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11.1.1 Equilibrium for nonlinear IVP

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MO2.8 MO2.10

The objective of this chapter is to explain how to solve the equilibrium condition when an IVP is governed by nonlinear differential equations. Root-finding methods, including Newton's method, will replace e.g. Gaussian elimination (in the linear case) when solving for equilibrium states. Of course, root-finding methods have many applications beyond finding equilibrium conditions for nonlinear IVP (and we will see one of these other uses when we discuss implicit time integration methods in Chapter 12).

As in the linear case, discussed in Section 10.1.6, an equilibrium condition is a state $\underline{u} = \underline{u}_{\text{eq}}$ that does not depend on time but satisfies the IVP. Since, $\mathrm{d}\underline{u}/\mathrm{d}t = \underline{0}$, then from our canonical IVP form in Equation (8.36),

$$\underline{f}\left(\underline{u}_{\text{eq}}, t\right) = 0 \tag{11.1}$$

Since $\underline{u}_{\text{eq}}$ does not depend on time, this means that the forcing $\underline{f}$ cannot have a dependence on time, $\underline{f} = \underline{f}\left(\underline{u}\right)$. If $\underline{f}\left(\underline{u}, t\right)$ actually depended on time, then we could not satisfy $\underline{f}\left(\underline{u}, t\right) = \underline{0}$ with a constant $\underline{u}$. So, our equilibrium condition for a nonlinear IVP really requires that $\underline{f}$ only depend on $\underline{u}$, and so the equilibrium condition is,

$$\underline{f}\left(\underline{u}_{\text{eq}}\right) = 0 \tag{11.2}$$

This is a nonlinear system of equations we now must solve. This type of problem is more broadly known as a root-finding problem, as $\underline{u}_{\text{eq}}$ is a root of the