The Bomber's Paradox

In this lecture we'll talk about some paradoxes based on reverse ω-sequences.

Josh Parsons, who was a fellow at Oxford until shortly before his untimely death, once told me about the following puzzle. (It is a version of Benardete's Paradox.)

There are infinitely many electronic bombs, $B_0, B_1, B_2, \ldots$, one for each natural number.

They are set to go off on the following schedule:

Bomb	When bomb is set to go off
B_0	12:00pm
B_1	11:30am
B_2	11:15am
$\vdots$	$\vdots$
B_k	$\frac{1}{2^k}$ hours after 11:00am
$\vdots$	$\vdots$

Our bombs are of a special kind: they target electronics. Should one of the bombs go off, it will instantaneously disable all nearby electronic devices, *including other bombs*. This means that a bomb goes off if and only if no bombs have gone off before it.

More specifically:

$$
\begin{array}{ll}
(0) & B_0 \text{ goes off if and only if, for each } n > 0,\ B_n \text{ fails to go off.} \\
(1) & B_1 \text{ goes off if and only if, for each } n > 1,\ B_n \text{ fails to go off.} \\
(2) & B_2 \text{ goes off if and only if, for each } n > 2,\ B_n \text{ fails to go off.} \\
\vdots & \vdots \\
(\mathbf{k}) & B_k \text{ goes off if and only if, for each } n > k,\ B_n \text{ fails to go off.} \\
(\mathbf{k+1}) & B_{k+1} \text{ goes off if and only if, for each } n > k+1,\ B_n \text{ fails to go off.}
\end{array}
$$

Will any bombs go off? If so, which ones? Here's a proof that bomb B_k can't go off:

> Suppose that bomb B_k goes off. It follows from statement (k) above that B_n must fail to go off for each $n > k$. This means, in particular, that B_{k+1} must have failed to go off. But it follows from statement $(k+1)$ that the only way for that to happen is for B_m to go off for some $m > k+1$. And that's impossible: we concluded earlier that B_n must fail to go off for each $n > k$.

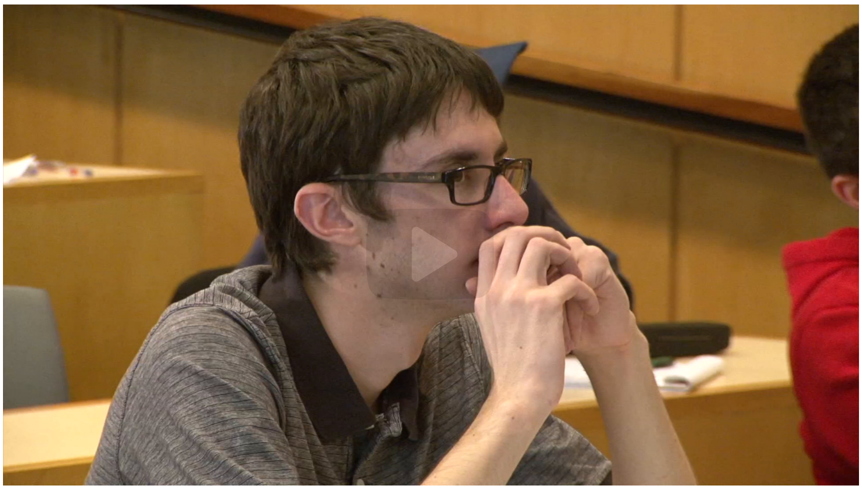
But wait! Here's a proof that bomb B_k must go off:

> Suppose that B_k fails to go off. It follows from statement (k) above that B_n must go off for some $n > k$, and the previous argument shows that that is impossible.

What's going on?

Video Review: The Bombers

Start of transcript. Skip to the end.



So now let's talk about the bomber paradox.

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So the bomber paradox, too, is a reverse omega sequence.

So just to remind you, what's going to happen

is that you have infinitely many people,

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Reverse omega sequence of logical statements.

Let's consider a reverse omega sequence of logical statements which can be true or false. Statem...

3