Problem 4

What is **the number of prime numbers** less than 1,000,000 with last digit 3 (such as 3,13,23,43,⋯).

Choose the closest number.

(Hint: **log(1000000) = 13.8155**)

(a) 10,000
(b) 19,000
(c) 39,000
(d) 47,000

Problem 4

Theorem (de la Vallée-Poussin, 1896)

A, B ≥ 1 **relatively prime**

$\pi_{A,B}$**(N)** = the number of prime numbers P of the form **P = A + K B** with **P ≤ N**

$$\lim_{N\to\infty} \frac{\pi_{A,B}(N)}{N/\log(N)} = \frac{1}{\phi(B)}$$

Charles Jean
de la Vallée-Poussin
(1866-1962)

https://en.wikipedia.org/wiki/Charles_Jean_de_la_Vall%C3%A9e-Poussin

Problem 4

- Use **de la Vallée-Poussin's Thm** to estimate # of prime numbers with last digit 3.

A=3, B=10, $\phi(10) = 4$

The number of prime numbers less than N=1,000,000 with last digit 3 is **approximately**

$$(N/\log(N)) \times (1/\phi(10)) \fallingdotseq 18{,}096.$$

Answer (b) 19,000

Charles Jean
de la Vallée-Poussin
(1866-1962)

Problem 4

N = 1,000,000

19,665 prime numbers ≤ N, with last digit 3

- (N/log(N)) × (1/ϕ(10)) ≒ **18,096**
- li(N) × (1/ϕ(10)) ≒ **19,657**

$$\mathrm{li}(x) = \int_0^x \frac{dt}{\log(t)}$$

Riemann Hypothesis!

Bernhard Riemann
(1826-1866)

https://en.wikipedia.org/wiki/Bernhard_Riemann