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5. Visualizing the gradient vector

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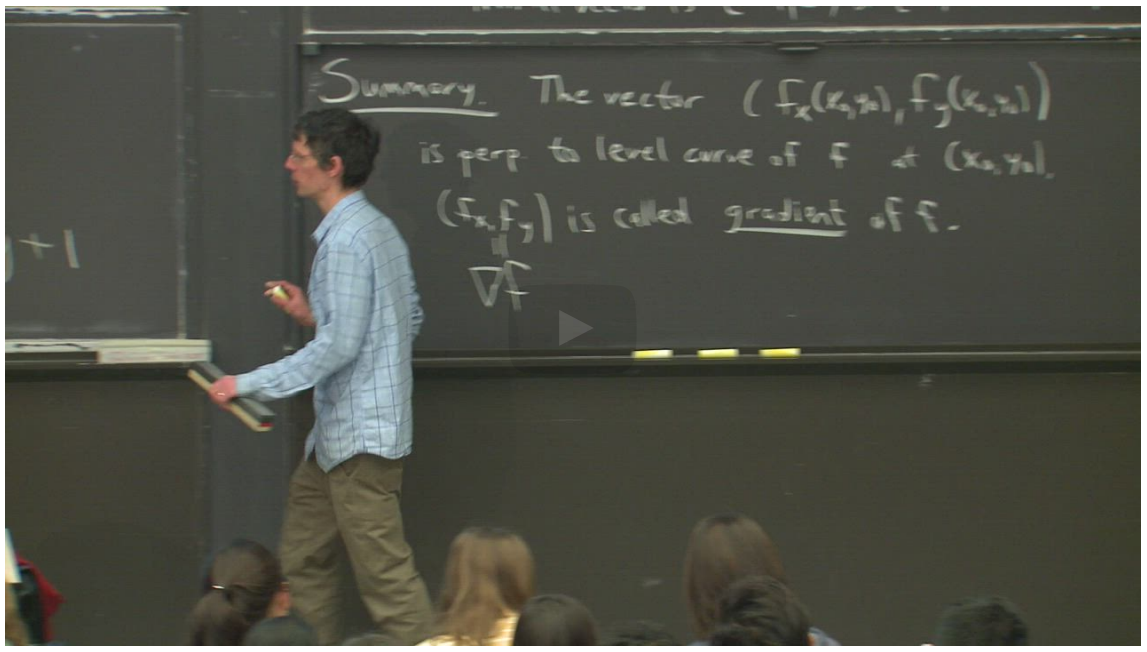
Lecture due Aug 18, 2021 20:30 IST Completed

Synthesize

Drawing the gradient



Start of transcript. Skip to the end.

PROFESSOR: So let's do one more example together.

And we'll look at a picture and see

that the gradient-- see what it means that the gradient is

normal to the level curves.

OK.

So the example is a function that we

worked with since early in the class,

x squared plus y squared.

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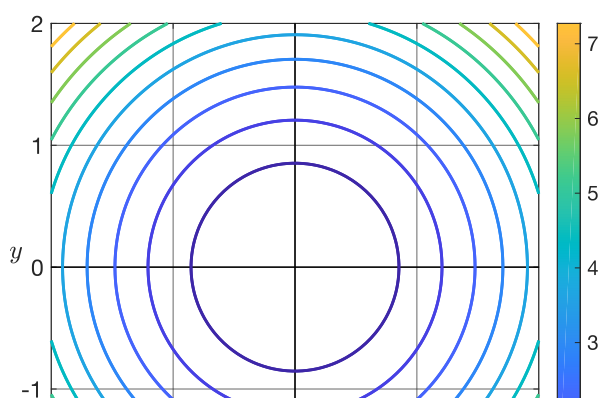
To visualize the gradient vector, we draw an arrow representing the vector $\nabla f\,(x, y)$ for several points (x, y). The arrow should start at the point (x, y) and point in the direction of $\nabla f\,(x, y)$. The resulting image is known as a gradient field. It is common to include some level curves of f as well.

