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6. The geometry of matrices as functions

Imagine evaluating an $m \times n$ matrix $\mathbf{A}$ on every vector in the input space $\mathbb{R}^n$, to get vectors in the output space $\mathbb{R}^m$. To visualize it, draw a shape in the input space, apply $\mathbf{A}$ to every point in the shape, and plot the output points in the output space.

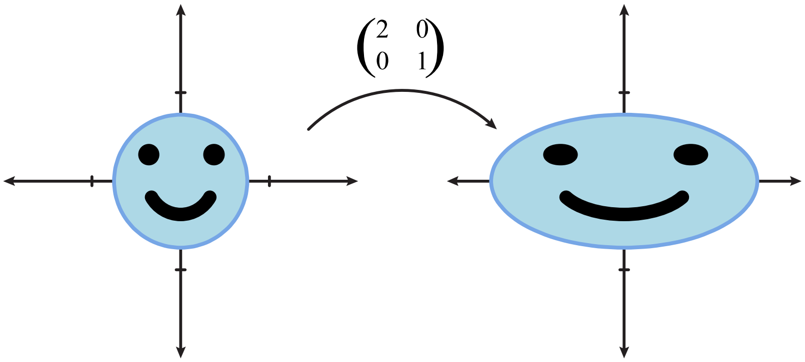
Problem 6.1 The matrix $\begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$ represents a function $\mathbf{f}$ from $\mathbb{R}^2$ to $\mathbb{R}^2$. Determine what it does to the *standard basis* vectors $\mathbf{e}_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, $\mathbf{e}_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ of $\mathbb{R}^2$. Then depict what happens to a smiley face of unit area under this matrix function.

Solution: We have

$$\mathbf{f}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \end{pmatrix}$$

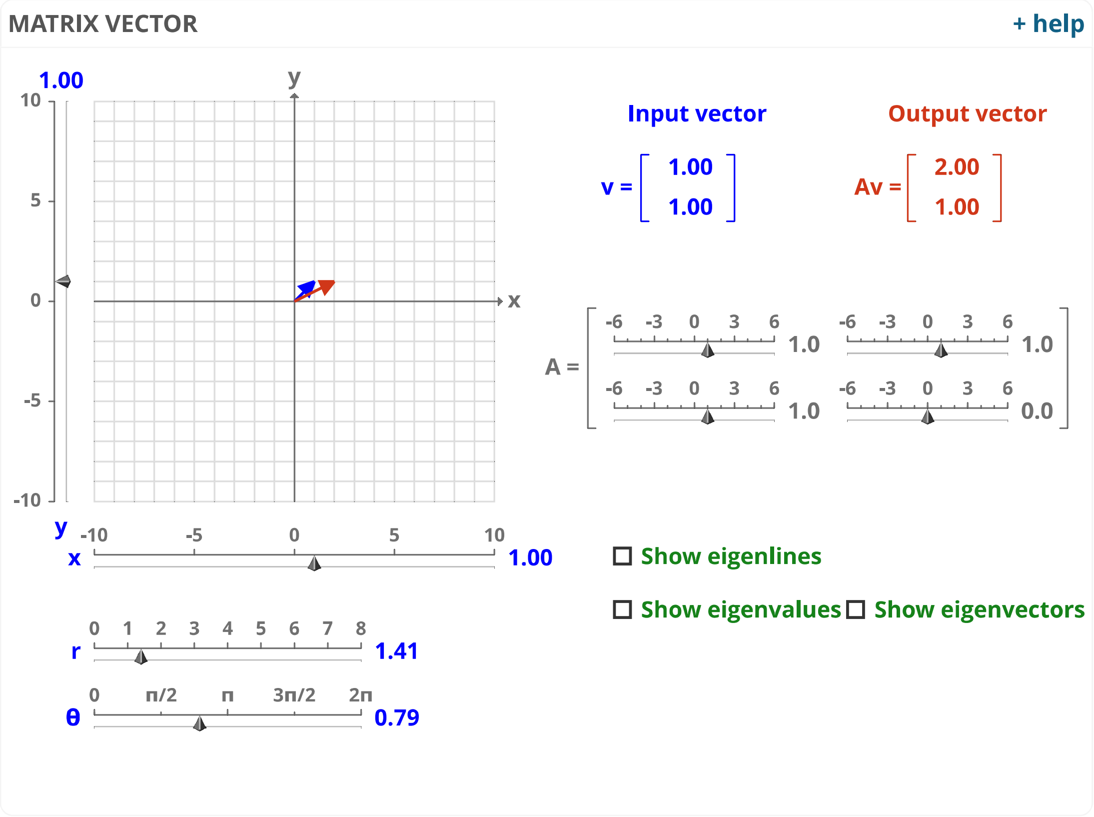
$$\mathbf{f}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

and the smiley with unit radius is stretched horizontally into a fat smiley of the same height.



Mathlet exploration

2/2 points (graded)



MATRIX VECTOR

Input vector

v = [1.00, 1.00]

Output vector

Av = [2.00, 1.00]

A = [1.0 1.0; 1.0 0.0]

x 1.00

y 1.00

r 1.41

θ 0.79

☐ Show eigenlines

☐ Show eigenvalues ☐ Show eigenvectors

mathlets.org

Use the mathlet above. Specify any arbitrary matrix $\mathbf{A}$ by changing the entries with the sliders. Then answer the following questions.

1. Where does the matrix $\mathbf{A}$ send the vector $\begin{pmatrix}1\\0\end{pmatrix}$?

   (*Hint: To create the input vector* $\begin{pmatrix}1\\0\end{pmatrix}$, *use the* $\boldsymbol{x}$ *and* $\boldsymbol{y}$ *sliders next to the vector graph output. Then try changing the entries of* $\mathbf{A}$ *and see what happens to the output vector.*)

   ● To the vector that is in the first column of the matrix $\mathbf{A}$. ✓

   ○ To the vector that is in the first row of the matrix $\mathbf{A}$.

   ○ To the vector that is in the second column of the matrix $\mathbf{A}$.

   ○ To the vector that is in the second row of the matrix $\mathbf{A}$.

2. Where does the matrix $\mathbf{A}$ sends the vector $\begin{pmatrix}0\\1\end{pmatrix}$?

   (*Hint: To create the input vector* $\begin{pmatrix}0\\1\end{pmatrix}$, *use the* $\boldsymbol{x}$ *and* $\boldsymbol{y}$ *sliders next to the vector graph output. Then try changing the entries of* $\mathbf{A}$ *and see what happens to the output vector.*)

   ○ To the vector that is in the first column of the matrix $\mathbf{A}$.

   ○ To the vector that is in the first row of the matrix $\mathbf{A}$.

● To the vector that is in the second column of the matrix $\mathbf{A}$. ✓

○ To the vector that is in the second row of the matrix $\mathbf{A}$.

Solution:

The matrix $\mathbf{A} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$ sends the vector $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ to the vector

$$\mathbf{A} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} a_{11} \\ a_{21} \end{pmatrix}$$

which is the first column of the matrix $\mathbf{A}$.

The matrix $\mathbf{A}$ sends the vector $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ to the vector

$$\mathbf{A} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} a_{12} \\ a_{22} \end{pmatrix}$$

which is the second column of the matrix $\mathbf{A}$.

Submit You have used 1 of 3 attempts

Answers are displayed within the problem

6. The geometry of matrices as functions

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The unit smiley
My first time encountering this. 4

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