



[Lecture 21: Introduction to
Generalized Linear Models;](#)

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9. More examples of Continuous
> Distributions

9. More examples of Continuous Distributions

Gamma, Inverse Gamma, and Inverse Gaussian Distribution

Examples of Continuous distributions

The following distributions form continuous exponential families of distributions with pdf:

► Gamma(a, b): $\frac{1}{\Gamma(a)b^a} y^{a-1} e^{-\frac{y}{b}}$;

► above: a : shape parameter, b : scale parameter

► reparametrize: $\mu = ab$: mean parameter

$$\frac{1}{\Gamma(a)} \left(\frac{a}{\mu}\right)^a y^{a-1} e^{-\frac{ay}{\mu}}.$$

► Inverse Gamma(α, β): $\frac{\beta^\alpha}{\Gamma(\alpha)} y^{-\alpha-1} e^{-\beta/y}$.

► Inverse Gaussian(μ, σ^2): $\sqrt{\frac{\sigma^2}{2\pi y^3}} e^{-\frac{\sigma^2(y-\mu)^2}{2\mu^2 y}}$.

Others: Chi-square, Beta, Binomial, Negative binomial distributions.

► 1:15 / 1:15

► 1.50x



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Practice: Gamma distribution as Exponential Family

1/1 point (graded)

Recall from the slides that the Gamma distribution can be reparameterized using the two parameters a , the shape parameter, and μ , the mean. The pdf looks like

$$f_{(a,\mu)}(y) = \frac{1}{\Gamma(a)} \left(\frac{a}{\mu}\right)^a y^{a-1} e^{-\frac{ay}{\mu}}$$

Let $\boldsymbol{\theta} = \begin{pmatrix} a \\ \mu \end{pmatrix}$ and rewrite this as the pdf of a 2-parameter exponential family. Enter $\boldsymbol{\eta}(\boldsymbol{\theta}) \cdot \mathbf{T}(\mathbf{y})$ below.

$\boldsymbol{\eta}(\boldsymbol{\theta}) \cdot \mathbf{T}(\mathbf{y}) =$

-a*y/mu+(a-1)*ln(y)

✓ Answer: -a*y/mu+(a-1)*ln(y)

STANDARD NOTATION

Solution:

$$\begin{aligned} f_{(a,\mu)}(y) &= \frac{1}{\Gamma(a)} \left(\frac{a}{\mu}\right)^a y^{a-1} e^{-\frac{ay}{\mu}} \\ &= \exp \left(\left(-\frac{ay}{\mu} + (a-1) \ln(y) \right) + \left(a \ln \left(\frac{a}{\mu} \right) - \ln(\Gamma(a)) \right) \right) \end{aligned}$$

Hence, we have $\boldsymbol{\eta}(\boldsymbol{\theta}) \cdot \mathbf{T}(\mathbf{y}) = \left(-\frac{ay}{\mu} + (a-1) \ln(y) \right)$, where possibly $\boldsymbol{\eta} = \begin{pmatrix} -\frac{a}{\mu} \\ a-1 \end{pmatrix}$ and $\mathbf{T}(\mathbf{y}) = \begin{pmatrix} y \\ \ln(y) \end{pmatrix}$. Here, $\boldsymbol{\eta}$ and \mathbf{T} are not unique since we can multiple $\boldsymbol{\eta}$ by an overall scalar and divide \mathbf{T} by the same.

On the other hand, $B(\boldsymbol{\theta}) = - \left(a \ln \left(\frac{a}{\mu} \right) - \ln(\Gamma(a)) \right)$.

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❗ Answers are displayed within the problem

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