Bayes' Law

We have considered two different constraints on rational belief: (1) one's *unconditional* credences can be represented by a probability function, and (2) one must respond to new information in accordance with one's *conditional* credences. Are there any constraints on how a subject's unconditional credences ought to be related to her conditional credences? Yes! It is natural to think that a rational subject should satisfy the following principle:

> **Bayes' Law**
> $p(AB) = p(A) \cdot p(B|A)$

Notice that whenever the subject assigns non-zero credence to A, Bayes' Law entails:

$$p(B|A) = \frac{p(AB)}{p(A)}$$

which allows one to determine the subject's conditional credences on the basis of her unconditional credences. More specifically, one can determine the subject's (conditional) credence in B given A by looking at her (unconditional) credence in A and her (unconditional) credence in A-and-B.

Problem 1

2/2 points (ungraded)

Consider a subject S whose unconditional credences regarding rain (R), and regarding a sudden drop in atmospheric pressure (D), are as follows:

$$\begin{aligned} p(R) = &\quad 0.2 \\ p(D) = &\quad 0.1 \\ p(RD) = &\quad 0.09 \end{aligned}$$

Use Bayes' Law to calculate $p(R|D)$ and $p(D|R)$.

$p(R|D)$=?

0.9

Answer: .9

0.9

$p(D|R)$=?

0.45

Answer: .45

0.45

Explanation

$$p(R|D) = \frac{p(RD)}{p(D)} = \frac{0.09}{0.1} = 0.9$$

$$p(D|R) = \frac{p(DR)}{p(R)} = \frac{p(RD)}{p(R)} = \frac{0.09}{0.2} = 0.45$$

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Problem 2

1/1 point (ungraded)

Now suppose that $p(RD) = 0.02$. How do the conditional probabilities $p(R|D)$ and $p(D|R)$ compare to the unconditional probabilities $p(R)$ and $p(D)$?

- ○ $p(R) > p(R|D)$ and $p(D) \geq p(D|R)$
- ○ $p(R) \leq p(R|D)$ and $p(D) \leq p(D|R)$
- ● $p(R) = p(R|D)$ and $p(D) = p(D|R)$
- ○ $p(R) > p(R|D)$ and $p(D) \leq p(D|R)$

Explanation

$$p\left(R|D\right)=\frac{p\left(RD\right)}{p\left(D\right)}=\frac{0.02}{0.1}=0.2=p\left(R\right)$$

$$p\left(D|R\right)=\frac{p\left(DR\right)}{p\left(R\right)}=\frac{p\left(RD\right)}{p\left(R\right)}=\frac{0.02}{0.2}=0.1=p\left(D\right)$$

So $p\left(R|D\right)=p\left(R\right)$ and $p\left(D|R\right)=p\left(D\right)$. As we'll see, this must always be the case when $p\left(RD\right)=p\left(R\right)\cdot p\left(D\right)$.

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Problem 3

1/1 point (ungraded)

Suppose that S is perfectly rational. (In particular, her credences are described by a probability function, she updates by conditionalizing on her evidence, and she respects Bayes' Law.)

Suppose, moreover, that she is certain of A (i.e. $p\left(A\right)=1$), and that she assigns a non-trivial probability to B (i.e. $p\left(B\right)\neq 0$).

Could learning B cause her to be less than certain about A?

- ◯ Yes.
- ◉ No.

✔

Explanation

No. Learning B cannot cause the subject to lose her certainty in A.

To see this, let us assume otherwise and use this assumption to prove a contradiction. We will assume, in particular, that:

$$p^{old}\left(A\right)=1 \qquad p^{new}\left(A\right)<1$$

where p^{old} is S's credence function before she learns b and p^{new} is S's credence function after she learns that B. Since S updates by conditionalizing, this gives us:

$$p^{old}\left(A\right)=1 \qquad p^{old}\left(A|B\right)<1$$

We now verify the following propositions, where p is a probability function and $\overline{X}$ is the negation of X:

1. $p(A) = 1$ entails $p(\overline{A}) = 0$

2. $p(A) = 1$ entails $p(\overline{A}B) = 0$

3. $p(A) = 1$ entails $p(AB) = p(B)$

Proposition (i) is an immediate consequence of the fact that $p(\overline{A}) = 1 - p(A)$, which we verified in earlier. To verify Proposition (ii), note that Additivity entails that $p(\overline{A}) = p(\overline{A}B) + p(\overline{A}\overline{B})$. But we know from Proposition (i) that $p(\overline{A})$ is zero, so $p(\overline{A}B)$ and $p(\overline{A}\overline{B})$ must both be zero as well (since probabilities are always non-negative real numbers). To verify Proposition (iii) note that Additivity entails that $p(B) = p(AB) + p(\overline{A}B)$. But we know from Proposition (ii) that $p(\overline{A}B)$ is zero, so it must be the case that $p(B) = p(AB)$. Since we are assuming that p^{old} is a probability function, we may conclude that $p^{old}(B) = p^{old}(AB)$. But note that Bayes' Law entails that $p^{old}(AB) = p^{old}(B) \cdot p^{old}(A|B)$. Since $p^{old}(B) = p^{old}(AB)$, this means that $p^{old}(B) = p^{old}(B) \cdot p^{old}(A|B)$. So as long as $p^{old}(B) \neq 0$, it must be the case that $p^{old}(A|B) = 1$, which contradicts the assumption that $p^{old}(A|B) < 1$.

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bullet 2 in problem 2 is not a wrong answer a mathematician would say
See title
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Problem 3: Does A and B need to be a simple probabilities (not conditional ones)?
Let's say p(A) = 1 = P(R|T) is the probability of raining when temperature T is in the 1 to 6 C° range. p(...
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