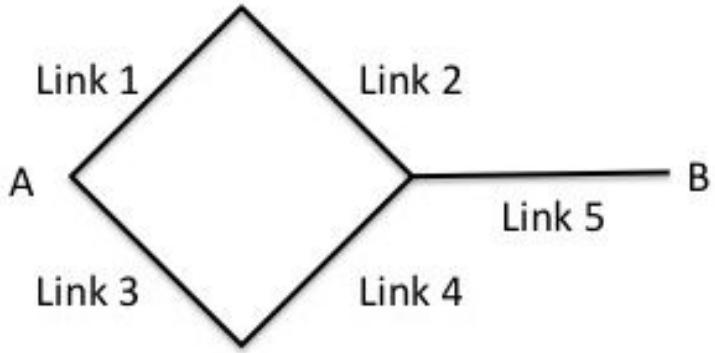
Problem 2: A reliability problem

(4/4 points)

Consider the communication network shown in the figure below and suppose that each link can fail with probability p. Assume that failures of different links are independent.



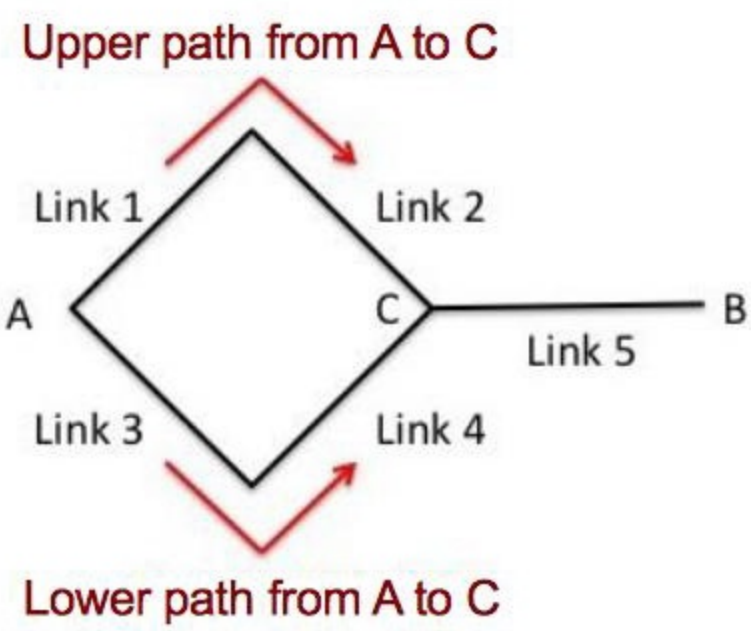
1. Assume that $p = 1/3$. Find the probability that there exists a path from A to B along which no link has failed. (Give a numerical answer.)

0.4609053 ✔ **Answer:** 0.46091

2. Given that exactly one link in the network has failed, find the probability that there exists a path from A to B along which no link has failed. (Give a numerical answer.)

0.8 ✔ **Answer:** 0.8

Answer:



Let E be the event that there exists an operational path from A to B. Note that the probability that the Upper path from A to C is operational is $(1-p)^2$. So the probability that the Upper path fails is $1-(1-p)^2$. Similarly, the Lower path fails with probability $1-(1-p)^2$ as well.

- Final Exam

1. We can break the problem down into two parts. In order for there to be an operational path from A to B, there must be an operational path from A to C, and Link 5 must be operational.

$$\begin{aligned}\mathbf{P}(E) &= \mathbf{P}(\text{there exists a path from } A \text{ to } C \text{ and Link 5 is operational}) \\ &= (1 - \mathbf{P}(\text{Upper path fails and Lower path fails})) \cdot \mathbf{P}(\text{Link 5 is operational}) \\ &= \left\{1 - \left[1 - (1-p)^2\right]^2\right\} \cdot (1-p).\end{aligned}$$

When $p = 1/3$, this gives us $\mathbf{P}(E) \approx 0.46091$.

2. Since all links are equally likely to fail and since exactly one link has failed, each link has the same probability $1/5$ of being the one that failed. There will be no path from A to B only in the case where the link that failed is Link 5, which happens with probability $1/5$. Therefore, the desired probability is $1 - 1/5 = 4/5$.

You have used 2 of 2 submissions

DISCUSSION

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