



## MITx: 6.041x Introduction to Probability - The Science of Uncertainty



Bookmarks

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## Unit overview

Lec. 8: Probability  
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Bayes rule vertical5



Bookmark

## Exercise: Independence and CDFs

(2/2 points)

a) Suppose that  $\mathbf{X}$  and  $\mathbf{Y}$  are independent. Is it true that their joint CDF satisfies  $F_{\mathbf{X},\mathbf{Y}}(x, y) = F_{\mathbf{X}}(x)F_{\mathbf{Y}}(y)$ , for all  $x$  and  $y$ ?

Yes ▾



Answer: Yes

b) Suppose that  $F_{\mathbf{X},\mathbf{Y}}(x, y) = F_{\mathbf{X}}(x)F_{\mathbf{Y}}(y)$ , for all  $x$  and  $y$ . Is it true that  $\mathbf{X}$  and  $\mathbf{Y}$  are independent?

Hint: Recall the formula  $f_{\mathbf{X},\mathbf{Y}}(x, y) = (\partial^2 / \partial x \partial y) F_{\mathbf{X},\mathbf{Y}}(x, y)$ .

Yes ▾



Answer: Yes

Answer:

a) Yes. We have

$$\begin{aligned}
 F_{\mathbf{X},\mathbf{Y}}(x, y) &= \mathbf{P}(\mathbf{X} \leq x, \mathbf{Y} \leq y) \\
 &= \int_{-\infty}^y \int_{-\infty}^x f_{\mathbf{X},\mathbf{Y}}(x, y) dx dy \\
 &= \int_{-\infty}^x f_{\mathbf{X}}(x) dx \int_{-\infty}^y f_{\mathbf{Y}}(y) dy \\
 &= F_{\mathbf{X}}(x)F_{\mathbf{Y}}(y).
 \end{aligned}$$

b) True. Using the formula in the hint, we find that

$$\begin{aligned}
 f_{\mathbf{X},\mathbf{Y}}(x, y) &= \frac{\partial^2}{\partial x \partial y} F_{\mathbf{X},\mathbf{Y}}(x, y) \\
 &= \frac{\partial^2}{\partial x \partial y} F_{\mathbf{X}}(x)F_{\mathbf{Y}}(y) \\
 &= \frac{\partial}{\partial x} F_{\mathbf{X}}(x) \frac{\partial}{\partial y} F_{\mathbf{Y}}(y) \\
 &= f_{\mathbf{X}}(x)f_{\mathbf{Y}}(y),
 \end{aligned}$$

and therefore we have independence.

**Lec. 10:  
Conditioning on a  
random variable;  
Independence;  
Bayes' rule**

Exercises 10 due Mar  
16, 2016 at 23:59 UTC

Standard normal  
table

Solved problems

**Problem Set 5**

Problem Set 5 due Mar  
16, 2016 at 23:59 UTC

Unit summary

*You have used 1 of 1 submissions*

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