13. Variance in Terms of the Canonical Parameter

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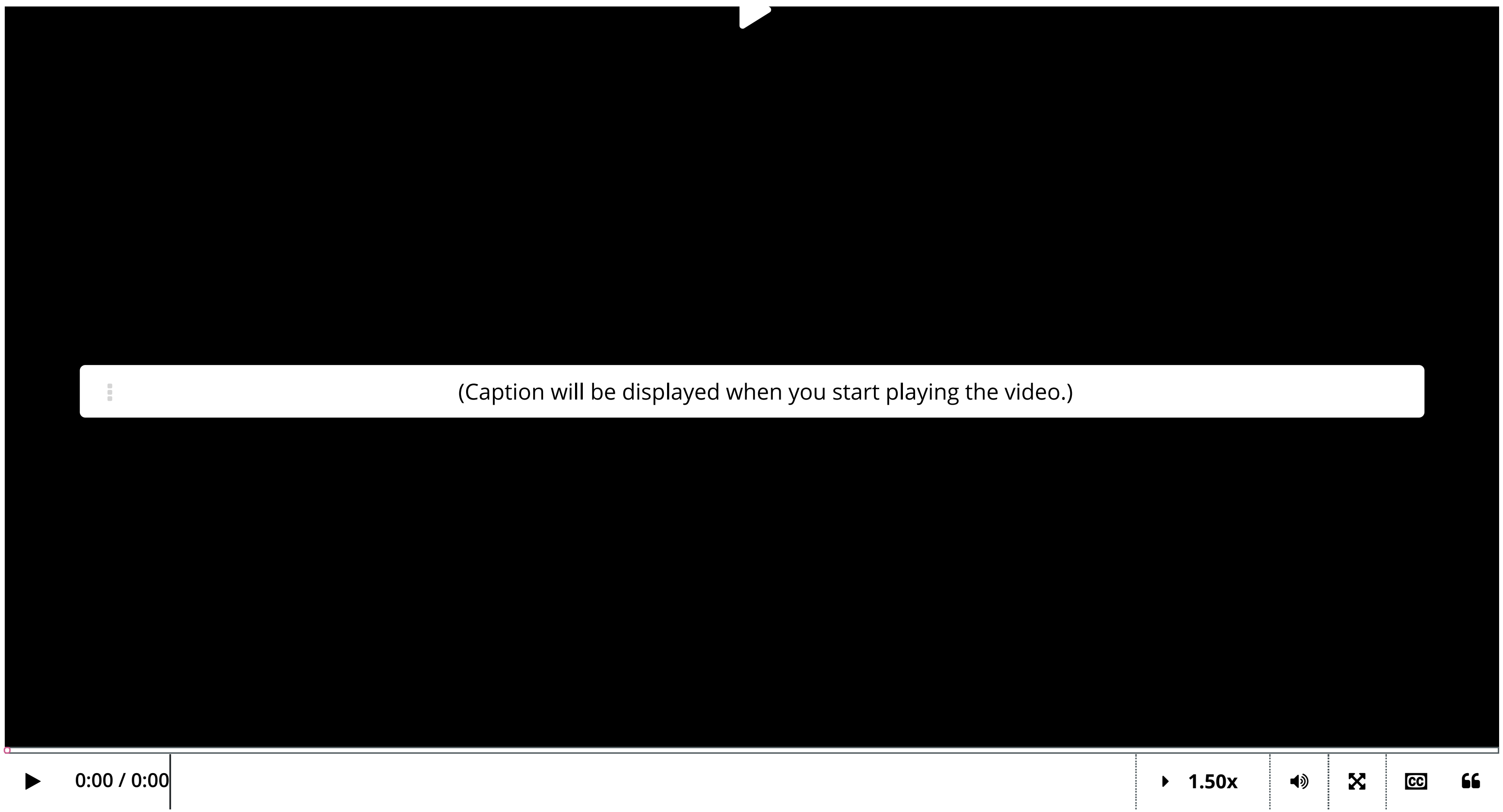

