1.5.2 Exploratory Quiz: Probability Density Functions in Action

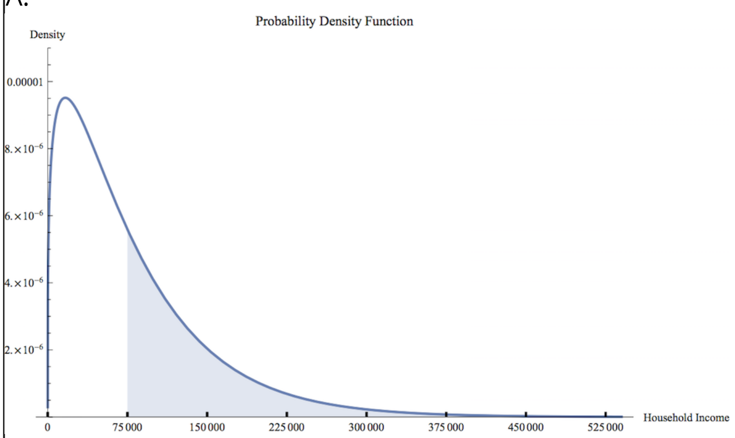
Bookmark this page

Question 1

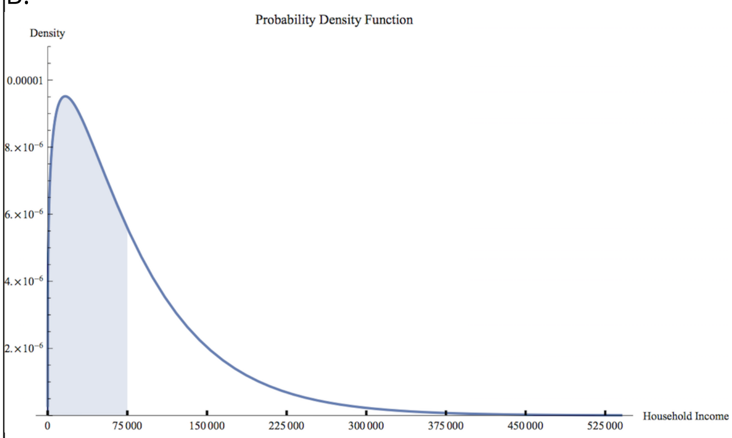
6/6 points (graded)

For each mathematical expression, find the image or images that visually represent the value of the expression. (Every expression has at least one image. One has two images.)

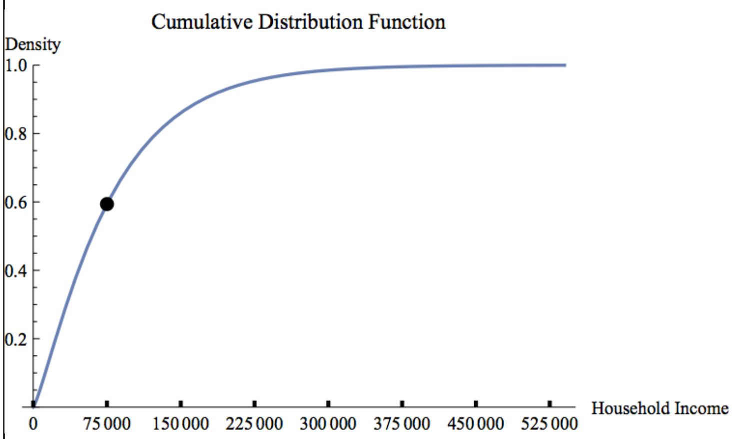
NOTE: The graphs are of probability density functions. The vertical axis represents the rescaled relative frequency, also called *density.*

Graph	Equation
A. 	○ $P(75000 < X < 150000)$ ○ $P(X < 75,000)$ ○ $P(X < 150000)$ ◉ $P(X > 75000)$ ✔ ○ None of the Above

B.

