7.3.3 Simple Examples

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Week 7 due Nov 20, 2023 01:42 IST Completed

7.3.3 Simple Examples

Video

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Dr. Robert van de Geijn: So now we're going

to walk you through a whole bunch of special matrices

and talk about what the inverse matrices of those matrices are.

And what you're going to see is that there's

a pattern of how you should think about this.

0:00 / 0:00 2.0x

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Reading Assignment

0 points possible (ungraded)

There is an important discussion regarding the inversion of Gauss transforms and how it relates transforming a matrix to an upper triangular matrix to LU factorization. We suggest you read the text!!!

Read Unit 7.3.3 of the notes. [LINK]

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Homework 7.3.3.4 - Typo in answer?

I believe there is a typo in the answer of Homework 7.3.3.4. The fourth bullet point says that "You take 2 times the first component and add it

Calculator

Homework 7.3.3.10
Hi, In the answer of homework 7.3.3.10, It has been written, matrix X is equal to x0*e0 + x1*e1 + ... but I think it must be e0*x0 + e1*x1 + ...(exch... 2

Homework 7.3.3.1

1/1 point (graded)

If I is the identity matrix, then $I^{-1} = I$.

True ✔ Answer: True

Answer: True
What is the matrix that undoes Ix? Well, $Ix = x$, so to undo it, you do nothing. But the matrix that does nothing is the identity matrix.
Check: $II = II = I$. Hence I equals the inverse of I.

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Video

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Dr. Robert van de Geijn: What do we have here?

I is the identity matrix.

What do we know about the identity matrix?

We know that I times x is equal to x.

Now what matrix has the property such that

if you apply it to x you get x back?

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Homework 7.3.3.2

1/1 point (graded)

$$\text{Compute} \begin{pmatrix} -1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & \frac{1}{3} \end{pmatrix}^{-1} =$$

$$\begin{pmatrix} -1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & \frac{1}{3} \end{pmatrix}$$

Calculator

$$\begin{pmatrix} 0 & 0 & \frac{1}{3} \end{pmatrix}$$

- [x] $\begin{pmatrix} -1 & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & 3 \end{pmatrix}$
- [ ] $\begin{pmatrix} -1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & \frac{1}{2} \end{pmatrix}$
- [ ] $\begin{pmatrix} \frac{1}{3} & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & -1 \end{pmatrix}$

✔

Answer: Question: What effect does applying $\begin{pmatrix} -1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & \frac{1}{3} \end{pmatrix}$ to a vector have? Answer: The first component is multiplied by -1, the second by 2 and the third by $1/3$. To undo this, one needs to take the result first resulting component and multiply it by -1, the second resulting component by $1/2$ and the third resulting component by 3. This motivates that

$$\begin{pmatrix} -1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & \frac{1}{3} \end{pmatrix}^{-1} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & 3 \end{pmatrix}.$$

Now we check:

$$\begin{pmatrix} -1 & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & 3 \end{pmatrix} \begin{pmatrix} -1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & \frac{1}{3} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

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Homework 7.3.3.3

1/1 point (graded)

Assume $\delta_j \neq 0$ for $0 \leq j < n$. Then

$$\begin{pmatrix} \delta_0 & 0 & \cdots & 0 \\ 0 & \delta_1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \delta_{n-1} \end{pmatrix}^{-1} = \begin{pmatrix} \frac{1}{\delta_0} & 0 & \cdots & 0 \\ 0 & \frac{1}{\delta_1} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \frac{1}{\delta_{n-1}} \end{pmatrix}.$$

True ✔ Answer: True

Answer: True by the insights of the previous exercise.

Calculator

Check:

$$\begin{pmatrix} \delta_0 & 0 & \cdots & 0 \\ 0 & \delta_1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \delta_{n-1} \end{pmatrix} \begin{pmatrix} \frac{1}{\delta_0} & 0 & \cdots & 0 \\ 0 & \frac{1}{\delta_1} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \frac{1}{\delta_{n-1}} \end{pmatrix} = \begin{pmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{pmatrix}$$

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Dr. Robert van de Geijn: So what you want to do

is you want to ask the question what effect does

applying a diagonal matrix to a vector have?

