# 1.2.2 Quiz: Equilibria of the Budworm Model with Predation

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Here's the budworm model, modified to include predation:

$$\frac{dP}{dt} = \frac{1}{2}P\left(1 - \frac{P}{q}\right) - \frac{P^2}{1+P^2}.$$

The first term represents the growth rate as given by the logistic model and the term $\frac{P^2}{1+P^2}$ represents the predation rate (in worms/year).

You'll want paper and pencil for this next part.

## Question 1

1/1 point (graded)

Consider the predation rate $R = \frac{P^2}{1+P^2}$ as a function of $P$. What happens to this rate as the population $P$ increases? (What does this mean biologicially?)

- ○ The predation rate increases without bound.
- ◉ The predation rate increases but eventually levels off near 1. ✔
- ○ The predation rate increases then decreases to 1.
- ○ The predation rate increases then decreases to 0.
- ○ None of the above.

**Hint:** Take an appropriate limit.

**Explanation**

The derivative of $R(P)$ is $\frac{2P}{(1+P^2)^2}$ which is positive for $P > 0$. Thus the predation rate $R$ is always increasing. To see what happens over time, we take the limit of $\frac{P^2}{1+P^2}$ as $P \to \infty$. This limit is $1$, thus the predation rate increases but eventually levels off to 1.

What does this mean? As Wes said, the birds can only eat so many worms/year. So even if the budworm population is huge, then the birds still have some saturation point, so the predation rate cannot increase without bound.

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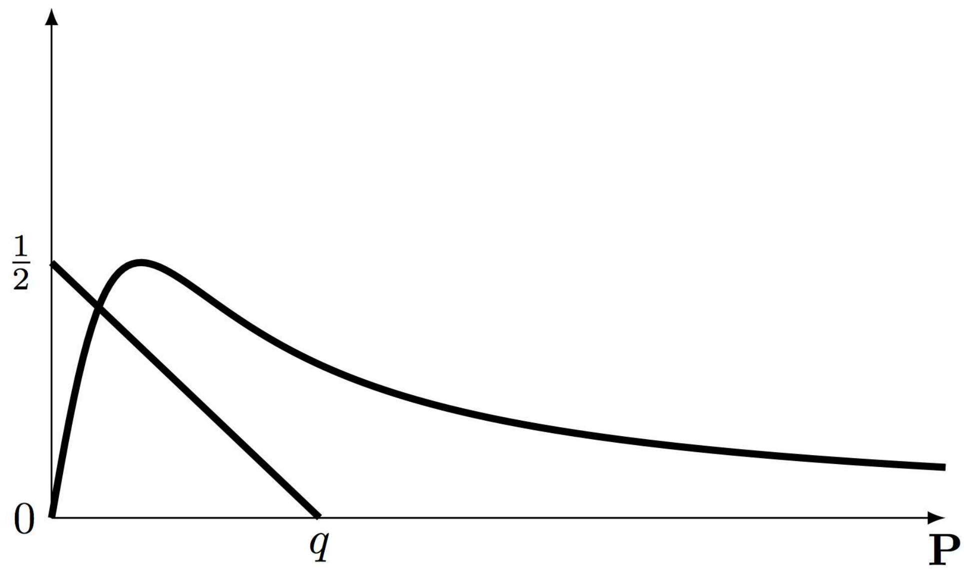
In order to identify equilibrium solutions, Wes factored out a $P$:

$$\frac{dP}{dt} = P\left[\frac{1}{2}\left(1 - \frac{P}{q}\right) - \frac{P}{1+P^2}\right]$$

What can we learn from this?

- $P = 0$ is one equilibrium solution.
- The other equilibrium solutions occur for $P$-values at which $\frac{1}{2}(1 - \frac{P}{q}) - \frac{P}{1+P^2} = 0$.

  This corresponds graphically to the intersection of the graphs of $y = \frac{1}{2}(1 - \frac{P}{q})$ and $y = \frac{P}{1+P^2}$.

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- For populations greater than the carrying capacity ($P > q$), the term $\frac{1}{2}\left(1-\frac{P}{q}\right)$ is negative, thus the whole factor $\left[\frac{1}{2}\left(1-\frac{P}{q}\right)-\frac{P}{1+P^2}\right]$ is negative.

  Since $P$ is always positive, this means

$$\frac{dP}{dt} = P\left[\frac{1}{2}\left(1-\frac{P}{q}\right)-\frac{P}{1+P^2}\right]$$

  is negative. This means the population decreases for $P > q$. (This makes biological sense: not only are the budworms above biological carry capacity but they are also being eaten by predator birds.)

- The horizontal axis intercept for $y = \frac{1}{2}(1-\frac{P}{q})$ is $P = q$ which is the carrying capacity. In what follows, we'll focus on populations less than or at carrying capacity. This means $\frac{1}{2}\left(1-\frac{P}{q}\right)$ is positive so we can think of this as a growth term, while $\frac{P}{1+P^2}$ is a death term (as it comes from the predation term).

