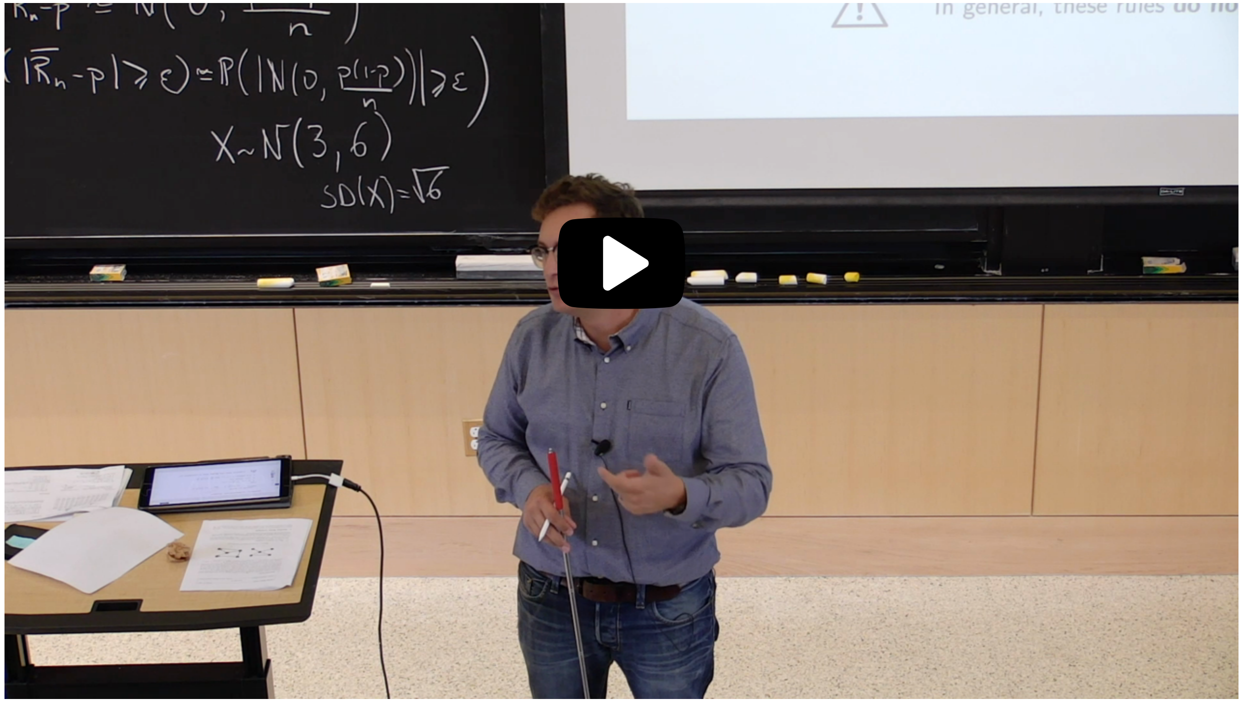
8. Operations on Sequences and Convergence

Addition, multiplicaton, division; Slutsky's Theorem; Continuous Mapping Thoerem

Start of transcript. Skip to the end.

Generating Speech Output



0:00 / 0:00 1.50x

OK.

So now I have ways to get limits.

From the law of large numbers I get the limit.

I get to almost sure in probability.

And from the central limit theorem I get another limit.

We're going to want to combine those results.

Maybe we're going to have one estimator

for the mean, for which we're going

Video

Download video file

Transcripts

Download SubRip (.srt) file

Download Text (.txt) file

We restate the theorems discussed in lecture below.

Addition, Multiplication, Division for Convergence almost surely and in probability :

Generating Speech Output

Addition, Multiplication, and Division preserves convergence almost surely ($\mathrm{a.s.}$) and convergence in probability ($\mathbf{P}$).

More precisely, assume

$$T_n \xrightarrow[n\to\infty]{\mathrm{a.s.}/\mathbf{P}} T \qquad \text{and} \qquad U_n \xrightarrow[n\to\infty]{\mathrm{a.s.}/\mathbf{P}} U$$

Then,

- $T_n + U_n \xrightarrow[n\to\infty]{\mathrm{a.s.}/\mathbf{P}} T + U$,
- $T_n U_n \xrightarrow[n\to\infty]{\mathrm{a.s.}/\mathbf{P}} TU$,
- If in addition, $U \neq 0$ a.s., then $\frac{T_n}{U_n} \xrightarrow[n\to\infty]{\mathrm{a.s.}/\mathbf{P}} \frac{T}{U}$.

Warning: In general, these rules **do not** apply to convergence in distribution (d).

Slutsky's Theorem

For convergence in distribution, the Slutsky's Theorem will be our main tool.

