5. Designing a Tuned Mass Damper

When the John Hancock tower was first built, it did not contain a tuned mass damper (TMD). It was only after occupants on the top floors complained of motion sickness that a consulting firm was hired to design and install a TMD to limit this undesirable swaying motion.

In this problem, we will consider a similar scenario where we have been hired to design a TMD to reduce the swaying motion of a building that is described by the non-dimensional parameters: $m_1 = 1,\ k_1 = 1,\ b_1 = 0.001$. We have been told that the TMD must have a non-dimensional mass of $m_2 = 0.05$ and the wind-forcing on the building has been measured to be of the form $F = \sin(\Omega t)$, with $\Omega \in [0.7, 1.3]$.

For a given forcing frequency, the amplitude of the building's oscillations will depend on how we choose the two parameters, k_2 and b_2, which determine the coupling between the TMD and the building. Our goal is to find the optimal values of k_2 and b_2 that will minimize the largest amplitude of the building's oscillations for forcing frequencies in the range $\Omega \in [0.7, 1.3]$.

Previously, we derived a four-dimensional, inhomogeneous system describing the coupling between the motion of a building and a TMD.

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{y}_1 \\ \dot{y}_2 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \frac{-(k_1+k_2)}{m_1} & \frac{k_2}{m_1} & \frac{-(b_1+b_2)}{m_1} & \frac{b_2}{m_1} \\ \frac{k_2}{m_2} & \frac{-k_2}{m_2} & \frac{b_2}{m_2} & \frac{-b_2}{m_2} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ y_1 \\ y_2 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \frac{F}{m_1} \\ 0 \end{pmatrix}$$

In order to find the optimal values of k_2 and b_2, we will need to solve this system and analyze its solutions for a variety of different parameters. As this would be very time consuming to do by hand, our first task is to write a MATLAB script that uses ODE45 to solve this system numerically.

Problem 1 (External resource) (1.0 / 1.0 points)

Modeling the tuned mass damper, without the damper

The script below is designed to solve the system of coupled oscillators using ODE45. Once completed, it will plot five periods of the steady state solution and a numeric value of the building's oscillation amplitude will be displayed. To complete the script, you must:

1. Enter the parameters describing the building and the TMD.
2. Define the variables **`x0`**, **`tspan`**, and **`A`**.
3. Enter an appropriate expression for the first argument of ODE45, which contains a homogeneous term and an inhomogeneous forcing term.

To test whether the script runs correctly, we will first look at the response to only one forcing frequency of $\Omega = 0.95$ in the case where the TMD has not been installed. Having no TMD installed corresponds to the parameters:

$$\begin{aligned} m_1 &= 1, & m_2 &= 0.05, \\ k_1 &= 1, & k_2 &= 0, \\ b_1 &= 0.001, & b_2 &= 0. \end{aligned}$$

Note that $k_2 = b_2 = 0$ and so there is no coupling between the TMD and the building. As we are only interested in the steady state solution, and not the transient behavior, we are free to use any initial conditions. For this test, choose

$$\mathbf{x}_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

and solve the system on the interval $t \in [0, 7000]$.

Your Script

Save Reset MATLAB Documentation (https://www.mathworks.com/help/)

```
%Define parameters
m1 = 1;
m2 = 0.05;
k1 = 1;
k2 = 0;
b1 = 0.001;
b2 = 0;
om = 0.95;

%Numerically solve DE
x0 = [0;0;0;0];
tspan = [0,7000];
```

```
A = [0,0,1,0;0,0,0,1;-(k1+k2)/m1,k2/m1,-(b1+b2)/m1,b2/m1;k2/m2,-k2/m2,b2/m2,-b2/m2];
[t,x] = ode45(@(t,x) A*x + [0;0;sin(om*t)/m1;0],tspan,x0);

%Plot steady state solution
lt = length(t);
per = 2*pi/om;
[~,idx] = min(abs(t-(t(end)-5*per)));
plot(t(idx:lt),x(idx:lt,1),'b','linewidth',3); hold on;
plot(t(idx:lt),x(idx:lt,2),'r','linewidth',3);
xlim([t(idx),t(lt)]); xlabel('$t$','interpreter','latex'); ylabel('$x(t)$','interpre
legend('Building','TMD','location','northeast'); title('Steady State Solution');
set(gca,'fontsize',25)
disp(['Amplitude of building''s oscillation: ',num2str(max(x(idx:lt,1)),4)]);

```

▶ Run Script ❓ ()

Previous Assessment: Correct

Submit ❓ ()

✅ **Correct definition of x0**

✅ **Correct definition of tspan**

✅ **Correct definition of A**

✅ **Correct numerical solution**

Play with your code

1/1 point (graded)

Let's now explore what happens when we couple the building to a TMD. Using MATLAB online, copy the script you used in problem 1 and re-run it using $k_2 = b_2 = 0.02$, while keeping all of the other parameters the same. You will now see a plot showing the coupled motion of the building and the TMD. Note that for this choice of k_2 and b_2, the addition of the TMD has caused the amplitude of the building's oscillations to decrease. Enter the value,

$$\frac{\text{amplitude with TMD}}{\text{amplitude without TMD}},$$

below.

0.8590211 ✔ **Answer:** 0.86

Solution:

$\frac{8.951}{10.42} \approx 0.86.$

Submit You have used 1 of 4 attempts

Answers are displayed within the problem

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- Play with your code assessment 4
  I got the expected amplitude in part 1 but did not get the same ratio. My amplitude in part 2 was reduce...
- function dxdydt = myodefun(t,xy) 6
- [~,idx] = min(abs(t-(t(end)-5*per))); 3
  Community TA
- Designing a Tuned Mass Damper - FIRST PART 5
  Dear staff or any other helper out there !! I am experiencing some problems regarding the time scale. Is ...
- [Staff] TMD Design simulation assessment 2

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