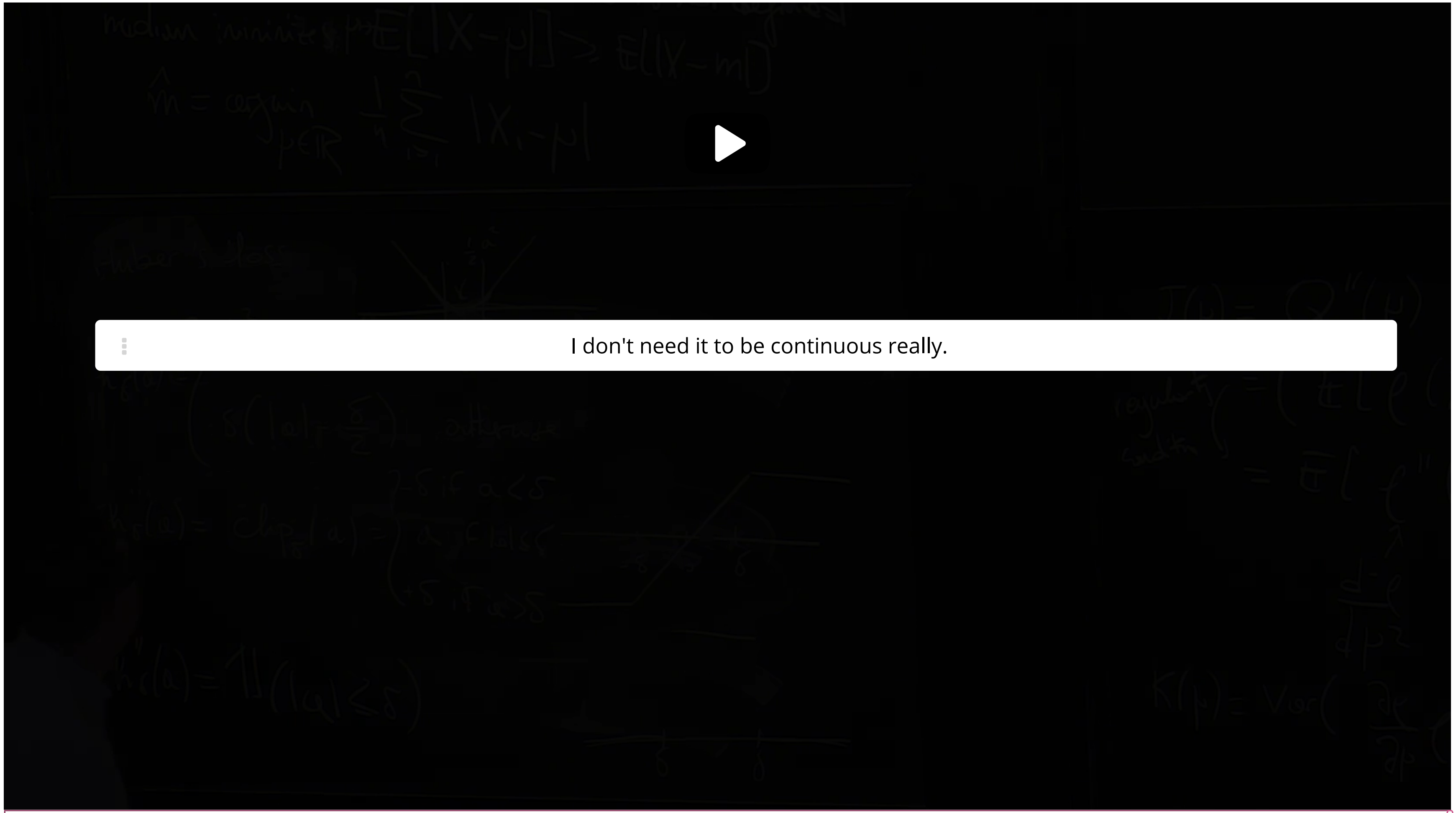
7. Robust Statistics and Huber's Loss

Motivation and Introduction to Huber's Loss



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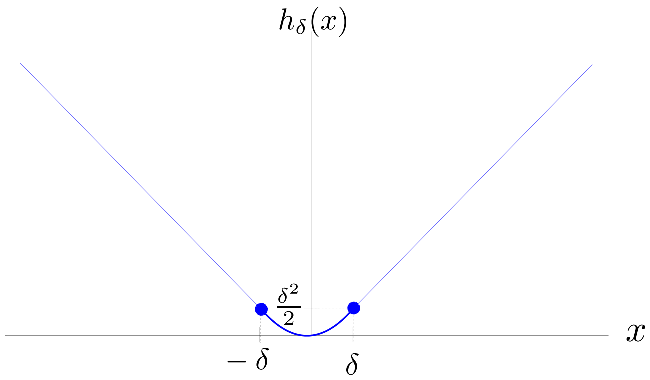
Huber's Loss

3/3 points (graded)

Huber's loss is defined to be

$$h_\delta(x) = \begin{cases} \frac{x^2}{2} & \text{if } |x| \le \delta \\ \delta\left(|x| - \delta/2\right) & \text{if } |x| > \delta \end{cases}$$



Let k denote the smallest integer such that the $\frac{d^k}{dx^k} h_\delta(x)$ is **not** a continuous function.

What is k?

2

Answer: 2

The function $\frac{d^k}{dx^k} h_\delta(x)$ is discontinuous at two points $x_1, x_2 \in \mathbb{R}$ where $x_1 < x_2$.

What are x_1 and x_2 in terms of δ?

$x_1 =$ -delta

Answer: -delta

$-\delta$

$x_2 =$ delta ✔ **Answer:** delta

δ

STANDARD NOTATION

Solution:

Observe that

$$\frac{\partial h_\delta}{\partial x}(x) = \begin{cases} x & \text{if } |x| < \delta \\ \delta & \text{if } x > \delta \\ -\delta & \text{if } x < -\delta, \end{cases}$$

which is a continuous function. However, the next derivative

$$\frac{\partial^2 h_\delta}{\partial^2 x}(x) = \begin{cases} 1 & \text{if } |x| < \delta \\ 0 & \text{if } |x| > \delta \end{cases}$$

has discontinuities at $x = \pm\delta$. In particular, $\frac{\partial^2 h_\delta}{\partial^2 x}(\pm\delta)$ is not defined. Therefore, for the first question, we conclude that $k = 2$. For the second question, $x_1 = -\delta$ and $x_2 = \delta$.

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Answers are displayed within the problem

Comparing Huber's Loss and the absolute value function

1/1 point (graded)

