



MITx: 6.041x Introduction to Probability - The Science of Uncertainty



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Exercise: Estimator properties

(4/4 points)

We estimate the unknown mean θ of a random variable X (where X has a finite and positive variance) by forming the sample mean $M_n = (X_1 + \cdots + X_n)/n$ of n i.i.d. samples X_i and then forming the estimator

$$\hat{\Theta} = M_n + \frac{1}{n}.$$

Is this estimator unbiased?

No ▼



Answer: No

Is this estimator consistent?

Yes ▼




Answer: Yes


- ▶ Unit 6: Further topics on random variables
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Unit overview


Lec. 18: Inequalities, convergence, and the Weak Law of Large Numbers

Exercises 18 due Apr 27, 2016 at 23:59 UTC 

Lec. 19: The Central Limit Theorem (CLT)

Exercises 19 due Apr 27, 2016 at 23:59 UTC 

Lec. 20: An introduction to classical statistics

Exercises 20 due Apr 27, 2016 at 23:59 UTC 

Consider now a different estimator, $\hat{\Theta}_n = X_1$, which ignores all but the first measurement.

Is this estimator unbiased?

Yes ▼



Answer: Yes

Is this estimator consistent?

No ▼



Answer: No


Answer:

We have $\mathbf{E}[\hat{\Theta}_n] = \theta + (1/n) \neq \theta$, so it is not unbiased. On the other hand, M_n converges (in probability) to θ , and $1/n$ converges to zero. So, their sum, $\hat{\Theta}_n = M_n + (1/n)$ also converges (in probability) to θ , and the estimator is consistent.

The second estimator is unbiased, because $\mathbf{E}[\hat{\Theta}_n] = \mathbf{E}[X_1] = \theta$. But it is not consistent. Its value stays the same (equal to X_1) for all n and therefore cannot converge to θ , unless X_1 is guaranteed to be equal to θ . But this is impossible since X has positive variance.

You have used 1 of 1 submissions

[Solved problems](#)[Additional theoretical material](#)[Problem Set 8](#)

Problem Set 8 due Apr 27, 2016
at 23:59 UTC 

[Unit summary](#)

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