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12.1.4 Unstable Growth of Forward Euler Method

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MO2.5 MO2.7

As we observed in the previous example, if the timestep Δt is too large, application of the Forward Euler method can lead to unstable growth of the numerical solution as the number of timesteps taken increases (i.e. as n increases) even though the exact solution does not have such unstable behavior. In this section, we will build some insight into why this is happening. We note that this unstable behavior can be theoretically explained in a more general manner, however, this more general theory is beyond the scope of this class.

The potential for unstable growth can be understood by looking at the simple exponential model problem discussed in Section 8.6. The differential equation in that case is

$$\frac{\mathrm{d}u}{\mathrm{d}t} = \lambda u \tag{12.4}$$

The solution to this problem starting from an initial condition of $u(0) = u^0$ gives the analytic solution,

$$u(t) = u^0 \exp(\lambda t) \tag{12.5}$$

Recall that Forward Euler to this problem gives the following method to find u^{n+1} from u^n:

$$u^{n+1} = u^n + \Delta t \lambda\, u^n \tag{12.6}$$

Figure 12.5 shows the application of Forward Euler to the exponential decay situation (i.e. $\lambda < 0$) in which the timestep $\Delta t = \Delta t_A$ is chosen so that $u^n = 0$ for all $n > 0$. We label this Forward Euler solution with $\Delta t = \Delta t_A$ as u_A^n. We can determine this Δt_A by applying the Forward Euler method and requiring $u_A^1 = 0$:

$$u_A^1 = u^0 + \Delta t_A \lambda\, u^0 \tag{12.7}$$

$$= (1 + \Delta t_A \lambda)\, u^0 \tag{12.8}$$

Thus, if $\Delta t_A = -1/\lambda$ then $u_A^1 = 0$ independent of the value of u^0. And, since $u_A^1 = 0$ then any following u_A^n will also be zero. For example, u_A^2 gives:

$$u_A^2 = u_A^1 + \Delta t_A \lambda\, u_A^1 = 0 \tag{12.9}$$

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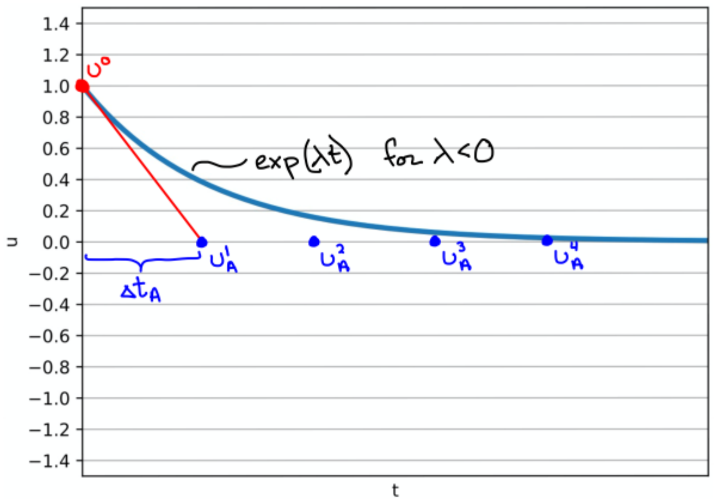

Figure 12.5: Forward Euler behavior for exponential decaying solution $\mathrm{d}u/\mathrm{d}t = \lambda u$ with $\lambda < 0$ and $\Delta t = \Delta t_A = -1/\lambda$ chosen so that $u_A^n = 0$ for $n > 0$.

Now, suppose we take a timestep which is twice Δt_A and call that $\Delta t_B = 2\Delta t_A = -2/\lambda$. This situation in shown in Figure 12.6. After one timestep, we can show that $u_B^1 = -u^0$, specifically,

$$u_B^1 = u^0 + \Delta t_B \lambda u^0 \tag{12.10}$$

$$= (1 + \Delta t_B \lambda)\, u^0 \tag{12.11}$$

$$= \left(1 - \frac{2}{\lambda}\lambda\right) u^0 \tag{12.12}$$

$$= (1 - 2)\ u^0 \tag{12.13}$$

$$\Rightarrow u_B^1 = -u^0 \tag{12.14}$$

Then, consider the next timestep that leads to:

$$u_B^2 = u_B^1 + \Delta t_B \lambda u_B^1 \tag{12.15}$$

$$= (1 + \Delta t_B \lambda)\ u_B^1 \tag{12.16}$$

$$= (1 + \Delta t_B \lambda)\ (-u^0) \tag{12.17}$$

$$= (-1)\ (-u^0) \tag{12.18}$$

$$\Rightarrow u_B^2 = u^0 \tag{12.19}$$

Thus, the second timestep returns back to u^0. This oscillation will continue for all n so that

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