7. Linearization

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Lecture due Sep 15, 2021 20:30 IST

Explore

This process of approximating a transformation with a linear function is called **linearization**. Linearizing a function at a point means computing the matrix that tells you how the output changes when the inputs change near that point.

The term "output" should be understood to mean all of the variables that depend on the input variables. In the robot arm example, the "input" was two numbers, the variables L and θ, and the "output" was two numbers, the x, y coordinate of the tip of the robot arm. Most of the other examples in this section will be of this same type: two inputs, and two outputs.

Linearization Steps

- **Step 1.** Find the equations that describes the relationship between input variables and output variables. Decide what should be the base point (x_0, y_0) (assuming there are two input variables x and y).
- **Step 2.** For each output variable P, do a linear approximation of $P(x_0 + \Delta x, y_0 + \Delta y)$. You will need to compute the partial derivatives evaluated at (x_0, y_0). Repeat for the other output variables.
- **Step 3.** Find the matrix that captures the linear approximations.

Example

Example 7.1 Suppose that the variables A and B depend on the variables x and y as follows.

$$A = xy \tag{5.111}$$

$$B = x^2 - y^2 \tag{5.112}$$

Compute the linearization of A, B at the point $(1, 2)$.

Step 1 We want to linearize the relationship $x, y \implies A, B$. In this case we were already given the equations for A and B in terms of x, y. So Step 1 is done.

Step 2 Next, we will do a linear approximation of $A(1 + \Delta x, 2 + \Delta y) - A(1, 2)$. This is given by

$$A(1 + \Delta x, 2 + \Delta y) - A(1, 2) \approx A_x(1, 2)\,\Delta x + A_y(1, 2)\,\Delta y, \tag{5.113}$$

where A_x and A_y are the partial derivatives of A. We can compute these:

$$A_x = y \tag{5.114}$$

$$A_y = x \tag{5.115}$$

So at the point $(1, 2)$ we have $A_x = 2$ and $A_y = 1$. Thus we obtain the approximation

$$A(1 + \Delta x, 2 + \Delta y) \approx 2\Delta x + \Delta y. \tag{5.116}$$

In a similar way, we obtain

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$$B(1+\Delta x, 2+\Delta y) \approx 2\Delta x - 4\Delta y. \tag{5.117}$$

This completes step 2.

Step 3 The last step is to find the matrix that describes the relationship $\Delta x, \Delta y \implies \Delta A, \Delta B$. We have shown

$$\begin{aligned}\Delta A &\approx 2\Delta x + \Delta y\\ \Delta B &\approx 2\Delta x - 4\Delta y\end{aligned}$$

This can also be written as

$$\begin{pmatrix}\Delta A\\ \Delta B\end{pmatrix} \approx \underbrace{\begin{pmatrix}2 & 1\\ 2 & -4\end{pmatrix}}_{\text{Linearization}}\begin{pmatrix}\Delta x\\ \Delta y\end{pmatrix} \tag{5.118}$$

Our final answer is the matrix $\begin{pmatrix}2 & 1\\ 2 & -4\end{pmatrix}$.

Practice Linearization

1 point possible (graded)
Suppose that the variables A and B depend on the variables x and y as

$$A = (x+y)^2 - 2y \tag{5.119}$$
$$B = (x-y)^2 \tag{5.120}$$

Find the linearization of A, B at the point $(0,1)$. Enter as a matrix.

(Enter a matrix using notation such as `[[a,b],[c,d]]`.)

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