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Problem Set A due Sep 13, 2021 20:30 IST Completed



Practice

7(a)

1/1 point (graded)

Use the method of least squares approximation to find the line that best fits the three points:

$$(1,1)$$
, $(1,3)$, $(2,4)$.

Solution:

We solve for a line of the form y = ax + b.

We start by computing the necessary sums:

$$\sum_{i=1}^{3} x_i = 1 + 1 + 2 = 4 \tag{4.251}$$

$$\sum_{i=1}^{3} x_i^2 = 1^2 + 1^2 + 2^2 = 6 (4.252)$$

$$\sum_{i=1}^{3} x_i y_i = 1(1) + 1(3) + 2(4) = 12$$
 (4.253)

$$\sum_{i=1}^{3} y_i = 1 + 3 + 4 = 8 \tag{4.254}$$

This gives rise to the following system that allows us to solve for a and b.

$$6a + 4b = 12 (4.255)$$

$$4a + 3b = 8 (4.256)$$

Multiplying the first equation by 2 and the second equation by 3 and subtracting, we get b=0. Thus a=2. The best fit line is y=2x.

Submit

You have used 1 of 10 attempts

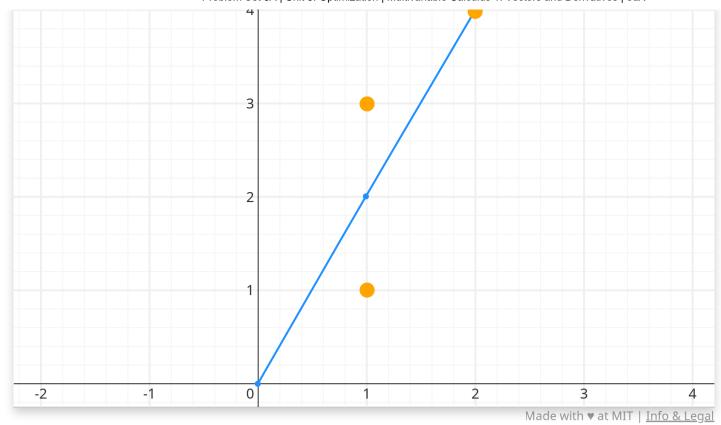
1 Answers are displayed within the problem

7(b)

1.0/1 point (graded)

Sketch the best fit line and three data points as a check. (Sketch your fit for $0 \le x \le 2$.)





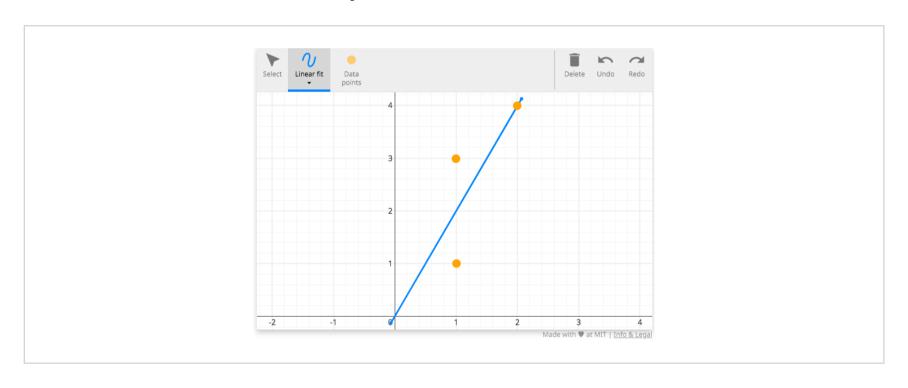
Answer: See solution.



Well done.

Solution:

The sketch of the three points and the line y=2x is shown below.



Submit

You have used 1 of 25 attempts

1 Answers are displayed within the problem

7(c)

2.0/2 points (graded)

Transform the data to find the best fit power law of the form $y=cx^d$ for the same data: (1,1), (1,3), (2,4).

Find \boldsymbol{c} and \boldsymbol{d} .

? INPUT HELP

Solution:

Trying to fit a power law to this data gives us a new set of data

$$(\ln(1), \ln(1)) = (0,0) \tag{4.257}$$

$$(\ln(1), \ln(3)) = (0, \ln(3))$$
 (4.258)

$$(\ln(2), \ln(4)) = (\ln(2), 2\ln(2))$$
 (4.259)

Next we compute the necessary sums:

$$\sum_{i=1}^{3} x_i = 0 + 0 + \ln(2) = \ln(2)$$
 (4.260)

$$\sum_{i=1}^{3} x_i^2 = 0 + 0 + (\ln(2))^2 = (\ln(2))^2$$
 (4.261)

$$\sum_{i=1}^{3} x_i y_i = 0 (0) + 0 (\ln (3)) + \ln (2) (2 \ln (2)) = 2(\ln (2))^2$$
 (4.262)

$$\sum_{i=1}^{3} y_{i} = 0 + \ln(3) + \ln(4) = \ln(3) + \ln(4)$$
 (4.263)

