4. Describing and visualizing bounded and unbounded regions

Lecture due Sep 13, 2021 20:30 IST Completed

Practice

Describing regions

In order to describe a region R, we generally have to describe the boundary of that region, and a rule for determining if a point is inside of the region or not.

The typical way to do this is with an inequality (or a system of inequalities).

Examples 4.1

1. The interior and boundary of a square region whose vertices lie at the points $(0,0)$, $(1,0)$, $(0,1)$, and $(1,1)$ is described by the set of points (x,y) that satisfies the system of inequalities

$$0 \leq x \leq 1 \tag{4.109}$$

$$0 \leq y \leq 1 \tag{4.110}$$

.

2. If we wish to consider a region R given by an ellipse whose major axis has length 4 along the x axis and minor axis lies along the y-axis with length 2, one way to do this is to describe R as the set of points that satisfy $x^2/4 + y^2 \leq 1$.

   Note that $x^2/4 + y^2 = 1$ exactly describes the boundary ellipse. The condition $x^2/4 + y^2 \leq 1$ is specifying a relationship that describes all points that lie inside of that ellipse.

Visualizing regions

To understand a region described by inequalities, first draw the boundary curves that are described by the associated system equalities, then determine what it means for a point to satisfy the inequality, typically by shading the region.

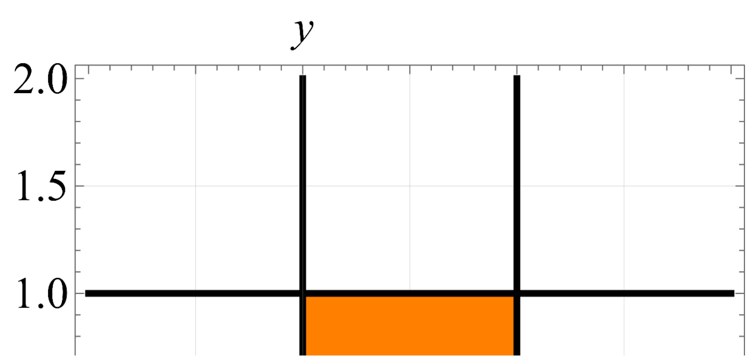
Example 4.2

To describe the square region:

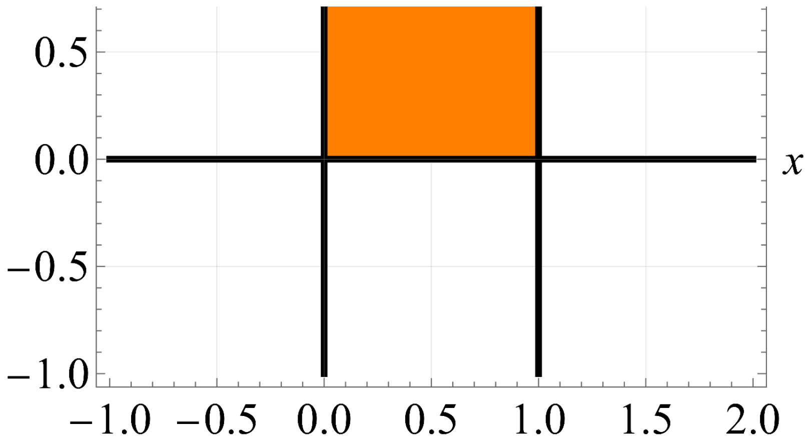
$$0 \leq x \leq 1 \tag{4.111}$$

$$0 \leq y \leq 1, \tag{4.112}$$

first draw the curves defining the boundary. The boundaries are the vertical lines $x = 0$ and $x = 1$, as well as the horizontal lines $y = 0$ and $y = 1$. The inequality $0 \leq x \leq 1$ specifies that the x values are restricted to the region between the two vertical lines. The inequality $0 \leq y \leq 1$ specifies that the y values are restricted to the region between the two horizontal lines.



