

MITx: 6.041x Introduction to Probability - The Science of Uncertainty



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Problem 5: Fire alarm

(4/4 points)

Consider a fire alarm that senses the environment constantly to figure out if there is smoke in the air and hence to conclude whether there is a fire or not. Consider a simple model for this phenomenon. Let Θ be the unknown true state of the environment: $\Theta=1$ means that there is a fire and $\Theta=0$ means that there is no fire. The signal observed by the alarm at time n is $X_n=\Theta+W_n$, where the random variable W_n represents noise. Assume that W_n is Gaussian with mean 0 and variance 1 and is independent of Θ . Furthermore, assume that for $i\neq j$, W_i and W_j are independent. Suppose that Θ is 1 with probability 0.1 and 0 with probability 0.9.

Give numerical answers for all parts below.

1. Given the observation $X_1=0.5$, calculate the posterior distribution of Θ . That is, find the conditional distribution of Θ given $X_1=0.5$.

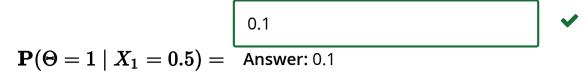
$$\mathbf{P}(\Theta = 0 \mid X_1 = 0.5) = \text{Answer: 0.9}$$

- Unit 6: Further topics on random variables
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Exam 2

Exam 2 due Apr 20, 2016 at 23:59 UTC

 Unit 8: Limit theorems and classical statistics



2. What is the LMS estimate of Θ given $X_1=0.5$?

$$\hat{ heta}_{LMS} =$$
 Answer: 0.1

3. What is the resulting conditional mean squared error of the LMS estimator given $X_1=0.5$?

0.09 **★ Answer:** 0.09

Answer:

1. By symmetry, $f_{W_1}(w) = f_{W_1}(-w)$. Using Bayes' rule,

$$egin{aligned} p_{\Theta|X_1}(0\mid 0.5) &= rac{p_{\Theta}(0)f_{X_1\mid\Theta}(0.5\mid 0)}{f_{X_1}(0.5)} \ &= rac{p_{\Theta}(0)f_{X_1\mid\Theta}(0.5\mid 0)}{p_{\Theta}(0)f_{X_1\mid\Theta}(0.5\mid 0) + p_{\Theta}(1)f_{X_1\mid\Theta}(0.5\mid 1)} \end{aligned}$$

$$egin{aligned} &= rac{0.9 \cdot f_{W_1} \left(0.5
ight)}{0.9 \cdot f_{W_1} \left(0.5
ight) + 0.1 \cdot f_{W_1} \left(-0.5
ight)} \ &= rac{0.9 \cdot f_{W_1} \left(0.5
ight)}{f_{W_1} \left(0.5
ight)} \ &= 0.9. \end{aligned}$$

Hence, $p_{\Theta\mid X_1}(1\mid 0.5)=0.1$.

2. From the posterior distribution found in part (1), we calculate

$$\hat{ heta}_{LMS} = \mathbf{E}[\Theta \mid X_1 = 0.5] = 0.1.$$

3. The conditional mean squared error of the LMS estimator is the conditional variance:

$$var(\Theta \mid X_1 = 0.5) = 0.9 \cdot 0.1 = 0.09.$$

You have used 2 of 2 submissions



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