3. Geometry proofs using coordinate free vectors

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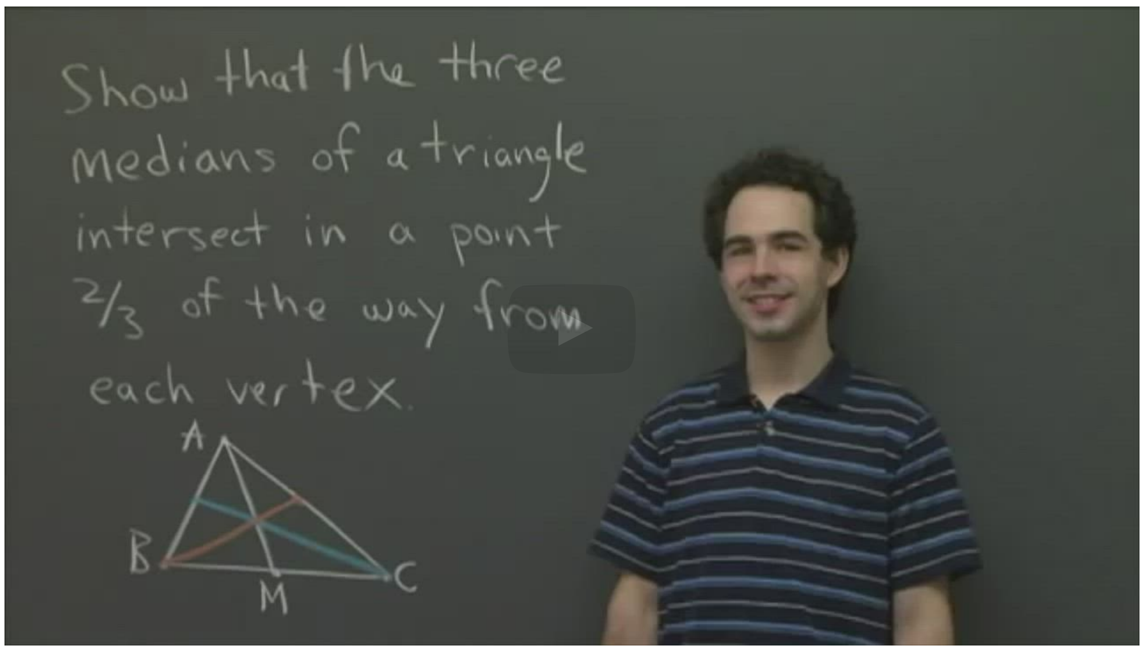
Worked example: Coordinate free proofs



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Start of transcript. Skip to the end.

PROFESSOR: Hi.

Welcome to recitation.

In lecture you've started learning about vectors.

Now vectors are going to be really important

throughout the whole of this course.

And I wanted to give you one problem

just to work with them in a slightly

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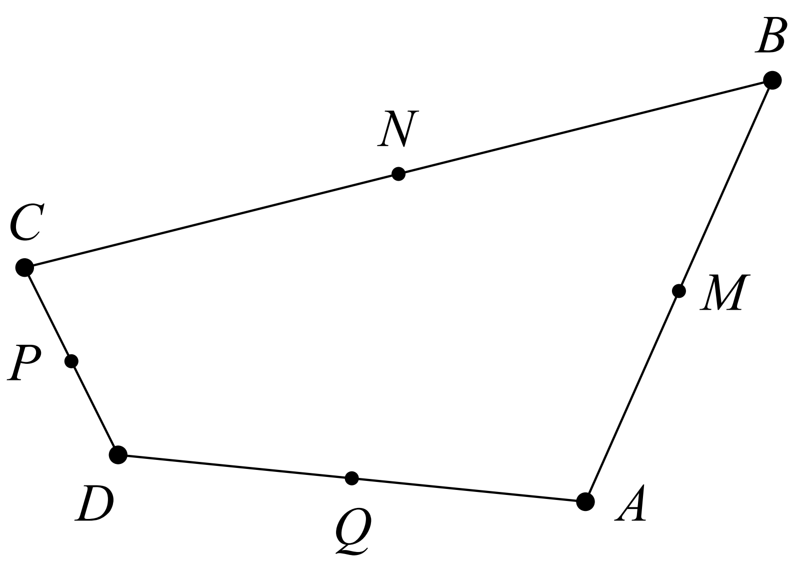
6.

