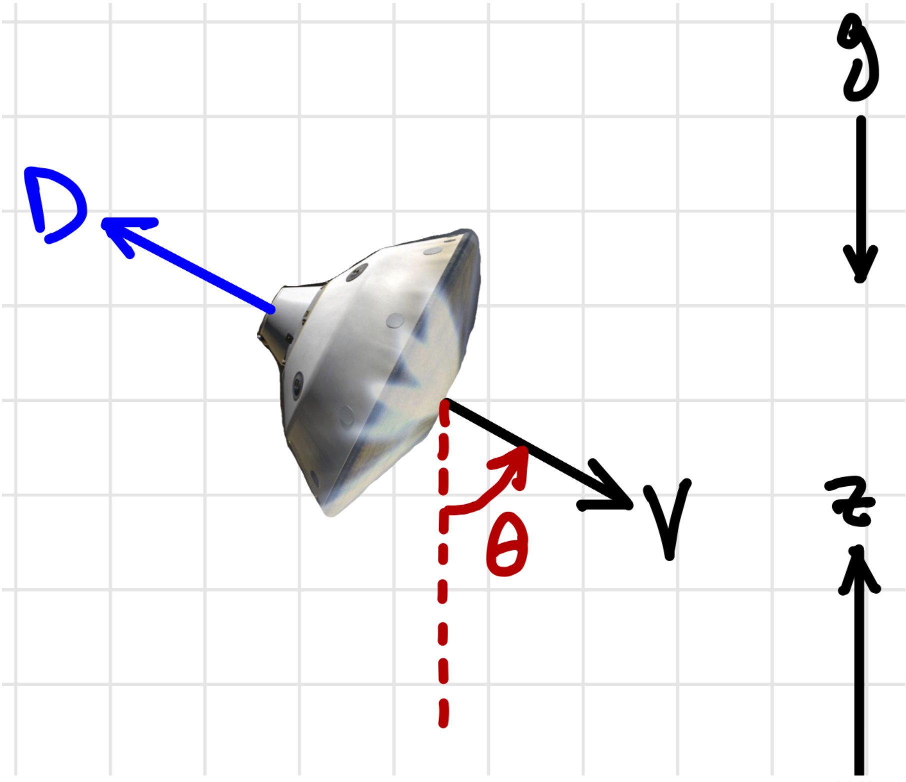


**Figure 14.1**: Martian lander showing its velocity $V$, drag force $D$, gravity $g$, and flight angle $\theta$.

Specifically, during the entry phase prior to the parachute being deployed, the model equations for the time rate of change of the lander's velocity $V$ and altitude $z$ were

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix} V \\ z \end{bmatrix} = \begin{bmatrix} g\cos\theta - D_l/m_l \\ -V\cos\theta \end{bmatrix} \tag{14.1}$$

where $m_l$ is the mass of the lander, and $D_l$ is the drag on the lander. The drag on the lander is estimated using a drag coefficient $C_{Dl}$,

$$D = \frac{1}{2}\rho_a V^2 A_l C_{Dl} \tag{14.2}$$

where $A_l$ is the projected area of the lander, and $\rho_a$ is the atmospheric density which is a function of altitude $z$. Even if we believe the model has all of the dominant effects (which it does not, but that would get even more complicated), there would still be uncertainty about many of the parameters used in the model. For example,

- The drag coefficient of the lander, $C_{Dl}$, will have significant uncertainty because of the extremely high speed which will involve complex fluid dynamics, including chemical dissociation and significant heat transfer.

- The initial velocity, $V(t_I)$, and entry flight angle, $\theta_e$, will have uncertainty as it will be difficult to control these to high precision.

- The Martian atmosphere is subject to localized weather which by itself can impact the density by $\pm 10\%$. So, let's consider the atmospheric density as being given by $\rho_a(z, f_\rho) = (1 + f_\rho)\, \rho_{\text{std}}(z)$ where $\rho_{\text{std}}(z)$ is a standard day model of the density of the Martian atmosphere and $f_\rho$ accounts for uncertainty (due to localized weather or other effects).

One way we can begin to quantify the impacts of these parameter uncertainties is to look at how the behavior of the lander changes when one of the parameter varies over the range of possible values. For example, let the nominal values of these parameters be,

$$C_{Dl} = 1.7, \quad V(t_I) = 5800\,\mathrm{m/s}, \quad \theta_e = 83^\circ, \quad f_\rho = 0$$

With the parachute deploying when $V = 470$ m/s, then for the nominal parameters this would occur at 9.75 km.

