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Questions 1 - 8

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In the first part of the problem set, we will delve more deeply into **auction theory**, which Sara introduced in lecture. We will demonstrate some auction theory properties by performing simulations of data. In these simulations, we will compare different schemes for auctions by varying the number of bidders and valuations. At an auction, **bidders** make offers to buy the goods, and a bidder's **valuation** is how much the bidder offers to pay for the good.

To start, try to understand the following R code. Run the code to test your understanding. We will start with the assumption that there are 2 bidders. We will simulate the auction 1000 times, resulting in 1000 valuations for these 2 bidders. Imagine that you are the person trying to sell a particular good, and that you are using R to figure out the perfect pricing and allocation scheme.

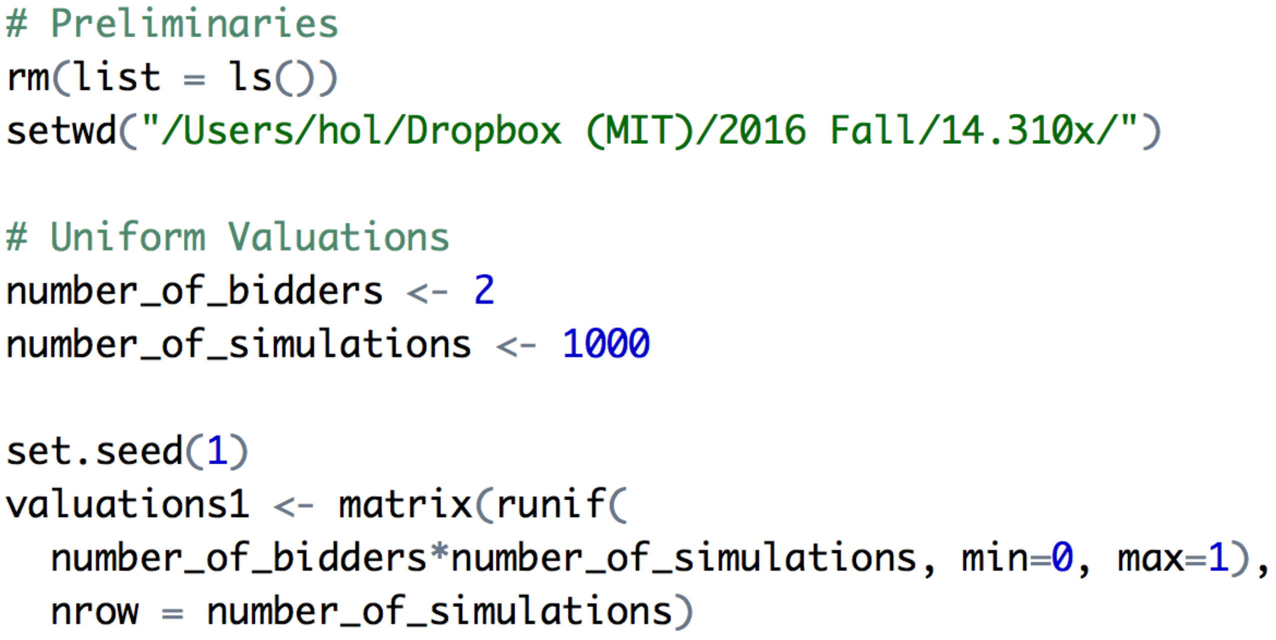
- Module 5: Moments of a Random Variable, Applications to Auctions, & Intro to Regression

Moments of a Distribution and Auctions
Finger Exercises due Oct 31, 2016 at 05:00 IST

Expectation, Variance, and an Introduction to Regression
Finger Exercises due Oct 31, 2016 at 05:00 IST

Module 5: Homework
Homework due Oct 24, 2016 at 05:00 IST

- Exit Survey



```
# Preliminaries
rm(list = ls())
setwd("/Users/hol/Dropbox (MIT)/2016 Fall/14.310x/")

# Uniform Valuations
number_of_bidders <- 2
number_of_simulations <- 1000

set.seed(1)
valuations1 <- matrix(runif(
  number_of_bidders*number_of_simulations, min=0, max=1),
  nrow = number_of_simulations)
```

Question 1

1/1 point (graded)

Try to figure out what the set.seed() command is doing, and then answer the following true or false question:

True or False: Even though we are simulating random numbers, the use of the set.seed() function allows us to have the same valuations each time we re-run this code.

