12. Decomposition along unit vectors

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Lecture due Aug 18, 2021 20:30 IST Completed

Synthesize

In the solution to the previous problem, you may have noticed that the vector we chose that was tangent to the inclined plane was unit length. This choice simplified our computation significantly!

The following problems ask you to think through the vector decomposition formulas when you choose a tangent direction that is unit length, and a perpendicular vector that is unit length.

Find a unit normal vector

1.0/1 point (graded)
Suppose that the vector $\hat{u} = \langle u_1, u_2 \rangle$ is unit length.

Find a unit length vector $\hat{w}$ that is normal to $\hat{u}$.

(Enter your answer in terms of u_1 and u_2. Enter the vector between square brackets. For example, type [u _ 1, u _ 2] for the vector $\langle u_1, u_2 \rangle$.)

$\hat{w} =$ [-u_2,u_1] ✔ **Answer:** [u_2, -u_1]

Solution:

A vector perpendicular to $\hat{u}$ must satisfy the equation

$$\langle u_1, u_2 \rangle \cdot \langle ?, ? \rangle = 0$$

One answer is $\langle u_2, -u_1 \rangle$. The norm of this vector is $u_2^2 + (-u_1)^2 = u_2^2 + (u_1)^2 = 1$ since we assume $\hat{u}$ has unit length.

(Note that the opposite vector $\langle -u_2, u_1 \rangle$ is also a valid answer. These are the only two unit vectors normal to $\hat{u}$.)

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Answers are displayed within the problem

Find the vector decomposition

2.0/2 points (graded)
Suppose that the vector $\hat{u} = \langle u_1, u_2 \rangle$ is unit length. Let $\hat{w}$ be the vector normal to $\hat{u}$ that you defined above.

Find the vectors $\vec{a}$ and $\vec{b}$ in the vector decomposition of a vector $\vec{v} = \vec{a} + \vec{b}$ where $\vec{a}$ is parallel to $\hat{u}$ and $\vec{b}$ is parallel to $\hat{w}$.

Express $\vec{a} = \lambda_1 \hat{u}$ and $\vec{b} = \lambda_2 \hat{w}$, where λ_1 and λ_2 are numbers.

(Express $\vec{a}$ and $\vec{b}$ in terms of $\vec{v}$, $\hat{u}$, and $\hat{w}$. Type `vecv` for $\vec{v}$. Type hatu for $\hat{u}$. Type `*` to denote the dot product of vectors, as well as the product between scalars and vectors. You may find it useful to use parentheses to help distinguish which product you want to happen first. For example, type `hatu*hatw` for the dot product $\hat{u} \cdot \hat{w}$, but `(vecv*vecv)*vecv` is a vector quantity that is equivalent to $|\vec{v}|^2 \vec{v}$.)

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$\vec{a} =$ (hatu*vecv)*hatu ✔ **Answer:** (vecv*hatu)*hatu

$\vec{b} =$ (hatw*vecv)*hatw ✔ **Answer:** (vecv*hatw)*hatw

Solution:

We apply the formula for $\vec{a}$ that we derived on the previous page:

$$\vec{a} = \frac{\hat{u} \cdot \vec{v}}{\hat{u} \cdot \hat{u}} \hat{u},$$

however, we notice that this formula simplifies because $\hat{u} \cdot \hat{u} = 1$. Thus we simplify this expression as

$$(\hat{u} \cdot \vec{v})\, \hat{u}$$

Note that to find the component $\vec{b}$ in the direction of the vector $\hat{w}$, we can apply this simplified formula direction since $\hat{w}$ is also of unit length! This tells us that

$$\vec{b} = (\hat{w} \cdot \vec{v})\, \hat{w}.$$

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Answers are displayed within the problem

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- ? Not able to submit problem solution — 2
  It is 3:21 PM PDT and all submit button are disabled now, Isn't the submission time is 8:00 PM PDT ?
- ? [STAFF] Find a unit normal vector (alternate solution - would this work?) — 4
- [staff] last problem has disappeared — 3
  I worked on a solution to the problem for about half an hour. When I came to submit my answer the submit button and save button w...
- [STAFF] typo in problem instructions for vector decomposition — 5
- ? Find a unit normal vector — 8
- ? Are formulas for vector b equivalent? — 5
  We have formula for vector b=v-a, but now we apply the same formula as for vector a=(v*u/u*u)*u. Would these two formulas give t...
- Unit vectors — 3
  It isn't explicitly stated that the "hat" vectors have unit length. That is, while the problem states that "hatu" is unit length, the proble...
- The second and third questions are messed up — 2
  The problem states (Express a and b in terms of v, uhat , and what. When I submit my answer I get the following message: Invalid In...

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