Well what this answer here summarizes is the fact

that if you take this diagonal matrix, and I'm not going to write the 0s

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Homework 7.3.3.4

1/1 point (graded)

$$\text{Find } \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ -2 & 0 & 1 \end{pmatrix}^{-1}$$

- $\begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ -2 & 0 & 1 \end{pmatrix}$

- $\begin{pmatrix} -1 & 0 & 0 \\ -1 & -1 & 0 \\ 2 & 0 & -1 \end{pmatrix}$

Calculator

$$\begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ -\frac{1}{2} & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 2 & 0 & 1 \end{pmatrix}$$

There is a typo in the below: It should be "You take 2 times the first component and add it to the *third* component.

Answer: What effect does applying $\begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ -2 & 0 & 1 \end{pmatrix}$ to a vector have? Answer:

- It takes first component and adds it to the second component.
- It takes -2 times the first component and adds it to the third component.

How do you undo this?

- You take first component and *subtract* it from the second component.
- You take 2 times first component and add it from the second component.

The Gauss transform that does this is given by

$$\begin{pmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 2 & 0 & 1 \end{pmatrix}.$$

Notice that the elements below the diagonal are negated: 1 becomes -1 and -2 becomes 2.
Check:

$$\begin{pmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 2 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ -2 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

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Homework 7.3.3.5

1/1 point (graded)

Assume that the matrices below are partitioned conformally so that the multiplications and comparison are legal.

$$\left(\begin{array}{c|c|c} I & 0 & 0 \\ \hline 0 & 1 & 0 \\ \hline 0 & l_{21} & I \end{array}\right)^{-1} = \left(\begin{array}{c|c|c} I & 0 & 0 \\ \hline 0 & 1 & 0 \\ \hline 0 & -l_{21} & I \end{array}\right)$$

True

Answer: True

Answer: True by the argument for the previous exercise.
Check:

Calculator

$$\left(\begin{array}{c|c|c} 0 & 1 & 0 \\ \hline 0 & -l_{21} & I \end{array}\right)\left(\begin{array}{c|c|c} 0 & 1 & 0 \\ \hline 0 & l_{21} & I \end{array}\right) = \left(\begin{array}{c|c|c} 0 & 1 & 0 \\ \hline 0 & 0 & I \end{array}\right).$$

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Dr. Robert van de Geijn: OK we're back.

What's going on here?

Well remember that this matrix actually represented a very specific kind

of transformation when applied to a vector.

If you look at this matrix and you apply it to a vector, then what happens?

The first component is left alone.

0:00 / 0:00 2.0x

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Homework 7.3.3.6

1/1 point (graded)

Assume that the matrices below are partitioned conformally so that the multiplications and comparison are legal.

$$\left(\begin{array}{c|c|c} L_{00} & 0 & 0 \\ \hline l_{10}^T & 1 & 0 \\ \hline L_{20} & 0 & I \end{array}\right)\left(\begin{array}{c|c|c} I & 0 & 0 \\ \hline 0 & 1 & 0 \\ \hline 0 & l_{21} & I \end{array}\right) = \left(\begin{array}{c|c|c} L_{00} & 0 & 0 \\ \hline l_{10}^T & 1 & 0 \\ \hline L_{20} & l_{21} & I \end{array}\right)$$

Always ✔ Answer: Always

Answer: Always Just multiply it out.

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Answers are displayed within the problem

Homework 7.3.3.7

Calculator

1/1 point (graded)

$$\text{Compute } \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}^{-1} =$$

- (selected) $\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$
- $\begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$
- $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$
- $\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$

✓

Answer: The thing to ask is "What does this matrix do when applied to a vector?" It swaps the top two elements. How do you "undo" that? You swap the top two elements. So, the inverse is

$$\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

Check:

$$\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

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Homework 7.3.3.8

1/1 point (graded)

$$\text{Compute } \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}^{-1} =$$

- $\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ \ldots \end{pmatrix}$

Calculator

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

- [ ] $\begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$

- [ ] $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$

- [x] $\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$

✓

Answer: The thing to ask is "What does this matrix do when applied to a vector?" This permutation rotates the rows down one row. How do you "undo" that? You need to rotate the rows up one row. So, the inverse is

$$\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}.$$

Check:

$$\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}\begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

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Homework 7.3.3.9

1/1 point (graded)

Let P be a permutation matrix.

