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4. The gradient

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Gradient definition

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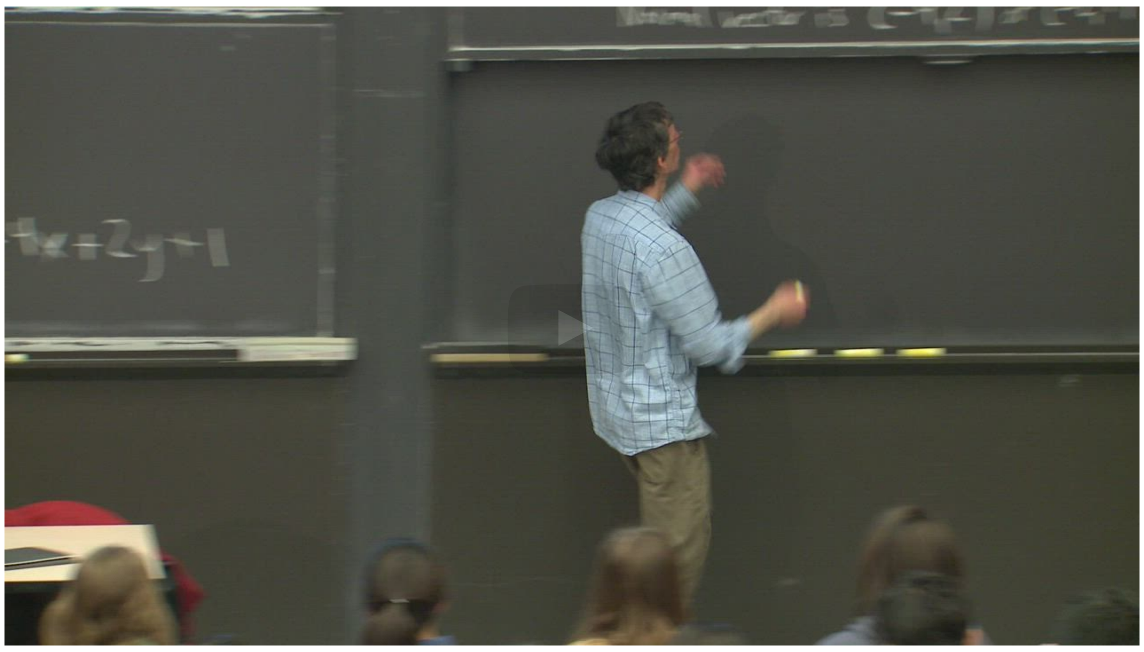

0:00 / 0:00 2.0x

PROFESSOR: So let's summarize what we learned.

It wasn't special for this function.

We figured out how to find the normal vector to level curves.

So here's a summary.

So where did it come from, negative 4, 2.

That negative 4, 2, those were the coefficients

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At any point (x_0, y_0), the vector $(f_x(x_0, y_0), f_y(x_0, y_0))$ is perpendicular to the level curve of f through (x_0, y_0).

Definition 4.1

The vector $\langle f_x, f_y \rangle$ is called the **gradient** of f.

The notation for the gradient of f is ∇f.

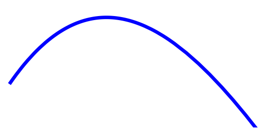
A note about notation

- The upside down triangle ∇ in ∇f is pronounced "del" or "nabla".
- An alternative notation for the gradient is $\operatorname{grad} f$.

A sketch of a proof using the tangent plane

We will look at a sketch of how to prove that the gradient is always normal to the level curve. To do this, let us consider a function $f(x, y)$ and restrict to a level curve $f(x, y) = c$.

Suppose that the point (x, y) lies on this level curve. That is $f(x, y) = c$.



