Exercise: LMS estimation error

(3/3 points)

As in the previous exercise, let Θ be the bias of a coin, i.e., the probability of Heads at each toss. We assume that Θ is uniformly distributed on $[0,1]$. Let K be the number of Heads in 9 independent tosses. We have seen that the LMS estimate of K is $\mathbf{E}[K \mid \Theta = \theta] = n\theta$.

a) Find the conditional mean squared error $\mathbf{E}\big[\big(K - \mathbf{E}[K \mid \Theta = \theta]\big)^2 \mid \Theta = \theta\big]$ if $\theta = 1/3$.

2 ✔ **Answer:** 2

b) Find the overall mean squared error of this estimation procedure.

3/2 ✔ **Answer:** 1.5

Answer:

a) This is the variance of the conditional distribution of K. Since the conditional distribution is binomial with parameters $n = 9$ and $\theta = 1/3$, the conditional variance is $9(1/3)(2/3) = 2$.

b) This is the average of the conditional variance, averaged over all possible values of the observation Θ, which has a uniform distribution:

$$\begin{aligned}\int_0^1 f_\Theta(\theta)\mathbf{var}(K \mid \Theta = \theta)\, d\theta &= \int_0^1 9\theta(1-\theta)\, d\theta \\ &= \left(9\frac{1}{2}\theta^2 - 9\frac{\theta^3}{3}\right)\Big|_0^1 \\ &= 4.5 - 3 \\ &= 1.5.\end{aligned}$$

You have used 1 of 2 submissions


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