**MITx: 15.053x Optimization Methods in Business Analytics**

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# PART A

Consider the LP:

$$\left.\begin{array}{ll} \textbf{max} & -10x_1 - 4x_2 \\ \textbf{s.t.:} & \\ (1) & 2x_1 + x_2 \leq 4 \\ (2) & x_1 + 2x_2 \leq 4 \\ (3) & x_1 + x_2 \geq 1 \\ (4) & x_1, x_2 \geq 0 \end{array}\right\}$$

Solve geometrically and also trace the simplex procedure steps graphically. What is the optimal objective value?

-4 ✓ **Answer:** -4

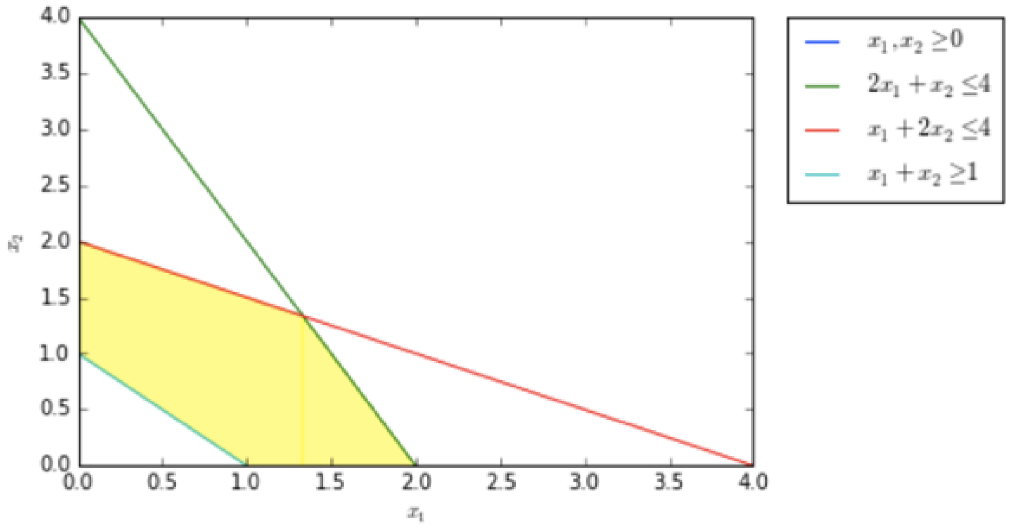
$-4$

## EXPLANATION

**Solution**

▸ Exit Survey

The extreme points are $(0,1),(1,0),(0,2),(2,0),(4/3,4/3)$ and the optimum value of $-4$ is achieved at $(0,1)$. Tracing out the simplex algorithm depends on where you start. If you happen to start from the extreme point $(2,0)$, the simplex algorithm will either trace $(1,0),(0,1)$ or $(4/3,4/3),(0,1)$ in order.



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## PART B

Starting with your graphical solution to the previous part, what is the shadow price for the third constraint?

-4

✓ **Answer:** -4

$-4$

## EXPLANATION

**Solution**

We first note that the optimal solution occurs when the third constraint holds with equality and when $x_1 = 0$.

The shadow price of the third constraint can be found by increasing the RHS of the third constraint from 1 to 2. Or it can be found by increasing the RHS of the third constraint from 1 to 1.1, and then dividing the total change in the optimal objective value by .1. If we do the latter, then the new optimal solution is obtained when $x_1 + x_2 = 1.1 \text{ and } x_1 = 0$. The solution is: $(x_1, x_2) = (0, 1.1)$ with objective value $-4.4$. Therefore, the shadow price is the new objective value minus the old objective value divided by change in RHS. That is, it is $\frac{(-4.4+4)}{0.10} = -4$.

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# PART C

Suppose that the objective function is changed to $z = -x_1 + cx_2$. Graphically determine the values of $c$ for which the solution found in part (a) remains optimal.

● $-1 \le c \le 0$ ✓

○ $c \le -1, c \ge 1$

○ $c \le 0$

○ $c \geq 1$

○ None of the above

EXPLANATION

**Solution**

The correct answer is:

$-1 \leq c \leq 0$

For $-1 \leq c \leq 0$, the extreme point (0, 1) is optimal. For $c = 0$ the optimum extreme points are (0,1) and (0,2). For $c > 0$, (0,1) is not optimal anymore. Similarly, for $c = -1$, (0,1) and (1,0) are both optimal. For $c < -1$, (0,1) isn't optimal anymore.