https://www.nist.gov/pml/nist-technical-note-1297/nist-tn-1297-appendix-law-propagation-uncertainty

National Institute of Standards and Technology
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(https://www.nist.gov/pml)

NIST TN 1297: Appendix A. Law of Propagation of Uncertainty

A.1 In many cases a measurand Y is not measured directly, but is determined from N other quantities $X_1, X_2, \ldots, X_N$ through a functional relation f:

$$Y = f(X_1,\ X_2,\ \ldots,\ X_N)\ .$$

(A-1)

Included among the quantities X_i are corrections (or correction factors) as described in subsection 5.2 (https://www.nist.gov/physical-measurement-laboratory/nist-tn-1297-5-combined-standard-uncertainty), as well as quantities that take into account other sources of variability, such as different observers, instruments, samples, laboratories, and times at which observations are made (e.g., different days). Thus the function f of Eq. (A-1) should express not simply a physical law but a measurement process, and in particular, it should contain all quantities that can contribute a significant uncertainty to the measurement result.

A.2 An estimate of the measurand or *output quantity* Y, denoted by y, is obtained from Eq. (A-1) using *input estimates* $x_1, x_2, \ldots, x_N$ for the values of the N *input quantities* $X_1, X_2, \ldots, X_N$. Thus the *output estimate* y, which is the result of the measurement, is given by

$$y = f(x_1,\ x_2,\ \ldots,\ x_N)\ .$$

(A-2)

A.3 The combined standard uncertainty of the measurement result y, designated by $u_c(y)$ and taken to represent the estimated standard deviation of the result, is the positive square root of the estimated variance $u_c^2(y)$ obtained from

(A-3)

$$u_c^2(y) = \sum_{i=1}^{N} \left(\frac{\partial f}{\partial x_i}\right)^2 u^2(x_i) + 2\sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \frac{\partial f}{\partial x_i}\frac{\partial f}{\partial x_j} u(x_i, x_j) .$$

Equation (A-3) is based on a first-order Taylor series approximation of $Y = f(X_1, X_2, \dots, X_N)$ and is conveniently referred to as the *law of propagation of uncertainty*. The partial derivatives $\partial f/\partial x_i$ (often referred to as *sensitivity coefficients*) are equal to $\partial f/\partial X_i$ evaluated at $X_i = x_i$; $u(x_i)$ is the standard uncertainty associated with the input estimate x_i; and $u(x_i, x_j)$ is the estimated covariance associated with x_i and x_j.

A.4 As an example of a Type A evaluation, consider an input quantity X_i whose value is estimated from n independent observations $X_{i,k}$ of X_i obtained under the same conditions of measurement. In this case the input estimate x_i is usually the sample mean

$$x_i = \bar{X}_i = \frac{1}{n} \sum_{k=1}^{n} X_{i,k} ,$$

(A-4)

and the standard uncertainty $u(x_i)$ to be associated with x_i is the estimated standard deviation of the mean

$$u(x_i) = s(\bar{X}_i) = \left(\frac{1}{n(n-1)} \sum_{k=1}^{n} (X_{i,k} - \bar{X}_i)^2\right)^{1/2} .$$

(A-5)

A.5 As an example of a Type B evaluation, consider an input quantity X_i whose value is estimated from an assumed rectangular probability distribution of lower limit a_- and upper limit a_+. In this case the input estimate is usually the expectation of the distribution

$$x_i = (a_+ + a_-)/2 ,$$

(A-6)

and the standard uncertainty $u(x_i)$ to be associated with x_i is the positive square root of the variance of the distribution

$$u(x_i) = a/\sqrt{3} ,$$

(A-7)

where $a = (a_+ - a_-)/2$ (see subsection 4.6 (https://www.nist.gov/physical-measurement-laboratory/nist-tn-1297-4-type-b-evaluation-standard-uncertainty)).

NOTE -- When x_i is obtained from an assumed distribution, the associated variance is appropriately written as $u^2(X_i)$ and the associated standard uncertainty as $u(X_i)$, but for simplicity, $u^2(x_i)$ and $u(x_i)$ are used. Similar considerations apply to the symbols $u_c{}^2(y)$ and $u_c(y)$.

Metrology (https://www.nist.gov/topic-terms/metrology) and Physics (https://www.nist.gov/topic-terms/physics)

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- ‹ NIST TN 1297: 8. References (https://www.nist.gov/pml/nist-technical-note-1297/nist-tn-1297-8-references)
- NIST TN 1297: Appendix B. Coverage Factors › (https://www.nist.gov/pml/nist-technical-note-1297/nist-tn-1297-appendix-b-coverage-factors)