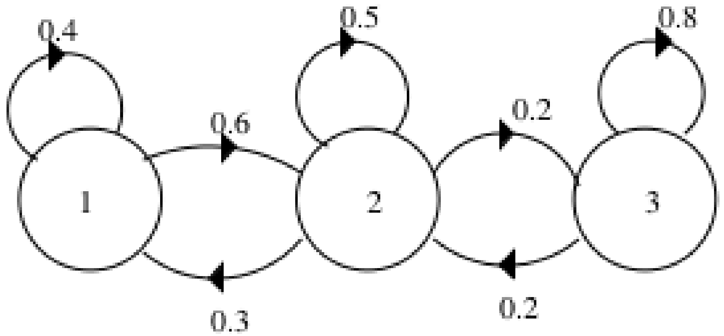
Exercise: Birth and death

(4/5 points)

Consider the Markov chain below. Let us refer to a transition that results in a state with a higher (respectively, lower) index as a birth (respectively, death). Calculate the following probabilities, assuming that when we start observing the chain, it is already in steady-state.



1. The steady-state probabilities for each state.

$\pi_1 =$ 0.2 ✔ **Answer:** 0.2

$\pi_2 =$ 0.4 **Answer:** 0.4

$\pi_3 =$ 0.4 **Answer:** 0.4

2. The probability that the first transition we observe is a birth.

0.2 **Answer:** 0.2

3. The probability that the first change of state we observe is a birth.

0.28 **Answer:** 0.36

Answer:

1. The local balance equations take the form $0.6\pi_1 = 0.3\pi_2$ and $0.2\pi_2 = 0.2\pi_3$. Together with the normalization equation, we get $\pi_1 = 1/5$, $\pi_2 = \pi_3 = 2/5$.

2. We observe a birth if (i) we are in state 1 and the next transition is from 1 to 2, or (ii) we are in state 2 and the next transition is from 2 to 3. Hence, the desired probability is $\pi_1 p_{12} + \pi_2 p_{23} = 1/5$.

3. Note that a self-transition is not a change of state. If the state is 1, which happens with probability $1/5$, the first change of state is certain to be a birth. If the state is 2, which happens with probability $2/5$, the next change of state is to either 1 or 3. The probability that it is to 3 (i.e., a birth) is $p_{23}/(p_{21} + p_{23}) = 0.2/(0.3 + 0.2) = 2/5$.

Finally, if the state is 3, the probability that the first change of state is a birth is equal to 0 since 3 is the highest state. Thus, the probability that the first change of state that we observe is a birth is equal to $(1/5)(1) + (2/5)(2/5) = 9/25$.

You have used 2 of 2 submissions