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Exercise: Multiple observations and unknowns

(4/4 points)

Let Θ_1 , Θ_2 , W_1 , and W_2 be independent standard normal random variables. We obtain two observations,

$$X_1 = \Theta_1 + W_1, \quad X_2 = \Theta_1 + \Theta_2 + W_2.$$

Find the MAP estimate $\hat{\theta} = (\hat{\theta}_1, \hat{\theta}_2)$ of (Θ_1, Θ_2) if we observe that $X_1 = 1$, $X_2 = 3$. (You will have to solve a system of two linear equations.)

 $\hat{\theta}_1 =$

1



Answer: 1

 $\hat{\theta}_2 =$

1



Answer: 1

Answer:

As usual, we focus on the exponential term in the numerator of the expression given by Bayes' rule. The prior contributes a term of the form

$$e^{-\frac{1}{2}(\theta_1^2 + \theta_2^2)}.$$

Conditioned on $(\Theta_1, \Theta_2) = (\theta_1, \theta_2)$, the measurements are independent. In the conditional universe, X_1 is normal with mean θ_1 , X_2 is normal with mean $\theta_1 + \theta_2$, and both variances are 1. Thus, the term $f_{X_1, X_2 | \Theta_1, \Theta_2}$ makes a contribution of the form

$$e^{-\frac{1}{2}(x_1 - \theta_1)^2} \cdot e^{-\frac{1}{2}(x_2 - \theta_1 - \theta_2)^2}.$$

Unit overview

Lec. 14:

Introduction to

Bayesian inference

Exercises 14 due Apr
06, 2016 at 23:59 UTCLec. 15: Linear
models with
normal noiseExercises 15 due Apr
06, 2016 at 23:59 UTC

Problem Set 7a

Problem Set 7a due
Apr 06, 2016 at 23:59
UTCLec. 16: Least
mean squares
(LMS) estimationExercises 16 due Apr
13, 2016 at 23:59 UTCLec. 17: Linear
least mean
squares (LLMS)
estimationExercises 17 due Apr
13, 2016 at 23:59 UTC

Problem Set 7b

Problem Set 7b due
Apr 13, 2016 at 23:59
UTC

Solved problems

Additional
theoretical
material

Unit summary

We substitute $\mathbf{x}_1 = \mathbf{1}$ and $\mathbf{x}_2 = \mathbf{3}$, and in order to find the MAP estimate, we minimize the expression

$$\frac{1}{2}(\theta_1^2 + \theta_2^2 + (\theta_1 - 1)^2 + (\theta_1 + \theta_2 - 3)^2).$$

Setting the derivatives (with respect to θ_1 and θ_2) to zero, we obtain:

$$\hat{\theta}_1 + (\hat{\theta}_1 - 1) + (\hat{\theta}_1 + \hat{\theta}_2 - 3) = 0, \quad \hat{\theta}_2 + (\hat{\theta}_1 + \hat{\theta}_2 - 3) = 0,$$

or

$$3\hat{\theta}_1 + \hat{\theta}_2 = 4, \quad \hat{\theta}_1 + 2\hat{\theta}_2 = 3.$$

Either by inspection, or by substitution, we obtain the solution $\hat{\theta}_1 = 1$, $\hat{\theta}_2 = 1$.

You have used 1 of 3 submissions

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