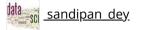




MITx: 18.6501x **Fundamentals of Statistics** 

<u>Help</u>



<u>Course</u> > <u>Final exam</u> > <u>Final Exam</u> > 3.

# 3.

## Setup:

Let X be a random variable with pdf given by

$$f_{\mu}\left(x
ight)=\left\{egin{array}{ll} 0, & x\leq0 \ & \ \dfrac{1}{\sqrt{2\pi x^{3}}}\mathrm{exp}\left(-\dfrac{\left(x-\mu
ight)^{2}}{2\mu^{2}x}
ight), & x>0\,. \end{array}
ight.$$

### Canonical Form

2/2 points (graded)

Show that  $\{f_{\mu}, \mu > 0\}$  belongs to the canonical exponential family of distributions by writing it in canonical form. Identify the canonical parameter  $\theta$ , the function  $b\left(\theta\right)$ , and take  $\phi=1$  (no need to identify the function c).

$$\theta = \begin{bmatrix} -1/(2*mu^2) & \checkmark \text{ Answer: -1/(2*mu^2)} \\ -\frac{1}{2\cdot\mu^2} & \end{cases}$$

Submit

You have used 1 of 3 attempts

• Answers are displayed within the problem

## Canonical Link

1/1 point (graded)

What is the canonical link  $g(\mu)$ ?

$$g\left(\mu
ight)=$$
 
$$-\frac{1}{2\cdot\mu^2}$$
 Answer: -1/(2\*mu^2)

STANDARD NOTATION

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You have used 1 of 3 attempts

**1** Answers are displayed within the problem

# **Expectation and Variance**

2.0/2 points (graded)

Compute the expected value  $\mathbb{E}[X]$  and the variance  $\mathsf{Var}(X)$  of X.

(You may enter in terms of  $\mu$  or the canonical parameter  $\theta$ .)

**STANDARD NOTATION** 

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You have used 1 of 3 attempts

• Answers are displayed within the problem

## Fisher Information

0/1 point (graded)

Compute the Fisher information  $I(\theta)$ .

(You may enter in terms of heta or  $\mu$ .)

 $I(\theta) = 1/\text{mu}^3$  **X Answer:** (-2\*theta)^(-3/2)

STANDARD NOTATION

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You have used 1 of 3 attempts

**1** Answers are displayed within the problem

### MLE

2.0/2 points (graded)

Let  $X_1, \ldots, X_n$  be n i.i.d. copies of X.

Compute the maximum likelihood estimator  $\hat{\mu}$  of  $\mu$  and the maximum likelihood estimator  $\hat{\theta}$  of  $\theta$ .

(Enter  $\operatorname{\textbf{barX\_n}}$  for  $\overline{X_n}$ . If applicable, enter  $\operatorname{\textbf{bar(X\_n^2)}}$  for  $\overline{X_n^2}$ . Note "barX\_n^2" represents  $(\overline{X_n})^2$ , NOT  $\overline{X_n^2}$ .)

$\hat{ heta} =$	-1/(2*(barX_n)^2)	<b>✓ Answer:</b> -1/(2*barX_n^2)

STANDARD NOTATION

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You have used 1 of 3 attempts

• Answers are displayed within the problem

# Asymptotic Distribution and Mean

2.0/2.0 points (graded)

In the next two problems, you will specify the asymptotic distribution of  $\sqrt{n}\,(\hat{ heta}- heta)$  and of  $\sqrt{n}\,(\hat{\mu}-\mu)$  .

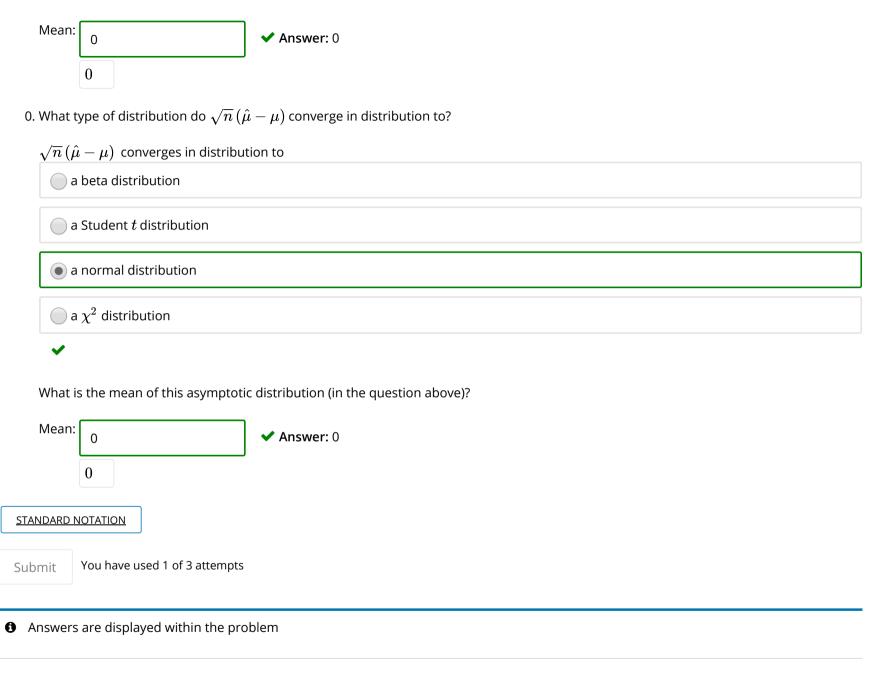
0. What type of distribution does  $\sqrt{n}\left(\hat{ heta}- heta
ight)$  converge in distribution to?

