Observation Theory

Script V12B

In this lecture, we have a closer look to properties of random errors.

In the previous lecture it was shown that the spread in the outcomes of repeated observations are caused by random errors.

These errors can be the result of many different – generally small – noise sources.

From the name you can already guess that the size of the random errors for a GIVEN measurement is unknown; the random error is different for every repeated observation.

Fortunately, we may generally describe the behaviour of random errors by means of a well-known statistical distribution.

In this lecture we will have a closer look at this.

We will start again with the laser distance measurements.

Let's zoom in to a few observations and the associated random errors, which we will denote as e

Where the i-th measurement is denoted with y_i and the mean of all measurements is denoted with the bar.

Now, let's go back to the observations we had and make a histogram of the random errors

Here, the height of each bar shows 'probability density' which means that the area of this

Bin represents the probability that the random error falls in the interval I to j.

Similarly, we can also make the histogram for the rope measurements,

Which had a lower precision.

By comparing the two histograms it is clear that in case of lower precision, there is a higher chance that the absolute random errors are larger, and consequently the probability that they are close to 0 is smaller.

From these histograms you may indeed recognize that the random errors seem to be normally distributed

Before we continue, we will now first briefly review the normal distribution.

It is one of the many statistical distributions that may describe the random nature of certain variables.

From experience we know that the normal distribution adequately describes the distribution of random errors.

The probability density function, of a normally distributed random variable y looks like this

This probability density function is completely specified by 2 parameters:

The mean

And the standard deviation, the latter being the the distance from the mean to the inflection point.

The probability that an outcome will deviate less than 1 standard deviation from the mean, as shown by the green area here

Is equal to 68.3%.

The notation used to indicate that y is normally distributed with a certain mean and standard deviation is shown here.

Note that the underlining of y is used to indicate that this is a random variable

The normal PDF is thus the continuous function, which describes the probability distribution of our random errors.

Here you can see the histograms we had before, but now with the fitted normal probability density functions.

But what is the interpretation of the standard deviation?

It is in fact the parameter describing the spread in the outcomes of the random errors.

In other words: the standard deviation is the measure for the precision.

To be more specific a small standard deviation, as we see in the top graph, means that it is more likely that random errors will be small, corresponding to a high precision.

A large standard deviation, on the other hand, means that it is more likely that random errors will be large, corresponding to a low precision, as we see below.

In the next video lecture we will look in more detail at how to determine the standard deviation and how to describe the precision of a set of measurements using a covariance matrix.