MATHEMATICS

How do I solve the following multivariable minimization optimization problem?

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2

$$\begin{array}{ll} \text{minimize} & b_1x_1 + b_2x_2 + b_3x_3 + b_4x_4 \\ \text{subject to} & x_1 + x_2 + x_3 + x_4 = 1 \end{array}$$

$$d(1)x(1) + d(2)x(2) + d(3)x(3) + d(4)x(4) \leq Dt$$

$$b(1) > b(2) > b(3) > b(4)$$

$$d(1) < d(2) < d(3) < d(4)$$

- The four variables $x(1)$ to $x(4)$ represent percentages and must add up to 1
- Is there a minimum for the objective function such that all the constraints are satisified? How to approach such a problem?

linear-algebra optimization maxima-minima lagrange-multiplier

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edited 17 hours ago

asked 2 days ago
Mohamed R
21 2

New contributor

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Please clarify your specific problem or provide additional details to highlight exactly what you need. As it's currently written, it's hard to tel
asking. – Community ♦ 2 days ago

Do you correctly formulate the problem? You want to minimize the sum, but you claim that it is equal to 1. – user376343 2 days ago

As it is stated, if the admissible region, S, is non-empty, the minimum is 1 and that value is attained at every point in S. Maybe you want to
– PierreCarre 2 days ago

@user376343 Hello, I have edited the problem. Does the new formulation make sense? – Mohamed R 23 hours ago

@PierreCarre Hello, I have edited the problem. Does the new formulation make sense? – Mohamed R 23 hours ago

1 Answer

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0

Calling

$$\begin{cases} f(x) = \sum_k b_k x_k^2 \\ r_1(x) = \sum_k x_k^2 - 1 \\ r_2(x,s) = \sum_k d_k x_k^2 - D + s^2 \end{cases}$$

we have the lagrangian

$$L(x,\lambda,\mu,s) = f(x) + \lambda r_1(x) + \mu r_2(x,s)$$

here s is a slack variable to transform the inequality into an equivalent equation.

The **stationary points** are the solutions for

$$\nabla L = \begin{cases} b_j x_j + \lambda x_j + \mu d_j x_j = 0, \quad j = 1,\cdots,4 \\ \sum_k x_k^2 - 1 = 0 \\ \sum_k d_k x_k^2 - D + s^2 = 0 \end{cases}$$

giving

$$\begin{bmatrix}
f & x_1^2 & x_2^2 & x_3^2 & x_4^2 & \lambda & \mu & s^2 & \text{condition} \\
b_1 & 1 & 0 & 0 & 0 & -b_1 & 0 & D-d_1 & D \ge d_1 \\
b_2 & 0 & 1 & 0 & 0 & -b_2 & 0 & D-d_2 & D \ge d_2 \\
b_3 & 0 & 0 & 1 & 0 & -b_3 & 0 & D-d_3 & D \ge d_3 \\
b_4 & 0 & 0 & 0 & 1 & -b_4 & 0 & D-d_4 & D \ge d_4 \\
\frac{b_2(d_1-D)}{d_1-d_2}+\frac{b_1(d_2-D)}{d_2-d_1} & \frac{d_2-D}{d_2-d_1} & \frac{d_1-D}{d_1-d_2} & 0 & 0 & \frac{b_1d_2-b_2d_1}{d_1-d_2} & \frac{b_2-b_1}{d_1-d_2} & 0 & d_1 \le D \le d_2 \\
\frac{b_3(d_1-D)}{d_1-d_3}+\frac{b_1(d_3-D)}{d_3-d_1} & \frac{d_3-D}{d_3-d_1} & 0 & \frac{d_1-D}{d_1-d_3} & 0 & \frac{b_1d_3-b_3d_1}{d_1-d_3} & \frac{b_3-b_1}{d_1-d_3} & 0 & d_1 \le D \le d_3 \\
\frac{b_3(d_2-D)}{d_2-d_3}+\frac{b_2(d_3-D)}{d_3-d_2} & 0 & \frac{d_3-D}{d_3-d_2} & \frac{d_2-D}{d_2-d_3} & 0 & \frac{b_2d_3-b_3d_2}{d_2-d_3} & \frac{b_3-b_2}{d_2-d_3} & 0 & d_2 \le D \le d_3 \\
\frac{b_4(d_1-D)}{d_1-d_4}+\frac{b_1(d_4-D)}{d_4-d_1} & \frac{d_4-D}{d_4-d_1} & 0 & 0 & \frac{d_1-D}{d_1-d_4} & \frac{b_1d_4-b_4d_1}{d_1-d_4} & \frac{b_4-b_1}{d_1-d_4} & 0 & d_1 \le D \le d_4 \\
\frac{b_4(d_2-D)}{d_2-d_4}+\frac{b_2(d_4-D)}{d_4-d_2} & 0 & \frac{d_4-D}{d_4-d_2} & 0 & \frac{d_2-D}{d_2-d_4} & \frac{b_2d_4-b_4d_2}{d_2-d_4} & \frac{b_4-b_2}{d_2-d_4} & 0 & d_2 \le D \le d_4 \\
\frac{b_4(d_3-D)}{d_3-d_4}+\frac{b_3(d_4-D)}{d_4-d_3} & 0 & 0 & \frac{d_4-D}{d_4-d_3} & \frac{d_3-D}{d_3-d_4} & \frac{b_3d_4-b_4d_3}{d_3-d_4} & \frac{b_4-b_3}{d_3-d_4} & 0 & d_3 \le D \le d_4
\end{bmatrix}$$

NOTE

We used x_k^2 to assure the positiveness. When $s=0$ indicates that $r_2(x,s)$ is actuating. The table of results should be interpreted according to the given condition at the last column.

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edited 57 mins ago

answered 10 hours ago

Cesareo

22.7k 14 11 31