Counting Prime Numbers (1)

Question Can we count prime numbers?

$\pi(N)$ = the number of prime numbers P with $\mathbf{P} \leq \mathbf{N}$

$\pi(10) = 4 \quad (2, 3, 5, 7)$

$\pi(100) = 25 \quad (2, 3, 5, 7, \cdots, 97)$

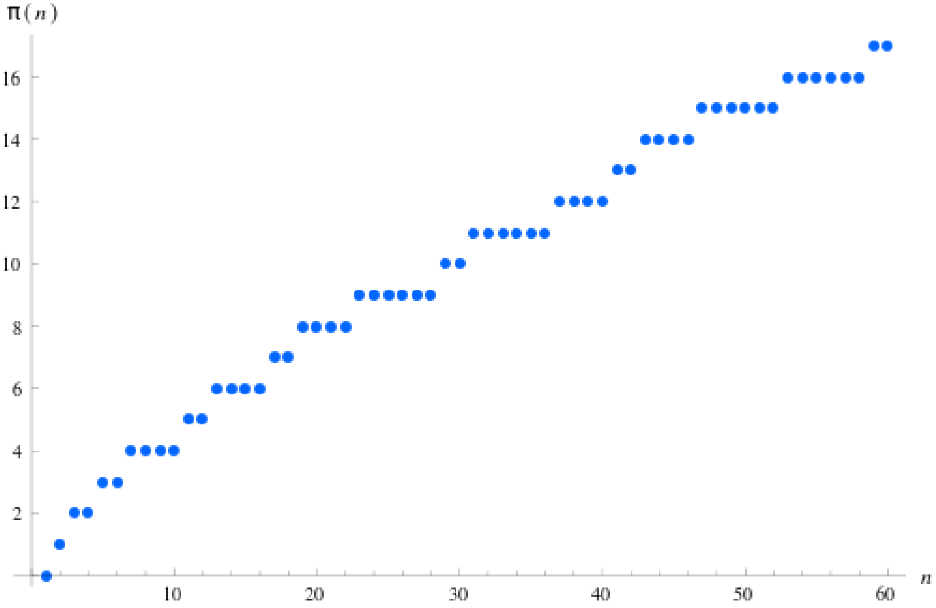
$\pi(1000) = 168$

$\pi(10000) = 1229$

$\pi(N) = ???$

Counting Prime Numbers (2)

$\pi(N) \fallingdotseq N/\log(N)$?



Adrien-Marie Legendre (1752-1833)

Carl Friedrich Gauss (1777-1855)

https://en.wikipedia.org/wiki/Prime-counting_function
https://en.wikipedia.org/wiki/Adrien-Marie_Legendre
https://en.wikipedia.org/wiki/Carl_Friedrich_Gauss

Counting Prime Numbers (3)

Prime Number Theorem (1896)

As N→∞, π(N) increases like N/log(N).

$$\lim_{N\to\infty} \frac{\pi(N)}{N/\log(N)} = 1$$

Jacques Salomon Hadamard (1865-1963)

Charles Jean de la Vallée-Poussin (1866-1962)

https://en.wikipedia.org/wiki/Jacques_Hadamard
https://en.wikipedia.org/wiki/Charles_Jean_de_la_Vall%C3%A9e-Poussin

Counting Prime Numbers (4)

- The proof of PNT is complex analytic. It requires detailed analysis of the **Riemann zeta function**.

$$\zeta(s) = \sum_{N=1}^{\infty} \frac{1}{N^s} = 1 + \frac{1}{2^s} + \frac{1}{3^s} + \frac{1}{4^s} + \frac{1}{5^s} + \frac{1}{6^s} + \cdots$$

- Today, PNT is successfully generalized to many directions (**Density Theorems** on prime numbers).