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Exercise: Exponential PDF

(2/2 points)

Let X be an exponential random variable with parameter $\lambda = 2$. Find the values of the following. Use 'e' for the base of the natural logarithm (e.g., enter `e^(-3)` for e^{-3}).

a) $\mathbf{E}[(3X+1)^2] =$ 17/2 ✔ **Answer:** 8.5

b) $\mathbf{P}(1 \le X \le 2) =$ e^(-2)-e^(-4) ✔ **Answer:** 0.11702

Answer:

a) By expanding the quadratic, using linearity of expectations, and the facts that $\mathbf{E}[X] = 1/\lambda$ and $\mathbf{E}[X^2] = 2/\lambda^2$, we have

$$\mathbf{E}[(3X+1)^2] = 9\mathbf{E}[X^2] + 6\mathbf{E}[X] + 1 = 9 \cdot \frac{2}{2^2} + 6 \cdot \frac{1}{2} + 1 = \frac{17}{2}.$$

b) We have seen that for $a > 0$, we have $\mathbf{P}(X \ge a) = e^{-\lambda a}$, so that $\mathbf{P}(X \le a) = 1 - e^{-\lambda a}$. Therefore,

$$\mathbf{P}(1 \le X \le 2) = \mathbf{P}(X \le 2) - \mathbf{P}(X \le 1) = (1 - e^{-4}) - (1 - e^{-2}) = e^{-2} - e^{-4}$$

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