Calculator Hide Notes

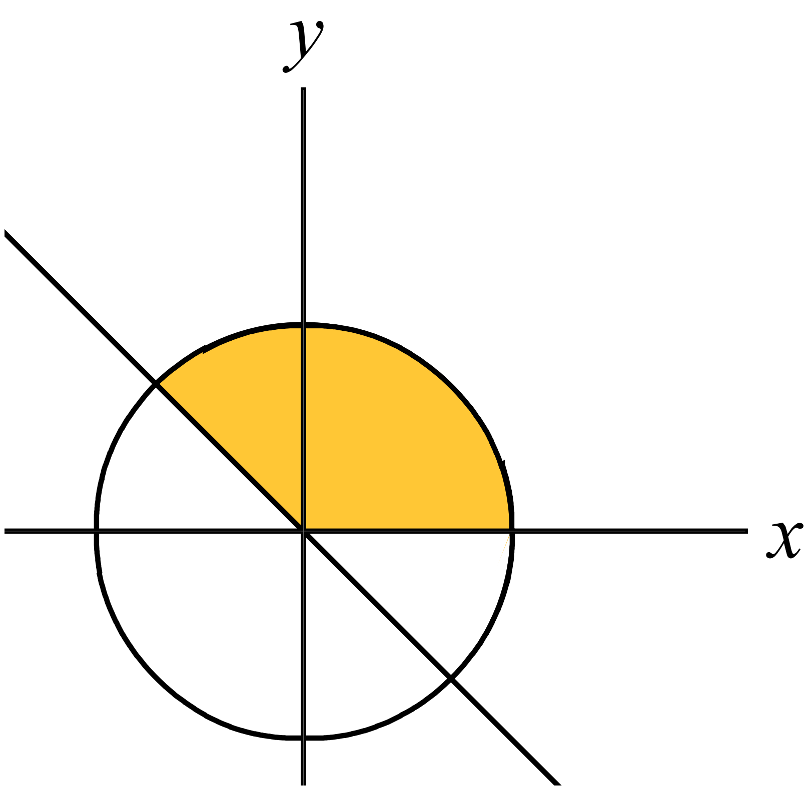


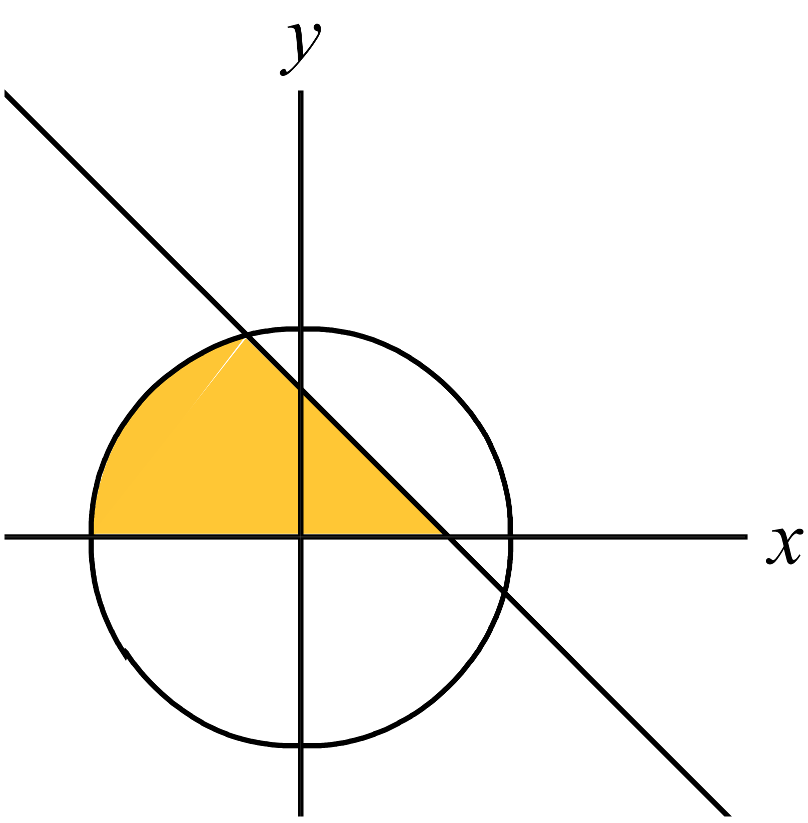
Practice with bounded regions 1

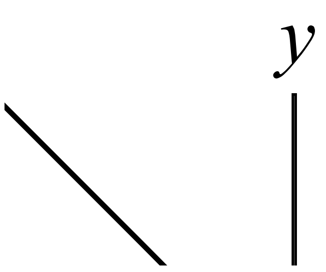
1/1 point (graded)

The bounded region R is defined by $y \geq 0$, $y + x \geq 1$, and $x^2 + y^2 \leq 2$.

