



## MITx: 6.041x Introduction to Probability - The Science of Uncertainty



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**Exam 1**

Exam 1 due Mar 09, 2016 at 23:59 UTC



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## Problem 2: A binary communication system - Part 1

(4/5 points)

A binary communication system is used to send one of two messages:

- (i) message A is sent with probability  $2/3$ , and consists of an infinite sequence of zeroes,
- (ii) message B is sent with probability  $1/3$ , and consists of an infinite sequence of ones.

The  $i$ th received bit is "correct" (i.e., the same as the transmitted bit) with probability  $3/4$ , and is "incorrect" (i.e., a transmitted 0 is received as a 1, and vice versa), with probability  $1/4$ . We assume that **conditioned on any specific message sent**, the received bits, denoted by  $Y_1, Y_2, \dots$  are independent.

**Note:** Enter numerical answers; do not enter '!' or combinations.

1. Find  $\mathbf{P}(Y_1 = 0)$ , the probability that the first bit received is 0.



Answer: 0.58333

- 2.

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Given that message A was transmitted, what is the probability that exactly 6 of the first 10 received bits are ones? (Answer with at least 3 decimal digits.)

✓ Answer: 0.01622

3. Find the probability that the first and second received bits are the same.

✓ Answer: 0.625

4. Given that  $Y_1, \dots, Y_5$  were all equal to 0, what is the probability that  $Y_6$  is also zero?

✓ Answer: 0.74897

5. Find the mean of  $K$ , where  $K = \min\{i : Y_i = 1\}$  is the index of the first bit that is 1.

✗ Answer: 3.11111

Answer:

1. Let event  $A$  be the case where message A is transmitted. Let event  $B$  be the case where message B is transmitted. Using the total probability theorem we find:

$$\begin{aligned} \mathbf{P}(Y_1 = 0) &= \mathbf{P}(A)\mathbf{P}(Y_1 = 0 \mid A) + \mathbf{P}(B)\mathbf{P}(Y_1 = 0 \mid B) \\ &= (2/3)(3/4) + (1/3)(1/4) \end{aligned}$$

$$= 7/12.$$

2. Given  $A$ , we can consider each bit sent as an independent Bernoulli trial with the probability of getting a 1 equal to  $1/4$ .

$$\mathbf{P}(Y_1 + Y_2 + \dots + Y_{10} = 6 \mid A) = \binom{10}{6} (1/4)^6 (3/4)^4 \approx 0.01622.$$

3. Let event  $C$  be the event that the first and second received bits are the same (i.e.,  $C = \{(Y_1, Y_2) = (0, 0)\} \cup \{(Y_1, Y_2) = (1, 1)\}$ ). Using the total probability theorem we find:

$$\begin{aligned} \mathbf{P}(C) &= \mathbf{P}(A)\mathbf{P}(C \mid A) + \mathbf{P}(B)\mathbf{P}(C \mid B) \\ &= (2/3)(9/16 + 1/16) + (1/3)(1/16 + 9/16) \\ &= 5/8. \end{aligned}$$

4. Using the total probability theorem:

$$\mathbf{P}(Y_1 = 0, \dots, Y_6 = 0) = (2/3)(3/4)^6 + (1/3)(1/4)^6$$

$$\mathbf{P}(Y_1 = 0, \dots, Y_5 = 0) = (2/3)(3/4)^5 + (1/3)(1/4)^5$$

$$\mathbf{P}(Y_6 = 0 \mid Y_1 = 0, \dots, Y_5 = 0) = \frac{\mathbf{P}(Y_1=0, \dots, Y_6=0)}{\mathbf{P}(Y_1=0, \dots, Y_5=0)} \approx 0.74897.$$

5. If message A (respectively, B) is transmitted, then  $K$  is geometric with parameter  $1/4$  (respectively,  $3/4$ ). Therefore, using the total expectation theorem:

$$\begin{aligned} \mathbf{E}[K] &= \mathbf{P}(A)\mathbf{E}[K \mid A] + \mathbf{P}(B)\mathbf{E}[K \mid B] \\ &= \frac{2}{3} \cdot \frac{1}{1/4} + \frac{1}{3} \cdot \frac{1}{3/4} \end{aligned}$$

$$= 28/9$$

*You have used 2 of 2 submissions*

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