3. Transformed Curves

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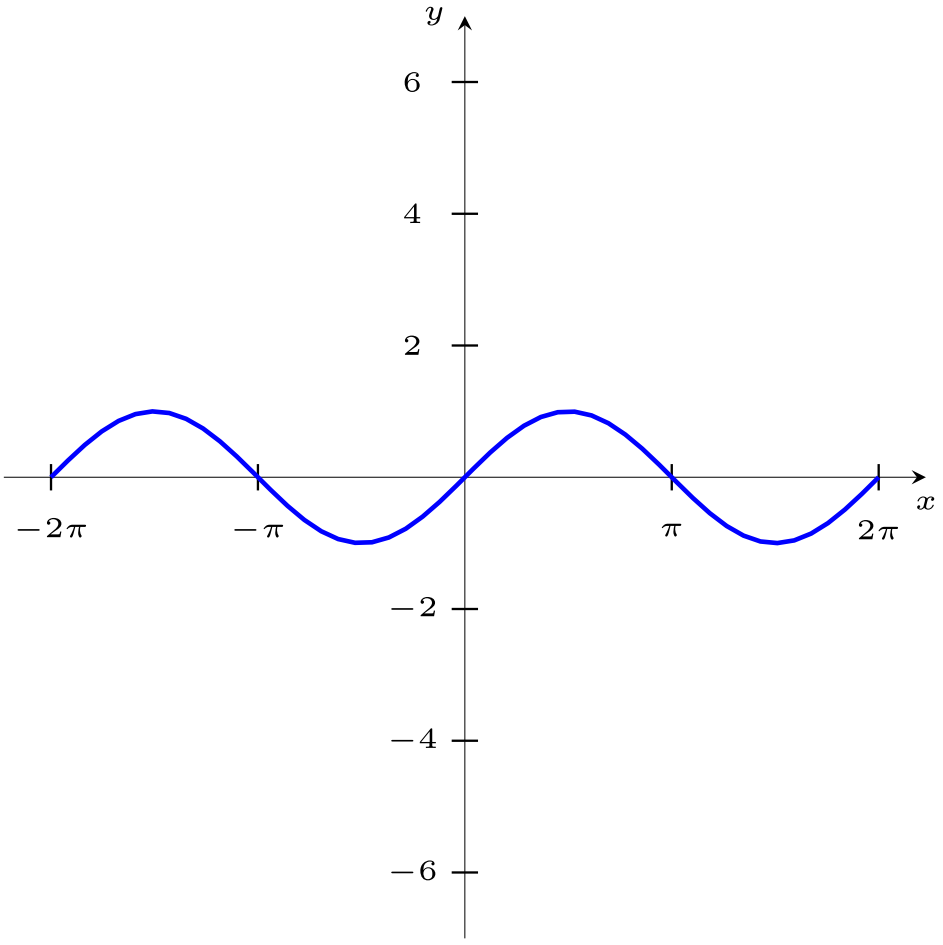
Lecture due Oct 5, 2021 20:30 IST

Practice

Parameterize a function

2/2 points (graded)

The image below shows the graph of $y = \sin x$ in the xy-plane for $-2\pi \leq x \leq 2\pi$. Find an equation for $\vec{r}\,(t) = \begin{pmatrix} x\,(t) \\ y\,(t) \end{pmatrix}$ whose trajectory is the curve shown for $-2\pi \leq t \leq 2\pi$.



(There is more than one correct answer. The answer boxes are graded together. This means e.g. you could have a correct $x\,(t)$, but if $y\,(t)$ is not correct, then both boxes will be marked as incorrect.)

$x\,(t) =$ t ✔ **Answer:** t

$y\,(t) =$ sin(t) ✔ **Answer:** sin(t)

? INPUT HELP

Solution:

Since we want $y = \sin x$, a simple option is $(t, \sin t)$. For every value of t, the point $(t, \sin t)$ is on the curve $y = \sin x$.

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Answers are displayed within the problem

