You are taking "Exam (Timed, No Correctness Feedback)" as a timed exam. Show more

End My Exam 22:48:18

5. Maximum along the boundary intuition

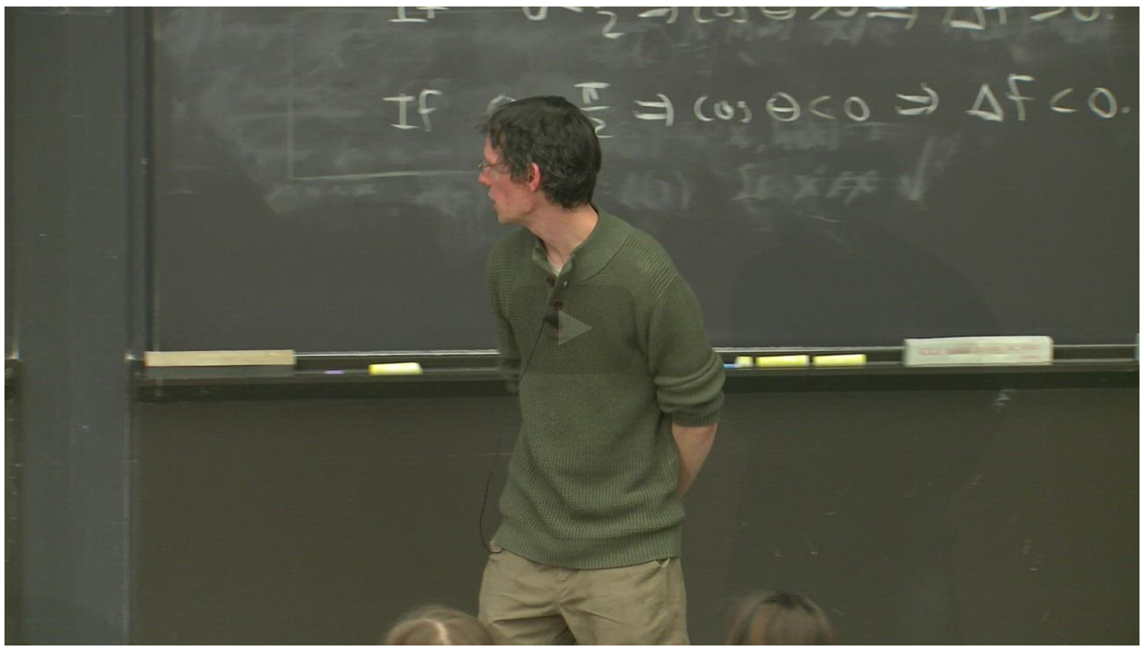
Bookmark this page

Lecture due Sep 13, 2021 20:30 IST Completed

Explore

Setting up the statement

Start of transcript. Skip to the end.



0:00 / 0:00 2.0x

PROFESSOR: So here's what we're going to do.

In a couple of minutes, I'm going to tell you

what this function f is.

We're going to write down the formula for it,

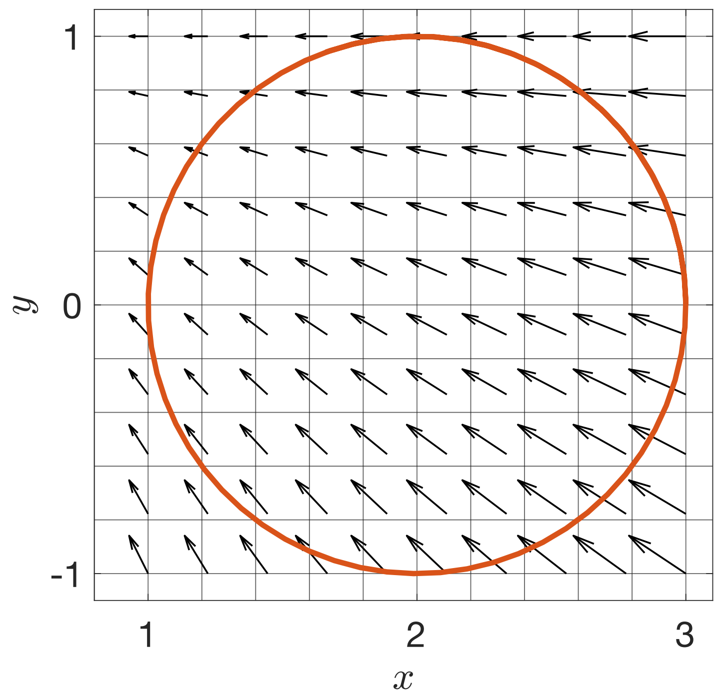
and then we're going to compute exactly where the maximum is.

Video
Download video file

Transcripts
Download SubRip (.srt) file
Download Text (.txt) file

We know that maximum of a function $f(x, y)$ over a curve occurs at the point where ∇f is perpendicular the boundary of the given region. The procedure we will illustrate here is how to find a vector that is perpendicular to the boundary curve of a given region R.

Recall the gradient field of a function $f(x, y)$ over the circular region R that we have been analyzing in the previous sections.



In our example, the region R is given by

$$(x-2)^2 + y^2 \leq 1 \qquad (4.157)$$

which has boundary

Calculator Hide Notes

$$(x-2)^2 + y^2 = 1. \tag{4.158}$$

We can think of the equation for the boundary as the level curve of some function $g(x, y)$ at height 1 where

$$g(x, y) = (x-2)^2 + y^2. \tag{4.159}$$

Because the equation for the boundary is a level curve of g, we know that ∇g is perpendicular to $g(x, y)$ at each point (x, y). The gradient of g is given by

$$\nabla g(x, y) = \langle 2x-4, 2y \rangle. \tag{4.160}$$

Putting it all together

1/1 point (graded)
(Try to think through the answer and make a guess. If you don't get it right the first time, watch the video below and answer again! You have 3 attempts.)

