Observation Theory

Script V22A – Properties of the functional model

Hello everyone!

So far in this week we learned about the two parts of the mathematical model for estimation problems: the functional model and the stochastic model.

The first one is a function that relates observations to unknowns, and the latter "stochastic model" describes statistical characteristics of the observational errors.

In this section, we want to have a closer look at the first part: "functional model" and discuss some of its important properties.

Please note that, in this lecture, we use some mathematical concepts from linear algebra.

Familiarity with these concepts is very helpful for following this lecture.

If you are not familiar with the basics of linear algebra (for example concepts like rank of a matrix, or vector dependency), and if you have not followed yet the first unit of this section, that is "the Intermezzo: recap on linear algebra", I suggest to pause this video, go back to the previous unit, and study the short recap on linear algebra, and then continue with this lecture.

We have learned so far about the linear functional models in the form of y, or expectation of y, is equal to Ax.

With this kind of models, the estimation problem can be seen as how to solve a system of observation equations with "m" equations (corresponding to m observations), and n unknowns.

An important question is whether this system of equations has a solution or not?

Is it solvable?

Or in other words, under what conditions does a solution exist?

To answer this question, let's have a closer look to the system of y equal to Ax.

What does it mean when we say a system of equations has a solution.

Let's assume a matrix A with m rows and n columns.

From linear algebra, we know that the multiplication of this matrix to a vector (for example the vector x with n unknowns), is a linear combination of the column vectors of A.

So we can conclude that a solution to this system of equations exists if and only if the vector y can be written as a linear combination of the column vectors of matrix A.

If this is the case, the vector y is an element of the column space (or range space) of matrix A.

Recall that the range space of A, denoted by "R(A)", is the space in which all vectors can be constructed as a linear combination of the columns of A.

This fact can be conceptually visualised as the following.

If the gray area here, shows the range space of A, the system of y-is-Ax has a solution only if the vector y belongs to the range space of A.

Systems of equations for which this holds are called “consistent” systems.

In other words, if the system is consistent, it means it has at least one solution.

Note that, because the vector of observations is m dimensional, as we have m observations, we can say that the consistency is guaranteed if the range space of A is equal to the entire m-dimensional space.

In that case, the m-dimensional vector y definitely belongs to R(A).

Again from linear algebra, we know that: in order to have a matrix “A” whose columns can span the whole m-dimensional space, the number of its independent column vectors (or we call it “rank” of the matrix A) should be equal to m: the number of observations.

In summary, in this video we learned that:

1) a solution to y-is-Ax exists if and only if y can be written as linear combination of the columns of A

(or when y is a member of the range space of A)

2) systems with at least one solution are called “consistent” systems

3) And consistency is guaranteed if the rank of A - or number of independent columns of A - is equal to the number of observations.

Now, to get a better sense of consistency and inconsistency, let’s look at some numerical examples in the next unit.