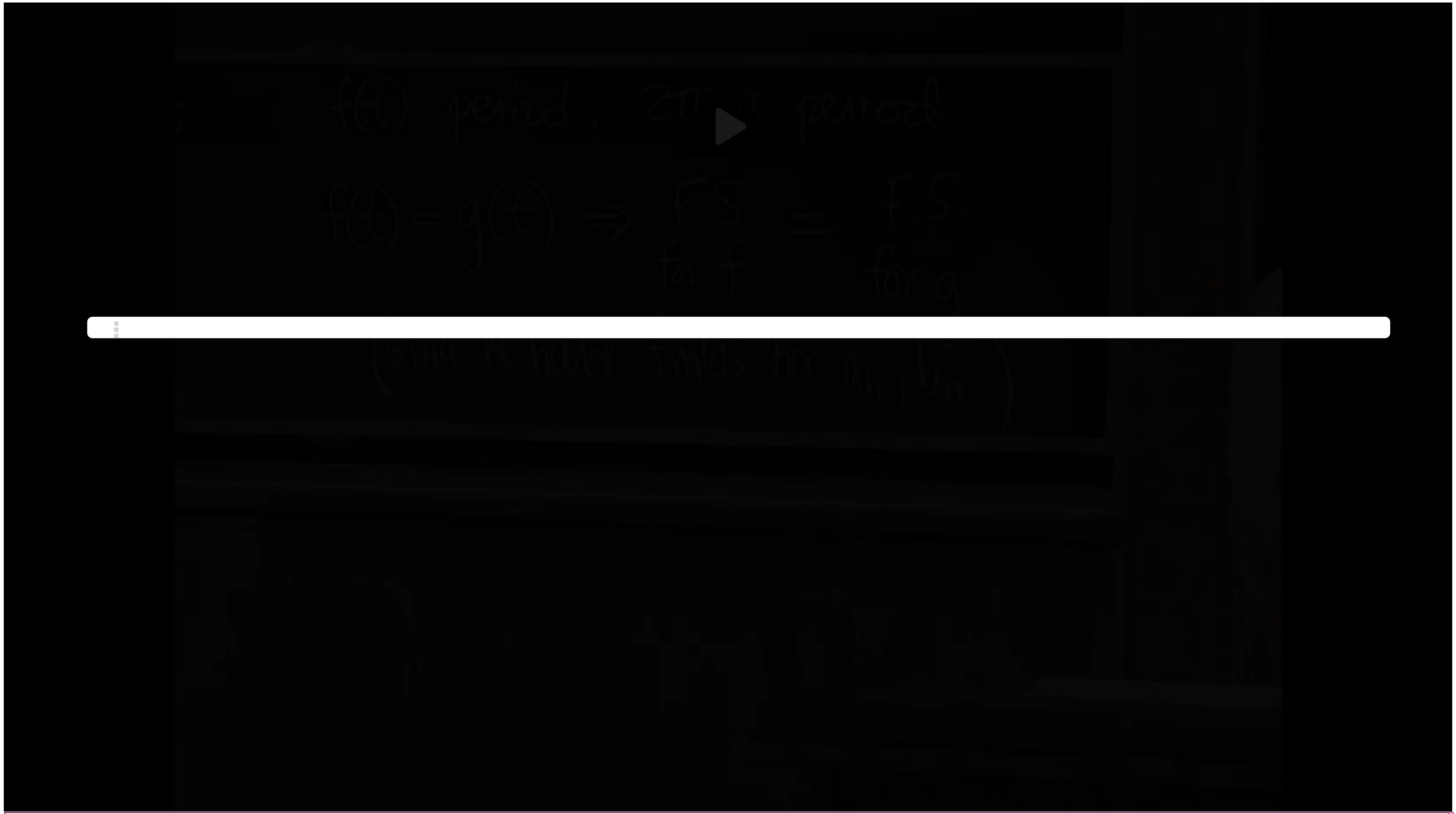
12. Uniqueness of Fourier series

Fourier series are unique



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Recall that for any function $f(t)$ that is periodic and has period 2π, the Fourier series is given by

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nt + b_n \sin nt),$$

where the formulas for the coefficients a_n and b_n are given by

$$\begin{aligned} \frac{a_0}{2} &= \frac{1}{2\pi} \int_{-\pi}^{\pi} f(t)\ dt = \frac{\langle f(t), 1\rangle}{\langle 1, 1\rangle}, \\ a_n &= \frac{1}{\pi} \int_{-\pi}^{\pi} f(t) \cos(nt)\ dt = \frac{\langle f(t), \cos(nt)\rangle}{\langle \cos nt, \cos nt\rangle}, \qquad n \geq 1 \\ b_n &= \frac{1}{\pi} \int_{-\pi}^{\pi} f(t) \sin(nt)\ dt = \frac{\langle f(t), \sin(nt)\rangle}{\langle \sin nt, \sin nt\rangle}, \qquad n \geq 1. \end{aligned}$$

By virtue of the fact that we have formulas for the coefficients (in terms of an inner product on functions with respect to an orthogonal basis of functions), a function has only one Fourier series. That is, if $f(t) = g(t)$, then the Fourier series for $f(t)$ is the same as the Fourier series for $g(t)$.

We will use this idea to come up with easier ways to compute Fourier series.

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Orthotonality is necessary for uniqueness?

Is it not sufficient to have a base, in linear algebra sense of base, to have an unique decomposition in component space? Do you need orthogonality? Orthogonality "only" sim... 3

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