2. A Simple Singular Covariance Matrix

Suppose $\mathbf{X}$ is a random vector, where $\mathbf{X} = (X^{(1)}, \ldots, X^{(d)})^T$, with mean $\mathbf{0}$ and covariance matrix $\mathbf{v}\mathbf{v}^T$, for some vector $\mathbf{v} \in \mathbb{R}^d$.

(a)

1/1 point (graded)
If $d > 1$, is the covariance matrix $\mathbf{v}\mathbf{v}^T$ invertible?

Hint: Compute the determinant for the case $d = 2$. That result will generalize to higher dimension.

- ( ) $\mathbf{v}\mathbf{v}^T$ is invertible.
- (•) $\mathbf{v}\mathbf{v}^T$ is **not** invertible.

✔

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Correct (1/1 point)

(b)

1/1 point (graded)

Let $\mathbf{u}$ be a vector in $\mathbb{R}^d$ such that $\mathbf{u} \perp \mathbf{v},$ i.e. $\mathbf{u} \cdot \mathbf{v} = \mathbf{u}^T\mathbf{v} = \mathbf{v}^T\mathbf{u} = 0.$

Find the variance of $\mathbf{u}^T\mathbf{X}$.

(If applicable, enter **trans(v)** for the transpose $\mathbf{v}^T$ of a vector $\mathbf{v}$, and **norm(v)** for the norm $||\mathbf{v}||$ of a vector $\mathbf{v}$.)

Var $(\mathbf{u}^T\mathbf{X}) =$ 0

STANDARD NOTATION

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Correct (1/1 point)

(c)

1/1 point (graded)

Let $\overline{\mathbf{v}} = \frac{\mathbf{v}}{||\mathbf{v}||}$ (i.e., $\overline{\mathbf{v}}$ is the normalized version of $\mathbf{v}$). What is the variance of $\overline{\mathbf{v}}^T\mathbf{X}$?

(If applicable, enter **trans(v)** for the transpose $\mathbf{v}^T$ of $\mathbf{v}$, and **norm(v)** for the norm $||\mathbf{v}||$ of a vector $\mathbf{v}$.)

Var $(\overline{\mathbf{v}}^T\mathbf{X}) =$ norm(v) ^2

STANDARD NOTATION

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✔ Correct (1/1 point)

(d)

1/1 point (graded)

Suppose we observe n independent copies of $\mathbf{X}$ and call them $\mathbf{X}_1, \ldots, \mathbf{X}_n$. What is the asymptotic distribution of $\overline{\mathbf{X}}_n = \frac{\sum_{i=1}^n \mathbf{X}_i}{n}$?

(Select all that apply.)

- [ ] $\sqrt{n}\left(\overline{\mathbf{X}}_n - \mathbf{0}\right) \xrightarrow[n\to\infty]{(d)} \mathcal{N}\left(\mathbf{0}, \mathbf{I}_d\right)$ where $\mathbf{I}_d$ is the identity matrix in R^d
- [x] $\sqrt{n}\left(\overline{\mathbf{X}}_n - \mathbf{0}\right) \xrightarrow[n\to\infty]{(d)} \mathcal{N}\left(\mathbf{0}, \mathbf{v}\mathbf{v}^T\right)$
- [ ] $\sqrt{n}\left(\overline{\mathbf{X}}_n - \mathbf{0}\right) \xrightarrow[n\to\infty]{(d)} \mathcal{N}\left(\mathbf{0}, \|\mathbf{v}\|^2\right)$

✔

Note on notation: In the choices above, $\mathcal{N}$ denotes a multivariate Gaussian distribution. In lecture and elsewhere, a multivariate Gaussian distribution in d dimension is also sometimes denoted with an extra subscript by $\mathcal{N}_d$. To be accurate, read the dimension from the arguments, i.e. the mean and the covariance matrix.

Submit You have used 1 of 2 attempts

✔ Correct (1/1 point)

(e)

2/2 points (graded)

Let $\mathbf{Y}_i = \overline{\mathbf{v}}\,(\overline{\mathbf{v}}^T \mathbf{X}_i)$, or equivalently $\overline{\mathbf{v}}\,(\overline{\mathbf{v}} \cdot \mathbf{X}_i) = (\overline{\mathbf{v}} \cdot \mathbf{X}_i)\,\overline{\mathbf{v}}$, where $\overline{\mathbf{v}} = \dfrac{\mathbf{v}}{\|\mathbf{v}\|}$ is the same as in part (c).

We will compare the asymptotic distribution of $\overline{\mathbf{X}}_n$ you obtain in part (d) to the asymptotic distribution of $\overline{\mathbf{Y}}_n$ where $\overline{\mathbf{Y}}_n = \dfrac{\sum_i^n \mathbf{Y}_i}{n}$.

What is the expectation $\mathbb{E}[\mathbf{Y}_i]$ of $\mathbf{Y}_i$?
(Choose all that apply.)

- [x] $\overline{\mathbf{v}}\overline{\mathbf{v}}^T \mathbb{E}[\mathbf{X}_i]$
- [x] $\mathbf{0}$ (the zero vector in $\mathbb{R}^d$)
- [ ] 0 (the real number zero)
- [ ] $\overline{\mathbf{v}}^T \mathbf{v}$

✔

Find the covariance matrix $\Sigma_{\mathbf{Y}_i}$ of $\mathbf{Y}_i$ in terms of the vector $\mathbf{v}$.

(If applicable, enter **trans(v)** for the transpose $\mathbf{v}^T$ of $\mathbf{v}$, and **norm(v)** for the norm $||\mathbf{v}||$ of a vector $\mathbf{v}$.)

$\Sigma_{\mathbf{Y}_i} =$ v*trans(v) ✔

(There is no answer box for the following question.)

Notice that $\mathbf{Y}_i$ is a scalar multiple of the vector $\mathbf{v}$ and hence lies on the same line as $\mathbf{v}$ no matter what value $\mathbf{X}_i$ takes. (Specifically, $\mathbf{Y}_i = (\overline{\mathbf{v}}^T \mathbf{X}_i)\,\overline{\mathbf{v}}$ is the projection of $\mathbf{X}_i$ onto the vector $\mathbf{v}$.) Use your answers for $\mathbb{E}[\mathbf{Y}_i]$ and $\Sigma_{\mathbf{Y}_i}$ to find the asymptotic distribution of $\overline{\mathbf{Y}}_n$. Compare this with the asymptotic distribution of $\overline{\mathbf{X}}_n$ from the previous part. What can you conclude about the asymptotic distribution of $\overline{\mathbf{X}}_n$?

STANDARD NOTATION

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Correct (2/2 points)

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? Is v v(t) an outer product in (a) and u(t) X an inner product in (b)?
Is v v(t) an outer product in (a) and u(t) X an inner product in (b)? Sorry I am weak in linear algebra and am having trouble interpreting the questions... 1

? Confused by notation in part e 2

Why is a covariance matrix not necessarily of rank 1? 3

? Part C
Is there any material I can read to solve part c? I got all others but not this one and although I'm sure is intuitive I can't get it right 4

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