

**A**

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**Tables of Fourier Series and Transform  
Properties**

**Table A.1** Properties of the continuous-time Fourier series
$$x(t) = \sum_{k=-\infty}^{\infty} C_k e^{jk\Omega t} \qquad C_k = \frac{1}{T} \int_{-T/2}^{T/2} x(t) e^{-jk\Omega t} dt$$

Property	Periodic function $x(t)$ with period $T = 2\pi/\Omega$	Fourier series $C_k$
Time shifting	$x(t \pm t_0)$	$C_k e^{\pm jk\Omega t_0}$
Time scaling	$x(\alpha t)$ , $\alpha > 0$	$C_k$ with period $\frac{T}{\alpha}$
Differentiation	$\frac{d}{dt}x(t)$	$jk\Omega C_k$
Integration	$\int_{-\infty}^t x(t) dt < \infty$	$\frac{1}{jk\Omega} C_k$
Linearity	$\sum_i \alpha_i x_i(t)$	$\sum_i \alpha_i C_{ik}$
Conjugation	$x^*(t)$	$C_{-k}^*$
Time reversal	$x(-t)$	$C_{-k}$
Modulation	$x(t)e^{jK\Omega t}$	$C_{k-K}$
Multiplication	$x(t)y(t)$	$\sum_{i=-\infty}^{\infty} C_{xi} C_{y(k-i)}$
Periodic convolution	$\int_T x(\theta)y(t-\theta)d\theta$	$TC_{xk}C_{yk}$
Symmetry	$x(t) = x^*(t)$ real	$\begin{cases} C_k = C_{-k}^*,  C_k  =  C_{-k} , \\ \operatorname{Re} C_k = \operatorname{Re} C_{-k}, \\ \operatorname{Im} C_k = -\operatorname{Im} C_{-k}, \\ \arg C_k = -\arg C_{-k} \end{cases}$
	$x(t) = x^*(t) = x(-t)$ real and even	$\begin{cases} C_k = C_{-k}, C_k = C_k^*, \\ \text{real and even} \end{cases}$
	$x(t) = x^*(t) = -x(-t)$ real and odd	$\begin{cases} C_k = -C_{-k}, C_k = -C_k^*, \\ \text{imaginary and odd} \end{cases}$
Parseval's theorem		$\frac{1}{T} \int_{-T/2}^{T/2}  x(t) ^2 dt = \sum_{k=-\infty}^{\infty}  C_k ^2$

**Table A.2** Properties of the continuous-time Fourier transform

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\omega) e^{j\omega t} d\omega \quad X(j\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$$

Property	Nonperiodic function $x(t)$	Fourier transform $X(j\omega)$
Time shifting	$x(t \pm t_0)$	$e^{\pm j\omega t_0} X(j\omega)$
Time scaling	$x(\alpha t)$	$\frac{1}{ \alpha } X\left(\frac{j\omega}{\alpha}\right)$
Differentiation	$\frac{d}{dt} x(t)$	$j\omega X(j\omega)$
Integration	$\int_{-\infty}^t x(t) dt$	$\frac{1}{j\omega} X(j\omega) + \pi X(j0) \delta(\omega)$
	$\int_{-\infty}^{\infty} x(t) dt$	$X(j0)$
Frequency integration	$2\pi x(0)$	$\int_{-\infty}^{\infty} X(j\omega) d\omega$
Linearity	$\sum_i \alpha_i x_i(t)$	$\sum_i \alpha_i X_i(j\omega)$
Conjugation	$x^*(t)$	$X^*(-j\omega)$
Time reversal	$x(-t)$	$X(-j\omega)$
Modulation	$x(t) e^{j\omega_0 t}$	$X(j\omega - j\omega_0)$
Multiplication	$x(t) y(t)$	$\frac{1}{2\pi} X(j\omega) \times Y(j\omega)$
Convolution	$x(t) * y(t)$	$X(j\omega) Y(j\omega)$
Symmetry	$x(t) = x^*(t)$ real	$\begin{cases} X(j\omega) = X^*(-j\omega), \\  X(j\omega)  =  X(-j\omega) , \\ \operatorname{Re} X(j\omega) = \operatorname{Re} X(-j\omega), \\ \operatorname{Im} X(j\omega) = -\operatorname{Im} X(-j\omega), \\ \arg X(j\omega) = -\arg X(-j\omega) \end{cases}$
	$x(t) = x^*(t) = x(-t)$ real and even	$\begin{cases} X(j\omega) = X(-j\omega), \\ X(j\omega) = X^*(j\omega), \\ \text{real and even} \end{cases}$
	$x(t) = x^*(t) = -x(-t)$ real and odd	$\begin{cases} X(j\omega) = -X(-j\omega), \\ X(j\omega) = -X^*(j\omega), \\ \text{imaginary and odd} \end{cases}$
Rayleigh's theorem	$E_x = \int_{-\infty}^{\infty}  x(t) ^2 dt = \frac{1}{2\pi} \int_{-\infty}^{\infty}  X(j\omega) ^2 d\omega$	

