15.3.9 Confidence Intervals for Probability Estimation

MO2.3 MO2.13 MO2.14 MO2.15

Now we develop the confidence interval for a probability estimates. This confidence interval is derived by re-writing the $\hat{p}_{\text{event}}$ probability estimate given in Equation (15.14) in terms of the mean of an indicator function x:

- $x = 0$: when the event did not occur for an instance
- $x = 1$: when the event did occur for an instance

With this indicator function, then we can prove that the sample mean of x is equal to $\hat{p}_{\text{event}}$:

$$\overline{x} = \frac{1}{N}\sum_{i=0}^{N-1} x_i = \frac{N_{\text{event}}}{N} = \hat{p}_{\text{event}} \tag{15.24}$$

Thus, we can use the properties of the sample mean estimate to derive the properties of the $\hat{p}_{\text{event}}$ estimate. Specifically,

- Since $\overline{x}$ is an unbiased estimate of μ_x, then $\hat{p}_{\text{event}}$ is an unbiased estimate of p_{event}.
- With some algebraic manipulations, the standard error for $\hat{p}_{\text{event}}$ can be shown to be:

$$\sigma_{\hat{p}}^2 \equiv E\left[(\hat{p}_{\text{event}} - p_{\text{event}})^2\right] = \frac{p_{\text{event}}\,(1 - p_{\text{event}})}{N} \tag{15.25}$$

- In practice, the standard error is estimated by assuming $p_{\text{event}} \approx \hat{p}_{\text{event}}$, such that,

$$\sigma_{\hat{p}}^2 \approx \frac{\hat{p}_{\text{event}}\,(1 - \hat{p}_{\text{event}})}{N} \tag{15.26}$$

- A 95% confidence interval for the probability estimate is then,

$$\hat{p}_{\text{event}} - 1.96\sigma_{\hat{p}} < p_{\text{event}} < \hat{p}_{\text{event}} + 1.96\sigma_{\hat{p}} \tag{15.27}$$

The following video discusses this confidence interval for a probability estimate.

The Python script used in this video (and several others) is available here.

Video on confidence intervals for probability estimates

Start of transcript. Skip to the end.

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intervals for our probability estimates?

Video