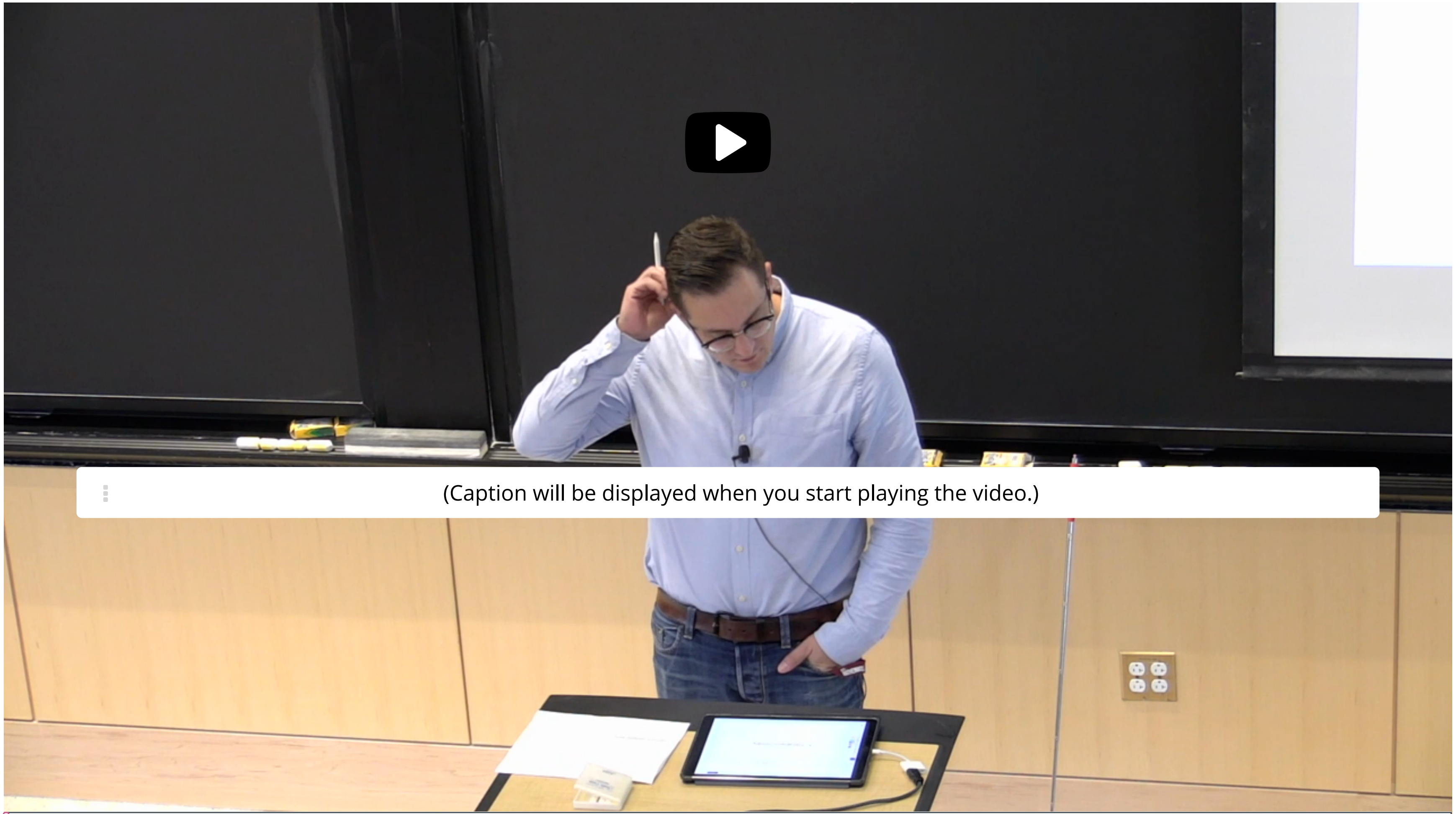
3. Unit Overview

What We Have Seen in Hypothesis Testing So Far...



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Now would be a good time to review Hypothesis Testing and its related terminology as we have seen in Lecture 6, Lecture 7, and Homework 3.

Hypothesis Testing Review I

1/1 point (graded)

In the next few problems, we use a similar (but not identical) set-up to Problem 6 in Homework 3.

