9. Harmonic frequency mathlet

The mathlet below helps you to visualize solutions to the differential equation

$$\ddot{x} + \omega_0^2 x = \omega_0^2 f\left(t\right).$$

In this applet the function $f\left(t\right)$ has a fixed angular frequency of 1. That is, the input signal is 2π-periodic. But, you can choose what 2π-periodic function it is:

- a sine wave $\sin\left(t\right)$,
- the square wave of period 2π,
- the sawtooth wave of period 2π, or
- a 2π-periodic impulse train.

You can adjust the natural frequency ω_0 of the system using the slider. Thus the slider adjusts the resonant frequency of the system.

You should have seen that with ω_0 near 1 the output resembles a frequency 1 sine wave. For ω_0 near 3 the dominant frequency in the output is 3, i.e. there are three peaks in the oscillation over one cycle of the square wave. Likewise for ω_0 near 5 the dominant frequency is 5.

We can explain this using Fourier series. The square wave has Fourier series

$$f(t) = \frac{4}{\pi} \sum_{n \text{ odd}} \frac{\sin nt}{n}.$$

Each term in the series affects the system. If the system has natural frequency (very) close to 3 then the $\sin 3t$ term resonates with a large amplitude. Thus, the response to that term is far larger than the response to any other term.

Hide

Identify the resonant frequencies

2/2 points (graded)

The mathlet below helps you to visualize solutions to the differential equation

$$\ddot{x} + x = f(\omega t).$$

Here the resonant or natural frequency of this system is 1 and is fixed. The slider changes the angular frequency of the input function $f(t)$. Use the mathlet to aid your answer to the following questions.

○ $\omega = 1, 3, 5, \ldots$

○ $\omega = 1, 2, 3, \ldots$

○ $\omega = 1, 1/3$.

○ $\omega = 1, 1/2, 1/3$.

○ $\omega = 1, 1/3, 1/5, \ldots$.

○ $\omega = 1, 1/2, 1/3, 1/4, \ldots$

✓

2. Choose the input $f(\omega t) = Sq(\omega t)$.

Which angular frequencies ω of the input are resonant with the system?

○ Only $\omega = 1$.

○ $\omega = 1, 3, 5, \ldots$

○ $\omega = 1, 2, 3, \ldots$

○ $\omega = 1, 1/3$.

○ $\omega = 1, 1/2, 1/3$.

● $\omega = 1, 1/3, 1/5, \ldots$.

○ $\omega = 1, 1/2, 1/3, 1/4, \ldots$

✓

Solution:

1. A particular solution to

$$\ddot{x} + x = \sin(\omega t) .$$

is given by

$$x_p = \frac{\sin \omega t}{1-\omega^2}, \qquad \omega \neq 1$$

When $\omega = 1$, the system is in resonance, and this is the only input frequency that will result in a resonant response.

2. A particular solution to

$$\ddot{x} + x = \mathrm{Sq}(\omega t) = \frac{4}{\pi} \sum_{n \text{ odd}} \frac{\sin(\omega n t)}{n}.$$

is given by

$$x_p = \frac{4}{\pi} \sum_{n \text{ odd}} \frac{\sin(\omega n t)}{(1-\omega^2 n^2)\, n}$$

When $\omega = 1$, the term $n = 1$ is in resonance with the system. However, there are more terms of resonance! When $n = 3$, and $\omega = 1/3$, the second term in the Fourier series is resonant with the system. Similarly for $n = 5$ and $\omega = 1/5$. Therefore, the response is resonant with the system whenever $\omega = 1/n$ for some odd, positive integer n.

