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9. Gradient connection to function height

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Lecture due Sep 13, 2021 20:30 IST Completed

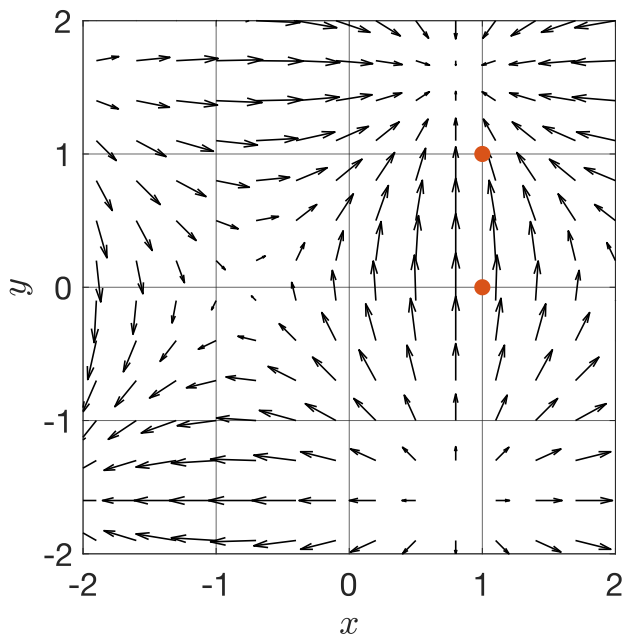
Explore

We have thought mostly about the gradient from the perspective that we are given level curves and want to understand the gradient. Now we want to go in the other direction. Suppose we are given the information about the gradient of a function, and now we want to understand something about the shape of that function. In other words, we want to understand the level curves from the gradient so that we can identify maxima and minima.

Gradient and function height 1

1/1 point (graded)

Consider the gradient field for a function $f(x,y)$ shown below. The (x,y) coordinates $(1,0)$ and $(1,1)$ are indicated by orange markers in the figure below.



Which of the following is true?

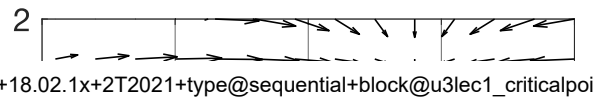
- (selected) $f(1,0) < f(1,1)$
- $f(1,0) > f(1,1)$
- $f(1,0) = f(1,1)$
- Not enough information to determine

Submit You have used 1 of 2 attempts

Gradient and function height 2

1/1 point (graded)

Consider the gradient field for the same function $f(x,y)$. This time, the (x,y) coordinates $(0,-1)$ and $(0,1)$ are indicated by orange markers in the figure below.



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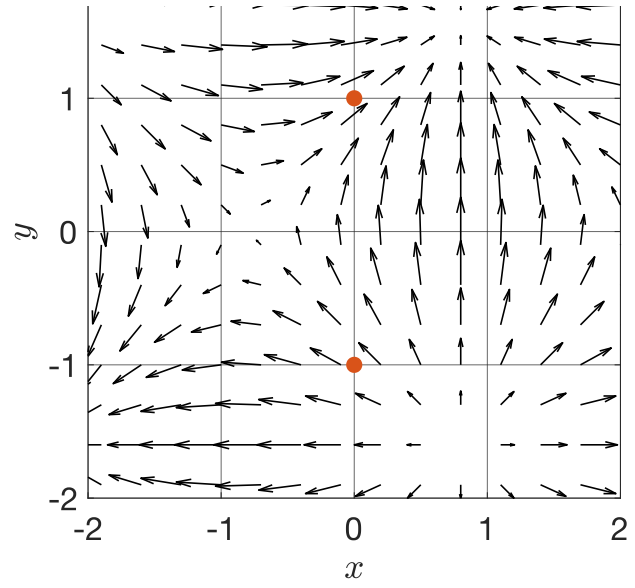


Which of the following is true?

- ◉ $f(0,-1) < f(0,1)$
- ○ $f(0,-1) > f(0,1)$
- ○ $f(0,-1) = f(0,1)$
- ○ Not enough information to determine

✔

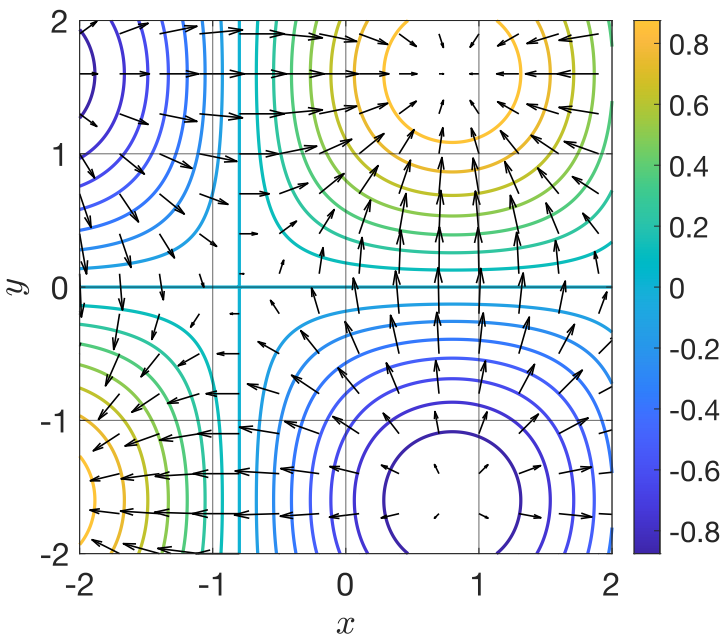
Solution:

From the gradient field, we can conclude that $f(0,-1) < f(0,1)$. We know that the gradient points in the direction of steepest increase of a function. Although the arrows at $(0,-1)$ do not point straight towards $(0,1)$, the vertical componenents of the gradient vectors are still in the upward direction. So the function is still increasing when we move from $(0,-1)$ to $(0,1)$, it is just not increasing as fast as possible.