$$\sqrt{n} \left( \hat{ heta} - heta 
ight)$$
 converges in distribution to

- a beta distribution
- igcup a Student t distribution
- a normal distribution
- $\bigcirc$  a  $\chi^2$  distribution

~

What is the mean of this asymptotic distribution (in the question above)?



# Asymptotic Variance

2.0/4.0 points (graded)
Continuing from the problem above,

0. Find the asymptotic variance  $V(\hat{\theta})$  of  $\hat{\theta}$  , i.e. the variance of the asymptotic distribution of  $\sqrt{n}(\hat{\theta}-\theta)$ .

0. Find the asymptotic variance  $V(\hat{\mu})$  of  $\hat{\theta}$ , i.e. the variance of the asymptotic distribution of  $\sqrt{n}(\hat{\mu}-\mu)$ .

$$V\left(\hat{\mu}
ight)=$$
  $\mu^3$  Answer: mu^3

STANDARD NOTATION

Submit

You have used 1 of 3 attempts

**1** Answers are displayed within the problem

#### One-sided Test

2.0/2.0 points (graded)

Construct a test for  $H_0: \mu=1$ , vs  $H_1: \mu<1$ . Give the formula of the (asymptotic) p-value of this test in terms of  $\hat{\mu}$ .

(Different reasonable answers will be accepted.)

(Enter **hatmu** for  $\hat{\mu}$ .

To avoid double jeopardy, you may enter **V** for the asymptotic variance of  $\hat{\mu}$  evaluated at  $\mu=\hat{\mu}$ .

If applicable, enter **Phi(z)** for the cdf  $\Phi$  (z) =  $\mathbf{P}$  ( $Z \le z$ ) of the standard normal variable, e.g. enter **Phi(0.1)** for  $\Phi$  (0.1); enter **q(alpha)** for the  $1-\alpha$  quantile  $q_{\alpha}$  of the standard normal distribution, i.e.  $\mathbf{P}$  ( $Z \le q_{\alpha}$ ) =  $1-\alpha$ ). For, example enter **q(0.01)** for  $q_{0.01}$ )

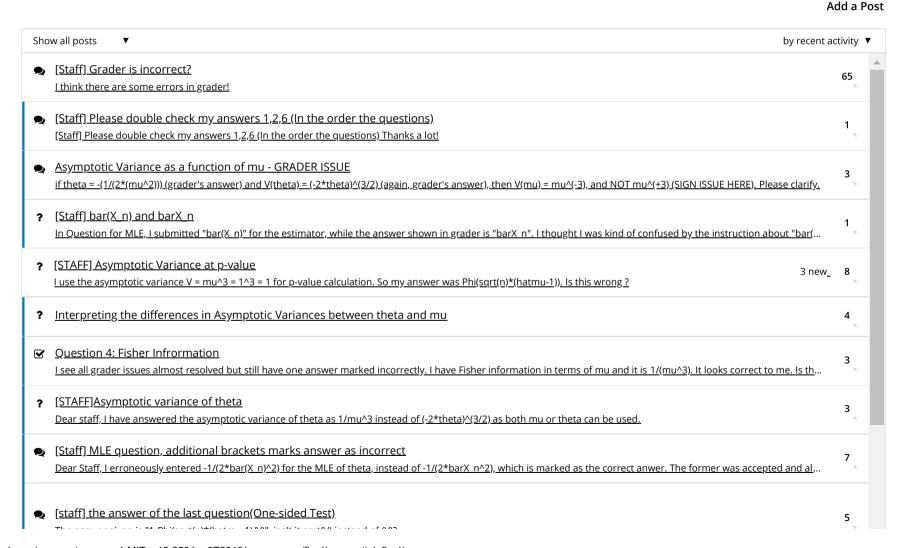
p-value= Phi((hatmu-1)/sqrt(V))  $\checkmark$  Answer: Phi(sqrt(n/V)\*(hatmu-1))

**1** Answers are displayed within the problem

## Error and Bug Reports/Technical Issues

**Topic:** Final exam: Final Exam / 3.

**Hide Discussion** 



Ine answer given is "i-rni(sqrt(η)\*(natmu-i)/v)", isn't it sqrt(v) instead or (v)/ε

[Staff] Something is wrong with the grader

Dear Staff, Please check the following: 1. My answer of variance is Var [X] = μ³ which is equivatent to Var [X] = μ³ = (-2 \* θ)³/2 2. My answer to Fisher's informati...

[Staff] My inputs of Var(X), I(theta) and V(hattheta) are equivalent to the answer provided but marked incorrect.

1 new 5

[Staff] Grader Accents only one form of answer?

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