Prove that the only prime triple is 3, 5, 7 [duplicate]

This question already has an answer here:

How can I prove that one of n, $n+2$, and $n+4$ must be divisible by three, for any $n \in \mathbb{N}$ *13 answers*

Prove that the only prime triple is $3, 5, 7$.
I tried proving using this method: Multiplication of 3 jumps back and forth between being an even and an odd number. Thus goes from odd to odd over an interval max size 6, and likewise from even to even. In either case these will have the following types: (even - odd - even - odd - even - odd) and (odd - even - odd - even - odd - even), with 3 being a multiple of the first instance in the combinations. In the both combinations, the first and fourth would be divisible by three, leaving only two possible primes left (two odd numbers). Thus the only triple prime can be a combination where three is considered a prime....
However, is this a formal proof? can anybody tell me how to do a proof for this questions?

(proof-writing)

asked Nov 2 '13 at 21:33
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marked as duplicate by ShreevatsaR, hardmath, Thomas Andrews, Nick Peterson, Gigili Jan 13 '14 at 5:23

This question has been asked before and already has an answer. If those answers do not fully address your question, please ask a new question.

8 You want to show that one of $x, x+2, x+4$ is always divisible by 3. That is what your argument is getting at. It could have been done more efficiently: x leaves remainder 0, 1, or 2 on division by 3. If it is 0, we are finished with this part of the argument. If it is 1, then $x+2$ is divisible by 3. If it is 2 then $x+4$ is. So one of our numbers must be 3. – André Nicolas Nov 2 '13 at 21:39

2 Here is a link to a question that asks the statement in André's comment. – Jay Nov 2 '13 at 21:47

1 Answer

Every non-negative integer n can be written in the form $n = 6x + k$, where x is a non-negative integer and $k \in \{0, 1, 2, 3, 4, 5\}$

$n = 6x, 6x + 2$, or $6x + 4 \implies 2 \mid n$, while $n = 6x$ or $6x + 3 \implies 3 \mid n$, which leaves only $n = 6x + 1$ and $n = 6x + 5 = 6(x + 1) - 1$ for integers greater than 3 and not divisible by 2 or 3, and as a special case for integers greater than 3 and not divisible by any smaller prime i.e. primes,

so for $p \in \mathbb{P}$ such that $p > 3$,

$p + 2 \in \mathbb{P} \implies p = 6x + 5 \implies p + 4 = 6(x + 1) + 3 \implies 3 \mid p + 4.$

edited Nov 15 '13 at 3:30 answered Nov 2 '13 at 23:45
Jaycob Coleman
188 4 22