3.2.6 Symmetric Matrices

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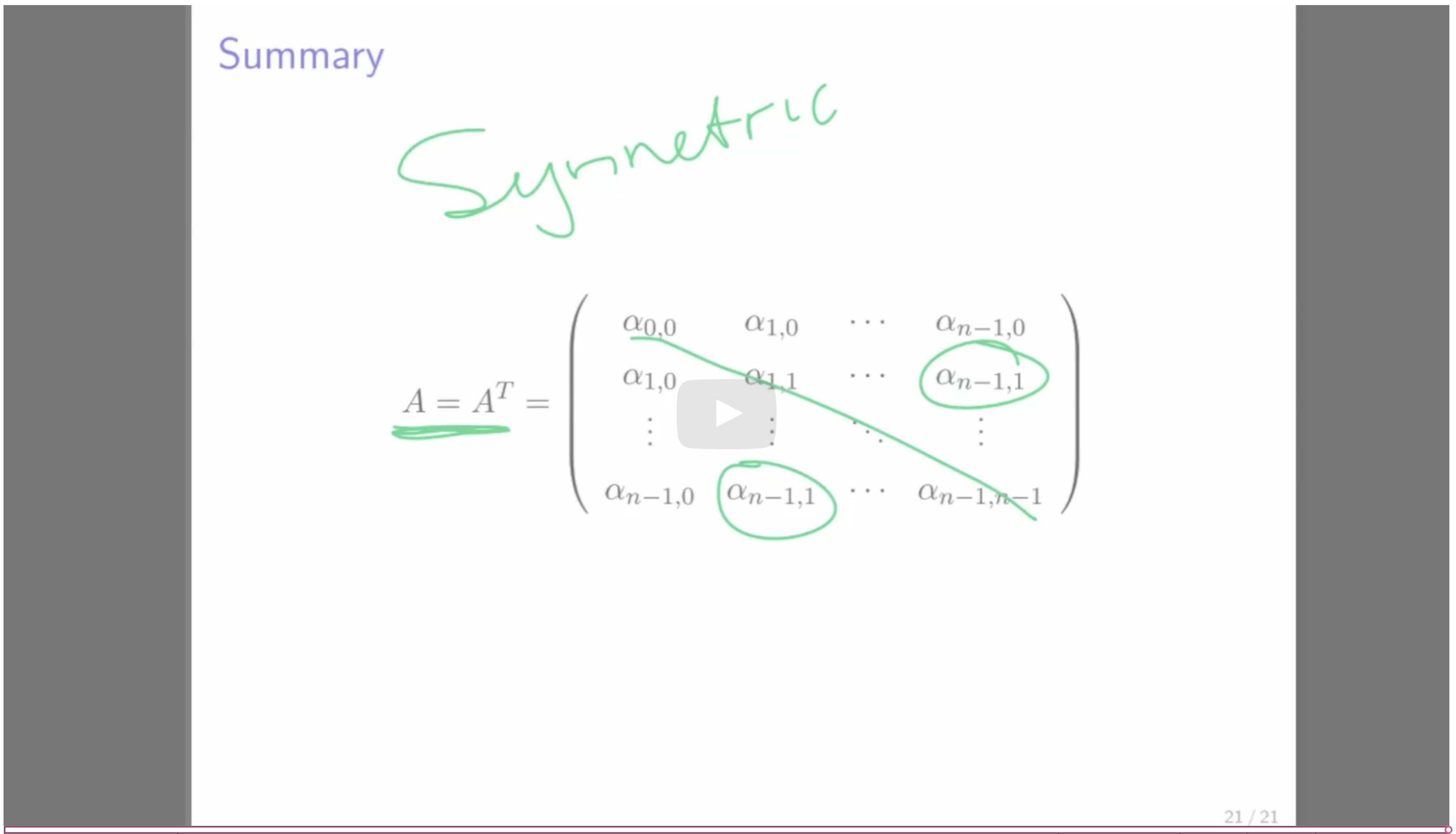
Week 3 due Oct 18, 2023 06:12 IST

3.2.6 Symmetric Matrices



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Reading Assignment

0 points possible (ungraded)

Read Unit 3.2.1 of the notes. [LINK]

- [x] Done

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Small Typo
The Reading Assignment says 'Read Unit 3.2.1 of the notes'. It should say to read unit 3.2.6.
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Homework 3.2.6.1

9/9 points (graded)
Assume the below matrices are symetric. Fill in the remaining elements.

$$\begin{pmatrix} 2 & \alpha_0 & -1 \\ -2 & 1 & -3 \\ \alpha_1 & \alpha_2 & -1 \end{pmatrix}; \begin{pmatrix} 2 & \beta_0 & \beta_1 \\ -2 & 1 & \beta_2 \\ -1 & 3 & -1 \end{pmatrix}; \begin{pmatrix} 2 & 1 & -1 \\ \gamma_0 & 1 & -3 \\ \gamma_1 & \gamma_2 & -1 \end{pmatrix}.$$

