DelftX: OT.1x Observation theory: Estimating the Unknown

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Exercises: Estimate vs. Estimator

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Understanding random/stochastic variables

1/1 point (ungraded)

In module 4.1, new terminology was introduced.

Of the names of the variables below, please indicate which ones are used to indicate a random/stochastic variable.

- [x] estimator
- [ ] estimate
- [ ] observation
- [x] observable

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✔ Correct (1/1 point)

Stochastic vs. deterministic variables

1/1 point (ungraded)

Theory:

It is proven in statistics that a linear function of a normally distributed variable/vector is again normally distributed. That is, if $\underline{y}$ is normally distrubted and $\underline{z}$ is a linear function of $\underline{y}$ (such as $\underline{z} = L\underline{y}$), then $\underline{z}$ is also normally distributed.

Based on this theory answer the following question.

Assume we observe the width of a canal five times and we want to estimate the width of the canal. The observables are $\underline{y}_1$, $\underline{y}_2$, $\underline{y}_3$, $\underline{y}_4$, and $\underline{y}_5$, and all are normally distributed. For this estimation problem, the observable vector $\underline{y}$, and the design matrix A can be written as

$$\underline{y} = \begin{bmatrix} \underline{y}_1 \\ \underline{y}_2 \\ \underline{y}_3 \\ \underline{y}_4 \\ \underline{y}_5 \end{bmatrix}, \quad A = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}.$$

Applying least squares estimation on this model provides us the estimator for the canal width as:

$$\underline{\hat{x}} = (A^T A)^{-1} A^T \underline{y}.$$

Which of the following statements is correct?

- (selected) $\hat{\underline{x}}$ is a stochastic variable, and it is normally distributed. ✔
- $\hat{\underline{x}}$ is a stochastic variable, but it is not necessarily normally distributed.
- $\hat{\underline{x}}$ is deterministic, and so it does not have any distribution

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Stochastic vs. deterministic variables (continued)

1/1 point (ungraded)

Now for the above question assume the observations are given as: $y_1 = 10.1$m, $y_2 = 10.15$m, $y_3 = 9.9$m, $y_4 = 10.2$m, and $y_5 = 10.1$m. Application of LS estimation gives the estimate $\hat{x}$ as

$$\hat{x} = (A^T A)^{-1} A^T \begin{bmatrix} 10.1 \\ 10.15 \\ 9.9 \\ 10.2 \\ 10.1 \end{bmatrix} = 10.09$$

Which of the following statements is correct?

- ( ) $\hat{x} = 10.09$ is stochastic, and it is normally distributed.
- (•) $\hat{x} = 10.09$ is deterministic, but it is a realization of the estimator $\underline{\hat{x}}$. ✔

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