## Question 2

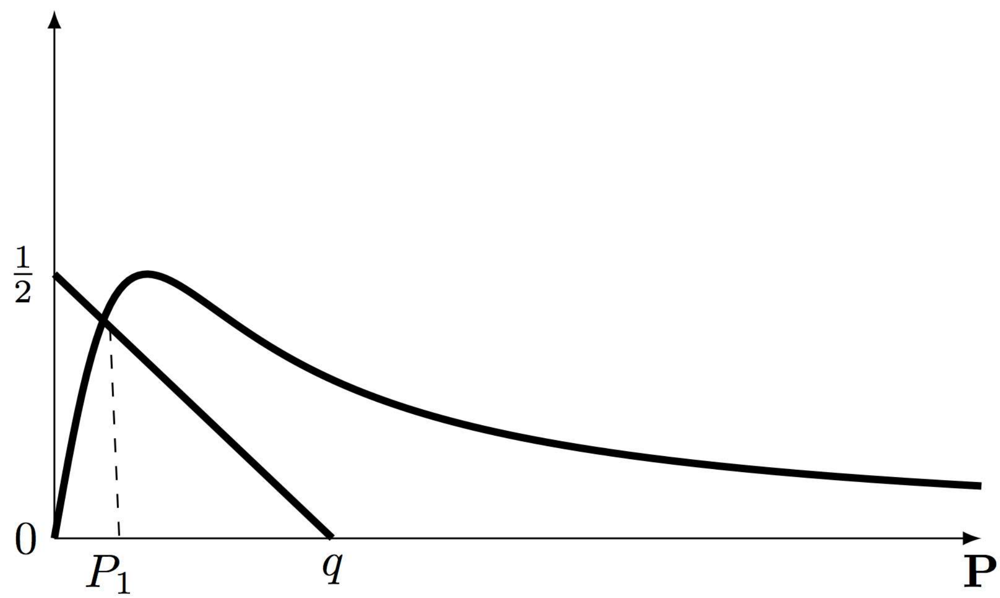
1/1 point (graded)

Let's consider the model when $q = 3$, meaning the carrying capacity is 3 million budworms

$$\frac{dP}{dt} = P\left[\frac{1}{2}\left(1 - \frac{P}{3}\right) - \frac{P}{1+P^2}\right]$$

Since $P$ is always positive, the sign of $\frac{dP}{dt}$ depends on the factor $\left[\frac{1}{2}\left(1 - \frac{P}{3}\right) - \frac{P}{1+P^2}\right]$.

The two terms in brackets are graphed below: $y = \frac{1}{2}(1 - \frac{P}{3})$ and $y = \frac{P}{1+P^2}$.

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The $P$ value at which the graphs intersect is an equilibrium solution, call it $P_1$.

Make a rough sketch of these graphs on a piece of paper. We will use this to analyze the stability of the equilibrium point $P_1$ by considering for which intervals of $P$ the derivative $\frac{dP}{dt}$ is positive.

When the line $y = \frac{1}{2}(1 - \frac{P}{3})$ is above the curve $y = \frac{P}{1+P^2}$, the quantity in brackets $[\frac{1}{2}(1 - \frac{P}{3}) - \frac{P}{1+P^2}]$ will be

- (selected) always positive, so $\frac{dP}{dt} > 0$ ✔
- always negative, so $\frac{dP}{dt} < 0$

- sometimes positive, sometimes negative, so $\frac{dP}{dt}$ changes sign
- None of the above.

**Explanation**

When the line $y = \frac{1}{2}(1 - \frac{P}{3})$ is above the curve $y = \frac{P}{1+P^2}$, the quantity in brackets $[\frac{1}{2}(1 - \frac{P}{3}) - \frac{P}{1+P^2}]$ is positive. (Since $P$ is always positive, this means

$$\frac{dP}{dt} = P\left[\frac{1}{2}\left(1 - \frac{P}{3}\right) - \frac{P}{1+P^2}\right]$$

is positive.)

This means $P$ will increase for those $P$-values. Indicate this by drawing right-handed arrows on the horizontal axis for the regions where the line is above the curve.

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## Question 3

1/1 point (graded)

When the line $y = \frac{1}{2}(1 - \frac{P}{3})$ is below the curve $y = \frac{P}{1+P^2}$, the quantity in brackets $[\frac{1}{2}(1 - \frac{P}{3}) - \frac{P}{1+P^2}]$ will be

- always positive, so $\frac{dP}{dt} > 0$
- always negative, so $\frac{dP}{dt} < 0$ ✔
- sometimes positive, sometimes negative, so $\frac{dP}{dt}$ changes sign
- None of the above.

