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Unit overview

**Lec. 11: Derived distributions**

Exercises 11 due Mar 30, 2016 at 23:59 UTC

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 Derived distributions vertical5


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## Exercise: A function of multiple r.v.'s

(2/2 points)

Suppose that  $\mathbf{X}$  and  $\mathbf{Y}$  are described by a joint PDF which is uniform inside the unit circle, that is, the set of points that satisfy  $x^2 + y^2 \leq 1$ . In particular, the joint PDF takes the value of  $1/\pi$  on the unit circle. Let  $Z = \sqrt{X^2 + Y^2}$ , which is the distance of the outcome  $(X, Y)$  from the origin. The PDF of  $Z$ , for  $z \in [0, 1]$ , takes the form  $f_Z(z) = az^b$ . Find  $a$  and  $b$ .

 $a =$ 

2



Answer: 2

 $b =$ 

1



Answer: 1

Answer:

Note that the set of points that satisfy  $x^2 + y^2 \leq z^2$  is a circle of radius  $z$ , has area  $\pi z^2$ , and probability  $z^2$ . Therefore,

$$F_Z(z) = \mathbf{P}(Z \leq z) = \mathbf{P}(X^2 + Y^2 \leq z^2) = z^2,$$

from which it follows that  $f_Z(z) = 2z$ .

*You have used 1 of 2 submissions*

**Lec. 12: Sums of independent r.v.'s; Covariance and correlation**

Exercises 12 due Mar 30, 2016 at 23:59 UTC

**Lec. 13: Conditional expectation and variance revisited; Sum of a random number of independent r.v.'s**

Exercises 13 due Mar 30, 2016 at 23:59 UTC

**Solved problems**

**Additional theoretical material**

**Problem Set 6**

Problem Set 6 due Mar 30, 2016 at 23:59 UTC

**Unit summary**

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