Dodecahedron

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In geometry, a **dodecahedron** (Greek δωδεκάεδρον, from δώδεκα *dōdeka* "twelve" + ἕδρα *hédra* "base", "seat" or "face") is any polyhedron with twelve flat faces, but usually a **regular dodecahedron** is meant, which is one of the five Platonic solids. It is composed of twelve regular pentagonal faces, with three meeting at each vertex, and is represented by the Schläfli symbol {5,3}. It has 20 vertices, 30 edges and 160 diagonals. Its dual polyhedron is the icosahedron, with Schläfli symbol {3,5}.

The pyritohedron is an irregular pentagonal dodecahedron, having the same topology as the regular one but pyritohedral symmetry. The rhombic dodecahedron has octahedral symmetry. There are a large number of other dodecahedra.

Contents

Regular dodecahedron

Dimensions

If the edge length of a regular dodecahedron is a, the radius of a circumscribed sphere (one that touches the dodecahedron at all vertices) is

$$r_u = a\frac{\sqrt{3}}{4}\left(1+\sqrt{5}\right) \approx 1.401258538 \cdot a$$ A179296

and the radius of an inscribed sphere (tangent to each of the dodecahedron's faces) is

$$r_i = a\frac{1}{2}\sqrt{\frac{5}{2}+\frac{11}{10}\sqrt{5}} \approx 1.113516364 \cdot a$$

while the midradius, which touches the middle of each edge, is

$$r_m = a\frac{1}{4}\left(3+\sqrt{5}\right) \approx 1.309016994 \cdot a$$

These quantities may also be expressed as

$$r_u = a\,\frac{\sqrt{3}}{2}\varphi$$

where φ is the golden ratio.

Note that, given a regular pentagonal dodecahedron of edge length one, r_u is the radius of a circumscribing sphere about a cube of edge length φ, and r_i is the apothem of a regular pentagon of edge length φ.

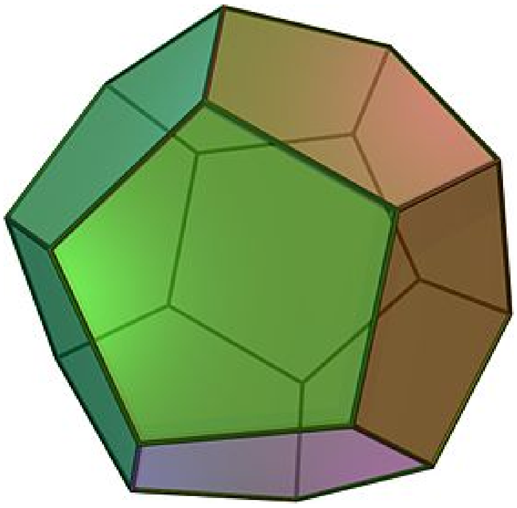
Area and volume

The surface area A and the volume V of a regular dodecahedron of edge length a are:

Two-dimensional symmetry projections

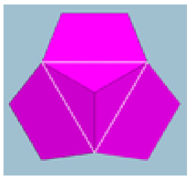
The *dodecahedron* has two special orthogonal projections, centered, on vertices and pentagonal faces, correspond to the A_2 and H_2 Coxeter planes.

Regular Dodecahedron

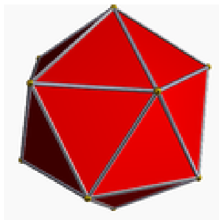


(Click here for rotating model)

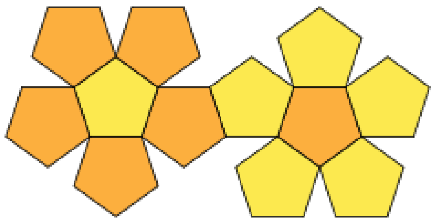
Type	Platonic solid
Elements	$F = 12$, $E = 30$ $V = 20$ ($\chi = 2$)
Faces by sides	12{5}
Conway notation	D gT
Schläfli symbols	{5,3}
Wythoff symbol	3 | 2 5
Coxeter diagram	
Symmetry	I_h, H_3, [5,3], (*532)
Rotation group	I, $[5,3]^+$, (532)
References	U_{23}, C_{26}, W_5
Properties	Regular convex
Dihedral angle	116.56505° = arccos(-1/√5)



