8. Second derivative test: General case

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Second derivative test

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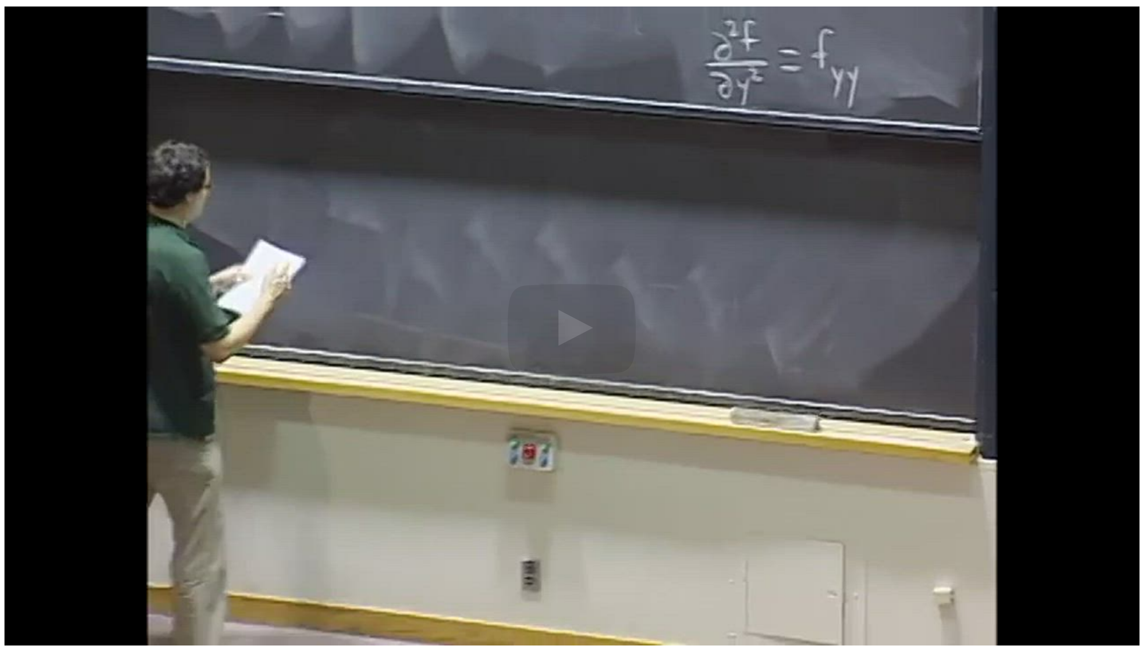

0:00 / 0:00 2.0x

PROFESSOR: What does the second derivative test say?

It says-- say that you have a critical point (x0, y0)

of a function of two variables of f,

and then let's compute the partial derivatives.

So let's call capital A the second derivative

with respect to x.

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A general function $f(x, y)$ has three distinct second partial derivatives: f_{xx}, $f_{xy} = f_{yx}$, and f_{yy}. The second derivative test uses these second derivatives to determine the type of critical point of $f(x, y)$.

Second derivative test

Let (x_0, y_0) be a critical point of $f(x, y)$. Define

$$A = f_{xx}(x_0, y_0), \qquad (4.66)$$

$$B = f_{xy}(x_0, y_0) = f_{yx}(x_0, y_0), \text{ and} \qquad (4.67)$$

$$C = f_{yy}(x_0, y_0). \qquad (4.68)$$

Case 1: If $AC - B^2 < 0$, then (x_0, y_0) is a **saddle point** .

Case 2: If $AC - B^2 > 0$, then there are two subcases.

- If $AC - B^2 > 0$ and $A > 0$, then (x_0, y_0) is a **local minimum** .
- If $AC - B^2 > 0$ and $A < 0$, then (x_0, y_0) is a **local maximum** .

Case 3: If $AC - B^2 = 0$, then the test is inconclusive.

Let's see how this test reduces to what we saw in the special case of a quadratic equation

$$w(x, y) = ax^2 + bxy + cy^2.$$

Computing the partial derivatives of $w(x,y)$ at its critical point $(0,0)$, we find

$$A = w_{xx}(0,0) = 2a, \quad (4.70)$$

$$B = w_{xy}(0,0) = b, \text{ and} \quad (4.71)$$

$$C = w_{yy}(0,0) = 2c. \quad (4.72)$$

Therefore

$$AC - B^2 = (2a)(2c) - b^2 = 4ac - b^2. \quad (4.73)$$

This rule is justified on the next page by connecting a general function $f(x,y)$ to a quadratic function $w(x,y)$ via the quadratic approximation.

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Prof. Auroux is really funny and lighthearted
if you are tempted to fast fwd or skip videos, think again. Prof. Auroux not just teaches well, he can be very funny as well :-) e.g. 3:25
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Case 3 clarificatoin
Case 3 on this page says that the test is inconclusive. Case 3 here is similar to case 2 on previous page, which says that the origin is...
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