F.2.6 Sample Exam Answers and Videos Questions 9

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Question 9

0 points possible (ungraded)

9. Let $L = \begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix}$, $U = \begin{pmatrix} 1 & -2 & -1 & 1 \\ 0 & 0 & 2 & 1 \end{pmatrix}$, and $b = \begin{pmatrix} 2 \\ -4 \end{pmatrix}$. Let $A = LU$.

   (a) (5 points) Solve $Lx = b$.

   (b) (5 points) Find a specific (particular) solution of $Ax = b$.

   (c) (1 points) Is b in the column space of A? Yes/No

   (d) (1 points) Is b in the column space of L? Yes/No

   (e) (5 points) Find two linearly independent solutions to $Ax = 0$.

   (f) (3 points) Give a general solution to $Ax = b$.

9. Let $L = \begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix}$, $U = \begin{pmatrix} 1 & -2 & -1 & 1 \\ 0 & 0 & 2 & 1 \end{pmatrix}$, and $b = \begin{pmatrix} 2 \\ -4 \end{pmatrix}$. Let $A = LU$.

   (a) (5 points) Solve $Lx = b$.

$$\begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} \psi_0 \\ \psi_1 \end{pmatrix} = \begin{pmatrix} 2 \\ -4 \end{pmatrix}$$

means that $\psi_0 = 2$ and $\psi_1 = (-4 - (-1)(2))/1 = -2$. Thus, $y = \begin{pmatrix} 2 \\ -2 \end{pmatrix}$.

   (b) (5 points) Find a specific (particular) solution of $Ax = b$.

$Ax = b$ means $L(Ux) = b$. So, if we first solve $Ly = b$, then we can instead solve $Ux = y$ for x. But from (a) we know that $y = \begin{pmatrix} 2 \\ -2 \end{pmatrix}$ solves $Ly = b$. Let's set $Ux = y$ up as an appended system:

$$\left(\begin{array}{cccc|c} \boxed{1} & -2 & -1 & 1 & 2 \\ 0 & 0 & \boxed{2} & 1 & -2 \end{array}\right)$$

One solution has the form $x_s = \begin{pmatrix} \boxed{\chi_0} \\ 0 \\ \boxed{\chi_2} \\ 0 \end{pmatrix}$. Solving for χ_0 and χ_2:

$$\begin{array}{cccccc} \chi_0 & -2(0) & -\chi_2 & +(0) & = & 2 \\ & & 2\chi_2 & +(0) & = & -2 \end{array}$$

So that $\chi_2 = -1$ and $\chi_0 = 2 + (-1) = 1$. So a particular solution is given by $x_s = \begin{pmatrix} \boxed{1} \\ 0 \\ \boxed{-1} \\ 0 \end{pmatrix}$.

   (c) (1 points) Is b in the column space of A? Yes/No Yes

   (d) (1 points) Is b in the column space of L? Yes/No Yes

   (e) (5 points) Find two linearly independent solutions to $Ax = 0$. $Ax = 0$ means $L(Ux) = 0$. So, if we first solve $Ly = 0$, then we can instead solve $Ux = y$ for x. But since L is nonsingular, $y = 0$, so all we need to do is solve $Ux = 0$. Let's set this up as an appended system:

$$\left(\begin{array}{cccc|c} \boxed{1} & -2 & -1 & 1 & 0 \\ 0 & 0 & \boxed{2} & 1 & 0 \end{array}\right)$$

Two linearly independent solutions have the form $x_{n_0} = \begin{pmatrix} \boxed{\chi_0} \\ 1 \\ \boxed{\chi_2} \\ 0 \end{pmatrix}$ and $x_{n_0} = \begin{pmatrix} \boxed{\chi_0} \\ 0 \\ \boxed{\chi_2} \\ 1 \end{pmatrix}$.

Solving the first for χ_0 and χ_2:

$$\begin{array}{cccccc} \chi_0 & -2(1) & -\chi_2 & +(0) & = & 0 \\ & & 2\chi_2 & +(0) & = & 0 \end{array}$$

So that $\chi_2 = 0$ and $\chi_0 = 0 + 2(1) = 2$. So the first vectors is given by $x_{n_0} = \begin{pmatrix} \boxed{2} \\ 1 \\ \boxed{0} \\ 0 \end{pmatrix}$.