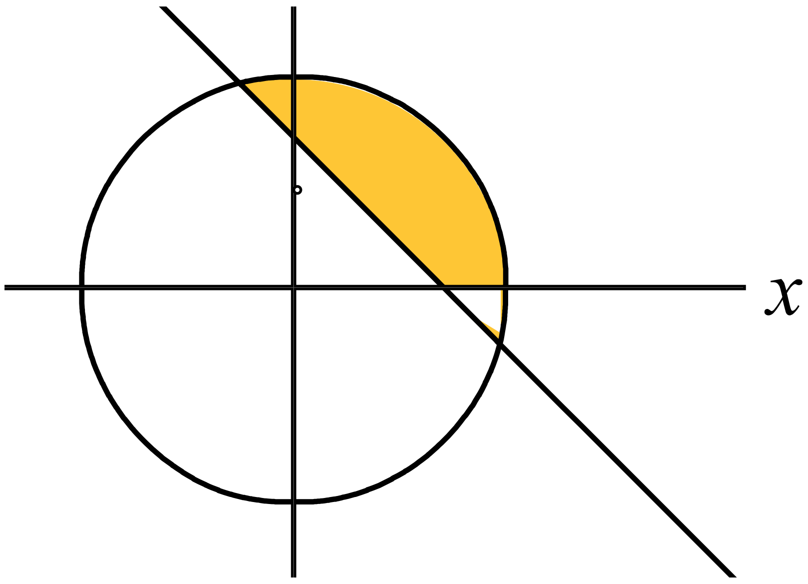
Which of the shaded orange regions is R?

