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[Lecture 8: Distance measures](#)

4. Introduction to Total Variation

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> Distance

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4. Introduction to Total Variation Distance

Definition of Total Variation Distance

Total variation distance between discrete measures



Assume that E is discrete (i.e., finite or countable). This includes Bernoulli, Binomial, Poisson,

Therefore X has a PMF (probability mass function):
 $\mathbb{P}_\theta(X = x) = p_\theta(x)$ for all $x \in E$,

$$p_\theta(x) \geq 0, \quad \sum_{x \in E} p_\theta(x) = 1$$

The total variation distance between \mathbb{P}_θ and $\mathbb{P}_{\theta'}$ is a simple function of the PMF's p_θ and $p_{\theta'}$:

$$\text{TV}(\mathbb{P}_\theta, \mathbb{P}_{\theta'}) = \frac{1}{2} \sum_{x \in E} |p_\theta(x) - p_{\theta'}(x)|.$$

▶ 6:18 / 6:18

▶ 1.50x



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Interpreting Total Variation Distance

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Recall from lecture that the **total variation distance** between two probability measures \mathbf{P}_θ and $\mathbf{P}_{\theta'}$ with sample space E is defined by

$$\text{TV}(\mathbf{P}_\theta, \mathbf{P}_{\theta'}) = \max_{A \subseteq E} |\mathbf{P}_\theta(A) - \mathbf{P}_{\theta'}(A)|$$

Let $X_1, \dots, X_n \stackrel{iid}{\sim} \mathbf{P}_{\theta^*}$ where $\theta^* \in \mathbb{R}$ is an unknown parameter. You construct a statistical model $(E, \{\mathbf{P}_\theta\}_{\theta \in \mathbb{R}})$ for your data. By analyzing your data, you are able to produce an estimator $\hat{\theta}$ such that the distributions $\mathbf{P}_{\hat{\theta}}$ and \mathbf{P}_{θ^*} are close in **total variation distance**. More precisely, you know that

$$\text{TV}(\mathbf{P}_{\hat{\theta}}, \mathbf{P}_{\theta^*}) \leq \epsilon,$$

where ϵ is a very small positive number.

Which of the following can you conclude about the distributions $\mathbf{P}_{\hat{\theta}}$ and \mathbf{P}_{θ^*} ? (Choose all that apply.)

☒ Let A be an event. Then $|\mathbf{P}_{\theta^*}(A) - \mathbf{P}_{\hat{\theta}}(A)| \leq \epsilon$

☒ Let $X \sim \mathbf{P}_{\theta^*}$, let $Y \sim \mathbf{P}_{\hat{\theta}}$, and suppose $a, b \in \mathbb{R}$ where $a \leq b$. Then $|\mathbf{P}_{\theta^*}(a \leq X \leq b) - \mathbf{P}_{\hat{\theta}}(a \leq Y \leq b)| \leq \epsilon$

☐ $|\theta^* - \hat{\theta}| \leq \epsilon$.



Solution:

Recall that by definition,

$$\text{TV}(\mathbf{P}_{\hat{\theta}}, \mathbf{P}_{\theta^*}) = \max_{A \subseteq E} |\mathbf{P}_{\hat{\theta}}(A) - \mathbf{P}_{\theta^*}(A)|$$

where the maximum is over all events A . Since we are given that $\text{TV}(\mathbf{P}_{\hat{\theta}}, \mathbf{P}_{\theta^*}) \leq \epsilon$, we conclude that $|\mathbf{P}_{\hat{\theta}}(A) - \mathbf{P}_{\theta^*}(A)| \leq \epsilon$ for every event A . Hence, the first choice is correct.

Let A be the event given by the interval (a, b) . Then,

$$|\mathbf{P}_{\theta^*}(a \leq X \leq b) - \mathbf{P}_{\hat{\theta}}(a \leq Y \leq b)| \leq \epsilon$$

is the same as saying $|\mathbf{P}_{\hat{\theta}}(A) - \mathbf{P}_{\theta^*}(A)| \leq \epsilon$. Thus, the second choice is true as well.

The third choice, " $|\theta^* - \hat{\theta}| \leq \epsilon$.", is incorrect. In general, even if distributions $\mathbf{P}_{\hat{\theta}}$ and \mathbf{P}_{θ^*} are close, there is no reason to expect the parameters θ^* and $\hat{\theta}$ to be close. To conclude that the estimated parameter is close to the true parameter given their distributions are close, we would need some assumptions on the map $\theta \mapsto \mathbf{P}_{\theta}$. No such assumption is given here.

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i Answers are displayed within the problem

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