

### MITx: 6.041x Introduction to Probability - The Science of Uncertainty

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## Lec. 4: Counting

Exercises 4 due Feb 24, 2016 at 23:59 UTC

Solved problems

#### **Problem Set 3**

Problem Set 3 due Feb 24, 2016 at 23:59 UTC

Unit 4: Discrete

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# Problem 4: A three-sided die

(4/4 points)

The newest invention of the 6.041x staff is a three-sided die. On any roll of this die, the result is 1 with probability 1/2, 2 with probability 1/4, and 3 with probability 1/4.

Consider a sequence of six independent rolls of this die.

1. Find the probability that exactly two of the rolls results in a 3.

$$\bullet \quad \binom{6}{2} \left(\frac{1}{4}\right)^2 \left(\frac{3}{4}\right)^4 \quad \checkmark$$

 $igcup {6 \choose 2} \left(rac{1}{4}
ight)^2$ 

random variables

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$$\qquad \binom{6}{2} \left(\frac{1}{4}\right)^2 \binom{6}{4} \left(\frac{3}{4}\right)^4$$

2. Given that exactly two of the six rolls resulted in a 1, find the probability that the first roll resulted in a 1. **Note:** Your answer should be a number. Do not enter '!' or combinations in your answer.

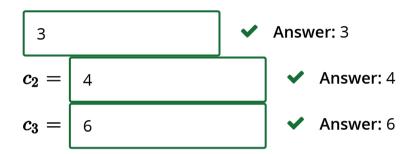
3. We are told that exactly three of the rolls resulted in a 1 and exactly three rolls resulted in a 2. Given this information, find the probability that the six rolls resulted in the sequence (1,2,1,2,1,2). **Note:** Your answer should be a number. Do not enter '!' or combinations in your answer.

4. The conditional probability that exactly k rolls resulted in a 3, given that at least one roll resulted in a 3, is of the form:

$$rac{1}{1-(c_1/c_2)^{c_3}}inom{c_3}{k}inom{1}{c_2}^kinom{c_1}{c_2}^{c_3-k},\quad ext{for } k=1,2,\ldots,6.$$

Find the values of the constants  $c_1$ ,  $c_2$ , and  $c_3$ :

 $c_1 =$ 



### Answer:

1. Each roll is an independent trial with probability 1/4 of resulting in a 3 (a "success"). The probability of exactly 2 successes in 6 trials is given by the binomial probabilities with  $n=6,\,k=2$ , and p=1/4:

$$\binom{6}{2} \left(\frac{1}{4}\right)^2 \left(\frac{3}{4}\right)^4$$

2.

The probability of obtaining a 1 on a single roll is 1/2, and the probability of obtaining a 2 or 3 on a single roll is also 1/2. For the purposes of solving this problem, we treat obtaining a 2 or a 3 as an equivalent result. We know that there are  $\binom{6}{2}$  ways of rolling exactly two 1's. Of these  $\binom{6}{2}$  ways, exactly  $\binom{5}{1}=5$  ways result in a 1 on the first roll, since we can place the other 1 in any of the five remaining rolls. The rest of the rolls must be either 2 or 3. Thus the probability that the first roll is a 1 given exactly two rolls resulted in a 1 is  $\frac{5}{\binom{6}{3}}=\frac{1}{3}$ .

3. We want to find

$$\mathbf{P}(121212 \mid ext{exactly three 1's and three 2's}) = \frac{\mathbf{P}(121212)}{\mathbf{P}( ext{exactly three 1's and three 2's})}$$

Any particular sequence of three 1's and three 2's will have the same probability:  $\left(\frac{1}{2}\right)^3\left(\frac{1}{4}\right)^3$ . There are  $\binom{6}{3}$  possible sequences with exactly three 1's and three 2's, of which exactly one sequence is 121212. Therefore,

$$\mathbf{P}(121212 \mid ext{exactly three 1's and three 2's}) = rac{\left(rac{1}{2}
ight)^3 \left(rac{1}{4}
ight)^3}{\left(rac{6}{3}
ight) \left(rac{1}{2}
ight)^3 \left(rac{1}{4}
ight)^3} = rac{1}{20}.$$

4. Let  $\boldsymbol{A}$  be the event that at least one roll results in a 3. Then,

$$\mathbf{P}(A) = 1 - \mathbf{P}(\text{no rolls resulted in a 3}) = 1 - \left(\frac{3}{4}\right)^6$$
.

Let  $oldsymbol{B}$  be the event that there were exactly  $oldsymbol{k}$  rolls that resulted in a 3, where

$$k \in \{1,2,\ldots,6\}$$
. Note that  $\mathbf{P}(B) = {6 \choose k} {\left(\frac{1}{4}\right)}^k {\left(\frac{3}{4}\right)}^{6-k}$ .

Note also that  $B \subset A$ . Thus, the desired probability is:

$$egin{align} \mathbf{P}(B \mid A) &= rac{\mathbf{P}(A \cap B)}{\mathbf{P}(A)} \ &= rac{\mathbf{P}(B)}{\mathbf{P}(A)} \ &= rac{1}{1 - (3/4)^6} inom{6}{k} inom{1}{4}^k inom{3}{4}^{6-k} ext{ for } k = 1, 2, \dots, 6. \end{align}$$

You have used 1 of 2 submissions

## DISCUSSION

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