Bookmarks

- 0. Getting Started
- 1. Introduction to Observation Theory
- **2. Mathematical model**
  - **Warming Up**
  - **2.1 Functional Model**
  - **2.2 Properties of Functional Models**
  - **2.3 Stochastic Model**
  - **Assessment**
    Graded Assignment due Feb 8, 2017 17:30 IST
  - **Q&A Forum**
  - **Feedback**
- 3. Least Squares Estimation (LSE)

2. Mathematical model > 2.3 Stochastic Model > Exercises: Stochastic model

Exercises: Stochastic model

Bookmark this page

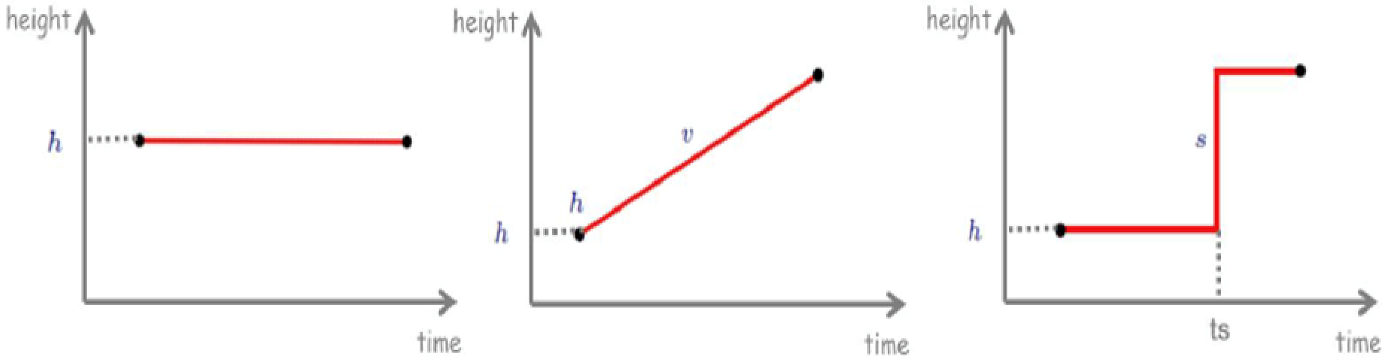
Airborne Laser Scanning of beach height

4/4 points (ungraded)

Consider a beach on the coast of the Netherlands. Airborne Laser Scanning is used every year to measure the beach's topography.

After processing, the data set from each acquistion is comprised of a grid of height measurements in meters. For this exercise, let's focus on one particular grid cell (i.e. one location on the beach). This particular grid cell's data set has the height observations from five consecutive years. The goal is to assess what is happening to the location's height over time. There are three scenarios, illustrated in the figure below. The height at this location may have been stable, may have been changing with constant velocity (in $[\mathrm{cm}/\mathrm{yr}]$, or may have been constant and then suddenly increased due to a sand suppletion (sand was dumped on the location to protect the beach).

- 4. Best Linear Unbiased Estimation (BLUE)
- Pre-knowledge Mathematics
- MATLAB Learning Content



Note the variables drawn in the figure, you will need them for the exercises!

Specify the functional model $E\{\underline{y}\}$ for a stable height scenario.

●

$$E\{\underline{y}\} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} [h]$$ ✓

○ $$E\{\underline{y}\} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} [h]$$

○ $$E\{\underline{y}\} = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{bmatrix} [h]$$

○ $$E\{\underline{y}\} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} [s]$$

Specify the functional model $E\{\underline{y}\}$ for a constant vertical velocity scenario.

● $E\{\underline{y}\} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 2 \\ 1 & 3 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} h \\ v \end{bmatrix}$ ✓

○ $E\{\underline{y}\} = \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 1 & 0 \\ 1 & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} h \\ s \end{bmatrix}$

○ $E\{\underline{y}\} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \\ 0 & 2 \\ 0 & 3 \\ 0 & 4 \end{bmatrix} \begin{bmatrix} h \\ v \end{bmatrix}$

○ $$E\{\underline{y}\} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \begin{bmatrix} v \end{bmatrix}$$

Specify the functional model $E\{\underline{y}\}$ for the scenario with constant height, followed by a sudden height increase after the 2nd year due to suppletion.

