Could we give up on the Axiom of Choice?

In order for V to count as an example of a non-measurable set, it has to exist. And, as I mentioned earlier, V cannot be shown to exist without the Axion of Choice:

> **Axiom of Choice**
> Every set of non-empty, non-overlapping sets has a choice set.

Because of this, one might be tempted to sidestep the phenomenon of non-measurability altogether, by giving up on the Axiom of Choice. Such a temptation might seem especially strong in light of the fact that the Axiom of Choice can be used to prove all sorts of bizarre results. We used it in Lecture 3 to find an incredible solution to a Hard Hat Problem, and we'll soon use it to prove one of the most perplexing, and famous, of all its consequences: the Banach-Tarski Theorem.

Unfortunately, there are regions of mathematics that would be seriously weakened without the Axiom of Choice. For example, there would be no way of proving that, for any two sets, either they have the same size or one is bigger than the other:

$$|A| = |B| \text{ or } |A| < |B| \text{ or } |B| < |A|$$

It is hard to know what to do about the Axiom of Choice. When you think about some of its bizarre consequences, you might be tempted to give it up. But when you're trying to work within certain areas of mathematics, you feel like you can't live without it. My own view is that we should learn to live with the Axiom of Choice. But there is no denying that it delivers strange results.

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