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Totally Ordered Set

A total order (or "totally ordered set," or "linearly ordered set") is a set plus a relation on the set (called a total order) that satisfies the conditions for a partial order plus an additional condition known as the comparability condition. A relation $\leq$ is a total order on a set S ("$\leq$ totally orders S") if the following properties hold.

1. Reflexivity: $a \leq a$ for all $a \in S$.

2. Antisymmetry: $a \leq b$ and $b \leq a$ implies $a = b$.

3. Transitivity: $a \leq b$ and $b \leq c$ implies $a \leq c$.

4. Comparability (trichotomy law): For any $a, b \in S$, either $a \leq b$ or $b \leq a$.

The first three are the axioms of a partial order, while addition of the trichotomy law defines a total order.

Every finite totally ordered set is well ordered. Any two totally ordered sets with k elements (for k a nonnegative integer) are order isomorphic, and therefore have the same order type (which is also an ordinal number).

SEE ALSO:
Order Isomorphic, Order Type, Partial Order, Relation, Total Order, Trichotomy Law, Well Ordered Set

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