The BSD Conjecture (1)

- The zeta function (L-function)

$$L(\mathrm{E}, s) = \left(\prod_{\mathrm{P}} \frac{1}{1 - (\mathrm{P} + 1 - \mathrm{N_P})\,\mathrm{P}^{-s} + \mathrm{P}^{-2s}} \right) \times (\text{Bad Factors})$$

  is a generalization of the Riemann zeta function:

$$\zeta(s) = \sum_{N=1}^{\infty} \frac{1}{N^s} = 1 + \frac{1}{2^s} + \frac{1}{3^s} + \frac{1}{4^s} + \frac{1}{5^s} + \frac{1}{6^s} + \cdots$$

- There are deep results and conjectures.

The BSD Conjecture (2)

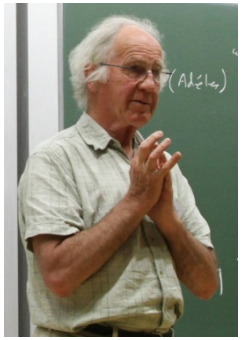
Two Fundamental Questions

- $E : Y^2 = X^3 + AX + B$.
  - How can we calculate $\mathrm{rank}\, E(\mathbb{Q})$?
  - Analytic properties of L(E,s) ?
- In 1960's, with the help of computers, Birch and Swinnerton-Dyer discovered a **mysterious connection** between them.

The BSD Conjecture (3)

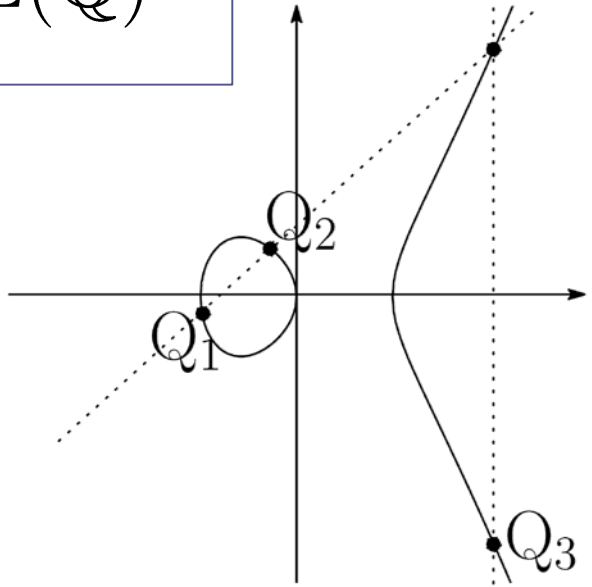
BSD Conjecture (unsolved)

$$\operatorname*{ord}_{s=1} L(\mathrm{E}, s) \;=\; \operatorname{rank} \mathrm{E}(\mathbb{Q})$$



Bryan John Birch (1931-)

Peter Swinnerton-Dyer (1927-)



https://en.wikipedia.org/wiki/Bryan_John_Birch
https://en.wikipedia.org/wiki/Peter_Swinnerton-Dyer

The BSD Conjecture (4)

- BSD Conj is known when

$$\operatorname*{ord}_{s=1} L(\mathrm{E}, s) \;\leq\; 1$$

- This striking result was proved combining
  - **Modularity** (proved in 2001)
  - **Gross-Zagier's formula** on L'(E,1)
  - Kolyvagin's **Euler Systems** (following the theory of Kummer and Iwasawa)

The BSD Conjecture (5)

- More recently, in 2014, Skinner, Urban, and Zhang proved BSD Conj when

$$\operatorname{rank} \mathrm{E}(\mathbb{Q}) \leq 1$$

under some technical assumptions.

Christopher McLean Skinner

Eric Jean-Paul Urban

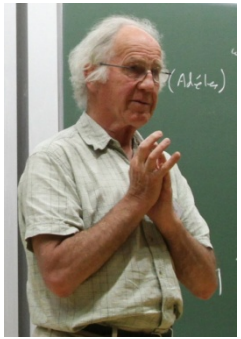
Wei Zhang

https://www.ias.edu/scholars/christopher-skinner
http://www.math.columbia.edu/~urban/
https://www.math.columbia.edu/~wzhang/

The BSD Conjecture (6)

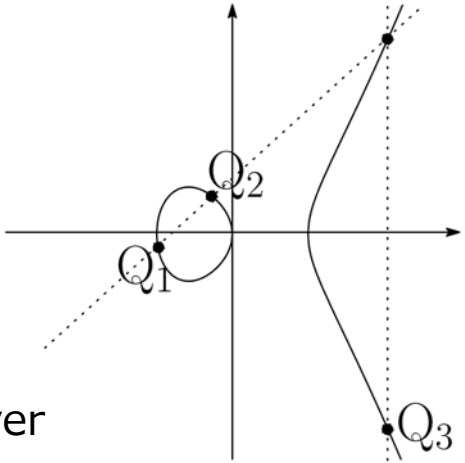
- BSD Conj is one of the most important problems in math.
- **Millennium Prize Problem**

Bernhard Riemann (1826-1866)



Bryan John Birch (1931-)

Peter Swinnerton-Dyer (1927-)



https://en.wikipedia.org/wiki/Bernhard_Riemann
http://www.claymath.org/