1.3.2 Exploratory Quiz: Projectile Motion

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This first quiz is for you to get more familiar with **doing and thinking about math online** and **the quiz format** and **dynamic graphing** we'll use in the course.

- Learning math means trying problems, doing computations, sketching graphs, making observations. Have a **paper and pen or pencil** ready as you work through a quiz.
- Most questions in the course are of the following types: **multiple choice** (one answer), **checkbox** (choose all correct answers) or **drag and drop** (matching style).
- **Think About It...** questions are open response questions, graded on completion only. They are places to reflect on previous problems or explore a new idea.
- **Quizzes** are graded for correctness, but you will have multiple attempts.
- **Exploratory quizzes**, like this one, are graded on completion only. They are meant to get you thinking about an idea before we've fully explained it.

Let's get started!

Peter presented a model for the height of a ball thrown straight up in the air. This is usually called a **projectile motion** model. He described the height of the ball above the ground as:

$$h(t) = 5 + 40t - 16t^2$$

where

- t is in seconds after being thrown in the air
- height is measured in feet above the ground.

We can use this model to answer some questions like how high the ball will reach.

As a reminder, the velocity of the ball at time t is $v(t) = h'(t)$ and the acceleration is at $a(t) = h''(t)$.

(**Side note on projectile motion:** there are variations on this model, for example, if the ball is not thrown straight up, but at an angle with the ground. We will not focus on those in this example.)

Correction

NOTE: In the video, there is an error in the image of the ball being thrown. The model Peter presents is of the height of a ball being thrown directly upwards: all motion is vertical. Thus the video image should be of a ball thrown directly upward, NOT at an angle. (This is specified in the introduction to the the first Exploratory quiz.)

(**NOTE:** If the ball is thrown at an angle like the boy throws the ball in the video, the model must be modified to account for the fact that now the velocity has both a vertical and horizontal component. The function for vertical height will still have the same quadratic form, due to gravity, but the initial velocity for that function is the vertical velocity. See for example, https://en.wikipedia.org/wiki/Projectile_motion.)

Question 1

1/1 point (graded)

When is the ball the highest?

- $t = 32$ seconds
- $t = 40$ seconds
- $t = \frac{40}{32}$ seconds ✔
- $t = \frac{32}{40}$ seconds
- None of the above

Explanation

The ball is highest when $h(t)$ is at a maximum. We know from calculus that a function which is differentiable can only have a maximum or minimum when $h'(t) = 0$ (a critical point). Here $h'(t) = 0$ when $40 - 32t = 0$, so $t = \frac{40}{32}$ is the only critical point. That doesn't necessarily guarantee this gives a maximum value of $h(t)$.
For that we look at the second derivative. We see the function is concave down at that point, using the fact that the second derivative, $h''(t) = -32$, is negative at that point (and everywhere in this case). So $h(t)$ is at a maximum when $t = \frac{40}{32}$ seconds, a little more than a minute after having been thrown up in the air.
Note: since projectile motion is quadratic, you could also solve this problem using what you know about maxima and minima of parabolas.

Submit

Answers are displayed within the problem

Question 2

1/1 point (graded)
The ball hits the ground at $t \approx 2.6$ seconds. (We find this by solving $h(t) = 0$ and choosing the positive t-value from the two solutions to this quadratic equation.) Which of the following would tell you the approximate speed of the ball when it hits the ground?

Select All that Apply

- [ ] $v(2.6)$
- [x] $|v(2.6)|$ ✔
- [ ] $v'(2.6)$
- [ ] $|v'(2.6)|$
- [ ] $h'(2.6)$
- [x] $|h'(2.6)|$ ✔
- [ ] $h''(2.6)$
- [ ] $|h''(2.6)|$
- [ ] None of the above.

✔

Explanation

Speed (how fast the ball is going) is the absolute value of velocity. So if v is the velocity at time t, we want $|v(2.6)|$. Velocity is the derivative of position, so this is the same as $|h'(2.6)|$.

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Answers are displayed within the problem

Question 3

1/1 point (graded)

Peter mentioned that we can use calculus to find the model $h(t)$, by starting with acceleration and integrating to get velocity, then again to get position. Let's go over this, but with a generic constant for gravity. We'll let g represent this constant.

For a positive gravity constant of g ft/sec^2, the acceleration of the ball is$-g$ ft/sec^2. (The negative reflects that gravity is pulling objects back to the ground.)

If we integrate $a(t) = -g$ with respect to t, then we get velocity:

$$v(t) = \int a(t)dt = -gt \text{ plus a constant.}$$

Since we have initial velocity of 40 ft/sec, that constant is 40, so

$$v(t) = -gt + 40.$$

Now let's do it again. Integrate $v(t) = -gt + 40$ with respect to t, and use the fact that the initial height is 5 feet, to get the position function $h(t)$:

Which of the following is correct? (Remember g is a positive constant representing gravity.)

