2. Nonzero concentration specified at endpoints

2(a)

5/5 points (graded)

Consider a thin rod of length 1 containing a saline solution. The left end point is placed in a large saline bath of concentration 1. The right end point is placed in a large bath of freshwater (concentration 0). The concentration u satsifies the following initial value problem.

$$\begin{aligned} \frac{\partial u}{\partial t} &= 2\frac{\partial^2 u}{\partial x^2}, \qquad 0 < x < 1, \quad t > 0 \\ u(x,0) &= x, \qquad 0 < x < 1 \end{aligned}$$

Note that the diffusion constant is 2 in this problem.

Identify the boundary conditions. Enter the value if known, and enter UNK if unknown.

For $t > 0$, $u(0,t) =$ 1

For $t > 0$, $\frac{\partial u}{\partial x}(0,t) =$ UNK

For $t > 0$, $u(1,t) =$ 0

For $t > 0$, $\frac{\partial u}{\partial x}(1,t) =$ UNK

What is the steady state solution (particular solution) $u_{st}(x)$, (the solution defined by $u(x,t) \to u_{st}(x)$ as $t \to \infty$)?

$u_{st}(x) =$ 1-x

$1 - x$

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✔ Correct (5/5 points)

2(b)

3/3 points (graded)

Use superposition to write $u(x,t) = u_{st}(x) + u_h(x,t)$.

Find the boundary conditions satisfied by u_h and then use separation of variables $u_{h,k}(x,t) = v_k(x)\, w_k(t)$ to find the eigenvalues λ_k and eigenfunctions $v_k(x)$. Only use the constant 2 from the PDE in finding $w_k(t)$.

For $k = 1, 2, 3, \ldots,$ $\lambda_k =$ -k^2*pi^2 ✔

$-k^2 \cdot \pi^2$

For $k = 1, 2, 3, \ldots,$ $v_k(x) =$ sin(k*pi*x) ✔

$\sin(k \cdot \pi \cdot x)$

For $k = 1, 2, 3, \ldots,$ $w_k(t) =$ e^(-2*k^2*pi^2*t) ✔

$e^{-2 \cdot k^2 \cdot \pi^2 \cdot t}$

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✔ Correct (3/3 points)

2(c)

1/1 point (graded)

Find the initial condition $u_h\left(x, 0\right)$.

$u_h\left(x, 0\right) =$ 2*x-1 ✔

$2 \cdot x - 1$

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✔ Correct (1/1 point)

2(d)

2/2 points (graded)

Find the appropriate periodic extension for $u_h\left(x, 0\right)$ to be able to solve for the Fourier coefficients

$$u_h\left(x, 0\right) = \sum c_k v_k\left(x\right), \qquad 0 < x < 1.$$

(Hint: You can use superposition of two known (manipulated) Fourier series to do so.)

Enter the coefficient c_k below.

For k odd $c_k =$ 0 ✔

For k even, $c_k =$ -4/(k*pi)

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Correct (2/2 points)

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discussion posted 10 days ago by sfisch

If anyone is still having trouble with this," Paul's Online Notes" has a particularly good section on Solving the Heat Equation (Section 9-5)

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1 response

jfrench (Staff)
7 days ago

Thanks for sharing!

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