13. Determinants

To each **square** matrix $\mathbf{A}$ is associated a number called the **determinant** :

$$\begin{aligned}\det\left(a\right) &:= a\\ \det\begin{pmatrix} a & b \\ c & d\end{pmatrix} &:= ad - bc\\ \det\begin{pmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3\end{pmatrix} &:= a_1 b_2 c_3 + a_2 b_3 c_1 + a_3 b_1 c_2 - c_1 b_2 a_3 - c_2 b_3 a_1 - c_3 b_1 a_2.\end{aligned}$$

Alternative notation for determinant: $|\mathbf{A}| = \begin{vmatrix} a & b \\ c & d\end{vmatrix}$. (This is a scalar, not a matrix!)

Why we care about determinants.

The inverse of a matrix $\mathbf{A}$ exists if and only if $\det\mathbf{A} \neq 0$.

How to compute determinants.

One way to compute determinants is with the aid of a computer algebra system like MATLAB. However, there are methods for computing the determinant of any matrix by hand as well.

Laplace expansion (**along the first row**) for a 3×3 determinant:

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3\end{vmatrix} = +a_1\begin{vmatrix} b_2 & b_3 \\ c_2 & c_3\end{vmatrix} - a_2\begin{vmatrix} b_1 & b_3 \\ c_1 & c_3\end{vmatrix} + a_3\begin{vmatrix} b_1 & b_2 \\ c_1 & c_2\end{vmatrix}.$$

The general rule leading to the formula above is this:

1. Move your finger along the entries in a row.

2. At each position, compute the **minor** , defined as the smaller determinant obtained by crossing out the row and the column through your finger; then multiply the minor by the number you are pointing at, and adjust the sign according to the checkerboard pattern

$$\begin{matrix} + & - & + \\ - & + & - \\ + & - & + \end{matrix}$$

   (the pattern always starts with $+$ in the upper left corner).

3. Add up the results.

There is a similar expansion for a determinant of any size, computed along any row or column.

Practice the Laplace expansion formula

1/1 point (graded)

Suppose that

$$\mathbf{A} = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$$

Calculate $\det \mathbf{A}$. Enter your answer as a number (both fractional and decimal are okay).

$\det \mathbf{A} =$ 0 ✔ **Answer:** 0

Solution:

Using Laplace expansion along the first row we see that

$$\begin{aligned}\det A &= 1 \cdot \det \begin{pmatrix} 5 & 6 \\ 8 & 9 \end{pmatrix} - 2 \cdot \det \begin{pmatrix} 4 & 6 \\ 7 & 9 \end{pmatrix} + 3 \cdot \det \begin{pmatrix} 4 & 5 \\ 7 & 8 \end{pmatrix} \\ &= -3 - 2\,(-6) + 3\,(-3) = 0.\end{aligned}$$

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Determinants and transpose matrices

1/1 point (graded)

The **transpose** of a matrix $\mathbf{A}$ is another matrix $\mathbf{A}^T$ whose columns are the rows of $\mathbf{A}$. For example, the matrix

$$\mathbf{B} = \begin{pmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{pmatrix}$$

is the transpose of the matrix $\mathbf{A}$ of the previous problem.

Calculate $\det \mathbf{B}$. Enter your answer as a number (both fractional and decimal are okay).

$\det \mathbf{B} =$ 0 ✔ **Answer:** 0

Solution:

The Laplace expansion along the first **column** of $\mathbf{B}$ is the same as the Laplace expansion along the first **row** of $\mathbf{A}$ (from the previous problem), so $\det \mathbf{B} = \det \mathbf{A} = 0$.
The same observation about the transpose of any square matrix shows that

$$\det \mathbf{A}^{\mathsf{T}} = \det \mathbf{A}.$$

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