# 9. Meaning of the constant term

Recall that the average value of a function $f(t)$ on an interval $(a,b)$ is defined to be the area under the curve over that interval, divided by the length of that interval:

$$\text{ave}(f) := \frac{\int_a^b f(t)\ dt}{b-a}.$$

The constant term of the Fourier series of $f$ is

$$\frac{a_0}{2} = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(t)\ dt = \frac{\int_{-\pi}^{\pi} f(t)\ dt}{\pi-(-\pi)},$$

which is the average value of $f$ on the interval $(-\pi,\pi)$.

**Example 9.1**

Find the constant term of the $2\pi$-periodic function

$$f(t) = \begin{cases} t, & 0 < t < \pi \\ 0, & -\pi < t < 0 \end{cases}.$$

**Solution:** The average value of this function over one period is given by the area under the curve, divided by the length of the interval, which in this case is $2\pi$. By sketching the function over the interval $-\pi < t < \pi$,

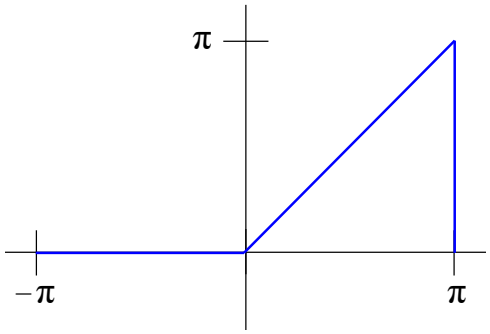


**Figure 4**: The graph of $f(t)$ over the interval $-\pi < t < \pi$.

we can compute the area under the curve directly to find the average value, which is

$$\frac{(1/2)\,\pi^2}{2\pi} = \frac{\pi}{4}.$$

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## Find the constant term, 1

1/1 point (graded)

Find the constant term of Fourier series for the following $2\pi$-periodic function defined by

$$f(t) = |t|, \qquad -\pi < t < \pi.$$

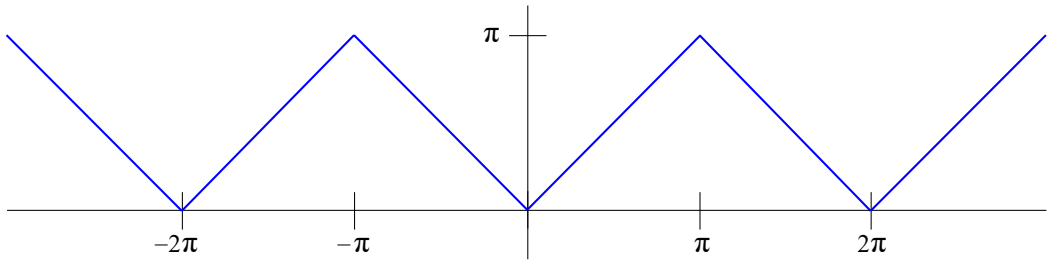
(This function is called the triangle wave.)

$\frac{a_0}{2} =$ pi/2 ✔ **Answer:** pi/2

$\frac{\pi}{2}$

**Solution:**

Sketch the function to determine the area under the graph over the interval $-\pi < t < \pi$.




The area under the curve is $\pi^2$, and the length of the interval is $2\pi$, so the average value of this function on this interval is $\pi/2$.

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Answers are displayed within the problem

## Find the constant term, 2

1/1 point (graded)

Find the constant term of Fourier series for the following $2\pi$ periodic function defined by

$$f(t) = t, \qquad -\pi < t < \pi.$$

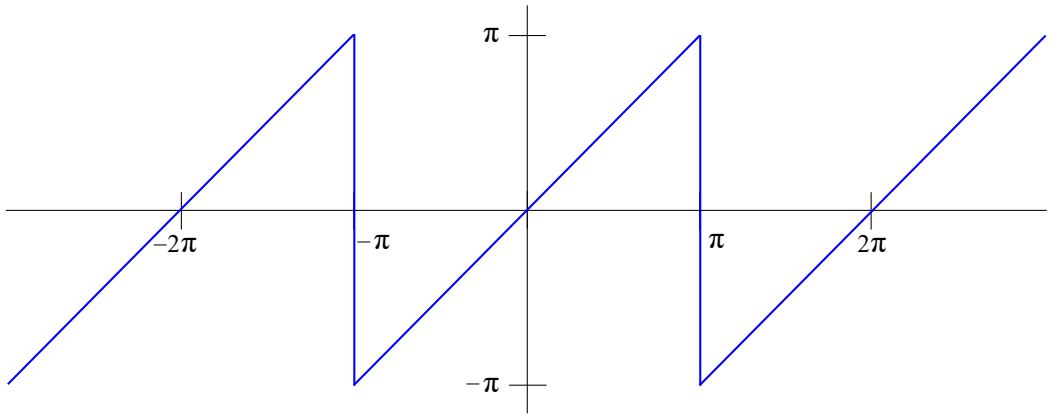
(This function is called the sawtooth wave.)