Submit You have used 2 of 3 attempts

Answers are displayed within the problem

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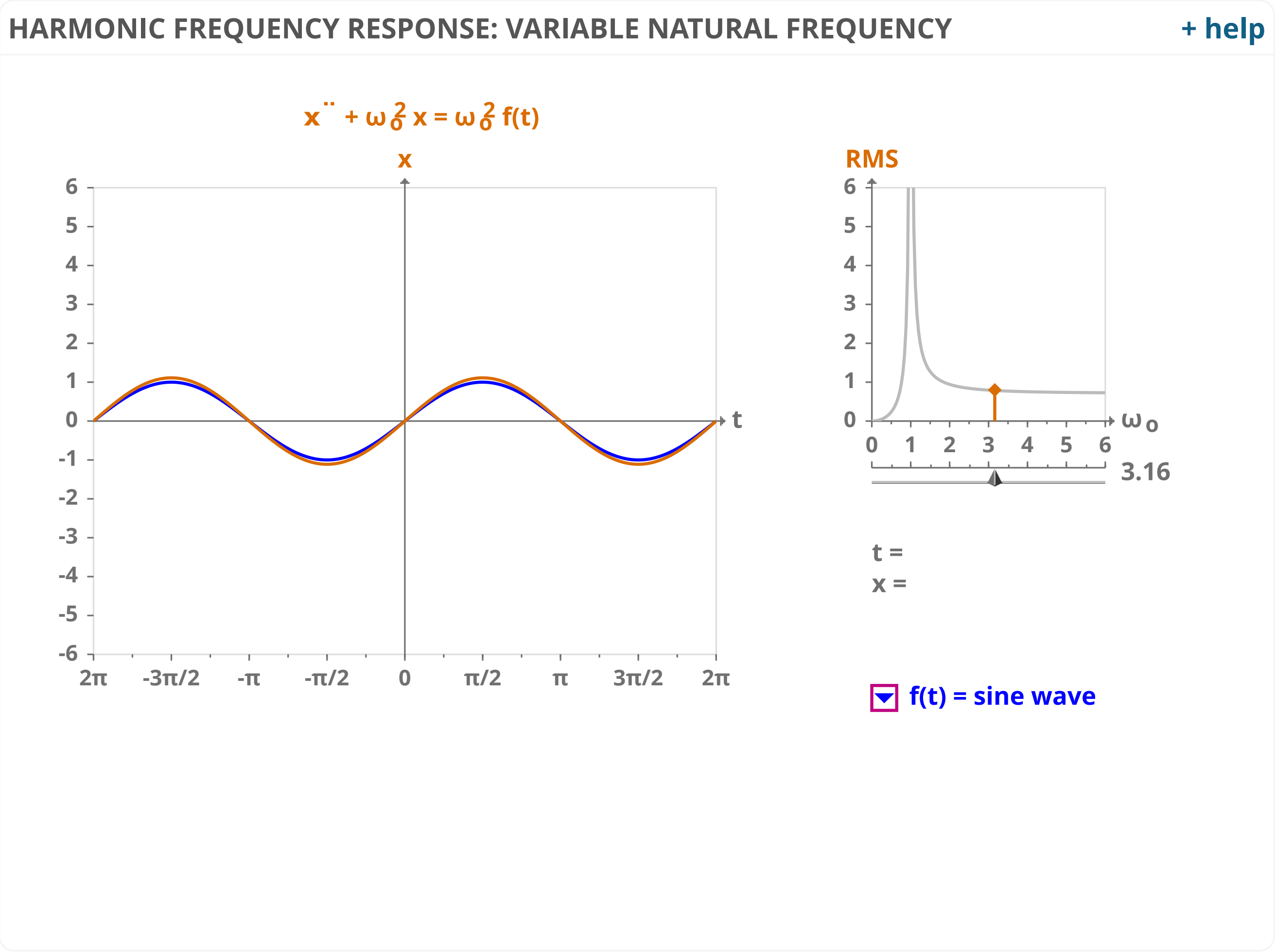

