

MITx: 6.008.1x Computational Probability and Inference

Heli

Ħ

Bookmarks

- **▶** Introduction
- Part 1: Probability and Inference
- Part 2: Inference in Graphical Models
- Part 3: Learning
 Probabilistic Models

Week 8: Introduction to Learning Probabilistic Models

Week 8: Introduction to
Parameter Learning Maximum Likelihood and
MAP Estimation
due Nov 10, 2016 03:30 IST

<u>Week 9: Parameter Learning</u>
- Naive Bayes Classification

Part 3: Learning Probabilistic Models > Week 10: Homework 7 > Homework Problem: Ising Model

Homework Problem: Ising Model

☐ Bookmark this page

Homework Problem: Ising Model

10/10 points (graded)

Consider the joint probability distribution of random variables $X_1, X_2, X_3 \in \{-1, 1\}$ that is parameterized by $(\alpha_{12}, \alpha_{13}, \alpha_{23})$ as follows:

$$p_{X_1,X_2,X_3}(x_1,x_2,x_3;\alpha_{12},\alpha_{13},\alpha_{23}) = \frac{1}{Z} \cdot e^{\alpha_{12}x_1x_2} \cdot e^{\alpha_{23}x_2x_3} \cdot e^{\alpha_{13}x_1x_3}, \tag{4.1}$$

where $Z=Z(\alpha_{12},\alpha_{13},\alpha_{23})$ is the required normalization constant (Z is also called the *partition function*).

ullet (a) If $lpha_{13}=0$, is it the case that $X_1\perp X_3|X_2$, i.e., X_1,X_2,X_3 form a Markov chain?



False

Week 9: Mini-project on **Email Spam Detection**

due Nov 17, 2016 03:30 IST

Week 10: Parameter **Learning - Finite Random Variables and Trees** due Nov 24, 2016 03:30 IST

Week 10: Structure Learning - Trees

due Nov 24, 2016 03:30 IST

Week 10: Homework 7

due Nov 24, 2016 03:30 IST

- ullet n i.i.d. samples $\left\{(x_1^{(i)},x_2^{(i)},x_3^{(i)})
 ight\}_{i=1}^n$ are drawn from $p_{X_1,X_2,X_3}(\cdot,\cdot,\cdot;lpha,0,lpha)$ for some unknown value α .
 - Determine the ML estimate $\hat{\alpha}_{\rm ML}$ of α in terms of m_{13} , which is defined to be

$$m_{13} riangleq rac{1}{2n} \sum_{i=1}^n (x_1^{(i)} x_2^{(i)} + x_2^{(i)} x_3^{(i)})$$

You should come up with a general expression for $\widehat{\alpha}$ in terms of m_{13} . However, we will only ask you to specify your answer in terms of a few specific numerical values for m_{13} . Please use natural log.

(Please be precise with at least 3 decimal places, unless of course the answer doesn't need that many decimal places. You could also put a fraction.)

When
$$m_{13}=0.3$$
, then $\widehat{lpha}_{
m ML}= \boxed{ 0.309519604203 }$

When
$$m_{13}=0.8$$
, then $\widehat{lpha}_{
m ML}=1.09861228867$

• Determine a general expression the function q such that the maximum log-likelihood of the entire sample set is equal to

$$g(m_{13},n),$$

where m_{13} is defined in the previous part, and n is the number of training data points.

You should come up with a general expression for g in terms of m_{13} and n. However, we will only ask you to specify your answer in terms of a few specific numerical values for m_{13} and n. Please use natural log.

(Please be precise with at least 3 decimal places, unless of course the answer doesn't need that many decimal places. You could also put a fraction.)

When
$$m_{13}=0.2$$
 and $n=17$, $g=\boxed{ ext{ -34.6658987478}}$

When
$$m_{13}=0.9$$
 and $n=15$, $g=\boxed{ ext{ -16.3526650088} }$

• We define a set of parameter values $\mathcal{M} = \{(0, \alpha, \alpha) | \alpha \in \mathbb{R}\} \cup \{(\alpha, 0, \alpha) | \alpha \in \mathbb{R}\} \cup \{(\alpha, \alpha, 0) | \alpha \in \mathbb{R}\}$ to describe all distributions of the form (4.1) where $\alpha_{ij} = 0$ for some i < j, and the other two parameters are equal.

Bob observes n i.i.d. samples $\left\{(x_1^{(i)}, x_2^{(i)}, x_3^{(i)})
ight\}_{i=1}^n$ and computes three values

$$m_{12} = rac{1}{2n} \sum_{i=1}^{N} (x_1^{(i)} x_3^{(i)} + x_2^{(i)} x_3^{(i)}),$$

$$m_{13} = rac{1}{2n} \sum_{i=1}^{N} (x_1^{(i)} x_2^{(i)} + x_2^{(i)} x_3^{(i)}),$$

$$m_{23} = rac{1}{2n} \sum_{i=1}^{N} (x_1^{(i)} x_2^{(i)} + x_1^{(i)} x_3^{(i)}).$$

Bob only tells you that

$$|m_{12}| < |m_{23}| < |m_{13}|,$$

and that he is performing an ML estimation over all possible models in set \mathcal{M} . Determine which of the three parameter estimates $(\hat{\alpha}_{12}, \hat{\alpha}_{23}, \hat{\alpha}_{13})$ Bob will set to zero.

