MITx: 6.041x Introduction to Probability - The Science of Uncertainty

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Exercise: Markov property

(2/3 points)

For each of the following definitions of the state X_n at time n ($n = 1, 2, \ldots$), determine whether the Markov property is satisfied.

1. X_n is a sequence of independent discrete random variables.

   Yes ▾ ✔ **Answer:** Yes

2. You have m distinct boxes, numbered 1 through m, each containing some tokens. On each token is written an integer from 1 to m. Each box contains at least one token, but different boxes may contain different numbers of tokens. A box may also contain multiple tokens with the same number. Assume that you know the distribution of tokens in each box.

   At time 0, you pick one box at random, say box i. You pick one of the tokens in box i randomly (each token in the box is equally likely to be chosen), read the corresponding number (say j), and put the token back in box i. At the next time slot, you pick one of the tokens in box j randomly (each token in the box is equally likely to be chosen) and repeat this process forever. At time n, you will be choosing tokens from some box. Let X_n be the number of this box.

No ▼ ✖ **Answer:** Yes

3. Alice and Bob take turns tossing a fair coin. Assume that tosses are independent. Whenever the result is Heads, Alice gives 1 dollar to Bob, and whenever it is Tails, Bob gives 1 dollar to Alice. Alice starts with A dollars and Bob starts with B dollars, for some positive integers A and B. They keep playing until one player goes broke. Let X_n be the amount of money that Alice has after the nth toss.

Yes ▼ ✔ **Answer:** Yes

Answer:

1. Yes. Trivially, the independence assumption ensures that we have

$$\mathbf{P}(X_{n+1} = j \mid X_n = i,\ X_{n-1} = i_{n-1}, \ldots, X_0 = i_0) = \mathbf{P}(X_{n+1} = j).$$

In this case, not only does the past not matter, but the present (X_n) also has no influence.

2.

Yes. The set of states is $\{1, 2, \ldots, m\}$. Given that $X_n = i$, the probabilities of what the next box will be at time $n+1$ depend only on the tokens in box i and not on the past values of $X_0, \ldots, X_{n-1}$ (i.e., the past boxes), and so the Markov property is satisfied.

3. Yes. The set of states is $\{0, 1, 2, 3, \ldots, A+B\}$. If $X_n = 0$ or $X_n = A+B$, the game stops (i.e., we stay in state 0 or $A+B$, respectively). If $1 \leq X_n \leq A+B-1$, then the value of X_n is the only knowledge needed in order to describe the probabilities of what X_{n+1} will be next: it will be $X_n + 1$ with probability $1/2$ and $X_n - 1$ with probability $1/2$. Hence, the Markov property is satisfied.

You have used 1 of 1 submissions