9. Procedure summary

Steps for solving

$$\frac{\partial \theta}{\partial t} = \nu \frac{\partial^2 \theta}{\partial x^2}, \qquad 0 < x < L,\ t > 0$$

with given boundary conditions and initial condition $\theta(x,0) = f(x)$.

1. Ignore initial condition and focus on boundary conditions.

2. If any boundary condition is nonzero, find a steady state solution $\Theta(x)$ which satisfies the given boundary conditions. Reduce problem to solving $\theta_h(x,t) = \theta(x,t) - \Theta(x)$ which has homogeneous boundary conditions and initial condition $\theta_h(x,0) = f(x) - \Theta(x)$.

3. Look up standard form of eigenvalues, eigenfunctions, and normal modes for the homogeneous cases already computed.

   Else, use separation of variables. That is, try $\theta_h(x,t) = v(x)\, w(t)$ (or $\theta(x,t) = v(x)\, w(t)$ if original problem is homogeneous) to find family of normal modes $\theta_n(x,t) = v_n(x)\, w_n(t)$.

4. Take linear combinations to get the general solution

$$\boxed{\theta_h(x,t) = b_1 w_1(t)\, v_1(x) + b_2 w_2(t)\, v_2(x) + b_3 w_3(t)\, v_3(x) + \cdots}$$

5. Extend the initial condition $f(x) - \Theta(x)$ to have the correct base period and even/odd properties in order to be able to solve for the Fourier coefficients

$$f(x) - \Theta(x) = b_1 v_1(x) + b_2 v_2(x) + b_3 v_3(x) + \cdots.$$

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