Module 3 Assessment - Part 2 (incl. MATLAB)

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You have 6 monthly measurements of the height of a point on a glacier. The measurements are obtained from a satellite laser altimeter.

Time [months]: $[0,\ 1,\ 2,\ 3,\ 4,\ 5]$

Observed heights [meters]: $[100.9,\ 99.6,\ 98.7,\ 99.9,\ 99.4,\ 99.5]$

We will consider two different models, with the following observation equations:

Model 1: $y_i = x_0 + x_1 t_i + x_2 t_i^2 = \sum_{p=0}^{2} x_p t_i^p$

Model 2: $y_i = x_0 + x_1 \cdot t_i + x_2 \cos\left(\frac{2\pi t_i}{12}\right)$

GLACIER MODEL - MATLAB EXERCISE (EXTERNAL RESOURCE)

Mid-survey

Feedback

- 4. Best Linear Unbiased Estimation (BLUE)
- 5. How precise is the estimate?
- Pre-knowledge Mathematics
- MATLAB Learning Content

Save Reset MATLAB Documentation (https://www.mathworks.com/help/)

```
%% times of observation [months]
t = [0 1 2 3 4 5]';
%% observed heights [m]
y = [100.9 99.6 98.7 99.9 99.4 99.5]';

%% number of observations
m  = length(t);


%% design matrices for the two models
A1 = [ones(1,m)', t, t.^2];
A2 = [ones(1,m)', t, cos(2*pi*t/12)];
A3 = [ones(1,m)', t, t.^2, t.^3, t.^4, t.^5];

%% what is the least squares solution for each of the models:
xhat1 = inv(A1'*A1)*(A1'*y)
xhat2 = inv(A2'*A2)*(A2'*y)
xhat3 = inv(A3'*A3)*(A3'*y)

%% what is the sum of squared residuals for each of the models:
eTe1  = (y - A1*xhat1)'*(y - A1*xhat1)
eTe2  = (y - A2*xhat2)'*(y - A2*xhat2)
eTe3  = (y - A3*xhat3)'*(y - A3*xhat3)

figure
plot(t,y,'xb')
hold on
plot(t,A1*xhat1,'r')
plot(t,A2*xhat2,'c')
plot(t,A3*xhat3,'g')
set(gca,'xlim',[0 5.1])
xlabel('time [months]')
ylabel('height [meter]')
legend('observations','model 1','model 2','model 3')
```

Run Submit for Assessment

Output

Glacier model - questions

4/4 points (graded)

What is the sum of squared residuals for model 1? [give your answer to 3 decimal places]

1.148 ✔ **Answer:** 1.148

1.148

What is the sum of squared residuals for model 2? [give your answer to 3 decimal places]

0.542 ✔ **Answer:** 0.542

0.542

Which model fits best to the observations?

model 2 ✔ **Answer:** model 2

Explanation

The sum of squared residuals with the second model is smaller, hence the model fits better.

What will be the value of the sum of squared residuals if we fit a fifth order polynomial? [give your answer to 3 decimal places]

0.000 ✔ **Answer:** 0.000

0.000

Explanation

The sum of squared residuals will be equal to 0, since this is a 'determined' system of equations, for which we have $\hat{x} = A^{-1}y$ and $\hat{y} = A\hat{x} = AA^{-1}y = y$, resulting in $\hat{e} = y - \hat{y} = 0$.

For an arbitrary set of observations y: will the solution change if we apply weighted least squares with $W = 4I_6$ compared to ordinary least squares?

no ✔ **Answer:** no

Explanation

$\hat{x} = (A^T(4I_6)A)^{-1}A^T(4I_6)y = \frac{1}{4}(A^TA)^{-1}A^T4y = (A^TA)^{-1}A^Ty$. Hence, the solution is not affected.

Submit You have used 1 of 3 attempts

✔ Correct (4/4 points)