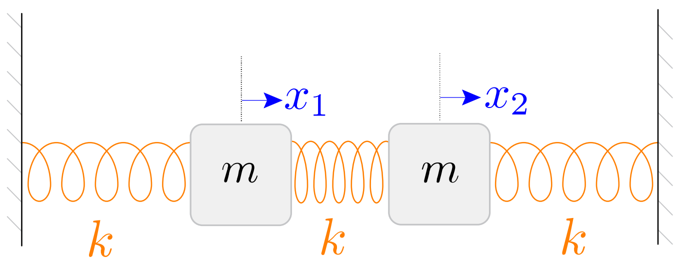
16. Mathlet: coupled oscillator

Initial conditions

4/4 points (graded)

Consider the same coupled oscillator as before.



Recall that the general solution of the positions of the masses are:

$$\begin{aligned} x_1 &= A_1 \cos(\omega t + \phi_1) + A_2 \cos(\sqrt{3}\omega t + \phi_2) \\ x_2 &= A_1 \cos(\omega t + \phi_1) - A_2 \cos(\sqrt{3}\omega t + \phi_2). \end{aligned}$$

(Note that $x_1 = x_2 = 0$ corresponds to when all three springs are relaxed.)

Which initial conditions will result in the masses oscillating in the pure sinusoidal mode with **angular frequency** ω and **amplitude** 1? Enter the values of $x_1(0)$, $y_1(0)$, $x_2(0)$, $y_2(0)$ in terms of the ω and the parameters ϕ_1, ϕ_2 below.

$x_1(0) =$ cos(phi_1) ✔ **Answer:** cos(phi_1)

$\cos(\phi_1)$

$x_2(0) =$ cos(phi_1) ✔ **Answer:** cos(phi_1)

$\cos(\phi_1)$

$y_1(0) =$ -omega*sin(phi_1) ✔ **Answer:** -omega*sin(phi_1)

$-\omega \cdot \sin(\phi_1)$

$y_2(0) =$ -omega*sin(phi_1) ✔ **Answer:** -omega*sin(phi_1)

$-\omega \cdot \sin(\phi_1)$

FORMULA INPUT HELP

Solution:

Recall the general solution is

$$\begin{aligned} x_1(t) &= A_1 \cos(\omega t + \phi_1) + A_2 \cos(\sqrt{3}\omega t + \phi_2) \\ x_2(t) &= A_1 \cos(\omega t + \phi_1) - A_2 \cos(\sqrt{3}\omega t + \phi_2). \end{aligned}$$

and the pure sinusoidal mode with angular frequency ω and amplitude 1 corresponds to $A_2 = 0$ and $A_1 = 1,$ hence

$$\begin{aligned} x_1(t) &= \cos(\omega t + \phi_1) \\ x_2(t) &= \cos(\omega t + \phi_1). \end{aligned}$$

Therefore, the initial positions are

$$x_1(0) = x_2(0) = \cos(\phi_1).$$

The velocities y_1, y_2 are the derivatives of x_1, x_2:

$$\begin{aligned} y_1(t) &= -\omega \sin(\omega t + \phi_1) \\ y_2(t) &= -\omega \sin(\omega t + \phi_1) \end{aligned}$$

Hence, the initial velocities are

$$\begin{aligned} y_1(0) &= -\omega \sin(\phi_1) \\ y_2(0) &= -\omega \sin(\phi_1). \end{aligned}$$

Submit You have used 1 of 3 attempts

Answers are displayed within the problem

Mathlet

The coupled oscillator is simulated in the mathlet below.

To repeat the example above, choose $k_1 = k_2 = k_3 = k$ and $m_1 = m_2 = m.$ Recall that $\omega^2 = \frac{k}{m}$.

