MITx: 6.041x Introduction to Probability - The Science of Uncertainty

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Problem 6: True or False

(4/5 points)

For each of the following statements, determine whether it is true (meaning, always true) or false (meaning, not always true). Here, we assume all random variables are discrete, and that all expectations are well-defined and finite.

1. Let X and Y be two binomial random variables.

   a) If X and Y are independent, then $X+Y$ is also a binomial random variable.

   False ✔ **Answer:** False

   b) If X and Y have the same parameters, n and p, then $X+Y$ is a binomial random variable.

   True ✘ **Answer:** False

   c) If X and Y have the same parameter p, and are independent, then $X+Y$ is a binomial random variable.

True ▾ ✔ **Answer:** True

2. Suppose that $\mathbf{E}[X]=0$. Then, $X=0$.

False ▾ ✔ **Answer:** False

3. Suppose that $\mathbf{E}[X^2]=0$. Then, $\mathbf{P}(X=0)=1$.

True ▾ ✔ **Answer:** True

Answer:

1. a) False. Intuitively, X corresponds to independent coin flips of a coin with a certain bias, and Y corresponds to independent coin flips of another coin, which need not have the same bias as the first coin. Throughout the overall number of coin flips, the bias is not kept constant, and so we are in a different situation from the one modeled by binomial random variables.

   For a concrete counter-example, suppose that X and Y are independent Bernoulli random variables, with parameters 0.9 and 0.1, respectively. In particular, they are each binomial with parameter $n=1$. The sum $X+Y$ takes values in $\{0,1,2\}$. So, if it were binomial, it would need to have a parameter n equal to 2. The parameter p of such a binomial would have to satisfy $\mathbf{E}[X+Y]=2p$. Since $\mathbf{E}[X+Y]=\mathbf{E}[X]+\mathbf{E}[Y]=0.9+0.1=1$, we would require $p=1/2$. This would then imply that $\mathbf{P}(X+Y=2)=p^2=1/4$. However, we can check that $\mathbf{P}(X+Y=2)=\mathbf{P}(X=1)\cdot\mathbf{P}(Y=1)=0.9\cdot 0.1\neq 1/4$. The contradiction shows that $X+Y$ is not binomial.

b) False. If X and Y have the same parameters, n and p, $X+Y$ is not necessarily a binomial random variable. For example, if the random variables X and Y are dependent such that $X=Y$, then the random variable $X+Y$ has zero probability at all odd values of n. Therefore, $X+Y$ is not binomial.

c) True. We may interpret $X+Y$ as the number, X, of Heads in some independent tosses of a coin, plus the number, Y, of Heads in some additional independent tosses of the same coin. Therefore, $X+Y$ is binomial.

2. False. Consider a random variable with

$$p_X(x)=\begin{cases}1/2, & \text{if } x=1,\\ 1/2, & \text{if } x=-1.\end{cases}$$

We have $\mathbf{E}[X]=0$, but X takes nonzero values.

3. True. Suppose that X satisfies $\mathbf{E}[X^2]=0$ but $\mathbf{P}(X=0)\neq 1$. Then, $\mathbf{P}(X=w)>0$ for some $w\neq 0$. It would follow that $\mathbf{E}[X^2]\geq w^2\cdot\mathbf{P}(X=w)>0$, which would contradict the assumption that $\mathbf{E}[X^2]=0$.

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