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4.

Setup:

Suppose you have observations X_1, X_2, X_3, X_4, X_5 which are i.i.d. draws from a Gaussian distribution with unknown mean μ and unknown variance σ^2 .

Given Facts:

You are given the following:

$$rac{1}{5}\sum_{i=1}^{5}X_{i}=0.90, \qquad rac{1}{5}\sum_{i=1}^{5}X_{i}^{2}=1.31$$

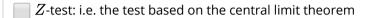
Choose a test

0.67/1 point (graded)

To test the null hypothesis $H_0: \mu=0$ versus the alternative hypothesis $H_1: \mu\neq 0$ using the data above, which of the following test(s) is appropriate?

(Choose all that apply.)









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You have used 2 of 3 attempts

1 Answers are displayed within the problem

Unbiased Sample Variance

1/1 point (graded)

Compute the **unbiased sample variance** S.

(Enter a numerical answer accurate to at least 2 decimal places.)

$$S = 0.625$$
 • Answer: 0.625

Solution:

Recall the unbiased sample variance is

$$S = rac{n}{n-1} \Biggl(rac{1}{n} \sum_{i=1}^{n} (X_i - \mu)^2 \Biggr)$$

$$= rac{n}{n-1} \Biggl(\overline{X_n^2} - \overline{X_n}^2 \Biggr)$$

$$= rac{5}{4} (1.31 - 0.9^2) = 0.625.$$

(Without the 5/4 factor, the biased variance is ~ 0.5 .)

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MLE

2/2 points (graded)

Give an estimate the mean μ and variance σ^2 . Use the maximum likelihood estimator.

$$\hat{\mu} = \begin{bmatrix} 0.9 \end{bmatrix}$$
 Answer: 0.9

$$\hat{\sigma}^2 =$$
 0.5 \checkmark Answer: 0.5

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T test

1/2 points (graded)

Find the value of the t-statistic for testing the hypotheses above:

$$H_0: \mu=0 \quad ext{versus} \quad H_1: \mu
eq 0$$

given this set of data.

(Enter a numerical answer accurate to at least 2 decimal places.)

 ${f t}-$ statistic:

1.1384199576606164

X Answer: 2.546

If we allow 5% of samples to wrongly reject H_0 when H_0 is in fact true, what can we conclude from the t-test?

 \bigcirc reject H_0

igcup accept H_0

lacksquare fail to reject H_0



Solution:

Recall that the t-distribution with d degrees of freedom is the law of a ratio

$$rac{Z}{\sqrt{V/d}} \quad ext{where} \quad Z \sim \mathcal{N}\left(0,1
ight) ext{ indep } V \sim \chi_d^2.$$

The t-statistic is the sample mean scaled by the square root of the sample size, devided by the sample variance, where the variance must be the unbiased estimator with $\frac{1}{n-1}$ normalization, and the t statistics has d=n-1 degrees of freedom. Hence, in this problem, the t-statistic is

$$T_n = rac{\sqrt{n} \overline{X}_n}{\sqrt{S_n^{
m unbiased}}} \, = \, rac{\sqrt{5}0.9}{\sqrt{0.625}} \, = \, 2.5456.$$

The rejection region for this two-sided test with confidence level lpha=0.05 is

$$\psi_lpha=\mathbf{1}\left(|T_5|>q_{lpha/2}
ight)=\mathbf{1}\left(|T_5|>2.78
ight)$$

where $q_{lpha/2}=2.78$ is the (1-lpha/2) quantile of t_4 distribution. We fail to reject the null hypothesis in this sample.

1 Answers are displayed within the problem

Confidence interval

2/2 points (graded)

Provide a non-asymptotic confidence interval [A,B] for μ that is symmetric around the sample mean and covers the true mean in 95% of samples.

(To avoid double jeopardy, enter your answer in terms of the (unbiased) sample variance S from above, or directly enter numerical answers accurate to at least 2 decimal places.)

(Be careful that A < B.)

Lower bound $A = \begin{bmatrix} -0.08162158073878045 \\ -0.08162158073878045 \end{bmatrix}$ Answer: 0.90-2.776445*sqrt(S)/sqrt(5)

Upper bound B = 1.8816215807387806 **Answer**: 0.90+2.776445*sqrt(S)/sqrt(5)

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You have used 1 of 3 attempts

Answers are displayed within the problem

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