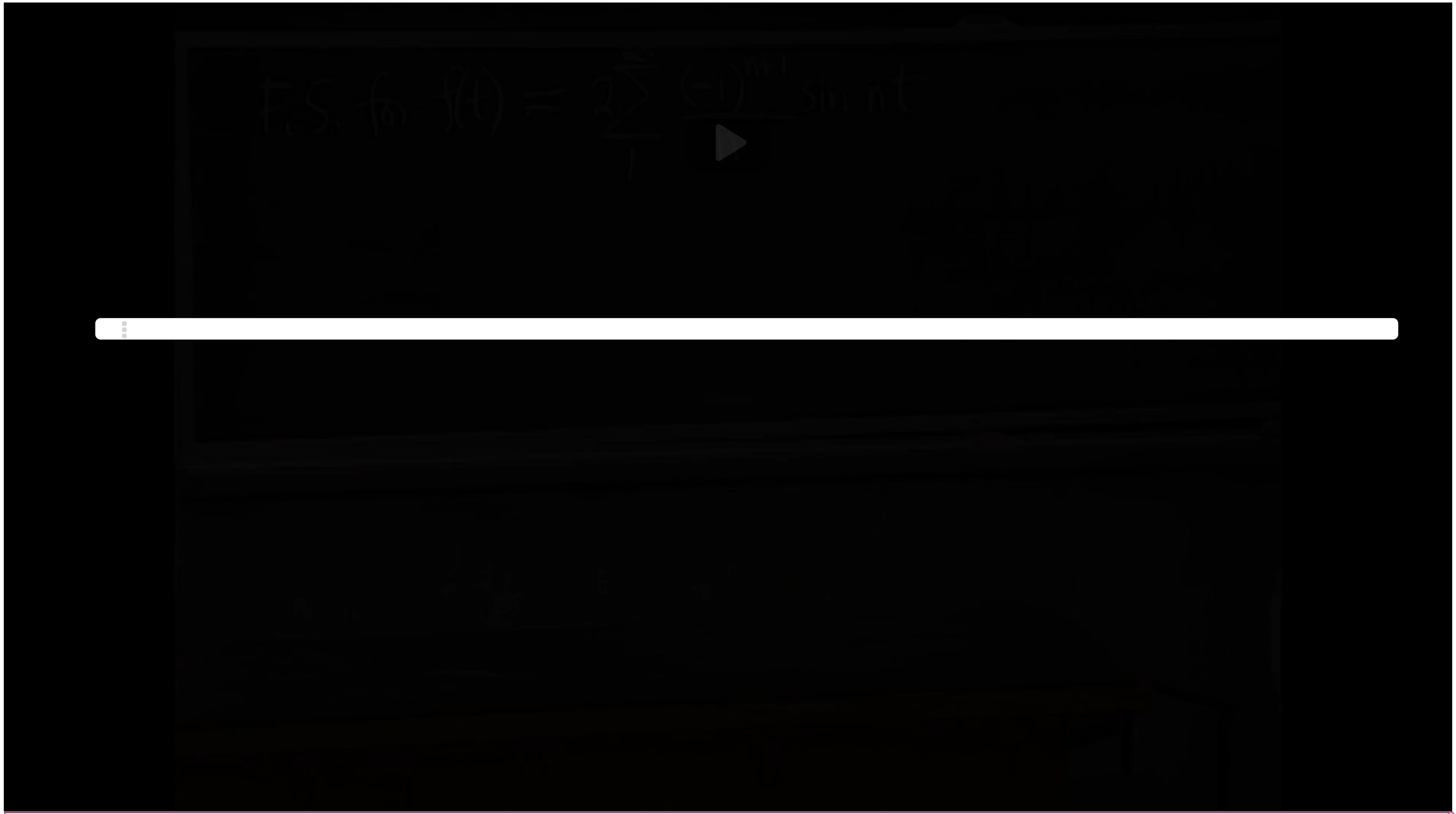
15. Worked example: odd periodic function

