E1.3.4 Exam Question 4

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Exam 1 due Oct 31, 2023 09:12 IST Completed

Question 4

11/11 points (graded)

1. $$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & -1 \end{pmatrix} =$$

1 ✔	0 ✔	0 ✔
Answer: 1	Answer: 0	Answer: 0
0 ✔	4 ✔	0 ✔
Answer: 0	Answer: 4	Answer: 0
0 ✔	0 ✔	1 ✔
Answer: 0	Answer: 0	Answer: 1

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

2. Let $D = \begin{pmatrix} \delta_0 & 0 & 0 \\ 0 & \delta_1 & 0 \\ 0 & 0 & \delta_2 \end{pmatrix}$ where δ_0, δ_1, and δ_2 are scalars. Compute DD (matrix D multiplied by itself).

$DD =$

- ○ $D = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$
- ○ $D = \begin{pmatrix} 2\delta_0 & 0 & 0 \\ 0 & 2\delta_1 & 0 \\ 0 & 0 & 2\delta_2 \end{pmatrix}$
- ● $D = \begin{pmatrix} \delta_0^2 & 0 & 0 \\ 0 & \delta_1^2 & 0 \\ 0 & 0 & \delta_2^2 \end{pmatrix}$
- ○ $D = \begin{pmatrix} \delta_0^1 & 0 & 0 \\ 0 & \delta_1^2 & 0 \\ 0 & 0 & \delta_2^3 \end{pmatrix}$

✔

$$DD = \begin{pmatrix} \delta_0^2 & 0 & 0 \\ 0 & \delta_1^2 & 0 \\ 0 & 0 & \delta_2^2 \end{pmatrix}$$

Calculator

3. For square matrix A define A^n as $A^0 = I$ (the identity) and $A^{n+1} = A^n A$ for $n \geq 0$.

3. For square matrix A define A^0 as $A^0 = I$ (the identity) and $A^{n+1} = A^n A$ for $n \geq 0$.

Let D again be defined as $D = \begin{pmatrix} \delta_0 & 0 & 0 \\ 0 & \delta_1 & 0 \\ 0 & 0 & \delta_2 \end{pmatrix}$, then $D^n = \begin{pmatrix} \delta_0^n & 0 & 0 \\ 0 & \delta_1^n & 0 \\ 0 & 0 & \delta_2^n \end{pmatrix}$ for $n \geq 0$.

Always ✔ Answer: Always

Answer:

Proof by induction.

Base Case: $n = 0$.

$$D^0 = I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} \delta_0^0 & 0 & 0 \\ 0 & \delta_1^0 & 0 \\ 0 & 0 & \delta_2^0 \end{pmatrix}.$$

Inductive Step. I.H.: Assume that $D^n = \begin{pmatrix} \delta_0^n & 0 & 0 \\ 0 & \delta_1^n & 0 \\ 0 & 0 & \delta_2^n \end{pmatrix}$.

Show that $D^{n+1} = \begin{pmatrix} \delta_0^{n+1} & 0 & 0 \\ 0 & \delta_1^{n+1} & 0 \\ 0 & 0 & \delta_2^{n+1} \end{pmatrix}$:

$$\begin{aligned} & D^{n+1} \\ = & \quad < \text{Definition of } A^{n+1} > \\ & D^n D \\ = & \quad < \text{I.H.} > \\ & \begin{pmatrix} \delta_0^n & 0 & 0 \\ 0 & \delta_1^n & 0 \\ 0 & 0 & \delta_2^n \end{pmatrix} \begin{pmatrix} \delta_0 & 0 & 0 \\ 0 & \delta_1 & 0 \\ 0 & 0 & \delta_2 \end{pmatrix} \\ = & \quad < \text{matrix-matrix multiplication} > \\ & \begin{pmatrix} \delta_0^{n+1} & 0 & 0 \\ 0 & \delta_1^{n+1} & 0 \\ 0 & 0 & \delta_2^{n+1} \end{pmatrix}. \end{aligned}$$

By the PMI the result holds for all $n \geq 0$.

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