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Exercise: Conditional variance definition

(4/5 points)

For each one of the following statements, indicate whether it is true or false.

(a) If $X=Y$ (i.e., the two random variables always take the same values), then $\mathbf{var}(X \mid Y)=0$.

True ✔ **Answer:** True

(b) If $X=Y$ (the two random variables always take the same values), then $\mathbf{var}(X \mid Y)=\mathbf{var}(X)$.

False ✔ **Answer:** False

(c) If Y takes on the value y, then the random variable $\mathbf{var}(X \mid Y)$ takes the value

$$\mathbf{E}\big[(X-\mathbf{E}[X \mid Y=y])^2 \mid Y=y\big].$$

True ✔ **Answer:** True

(d) If Y takes on the value y, then the random variable $\mathbf{var}(X \mid Y)$ takes the value

$$\mathbf{E}\big[(X-\mathbf{E}[X \mid Y])^2 \mid Y=y.\big]$$

False ✘ **Answer:** True

(e) If Y takes on the value y, then the random variable $\mathbf{var}(X \mid Y)$ takes the value

$$\mathbf{E}\big[(X - \mathbf{E}[X])^2 \mid Y = y.\big]$$

False ▾ ✔ **Answer:** False

Answer:
(a) Conditioned on Y, X is deterministic, and $\mathbf{var}(X \mid Y = y) = 0$. This implies that the random variable $\mathbf{var}(X \mid Y)$ is identically equal to zero. Thus, the statement is true.

(b) False, because the previous statement is true.

(c) This statement is just the definition of the numerical value of the conditional variance. We are in a universe where the event $Y = y$ is known to have occurred, and every expectation is replaced by the corresponding conditional expectation.

(d) The outer expectation places us in a universe where $Y = y$. Given this information, the value of the random variable $\mathbf{E}[X \mid Y]$ becomes a known quantity, equal to $\mathbf{E}[X \mid Y = y]$. Thus, this statement is equivalent to the preceding one and is true.

(e) This is false, because all expectations should be conditional on the universe ($Y = y$) within which we are working. For a concrete counterexample, suppose that X is zero-mean and that $Y = X$. Then, as in part (a), $\mathbf{var}(X \mid Y = y) = 0$. On the other hand, since $\mathbf{E}[X] = 0$, we have

$$\mathbf{E}\big[(X - \mathbf{E}[X])^2 \mid Y = y\big] = \mathbf{E}[X^2 \mid Y = y] = \mathbf{E}[Y^2 \mid Y = y] = y^2.$$

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