



## MITx: 6.041x Introduction to Probability - The Science of Uncertainty



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## Problem 5: Hypothesis test between two normals

(4/4 points)

Conditioned on the result of an unbiased coin flip, the random variables  $T_1, T_2, \dots, T_n$  are independent and identically distributed, each drawn from a common normal distribution with mean zero. If the result of the coin flip is Heads this normal distribution has variance **1**, otherwise it has variance **4**. Based on the observed values  $t_1, t_2, \dots, t_n$ , we use the MAP rule to decide whether the normal distribution from which they were drawn has variance **1** or variance **4**. The MAP rule decides that the underlying normal distribution has variance **1** if and only if

$$\left| c_1 \sum_{i=1}^n t_i^2 + c_2 \sum_{i=1}^n t_i \right| < 1.$$

Find the values of  $c_1 \geq 0$  and  $c_2 \geq 0$  such that this is true. Express your answer in terms of  $n$ , and use 'ln' to denote the natural logarithm function, as in 'ln(3)'.

 $c_1 =$  


Answer: 3/(8\*n\*ln(2))

 $c_2 =$  



Answer: 0

on random variables


## ▼ Unit 7: Bayesian inference

### Unit overview


#### Lec. 14: Introduction to Bayesian inference

Exercises 14 due Apr 06, 2016 at 23:59 UTC 


#### Lec. 15: Linear models with normal noise

Exercises 15 due Apr 06, 2016 at 23:59 UTC 


#### Problem Set 7a

Problem Set 7a due Apr 06, 2016 at 23:59 UTC 


#### Lec. 16: Least mean squares (LMS) estimation

Exercises 16 due Apr 13, 2016 at 23:59 UTC 

#### Lec. 17: Linear least mean squares (LLMS) estimation

Exercises 17 due Apr 13, 2016 at 23:59 UTC 

#### Problem Set 7b

Problem Set 7b due Apr 13, 2016 at 23:59 UTC 

### Solved problems

Answer:

Let  $\Theta = 0$  denote that the observations  $t_1, t_2, \dots, t_n$  were generated from a normal distribution with variance 1, and let  $\Theta = 1$  denote that they were generated from a normal distribution with variance 4. For simplicity, let us use the notation  $N(t_1, \dots, t_n; 0, \sigma^2)$  to denote the joint PDF of  $n$  i.i.d. normal random variables with mean 0 and variance  $\sigma^2$ , evaluated at  $t_1, \dots, t_n$ .

Therefore, given the observations  $t_1, \dots, t_n$ , the posterior probability that the samples are generated from a normal distribution with variance 1 is

$$\mathbf{P}(\Theta = 0 \mid T_1 = t_1, \dots, T_n = t_n) = \frac{(1/2) \cdot N(t_1, \dots, t_n; 0, 1)}{(1/2) \cdot N(t_1, \dots, t_n; 0, 1) + (1/2) \cdot N(t_1, \dots, t_n; 0, 4)}.$$

Similarly, the probability that the samples are generated from a normal distribution with variance 4 is given by

$$\mathbf{P}(\Theta = 1 \mid T_1 = t_1, \dots, T_n = t_n) = \frac{(1/2) \cdot N(t_1, \dots, t_n; 0, 4)}{(1/2) \cdot N(t_1, \dots, t_n; 0, 1) + (1/2) \cdot N(t_1, \dots, t_n; 0, 4)}.$$

The MAP rule favors  $\Theta = 0$  if the following inequality holds:

$$\mathbf{P}(\Theta = 0 \mid T_1 = t_1, \dots, T_n = t_n) > \mathbf{P}(\Theta = 1 \mid T_1 = t_1, \dots, T_n = t_n)$$

## Additional theoretical material

### Unit summary

- ▶ Exam 2
- ▶ Unit 8: Limit theorems and classical statistics
- ▶ Unit 9: Bernoulli and Poisson processes
- ▶ Unit 10: Markov chains
- ▶ Exit Survey
- ▶ Final Exam

We notice that the denominators in the expressions for  $\mathbf{P}(\Theta = 0 \mid \dots)$  and  $\mathbf{P}(\Theta = 1 \mid \dots)$  are the same, so it suffices to compare the numerators. Therefore, the MAP rule favors  $\Theta = 0$  if the following inequality holds:

$$N(t_1, \dots, t_n; 0, 1) > N(t_1, \dots, t_n; 0, 4)$$

$$\prod_{i=1}^n \frac{1}{\sqrt{2\pi \cdot 1}} e^{-\frac{t_i^2}{2 \cdot 1}} > \prod_{i=1}^n \frac{1}{\sqrt{2\pi \cdot 4}} e^{-\frac{t_i^2}{2 \cdot 4}}.$$

With a little bit of algebra, we obtain

$$\left| \frac{3}{8} \sum_{i=1}^n t_i^2 \right| < n \cdot \ln(2).$$

*You have used 1 of 2 submissions*

## DISCUSSION

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