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1. Practice with critical points

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Recitation due Sep 13, 2021 20:30 IST Completed

Practice

Optimization problem 1(a)

2.0/2 points (graded)
Let's consider the function $f(x,y) = x^2 + x + y^2$. Let R denote the square $-10 \le x \le 10$ and $-10 \le y \le 10$. We'll find the absolute minimum of f on R. We learned in class that either the minimum occurs at a critical point or it occurs on the boundary.

Find all the critical points of f that are inside R. (Evaluate f at each critical point.)

(Enter points as ordered pairs surrounded by round parentheses: `(a,b)`.)

Critical point: (-1/2,0) ✔ **Answer:** (-1/2,0)

Function value: -1/4 ✔ **Answer:** -1/4

? INPUT HELP

Solution:

The critical points are the points (x,y) where $\nabla f(x,y) = \langle 0,0 \rangle$.

Taking partial derivatives to find the gradient, we find

$$f_x(x,y) = 2x+1 \quad (4.141)$$
$$f_y(x,y) = 2y \quad (4.142)$$

Setting both equal to zero we obtain the critical point $(-1/2, 0)$. At this critical point, the value of the function is $-1/4$.

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Answers are displayed within the problem

Optimization problem 1(b)

1/1 point (graded)
Let $f(x,y) = x^2 + x + y^2$. Let R denote the square $-10 \le x \le 10$ and $-10 \le y \le 10$.

Sketch the boundary of R on a piece of paper. Are there any points on the boundary of R where f attains a smaller value than the smallest value you found in a.)?

- [ ] yes
- [x] no

✔

Solution:

Note that the boundaries are defined by the lines:

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$$x = \pm 10, \quad -10 \le y \le 10, \qquad y = \pm 10, \quad -10 \le x \le 10$$

To find where f attains its maximum and minimum along these boundary lines, we substitute values for x and y as needed and solve a single variable calculus problem.

- Along the boundary $x = 10$ and $-10 \le y \le 10$, the function is equal to $f(10, y) = 110 + y^2$, which has critical points where $y = 0$. We also check the boundaries.

$$f(10,0) = 110 \quad (4.143)$$
$$f(10,10) = 210 \quad (4.144)$$
$$f(10,-10) = 210 \quad (4.145)$$

  Therefore the minimum along this edge is 110, which is significantly larger than the local minimum $-1/4$ which we found at the critical point.

- Along the boundary $x = -10$ and $-10 \le y \le 10$, the function is equal to $f(-10, y) = 90 + y^2$, which has critical points where $y = 0$. We also check the boundaries.

$$f(-10,0) = 90 \quad (4.146)$$
$$f(-10,10) = 190 \quad (4.147)$$
$$f(-10,-10) = 190 \quad (4.148)$$

  Therefore the minimum along this edge is 90, which is still significantly larger than the local minimum $-1/4$ which we found at the critical point.

- Along the boundary $y = 10$ and $-10 \le x \le 10$, the function is equal to $f(x, 10) = x^2 + x + 100$, which has critical points when $x = -1/2$. We also check the boundaries.

$$f(-1/2,10) = 99.75 \quad (4.149)$$
$$f(10,10) = 210 \quad (4.150)$$
$$f(-10,10) = 190 \quad (4.151)$$

  Therefore the minimum along this edge is 99.75, which is still significantly larger than the local minimum $-1/4$ which we found at the critical point.

- Along the boundary $y = -10$ and $-10 \le x \le 10$, the function is equal to $f(x, -10) = x^2 + x + 100$, which has critical points when $x = -1/2$. We also check the boundaries.

$$f(-1/2,-10) = 99.75 \quad (4.152)$$
$$f(10,-10) = 210 \quad (4.153)$$
$$f(-10,-10) = 190 \quad (4.154)$$

  Therefore the minimum along this edge is 99.75, which is still significantly larger than the local minimum $-1/4$ which we found at the critical point.

