1. Resonance

1-1

1/1 point (graded)

Pure resonance occurs when an undamped system is forced at the same frequency as (one of) its natural frequencies. Select all of the following cases in which pure resonance occurs. You never need to compute the full Fourier series for this, only at most one coefficient (or one sine or cosine coefficient with the same frequency).

Assume in all cases that $F(t)$ is odd with period 2π.

- [ ] (a) $2\ddot{x}+10x=F(t)$; where F is odd.
- [x] (b) $3\ddot{x}+27x=F(t)$; where $F(t)=1$ on $0<t<\pi$, and F is odd.
- [x] (c) $\ddot{x}+4x=F(t)$; where $F(t)>0$ on $0<t<\pi/2$, $F(t)=0$ on $\pi/2<t<\pi$, and F is odd.
- [x] (d) $2\ddot{x}+50x=F(t)$; where $F(t)=\pi t-t^2$ on $0<t<\pi$, and F is odd
- [ ] (e) $2\ddot{x}+(0.1)\dot{x}+18x=F(t)$; where $F(t)=2t$ on $0<t<\pi$, and F is odd.

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Solution:

(a) (No pure resonance.) Because $F(t)$ is odd of period 2π, its Fourier series takes the form

$$F(t) = \sum_{n=1}^{\infty} b_n \sin(nt).$$

The differential equation $2\ddot{x} + 10x = F(t)$ has characteristic equation $P(r) = 2r^2 + 10 = 0$, which has roots $r = \pm i\sqrt{5}$. Therefore the only input frequency that will give rise to an unbounded response is $\sqrt{5}$. Because the input frequencies are $n = 1, 2, 3, \ldots$, there is no pure resonance.

(b) (Yes, there is pure resonance.) In this problem, $F(t) = \mathrm{Sq}(t)$, the standard square wave, which has Fourier series

$$\mathrm{Sq}(t) = \frac{4}{\pi}\left(\sin(t) + \frac{1}{3}\sin(3t) + \cdots\right).$$

The differential equation $3\ddot{x} + 27x = F(t)$ has characteristic polynomial $P(r) = 3r^2 + 27$, which has roots $r = \pm 3i$. Therefore there is pure resonance since the coefficient b_3 in the Fourier series is nonzero.

(c) (Yes, there is pure resonance.) The differential equation $\ddot{x} + 4x = F(t)$ has characteristic polynomial $P(r) = r^2 + 4$, which has roots $r = \pm 2i$. Therefore all we need to determine is if the coefficient b_2 is nonzero.

$$b_2 = \frac{2}{\pi}\int_0^{\pi} F(t)\sin(2t)\,dt = \frac{2}{\pi}\int_0^{\pi/2} F(t)\sin(2t)\,dt > 0$$

The last integral is greater than zero because both $F(t) > 0$ and $\sin(2t) > 0$ for $0 < t < \pi/2$.

(d) (Yes, there is pure resonance.) The differential equation $2\ddot{x} + 50x = F(t)$ has characteristic polynomial $P(r) = 2r^2 + 50$, which has roots $r = \pm 5i$. Therefore we only need to determine if b_5 is nonzero in the Fourier series for $F(t)$.

$$b_5 = \frac{2}{\pi}\int_0^{\pi} F(t)\sin(5t)\,dt = \frac{2}{\pi}\int_0^{\pi} (\pi t - t^2)\sin(5t)\,dt \neq 0,$$

where we compute the last integral numerically or by elaborate integration by parts. Therefore the resonant term is present in the input signal, so there is pure resonance.

(e) (No pure resonance.) The characteristic polynomial of the differential equation $2\ddot{x}+(0.1)\,\dot{x}+18x=F(t)$ is $P(r)=2r^2+0.1r+18.$ For all input frequencies ω,

$$P(i\omega)=18-2\omega^2+0.1i\omega\neq 0.$$

Therefore there is no pure resonance. (Pure resonance only occurs when the roots of the characteristic polynomial are purely imaginary. This doesn't happen with a damping term.)

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Answers are displayed within the problem

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[STAFF] 1 week extension

Hi, 18.033 ends at May 27 and I'm still working on it. Is it possible to extend one more week for this unit? Thanks 6