

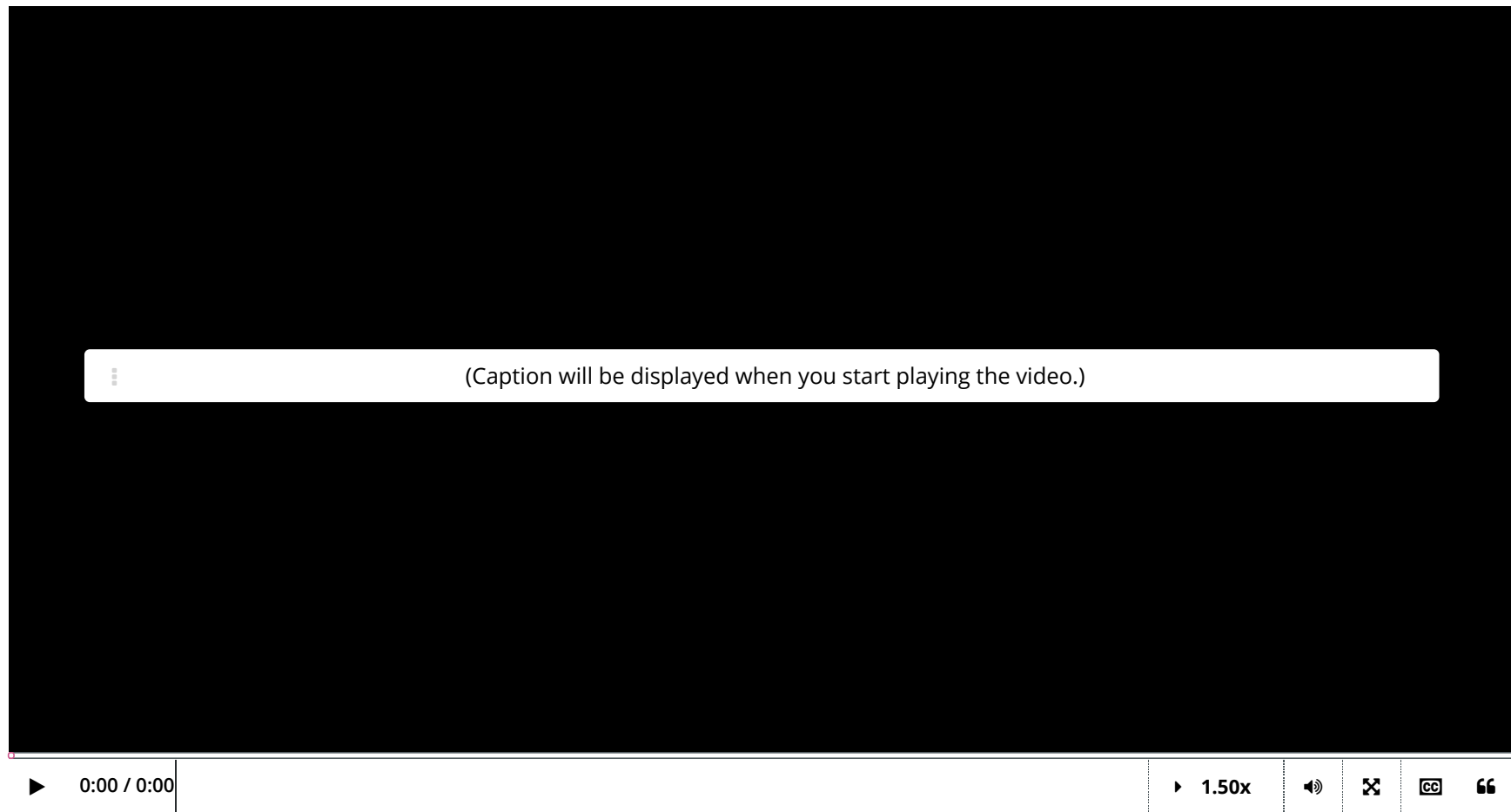


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10. Prediction Error

Prediction Error





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Estimating the variance

1/1 point (graded)

Use same setup as in the previous problems.

Setup as in the previous problems

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Recall the formula for the mean prediction error $\mathbb{E}\|\mathbf{Y} - \mathbb{X}\hat{\boldsymbol{\beta}}\|_2^2$ in terms of n , p and σ^2 (given in the solution of the previous problem). This suggests a formula for an unbiased estimator of σ^2 :

$$\hat{\sigma}^2 = \frac{1}{n-p} \|\mathbf{Y} - \mathbb{X}\hat{\boldsymbol{\beta}}\|_2^2.$$

Assume that based on your findings, the **residual sum** $\|\mathbf{Y} - \mathbb{X}\hat{\boldsymbol{\beta}}\|_2^2$ evaluates to 104.9. Compute $\hat{\sigma}^2$, to the nearest thousandths (10^{-3}) digit.

✓ Answer: 0.105

Solution:

The formula calls for $n = 1000$, $p = 2$ and $\|\mathbf{Y} - \mathbb{X}\hat{\boldsymbol{\beta}}\|_2^2 = 104.9$. Plugging these numbers in gives $\hat{\sigma}^2 = \frac{104.9}{1000-2} \approx 0.105$.

You have used 1 of 3 attempts

i Answers are displayed within the problem

Properties of LSE

► LSE = MLE

► Distribution of $\hat{\beta}$: $\hat{\beta} \sim N\left(\beta, \sigma^2 (X^T X)^{-1}\right)$

► Quadratic risk of $\hat{\beta}$: $\mathbb{E} \left[\|\hat{\beta} - \beta\|_2^2 \right] = \sigma^2 \text{tr} \left((X^T X)^{-1} \right).$

► Prediction error: $\mathbb{E} \left[\|Y - X\hat{\beta}\|_2^2 \right] = \sigma^2 (n - p).$

► Unbiased estimator of σ^2 : $\hat{\sigma}^2 = \frac{\|Y - X\hat{\beta}\|_2^2}{n - p} = \frac{1}{n - p} \sum_{i=1}^n \hat{\varepsilon}_i^2$

Theorem

► $(n - p) \frac{\hat{\sigma}^2}{\sigma^2} \sim \chi_{n-p}^2.$

► $\hat{\beta} \perp \hat{\sigma}^2.$ (Cochran's Theorem)

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