4.1.7 Problem Set: Description of IVP test problems

The following is a description of the test problems used in `IVP_scenarios.py`.

`test0` numerically solves the following problem:

$$\frac{\mathrm{d}u}{\mathrm{d}t} = 2 \tag{4.20}$$

$$t_I = 1, \quad t_F = 2, \quad u\left(t_I\right) = 3\,\frac{1}{3} \tag{4.21}$$

which has the exact solution

$$u\left(t\right) = 1\,\frac{1}{3} + 2t \tag{4.22}$$

Since this is a linear solution in t, then any method which is first-order accurate or better will solve this problem exactly. Thus, the maximum error reported will be zero (to within machine precision).

`test1` numerically solves the following problem:

$$\frac{\mathrm{d}u}{\mathrm{d}t} = 1 - t + 6t^2 - \frac{2}{3}t^3 \tag{4.23}$$

$$t_I = 1, \quad t_F = 2, \quad u\left(t_I\right) = 12\,\frac{1}{3} \tag{4.24}$$

which has the exact solution

$$u\left(t\right) = 10 + t - \frac{1}{2}t^2 + 2t^3 - \frac{1}{6}t^4 \tag{4.25}$$

Since this is a quartic solution in t, then any method which is fourth-order accurate or better will solve this problem exactly (e.g.. RK4) to within machine precision. However, for methods with lower accuracy (e.g. Forward Euler), the numerical solution will not be exact.

`test2` combines `test0` and `test1` into a system of two equations.

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix} u_0 \\ u_1 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 - t + 6t^2 - \frac{2}{3}t^3 \end{bmatrix} \tag{4.26}$$

$$t_I = 1, \quad t_F = 2, \quad u\left(t_I\right) = \left[3\,\frac{1}{3}, 12\,\frac{1}{3}\right] \tag{4.27}$$

Since the equations for u_0 and u_1 do not depend on each other, the exact solutions are respectively from `test0` and `test1`. And, if implemented correctly, the numerical results will be identical to the results when the method(s) are run with `test0` and `test1`.

`test3` numerically solves the following problem:

$$\frac{\mathrm{d}u}{\mathrm{d}t} = -u/10 + t \tag{4.28}$$

$$t_I = 0, \quad t_F = 4.5, \quad u(t_I) = 1 \tag{4.29}$$

which has the exact solution

$$u(t) = 10t + 101e^{-t/10} - 100 \tag{4.30}$$

This is the first test which involves a right-hand side f which depends on the state u.

`test4` numerically solves the following problem system of two equations.

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix} u_0 \\ u_1 \end{bmatrix} = \begin{bmatrix} -u_0^2 \\ -u_1 + 1/u_0 \end{bmatrix} \tag{4.31}$$

$$t_I = 0, \quad t_F = 1, \quad u(t_I) = [2, 1] \tag{4.32}$$

which has the exact solution

$$u_0(t) = \frac{2}{2t+1} \tag{4.33}$$

$$u_1(t) = t + \frac{3}{2}e^{-t} - \frac{1}{2} \tag{4.34}$$

This system introduces both nonlinearity (from $-u_0^2$ and $1/u_0$ terms) and a coupled system (since f_1 depends on both u_0 and u_1).

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