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4. Uniform random variables

Expectation, variance and probabilities

4/4 points (graded)

Let X be a uniform random variable in the interval $[2, 8.5]$. Find the following quantities (if needed, round to the nearest 10^{-4}):

$\mathbb{E}[X] =$ 5.25 ✔

$\text{Var}[X] =$ 3.520833 ✔

$\mathbf{P}(X > 4) =$ 0.6923076923076923 ✔

$\mathbf{P}(\log(X) \leq 1) =$ 0.11050489668600694 ✔

(Note that the logarithm is the natural one to base $\mathbf{e}$)

STANDARD NOTATION

Submit You have used 3 of 4 attempts

✔ Correct (4/4 points)

Two independent copies

3/3 points (graded)

Let U, V be i.i.d. random variables uniformly distributed in $[0,1]$. Compute the following quantities:

$\mathbb{E}[|U-V|] =$ 1/3 ✔ **Answer:** 1/3

$\mathbf{P}(U = V) =$ 0 ✔ **Answer:** 0

$\mathbf{P}(U \leq V) =$ 1/2 Answer: 1/2

STANDARD NOTATION

Solution:

For the first quantity, we write the joint expectation as an iterated expectation and conditional expectation,

$$\mathbb{E}[|U - V|] = \mathbb{E}[\mathbb{E}[|U - V||V]].$$

By independence, we can compute the inner expectation as

$$\begin{aligned}
\mathbb{E}[|U - V||V = v] &= \int_0^1 |u - v|\, du \\
&= \int_0^v (v - u)\, du + \int_v^1 (u - v)\, du \\
&= \left[vu - \frac{1}{2}u^2\right]_0^v + \left[\frac{1}{2}u^2 - vu\right]_v^1 = v^2 - \frac{1}{2}v^2 + \frac{1}{2} - v - \frac{1}{2}v^2 + v^2 \\
&= v^2 - v + \frac{1}{2},
\end{aligned}$$

so

$$\mathbb{E}\left[\,|U-V|\,\right] = \mathbb{E}\left[V^2 - V + \frac{1}{2}\right] = \frac{1}{3} - \frac{1}{2} + \frac{1}{2} = \frac{1}{3}.$$

For the probability $\mathbf{P}\left(U = V\right)$, just write this as double expectation as well and notice that

$$\mathbf{P}\left(U=V\right) = \mathbb{E}\left[\mathbb{E}\left[\mathbf{1}\left(U=V\right)|V\right]\right] = \mathbb{E}\left[0\right] = 0,$$

because the probability of a uniform random variable being equal to any fixed number between 0 and 1 is zero.

For $\mathbf{P}\left(U \leq V\right)$, write it again as a double expectation,

$$\mathbf{P}\left(U \leq V\right) = \mathbb{E}\left[\mathbb{E}\left[\mathbf{1}\left(U \leq V\right)|V\right]\right] = \mathbb{E}\left[\mathbf{P}\left(U \leq V\right)|V\right] = \mathbb{E}\left[V\right] = \frac{1}{2}.$$

Alternatively, this can also be seen by symmetry of the two variables, i.e., $P\left(U \leq V\right) = P\left(V \leq U\right)$ and either one of the two must be true, counting double the zero-set of $\mathbf{P}\left(U = V\right)$.
: Uniform PDF in Lecture 8, *Probability density functions*.

Submit

You have used 2 of 3 attempts

Answers are displayed within the problem

Maximum and sum of independent copies

1/1 point (graded)

