Icosahedron

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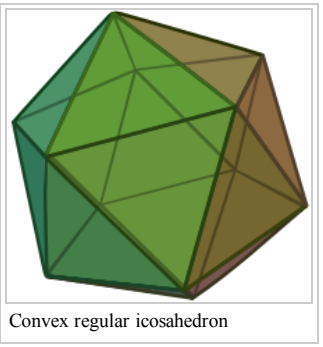
In geometry, an **icosahedron** (/ˌaɪkɵsəˈhiːdrən/ or /aɪˌkɒsəˈhiːdrən/) is a polyhedron with 20 faces. The name comes from Greek εἴκοσι *(eíkosi)*, meaning "twenty", and ἕδρα *(hédra)*, meaning "seat". The plural can be either "icosahedra" (-/drə/) or "icosahedrons".



Convex regular icosahedron

There are many kinds of icosahedron, with some being more symmetrical than others. The most well known is the regular convex or Platonic icosahedron.

Contents

Regular icosahedra

The most symmetrical are the two kinds of regular icosahedron. Each has 20 equilateral triangle faces with five meeting at each of its twelve vertices.

Convex regular icosahedron

The convex regular icosahedron is one of the five regular Platonic solids and is represented by its Schläfli symbol {3, 5}.

The dual polyhedron is the regular dodecahedron {5, 3} having three regular pentagonal faces around each vertex.

Great icosahedron

The great icosahedron is one of the four regular star Kepler-Poinsot polyhedra. Its Schläfli symbol is {3, 5/2}.

The dual polyhedron is the great stellated dodecahedron {5/2, 3}, having three regular star pentagonal faces around each vertex.

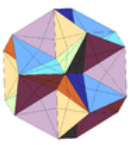
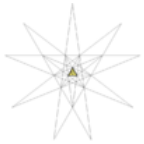
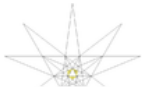
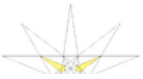
Stellated icosahedra

Stellation is the process of extending the faces or edges of a polyhedron until they meet to form a new polyhedron. It is done symmetrically so that the rsulting figure retains the overall symmetry of the parent figure.

In their book The fifty nine icosahedra, Coxeter et al. enumerated 58 such stellations of the regular icosahedron.

Of these, many have a single face in each of the 20 face planes and so are also icosahedra. The great icosahedron is among them.

Other stellations have more than one face in each plane or form compounds of simpler polyhedra. These are not strictly icosahedra, although they are often referred to as such.

Notable stellations of the icosahedron									
Regular	Uniform duals			Regular compounds			Regular star	Others	
(Convex) icosahedron	**Small triambic icosahedron**	**Medial triambic icosahedron**	**Great triambic icosahedron**	**Compound of five octahedra**	**Compound of five tetrahedra**	**Compound of ten tetrahedra**	**Great icosahedron**	**Excavated dodecahedron**	**Final stellation**
									
									

The stellation process on the icosahedron creates a number of related polyhedra and compounds with icosahedral symmetry.

Pyritohedral symmetry

A *regular icosahedron* can be constructed with pyritohedral symmetry, and is called a **snub octahedron** or **snub tetratetrahedron** or **snub tetrahedron**. this can be seen as an alternated truncated octahedron. If all the triangles are equilateral, the symmetry can also be distinguished by colouring the 8 and 12 triangle sets differently.

Pyritohedral symmetry has the symbol (3*2), $[4,3^+]$, with order 24. Tetrahedral symmetry has the symbol (332), $[3,3]^+$, with order 12. These lower symmetries allow geometric distortions from 20 equilateral triangular faces, instead having 8 equilateral triangles and 12 congruent isosceles triangles.

Cartesian coordinates

The coordinates of the 12 vertices can be defined by the vectors defined by all the possible cyclic permutations and sign-flips of coordinates of the form (2, 1, 0). These coordinates represent the truncated octahedron with alternated vertices deleted.

This construction is called a *snub tetrahedron* in its regular icosahedron form, generated by the same operations carried out starting with the vector (φ, 1, 0), where φ is the golden ratio.[1]

Jessen's icosahedron

In **Jessen's icosahedron**, sometimes called **Jessen's orthogonal icosahedron**, the 12 isosceles faces are arranged differently such that the figure is non-convex. It has right dihedral angles.

It is scissors congruent to a cube, meaning that it can be sliced into smaller polyhedral pieces that can be rearranged to form a solid cube.

Other symmetries

Rhombic icosahedron

The rhombic icosahedron is a zonohedron made up of 20 congruent rhombs. It can be derived from the rhombic triacontahedron by removing 10 middle faces. Even though all the faces are congruent, the rhombic icosahedron is not face-transitive.

Tetrahedral colouring

20 triangles can also be arranged with tetrahedral symmetry (332), $[3,3]^+$, seen as the 8 triangles marked (colored) in alternating pairs of four, with order 12. These symmetries offer Coxeter diagrams: and  respectfully, each representing the lower symmetry to the regular icosahedron  , (*532), [5,3] icosahedral symmetry of order 120.

Pyramid and prism symmetries

Common icosahedra with pyramid and prism symmetries include:

- 19-sided pyramid (plus 1 base = 20).
- 18-sided prism (plus 2 ends = 20).
- 9-sided antiprism (2 sets of 9 sides + 2 ends = 20).
- 10-sided bipyramid (2 sets of 10 sides = 20).
- 10-sided trapezohedron (2 sets of 10 sides = 20).

Johnson solids

Several Johnson solids are icosahedra:[2]

Construction from the vertices of a truncated octahedron, showing internal rectangles with edge length ratios of 2:1.

Pyritohedral and tetrahedral symmetries	
Four views of an icosahedron with tetrahedral symmetry, with eight equilateral triangles (red and yellow), and 12 blue isosceles triangles. Yellow and red triangles are the same color in pyritohedral symmetry.	
Coxeter diagrams	(pyritohedral) (tetrahedral)
Schläfli symbol	s{3,4} sr{3,3} or
Faces	20 triangles: 8 equilateral 12 isosceles
Edges	30 (6 short + 24 long)
Vertices	12
Symmetry group	T_h, $[4,3^+]$, (3*2), order 24
Rotation group	T_d, $[3,3]^+$, (332), order 12
Dual polyhedron	Pyritohedron
Properties	convex
Net	

J22	J35	J36	J59	J60	J92
Gyroelongated triangular cupola	Elongated triangular orthobicupola	Elongated triangular gyrobicupola	Parabiaugmented dodecahedron	Metabiaugmented dodecahedron	Triangular hebesphenorotunda
16 triangles 3 squares 1 hexagon	8 triangles 12 squares	8 triangles 12 squares	10 triangles 10 pentagons	10 triangles 10 pentagons	13 triangles 3 squares 3 pentagons 1 hexagon

References

1. ^ John Baez (September 11, 2011). "Fool's Gold" (http://math.ucr.edu/home/baez/golden.html).
2. ^ Icosahedron (http://mathworld.wolfram.com/Icosahedron.html) on Mathworld.