5. Quadratic Risk of Estimators

Quadratic Risk of Estimators

like.

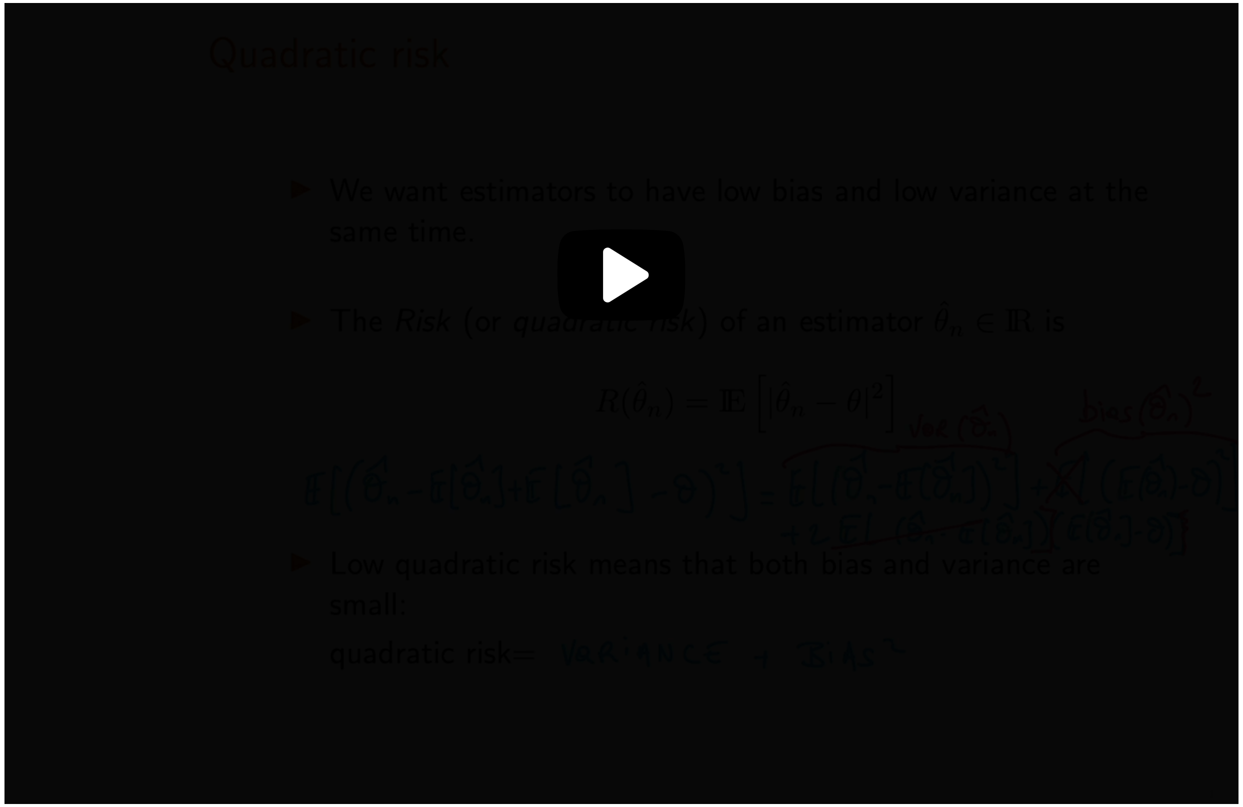
Take blah to be an estimator.

Compute its bias.

Compute its variance.

Compute its quadratic risk.

Well, hopefully, if you compute the bias



and you compute the variance, it's

going to cost you much to compute the quadratic risk.

OK.

Are there any questions?

7:22 / 7:22

1.50x

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Find the Quadratic Risk

1/1 point (graded)

Let $\hat{\theta}_n$ denote an estimator for a true parameter θ. The **quadratic risk** of $\hat{\theta}_n$ is defined to be

$$\mathbb{E}\left[\left(\hat{\theta}_n - \theta\right)^2\right].$$

As in the previous problem on variance, let $X_1, \ldots, X_n \stackrel{iid}{\sim} \mathcal{U}([a, a+1])$ where a is an unknown parameter. What is the quadratic risk of the estimator $\overline{X}_n - \frac{1}{2}$?

Quadratic risk : 1/(12*n) ✔ **Answer:** 1/(12*n)

$\frac{1}{12 \cdot n}$

Solution:

Recall that

$$\text{quadratic risk} = \text{variance} + \text{bias}^2.$$

We showed in a previous question that this estimator is unbiased. Also note that $\text{Var}\left(\overline{X}_n\right) = \text{Var}\left(\overline{X}_n - \frac{1}{2}\right) = \frac{1}{12n}$. Hence, the quadratic risk is also $\frac{1}{12n}$.

Submit You have used 1 of 3 attempts

Answers are displayed within the problem

Properties of Estimators

1/1 point (graded)

Let $\hat{\theta}_n$ denote an estimator for a true parameter θ. Here n denotes the sample size. Which of the following properties of $\hat{\theta}_n$ would ensure that $\hat{\theta}_n$ converges in probability to θ as $n \to \infty$? (Choose all that apply.)

- [x] $\hat{\theta}_n$ is consistent.
- [ ] $\hat{\theta}_n$ is unbiased.
- [x] The quadratic risk of $\hat{\theta}_n$ goes to 0 as $n \to \infty$.
- [ ] The variance of $\hat{\theta}_n$ goes to 0 as $n \to \infty$.

Solution:

The first choice is correct, because by definition, consistency implies that the estimator $\hat{\theta}_n \to \theta$ as $n \to \infty$. The third choice, "The quadratic risk of $\hat{\theta}_n$ goes to 0 as $n \to \infty$.", is correct because if the quadratic risk $\mathbb{E}\left[(\hat{\theta}_n - \theta)^2\right] \to 0$ then $\hat{\theta}_n \to \theta$ in L^2. By the properties of convergence, this implies that $\hat{\theta}_n \to \theta$ in probability.

Recall: Refer to Chapter 1 to review the relationship between the different types of convergence.

The second choice, "$\hat{\theta}_n$ is unbiased.", is incorrect. We give an example that shows that this choice is incorrect. Note that if $X_1, \ldots, X_n \stackrel{iid}{\sim} N(\mu, 1)$, then $\hat{\theta}_n := X_1$ is an unbiased estimator for μ because $\mathbb{E}[X_1] = \mu$. However, it is not consistent: $X_1 - \mu$ does not tend to 0 as $n \to \infty$.

Using this same example, we can also see that the fourth choice "The variance of $\hat{\theta}_n$ goes to 0 as $n \to \infty$." is incorrect. The estimator $\hat{\theta}_n := 0$ (as an estimator for μ) has variance 0 for all n. If $\mu \neq 0$, then $\hat{\theta}_n - \mu = -\mu$, which is constant for all n and does not converge to 0.

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Answers are displayed within the problem

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Properties of Estimation and Chebyshev's Inequality 4

Understanding why the Quadratic Risk/Mean Squared Error Equals Variance(estimator) + (Bias(estimator)^2 1

- Quadratic risk and Mean Squared Error — 3
  Are they the same thing?
- ? What's the use of the quadratic risk when its expression involves the unknown? — 2
  While the expression in the example is a happy one, in which I know the variance does not depend on the parameter... ...the example may be differe...
- Properties of Estimators - problem #3 — 2

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