



## MITx: 6.041x Introduction to Probability - The Science of Uncertainty



Bookmarks

- ▶ Unit 0: Overview
- ▶ Entrance Survey
- ▶ Unit 1: Probability models and axioms
- ▶ Unit 2: Conditioning and independence
- ▶ Unit 3: Counting
- ▶ Unit 4: Discrete random variables
- ▶ Exam 1
- ▼ Unit 5: Continuous random variables

Unit 5: Continuous random variables > Problem Set 5 > Problem 5 Vertical: A joint PDF on a triangular region

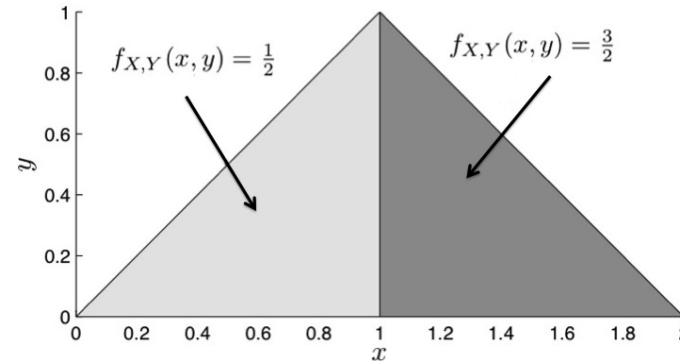


Bookmark

### Problem 5: A joint PDF on a triangular region

(7/7 points)


This figure below describes the joint PDF of the random variables  $\mathbf{X}$  and  $\mathbf{Y}$ . These random variables take values in  $[0, 2]$  and  $[0, 1]$ , respectively. At  $\mathbf{x} = 1$ , the value of the joint PDF is  $\mathbf{1/2}$ .




1. Are  $\mathbf{X}$  and  $\mathbf{Y}$  independent?

## Unit overview


## Lec. 8: Probability density functions

Exercises 8 due Mar 18, 2016 at 23:59 UTC 

## Lec. 9: Conditioning on an event; Multiple r.v.'s

Exercises 9 due Mar 18, 2016 at 23:59 UTC 


## Lec. 10: Conditioning on a random variable; Independence; Bayes' rule

Exercises 10 due Mar 18, 2016 at 23:59 UTC 

## Standard normal table

## Solved problems

## Problem Set 5

Problem Set 5 due Mar 18, 2016 at 23:59 UTC 

## Unit summary

- ▶ Unit 6: Further topics on random variables
- ▶ Unit 7: Bayesian inference

☐ Yes

☒ No 

2. Find  $f_X(x)$ . Express your answers in terms of  $x$  using standard notation .

If  $0 < x < 1$ ,

$$f_X(x) = \boxed{x/2}$$

 Answer:  $x/2$

If  $1 < x < 2$ ,

$$f_X(x) = \boxed{3 - 3x/2}$$

 Answer:  $(-3/2)x+3$

3. Find  $f_{Y|X}(y | 0.5)$ .

If  $0 < y < 1/2$ ,

$$f_{Y|X}(y | 0.5) = \boxed{2}$$

 Answer: 2

4. Find  $f_{X|Y}(x | 0.5)$ .

- ▶ Exam 2
- ▶ Unit 8: Limit theorems and classical statistics
- ▶ Unit 9: Bernoulli and Poisson processes
- ▶ Unit 10: Markov chains
- ▶ Exit Survey
- ▶ Final Exam

If  $1/2 < x < 1$ ,

$$f_{X|Y}(x | 0.5) = \boxed{1/2} \quad \checkmark \text{ Answer: 0.5}$$

If  $1 < x < 3/2$ ,

$$f_{X|Y}(x | 0.5) = \boxed{3/2} \quad \checkmark \text{ Answer: 1.5}$$

5. Let  $R = XY$  and let  $A$  be the event  $\{X < 0.5\}$ . Evaluate  $\mathbf{E}[R | A]$ .

$$\mathbf{E}[R | A] = \boxed{1/16} \quad \checkmark \text{ Answer: 0.0625}$$

Answer:

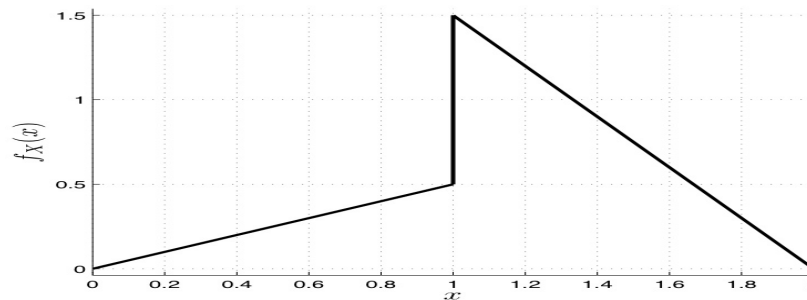
1. In order for  $X$  and  $Y$  to be independent, the value of  $X$  should not give any information about  $Y$ . But if  $X$  is smaller than some  $\epsilon > 0$ , then we can infer that  $Y < \epsilon$ .

