12.2.2 Simple Examples (continued)

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Week 12 due Dec 29, 2023 10:42 IST Completed

12.2.2 Simple Examples (continued)

Reading Assignment

0 points possible (ungraded)
Read Unit 12.2.2 of the notes. [LINK to Week12.pdf]

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Homework 12.2.2.4

1/1 point (graded)

Consider the diagonal matrix $\begin{pmatrix} \delta_0 & 0 & \cdots & 0 \\ 0 & \delta_1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \delta_{n-1} \end{pmatrix}$.

Eigenpairs for this matrix are given by $(\delta_0, e_0), (\delta_1, e_1), \cdots, (\delta_{n-1}, e_{n-1})$, where e_j equals the jth unit basis vector.

Always ✔ Answer: Always

We will simply say "by examination" this time and get back to a more elegant solution in Unit 12.3.3 .

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ℹ Answers are displayed within the problem

Video 12.2.2 Part 5

as is any multiple of that unit basis vector.

OK, so now let's make things a little bit more difficult.

Why don't you contemplate wha... eigenvalues of an upper trianqu...

Calculator

matrix are, and then find representative corresponding eigenvectors.

You will see me in the next video.

0:00 / 0:00 2.0x

End of transcript. Skip to the start.

Video

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Homework 12.2.2.5

10.0/10.0 points (graded)

Which of the following are eigenpairs (λ, x) of the 2×2 triangular matrix:

$$\begin{pmatrix} 3 & 1 \\ 0 & -1 \end{pmatrix} x = \lambda x,$$

where $x \neq 0$.

(Mark all correct answers.)

- [x] $(-1, \begin{pmatrix} -1 \\ 4 \end{pmatrix})$.
- [ ] $(1/3, \begin{pmatrix} 1 \\ 0 \end{pmatrix})$.
- [x] $(3, \begin{pmatrix} 1 \\ 0 \end{pmatrix})$.
- [ ] $(-1, \begin{pmatrix} 1 \\ 0 \end{pmatrix})$.
- [x] $(3, \begin{pmatrix} -1 \\ 0 \end{pmatrix})$.
- [ ] $(-1, \begin{pmatrix} 3 \\ -1 \end{pmatrix})$.

Answers a), c), and e) are all correct, since for all $Ax = \lambda x$.

Answers b), d), and f) are not correct because they don't satisfy $Ax = \lambda x$.

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Answers are displayed within the problem

Video 12.2.2 Part 6

0:00 / 0:00 2.0x

then you end up with the equation 4, 1; 0, 0, times chi 0, chi 1, equals 0, 0.

Now this is in row echelon form.

Therefore, you pick the free variable to be equal to 1,

and you look for vectors of the form 1.

Something like that.

And if you then look at this right here, it's not hard to see that minus 1/4

works.

And what you get are the eigenpairs.

Let's see, 3 comma 1, 0, and minus 1.

And the scalar multiple of that is also an eigenvector.

And therefore, why don't we instead pick minus 1, 4 for our eigenpair.

That's just a nicer vector, because it only involves integers.

Video

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Homework 12.2.2.6

10.0/10.0 points (graded)

Consider the upper triangular matrix $U = \begin{pmatrix} v_{0,0} & v_{0,1} & \cdots & v_{0,n-1} \\ 0 & v_{1,1} & \cdots & v_{1,n-1} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & v_{n-1,n-1} \end{pmatrix}$.

The eigenvalues of this matrix are $v_{0,0}, v_{1,1}, \ldots, v_{n-1,n-1}$.

Always ✔ Answer: Always

The upper triangular matrix $U - v_{i,i}I$ has a zero on the diagonal. Hence it is singular. Because it is singular, $v_{i,i}$ is an eigenvalue. This is true for $i = 0, \ldots, n-1$.

