

EdX and its Members use cookies and other tracking technologies for performance, analytics, and marketing purposes. By using this website, you accept this use. Learn more about these technologies in the [Privacy Policy](#).



[Unit 1 Linear Classifiers and](#)  
[Course](#) > [Generalizations \(2 weeks\)](#) > [Homework 1](#) > 2. Perceptron Performance

## 2. Perceptron Performance

In class we initialized the perceptron algorithm with  $\theta = 0$ . In this problem we will also explore other initialization choices.

### 2. (a)

2.0/2 points (graded)

The following table shows a data set and the number of times each point is misclassified during a run of the perceptron algorithm (**with offset**  $\theta_0$ ).  $\theta$  and  $\theta_0$  are initialized to zero.

$i$	$x^{(i)}$	$y^{(i)}$	times misclassified
1	[-4, 2]	+1	1
2	[-2, 1]	+1	0

3	$[-1, -1]$	-1	2
4	$[2, 2]$	-1	1
5	$[1, -2]$	-1	0

Write down the state of  $\theta$  and  $\theta_0$  after this run has completed (note, the algorithm may not yet have converged). Enter  $\theta$  as a list  $[\theta_1, \theta_2]$  and  $\theta_0$  as a single number in the following boxes.

Please enter  $\theta$ :

✓ Answer:  $[-4, 2]$

Please enter  $\theta_0$  :

✓ Answer: -2

### Solution:

- Since perceptron update rule updates  $\theta$  simply by adding  $x^{(i)}y^{(i)}$ , the resulting  $\theta$  should be the summation of all mistakes.
- Additional Insight: since perceptron update rule is additively associative, doing updates in any order would lead to the same result.

Submit

You have used 1 of 3 attempts

**i** Answers are displayed within the problem

## 2. (b)

2/2 points (graded)

Provide one example of a different initialization of  $\theta$  such that the perceptron algorithm with this initialization would not produce any mistakes during a run through the data.

$[\theta_1, \theta_2]$ :   Answer: See solution

$\theta_0$ :   Answer: See solution

### Solution:

The answer  $(\theta, \theta_0)$  should be such that:

- $-4\theta_1 + 2\theta_2 + \theta_0 > 0$
- $-2\theta_1 + \theta_2 + \theta_0 > 0$
- $-\theta_1 - \theta_2 + \theta_0 < 0$
- $2\theta_1 + 2\theta_2 + \theta_0 < 0$

- $1\theta_1 - 2\theta_2 + \theta_0 < 0$

For instance, any line strictly inside the boundry  $x_2 = x_1$  and  $x_2 = x_1 + 3$  are valid solutions.

Submit

You have used 2 of 3 attempts

**i** Answers are displayed within the problem

## 2. (c)

3.0/3 points (graded)

The theorem from question 1. (e) provides an upper bound on the number of steps of the Perceptron algorithm and implies that it indeed converges. In this question, we will show that the result still holds even when  $\theta$  is not initialized to 0.

In other words: Given a set of training examples that are linearly separable through the origin, show that the initialization of  $\theta$  does not impact the perceptron algorithm's ability to eventually converge.

To derive the bounds for convergence, we assume the following inequalities holds:

- There exists  $\theta^*$  such that  $\frac{y^{(i)} (\theta^* x^{(i)})}{\|\theta^*\|} \geq \gamma$  for all  $i = 1, \dots, n$  and some  $\gamma > 0$
- All the examples are bounded  $\|x^{(i)}\| \leq R, i = 1, \dots, n$

If  $\theta$  is initialized to 0, we can show by induction that:

$$\theta^{(k)} \cdot \frac{\theta^*}{\|\theta^*\|} \geq k\gamma$$

For instance,

$$\theta^{(k+1)} \cdot \frac{\theta^*}{\|\theta^*\|} = (\theta^{(k)} + y^{(i)} x^{(i)}) \cdot \frac{\theta^*}{\|\theta^*\|} \geq (k+1)\gamma$$

If we initialize  $\theta$  to a general (not necessarily 0)  $\theta^{(0)}$ , then:

$$\theta^{(k)} \cdot \frac{\theta^*}{\|\theta^*\|} \geq a + k\gamma$$

Determine the formulation of  $a$  in terms of  $\theta^*$  and  $\theta^{(0)}$ :

**Important:** Please enter  $\theta^*$  as `theta^{star}` and  $\theta^{(0)}$  as `theta^{0}`, and use `norm(...)` for the vector norm  $\|\dots\|$ .

