

## MITx: 6.041x Introduction to Probability - The Science of Uncertainty



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## Exam 1

Exam 1 due Mar 09, 2016 at 23:59 UTC

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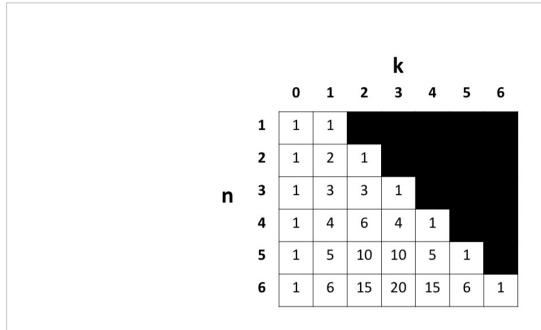
## Problem 3: A six-sided die

(3/3 points)

A fair, 6-sided die is rolled 6 times independently. Assume that the results of the different rolls are independent. Let  $(a_1,\ldots,a_6)$  denote a typical outcome, where each  $a_i$  belongs to  $\{1,\ldots,6\}$ .

**Note:** Enter numerical answers; do not enter '!' or combinations. The following table for  $\binom{n}{k}$  up to n=6 has been provided for your convenience:

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1. Find the probability that the results of the 6 rolls are all different. (Answer with at least 3 decimal digits.)

For any outcome  $\omega=(a_1,\ldots,a_6)$ , let  $R(\omega)$  be the **set**  $\{a_1,\ldots,a_6\}$ ; this is the set of numbers that showed up at least once in the different rolls. For example, if  $\omega=(2,2,5,2,3,5)$ , then  $R(\omega)=\{2,3,5\}$ .

2. Find the probability that  $R(\omega)$  has exactly two elements. (Answer with at least 3 decimal digits.)

0.01993313

**✓ Answer:** 0.01993

3. Find the probability that  $R(\omega)$  has exactly three elements.

0.2314815

**~** 

**Answer:** 0.23148

## Answer:

1. Let A be the event where the results of the 6 rolls are all the same. Since all outcomes are equally likely,  $\mathbf{P}(A) = \frac{|A|}{|\Omega|}$ .

$$|A|=6! \ |\Omega|=6^6 \ P(A)=rac{6!}{6^6}=5/324pprox 0.01543$$

2. Let B be the event where  $R(\omega)$  has exactly two elements. We can find the number of elements in B by first choosing a pair of distinct numbers for  $R(\omega)$  – there are  $\binom{6}{2}$  choices. For each pair, we can then count the number of ways they can be assigned to a sequence of length 6 that consists of only those two numbers and has at least one of each. We see that there are  $\binom{6}{k}$  ways of constructing a sequence that consists of k repetitions of the first number, and k0 repetitions of the second number, where k1 k2 k3. Therefore,

$$egin{aligned} |B| &= inom{6}{2} \sum_{k=1}^5 inom{6}{k} \ |\Omega| &= 6^6 \ P(B) &= rac{|B|}{|\Omega|} = 155/7776 pprox 0.01993 \end{aligned}$$

3. Let C be the event where  $R(\omega)$  has exactly three elements. We can find the number of elements in C by first choosing a triple of distinct numbers for  $R(\omega)$  – there are  $\binom{6}{3}$  choices. For each triple, we can then count the number of ways they can be assigned to a sequence of length 6 that consists of only those three numbers and has at least one of each. We see that there are  $\binom{6}{k}\binom{6-k}{l}$  ways of constructing a sequence that consists of k repetitions of the first number, l repetitions of the second number, and l0 and l1 and l2 and l3 and l4 and l5 and l6 and l7 and l8. Therefore,

$$|C|=inom{6}{3}\sum_{k=1}^4\sum_{l=1}^{5-k}inom{6}{k}inom{6-k}{l}$$
  $|\Omega|=6^6$ 

By numerically evaluating the various entries in the formula for |C|, we find that:

$$P(C) = rac{|C|}{|\Omega|} = 25/108 pprox 0.23148$$

You have used 1 of 2 submissions

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