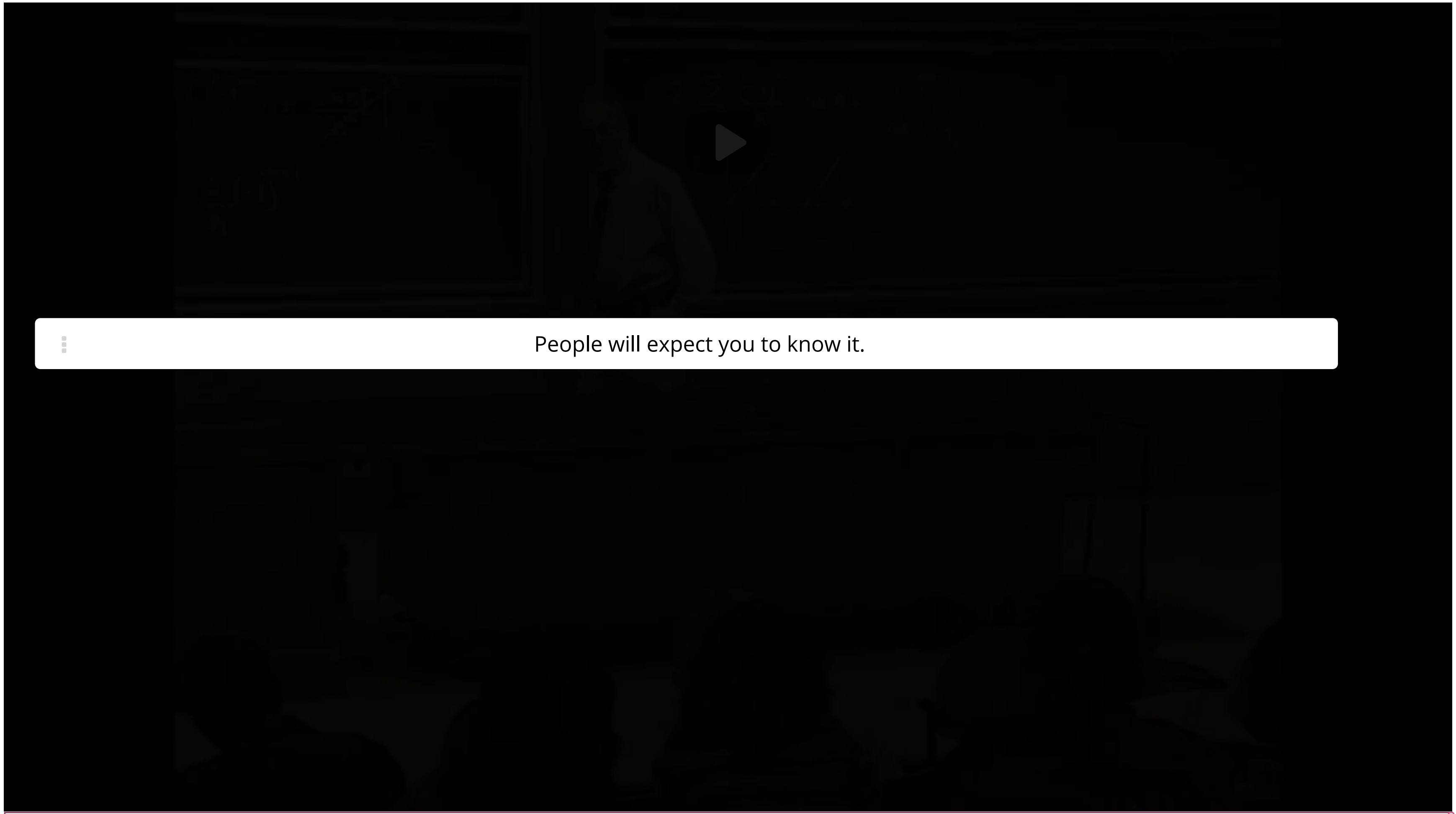
5. Convergence of a Fourier series

Convergence at jump discontinuities



7:02 / 7:02 1.50x

Video

[Download video file]

Transcripts

[Download SubRip (.srt) file]

[Download Text (.txt) file]

Definition 5.1 A periodic function f of period $2L$ is called **piecewise differentiable** if

- at points where the derivative exists it is bounded, (that is, there is an M such that $|f'(t)| \leq M < \infty$ at all t for which $f'(t)$ exists),

- there are at most finitely many points in $[-L, L)$ where $f'(t)$ does not exist, and
- at each such point τ, the left limit $f(\tau^-) := \lim_{t\to\tau^-} f(t)$ and right limit $f(\tau^+) := \lim_{t\to\tau^+} f(t)$ exist (although they might be unequal, in which case we say that f has a **jump discontinuity** at τ).

Theorem 5.2 If f is a piecewise differentiable periodic function, then the Fourier series of f (with the a_n and b_n defined by the Fourier coefficient formulas)

- converges to $f(t)$ at values of t where f is continuous, and
- converges to $\frac{f(t^-)+f(t^+)}{2}$ where f has a jump discontinuity.

Example 5.3 The left limit $\mathrm{Sq}(0^-) = -1$ and right limit $\mathrm{Sq}(0^+) = 1$ average to 0. The Fourier series

$$\frac{4}{\pi}\left(\sin t + \frac{\sin 3t}{3} + \frac{\sin 5t}{5} + \cdots\right)$$

evaluated at $t = 0$ converges to 0 too.

(Note that the examples here are for 2π-periodic functions, but this theorem on convergence holds for periodic functions of any period, which we begin to discuss on the next page.)

Practice 1

1/1 point (graded)
Consider the 2π-periodic function

$$f(t) = \begin{cases} t & 0 < t < \pi \\ 0 & -\pi < t < 0 \end{cases}.$$

Let $g(t)$ be the Fourier series for $f(t)$. What is $g(\pi)$?

$g(\pi) =$ pi/2 ✔ **Answer:** pi/2

$\frac{\pi}{2}$

FORMULA INPUT HELP

Solution:

The function $f(t)$ has a jump discontinuity at π. Note that $f(\pi^-) = \pi$ and $f(\pi^+) = 0$, therefore

$$g(\pi) = \frac{f(\pi^-) + f(\pi^+)}{2} = \frac{\pi + 0}{2} = \frac{\pi}{2}.$$

Submit You have used 1 of 5 attempts

Answers are displayed within the problem

Practice 2

1/1 point (graded)

Let $U(t)$ be the Fourier series of the 2π-periodic triangle wave

$$T(t) = |t|, \quad -\pi < t < \pi.$$

Find the value of $U(\pi)$.

$U(\pi) =$ pi ✔ **Answer:** pi

π

FORMULA INPUT HELP

Solution:

This function is continous, so the value of the Fourier series is the value of the function at these points, which is π.

Submit You have used 1 of 5 attempts

Answers are displayed within the problem

5. Convergence of a Fourier series

Hide Discussion

Topic: Unit 1: Fourier Series / 5. Convergence of a Fourier series

Add a Post

Show all posts ▼ by recent activity ▼

g(pi)
Please, This is a Odd or Even function? In mt point is a Even function, so you have only the an coeficiente, ok? Thanks, Ricardo 6

How to enter the sigma notation?
In order to answer the question 1 and 2, we need to enter the series which contains sigma notation, how I am supposed to enter it? 2

Piecewise differentiability - the first point 3

Learn About Verified Certificates