1. Planes

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Problem Set B due Sep 15, 2021 20:30 IST

Practice

Normal to Plane 1

1/1 point (graded)

The equation $2x + y - 2z = 0$ describes a plane $\mathcal{Q}$ in three dimensions. Find a vector that is normal to $\mathcal{Q}$.

(Enter a vector using notation such as `[a,b]`.)

$v =$ [2,1,-2] ✓ **Answer:** [2, 1, -2]

? INPUT HELP

Solution:

We can recognize the equation for $\mathcal{Q}$ as a "hidden dot product":

$$2x + y - 2z = 0 \qquad \text{same as} \qquad \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} = 0 \tag{5.206}$$

Therefore, the points of $\mathcal{Q}$ are all points (x, y, z) such that the vector from the origin to (x, y, z) is perpendicular to $\begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}$. Since the origin belongs to the plane, we conclude that the vector $\begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}$ will be normal to $\mathcal{Q}$.

Alternative approach: It is also possible to find the normal vector by taking the partial derivatives of $f(x, y, z) = 2x - y + z$. As was the case in two dimensions, the gradient of f will be the normal vector.

Therefore another route to the same answer is the computation $\nabla f = \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}$.

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Answers are displayed within the problem

Normal to Plane 2

1/1 point (graded)

The equation $2x + y - 2z = 8$ describes a plane $\mathcal{P}$ in three dimensions. Find a vector that is normal to $\mathcal{P}$.

(Enter a vector using notation such as `[a,b]`.)

$v =$ [2,1,-2] ✓ **Answer:** [2, 1, -2]

? INPUT HELP

Solution:

The plane $\mathcal{P}$ is parallel to the plane $\mathcal{Q}$ from the previous question. One way of seeing

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The plane $\mathcal{P}$ is parallel to the plane $\mathcal{Q}$ from the previous question. One way of seeing this is that (x, y, z) belongs to $\mathcal{P}$ if and only if $(x, y-8, z)$ belongs to $\mathcal{Q}$.

Therefore the normal vector to $\mathcal{P}$ is the same as the normal vector to $\mathcal{Q}$, which we found to be $\begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}$.

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Distance 1

1/1 point (graded)

The equation $2x + y - 2z = 8$ describes a plane $\mathcal{P}$ in three dimensions. Find the distance between the origin and $\mathcal{P}$. This distance is defined to be the distance between the origin and the closest point belonging to $\mathcal{P}$.

Distance: 8/3 ✓ **Answer:** 8/3

Solution:

In the above problem, we saw that that the normal vector to $\mathcal{P}$ is $\vec{n} = \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}$. The shortest distance from the origin is therefore the length of $\lambda\vec{n}$ where λ is chosen such that $\lambda\vec{n}$ is in $\mathcal{P}$.

It may help to sketch a picture: draw coordinate axes and a plane, and draw a line segment representing the shortest distance from the origin to the plane. You will see that this distance is along the normal vector $\vec{n}$.

We can solve for λ from the equation

$$\vec{n} \cdot (\lambda\vec{n}) = 8. \qquad (5.207)$$

Since $\vec{n} \cdot \vec{n} = 9$, we have $\lambda = 8/9$. Finally, the length of $\frac{8}{9}\vec{n}$ is equal to $\frac{8}{3}$.

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Distance 2

1/1 point (graded)

The equation $2x + 4y - 4z = 40$ describes a plane $\mathcal{R}$ in three dimensions. What is the distance between the point $U = (1, 2, 3)$ and $\mathcal{R}$? (Again, "distance" means "shortest distance" in this context).

Distance: 7 ✓ **Answer:** 7

Solution:

The normal vector to $\mathcal{R}$ is $\vec{n} = \begin{pmatrix} 2 \\ 4 \\ -4 \end{pmatrix}$. The shortest distance from U to $\mathcal{R}$ is the length of the vector $\lambda\vec{n}$ where λ is chosen such that $U + \lambda\vec{n}$ is in $\mathcal{R}$. We can solve for λ from the equation

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$$\vec{n} \cdot (U + \lambda\vec{n}) = 40. \tag{5.208}$$

Since $\vec{n} \cdot \vec{n} = 36$, and $\vec{n} \cdot U = -2$, we solve $-2 + 36\lambda = 40$. This gives $\lambda = 7/6$. Finally, the length of $\frac{7}{6}\vec{n}$ is equal to 7.

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Distance 3

1/1 point (graded)

Define the distance between two planes to be the shortest distance between any two points, one in each plane. What is the distance between the planes $4x - 2y + 4z = 0$ and $4x - 2y + 4z = 1$?

1/6 ✓ **Answer:** 1/6

Solution:

Recall that the normal vector to both planes is $\vec{n} = \begin{pmatrix} 4 \\ -2 \\ 4 \end{pmatrix}$. The shortest distance is therefore the smallest multiple of $\vec{n}$ that we need to add to get from one plane to the other. Let $\vec{p}$ be any point in the plane $4x - 2y + 4z = 0$. We need to find the smallest λ such that $\vec{p} + \lambda\vec{n}$ belongs to the $= 1$ plane. In other words, we need to solve

$$\vec{n} \cdot (\vec{p} + \lambda\vec{n}) = 1. \tag{5.209}$$

Since $\vec{n} \cdot \vec{p} = 0$, the equation becomes

$$\lambda\,(\vec{n} \cdot \vec{n}) = 1. \tag{5.210}$$

Therefore $\lambda = \frac{1}{|\vec{n}|^2}$. In this case, $|\vec{n}|^2 = 36$, so $\lambda = \frac{1}{36}$. Finally, the length of $\lambda\vec{n}$ is equal to $\frac{1}{6}$.

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Distance 4

1/1 point (graded)

For $k = 0, 1, 2$, let $\mathcal{P}_k$ be the plane described by $ax + by + cz = k$ for nonzero a, b, c. True or false: the distance between $\mathcal{P}_0$ and $\mathcal{P}_2$ is twice the distance between $\mathcal{P}_0$ and $\mathcal{P}_1$.

● true

○ false

✓

Solution:

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Yes, the distance between $\mathcal{P}_k$ and $\mathcal{P}_{k+1}$ for any k is equal to $\frac{1}{|\vec{n}|}$ where $\vec{n}$ is the vector $\begin{pmatrix} a \\ b \\ c \end{pmatrix}$.

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1. Planes

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? Distance 1 [SPOILER]
Why when calculating λ a dot product is needed? Also can the problem be solved if one minimizes D=√x^2+y^2+z^2 within the plane? 2

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