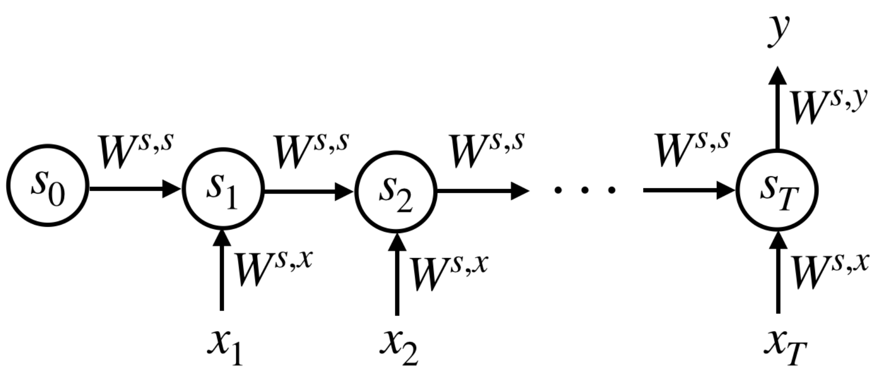
Problem 6



Recurrent neural networks (RNN) can be used as classification models for time series data. Here we have a simple RNN as shown in the figure above, where

$$s_t = f_1\left(W^{s,s} s_{t-1} + W^{s,x} x_t\right), \quad t = 1, 2, \ldots, T$$

and

$$y = f_2\left(W^{s,y} s_T + W_0\right)$$

We assume all offsets are 0 except W_0 for the final output layer and we decide the two activation functions to be:

$$f_1(z) = \mathrm{RELU}(z) = max(0, z)$$

and

$$f_2(z) = \mathrm{sign}(z) = \begin{cases} 1, & if \quad z \geq 0 \\ 0, & if \quad z < 0 \end{cases}$$

Note that the $\mathrm{RELU}(z)$ can be applied elementwise if z is a vector.

Suppose we want to apply this model to classify sentences into different categories (e.g. positive/negative sentiment), we need to encode each word in a sentence into a vector as the input x_t to the model. One way to do this is to represent the tth word as a column vector of length $|V|$, where V is the set of the entire vocabulary. The ith element of x_t is 1 if the word is the ith word in the vocabulary and all other elements are zero.

6. (1)

2.0/2 points (graded)

We first explore a simple scenario where our vocabulary contains only 2 words, $V = \{A, B\}$. Let $s_t \in \mathbb{R}^2$ and we set the initial state s_0 and the weights before the last layer as follows:

$$s_0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad W^{s,s} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}, \quad W^{s,x} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Now given 3 training sentences: AA, ABB, BAA

Encode each of them into a sequence of vectors. As an example, the sentence AA is encoded as $x^{(1)} = (x_1^{(1)}, x_2^{(1)})$, where $x_1^{(1)} = x_2^{(1)} = [1, 0]^T$.

(To enter the sequence above, type `[[1,0],[1,0]]`.)

Now encode the other 2 sentences into $x^{(2)}$ and $x^{(3)}$.

$x^{(2)} =$ [[1,0],[0,1],[0,1]] ✔ **Answer:** [[1,0],[0,1],[0,1]]

$x^{(3)} =$ [[0,1],[1,0],[1,0]] ✔ **Answer:** [[0,1],[1,0],[1,0]]

Solution:

A is encoded as $[1, 0]^T$ and B is encoded as $[0, 1]^T$, so we have

$$x^{(2)} = (x_1^{(2)}, x_2^{(2)}, x_3^{(2)}) = ([1, 0]^T, [0, 1]^T, [0, 1]^T)$$

$$x^{(3)} = (x_1^{(3)}, x_2^{(3)}, x_3^{(3)}) = ([0,1]^T, [1,0]^T, [1,0]^T)$$

Submit You have used 1 of 5 attempts

Answers are displayed within the problem

6. (2)

3.0/3 points (graded)

Now compute the final hidden state $s_T^{(1)}$,$s_T^{(2)}$,$s_T^{(3)}$ for each of the three senteces AA, ABB, BAA in this RNN.

(Enter `[0,0]` for $S_T = [0,0]^T$.)

$s_T^{(1)} =$ [0,0] **Answer:** [0,0]

$s_T^{(2)} =$ [0,2] **Answer:** [0,2]

$s_T^{(3)} =$ [0,1] **Answer:** [0,1]

Solution:

Using $s_t = RELU\left(W^{s,s} s_{t-1} + W^{s,x} x_t\right)$, we can compute

$$s_T^{(1)} = [0,0]^T$$
$$s_T^{(2)} = [0,2]^T$$
$$s_T^{(3)} = [0,1]^T$$

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Answers are displayed within the problem

6. (3)

0/1 point (graded)

Fixing s_0, $W^{s,s}$, $W^{s,x}$ and by only learning the linear classifier in the final layer, can this RNN separate the 3 examples regardless of how they were labeled?

- Yes ✗
- No ✓

Solution:

No. As the final layer is a linear classifier and $s_T^{(1)}$, $s_T^{(2)}$, $s_T^{(2)}$ are collinear points, they are not linear seperable in general. A concrete example is when $y^{(1)} = y^{(2)} = 1$ and $y^{(3)} = -1$.

Submit You have used 2 of 3 attempts

Answers are displayed within the problem

6. (4)

1/1 point (graded)

A simpler model to classify sentences is to represent the entire sentence into a vector z and apply a linear model on z, i.e.

$$y = sign\left(W^{z,y} z\right)$$

The vector z has length $|V|$ and the ith element of z is the count of how many times the ith word appears in the sentence. For example, the sentence ABA with $V = \{A, B\}$ will be encoded as $z = [2,1]^T$. If we want the RNN we described earlier to match the output of this linear model given any input sentences, Which of the following is a possible setting of the weights and initial state s_0 of the RNN? Check all that apply.

