7. Practice

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Lecture due Sep 13, 2021 20:30 IST Completed

Practice

In the sequence of problems on this page, we will walk you through the process of finding the maximum value of the function $f(x,y) = xy^2 - x^3/3$ on the region R defined by $0 \le x \le 2, 0 \le y \le 2$.

We will go about this in four steps:

1. Find the critical point.

2. Identify the type of critical point.

3. Find the equation of the function along each boundary edge.

4. Find the absolute maximum by comparing maximal values along the boundary and the value at the critical point.

Step 1. Find the critical point

1.0/1 point (graded)
This function $f(x,y) = xy^2 - x^3/3$ has one critical point on the region R defined by $0 \le x \le 2, 0 \le y \le 2$.

Find this critical point.

(Enter the point in the plane as an ordered pair surrounded by round parentheses: `(a, b)` .)

(0,0)

✔ **Answer:** (0,0)

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Solution:

First we check for critical points.

$$f_x(x,y) = y^2 - x^2 \quad (4.128)$$
$$f_y(x,y) = 2xy \quad (4.129)$$

Setting the partial derivatives equal to zero we see that the critical point is $(0,0)$, which is on our boundary. The value of the function there is 0.

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Answers are displayed within the problem

Step 2. Identify the type of critical point

1/1 point (graded)
Use the second derivative test to identify the type of critical point.

- Local maximum
- Local minimum
- Saddle point

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Cannot be determined

Solution:

We start by computing the second derivatives.

$$f_{xx}(x,y) = -2x \quad (4.130)$$
$$f_{xy}(x,y) = 2y \quad (4.131)$$
$$f_{yy}(x,y) = 2x \quad (4.132)$$

At the origin, all the numbers $A = B = C = 0$ thus the second derivative test is inconclusive.

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Step 3. Find the equation for the function along each boundary

8/8 points (graded)

Restrict the function $f(x,y) = xy^2 - x^3/3$ to each edge of the region R. Find an equation in either x alone or y alone as specified.

$f(x,0) =$ -x^3/3 **Answer:** -x^3/3

$-\frac{x^3}{3}$

$f(x,2) =$ 4*x-x^3/3 **Answer:** 4*x-x^3/3

$4 \cdot x - \frac{x^3}{3}$

$f(0,y) =$ 0 **Answer:** 0

0

$f(2,y) =$ 2*y^2-8/3 **Answer:** 2*y^2-8/3

$2 \cdot y^2 - \frac{8}{3}$

Evaluate the function $f(x,y) = xy^2 - x^3/3$ at the four corners.

$f(0,0) =$ 0 **Answer:** 0

$f(0,2) =$ 0 **Answer:** 0

$f(2,0) =$ -8/3 **Answer:** -8/3

$f(2,2) =$ 16/3 **Answer:** 16/3

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Solution:

We identify the function along each edge of the boundary by plugging in the

$$
\begin{aligned}
f(0,y) &= 0 && (4.133)\\
f(2,y) &= 2y^2 - 8/3 && (4.134)\\
f(x,0) &= -x^3/3 && (4.135)\\
f(x,2) &= 4x - x^3/3 && (4.136)
\end{aligned}
$$

We plug in the values of each corner to find the values.

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Answers are displayed within the problem

Step 4. Find the absolute maximum value

1/1 point (graded)

16/3 **Answer:** 16/3

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Solution:

We have formulas for the function along each boundary line.

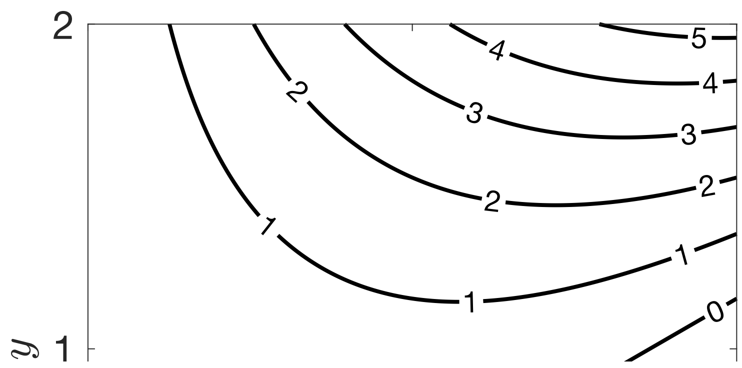
$$
\begin{aligned}
f(0,y) &= 0 && (4.137)\\
f(2,y) &= 2y^2 - 8/3 && (4.138)\\
f(x,0) &= -x^3/3 && (4.139)\\
f(x,2) &= 4x - x^3/3 && (4.140)
\end{aligned}
$$

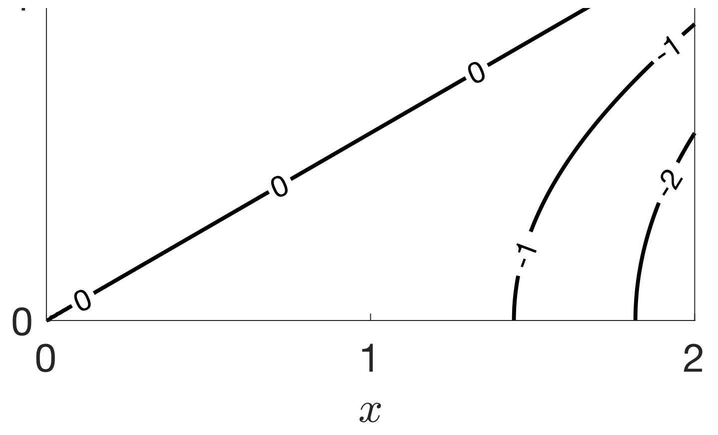
We optimize along each edge.

- The first edge gives us the value 0 along the entire edge.
- The second edge gives us $f'(2,y) = 4y = 0$ gives us the point $(2,0)$, which has value $f(2,0) = -8/3 < 0$. Thus is not a maximum.
- The third edge gives us $f'(x,0) = -x^2 = 0$ gives us the point $(0,0)$ again, which has value 0.
- The fourth edge gives us $f'(x,2) = 4 - x^2 = 0$ gives us the point $(2,2)$, which has value $16/3$.

Therefore the maximum occurs at the upper corner $(2,2)$ with maximum value $16/3$.

We can see that this location makes sense as the location for the maximum based on an image of the level curves on this region.





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