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### 6.3.3 Solving $Ux = b$ (Back substitution)

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Week 6 due Nov 13, 2023 12:12 IST

### 6.3.3 Solving $Ux = b$ (Back substitution)

**Algorithm:**  $[b] := \text{UTRSV\_UNB\_VAR1}(U, b)$

**Partition**  $U \rightarrow \left( \begin{array}{c|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array} \right), b \rightarrow \left( \begin{array}{c} b_T \\ \hline b_B \end{array} \right)$

**where**  $U_{BR}$  is  $0 \times 0$ ,  $b_B$  has 0 rows

**while**  $m(U_{BR}) < m(U)$  **do**

**Repartition**

$$\left( \begin{array}{c|c} U_{TL} & U_{TR} \\ \hline 0 & U_{BR} \end{array} \right) \rightarrow \left( \begin{array}{c|c|c} U_{00} & u_{01} & U_{02} \\ \hline 0 & v_{11} & u_{12}^T \\ \hline 0 & 0 & U_{22} \end{array} \right), \left( \begin{array}{c} b_T \\ \hline b_B \end{array} \right) \rightarrow \left( \begin{array}{c} b_0 \\ \hline \beta_1 \\ \hline b_2 \end{array} \right)$$

$\beta_1 := \beta_1 - u_{12}^T b_2$  ← dots

$\beta_1 := \beta_1 / v_{11}$  ←

**Continue with**

$$\left( \begin{array}{c|c} U_{TL} & U_{TR} \\ \hline 0 & U_{BR} \end{array} \right) \leftarrow \left( \begin{array}{c|c|c} U_{00} & u_{01} & U_{02} \\ \hline 0 & v_{11} & u_{12}^T \\ \hline 0 & 0 & U_{22} \end{array} \right), \left( \begin{array}{c} b_T \\ \hline b_B \end{array} \right) \leftarrow \left( \begin{array}{c} b_0 \\ \hline \beta_1 \\ \hline b_2 \end{array} \right)$$

**endwhile**

the last element of the top part of b.

What do we do there?

We update it as such.

And what you notice is that that's a dot product.

As a matter of fact, we have this  
laff.dots routine

that would be just perfect for that computation.

And then we have to divide beta 1 by epsilon 11, which

is the current coefficient in U that's exposed.

Now we have an algorithm for solving  $Ux = b$ , which is the same as back substitution.

 4:51 / 5:06
  2.0x
 




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## Reading Assignment

0 points possible (ungraded)

Read Unit 6.3.3 of the notes. [[LINK](#)]

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✓

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✓ Correct

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Calculator

? Error in HW code

When I run my Matlab code for Utrsv\_unb\_var1, I get the following error: Error using FLA\_Cont\_with\_3x1\_to\_2x1 input matrices must have the sa...

5

## Homework 6.3.3.1

3/3 points (graded)

**Algorithm:**  $[b] := \text{UTRSV\_UNB\_VAR1}(U, b)$

**Partition**  $U \rightarrow \left( \begin{array}{c|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array} \right), b \rightarrow \left( \begin{array}{c} b_T \\ b_B \end{array} \right)$

**where**  $U_{BR}$  is  $0 \times 0$ ,  $b_B$  has 0 rows

**while**  $m(U_{BR}) < m(U)$  **do**

**Repartition**

$\left( \begin{array}{c|c} U_{TL} & U_{TR} \\ \hline 0 & U_{BR} \end{array} \right) \rightarrow \left( \begin{array}{c|c|c} U_{00} & u_{01} & U_{02} \\ \hline 0 & v_{11} & u_{12}^T \\ \hline 0 & 0 & U_{22} \end{array} \right), \left( \begin{array}{c} b_T \\ b_B \end{array} \right) \rightarrow \left( \begin{array}{c} b_0 \\ \beta_1 \\ b_2 \end{array} \right)$

