MITx: 6.041x Introduction to Probability - The Science of Uncertainty

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Problem 3: The PDF of the maximum

(3/3 points)

Let X and Y be independent random variables, each uniformly distributed on the interval $[0, 1]$.

1. Let $Z = \max\{X, Y\}$. Find the PDF of Z. Express your answer in terms of z using standard notation .

   For $0 < z < 1$, $f_Z(z) =$ 2*z ✔ **Answer:** 2*z

2. Let $Z = \max\{2X, Y\}$. Find the PDF of Z. Express your answer in terms of z using standard notation .

   For $0 < z < 1$, $f_Z(z) =$ z ✔ **Answer:** z

   For $1 < z < 2$, $f_Z(z) =$ 1/2 ✔ **Answer:** 0.5

Answer:
Recall that for a random variable U distributed uniformly on the interval $[0, 1]$, its CDF is given by

$$F_U(u) = \begin{cases} 0, & \text{if } u < 0, \\ u, & \text{if } 0 \le u \le 1, \\ 1, & \text{if } u > 1. \end{cases}$$

1. Let $Z = \max\{X, Y\}$. For $z \in (0, 1)$,

$$\begin{aligned} F_Z(z) &= \mathbf{P}(Z \le z) \\ &= \mathbf{P}(X \le z \text{ and } Y \le z) \\ &= F_X(z) F_Y(z) \\ &= z^2 \end{aligned}$$

   Hence, $f_Z(z) = 2z$ for $z \in (0, 1)$.

2. Let $Z = \max\{2X, Y\}$.

$$F_Z(z) = \mathbf{P}(Z \le z) = \mathbf{P}(2X \le z \text{ and } Y \le z) = F_X(z/2) F_Y(z).$$

   Hence, for $0 < z < 1$, $F_Z(z) = (z/2) \cdot z = z^2/2$ and $f_Z(z) = z$.
   For $1 < z < 2$, $F_Z(z) = (z/2) \cdot 1 = z/2$, and $f_Z(z) = 1/2$.

You have used 2 of 2 submissions

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