5. Swimming in Iceland

Problem Set B due Aug 18, 2021 20:30 IST Completed

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5(a)

2.0/2 points (graded)
You are swimming along the surface of a large natural hot water spring. The temperature is hottest near the geothermal heat sources, and cools off inversely proportional to the distance from the heat source as you move away. The hot spring you have found has two heat sources. One is located below $x = 0$, $y = 0$. The other is located at $x = -10$, $y = 20$.

You do a quick approximation of the temperature of this hot spring to be

$$T(x,y) = \frac{450}{\sqrt{x^2+y^2+1}} + \frac{420}{\sqrt{(x+10)^2+(y-20)^2+1}}$$

where temperature T is measured in Celsius and x and y along the surface of the hot water spring are measured in meters.

You enter the pool on the edge $(20, 20)$ where it is 30 degrees Celsius.

What is the gradient of the temperature at that point?

(Enter answer as a vector quantity with 4 decimal places of accuracy in each component surrounded by square brackets: `[a,b]`.)

[-0.8628929, -0.397003] ✔ **Answer:** [-0.8629, -0.3970]

At the given point, what rate does the temperature rise per unit distance in that direction?

0.9498397 degrees Celsius per meter ✔ **Answer:** 0.9498

Solution:

To figure out the direction the temperature increases fastest at the point $(20, 20)$ we compute the gradient.

$$T_x(x,y) = \frac{-450x}{\sqrt{x^2+y^2+1}^3} + \frac{-420(x+10)}{\sqrt{(x+10)^2+(y-20)^2+1}^3} \tag{3.159}$$

$$T_x(20,20) = \frac{-450(20)}{\sqrt{400+400+1}^3} + \frac{-420(20+10)}{\sqrt{(30)^2+1}^3} \approx -0.8629 \tag{3.160}$$

$$T_y(x,y) = \frac{-450y}{\sqrt{x^2+y^2+1}^3} + \frac{-420(y-20)}{\sqrt{(x+10)^2+(y-20)^2+1}^3} \tag{3.161}$$

$$T_y(20,20) = \frac{-450(20)}{\sqrt{400+400+1}^3} + \frac{-420(0)}{\sqrt{(30)^2+1}^3} \approx -0.3970 \tag{3.162}$$

Thus the temperature increases most quickly in the direction $\langle -0.8629, -0.3970 \rangle$.

The magnitude of the gradient here is the "slope" or how quickly the temperature is ri

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travelled. $|\langle -0.8629, -0.3970\rangle| \approx 0.9498$

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Answers are displayed within the problem

5(b)

3.0/3 points (graded)

You swim in this direction and continue in a straight line until the temperature reaches a balmy 40 degrees Celsius. *Approximately* how far must you swim?

10.52809 meters ✔ **Answer:** 10.5285

What is the position you swim to?

(Enter the point as an ordered pair, separated by commas, and surrounded by round parentheses: `(a,b)`. Each component should be entered to 2 decimal places.)

(10.44, 15.60) ✔ **Answer:** (10.46,15.61)

What is the actual temperature there? (Answer to the nearest degree.)

44 degrees Celsius ✔ **Answer:** 43.96

Solution:

You want to travel in the direction $\langle -0.8629, -0.3970\rangle = \nabla T$ until the temperature has increased by 10 degrees celsius. To get the units right, we first need to convert this direction to a unit vector: $\langle -0.9085, -0.4180\rangle = \frac{\nabla T}{|\nabla T|}$.

Now we can use linear approximation to find the distance we need to travel to increase the temperature by 10.

$$10 = \Delta T \approx T_x\left(-0.9085d\right) + T_y\left(-0.4180d\right) = \nabla T \cdot \frac{\nabla T}{|\nabla T|} d = |\nabla T| d$$

Therefore $d = 10/|\nabla T| \approx 10.5$ meters.

Your position after swimming is $\left(20 - 0.9085\left(10.5\right), 20 - 0.4180\left(10.5\right)\right) \approx (10.46, 15.61)$.

