11. Practice problems

Practice with no damping

1/1 point (graded)

Let $f(t)$ be the odd square wave of period 2π with $f(t) = 1$ for $0 < t < \pi$.

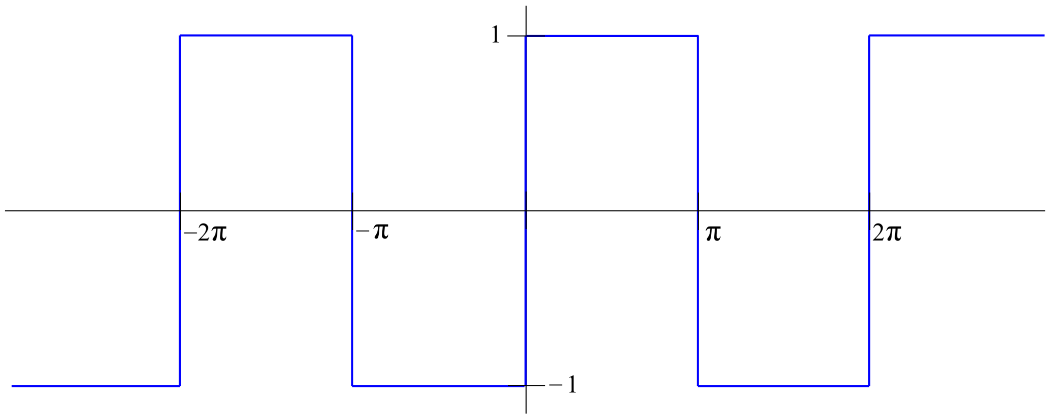

Figure 16: The 2π-periodic square wave.

Use Fourier series to solve the DE

$$\ddot{x} + 9.1x = f(t). \tag{2.4}$$

Enter the sum of the two largest (dominant) terms of the steady periodic response.

4.244131816*sin(3*t)+0.157190067*sin(t)

$$4.244131816 \cdot \sin(3 \cdot t) + 0.157190067 \cdot \sin(t)$$

FORMULA INPUT HELP

Submit You have used 2 of 7 attempts

Correct (1/1 point)

Practice with larger damping

1/1 point (graded)

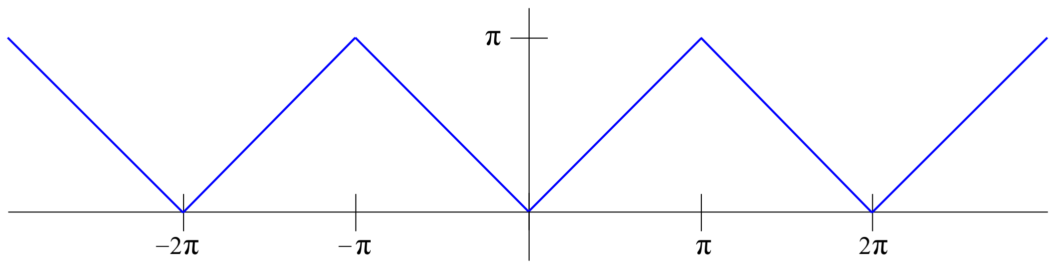

Figure 17: The even 2π-periodic triangle wave.

Let $f(t)$ be the triangle wave shown in the figure above. Solve the differential equation

$$\ddot{x} + 2\dot{x} + 9x = f(t).$$

The steady state response has the form

$$x_p = d_0 + \sum_{n \geq 1} d_n \cos\left(nt - \phi_n\right).$$

Identify the first two largest terms of the steady periodic response.

- [x] The constant term, d_0.
- [x] $d_1 \cos\left(t - \phi_1\right)$
- [ ] $d_3 \cos\left(3t - \phi_3\right)$
- [ ] $d_5 \cos\left(5t - \phi_5\right)$

✔

Solution:

Since $f\left(t\right)$ is an even function, its Fourier series has only cosine terms. The average value $a_0/2$ is $\pi/2$. For $n \geq 1$ (actually we could have used this for $n = 0$ too), the coefficient a_n is given by

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f\left(t\right) \cos nt \, dt = \frac{2}{\pi} \int_{0}^{\pi} t \cos nt \, dt,$$

which can be computed by integration by parts. The result is

$$f\left(t\right) = \frac{\pi}{2} - \frac{4}{\pi}\left(\cos t + \frac{\cos 3t}{3^2} + \frac{\cos 5t}{5^2} + \dots\right).$$

We follow the same steps as in the example in the previous note.

