9.4.2 Subspaces

Week 9 due Dec 9, 2023 18:12 IST Completed

9.4.2 Subspaces

Notice: Second video at end of this unit

Video

What about the empty set?

Is it a subspace of Rn?

Well, go have a look.

Go think about this.

And the answer is no.

Why?

Because 0 is not an element of the empty set.

The empty set has nothing in it.

OK, well what about the set that only has the zero vector in it?

And the answer, go think about that one.

OK, you're back.

And the answer this time is yes!

Why because certainly 0 is an element of S, if we call this set S.

And if x and y are an element of S, and then it is the case that x plus y

is equal to 0 plus 0, because x and y

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Reading Assignment

0 points possible (ungraded)

Read Unit 9.4.2 of the notes. [LINK]

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Question on Homework 9.4.2.1 (5)

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Hi all, For the fifth subquestion, I find it a bit hard to understand why x + y is still in S? For example, how to show that (x_0 + y_0)*(x_0 + y+0 - x_...

Homework 9.4.2.1

5/5 points (graded)

Which of the following subsets of $\mathbb{R}^3$ are subspaces of $\mathbb{R}^3$?

1. The plane of vectors $x = \begin{pmatrix} \chi_0 \\ \chi_1 \\ \chi_2 \end{pmatrix}$ such that $\chi_0 = 0$. In other words, the set of all vectors $\left\{ x \middle| x = \begin{pmatrix} 0 \\ \chi_1 \\ \chi_2 \end{pmatrix} \right\}$.

   TRUE ✔ Answer: TRUE

   - $x + y$ is in the set: If x and y are in the set, then $x = \begin{pmatrix} 0 \\ \chi_1 \\ \chi_2 \end{pmatrix}$ and $y = \begin{pmatrix} 0 \\ \psi_1 \\ \psi_2 \end{pmatrix}$. But then $x + y = \begin{pmatrix} 0 \\ \chi_1 + \psi_1 \\ \chi_2 + \psi_2 \end{pmatrix}$ is in the set.
   - αx is in the set: If x is in the set and $\alpha \in \mathbb{R}$, then $\alpha x = \begin{pmatrix} 0 \\ \alpha\chi_1 \\ \alpha\chi_2 \end{pmatrix}$ is in the set.

2. Similarly, the plane of vectors x with $\chi_0 = 1$: $\left\{ x \middle| x = \begin{pmatrix} 1 \\ \chi_1 \\ \chi_2 \end{pmatrix} \right\}$.

   FALSE ✔ Answer: FALSE

   No. 0 is not in the set and hence this cannot be a subspace.

3. $\left\{ x \middle| x = \begin{pmatrix} \chi_0 \\ \chi_1 \\ \chi_2 \end{pmatrix} \wedge \chi_0\chi_1 = 0 \right\}$. (Recall, $\wedge$ is the logical "and" operator.)

   FALSE ✔ Answer: FALSE

   $x + y$ is not in the set if $x = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ and $y = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$.

4. $\left\{ x \middle| x = \alpha \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + \beta \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix} \text{ where } \alpha, \beta \in \mathbb{R} \right\}$.

   TRUE ✔ Answer: TRUE

   Again, this is a matter of showing that if x and y are in the set and $\alpha \in \mathbb{R}$ then $x + y$ and αx are in the set.

5. $\left\{ x \middle| x = \begin{pmatrix} \chi_0 \\ \chi_1 \\ \chi_2 \end{pmatrix} \wedge \chi_0 - \chi_1 + 3\chi_2 = 0 \right\}$.

   TRUE ✔ Answer: TRUE

   Once again, this is a matter of showing that if x and y are in the set and $\alpha \in \mathbb{R}$ then $x + y$ and αx are in the set.

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Homework 9.4.2.2

1/1 point (graded)
The empty set, $\emptyset$, is a subspace of $\mathbb{R}^n$.

FALSE ✔ Answer: FALSE

0 (the zero vector) is not an element of $\emptyset$.

Notice that the other two conditions **are** met: "If $u, w \in \emptyset$ then $u + w \in \emptyset$" is *true* because $\emptyset$ is empty. Similarly "If $\alpha \in \mathbb{R}$ and $v \in \emptyset$ then $\alpha v \in \emptyset$" is *true* because $\emptyset$ is empty. This is kind of subtle.

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Homework 9.4.2.3

1/1 point (graded)
The set $\{0\}$ where 0 is a vector of size n is a subspace of $\mathbb{R}^n$.

