4.6.1 Homework

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Week 4 due Oct 24, 2023 19:42 IST

4.6.1 Homework

Reading Assignment

0 points possible (ungraded)
Read Unit 4.6.1 of the notes. [LINK]

- [x] Done

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Hello, I am confused about a discrepancy between an explanation of what variables represent in the question itself, and in the explanation for th...

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Homework 4.6.1.1

1/1 point (graded)
Let $A \in \mathbb{R}^{m \times n}$ and $x \in \mathbb{R}^n$. Then $(Ax)^T = x^T A^T$.

Always ✔ Answer: Always

Answer: Always

$$
\begin{aligned}
& (Ax)^T \\
= \quad & \langle \text{ Partition } A \text{ into rows } \rangle \\
& \left(\left(\begin{array}{c} \tilde{a}_0^T \\ \hline \tilde{a})_1^T \\ \hline \vdots \\ \hline \tilde{a})_{m-1}^T \end{array} \right) x \right)^T \\
= \quad & \langle \text{ Matrix-vector multiplication } \rangle \\
& \left(\begin{array}{c} \tilde{a}_0^T x \\ \hline \tilde{a}_1^T x \\ \hline \vdots \end{array} \right.
\end{aligned}
$$

Calculator

$$\left(\tilde{a}_{m-1}^T x \right)$$

$$= \quad < \text{transpose the column vector} >$$

$$\left(\tilde{a}_0^T x \,\middle|\, \tilde{a}_1^T x \,\middle|\, \cdots \,\middle|\, \tilde{a}_{m-1}^T x \right)$$

$$= \quad < \text{dot product commutes} >$$

$$\left(x^T \tilde{a}_0 \,\middle|\, x^T \tilde{a}_1 \,\middle|\, \cdots \,\middle|\, x^T \tilde{a}_{m-1} \right)$$

$$= \quad < \text{special case of matrix-matrix multiplication} >$$

$$x^T \left(\tilde{a}_0 \,\middle|\, \tilde{a}_1 \,\middle|\, \cdots \,\middle|\, \tilde{a}_{m-1} \right)$$

$$= \quad < \text{transpose the matrix} >$$

$$x^T \left(\begin{array}{c} \tilde{a}_0^T \\ \hline \tilde{a})_1^T \\ \hline \vdots \\ \hline \tilde{a})_{m-1}^T \end{array} \right)^T$$

$$= \quad < \text{unpartition the matrix} >$$

$$x^T A^T$$

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Answers are displayed within the problem

Homework 4.6.1.2

1/1 point (graded)

Our laff library has a routine

```
laff_gemv( trans, alpha, A, x, beta, y )
```

that has the following property

- `y = laff_gemv( 'No transpose', alpha, A, x, beta, y)` computes $y := \alpha A x + \beta y$.
- `y = laff_gemv( 'Transpose', alpha, A, x, beta, y)` computes $y := \alpha A^T x + \beta y$.

The routine works regardless of whether x and/or y are column and/or row vectors. Our library does NOT include a routine to compute $y^T = x^T A$. What call could you use to compute $y^T := x^T A$ if y^T is stored in yt and x^T in xt?

- [ ] `laff_gemv( 'No transpose', 1.0 , A, xt, 0.0, yt)`
- [ ] `laff_gemv( 'No transpose', 1.0 , A, xt, 1.0, yt)`
- [ ] `laff_gemv( 'Transpose', 1.0 , A, xt, 1.0, yt)`
- [x] `laff_gemv( 'Transpose', 1.0 , A, xt, 0.0, yt)`

✔

Answer: `laff_gemv( 'Transpose', 1.0, A, xt, 0.0, yt )` computes $y := A^T x$, where y is stored in `yt` and x is stored in `xt`.

To understand this, transpose both sides: $y^T = (A^T x)^T = x^T A^{T^T} = x^T A$.

For this reason, our `laff` library does not include a routine to compute $y^T := \alpha x^T A + \beta y^T$.

