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1.3.5 Quiz: (Optional) Qualitative Analysis of the Fishing Model

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Peter just showed how to use the graph of $\frac{dP}{dt}$ versus P to do a **qualitative analysis** of the differential equation $\frac{dP}{dt} = \frac{1}{10}P(1 - \frac{P}{40000}) - \alpha$, in the case of $\alpha = 0$ and a 'small' α.

Now it's your turn to do the same for 'large' α and the critical α.

Question 1

7/7 points (graded)

For each of the three different cases, match the graph of $\frac{dP}{dt}$ versus P with the qualitative analysis of $\frac{dP}{dt}$ (the qualitative sketch of solutions $P(t)$).

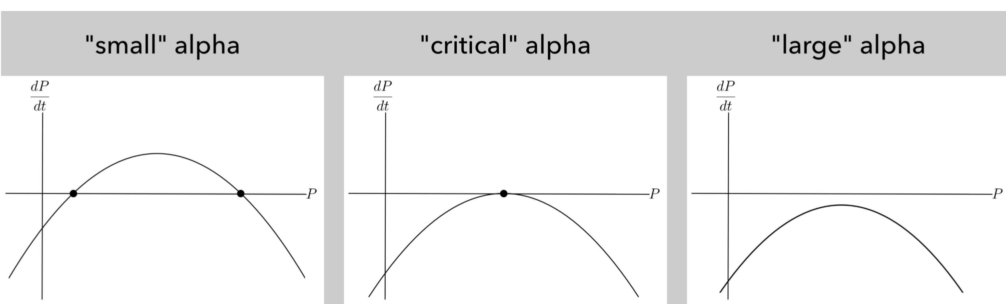

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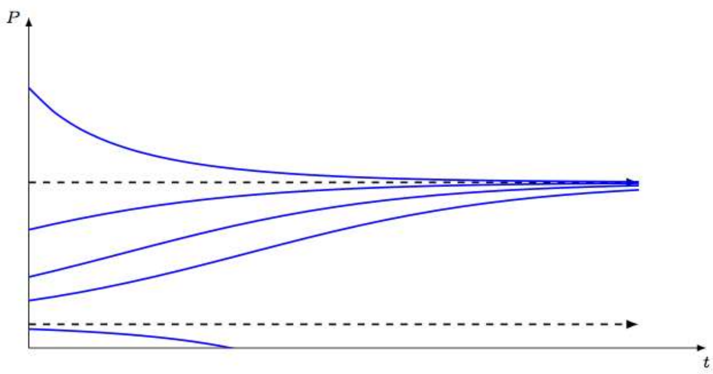
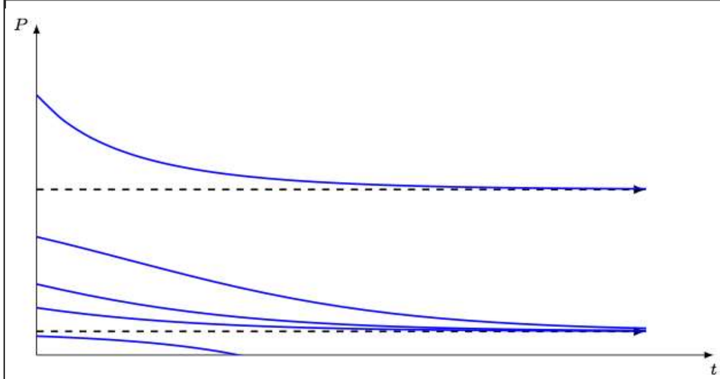
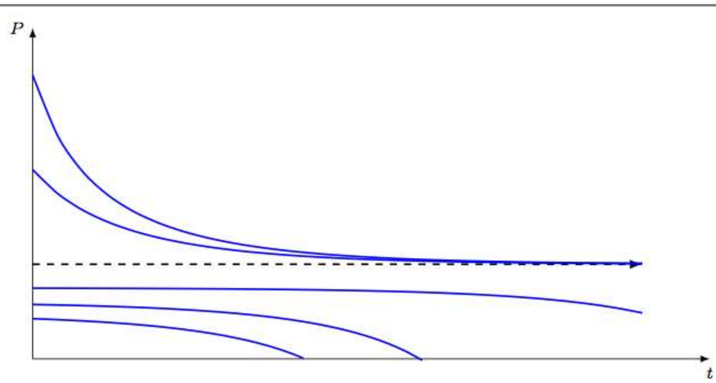
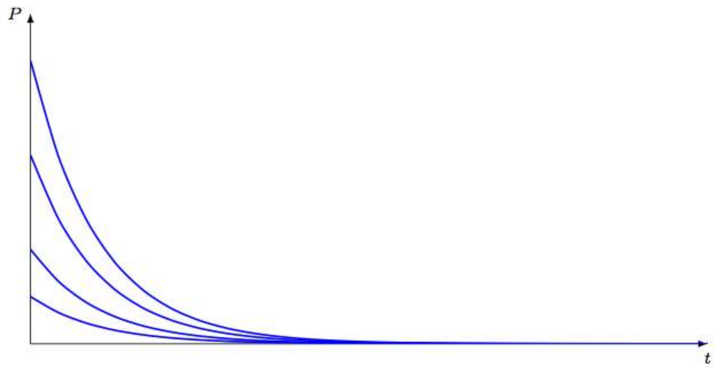
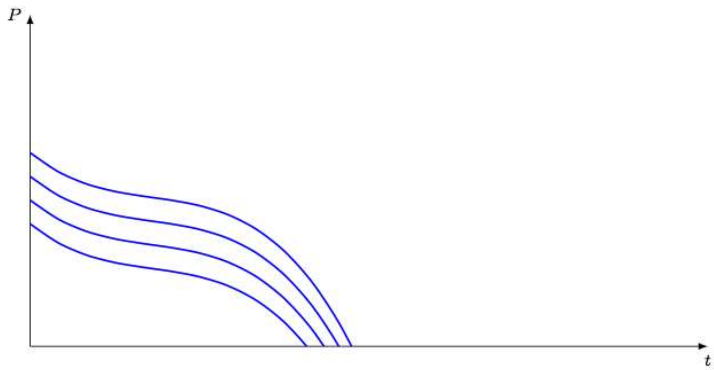
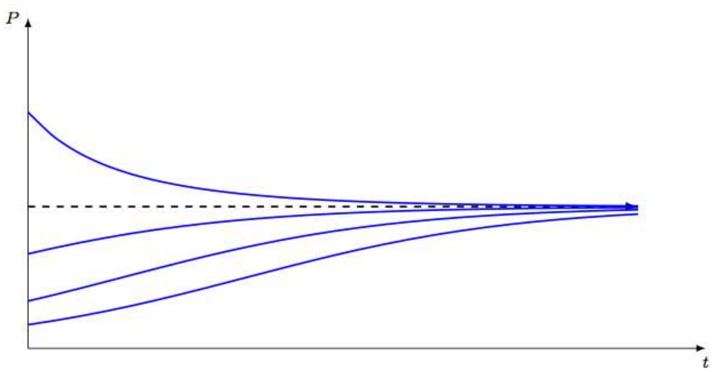
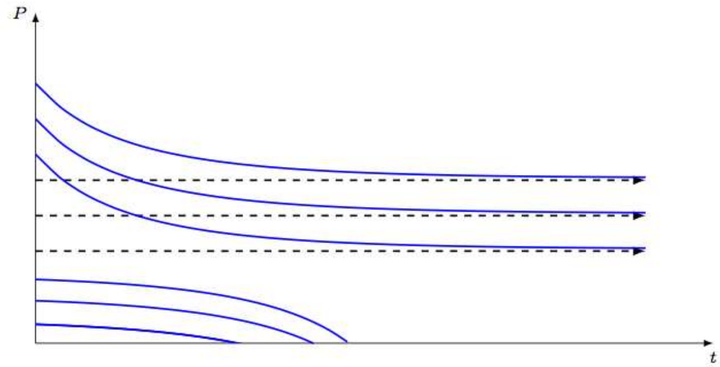
Image Description

