MITx: 6.041x Introduction to Probability - The Science of Uncertainty

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Problem 4: A three-sided die

(4/4 points)

The newest invention of the 6.041x staff is a three-sided die. On any roll of this die, the result is 1 with probability 1/2, 2 with probability 1/4, and 3 with probability 1/4.

Consider a sequence of six independent rolls of this die.

1. Find the probability that exactly two of the rolls results in a 3.

● $\binom{6}{2}\left(\frac{1}{4}\right)^2\left(\frac{3}{4}\right)^4$ ✓

○ $\binom{6}{2}\left(\frac{1}{4}\right)^2$

random variables

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○ $\binom{6}{2}\left(\frac{1}{4}\right)^2\binom{6}{4}\left(\frac{3}{4}\right)^4$

○ $\binom{6}{2}\left(\frac{1}{4}\right)^4\left(\frac{3}{4}\right)^2$

2. Given that exactly two of the six rolls resulted in a 1, find the probability that the first roll resulted in a 1. **Note:** Your answer should be a number. Do not enter '!' or combinations in your answer.

0.3333333 ✓ **Answer:** 0.33333

3. We are told that exactly three of the rolls resulted in a 1 and exactly three rolls resulted in a 2. Given this information, find the probability that the six rolls resulted in the sequence $(1, 2, 1, 2, 1, 2)$. **Note:** Your answer should be a number. Do not enter '!' or combinations in your answer.

0.05 ✓ **Answer:** 0.05

4. The conditional probability that exactly k rolls resulted in a 3, given that at least one roll resulted in a 3, is of the form:

$$\frac{1}{1-(c_1/c_2)^{c_3}}\binom{c_3}{k}\left(\frac{1}{c_2}\right)^{k}\left(\frac{c_1}{c_2}\right)^{c_3-k}, \qquad \text{for } k=1,2,\ldots,6.$$

Find the values of the constants c_1, c_2, and c_3:

$c_1 =$

3 ✓ **Answer:** 3

$c_2 =$ 4 ✓ **Answer:** 4

$c_3 =$ 6 ✓ **Answer:** 6

Answer:

1. Each roll is an independent trial with probability 1/4 of resulting in a 3 (a “success"). The probability of exactly 2 successes in 6 trials is given by the binomial probabilities with $n=6$, $k=2$, and $p=1/4$:

$$\binom{6}{2}\left(\frac{1}{4}\right)^2\left(\frac{3}{4}\right)^4$$

2.

The probability of obtaining a 1 on a single roll is 1/2, and the probability of obtaining a 2 or 3 on a single roll is also 1/2. For the purposes of solving this problem, we treat obtaining a 2 or a 3 as an equivalent result. We know that there are $\binom{6}{2}$ ways of rolling exactly two 1's. Of these $\binom{6}{2}$ ways, exactly $\binom{5}{1} = 5$ ways result in a 1 on the first roll, since we can place the other 1 in any of the five remaining rolls. The rest of the rolls must be either 2 or 3. Thus the probability that the first roll is a 1 given exactly two rolls resulted in a 1 is $\frac{5}{\binom{6}{2}} = \frac{1}{3}$.

3. We want to find

$$\mathbf{P}(121212 \mid \text{exactly three 1's and three 2's}) = \frac{\mathbf{P}(121212)}{\mathbf{P}(\text{exactly three 1's and three 2's})}.$$

Any particular sequence of three 1's and three 2's will have the same probability: $\left(\frac{1}{2}\right)^3\left(\frac{1}{4}\right)^3$. There are $\binom{6}{3}$ possible sequences with exactly three 1's and three 2's, of which exactly one sequence is 121212. Therefore,

$$\mathbf{P}(121212 \mid \text{exactly three 1's and three 2's}) = \frac{\left(\frac{1}{2}\right)^3\left(\frac{1}{4}\right)^3}{\binom{6}{3}\left(\frac{1}{2}\right)^3\left(\frac{1}{4}\right)^3} = \frac{1}{20}.$$

4. Let A be the event that at least one roll results in a 3. Then,

$$\mathbf{P}(A) = 1 - \mathbf{P}(\text{no rolls resulted in a 3}) = 1 - \left(\frac{3}{4}\right)^6.$$

Let B be the event that there were exactly k rolls that resulted in a 3, where $k \in \{1, 2, \ldots, 6\}$. Note that $\mathbf{P}(B) = \binom{6}{k}\left(\frac{1}{4}\right)^{k}\left(\frac{3}{4}\right)^{6-k}$.

Note also that $B \subset A$. Thus, the desired probability is:

$$\begin{aligned} \mathbf{P}(B \mid A) &= \frac{\mathbf{P}(A \cap B)}{\mathbf{P}(A)} \\ &= \frac{\mathbf{P}(B)}{\mathbf{P}(A)} \\ &= \frac{1}{1-(3/4)^6}\binom{6}{k}\left(\frac{1}{4}\right)^{k}\left(\frac{3}{4}\right)^{6-k} \text{ for } \quad k = 1, 2, \ldots, 6. \end{aligned}$$

You have used 1 of 2 submissions

DISCUSSION

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