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Course › Unit 5 Bayesian statistics › Lecture 17: Introduction to Bayesian Statistics › 8. Warm-up / Review: Proportionality

8. Warm-up / Review: Proportionality

Distributions with One Parameter

6/6 points (graded)

Match each of the proportionality expressions below to the corresponding well-known distribution, and supply the missing parameters. The variable of interest is θ. In entering the expressions for the parameters, only the variables a, b, or c may be used.

In this problem, the distribution **Geom** (p) is assumed to be over the nonnegative integers. The more explicit specification for the geometric distribution is the number of failure until the first success in a sequence of i.i.d. Bernoulli(p) Trials.

$\pi(\theta) \propto a^{1-\theta}(1-a)^{\theta}$ (for $\theta \in \{0,1\}$, and it is known that $a \in (0,1)$)

● Ber (p)

○ Exp (λ)

○ Poiss (λ)

○ Geom (p)

✓

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$\text{parameter} =$

1-a

✓ **Answer:** (1-a)+b*0+c*0

$1 - a$

$\pi(\theta) \propto c^{a\theta+b}$ (for $\theta \in \mathbb{N} \cup \{0\}$, and it is known that $a \in (0,1)$)

○ Ber (p)

○ Exp (λ)

○ Poiss (λ)

● Geom (p)

✓

$\text{parameter} =$

1-c^a

✓ **Answer:** 1-c^a+b*0

$1 - c^a$

$\pi(\theta) \propto 100e^{a\theta+b}$ (for $\theta \geq 0$, and it is known that $a < 0$)

○ Ber (p)

● Exp (λ)

○ Poiss (λ)

○ Geom (p)

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✓

$\text{parameter} =$

-a

✓ **Answer:** -a+b*0+c*0

$-a$

STANDARD NOTATION

Solution:

- It must be the Bernoulli distribution as this is the only distribution among our choices that has the binary support $\{0,1\}$. The Bernoulli parameter p represents the probability of $\theta = 1$. If we write $f(\theta) = a^{1-\theta}(1-a)^{\theta}$, we get $f(0) = a$ and $f(1) = 1-a$, so the normalization constant is $a + (1-a) = 1$, and we thus have $\pi(0) = a$, $\pi(1) = 1-a$. Hence the parameter is $\boxed{p = 1-a}$.
- It must be the geometric distribution. Our un-normalized PMF $f(\theta) = c^{a\theta+b}$ is characterized by $f(0) = c^b$ and $\frac{f(\theta+1)}{f(\theta)} = c^a$, which define a geometric distribution. The PMF $g(x)$ of the geometric distribution $\textsf{Geom}(p)$ satisfies $\frac{g(x+1)}{g(x)} = 1-p$, thus equating gives $c^a = 1-p$, or that $\boxed{p = 1-c^a}$.
- This is a continuous version of the second item and features a linearly increasing exponent, which implies that it must be the exponential distribution. The PMF $g(x)$ of the exponential distribution $\textsf{Exp}(\lambda)$ satisfies $\frac{g(x+1)}{g(x)} = e^{-\lambda}$. Computing this quantity for the distribution with un-normalized PMF $100e^{a\theta+b}$ gives e^a, so equating gives $e^{-\lambda} = e^a$, equivalent to $\boxed{\lambda = -a}$.

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You have used 3 of 3 attempts

Answers are displayed within the problem

Distributions with Two Parameters

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graded)

Match each of the proportionality expressions below to the corresponding well-known distribution, and then compute the values of the parameter(s) of the distribution in terms of the given a, b, and/or c. The variable of interest is θ. Express the parameters in the order of which they appear in the expression. In entering the expressions for the parameters, only the variables a, b, or c may be used.

In this problem, the distribution $\mathsf{N}\left(\mu,\sigma^2\right)$ has parameters μ and σ^2.

$\pi\left(\theta\right) \propto c$ (for $\theta \in [a,b]$ where $a,b \in \mathbb{R}$, $a < b$)

● $\mathsf{Unif}\left([\alpha,\beta]\right)$

○ $\mathsf{N}\left(\mu,\sigma^2\right)$

○ $\mathsf{Binom}\left(n,p\right)$

○ $\mathsf{Beta}\left(\alpha,\beta\right)$

✓

left parameter $=$

a

✓ **Answer:** a+b*0+c*0

a

right parameter $=$

b

✓ **Answer:** a*0+b+c*0

b

$\pi\left(\theta\right) \propto \theta^a\left(c - c\theta\right)^b$ (for $\theta \in [0,1]$ where $a,b > -1$)

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○ Unif $([\alpha, \beta])$

○ N (μ, σ^2)

○ Binom (n, p)

● Beta (α, β)

