10. Multivariate Central Limit Theorem

Note: The following exercise will be presented in the video that follows. We encourage you to attempt it before watching the video.

Vector Version of the Central Limit Theorem

1/1 point (graded)

Let $\mathbf{X}$ be a random vector of dimension $d \times 1$ and let μ and Σ be its mean and covariance. Let $\mathbf{X}_1, \ldots, \mathbf{X}_n$ be i.i.d. copies of $\mathbf{X}$. Let $\overline{\mathbf{X}}_n \triangleq \frac{1}{n}\sum_{i=1}^{n} \mathbf{X}_i$.

Based on your knowledge of the central limit theorem for a single random variable, select from the following the correct shift and scale factor for $\overline{\mathbf{X}}_n$ so that $\overline{\mathbf{X}}_n$ could potentially converge to the Gaussian random vector $\mathcal{N}(0, I_{d\times d})$.

- $\sqrt{d} \cdot \Sigma^{-\frac{1}{2}} \left(\overline{\mathbf{X}}_n - \mu\right)$
- $\sqrt{d} \cdot \Sigma^{-1} \left(\overline{\mathbf{X}}_n - \mu\right)$
- $\sqrt{n} \cdot \Sigma^{-1} \left(\overline{\mathbf{X}}_n - \mu\right)$
- $\sqrt{n} \cdot \Sigma^{-\frac{1}{2}} \left(\overline{\mathbf{X}}_n - \mu\right)$
- None of the above

Solution:

The shift of $\mathbf{X}$ by μ is the correct shift that needs to be applied in order to center the random vector.

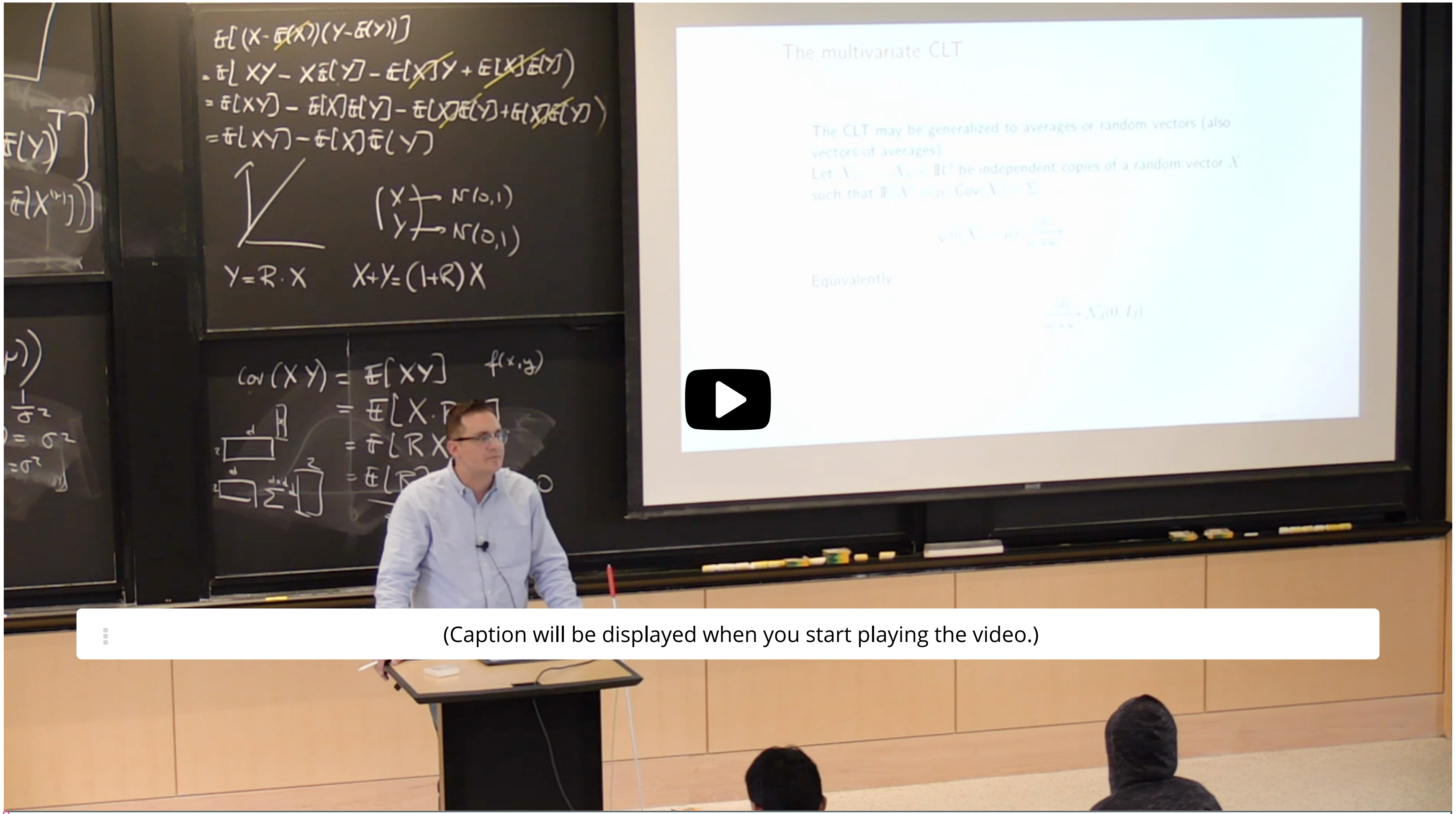
The scaling factor should be $\sqrt{n}\Sigma^{-\frac{1}{2}}$ because it mimics the single variable CLT case most closely. In particular, the division by $\sqrt{\sigma^2}$ in the single variable CLT case is being taken care of by the inverse of the square root of Σ.

Note: Of course, this is only a heuristic discussion that is meant to test how you can potentially generalize the single variable CLT. This is not a proof and the solution is also written as guesswork.

Submit You have used 1 of 3 attempts

Answers are displayed within the problem

Multivariate Central Limit Theorem



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(Optional) Multivariate Convergence in Distribution and Proof of Multivariate CLT

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A covariance matrix is a positive definite matrix and vice-versa. Why?

discussion posted 3 days ago by mbh038

Wow.

'Any covariance matrix is a positive definite matrix, any positive definite matrix is a covariance matrix'.

Anyone care to explain why this is so?

This post is visible to everyone.

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1 response

markweitzman (Community TA)
3 days ago

See: Positive definite Real Symmetric Matrix and its Eigenvalues

If I understood correctly, we also assume here and in the rest of the course that variances (diagonal elements) are non-zeros. Since they already were greater or equal to zero (e.g., because they are variances) now we know that they are positive.

We can diagonalize Σ. When a matrix is in a diagonal form all its eigenvalues are on the diagonal. And in our case they all are positive, so the matrix is, obviously, positive definite.

Reverse (if a matrix A is positive definite than it is a covariance matrix) I believe can be proved in the same way. Lets diagonalize A and get some matrix Σ_Y . Now Σ_Y is a covariance matrix of a certain random vector $Y_1, \ldots, Y_n$, so (since the transformation is done by using orthogonal, i.e. invertable, matrices) now we can trace bake the origin of our $Y_1, \ldots, Y_n$ and find the original $X_1, \ldots, X_n$ and their covariance matrix Σ_X.

posted a day ago by **Alexander_Andrianov**

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