




MITx: 6.041x Introduction to Probability - The Science of Uncertainty



Bookmarks

- ▶ Unit 0: Overview
- ▶ Entrance Survey
- ▶ Unit 1: Probability models and axioms
- ▶ Unit 2: Conditioning and independence
- ▶ Unit 3: Counting
- ▶ Unit 4: Discrete random variables
- ▼ Exam 1

Exam 1

Exam 1 due Mar 09, 2016 at 23:59 UTC 

Exam 1 > Exam 1 > Exam 1 vertical3

 Bookmark

Problem 3: A six-sided die

(3/3 points)

A fair, 6-sided die is rolled 6 times independently. Assume that the results of the different rolls are independent. Let (a_1, \dots, a_6) denote a typical outcome, where each a_i belongs to $\{1, \dots, 6\}$.

Note: Enter numerical answers; do not enter '!' or combinations. The following table for $\binom{n}{k}$ up to $n = 6$ has been provided for your convenience:

- ▶ Unit 5: Continuous random variables
- ▶ Unit 6: Further topics on random variables
- ▶ Unit 7: Bayesian inference
- ▶ Exam 2
- ▶ Unit 8: Limit theorems and classical statistics

		k						
		0	1	2	3	4	5	6
n	1	1	1					
	2	1	2	1				
	3	1	3	3	1			
	4	1	4	6	4	1		
	5	1	5	10	10	5	1	
	6	1	6	15	20	15	6	1

1. Find the probability that the results of the 6 rolls are all different. (Answer with at least 3 decimal digits.)



Answer: 0.01543

For any outcome $\omega = (a_1, \dots, a_6)$, let $R(\omega)$ be the **set** $\{a_1, \dots, a_6\}$; this is the set of numbers that showed up at least once in the different rolls. For example, if $\omega = (2, 2, 5, 2, 3, 5)$, then $R(\omega) = \{2, 3, 5\}$.

2.

Find the probability that $R(\omega)$ has exactly two elements. (Answer with at least 3 decimal digits.)



Answer: 0.01993

3. Find the probability that $R(\omega)$ has exactly three elements.



Answer: 0.23148

Answer:

1. Let A be the event where the results of the 6 rolls are all the same. Since all outcomes are equally likely, $P(A) = \frac{|A|}{|\Omega|}$.

$$|A| = 6!$$

$$|\Omega| = 6^6$$

$$P(A) = \frac{6!}{6^6} = 5/324 \approx 0.01543$$

2. Let B be the event where $R(\omega)$ has exactly two elements. We can find the number of elements in B by first choosing a pair of distinct numbers for $R(\omega)$ – there are $\binom{6}{2}$ choices. For each pair, we can then count the number of ways they can be assigned to a sequence of length 6 that consists of only those two numbers and has

at least one of each. We see that there are $\binom{6}{k}$ ways of constructing a sequence that consists of k repetitions of the first number, and $6 - k$ repetitions of the second number, where $1 \leq k \leq 5$. Therefore,

$$|B| = \binom{6}{2} \sum_{k=1}^5 \binom{6}{k}$$

$$|\Omega| = 6^6$$

$$P(B) = \frac{|B|}{|\Omega|} = 155/7776 \approx 0.01993$$

3. Let C be the event where $R(\omega)$ has exactly three elements. We can find the number of elements in C by first choosing a triple of distinct numbers for $R(\omega)$ – there are $\binom{6}{3}$ choices. For each triple, we can then count the number of ways they can be assigned to a sequence of length 6 that consists of only those three numbers and has at least one of each. We see that there are $\binom{6}{k} \binom{6-k}{l}$ ways of constructing a sequence that consists of k repetitions of the first number, l repetitions of the second number, and $6 - k - l$ repetitions of the third number, where $1 \leq k \leq 4$ and $1 \leq l \leq 5 - k$. Therefore,

$$|C| = \binom{6}{3} \sum_{k=1}^4 \sum_{l=1}^{5-k} \binom{6}{k} \binom{6-k}{l}$$

$$|\Omega| = 6^6$$

By numerically evaluating the various entries in the formula for $|C|$, we find that:

$$P(C) = \frac{|C|}{|\Omega|} = 25/108 \approx 0.23148$$

You have used 1 of 2 submissions

© All Rights Reserved



© edX Inc. All rights reserved except where noted. EdX, Open edX and the edX and Open EdX logos are registered trademarks or trademarks of edX Inc.

POWERED BY
OPENedX