✓

left parameter $=$

a+1 ✓ **Answer:** a+1+b*0+c*0

$a + 1$

right parameter $=$

b+1 ✓ **Answer:** b+1+a*0+c*0

$b + 1$

$\pi(\theta) \propto e^{a\theta^2 + b\theta + c}$ (for $\theta \in \mathbb{R}$, and it is known that $a < 0$)

○ Unif $([\alpha, \beta])$

● N (μ, σ^2)

○ Binom (n, p)

○ Beta (α, β)

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✓

$\text{left parameter} =$

-b/(2*a)

✓ **Answer:** -b/(2*a)+c*0

$-\frac{b}{2\cdot a}$

$\text{right parameter} =$

-1/(2*a)

✓ **Answer:** -1/(2*a)+b*0+c*0

$-\frac{1}{2\cdot a}$

STANDARD NOTATION

Solution:

- We are given a distribution that is flat over a given finite interval over the real line, which implies that we have a uniform distribution. The parameters of a uniform distribution are the bounds of the interval. Here, they are a and b, so these are also the parameters of the distribution, giving $\text{Unif}\,(a,b)$.

- Rewriting by dividing the distribution by c^b (which is a constant multiplier) gives $f\,(\theta) = \theta^a\,(1-\theta)^b$. This resembles the form of a Beta distribution, as discussed in lecture, with parameters $\boxed{\alpha = a+1}$ and $\boxed{\beta = b+1}$.

- We have a support over the real line, so a normal distribution is our only choice here. The standard form of a normal distribution is $\frac{1}{\sqrt{2\pi\sigma^2}}e^{-\frac{(x-\mu)^2}{2\sigma^2}}$. Our variable of interest is x, so we may drop the left multiplier, ending up with $e^{-\frac{(x-\mu)^2}{2\sigma^2}}$. Now the exponent is a quadratic in x : $-\frac{x^2}{2\sigma^2}+\frac{\mu}{\sigma^2}x-\frac{\mu^2}{\sigma^2}$. Equating the coefficient with x^2 gives $a = -\frac{1}{2\sigma^2}$, or that $\boxed{\sigma^2 = -\frac{1}{2a}}$. Next, equating the coefficient of x gives $b = \frac{\mu}{\sigma^2} = \frac{\mu}{-\frac{1}{2a}}$. Hence $\boxed{\mu = -\frac{b}{2a}}$.

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You have used 3 of 3 attempts

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Answers are displayed within the problem

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Last part, final distribution

question posted 6 days ago by **mbh038**

I thought we were allowed to use a, b or c? Given that, then two answers are possible for one of the entries, I think. I tried both. Only one of them is marked as correct.

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2 responses

SuhailWali
5 days ago

and the answer is indeed made up of a,b ,or c only.

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sudarsanvsr_mit (Staff)
5 days ago

Can you please point to which part of which problem on this page is it that you are talking about? I want to make sure our grader is programmed correctly...

I meant that (I thought) that the expression for μ in the last part of the last question could be written in two ways using just a, b or c. But I had not taken into account that the expression we were to start from is only a proportionality, not an equality.

Thanks for replying.

posted 4 days ago by **mbh038**

No problem. Glad you figured it out.

posted 3 days ago by **sudarsanvsr_mit** (Staff)

I am sorry if I am revealing too much information (please delete if required), but I can't find out any mistake in my following solution (and there is only one attempt left) for $\mathcal{N}\left(\mu,\sigma^2\right)$, although rejected by the grader, any idea where I am doing wrong? thanks in advance for helping.

Deleted by MW-CTA

posted about 9 hours ago by **sandipan_dey**

sandipan_rey, remember that it is a proportionality, you have to simplify your expression ;)

posted about 8 hours ago by **vascomfmneves**

Please respect the honor policy, and not post solutions whether correct or incorrect before the due date of the exercise.

posted about 6 hours ago by **markweitzman** (Community TA)

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sure @markweitzmann.

@vascomfmneves, even though it's a proportionality, the terms including μ and σ must be there right (since they are the parameters, or in Bayesian term the variables)? I thought we can only get rid of only the constants for proportionality, not the parameters.

For example, $f\left(\mu,\sigma^2\right) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(x-\mu)^2/2\sigma^2} \propto \frac{1}{\sqrt{\sigma^2}} e^{-(x-\mu)^2/2\sigma^2}$ but $f\left(\mu,\sigma^2\right)$ is NOT proportional to $e^{-(x-\mu)^2/2\sigma^2}$, since $\sqrt{\sigma^2}$ in the denominator is NOT a constant, that's what I thought why the expression can't be simplified further (because all terms contain σ^2), is not that correct? if not, why?

[EDIT] I see the issue now, for all the problems here, we need to ignore the parts not involving θ, because proportionality can give rise to some hidden constants that we may miss out) and should not have any equation from those parts, thanks everyone.

posted about 5 hours ago by **sandipan_dey**

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