4.2.2 Transposing a Partitioned Matrix

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Week 4 due Oct 24, 2023 19:42 IST

4.2.2 Transposing a Partitioned Matrix

Important

Around 1:20 in the below video, the slide that shows the transposed matrix is completely wrong (the indices of the blocks are wrong...) Here is how it should be:

Let $A \in \mathbb{R}^{m \times n}$ be partitioned as follows:

$$A = \left(\begin{array}{c|c|c|c} A_{0,0} & A_{0,1} & \cdots & A_{0,N-1} \\ \hline A_{1,0} & A_{1,1} & \cdots & A_{1,N-1} \\ \hline \vdots & \vdots & & \vdots \\ \hline A_{M-1,0} & A_{M-1,1} & \cdots & A_{M-1,N-1} \end{array}\right),$$

where $A_{i,j} \in \mathbb{R}^{m_i \times n_j}$. Then

$$A^T = \left(\begin{array}{c|c|c|c} A_{0,0}^T & A_{1,0}^T & \cdots & A_{M-1,0}^T \\ \hline A_{0,1}^T & A_{1,1}^T & \cdots & A_{M-1,1}^T \\ \hline \vdots & \vdots & & \vdots \\ \hline A_{0,N-1}^T & A_{1,N-1}^T & \cdots & A_{M-1,N-1}^T \end{array}\right).$$

Similarly, later, in "special cases",

Each submatrix is a scalar. If

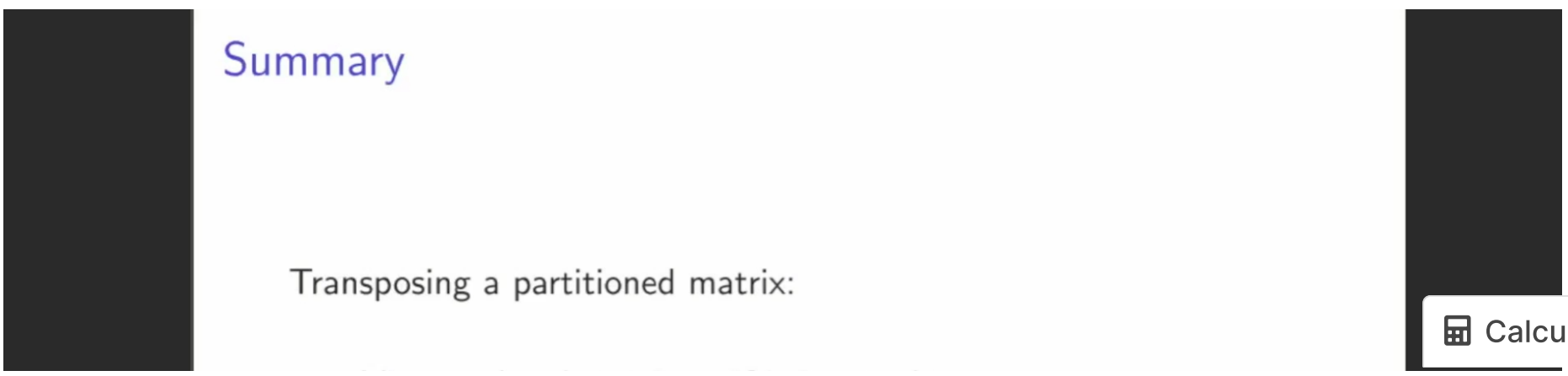
$$A = \left(\begin{array}{c|c|c|c} \alpha_{0,0} & \alpha_{0,1} & \cdots & \alpha_{0,N-1} \\ \hline \alpha_{1,0} & \alpha_{1,1} & \cdots & \alpha_{1,N-1} \\ \hline \vdots & \vdots & & \vdots \\ \hline \alpha_{M-1,0} & \alpha_{M-1,1} & \cdots & \alpha_{M-1,N-1} \end{array}\right)$$

