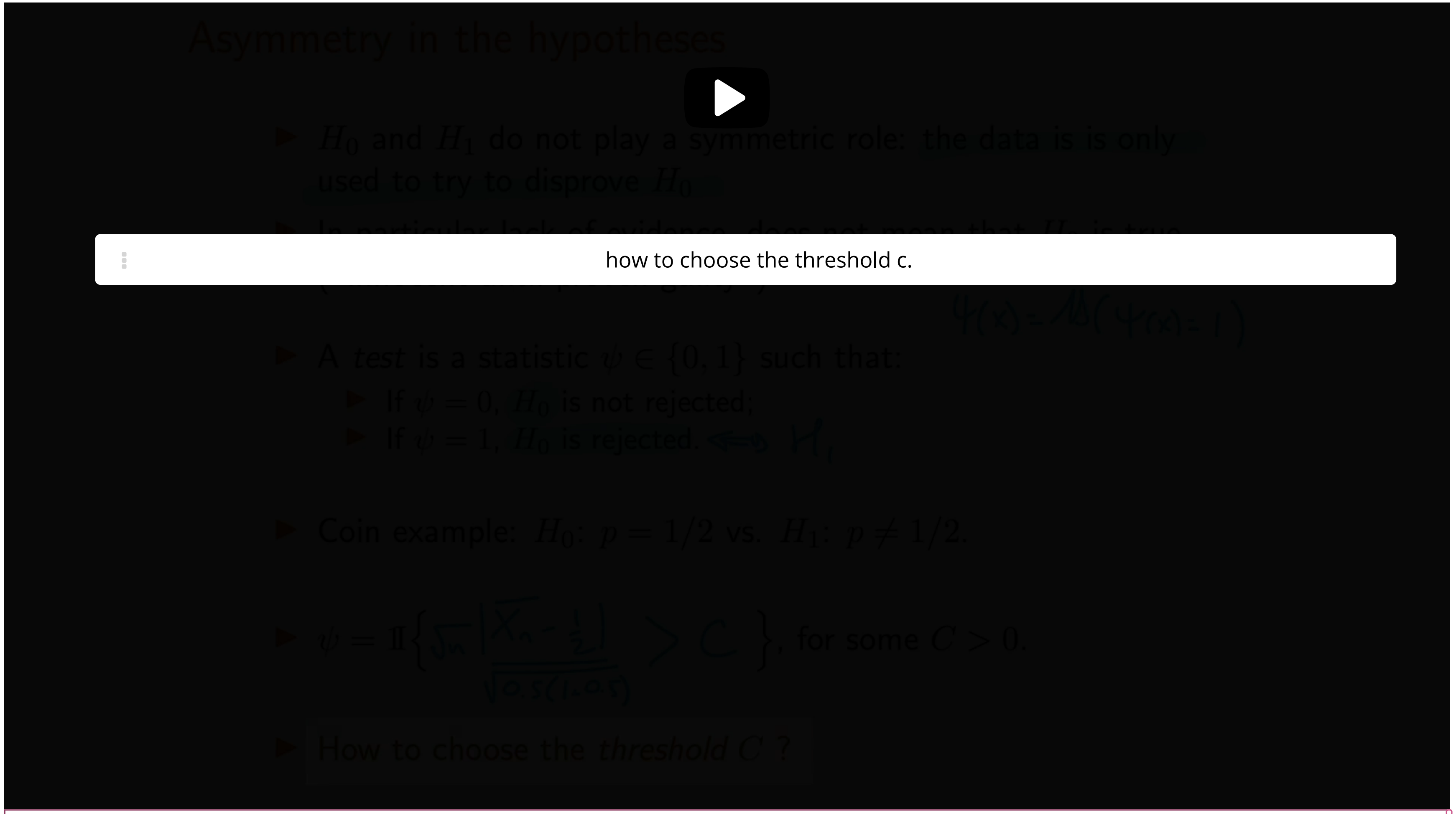
12. Statistical Tests

Statistical Tests



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Which Statistics are Tests?

1/1 point (graded)

Recall that a **statistic** is, intuitively speaking, a function that can be computed from the data.

A **(statistical) test** is an **statistic** whose output is **always** either 0 or 1, and like an estimator, does not depend explicitly on the value of true unknown parameter.

Let $X_1, \ldots, X_n \overset{iid}{\sim} \mathrm{Ber}(\theta)$ for some unknown parameter $\theta \in (0,1)$. Which of the following statistics are also tests?

