Exercise: CLT for the binomial

(3/3 points)

Let X be binomial with parameters $n = 49$ and $p = 1/10$.

4.9

The mean of X is: **Answer:** 4.9

2.1

The standard deviation of X is: **Answer:** 2.1

The CLT, together with the $1/2$-correction, suggests that

0.1644901

$\mathbf{P}(X = 6) \approx$ **Answer:** 0.1623

You may want to refer to the normal table .

Note: In this case, the CLT may not provide a great approximation. The range of values that X is likely to take is quite narrow, so that its PMF consists of only a few entries of substantial size. But, regardless, we can still calculate what the CLT suggests.

Answer:
We have $\mathbf{E}[X]=np=4.9$, and

$$\mathrm{var}(X)=np(1-p)=49\cdot\frac{1}{10}\cdot\frac{9}{10}=\frac{49\cdot 9}{10^2},$$

so that the standard deviation of X is $21/10=2.1$.

The standardized version of X is $(X-4.9)/2.1$. Thus,

$$\begin{aligned}\mathbf{P}(X=6) &= \mathbf{P}(5.5<X<6.5)=\mathbf{P}\left(\frac{5.5-4.9}{2.1}\le\frac{X-4.9}{2.1}\le\frac{6.5-4.9}{2.1}\right)\\ &\approx \Phi(0.76)-\Phi(0.29)\approx 0.7764-0.6141=0.1623.\end{aligned}$$

For comparison, the answer calculated by using the binomial PMF directly is

$$\mathbf{P}(X=6)=\binom{49}{6}(0.1)^6(0.9)^{49-6}\approx 0.1507.$$

You have used 1 of 2 submissions