Image Description for answer graphs A-G

Graphs	Cases
 View Larger Image	small: α < 1000 ▼ **Answer:** small: α < 1000
 View Larger Image	None of the Above ▼ **Answer:** None of the Above
 View Larger Image	critical: α = 1000 ▼ **Answer:** critical: α = 1000

 View Larger Image	None of the Above ▼ ✔ **Answer:** None of the Above
 View Larger Image	large: α > 1000 ▼ ✔ **Answer:** large: α > 1000
 View Larger Image	None of the Above ▼ ✔ **Answer:** None of the Above



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None of the Above ▼

Answer: None of the Above

Explanation

For small values of α, the differential equation has two equilibrium solutions, a stable equilibrium at a high value of P and an unstable equilibrium at a low value of P. Solution curves with initial condition $P(0)$ greater than the larger equilibrium will decrease toward the upper asymptote, solution curves with initial condition $P(0)$ between the two equilibria will tend toward the upper asymptotes, and solution curves with initial condition $P(0)$ less than the larger equilibrium will tend toward $P = 0$.

Note: We can see from the graph of $\frac{dP}{dt}$ versus P that for solutions with initial condition less than the larger equilibrium, the rate of change of P becomes more and more negative as P decreases. This means the solution curves are concave down, hence there is no horizontal asymptote at $P = 0$.

For the critical value of α, the differential equation has a semi-stable equilibrium at $P = 20,000$. Solution curves with initial condition above this value will decrease to this asymptote, while solutions curves with initial condition below this value will decrease to $P = 0$, extinction. The curves will decrease most steeply for high and low values of P since $\frac{dP}{dt}$ is most negative there.

For large values of α, the population decreases to $P = 0$ no matter what the starting value of P is since $\frac{dP}{dt}$ is always negative. We can see from the graph of $\frac{dP}{dt}$ versus P that as P decreases, the derivative $\frac{dP}{dt}$ eventually becomes increasingly negative. This means the solution curves are concave down as they hit $P = 0$. Each of these situations describes a different possible behavior for the solutions of the differential equation, depending on the α level. Close to the value $\alpha = 1000$, small changes in α may lead to major changes in behavior of solutions. This is an example of a bifurcation, with critical value $\alpha = 1000$.

Submit

You have used 2 of 5 attempts

Answers are displayed within the problem

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