Problem 2

$\phi(1) = \square \quad \phi(2) = \square \quad \phi(3) = \square$

$\phi(4) = \square \quad \phi(6) = \square \quad \phi(12) = \square$

$\phi(1)+\phi(2)+\phi(3)+\phi(4)+\phi(6)+\phi(12)$

$= \square$

Euler's Totient Function

$\boldsymbol{\phi}$**(N)** = the number of **1 ≤ K ≤ N**

such that K and N are

relatively prime

(GCD(K,N) = 1)

Leonhard
Euler
(1707-1783)

https://en.wikipedia.org/wiki/Leonhard_Euler

Problem 2

$N = 12 = 2\times2\times3$

$\phi(\mathbf{12})$ = # of $\mathbf{1} \le \mathbf{K} \le \mathbf{12}$ such that K and 12 are **relatively prime**

= # of $\mathbf{1} \le \mathbf{K} \le \mathbf{12}$ such that K is **neither divisible by 2 nor 3**

1 2 3 4 **5** 6 **7** 8 9 10 **11** 12

$\Rightarrow \phi(12) = 4$

Leonhard Euler (1707-1783)

Problem 2

Answer

$$\mathbf{1} \Rightarrow \phi(1) = 1$$

$$\mathbf{1}\ 2 \Rightarrow \phi(2) = 1$$

$$\mathbf{1}\ \mathbf{2}\ 3 \Rightarrow \phi(3) = 2$$

$$\mathbf{1}\ 2\ \mathbf{3}\ 4 \Rightarrow \phi(4) = 2$$

$$\mathbf{1}\ 2\ 3\ 4\ \mathbf{5}\ 6 \Rightarrow \phi(6) = 2$$

$$\phi(1)+\phi(2)+\phi(3)+\phi(4)$$
$$+\phi(6)+\phi(12)$$
$$= \mathbf{1+1+2+2+2+4 = 12}$$

Leonhard
Euler
(1707-1783)

Problem 2

Theorem

The sum of $\phi(K)$, where K divides N, is equal to N.

Example

- $\phi(1)+\phi(2)+\phi(5)+\phi(10)=10$
- $\phi(1)+\phi(2)+\phi(3)+\phi(4)+\phi(6)+\phi(12) = 12$

Leonhard Euler (1707-1783)