Example with a transformation of one random variable

Suppose X is an exponential random variable with $E(X) = 3$ and suppose $Y = 5X + 7$

① Then $E(Y) = E(5X+7) = 5E(X) + 7 = (5)(3) + 7 = 22$

② $F_Y(a) = P(Y \leq a) = P(5X + 7 \leq a)$

for $a > 7$

$$= P\left(X \leq \frac{a-7}{5}\right)$$

$$= 1 - e^{-((a-7)/5)(1/3)}$$

$$f_Y(y) = \frac{\partial}{\partial y}\left(1 - e^{-((y-7)/5)(1/3)}\right) = \frac{\partial}{\partial y}\left(1 - e^{-(1/15)(y-7)}\right)$$

$$= \frac{1}{15} e^{-(1/15)(y-7)}$$

$$E(Y) = \int_7^\infty (y)\left(\frac{1}{15}\right) e^{-(1/15)(y-7)} \, dy = 22.$$