Observation Theory

Script V12C – Precision and Covariance Matrix

In this video we will have a look at the definition of a covariance matrix, which is a measure for the precision of our observations.

We finished the previous lecture with the distributions of random errors for an example with high precision and a situation with low precision; here we see the corresponding histograms.

Recall the important property that: on average the random errors are zero.

Hence, the mean is zero.

What remains to be shown is how to calculate the standard deviation based on a large set of repeated measurements.

This figure shows some realisations of the random errors and the red dotted lines are the deviations from the mean.

The square of the standard deviation, which is called the VARIANCE, is defined by this equation:

Where first we recognise the squared deviations from the mean:

And we see that the variance is approximately equal to the average of these squared deviations:

Now remember that the mean error will be zero:

And it cancels out:

Furthermore, the individual errors are equal to the distance from individual observations to their mean.

Now by plugging in this equation, we get this result

We can now see that in fact the variance of the random errors is equal to that of the observations.

The standard deviation follows as the square root of the variance.

Hence, if you need to determine the precision of a certain instrument or measurement technique, the procedure is to first carry out a large number “m” of calibration measurements and calculate the standard deviation using this equation.

You can then use this standard deviation to describe the precision of future measurements.

So, let’s assume we know the precision that we will get with different instruments to measure the canal width.

And furthermore, let’s assume we will collect 3 measurements; the corresponding random errors are collected in a vector.

Even before taking the measurements we know the precision of this vector e.

The covariance matrix describes the precision of the 3 observations.

The covariance matrix is defined by the shown matrix in which the diagonal elements are the variances of the 3 random errors.

To keep the notation simple, only the index of the corresponding error is used as a subscript.

The off-diagonal elements are the covariances.

For example …

This element is the covariance between the first and second random error.

An important property of covariance matrices is that they are SYMMETRIC.

In most situations, different observations and thus different random errors are independent, which means that they are uncorrelated and their covariances are therefore all equal to zero.

Consequently the covariance matrix is then diagonal.

We will now show an example.

From the canal width measurements we know that measurements with different instruments have different precisions.

Assume we collect 6 measurements with associated random errors

3 laser distance measurements with standard deviation sigma-l, and 3 rope measurements with standard deviation sigma-r.

All measurements are independent.

The covariance matrix will now be equal to this 6x6 matrix

Where we can clearly identify the two submatrices linked to the two different types of measurements.

In the exercises you will look at more examples.

In the next video we will then look at the case that the covariances are not equal to zero.