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3. Choosing a Prior

Choosing a Prior

Examples

to p.

$$\pi(p|X_1, \dots, X_n) \propto p^{\sum_{i=1}^n X_i} (1-p)^{n - \sum_{i=1}^n X_i} = L_n(X_1, \dots, X_n | p)$$

i.e., the posterior distribution is

$$\text{Beta}\left(1 + \sum_{i=1}^n X_i, 1 + n - \sum_{i=1}^n X_i\right)$$

► If $\pi(\theta) = 1, \forall \theta \in \mathbb{R}$ and given $X_1, \dots, X_n | \theta \stackrel{i.i.d.}{\sim} \mathcal{N}(\theta, 1)$:

$$\pi(\theta|X_1, \dots, X_n) \propto \exp$$

i.e., the posterior distribution is

▶ 9:32 / 9:32

▶ 1.50x



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Uniform Priors: True or False

1/1 point (graded)

Select from the following statements the **true** ones for uniform priors. (In this question, we also allow *improper* priors.)

☐ They can be defined only on parameter sets Θ with a finite number of possible values.

☐ They should integrate to 1 (or if the distribution is discrete; should sum to 1)

☒ They reflect an equal belief in each possible hypothesis.

☒ The maximum a-posteriori and maximum likelihood estimators when using such a prior would always be the same.



Solution:

- The first choice is false. As discussed in the lecture, they can be defined on infinite sets or even non-discrete distributions with an uncountably infinite number of possible parameter values.
- The second choice is also false. If $\pi(\cdot)$ is improper, then it will definitely not integrate to 1 by definition.
- The third choice is correct. A uniform prior reflects an "equal" belief in each of the possible hypothesis.
- The last choice is also correct. Recall that the maximum-a-posterior estimator maximizes $\pi(\theta|X_1, \dots, X_n)$ while the maximum likelihood estimator maximizes $L_n(X_1, \dots, X_n|\theta)$, both taken as functions of θ . By Bayes' rule, we have that

$$\pi(\theta|X_1, \dots, X_n) \propto L_n(X_1, \dots, X_n|\theta) \pi(\theta) \propto L_n(X_1, \dots, X_n|\theta),$$

where the first proportionality is due to Bayes' rule and the second proportionality is due to $\pi(\cdot)$ being uniform. The two statistics, when taken as a function of θ , are therefore identical up to a constant of proportionality. Hence, while the maximum values might be different, the value of θ attaining the maximum for both quantities are the same.

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Beta Distribution: True or False

1/1 point (graded)

One specific prior discussed in the previous lecture is the Beta distribution, which was then demonstrated in a scenario with a Bernoulli statistical model. **Which of the following statements is/are true about the Beta distribution**, written as $\text{Beta}(\alpha, \beta) \propto p^{\alpha-1}(1-p)^{\beta-1}$?

- ☒ The Beta distribution is very suited to models where our parameter represents a probability due to its support being $[0, 1]$.
- ☐ The Beta distribution is very suited to models where our parameter represents a probability due to its maximum always being close to $\frac{1}{2}$.
- ☒ The Beta distribution is very suited to models where our parameter represents a probability because multiplying it by p or $1 - p$ simply involves incrementing the respective parameter.



Solution:

The first and third statements are correct.

- **The first statement is correct.** The Beta distribution indeed has support on the interval $[0, 1]$, which is also the range of possible probabilities. Thus, using the Beta distribution to model possible parameters p would allow us to exactly cover the feasible range.
- **The second statement is incorrect.** The Beta family of distributions is very flexible and does not constrain us to symmetric shapes (which happens if we instead use a Gaussian prior). Indeed, if you recall the calculation of modes from the previous lecture, the mode can be at 0 or 1 for certain special cases. In the general case $\alpha > 1, \beta > 1$, the mode is at $\frac{\alpha-1}{\alpha+\beta-2}$, which could range anywhere in $(0, 1)$ depending on α and β .
- **The third statement is correct.** Mathematically, it is easy to see that multiplying the PDF of a Beta distribution by p or $1 - p$ increments the α or β coefficient, respectively, by 1. In practical terms, it is very common in statistical applications, as you've seen in the previous lecture, to multiply the likelihood function by either p or $1 - p$ depending on the outcome of a binary trial.

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