Uniformity

The Lebesgue Measure is one kind of measure on the Borel Sets. But there are others.

A function on the Borel Sets is said to be a **measure** if and only if it satisfies Countable Additivity and Non-Negativity (and for it to assign the value 0 to the empty set).

Accordingly, the Lebesgue Measure is a measure with the additional property of satisfying Length on Segments.

This is important because it makes the Lebesgue Measure *uniform*.

Intuitively speaking, what it means for a measure to be uniform is for it to be such that the measure of a set always remains unchanged when the set is moved to a different location within the real line.

Formally speaking, a measure μ is uniform if and only if:

> **Uniformity**
> Let $\mu(A)$ be well-defined, and let A^c be the result of translating A by $c \in \mathbb{R}$. Then $\mu(A^c)$ is well-defined, and equal to $\mu(A)$.
>
> (The **translation** of A by c is the result of adding c to each member of A. In other words: $A^c = \{x + c : x \in A\}$. Intuitively, you can think of the translation of A by c as the result of rigidly moving A c units to the right.)

Here is an example. The Lebesgue Measure of $[0, \frac{1}{4}]$ is $\frac{1}{4}$. So the uniformity of Lebesgue Measure entails that the Lebesgue Measure of $[0, \frac{1}{4}]^{\frac{1}{2}}$ (which is the result of translating $[0, \frac{1}{4}]$ by $\frac{1}{2}$) must also equal $\frac{1}{4}$.

Uniformity is such a natural property that it is tempting to take it for granted. But there are all sorts of measures that do not satisfy Uniformity.

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We'll talk about one of them next.

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