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2.3.2 Exploratory Quiz: What happens when there is fishing?

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Here's the predator-prey system modified to include the effect of fishing. We assume that some proportion e of each population is caught in the nets. How do we capture this in the differential equation?

A general framework that can be useful for differential equations is " rate in - rate out". Let's do this for the sardine population. Without fishing, the "rate in" of sardines is $\mathbf{a}S$, because they grow at a rate proportional to the population. The "rate out" is $\mathbf{b}SM$, the rate of predation by marlin. When we include fishing, this creates another term in the "rate out", the rate of sardines removed due to fishing. Because the fishing is proportional to the amount of sardines, this "rate out" term is eS. Thus for sardines we have

$$\frac{dS}{dt} = \mathbf{a}S - \mathbf{b}SM - eS$$

Marlin are also caught by nets so their differential equation is affected similarly. We have

$$\frac{dS}{dt} = \mathbf{a}S - \mathbf{b}SM - eS$$

$$\frac{dM}{dt} = -\mathbf{c}M + \mathbf{d}SM - eM$$

We can rearrange this system by grouping like terms:

$$\frac{dS}{dt} = (a - e)S - \mathbf{b}SM$$

$$\frac{dM}{dt} = -(c + e)M + \mathbf{d}SM$$

Why do this? Notice this gets the system in the predator-prey form we looked at previously, just with different coefficients. This means we can use what we know about solution trajectories for predatory-prey systems. In particular, we know the trajectories are closed cycles and the average value of the populations along a cycle is exactly the non-zero equilibrium point of the system:

$$(\overline{S}, \overline{M}) = (\frac{c+e}{d}, \frac{a-e}{b}).$$

Let's explore the effect of fishing now. We'll use the parameter values of $a, b, c,$ and d from our previous model: $\mathbf{a} = 0.5, \mathbf{b} = 0.4, \mathbf{c} = 0.2, \mathbf{d} = 0.03$. For each of the following situations, determine whether the average values $\overline{S}$ and $\overline{M}$ increase, decrease or remain unchanged. You may use the expression for the equilibrium point or visualize it with the following dynamic graph of the non-axis nullclines.

Open the Dynamic Graph on nullclines in Desmos

> **Note:** The fishing effort is represented by the letter f (not e) in the dynamic graph.

Question 1

2/2 points (graded)

What happens when we change from no fishing to some fishing, $e = 0$ to $e = .2$?

$\overline{S}$ increases ▼ ✔ **Answer:** increases and $\overline{M}$ decreases ▼ ✔ **Answer:** decreases .

$\overline{S}$ increases, $\overline{M}$ decreases

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Question 2

2/2 points (graded)

What happens when we change from some fishing to no fishing, $e = .2$ to $e = 0$?

$\overline{S}$ decreases ▼ ✔ **Answer:** decreases and $\overline{M}$ increases ▼ ✔ **Answer:** increases .

A: $\overline{S}$ decreases, $\overline{M}$ increases

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Question 3

2/2 points (graded)
What happens when we change from some fishing to less fishing, $e = .2$ to $e = .1$?

$\overline{S}$ decreases ▼ ✔ **Answer:** decreases and $\overline{M}$ increases ▼ ✔ **Answer:** increases .

A: $\overline{S}$ decreases, $\overline{M}$ increases

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Question 4

1/1 point (graded)
Which of the situations above correspond most closely to the reduced fishing that happened during World War I (1914-1918)

- a. No fishing to some fishing $e = 0$ to $e = .2$
- b. Some fishing to no fishing $e = .2$ to $e = 0$
- c. Some fishing to less fishing $e = .2$ to $e = .1$ ✔
- d. None of the above.

Explanation
Choice (c). During World War I there was a decrease in fishing level, but still some fishing.

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Question 5

1/1 point (graded)
Are the changes to the average values of the marlin and sardine populations consistent with D'Ancona's observation that the proportion of predator fish increased World War I (1914-1918)?

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yes, since M increases if e decreases
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Thank you for your response.

Explanation
The model predicts that the average size of the population of marlin (predators) over the length of a cycle will increase and the average population of sardine (prey) will decrease. This is consistent with D'Ancona's observation: the percentage of predator fish in each catch increased.

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Question 6

1/1 point (graded)
Consider the model which includes fishing:

$$\frac{dS}{dt} = \mathbf{a}S - \mathbf{b}SM - eS$$

$$\frac{dM}{dt} = -\mathbf{c}M + \mathbf{d}SM - eM$$

Recall that e is the fishing parameter, so eS is the rate out of sardines due to fishing, and similarly for eM.

Which of the following reason(s) explain why we should restrict fishing levels to $e \leq a$? (Choose all that apply.)

- [x] If $e > a$, then the fishing rate is greater than the reproduction rate of the sardines, so the sardine population would be overfished.
- [ ] If $e > a$, then the fishing rate is greater than the reproduction rate of the marlin, so the marlin population would be overfished.
- [ ] If $e > a$, then the model predicts the average size of the sardine population would be negative which does not make physical sense.

- [x] If $e > a$, then the model predicts the average size of the marlin population would be negative which does not make physical sense. ✔

- [ ] There is no good reason. The model makes sense for all positive values of e.

✔

Explanation

Choice A is correct. When $e > a$, the derivative $\frac{dS}{dt} = \mathbf{a}S - \mathbf{b}SM - eS = (a - e)S - \mathbf{b}SM$ is always negative, so the sardine population is decreasing over time. Another way to say this is that since $e > a$, the rate of fishing is greater than the rate of reproduction of sardines.
Choice D is correct. The average values are the coordinates of the equilibrium point

$$(\overline{S}, \overline{M}) = (\frac{c+e}{d}, \frac{a-e}{b}).$$

If $e > a$, the predicted average value for marlin would be negative, which does not make physical sense. What it means in practice is that at some point during the cycle, the marlin population will reach zero. This is probably because the sardine have been fished to a low-enough level that there are not enough to feed the marlin.

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