You are taking "Exam (Timed, No Correctness Feedback)" as a timed exam. Show more

End My Exam 43:26:42

4. Types of critical points

Bookmark this page

Problem Set A due Sep 13, 2021 20:30 IST Completed

Practice

5(a)

4.0/4 points (graded)

Let $g(x,y) = xy - \frac{x^2}{2} - \frac{y^3}{3} + 2.$

Find the critical points of g, and classify each as a local minimum, local maximum, or saddle point.

(In the first column, enter a critical point between round parentheses, e.g. `(a,b)`. In the second column, enter the word `min` for a local minimum, `max` for a local maximum, or `saddle` for a saddle point.)

Critical point	Type of critical point
(0,0)	saddle
(1,1)	max

? INPUT HELP

Solution:

We compute the partial derivatives, and set them equal to zero to find the critical points.

$$g_x(x,y) = y - x = 0 \quad (4.246)$$
$$g_y(x,y) = x - y^2 = 0 \quad (4.247)$$

Plugging $x = y$ into the equation for the partial y derivative we find that $x = 0$ or $x = 1$. Since $x = y$, we find two critical points, $(0,0)$ and $(1,1)$.

To classify them, we look at the second derivatives.

$$A = h_{xx}(x,y) = -1 \quad (4.248)$$
$$B = h_{xy}(x,y) = 1 \quad (4.249)$$
$$C = h_{yy}(x,y) = -2y \quad (4.250)$$

- Applying the second derivative test at the point $(0,0)$, we find that $AC - B^2 = 0 - 1 < 0$. Thus the origin is a saddle point.
- Applying the second derivative test at the point $(1,1)$, we find that $AC - B^2 = (-1)(-2) - 1 = 1 > 0$, and $A = -1 < 0$. Thus $(1,1)$ is local maximum.

Submit You have used 1 of 5 attempts

Answers are displayed within the problem

5(b)

2/2 points (graded)

Does $g(x,y) = xy - x^2/2 - y^3/3 + 2$ attain a maximum on the region R, where *R* ... a

Calculator Hide Notes

- [x] yes
- [ ] no

✔

Does $g(x, y) = xy - x^2/2 - y^3/3 + 2$ attain a minimum on the region R, where R is the first quadrant?

- [ ] yes
- [x] no

✔

Solution:

First we evaluate the function at the critical points so we have values of comparison: Note that $g(0, 0) = 2$ and $g(1, 1) = 13/6 > 2$.

Next we check the behavior along the boundary lines $x = 0$ and $y = 0$ and compare to the values at the critical points.

When $x = 0$, $g(0, y) = -y^3/3 + 2$. This function has critical point $g'(0, y) = -y^2 = 0$ at the origin. We know this is a saddle point, and the height of the function there is 2. Additionally, $g(0, y)$ is decreasing to negative infinity as y increases. So the maximum along the boundary is 2 and it has no minimum.

When $y = 0$, $g(x, 0) = -x^2/2 + 2$. This function has a critical point $g'(x, 0) = -x = 0$ again at the origin. We know this is a saddle point, and the height of the function there is 2. Additionally, $g(x, 0)$ is decreasing to negative infinity as x increases. So the maximum along the boundary is 2 and it has no minimum.

Therefore the only possible maximum is at the critical point $(1, 1)$, which has larger value than the saddle point at $(0, 0)$ along the boundary. This function does not have a minimum as it decreases without bound as x and y increase.

Submit You have used 1 of 1 attempt

Answers are displayed within the problem

5(c)

4.0/4 points (graded)

Find the maximum and minimum points of the function $g(x, y) = xy - x^2/2 - y^3/3 + 2$ on the region R, where R is the square $0 \leq x \leq 2$, $0 \leq y \leq 2$.

Identify the point where $g(x, y)$ attains its maximum value. (Enter point as an ordered pair surrounded by round parentheses: `(a,b)`. Enter numerical value to 2 decimal places.)

(1,1) ✔ **Answer:** (1,1)

The maximum value is: 13/6 ✔ **Answer:** 13/6

Identify the point where $g(x, y)$ attains its minimum value. (Enter point as an ordered pair surrounded by round parentheses: `(a,b)`. Enter numerical value to 2 decimal places.)

(0,2) ✔ **Answer:** (0,2)

Calculator Hide Notes

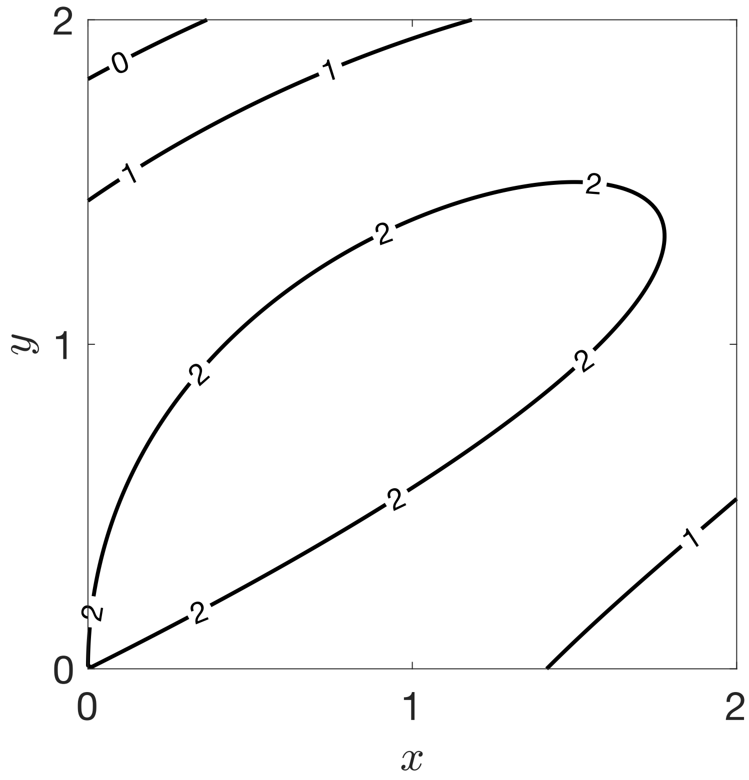
The minimum value is: -2/3 ✔ **Answer:** -2/3

? INPUT HELP

Solution:

From the previous problem, we know the maximum occurs at the critical point $(1,1)$. We also know that the minimum will occur along the boundary. Based on the shape of the function, we suspect it will occur where $x=0$ and y is as large as possible. That is at the point $(0,2)$.

