10.4.2 Finding the Best Solution

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Week 10 due Dec 16, 2023 07:42 IST

10.4.2 Finding the Best Solution

Video

Start of transcript. Skip to the end.

Dr. Robert van de Geijn: Let's have a look

at what we mean by the best solution and how to find it.

To do so, we're going to simplify the problem slightly more.

We're instead going to look at putting a line through three points,

while before, we were looking at a line

0:00 / 0:00 2.0x

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Reading Assignment

0 points possible (ungraded)

Read Unit 10.4.2 of the notes. [LINK]

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Question

Hi Professor I try the following thing to solve Ax=b: Assume b is not in the column space of A, and x does not exist. b = z + w where w is the vec... 2

Clarification on (A^T*A)^-1

To go from A^T * A * x = A^T * b to (A^T * A)^-1 * A^T * b = x You did some algebra to divide x out to the other side, then flipped 1/(A^T * A) int

Calculator

[Homework 10.4.2.3 - Different Approach](#) 9
Looking at the answer, I feel I wouldn't have reached that line of thought. Perhaps with a little bit more "brain flexing" I'll get there. But a few thin...

[What does it mean to project a plane onto a line?](#) 2
I got stuck right at the beginning on question 10.4.3.2 because I had trouble working out what it meant to project a plane onto a line. I wonder if ...

? [Homework 10.4.2.3](#) 4
In Homework 10.4.2.3, would matrix [1 -1 -1 1] also work? It is a scalar multiple of the answer given.

Homework 10.4.2.1

31/31 points (graded)

Consider $A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{pmatrix}$ and $b = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$.

Answer the following questions related to the computation of the approximate solution, in the least squares sense, of $Ax \approx b$

b is in the column space of A, $\mathcal{C}(A)$.

FALSE ✔ Answer: FALSE

Solution: Row echelon form: $\left(\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & -2 \end{array}\right)$. This means that the last row is inconsistent, hence b is not in the column space.

$A^T b =$
1 ✔ Answer: 1
1 ✔ Answer: 1

$A^T A =$
2 ✔ Answer: 2 | 1 ✔ Answer: 1
1 ✔ Answer: 1 | 2 ✔ Answer: 2

$(A^T A)^{-1} =$
2/3 ✔ Answer: 2/3 | -1/3 ✔ Answer: -1/3
-1/3 ✔ Answer: -1/3 | 2/3 ✔ Answer: 2/3

Best approximate solution:

$\hat{x} = (A^T A)^{-1} A^T b =$
1/3 ✔ Answer: 1/3
1/3 ✔ Answer: 1/3

Projection of b onto the column space:

1/3 ✔ Answer: 1/3
1/3 ✔ Answer: 1/3

Calculator

$\hat{b} = A\hat{x} =$ 1/3 Answer: 1/3

2/3 Answer: 2/3

$b - \hat{b} =$

2/3 Answer: 2/3

2/3 Answer: 2/3

-2/3 Answer: -2/3

Pseudo inverse:

$A^\dagger = (A^T A)^{-1} A^T =$

2/3 Answer: 2/3	-1/3 Answer: -1/3	1/3 Answer: 1/3
-1/3 Answer: -1/3	2/3 Answer: 2/3	1/3 Answer: 1/3

$\hat{x} = A^\dagger b =$

1/3 Answer: 1/3

1/3 Answer: 1/3

$A^\dagger A = (A^T A)^{-1} A^T A =$

1 Answer: 1	0 Answer: 0
0 Answer: 0	1 Answer: 1

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Answers are displayed within the problem

Homework 10.4.2.2

31/31 points (graded)

Consider $A = \begin{pmatrix} 1 & -1 \\ 1 & 0 \\ 1 & 1 \end{pmatrix}$ and $b = \begin{pmatrix} 4 \\ 5 \\ 9 \end{pmatrix}$.

Answer the following questions related to the computation of the approximate solution, in the least squares sense, of $Ax \approx b$

b is in the column space of A, $\mathcal{C}(A)$.

FALSE Answer: FALSE

Solution: Row echelon form: $\left(\begin{array}{cc|c} 1 & -1 & 4 \\ 0 & 1 & 1 \\ 0 & 0 & 3 \end{array}\right)$. This means that the last row is inconsistent, hence b is not in the column space.

Calculator

$A^T b =$

18	Answer: 18
5	Answer: 5

$A^T A =$

3	Answer: 3	0	Answer: 0
0	Answer: 0	2	Answer: 2

$(A^T A)^{-1} =$

1/3	Answer: 1/3	0	Answer: 0
0	Answer: 0	1/2	Answer: 1/2

Best approximate solution:

$\hat{x} = (A^T A)^{-1} A^T b =$

6	Answer: 6
5/2	Answer: 5/2

Projection of b onto the column space:

$\hat{b} = A\hat{x} =$

7/2	Answer: 7/2
6	Answer: 6
17/2	Answer: 17/2

$b - \hat{b} =$

1/2	Answer: 1/2
-1	Answer: -1
1/2	Answer: 1/2

Pseudo inverse:

$A^\dagger = (A^T A)^{-1} A^T =$

1/3 Answer: 1/3	1/3 Answer: 1/3	1/3 Answer: 1/3
-1/2 Answer: -1/2	0 Answer: 0	1/2 Answer: 1/2

$\hat{x} = A^\dagger b =$

6	Answer: 6
5/2	Answer: 5/2

Calculator

$(A^T A)^{-1} A^T A =$

1	✔ Answer: 1	0	✔ Answer: 0
0	✔ Answer: 0	1	✔ Answer: 1

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Answers are displayed within the problem

Homework 10.4.2.3

4/4 points (graded)

What 2×2 matrix A projects the x-y plane onto the line $x + y = 0$?

