12.3.3 Diagonalization, Again

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Week 12 due Dec 29, 2023 10:42 IST Completed

12.3.3 Diagonalization, Again

Video

Start of transcript. Skip to the end.

Dr. Robert van de Geijn: A square matrix A that's n by n,

can only be diagonalized if A has n linearly independent eigenvectors.

Often when you try to prove an if and only if, you prove it in one direction

first, and then in the other direction.

So what we're going to start with is,

0:00 / 0:00 2.0x

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Reading Assignment

0 points possible (ungraded)

Read Unit 12.3.3 of the notes. [LINK]

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Calculator

Homework 12.3.3.1

10.0/10.0 points (graded)

Consider $A = \begin{pmatrix} 2 & 1 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 2 \end{pmatrix}$.

- The algebraic multiplicity of $\lambda = 2$ is

  4 ✔ Answer: 4

  4.

- The geometric multiplicity of $\lambda = 2$ is

  2 ✔ Answer: 2

  2.

- The following vectors are linearly independent eigenvectors associated with $\lambda = 2$:

$$\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}.$$

  TRUE ✔ Answer: TRUE

  Just multiply out.

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Homework 12.3.3.2

1/1 point (graded)

Let $A \in \mathbb{A}^{n \times n}$, $\lambda \in \Lambda(A)$, and S be the set of all vectors x such that $Ax = \lambda x$. Finally, let λ have algebraic multiplicity k (meaning that it is a root of multiplicity k of the characteristic polynomial).

The dimension of S is k ($\dim(S) = k$).

Sometimes ✔ Answer: Sometimes

An example of where it is true:

$$A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

This matrix has 1 as its only eigenvalue, and it has algebraic multiplicity two. But both $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ are corresponding eigenvectors and hence $S = \mathbb{R}^2$, which has dimension 2.

An example of where it is false:

Calculator

$$A = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$$

This matrix has 1 as its only eigenvalue, and it has algebraic multiplicity two. Now, to find the eigenvectors we consider

$$\begin{pmatrix} 1-1 & 1 \\ 0 & 1-1 \end{pmatrix} = \begin{pmatrix} 0 & \boxed{1} \\ 0 & 0 \end{pmatrix}.$$

It is in row echelon form and has one pivot. Hence, the dimension of its null space is $2 - 1 = 1$. Since S is this null space, its dimension equals one.

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