The ABC Conjecture and Beyond (1)

- The **ABC Conjecture** was proposed by Oesterlé and Masser in 1980's.
- It is related to another deep conjecture on elliptic curves, the **Szpiro's Conjecture**.

Joseph Oesterlé
(1954-)

David William Masser
(1948-)

Lucien Szpiro
(1941-)

https://en.wikipedia.org/wiki/Joseph_Oesterl%C3%A9
https://en.wikipedia.org/wiki/David_Masser
https://en.wikipedia.org/wiki/Lucien_Szpiro

2 3 5 7 11 13 17 19 23 29 31 37 41 43 47 53 59 61 67 71 73 79 83 89 97 101 103 107 109 113 127 131 137 139 149 151 157 163 167 173 179 181 191 193 197 199

The ABC Conjecture and Beyond (2)

ABC triple

ABC Conj concerns (A, B, C) s.t.

- **A + B = C**,
- A and B are **relatively prime** (GCD(A,B)=1)

Radical **R** = (product of P dividing A×B×C)

Example (A, B, C) = (4, 21, 25)

R = 2×3×5×7 = 210

The ABC Conjecture and Beyond (3)

- The following simplified version (explicit, weak) was proposed by Baker.

ABC Conjecture (Baker)

For any ABC triples (A, B, C) with radical R,

$$C < R^{1.75}$$

Example (A, B, C) = (4, 21, 25)

$$25 < 210^{1.75} = 11584.67$$

Alan Baker
(1939-)

https://en.wikipedia.org/wiki/Alan_Baker_(mathematician)

The ABC Conjecture and Beyond (4)

- Original version (inexplicit, strong):

ABC Conjecture (Oesterlé-Masser, 1980's)
For any real number $K>0$,

$$C < R^{1+K}$$

except for finitely many (A, B, C).

- (Baker) $C < R^{1.75}$ **for all** ABC triples

https://en.wikipedia.org/wiki/Abc_conjecture