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6. Total Variation Distance for
> Continuous Distributions

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6. Total Variation Distance for Continuous Distributions

Total Variation Distance for Continuous Distributions

and to actually compute this maximum over a , OK?

Assume that E is continuous. This includes Gaussian, Exponential.

Assume that X has a density $\mathbb{P}_a(X \in A) = \int_A f_a(x) dx$ for all

The total variation distance between \mathbb{P}_a and $\mathbb{P}_{a'}$ is a simple function of the densities f_a and $f_{a'}$.

$$TV(\mathbb{P}_a, \mathbb{P}_{a'}) = \frac{1}{2} \int |f_a(x) - f_{a'}(x)| dx$$

Handwritten on chalkboard:

$$A = \{x \in E : p_0(x) \geq p_1(x)\}$$

$$\sum_{x: p_0(x) \geq p_1(x)} |p_0(x) - p_1(x)| = |\mathbb{P}_0(A) - \mathbb{P}_1(A)|$$

If I sum the two equations:

$$\frac{1}{2} \sum_{x \in E} |p_0(x) - p_1(x)| = \frac{1}{2} |\mathbb{P}_0(A) - \mathbb{P}_1(A)| + \frac{1}{2} |\mathbb{P}_1(A) - \mathbb{P}_0(A)|$$

$$A \leadsto \{x \in E : f_0(x) \geq f_1(x)\}$$

▶ 5:01 / 5:01

▶ 1.50x 🔊 🗒 📄 🗑

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Let \mathbf{P} and \mathbf{Q} be probability distributions on a **continuous** sample space E with probability density functions f and g . Then, the total variation distance between \mathbf{P} and \mathbf{Q}

$$\text{TV}(\mathbf{P}, \mathbf{Q}) = \max_{A \subset E} |\mathbf{P}(A) - \mathbf{Q}(A)|,$$

