2.4.3 It Goes Both Ways

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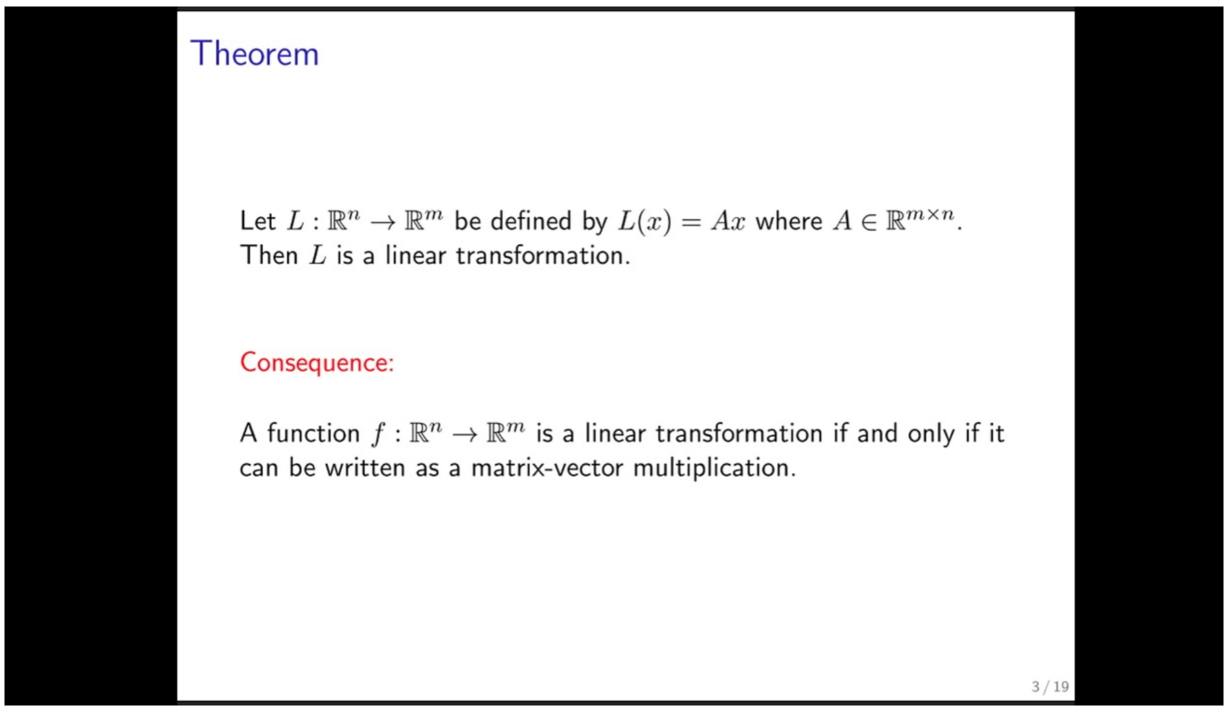
2.4.3 It Goes Both Ways



1:12 / 5:04 2.0x

or is not a linear transformation.

So here is what I just said as a theorem.

If we have a vector function from Rn to Rm defined by l of x is equal to A

times x, then l is a linear transformation.

We're going to leave the proof of this as an exercise.

But what this shows is that l is a linear transformation if and only if

it can be represented by a matrix A and l of x is equal to A times x.

Now let's see how we can exploit that.

Last week, we asked the question, is the function that transforms the

vector chi 0, chi 1 into the vector chi 0 plus chi 1 for the first

component, and chi 0 for the second component a linear transformation.

Here's an alternative proof.

