

Ţ <u>Help</u>

sandipan\_dey ~

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★ Course / Week 5: Matrix- Matrix Multiplication / 5.2 Observations

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**5.2.4 Matrix-Matrix Multiplication with Special Matrices** 

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Week 5 due Nov 6, 2023 22:42 IST

# 5.2.4 Matrix-Matrix Multiplication with Special Matrices

No introductory video

## Reading Assignment

0 points possible (ungraded)
Read Unit 5.2.4 of the notes. [LINK]



Done



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✓ Correct

#### Discussion

**Topic:** Week 5 / 5.2.4

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? triangular and diagonal matrix

Hi Maggie: Regarding Homework 5.2.4.12, could you please help to provide an example matrix which is a triangular but is not diagonal matrix? T...

## Homework 5.2.4.1

21/21 points (graded)

Compute

$$\bullet \begin{pmatrix} 1 & -2 & -1 \\ 2 & 0 & 2 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$
 Answer: 1

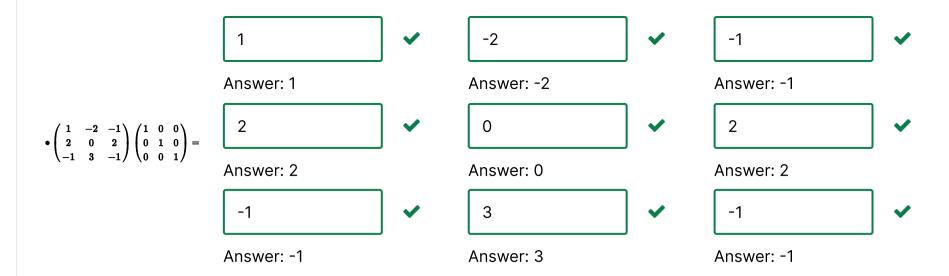
$$\bullet \begin{pmatrix} 1 & -2 & -1 \\ 2 & 0 & 2 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \begin{bmatrix} -2 & & \checkmark \text{ Answer: -2} \\ & & \checkmark \text{ Answer: 0} \end{bmatrix}$$

$$\bullet \begin{pmatrix} 1 & -2 & -1 \\ 2 & 0 & 2 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$$
 Answer: -1

Answer: 2

Answer: 0

Answer: 2



#### Explanation

$$\bullet \left(\begin{array}{rrr} 1 & -2 & -1 \\ 2 & 0 & 2 \end{array}\right) \left(\begin{array}{r} 1 \\ 0 \\ 0 \end{array}\right) = \left(\begin{array}{r} 1 \\ 2 \end{array}\right)$$

$$\bullet \left(\begin{array}{rrr} 1 & -2 & -1 \\ 2 & 0 & 2 \end{array}\right) \left(\begin{array}{r} 0 \\ 1 \\ 0 \end{array}\right) = \left(\begin{array}{r} -2 \\ 0 \end{array}\right)$$

$$\bullet \left(\begin{array}{rrr} 1 & -2 & -1 \\ 2 & 0 & 2 \end{array}\right) \left(\begin{array}{r} 0 \\ 0 \\ 1 \end{array}\right) = \left(\begin{array}{r} -1 \\ 2 \end{array}\right)$$

$$\bullet \left(\begin{array}{rrr} 1 & -2 & -1 \\ 2 & 0 & 2 \end{array}\right) \left(\begin{array}{rrr} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array}\right) = \left(\begin{array}{rrr} 1 & -2 & -1 \\ 2 & 0 & 2 \end{array}\right)$$

$$\bullet \left(\begin{array}{rrr} 1 & -2 & -1 \\ 2 & 0 & 2 \\ -1 & 3 & -1 \end{array}\right) \left(\begin{array}{rrr} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array}\right) = \left(\begin{array}{rrr} 1 & -2 & -1 \\ 2 & 0 & 2 \\ -1 & 3 & -1 \end{array}\right)$$

Answer: There are at least two things to notice:

- 1. The first three results provide the columns for the fourth result. The fourth result provides the first two rows of the fifth result.
- 2. Multiplying the matrix from the right with the identity matrix does not change the matrix.

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Answers are displayed within the problem

#### Homework 5.2.4.2

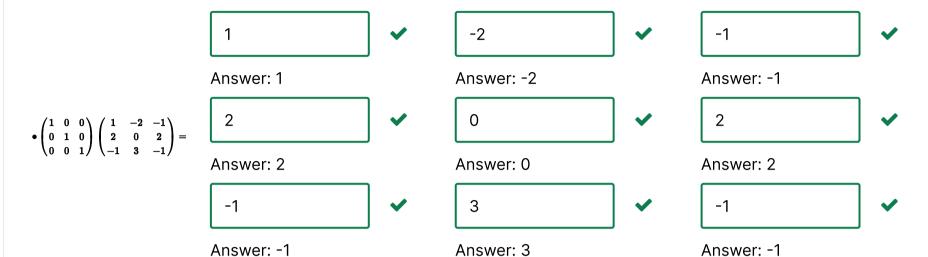
18/18 points (graded) Compute

$$\bullet \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} = 
\begin{bmatrix} 1 & & \checkmark & \text{Answer: 1} \\ 2 & & \checkmark & \text{Answer: 2} \\ & & \checkmark & \text{Answer: -1} \end{bmatrix}$$

$$\bullet \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} -2 \\ 0 \\ 3 \end{pmatrix} = \begin{bmatrix} -2 \\ 0 \\ 3 \end{bmatrix}$$
Answer: -2