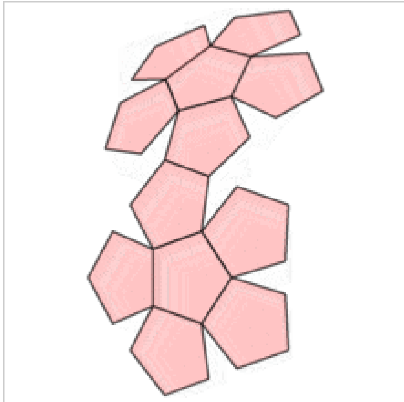
5.5.5
(Vertex figure)



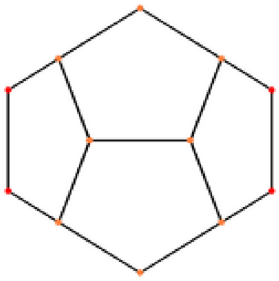
Icosahedron
(dual polyhedron)



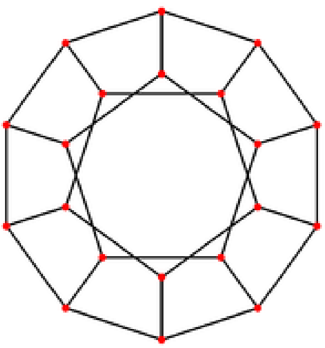
Net


Animation of a net of a dodecahedron being folded

Orthogonal projections

Centered by	Vertex	Edge	Face
Image			
Projective symmetry	[[3]] = [6]	[2]	[[5]] = [10]

In perspective projection, viewed above a pentagonal face, the dodecahedron can be seen as a linear-edged schlegel diagram, or stereographic projection as a spherical polyhedron. These projections are also used in showing the four-dimensional 120-cell, a regular 4-dimensional polytope, constructed from 120 dodecahedra, projecting it down to 3-dimensions.

Projection	Orthogonal projection	Perspective projection	
		Schlegel diagram	Stereographic projection
Dodecahedron			
Dodecaplex (120-cell)			

Spherical tiling

The dodecahedron can also be represented as a spherical tiling.

Orthographic projection

Cartesian coordinates

The following Cartesian coordinates define the vertices of a dodecahedron centered at the origin and suitably scaled and oriented:[1]

$(\pm 1, \pm 1, \pm 1)$
$(0, \pm 1/\varphi, \pm\varphi)$
$(\pm 1/\varphi, \pm\varphi, 0)$
$(\pm\varphi, 0, \pm 1/\varphi)$

where $\varphi = (1 + \sqrt{5}) / 2$ is the golden ratio (also written τ) ≈ 1.618. The edge length is $2/\varphi = \sqrt{5} - 1$. The containing sphere has a radius of $\sqrt{3}$.

Vertex coordinates:

- The orange vertices lie at $(\pm 1, \pm 1, \pm 1)$ and form a cube (dotted lines).
- The green vertices lie at $(0, \pm 1/\varphi, \pm\varphi)$ and form a rectangle on the y–z plane.
- The blue vertices lie at $(\pm 1/\varphi, \pm\varphi, 0)$ and form a rectangle on the x–y plane.
- The pink vertices lie at $(\pm\varphi, 0, \pm 1/\varphi)$ and form a rectangle on the x–z plane.

The distance between adjacent vertices is $2/\varphi$, and the distance from the origin to any vertex is $\sqrt{3}$.
$\varphi = (1 + \sqrt{5}) / 2$ is the golden ratio.

Properties

- The dihedral angle of a dodecahedron is 2 arctan(φ) or approximately 116.5650512 degrees (where again $\varphi = (1 + \sqrt{5}) / 2$, the golden ratio). A137218
- If the original dodecahedron has edge length 1, its dual icosahedron has edge length φ.
- If the five Platonic solids are built with same volume, the dodecahedron has the shortest edges.
- It has 43,380 nets.
- The map-coloring number of a regular dodecahedron's faces is 4.
- The distance between the vertices on the same face not connected by an edge is φ times the edge length.

Space filling with cube and bilunabirotunda

Regular dodecahedra fill the space with cubes and bilunabirotundae, Johnson solid 91, in the ratio of 1 to 1 to 3.[2][3] The dodecahedra alone make a lattice of edge-to-edge pyritohedra. The bilunabirotundae fill the rhombic gaps. Each cube meets six bilunabirotundae in three orientations.

