Data Analysis: Statistical Modeling and Computation in Applications

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Exercises due Nov 10, 2021 17:29 IST Completed

Partial Autocorrelation

Let X_0,\ldots,X_n be a stationary time series. Recall that the autocorrelation function of the series at lag h is defined

$$ho_{X}\left(h
ight)=\mathsf{Corr}\left(X_{h},X_{0}
ight)=\mathbf{E}\left[\left(X_{h}-\mathbf{E}\left[X_{0}
ight]
ight)\left(X_{0}-\mathbf{E}\left[X_{0}
ight]
ight)
ight]/\mathsf{Var}\left(X_{0}
ight).$$

The partial autocorrelation between X_h and X_0 is the correlation between X_h and X_0 with the correlation due to the intermediate terms of the series X_1,\ldots,X_{h-1} removed. That is, the correlation between X_h and X_0 after the intermediate terms of the series X_1,\ldots,X_{h-1} have been partialled-out. Formally, the partial autocorrelation of time series $oldsymbol{X_t}$ at lag $oldsymbol{h}$ is

$$lpha_{X}\left(h
ight)\!:=\!\!\operatorname{\mathsf{Corr}}\!\left(X_{h}-\hat{X}_{h}^{\mathsf{lin}_{h-1}},\;X_{0}-\hat{X}_{0}^{\mathsf{lin}_{h-1}}
ight)\!,$$

where $\hat{X}_h^{\mathsf{lin}_{h-1}}$ is the linear regression (projection) of X_h on X_1,\ldots,X_{h-1} , and $\hat{X}_0^{\mathsf{lin}_{h-1}}$ is the linear regression (projection) of X_0 on X_1,\dots,X_{h-1} . That is, $lpha_X\left(h
ight)$ is the correlation between X_h and X_0 with the best linear predictions $\hat{X}_h^{\mathsf{lin}_{h-1}}$ of X_h and $\hat{X}_0^{\mathsf{lin}_{h-1}}$ of X_0 based on the intermediate terms of the series X_1, \dots, X_{h-1} removed from X_h and X_0 respectively.

A convenient method to compute the partial autocorrelation $lpha_X(h)$ is to use the Frisch-Waugh-Lovell theorem. Specifically, FWL theorem says that $lpha_X\left(h
ight)$ is the regression coefficient on regressor X_{t-h} in the regression of X_t on the set of regressors X_{t-1}, \ldots, X_{t-h} : If

$$X_t = \phi_1 X_{t-1} + \cdots + \phi_h X_{t-h} + ilde{X}_t, \quad ext{where} \quad \mathbf{E}\left[X_{t-j} ilde{X}_t
ight] = 0, \quad j = 1, \ldots, h,$$

is the best linear prediction of X_t in the linear span of X_{t-1},\dots,X_{t-h} (i.e., the linear regression), then $lpha_X\left(h
ight)=\phi_h$. The regression coefficients ϕ_1,\ldots,ϕ_h can be estimated by e.g., method of moments with Yule-Walker equations.

The partial autocorrelation function $lpha_X\left(h
ight)$ is used to determine the order of the $\mathsf{AR}\left(p_0
ight)$ process:

$$X_t = \phi_1 X_{t-1} + \dots + \phi_{p_0} X_{t-p_0} + W_t, \qquad W_t \sim \mathsf{WN}\left(\sigma_W^2
ight), \quad \mathbf{E}\left[X_t
ight] = 0.$$

We have seen that the autocorrelation function $\gamma_X\left(h
ight)
eq 0$ for all time lags $h>p_0$. This happens because X_{t-j} for $j=1,\ldots,p_0$ are linearly related to X_{t-h} through the recursive definition of the process. This means that we cannot use the autocovariance function to determine the order of the process.

Recap of AR model

2/2 points (graded)

Consider $\mathsf{AR}\left(1\right)$ process $X_{t} = \phi X_{t-1} + W_{t}$, $|\phi| < 1$.

Is X_t stationary?



True

False

Which one of the following is the correct formula of $\gamma_X(h)$?

$$\bigcirc \ \gamma_X\left(h
ight) = \phi^{h-1}\sigma_W^2/\left(1-\phi^2
ight)$$

$$\bigcirc \ \gamma_X\left(h
ight) = \phi^{h-1}\sigma_W^2/\phi^2$$

$$\bigcirc \ \gamma_X\left(h
ight) = \phi^h\sigma_W^2/\phi^2$$

$$\bigcirc \ \gamma_{X}\left(h
ight)=\phi^{h}\sigma_{W}^{2}/\left(1-\phi^{2}
ight)$$



Solution:

Write

$$egin{array}{ll} X_t &= \phi X_{t-1} + W_t \ &= \phi \left[\phi X_{t-2} + W_{t-1}
ight] + W_t \ &dots \ &= \sum_{j=0}^1 \phi^j W_{t-j} + \phi^2 X_{t-2} \ &dots \ &= \sum_{j=0}^{h-1} \phi^j W_{t-j} + \phi^h X_{t-h} &dots \ &= \sum_{j=0}^\infty \phi^j W_{t-j} \end{array}$$

From the last line, we find that ${\sf Var}(X_t)=\sigma_W^2\frac{1}{1-\phi^2}$ by taking the variance on the right hand side, noting that white noise terms are uncorrelated and summing the geometric series with the common ration ϕ^2 (by the property of squaring a constant taken outside of the variance).

