13. Summary

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Summarize

Big Picture

Many questions and equations involving vectors can be rewritten in terms of “hidden" dot products. These hidden dot products can give us insights into the geometry of what is happening.

The main example in this lecture is the line defined by

$$ax + by = c \tag{3.74}$$

for some constants a, b, and c. This equation can be rewritten in terms of a dot product

$$\langle a, b\rangle \cdot \langle x, y\rangle = c. \tag{3.75}$$

Because we know that the line $ax + by = c$ is parallel to the line $ax + by = 0$, which is defined by the dot product equation

$$\langle a, b\rangle \cdot \langle x, y\rangle = 0. \tag{3.76}$$

We know the vector $\langle a, b\rangle$ is perpendicular to any line of the form $ax + by = c$.

Mechanics

1. If two vectors $\vec{v}$ and $\vec{w}$ are **perpendicular**, then

$$\vec{v} \cdot \vec{w} = 0. \tag{3.77}$$

   Similarly, if $\vec{v} \cdot \vec{w} = 0$, then $\vec{v}$ and $\vec{w}$ are perpendicular.

2. If two vectors $\vec{v}$ and $\vec{w}$ are **parallel**, then there exists some constant $\lambda \neq 0$ such that

$$\vec{v} = \lambda \vec{w}. \tag{3.78}$$

3. Given any vectors $\vec{v}$ and $\vec{a}$, we can **decompose** the vector $\vec{v}$ into a sum of components, one tangent to $\vec{a}$ and one perpendicular to $\vec{a}$. Let $\vec{b}$ be perpendicular to $\vec{a}$, then

$$\vec{v} = \left(\frac{\vec{v} \cdot \vec{a}}{\vec{a} \cdot \vec{a}}\right)\vec{a} + \left(\frac{\vec{v} \cdot \vec{b}}{\vec{b} \cdot \vec{b}}\right)\vec{b}.$$

Ask Yourself

Can the dot product of two vectors be a negative number?

Yes, the dot product $\vec{v} \cdot \vec{w} = |v||w|\cos\theta$, so the dot product is negative whenever $\cos\theta$ is negative. This happens for obtuse angles θ.

Calculator Hide Notes

One may think of the dot product as measuring “agreement" between vectors. When the vectors point in the same general direction, the dot product is positive. If they point in generally opposing directions, the dot product is negative. And when they are perpendicular, the dot product is zero.

Hide

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an extension would be helpful.
Yes,an extension would be helpful. 1

[STAFF] typo in "Ask Yourself"
I think there is a typo in the beginning of "Ask Yourself" : " Yes, the dot prouct " instead of " Yes, the dot product " 6

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