1. Companion system

Find the companion matrix

1.0/1 point (graded)

Consider the differential equation

$$D^3x - 2D^2x - Dx + 2x = 0, \qquad D = \frac{d}{dt}.$$

Convert the equation into a first order system of the form $\dot{\mathbf{x}} = \mathbf{A}\mathbf{x}$ where $\mathbf{x} = \begin{pmatrix} x \\ \dot{x} \\ \ddot{x} \end{pmatrix}$. Find $\mathbf{A}$.

(Enter as matrix between square brackets, entries in each row separated by commas, rows separated by semicolons: e.g. type **[-3, 1, 0; 4, 0 ,0; 5, 1, 1]** for the matrix $\begin{pmatrix} -3 & 1 & 0 \\ 4 & 0 & 0 \\ 5 & 1 & 1 \end{pmatrix}$.)

$\mathbf{A} =$ [0,1,0;0,0,1;-2,1,2]

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Find the eigenvalues

1.0/1 point (graded)

Find the eigenvalues of the matrix $\mathbf{A}$ above.

(Enter as a comma separated list of numbers: e.g. -3, 4, 5.)

2,-1,1

Answer: 2,1,-1

Solution:

The characteristic polynomial of the companion matrix matches that of the differential equation, so it is given by

$$\lambda^3 - 2\lambda^2 - \lambda + 2 = (\lambda - 2)(\lambda - 1)(\lambda + 1),$$

and the eigenvalues are 2, 1 and -1.

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Answers are displayed within the problem

Find the eigenvectors

3/3 points (graded)

For each eigenvalue of the matrix $\mathbf{A}$ above, find an associated nonzero eigenvector.

Note that $\mathbf{A}$ has three eigenvalues such that $a > b > c$. Enter the eigenvectors in order, so that the eigenvector associated to the largest eigenvalue goes into the first box, the eigenvector associated to the smallest eigenvalue goes into the third answer box, and the middle eigenvector is associated with the middle eigenvalue.

(Enter as column vectors, entries separated by semicolons: e.g. [-3; 4; 5].)

[1;2;4]

Answer: [1;2;4]

[1;1;1]

Answer: [1;1;1]

[1;-1;1]

Answer: [1;-1;1]

Solution:

For each eigenvalues we find the corresponding eigenspace. Since each eigenvalue has multiplicity one, each eigenspace is one-dimensional.

A nonzero eigenvector corresponding to eigenvalue $\lambda = 2$ is a (nonzero) vector in the nullspace of

$$\begin{pmatrix} -2 & 1 & 0 \\ 0 & -2 & 1 \\ -2 & 1 & 0 \end{pmatrix},$$

which is any multiple of the vector $\begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix}$.

A nonzero eigenvector corresponding to eigenvalue $\lambda = 1$ is a (nonzero) vector in the nullspace of

$$\begin{pmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ -2 & 1 & 1 \end{pmatrix},$$

which is any nonzero multiple of the vector $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$.

A nonzero eigenvector corresponding to eigenvalue $\lambda = -1$ is a (nonzero) vector in the nullspace of

$$\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ -2 & 1 & 3 \end{pmatrix},$$

which is any nonzero multiple of the vector $\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$.

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Solve the original differential equation

1/1 point (graded)

Solve the original differential equation for x which satisfies the following initial conditions:

$$\begin{aligned} x &= 1 \\ \dot{x} &= -1 \\ \ddot{x} &= 0. \end{aligned}$$

$x(t) =$

-exp(2*t)/3+exp(t)/2+5*exp(-t)/6

Answer: -e^(2*t)/3+e^t/2+5*e^(-t)/6

$$-\frac{\exp(2\cdot t)}{3} + \frac{\exp(t)}{2} + \frac{5\cdot\exp(-t)}{6}$$

Solution:

The general solution takes the form

$$\begin{pmatrix} x(t) \\ \dot{x}(t) \\ \ddot{x}(t) \end{pmatrix} = c_1 \begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix} e^{2t} + c_2 \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} e^{t} + c_3 \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} e^{-t}.$$

We use the initial conditions to get 3 equations in the 3 unknown constants c_1, c_2, and c_3:

$$\begin{aligned} c_1 + c_2 + c_3 &= 1 \\ 2c_1 + c_2 - c_3 &= -1 \\ 4c_1 + c_2 + c_3 &= 0. \end{aligned}$$

Solving this linear system by elimination and back substitution we find

$$\begin{aligned} c_1 &= -1/3 \\ c_2 &= 1/2 \\ c_3 &= 5/6. \end{aligned}$$

Therefore $x(t) = -e^{2t}/3 + e^{t}/2 + 5e^{-t}/6$.

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You have used 1 of 5 attempts

Answers are displayed within the problem

1. Companion system

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