

<u>Course</u> > <u>Unit 6 Linear Regression</u> > <u>Lecture 19: Linear Regression 1</u> > 7. Linear Regression - Basic Setup

7. Linear Regression - Basic Setup Linear Regression: The Function for Conditional Expectation of Y Given a value x





In **Linear Regression** , we will work with the assumption that the regression function $u\left(x\right):=\mathbb{E}\left[Y|X=x\right]$ is linear, so that

$$\nu \left(x\right) =a+bx$$

for some pair (a, b).

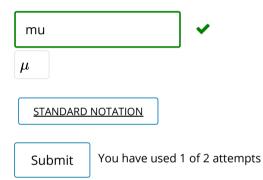
In this unit, we will be studying the **Least Squares Estimator** . It is an estimator (\hat{a},\hat{b}) so that $\hat{Y}=\hat{a}+\hat{b}X$ is "close" (in some distance metric) to the actual Y as often as possible.

A Minimization Problem

1/1 point (graded)

Let X be an arbitrary random variable, with mean μ and variance σ^2 . In terms of μ and σ^2 , which scalar k is the unique minimizer of the function $f(k) = \mathbb{E}[(X-k)^2]$?

Hint: Write f(k) as a quadratic in k.



✓ Correct (1/1 point)

An Estimator

1/1 point (graded)

Let (X,Y) be a pair of random variables for which the regression function $u\left(x
ight)=\mathbb{E}\left[Y|X=x
ight]$ takes the form

$$u\left(x
ight) =a+bx$$

for some pair of real numbers (a, b).

What is a random variable \hat{Y} that is a function of X that minimizes

$$\mathbb{E}\left[\left(Y-\hat{Y}
ight)^{2}|X=x
ight]$$

over all possible choices of \hat{Y} and for all x? Enter your answer in terms of a, b and the random variable X (capital letter "X").

(Remark: for a clean, quick solution, it may be helpful to review the law of iterated expectations: $\mathbb{E}_{X,Y}\left[\cdot\right] = \mathbb{E}_{X}\left[\mathbb{E}_{Y}\left[\cdot\mid X\right]\right]$, where $\mathbb{E}_{Y}\left[\cdot\mid X\right]$ denotes the conditional expectation, which is a random variable. Use the insight from the previous exercise.)



Solution:

For each realization x of X, the previous exercise tells us that $\mathbb{E}_Y[(Y-\hat{y})^2|X=x]$ is minimized by $\hat{y}=\nu(x)$. Since $\nu(x)=a+bx$ is a minimizer for each choice of x, $\nu(X)$ is a minimizer over all choices of \hat{Y} .

These two exercises verify that the Least Squares Estimator is consistent in the following sense: **using the actual distribution on** (X,Y), **the true pair** (a,b) **itself is a least squares estimator.** It may or may not be unique; we will address this in the following sections.

Submit You have used 1 of 3 attempts

• Answers are displayed within the problem

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