Exercises: Overall Model Test

Computing the critical value

12/12 points (ungraded)

Assume we have have m observations in the vector y, and n number of unknown parameters in the vector x. The observables are normally distributed and the linear system of observation-equations has been given as

$$\mathrm{E}\{\underline{y}\} = Ax, \quad \mathrm{D}\{\underline{y}\} = Q_{yy}.$$

The BLU estimate of x has been given as $\hat{x}$, resulting in the BLU estimate of residual vector as

$$\hat{e} = y - A\hat{x}.$$

For each of the following situation, compute the critical value for the overall model test, and determine whether the test is accepted or not.

Case 1:

$m = 20$, $n = 2$, $\alpha = 0.05$, and $\hat{e}^T Q_{yy}^{-1} \hat{e} = 30$.

What is the OMT critical value K_α (upto 2 decimal places)?

28.87

✔ **Answer:** 28.869

28.87

Is the OMT accepted or not?

- Accepted
- Rejected ✔

Answer
Correct: $30 < k_\alpha = 28.869$, so the test is rejected

Case 2:
$m = 82$, $n = 2$, $\alpha = 0.05$, and $\hat{e}^T Q_{yy}^{-1} \hat{e} = 100$.

What is the OMT critical value K_α (upto 2 decimal places)?

101.88 ✔ **Answer:** 101.88

101.88

Is the OMT accepted or not?

- Accepted ✔
- Rejected

Answer
Correct: $100 \leq k_\alpha = 101.88$, so the test is accepted

Case 3:

$m = 20$, $n = 3$, $\alpha = 0.05$, and $\hat{e}^T Q_{yy}^{-1} \hat{e} = 25$.

What is the OMT critical value K_α (upto 2 decimal places)?

27.59 ✔ **Answer:** 27.587

27.59

Is the OMT accepted or not?

- (•) Accepted ✔
- ( ) Rejected

Answer
Correct: $25 \leq k_\alpha = 27.587$, so the test is accepted

Case 4:

$m = 20$, $n = 10$, $\alpha = 0.05$, and $\hat{e}^T Q_{yy}^{-1} \hat{e} = 15$.

What is the OMT critical value K_α (upto 2 decimal places)?

18.31 ✔ **Answer:** 18.307

18.31

Is the OMT accepted or not?

- (•) Accepted ✔
- ( ) Rejected

Answer
Correct: $15 \leq k_\alpha = 18.31$, so the test is accepted

Case 5:
$m = 20$, $n = 2$, $\alpha = 0.001$, and $\hat{e}^T Q_{yy}^{-1} \hat{e} = 44$.

What is the OMT critical value K_α (upto 2 decimal places)?

42.31 ✔ **Answer:** 42.312

42.31

Is the OMT accepted or not?

- ( ) Accepted
- (•) Rejected ✔

Answer
Correct: $44 \lneq k_\alpha = 42.31$, so the test is rejected

Case 6:
$m = 20$, $n = 2$, $\alpha = 0.05$, $Q_{yy} = 4I_m$ (I_m is the identity matrix) and $\hat{e}^T\hat{e} = 108$.

What is the OMT critical value K_α (upto 2 decimal places)?

28.87 ✔ **Answer:** 28.869

28.87

Is the OMT accepted or not?

- (•) Accepted ✔
- ( ) Rejected

Answer
Correct: $T = \frac{108}{4} = 27$, so $T \leq k_\alpha = 28.87$. The test is accepted

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✔ Correct (12/12 points)

Properties of overall model test

4/4 points (ungraded)

For each of the following statements, determine whether it is true or false.

1. The overall model test statistics $\underline{T} = \underline{\hat{e}}^{\mathrm{T}} Q_{yy}^{-1} \underline{\hat{e}}$ follows the χ^2-distribution, regardless of what the distribution of observations is.

- True
- False ✔

Answer

Correct:

The OMT test statistics follows the χ^2-distribution only when the observations are normally distributed.

2. The larger the value of the level of significance α, the higher the chance to falsely accept the overall model test.

- True
- False ✔

Answer

Correct:
α is the probability of false rejection of the test. The larger the α, the higher the chance to falsely reject the test, and the lower the chance to falsely accept the test.

3. Assuming the same level of significance α and the same functional model, a larger number of observations results in a smaller crtical value.

- True
- False ✔

Answer
Correct: More observations result in more degrees of freedom, and so a larger critical value.

4. Assuming the same level of significance α and the same number of observations m, a functional model with fewer unknowns has a lower chance to be falsely rejected with the overall model test.

- True
- False ✔

Answer
Correct: The probability of false rejection is α which is constant in this case.

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✔ Correct (4/4 points)