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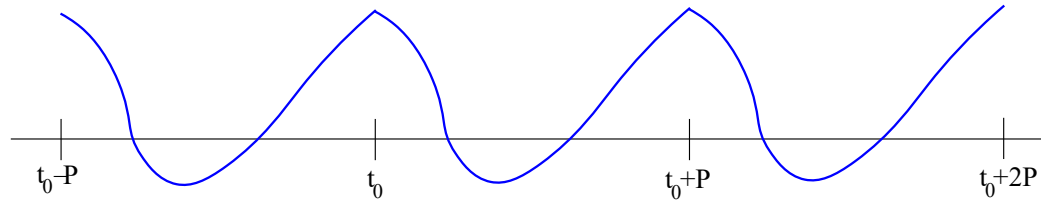
## 4. Periodic functions

Because

$$\sin(t + 2\pi) = \sin t, \quad \cos(t + 2\pi) = \cos t,$$

hold for all  $t$ , the functions  $\sin t$  and  $\cos t$  are called periodic with period  $2\pi$ . In general a function  $f(t)$  defined for all real  $t$  is **periodic of period  $P$**  if  $f(t + P) = f(t)$  for all  $t$ .

There are many such functions beyond the sinusoidal functions. To construct one, divide the real line into intervals of length  $P$ , start with any function defined on one such interval  $[t_0, t_0 + P)$ , and then copy its values in the other intervals. The entire graph consists of horizontally shifted copies of the width  $P$  graph.



**Figure 1:** A periodic function of period  $P$  created by copying and shifting the graph over the interval  $[t_0, t_0 + P)$ .

For now we focus on functions with period  $P = 2\pi$ , which are defined over the interval  $[-\pi, \pi)$ .

**Question 4.1** Is  $\sin 3t$  periodic of period  $2\pi$ ?

**Answer**

The **shortest** period is  $2\pi/3$ , but  $\sin 3t$  is also periodic with period any positive integer multiple of  $2\pi/3$ , including  $3(2\pi/3) = 2\pi$ :

$$\sin(3(t + 2\pi)) = \sin(3t + 6\pi) = \sin 3t.$$

So the answer is yes.

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## Base period

1/1 point (graded)

The **base period** is the smallest common period of a collection of functions. Which of the following collection of functions have a base period equal to  $2\pi$ ?

☒  $\cos(x), \cos(3x), \cos(5x)$

☐  $\cos(2x), \cos(4x), \cos(8x)$



☐  $\cos(x), \cos(x/2), \cos(x/4)$

☐  $\cos(\pi x), \cos(2\pi x), \cos(3\pi x)$



### Solution:

- The smallest period of  $\cos(x)$  is  $2\pi$ . The smallest period of  $\cos(3x)$  is  $2\pi/3$ , but  $2\pi$  is also a period. Similarly, the smallest period of  $\cos(5x)$  is  $2\pi/5$ , but  $2\pi$  is also a period. Therefore  $2\pi$  is the smallest common period.
- The smallest common period of  $\cos(2x), \cos(4x), \cos(8x)$  is  $\pi$ ; note this collection also has period  $2\pi$  since  $2\pi$  is an integer multiple of  $\pi$ , but the base period is  $\pi$ .
- The smallest common period of  $\cos(x), \cos(x/2), \cos(x/4)$  is  $8\pi$ . This collection does not have period  $2\pi$  since, for example,  $\cos(0/4) = 1 \neq \cos(2\pi/4) = 0$ .
- The smallest common period of  $\cos(\pi x), \cos(2\pi x), \cos(3\pi x)$  is  $2$ . No function in this collection has period  $2\pi$ . In particular  $\cos((2\pi)0) = 1 \neq \cos((2\pi)(2\pi)) \approx 0.77$ .

