

MITx: 6.041x Introduction to Probability - The Science of Uncertainty



Unit 0: Overview

- EntranceSurvey
- Unit 1: Probability models and axioms
- Unit 2: Conditioning and independence
- Unit 3: Counting
- Unit 4: Discrete random variables
- Exam 1
- Unit 5: Continuous random variables

Unit overview

Lec. 8: Probability density functions

Exercises 8 due Mar 16, 2016 at 23:59 UT 🗗

Lec. 9: Conditioning on an event; Multiple r.v.'s Unit 5: Continuous random variables > Lec. 10: Conditioning on a random variable; Independence; Bayes' rule > Lec 10 Conditioning on a random variable Independence Bayes rule vertical2

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Exercise: Expected value rule and total expectation theorem

(5/8 points)

Let X, Y, and Z be jointly continuous random variables. Assume that all conditional PDFs and expectations are well defined. E.g., when conditioning on X=x, assume that x is such that $f_X(x)>0$. For each one of the following formulas, state whether it is true for all choices of the function g or false (i.e., not true for all choices of g).

$$^{1.}\mathbf{E}ig[g(Y)\,|\,X=xig]=\int\!g(y)f_{Y|X}(y\,|\,x)\,dy$$

2.
$$\mathbf{E}ig[g(y) \mid X = xig] = \int g(y) f_{Y\mid X}(y\mid x) \, dy$$
False • Answer: False

$$^{3.}\mathbf{E}ig[g(Y)ig] = \int\!\mathbf{E}ig[g(Y)\,|\,Z=zig]\,f_Z(z)\,dz$$

$$^{4.}\mathbf{E}ig[g(Y)\,|\,X=x,Z=zig]=\int\!g(y)f_{Y|X,Z}(y\,|\,x,z)\,dy$$

5.
$$\mathbf{E}ig[g(Y)\,|\,X=xig]=\int\!\mathbf{E}ig[g(Y)\,|\,X=x,Z=zig]\,f_{Z|X}(z\,|\,x)\,dz$$

Exercises 9 due Mar 16, 2016 at 23:59 UT

Lec. 10: Conditioning on a random variable; Independence; Bayes' rule

Exercises 10 due Mar 16, 2016 at 23:59 UT

Standard normal table

Solved problems

Problem Set 5

Problem Set 5 due Mar 16, 2016 at 23:59 UT 🗹

Unit summary

 Unit 6: Further topics on random variables

6.
$$\mathbf{E} \big[g(X,Y) \, | \, Y=y \big] = \mathbf{E} \big[g(X,y) \, | \, Y=y \big]$$

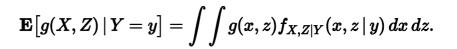
7.
$$\mathbf{E}[g(X,Y) \mid Y=y] = \mathbf{E}[g(X,y)]$$

True \bullet Answer: False

8.
$$\mathbf{E}ig[g(X,Z)\,|\,Y=yig]=\int\!g(x,z)f_{X,Z|Y}(x,z\,|\,y)\,dy$$

Answer:

- 1. True. This is the usual expected value rule, applied to a conditional model where we are given that $\boldsymbol{X} = \boldsymbol{x}$.
- 2. False. Here the quantity inside the expectation, g(y), is a number (not a random variable). The left-hand side is a function of y, whereas on the right-hand side, y, is a dummy variable that gets integrated away. So, the formula is wrong on a purely syntactical basis (the left-hand side depends on y, while the right-hand side does not).
- 3. True. This is the total expectation theorem, where we condition on the events $oldsymbol{Z}=oldsymbol{z}$.
- 4. True. This is the usual expected value rule, applied to a conditional model where we are given that $\pmb{X} = \pmb{x}$ and $\pmb{Z} = \pmb{z}$.
- 5. True. This is the same total expectation theorem as in the third part, except that everything is calculated within a conditional model in which event $\boldsymbol{X}=\boldsymbol{x}$ is known to have occurred.
- 6. True. When we condition on Y=y, we know the value of Y, and we can replace g(X,Y) by g(X,y).
- 7. False. Given that Y=y, we need to somehow take into account the conditional distribution of X, whereas the right-hand side is determined by the unconditional PDF of X.
- 8. False. The left-hand side is a function of y, whereas the right-hand side (after y is integrated out) is a function of x and z. The correct form (expected value rule, in a conditional model) is:



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