13.1.3 Conditions for a minimimum in one dimension

MO2.11

Now let's consider a more general function, $J(a)$, which we would like to find the minimum value of over some subject of $a \in S$. For example, suppose S is the range of values from $a_{\text{left}} < a < a_{\text{right}}$. The function being minimized $J(a)$ is known as the *objective function*. We will assume that the objective function is smooth so that any derivatives of J with respect to a that we are interested in will be continuous. Such a function is sketched in Figure 13.3.

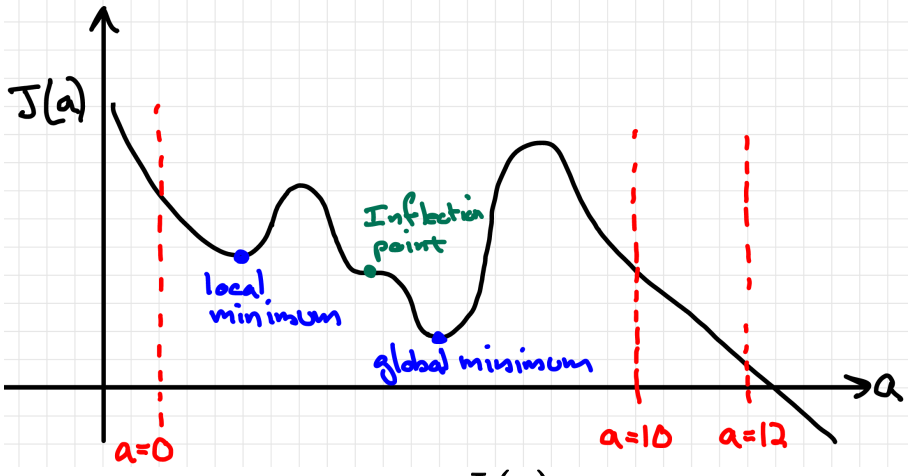

Figure 13.3: Example of $J(a)$ showing local and global minimums

Note that while we only will discuss minimization, the same ideas directly apply to maximization of a function, $f(a)$, by simply multiplying the function by -1 such that $J(a) = -f(a)$. By doing this, then minimizing $J(a)$ is equivalent to maximizing $f(a)$.

We now introduce some fundamental concepts for optimization problems:

- Suppose that for this function, we are considering only the range $0 < a < 10$. Then we see that the *global minimum* occurs at $a \approx 6$. At this location, $\mathrm{d}J/\mathrm{d}a = 0$ and $\mathrm{d}^2J/\mathrm{d}a^2 \geq 0$.
- There is also what is referred to as a *local minimum* at $a \approx 2$. This local minimum also satisfies the properties the $\mathrm{d}J/\mathrm{d}a = 0$ and $\mathrm{d}^2J/\mathrm{d}a^2 \geq 0$. However, its value is not the global minimum since a lower value of J is achieved at $a \approx 6$.
- An *inflection point*, which occurs at $a \approx 4$, is a point at which the second derivative changes sign (and hence at the inflection point the second derivative is zero). In general, an inflection point can have non-zero first derivative, though for the situation shown in the figure we have drawn the inflection point with $J'(a) = 0$.

Discussions

Posting in discussic

All posts sorted by recent activity

‹ Previous Next ›