8. The One-Dimensional Delta Method

Applying Linear Functions to a Random Sequence

3/3 points (graded)

Let $(Z_n)_{n\geq 1}$ be a sequence of random variables such that

$$\sqrt{n}\,(Z_n - \theta) \xrightarrow[n\to\infty]{(d)} Z$$

for some $\theta \in \mathbb{R}$ and some random variable Z.

Let $g(x) = 5x$ and define another sequence by $Y_n = g(Z_n)$.

The sequence $\sqrt{n}\,(Y_n - g(\theta))$ converges. In terms of Z, what random variable does it converge to?

$$\sqrt{n}\left(Y_n - g(\theta)\right) \xrightarrow[n\to\infty]{(d)} Y.$$

(Answer in terms of Z)

$Y =$ 5*Z ✔ **Answer:** 5*Z

$5 \cdot Z$

What theorem did we invoke to compute Y?
(There can be more than 1 acceptable answers.)

- Laws of large number
- Central Limit theorem
- Slutsky theorem ✔
- Continuous mapping theorem

✔

If $\textsf{Var}\left(Z\right) = \sigma^2$, what is $\textsf{Var}\left(Y\right)$? This is the asymptotic variance of $\left(Y_n\right)_{n\geq 1}$.
(Answer in terms of σ^2.)

$\textsf{Var}\left(Y\right) =$ 25*sigma^2 ✔ **Answer:** 25*sigma^2

$25 \cdot \sigma^2$

STANDARD NOTATION

Solution:

1.
$$\sqrt{n}\left(Y_n - g(\theta)\right) = \sqrt{n}\left(g(Z_n) - g(\theta)\right) = \sqrt{n}\left(5Z_n - 5\theta\right)$$
$$= 5\left(\sqrt{n}\left(Z_n - \theta\right)\right) \xrightarrow[n\to\infty]{(d)} 5Z$$

by the continuous mapping theorem because $5\left(\sqrt{n}\left(Z_n - \theta\right)\right)$ is a linear and hence continuous function of Z_n in the last step. Alternatively, since we were given that $\sqrt{n}\left(Z_n - \theta\right) \xrightarrow[n\to\infty]{(d)} Z$, and 5 converges trivially in probability to itself, we can also use Slutsky theorem to conclude.

2. Since $Y = 5Z$, Var $(Y) = 25$Var $(Z) = 25\sigma^2$.

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You have used 1 of 2 attempts

Answers are displayed within the problem

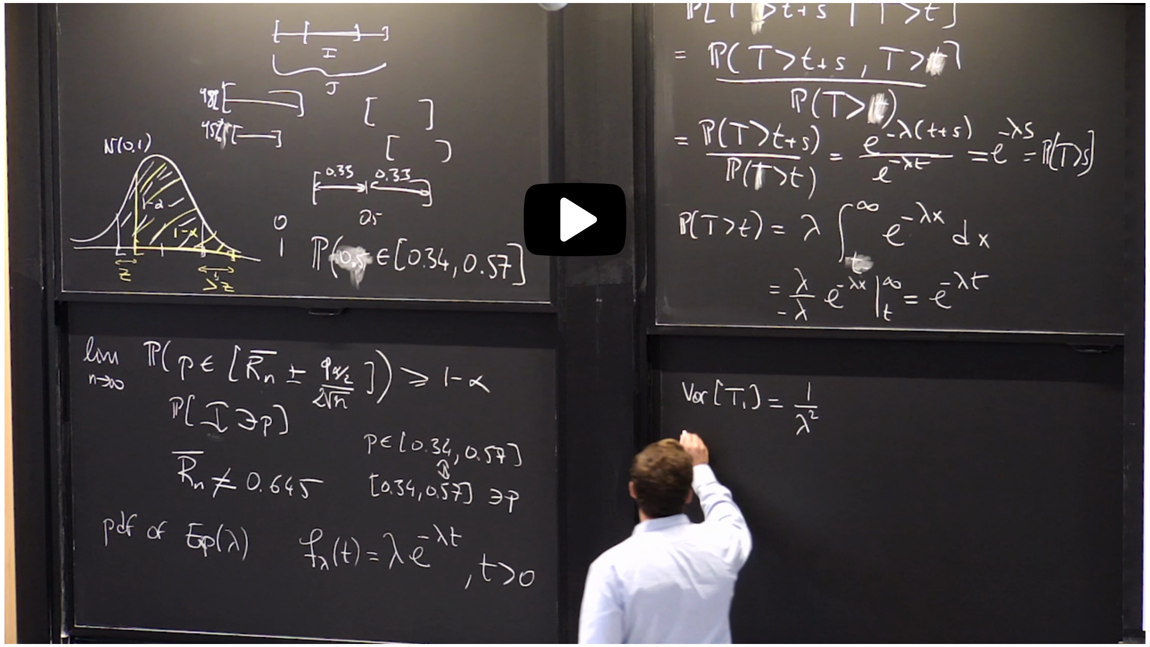
Video note: In the video below, there is an important misprint at roughly 1:26, which will be corrected in the video on the next page. The Central limit theorem applied to $\overline{T}_n$ should read

$$\sqrt{n}\left(\overline{T}_n - \frac{1}{\lambda}\right) \xrightarrow[n\to\infty]{(d)} \mathcal{N}\left(0, \frac{1}{\lambda^2}\right).$$

the Delta Method

The problem is that this is not something of the form estimator

of lambda minus lambda.



Right?

What I would want to see is something

that looks like square root of n one

over T n bar, which is actually my lambda hat,

minus lambda converges to some Gaussian

as n goes to infinity in distribution.

Maybe zero and some sigma squared here.

Right?

That's what I want to see.

Because once I know how to do this,

then I can start unpacking my confidence intervals

0:00 / 0:00 1.50x

Video

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(Optional) Proof of the Delta Method

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? Delta method
Hello, Does anyone know as to why this is called the delta method? Thanks
2

Error in video at 1:31
2

There is a bug in question 2
The question 2 is a single-select question, but may accepts many answers as valid.
2

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