- $h(t) = 5 + 40t - gt^2$
- $h(t) = 5 + 40 - gt^2$
- $h(t) = 5 + 40t - \frac{1}{2}gt^2$ ✔
- $h(t) = 5 + 20t - \frac{1}{2}gt^2$

Explanation

We compute the integral (antiderivative) $\int v(t)\,dt$ and get

$$\int -gt + 40\,dt = -g\frac{1}{2}t^2 + 40t \text{ plus a constant.}$$

Since we have $h(0) = 5$, this means the constant is 5. So the position function is $h(t) = 5 + 40t - \frac{1}{2}gt^2$.

Submit

Answers are displayed within the problem

Question 4: Think About It

1/1 point (graded)

On the moon, gravity is less strong than on earth, about 5:3 feet per second squared, so acceleration is -5.3 feet per second squared. (See "What is Gravity?" from NASA (accessible version))

Let's suppose we are on moon, but the rest of situation is the same: we throw a ball directly upward from a starting height of 5 feet above the ground, and with an initial velocity of 40 feet per second. Again, we assume the only force on the ball is gravity of the moon. How do you think the different gravity of the moon will affect how high the ball will go? the time it takes the ball to reach its highest point? when it will land?

```
the ball will go higher, it will go 5 + 800/5.3 ft.
the time it takes the ball to reach its highest point will
increase, it will be 40/5.3 sec.
it will take longer time to land 40/5.3 + sqrt(1653)/5.3
sec.
```

Thank you for your response.

Explanation

The answer to this question will be revealed as you complete the next few questions.

Submit

Answers are displayed within the problem

Question 5

1/1 point (graded)

One way to explore this question is to think about how the change in gravity g changes the model.

Here's the projectile motion model for the height of a ball thrown straight up at 40 feet per second, from 5 feet above the surface (of the planet, moon, etc.) where gravity is the constant g:

$$h(t) = 5 + 40t - \frac{1}{2}gt^2.$$

We can graph this for different values of g, like $g = 32$ (Earth) and $g = 5.3$ (moon), and compare.

In this course, we'll be using the Desmos graphing tool to display interactive graphs. (If you do not want to use Desmos to plot your own graphs, you can plot this graph for the different values of g on a graphing calculator.)

Open a Desmos graph of $h(t)$.

The graph has $g = 32$, but you can decrease that gravity constant to 5.3 and observe how the graph of $h(t)$ changes.

How does changing g affect the maximum height of the graph? the value of t at which it reaches that height? the intercepts of the graph?

Describe these changes as specifically as you can. How do your observations fit with your previous answer?

```
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sec.
```

Thank you for your response.

Explanation

We observe that as we decrease g the highest point of the graph of $h(t)$ (the maximum) increases and the t-value of the maximum also increases, as well as the horizontal axis intercepts (the times t when the height is zero).

That fits common intuition --- less gravity on the moon means less "pull" on the object so the ball is not "pulled down" to the ground as fast as on Earth. This means it can travel higher and longer before falling back to the ground.

Submit

Answers are displayed within the problem

Question 6

1/1 point (graded)

You probably observed or guessed that on the moon, because there is less gravity (5.3 ft/s^2 versus $32 ft/s^2$ on the Earth), the ball will go higher and will take more time to get there compared to on Earth.

To confirm this, use calculus to solve for the **maximum height of a ball thrown upwards from 5 feet at 40 ft/sec** in terms of the positive constant g.

Here's the model again:

$$h(t) = 5 + 40t - \frac{1}{2}gt^2.$$

- maximum height is $h = \frac{40}{g}$
- maximum height is $h = 5 - \frac{800}{g}$

- (selected) maximum height is $h = 5 + \frac{800}{g}$ ✔
- maximum height is $h = 5 + 800g$
- None of the above

Explanation

Taking the derivative with respect to t, we find that $h'(t) = 40 - gt$. This is zero when $t = \frac{40}{g}$. The height reaches its maximum at this time, since the graph of $h(t)$ is concave down $(h''(t) = -g < 0$). Plugging $t = \frac{40}{g}$ in to $h(t)$, we get

$$h = 5 + 40\left(\frac{40}{g}\right) - \frac{1}{2} \cdot g \cdot \left(\frac{40}{g}\right)^2$$
$$= 5 + \frac{800}{g}.$$

Submit

Answers are displayed within the problem

Question 7: Think About It

1/1 point (graded)

Using the expression you chose above, check that as you decrease g the maximum height of the ball increases. Feel free to record anything else you notice.

```
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```

✔

Thank you for your response.

Submit

✔ Correct (1/1 point)

English ▼