An odd, periodic example



7:20 / 7:20 1.50x

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Find the Fourier series of the 2π-periodic **sawtooth** wave

$$f(t) = t, -\pi < t < \pi.$$

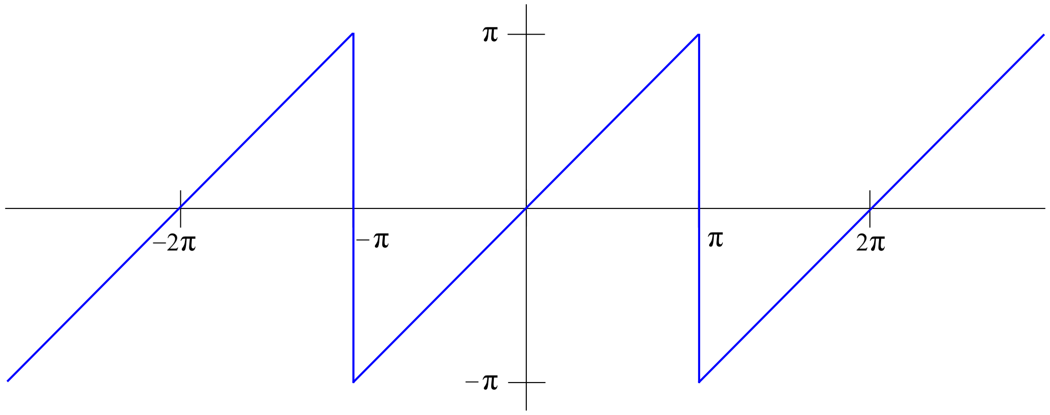


Figure 5: The sawtooth wave.

Solution: The function is odd, therefore $a_n = 0$.

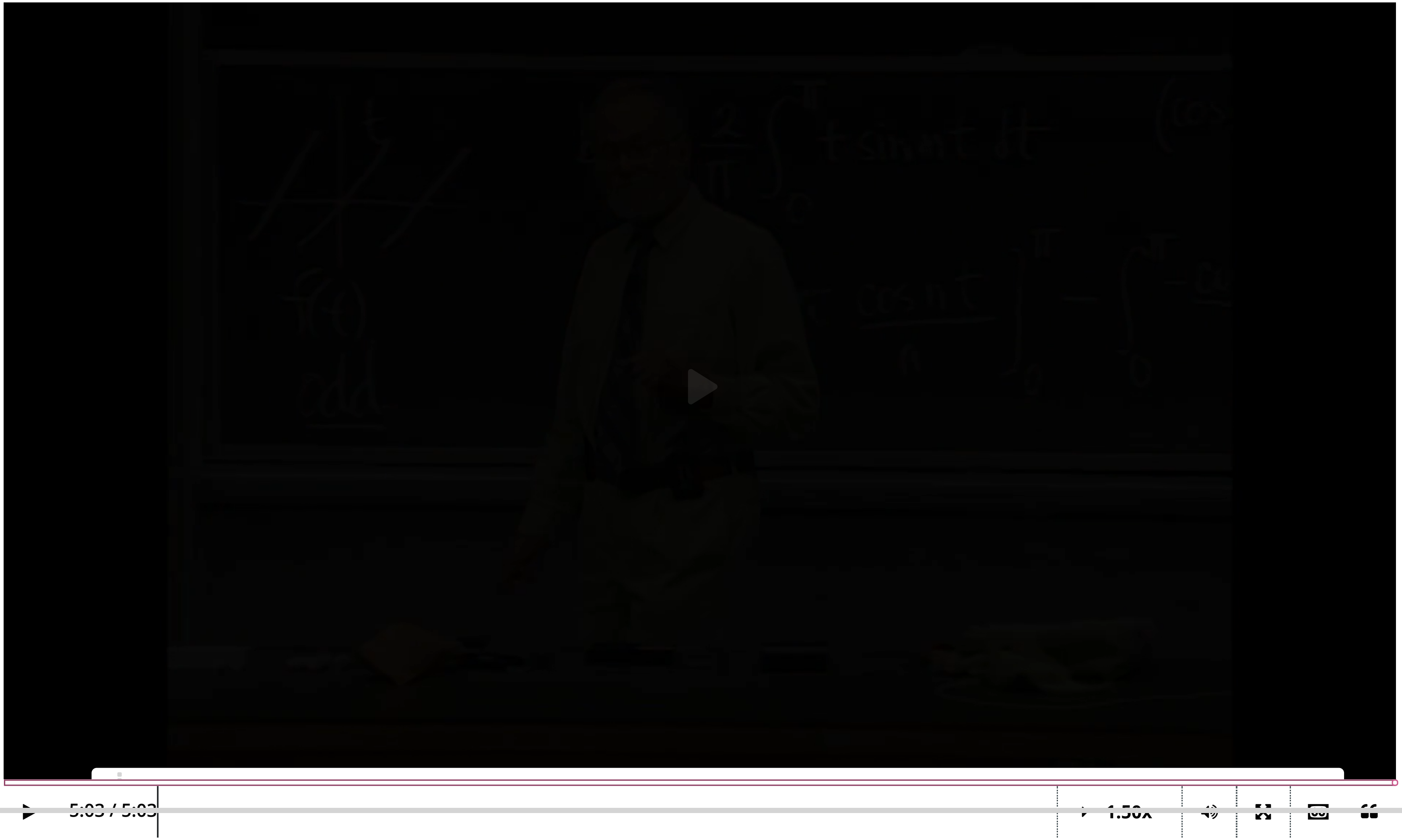
Using our simplified formula for b_n, we find

