Random Walk With Drift

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Use a biased coin to simulate a random walk of 30 steps on the line. If the coin comes up heads (H), take one step to the right, if it comes up tails (T), take one step left. After 30 steps, note the final position. Take the probability P(H) = 0.75. Plot a sample path of the random walk for 30 time steps. Make a histogram of the final position for 1 million such random walks. Compute the sample mean and the sample variance.

Brownian Motion can be recognised from a proposed Drunk's trek. GEOMETRIC DISPLAYS MAY TAKE SOME TIME TO EXHIBIT

```
n<-30 # 30 steps in drunkard's walk.
prob<-0.75# probability of heads via binomial simulation.
size<-1e6
u<-runif(n)
u<prob
```

```
##  [1] FALSE  TRUE FALSE  TRUE  TRUE FALSE  TRUE  TRUE  TRUE  TRUE  TRUE
## [12]  TRUE  TRUE  TRUE  TRUE FALSE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE
## [23]  TRUE FALSE  TRUE  TRUE  TRUE  TRUE  TRUE FALSE
```

```
staggerstep<-function(size,n,prob) sum(runif(n)<prob) #action catering movement staggers.
sample<-replicate(size,staggerstep(size, n,prob)) # 1e6 trials for drunk.
hist(sample,100,col="green")
# E(x)=n*p ; Var(x)=n*p*(1-p)
30*0.75 # analytic formula for binomial expectation.
```

```
## [1] 22.5
```

```
mean(sample) # resultis consistent with analytic formula of binomial expectation.
```

```
## [1] 22.49858
```

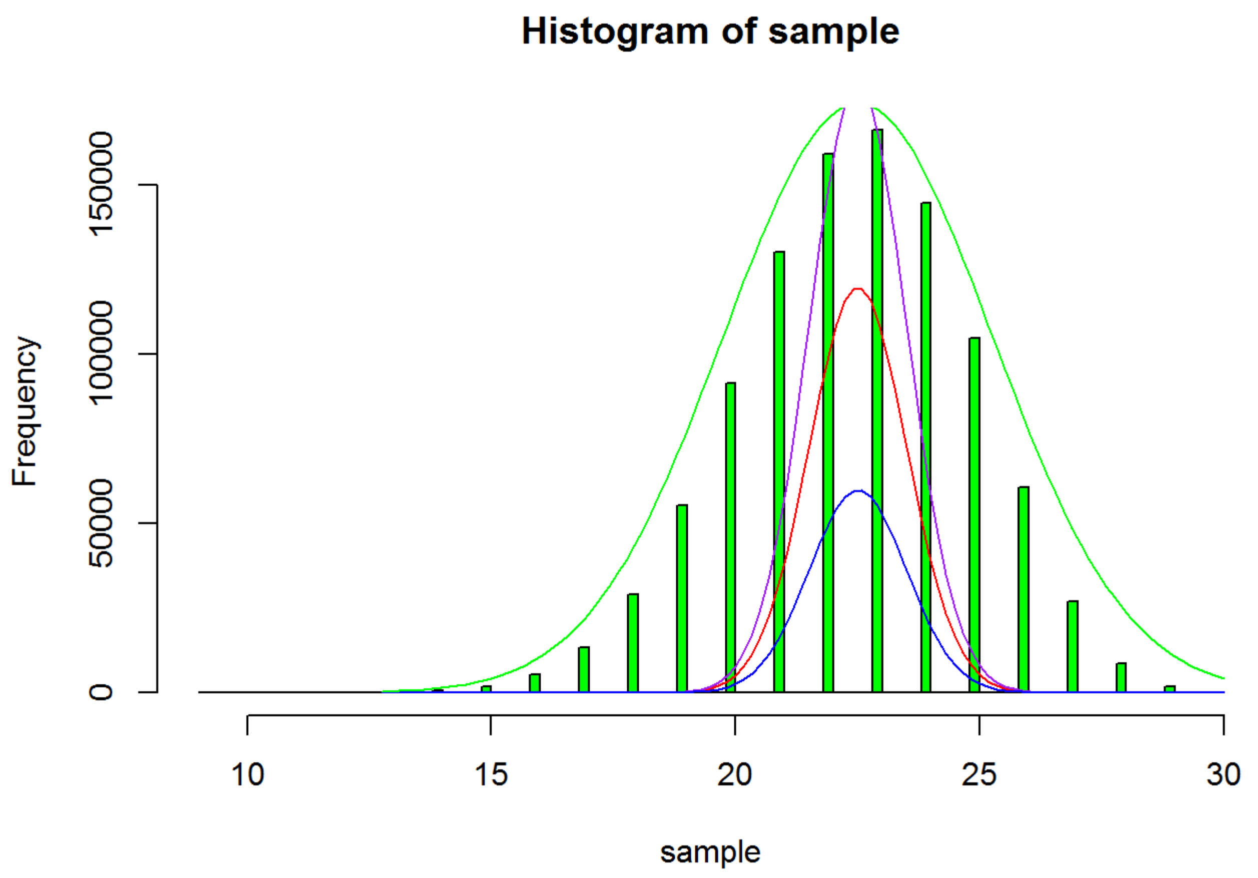
```
30*0.75*0.25 # analytic formula for binomial variance.
