3. Intersection of planes

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Problem Set A due Sep 15, 2021 20:30 IST

Practice

Two Planes

6/6 points (graded)

Let $\mathcal{P}_1$ be the plane $x + y + z = 6$ and $\mathcal{P}_2$ be the plane $3x + 2y + 2z = 14$.

Find two distinct points (x_0, y_0, z_0) and (x_1, y_1, z_1) that belong to the intersection of $\mathcal{P}_1$ and $\mathcal{P}_2$.

$x_0 =$ 2 ✔ **Answer:** 2

$y_0 =$ 2 ✔ **Answer:** 4

$z_0 =$ 2 ✔ **Answer:** 0

$x_1 =$ 2 ✔ **Answer:** 2

$y_1 =$ 1 ✔ **Answer:** 0

$z_1 =$ 3 ✔ **Answer:** 4

Solution:

The intersection of the two planes consists of all points x, y, z that satisfy both equations:

$$x + y + z = 6 \quad (5.173)$$
$$3x + 2y + 2z = 14 \quad (5.174)$$

There are several ways of finding solutions to this system. One method is to just set variables equal to 0 until the system has a unique solution. If we set $z = 0$ then we get the system

We can start by multiplying the first equation by 2:

$$x + y = 6 \quad (5.175)$$
$$3x + 2y = 14 \quad (5.176)$$

Now we proceed with elimination. We can subtract three of the first equation from the second to obtain:

$$x + y = 6 \quad (5.177)$$
$$-y = -4 \quad (5.178)$$

So we obtain $y = 4$. Then the second equation says $x = 2$. Thus the point $(2, 4, 0)$ belongs to both planes.

In a similar way, we can set $y = 0$ and simplify, finding that the point $(2, 0, 4)$ also belongs to both planes.

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Find the planes 1

2/2 points (graded)

Find the equations for two planes $\mathcal{P}_1$ and $\mathcal{P}_2$ such that the intersection of $\mathcal{P}_1$ and $\mathcal{P}_2$ contains the points $(0,0,0)$ and $(1,3,1)$. Your answer must have $\mathcal{P}_1 \neq \mathcal{P}_2$.

Your answer is given partial credit if the points belong to one but not both of the planes.

(Enter an equation using notation such as `x+y+1=4*z`.)

Equation for $\mathcal{P}_1$: x+y=4*z ✔ **Answer:** x + y - 4*z = 0

Equation for $\mathcal{P}_2$: x=z ✔ **Answer:** -4*x + y + z = 0

Solution:

We need to find numbers a, b, c, d such that the equation

$$ax + by + cz = d \tag{5.179}$$

is satisfied for $(x,y,z) = (0,0,0)$ and $(1,3,1)$. Substituting $(0,0,0)$ tells us that d will have to equal 0. Then, substituting $(1,3,1)$ tells us

$$a + 3b + c = 0 \tag{5.180}$$

Since this is just one equation with three unknowns, there are many solutions. We might be tempted to set some variables equal to 0, but this would lead to the “Plane" $0 = 0$ which is not a plane at all.

Instead, we can set some variables equal to 1. If we set a and b to 1, then we get $c = -4$. Therefore, one valid choice of $\mathcal{P}_1$ is

$$x + y - 4z = 0 \tag{5.181}$$

If we set b and c to 1, then we get $a = -4$. Therefore, we can make $\mathcal{P}_2$ have the equation

$$-4x + y + z = 0 \tag{5.182}$$

There are many other correct answers.

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Find the planes 2

2/2 points (graded)

Find the equations for two planes $\mathcal{P}_1$ and $\mathcal{P}_2$ such that the intersection of $\mathcal{P}_1$ and $\mathcal{P}_2$ contains the points $(-4,1,1)$ and $(2,2,0)$. Your answer must have $\mathcal{P}_1 \neq \mathcal{P}_2$.

(Enter an equation using notation such as `x+y+1=4*z`.)

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Equation for $\mathcal{P}_1$: x+y+7*z=4 ✔ **Answer:** y + z = 2

Equation for $\mathcal{P}_2$: x+6*z=2 ✔ **Answer:** x + 6*z = 2

Solution:

As before, we need to find numbers a, b, c, d such that the equation

$$ax + by + cz = d \tag{5.183}$$

is satisfied for $(x, y, z) = (-4, 1, 1)$ and $(2, 2, 0)$. Substituting $(x, y, z) = (-4, 1, 1)$ gives us the equation

$$-4a + b + c = d. \tag{5.184}$$

And substituting $(2, 2, 0)$ gives us the equation.

$$2a + 2b = d. \tag{5.185}$$

Thus we have a system of two equations in four unknowns:

$$-4a + b + c - d = 0quad2a + 2b - d = 0 \tag{5.186}$$

As in the previous problems, we can find solutions by choosing values for the extra variables (while taking care to avoid the solution $(a, b, c, d) = (0, 0, 0, 0)$).

One option is to set $a = 0$ and $b = 1$ to get the system:

$$c - d = -1 \qquad -d = -2 \tag{5.187}$$

Which has solution $c = 1$ and $d = 2$. Therefore an option for $\mathcal{P}_1$ is

$$y + z = 2 \tag{5.188}$$

To get $\mathcal{P}_2$ we can set $a = 1$ and $b = 0$ to obtain the system:

$$c - d = 4 \qquad -d = -2 \tag{5.189}$$

This system has solution $c = 6$ and $d = 2$. Therefore an option for $\mathcal{P}_2$ is

$$x + 6z = 2 \tag{5.190}$$

There are many other correct answers.

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[STAFF] Typo in solution
There are missing line breaks: after the first "0" in Eq (5.186) after the "-1" in Eq (5.187) after the "4" in Eq (5.189)
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