8. Autoregressive model as linear process

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We can also write autoregressive model as linear process. Let us consider AR(1) first:

$$\begin{aligned} X_t &= W_t + \psi_1 X_{t-1} \\ &= \sum_{j=0}^{\infty} \psi_1^j W_{t-j} \end{aligned}$$

This is a special form of the linear process model **if the infinite sum exist**. In another word, we need $\sum_{j=0}^{\infty} \psi_1^j < \infty$ for the time series to be well-defined.

We also have a more general result that AR(p) is stationary and causal if the corresponding linear process converges.

ARMA model as linear process

2/2 points (graded)

Consider an ARMA model of parameter $1,1$ $X_t = ARMA\,(1,1)$ defined as the following:

$$X_t = 0.5X_{t-1} + W_t + 0.5W_{t-1}$$

Can you write X_t as a linear process?

- (●) True
- ( ) False

✔

Is X_t causal?

- (●) True
- ( ) False

✔

Solution:

We can expand X_t by expressing X_{t-1} with X_{t-2}:

$$\begin{aligned} X_t &= 0.5X_{t-1} + W_t + 0.5W_{t-1} \\ &= 0.5\left(0.5X_{t-2} + W_{t-1} + 0.5W_{t-2}\right) + W_t + 0.5W_{t-1} \\ &= 0.5^2 X_{t-2} + W_t + W_{t-1} + 0.5^2 W_{t-2} \\ &= W_t + \sum_{j=0}^{\infty} 0.5^{j-1} W_{t-j} \end{aligned}$$

this linear process converges and only the coefficient of the historical terms are non-zero.

In fact, more generally speaking, any ARMA model of parameter p and q (X_t=ARMA(p,q)), X_t is causal if and only if there exists a converging linear process such that $X_t = \sum_{j=0}^{\infty} \psi_j W_{t-j}$.

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