```

```
## [1] 5.625
```

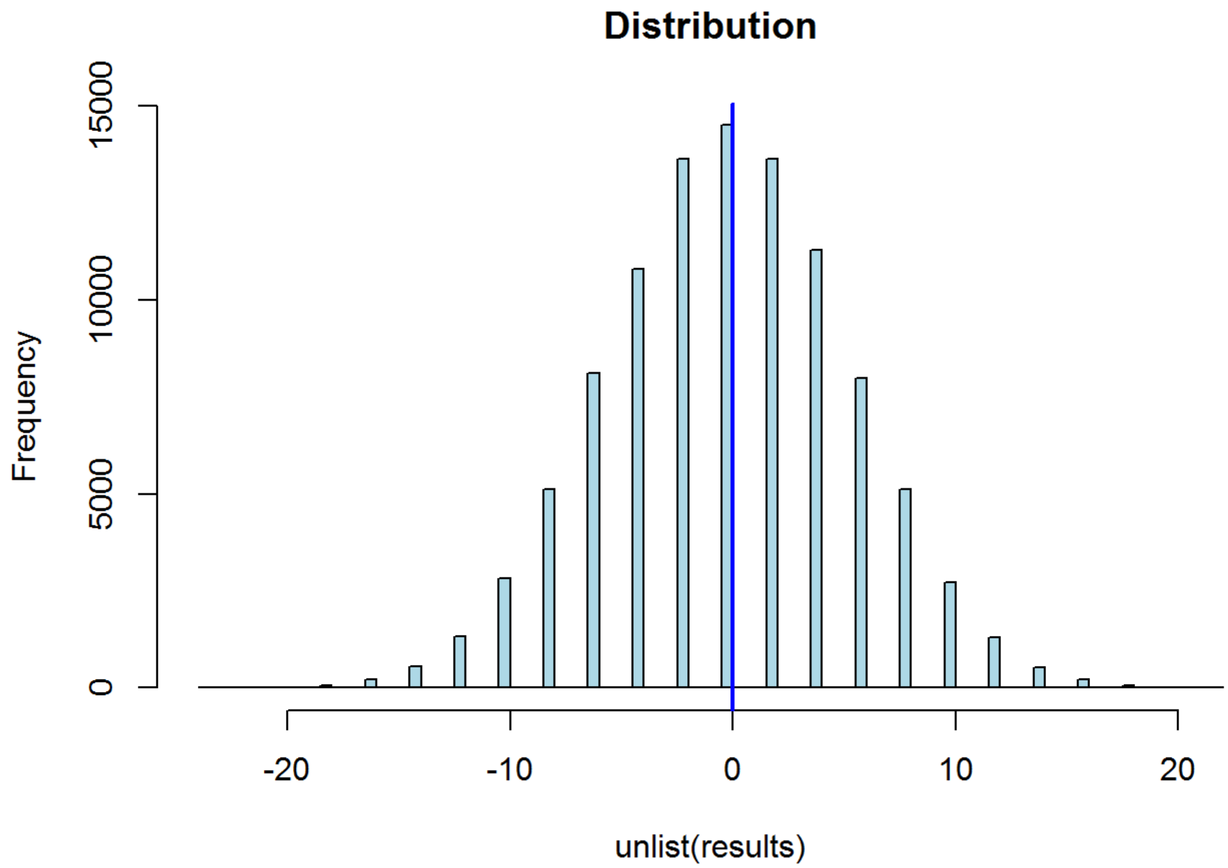
```
mean(sample)*(1-prob) # result consistent with analytic formula for binomial variance.
