15.2.1 Probability Density Functions (PDFs) and Random Variables

Discussions

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MO2.1 MO2.13

A variable whose value depends on the outcome of a random phenomenon is known as a *random variable*. In the projectile example, the input parameters V_0 and θ_0 are random variables because we have said they have randomness. And, as a result, the impact location x_f is also a random variable (since it depends on these random variables).

A typical notation used for a random variable is capital letters. For example, the impact location would be referred to as X_f and the meaning is that X_f does not have a precise value but rather may take on a range of values. We say that X_f has a distribution of values.

A common way to describe the distribution of a random variable is through its probability density function (or PDF for short). Let the PDF of a variable X be $f(x)$ then the probability that $x - \mathrm{d}x/2 < X < x + \mathrm{d}x/2$ is equal to $f(x)\,\mathrm{d}x$, i.e.,

$$P\left(x - \frac{\mathrm{d}x}{2} < X < x + \frac{\mathrm{d}x}{2}\right) = f(x)\ \mathrm{d}x \tag{15.2}$$

Thus, the PDF is a measure of how likely the value of a random variable is. From this definition of the PDF, the probability that $X \leq x$ is related to $f(x)$ through the following integral,

$$P(X \leq x) = \int_{-\infty}^{x} f(\xi)\ \mathrm{d}\xi \tag{15.3}$$

When we consider all possible values, i.e. $x \to +\infty$, then the probability $P(X \leq +\infty)$ must be one. This means that the integral of $f(x)$ over the entire real numbers must be one,

$$\int_{-\infty}^{+\infty} f(\xi)\ \mathrm{d}\xi = 1 \tag{15.4}$$

Often, we are interested in the probability that X lies between x_a and x_b. That can be found as follows,

$$P(x_a \leq X \leq x_b) = \int_{x_a}^{x_b} f(\xi)\ \mathrm{d}\xi \tag{15.5}$$

The following video discusses PDFs and how to use the `matplotlib hist` command to plot them from an array of data.

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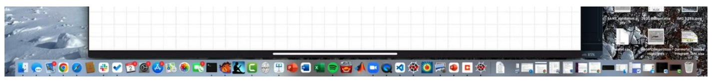

And so X would be a random variable.