12.3.1 Eigenvalues and Eigenvectors of n x n Matrices: Special Cases

Week 12 due Dec 29, 2023 10:42 IST Completed

12.3.1 Eigenvalues and Eigenvectors of n x n Matrices: Special Cases

Video 12.3.1 Part 1

Start of transcript. Skip to the end.

Dr. Robert van de Geijn: So now that we've examined 2 by 2 matrices,

and 3 by 3 matrices, and how to find the eigenvalues and eigenvectors of those,

let's move on to the general case where we have n by n matrices.

We've seen the general case a little bit

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Homework 12.3.1.1

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Let $A \in \mathbb{R}^{n \times n}$ be a diagonal matrix: $A = \begin{pmatrix} \alpha_{0,0} & 0 & 0 & \cdots & 0 \\ 0 & \alpha_{1,1} & 0 & \cdots & 0 \\ 0 & 0 & \alpha_{2,2} & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & \alpha_{n-1,n-1} \end{pmatrix}$.

Then e_i is an eigenvector associated with eigenvalue $\alpha_{i,i}$.

TRUE ✔ Answer: TRUE

Just multiply it out. Without loss of generality (which means: take as a typical case), let $i = 1$. Then

$$\begin{pmatrix} \alpha_{0,0} & 0 & 0 & \cdots & 0 \\ 0 & \alpha_{1,1} & 0 & \cdots & 0 \\ 0 & 0 & \alpha_{2,2} & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & \alpha_{n-1,n-1} \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix} = \alpha_{1,1} \begin{pmatrix} 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix}$$

Here is another way of showing this, leveraging our notation: Partition

$$A = \left(\begin{array}{c|c|c} A_{00} & 0 & 0 \\ \hline 0 & \alpha_{11} & 0 \\ \hline 0 & 0 & A_{22} \end{array}\right) \quad \text{and} \quad e_j = \left(\begin{array}{c} 0 \\ \hline 1 \\ \hline 0 \end{array}\right),$$

where α_{11} denotes diagonal element $\alpha_{j,j}$. Then

$$\left(\begin{array}{c|c|c} A_{00} & 0 & 0 \\ \hline 0 & \alpha_{11} & 0 \\ \hline 0 & 0 & A_{22} \end{array}\right) \left(\begin{array}{c} 0 \\ \hline 1 \\ \hline 0 \end{array}\right) = \left(\begin{array}{c} 0 \\ \hline \alpha_{11} \\ \hline 0 \end{array}\right) = \alpha_{11} \left(\begin{array}{c} 0 \\ \hline 1 \\ \hline 0 \end{array}\right)$$

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Video 12.3.1 Part 2

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Dr. Robert van de Geijn: So here

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answer.

You partition the matrix where you identify

the i-th entry on the diagonal.

And we'll call that alpha 1 1.

And if you then take the unit basis vector

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Homework 12.3.1.2

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Let $A = \left(\begin{array}{c|c|c} A_{00} & a_{01} & A_{02} \\ \hline 0 & \alpha_{11} & a_{12}^T \\ \hline 0 & 0 & A_{22} \end{array} \right)$, where A_{00} is square. Then α_{11} is an eigenvalue of A and $\left(\begin{array}{c} -(A_{00} - \alpha_{11} I)^{-1} a_{01} \\ 1 \\ 0 \end{array} \right)$ is a corresponding eigenvector (provided $A_{00} - \alpha_{11} I$ is nonsingular).

TRUE ✔ Answer: TRUE

What we are going to show is that $(A - \alpha_{11} I)\, x = 0$ for the given vector.

$$\left(\left(\begin{array}{c|c|c} (A_{00} - \alpha_{11} I) & a_{01} & A_{02} \\ \hline 0 & 0 & a_{12}^T \\ \hline 0 & 0 & (A_{22} - \alpha_{11} I) \end{array} \right) \left(\begin{array}{c} -(A_{00} - \alpha_{11} I) a_{01} \\ 1 \\ 0 \end{array} \right) \right)$$

$$= \left(\begin{array}{c} (A_{00} - \alpha_{11} I)\left[-(A_{00} - \alpha_{11} I)^{-1} a_{01}\right] + a_{01} + 0 \\ 0 + 0 + 0 \\ 0 \end{array} \right) = \left(\begin{array}{c} -a_{01} + a_{01} + 0 \\ 0 \\ 0 \end{array} \right) = \left(\begin{array}{c} 0 \\ \hline 0 \\ \hline 0 \end{array} \right)$$

A more constructive way of verifying the result is to notice that clearly

$$\left(\begin{array}{c|c|c} (A_{00} - \alpha_{11} I) & a_{01} & A_{02} \\ \hline 0 & 0 & a_{12}^T \end{array} \right.$$

Calculator

$$\left(\begin{array}{c|c|c} 0 & 0 & (A_{22} - \alpha_{11} I) \end{array}\right)$$

is singular since if one did Gaussian elimination with it, a zero pivot would be encountered exactly where the 0 in the middle appears. Now, consider a vector of form $\left(\begin{array}{c} x_0 \\ \hline 1 \\ \hline 0 \end{array}\right)$. Then

$$\left(\begin{array}{c|c|c} (A_{00} - \alpha_{11} I) & a_{01} & A_{02} \\ \hline 0 & 0 & a_{12}^T \\ \hline 0 & 0 & (A_{22} - \alpha_{11} I) \end{array}\right) \left(\begin{array}{c} x_0 \\ \hline 1 \\ \hline 0 \end{array}\right) = \left(\begin{array}{c} 0 \\ \hline 0 \\ \hline 0 \end{array}\right)$$

means that $(A_{00} - \alpha_{11} I) x_0 + a_{01} = 0$. (First component on both sides of the equation.) Solve this to find that $x_0 = -(A_{00} - \alpha_{11} I)^{-1} a_{01}$.

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Answers are displayed within the problem

Video 12.3.1 Part 3

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Dr. Robert van de Geijn: And here's the answer.

If you take A minus alpha11I and you do that for the partitioned matrix,

you get this.

What you would like to show is that this vector is in the null space.

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Homework 12.3.1.3

1/1 point (graded)

The eigenvalues of a triangular matrix can be found on its diagonal.

TRUE ✓

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