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1. Lecture 3

The following can be done after Lecture 3.

3-1

5/5 points (graded)

Each matrix below is the augmented matrix for a system of linear equations. For which matrices is the system consistent?

- [ ] $\left(\begin{array}{cccc|c} 0 & 1 & 2 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$
- [ ] $\left(\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 8 \end{array}\right)$
- [x] $\left(\begin{array}{cccc|c} 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 3 & 3 & 1 \\ 0 & 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right)$ ✔
- [x] $\left(\begin{array}{cccc|c} 1 & 0 & 0 & 2 & 8 \\ 0 & 1 & 0 & 5 & 5 \\ 0 & 0 & 1 & 6 & 6 \\ 0 & 1 & 0 & 3 & 1 \end{array}\right)$ ✔

$$\left(\begin{array}{cccc|c} 0 & 1 & 1 & 4 & 0 \\ 0 & 0 & 1 & 5 & 0 \\ 0 & 0 & 0 & 0 & 5 \end{array}\right)$$

Solution:

Both $\left(\begin{array}{cccc|c} 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 3 & 3 & 1 \\ 0 & 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right)$ and $\left(\begin{array}{cccc|c} 1 & 0 & 0 & 2 & 8 \\ 0 & 1 & 0 & 5 & 5 \\ 0 & 0 & 1 & 6 & 6 \\ 0 & 1 & 0 & 3 & 1 \end{array}\right)$ are consistent.

To check consistency, put the matrix in row-echelon form; then the system is inconsistent if and only if there is a pivot in the augmented column (the last column).

Matrix $\left(\begin{array}{cccc|c} 0 & 1 & 2 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$ is in row-echelon form already, and has a pivot in the lower right corner, so the system is inconsistent.

Matrix $\left(\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 8 \end{array}\right)$ is in row-echelon form, and has a pivot in the lower right corner, so the system is inconsistent.

Matrix $\left(\begin{array}{cccc|c} 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 3 & 3 & 1 \\ 0 & 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right)$ is not in row-echelon form, but is so after interchanging rows to move the all-zero row to the bottom: $\left(\begin{array}{cccc|c} 0 & 1 & 3 & 3 & 1 \\ 0 & 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$. There is no pivot in the last column of the resulting matrix (the 1 in that last column is not the pivot of its row), so the system is consistent.

Matrix $\left(\begin{array}{cccc|c} 1 & 0 & 0 & 2 & 8 \\ 0 & 1 & 0 & 5 & 5 \\ 0 & 0 & 1 & 6 & 6 \\ 0 & 1 & 0 & 3 & 1 \end{array}\right)$ is not in row-echelon form, but is so after adding -1 times the second row to the fourth row: $\left(\begin{array}{cccc|c} 1 & 0 & 0 & 2 & 8 \\ 0 & 1 & 0 & 5 & 5 \\ 0 & 0 & 1 & 6 & 6 \\ 0 & 0 & 0 & -2 & -4 \end{array}\right)$. The resulting matrix has no pivots in the augmented column, hence is consistent.

Matrix $\left(\begin{array}{cccc|c} 0 & 1 & 1 & 4 & 0 \\ 0 & 0 & 1 & 5 & 0 \\ 0 & 0 & 0 & 0 & 5 \end{array}\right)$ is in row-echelon form, and has a pivot in the last column (the last 5), so the system is inconsistent.

Submit You have used 1 of 5 attempts

Answers are displayed within the problem

3-2

1/1 point (graded)

Which vector subspaces do row reduction operations preserve? (Check all that apply.)

- [ ] Column Space
- [x] Nullspace ✔

✔

Solution:

Row reduction operations preserve the nullspace of a matrix, which is why when we are finding bases for it, we take the basis of the nullspace of the row echelon form of the matrix. However, row reduction operations do not preserve the column space of a matrix and for this reason we have to go back to the original matrix to find the basis for its column space.

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Answers are displayed within the problem

3-3

1/1 point (graded)

A 3×4 matrix has rank $2.$ What are all the possible values for its nullity (the dimension of its nullspace). (Check all that apply.)

