Problem 3

Choose all the integers which is written as the sum of two squares in **two different ways**. (Multiple choices)

45 52 85 116 145

Answer 85 145

$85 = 2^2 + 9^2 = 6^2 + 7^2$

$145 = 1^2 + 12^2 = 8^2 + 9^2$

- How can we find these equalities?

Problem 3

Theorem

If $P \neq Q$ and $P, Q \equiv 1 \pmod 4$,
$N = P \times Q$ is written as the **sum of two squares** in **two different ways**.

Proof

$P = A^2 + B^2 \quad Q = C^2 + D^2$

$$(A^2+B^2)(C^2+D^2) = (AC+BD)^2+(AD-BC)^2$$
$$= (AD+BC)^2+(AC-BD)^2$$

Problem 3

$45 = 3^2 \times 5 = 3^2 \times (1^2+2^2)$

$52 = 2^2 \times 13 = 2^2 \times (2^2+3^2)$

$85 = 5 \times 17 = (1^2+2^2)(1^2+4^2)$

$116 = 2^2 \times 29 = 2^2 \times (2^2+5^2)$

$145 = 5 \times 29 = (1^2+2^2)(2^2+5^2)$