2. White noise model

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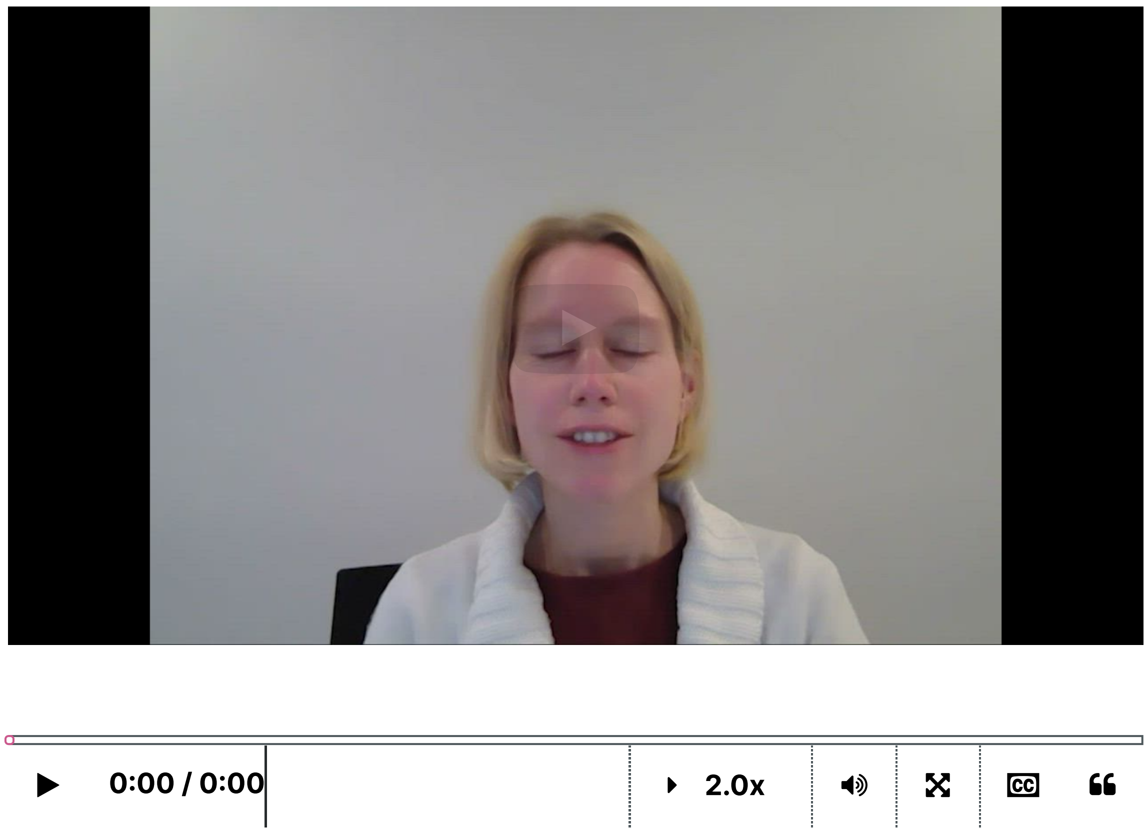
Exercises due Nov 10, 2021 17:29 IST Completed

White noise, Autoregressive, Random Walk Models



Start of transcript. Skip to the end.

Prof Jegelka: In today's lecture, we

will continue our journey into time series analysis,

and while last time we talked about more

of an exploratory analysis, today we'll

talk about statistical models and how

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Video note: At 15:05, the random walk with drift is $X_t = t\delta + X_0 + \sum_{s=1}^{t} W_s$.

The simplest time series model is the **white noise process** $\{W_t\}_t$ of random variables that have zero mean, the same variance σ_W^2, and zero correlations:

$$\begin{aligned}
\mu_W(t) &= \mathbf{E}[W_t] = 0 \\
\gamma_W(t,t) &= \text{Var}(X_t) = \sigma_W^2 \\
\gamma_W(t,s) &= \text{Cov}(X_t, X_s) = 0, \quad \text{for} \quad t \neq s.
\end{aligned}$$

What does white noise look like

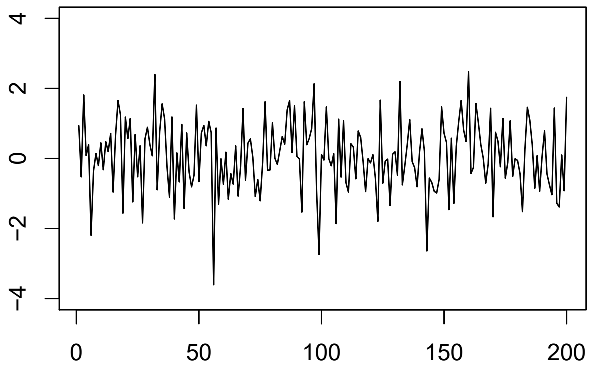
1/1 point (graded)

Which plots below show the path of a white noise process and its acf function?

