Bookmark

Exercise: Independence

(5/5 points)

Let X, Y, and Z be discrete random variables.

a) Suppose that Z is identically equal to 3, i.e., $\mathbf{P}(Z=3)=1$. Is X guaranteed to be independent of Z?

Yes ✔ **Answer:** Yes

b) Would either of the following be an appropriate definition of independence of the pair (X,Y) from Z?

- $p_{X,Y,Z}(x,y,z)=p_X(x)\,p_Y(y)\,p_Z(z)$, for all x,y,z

  No ✔ **Answer:** No

- $p_{X,Y,Z}(x,y,z)=p_{X,Y}(x,y)\,p_Z(z)$, for all x,y,z

  Yes ✔ **Answer:** Yes

c) Suppose that X,Y,Z are independent. Is it true that X and Y are independent?

Yes ✔ **Answer:** Yes

d) Suppose that X,Y,Z are independent. Is it true that (X,Y) is independent from Z?

Yes ✔ **Answer:** Yes

Answer:

a) Since Z is deterministic, the value of Z does not provide any information, and so, intuitively, we have independence. For a formal argument, suppose that $z\neq 3$. Then, $p_{X,Z}(x,z)=0=p_X(x)p_Z(z)$. And for $z=3$, $p_{X,Z}(x,3)=\mathbf{P}(X=x,Z=3)=\mathbf{P}(X=x)=\mathbf{P}(X=x)\cdot 1=p_X(x)p_Z(3)$, so that the definition of independence is satisfied.

b) The second definition is correct, because it says that events of the form $\{X=x \text{ and } Y=y\}$ are independent from events of the form $\{Z=z\}$. On the other hand, the first imposes the stronger requirement that X is also independent of Y.

c) Intuitively, since X,Y,Z are independent, none of the random variables provides information about the others. For a formal argument,

$$p_{X,Y}(x,y)=\sum_z p_{X,Y,Z}(x,y,z)=\sum_z p_X(x)p_Y(y)p_Z(z)=p_X(x)p_Y(y)\sum_z p_Z(z)=p_X(x)p$$

d) Intuitively, the value of the pair (X,Y) provides no information about the random variable Z. We will verify that the appropriate definition of independence of (X,Y) from Z from part (b) is satisfied. We first use independence of X, Y, Z, and then the fact, from part (c), that $p_X(x)p_Y(y)=p_{X,Y}(x,y)$, to obtain

$$p_{X,Y,Z}(x,y,z) = p_X(x)p_Y(y)p_Z(z) = p_{X,Y}(x,y)p_Z(z),$$

as desired.

You have used 1 of 1 submissions