MATHEMATICS

Bayes' rule with 3 variables

Asked 4 years, 6 months ago Active 4 years, 6 months ago Viewed 13k times

18

7

I have been using Sebastian Thrun's course on AI and I have encountered a slightly difficult problem with probability theory.

He poses the following statement:

$$P(R \mid H, S) = \frac{P(H \mid R, S)\ P(R \mid S)}{P(H \mid S)}$$

I understand he used Bayes' Rule to get the RHS equation, but fail to see how he did this. If somebody could provide a breakdown of the application of the rule in this problem that would be great.

probability bayes-theorem

edited May 14 '15 at 5:51
Mike Pierce
14.2k 10 43 93

asked May 14 '15 at 5:48
slyaer
181 1 1 3

2 Answers

27

Taking it one step at a time:

$$\begin{aligned}
\mathsf{P}(R \mid H, S) &= \frac{\mathsf{P}(R, H, S)}{\mathsf{P}(H, S)} \\
&= \frac{\mathsf{P}(H \mid R, S)\,\mathsf{P}(R, S)}{\mathsf{P}(H, S)} \\
&= \frac{\mathsf{P}(H \mid R, S)\,\mathsf{P}(R \mid S)\,\mathsf{P}(S)}{\mathsf{P}(H, S)} \\
&= \frac{\mathsf{P}(H \mid R, S)\,\mathsf{P}(R \mid S)}{\mathsf{P}(H \mid S)}\frac{\mathsf{P}(S)}{\mathsf{P}(S)} \\
&= \frac{\mathsf{P}(H \mid R, S)\ \mathsf{P}(R \mid S)}{\mathsf{P}(H \mid S)}
\end{aligned}$$

answered May 14 '15 at 8:24

Graham Kemp
95.1k 4 38 87

9

You don't really need Bayes' Theorem. Just apply the definition of conditional probability in two ways. Firstly,

$$P(R \mid H, S) = \frac{P(R, H \mid S)}{P(H \mid S)}$$

$$\therefore \quad P(R, H \mid S) = P(R \mid H, S)P(H \mid S).$$

Secondly,

$$P(H \mid R, S) = \frac{P(R, H \mid S)}{P(R \mid S)}$$

$$\therefore \quad P(R, H \mid S) = P(H \mid R, S)P(R \mid S).$$

Combine these two to get the result.

answered May 14 '15 at 8:10

Mick A
9,203 2 9 26

1 I would vote up, but alas, I do not have enough reputation to vote up an answer to my own question :/ – slyaer May 14 '15 at 9:55

How to you get from $P(R|H,S)$ to $P(R,H|S)/P(H|S)$? – Mo Prog Jul 19 at 16:35

@MoProg From the definition of conditional probability. Just as you can have $P(A \mid B) = P(A, B)/P(B)$, which is from the standard definition, you can also have $P(A \mid B, C) = P(A, B \mid C)/P(B \mid C)$, for events A, B, C. – Mick A Jul 19 at 22:50

Sorry @MickA, but I still don't understand how you use the definition of conditional probablity $P(X|Y) = P(X,Y)/P(Y)$ to get from $P(A|B,C)$ to $P(A,B|C)/P(B|C)$. If I use the definition of conditional probability I would have $P(A|B,C) = P(A,B,C)/P(B,C)$, because I will substitute $Y = B, C$. – Mo Prog Jul 22 at 7:57

1 @MoProg We can derive it this way: $\frac{P(A,B|C)}{P(B|C)} = \frac{P(A,B,C)/P(C)}{P(B,C)/P(C)} = \frac{P(A,B,C)}{P(B,C)} = P(A|B,C)$. – Mick A Jul 22 at 13:59