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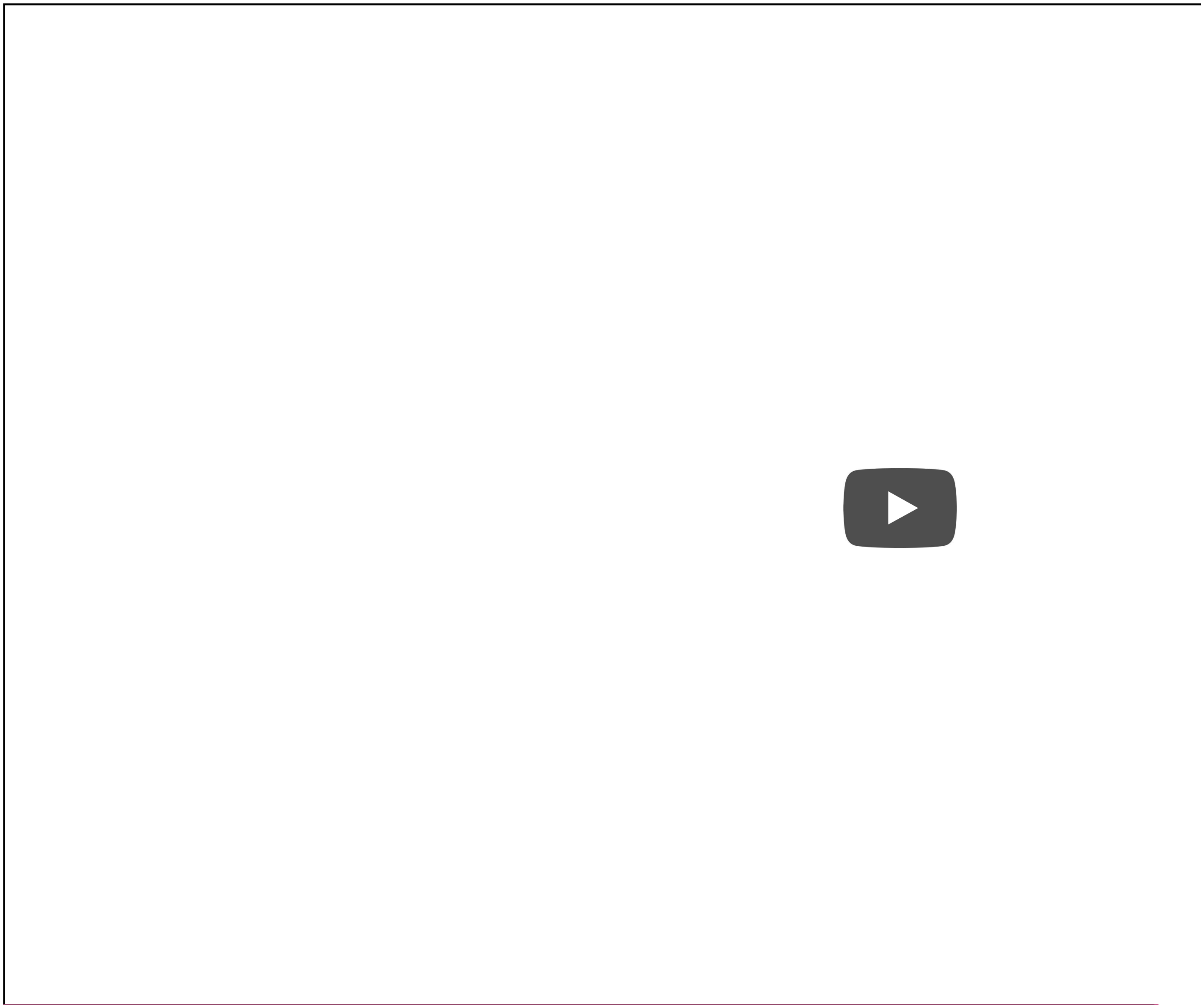
16. Singular matrices
Singular matrices



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And there is no way A inverse can recover it.

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Matrices that have no inverse are called **singular.** The theorem for invertible matrices can be stated in terms of the complementary conditions for singular matrices. (It's essentially the same theorem, so this is really just a review.)

