MITx: 6.041x Introduction to Probability - The Science of Uncertainty

Exercise: Checkout counter

(2/2 points)

Consider our checkout counter example. Assume that there are two types of customers who arrive according to independent Bernoulli processes with rates $p_1 \in (0,1)$ and $p_2 \in (0,1)$, respectively. The overall arrival process of all customers follows a merged Bernoulli process of the two separate Bernoulli processes. All customers who arrive join a single queue, which has a capacity of 10 customers. We are interested in making predictions about the length of the queue at any point in time.

For each of the following parts, choose the correct statement.

1. Assume that service times are not type-dependent and are modelled as independent geometric random variables with parameter $q \in (0,1)$ for all customers in the queue.

- [ ] One can model this queue using the same transition probability graph as in the previous video with $p = (p_1 + p_2)/2$ and q.
- [x] One can model this queue using the same transition probability graph as in the previous video with $p = 1 - (1 - p_1)(1 - p_2)$ and q. ✔

- One can model this queue using the same transition probability graph as in the previous video with some other appropriate choice of p and q.
- There are no values of p and q for which one can model the queue using the same transition probability graph as in the previous video.

2. Assume now that service times are type-dependent and are modelled as independent geometric random variables with parameters $q_1 \in (0,1)$ and $q_2 \in (0,1)$, respectively, for the two types of customers.

- One can model the queue using the same transition probability graph as in the previous video with $p = (p_1 + p_2)/2$ and $q = (q_1 + q_2)/2$.
- One can model the queue using the same transition probability graph as in the previous video with $p = 1 - (1 - p_1)(1 - p_2)$ and $q = (p_1 q_1 + p_2 q_2)/(p_1 + p_2)$.
- One can model this queue using the same transition probability graph as in the previous video with some other appropriate choice of p and q.
- (selected) There are no values of p and q for which one can model the queue using the same transition probability graph as in the previous video. ✔

Answer:

1. Option 2 is correct. The value of p corresponds to the arrival probability of the merged Bernoulli process.

2. Option 4 is correct. The value of p needs to correspond to the arrival probability of the merged Bernoulli process, as in part (1), which rules out Option 1. Given that there is a customer in the queue, the probability that she is of the first type is $p_1/(p_1+p_2)$, and in that case the service time will a geometric random variable with parameter q_1. Similarly, the probability that she is of the second type is $p_2/(p_1+p_2)$, and in that case the service time will a geometric random variable with parameter q_2. But one cannot combine these two cases and argue that the unconditional service time for any customer starting to be served will be a geometric random variable with the prorated parameter q (the weighted sum of two independent geometric r.v.'s is in general not a geometric r.v.) and so Option 2 cannot be correct. Finally, in order for Option 3 to be correct, one would need to find a value of q for which the previous weighted service time would be a geometric random variable, and this can't be the case. Hence, there are no values of p and q for which one can model the queue using the same transition probability graph as in the previous video.

You have used 2 of 2 submissions