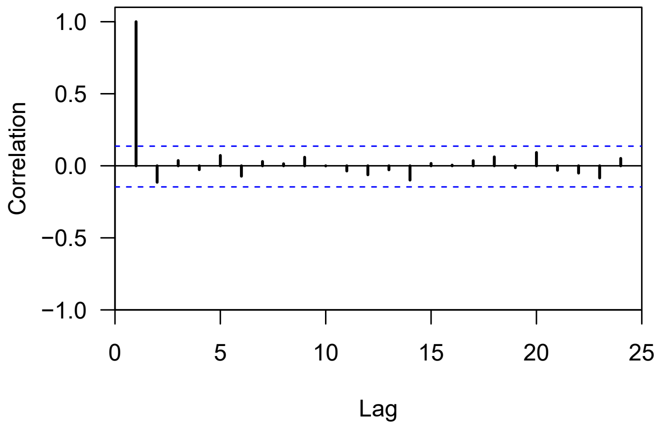
(Note that the horizontal blue dashed line denote the range of estimation error around zero correlation, so that correlation values falling in between the two blue dashed line can not be distinguished from zero up to estimation error.)

●

(a):

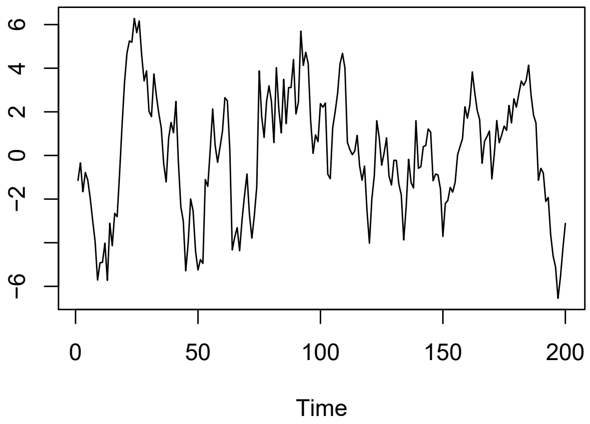


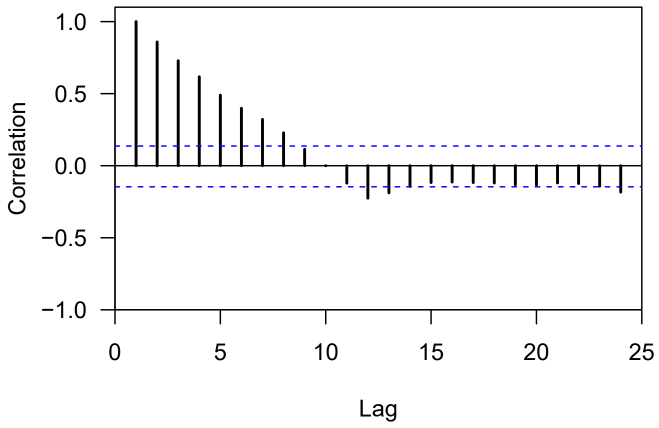
Time


Correlation
1.0
0.5
0.0
−0.5
−1.0
0
5
10
15
20
25
Lag

○

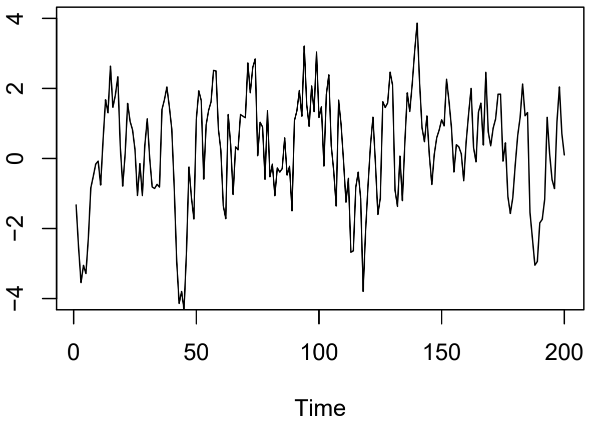
(b):