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You have used 1 of 3 attempts

**i** Answers are displayed within the problem

### Graph the function, 1

1/1 point (graded)

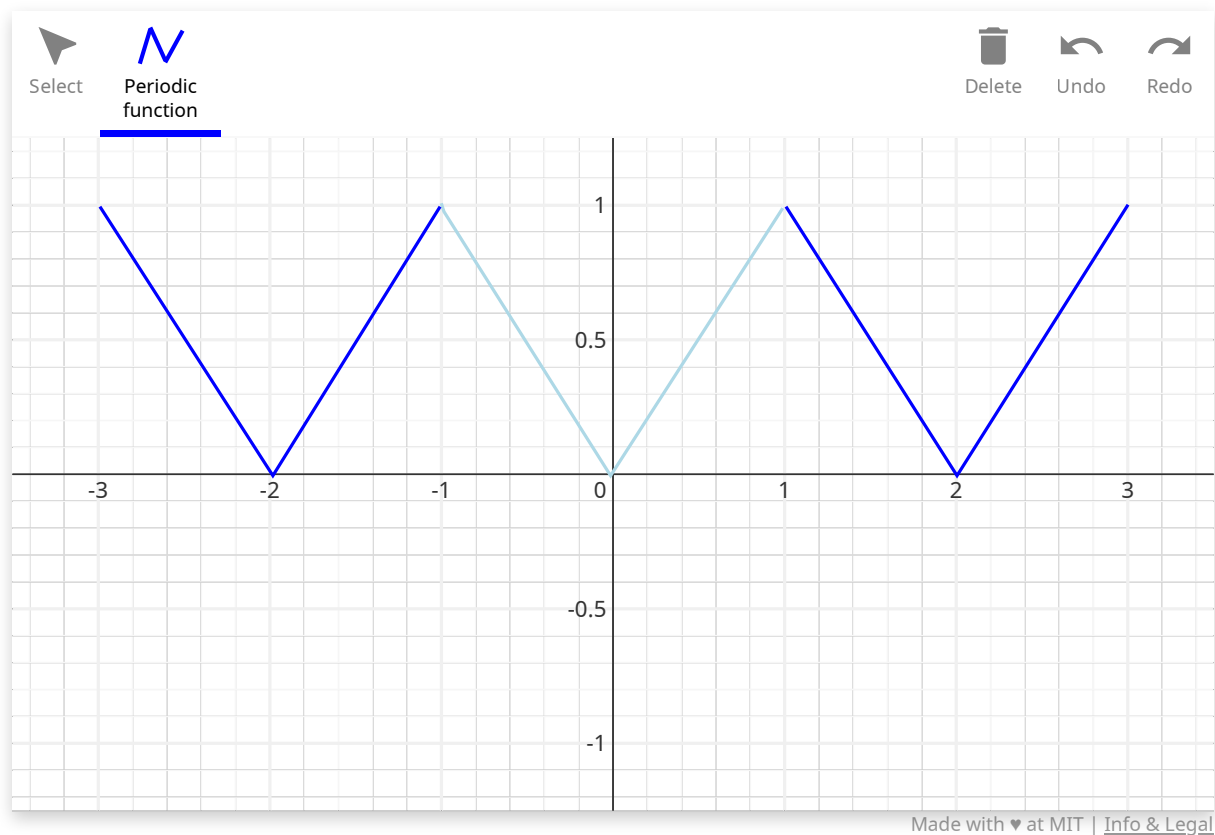
Graph the 2-periodic function on the interval  $[-3, 3]$ , which is defined on the interval  $[-1, 1]$  by

$$f(x) = \begin{cases} x & 0 \leq x \leq 1 \\ -x & -1 \leq x \leq 0 \end{cases}.$$



The function is drawn below in light blue. Your sketch should show the function drawn on the interval from  $[-3, 3]$  and cover the sketch of the function over the interval  $[-1, 1]$ .

*Note that to Delete, you must first use the select tool to select the item you want to delete, and then use the delete button on the SketchResponse tool or on your computer to delete the selected item.*



Answer: .

