Exercise: The mean-squared error

(1/1 point)

In this exercise we want to understand a little better the formula

$$\frac{1}{\sum_{i=0}^{n} \frac{1}{\sigma_i^2}}$$

for the mean squared error by considering two alternative scenarios.

In the first scenario, $\Theta \sim N(0,1)$ and we observe $X = \Theta + W$, where $W \sim N(0,1)$ is independent of Θ.

In the second scenario, the prior information on Θ is extremely inaccurate: $\Theta \sim N(0, \sigma_0^2)$, where σ_0^2 is so large that it can be treated as infinite. But in this second scenario we obtain two observations of the form $X_i = \Theta + W_i$, where the W_i are standard normals, independent of each other and of Θ.

The mean squared error is

- ( ) smaller in the first scenario.
- ( ) smaller in the second scenario.
- (•) the same in both scenarios. ✔

Answer:
We use the formula for the mean squared error. For the second scenario, we set $\sigma_0^2 = \infty$. In the first scenario, we obtain

$$\frac{1}{\frac{1}{1}+\frac{1}{1}} = \frac{1}{2},$$

and in the second scenario, we obtain the same mean squared error:

$$\frac{1}{\frac{1}{\infty}+\frac{1}{1}+\frac{1}{1}} = \frac{1}{2}.$$

This suggests the following interpretation: the prior information on Θ in the first scenario is, in a loose sense, exactly as informative as having no useful prior information but one more observation, as in the second scenario.

You have used 1 of 1 submissions