$\beta_1 := \beta_1 - u_{12}^T b_2$

$\beta_1 := \beta_1 / v_{11}$

**Continue with**

$\left( \begin{array}{c|c} U_{TL} & U_{TR} \\ \hline 0 & U_{BR} \end{array} \right) \leftarrow \left( \begin{array}{c|c|c} U_{00} & u_{01} & U_{02} \\ \hline 0 & v_{11} & u_{12}^T \\ \hline 0 & 0 & U_{22} \end{array} \right), \left( \begin{array}{c} b_T \\ b_B \end{array} \right) \leftarrow \left( \begin{array}{c} b_0 \\ \beta_1 \\ b_2 \end{array} \right)$

**endwhile**

With pencil and paper, side-by-side, solve the upper triangular linear system

$$-2\chi_0 - \chi_1 + \chi_2 = 6$$

$$-3\chi_1 - 2\chi_2 = 9$$

$$\chi_2 = 3$$

via back substitution and by executing the above algorithm with inputs

$$U = \begin{pmatrix} -2 & -1 & 1 \\ 0 & -3 & -2 \\ 0 & 0 & 1 \end{pmatrix} \text{ and } b = \begin{pmatrix} 6 \\ 9 \\ 3 \end{pmatrix}.$$

$\chi_0 =$   ✓ Answer: 1

$\chi_1 =$   ✓ Answer: -5

$\chi_2 =$   ✓ Answer: 3

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**i** Answers are displayed within the problem

## Homework 6.3.3.2

1/1 point (graded)

**Algorithm:**  $[b] := \text{UTRSV\_UNB\_VAR1}(U, b)$

**Partition**  $U \rightarrow \left( \begin{array}{c|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array} \right), b \rightarrow \left( \begin{array}{c} b_T \\ b_B \end{array} \right)$

Calculator

$$\left( \begin{array}{c|c} U_{BL} & U_{BR} \end{array} \right) \quad \left( \begin{array}{c} b_B \end{array} \right)$$

where  $U_{BR}$  is  $0 \times 0$ ,  $b_B$  has 0 rows

while  $m(U_{BR}) < m(U)$  do

Repertition

$$\left( \begin{array}{c|c} U_{TL} & U_{TR} \\ \hline 0 & U_{BR} \end{array} \right) \rightarrow \left( \begin{array}{c|c|c} U_{00} & u_{01} & U_{02} \\ \hline 0 & v_{11} & u_{12}^T \\ \hline 0 & 0 & U_{22} \end{array} \right), \quad \left( \begin{array}{c} b_T \\ \hline b_B \end{array} \right) \rightarrow \left( \begin{array}{c} b_0 \\ \hline \beta_1 \\ \hline b_2 \end{array} \right)$$


---


$$\beta_1 := \beta_1 - u_{12}^T b_2$$

$$\beta_1 := \beta_1 / v_{11}$$


---

Continue with

$$\left( \begin{array}{c|c} U_{TL} & U_{TR} \\ \hline 0 & U_{BR} \end{array} \right) \leftarrow \left( \begin{array}{c|c|c} U_{00} & u_{01} & U_{02} \\ \hline 0 & v_{11} & u_{12}^T \\ \hline 0 & 0 & U_{22} \end{array} \right), \quad \left( \begin{array}{c} b_T \\ \hline b_B \end{array} \right) \leftarrow \left( \begin{array}{c} b_0 \\ \hline \beta_1 \\ \hline b_2 \end{array} \right)$$

endwhile

Implement the algorithm in the above figure.

- `[ b_out ] = Utrsv_unb_var1( U, b )`

You can check that it computes the right answer with the following script:

- `test_Utrsv_unb_var1.m` (In LAFF-2.0xM/Programming/Week06/ )

Unfortunately, PictureFLAME does not work for this problem.

This script exercises the function by starting with matrix

```
U = [
    2    0    1    2
    0   -1    2    1
    0    0    1   -1
    0    0    0   -2
]
```

Next, it solves  $Ux = b$  with the right-hand size vector

```
b = [
    2
    4
    3
    2
]
```

by calling

```
x = Utrsv_unb_var1( U, b )
```

Finally, it checks if  $x$  indeed solves  $Ux = b$  by computing

```
b - U * x
```

which should yield a zero vector of size four.

☒ Done/Skip



Here is our implementations of the function:

- `Utrsv_unb_var1.m`

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Answers are displayed within the problem

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Calculator