8. For an $m \times n$ matrix A and an $n \times m$ matrix B,

$$(AB)^T = A^T B^T.$$

Always/Sometimes/Never

Answer:

Sometimes

- When confronted with this kind of question, it pays to first thing to yourself "What if I pick somethings really simple?"
  The simplest thing one can pick is $m = n = 1$ and then you can see what happens if you take $A = \begin{pmatrix} 0 \end{pmatrix}$ and $B = \begin{pmatrix} 0 \end{pmatrix}$.
  You quickly conclude that for this example $(AB)^T = A^T B^T$ (both sides evaluate to $\begin{pmatrix} 0 \end{pmatrix}$.
- Now think a little further: What if $m = n = 1$ and $A = \begin{pmatrix} \alpha \end{pmatrix}$ and $B = \begin{pmatrix} \beta \end{pmatrix}$, where α and β are arbitrary scalars. You notice that $AB = \begin{pmatrix} \alpha \end{pmatrix}\begin{pmatrix} \beta \end{pmatrix} = \begin{pmatrix} \alpha\beta \end{pmatrix}$ and $A^T B^T = \begin{pmatrix} \alpha \end{pmatrix}^T \begin{pmatrix} \beta \end{pmatrix}^T = \begin{pmatrix} \alpha \end{pmatrix}\begin{pmatrix} \beta \end{pmatrix} = \begin{pmatrix} \alpha\beta \end{pmatrix}$. So now you start thinking that perhaps the answer may be "always".
- But then you think to yourself: AB should be a matrix that is $m \times m$ and $A^T B^T$ is a matrix that is $n \times n$ (because A^T is $n \times m$ and B^T is $m \times n$. So, if you pick $m \neq n$ and you are guaranteed to have an example where $AB \neq A^T B^T$.
- But then you think to yourself: What if $m = n$? If you pick almost any matrix where $A \neq B$ you will find that $(AB)^T \neq A^T B^T$.