Problem 1: Normal random variables

(5/5 points)

Let X and Y be normal random variables with means 0 and 2, respectively, and variances 1 and 9, respectively. Find the following, using the standard normal table . Express your answers to an accuracy of 4 decimal places.

1. $\mathbf{P}(X > 0.75) =$ 0.2266274 ✔ **Answer:** 0.2266

2. $\mathbf{P}(X \leq -1.25) =$ 0.1056498 ✔ **Answer:** 0.1056

3. Let $Z = (Y - 3)/4$. Find the mean and the variance of Z.

$\mathbf{E}[Z] =$ -1/4 ✔ **Answer:** -0.25

$\mathbf{var}(Z) =$ 9/16 ✔ **Answer:** 0.5625

4.

$\mathbf{P}(-1 \leq Y \leq 2) =$ 0.3413447 ✔ **Answer:** 0.3413

Answer:

1. X is a standard normal, so by using the normal table, we have
   $\mathbf{P}(X \leq 0.75) = \Phi(0.75) \approx 0.7734$.
   Hence, $\mathbf{P}(X > 0.75) = 1 - \mathbf{P}(X \leq 0.75) \approx 1 - 0.7734 = 0.2266$.

2. Since the distribution of X is symmetric with respect to 0,
   $\mathbf{P}(X \leq -1.25) = \mathbf{P}(X \geq 1.25) = 1 - \Phi(1.25) \approx 1 - 0.8944 = 0.1056.$

3. We have $\mathbf{E}[Z] = \frac{1}{4}\mathbf{E}[Y] - \frac{3}{4} = \frac{1}{4} \cdot 2 - \frac{3}{4} = -\frac{1}{4}$,
   and $\mathbf{var}(Z) = \frac{1}{4^2}\mathbf{var}(Y) = \frac{1}{16} \cdot 9 = \frac{9}{16}$.

4. By standardizing Y and using the normal table, we have

$$\begin{aligned}
\mathbf{P}(-1 \leq Y \leq 2) &= \mathbf{P}\left(\frac{-1-2}{3} \leq \frac{Y-2}{3} \leq \frac{2-2}{3}\right) \\
&= \mathbf{P}(-1 \leq Z \leq 0) \\
&= \mathbf{P}(0 \leq Z \leq 1) \\
&= \mathbf{P}(Z \leq 1) - \mathbf{P}(Z \leq 0) \\
&= \Phi(1) - \Phi(0) \\
&\approx 0.8413 - 0.5 \\
&= 0.3413,
\end{aligned}$$

where Z is a standard normal random variable.

You have used 1 of 3 submissions

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