2. Review: Cumulative Distribution Functions

Warmup: Integration limits of CDF

3/3 points (graded)

The cumulative distribution function $\Phi : \mathbb{R} \to \mathbb{R}$ of the standard normal $\mathcal{N}(0,1)$ can be written as

$$\Phi(z) = \int_{A}^{B(z)} \frac{1}{\sqrt{2\pi}} e^{C(x)}\, dx$$

where $B(z)$ is a function of z and $C(x)$ is a function of x. Write down the integration limits $A, B(z)$, as well as the function $C(x)$ in the integrand.

Enter inf for ∞.

$A =$ -inf

$-inf$

✔ **Answer:** -inf

$B =$ z ✔ **Answer:** z

z

$C =$ -x^2/2 ✔ **Answer:** -x^2/2

$-\frac{x^2}{2}$

STANDARD NOTATION

Solution:

Recall the cdf of a distribution $\mathbf{P}$ is a function F such that

$$F(x) = P(X \leq x), \quad X \sim \mathbf{P}.$$

The density of a standard Gaussian is given by $\phi(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$. Therefore, the CDF of $\mathcal{N}(0,1)$ is given by

$$\Phi(z) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} e^{-x^2/2}\, dx.$$

Hence $A = -\infty$, $B = z$, and $C = -x^2/2$.

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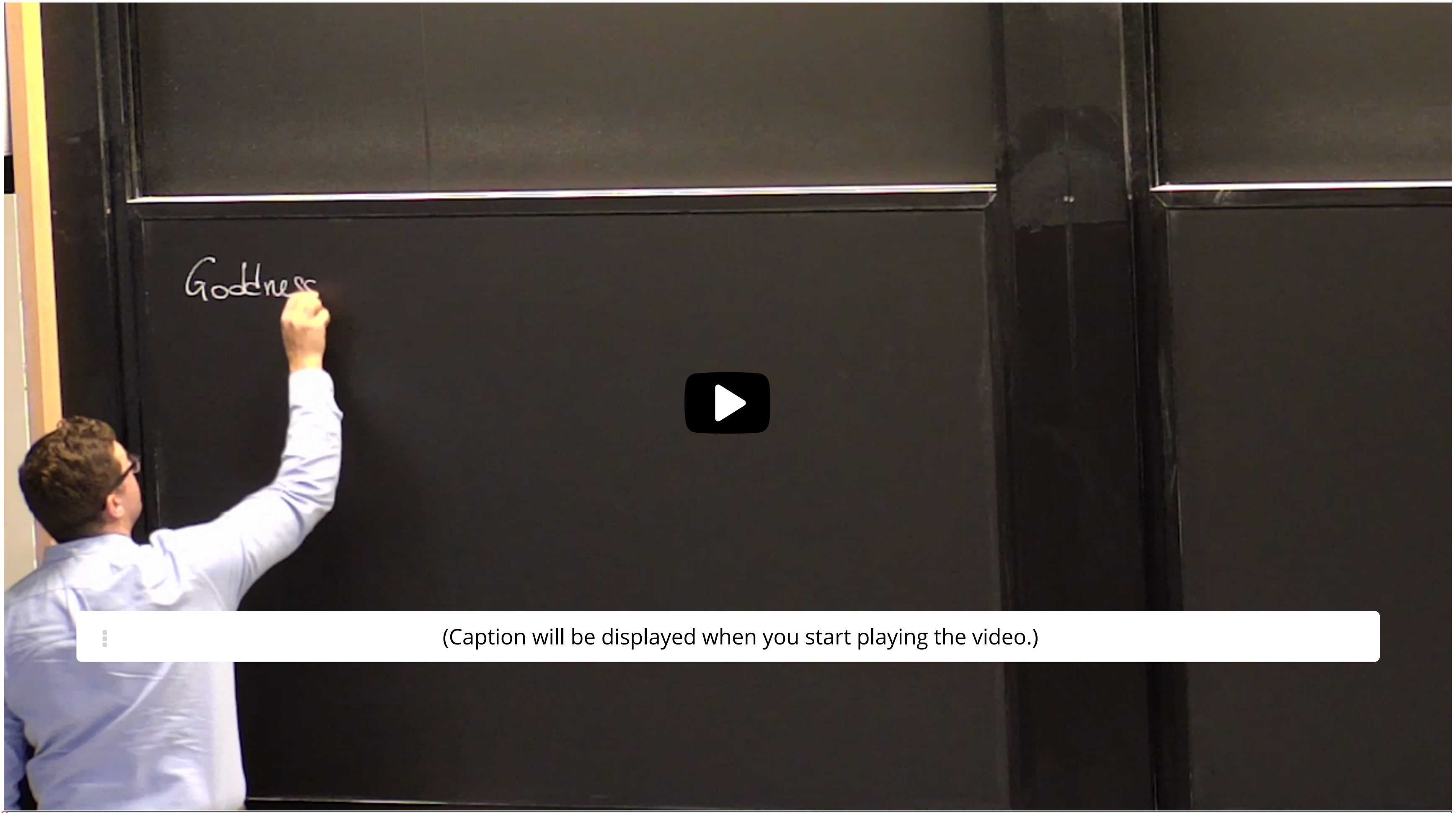
Answers are displayed within the problem

