



## MITx: 6.041x Introduction to Probability - The Science of Uncertainty



Bookmarks

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Bookmark

## Problem 1: Defective coin

(3/3 points)

A defective coin minting machine produces coins whose probability of Heads is a random variable  $Q$  with PDF

$$f_Q(q) = \begin{cases} 3q^2, & \text{if } q \in [0, 1], \\ 0, & \text{otherwise.} \end{cases}$$

A coin produced by this machine is tossed repeatedly, with successive tosses assumed to be independent. Let  $A$  be the event that the first toss of this coin results in Heads, and let  $B$  be the event that the second toss of this coin results in Heads.

1.

 $P(A) =$ 

3/4



Answer: 0.75

(Your answer should be a number.)


2. Find the conditional PDF of  $Q$  given event  $A$ . Express your answer in terms of  $q$  using standard notation .

- ▶ Unit 6: Further topics on random variables


## ▼ Unit 7: Bayesian inference

### Unit overview


#### Lec. 14: Introduction to Bayesian inference

Exercises 14 due Apr 06, 2016 at 23:59 UTC 


#### Lec. 15: Linear models with normal noise

Exercises 15 due Apr 06, 2016 at 23:59 UTC 


#### Problem Set 7a

Problem Set 7a due Apr 06, 2016 at 23:59 UTC 

#### Lec. 16: Least mean squares (LMS) estimation

Exercises 16 due Apr 13, 2016 at 23:59 UTC 

#### Lec. 17: Linear least mean squares (LLMS) estimation

Exercises 17 due Apr 13, 2016 at 23:59 UTC 

#### Problem Set 7b

For  $0 \leq q \leq 1$ ,  $f_{Q|A}(q) =$

✓ Answer:  $4 \cdot q^3$

3.

$\mathbf{P}(B | A) =$

✓ Answer: 0.8

(Your answer should be a number.)

Answer:

1. To calculate  $\mathbf{P}(A)$ , we use the continuous version of the total probability theorem:

$$\mathbf{P}(A) = \int_0^1 \mathbf{P}(A | Q = q) f_Q(q) dq = \int_0^1 q \cdot (3q^2) dq = \left[ \frac{3}{4} q^4 \right]_0^1 = \frac{3}{4}.$$

2. Using Bayes' rule,

$$\begin{aligned} f_{Q|A}(q) &= \frac{\mathbf{P}(A | Q = q) f_Q(q)}{\mathbf{P}(A)} \\ &= \begin{cases} \frac{q \cdot (3q^2)}{3/4}, & \text{if } 0 \leq q \leq 1, \\ 0, & \text{otherwise,} \end{cases} \\ &= \begin{cases} 4q^3, & \text{if } 0 \leq q \leq 1, \\ 0, & \text{otherwise.} \end{cases} \end{aligned}$$

Problem Set 7b due Apr 13,  
2016 at 23:59 UTC



Solved problems

Additional theoretical  
material

Unit summary

- ▶ Exam 2
- ▶ Unit 8: Limit theorems and classical statistics
- ▶ Unit 9: Bernoulli and Poisson processes
- ▶ Unit 10: Markov chains
- ▶ Exit Survey
- ▶ Final Exam

3.

$$\begin{aligned}
 \mathbf{P}(B \mid A) &= \int_0^1 \mathbf{P}(B \mid A, Q = q) f_{Q|A}(q) dq \\
 &= \int_0^1 \mathbf{P}(B \mid Q = q) f_{Q|A}(q) dq \\
 &= \int_0^1 q(4q^3) dq \\
 &= 4/5.
 \end{aligned}$$

The second equality holds because for a given value  $q$  of  $Q$ , the events  $A$  and  $B$  are (conditionally) independent.

*You have used 2 of 2 submissions*

Printable problem set available here .

## DISCUSSION

Click "Show Discussion" below to see discussions on this problem.



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