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7. Test linear approximation against level curves

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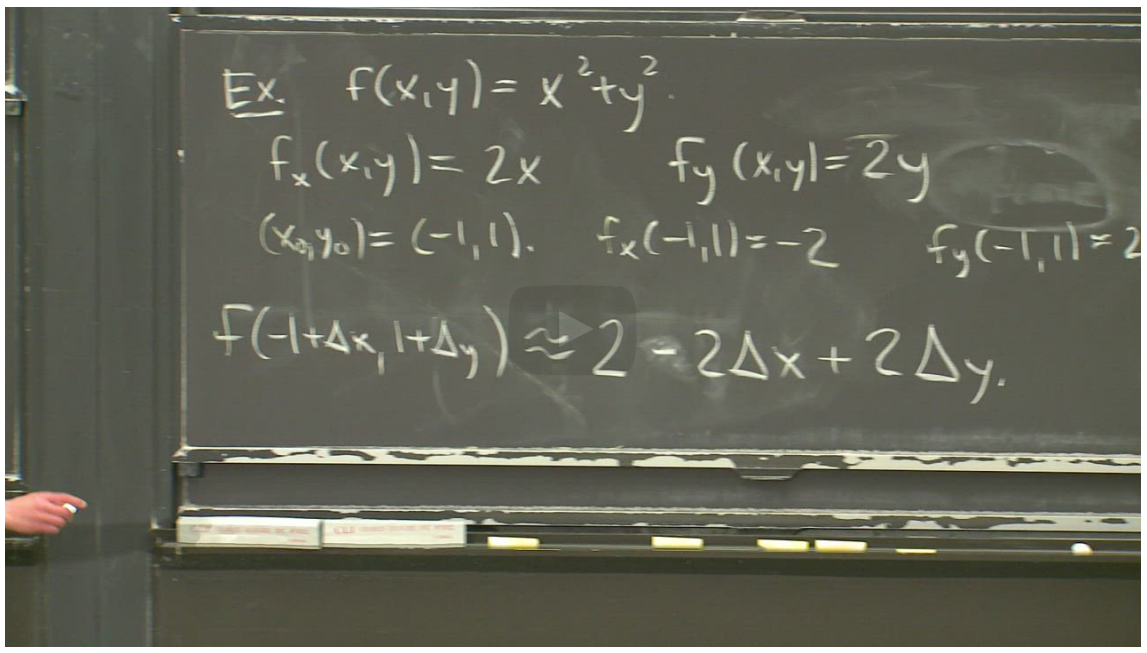
Lecture due Aug 4, 2021 20:30 IST Completed

Reflect

Testing approximations



0:00 / 0:00 2.0x

Start of transcript. Skip to the end.

PROFESSOR: OK.

So that's the linear approximation of the function

f around this point (negative 1 positive 1).

And to see how it works, I'm going

to show us all a picture of the level curves of this function.

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Recall the linear approximation of $f(x,y)=x^2+y^2$ near $(x_0,y_0)=(-1,1)$ found in the previous example:

$$f(-1+\Delta x, 1+\Delta y)\approx 2-2\Delta x+2\Delta y.$$

Here is an image of the level curves of $f(x,y)$. We will use the linear approximation to estimate the value of the function at nearby points and compare with the value of the function.