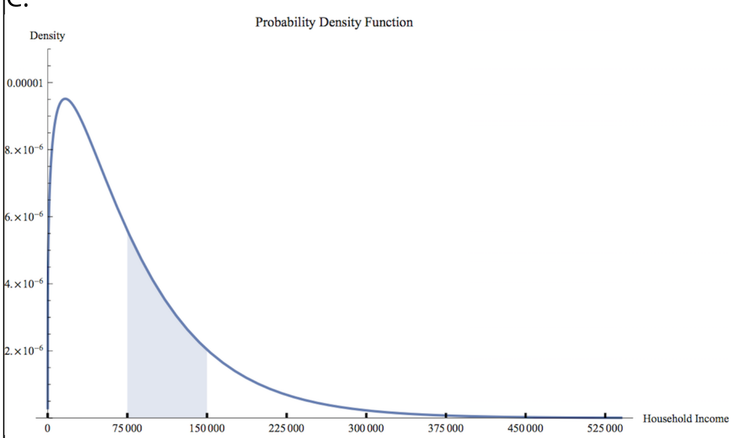


AND

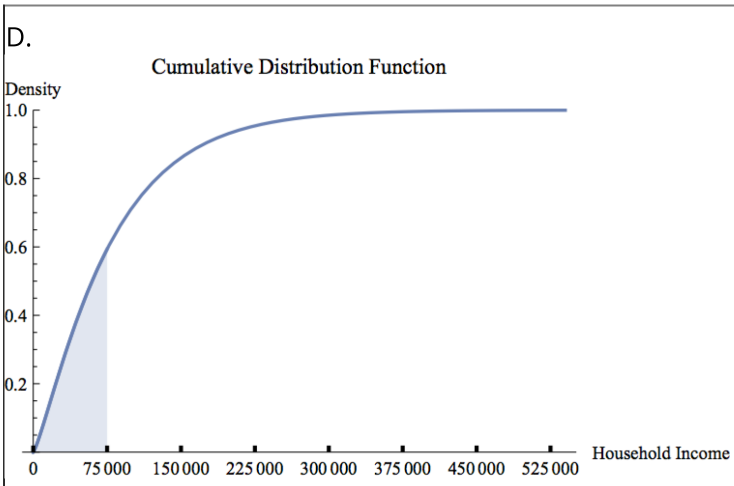
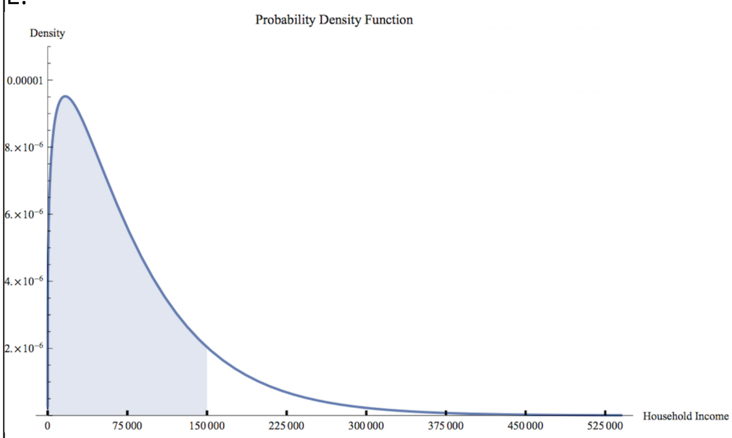
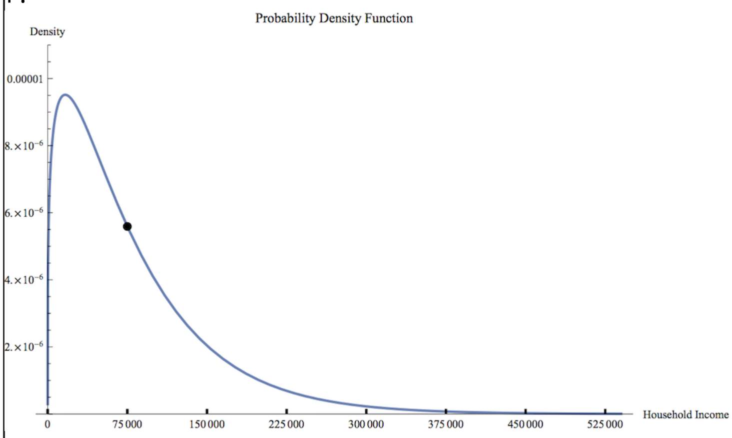


- $P(75000 < X < 150000)$
- $P(X < 75,000)$ ✔
- $P(X < 150000)$
- $P(X > 75000)$
- None of the Above

C.



- $P(75000 < X < 150000)$ ✔
- $P(X < 75,000)$
- $P(X < 150000)$
- $P(X > 75000)$
- None of the Above

D. 	○ $P(75000 < X < 150000)$ ○ $P(X < 75,000)$ ○ $P(X < 150000)$ ○ $P(X > 75000)$ ◉ None of the Above ✔
E. 	○ $P(75000 < X < 150000)$ ○ $P(X < 75,000)$ ◉ $P(X < 150000)$ ✔ ○ $P(X > 75000)$ ○ None of the Above
F. 	○ $P(75000 < X < 150000)$ ○ $P(X < 75,000)$ ○ $P(X < 150000)$ ○ $P(X > 75000)$ ◉ None of the Above ✔

The probability density function has total area under the curve equal to one and increases or decreases depending on the number of households with incomes at a certain level (for our example). Probabilities correspond to areas under the curve of the probability density function.

