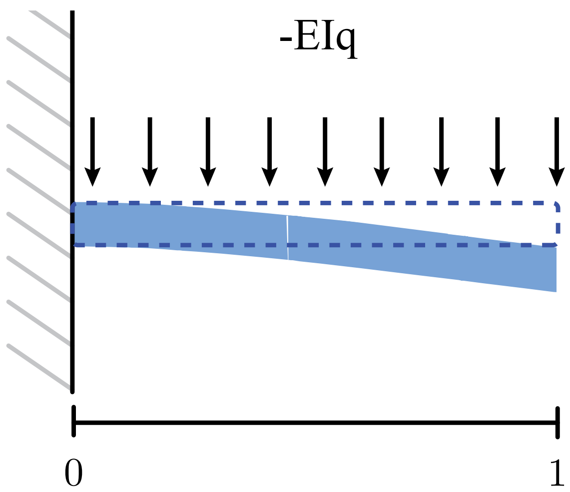
10. Worked example: solving the beam equation

Question 10.1



Find the vertical displacement for the beam in the image above. In other words, solve the following boundary value problem:

$$EI\frac{d^4v}{dx^4}(x) = -EIq,\ x \in [0,1] \tag{3.26}$$

$$v(0) = 0 \tag{3.27}$$

$$\frac{dv}{dx}(0) = 0 \tag{3.28}$$

$$\frac{d^2v}{dx^2}(1) = 0 \tag{3.29}$$

$$\frac{d^3v}{dx^3}(1) = 0 \tag{3.30}$$

Show worked solution

We integrate our initial differential equation four times to get

$$v(x) = -\frac{1}{24}qx^4 + ax^3 + bx^2 + cx + d \tag{3.31}$$

By our first two boundary conditions, we get that $d = c = 0$. So now we have

$$v(x) = -\frac{1}{24}qx^4 + ax^3 + bx^2 \tag{3.32}$$

Taking the second derivative gives

$$\frac{d^2v}{dx^2}(x) = -\frac{1}{2}qx^2 + 6ax + 2b \tag{3.33}$$

and the third gives

$$\frac{d^3v}{dx^3}(x) = -qx + 6a \tag{3.34}$$

From $\frac{d^3v}{dx^3}(1) = 0$ we get that

$$a = \frac{q}{6} \tag{3.35}$$

we plug this back into the second derivative at $x = 1$ to get

$$0 = -\frac{1}{2}q + 6\frac{q}{6} + 2b \tag{3.36}$$

or

$$b = -\frac{q}{4} \tag{3.37}$$

and our final solution is

$$v\left(x\right) = -\frac{q}{24}x^4 + \frac{q}{6}x^3 - \frac{q}{4}x^2 \tag{3.38}$$

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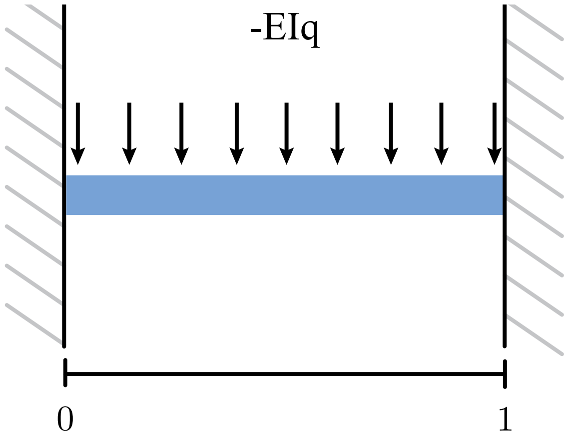
Practice with beam equation, given boundary conditions

1/1 point (graded)

A horizontal beam is fixed into a wall at both ends, and there is a uniform distributed load $-EIq$ along the beam. Find the deflection $v\left(x\right)$ of the beam under this distributed load.



(Try plotting your solution in any graphing software to test that it satisfies the boundary conditions.)

$v(x) =$ -q/24*x^4+q/12*x^3-q/24*x^2 ✔ **Answer:** -q*x^4/24+q*x^3/12-q*x^2/24

$-\frac{q}{24} \cdot x^4 + \frac{q}{12} \cdot x^3 - \frac{q}{24} \cdot x^2$

FORMULA INPUT HELP

Solution:

Both ends fixed means that $v(0) = 0$, $\frac{dv}{dx}(0) = 0$, $v(1) = 0$, and $\frac{dv}{dx}(1) = 0$.

