MITx: 14.310x Data Analysis for Social Scientists

Help

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Question 1 - 5

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Suppose that X_i i.i.d. $U[0, \theta]$. You want to build a 90% confidence interval for θ. To do so, you will need an estimator for θ and you will need to know the estimator's distribution. Let's consider $\hat{\theta} = \frac{n+1}{n} X_{(n)}$. (Remember that $X_{(n)}$ is the nth order statistic.) This estimator is a variant on the MLE. We have used the n^{th} order statistic, which is the MLE, but multiplied it by $\frac{n+1}{n}$ to remove its bias. Its PDF is $\frac{n^{n+1}}{(n+1)^n} \frac{x^{n-1}}{\theta^n}$ for $x \in [0, \frac{n+1}{n}\theta]$ and 0 otherwise.

Question 1

1/1 point (graded)

What is the value a such that 5% of the distribution of $\hat{\theta}$ is to the left of a? (It will be a function of n and θ.)

- a. It is given by $\sqrt[n]{0.1}\frac{n}{n+1}\theta$
- b. It is given by $\sqrt[n]{0.05}\frac{n}{n+1}\theta$
- c. It is given by $\sqrt[n]{0.1}\frac{n+1}{n}\theta$

- Module 5: Moments of a Random Variable, Applications to Auctions, & Intro to Regression
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Assessing and Deriving Estimators
Finger Exercises due Nov 14, 2016 at 05:00 IST

Confidence Intervals and Hypothesis Testing
Finger Exercises due Nov 14, 2016 at 05:00 IST

Module 7: Homework
Homework due Nov 07, 2016 at 05:00 IST

d. It is given by $\sqrt[n]{0.05}\frac{n+1}{n}\theta$ ✔

Explanation

We should find the value of a such that:

$$Pr(\hat{\theta} \leq a) = 0.05$$

$$\int_0^a \frac{n^{n+1}}{(n+1)^n}\frac{x^{n-1}}{\theta^n}dx = 0.05$$

$$\frac{n^{n+1}}{(n+1)^n}\frac{a^n}{n\theta^n} = 0.05$$

$$a(\theta, n) = \sqrt[n]{0.05}\frac{n+1}{n}\theta$$

Submit You have used 1 of 2 attempts

✔ Correct (1/1 point)

Question 2

1/1 point (graded)

- Exit Survey

What is the value b such that 5% of the distribution of is to the right of b?

- a. It is given by $\sqrt[n]{0.9}\frac{n+1}{n}\theta$
- b. It is given by $\sqrt[n]{0.9}\frac{n}{n+1}\theta$
- c. It is given by $\sqrt[n]{0.95}\frac{n+1}{n}\theta$ ✔
- d. It is given by $\sqrt[n]{0.95}\frac{n}{n+1}\theta$

Explanation

We should find the value of b such that:

$$Pr(\hat{\theta} \geq b) = 0.05$$

$$\int_b^{\frac{n+1}{n}\theta} \frac{n^{n+1}}{(n+1)^n}\frac{x^{n-1}}{\theta^n}dx = 0.05$$

$$\frac{n^{n+1}}{(n+1)^n}\left(\frac{(n+1)^n\theta^n}{n^{n+1}\theta^n} - \frac{b^n}{n\theta^n}\right) = 0.05$$

$$b(\theta, n) = \sqrt[n]{0.95}\frac{n+1}{n}\theta$$

Submit You have used 1 of 2 attempts

Correct (1/1 point)

Question 3

1/1 point (graded)

Using those values that you found above, what is a probability statement of the form $P(a < \hat{\theta} < b) = .90$ as a function of n and θ.

- a. It is $P\left(\sqrt[n]{0.05}\frac{n+1}{n}\theta \leq \hat{\theta} \leq \sqrt[n]{0.95}\frac{n+1}{n}\theta\right) = 0.9$
- b. It is $P\left(\sqrt[n]{0.1}\frac{n+1}{n}\theta \leq \hat{\theta} \leq \sqrt[n]{0.9}\frac{n+1}{n}\theta\right) = 0.9$
- c. It is $P\left(\sqrt[n]{0.05}\frac{n+1}{n}\theta \leq \hat{\theta} \leq \sqrt[n]{0.9}\frac{n+1}{n}\theta\right) = 0.9$
- d. It is $P\left(\sqrt[n]{0.1}\frac{n+1}{n}\theta \leq \hat{\theta} \leq \sqrt[n]{0.95}\frac{n+1}{n}\theta\right) = 0.9$

Explanation

We know that $P(a \leq \hat{\theta} \leq b) = 0.9$. So we plug in the values for $a(\theta, n)$ and $b(\theta, n)$ that we found above.

Submit You have used 1 of 2 attempts

✔ Correct (1/1 point)

Question 4

1/1 point (graded)

If you rearrange the quantities in the probability statement so that θ is alone in the middle, bracketed by functions of the random sample and known quantities, what would be this probability statement?

- a. It is $P\left(\frac{\hat{\theta}}{\sqrt[n]{0.05}\frac{n}{n+1}} \leq \theta \leq \frac{\hat{\theta}}{\sqrt[n]{0.95}\frac{n}{n+1}}\right) = 0.9$
- b. It is $P\left(\frac{\hat{\theta}}{\sqrt[n]{0.95}\frac{n}{n+1}} \leq \theta \leq \frac{\hat{\theta}}{\sqrt[n]{0.05}\frac{n}{n+1}}\right) = 0.9$ ✔
- c. It is $P\left(\frac{\hat{\theta}}{\sqrt[n]{0.9}\frac{n}{n+1}} \leq \theta \leq \frac{\hat{\theta}}{\sqrt[n]{0.1}\frac{n}{n+1}}\right) = 0.9$
- d. It is $P\left(\frac{\hat{\theta}}{\sqrt[n]{0.95}\frac{n}{n+1}} \leq \theta \leq \frac{\hat{\theta}}{\sqrt[n]{0.1}\frac{n}{n+1}}\right) = 0.9$

Explanation
This is your 90% confidence interval for θ. (If you were to draw 100 different samples and construct 100 such confidence intervals, you would expect 90 of them to contain the true value of θ.)

Submit You have used 1 of 2 attempts

Correct (1/1 point)

We are going to show this in R. We have provided you with this code that demonstrates this.

Question 5

1/1 point (graded)
In the code, there are three symbols standing in for specific values: XXX, YYY, and ZZZ. Which of the following values correspond to XXX, YYY, and ZZZ?

- a. XXX$=n$, YYY$=2$, ZZZ$=|$
- b. XXX$=\theta$, YYY$=1$, ZZZ = &
- c. XXX$=\theta$, YYY$=2$, ZZZ = &

d. XXX$= n$, YYY$= 2$, ZZZ = &

e. XXX$= \theta$, YYY$= 1$, ZZZ$= |$

Explanation

XXX corresponds to the maximum value of the uniform distribution we are simulating, which should be θ. We are calculating the maximum value through the columns since each column represents a different sample size of size n. Thus, we should use the apply function over the columns. This implies that YYY$= 2$. Finally, our confidence interval contains the real value of θ if $\theta \geq \frac{\hat{\theta}}{\sqrt[n]{0.95}\frac{n}{n+1}}$ and $\theta \leq \frac{\hat{\theta}}{\sqrt[n]{0.05}\frac{n}{n+1}}$. Thus, ZZZ should be equal to &.

Submit You have used 1 of 2 attempts

Correct (1/1 point)

We invite you to run this code to see that it is true that in 90% of the simulated samples this confidence interval (CI) contains the real value of θ. You can play with the code, changing both the value of θ and the sample size.

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