

More Advanced Reciprocity Laws (1)

- The Quadratic Reciprocity Law is one of the most beautiful laws of prime numbers.
- It has many applications. We can generalize Fermat's Thm on Sums of Two Squares.
- **Class Field Theory** is a further generalization of QRL.

More Advanced Reciprocity Laws (2)

- Class Field Theory was established by Weber, Hilbert, Takagi, Artin by the beginning of the 20th century.



Heinrich Martin
Weber
(1842-1913)



David Hilbert
(1862-1943)



Teiji Takagi
(1875-1960)



Emil Artin
(1898-1962)

https://en.wikipedia.org/wiki/Heinrich_Martin_Weber

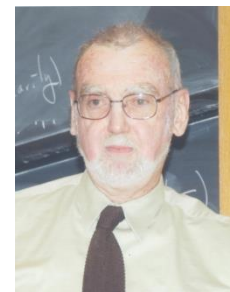
https://en.wikipedia.org/wiki/David_Hilbert

https://en.wikipedia.org/wiki/Teiji_Takagi

https://en.wikipedia.org/wiki/Emil_Artin

More Advanced Reciprocity Laws (3)

- More recently, Class Field Theory has been further generalized using **modular forms** and **automorphic forms** (Langlands's program).
- Wiles's proof of Fermat's Last Thm is considered as establishing new Reciprocity Laws for **elliptic curves**.



Robert Phelan
Langlands
(1936-)



Andrew John
Wiles
(1953-)

https://en.wikipedia.org/wiki/Robert_Langlands

https://en.wikipedia.org/wiki/Andrew_Wiles

Summary of Week 3

- Gauss and his Reciprocity Laws
 - ◆ The Quadratic Reciprocity Law (QRL)
 - ◆ Legendre symbols
- Primitive Roots of Unity
- Euler's Criterion, Eisenstein's Lemma
- Proof of the Quadratic Reciprocity Law
- More Advanced Reciprocity Laws
(Langlands's Program)

Plan of Week 4

We will learn applications of prime numbers to Cryptography. Recently, geometric objects such as elliptic curves are also applied to Cryptography.

Let's explore the practical world of prime numbers.

See you next week!



Ronald Linn
Rivest
(1947-)