then

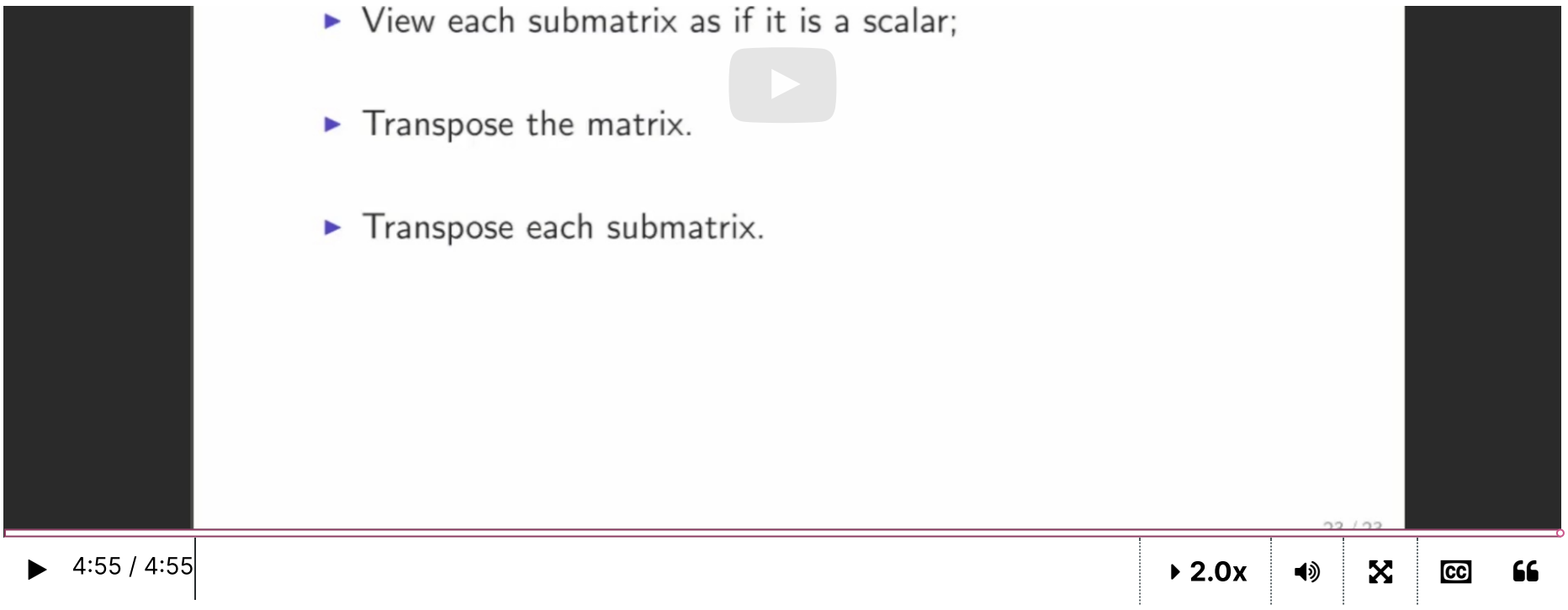
$$A^T = \left(\begin{array}{c|c|c|c} \alpha_{0,0}^T & \alpha_{1,0}^T & \cdots & \alpha_{M-1,0}^T \\ \hline \alpha_{0,1}^T & \alpha_{1,1}^T & \cdots & \alpha_{M-1,1}^T \\ \hline \vdots & \vdots & & \vdots \\ \hline \alpha_{0,N-1}^T & \alpha_{1,N-1}^T & \cdots & \alpha_{M-1N-1}^T \end{array}\right) = \left(\begin{array}{cccc} \alpha_{0,0} & \alpha_{1,0} & \cdots & \alpha_{M-1,0} \\ \alpha_{0,1} & \alpha_{1,1} & \cdots & \alpha_{M-1,1} \\ \vdots & \vdots & & \vdots \\ \alpha_{0,N-1} & \alpha_{1,N-1} & \cdots & \alpha_{M-1,N-1} \end{array}\right).$$

The more I look at it, the more I notice how much I messed up the indices in this particular video... I hate indices...

