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Unit overview

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Solved problems

Additional theoretical material

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Unit summary

Bookmark

Exercise: Nonmonotonic functions

(4/4 points)
Suppose that X is a continuous random variable and that $Y = X^4$. Then, for $y \geq 0$, we have

$$f_Y(y) = ay^b f_X(-cy^d) + ay^b f_X(cy^d),$$

for some a, b, d, and some $c > 0$. Find a, b, c, and d.

$a =$ 1/4 ✔ **Answer:** 0.25

$b =$ -3/4 ✔ **Answer:** -0.75

$c =$ 1 ✔ **Answer:** 1

$d =$ 1/4 ✔ **Answer:** 0.25

Answer:
We have, for $y \geq 0$,

$$F_Y(y) = \mathbf{P}(Y \leq y) = \mathbf{P}(X^4 \leq y) = \mathbf{P}(-y^{1/4} \leq X \leq y^{1/4}) = F_X(y^{1/4}) - F_X(-y^{1/}$$

By differentiating, and using also the chain rule, we obtain

$$f_Y(y) = f_X(y^{1/4}) \cdot \frac{1}{4} \cdot y^{-3/4} + f_X(-y^{1/4}) \cdot \frac{1}{4} \cdot y^{-3/4}.$$

Therefore, $a = 1/4$, $b = -3/4$, $c = 1$, and $d = 1/4$.

You have used 1 of 2 submissions