Co Prime Numbers less than N

I need to find all the numbers that are coprime to a given N and less than N. Note that N can be as large as 10^9. For example, numbers coprime to 5 are $1, 2, 3, 4$.

I want an efficient algorithm to do it. Can anyone help?

(algorithms) (prime-numbers)

edited May 18 '15 at 10:11
George Simpson
3,965 3 15 38

asked Dec 29 '13 at 7:13
user3001932
418 5 18

Do you really need to find all the numbers themselves? Or the number of such numbers? – mathlove Dec 29 '13 at 7:25

@mathlove I need to find all the numbers themseleves – user3001932 Dec 29 '13 at 7:26

1 Do you know the factors of N? Then you can use a sieve and you will need memory of 10^9 bits or 125 Megabytes. Time required will be proportional to $N \times k$ where k is the number of primes that divide N. Not sure you can do it any more efficiently in the general case. – user44197 Dec 29 '13 at 7:28

@user44197 Can u please provide an algo for doing so.Suppose i have calculated factors and they are stored in array named say factor and sieve to find all primes storing primes in array say prime. – user3001932 Dec 29 '13 at 7:31

@user3001932: Please see my answer. I am sure you can find ways to make it more efficient. – user44197 Dec 29 '13 at 7:43

4 Answers

Once you find the prime factors $p_1, \ldots, p_k$ of N, you could use a sieve: start with $1 \ldots N-1$, delete all multiples of p_1, then all multiples of p_2, etc. What's left is coprime to N.

answered Dec 29 '13 at 7:30
Robert Israel
192k 8 118 304

This is a complete algorithm no matter how it seems to be difficult. – mathlove Dec 29 '13 at 7:32

What you want is Euler's totient function. You'll find a formula there.

answered Dec 29 '13 at 7:20
mathlove
58.2k 8 55 148

1 I think user3001932 wants to find the numbers themselves, not just how many. – Robert Israel Dec 29 '13 at 7:23

I think that mathlove has provided the answer. However, you need complete factorization of N to do the calculation. If, however, you do not have all the prime factors of N then you have stumbled on what is considered a very difficult problem. But factoring numbers $< 10^9$ is trivial. – user44197 Dec 29 '13 at 7:24

@RobertIsrael: Enumerating all those will take a long time especially of N is a product of two large primes. – user44197 Dec 29 '13 at 7:25

@RobertIsrael yeah..u are right. I need to find all the numbers not their count – user3001932 Dec 29 '13 at 7:27

I assume you know how to implement sets. We will only add numbers $< N$ to the set so any bit representation is okay.

If you know that the primes dividing N then the following works:

```
S={} // Empty sets
for p in primes dividing $N$
   add p,2p,3p... to S
end for
```

If you do not know the factors of N then here is one not very efficient (but not terribly inefficient) way to do it.

```
S={} // Empty set
for k=2 to N
    if k is not in the set S then
          if gcd(k,N) > 1 then
             Add the following to S: k, 2k, 3k, 4k
           end if
     end if
 end for
```

Now all the elements in S are *not* co prime to N, so its complement is the numbers you want

edited Dec 29 '13 at 7:48

answered Dec 29 '13 at 7:41

user44197
8,276 8 14

First find the factors of it by factorisation second get the prime numbers involved in that say $(a, b, \ldots)$. E.g., $18 = (2^1)(3^2)$. The prime factors are 2 and 3. Third formula:

$$\text{No. of coprimes to } N = N(1 - 1/a)(1 - 1/b)\cdots$$

edited May 18 '15 at 9:36

Casteels
8,874 4 18 32

answered May 18 '15 at 9:09

Sabareesh Muralidharan
1 1

But this just finds the number of numbers less than N that are coprime to N. OP wants the actual numbers coprime to N. – Casteels May 18 '15 at 9:37