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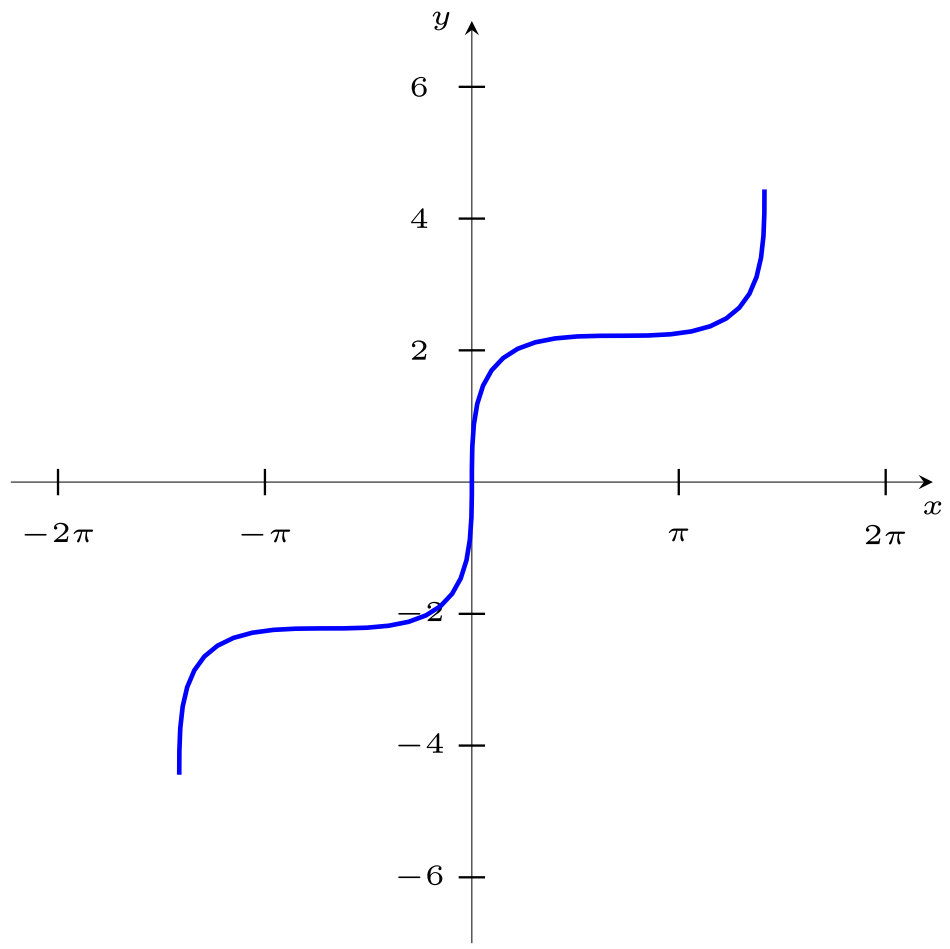
Rotate a sine wave

2/2 points (graded)

The image below shows a rotated sine wave that runs along the line $y = x$. Find an equation for $\vec{r}\,(t) = \begin{pmatrix} x\,(t) \\ y\,(t) \end{pmatrix}$ whose trajectory is the curve shown for $-2 \leq t \leq 2$.

Hint: write $\vec{r}\,(t)$ as a sum of two vectors, one that is parallel to $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ and one that is perpindicular to $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$.



$x\,(t) =$ (t-sin(t))/sqrt(2) ✔ **Answer:** (t - sin(t))/sqrt(2)

$y\,(t) =$ (t+sin(t))/sqrt(2) ✔ **Answer:** (t + sin(t))/sqrt(2)

? INPUT HELP

Solution:

The particle's position may be described as the sum of two vectors: $\vec{r} = \overrightarrow{v_1} + \overrightarrow{v_2}$.

First, for a given t, we should move a distance of t in the direction $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$. Since $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ has a length of $\sqrt{2}$ this means $\overrightarrow{v_1} = \frac{t}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$.

Second, for a given t, we should move a distance of $\sin t$ in the perpindicular direction, $\begin{pmatrix} -1 \\ 1 \end{pmatrix}$. Thus $\overrightarrow{r_2} = \frac{\sin t}{\sqrt{2}} \begin{pmatrix} -1 \\ 1 \end{pmatrix}$.

In total we have

$$\vec{r} = \overrightarrow{v_1} + \overrightarrow{v_2}$$
$$= \frac{t}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix} + \frac{\sin t}{\sqrt{2}} \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

Calculator Hide Notes

$$= \frac{t}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix} + \frac{\sin t}{\sqrt{2}}\begin{pmatrix}-1\\1\end{pmatrix} \tag{6.104}$$

$$= \begin{pmatrix}\frac{t-\sin t}{\sqrt{2}}\\ \frac{t+\sin t}{\sqrt{2}}\end{pmatrix} \tag{6.105}$$

Therefore,

$$x(t) = \frac{t-\sin t}{\sqrt{2}} \tag{6.106}$$

$$y(t) = \frac{t+\sin t}{\sqrt{2}} \tag{6.107}$$

One can also obtain this answer by applying the rotation matrix $R_{\pi/4} = \begin{pmatrix}1/\sqrt{2} & -1/\sqrt{2}\\ 1/\sqrt{2} & 1/\sqrt{2}\end{pmatrix}$ to the $\vec{r}(t) = \begin{pmatrix}t\\ \sin t\end{pmatrix}$.

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Answers are displayed within the problem

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Typo in problem "Rotate a sine wave"? 2

I think t lies between -2 pi and 2 pi, not between -2 and 2.

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