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11. Using Slutsky Theorem: Plug-in

Confidence Interval by Plug-in

But this one's simpler.

And if you're going to take n goes to infinity,

you might as well use all the tools that you know about this.

Now, of course, if n is not infinite,

if n is for any finite n, you're sort of

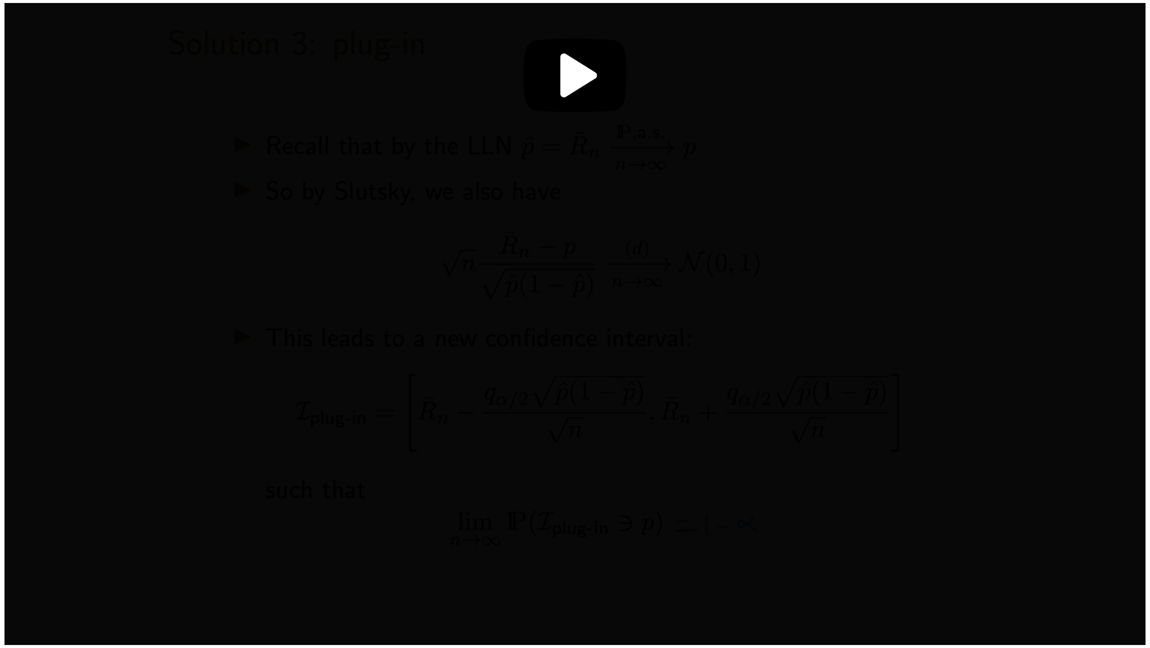
accumulating the amount of approximations

that you're making.

So you're relying twice on asymptotics.

And if you don't want to do this,

you might want to stick to [? iSolve. ?]



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Convergences of different quantities

3/3 points (graded)

As in lecture, recall that $R_1, \ldots, R_n \stackrel{iid}{\sim} \text{Ber}(p)$ for some unknown parameter p, and we estimate p using the estimator $\hat{p} = \overline{R}_n = \frac{1}{n}\sum_{i=1}^{n} R_i$.

As in the methods before, our starting point is the following result of the central limit theorem:

$$\lim_{n\to\infty} \mathbf{P}\left(\left|\sqrt{n}\frac{\overline{R}_n - p}{\sqrt{p(1-p)}}\right| < q_{\alpha/2}\right) = 1-\alpha.$$

Choose the correct convergence statement for each quantity below:
(Choose all that apply for each column.)
Note: In the third and fourth choices below, “is approximated by (in distribution)", means that the CDFs are close; i.e. $\lim_{n\to\infty} F_n(x) - G_n(x) \to 0$, where F_n is the CDF of the RV in the question and G_n is the CDF of the normal distribution with mean p and the written variance, e.g. $\mathcal{N}\left(p, \frac{p(1-p)}{n}\right)$.