 Block model		Lattice of dodecahedra	6 bilunabirotundae around a cube

Geometric relations

The *regular dodecahedron* is the third in an infinite set of truncated trapezohedra which can be constructed by truncating the two axial vertices of a pentagonal trapezohedron.

The stellations of the dodecahedron make up three of the four Kepler–Poinsot polyhedra.

A rectified dodecahedron forms an icosidodecahedron.

The regular dodecahedron has icosahedral symmetry I_h, Coxeter group [5,3], order 120, with an abstract group structure of $A_5 \times Z_2$.

Icosahedron vis-à-vis dodecahedron

When a dodecahedron is inscribed in a sphere, it occupies more of the sphere's volume (66.49%) than an icosahedron inscribed in the same sphere (60.54%).

A regular dodecahedron with edge length 1 has more than three and a half times the volume of an icosahedron with the same length edges (7.663... compared with 2.181...), which is approximately 3.51246117975, or in real terms: $(3/5)(3\varphi + 1)$ or $(1.8\varphi + .6)$.

A regular dodecahedron has 12 faces and 20 vertices, whereas a regular icosahedron has 20 faces and 12 vertices. Both have 30 edges.

Nested cube

A cube can embed within a regular dodecahedron, affixed to eight of its equidistant vertices, in five different positions.[4] In fact, five cubes may overlap and interlock inside the dodecahedron to result in the compound of five cubes.

The ratio of the edge of a regular dodecahedron to the edge of a cube embedded inside such a dodecahedron is $1 : \varphi$; or $\varphi - 1 : 1$.

The ratio of a regular dodecahedron's volume to the volume of a cube embedded inside such a dodecahedron is $1 : 2/(2 + \varphi)$; or $1 + \varphi/2 : 1$. Another useful ratio is $5+\sqrt{5} : 4$.

For example, an embedded cube with a volume of 64 (and edge length of 4), will nest within a dodecahedron of volume $64 + 32\varphi$ (and edge length of $4\varphi - 4$).

Thus, the difference in volume between the encompassing dodecahedron and the enclosed cube is always one half the volume of the cube times φ (*i.e.*, the golden mean).

From these ratios derive simple formulas for a regular dodecahedron's volume of edge length *a* using the golden mean:

$$V = (a\varphi)^3 \cdot (1/4)(5 + \sqrt{5})$$

$$V = (1/4)(14\varphi + 8)a^3$$

The dodecahedron's golden frame

Golden ratio rectangles of ratio φ + 1 : 1 and φ to 1 also fit perfectly within a regular dodecahedron.[5] In proportion to this golden rectangle, an enclosed cube's edge is φ, when the long length of the rectangle is φ + 1 (or φ^2) and the short length is 1 (the edge shared with the dodecahedron).

In addition, the center of each face of the dodecahedron form three intersecting golden rectangles.[6]

As a graph

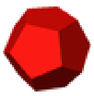
A Hamiltonian cycle in a dodecahedron.

The skeleton of the dodecahedron—the vertices and edges—form a graph. This graph can also be constructed as the generalized Petersen graph *G*(10, 2). The high degree of symmetry of the polygon is replicated in the properties of this graph, which is distance-transitive, distance-regular, and symmetric. The automorphism group has order 120. The vertices can be colored with 3 colors, as can the edges, and the diameter is 5.[7]

The dodecahedral graph is Hamiltonian—there is a cycle containing all the vertices. Indeed, this name derives from a mathematical game invented in 1857 by William Rowan Hamilton, the icosian game. The game's object was to find a Hamiltonian cycle along the edges of a dodecahedron.

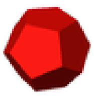
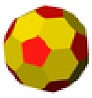
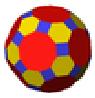
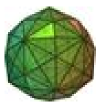
Related polyhedra and tilings

The regular dodecahedron is topologically related to a series of tilings by vertex figure n^3.

