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6.

Setup: All problems on this page use the following setup.

Let $\mathcal{X} = \{1, \dots, k\}$ be a sample space of k possible outcomes of an experiment, and $(\mathcal{X}, \{P_{\theta}\}_{\theta \in \Theta})$ be a statistical model for this experiment.

Suppose that this model has d parameters, so that $\Theta \subset \mathbb{R}^d$.

Suppose that the Fisher information matrix $I(\theta)$ of this statistical model is nonsingular (i.e. invertible) and that the MLE $\hat{\theta}_n$ has the Gaussian asymptotic distribution

$$\sqrt{n}\left(\hat{ heta}_{n}- heta
ight) \stackrel{ ext{in }(d) ext{ under }P_{ heta}}{\longrightarrow} \mathcal{N}\Big(0,I^{-1}\left(heta
ight)\Big)$$

for every $\theta \in \Theta$.

Let X_1,\ldots,X_n be i.i.d. observations from repeated trials of the experiment. Let

$$N_j = \sum_{i=1}^n \mathbf{1} \left(X_i = j
ight)$$

be the counts of the number each outcome is observed in the data.

Dimension of the Parameter

0/1 point (graded)

What must be true about d, the dimension of the model?

- $\bigcirc d = 1$
- $\bigcirc d = k-1$
- $\bigcirc d \leq k-1$
- $left d \geq k-1$



Solution:

Since the asymptotic distribution of the MLE is non-degenerate, we must have $d \leq k$.

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You have used 2 of 3 attempts

1 Answers are displayed within the problem

Degrees of Freedom

1/1 point (graded)

Consider the following χ^2 test statistics:

$$T_{n} \; = \; n \sum_{i=1}^{k} rac{\left(N_{i}/n - P_{ heta}\left(i
ight)
ight)^{2}}{P_{ heta}\left(i
ight)}.$$

Under the null hypothesis

$$H_0: heta = heta^0$$

what is the number of degrees of freedom of the asymptotic distribution, which is a χ^2 distribution, of T_n ?

Degrees of freedom:

k-1	

✓ Answer: k-1

k-1

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Alternate Chi Square Test

1/1 point (graded)

Now, consider the alternate χ^2 test statistic below:

$$\widetilde{T_n} \; = \; n \sum_{i=1}^k rac{\left(N_i/n - P_{\hat{ heta}}\left(i
ight)
ight)^2}{P_{\hat{ heta}}\left(i
ight)}.$$

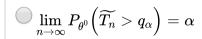
(The difference between T_n and $\widetilde{T_n}$ is that heta is changed to $\hat{ heta}$ as the parameter for P.)

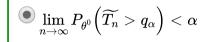
Suppose that q_{lpha} is chosen such that

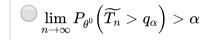
$$\lim_{n o\infty}P_{ heta^0}\Bigl(T_n>q_lpha\Bigr)=lpha$$

where $T_n=n\sum_{i=1}^krac{\left(N_i/n-P_ heta(i)
ight)^2}{P_ heta(i)}$ was defined as in the previous problem.

Which of the following must be true?







None of the above



Solution:

- Under the null hypothesis, $T_n^{(j=0)} \xrightarrow[n \to \infty]{(d)} \chi_{k-1}^2$ and coincides with the Wald's test of the multinomial statistical model. This follows from the discussion of Lecture 15.
- By estimating the parameter θ with MLE in the test statistic, we obtain an asymptotic χ^2 distribution with **fewer** degrees of freedom:

$$T_n^{(j)} \xrightarrow[n o \infty]{(d)} \chi^2_{k-1-j}.$$

Therefore, the threshold $q_{\alpha}^{(j)}$ of the test $\psi_j\left(q_{\alpha}^{(j)}\right)=\mathbf{1}\left(T_n^{(j)}>q_{\alpha}^{(j)}\right)$ with asymptotic level α **decreases** with the number j of parameters that are estimated from data. Consequently, $\psi_{j+1}\left(q\right)$ is more **conservative** (rejects less often) than $\psi_j\left(q\right)$.

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You have used 2 of 3 attempts

Answers are displayed within the problem
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10 = k.1.2) k.1.3) 1st choice

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