PurdueX: 416.2x Probability: Distribution Models & Continuous Random Variables

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Unit 9: Quiz

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Unit 9: Quiz

The problems in the quiz will be automatically graded.

You are given only **ONE** attempt for each problem. You will be able to see the solution after you submit your answers.

If the answer is numeric and has more than 4 decimal places, just give your answers to **4 decimal places (0.0001 accuracy)** unless otherwise stated.

e.g. for 0.123456... you just need to round it up to 0.1235

Problem 1

2/2 points (graded)

1. Consider a pair of random variables X, Y with constant joint density on the quadrilateral with vertices located at the points $(0,0)$, $(3,0)$, $(5,2)$, $(0,2)$.

1a. Find $P(X > Y)$.

3/4

✓ **Answer:** 0.75

L9.5: Practice

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Quiz

- Unit 10: Normal Distribution and Central Limit Theorem (CLT)
- Unit 11: Covariance, Conditional Expectation, Markov and Chebychev Inequalities
- Unit 12: Order Statistics, Moment Generating Functions, Transformation of RVs

1b. Find $P(X+Y \le 3)$.

1/2

✓ **Answer:** 0.5

Explanation

1a. Since the joint density is constant, one possible method is to use the areas of the regions under study. The area of the whole region is 8, and the area where $X > Y$ is 6. Thus $P(X>Y) = 6/8 = 3/4$. Another possible method is to integrate: $\int_0^2 \int_y^{y+3} 1/8\,dx\,dy = \int_0^2 3/8\,dy = 3/4$.

1b. Since the joint density is constant, one possible method is to use the areas of the regions under study. The area of the whole region is 8, and the area where $X+Y \le 3$ is 4. Thus $P(X+Y \le 3) = 4/8 = 1/2$. Another possible method is to integrate: $\int_0^2 \int_0^{3-y} 1/8\,dx\,dy = \int_0^2 (3-y)/8\,dy = 1/2$.

Submit You have used 1 of 1 attempt

✓ Correct (2/2 points)

Problem 2

1/1 point (graded)

2. Suppose X and Y have a constant joint density on the square with vertices $(0,0)$, $(0,5)$, $(5,5)$, $(5,0)$. Find $\mathbb{E}(\max(X,Y))$.

10/3

✓ **Answer:** 3.333

Explanation

2. One method is to write $Z = \max{(X, Y)}$. For $0 < z < 5$, we have $F_Z(a) = P(Z \le a) = a^2/25$, so $f_Z(z) = 2z/25$, and thus $\mathbb{E}(\max{(X, Y)}) = \mathbb{E}(Z) = \int_0^5 (z)(2z/25)\,dz = 10/3$.

Another method is to note that $\max{(X, Y)} = Y$ when $Y > X$, and $\max{(X, Y)} = X$ when $Y < X$. Thus, we get

$$\mathbb{E}(\max(X, Y)) = \int_0^5 \int_0^x (x)(1/25)\,dy\,dx + \int_0^5 \int_0^y (y)(1/25)\,dx\,dy$$

$$= \int_0^5 x^2/25\,dx + \int_0^5 y^2/25\,dy = 5/3 + 5/3 = 10/3.$$

Submit You have used 1 of 1 attempt

✓ Correct (1/1 point)

Problem 3

1/1 point (graded)

3. Suppose X has density $f_X(x) = 1/2$ for $0 < x < 2$, and $f_X(x) = 0$ otherwise. Suppose that Y has density $f_Y(y) = e^{-y}$ for $y > 0$, and $f_Y(y) = 0$ otherwise. Also suppose that X and Y are independent. Find $P(Y > X)$.

