1.4.2 Quiz: What Can Happen to the Budworm Population?

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Let's think back on the different possible behaviors of the population we saw. Here are the graphs of the three qualitatively different situations that can happen when we vary q, the carrying capacity. Here is the case of "large" q. (The other two are shown below.)

> **Note:** In the case of three non-equilibrium solutions, P_1 is our notation for the smallest equilibrium, P_2 for the middle one and P_3 for the largest equilibrium. We are not saying that P_1 and P_2 are the same exact values as they were in the case of small q or critical q shown earlier.

"Large" q

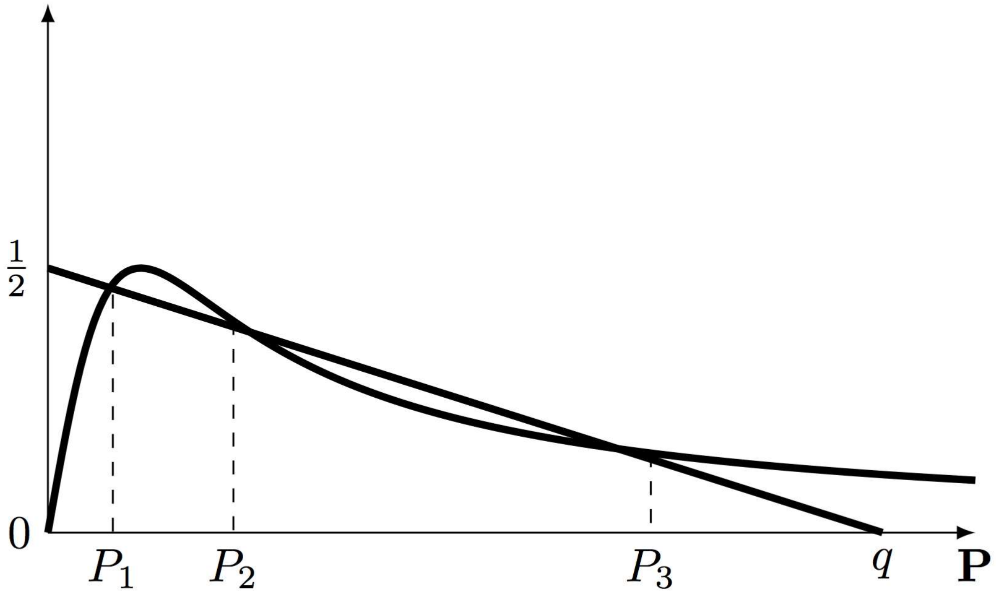

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Image Description

Here are the cases of the critical q_* and the small q.

> **Note:** A mathematical biologist would likely not focus on the specific threshold q_* as much as the fact that such a threshold q exists. What is of interest is what happens below versus above this threshold q.

