8.2.8 Neurons

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A neuron is a cell in animals that transmits and stores information. A key aspect of neuron behavior is known as a spike, which is a rapid increase its voltage that can occur with sufficient stimuli. Differential equation models have been used to describe the behavior of neurons. In a series of papers in 1952, Alan Hodgkin and Andrew Huxley developed a model that approximates the electrical characteristics of excitable cells such as neurons by studying the squid giant axon (an axon is part of a neuron as shown in Figure 8.8). The so-called Hodgkin–Huxley model is a set of four nonlinear differential equations. For this landmark work, Hodgkin and Huxley received the 1963 Nobel Prize in Physiology or Medicine.

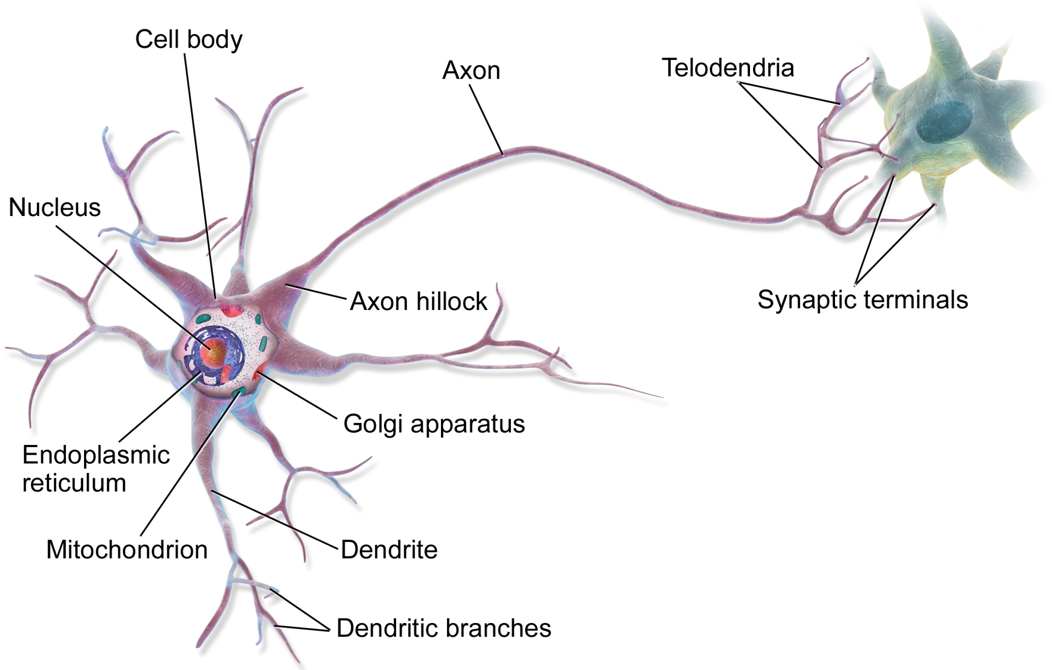

Figure 8.8: Anatomy of a neuron. By BruceBlaus - Own work, CC BY 3.0

In these notes, we will consider a simplified form of the Hodgkin-Huxley model known as the FitzHugh-Nagumo model. This model has two states:

- $V(t)$: this state represents the voltage in the neuron.
- $W(t)$: this a recovery variable that provides a slow negative feedback that drives the voltage down toward its unexcited state.

The differential equations of the FitzHugh-Nagumo model can be written as:

$$\frac{dV}{dt} = \frac{1}{\tau_V}[F(V) - W] + I(t) \tag{8.32}$$

$$\frac{dW}{dt} = \frac{1}{\tau_W}(\alpha V - W) \tag{8.33}$$

where:

- $F(V) = V(V - V_s)(1 - V)$ and V_s is a constant. Note that $F(V)$ switches sign at V_s. Specifically, for $0 < V < V_s$ then $F(V) < 0$ while $V_s < V < 1$ then $F(V) > 0$. As a result, $0 < V < V_s$, F will tend to decrease V back towards 0. We think of this as a stabilizing effect on V. While for $V_s < V < 1$, F will increase V.
- $I(t)$ is the stimulus (current) that excites the neuron voltage.

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- $I(t)$ is the stimulus (current) that excites the neuron voltage.

- τ_V and τ_W are timescales at which the voltage state V and the recovery state W respond with.

- α is a constant that relates the existence of V to the production rate of W.

As an example, consider the following parameter values,

$$V_s = 0.25, \quad \tau_V = 0.05\,\text{ms}, \quad \tau_W = 10\,\text{ms}, \quad \alpha = 1.25 \tag{8.34}$$

We will consider cases with no current stimulus, $I(t) = 0$. The initial condition will have non-zero voltages $V(0) \neq 0$, while the recovery state is zero $W(0) = 0$. Specifically, Figure 8.9 shows the results for $V(0) = 0.2$ and $V(0) = 0.3$. In the $V(0) = 0.2$ case, the voltage immediately decreases which is expected since $F(V) < 0$ since $V < V_s$. However, as the $V(0) = 0.3$ case starts above V_s, then $F(V) > 0$ and the voltage shows a rapid increase and peaks at approximately $V = 0.9$ at about $t = 1$ ms. Thus, this small difference in initial voltage between the two cases results in significant differences in voltage levels and is a common feature of neurons and other excitable cells. Following the spike, the voltage then rapidly decreases and becomes somewhat negative before slowly returning towards zero for longer times (roughly 10 ms).

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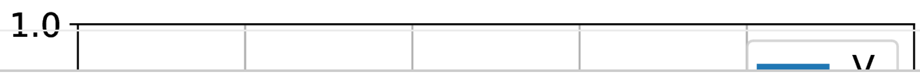

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