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7. Practice Exam

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Let c be the curve $x^2 + xy + y^2 = 3$. (This is not the same curve as in the previous problem.) Let $f(x, y) = x$. Find the point of the curve c where f is maximal.

[2,-1] ✔ **Answer:** (2,-1)

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Solution:

Let $g(x, y) = x^2 + xy + y^2$. We seek to maximize the target function $f(x, y) = x$ subject to the constraint $g(x, y) = 3$. Setting up the Lagrange equation

$$\nabla g = \lambda \nabla f$$

gives

$$\begin{pmatrix} 2x + y \\ x + 2y \end{pmatrix} = \lambda \begin{pmatrix} 1 \\ 0 \end{pmatrix}.$$

This becomes the two equations

$$\begin{aligned} 2x + y &= \lambda, \\ x + 2y &= 0. \end{aligned}$$

The second equation tells us that $x = -2y$. Substituting this into the constraint equation $x^2 + xy + y^2 = 3$ then gives

$$\begin{aligned} 3 = x^2 + xy + y^2 &= (-2y)^2 + (-2y)\, y + y^2 \\ &= 4y^2 - 2y^2 + y^2 \\ &= 3y^2 \\ &\implies y = \pm 1. \end{aligned}$$

$$(x, y) = \pm (2, -1).$$

Evaluating the target function $f(x, y) = x$ at these two points gives that the maximum must either be $f(2, -1) = 2$ or $f(-2, 1) = -2$. Therefore the maximum value is 2 and it is attained at the point $\boxed{(2, -1).}$

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Answers are displayed within the problem

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