$a =$

✓ **Answer:** `theta^{0}*theta^{star} / norm(theta^{star})`

If  $\theta$  is initialized to 0, we can show by induction that:

$$\|\theta^{(k)}\|^2 \leq kR^2$$

For instance,

$$\|\theta^{(k+1)}\|^2 \leq \|\theta^{(k)} + y^{(i)} x^{(i)}\|^2 \leq \|\theta^{(k)}\|^2 + R^2$$

If we initialize  $\theta$  to a general (not necessarily 0)  $\theta^{(0)}$ , then:

$$\|\theta^{(k)}\|^2 \leq kR^2 + c^2$$

Determine the formulation of  $c^2$  in terms of  $\theta^{(0)}$ :

$$c^2 = \text{norm}(\theta^{(0)})^2$$

✓ Answer:  $\text{norm}(\theta^{(0)})^2$

From the above inequality, we can derive the inequality  $\|\theta^{(k)}\| \leq c + \sqrt{k}R$  by applying the following inequality:

$$\sqrt{x^2 + y^2} \leq \sqrt{(x + y)^2} \text{ if } x, y > 0.$$

If  $\theta$  is initialized to 0, we then use the fact that  $1 \geq \frac{\theta^{(k)}}{\|\theta^{(k)}\|} \cdot \frac{\theta^*}{\|\theta^*\|}$  to get the upper bound  $k \leq \frac{R^2}{\gamma^2}$ .

In the case where we initialize  $\theta$  to a general  $\theta^{(0)}$ , use the inequality for  $\theta^{(k)} \cdot \frac{\theta^*}{\|\theta^*\|}$  above and the inequality  $\|\theta^{(k)}\| \leq c + \sqrt{k}R$  to derive a bound on the number of iterations  $k$ .

**Hint:** Use the larger root of a quadratic equation to obtain the upper bound.

**Note:** Give your answer in terms of  $a, c, R, \gamma$  (enter the latter as gamma).

$k \leq$

$$(R^2/2 + R \cdot \sqrt{R^2 - 4 \cdot a \cdot \gamma + 4 \cdot c \cdot \gamma})/2 - a \cdot \gamma + c \cdot \gamma / \gamma^2$$



**Answer:**  $(R \cdot \sqrt{R^2 + 4 \cdot (c - a) \cdot \gamma} + R^2 + 2 \cdot (c - a) \cdot \gamma) / (2 \cdot \gamma^2)$

$$\frac{\frac{R^2}{2} + \frac{R \cdot \sqrt{R^2 - 4 \cdot a \cdot \gamma + 4 \cdot c \cdot \gamma}}{2} - a \cdot \gamma + c \cdot \gamma}{\gamma^2}$$

STANDARD NOTATION

**Solution:**

The first bound follows by recursion of  $\theta^k \cdot \frac{\theta^*}{\|\theta^*\|} \geq \theta^{k-1} \cdot \frac{\theta^*}{\|\theta^*\|} + \gamma$ .

The second bound follows by recursion of  $\|\theta^k\|^2 \leq \|\theta^{k-1}\|^2 + R^2$ .

The final bound is obtained by solving the inequality  $1 \geq \theta^k \cdot \frac{\theta^*}{\|\theta^k\| \|\theta^*\|} \geq \frac{a + k\gamma}{c + \sqrt{k}R}$ , i.e.  $a + k\gamma - c \leq \sqrt{k}R$ .