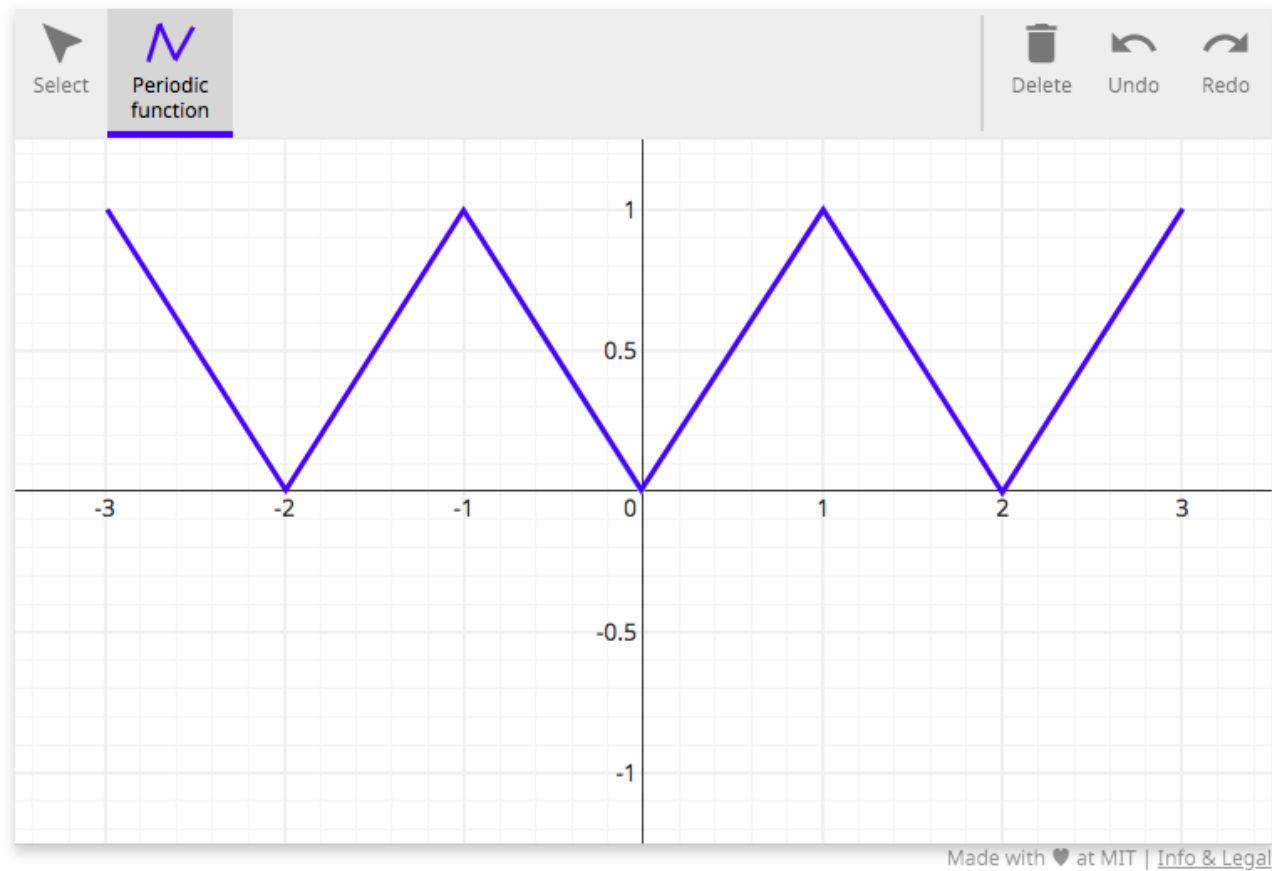


Good job!

**Solution:**

The function is shown below. It is known as a triangle wave.