$P^{-1} = P$

Sometimes ✓ Answer: Sometimes

Answer: Sometimes The last two exercises provide and example and a counter example.

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Homework 7.3.3.10

1/1 point (graded)

Calculator

Let P be a permutation matrix.

$P^{-1} = P^T$

Always

✔ Answer: Always

Answer: Always (Note: it took me a good hour to think of the below explanation. The simple thing would have been to simply verify the result.)

What "action" does applying the matrix have when applied to a vector? It permutes the components. Now, it would seem that to "un"permute the vector, one probably has to apply another permutation. So we are looking for a permutation.

Let's let $p = \begin{pmatrix} k_0 & k_1 & \cdots & k_{n-1} \end{pmatrix}^T$ be the permutation vector such that $P = P(p)$. Then

$$P = \begin{pmatrix} e_{k_0}^T \\ e_{k_1}^T \\ \vdots \\ e_{k_{n-1}}^T \end{pmatrix} \quad \text{and} \quad Px = \begin{pmatrix} \chi_{k_0} \\ \hline \chi_{k_1} \\ \hline \vdots \\ \hline \chi_{k_{n-1}} \end{pmatrix}.$$

Let's say that B is the inverse of P. Then we want to choose B so that

$$\underbrace{\underbrace{\begin{pmatrix} \chi_0 \\ \hline \chi_1 \\ \hline \vdots \\ \hline \chi_{n-1} \end{pmatrix}}_{\chi_0 e_0 + \chi_1 e_1 + \cdots + \chi_{n-1} e_{n-1}}}_{\chi_{k_0} e_{k_0} + \chi_{k_1} e_{k-1} + \cdots + \chi_{k_{n-1}} e_{k_{n-1}}} = BPx = \underbrace{\begin{pmatrix} b_0 \mid b_1 \mid \cdots \mid b_{n-1} \end{pmatrix} \begin{pmatrix} \chi_{k_0} \\ \hline \chi_{k_1} \\ \hline \vdots \\ \hline \chi_{k_{n-1}} \end{pmatrix}}_{\chi_{k_0} b_0 + \chi_{k_1} b_1 + \cdots + \chi_{k_{n-1}} b_{n-1}}.$$

Hmmm, but if you pick $b_j = e_{k_j}$, the left- and the right-hand sides are equal. Thus,

$$P^{-1} = B = \begin{pmatrix} e_{k_0} \mid e_{k_1} \mid \cdots \mid e_{k_{n-1}} \end{pmatrix} = \begin{pmatrix} e_{k_0}^T \\ e_{k_1}^T \\ \vdots \\ e_{k_{n-1}}^T \end{pmatrix}^T = P^T.$$

It is easy to check this by remembering that $P^{-1}P = PP^{-1} = I$ has to be true:

$$\begin{aligned} PP^T &= \begin{pmatrix} e_{k_0}^T \\ e_{k_1}^T \\ \vdots \\ e_{k_{n-1}}^T \end{pmatrix} \begin{pmatrix} e_{k_0}^T \\ e_{k_1}^T \\ \vdots \\ e_{k_{n-1}}^T \end{pmatrix}^T = \begin{pmatrix} e_{k_0}^T \\ e_{k_1}^T \\ \vdots \\ e_{k_{n-1}}^T \end{pmatrix} \begin{pmatrix} e_{k_0} \mid e_{k_1} \mid \cdots \mid e_{k_{n-1}} \end{pmatrix} \\ &= \begin{pmatrix} e_{k_0}^T e_{k_0} & e_{k_0}^T e_{k_1} & \cdots & e_{k_0}^T e_{k_{n-1}} \\ e_{k_1}^T e_{k_0} & e_{k_1}^T e_{k_1} & \cdots & e_{k_1}^T e_{k_{n-1}} \\ \vdots & \vdots & \ddots & \vdots \\ e_{k_{n-1}}^T e_{k_0} & e_{k_{n-1}}^T e_{k_1} & \cdots & e_{k_{n-1}}^T e_{k_{n-1}} \end{pmatrix} = \begin{pmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{pmatrix} = I. \end{aligned}$$

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Calculator

0:00 / 0:00 2.0x

Dr. Robert van de Geijn: OK I'm back.