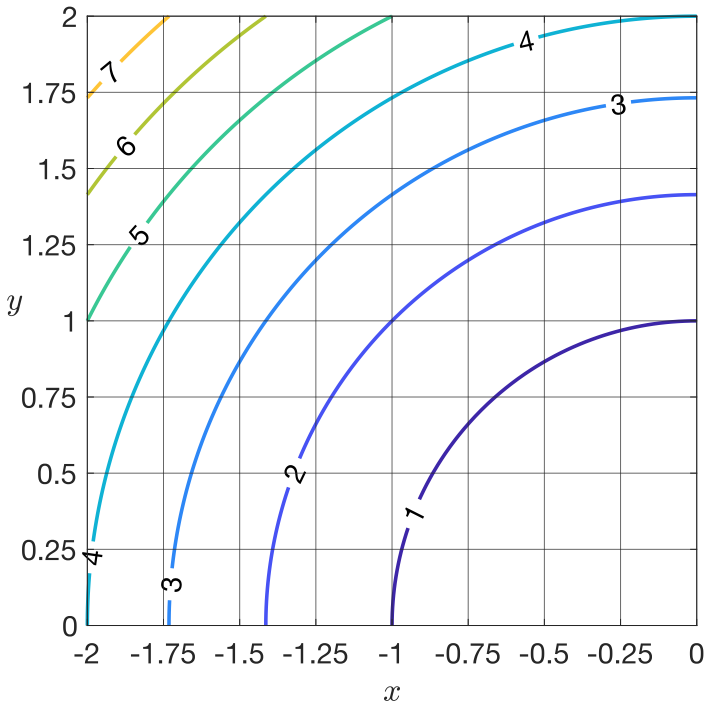

Figure 4: Level curves of $f(x,y)=x^2+y^2$ near $(-1,1)$.

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Δx and Δy The approximation $f(x+\Delta x, y+\Delta y)$

Δx and Δy	The approximation $f(x+\Delta x, y+\Delta y)$	Good or bad?
$\Delta x = 0$, $\Delta y = 0$	$f(-1,1) \approx 2$	✓
$\Delta x = 0$, $\Delta y = 1/2$	$f(-1,3/2) \approx 3$	✓
$\Delta x = 15$, $\Delta y = 0$	$f(-1+15,1) \approx -28$	x
$\Delta x = 1/4$, $\Delta y = 1/4$	$f(-1+1/4,1+1/4) \approx 2$	✓

For the choices that do not work, what is it about Δx (or Δy) that makes it a bad choice?

Remark 7.1 Linear approximation is only good when Δx and Δy are small, so let's try this experiment again but zoomed in closer to our point.

Test approximation against level curves

9/9 points (graded)

Here is a zoomed in image of the function $f(x,y) = x^2 + y^2$ near the point $(-1,1)$.

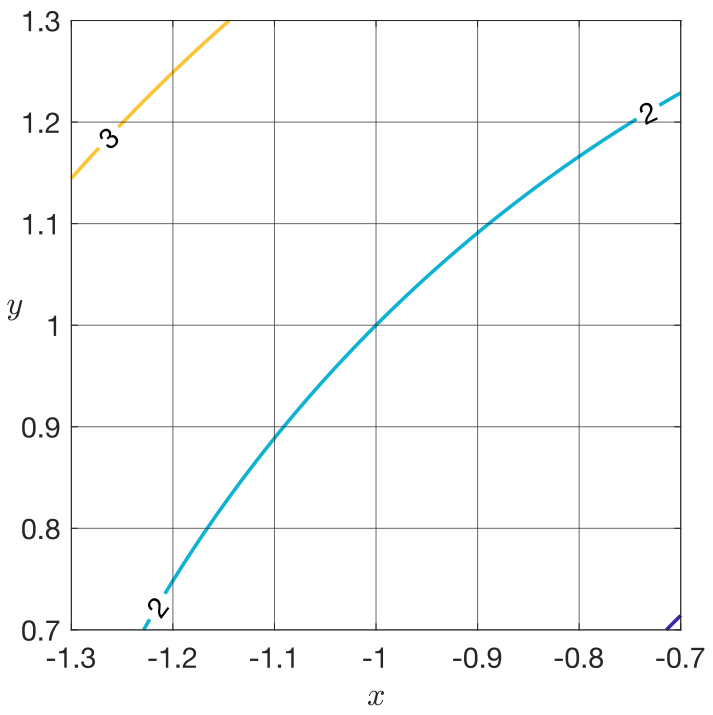

Figure 5: Small region around $(-1,1)$ of the level curves of $f(x,y) = x^2 + y^2$.