```

```
## [1] 5.624646
```

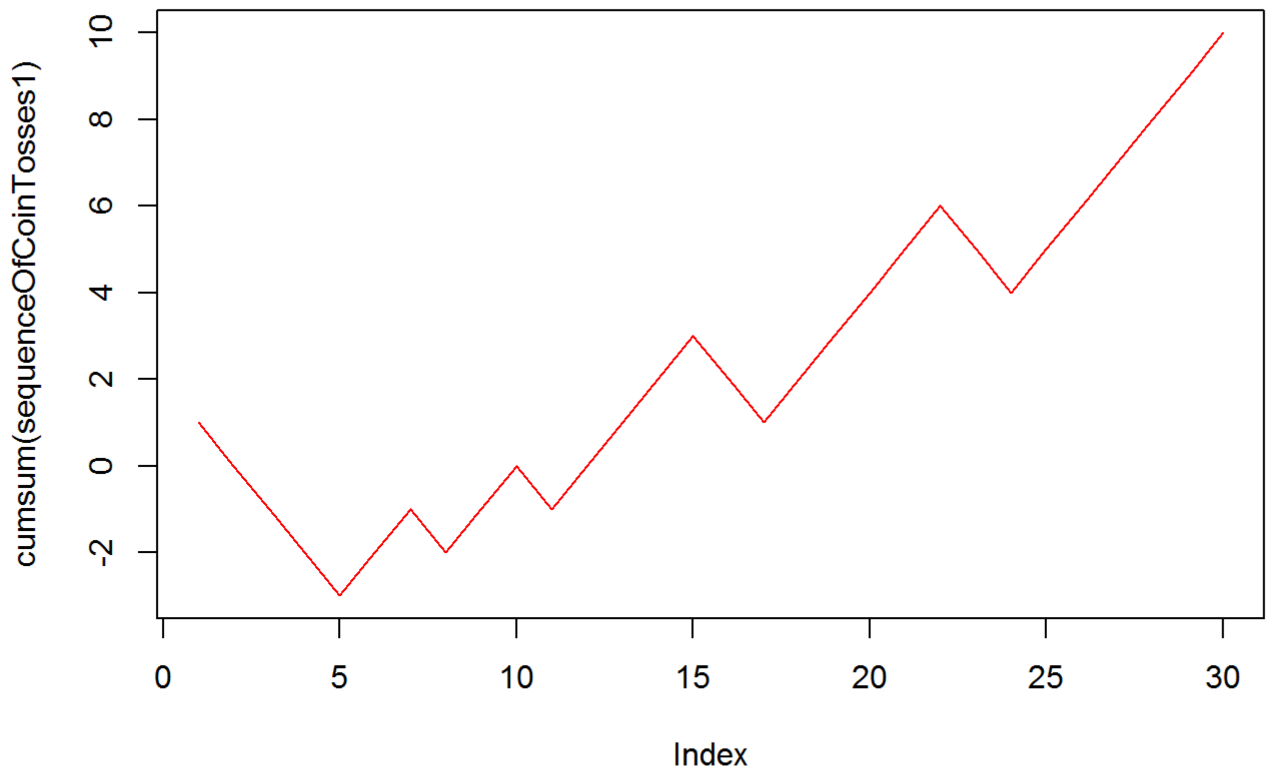
```
curve(300000*dnorm(x,22.5,1),12.75,30,col="red",add=T)
curve(450000*dnorm(x,22.5,1),12.75,30,col="purple",add=T)
curve(150000*dnorm(x,22.5,1),12.75,30,col="blue",add=T)
curve(1.2e6*dnorm(x,22.5,2.75),12.75,30,col="green",add=T)
```