Answer: 0

$$\bullet \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} -1 \\ 2 \\ -1 \end{pmatrix} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$$
Answer: -1
Answer: -1
Answer: -1



#### Explanation

$$\bullet \left(\begin{array}{ccc} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array}\right) \left(\begin{array}{c} 1 \\ 2 \\ -1 \end{array}\right) = \left(\begin{array}{c} 1 \\ 2 \\ -1 \end{array}\right)$$

$$\bullet \left( \begin{array}{ccc} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array} \right) \left( \begin{array}{c} -2 \\ 0 \\ 3 \end{array} \right) = \left( \begin{array}{c} -2 \\ 0 \\ 3 \end{array} \right)$$

$$\bullet \left(\begin{array}{ccc} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array}\right) \left(\begin{array}{c} -1 \\ 2 \\ -1 \end{array}\right) = \left(\begin{array}{c} -1 \\ 2 \\ -1 \end{array}\right)$$

$$\bullet \left(\begin{array}{ccc} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array}\right) \left(\begin{array}{ccc} 1 & -2 & -1 \\ 2 & 0 & 2 \\ -1 & 3 & -1 \end{array}\right) = \left(\begin{array}{ccc} 1 & -2 & -1 \\ 2 & 0 & 2 \\ -1 & 3 & -1 \end{array}\right)$$

Answer: There are at least three things to notice:

- 1. The first three results provide the columns for the fourth result.
- 2. Multiplying the matrix from the left with the identity matrix does not change the matrix.
- 3. This homework and the last homework yield the same result.

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Answers are displayed within the problem

#### Homework 5.2.4.3

1/1 point (graded)

Let  $A \in \mathbb{R}^{m imes n}$  and let I denote the identity matrix of appropriate size.

$$AI = IA = A$$

**Always** ✓ Answer: Always

Explanation

Transcripted in final section of this week

Answer: Always

Partition A and I by columns:

$$A = \left(\begin{array}{c|c} a_0 & a_1 & \cdots & a_{n-1} \end{array}\right)$$
 and  $I = \left(\begin{array}{c|c} e_0 & e_1 & \cdots & e_{n-1} \end{array}\right)$ 

and recall that  $e_j$  equals the jth unit basis vector.

AI = A:

AI= < Partition I by columns >  $A\left(\begin{array}{c|c}e_0 & e_1 & \cdots & e_{n-1}\end{array}\right)$ = < Partitioned matrix-matrix multiplication >  $\left(\begin{array}{c|c} Ae_0 & Ae_1 & \cdots & Ae_{n-1} \end{array}\right)$ =  $\langle a_j = Ae_j \rangle$  $\left(\begin{array}{c|c} a_0 & a_1 & \cdots & a_{n-1} \end{array}\right)$ = < Partition A by columns >  $\boldsymbol{A}$ 

IA = A:

$$IA$$

$$= \langle \text{ Partition } A \text{ by columns } \rangle$$
 $I\left(a_0 \mid a_1 \mid \cdots \mid a_{n-1}\right)$ 

$$= \langle \text{ Partitioned matrix-matrix multiplication } \rangle$$
 $\left(Ia_0 \mid Ia_1 \mid \cdots \mid Ia_{n-1}\right)$ 

$$= \langle Ix = x \rangle$$
 $\left(a_0 \mid a_1 \mid \cdots \mid a_{n-1}\right)$ 

$$= \langle \text{ Partition } A \text{ by columns } \rangle$$
 $A$ 

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#### Answers are displayed within the problem

## Homework 5.2.4.4

12/12 points (graded)

Compute

$$\bullet \begin{pmatrix} 1 & -2 & -1 \\ 2 & 0 & 2 \end{pmatrix} \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix} =$$

$$4 \qquad \qquad \bullet \text{Answer: 2}$$

$$\bullet \text{Answer: 4}$$

$$\bullet \begin{pmatrix} 1 & -2 & -1 \\ 2 & 0 & 2 \end{pmatrix} \begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix} = \begin{bmatrix} 2 \\ 0 \\ \end{bmatrix}$$
 Answer: 2

$$\bullet \begin{pmatrix} 1 & -2 & -1 \\ 2 & 0 & 2 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix} = \begin{bmatrix} 3 \\ 6 \end{bmatrix}$$
 Answer: 3