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**i** Answers are displayed within the problem

Graph the function, 2

1/1 point (graded)

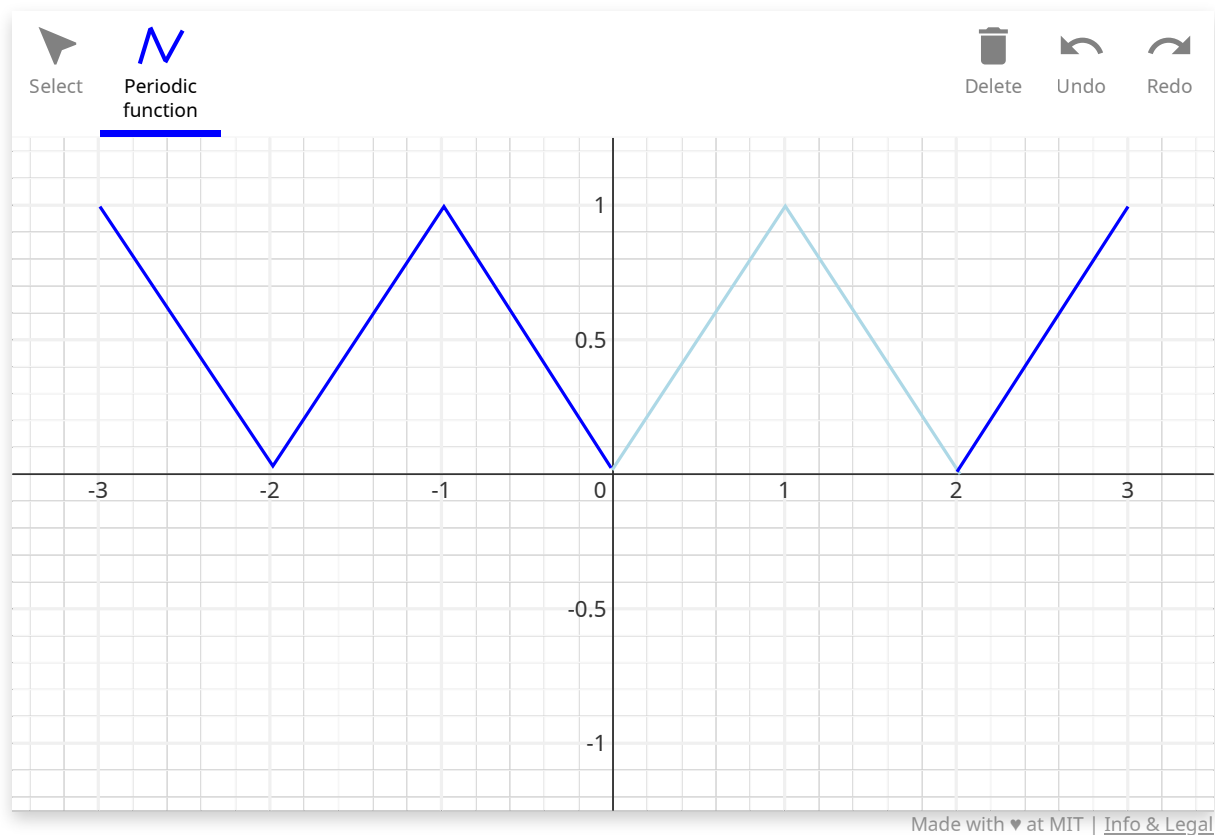
Graph the 2-periodic function on the interval  $[-3, 3]$ , which is defined on the interval  $[0, 2]$  by



$$f(x) = \begin{cases} x & 0 \leq x \leq 1 \\ 2 - x & 1 \leq x \leq 2 \end{cases}.$$

The function is drawn below in light blue. Your sketch should show the function drawn on the interval from  $[-3, 3]$  and cover the sketch of the function over the interval  $[0, 2]$ .