Critical q

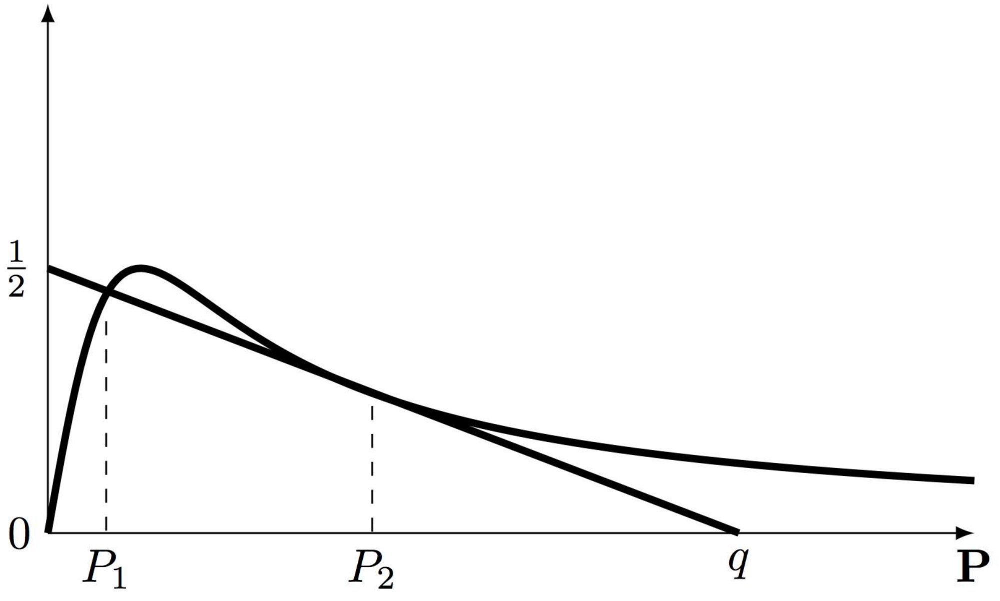

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Image Description

"Small" q

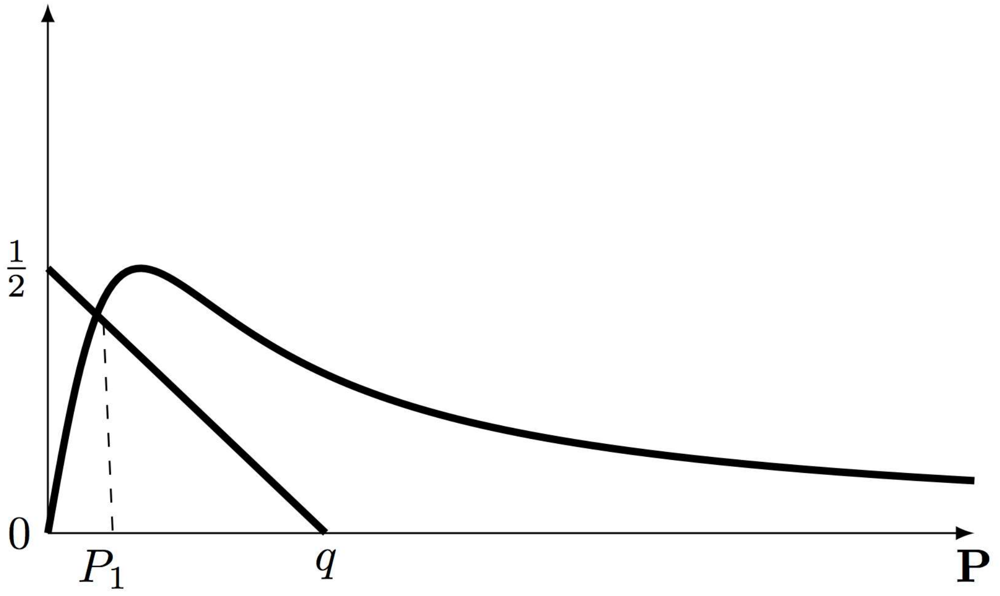

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Image Description

Question 1

1/1 point (graded)

When the carrying capacity q is larger than the critical value q_*, there are four equilibrium: $P = 0$ and the three corresponding to the three points of intersection of the curve and line.