Prove using vector methods (without components) that the midpoints of the sides of a quadrilateral in the plane form a parallelogram.

The first step is to draw a diagram. We have drawn a diagram and labeled the four corners A, B, C and D counterclockwise around the quadrilateral. We have then labeled the midpoints M, N, P, and Q starting with M as the midpoint of the vector $\overrightarrow{AB}$.



The next step it to identify what it is we need to show in order to prove that the shape connecting the midpoints is in fact a parallelogram.

In this problem, the easiest thing for us to show is that the opposite sides of the quad

connecting the midpoints have equal length. Write this out in coordinate free vector n

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connecting the midpoints have equal length. Write this out in coordinate free vector notation, and try to work out the solution. You can read our solution below.

Hint

Use the fact that there are two ways to write the diagonals as vector sums of the original quadrilateral.

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Full worked solution

We know that we can traverse the quadrilateral in either direction to write the diagonal vector $\overrightarrow{DB}$. Let us start by showing that $\overrightarrow{PN} = \overrightarrow{QM}$.

1. $\overrightarrow{DB} = \overrightarrow{DA} + \overrightarrow{AB} = \overrightarrow{DC} + \overrightarrow{CB}$ Definition of vector addition.

2. $\overrightarrow{PN} = \overrightarrow{PC} + \overrightarrow{CN}$ Definition of vector addition.

3. $\overrightarrow{PC} = \frac{1}{2}\overrightarrow{DC}$, $\overrightarrow{CN} = \frac{1}{2}\overrightarrow{CB}$ Definition of midpoint.

4. Plugging 3. into 2 we get that

$$\overrightarrow{PN} = \frac{1}{2}\overrightarrow{DC} + \frac{1}{2}\overrightarrow{CB} = \frac{1}{2}(\overrightarrow{DC} + \overrightarrow{CB}) \quad (3.70)$$

$$= \frac{1}{2}(\overrightarrow{DA} + \overrightarrow{AB}) \qquad \text{Substituting from 1.)} \quad (3.71)$$

$$= \overrightarrow{QA} + \overrightarrow{AM} \quad \text{(Definition of midpoint)} \quad (3.72)$$

$$= \overrightarrow{QM} \quad (3.73)$$

We've shown now that $\overrightarrow{PN} = \overrightarrow{QM}$.

A similar argument using the fact that the other diagonal of the quadrilateral is $\overrightarrow{AC} = \overrightarrow{AB} - \overrightarrow{CB} = \overrightarrow{DC} - \overrightarrow{DA}$ will show that $\overrightarrow{QP} = \overrightarrow{MN}$.

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7.

Prove using vector methods (without components) that the diagonals of a parallelogram bisect each other.

Hint for one approach

Let X and Y be the midpoints of the two diagonals; show $X = Y$.

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Post your solution in the forum, or comment on another learner's solution in the forum!

8.

Label the four vertices of a parallelogram in counterclockwise order as OPQR. Prove that the line segment from O to the midpoint of PQ intersects the diagonal PR in a point X that is $1/3$ of the way fro

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Hint

Let $\vec{A} = \overrightarrow{OP}$, and $\vec{B} = \overrightarrow{OR}$; express everything in terms of $\vec{A}$ and $\vec{B}$.

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Post your solution in the forum, or comment on another learner's solution in the forum!

3. Geometry proofs using coordinate free vectors

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My answers to Q7 and Q8 As I used the hints, my answer for question 8 has $\vec{A}$ and $\vec{B}$. ---------- Q7 -- I labeled the four vertices of my parallelogram in clo...	1
re: proof in the video one thing i did notice was that the solution was by way of three equations (OA, OM, OP) and followed by substitutions. i found m...	2
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[Staff] typo? I believe the statement: *the easiest thing for us to show is that the opposite sides of the quadrilateral formed by connecting the...	2
7.solution For the parallelogram ABCD, X is the midpoint of AC and Y is the midpoint of BD. The origin O is anywhere: OX = 0.5 * (OA + OC) ...	2
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