● True ✓

○ False

Explanation

The set.seed() command ensures that we can replicate the same random numbers (and therefore bidder valuations) each time we run this piece of code. This is particularly important because otherwise we will have a different set of valuations each time we run this code.

Submit You have used 1 of 1 attempts

✓ Correct (1/1 point)

Question 2

1/1 point (graded)

Let us consider the **posted price model.** In lecture, we saw that the expected revenue when there are posted prices is given by: $pPr(V_i \geq p)$ for at least one i, where p is the posted price, and v_i is individual i's valuation of the good. Then, the expected revenue is equal to:
$p(1 - Pr(v_{(N)} < p)) = p(1 - F(p)^N)$

What is the optimal price in the case of two bidders, and a U[0,1] distribution for valuations?

○ $\frac{1}{\sqrt[3]{2}}$

○ $\frac{1}{\sqrt[3]{3}}$

● $\frac{1}{\sqrt{3}}$ ✓

○ $\frac{1}{\sqrt{2}}$

Explanation

We maximize the expected revenue to find the optimal price. As stated in lecture, under the assumption that the bidders' evaluations are drawn from a uniform [0,1] distribution, the expected revenue above is equal to $p(1-p^N)$. To maximize the expected revenue, take the derivative of the expected revenue and set it equal to zero. Then solve for p. As shown in lecture, assuming a U[0,1] distribution, we know that the optimal posted price is $p^* = \left(\frac{1}{N+1}\right)^{\frac{1}{N}}$. By substituting $N = 2$, we have that $p^* = \frac{1}{\sqrt{3}}$.

Submit

You have used 1 of 2 attempts

✓ Correct (1/1 point)

Question 3

1/1 point (graded)

Using the same scenario from Question 2, what is the expected revenue for the seller?

● $\frac{2}{3\sqrt{3}}$ ✓

○ $\frac{2}{3\sqrt[3]{2}}$

○ $\frac{2}{3\sqrt[3]{3}}$

○ $\frac{3}{2\sqrt[3]{2}}$

Explanation

In lecture, the expected profit under the optimal price is shown to be $E\left(\pi(p^*)\right) = \left(\frac{1}{N+1}\right)^{\frac{1}{N}} \frac{N}{N+1}$. In this case, we substitute N=2, which results in expected revenue $\frac{2}{3\sqrt{3}}$.

Submit

You have used 1 of 2 attempts

✓ Correct (1/1 point)

Question 4

1/1 point (graded)

Now, we will use the R code above to test whether these predictions hold.

First, let's find the maximum valuation among all 1000 simulations among the two bidders. Name the function in R that allows you to get this value. The function you use should return the maximum valuation when you run $funcName(valuations1)$.

Please enter only the function name (what you typed for $funcName$ *-- no arguments or parentheses!*

max

✓ **Answer:** max

Explanation

The function in R that allows you to get the maximum of a vector is **max**.

