PART A

(1/1 point)

A *combinatorial auction* is an auction in which participants can place bids on sets of items, instead of placing bids on individual items. A combinatorial auction is useful in many situations. For example, consider the problem of an airline company buying takeoff and landing slots at an airport: clearly, the value of a single slot may be small if the slot is taken by itself, but the value may be much larger if several slots can be bought at the same time, allowing the company to setup flight routes according to the desired timetable. Thus, the airport wants to sell its available slots to airline companies maximizing its own profit (i.e. the total value at which the slots are sold), allowing airlines to bid on sets of items and choosing the most profitable combination of bids among the received ones. Many other examples exist. In this problem, we study a simple formulation for a combinatorial auction.

Consider a set composed by 5 items, labeled for simplicity: $\{1, 2, 3, 4, 5\}$. We auction off these items and receive the following bids, where each bid is placed on a subset of the items and assigns a value to the whole subset:

- Bid 1: subset $\{1, 5\}$ valued at 10.
- Bid 2: subset $\{1, 2, 4\}$ valued at 20.
- Bid 3: subset $\{3\}$ valued at 8.
- Bid 4: subset $\{5\}$ valued at 4.
- Bid 5: subset $\{2, 4\}$ valued at 15.

- Bid 6: subset $\{2, 3, 4, 5\}$ valued at 30.
- Bid 7: subset $\{1, 2, 3\}$ valued at 18.

Let x_j be 1 if the j-th bid is accepted, 0 if it is not. These are all the variables we need.

Formulate an integer program to choose the subset of bids that maximizes profit for the auctioneer, i.e., the total value for which the items are sold is maximum. We remark that each item can be sold at most once, and that bids cannot be split; that is: a bid for items $\{1, 2\}$ can only be accepted if both item 1 and 2 are available. Which integer program below properly models the situation?

$$\left.\begin{aligned} \max \quad & 10x_1 + 20x_2 + 8x_3 + 4x_4 + 15x_5 + 30x_6 + 18x_7 \\ \text{s.t.:} \quad & \\ & x_1 + x_2 + x_7 \leq 1 \\ & x_2 + x_5 + x_6 + x_7 \leq 1 \\ & x_3 + x_6 + x_7 \leq 1 \\ & x_2 + x_5 + x_6 \leq 1 \\ & x_1 + x_4 + x_6 \leq 1 \\ & x_1, x_2, x_3, x_4, x_5, x_6, x_7 \in \{0, 1\} \end{aligned}\right\}$$

$$\left.\begin{array}{rr} \max & 10x_1 + 20x_2 + 8x_3 + 4x_4 + 15x_5 + 30x_6 + 18x_7 \\ \text{s.t.:} & \\ & x_1 + x_2 + x_7 \geq 1 \\ & x_2 + x_5 + x_6 + x_7 \geq 1 \\ & x_3 + x_6 + x_7 \geq 1 \\ & x_2 + x_5 + x_6 \geq 1 \\ & x_1 + x_4 + x_6 \geq 1 \\ & x_1, x_2, x_3, x_4, x_5, x_6, x_7 \in \{0, 1\} \end{array}\right\}$$

$$\left.\begin{array}{rr} \max & 10x_1 + 20x_2 + 8x_3 + 4x_4 + 15x_5 + 30x_6 + 18x_7 \\ \text{s.t.:} & \\ & x_1 + x_5 \leq 1 \\ & x_1 + x_2 + x_4 \leq 1 \\ & x_3 \leq 1 \\ & x_5 \leq 1 \\ & x_2 + x_4 \leq 1 \\ & x_1, x_2, x_3, x_4, x_5, x_6, x_7 \in \{0, 1\} \end{array}\right\}$$

You have used 1 of 2 submissions

PART B

(1/1 point)
Suppose now that we are still auctioning off the set of items $\{1, 2, 3, 4, 5\}$, but now we have two copies each of items 1, 2, 3. In other words, the set of items for auctions looks like this: $\{1, 1, 2, 2, 3, 3, 4, 5\}$. The set of received bids does not change, and each bid can only be accepted at most once. Which IP properly models the situation?

