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Exercise: Natural estimators

(3/3 points)

The random variables X_i are i.i.d. and satisfy $\mathbf{E}[X_i^2] = \theta$. Use a natural estimator to calculate an estimate of θ based on the values $X_1 = 1, X_2 = 3, X_3 = -1, X_4 = 2, X_5 = 0$.

3 ✔

Answer: 3

In order to calculate confidence intervals around your estimator, you need information on the variance of your estimator. This variance is determined by $\mathbf{E}[X_i^2]$ and $\mathbf{E}[X_i^a]$ for some other power a. What is the value of a?

$a =$ 4 ✔

Answer: 4

If you do not have any prior knowledge about the value of $\mathbf{E}[X_i^a]$, can you estimate it based on the available data?

Yes ▾ ✔ **Answer:** Yes

Answer:
A natural estimator is

$$\frac{1}{5}\sum_{i=1}^{5} X_i^2 = \frac{1}{5}(1+9+1+4+0) = 3.$$

To find the variance of the estimator, you need the variance of X_i^2. Since $\mathbf{var}(X_i^2) = \mathbf{E}[X_i^4] - \left(\mathbf{E}[X_i^2]\right)^2$, you need to know $\mathbf{E}[X_i^4]$. This quantity can be estimated using the natural estimator

$$\frac{1}{n}\sum_{i=1}^{n} X_i^4.$$

You have used 2 of 2 submissions