3. Review: modeling two species populations

Modeling two populations

MIT180312016-V023100

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The basic model for two interacting populations is the 2×2 autonomous system of **Lotka-Volterra equations** :

$$\begin{aligned}\dot{x} &= \left(ax - bx^2\right) \pm cxy \\ \dot{y} &= \left(dy - hy^2\right) \pm fxy.\end{aligned}$$

where the equations govern the rates of change of the populations x and y respectively, and $a,\ b,\ c,\ d,\ f,\ h$ are the 6 parameters of the system. Note that here we have written the equations so that the parameters are positive.

In each equation, the two terms in the brackets model the **logistic** growth of a single population in the absence of the other.

Interactions between the two populations

The final term in each equation is proportional to the product $xy,$ and models the effect of interaction between x and y on the respective population. For example, in the first equation, which governs the rate of change of the population $x,$ if the term proportional to xy is positive, then $\dot{x}$ is increased. This means interactions are beneficial to the x population.

The signs of the xy terms in the two equations determine which of the following scenarios are modeled.

1. **Competition**:

$$\begin{aligned}\dot{x} &= \left(ax - bx^2\right) - cxy \\ \dot{y} &= \left(dy - hy^2\right) - fxy.\end{aligned}$$

   If the terms proportional to xy in both equations are negative, then interactions decrease the rates of change of both x and y. This models two species mutually harmful to one another. An example is moose and deer competing for vegetation in the same habitat.

2. **Mutualism**:

$$\begin{aligned}\dot{x} &= \left(ax - bx^2\right) + cxy \\ \dot{y} &= \left(dy - hy^2\right) + fxy.\end{aligned}$$

   If the xy terms in both equations are positive, then interactions increase the rates of change of both x and y. This models two species mutually beneficial to one another. An example is human and the bacteria living in our digestive system.

3. **Predator-prey**:

$$\begin{aligned} \dot{x} &= \left(ax - bx^2\right) - cxy \\ \dot{y} &= \left(dy - hy^2\right) + fxy. \end{aligned}$$

If the signs of the xy terms in the two equations are different, then interaction benefits one population but harms the other. This models a predator species and a prey species. An example is wolves and deer, with the deer being a food source for the wolves.

Identify the type of population model

0 points possible (ungraded, results hidden)
(Note this problem is for review and has zero weight towards your grade.)

Which of the following scenarios is modeled by the system

$$\begin{aligned} \dot{x} &= x - x^2 - 2xy \\ \dot{y} &= 3y - 2y^2 + xy \end{aligned}$$

- ○ Competition.
- ○ Mutualism.
- ○ Predator-prey with x the predator and y the prey.
- ◉ Predator-prey with y the predator and x the prey.

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Identify the critical points

2 points possible (graded, results hidden)
(This problem and the following count towards your grade.)

The system

$$\begin{aligned}\dot{x} &= x - x^2 - axy \\ \dot{y} &= 3y - 2y^2 - bxy\end{aligned}$$

has four critical points when $ab \neq 2$. Three of them are $(0,0)$, $(0,3/2)$, $(1,0)$. Find the x and y coordinates of the fourth critical point. (For the 4 critical points to be distinct, please assume that $b \neq 3$ and $a \neq 2/3$.)

x coordinate: (2-3*a)/(2-a*b)

$\frac{2-3\cdot a}{2-a\cdot b}$

y coordinate: (3-b)/(2-a*b)

$\frac{3-b}{2-a\cdot b}$

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