At this point, you can square both sides and solve the quadratic equation to get the upper bound.

Alternatively, solve the quadratic equation for  $\sqrt{k}$  and square the answer to get the desired upper bound:

$$k \leq \frac{(R + \sqrt{R^2 - 4\gamma(a - c)})^2}{4\gamma^2}$$

Submit

You have used 2 of 3 attempts

**i** Answers are displayed within the problem

## 2. (d)

2/2 points (graded)

Since the convergence of the perceptron algorithm doesn't depend on the initialization, the end performance on the training set must be the same. Are the resulting  $\theta$ 's the same regardless of the initialization?

☐ Yes

☒ No ✓

Does this necessarily imply that the performance on a test set is the same?



☐ Yes☒ No ✓**Solution:**

- Any distinct  $\theta$  that can separate the data are valid solutions, so there are infinitely many different valid correct  $\theta$  in general given that the data can be separated by more than 1 line.
- Two different  $\theta$  would always make different predictions for a testing data point between the two lines, so the testing performance is always different for a testing dataset that contains exactly this point.

Submit

You have used 3 of 3 attempts

**i** Answers are displayed within the problem

## Discussion













[Hide Discussion](#)

**Topic:** Unit 1 Linear Classifiers and Generalizations (2 weeks):Homework 1 / 2. Perceptron Performance

[Add a Post](#)

Show all posts ▼

by recent activity ▼

 <u>2(a) and 2(b) Python Program Issue due to Incorrect Understanding of Math</u>	13
<u>For question 2(a) I calculated the values for theta and theta_0 by hand, both answers were wrong. I assumed this was an algebraic issue, so I wrote...</u>	
 <u>Unable to submit anything</u>	2
<u>Amazingly, except for this post. Please help.</u>	
 <u>c) the corrector refused my answer saying <math>R^2</math> cannot be accepted</u>	2
<u>c) the corrector refused my answer saying <math>R^2</math> cannot be accepted. I was right at the beginning. Very unfair. Improve!</u>	
 <u>2.(c) Don't understand the question.</u>	16
 <u>Community TA</u>	
 <u>correct answers but not able to submit</u>	2
<u>Hello support team; I have entered correct answer in correct format for question 2c (first and second). However, the page was not letting me submit...</u>	
 <u>Problem 2(C) Where to start?!</u>	7
<u>The one day extension on Homework 1 gives me the extra time to attempt problem # 2(c). That is the good news, bad news is I am clueless! If so...</u>	
 <u>Grader 2b</u>	18
<u>Is grader accepting any right solution there? My initialization does not make any mistake in a run but marked wrong</u>	
 <u>[staff] marks in progress bar did not update to 100% though I got all correct (1,2,3 questions)</u>	4
<u>I have got all questions in (1,2,3) correct, but the grade shows I did not get 100%; I think marks for 2c which I recently attempted and successfully...</u>	
 <u>2a point 4 never misclassified</u>	7
<u>I made a python program that runs the whole algorithm and plots all the points and the line, the algorithm runs until there is no more misclassification...</u>	
 <u>Is there a mathematical approach or are there any other approaches to solving question (2a)?</u>	11
<u>Hey all! I finished the question already and got the correct answer. However, I had to write my own code on python and use breakpoints to get the theta...</u>	
 <u>Q2(c) Invalid Input: star, theta not permitted in answer as a variable</u>	4
<u>"Invalid Input: star, theta not permitted in answer as a variable" --I am getting the above comment from the grader. I followed the instruction: "I..."</u>	

🗨 Lost

7

I don't really know where to start thinking this problem. What does gamma has to do with the algorithm? I lack an intuitive understanding of the ...

🗨 k <= ?

2

I see a lot of mates have struggled specifically with  $c^2$ . I haven't tested it yet but I think at this point I got it, and I'm struggling with filling the ine...

© All Rights Reserved