Expected value of max/min of random variables

I am trying to solve the following problem.

Let there be n urns and let us have k balls. Assume we put every ball into one of the urns with uniform probability. Denote by X_i the random variable counting the number of balls in urn i. If $X = min\{X_1, \ldots, X_n\}$, what is $E[X]$?

As a more general question, one could ask: what is the expected value of the minimum of some equally distributed random variables?

I do not see any way of solving it besides using the definition of expected value which results in a nasty expression.

I believe there is some better technique for approaching this kinds of problems.

Anyone happens to know how?

(probability-theory)

asked May 1 '11 at 23:34

Jernej
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I would expect this particular question to be difficult. It may be easier in cases where the random variables are identically *and independently* distributed, using order statistics methods, but they are not independent here. – Henry May 2 '11 at 0:02

So in the limit where n is large (lots of urns) and $k = \alpha n$, the number of balls in urn i will be approximately Poisson with mean α. Furthermore if n is large then the counts in the different boxes will be approximately independent. So you can probably get an approximate answer starting from this using order statistics methods, as Henry suggested. – Michael Lugo May 2 '11 at 0:28

It seems that for $n = 2$, $E\left[\min(X_i)\right]$ may be $k\left(1/2 - \binom{k-1}{\lfloor k/2 \rfloor}/2^k\right)$. I would not expect this to get easier in general. – Henry May 2 '11 at 0:31

Perhaps it does get easier. For $n = 2$ and large k it seems that $\dfrac{k}{2} - \sqrt{\dfrac{k}{2\pi}}$ is a reasonable approximation. In general, $\dfrac{k}{n}$ is clearly an upper bound, so perhaps there is a general approximation. – Henry May 2 '11 at 0:51