1. Flux zero

1(a)

1/1 point (graded)

The saline concentration u in a thin metal tube of length 1 containing the solution satisfies the diffusion equation

$$\frac{\partial u}{\partial t} = 2\frac{\partial^2 u}{\partial x^2}, \qquad 0 < x < 1, \quad t > 0.$$

(Note that the diffusion constant is not 1 in this problem.)

Assume flux of saline at the boundary is zero, that is, assume $\frac{\partial u}{\partial x} = 0$ at the boundary. Thus the initial and boundary conditions in this situation are

$$\begin{aligned} \frac{\partial u}{\partial x}(0,t) &= \frac{\partial u}{\partial x}(1,t) = 0, \qquad t > 0 \\ u(x,0) &= x, \qquad 0 < x < 1 \end{aligned}$$

Use separation of variables to look for solutions of the form $v(x)\, w(t)$. **Use the diffusion constant 2 only in finding $w(t)$.**

Determine a basis of normalized functions (amplitude 1) that spans the space of possible solutions $u_k(x,t) = v_k(x)\, w_k(t)$.

$u_k(x,t) = v_k(x)\, w_k(t) =$ cos(k*pi*x)*e^(-2*k^2*pi^2*t)

$\cos(k \cdot \pi \cdot x) \cdot e^{-2 \cdot k^2 \cdot \pi^2 \cdot t}$

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1(b)

1/1 point (graded)
Find the Fourier series for

$$u(x,0) = x = \sum a_k v_k(x), \qquad 0 < x < 1.$$

(Enter the first 4 nonzero terms in the series.)

$u(x,0) =$ 1/2-4/pi^2*cos(pi*x)-4/pi^2/9*cos(3*pi*x)-4/pi^2/25*cos(5*pi*x)

$\frac{1}{2} - \frac{4}{\pi^2} \cdot \cos(\pi \cdot x) - \frac{4}{\pi^2 \cdot 9} \cdot \cos(3 \cdot \pi \cdot x) - \frac{4}{\pi^2 \cdot 25} \cdot \cos(5 \cdot \pi \cdot x)$

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1(c)

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Find the solution $u(x,t)$ as a linear combination of $u_k(x,t)$.

(In the answer box, type in the first three nonzero terms in the series expression.)

$u(x,t) =$ 1/2-4/pi^2*cos(pi*x)*e^(-2*pi^2*t)-4/pi^2/9*cos(3*pi*x)*e^(-18*pi^

$$\frac{1}{2} - \frac{4}{\pi^2} \cdot \cos(\pi \cdot x) \cdot e^{-2 \cdot \pi^2 \cdot t} - \frac{4}{\pi^2 \cdot 9} \cdot \cos(3 \cdot \pi \cdot x) \cdot e^{-18 \cdot \pi^2 \cdot t} - \frac{4}{\pi^2 \cdot 25} \cdot \cos(5 \cdot \pi \cdot x) \cdot e^{-50 \cdot \pi^2 \cdot t}$$

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1(d)

1/1 point (graded)

What is the steady state solution $u_{st}(x)$? (The solution defined by $u(x,t) \to u_{st}(x)$ as $t \to \infty$.)

$u_{st}(x,t) =$ 1/2

$$\frac{1}{2}$$

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1(e)

1/1 point (graded)

Estimate to 2 significant figures, the time T it takes to be within 1% of the steady solution.

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(You can use a computer for an accurate answer, but may also do a rough estimate by hand (and calculator).)

$T =$ 0.22

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part e

question posted about 24 hours ago by XaviHaz

Any hint for part e, please! I've been using, in a specific point in space xs, |u(xs,t) - Ust| < = 1/100*Ust

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1 response

sandipan_dey

about 5 hours ago

Actually while diffusion, the temperature at different points are different at different time instants (until the steady state appears), i obtained an animation like the following to keep track of how the temperature u at different x are changing at different time t (the initial state corresponds to the time t=-0.005, where the diffusion starts at t = 0 according to the equation obtained till part (d)).

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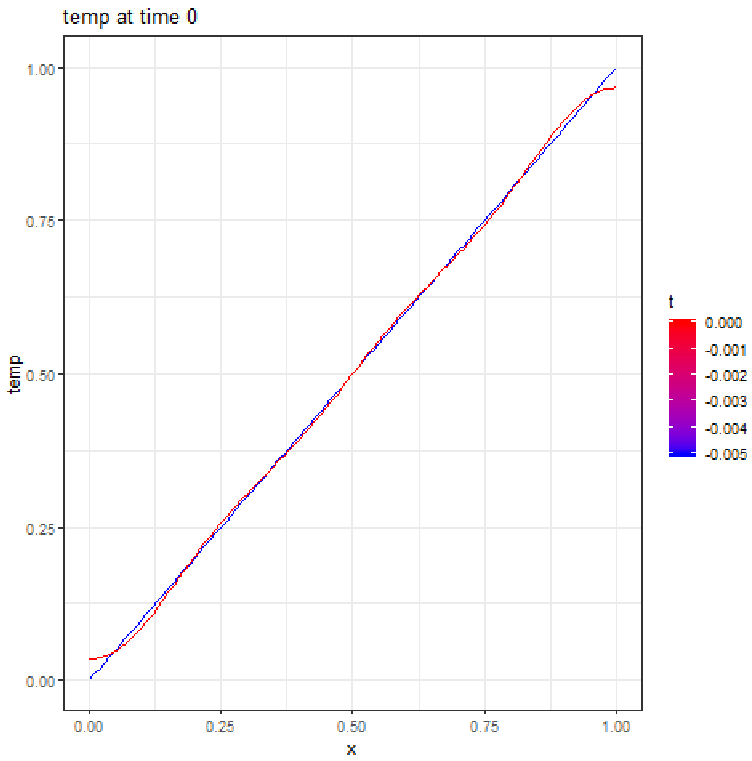

