F.3.4 Final Questions 7-8

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? QUESTION 7, "basis for the column space"?
Why is (d) (1, -2) and (2, -5) correct? I read the answer key but am still confused? How does the span of these vectors equal the column space o... 2

QUESTION 7

10.0/10.0 points (graded)

Consider the matrix

$$A = \begin{pmatrix} 1 & -1 & 3 & 2 \\ -2 & 3 & -6 & -5 \end{pmatrix}.$$

Reduce A to row echelon form.

1 ✓	-1 ✓	3 ✓	2 ✓
Answer: 1	Answer: -1	Answer: 3	Answer: 2
0 ✓	1 ✓	0 ✓	-1 ✓
Answer: 0	Answer: 1	Answer: 0	Answer: -1

Which of the following form a basis for the column space of A: (Mark all)

- [ ] $\begin{pmatrix} 1 \\ -2 \end{pmatrix}$ and $\begin{pmatrix} 3 \\ -6 \end{pmatrix}$.
- [ ] $\begin{pmatrix} 1 \\ -2 \end{pmatrix}$, $\begin{pmatrix} 3 \\ -6 \end{pmatrix}$, and $\begin{pmatrix} 3 \\ -6 \end{pmatrix}$.
- [x] $\begin{pmatrix} 1 \\ -2 \end{pmatrix}$ and $\begin{pmatrix} -1 \\ 3 \end{pmatrix}$.
- [x] $\begin{pmatrix} 1 \\ -2 \end{pmatrix}$ and $\begin{pmatrix} 2 \\ -5 \end{pmatrix}$.

✓

Which of the following vectors are in the row space of A: (Mark all)

- [ ] $\begin{pmatrix} 1 & -1 & 3 & 2 \end{pmatrix}$
- [x] $\begin{pmatrix} 1 \\ -1 \\ \ldots \end{pmatrix}$

Calculator

$\begin{pmatrix} 3 \\ 2 \end{pmatrix}$

- [x] $\begin{pmatrix} -2 \\ 3 \\ -6 \\ -5 \end{pmatrix}$

- [x] $\begin{pmatrix} 0 \\ 1 \\ 0 \\ -1 \end{pmatrix}$

Which of the following form a basis for the the null space of A. (Mark all)

- [ ] $\begin{pmatrix} 1 \\ -1 \\ 3 \\ 2 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix}$

- [ ] $\begin{pmatrix} -3 \\ 0 \\ 1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} -1 \\ 0 \\ 3 \\ 0 \end{pmatrix}$

- [x] $\begin{pmatrix} -3 \\ 0 \\ 1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} -1 \\ 1 \\ 0 \\ 1 \end{pmatrix}$

(a) Which of the following form a basis for the column space of A:

- Reduce A to row echelon form.
  Answer:
  (2 x 4 array of boxes here)
  $\begin{pmatrix} 1 & -1 & 3 & 2 \\ 0 & 1 & 0 & -1 \end{pmatrix}$
- $\begin{pmatrix} 1 \\ -2 \end{pmatrix}$ and $\begin{pmatrix} 3 \\ -6 \end{pmatrix}$.
  True/False
  False. These two vectors are linearly dependent.
- $\begin{pmatrix} 1 \\ -2 \end{pmatrix}$, $\begin{pmatrix} 3 \\ -6 \end{pmatrix}$, and $\begin{pmatrix} 3 \\ -6 \end{pmatrix}$.
  True/False
  False. The span of these vectors equals the column space of A, but they are not linearly independent and hence not a basis.
- $\begin{pmatrix} 1 \\ -2 \end{pmatrix}$ and $\begin{pmatrix} -1 \\ 3 \end{pmatrix}$.
  True/False
  True. The span of these vectors equals the column space of A *and* they are linearly independent.
- $\begin{pmatrix} 1 \\ -2 \end{pmatrix}$ and $\begin{pmatrix} 2 \\ -5 \end{pmatrix}$. True/False
  True. It is not the set of vectors that you get from the procedure was given, but the span of these vectors equals the column space of A and they are linearly independent.
  The point: The solution is not unique.

(b) Which of the following vectors are in the row space of A:

- $\begin{pmatrix} 1 & -1 & 3 & 2 \end{pmatrix}$
  True/False
  False: it should be a column vector
- $\begin{pmatrix} 1 \\ -1 \\ 3 \\ 2 \end{pmatrix}$
  True/False
  True

Calculator

- $\begin{pmatrix} -2 \\ 3 \\ -6 \\ -5 \end{pmatrix}$
True/False
True

- $\begin{pmatrix} 0 \\ 1 \\ 0 \\ -1 \end{pmatrix}$
True/False
True

(c) Which of the following form a basis for the the null space of A.