## **B**

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### **Tables of Fourier Series and Transform of Basis Signals**

**Table B.1** The Fourier transform and series of basic signals

Signal $x(t)$	Transform $X(j\omega)$	Series $C_k$
1	$2\pi\delta(\omega)$	$C_0 = 1, C_{k \neq 0} = 0$
$\delta(t)$	1	$C_k = \frac{1}{T}$
$u(t)$	$\frac{1}{j\omega} + \pi\delta(\omega)$	—
$u(-t)$	$-\frac{1}{j\omega} + \pi\delta(\omega)$	—
$e^{j\Omega t}$	$2\pi\delta(\omega - \Omega)$	$C_1 = 1, C_{k \neq 1} = 0$
$\sum_{k=-\infty}^{\infty} C_k e^{jk\Omega t}$	$2\pi \sum_{k=-\infty}^{\infty} C_k \delta(\omega - k\Omega)$	$C_k$
$\cos \Omega t$	$\pi[\delta(\omega - \Omega) + \delta(\omega + \Omega)]$	$C_1 = C_{-1} = \frac{1}{2}, C_{k \neq \pm 1} = 0$
$\sin \Omega t$	$\frac{\pi}{j}[\delta(\omega - \Omega) - \delta(\omega + \Omega)]$	$C_1 = -C_{-1} = \frac{1}{2j}, C_{k \neq \pm 1} = 0$
$\frac{1}{\alpha^2 + t^2}$	$e^{-\alpha \omega }$	$\frac{1}{T} e^{-\frac{2\pi\alpha k }{T}}$
Rectangular (Fig. 2.16c)	$A\tau \frac{\sin(\omega\tau/2)}{\omega\tau/2}$	$\frac{A}{q} \frac{\sin(k\pi/q)}{k\pi/q}$
Triangular (Fig. 2.21a)	$\frac{A\tau}{2} \frac{\sin^2(\omega\tau/4)}{(\omega\tau/4)^2}$	$\frac{A}{2q} \frac{\sin^2(k\pi/2q)}{(k\pi/2q)^2}$
Trapezoidal (Fig. 2.30)	$A\tau \frac{\sin(\omega\tau/2)}{\omega\tau/2} \frac{\sin(\omega\tau_s/2)}{\omega\tau_s/2}$	$\frac{A}{q} \frac{\sin(k\pi/q)}{k\pi/q} \frac{\sin(k\pi/q_s)}{k\pi/q_s}$
Ramp (Fig. 2.34b)	$\frac{A}{j\omega} \left[ \frac{\sin(\omega\tau/2)}{\omega\tau/2} e^{j\frac{\omega\tau}{2}} - 1 \right]$	$\frac{A}{j2\pi k} \left[ \frac{\sin(k\pi/q)}{k\pi/q} e^{j\frac{k\pi}{q}} - 1 \right]$
Ramp (Fig. 2.34c)	$\frac{A}{j\omega} \left[ 1 - \frac{\sin(\omega\tau/2)}{\omega\tau/2} e^{-j\frac{\omega\tau}{2}} \right]$	$\frac{A}{j2\pi k} \left[ 1 - \frac{\sin(k\pi/q)}{k\pi/q} e^{-j\frac{k\pi}{q}} \right]$
$\frac{\sin \alpha t}{\alpha t}$	$\begin{cases} \frac{\pi}{\alpha}, &  \omega  < \alpha \\ 0, &  \omega  > \alpha \end{cases}$	$\begin{cases} \frac{\pi}{\alpha T}, &  k  < \frac{\alpha T}{2\pi} \\ 0, &  k  > \frac{\alpha T}{2\pi} \end{cases}$
$e^{-\alpha t} u(t),$ $\text{Re } \alpha > 0$	$\frac{1}{\alpha + j\omega}$	$\frac{1}{\alpha T + j2\pi k}$
$te^{-\alpha t} u(t),$ $\text{Re } \alpha > 0$	$\frac{1}{(\alpha + j\omega)^2}$	$\frac{T}{(\alpha T + j2\pi k)^2}$