And the question you should ask yourself is

what is it that this permutation matrix does?

Well it swaps the first row with the second row.

So you would think that in order to undo that you

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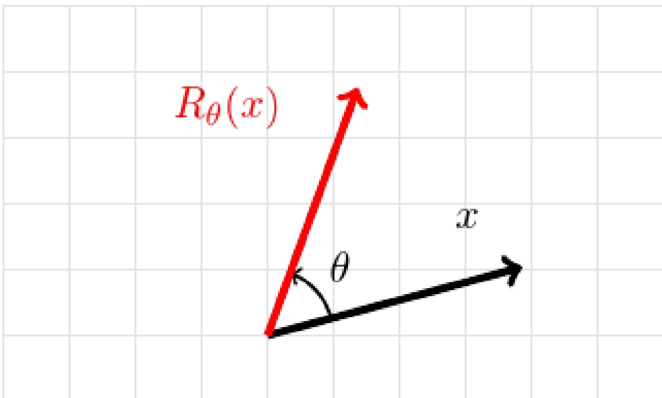
Homework 7.3.3.11

1/1 point (graded)

Recall from Week 2 how $R_\theta(x)$ rotates a vector x throught angle θ:



R_θ is represented by the matrix

$$R = \begin{pmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{pmatrix}.$$

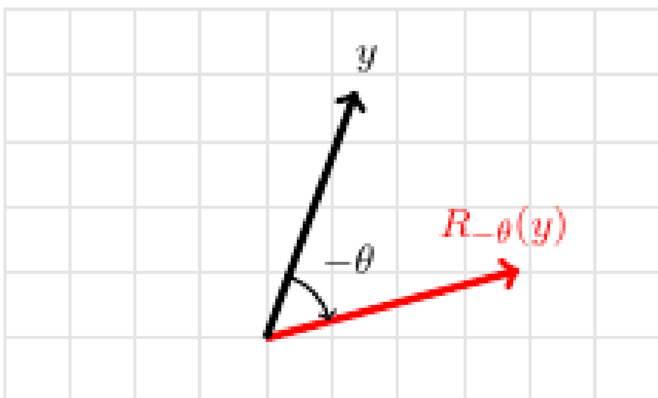
What transformation will "undo" this rotation through angle θ? (Mark all correct answers)

- [x] $R_{-\theta}(x)$
- [x] Ax, where $A = \begin{pmatrix} \cos(-\theta) & -\sin(-\theta) \\ \sin(-\theta) & \cos(-\theta) \end{pmatrix}$
- [x] Ax, where $A = \begin{pmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{pmatrix}$

Explanation

Answer: (a), (b), (c)

Well, if $y = R_\theta(x)$, then y must be rotated through angle $-\theta$ to transform it back into x:



Calculator

So, the inverse function for $R_\theta(x)$ is $R_{-\theta}(x)$. The matrix that represents R_θ is given by

$$\left(\begin{array}{cc} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{array} \right).$$

Thus, the matrix that represents $R_{-\theta}$ is given by

$$\left(\begin{array}{cc} \cos(-\theta) & -\sin(-\theta) \\ \sin(-\theta) & \cos(-\theta) \end{array} \right).$$

But we remember from trigonometry that $\cos(-\theta) = \cos(\theta)$ and $\sin(-\theta) = -\sin(\theta)$. This would mean that

$$\left(\begin{array}{cc} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{array} \right)^{-1} = \left(\begin{array}{cc} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{array} \right).$$

Let us check!

$$\begin{aligned} &\left(\begin{array}{cc} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{array} \right) \left(\begin{array}{cc} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{array} \right) \\ &\quad = \left(\begin{array}{cc} \cos(\theta)\cos(\theta) + \sin(\theta)\sin(\theta) & -\cos(\theta)\sin(\theta) + \sin(\theta)\cos(\theta) \\ -\sin(\theta)\cos(\theta) + \cos(\theta)\sin(\theta) & \sin(\theta)\sin(\theta) + \cos(\theta)\cos(\theta) \end{array} \right) = \left(\begin{array}{cc} 1 & 0 \\ 0 & 1 \end{array} \right). \end{aligned}$$

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Dr. Robert van de Geijn: We're back.

