

## L9 PROBLEM 2 (5/5 points)

For the following programs, fill in the best-case and the worst-case number of steps it will take to run each program.

For these questions, you'll be asked to write a mathematical expression. Use +, -, / signs to indicate addition, subtraction, and division. Explicitly indicate multiplication with a \* (ie say "6\*n" rather than "6n"). Indicate exponentiation with a caret (^) (ie "n^4" for  $n^4$ ). Indicate base-2 logarithms with the word log2 followed by parenthesis (ie "log2(n)").

## 1. Program 1:

```
def program1(x):
    total = 0
    for i in range(1000):
        total += i

    while x > 0:
        x -= 1
        total += x

    return total
```

What is the number of steps it will take to run Program 1 in the best case? Express your answer in terms of  $n$ , the size of the input  $x$ .

**Answer:** 3003

What is the number of steps it will take to run Program 1 in the worst case? Express your answer in terms of  $n$ , the size of the input  $x$ .

**Answer:** 5\*n + 3003

**EXPLANATION:**

In the best case scenario,  $x$  is less than or equal to 0. We first execute the assignment `total = 0` for one step. Next we execute the `for i in range(1000)` loop. This loop is executed 1000 times and has three steps (one for the assignment of `i` each time through the loop, as well as two for the `+=` operation) on each iteration. We next check if `x > 0` - it is not so we do not enter the loop. Adding one more step for the return statement, in the best case we execute  $1 + 3 \cdot 1000 + 1 + 1 = 3003$  steps.

In the worst case scenario,  $x$  is a large positive number. In this case, we first execute the assignment `total = 0` for one step. Next we execute the first loop 1000 times (3000 total steps), then we execute the second loop (`while x > 0`)  $n$  times. This loop has five steps (one for the conditional check, `x > 0`, and two each for the `--`

and `+=` operations). When we finally get to the point where `x = 0`, we execute the conditional check `x > 0` one last time - since it is not, we do not enter the loop. Adding one more step for the return statement, in the worst case we execute  $1 + 3 \cdot 1000 + 5 \cdot n + 1 + 1 = 5 \cdot n + 3003$  steps.

## 2. Program 2:

```
def program2(x):
    total = 0
    for i in range(1000):
        total = i

    while x > 0:
        x /= 2
        total += x

    return total
```

What is the number of steps it will take to run Program 2 in the best case? Express your answer in terms of  $n$ , the size of the input `x`.

**Answer:** 2003

What is the number of steps it will take to run Program 2 in the worst case? Express your answer in terms of  $n$ , the size of the input `x`.

**Answer:**  $5 \cdot \log_2(n) + 2008$

### EXPLANATION:

In the best case scenario, `x` is less than or equal to 0. We first execute the assignment `total = 0` for one step. Next we execute the `for i in range(1000)` loop. This loop is executed 1000 times and has two steps (one for the assignment of `i` each time through the loop, as well as one for the `=` operation) on each iteration. We next check if `x > 0` - it is not so we do not enter the loop. Adding in one step for the return statement, in the best case we execute  $1 + 2 \cdot 1000 + 1 + 1 = 2003$  steps.

In the worst case scenario, `x` is a large positive number. In this case we first execute the assignment `total = 0` for one step, then we execute the first loop 1000 times (2000 total steps). Finally execute the second loop (`while x > 0`)  $\log_2(n) + 1$  times. **This is tricky!** Because we divide `x` by 2 every time through the loop, we only execute this loop a logarithmic number of times.  $\log_2(n)$  divisions of `x` by 2 will get us to `x = 1`; we'll need one more division to get `x <= 0`. Don't worry if you couldn't get this fact; we'll go through it a few more times in this unit.

This while loop has five steps (one for the conditional check, `x > 0`, and two each for the `-=` and `+=` operations). When we finally get to the point where `x = 0`, we execute the conditional check `x > 0` one last time - since it is not, we do not enter the loop. Adding in one step for the return statement, in the worst case we execute  $1 + 2 \cdot 1000 + 5 \cdot (\log_2(n) + 1) + 1 + 1 = 5 \cdot \log_2(n) + 2008$  steps.

## 3. Program 3:

```
def program3(L):
    totalSum = 0
    highestFound = None
    for x in L:
        totalSum += x

    for x in L:
        if highestFound == None:
            highestFound = x
        elif x > highestFound:
            highestFound = x

    return (totalSum, highestFound)
```

What is the number of steps it will take to run Program 3 in the best case? Express your answer in terms of  $n$ , the number of elements in the list `L`.



**Answer:** 3

What is the number of steps it will take to run Program 3 in the worst case? Express your answer in terms of  $n$ , the number of elements in the list `L`.



**Answer:**  $7 \cdot n + 2$

#### EXPLANATION:

In the best case scenario, `L` is an empty list. Thus we execute only the first two assignment statements, then the return statement. Therefore in the best case we execute 3 steps. Note that since the list is empty, no assignments are performed in the `for x in L` lines.

In the worst case scenario, `L` is a list with its elements sorted in increasing order (eg, `[1, 3, 5, 7, ...]`). In this case we execute the first two assignment statements (2 steps). Next we execute the first loop  $n$  times. This first loop has three steps (one for the assignment of `x` each time through the loop, as well as two for the `+=` operation), adding  $3 \cdot n$  steps.

Finally we execute the second loop  $n$  times. The first time we execute this loop, we perform 3 steps - one for the assignment of `x`; then we run the check `if highestFound == None`, and finding it to be True, we execute the assignment `highestFound = x`.

The next  $(n-1)$  times we execute this loop, we perform 4 steps: one for the assignment of `x`, then we run the check `if highestFound == None`, and finding it to be False, we run the check `elif x > highestFound`. Since this is always True (the list is sorted in **increasing** order), we execute the assignment `highestFound = x`. Therefore in the second loop we execute  $3 + (n-1) \cdot 4 = 3 + 4 \cdot n - 4 = 4 \cdot n - 1$  steps.

Finally we execute the return statement, which is one more step.

Pulling this all together, we can see that in the worst case we execute  $2 + 3 \cdot n + 4 \cdot n - 1 + 1 = 7 \cdot n + 2$  steps.

Reminder: You do not lose points for trying a problem multiple times, nor do you lose points if you hit "Show Answer". If this problem has you stumped after you've tried it a few times, feel free to reveal the solution.

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