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1.1.2 A Population Model for Budworms

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Consider the logistic model Wes presented for budworm population:

$$\frac{dP}{dt} = rP\left(1 - \frac{P}{q}\right)$$

- P represents the population of budworms, in millions;
- t is time, in years;
- q is the carrying capacity (in millions of budworms).
- r is the reproduction rate parameter (with units $\frac{1}{years}$)

The big question of this section is: How does the carrying capacity of the budworms effect how the population grows? This carrying capacity is highly related to the foliage of the fir trees, the food source of the budworms.

In order to work with a more concrete example, we will suppose that a field ecologist has measured the reproduction rate constant r, obtaining $r = \frac{1}{2}$.

Thus the model is:

$$\frac{dP}{dt} = \frac{1}{2}P\left(1 - \frac{P}{q}\right).$$

Notes on the model:

- To get a sense of scale, $q = 1$ would mean a carrying capacity of one million. A single spruce tree can support 3000 budworms, so this would be appropriate for a forest of about 300 trees.

- To make sense of $r = \frac{1}{2}$ with units of $\frac{1}{year}$, consider what happens when P is much lower than the capacity. Then the worms approximately obey a differential equation $\frac{dP}{dt} = rP$ which has exponential $P(t) = Ce^{rt}$ as solutions. If $r = \frac{1}{2}$, then in the course of one year we'd expect population to increase by a factor of $e^{1/2} \approx 1.65$. (Note this is a bit low in reality: budworm populations are easily capable of tripling in the course of a year.)

- This model and those that follow have been simplified from their original form. This is so we can focus on questions of bifurcation and outbreak. Technically, the variable P in this model is not an absolute number of budworms, but a scaled population relative to the maximum predation rate of the birds that eat budworms. But this is not a can of [bud]worms worth opening - for this module, it's sufficient to think of P as representing population in millions.