1. Composite Linearization

Recitation due Sep 15, 2021 20:30 IST

Synthesize

Linearization

1/1 point (graded)

Suppose that the variables A and B depend on the variables x and y as follows.

$$A = xy \tag{5.146}$$

$$B = x^2 - y^2 \tag{5.147}$$

In lecture, we computed the linearization at the point $(1, 2)$.

Compute the linearization of the the transformation $x, y \implies A, B$ at the point $(1, 1)$.

(Enter a matrix using notation such as `[[a,b],[c,d]]`.)

[[1,1],[2,-2]] ✔ **Answer:** [[1,1],[2,-2]]

Solution:

$$\begin{pmatrix} 1 & 1 \\ 2 & -2 \end{pmatrix} \tag{5.148}$$

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Now suppose that we cannot control x, y directly, but we can control the polar coordinates, that is, the values of r, θ. Recall that

$$x = r\cos\theta \tag{5.149}$$

$$y = r\sin\theta \tag{5.150}$$

There are two ways of computing the linearization of the composite transformation $r, \theta \implies A, B$. First, we will do it directly, and second, we will use matrix multiplication.

New transformation

2/2 points (graded)

Find the formulas for A and B in terms of r and θ.

$A =$ r^2*sin(2*theta)/2 ✔ **Answer:** r^2*sin(theta)*cos(theta)

$B =$ r^2*cos(2*theta) ✔ **Answer:** r^2*cos(theta)^2 - r^2*sin(theta)^2

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Solution:

$A = r^2 \sin\theta\cos\theta$

$B = r^2\cos^2\theta - r^2\sin^2\theta$

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New linearization

1/1 point (graded)
Using the formulas you found, compute the linearization of the relationship $r, \theta \implies A, B$ at the point $(r,\theta) = (\sqrt{2}, \pi/4)$. Enter using exact expressions, or round to four decimal places.

(Enter a matrix using notation such as `[[a,b],[c,d]]`.)

[[sqrt(2),0],[0,-4]] **Answer:** [[sqrt(2), 0],[0, -4]]

Solution:

The matrix of partial derivatives is:

$$\begin{pmatrix} 2r\cos(\theta)\sin(\theta) & r^2\cos^2(\theta) - r^2\sin^2(\theta) \\ 2r\cos^2(\theta) - 2r\sin^2(\theta) & -4r^2\cos(\theta)\sin(\theta) \end{pmatrix} \tag{5.151}$$

Evaluating at the point $(r,\theta) = (\sqrt{2}, \pi/4)$ we obtain,

$$\begin{pmatrix} \sqrt{2} & 0 \\ 0 & -4 \end{pmatrix} \tag{5.152}$$

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Linearize twice

1/1 point (graded)
Now we will use matrix multiplication instead. First compute the linearization of the the transformation $r, \theta \implies x, y$ at the point $(r,\theta) = (\sqrt{2}, \pi/4)$. Enter using exact expressions, or round to four decimal places.

(Enter a matrix using notation such as `[[a,b],[c,d]]`.)

[[1/sqrt(2),-1],[1/sqrt(2),1]] **Answer:** [[1/sqrt(2), -1],[1/sqrt(2), 1]]

Solution:

Evaluating the matrix of partial derivatives at $(r,\theta) = (\sqrt{2}, \pi/4)$ we obtain:

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$$\begin{pmatrix} \frac{1}{\sqrt{2}} & -1 \\ \frac{1}{\sqrt{2}} & 1 \end{pmatrix}. \tag{5.153}$$

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Two Transforms

1/1 point (graded)

Let M_1 be the linearization of the transformation $r, \theta \implies x, y$ at the point $(r, \theta) = (\sqrt{2}, \pi/4)$. Let M_2 be the linearization of the transformation $x, y \implies A, B$ at the point $(x, y) = (1, 1)$. Which of the following is the linearization of the transformation $r, \theta \implies A, B$ at the point $(r, \theta) = (\sqrt{2}, \pi/4)$?

- ○ $M_1 M_2$
- ● $M_2 M_1$
- ○ Neither

✔

Solution:

It is $M_2 M_1$, because we first apply M_1 to the input ($r, \theta \to x, y$), then apply M_2 ($x, y \to A, B$), and the result is the composition of both transformations. (It is possible to justify this reasoning rigorously; this is known as the multivariate chain rule.)

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1. Composite Linearization

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