$$\begin{aligned} b_n &= \frac{2}{\pi}\int_0^\pi t\sin(nt)\ dt \\ &= \frac{2}{\pi}\left[\frac{-t\cos(nt)}{n}\bigg|_0^\pi - \int_0^\pi \frac{-\cos(nt)}{n}\ dt\right] \\ &= \frac{2}{\pi}\left[-\frac{\pi(-1)^n}{n} + \frac{\sin(nt)}{n^2}\bigg|_0^\pi\right] \\ &= \frac{2(-1)^{n+1}}{n}. \end{aligned}$$

Therefore the Fourier series for the sawtooth wave is

$$\sum_{n=1}^{\infty} \frac{2(-1)^{n+1}}{n} \sin(nt).$$

Graphical intuition



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Practice problem

3/3 points (graded)

Find the Fourier series of the 2π-periodic **triangle wave** , which is defined by

$$T(t) := \begin{cases} t, & \text{if } 0 < t < \pi, \\ -t & \text{if } -\pi < t < 0. \end{cases}$$

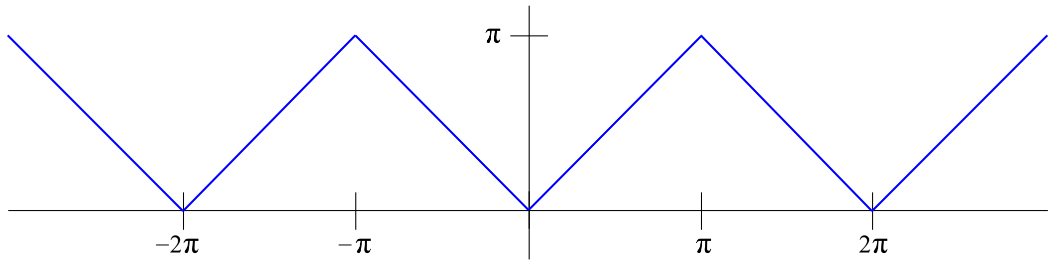

Figure 6: The triangle wave.

Note that this function is even, therefore $b_n = 0$.

$\frac{a_0}{2} =$ pi/2 ✔ **Answer:** pi/2

$\frac{\pi}{2}$

a_n, n odd = -4/(pi*n^2) ✔ **Answer:** -4/(pi*n^2)

$-\frac{4}{\pi \cdot n^2}$

a_n, n even = 0 ✔ **Answer:** 0

0

FORMULA INPUT HELP

Solution:

The triangle wave is an even function, therefore the $b_n = 0$.

We use the fact that $a_0/2$ is the average value of the function on the interval $-\pi < t < \pi$, which is $\pi/2$.

To compute the rest of the terms, we do so directly from the simplified formulas:

$$a_n = \frac{2}{\pi}\int_0^{\pi} t\cos(nt)\,dt = \frac{2}{\pi}\left(\frac{(-1)^n - 1}{n^2}\right) = \begin{cases} \frac{-4}{\pi n^2} & n \text{ odd,} \\ 0 & n \text{ even.} \end{cases}$$

Therefore the Fourier series is given by

$$\frac{\pi}{2} - \frac{4}{\pi}\sum_{n \text{ odd}} \frac{\cos(nt)}{n^2}.$$

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Answers are displayed within the problem

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	a_n odd I think my answer is correct. Could you check it ?	5
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