

<u>Help</u>

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1. Planes

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9/3/2021

Problem Set B due Sep 15, 2021 20:30 IST



Practice

Normal to Plane 1

1/1 point (graded)

The equation 2x+y-2z=0 describes a plane ${\cal Q}$ in three dimensions. Find a vector that is normal to ${\cal Q}$.

(Enter a vector using notation such as [a,b].)

? INPUT HELP

Solution:

We can recognize the equation for ${oldsymbol{\mathcal{Q}}}$ as a "hidden dot product":

$$2x+y-2z=0$$
 same as $\begin{pmatrix} 2\\1\\-2 \end{pmatrix} \cdot \begin{pmatrix} x\\y\\z \end{pmatrix} = 0$ (5.206)

Therefore, the points of ${\cal Q}$ are all points (x,y,z) such that the vector from the origin to (x,y,z) is perpendicular

to
$$\begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}$$
. Since the origin belongs to the plane, we conclude that the vector $\begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}$ will be normal to $\mathcal Q$.

Alternative approach: It is also possible to find the normal vector by taking the partial derivatives of f(x,y,z)=2x-y+z. As was the case in two dimensions, the gradient of f will be the normal vector.

Therefore another route to the same answer is the computation $abla f = egin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}$.

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You have used 1 of 4 attempts

Answers are displayed within the problem

Normal to Plane 2

1/1 point (graded)

The equation 2x + y - 2z = 8 describes a plane ${\mathcal P}$ in three dimensions. Find a vector that is normal to ${\mathcal P}$.

(Enter a vector using notation such as [a,b].)

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Solution:

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Ine plane ho is parallel to the plane $\red z$ from the previous question. One way of seeing this is that (x,y,z) belongs to \mathcal{P} if and only if (x, y - 8, z) belongs to \mathcal{Q} .

Therefore the normal vector to \mathcal{P} is the same as the normal vector to \mathcal{Q} , which we found to be $\begin{pmatrix} 1 \\ -2 \end{pmatrix}$.

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You have used 1 of 4 attempts

1 Answers are displayed within the problem

Distance 1

1/1 point (graded)

The equation 2x+y-2z=8 describes a plane ${\cal P}$ in three dimensions. Find the distance between the origin and ${\cal P}$. This distance is defined to be the distance between the origin and the closest point belonging to ${\cal P}$.

Distance:

8/3

✓ Answer: 8/3

Solution:

In the above problem, we saw that that the normal vector to ${\cal P}$ is $ec n=egin{pmatrix}2\\1\\-2\end{pmatrix}$. The shortest distance from the origin is therefore the length of $\lambda ec{n}$ where λ is chosen such that $\lambda ec{n}$ is in ${\cal P}$

It may help to sketch a picture: draw coordinate axes and a plane, and draw a line segment representing the shortest distance from the origin to the plane. You will see that this distance is along the normal vector \vec{n} .

We can solve for λ from the equation

$$\vec{n} \cdot (\lambda \vec{n}) = 8. \tag{5.207}$$

Since $\vec{n}\cdot\vec{n}=9$, we have $\lambda=8/9$. Finally, the length of $\frac{8}{9}\vec{n}$ is equal to $\frac{8}{3}$

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You have used 1 of 4 attempts

Answers are displayed within the problem

Distance 2

1/1 point (graded)

The equation 2x+4y-4z=40 describes a plane ${\cal R}$ in three dimensions. What is the distance between the point U=(1,2,3) and \mathcal{R} ? (Again, "distance" means "shortest distance" in this context).

Distance:

✓ Answer: 7

Solution:

The normal vector to $\mathcal R$ is $ec n=egin{pmatrix}2\\4\\-4\end{pmatrix}$. The shortest distance from U to $\mathcal R$ is the length of the vector $\lambdaec n$

where λ is chosen such that $U + \lambda \vec{n}$ is in \mathcal{R} . We can solve for λ from the equation

■ Calculator

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$$\vec{n} \cdot (U + \lambda \vec{n}) = 40. \tag{5.208}$$

Since $\vec{n} \cdot \vec{n} = 36$, and $\vec{n} \cdot U = -2$, we solve $-2 + 36\lambda = 40$. This gives $\lambda = 7/6$. Finally, the length of $\frac{7}{6}\vec{n}$ is equal to 7.

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You have used 1 of 4 attempts

1 Answers are displayed within the problem

Distance 3

1/1 point (graded)

Define the distance between two planes to be the shortest distance between any two points, one in each plane. What is the distance between the planes 4x - 2y + 4z = 0 and 4x - 2y + 4z = 1?