**Table B.1** The Fourier transform and series of basic signals (*Contd.*)

$\frac{t^{n-1}}{(n-1)!} e^{-\alpha t} u(t)$ , Re $\alpha > 0$	$\frac{1}{(\alpha + j\omega)^n}$	$\frac{T^{n-1}}{(\alpha T + j2\pi k)^n}$
$e^{-\alpha t }$ , $\alpha > 0$	$\frac{2\alpha}{\alpha^2 + \omega^2}$	$\frac{2\alpha T}{\alpha^2 T^2 + 4\pi^2 k^2}$
$e^{-\alpha^2 t^2}$	$\frac{\sqrt{\pi}}{\alpha} e^{-\frac{\omega^2}{4\alpha^2}}$	$\frac{\sqrt{\pi}}{\alpha T} e^{-\frac{\pi^2 k^2}{\alpha^2 T^2}}$

$C_k$  corresponds to  $x(t)$  repeated with period  $T$ ,  $\tau$  and  $\tau_s$  are durations,  $q = \frac{T}{\tau}$ , and  $q_s = \frac{T}{\tau_s}$ .

**Table B.2** The Fourier transform and series of complex signals

Signal $y(t)$	Transform $Y(j\omega)$	Series $C_k$
Burst of $N$ pulses with known $X(j\omega)$	$X(j\omega) \frac{\sin(\omega NT/2)}{\sin(\omega T/2)}$	$\frac{1}{T_1} X\left(j\frac{2k\pi}{T_1}\right) \frac{\sin(k\pi/q_2)}{\sin(k\pi/Nq_2)}$
Rectangular pulse-burst (Fig. 2.47)	$A\tau \frac{\sin(\omega\tau/2)}{\omega\tau/2} \frac{\sin(\omega NT/2)}{\sin(\omega T/2)}$	$\frac{A}{q_1} \frac{\sin(k\pi/q_1)}{k\pi/q_1} \frac{\sin(k\pi/q_2)}{\sin(k\pi/Nq_2)}$
Triangular pulse-burst	$\frac{A\tau}{2} \frac{\sin^2(\omega\tau/4)}{(\omega\tau/4)^2} \frac{\sin(\omega NT/2)}{\sin(\omega T/2)}$	$\frac{A}{2q_1} \frac{\sin^2(k\pi/2q_1)}{(k\pi/2q_1)^2} \frac{\sin(k\pi/q_2)}{\sin(k\pi/Nq_2)}$
Sinc-shaped pulse-burst	$\begin{cases} \frac{A\pi}{\alpha} \frac{\sin(\omega NT/2)}{\sin(\omega T/2)}, &  \omega  < \alpha \\ 0, &  \omega  > \alpha \end{cases}$	$\begin{cases} \frac{A\pi}{\alpha T_1} \frac{\sin(k\pi/q_2)}{\sin(k\pi/Nq_2)}, &  k  < \frac{\alpha T_1}{2\pi} \\ 0, &  k  > \frac{\alpha T_1}{2\pi} \end{cases}$

$C_k$  corresponds to  $y(t)$  repeated with period  $T_1$ ,  $\tau$  is pulse duration,  $T$  is period of pulse in the burst,  $T_1$  is period of pulse-bursts in the train,  $q_1 = \frac{T_1}{\tau}$ , and  $q_2 = \frac{T_1}{NT}$ .