$\overline{R}_n$:

$\sqrt{n}\left(\overline{R}_n - p\right)$:

$\sqrt{n}\frac{\overline{R}_n - p}{\sqrt{p(1-p)}}$:

[ ] $\xrightarrow[n\longrightarrow\infty]{(d)} \mathcal{N}(0,1)$	[ ] $\xrightarrow[n\longrightarrow\infty]{(d)} \mathcal{N}(0,1)$	[x] $\xrightarrow[n\longrightarrow\infty]{(d)} \mathcal{N}(0,1)$
[ ] $\xrightarrow[n\longrightarrow\infty]{(d)} \mathcal{N}(0,p(1-p))$	[x] $\xrightarrow[n\longrightarrow\infty]{(d)} \mathcal{N}(0,p(1-p))$	[ ] $\xrightarrow[n\longrightarrow\infty]{(d)} \mathcal{N}(0,p(1-p))$
[x] is approximated by (in distribution) $\mathcal{N}(p, \frac{p(1-p)}{n})$	[ ] is approximated by (in distribution) $\mathcal{N}(p, \frac{p(1-p)}{n})$	[ ] is approximated by (in distribution) $\mathcal{N}(p, \frac{p(1-p)}{n})$
[ ] is approximated by (in distribution) $\mathcal{N}(p, \frac{p(1-p)}{\sqrt{n}})$	[ ] is approximated by (in distribution) $\mathcal{N}(p, \frac{p(1-p)}{\sqrt{n}})$	[ ] is approximated by (in distribution) $\mathcal{N}(p, \frac{p(1-p)}{\sqrt{n}})$
[x] $\xrightarrow[n\longrightarrow\infty]{(\mathbf{P})} p$	[ ] $\xrightarrow[n\longrightarrow\infty]{(\mathbf{P})} p$	[ ] $\xrightarrow[n\longrightarrow\infty]{(\mathbf{P})} p$
[ ] $\xrightarrow[n\longrightarrow\infty]{(\mathbf{P})} 1$	[ ] $\xrightarrow[n\longrightarrow\infty]{(\mathbf{P})} 1$	[ ] $\xrightarrow[n\longrightarrow\infty]{(\mathbf{P})} 1$
[ ] $\xrightarrow[n\longrightarrow\infty]{(\mathbf{P})} \sqrt{p(1-p)}$	[ ] $\xrightarrow[n\longrightarrow\infty]{(\mathbf{P})} \sqrt{p(1-p)}$	[ ] $\xrightarrow[n\longrightarrow\infty]{(\mathbf{P})} \sqrt{p(1-p)}$
✔	✔	✔

Solution:

1.
   - $\overline{R}_n \xrightarrow[n\longrightarrow\infty]{(\mathbf{P})} \mathbb{E}\left[\overline{R}_n\right] = p$ by the (weak) law of large number.
   - $\overline{R}_n \xrightarrow[n\longrightarrow\infty]{(d)} \mathcal{N}\left(\mathbb{E}\left[\overline{R}_n\right], \text{Var}\left(\overline{R}_n\right)\right) = \mathcal{N}\left(p, \frac{p(1-p)}{n}\right)$ by the CLT.

2.

$\sqrt{n}\left(\overline{R}_n - p\right) \xrightarrow[n\longrightarrow\infty]{(d)} \mathcal{N}\left(\mathbb{E}\left[\sqrt{n}\left(\overline{R}_n - p\right)\right], n\mathsf{Var}\left(\overline{R}_n\right)\right) = \mathcal{N}\left(0, p\left(1-p\right)\right)$ by the CLT. Note that with an asymptotic variance that does not depend on n, $\sqrt{n}\left(\overline{R}_n - p\right)$ does not converge in probability to a constant.

3. $\sqrt{n}\frac{\overline{R}_n - p}{\sqrt{p\left(1-p\right)}} \xrightarrow[n\longrightarrow\infty]{(d)} \mathcal{N}\left(0,1\right))$ by the CLT. This is a recaling of the convergence statement immediately above.

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Answers are displayed within the problem

Convergences of different quantities (continued)

3/3 points (graded)

This is a continuation of the previous problem. Choose all that apply for each column below.