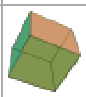
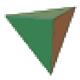
Spherical Polyhedra	**Polyhedra**			**Euclidean**	**Hyperbolic tilings**			
{2,3}	{3,3}	{4,3}	 **{5,3}**	{6,3}	{7,3}	{8,3}	...	(∞,3}

The dodecahedron can be transformed by a truncation sequence into its dual, the icosahedron:

Family of uniform icosahedral polyhedra

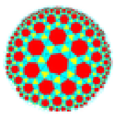
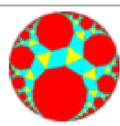
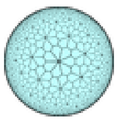
Symmetry: [5,3], (*532)							**[5,3]$^+$, (532)**
							
{5,3}	t{5,3}	r{5,3}	2t{5,3}=t{3,5}	2r{5,3}={3,5}	rr{5,3}	tr{5,3}	sr{5,3}
Duals to uniform polyhedra							
							
V5.5.5	V3.10.10	V3.5.3.5	V5.6.6	**V3.3.3.3.3**	V3.4.5.4	V4.6.10	V3.3.3.3.5

Uniform octahedral polyhedra

Symmetry: [4,3], (*432)							$[4,3]^+$ (432)	$[1^+,4,3] = [3,3]$ (*332)		$[3^+,4]$ (3*2)
{4,3}	t{4,3}	r{4,3} r{$3^{1,1}$}	t{3,4} t{$3^{1,1}$}	{3,4} {$3^{1,1}$}	rr{4,3} s_2{3,4}	tr{4,3}	sr{4,3}	h{4,3} {3,3}	h_2{4,3} t{3,3}	s{3,4} s{$3^{1,1}$}
		=	=	=				= or	= or	=
										
Duals to uniform polyhedra										
$V4^3$	$V3.8^2$	$V(3.4)^2$	$V4.6^2$	$V3^4$	$V3.4^3$	V4.6.8	$V3^4.4$	$V3^3$	$V3.6^2$	$V3^5$
										

The regular dodecahedron is a member of a sequence of otherwise non-uniform polyhedra and tilings, composed of pentagons with face configurations (V3.3.3.3.*n*). (For *n* > 6, the sequence consists of tilings of the hyperbolic plane.) These face-transitive figures have (n32) rotational symmetry.

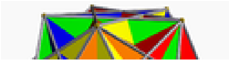
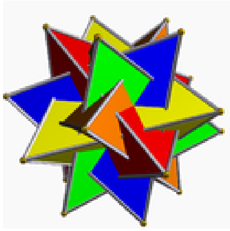
Dimensional family of snub polyhedra and tilings: *3.3.3.3.n*

Symmetry n32 $[n,3]^+$	Spherical				Euclidean	Compact hyperbolic		Paracompact
	232 $[2,3]^+$ D_3	332 $[3,3]^+$ T	432 $[4,3]^+$ O	532 $[5,3]^+$ I	632 $[6,3]^+$ P6	732 $[7,3]^+$	832 $[8,3]^+$...	∞32 $[\infty,3]^+$
Snub figure	3.3.3.3.2	3.3.3.3.3	3.3.3.3.4	3.3.3.3.5	3.3.3.3.6	 3.3.3.3.7	3.3.3.3.8	 3.3.3.3.∞
Coxeter Schläfli	sr{2,3}	sr{3,3}	sr{4,3}	sr{5,3}	sr{6,3}	sr{7,3}	sr{8,3}	sr{∞,3}
Snub dual figure	V3.3.3.3.2	**V3.3.3.3.3**	 V3.3.3.3.4	V3.3.3.3.5	V3.3.3.3.6	 V3.3.3.3.7	V3.3.3.3.8	V3.3.3.3.∞
Coxeter								

Vertex arrangement

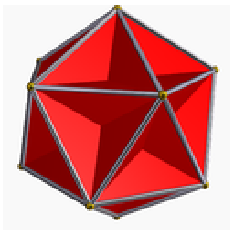
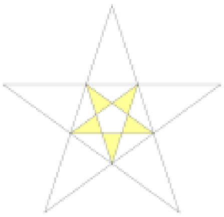
The dodecahedron shares its vertex arrangement with four nonconvex uniform polyhedra and three uniform polyhedron compounds.

Five cubes fit within, with their edges as diagonals of the dodecahedron's faces, and together these make up the regular polyhedral compound of five cubes. Since two tetrahedra can fit on alternate cube vertices, five and ten tetrahedra can also fit in a dodecahedron.

