2.1 Quiz: Pendulum Model

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Recall our pendulum model:

$$\frac{d^2\theta}{dt^2} = -\frac{g}{l}\sin(\theta)$$

Question 1: Think About It...

1/1 point (graded)

How does the ideal pendulum model differ from a real-world pendulum, like in a clock? What assumptions are not completely valid?

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The rod may bend or stretch.

The entire mass m of the pendulum is not concentrated at
the center of the bob. The rod has  some mass.
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Thank you for your response.

Explanation

A clock pendulum may experience some friction or air resistance and its rod is not mass-less. Some of these assumptions mean this model is not a perfect model. However, it still may provide us some interesting insights.

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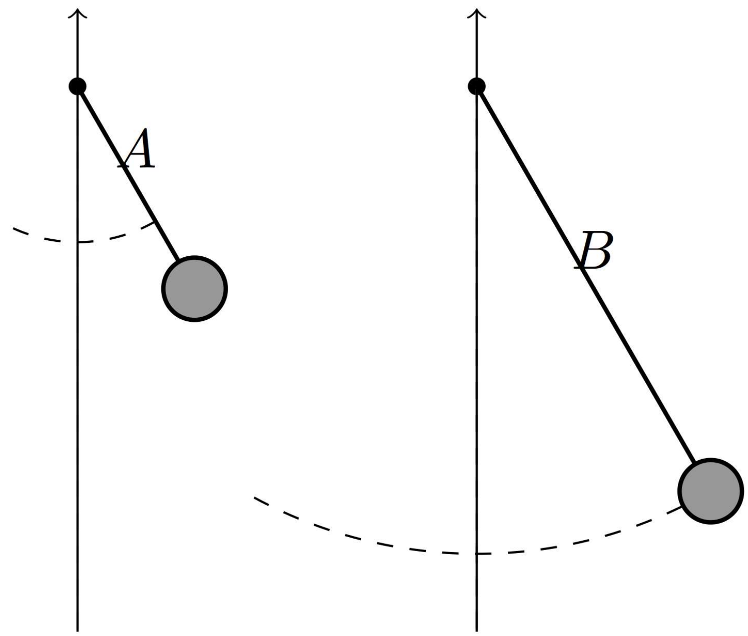
Answers are displayed within the problem

Question 2

1/1 point (graded)

Let's consider two different pendulums at a specific instant in time at which both pendulums are at an angle of θ_0 away from the vertical. Pendulum A has a rod of length $l = 1$, and pendulum B has a rod of length $l = 2$. For which pendulum, if any, is the absolute value of the angular acceleration $\left|\frac{d^2\theta}{dt^2}\right|$ greatest at this moment in time?



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- [x] The value of $\left|\frac{d^2\theta}{dt^2}\right|$ is greater for the shorter pendulum A than for the longer pendulum B. ✔
- [ ] The value of $\left|\frac{d^2\theta}{dt^2}\right|$ is greater for the longer pendulum B than for the shorter pendulum A.
- [ ] The value of $\left|\frac{d^2\theta}{dt^2}\right|$ is the same for the shorter pendulum A as for the longer pendulum B.
- [ ] We cannot tell which pendulum has the greater value of $\left|\frac{d^2\theta}{dt^2}\right|$ until we select units and have a value for g.

Explanation

We know that $\frac{d^2\theta}{dt^2} = -\frac{g}{l}\sin(\theta_0)$.

For pendulum A, $\frac{d^2\theta}{dt^2} = -\frac{g}{1}\sin(\theta_0)$.

For pendulum B, $\frac{d^2\theta}{dt^2} = -\frac{g}{2}\sin(\theta_0)$.

In other words, the angular acceleration $\frac{d^2\theta}{dt^2}$ for the shorter pendulum A is two times the acceleration for the longer pendulum B, so the absolute value is larger for A.

Submit

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Question 3

1/1 point (graded)

The function $\theta(t)$ describing how the angle of the pendulum's swing changes over time satisfies the differential equation

$$\frac{d^2\theta}{dt^2} = -\frac{g}{l}\sin(\theta)$$

Which of the functions below, if any, is a solution to this differential equation?

(Hint: You do not need to solve the differential equation; just test whether each function is a solution or not.)

- $\theta(t) = \frac{g}{l}\sin(\theta)$
- $\theta(t) = \frac{g}{l}\sin(t)$
- $\theta(t) = \sin(\sqrt{\frac{g}{l}}t)$
- $\theta(t) = -\sin(\sqrt{\frac{g}{l}}t)$
- None of the above. ✔

Explanation

We calculate $\frac{d^2\theta}{dt^2}$ and $-\frac{g}{l}\sin(\theta)$ for each of the possible responses and see if they are equal.

For example, if we define $\theta(t) = \frac{g}{l}\sin(t)$ then $\frac{d^2\theta}{dt^2} = -\frac{g}{l}\sin(t)$. This looks promising, until we realize that

$$-\frac{g}{l}\sin(\theta) = -\frac{g}{l}\sin\left(\frac{g}{l}\sin(t)\right)$$

when $\theta(t) = \frac{g}{l}\sin(t)$.

Since $-\frac{g}{l}\sin(\theta) \neq -\frac{g}{l}\sin(t)$,this choice of $\theta(t)$ is not a solution.

In fact, none of the definitions of $\theta(t)$ suggested above solves the differential equation given. (Note that choice (a) does not make sense, because θ is in the definition of the function θ.)

Solving the differential equation is beyond the scope of this course; the solution is not in terms of elementary functions.

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Answers are displayed within the problem

Question 4

1/1 point (graded)

In the next section, we will further simplify our model by assuming θ is a small angle.

Which of the following is the linear approximation of $y = \sin(\theta)$ at $\theta = 0$? (By linear approximation, we mean the tangent line approximation to the function $y = \sin(\theta)$ when $\theta = 0$.)

You can add an optional tip or note related to the prompt like this.

- ○ $y = 0$, or $\sin\theta \approx 0$ for small θ
- ◉ $y = \theta$, or $\sin\theta \approx \theta$ for small θ ✔
- ○ $y = \theta + 1$, or $\sin\theta \approx \theta + 1$ for small θ
- ○ $y = \theta\cos(\theta)$, or $\sin\theta \approx \theta\cos(\theta)$ for small θ

None of the above.

Explanation

The function $y = \theta\cos(\theta)$ is not linear, so we can eliminate this option immediately.
The graph of the sine function goes through the point $(0, 0)$. When $\theta = 0$, the slope of the graph of $\sin(\theta)$ will be $\cos(0) = 1$. Thus, a good linear approximation would be a line through the origin with slope 1. This is the line $y = \theta$.
You can observe this is a good approximation using a scientific calculator. When θ is small, we can see the value of $\sin(\theta)$ is very close to the value of θ.

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Answers are displayed within the problem

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