MITx: 6.041x Introduction to Probability - The Science of Uncertainty

Problem 2: 13 cards in a deck

(4/4 points)

A player is randomly dealt a sequence of 13 cards from a standard 52-card deck. All sequences of 13 cards are equally likely. In an equivalent model, the cards are chosen and dealt one at a time. When choosing a card, the dealer is equally likely to pick any of the cards that remain in the deck.

1. What is the probability the 13th card dealt is a King? **Note:** Your answer should be a number. Do not enter '!' or combinations in your answer.

   0.07692308 ✔ **Answer:** 0.07692

2. Find the probability of the event that the 13th card dealt is the first King dealt. Identify the correct expression.

   ○ $13 \cdot \frac{4\binom{48}{12}}{\binom{52}{13}}$

- $13 \cdot \frac{\binom{48}{12}}{\binom{52}{13}}$
- $\frac{1}{13} \cdot \frac{4\binom{48}{12}}{\binom{52}{13}}$ ✔
- $\frac{1}{13} \cdot \frac{\binom{48}{13}}{\binom{52}{13}}$

Answer:

1. Since we are not told anything about the first 12 cards that are dealt, the probability that the 13th card dealt is a King is the same as the probability that the first card dealt, or in fact that any particular card dealt, is a King, which is $4/52 = 1/13$.

2. The probability that the 13th card is the first King to be dealt is the probability that out of the 13 cards to be dealt, exactly one was a King, and that the King was dealt last. Using the multiplication rule,

$$\begin{aligned}&\mathbf{P}(\text{13th card is first King dealt})\\ &= \mathbf{P}(\text{1 King out of 13 cards})\mathbf{P}(\text{King is last card dealt} \mid \text{1 King out of 13 cards}).\end{aligned}$$

Now, given that exactly one King was dealt in the 13 cards, the probability that the King was dealt last is just $1/13$, since each “position" is equally likely. Hence,

$$\mathbf{P}(\text{King is last card dealt} \mid 1 \text{ King out of 13 cards}) = \frac{1}{13}.$$

Thus, it remains to calculate the probability that there was exactly one King in the 13 cards dealt. To calculate this probability, we count the number of “favorable" outcomes and divide by the total number of possible outcomes.

We first count the number of favorable outcomes, namely those with exactly one King in the 13 cards dealt. We can choose the King in 4 ways (from the 4 suits), and we can choose the other 12 cards in $\binom{48}{12}$ ways out of the remaining 48 cards that are not Kings. Therefore, there are $4 \cdot \binom{48}{12}$ favorable outcomes. There are $\binom{52}{13}$ total possible outcomes (here, an “outcome" is a subset of size 13), so

$$\mathbf{P}(1 \text{ King out of 13 cards}) = \frac{4 \cdot \binom{48}{12}}{\binom{52}{13}}.$$

Combining this with the conditional probability that the King was dealt last, given that exactly one King was dealt, which we found earlier to be $1/13$, the desired probability is $\frac{1}{13} \cdot \frac{4\binom{48}{12}}{\binom{52}{13}}$.

For an alternative solution, we argue as in Example 1.10 in the textbook. The probability that the first card is not a King is $48/52$. Given that, the probability that the second is not a King is $47/51$. We continue similarly until the 12th card. The probability that the 12th card is not a King, given that none of the preceding 11 were Kings, is $37/41$. (There are $52-11=41$ cards left, and $48-11=37$ of them are not Kings.) Finally, the conditional probability that the 13th card is a King, given that none of the preceding 12 were Kings, is $4/40$. The desired probability is therefore $\frac{48\cdot 47\cdots 37\cdot 4}{52\cdot 51\cdots 41\cdot 40}$. This expression can be manipulated to see that it is equal to the answer we derived earlier.

You have used 2 of 2 submissions

DISCUSSION

Click “Show Discussion" below to see discussions on this problem.