Answer: 0

Answer: -6

Explanation

$$\bullet \left(\begin{array}{ccc} 1 & -2 & -1 \\ 2 & 0 & 2 \end{array}\right) \left(\begin{array}{c} 2 \\ 0 \\ 0 \end{array}\right) = \left(\begin{array}{c} 2 \\ 4 \end{array}\right)$$

Answer: 4

$$\bullet \left(\begin{array}{ccc} 1 & -2 & -1 \\ 2 & 0 & 2 \end{array}\right) \left(\begin{array}{c} 0 \\ -1 \\ 0 \end{array}\right) = \left(\begin{array}{c} 2 \\ 0 \end{array}\right)$$

$$\bullet \left(\begin{array}{cc} 1 & -2 & -1 \\ 2 & 0 & 2 \end{array}\right) \left(\begin{array}{c} 0 \\ 0 \\ -3 \end{array}\right) = \left(\begin{array}{c} 3 \\ -6 \end{array}\right)$$

$$\bullet \left(\begin{array}{rrr} 1 & -2 & -1 \\ 2 & 0 & 2 \end{array}\right) \left(\begin{array}{rrr} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -3 \end{array}\right) = \left(\begin{array}{rrr} 2 & 2 & 3 \\ 4 & 0 & -6 \end{array}\right)$$

Answer: Notice the relation between the above problems.

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Answers are displayed within the problem

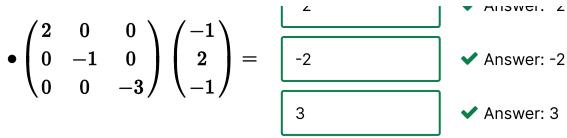
#### Homework 5.2.4.5

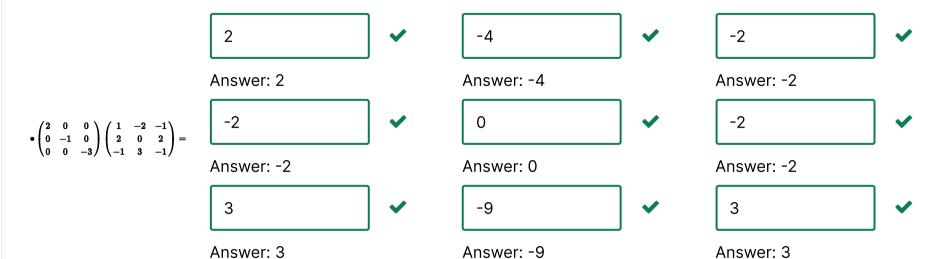
18/18 points (graded)

• 
$$\begin{pmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -3 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} = \begin{bmatrix} 2 \\ -2 \\ 3 \end{bmatrix}$$
 Answer: 2

Answer: 2

• 
$$\begin{pmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -3 \end{pmatrix} \begin{pmatrix} -2 \\ 0 \\ 3 \end{pmatrix} = \begin{bmatrix} -4 \\ 0 \\ -9 \end{bmatrix}$$
 Answer: -4 Answer: -4 Answer: -9





## Explanation

$$\bullet \begin{pmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -3 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} = \begin{pmatrix} 2 \\ -2 \\ 3 \end{pmatrix}$$

$$\bullet \begin{pmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -3 \end{pmatrix} \begin{pmatrix} -2 \\ 0 \\ 3 \end{pmatrix} = \begin{pmatrix} -4 \\ 0 \\ -9 \end{pmatrix}$$

$$\bullet \begin{pmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -3 \end{pmatrix} \begin{pmatrix} -1 \\ 2 \\ -1 \end{pmatrix} = \begin{pmatrix} -2 \\ -2 \\ 3 \end{pmatrix}$$

$$\bullet \begin{pmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -3 \end{pmatrix} \begin{pmatrix} 1 & -2 & -1 \\ 2 & 0 & 2 \\ -1 & 3 & -1 \end{pmatrix} = \begin{pmatrix} 2 & -4 & -2 \\ -2 & 0 & -2 \\ 3 & -9 & 3 \end{pmatrix}$$

Answer: Notice the relation between the above problems.

**Submit** 

Answers are displayed within the problem

#### Homework 5.2.4.6

1/1 point (graded)

Let  $A\in\mathbb{R}^{m imes n}$  and let D denote the diagonal matrix with diagonal elements  $\delta_0,\delta_1,\cdots,\delta_{n-1}$ . Partition A by columns :

$$A=\left(egin{array}{c|c} a_0 & a_1 & \ldots & a_{n-1} \end{array}
ight).$$

$$AD = \left(egin{array}{c|c} \delta_0 a_0 & \delta_1 a_1 & \dots & \delta_{n-1} a_{n-1} \end{array}
ight).$$