Submit You have used 2 of 2 attempts

Answers are displayed within the problem

In the figure below, we include the level curves of the function $f(x,y)$ whose gradient field we were looking at in the previous questions.



Observations and summary

- The direction of the gradient ∇f is perpendicular to the level curves.

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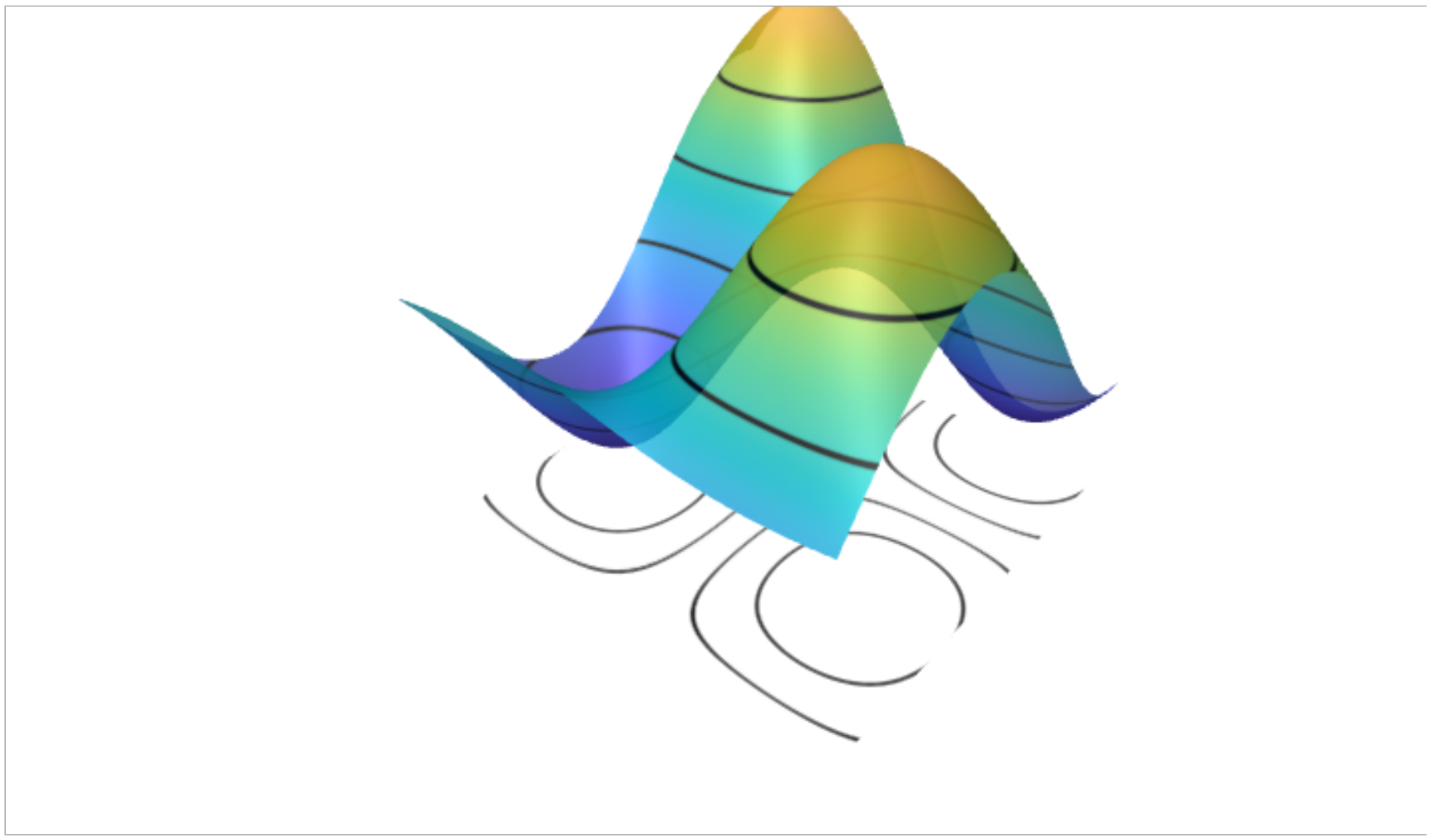
- The magnitude of the gradient $|\nabla f|$ is larger where the level curves are closer together.
- The slope of the 3D graph of the surface $z = f(x, y)$ is steeper in locations where the gradient is large and the level curves are close together.

In the figure below, we plot some level curves on top of the 3D function. The level curves are also shown on a plane below the 3D surface.

▶ **Hills with contours**



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