Great stellated dodecahedron	Small ditrigonal icosidodecahedron	Ditrigonal dodecadodecahedron	Great ditrigonal icosidodecahedron
 Compound of five cubes	 Compound of five tetrahedra	Compound of ten tetrahedra	

Stellations

The 3 stellations of the dodecahedron are all regular (nonconvex) polyhedra: (Kepler–Poinsot polyhedra)

	0	1	2	3
Stellation	*Dodecahedron*	 Small stellated dodecahedron	 Great dodecahedron	Great stellated dodecahedron
Facet diagram				

Pyritohedron

A **pyritohedron** is a dodecahedron with pyritohedral (T_h) symmetry. Like the regular dodecahedron, it has twelve identical pentagonal faces, with three meeting in each of the 20 vertices. However, the pentagons are not necessarily regular, so the structure normally has no fivefold symmetry axes. Its 30 edges are divided into two sets – containing 24 and 6 edges of the same length.

Although regular dodecahedra do not exist in crystals, the distorted, pyritohedron form occurs in the crystal pyrite, and it may be an inspiration for the discovery of the regular Platonic solid form.

Crystal pyrite

Its name comes from one of the two common crystal forms of pyrite, the other one being cubical.

Cubic pyrite	Pyritohedral	Ho-Mg-Zn quasicrystal

Cartesian coordinates

The coordinates of the eight vertices of the original cube are:

$(\pm 1, \pm 1, \pm 1)$

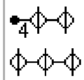
The coordinates of the 12 vertices of the cross-edges are:

$(0, \pm(1 + h), \pm(1 - h^2))$
$(\pm(1 + h), \pm(1 - h^2), 0)$
$(\pm(1 - h^2), 0, \pm(1 + h))$

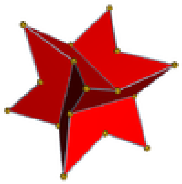
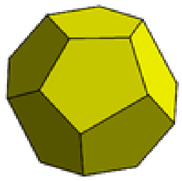
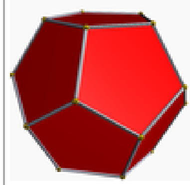
where h is the height of the wedge-shaped "roof" above the faces of the cube. When $h = 1$, the six cross-edges degenerate to points and a rhombic dodecahedron is formed. When $h = 0$, the cross-edges are absorbed in the facets of the cube, and the pyritohedron reduces to a cube. When $h = (\sqrt{5} - 1)/2$, the inverse of the golden ratio, the original edges of the cube are absorbed in the facets of the wedges, which become co-planar, resulting in a regular dodecahedron.

Geometric freedom

The pyritohedron has a geometric degree of freedom with limiting cases of a cubic convex hull at one limit of colinear edges, and a rhombic dodecahedron as the other limit as 6 edges are degenerated to length zero. The regular dodecahedron represents a special intermediate case where all edges and angles are equal.

Pyritohedron	
A pyritohedron has 30 edges, divided into two lengths: 24 and 6 in each group.	
Face polygon	irregular pentagon
Coxeter diagrams	
Faces	12
Edges	30 (6+24)
Vertices	20 (8+12)
Symmetry group	T_h, $[4,3^+]$, (3*2), order 24
Rotation group	T, $[3,3]^+$, (332), order 12
Dual polyhedron	Pseudoicosahedron
Properties	convex, face transitive
Net	

Special cases of the pyritohedron

1 : 1	1 : 1	2 : 1	1.3092... : 1	1 : 1	0 : 1
		$h = 0$		$h = (\sqrt{5} - 1)/2$	$h = 1$
Regular star, great stellated dodecahedron, with pentagons distorted into regular pentagrams	 Concave pyritohedral dodecahedron	A cube can be divided into a pyritohedron by bisecting all the edges, and faces in alternate directions.	 The geometric proportions of the pyritohedron in the Weaire–Phelan structure	 A regular dodecahedron is an intermediate case with equal edge lengths.	 A rhombic dodecahedron is the limiting case with the 6 crossedges reducing to length zero.

Rhombic dodecahedra

The rhombic dodecahedron is a zonohedron with twelve rhombic faces and octahedral symmetry. It is dual to the quasiregular cuboctahedron (an Archimedean solid) and occurs in nature as a crystal form.[8] The rhombic dodecahedron packs together to fill space.

Rhombic dodecahedron

The rhombic dodecahedron has several stellations, the first of which is also a spacefiller.

