MATHEMATICS

Find the number of ways the commander can be chosen

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1

Here is the question I'm trying to solve:

n soldiers standing in a line are divided into several non-empty units and then a commander is chosen for each unit. Count the number of ways this can be done.

My approach: Let $C(x)$ be the required generating function. Considering the type A structures on the non empty intervals as $a_k = 1$, the generating function is given as $A(x) = \sum_{k\geq 1} 1 \cdot x^k$ which gives, $A(x) = \frac{x}{1-x}$.

Next the type B structures are given by $b_k = \binom{k}{1}$ giving the generating function $B(x) = \sum_{k\geq 1} \binom{k}{1} \cdot x^k$ which gives, $B(x) = \frac{x}{(1-x)^2}$.

Now $C(x) = B(A(x))$

Solving for $C(x)$ I get

$C(x) = \frac{x\cdot(1-x)}{(1-2x)^2}$

Is my approach correct? How do I find the coefficient of x^n in $C(x)$?

Edit: Any hints on where I'm going wrong here? Because the expected answer doesn't match the answer I have calculated. Any help is appreciated.

combinatorics generating-functions

Share Cite Edit Follow Flag edited Sep 6 at 13:07 asked Sep 6 at 10:34

lettuce
77 5

Is the commander one of the soldiers in the unit? – Henry Sep 6 at 10:38

Yes the commander is one of the soldiers in the unit. – lettuce Sep 6 at 10:44

It looks as if the coefficient from your generating function may be $a_n = (n+1)2^{n-2}$ at least for $n \geq 1$. You can derive this from your generating function since its denominator suggests $a_n = 4a_{n-1} - 4a_{n-2}$. Manual calculation from your original question confirms this starts $1, 3, 8, \ldots$ – Henry Sep 6 at 10:49

Thanks Also is my approach correct? Because the expected answer is different. – lettuce Sep 6 at 10:55

1 The expression I calculated doesn't hold for n = 4, for example. The expected value is 21 but my expression gives 20. However, it matches the expected values till n = 3. – lettuce Sep 6 at 13:19

2 Answers

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2

I think your generating function may be wrong. In particular when $n = 4$, I think it gives 20 when I can count 21 cases

I think if $b_{n,k}$ is the number of choices when you have n soldiers and the first group has k individuals then you can consider adding an additional soldier at the beginning so you can say

$$b_{n+1,k+1} = \frac{k+1}{k} b_{n,k}$$

for $k \geq 1$ while

$$b_{n+1,1} = \sum_{k=1}^{n} b_{n,k}$$

which leads to results like

- $b_{n,k} = k\, b_{n+1-k,1}$
- $b_{n+1,1} = b_{n,1} + \sum\limits_{m=1}^{n} b_{m,1}$
- $b_{n+1,1} = 3b_{n,1} - b_{n-1,1}$
- sine the number you want $a_n = b_{n+1,1}$:

$$a_{n,1} = 3a_{n-1,1} - a_{n-2,1}$$

Since the numbers are $1, 3, 8, 21, \ldots$ when $n = 1, 2, 3, 4, \ldots$, this leads to a generating function of the form $\dfrac{x}{1-3x+x^2}$ if you think the answer is 0 when $n = 0$, or of the form $\dfrac{1-2x+x^2}{1-3x+x^2}$ if you think the answer is 1 when $n = 0$.

This is a second order recurrence and can be solved related to $\frac{3 \pm \sqrt{5}}{2}$ but can also be written by saying the coefficient of x^n is $\text{Fib}(2n)$.

Share Cite Edit Follow Flag edited Sep 6 at 15:13

answered Sep 6 at 13:40
Henry
136k 9 108 216

Thanks for your help! I understand it now. – lettuce Sep 6 at 14:38

Could you please explain how you reached the listed results? – lettuce Sep 6 at 15:02

For example $b_{n,3} = \frac{3}{2} b_{n-1,2}$ and $b_{n-1,2} = \frac{2}{1} b_{n-1,1}$ so $b_{n,3} = 3b_{n-2,1}$ – Henry Sep 6 at 15:12

Please correct me if i am wrong , you said that " when i count 21 cases " . So , i decided to count so as to reach same value but i could not . My procedure was if $n=4$,then there are 4 ways to divide it into non-empty sets such that $\{1-1-1-1\}$, $\{1-3\}$, $\{2-2\}$, $\{2-1-1\}$, we must also select a commander in each unit . I thought that we can calculate them such that 1 ways for $\{1-1-1-1\}$, $4 \times C(3,1) = 12$ for $\{1-3\}$, $(6 \times 2 \times 2)/2 = 12$ for $\{2-2\}$, $(6 \times 2 \times 2)/2 = 12$ for $\{2-1-1\}$, so $1+12+12+12 = 37$. Wht i missng ? – Bulbasaur Sep 6 at 15:22

@ Your $\{1-1-1-1\}, \{1-3\}, \{2-2\}, \{2-1-1\}$ generate $1 + 3 \times 2 + 2^2 + 2 \times 3 = 17$ but you are missing $\{4\}$ with 4 more possibilities making 21 – Henry Sep 6 at 15:32

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2

We use

$$\frac{1}{(1-2x)^2} = 1 + 2(2x) + 3(2x)^2 + \dots.$$

Hence

$$C(x) = (x - x^2)(1 + 2(2x) + 3(2x)^2 ..)$$

So, coefficient of x^n is $n.2^{n-1} - (n-1)2^{n-2}$.

Hence the answer is $(n+1)2^{n-2}$

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answered Sep 6 at 10:59
Ilovemath
1,158 12