1. Differentials

Bookmark this page

Recitation due Oct 5, 2021 20:30 IST

Practice

Chain Rule Problem

1:05 / 1:05 2.0x

So in part a, we just want to compute the total differential
dz in terms of dx and dy.
So u and v aren't going to enter into the picture.
And then in part b, we're going to compute
the partial derivative, partial z, partial u,
in two different ways.
First, we're going to compute it using the chain rule,
and then we're going to compute it using total differentials.
And so we'll substitute in some of the work
that we had in a to solve that part.
So why don't I pause--
why don't you pause the video now, and work on the problem.

Video
Download video file

Transcripts
Download SubRip (.srt) file
Download Text (.txt) file

Differentials and Chain Rule

2/2 points (graded)
Suppose

$$z = x^2 + y^2 \tag{6.213}$$

and

$$x = u^2 - v^2, \qquad y = uv \tag{6.214}$$

(a) Write the total differential dz in terms of dx, dy.

(Don't forget to use `*` when multiplying differentials.)

$dz =$ 2*x*dx+2*y*dy ✔ **Answer:** 2*x*dx + 2*y*dy

? INPUT HELP

(b) Compute $\dfrac{\partial z}{\partial u}$. Your answer may contain u, v, x and/or y (the grader will make the substitution $x = u^2 - v^2$ and $y = uv$ when checking your answer). Check your answer in two ways: using the chain rule and differentials.

∂z

Calculator Hide Notes

$\frac{\partial z}{\partial u} =$ 4*x*u+2*y*v ✔ **Answer:** 4*u*x + 2*v*y

? INPUT HELP

The answer is given in the following video.

Solution:

Using differentials, we have

$$dz = z_x dx + z_y dy. \tag{6.215}$$

Therefore,

$$dz = 2x dx + 2y dy \tag{6.216}$$

Then, using the chain rule, we have

$$\frac{\partial z}{\partial u} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial u} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial u}. \tag{6.217}$$

Substituting the appropriate partial derivatives, we obtain

$$\frac{\partial z}{\partial u} = 4ux + 2vy. \tag{6.218}$$

Alternatively, one may use total differentials. The general strategy is to make the necessary substitutions in the expression for dz, and then extract the coefficient on du, which must be the desired partial derivative.

From above, we have

$$dz = 2x dx + 2y dy \tag{6.219}$$

From $x = u^2 - v^2$ and $y = uv$ we have

$$dx = 2u du - 2v dv \tag{6.220}$$

and

$$dy = v du + u dv. \tag{6.221}$$

Now substituting the values for dx and dy into dz we obtain

$$dz = 2x\left(2u du - 2v dv\right) + 2y\left(v du + u dv\right) \tag{6.222}$$

Collecting like terms:

$$dz = \left(4xu + 2vy\right) du + \left(2yu - 4xv\right) dv \tag{6.223}$$

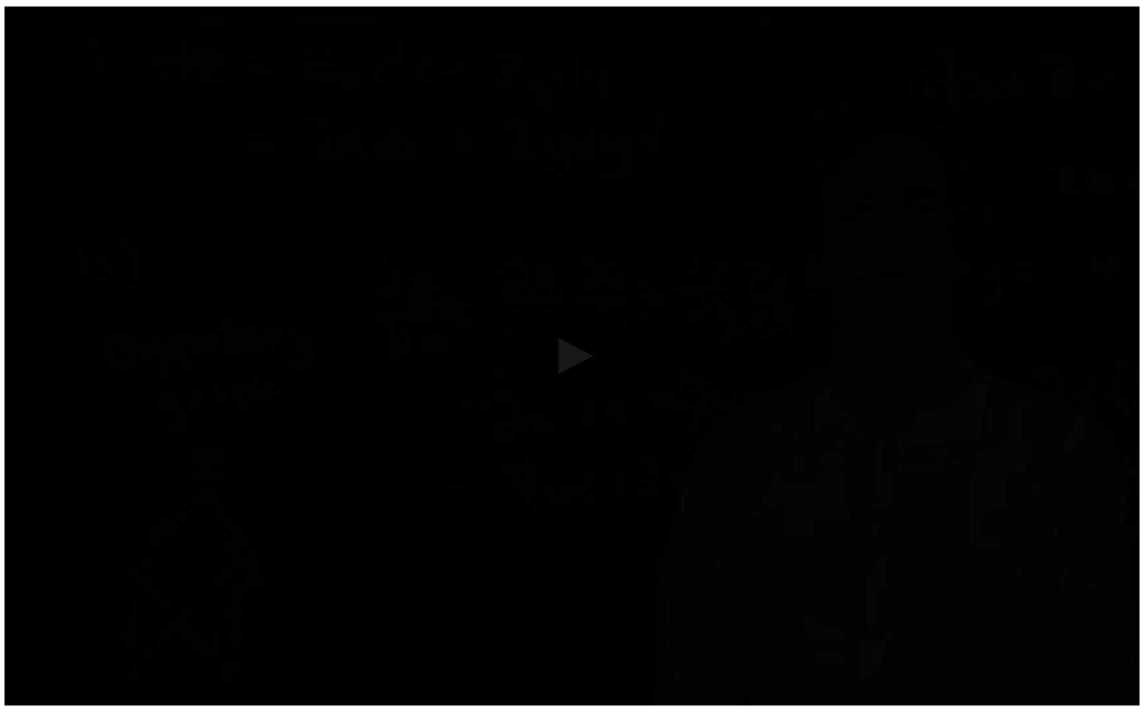
Thus, we obtain $\frac{dz}{du} = 4xu + 2vy.$

Calculator Hide Notes

Submit You have used 1 of 5 attempts

Answers are displayed within the problem

Chain Rule Problem Solution



sides

is essentially the same expression.

So that means, if we want to compute parcels z partial u,

then that's just equal to this coefficient here.

So we get that partial z partial u is 4xu plus 2--

that should be v. One of those is an x.

Let's see.

So what where did this come from?

Yeah, one of those is an x, sorry.

STUDENT: It's a y.

PROFESSOR: It's a y, 2vy.

Now, just as a sanity check, why don't we go back

to the middle of the board.

And we'll see that we got the same thing.

10:21 / 10:21 2.0x

Video
Download video file

Transcripts
Download SubRip (.srt) file
Download Text (.txt) file

Chain Rule Problem 2

1/1 point (graded)

Suppose $f\left(x,y,z\right)$ is a function such that $\nabla f = \begin{pmatrix} xy+z \\ x^2+5y \\ -x \end{pmatrix}$. Suppose that x, y, and z all depend on t via $x=t, y=t^2, z=t^3$. Find $\dfrac{df}{dt}$.

$\dfrac{df}{dt} =$ 11*t^3 ✔ **Answer:** 11*t^3

? INPUT HELP

Solution:

By the chain rule, we have

$$\frac{df}{dt} = \frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial y}\frac{dy}{dt} + \frac{\partial f}{\partial z}\frac{dz}{dt} \qquad (6.224)$$

We are given:

$$\frac{\partial f}{\partial x} = xy+z, \quad \frac{\partial f}{\partial y} = x^2+5y, \quad \frac{\partial f}{\partial z} = -x$$

Calculator Hide Notes

Then, from $x = t, y = t^2, z = t^3$, we obtain

$$\frac{\partial f}{\partial x} = 2t^3, \quad \frac{\partial f}{\partial y} = 6t^2, \quad \frac{\partial f}{\partial z} = -t \tag{6.226}$$

and

$$\frac{dx}{dt} = 1, \quad \frac{dy}{dt} = 2t, \quad \frac{dz}{dt} = 3t^2 \tag{6.227}$$

Therefore,

$$\frac{df}{dt} = (2t^3)(1) + (6t^2)(2t) + (-t)(3t^2) = 11t^3 \tag{6.228}$$

Submit You have used 1 of 5 attempts

Answers are displayed within the problem

1. Differentials

Hide Discussion

Topic: Unit 5: Curves and Surfaces / 1. Differentials

Add a Post

Show all posts by recent activity

There are no posts in this topic yet.

×

< Previous Next >

edX

edX

Calculator Hide Notes