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3. Practice with Lagrange Multipliers

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Problem Set B due Sep 13, 2021 20:30 IST Completed

Practice

3 (a)

4/4 points (graded)

Set up a system of Lagrange multipliers to maximize $f(x,y) = x^2 - 6x + y^2 - 6y + xy$ subject to $x^2 + y^2 = 50$.

(Make multiplication explicit by typing `*`. For example, type `x*y` for xy.)

2*x-6+y $= \lambda *$ 2*x

Answer: 2*x+y-6 **Answer:** 2*x

2*y-6+x $= \lambda *$ 2*y

Answer: x+2*y-6 **Answer:** 2*y

? INPUT HELP

Solution:

The Lagrange multiplier system reduces to the following

$$2x - 6 + y = \lambda 2x \quad (4.293)$$

$$2y - 6 + x = \lambda 2y \quad (4.294)$$

Submit You have used 1 of 5 attempts

Answers are displayed within the problem

3 (b)

2.0/2 points (graded)

Now find the where the function $f(x,y) = x^2 - 6x + y^2 - 6y + xy$ attains its global maximum and global minimum on the domain $x^2 + y^2 \leq 50$. Note the inequality.

(Enter points in the plane as two numbers comma separated between round parentheses, e.g. `(0,0)`.)

The maximum occurs at the point

$(x,y) =$ (-5,-5) **Answer:** (-5,-5)

The minimum occurs at the point

$(x,y) =$ (2,2) **Answer:** (2,2)

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Solution:

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We need to

1. find the critical points of $f(x, y)$,
2. solve the Lagrange multiplier problem in part (a)
3. determine the global maximum and minimum on the given region

We will start by finding critical points by computing partial derivatives and setting them to 0.

$$\begin{aligned} f_x(x, y) &= 2x - 6 + y = 0 & (4.295) \\ f_y(x, y) &= 2y - 6 + x = 0 & (4.296) \end{aligned}$$

Solving this linear system for x and y we find $x = 2$ and $y = 2$. Thus there is one critical point at $(2, 2)$ which is within the specified region since $2^2 + 2^2 = 8 < 50$.

Next we will solve the Lagrange multiplier problem. We will start by subtracting one equation from the other in our Lagrange system

$$\begin{aligned} 2x - 6 + y &= \lambda 2x & (4.297) \\ -(2y - 6 + x &= \lambda 2y) & (4.298) \\ 2x + y - 2y - x &= 2\lambda(x - y) & (4.299) \\ (x - y) &= 2\lambda(x - y) & (4.300) \end{aligned}$$

This equation is true if $x = y$ or if $\lambda = 1/2$.

First if $x = y$, then we are in the case $x^2 + x^2 = 50$ which implies that $x = \pm 5$. This gives us two more potential maxima and minima: $(5, 5)$ and $(-5, -5)$.

If $\lambda = 1/2$, then we have a system

$$\begin{aligned} 2x - 6 + y &= x & (4.301) \\ 2y - 6 + x &= y & (4.302) \end{aligned}$$

which gives us $y = 6 - x$. Plugging into the constraint equation this gives us

$$\begin{aligned} x^2 + (6 - x)^2 &= 50 & (4.303) \\ x^2 + 36 - 12x + x^2 &= 50 & (4.304) \\ 2x^2 - 12x - 14 &= 0 & (4.305) \\ 2(x - 7)(x + 1) &= 0 & (4.306) \end{aligned}$$

This gives us two more candidates: $(7, -1)$ and $(-1, 7)$.

At this point we plug in our candidates for maximum and minimum into our function $f(x, y)$ to find the absolute maximum and minimum on the region.

We find that the maximum occurs at the boundary point $(-5, -5)$, with value 135, and the minimum occurs at the critical point $(2, 2)$ with value -12.

Submit You have used 1 of 3 attempts

Answers are displayed within the problem

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