2.3.1 What is the Principle of Mathematical Induction?

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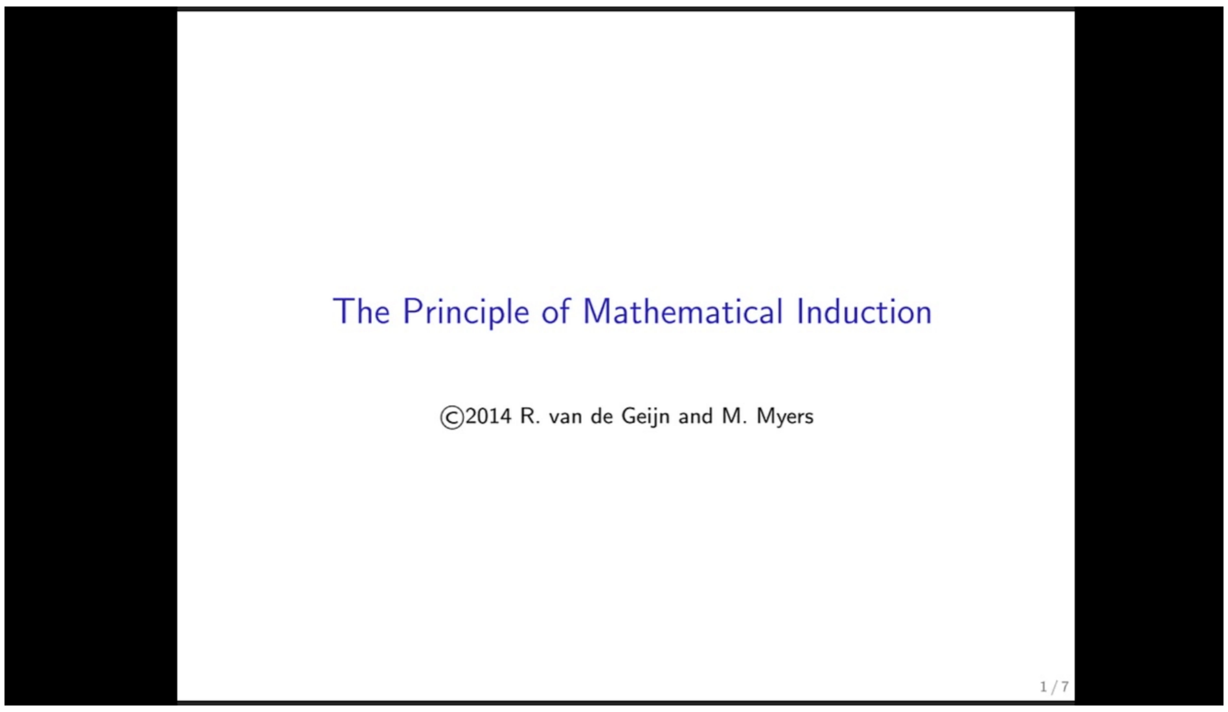

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mathematical induction.

We're going to give you some practice proving various things via

mathematical induction.

What's the principle of mathematical induction?

Well, there are actually several different ways of doing

mathematical induction.

And what we're going to do is sometimes called weak induction.

So what do we have?

If one can show that some property holds for k equal to some base case.

In a previous example, this base case was equal to 1.

And we can show that if it holds for k equals K, where k is greater than or

equal to that base case, then it also holds for the next little k.

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Inductive set

question posted 2 years ago by **jchen9619**

What is the definition of a set that can be defined inductively?

Calculator

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1 response

maggiemyers (Staff)
about a year ago

From Cornell notes:

"An inductively defined set is a set where the elements are constructed by a finite number of applications of a given set of rules for creating more complicated objects from simpler ones."

My thinking:
Sets are collections. Some of theses sets can be "ordered" so that the entire collection can be described by rules (3 clauses.) They are to describe set S, specify

- one or more elements of S (basis clause). one or more rules to
- construct elements of S from existing elements of S (Inductive clause)
- that no other elements are in S (closure or extent clause).

While sets are collections where order doesn't matter, this ordering allows us to describe the contents of the set and then create arguments about it.

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Comments by Maggie in response to a posting on the discussion board

If mathematical induction intimidates you, have a look at in the enrichment (Unit 2.5.2) :Puzzles and Paradoxes in Mathematical Induction", by Adam Bjorndahl.

Here is my take on Induction. I am actually extending it beyond the proofs we do.

If you want to prove something holds for all members of a set that can be defined inductively, then you would use mathematical induction. You may recall a set is a collection and as such the order of its members is not important. However, some sets do have a natural ordering that can be used to describe the membership. This is especially valuable when the set has an infinite number of members, for example, natural numbers. Sets for which the membership can be described by suggesting there is a first element (or small group of firsts) then from this first you can create another (or others) then more and more by applying a rule to get another element in the set are our focus here. If all elements (members) are in the set because they are either the first (basis) or can be constructed by applying "The" rule to the first (basis) a finite number of times, then the set can be inductively defined.

So for us, the set of natural numbers is inductively defined. As a computer scientist you would say 0 is the first a Calculator
rule is to add one to get another element. So 0,1,2,3.... are members of the natural numbers. 10 is a member of natura

numbers because you can find it by adding 1 to 0 ten times to get it.

So, the Principle of Mathematical induction proves that something is true for all of the members of a set that can be defined inductively. If this set has an infinite number of members, you couldn't show it is true for each of them individually. The idea is if it is true for the first(s) and it is true for any constructed member(s) no matter where you are in the list, it must be true for all. Why? Since we are proving things about natural numbers, the idea is if it is true for 0 and the next constructed, it must be true for 1 but then its true for 2, and then 3 and 4 and 5 ...and 10 and ...10000 and 10001, etc (all natural numbers) This is only because of the special ordering we can put on this set so we can know there is a next one for which it must be true. People often picture this rule by thinking of climbing a ladder or pushing down dominoes. If you know you started and you know where ever you are the next will follow then you must make it through all (even if there are an infinite number).

That is why to prove something using the Principle of Mathematical Induction you must show what you are proving holds at a start and then if it holds (assume it holds up to some point) then it holds for the next constructed element in the set. With these two parts shown, we know it must hold for all members of this inductively defined set.

You can find many examples of how to prove using PMI as well as many examples of when and why this method of proof will fail all over the web. Notice it only works for statements about sets "that can be defined inductively". Also notice subsets of natural numbers can often be defined inductively. For example, if I am a mathematician I may start counting at 1. Or I may decide that the statement holds for natural numbers ≥ 4 so I start my base case at 4.

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