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☆ Course / Unit 2: Geometry of Derivatives / Lecture 7: Directional derivatives



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**Summarize** 

## Big Picture

When we are considering a function of 2 (or more) variables, it is natural to wonder what the slope of the graph of this function is. However, the function has a slope that may be different depending on the direction you move away from this point. This geometric notion of the slope of the graph along a particular direction is the **directional derivative**, and can be computed with a dot product with the gradient of the function.

#### Mechanics

#### **Directional derivatives definition**

#### **Definition 11.1**

The **directional derivative** of a function f(x,y) in the direction of the unit vector  $\hat{u}$  at the point (x,y) is given by

$$D_{\hat{u}}f\left( x,y
ight) =
abla f\cdot\hat{u}.$$

For the directional derivative along any non-zero vector  $ec{v}$ , we use  $D_{ec{v}} = D_{ec{v}/|ec{v}|}$  .

#### Extension to higher dimension: Directional derivatives

In n dimensions, the definition of the directional derivative is the same. We would have an n-dimensional unit vector  $\hat{u}=\langle u_1,u_2,\ldots,u_n\rangle$ . Then

$$egin{array}{lcl} D_{\hat{u}}f\left(x_{1},x_{2},\ldots,x_{n}
ight) & = & 
abla f\left(x_{1},x_{2},\ldots,x_{n}
ight) \cdot \hat{u} \ \\ & = & \left\langle f_{x_{1}},f_{x_{2}},\ldots,f_{x_{n}}
ight
angle \cdot \left\langle u_{1},u_{2},\ldots,u_{n}
ight
angle \ \\ & = & f_{x_{1}}\left(x_{1},x_{2},\ldots,x_{n}
ight)u_{1} + f_{x_{2}}\left(x_{1},x_{2},\ldots,x_{n}
ight)u_{2} + \cdots + f_{x_{n}}\left(x_{1},x_{2},\ldots,x_{n}
ight)u_{n}. \end{array}$$

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## Directional derivatives given an angle

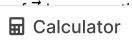
Let heta be an angle measured from the positive x-axis.The rate of change of f(x,y) in the direction of the angle heta is given by

$$D_{\hat{u}}f\left(x,y
ight)=f_{x}\cos heta+f_{y}\sin heta$$

This is the same as the directional derivative of f in the direction of  $\hat{u} = \langle \cos \theta, \sin \theta \rangle$ .

## **Directional derivatives with non-unit vectors**

Given a vector  $ec{v}$  whose magnitude is not 1, we can obtain a unit vector in the direction



$$\hat{m{u}} = rac{1}{|ec{m{v}}|} ec{m{v}}.$$

Then the directional derivative of  $m{f}$  in the direction of  $m{ec{v}}$  is

$$D_{ec{u}}f\left( x,y
ight) =rac{
abla f\cdot ec{v}}{\leftert ec{v}
ightert }.$$

## Direction of maximal change in f

We can write the directional derivative as the dot product

$$D_{\hat{u}}f\left(x,y
ight)=
abla f\left(x,y
ight)\cdot\hat{u}=\left|
abla f
ight|\cos heta$$

where heta is the angle between abla f and  $\hat{m{u}}$ . This quantity is maximized when  $m{ heta}=m{0}$ , which implies that **the gradient** is the direction of the maximum rate of change of  $oldsymbol{f}$  .

## Ask Yourself

Is the directional derivative a vector or a scalar?

It's a scalar.

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#### → Why is the directional derivative useful?

We have seen that, when we move in the horizontal direction by a small amount  $\Delta x$ , linear approximation can tell us how f will change. In fact, f will change by  $\frac{\partial f}{\partial x}\cdot\Delta x$ . But there is nothing special about moving horizontally - sometimes we wish to move in an arbitrary direction. Directional derivatives give us a compact expression for the resulting change in f. If we move from the point  $(x_0,y_0)$  in the direction  $ec{u}$  by a distance of  $\Delta s$ , then the change in f will be  $D_{\hat{u}}f(x_0,y_0)\cdot \Delta s$ .

In summary: if we move the input to  $m{f}$  along the direction of  $m{ec{v}}$ , then  $m{D}_{m{ec{v}}}m{f}$  tells us how the function changes up to linear approximation.

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#### What does it mean if the directional derivative in a certain direction is zero?

This means that, up to linear approximation, moving in that direction doesn't change the value of f. For example, if  $f(x,y)=x^2+y^2$  , then at the point (3,4) , if we use  $\hat{u}=(-4/5,3/5)$  then  $D_{\hat{u}}f(3,4)=0$  . This means that moving from (3,4) in the direction (-4/5,3/5) will keep the value of x^2 + y^2 approximately constant.

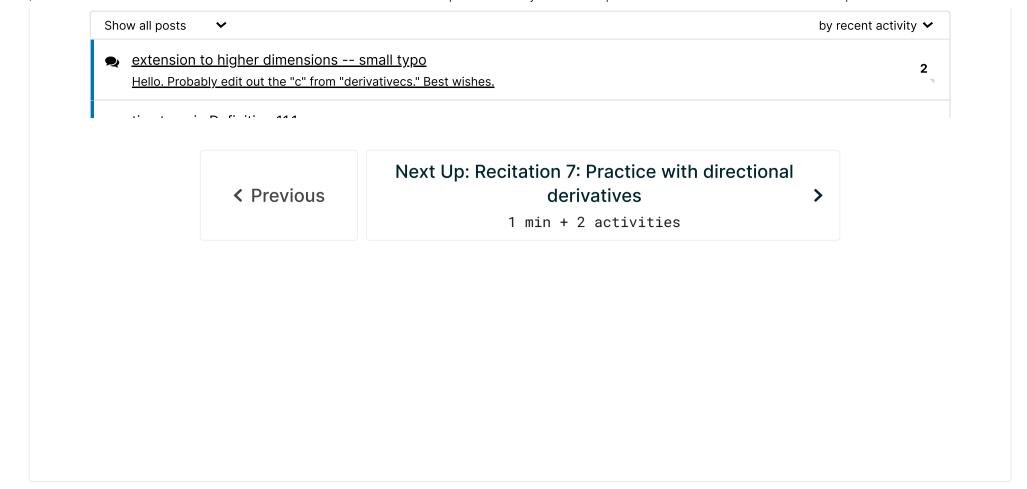
Concretely, f(3,4)=25 and f(3-4/5,4+3/5)=26, not much of a change! In contrast, if we move in the direction (1,0) then the new value of f would be f(3+1,4)=32, a big change!

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## 11. Summary

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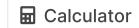














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