11. Dimension vs Units

Lecture due Aug 4, 2021 20:30 IST Completed

Explore

We've said that the world around us is multivariable. But how do we actually use functions of many variables to represent the world? Can any function really exist in the real world? We'll explore these questions and more by learning the basics of a problem solving technique called **dimensional analysis** .

Dimension defined

Dimension and **units** are related, but different, concepts.

Physical quantities are measured in **units** . The **dimension** of a physical quantity is independent of the units used in measuring it.

> **Example 11.1** Grams and kilograms are both **units** of **mass**. Mass is what we will call the **dimension** .

Fundamental dimensions

Here are some common fundamental dimensions.

1. length L
2. mass M
3. time T
4. temperature K
5. charge C

All dimensions can be described in terms of these fundamental dimensions. How do we describe the dimension of a quantity that is not one of these fundamental dimensions? We find the dimension as a product of powers of these fundamental dimensions.

> **Example 11.2** Velocity is a rate of change, measuring a change in length over a change in time. Thus its dimension is L/T, Length per Time.

> **Example 11.3** Forces are measured in Newtons N. But N is not a dimension it is a unit. To understand its dimension, we need to determine which fundamental dimensions are used to describe it.
>
> Newton's second law tells us $F = ma$, where m is mass and a is acceleration. The dimension of m is Mass (e.g. kilograms), and the dimension of a is Length per Time Squared (e.g. meters/second ^{2}). Thus, the dimension of $F = ma$ is $(M)\left(L/T^2\right) = ML/T^2$.

Identify terms with the same dimension

1/1 point (graded)

When trying to determine if a formula involving quantities is valid, one must make sure Calculator n Hide Notes

terms in the equation are the same. This is just like what we do with units.

terms in the equation are the same. This is just like what we do with units.

Identify the terms below that have the same dimension as force F.

Let the following variables represent the quantities described.

- m has dimension of mass
- t has dimension of time
- x has dimension of length
- a is acceleration
- v is velocity
- p is momentum

(You may need to look up the units/dimension of these quantities. That's OK!)

- [x] ma
- [ ] mv
- [x] $\frac{dm}{dt}v$
- [ ] p
- [x] $\frac{dp}{dt}$

✔

Solution:

Let's go through each choice.

- We used the fact that ma had the same dimension to determine the dimension of F to begin with.
- We know that mv is momentum, which has dimension ML/T. This is not a correct choice.
- The dimension of $\frac{dm}{dt}v$ is $(M/T)\,(L/T) = ML/T^2$, which is correct.
- Momentum p has dimension of ML/T. This is not a correct choice.
- Differentiating with respect to time, we get that the dimension of $\left[\frac{dp}{dt}\right] = ML/T^2$, which is the same as the dimension of $[N]$. In fact, the more general formula for force is

$$F = \frac{dp}{dt} = \frac{d}{dt}mv = \frac{dm}{dt}v + ma.$$

Submit You have used 1 of 5 attempts

Answers are displayed within the problem

Find the equation

Calculator Hide Notes

2/2 points (graded)

Find numbers k_1 and k_2 so that both sides of the formula $E = m^{k_1} c^{k_2}$ have the same dimension, where m is mass and c is the speed of light (a velocity).

(Note that energy has units of Joules J, which can be expressed in terms of fundamental dimension as ML^2/T^2.)

$k_1 =$ 1 ✔ **Answer:** 1

$k_2 =$ 2 ✔ **Answer:** 2

Solution:

Energy has units of Joules J, which can be expressed in terms of fundamental dimension as ML^2/T^2. Therefore $k_1 = 1$ will give us a dimension of mass, and $k_2 = 2$ will give us the dimension L^2/T^2.

Submit You have used 1 of 5 attempts

Answers are displayed within the problem

Remark 11.4 Dimensional analysis has limitations. Note that if you try to find a formula for kinetic energy, dimensional analysis will not be able to identify the dimensionless 1/2 in front of the equation for kinetic energy $E = 1/2mv^2$. All dimensional analysis can tell you is whether or not the two sides might be proportional, but will not give you equality without further understanding of the physical properties underlying the mathematical model.

If you are interested in exploring dimensional analysis from a more advanced perspective, you might be interested in watching the following video.

Dimensional Analysis

11. Dimension vs Units

Hide Discussion

Topic: Unit 1: Functions of two variables / 11. Dimension vs Units

Add a Post

Show all posts by recent activity

- Interesting finding in "Identify terms with the same dimension" 1
  I really enjoyed this unexpected finding from this question. Made me look on a force from an interesting point of view.
- [Staff] Sentence issue? 7
- Typo in solution to "Identify terms with the same dimension"? 3
  In the last line of the solution, I think there should be an equals sign between the last two terms instead of the existing plus sign.
- Identify terms with the same dimension 8
  My physics is a bit rusty, but I believe the derivative should be velocity w.r.t. time, not the mass w.r.t. time.

< Previous Next >

Calculator Hide Notes