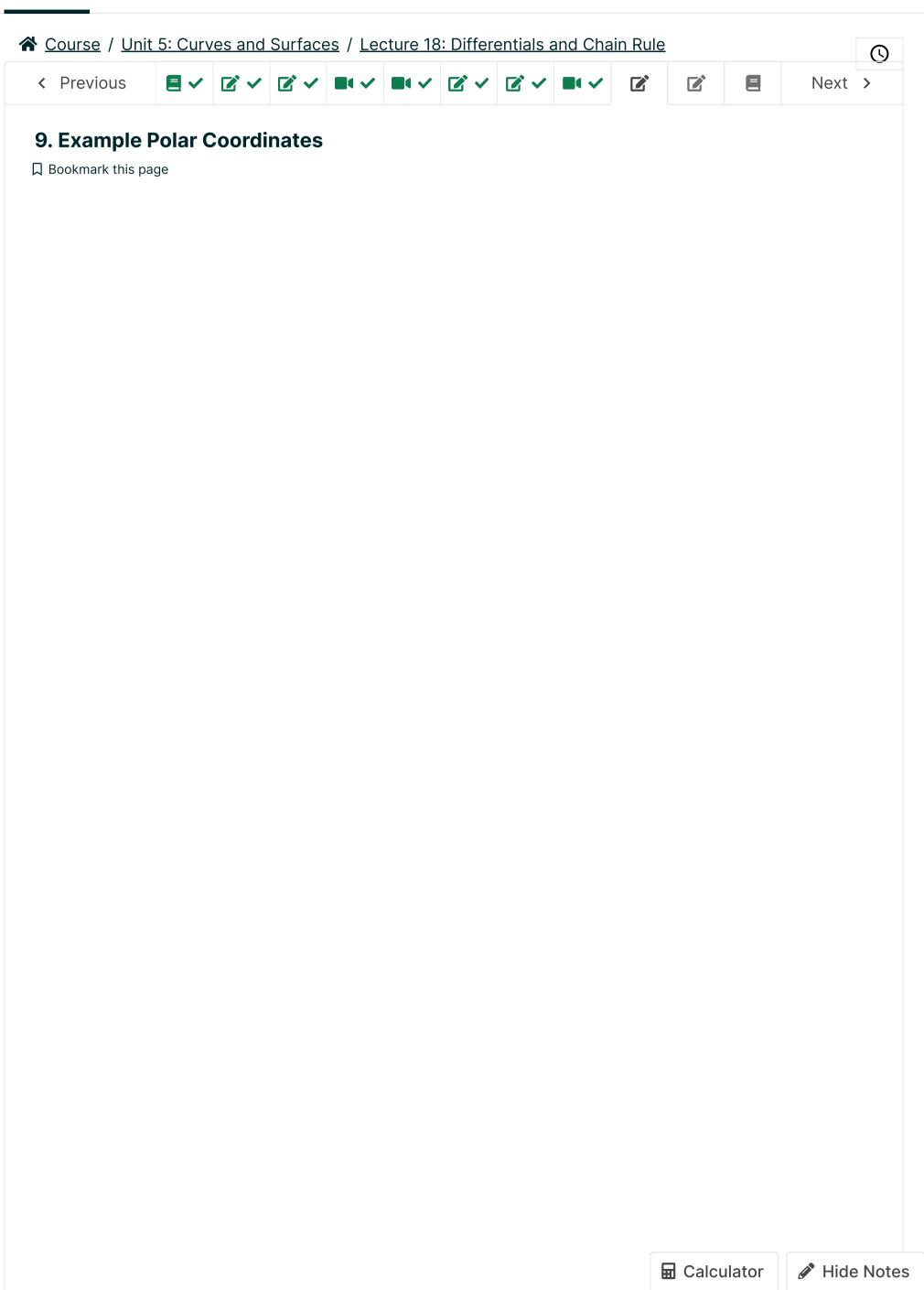


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Lecture due Oct 5, 2021 20:30 IST



Synthesize

Chain Rule Polar Coordinates



you can plug these and get the function of r and theta. And then you can ask yourself, well, what is partial f over partial r? And that's going to be-- well, you want to take partial f, partial x, partial Х, partial r plus partial f, partial y, partial partial r. And so that will end up being, actually, f sub x times cosine theta plus f sub y times sine theta. And you can do the same thing to find partial f, partial theta. So you can express derivatives

depends on x and y, then, in fact,

or in terms of r and theta with simple relations between them.

either in terms of x and y

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A useful application of the chain rule would be when we need to switch between rectangular and polar coordinates.

Suppose a quantity f varies in the plane with x and y. Perhaps we already know $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$, but what we really want to know are the partial derivatives of f with respect to the polar coordinates f and f. The chain rule gives us a way to find these partial derivatives without writing f explicitly in terms of f and f.

In particular, the chain rule tells us that

$$\frac{\partial f}{\partial r} = \frac{\partial f}{\partial x} \frac{\partial x}{\partial r} + \frac{\partial f}{\partial y} \frac{\partial y}{\partial r}.$$
(6.179)

This equation follows from the "more variables" version of the chain rule.

