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1. Lecture 1

The following can be done after Lecture 1.

1-1

5/5 points (graded)

The matrix

$$\begin{pmatrix} 1 & 9 & 9 & 0 & 1 & 1 & 1 & 8 \\ 0 & 0 & 7 & 0 & 7 & 0 & 3 & 6 \\ 0 & 0 & 0 & 0 & 0 & 0 & 6 & 6 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

is in row-echelon form. Which of its columns are pivot columns? (Recall that columns are numbered from left to right starting with the number 1.)

☑ Column 1 ✓

☐ Column 2

☑ Column 3 ✓

☐ Column 4

☐ Column 5

☐ Column 6

☑ Column 7 ✓

☐ Column 8

✓

Solution:

Columns 1, 3, and 7.
Row 1 has a pivot in column 1. Row 2 has a pivot in column 3. Row 3 has a pivot in column 7. Rows 4 and 5 have no pivot. So the pivot columns, the columns that contain a pivot, are columns 1, 3, and 7.

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You have used 1 of 5 attempts

Answers are displayed within the problem

1-2

5/5 points (graded)

Which of the following matrices are in row-echelon form?

(Our definition of row-echelon form does not require pivots to be 1.)

☑ $$\begin{pmatrix} 0 & 1 & 2 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$ ✓

☑ $$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 8 \end{pmatrix}$$ ✓

☐ $$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 3 & 3 & 1 \\ 0 & 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

☐ $$\begin{pmatrix} 1 & 0 & 0 & 2 & 8 \\ 0 & 1 & 0 & 5 & 5 \\ 0 & 0 & 1 & 6 & 6 \\ 0 & 1 & 0 & 3 & 1 \end{pmatrix}$$

☑ $$\begin{pmatrix} 0 & 1 & 1 & 4 & 0 \\ 0 & 0 & 1 & 5 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ ✓

✓

Solution:

$$\begin{pmatrix} 0 & 1 & 2 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 8 \end{pmatrix}, \begin{pmatrix} 0 & 1 & 1 & 4 & 0 \\ 0 & 0 & 1 & 5 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}.$$

Row-echelon form means

1. All the zero rows (if any) are grouped at the bottom of the matrix.
2. Each pivot lies farther to the right than the pivots of higher rows.

Matrix $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 3 & 3 & 1 \\ 0 & 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix}$ violates the first condition. Matrix $\begin{pmatrix} 1 & 0 & 0 & 2 & 8 \\ 0 & 1 & 0 & 5 & 5 \\ 0 & 0 & 1 & 6 & 6 \\ 0 & 1 & 0 & 3 & 1 \end{pmatrix}$ violates the second condition. The others are OK.

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You have used 1 of 5 attempts

Answers are displayed within the problem

1-3

5/5 points (graded)

Which of the following matrices are in **reduced** row-echelon form?

☐ $$\begin{pmatrix} 0 & 1 & 5 & 3 & 0 \\ 0 & 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

☐ $$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 8 & 6 & 1 \\ 0 & 0 & 1 & 4 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

☐ $$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 4 \end{pmatrix}$$

☑ $$\begin{pmatrix} 0 & 1 & 7 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$ ✓

☐ $$\begin{pmatrix} 1 & 0 & 0 & 2 & 8 \\ 0 & 1 & 0 & 3 & 3 \\ 0 & 0 & 1 & 6 & 6 \\ 0 & 1 & 0 & 3 & 4 \end{pmatrix}$$

✓

Solution:

Only $\begin{pmatrix} 0 & 1 & 7 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$.

A matrix is in reduced row-echelon form if all of the following hold:

1. The matrix is in row-echelon form.
2. In each nonzero row, the pivot is a 1.
3. In each pivot column, all the entries are 0 except for the pivot itself.

Matrices $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 8 & 6 & 1 \\ 0 & 0 & 1 & 4 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix}$ and $\begin{pmatrix} 1 & 0 & 0 & 2 & 8 \\ 0 & 1 & 0 & 3 & 3 \\ 0 & 0 & 1 & 6 & 6 \\ 0 & 1 & 0 & 3 & 4 \end{pmatrix}$ violate the first condition.

Matrix $\begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 4 \end{pmatrix}$ violates the second condition.

Matrix $\begin{pmatrix} 0 & 1 & 5 & 3 & 0 \\ 0 & 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$ violates the third condition (the third column is a pivot column, but has a 5 in addition to the pivot).

