

MITx: 6.041x Introduction to Probability - The Science of Uncertainty

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■ Bookmark

Problem 1: The PDF of exp(X)

(6/6 points)

Let X be a random variable with PDF f_X . Find the PDF of the random variable $Y=e^X$ for each of the following cases:

1. For general f_X , when y>0, $f_Y(y)=$

- $\circ \qquad f_X\left(rac{e^y}{y}
 ight)$
- $\qquad f_X\left(\frac{\ln y}{y}\right)$
- $\bullet \qquad \frac{f_X(\ln y)}{y} \quad \checkmark$
 - none of the above

Unit 6: Further topics on random variables

Unit overview

Lec. 11: Derived distributions

Exercises 11 due Mar 30, 2016 at 23:59 UTC

Lec. 12: Sums of independent r.v.'s; Covariance and correlation

Exercises 12 due Mar 30, 2016 at 23:59 UTC

Lec. 13: Conditional expectation and variance revisited; Sum of a random number of independent r.v.'s

Exercises 13 due Mar 30, 2016 at 23:59 UTC

Solved problems

Additional theoretical material

Problem Set 6

Problem Set 6 due Mar 30, 2016 at 23:59 UTC

Unit summary

$$^{2.}$$
 When $f_X(x) \ = \ egin{cases} 1/3, & ext{if } -2 < x \leq 1, \ 0, & ext{otherwise}, \end{cases}$

we have
$$f_Y(y) \ = \ egin{cases} g(y), & ext{if } a < y \leq b, \ 0, & ext{otherwise.} \end{cases}$$

Give a formula for g(y) and the values of a and b using standard notation . (In your answers, you may use the symbol 'e' to denote the base of the natural logarithm.)

3. When
$$f_X(x) \ = \ egin{cases} 2e^{-2x}, & ext{if } x>0, \ 0, & ext{otherwise}, \end{cases}$$

we have
$$f_Y(y) = egin{cases} g(y), & ext{if } a < y, \ 0, & ext{otherwise.} \end{cases}$$

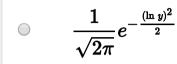
Give a formula for g(y) and the value of a using the standard notation .

Unit 7: Bayesian inference

- ▶ Exam 2
- Unit 8: Limit theorems and classical statistics
- Unit 9: Bernoulli and Poisson processes
- Unit 10: Markov chains
- Exit Survey
- ▶ Final Exam



4. When X is a standard normal random variable, we have, for y>0, $f_Y(y)=$



$$\sim rac{1}{\sqrt{2\pi}}e^{-rac{(\ln y)^2}{2y}}$$

$$\qquad \frac{1}{\sqrt{2\pi}}\frac{e^{-\frac{\ln y}{2}}}{y}$$

onone of the above

Answer:

1.

Since $Y=e^X$ is a strictly monotonic function of the continuous random variable X, we can apply the corresponding formula for derived distributions to obtain $f_Y(y)=rac{f_X(\ln y)}{|y|}$. Note that $Y=e^X>0$ and so the PDF of Y is nonzero only for y>0. Thus, we can remove the absolute value in the denominator to arrive at the simpler expression $f_Y(y)=rac{f_X(\ln y)}{y}$ for y>0.

2. By applying the derived distribution formula, we have

$$f_Y(y) \ = \ egin{cases} 1/(3y), & ext{if } e^{-2} < y \leq e, \ 0, & ext{otherwise}. \end{cases}$$

3. By applying the derived distribution formula, we have

$$f_Y(y) \ = \ egin{cases} 2/(y^3), & ext{if } 1 < y, \ 0, & ext{otherwise.} \end{cases}$$

 $^{4.}$ We apply the result found in part (1), with the specific PDF $f_X(x)=rac{1}{\sqrt{2\pi}}e^{-rac{x^2}{2}}$, to obtain

$$f_Y(y)=rac{1}{\sqrt{2\pi}}rac{e^{-rac{ ext{ln}^2y}{2}}}{y},\, ext{for}\,\,y>0.$$

You have used 2 of 2 submissions

Printable problem set available here.

DISCUSSION

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