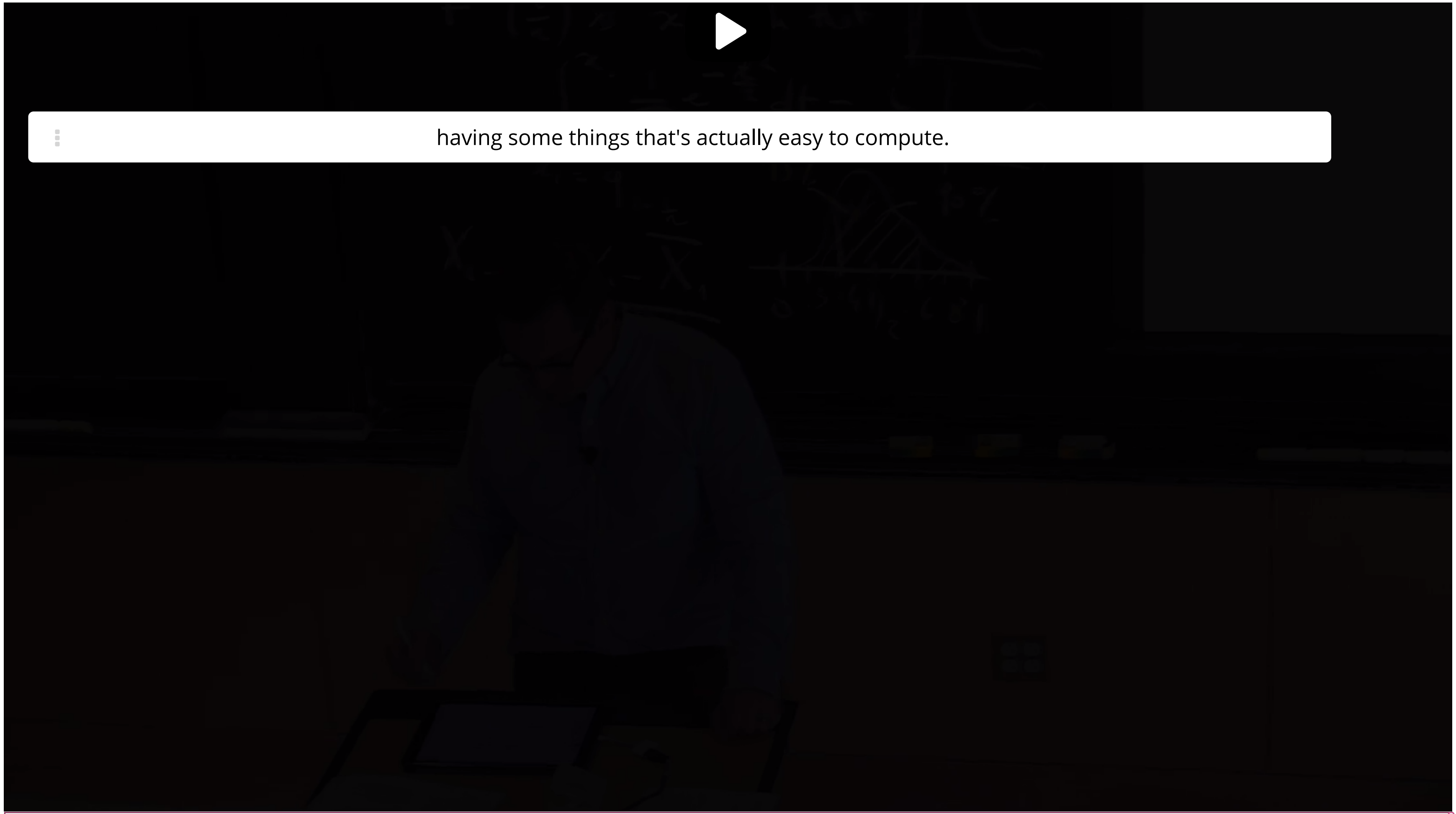
5. The Prior Distribution

Clinical Trial Examples, Source of Priors



3:53 / 3:53 1.50x

Video
[Download video file](#)

Transcripts
Download SubRip (.srt) file
Download Text (.txt) file

Designing Priors for an Experiment: Coin tossing

1/1 point (graded)

In a coin tossing experiment, let $Y_1, Y_2, \ldots, Y_n$ be i.i.d. random variables corresponding to the toss of the same coin whose bias θ is unknown. Assume that the model is

$$\mathbf{P}_{Y|\theta} = \mathrm{Ber}\left(\theta\right),$$

and that the coin lands heads or tails with some positive probability. We want to design a prior for the unknown bias, in order to estimate the bias of the coin.

Which one of the following distributions would be a realistic candidate to model the prior on the set of all possible biases Θ?

- ○ A Bernoulli distribution.
- ○ An exponential distribution.
- ◉ A uniform distribution.
- ○ A Gaussian distribution.

✔

Solution:

The answer is the uniform distribution.

Since the bias, θ is a parameter for the coin tossing, we have, $\Theta = [0,1]$. Namely, we need a distribution supported on the set $[0,1]$.

- The Bernoulli distribution may not be used as it only allows the bias to be 0 or 1, which means that the coin either always lands heads or tails, whereas we assumed that the coin lands heads or tails with some positive probability.
- Since the exponential distribution is supported on $(0,\infty)$, it cannot be a correct candidate for the prior.
- A uniform distribution is a possible candidate; which reflects that we believe equally for each value of the $\theta \in [0,1]$.