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Answers are displayed within the problem

5(c)

3.0/3 points (graded)

Your sense of direction isn't the best. Instead of swimming in the direction of the gradient, you swim in the direction $\langle -1, 0\rangle$. How fast is the temperature changing the moment you begin swimming (i.e. at the point $(20, 20)$)? (Give at least 2 decimal places.)

0.8628929 degrees Celsius per meter ✔ **Answer:** 0.8629

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You swim 9 meters in the direction $\langle -1, 0 \rangle$. What is the *actual* temperature where you end up? (Enter to the nearest degree.)

40 degrees Celsius ✔ **Answer:** 39.6733

Which direction should you now start swimming so that the temperature does not change?

(Enter answer as a vector quantity to 4 decimal places in each component surrounded by square brackets: `[a,b]` .)

[0.7546347, -1.3642] ✔ **Answer:** [-0.7546, 1.3642]

Solution:

The rate of change in the direction $\langle -1, 0 \rangle$ is the directional derivative

$$\begin{aligned} D_{\langle -1,0\rangle} T(20,20) &= \nabla T(20,20) \cdot (-1,0) && (3.163) \\ &= -T_x(20,20) = 0.8629 \text{ degrees Celsius per meter} && (3.164) \end{aligned}$$

If you swim 9 meters in the negative x direction, you end up at $(11, 20)$. The temperature at this point is 39.6 degrees Celsius.

We compute the gradient at the point $(11, 20)$. Then we find a vector normal to this vector.

$$\begin{aligned} T_x(x,y) &= \frac{-450x}{\sqrt{x^2+y^2+1}^3} + \frac{-420(x+10)}{\sqrt{(x+10)^2+(y-20)^2+1}^3} && (3.165) \\ T_x(11,20) &= \frac{-450(11)}{\sqrt{121+400+1}^3} + \frac{-420(21)}{\sqrt{(21)^2+1}^3} \approx -1.3642 && (3.166) \\ T_y(x,y) &= \frac{-450y}{\sqrt{x^2+y^2+1}^3} + \frac{-420(y-20)}{\sqrt{(x+10)^2+(y-20)^2+1}^3} && (3.167) \\ T_y(11,20) &= \frac{-450(20)}{\sqrt{121+400+1}^3} + 0 \approx -0.7546 && (3.168) \end{aligned}$$

Therefore one answer is $\langle -0.7546, 1.3642 \rangle$. (Another answer is the opposite direction. Or any other scalar multiple of this direction.)

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Answers are displayed within the problem

5. Swimming in Iceland

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[STAFF] Grader bug for 5(c)
Please check my answer 5(c) third part since the grader has marked my wrong answer as right! 6

[Staff] 5(b)2 Grading Error
I used all my five attempts on 5(b)2, but after seeing the correct answer, I am pretty sure that I have submi... 23

[Staff] PS2B 5(c) Third Question Grading

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[Staff] PS2B 5(c) Third Question Grading 2

? [Staff] Question 5(a) 10
Hi, Im pressed against the deadline. I've done my calculations a couple of times and arrived at the same answer, can you please t...

5.b Starting temperature 1
Use temperature of 30° celsius at (20,20), as given, not the actual temparature of 29.89°. In other words, also round to nearest d...

Misguided emphasis on rounding 7
I find this emphasis on rounding misguided. I believe a good principle for this course would be that any answer within some parti...

[Staff] PS2B 5(a) First Question 1
Just a suggestion to modify the question to clarify a bit more: What is the gradient of the temperature **at that point**?

[STAFF] Question 5a not appearing 3
Error Message: Could not format HTML for problem. Contact course staff in the discussion forum for assistance. Anyone else get...

[Staff] Typo in Solution to 5(a) 3

5a negative vector and "rise" in temp in that direction 4
I have the first part correct with two negative coordinates. I'm very confused how that translates to a "rise" in temperature in tha...

? Question 5(b) is weird 6

5.b Precison of provided numbers. 6
The question says it's 30 degree at the location, but if you plug in the numbers, you get 29.892196897803558 from which I used...

Some python code and a map of the Hot Spring 5
To help me with the calculations, I wrote a few lines of python that one may find useful. So not to spoil the exercise, in the code ...
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