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7.

Setup:

Suppose you observe an i.i.d. sample $X_1, \ldots, X_n$ of a Bernoulli random variable X with unknown parameter $\theta_0 = \mathbf{P}(X=1)$.

We use the Bayesian approach to statistical inference and model the unknown parameter θ_0 as a random variable Θ (defined jointly with X). That is, let the conditional distribution of X given Θ be

$$X|\Theta \sim \mathsf{Ber}(\Theta)$$

and let

$$\pi_\Theta(\theta) = 3\theta^2 \mathbf{1}(\theta \in [0,1])$$

be the prior distribution of Θ.

Let P denote the probabilities implied by the Bayesian model.

Useful Facts:

The $\mathsf{Beta}(\alpha, \beta)$ distribution has pdf

$$f(t) = \frac{t^{\alpha-1}(1-t)^{\beta-1}}{B(\alpha,\beta)} \quad \text{where} \quad B(\alpha,\beta) = \int_0^1 t^{\alpha-1}(1-t)^{\beta-1}\,dt,$$

and

$$\begin{aligned} \text{Mean} &= \frac{\alpha}{\alpha+\beta} \\ \text{Variance} &= \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)} \\ \text{Mode} = \text{argmax} f &= \frac{\alpha-1}{\alpha+\beta-2} \text{ for } \alpha,\beta > 1. \end{aligned}$$

The **mode** is the value of the random variable at which the pdf attains its **maximum**.

Posterior Distribution

3.0/3.0 points (graded)

The posterior distribution $\pi_{\Theta|X_1,...,X_n}$ of Θ given the sample $X_1, \ldots, X_n$ is a Beta distribution Beta (a,b). Specifiy the parameters a and b below.

(Enter **barX_n** for $\overline{X}_n$.)

$a =$ n*barX_n +3 ✔ **Answer:** 3+ n*barX_n

$b =$ n-n*barX_n +1 ✔ **Answer:** n +1 - n*barX_n

Submit You have used 1 of 3 attempts

Answers are displayed within the problem

Bayes Estimator and its Limit

2/2 points (graded)

What is the mean $\hat{\theta}_n^{\text{Bayes}}$ of the posterior distribution in terms of $\overline{X}_n$?

(Enter **barX_n** for $\overline{X}_n$).

$\hat{\theta}^{\text{Bayes}} = \mathbb{E}[\Theta|\vec{X}_n] =$ (n*barX_n +3)/(n+4) ✔ **Answer:** (3+ n*barX_n)/(n + 4)

The estimator $\hat{\theta}_n^{(\pi)}$ converges in probability to a constant as $n \to \infty$? What is this limiting constant?

(Enter in terms of the true parameter θ_0.)

$\hat{\theta}_n^{(\pi)} \xrightarrow{\text{in } \mathbb{P}}$ theta_0 ✔ **Answer:** theta_0

θ_0

STANDARD NOTATION

Submit You have used 1 of 3 attempts

Answers are displayed within the problem

MAP Estimator

1/1 point (graded)

What is the maximum a posteriori (MAP) estimator $\hat{\theta}_n^{\text{MAP}}$ of θ_0 in terms of $\overline{X}_n$? The maximum a posteriori (MAP) estimator is the value of θ where the posterior distribution is maximum, i.e. $\hat{\theta}^{\text{MAP}} = \arg\max_\theta \pi_{\Theta|\vec{X}_n}(\theta)$.

(Enter **barX_n** for $\overline{X}_n$).

$\hat{\theta}^{\text{MAP}} = \arg\max_\theta \pi_{\Theta|\vec{X}_n}(\theta) =$ (n*barX_n +2)/(n+2) ✔ **Answer:** (2+ n*barX_n)/(n + 2)

What is the limit in probability of $\hat{\theta}_n^{\text{MAP}}$ as $n \to \infty$? (There is no answer box for this question.)

STANDARD NOTATION

Submit You have used 1 of 3 attempts

Answers are displayed within the problem

MLE

2/2 points (graded)

What is the MLE $\hat{\theta}^{\text{MLE}}$ of θ in terms of $\overline{X}_n$?

(Enter **barX_n** for the mean $\overline{X}_n$.)

$\hat{\theta}^{\text{MLE}} =$ barX_n ✔ **Answer:** barX_n

The MLE $\hat{\theta}_n^{\text{MLE}}$ converges in probability as $n \to \infty$ to what constant?

$\hat{\theta}_n^{\text{MLE}} \xrightarrow{\text{in } \mathbb{P}}$ theta_0 ✔ **Answer:** theta_0

θ_0

STANDARD NOTATION

Submit You have used 1 of 3 attempts

Answers are displayed within the problem

Ungraded Questions for Thought

The following questions do not have answer boxes and are for you to explore.

1. What is the Fisher information $\mathcal{I}(\theta)$?

2. What is the variance $\mathsf{Var}\left(\Theta|\vec{X}_n\right)$ of the posterior distribution?

3. What is the limit of $n\mathsf{Var}\left(\Theta|\vec{X}_n\right)$ in terms of the Fisher information $\mathcal{I}(\theta)$?

Error and Bug Reports/Technical Issues

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[STAFF] Grade calculation
Hi, it seems that my total grade and the number of wrong answer are mismatched. Can you check my grade again please ? 2

[Staff] Midterm2 grading time
It is about an hour over the end time of submission of midterm2. I can't see the grading on the same. Also, the submission is still open and anybody can still submit. Some... 3 new_

Final score out?
I think the final verdict is out in the progress bar. Still, the tick marks are yet to come out in the exam section. 3

[Staff] when will final exam be released?
Do we have a date for the final release please? 3

[Polling] What are your answers for this page?
1) a = n*barX_n +3; b = n+1-n*barX_n 2) bayes = (n*barX_n +3)/(n+4) ; converge to theta_0 as n to inf 3) map = (n*barX_n +2)/(n+2) 4) MLE = barX_n ; converge to theta_0 as... 11

Mistakes during exam/assignments
This post has probably no real learning value and is mostly a rant, but I'm still curious to know how many of you guys makes some really stupid mistakes while solving exa...
12

Possible resolution MAP Estimator
(Of course I could be totally wrong, please correct me in case) [Edited name of variables] # Load the needed Julia packages using SymPy ## Map estimator θ, x, Σx², Σx, Πx ...
5

Exam still open after deadline..
It's 2:58 am 28/Nov Paris time (baby Waked up.. :-( ) but the midterm 2, instead of showing answers, is still open...
8

Scores
No scores, no answers. Is this working?
2

Logged back in & still able to see exam questions
Just wanted to highlight that it seems like the timer hasn't ended yet (don't see any). I've been unable to hit submit (no changes) - hope that there won't be any issues for ...
2

how can i enter theta_0 in Bayes Estimator and its Limit
how can i enter theta_0 in Bayes Estimator and its Limit
2

Deleted by MW-CTA
Dear TA, **Deleted by MW-CTA**
2

[NOTE FROM STAFF] Enter theta_0 for θ_0

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