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Practice: The Mean and Variance of Binomial Distribution

2/2 points (graded)

Recall that the pmf of a Binomial distribution $\text{Binom}\,(n, p)$, with known n can be written as:

$$f_\theta(y) \;=\; \exp\left(\frac{y\theta - b(\theta)}{\phi} + c(y,\phi)\right).$$

Refer to the answer of $b(\theta)$ and ϕ to the problem *Practice: Binomial Distribution as a Canonical Exponential Family* 2 pages before this one.

Compute $b'(\theta)$.

$b'(\theta) =$ n*e^theta/(1+e^theta) ✔ **Answer:** n*e^theta/(1+e^theta)

$\frac{n\cdot e^\theta}{1+e^\theta}$

(Is this equal to $\mathbb{E}[Y]$?)

Compute $\phi b''(\theta)$.

$\phi b''(\theta) =$ n*e^theta/(1+e^theta)^2 ✔ **Answer:** n*e^theta/(1+e^theta)^2

$\frac{n\cdot e^\theta}{(1+e^\theta)^2}$

Note: Express your answers in terms of the canonical parameter θ.

STANDARD NOTATION

Solution:

Recall

$$b(\theta) \;=\; n\ln(1+e^\theta).$$

Taking the derivative gives

$$b'(\theta) \;=\; \frac{db}{d\theta}(\theta) \;=\; \frac{ne^\theta}{1+e^\theta}.$$

Recall that $\theta = \ln\left(\frac{p}{1-p}\right)$ so $e^{\theta} = \frac{p}{1-p}$. Plugging this in to the equation above gives

$$b'(\theta(p)) = \frac{ne^{\theta}}{1+e^{\theta}} = np$$

which is, as expected, equal to $\mathbb{E}[Y]$ where $Y \sim \textsf{Binom}(n,p)$.

Take the second derivative of $b(\theta)$:

$$\begin{aligned} b''(\theta) &= \frac{db}{d\theta}\frac{ne^{\theta}}{1+e^{\theta}} \\ &= n\frac{e^{\theta}(1+e^{\theta}) - (e^{\theta})e^{\theta}}{(1+e^{\theta})^2} \\ &= n\frac{e^{\theta}}{(1+e^{\theta})^2} \end{aligned}$$

Recall that $\phi = 1,$ so $\phi b''(\theta) = b''(\theta)$. Rewriting $\phi b''(\theta)$ in terms of p gives $\phi b''(\theta(p)) = np(1-p),$ which is indeed the variance of a binomial variable $Y \sim \textsf{Binom}(n,p)$.

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Answers are displayed within the problem

The Log-Partition Function b

4/4 points (graded)

For each proposed function b shown below, indicate (based only on its second derivative) whether it could potentially be a log-partition function of some exponential family **with dispersion** $\phi = 1$.

- $b(\theta) = \theta^2 - 2\theta + 1$

- [x] Valid
- [ ] Invalid

✔

- $b(\theta) = \sqrt{\theta}$

- [ ] Valid
- [x] Invalid

✔

- $b(\theta) = \ln\theta$

- [ ] Valid
- [x] Invalid

✔

- $b(\theta) = \theta$

- [x] Valid
- [ ] Invalid

✔

Solution:

Recall that in a canonical exponential family, $b''(\theta) \cdot \phi = \text{Var}(Y)$ in lecture, which is always non-negative, i.e. $b(\theta)$ must be convex. Not all of the functions listed satisfy this property.

- Yes. Since the second derivative is positive, it is convex and therefore valid.
- No. Since the second derivative is negative, it is not convex and therefore invalid.
- No. Since the second derivative is negative, it is not convex and therefore invalid.
- Yes. Since the second derivative is non-negative, it is convex and therefore valid.

Submit You have used 1 of 1 attempt

Answers are displayed within the problem

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