The National Assessment of Educational Progress tested a simple random sample of n thirteen year old students in both 2004 and 2008 and recorded each student's score. The standard deviation in 2004 was 39. In 2008, the standard deviation was 38.

Your goal as a statistician is to assess whether or not there were statistically significant changes in the average test scores of students from 2004 to 2008. To do so, you make the following modeling assumptions regarding the test scores:

- $X_1, \ldots, X_n$ represent the scores in 2004.
- $X_1, \ldots, X_n$ are iid Gaussians with standard deviation 39.
- $\mathbb{E}[X_1] = \mu_1$, which is an unknown parameter.
- $Y_1, \ldots, Y_n$ represent the scores in 2008.
- $Y_1, \ldots, Y_n$ are iid Gaussians with standard deviation 38.
- $\mathbb{E}[Y_1] = \mu_2$, which is an unknown parameter.
- $X_1, \ldots, X_n$ are independent of $Y_1, \ldots, Y_n$.

You define your hypothesis test in terms of the null $H_0 : \mu_1 = \mu_2$ (signifying that there were not significant changes in test scores) and $H_1 : \mu_1 \neq \mu_2$.

The test given above is a

- One-sided, two-sample test.
- Two-sided, two-sample test. ✔
- One-sided, one-sample test.

Two-sided, one-sample test.

Solution:

The test above is a **Two-sided, two-sample test.** This is because there are two samples: $X_1, \ldots, X_n$ and $Y_1, \ldots, Y_n$. This is also a two-sided test, because we are testing whether or not $\mu_1 = \mu_2$ or $\mu_1 \neq \mu_2$.

It would be a one-sided test if, for instance, H_1 were defined to be $\mu_1 > \mu_2$. In this situation, we would only be testing whether or not μ_1 is larger than μ_2.

Submit You have used 1 of 2 attempts

Answers are displayed within the problem

Hypothesis Testing Review II

2/2 points (graded)

Let us use the same set-up as in the previous problem. Assuming that the null hypothesis $H_0 : \mu_1 = \mu_2$ holds, which of the following are true about the distribution of the statistic

$$\sqrt{n}\frac{\overline{X}_n - \overline{Y}_n}{\sqrt{38^2 + 39^2}}?$$

(Choose all that apply.)

- [x] The distribution is standard Gaussian.
- [ ] The distribution is known, but changes depending on n.

The distribution cannot be determined.

Suppose now that the variance σ_X^2 of $X_1,\ldots,X_n$ and the variance σ_Y^2 of $Y_1,\ldots,Y_n$ are unknown. Let $\widehat{\sigma_X}^2$ denote the sample variance of $X_1,\ldots,X_n$, and let $\widehat{\sigma_Y}^2$ denote the sample variance of $Y_1,\ldots,Y_n$. Replacing the true variances with the sample variances, we construct the statistic

$$\sqrt{n}\frac{\overline{X}_n-\overline{Y}_n}{\sqrt{\widehat{\sigma_X}^2+\widehat{\sigma_Y}^2}}.$$

Still assuming that $H_0 : \mu_1 = \mu_2$ holds, which of the following are true about the distribution of this statistic? (Choose all that apply.)

- [ ] The distribution is a standard Gaussian for $n = 10$.
- [ ] The distribution is a standard Gaussian for all $n \in \mathbb{N}$.
- [x] By Slutsky's theorem, as $n \to \infty$, its distribution converges to standard Gaussian.

Solution:

Recall that a linear combination of independent Gaussian random variables is again a Gaussian. Therefore, it suffices to determine the mean and variance of the given statistic. (Already, we see that the response **The distribution cannot be determined.** is incorrect.)

By linearity of expectation,

$$\mathbb{E}\left[\sqrt{n}\frac{\overline{X}_n - \overline{Y}_n}{\sqrt{38^2+39^2}}\right] = \sqrt{\frac{n}{38^2+39^2}}\left(\mathbb{E}\left[\overline{X}_n\right] - \mathbb{E}\left[\overline{Y}_n\right]\right) = 0.$$

Next, by independence, the variance is additive:

$$\mathrm{Var}\left(\sqrt{n}\frac{\overline{X}_n - \overline{Y}_n}{\sqrt{38^2+39^2}}\right) = \frac{1}{n\left(38^2+39^2\right)}\left(\sum_{i=1}^{n}\mathrm{Var}\left(X_i\right) + \sum_{i=1}^{n}\mathrm{Var}\left(Y_i\right)\right) = 1.$$

Hence, the first response **The distribution is standard Gaussian.** is correct. Note that the distribution does not depend on the sample size n.