6
4
2
0
−2
−6
0
50
100
150
200
Time

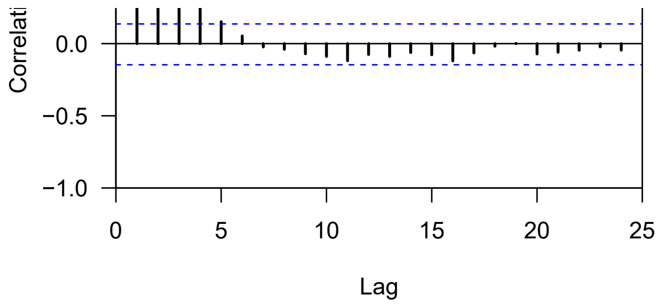

Correlation
1.0
0.5
0.0
−0.5
−1.0
0
5
10
15
20
25
Lag

○

(c):


4
2
0
−2
−4
0
50
100
150
200
Time


ion
1.0
0.5



✓

Solution:

Figure (a) contains a sample path and an estimated acf function of the white noise process. Figures (b) and (c) contain paths and acf functions of stationary time series with correlated terms. As the three figures illustrate, it may be difficult to detect stochastic dependence or lack there of only by looking at the sample path of the process (the paths of all three time series look somewhat similar). However, the acf function clearly indicates that for the times series in (a), there is no correlation (up to estimation error indicated by the blue dashed lines) between the terms of the series at different lags. The acf function in figure (b) indicates that there is correlation between adjacent in time terms of the series (which is not possible for white noise). The acf function in figure (c) indicates that there are correlations between 5 or 6 consecutive terms of the series.

Submit You have used 1 of 3 attempts

Answers are displayed within the problem

Property of white noise model

1/1 point (graded)

Is the white noise time series stationary?

● Yes

○ No

✓

Solution:

The white noise process is a weakly stationary time series because it has a constant marginal mean function and its autocovariance is zero, therefore does not depend at all on the two time stamps $t \neq s$ that are distinct. The white noise series need not be strongly stationary unless explicitly assumed to have this strong property.

Submit You have used 1 of 1 attempt

Answers are displayed within the problem

The white noise model is not very interesting. In particular, it has no stochastic dependencies (correlations). The purpose of the white noise model is to model the “best" case residuals that contain no information, after we fit a good time series model to the data and subtract the fitted values for the data. That is, we will be completely satisfied with a time series model for a given dataset, if the residuals of that model contain no further information about the dependencies in the data. This would mean that our statistical model for the data captures all the stochastic dependence exhibited in the data, which we can harness for e.g., predicting future observation.

To detect a white noise time series, we first check that the series is stationary (e.g. by plotting it and making sure there is no trend or seasonal variation or exploding variance) and then look at the autocovariance function to detect stochastic dependencies in the data. Of course, we don't know the true autocovariance function of the process that generated the data and have to estimate it with $\hat{\gamma}_W\left(h\right)$. Under appropriate technical conditions, the distribution of the estimator is

$$\hat{\gamma}_W\left(h\right) \sim N\left(0, \frac{\sigma_W^2}{n}\right)$$

which means that we do not expect to see the theoretical acf function exactly as our estimate, but only approximately up to estimation error.

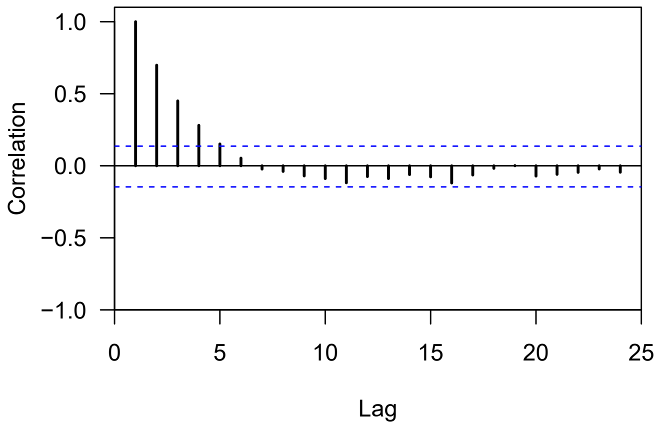
Property of white noise model

1/1 point (graded)

