2. Review and Likelihood of a Gaussian Distribution

Concept Check: Likelihoods of a Bernoulli, a Poisson, and a Gaussian Distribution

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Is the Likelihood Discrete or Continuous?

2/2 points (graded)

Setup:

Consider a **discrete** statistical model $M_1 = (\mathbb{Z}, \{\mathbf{P}_\theta\}_{\theta\in\mathbb{R}})$ and a **continuous** statistical model $M_2 = (\mathbb{R}, \{Q_\theta\}_{\theta\in\mathbb{R}})$. Let p_θ denote the pmf of $\mathbf{P}_\theta$, and let q_θ denote the pdf of Q_θ. Assume that p_θ and q_θ both vary continuously with the parameter θ.

Let $x_1, \ldots, x_n$ be fixed natural numbers and $y_1, \ldots, y_n$ be fixed real numbers. Let $(L_1)_n$ denote the likelihood of the discrete model M_1, and let $(L_2)_n$ denote the likelihood of the continuous model M_2. Keeping $x_1, \ldots, x_n$ and $y_1, \ldots, y_n$ fixed, let's think of $(L_1)_n(x_1, \ldots, x_n, \theta)$ and $(L_2)_n(y_1, \ldots, y_n, \theta)$ as functions of θ.

Question

Decide whether the following claims about $(L_1)_n$ and $(L_2)_n$ are true or false.

The map $\theta \mapsto (L_1)_n(x_1, \ldots, x_n, \theta)$ is a continuous function of θ.

- (•) True
- ( ) False

The map $\theta \mapsto (L_2)_n(y_1, \ldots, y_n, \theta)$ is a continuous function of θ.

- (•) True
- ( ) False

Solution:

Observe that

$$(L_1)_n(x_1,\ldots,x_n,\theta) = \prod_{i=1}^{n} p_\theta(x_i), \quad (L_2)_n(y_1,\ldots,y_n,\theta) = \prod_{i=1}^{n} q_\theta(y_i).$$

We are given that p_θ and q_θ are both continuous function of the parameter $\theta \in \mathbb{R}$. Since products of continuous functions are continuous, this implies that the maps $\theta \mapsto (L_1)_n(x_1,\ldots,x_n,\theta)$ and $\theta \mapsto (L_2)_n(y_1,\ldots,y_n,\theta)$ are continuous functions of the parameter $\theta \in \mathbb{R}$.

Remark: It may be confusing that even the likelihood of a discrete statistical model can be continuous. However, considering the likelihood of a Bernoulli (derived in a previous question),

$$L(x_1,\ldots,x_n,p) = \prod_{i=1}^{n} p^{x_i}(1-p)^{1-x_i} = p^{\sum_{i=1}^n x_i}(1-p)^{n-\sum_{i=1}^n x_i}.$$

we can clearly see that the above varies continuously as a function of the *parameter*. This is also true for a host of other discrete models (for example, the Poisson model).

Submit You have used 1 of 1 attempt

Answers are displayed within the problem

Quiz: Likelihood of a Gaussian Statistical Model

3/3 points (graded)

Let $X_1,\ldots,X_n \stackrel{iid}{\sim} N(\mu^*,(\sigma^*)^2)$ for some unknown $\mu^* \in \mathbb{R}, (\sigma^*)^2 > 0$. You construct the associated statistical model $\left(\mathbb{R}, \{N(\mu,\sigma^2)\}_{(\mu,\sigma^2)\in\mathbb{R}\times(0,\infty)}\right)$.

The likelihood of this model can be written

$$L_n\left(x_1,\ldots,x_n,(\mu,\sigma^2)\right) = \frac{1}{\left(\sigma\sqrt{2\pi}\right)^C}\exp\left(-\frac{1}{A}\sum_{i=1}^{C} B_i\right)$$

where A depends on σ, B_i depends on μ and x_i. Find A,B_i and C.

(Choose a B_i that has coefficient 1 for the highest degree term in x_i.)

(Type **sigma** for σ, **mu** for μ, and **x_i** for x_i.)

$A =$ 2*sigma^2

$2\cdot\sigma^2$

$B_i =$ x_i^2+mu^2-2*x_i*mu

$x_i^2+\mu^2-2\cdot x_i\cdot\mu$

$C =$ n

n

STANDARD NOTATION

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Correct (3/3 points)

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? Should we consider cases where there may be an indicator function based on the parameter?
The prof uses a distribution such as this in one of the next few lectures and it leads me to one answer I think but it doesn't seem to be in the spirit of the question. 5

[STAFF] Possible Error in Solution for Gaussian question
I believe the second term in the solution, after "the," is a possibly confusing typo. **edit**: to clarify in title that this refers to Gaussian question. 4