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Course > Midterm Exam (1 week) > Midterm Exam 1 > Problem 4

Problem 4

For simplicity, suppose our rating matrix is a 2x2 matrix and we are looking for a rank-1 solution UV^T so that user and movie features U and V are both 2x1 matrices. The observed rating matrix has only a single entry:

$$Y = \begin{bmatrix} ? & 1 \\ ? & ? \end{bmatrix} \tag{6.4}$$

In order to learn user/movie features, we minimize

$$J(U,V) = \left(\frac{1}{2} \sum_{(a,i) \in D} (Y_{ai} - [UV^T]_{ai})^2\right) + \lambda (U_1^2 + V_1^2)$$
(6.5)

where U_1 and V_1 are the first components of the vectors U and V respectively (if $U=[u_1,u_2]$, then $U_1=u_1$), the set D is just the observed entries of the matrix Y, in this case just (1,2).

Note that the regularization we use applies only to the first coordinate of user/movie features . We will see how things get a bit tricky with this type of partial regularization.

Correction Note (July 30 21:00UTC): An earlier version does not include the clarifcation "where U_1 and V_1 are the first components of the vectors U and V respectively."

Correction Note (Aug 4 03:00UTC): Added an example of what U_1 means: if $U=[u_1,u_2]$, then $U_1=u_1$).

4. (1)

1.0/1 point (graded)

If we initialize $U=[u \quad 1]^T$, for some u>0 , what is the solution to the vector $V=[v_1 \quad v_2]^T$ as a function of λ and u?

(Enter V as a vector, enclosed in square brackets, and components separated by commas, e.g. type <code>[u,lambda+1]</code> if $V = \begin{bmatrix} u & \lambda + 1 \end{bmatrix}^T$.)

$$V = \begin{bmatrix} 0,1/u \end{bmatrix}$$
 \checkmark Answer: $\begin{bmatrix} 0,1/u \end{bmatrix}$

STANDARD NOTATION

Solution:

Notice that J only regularizes on the first coordinate. Thus, we only want to minimize $J\left(v_1,v_2\right)=rac{1}{2}(1-uv_2)^2+\lambda\left(v_1^2+u^2
ight)$ given that $V=\left[v_1,v_2\right]^T$. We can see that J is minimized when $v_1=0,v_2=rac{1}{u}$. Therefore,

$$V = \left[0, \frac{1}{u}\right]^T. \tag{6.6}$$

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You have used 1 of 3 attempts

1 Answers are displayed within the problem

4. (2)

0/1 point (graded)

What is the resulting value of $J\left(U,V
ight)$ as a function of λ and u?

(Type lambda for λ).

0

X Answer: (lambda*u^2)

0

STANDARD NOTATION

Solution:

Notice that J only regularizes on the first coordinate. Therefore,

$$J(U, V) = \frac{1}{2} \sum_{(a,i) \in D} (Y_{ai} - [UV^T]_{ai})^2 + \lambda (U_1^2 + V_1^2)$$

$$= \frac{1}{2} (1 - u_1 v_2)^2 + \lambda (u_1^2 + v_1^2)$$

$$= \frac{1}{2} \left(1 - u \cdot \frac{1}{u} \right)^2 + \lambda (u^2 + 0^2)$$

$$= \lambda u^2$$

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1 Answers are displayed within the problem

4. (3)

1/1 point (graded)

If we continue to iteratively solve for U and V, what would U and V converge to?

- ullet U goes to [0,1], V goes to $[0,\infty]$
- ullet U goes to [0,0], V goes to [0,0]

- ullet U goes to [0,1], V goes to [0,0]
- ullet U goes to $[0,\infty]$, V goes to [1,0]

Correction Note (July 29): In an earlier version, V was missing in all choices.

Solution:

The regularization error is minimized when u_1 and v_1 are 0. Over many iterations, u_1 will eventually converge to zero. The squared error term is $\frac{1}{2}(1-u_1v_2)^2$ is minized when $u_1v_2=1$. Since u_1 converges to 0, v_2 diverges to ∞ .