Always ✓ ✓ Answer: Always

Explanation

Transcripted in final section of this week

Answer: Always

$$\left(\begin{array}{c|c|c}
a_0 & a_1 & \cdots & a_{n-1}
\end{array}\right) \left(\begin{array}{c|c|c}
\delta_0 & 0 & \cdots & 0 \\
\hline
0 & \delta_1 & \cdots & 0 \\
\hline
\vdots & \vdots & \ddots & \vdots \\
\hline
0 & 0 & \vdots & \delta_{n-1}
\end{array}\right)$$

$$\begin{pmatrix} a_0 \delta_0 & a_1 \delta_1 & \cdots & a_{n-1} \delta_{n-1} \\
= & \langle x\beta = \beta x \rangle \\
\delta_0 a_0 & \delta_1 a_1 & \cdots & \delta_{n-1} a_{n-1} \end{pmatrix}$$

Submit

Answers are displayed within the problem

## Homework 5.2.4.7

1/1 point (graded)

Let  $A\in\mathbb{R}^{m imes n}$  and let D denote the diagonal matrix with diagonal elements  $\delta_0,\delta_1,\cdots,\delta_{m-1}$ . Partition A by rows :

$$A = egin{pmatrix} ilde{a}_0^T \ ilde{a}_1^T \ dots \ ilde{ar{a}}_{m-1}^T \end{pmatrix}.$$

$$DA = egin{pmatrix} rac{\delta_0 ilde{a}_0^T}{\delta_1 ilde{a}_1^T} \ dots \ \hline rac{dots}{\delta_{m-1} ilde{a}_{m-1}^T} \end{pmatrix}.$$

Always ~

✓ Answer: Always

Explanation

Transcripted in final section of this week

Answer: Always

$$DA = \begin{pmatrix} \begin{array}{c|ccc} \delta_0 & 0 & \cdots & 0 \\ \hline 0 & \delta_1 & \cdots & 0 \\ \hline \vdots & \vdots & \ddots & \vdots \\ \hline 0 & 0 & \vdots & \delta_{m-1} \\ \end{array} \end{pmatrix} \begin{pmatrix} \begin{array}{c|ccc} \widetilde{a}_0^T \\ \hline \widetilde{a}_1^T \\ \hline \vdots \\ \hline \widetilde{a}_{m-1}^T \\ \end{array} \end{pmatrix} = \begin{pmatrix} \begin{array}{c|ccc} \delta_0 \widetilde{a}_0^T \\ \hline \delta_1 \widetilde{a}_1^T \\ \hline \vdots \\ \hline \delta_{m-1} \widetilde{a}_{m-1}^T \\ \end{array} \end{pmatrix}$$

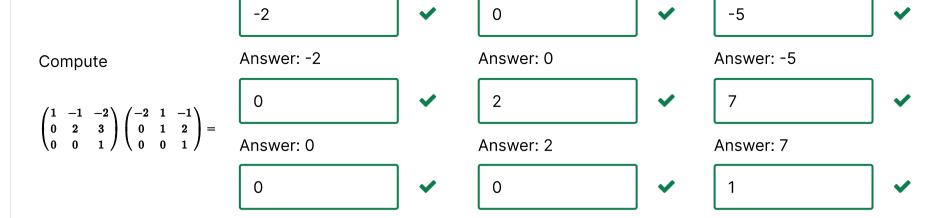
by simple application of partitioned matrix-matrix multiplication.

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**1** Answers are displayed within the problem

#### Homework 5.2.4.8

9/9 points (graded)



Answer: 0

Answer: 1

$$\begin{pmatrix} -2 & 0 & -5 \\ 0 & 2 & 7 \\ 0 & 0 & 1 \end{pmatrix}$$

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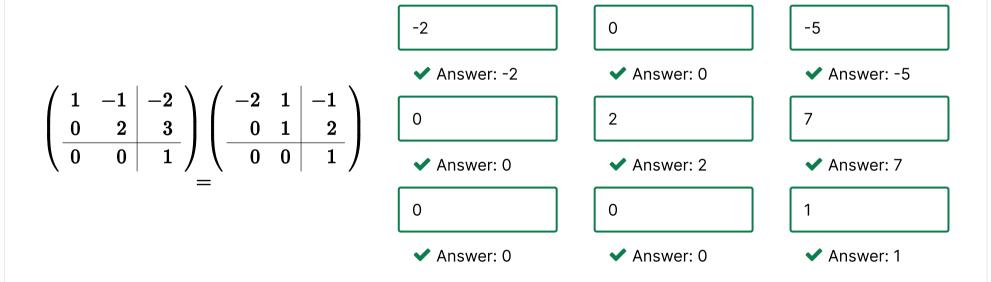
• Answers are displayed within the problem

Answer: 0

#### Homework 5.2.4.9

9/9 points (graded)

Compute the following, using what you know about partitioned matrix-matrix multiplication:



