# 9. Worked example

Bookmark this page

**Reflect**

**Example 9.1**

Maximize the function $f(x,y) = x^2 + y^2$ along the curve $x^2y + y = 4$.

The Lagrange multiplier method tells us that to maximize $f$ along the level curve $g(x,y) = 4$, we must find the locations where $\nabla f = \lambda \nabla g$. Here $g(x,y) = x^2y + y$. The system becomes

$$2x = \lambda 2xy \quad (4.181)$$

$$2y = \lambda (x^2 + 1) \quad (4.182)$$

If $x \neq 0$, the first equation becomes $1 = \lambda y$. Plugging in $1/y$ for $\lambda$ in the second equation gives

$$2y = (x^2 + 1)/y \quad (4.183)$$

$$2y^2 = x^2 + 1 \quad (4.184)$$

Plugging into our constraint curve, we get

$$g(x,y) = x^2y + y = (x^2+1)y \quad (4.185)$$

$$= (2y^2)y = 4 \quad (4.186)$$

$$y^3 = 2 \quad (4.187)$$

$$y = 2^{1/3} \quad (4.188)$$

Solving for $x$ this gives

$$(x^2+1)2^{1/3} = 4 \quad (4.189)$$

$$x = \pm\sqrt{2^{5/3} - 1} \quad (4.190)$$

Thus we find two points on the boundary $(\pm\sqrt{2^{5/3}-1}, 2^{1/3})$.

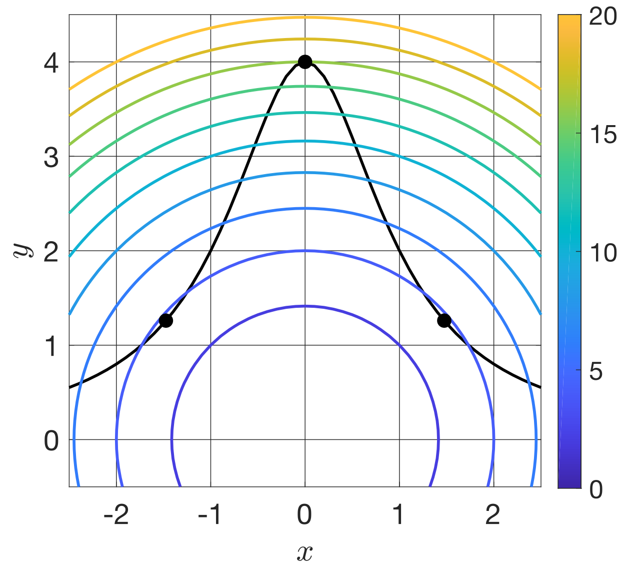
Note that we need to consider the case $x = 0$. In this case, $y = 4$. Thus we must consider the value of the function at the three points $(\sqrt{2^{5/3}-1}, 2^{1/3})$, $(-\sqrt{2^{5/3}-1}, 2^{1/3})$, and $(0,4)$.

- $f(0,4) = 16$
- $$f(\sqrt{2^{5/3}-1}, 2^{1/3}) = \sqrt{2^{5/3}-1}^2 + (2^{1/3})^2 \quad (4.191)$$
  $$= 2^{5/3} - 1 + 2^{2/3} \approx 3.76 < 16 \quad (4.192)$$

  Therefore the minimum value occurs at the two points $(\pm\sqrt{2^{5/3}-1}, 2^{1/3})$.

  The three points are marked on the plot below, which shows the curve $x^2y + y = 4$ in black along with the level curves of $f(x,y) = x^2 + y^2$. Note that from the image, you can see that the two points $(\pm\sqrt{2^{5/3}-1}, 2^{1/3})$ are absolute minima, while the point $(0,4)$ is a local maximum.

Calculator Hide Notes




## 9. Worked example

**Topic:** Unit 3: Optimization / 9. Worked example

Hide Discussion

**Add a Post**

Show all posts | by recent activity

- What kind of constraint is g(x,y) — 5
- The case when y = 0 — 2
- Why is (0,4) not an absolute max? — 2
- How to Differentiate between Local Maximum and Minimum? — 3
  I get a little confused. Without drawing the graph, how can I differentiate between whether the point is local maxium or minum? Like ...
- Example 9.1 Constraint curve case x = 0 — 3

< Previous Next >



# edX

About
Affiliates
edX for Business
Open edX
Careers

Calculator Hide Notes

News

## Legal

Terms of Service & Honor Code
Privacy Policy
Accessibility Policy
Trademark Policy
Sitemap

## Connect

Blog
Contact Us
Help Center
Media Kit
Donate

GET IT ON Google play
Download on the App Store



Calculator Hide Notes