Here $c^{|V|}$ stands for a vector of length $|V|$ in which every element is c (e.g. $[1,1,1]^T$ if $c = 1$ and $|V| = 3$), $I_{|V|}$ stands for the identity matrix of size $|V|$.

- [ ] $s_0 = 1^{|V|}$, $W^{s,s} = I_{|V|}$, $W^{s,x} = I_{|V|}$,$W^{s,y} = -W^{z,y}$, $W_0 = \sum_i W_i^{z,y}$
- [x] $s_0 = 1^{|V|}$, $W^{s,s} = I_{|V|}$, $W^{s,x} = I_{|V|}$,$W^{s,y} = W^{z,y}$, $W_0 = -\sum_i W_i^{z,y}$ ✔

- [ ] $s_0 = 0^{|V|}, W^{s,s} = I_{|V|}, W^{s,x} = -I_{|V|}, W^{s,y} = -W^{z,y}, W_0 = 0$
- [x] $s_0 = 0^{|V|}, W^{s,s} = I_{|V|}, W^{s,x} = I_{|V|}, W^{s,y} = W^{z,y}, W_0 = 0$ ✔

✔

Solution:

By setting $W^{s,s} = I_{|V|}$ and $W^{s,x} = I_{|V|}$, we have $s_t = RELU\left(s_{t-1} + x_t\right)$.
If we initialize $s_0 = 0^{|V|}$, then s_T will be the same as z,
thus $W^{s,y} = W^{z,y}$ and $W_0 = 0$.
If we initialize $s_0 = 1^{|V|}$,
then $s_T = z + 1^{|V|}$.
To make $W^{z,y}z = W^{s,y}\left(z + 1^{|V|}\right) + W_0$,
we have $W^{s,y} = W^{z,y}$ and $W_0 = -W^{s,y} \cdot 1^{|V|} = -\sum_i W_i^{z,y}$

Submit You have used 1 of 3 attempts

Answers are displayed within the problem

6. (5)

1/1 point (graded)

Now suppose we want to use indicators instead of counts for the vector z. That is the ith element of z will be 1 if the ith word appears anywhere in the sentence. Which of the following is a possible setting of the weights and initial state s_0 of the RNN? Check all that apply.

Note (Sept 8): In the choices below, W_0 is a scalar, and the summation $\sum_i W_i^{z,y}$ over i is summing over the elements of the vector $W^{z,y}$.

- [ ] $s_0 = 1^{|V|}, W^{s,s} = I_{|V|}, W^{s,x} = -I_{|V|}, W^{s,y} = W^{z,y}, W_0 = -\sum_i W_i^{z,y}$
- [x] $s_0 = 1^{|V|}, W^{s,s} = I_{|V|}, W^{s,x} = -I_{|V|}, W^{s,y} = -W^{z,y}, W_0 = \sum_i W_i^{z,y}$ ✔
- [ ] $s_0 = 0^{|V|}, W^{s,s} = I_{|V|}, W^{s,x} = -I_{|V|}, W^{s,y} = -W^{z,y}, W_0 = 0$
- [ ] $s_0 = 0^{|V|}, W^{s,s} = I_{|V|}, W^{s,x} = I_{|V|}, W^{s,y} = W^{z,y}, W_0 = 0$

✔

Solution:

As the RELU activation function will map all non-positive input to 0, so whenever a word appears, we can minus the corresponding state by 1. If we set the initial state to be 1, representing a word does not appear, then as long as a word appears, no matter how many times, the state will be 0.
With this idea, we choose $s_0 = 1^{|V|}$, $W^{s,s} = I_{|V|}$ and $W^{s,x} = -I_{|V|}$. By doing so, we have $s_T = 1^{|V|} - z$.
To make $W^{z,y}z = W^{s,y}(1^{|V|} - z) + W_0$,
We have $W^{s,y} = -W^{z,y}$ and $W_0 = \sum_i W_i^{z,y}$.

Submit You have used 1 of 3 attempts

Answers are displayed within the problem

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? [STAFF] Is the extension for Everybody?
Hello staff, May I please ask if the 12-hour exam extension is for everybody? If yes would the timer be adjusted to reflect that? Seeing that I have ... 8

? I have been informed that there was a extension given but the timer on my page does not reflect that
I opened the exam page this morning (i.e. approximately 10 hours ago). Right now the timer on the upper right corner of the webpage shows tha... 6

Could you please help me to reset the attempt for Q6(4)?
I accidentally double clicked submitted Q 6(4) which was a wrong answer. Could you please help me to reset the attempt? Thank you very much!! 4

[staff] typo in intro description for Problem 6
1 of the word -> 1 if the word 1

? 6.1 clarification.
Can you please clarify this statement. I am having a hard time understanding it as the grammar in the highlighted part is off: The i th element of ... 4 new_

Sign function
--- edited --- 3

	NOT in course lectures Most of the content in the exam was not presented in the course.	4
?	[Staff] 6(4): need confirmation as for the statement in 6(4), is y=sign(W z,y z) correct? i'm wondering if W 0 is missing.	2
?	6. (4) 6. (5) Need explanation in notation.	4
?	Fixing = to leave as was defined (e.g. Firmly fixed) or to correct/amend ? Fixing = to leave as was defined (e.g. Firmly fixed) or to correct/amend ?	1
?	[Edited to remove content] [Edited to remove content. We have emailed you]	3
	Number of Attempts for 6.(3) The question has only one attempt. Should there be three? Thanks.	2
	Number of attempts in 6.3 Right after I clicked submit on 6.3 I noticed I had clicked in the wrong answer (not the one I wanted). I know this is my mistake, but every questio...	3