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Lecture 11: Fisher Information, Asymptotic Normality of MLE;

Course > Unit 3 Methods of Estimation > Method of Moments

> 3. Fisher Information

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3. Fisher Information **Fisher Information: Definitions**



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Let $(\mathbb{R},\{\mathbf{P}_{\theta}\}_{\theta\in\mathbb{R}})$ denote a continuous statistical model. Let $f_{\theta}\left(x
ight)$ denote the pdf (probability density function) of the continuous distribution \mathbf{P}_{θ} . Assume that $f_{\theta}\left(x\right)$ is twice-differentiable as a function of the parameter θ .

In the next few problems, you will derive the formula

$$\mathcal{I}\left(heta
ight)=\int_{-\infty}^{\infty}rac{\left(rac{\partial f_{ heta}\left(x
ight)}{\partial heta}
ight)^{2}}{f_{ heta}\left(x
ight)}\,dx$$

using the definition $\mathcal{I}(\theta) = \mathsf{Var}(\ell'(\theta))$ and the basic formula $\mathsf{Var}(X) = \mathbb{E}[X^2] - \mathbb{E}[X]^2$ for any random variable X.

For computations, it is sometimes convenient to use the above formula for the Fisher information.

Note: The derivation in the next set of problems is presented as a proof in the video that follows, but we encourage you to attempt these problems before watching the video.

Deriving a Useful Formula for the Fisher Information I

2/2 points (graded)

Let $(\mathbb{R}, \{\mathbf{P}_{\theta}\}_{\theta \in \mathbb{R}})$ denote a statistical model for a continuous distribution \mathbf{P}_{θ} . Let f_{θ} denote the pdf (probability density function) of the continuous distribution \mathbf{P}_{θ} . Recall that

$$\int_{-\infty}^{\infty}f_{ heta}\left(x
ight)\,dx=1$$

for all $heta \in \mathbb{R}$.

For the next two questions, assume that you are allowed to interchange derivatives and integrals.

What is

$$\int_{-\infty}^{\infty} \frac{\partial}{\partial \theta} f_{ heta}\left(x
ight) dx$$
 ?

0 **✓** Aı

✓ Answer: 0.0

What is

$$\int_{-\infty}^{\infty} \frac{\partial^2}{\partial \theta^2} f_{\theta}\left(x\right) dx$$
 ?

0

✓ Answer: 0.0

STANDARD NOTATION

Solution:

Since we know $\int_{-\infty}^{\infty}f_{ heta}\left(x
ight)\,dx=1$, this implies that

$$rac{\partial}{\partial heta}\int_{-\infty}^{\infty}f_{ heta}\left(x
ight)\,dx=rac{\partial}{\partial heta}1=0.$$

Since we are allowed to interchange the integral and derivative, this implies that

$$\int_{-\infty}^{\infty}rac{\partial}{\partial heta}f_{ heta}\left(x
ight)\,dx=0.$$

Similarly for the second derivative,

$$\int_{-\infty}^{\infty}rac{\partial^{2}}{\partial heta^{2}}f_{ heta}\left(x
ight)\,dx=rac{\partial^{2}}{\partial heta^{2}}\int_{-\infty}^{\infty}f_{ heta}\left(x
ight)\,dx=rac{\partial^{2}}{\partial heta^{2}}1=0.$$

Remark: If f is "nice enough," analytically speaking, then we can rigorously justify interchanging the integral and derivative.

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You have used 1 of 2 attempts

• Answers are displayed within the problem

Deriving a Useful Formula for the Fisher Information II

1/1 point (graded)

As before, let f_{θ} denote the pdf (probability density function) of the continuous distribution \mathbf{P}_{θ} . By definition,

$$\ell(\theta) = \ln L_1(X, \theta) = \ln f_{\theta}(X)$$

where $X \sim \mathbf{P}_{\theta}$. Differentiating, we see

$$\ell'\left(heta
ight)=rac{\partial}{\partial heta}\mathrm{ln}\,f_{ heta}\left(X
ight)=rac{rac{\partial}{\partial heta}f_{ heta}\left(X
ight)}{f_{ heta}\left(X
ight)}.$$

What is

$$\mathbb{E}\left[\ell'\left(heta
ight)
ight]=\mathbb{E}\left[rac{rac{\partial}{\partial heta}f_{ heta}\left(X
ight)}{f_{ heta}\left(X
ight)}
ight]?$$

0

✓ Answer: 0.0

(Note that $X \sim \mathbf{P}_{ heta}$.)

STANDARD NOTATION

Solution:

Observe that

$$\mathbb{E}\left[rac{rac{\partial}{\partial heta}f_{ heta}\left(X
ight)}{f_{ heta}\left(X
ight)}
ight]=\int_{-\infty}^{\infty}\left(rac{rac{\partial}{\partial heta}f_{ heta}\left(x
ight)}{f_{ heta}\left(x
ight)}
ight)f_{ heta}\left(x
ight)\,dx=\int_{-\infty}^{\infty}rac{\partial}{\partial heta}f_{ heta}\left(x
ight)\,dx=0,$$

by the computation in the previous question.

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1 Answers are displayed within the problem

Deriving a Useful Formula for the Fisher Information III

1/1 point (graded)

As before, let f_{θ} denote the pdf (probability density function) of the continuous distribution \mathbf{P}_{θ} . By definition,

$$\ell(\theta) = \ln L_1(X, \theta) = \ln f_{\theta}(X)$$

where $X \sim \mathbf{P}_{\theta}$.