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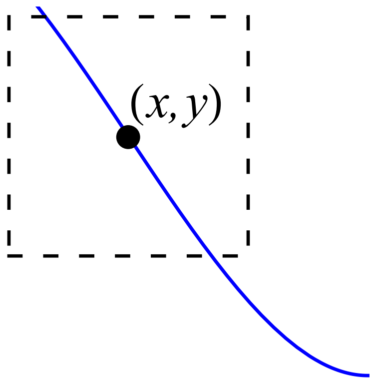
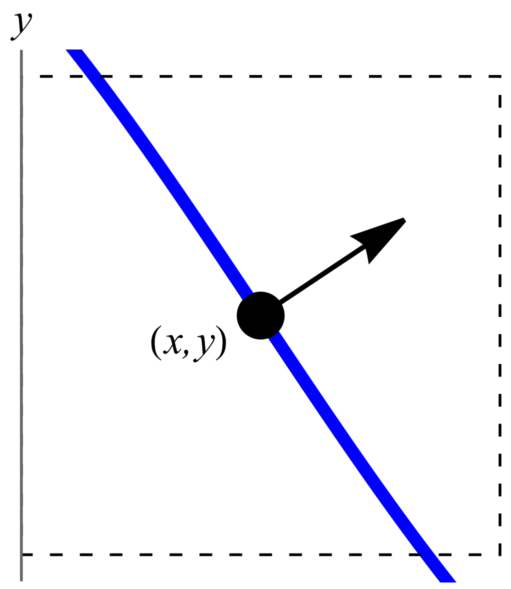


A point (x, y) on a level curve $f(x, y) = c$.

In a small neighborhood of (x, y), the function $f(x, y)$ is well approximated by its tangent plane. And the level curve is approximately a straight line.



We know that the level curve of the straight line tangent approximation is normal to the vector $\langle f_x(x, y), f_y(x, y) \rangle$. This vector is the gradient, thus the gradient is normal to the level curve of the original function as well.

Because this argument holds for any point on any level curve, it is true for all points on all level curves.

Hide

Find the gradient from the function

3.0/3 points (graded)

Note that the names of the variables don't change the form of the gradient. A function $f(x, t)$ has gradient $\nabla f = \langle f_x, f_t \rangle$.

Compute the gradient of the function $f(x, t) = \sin(x - t)$.

(Enter vectors surrounded by square brackets: e.g. `[a,b]` . See help button for more help entering functions of x and t.)

$\nabla f =$ [cos(x-t), -cos(x-t)] ✔ **Answer:** [cos(x-t),-cos(x-t)]

Compute the gradient at the point $(x, t) = (\pi/2, \pi/4)$.

(Enter vectors surrounded by square brackets: e.g. `[a,b]` .)

[1/sqrt(2),-1/sqrt(2)] ✔ **Answer:** [sqrt(2)/2,-sqrt(2)/2]

Compute the gradient at the point $(x, t) = (\pi/2, 0)$.

(Enter vectors surrounded by square brackets: e.g. `[a,b]` .)

[0,0] ✔ **Answer:** [0,0]

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? INPUT HELP

Solution:

The gradient of $f(x,t)$ is

$$\nabla f = \langle \frac{\partial}{\partial x}\sin(x-t), \frac{\partial}{\partial t}\sin(x-t)\rangle = \langle \cos(x-t), -\cos(x-t)\rangle$$

At $(x,t) = (\pi/2, \pi/4)$,

$$\nabla f(\pi/2, \pi/4) = \langle \cos(\pi/4), -\cos(\pi/4)\rangle = \langle \sqrt{2}/2, -\sqrt{2}/2\rangle.$$

At $(x,t) = (\pi/2, 0)$,

$$\nabla f(\pi/2, 0) = \langle \cos(\pi/2), -\cos(\pi/2)\rangle = \langle 0, 0\rangle.$$

Submit You have used 1 of 4 attempts

Answers are displayed within the problem

Gradient zero?

1/1 point (graded)
What does it mean for a gradient to be zero at a point?

- (x) The tangent plane is horizontal.
- ( ) The tangent plane is undefined.
- ( ) Nothing, more information is needed.

