Overview

The Statland Games in Averagemont

Expectation is one of the most useful concepts in statistics, not only in its own right but also because it is the key to defining many other useful quantities, such as standard deviation and correlation. Linearity often lets us break complicated expectations into simpler pieces.

Expectation is the most widely used notion of average in statistics, because of its intuitive interpretations and convenient properties. For both discrete and continuous random variables, expectation can be thought of as a weighted average of the possible values of the random variable, e.g., we can average the possible outcomes for John Smith's jumps with weights given by the probabilities of the possible outcomes.

Linearity is the most important property of expectation. It may seem counterintuitive at first that linearity holds in such generality, but Sylwia showed Colin an intuitive way to think about it, when they were studying Lyra Loopski's skiloop data.

The law of large numbers (LLN) and central limit theorem (CLT) are powerful results about the sample mean of a large number of independent random variables, each following the same distribution. Under some conditions, the LLN says that the sample mean is likely to be close to the theoretical expectation and the CLT says that the sample mean will be approximately Normal. LLN and CLT are amazingly general theorems but, as John Smith alluded to, their statements do have assumptions. It is crucial to check the assumptions of a theorem rather than unthinkingly apply it.

Learning Objectives

In this section, you will:

- Define the expectation for both discrete and continuous random variables, as a notion of average
- Define the variance and standard deviation for both discrete and continuous random variables, as notions of spread or variability
- Use linearity to break the expected value of a sum into a bunch of separate expectations

- Use LOTUS (the Law of the Unconscious Statistician) to help find the expected value of a function of a random variable
- Work with the Geometric, Negative Binomial, and Poisson distributions
- Explore two of the most famous and useful theorems in all of probability: the law of large numbers and the central limit theorem