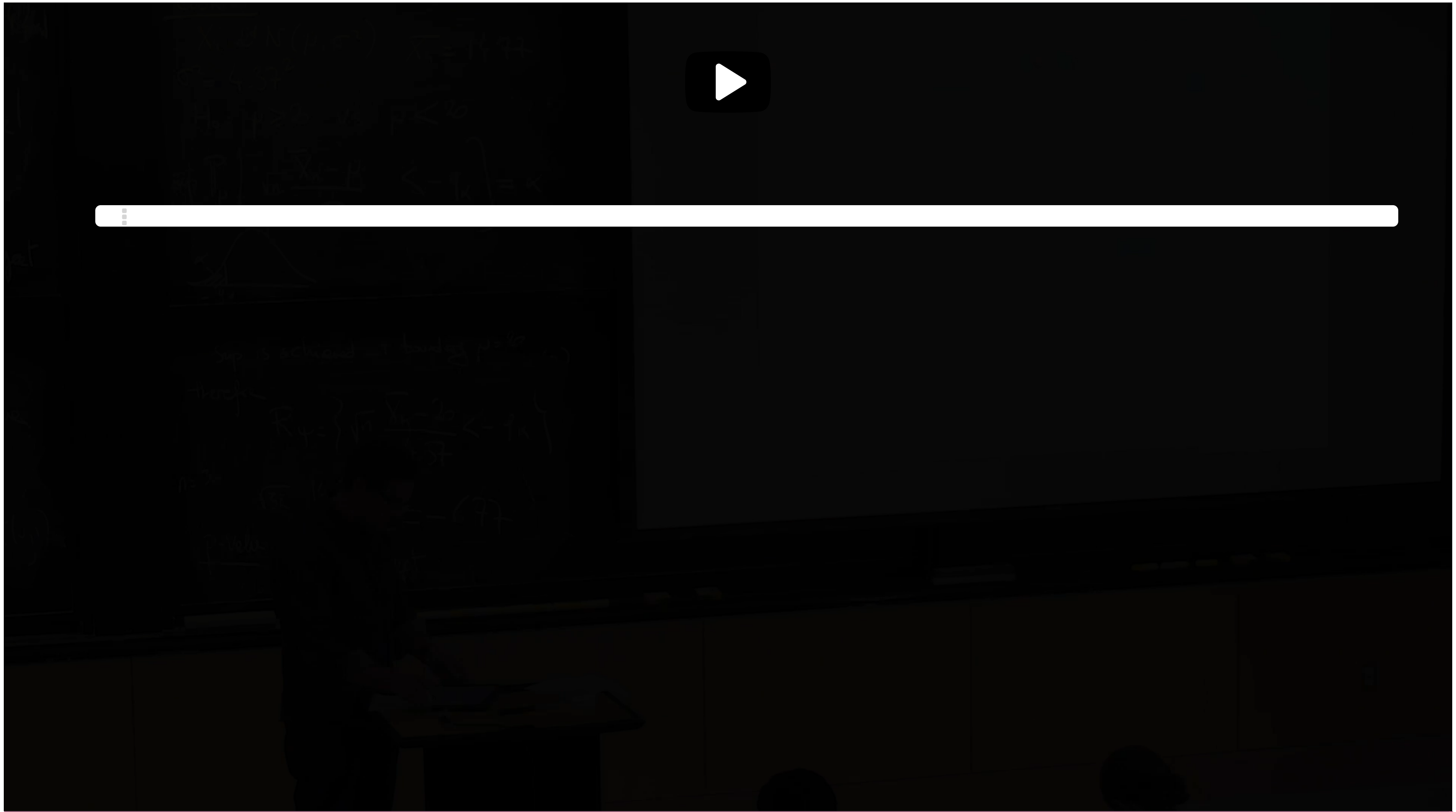
9. Are there at Least 20 chocolate Chips on a Cookie?

Worked Example 2: the P-value of a One-Sided Test



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Computing p-values II: Counting Chocolate Chips Examples

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Students are asked to count the number of chocolate chips in 15 cookies for a class activity. They found that the cookies on **average** had **16.5** chocolate chips with a **standard deviation** of **5.2** chocolate chips. The packaging for these cookies claims that there are at least 20 chocolate chips per cookie.