Submit

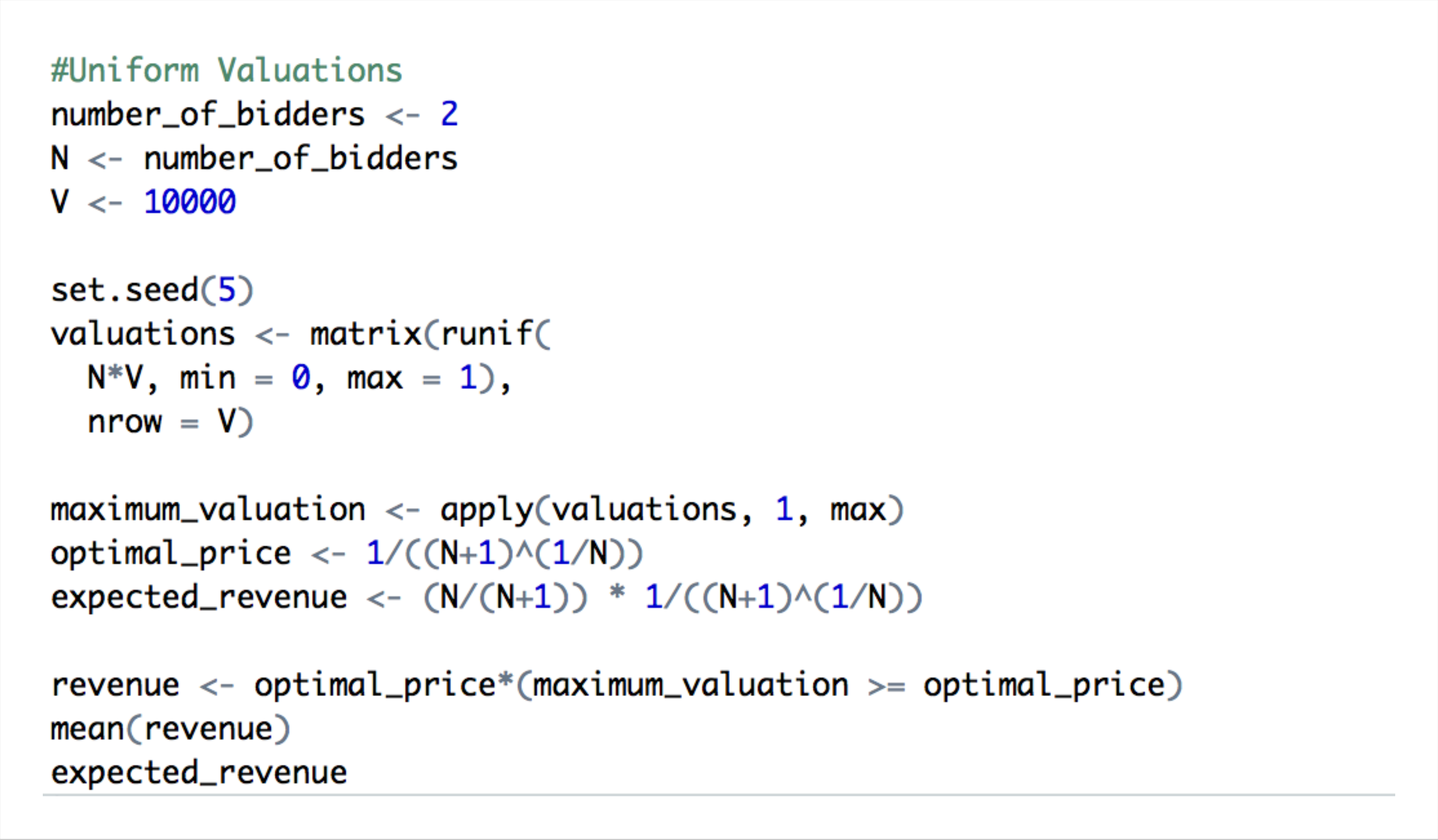
You have used 1 of 2 attempts

✓ Correct (1/1 point)

Question 5

1/1 point (graded)

Take a look at the following R code that calculates the analytic solution to the expected revenue, and compare it with the one coming from the simulation.



```
#Uniform Valuations
number_of_bidders <- 2
N <- number_of_bidders
V <- 10000

set.seed(5)
valuations <- matrix(runif(
  N*V, min = 0, max = 1),
  nrow = V)

maximum_valuation <- apply(valuations, 1, max)
optimal_price <- 1/((N+1)^(1/N))
expected_revenue <- (N/(N+1)) * 1/((N+1)^(1/N))

revenue <- optimal_price*(maximum_valuation >= optimal_price)
mean(revenue)
expected_revenue
```

What variable captures the number of simulations we are using in the code?

Please enter ONLY the name of the variable without any additional text. Make sure that the capitalization matches the code!

V

✓ **Answer:** V

Explanation

If you take a look at the code then you can see that the number of simulations is captured through the variable that we have called **V**.