Best to read the "Related Reading" for this unit...



Calculator



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Reading Assignment

0 points possible (ungraded)

Read Unit 4.2.2 of the notes. [LINK]

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Problems Reduced to Subproblems

I can't help but comment that this "slicing and dicing" method of parsing a linear transformation is similar in spirit to a divide and conquer approa... 2

Homework 4.2.2.1

1/1 point (graded)

Show, step-by-step, how to transpose: $\left(\begin{array}{ccc|c} 1 & -1 & 3 & 2 \\ 2 & -2 & 1 & 0 \\ \hline 0 & -4 & 3 & 2 \end{array}\right)$

Steps:

$$1.\ \left(\begin{array}{cc|c} \begin{pmatrix} 1 & 2 \\ -1 & -2 \\ 3 & 1 \end{pmatrix} & \begin{pmatrix} 0 \\ -4 \\ 3 \end{pmatrix} \\ \hline \begin{pmatrix} 2 & 0 \end{pmatrix} & \begin{pmatrix} 2 \end{pmatrix} \end{array}\right) = \left(\begin{array}{cc|c} 1 & 2 & 0 \\ -1 & -2 & -4 \\ 3 & 1 & 3 \\ \hline 2 & 0 & 2 \end{array}\right)$$

Calculator

2. $$\left(\begin{array}{c|c} \begin{pmatrix} 1 & -1 & 3 \\ 2 & -2 & 1 \end{pmatrix} & \begin{pmatrix} 2 \\ 0 \end{pmatrix} \\ \hline (0 \quad -4 \quad 3) & (2) \end{array}\right)^T$$

3. $$\left(\begin{array}{c|c} \begin{pmatrix} 1 & -1 & 3 \\ 2 & -2 & 1 \end{pmatrix}^T & (0 \quad -4 \quad 3)^T \\ \hline \begin{pmatrix} 2 \\ 0 \end{pmatrix}^T & (2)^T \end{array}\right)$$

List the correct order of the steps taken above. (Answer Format: x,x,x where x is a step number)

2,3,1

✔ Answer: 2,3,1 **or** 2, 3, 1

Explanation

- Step 2: $\left(\begin{array}{c|c} A_{00} & A_{01} \\ \hline A_{10} & A_{11} \end{array}\right)^T$
- Step 3: $\left(\begin{array}{c|c} A_{00}^T & A_{10}^T \\ \hline A_{01}^T & A_{11}^T \end{array}\right)$
- Step 1: Explicitly transpose the individual matrices.

Here

$$\begin{array}{c|c} A_{00} = \begin{pmatrix} 1 & -1 & 3 \\ 2 & -2 & 1 \end{pmatrix} & A_{01} = \begin{pmatrix} 2 \\ 0 \end{pmatrix} \\ \hline A_{10} = (0 \quad -4 \quad 3) & A_{11} = (2) \end{array}$$

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Answers are displayed within the problem

Homework 4.2.2.2

6/6 points (graded)

$(3)^T = (3)$

TRUE ✔ Answer: TRUE

$$\begin{pmatrix} 3 \\ \hline 1 \\ \hline 1 \\ \hline 8 \end{pmatrix}^T = \begin{pmatrix} 8 \\ \hline 1 \\ \hline 1 \\ \hline 3 \end{pmatrix}$$

FALSE ✔ Answer: FALSE

$$\left(\begin{array}{c|c|c|c} 3 & 1 & 1 & 8 \end{array}\right)^T = \begin{pmatrix} 3 \\ \hline 1 \\ \hline 1 \\ \hline 8 \end{pmatrix}$$

TRUE ✔ Answer: TRUE

Calculator

$$\left(\begin{array}{c|c|c|c} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \end{array}\right)^T = \left(\begin{array}{cccc} 1 & 2 & 3 & 4 \\ \hline 5 & 6 & 7 & 8 \\ \hline 9 & 10 & 11 & 12 \end{array}\right)$$

