The Two-Envelope Paradox

Introducing the Two-Envelope Paradox

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Start of transcript. Skip to the end.

I brought with me two envelopes.

And I don't know if you can see with a light.

But I filled them both with money.

OK.

So I'm going to tell you something

about how I filled them I picked a number N, a positive integer

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“Free Money!”, reads a sign near the town square. Next to the sign is small man in a white suit. Next to the man is a table, with two envelopes on top. “This must be a joke!” you think to yourself as you walk by. “Who could possibly be giving out free money?” A few days later, however, a friend informs you that the sign is for real. The small man in the white suit is actually giving out free money. “There must be some kind of catch!” you say. “How does it work?”

“It’s very simple,” your friend explains. “The man has placed a check in each envelope. We don’t know how much money the checks are made out for—it’s different each day. But we do know about the method that the man uses to fill out the checks. The first check is made out for n dollars, where n is a natural number chosen at random. The second check is made out for $2n$ dollars. Unfortunately, you won’t know which of the two checks has the larger amount and which one has the smaller one.”

“That’s it?” you ask, incredulous. “That’s it!” your friend replies. “All you need to do is pick one of the envelopes, and you’ll get to keep the check. I played the game last week, and I ended up with a check for $1,834,288. I took it to the bank, and cashed it without a hitch. Now I’m looking to buy a beach house. It’s a shame the man won’t let you play more than once!”

Unable to help yourself any longer, you decide to pay a visit to the man in the white suit. “Welcome!” he says, with a big smile. “Select one of these envelopes, and its contents will be yours.” After a moment’s hesitation, you reach for the envelope on the left. How wonderful it would be to have your very own beach house! As you are about to open the envelope, the man interjects: “Are you interested in mathematics?”

“Why, yes. I am,” you say. “Why do you ask?”

“You’ve selected the envelope on the left, but I’ll give you a chance to change your mind. Think about it for a moment,” the man adds with a smile, “it might be in your interests to do so.” You consider the issue for a moment, and conclude that the man right: you should switch! For you reason as follows:

Even though your envelope remains sealed, you know it contains a certain amount of money: k dollars, say. If k is odd, you should definitely switch.

What about the case in which k is even? In that case the envelope on the right must contain either $2k$ dollars or $k/2$ dollars. These outcomes ought to have equal probability, since the initial number n was selected at random. And if the outcomes have equal probability, you should switch. For although it is true that if you switch you are just as likely to gain money as you are to lose money, what you'll gain if you gain is more than what you'll lose if you lose. More specifically: if you gain, you'll gain k dollars, and if you lose you'll lose $k/2$ dollars.

We can put this reasoning more formally by using the notion of expected value, which was introduced in Lecture 4:

$$EV\,(\text{switch}) = k/2 \cdot 0.5 + 2k \cdot 0.5 = 5/4 \cdot k$$

And $5/4 \cdot k$ is always larger than k. In contrast, the expected value of staying is just k. So the Principle of Expected Value Maximization entails that you should switch!

"All right!" you cry, "I'll switch!" With a trembling hand, you place the envelope you had been holding back on the table, still unopened, and pick up the other. As you prepare to open it, the man in the white suit interrupts: "Excuse me, but are you sure you don't want to switch again? It might be in your interests to do so…" It is at that point that the problem dawns on you. The exact same reasoning that led you to switch the first time could be used to argue that you should switch again, and go back to your original selection.

Problem 1

1/1 point (ungraded)
When I speak of "probability" and "chance" above, should I be understood as talking about subjective probability or objective probability?

- (•) Subjective probability
- ( ) Objective probability
- ( ) Both

✔

Explanation

What I have in mind is subjective probability and, more specifically, the credences that you ought to have, given the information at your disposal.

Since there is a fact of the matter about how much money each of the envelopes contains, the objective probability of getting $k/2$ dollars if you switch must be either 0 or 1, rather than 50%, as I suggest in the text. So the difference between objective probability and subjective probability definitely matters.

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I see no real Paradox in the two-envelope story

discussion posted 2 days ago by zeszes6

Unfortunately I do not see a paradox in the two-envelope story, it could be I am missing something still, but I see only some problems in the reasoning. Let's name the first envelope "1", and the second envelope, which contains double the money, "2". As long as we have not opened an envelope we have no clue if the left envelope is "1" or "2". Let's define P(1,odd) as the probability that the left envelope is envelope "1", and so on. Likewise we define P(1) to be the chance that envelope "1" is the left envelope. Same idea for P(2). We now find: P(1,odd) = P(odd|1) x P(1) = 0.5 x 0.5 = 0.25 P(1,even) = P(even|1) x P(1) = 0.5 x 0.5 = 0.25 P(2,odd) = P(odd|2) x P(2) = 0 x 0.5 = 0 P(2,even) = P(even|2) x P(2) = 1 x 0.5 = 0.5 Since P(1,odd) and P(1,even) are mutually exclusive events, we find: P(1) = P(1,odd) + P(1,even) = 0.25 + 0.25 = 0.5 Same reasoning gives: P(2) = 0 + 0.5 = 0.5 If we take this reasoning and compare it with the reasoning done in the text, we see that in case the left envelope would contain an even number it would be unwise to change the envelope, since we have a better chance we have the envelope with the most money. So the first flaw I see in the text is "These outcomes ought to have equal probability". In the calculation of the expected value I see two extra flaws. First: since we have no clue if the money in the envelope is odd or even, we should take all possibilities above into account. Second: we only know for certain now that envelope "1" contains a certain amount k and envelope "2" contains amount 2k. Taking those two ideas into account the correct calculation for the expected value becomes: EV(switch) = P(1,odd).2k + P(1,even).2k + P(2,odd).k + P(2,even).k = 0.25 x 2k + 0.25 x 2k + 0 x k

