2. Lecture 2

The following can be done after Lecture 2.

2-1

5.0/5.0 points (graded)

What is the smallest period of $|\sin 4\pi t|$?

Note: You must use a star to denote multiplication; e.g. $7*x=7x$. Use $\wedge$ to denote exponentiation; e.g. $e\wedge x=e^x$. A slash denotes division; e.g. $1/2=0.5$. Please type pi rather than a numerical approximation for π.

1/4

$\frac{1}{4}$

Submit

You have used 1 of 10 attempts

2-2

10.0/10.0 points (graded)

Let $f(t)$ be the periodic function of period 6 such that $f(t)=|t|$ for $-3\leq t<3$. Find the number a_0 such that $a_0/2$ is the constant term of the Fourier series of f.

Note: You must use a star to denote multiplication; e.g. $7 * x = 7x$. Use $\wedge$ to denote exponentiation; e.g. $e \wedge x = e^x$. A slash denotes division; e.g. $1/2 = 0.5$. Please type pi rather than a numerical approximation for π.

3

Answer: 3

3

Solution:

$3.$

The Fourier coefficient formula for period $2L$ with $L = 3$ says that

$$\begin{aligned} a_0 &= \frac{1}{3}\int_{-3}^{3} f(t)\, dt \\ &= \frac{1}{3}\int_{-3}^{3} |t|\, dt \\ &= \frac{2}{3}\int_{0}^{3} t\, dt \\ &= \frac{2}{3}\cdot \frac{t^2}{2}\Big|_0^3 \\ &= 3. \end{aligned}$$

(This makes sense, since $a_0/2 = 3/2$ is the average value of this function.)

Submit

You have used 2 of 10 attempts

Answers are displayed within the problem

2-3

15.0/15.0 points (graded)

Let f be a period 2π function such that

$$f(t) = \begin{cases} -2, & \text{if} \quad -2\pi/3 < t < \pi/3 \\ 2, & \text{if} \quad \pi/3 < t < 4\pi/3 \end{cases}$$

Find the coefficient of $\sin 5t$ in the Fourier series of $f(t)$.

(**Hint:** The standard square wave $\mathrm{Sq}(t)$, defined as the period 2π odd function such that $\mathrm{Sq}(t) = 1$ for $0 < t < \pi$, has Fourier series $\dfrac{4}{\pi} \displaystyle\sum_{n \geq 1,\ \text{odd}} \dfrac{\sin nt}{n}$. Express $f(t)$ in terms of Sq.)

4/(5*pi)

✔ **Answer:** 4/(5*pi)

$\frac{4}{5 \cdot \pi}$

Solution:

$4/(5\pi)$.

The function $f(t)$ is a horizontally shifted and vertically scaled version of the standard square wave. Specifically, if $t = u + \pi/3$, then $f(t) = 2\mathrm{Sq}(u)$.

$$\begin{aligned} f(t) &= 2\mathrm{Sq}(u) \\ &= 2\mathrm{Sq}(t - \pi/3) \\ &= \frac{8}{\pi} \sum_{\$n \geq 1\$,\ \text{odd}} \frac{\sin n(t - \pi/3)}{n} \\ &= \frac{8}{\pi} \sum_{\$n \geq 1\$,\ \text{odd}} \frac{\sin(nt - n\pi/3)}{n}. \end{aligned}$$

Apply the subtraction formula for $\sin$: then only the $n = 5$ term contributes to the coefficient of $\sin 5t$. That $n = 5$ term is

$$\frac{8}{\pi}\frac{\sin(5t-5\pi/3)}{5} = \frac{8}{5\pi}\left(\sin 5t \cos\frac{5\pi}{3} - \cos 5t \sin\frac{5\pi}{3}\right),$$

so the coefficient of $\sin 5t$ is

$$\frac{8}{5\pi}\cos\frac{5\pi}{3} = \frac{4}{5\pi}.$$

Submit You have used 3 of 100 attempts

Answers are displayed within the problem

2-4

5.0/5.0 points (graded)

Let f be the period 2 function such that $f(t) = t+5$ for $t \in [-1,1)$, and let g be its Fourier series. What is $g(1/2)$?

5.5

Answer: 11/2

5.5

Solution:

Since f is continuous at $1/2$, we have $g(1/2) = f(1/2) = 11/2$.

Submit You have used 7 of 100 attempts

Answers are displayed within the problem

2-5

5.0/5.0 points (graded)

Let f be the period 2 function such that $f(t) = t + 5$ for $t \in [-1, 1)$, and let g be its Fourier series. What is $f(1)\, g(1)$?