FALSE ✔ Answer: FALSE

$$\left(\begin{array}{c|c|c} 1 & 5 & 9 \\ 2 & 6 & 10 \\ 3 & 7 & 11 \\ 4 & 8 & 12 \end{array}\right)^T = \left(\begin{array}{cccc} 1 & 2 & 3 & 4 \\ \hline 5 & 6 & 7 & 8 \\ \hline 9 & 10 & 11 & 12 \end{array}\right)$$

TRUE ✔ Answer: TRUE

$$\left(\left(\begin{array}{cc|cc} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ \hline 9 & 10 & 11 & 12 \end{array}\right)^T\right)^T = \left(\begin{array}{cc|cc} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ \hline 9 & 10 & 11 & 12 \end{array}\right)$$

TRUE ✔ Answer: TRUE

Explanation

1. $\begin{pmatrix} 3 \end{pmatrix} \quad \begin{pmatrix} 3 \end{pmatrix}^T = \begin{pmatrix} 3^T \end{pmatrix} \begin{pmatrix} 3 \end{pmatrix}$
2. $\begin{pmatrix} 3 \\ \hline 1 \\ \hline 1 \\ \hline 8 \end{pmatrix} \quad \begin{pmatrix} 3 \\ \hline 1 \\ \hline 1 \\ \hline 8 \end{pmatrix}^T = \left(\begin{array}{c|c|c} \begin{pmatrix} 3 \\ 1 \end{pmatrix}^T & \begin{pmatrix} 1 \end{pmatrix}^T & \begin{pmatrix} 8 \end{pmatrix}^T \end{array}\right) = \left(\begin{array}{cc|c|c} 3 & 1 & 1 & 8 \end{array}\right)$
3. $\left(\begin{array}{cc|c|c} 3 & 1 & 1 & 8 \end{array}\right) \quad \begin{pmatrix} \begin{pmatrix} 3 & 1 \end{pmatrix}^T \\ \hline \begin{pmatrix} 1^T \end{pmatrix} \\ \hline \begin{pmatrix} 8^T \end{pmatrix} \end{pmatrix} = \begin{pmatrix} 3 \\ \hline 1 \\ \hline 1 \\ \hline 8 \end{pmatrix}$
4. $\left(\begin{array}{c|c|c|c} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \end{array}\right) \quad \begin{pmatrix} 1 & 5 & 9 \\ \hline 2 & 6 & 10 \\ \hline 3 & 7 & 11 \\ \hline 4 & 8 & 12 \end{pmatrix}$
5. $\left(\begin{array}{c|c|c} 1 & 5 & 9 \\ 2 & 6 & 10 \\ 3 & 7 & 11 \\ 4 & 8 & 12 \end{array}\right) \quad \left(\begin{array}{c|c|c|c} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \end{array}\right)$
6. $\left(\begin{array}{cc|cc} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ \hline 9 & 10 & 11 & 12 \end{array}\right) \quad \begin{pmatrix} \begin{pmatrix} 1 & 2 \\ 5 & 6 \end{pmatrix}^T & \begin{pmatrix} 3 & 4 \\ 7 & 8 \end{pmatrix}^T \\ \hline \begin{pmatrix} 9 & 10 \end{pmatrix}^T & \begin{pmatrix} 11 & 12 \end{pmatrix}^T \end{pmatrix} = \left(\begin{array}{cc|c} 1 & 5 & 9 \\ 2 & 6 & 10 \\ \hline 3 & 7 & 11 \\ 4 & 8 & 12 \end{array}\right)$

$\left(\left(\begin{array}{cc|cc} 1 & 2 & 3 & 4 \end{array}\right)^T\right)^T \quad \left(\begin{array}{cc|cc} 1 & 2 & 3 & 4 \end{array}\right)$

Calculator

7. $\left(\left(\begin{array}{cc|cc} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ \hline 9 & 10 & 11 & 12 \end{array}\right)\right)$ $\left(\begin{array}{cc|cc} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ \hline 9 & 10 & 11 & 12 \end{array}\right)$ (You are transposing twice...)

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Answers are displayed within the problem

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