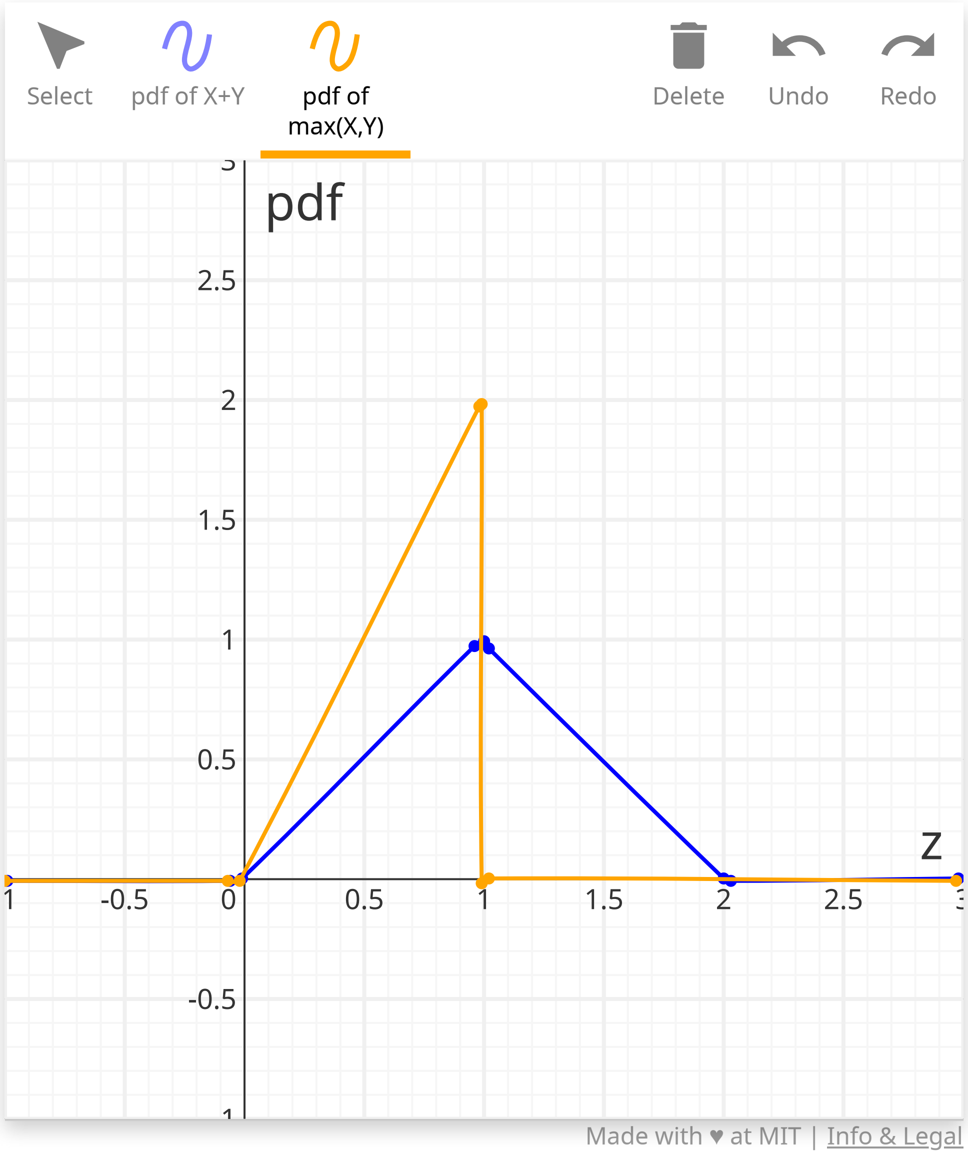
Let X, Y be independent random variables uniformly distributed in $[0, 1]$. In the graph below, sketch

1. the probability density $f_{X+Y}(z)$ of $X + Y$;

2. the probability density $f_{\max(X,Y)}(z)$ of $\max(X, Y)$.

(Be sure to sketch on the **entire domain** shown on the graph.)

Drawing tip: The spline tool draws a smooth curve connecting the points you click. To draw sharp corners, click on the point where the corner would be, then click again very close to it, and then continue onto the next point of your function.



Answer: See solution.

STANDARD NOTATION

Solution:

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The density of $X+Y$ is given by the convolution of the density of a uniform random variable,

$$f(x) = \mathbf{1}[0,1] = \begin{cases} 1, & \text{if } x \in [0,1] \\ 0, & \text{otherwise} \end{cases}$$