You will need this next week!!!

Calculator

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Homework 4.6.1.3

12/12 points (graded)

Let $A = \begin{pmatrix} 1 & -1 \\ 1 & -1 \end{pmatrix}$. Compute

$A^2 =$

0	✔ Answer: 0	0	✔ Answer: 0
0	✔ Answer: 0	0	✔ Answer: 0

$A^3 =$

0	✔ Answer: 0	0	✔ Answer: 0
0	✔ Answer: 0	0	✔ Answer: 0

For $k > 1$, $A^k =$

0	✔ Answer: 0	0	✔ Answer: 0
0	✔ Answer: 0	0	✔ Answer: 0

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Answers are displayed within the problem

Homework 4.6.1.4

16/16 points (graded)

Let $A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$. Compute

$A^2 =$

1	✔ Answer: 1	0	✔ Answer: 0
0	✔ Answer: 0	1	✔ Answer: 1

$A^3 =$

0	✔ Answer: 0	1	✔ Answer: 1
1	✔ Answer: 1	0	✔ Answer: 0

For $k \geq 0$, $A^{2k} =$

1	✔ Answer: 1	0	✔ Answer: 0
0	✔ Answer: 0	1	✔ Answer: 1

For $k \geq 0$, $A^{2k+1} =$

0	✔ Answer: 0	1	✔ Answer: 1
1	✔ Answer: 1	0	✔ Answer: 0

Calculator

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Answers are displayed within the problem

Homework 4.6.1.5

16/16 points (graded)

Let $A = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$. Compute

$A^2 =$

-1 ✔ Answer: -1 | 0 ✔ Answer: 0

0 ✔ Answer: 0 | -1 ✔ Answer: -1

$A^3 =$

0 ✔ Answer: 0 | 1 ✔ Answer: 1

-1 ✔ Answer: -1 | 0 ✔ Answer: 0

For $k \geq 0$, $A^{4k} =$

1 ✔ Answer: 1 | 0 ✔ Answer: 0

0 ✔ Answer: 0 | 1 ✔ Answer: 1

For $k \geq 0$, $A^{4k+1} =$

0 ✔ Answer: 0 | -1 ✔ Answer: -1

1 ✔ Answer: 1 | 0 ✔ Answer: 0

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Answers are displayed within the problem

Homework 4.6.1.6

1/1 point (graded)

Let A be a square matrix. If $AA = 0$ (the zero matrix) then A is a zero matrix. (AA is often written as A^2.)

FALSE ✔ Answer: FALSE

Answer: False!

$$\begin{pmatrix} 1 & 1 \\ -1 & -1 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ -1 & -1 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}.$$

This may be counter intuitive since if α is a scalar, then $\alpha^2 = 0$ only if $\alpha = 0$.
So, one of the points of this exercise is to make you skeptical about "facts" about scalar multiplications that you may try to transfer to matrix-matrix multiplication.

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Calculator

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Homework 4.6.1.7

1/1 point (graded)

There exists a real value matrix A such that $A^2 = -I$. (Recall: I is the identity)

TRUE ✔ Answer: TRUE

Homework 4.6.1.4 There exists a real valued matrix A such that $A^2 = -I$. (Recall: I is the identity)

True/False

Answer: True! Example: $A = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$.

This may be counter intuitive since if α is a real scalar, then $\alpha^2 \neq -1$.

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Answers are displayed within the problem

Homework 4.6.1.8

1/1 point (graded)

There exists a matrix A that is not diagonal such that $A^2 = I$.

TRUE ✔ Answer: TRUE

Answer: True! An examples of a matrices A that is not diagonal yet $A^2 = I$: $A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$.

This may be counter intuitive since if α is a real scalar, then $\alpha^2 = 1$ only if $\alpha = 1$ or $\alpha = -1$. Also, if a matrix is 1×1, then it is automatically diagonal, so you cannot look at 1×1 matrices for inspiration for this problem.

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