Answer:

$$\begin{pmatrix}
1 & -1 & | & -2 \\
0 & 2 & | & 3 \\
\hline
0 & 0 & | & 1
\end{pmatrix}
\begin{pmatrix}
-2 & 1 & | & -1 \\
0 & 1 & | & 2 \\
\hline
0 & 0 & | & 1
\end{pmatrix}$$

$$= \begin{pmatrix}
\begin{pmatrix}
1 & -1 \\
0 & 2
\end{pmatrix}
\begin{pmatrix}
-2 & 1 \\
0 & 1
\end{pmatrix}
+ \begin{pmatrix}
-2 \\
3
\end{pmatrix}
\begin{pmatrix}
0 & 0
\end{pmatrix}
\begin{pmatrix}
1 & -1 \\
0 & 2
\end{pmatrix}
\begin{pmatrix}
-1 \\
2
\end{pmatrix}
+ \begin{pmatrix}
-2 \\
3
\end{pmatrix}
(1)$$

$$\begin{pmatrix}
0 & 0
\end{pmatrix}
\begin{pmatrix}
-2 & 1 \\
0 & 1
\end{pmatrix}
+ (1) \begin{pmatrix}
0 & 0
\end{pmatrix}
\begin{pmatrix}
-1 \\
2
\end{pmatrix}
+ (1)(1)$$

$$= \begin{pmatrix}
\begin{pmatrix}
-2 & 0 \\
0 & 2
\end{pmatrix}
+ \begin{pmatrix}
0 & 0 \\
0 & 0
\end{pmatrix}
\begin{pmatrix}
-3 \\
4
\end{pmatrix}
+ \begin{pmatrix}
-2 \\
3
\end{pmatrix}
= \begin{pmatrix}
-2 & 0 & | & -5 \\
0 & 2 & | & 7
\end{pmatrix}$$

$$\blacksquare Calculator$$

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Answers are displayed within the problem

### Homework 5.2.4.10

1/1 point (graded)

Let  $U,R\in\mathbb{R}^{n imes n}$  be uppertriangular matrices.

The product UR is an upper triangular matrix.

**Always** 

✓ Answer: Always

Explanation

Answer: Always We will prove this by induction on n, the size of the square matrices.

Base case: n = 1. If  $U, R \in \mathbb{R}^{1 \times 1}$  then they are scalars. (Scalars are inherently upper triangular since they have no elements below the diagonal!). But then UR is also a scalar, which is an upper triangular matrix. Thus the result is true for n=1.

**Inductive Step:** Induction Hypothesis (I.H.): Assume the result is true for n = N, where  $N \ge 1$ .

We will show the result is true for n = N + 1.

Let U and R be  $n \times n$  upper triangular matrices with n = N + 1. We can partition

$$U = \begin{pmatrix} U_{00} & u_{01} \\ \hline 0 & v_{11} \end{pmatrix} \quad \text{and} \quad R = \begin{pmatrix} R_{00} & r_{01} \\ \hline 0 & \rho_{11} \end{pmatrix},$$

where  $U_{00}$  and  $R_{00}$  are  $N \times N$  matrices and are upper triangular themselves. Now,

$$UR = \begin{pmatrix} U_{00} & u_{01} \\ \hline 0 & v_{11} \end{pmatrix} \begin{pmatrix} R_{00} & r_{01} \\ \hline 0 & \rho_{11} \end{pmatrix}$$

$$= \begin{pmatrix} U_{00}R_{00} + u_{01}0 & U_{00}r_{01} + u_{01}\rho_{11} \\ \hline 0R_{00} + v_{11}0 & 0r_{01} + v_{11}\rho_{11} \end{pmatrix} = \begin{pmatrix} U_{00}R_{00} & U_{00}r_{01} + u_{01}\rho_{11} \\ \hline 0 & v_{11}\rho_{11} \end{pmatrix}.$$

By the I.H.,  $U_{00}R_{00}$  is upper triangular. Hence,

$$UR = \begin{pmatrix} U_{00}R_{00} & U_{00}r_{01} + u_{01}\rho_{11} \\ \hline 0 & v_{11}\rho_{11} \end{pmatrix}$$

is upper triangular.

By the Principle of Mathematical Induction (PMI), the result holds for all n.

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Answers are displayed within the problem

### Homework 5.2.4.11

1/1 point (graded)

The product of an  $n \times n$  lower triangular matrix times an  $n \times n$  lower triangular matrix is a lower triangular matrix.

Always ~

✓ Answer: Always

#### Explanation

#### Always!

We prove this by induction on n, the size of the square matrices.

Let A and B be lower triangular matrices.

**Base case:** n = 1. If  $A, B \in \mathbb{R}^{1 \times 1}$  then they are scalars. (Scalars are inherently lower triangular since they have no elements below the diagonal!). But then AB is also a scalar, which is an lower triangular matrix. Thus the result is true for n = 1.

**Inductive Step:** Induction Hypothesis (I.H.): Assume the result is true for n = N, where  $N \ge 1$ .

We will show the result is true for n = N + 1.

