4. Matrices and Rotation

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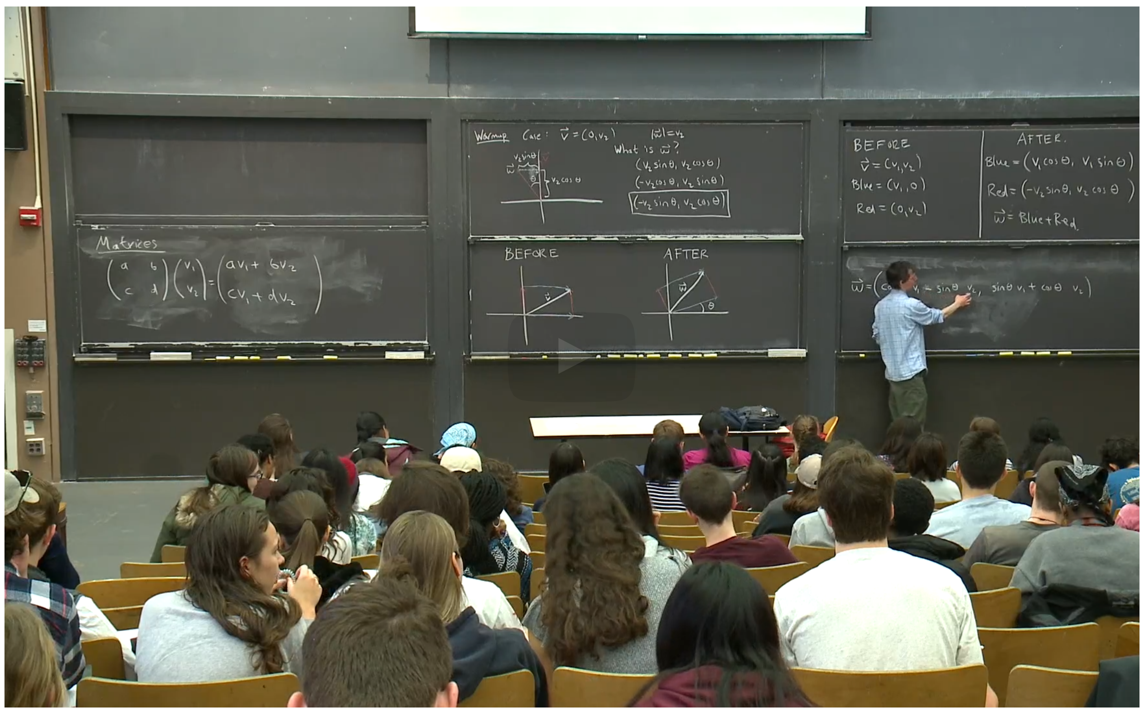
Lecture due Sep 15, 2021 20:30 IST

Explore

Matrix Multiplication



▶ 2:06 / 4:18 ▸ 2.0x

v1 plus a number of times v2 for the first component

and a number of times v1 plus a number times

v2 for the second component.

If you're on the lookout for that, you'll see it a lot.

This is a little bit of a rigged example,

but there is an example on the board.

Where do we see over here?

So pick your favorite angle, theta pi over 6.

This is a number of times v1 plus a number of times v2.

That's the first component, a number of times

v1 plus a number times v2.

So that's a matrix.

Whenever we see that, we know that

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Matrices and Rotation

There is a matrix hiding in this rotation problem! Can you spot it? Let's review 2×2 matrix multiplication. To multiply a matrix

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \tag{5.10}$$

by a vector

$$\begin{pmatrix} v_1 \\ v_2 \end{pmatrix}, \tag{5.11}$$

we perform the operation,

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} av_1 + bv_2 \\ cv_1 + dv_2 \end{pmatrix}. \tag{5.12}$$

Notice that the result in this case is a vector with x-component $av_1 + bv_2$ and y-component $cv_1 + dv_2$.

Take a look at the form of the components. When we see an expression of the form a

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of a matrix multiplication. These types of expressions come up a lot in science and engineering. One example is the rotation of a vector! The vector $\vec{w} = \begin{pmatrix} w_1 \\ w_2 \end{pmatrix}$ obtained from rotating $\vec{v}$ can be written as the following matrix equation:

$$\begin{pmatrix} w_1 \\ w_2 \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}. \tag{5.13}$$

Clockwise 1

2/2 points (graded)

Given a vector $\vec{v} = \begin{pmatrix} a \\ b \end{pmatrix}$ let $\vec{u}$ be the vector obtained by rotating $\vec{v}$ **clockwise** by an angle t.

Find the coordinates of $\vec{u} = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$. Your answer will contain a, b, and t.

$u_1 =$ a*cos(t) + b*sin(t) ✔

$u_2 =$ -a*sin(t) + b*cos(t) ✔

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✔ Correct (2/2 points)

Clockwise 2

1/1 point (graded)

As in the previous problem, let $\vec{u}$ be the vector obtained by rotating $\vec{v} = \begin{pmatrix} a \\ b \end{pmatrix}$ **clockwise** by an angle t.

There is a 2×2 matrix M such that $\vec{u} = M\vec{v}$. Find M.

Your answer will contain t.

(Enter a matrix using notation such as `[[a,b],[c,d]]`.)

$M =$ [[cos(t), sin(t)],[-sin(t), cos(t)] ✔ **Answer:** [[cos(t), sin(t)],[-sin(t), cos(t)]]

Solution:

We found in the previous problem that

$$u_1 = a\cos(t) + b\sin(t) \tag{5.14}$$
$$u_2 = -a\sin(t) + b\cos(t) \tag{5.15}$$

The first row is "a number times a" plus "a number times b". The two numbers are $\cos(t)$ and $\sin(t)$. Therefore, the first row of the matrix is $\cos(t), \sin(t)$. Similarly we obtain the second row of the matrix.

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Answers are displayed within the problem

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[Staff] Trig Identities in the First Problem

discussion posted about 14 hours ago by **alan_driscoll**

> *We may simplify further using the formulas* $\sin(-t) = -\sin(t)$ *and* $\cos(-t) = -\cos(t)$.

The second identity should be $\cos(-t) = +\cos(t)$, right?

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1 response

jfrench (Staff)
about 14 hours ago

Yup!!

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