



MITx: 6.041x Introduction to Probability - The Science of Uncertainty



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Bookmark

Problem 3: The PDF of the maximum

(3/3 points)

Let \mathbf{X} and \mathbf{Y} be independent random variables, each uniformly distributed on the interval $[0, 1]$.

1. Let $\mathbf{Z} = \max\{\mathbf{X}, \mathbf{Y}\}$. Find the PDF of \mathbf{Z} . Express your answer in terms of z using standard notation .

For $0 < z < 1$, $f_Z(z) =$

2*z

✓ Answer: 2*z

2. Let $\mathbf{Z} = \max\{2\mathbf{X}, \mathbf{Y}\}$. Find the PDF of \mathbf{Z} . Express your answer in terms of z using standard notation .

For $0 < z < 1$, $f_Z(z) =$

z

✓ Answer: z

For $1 < z < 2$, $f_Z(z) =$

1/2

✓ Answer: 0.5


Answer:

Recall that for a random variable \mathbf{U} distributed uniformly on the interval $[0, 1]$, its CDF is given by


▼ Unit 6: Further topics on random variables

Unit overview


Lec. 11: Derived distributions

Exercises 11 due Mar 30, 2016
at 23:59 UTC 

Lec. 12: Sums of independent r.v.'s; Covariance and correlation

Exercises 12 due Mar 30, 2016
at 23:59 UTC 


Lec. 13: Conditional expectation and variance revisited; Sum of a random number of independent r.v.'s

Exercises 13 due Mar 30, 2016
at 23:59 UTC 

Solved problems

Additional theoretical material

Problem Set 6

Problem Set 6 due Mar 30, 2016
at 23:59 UTC 

Unit summary

$$F_U(u) = \begin{cases} 0, & \text{if } u < 0, \\ u, & \text{if } 0 \leq u \leq 1, \\ 1, & \text{if } u > 1. \end{cases}$$

1. Let $Z = \max\{X, Y\}$. For $z \in (0, 1)$,

$$\begin{aligned} F_Z(z) &= \mathbf{P}(Z \leq z) \\ &= \mathbf{P}(X \leq z \text{ and } Y \leq z) \\ &= F_X(z)F_Y(z) \\ &= z^2 \end{aligned}$$

Hence, $f_Z(z) = 2z$ for $z \in (0, 1)$.

2. Let $Z = \max\{2X, Y\}$.

$$F_Z(z) = \mathbf{P}(Z \leq z) = \mathbf{P}(2X \leq z \text{ and } Y \leq z) = F_X(z/2)F_Y(z).$$

Hence, for $0 < z < 1$, $F_Z(z) = (z/2) \cdot z = z^2/2$ and $f_Z(z) = z$.
For $1 < z < 2$, $F_Z(z) = (z/2) \cdot 1 = z/2$, and $f_Z(z) = 1/2$.

You have used 2 of 2 submissions

- ▶ Unit 7: Bayesian inference
- ▶ Exam 2
- ▶ Unit 8: Limit theorems and classical statistics
- ▶ Unit 9: Bernoulli and Poisson processes
- ▶ Unit 10: Markov chains
- ▶ Exit Survey
- ▶ Final Exam

DISCUSSION

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