MATHEMATICS

Why does this approximation for 8 work?

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1

I previously saw that peculiarly $\frac{987654321}{123456789} \approx 8$. I was wondering if there was any significance to it i.e. if there is any way to derive this approximation (aside from long division).

I already shared my solution. I would love to see other interesting methods (if any).

real-analysis approximation

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asked 27 mins ago
Alan Abraham
2,625 3 14

I see now it was already raised here, here and here – user 3 mins ago

3 Answers

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2

We can express 987654321 as approximately $\sum_{k=-9}^{\infty} -k \cdot 10^{-k-1}$ and 123456789 as approximately $\sum_{k=1}^{\infty} k \cdot 10^{9-k}$

If we take

$$f(x) = \sum_{k=-9}^{\infty} x^k = \frac{x^{-9}}{1-x}$$

then

$$f'(x) = \sum_{k=-9}^{\infty} kx^{k-1} = \frac{-9x^{-10}+10x^{-9}}{(1-x)^2}$$

$$\implies f'(x) = \sum_{k=-9}^{\infty} kx^{k-1} = \frac{-9x^{-10}+10x^{-9}}{(1-x)^2}$$

$$\implies x^2 f'(x) = \sum_{k=-9}^{\infty} kx^{k+1} = \frac{-9x^{-8}+10x^{-7}}{(1-x)^2}$$

$$\implies -10^2 \cdot f'\left(\frac{1}{10}\right) = \sum_{k=-9}^{\infty} -k \cdot 10^{-k-1} = \frac{8 \cdot 10^{10}}{81}$$

Similarly, if we take

$$g(x) = \sum_{k=0}^{\infty} x^k = \frac{1}{1-x}$$

then

$$g'(x) = \sum_{k=1}^{\infty} k \cdot x^{k-1} = \frac{1}{(1-x)^2}$$

$$\implies x^{-8} g'(x) = \sum_{k=1}^{\infty} k \cdot x^{k-9} = \frac{x^{-9}}{(1-x)^2}$$

$$\implies 10^8 \cdot g'\left(\frac{1}{10}\right) = \sum_{k=1}^{\infty} k \cdot 10^{9-k} = \frac{10^{10}}{81}$$

Hence, from our approximations for 987654321 and 123456789, it follows that $\frac{987654321}{123456789} \approx 8$

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Alan Abraham
2,625 3 14

1

More generally, in base b, the number with digits decreasing from $b-1$ to 1 is

$$N = \sum_{i=0}^{b-2} (i+1) b^i = \frac{b-2}{(b-1)^2} b^b + \frac{1}{(b-1)^2}$$

while the number with digits increasing from 1 to $b-1$ is

$$D = \sum_{i=0}^{b-2} (b-1-i) b^i = \frac{b^b}{(b-1)^2} - \frac{b^2-b+1}{(b-1)^2}$$

For large b, the dominant terms are those with b^b, so

$$\frac{N}{D} \sim b-2$$

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answered 11 mins ago
Robert Israel
408k 24 291 588

Very nice generalization! – user 10 mins ago

1

We have that in base 8

- $987654321 = 7267464261_8$

- $123456789 = 726746425_8$

then

$$987654321 - 8 \cdot 123456789 = 7267464261_8 - 10_8 \cdot 726746425_8 = 11_8 = 9$$

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answered 17 mins ago

user

138k 12 67 126