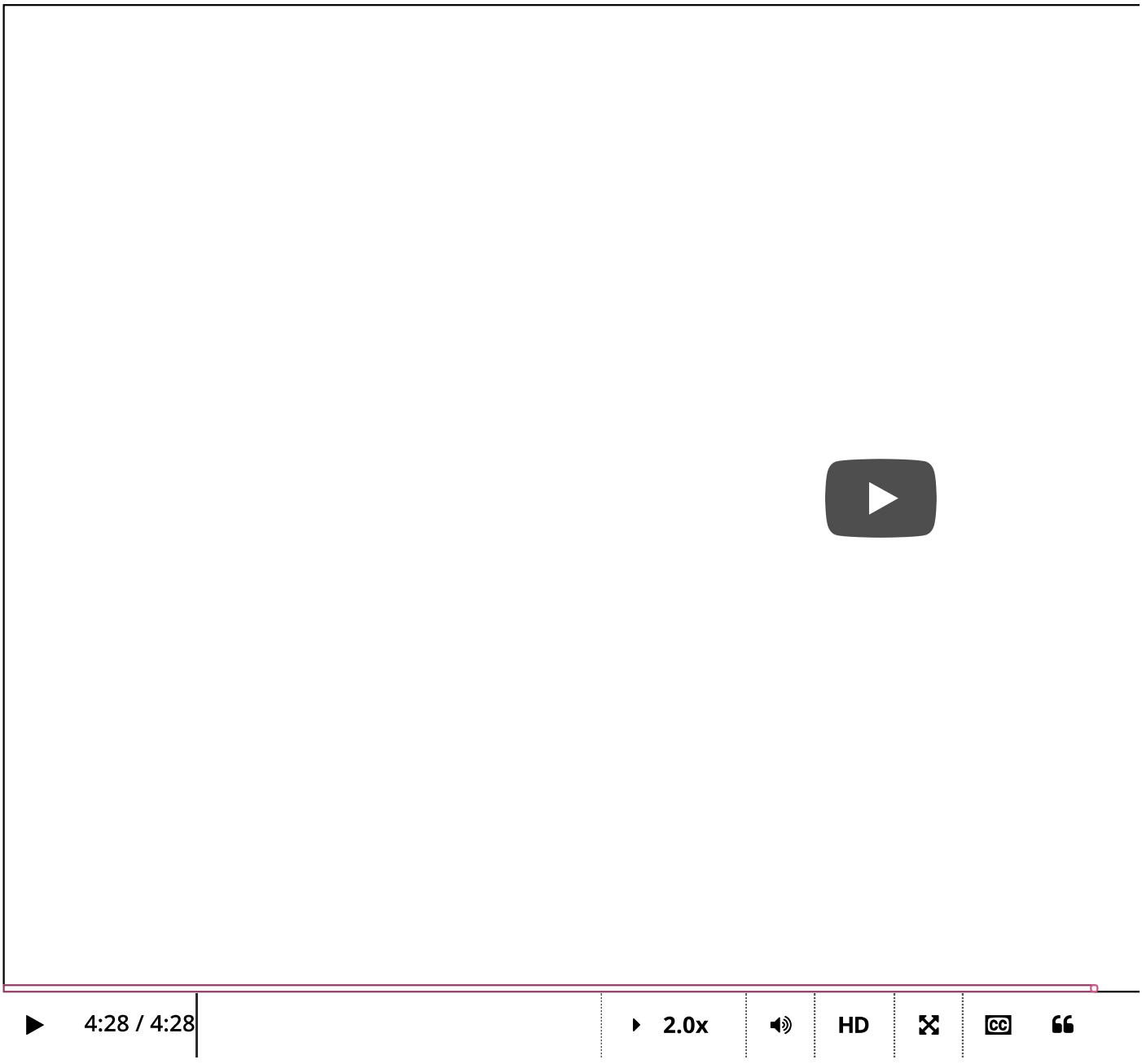
5. Numerically solving higher order ODEs
Passing to a first order system



Problem 2 (External resource) (1.0 points possible)

$$\ddot{x} + 2\dot{x} + x = 0, \quad x(0) = 0, \dot{x}(0) = 1,$$

on the interval $t \in [0, 5]$. You can verify that the analytic solution of this IVP is $x(t) = te^{-t}$. Remember that in order to use `ODE45`, you will first need to express this second-order DE as a system of two first-order DEs in the form $\dot{\mathbf{x}} = \mathbf{A}\mathbf{x}$. To complete the script, you must:

1. Correctly define the variables `x0`, `tspan`, and `A`.
2. Enter an appropriate expression for the first argument of `ODE45`.
3. Define a column vector `xTrue` that contains the values of the analytic solution at the times in the vector `t` returned by `ODE45`. (Hint: You will need to use the `.*` operator to perform element-wise multiplication. You can review this in Recitation 4: Vector and Matrix Operations.)

As in problem 1, if your script runs correctly you will see a plot comparing the analytic and numerical solutions, a plot showing the relative error between the two solutions, and the time it took `ODE45` to compute the numerical solution. In the first plot it is easy to see that `ODE45` is not using a constant time step. It begins by using a small time step but then increases the time step to reduce the number of computations while keeping the error bounded.

Your Script

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```
%Numerically solve DE and time how long it takes
x0 = [0;1];
tspan = [0,5];
A = [0,1;-1,-2];
tic;
[t,x] = ode45(@(t,x) [x(2);-x(1)-2*x(2)],tspan,x0);
timeElapsed = toc;
disp(['It took ODE45 ',num2str(timeElapsed,3), ' seconds to compute the solution'

%Enter analytic solution (Hint: it is in the text above.)
xTrue = t.*exp(-t);

%Plot results
%Do not edit the code below.
figure(1)
plot(t,x(:,1),'bo','markersize',10); hold on;
plot(t,xTrue,'r','linewidth',3);
legend('Numerical Solution','Exact Solution','location','northeast');
xlabel('$t$','interpreter','latex'); ylabel('$x(t)$','interpreter','latex')
title('Comparison of Solutions','interpreter','latex')
set(gca,'fontsize',25)

figure(2)
```

```
23 figure(2)
24 plot(t,abs(x(:,1)-xTrue)./xTrue,'r','linewidth',3);
25 xlabel('$t$','interpreter','latex'); ylabel('$|x(t)-x_{true}|/x_{true}$','interpr
26 title('Relative Error','interpreter','latex')
27 set(gca,'fontsize',25)
28
```

▶ Run Script ()

Assessment: Correct

Submit ()

- Correct definition of x0
- Correct definition of tspan
- Correct definition of A
- Correct definition of xTrue
- Correct numerical solution

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