Which of the following choices of Δx and Δy does the linear approximation give a good approximation?

For the choices that do not work, figure out whether Δx makes it a bad choice, or Δy makes it a bad choice. For the following choices of Δx and Δy, determine the linear approximation of f, the actual value, and the error.

Definition 7.2 The **error** is defined as the ratio

$$\frac{(\text{actual value}) - (\text{approximate value})}{\text{actual value}}$$

Round your answers to 4 decimal places.

Δx and Δy	The approximation of $f(x+\Delta x, y+\Delta y)$	Actual value of $f(x+\Delta x, y+\Delta y)$	Error
	$f(-1,1.1) \approx$	$f(-1,1.1) =$	

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$\Delta x = 0$, $\Delta y = 0.1$	2.21	2.2	0.004524887
	✔ **Answer:** 2.2	✔ **Answer:** 2.21	✔ **Answer:** 0.0045
	$f(-0.9, 1) \approx$	$f(-0.9, 1) =$	
$\Delta x = 0.1$, $\Delta y = 0$	1.81	1.8	0.005524862
	✔ **Answer:** 1.8	✔ **Answer:** 1.81	✔ **Answer:** 0.0055
	$f(-0.9, 1.1) \approx$	$f(-0.9, 1.1) =$	
$\Delta x = 0.1$, $\Delta y = 0.1$	2	2.02	0.00990099
	✔ **Answer:** 2	✔ **Answer:** 2.02	✔ **Answer:** 0.0099

Solution:

$$\begin{aligned} f(-1+\Delta x, 1+\Delta y) &\approx 2 - 2\Delta x + 2\Delta y \\ f(-1, 1.1) &\approx 2 + 2(0.1) = 2.2 \\ f(-0.9, 1) &\approx 2 - 2(0.1) = 1.8 \\ f(-0.9, 1.1) &\approx 2 - 2(0.1) + 2(0.1) = 2 \end{aligned}$$

Note that the actual values are

$$\begin{aligned} f(-1, 1.1) &= 1 + 1.21 = 2.21 \\ f(-0.9, 1) &= 0.81 + 1 = 1.81 \\ f(-0.9, 1.1) &= 0.81 + 1.21 = 2.02 \end{aligned}$$

So the errors are 0.0045, 0.0055, and 0.0099 respectively.

Note that with small values of Δx and Δy, the approximate values differ by much smaller amounts.

Submit You have used 5 of 5 attempts

Answers are displayed within the problem

Take aways

Why do we need two partial derivatives f_x and f_y?

If you only knew $f_x(-1, 1)$, you could make a good guess about $f(-0.9, 1)$, but not $f(-1, 0.9)$. Similarly, if you knew $f_y(-1, 1)$, you could make a good guess about $f(-1, 0.9)$, but not $f(-0.9, 1)$. But if we know both f_x and f_y, we get a nice approximate description of how f behaves in all directions!

Note that the linear approximation at (x_0, y_0) only works well for points close to (x_0, y_0).

7. Test linear approximation against level curves

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Topic: Unit 1: Functions of two variables / 7. Test linear approximation against level curves

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? Staff: Is grader grading correctly? 6
Hello! I would like to confirm whether the grader is grading correctly? I'm trying to input the answer obtained via the definition of the...

? Linear approximation
I have tried this question several times but I keep getting the wrong answers even for the actual values. Fo g.

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confused about f(-1, 1.1), f(-0.9, 1), f(-0.9,1.1) I didn't understand if these points are going to be used anywhere at all? Can anyone explain?	3
Grade mistake? I feel like the grading for my answer to the error in the last question is wrong, can you take a look? Thanks!	5
Error sign and raw vs. relative error In sciences, the relative error is taken as the (experimental value - actual/theoretical value) / actual/theoretical value. This makes se...	2
misleading grading error? hi. i made a mistake on one of my answers and the grader gave credit for the incorrect answer as well as a red 'x' for a different part ...	3
[staff] typo	2

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