(Recall that $\mathbf{1}(A)$ is the indicator defined as follows: $\mathbf{1}(A) = \begin{cases} 1 & \text{if } A \text{ occurs} \\ 0 & \text{otherwise} \end{cases}$.

(Choose all that apply.)

- [ ] $\overline{X}_n$
- [x] $\mathbf{1}(\overline{X}_n > 0.5)$
- [x] $\mathbf{1}(|\overline{X}_n - 0.5| > 0.01)$
- [ ] $\mathbf{1}(|\overline{X}_n - \theta| > 0.5)$
- [x] $\mathbf{1}(\overline{X}_n \text{ is a rational number})$

✔

Solution:

We examine the choices in order.

- $\overline{X}_n := \frac{1}{n}\sum_{i=1}^{n} X_i$ is **not** a statistical test, because the sample average is not **always** either 0 or 1.
- $\mathbf{1}(\overline{X}_n > 0.5)$ is a statistical test. Its expression only depends on the sample (and not the true parameter), and since it is an indicator, it takes values only in $\{0,1\}$.

- $\mathbf{1}\left(|\overline{X}_n - 0.5| > 0.01\right)$ is a statistical test. Its expression only depends on the sample (and not the true parameter), and since it is an indicator, it takes values only in $\{0,1\}$.
- $\mathbf{1}\left(|\overline{X}_n - \theta| > 0.5\right)$ is **not** a statistical test because its output depends on the unknown parameter θ.
- $\mathbf{1}\left(\overline{X}_n \text{is a rational number}\right)$ is a statistical test. Its expression only depends on the sample (and not the true parameter), and since it is an indicator, it takes values only in $\{0,1\}$. This is a rather bizarre test, but it does satisfy all required properties.

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Answers are displayed within the problem

Applying a Statistical Test on a Data Set

1/1 point (graded)

Let $X_1, \ldots, X_n \stackrel{iid}{\sim} N(\mu, 1)$ where μ is an unknown parameter. You are interested in answering the **question of interest**: "**Does $\mu = 0$?**". To do so you construct the **null hypothesis** $H_0 : \mu = 0$ and the **alternative hypothesis** $H_1 : \mu \neq 0$.

You design the test

$$\psi = \mathbf{1}\left(\sqrt{n}\left|\overline{X}_n\right| > 0.25\right).$$

If $\psi = 1$, you will **reject** the null hypothesis, and if $\psi = 0$, you will **fail to reject**. For simplicity, we will set the sample size to be $n = 7$.

On which of the following data sets would you reject the null hypothesis?
(Choose all that apply. Feel free to use computational tools.)

- [x] $-1.0, -0.8, -2.9, 1.4, 0.3, -0.8, 1.4$
- [x] $-1.7, -0.1, -0.2, 0.3, 0.3, -0.9, -0.03$

- [ ] $-0.2, 0.6, 1.1, -0.9, 0.1, -1.2, 1.1$

✔

Solution:

We examine the choices in order.

- The first choice is correct. For this data set, we compute $\sqrt{7}\,\overline{X}_7 \approx -0.9072$. Since $|-0.9072| > 0.25$, we reject.
- The second choice is correct. For this data set, we compute $\sqrt{7}\,\overline{X}_7 \approx -0.8768$. Since $|-0.8768| > 0.25$, we reject.
- The third choice is incorrect. For this data set, we compute $\sqrt{7}\,\overline{X}_7 \approx -0.2267$. Since $|0.2267| \leq 0.25$, we fail to reject.

Remark 1: It is useful to keep in mind the following mnemonic,

$$\psi = 0 \Rightarrow H_0$$
$$\psi = 1 \Rightarrow H_1.$$

Of course, the implications above are informal and should not be taken literally. To be precise, we say that if $\psi = 0$, we fail to reject H_0, and if $\psi = 1$, then we reject H_0 in favor of H_1.

Remark 2: If we assume the null hypothesis $H_0 : \mu = 0$, then since the variance is known to be 1, the CLT guarantees that

$$\sqrt{n}\,\overline{X}_n \xrightarrow[n\to\infty]{(d)} \mathcal{N}(0,1)\,.$$

The quantiles of $\mathcal{N}(0,1)$ can be understood using tables or computational software, so if n is very large, then we can approximate the probability of our test ψ **rejecting** or **failing to reject** under the null hypothesis. This concept will be further explored in the next page where we explore the “type 1" and “type 2 error" of a test.

You have used 1 of 2 attempts

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