Solving the differential equation

$$EI\frac{d^4 v}{dx^4} = -EIq$$

by integrating 4 times we find

$$\begin{aligned}
\frac{d^3v}{dx^3}(x) &= -qx + a \\
\frac{d^2v}{dx^2}(x) &= -qx^2/2 + ax + b \\
\frac{dv}{dx}(x) &= -qx^3/6 + ax^2/2 + bx + c \\
v(x) &= -qx^4/24 + ax^3/6 + bx^2/2 + cx + d
\end{aligned}$$

Plugging in the boundary conditions at $x = 0$ we obtain

$$\begin{aligned}
v(0) &= d = 0, \\
\frac{dv}{dx}(0) &= c = 0.
\end{aligned}$$

Next we apply the boundary conditions at $x = 1$,

$$\begin{aligned}
v(1) &= -q/24 + a/6 + b/2 = 0 \\
\frac{dv}{dx}(1) &= -q/6 + a/2 + b = 0
\end{aligned}$$

which leads to a system of two equations with two unknowns. Eliminating b from the equation we find

$$\begin{aligned}
-q/12 + a/6 &= 0 \\
a &= q/2.
\end{aligned}$$

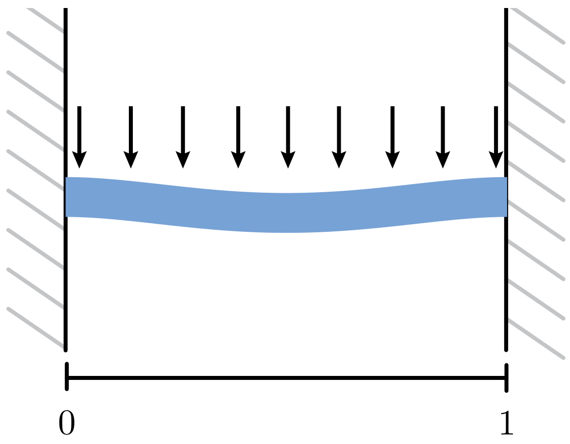
Solving for b we find

$$\begin{aligned}
-q/6 + q/4 + b &= 0 \\
-2q/12 + 3q/12 + b &= 0 \\
b &= -q/12
\end{aligned}$$

Therefore the solution is

$$v\left(x\right) = -qx^4/24 + qx^3/12 - qx^2/24.$$

The graph of the deflection is shown below.



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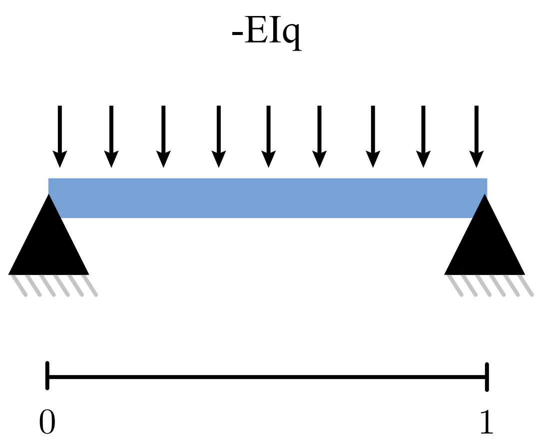
You have used 1 of 4 attempts

Answers are displayed within the problem

Solve the complete problem, 1

1/1 point (graded)

Identify the boundary conditions indicated by the diagram of the horizontal beam with distributed load $q_y\left(x\right) = -EIq$.



Then use those boundary conditions to find the equation for the deflection of the beam $v(x)$.

(Try plotting your solution in infinitesimal or other graphing software to test that it satisfies the boundary conditions.)

$v(x) =$ -q/24*x^4+q/12*x^3-q/24*x ✔ **Answer:** -q*x^4/24+q*x^3/12-q*x/24

$-\frac{q}{24} \cdot x^4 + \frac{q}{12} \cdot x^3 - \frac{q}{24} \cdot x$

FORMULA INPUT HELP

Solution:

Both ends pinned means that $v(0) = 0$, $\frac{d^2v}{dx^2}(0) = 0$, $v(1) = 0$, and $\frac{d^2v}{dx^2}(1) = 0$.

