4. Writing systems in matrix form

A system of 3 equations with 3 unknowns,

$$\begin{aligned} 8y - 4z &= 0 \\ x - y - 4z &= 1 \\ -x + 5y + 2z &= 0, \end{aligned}$$

can be written in matrix form as $\mathbf{Ax} = \mathbf{b}$:

$$\underset{\mathbf{A}}{\begin{pmatrix} 0 & 8 & -4 \\ 1 & -1 & -4 \\ -1 & 5 & 2 \end{pmatrix}} \underset{\mathbf{x}}{\begin{pmatrix} x \\ y \\ z \end{pmatrix}} = \underset{\mathbf{b}}{\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}}$$

We can think of multiplying the vector $\mathbf{x}$ and the matrix $\mathbf{A}$ in 2 equivalent ways as follows:

1. To multiply A and $\mathbf{x}$ to get a vector $\mathbf{b}$, the ith entry of $\mathbf{b}$ is the dot product of the ith row of A with $\mathbf{x}$.

$$\begin{pmatrix} 0 & 8 & -4 \\ 1 & -1 & -4 \\ -1 & 5 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 8y - 4z \\ x - y - 4z \\ -1x + 5y + 2z \end{pmatrix}$$

   This explains why the matrix form is equivalent to the original system of equations.

2. The product $\mathbf{Ax}$ can be expressed as a linear combination of the columns of $\mathbf{A}$:

$$\begin{pmatrix} 0 & 8 & -4 \\ 1 & -1 & -4 \\ -1 & 5 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = x \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} + y \begin{pmatrix} 8 \\ -1 \\ 5 \end{pmatrix} + z \begin{pmatrix} -4 \\ -4 \\ 2 \end{pmatrix}.$$

Notice that the coefficients x, y, and z in this linear combination are the entries of the vector $\mathbf{x}$.

Matrix vector multiplication, dot product with rows

1/1 point (graded)

Find the missing 3rd entry c of the vector given by the matrix vector product

$$\begin{pmatrix} -1 & 1 & 0 & 0 \\ 1 & -2 & 1 & 0 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 1 & -1 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ c \\ -1 \end{pmatrix}$$

$c =$ 0 ✔

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✔ Correct (1/1 point)

Matrix vector multiplication, combinations of columns viewpoint

3/3 points (graded)

Find a nonzero vector $\mathbf{x} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ such that $\mathbf{Ax} = \mathbf{0}$ where $\mathbf{A}$ is the matrix

$$\mathbf{A} = \begin{pmatrix} 1 & 4 & 3 \\ -2 & -8 & 1 \\ 0 & 0 & 4 \\ 1 & 4 & 2 \end{pmatrix}$$

(Hint: the second column is 4 times the first column.)

$x =$ 4 ✔

$y =$ -1 ✔

$z =$ 0 ✔

Submit You have used 1 of 5 attempts

✔ Correct (3/3 points)

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- Writing systems in matrix form. — 2
  When written in matrix form, there are 3 possible solutions for a linear system of equations. 1.) No soluti...
- Answer for the second question: "Matrix vector multiplication, combinations of columns viewpoint" — 2
  What does this explanation for the 2nd question mean? "The third column is independent since it has a n...
- [Latex issue] Matrix A in solution not displaying correctly — 2

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