*Note that to Delete, you must first use the select tool to select the item you want to delete, and then use the delete button on the SketchResponse tool or on your computer to delete the selected item.*



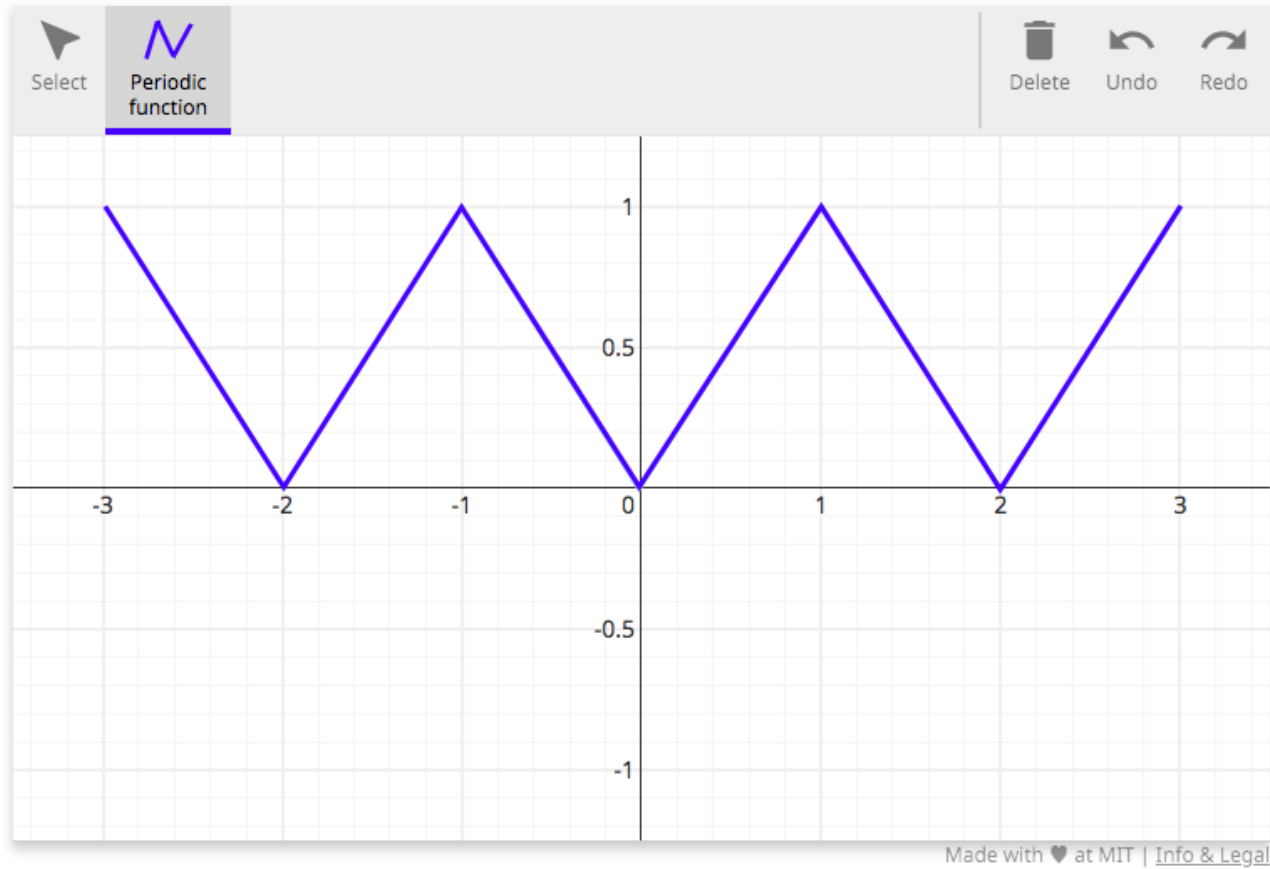
Answer..



Good job!

**Solution:**


Observe that this is the same function as the one in the problem above. Only the defining interval is different.



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 Answers are displayed within the problem

## 4. Periodic functions




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