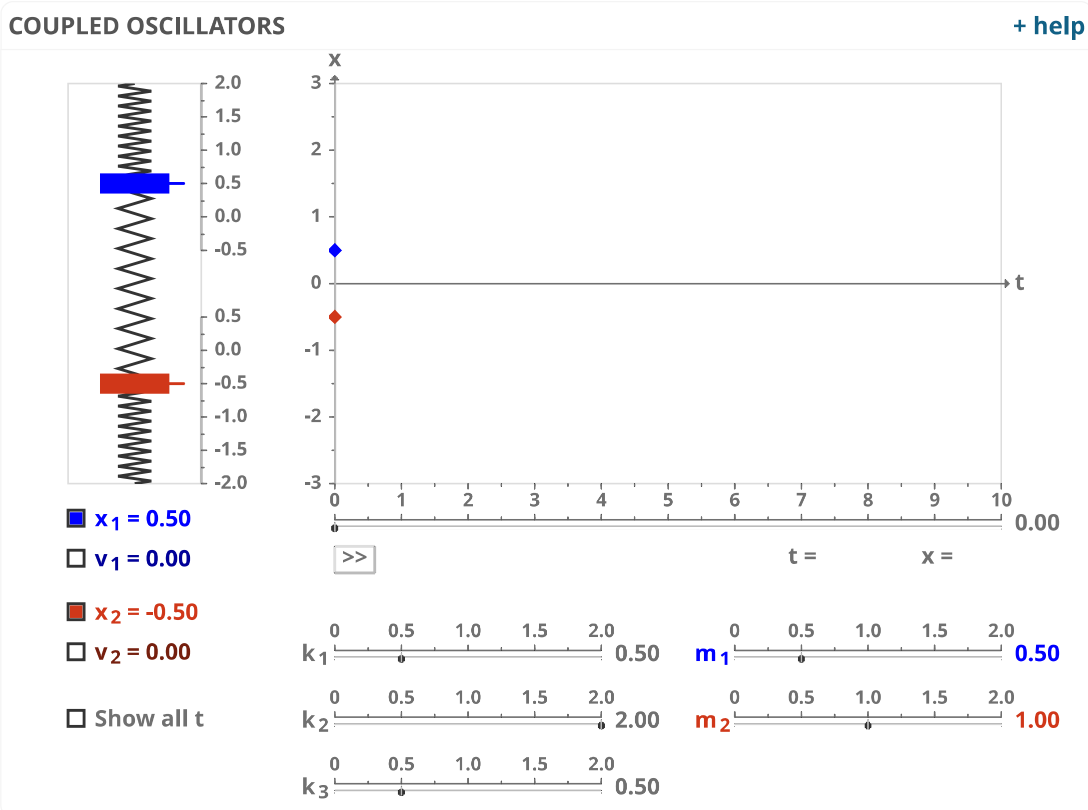
To adjust initial positions $x_1, x_2,$ drag the masses on the diagram; to adjust initial velocities, check the boxes next to v_1 or v_2 and drag the end of the vertical bar on the left of the mass. The graph on the right displays the evolution of any of $x_1(t), x_2(t), v_1(t), v_2(t)$ that you have checked on the left.

Choose initial conditions that result in the pure sinusoidal modes and watch the action.

Then explore solutions resulting from other initial conditions. These solutions are linear combinations of the two pure sinusoidal modes. Because the pure sinusoids are at different frequencies, their linear combinations look chaotic. In fact, because $\sqrt{3}$ is an irrational number, there are no periodic solutions except for the two pure sinusoidal modes.

To stay within the simplifying assumptions of our model, choose initial x_1 and x_2 to be between -0.5 and 0.5, and keep $v_1 = v_2 = 0$. The two masses will not collide and the spring will not be too compressed. If you are outside this range, our model does not work, and you may see non-physical phenomena where the masses cross each other without interacting.

In the graph on the right, you will see that graphs of $x_1(t)$ and $x_2(t)$ intersect. Note that each of these functions are displayed relative to the equilibrium positions of the corresponding mass, and that intersections of these graphs correspond the two masses being at the same displacements away from their respective equilibrium positions, rather than the two masses colliding physically.



Explore the system with different masses and spring constants by adjusting the respective sliders!

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? Multiple correct answers? 2
So I will try to describe my concern and phrase my question as to not give away any clues to the answer. I should think that there would be an infinite amount of solut...

[Staff] Initial conditions problem 8
Community TA

Pure sinusoidal mode? 2
What is pure sinusoidal mode? What has that to do with initial conditions mathematically? I look at the other posts and still have no clue.

? Coupled oscillator: How does one go about solving this one? 2
Hey guys, I was wondering how one solves this question. I played with the Mathlet, and found conditions that seemed to be correct, but I haven't figured out a way to ...

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