Suppose $X_n \backsim Poisson(n)$, Show that $\sqrt{X_n} - \sqrt{n} \Rightarrow N(0, 1/4)$

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Suppose $X_n \backsim Poisson(n)$, Show that $\sqrt{X_n} - \sqrt{n} \Rightarrow N(0, 1/4)$.

I already know that $(X_n - n)/\sqrt{n} \Rightarrow N(0, 1)$. How to do next to go through the proof?

(probability) (probability-theory) (probability-distributions)

asked 37 mins ago

J.Mike
31 6

2 Answers

Set $Y_n = \frac{X_n}{n}$, then you already know $\sqrt{n}(Y_n - 1) \to \mathcal{N}(0, 1)$.

Setting $g(x) = \sqrt{n}$, the delta method yields

$$\sqrt{n}(g(Y_n) - g(1)) \to \mathcal{N}(0, g'(1)^2).$$

Now note that $g'(x) = \frac{1}{2\sqrt{x}}$, i.e. $g'(1)^2 = \frac{1}{4}$ and

$$\sqrt{n}(g(Y_n) - g(1)) = \sqrt{n}\left(\sqrt{\frac{X_n}{n}} - 1\right) = \sqrt{X_n} - \sqrt{n}.$$

answered 28 mins ago

Dominik

15.9k 15 40

Hint: Apply the Delta method with $g(x)=\sqrt{x}$, on

$$(X_n-n)/\sqrt{n}=\sqrt{n}\left(\frac{X_n}{n}-1\right)\xrightarrow{d}\mathcal{N}(0,1)$$

answered 30 mins ago

Jimmy R.
28.8k 4 17 49