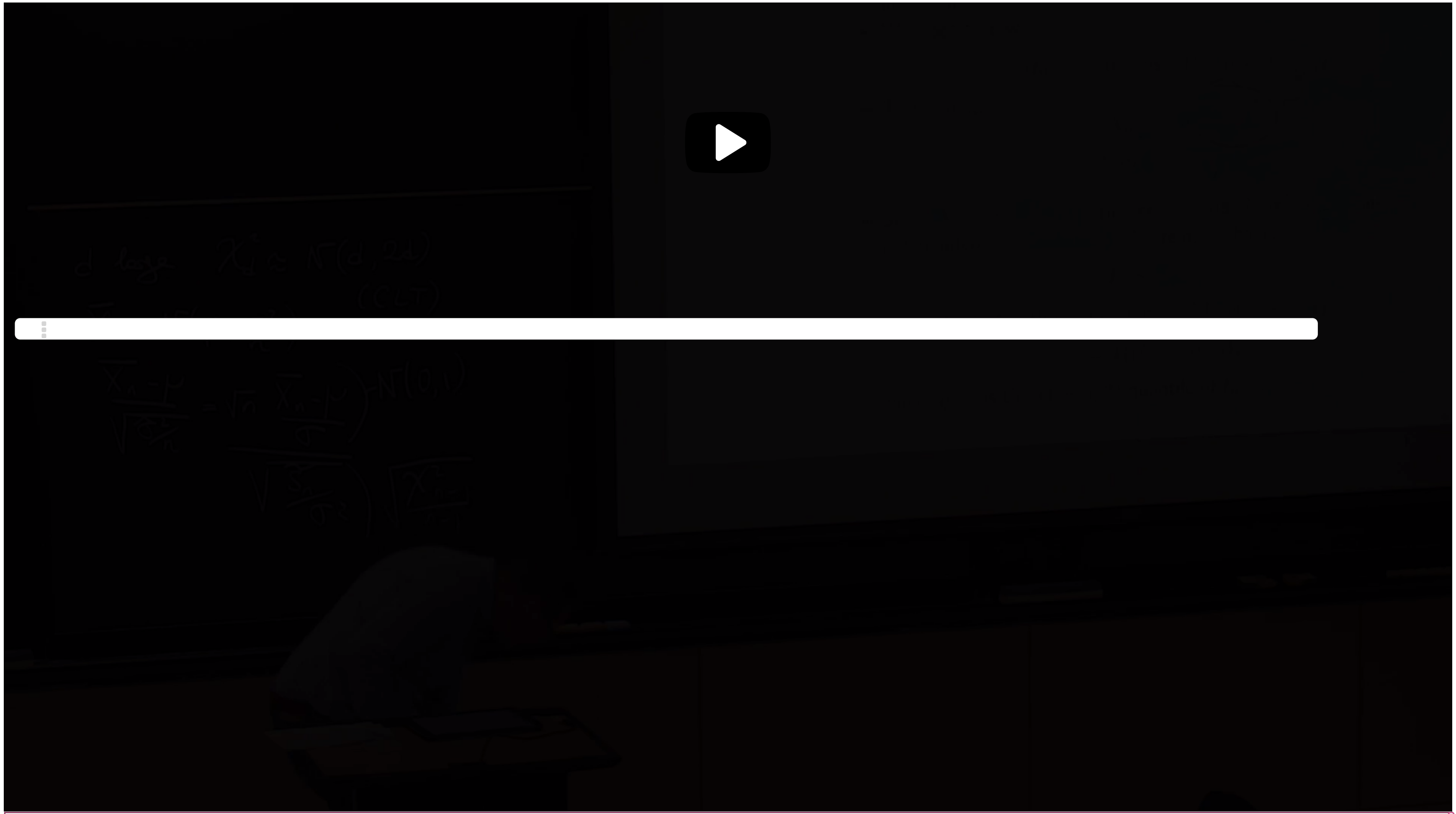
10. The Student's T Test (T Test)

