Exercise: Counting partitions

(3/3 points)

We have 9 distinct items and three persons. Alice is to get 2 items, Bob is to get 3 items, and Charlie is to get 4 items.

1. As just discussed, this can be done in $\frac{a!}{b!\,3!\,4!}$ ways. Find a and b.

   $a =$ 9 — Answer: 9

   $b =$ 2 — Answer: 2

2. A different way of generating the desired partition is as follows. We first choose 2 items to give to Alice. This can be done in $\binom{c}{d}$ different ways. Find c and d. (There are 2 possible values of d that are correct. Enter the smaller value.)

   $c =$ 9 — Answer: 9

   $d =$ 2 — Answer: 2

3. Having given 2 items to the Alice, we now give 3 items to Bob. This can be done in $\binom{e}{f}$ ways. Find e and f. (There are 2 possible values of f that are correct. Enter the smaller value.)

   $e =$ 7 — Answer: 7

   $f =$ 3 — Answer: 3

Verify that the answer from part 1 agrees with the answer that you get by combining parts 2 and 3.

Answer:

1. By the multinomial formula, $a = 9$ and $b = 2$.

2. We want the number of ways of choosing 2 items out of 9 items. This is the number of 2-element subsets of a 9-element set, so that $c = 9$ and $d = 2$.

3. We have 7 remaining items out of which we need to choose 3. Hence, $e = 7$ and $f = 3$.

From part 1, the number of ways of splitting up the 9 items between Alice, Bob, and Charlie in the specified manner is $\frac{9!}{2!3!4!}$.

In parts 2 and 3, we calculate this answer in a different way. Let us now verify that the two methods produce the same answer.

From part 2, we can first give Alice her 2 items in $\binom{9}{2} = \frac{9!}{2!7!}$ ways. Then, from part 3, we can give Bob his 3 items from the remaining 7 items in $\binom{7}{3} = \frac{7!}{3!4!}$ ways. Finally, Charlie's 4 items are exactly the 4 items that remain, so there is only 1 way to give him his items. Combining these steps, we have a total of

$$\frac{9!}{2!7!} \cdot \frac{7!}{3!4!} \cdot 1 = \frac{9!}{2!3!4!}$$

ways, which agrees with the answer from part 1.

You have used 1 of 2 submissions