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Let C be the curve defined by the equation

$$x^3 - 2x + y^2 = 3.$$

Notice that the point $(1, 2)$ is on the curve C. Suppose that we put our finger down on the point $(1, 2)$ and slide it along the curve C so that the y coordinate gradually increases from 2 to 2.01. Which of the following is the best approximation for the point where we end up:

- ○ $(1.01, 2.01)$
- ○ $(1, 2.01)$
- ○ $(0.99, 2.01)$
- ○ $(0.98, 2.01)$
- ○ $(0.97, 2.01)$
- ◉ $(0.96, 2.01)$ ✔

Solution:

To approximate the point on C where $y = 2.01$ and x is approximately 1, we replace the curve by its linear approximation at the point $(1, 2)$. Letting $f(x, y) = x^3 - 2x + y^2$, we compute

$$\begin{aligned} f(x, y) = x^3 - 2x + y^2 &\implies f(1, 2) = 3, \\ f_x(x, y) = 3x^2 - 2 &\implies f(1, 2) = 1, \\ f_y(x, y) = 2y &\implies f(1, 2) = 4. \end{aligned}$$

Therefore the linear approximation at $(1, 2)$ is

$$f(x, y) \approx 3 + 1(x - 1) + 4(y - 2).$$

We are seeking for x such that $f(x, 2.01) = 3$; that gives

$$3 = f(x, 2.01) \approx 3 + 1(x - 1) + 4(2.01 - 2) = 3 + (x - 1) + 0.04.$$

Solving for x then gives $x = 1 - 0.04 = 0.96$. Hence the best approximation is $\boxed{(0.96, 2.01).}$

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Answers are displayed within the problem

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