The T Test - One Sample, Two-Sided



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Video

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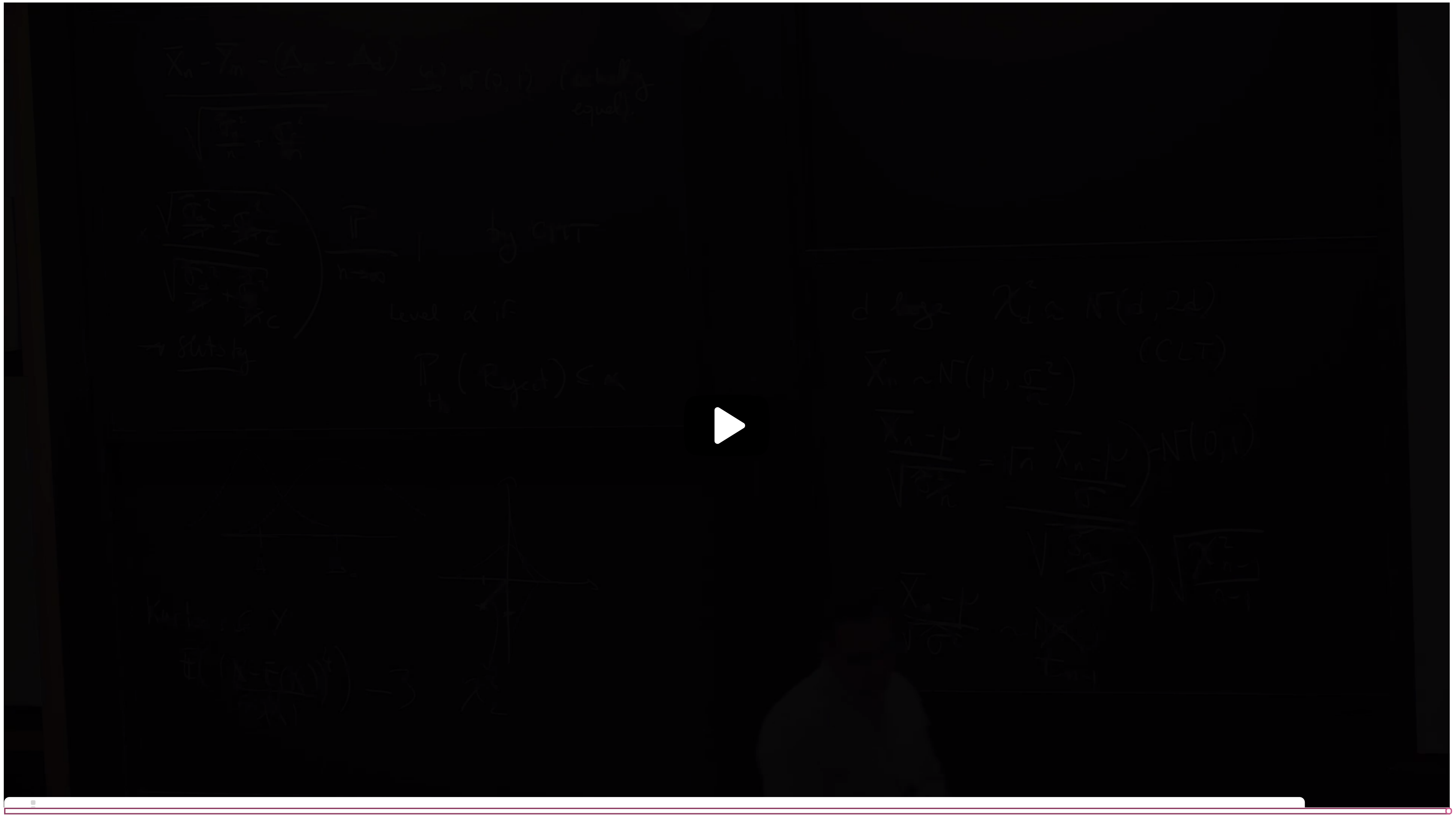
Video note: In the videos on this page and on Slides 16 and 17, the test statistic T_n is missing a scaling factor. The correct equation for T_n is

$$T_n = \frac{\sqrt{n}\,(\overline{X}_n - \mu_0)}{\sqrt{\widetilde{S}_n}}.$$

The unfilled and typed slides have both been corrected.

One Sample, One-Sided T Test



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Concept Check: Student's T Distribution

3/3 points (graded)
Consider the statistic

$$T_n := \sqrt{n}\left(\frac{\overline{X}_n - \mu}{\sqrt{\frac{1}{n-1}\sum_{i=1}^{n}\left(X_i - \overline{X}_n\right)^2}}\right).$$

For all $n \geq 2$, the distribution of T_n is a standard Gaussian $\mathcal{N}(0,1)$.

- ( ) True
- (•) False

✔

As $n \to \infty$, what does

$$\frac{1}{n-1}\sum_{i=1}^{n}\left(X_i - \overline{X}_n\right)^2$$

converge to...