What we're going to do is we're going to

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Reading Assignment

0 points possible (ungraded)

Read Unit 2.4.3 of the notes. [LINK]

- [x] Done

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Proof
I have written a detailed proof for this part but i dont know how to upload it in the discussion forum in case someone is interested to see it. You t... 2

Calculator

Homework 2.4.3.1

1/1 point (graded)
Which is the linear transformation that corresponds to the matrix

$$\begin{pmatrix} 2 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 \end{pmatrix}$$

- $f(\begin{pmatrix} \chi_0 \\ \chi_1 \\ \chi_2 \\ \chi_3 \end{pmatrix}) = \begin{pmatrix} \chi_0 + \chi_2 \\ \chi_1 + \chi_3 \end{pmatrix}$
- $f(\begin{pmatrix} \chi_0 \\ \chi_1 \\ \chi_2 \\ \chi_3 \end{pmatrix}) = \begin{pmatrix} 2\chi_0 + \chi_1 \\ \chi_1 \\ \chi_0 \\ \chi_0 + \chi_1 \end{pmatrix}$
- $f(\begin{pmatrix} \chi_0 \\ \chi_1 \\ \chi_2 \\ \chi_3 \end{pmatrix}) = \begin{pmatrix} 2\chi_0 \\ \chi_1 \end{pmatrix}$
- (selected) $f(\begin{pmatrix} \chi_0 \\ \chi_1 \\ \chi_2 \\ \chi_3 \end{pmatrix}) = \begin{pmatrix} 2\chi_0 + \chi_1 - \chi_3 \\ \chi_2 - \chi_3 \end{pmatrix}$
- none of the above

✓

Explanation

$$\textbf{Answer:} \quad f(\begin{pmatrix} \chi_0 \\ \chi_1 \\ \chi_2 \\ \chi_3 \end{pmatrix}) = \begin{pmatrix} 2 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 \end{pmatrix} \begin{pmatrix} \chi_0 \\ \chi_1 \\ \chi_2 \\ \chi_3 \end{pmatrix} = \begin{pmatrix} 2\chi_0 + \chi_1 - \chi_3 \\ \chi_2 - \chi_3 \end{pmatrix}.$$

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Answers are displayed within the problem

Homework 2.4.3.2

1/1 point (graded)
Which is the linear transformation that corresponds to the matrix

$$\begin{pmatrix} 2 & 1 \\ 0 & 1 \\ 1 & 0 \\ 1 & 1 \end{pmatrix}$$

- $f(\begin{pmatrix} \chi_0 \\ \cdot \end{pmatrix}) = \begin{pmatrix} \chi_0 + \chi_2 \\ \cdot \end{pmatrix}$

Calculator

$\chi_1 \quad \chi_1 + \chi_3$

- [x] $f\left(\begin{pmatrix} \chi_0 \\ \chi_1 \end{pmatrix}\right) = \begin{pmatrix} 2\chi_0 + \chi_1 \\ \chi_1 \\ \chi_0 \\ \chi_0 + \chi_1 \end{pmatrix}$
- [ ] $f\left(\begin{pmatrix} \chi_0 \\ \chi_1 \end{pmatrix}\right) = \begin{pmatrix} 2\chi_0 \\ \chi_1 \end{pmatrix}$
- [ ] $f\left(\begin{pmatrix} \chi_0 \\ \chi_1 \end{pmatrix}\right) = \begin{pmatrix} 2\chi_0 + \chi_1 - \chi_3 \\ \chi_2 - \chi_3 \end{pmatrix}$
- [ ] none of the above

✔

Explanation

Answer: $f\left(\begin{pmatrix} \chi_0 \\ \chi_1 \end{pmatrix}\right) = \begin{pmatrix} 2 & 1 \\ 0 & 1 \\ 1 & 0 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} \chi_0 \\ \chi_1 \end{pmatrix} = \begin{pmatrix} 2\chi_0 + \chi_1 \\ \chi_1 \\ \chi_0 \\ \chi_0 + \chi_1 \end{pmatrix}.$

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Homework 2.4.3.3

1/1 point (graded)

Let f be a vector function such that $f\left(\begin{pmatrix} \chi_0 \\ \chi_1 \end{pmatrix}\right) = \begin{pmatrix} \chi_0^2 \\ \chi_1 \end{pmatrix}$ then

- [ ] (a) f is a linear transformation.
- [x] (b) f is not a linear transformation.
- [ ] (c) Not enough information is given to determine whether or not f is a linear transformation.