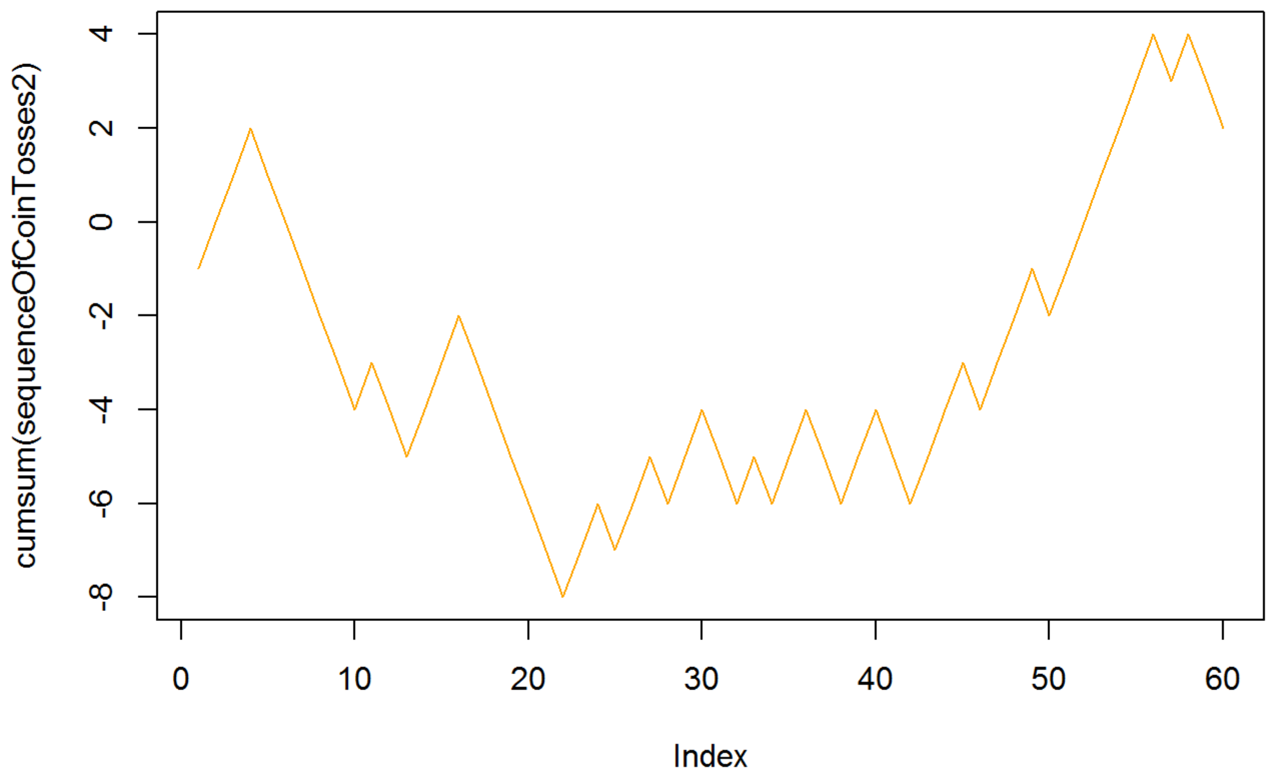

```
# OBSERVING NOW A MORE DIRECT APPROACH WITH FAIRNESS FOR GENERALITY.
 # Empty list creation to store results
 results <- list()
 for(i in 1:100000) {
     coinTosses   <- cumsum(sample(c(-1,1), 30, replace = TRUE))
     results[[i]] <- coinTosses[length(coinTosses)]