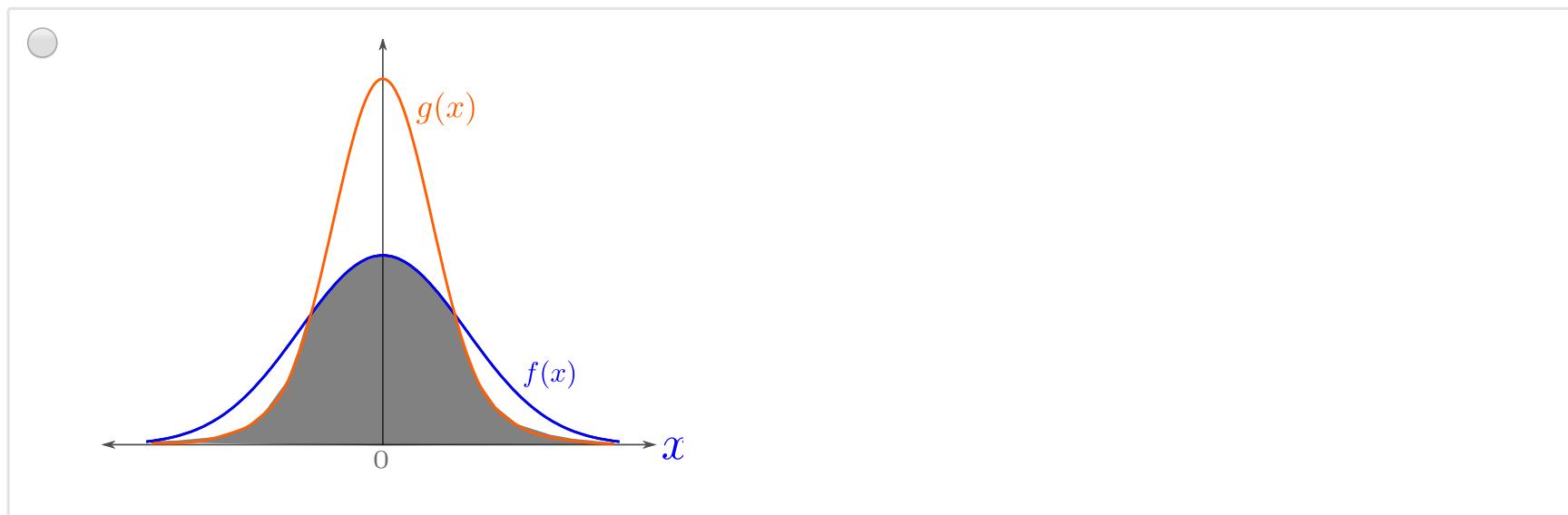
can be computed as

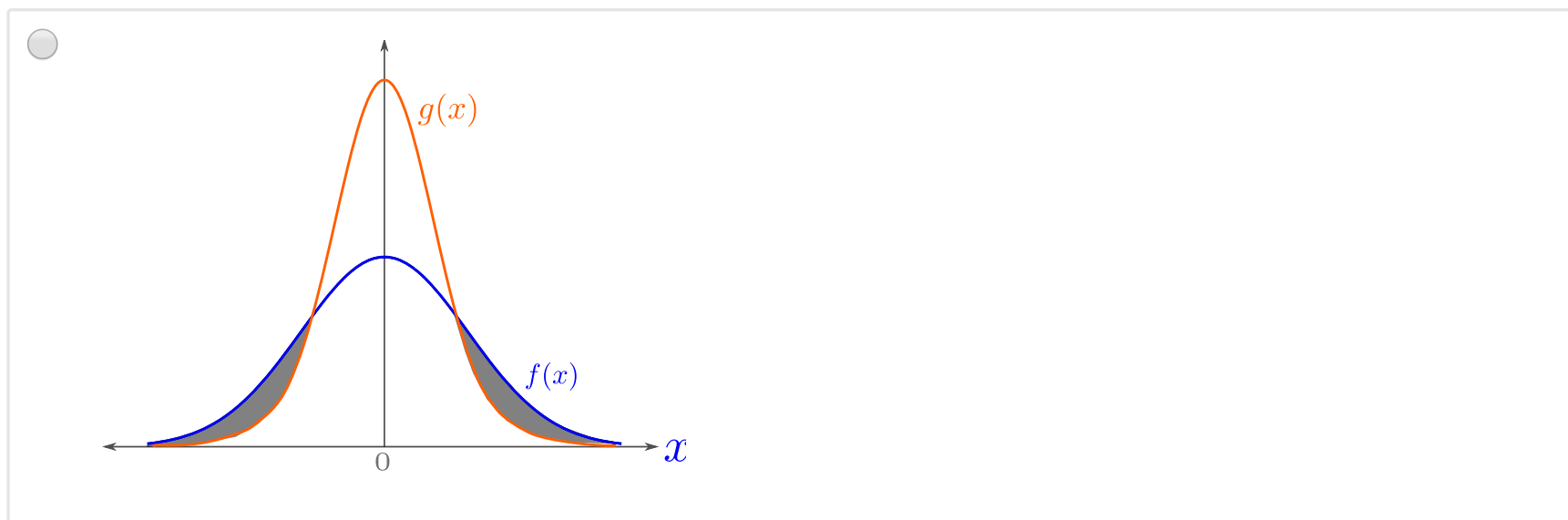
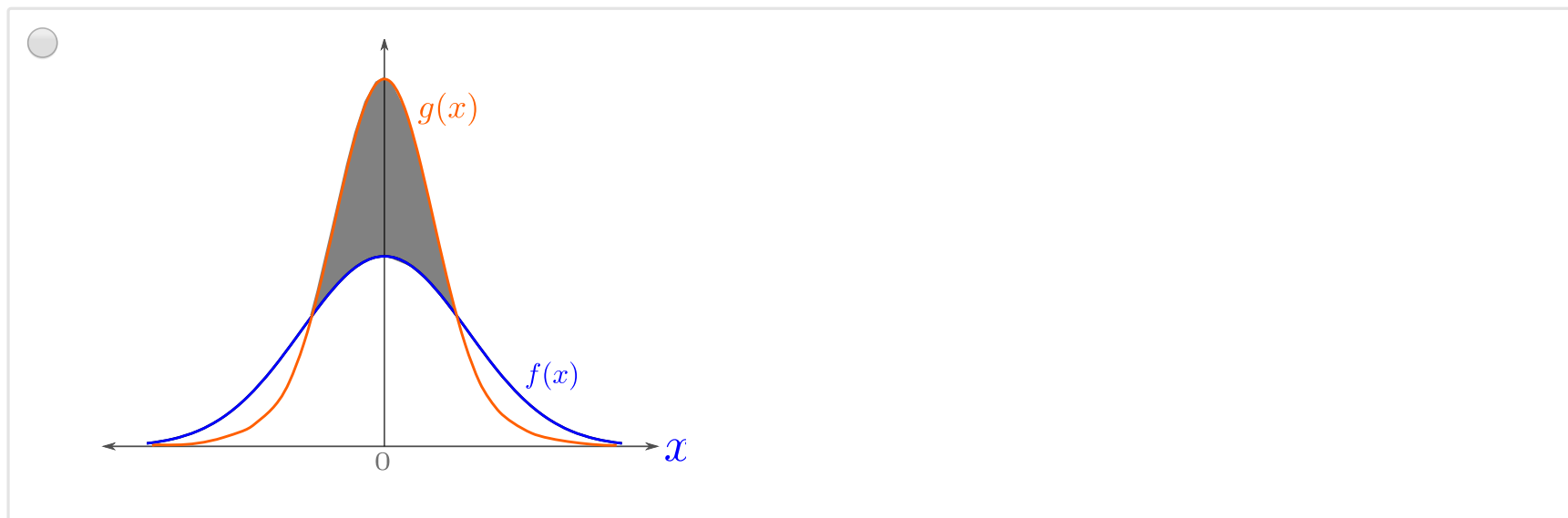
$$\text{TV}(\mathbf{P}, \mathbf{Q}) = \frac{1}{2} \int_{x \in E} |f(x) - g(x)|.$$

