11. Summary

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Summarize

Big Picture

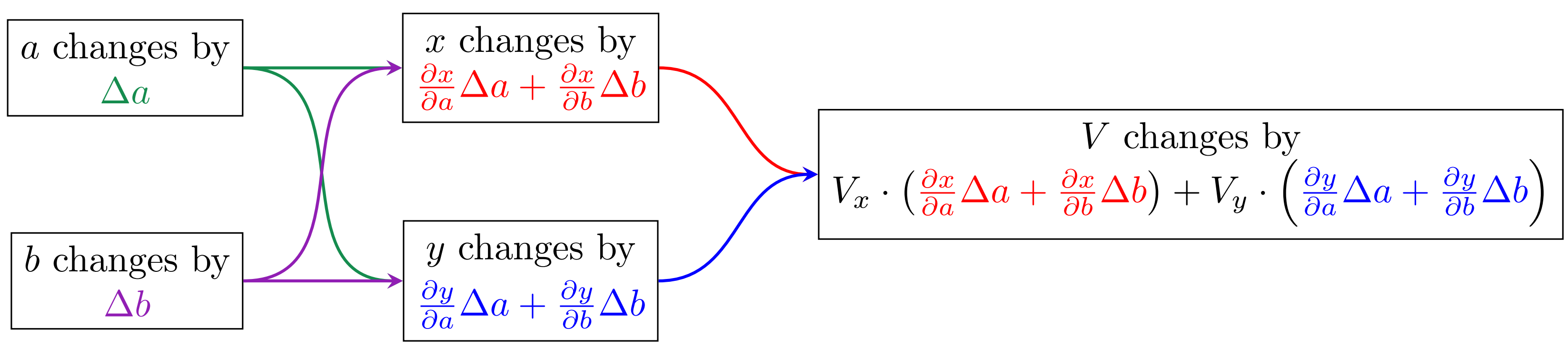
Differentials give us a quick way of understanding how small changes in inputs will affect the change in outputs. By "differentiating" an equation, we obtain a relationship between the differentials, which expresses the linear approximation formula. These differentials dx can be manipulated in a similar way to manipulating Δx's, but they carry extra meaning.

The **total differential** of a variable $w = f(x, y)$ is an equation for dw in terms of all of the differentials of its inputs dx and dy. It is the same as the linear approximation formula, but written with dx, dy instead of $\Delta x, \Delta y$ (and with $=$ instead of $\approx$).

The **multivariable chain rule** is needed when we need to differentiate a function whose inputs are controlled by other variables. The chain rule can be understood in terms of differentials, or in terms of the chain of transformations that bring the input to the output.

Mechanics

The general principle underlying the chain rule is always the same: we get the derivative of a given output with respect to a given input by summing up all the paths from the chosen input to the chosen output, weighted by the product of the appropriate partial derivatives. In this context a "path from the input to the output" is the sequence of transformations that bring the input to the output. For example, if $V = f(x, y)$ and $x = x(a, b)$ then there is a path from $a \to x \to V$ because changing a causes x to change, which causes V to change.



Ask Yourself

When should we use the straight d or the curly d?

The curly d (∂) means partial derivative, so when you are taking a partial derivative you use a curly d such as $\frac{\partial z}{\partial x}$.

The straight d (d) has multiple uses.

First, we always use the straight d for differentials. Even for functions of multiple variables, such as $z = f(x, y)$, we still use straight d's for its total differential: $dz = z_x dx + z_y dy$.

Second, we also use the straight d for the derivative of single-variable functions, such as $y = f(x)$, where the derivative is written $\frac{dy}{dx}$. This second usage is really the same as the first usage, since for a function of one variable $y = f(x)$, its total differential is given by $dy = y'(x)\ dx$, so $\frac{dy}{dx} = y$

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We also always use the straight d when doing integrals, such as $\int_a^b f(x)\,dx$. Even when we do multivariable integrals, you will see that we continue to use straight d's.

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