Playing Cards Card Games Probability (statistics)

What is the probability that in a shuffled deck of cards a king is next to a 3?

5 Answers

Michael Lamar, Assistant Professor of Statistics
Updated 23 Dec 2015

Let's call the four suits 1, 2, 3, and 4 rather than Spades, Hearts, Clubs, and Diamonds so that we can index things easier.

Let's call the event with the king of the ith suit is next to the 3 of the jth suit A_{ij} for $i, j \in \{1, 2, 3, 4\}$.

Now we are interested in finding:

$$P\left(\bigcup_{i=1}^{4} \bigcup_{j=1}^{4} A_{ij}\right)$$

To do so, we use the inclusion/exclusion principle. We first add together the probabilities of all sixteen events, A_{ij}.Thankfully, they are all the same, so we only need to find one of them and then multiply by sixteen. The probability of king of suit 1 being next to three of suit 1 is simple to find. If the king is immediately followed by the three, then there are 51! such orderings. (Just imagine that the two cards were glued together in that order so that the deck effectively had just 51 cards.) Similarly there are 51! orderings in which the three is immediately followed by the king. There are a total of 52! total arrangements, so the probability is 1/26. Now there are 16 events in our union, so from step one of the calculation, we get:

$$p_1 = \frac{16}{26} \approx 0.6154$$

For step 2, we must subtract the probabilities of all the pairwise intersections of the form $A_{ij} \cap A_{mn}$. This problem is a little nastier since there are many such intersections. But that's not all. They don't all have the same probability either. For example, the probability of king suit 1 being followed by 3 suit 1 and king suit 1 being followed by three suit 2 is zero. However, the order could be three suit 1, king suit 1, three suit 2 so P($A_{11} \cap A_{12}) \neq 0$. But it isn't the same as P($A_{11} \cap A_{22}$). So there are $\binom{16}{2} = 120$ ways to pick two events to intersect, but those that have a match in one of the two places in the subscript have a different probability than those with no such match. So we need to figure out the two probabilities separately and then figure out how many of each type there are. That's not actually a very difficult task, but it takes a little work. If I did it right, there are 72 ways to intersect so that none of the cards in the two events overlap and the probability of each such intersection is $4 \cdot \frac{50!}{52!}$ while there are 48 intersections in which one of the cards is duplicated and the probability of each such intersection is $2 \cdot \frac{50!}{52!}$. So after step two, the result looks like

$$p_2 = \frac{16}{26} - 72{\cdot}4 \cdot \frac{50!}{52!} - 48{\cdot}2 \cdot \frac{50!}{52!} \approx 0.4706$$

For step 3, we must add back the probability of all the intersections of the form:

$$A_{ij} \cap A_{mn} \cap A_{st}$$

There are even more of them, and there are now several different cases to account for. At this point, we'd have to be pretty masochistic to want to continue knowing that we will have to continue on to step 4 and beyond. But if you really want to get an exact answer, that would be a way to do it. You won't have to do all 16 steps because it is, for example, impossible for the king of spades to be adjacent to 3 different threes. But you do have several more steps to account for.

The good news is that the further down the path we are willing to walk, the better our estimate will be. The bad news is that the further we want to walk, the more difficult the path becomes. It seems clear to me that it is strongly preferable to just do the Monte Carlo simulation. It isn't too hard to write. Here's a quick Octave/Matlab script to do it. The resulting output is a 95% confidence interval for the true probability in question. My code, running on an ancient laptop for about 5 minutes, performed 10 million pseudo-iid shuffles and gave the result that the probability is in the interval:

(0.48583, 0.48645)

CODE

```
trials = 1e67;  %number of iid trials

deck = [zeros(4,1); ones(4,1); 3*ones(44,1)];  %Denote king by 0, three by 1, and all else by 3 so that any difference between adjacent cards of 1 or -1 means a king is next to three.  All other possible differences are 0, 2, 3, -2, and -3 means some other pair are adjacent.

successes = 0;  %count the number of trials with kings and threes next to each other

for i=1:trials %loop over trials

    r = randperm(n_cards);  %shuffle the deck

    successes = successes + any( abs( diff( deck(r) ) )==1 ); %check for any adjacent kings and threes which will have the difference equal to one in absolute value

end

p_est = successes/trials; %estimate the probability

stdev_p = sqrt(p_est*(1-p_est)/trials);  %estimate the standard deviation

ci = p_est +[-1 1]*1.96*stdev_p;  %approximate 95% confidence interval

disp(ci) %display the result
```


Related Questions

More Answers Below

What is the probability that the 47th card, drawn from a well-shuffled new deck, will be a diamond? And why?

What is the probability that in two shuffled decks, at least one card is in the same position in both decks?

