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Lecture 5: Delta Method and

12. Interpretation of the frequentist

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> Confidence Interval

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# 12. Interpretation of the frequentist Confidence Interval Frequentist Interpretation of a Confidence Interval

we're not really going to talk about them except for their connections to hypothesis testing.

So next week, we'll start hypothesis testing in a slightly mild way, mostly coming from--

so you will have some exercises that like are soft and connect them to confidence intervals.

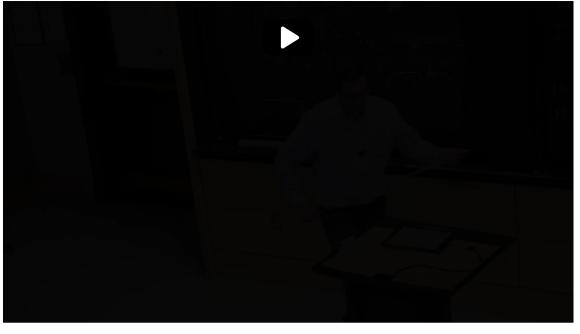
There's going to be a lot of terminology.

We'll talk about type I, type II error, one-sided, two-sided tests.

We're going to talk about power.

We're going to talk about p values.

\\/\_\\_\_\_\_



we're going to see a punch of new words.

So make sure that you're here, or make sure that you take a look at the notes.

Again, office hours this afternoon are actually right upstairs in 2-290.

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## Frequentist Interpretation of a Confidence Interval

1/1 point (graded)

In a particular experiment, you gather data in the form of a sample  $X_1,\ldots,X_n\stackrel{iid}{\sim}P_{\theta}$ , and construct a confidence interval  $\mathcal I$  with level 90% for the true (unknown) parameter  $\theta$ .

After conducting the experiment, there are two possibilities:

•  $\mathcal{I}$  contains  $\theta$  (We refer to this as a **success**.)

•  $\mathcal{I}$  does not contain  $\theta$  (We refer to this as a **failure**.)

Suppose you repeat the experiment above T total times, and assume that the experiments are jointly independent. Moreover, the value of the unknown parameter  $\theta$ , is always assumed to be the same. After conducting these T experiments, you will have constructed T confidence intervals  $\mathcal{I}_1, \mathcal{I}_2, \ldots, \mathcal{I}_T$ .

As T grows very large, what percentage of experiments do you expect to be successes?

90 **✓ Answer**: 90 %

#### Solution:

By the definition of confidence interval, we know that for the j-th experiment ( $1 \le j \le T$ ) that

$$P(\mathcal{I}_j 
i heta) = 90\%.$$

Consider the indicator random variables  $\mathbf{1} (\theta \in \mathcal{I}_1)$ ,  $\mathbf{1} (\theta \in \mathcal{I}_2)$ ,...,  $\mathbf{1} (\theta \in \mathcal{I}_T)$ . Since the experiments are jointly independent, this means that  $\mathbf{1} (\theta \in \mathcal{I}_1)$ ,  $\mathbf{1} (\theta \in \mathcal{I}_2)$ ,...,  $\mathbf{1} (\theta \in \mathcal{I}_T)$  are independent. Moreover, for all j, the random variable  $\mathbf{1} (\theta \in \mathcal{I}_T)$  is Bernoulli because it can only take value 0 or 1. It follows that  $\mathbf{1} (\theta \in \mathcal{I}_1)$ ,  $\mathbf{1} (\theta \in \mathcal{I}_2)$ ,...,  $\mathbf{1} (\theta \in \mathcal{I}_T)$  are identically distributed, because for all j,

$$P\left(\mathbf{1}\left( heta\in\mathcal{I}_{j}
ight)=1
ight)=P\left(\mathcal{I}_{j}
ightarrow heta
ight)=90\%.$$

In summary,  $\mathbf{1}$   $(\theta \in \mathcal{I}_1)$ ,  $\mathbf{1}$   $(\theta \in \mathcal{I}_2)$ , ...,  $\mathbf{1}$   $(\theta \in \mathcal{I}_T) \stackrel{iid}{\sim} \mathrm{Ber}\,(0.9)$ . By the strong law of large numbers,

$$\lim_{T o \infty} rac{\sum_{j=1}^{T} \mathbf{1} \left( heta \in \mathcal{I}_j 
ight)}{T} = \mathbb{E} \left[ \mathbf{1} \left( heta \in \mathcal{I}_j 
ight) 
ight] = 0.9$$

almost surely. Since

$$rac{\sum_{j=1}^{T}\mathbf{1}\left( heta\in\mathcal{I}_{j}
ight)}{T}=rac{ ext{Number of successes}}{ ext{Total number of experiments}},$$

the correct response is 90%.

Submit

You have used 1 of 1 attempt

• Answers are displayed within the problem

## Discussion

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