One student thinks this number is unreasonably high since the average they found is significantly lower. Another student claims the difference might be due to chance.

As a statistician, you decide to approach this question with the tools of hypothesis testing. You make the following modeling assumptions on the cookies:

- $X_1, \ldots, X_n$ are iid Gaussian random variables,
- $\sqrt{\mathrm{Var}\left(X_1\right)} = 5.2$, and
- $\mathbb{E}\left[X_1\right] = \mu$ is an unknown parameter.

You define the hypotheses as follows

$$H_0 : \mu \geq 20, \quad H_1 : \mu < 20.$$

and specify the test

$$\psi_n := \mathbf{1}\left(\sqrt{n}\frac{\overline{X}_n - 20}{5.2} < -q_\eta\right),$$

where q_η is the $1-\eta$ quantile of a standard Gaussian. (Note that if $Z \sim N\left(0,1\right)$, then $P\left(Z < -q_\eta\right) = P\left(Z > q_\eta\right) = \eta$. Also, since this is a **one-sided test**, we will not use an absolute value to define our test statistic.)

For this a one-sided test, the p-value is still defined to be the smallest level at which ψ_n rejects H_0 on a given data set.

Hint: If $\mu = 20$ and $X_1, \ldots, X_n \overset{iid}{\sim} N\left(\mu, 5.2^2\right)$, the given test statistic is a standard Gaussian:

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$$\sqrt{n}\left(\frac{\overline{X}_n - 20}{5.2}\right) \sim N(0,1).$$

The above holds for *any* value of n, not just asymptotically.

For this test and the observed sample mean $\overline{X}_n = 16.5$, what is the associated p-value? (You are encouraged to use computational tools or a table.)

0.0045694265537349520 ✔ **Answer:** 0.00466

Solution:

For notational convenience, let $\mathbf{P}_\mu$ denote the distribution $N(\mu, 5.2^2)$. Recall that the level α is a bound on the type 1 error. *i.e.*, α is a level of ψ if

$$\alpha_\psi(\mu) = \mathbf{P}_\mu(T_n < -q_\eta) \le \alpha \quad \text{for all } \mu \ge 20,$$

where

$$T_n \;=\; \sqrt{n}\frac{\overline{X}_n - 20}{5.2}.$$

Observe that if $X_1, \ldots, X_n \sim P_\mu$ and $\mu > 20$, then

$$\begin{aligned} T_n &= \sqrt{n}\frac{\overline{X}_n - \mu + (\mu - 20)}{5.2} \\ &\sim Z + \frac{\sqrt{n}}{5.2}(\mu - 20). \end{aligned}$$

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In particular, the distribution of T_n is normal with mean shifted to the **right** of $\mathcal{N}(0,1)$. Comparing the tails visually (as in previous problems) shows the inequality

$$\mathbf{P}_\mu\left(T_n < -q_\eta\right) < \mathbf{P}_{20}\left(T_n < -q_\eta\right) = \eta.$$

Therefore, $\mu = 20$ is the 'worst-case' possibility under the null, and ψ is a test of level η. To compute the p-value, we just need to find the smallest possible η such that ψ rejects H_0. Hence, we set

$$q_\eta = \sqrt{15}\left(\frac{16.5-20}{5.2}\right) \approx -2.6068$$

and compute

$$P\left(Z < -\sqrt{15}\left(\frac{16.5-20}{5.2}\right)\right) = P\left(Z > \sqrt{15}\left(\frac{16.5-20}{5.2}\right)\right) \approx 0.0047$$

where $Z \sim N(0,1)$. This gives a p-value of ≈ 0.0047 or roughly 0.5 $\%$.

Remark: A p-value less than 1 $\%$ indicates that observing a sample mean smaller than 16.5 is a less than 1 $\%$ chance event if $\mu = 20$ (which is the worst-case scenario under H_0). This indicates a fairly rare event, so it seems reasonable, given our modeling assumptions, to doubt the second student's claim that the low number of chocolate chips was due to chance.

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Answers are displayed within the problem

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