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Linear combinations of normal random variables

One property that makes the normal distribution extremely tractable from an analytical viewpoint is its closure under linear combinations: the linear combination of two independent random variables having a normal distribution also has a normal distribution. The following sections present a multivariate generalization of this elementary property and then discuss some special cases.

Linear transformation of a multivariate normal random vector

A linear transformation of a multivariate normal random vector also has a multivariate normal distribution, as illustrated by the following proposition.

> **Proposition** Let be a multivariate normal random vector with mean and covariance matrix . Let be an real vector and an full-rank real matrix. Then the random vector defined by
>
> has a multivariate normal distribution with mean

> and covariance matrix

> Proof

The following examples present some important special cases of the above property.

Example 1 - Sum of two independent normal random variables

The sum of two independent normal random variables has a normal distribution, as stated in the following:

> **Example** Let be a random variable having a normal distribution with mean and variance . Let be a random variable, independent of , having a normal distribution with mean and variance . Then, the random variable defined as:
>
> has a normal distribution with mean
>
> and variance

> Proof

Example 2 - Sum of more than two mutually independent normal random variables

The sum of more than two independent normal random variables also has a normal distribution, as shown in the following example.

Example Let be mutually independent normal random variables, having means and variances . Then, the random variable defined as

has a normal distribution with mean

and variance

Proof

Example 3 - Linear combinations of mutually independent normal random variables

The properties illustrated in the previous two examples can be further generalized to linear combinations of mutually independent normal random variables.

Example Let be mutually independent normal random variables, having means and variances . Let be constants. Then, the random variable defined as

has a normal distribution with mean

and variance

Proof

Example 4 - Linear transformation of a normal random variable

A special case of the above proposition obtains when has dimension (i.e., it is a random variable).

Example Let be a normal random variable with mean and variance . Let and be two constants (with). Then the random variable defined by

has a normal distribution with mean

and variance

Proof

Example 5 - Linear combinations of mutually independent normal random vectors

The property illustrated in Example 3 can be generalized to linear combinations of mutually independent normal random vectors.

Example Let be mutually independent normal random vectors, having means and covariance matrices . Let be real full-rank matrices. Then, the random vector defined as

has a normal distribution with mean

and covariance matrix

Proof

Solved exercises

Below you can find some exercises with explained solutions.

Exercise 1

Let

be a multivariate normal random vector with mean

and covariance matrix

Find the distribution of the random variable defined as

Solution

Exercise 2

Let , ..., be mutually independent standard normal random variables. Let be a constant. Find the distribution of the random variable defined as

Solution

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