0.4323324

✓ **Answer:** 0.4323

Explanation

3. We have

$$P(Y > X) = \int_0^2 \int_x^\infty (1/2)(e^{-y})\,dy\,dx$$
$$= \int_0^2 (1/2)(e^{-x})\,dx = (1 - e^{-2})/2 = 0.4323.$$

Submit You have used 1 of 1 attempt

✓ Correct (1/1 point)

Problem 4

0/1 point (graded)

4. On Halloween you are visiting a haunted corn maze. Let X denote the time (in minutes) until you hear someone scream. Let Y denote the time (in minutes) until you get scared by a ghost. Suppose that X and Y are independent Exponential random variables, with $\mathbb{E}(X) = 1/2$ and $\mathbb{E}(Y) = 1/3$. Find the probability that, within the next 1 minute, you don't hear anybody scream and you don't get scared by a ghost. In other words, find $P(\min(X, Y) > 1)$, or equivalently, $P(X > 1 \;\&\; Y > 1)$.

0.4345982 ✗ **Answer:** 0.006738

Explanation

4. We have $P(X > 1 \;\&\; Y > 1) = \int_1^\infty \int_1^\infty (2e^{-2x})(3e^{-3y})\,dy\,dx$.

Or, since X and Y are independent, you might choose to write

$$P(X > 1 \;\&\; Y > 1) = P(X > 1)P(Y > 1)$$
$$= \left(\int_1^\infty 2e^{-2x}\,dx\right)\left(\int_1^\infty 3e^{-3y}\,dy\right).$$

Either way, you get $(e^{-2)})(e^{-3}) = e^{-5} = 0.006738$.

Submit You have used 1 of 1 attempt

✖ Incorrect (0/1 point)

Problem 5

4/4 points (graded)

5a. In the scenario from question **4**, start listening for screams at 11:56 PM. Given that nobody has screamed by 11:59 PM, use integration to compute the conditional probability that nobody screams by midnight. I.e., find $P(X > 4 \mid X > 3)$.

0.1353353 ✓ **Answer:** 0.1353353

5b. Does your solution agree with what you would find, if you had (instead) just used the memoryless property of Exponential random variables? In other words, was your conditional probability equal to $P(X > 1)$?

0.1353353 ✓ **Answer:** 0.1353353

5c. To convince yourself that the memoryless property is something special for Exponential random variables, we could demonstrate that it does not hold for other kinds of random variables. For instance, if U is a Continuous Uniform random variable on $[0, 10]$, show that $P(U > 4 \mid U > 3)$ is not equal to $P(U > 1)$.

$P(U > 4 \mid U > 3) =$ 6/7 ✓ **Answer:** 0.8571429

It is not equal to $P(U > 1)$, because

$P(U > 1) =$ 9/10 ✓ **Answer:** 0.9

Explanation

5a. We have $P(X > 4 \mid X > 3) = \frac{P(X>4 \ \& \ X>3)}{P(X>3)} = \frac{P(X>4)}{P(X>3)} = \frac{e^{-2(4)}}{e^{-2(3)}} = \frac{e^{-8}}{e^{-6}} = e^{-2}$.

5b. Yes, we also have $P(X > 1) = e^{-2(1)} = e^{-2}$.

5c. We have $P(U > 4 \mid U > 3) = \frac{P(U>4 \ \& \ U>3)}{P(U>3)} = \frac{P(U>4)}{P(U>3)} = \frac{6/10}{7/10} = 6/7$. This is not equal to $P(U > 1)$, because $P(U > 1) = 9/10$.

Submit You have used 1 of 1 attempt

✓ Correct (4/4 points)

Problem 6

1/1 point (graded)

6. Finally it is almost 2 AM and you are scared and tired. You think it is time to leave the haunted corn maze, but none of your friends are around, when you get back to your car. You came to the corn maze with two friends, Alejandro and Brenda. Let X denote the time (in minutes) until Alejandro arrives at the car and let Y denote the time (in minutes) until Brenda arrives at the car. Since you and

Alejandro and Brenda all got hopelessly lost from each other during the night at the corn maze, you can assume X and Y are independent Exponential random variables, which each have mean 5, i.e., $\mathbb{E}(X) = \mathbb{E}(Y) = 5$.

