HarvardX: CalcAPL1x Calculus Applied!

Help

Course › Section 10: E = mc²: Taylor Approximation and the Energy Equation (OPTIONAL) ›
1.3 Making Sense of the First Few Terms of the Approximation ›
1.3.2 Quiz: Making Sense of the First Few Terms of the Approximation

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Question 1

1/1 point (graded)

In the first video, Mboyo mentioned that $E = m_0 c^2$ is a Taylor approximation to the complete energy equation, considered as a function of $\frac{v}{c}$:

$$E(\frac{v}{c}) = \frac{m_0 c^2}{\sqrt{1 - v^2/c^2}}.$$

The **nth order Taylor approximation** is the polynomial which includes all terms with degree $\leq n$ from the Taylor series of the function.

Which of the following are true? (Choose all that are correct.)

- ☑ $E = m_0 c^2$ is the zeroth order Taylor approximation for the complete energy equation. ✓
- ☑ $E = m_0 c^2$ is the first order Taylor approximation for the complete energy equation. ✓
- ☐ $E = m_0 c^2$ is the second order Taylor approximation for the complete energy equation.
- ☐ $E = m_0 c^2$ is the third order Taylor approximation for the complete energy equation.

✓

Explanation

The approximation $E = m_0 c^2$ has only a constant term, so it's therefore a zero order approximation.

We could also consider it a first order approximation because the first degree (or linear) term of the expansion is zero.
It is not a second order or higher approximation since the Taylor series expansion has a second degree term.

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ℹ Answers are displayed within the problem

Question 2

1/1 point (graded)

The Taylor series of the energy equation up to degree two term is $m_0c^2 + \frac{1}{2}m_0v^2$. Find the **third** degree term in the Taylor series for the energy equation. Use the strategy we did before:

- Find the third degree term of the expansion for $f(x) = \frac{1}{\sqrt{1-x^2}}$ around the center $x = 0$
- Then multiply by m_0c^2 and substitute in $x = \frac{v}{c}$.

● 0 ✓

○ $\frac{1}{3}m_0c^2(\frac{v}{c})^3$

○ $\frac{1}{6}m_0c^2(\frac{v}{c})^3$

○ $m_0c^2(\frac{v}{c})^3$

○ None of the above.

Explanation

The third derivative of $f(x) = \frac{1}{\sqrt{1-x^2}}$ is $f^{(3)}(x) = 3x(3x^2+2)(1-x^2)^{-7/2}$. This is zero when evaluated at $x = 0$, which makes the third degree term 0. Thus, the third order Taylor approximation of the energy equation is $m_0c^2 + \frac{1}{2}m_0v^2$.

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Question 3

1/1 point (graded)

The Taylor series of the energy equation up to degree two term is $m_0c^2 + \frac{1}{2}m_0v^2$. Find the **fourth** degree term in the Taylor series for the energy equation. Use the strategy we did before:

- Find the fourth degree term of the expansion for $f(x) = \frac{1}{\sqrt{1-x^2}}$ around the center $x = 0$
- Then multiply by m_0c^2 and substitute in $x = \frac{v}{c}$.

○ 0

○ $\frac{1}{24}m_0c^2(\frac{v}{c})^4$

○ $\frac{3}{4}m_0c^2(\frac{v}{c})^4$

● $\frac{3}{8}m_0c^2(\frac{v}{c})^4$

✓

○ $9m_0c^2(\frac{v}{c})^4$

○ None of the above.

Explanation

The fourth derivative of $f(x) = \frac{1}{\sqrt{1-x^2}}$ is $f^{(4)}(x) = (24x^4 + 72x^2 + 9)(1 - x^2)^{-9/2}$. Evaluated at $x = 0$, this is 9. Divided by $4!$, we get that the fourth degree term is $3/8m_0v^4/c^2$. Thus, the fourth order Taylor approximation of the energy equation is $m_0c^2 + \frac{1}{2}m_0v^2 + \frac{3}{8}m_0c^2\frac{v^4}{c^4}$.

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Question 4

1/1 point (graded)

Consider an an object moving at speed v. For which of the following situations will the the fourth order Taylor approximation of the energy-mass equation give a reasonable approximation for the energy of this object? (Remember, the speed of light c is approximately 300,000 km/sec.)

- ☑ An object at rest ($v = 0$) ✓
- ☑ A person running at 6 miles per hour ($v \approx .003$ km/s) ✓
- ☑ The fastest rocket-powered airplane (as of 2017) $(v = 121$ km/sec$)$ ✓
- ☐ Something moving at half the speed of light $(v = \frac{1}{2}c)$ (like an electron)
- ☐ Something moving at 99% of the speed of light (like an accelerated electron)
- ☐ A particle of light ($v = c$)

✓

Explanation

The approximation is reasonable if $\frac{v}{c}$ is very close to zero, that is if v small compared to c. This is true of the first three situations, and in fact, the approximation will be exact in the case of $v = 0$ since the higher order terms will all be zero. What happens if $v = c$? See the next question.

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Question 5

1/1 point (graded)

We've been talking about the energy of objects moving at different speeds.

According to Einstein's energy equation, what speeds are possible for a moving object?

In other words, consider the equation as a function of v, the speed of the object,

$$E(v) = \frac{m_0 c^2}{\sqrt{1 - \frac{v^2}{c^2}}}.$$

For which speeds v does this function make sense?

○ All $v \geq 0$

○ All $v \geq 0$ except $v = c$

○ $0 < v < c$

● $0 \leq v < c$ ✓

○ $0 \leq v \leq c$

Explanation

The domain of this function is $-c < v < c$. This is because $1 - \frac{v^2}{c^2}$ must be nonnegative in order to take a square root of the quantity. This implies $\frac{v^2}{c^2} \leq 1$, or $|v| \leq c$. Furthermore, $v \neq c$, since that would create a denominator of zero.
In physics we don't consider negative speeds, so this function is valid for $0 <= v < c$. This reflects the fact that in physics the maximum speed of anything is the speed of light, c, but only massless particles can have a speed exactly equal to c.

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Question 6

1/1 point (graded)

What happens to $E(v)$ as the speed v approaches c, the speed of light?

○ $E(v) \to 0$

○ $E(v) \to m_0 c^2$

● $E(v) \to \infty$ ✓

○ None of the above.

Explanation

$E(v) \to \infty$. We compute $\lim_{v \to c} E(v)$ and get ∞. Note if you plot the function, you will see that there is a vertical asymptote at $v = c$, and $E(v)$ approaches positive infinity. This reflects the fact that in physics the maximum speed of anything is the speed of light, c, but only massless particles can have a speed exactly equal to c. So the plot shows that v approaches c, but is never equal to it (this equation for energy is only valid for particles with mass.)

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🛈 Answers are displayed within the problem

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