- [ ] 0
- [ ] 1
- [x] 2 ✔
- [ ] 3
- [ ] 4

✔

Solution:

The rank-nullity theorem gives

$$\begin{aligned} \textbf{rank} + \dim(NS) &= \#\,\text{columns} \\ 2 + \dim(NS) &= 4 \\ \dim(NS) &= 2. \end{aligned}$$

Submit

You have used 1 of 3 attempts

Answers are displayed within the problem

3-4

1/1 point (graded)

Let $\mathbf{M} = \begin{pmatrix} 1 & a & b \\ 0 & 1 & c \\ 0 & 0 & 1 \end{pmatrix}$. What is the upper right corner entry of $\mathbf{M}^{-1}$?

(Express your answer in terms of a, b and c.)

-b+c*a

✔ **Answer:** a*c-b

$-b + c \cdot a$

FORMULA INPUT HELP

Solution:

To find the inverse, apply Gauss–Jordan elimination to $\left(\begin{array}{c|c} \mathbf{M} & \mathbf{I} \end{array}\right)$:

$$\left(\begin{array}{ccc|ccc} 1 & a & b & 1 & 0 & 0 \\ 0 & 1 & c & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right) \to \left(\begin{array}{ccc|ccc} 1 & 0 & b-ac & 1 & -a & 0 \\ 0 & 1 & c & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right) \to \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -a & ac-b \\ 0 & 1 & 0 & 0 & 1 & -c \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right),$$

so the inverse is $\begin{pmatrix} 1 & -a & ac-b \\ 0 & 1 & -c \\ 0 & 0 & 1 \end{pmatrix}$. The upper right entry is $ac-b$.

Submit You have used 1 of 5 attempts

Answers are displayed within the problem

3-5

10.0/10.0 points (graded)

There is a vector $\mathbf{v}$ in the column space of $\begin{pmatrix} 1 & 2 \\ 1 & 3 \\ 5 & 6 \\ 2 & 7 \end{pmatrix}$ whose last two coordinates are 3 and -8. Find the first coordinate of $\mathbf{v}$.

-1

Answer: -1

Solution:

The answer is -1.

To say that $\mathbf{v}$ is in the column space means that $\mathbf{v}$ is a linear combination of the columns:

$$\mathbf{v} = x\begin{pmatrix} 1 \\ 1 \\ 5 \\ 2 \end{pmatrix} + y\begin{pmatrix} 2 \\ 3 \\ 6 \\ 7 \end{pmatrix}$$

for some numbers x and y. For this to have the specified last two coordinates, we must have

$$\begin{aligned} 5x + 6y &= 3 \\ 2x + 7y &= -8. \end{aligned}$$

Solving this system leads to $x = 3$ and $y = -2$, so

$$\mathbf{v} = 3\begin{pmatrix} 1 \\ 1 \\ 5 \\ 2 \end{pmatrix} - 2\begin{pmatrix} 2 \\ 3 \\ 6 \\ 7 \end{pmatrix} = \begin{pmatrix} -1 \\ -3 \\ 3 \\ -8 \end{pmatrix}.$$

Its first coordinate is -1.

Submit You have used 2 of 5 attempts

Answers are displayed within the problem

3-6

10.0/10.0 points (graded)

What is the dimension of the span of the vectors $\begin{pmatrix} 0 \\ -1 \\ -2 \end{pmatrix}, \begin{pmatrix} 3 \\ 2 \\ 7 \end{pmatrix}, \begin{pmatrix} -2 \\ 3 \\ 4 \end{pmatrix}, \begin{pmatrix} 5 \\ 4 \\ 13 \end{pmatrix}$?

2

✔ **Answer:** 2

2

Solution:

The answer is 2.