- $\begin{pmatrix} 1 \\ -1 \\ 3 \\ 2 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix}$
True/False
False
- $\begin{pmatrix} -1/3 \\ 0 \\ 1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} -1 \\ 0 \\ 3 \\ 0 \end{pmatrix}$
True/False
False. They are not linearly independent nor do they span the null space.
- $\begin{pmatrix} -3 \\ 0 \\ 1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} -1 \\ 1 \\ 0 \\ 1 \end{pmatrix}$
True/False
True. They result from the suggested procedure.

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Answers are displayed within the problem

Question 8

10.0/10.0 points (graded)

Consider $A = \begin{pmatrix} 2 & 3 \\ 2 & 1 \end{pmatrix}$

1. The largest eigenvalue (in magnitude) is:

   4 ✔ Answer: 4

   The smallest eigenvalue (in magnitude) is:

   -1 ✔ Answer: -1

$$\begin{aligned} \det(A - \lambda I) &= \det\left(\begin{pmatrix} 2-\lambda & 3 \\ 2 & 1-\lambda \end{pmatrix}\right) \\ &= (2-\lambda)(1-\lambda) - (3)(2) \\ &= \lambda^2 - 3\lambda + 2 - 6 \\ &= \lambda^2 - 3\lambda - 4 \\ &= (\lambda - 4)(\lambda + 1) \end{aligned}$$

   Thus, the roots of the characteristic polynomial are 4 and -1, which are then the eigenvalues of A.

2. Which of the following is an eigenvector associated with the largest eigenvalue of A (in magnitude): (Mark all)

Calculator

- [x] $\begin{pmatrix} 3 \\ 2 \end{pmatrix}$
- [ ] $\begin{pmatrix} 3 \\ -2 \end{pmatrix}$
- [ ] $\begin{pmatrix} -1 \\ 1 \end{pmatrix}$

✔

To find an eigenvector associated with $\lambda = 4$: Consider

$$A - \lambda = \begin{pmatrix} 2-4 & 3 \\ 2 & 1-4 \end{pmatrix} = \begin{pmatrix} -2 & 3 \\ 2 & -3 \end{pmatrix}$$

By examination, the vector $\begin{pmatrix} 3 \\ 2 \end{pmatrix}$ is an eigenvector. So is any multiple of this vector.

3. Which of the following is an eigenvector associated with the smallest eigenvalue of A (in magnitude): (Mark all)

- [ ] $\begin{pmatrix} 3 \\ 2 \end{pmatrix}$
- [ ] $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$
- [x] $\begin{pmatrix} -1 \\ 1 \end{pmatrix}$

✔

To find an eigenvector associated with $\lambda = -1$: Consider

$$A - \lambda = \begin{pmatrix} 2-(-1) & 3 \\ 2 & 1-(-1) \end{pmatrix} = \begin{pmatrix} 3 & 3 \\ 2 & 2 \end{pmatrix}$$

By examination, the vector $\begin{pmatrix} -1 \\ 1 \end{pmatrix}$ is an eigenvector. So is any multiple of this vector.

4. Consider the matrix $B = 2A$.

- The largest eigenvalue of B is

  8 ✔ Answer: 8

- The smallest eigenvalue of B is

  -2 ✔ Answer: -2

- Which of the following is an eigenvector associated with the largest eigenvalue of B (in magnitude): (Mark all)

  - [x] $\begin{pmatrix} 3 \\ 2 \end{pmatrix}$
  - [ ] $\begin{pmatrix} 3 \\ -2 \end{pmatrix}$

Calculator

- [ ] $\begin{pmatrix} -1 \\ 1 \end{pmatrix}$

✔

If $Ax = \lambda x$ then $(2A)\,x = (2\lambda x)$ and hence the eigenvectors of A are the eigenvectors of B. Thus, the answers are the same as for the same question asked with matrix A.

- Which of the following is an eigenvector associated with the smallest eigenvalue of B (in magnitude): (Mark all)

- [ ] $\begin{pmatrix} 3 \\ 2 \end{pmatrix}$
- [ ] $\begin{pmatrix} 3 \\ -2 \end{pmatrix}$
- [x] $\begin{pmatrix} -1 \\ 1 \end{pmatrix}$

✔

If $Ax = \lambda x$ then $(2A)\,x = (2\lambda x)$ and hence the eigenvectors of A are the eigenvectors of B. Thus, the answers are the same as for the same question asked with matrix A.

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Answers are displayed within the problem

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