(This one is tricky. Hint: find a vector on that line...)

$A =$

1/2	✔ Answer: .5	-1/2	✔ Answer: -.5
-1/2	✔ Answer: -.5	1/2	✔ Answer: .5

Answer: Notice that we first need a vector that satisfies the equation: $x = 1$, $y = -1$ satisfies the equation, so all points on the line are in the column space of the matrix $A = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$.
Now the matrix that projects onto the column space of a matrix A is given by

$$\begin{aligned} A(A^T A)^{-1} A^T &= \begin{pmatrix} 1 \\ -1 \end{pmatrix} (\begin{pmatrix} 1 \\ -1 \end{pmatrix}^T \begin{pmatrix} 1 \\ -1 \end{pmatrix})^{-1} \begin{pmatrix} 1 \\ -1 \end{pmatrix}^T \\ &= \begin{pmatrix} 1 \\ -1 \end{pmatrix} (2)^{-1} \begin{pmatrix} 1 \\ -1 \end{pmatrix} \\ &= \frac{1}{2} \begin{pmatrix} 1 \\ -1 \end{pmatrix} \begin{pmatrix} 1 \\ -1 \end{pmatrix}^T \\ &= \frac{1}{2} \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} \\ &= \begin{pmatrix} \frac{1}{2} & \frac{1}{-2} \\ \frac{1}{-2} & \frac{1}{2} \end{pmatrix} \end{aligned}$$

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Answers are displayed within the problem

Homework 10.4.2.4

2/2 points (graded)

Find the line, $y = \gamma_0 + \gamma_1 x$, that best fits the following data:

x	y
-1	2
1	-3
0	0
2	-5

Calculator

$y = \gamma_0 + \gamma_1 x =$ -0.3 ✔ Answer: -3/10 $+$ -2.4 ✔ Answer: -24/10 x

Answer Let $y = \gamma_0 + \gamma_1 x$ be the straight line, where γ_0 and γ_1 are to be determined. Then

$$\begin{aligned} \gamma_0 + \gamma_1(-1) &= 2 \\ \gamma_0 + \gamma_1(\ \ 1) &= -3 \\ \gamma_0 + \gamma_1(\ \ 0) &= 0 \\ \gamma_0 + \gamma_1(\ \ 2) &= -5 \end{aligned}$$

which in matrix notation means that we wish to approximately solve $Ac = b$ where

$$A = \begin{pmatrix} 1 & -1 \\ 1 & 1 \\ 1 & 0 \\ 1 & 2 \end{pmatrix}, \quad c = \begin{pmatrix} \gamma_0 \\ \gamma_1 \end{pmatrix}, \quad \text{and} \quad b = \begin{pmatrix} 2 \\ -3 \\ 0 \\ -5 \end{pmatrix}.$$

The solution to this is given by $c = (A^T A)^{-1} A^T b$.

$$\begin{aligned} A^T A &= \begin{pmatrix} 1 & -1 \\ 1 & 1 \\ 1 & 0 \\ 1 & 2 \end{pmatrix}^T \begin{pmatrix} 1 & -1 \\ 1 & 1 \\ 1 & 0 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 4 & 2 \\ 2 & 6 \end{pmatrix} \\ (A^T A)^{-1} &= \frac{1}{(4)(6)-(2)(2)} \begin{pmatrix} 6 & -2 \\ -2 & 4 \end{pmatrix} = \frac{1}{20} \begin{pmatrix} 6 & -2 \\ -2 & 4 \end{pmatrix} \\ A^T b &= \begin{pmatrix} 1 & -1 \\ 1 & 1 \\ 1 & 0 \\ 1 & 2 \end{pmatrix}^T \begin{pmatrix} 2 \\ -3 \\ 0 \\ -5 \end{pmatrix} = \begin{pmatrix} -6 \\ -15 \end{pmatrix} \\ (A^T A)^{-1} A^T b &= \frac{1}{20} \begin{pmatrix} 6 & -2 \\ -2 & 4 \end{pmatrix} \begin{pmatrix} -6 \\ -15 \end{pmatrix} = \frac{1}{20} \begin{pmatrix} -6 \\ -48 \end{pmatrix} \end{aligned}$$

which I choose not to simplify.
So, the desired coefficients are given by $\gamma_0 = -3/10$ and $\gamma_1 = -12/5$.

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Answers are displayed within the problem

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Calculator