6. Equations of lines and normal vectors

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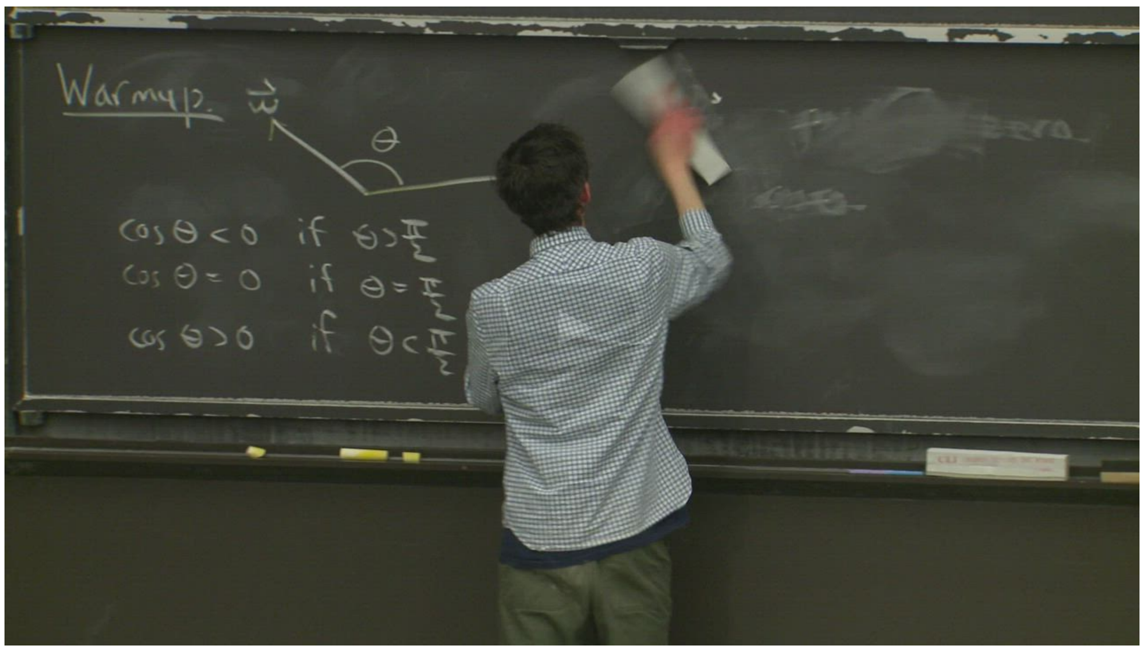
Summarize

Summary



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Start of transcript. Skip to the end.

PROFESSOR: So I use particular numbers,

but the same story works if you have the line-- if you

have ax plus by plus c.

So I'll just record.

The same reasoning shows that if I have a line

ax plus by plus c equals 0, then the vector (a, b)

is perpendicular to the line.

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The vector $\langle a, b\rangle$ is normal to the line $ax + by + c = 0$.

To see that this is true:

1. Pick a point on the line.
2. Draw a copy of $\langle a, b\rangle$ starting at that point.
3. Observe that the line and the vector are perpendicular.

Note that the line $ax + by + c = 0$ is parallel to the line $ax + by = 0$. The equation $ax + by = 0$ is equivalent to $\langle x, y\rangle \cdot \langle a, b\rangle = 0$, which exactly tells us that the vector $\langle a, b\rangle$ is perpendicular to the line $ax + by = 0$. Therefore, if you draw the vector $\langle a, b\rangle$ starting at any point along the parallel line $ax + by + c = 0$, it will also be perpendicular to this line! The two vectors are parallel, and the two lines are parallel. Thus the line $ax + by + c = 0$ and vector $\langle a, b\rangle$ must also be orthogonal.

Definition 6.1 A synonym for the word **perpendicular** is the word **normal**.

Remark 6.2 We typically use the word perpendicular in the context of 2-dimensional vectors, and use the word normal in the context of vectors of dimension 3 and higher. These two words will be used interchangeably in this course.

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Spoiler: Normal in higher dimensions

Two vectors are **normal** if their dot product is zero.

Hide

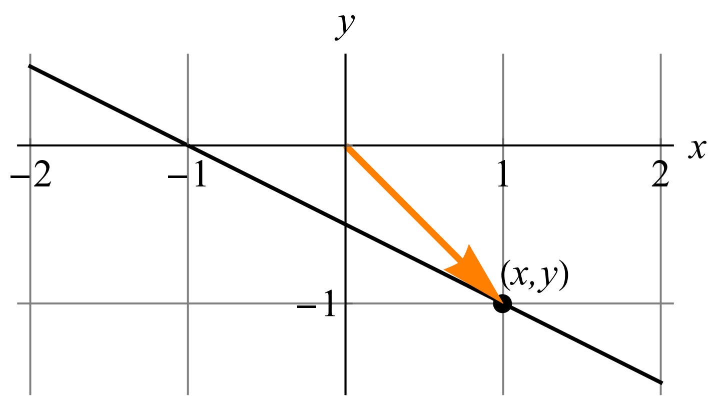
Explore lines not through the origin further

Remark 6.3

Note that the process we describe above is geometric. We cannot use a *hidden dot product* to determine that $\langle 1, 2\rangle$ is perpendicular to the line $x + 2y = -1$.

However, we can describe the equation for this line in terms of a dot product equation. Let's try to understand what the dot product equation is telling us in the case of the line $x + 2y = -1$. Written in terms of a dot product, this is $\langle 1, 2\rangle \cdot \langle x, y\rangle = -1$. The first thing to notice is that the vector $\langle 1, 2\rangle$ and $\langle x, y\rangle$ are not perpendicular!