+ 0.5 x k = 1.5k This is perfectly in line with what we want: the amount of money we expect to receive on average would be 1.5k if we switch. The amount if we just pick an envelope randomly without doing the switch, would also be 1.5k.

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1 response

tschrans
2 days ago

Hard to read if you don't use the useful newline insertions.

Unfortunately I do not see a paradox in the two-envelope story, it could be I am missing something still, but I see only some problems in the reasoning.
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Likewise we define P(1) to be the chance that envelope "1" is the left envelope.
Same idea for P(2).
We now find:
P(1,odd) = P(odd|1) x P(1) = 0.5 x 0.5 = 0.25
P(1,even) = P(even|1) x P(1) = 0.5 x 0.5 = 0.25
P(2,odd) = P(odd|2) x P(2) = 0 x 0.5 = 0
P(2,even) = P(even|2) x P(2) = 1 x 0.5 = 0.5
Since P(1,odd) and P(1,even) are mutually exclusive events, we find: P(1) = P(1,odd) + P(1,even) = 0.25 + 0.25 = 0.5
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This is perfectly in line with what we want: the amount of money we expect to receive on average would be 1.5k if we switch.
The amount if we just pick an envelope randomly without doing the switch, would also be 1.5k.

posted 2 days ago by **zeszes6**

You should say Fortunately you don't see a paradox.

I've noticed in this class that there aren't really too many real paradoxes, just (intentionally) created confusion and incomplete situations to make it seem like paradoxes.

The problem is real simple. You have to envelops E_1 and E_2 each with an amount of money M_1 and M_2 you can earn. And M_1 is the money in envelope 1, while M_2 is in the second envelope. Choosing and envelope doesn't change the amount in the envelope.

So we have

$P(M_1|E_1) = 1$ and $P(M_2|E_1) = 0$. So the probability of getting M_1 under the condition of choosing envelope 1 is 100% and 0% for getting M_2.

Similarly

$P(M_1|E_2) = 0$ and $P(M_2|E_2) = 1$

What you also know is that one of the amounts is larger (double the other) so

$P(M_1 = 2M_2) = P(M_1 = \frac{1}{2}M_2) = 0.5$ or think of it as the factor of 2 doesn't really matter, it just changes the expected value

$P(M_1 < M_2) = P(M_1 > M_2) = 0.5$

But is that really correct? No, the fact that you don't know which has the larger value doesn't mean that it is random. It is predetermined so what you really have is either $P(M_1 > M_2) = 1$ or $P(M_1 > M_2) = 0$, which is not the same as $P(M_1 < M_2) = P(M_1 > M_2) = 0.5$ It's a little bit like the problem in the notes on the coin flip for which the results is predetermined at the beginning of the universe. The fact that you don't know the result doesn't mean it's random with probability 50%.

So if you say you pick the envelope randomly, then the expected value is $\frac{M_1+M_2}{2}$ and it's fixed. It doesn't change because you picked on or the other envelope.

The expected value doesn't change by switching and it doesn't change once you've picked an envelope. So you have $EV(switch) = \frac{M_1+M_2}{2}$ and not what is stated in the notes.

What does change when you switch is whether you have M_1 or M_2.

Like most so called paradoxes, it's just confusion and incomplete analysis.

posted 2 days ago by **tschrans**

•••

@tschrans
"You should say Fortunately you don't see a paradox."
I agree more or less with you tschrans, depending on the point of view however.
For the ones that claim it is a true paradox I think that's kind of unfortunate.
For myself however it's also unfortunate, since I was expecting that when problems are stated in this course which may appear as a paradox but clearly are not, at least the answer to where the reasoning went wrong would be explained in the course itself.
Or also satisfying to me would be if 'real' paradoxes were stated which many geniuses already stumbled upon without being able to come up with a clear explanation.
I am happy however to see there are more critical beings here who noticed the same.

posted a day ago by **zeszes6**

•••

I feel your same pain and disappointment. That many of the seemingly paradoxes in the class are just poorly worded and incomplete description. Let me give you an example of a poorly worded problem that seems to be a paradox, but actually is not.

1. I give you two numbers x and y
2. I give you an algebraic relationship between the the numbers $x + y = 0$

What are the values of x and y?

Note this is the same construct as the cube problem.

1. You have cubes of length l
2. The relationship is that $0 < l \leq 1$

What is the probability that $0 < l < 0.5$

In the lectures we conclude from the cube problem that the indifference theorem is crippled.

Similarly from my example I have to conclude following the same logic as in the lectures that algebra is crippled.

posted a day ago by **tschrans**

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