11.2.3 Projection onto a Subspace

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11.2.3 Projection onto a Subspace

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Homework 11.2.3.1

7/7 points (graded)

Consider $A = \begin{pmatrix} 1 & 1 \\ 1 & -1 \\ -2 & 4 \end{pmatrix}$ and $b = \begin{pmatrix} 1 \\ 2 \\ 7 \end{pmatrix}$.

1. Find the projection of b onto the column space of A.

2.090909 ✔ Answer: 92/44

-1.272727 ✔ Answer: -56/44

5.909091 ✔ Answer: 260/44

The formula for the projection, when A has linearly independent columns, is

$$A(A^T A)^{-1} A^T b.$$

Now

$$\left(\begin{pmatrix} 1 & 1 \\ 1 & -1 \\ -2 & 4 \end{pmatrix}^T \begin{pmatrix} 1 & 1 \\ 1 & -1 \\ -2 & 4 \end{pmatrix} \right) \begin{pmatrix} 6 & 8 \end{pmatrix}$$

Calculator

$$A^TA = \begin{pmatrix} 1 & -1 \\ -2 & 4 \end{pmatrix}\begin{pmatrix} 1 & -1 \\ -2 & 4 \end{pmatrix} = \begin{pmatrix} 6 & -8 \\ -8 & 18 \end{pmatrix}$$

$$(A^TA)^{-1} = \frac{1}{(6)(18)-(-8)(-8)}\begin{pmatrix} 18 & 8 \\ 8 & 6 \end{pmatrix} = \frac{1}{44}\begin{pmatrix} 18 & 8 \\ 8 & 6 \end{pmatrix}$$

$$A^Tb = \begin{pmatrix} 1 & 1 \\ 1 & -1 \\ -2 & 4 \end{pmatrix}^T\begin{pmatrix} 1 \\ 2 \\ 7 \end{pmatrix} = \begin{pmatrix} -11 \\ 27 \end{pmatrix}$$

$$(A^TA)^{-1}A^Tb = \frac{1}{44}\begin{pmatrix} 18 & 8 \\ 8 & 6 \end{pmatrix}\begin{pmatrix} -11 \\ 27 \end{pmatrix} = \frac{1}{44}\begin{pmatrix} 18 \\ 74 \end{pmatrix}$$

$$A(A^TA)^{-1}A^Tb = \frac{1}{44}\begin{pmatrix} 1 & 1 \\ 1 & -1 \\ -2 & 4 \end{pmatrix}\begin{pmatrix} 18 \\ 74 \end{pmatrix} = \frac{1}{44}\begin{pmatrix} 92 \\ -56 \\ 260 \end{pmatrix}$$

which I choose not to simplify...

2. Split b into $z + w$ where z is in the column space and w is perpendicular (orthogonal) to that space.

$w =$ -1.090909 ✔ Answer: 1- 92/44

3.272727 ✔ Answer: 2 + 56/44

1.090909 ✔ Answer: 7 - 260/44

Notice that $z = A(A^TA)^{-1}A^Tb$ so that

$$w = b - A(A^TA)^{-1}A^Tb = \begin{pmatrix} 1 \\ 2 \\ 7 \end{pmatrix} - \frac{1}{44}\begin{pmatrix} 92 \\ -56 \\ 260 \end{pmatrix}.$$

3. Which of the four subspaces $C(A)$, $R(A)$, $\mathcal{N}(A)$, $\mathcal{N}(A^T)$ contains w?

- [ ] $C(A)$
- [ ] $R(A)$
- [ ] $\mathcal{N}(A)$
- [x] $\mathcal{N}(A^T)$

✔

This vector is orthogonal to the column space and therefore is in the left null space of A: $w \in \mathcal{N}(A^T)$.

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