4. Maximum Likelihood Estimator for Curved Gaussian

(a)

1.0/1 point (graded)

Note: *To avoid too much double jeopardy, the solution to part (a) will be available once you have either answered it correctly or reached the maximum number of attempts.*

Let $X_1, \ldots, X_n$ be n i.i.d. random variables with distribution $\mathcal{N}(\theta, \theta)$ for some unknown $\theta > 0$.

Compute the maximum likelihood estimator $\hat{\theta}$ for θ in terms of the sample averages of the linear and quadratic means, i.e. $\overline{X}_n$ and $\overline{X_n^2}$.

(Enter **barX_n** for $\overline{X}_n$ and **bar(X_n^2)** for $\overline{X_n^2}$. Note that **barX_n^2** represents $\left(\overline{X}_n\right)^2$, and is **not** equal to **bar(X_n^2)** with the brackets.

$\hat{\theta} =$ (-1+sqrt(1+4*bar(X_n^2)) ✔

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STANDARD NOTATION

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(b)

4/4 points (graded)

We want to compute the asymptotic variance of $\hat{\theta}$ via two methods.

In this problem, we apply the Central Limit Theorem and the 1-dimensional Delta Method. We will compare this with the approach using the Fisher information next week.

First, compute the limit and asymptotic variance of $\overline{X_n^2}$.

The limit to which $\overline{X_n^2}$ converges in probability, also known as its $\mathbf{P}$ **-limit**, is

$\overline{X_n^2} \xrightarrow[n\to\infty]{\mathbf{P}}$ theta^2+theta ✔

$\theta^2 + \theta$

The asymptotic variance $V\left(\overline{X_n^2}\right)$ of $\overline{X_n^2}$, which is equal to $\mathsf{Var}\left(X_1^2\right)$, is

$V\left(\overline{X_n^2}\right) = \mathsf{Var}\left(X_1^2\right) =$ 4*theta^3+2*theta^2 ✔

$4 \cdot \theta^3 + 2 \cdot \theta^2$

Now, write $\hat{\theta}$ as the function of $\overline{X_n^2}$ you found in part (a),

$$\hat{\theta} = g\left(\overline{X}_n^2\right)$$

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and give its first derivative, $g'(x)$,

$g'(x) =$ 1/sqrt(1+4*x) ✔

$\frac{1}{\sqrt{1+4\cdot x}}$

What can you conclude about the asymptotic variance $V(\hat{\theta})$ of $\hat{\theta}$?

$V(\hat{\theta}) =$ 2*theta^2/(1+2*theta) ✔

$\frac{2\cdot\theta^2}{1+2\cdot\theta}$

STANDARD NOTATION

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✔ Correct (4/4 points)

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Part (b) asymptotic variance

discussion posted 22 minutes ago by sandipan_dey

I really liked the problem very much and also interested to know about the thought-process to come up with a function $g(.)$ s.t. $g(\theta^2 + \theta) = \theta$ ... structing the problem.

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It's nice to notice that the quadratic $\theta^2 + \theta = 1$ has the +ve root $\frac{-1+\sqrt{5}}{2} = g(1)$.

Question: Why asymptotic variance of $\bar{X_n^2}$ i.e. $Var(\bar{X_n^2}) = Var(X_1^2)$, whereas $Var(\bar{X_n^2}) = Var\left(\frac{1}{n}.\sum_{i=1}^{n} X_i^2\right) = \frac{1}{n}Var(X_i^2)$? Is it because when $n \to \infty$ the (sampling) distribution of $\bar{X_n^2}$ tends to be same as the (population) distribution of X_i^2?

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