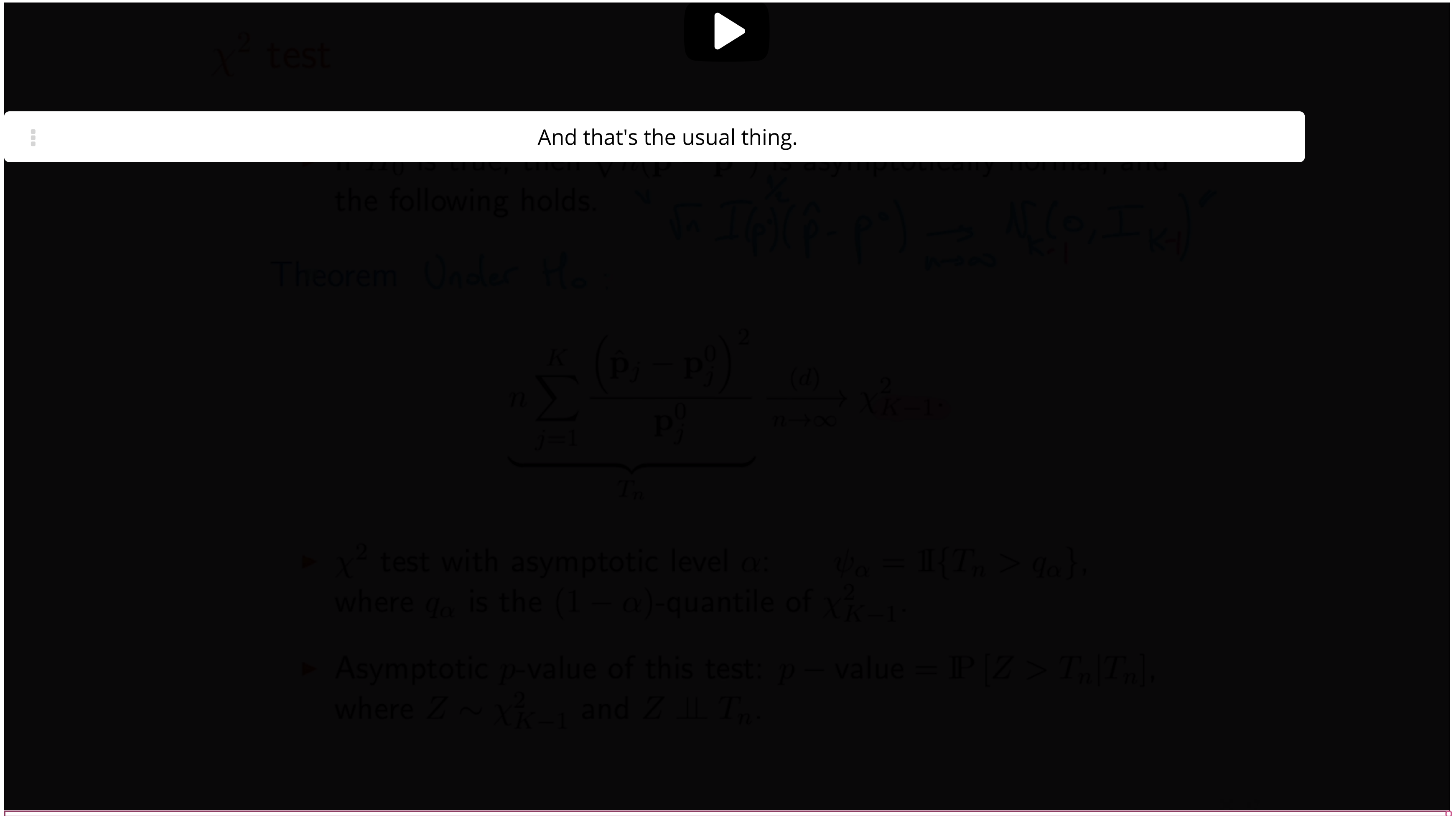
10. The Chi-Squared Test - A Few Thoughts

The Correct Number of Degrees of Freedom Matters in the Chi-Squared Test



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The Chi-Squared Test for Two Modalities

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Note: This problem is presented in the following video, but we encourage you to try it out (or think about it) before watching the video.

Consider the χ^2 test statistic for $K = 2$:

$$T_n = n \sum_{j=1}^{2} \frac{\left(\hat{p}_j - p_j^0\right)^2}{p_j^0}.$$

We can use this statistic in a chi-squared test with 1 degree of freedom to determine, with an asymptotic level α, whether the observed iid samples follow the distribution $\mathrm{Ber}\left(p_2^0\right)$ under the null hypothesis H_0, with the sample space being the two values $a_1 = 0$ and $a_2 = 1$. The chi-squared test with asymptotic level α is

$$\mathbf{1}\left\{T_n > q_\alpha\right\},$$

where q_α is the $(1-\alpha)$-quantile of the chi-squared distribution with 1 degree of freedom.

Is the following statement true or false? "This test is identical (asymptotically) to Wald's test of the Bernoulli statistical model with parameter p, null hypothesis $H_0 : p = p_2^0$ and alternative hypothesis $H_1 : p \neq p_2^0$, where p_2^0, as defined above, is the probability of $a_2 = 1$ under the null hypothesis."

