14. Symmetric matrices and their diagonalization

Note that because the eigenvalues of a real symmetric matrix are always complete, a real symmetric matrix is always diagonalizable. More it turns out is true for symmetric matrices.

Fun Fact 1 If $\mathbf{A}$ is a symmetric, real $n \times n$ matrix, its eigenvalues are real.

Two vectors $\mathbf{v}$ and $\mathbf{w}$ are **orthogonal** if their dot product is zero. The dot product of the two vectors $\mathbf{v}$ and $\mathbf{w}$ can be written as $\mathbf{v}^T\mathbf{w}$ using ordinary matrix multiplication.

Fun Fact 2 If $\mathbf{A}$ is a symmetric, real $n \times n$ matrix, the eigenvectors of any two distinct eigenvalues are orthogonal.

In particular, this means that we can find a basis for the eigenspaces that are all mutually orthogonal. This is because any repeated eigenvector has a complete eigenspace, and there is an algorithm, called the Gram–Schmidt algorithm, that allows us to find an orthogonal set of vectors.

Why do we care about orthogonal vectors?

Let $\mathbf{v}_1, \ldots, \mathbf{v}_n$ be a collection of eigenvectors of $\mathbf{A}$ that are pairwise orthogonal, that is

$$\mathbf{v}_i^T\mathbf{v}_j = 0 \qquad \text{if } i \neq j.$$

Normalize these vectors so that each of them has length one, that is

$$\mathbf{v}_i^T \mathbf{v}_i = 1.$$

Once the vectors are normalized in this manner, they are said to be **orthonormal** .

Let $\mathbf{S}$ be the matrix whose columns are these orthonormal vectors. Then we say that $\mathbf{S}$ is an **orthogonal matrix** , and has the special property that $\mathbf{S}^{-1} = \mathbf{S}^T$. This is convenient because computing inverses is often computationally intensive, but the transpose is simple.

In particular, the diagonalization of the matrix $\mathbf{A}$ can be expressed as $\mathbf{A} = \mathbf{S}\mathbf{D}\mathbf{S}^T$.

Remark 14.1 If you use the MATLAB command `[S,D] = eig(A)` to find the eigenvectors of a matrix $\mathbf{A}$, the eigenvectors it finds will be unit length, and orthogonal if possible.

---

14. Symmetric matrices and their diagonalization

Hide Discussion

Topic: Unit 2: Linear Algebra, Part 2 / 14. Symmetric matrices and their diagonalization

Add a Post

Show all posts ▼ by recent activity ▼

There are no posts in this topic yet.

✖

Learn About Verified Certificates