Exercises: What is an "estimation problem"?

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Equations for the canal width problem

1/1 point (ungraded)

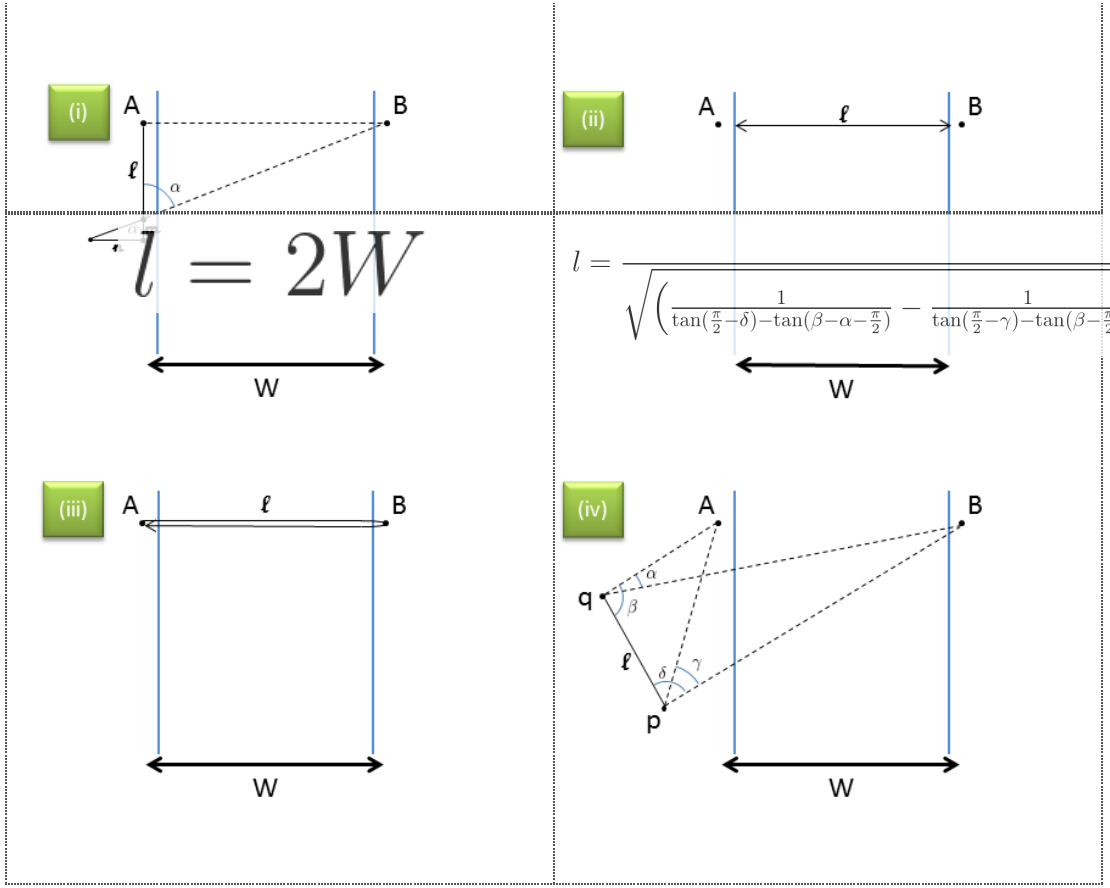
PROBLEM

Connect the four measurement set-ups and with their associated equations.

$l = \frac{m}{n}$ $l = W$

Correct, see feedback after finishing this exercise.

- MATLAB Learning Content



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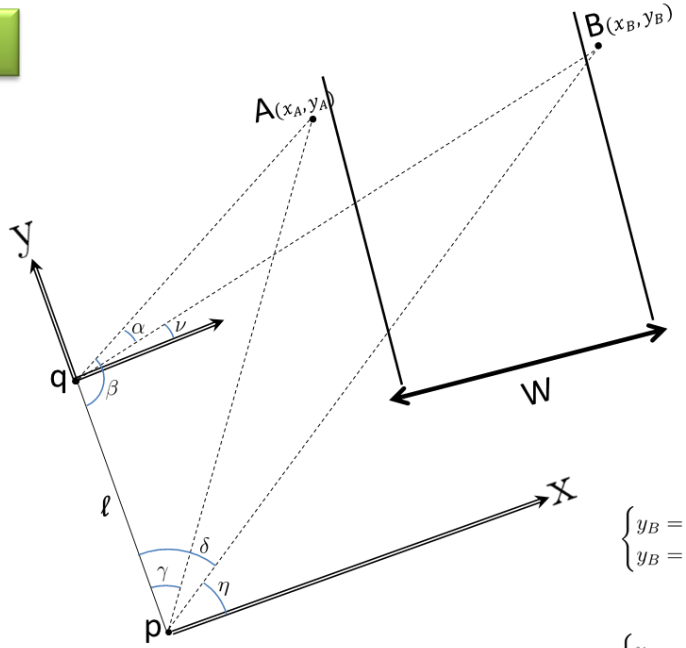
Feedback