Solving the second for χ_0 and χ_2:

$$\begin{array}{cccccc} \chi_0 & -2(0) & -\chi_2 & +(1) & = & 0 \\ & & 2\chi_2 & +(1) & = & 0 \end{array}$$

Calculator

So that $\chi_2 = -1/2$ and $\chi_0 = 0 + (-1/2) - 1 = -3/2$. So the second vector is given by

$$x_{n_2} = \begin{pmatrix} \boxed{-3/2} \\ 0 \\ \boxed{-1/2} \\ 1 \end{pmatrix}.$$

(f) (3 points) Give a general solution to $Ax = b$.

$$x = x_s + \alpha_0 x_{n_0} + \alpha_1 x_{n_1}.$$

(Plug in the vectors from (b) and (e).

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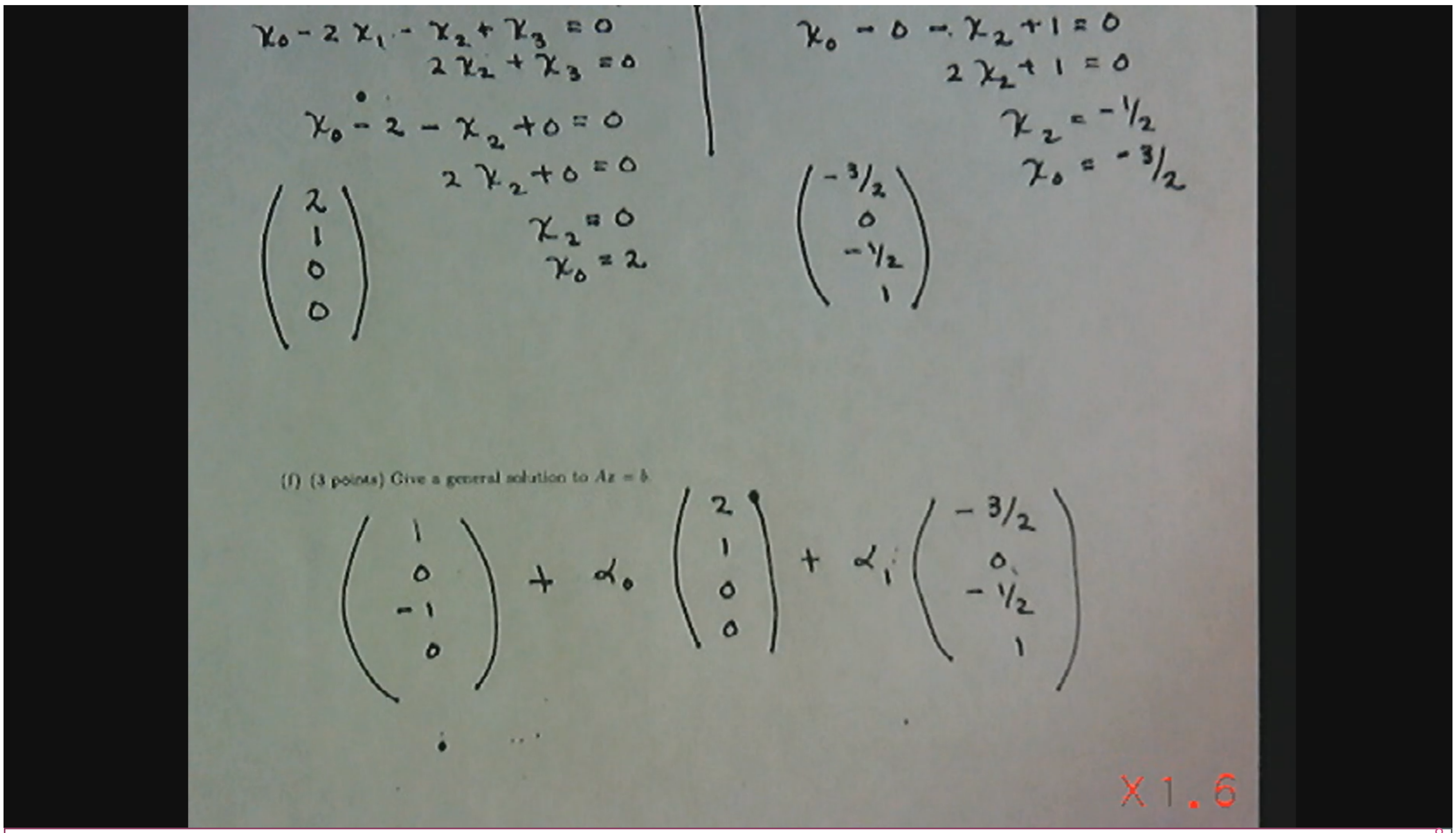
Answers are displayed within the problem

Question 9



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