And the first thing we can observe is that look, the rotation through angle

theta can be undone by rotating through an angle minus theta.

So we know that the inverse of the transformation R sub theta

is the transformation R sub minus theta.

We also know that R sub theta of x for

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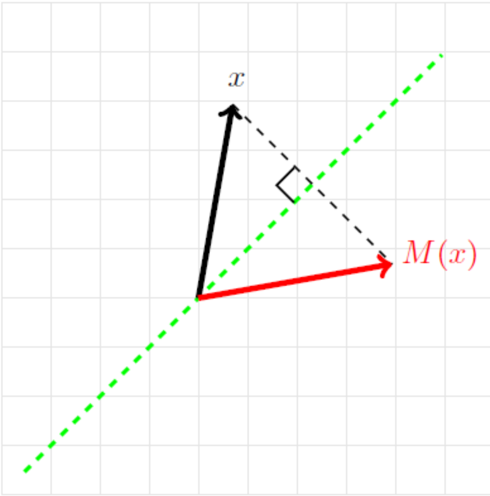
Calculator

Homework 7.3.3.12

1/1 point (graded)

Consider a reflection with respect to the 45 degree line:

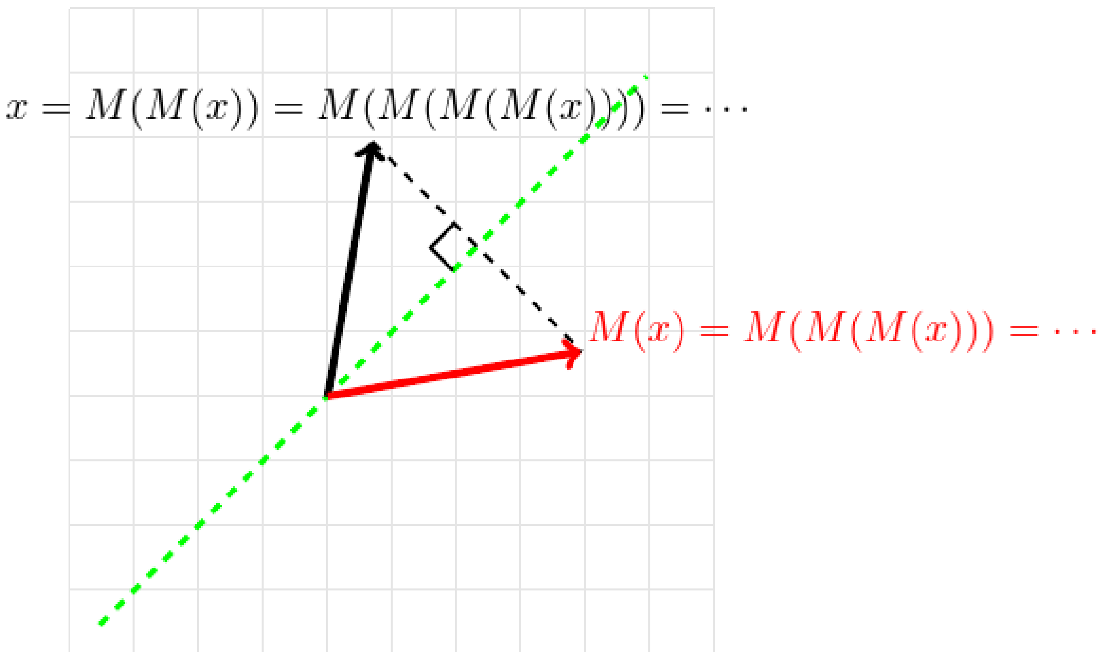


and let *A* be the matrix that represent the reflection *M(x)*.

- ( ) $A^{-1} = -A$
- (•) $A^{-1} = A$
- ( ) $A^{-1} = I$
- ( ) All of the above.

✔

Answer: (b) $A^{-1} = A$: Mirror twice, and you should get the original answer back. (The angle doesn't need to be 45 degrees. This is true for any reflection.)



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