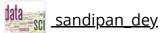


<u>Help</u>





Unit 1: Probability, Counting, and

<u>Course</u> > <u>Story Proofs</u>

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# 1.5 Story Proofs Unit 1: Probability and Counting

Adapted from Blitzstein-Hwang Chapter 1.

A *story proof* is a proof by interpretation. For counting problems, this often means counting the same thing in two different ways, rather than doing tedious algebra. A story proof often avoids messy calculations and goes further than an algebraic proof toward *explaining* why the result is true. The word "story" has several meanings, some more mathematical than others, but a story proof (in the sense in which we're using the term) is a fully valid mathematical proof. Here are some examples of story proofs, which also serve as further examples of counting.

#### Example 1.5.1 (The team captain).

For any positive integers n and k with k < n,

$$ninom{n-1}{k-1}=kinom{n}{k}.$$

This is again easy to check algebraically, using the fact that m! = m(m-1)! for any positive integer m, but a story proof is more insightful.

## Story proof

Consider a group of n people, from which a team of k will be chosen, one of whom will be the team captain. To specify a possibility, we could first choose the team captain and then choose the remaining k-1 team members; this gives the left-hand side. Equivalently, we could first choose the k team members and then choose one of them to be captain; this gives the right-hand side.

## Example 1.5.2 (Vandermonde's identity).

A famous relationship between binomial coefficients, called Vandermonde's identity, says that

$$inom{m+n}{k} = \sum_{j=0}^k inom{m}{j} inom{n}{k-j}.$$

This identity will come up several times in this course. Trying to prove it with a brute force expansion of all the binomial coefficients would be a might nate. But a story proves the result elegantly and makes it clear why the identity holds.

#### Story proof

Consider a group of m peacocks and n toucans, from which a set of size k birds will be chosen. There are  $\binom{m+n}{k}$  possibilities for this set of birds. If there are j peacocks in the set, then there must be k-j toucans in the set. The right-hand side of Vandermonde's identity sums up the cases for j.

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