2. Measurement error

Review

The example below is a problem that was posed on the first problem set of the class when Larry taught it. We did not present this problem to you in homework, but instead, offer it for you here to try before watching the video.

The problem is ungraded.

What is the error?

0 points possible (ungraded)

6. We experimentally measured two quantities called x and y. We determined that x is equal to 1 to within a measurement error of 0.01 and y is equal to 9 to within a measurement error of 0.01. The quantity that we really care about is

$$z = \frac{x}{x+y}.$$

If we plug in $x = 1$ and $y = 9$, then we get $z = 0.1$ But there are measurement errors: x is not exactly 1 and y is not exactly 9, and so we can't say that z is exactly 0.1 There is some measurement error in z.

What is the order of magnitude of the measurement error in z? Is it best described as 10^{-2}, 10^{-3}, or 10^{-4}? (Make sure you can explain your reasoning.)

- ( ) 10^{-2}
- (•) 10^{-3}
- ( ) 10^{-4}

✔

Our experiment has suffered budget cuts, and we don't have the money to do it as accurately as in the original plan. We can either measure x to within an error of 0.01 and y to within an error of 0.03, or we can measure x to within an error of 0.03 and y to within an error of 0.01. The thing we really care about is $z = \frac{x}{x+y}$, and we want to approximate z as accurately as we can. We know from experience that x will be fairly close to 1 and y will be fairly close to 9. Which plan is better?

- (•) measure x to within an error of 0.01 and y to within an error of 0.03
- ( ) measure x to within an error of 0.03 and y to within an error of 0.01
- ( ) both approaches lead to the same error in z

✔

Solution:

The solution is in the video and text below.

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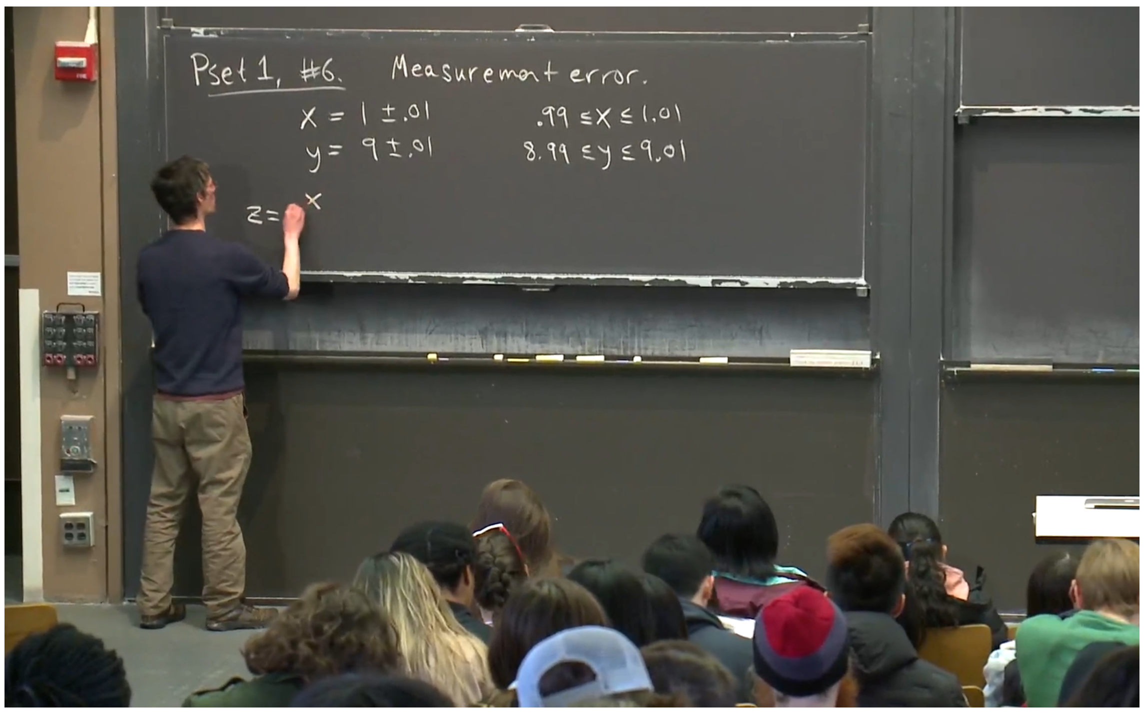
Answers are displayed within the problem

Problem solution and discussion



1:10 / 9:53 2.0x

1.

