edX

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Definition of Expected Value

Now that you know how expected value works, I can give you a more precise definition.

The **expected value** of an option A is the weighted average of the value of the outcomes that A might lead to, with weights determined by the probability of the relevant state of affairs, given that you choose A.

Formally:

$$EV(A) = v(AS_1) \cdot p(S_1|A) + v(AS_2) \cdot p(S_2|A) + \ldots + v(AS_n) \cdot p(S_n|A)$$

where $S_1, S_2, \ldots S_n$ is any list of (exhaustive and mutually exclusive) states of the world, $v(AS_i)$ is the value of being in a situation in which you've chosen A and S_i is the case, and $p(S|A)$ is the probability of S, given that you choose A.

Here is how to apply this formula in the case of the evil teacher.

Recall that there are two relevant states of affairs: easy exam (E) and hard exam (H). So the expected value of drinking (D) and studying (S) should be calculated as follows:

$$\begin{array}{rcccccccccc} EV(D) & = & v(DE) & \cdot & p(E|D) & + & v(DH) & \cdot & p(H|D) & & \\ & = & 35 & \cdot & 0.3 & + & (-25) & \cdot & 0.7 & = & -7 \end{array}$$

$$\begin{array}{rcccccccccc} EV(S) & = & v(SE) & \cdot & p(E|S) & + & v(SH) & \cdot & p(H|S) & & \\ & = & 18 & \cdot & 0.9 & + & 18 & \cdot & 0.1 & = & 18 \end{array}$$

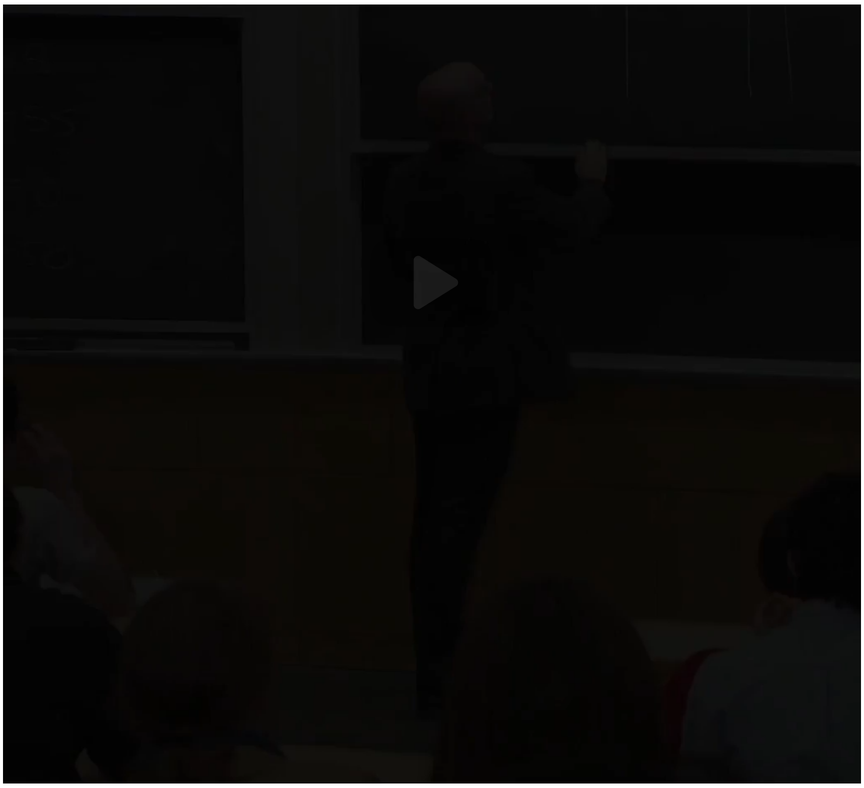
Video Review: An Expected Value Calculation

is do whatever maximizes expected value.

Since studying maximizes expected value,

standard decision theory says: study.

And I hope that sounds



eminently sensible to everyone.

In this case, it works perfectly.

But now for the bad news.

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Problem 1

3/3 points (ungraded)

A fair coin will be tossed, and you must choose between the following two bets:

B_1	B_2
\$1000 if Heads; $-$\$200 if Tails.	\$100 Heads; \$50 if Tails.

(Assume the degree to which you value a given outcome corresponds to the amount of money you end up with. So, for example, you assign value 1000 to an outcome in which you receive \$1000, and value -200 to an outcome in which you pay \$200.)

What is the expected value of accepting B_1 ?

400

✔ **Answer:** 400

400

What is the expected value of accepting B_2?

75

✔ **Answer:** 75

75

Which of the two bets should you accept, according to the Principle of Expected Value Maximization?

- (●) You should accept B_1
- ( ) You should accept B_2

✔

Explanation
There are four possible outcomes, depending on whether you pick B_1 or B_2 and on whether the coin lands Heads or Tails:

	Coin lands Heads	**Coin lands Tails**
You take bet B_1	$B_1\ H$	$B_1\ T$
You take bet B_2	$B_2\ H$	$B_2\ T$

And we know that the value of each of these outcomes is as follows:

$$\begin{aligned} v\left(B_1\ H\right) &= 1000 \qquad & v\left(B_1\ T\right) &= -200 \\ v\left(B_1\ H\right) &= 100 \qquad & v\left(B_1\ T\right) &= 50 \end{aligned}$$

The expected values of B_1 and B_2 can be characterized on the basis of these outcomes:

$$\begin{aligned} EV\left(B_1\right) = &\quad v\left(B_1 H\right)\cdot p\left(H|B_1\right) + v\left(B_1 T\right)\cdot p\left(T|B_1\right) \\ EV\left(B_2\right) = &\quad v\left(B_2 H\right)\cdot p\left(H|B_2\right) + v\left(B_2 T\right)\cdot p\left(T|B_2\right) \end{aligned}$$

Since the coin is fair, we can fill in numerical values as follows:

$$\begin{aligned} EV\left(B_1\right) &= 1000\cdot 0.5 + (-200)\cdot 0.5 &= 400 \\ EV\left(B_2\right) &= (100\cdot 0.5) + (50\cdot 0.5) &= 75 \end{aligned}$$

Since $400 > 75$, the expected value of accepting B_1 is greater than the expected value of B_2. So the Principle of Expected Value Maximization entails that you should accept B_1.

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Answers are displayed within the problem

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What about investment?
Problem 1 is quite interesting, as it makes you think of what would be the best criteria for selecting b... 6