Find $P(X < \frac{1}{2}Y)$, i.e., find the probability that your waiting time for Alejandro is less than half of your waiting time for Brenda.

[[Hint: If you prefer, equivalently, you can find $P(2X < Y)$. Just draw the region where $2X < Y$ in the plane, and then integrate the joint density of X and Y over that region.]]

$P(X < \frac{1}{2}Y) =$ 1/3 ✓ **Answer:** 0.333

Explanation

6. We compute $P(2X < Y) = \int_0^\infty \int_{2x}^\infty \frac{1}{5} e^{-(1/5)x} \frac{1}{5} e^{-(1/5)y} \, dy dx$

$$= \int_0^\infty \frac{1}{5} e^{-(3/5)x} \, dx = 1/3.$$

Submit You have used 1 of 1 attempt

✓ Correct (1/1 point)

Problem 7

1/1 point (graded)

7. Consider independent exponential random variables X and Y that each have average 1. Find $P(|X - Y| > 1)$.

[Hint: Equivalently, find $P(X - Y > 1 \text{ or } Y - X > 1)$, which is $P(X - Y > 1) + P(Y - X > 1)$, since the corresponding regions are disjoint, i.e., are nonoverlapping.]

$P(|X - Y| > 1) =$ 0.3678794 ✓ **Answer:** 0.3679

Explanation

7. We have $P(X - Y > 1) = \int_0^\infty \int_{y+1}^\infty e^{-x} e^{-y}\,dx\,dy$. For the inner integral, we focus on the x part of the integrand, and we get: $\int_{y+1}^\infty e^{-x}\,dx = e^{-(y+1)}$. So then we have
$P(X - Y > 1) = \int_0^\infty e^{-y} e^{-(y+1)}\,dy = \int_0^\infty e^{-2y-1}\,dy = (1/2)e^{-1}$.
Similarly (just switching the roles of x and y), we get $P(Y - X > 1) = (1/2)e^{-1}$.
So altogether we have $P(|X - Y| > 1) = (1/2)e^{-1} + (1/2)e^{-1} = e^{-1} = 0.3679$.

Submit You have used 1 of 1 attempt

✓ Correct (1/1 point)

Problem 8

6/6 points (graded)

8. Elena just got engaged to be married. She posts a message about the engagement on Facebook. Three of her friends, Alicia, Barbara, and Charlene, will click “like" on her post. Use X, Y, and Z (respectively) to denote the waiting times until Alicia, Barbara, and Charlene click “like" on this post, and assume that these three random variables are independent. Assume each of the random variables is an Exponential random variable that has an average of 2 minutes.

8a. Find $P(X < 1)$.

0.3934693 ✓ **Answer:** 0.3935

8b. Use your answer to **8a** to find the probability that all 3 friends “like" the post within 1 minute.

0.06091618 ✓ **Answer:** 0.0609

8c. Use your answer to **8a** to find the probability that none of the 3 friends “like" the post within 1 minute.

0.2231302 ✓ **Answer:** 0.2231

8d. Use your answer to **8a** to find the probability that exactly 1 of the 3 friends “likes" the post within 1 minute.

0.4342478 ✓ **Answer:** 0.4342

8e. Use your answer to **8a** to find the probability that exactly 2 of the 3 friends “like" the post within 1 minute.

0.2817058 ✓ **Answer:** 0.2817

8f. Let V denote the number of friends (among these 3) who “like" the post within 1 minute. Then V is a discrete random variable. What kind of random variable is V? [Hint: In **8b**, we have $P(V = 3)$; in **8c**, we have $P(V = 0)$; in **8d**, we have $P(V = 1)$; in **8e**, we have $P(V = 2)$. Your answers in **8b**, **8c**, **8d**, **8e** should sum to 1.]

