12. Interpretation of the frequentist Confidence Interval

Frequentist Interpretation of a Confidence Interval

We're not really going to talk about them

except for their connections to hypothesis testing.

So next week, we'll start hypothesis

testing in a slightly mild way, mostly coming from--

so you will have some exercises that like are soft

and connect them to confidence intervals.

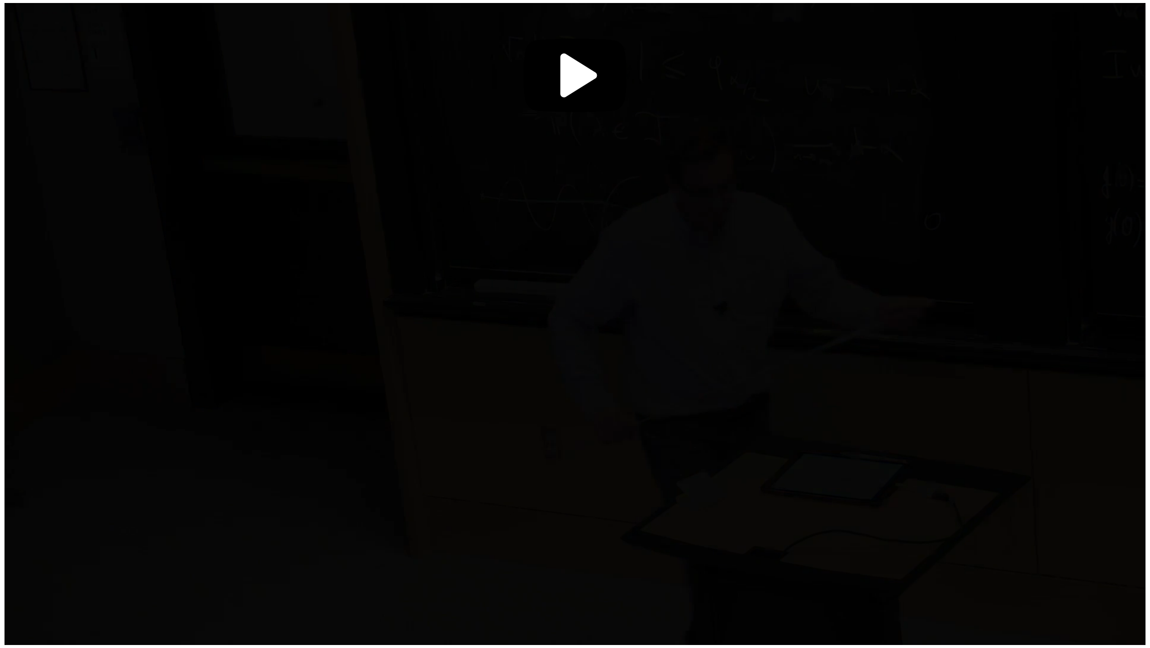
There's going to be a lot of terminology.

We'll talk about type I, type II error,

one-sided, two-sided tests.

We're going to talk about power.

We're going to talk about p values.



We're going to see a bunch of new words.
So make sure that you're here, or make sure
that you take a look at the notes.
Again, office hours this afternoon
are actually right upstairs in 2-290.

5:59 / 5:59 1.50x

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Frequentist Interpretation of a Confidence Interval

1/1 point (graded)

In a particular experiment, you gather data in the form of a sample $X_1, \ldots, X_n \stackrel{iid}{\sim} P_\theta$, and construct a confidence interval $\mathcal{I}$ with level 90% for the true (unknown) parameter θ.

After conducting the experiment, there are two possibilities:

- $\mathcal{I}$ contains θ (We refer to this as a **success**.)

- $\mathcal{I}$ does not contain θ (We refer to this as a **failure**.)

Suppose you repeat the experiment above T total times, and assume that the experiments are jointly independent. Moreover, the value of the unknown parameter θ, is always assumed to be the same. After conducting these T experiments, you will have constructed T confidence intervals $\mathcal{I}_1, \mathcal{I}_2, \ldots, \mathcal{I}_T$.

As T grows very large, what percentage of experiments do you expect to be successes?

90

✔ **Answer:** 90 $\%$

Solution:

By the definition of confidence interval, we know that for the j-th experiment ($1 \leq j \leq T$) that

$$P(\mathcal{I}_j \ni \theta) = 90\%.$$

Consider the indicator random variables $\mathbf{1}(\theta \in \mathcal{I}_1), \mathbf{1}(\theta \in \mathcal{I}_2), \ldots, \mathbf{1}(\theta \in \mathcal{I}_T)$. Since the experiments are jointly independent, this means that $\mathbf{1}(\theta \in \mathcal{I}_1), \mathbf{1}(\theta \in \mathcal{I}_2), \ldots, \mathbf{1}(\theta \in \mathcal{I}_T)$ are independent. Moreover, for all j, the random variable $\mathbf{1}(\theta \in \mathcal{I}_T)$ is Bernoulli because it can only take value 0 or 1. It follows that $\mathbf{1}(\theta \in \mathcal{I}_1), \mathbf{1}(\theta \in \mathcal{I}_2), \ldots, \mathbf{1}(\theta \in \mathcal{I}_T)$ are identically distributed, because for all j,

$$P(\mathbf{1}(\theta \in \mathcal{I}_j) = 1) = P(\mathcal{I}_j \ni \theta) = 90\%.$$

In summary, $\mathbf{1}(\theta \in \mathcal{I}_1), \mathbf{1}(\theta \in \mathcal{I}_2), \ldots, \mathbf{1}(\theta \in \mathcal{I}_T) \overset{iid}{\sim} \mathrm{Ber}(0.9)$. By the strong law of large numbers,

$$\lim_{T \to \infty} \frac{\sum_{j=1}^{T} \mathbf{1}(\theta \in \mathcal{I}_j)}{T} = \mathbb{E}[\mathbf{1}(\theta \in \mathcal{I}_j)] = 0.9$$

almost surely. Since

$$\frac{\sum_{j=1}^{T} \mathbf{1}\left(\theta \in \mathcal{I}_j\right)}{T} = \frac{\text{Number of successes}}{\text{Total number of experiments}},$$

the correct response is 90%.

Submit You have used 1 of 1 attempt

Answers are displayed within the problem

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