4. Fast Fourier Transform

The **Fast Fourier Transform (FFT)** is an algorithm originated by Cooley and Tukey in 1965 for computing the FDFT very efficiently.

Note that if $\mathbf{A}$ is an $N \times N$ matrix, and y is $N \times 1$, to compute $\mathbf{A}y$ requires N^2 scalar multiplications in general.

We reduce this by taking advantage of the symmetries in $\mathbf{F}_N$.

Suppose that $N = 2^j$ for j a positive integer.

FFT is a divide and conquer algorithm, which exploits the fact that

$$\mathbf{F}_N y = \begin{pmatrix} \mathbf{I}_{N/2} & \mathbf{D}_{N/2} \\ \mathbf{I}_{N/2} & -\mathbf{D}_{N/2} \end{pmatrix} \begin{pmatrix} \mathbf{F}_{N/2} & 0 \\ 0 & \mathbf{F}_{N/2} \end{pmatrix} \begin{pmatrix} y_1 \\ y_3 \\ \vdots \\ y_{N/2-1} \\ y_2 \\ \vdots \\ y_N/2 \end{pmatrix},$$

where $\mathbf{I}_{N/2}$ is the $N/2 \times N/2$ identity matrix, and $\mathbf{D}_{N/2}$ is the diagonal matrix

$$\mathbf{D}_{N/2} = \begin{pmatrix} 1 & & & & \\ & \overline{\omega} & & & \\ & & \overline{\omega}^2 & & \\ & & & \ddots & \\ & & & & \overline{\omega}^{N/2} \end{pmatrix}$$

We continue to apply the same rule to the two $\mathbf{F}_{N/2}$ matrices to get a formula for $\mathbf{F}_N$ involving four $\mathbf{F}_{N/4}$ matrix multiplications instead.

Continue.

Result We can compute $\mathbf{F}_N y$ with only $\frac{1}{2}Nj = \frac{1}{2}N \log N$ multiplications!

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? FFT Rule

Please, can you show us at least the rule for FN/4? 2

Motivation? 2

While I'm familiar with the DFT and FFT, and this is a nice review, I'm not sure I'm following how this is relating to the goal of learning to solve PDEs using FS? I suspect you are ...

[staff] possible errors: Fast Fourier Transform 8

for the reordered y column vector on the right, perhaps the index N/2 in the middle should be N-2 and the index N/2-1 at the end of the list should be N-1. Also, for the D_N/2...

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