6. A confidence interval for uniform distributions

(a)

2/2 points (graded)

Let $X_1, \ldots, X_n$ be i.i.d. uniform random variables in $[0, \theta]$, for some $\theta > 0$. Denote by

$$M_n = \max_{i=1,\ldots,n} X_i.$$

Compute the following probabilities:

$\mathbf{P}(M_n \geq \theta) =$ 0 ✔

0

For all $0 \leq t \leq \theta$:

$\mathbf{P}\left(M_n \leq \theta - t\right) =$ (1-t/theta)^n ✔

(*Food for thought:* What can you conclude?)

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✔ Correct (2/2 points)

(b)

2/2 points (graded)

Compute the cumulative distribution function $F_n\left(t\right)$ of $n\left(1 - M_n/\theta\right)$ for fixed $t \in \left[0, n\right]$ and any positive integer n.

$F_n\left(t\right) =$ 1-(1-t/n)^n ✔

Compute the following limit.

$\lim_{n\to\infty} F_n\left(t\right) =$ 1-e^(-t) ✔

(*Food for thought:* Again, What can you conclude?)

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✔ Correct (2/2 points)

(c)

2/2 points (graded)

Next, we will use the previous question to find an interval $\mathcal{I}$ of the form $\mathcal{I} = [M_n, M_n + c]$, that does not depend on θ and such that

$$\mathbf{P}[\mathcal{I} \ni \theta] \to .95, \text{ as } n \to \infty.$$

The strategy now is to use a plug-in estimator for θ to replace it in the expression for c. Parts (a) and (b) suggest that we use c of the form $\left(\frac{t}{n}\right) M_n$, where t ought to equal a certain value in order for $\mathbf{P}[\mathcal{I} \ni \theta] \to .95$. What is the appropriate numerical value of t?

$t =$ 2.995732273553991 ✔

Why can we use a plugin-estimator for the asymptotic confidence interval?

- By the Delta Method, the asymptotic variance scales with the square of the first derivative of the plugin function.
- By Slutsky's Theorem, we can combine convergence in distribution of Y_n and in probability of Z_n if Z_n converges to a constant.
- By the Central Limit Theorem, the plugin variable will again be normally distributed.

✔

STANDARD NOTATION

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✔ Correct (2/2 points)

(d)

1/1 point (graded)

Compute the bias of M_n as an estimator of θ.

$\mathbb{E}[M_n] - \theta =$ -theta/(n+1) ✔

$-\frac{\theta}{n+1}$

STANDARD NOTATION

Submit You have used 1 of 3 attempts

✔ Correct (1/1 point)

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i got this error message for 6a... 1

? Delta Method
Hello Staff, a silly question. Part c) of the problem refers (in one option) to the Delta Method, which is subject of lecture 5. I was supposed that lectures and homeworks are to ... 1

ving a hard time understanding the estimation for c
I took for granted the term was t/n*Mn, and solved it. However, I don't get from a and b how to reach that one. 17

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💬	"Parts (a) and (b) suggest that we use c of the form (t/n)*M_n", how? don't get this. "Parts (a) and (b) suggest that we use c of the form (t/n)*M_n", why? don't get this.	2
💬	Should 6a box 2 depend on n. I keep putting in what I think is correct with a dependence on n, however nothing seems to work. Is there a clue I'm missing here?	2
?	limit part b I got the first part right, so I can see the way Fn is expressed but I can't get the limit right (in my case I got a single digit result). Is it expected to depend on any variable and tha...	6
?	Clarification on notation used in part b	2
?	Stuck again on the interval question....	5
?	Help on Part B I got part a right, but already got 2 attempts incorrect with part b, and if I don't get this right, I cannot continue with the rest of the problems! I approached this problem the s...	4
💬	Solutions. Hello, I'm wondering if it's possible to see the solutions, even if you get the correct answer. Because some of these, it seems like it should be right, but you don't really know a...	2

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