$\frac{a_0}{2} =$ 0 ✔ **Answer:** 0

0

**Solution:**

First we sketch the function.




The area of this odd function over the interval is zero, therefore the average value is $0$.

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## Find the constant term, 3

1/1 point (graded)

Find the constant term of Fourier series for the following $2\pi$-periodic function.

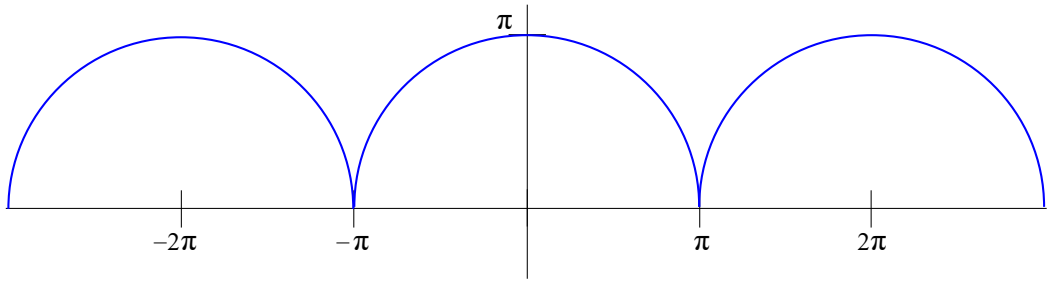
$$f(t) = \sqrt{\pi^2 - t^2}, \qquad -\pi < t < \pi.$$

$\frac{a_0}{2} =$ 2.467401 ✔ **Answer:** pi^2/4

$2.467401$

**Solution:**

The graph of this function is a semicircle of radius [mathjaxinline]\pi[/mathjaxinline] centered at zero, and repeated periodically.




The area under this curve from [mathjaxinline]-\pi < t< \pi[/mathjaxinline] is the area of half a circle of radius [mathjaxinline]\pi[/mathjaxinline], which is [mathjaxinline]\pi ^3/2[/mathjaxinline]. Dividing by the length of the interval, which is [mathjaxinline]2\pi[/mathjaxinline], we get the average value, which is [mathjaxinline]\pi ^2/4[/mathjaxinline].

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Answers are displayed within the problem

## Find the constant term, 4

1/1 point (graded)

Find the constant term of Fourier series for the following [mathjaxinline]2\pi[/mathjaxinline]-periodic function.

[mathjax]f(t) = t^2, \qquad -\pi < t < \pi .[/mathjax]

[mathjaxinline]\displaystyle \frac{a_0}{2} =[/mathjaxinline] pi^2/3 **Answer:** pi^2/3

\(\)

**Solution:**

The graph of this function is a seen below.

The area under this curve from [mathjaxinline]-\pi < t< \pi[/mathjaxinline] is

[mathjax]\int _{-\pi }^{\pi } t^2 \, dt = \left. \frac{t^3}{3}\right|_{-\pi }^{\pi } = \frac{2\pi ^3}{3}.[/mathjax]

Dividing by the length of the interval, which is [mathjaxinline]2\pi[/mathjaxinline], we get the average value, which is [mathjaxinline]\pi ^2/3[/mathjaxinline].

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Answers are displayed within the problem

# 9. Meaning of the constant term

**Topic:** Unit 1: Fourier Series / 9. Meaning of the constant term

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? Example 9.1?

if I integrate f(t) from [-pi, pi], the integral from [-pi, 0], is 0 since f is zero on that interval. The remaining integral from [0, pi] of t is t^2/2 giving pi^2/2. Divide by pi I get pi/2, no...

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