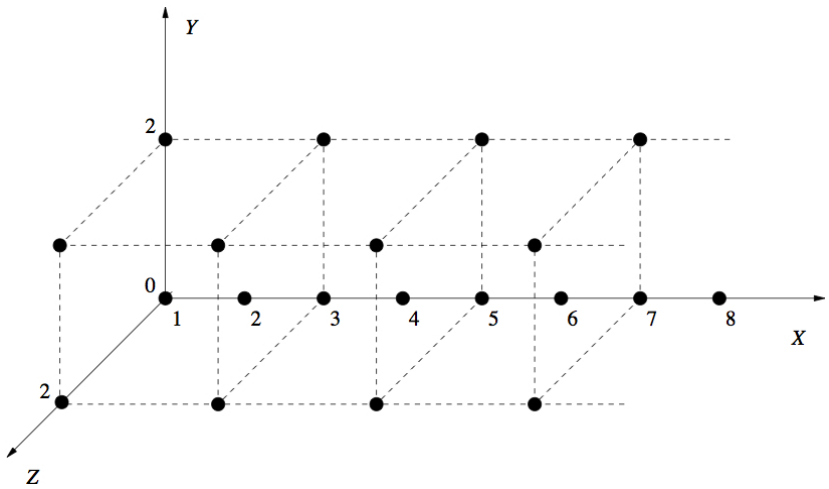
Problem 5: Joint PMF calculations - Part 1

(2/4 points)

Consider three random variables X, Y, and Z, associated with the same experiment. The random variable X is geometric with parameter $p \in (0,1)$. If X is even, then Y and Z are equal to zero. If X is odd, (Y,Z) is uniformly distributed on the set $S = \{(0,0),(0,2),(2,0),(2,2)\}$. The figure below shows all the possible values for the triple (X,Y,Z) that have $X \leq 8$. (Note that the X axis starts at 1 and that a complete figure would extend indefinitely to the right.)



1. Answer the following with “Yes" or “No":

a) Are Y and Z independent?

No ▾ ✔ **Answer:** No

b)Given that $Z = 2$, are X and Y independent?

Yes ▾ ✔ **Answer:** Yes

c) Given that $Z = 0$, are X and Y independent?

Yes ▾ ✖ **Answer:** No

d) Given that $Z = 2$, are X and Z independent?

No ▾ ✖ **Answer:** Yes

Answer:

1. a) No. If $Y = 2$, then Z is equally likely to be 0 or 2. However, if $Y = 0$, then Z is more likely to be 0.

   b) Yes. Let us work in the conditional model, where Z is known to be equal to 2. If we are further given that $X = x$, then x is necessarily odd and Y is equally likely to be 0 or 2. Thus, the conditional PMF of Y given $X = x$ (in this conditional model) does

not depend on x, and this is equivalent to independence of Y from X, in the conditional model.

c) No. Let us work in the conditional model, where Z is known to be equal to 0. If we are further given that $X = 1$, then Y is equally likely to be 0 or 2, whereas if we are further given that $X = 2$, then Y must be equal to 0. Thus, within the conditional model, knowledge of X affects the distribution of Y.

d) Yes. Within the conditional model where $Z = 2$, the probability that $Z = 2$ is always 1, and is not affected by the value x of X. (This is an instance of the more general fact that an event that has probability 1 is independent of every other event.)

You have used 1 of 1 submissions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