MITx: 6.008.1x Computational Probability and Inference

Hel





# Practice Problem: Computing the Normalization Constant Solution

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## PRACTICE PROBLEM: COMPUTING THE NORMALIZATION CONSTANT (SOLUTION)

It turns out that once we know the potential functions, the normalization constant $Z$ becomes fixed since the distribution needs to sum to 1. Let's show this for a simple case. Consider a two node graphical model with an edge between the two nodes corresponding to

$$p_{X_1,X_2}(x_1,x_2) = \frac{1}{Z}\phi_1(x_1)\phi_2(x_2)\psi_{12}(x_1,x_2).$$

Suppose that we are given what the potential functions are. Show what $Z$ is equal to as a function of $\phi_1$, $\phi_2$, and $\psi_{12}$.

*Hint:* Sum both sides over all values of $x_1$ and all values of $x_2$. What is $\sum_{x_1}\sum_{x_2} p_{X_1,X_2}(x_1,x_2)$ equal to?

Because knowing the potentials fixes what the value of $Z$ is, often times we'll omit writing $Z$ and instead write

$$p_{\underline{X}}(\underline{x}) \propto \prod_{i \in V} \phi_i(x_i) \prod_{(i,j) \in E} \psi_{ij}(x_i, x_j),$$

where "$\propto$" means "proportional to".

**Solution:** We have

$$\begin{aligned} 1 &= \sum_{x_1} \sum_{x_2} p_{X_1,X_2}(x_1, x_2) \\ &= \sum_{x_1} \sum_{x_2} \frac{1}{Z} \phi_1(x_1) \phi_2(x_2) \psi_{12}(x_1, x_2) \\ &= \frac{1}{Z} \sum_{x_1} \sum_{x_2} \phi_1(x_1) \phi_2(x_2) \psi_{12}(x_1, x_2), \end{aligned}$$

so

$$Z = \sum_{x_1} \sum_{x_2} \phi_1(x_1) \phi_2(x_2) \psi_{12}(x_1, x_2).$$

In general for a graphical model with graph $G = (V, E)$ and factorization

$$p_{X_1,\ldots,X_n}(x_1,\ldots,x_n) = \frac{1}{Z}\prod_{i\in V}\phi_i(x_i)\prod_{(i,j)\in E}\psi_{ij}(x_i,x_j),$$

using the same reasoning as above,

$$Z = \sum_{x_1}\cdots\sum_{x_n}\prod_{i\in V}\phi_i(x_i)\prod_{(i,j)\in E}\psi_{ij}(x_i,x_j).$$