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9. Worked Examples on Total  
> Variation Distance Continued

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## 9. Worked Examples on Total Variation Distance Continued

**Note:** The following exercises will be presented in lecture, but we encourage you to attempt these yourselves first.

### Computing Total Variation IV

1/1 point (graded)

So far, we have defined the total variation distance to be a distance  $\text{TV}(\mathbf{P}, \mathbf{Q})$  between **two probability measures  $\mathbf{P}$  and  $\mathbf{Q}$** . However, we will also refer to the total variation distance between **two random variables** or between **two pdfs** or **two pmfs**, as in the following.

Compute  $\text{TV}(X, X + a)$  for any  $a \in (0, 1)$ , where  $X \sim \text{Ber}(0.5)$ .

$\text{TV}(X, X + a) =$

✓ Answer: 1

#### Solution:

Since  $a \in (0, 1)$ ,  $X$  and  $X + a$  have no support points where both pmf's are non-zero. Therefore, the total variation distance is equal to 1.

Submit

You have used 1 of 3 attempts

**i** Answers are displayed within the problem

## Computing Total Variation V

1/1 point (graded)

Compute  $\text{TV}(2\sqrt{n}(\bar{X}_n - 1/2), Z)$  where  $X_i \stackrel{i.i.d}{\sim} \text{Ber}(0.5)$  and  $Z \sim \mathcal{N}(0, 1)$ .

$\text{TV}(2\sqrt{n}(\bar{X}_n - 1/2), Z) =$   **✓ Answer: 1**

**Solution:**

Let  $\mathbf{P}$  and  $\mathbf{Q}$  denote the probability measures of  $2\sqrt{n}(\bar{X}_n - 1/2)$  and  $Z$ , respectively. Recall the total variation distance is defined as

$$\max_{A \subseteq E} |\mathbf{P}(A) - \mathbf{Q}(A)|$$

Let  $B \triangleq \{a_i = 2\sqrt{n}(\frac{i}{n} - \frac{1}{2}) \mid i = 0, 1, \dots, n\}$  be set of  $n + 1$  points where the pmf of  $2\sqrt{n}(\bar{X}_n - 1/2)$  is non-zero.

Consider the set  $A = \mathbb{R} \setminus B (= R \cap B^c)$ . For this set,  $\mathbf{P}(A) = 0$  and  $\mathbf{Q}(A) = 1$ . Therefore,  $|\mathbf{P}(A) - \mathbf{Q}(A)| = 1$ . We know from a previous problem that the total variation distance is upper bounded by 1 for any two distributions. Since we have produced a set where this bound is met with equality,  $\text{TV}(2\sqrt{n}(\bar{X}_n - 1/2), Z) = 1$ .

Submit

You have used 1 of 3 attempts

**i** Answers are displayed within the problem

## Worked Examples on Total Variation Distance Continued

Handwritten notes on a blackboard background showing calculations for Total Variation Distance and Central Limit Theorem.

**Left side:**

$$f(x) = e^{-x} \mathbb{1}_{\{x \geq 0\}}, \quad g(x) = \mathbb{1}_{\{0 \leq x < 1\}}$$

$$\frac{1}{2} \int |e^{-x} \mathbb{1}_{\{x \geq 0\}} - \mathbb{1}_{\{0 \leq x < 1\}}| dx$$

$$= \frac{1}{2} \int_0^1 |e^{-x} - 1| dx + \frac{1}{2} \int_1^\infty e^{-x} dx$$

$$= \frac{1}{2} + \frac{1}{2} \int_0^1 (1 - e^{-x}) dx + \frac{1}{2} \int_1^\infty e^{-x} dx$$

$$= \frac{1}{2} + \frac{1}{2} \left( x - e^{-x} \right) \Big|_0^1 - \frac{1}{2} e^{-x} \Big|_1^\infty = \frac{1}{2} + \frac{1}{2} (1 - 1 + 1) - \frac{1}{2} (0 - e^{-1}) = \frac{1}{2} + \frac{1}{2} = 1$$

**Right side:**

$$TV = \frac{1}{2} \left( \left| \frac{1}{2} - 0 \right| + \left| \frac{1}{2} - \frac{1}{2} \right| + \left| \frac{1}{2} - 0 \right| \right) = \frac{1}{2}$$

**Bottom left:**

$$X = \begin{cases} 0 & \text{w.p. } \frac{1}{2} \\ 1 & \text{w.p. } \frac{1}{2} \end{cases}, \quad Y = \begin{cases} 0 & \text{w.p. } \frac{1}{2} \\ 1 & \text{w.p. } \frac{1}{2} \end{cases}$$

disjoint support

$t = \{0, 1\}$

$P(X \in \{0, 1\}) = 1$

$P(Y \in \{0, 1\}) = 1$

$P(X \in \{0, 1\}) = 1$

$P(Y \in \{0, 1\}) = 1$

**Bottom right:**

When taking  $2\sqrt{n}(\bar{X} - \frac{1}{2})$

$\bar{X} = \frac{1}{n} \sum_{i=1}^n X_i$

Standard size  $n+1$  (discrete)

$S_n = \text{sum of } 2\sqrt{n}(\bar{X}_n - \frac{1}{2})$

$A = S_n$

$P(2\sqrt{n}(\bar{X}_n - \frac{1}{2}) \in A) = 1$

the supports are disjoint.

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? [TV=1 for continuous and discrete random variables](#)

3

Can the last example be extended to all TVs between continuous and discrete random variables? I mean is  $TV = 1$  always when I consider continuous and discrete random var...

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