EXERCISE: BAYES' RULE AND THE FALSE-POSITIVE PUZZLE (4/4 points)

A test for a certain rare disease is assumed to be correct 95% of the time: if a person has the disease, the test result is positive with probability 0.95, and if the person does not have the disease, the test result is negative with probability 0.95. A person drawn at random from a certain population has probability 0.001 of having the disease.

1. Find the probability that a random person tests positive.

0.05090 ✔ **Answer:** 0.0509

2. Given that the person just tested positive, what is the probability he actually has the disease?

0.01866405 ✔ **Answer:** 0.01866

Answer:
Let A be the event that the person has the disease, and B the event that the test result is positive.

1. The desired probability is

$$\mathrm{P}(B) = \mathrm{P}(A)\mathrm{P}(B \mid A) + \mathrm{P}(A^c)\mathrm{P}(B \mid A^c) = 0.001 \cdot 0.95 + 0.999 \cdot 0.05 = 0.05$$

2. The desired probability is

$$\mathrm{P}(A \mid B) = \frac{\mathrm{P}(A)\mathrm{P}(B \mid A)}{\mathrm{P}(B)} = \frac{0.001 \cdot 0.95}{0.0509} \approx 0.01866.$$

Note that even though the test was assumed to be fairly accurate, a person who has tested positive is still very unlikely (probability less than 2%) to have the disease. The explanation is that when testing 1000 people, we expect about 1 person to have the disease (and most likely test positive), but also expect about $1000 \cdot 0.999 \cdot 0.05 \approx 50$ people to test positive without having the disease. Hence, when we see a positive test, it is about 50 times more likely to correspond to one of the 50 false positives.

You have used 1 of 2 submissions