And there is a measurement error of plus or minus 0.1.

Just to be clear what that means,

it means that x is somewhere in between 1.01 and 0.99.

And then we measured something called y.

And we found that y is approximately 9.

And there is a measurement error of 0.01.

And so that means that y is in between 8.99 and 9.01.

OK, but these things that we were

able to measure experimentally are not the thing

that we really care about, which is harder to measure.

And so the thing we really care about is called z.

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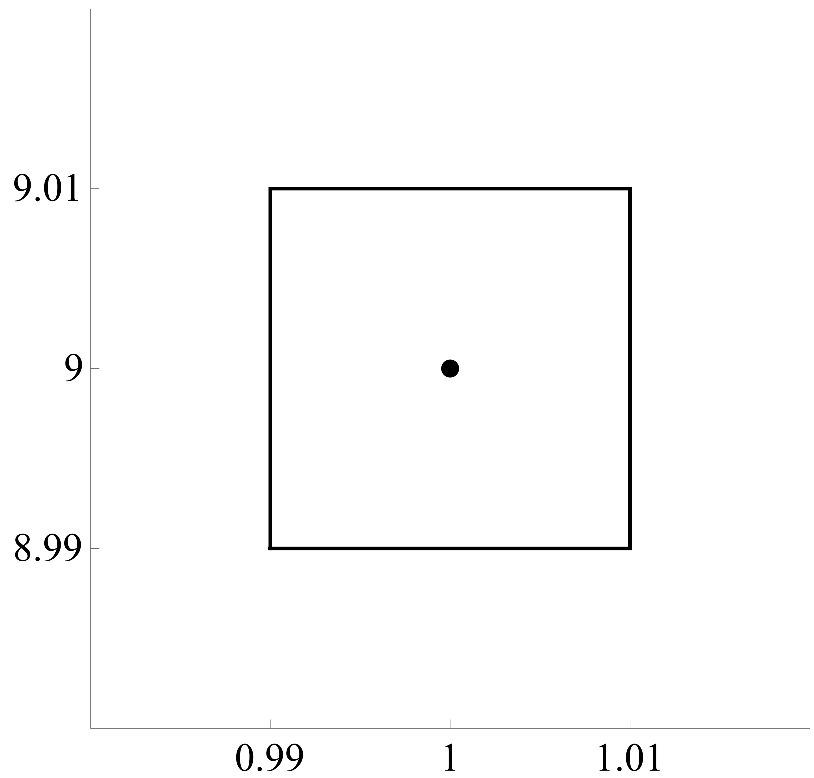
- The measurement error for x being $x = 1 \pm 0.01$ is telling us that $0.99 \leq x \leq 1.01$.
- The measurement error for y being $y = 9 \pm 0.01$ is telling us that $8.99 \leq x \leq 9.01$.

We want to find the error in $z = \dfrac{x}{x+y}$.

First, write $f(x, y) = \dfrac{x}{x+y}$. We know that $f(1, 9) = \dfrac{1}{1+9} = 0.1$.

In terms of a picture, this is telling us that we are studying the values of the function $f(x, y)$ on the box $0.99 \leq x \leq 1.01$ and $8.99 \leq y \leq 9.01$. We know the value at the center point is 0.1.



If we could draw the level curves, we could solve this easily. The image below shows o region.

