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[STAFF] Overlapped symbols & math processing error in the solution to the problem "Is the Direction of Largest Empirical Variance Unique?"

discussion posted 13 days ago by sandipan_dey

The solution provided for the problem "Is the Direction of Largest Empirical Variance Unique?" contains many overlapped symbols & math processing errors, can you please correct?

Related to: (Optional) Unit 8 Principal component analysis:(Optional) Lecture 23: Principal Component Analysis / 7. Largest Eigenvalue and Principal Directions
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2 responses

sudarsanvsr_mit (Staff)
10 days ago

Hi sandipan_dey, this must be a mathjax processing error on your browser. Did it get corrected automatically?

No, it's still the same as it was before.

posted 8 days ago by sandipan_dey

+1

posted 4 days ago by markarjaybajo

Add a comment

sudarsanvsr_mit (Staff)
7 days ago

If anyone still has a rendering issue.

Solution:

The correct answer is **"No"**. First observe that if $\mathbf{w}$ is a unit vector, then $P^T\mathbf{w}$ is also a unit vector. This is because

$$\begin{aligned} \|P^T\mathbf{w}\|_2^2 &= (P^T\mathbf{w})^T P^T\mathbf{w} \\ &= \mathbf{w}^T PP^T\mathbf{w} \\ &= \mathbf{w}^T\mathbf{w} \\ &= \|\mathbf{w}\|_2^2 \\ &= 1. \end{aligned}$$

To go from the second to third line, we used that $PP^T = I_d$ and associativity of matrix multiplication.

Next, note that by the given decomposition

$$\mathbf{w}^T S\mathbf{w} = \mathbf{w}^T PDP^T\mathbf{w} = (P^T\mathbf{w})^T D(P^T\mathbf{w}).$$

But as $\mathbf{w}$ ranges over all unit vectors, we know that $P^T\mathbf{w}$ also ranges over all unit vectors. So if there exists $\mathbf{w} \neq \mathbf{v}_1$ such that $\mathbf{w}^T S\mathbf{w} = \lambda_1$, there must exist $\mathbf{b} \neq P^T\mathbf{v}_1 = (1, 0, \ldots, 0)^T$ such that $\mathbf{b}^T D\mathbf{b} = \lambda_1$. Observe that by matrix multiplication,

$$\mathbf{b}^T D\mathbf{b} = \sum_{i=1} \lambda_i (\mathbf{b}^i)^2 \leq \lambda_1(\mathbf{b}^1)^2 + \lambda_2(1 - \mathbf{b}_1^2).$$

We also used that $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_d \geq 0$ and $0 \leq \mathbf{b}_j^2 \leq 1 - \mathbf{b}_1^2$ for all $j \neq 1$. Suppose $\mathbf{b}^1 \neq 1$ (so that $\mathbf{b} \neq \mathbf{v}_1$ and $1 - \mathbf{b}_1^2 > 0$), then we have

$$\mathbf{b}^T D\mathbf{b} \leq \lambda_1(\mathbf{b}^1)^2 + \lambda_2(1 - \mathbf{b}_1^2) < \lambda_1(\mathbf{b}^1)^2 + \lambda_1(1 - \mathbf{b}_1^2) = \lambda_1,$$

where we used the strict inequality $\lambda_1 > \lambda_2$. Therefore, the equality case is **only** possible if $\mathbf{b} = (1, 0, \ldots, 0)^T$. Hence, we must also have $\mathbf{w} = \mathbf{v}_1$ if equality holds.

Thank you!

posted 4 days ago by markarjaybajo

Thank you!

posted 3 days ago by Alexander_Andrianov

It looks like there is a mistake in the solution closer to the end:

Should not it be

"Suppose $\mathbf{b}^1 \neq 1$ (so that $\mathbf{b} \neq P^T\mathbf{v}_1$ and $1 - b_1^2 > 0$)..."

instead of

"...(so that $\mathbf{b} \neq \mathbf{v}_1$ and..."

?

posted 3 days ago by Alexander_Andrianov

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