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Are functions of independent variables also independent?

It's a really simple question. However I didn't see it in books and I tried to find the answer on the web but failed.

If I have two independent random variables, X_1 and X_2, then I define two other random variables Y_1 and Y_2, where $Y_1 = f_1(X_1)$ and $Y_2 = f_2(X_2)$.

Intuitively, Y_1 and Y_2 should be independent, and I can't find a counter example, but I am not sure. Could anyone tell me whether they are independent? Does it depend on some properties of f_1 and f_2?

Thank you.

(probability-theory)

edited Nov 16 '14 at 15:15
Jack
12.6k 9 39 111

asked Nov 3 '10 at 10:20
LLS
185 1 7

Not seen in the books?? – Did Nov 16 '14 at 15:16

3 Answers

For any two (measurable) sets A_i, $i = 1, 2$, $Y_i \in A_i$ if and only if $X_i \in B_i$, where B_i are the sets $\{ s : f_i(s) \in A_i \}$. Hence, since the X_i are independent, $\mathrm{P}(Y_1 \in A_1, Y_2 \in A_2) = \mathrm{P}(Y_1 \in A_1)\mathrm{P}(Y_2 \in A_2)$. Thus, the Y_i are independent (which is intuitively clear anyway). [We have used here that random variables Z_i, $i = 1, 2$, are independent if and only if $\mathrm{P}(Z_1 \in C_1, Z_2 \in C_2) = \mathrm{P}(Z_1 \in C_1)\mathrm{P}(Z_2 \in C_2)$ for any two measurable sets C_i.]

edited Nov 3 '10 at 14:07

answered Nov 3 '10 at 13:42
Shai Covo
19k 1 19 42

I had no idea about the theorem of measurable sets and independence. Anyway, it seems to be a valid proof. (But I have no idea what the measurable sets are) – LLS Nov 5 '10 at 11:54

On the one hand, my answer also assumes that the functions f_i are measurable. On the other hand, the use of the prefix "measurable" (for sets/functions) may be omitted in an introductory setting. – Shai Covo Nov 5 '10 at 12:30

Yes, they are independent.

If you are studying rigorous probability course with sigma-algebras then you may prove it by noticing that the sigma-algebra generated by $f_1(X_1)$ is smaller than the sigma-algebra generated by X_1, where f_1 is borel-measurable function.

If you are studying an introductory course - then just remark that this theorem is consistent with our intuition: if X_1 does not contain info about X_2 then $f_1(X_1)$ does not contain info about $f_2(X_2)$.

answered Nov 3 '10 at 10:45
Roah
737 5 11

1 Thank you very much. I am studying an introductory course and it seems to be a little hard for me to get things too serious. – LLS Nov 5 '10 at 11:57

Yes, they are independent.

The previous answers are sufficient and rigorous. On the other hand, it can be restated as followed. Assume they are discrete random variable.

$Pr[Y_1 = f_1(X_1) \wedge Y_2 = f_2(X_2)] = Pr[X_1 \in f_1^{-1}(Y_1) \wedge X_2 \in f_2^{-1}(Y_2)]$
$= Pr[X_1 \in A_1 \wedge X_2 \in A_2]$

and we spend it by probability mass function derived

$= \sum_{x_1 \in A_1 \wedge x_2 \in A_2} Pr(x_1, x_2) = \sum_{x_1 \in A_1 \wedge x_2 \in A_2} Pr(x_1)Pr(x_2)$

Here we use the independency of X_1 and X_2, and we shuffle the order of summation

$= \sum_{x_1 \in A_1} Pr(x_1) * \sum_{x_2 \in A_2} Pr(x_2) = Pr[X_1 \in f_1^{-1}(Y_1)] * Pr[X_2 \in f_2^{-1}(Y_2)]$
$= Pr[Y_1 = f_1(X_1)]Pr[Y_2 = f_2(X_2)]$

Here we show the function of independent random variable is still independent

answered Nov 24 '14 at 3:21

Fang-Yi Yu
11 1