**C**

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**Tables of Hilbert Transform and Properties**

**Table C.1** Properties of the Hilbert transform

$$y(t) = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{\hat{y}(\theta)}{\theta - t} d\theta \qquad \hat{y}(t) = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{y(\theta)}{t - \theta} d\theta$$

Property	Function $y(t)$	Transform $\hat{y}(t)$
Filtering	$y(t)$ is constant	$\hat{y}(t)$ is zero
	is real	is real
	is even (odd)	is odd (even)
Causality	If $y(t)$ is causal with known transform $Y(j\omega) = Y_r(\omega) + jY_i(\omega)$ , then: $Y_r(\omega)$	$Y_i(\omega)$
	$Y_i(\omega)$	$-Y_r(\omega)$
Linearity	$\sum_i \alpha_i y_i(t)$	$\sum_i \alpha_i \hat{y}_i(t)$
Time shifting	$y(t \pm \theta)$	$\hat{y}(t \pm \theta)$
Time reversal	$y(-t)$	$-\hat{y}(-t)$
Scaling	$y(at)$	$\hat{y}(at)$
	$y(-at)$	$-\hat{y}(-at)$
Multiple transform	$\hat{y}(t)$	$-y(t)$
		$\mathcal{H}^3 y(t) = \mathcal{H}^{-1} y(t)$
		$\mathcal{H}^4 y(t) = y(t)$
		$\mathcal{H}^n y(t) \xleftrightarrow{\mathcal{F}} [-j \operatorname{sgn}(\omega)]^n Y(j\omega)$
Differentiation	$\frac{d}{dt} y(t)$	$\frac{d}{dt} \hat{y}(t)$
Integration	$\int_a^b y(t) dt$ , $a$ and $b$ are constants	0
Convolution	$y_1(t) * y_2(t)$	$y_1(t) * \hat{y}_2(t)$
Autocorrelation	$y(t) * y(t)$	$\widehat{y(t) * y(t)}$
Multiplication	$ty(t)$	$t\hat{y}(t) + \frac{1}{\pi} \int_{-\infty}^{\infty} y(t) dt$



**Table C.2** Useful relations between  $y(t)$  and its Hilbert transform  $\hat{y}(t)$ 

Property	Relation
Orthogonality	$\int_{-\infty}^{\infty} y(t)\hat{y}(t)dt = 0$
Integration	$\int_{-\infty}^{\infty} y_1(t)\hat{y}_2(t)dt = \int_{-\infty}^{\infty} \hat{y}_1(t)y_2(t)dt$ $\int_{-\infty}^{\infty} y_1(t)y_2(t)dt = \int_{-\infty}^{\infty} \hat{y}_1(t)\hat{y}_2(t)dt$
Energy	$\int_{-\infty}^{\infty} y^2(t)dt = \int_{-\infty}^{\infty} \hat{y}^2(t)dt$ $\int_{-\infty}^{\infty}  y(t) ^2(t)dt = \int_{-\infty}^{\infty}  \hat{y}(t) ^2dt$
Autocorrelation	$\int_{-\infty}^{\infty} y(t)y(t-\theta)dt = \int_{-\infty}^{\infty} \hat{y}(t)\hat{y}(t-\theta)dt$