ℹ Remark about the solution for the sporty team (rope): this team is measuring the actual width of canal and therefore W is different. Remark about the solution for the lazy guy (laser): the measurement principle is to measure the two-way distance travelled by the laser, hence we have l=2W. You may have noticed in the video that the readings on the display are in fact already divided by 2.

Hide the worked out solution for the nerd team and for the cool team (in case you are curious).

Solution of triangulation method (cool team)

(iv)



Measured: $l, \alpha, \beta, \gamma, \delta$
Unknown: W

$$\begin{cases} y_B = x_B \tan\eta \\ y_B = x_B \tan\nu + l \end{cases} \Rightarrow \quad (\tan\eta - \tan\nu)x_B = l$$

$$\Rightarrow x_B = \frac{l}{\tan\eta - \tan\nu}$$

$$\begin{cases} y_A = x_A \tan(\frac{\pi}{2} - \gamma) \\ y_A = x_A \tan(\alpha + \nu) + l \end{cases} \Rightarrow \quad \left(\tan(\frac{\pi}{2} - \gamma) - \tan(\alpha + \nu)\right) x_A = l$$

$$\Rightarrow x_A = \frac{l}{\tan(\frac{\pi}{2} - \gamma) - \tan(\alpha + \nu)}$$

$$W = \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2}$$
$$= \sqrt{\left(\frac{l}{\tan\eta - \tan\nu} - \frac{l}{\tan(\frac{\pi}{2} - \gamma) - \tan(\alpha + \nu)}\right)^2 + \left(\frac{l\tan\eta}{\tan\eta - \tan\nu} - \frac{l\tan(\frac{\pi}{2} - \gamma)}{\tan(\frac{\pi}{2} - \gamma) - \tan(\alpha + \nu)}\right)^2}$$

$$W = \sqrt{l^2\left(\frac{1}{S} - \frac{1}{Q}\right)^2 + l^2\left(\frac{\tan\eta}{S} - \frac{\tan(\frac{\pi}{2} - \gamma)}{Q}\right)^2}$$
$$= l\sqrt{\left(\frac{1}{S} - \frac{1}{Q}\right)^2 + \left(\frac{\tan\eta}{S} - \frac{\tan(\frac{\pi}{2} - \gamma)}{Q}\right)^2}$$

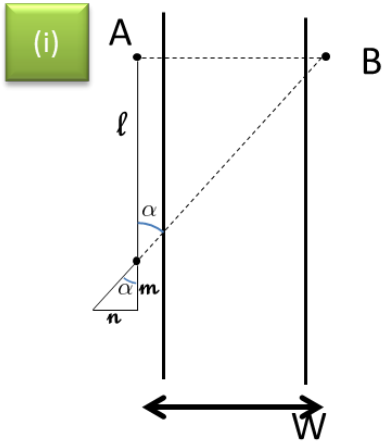
$$l = \frac{W}{\left(\frac{1}{S} - \frac{1}{Q}\right)^2 + \left(\frac{\tan\eta}{S} - \frac{\tan(\frac{\pi}{2} - \gamma)}{Q}\right)^2}$$

where

$$S = \tan\eta - \tan\nu$$
$$= \tan(\frac{\pi}{2} - \delta) - \tan(\beta - \alpha - \frac{\pi}{2})$$
$$Q = \tan(\frac{\pi}{2} - \gamma) - \tan(\alpha + \nu)$$
$$= \tan(\frac{\pi}{2} - \gamma) - \tan(\beta - \frac{\pi}{2})$$

Solution of boy-scout method (nerd-team)

(i)



$$m = 60 \text{ cm}$$
$$n = 50 \text{ cm}$$

$$\begin{cases} l = \frac{W}{\tan\alpha} \\ \tan\alpha = \frac{50}{60} \end{cases} \Rightarrow \quad l = \frac{6}{5}W$$