Negative Binomial Series

The Series which arises in the Binomial Theorem for Negative integral n,

$$(x + a)^{-n} = \sum_{k=0}^{\infty} \binom{-n}{k} x^k a^{-n-k}$$

$$= \sum_{k=0}^{\infty} (-1)^k \binom{n+k-1}{k} x^k a^{-n-k}.$$

For $a = 1$, the negative binomial series simplifies to

$$(x+1)^{-n} = 1 - nx + \tfrac{1}{2}n(n+1)x^2 - \tfrac{1}{6}n(n+1)(n+2) + \ldots.$$

See also Binomial Series, Binomial Theorem