Stabilizer

Let G be a permutation group on a set Ω and x be an element of Ω. Then

$$G_x = \{g \in G : g(x) = x\} \quad (1)$$

is called the stabilizer of x and consists of all the permutations of G that produce group fixed points in x, i.e., that send x to itself. For example, the stabilizer of 1 and of 2 under the permutation group $\{(1)(2)(3)(4), (12)(3)(4), (1)(2)(34), (12)(34)\}$ is both $\{(1)(2)(3)(4), (1)(2)(34)\}$, and the stabilizer of 3 and of 4 is $\{(1)(2)(3)(4), (12)(3)(4)\}$.

More generally, the subset of all images of $x \in \Omega$ under permutations of the group G

$$G(x) = \{g(x) : g \in G\} \quad (2)$$

is called the group orbit of x in G.

A group's action on an group orbit through x is transitive, and so is related to its isotropy group. In particular, the cosets of the isotropy subgroup correspond to the elements in the orbit,

$$G(x) \sim G/G_x, \quad (3)$$

where $G(x)$ is the orbit of x in G and G_x is the stabilizer of x in G. This immediately gives the identity

$$|G| = |G_x|\,|G(x)|, \quad (4)$$

where $|G|$ denotes the order of group G (Holton and Sheehan 1993, p. 27).

SEE ALSO:
Group Action, Group Fixed Point, Group Orbit, Permutation Group

REFERENCES:
Holton, D. A. and Sheehan, J. Ch. 6 in *The Petersen Graph.* Cambridge, England: Cambridge University Press, p. 26, 1993.

Referenced on Wolfram|Alpha: Stabilizer

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