Exercise: Using the formula for the monotonic case

(6/6 points)

The random variable X is exponential with parameter $\lambda = 1$. The random variable Y is defined by $Y = g(X) = 1/(1+X)$.

a) The inverse function h, for which $h(g(x)) = x$, is of the form $ay^b + c$. Find a, b, and c.

$a =$ 1 ✔ **Answer:** 1

$b =$ -1 ✔ **Answer:** -1

$c =$ -1 ✔ **Answer:** -1

b) For $y \in (0,1]$, the PDF of Y is of the form $f_Y(y) = y^a e^{(b/y)+c}$. Find a, b, and c.

$a =$ -2 ✔ **Answer:** -2

$b =$ -1 ✔ **Answer:** -1

$c =$ 1 ✔ **Answer:** 1

Answer:
a) If x and y obey the relation $y = g(x) = 1/(1+x)$, then $y + yx = 1$, so that

$$x = h(y) = \frac{1-y}{y} = \frac{1}{y} - 1.$$

Note that we are interested in $x \geq 0$ which restricts y to the range $(0,1]$. Notice also that the functions g and h are monotonically decreasing on the relevant ranges of values.

b) Note that

$$\frac{dh}{dy}(y) = -\frac{1}{y^2}.$$

Therefore,

$$f_Y(y) = f_X\left(h(y)\right)\left|\frac{dh}{dy}(y)\right| = e^{-(1/y)+1} \cdot \frac{1}{y^2}.$$

You have used 1 of 2 submissions