The differential equation

$$EI\frac{d^4v}{dx^4} = -EIq$$

has a general solution of the form

$$v(x) = -qx^4/24 + ax^3 + bx^2 + cx + d.$$

Plugging in the boundary conditions at $x = 0$ we obtain

$$\begin{aligned} v(0) &= d = 0 \\ \frac{d^2v}{dx^2}(0) &= 2b = 0 \end{aligned}$$

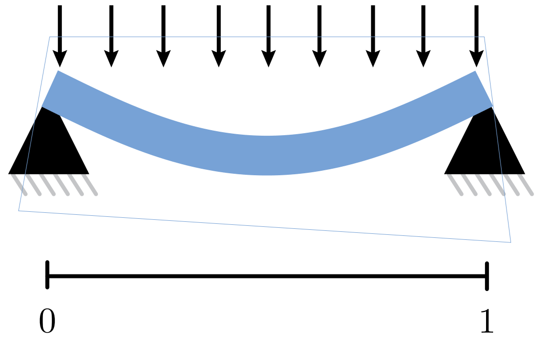
Next we apply the boundary conditions at $x = 1$

$$\begin{aligned} \frac{d^2v}{dx^2}(1) &= -q/2 + 6a = 0 \\ v(1) &= -q/24 + q/12 + c = 0 \end{aligned}$$

Therefore the solution is

$$v(x) = -qx^4/24 + qx^3/12 - qx/24$$

The graph of the deflection is shown below.



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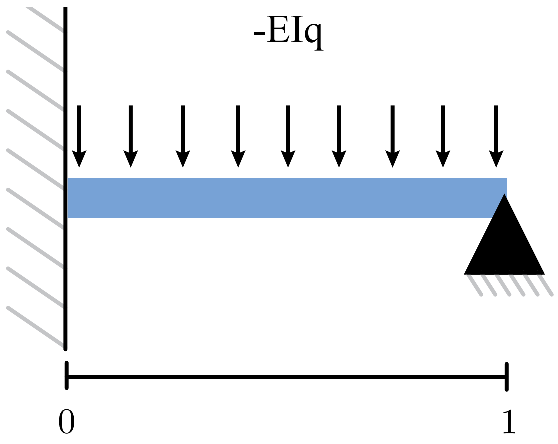
Solve the complete problem, 2

1/1 point (graded)

Identify the boundary conditions indicated by the diagram of the horizontal beam with constant distributed load $q_y(x) = -EIq$.



Then use those boundary conditions to find the equation for the deflection of the beam $v(x)$.

(Try plotting your solution in infinitesimal or other graphing software to test that it satisfies the boundary conditions.)

$v(x) =$ -q/24*x^4+5*q/48*x^3-q/16*x^2 ✔ **Answer:** -q*x^4/24+5*q*x^3/48-3*q*x^2/48

$-\frac{q}{24} \cdot x^4 + \frac{5 \cdot q}{48} \cdot x^3 - \frac{q}{16} \cdot x^2$

FORMULA INPUT HELP

Solution:

The left end fixed means that $v(0) = 0$, $\frac{dv}{dx}(0) = 0$. The right end pinned means $v(1) = 0$, and $\frac{d^2v}{dx^2}(1) = 0$.

The differential equation

$$EI\frac{d^4v}{dx^4} = -EIq$$

has a general solution of the form

$$v(x) = -qx^4/24 + ax^3 + bx^2 + cx + d.$$

Plugging in the boundary conditions at $x = 0$ we obtain

$$\begin{aligned} v(0) &= d = 0 \\ \frac{dv}{dx}(0) &= c = 0 \end{aligned}$$

Next we apply the boundary conditions at $x = 1$

$$\begin{aligned} \frac{d^2v}{dx^2}(1) &= -q/2 + 6a + 2b = 0 \\ v(1) &= -q/24 + a + b = 0 \end{aligned}$$

which leads to a system of two equations with two unknowns. Eliminating b from the equation we find

$$\begin{aligned} \frac{d^2v}{dx^2}(1) &= -q/2 + 6a + 2b = 0 \\ 2v(1) &= -q/12 + 2a + 2b = 0 \\ -5q/12 + 4a &= 0 \\ a &= 5q/48 \end{aligned}$$

Solving for b we find

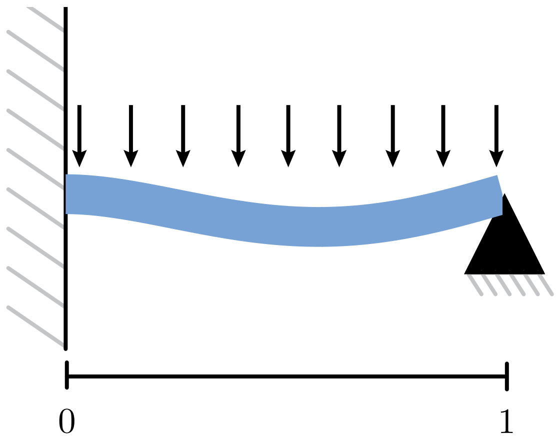
$$-q/24 + 5q/48 + b = 0$$

$$b = -3q/48$$

Therefore the solution is

$$v(x) = -qx^4/24 + 5qx^3/48 - 3qx^2/48$$

The graph of the deflection is shown below.



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