4.

Setup:

Suppose you have observations X_1, X_2, X_3, X_4, X_5 which are i.i.d. draws from a Gaussian distribution with unknown mean μ and unknown variance σ^2.

Given Facts:

You are given the following:

$$\frac{1}{5}\sum_{i=1}^{5} X_i = 0.90, \qquad \frac{1}{5}\sum_{i=1}^{5} X_i^2 = 1.31$$

Choose a test

0.67/1 point (graded)

To test the null hypothesis $H_0 : \mu = 0$ versus the alternative hypothesis $H_1 : \mu \neq 0$ using the data above, which of the following test(s) is appropriate?

(Choose all that apply.)

- [x] t-test ✱
- [ ] Z-test: i.e. the test based on the central limit theorem
- [x] Wald's test

✱

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Answers are displayed within the problem

Unbiased Sample Variance

1/1 point (graded)

Compute the **unbiased sample variance** S.

(Enter a numerical answer accurate to at least 2 decimal places.)

$S =$ 0.625 ✔ **Answer:** 0.625

Solution:

Recall the unbiased sample variance is

$$\begin{aligned} S &= \frac{n}{n-1}\left(\frac{1}{n}\sum_{i=1}^{n}(X_i-\mu)^2\right) \\ &= \frac{n}{n-1}\left(\overline{X_n^2}-\overline{X_n}^2\right) \\ &= \frac{5}{4}(1.31-0.9^2) = 0.625. \end{aligned}$$

(Without the $5/4$ factor, the biased variance is ~ 0.5.)

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MLE

2/2 points (graded)
Give an estimate the mean μ and variance σ^2. Use the maximum likelihood estimator.

$\hat{\mu} =$ 0.9 **Answer:** 0.9

$\hat{\sigma}^2 =$ 0.5 **Answer:** 0.5

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T test

1/2 points (graded)
Find the value of the t-statistic for testing the hypotheses above:

$$H_0 : \mu = 0 \quad \text{versus} \quad H_1 : \mu \neq 0$$

given this set of data.

(Enter a numerical answer accurate to at least 2 decimal places.)

$t-$ statistic: 1.1384199576606164 ✖ **Answer:** 2.546

If we allow 5% of samples to wrongly reject H_0 when H_0 is in fact true, what can we conclude from the t-test?

- ○ reject H_0
- ○ accept H_0
- ◉ fail to reject H_0

✔

Solution:

Recall that the t-distribution with d degrees of freedom is the law of a ratio

$$\frac{Z}{\sqrt{V/d}} \quad \text{where} \quad Z \sim \mathcal{N}(0,1) \text{ indep } V \sim \chi^2_d.$$

The t-statistic is the sample mean scaled by the square root of the sample size, devided by the sample variance, where the variance must be the unbiased estimator with $\frac{1}{n-1}$ normalization, and the t statistics has $d = n-1$ degrees of freedom. Hence, in this problem, the t-statistic is

$$T_n = \frac{\sqrt{n}\overline{X}_n}{\sqrt{S_n^{\text{unbiased}}}} = \frac{\sqrt{5}0.9}{\sqrt{0.625}} = 2.5456.$$

The rejection region for this two-sided test with confidence level $\alpha = 0.05$ is

$$\psi_\alpha = \mathbf{1}\left(|T_5| > q_{\alpha/2}\right) = \mathbf{1}\left(|T_5| > 2.78\right)$$

where $q_{\alpha/2} = 2.78$ is the $(1-\alpha/2)$ quantile of t_4 distribution. We fail to reject the null hypothesis in this sample.

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Answers are displayed within the problem

Confidence interval

2/2 points (graded)

Provide a non-asymptotic confidence interval $[A, B]$ for μ that is symmetric around the sample mean and covers the true mean in 95% of samples.

(To avoid double jeopardy, enter your answer in terms of the (unbiased) sample variance S from above, or directly enter numerical answers accurate to at least 2 decimal places.)

(Be careful that $A < B$.)

Lower bound $A =$ -0.08162158073878045 ✔ **Answer:** 0.90-2.776445*sqrt(S)/sqrt(5)

-0.08162158073878045

Upper bound $B =$ 1.8816215807387806 ✔ **Answer:** 0.90+2.776445*sqrt(S)/sqrt(5)

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Answers are displayed within the problem

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