Let A and B be  $n \times n$  lower triangular matrices with n = N + 1. We can partition

$$A = \left( egin{array}{c|c} A_{00} & 0 \ \hline a_{10}^T & lpha_{11} \end{array} 
ight) \quad ext{and} \quad B = \left( egin{array}{c|c} B_{00} & 0 \ \hline b_{10}^T & eta_{11} \end{array} 
ight),$$

where  $A_{00}$  and  $B_{00}$  are  $N \times N$  matrices and are lower triangular themselves. Now,

$$\begin{array}{lll} AB & = & \left( \begin{array}{c|c|c} A_{00} & 0 \\ \hline a_{10}^T & \alpha_{11} \end{array} \right) \left( \begin{array}{c|c} B_{00} & 0 \\ \hline b_{10}^T & \beta_{11} \end{array} \right) \\ & = & \left( \begin{array}{c|c} A_{00}B_{00} + 0b_{10}^T & A_{00}0 + 0\beta_{11} \\ \hline a_{10}^TB_{00} + \alpha_{11}b_{10}^T & a_{10}^T0 + \alpha_{11}\beta_{11} \end{array} \right) = \left( \begin{array}{c|c} A_{00}B_{00} & 0 \\ \hline a_{10}^TB_{00} + \alpha_{11}b_{10}^T & \alpha_{11}\beta_{11} \end{array} \right). \end{array}$$

By the I.H.,  $A_{00}B_{00}$  is lower triangular. Hence,

$$AB = \left(egin{array}{c|c} A_{00}B_{00} & 0 \ \hline a_{10}^TB_{00} + lpha_{11}b_{10}^T & lpha_{11}eta_{11} \end{array}
ight).$$

is lower triangular.

By the Principle of Mathematical Induction (PMI), the result holds for all n.

**Submit** 

Answers are displayed within the problem

## Homework 5.2.4.12

1/1 point (graded)

The product of an  $n \times n$  lower triangular matrix times an  $n \times n$  upper triangular matrix is a diagonal matrix.

Sometimes ~

Answer: Sometimes

### Explanation

Diagonal matrices are both upper and lower triangular. Multiply them together, and you get a diagonal matrix. But take any lower triangular matrix that is not diagonal and multiply it by an upper triangular matrix (diagonal or not), and you don't get a diagonal matrix.

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• Answers are displayed within the problem

### Homework 5.2.4.13

1/1 point (graded) Let  $A \in \mathbb{R}^{m \times n}$ .

 ${\it A}^T{\it A}$  is symmetric.

Always ~

✓ Answer: Always

Explanation

Answer: Always

$$(A^T A)^T = A^T (A^T)^T = A^T A.$$

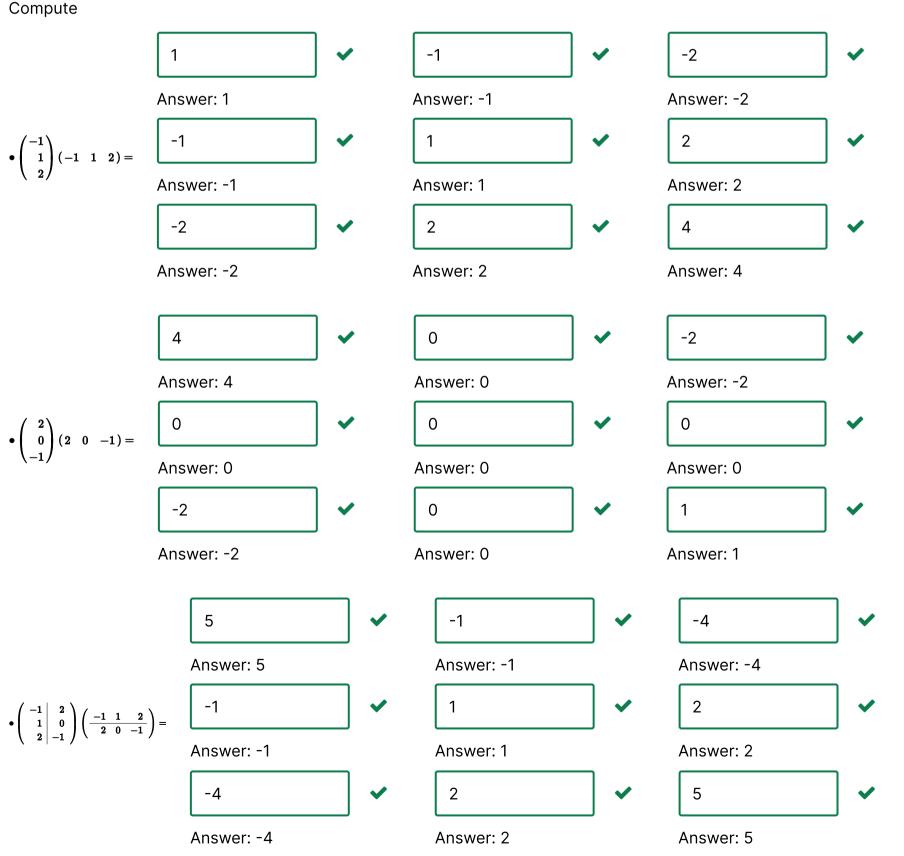
Hence,  $A^TA$  is symmetric.