✔

Explanation

Answer: (b): To compute a possible matrix that represents f consider:

$$f\left(\begin{pmatrix} 1 \\ 0 \end{pmatrix}\right) = \begin{pmatrix} 1^2 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \text{and} \quad f\left(\begin{pmatrix} 0 \\ 1 \end{pmatrix}\right) = \begin{pmatrix} 0^2 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}.$$

Thus, *if* f is a linear transformation, then $f(x) = Ax$ where $A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$. Now,

Calculator

$$Ax = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \chi_0 \\ \chi_1 \end{pmatrix} = \begin{pmatrix} \chi_0 \\ \chi_1 \end{pmatrix} \neq \begin{pmatrix} \chi_0 \\ \chi_1 \end{pmatrix} = f\left(\begin{pmatrix} \chi_0 \\ \chi_1 \end{pmatrix}\right) = f(x).$$

Hence f is *not* a linear transformation since $f(x) \neq Ax$.

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Answers are displayed within the problem

Homework 2.4.3.4

2/2 points (graded)

Let f be a vector function such that $f\left(\begin{pmatrix} \chi_0 \\ \chi_1 \\ \chi_2 \end{pmatrix}\right) = \begin{pmatrix} \chi_0 \\ 0 \\ \chi_2 \end{pmatrix}$ then

- (selected, correct) f is a linear transformation.
- f is not a linear transformation.
- Not enough information is given to determine whether or not f is a linear transformation.

Let f be a vector function such that $f\left(\begin{pmatrix} \chi_0 \\ \chi_1 \end{pmatrix}\right) = \begin{pmatrix} \chi_0^2 \\ 0 \end{pmatrix}$ then

- f is a linear transformation.
- (selected, correct) f is not a linear transformation.
- Not enough information is given to determine whether or not f is a linear transformation.

Explanation

- $f\left(\begin{pmatrix} \chi_0 \\ \chi_1 \\ \chi_2 \end{pmatrix}\right) = \begin{pmatrix} \chi_0 \\ 0 \\ \chi_2 \end{pmatrix}$.

  Answer: True To compute a possible matrix that represents f consider:

$$f\left(\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}\right) = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad f\left(\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}\right) = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \quad \text{and} \quad f\left(\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}\right) = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}.$$

  Thus, *if* f is a linear transformation, then $f(x) = Ax$ where $A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$. Now,

$$Ax = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ \end{pmatrix} \begin{pmatrix} \chi_0 \\ \chi_1 \\ \end{pmatrix} = \begin{pmatrix} \chi_0 \\ 0 \\ \end{pmatrix} = f\left(\begin{pmatrix} \chi_0 \\ \chi_1 \\ \end{pmatrix}\right) = f(x).$$

Calculator

Hence f is a linear transformation since $f(x) = Ax$.

- $f(\begin{pmatrix} \chi_0 \\ \chi_1 \end{pmatrix}) = \begin{pmatrix} \chi_0^2 \\ 0 \end{pmatrix}$.

  Answer: False To compute a possible matrix that represents f consider:

$$f(\begin{pmatrix} 1 \\ 0 \end{pmatrix}) = \begin{pmatrix} 1^2 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \text{and} \quad f(\begin{pmatrix} 0 \\ 1 \end{pmatrix}) = \begin{pmatrix} 0^2 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}.$$

  Thus, *if* f is a linear transformation, then $f(x) = Ax$ where $A = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$. Now,

$$Ax = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \chi_0 \\ \chi_1 \end{pmatrix} = \begin{pmatrix} \chi_0 \\ 0 \end{pmatrix} \neq \begin{pmatrix} \chi_0^2 \\ 0 \end{pmatrix} = f\left(\begin{pmatrix} \chi_0 \\ \chi_1 \end{pmatrix}\right) = f(x).$$

  Hence f is *not* a linear transformation since $f(x) \neq Ax$.

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