1. Lecture 4

The following can be done after Lecture 4.

Please enter solutions in terms of π rather than numerical approximations to guarantee a correct grading. Simply type **pi** into the answer box and treat as any other variable, using * to denote multiplication, / to denote division, and $\wedge$ to denote exponents.

4-1

15.0/15.0 points (graded)

For a fixed real number c, how many solutions to $\ddot{y} = cy$ satisfy the conditions $y(0) = 0$ and $y(1) = 1$?

- ( ) Zero.
- ( ) Exactly one.
- ( ) Infinitely many.
- (•) Either zero or one, depending on the value of c.
- ( ) Either zero or infinitely many, depending on the value of c.
- ( ) Either one or infinitely many, depending on the value of c.

None of the above.

Solution:

Either zero or one, depending on the value of c.

Case 1: $c > 0$. The general solution to $\ddot{y} = cy$ is $y = ae^{\sqrt{c}t} + be^{-\sqrt{c}t}$ The conditions amount to the linear system

$$\begin{aligned} a + b &= 0 \\ ae^{\sqrt{c}} + be^{-\sqrt{c}} &= 1. \end{aligned}$$

Since $\det \begin{pmatrix} 1 & 1 \\ e^{\sqrt{c}} & e^{-\sqrt{c}} \end{pmatrix} \neq 0$, there is a unique solution.

Case 2: $c = 0$. Then the general solution is $a + bt$, and the boundary conditions say

$$\begin{aligned} a &= 0 \\ a + b &= 1, \end{aligned}$$

which has a unique solution $(a, b) = (0, 1)$; so the unique $y(t)$ is t.

Case 3: $c < 0$. If we write $c = -\omega^2$ for some $\omega > 0$, then the roots of the characteristic polynomial are $\pm i\omega$, and the general solution is $a\cos\omega t + b\sin\omega t$. The first boundary condition says $a = 0$, so $v = b\sin\omega t$. The second boundary condition then says $b\sin\omega = 1$. If $\sin\omega = 0$ (i.e., if $\omega = n\pi$ for some positive integer n), there is no solution; if $\sin\omega \neq 0$, there is a unique solution.

Conclusion: If $c = -(n\pi)^2$ for some positive integer n, there is no solution; otherwise there is exactly one solution.

Submit You have used 2 of 3 attempts

Answers are displayed within the problem

4-2

15.0/15.0 points (graded)

For a fixed real number c, how many solutions to $\ddot{y} = cy$ satisfy the conditions $y(0) = 0$ and $y'(1) = 0$?

- Zero.
- Exactly one.
- Infinitely many.
- Either zero or one, depending on the value of c.
- Either zero or infinitely many, depending on the value of c.
- Either one or infinitely many, depending on the value of c.
- None of the above.

Solution:

Either one or infinitely many, depending on the value of c.

Case 1: $c > 0$. The general solution to $\ddot{y} = cy$ is $y = ae^{\sqrt{c}t} + be^{-\sqrt{c}t}$ The conditions amount to the linear system

$$\begin{aligned} a + b &= 0 \\ a\sqrt{c}e^{\sqrt{c}} - b\sqrt{c}e^{-\sqrt{c}} &= 0. \end{aligned}$$

The first equation says $b = -a$, so the second equation becomes

$$\begin{aligned} a\sqrt{c}e^{\sqrt{c}} + a\sqrt{c}e^{-\sqrt{c}} &= 0 \\ e^{\sqrt{c}} + e^{-\sqrt{c}} &= 0 \quad \text{or} \quad a = 0 \end{aligned}$$

Note that this only holds in the case that $a = 0$. Therefore there is one solution.

Case 2: $c = 0$. Then the general solution is $a + bt$, and the boundary conditions say

$$\begin{aligned} a &= 0 \\ b &= 0, \end{aligned}$$

which has a unique solution $(a, b) = (0, 0)$; so the unique $y(t)$ is 0.

Case 3: $c < 0$. If we write $c = -\omega^2$ for some $\omega > 0$, then the roots of the characteristic polynomial are $\pm i\omega$, and the general solution is $a \cos \omega t + b \sin \omega t$. The first boundary condition says $a = 0$, so $v = b \sin \omega t$. The second boundary condition then says $b\omega \cos \omega = 0$. If $\cos \omega \neq 0$ (i.e., if $\omega \neq (2n+1)\,\pi/2$ for some positive integer n), there is one solution, the zero solution when $b = 0$; if $\cos \omega = 0$, (i.e., if $\omega \neq (2n+1)\,\pi/2$ for some positive integer n), is are infinitely many solutions $v = b \sin \omega t$ for b any constant.

Conclusion: If $c = -((2n+1)\,\pi/2)^2$ for some positive integer n, there are infinitely many solutions; otherwise there is one solution (the zero solution).

Submit You have used 1 of 3 attempts

Answers are displayed within the problem

4-3

5/5 points (graded)

Consider a horizontal beam of length L with elasticity E and moment of inertia I that carries a load transverse to the beam axis (x). It is pinned (on a hinge) at the left ($x = 0$) end with an applied torque (applied bending moment) Q in the positive direction. It is pinned (on a hinge) at the right end ($x = L$).

Let $u(x)$ denote the vertical deflection of the beam.

Which of the boundary conditions are zero? (Choose all that apply. Both columns are graded separately.)

☑ $v(0)$	☑ $v(L)$
☐ $\frac{dv}{dx}(0)$	☐ $\frac{dv}{dx}(L)$
☐ $\frac{d^2v}{dx^2}(0)$	☑ $\frac{d^2v}{dx^2}(L)$
☐ $\frac{d^3v}{dx^3}(0)$	☐ $\frac{d^3v}{dx^3}(L)$
☐ $\frac{d^4v}{dx^4}(0)$	☐ $\frac{d^4v}{dx^4}(L)$

Solution:

The left end point at $x = 0$ is pinned on a hinge, so $v(0) = 0$. There is an applied torque so we know that $\frac{d^2v}{dx^2}(0)$ is not zero, but instead is determined by the applied torque.

The right end point at $x = L$ is pinned on a hinge with no applied moment. Therefore we know that $v(L) = 0$ and $\frac{d^2v}{dx^2}(L) = 0$.

Submit You have used 1 of 1 attempt

Answers are displayed within the problem

1. Lecture 4

Topic: Unit 2: Boundary value problems and PDEs / 1. Lecture 4

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? [Staff] The zero solution
Should we count the zero solution in the number of solutions?
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