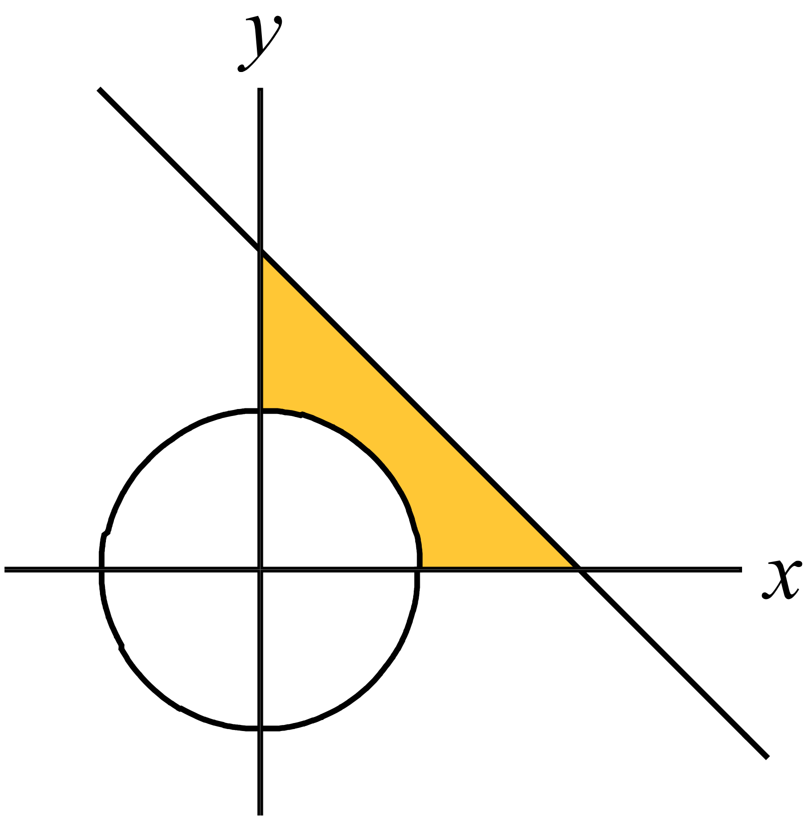






Calculator Hide Notes

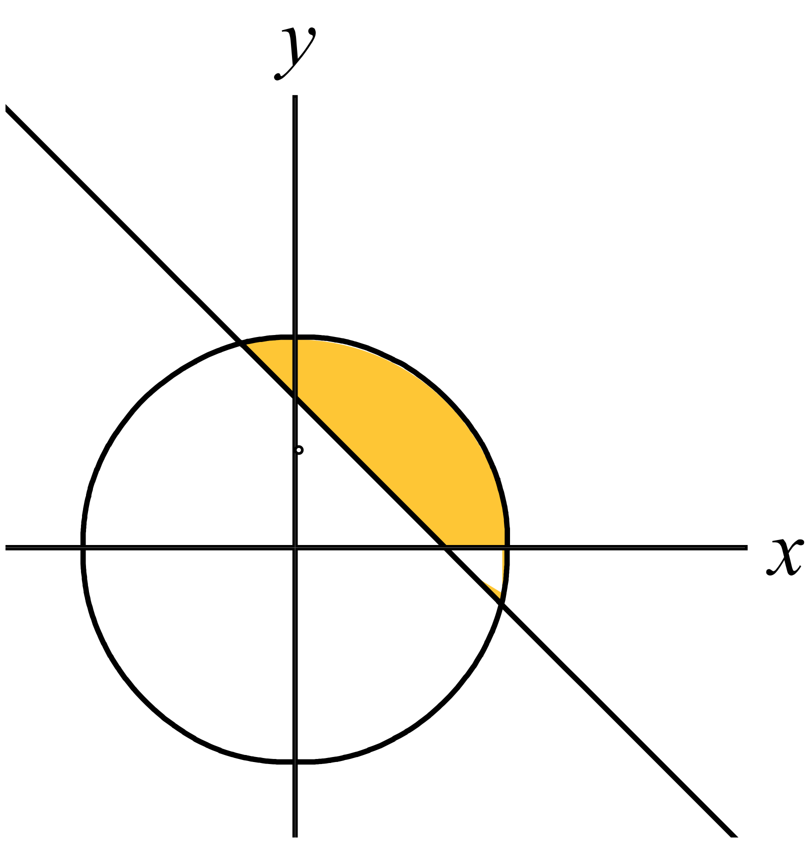




None of the above

Solution:

The region must be restricted to quadrants 1 and 2 because of the equation $y \geq 0$. All of the regions satisfy this inequality. The line $x + y \geq 1$ is the region above the line of slope -1 that intersects the y-axis at 1. The only region that satisfies this is the third option. And we know that the region should also lie inside of the circle of radius $\sqrt{2}$, and this region does in fact lie above the line but within this circular region in the first quadrant.



