1.3.1 Exploratory Quiz: What Effect does Fishing have on Population?

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In this section, we'll look at how the fish population model changes when we include fishing. We'll begin with an exploratory quiz to develop some conjectures about how fishing might affect the population.

Here is the modified model Wes introduced:

$$\frac{dP}{dt} = \frac{1}{10}P(1 - \frac{P}{40000}) - \alpha$$

Recall the parameter α is fishing rate, a constant rate of α fish per year, where $\alpha > 0$.

Note: This is different than in the case of Population Dynamics, where we assumed that the rate of fishing *e* was proportional to the population size. Here we're assuming a constant rate, as when fishing is controlled by permit.

Question 1: Think About It...

1/1 point (graded)

Intuitively, what effect do you think a large value of α will have on the fish population in the long run?

```
Large value of alpha will eventually reduce the population
size to zero.
```

Thank you for your response.

If the fishing rate is too large, the fish population would not reproduce fast enough to offset the fishing and would therefore die out.

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Answers are displayed within the problem

Question 2

1/1 point (graded)

When $\frac{dP}{dt} = \frac{1}{10}P\left(1 - \frac{P}{40000}\right)$, the graph of $\frac{dP}{dt}$ versus P is a parabola. We can modify the model to include a positive fishing rate α, to get:

$$\frac{dP}{dt} = \frac{1}{10}P(1 - \frac{P}{40000}) - \alpha.$$

How does this change the graph of $\frac{dP}{dt}$ versus P?

- ( ) There is no change in the graph.
- ( ) The graph shifts to the left.
- ( ) The graph shifts to the right.
- ( ) The graph shifts up.
- (•) The graph shifts down. ✔
- ( ) None of the above.

Explanation

The height of each point in the graph of $\frac{dP}{dt}$ versus P indicates the value of $\frac{dP}{dt}$. The number α of fish caught must be positive, so subtracting α reduces the value of $\frac{dP}{dt}$. This shifts the graph down by α units.

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Recall that the points at which $\frac{dP}{dt} = 0$ are the horizontal intercepts of the graph of $\frac{dP}{dt}$ vs. P and also the equilibrium solutions of the system. Recall that the points where $\frac{dP}{dt} = 0$ are the horizontal axis intercepts of the graph of $\frac{dP}{dt}$ as a function of P. These are the equilibrium solutions of the system, since the rate of change is zero ($\frac{dP}{dt} = 0$).

Follow the link to the interactive graph. Use it to answer the questions about what happens as you increase α, the fishing rate. (**Note:** we use "a" to represent α in the desmos graph.

Question 3

1/1 point (graded)

When there is no fishing ($\alpha = 0$), the differential equation has two equilibrium solutions. When α is less than 100, how many equilibrium solutions are there? Enter a numerical value.

2

✔ **Answer:** 2

2

Explanation

When $\alpha > 0$ but small, there are still two equilibrium solutions. One is near $P = 0$ and one is near $P = 40,000$. This means a small amount of fishing (a small increase of the parameter α from 0) does not change the expected behavior of the system.

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Question 4: Think About It...

1/1 point (graded)

Does the system have the same number of equilibrium solutions for all values of α that you were able to explore? Record your observations below.

How might this relate to the idea of **bifurcation:** *'a major change in the expected qualitative dynamical behavior of a system in response to changing a parameter'*?

```
After a threshold value of alpha there will be a single
equilibrium.
```

✔

Thank you for your response.

The number of equilibria equals the number of points at which the graph intersects the P axis. For values of α greater than $1,000$, the graph does not intersect the P axis at all. When $\alpha = 1,000$ the graph intersects the P axis at a single point. So the number of equilibria might be $0, 1,$ or 2 depending on the value of α.

Near $\alpha = 1,000$, a small change in alpha may result in a sudden major change in the expected behavior of the system.

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Question 5: Think About It...

1/1 point (graded)

For each situation with a different number of equilibria, sketch a graph of $\frac{dP}{dt}$ vs P. Analyze the stability of the equilibrium solutions using the graph by sketching arrows on the P axis as we did in the example with no fishing $(\alpha = 0)$.

Record your observations below.

```
at alpha=1000, one equilibrium point, which will be
semistable.
at alpha > 1000, no equilibrium point.
```

Thank you for your response.

We know that P is increasing when the graph of $\frac{dP}{dt}$ is above the P axis and decreasing when the graph is below. This makes it easy to sketch in arrows showing the direction of change in population on the P axis, as shown in next section's video.

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