Let (x, y) be a point on the parallel line $x + 2y = -1$. Then the vector $\langle x, y\rangle$ is a vector that points from the origin to the point (x, y) on the line.



Observe that this vector is not parallel to the line! So it makes sense that the dot product of $\langle x, y\rangle$ with $\langle 1, 2\rangle$ is nonzero.

$$\langle 1, 2\rangle \cdot \langle x, y\rangle = -1$$

To better understand this dot product equation, we rewrite it in terms of the angle between the two vectors:

$$\begin{aligned} \langle 1, 2\rangle \cdot \langle x, y\rangle &= -1 \\ |\langle 1, 2\rangle||\langle x, y\rangle| \cos\theta &= -1 \end{aligned}$$

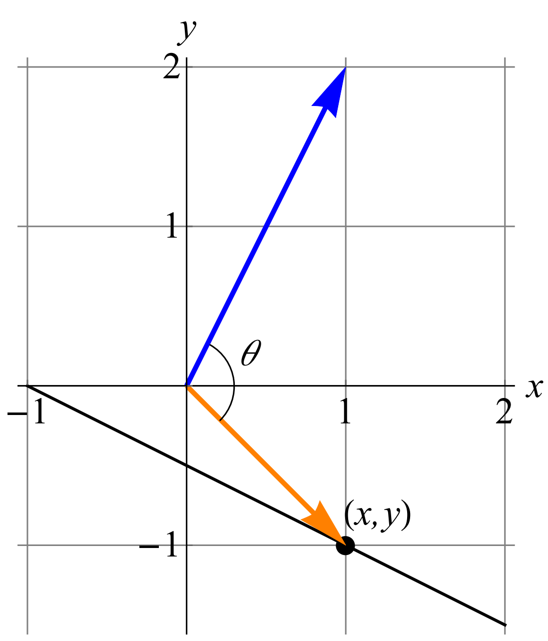


Figure 16: The vector $\langle 1, 2\rangle$ in blue and the vector $\langle x, y\rangle$ in orange both starting from the origin, where (x, y) is a point on the line $x + 2y = -1$ in the fourth qu

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The quantity $|\langle x, y\rangle| \cos\theta$ is the component of the vector $\langle x, y\rangle$ that points in the direction $\langle 1, 2\rangle$. That is, it is (plus or minus) the length of the line connecting the origin to the line $x + 2y = -1$.

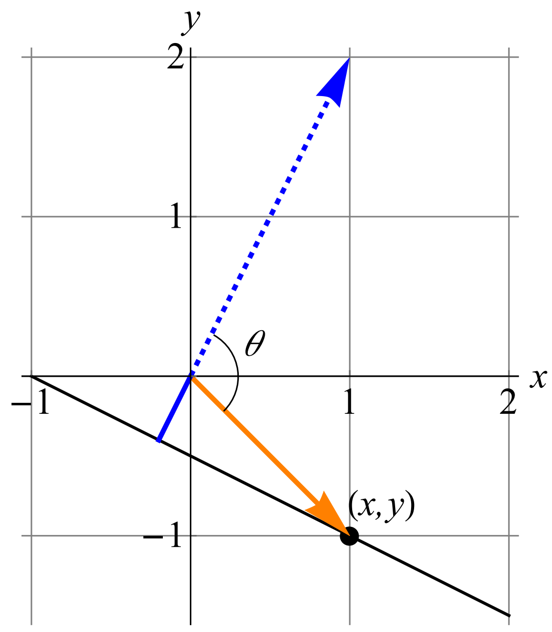

Figure 17: The length of the blue line can be determined by the length of the blue dotted vector $\langle 1, 2\rangle$ and the orange vector $\langle x, y\rangle$, and the angle between them, where (x, y) is any point on the line $x + 2y = -1$.

What is special about this line? It is exactly the shortest line connecting the origin to any point on the line $x + 2y = -1$. Why is that? Because this line segment is perpendicular to the line as we have discovered, and any other line will not create a right triangle and will necessarily be longer than this perpendicular distance.

By rearranging the dot product formula, we can determine explicitly the distance between the origin and the line to be $1/\sqrt{5}$.

$$\underbrace{|\langle 1,2\rangle|}_{\sqrt{5}}\ \underbrace{|\langle x,y\rangle|\cos\theta}_{\pm\text{length of blue line}} = -1$$

$$|\langle x,y\rangle|\cos\theta = \frac{-1}{\sqrt{5}}$$

Conclusion: The dot product of the normal vector to a line with the vector connecting the origin to any point on that line can be described as a dot product that can be used to determine the shortest distance from the origin to the line.

6. Equations of lines and normal vectors

Hide Discussion

Topic: Unit 2: Geometry of Derivatives / 6. Equations of lines and normal vectors

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- Small correction: abs needed to get distance of line to origin 2
- F for student who raised hand at 1:36 :') 3
  Respects are due.
- ? Observe that this vector is not parallel to the line! So it makes sense that the dot product of $\langle x,y\rangle$ with $\langle 1,2\rangle$ is nonzero. 3
  shouldn't that be **perpendicular** instead of parallel?
- When ax +by does not equal 0 6
  I don't understand how to get the length of the blue line and how it has been done above.
- [Staff] Odd definition 6.1 or typo? Normal isn't Orthogonal 6
  Community TA
- ? [Staff] Remark 6.3 3
  Remark 6.3 says We cannot use a hidden dot product to determine that <1,2> is perpendicular to the line x+2y=-1. But we have done...

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