Exercise: Random Variables

(4/4 points)
Consider the following probability space:

```
prob_space = {'cat': 0.2, 'dog':0.7, 'shark':0.1}
```

Let's define a random variable X that maps 'cat' and 'dog' both to 5, and 'shark' to 7.

- What is the set of values that X can take on? Express your answer as a Python set.

set([5,7]) ✔ **Answer:** {5, 7}

- Express the probability table (also called the probability mass function) p_X of random variable X as a Python dictionary. (Your answer should be the Python dictionary itself, and *not* the dictionary assigned to a variable, so please do not include, for instance, "`prob_space =`" before specifying your answer. You can use fractions. If you use decimals instead, please be accurate and use at least 5 decimal places.)

{5: 0.9, 7: 0.1} ✔ **Answer:** {5: 0.9, 7: 0.1}

- Suppose we have a different random variable Y, where $Y = 5$ with probability 0.9 and $Y = 7$ with probability 0.1. Do X and Y have the same probability mass function, i.e., does $p_X(a) = p_Y(a)$ for $a \in \{5, 7\}$?

  - (•) Yes ✔
  - ( ) No

- Random variable Y does not have to be associated with the same underlying probability space as X.

  - (•) True ✔
  - ( ) False

Solution:

- What is the set of values that X can take on? Express your answer as a Python set.

  We see that the outcomes get relabeled to 5 and 7, i.e., the alphabet for X is **{5, 7}**.

- Express the probability table (also called the probability mass function) p_X of random variable X as a Python dictionary.

The probability that $X = 5$ corresponds to the event {cat, dog}, which has probability 0.2 + 0.7 = 0.9.

The probability that $X = 7$ corresponds to the event {shark}, which has probability 0.1.

Thus, expressing the PMF as a Python dictionary, we get: **{5: 0.9, 7: 0.1}**

- Suppose we have a different random variable Y, where $Y = 5$ with probability 0.9 and $Y = 7$ with probability 0.1. Do X and Y have the same probability mass function, i.e., does $p_X(a) = p_Y(a)$ for $a \in \{5, 7\}$?

  Yes: Just compare the answer to the previous part to the PMF for Y.

- Random variable Y does not have to be associated with the same underlying probability space as X.

  True: Just because random variables X and Y have the same probability distribution does not mean that they have to be associated with the same probability space.

  To give a different example that makes this more clear:

  Consider a probability space represented in Python by `{'heads': 1/2, 'tails': 1/2}` (a fair coin flip) and let random variable U map 'heads' to 1 and 'tails' to 0.

  Consider a different probability space represented in Python by `{1: 1/6, 2: 1/6, 3: 1/6, 4: 1/6, 5: 1/6, 6: 1/6}` (a fair six-sided die roll) and let random variable V map even rolls to 1 and odd rolls to 0.

  U and V have the same PMF but they are associated with different probability spaces.

You have used 1 of 5 submissions