○ Bernoulli random variable

● Binomial random variable ✓

○ Geometric random variable

○ Poisson random variable

Explanation

8a. We have $P(X < 1) = F_X(1) = 1 - e^{-(1/2)(1)} = 0.3935.$

8b. The probability is

$$P(X < 1, Y < 1, Z < 1) = P(X < 1)P(Y < 1)P(Z < 1)$$
$$= (1 - e^{-(1/2)(1)})^3 = 0.0609.$$

8c. The probability is

$$P(X > 1, Y > 1, Z > 1) = P(X > 1)P(Y > 1)P(Z > 1) \qquad = (e^{-(1/2)(1)})^3 = 0.2231.$$

8d. The probability is $3(1 - e^{-(1/2)(1)})^1(e^{-(1/2)(1)})^2 = 0.4342.$

8e. The probability is $3(1 - e^{-(1/2)(1)})^2(e^{-(1/2)(1)})^1 = 0.2817.$

8f. The random variable V is a Binomial random variable with parameters $n = 3$ and $p = 1 - e^{-1/2} = 0.3935.$

Submit You have used 1 of 1 attempt

✓ Correct (6/6 points)

Problem 9

5/5 points (graded)

9. Suppose that U and V have joint probability density function $f_{U,V}(u,v) = 9e^{-3u-3v}$ for u and v positive, and $f_{U,V}(u,v) = 0$ otherwise.

9a. Are U and V independent?

● Yes ✓

○ No

9b. What is the density of U? Compute $f_U(1)$.

$f_U(1) =$ 0.1493612 ✓ **Answer:** 0.1493612

9c. If we define $X = U + V$, what kind of random variable is X?

○ Uniform random variable

○ Exponential random variable

● Gamma random variable ✓

○ Beta random variable

What is the density of X? Compute $f_X(1)$.

$f_X(1) =$ 0.4480836 ✓ **Answer:** 0.4480836

9d. Can you find $P(X \leq 1/2)$?

0.4421746 ✓ **Answer:** 0.4421746

Explanation

9a. Yes! The random variables U and V are independent, because $f_{U,V}(u,v)$ can be factored into u and v parts.

9b. By symmetry, we have $f_U(u) = 3e^{-3u}$ for u positive, and $f_U(u) = 0$ otherwise.

9c. The random variable X is a Gamma random variable with parameters $r = 2$ and $\lambda = 3$, so the density of X is $f_X(x) = 9xe^{-3x}$ for $x > 0$, and $f_X(x) = 0$ otherwise.

9d. We have

$$P(X \le 1/2) = F_X(1/2)$$
$$= 1 - e^{-(3)(1/2)}(1 + (3)(1/2)) = 1 - (5/2)e^{-3/2}.$$

Or we could calculate

$$P(X \le 1/2) = \int_0^{1/2} 9xe^{-3x}\,dx = 1 - (5/2)e^{-3/2}.$$

Submit You have used 1 of 1 attempt

✓ Correct (5/5 points)

Problem 10

4/4 points (graded)

10. Same setup as #9.

10a. What is $P(U > V)$? Hint: You shouldn't have to calculate anything to solve this.

1/2 ✓ **Answer:** 0.5

10b. Let $Y = 0$ if $U > 1/10$ and $V > 1/10$.

Let $Y = 2$ if $U \le 1/10$ and $V \le 1/10$.

Let $Y = 1$ if $U > 1/10$ and $V \le 1/10$, or if $U \le 1/10$ and $V > 1/10$.

In other words, let Y count how many of the variables U and/or V are less than $1/10$. What kind of random variable is Y?

● Binomial random variable ✓

○ Geometric random variable

○ Gamma random variable

○ Beta random variable

What are the parameters of Y?

$n =$ 2 ✓ **Answer:** 2 $p =$ 0.2591818 ✓

Answer: 0.2592

Explanation

10a. We have
$P(U > V) + P(U = V) + P(U < V) = 1$, and
$P(U = V) = 0$, and $P(U > V) = P(U < V)$,
so it must be the case that $P(U > V) = 1/2$.