**Table C.3** The Hilbert transform of analytic signals

Property	Signal	Transform
Analytic signal	$y_a(t) = y(t) + j\hat{y}(t)$	$-jy_a(t) = \hat{y}(t) - jy(t)$
Multiplication	$y_{a1}(t)y_{a2}(t)$	$-jy_{a1}(t)y_{a2}(t)$ $= \hat{y}_{a1}(t)y_{a2}(t)$ $= y_{a1}(t)\hat{y}_{a2}(t)$
Power $n$	$y_a^n(t)$	$-jy_a^n(t)$
Real product	$\text{Re}[y_{a1}(t)y_{a2}(t)]$	$\text{Im}[y_{a1}(t)y_{a2}(t)]$
Imaginary product	$\text{Im}[y_{a1}(t)y_{a2}(t)]$	$-\text{Re}[y_{a1}(t)y_{a2}(t)]$

**Table C.4** Products of the analytic signals

Product	Relation
$y_{a1}(t)y_{a2}(t)$	$= j\hat{y}_{a1}(t)y_{a2}(t) = jy_{a1}(t)\hat{y}_{a2}(t)$
$\hat{y}_{a1}(t)y_{a2}(t)$	$= -jy_{a1}(t)y_{a2}(t) = y_{a1}(t)\hat{y}_{a2}(t)$

Table C.5 The Hilbert transform of basic signals

Signal	Transform
$\delta(x)$	$\frac{1}{\pi x}$
$\delta'(x)$	$-\frac{1}{\pi x^2}$
$\cos x$	$\sin x$
$\sin x$	$-\cos x$
$\cos(ax)J_n(bx) \quad [0 < b < a]$	$-\sin(ax)J_n(bx)$
$\frac{\sin x}{x}$	$\frac{\sin^2 x/2}{x/2}$
$e^{jx}$	$j e^{jx}$
$\frac{a}{x^2+a^2} \quad [a > 0]$	$\frac{x}{x^2+a^2}$
$\frac{ab-x^2}{(ab-x^2)^2+x^2(a+b)^2}$	$\frac{x(a+b)}{(ab-x^2)^2+x^2(a+b)^2}$
$\delta(x-a) - \delta(x+a)$	$\frac{2}{\pi} \frac{a}{x^2-a^2}$
$\delta(x-a) + \delta(x+a)$	$\frac{2}{\pi} \frac{x}{x^2-a^2}$
$e^{-x} \quad [x > 0]$	$\frac{1}{\pi} e^{-x} \text{Ei}(x)$
$\text{sign } x e^{- x }$	$\frac{1}{\pi} [e^{-x} \text{Ei}(x) + e^x \text{Ei}(-x)]$
$e^{- x }$	$\frac{1}{\pi} [e^{-x} \text{Ei}(x) - e^x \text{Ei}(-x)]$
$ x ^{v-1} \quad [0 < \text{Re } v < 1]$	$\cot\left(\frac{\pi v}{2}\right)  x ^{v-1} \text{sign } x$
$ x ^{v-1} \text{sign } x \quad [0 < \text{Re } v < 1]$	$-\tan\left(\frac{\pi v}{2}\right)  x ^{v-1}$
$\text{sign } x$	$-\infty$
Even rectangular pulse $u(x+\tau) - u(x-\tau)$	$\frac{1}{\pi} \ln \left  \frac{x+\tau}{x-\tau} \right $
Odd rectangular pulse $-u(x+\tau) + 2u(x) - u(x-\tau)$	$-\frac{1}{\pi} [\ln \left  \frac{x+\tau}{x} \right  + \ln \left  \frac{x-\tau}{x} \right ]$
Ramp pulse $\frac{1}{\tau}(\tau-x)[u(x) - u(x-\tau)]$	$\frac{1}{\pi} \left[ \frac{x-\tau}{\tau} \ln \left  \frac{x-\tau}{x} \right  + 1 \right]$
Triangular pulse $\frac{1}{\tau}(\tau- x )[u(x+\tau) - u(x-\tau)]$	$\frac{1}{\pi} \left[ \frac{x+\tau}{\tau} \ln \left  \frac{x+\tau}{x} \right  + \frac{x-\tau}{\tau} \ln \left  \frac{x-\tau}{x} \right  \right]$
Sawtooth pulse $\frac{1}{\tau}(\tau- x )[-u(x+\tau) + 2u(x) - u(x-\tau)]$	$\frac{1}{\pi} \left[ \frac{x-\tau}{\tau} \ln \left  \frac{x-\tau}{x} \right  - \frac{x+\tau}{\tau} \ln \left  \frac{x+\tau}{x} \right  + 2 \right]$

