MITx: 14.310x Data Analysis for Social Scientists

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Questions 1 - 4

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Question 1

1/1 point (graded)

A manufacturer receives a shipment of 100 parts from a vendor. The shipment will be unacceptable if more than five of the parts are defective. The manufacturer is going to randomly select K parts from the shipment for inspection, and the shipment will be accepted if no defective parts are found in the sam of sample of size K.

How large does K have to be to ensure that the probability that the manufacturer accepts an unacceptable shipment is less than 0.1? Hint: We recommend that use R to plug in different values of K.

- a. 12
- b. 22
- c. 32 ✔
- d. 42

Explanation

Let's denote by X the number of defective parts in the sample. Then, we have that $X \sim \textbf{hypergeometric}(N = 100, M, K)$ where M is the number of defectives in the shipment and K equals the sample size chosen by the manufacturer. If there are 6 or more defectives in the shipment, the the probability that the shipment is accepted ($X = 0$) is at most:

$$P(X = 0|M = 100, N = 6, K) = \frac{\binom{6}{0}\binom{94}{K}}{\binom{100}{K}} = \frac{(100-K)\cdots(100-K-5)}{100\cdots 95}$$

You can simulate this in R and vary the number of K. We find that $P(X = 0) = 0.10056$ for $K = 31$ and $P(X = 0) = 0.09182$ for $K = 32$. Then, the sample size must be at least 32.

Submit You have used 1 of 2 attempts

Question 2

1/1 point (graded)

Now suppose that the manufacturer decides to accept the shipment if there is at most one defective part in the sample. How large does K have to be to ensure that the probability that the manufacturer accepts an unacceptable shipment is less than 0.1?

51 ✔ **Answer:** 51

51

Explanation

Now we have that $P(accept\ shipment) = P(X = 0\ or\ 1)$, and, for 6 or more defectives, the probability is at most:

$$P(X = 0\ or\ 1 | M = 100, N = 6, K) = \frac{\binom{6}{0}\binom{94}{K}}{\binom{100}{K}} + \frac{\binom{6}{1}\binom{94}{K-1}}{\binom{100}{K}}$$

We can simulate this in R and vary K and we have that $P(X = 0\ or\ 1) = 0.10220$ for $K = 50$ and $P(X = 0\ or\ 1) = 0.09331$ for $K = 51$. Then, the sample size must be at least 51.

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Question 3

1/1 point (graded)

A man with n keys wants to open his door and tries the keys at random. Exactly one key will open the door.

What is the mean number of trials to open the door if unsuccessful keys are not eliminated for further selections?

- a. The expected value is given by $n - 1$ ✔
- b. The expected value is given by $n - 2$

c. The expected value is given by $\frac{n-1}{n+1}$

d. The expected value is given by n

Explanation

This just corresponds to a geometric distribution with $q = \frac{n-1}{n}$ and $p = \frac{1}{n}$. Then, we have that $\mathbb{E}\left[X\right] = \frac{n-1}{n} / \frac{1}{n} = n - 1$.

Submit You have used 1 of 2 attempts

Correct (1/1 point)

Question 4

1/1 point (graded)

Let the number of chocolate chips in a certain type of cookie have a Poisson distribution. We want the probability that a randomly chosen cookie has at least two chocolate chips to be greater than 0.99. For which of the following values of the mean of the distribution is this condition assured. (Please select all that apply!)

Hint: You may wish to try different values in R when solving this problem if you have having trouble solving the relevant equations otherwise.

- [ ] a. 6
- [x] b. 7 ✔
- [x] c. 8 ✔
- [x] d. 9 ✔

✔

Explanation

We have that $X \sim Poisson(\lambda)$. We want $P(X \geq 2) \geq 0.99$, that is:

$$P(X \leq 1) = e^{-\lambda} + \lambda e^{-\lambda} \leq 0.01.$$

If we simulate this in R we find that this condition is assured for $\lambda \geq 6.64$.

Submit You have used 1 of 2 attempts

✔ Correct (1/1 point)