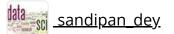


<u>Help</u>





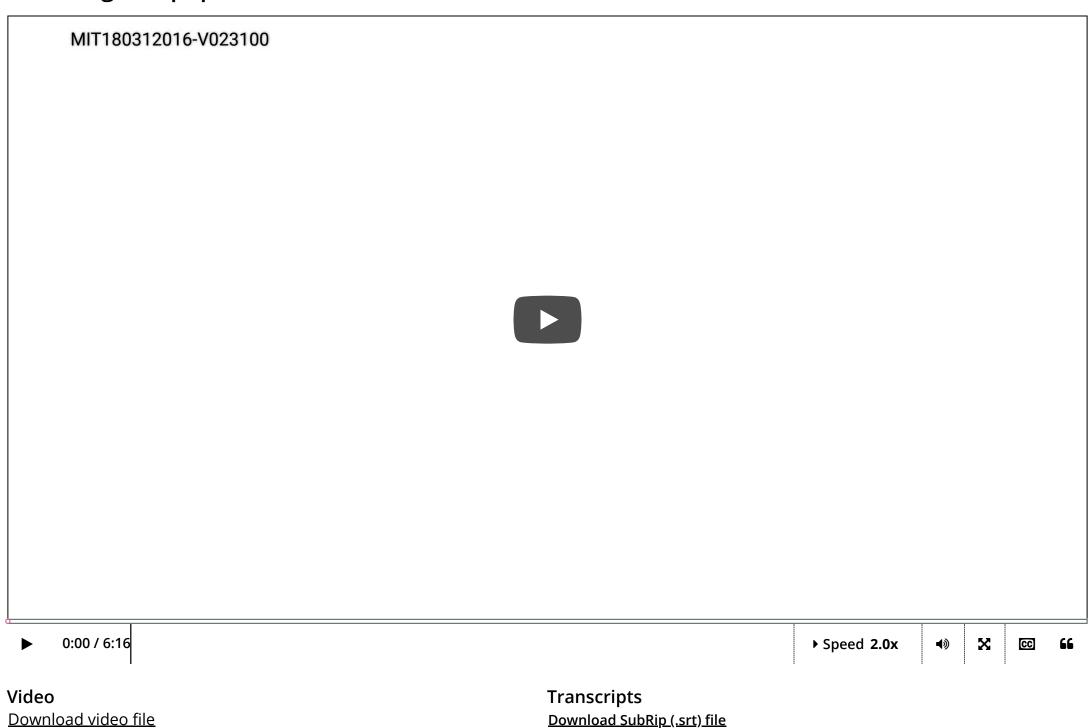
Final project: Applications to <u>Course</u> > <u>nonlinear differential equations</u> <u>Project 1: Review of nonlinear</u>

> populations models

3. Review: modeling two species

> populations

3. Review: modeling two species populations Modeling two populations



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The basic model for two interacting populations is the 2×2 autonomous system of **Lotka-Volterra equations**:

$$\dot{x} = (ax - bx^2) \pm cxy$$

$$\dot{y} = \left(dy - hy^2\right) \pm fxy.$$

where the equations govern the rates of change of the populations x and y respectively, and a, b, c, d, f, h are the 6 parameters of the system. Note that here we have written the equations so that the parameters are positive.

In each equation, the two terms in the brackets model the **logistic** growth of a single population in the absence of the other.

Interactions between the two populations

The final term in each equation is proportional to the product xy, and models the effect of interaction between x and y on the respective population. For example, in the first equation, which governs the rate of change of the population x, if the term proportional to xy is positive, then \dot{x} is increased. This means interactions are beneficial to the x population.

The signs of the xy terms in the two equations determine which of the following scenarios are modeled.

1. Competition:

$$\dot{x} = (ax - bx^2) - cxy$$

$$egin{array}{lll} \dot{x} & = & \left(ax-bx^2
ight)-cxy \ & & \ \dot{y} & = & \left(dy-hy^2
ight)-fxy. \end{array}$$

If the terms proportional to xy in both equations are negative, then interactions decrease the rates of change of both x and y. This models two species mutually harmful to one another. An example is moose and deer competing for vegetation in the same habitat.

2. Mutualism:

$$\dot{x} = \left(ax - bx^2
ight) + cxy$$

$$egin{array}{lcl} \dot{x} & = & \left(ax-bx^2
ight)+cxy \ & \ \dot{y} & = & \left(dy-hy^2
ight)+fxy. \end{array}$$

If the xy terms in both equations are positive, then interactions increase the rates of change of both x and y. This models two species mutually beneficial to one another. An example is human and the bacteria living in our digestive system.

5/24/2018

3. **Predator-prey**:

$$\dot{x} = (ax - bx^2) - cxy$$

$$egin{array}{lll} \dot{x} &=& \left(ax-bx^2
ight)-cxy \ & & \ \dot{y} &=& \left(dy-hy^2
ight)+fxy. \end{array}$$

If the signs of the xy terms in the two equations are different, then interaction benefits one population but harms the other. This models a predator species and a prey species. An example is wolves and deer, with the deer being a food source for the wolves.

Identify the type of population model

0 points possible (ungraded, results hidden) (Note this problem is for review and has zero weight towards your grade.)

Which of the following scenarios is modeled by the system

$$\dot{x} = x - x^2 - 2xy$$

$$\dot{y} = 3y - 2y^2 + xy$$

	Compe	tition.
--	-------	---------

Mutualism.

lacktriangle Predator-prey with $m{x}$ the predator and $m{y}$ the prey.

lacktriangle Predator-prey with $oldsymbol{y}$ the predator and $oldsymbol{x}$ the prey.

Submit

You have used 2 of 2 attempts

Identify the critical points

2 points possible (graded, results hidden) (This problem and the following count towards your grade.)

The system

$$egin{array}{lll} \dot{x}&=&x-x^2-axy \ \dot{y}&=&3y-2y^2-bxy \end{array}$$

has four critical points when $ab \neq 2$. Three of them are (0,0), (0,3/2), (1,0). Find the x and y coordinates of the fourth critical point. (For the 4 critical points to be distinct, please assume that $b \neq 3$ and $a \neq 2/3$.)

 \boldsymbol{x} coordinate:

(2-3*a)/(2-a*b)

 $\frac{2-3\cdot a}{2-a\cdot b}$

y coordinate:

(3-b)/(2-a*b)

3-b $\overline{2-a\cdot b}$

FORMULA INPUT HELP

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You have used 1 of 5 attempts

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