- [ ] The number μ (weakly)
- [x] The number σ^2 (weakly)
- [ ] The distribution $\mathcal{N}(0,1)$
- [ ] The distribution χ^2_{n-1}

✔

As $n \to \infty$, the statistic T_n converges in distribution to

- [x] $\mathcal{N}(0,1)$
- [ ] $\mathcal{N}(\mu,\sigma^2)$
- [ ] χ^2_{n-1}
- [ ] χ^2_n

✔

Solution:

The definition of the student's T distribution with $n-1$ degrees of freedom is that it is given by the distribution of $\frac{Z}{\sqrt{V/(n-1)}}$ where $Z \sim \mathcal{N}(0,1)$, $V \sim \chi^2_{n-1}$ and Z and V are independent. Since we are dividing by V, a χ^2 random variable, then T_n will not have the same distribution as $\mathcal{N}(0,1)$ for all $n \geq 2$.

By the law of large numbers and Slutsky's lemma,

$$\frac{1}{n-1}\sum_{i=1}^{n}\left(X_i-\overline{X}_n\right)^2=\frac{n}{n-1}\left[\left(\frac{1}{n}\sum_{i=1}^{n}X_i^2\right)-\left(\overline{X}_n\right)^2\right]\to\sigma^2$$

in probability.

By the central limit theorem,

$$\sqrt{n}\left(\frac{\overline{X}_n-\mu}{\sigma}\right)\to\mathcal{N}\left(0,1\right).$$

Hence, by the law of large numbers and Slutsky's theorem,

$$\sqrt{n}\left(\frac{\overline{X}_n-\mu}{\sqrt{\frac{1}{n-1}\sum_{i=1}^{n}\left(X_i-\overline{X}_n\right)^2}}\right)\xrightarrow[n\to\infty]{(d)}\mathcal{N}\left(0,1\right).$$

Submit You have used 2 of 3 attempts

Answers are displayed within the problem

Quantiles of the T Distribution:

The $(1-\alpha)$-quantile of the t_{n-1} (corresponding to a one-sided test with statistic T_n) can be computed using standard computational tools such as R. One can also find online tables for the quantiles via a simple Google search, which yields results such as this, this, and this.

As a reminder, in this class the $(1-\alpha)$ quantile of the distribution of a random variable T is the number q_α such that

$$P(T \leq q_\alpha) = 1 - \alpha.$$

Concept Check: T Test

1/1 point (graded)

Let $X_1, \ldots, X_n \overset{iid}{\sim} \mathcal{N}(\mu^*, \sigma^2)$ for some unknown $\mu^* \in \mathbb{R}$ and $\sigma^2 > 0$. You want to decide between the following null and alternative hypotheses on the mean of $X_1, \ldots, X_n$:

$$\begin{aligned} H_0 &: \mu^* = 0 \\ H_1 &: \mu^* \neq 0. \end{aligned}$$

To do so, you define the student's T statistic

$$T_n = \sqrt{n} \frac{\overline{X}_n}{\sqrt{\widetilde{S}_n}}$$

where

$$\widetilde{S}_n = \frac{1}{n-1} \sum_{i=1}^{n} \left(X_i - \overline{X}_n\right)^2$$

is the unbiased sample variance.

The student's T test of level α is specified by

$$\psi_\alpha = \mathbf{1}\left(|T_n| > q_{\alpha/2}\right)$$

where $q_{\alpha/2}$ is the unique number such that $P\left(T_n < q_{\alpha/2}\right) = 1 - \frac{\alpha}{2}$.

Which of the following are true about the student's T test? (Choose all that apply.)

- [ ] The statistic T_n is distributed as a standard Gaussian.
- [x] The test requires the data $X_1, \ldots, X_n$ to be Gaussian.
- [x] The distribution of T_n is pivotal, *i.e.*, its quantiles may be found in tables.
- [x] The test is non-asymptotic. That is, for any fixed n, we can compute the level of our test rather than the *asymptotic* level.

✔

Solution:

We examine the choices in order.

- The first choice is incorrect. Due to the fact that T_n has the sample variance $\widehat{S}_n$ in the denominator and not the *true* variance σ^2, the statistic T_n will **not** be standard Gaussian.
- The second choice is correct. It is a key assumption that the data is Gaussian. Otherwise, the test statistic T_n will not necessarily follow the student's T distribution and, hence, may not even be pivotal.
- The third choice is correct. For any fixed n, we may find the quantiles of the student's T distribution in tables. Since the distribution does not depend on the value of the true parameter, the test statistic T_n is indeed pivotal.
- The last choice is also correct. As stated in the previous bullet, for any fixed n, the quantiles of the student's T distribution may be found in tables. Hence, we can find the non-asymptotic level of this test.

Remark: Assuming the data is Gaussian, the student's T test is useful in situations where the sample size is not very large, since the level may be precisely quantified even for small n.

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You have used 1 of 2 attempts

Answers are displayed within the problem

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