

## **DelftX:** OT.1x Observation theory: Estimating the Unknown

Help

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### Assessment

Graded Assignment due Feb 8, 2017 17:30 IST

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# Module 3 Assessment - Part 2 (incl. MATLAB)

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You have 6 monthly measurements of the height of a point on a glacier. The measurements are obtained from a satellite laser altimeter.

Time [months]: [0, 1, 2, 3, 4, 5]

Observed heights [meters]: [100.9, 99.6, 98.7, 99.9, 99.4, 99.5]

We will consider two different models, with the following observation equations:

Model 1: 
$$y_i=x_0+x_1t_i+x_2t_i^2=\sum_{p=0}^2x_pt_i^p$$

Model 2: 
$$y_i = x_0 + x_1 \cdot t_i + x_2 \cos \left( rac{2\pi t_i}{12} 
ight)$$

GLACIER MODEL - MATLAB EXERCISE (EXTERNAL RESOURCE)

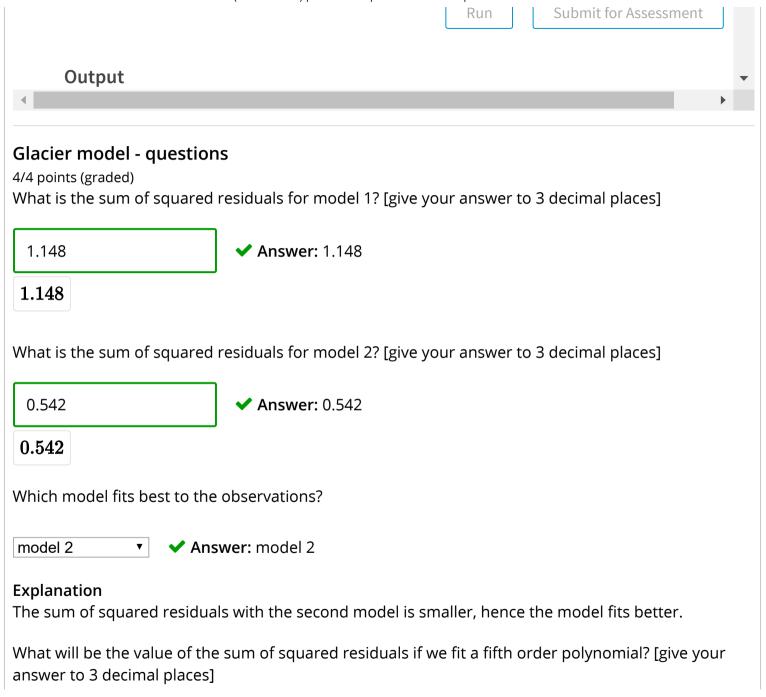
### Mid-survey

#### **Feedback**

- 4. Best Linear Unbiased Estimation (BLUE)
- ▶ 5. How precise is the estimate?
- Pre-knowledgeMathematics
- MATLAB Learning Content

```
Save C Reset MATLAB Documentation (https://www.mathworks.com/help/)
```

```
1 %% times of observation [months]
   2 t = [0 1 2 3 4 5]';
   3 % observed heights [m]
    4 v = [100.9 99.6 98.7 99.9 99.4 99.5]';
    5
   6 % number of observations
   7 m = length(t);
 10 % design matrices for the two models
11 A1 = [ones(1,m)', t, t.^2];
12 A2 = [ones(1,m)', t, cos(2*pi*t/12)];
13 A3 = [ones(1,m)', t, t.^2, t.^3, t.^4, t.^5];
 14
15 % what is the least squares solution for each of the models:
16 \times 16 \times 10^{+4} \times 10^{+4} \times 10^{-4} \times 10^{
|17| \text{ xhat2} = \text{inv}(A2'*A2)*(A2'*y)
18 xhat3 = inv(A3'*A3)*(A3'*y)
 19
20 % what is the sum of squared residuals for each of the models:
|21| eTe1 = (y - A1*xhat1)'*(y - A1*xhat1)
22 eTe2 = (y - A2*xhat2)'*(y - A2*xhat2)
23 eTe3 = (y - A3*xhat3)'*(y - A3*xhat3)
 24
25 figure
26 plot(t,y,'xb')
27 hold on
28 plot(t,A1*xhat1,'r')
29 plot(t,A2*xhat2,'c')
30 plot(t,A3*xhat3,'g')
31 set(gca,'xlim',[0 5.1])
32 xlabel('time [months]')
33 ylabel('height [meter]')
34 legend('observations','model 1','model 2','model 3')
```



0.000 **✓** Answer: 0.000 **0.000** 

## **Explanation**

The sum of squared residuals will be equal to 0, since this is a 'determined' system of equations, for which we have  $\hat{x}=A^{-1}y$  and  $\hat{y}=A\hat{x}=AA^{-1}y=y$ , resulting in  $\hat{e}=y-\hat{y}=0$ .

For an arbitrary set of observations  $m{y}$ : will the solution change if we apply weighted least squares with  $m{W}=m{4I_6}$  compared to ordinary least squares?

## **Explanation**

 $\hat{x} = (A^T(4I_6)A)^{-1}A^T(4I_6)y = \frac{1}{4}(A^TA)^{-1}A^T4y = (A^TA)^{-1}A^Ty$ . Hence, the solution is not affected.

Submit

You have used 1 of 3 attempts

✓ Correct (4/4 points)

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