The cumulative distribution function shows the probability that (in our example) a randomly selected household's income is less than the independent variable. Thus, it is always increasing. Probabilities correspond to the height of a point on the graph of the cumulative distribution function.

The height of a point on the probability density function is related to the number of households with income at that level; it does not tell us the probability of a household having that income. We have not attempted to interpret the area under the curve of the cumulative distribution function.

Submit

Answers are displayed within the problem

Question 2

1/1 point (graded)

Which of the following are valid ways to compute the probability that a randomly chosen household income X is greater than a dollars?

- [ ] Compute $f(a)$, where f is the probability density function for X.
- [ ] Compute $\int_0^a f(t)\,dt$, where f is the probability density function for X.
- [x] Compute $\int_a^\infty f(t)\,dt$, where f is the probability density function for X. ✔
- [ ] Compute $F(a)$, where F is the cumulative distribution function for X.
- [x] Compute $1 - F(a)$, where F is the cumulative distribution function for X. ✔

✔

Explanation

Answer: To use the probability density function $f(x)$ to find $P(X > a)$, we would compute the integral $\int_a^\infty f(x)\,dx$.

Alternately, since the total area under the graph of f on $[0, \infty)$ is 1, we can compute $P(X > a)$, the area from a to infinity, as 1 minus the $P(X < a)$, the area from 0 to a. We can write this in terms of the cumulative distribution function $F(x)$. Since $F(a) = \int_0^a f(x)\,dx = P(X < a)$, $P(X > a) = 1 - F(a)$.

Submit

Answers are displayed within the problem

Question 3

1/1 point (graded)

We can use our model to get an estimate how many US households earning some income made more than a million dollars in 2013.

Use the following information to make an estimate.

- Using a computer, we can compute $\int_{1{,}000{,}000}^{\infty} f(x)\,dx \approx 2.9 \times 10^{-7}$.
- There were about 124 million households in the US in 2013. (Note: this includes households earning non-positive income, but you can still use this to make an overestimate.)

Which of the following is the best estimate according to our model?

- [x] The model estimates that the chance that a randomly chosen US household earning some income makes more than 1 million dollars is almost zero. ✔
- [ ] The model estimates that there are no households in 2013 making above 1 million dollars.
- [x] The model estimates that there are about 36 households making above 1 million dollars. ✔
- [ ] None of the above.

✔

Explanation

The probability that a randomly chosen US household earning some income has an annual income of more than a million dollars is very close to zero, so choice (a) is correct.
But this doesn't mean there are no households making about 1 million. That's because there are a lot of households in the US.
To estimate how many, we multiply the probability we found by the total number of households in the US. We get:

$$\begin{aligned}(124 \times 10^6) \cdot (2.9 \times 10^{-7}) &= 124 \times 2.9 \times 10^{-1} \\ &= 35.96.\end{aligned}$$

which translates to about 36 households. (Note:Since 124 million is all US house holds, not just those earning some income, this gives an overestimate.)
In doing this computation, we're treating the probability of having a certain income level as the fraction of total households at a certain income level. Thinking this way, it makes sense to multiply. If we knew the exact number of households at that level, we could divide it by the total number of households to find the fraction of households at that level of income.
This is only an estimate, however, because the probability is only an estimate of the sample data. We don't know the actual fraction of total households at that level. We'd need to have a survey of all households to know that exact fraction.

Submit

Answers are displayed within the problem

Question 4

1/1 point (graded)
The **median** of a set of data cuts the data in half, separating the higher half of the data from the lower half.

We define median for a continuous model similarly. The **median of a continuous distribution** is the value such that such that a randomly chosen X has a 50% chance of being higher than that value and a 50% chance of being lower.

Which of the following do you think would compute the *median*, m, *of the probability density function* $f(x)$? (We'll come back to this question in the next video.)

- Compute $m = \int_0^{0.5} f(x)\,dx$
- Find the value m such that $\int_0^m f(x)\,dx = 0.5$ ✔
- Find the value m such that $\int_0^m f(x)\,dx = 50$
- None of the above.

Explanation
We want the value m such that there is a $50\% = 0.5$ probability of a randomly chosen US household having an annual income of less than or equal to m. This is the value m with $\int_0^m f(x)\,dx = 0.5$.

Submit

Answers are displayed within the problem

Question 5: Think About It...

1/1 point (graded)
Can you think of a way to use the probability density function to compute the mean annual income? Record your ideas below.

```
E(X) = I_{0}^{m} x*f(x) dx
```

✔
Thank you for your response.

Explanation
Watch the next video and text that follows to learn more.

Submit

Answers are displayed within the problem

Learn About Verified Certificates

English ▼

POWERED BY

Download on the App Store

ANDROID APP ON Google play