Another important rhombic dodecahedron has twelve faces congruent to those of the rhombic triacontahedron, i.e. the diagonals are in the ratio of the golden ratio. It is also a zonohedron and was described by Bilinski in 1960.[9] This figure is another spacefiller, and can also occur in non-periodic spacefillings along with the rhombic triacontahedron, the rhombic icosahedron and rhombic hexahedra.[10]

Other dodecahedra

There are 6,384,634 topologically distinct *convex* dodecahedra, excluding mirror images, having at least 8 vertices.[12] (Two polyhedra are "topologically distinct" if they have intrinsically different arrangements of faces and vertices, such that it is impossible to distort one into the other simply by changing the lengths of edges or the angles between edges or faces.)

Topologically identical[11] irregular dodecahedra

The truncated pentagonal trapezohedron has D_{5d} dihedral symmetry.	The pyritohedron has T_h tetrahedral symmetry.

Topologically distinct dodecahedra include:

- Pentagonal dodecahedra:
  - Regular dodecahedron, 12 pentagonal faces, I_h symmetry, order 120
  - Pentagonal truncated trapezohedron – same topology as regular form, but D_{5d} symmetry, order 20
  - "Pyritohedron" – same topology as regular form, but T_h symmetry, order 12
- Uniform polyhedra:
  - Decagonal prism – 10 squares, 2 decagons, D_{10h} symmetry, order 40
  - Pentagonal antiprism – 10 equilateral triangles, 2 pentagons, D_{5d} symmetry, order 20
- Johnson solids (regular faced):

  - Pentagonal cupola – 5 triangles, 5 squares, 1 pentagon, 1 decagon, C_{5v} symmetry, order 10
  - Snub disphenoid – 12 triangles, D_{2d}, order 8
  - Elongated square dipyramid – 8 triangles and 4 squares, D_{4h} symmetry, order 16
  - Metabidiminished icosahedron – 10 triangles and 2 pentagons, C_{2v} symmetry, order 4
- Congruent irregular faced: (face-transitive)
  - Hexagonal bipyramid – 12 isosceles triangles, dual of hexagonal prism, D_{6h} symmetry, order 24
  - Hexagonal trapezohedron – 12 kites, dual of hexagonal antiprism, D_{6d} symmetry, order 24
  - Triakis tetrahedron – 12 isosceles triangles, dual of truncated tetrahedron, T_d symmetry, order 24
  - Rhombic dodecahedron (mentioned above) – 12 rhombi, dual of cuboctahedron, O_h symmetry, order 48
- Other irregular faced:
  - Hendecagonal pyramid – 11 isosceles triangles and 1 hendecagon, C_{11v}, order 11
  - Trapezo-rhombic dodecahedron – 6 rhombi, 6 trapezoids – dual of triangular orthobicupola, D_{3h} symmetry, order 12
  - Rhombo-hexagonal dodecahedron or *Elongated Dodecahedron* – 8 rhombi and 4 equilateral hexagons, D_{4h} symmetry, order 16

History and uses

Dodecahedral objects have found some practical applications, and have also played a role in the visual arts and in philosophy.

Roman dodecahedron

Iamblichus states that Hippasus, a Pythagorean, perished in the sea, because he boasted that he first divulged "the sphere with the twelve pentagons." [13] In *Theaetetus*, a dialogue of Plato, Plato was able to prove that there are just five uniform regular solids; they later became known as the platonic solids. Timaeus (c. 360 B.C.), as a personage of Plato's dialogue, associates the other four platonic solids with the four classical elements, adding that there is a fifth solid pattern which, though commonly associated with the dodecahedron, is never directly mentioned as such; "this God used in the delineation of the universe."[14] Aristotle also postulated that the heavens were made of a fifth element, which he called aithêr (*aether* in Latin, *ether* in American English).

Dodecahedra have been used as dice and probably also as divinatory devices. During the hellenistic era, small, hollow bronze Roman dodecahedra were made and have been found in various Roman ruins in Europe. Their purpose is not certain.

Ho-Mg-Zn quasicrystal

In 20th-century art, dodecahedra appear in the work of M.C. Escher, such as his lithographs *Reptiles* (1943) and *Gravitation* (1952). In Salvador Dalí's painting *The Sacrament of the Last Supper* (1955), the room is a hollow dodecahedron.

In modern role-playing games, the dodecahedron is often used as a twelve-sided die, one of the more common polyhedral dice. Some quasicrystals have dodecahedral shape (see figure). Some regular crystals such as garnet and diamond are also said to exhibit "dodecahedral" habit, but this statement actually refers to the rhombic dodecahedron shape.[8]

Omnidirectional sound source

Immersive media, a Camera manufacturing company, has made the Dodeca 2360 camera, the world's first 360°, full motion camera which captures high-resolution video from every direction simultaneously at more than 100 million pixels per second or 30 frames per second. It is based on dodecahedron.