TRUE ✔ Answer: TRUE

- 0 (the zero vector) is an element of $\{0\}$.
- If $u, w \in \{0\}$ then $(u + w) \in \{0\}$: this is *true* because if $u, w \in \{0\}$ then $v = w = 0$ and $v + w = 0 + 0 = 0$ is an element of $\{0\}$.
- If $\alpha \in \mathbb{R}$ and $v \in \{0\}$ then $\alpha v \in \{0\}$: this is *true* because if $v \in \{0\}$ then $v = 0$ and for any $\alpha \in \mathbb{R}$ it is the case that $\alpha v = \alpha 0 = 0$ is an element of $\{0\}$.

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Homework 9.4.2.4

1/1 point (graded)
The set $S \subset \mathbb{R}^n$ described by

$$\{x \mid \|x\|_2 < 1\}$$

is a subspace of $\mathbb{R}^n$.

(Recall that $\|x\|_2$ is the Euclidean length of vector x so this describes all elements with length less than or equal to one.)

FALSE ✔ Answer: FALSE

Pick any vector $v \in S$ such that $v \neq 0$. Let $\alpha > 1/\|v\|_2$. Then

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$$\|\alpha v\|_2 = \alpha \|v\|_2 > (1/\|v\|_2)\,\|v\|_2 = 1$$

and hence $\alpha v \notin S$.

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Homework 9.4.2.5

1/1 point (graded)
The set $S \subset \mathbb{R}^n$ described by

$$\left\{ \begin{pmatrix} \nu_0 \\ 0 \\ \vdots \\ 0 \end{pmatrix} \middle| \, \nu_0 \in \mathbb{R} \right\}$$

is a subspace of $\mathbb{R}^n$.

TRUE ✔ Answer: TRUE

- $0 \in S$: (pick $\nu_0 = 0$).
- If $u, w \in S$ then $(u + w) \in S$: Pick $u, w \in S$. Then for some ν_0 and some ω_0

$$v = \begin{pmatrix} \nu_0 \\ 0 \\ \vdots \\ 0 \end{pmatrix} \quad \textbf{and} \quad w = \begin{pmatrix} \omega_0 \\ 0 \\ \vdots \\ 0 \end{pmatrix}.$$

But then $v + w = \begin{pmatrix} \nu_0 \\ 0 \\ \vdots \\ 0 \end{pmatrix} + \begin{pmatrix} \omega_0 \\ 0 \\ \vdots \\ 0 \end{pmatrix} = \begin{pmatrix} \nu_0 + \omega_0 \\ 0 \\ \vdots \\ 0 \end{pmatrix}$, which is also in S.

- If $\alpha \in \mathbb{R}$ and $v \in S$ then $\alpha v \in S$: Pick $\alpha \in \mathbb{R}$ and $v \in S$. Then for some ν_0

$$v = \begin{pmatrix} \nu_0 \\ 0 \\ \vdots \\ 0 \end{pmatrix}.$$

But then $\alpha v = \begin{pmatrix} \alpha\nu_0 \\ 0 \\ \vdots \\ 0 \end{pmatrix}$, which is also in S.

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Answers are displayed within the problem

Homework 9.4.2.6

1/1 point (graded)

The set $S \subset \mathbb{R}^n$ described by

$$\{\nu e_j \mid \nu \in \mathbb{R}\},$$

where j is fixed and e_j is a unit basis vector, is a subspace.

TRUE ✔ Answer: TRUE

red True

- $0 \in S$: (pick $\nu = 0$).
- If $u, w \in S$ then $(u + w) \in S$: Pick $u, w \in S$. Then for some ν and some ω, $v = \nu e_j$ and $w = \omega e_j$. But then $v + w = \nu e_j + \omega e_j = (\nu + \omega)\, e_j$, which is also in S.
- If $\alpha \in \mathbb{R}$ and $v \in S$ then $\alpha v \in S$: Pick $\alpha \in \mathbb{R}$ and $v \in S$. Then for some ν, $v = \nu e_j$. But then $\alpha v = \alpha\,(\nu e_j) = (\alpha\nu)\, e_j$, which is also in S.

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Homework 9.4.2.7

1/1 point (graded)

The set $S \subset \mathbb{R}^n$ described by

$$\{\chi a \mid \chi \in \mathbb{R}\},$$

where $a \in \mathbb{R}^n$, is a subspace.

TRUE ✔ Answer: TRUE

- $0 \in S$: (pick $\chi = 0$).
- If $v, w \in S$ then $(u + w) \in S$: Pick $v, w \in S$. Then for some ν and some ω, $v = \nu a$ and $w = \omega a$. But then $v + w = \nu a + \omega a = (\nu + \omega)\, a$, which is also in S.
- If $\alpha \in \mathbb{R}$ and $v \in S$ then $\alpha v \in S$: Pick $\alpha \in \mathbb{R}$ and $v \in S$. Then for some ν, $v = \nu a$. But then $\alpha v = \alpha\,(\nu a) = (\alpha\nu)\, a$, which is also in S.