The span of these vectors is the same as the column space of

$$\begin{pmatrix} 0 & 3 & -2 & 5 \\ -1 & 2 & 3 & 4 \\ -2 & 7 & 4 & 13 \end{pmatrix}.$$

To find a basis, put this matrix in row-echelon form. First, interchange the first two rows:

$$\begin{pmatrix} -1 & 2 & 3 & 4 \\ 0 & 3 & -2 & 5 \\ -2 & 7 & 4 & 13 \end{pmatrix}.$$

Next add -2 times the first row to the third row:

$$\begin{pmatrix} -1 & 2 & 3 & 4 \\ 0 & 3 & -2 & 5 \\ 0 & 3 & -2 & 5 \end{pmatrix}.$$

Finally, add -1 times the second row to the third row:

$$\begin{pmatrix} -1 & 2 & 3 & 4 \\ 0 & 3 & -2 & 5 \\ 0 & 0 & 0 & 0 \end{pmatrix}.$$

The pivot columns are the first two columns, so the first two columns of the original matrix form a basis for the original column space. Thus the dimension is 2.

Submit You have used 1 of 2 attempts

Answers are displayed within the problem

3-7

15.0/15.0 points (graded)

Evaluate $\det \begin{pmatrix} 1 & 8 & 0 & 3 \\ 0 & 4 & 2 & 1 \\ 0 & 0 & 3 & 7 \\ 0 & 0 & 0 & -3 \end{pmatrix}$.

-36 **Answer:** -36

-36

Solution:

The answer is -36.

The matrix is upper triangular, so the determinant is the product of the diagonal entries: $(1)(4)(3)(-3) = -36$.

Submit

Answers are displayed within the problem

3-8

5.0/5.0 points (graded)

Check all the statements below that are true for the matrix

$$\mathbf{A} := \begin{pmatrix} 1 & 2 & 3 \\ 50 & 60 & 70 \\ 51 & 62 & 73 \end{pmatrix}.$$

(Hint: row operations do not change whether $\det \mathbf{A} = 0$.)

- [x] $\det \mathbf{A} = 0$ ✔
- [x] $\mathbf{Ax} = \mathbf{0}$ has a nonzero solution. ✔
- [x] $\mathrm{rank}(\mathbf{A}) < 3$ ✔
- [ ] $\mathrm{CS}(\mathbf{A}) = \mathbb{R}^3$
- [ ] For each vector $\mathbf{b}$, the system $\mathbf{Ax} = \mathbf{b}$ has exactly one solution.
- [ ] $\mathbf{A}^{-1}$ exists.
- [ ] The reduced row-echelon form $\mathtt{rref}(\mathbf{A})$ of $\mathbf{A}$ equals the identity matrix $\mathbf{I}$.

✔

Solution:

The answers are that $\det \mathbf{A} = 0$, $\mathbf{Ax} = \mathbf{0}$ has a nonzero solution, and $\mathrm{rank}(\mathbf{A}) < 3$.

Adding -1 times the first row to the third, and then adding -1 times the second row to the third makes the third row all 0, so the determinant becomes 0. Thus the determinant must have been 0 to begin with. Thus $\det \mathbf{A} = 0$ is true.

Since $\det \mathbf{A} = 0$, some dimensions must be getting crushed, so $\mathbf{Ax} = \mathbf{0}$ has a nonzero solution. Thus the second option is true. Also, since some of the $\mathbf{3}$ input dimensions are crushed, the image is less than $\mathbf{3}$-dimensional, so $\mathbf{rank}(\mathbf{A}) < \mathbf{3}$, i.e., the third option is true.

The image $\mathrm{CS}(\mathbf{A})$ has dimension less than $\mathbf{3}$, so $\mathrm{CS}(\mathbf{A}) \neq \mathbb{R}^{\mathbf{3}}$. Thus the forth option is false. If $\mathbf{b}$ is a vector of $\mathbb{R}^{\mathbf{3}}$ that is not in $\mathrm{CS}(\mathbf{A})$, then $\mathbf{Ax} = \mathbf{b}$ has no solution, so the fifth option is false. Since $\det \mathbf{A} = 0$, the inverse $\mathbf{A}^{-1}$ does not exist. Thus the sixth option is false. The matrix $\mathtt{rref}(\mathbf{A})$ is obtained from $\mathbf{A}$ by row operations, so its determinant is 0 too, so $\mathtt{rref}(\mathbf{A})$ cannot be $\mathbf{I}$. Thus the last option is false.

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Answers are displayed within the problem

1. Lecture 3

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