4. Minimize the deviation function

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Lecture due Sep 13, 2021 20:30 IST Completed

Explore

Partial derivatives

2/2 points (graded)

Given the function

$$D(a,b) = \sum_{i=1}^{n} [y_i - (ax_i + b)]^2$$

compute the partial derivatives with respect to a and b. Enter your answer in terms of a, b, x_i, and y_i. (To enter x_i and y_i, use x_i and y_i, respectively.)

$\frac{\partial D}{\partial a} = \sum_{i=1}^{n}$ 2*(-x_i)*(y_i-(a*x_i+b)) ✔

$2 \cdot (-x_i) \cdot (y_i - (a \cdot x_i + b))$

$\frac{\partial D}{\partial b} = \sum_{i=1}^{n}$ (-2)*(y_i-(a*x_i+b)) ✔

$(-2) \cdot (y_i - (a \cdot x_i + b))$

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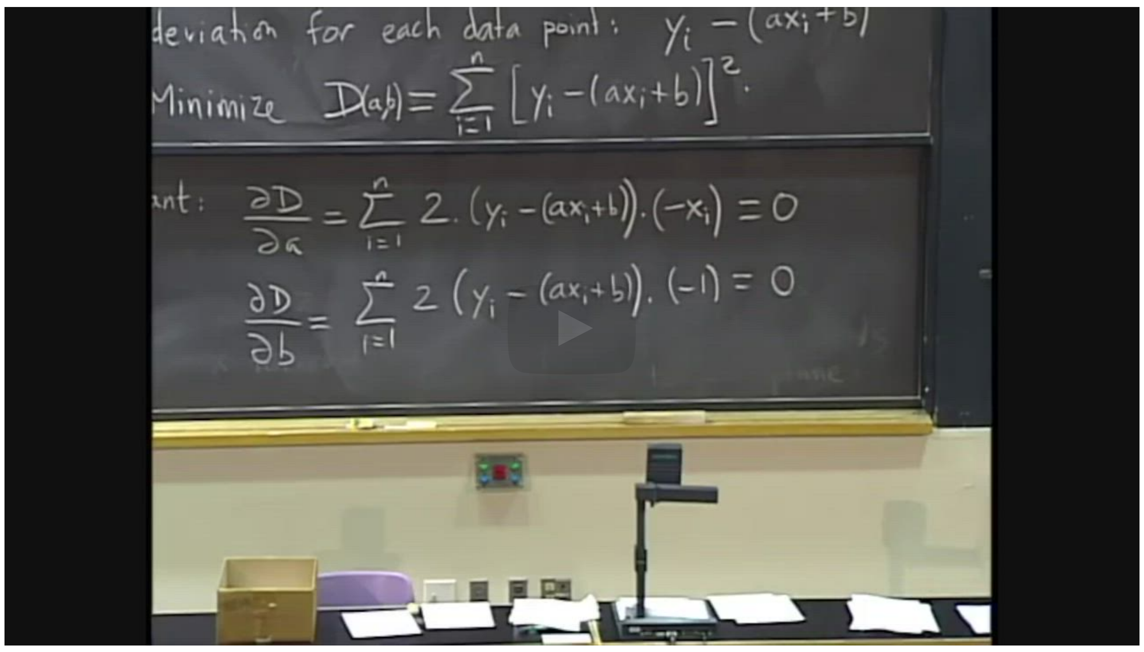
In the last problem, you computed $\frac{\partial D}{\partial a}$ and $\frac{\partial D}{\partial b}$. We now need to find the critical points of D by setting these expressions equal to zero.

Rearranging the formula



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PROFESSOR: So that's the equations we have to solve.

Well, let's re-organize this a little bit.

So the first equation.

So see, there's a's and there's b's in these equations.

I'm going to just look at the coefficients of a and b.

If you have good eyes, you can see

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Setting the partial derivatives equal to zero and rearranging the terms results in the following system of linear equations in a and b:

$$\left(\sum_{i=1}^{n} x_i^2\right) a + \left(\sum_{i=1}^{n} x_i\right) b = \sum_{i=1}^{n} x_i y_i \tag{4.201}$$

$$\left(\sum_{i=1}^{n} x_i\right) a + nb = \sum_{i=1}^{n} y_i. \tag{4.202}$$

We would then solve the above system for a and b. Let's see how to do this with an example.

Example 4.1

Suppose the data points we want to fit are $(-2,-1)$, $(2,3)$, $(0,1)$, and $(4,2)$. We can label these points to correspond to our formula:

$$\begin{aligned} (x_1, y_1) &= (-2,-1) \\ (x_2, y_2) &= (2,3) \\ (x_3, y_3) &= (0,1) \\ (x_4, y_4) &= (4,2)\,. \end{aligned}$$

We have 4 data points, so in our case, $n=4$. Now, we want to plug these into our system of equations above and solve for a and b. To do this, we will need to compute the following quantities:

$$\begin{aligned} \sum_{i=1}^{4} x_i^2 &= (-2)^2 + 2^2 + 0^2 + 4^2 = 24 \\ \sum_{i=1}^{4} x_i &= (-2) + 2 + 0 + 4 = 4 \\ \sum_{i=1}^{4} x_i y_i &= (-2)(-1) + (2)(3) + (0)(1) + (4)(2) = 16 \\ \sum_{i=1}^{4} y_i &= (-1) + 3 + 1 + 2 = 5. \end{aligned}$$

Then our system of equations becomes

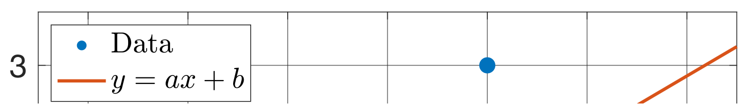
$$24a + 4b = 16 \tag{4.203}$$

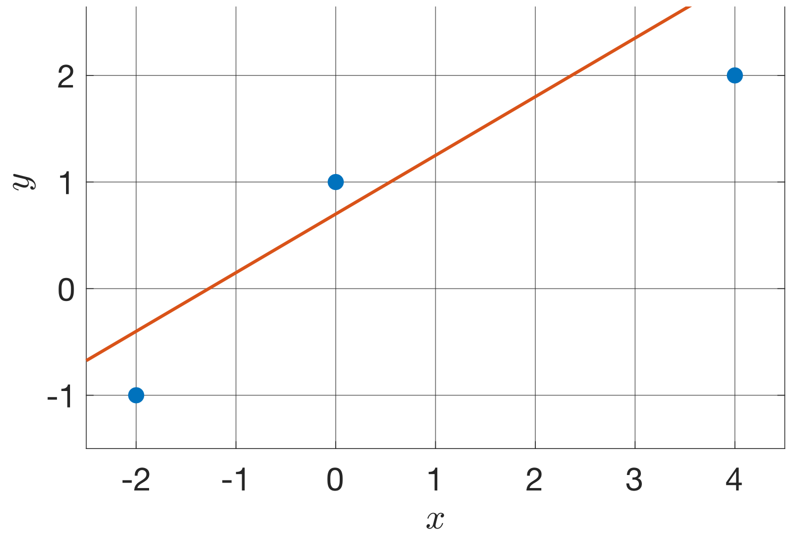
$$4a + 4b = 5. \tag{4.204}$$

Solving this system by elimination or substitution gives $a = \dfrac{11}{20}$ and $b = \dfrac{7}{10}$. So the best fit line for our data points is

$$y = \frac{11}{20}x + \frac{7}{10}. \tag{4.205}$$

The figure below shows our data points along with this best fit line.





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4.204
I understand where the coefficients for 4.203 come from, but I don't understand where the first two coefficients for 4.204 come from.
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