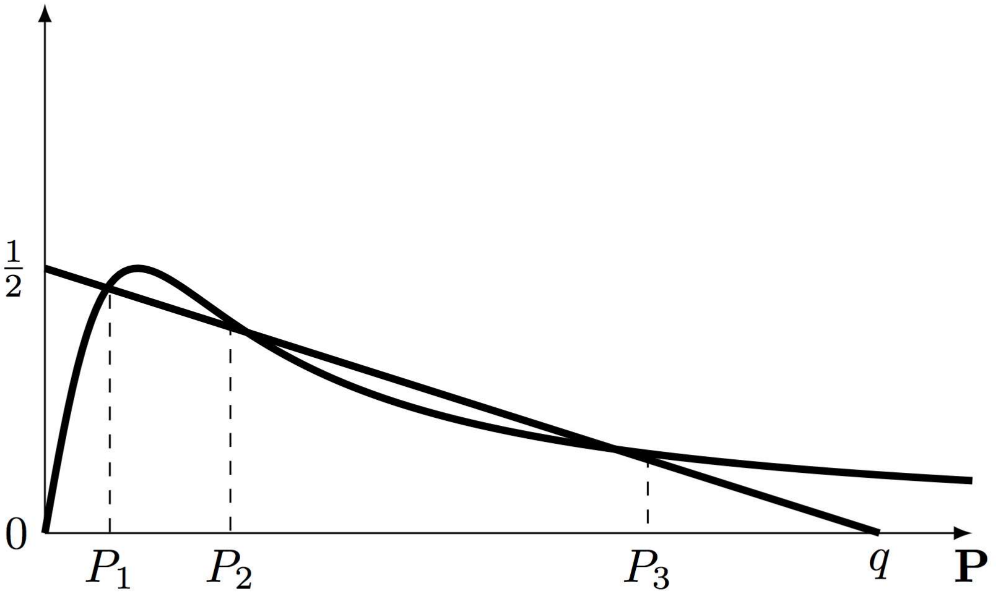

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Image Description

Using the graph above, estimate about how much larger the equilibrium population P_3 is compared to the equilibrium population P_1.

○ About twice as large.

○ About five times as large.

● About ten times as large. ✓

○ About a hundred times as large.

○ None of the above.

Explanation

When the bifurcation occurs, we notice that the largest of the three non-zero equilibria is about ten times larger than the smallest of the three non-zero equilibria. We see this since the location of P_3 on the P-axis is about 10 times as far from 0 as the location of P_1. Hence, the possibility of having a year in which budworms are 10 times more prevalent than they normally would be.

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You have used 1 of 3 attempts

Answers are displayed within the problem

Question 2

1/1 point (graded)

If the carrying capacity q is larger than the critical value q_*, there are four equilibrium: $P = 0$ and the three corresponding to the three points of intersection of the curve and line.

Assume we start with some quantity of budworms ($P \neq 0$). According to the model, which of the following are possible long-term behaviors of the budworm population? If there is more than one, choose all such.

☐ Go extinct

☑ Increase toward the smallest non-zero equilibrium solution, P_1 ✓

☑ Decrease toward the smallest non-zero equilibrium solution, P_1 ✓

☐ Increase toward the middle non-zero equilibrium solution, P_2

☐ Decrease toward the middle non-zero equilibrium solution, P_2

☑ Increase toward the largest non-zero equilibrium solution, P_3 ✓

☑ Decrease toward the largest non-zero equilibrium solution, P_3 ✓

☐ None of the above.

✓

Explanation

Choices B, C and E, F are correct.

$P = 0$ (extinction) and the equilibrium at P_2 are both unstable equilibria so the population will never tend toward these.

The smallest and largest non-zero equilibrium, P_1 and P_3 are both stable. Depending on the size of the starting population P, the population will tend toward P_1 or P_3.

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Question 3

1/1 point (graded)
According to this model, will the budworms ever have the possibility of going extinct? (You may find it helpful to look at the possible outcomes for each level of q, as discussed in previous sections.)

$$\frac{dP}{dt} = \frac{1}{2}P\left(1 - \frac{P}{q}\right) - \frac{P^2}{1+P^2}$$

Choose all that apply.

☑ Never, regardless the value of q. ✓

☐ Yes, for "small" q values (less than q_*)

☐ Yes, for the "critical" q value, q_*

☐ Yes, for "large" q values (greater than q_*)

☐ None of the above

✓

Explanation
Never, regardless the value of q. The equilibrium of $P = 0$ is always unstable, regardless of the value of q. So the budworm population will never go extinct according to this model.

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Answers are displayed within the problem

Question 4

1/1 point (graded)
According to this model, will the budworms ever approach their carrying capacity q? (You may find it helpful to look at the possible outcomes for each level of q, as discussed in previous sections.)

$$\frac{dP}{dt} = \frac{1}{2}P\left(1 - \frac{P}{q}\right) - \frac{P^2}{1 + P^2}$$

☑ Never, regardless the value of q. ✓

☐ Yes, for "small" q values (less than q_*)

☐ Yes, for the "critical" q value, q_*

☐ Yes, for "large" q values (greater than q_*)

☐ None of the above

✓

Explanation
Never, regardless the value of q. In all cases, we see that the population approaches one of the stable or semi-stable equilibrium points, and these occur at P-values less than $P = q$, the carrying capacity, which is represented by the horizontal axis intercept of the line. So the budworm population will never approach carrying capacity according to this model.