Review



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Review: Cumulative Distribution Function

1/1 point (graded)

Let X and Y be real-valued random variables, both distributed according to a distribution $\mathbf{P}$. (We make no assumption about their joint distribution). Let F denote the **cdf** of $\mathbf{P}$.

Which of the following are true about the cdf F? (Choose all that apply.)

- [ ] $P(X \leq t)$ and $P(Y \leq t)$ are random variables.
- [x] For all $t \in \mathbb{R}$, $F(t) = P(X \leq t)$ and $F(t) = P(Y \leq t)$.
- [ ] $F(t) = P(X \leq t) = P(Y \leq t)$ only if X and Y are independent.
- [x] $\lim_{t \to \infty} F(t) = 1$.
- [x] $\lim_{t \to -\infty} F(t) = 0$.
- [ ] $\int_{-\infty}^{\infty} F(t)\, dt = 1$

✔

Solution:

We examine the choices in order.

- The first choice is incorrect. The probability of an event is a real number, not a random variables, so both $\mathbf{P}(X \leq t)$ and $\mathbf{P}(Y \leq t)$ are deterministic numbers.
- The second choice is correct. The joint distribution of X and Y is irrelevant– so long as X and Y have the same distribution, it will be true that $\mathbf{P}(X \leq t) = \mathbf{P}(Y \leq t)$. By definition, both of these quantities are equal to $F(t)$.
- The third choice is incorrect. As stated in the previous bullet, regardless of the joint distribution of X and Y, as long as they are identically distributed, it is true that $\mathbf{P}(X \leq t) = \mathbf{P}(Y \leq t)$.

- The fourth choice is correct. Observe that $\mathbf{P}(X \leq \infty) = 1$ because X is real-valued. Moreover, F is an increasing function of t. Therefore, $\lim_{t \to \infty} F(t) = \mathbf{P}(X \leq \infty) = 1$.

- The fifth choice is correct, since $\lim_{t \to -\infty} F(t) = \mathbf{P}(X \leq -\infty) = 0$.

- The final choice is incorrect. The statement is true for the **pdf** not the cdf:

$$\int_{-\infty}^{\infty} f(t)\,dt = 1 \qquad \text{if } f(t) \text{ is pdf of } X.$$

Since the cdf $F(t)$ has limit $\lim_{t \to \infty} F(t) = 1,$ the integral over $\mathbb{R}$ of $F(t)$ diverges.

Submit

You have used 1 of 2 attempts

Answers are displayed within the problem

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