MATHEMATICS

# Irrational numbers and Borel Sets

Asked 6 years, 9 months ago Active 6 years, 9 months ago Viewed 4k times

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Is the set of all irrational numbers in [0; 1] a Borel set? If yes, what is its Lebesgue measure? I have been trying to answer this for a long time now. I know that the set of rational numbers is infinitely countable but I am having trouble with the proof of this question. I am not necessarily looking for an answer just confirmation that I am doing the correct thing.

I have already tried the following: The set of rationals is countable between 0 and 1 and is therefore a Borel Set. Since it is countable its lebesgue measure is 0

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calculus probability

edited Oct 15 '13 at 12:55

asked Oct 15 '13 at 12:42

Gordon Wilson
**119** 2 4

1 Your question would be better if you included some of the things you have tried when you were thinking about the problem. That would help others give more focused answers. – Carl Mummert Oct 15 '13 at 12:46

## 2 Answers

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The set of all rational numbers in $[0, 1]$ is countable and hence a Borel set. Therefore, also its complement is a Borel set.

The Lebesgue measure of $[0, 1]$ is 1, the lebesgue measure of all rational numbers in $[0, 1]$ is 0 since it is countable. Therefore,...

answered Oct 15 '13 at 12:46

Stefan Hansen
**22.8k** 7 45 75

-1

If you have thought about this, then I am sure you have the ability to answer it already! First, is the set of rationals a Borel set? What is the Lebesgue measure of the set of rationals?

answered Oct 15 '13 at 12:45

Carl Mummert
**72.5k** 8 148 270

The set of rationals is a Borel set, so its complement is also a Borel set, correct? The lebesgue measure of the rationals in (0,1) is 0 so the lebesgue measure for the irrationals is also O, correct? – Gordon Wilson Oct 15 '13 at 12:53

Gordon Wilson Oct [illegible] at 12:[illegible]

Could you explain your reasoning in the second sentence of your comment? – Carl Mummert Oct 15 '13 at 12:53