MITx: 6.041x Introduction to Probability - The Science of Uncertainty

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Problem 1: Defective coin

(3/3 points)

A defective coin minting machine produces coins whose probability of Heads is a random variable Q with PDF

$$f_Q(q) = \begin{cases} 3q^2, & \text{if } q \in [0,1], \\ 0, & \text{otherwise.} \end{cases}$$

A coin produced by this machine is tossed repeatedly, with successive tosses assumed to be independent. Let A be the event that the first toss of this coin results in Heads, and let B be the event that the second toss of this coin results in Heads.

1. $\mathbf{P}(A) =$ 3/4 **Answer:** 0.75

   (Your answer should be a number.)

2. Find the conditional PDF of Q given event A. Express your answer in terms of q using standard notation .

For $0 \leq q \leq 1$, $f_{Q|A}(q) =$ 4*q^3 **Answer:** 4*q^3

3.

$\mathbf{P}(B \mid A) =$ 4/5 **Answer:** 0.8

(Your answer should be a number.)

Answer:

1. To calculate $\mathbf{P}(A)$, we use the continuous version of the total probability theorem:

$$\mathbf{P}(A) = \int_0^1 \mathbf{P}(A \mid Q = q) f_Q(q)\, dq = \int_0^1 q \cdot (3q^2)\, dq = \left[\frac{3}{4}q^4\right]_0^1 = \frac{3}{4}.$$

2. Using Bayes' rule,

$$\begin{aligned} f_{Q|A}(q) &= \frac{\mathbf{P}(A \mid Q = q) f_Q(q)}{\mathbf{P}(A)} \\ &= \begin{cases} \dfrac{q \cdot (3q^2)}{3/4}, & \text{if } 0 \leq q \leq 1, \\ 0, & \text{otherwise,} \end{cases} \\ &= \begin{cases} 4q^3, & \text{if } 0 \leq q \leq 1, \\ 0, & \text{otherwise.} \end{cases} \end{aligned}$$

Problem Set 7b due Apr 13, 2016 at 23:59 UTC

Solved problems

Additional theoretical material

Unit summary

- Exam 2
- Unit 8: Limit theorems and classical statistics
- Unit 9: Bernoulli and Poisson processes
- Unit 10: Markov chains
- Exit Survey
- Final Exam

3.

$$\begin{aligned}
\mathbf{P}(B \mid A) &= \int_0^1 \mathbf{P}(B \mid A, Q=q) f_{Q|A}(q)\, dq \\
&= \int_0^1 \mathbf{P}(B \mid Q=q) f_{Q|A}(q)\, dq \\
&= \int_0^1 q(4q^3)\, dq \\
&= 4/5.
\end{aligned}$$

The second equality holds because for a given value q of Q, the events A and B are (conditionally) independent.

You have used 2 of 2 submissions

Printable problem set available here .

DISCUSSION

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