Exercises: Least Squares Estimation

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Truth about least squares

2/2 points (ungraded)

Which of the following statements is correct? *Select all correct statements*

- [ ] Least squares Estimation takes the minimum of the squared observations.
- [ ] Least squares Estimation takes the minimum of the unknowns.
- [ ] The least squares solution of a system of observation equations minimizes the differences between the observations and the unknowns.
- [x] The least squares solution of a system of observation equations minimizes the differences between the observations $\boldsymbol{y}$ and the adjusted observations $\hat{\boldsymbol{y}}$.
- [x] The adjusted observations $\hat{\boldsymbol{y}}$ always fit perfectly to the $\boldsymbol{y} = \boldsymbol{A}\boldsymbol{x}$ model.

✔

Which of the following statements is correct? *Select all correct statements*

- [x] If a system of observation equations has m observations, then $e^T e = e_1^2 + e_2^2 + \ldots + e_m^2$.
- [x] The norm of a vector represents the length of that vector.
- [ ] The norm of a vector is the squared length of that vector.
- [x] The normal matrix is always a square matrix.

✔

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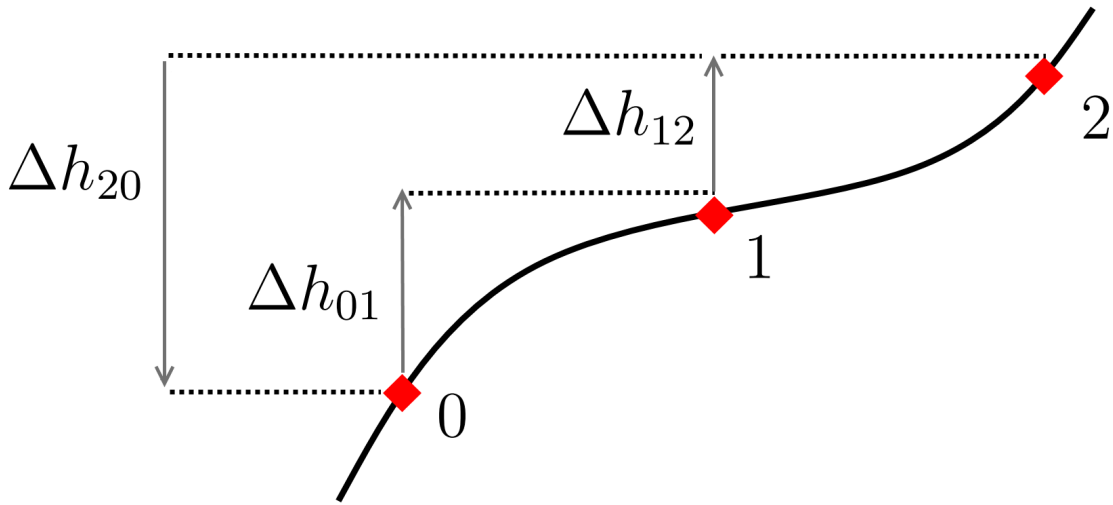
✔ Correct (2/2 points)



The above figure shows the scheme of a leveling campaign. The observed height difference between locations i and j is given by:

$$h_{ij} = h_j - h_i$$

In this case assume that h_0 is known and equal to zero ($h_0 = 0$).

Design matrix

1/1 point (ungraded)

For the given observation vector $y = [h_{01} \quad h_{12} \quad h_{20}]^T$, which of the following matrices is the correct design matrix?

- ◉ $\begin{bmatrix} 1 & 0 \\ -1 & 1 \\ 0 & -1 \end{bmatrix}$ ✔
- ○ $\begin{bmatrix} 0 & 1 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & -1 \end{bmatrix}$
- ○ $\begin{bmatrix} 1 & 0 \\ -1 & 1 \\ 0 & 1 \end{bmatrix}$

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✔ Correct (1/1 point)

Least square solution

4/4 points (ungraded)

For the same leveling campaign the following observation vector is given: $y = [2.2 \;\; 4.3 \;\; -6.7]^T$

Is the system of equations consistent?

no ✔ **Answer:** no

Explanation
No, we cannot find a solution for h_1 and h_2 such that all 3 observed height differences are obtained. For that we would need e.g. that $y_1 = 2.2 = h_1$ and $y_3 = -6.7 = -h_2$. But then $h_2 - h_1 = 6.7 - 2.2 = 4.5 \neq y_2 = 4.3$.

Apply least squares estimation to estimate the two unknown heights, $\hat{x} = [\hat{h}_1 \quad \hat{h}_2]^T$. [Give your answer to 2 decimal places, e.g. 1.00]

$\hat{h}_1 =$

2.27 ✔ **Answer:** 2.27

2.27

$\hat{h}_2 =$

6.63 ✔ **Answer:** 6.63

6.63

Which of the following is the correct vector with residuals?

- $\hat{e} = [-0.07 \quad -0.07 \quad -0.07]^T$ ✔

- $\hat{e} = [2.27 \;\; 4.37 \;\; -6.63]^T$
- $\hat{e} = [-0.20 \;\; -0.20 \;\; -0.20]^T$
- $\hat{e} = [0.33 \;\; 0.33 \;\; -0.67]^T$

Explanation

We have that $\hat{x} = [2.27 \;\; 6.63]^T$, $\hat{y} = A\hat{x} = [2.27 \;\; 4.37 \;\; -6.63]^T$, and $\hat{e} = y - \hat{y} = y - A\hat{x} = [-0.07 \;\; -0.07 \;\; -0.07]^T$.

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✔ Correct (4/4 points)