Therefore the smallest value of the function along the boundary is 90, and occurs at the point $(-10, 0)$. This is much larger than the minimum value inside the region.

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Optimization problem 1(c)

1/1 point (graded)

As above, let $f(x, y) = x^2 + x + y^2$. Let R denote the square $-10 \le x \le 10$ and

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What is the absolute minimum value of f on R?

-1/4

Answer: -1/4

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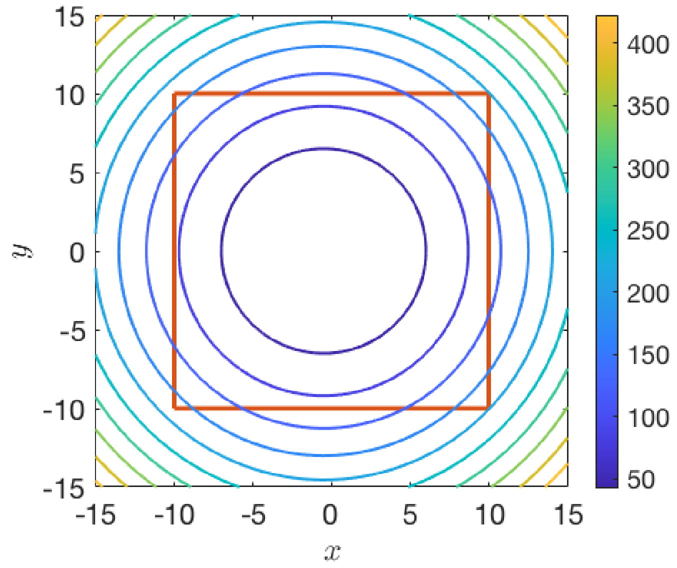
Solution:

The minimum value of the function occurs on the interior of the region and is $-1/4$.

Here is a graphic of the level curves of this function.



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Optimization problem 2

1/1 point (graded)

What is the absolute minimum value of $f(x, y) = x^4 + y^4 - xy$ on the region R, where R is the square region $-20 \leq x \leq 20$ and $-20 \leq y \leq 20$?

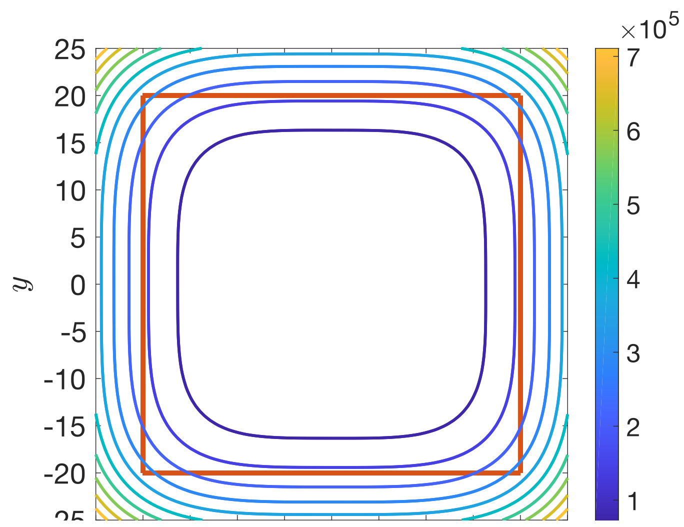
-1/8

Answer: -1/8

? INPUT HELP

Solution:

Again the absolute minimum occurs at the critical point within the region.



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-25 -20 -15 -10 -5 0 5 10 15 20 25

x

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1. Practice with critical points

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- [STAFF] wording improvement: "absolute minimum" → global minimum 2
- Helpful resource to supplement lectures 2
  Can't speak for the video [here][1], but section 2.10 in the textbook which it corresponds to (Worldwide Multivariable Calculus by Da...
- STAFF - html does not display 3
  html does not display

Previous Next

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