2. Linear Regression with Closed Form Solution

After seeing the problem, your classmate Alice immediately argues that we can apply a linear regression model, as the labels are numbers from 0-9, very similar to the example we learned from the lecture. Though being a little doubtful, you decide to have a try and start simple by using the raw pixel values of each image as features.

Alice wrote a skeleton code `run_linear_regression_on_MNIST` in main.py, but she needs your help to complete the code and make the model work.

Closed Form Solution of Linear Regression

5/5 points (graded)
To solve the linear regression problem, you recall the linear regression has a closed form solution:

$$\theta = (X^T X + \lambda I)^{-1} X^T Y$$

Write a function `closed_form` that computes this closed form solution given the features X, labels Y and the regularization parameter λ.

Available Functions: You have access to the NumPy python library as `np`; No need to import anything.

```
def closed_form(X, Y, lambda_factor):
    """
    Computes the closed form solution of linear regression with L2 regularization

    Args:
        X - (n, d + 1) NumPy array (n datapoints each with d features plus the bias feature in the first dimension)
        Y - (n, ) NumPy array containing the labels (a number from 0-9) for each
            data point
        lambda_factor - the regularization constant (scalar)
```

```
    Returns:
        theta - (d + 1, ) NumPy array containing the weights of linear regression. Note that theta[0]
        represents the y-axis intercept of the model and therefore X[0] = 1
    """
    # YOUR CODE HERE
    return np.linalg.inv(X.T@X + lambda_factor*np.eye(X.shape[1]))@(X.T@Y)
```

Press ESC then TAB or click outside of the code editor to exit

Correct

Test results

Hide output

CORRECT

Test: output

Output:

```
X: [[0.51261794 0.70242204]
 [0.74943563 0.74858527]
 [0.89390655 0.89801078]
 [0.09047841 0.29687843]
 [0.50363122 0.64924333]
 [0.10424944 0.71019987]
 [0.93943595 0.95494293]
 [0.92742283 0.14598462]
 [0.45884661 0.26049004]
 [0.91260513 0.35158453]
 [0.56993027 0.20429471]]
Y: [0.96510048 0.85659104 0.77623373 0.34989444 0.34228261 0.22196316
 0.32790104 0.85882415 0.98712346 0.98605459 0.73846034]
lambda_factor: 0.8373422026379005
Submission output: [0.70702669 0.21486756]
```

Test: size

Output:

```
Output size: (3,)
```

Hide output

Submit You have used 1 of 20 attempts

✔ Correct (5/5 points)

Test Error on Linear Regression

1/1 point (graded)
Apply the linear regression model on the test set. For classification purpose, you decide to round the predicted label into numbers 0-9.

Note: For this project we will be looking at the error rate defined as the fraction of labels that don't match the target labels, also known as the "gold labels" or ground truth. In other contexts, you might want to consider other performance measures we've learned about in this class, including precision and recall.

Please enter the **test error** of your linear regression algorithm for different λ (copy the output from the `main.py` run).

$\text{Error}|_{\lambda=1} =$ 0.744 ✔

$\text{Error}|_{\lambda=0.1} =$ 0.7442 ✔

$\text{Error}|_{\lambda=0.01} =$ 0.744 ✔

Submit You have used 1 of 20 attempts

✔ Correct (1/1 point)

What went Wrong?

1/1 point (graded)
Alice and you find that no matter what lambda factor you try, the test error is large. With some thinking, you realize that something is wrong with this approach.

- [ ] Gradient descent should be used instead of the closed form solution.

- [x] The loss function related to the closed-form solution is inadequate for this problem.
- [ ] Regularization should not be used here.

✔

Submit You have used 1 of 2 attempts

✔ Correct (1/1 point)

Discussion

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Topic: Unit 2 Nonlinear Classification, Linear regression, Collaborative Filtering (2 weeks):Project 2: Digit recognition (Part 1) / 2. Linear Regression with Closed Form Solution

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? Problem with last question — 2
1. I don't know what is loss function related to closed form solution, probably there is one but I don't remember this from lectures. 2. When I remove regularization from this ...

32 bit address space appears to be not enough. — 3
I am still using Windows XP 32-bit. On my local machine, the code produced the message "MemoryError". With no changes to the code, I get "CORRECT" when I submit my cod...

Assesment not available to non-upgraded people — 1
I understand that people not paying upgrade have no access to final certificate, not get grader explanations and more. But I think that to block completely the project explana...

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