

10. Let A have an inverse and let $\beta \neq 0$. Prove that $(\beta A)^{-1} = \frac{1}{\beta} A^{-1}$.

Show ① $(\beta A) \left(\frac{1}{\beta} A^{-1} \right) = I$

② $\frac{1}{\beta} A^{-1} (\beta A) = I$

$$\begin{aligned} \text{① } & (\beta A) \left(\frac{1}{\beta} A^{-1} \right) \\ &= \beta \left(A \cdot \frac{1}{\beta} \right) A^{-1} \\ &= \beta \left(\frac{1}{\beta} A \right) A^{-1} \\ &= \left(\beta \cdot \frac{1}{\beta} \right) \cdot (A \cdot A^{-1}) \\ &= 1 I = I \end{aligned}$$

$$\begin{aligned} \text{② } & \left(\frac{1}{\beta} A^{-1} \right) (\beta A) \\ &= \frac{1}{\beta} (A^{-1} \beta) A \\ &= \frac{1}{\beta} (\beta A^{-1}) A \\ &= \left(\frac{1}{\beta} \beta \right) (A^{-1} A) \\ &= 1 I = I \end{aligned}$$

$$\text{So } (\beta A)^{-1} = \frac{1}{\beta} A^{-1}$$