## D

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### Mathematical Formulas

#### Limits:

- ▷  $\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \lim_{x \rightarrow a} \frac{\partial f(x)/\partial x}{\partial g(x)/\partial x} \quad (\text{L'Hospital's rule})$
- ▷  $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$
- ▷  $\lim_{x \rightarrow 0} \frac{\sin Nx}{\sin x} = N$
- ▷  $\int_0^\infty \sin bx \, dx = \lim_{\alpha \rightarrow 1} \int_0^\infty x^{\alpha-1} \sin bx \, dx = \left. \frac{\Gamma(\alpha)}{b^\alpha} \sin \frac{\alpha\pi}{2} \right|_{\alpha=1} = \frac{1}{b}$

#### Trigonometric identities:

- ▷  $e^{jx} = \cos x + j \sin x \quad (\text{Euler's formula})$
- ▷  $e^{(\alpha+jx)} = e^\alpha (\cos x + j \sin x)$
- ▷  $\cos x = \frac{e^{jx} + e^{-jx}}{2}$
- ▷  $\sin x = \frac{e^{jx} - e^{-jx}}{2j}$
- ▷  $\cos^2 x + \sin^2 x = 1$
- ▷  $\cos^2 x - \sin^2 x = \cos 2x$
- ▷  $2 \cos x \sin x = \sin 2x$
- ▷  $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$
- ▷  $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$
- ▷  $\cos x \cos y = \frac{1}{2}[\cos(x+y) + \cos(x-y)]$
- ▷  $\sin x \sin y = \frac{1}{2}[\cos(x-y) - \cos(x+y)]$

- ▷  $\sin x \cos y = \frac{1}{2}[\sin(x+y) + \sin(x-y)]$
- ▷  $\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$
- ▷  $\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$
- ▷  $\cos x + \cos y = 2 \cos \frac{x+y}{2} \cos \frac{x-y}{2}$
- ▷  $\cos x - \cos y = -2 \sin \frac{x+y}{2} \sin \frac{x-y}{2}$
- ▷  $\sin x \pm \sin y = 2 \sin \frac{x \pm y}{2} \cos \frac{x \mp y}{2}$
- ▷  $a \cos x + b \sin x = r \sin(x + \varphi) = r \cos(x - \psi),$   
 $r = \sqrt{a^2 + b^2}, \sin \varphi = \frac{a}{r}, \cos \varphi = \frac{b}{r}, \sin \psi = \frac{b}{r}, \cos \psi = \frac{a}{r}$
- ▷  $\frac{d}{dx} \arcsin x = \frac{1}{\sqrt{1-x^2}}$
- ▷  $\frac{d}{dx} \arccos x = -\frac{1}{\sqrt{1-x^2}}$
- ▷  $\frac{d}{dx} \arctan x = \frac{1}{1+x^2}$
- ▷  $\frac{d}{dx} \operatorname{arccot} x = -\frac{1}{1+x^2}$