Probability (statistics): How many times would one have to shuffle a pack of cards repeatedly to get the same configuration?

A card is drawn from a regular deck, and it is a face card. What is the probability that the card is red?

A hand consists of 5 cards (52 card deck). What is the probability of getting 2 aces and 2 kings?

Carter McClung, High School Stat Teacher
Written 4 Dec 2015

Yikes, this one's a beating.

I'm going to simplify and get an answer close to the actual one. I'm going to assume that no king is the top or the bottom card, and there are never two in a row. This will make this a slight overestimation.

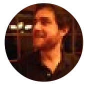
You have four kings in the deck. With my assumptions, that leaves 8 places that a 3 could go.

There are 48 possible places for a 3 to go. Let's calculate the probability of the 3 **NOT** being next to a king.

$$\frac{40}{48} * \frac{39}{47} * \frac{38}{46} * \frac{37}{45} = \frac{40!/36!}{48!/44!} \approx .47$$

This means the probability it **IS** next to a king is around .53. With my false assumptions, it could probably be as low as .5.

267 Views · View Upvotes



Anatole Ginsberg, semi-employed math teacher
Updated 10 Dec 2015

Edit: My answer below is incomplete. There doesn't seem to be any straightforward way of doing this without a whole long list of operations (mainly, counting the "repeats").

I did run a simulation and found the answer should be approximately **0.494**

This special deck has 51 cards.
And 51! permutations. All of these will have a 3 next to K.

However, each 3 could be paired with each K, and in both positions.

So there are $4 \cdot 3 \cdot 2 = 24$ "special decks" to look at.
(4x3 is the ways we can pair each 3 to a K, and x2 is to flip their position top/bottom)

$24 \cdot 51!$ ways to shuffle a deck of cards and have the 3 next to K.

And there are 52! ways to shuffle a deck of cards.

$$\frac{24 \cdot 51!}{52!} = \frac{6}{13} \approx 0.462$$

Steve Waddington, Network Engineer, CIO and CEO. Current Gemologist
Written 4 Dec 2015

Lets look at a single 3 first; so for any 3 there is 8 ways out of 51 positions it could be next to a king - 4 on top and 4 underneath.

However, if the 3 is the top or the bottom card, then there are only 4 ways out of 51 positions that a king can be next to it. There are 2 ways out of 52 positions that the 3 could be the top or bottom card.

Related Questions

What is the probability that the 47th card, drawn from a well-shuffled new deck, will be a diamond? And why?

What is the probability that in two shuffled decks, at least one card is in the same position in both decks?

Probability (statistics): How many times would one have to shuffle a pack of cards repeatedly to get the same configuration?

A card is drawn from a regular deck, and it is a face card. What is the probability that the card is red?

A hand consists of 5 cards (52 card deck). What is the probability of getting 2 aces and 2 kings?

What is the probability of getting 4 Red cards and 1 Black card from a deck of cards?

A poker hand consists of 5 cards. It will come from a well-shuffled deck of cards. What is the probability of having 4 Queens and any face car...

Two cards are drawn from a deck of 52 cards. What is

That give us 50 ways in 52 there can be an 8 in 51 chance of a king next to the 3 and 2 ways in 52 there can be a 4 in 51 chance of a king next to the 3.

So I think we get:

((50/52) x (8/51)) + ((2/52) x (4/51)) = 0.1509

0.1509 x 4 = 0.6037

I am not at all sure if that is right though.

Paul Hsieh, Master in Mathematics from the University of Toronto.
Written 7 Dec 2015

I have no idea how to do this question easily. This just seems like it boils down to busy work.

Basically what you need to do is break down the number of cases of the king being next to *each other,* as well as being the first or last card of the deck. There are enough such cases, that I would get annoyed at having to do such a question.

After you've enumerated those cases identify the positions where you can place a 3 next to one of the kings in each case and form the inverse ratio of "no 3s next to any Kings", (since that will be easier), then just complement that to get the desired ratio. Then weight them according to the quantity of each case, and form your final ratio.

If you find some clever faster or more efficient way to do this, I would like to know.

The truth is, even in doing this, I would have a hard time trusting my own calculations -- I would probably also write computer simulation that just did this monte carlo style, if for no other reason, to check a result calculated by hand.

the probability that one card is kings and second is queen if the first card is replaced ...

Three cards are drawn at random from a well-shuffled deck of cards. What is the probability of drawing an ace, a king, and a jack?

What's the probability that after shuffling a deck of cards, no two cards of the same rank or same suit are adjacent to each other?

Related Questions

What is the probability that the 47th card, drawn from a well-shuffled new deck, will be a diamond? And why?

What is the probability that in two shuffled decks, at least one card is in the same position in both decks?

Probability (statistics): How many times would one have to shuffle a pack of cards repeatedly to get the same configuration?

