



## MITx: 6.041x Introduction to Probability - The Science of Uncertainty



Bookmarks

- ▶ Unit 0: Overview
- ▶ Entrance Survey
- ▶ Unit 1: Probability models and axioms
- ▶ Unit 2: Conditioning and independence
- ▶ Unit 3: Counting
- ▶ Unit 4: Discrete random variables
- ▶ Exam 1
- ▶ Unit 5: Continuous random variables

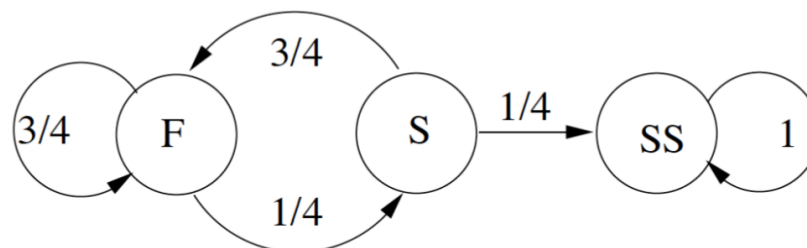
Bookmark

Unit 10: Markov chains > Lec. 26: Absorption probabilities and expected time to absorption > Lec 26  
Absorption probabilities and expected time to absorption vertical4

## Exercise: Time until consecutive successes

(3/3 points)

Consider a sequence,  $X_n$ , of independent Bernoulli random variables with common success probability  $p = 1/4$ . Let  $T$  be the first time at which we have a success immediately following a previous success; that is,  $T = \min\{n : X_n = X_{n-1} = \text{success}\}$ . We are interested in  $\mathbf{E}[T]$ . We model this problem using the following Markov chain:




The state  $S$  denotes a success, state  $F$  denotes a failure, and state  $SS$  is an absorbing state denoting the event that we have obtained two successes in a row. Calculate the numerical values of the following quantities.

1.


- ▶ Unit 6: Further topics on random variables
- ▶ Unit 7: Bayesian inference
- ▶ Exam 2
- ▶ Unit 8: Limit theorems and classical statistics
- ▶ Unit 9: Bernoulli and Poisson processes
- ▼ **Unit 10: Markov chains**

#### Unit overview

##### Lec. 24: Finite-state Markov chains

Exercises 24 due May 18, 2016  
at 23:59 UTC 

##### Lec. 25: Steady-state behavior of Markov chains

Exercises 25 due May 18, 2016  
at 23:59 UTC 

$$\mu_S = \mathbf{E}[T \mid X_0 = S] = \boxed{16} \quad \checkmark \text{ Answer: 16}$$

2.

$$\mu_F = \mathbf{E}[T \mid X_0 = F] = \boxed{20} \quad \checkmark \text{ Answer: 20}$$

3.

$$\mathbf{E}[T] = \boxed{19} \quad \checkmark \text{ Answer: 19}$$

Answer:


$\mu_S = \mathbf{E}[T \mid X_0 = S]$  and  $\mu_F = \mathbf{E}[T \mid X_0 = F]$  are the expected times to absorption starting from states  $S$  and  $F$ , respectively. We have the following system of equations:

$$\begin{aligned}\mu_S &= 1 + \frac{3}{4}\mu_F \\ \mu_F &= 1 + \frac{3}{4}\mu_F + \frac{1}{4}\mu_S,\end{aligned}$$


and so  $\mu_S = 16$  and  $\mu_F = 20$ . Using the total expectation theorem, we have

$$\begin{aligned}\mathbf{E}[T] &= \mathbf{P}(X_0 = F) \cdot \mathbf{E}[T \mid X_0 = F] + \mathbf{P}(X_0 = S) \cdot \mathbf{E}[T \mid X_0 = S] \\ &= \frac{3}{4} \cdot 20 + \frac{1}{4} \cdot 16 \\ &= 19.\end{aligned}$$

**Lec. 26: Absorption probabilities and expected time to absorption**

Exercises 26 due May 18, 2016 at 23:59 UTC 

**Solved problems****Problem Set 10**

Problem Set 10 due May 18, 2016 at 23:59 UTC 

► Exit Survey

*You have used 1 of 2 submissions*

© All Rights Reserved



© edX Inc. All rights reserved except where noted. EdX, Open edX and the edX and Open EdX logos are registered trademarks or trademarks of edX Inc.

POWERED BY  
OPENedX



