

Unit 2: Boundary value problems

2. Nonzero concentration specified

Course > and PDEs

> Recitation 5 (with MATLAB) > at endpoints

2. Nonzero concentration specified at endpoints

2(a)

5/5 points (graded)

Consider a thin rod of length 1 containing a saline solution. The left end point is placed in a large saline bath of concentration 1. The right end point is placed in a large bath of freshwater (concentration 0). The concentration u satisfies the following initial value problem.

$$egin{array}{ll} rac{\partial u}{\partial t} &=& 2rac{\partial^2 u}{\partial x^2}, & & 0 < x < 1, \quad t > 0 \ u\left(x,0
ight) &=& x, & 0 < x < 1 \end{array}$$

Note that the diffusion constant is 2 in this problem.

Identify the boundary conditions. Enter the value if known, and enter UNK if unknown.

What is the steady state solution (particular solution) $u_{st}\left(x\right)$, (the solution defined by $u\left(x,t\right) o u_{st}\left(x\right)$ as $t o \infty$)?

$$u_{st}\left(x
ight) = extstyle{igsquare}$$
 1-x

FORMULA INPUT HELP

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✓ Correct (5/5 points)

2(b)

3/3 points (graded)

Use superposition to write $u\left(x,t
ight)=u_{st}\left(x
ight)+u_{h}\left(x,t
ight).$

Find the boundary conditions satisfied by u_h and then use separation of variables $u_{h,k}\left(x,t\right)=v_k\left(x\right)w_k\left(t\right)$ to find the eigenvalues λ_k and eigenfunctions $v_k\left(x\right)$. Only use the constant 2 from the PDE in finding $w_k\left(t\right)$.

For
$$k=1,2,3,\ldots$$
, $v_k\left(x\right)=oxed{\sin(k^*\operatorname{pi}^*\!x)}$

For
$$k=1,2,3,\ldots$$
, $w_k\left(t
ight)=$ $e^{\left(-2*k^2+\pi^2\cdot t^2\right)}$ $e^{-2\cdot k^2\cdot \pi^2\cdot t}$

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✓ Correct (3/3 points)

2(c)

1/1 point (graded)

Find the initial condition $u_h(x,0)$.

$$u_h\left(x,0
ight)= oxedsymbol{2^{\star}}_{2 \cdot x-1}$$

FORMULA INPUT HELP

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✓ Correct (1/1 point)

2(d)

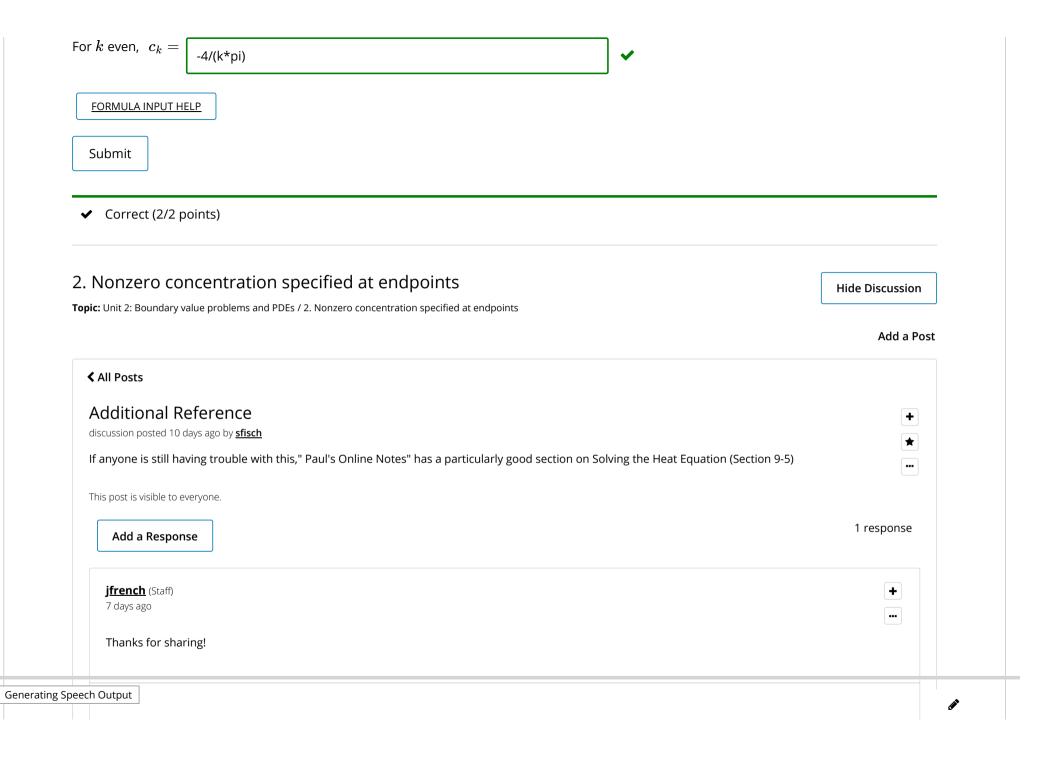
2/2 points (graded)

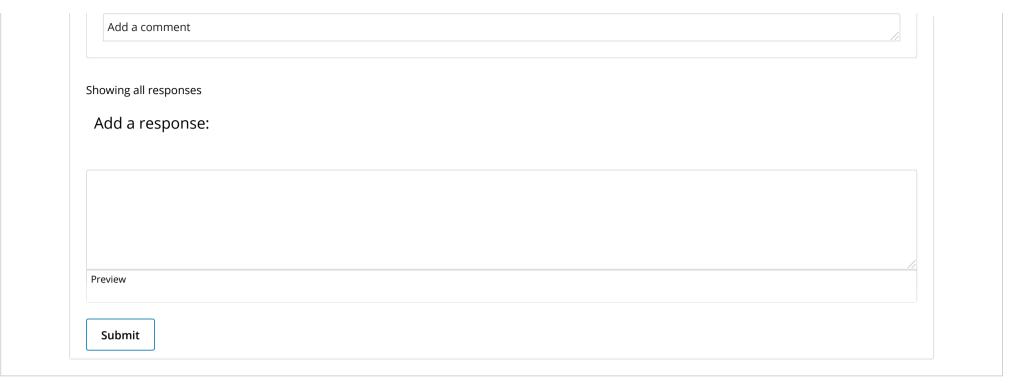
Find the appropriate periodic extension for $u_h\left(x,0\right)$ to be able to solve for the Fourier coefficients

$$u_{h}\left(x,0
ight) =\sum c_{k}v_{k}\left(x
ight) ,\qquad 0< x<1.$$

(Hint: You can use superposition of two known (manipulated) Fourier series to do so.)

Enter the coefficient c_k below.





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