Exercise: Mutual vs Pairwise Independence

(1/1 point)

Suppose random variables X and Y are independent, where X is 1 with probability 1/2, and -1 otherwise. Similarly, Y is also 1 with probability 1/2, and -1 otherwise. In this case, we say that X and Y are *identically distributed* since they have the same distribution (remember, just because they have the same distribution doesn't mean that they are the same random variable — here X and Y are independent!). Note that often in this course, we'll be seeing random variables that are independent and identically distributed (i.i.d.).

Suppose we have another random variable Z that is the product of X and Y, i.e., $Z = XY$.

Select all of the following that are true:

- [x] The distributions p_X, p_Y, and p_Z are the same. ✔
- [x] The joint distributions $p_{X,Y}$, $p_{X,Z}$, and $p_{Y,Z}$ are the same. ✔
- [x] X, Y, and Z are pairwise independent. ✔
- [ ] X, Y, and Z are mutually independent.

Solution:

The fastest way to solve this problem is to realize that it's actually the same problem as in the previous video where we had X and Y independent and identically distributed as $\text{Bernoulli}(1/2)$, and Z was the exclusive-or (XOR) of X and Y. All we did was slightly change the labels of the outcomes to get this problem! Notice that Z takes on value -1 precisely when X and Y are different, and 1 otherwise. Hopefully that should sound like XOR. Basically -1 is what used to be 1, and 1 is what used to be 0. The problem is otherwise the same and the identical reasoning used in the video can be used here, so we won't actually spell out the solution again in detail (it's in the video!).

As a reminder, you can check that p_X, p_Y, and p_Z are each going to have 1/2-1/2 chance of being either 1 or -1, so they have the same distribution, and when we look at any pair of the random variables, they are going to appear independent with (1, 1), (1, -1), (-1, 1), and (-1, -1) equally likely so the pairs of random variables also have the same distribution. However, as before, when we look at all three random variables, they are not mutually independent!

You have used 2 of 5 submissions