MATHEMATICS

Solving non homogeneous recurrence relation

Asked 8 years, 2 months ago Active 6 years, 11 months ago Viewed 21k times

8

2

I am having a hard time understanding these questions. I know I need to find the associated homogeneous recurrence relation first, then its characteristic equation. I cant figure out how to find the particular solution to the non homo recurrence relation though.

Ex:

$$a_n = 4a_{n-1} + 4a_{n-2} + (n+1)2^n$$

My characteristic equation is $r^2 - 4r - 4 = 0$ and $r = 2(1+\sqrt{2}), r = 2(1-\sqrt{2})$. Next I need to guess some equation for my $f(n) = (n+1)2^n$ and plug it into the original to find some constants,, I am having the trouble here,, I dont understand how to come up with these guess equations.

I know the theorem that says the general solution (of the non homo recurrence relation) is the general solution of the associated recurrence relation + the particular solution: $a_n = a_n^{(h)} + a_n^{(p)}$

So far I have $a_n = A2(1+\sqrt{2})^n + B2(1-\sqrt{2})^n + a_n^{(p)}$

discrete-mathematics recurrence-relations homogeneous-equation

Share Cite Edit Follow Flag

asked Jun 21 '13 at 1:45

htcSpt

81 1 1 2

do you know generating functions? – Alex Jun 21 '13 at 3:36

math.stackexchange.com/questions/393993 – Alex Jun 21 '13 at 3:38

You can assume $a_n^p = (A+Bn)2^n$. Here is a related problem. – Mhenni Benghorbal Jun 21 '13 at 6:44

2 Answers

Active Oldest Votes

6

Forget all this, use generating functions directly. Define $A(z) = \sum_{n \geq 0} a_n z^n$, write:

$$a_{n+2} = 4a_{n+1} + 4a_n + 8(n+3) \cdot 2^n$$

Multiply by z^n, sum over $n \geq 0$, recognize:

$$\begin{aligned}
\sum_{n \geq 0} a_{n+r} z^n &= \frac{A(z) - a_0 - a_1 z - \ldots - a_{r-1} z^{r-1}}{z^r} \\
\sum_{n \geq 0} 2^n z^n &= \frac{1}{1 - 2z} \\
\sum_{n \geq 0} n 2^n &= z \frac{\mathrm{d}}{\mathrm{d}z} \frac{1}{1 - 2z} \\
&= \frac{2z}{(1 - 2z)^2}
\end{aligned}$$

and get:

$$\frac{A(z) - a_0 - a_1 z}{z^2} = 4 \frac{A(z) - a_0}{z} + 4A(z) + \frac{16z}{(1 - 2z)^2} + \frac{3}{1 - 2z}$$

This gives, written as partial fractions (partially):

$$A(z) = \frac{1 - 2a_0 - 2(a_1 - 1)z}{2(1 - 2z^2)} + \frac{1}{2(1 - 2z)}$$

The $(1 - 2z)^{-1}$ gives rise to a 2^n in the solution, while you can write:

$$\begin{aligned}
\frac{1}{1 - 2z^2} &= \sum_{n \geq 0} 2^n z^{2n} \\
\frac{z}{1 - 2z^2} &= \sum_{n \geq 0} 2^n z^{2n+1}
\end{aligned}$$

Thus you get expressions for even/odd indices. Or you could split that into partial fractions too, and mess with the resulting irrationals.

If you are simply interested in *a* particular solution, pick any easy values, like $a_0 = 0$ and $a_1 = 1$, and expand the above.

Share Cite Edit Follow Flag

answered Apr 29 '14 at 17:13
vonbrand
26.2k 6 37 70

3

Make sure you have enough functions to span the non-homogeneous term. In this case, the non-homogeneous term is $n2^n + 2^n$, so I would guess $a_n^{(p)} = An2^n + B2^n$. The reason this is enough is because the space of functions spanned by $\{n2^n, 2^n\}$ is invariant under the *next* operation, and it intersects the space of homogeneous solutions only at 0.

Share Cite Edit Follow Flag

edited Jun 24 '13 at 8:25

answered Jun 21 '13 at 1:59
Tunococ
9,554 25 36

Hmmm, sounds foreign to me,, Is there a general way of finding an^(p)? Do you always just stick a constant to each term and see if you can solve it? – htcSpt Jun 21 '13 at 2:15

There is a general way, by means of generating functions or Z transformation, or variation of parameters. But I would say this method of undetermined coefficients is much simpler in this case. – Tunococ Jun 21 '13 at 2:17