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You have used 1 of 3 attempts

ℹ Answers are displayed within the problem

Question 5: Think About It...

1/1 point (graded)
We've been looking at the long term behavior in situations where the carrying capacity q is fixed.

In reality, the carrying capacity fluctuates with time as well (the foliage has a good or bad year). This affects the possible population outcomes: we might change from a situation where we have one non-zero low-level equilibrium to one with three non-zero equilibria, one of which is quite high. This is the outbreak situation.

Suppose the budworm population has an outbreak, meaning there is a third stable equilibrium which is much larger. Why might the carrying capacity decrease to a small enough q-value that there is only one non-zero equilibrium? What would the effect be on the budworms population?

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Lack of resources, foods.
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✓

Thank you for your response.

A decrease in carrying capacity (fir foliage) will result in the system returning to the state of having only the one stable low-level equilibrium.

This decrease in foliage could be because of a long winter where fir growth is slower. (According to the Forestry Commission in England's website:, *"As most conifers do not have a fixed growing season, in the right conditions they will continue to grow throughout the winter months. They do speed up, however, in the spring."*)

The decrease in foliage could also be because the outbreak budworm population is so large that the budworms eat almost all available foliage. (According to a US Dept. of Agriculture report, *"Once a spruce budworm outbreak begins, it usually continues until the larvae consume much of the available foliage."*)

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You have used 1 of 2 attempts

ℹ Answers are displayed within the problem

Question 6

1/1 point (graded)

In the case when q is larger than the critical q_*, we have three non-zero equilbria, the smallest of which we called P_1.

Wes said that this smallest equilibrium is "sort of the low-level population that you would have equilibrated to had q been small."

Use the dynamic Desmos graph here to investigate this. How does the location of the smallest equilibrium change when q increases from less than q_* to greater than q_*? What does this mean biologically? (You can think of q as representing the amount of fir foliage available for budworms to eat.)

Choose the best response.

Graph Description: Graph of $y = \frac{1}{2}(1 - \frac{P}{q})$. and $y = \frac{P}{1+P^2}$ with the parameter q ranges from .1 to 10 and starts at 7.4, the x-axis is labeled P, but no label on y axis, the plot range is (-1,11) and (-.1,1).

Desmos Hint: By clicking on either equation on the left, you can see the intersection points highlighted (as well as max and min of the graph). Hovering the cursor over a point will show its' coordinates.

○ As q increases, the location of the smallest equilibrium is unchanged. Even though there is more food to eat, there are also more predators to eat worms.

● As q increases, the location of the smallest equilibrium increases, but only slightly. The increase makes sense because there is more food to eat. ✓

○ As q increases, the location of the smallest equilibrium increases significantly. The increase makes sense because there is more food to eat.

○ As q increases, the location of the smallest equilibrium decreases, but only slightly. The increase makes sense because there is less food to eat.

○ As q increases, the location of the smallest equilibrium decreases significantly. The increase makes sense because there is less food to eat.

○ None of the above.

Explanation

The location of the smallest equilibrium increases, but only slightly. This means in the case of a large carrying capacity, if the budworms tend toward the low-level stable population, it will be slightly larger than if q had been small. The fact that it is slightly larger should not be surprising biologically, as there is more food to feed on than in the small q case.

Intuitively, the smallest value does not change much with variation in carrying capacity because most of the “negative force” which acts to slow budworm growth just below the smallest value comes from predation, not limited resources. So more food resources will not significantly increase the growth rate of budworms below the lowest equilibrium value; there are so few budworms at that level that there are still plenty of resources to go around.

Submit You have used 1 of 3 attempts

Answers are displayed within the problem

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