Calculator Hide Notes

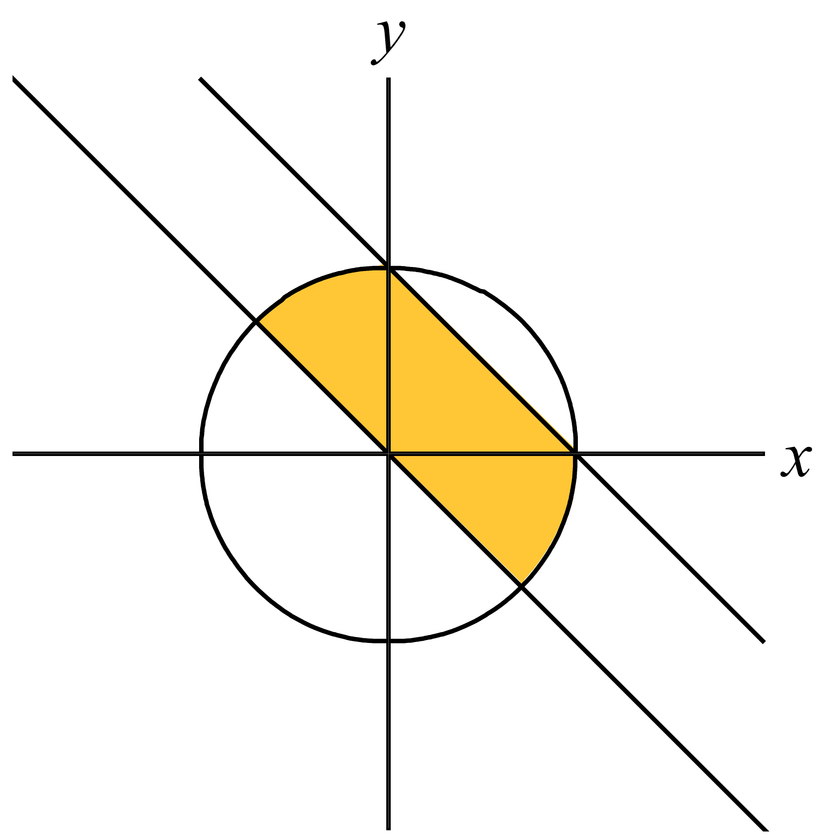
Submit You have used 1 of 3 attempts

Answers are displayed within the problem

Practice with bounded regions 2

1/1 point (graded)

Select the inequalities below that together define the bounded region shown. Note that the circle has radius 1.



- [ ] $x \geq 0$
- [ ] $y \geq 0$
- [x] $x + y \geq 0$
- [ ] $x + y \leq 0$
- [ ] $x + y \geq 1$
- [x] $x + y \leq 1$
- [ ] $x^2 + y^2 \geq 1$
- [x] $x^2 + y^2 \leq 1$
- [ ] $x + y \leq x^2 + y$

Solution:

The region inside the unit circle is $x^2 + y^2 \leq 1$. The region above the line $y = -x$ is given by the inequality $y \geq -x$, which can be rewritten as $y + x \geq 0$. The region below the line $y = 1 - x$ is given by the inequality $y \leq 1 - x$, which is the same as $x + y \leq 1$.

Calculator Hide Notes

Therefore these three inequalities completely define the region.

Submit You have used 1 of 3 attempts

Answers are displayed within the problem

Bounded or unbounded?

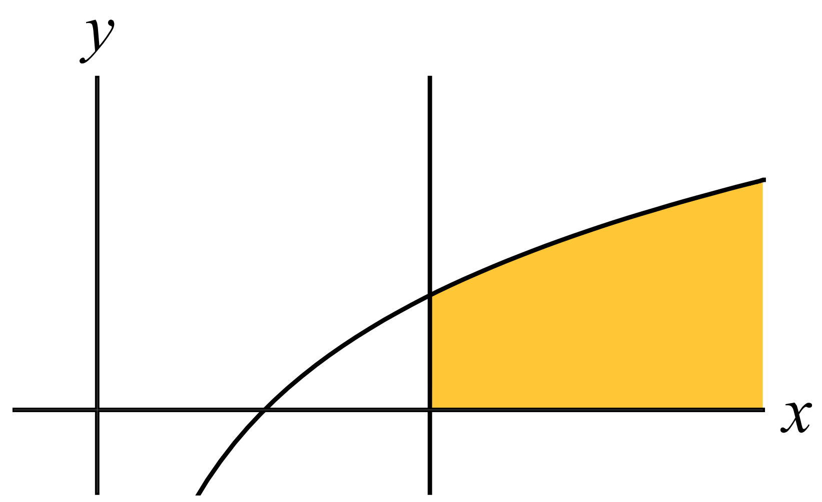
1/1 point (graded)
Consider the region R defined by the inequalities $x \geq 2$, $y \geq 0$, and $y \leq \ln(x)$.

This region is

- bounded
- unbounded

Solution:

The region $x \geq 2$ is the region to the right of the line $x = 2$. The region $y \geq 0$ is the region above the x-axis. The region $y \leq \ln(x)$ is the region below the curve $y = \ln(x)$. This region is unbounded.



Submit You have used 1 of 1 attempt

Answers are displayed within the problem

4. Describing and visualizing bounded and unbounded regions

Hide Discussion

Topic: Unit 3: Optimization / 4. Describing and visualizing bounded and unbounded

< Previous Next >

Calculator Hide Notes

edX

[About](#)
[Affiliates](#)
[edX for Business](#)
[Open edX](#)
[Careers](#)
[News](#)

Legal

[Terms of Service & Honor Code](#)
[Privacy Policy](#)
[Accessibility Policy](#)
[Trademark Policy](#)
[Sitemap](#)

Connect

[Blog](#)
[Contact Us](#)
[Help Center](#)
[Media Kit](#)
[Donate](#)

GET IT ON Google play

Download on the App Store

Calculator Hide Notes