11.3.2 Orthonormal Vectors

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Week 11 due Dec 22, 2023 21:12 IST Completed

11.3.2 Orthonormal Vectors

At 2:11 in the first video, Robert writes square root of 2 as the length of the vector when it should be square root of 5. Please make a note of it!

Video 11.3.2 Part 1

Start of transcript. Skip to the end.

0:00 / 0:00 2.0x

Dr. Robert van de Geijn: Let's first look at what orthonormal vectors are.

So here we're given k vectors.

Let's call them q0 through qk minus 1.

They're all vectors in Rm.

We're going to say that these vectors are mutually orthonormal.

If all i and j, the inner product of those two vectors is always 0.

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Reading Assignment

0 points possible (ungraded)

Read Unit 11.3.2 of the notes. [LINK]

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Calculator

Homework 11.3.2.1

2/2 points (graded)

1. The vectors $\begin{pmatrix} -\sin(\theta) \\ \cos(\theta) \end{pmatrix}, \begin{pmatrix} \cos(\theta) \\ \sin(\theta) \end{pmatrix}$ are mutually orthonormal.

   TRUE ✔

2. The vectors $\begin{pmatrix} \sin(\theta) \\ \cos(\theta) \end{pmatrix}, \begin{pmatrix} \cos(\theta) \\ -\sin(\theta) \end{pmatrix}$ are mutually orthonormal. True/False

   TRUE ✔

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Video 11.3.2 Part 2

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Dr. Robert van de Geijn: So the answer turns out to be yes.

First of all, if you take the dot product of these two vectors,

what do you get?

You get minus the sine of theta, times the cosine of theta,

plus the cosine of theta, times the sine of theta.

0:00 / 0:00 2.0x

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Homework 11.3.2.2

1/1 point (graded)

Let $q_0, q_1, \ldots, q_{k-1} \in \mathbb{R}^m$ be a set of orthonormal vectors and let $Q = \left(q_0 \mid q_1 \mid \cdots \mid q_{k-1} \right)$.

Then $Q^T Q = I$.

TRUE ✔ Answer: TRUE

$$Q^T Q = \left(q_0 \mid q_1 \mid \cdots \mid q_{k-1} \right)^T \left(q_0 \mid q_1 \mid \cdots \mid q_{k-1} \right) =$$

$$\Big(\; q_0^T \; \Big)$$

Calculator

$$= \left(\begin{array}{c} q_1^T \\ \hline \vdots \\ \hline q_{k-1}^T \end{array} \right) \left(\begin{array}{c|c|c|c} q_0 & q_1 & \cdots & q_{k-1} \end{array} \right)$$

$$= \left(\begin{array}{c|c|c|c} q_0^T q_0 & q_0^T q_1 & \cdots & q_0^T q_{k-1} \\ \hline q_1^T q_0 & q_1^T q_1 & \cdots & q_1^T q_{k-1} \\ \hline \vdots & \vdots & & \vdots \\ \hline q_{k-1}^T q_0 & q_{k-1}^T q_1 & \cdots & q_{k-1}^T q_{k-1} \end{array} \right) = \left(\begin{array}{c|c|c|c} 1 & 0 & \cdots & 0 \\ \hline 0 & 1 & \cdots & 0 \\ \hline \vdots & \vdots & & \vdots \\ \hline 0 & 0 & \cdots & 1 \end{array} \right)$$

$$\text{since } q_i^T q_j = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{otherwise.} \end{cases}$$

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Answers are displayed within the problem

Video 11.3.2 Part 3

0:00 / 0:00 2.0x

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Robert van de Geijn: And the answer is that this is true.

How do we show that?

Just go through it.

If you multiply Q transpose times Q, that's

the same as exposing the columns of Q in each of these matrices.

If you then transpose the first one you

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Homework 11.3.2.3

1/1 point (graded)

Let $Q \in \mathbb{R}^{m \times k}$ (with $k \leq m$) and $Q^T Q = I$. Partition

$$Q = \left(\begin{array}{c|c|c|c} q_0 & q_1 & \cdots & q_{k-1} \end{array} \right).$$

Then $q_0, q_1, \ldots, q_{k-1}$ are mutually orthonormal vectors.

TRUE ✔ Answer: TRUE

Calculator

$$
\begin{aligned}
Q^T Q &= \left(\begin{array}{c|c|c|c} q_0 & q_1 & \cdots & q_{k-1} \end{array} \right)^T \left(\begin{array}{c|c|c|c} q_0 & q_1 & \cdots & q_{k-1} \end{array} \right) \\
&= \left(\begin{array}{c} q_0^T \\ \hline q_1^T \\ \hline \vdots \\ \hline q_{k-1}^T \end{array} \right) \left(\begin{array}{c|c|c|c} q_0 & q_1 & \cdots & q_{k-1} \end{array} \right) \\
&= \left(\begin{array}{c|c|c|c} q_0^T q_0 & q_0^T q_1 & \cdots & q_0^T q_{k-1} \\ \hline q_1^T q_0 & q_1^T q_1 & \cdots & q_1^T q_{k-1} \\ \hline \vdots & \vdots & & \vdots \\ \hline q_{k-1}^T q_0 & q_{k-1}^T q_1 & \cdots & q_{k-1}^T q_{k-1} \end{array} \right) \\
&= \left(\begin{array}{c|c|c|c} 1 & 0 & \cdots & 0 \\ \hline 0 & 1 & \cdots & 0 \\ \hline \vdots & \vdots & & \vdots \\ \hline 0 & 0 & \cdots & 1 \end{array} \right).
\end{aligned}
$$

Hence $q_i^T q_j = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{otherwise.} \end{cases}$

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Answers are displayed within the problem

Video 11.3.2 Part 4

Start of transcript. Skip to the end.

Dr. Robert van de Geijn: And the answer is that this is true.

How do we know that?

Well, if you look at Q transpose Q, you get

that, which is then equal to that, which is then

equal to that, which is then equal to

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