The level curves of the function $g(x,y)$ are shown in the image below.



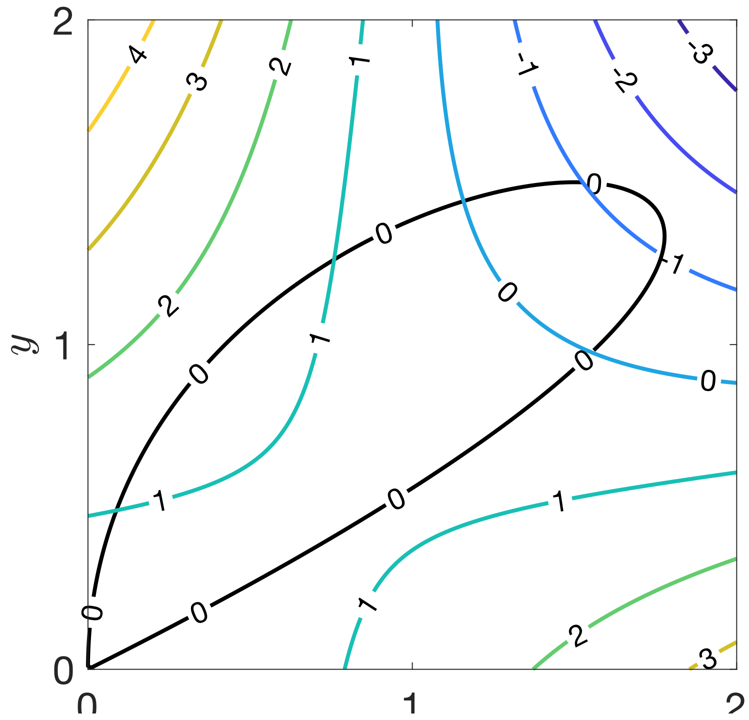
Submit You have used 2 of 10 attempts

Answers are displayed within the problem

6

4/4 points (graded)

We wish to find the maximum and minimum points of the function $f(x,y) = x^2/3 + y^2/4 - 3xy + x + 2y$ subject to the constraint equations $g(x,y) = xy - x^2/2 - y^3/3 = 0$, $x \geq 0$, $y \geq 0$.



Calculator Hide Notes

x

Solving the Lagrange multiplier problem, we obtain the following approximate locations where the $\nabla f = \lambda \nabla g$: `(0.359229, 0.932835), (0.843235, 0.460127), (1.73080, 1.43761)` .

Identify the point where $f(x, y)$ attains its maximum value along $g(x, y) = 0$.

- (x) (0.359229, 0.932835)
- ( ) (0.843235, 0.460127)
- ( ) (1.73080, 1.43761)

✔

What is the maximum value (approximately) along the constraint curve? (Enter numerical value to 2 decimal places.)

1.480155 ✔ **Answer:** 1.48

Identify the point where $f(x, y)$ attains its minimum value along $g(x, y) = 0$.

- ( ) (0.359229, 0.932835)
- ( ) (0.843235, 0.460127)
- (x) (1.73080, 1.43761)

✔

What is the minimum value (approximately) along the constraint curve? (Enter numerical value to 2 decimal places.)

-1.343389 ✔ **Answer:** -1.34

Solution:

We use the level curves to approximate by eye where the maximum and minimum should occur and compare to the numerical values provided. Plugging into the equation we find that the maximum is about 1.5 and the minimum is about -1.34.

Submit You have used 1 of 2 attempts

Answers are displayed within the problem

4. Types of critical points

Hide Discussion

Topic: Unit 3: Optimization / 4. Types of critical points

Add a Post

Show all posts by recent activity

Solving PS3A 5(a) using Sympy 1

[Staff] Error in Solution to 5(b)

Calculator Hide Notes

I believe that the partial function g(x) = -x^2 + 2 should be g(x) = -x^2/2 + 2 in the solution.

Q1
Hi! I feel very dumb right now seeing all of your questions here and comparing to mine but here it goes: I found the derivatives of g t... 3

[Staff] Help please. Concept check!!
Hi Staff, This is regarding Critical points and the bounded region, particularly questions 5a and 5c. I did get 5a correct but I am stum... 3

Clarification first quadrant
After a little bit of research (https://mathworld.wolfram.com/Quadrant.html), first quadrant seems to mean: x>0 and y>0. The (0,0) p... 9

[Staff] Error message
I'm getting this error message: **Could not format HTML for problem. Contact course staff in the discussion forum for assistance.** 3

[Staff] Q6: constraint g(x,y)=0
Are you sure g(x,y) = 0 and not 2? Plugging in the given (x,y) values gives a result of 2. 4

Clarification 5b)
I might have misunderstood the question, but does it mean attaining a minimum only due to a critical point? Because in the region R ... 4

Extension for Unit 3 would be much appreciated:)
Hi, as mentioned in the announcement on the course main page an extension would be really really appreciated :) Many thanks in ad... 3

< Previous Next >