10b. The random variable Y is a Binomial random variable with parameters $n = 2$ and
$p = 1 - e^{-(3)(1/10)} = 0.2592$.

Submit You have used 1 of 1 attempt

✓ Correct (4/4 points)

Problem 11

4/4 points (graded)

11. Suppose that Alfredo, Bruno, and Charlie wait for their (respective) girlfriends outside of their classes, and their girlfriends are all in different classes, so their waiting times are independent. Let X, Y, and Z denote their (respective) waiting times, given in minutes. Suppose these waiting times are each Exponential random variables, with average of 5 minutes each.

Now define $W = X + Y + Z$, i.e., their total waiting time, given in minutes.

11a. What kind of random variable is W?

○ Uniform random variable

○ Exponential random variable

● Gamma random variable ✓

○ Beta random variable

What is the density of W? Compute $f_W(1)$.

$f_W(1) =$ 0.003274923 ✓ **Answer:** 0.003274923

11b. What is $\mathrm{Var}(W)$? (You do not need to calculate any integrals. You can simply find this by the general formula for the variance of the sum of independent random variables, and you should check that your answer agrees with what you know about the variance of the type of random variable that W is.)

75 ✓ **Answer:** 75

11c. Notice that $W/60$ is their total waiting time, given in hours. What is $\mathrm{Var}(W/60)$?

3/144 ✓ **Answer:** 0.02083333

Explanation

11a. The random variable W is a Gamma random variable with $r = 3$ and $\lambda = 1/5$. The density of W is $f_W(w) = \frac{(1/5)^3 w^2}{2} e^{-(1/5)w}$ for $w > 0$, and $f_W(w) = 0$ otherwise.

11b. The variance of W is $\mathrm{Var}(W) = r/\lambda^2 = \frac{3}{(1/5)^2} = 75$.

11c. The variance of $W/60$ is $\mathrm{Var}(W/60) = 75(1/60)^2 = 1/48$.

Submit You have used 1 of 1 attempt

✓ Correct (4/4 points)

Problem 12

2/2 points (graded)

12. Some students take an examination in a course at Purdue. Let X denote the percent of students who pass the examination. Suppose that X is a Beta random variable with $\alpha = 8$ and $\beta = 2$.

12a. What is the expected percentage of students who pass the exam? I.e., what is $\mathbb{E}(X)$?

4/5 ✓ **Answer:** 0.8

12b. What is the probability density function $f_X(x)$ of X? Compute $f_X(0.9)$.

$f_X(0.9) =$ 3.443738 ✓ **Answer:** 3.443738

12c. Can you verify that $f_X(x)$ is a valid probability density function?

Explanation

12a. Using the formula for the expected value of a Beta random variable, we have $\mathbb{E}(X) = \alpha/(\alpha+\beta) = \frac{8}{8+2} = \frac{8}{10}$; or, if you prefer to calculate:
$\mathbb{E}(X) = \int_0^1 (x) \frac{9!}{7!1!} x^7 (1-x)^1 \, dx = 4/5.$

12b. We see that $f_X(x) = \frac{9!}{7!1!} x^7 (1-x)^1$ for $0 \leq x \leq 1$, and $f_X(x) = 0$ otherwise.

12c. Yes! The function $f_X(x)$ is always nonnegative, and we have $\int_0^1 \frac{9!}{7!1!} x^7 (1-x)^1 \, dx = 1$.

Submit You have used 1 of 1 attempt

✓ Correct (2/2 points)

Problem 13

2/2 points (graded)

13. Same setup as #12.

13a. Find $P(X > 0.90)$, i.e., the probability that at least 90% of students pass the exam.

0.225159 ✓ **Answer:** 0.2252

13b. Find $P(X > 0.90 \mid X > 0.80)$.

0.3993651 ✓ **Answer:** 0.3994

Explanation

13a. We have $P(X > 0.90) = \int_{0.90}^{1} \frac{9!}{7!1!} x^7 (1-x)^1 \, dx = 0.2252$.