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Homework 9.4.2.8

1/1 point (graded)

The set $S \subset \mathbb{R}^n$ described by

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$$\{\chi_0 a_0 + \chi_1 a_1 \mid \chi_0, \chi_1 \in \mathbb{R}\},$$

where $a_0, a_1 \in \mathbb{R}^n$, is a subspace.

TRUE ✔ Answer: TRUE

- $0 \in S$: (pick $\chi_0 = \chi_1 = 0$).
- If $v, w \in S$ then $(u + w) \in S$: Pick $v, w \in S$. Then for some $\nu_0, \nu_1, \omega_0, \omega_1 \in \mathbb{R}$, $v = \nu_0 a_0 + \nu_1 a_1$ and $w = \omega_0 a_0 + \omega_1 a_1$. But then $v + w = \nu_0 a_0 + \nu_1 a_1 + \omega_0 a_0 + \omega_1 a_1 = (\nu_0 + \omega_0) a_0 + (\nu_1 + \omega_1) a_1$, which is also in S.
- If $\alpha \in \mathbb{R}$ and $v \in S$ then $\alpha v \in S$: Pick $\alpha \in \mathbb{R}$ and $v \in S$. Then for some $\nu_0, \nu_1 \in \mathbb{R}$, $v = \nu_0 a_0 + \nu_1 a_1$. But then $\alpha v = \alpha(\nu_0 a_0 + \nu_1 a_1) = (\alpha\nu_0) a_0 + (\alpha\nu_1) a_1$, which is also in S.

What this means is that the set of all linear combinations of two vectors is a subspace.

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Homework 9.4.2.9

1/1 point (graded)
The set $S \subset \mathbb{R}^n$ described by

$$\left\{ \left(\begin{array}{c|c} a_0 & a_1 \end{array} \right) \begin{pmatrix} \chi_0 \\ \chi_1 \end{pmatrix} \middle| \chi_0, \chi_1 \in \mathbb{R} \right\},$$

where $a_0, a_1 \in \mathbb{R}^n$, is a subspace.

TRUE ✔ Answer: TRUE

- $0 \in S$: (pick $\chi_0 = \chi_1 = 0$).
- If $v, w \in S$ then $(v + w) \in S$: Pick $v, w \in S$. Then for some $\nu_0, \nu_1, \omega_0 \omega_1 \in \mathbb{R}$,

$$v = \left(\begin{array}{c|c} a_0 & a_1 \end{array} \right) \begin{pmatrix} \nu_0 \\ \nu_1 \end{pmatrix} \quad \textbf{and} \quad w = \left(\begin{array}{c|c} a_0 & a_1 \end{array} \right) \begin{pmatrix} \omega_0 \\ \omega_1 \end{pmatrix}.$$

  But then

$$\begin{array}{rcl} \begin{array}{l} v \\ +w \end{array} & = & \left(\begin{array}{c|c} a_0 & a_1 \end{array} \right) \begin{pmatrix} \nu_0 \\ \nu_1 \end{pmatrix} + \left(\begin{array}{c|c} a_0 & a_1 \end{array} \right) \begin{pmatrix} \omega_0 \\ \omega_1 \end{pmatrix} \\ & = & \left(\begin{array}{c|c} a_0 & a_1 \end{array} \right) \left(\begin{pmatrix} \nu_0 \\ \nu_1 \end{pmatrix} + \begin{pmatrix} \omega_0 \\ \omega_1 \end{pmatrix} \right) \\ & = & \left(\begin{array}{c|c} a_0 & a_1 \end{array} \right) \begin{pmatrix} \nu_0 + \omega_0 \\ \nu_1 + \omega_1 \end{pmatrix}, \end{array}$$

  which is also in S.
- If $\alpha \in \mathbb{R}$ and $v \in S$ then $\alpha v \in S$: Pick $\alpha \in \mathbb{R}$ and $v \in S$. Then for some $\nu_0, \nu_1 \in \mathbb{R}$, $v = \left(\begin{array}{c|c} a_0 & a_1 \end{array} \right) \begin{pmatrix} \upsilon_0 \\ \upsilon_1 \end{pmatrix}$. But then

$$\alpha v = \alpha \left(\begin{array}{c|c} a_0 & a_1 \end{array} \right) \begin{pmatrix} \upsilon_0 \\ \upsilon_1 \end{pmatrix} = \left(\begin{array}{c|c} a_0 & a_1 \end{array} \right) \alpha \begin{pmatrix} \upsilon_0 \\ \upsilon_1 \end{pmatrix} = \left(\begin{array}{c|c} a_0 & a_1 \end{array} \right) \begin{pmatrix} \alpha\upsilon_0 \\ \alpha\upsilon_1 \end{pmatrix},$$

Calculator

which is also in S.

