



MITx: 6.041x Introduction to Probability - The Science of Uncertainty



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Problem 7: Sum of a random number of r.v.'s

(4/4 points)

A fair coin is flipped independently until the first Heads is observed. Let K be the number of Tails observed **before** the first Heads (note that K is a random variable). For $k = 0, 1, 2, \dots, K$, let X_k be a continuous random variable that is uniform over the interval $[0, 3]$. The X_k 's are independent of one another and of the coin flips. Let the random variable X be defined as the sum of all the X_k 's generated before the first Heads. That is, $X = \sum_{k=0}^K X_k$. Find the mean and variance of X . You may use the fact that the mean and variance of a geometric random variable with parameter p are $1/p$ and $(1-p)/p^2$, respectively.

 $\mathbf{E}[X] =$

3



Answer: 3

 $\mathbf{var}(X) =$

6



Answer: 6


Answer:

Since X_k is uniform over $[0, 3]$, we have $\mathbf{E}[X_k] = 3/2$ and $\mathbf{var}(X_k) = 3^2/12 = 3/4$.


▼ Unit 6: Further topics on random variables

Unit overview


Lec. 11: Derived distributions

Exercises 11 due Mar 30, 2016 at 23:59 UTC 

Lec. 12: Sums of independent r.v.'s; Covariance and correlation

Exercises 12 due Mar 30, 2016 at 23:59 UTC 


Lec. 13: Conditional expectation and variance revisited; Sum of a random number of independent r.v.'s

Exercises 13 due Mar 30, 2016 at 23:59 UTC 

Solved problems

Additional theoretical material

Problem Set 6

Problem Set 6 due Mar 30, 2016 at 23:59 UTC 

Unit summary

As the problem states, K is the number of Tails before the first Heads. Therefore, K can be interpreted as the number of failures until the first success, which is not quite a geometric random variable as we have defined it. However, if we add 1 to K , this accounts for the last trial (which is a success), and $K + 1$ can then be interpreted as the number of trials until the first success. Hence, the random variable $N = K + 1$ is geometrically distributed with parameter $p = 1/2$. Thus, $\mathbf{E}[K + 1] = \mathbf{E}[N] = 2$ and $\mathbf{var}(K + 1) = \mathbf{var}(N) = 2$.

Since $X = \sum_{k=0}^K X_k$ is the sum of a random number of independent and identically distributed random variables, we have

$$\mathbf{E}[X] = \mathbf{E}[X_1]\mathbf{E}[K + 1] = 3,$$

and

$$\mathbf{var}(X) = \mathbf{var}(X_1)\mathbf{E}[K + 1] + (\mathbf{E}[X_1])^2\mathbf{var}(K + 1) = \frac{3}{4} \cdot 2 + \frac{9}{4} \cdot 2 = 6.$$

You have used 2 of 2 submissions

DISCUSSION

Click "Show Discussion" below to see discussions on this problem.

- ▶ Unit 7: Bayesian inference
- ▶ Exam 2
- ▶ Unit 8: Limit theorems and classical statistics
- ▶ Unit 9: Bernoulli and Poisson processes
- ▶ Unit 10: Markov chains
- ▶ Exit Survey
- ▶ Final Exam

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