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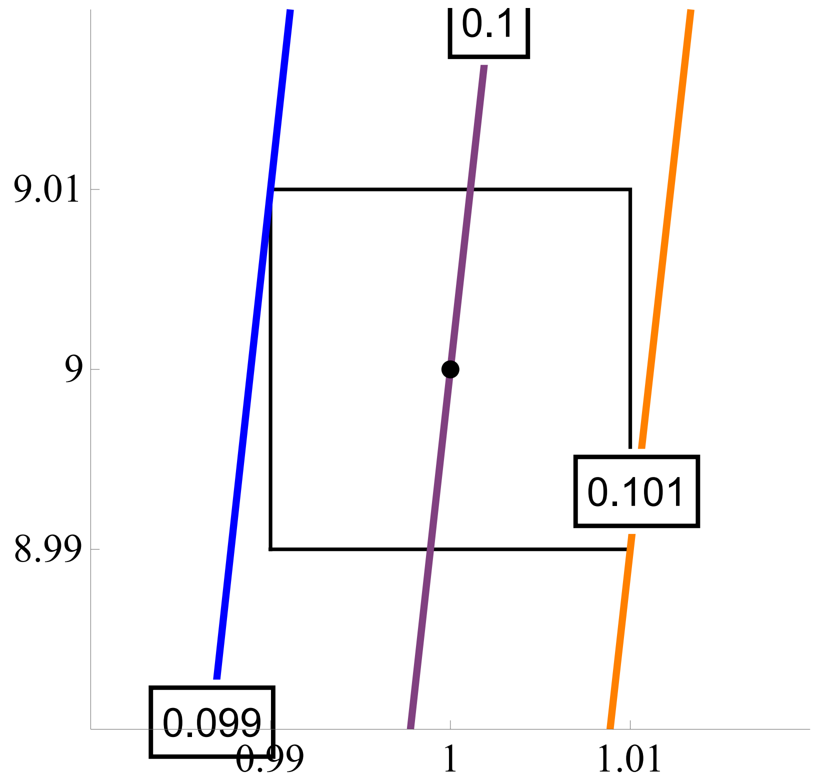

We can see that the smallest value of $f(x,y)$ occurs in the upper left corner where the function value is 0.099 and the largest value is in the lower right corner, where the function value is 0.101. Because we are studying this function on a small region, the linear approximation is quite accurate.

$$f(1+\underbrace{\Delta x}_{\text{Error in } x}, 9+\underbrace{\Delta y}_{\text{Error in } y}) \approx \underbrace{f(1,9)}_{0.1} + \underbrace{f_x(1,9)\,\Delta x + f_y(1,9)\,\Delta y}_{\text{Error in } z} \quad (7.7)$$

$$\approx 0.1 + 0.09\Delta x - 0.01\Delta y \quad (7.8)$$

- The error in z will be as large as possible if $\Delta x = 0.01$ and $\Delta y = -0.01$.

$$\text{When } \Delta x = 0.01 \text{ and } \Delta y = -0.01, \quad \text{error} = 0.001$$

- The error in z will be most negative if $\Delta x = -0.01$ and $\Delta y = 0.01$.

$$\text{When } \Delta x = -0.01 \text{ and } \Delta y = 0.01, \quad \text{error} = -0.001$$

Therefore the error is 0.001, or 10^{-3}.

Let's look at the error for different error measurements.

$$\Delta z \approx 0.09\Delta x - 0.01\Delta y \quad (7.9)$$

- If $\Delta x = \pm 0.01$ and $\Delta y = \pm 0.03$, we get a maximum error of

$$|\Delta z| \leq 0.09\,(0.01) - 0.01\,(-0.03) = 0.0012 \quad (7.10)$$

- If $\Delta x = \pm 0.03$ and $\Delta y = \pm 0.01$, we get a maximum error of

$$|\Delta z| \leq 0.09\,(0.03) - 0.01\,(-0.01) = 0.0028 \quad (7.11)$$

Therefore if budget cuts are forcing us to measure one quantity with less accuracy, we should choose to measure x with smaller error, and allow the larger error in our measurement of y.

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