α_0 -2 ✔ Answer: -2

α_1 -1 ✔ Answer: -1

α_2 -3 ✔ Answer: -3

β_0 -2 ✔ Answer: -2

β_1 -1 ✔ Answer: -1

β_2 3 ✔ Answer: 3

γ_0 1 ✔ Answer: 1

γ_1 -1 ✔ Answer: -1

γ_2 -3 ✔ Answer: -3

Explanation

$$\begin{pmatrix} 2 & -2 & -1 \\ -2 & 1 & -3 \\ -1 & -3 & -1 \end{pmatrix}; \begin{pmatrix} 2 & -2 & -1 \\ -2 & 1 & 3 \\ -1 & 3 & -1 \end{pmatrix}; \begin{pmatrix} 2 & 1 & -1 \\ 1 & 1 & -3 \\ -1 & -3 & -1 \end{pmatrix}$$

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Answers are displayed within the problem

Homework 3.2.6.2

1/1 point (graded)
A triangular matrix that is also symmetric is, in fact, a diagonal matrix.

Always ✔ Answer: Always

Explanation

Answer: Always

Without loss of generality is a common expression in mathematical proofs. It is used to argue a specific case that is easier to prove but all other cases can be argued using the same strategy

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a specific case that is easier to prove but all other cases can be argued using the same strategy. Thus, given a proof of the conclusion in the special case, it is easy to adapt it to prove the conclusion in all cases. It is often abbreviated as "W.l.o.g.".

W.l.o.g., let A be both symmetric and lower triangular. Then

$$\alpha_{i,j} = 0 \text{ if } i < j$$

since A is lower triangular. But $\alpha_{i,j} = \alpha_{j,i}$ since A is symmetric. We conclude that

$$\alpha_{i,j} = \alpha_{j,i} = 0 \text{ if } i < j.$$

But this means that $\alpha_{i,j} = 0$ if $i \neq j$, which means A is a diagonal matrix.

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Answers are displayed within the problem

Homework 3.2.6.3

1/1 point (graded)

Algorithm: $[A] := \text{SYMMETRIZE\_FROM\_LOWER\_TRIANGLE}(A)$

Partition $A \rightarrow \left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right)$

where A_{TL} is 0×0

while $m(A_{TL}) < m(A)$ **do**

Repartition

$$\left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c|c|c} A_{00} & a_{01} & A_{02} \\ \hline a_{10}^T & \alpha_{11} & a_{12}^T \\ \hline A_{20} & a_{21} & A_{22} \end{array}\right)$$

where α_{11} is 1×1

(set a_{01}'s components to their symmetric parts below the diagonal)

$a_{01} := (a_{10}^T)^T$

Continue with

$$\left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{cc|c} A_{00} & a_{01} & A_{02} \\ a_{10}^T & \alpha_{11} & a_{12}^T \\ \hline A_{20} & a_{21} & A_{22} \end{array}\right)$$

endwhile

In the above algorithm, one can replace $a_{01} = (a_{10}^T)^T$ by $a_{12}^T = (a_{21})^T$.

Always ✔ Answer: Always

Explanation

- $a_{01} = (a_{10}^T)^T$ sets the elements above the diagonal to their symmetric counterparts, one column at a time.
- $a_{12}^T = a_{21}^T$ sets the elements to the right of the diagonal to their symmetric counterparts, one row at a time.

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Answers are displayed within the problem

Homework 3.2.6.4

1/1 point (graded)

Consider the following algorithm.

Algorithm: $[A] := \text{SYMMETRIZE\_ FROM\_?????\_TRIANGLE}(A)$

Partition $A \rightarrow \left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right)$

where A_{TL} is 0×0

while $m(A_{TL}) < m(A)$ **do**

Repartition

$$\left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c|c|c} A_{00} & a_{01} & A_{02} \\ \hline a_{10}^T & \alpha_{11} & a_{12}^T \\ \hline A_{20} & a_{21} & A_{22} \end{array}\right)$$

where α_{11} is 1×1

Continue with

$$\left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c|c|c} A_{00} & a_{01} & A_{02} \\ \hline a_{10}^T & \alpha_{11} & a_{12}^T \\ \hline A_{20} & a_{21} & A_{22} \end{array}\right)$$

endwhile

What commands may be introduced between the lines in order to "symmetrize A assuming that only upper triangular part is stored.

(Check all that apply)

- [x] $a_{21} := (a_{12}^T)^T$
- [ ] $a_{11} := 0$
- [x] $a_{10}^T := (a_{01})^T$
- [ ] $a_{10}^T := 0$
- [ ] $a_{12}^T := (a_{21})^T$

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Answers are displayed within the problem

Homework 3.2.6.5

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1/1 point (graded)
Implement functions

- `Symmetrize_from_lower_triangle_unb`
- `Symmetrize_from_upper_triangle_unb`

(Implement as many as you enjoy implementing and/or until you "get the point". Then move on. We suggest you implement at least one of these.)

Some links that will come in handy:

- Spark (alternatively, open the file `LAFF-2.0xM/Spark/index.html`)
- PictureFLAME (alternatively, open the file `LAFF-2.0xM/PictureFLAME/PictureFLAME.html`)

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Correct (1/1 point)

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