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3. Review the exponential response

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3. Review the exponential response formula

Recall that the exponential response formula gives us a quick method for finding the particular solution to any linear, constant coefficient, differential equations whose input can be expressed in terms of an exponential function.

The exponential response formula (ERF): Let P be a polynomial with real, constant coefficients, $D=\frac{d}{dt}$ a differential operator, and r a (real or complex) number. If $P(r) \neq 0$, then a particular solution to the inhomogeneous differential equation

$$P\left(D
ight)y=e^{rt} \qquad ext{is given by} \qquad y_p=rac{e^{rt}}{P\left(r
ight)}.$$

Caveat

Caveat: If
$$P\left(r
ight)=P'\left(r
ight)=P''\left(r
ight)=\ldots=P^{\left(k-1
ight)}\left(r
ight)=0$$
 , but $P^{\left(k
ight)}\left(r
ight)
eq0$, then a particular solution to $P\left(D
ight)y=e^{rt}$ is given by

$$y_p = rac{t^k e^{rt}}{P^{(k)}(r)}.$$

<u>Hide</u>

Sinusoidal input:

$$P(D) x = \cos(\omega t)$$

is the real part of

$$P\left(D\right)z=e^{i\omega t}$$
 ;

$$P(D) x = \sin(\omega t)$$

is the imaginary part of

$$P(D)z = e^{i\omega t}$$
.

Therefore

- ullet a particular solution to $P\left(D
 ight)x=\cos\left(\omega t
 ight)$ is given by $x_p=\mathrm{Re}\left[rac{e^{i\omega t}}{P\left(i\omega
 ight)}
 ight]$;
- ullet a particular solution to $P\left(D
 ight)x=\sin\left(\omega t
 ight)$ is given by $x_p=\mathrm{Im}\left[rac{e^{i\omega t}}{P\left(i\omega
 ight)}
 ight].$

Review ERF

1/1 point (graded)

Use ERF to find a particular solution to the differential equation

$$(D^3 + D + 3) x = \cos 2t.$$

$$x_p = | (\cos(2*t)-2*\sin(2*t))/15$$

✓ Answer: cos(2*t)/15-2*sin(2*t)/15

$$\cos(2{\cdot}t){-}2{\cdot}\sin(2{\cdot}t)$$

15

FORMULA INPUT HELP

Solution:

A particular solution is given by finding the real part of

$$z_p = rac{e^{i2t}}{\left(i2
ight)^3 + \left(i2
ight) + 3} = rac{e^{i2t}}{3 - i6}.$$

Therefore

$$x_p = \operatorname{Re}\left[rac{e^{i2t}}{3-i6}
ight] = \operatorname{Re}\left[rac{\left(\cos\left(2t
ight) + i\sin\left(2t
ight)
ight)\left(3+i6
ight)}{45}
ight] = rac{\cos2t}{15} - rac{2\sin2t}{15}.$$

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