Exercises: Solvability of a system of equations

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Consistency of the functional model

1/1 point (ungraded)

We observe an unknown distance m times. Construct the functional model (i.e., a system of observation-equations) in order to estimate the unknown distance. Which of the following statements is true? (more than one statement may be true!).

- [ ] A: if $m > 1$ the system cannot be consistent
- [ ] B: if $m > 1$ the system is consistent
- [x] C: if $m > 1$ the system might be inconsistent
- [x] D: if $m = 1$ the system is consistent

Explanation

The rank of the A-matrix is 1.

If $m > 1$ the system may or may not be consistent depending on the actual observations: only if all observations are identical, the system is consistent.

If $m = 1$, the rank is equal to m, and the system is thus consistent.

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✔ Correct (1/1 point)

Consistency

1/1 point (ungraded)

Which of the following systems of equations is always consistent?

- ○ $\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} x$
- ○ $\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix} x$
- ◉ $\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix} x$ ✔
- ○ $\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1.5 & 4.5 \\ 1.1 & 3.3 \end{bmatrix} x$

Answer

Correct: $\mathbf{rank}(A) = 2 = m$, so the consistency is guaranteed.

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✔ Correct (1/1 point)

Rank and Consistency

2/2 points (ungraded)

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 3 & 6 \\ 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

What is $\mathbf{rank}(A)$?

2 ✔

2

Does this mean the system is inconsistent?

- yes

- (selected) no, e.g. $\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 15 \\ 5 \\ 11 \end{bmatrix}$ results in a consistent system ✔
- no, it is always consistent

Answer

Correct: $x_1 = 1$ and $x_2 = 2$ is the solution for this system, so the system is consistent.

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✔ Correct (2/2 points)

A-matrix

1/1 point (ungraded)

An object is travelling with constant speed along a straight line. The distance to the object measured at t_1 = 0 s: $y_1 = 1$ m, and at $t_2 = 1$ s: $y_2 = 2$ m. We would like to know the position on the line at t_1 and the velocity, x_0 and v respectively. What is the correct A-matrix?

- (selected) $A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ ✔

- $A = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$
- $A = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$
- $A = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

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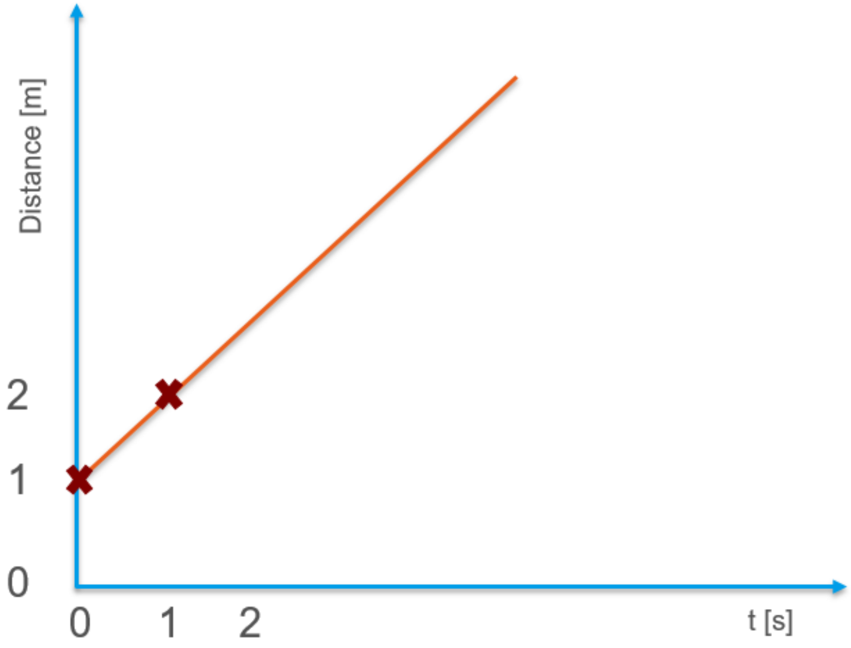
✔ Correct (1/1 point)

Consistency

5/5 points (ungraded)



The figure shows the two distance observations corresponding to the previous problem.

Is this system consistent?

- No
- Yes ✔

Explanation

$\mathrm{rank}(A) = 2 = m$

What is the solution for $x = \begin{bmatrix} x_0 \\ v \end{bmatrix}$

x_0 :

1

✔ **Answer:** 1

1

v :

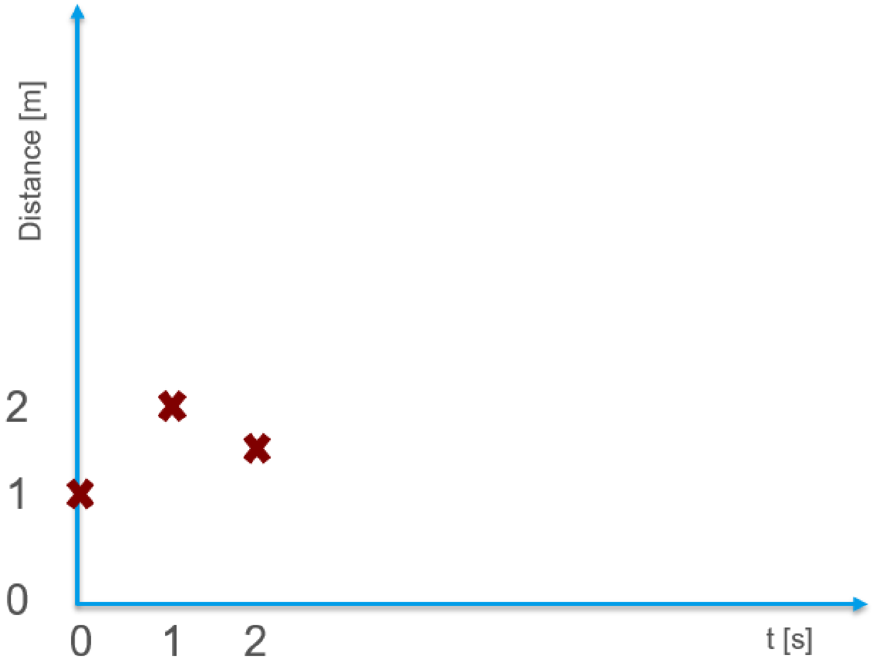
1

✔ **Answer:** 1

1

Explanation

To see this, fill in $x = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ in the system of equations! (This solution can be simply computed as $A^{-1}y$.)



A third measurement is made at t_3: $y_3 = 1.5$m. Is the resulting system consistent?

- No ✔
- Yes

For what value of y_3 would the system be consistent (the other values remain unchanged)?

3 ✔ **Answer:** 3

3

Explanation

For $y_3 = 3$ the 3 observations would be on a line, and the system would be indeed consistent.

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✔ Correct (5/5 points)