Example 5.1

Let's return to the example of a paraboloid.

$$\begin{aligned} f\,(x, y) &= x^2 + y^2 \\ f_x = 2x & \qquad f_y = 2y \\ \nabla f &= \langle 2x, 2y \rangle \end{aligned}$$

Here is a plot of the level curves of $f\,(x, y)$.



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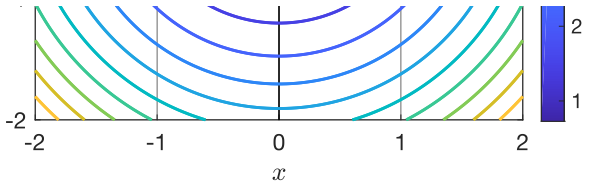


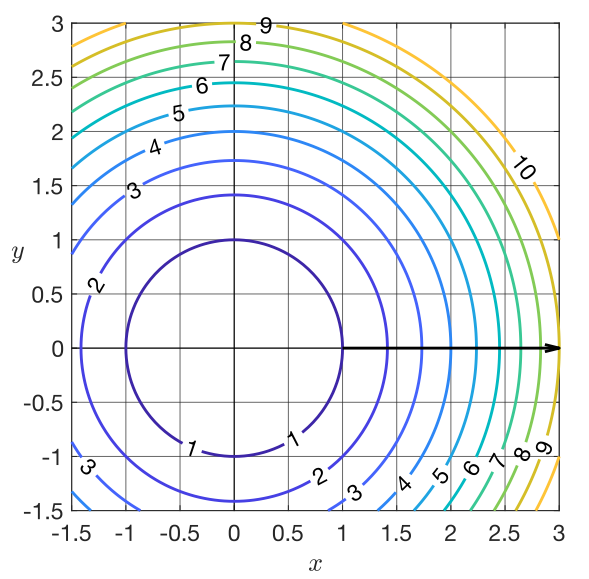
We compute the gradient of f at some points.

$$\nabla f(1,0) = \langle 2(1), 0\rangle = \langle 2, 0\rangle \tag{3.88}$$

$$\nabla f(1,1) = \langle 2(1), 2(1)\rangle = \langle 2, 2\rangle \tag{3.89}$$

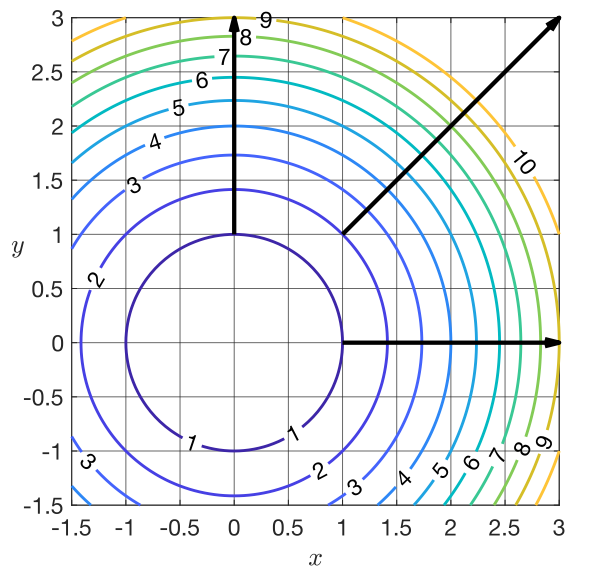
$$\nabla f(0,1) = \langle 0, 2(1)\rangle = \langle 0, 2\rangle \tag{3.90}$$

To draw the gradient vector at the point $(x, y) = (1, 0)$, we draw the vector $\langle 2, 0\rangle$ on our plot of level curves starting from the point $(1, 0)$.



Note that the gradient $\langle 2, 0\rangle$ is perpendicular to the level curve of $f(x, y) = 1$ at the point $(1, 0)$.

