Course > Unit 3: ... > 6 Deco... > 3. Gen...

3. General solution to inhomogeneous systems

Introduction to inhomogeneous system

4:02 / 4:22 2.0x

Video

Download video file

Transcripts

Download SubRip (.srt) file
Download Text (.txt) file

Recall that an **inhomogeneous** first order $n \times n$ linear system of ODEs is

$$\dot{\mathbf{x}} \quad = \quad \mathbf{A}\mathbf{x} + \mathbf{r}(t)$$

where $\mathbf{A}$ is an $n \times n$ matrix, $\mathbf{r}(t)$ is a vector in n-dimensional space, and they both depend only on the independent variable t.

The general solution to such an inhomogeneous system is

$$\mathbf{x}(t) \quad = \quad \underbrace{\mathbf{x}_h(t)}_{\text{homogeneous}} \quad + \quad \underbrace{\mathbf{x}_p(t)}_{\text{particular}} \ ,$$

where $\mathbf{x}_h$ is the general solution to the associated homogeneous system:

$$\dot{\mathbf{x}}_h \quad = \quad \mathbf{A}\mathbf{x}_h,$$

and $\mathbf{x}_p$ is one particular solution satisfying the full inhomogeneous equation:

$$\dot{\mathbf{x}}_p \quad = \quad \mathbf{A}\mathbf{x}_p + \mathbf{r}.$$

This is due to the linearity of the system and the superposition principle.

As before in this course, we will restrict ourselves to the case when $\mathbf{A}$ is **constant**.

Review: companion system

2/2 points (graded)
For the second order single ODE

$$\ddot{x} + x \quad = \quad \tan(t),$$

find the companion system with new variable $y = \dot{x}$:

$$\dot{\mathbf{x}} \;=\; \mathbf{A}\mathbf{x} + \mathbf{r} \qquad \text{where } \mathbf{x} = \begin{pmatrix} x \\ y \end{pmatrix},$$

by entering the companion matrix $\mathbf{A}$ and the vector $\mathbf{r}$ below

(Enter **[a,b;c,d]** for the matrix $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$. That is, use commas to separate entries within the same row and **semicolon** to separate rows, and **square bracket** the entire matrix. Similarly for larger matrices.)

$\mathbf{A} =$ [0,1;-1,0] ✔ **Answer:** [0,1;-1,0]

$\mathbf{r} =$ [0;tan(t)] ✔ **Answer:** [0;tan(t)]

Solution:

Let $y = \dot{x}$, then the companion system is

$$\begin{aligned} \dot{x} &= y \\ \dot{y} &= -x + \tan(t). \end{aligned}$$

In matrix form, this is

$$\begin{aligned} \dot{\mathbf{x}} \;=\; \mathbf{A}\mathbf{x} + \mathbf{r} \qquad \text{where } \mathbf{x} &= \begin{pmatrix} x \\ y \end{pmatrix}, \\ \mathbf{A} &= \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \\ \mathbf{r}(t) &= \begin{pmatrix} 0 \\ \tan(t) \end{pmatrix}. \end{aligned}$$

Submit You have used 1 of 3 attempts

Answers are displayed within the problem

3. General solution to inhomogeneous systems

Hide Discussion

Topic: Unit 3: Solving systems of first order ODEs using matrix methods / 3. General solution to inhomogeneous systems

Add a Post

Show all posts ▼ by recent activity ▼

There are no posts in this topic yet.

✖

Learn About Verified Certificates