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**1** Answers are displayed within the problem

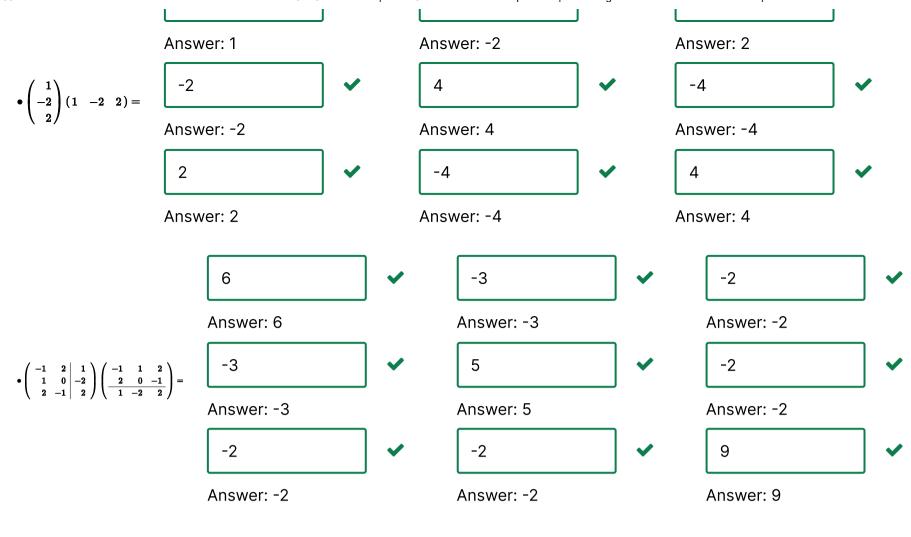
#### Homework 5.2.4.14

45/45 points (graded)



2

-2



$$\bullet \left(\begin{array}{c} -1\\1\\2 \end{array}\right) \left(\begin{array}{cccc} -1&1&2 \end{array}\right) = \left(\begin{array}{ccccc} 1&-1&-2\\-1&1&2\\-2&2&4 \end{array}\right).$$

$$\bullet \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix} \begin{pmatrix} 2 & 0 & -1 \end{pmatrix} = \begin{pmatrix} 4 & 0 & -2 \\ 0 & 0 & 0 \\ -2 & 0 & 1 \end{pmatrix}.$$

$$\bullet \begin{pmatrix} -1 & 2 \\ 1 & 0 \\ 2 & -1 \end{pmatrix} \begin{pmatrix} \frac{-1}{2} & 1 & 2 \\ \frac{-1}{2} & 0 & -1 \end{pmatrix} = \begin{pmatrix} 1 & -1 & -2 \\ -1 & 1 & 2 \\ -2 & 2 & 4 \end{pmatrix} + \begin{pmatrix} 4 & 0 & -2 \\ 0 & 0 & 0 \\ -2 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 5 & -1 & -4 \\ -1 & 1 & 2 \\ -4 & 2 & 5 \end{pmatrix}.$$

$$\bullet \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix} \begin{pmatrix} 1 & -2 & 2 \end{pmatrix} = \begin{pmatrix} 1 & -2 & 2 \\ -2 & 4 & -4 \\ 2 & -4 & 4 \end{pmatrix}.$$

$$\bullet \left(\begin{array}{cc|cc} -1 & 2 & 1 \\ 1 & 0 & -2 \\ 2 & -1 & 2 \end{array}\right) \left(\begin{array}{cc|cc} -1 & 1 & 2 \\ 2 & 0 & -1 \\ \hline 1 & -2 & 2 \end{array}\right) =$$

Answer:

$$\begin{pmatrix} -1 & 2 \\ 1 & 0 \\ 2 & -1 \end{pmatrix} \begin{pmatrix} -1 & 1 & 2 \\ 2 & 0 & -1 \end{pmatrix} + \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix} \begin{pmatrix} 1 & -2 & 2 \\ 2 \end{pmatrix}$$

$$= \begin{pmatrix} 5 & -1 & -4 \\ -1 & 1 & 2 \\ -4 & 2 & 5 \end{pmatrix} + \begin{pmatrix} 1 & -2 & 2 \\ -2 & 4 & -4 \\ 2 & -4 & 4 \end{pmatrix} = \begin{pmatrix} 6 & -3 & -2 \\ -3 & 5 & -2 \\ -2 & -2 & 9 \end{pmatrix}.$$

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### Answers are displayed within the problem

#### Homework 5.2.4.15

1/1 point (graded) Let  $x \in \mathbb{R}^n$ .

 $oldsymbol{x}oldsymbol{x}^T$  is symmetric.

Always 🗸

✓ Answer: Always

#### Explanation

Answer: Always

**Proof 1:** Since  $A^TA$  is symmetric for any matrix  $A \in \mathbb{R}^{m \times n}$  and vector  $A = x^T \in \mathbb{R}^n$  is just the special case where the matrix is a vector.