 }
 # Unlist the list and create a histogram.
hist(unlist(results), main = "Distribution",col = "lightblue", breaks = 100)
 # Vertical line at 0, of breadth 2 to exhibit the expectation of the distribution
abline(v = 0, col = "blue", lwd = 2)
```



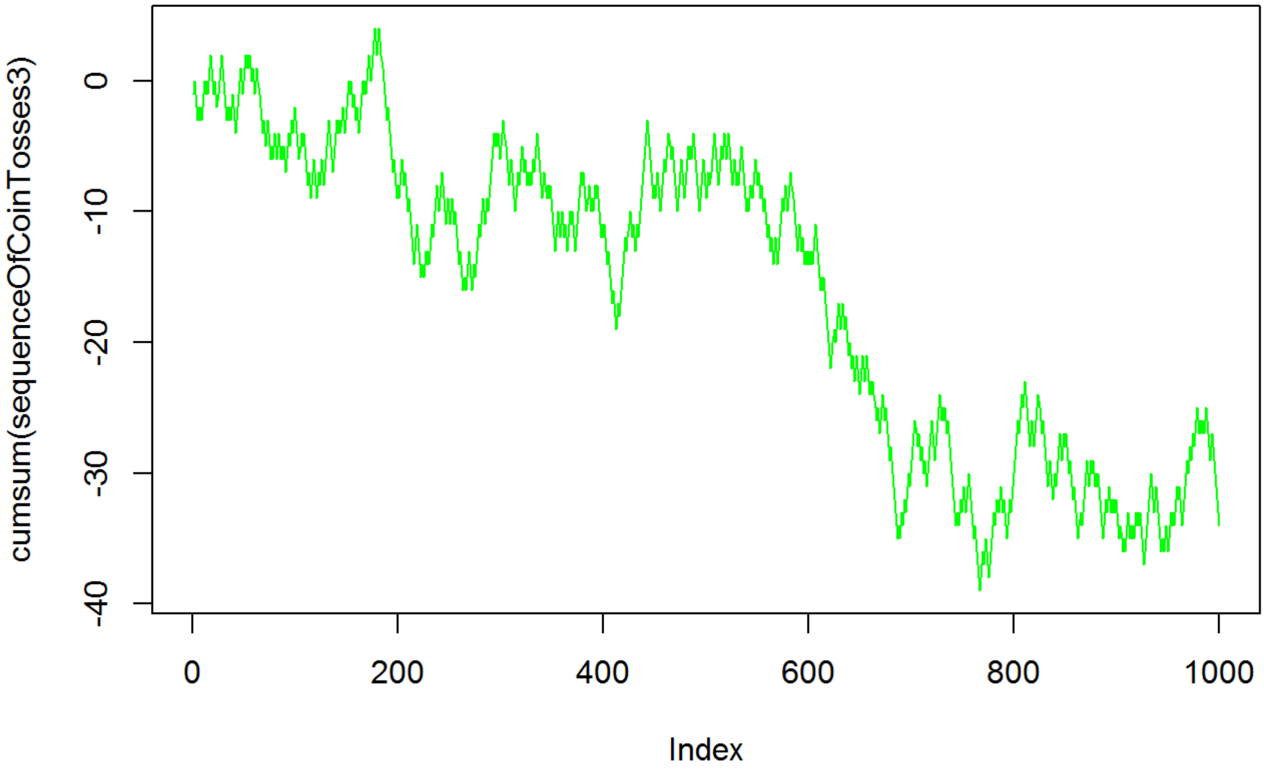

```
sequenceOfCoinTosses1 <- sample(c(-1,1), 30, replace = TRUE)
sequenceOfCoinTosses2 <- sample(c(-1,1), 60, replace = TRUE)
sequenceOfCoinTosses3 <- sample(c(-1,1), 1000, replace = TRUE)
plot(cumsum(sequenceOfCoinTosses1), type = 'l',col="red")
```




```
plot(cumsum(sequenceOfCoinTosses2), type = 'l',col="orange")
```

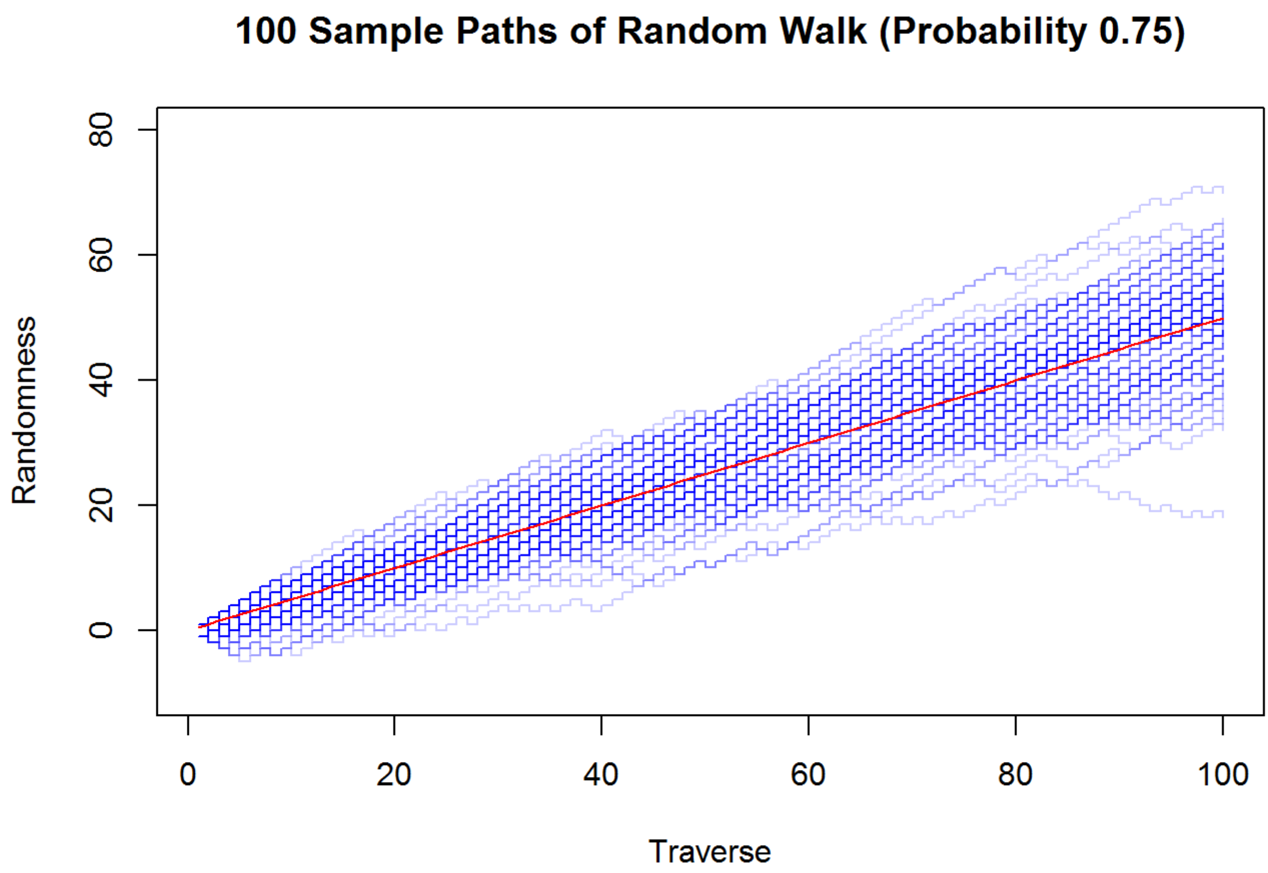


```
plot(cumsum(sequenceOfCoinTosses3), type = 'l', col="green")
```




```
# Recognised is fractal brownian motion from a generic process, hence contemplating the proba
bility of such with biasness.

# 100 Sample Paths of Random Walk with Probability 0.75 with a more general geometry
plot(c(1, 100), c(-10, 80), type = "n", xlab = "Traverse",
     ylab = "Randomness")
Zn = vector()
for (i in 1:100) {
  X = rbinom(n = 100, size = 1, prob = 0.75)
  Z = cumsum(2 * X - 1)
  lines(1:100, c(Z), type = "s", col = rgb(0, 0,
                                           1, alpha = 0.2))
  Zn[i] = Z[100]
}
curve(x * (2 * 0.75 - 1), add = TRUE, col = "red")
title("100 Sample Paths of Random Walk (Probability 0.75)")
```




```
# CONCLUSION: A sample size of 1e6 simulations of the like random variables implies the Centr
al Limit Theorem. Such is known with higher number experiments of binomial processes. As well
from observation of simulations with "Sequence of Coin Tosses" functions with fairness (being
binomial), the higher the mount of trials, leads to more "randomness" in successive geometric
figures.
```