$\sqrt{\overline{R}_n\left(1-\overline{R}_n\right)}$:

$\frac{\sqrt{\overline{R}_n\left(1-\overline{R}_n\right)}}{\sqrt{p\left(1-p\right)}}$:

$\left(\sqrt{n}\frac{\overline{R}_n - p}{\sqrt{p\left(1-p\right)}}\right)\left(\frac{\sqrt{p\left(1-p\right)}}{\sqrt{\overline{R}_n\left(1-\overline{R}_n\right)}}\right)$:

☐ $\xrightarrow[n\longrightarrow\infty]{(d)} \mathcal{N}(0,1)$	☐ $\xrightarrow[n\longrightarrow\infty]{(d)} \mathcal{N}(0,1)$	☑ $\xrightarrow[n\longrightarrow\infty]{(d)} \mathcal{N}(0,1)$
☐ $\xrightarrow[n\longrightarrow\infty]{(d)} \mathcal{N}(0,p(1-p))$	☐ $\xrightarrow[n\longrightarrow\infty]{(d)} \mathcal{N}(0,p(1-p))$	☐ $\xrightarrow[n\longrightarrow\infty]{(d)} \mathcal{N}(0,p(1-p))$
☐ is approximated by (in distribution) $\mathcal{N}(p, \frac{p(1-p)}{n})$	☐ is approximated by (in distribution) $\mathcal{N}(p, \frac{p(1-p)}{n})$	☐ is approximated by (in distribution) $\mathcal{N}(p, \frac{p(1-p)}{n})$
☐ is approximated by (in distribution) $\mathcal{N}(p, \frac{p(1-p)}{\sqrt{n}})$	☐ is approximated by (in distribution) $\mathcal{N}(p, \frac{p(1-p)}{\sqrt{n}})$	☐ is approximated by (in distribution) $\mathcal{N}(p, \frac{p(1-p)}{\sqrt{n}})$
☐ $\xrightarrow[n\longrightarrow\infty]{(\mathbf{P})} p$	☐ $\xrightarrow[n\longrightarrow\infty]{(\mathbf{P})} p$	☐ $\xrightarrow[n\longrightarrow\infty]{(\mathbf{P})} p$
☐ $\xrightarrow[n\longrightarrow\infty]{(\mathbf{P})} 1$	☑ $\xrightarrow[n\longrightarrow\infty]{(\mathbf{P})} 1$	☐ $\xrightarrow[n\longrightarrow\infty]{(\mathbf{P})} 1$
☑ $\xrightarrow[n\longrightarrow\infty]{(\mathbf{P})} \sqrt{p(1-p)}$	☐ $\xrightarrow[n\longrightarrow\infty]{(\mathbf{P})} \sqrt{p(1-p)}$	☐ $\xrightarrow[n\longrightarrow\infty]{(\mathbf{P})} \sqrt{p(1-p)}$
✔	✔	✔

Solution:

1. $\sqrt{\overline{R}_n\left(1-\overline{R}_n\right)} \xrightarrow[n\longrightarrow\infty]{\mathbf{P}} \sqrt{p(1-p)}$ by the continuous mapping theorem.
2. $\dfrac{\sqrt{\overline{R}_n\left(1-\overline{R}_n\right)}}{\sqrt{p(1-p)}} \xrightarrow[n\longrightarrow\infty]{\mathbf{P}} 1$ since constant multiple of sequences that converge in probability still converge in probability.

3. $$\left(\sqrt{n}\frac{\overline{R}_n - p}{\sqrt{p(1-p)}}\right)\left(\frac{\sqrt{p(1-p)}}{\sqrt{\overline{R}_n\left(1-\overline{R}_n\right)}}\right) \xrightarrow[n\longrightarrow\infty]{(d)} \mathcal{N}(0,1)$$ by Slutsky.

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You have used 1 of 3 attempts

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Convergence in probability 4

? More clarification on "is approximated by (in distribution)", means that the CDFs are close" 3
What does it actually mean? Does it have something to do with convergence in distribution?

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