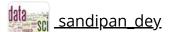


<u>Help</u>





Final project: Applications to

Course > nonlinear differential equations

Project 1: Review of nonlinear

> populations models

> 4. Review of linearization techniques

4. Review of linearization techniques

Recall that the best linear approximation of a function of 1 variable f(x) near a point x=a is

$$f(x) \approx f(a) + f'(a)(x-a).$$

Recall that for a function of 2 variables f(x,y), the best linear approximation near a point (x,y)=(a,b) is

$$f(x,y) \hspace{0.2cm} pprox \hspace{0.2cm} f(a,b) + rac{\partial f}{\partial x}(a,b) \hspace{0.1cm} (x-a) + rac{\partial f}{\partial y}(a,b) \hspace{0.1cm} (y-b)$$

In the course *Differential equations: 2x2 systems*, we saw that to linearize a 2×2 system of autonomous equations near a critical point (x,y)=(a,b),

$$\dot{x} = f(x, y)$$

$$\dot{y} = g(x,y)$$

we need to replace **both** f(x,y) and g(x,y) by the best linear approximations near (a,b), which are given by

$$\left|f(x,y)
ight|pprox \left|f(a,b)+rac{\partial f}{\partial x}
ight|_{(a,b)}\left(x-a
ight)+rac{\partial f}{\partial y}
ight|_{(a,b)}\left(y-b
ight) = \left|f(a,b)+f_x(a,b)
ight.\left(x-a
ight)+f_y(a,b)\left(y-b
ight)$$

$$g(x,y) \quad pprox \quad g(a,b) + rac{\partial g}{\partial x}igg|_{(a,b)} \; (x-a) + rac{\partial g}{\partial y}igg|_{(a,b)} \; (y-b) \, = \, g(a,b) + g_x(a,b) \; (x-a) + g_y(a,b) \; (y-b).$$

Using the definition of matrix multiplication, the best linear approximation of the vector-valued function $egin{pmatrix} f(x,y) \\ g(x,y) \end{pmatrix}$ near the point (x,y)=(a,b) is

$$egin{pmatrix} f(x,y) \ g(x,y) \end{pmatrix} pprox egin{pmatrix} f(a,b) \ g(a,b) \end{pmatrix} + egin{pmatrix} f_x & f_y \ g_x & g_y \end{pmatrix} igg|_{\mathbf{a}} (\mathbf{x}-\mathbf{a})\,, \qquad ext{where } \mathbf{x} = igg(x \ y \end{pmatrix}, \quad \mathbf{a} = igg(a \ b \end{pmatrix}.$$

We can abbreviate this notation as

$$f(x) \approx f(a) + J(a)(x - a),$$

where

$$\mathbf{f}(\mathbf{x}) = egin{pmatrix} f(x,y) \ g(x,y) \end{pmatrix},$$

$$\mathbf{f}(\mathbf{a}) = egin{pmatrix} f(a,b) \ g(a,b) \end{pmatrix},$$

$${f J} = egin{pmatrix} f_x & f_y \ g_x & g_y \end{pmatrix},$$

and the matrix ${f J}({f a})$ is the matrix of partial derivatives evaluated at ${f a}=egin{pmatrix} a \\ b \end{pmatrix}$.

The matrix ${f J}$ is called the **Jacobian matrix** .

Stability of fourth critical point

2 points possible (graded, results hidden)

We are modeling two populations via the model used on the previous page

$$\dot{x} = x - x^2 - axy$$

$$\dot{y} = 3y - 2y^2 - bxy.$$

Find conditions on the parameters $m{a}$ and $m{b}$ so that the critical point you found on the previous page is a stable critical point.

$$0 < a < \boxed{}_{2/3}$$

FORMULA INPUT HELP

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You have used 1 of 5 attempts

Identify the long term behavior, case 1

2 points possible (graded, results hidden)

Identify the long term population (x_{∞} , y_{∞}) of the system below, assuming that the starting populations are both positive.

$$\dot{x} = x - x^2 - 0.5xy$$

$$\dot{y} = 3y - 2y^2 - 2xy.$$

(If the long term behavior is oscillatory, or there are two possible outcomes, enter none in both answer boxes.)

$$x_{\infty} =$$

$$0.5$$

0.5

$$oldsymbol{y_{\infty}} = oldsymbol{f 1}$$

FORMULA INPUT HELP

Submit

You have used 1 of 5 attempts

Identify the long term behavior, case 2

2 points possible (graded, results hidden)

Identify the long term population (x_∞ , y_∞) of the system below, assuming that the starting populations are both positive.

$$\dot{x} = x - x^2 - xy$$

$$\dot{y} = 3y - 2y^2 - xy.$$

4. Review of linearization techniques Project 1: Review of nonlinear populations models 18.033x Courseware edX		
(If the long term behavior is oscillatory, or there are two possible outcomes, enter none in both answer boxes.) $x_\infty = egin{bmatrix} 0 & & & & & & & & & & & & & & & & & & $		
$y_{\infty} = \boxed{\frac{3}{2}}$ FORMULA INPUT HELP		
Submit You have used 1 of 5 attempts		
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