What this means is that the set of all linear combinations of two vectors is a subspace, except expressed as a matrix-vector multiplication. In other words, this exercise is simply a restatement of the previous exercise.

We are going somewhere with this!

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Homework 9.4.2.10

1/1 point (graded)

The set $S \subset \mathbb{R}^n$ described by

$$\left\{ Ax \mid x \in \mathbb{R}^2 \right\},$$

where $A \in \mathbb{R}^{n \times 2}$, is a subspace.

TRUE ✔ Answer: TRUE

True/False

Answer: True

- $0 \in S$: (pick $x = 0$).
- Now here we need to use different letters for x and y, since x is already being used. If $v, w \in S$ then $(v + w) \in S$: Pick $v, w \in S$. Then for some $x, y \in \mathbb{R}^2$, $v = Ax$ and $w = Ay$. But then $v + w = Ax + Ay = A(x + y)$, which is also in S.
- If $\alpha \in \mathbb{R}$ and $v \in S$ then $\alpha v \in S$: Pick $\alpha \in \mathbb{R}$ and $v \in S$. Then for some $x \in \mathbb{R}^2$, $v = Ax$. But then $\alpha v = \alpha(Ax) = A(\alpha x)$, which is also in S since $\alpha x \in \mathbb{R}^2$.

What this means is that the set of all linear combinations of two vectors is a subspace, except expressed even more explicitly as a matrix-vector multiplication. In other words, this exercise is simply a restatement of the previous two exercises. Now we are getting somewhere!

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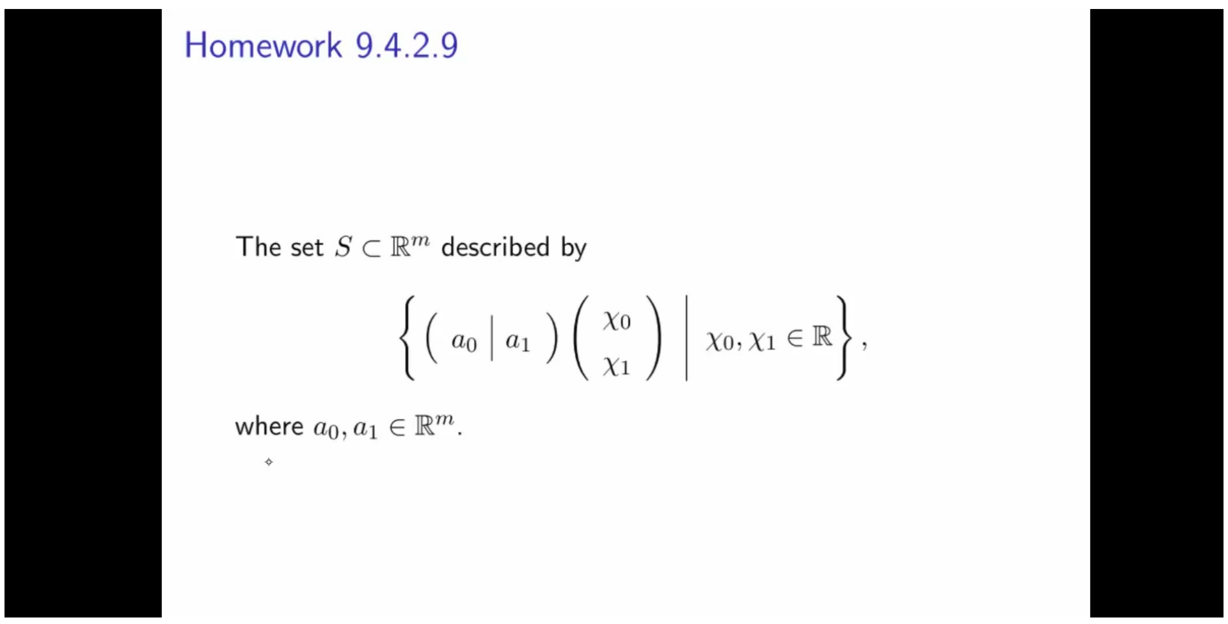
Video



What does this here say?

Given two vectors a0 and a1, look at the set

of all vectors that are linear combinations of those two vectors.

And it turns out that that's a subspace.

And you can prove this in a very similar way.

As the last one, I'll just let you look at the proof that

came with the homework.

This here is just a rewording of that.

Notice that this here says the set of all vectors, Chi 0 times a0

plus Chi 1 times a1, because this is just

Calculator

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2:15 / 3:19 2.0x

viewing these vectors as the columns of a matrix,

multiplying times any vector of size 2 with components Chi 0

and Chi 1, which is exactly how this

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