2. Discrete random variables

Normalization constant for the Poisson distribution

1/1 point (graded)

The probability mass function (pmf) of a **Poisson distribution** with parameter λ is given by

$$\mathrm{Poi}\,(\lambda) = \frac{c\lambda^k}{k!}, \quad k = 0, 1, 2, \ldots.$$

Compute the value of c.

$c =$ exp(-lambda) ✔ **Answer:** exp(-lambda)

$\exp(-\lambda)$

STANDARD NOTATION

Solution:

In order to obtain a probability distribution, we must have

$$c\sum_{k=0}^{\infty} \frac{\lambda^k}{k!} = 1. \tag{1.1}$$

But

$$\sum_{k=0}^{\infty} \frac{\lambda^k}{k!} = \exp(\lambda) \tag{1.2}$$

by the series definition of the exponential function. Hence,

$$c = \exp(-\lambda).$$

: Probability axioms in lecture 1, *Probability models and axioms*.

Submit You have used 1 of 2 attempts

Answers are displayed within the problem

Moments of Bernoulli variables

3/3 points (graded)

Recall that a **Bernoulli random variable with parameter** p is a random variable that takes the value 1 with probability p, and the value 0 with probability $1-p$.

Let X be a Bernoulli random variable with parameter 0.7. Compute the **expectation values** of X^k, denoted by $\mathbb{E}[X^k]$, for the following three values of k: $k = 1, 4$, and 3203.

$\mathbb{E}[X] =$ 0.7 ✔ **Answer:** 0.7

$\mathbb{E}[X^4] =$ 0.7 ✔ **Answer:** 0.7

$\mathbb{E}\left[X^{3203}\right] =$ 0.7 ✔ **Answer:** 0.7

STANDARD NOTATION

Solution:

Remember, the expectation of a discrete random variable is

$$\mathbb{E}\left[X\right] = \sum_{j \in \text{range}(X)} j\,\mathbf{P}\left(X = j\right),$$

while the higher moments are

$$\mathbb{E}\left[X^n\right] = \sum_{j \in \text{range}(X)} j^n\,\mathbf{P}\left(X = j\right),$$

For a Bernoulli random variable with parameter p, the range is $\{0, 1\}$, and $0^k = 0$, $1^k = 1$ for all $k \geq 1$, so all moments are equal to the first one,

$$\mathbb{E}\left[X\right] = 0 \times (1-p) + 1 \times p = p,$$

and we get the result by plugging in $p = 0.7$.
: Expectation in lecture 5, *Probability mass functions and expectations*.

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Answers are displayed within the problem

Variance of Bernoulli variables

3/3 points (graded)

Let X be a Bernoulli random variable with parameter $p \in [0,1]$. Compute the **variance** of X, which is denoted by $\mathrm{Var}[X]$.

$\mathrm{Var}[X] =$ p*(1-p) ✔ **Answer:** p*(1-p)

$p \cdot (1-p)$

What value(s) of the parameter p maximize the variance? What values minimize it?

(For each question, enter the values of p as a list of **numbers**, separated by commas. For example, to enter the set $\{0.2, 0.3\}$, type **0.2, 0.3**.)

The values of p for which $\mathrm{Var}[X]$ is minimized: 0, 1 ✔ **Answer:** 0, 1

The values of p for which $\mathrm{Var}[X]$ is maximized: 1/2 ✔ **Answer:** 1/2

STANDARD NOTATION

Solution:

Recall from the previous exercise that $\mathbb{E}\left[X^n\right] = p$ for all positive integers n. Therefore, the variance is

$$\mathsf{Var}\left[X\right] = \mathbb{E}\left[X^2\right] - \mathbb{E}[X]^2 = p - p^2 = p\left(1-p\right).$$

This is a quadratic polynomial with negative leading factor, hence it does not attain a global minimum on $\mathbb{R}$. For the range $p \in [0,1]$ in question, its minima are attained at both boundary points $p = 0$ and $p = 1$. Its maximum can be found by differentiating and setting the derivative equal to zero. It occurs at $p = \frac{1}{2}$.
: Variance in lecture 6, *Variance; conditioning on an event; multiple random variable*.

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Answers are displayed within the problem

Sum of Bernoulli variables

1/1 point (graded)
Given n i.i.d. realizations $X_1, \ldots, X_n \sim \mathrm{Ber}\left(p\right)$, what is the distribution of $\sum_{i=1}^n X_i$?

- Poisson with parameter pn
- Gamma with parameters n and p

- (•) Binomial with parameters n and p ✔
- ( ) Bernoulli with parameter pn

STANDARD NOTATION

Solution:

We know from probability theory that $\sum_{i=1}^{n} X_i$ follows a Binomial distribution with parameters n and p.

Submit You have used 1 of 3 attempts

Answers are displayed within the problem

Discrete uniform random variables

2/2 points (graded)

Recall that a **uniform random variable** is a random variable that takes values with equal probability,

Let X be a uniform random variable in the finite set $\{1, 2, \ldots, 20\}$.

Compute the following quantities.

The probability that X is an even number:

$\mathbf{P}\left(X \text{ is an even number}\right) =$ 1/2 ✔ **Answer:** 1/2

The probability that X is a prime number:

$\mathbf{P}\left(X \text{ is a prime number}\right) =$ 2/5 ✔ **Answer:** 2/5

STANDARD NOTATION

Solution:

There are 10 even numbers in $\{1, \ldots, 20\}$, therefore

$$\mathbf{P}\left(X \text{ is an even number}\right) = \frac{10}{20} = \frac{1}{2}.$$

There are 8 prime numbers in $\{1, \ldots, 20\}$, (namely $\{2, 3, 5, 7, 11, 13, 17, 19\}$, so

$$\mathbf{P}\left(X \text{ is a prime number}\right) = \frac{8}{20} = \frac{2}{5}.$$

:

Submit You have used 1 of 2 attempts

Answers are displayed within the problem

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[STAFF] Is it possible to get credit for the Variance of the Bernoulli added retroactively? I have the right answer, but the formatting is off. It seems to me the grader should be able to accept this. Is it possible to still receive credit for the rig...	4
How do you denote factorials in Standard Notation? How can you write k! in Standard Notation?	3
1 is not a prime number To correctly answer the above question, consider that 1 is not a prime number.	4
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Printing problem sets Hi - Is there a way to elegantly print the whole problem set? Best I can do is printing web page but that includes discussion etc. Ideally it would be HT...	2

the value of c

I think the phrase "value of c" is implying that c is a scalar when in fact it needs to be expressed in certain given parameters. I think this should have ...

4

Standard notation - Difference between exp() and e^()

Hello, I used the e^(...) standard notation to answer a question, and it was wrong... the given solution is the same I wrote, but with exp() notation. A c...

2

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