- $\hat{m{lpha}}_{12}$
- \hat{lpha}_{23}
- â₁₃ ✓

Submit

You have used 4 of 5 attempts

✓ Correct (10/10 points)

Discussion

Topic: Homework 7 / Homework Problem: Ising Model

Hide Discussion

Add a Post

help in getting Z

question posted about 20 hours ago by javiernl

Can anybody help me to get Z (I know that most part of you can)? I tried to sum the expressions for all of the possible combinations of x1, x2...

This post is visible to everyone.

+ Expand discussion

MLE

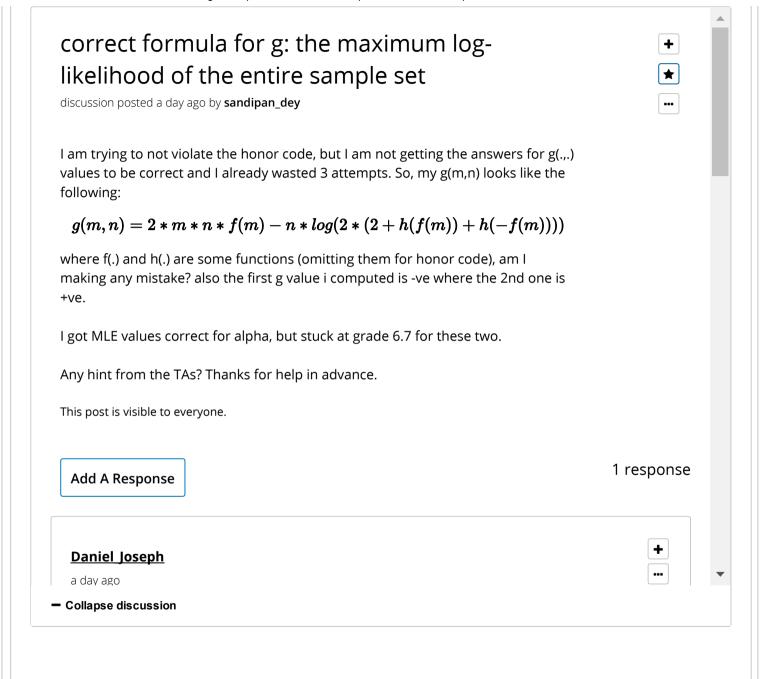
question posted 4 days ago by wcrfrench

In previous examples, the MLE estimates stems from the multiplication of the individual operations.

Therefore does the summation arise in M13...

This post is visible to everyone.

+ Expand discussion



General Expression g

discussion posted a day ago by Aeschbacher

For the equation:

$$m_{13} riangleq rac{1}{2n} \sum_{i=1}^{n} (x_1^{(i)} x_2^{(i)} + x_2^{(i)} x_3^{(i)})$$

I'm confused how we can get \$m_{13}...

This post is visible to everyone.

+ Expand discussion

What does this mean?

discussion posted 5 days ago by seanedXacc

pX1,X2,X3(\cdot , \cdot , \cdot ; α ,0, α)? Does it mean that there is no node 12, 23, 32 because x2 occurs with P(0)?

This post is visible to everyone.

+ Expand discussion

is there a grading issue?

discussion posted 5 days ago by **Stef2970**

It looks like I found a possible formula for the ML estimates alfa_hat values for m being 0.3 and 0.8 as both my answers are graded ok. I also...

This post is visible to everyone.

+ Expand discussion

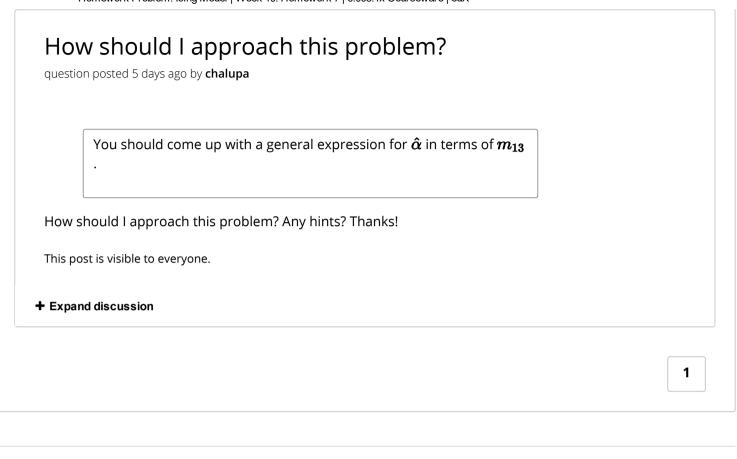
Test Answer

discussion posted 3 days ago by Daniel_Joseph

I submitted my answers and am getting some red checks. Upon further review, I believe that I made a minor mistake in the derivation of my formulas....

This post is visible to everyone.

+ Expand discussion



© All Rights Reserved



© 2016 edX Inc. All rights reserved except where noted. EdX, Open edX and the edX and Open EdX logos are registered trademarks or trademarks of edX Inc.

















