

Course > Midterm Exam 1 > Midterm Exam 1 > 3.

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3. Setup:

We continue with the problem on the previous page.

Recall the lifetime (in months) of a battery is modeled by a random variable X that has pdf

$$f_{ heta}\left(x
ight)=K heta^{x}\mathbf{1}\left(x>0
ight) \qquad ext{where }K=\ln\left(1/ heta
ight)$$

for an unknown parameter $\theta \in (0,1)$.

Assume instead that we cannot actually observe the lifetime of the batteries. Instead, we only observe if the battery is still working after τ months for some known τ to be chosen later (this is called **censored data**).

Let Y_1, \ldots, Y_n be our observations where $Y_i = \mathbf{1}\left(X_i > \tau\right)$ indicates that the ith battery is still working after τ months. Our goal is to estimate $\theta \in (0,1)$ (the parameter for the pdf of X) based on this new data.

Distribution of Y

2/2 points (graded)

What kind of distribution does $Y=\mathbf{1}\left(X> au
ight)$ follow?

Standard Gaussian

igcup Gaussian with mean $\mu
eq 0$

Bernoulli

Cannot be determined



What is $\mathbb{E}\left[Y_i
ight]$?

 $\mathbb{E}\left[Y_i
ight] =$ theta^tau lacktriangle Answer: theta^tau

STANDARD NOTATION

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You have used 1 of 3 attempts

• Answers are displayed within the problem

Statistical Model

1/1 point (graded)

What is the statistical model $(E,\mathbf{P}_{ heta\in\Theta})$ associated to this experiment?

 $igcup \left(\{0,1\}, \mathsf{Ber}(heta^ au)_{ au \in (0,1)})
ight)$

 $\frac{\left(\left(-\infty,\infty\right),\mathcal{N}\left(0,\theta^{\tau}\right)_{\theta\in\left(0,1\right)}\right)}{\left(\left(-\infty,\infty\right),\mathcal{N}\left(\mu,\theta^{\tau}\right)_{\tau\in\left(0,1\right)}\right)}$

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MLE

2.0/2.0 points (graded)

Find the maximum likelihood estimator $\tilde{\theta}$ of θ based on the censored data.

(Enter $\mathbf{barY_n}$ for $\overline{Y_n}$ and \mathbf{tau} for au.)

(barY_n)^(1/tau)

✓ Answer: barY n^(1/tau)

STANDARD NOTATION

Solution:

the maximum likelihood estimator $ilde{ heta}$ of heta based on the censored data is given by $\hat{ heta}=\left(\overline{Y_n}\right)^{rac{1}{ au}}$.

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Consistency

1/1 point (graded)

Is $\tilde{\theta}$ a consistent estimator? If yes, enter the limit in probability; that is, enter the constant C such that $\tilde{\theta} \xrightarrow[n \to \infty]{\mathbf{P}} C$ (where the convergence is in probability). If no, enter **-999**.

$$C=egin{pmatrix} ext{theta} & \ ext{ } & \ ext{ }$$

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• Answers are displayed within the problem

Asymptotic Variance

3.0/3.0 points (graded)

The quantity $\sqrt{n}\,(\tilde{\theta}-\theta)$ converges in distribution to $\mathcal{N}\,(0,\tilde{\sigma^2})$. Find the asymptotic variance $\tilde{\sigma^2}$.

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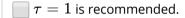
1 Answers are displayed within the problem

Different Tau's

1/1 point (graded)

Previous studies indicate that heta>1/3. Would you recommend using au=1 or au=2? Select the correct reasoning.

(Choose all that apply.)



ullet au = 2 is recommended.

 $lap{/}$ The recommended au value should give a smaller asymptotic variance.

The recommended au value should give a larger asymptotic variance.

~

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