1. Linear algebra

Reduced row echelon form and rank

2/2 points (graded)

Find the reduced row echelon form of the matrix

$$\mathbf{A} = \begin{pmatrix} 0 & 0 & -2 & 0 & 7 & 12 \\ 2 & 4 & -10 & 6 & 12 & 28 \\ 2 & 4 & -5 & 6 & -5 & -1 \end{pmatrix}.$$

(Enter the matrix in MATLAB notation. That is, enter the coordinates between square brackets separated by commas, with semicolons at the end of each row: e.g. type **[1, 0, 0; 0, 1, 0; 0, 0, 1]** for the 3×3 identity matrix.)

$\mathbf{rref}(\mathbf{A}) =$ [1,2,0,3,0,7;0,0,1,0,0,1;0,0 ✔ **Answer:** [1, 2, 0, 3, 0, 7; 0, 0, 1, 0, 0, 1; 0, 0, 0, 0, 1, 2]

How many pivot columns does $\mathbf{rref}\mathbf{A}$ have? 3 ✔ **Answer:** 3

Solution:

Either using MATLAB Online, or by performing a series of row operations, we find that

$$\begin{pmatrix} 0 & 0 & -2 & 0 & 7 & 12 \\ 2 & 4 & -10 & 6 & 12 & 28 \\ 2 & 4 & -5 & 6 & -5 & -1 \end{pmatrix} \to \begin{pmatrix} 2 & 4 & -10 & 6 & 12 & 28 \\ 0 & 0 & -2 & 0 & 7 & 12 \\ 2 & 4 & -5 & 6 & -5 & -1 \end{pmatrix} \to \begin{pmatrix} 2 & 4 & -10 & 6 & 12 & 28 \\ 0 & 0 & -2 & 0 & 7 & 12 \\ 0 & 0 & 5 & 0 & -17 & -29 \end{pmatrix}$$

$$\to \begin{pmatrix} 1 & 2 & -5 & 3 & 6 & 14 \\ 0 & 0 & -2 & 0 & 7 & 12 \\ 0 & 0 & 0 & 0 & 0.5 & 1 \end{pmatrix} \to \begin{pmatrix} 1 & 2 & -5 & 3 & 0 & 2 \\ 0 & 0 & -2 & 0 & 0 & -2 \\ 0 & 0 & 0 & 0 & 1 & 2 \end{pmatrix} \to \begin{pmatrix} 1 & 2 & 0 & 3 & 0 & 7 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 2 \end{pmatrix}.$$

The number of pivots in $\mathbf{rref}(\mathbf{A})$ is 3.

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Answers are displayed within the problem

Find the dimension of the nullspace 1

2/2 points (graded)
Find a the dimension of the nullspace of the matrix

$$\mathbf{B} = \begin{pmatrix} -3 & 6 & -1 & 1 & 7 \\ 1 & -2 & 2 & 3 & -1 \\ 2 & -4 & 5 & 8 & -4 \end{pmatrix}.$$

$\dim(\mathrm{NS}(\mathbf{B})) =$ 2 ✔ Answer: 2

What the number of pivots in $\mathtt{rref}\mathbf{B}$? 3 ✔ Answer: 3

Solution:

To find the dimension of nullspace of $\mathbf{B}$ and its rank, we find a row echelon form:

$$\begin{pmatrix} -3 & 6 & -1 & 1 & 7 \\ 1 & -2 & 2 & 3 & -1 \\ 2 & -4 & 5 & 8 & -4 \end{pmatrix} \to \begin{pmatrix} 1 & -2 & 1/3 & -1/3 & -7/3 \\ 0 & 0 & 1 & 2 & 4/5 \\ 0 & 0 & 1 & 2 & 2/13 \end{pmatrix} \to \begin{pmatrix} 1 & -2 & 0 & -1 & 0 \\ 0 & 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

This row echelon form has 5 columns: 3 pivot columns and 2 non-pivot columns. Therefore the dimension of the nullspace is 2.

Submit You have used 1 of 4 attempts

Answers are displayed within the problem

Find the dimension of the nullspace 2

2/2 points (graded)
Find a the dimension of the nullspace of the matrix

$$\mathbf{C} = \begin{pmatrix} 2 & 2 & -1 & 0 & 1 \\ -1 & -1 & 2 & -3 & 1 \\ 1 & 1 & -2 & 0 & -1 \\ 0 & 0 & 1 & 1 & 1 \end{pmatrix}.$$

$\mathbf{dim(NS(C))} =$ 2 ✔ Answer: 2

What is the number of pivots of $\mathbf{rref C}$? 3 ✔ Answer: 3

Solution:

We start by finding a row echelon form by hand or by computer:

$$\begin{pmatrix} 2 & 2 & -1 & 0 & 1 \\ -1 & -1 & 2 & -3 & 1 \\ 1 & 1 & -2 & 0 & -1 \\ 0 & 0 & 1 & 1 & 1 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 1 & -2 & 0 & -1 \\ 0 & 0 & 3 & 0 & 3 \\ 0 & 0 & 0 & -3 & 0 \\ 0 & 0 & 1 & 1 & 1 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

This row echelon form of $\mathbf{C}$ has 3 pivot columns and 2 nonpivot columns. Therefore the dimension of the nullspace is 2.

In MATLAB, plug in matrix, apply **rref**. Count the number of pivot columns, and the number of nonpivot columns, which is equal to the dimension of the nullspace.

Submit You have used 1 of 4 attempts

Answers are displayed within the problem

1. Linear algebra

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- Rank — definition is not given in lectures for rank ? where can i find it ? — 5
- [Staff] Errors in intermediate step in solution to "Find the dimension of the nullspace 1" — 4
- MATLAB console for the course.

Since the use of MATLAB is allowed for H/W problems, it will be good/convenient if a link to MATLAB console is provided in this cou... 6

Warning on Part 1
Start by rearranging the Matrix in order, top to bottom, original row 2, 1, 3, or you'll end up with nasty fractions in the right two col... 1

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