HARMONIC FREQUENCY RESPONSE: VARIABLE NATURAL FREQUENCY

$\ddot{x} + \omega_0^2 x = \omega_0^2 f(t)$

f(t) = sine wave

ω_0 3.16

t =

x =

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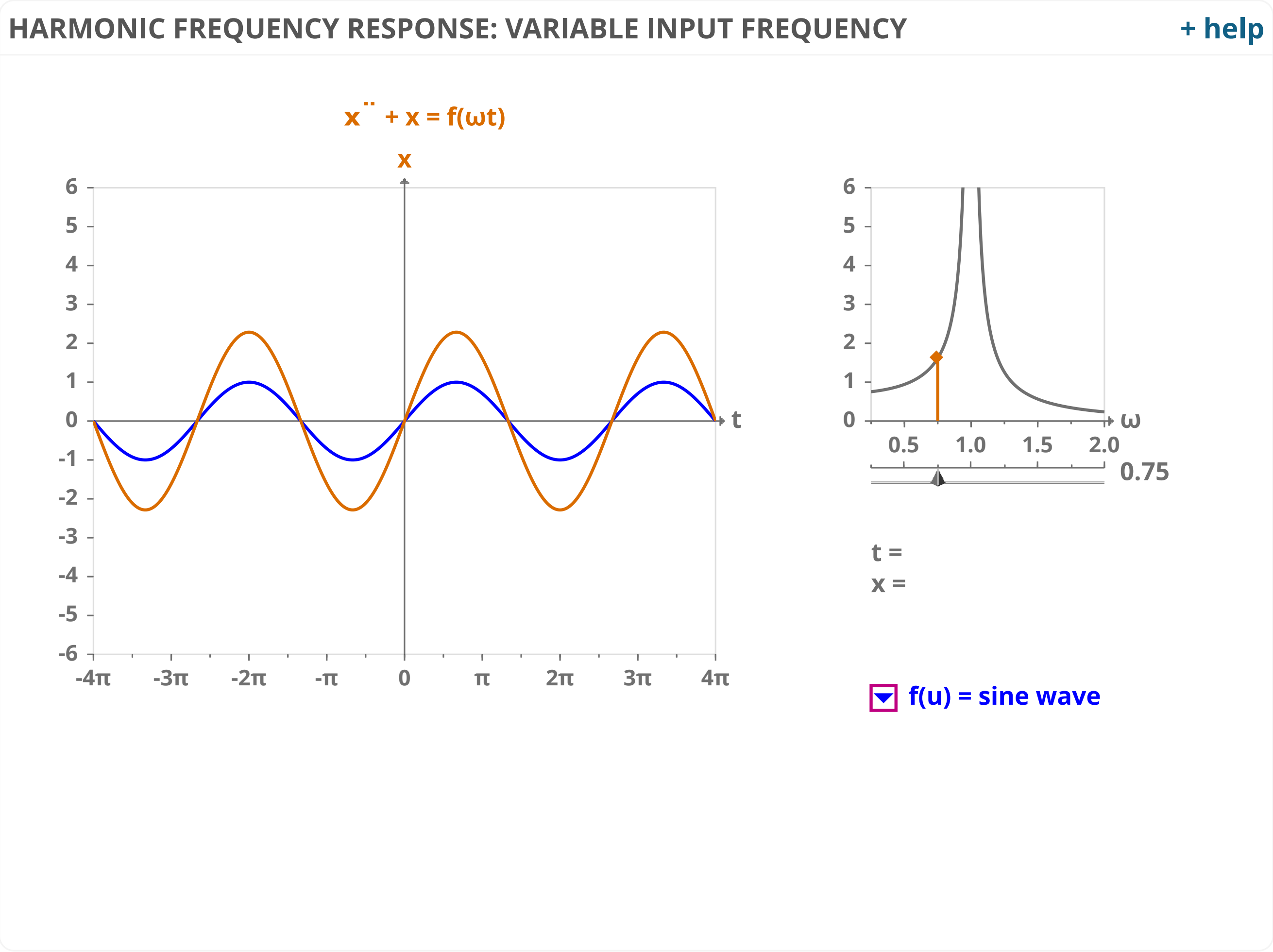
Choose $f(t)$ to be the sine wave. Look at what happens as you change ω_0. Why does the amplitude of the response go to infinity when $\omega_0 = 1$?

Now choose $f(t)$ to be the square wave. Notice that the amplitude of the response becomes infinite at ω_0 = 1, 3 or 5.

Question 9.1 As ω_0 gets close to $1, 3,$ or 5 what is the dominant frequency in the output?

Answer



HARMONIC FREQUENCY RESPONSE: VARIABLE INPUT FREQUENCY

$\ddot{x} + x = f(\omega t)$

ω = 0.75

t =
x =

f(u) = sine wave

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1. Choose the input $f(\omega t) = \sin(\omega t)$.

Which angular frequencies ω of the input are resonant with the system?

● Only $\omega = 1$.