MITx: 15.053x Optimization Methods in Business Analytics

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Lecture 1
Lecture questions due Sep 13, 2016 at 19:30 IST

Recitation

Problem Set 1
Homework due Sep 13, 2016 at 19:30 IST

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PART A

(1/1 point)

This problem asks you to formulate a 3-variable linear program in three different ways (four ways if you also count the algebraic formulation). Both of the first two ways are fairly natural. The third way is a bit obscure. And the algebraic formulation may seem overly complex. In practice, there are advantages to formulating linear programs in different ways. And there are huge advantages in the algebraic formulation. (One can express huge problems efficiently on a computer using a modeling language, such as Julia/JuMP, which is based on the algebraic formulation.) In addition, formulating an LP in multiple ways provides insight into the LP models.

Accessories Co. is producing three kinds of covers for Apple products: one for the iPod, one for the iPad, and one for the iPhone. The company's production facilities are such that if they devote the entire production to iPod covers, they can produce 7000 of them in one day. If they devote the entire production to iPhone covers, they can produce 5000 of them in one day. If they devote the entire production to iPad covers, they can produce 3000 of them in one day. The production schedule is one week (5 working days), and the week's production must be stored before distribution. Storing 1000 iPod covers (packaging included) takes up 40 cubic feet of space. Storing 1000 iPhone covers (packaging included) takes up 55 cubic feet of space. Storing 1000 iPad covers (packaging included) takes up 210 cubic feet of space. The total storage space available is 6000 cubic feet. Due to commercial agreements with Apple, Accessories Co. has to deliver at least 5000 iPod covers and at least 6000 iPad covers per week in order to strengthen the product's diffusion. The marketing department estimates that the weekly demand for iPod covers, iPhone, and iPad covers does not exceed 6000, 15000, and 8000 units.

Therefore, the company does not want to produce more than these amounts for iPod, iPhone, and iPad covers. Finally, the net profit per each iPod cover, iPhone cover, and iPad cover is $4, $6, and $10, respectively.

The aim is to determine a weekly production schedule that maximizes the total net profit.

Write a linear programming formulation for the problem. For this first formulation, the decision variables should represent the proportion of time spent each day on producing each of the two items:

- $x_1 =$ proportion of time devoted each day to iPod cover production
- $x_2 =$ proportion of time devoted each day to iPhone cover production
- $x_3 =$ proportion of time devoted each day to iPad cover production

Choose the correct constraint(s) from below:

☑ $x_1 + x_2 + x_3 \leq 1$

☐ $x_1 + x_2 + x_3 \leq 5$

☐ $40x_1 + 55x_2 + 210x_3 \leq 6000$

☑ $1400x_1 + 1375x_2 + 3150x_3 \leq 6000$

☑ $35000x_1 \geq 5000$

☐ $7000x_1 \geq 5000$

☑ $15000x_3 \geq 6000$

☐ $3000x_3 \geq 6000$

☑ $35000x_1 \leq 6000$

☐ $7000x_1 \leq 6000$

☑ $25000x_2 \leq 15000$

☐ $5000x_2 \leq 15000$

☑ $15000x_3 \leq 8000$

☐ $3000x_3 \leq 8000$

☑ $x_1, x_2, x_3 \geq 0$

✓

You have used 1 of 3 submissions

PART B

(1/1 point)

Write a second linear programming formulation for the problem. For this second formulation, the decision variables should represent the number of items of each type produced over the week:

- $y_1 =$ number of iPod covers produced over the week
- $y_2 =$ number of iPhone covers produced over the week
- $y_3 =$ number of iPad covers produced over the week

The problem data is the same but you must make sure that everything matches the new decision variables.

Choose the correct constraint(s) from below:

☐ $4y_1 + 6y_2 + 10y_3 \leq 6000$

☐ $4y_1 + 6y_2 + 10y_3 \geq 5$

☐ $7000y_1 + 5000y_2 + 3000y_3 \leq 5$

☑ $\frac{1}{7000}y_1 + \frac{1}{5000}y_2 + \frac{1}{3000}y_3 \leq 5$

☐ $40y_1 + 55y_2 + 210y_3 \leq 6000$

☑ $0.040y_1 + 0.055y_2 + 0.210y_3 \leq 6000$

☐ $0.040y_1 + 0.055y_2 + 0.210y_3 \leq 60$

☐ $y_1 \geq 6000$

☑ $y_1 \geq 5000$

☑ $y_1 \leq 6000$

☑ $y_3 \geq 6000$

☐ $y_3 \geq 5000$

☐ $y_2 \geq 15000$

☑ $y_2 \leq 15000$

☑ $y_3 \leq 8000$

☐ $y_3 \geq 8000$

☐ $y_1 + y_2 + y_3 \geq 0$

☑ $y_1, y_2, y_3 \geq 0$

✓

You have used 1 of 3 submissions

PART C

(1/1 point)

Write a third linear programming formulation for the problem. Assume that each working day has 8 working hours. For this third formulation, the decision variables should be:

- $z_1 =$ number of hours devoted to the production of iPod covers in one week
- $z_2 =$ number of hours devoted to the production of iPhone covers in one week
- $z_3 =$ total number of production hours employed during the week

The problem data is the same but you must make sure that everything matches the new decision variables.