Now, my question is: how to aggregate the temperatures of the tube (for different xs) at a particular time for (e), to compute a single value and compare it with the steady state temperature (e.g., mean does not work)?

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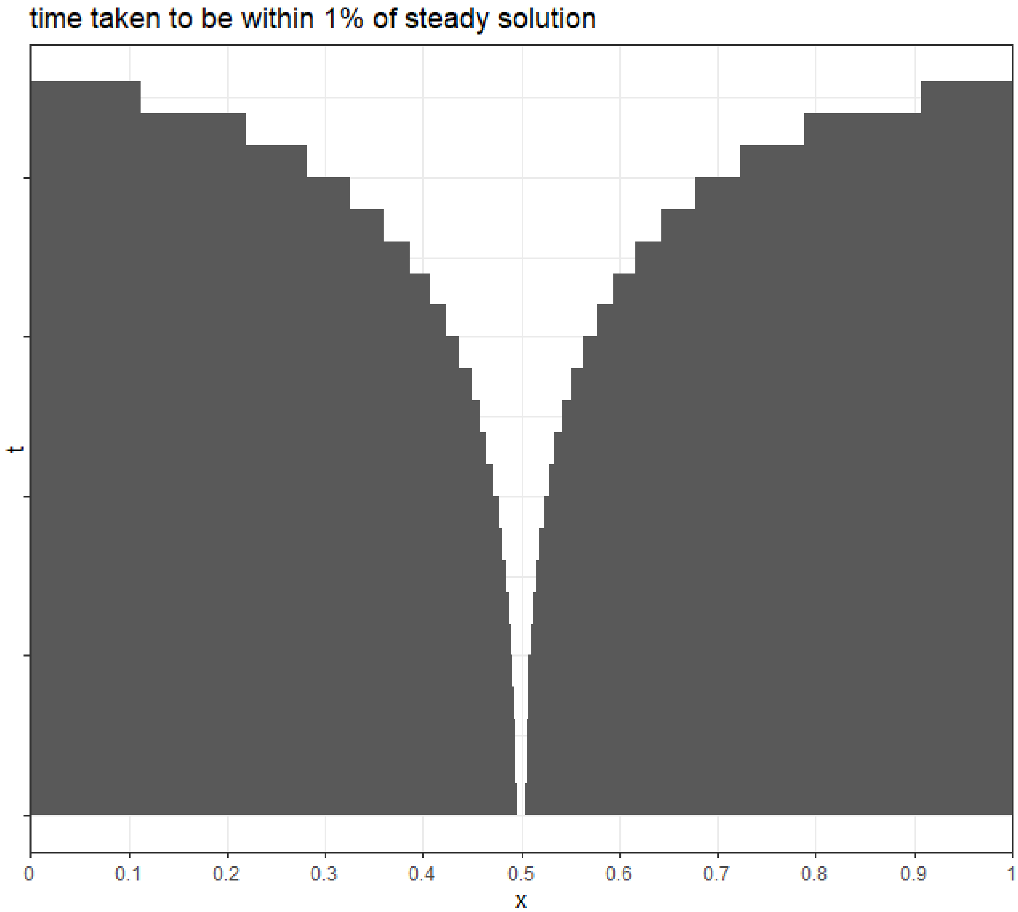
UPDATE

The below plot i obtained shows the time taken to be within 1% of the steady solution for different x values (time ticks omitted). The grader accepts a value in the extremes.



Also, from the above figure, it seems that there is a relation between the time taken and the initial difference of temperature of a position x with the steady solution, may be something like $t = O(\sqrt{\epsilon})$ where $\epsilon = \frac{|u(x,0)-u(x,t_s)|}{u(x,t_s)}$ (here t_s is the time after which the steady solution is obtained, assuming $u\left(x, t_s\right) \neq 0$ here), can there a theoretical upper-bound on the time to arrive at the steady solution be established as a function of the initial temperature difference? Any reference along the line will be helpful.

Also, this reminds of Newton's law of cooling which states $\frac{d\theta}{dt} \propto (\theta - \theta_0)$, is this related in some way to the diffusion time taken as per the heat

n? thanks in advance.

posted about 5 hours ago by **sandipan_dey**

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