✓

Solution:

If the gradient is zero at (x_0, y_0) if and only if $f_x(x_0, y_0)$ and $f_y(x_0, y_0)$ are both zero. By definition, the tangent plane at such points is given by the equation $f(x,y) = f(x_0, y_0)$, which is a horizontal plane.

Submit You have used 2 of 2 attempts

Answers are displayed within the problem

Find the gradient from a tangent plane

1.0/1 point (graded)
Suppose that at the point $(2,-1)$ the tangent plane of a function $f(x,y)$ is given by the equation $g(x,y) = 3 - x/2 + y/3$.

Compute the gradient of $f(x,y)$ at the point $(2,-1)$.

$\nabla f(2,-1) =$ [-1/2,1/3] ✓ **Answer:** [-1/2,1/3]

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Solution:

Recall that the tangent plane at the point $(2,-1)$ is given by

$$z = 3 - x/2 + y/3 = f(2,-1) + f_x(2,-1)(x-2) + f_y(2,-1)(y-(-1)) \quad (3.86)$$
$$= f(2,-1) - 2f_x(2,-1) + f_y(2,-1) + f_x(2,-1)\,x + f_y(2,-1)\,y \quad (3.87)$$

Thus the gradient is $\langle f_x(2,-1), f_y(2,-1)\rangle = \langle -1/2, 1/3\rangle$.

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Answers are displayed within the problem

Given a gradient, find the function

2/2 points (graded)
The gradient of a function is the vector $\langle y^2, 2xy\rangle$. Which of the following could be the function?

(Choose all that apply.)

- [ ] $2xy$
- [ ] y^2
- [ ] $2xy^2$
- [x] xy^2
- [x] xy^2+1
- [x] $xy^2-11100$
- [ ] xy^2+x-y^3

The gradient of a function is the vector $\langle\sqrt{x}, 1\rangle$. Which of the following could be the function?

(Choose all that apply.)

- [ ] $\sqrt{x}$
- [ ] 1
- [ ] $\frac{2}{3}x^{3/2}$
- [ ] $\frac{2}{3}x^{3/2}+1$
- [x] $\frac{2}{3}x^{3/2}+y$
- [x] $2 \ldots$

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$\frac{2}{3}x^{3/2} + y + 4$

$\frac{2}{3}x^{3/2} + y + xy^3 - 3$

Solution:

$\frac{\partial h}{\partial x} = y^2$ $\frac{\partial h}{\partial y} = 2xy$ Thus $h(x, y) = xy^2 + C$

$\frac{\partial k}{\partial x} = \sqrt{x}$ $\frac{\partial k}{\partial y} = 1$ Thus $k(x, y) = \frac{2}{3}x^{3/2} + y + C$

Submit You have used 1 of 4 attempts

Answers are displayed within the problem

4. The gradient

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[STAFF] "Gradient zero?" 2
Tiny correction: In the first sentence of the solution to the "Gradient zero?" question the first "If" and the second "both" are superflu...

Another way to find the gradient from the tangent plane 2
The level curve of the tangent plane near the point is a straight line. We know from our earlier lessons that a normal to this line are th...

[Staff]Error in the box of the sketch of proof. 2
In the final, the text says *We know that the level curve of the straight line tangent approximation is*, but followed by the gradient ve...

Gradient from tangent plane - A little correction might be required in the solution 4

A direct method for finding f(x,y) from grad(f)=<p(x,y),q(x,y)> 3

multi-dimensional slope 4
So is the gradient like a multi-dimensional slope, where the slope in each direction is given by the partial derivative for that direction...

[STAFF] Error in solution to "Find the gradient from a tangent plane" 3
It should be : z = 3 -x/2 +y/3 = f(2,-1) +fx(2,-1)(x-2) +fy(2,-1)(y+1) = f(2,-1) -1/2(x-2) +1/3(y+1) = f(2,-1)+1+1/3 -x/2 +y/3 The gradien...

[STAFF] Typo at "Find the gradient from a tangent plane" 2

[STAFF] Typo in solution to Gradient zero? 2

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