Choose the correct constraint(s) from below:

☑ $z_1 \leq 40$

☐ $z_1 \geq 40$

☑ $z_2 \leq 40$

☐ $z_2 \geq 40$

☑ $35z_1 + 34.375z_2 + 78.75(z_3 - z_1 - z_2) \leq 6000$

☐ $35z_1 + 34.375z_2 \leq 6000$

☑ $875z_1 \geq 5000$

☐ $875z_1 \leq 5000$

☑ $375(z_3 - z_1 - z_2) \geq 6000$

☐ $375(z_3 - z_1 - z_2) \leq 6000$

☑ $875z_1 \leq 6000$

☑ $625z_2 \leq 15000$

☐ $625z_2 \geq 15000$

☐ $375(z_3 - z_1 - z_2) \geq 8000$

☑ $375(z_3 - z_1 - z_2) \leq 8000$

☑ $z_1, z_2 \geq 0$

☐ $z_1, z_2, z_3 \geq 0$

✓

You have used 3 of 3 submissions

PART D

(1/1 point)

What is the relationship between the variables z_1, z_2, z_3 of part (c) and the variables x_1, x_2, x_3 of part (a) of this problem? Give a formula to compute z_1, z_2, z_3 from x_1, x_2, x_3.

☑ $z_1 = 40x_1$

☑ $z_2 = 40x_2$

☐ $z_1 + z_2 + z_3 = 40(x_1 + x_2 + x_3)$

☐ $z_3 - z_1 - z_2 = 40(x_1 + x_2 + x_3)$

☐ $z_3 - z_1 - z_2 = 40x_2$

☑ $z_3 = 40(x_1 + x_2 + x_3)$

✓

You have used 1 of 3 submissions

PART E

(1/1 point)

Solve the problem using Excel Solver or using OpenSolver, following the guidelines given in the Excel Spreadsheet ps1_p2.xlsx. *Google sheets version available here.* Pay attention to the formulation in the Excel Workbook: it is similar to the one required for part (b), but it is not exactly the same.

What is the profit under optimal production? Express your answer using one digit to the right of the decimal point.

148570 ✓

148570

You have used 2 of 5 submissions

PART F

(1/1 point)

Write an algebraic formulation of the weekly production schedule problem described above using the following notation:

- T is the number of product types.

- x_j is the number of days devoted to the production of products of type j.
- p_j is the number of items of type j that can be manufactured in one day, assuming that the process is devoted to products of type j.
- P is the number of production days in one week.
- s_j is the storage space required by an item of type j.
- S is the total storage space available for the week's production.
- r_j is the unit profit for each product of type j.
- d_j is the weekly maximum demand for an item of type j.

Note: in this more general version of the problem we do not impose a minimum weekly production for one of the items, as was the case for iPod and iPad smart covers in the initial problem description. The formulation will therefore be slightly different.

Choose the correct objective function

● MAX$\sum_{j=1}^{T} r_j p_j x_j$ ✓

○ MAX$\sum_{j=1}^{T} x_j$

○ MAX$\sum_{j=1}^{T} p_j x_j$

○ MAX$\sum_{j=1}^{T} r_j + p_j + x_j$

○ MAX$\sum_{j=1}^{T} r_j p_j$

You have used 1 of 2 submissions

PART G

(1/1 point)

Choose all the necessary constraint(s).

☐ $\sum_{j=1}^{T} x_j \leq S$

☑ $\sum_{j=1}^{T} x_j \leq P$

☐ $\sum_{j=1}^{T} x_j \geq P$

☑ $\sum_{j=1}^{T} x_j s_j p_j \leq S$

☐ $\sum_{j=1}^{T} r_j + p_j + x_j \leq S$

☐ $p_j x_j \geq d_j, \forall j = 1, \ldots, T$

☑ $p_j x_j \leq d_j, \forall j = 1, \ldots, T$

☐ $r_j p_j \geq 0, \forall j = 1, \ldots, T$

☑ $x_j \geq 0, \forall j = 1, \ldots, T$

☐ $s_j \geq 0, \forall j = 1, \ldots, T$

✓

You have used 1 of 3 submissions