7. The inner product for periodic functions of arbitrary period

Adapt the definition of the inner product to the case of functions f and g of period $2L$:

$$\langle f, g \rangle := \int_{-L}^{L} f(t)\, g(t)\ dt.$$

(This conflicts with the earlier definition of $\langle f, g \rangle$, for functions for which both make sense, so perhaps it would be better to write $\langle f, g \rangle_L$ for the new inner product, but we won't bother to do so.)

The same calculations as before show that the functions

$$1, \cos\frac{\pi t}{L}, \cos\frac{2\pi t}{L}, \cos\frac{3\pi t}{L}, \ldots, \sin\frac{\pi t}{L}, \sin\frac{2\pi t}{L}, \sin\frac{3\pi t}{L}, \ldots$$

form an orthogonal basis for the vector space of “all" periodic functions of period $2L$, with

$$\begin{aligned} \langle 1,1\rangle &= 2L \\ \left\langle \cos\frac{n\pi t}{L}, \cos\frac{n\pi t}{L}\right\rangle &= L \\ \left\langle \sin\frac{n\pi t}{L}, \sin\frac{n\pi t}{L}\right\rangle &= L \end{aligned}$$

(the average value over the whole period of $\cos^2 \omega t$ is $1/2$ for any ω, and the average value of $\sin^2 \omega t$ is $1/2$ too).

This gives another way to derive the Fourier coefficient formulas for functions of period $2L$.

Compute the inner product

1/1 point (graded)

Let $T(t)$ be the periodic function of period 2 such that $T(x) = |x|$ for $-1 \leq x \leq 1$; this is called a **triangle wave.** Let $\mathrm{Sq}(x)$ denote the **odd** square wave of period 2. Compute $\langle T, \mathrm{Sq}\rangle$ directly from the integral definition.

$\langle T, \mathrm{Sq}\rangle =$ 0 ✔ **Answer:** 0

Solution:

We see that

$$\begin{aligned} \langle T, \mathrm{Sq}\rangle &= \int_{-1}^{1} |x|\,\mathrm{Sq}(x)\,dx = \int_{-1}^{0} -(-x)\,dx + \int_{0}^{1} x dx \\ &= \int_{-1}^{1} x dx = 0. \end{aligned}$$

Hence, the triangle wave and square wave are orthogonal.

Note that this makes sense because the triangle wave is even periodic, and the square wave is odd periodic. So we would anticipate that every term in one Fourier series is orthogonal to the terms in the other.

Submit You have used 1 of 5 attempts

Answers are displayed within the problem

7. The inner product for periodic functions of arbitrary period

Hide Discussion

Topic: Unit 1: Fourier Series / 7. The inner product for periodic functions of arbitrary period

Add a Post

Show all posts ▼ by recent activity ▼

? Doubt
I did this exercise but I don't know if my reasoning is right. The answer could be related to the orthogonality of T and Sq. Thanks! 1

Learn About Verified Certificates