○ $$E\{\underline{y}\} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 2 \\ 1 & 3 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} h \\ s \end{bmatrix}$$

● $$E\{\underline{y}\} = \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 1 & 1 \\ 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} h \\ s \end{bmatrix}$$ ✓

○

$$E\{\underline{y}\} = \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 1 & 1 \\ 1 & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} h \\ s \end{bmatrix}$$

○

$$E\{\underline{y}\} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \\ 1 \end{bmatrix} [h]$$

Now we need to specify what the stochastic model is. Let's say the standard deviation of the laser scanner used is $\sigma = 1[\mathrm{cm}]$. Now imagine that after the 2nd year, the surveying project's budget was increased, and they were able to upgrade to a more precise laser scanner. This improved laser scanner had a standard deviation of $\sigma = 0.5[\mathrm{cm}]$.

Specify which stochastic model $D\{\underline{y}\} = Q_{yy}$ corresponds to this problem. Note: you may assume that the observations are independent of each other.

○

$$Q_{yy} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

○

$$Q_{yy} = \begin{bmatrix} .5^2 & 0 & 0 & 0 & 0 \\ 0 & .5^2 & 0 & 0 & 0 \\ 0 & 0 & .5^2 & 0 & 0 \\ 0 & 0 & 0 & .5^2 & 0 \\ 0 & 0 & 0 & 0 & .5^2 \end{bmatrix}$$

○

$$Q_{yy} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & .5 & 0 & 0 \\ 0 & 0 & 0 & .5 & 0 \\ 0 & 0 & 0 & 0 & .5 \end{bmatrix}$$

● $$Q_{yy} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & .5^2 & 0 & 0 \\ 0 & 0 & 0 & .5^2 & 0 \\ 0 & 0 & 0 & 0 & .5^2 \end{bmatrix}$$ ✓

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✓ Correct (4/4 points)

Observing an object at constant velocity

3/3 points (ungraded)

Consider an object that is moving in a straight line at a constant but unknown speed v. It started at the origin $y = 0$ at $t = 0$. Uncorrelated observations y_i of the distance travelled by the object from $y = 0$ have been made at corresponding time instants $t_i = i$ seconds, where $i = 1, 2, \ldots m$. The precision of the observations is given by $\sigma_{y_i} = i \cdot \sigma$.

Specify the functional model.

○ $$E\{\begin{bmatrix}\underline{y}_1\\\underline{y}_2\\\vdots\\\underline{y}_m\end{bmatrix}\}=\begin{bmatrix}1\\1\\\vdots\\1\end{bmatrix}v$$

● $$E\{\begin{bmatrix}\underline{y}_1\\\underline{y}_2\\\vdots\\\underline{y}_m\end{bmatrix}\}=\begin{bmatrix}1\\2\\\vdots\\m\end{bmatrix}v$$ ✓

○ $$E\{\begin{bmatrix}\underline{y}_1\\\underline{y}_2\\\vdots\\\underline{y}_m\end{bmatrix}\}=\begin{bmatrix}0\\1\\\vdots\\m\end{bmatrix}v$$

Specify the stochastic model.

○ $$D\{\begin{bmatrix}\underline{y}_1\\ \underline{y}_2\\ \vdots\\ \underline{y}_m\end{bmatrix}\} = \begin{bmatrix}1^2 & 0 & \dots & 0\\ 0 & 2^2 & & \vdots\\ \vdots & & \ddots & 0\\ 0 & \dots & 0 & m^2\end{bmatrix}$$

● $$D\{\begin{bmatrix}\underline{y}_1\\ \underline{y}_2\\ \vdots\\ \underline{y}_m\end{bmatrix}\} = \sigma^2\begin{bmatrix}1^2 & 0 & \dots & 0\\ 0 & 2^2 & & \vdots\\ \vdots & & \ddots & 0\\ 0 & \dots & 0 & m^2\end{bmatrix}$$ ✓

○ $$D\{\begin{bmatrix}\underline{y}_1\\ \underline{y}_2\\ \vdots\\ \underline{y}_m\end{bmatrix}\} = \sigma^2\begin{bmatrix}1 & 0 & \dots & 0\\ 0 & 2 & & \vdots\\ \vdots & & \ddots & 0\\ 0 & \dots & 0 & m\end{bmatrix}$$

○ $$D\{\begin{bmatrix}\underline{y}_1\\ \underline{y}_2\\ \vdots\\ \underline{y}_m\end{bmatrix}\} = \begin{bmatrix}\sigma^2 & 0 & \dots & 0\\ 0 & \sigma^2 & & \vdots\\ \vdots & & \ddots & 0\\ 0 & \dots & 0 & \sigma^2\end{bmatrix}$$

What is the redundancy of this system?

● $m-1$ ✓

○ m

○ 0

○ 1

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✓ Correct (3/3 points)