Theorem 16.1 For a square matrix $\mathbf{A}$, the following are equivalent:

1. $\det \mathbf{A} = 0$
2. $\mathrm{NS}(\mathbf{A})$ is larger than $\{\mathbf{0}\}$ (i.e., $\mathbf{Ax} = \mathbf{0}$ has a nonzero solution)
3. $\mathbf{rank}(\mathbf{A}) < n$ (image has dimension less than n)
4. $\mathrm{CS}(\mathbf{A})$ is smaller than $\mathbb{R}^n$ (image is not the whole space $\mathbb{R}^n$)
5. The system $\mathbf{Ax} = \mathbf{b}$ has no solutions for some vectors $\mathbf{b}$, and infinitely many solutions for other vectors $\mathbf{b}$.
6. $\mathbf{A}^{-1}$ does not exist.
7. $\mathtt{rref}(\mathbf{A}) \neq \mathbf{I}$

Singular matrices concept check

1/1 point (graded)
Suppose $\mathbf{A}$ is any matrix, and $\mathbf{A}$, $\mathbf{B}$, and $\mathbf{C}$ are three $n \times n$ matrices. **True or False?** If $\mathbf{AB} = \mathbf{AC}$, then $\mathbf{B} = \mathbf{C}$.

- True
- False ✔

Solution:

False. Suppose that $\mathbf{A}$ is the zero matrix, then $\mathbf{AB} = \mathbf{AC}$ for all $n \times n$ matrices $\mathbf{B}$ and $\mathbf{C}$. (Moreover, if $\det\mathbf{A} = 0$ we can always find distinct $\mathbf{B}$ and $\mathbf{C}$ such that $\mathbf{AB} = \mathbf{AC}$. As a challenge problem, try to construct such matrices $\mathbf{B}$ and $\mathbf{C}$ given a singular matrix $\mathbf{A}$.) If $\mathbf{A}$ is invertible, however; then multiplying by $\mathbf{A}^{-1}$ we see that $\mathbf{B} = \mathbf{C}$.

This problem demonstrates one of the many properties true of invertible matrices, but not of matrices in general.

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You have used 1 of 1 attempt

Answers are displayed within the problem

Now let's explain the consequences of $\det\mathbf{A} = 0$.

1. The input space is flattened by $\mathbf{A}$. This means that some input dimensions are getting crushed, so $\mathrm{NS}(\mathbf{A})$ is larger than $\{\mathbf{0}\}$ (at least 1-dimensional), and the image is smaller than the n-dimensional input space: $\mathbf{rank}(\mathbf{A}) < n$.

2. In particular, the image $\mathrm{CS}(\mathbf{A})$ is not all of $\mathbb{R}^n$.

3. If $\mathbf{b}$ is not in $\mathrm{CS}(\mathbf{A})$, then $\mathbf{Ax} = \mathbf{b}$ has no solution.

4. If $\mathbf{b}$ is in $\mathrm{CS}(\mathbf{A})$, then $\mathbf{Ax} = \mathbf{b}$ has the same number of solutions as $\mathbf{Ax} = \mathbf{0}$, i.e., infinitely many since $\dim \mathrm{NS}(\mathbf{A}) \geq 1$.

5. The associated linear transformation $\mathbf{f}$ is not a 1-to-1 correspondence (it maps many vectors to $\mathbf{0}$); thus $\mathbf{f}^{-1}$ does not exist, so $\mathbf{A}^{-1}$ does not exist. Row operations do not change the condition $\det\mathbf{A} = 0$, so $\det(\mathtt{rref}(\mathbf{A})) = 0$, so definitely $\mathtt{rref}(\mathbf{A}) \neq \mathbf{I}$. (In fact, $\mathtt{rref}(A)$ must have at least one 0 along the diagonal.)

Problem 16.2 Devise a test for deciding whether a homogeneous square system $\mathbf{Ax} = \mathbf{0}$ has a nonzero solution.

Solution

Compute $\det \mathbf{A}$. If $\det \mathbf{A} = 0$, there exists a nonzero solution. If $\det \mathbf{A} \neq 0$, then $\mathbf{A}\mathbf{x} = \mathbf{0}$ has only the zero solution.

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