The popular puzzle game Megaminx is in the shape of a dodecahedron.

In the children's novel *The Phantom Tollbooth*, the Dodecahedron appears as a character in the land of Mathematics. Each of his faces wears a different expression—*e.g.* happy, angry, sad—which he swivels to the front as required to match his mood.

Dodecahedron is the name of an avant-garde black metal band from Netherlands.[15]

A climbing wall consisting of three dodecahedral pieces

Shape of the universe

Various models have been proposed for the global geometry of the universe. In addition to the primitive geometries, these proposals include the Poincaré dodecahedral space, a positively curved space consisting of a dodecahedron whose opposite faces correspond (with a small twist). This was proposed by Jean-Pierre Luminet and colleagues in 2003[16][17] and an optimal orientation on the sky for the model was estimated in 2008.[18]

In Bertrand Russell's 1954 short story "THE MATHEMATICIAN'S NIGHTMARE: The Vision of Professor Squarepunt," the number 5 said: "I am the number of fingers on a hand. I make pentagons and pentagrams. And but for me dodecahedra could not exist; and, as everyone knows, the universe is a dodecahedron. So, but for me, there could be no universe."

See also

- 120-cell: a regular polychoron (4D polytope) whose surface consists of 120 dodecahedral cells.
- Pentakis dodecahedron
- Snub dodecahedron
- Truncated dodecahedron

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10. ^ Lord, K.; *Tilings, coverings, clusters and quasicrystals* (http://met.iisc.ernet.in/~lord/webfiles/tcq.html)
11. ^ Polyhedra are "topologically identical" if they have the same intrinsic arrangement of faces and vertices, such that one can be distorted into the other simply by changing the lengths of edges or the angles between edges or faces.
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External links

Wikimedia Commons has media related to ***Regular dodecahedra***.

- Weisstein, Eric W., "Dodecahedron" (http://mathworld.wolfram.com/Dodecahedron.html), *MathWorld*.
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- Richard Klitzing, 3D convex uniform polyhedra, o3o5x – doe (http://www.bendwavy.org/klitzing/dimensions/polyhedra.htm)
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- The Uniform Polyhedra (http://www.mathconsult.ch/showroom/unipoly/)
- Origami Polyhedra (http://www.flickr.com/photos/pascalin/sets/72157594234292561/) – Models made with Modular Origami
- Dodecahedron (http://polyhedra.org/poly/show/3/dodecahedron) – 3-d model that works in your browser
- Virtual Reality Polyhedra (http://www.georgehart.com/virtual-polyhedra/vp.html) The Encyclopedia of Polyhedra
  - VRML models
  1. Regular dodecahedron (http://www.georgehart.com/virtual-polyhedra/vrml/dodecahedron.wrl) regular
  2. Rhombic dodecahedron (http://www.georgehart.com/virtual-polyhedra/vrml/rhombic_dodecahedron.wrl) quasiregular
  3. Decagonal prism (http://www.georgehart.com/virtual-polyhedra/vrml/decagonal_prism.wrl) vertex-transitive
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  5. Hexagonal dipyramid (http://www.georgehart.com/virtual-polyhedra/vrml/hexagonal_dipyramid.wrl) face-transitive
  6. Triakis tetrahedron (http://www.georgehart.com/virtual-polyhedra/vrml/triakistetrahedron.wrl) face-transitive
  7. hexagonal trapezohedron (http://www.georgehart.com/virtual-polyhedra/vrml/hexagonal_trapezohedron.wrl) face-transitive
  8. Pentagonal cupola (http://www.georgehart.com/virtual-polyhedra/vrml/pentagonal_cupola_(J5).wrl) regular faces
- K.J.M. MacLean, A Geometric Analysis of the Five Platonic Solids and Other Semi-Regular Polyhedra (http://www.kjmaclean.com/Geometry/GeometryHome.html)
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- Stella: Polyhedron Navigator (http://www.software3d.com/Stella.php): Software used to create some of the images on this page.
- How to make a dodecahedron from a Styrofoam cube (http://video.fc2.com/content/20141015mMG9QR5R)

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