- (●) True
- ( ) False

✔

Solution:

The answer is true. Wald's test in the above statement is:

$$\mathbf{1}\left\{n \frac{\left(\hat{p}_2 - p_2^0\right)^2}{p_2^0\left(1 - p_2^0\right)} > q_\alpha\right\},$$

where q_α is the $(1-\alpha)$-quantile of the chi-squared distribution with 1 degree of freedom. The chi-squared test statistic can be re-written as:

$$
\begin{aligned}
T_n &= n\sum_{j=1}^{2}\frac{\left(\hat{p}_j - p_j^0\right)^2}{p_j^0} \\
&= n\frac{\left(\hat{p}_1 - p_1^0\right)^2}{p_1^0} + n\frac{\left(\hat{p}_2 - p_2^0\right)^2}{p_2^0} \\
&= n\frac{\left((1-\hat{p}_2) - (1-p_2^0)\right)^2}{1-p_2^0} + n\frac{\left(\hat{p}_2 - p_2^0\right)^2}{p_2^0} \\
&= n\frac{\left(\hat{p}_2 - p_2^0\right)^2\left(p_2^0 + 1 - p_2^0\right)}{p_2^0\left(1-p_2^0\right)} \\
&= n\frac{\left(\hat{p}_2 - p_2^0\right)^2}{p_2^0\left(1-p_2^0\right)},
\end{aligned}
$$

which is the same as the test statistic for Wald's test.

Submit You have used 1 of 1 attempt

Answers are displayed within the problem

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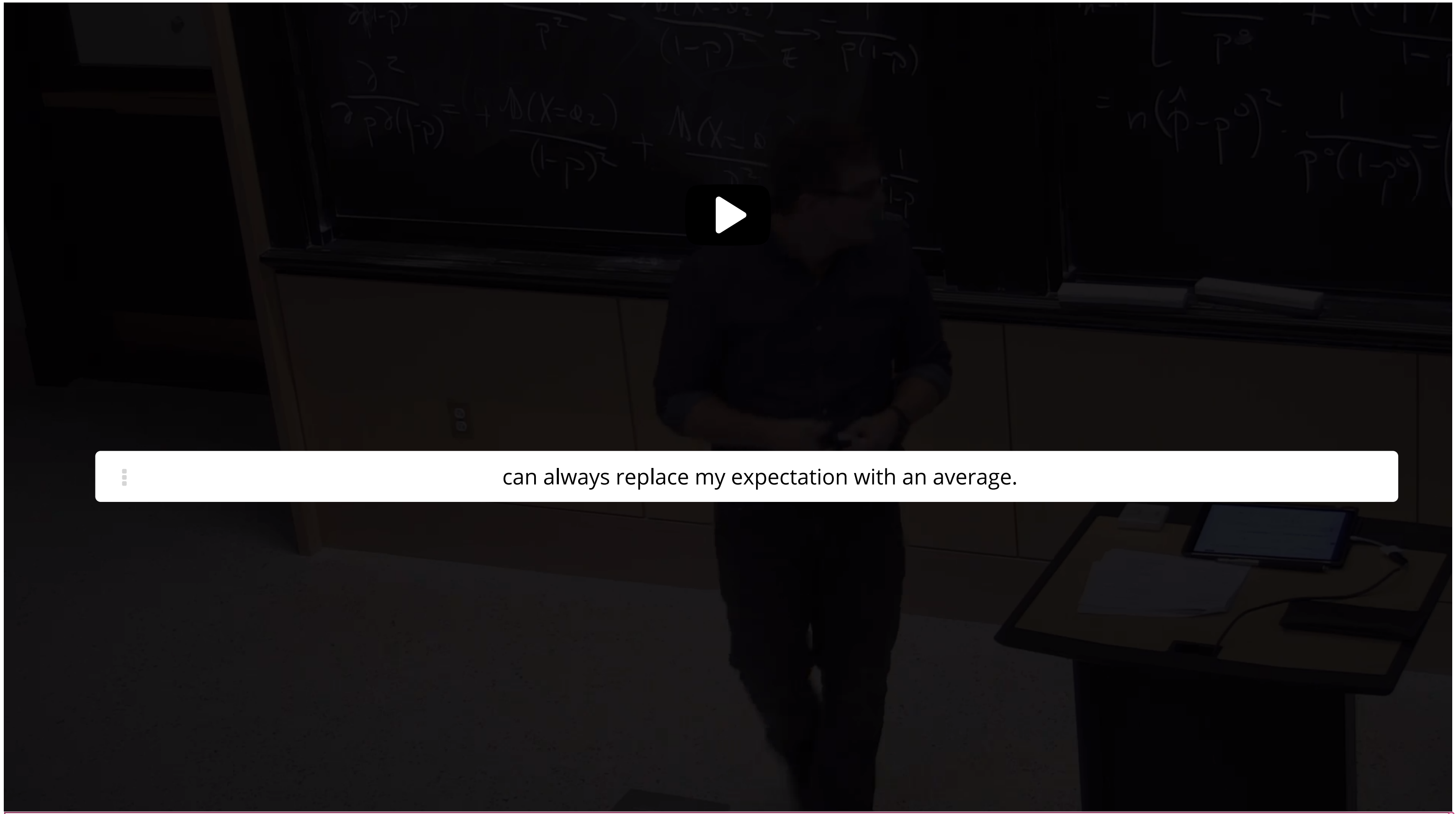

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