9. Why linearization is useful

Lecture due Sep 15, 2021 20:30 IST

Discuss

Sensitivity Analysis

As in the robot arm example, sometimes the variables you *can* control have a non-straightforward effect on the variables you *wish to* control. Linearization gives you a back-of-the-envelope method for saying what happens to the outputs for various changes in the inputs. For example, suppose we have a relationship $x, y \implies A, B$, and we compute the linearization of it near (x_0, y_0) and we get $\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$. Then we can immediately answer questions such as:

1. If x increases by ≈ 0.1 then what happens?

   Solution: Assuming y stays the same, we would see A and B both increase by ≈ 0.1. This is because $\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 0.1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0.1 \\ 0.1 \end{pmatrix}$.

2. If y decreases by ≈ 0.1 then what happens?

   Solution: Assuming x stays the same, we would see A stay about the same and B would decrease by ≈ 0.1. This is because $\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 0 \\ -0.1 \end{pmatrix} = \begin{pmatrix} 0 \\ -0.1 \end{pmatrix}$.

The analysis performed above is called a “sensitivity analysis." It tells us exactly how sensitive the output variables are to changes in each input variable near the point (x_0, y_0). In the above example, we can say that A was only sensitive to changes in x, but B was sensitive to changes in both x and y. We can also solve the "inverse question", such as:

What would it take to increase A by ≈ 0.1 while holding B the same?

Solution: This time we know what the output of $\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$ should be, $\begin{pmatrix} 0.1 \\ 0 \end{pmatrix}$. Now we can use our tools for solving for $\vec{u}$ in $\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}\vec{u} = \begin{pmatrix} 0.1 \\ 0 \end{pmatrix}$. Using elimination, or the inverse matrix, we see that $\vec{u} = \begin{pmatrix} 0.1 \\ -0.1 \end{pmatrix}$. Therefore, we should increase x by 0.1 and decrease y by 0.1 to produce the desired effect.

Practice Sensitivity

2/2 points (graded)
What would happen if we decrease x by 0.1 and increase y by 0.1? Use the linearization to answer.

A would increase by -0.1 ✔ **Answer:** -0.1

B would increase by 0 ✔ **Answer:** 0

Solution:

A would increase by -0.1 and B would increase by 0. This is because

$$\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} -0.1 \\ 0.1 \end{pmatrix} = \begin{pmatrix} -0.1 \\ 0 \end{pmatrix} \qquad (5.142)$$

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Answers are displayed within the problem

Practice Sensitivity 2

2/2 points (graded)
What would it take to increase A by 0.1 and decrease B by 0.1?

Increase x by 0.1 ✔ **Answer:** 0.1

Increase y by -0.2 ✔ **Answer:** -0.2

Solution:

Increase x by 0.1 and decrease y by 0.2. This is because

$$\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} \vec{u} = \begin{pmatrix} 0.1 \\ -0.1 \end{pmatrix} \tag{5.143}$$

has the solution $\vec{u} = \begin{pmatrix} 0.1 \\ -0.2 \end{pmatrix}$.

Submit You have used 3 of 7 attempts

Answers are displayed within the problem

Knowing the linearization can give further insight into what is going on near the base point. Suppose we are studying variables A and B that depend on the variables x and y as

$$A = (x+y-2)^2 - y \tag{5.144}$$

$$B = x - y^2 \tag{5.145}$$

Just looking at the formula, it's hard to get a feeling for what this transformation is doing. Suppose we need to know the behavior of this transformation near the point $(0, 2)$.

In search of understanding, we might try computing the linearization at that point. It turns out to be $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$.

You may recognize this matrix, because it is the matrix that rotates a vector by $\pi/2$ counter-clockwise. This tells us that near the point $(2, 0)$ the vector $\begin{pmatrix} \Delta A \\ \Delta B \end{pmatrix}$ is obtained from the vector $\begin{pmatrix} \Delta x \\ \Delta y \end{pmatrix}$ by this counter-clockwise rotation. This gives quite a bit of insight into what is going on near that point.

In summary, linearization is useful because we often have an easier time analyzing the behavior of a matrix (linear functions) rather than the behavior of a more complicated function.

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