5. Complex eigenvalues

Rotation matrix

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Since eigenvalues are roots of a polynomial with real coefficients, they can be complex, and when they are, they come in pairs of complex conjugates.

Example 5.1 Find the eigenvalues of the matrix $\mathbf{A} = \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$.

(This matrix represents the function that rotates any vector in $\mathbb{R}^3$ by 90° counterclockwise about the z-axis.)

Solution :

$$\begin{array}{rcl} \det(\mathbf{A} - \lambda \mathbf{I}) & = & \begin{vmatrix} -\lambda & -1 & 0 \\ 1 & -\lambda & 0 \\ 0 & 0 & 1-\lambda \end{vmatrix} \\ & = & (\lambda^2+1)(1-\lambda) \\ & = & -(\lambda+i)(\lambda-i)(\lambda-1) \\ \Longrightarrow \qquad \lambda & = & \pm i,\ 1. \end{array}$$

Hence, the eigenvalues are $\lambda = \pm i, 1,$ (each with multiplicity 1). Note that both i and its conjugate $-i$ are eigenvalues.

Theorem 5.2 For any $n \times n$ matrix, the total number of complex eigenvalues, counted with multiplicity, is n.

Note that complex eigenvalues include real eigenvalues.

Proof

We apply the fundamental theorem of algebra to the characteristic polynomial of $\mathbf{A}$, which is a degree n polynomial with real coefficients.

The fundamental theorem of algebra: Every nonzero degree n polynomial with complex coefficients has exactly n complex roots when counted with multiplicity.

Remember real numbers are also complex numbers, so this implies in particular that every nonzero degree n polynomial with real coefficients, e.g. the characteristic polynomial of $\mathbf{A}$, has exactly n complex roots when counted with multiplicity.

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