Using the previous question, which of the following are equal to $\mathsf{Var}\left(\ell'\left(heta
ight)
ight) = \mathsf{Var}\left(rac{\partial}{\partial heta} \ln f_{ heta}\left(X
ight)
ight)$? (Choose all that apply.)

 $\mathcal{I}(\theta)$

$$\mathbb{E}\left[\ell'\left(heta
ight)
ight]$$

$$\int_{-\infty}^{\infty} rac{\left(rac{\partial}{\partial heta} f_{ heta}(x)
ight)^2}{f_{ heta}(x)} \, dx$$



Solution:

We consider the choices in order.

- By definition, $\mathcal{I}(\theta) = \mathsf{Var}(\ell'(\theta))$, so the first answer choice $\mathcal{I}(\theta)$ is correct.
- By the previous question, $\mathbb{E}\left[\ell'\left(\theta\right)\right]=0$, so this answer choice is incorrect.
- By definition of variance,

$$\mathsf{Var}\left(\ell'\left(heta
ight)
ight) = \mathbb{E}\left[\ell'\left(heta
ight)^{2}
ight] - \mathbb{E}\left[\ell'\left(heta
ight)
ight]^{2},$$

and $\mathbb{E}\left[\ell'\left(\theta\right)\right]=0$, by the previous question. Hence, $\mathbb{E}\left[\left(\ell'\left(\theta\right)\right)^{2}\right]=\mathsf{Var}\left(\ell'\left(\theta\right)\right)$, and so the answer choice $\mathbb{E}\left[\left(\ell'\left(\theta\right)\right)^{2}\right]$ is correct.

• The last choice $\int_{-\infty}^{\infty} \frac{\left(\frac{\partial}{\partial \theta} f_{\theta}(x)\right)^2}{f_{\theta}(x)} \, dx$ is correct because, using the previous bullet,

$$egin{aligned} \mathsf{Var}\left(\ell'\left(heta
ight)
ight) &= \mathbb{E}\left[\left(\ell'\left(heta
ight)
ight)^2
ight] \ &= \mathbb{E}\left[\left(rac{rac{\partial}{\partial heta}f_{ heta}\left(X
ight)}{f_{ heta}\left(X
ight)}
ight)^2
ight] \end{aligned}$$

$$=\int_{-\infty}^{\infty}rac{\left(rac{\partial}{\partial heta}f_{ heta}\left(x
ight)
ight)^{2}}{f_{ heta}\left(x
ight)}\,dx.$$

Remark: A convenient way to compute the Fisher information is to use the fourth answer choice, which gives the useful formula

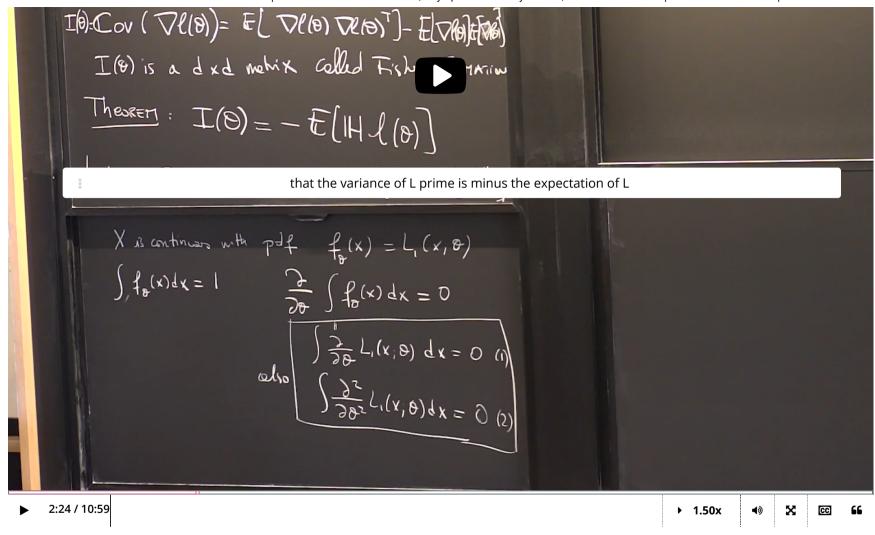
$$\mathcal{I}\left(heta
ight)=\int_{-\infty}^{\infty}rac{\left(rac{\partial}{\partial heta}f_{ heta}\left(x
ight)
ight)^{2}}{f_{ heta}\left(x
ight)}\,dx.$$

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You have used 1 of 3 attempts

1 Answers are displayed within the problem

Proof of Fisher Information Equivalent Formulas for 1 Dimension



Video

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Definition of Fisher Information

Let $heta\in\Theta\subset\mathbb{R}^d$ and let $\left(E,\{\mathbf{P}_{ heta}\}_{ heta\in\Theta}
ight)$ be a statistical model. Let $f_{ heta}\left(\mathbf{x}
ight)$ be the pdf of the distribution $\mathbf{P}_{ heta}$. Then, the Fisher information of the statistical model is

$$\mathcal{I}\left(heta
ight) = \mathsf{Cov}\left(
abla\ell\left(heta
ight)
ight) = -\mathbb{E}\left[\mathbf{H}\ell\left(heta
ight)
ight],$$

where $\ell(\theta) = \ln f_{\theta}(\mathbf{X})$.

The definition when the distribution has a pmf $p_{\theta}(\mathbf{x})$ is also the same, with the expectation taken with respect to the pmf.

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