13b. We have $P(X > 0.90 \mid X > 0.80) = \frac{P(X>0.90 \ \& \ X>0.80)}{P(X>0.80)} = \frac{P(X>0.90)}{P(X>0.80)}$. The numerator, as in part a, is 0.2252. The denominator is $\int_{0.80}^{1} \frac{9!}{7!1!} x^7 (1-x)^1 \, dx = 0.5638$. Putting these results together, the conditional probability is $P(X > 0.90 \mid X > 0.80) = \frac{0.2252}{0.5638} = 0.3994$.

Submit You have used 1 of 1 attempt

✓ Correct (2/2 points)

Problem 14

1/1 point (graded)

14. In a certain town in Oregon, the percentage of rainy days during a given time period is modelled by a Beta random variable X with $\alpha = 2$ and $\beta = 20$.

Find $P(X < 0.15)$. Hint: Use the u-substitution $u = x - 1$.

0.8449619 ✓ **Answer:** 0.8450

Explanation

14. We have

$$P(X < 0.15) = \int_0^{0.15} \frac{21!}{1!19!} x^1 (1-x)^{19}\, dx$$
$$= \int_{0.85}^{1} \frac{21!}{1!19!} (1-u)^1 u^{19}\, du = 0.8450.$$

Submit You have used 1 of 1 attempt

✓ Correct (1/1 point)

Problem 15

7/7 points (graded)

15. Review question:

15a. Is the sum of two independent Bernoulli random variables (with the same parameters p) also a Bernoulli random variable?

○ Yes

● No ✓

If not, what kind of random variable is the sum?

15b. Is the sum of two independent Binomial random variables (with the same parameters p) also a Binomial random variable?

● Yes ✓

○ No

If not, what kind of random variable is the sum?

15c. Is the sum of two independent Geometric random variables (with the same parameters p) also a Geometric random variable?

○ Yes

● No ✓

If not, what kind of random variable is the sum?

15d. Is the sum of two independent Negative Binomial random variables (with the same parameters p) also a Negative Binomial random variable?

● Yes ✓

○ No

If not, what kind of random variable is the sum?

15e. Is the sum of two independent Poisson random variables also a Poisson random variable?

● Yes ✓

○ No

If not, what kind of random variable is the sum?

15f. Is the sum of two independent Exponential random variables (with the same parameters λ) also an Exponential random variable?

○ Yes

● No ✓

If not, what kind of random variable is the sum?

15g. Is the sum of two independent Gamma random variables (with the same parameters λ) also an Gamma random variable?

● Yes ✓

○ No

If not, what kind of random variable is the sum?

Explanation

15a. No! The sum of two independent Bernoulli random variables (with the same parameters p) is a Binomial random variable with parameters $n = 2$ and p.

15b. Yes! The sum of two independent Binomial random variables (with the same parameters p) is a Binomial random variable too. The value of n is the sum of the values of the n's from the two original Binomial random variables. The value of p is the same as for those original Binomial random variables.

15c. No! The sum of two independent Geometric random variables (with the same parameters p) is a Negative Binomial random variable with parameters $r = 2$ and p.

15d. Yes! The sum of two independent Negative Binomial random variables (with the same parameters p) is a Negative Binomial random variable too. The value of r is the sum of the values of the r's from the two original Negative Binomial random variables. The value of p is the same as for those original Negative Binomial random variables.

15e. Yes! The sum of two independent Poisson random variables is a Poisson random variable too. The value of λ is the sum of the values of the λ's from the two Poisson random variables.

15f. No! The sum of two independent Exponential random variables (with the same parameters λ) is a Gamma random variable with parameters $r = 2$ and λ.

15g. Yes! The sum of two independent Gamma random variables (with the same parameters λ) is a Gamma random variable too. The value of r is the sum of the values of the r's from the two original Gamma random variables. The value of λ is the same as for those original Gamma random variables.

Submit You have used 1 of 1 attempt

✓ Correct (7/7 points)