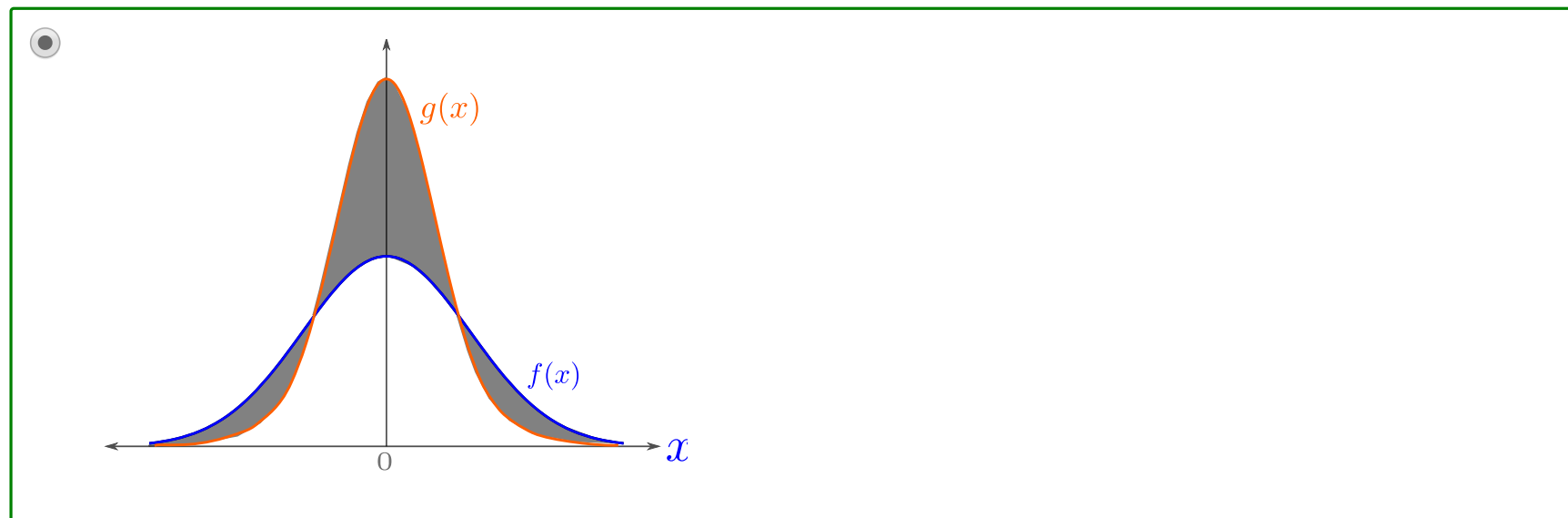
Graphical Interpretation of Total Variation

1/1 point (graded)

Let $X \sim \mathbf{P}$ and $Y \sim \mathbf{Q}$ be Gaussian random variables with mean 0. Let f denote the probability density function of X and g denote the density of Y . Which answer is a correct graphical interpretation of $2\text{TV}(\mathbf{P}, \mathbf{Q})$, 2 times the total variation distance between \mathbf{P} and \mathbf{Q} ?



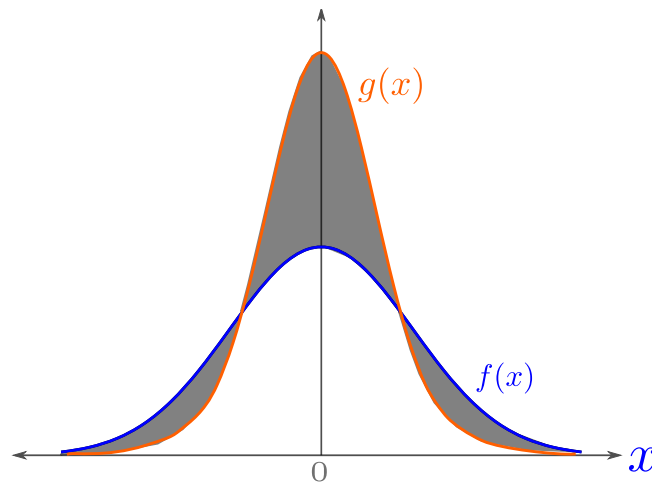


**Solution:**

Recall the formula for total variation when both distributions are continuous:

$$\text{TV}(\mathbf{P}, \mathbf{Q}) = \frac{1}{2} \int_{\mathbb{R}} |f(x) - g(x)| dx$$

The integral on the right hand side is precisely the (unsigned) area **between** the densities f and g :

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You have used 1 of 2 attempts

i Answers are displayed within the problem

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