

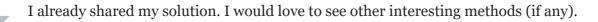
## Why does this approximation for 8 work?

Asked today Active today Viewed 25 times



I previously saw that peculiarly  $\frac{987654321}{123456789} \approx 8$ . I was wondering if there was any significance to it i.e. if there is any way to derive this approximation (aside from long division).



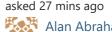




real-analysis approximation



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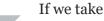
I see now it was already raised <a href="here">here</a>, <a href="here">here</a> and <

3 Answers





We can express 987654321 as approximately  $\sum_{k=-9}^\infty -k\cdot 10^{-k-1}$  and 123456789 as approximately  $\sum_{k=1}^\infty k\cdot 10^{9-k}$ 





$$f(x) = \sum_{k=-9}^{\infty} x^k = \frac{x^{-9}}{1-x}$$

then

$$f'(x) = \sum_{k=-9}^{\infty} kx^{k-1} = \frac{-9x^{-10} + 10x^{-9}}{(1-x)^2}$$

$$\implies f'(x) = \sum_{k=-9}^{\infty} kx^{k-1} = \frac{-9x^{-10} + 10x^{-9}}{(1-x)^2}$$

$$\implies x^2 f'(x) = \sum_{k=-9}^{\infty} kx^{k+1} = \frac{-9x^{-8} + 10x^{-7}}{(1-x)^2}$$

$$\implies -10^2 \cdot f'\left(\frac{1}{10}\right) = \sum_{k=-9}^{\infty} -k \cdot 10^{-k-1} = \frac{8 \cdot 10^{10}}{81}$$

Similarly, if we take

$$g(x) = \sum_{k=0}^\infty x^k = rac{1}{1-x}$$

then

$$g'(x) = \sum_{k=1}^{\infty} k \cdot x^{k-1} = \frac{1}{(1-x)^2}$$

$$\implies x^{-8}g'(x) = \sum_{k=1}^{\infty} k \cdot x^{k-9} = \frac{x^{-9}}{(1-x)^2}$$

$$\implies 10^8 \cdot g'\left(\frac{1}{10}\right) = \sum_{k=1}^{\infty} k \cdot 10^{9-k} = \frac{10^{10}}{81}$$

Hence, from our approximations for 987654321 and 123456789, it follows that  $\frac{987654321}{123456789} \approx 8$ 

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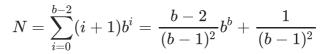
answered 27 mins ago





More generally, in base b, the number with digits decreasing from b-1 to 1 is







while the number with digits increasing from 1 to b-1 is

$$D = \sum_{i=0}^{b-2} (b-1-i)b^i = rac{b^b}{(b-1)^2} - rac{b^2-b+1}{(b-1)^2}$$

For large b, the dominant terms are those with  $b^b$ , so

$$rac{N}{D} \sim b-2$$

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answered 11 mins ago



Robert Israel

4 291

588

Very nice generalization! – user 10 mins ago



We have that in base 8

 $\bullet$  987654321 = 7267464261<sub>8</sub>



•  $123456789 = 726746425_8$ 



then

 $987654321 - 8 \cdot 123456789 = 7267464261_8 - 10_8 \cdot 726746425_8 = 11_8 = 9$ 

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answered 17 mins ago



user

381

12 67 126