EdX and its Members use cookies and other tracking technologies for performance, analytics, and marketing purposes. By using this website, you accept this use. Learn more about these technologies in the <u>Privacy Policy</u>.





Lecture 9: Introduction to Maximum

2. Review and Likelihood of a

Course > Unit 3 Methods of Estimation > Likelihood Estimation

> Gaussian Distribution

Currently enrolled in **Audit Track** (expires December 25, 2019) <u>Upgrade (\$300)</u>

2. Review and Likelihood of a Gaussian Distribution Concept Check: Likelihoods of a Bernoulli, a Poisson, and a Gaussian Distribution



Video

Download video file

Transcripts

<u>Download SubRip (.srt) file</u>

<u>Download Text (.txt) file</u>

Is the Likelihood Discrete or Continuous?

2/2 points (graded)

Setup:

Consider a **discrete** statistical model $M_1=(\mathbb{Z},\{\mathbf{P}_{\theta}\}_{\theta\in\mathbb{R}})$ and a **continuous** statistical model $M_2=(\mathbb{R},\{Q_{\theta}\}_{\theta\in\mathbb{R}})$. Let p_{θ} denote the pmf of \mathbf{P}_{θ} , and let q_{θ} denote the pdf of Q_{θ} . Assume that p_{θ} and q_{θ} both vary continuously with the parameter θ .

Let x_1,\ldots,x_n be fixed natural numbers and y_1,\ldots,y_n be fixed real numbers. Let $(L_1)_n$ denote the likelihood of the discrete model M_1 , and let $(L_2)_n$ denote the likelihood of the continuous model M_2 . Keeping x_1,\ldots,x_n and y_1,\ldots,y_n fixed, let's think of $(L_1)_n$ (x_1,\ldots,x_n,θ) and $(L_2)_n$ (y_1,\ldots,y_n,θ) as functions of θ .

Question

Decide whether the following claims about $(L_1)_n$ and $(L_2)_n$ are true or false.

The map $\theta\mapsto (L_1)_n\,(x_1,\ldots,x_n,\theta)$ is a continuous function of θ .

True





The map $heta \mapsto (L_2)_n \, (y_1, \dots, y_n, heta)$ is a continuous function of heta.







Solution:

Observe that

$$\left(L_{1}
ight)_{n}\left(x_{1},\ldots,x_{n}, heta
ight)=\prod_{i=1}^{n}p_{ heta}\left(x_{i}
ight),\quad\left(L_{2}
ight)_{n}\left(y_{1},\ldots,y_{n}, heta
ight)=\prod_{i=1}^{n}q_{ heta}\left(y_{i}
ight).$$

We are given that p_{θ} and q_{θ} are both continuous function of the parameter $\theta \in \mathbb{R}$. Since products of continuous functions are continuous, this implies that the maps $\theta \mapsto (L_1)_n (x_1, \dots, x_n, \theta)$ and $\theta \mapsto (L_2)_n (y_1, \dots, y_n, \theta)$ are continuous functions of the parameter $\theta \in \mathbb{R}$.

Remark: It may be confusing that even the likelihood of a discrete statistical model can be continuous. However, considering the likelihood of a Bernoulli (derived in a previous question),

$$L\left(x_{1},\ldots,x_{n},p
ight)=\prod_{i=1}^{n}p^{x_{i}}(1-p)^{1-x_{i}}=p^{\sum_{i=1}^{n}x_{i}}(1-p)^{n-\sum_{i=1}^{n}x_{i}}.$$

we can clearly see that the above varies continuously as a function of the *parameter*. This is also true for a host of other discrete models (for example, the Poisson model).

Submit

You have used 1 of 1 attempt

1 Answers are displayed within the problem

Ouiz: Likelihood of a Gaussian Statistical Model

3/3 points (graded)

Let $X_1,\ldots,X_n \overset{iid}{\sim} N\left(\mu^*,(\sigma^*)^2\right)$ for some unknown $\mu^* \in \mathbb{R},(\sigma^*)^2>0$. You construct the associated statistical model $\left(\mathbb{R},\left\{N\left(\mu,\sigma^2\right)\right\}_{(\mu,\sigma^2)\in\mathbb{R}\times(0,\infty)}\right)$.

The likelihood of this model can be written

$$L_n\left(x_1,\ldots,x_n,(\mu,\sigma^2)
ight) = rac{1}{\left(\sigma\sqrt{2\pi}
ight)^C} \mathrm{exp}\left(-rac{1}{A}\sum_{i=1}^C B_i
ight)$$

where A depends on σ , B_i depends on μ and x_i . Find A_iB_i and C.

(Choose a B_i that has coefficient 1 for the highest degree term in x_i .)

(Type **sigma** for σ , **mu** for μ , and **x_i** for x_i .)

$$B_i = oxed{oxed{x_i^2 + mu^2 - 2^*x_i^*mu}}$$

STANDARD NOTATION

Submit

You have used 1 of 3 attempts

✓ Correct (3/3 points)

Discussion

Hide Discussion

Topic: Unit 3 Methods of Estimation:Lecture 9: Introduction to Maximum Likelihood Estimation / 2. Review and Likelihood of a Gaussian Distribution

Add a Post

Show all posts

Should we consider cases where there may be an indicator function based on the parameter?
The prof uses a distribution such as this in one of the next few lectures and it leads me to one answer I think but it doesn't seem to be in the spirit of the question.

STAFF] Possible Error in Solution for Gaussian question
I believe the second term in the solution, after "the," is a possibly confusing typo, **edit**: to clarify in title that this refers to Gaussian question.

Audit Access Expires Dec 24, 2019

You lose all access to this course, including your progress, on Dec 24, 2019.

Upgrade by Nov 4, 2019 to get unlimited access to the course as long as it exists on the site. **Upgrade now**

Learn About Verified Certificates

© All Rights Reserved