MATHEMATICS

Asymptotic variance of MLE of normal distribution.

Asked 3 years, 7 months ago Active 4 days ago Viewed 1k times

0

1

I am trying to explicitly calculate (without using the theorem that the asymptotic variance of the MLE is equal to CRLB) the asymptotic variance of the MLE of variance of normal distribution, i.e.:

$$\hat{\sigma}^2 = \frac{1}{n}\sum_{i=1}^{n}(X_i - \hat{\mu})^2$$

I have found that:

$$\mathrm{Var}(\hat{\sigma}^2) = \frac{2\sigma^4}{n}$$

and so the limiting variance is equal to $2\sigma^4$, but how to show that the limiting variance and asymptotic variance coincide in this case?

probability statistics asymptotics statistical-inference estimation

edited Apr 5 '16 at 11:53 Jean-Claude Arbaut 17.5k 6 40 66

asked Apr 5 '16 at 10:37 marco11 784 5 19

For starters,

$$\hat{\sigma}^2 = \frac{1}{n}\sum_{i=1}^{n}(X_i - \bar{X}_i)^2.$$

– Math1000 Apr 5 '16 at 11:19

1 Sorry for a stupid typo and thank you for letting me know, corrected. – marco11 Apr 5 '16 at 11:41

2 Answers

0

You can use the following relation

Limiting Variance $\geq$ Asymptotic Variance $\geq CRLB_{n=1}$

Now calculate the CRLB for $n = 1$ (where n is the sample size), it'll be equal to $2\sigma^4$ which is the Limiting Variance. Therefore Asymptotic Variance also equals $2\sigma^4$.

answered Oct 13 at 16:27

Vishaal Sudarsan

441 2 7

0

From the asymptotic normality of the MLE and linearity property of the Normal r.v

$$\sqrt{n}\left(\hat{\sigma}_n^2 - \sigma^2\right) \xrightarrow{D} \mathcal{N}\left(0, \frac{2\sigma^4}{n}\right)$$
$$\left(\hat{\sigma}_n^2 - \sigma^2\right) \xrightarrow{D} \mathcal{N}\left(0, \frac{2\sigma^4}{n^2}\right)$$
$$\hat{\sigma}_n^2 \xrightarrow{D} \mathcal{N}\left(\sigma^2, \frac{2\sigma^4}{n}\right), \qquad n \to \infty$$

edited Apr 5 '16 at 12:38

answered Apr 5 '16 at 11:48

V. Vancak

12.2k 3 10 30

Thank you, but is it possible to do it without starting with asymptotic normality of the mle? – marco11 Apr 5 '16 at 12:04