# 11.2.3 Newton-Raphson method

MO2.10

The Newton-Raphson method (often refered to just as Newton's method) fits a tangent line to the point $(x^k, r(x^k))$ on the graph of $r$, and defines $x^{k+1}$ at the intersection of this tangent line with the $x$ axis. The slope of this graph at the point $x^k$ is $r'(x^k)$, and so the tangent line at $(x^k, r(x^k))$ satisfies the following equation,

$$r_{\tan}(x) = r(x^k) + (x - x^k)\, r'(x^k),$$

Then, finding where this tangent intersects the $x$ axis, defines the Newton iterate: $r_{\tan}(x^{k+1}) = 0$. Specifically,

$$x^{k+1} = x^k - \frac{r(x^k)}{r'(x^k)}. \tag{11.7}$$

An alternative (but equivalent) derivation is to perform a first-order Taylor series approximation of $r(x)$ about the current iterate $x^k$:

$$r(x^k + \Delta x) \approx r(x^k) + r'(x^k)\,\Delta x$$

and then set this Taylor series approximation to zero to find $\Delta x$.

$$r(x^k) + r'(x^k)\,\Delta x = 0 \Rightarrow \Delta x = -\frac{r(x^k)}{r'(x^k)}$$

And then update $x^k$ by $\Delta x$:

$$x^{k+1} = x^k + \Delta x = x^k - \frac{r(x^k)}{r'(x^k)}$$

which is the same result as Equation (11.7).

Convergence for Newton's method is very fast, when it occurs. Under some (reasonable) conditions, the error $\epsilon^k \equiv |x^* - x^k|$ obeys the following inequality:

$$|\epsilon^{k+1}| \le C(\epsilon^k)^2,$$

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