A card is drawn from a regular deck, and it is a face card. What is the probability that the card is red?

A hand consists of 5 cards (52 card deck). What is the probability of getting 2 aces and 2 kings?

What is the probability of getting 4 Red cards and 1 Black card from a deck of cards?

A poker hand consists of 5 cards. It will come from a well-shuffled deck of cards. What is the probability of having 4 Queens and any face car...

Two cards are drawn from a deck of 52 cards. What is

103 Views · View Upvotes · Answer requested by Tariq Rashid

Related Questions

What is the probability of getting 4 Red cards and 1 Black card from a deck of cards?

Two cards are drawn from a deck of 52 cards. What is the probability that one card is kings and second is queen if the first card is replaced ...

Three cards are drawn at random from a well-shuffled deck of cards. What is the probability of drawing an ace, a king, and a jack?

What's the probability that after shuffling a deck of cards, no two cards of the same rank or same suit are adjacent to each other?

If 4 cards are drawn from a well shuffled deck. what is the probability of getting at least one diamond?

What is probability of getting both king when two card are drawn from deck of card?

A deck of ordinary cards is shuffled and 13 cards are dealt. What is the probability that the last card dealt is an ace?

You draw 3 cards from a standard deck of 52 cards. what is the probability of getting 3 red cards?

How can I rate the shuffling of a card deck?

A poker hand consists of 5 cards, the order is conventionally disregarded from a well-shuffled deck of 52 cards. What is the probability of ha...

What is the probability of picking 13 cards from 2 standard decks of 53 cards (including Joker) shuffled together such that we get exactly 1 i...

the probability that one card is kings and second is queen if the first card is replaced ...

Three cards are drawn at random from a well-shuffled deck of cards. What is the probability of drawing an ace, a king, and a jack?

What's the probability that after shuffling a deck of cards, no two cards of the same rank or same suit are adjacent to each other?

Related Questions

What is the probability that the 47th card, drawn from a well-shuffled new deck, will be a diamond? And why?

What is the probability that in two shuffled decks, at least one card is in the same position in both decks?

Probability (statistics): How many times would one have to shuffle a pack of cards repeatedly to get the same configuration?

A card is drawn from a regular deck, and it is a face card. What is the probability that the card is red?

A hand consists of 5 cards (52 card deck). What is the probability of getting 2 aces and 2 kings?

What is the probability of getting 4 Red cards and 1 Black card from a deck of cards?

A poker hand consists of 5 cards. It will come from a well-shuffled deck of cards. What is the probability of having 4 Queens and any face car...

Two cards are drawn from a deck of 52 cards. What is

A coin is flipped, a die is rolled, and a card is drawn from a standard deck of cards. What is the probability of the following event: P (tail...

If I had two complete decks of cards shuffled randomly, what is the probability that two randomly dealt cards will be a pair?

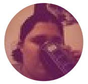
Suppose that 33 cards are drawn from a well-shuffled deck of 52 cards. What is the probability that all 33 are black?

the probability that one card is kings and second is queen if the first card is replaced ...

Three cards are drawn at random from a well-shuffled deck of cards. What is the probability of drawing an ace, a king, and a jack?

What's the probability that after shuffling a deck of cards, no two cards of the same rank or same suit are adjacent to each other?

What is the hardest thing you have ever done?

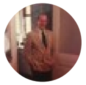


Becca Makin, autistic writer and mother of one
Written Dec 9, 2013

"Getting Over" my attempted murder by an Ex. About a year after my ex had tried to choke me to death (staying a good 5 months with him even after that because as far as I knew I had no where else to go), my mother told me I had to stop wallowing. That I needed to shut up about it, to get over it because life moves on, I wasn't dead and thus it needed to be put to rest. At that time

Read More

Psychology of Everyday Life: What are some of the greatest examples of absence of mind?



Jay Bazzinotti, Seeking my destiny; 4 patents, 3 books, 2 degrees, 24 countries, 46 states
Updated Nov 20, 2015

Read More

Related Questions

What is the probability that the 47th card, drawn from a well-shuffled new deck, will be a diamond? And why?

What is the probability that in two shuffled decks, at least one card is in the same position in both decks?

Probability (statistics): How many times would one have to shuffle a pack of cards repeatedly to get the same configuration?

A card is drawn from a regular deck, and it is a face card. What is the probability that the card is red?

A hand consists of 5 cards (52 card deck). What is the probability of getting 2 aces and 2 kings?

What is the probability of getting 4 Red cards and 1 Black card from a deck of cards?

A poker hand consists of 5 cards. It will come from a well-shuffled deck of cards. What is the probability of having 4 Queens and any face car...

Two cards are drawn from a deck of 52 cards. What is