**Explanation**

Where the line $y = \frac{1}{2}(1 - \frac{P}{3})$ is below the curve $y = \frac{P}{1+P^2}$, the quantity in brackets $[\frac{1}{2}(1 - \frac{P}{3}) - \frac{P}{1+P^2}]$ will be negative. Since $P$ is always positive, this means

$$\frac{dP}{dt} = P\left[\frac{1}{2}\left(1 - \frac{P}{3}\right) - \frac{P}{1+P^2}\right]$$

is negative.
This means $P$ will decrease for those $P$-values. Indicate this by drawing left-handed arrows on the horizontal axis for the regions where the line is below the curve.

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## Question 4

1/1 point (graded)
Use your work above to determine the stability of the non-zero equilibrium point $P_1$ (which occurs at the $P$-value for the point of intersection).

- $P_1$ is a unstable equilibrium solution.
- $P_1$ is a stable equilibrium solution. ✔
- $P_1$ is a semi-stable equilibrium solution.
- There is not enough information.

**Explanation**
Stable. See the next video for the answer.

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Answers are displayed within the problem

## Question 5

1/1 point (graded)
Use your work above to determine whether the equilibrium $P = 0$ is stable or unstable. (You only need to consider $P > 0$.)

- $P = 0$ is an unstable equilibrium solution. ✔

- ○ $P = 0$ is a stable equilibrium solution..
- ○ There is not enough information.

**Explanation**
Unstable. For any positive population values near zero, the solution curve $P$ increases away from 0, hence $P = 0$ is unstable.

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## Question 6

1/1 point (graded)
When the carrying capacity is $q = 3$, there is one equilibrium other than $P = 0$. Assuming we start with some quantity of budworms, which of the following are possible long-term behaviors of the budworm population? If there is more than one, choose all such.

- [ ] Go extinct
- [ ] Increase toward the non-zero equilibrium solution $P_1$ which is equal to the carrying capacity
- [x] Increase toward the non-zero equilibrium solution $P_1$ which is less than the carrying capacity ✔
- [ ] Decrease toward the non-zero equilibrium solution $P_1$ which is equal than the carrying capacity
- [x] Decrease toward the non-zero equilibrium solution $P_1$ which is less than the carrying capacity ✔
- [ ] None of the above.

✔

**Explanation**
The non-zero equilibrium $P_1$ is stable, so for starting $P$ values below it or above, the population will tend toward $P_1$. We also see that $P_1 < q$, since it occurs to the left of $q = 3$. Thus the only possible long term outcomes for populations are to increase or decrease toward $P_1$ which is less than the carrying capacity.
**Note:** $P = 0$ is an unstable equilibrium so the population will never tend toward it; that is, the population will never go extinct. The non-zero equilibrium is stable, so for starting P values below it or above, the population will tend toward it. Thus that is the only possible long term behavior of the system.

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## Question 7: Think About It...

1/1 point (graded)
Let’s consider the effect of increasing $q$, the carrying capacity. This is biologically plausible: the carrying capacity gives a sense of the overall fitness of foliage (the budworms food source) and can vary gradually year to year.

You can use the Desmos graph here. How many equilibrium solution(s) are there? Does it depend on the size of $q$? Record your observations, being as specific as possible.

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When q is high (>7.5) there are 2 equillibriums otherwise
there is one.
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Thank you for your response.

The answers are discussed in the next two sub-sections.

Submit You have used 1 of 2 attempts

Answers are displayed within the problem

## Question 8: Think About It...

1/1 point (graded)
Equilibrium solutions are the $P$-values of the intersection points of the line and the factor from the predation term.

What properties of the graph of $y = \frac{P}{1+P^2}$ allow there to be the number of equilibrium solutions you found in the previous question?

Suppose we replaced the $y = \frac{P}{1+P^2}$ curve with another continuous and differentiable function. What qualitative properties would the new function need to allow for more equilibria than you found in the previous problem? Try to draw such a curve.

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The function should be convex for q > q0, a threshold
capacity.
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Thank you for your response.

For there to be more than one nonzero equilibrium, the line $y = \frac{1}{2}(1 - \frac{P}{q})$ has to intersect the plot of the other function more than once. For large enough $q$, multiple intersections occur because the plot of $y = \frac{P}{1+P^2}$ is concave down for values of $P$ slightly less than $P_1$ (first intersection point). The graph of $y = \frac{P}{1+P^2}$ curves down to intersect the line again.

A third intersection point occurs because the plot of $y = \frac{P}{1+P^2}$ becomes concave up as $P$ keeps increasing. For $P$ values just greater than the second intersection point, the slope of this curve is more negative than for the line, but for even greater $P$, the curve slopes down more gradually than the line, allowing the line to intersect it a third time. If we wanted more than three intersection points, we would need a curve which changes concavity more than twice.

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