Choose the plot of the estimated acf for a white noise process:

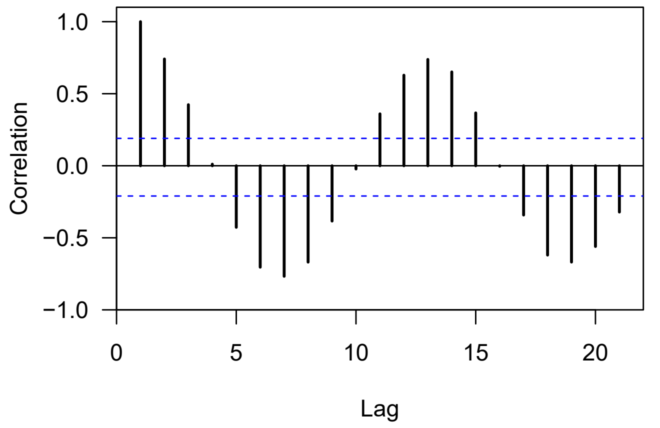
○

(a):



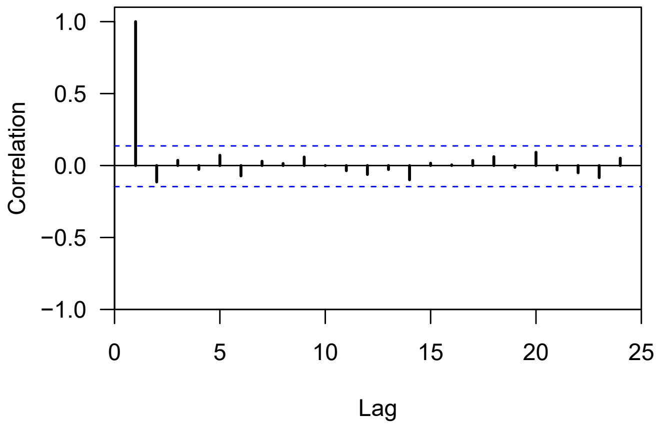
○

(b):



●

(c):



✓

Solution:

Figure (c) contains an estimated acf of a white noise time series. The acf functions in Figures (a) and (b) show correlation between adjacent terms of he time series which is not the case for white noise.

Submit You have used 1 of 1 attempt

Answers are displayed within the problem

Property of white noise model

2/2 points (graded)

Let $\{W_t\}_t$ and $\{V_t\}_t$ be two mutually independent white noise time series.

Is the series $Y_t = W_t + V_{t-1}$ a white noise?

● Yes

○ No

✓

Is the series $Z_t = W_t + W_{t-1}$ a white noise?

○ Yes

● No

✓

Solution:

Series Y is white noise. To check this, we verify the definition:

$$\begin{aligned}
\mathbf{E}\left[Y_t\right] &= \mathbf{E}\left[W_t + V_t\right] = \mathbf{E}\left[W_t\right] + \mathbf{E}\left[V_t\right] = 0 + 0 \\
\text{Var}\left(Y_t\right) &= \text{Var}\left(W_t + V_t\right) = \text{Var}\left(W_t\right) + \text{Var}\left(V_t\right) \quad \text{by independence} \\
&= \sigma_W^2 + \sigma_V^2 \\
\text{Cov}\left(Y_t, Y_s\right) &= \text{Cov}\left(W_t + V_t, W_s + V_s\right) \quad \text{for } s \neq t \\
&= \text{Cov}\left(W_t, W_s\right) + \text{Cov}\left(W_t, V_s\right) + \text{Cov}\left(V_t, W_s\right) + \text{Cov}\left(V_t, V_s\right) \\
&= 0 + 0 + 0 + 0 \qquad \text{by independence and WN property}
\end{aligned}$$

Series Z is not a white noise because its autocovariance is not zero at lag 1:

$$\begin{aligned}
\text{Cov}\left(Z_t, Z_{t+1}\right) &= \text{Cov}\left(W_t + W_{t-1}, W_{t+1} + W_t\right) \\
&= \text{Cov}\left(W_t, W_{t+1}\right) + \text{Cov}\left(W_t, W_t\right) + \text{Cov}\left(W_{t-1}, W_{t+1}\right) + \text{Cov}\left(W_{t-1}, W_t\right)
\end{aligned}$$

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