Submit You have used 1 of 2 attempts

✓ Correct (1/1 point)

Question 6

1/1 point (graded)

Now, perform this exercise for different numbers of simulations: 10, 100, 1000, and 10000. As you increase the number of simulations, does the mean of the numeric revenue vector coincide more or less with the analytic solution?

● It coincides more ✓

○ It coincides less

Explanation

As you increase the number of simulations, you should expect the mean of the simulated data to coincide more with the analytic solution. In general, when you have a greater number of simulations, it is more likely that the values will represent the exact PDF well.

Submit You have used 1 of 1 attempts

✓ Correct (1/1 point)

Question 7

1/1 point (graded)

Now, we will compare the results we just computed, which hold for the posted price model, with the results we would get from an **auction** . Let's consider an English auction, where the buyers' optimal strategy is to stay in the bidding until $p = V_i$ and then leave once $p > V_i$. As shown in lecture, the equilibrium price in this case is the second highest valuation.

What is the expected revenue when there are two bidders $(N = 2)$? Again, assume that bidders' valuations follow a uniform [0,1] distribution.

○ For 2 bidders the expected revenue of an English Auction is $\frac{2}{3}$

● For 2 bidders the expected revenue of an English Auction is $\frac{1}{3}$ ✓

○ For 2 bidders the expected revenue of an English Auction is 1

○ For 2 bidders the expected revenue of an English Auction is $\frac{1}{4}$

Explanation

The expected revenue in this case is equal to the expected vale of the second highest valuation which is given by:

$$\int_0^1 vN(N-1)F(v)^{N-2}(1-F(v))dv = N(N-1)\int_0^1 v^{N-1} - v^N dv = \frac{N-1}{N+1}$$

where N corresponds to the number of bidders. Thus, with two bidders we should have that this expected revenue is equal to $\frac{1}{3}$.

Submit You have used 1 of 2 attempts

✓ Correct (1/1 point)

Question 8

1/1 point (graded)

What is the minimum number of bidders such that a buyer prefers to sell the good in an English Auction rather than at a posted price auction?

Note: You can do this in two different ways: one is to solve the question mathematically (difficult!), and the other one is to use the simulation in R to answer the question. To use the simulation in R, you will need to write code that computes the expected revenue in an English Auction and the expected revenue in a posted price auction given some number of bidders. You can then compare the two expected revenues for different numbers of bidders.

○ You will need at least 1 bidder.

○ You will need at least 2 bidders.

