

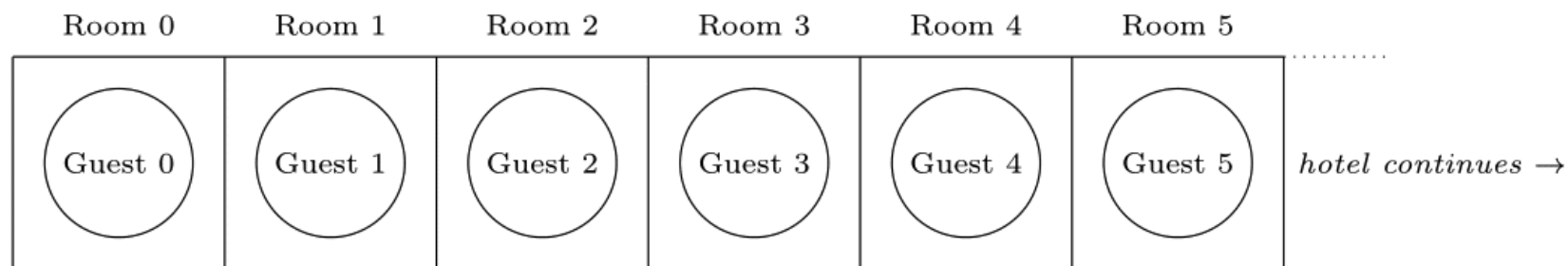
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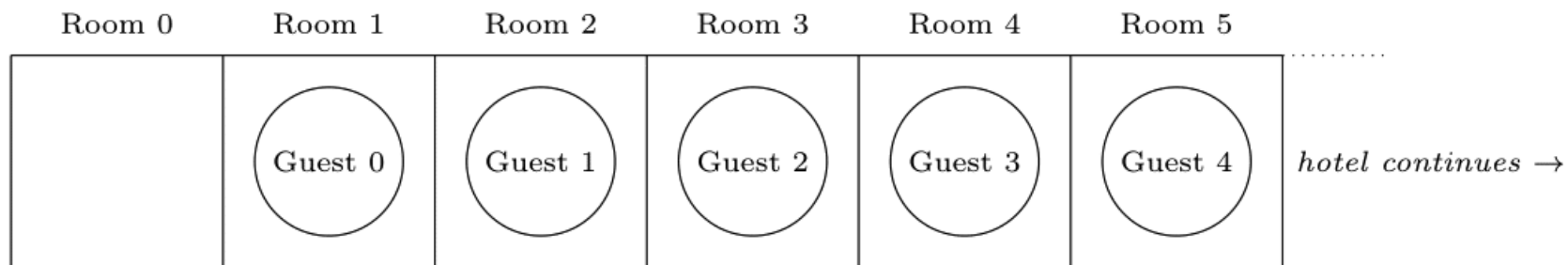
Hilbert's Hotel

Hilbert's Hotel has infinitely many rooms, one for each natural number: $0, 1, 2, 3, \dots$, and every room is occupied: Guest 0 is staying in room 0, Guest 1 is staying in room 1, and so forth:



When a *finite* hotel is completely full, there is no way of accommodating additional guests. In an *infinite* hotel, however, the fact that the hotel is full is not an impediment to accommodating extra guests.

Suppose, for example, that Oscar shows up without a reservation. If the hotel manager wants to accommodate him, all she needs to do is make the following announcement: "Would guest n kindly relocate to room $n + 1$ ". Assuming everyone abides by this request, Guest 0 will end up in room 1, Guest 1 will end up in room 2, and so forth. Under the new arrangement each of the original guests has her own room, and room 0 is available for Oscar!

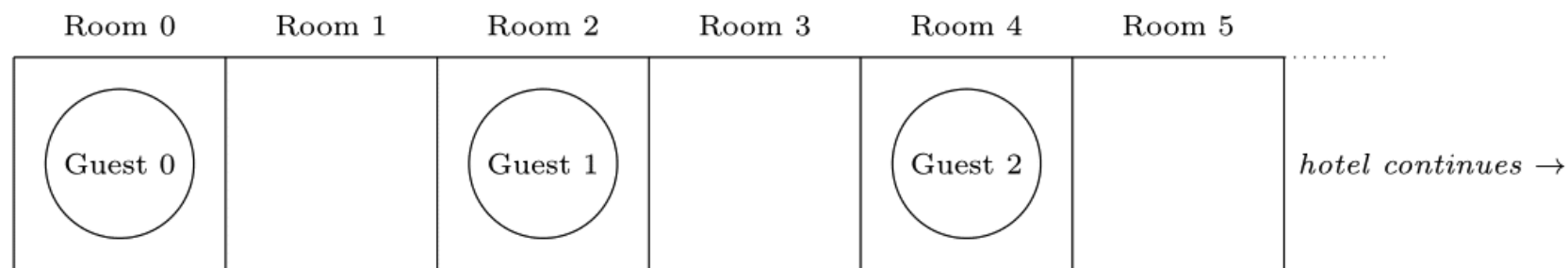


“But what about the guest in the *last* room” —you might wonder— “what about the guest in the room with the *biggest* number?”

The answer, of course, is that there is no such thing as the last room, because there is no such thing as the biggest natural number. Every room in Hilbert’s Hotel is followed by another.

Now suppose 5 billion new guests arrive at our completely full but infinite hotel. We can accommodate them too! This time the manager’s announcement is this: “Would guest n kindly relocate to room $n + 5$ billion”. In a similar way, we can accommodate *any* finite number of new guests!

We can even accommodate *infinitely* many new guests. For suppose the manager announced “if you are in room n , please relocate to room $2n$ ”. Assuming everyone complies, this will free infinitely many rooms while accommodating all of the hotel’s original guests:



Video Review: Hilbert's Hotel



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