- Finally, a Gaussian distribution is, again, not a possible candidate, since it has a support of $(-\infty, \infty)$.

Submit You have used 1 of 2 attempts

Answers are displayed within the problem

Scientist in the Desert

1/1 point (graded)

Suppose that a crazy scientist is traveling with his car in the desert. All of a sudden, he realizes that the engine's oil lamp is broken. Hence, he needs to estimate the remaining oil level to determine whether he can continue driving or should instead seek help. Since he has studied Bayesian statistics before, he wants to set up an experiment to estimate the oil level of his car, but for this, he needs a prior distribution on the oil level. Let us denote the density of the prior on the oil level by $f_o(x)$. The scientists knows that at any given time, the oil level x is guaranteed to be a real number in the interval $[0, 100]$.

Which properties should $f_o(\cdot)$ obey in order to result in a valid probability distribution and also reflect the scientist's knowledge of the car? (Choose all that apply.)

- [x] $f_o(x) \geq 0$, for every $x \in [0, 100]$.
- [x] $\int_0^{100} f_o(x)\ dx = 1.$
- [x] $\int_{-\infty}^{0} f_o(x)\ dx = 0.$
- [x] $\int_{100}^{\infty} f_o(x)\ dx = 0.$
- [ ] $\max_{x\in[0,100]} f_o(x) = f_o(50)$,namely, $f_o(\cdot)$ attains its maximum, in the midpoint of the interval.

- [ ] $f_o(\cdot)$ should be a decreasing function, on $[0, 50)$, and on $(50, \infty]$.

✔

Solution:

First of all, independent of the experiment, the conditions for $f_o(\cdot)$ to be a valid p.d.f., it should be non-negative on its support and must integrate to one on its support. The first two choices satisfy these conditions.

Furthermore, we must also have,

$$\int_{-\infty}^{0} f_o(x)\ dx = 0 = \int_{100}^{\infty} f_o(x)\ dx,$$

since the oil level is between $[0, 100]$.

The last two parts are not necessarily properties of a probability distribution. As a counterexample, the truncated normal distribution centered at $x = 40$ satisfies neither of the last two properties.

Submit You have used 1 of 3 attempts

Answers are displayed within the problem

Prior Design for a Gambler

1/1 point (graded)

While on your way to work, you see a gambler who invites people to bet on a coin that he is repeatedly tossing. You suspect that his coin is biased and that he is cheating people out of their money; in particular, you think that it is more likely for the coin to be biased one way or the other, and that it is unlikely that the coin is fair. In order to further understand this, you decide to model the bias of the coin, θ, using Bayesian statistics.

You have a **prior belief** is that the gambler's coin is *biased* (remember that the bias can be either way: either towards Heads or Tails). One reasonable criterion for the prior to reflect the belief stated above is for $\pi(\theta)$ to attain its minimum at $1/2$, be *strictly decreasing* from $[\epsilon, \frac{1}{2}]$, and lastly *strictly increasing* from $[\frac{1}{2}, 1-\epsilon]$. Which of the following priors satisfies this given criterion? (Choose all that apply.)

(Note: for each answer choice, the denominator Z is chosen to make sure that the integral of the density over its support is equal to 1. Moreover, we restrict ourself to the bias values in the interval $[\epsilon, 1-\epsilon]$, rather than the actual interval, $(0,1)$. Here we assume that ϵ is a very small number, e.g. $\epsilon = 10^{-5}$. You may want to use software to graph the given functions.)

- [ ] $\pi(\theta) \sim \mathrm{Unif}[\epsilon, 1-\epsilon]$
- [x] $\pi(\theta) = \frac{\theta^2+(1-\theta)^2}{Z}, \theta \in [\epsilon, 1-\epsilon]$
- [ ] $\pi(\theta) = \frac{\theta(1-\theta)}{Z}, \theta \in [\epsilon, 1-\epsilon]$
- [x] $\pi(\theta) = \frac{1/\theta+1/(1-\theta)}{Z}, \theta \in [\epsilon, 1-\epsilon]$

✔

Solution:

- As the uniform distribution weights each possibility equally, it cannot be a correct choice.
- Note that

$$f(\theta) = \theta^2 + (1-\theta)^2 = 2\theta^2 - 2\theta + 1 \implies f'(\theta) = 4\theta - 2.$$

  Setting the first derivative equal to 0, we obtain that, $f(\theta)$ attains its minimum at $1/2$, and so does $\pi(\theta)$. It also implies that $\pi(\theta)$ is indeed decreasing in $[\epsilon, \frac{1}{2}]$ and increasing in $[\frac{1}{2}, 1-\epsilon]$.

- As discussed in the lecture, the function $\theta(1-\theta)$ obtains its **maximum** at $\theta = 1/2$, so it cannot be a suitable prior for this experiment.
- Finally, as

$$\pi(\theta) \propto \frac{1}{\theta(1-\theta)},$$

and $f(\theta) = \theta(1-\theta)$ attains its maximum at $1/2$, $\pi(\theta)$ attains its minimum at $1/2$. In addition, $\pi(\theta)$ is decreasing in $[\epsilon, \frac{1}{2}]$ and increasing in $[\frac{1}{2}, 1-\epsilon]$, as desired.

Submit You have used 1 of 3 attempts

Answers are displayed within the problem

Discussion

Hide Discussion

Topic: Unit 5 Bayesian statistics:Lecture 17: Introduction to Bayesian Statistics / 5. The Prior Distribution

Add a Post

Show all posts ▼ by recent activity ▼

There are no posts in this topic yet.

✖