Data Analysis: Statistical Modeling and Computation in Applications

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sandipan_dey ~

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Exercises due Oct 20, 2021 17:29 IST Completed

Centrality Measures



Start of transcript. Skip to the end.

Prof Uhler: OK, welcome back to the second lecture

in this networks module.

So we already discussed a little bit this module,

got some intuition on actually how to

important nodes in a network.

0:00 / 0:00

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cc **66**

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Video

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Transcripts

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Video note: At 3:57, the convention for the definitions of $k_i^{
m in}$ and $k_i^{
m out}$ are different the one stated below.

Video note: At 7:00, the definition of betweenness centrality uses a normalization factor of n^{-2} , in the exercises below, the more common definition with no normalization factor will be used (shown in the last unit).

The **degree centrality** measures the importance of nodes in terms of the degree of a node. For a directed graph, define the in-degree k_i^{in} and the out-degree k_i^{out} of a node to be the sum over the ith column or the row of the adjacency matrix respectively:

$$k_i^{ ext{in}} = \sum_{j} A_{ji}, \quad k_i^{ ext{out}} = \sum_{j} A_{ij}.$$

Note that (unlike in the slides) we use the convention where $A_{ij}=1$ indicates an edge going from node i to node j. The convention used can differ in different fields applications.

The degree centrality only captures importance up to one-hop neighbors of a node. Depending upon the application, this may not be representative of the importance of a node in the overall graph.

High Degrees

2/2 points (graded)

Let an undirected graph have n nodes. Let the edges be selected according to the following random model: every possible edge (including self loop) is present with probability p independent of every other edge.

There is an inequality, known as the *Markov inequality*, that gives an upper bound on the tail probability of a non-negative random variable:

Let X be a nonnegative random variable and $\epsilon>0$, then

$$P\left(X \geq \epsilon
ight) \leq rac{\mathbb{E}\left[X
ight]}{\epsilon}.$$

(You may recall this from the course Probablity-The Science of Uncertainty and Data.)

1. Using Markov inequality, obtain an upper bound on the probability that, for any given node, there are at least n-1 edges connected to this node in this graph.



2. Now, find the exact probability that, for any given node, there are at least n-1 edges connected to this node in this graph.

Solution:

1. Let $oldsymbol{X}$ represent the random variable that is the number of edges connected to a node in this graph generation model. $m{X}$ is a binomial random variable with parameters $m{n}, m{p}$. The expected value of a binomial random variable is np. Therefore,

$$P\left(X\geq n-1
ight)\leq rac{np}{n-1}.$$

2. This probability is equal to

$$egin{aligned} P\left(X \geq n-1
ight) &= P_{\mathrm{Binomial}\left(n,p
ight)}\left(n-1
ight) + P_{\mathrm{Binomial}\left(n,p
ight)}\left(n
ight) \ &= n \cdot p^{n-1}\left(1-p
ight) + p^{n}. \end{aligned}$$

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You have used 1 of 3 attempts

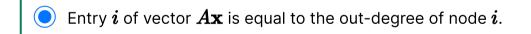
1 Answers are displayed within the problem

A Matrix Equation – Preparation for Eigenvector Centrality

2/2 points (graded)

Let A be an adjacency matrix of size n imes n. Assume that the graph is an unweighted graph. Use the convention that $A_{ij}=1$ indicates an edge going from node i to node j.

Let **x** be an all-ones vector of size $n \times 1$. What does entry **i** of the vector A**x** represent?

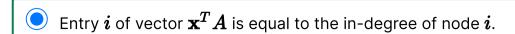


Entry i of vector $A\mathbf{x}$ is equal to the in-degree of node i.



Let \mathbf{x}^T be an all-ones vector of size $1 \times n$. What does entry i of the vector $\mathbf{x}^T A$ represent?

Entry i of vector $\mathbf{x}^T A$ is equal to the out-degree of node i.





Solution:

This follows from the definition of A for an unweighted graph.

