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10.4.1 Gaussian elimination on a small system

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A core "building block" of CSE is solving systems of linear algebraic equations, like $Ku = f$. When the system is small, this can be done by hand, e.g., by isolating an unknown in some equation, and substituting the resulting expression in the other equations – repeat until only one unknown remains. The systematic way to perform such operations in a computer-friendly way is called Gaussian elimination, also known as row reduction.

The idea of Gaussian elimination is to perform "row operations" on the system, like replacing an equation by itself plus a number time another equation. Such operations do not change the original system, hence they do not change its solution, and their goal is to progressively remove terms in the system of equations until it becomes easy to solve. These operations are called "row operations" because they can be implemented at the level of the matrix of the system (here K): taking linear combinations of equations corresponds to taking linear combinations of rows of the matrix. Note that the right-hand side f needs to be subjected to the same combinations.

Take the example of the 3-by-3 K matrix of the previous section, with a right-hand side f with $f_0 = f_1 = f_2 = 1$:

$$\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} u_0 \\ u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \tag{10.5}$$

It is convenient to number the rows by means of an index $i = 0, 1, 2$, and columns by means of an index $j = 0, 1, 2$, so

$$K = \begin{bmatrix} K_{00} & K_{01} & K_{02} \\ K_{10} & K_{11} & K_{12} \\ K_{20} & K_{21} & K_{22} \end{bmatrix}. \tag{10.6}$$

We say that the three entries K_{00}, K_{11}, K_{22} are on the diagonal of K.

Elimination proceeds by removing the nonzero entries in the first column ($j = 0$) below the top-left entry at $i = j = 0$, i.e., below the diagonal. There is one such entry, the -1 in row $i = 1$ and column $j = 0$. In

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itself plus $\frac{}{3}$ of the second row. This changes the
third row, and the third entry of the right-hand side,