The density g of $X+Y$ therefore is

$$\begin{aligned} g(z) = & \int_{\mathbb{R}} f(x)\, f(z-x)\; dx \\ = & \int_{\mathbb{R}} \mathbf{1}(x \in [0,1])\, \mathbf{1}(z-x \in [0,1])\; dx \\ = & \int_0^1 \mathbf{1}(z-1 \le x \le z)\; dx \\ = & \mathbf{1}(z \le 2) \int_{\max\{0,z-1\}}^{\min\{1,z\}} dx \\ = & \begin{cases} 0, & z < 0 \\ z, & 0 < z < 1 \\ 2-z, & 1 < z < 2 \\ 0, & z > 2. \end{cases} \end{aligned}$$

For the density of $\max\{X,Y\}$, first note that it is supported in $[0,1]$. Now, first compute the cdf on that interval:

$$\mathbf{P}(\max\{X,Y\} \le y) = \quad \mathbf{P}(X \le y)\, \mathbf{P}(Y \le y) \quad \text{(by independence)}$$

$$= \quad t^2.$$

Hence, the density h of $\max\{X,Y\}$ is given by

$$h(z) = \begin{cases} 0, & z < 0 \\ 2z, & 0 \le z \le 1 \\ 0, & z > 1. \end{cases}$$

Submit You have used 3 of 10 attempts

Answers are displayed within the problem

Maximum of uniform random variables

2/2 points (graded)

Let $U_1, \ldots, U_n$ be i.i.d. random variables uniformly distributed in $[0,1]$ and let $M_n = \max_{1 \le i \le n} U_i$.

Find the cdf of M_n, which we denote by $G(t)$, for $t \in [0,1]$.

For $t \in [0,1]$,

$G(t) =$ t^n ✔ **Answer:** t^n

t^n

Now, let $F_n(t)$ denote the cdf of $n(1-M_n)$; for $t > 0$, compute

$\lim_{n\to\infty} F_n(t) =$ 1-exp(-t) ✔ **Answer:** 1-exp(-t)

$1-\exp(-t)$

STANDARD NOTATION

Solution:

First, we compute the cdf. Let $t \in [0,1]$. Then,

$$\mathbf{P}(M_n \leq t) = \mathbf{P}\left(\max_{i=1,\ldots,n} U_i \leq t\right) = \mathbf{P}\left(\cap_{i=1}^n \{U_i \leq t\}\right) = \prod_{i=1}^n \mathbf{P}(U_i \leq t) = t^n,$$

where we used the independence of the U_i to write the intersection as a product.

Now,

$$\mathbf{P}(n(1-M_n) \leq t) = \mathbf{P}\left(1 - M_n \leq \frac{t}{n}\right) = \mathbf{P}\left(M_n \geq 1 - \frac{t}{n}\right)$$

$$= \quad 1 - \mathbf{P}\left(M_n < 1 - \frac{t}{n}\right) = 1 - \left(1 - \frac{t}{n}\right)^n \xrightarrow{n \to \infty} 1 - \mathbf{e}^{-t}.$$

Hence, $n(1 - M_n)$ converges in distribution to $\mathrm{Exp}(1)$.

Submit You have used 2 of 3 attempts

Answers are displayed within the problem

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Not getting the pdf of Max(X,Y).

discussion posted 4 days ago by sourabhXIII

Can someone please give me some hint. Spent lot of time but couldn't get it right. Got the PDF as derivative of CDF; plotted it but grader disagrees. :(

Guide: http://www.stankova.net/statistics_2012/lecture_12.pdf page 26.

This post is visible to everyone.

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3 responses

mrBB
4 days ago

Not sure what to add to the slide you refer to. Make sure you draw the pdf over it's entire (visible) domain, i.e. from -1 to 3.

I am clueless, how wrong this straightline can be.

posted 4 days ago by **sourabhXIII**

Add a comment

GiacomoDemarie
4 days ago

To find a PDF of max{X,Y}, start evaluating the CDF.

CDF(z) = P(z) = Prob(Z < z) = Prob(max{X,Y} < z).

Then think which condition must be satisifed so that max{X,Y} is less than z.

Hope this helps.

I get the CDF as `Prob(X<z)P(Y<z)` Then differentiate it to get the PDF. How wrong my drawing of a straight line can be.

posted 4 days ago by **sourabhXIII**

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sourabhXIII
4 days ago

Finally done with it. It was a classic stupidity from my end! Thank you guys.

Good to hear you figured it out!

posted 4 days ago by **mrBB**

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