Let $(T_n)\,, (U_n)$ be two sequences of r.v., such that:

Generating Speech Output

- $T_n \xrightarrow[n\to\infty]{(d)} T$
- $U_n \xrightarrow[n\to\infty]{\mathbf{P}} u$

where T is a r.v. and u is a given real number (deterministic limit: $\mathbf{P}(U = u) = 1$). Then,

- $T_n + U_n \xrightarrow[n\to\infty]{(d)} T + u$,
- $T_n U_n \xrightarrow[n\to\infty]{(d)} Tu$,
- If in addition, $u \neq 0$, then $\dfrac{T_n}{U_n} \xrightarrow[n\to\infty]{(d)} \dfrac{T}{u}$.

Continuous Mapping Theorem

If f is a continuous function:

$$T_n \xrightarrow[n\to\infty]{\text{a.s.}/\mathbf{P}/(d)} T \quad \Rightarrow \quad f(T_n) \xrightarrow[n\to\infty]{} f(T).$$

Convergence in distribution

Generating Speech Output

3/4 points (graded)

Let X_n be a sequence of random variables that are converging **in probability** to another random variable X. Let Y_n be a sequence of random variables that are converging **in probability** to another random variable Y.

For each of the statements below, choose true ("This statement is always true") or false ("This statement is sometimes false"). Keep in mind that "convergence in probability" is stronger than "convergence in distribution".

- $X_n + Y_n \longrightarrow X + Y$ in distribution.

  - (•) True
  - ( ) False

- $X_n Y_n \longrightarrow XY$ in distribution.

  - (•) True
  - ( ) False

- $X_n / Y_n \longrightarrow X/Y$ in distribution, provided Y is constant.

  - (•) True

Generating Speech Output

False

- $X_n^2 - 2X_n + 5 \longrightarrow X^2 - 2X + 5$ in distribution.

True

False

Solution:

- True. Sums of sequences that converge in probability converge in probability, and convergence in probability implies convergence in distribution.
- True. Since both X_n and Y_n converge in probability to X and Y respectively, $X_n Y_n$ converges in probability, and hence in distribution, to XY.
- False. Even though Y_n converges to a constant, this constant can very well be 0, in which case we do not have the desired convergence.
- True. This is a consequence of continuous mapping theorem, since the function $g(x) = x^2 - 2x + 5$ is continuous, $X_n \longrightarrow X$ in distribution implies $g(X_n) \longrightarrow g(X)$ in distribution.

Generating Speech Output ave used 2 of 2 attempts

Answers are displayed within the problem

Applying Slutsky's and the Continuous Mapping theorems

1/1 point (graded)
Given the following:

- $Z_1, Z_2, \ldots, Z_n, \ldots$ is a sequence of random variables that converge in distribution to another random variable Z;
- $Y_1, Y_2, \ldots, Y_n, \ldots$ is a sequence of random variables each of which takes value in the interval $(0,1)$, and which converges in probability to a constant c in $(0,1)$;
- $f(x) = \sqrt{x(1-x)}$.

Does $Z_n \dfrac{f(Y_n)}{f(c)}$ converge in distribution? If yes, enter the limit in terms of Z, Y and c; if no, enter **DNE**.

$Z_n \dfrac{f(Y_n)}{f(c)} \xrightarrow{\mathrm{d}}$ Z **Answer:** Z

Z

Solution:

Generating Speech Output

Since f is continuous in $(0,1)$, $f(Y_n)$ converges in probability to $f(c)$ by the continous mapping theorem. Since $f(c)$ is a constant, we have $\frac{f(Y_n)}{f(c)}$ converges in probability to 1. Finally, since Z_n converges in distribution to Z and $\frac{f(Y_n)}{f(c)}$ converges in probability to a constant, by Slutsky's Theorem, $Z_n \dfrac{f(Y_n)}{f(c)}$ converges in distribution to Z.

Submit You have used 1 of 3 attempts

Answers are displayed within the problem

Discussion

Hide Discussion

Topic: Unit 1 Introduction to statistics:Lecture 2: Probability Redux / 8. Operations on Sequences and Convergence

Add a Post

Show all posts ▼ by recent activity ▼

Last problem: how to start
I know from the recitations that to prove convergence on distribution I have to prove that the CDF of the sequence tends to CDF of some other distri... 3

[STAFF] How much of lectures 2.7 and 2.8 will be important in the rest of the course?
Struggling a bit with Convergence and associated content. Prof mentioned this isn't something particularly important for the class and it won't be inc... 2

[STAFF] Please consider a deadline extension because of the 6.86x Final
Dear 18.6501x Team, Would it be possible to postpone the deadlines for unit 0 and unit 1 by a few days since many students like myself are completi... 5

Generating Speech Output

[STAFF] Question 1 - first two parts solution review

Is the solution for the first two parts correct? In the question it is given that the R.V Y_n converges to R.V Y, in probability. However, in the lecture, it a... 6

[STAFF] - Is the grader working properly for the Convergence in Distribution Problems? 9
It seems everything is the opposite from what I gleaned from the lecture.

Typo in the heading 3
Addition, multiplicaton, division; Slutsky's Theorem; Continuous Mapping Thoerem...We get the idea but would great to move the letters around in o...

Staff 3
Hi, I would like to know when will Unit 2 be released. A few days ago I saw it was available but now it's gone. Thanks!

Learn About Verified Certificates

Generating Speech Output