9. Conservative Bound

Confidence Interval using a Conservative Bound

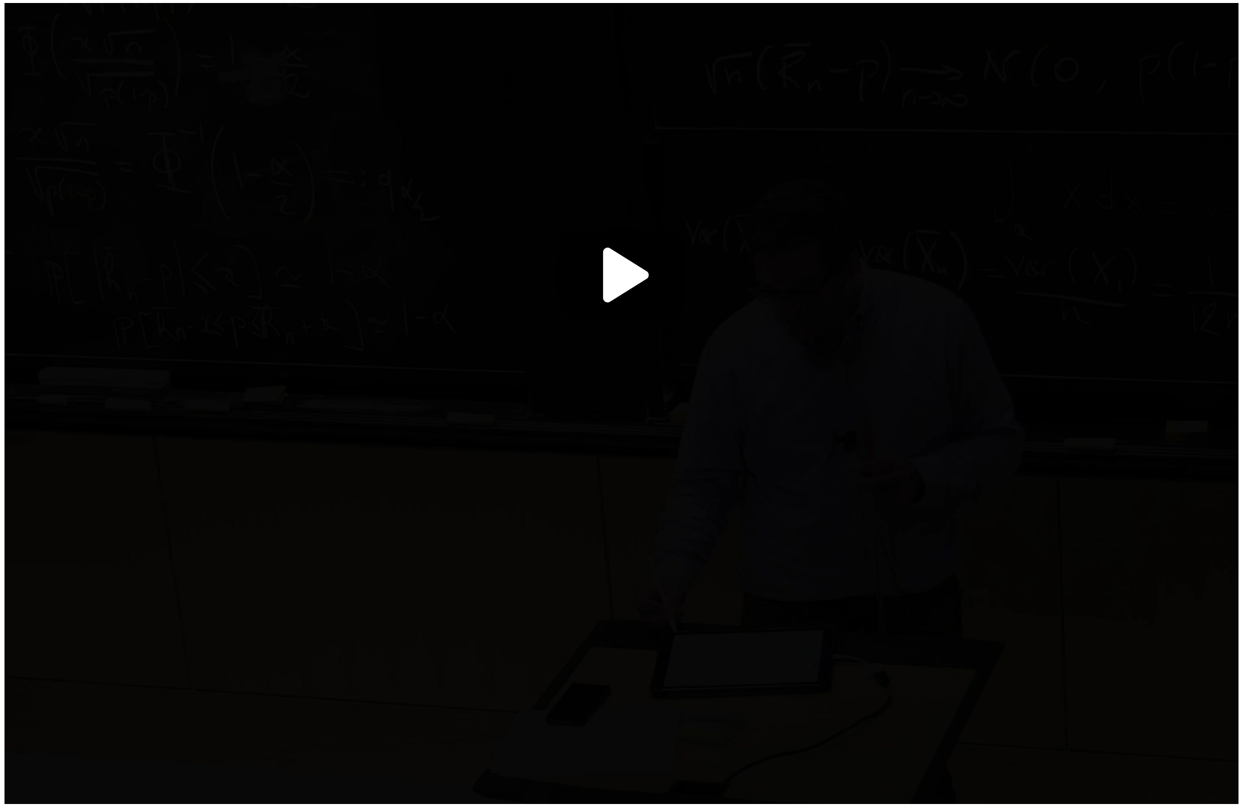
And I already knew that this was equal to 1 minus alpha

when I kept p 1 minus p.

OK.

So first way to do it, if you have an upper bound,

so if your bounds depend on your



unknown p,

just try to find something which contains

all the possible values or your unknown theta in general,

just get something which contains

all the possible values that it can take

and make it just more conservative but no more

than what's needed.

First solution.

4:10 / 4:10 1.50x

Video

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Conservative bound

1/1 point (graded)

As in the video above, let $R_1, \ldots, R_n \stackrel{iid}{\sim} \textsf{Ber}(p)$ for some unknown parameter p. We estimate p using the estimator $\hat{p} = \overline{R}_n = \frac{1}{n}\sum_{i=1}^{n} R_i$.

Recall that by the central limit theorem, for any p, $(0 < p < 1)$:

$$\lim_{n\to\infty} \mathbf{P}\left(\left|\sqrt{n}\frac{\overline{R}_n - p}{\sigma_p}\right| < q_{\alpha/2}\right) = \lim_{n\to\infty} \mathbf{P}\left(\overline{R}_n - q_{\alpha/2}\frac{\sigma_p}{\sqrt{n}} < p < \overline{R}_n + q_{\alpha/2}\frac{\sigma_p}{\sqrt{n}}\right) = 1 - \alpha$$

where $\sigma_p = \sqrt{p(1-p)}$.

To construct a confidence interval, we need to replace σ_p above by a number c that does not depend on the unknown parameter p.

Which of the following conditions on c will guarantee that for all p in $(0,1)$,

$$\lim_{n\to\infty} \mathbf{P}\left(\left|\sqrt{n}\frac{\overline{R}_n - p}{c}\right| < q_{\alpha/2}\right) \geq 1 - \alpha?$$

(Choose all that apply.)

- [x] $c \geq \sigma_p$ for **all** p
- [ ] $c \geq \sigma_p$ for **some** p
- [x] $c = \max_p (\sigma_p)$
- [ ] $c \leq \sigma_p$ for **all** p
- [ ] $c \leq \sigma_p$ for **some** p
- [ ] $c = \min_p (\sigma_p)$

✔

Solution:

Any number c such that

$$\left(\overline{R}_n - q_{\alpha/2}\frac{c}{\sqrt{n}}, \overline{R}_n + q_{\alpha/2}\frac{c}{\sqrt{n}}\right) \supseteq \left(\overline{R}_n - q_{\alpha/2}\frac{\sigma_p}{\sqrt{n}}, \overline{R}_n + q_{\alpha/2}\frac{\sigma_p}{\sqrt{n}}\right) \qquad \text{for all } p$$

will give the required probability for all p. Hence any $c \geq \max_p (\sigma_p)$ works.

Note: In this example, since $\sigma_p = \sqrt{p(1-p)},\ \max_p(\sigma_p) = \max_p\left(\sqrt{p(1-p)}\right) = 1/2.$

Submit You have used 1 of 2 attempts

Answers are displayed within the problem

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