Step 1: Solving for the individual components, we need to solve

$$\ddot{x}_n + 2\dot{x}_n + 9x_n = \cos nt \tag{2.7}$$

for each n. If $n = 0$ we get $x_{n,p} = \frac{1}{9}$.

For $n \geq 1$ we have the complex replacement ODE:

$$\ddot{z}_n + 2\dot{z}_n + 9z_n = e^{int},$$

where $x_n = \mathrm{Re}\,(z_n)$. By the Exponential Response formula:

$$z_{n,p} = \frac{e^{int}}{9 - n^2 + 2ni}.$$

The gain is $1/|9 - n^2 + 2in| = 1/\sqrt{(9-n^2)^2 + 4n^2}$. Let $R_n = \sqrt{(9-n^2)^2 + 4n^2}$ to simplify notation.

Thus, $z_{n,p} = \dfrac{e^{int}}{9 - n^2 + 2in}$, which implies that $x_{n,p} = \dfrac{\cos\,(nt - \phi_n)}{R_n}$.

Step 2: Superposition.

To make things easier in step one we did not include the Fourier coefficients of the input in the DE $\ddot{x}_n + 2\dot{x}_n + 9x_n = \cos nt$. To use superposition we need to include them here:

$$x_{\mathrm{sp}}\,(t) = \frac{\pi}{18} - \frac{4}{\pi}\left(\frac{\cos\,(t - \phi_1)}{R_1} + \frac{\cos\,(3t - \phi_3)}{3^2 R_3} + \frac{\cos\,(5t - \phi_5)}{5^2 R_5} + \dots\right),$$

with the formulas for R_n as above.

The amplitudes of each of the terms in the last line are:

$$\frac{\pi}{18} \approx 0.175,$$

$$\frac{4}{\pi}\left(\frac{1}{\sqrt{\left(9-1^2\right)^2+4\left(1^2\right)}}\right) \approx 0.154,$$

$$\frac{4}{\pi}\left(\frac{1}{3^2\sqrt{\left(9-3^2\right)^2+4\left(3^2\right)}}\right) \approx 0.024$$

$$\frac{4}{\pi}\left(\frac{1}{5^2\sqrt{\left(9-5^2\right)^2+4\left(5^2\right)}}\right) \approx 0.003$$

Therefore the two largest terms are the $n=0$ and $n=1$ terms.

Submit You have used 1 of 7 attempts

Answers are displayed within the problem

11. Practice problems

Hide Discussion

Topic: Unit 1: Fourier Series / 11. Practice problems

Add a Post

All Posts

Practice with larger damping

discussion posted 29 days ago by DBCroatia

Well you are asking for first two largest terms. By clicking to the three of them I've got 0.75 points which is more than when I was clicking to the two terms.

Now I'm going to click them all just to see what's going to happen :) That's because I have three more attempts Wish me luck

This post is visible to everyone.

Add a Response

2 responses

DBCroatia
29 days ago

Party time :) Now I have less points than before but I still have three marked answer despite that we are looking for two of them. and I still have two attempts to lose by having a lots of fun :)

I will put those three because I've been try all combinations with two solutions

posted 29 days ago by **DBCroatia**

@ DBCroatia

I like your spunkiness!

posted 15 days ago by **FFoulenIT**

Add a comment

Steve_Nicodemus (Community TA)
28 days ago

You could try all combinations of 2 since $\binom{4}{2} = 6$. On the other hand, it would probably be more useful to figure out the gain in combination with the coefficients of a triangle wave, or even just think of which terms are likely to be the largest.

It is interesting to look for two terms and by clicking on three answers I get more points than for two answers. How ican it be possible?

posted 26 days ago by **DBCroatia**

Add a comment

Showing all responses

Add a response:

Preview

Submit