1.2.3 Equilibrium Points and Stability

Bookmark this page

Here is some of the terminology that Wes used in discussing the fish population and the differential equation model.

- An **equilibrium solution** to a differential equation is a constant solution, one that makes $dP/dt = 0$ for all time. In this case, $P = 0$ and $P = 40,000$ are equilibrium solutions of $\frac{dP}{dt} = \frac{1}{10}P\left(1 - \frac{P}{40000}\right)$.

- An equilibrium solution is **stable** if for starting populations near that equilibrium solution, the population will tend back toward the equilibrium solution. In other words, for starting populations a little below a stable equilibrium, the population will increase towards the equilibrium value. For starting populations a little above a stable equilibrium, the population will decrease towards the equilibrium.

- An equilibrium solution is **unstable** if for any starting population near the equilibrium solution, the population will tend away from the equilibrium solution. In other words, for starting populations a little below an unstable equilibrium the population will decrease, moving away from the equilibrium. For starting populations a little above an unstable equilibrium the population will increase, moving away from equilibrium.

- If some starting population values near the equilibrium solution tend away from the equilibrium point while others tend toward it, we call this **semistable**. (Note: some sources just call this unstable.)

It can be useful to analyze stability using the graph of the derivative of the population, $\frac{dP}{dt}$ versus P, as Wes did.

Here's one example. The solution $P(t) = 0$ is an equilibrium solution. Is it stable? In order for $P(t) = 0$ to be stable by our definition, populations starting near 0 would have to decrease in size toward $P = 0$. However, for small positive values of $P(t)$, the model predicts that the population will increase (since the graph of $\frac{dP}{dt}$ is positive near $P = 0$). Thus $P(t) = 0$ is not stable.

To classify the equilibrium solution $P(t) = 0$ as unstable or semi-stable, we need to look at small negative values of P. These don't make biological sense, but mathematically we can look at the graph of $\frac{dP}{dt}$ and see that P decreases for values less than 0. Thus for P-values on either side of $P(t) = 0$, the 'population' P will move away from 0. Hence, we call $P(t) = 0$ unstable.

Learn About Verified Certificates

English ▼