Recall Huber's loss $h_\delta(x)$ as defined in the previous problem. The absolute value function is defined to be $|x|$.

Which of the following statements are true? (Choose all that apply.)

- [ ] Both Huber's loss and the absolute value are differentiable everywhere.
- [x] For $x > 0$ sufficiently large, both Huber's loss and the absolute value are both linear functions.
- [ ] In the intervals where $h_\delta(x)$ is a linear function, both Huber's loss and the absolute value function have the same slope.
- [x] Both Huber's loss and the absolute value function are convex.

Solution:

We examine the choices in order.

- "Both Huber's loss and the absolute value are differentiable everywhere." is incorrect. It is true that Huber's loss is differentiable everywhere. However, $|x|$ is not differentiable at $x = 0$.
- "For $x > 0$ sufficiently large, both Huber's loss and the absolute value are both linear functions." is correct. This is certainly true for the absolute function, as $|x| = x$ if $x > 0$. Moreover, if $x > \delta$, then we have $h_\delta(x) = \delta(x - \delta/2)$ which is also a linear function.
- "In the intervals where $h_\delta(x)$ is a linear function, both Huber's loss and the absolute value function have the same slope." is incorrect. For example, if $x > \delta$, then $|x|$ has slope $+1$. However, $h_\delta(x)$ has slope δ, which is not necessarily equal to 1.
- "Both Huber's loss and the absolute value function are convex." is correct. This is evident from the graphs of $|x|$ and $h_\delta(x)$.

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Answers are displayed within the problem

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