5. Justification

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Justify the Chain Rule

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And now if I take delta t very small,
then this guy tends to the differential--
sorry, tends to the derivative df dt.
Remember the definition of df dt is
the limit of this ratio when the time interval delta
t tends to 0.
So that means if I get-- if I choose
smaller and smaller values of delta t,
then these ratios of numbers will actually
tend to some value and that value is the derivative.
So similarly, here delta x over delta
t, when delta t is very small, will
tend to the derivative dx dt.
And similarly for the others.
So-- so that means in particular--
so if we take the limit as delta t tends

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From now on, we will sometimes just use the phrase "chain rule" instead of "multivariable chain rule."

Why is the chain rule true?

On the previous page, we claimed that you could "divide everything by dt" to obtain the chain rule. To justify this requires clearly interpreting differentials. However, the justifications given here are not entirely rigorous, and are just meant to give you an idea of what is going on when we write equations with differentials.

1st attempt

One justification is that, if x, y, and z are all functions of t, then we can compute dx, dy, and dz directly using single-variable calculus.

$$dx = x'(t)\ dt, \quad dy = y'(t)\ dt, \quad dz = z'(t)\ dt. \qquad (6.128)$$

By substitution, we obtain the "chain rule" statement made on the previous page.

$$df = f_x x'(t)\ dt + f_y y'(t)\ dt + f_z z'(t)\ dt \qquad (6.129)$$

$$= (f_x x'(t) + f_y y'(t) + f_z z'(t))\ dt \qquad (6.130)$$

Now we have an equation for df in terms of only t and dt. The coefficient on dt must be the derivative of f with respect to t.

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2nd attempt

Another way to convince ourselves of the chain rule is to replace the d's by Δ's. This removes any ambiguity about the meaning.

We know from linear approximation that

$$\Delta f \approx f_x\,\Delta x + f_y\,\Delta y + f_z\,\Delta z \tag{6.131}$$

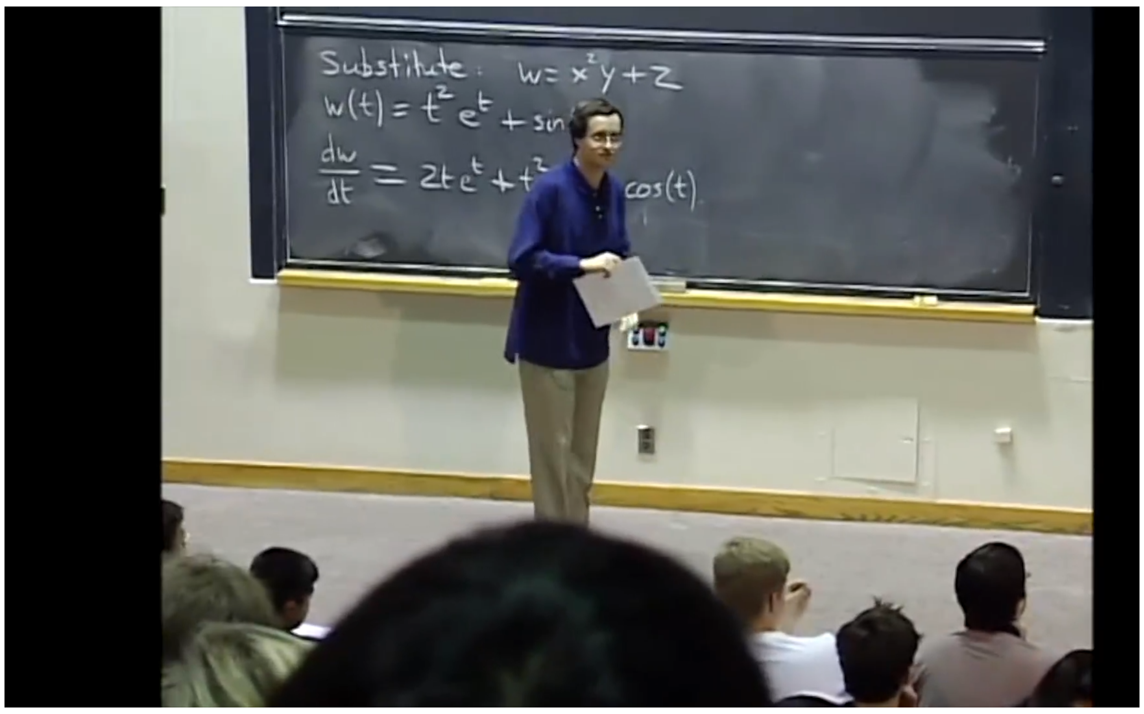
Dividing by Δt gives

$$\frac{\Delta f}{\Delta t} \approx \frac{f_x\,\Delta x + f_y\,\Delta y + f_z\,\Delta z}{\Delta t} \tag{6.132}$$

$$\frac{\Delta f}{\Delta t} \approx f_x\frac{\Delta x}{\Delta t} + f_y\frac{\Delta y}{\Delta t} + f_z\frac{\Delta z}{\Delta t} \tag{6.133}$$

Now if we imagine Δt moving towards zero, we can replace all Δ's with d's to get a correct statement.

Example of the Chain Rule



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And that's the same answer as over there.

So I ended up writing it, maybe I wrote slightly more here

but actually the amount of calculations

really was pretty much the same.

OK, any questions about that?

Yes.

What kind of object is w?

So well I was intending, you can think of w

as just another variable that's given as a function of x, y,

and z, for example.

So you have a function of x, y, z defined by this formula.

And I call it w, I call its value w.

So that then I can substitute t instead of x, y, z.

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Example 5.1 Let's look at an example of the chain rule in practice. Suppose we have a quantity w that depends on x, y, and z as

$$w = x^2 y + z \tag{6.134}$$

Now suppose we can't control x, y, and z directly, but they each depend on the parameter t:

$$x(t) = t \tag{6.135}$$

$$y(t) = e^t$$

$$z(t) = \sin t \tag{6.137}$$

Changing the variable t will cause w to change, and we would like to know the corresponding rate of change $\frac{dw}{dt}$.

The chain rule tells us that

$$\frac{dw}{dt} = \underbrace{(2xy)}_{w_x}\frac{dx}{dt} + \underbrace{x^2}_{w_y}\frac{dy}{dt} + \underbrace{1}_{w_z}\frac{dz}{dt} \tag{6.138}$$

Now substituting in the formulas for x, y, and z gives a final answer:

$$\frac{dw}{dt} = 2te^t + t^2e^t + \cos t \tag{6.139}$$

Checking the answer with the old way

Since the above method used a new technique (the chain rule) let's make sure we get the same answer using our old methods. Namely, in this case, we can write down the full formula for w as a function of t and take a single-variable derivative.

$$w = x^2y + z \tag{6.140}$$

$$w(t) = t^2e^t + \sin t \tag{6.141}$$

$$w'(t) = 2te^t + t^2e^t + \cos t \tag{6.142}$$

Indeed, the answers match.

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