MITx: 6.041x Introduction to Probability - The Science of Uncertainty







# Exercise: Estimator properties

(4/4 points)

We estimate the unknown mean $\theta$ of a random variable $X$ (where $X$ has a finite and positive variance) by forming the sample mean $M_n = (X_1 + \cdots + X_n)/n$ of $n$ i.i.d. samples $X_i$ and then forming the estimator

$$\widehat{\Theta} = M_n + \frac{1}{n}.$$

Is this estimator unbiased?

No ✔ **Answer:** No

Is this estimator consistent?

Yes ✔ **Answer:** Yes

Consider now a different estimator, $\widehat{\Theta}_n = X_1$, which ignores all but the first measurement.

Is this estimator unbiased?

Yes ▼ ✔ **Answer:** Yes

Is this estimator consistent?

No ▼ ✔ **Answer:** No

Answer:
We have $\mathbf{E}[\widehat{\Theta}_n] = \theta + (1/n) \neq \theta$, so it is not unbiased. On the other hand, $M_n$ converges (in probability) to $\theta$, and $1/n$ converges to zero. So, their sum, $\widehat{\Theta}_n = M_n + (1/n)$ also converges (in probability) to $\theta$, and the estimator is consistent.

The second estimator is unbiased, because $\mathbf{E}[\widehat{\Theta}_n] = \mathbf{E}[X_1] = \theta$. But it is not consistent. Its value stays the same (equal to $X_1$) for all $n$ and therefore cannot converge to $\theta$, unless $X_1$ is guaranteed to be equal to $\theta$. But this is impossible since $X$ has positive variance.

*You have used 1 of 1 submissions*