For the next question, we consider the statistic

$$\sqrt{n}\frac{\overline{X}_n - \overline{Y}_n}{\sqrt{\widehat{\sigma_X}^2 + \widehat{\sigma_Y}^2}}.$$

Since $\widehat{\sigma_X}$ and $\widehat{\sigma_Y}$ are random variables, the distribution of the above cannot be standard Gaussian for any fixed n. However, we know that

$$\widehat{\sigma_X} \xrightarrow{n\to\infty} \sigma_X, \quad \widehat{\sigma_Y} \xrightarrow{n\to\infty} \sigma_Y.$$

Therefore, Slutsky's theorem applies because

$$\sqrt{n}\frac{\overline{X}_n - \overline{Y}_n}{\sqrt{\sigma_X^2 + \sigma_Y^2}} \sim \mathcal{N}\left(0,1\right).$$

We conclude that

$$\sqrt{n}\frac{\overline{X}_n - \overline{Y}_n}{\sqrt{\widehat{\sigma_X}^2 + \widehat{\sigma_Y}^2}} \xrightarrow[(d)]{n\to\infty} \mathcal{N}(0,1).$$

Hence, the third choice **By Slutsky's theorem, as $n \to \infty$, its distribution converges to standard Gaussian.** is correct.

Submit You have used 1 of 3 attempts

Answers are displayed within the problem

Hypothesis Testing Review III

2/2 points (graded)

We use the same statistical set-up as in the previous two problems, and now we assume that the true variances are known: $\sigma_X^2 = 39^2$ and $\sigma_Y^2 = 38^2$. Accordingly, you design the test

$$\psi = \mathbf{1}\left(\sqrt{n}\left|\frac{\overline{X}_n - \overline{Y}_n}{\sqrt{38^2 + 39^2}}\right| \geq q_{\eta/2}\right).$$

where q_η represents the $1-\eta$ quantile of a standard Gaussian.

The level of this test is... (Choose all that apply.)

- [x] Asymptotic
- [x] Non-asymptotic

✔

The level of this test is ... (Choose all that apply.)

- [x] η
- [ ] $1 - \eta$
- [x] The probability of making a type 1 error under H_0.
- [ ] The probability of making a type 2 error under H_0.

✔

Solution:

From the previous problems, we know that if $H_0 : \mu_1 = \mu_2$ holds, then

$$\sqrt{n}\frac{\overline{X}_n - \overline{Y}_n}{\sqrt{38^2 + 39^2}} \sim \mathcal{N}(0,1)$$

for all $n \geq 1$. Since we know the distribution of the test statistic for all values of n, the test given has a **non-asymptotic** level η, and therefore also has an **asymptotic** level η.

Recall that the level of a test is the maximum probability of error assuming the null hypothesis. If the null hypothesis is true, then $\mu_1 = \mu_2$, and we conclude that

$$\sqrt{n}\frac{\overline{X}_n - \overline{Y}_n}{\sqrt{38^2 + 39^2}} \sim \mathcal{N}(0,1).$$

