

Problem 2

Choose all the prime numbers which are the **sums of two squares**.

$$71 \quad 101 \quad 401 \quad 2017$$

$$2^{74207281} - 1$$

Fermat's Thm on Sums of Two Squares

A prime number P is a **sum of two squares** if and only if

$$P = 2 \quad \text{or} \quad P \equiv 1 \pmod{4}.$$



Pierre de Fermat
(1607?-1665)

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$$71 = 17 \times 4 + 3 \equiv 3 \pmod{4}$$

\Rightarrow 17 is **not** a sum of two squares.

$$101 \equiv 1 \pmod{4}$$

\Rightarrow 101 is a **sum of two squares**.

$$(101 = 1^2 + 10^2)$$

$$401 \equiv 1 \pmod{4}$$

\Rightarrow 401 is a **sum of two squares**.

$$(401 = 1^2 + 20^2)$$

$$2017 \equiv 1 \pmod{4}$$

\Rightarrow 2017 is a **sum of two squares**.

$$(2017 = 9^2 + 44^2)$$



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$$\begin{aligned} 2^{74207281} - 1 \\ &= 4 \times 2^{74207279} - 1 \\ &\equiv -1 \pmod{4} \\ &\equiv 3 \pmod{4} \end{aligned}$$

$\Rightarrow 2^{74207281} - 1$ is **not** a sum of two squares.



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