An arithmetical language

The first step in our proof of Gödel's Theorem is to get clear about the language we'll be working with. That will be the aim of the present section.

An **arithmetical language** is a language in which one can talk about the natural numbers and their two basic operations, addition and multiplication. Here we will be working with an especially simple arithmetical language, L, which is (nearly) as simple as it can be while still being rich enough for Gödel's Theorem to hold. The basic ingredients of L are symbols of three different kinds:

Arithmetical Symbol	Meaning
0	names the number zero
1	names the number one
$+$	expresses addition
$\times$	expresses multiplication
$\wedge$	expresses exponentiation

Logical Symbol	Meaning
$=$	expresses identity
$\neg$	expresses negation
$\&$	expresses conjunction
$\forall$	expresses universal quantification
x_n (for $n \in \mathbb{N}$)	[variable]

Auxiliary Symbol	Name
(	left parenthesis
)	right parenthesis

(One way in which L is not as simple as it could be is that the exponentiation symbol "$\wedge$" is not necessary to prove the result. Gödel showed that, with some effort, exponentiation can be defined using "$+$" and "$\times$". I include the exponentiation symbol here because it makes things much simpler.)

Discussion

Hide Discussion

Topic: Week 10 / An arithmetical language

Add a Post

Show all posts by recent activity

There are no posts in this topic yet.