Furthermore, from the second to last line, we find that

$$\mathsf{Cov}\left(X_{t}, X_{t-h}
ight) = \mathsf{Cov}\left(\sum_{j=0}^{h-1} \phi^{j} W_{t-j} + \phi^{h} X_{t-h}, X_{t-h}
ight) = \phi^{h} \mathsf{Cov}\left(X_{t-h}, X_{t-h}
ight) = \phi^{h} rac{\sigma_{W}^{2}}{1-\phi^{2}}.$$

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1 Answers are displayed within the problem

PACF of AR(1)

4/4 points (graded)

Consider $\mathsf{AR}\left(1\right)$ process $X_{t}=0.5X_{t-1}+W_{t}$. Compute the PACF $lpha_{X}\left(h\right)$ for lag h= 0,1,2 and 3.

Compute α_X (0).

1 Answer: 1

Compute α_X (1).

0.5 **✓ Answer:** 0.5

Compute α_X (2).

0 **✓ Answer:** 0

Compute α_X (3).

0 **✓ Answer:** 0

Solution:

Consider any $\mathsf{AR}\left(1\right)$ process $X_{t} = \phi X_{t-1} + W_{t}$, $|\phi| < 1$.

The coefficient of regression of X_t on itself is always 1, so $\alpha_X(0)=1$.

Next, from the definition of the autoregressive process, we see that ϕ is the regression coefficient of X_t on X_{t-1} because the contemporary white noise term W_t is orthogonal to the past observation X_{t-1} . That is, $\alpha_X(1) = \phi = 0.5$ in this case.

Furthermore, we can write the AR(1) process as an AR(h) process:

$$X_t = \phi X_{t-1} + 0 \cdot X_{t-2} + \cdot + 0 \cdot X_{t-h} + W_t$$

and note that the contemporaneous white noise term W_t is orthogonal to all previous realizations of the process (X_{t-1},\ldots,X_{t-h}) . Hence, the coefficients in the expression above $(\phi,0,\ldots,0)$ are in fact least squares coefficients in the regression of X_t on X_{t-1},\ldots,X_{t-h} . Therefore, $\alpha_X(h)=0$ for all h>1.

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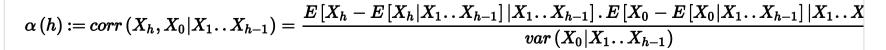
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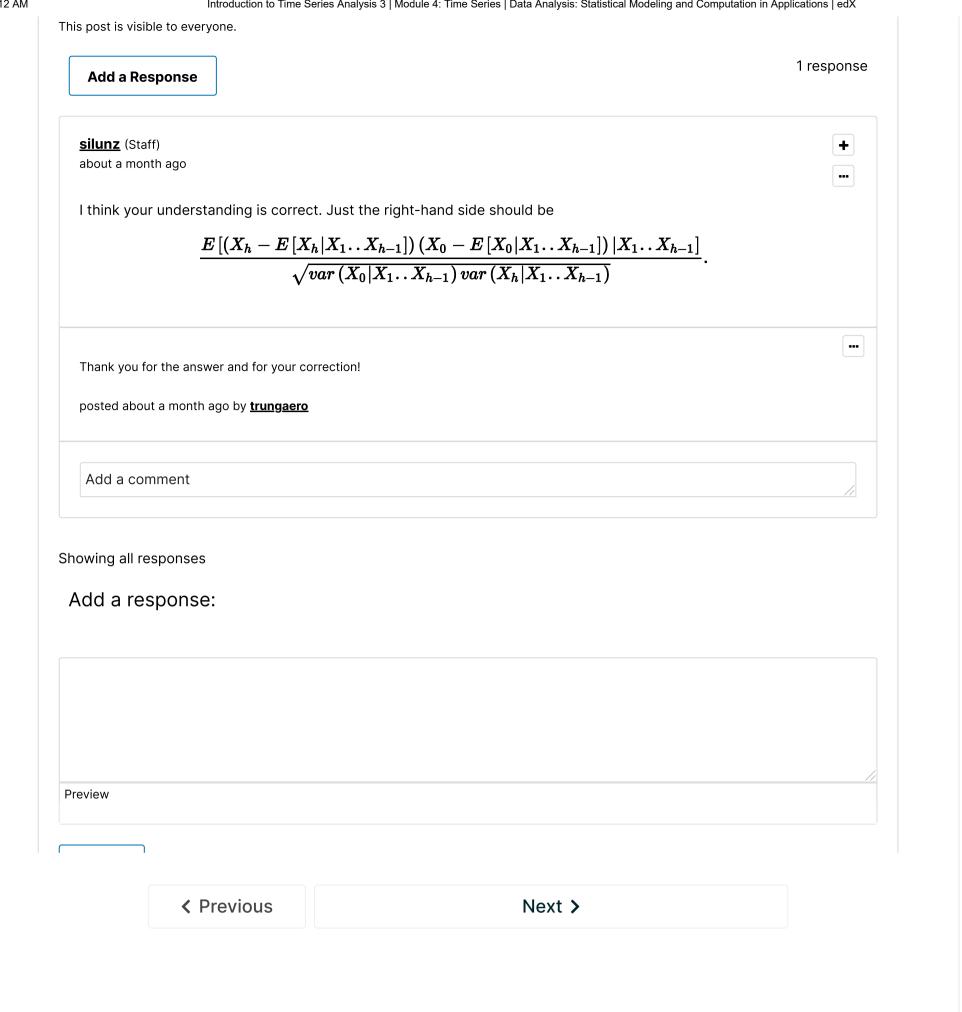
[STAFF] Partial autocorrelation definition?

discussion posted 2 months ago by **trungaero**

I am quite still confused about PCAF. In the same language to define ACF, is it correct to define PCAF as the conditional correlation between Xh and X0 conditioning on any Xt in between, or:



In the probability language, I understand that we try to figure out the conditional independence between X0 and Xh in the conditional world. And the use of linear regression is just for estimating the conditional expectation $E[X_h|X_1..X_{h-1}]$. Please correct my understanding if it's wrong? Thanks.



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