Exercise: Conditional variance II

(3/3 points)

The random variable Q is uniform on $[0,1]$. Conditioned on $Q=q$, the random variable X is Bernoulli with parameter q.

(a) The conditional variance, $\mathbf{var}(X \mid Q)$, is equal to:

- $1/4$
- $q(1-q)$
- $Q(1-Q)$ ✔
- q^2
- Q^2

(b) Recall that a uniform random variable on $[0,1]$ has a variance of $1/12$ and also satisfies $\mathbf{E}[Q^2]=1/3$. Then:

$\mathbf{var}\big(\mathbf{E}[X \mid Q]\big) =$ 1/12 ✔ **Answer:** 0.08333

$\mathbf{E}\big[\mathbf{var}(X \mid Q)\big] =$ 1/6 ✔ **Answer:** 0.16667

Answer:
(a) We know that $\mathbf{var}(X \mid Q=q)=q(1-q)$, for all $q \in [0,1]$, which translates into the abstract statement $\mathbf{var}(X \mid Q)=Q(1-Q)$.

(b) Since $\mathbf{E}[X \mid Q]=Q$, we have $\mathbf{var}\big(\mathbf{E}[X \mid Q]\big)=\mathbf{var}(Q)=1/12$.

Since $\mathbf{var}(X \mid Q) = Q(1-Q)$, we have

$$\mathbf{E}\big[\mathbf{var}(X \mid Q)\big] = \mathbf{E}\big[Q(1-Q)\big] = \mathbf{E}[Q] - \mathbf{E}[Q^2] = \frac{1}{2} - \frac{1}{3} = \frac{1}{6}.$$

You have used 1 of 2 submissions