Only matrix $\begin{pmatrix} 0 & 1 & 7 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$ satisfies all the conditions.

Submit You have used 1 of 5 attempts

ℹ Answers are displayed within the problem

1-4

5/5 points (graded)

How many free variables are there in the following (consistent) system?

$$\begin{aligned} 2x + y + z + 4u + 3v + 2w &= 8 \\ 9z + 5u + 2v + 2w &= 4 \\ 2u + 2v + 6w &= 1 \\ 2w &= 8 \end{aligned}$$

(Enter a number in the answer box below.)

2

✓ **Answer:** 2

2

Solution:

2.

The augmented matrix is $\left(\begin{array}{cccccc|c} 2 & 1 & 1 & 4 & 3 & 2 & 8 \\ 0 & 0 & 9 & 5 & 2 & 2 & 4 \\ 0 & 0 & 0 & 2 & 2 & 6 & 1 \\ 0 & 0 & 0 & 0 & 0 & 2 & 8 \end{array}\right)$. Columns 1, 3, 4, 6 are pivot columns. The free variables correspond to non-pivot columns, excluding the augmented column: these are columns 2 and 5. So the number of free variables is 2.

Submit

You have used 2 of 3 attempts

🛈 Answers are displayed within the problem

1-5

5/5 points (graded)

Let $\mathbf{f}$ be the function from $\mathbb{R}^3$ to $\mathbb{R}^2$ represented by the matrix $\begin{pmatrix} 2 & 3 & 5 \\ 7 & 11 & 13 \end{pmatrix}$, and let $\mathbf{v} = \begin{pmatrix} 3 \\ -5 \\ 4 \end{pmatrix}$. What is the second coordinate of $\mathbf{f}(\mathbf{v})$?

18

✓ **Answer:** 18

Solution:

18.

We have $\mathbf{f}(\mathbf{v}) = \begin{pmatrix} 2 & 3 & 5 \\ 7 & 11 & 13 \end{pmatrix} \begin{pmatrix} 3 \\ -5 \\ 4 \end{pmatrix} = \begin{pmatrix} 11 \\ 18 \end{pmatrix}$, whose second coordinate is 18.

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You have used 1 of 5 attempts

Answers are displayed within the problem

1-6

5/5 points (graded)

Let $\mathbf{f} : \mathbb{R}^2 \to \mathbb{R}^2$ be reflection about the line $y = -x$. What matrix represents $\mathbf{f}$? (Hint: Draw what $\mathbf{f}$ does to $\mathbf{e}_1$ and $\mathbf{e}_2$.)

○ $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$

○ $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

○ $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$

○ $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

○ $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$

○ $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

● $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$ ✓

○ None of the above.

Solution:

The answer is $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$.

If $\mathbf{A}$ is the desired matrix, then

$$\begin{aligned} \text{(first column of } \mathbf{A}\text{)} &= \mathbf{f}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \end{pmatrix} \\ \text{(second column of } \mathbf{A}\text{)} &= \mathbf{f}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \end{pmatrix}. \end{aligned}$$

Submit You have used 2 of 4 attempts

ℹ Answers are displayed within the problem

1-7

5/5 points (graded)

What is a geometric description of the function represented by the matrix $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$?

- ● Clockwise rotation by 90° ✓
- ○ Counterclockwise rotation by 90°
- ○ Rotation by 180°
- ○ Reflection in the x-axis
- ○ Reflection in the y-axis
- ○ Reflection in the line $y = x$
- ○ Reflection in the line $y = -x$
- ○ None of the above

Solution:

The answer is clockwise rotation by 90°.

This can be seen by plugging in some vectors: for example, the linear transformation maps $(2, 1)$ to $(1, -2)$. In general, if the input vector is (a, b), the output vector $(b, -a)$ has the same length and is perpendicular to the input vector (the dot product is 0), and the direction of rotation can be seen on examples.

Alternatively, the matrix representing counterclockwise rotation by θ is $\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$; here we have the special case $\theta = -\pi/2$.

Submit

You have used 1 of 4 attempts

ℹ Answers are displayed within the problem

1. Lecture 1

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☑ 1-5: Coordinate 2

What does coordinate mean in this context? I tried looking up the previous lectures and notes but could…

? 1-6 3

💬 Problem 1-4 3

In case somebody is wondering what a consistent system means, the answer is [here][1]. Short answer: I…

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