● You will need at least 3 bidders. ✓

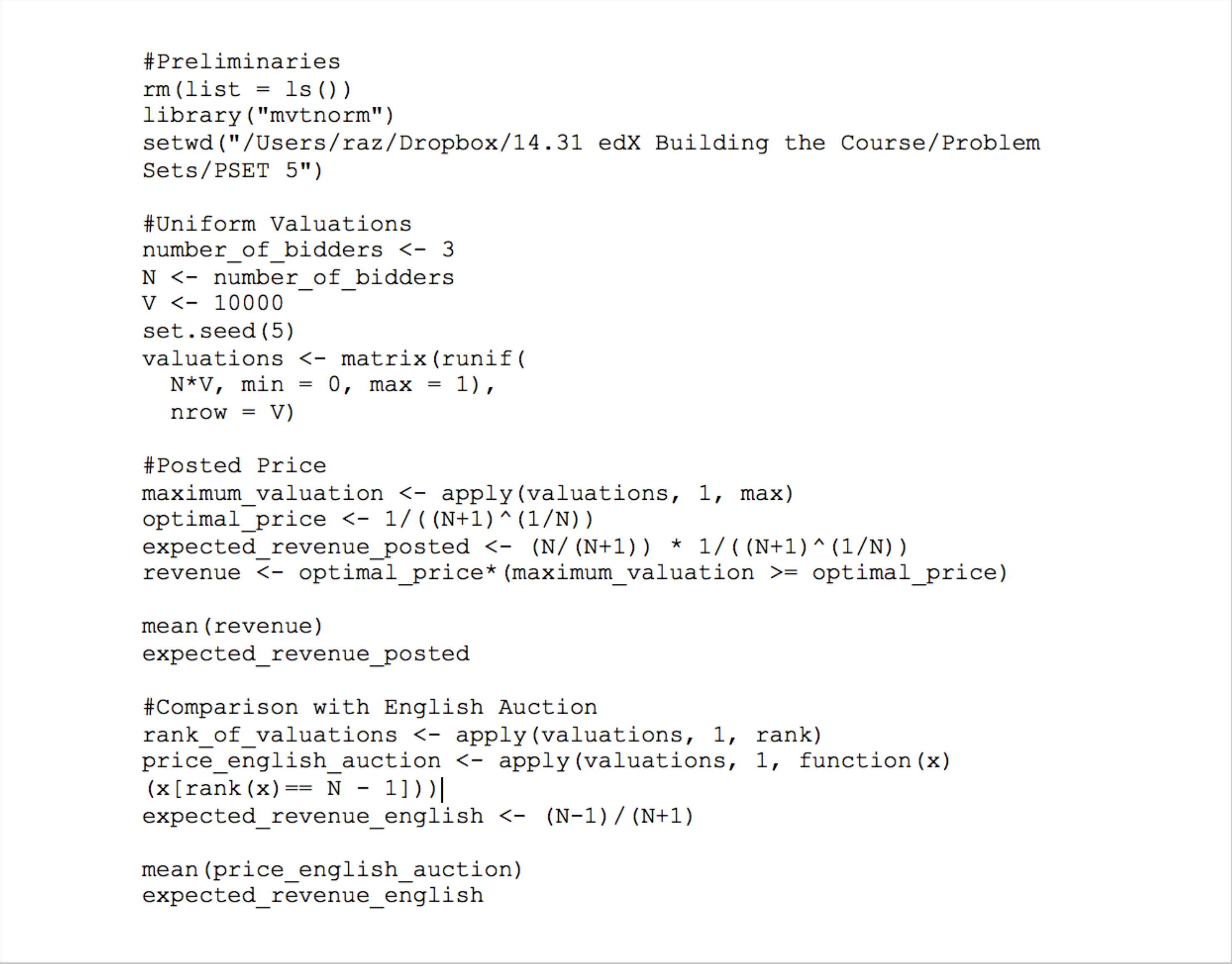
○ You will need at least 4 bidders.

Explanation

You can solve this question either mathematically or using the simulations.

Mathematically, we have that $\frac{N-1}{N+1} > \left(\frac{1}{N+1}\right)^{\frac{1}{N}} \frac{N}{N+1}$. We solve for N to find the number of bidders where the two models will result in equal expected revenues.

We can also solve this problem using a simulation in R. Example code is below. By trying out different numbers of bidders (1 bidder, 2 bidders, 3 bidders, 4 bidders), we see that a buyer will start to prefer an English auction to a posted price auction when $N \geq 3.$



```
#Preliminaries
rm(list = ls())
library("mvtnorm")
setwd("/Users/raz/Dropbox/14.31 edX Building the Course/Problem
Sets/PSET 5")

#Uniform Valuations
number_of_bidders <- 3
N <- number_of_bidders
V <- 10000
set.seed(5)
valuations <- matrix(runif(
  N*V, min = 0, max = 1),
  nrow = V)

#Posted Price
maximum_valuation <- apply(valuations, 1, max)
optimal_price <- 1/((N+1)^(1/N))
expected_revenue_posted <- (N/(N+1)) * 1/((N+1)^(1/N))
revenue <- optimal_price*(maximum_valuation >= optimal_price)

mean(revenue)
expected_revenue_posted

#Comparison with English Auction
rank_of_valuations <- apply(valuations, 1, rank)
price_english_auction <- apply(valuations, 1, function(x)
(x[rank(x)== N - 1]))
expected_revenue_english <- (N-1)/(N+1)

mean(price_english_auction)
expected_revenue_english
```

Submit

You have used 1 of 2 attempts

✓ Correct (1/1 point)