In other words,  $f_{Y|X}(y | 0.5) \neq f_Y(y)$ . Therefore,  $X$  and  $Y$  are not independent.

2. Using the formula  $f_X(x) = \int f_{X,Y}(x, y) dy$ , we have

$$\begin{aligned}
 f_X(x) &= \begin{cases} \int_0^x \frac{1}{2} dy, & \text{if } 0 < x \leq 1, \\ \int_0^{2-x} \frac{3}{2} dy, & \text{if } 1 < x < 2, \\ 0, & \text{otherwise,} \end{cases} \\
 &= \begin{cases} x/2, & \text{if } 0 < x \leq 1, \\ -3x/2 + 3, & \text{if } 1 < x < 2, \\ 0, & \text{otherwise.} \end{cases}
 \end{aligned}$$

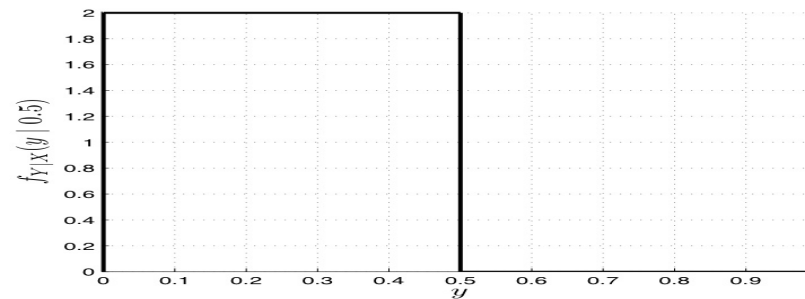
A plot of the PDF is shown below:



3. Given that  $X = 0.5$ ,  $Y$  is uniformly distributed between 0 and  $1/2$ . Thus,

$$f_{Y|X}(y | 0.5) = \begin{cases} 2, & \text{if } 0 \leq y \leq 1/2, \\ 0, & \text{otherwise.} \end{cases}$$

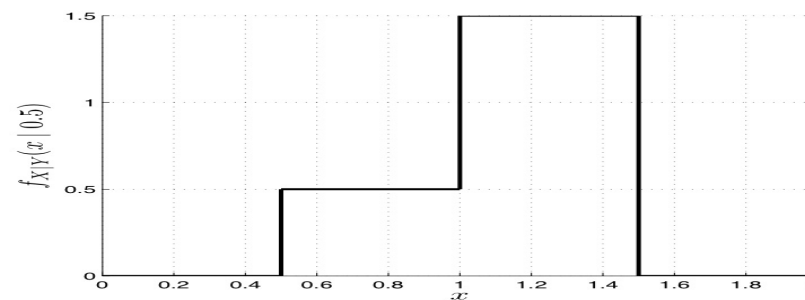
A plot of the conditional PDF is shown below:



4. Given that  $Y = 0.5$ , the conditional distribution of  $X$  is piecewise constant:

$$f_{X|Y}(x | 0.5) = \begin{cases} 1/2, & \text{if } 1/2 \leq x \leq 1, \\ 3/2, & \text{if } 1 < x \leq 3/2, \\ 0, & \text{otherwise.} \end{cases}$$

A plot of the conditional PDF is shown below:



5. Under event  $A$ , the pair  $(X, Y)$  takes values in a triangular region with sides of length  $1/2$ , and area  $1/8$ . The conditional point PDF is uniform, so that  $f_{X,Y|A}(x, y) = 8$  on that set. The conditional expectation is

$$\begin{aligned}\mathbf{E}[R \mid A] &= \mathbf{E}[XY \mid A] \\&= \int \int xy f_{X,Y|A}(x, y) \, dx \, dy \\&= \int_0^{0.5} \int_y^{0.5} 8xy \, dx \, dy \\&= 1/16.\end{aligned}$$

*You have used 3 of 3 submissions*

## DISCUSSION

Click "Show Discussion" below to see discussions on this problem.

© All Rights Reserved



© edX Inc. All rights reserved except where noted. EdX, Open edX and the edX and Open EdX logos are registered trademarks or trademarks of edX Inc.

POWERED BY  
**OPENedX**

