



Bookmarks



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Problem 6: Correlation coefficients

(6/6 points)

Consider the random variables X , Y and Z , which are given to be pairwise uncorrelated (i.e., X and Y are uncorrelated, X and Z are uncorrelated, and Y and Z are uncorrelated). Suppose that

- $\mathbf{E}[X] = \mathbf{E}[Y] = \mathbf{E}[Z] = 0$,
- $\mathbf{E}[X^2] = \mathbf{E}[Y^2] = \mathbf{E}[Z^2] = 1$,
- $\mathbf{E}[X^3] = \mathbf{E}[Y^3] = \mathbf{E}[Z^3] = 0$,
- $\mathbf{E}[X^4] = \mathbf{E}[Y^4] = \mathbf{E}[Z^4] = 3$.

Let $W = a + bX + cX^2$ and $V = dX$, where a, b, c , and d are constants, all greater than 0.

Find the correlation coefficients $\rho(X - Y, X + Y)$, $\rho(X + Y, Y + Z)$, $\rho(X, Y + Z)$ and $\rho(W, V)$.

1.

$$\rho(X - Y, X + Y) =$$

0



Answer: 0

2.

$$\rho(X + Y, Y + Z) =$$


1/2




Answer: 0.5

Unit overview


Lec. 11: Derived distributions

Exercises 11 due Mar 30, 2016 at 23:59 UTC 

Lec. 12: Sums of independent r.v.'s; Covariance and correlation

Exercises 12 due Mar 30, 2016 at 23:59 UTC 


Lec. 13: Conditional expectation and variance revisited; Sum of a random number of independent r.v.'s

Exercises 13 due Mar 30, 2016 at 23:59 UTC 

Solved problems

Additional theoretical material

Problem Set 6

Problem Set 6 due Mar 30, 2016 at 23:59 UTC 

Unit summary

- ▶ Unit 7: Bayesian inference
- ▶ Exam 2
- ▶ Unit 8: Limit theorems and classical statistics

3.

$$\rho(X, Y + Z) =$$

0

✓ Answer: 0

4. $\rho(W, V) =$

☐ $\frac{b}{\sqrt{b^2 + c^2}}$

☐ $\frac{b^2}{\sqrt{b^2 + 2c^2}}$

☐ $\frac{bd}{\sqrt{b^2 + 2c^2}}$

☒ $\frac{b}{\sqrt{b^2 + 2c^2}}$

✓

Answer:

1. We have

$$\begin{aligned}
 \text{cov}(X - Y, X + Y) &= \mathbf{E}[(X - Y)(X + Y)] - \mathbf{E}[X - Y]\mathbf{E}[X + Y] \\
 &= \mathbf{E}[X^2 - Y^2] - 0 \\
 &= \mathbf{E}[X^2] - \mathbf{E}[Y^2] \\
 &= 0.
 \end{aligned}$$

- ▶ Unit 9: Bernoulli and Poisson processes
- ▶ Unit 10: Markov chains
- ▶ Exit Survey
- ▶ Final Exam

Hence, $\rho(X - Y, X + Y) = 0$.

2. Since X and Y are uncorrelated, with zero means, we have $\mathbf{E}[XY] = \text{cov}(X, Y) = 0$. Similarly, we have $\mathbf{E}[XZ] = 0$ and $\mathbf{E}[YZ] = 0$. Hence,

$$\begin{aligned}\text{cov}(X + Y, Y + Z) &= \mathbf{E}[(X + Y)(Y + Z)] - \mathbf{E}[X + Y]\mathbf{E}[Y + Z] \\ &= \mathbf{E}[XY + XZ + Y^2 + YZ] \\ &= \mathbf{E}[Y^2] \\ &= 1.\end{aligned}$$

Also,

$$\begin{aligned}\text{var}(X + Y) &= \mathbf{E}[(X + Y)^2] - (\mathbf{E}[X + Y])^2 \\ &= \mathbf{E}[X^2 + 2XY + Y^2] - 0 \\ &= 2.\end{aligned}$$

Similarly, $\text{var}(Y + Z) = 2$.

$$\text{Therefore, } \rho(X + Y, Y + Z) = \frac{\text{cov}(X + Y, Y + Z)}{\sqrt{\text{var}(X + Y)\text{var}(Y + Z)}} = \frac{1}{2}.$$

3.

$$\begin{aligned}\text{cov}(X, Y + Z) &= \mathbf{E}[X(Y + Z)] - \mathbf{E}[X]\mathbf{E}[Y + Z] \\ &= \mathbf{E}[XY + YZ] - 0 \\ &= 0.\end{aligned}$$

Hence, $\rho(X, Y + Z) = 0$.

4.

$$\begin{aligned}
\text{cov}(W, V) &= \mathbf{E}[WV] - \mathbf{E}[W]\mathbf{E}[V] \\
&= \mathbf{E}[adX + bdX^2 + cdX^3] - 0 \\
&= bd, \\
\text{var}(W) &= \mathbf{E}[W^2] - (\mathbf{E}[W])^2 \\
&= \mathbf{E}[a^2 + 2abX + (2ac + b^2)X^2 + 2bcX^3 + c^2X^4] - (a + b\mathbf{E}[X] + c\mathbf{E}[X^2])^2 \\
&= (a^2 + 2ac + b^2 + 3c^2) - (a^2 + 2ac + c^2) \\
&= b^2 + 2c^2, \\
\text{var}(V) &= \mathbf{E}[d^2X^2] - (\mathbf{E}[dX])^2 = d^2.
\end{aligned}$$

$$\text{Hence, } \rho(W, V) = \frac{bd}{\sqrt{d^2(b^2 + 2c^2)}} = \frac{b}{\sqrt{b^2 + 2c^2}}.$$

You have used 1 of 2 submissions

DISCUSSION

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