Similarly, we draw each gradient vector starting from the point at which the gradient is evaluated.

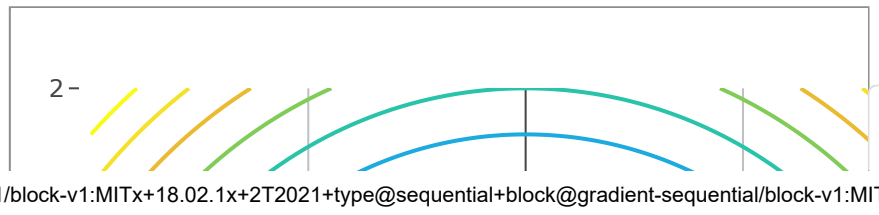


Pro tip: You must enable 3rd party cookies so that you can interact with the mathlet below.

- Click anywhere to see the gradient vector at the location of your click.
- Try changing the function using the dropdown menu.

▸ **Gradients**

Equation 1 $z = f(x, y) = \dfrac{x^2 + y^2}{4}$



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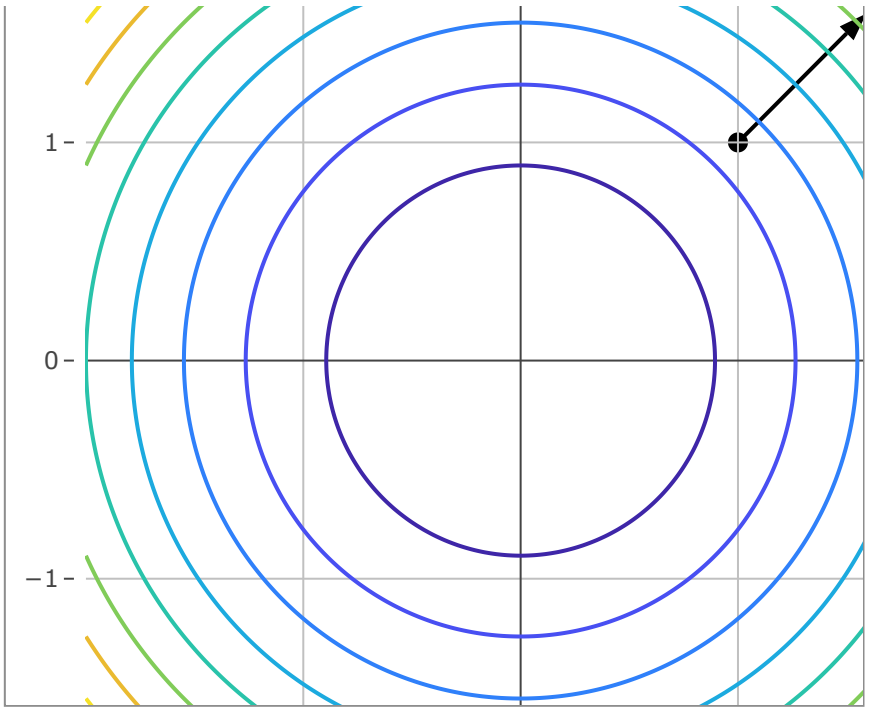

Food for thought:

1. What is the relationship between the direction of the gradient and the level curves?

2. What is the relationship between the magnitude of the gradient and the level curves?

You may use the mathlet to help check your answers to the problem below. Note that equation 1 is a scaled version of the function you are working with below.

PLEASE RATE THIS MATHLET
(Use a one star to five star rating scale.)