Next, we have

$$x = r\cos\theta \tag{6.180}$$

$$y = r\sin\theta \tag{6.181}$$

Therefore we have

$$rac{\partial x}{\partial r} = \cos heta$$

$$\frac{\partial y}{\partial r} = \sin \theta \tag{6.183}$$

So if we know $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ we can write $\frac{\partial f}{\partial r}$ as:

$$\frac{\partial f}{\partial r} = \underbrace{\frac{\partial f}{\partial x}}_{f_x} \cos \theta + \underbrace{\frac{\partial f}{\partial y}}_{f_y} \sin \theta \tag{6.184}$$

In a similar way it is possible to write $\frac{\partial f}{\partial \theta}$ in terms of $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$.

Example 9.1 Let f(x,y)=xy. Suppose we would like to know $\frac{\partial f}{\partial r}$, that is, the derivative of f with respect to f. It is straightforward to compute the partial derivatives with respect to f and f.

$$\frac{\partial f}{\partial x} = y \tag{6.185}$$

$$\frac{\partial f}{\partial y} = x \tag{6.186}$$

Then we will use the formula

$$\frac{\partial f}{\partial r} = \underbrace{\frac{\partial f}{\partial x}}_{f_x} \cos \theta + \underbrace{\frac{\partial f}{\partial y}}_{f_y} \sin \theta \tag{6.187}$$

All that remains is to rewrite $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ in terms of r and θ :

$$\frac{\partial f}{\partial x} = r \sin \theta \tag{6.188}$$

$$\frac{\partial f}{\partial u} = r \cos \theta \tag{6.189}$$

Then we have

$$\frac{\partial f}{\partial r} = r \sin \theta \cos \theta + r \cos \theta \sin \theta \tag{6.190}$$

This answer can be further simplified to

$$\frac{\partial f}{\partial r} = r \sin{(2\theta)} \tag{6.191}$$

This simplified expression can lend us some insight about the function f(x,y). For example, notice that $\sin{(2\theta)}$ is positive for $0<\theta<\pi/2$ and negative for $\pi/2<\theta<\pi$. It follows that if r is increased within quadrant 1 then f will increase. But if r is increased within quadrant 1, then 10 will decrease.

We can also see from $\frac{\partial f}{\partial r}=r\sin{(2\theta)}$ that, as r increases, the quantity f will increase most rapidly when $\theta=\pi/4$. In other words, moving out from the origin, f increases most rapidly along the line y=x.

1/1 point (graded)

Suppose a quantity f varies in the plane with x and y. Which of the following is the correct expression for $\frac{\partial f}{\partial \theta}$ in terms of $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$?

$$rac{\partial f}{\partial heta} = rac{\partial f}{\partial x} an heta + rac{\partial f}{\partial y} an heta$$

$$rac{\partial f}{\partial heta} = rac{\partial f}{\partial x} r \cos heta + rac{\partial f}{\partial y} r \sin heta$$

$$rac{\partial f}{\partial heta} = -rac{\partial f}{\partial x}r\sin heta + rac{\partial f}{\partial y}r\cos heta$$

$$rac{\partial f}{\partial heta} = rac{\partial f}{\partial x} {\cos heta} + rac{\partial f}{\partial y} {\sin heta}$$



Solution:

By the chain rule, we have

$$\frac{\partial f}{\partial \theta} = \frac{\partial f}{\partial x} \frac{\partial x}{\partial \theta} + \frac{\partial f}{\partial y} \frac{\partial y}{\partial \theta}$$
 (6.192)

From

$$x = r\cos\theta \tag{6.193}$$

$$y = r\sin\theta \tag{6.194}$$

we can find

$$\frac{\partial x}{\partial \theta} = -r \sin \theta \tag{6.195}$$

$$\frac{\partial y}{\partial \theta} = r \cos \theta \tag{6.196}$$

By substitution, we obtain the answer:

$$\frac{\partial f}{\partial \theta} = -\frac{\partial f}{\partial x} r \sin \theta + \frac{\partial f}{\partial y} r \cos \theta \tag{6.197}$$

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You have used 1 of 2 attempts

1 Answers are displayed within the problem

Polar coordinates chain rule 2

1/1 point (graded)

Let f(x,y)=xy. Using the answer to the previous problem, what is $rac{\partial f}{\partial heta}$?

Answer in terms of $m{r}$ and $m{ heta}$. Type theta for $m{ heta}$.

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Solution:

The partial derivatives of $m{f}$ are

$$\frac{\partial f}{\partial x} = y \tag{6.198}$$

$$\frac{\partial f}{\partial y} = x \tag{6.199}$$

Writing these in terms of $m{r}$ and $m{ heta}$ we have

$$\frac{\partial f}{\partial x} = r \sin \theta \tag{6.200}$$

$$\frac{\partial f}{\partial y} = r \cos \theta \tag{6.201}$$

From the previous problem, we have

$$\frac{\partial f}{\partial \theta} = -\frac{\partial f}{\partial x} r \sin \theta + \frac{\partial f}{\partial y} r \cos \theta \tag{6.202}$$

Therefore, by substitution we get the answer:

$$\frac{\partial f}{\partial \theta} = -r^2 \sin^2 \theta + r^2 \cos^2 \theta \tag{6.203}$$

This answer can be further simplified to

$$\frac{\partial f}{\partial \theta} = r^2 \cos(2\theta) \,. \tag{6.204}$$

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You have used 1 of 5 attempts

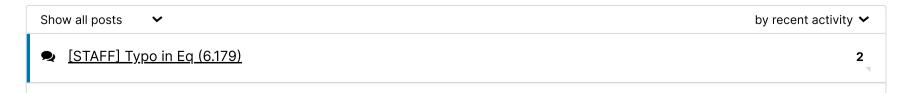
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9. Example Polar Coordinates

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