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You have used 1 of 3 attempts

1 Answers are displayed within the problem

4. (4)

3/3 points (graded)

Not all rating matrices Y can be reproduced by UV^T when we restrict the dimensions of U and V to be 2 imes 1.

For each matrix below, answer "Yes" or "No" according to whether it can be reproduced by such U and V of size 2 imes 1.

$$Y = \left[egin{array}{cc} 1 & -1 \ -1 & 1 \end{array}
ight]$$

yes

no

$$Y = egin{bmatrix} 1 & 0 \ 0 & 1 \end{bmatrix}$$

yes

● no

$$Y = \left[egin{array}{cc} 1 & 1 \ -1 & -1 \end{array}
ight]$$

yes

no

Solution:

In order for matrix Y to be reproduced by UV^T we must have $[u_1,u_2]^T imes [v_1,v_2] = Y$. For the second matrix, this would require $u_1 imes v_1 = 1$, $u_1 imes v_2 = 0$, $u_2 imes v_1 = 0$, $u_2 imes v_2 = 1$, which has no solution. The first matrix can be represented as $[1-1]^T imes [1-1]$ and the third can be represented as $[1-1]^T imes [11]$.

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You have used 1 of 3 attempts

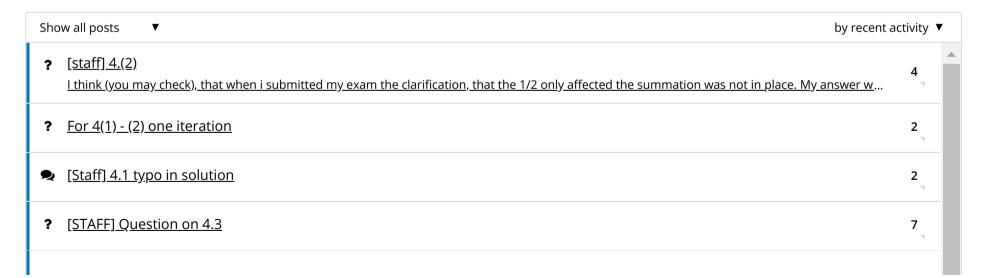
1 Answers are displayed within the problem

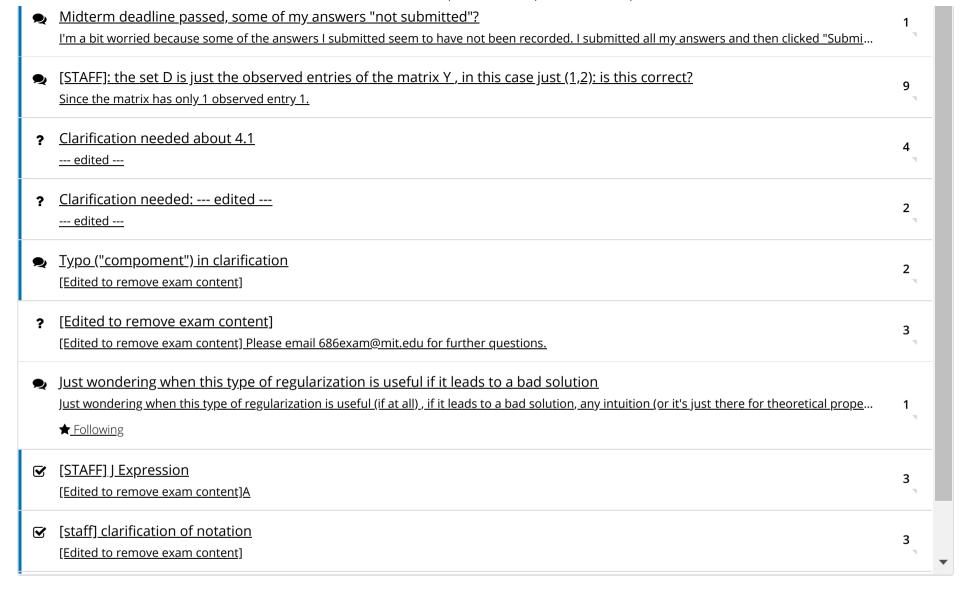
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