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### 3. Optimization

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Calculator



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Part B due Oct 5, 2021 20:30 IST



Practice

Minimize the distance to a sphere

3/3 points (graded)  
Let  $\mathcal{S}$  be the unit sphere around the origin, defined by  $x^2 + y^2 + z^2 = 1$ . Find the point of  $\mathcal{S}$  which is closest to the point  $(2, 2, 1)$ .

Find the coordinates  $(x, y, z)$ .

$x =$

✓ Answer: 2/3

$y =$

✓ Answer: 2/3

$z =$

✓ Answer: 1/3

? INPUT HELP

Solution:

Assume that the point  $(x, y, z)$  on the unit sphere makes the squared distance function

$$f(x, y, z) = (x - 2)^2 + (y - 2)^2 + (z - 1)^2$$

achieve its minimal value. Then by the Lagrangian multiplier, we have

$$\nabla ((x - 2)^2 + (y - 2)^2 + (z - 1)^2) = \lambda \nabla (x^2 + y^2 + z^2).$$

This is equivalent to

$$x - 2 = \lambda x, \quad y - 2 = \lambda y, \quad z - 1 = \lambda z.$$

Rearranging this gives

$(1 - \lambda) x = 2,$

(6.294)

$(1 - \lambda) y = 2,$

(6.295)

$(1 - \lambda) z = 1.$

(6.296)

Squaring and summing together we find Hence

$$(1 - \lambda)^2 \underbrace{(x^2 + y^2 + z^2)}_{=1} = 2^2 + 2^2 + 1^2$$

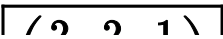
(6.297)

$$(1 - \lambda)^2 = 9$$

(6.298)

which implies that  $1 - \lambda = \pm 3$ . Therefore the point we are looking for is either  $(\frac{2}{3}, \frac{2}{3}, \frac{1}{3})$  or  $(-\frac{2}{3}, -\frac{2}{3}, -\frac{1}{3})$ .

According to the geometric condition, the fact that we are looking for the minimum ar



$\left(\frac{4}{3}, \frac{4}{3}, \frac{1}{3}\right)$  is the answer.

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You have used 1 of 5 attempts

**i** Answers are displayed within the problem


### 3. Optimization

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 [Staff] [Unit sphere vs ellipsoid](#)

In "[Minimize the distance to an ellipsoid](#)," the title says ellipsoid, and the equation is an ellipsoid, but the text says "unit sphere."

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