Simple, yet mighty

Our language L turns out to be extraordinarily powerful.

It can be used to formulate not only mundane arithmetical claims like "$2+3=5$", but also claims that have taken us hundreds of years to prove, like Fermat's Last Theorem, and hypotheses that are yet to be proved, like Goldbach's Conjecture. (Remember that formulating a hypothesis is not the same as proving the hypothesis, and that all we are talking about here is the fact that complex mathematical claims can be *formulated* in L.)

The fact that L is such an expressive language guarantees that if we were able to construct a Turing Machine that outputted every truth of L (and no falsehoods), we would have succeeded in concentrating a huge wealth of mathematical knowledge in a finite list of lines of code. Implicit in our program would be not just all the arithmetic we learned in school, but also remarkable mathematical results. If we had a machine that outputted all and only the true sentences in L, then—since we can express, for example, Goldbach's Conjecture in L—it would give us way of knowing for certain whether Goldbach's Conjecture is true.

Alas, Gödel's Theorem shows that the dream of constructing such a machine is not to be.

Gödel's Theorem is a pretty robust result, which doesn't depend on the details of one's arithmetical language.

All that is required to prove the theorem is that one's language be rich enough for the following lemma to be provable:

> **Lemma**
> The language contains a formula (abbreviated "$\text{Halt}(k)$") which is true if and only if the kth Turing Machine halts on input k.

In an Appendix at the end of this chapter I show that our simple language L is indeed rich enough for the lemma to hold.

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