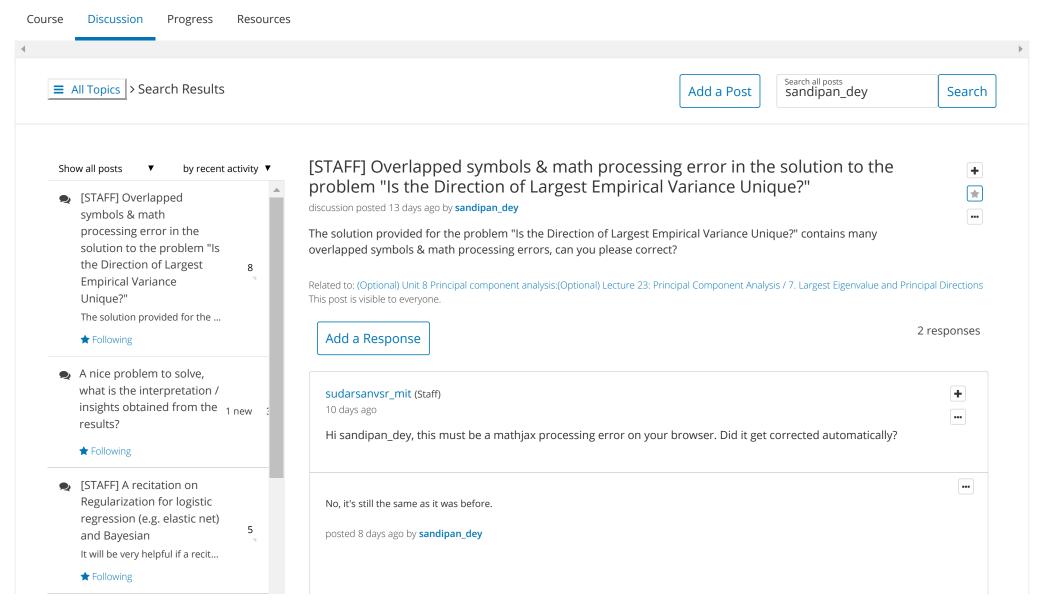
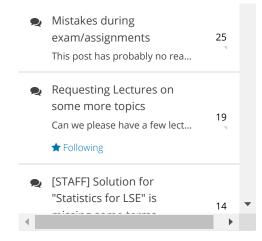
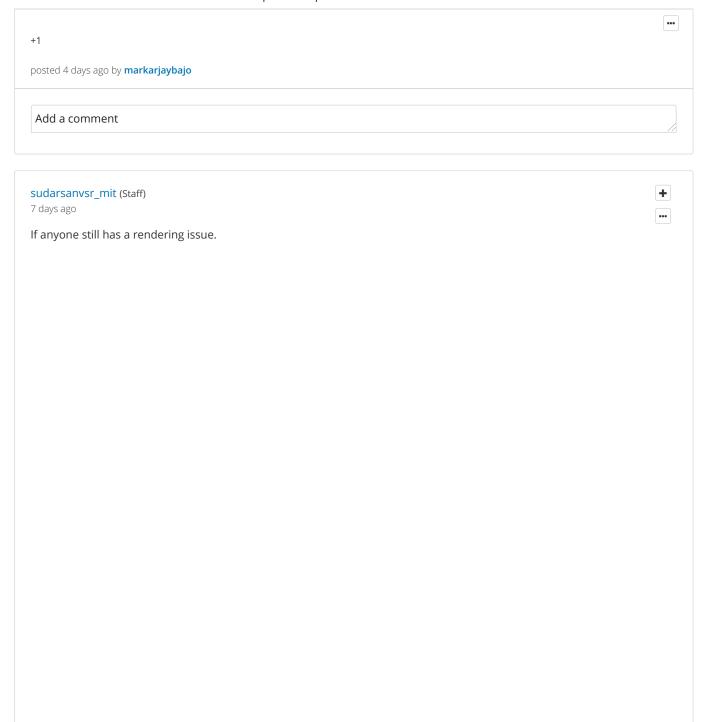
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Solution:

The correct answer is "No". First observe that if ${f w}$ is a unit vector, then $P^T{f w}$ is also a unit vector. This is because

$$||P^T \mathbf{w}||_2^2 = (P^T \mathbf{w})^T P^T \mathbf{w}$$
$$= \mathbf{w}^T P P^T \mathbf{w}$$
$$= \mathbf{w}^T \mathbf{w}$$
$$= ||\mathbf{w}||_2^2$$
$$= 1.$$

To go from the second to third line, we used that $PP^T=I_d$ and associativity of matrix multiplication.

Next, note that by the given decomposition

$$\mathbf{w}^T S \mathbf{w} = \mathbf{w}^T P D P^T \mathbf{w} = (P^T \mathbf{w})^T D (P^T \mathbf{w}).$$

But as \mathbf{w} ranges over all unit vectors, we know that $P^T\mathbf{w}$ also ranges over all unit vectors. So if there exists $\mathbf{w} \neq \mathbf{v}_1$ such that $\mathbf{w}^T S \mathbf{w} = \lambda_1$, there must exist $\mathbf{b} \neq P^T \mathbf{v}_1 = (1, 0, \dots, 0)^T$ such that $\mathbf{b}^T D \mathbf{b} = \lambda_1$. Observe that by matrix multiplication,

$$\mathbf{b}^T D \mathbf{b} = \sum_{i=1} \lambda_i (\mathbf{b}^i)^2 \leq \lambda_1 (\mathbf{b}^1)^2 + \lambda_2 (1 - \mathbf{b}_1^2).$$

We also used that $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_d \geq 0$ and $0 \leq \mathbf{b}_j^2 \leq 1 - \mathbf{b}_1^2$ for all $j \neq 1$. Suppose $\mathbf{b}^1 \neq 1$ (so that $\mathbf{b} \neq \mathbf{v}_1$ and $1 - \mathbf{b}_1^2 > 0$), then we have

$$\mathbf{b}^T D \mathbf{b} \leq \lambda_1 (\mathbf{b}^1)^2 + \lambda_2 (1 - \mathbf{b}_1^2) < \lambda_1 (\mathbf{b}^1)^2 + \lambda_1 (1 - \mathbf{b}_1^2) = \lambda_1,$$

where we used the strict inequality $\lambda_1 > \lambda_2$. Therefore, the equality case is **only** possible if $\mathbf{b} = (1, 0, \dots, 0)^T$. Hence, we must also have $\mathbf{w} = \mathbf{v}_1$ if equality holds.

•••

Thank you!

posted 4 days ago by markarjaybajo

•••

Thank you!

posted 3 days ago by Alexander_Andrianov

It looks like there is a mistake in the solution closer to the end: Should not it be "Suppose $\mathbf{b}^1
eq 1$ (so that $\mathbf{b}
eq P^T \mathbf{v}_1$ and $1 - b_1^2 > 0$)..." instead of "...(so that $\mathbf{b}
eq \mathbf{v}_1$ and..." posted 3 days ago by Alexander_Andrianov Add a comment Showing all responses Add a response: Preview Submit

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