1/6

✓ Answer: 1/6

Solution:

Recall that the normal vector to both planes is $ec{n}=egin{pmatrix}4\\-2\\4\end{pmatrix}$. The shortest distance is therefore the smallest

multiple of \vec{n} that we need to add to get from one plane to the other. Let \vec{p} be any point in the plane 4x-2y+4z=0. We need to find the smallest λ such that $\vec{p}+\lambda\vec{n}$ belongs to the =1 plane. In other words, we need to solve

$$\vec{n} \cdot (\vec{p} + \lambda \vec{n}) = 1. \tag{5.209}$$

Since $ec{n}\cdotec{p}=0$, the equation becomes

$$\lambda \left(\vec{n} \cdot \vec{n} \right) = 1. \tag{5.210}$$

Therefore $\lambda=rac{1}{|\vec{n}|^2}.$ In this case, $|\vec{n}|^2=36$, so $\lambda=rac{1}{36}.$ Finally, the length of $\lambda \vec{n}$ is equal to $rac{1}{6}.$

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You have used 2 of 4 attempts

1 Answers are displayed within the problem

Distance 4

1/1 point (graded)

For k=0,1,2, let \mathcal{P}_k be the plane described by ax+by+cz=k for nonzero a,b,c. True or false: the distance between \mathcal{P}_0 and \mathcal{P}_2 is twice the distance between \mathcal{P}_0 and \mathcal{P}_1 .



true



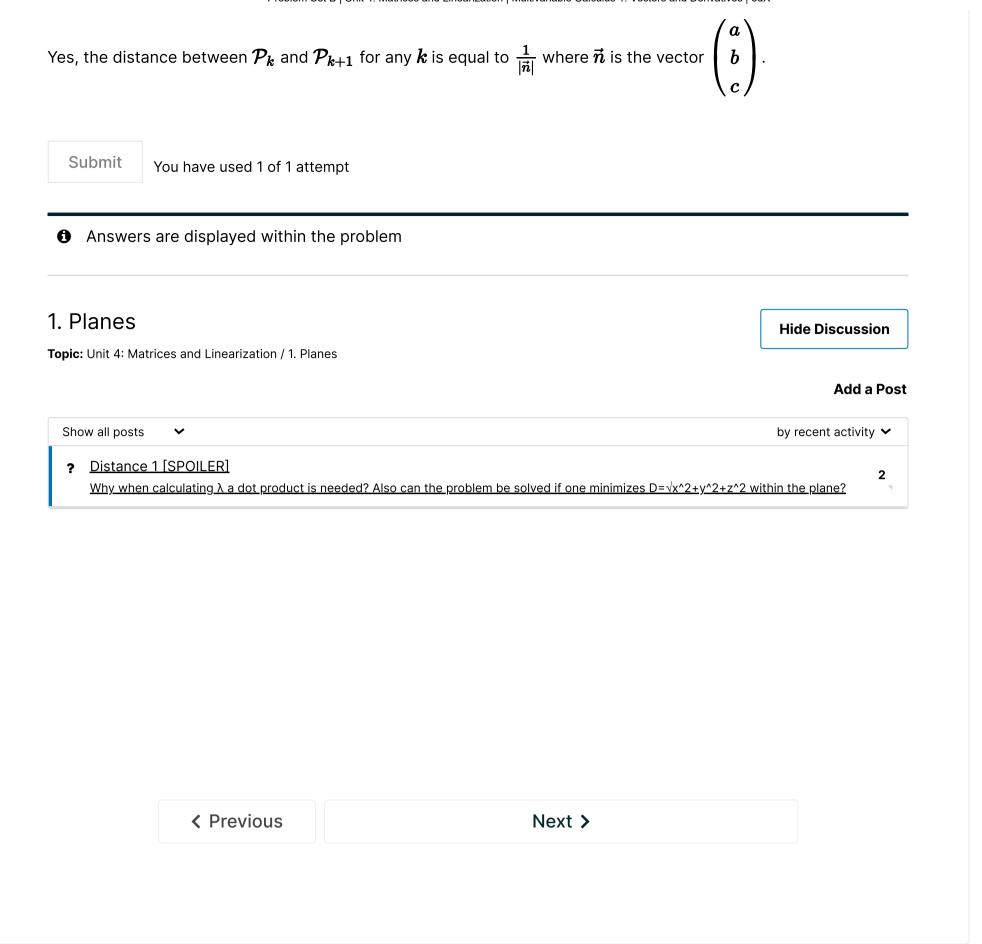
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Solution:



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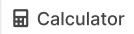
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