MITx: 14.310x Data Analysis for Social Scientists

Help

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Question 6 - 9

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Now suppose X_i i.i.d. $N(\mu, 4)$ and $n = 25$. We want to test $H_0 : \mu = 0$ vs. $H_a : \mu \neq 0$.

Question 6

1/1 point (graded)

What test statistic would you propose using?

- a. The sample mean ✔
- b. The sample variance
- c. The maximum
- d. The minimum

Explanation

Since μ is the mean of the distribution, the most natural test statistic is the sample mean.

- Module 5: Moments of a Random Variable, Applications to Auctions, & Intro to Regression
- Module 6: Special Distributions, the Sample Mean, the Central Limit Theorem, and Estimation
- **Module 7: Assessing and Deriving Estimators - Confidence Intervals, and Hypothesis Testing**

Assessing and Deriving Estimators
Finger Exercises due Nov 14, 2016 at 05:00 IST

Confidence Intervals and Hypothesis Testing
Finger Exercises due Nov 14, 2016 at 05:00 IST

Module 7: Homework
Homework due Nov 07, 2016 at 05:00 IST

Submit You have used 1 of 2 attempts

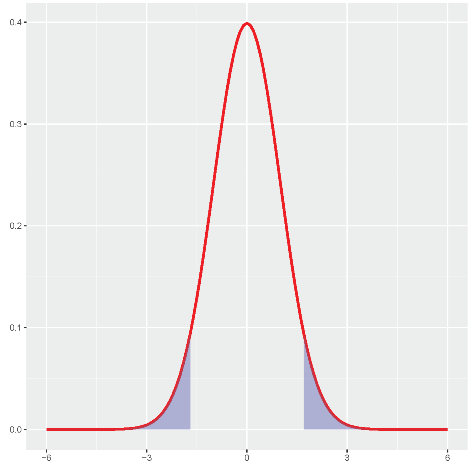
Correct (1/1 point)

Question 7

1/1 point (graded)

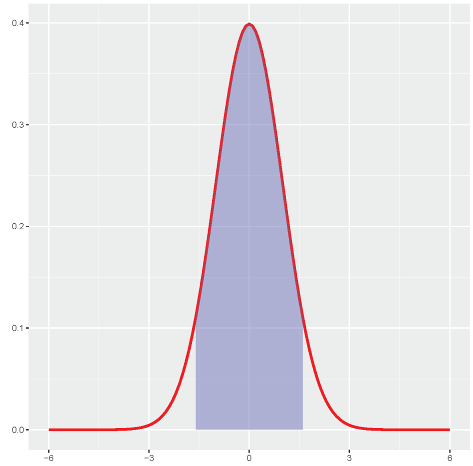
What shape will the critical region have? In other words, for what values of the test statistic would we want to reject the null in favor of the alternative? (Note: Shaded area corresponds to rejection)

a. It is given by:

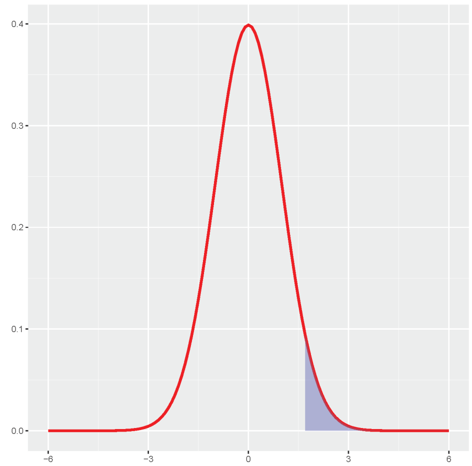


- Exit Survey

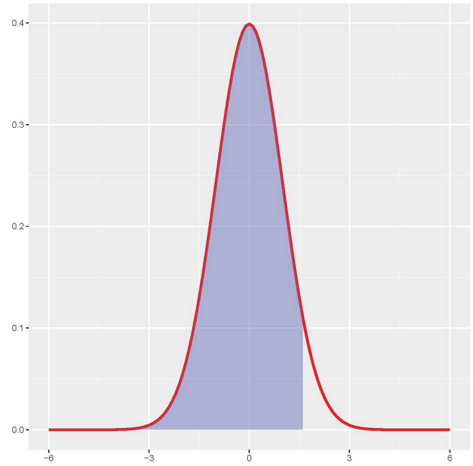
b. It is given by:



c. It is given by:



d. It is given by:



Explanation

The critical region, i.e., the region in which we reject $H_0 : \mu = 0$ will be comprised by the union of the left and right tail of the distribution. In other words, there will be some value k such that we reject the null if $X < -k$ or $X > k$. Note that we are using the fact that this is a symmetric test.

Submit You have used 1 of 2 attempts

✔ Correct (1/1 point)

Question 8

0/1 point (graded)

Compute α (the probability that we reject the null hypothesis when the null is true) as a function of the critical value(s). (A critical value is the boundary between the critical region and the rest of the sample space. In the example in class, we denoted the critical value k.)

- a. It is given by $2\left(1-\Phi\left(\frac{2k}{5}\right)\right)$
- b. It is given by $2\left(1-\Phi\left(\frac{5k}{2}\right)\right)$
- c. It is given by $\left(1-\Phi\left(\frac{5k}{2}\right)\right)$
- d. It is given by $\left(1-\Phi\left(\frac{2k}{5}\right)\right)$ ✖

Explanation

α is the probability that we reject the null hypothesis when the null is true:

$$\alpha = P(\overline{X} \leq -k|\mu = 0) + P(\overline{X} \geq k|\mu = 0).$$

Under the null hypothesis $\overline{X} \sim N\left(0, \frac{4}{25}\right)$. Hence $\alpha = \Phi\left(-\frac{5k}{2}\right) + 1 - \Phi\left(\frac{5k}{2}\right) = 2\left(1-\Phi\left(\frac{5k}{2}\right)\right)$.

Submit You have used 2 of 2 attempts

✖ Incorrect (0/1 point)

Question 9

0/1 point (graded)

Is there a connection between the procedure for finding α and the construction of a confidence interval for μ?

- a. Yes
- b. No ✖

Explanation

Yes, there is a connection. A $(1-\alpha)$ confidence interval for μ from a normal population of known variance is such that $P(\mu \in CI_{1-\alpha}) = 1-\alpha.$ The critical values for a test of size α that the parameter $\mu = 0$ are the boundaries for the $1-\alpha$ confidence interval centered at $\mu = 0$.

Submit You have used 1 of 1 attempt

✖ Incorrect (0/1 point)