



MITx: 6.041x Introduction to Probability - The Science of Uncertainty



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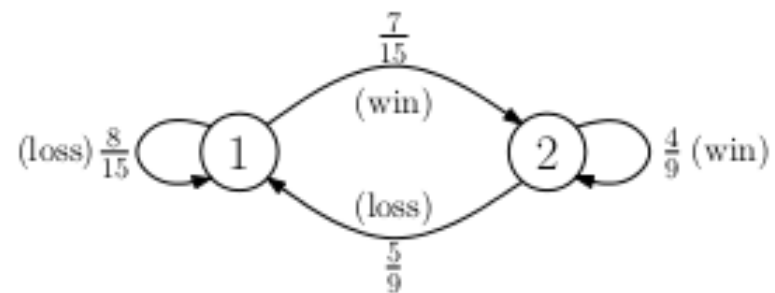
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Exercise: Frequency interpretations

(2/2 points)

Jack is a gambler who pays for his MIT tuition by spending weekends in Las Vegas. His latest game of choice is blackjack, which he plays using a fixed strategy. However, at this special blackjack table, the dealer uses one of 2 decks of cards for each hand. Using his fixed strategy, Jack wins with probability $\frac{7}{15}$ when deck #1 is used and with probability $\frac{4}{9}$ when deck #2 is used. Whenever deck #1 is used, if Jack wins, the dealer switches to deck #2 for the next hand, and if Jack loses, the dealer keeps using deck #1 for the next hand. Whenever deck #2 is used, if Jack wins, the dealer keeps using deck #2 for the next hand, and if Jack loses, the dealer switches to deck #1 for the next hand.


Jack's wins and losses can be modeled as the transitions of the following Markov chain, whose states correspond to the particular deck being used.




- ▶ Unit 6: Further topics on random variables
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- ▼ **Unit 10: Markov chains**

Unit overview

Lec. 24: Finite-state Markov chains

Exercises 24 due May 18, 2016
at 23:59 UTC 

Lec. 25: Steady-state behavior of Markov chains

Exercises 25 due May 18, 2016
at 23:59 UTC 

What is Jack's long-term probability of winning?

0.45652174

✓ Answer: 0.45652

Answer:

The long-term frequency of winning can be found as the sum of the long-term frequency of transitions from state 1 to state 2 and from state 2 back to itself. These can be found from the steady-state probabilities π_1 and π_2 , which are known to exist as the chain is aperiodic and recurrent. The local balance and normalization equations are

$$\begin{aligned}\frac{7}{15}\pi_1 &= \frac{5}{9}\pi_2 \\ \pi_1 + \pi_2 &= 1,\end{aligned}$$


which lead to the solution $\pi_1 = 25/46$ and $\pi_2 = 21/46$.

The probability of winning can now be found as


$$\pi_1 p_{12} + \pi_2 p_{22} = \frac{25}{46} \cdot \frac{7}{15} + \frac{21}{46} \cdot \frac{4}{9} = \frac{21}{46} = \pi_2,$$

and so for this particular game of blackjack, the long-term probability of winning also happens to equal the steady-state probability of being in state 2.

Lec. 26: Absorption probabilities and expected time to absorption

Exercises 26 due May 18, 2016 at 23:59 UTC 

Solved problems**Problem Set 10**

Problem Set 10 due May 18, 2016 at 23:59 UTC 

► Exit Survey

You have used 1 of 2 submissions

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