3 5 7 11 13 17 19 23 29 31 37 41 43 47 53 59 61 67 71 73 79 83 89 97 101 103 107 109 113 127 131 137 139 149 151 157 163 167 173 179 181 191 193 197 199

Problem 3

Choose all the integers which is written as the sum of two squares in **two different** ways. (Multiple choices)
45 52 85 116 145

Answer 85 145
$$85 = 2^2 + 9^2 = 6^2 + 7^2$$
 $145 = 1^2 + 12^2 = 8^2 + 9^2$

> How can we find these equalities?

Problem 3

Theorem

If $P \neq Q$ and $P, Q \equiv 1 \pmod{4}$, $N = P \times Q$ is written as the **sum of two squares** in **two different ways**.

Proof

$$P=A^{2}+B^{2}$$
 $Q=C^{2}+D^{2}$
 $(A^{2}+B^{2})(C^{2}+D^{2}) = (AC+BD)^{2}+(AD-BC)^{2}$
 $= (AD+BC)^{2}+(AC-BD)^{2}$

Problem 3

$$45 = 3^{2} \times 5 = 3^{2} \times (1^{2}+2^{2})$$

 $52 = 2^{2} \times 13 = 2^{2} \times (2^{2}+3^{2})$
 $85 = 5 \times 17 = (1^{2}+2^{2})(1^{2}+4^{2})$
 $116 = 2^{2} \times 29 = 2^{2} \times (2^{2}+5^{2})$
 $145 = 5 \times 29 = (1^{2}+2^{2})(2^{2}+5^{2})$