Sea level example

In this lecture we will look at two examples based on sea level observations for which we will step-by-step set-up the functional model.

This figure shows 10 years of monthly tide gauge observations in Ijmuiden, the Netherlands. From the graph you may be able to tell that the sea level rise seems to follow a linear trend - and in addition to that there is an annual oscillation – but we will ignore this for the moment.

Our goal is thus to set-up the functional model where we consider only the linear trend of sea level rise for now, as indicated by the green line. As a first step we need to identify the observations, the known and the unknown parameters.

Obviously, the observations are the tide gauge measurements, which are taken once a month.

The times at which these observations are taken are treated as known, and therefore deterministic parameters, and they are expressed in years. What are then the unknown parameters for this linear trend model?

Firstly, one unknown is the initial sea level at t_0, as indicated along the vertical axis. Secondly, the parameter that we are really interested in - namely, the change of sea level delta_L over a certain time interval.

We will call this the rate of change r and it is expressed in millimeters per year.

We now have all the ingredients we need to set-up the observation equations, which we will derive first for a single observation at time t_i.

The expected value of this observable will be equal to the initial sea level l_0, plus the rate of change r, multiplied by the time interval between t_i and t_0.

As we have monthly observations, for the first observation the time interval will therefore be: one-twelfth, since our time is in units of 1 year. For the second observation it is two-twelfth, and so on and so forth.

The system of m observation equations becomes as follows: where on each row we recognise the observation equations for time t_i.

For instance here we highlighted the equation for the second observation.

This system can be re-written as a matrix-vector product separating the knowns and unknowns as shown on the right-hand side of the equation.

So, this is our final functional model, where we distinguish the expectation of the vector of observables, the design matrix A and the vector x with unknown parameters.

As a second example, assume now we would again have ten years of monthly tide gauge observations, but now in the absence of a linear trend in sea level rise. Here we are only interested in the annual signal, modeled by a sine as shown by the green line.

Compared to the previous example, an additional known parameter is now the period T, since we are looking for an annual signal. Hence, capital T will be equal to 1 year. The unknown parameters are then again the sea level at t_0, which we will call l_0

plus the amplitude a of the annual signal indicated on the left-hand side of the graph.

The observation equation for this model becomes the sum of the unknown initial sea level l0 plus the sine-term multiplied with the unknown amplitude.

Since the period T is equal to 1 the observation equation simplifies to this one, based on which the functional model follows as this system of equations, where each row is equal the observation equation shown above with corresponding time interval.

This can also be written as a matrix-vector product as shown on the right-hand side such that we again recognise the different constituents of the functional model

You will now derive the functional model where you will be considering the combination of a linear trend and an annual signal as shown with the green graph in this plot. And this is one of the exercises.

Now that we know how the functional model is defined and can be set-up for a particular estimation problem, in the following lecture we will look at specific properties of the functional model.