#### Proof 2:

$$xx^{T} = \begin{pmatrix} \chi_{0} \\ \chi_{1} \\ \vdots \\ \chi_{n-1} \end{pmatrix} \begin{pmatrix} \chi_{0} \\ \chi_{1} \\ \vdots \\ \chi_{n-1} \end{pmatrix}^{T} = \begin{pmatrix} \chi_{0} \\ \chi_{1} \\ \vdots \\ \chi_{n-1} \end{pmatrix} \begin{pmatrix} \chi_{0} & \chi_{1} & \cdots & \chi_{n-1} \end{pmatrix}$$

$$= \begin{pmatrix} \chi_{0}\chi_{0} & \chi_{0}\chi_{1} & \cdots & \chi_{0}\chi_{n-1} \\ \vdots & \vdots & & \vdots \\ \chi_{n-1}\chi_{0} & \chi_{n-1}\chi_{1} & \cdots & \chi_{n-1}\chi_{n-1} \end{pmatrix}.$$

Since  $\chi_i \chi_j = \chi_j \chi_i$ , the (i, j) element of  $xx^T$  equals the (j, i) element of  $xx^T$ . This means  $xx^T$  is symmetric.

**Proof 3:**  $(xx^T)^T = (x^T)^T x^T = xx^T$ .

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#### Answers are displayed within the problem

#### Homework 5.2.4.16

1/1 point (graded)

Let  $A \in \mathbb{R}^{n \times n}$  be symmetric and  $x \in \mathbb{R}^n$ .

 $A + xx^T$  is symmetric.

Always 🗸

Answer: Always

#### Explanation

If matrices  $A,B\in\mathbb{R}^{n\mathbf{x}n}$  are symmetric, then A+B is symmetric since  $(A+B)^T=A^T+B^T=A+B$ . In this case,  $B=xx^T$ .

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## **1** Answers are displayed within the problem

## Homework 5.2.4.17

1/1 point (graded) Let  $A \in \mathbb{R}^{m imes n}$ 

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#### $\boldsymbol{A}\boldsymbol{A^{\prime}}$ is symmetric.

Always ✓ Answer: Always

#### Explanation

Answer: Always

**Proof 1:**  $(AA^{T})^{T} = (A^{T})^{T}A^{T} = AA^{T}$ .

**Proof 2:** We know that  $A^TA$  is symmetric. Take  $B = A^T$ . Then  $AA^T = B^TB$  and hence  $AA^T$  is symmetric.

Proof 3:

$$AA^{T} = \left( \begin{array}{c|c} a_{0} & a_{1} & \cdots & a_{n-1} \end{array} \right) \left( \begin{array}{c|c} a_{0} & a_{1} & \cdots & a_{n-1} \end{array} \right)^{T}$$

$$= \left( \begin{array}{c|c} a_{0} & a_{1} & \cdots & a_{n-1} \end{array} \right) \left( \begin{array}{c|c} \frac{a_{0}^{T}}{a_{1}^{T}} \\ \hline \vdots \\ \hline a_{n-1}^{T} \end{array} \right)$$

$$= a_{0}a_{0}^{T} + a_{1}a_{1}^{T} + \cdots + a_{n-1}a_{n-1}^{T}.$$

But each  $a_j a_j^T$  is symmetric (by a previous exercise) and adding symmetric matrices yields a symmetric matrix. Hence,  $AA^T$  is symmetric.

#### Proof 4:

Proof by induction on n.

Base case:  $A = \begin{pmatrix} a_0 \end{pmatrix}$ , where  $a_0$  is a vector. Then  $AA^T = a_0a^T$ . But we saw in an earlier homework that if x is a vector, then  $xx^T$  is symmetric.

Induction Step: Assume that  $AA^T$  is symmetric for matrices with n=N columns, where  $N\geq 1$ . We will show that  $AA^T$  is symmetric for matrices with n=N+1 columns. Let A have N+1 columns.

$$AA^{T}$$

$$= \langle \text{ Partition } A \rangle$$

$$\begin{pmatrix} A_{0} \mid a_{1} \end{pmatrix} \begin{pmatrix} A_{0} \mid a_{1} \end{pmatrix}^{T}$$

$$= \langle \text{ Transpose partitioned matrix } \rangle$$

$$\begin{pmatrix} A_{0} \mid a_{1} \end{pmatrix} \begin{pmatrix} \frac{A_{0}^{T}}{a_{1}^{T}} \end{pmatrix}$$

$$= \langle \text{ Partitioned matrix-matrix multiplication } \rangle$$

$$A_{0}A_{0}^{T} + a_{1}a_{1}^{T}$$

Now, by the I.H.  $A_0A_0^T$  is symmetric. From a previous exercise we know that  $xx^T$  is symmetric and hence  $a_1a_1^T$  is. From another exercise we know that adding symmetric matrices yields a symmetric matrix.

By the Principle of Mathematical Induction (PMI), the result holds for all n.

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#### Answers are displayed within the problem

#### Homework 5.2.4.18

1/1 point (graded)

Let  $A,B\in\mathbb{R}^{n imes n}$  be symmetric matrices.

 $m{AB}$  is symmetric.

Sometimes ~

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