**Hyperbolic identities:**

- ▷  $\sinh x = -\sinh(-x) = \pm \sqrt{\cosh^2 x - 1} = \pm \sqrt{\frac{1}{2}(\cosh 2x - 1)} = \frac{e^x - e^{-x}}{2}$
- ▷  $\cosh x = \cosh(-x) = \sqrt{\sinh^2 x + 1} = \sqrt{\frac{1}{2}(\cosh 2x + 1)} = 2 \cosh^2 \frac{x}{2} - 1 = \frac{e^x + e^{-x}}{2}$
- ▷  $\tanh x = \frac{\sinh x}{\cosh x} = \frac{e^x - e^{-x}}{e^x + e^{-x}}$
- ▷  $\coth x = \frac{\cosh x}{\sinh x} = \frac{e^x + e^{-x}}{e^x - e^{-x}}$
- ▷  $\cosh^2 x - \sinh^2 x = 1$
- ▷  $\cosh x + \sinh x = e^x$
- ▷  $\cosh x - \sinh x = e^{-x}$

**Exponents:**

- ▷  $e^{\ln x} = x$
- ▷  $\frac{e^x}{e^y} = e^{x-y}$
- ▷  $e^x e^y = e^{x+y}$
- ▷  $(e^x)^\alpha = e^{\alpha x}$

**Logarithms:**

- ▷  $\ln e^x = x$
- ▷  $\ln \frac{x}{y} = \ln x - \ln y$
- ▷  $\ln \alpha x = \ln \alpha + \ln x$
- ▷  $\ln x^\alpha = \alpha \ln x$

**Extension to series:**

- ▷  $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots + (-1)^n \frac{x^{2n+1}}{(2n+1)!} + \dots$
- ▷  $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots + (-1)^n \frac{x^{2n}}{(2n)!} + \dots$
- ▷  $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots + \frac{x^n}{n!} + \dots$
- ▷  $e^{jz \cos \phi} = \sum_{n=-\infty}^{\infty} j^n J_n(z) e^{jn\phi}$

**Series:**

- ▷  $\sum_{n=0}^{N-1} x^n = \frac{1-x^N}{1-x}, \quad x \neq 1 \quad (\text{by geometric progression})$
- ▷  $\sum_{n=0}^{N-1} e^{\alpha n} = \frac{1-e^{\alpha N}}{1-e^{\alpha}}$
- ▷  $\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}, \quad |x| < 1$
- ▷  $\sum_{k=1}^{\infty} \frac{\sin^2(k\pi/q)}{k^2} = \frac{\pi^2(q-1)}{2q^2}$

**Indefinite integrals:**

- ▷  $\int f'(x)g(x)dx = f(x)g(x) - \int f(x)g'(x)dx \quad (\text{integration by parts})$
- ▷  $\int f(x)dx = \int f[g(y)]g'(y)dy \quad [x = g(y)] \quad (\text{change of variable})$
- ▷  $\int \frac{dx}{x} = \ln |x|$
- ▷  $\int e^x dx = e^x$
- ▷  $\int a^x dx = \frac{a^x}{\ln a}$
- ▷  $\int \sin x dx = -\cos x$
- ▷  $\int \cos x dx = \sin x$

- ▷  $\int x \begin{Bmatrix} \sin x \\ \cos x \end{Bmatrix} dx = \begin{Bmatrix} \sin x \\ \cos x \end{Bmatrix} \mp x \begin{Bmatrix} \cos x \\ \sin x \end{Bmatrix}$
- ▷  $\int \frac{dx}{x(ax^r+b)} = \frac{1}{rb} \ln \left| \frac{x^r}{ax^r+b} \right|$
- ▷  $\int \sin^2 x \, dx = -\frac{1}{4} \sin 2x + \frac{x}{2}$
- ▷  $\int \cos^2 x \, dx = \frac{1}{4} \sin 2x + \frac{x}{2}$
- ▷  $\int x e^{\alpha x} dx = e^{\alpha x} \left( \frac{x}{\alpha} - \frac{1}{\alpha^2} \right)$
- ▷  $\int x^2 e^{\alpha x} dx = e^{\alpha x} \left( \frac{x^2}{\alpha} - \frac{2x}{\alpha^2} + \frac{2}{\alpha^3} \right)$
- ▷  $\int x^\lambda e^{\alpha x} dx = \frac{1}{\alpha} x^\lambda e^{\alpha x} - \frac{\lambda}{\alpha} \int x^{\lambda-1} e^{\alpha x} dx$
- ▷  $\int e^{-(ax^2+bx+c)} dx = \frac{1}{2} \sqrt{\frac{\pi}{a}} e^{\frac{b^2-4ac}{4a}} \operatorname{erf} \left( x\sqrt{a} + \frac{b}{2\sqrt{a}} \right)$
- ▷  $\int \frac{1}{x} e^{\alpha x} dx = \operatorname{Ei}(\alpha x) \quad [\alpha \neq 0]$
- ▷  $\int \frac{1}{\sqrt{x}} e^{-\alpha x} dx = \sqrt{\frac{\pi}{\alpha}} \operatorname{erf}(\sqrt{\alpha x}) \quad [\alpha > 0]$
- ▷  $\int \frac{e^x}{x^2+\alpha^2} dx = \frac{1}{\alpha} \operatorname{Im}[e^{j\alpha} \operatorname{Ei}(x-j\alpha)]$