Therefore, the level is given by

$$P\left(|Z| > q_{\eta/2}\right) = \eta$$

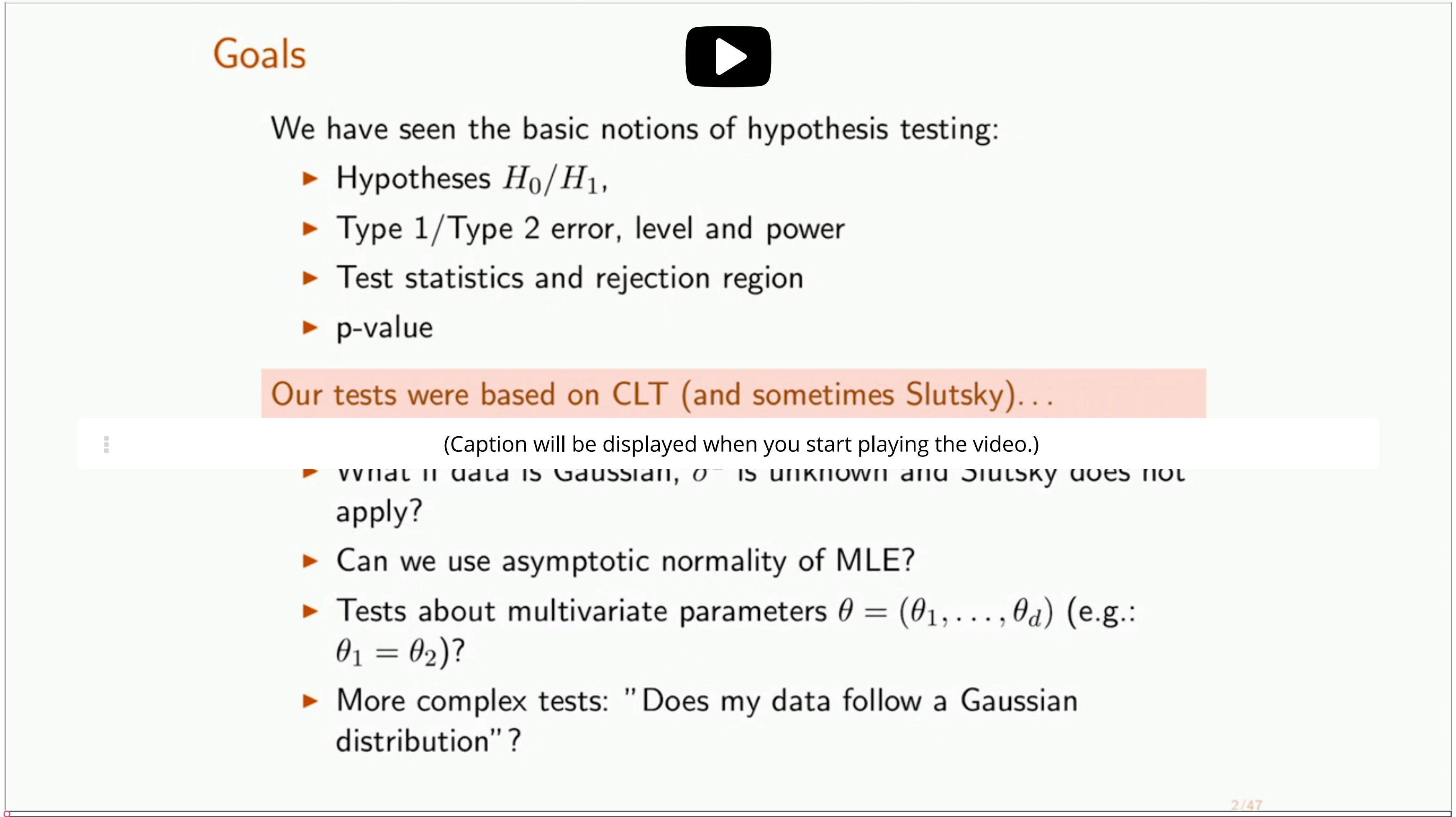
where $Z \sim \mathcal{N}\left(0,1\right)$. Equality holds above by the symmetry of the distribution $\mathcal{N}\left(0,1\right)$.

Hence, for the second question, the correct responses are η and **The probability of making a type 1 error under** H_0.

Submit You have used 3 of 3 attempts

Answers are displayed within the problem

What We Will See in Hypothesis Testing this Chapter...



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Hypothesis Testing Review III

discussion posted about 2 hours ago by sandipan_dey

Hypothesis Testing Review III - part 1, is the grader correct (in asymptotic / non-asymptotic) ? **Deleted by MW-CTA**

This post is visible to everyone.

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1 response

markweitzman (Community TA)
about an hour ago

The grader is correct. I edited your post as it basically leads to the answer before the due date of the exercise. To answer this question correctly, go back and review carefully the definitions of Asymptotic and Non-Asymptotic. They do not have the usual common sense English language meanings (and indeed I criticized the language earlier in the course). One way to think about the definitions is that asymptotic holds in the limit as sample size goes to infinity. Non-asymptotic means on the other hand that the test is valid for finite sample size.

Thank you for the explanation @markweitzman, I was interpreting the terms as logical predicates (tautology / inconsistency), but they are to be thought in terms of statistical definitions.

posted about a minute ago by sandipan_dey

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