

Problem 3

Count # of lattice points with **even X-coord** in the interior of the triangle $(0,0)-(7,0)-(7,5)$.
Count # lattice points in the interior of $(0,0)-(7/2, 0)-(7/2, 5/2)$.

(Background) **Eisenstein's Lemma**

➤ Is 5 **QR** or **non-QR** (mod 7)?



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Eisenstein's Lemma

$M = \#$ of lattice points with **even**
X-coord in the interior of
 $(0,0)-(7,0)-(7,5)$.

$N = \#$ of lattice points in
 $(0,0)-(7/2, 0)-(7/2, 5/2)$.

Then

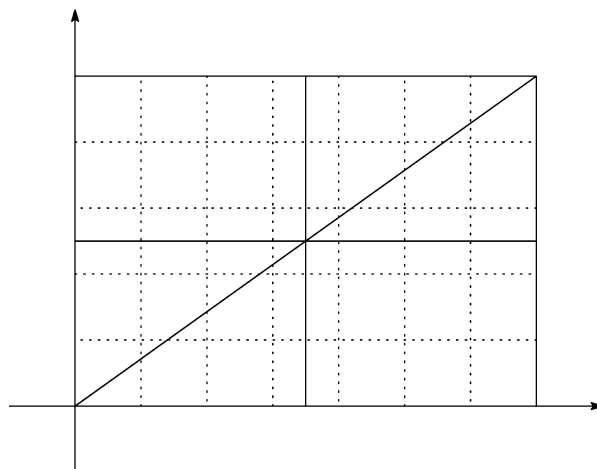
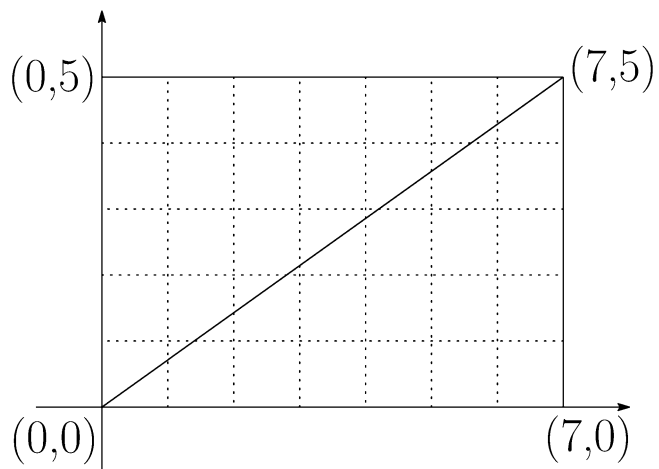
$$M \equiv N \pmod{2}.$$



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➤ Count # of lattice points.



Answer $M = 7, N = 3$



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➤ Confirm Eisenstein's Lemma

$$M = 7, N = 3$$

$$M \equiv N \pmod{2}$$

M, N are **odd**

\Leftrightarrow **5 is not QR (mod 7)**



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