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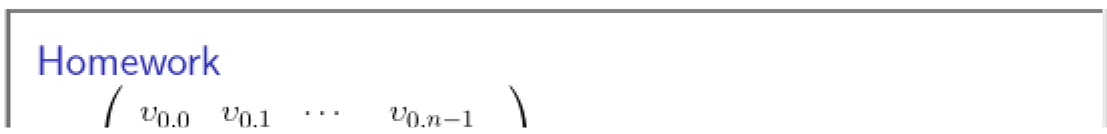
Answers are displayed within the problem

In the video, a few seconds in, the slide has an error (λ is subtracted from all elements above the diagonal).

Corrected slide:



Calculator

$$\begin{pmatrix} 0 & \upsilon_{1,1} & \cdots & \upsilon_{1,n-1} \\ \vdots & \vdots & \ddots & \cdots \\ 0 & 0 & \cdots & \upsilon_{n-1,n-1} \end{pmatrix} x = \lambda x$$

$$\left[\begin{pmatrix} \upsilon_{0,0} & \upsilon_{0,1} & \cdots & \upsilon_{0,n-1} \\ 0 & \upsilon_{1,1} & \cdots & \upsilon_{1,n-1} \\ \vdots & \vdots & \ddots & \cdots \\ 0 & 0 & \cdots & \upsilon_{n-1,n-1} \end{pmatrix} - \lambda I \right] x = 0$$

$$\begin{pmatrix} \upsilon_{0,0} - \lambda & \upsilon_{0,1} & \cdots & \upsilon_{0,n-1} \\ 0 & \upsilon_{1,1} - \lambda & \cdots & \upsilon_{1,n-1} \\ \vdots & \vdots & \ddots & \cdots \\ 0 & 0 & \cdots & \upsilon_{n-1,n-1} - \lambda \end{pmatrix} x = 0$$

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Video 12.2.2 Part 7

Start of transcript. Skip to the end.

Dr. Robert van de Geijn: What you have to do

is subtract lambda from the diagonal, and you get this right here.

And then you ask yourself the question, under what circumstances

is an upper triangular matrix singular?

Notice that if an upper triangular matrix has a 0 on the diagonal,

0:00 / 0:00 2.0x

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Homework 12.2.2.7

4/4 points (graded)

Consider $A = \begin{pmatrix} 1 & 3 \\ 3 & 1 \end{pmatrix}$

- The eigenvalue *largest in magnitude* is

4 ✔ Answer: 4

- Which of the following are eigenvectors associated with this largest eigenvalue (in magnitude):

☐ $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$

☑ $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

Calculator

- [x] $\begin{pmatrix} 2 \\ 2 \end{pmatrix}$
- [ ] $\begin{pmatrix} -1 \\ 2 \end{pmatrix}$

- The eigenvalue *smallest in magnitude* is

-2 ✔ Answer: -2

- Which of the following are eigenvectors associated with this smallest eigenvalue (in magnitude):

- [x] $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$
- [ ] $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$
- [ ] $\begin{pmatrix} 2 \\ 2 \end{pmatrix}$
- [ ] $\begin{pmatrix} -1 \\ 2 \end{pmatrix}$

For the most part, you just take the solutions and plug them in to verify.

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Answers are displayed within the problem

Homework 12.2.2.8

10.0/10.0 points (graded)

Consider $A = \begin{pmatrix} -3 & -4 \\ 5 & 6 \end{pmatrix}$

- The eigenvalue *largest in magnitude* is

2 ✔ Answer: 2

- The eigenvalue *smallest in magnitude* is

1 ✔ Answer: 1

For the most part, you just take the solutions and plug them in to verify.

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Answers are displayed within the problem

Calculator

Video 12.2.2 Part 8

Start of transcript. Skip to the end.

Dr. Robert van de Geijn: Let's do another example.

Again it's a 2 by 2 matrix.

We should be able to go through the same procedure.

We subtract off lambda from the diagonal.

We recall that we need to look for the determinant of the matrix.

0:00 / 0:00 2.0x

Video

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Homework 12.2.2.9

1/1 point (graded)

Consider $A = \begin{pmatrix} 2 & 2 \\ -1 & 4 \end{pmatrix}$. Which of the following are the eigenvalues of A:

- 4 and 2.
- $3 + i$ and 2.
- $3 + i$ and $3 - i$. (selected)
- $2 + i$ and $2 - i$.

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