Submit

You have used 1 of 1 attempt

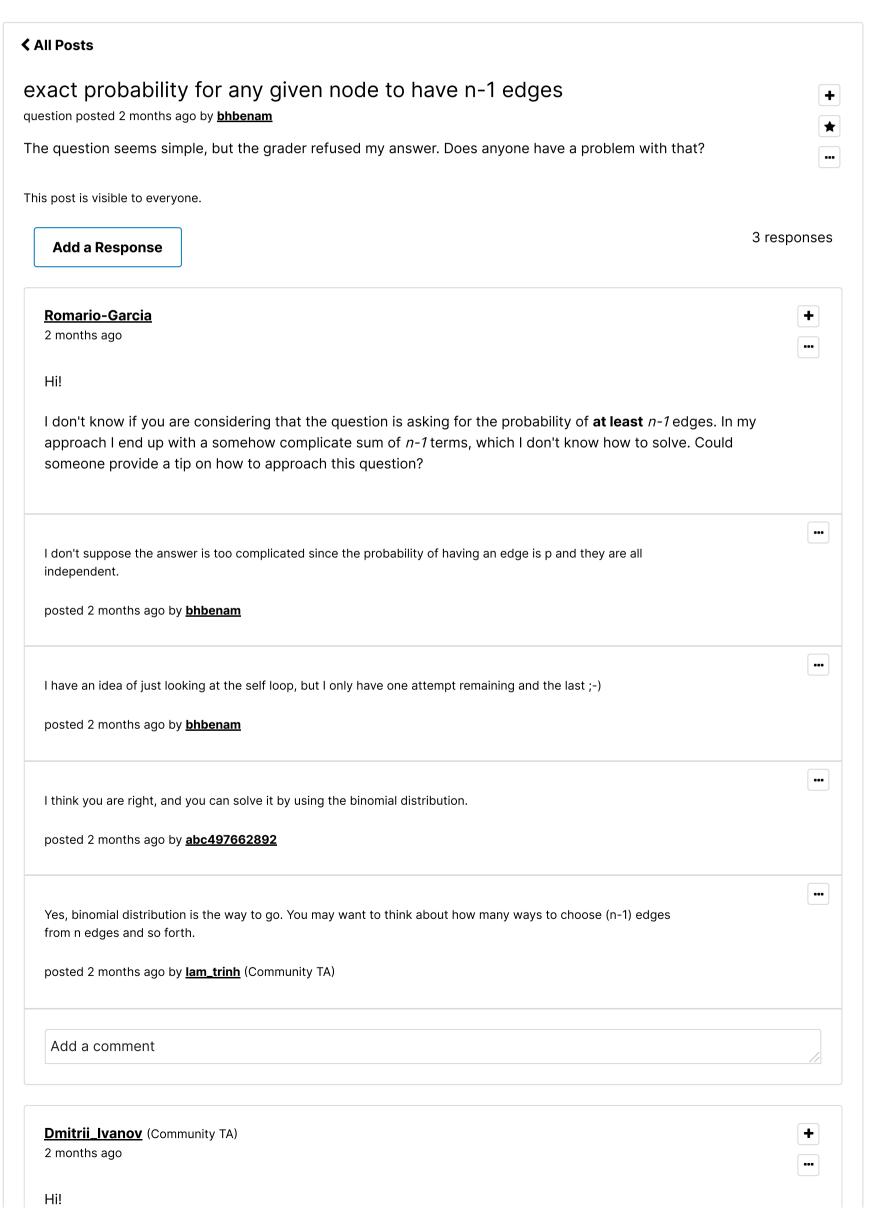
1 Answers are displayed within the problem

Discussion

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Topic: Module 3: Network Analysis: Graph Centrality Measures / 3. Centrality Measures – Introduction

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| graph allows m | graph with n nodes, the highest degree is n-1 for a given node in the graph, right? Unless the ulti edges between a pair of node and self loop in the graph, then the probability of a node with n s my understanding correct? | •• |
|-----------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----|
| posted 2 month | ns ago by <u>apcshark</u> | |
| you forgot self | edge, which in this case is allowed | • |
| posted 2 month | ns ago by <u>SunPenguin</u> | |
| Add a comm | nent | |
| | | |
| yz2001zzx 2 months ago | | + |
| P(x>=n-1), yo | time (stuck in here for about 1hr)I realized that there are just two meaningful PMF terms for but just decompose P(x>=n-1) (remember the binomial is a discrete distribution) into two terms and and you will get the answer. | ••• |
| | | |
| that means it's | ndle "for any given node" in these two questions? Once you have an edge selected for one node, at the same time also selected for another node, right? ns ago by graftedlife | •• |
| that means it's | at the same time also selected for another node, right? ns ago by graftedlife | ••• |
| that means it's posted 2 month | at the same time also selected for another node, right? ns ago by graftedlife | - |
| that means it's posted 2 month Add a comm | at the same time also selected for another node, right? ns ago by graftedlife nent | |
| that means it's posted 2 month Add a comm | at the same time also selected for another node, right? In s ago by graftedlife In ent In onses | |
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