$$\left.\begin{array}{rr} \max & 10x_1 + 20x_2 + 8x_3 + 4x_4 + 15x_5 + 30x_6 + 18x_7 \\ \text{s.t.:} & \\ & x_1 + x_2 + x_7 \geq 2 \\ & x_2 + x_5 + x_6 + x_7 \geq 2 \\ & x_3 + x_6 + x_7 \leq 1 \\ & x_2 + x_5 + x_6 \leq 1 \\ & x_1 + x_4 + x_6 \leq 1 \\ & x_1, x_2, x_3, x_4, x_5, x_6, x_7 \in \{0, 1\} \end{array}\right\}$$

$$\left.\begin{array}{ll} \max & 10x_1 + 20x_2 + 8x_3 + 4x_4 + 15x_5 + 30x_6 + 18x_7 \\ \text{s.t.:} & \\ & x_1 + x_2 + x_7 \leq 2 \\ & x_2 + x_5 + x_6 + x_7 \leq 2 \\ & x_3 + x_6 + x_7 \leq 2 \\ & x_2 + x_5 + x_6 \leq 1 \\ & x_1 + x_4 + x_6 \leq 1 \\ & x_1, x_2, x_3, x_4, x_5, x_6, x_7 \in \{0,1\} \end{array}\right\}$$

$$\left.\begin{array}{ll} \max & 10x_1 + 20x_2 + 8x_3 + 4x_4 + 15x_5 + 30x_6 + 18x_7 \\ \text{s.t.:} & \\ & x_1 + x_2 + x_7 \leq 2 \\ & x_2 + x_3 + x_4 + x_7 \leq 1 \\ & x_3 + x_6 + x_7 \leq 1 \\ & x_2 + x_5 + x_7 \leq 2 \\ & x_1 + x_4 + x_6 \leq 2 \\ & x_1, x_2, x_3, x_4, x_5, x_6, x_7 \in \{0,1\} \end{array}\right\}$$

$$\left.\begin{array}{ll} \max & 10x_1 - 20x_2 - 8x_3 - 4x_4 - 15x_5 - 30x_6 - 18x_7 \\ \text{s.t.:} & \\ & x_1 + x_5 \le 2 \\ & x_1 + x_2 + x_4 \le 2 \\ & x_3 \le 2 \\ & x_5 \le 1 \\ & x_2 + x_4 \le 1 \\ & x_1, x_2, x_3, x_4, x_5, x_6, x_7 \in \{0,1\} \end{array}\right\}$$

You have used 1 of 2 submissions

PART C

(1/1 point)

Write an algebraic formulation for the problem of maximizing profit of the auctioneer, using the following notation: $N = \{1, \ldots, n\}$ is the set of auctioned items, each item is available with multiplicity $\lambda_i \geq 1$ (i.e., there are λ_i copies of item $i, i = 1, \ldots, n)$, and we received b bids, where each bid consists of a subset $S_j \subseteq N$ and a corresponding value $p_j, j = 1, \ldots, b$. Which LP choice properly models the situation?

Recall that ":" means such that

$$\left.\begin{array}{ll}\max & \sum_{j=1}^{b} S_j x_j \\ \text{s.t.:} & \\ & \sum_{j:i\in S_j} x_j \leq \lambda_i, \forall i = 1,\ldots,n \\ & x_j \in \{0,1\}, \forall j = 1,\ldots,b\end{array}\right\}$$

$$\left.\begin{array}{ll}\max & \sum_{j=1}^{b} p_j x_j \\ \text{s.t.:} & \\ & \sum_{j:i\in b} x_j \leq \lambda_i, \forall i = 1,\ldots,n \\ & x_j \in \{0,1\}, \forall j = 1,\ldots,b\end{array}\right\}$$

$$\left.\begin{array}{ll}\max & \sum_{j=1}^{b} p_j x_j \\ \text{s.t.:} & \\ & \sum_{j:i\in S_j} p_j \leq \lambda_i, \forall i = 1,\ldots,n \\ & x_j \in \{0,1\}, \forall j = 1,\ldots,b\end{array}\right\}$$

$$\left.\begin{array}{lr} \max & \sum_{j=1}^{b} p_j x_j \\ \text{s.t.:} & \\ & \sum_{j:i \in S_j} x_j \leq \lambda_i, \forall i = 1, \ldots, n \\ & x_j \in \{0,1\}, \forall j = 1, \ldots, b \end{array}\right\}$$

You have used 1 of 2 submissions

PART D

(1/1 point)

Using the data provided in pset3_p3.xlsx, solve the combinatorial auction problem.

What is the optimal revenue? Error checking hint: the optimal value is between 85 and 95.

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You have used 1 of 3 submissions