Important: We account for observations using conditioning. It turns out that often we can solve inference problems *without* using random variables at all and only using events. In this sequence on "Conditioning on Events", to solve the problems presented, you should do them *without* using our machinery for random variables from earlier.

Of course, later on in the course and even beyond the course, depending on the inference problem you're trying to solve, you may find it easier to use events and not random variables, or random variables and not events, or both random variables and events. But for now, let's make sure you can use events and not random variables!

Exercise: Conditioning on Events

(4 points possible)

The six possible outcomes of a fair die roll are all equally likely.

- If we are told that the outcome of a roll is even, what is the probability that the outcome is 6? (Please be precise with at least 3 decimal places, unless of course the answer doesn't need that many decimal places. You could also put a fraction.)

1/3

? **Answer:** 1/3

Now suppose we roll two fair six-sided dice. Let $\mathcal{A}$ denote the event that the outcome of the roll of first die is an even number, and let $\mathcal{B}$ denote the event that the outcome of the second die roll is 3.

- Determine $\mathbb{P}(\mathcal{A} \cap \mathcal{B})$. To do this, first figure out what outcomes are contained in $\mathcal{A} \cap \mathcal{B}$. (Please be precise with at least 3 decimal places, unless of course the answer doesn't need that many decimal places. You could also put a fraction.)

1/12

? **Answer:** 1/12

- Determine $\mathbb{P}(\mathcal{A} \cup \mathcal{B})$. To do this, first figure out what outcomes are contained in $\mathcal{A} \cup \mathcal{B}$. (Please be precise with at least 3 decimal places, unless of course the answer doesn't need that many decimal places. You could also put a fraction.)

7/12

? **Answer:** 7/12

- Determine $\mathbb{P}(\mathcal{A}|\mathcal{B})$. (Please be precise with at least 3 decimal places, unless of course the answer doesn't need that many decimal places. You could also put a fraction.)

1/2

? **Answer:** 1/2

Solution:

The six possible outcomes of a fair die roll are all equally likely.

- If we are told that the outcome of a roll is even, what is the probability that the outcome is 6?

Towards Infinity in Modeling Uncertainty (Week 4)
Exercises due Oct 13, 2016 at 02:30 IST

Homework 3 (Week 4)
Homework due Oct 13, 2016 at 02:30 IST

Solution: A suitable sample space for this problem is $\Omega = \{1, \ldots, 6\}$, and each constituent outcome has probability $1/6$. Let $\mathcal{A}$ denote the event that the outcome of the roll is even, and let $\mathcal{B}$ denote the event that the outcome is 6. Hence, $\mathcal{A} = \{2, 4, 6\}$ and $\mathcal{B} = \{6\}$. Then

$$\mathbb{P}(\mathcal{B}|\mathcal{A}) = \frac{\mathbb{P}(\mathcal{B} \cap \mathcal{A})}{\mathbb{P}(\mathcal{A})} = \frac{\mathbb{P}(\mathcal{B})}{\mathbb{P}(\mathcal{A})} = \frac{1/6}{3/6} = \frac{1}{3}.$$

where to obtain the second equality we have used that $\mathcal{B} \subset \mathcal{A}$ so $\mathcal{B} \cap \mathcal{A} = \mathcal{B}$.

Now suppose we roll two fair six-sided dice. Let $\mathcal{A}$ denote the event that the outcome of the roll of first die is an even number, and let $\mathcal{B}$ denote the event that the outcome of the second die roll is 3.

Solution: A suitable sample space for this problem is $\Omega = \{1, \ldots, 6\} \times \{1, \ldots, 6\}$, which corresponds to the 36 possible outcomes of a pair of dice rolls. In this notation, an outcome $(i, j) \in \Omega$ corresponds to getting i on the first die roll and j on the second. Since all the outcomes are equally likely, each outcome has a probability of $1/36$.

- The set $\mathcal{A} \cap \mathcal{B}$ corresponds to the event that the first die roll is an even number and the second die roll is a 3. Hence, $\mathcal{A} \cap \mathcal{B} = \{(2, 3), (4, 3), (6, 3)\}$. Adding the probabilities of these outcomes, we obtain $\mathbb{P}(\mathcal{A} \cap \mathcal{B}) = 3/36 = 1/12$.

- The set $\mathcal{A} \cup \mathcal{B}$ corresponds to the event that the first die rolls is an even number, or the second die roll is a 3, or both. One way to compute $\mathbb{P}(\mathcal{A} \cup \mathcal{B})$ is to list out all of the outcomes in $\mathcal{A} \cup \mathcal{B}$. However, as an alternative, we can use

$$\mathbb{P}(\mathcal{A} \cup \mathcal{B}) = \mathbb{P}(\mathcal{A}) + \mathbb{P}(\mathcal{B}) - \mathbb{P}(\mathcal{A} \cap \mathcal{B}) = \frac{3}{6} + \frac{1}{6} - \frac{1}{12} = \frac{7}{12},$$

where we have used that $\mathbb{P}(\mathcal{A}) = 3/6$ and $\mathbb{P}(\mathcal{B}) = 1/6$, which we obtain by counting appropriate outcomes.

- Intuitively, conditioning on an event involving only the second die should not change the probability of events involving only the first die, i.e., such events are independent. Our analysis formally verifies this in the particular events of interest:

$$\mathbb{P}(\mathcal{A}|\mathcal{B}) = \frac{\mathbb{P}(\mathcal{A} \cap \mathcal{B})}{\mathbb{P}(\mathcal{B})} = \frac{1/12}{1/6} = \frac{1}{2} = \mathbb{P}(\mathcal{A}).$$

You have used 0 of 5 submissions