Let's suppose that the following ranges of values are all plausible for the four parameters,

- $1.5 \leq C_{Dl} \leq 1.9$
- $5500\,\mathrm{m/s} \leq V(t_I) \leq 6100\,\mathrm{m/s}$
- $80^\circ \leq \theta_e \leq 86^\circ$
- $-0.1 \leq f_\rho \leq 0.1$

Figures 14.2 – 14.5 show the impact of varying one of

Figures 14.2 – 14.6 show the impact of varying one of these parameters while holding the other parameters at the nominal values. Note the different

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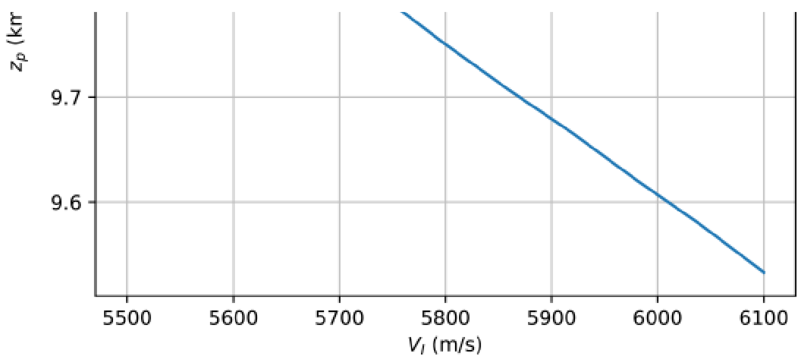
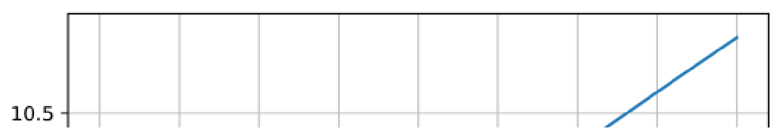



**Figure 14.4**: Impact of $V(t_I)$ variation on parachute deployment altitude $z_p$.



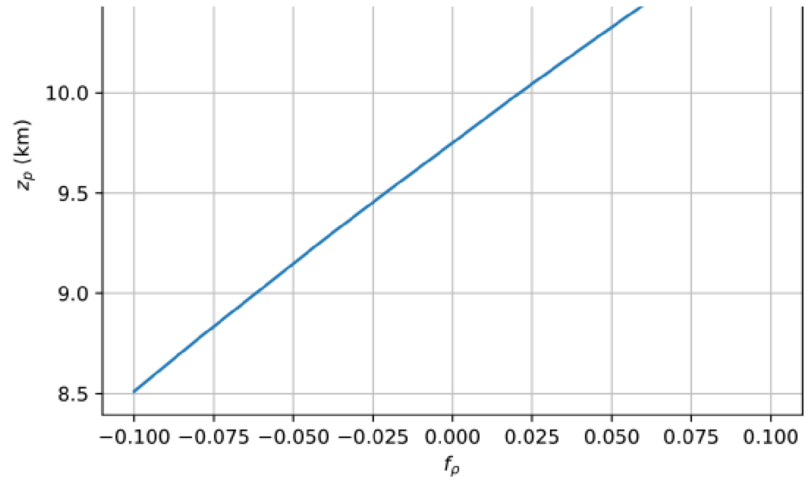



**Figure 14.5**: Impact of $f_\rho$ variation on parachute deployment altitude $z_p$.

Suppose that the parachute needs to open by 9 km to have a high likelihood that the lander can decelerate sufficiently before reaching the Martian surface (please note this value of 9 km is an arbitrary value and only being used for illustrative purposes). Then we could ask: what fraction of the possible $C_{Dl}$ values would the parachute deploy below 9 km? From Figure 14.2, we can observe that $z_p \leq 9$ km for $C_{Dl} < 1.59$ approximately. Thus, the fraction of $C_{Dl}$ values for which $z_p < 9$ km is $(1.59 - 1.5) / (1.9 - 1.5) \approx 0.23$ or $23\%$ of the entire $C_{Dl}$ range of values. Similarly, for the other parameters, $z_p < 9$ km when

- $\theta_e < 82.5^\circ$ which amounts to about $42\%$ of the possible $\theta_e$ range.
- $f_\rho < -0.0625$ which amounts to about $19\%$ of the possible $f_\rho$ range.
- No values of $V(t_I)$ in the stated range give $z_p < 9$ km.

Now, let's dip our toe a little into the world of probability and ask what is the probability of $z_p < 9$ km? We will get a little more precise about what we mean by probability in the next chapter. But for now, we will define the probability of an event occurring (e.g. the parachute opening below 9 km) as the fraction of all possible events during which the particular event occurs. So, in our Martian lander example, we need to first quantify the probability of the different values of each of the parameters. For simplicity to start, suppose that only $C_{Dl}$ is varying while $\theta_e$, $V(t_I)$, and $f_\rho$ are all at their nominal values. Let's consider two different situations for $C_{Dl}$.

#### Uniform distribution of drag coefficient

Suppose that all of the values of $C_{Dl}$ from 1.5 to 1.9 are equally likely to occur. Then the probability is exactly the fraction of values of $C_{Dl}$ for which

$z_p < 9$ km. We have already calculated this above to be approximately $23\%$.

### 10-40-40-10 distribution of drag coefficient