

[Course](#) > [The Higher Infinite](#) > [Ordinals](#) > Well-Ordering Finite and Infinite Sets

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## Well-Ordering Finite and Infinite Sets

As long as a set has more than one element, it can be well-ordered in multiple ways.

Notice, however, that the *well-orderings* of a *finite* set are always isomorphic to one another. For example, the finite set  $\{a, b, c\}$  has six different well-orderings:

$$\begin{array}{lll} a < b < c & b < a < c & c < a < b \\ a < c < b & b < c < a & c < b < a \end{array}$$

But they all fall under a single well-order type, the type corresponding to the shape " $|||$ ". This is true in general: every well-ordering of an  $n$ -element finite set is isomorphic to any other.

In contrast, there are always non-isomorphic ways of well-ordering an infinite set. We saw an example of this above, when we noted that there are well-orderings of the set of natural numbers with each of following two shapes:

$$\begin{array}{l} |||| \dots \\ |||| \dots | \end{array}$$

There are also well-orderings of the set of natural numbers with each of the following shapes:

$$\begin{array}{c}
||||| \dots || \\
||||| \dots ||| \\
||||| \dots |||| \\
\vdots \\
||||| \dots ||||| \dots \\
||||| \dots ||||| \dots | \\
||||| \dots ||||| \dots || \\
||||| \dots ||||| \dots ||| \\
\vdots \\
||||| \dots ||||| \dots ||||| \dots \\
||||| \dots ||||| \dots ||||| \dots | \\
\vdots
\end{array}$$

Note that even though each of these shapes corresponds to a different well-order type, the sets that are ordered by well-orderings of these types *all have the same cardinality*: they all have the cardinality of the natural numbers.

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## Video: Extending the Order Types



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## Problem 1

1/1 point (ungraded)

Which of the following orderings of the natural numbers, if any, has the following well-ordering shape:

|||| ... |||

☒  $3 < 4 < 5 < \dots 0 < 1 < 2$

☐  $0 < 1 < 2 < \dots 0 < 1 < 2$

☒  $3 < 4 < 5 < \dots 1 < 2 < 0$

☒  $3 < 4 < 5 < \dots 2 < 1 < 0$



**Explanation**

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**i** Answers are displayed within the problem

## Problem 2

1/1 point (ungraded)

Which of the following orderings of the natural numbers, if any, has the following well-ordering shape:

|||| ... |||| ...

☐  $1 < 2 < 3 < \dots 0$

☒  $0 < 2 < 4 < \dots 1 < 3 < 5 < \dots$

☒  $1 < 3 < 5 < \dots 0 < 2 < 4 < \dots$

☐  $1 < 2 < 3 < \dots 0 < 2 < 4 < \dots$



### Explanation

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 Answers are displayed within the problem


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