1.4 Unit 1 Homework Problems

Unit 1: Probability and Counting

Adapted from Blitzstein-Hwang Chapter 1.

HOMEWORK PROBLEMS.

Recall from the syllabus that the **homework problems are graded on correctness**, are worth a larger percentage of your grade, and have fewer attempts. The show answer button will appear on homework problems after you have used up all your attempts.

Problem 1a

1/1 point (graded)
(a) How many 7-digit phone numbers are possible, assuming that the first digit can't be a 0 or a 1?

8000000

8000000

Submit You have used 2 of 5 attempts

Problem 1b

1/1 point (graded)
(b) Re-solve the previous problem, except now assume also that the phone number is not allowed to start with 911.
In the USA, 911 is reserved for emergency use. It would not be desirable for the system to wait to see whether more digits were going to be dialed after someone has dialed 911.

7990000

7990000

Submit You have used 1 of 5 attempts

For Problem 2

Two chess players, A and B, are going to play 7 games. Each game has three possible outcomes: a win for A (which is a loss for B), a draw (tie), and a loss for A (which is a win for B). A win is worth 1 point, a draw is worth 0.5 points, and a loss is worth 0 points.

Problem 2a

1/1 point (graded)

(a) How many possible outcomes for the individual games are there, such that overall player A ends up with 3 wins, 2 draws, and 2 losses?

210 **Answer:** 210

210

Solution:

Writing W for win, D for draw, and L for loss (for player A), an outcome of the desired form is any permutation of WWWDDLL. So there are

$$\frac{7!}{3!2!2!} = 210$$

possible outcomes of the desired form.

Submit You have used 1 of 5 attempts

Answers are displayed within the problem

Problem 2b

1/1 point (graded)

(b) How many possible outcomes for the individual games are there, such that A ends up with 4 points and B ends up with 3 points?

357

✔ **Answer:** 357

357

Solution:

To end up with 4 points, A needs to have one of the following results: (i) 4 wins and 3 losses; (ii) 3 wins, 2 draws, and 2 losses; (iii) 2 wins, 4 draws, and 1 loss; or (iv) 1 win and 6 draws. Reasoning as in (a) and adding up these possibilities, there are

$$\frac{7!}{4!3!}+\frac{7!}{3!2!2!}+\frac{7!}{2!4!1!}+\frac{7!}{1!6!}=357$$

possible outcomes of the desired form.

Submit

You have used 2 of 5 attempts

Answers are displayed within the problem

Problem 2c

1/1 point (graded)

(c) Now assume that they are playing a best-of-7 match, where the match will end when either player has 4 points or when 7 games have been played, whichever is first. For example, if after 6 games the score is 4 to 2 in favor of A, then A wins the match and they don't play a 7th game. How many possible outcomes for the individual games are there, such that the match lasts for 7 games and A wins by a score of 4 to 3?

267

✔ **Answer:** 267

267

Solution:

For the desired outcomes, either (i) player A is ahead 3.5 to 2.5 after 6 games and then draws game 7, or (ii) the match is tied (3 to 3) after 6 games and then player A wins game 7. Reasoning as in (b), there are

$$\frac{6!}{3!1!2!}+\frac{6!}{2!3!1!}+\frac{6!}{1!5!}=126$$

possibilities of type (i) and

$$\frac{6!}{3!3!}+\frac{6!}{2!2!2!}+\frac{6!}{1!4!1!}+1=141$$

possibilities of type (ii), so overall there are

$$126 + 141 = 267$$

possible outcomes of the desired form.

Submit You have used 4 of 5 attempts

Answers are displayed within the problem

Problem 3

1/1 point (graded)

Three people get into an empty elevator at the first floor of a building that has 10 floors. Each presses the button for their desired floor (unless one of the others has already pressed that button). Assume that they are equally likely to want to go to floors 2 through 10 (independently of each other). What is the probability that the buttons for 3 consecutive floors are pressed?

14/243 **Answer:** 0.0576

$\frac{14}{243}$

Solution:

The number of possible outcomes for who is going to which floor is 9^3. There are 7 possibilities for which buttons are pressed such that there are 3 consecutive floors: $(2,3,4),(3,4,5),\ldots,(8,9,10)$. For each of these 7 possibilities, there are $3!$ ways to choose who is going to which floor. So by the naive definition, the probability is

$$\frac{3!\cdot 7}{9^3} = \frac{42}{729} = \frac{14}{243} \approx 0.0576.$$

Submit You have used 2 of 5 attempts

Answers are displayed within the problem

FOR PROBLEM 4

For each part, fill in the blanks with one of the options. In (a) and (b), the order in which people are chosen doesn't matter.

Problem 4a

1/1 point (graded)

(a) The number of ways to choose 5 people out of 10 is greater than (>) ▼ ✔ **Answer:** greater than (>) the number of ways to choose 6 people out of 10.

Solution:
Greater than (>). Using the fact that $n! = n \cdot (n-1)!$, we see that $\binom{10}{5} = \frac{10!}{5!5!} > \binom{10}{6} = \frac{10!}{4!6!}$ reduces to $6 > 5$. In general, $\binom{n}{k}$ is maximized at $k = n/2$ when n is even.

Submit You have used 1 of 1 attempt

Answers are displayed within the problem

Problem 4b

0/1 point (graded)

(b) The number of ways to break 10 people into 2 teams of 5 is greater than (>) ▼ ✘ **Answer:** less than (<) the number of ways to break 10 people into a team of 6 and a team of 4

Solution:
Less than (<). The righthand side is $\binom{10}{6}$ since the choice of the team of 6 determines the team of 4. But the lefthand side is $\frac{1}{2}\binom{10}{5}$ since choosing a team of 5 is equivalent to choosing the complementary 5 people. The inequality then reduces to $3 < 5$.

Submit You have used 1 of 1 attempt

Answers are displayed within the problem

Problem 4c

1/1 point (graded)

(c) The probability that all 3 people in a group of 3 were born on January 1 is less than (<) ▼ ✔ **Answer:** less than (<) the probability that in a group of 3 people, 1 was born on each of January 1, 2, and 3.

Solution:
Less than (<). The righthand side is 6 times as large as the lefthand side, since there are $3!$ ways the righthand event can occur, but only 1 way that the people could all be born on January 1.

Submit You have used 1 of 1 attempt

Answers are displayed within the problem

Problem 5

0/1 point (graded)
Martin and Gale play an exciting game of "toss the coin," where they toss a fair coin until the pattern HH occurs (two consecutive Heads) or the pattern TH occurs (Tails followed immediately by Heads). Martin wins the game if and only if HH occurs before TH occurs.

Which of the following statements is correct?

- (selected) Martin and Gale are equally likely to win, because their two patterns show up equally often when two coins are flipped. ✖
- Martin is less likely to win because as soon as Tails is tossed, TH will definitely occur before HH. ✔
- Martin is less likely to win because getting two heads in a row is less likely than getting tails and heads.

Solution:
Consider the first toss. If it's Tails, we're *guaranteed* to see TH before we see HH. If it's Heads, we could still see either TH or HH first. Hence, the probability of HH occurring sooner than TH is less than $1/2$.
Alternatively, consider the first two tosses. If it's HH, then Martin has already won. If it's TH, then Gale has already won. If it's HT or TT, then Gale will eventually win (she will win the next time that an H appears). So

$$P(\text{Martin wins}) = \frac{1}{4} < \frac{1}{2}.$$

Submit You have used 1 of 1 attempt

Answers are displayed within the problem

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