20

✔ **Answer:** 20

20

Solution:

$20.$

Since f is of period 2, we have $f(1) = f(-1) = 5 + (-1) = 4$. Since f has a jump discontinuity at 1,

$$g(1) = \frac{f(1^-) + f(1^+)}{2} = \frac{f(1^-) + f((-1)^+)}{2} = \frac{6+4}{2} = 5.$$

Thus $f(1)\, g(1) = 4 \cdot 5 = 20.$

Submit You have used 2 of 100 attempts

Answers are displayed within the problem

2-6

10/10 points (graded)

By an antiderivative of a piecewise differentiable function f, we mean a continuous function F such that $F'(t)$ exists and equals $f(t)$ at any point t where f is continuous. Which of the following functions $f(t)$ have a periodic antiderivative?

- [ ] (a) The constant function 1.
- [ ] (b) $\sin^2 t$.
- [ ] (c) $\cos^2 t$.
- [x] (d) The square wave $\operatorname{Sq}(t)$, defined as the period 2π function such that $\operatorname{Sq}(t) = 1$ for $0 < t < \pi$ and $\operatorname{Sq}(t) = -1$ for $-\pi < t < 0$.
- [ ] (e) The period 2 triangle wave $f(t)$ such that $f(t) = |t|$ for $-1 \leq t \leq 1$.

Solution:

Only (d) has a periodic antiderivative.

A piecewise differentiable periodic function f has a periodic antiderivative if and only if the constant term in its Fourier expansion is 0. That constant term is also the average value of f over one complete period. The functions in (a), (b), (c), (e) are everywhere nonnegative, and are positive on an open interval, so each has a positive average value. On the other hand, in (d) the average value is 0 by symmetry.

Submit You have used 2 of 5 attempts

Answers are displayed within the problem

2-7

10.0/10.0 points (graded)

Let $T(t)$ be the triangle wave of period 2 given by $T(t) = |t|$ for $-1 \leq t \leq 1$. Let $h(t)$ be the periodic function of period 2 such that $h(t) = t(|t| - 1)$ for $-1 \leq t \leq 1$. Given that the Fourier series of $T(t)$ is

$$T(t) = \frac{1}{2} - \sum_{n\geq 1,\ \text{odd}} \frac{4}{n^2\pi^2} \cos n\pi t,$$

find the coefficient of $\sin 3\pi t$ in the Fourier series of $h(t)$.

-0.00955601

✔ **Answer:** -8/(27*pi^3)

-0.00955601

Solution:

$-8/(27\pi^3)$.

For $t \in (0,1)$, we have $h(t) = t(t-1)$ and $h'(t) = 2t - 1 = 2(T(t) - 1/2)$. for $t \in (-1,0)$, we have $h(t) = t(-t-1)$ and $h'(t) = -2t - 1 = 2|t| - 1 = 2(T(t) - 1/2)$. Thus $h(t)$ is an antiderivative of

$$2(T(t) - 1/2) = -\sum_{n\geq 1,\ \text{odd}} \frac{8}{n^2\pi^2} \cos n\pi t.$$

Integrating term wise shows that the Fourier series of $h(t)$ is

$$C - \sum_{n\geq 1,\ \text{odd}} \frac{8}{n^3\pi^3} \sin n\pi t.$$

for some constant C. In particular, the coefficient of $\sin 3\pi t$ is $-8/(3^3\pi^3) = -8/27\pi^3$.

Submit

You have used 2 of 10 attempts

Answers are displayed within the problem

2. Lecture 2

Topic: Unit 1: Fourier Series / 2. Lecture 2

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Comment on form of answers to 2-4 and 2-5
I was trying to figure out how to enter a sequence into one of the answer boxes, and finally figured out that it is just asking for values at the given times.
Community TA
8

Are the graders working for 2-3 and 2-5?
I believe that my answers (esp for 2-5) should be correct as I've also evaluated them numerically? Are the graders working? Thanks.
6

Definition of Square Wave , Q2-3
In Q 2-3, the Sq wave is defined as t=1 over the interval 0 to pi. I guess I'm used to seeing Sq wave as t=1 over the interval 0 to pi AND t=-1 over the interval -pi to 0. Are these ...
2

help for 2-5
I don't understand what does f(1)g(1) mean, I tried to solve it as a product, and my answer still being wrong.
6

Part 2-6 Antiderivative
Does a periodic function has to be continuous at the end point for a periodic antiderivative to exist? Thanks,
4

Help with part 2-7
10

Convergence of solution
I was able to get the right answer with the help of the hint but on graphing the solution with the help of Kmplot the fourier series looks nothing like the function if graphed up ...
2