The function we have been analyzing is given by

$$f(x, y) = 5 - x^2 - (y-1)^2 \tag{4.161}$$

which has gradient

$$\nabla f(x, y) = \langle -2x, -2y+2 \rangle. \tag{4.162}$$

We know that the maximum of f over the region R occurs when ∇f is perpendicular to the boundary of R. We also know that ∇g is perpendicular to $(x-2)^2 + y^2 = 1$ at each point (x, y). Based on these facts, which of the following relations must be true at the maximum point?

- ( ) $\nabla f \cdot \nabla g = 0$
- ( ) $\nabla f \cdot \langle x, y \rangle = 0$
- (•) $\nabla f = \lambda \nabla g$ for some scalar λ
- ( ) $\nabla f = \lambda \langle x, y \rangle$ for some scalar λ

✓

Solution:

Take a moment to consider the things we know:

- ∇g is perpendicular to the boundary at every point
- ∇f is perpendicular to the boundary at the maximum point

These two ideas together tell us that the two vectors ∇f and ∇g are going to be in the same (or opposite) direction at the maximum point. Therefore

Calculator Hide Notes

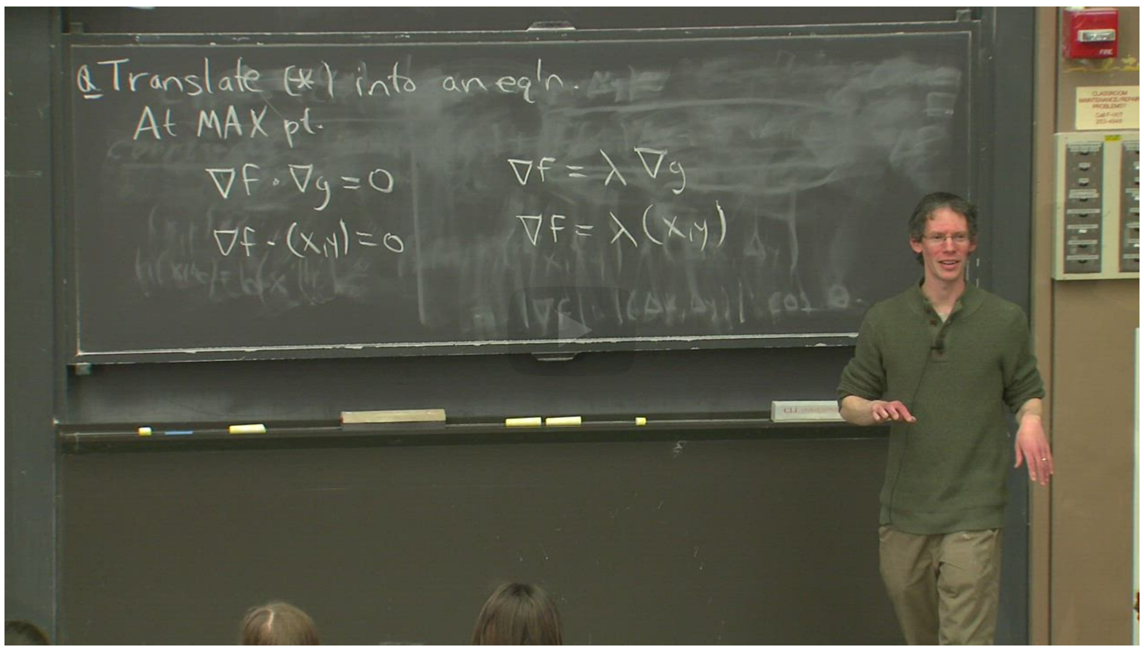
$$\nabla f = \lambda \nabla g$$

for some scalar λ.

Submit You have used 1 of 3 attempts

Answers are displayed within the problem

A hint to get started



Start of transcript. Skip to the end.

PROFESSOR: So let's talk about it a little more.

So I'm going to review the situation here.

But as I'm doing that, if you think of any questions that

would help, please ask.

Yeah?

STUDENT: So right now, we're just

0:00 / 0:00 2.0x

Video
Download video file

Transcripts
Download SubRip (.srt) file
Download Text (.txt) file

5. Maximum along the boundary intuition

Hide Discussion

Topic: Unit 3: Optimization / 5. Maximum along the boundary intuition

Add a Post

Show all posts by recent activity

- ? "We know that maximum of a function f(x,y) over a curve occurs at the point where grad(f) is perpendicular the boundary of the given region." 3
  Aren't there technically at least 2 points where grad(f) is perpendicular to f(x,y), i.e. one which maximises f(x,y) and a second one w...
- my guess 1
  i figured the two vectors would point in the same direction, but would have different magnitudes. but y/x for each of them would be t...
- Gradient and Level curves 2
  Could someone kindly share a link of the discussion on why the gradient is always perpendicular to the level curve, I'm trying to get ...
- [STAFF] Putting it all together typo 2

< Previous Next >

Calculator Hide Notes

edX

[About](#)
[Affiliates](#)
[edX for Business](#)
[Open edX](#)
[Careers](#)
[News](#)

Legal

[Terms of Service & Honor Code](#)
[Privacy Policy](#)
[Accessibility Policy](#)
[Trademark Policy](#)
[Sitemap](#)

Connect

[Blog](#)
[Contact Us](#)
[Help Center](#)
[Media Kit](#)
[Donate](#)

GET IT ON Google play
Download on the App Store

Calculator Hide Notes