RESULTS

☆	**0%**
☆ ☆	**0%**
☆ ☆ ☆	**2%**
☆ ☆ ☆ ☆	**15%**
☆ ☆ ☆ ☆ ☆	**82%**

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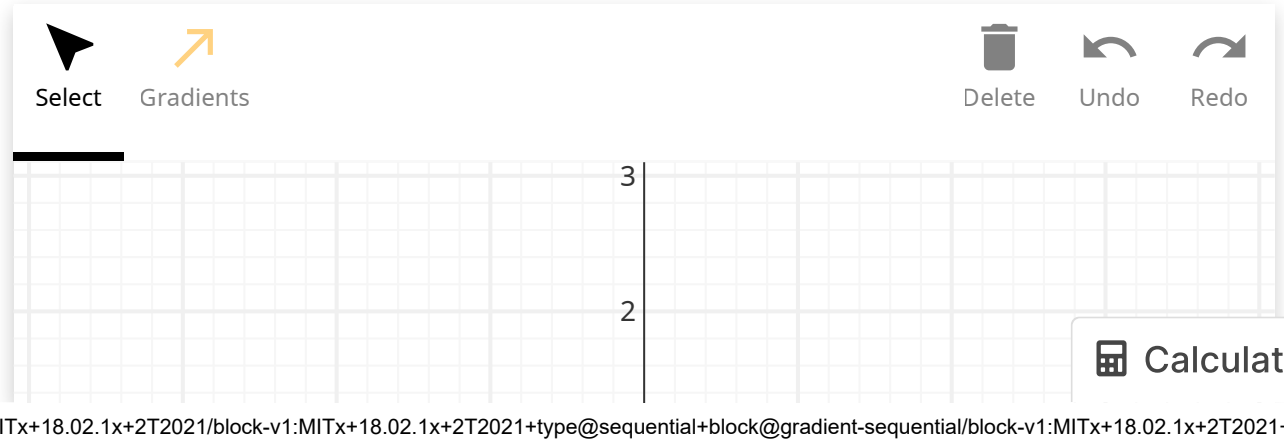
Results gathered from 448 respondents.

FEEDBACK
Your response has been recorded

Sketch the gradients

1.0/1 point (graded)
Continuing with the paraboloid $f(x,y) = x^2 + y^2$, sketch the gradients $\nabla f(-1/2, 0)$ and $\nabla f(1, -1)$.



Answer: See solution.

✔

Well done

Pro tip: For the best experience with this sketching tool, close the update banner.

Solution:

The gradient is the vector $\langle 2x, 2y \rangle$. Thus at

$$\nabla f(-1/2, 0) = \langle -1, 0 \rangle \tag{3.91}$$
$$\nabla f(1, -1) = \langle 2, -2 \rangle \tag{3.92}$$

To draw these gradient vectors, we must make sure they start at the correct point.

The gradient $\nabla f(-1/2, 0)$ is drawn as the vector starting at the point $(-1/2, 0)$ and ending at the point $(-3/2, 0)$. Similarly $\nabla f(1, -1)$ is the vector starting at the point $(1, -1)$ and ending at the point $(3, -3)$.

Submit You have used 2 of 10 attempts

Answers are displayed within the problem

5. Visualizing the gradient vector

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- Magnitude of gradient — 3
  What is the relationship between the magnitude of the gradient and the level curves? I am not able to find its answer. What do the m...
- [STAFF] Tolerance for Sketch the Gradients. PLease check my answer. — 5
  I keep getting the error mesdsdage "The gradient evaluated at the point (-0.5,0) does not end at the correct location." I followed the ...
- [Staff]Please check my answer for the sketch problem. — 7
  I think my answer is correct. After a few attemps, I still don't know why it was marked incorrect. Can you please check my answer? T...
- Beautiful mathlet, but I could see only about 90% of each function in analytic form — 3
  not sure if it's just me, but I do use chrome; the rest of the mathlet, and, most importantly, the pictorial part is 100% clear and very in...
- Magnitude of normal vectors — 3
  I still do not get the definition of gradient. They are normal vectors to the level curve if I understand it correctly. Are they the same a...
- [Staff] Gradients — 2
  I am not above to see the complete expression, f(x,y), for each of Eqns 2,5,6,7 and 8.

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