

### MITx: 6.041x Introduction to Probability - The Science of Uncertainty

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# Exercise: Discrete unknowns

(5/5 points)

Let  $\Theta_1$  and  $\Theta_2$  be some unobserved Bernoulli random variables and let X be an observation. Conditional on X=x, the posterior joint PMF of  $\Theta_1$  and  $\Theta_2$  is given by

$$p_{\Theta_1,\Theta_2|X}( heta_1, heta_2\mid x) = egin{cases} 0.26, & ext{if $ heta_1=0, heta_2=0$,} \ 0.26, & ext{if $ heta_1=0, heta_2=1$,} \ 0.21, & ext{if $ heta_1=1, heta_2=0$,} \ 0.27, & ext{if $ heta_1=1, heta_2=1$,} \ 0, & ext{otherwise.} \end{cases}$$

We can view this as a hypothesis testing problem where we choose between four alternative hypotheses: the four possible values of  $(\Theta_1, \Theta_2)$ .

a) What is the estimate of  $(\Theta_1, \Theta_2)$  provided by the MAP rule?

(1,1) ▼ **Answer:** (1,1)

b) Once you calculate the estimate  $(\hat{\theta}_1, \hat{\theta}_2)$  of  $(\Theta_1, \Theta_2)$ , you may report the first component,  $\hat{\theta}_1$ , as your estimate of  $\Theta_1$ . With this procedure, your estimate of  $\Theta_1$  will be

1 ▼ **✓** Answer: 1

c) What is the probability that  $\Theta_1$  is estimated incorrectly (the probability of error) when you use the procedure in part (b)?

0.52 **✓ Answer:** 0.52

d) What is the MAP estimate of  $\Theta_1$  based on X, that is, the one that maximizes  $p_{\Theta_1|X}(\theta_1\mid x)$ ?

0 **▼ ✓ Answer:** 0

#### Unit overview

## Lec. 14: Introduction to Bayesian inference

Exercises 14 due Apr 06, 2016 at 23:59 UT

# Lec. 15: Linear models with normal noise

Exercises 15 due Apr 06, 2016 at 23:59 UT

#### Problem Set 7a

Problem Set 7a due Apr 06, 2016 at 23:59 UTC

## Lec. 16: Least mean squares (LMS) estimation

Exercises 16 due Apr 13, 2016 at 23:59 UT

## Lec. 17: Linear least mean squares (LLMS) estimation

Exercises 17 due Apr 13, 2016 at 23:59 UT

#### Problem Set 7b

Problem Set 7b due Apr 13, 2016 at 23:59 UTC

## Solved problems

Additional theoretical material

**Unit summary** 

e) The moral of this example is that an estimate of  $\Theta_1$  obtained by identifying the maximum of the joint PMF of all unknown random variables is

can be different from ▼



**Answer:** can be different from

the MAP estimate of  $\Theta_1$ .

#### Answer:

- a) The posterior is largest when  $(\theta_1, \theta_2) = (1, 1)$ .
- b) The corresponding estimate of  $\Theta_1$  is the first component of (1,1), which is 1.
- c) The probability of error is the posterior probability that  $\Theta_1=0$ , which is 0.26+0.26=0.52.
- d) The posterior PMF of  $\Theta_1$  is the marginal (posterior) PMF obtained from the joint posterior PMF:

$$p_{\Theta_1|X}(0 \mid x) = 0.26 + 0.26 = 0.52,$$

$$p_{\Theta_1|X}(1\mid x) \; = \; 0.21 + 0.27 = 0.48.$$

Hence, the MAP estimate is  $\hat{ heta}_1=0$ .

e) These can be different, as illustrated by parts (b) and (d).

You have used 1 of 1 submissions

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