**Definite integrals:**

- ▷  $\int_{-\infty}^{\infty} \frac{\sin \alpha x}{x} dx = \pi$
- ▷  $\int_{-\infty}^{\infty} e^{-\alpha x^2} dx = \sqrt{\frac{\pi}{\alpha}}$
- ▷  $\int_{-\infty}^{\infty} x^2 e^{-\alpha x^2} dx = \sqrt{\pi} \alpha^{-3/2}$
- ▷  $\int_0^{\infty} \frac{\sin \alpha x}{x} dx = \frac{\pi}{2} \operatorname{sgn} \alpha$
- ▷  $\int_0^{\infty} \frac{\sin^2 \alpha x}{x^2} dx = \frac{\pi \alpha}{2}$
- ▷  $\int_{-\infty}^{\infty} \frac{\sin^4 \alpha x}{x^2} dx = \frac{\pi \alpha}{2}$
- ▷  $\int_0^{\infty} \frac{dx}{\alpha^2+x^2} = \frac{\pi}{2\alpha}$
- ▷  $\int_0^{\infty} x^{\alpha-1} \sin bx \, dx = \frac{\Gamma(\alpha)}{b^\alpha} \sin \frac{\alpha\pi}{2}$

$$\triangleright \int_0^{\infty} \sin bx \, dx = \frac{1}{b}$$

$$\triangleright \int_{-\infty}^{\infty} \frac{\sin x}{x(x-\alpha)} dx = \frac{\pi}{\alpha} (\cos \alpha - 1), \quad \alpha \text{ is real}$$

$$\triangleright \int_{-\infty}^{\infty} \frac{\cos(ax)}{b^2 - x^2} dx = \frac{\pi}{2b} \sin(ab), \quad a, b > 0$$

**Special functions:**

$$\triangleright \operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt \quad (\text{Error function})$$

$$\triangleright \operatorname{Ei}(x) = - \int_{-x}^{\infty} \frac{e^{-t}}{t} dt = \int_{-\infty}^x \frac{e^t}{t} dt \quad [x < 0] \quad (\text{Exponential-integral function})$$

$$\triangleright \operatorname{Ei}(x) = e^x \left[ \frac{1}{x} + \int_0^{\infty} \frac{e^{-t}}{(x-t)^2} dt \right] \quad [x > 0] \quad (\text{Exponential-integral function})$$

$$\triangleright S(x) = \frac{2}{\sqrt{2\pi}} \int_0^x \sin t^2 dt = \int_0^x \sin \frac{\pi t^2}{2} dt \quad (\text{Fresnel integral})$$

$$\triangleright C(x) = \frac{2}{\sqrt{2\pi}} \int_0^x \cos t^2 dt = \int_0^x \cos \frac{\pi t^2}{2} dt \quad (\text{Fresnel integral})$$

$$\triangleright J_n(z) = \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{-j(n\theta - z \sin \theta)} d\theta \quad (\text{Bessel function of the first kind})$$

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