Homework Problem: Ising Model

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10/10 points (graded)

Consider the joint probability distribution of random variables $X_1, X_2, X_3 \in \{-1, 1\}$ that is parameterized by $(\alpha_{12}, \alpha_{13}, \alpha_{23})$ as follows:

$$p_{X_1,X_2,X_3}(x_1, x_2, x_3; \alpha_{12}, \alpha_{13}, \alpha_{23}) = \frac{1}{Z} \cdot e^{\alpha_{12} x_1 x_2} \cdot e^{\alpha_{23} x_2 x_3} \cdot e^{\alpha_{13} x_1 x_3}, \quad (4.1)$$

where $Z = Z(\alpha_{12}, \alpha_{13}, \alpha_{23})$ is the required normalization constant (Z is also called the *partition function*).

- **(a)** If $\alpha_{13} = 0$, is it the case that $X_1 \perp X_3 | X_2$, i.e., X_1, X_2, X_3 form a Markov chain?

  - (●) True ✔
  - ( ) False

- n i.i.d. samples $\left\{\left(x_1^{(i)}, x_2^{(i)}, x_3^{(i)}\right)\right\}_{i=1}^{n}$ are drawn from $p_{X_1,X_2,X_3}(\cdot,\cdot,\cdot;\alpha,0,\alpha)$ for some unknown value α.

  - Determine the ML estimate $\hat{\alpha}_{\mathrm{ML}}$ of α in terms of m_{13}, which is defined to be

$$m_{13} \triangleq \frac{1}{2n}\sum_{i=1}^{n}\left(x_1^{(i)}x_2^{(i)} + x_2^{(i)}x_3^{(i)}\right)$$

You should come up with a general expression for $\hat{\alpha}$ in terms of m_{13}. However, we will only ask you to specify your answer in terms of a few specific numerical values for m_{13}. Please use natural log.

(Please be precise with at least 3 decimal places, unless of course the answer doesn't need that many decimal places. You could also put a fraction.)

When $m_{13} = 0.3$, then $\hat{\alpha}_{\mathrm{ML}} =$ 0.309519604203 ✔

When $m_{13} = 0.8$, then $\hat{\alpha}_{\mathrm{ML}} =$ 1.09861228867 ✔

  - Determine a general expression the function g such that the maximum log-likelihood of the entire sample set is equal to

$$g(m_{13}, n),$$

where m_{13} is defined in the previous part, and n is the number of training data points.

You should come up with a general expression for g in terms of m_{13} and n. However, we will only ask you to specify your answer in terms of a few specific numerical values for m_{13} and n. Please use natural log.

(Please be precise with at least 3 decimal places, unless of course the answer doesn't need that many decimal places. You could also put a fraction.)

When $m_{13} = 0.2$ and $n = 17$, $g =$ -34.6658987478 ✔

When $m_{13} = 0.9$ and $n = 15$, $g =$ -16.3526650088 ✔

- We define a set of parameter values $\mathcal{M} = \{(0, \alpha, \alpha) | \alpha \in \mathbb{R}\} \cup \{(\alpha, 0, \alpha) | \alpha \in \mathbb{R}\} \cup \{(\alpha, \alpha, 0) | \alpha \in \mathbb{R}\}$ to describe all distributions of the form (4.1) where $\alpha_{ij} = 0$ for some $i < j$, and the other two parameters are equal.

  Bob observes n i.i.d. samples $\left\{(x_1^{(i)}, x_2^{(i)}, x_3^{(i)})\right\}_{i=1}^{n}$ and computes three values

$$m_{12} = \frac{1}{2n}\sum_{i=1}^{N}(x_1^{(i)}x_3^{(i)} + x_2^{(i)}x_3^{(i)}),$$

$$m_{13} = \frac{1}{2n}\sum_{i=1}^{N}(x_1^{(i)}x_2^{(i)} + x_2^{(i)}x_3^{(i)}),$$

$$m_{23} = \frac{1}{2n}\sum_{i=1}^{N}(x_1^{(i)}x_2^{(i)} + x_1^{(i)}x_3^{(i)}).$$

Bob only tells you that

$$|m_{12}| < |m_{23}| < |m_{13}|,$$

and that he is performing an ML estimation over all possible models in set $\mathcal{M}$. Determine which of the three parameter estimates $(\hat{\alpha}_{12}, \hat{\alpha}_{23}, \hat{\alpha}_{13})$ Bob will set to zero.

- $\hat{\alpha}_{12}$
- $\hat{\alpha}_{23}$
- $\hat{\alpha}_{13}$ ✔

Submit You have used 4 of 5 attempts

✔ Correct (10/10 points)

Discussion

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help in getting Z

question posted about 20 hours ago by **javiernl**

Can anybody help me to get Z (I know that most part of you can)? I tried to sum the expressions for all of the possible combinations of x1, x2...

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MLE

question posted 4 days ago by **wcrfrench**

In previous examples, the MLE estimates stems from the multiplication of the individual operations.

Therefore does the summation arise in M13...

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correct formula for g: the maximum log-likelihood of the entire sample set

discussion posted a day ago by **sandipan_dey**

I am trying to not violate the honor code, but I am not getting the answers for g(.,.) values to be correct and I already wasted 3 attempts. So, my g(m,n) looks like the following:

$$g(m,n) = 2 * m * n * f(m) - n * log(2 * (2 + h(f(m)) + h(-f(m))))$$

where f(.) and h(.) are some functions (omitting them for honor code), am I making any mistake? also the first g value i computed is -ve where the 2nd one is +ve.

I got MLE values correct for alpha, but stuck at grade 6.7 for these two.

Any hint from the TAs? Thanks for help in advance.

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1 response

Daniel_Joseph

a day ago

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General Expression g

discussion posted a day ago by **Aeschbacher**

For the equation:

$$m_{13} \triangleq \frac{1}{2n} \sum_{i=1}^{n} (x_1^{(i)} x_2^{(i)} + x_2^{(i)} x_3^{(i)})$$

I'm confused how we can get $m_{13}...

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What does this mean?

discussion posted 5 days ago by **seanedXacc**

pX1,X2,X3(·,·,·;α,0,α)? Does it mean that there is no node 12, 23, 32 because x2 occurs with P(0)?

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is there a grading issue ?

discussion posted 5 days ago by **Stef2970**

It looks like I found a possible formula for the ML estimates alfa_hat values for m being 0.3 and 0.8 as both my answers are graded ok. I also...

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Test Answer

discussion posted 3 days ago by **Daniel_Joseph**

I submitted my answers and am getting some red checks. Upon further review, I believe that I made a minor mistake in the derivation of my formulas....

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How should I approach this problem?

question posted 5 days ago by **chalupa**

> You should come up with a general expression for $\hat{\alpha}$ in terms of m_{13} .

How should I approach this problem? Any hints? Thanks!

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