We solve the system:

$$(\ln(2))^2 a + \ln(2) b = 2(\ln(2))^2 \tag{4.264}$$

$$\ln(2) a + 3b = \ln(3) + \ln(4)$$
 (4.265)

We divide through the first equation by $\ln{(2)}$ and subtract from the second equation to get

$$2b = \ln(3) \longrightarrow b = \ln(3)/2.$$

Solving for a this gives $a=(2-rac{\ln 3}{\ln 4}).$

This gives the best fit line $\ln{(y)}=(2-\frac{\ln{3}}{\ln{4}})\ln{(x)}+\ln{(3)}/2$. Exponentiating we get $y=\pm\sqrt{3}x^{2-\ln{3}/\ln{4}}$. Because all of our data is positive, we can assume the best fit is with the positive multiplicative constant $\sqrt{3}$.

Submit

You have used 1 of 15 attempts

1 Answers are displayed within the problem

7(d)

2.0/2 points (graded)

Transform the data to find the best fit exponential curve of the form $y=ce^{kx}$ for the same data: (1,1), (1,3), (2,4).

Find c and k.

? INPUT HELP

Solution:

Trying to fit a power law to this data gives us a new set of data

$$(1, \ln(1)) = (1, 0) \tag{4.266}$$

$$(1, \ln(3)) = (1, \ln(3))$$
 (4.267)

$$(2, \ln(4)) = (2, 2\ln(2))$$
 (4.268)

Next we compute the necessary sums:

$$\sum_{i=1}^{3} x_i = 1 + 1 + 2 = 4 \tag{4.269}$$

$$\sum_{i=1}^{3} x_i^2 = 1^2 + 1^2 + 2^2 = 6 (4.270)$$

$$\sum_{i=1}^{3} x_i y_i = 1(0) + 1(\ln(3)) + 2(\ln(4)) = \ln(3) + 2\ln(4)$$
 (4.271)

$$\sum_{i=1}^{3} y_{i} = 0 + \ln(3) + \ln(4) = \ln(3) + \ln(4)$$
 (4.272)

We solve the system:

$$6a + 4b = \ln(3) + 2\ln(4) \tag{4.273}$$

$$4a + 3b = \ln(3) + \ln(4)$$
 (4.274)

Multiplying the first equation by 2 and the second equation by 3 and subtracting, we get

$$12a + 8b = 2\ln(3) + 4\ln(4) \tag{4.275}$$

$$12a + 9b = 3\ln(3) + 3\ln(4) \tag{4.276}$$

$$b = \ln(3) - \ln(4) = \ln(3/4)$$
 (4.277)

Solving for a this gives

$$4a + 3(\ln 3 - \ln 4) = \ln (3) + \ln (4)$$
 (4.278)

$$4a = -2\ln(3) + 4\ln(4) \tag{4.279}$$

$$a = \ln(4) - \ln(3)/2 \tag{4.280}$$

Transforming back we get a=k, and $b=\ln{(3/4)}=\ln{(c)}$. Thus c=3/4.

This gives the best fit exponential using this method is $y=3/4e^{\ln(4/\sqrt{3})x}$.

Submit

You have used 1 of 15 attempts

Answers are displayed within the problem

5. Data fitting

Topic: Unit 3: Optimization / 5. Data fitting

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<u> 7(b)</u>

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Please reformulate the question statement to state that the "best fit" to sketch is within the domain shown

Calculator



| ∀ | Q2-3 Can someone please explain the exact method for finding c and d in power law and c and k in exponential cause I've read about it on | 2 |
|----------|---|---|
| € | [STAFF] question about explanation in answer to 7(c) | 3 |
| Q | Matlab So I tried to throw all those data points into matlab and try to find the coefficients using curve fitting toolbox, somehow, the value ob | 7 |
| ? | Deadline extension Is it possible to get maybe 2 more days extension on the deadline for Unit 3? We are starting both grade school and university right | 4 |
| ? | [Staff] Acknowledgment/Question about final exam. Thanks to RStudio IDE, I was able to avoid having to solve the last two questions 7c and 7d by hand. I believe I'd be willing to try | 3 |
| R | SketchInput not loading in Firefox If the same happens to you try Chrome - it worked there for me. | 2 |
| ∀ | Stumped on 7c Are we supposed to take the sum of squares of logarithms? I'm not sure if I'm doing it right. | 8 |
| 2 | [Staff] Garbled Equation | 2 |
| ∀ | [Staff] 7.(c) Grader for coeff d ≜ Community TA | 5 |

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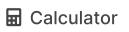
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