



Bookmarks

- ▶ Unit 0: Overview
- ▶ Entrance Survey
- ▶ Unit 1: Probability models and axioms
- ▶ Unit 2: Conditioning and independence
- ▶ Unit 3: Counting
- ▶ Unit 4: Discrete random variables
- ▶ Exam 1
- ▶ Unit 5: Continuous random variables
- ▼ Unit 6: Further topics on random variables

Unit overview

Lec. 11: Derived distributions

Exercises 11 due Mar 30, 2016 at 23:59 UTC

Unit 6: Further topics on random variables > Lec. 12: Sums of independent r.v.'s; Covariance and correlation > Lec 12 Sums of independent r v s Covariance and correlation vertical1



Bookmark

## Exercise: Continuous convolution

(2/2 points)

When calculating the convolution of two PDFs, one must be careful to use the appropriate limits of integration. Suppose that  $\mathbf{X}$  and  $\mathbf{Y}$  are nonnegative random variables. In particular,  $f_{\mathbf{X}}(x)$  is equal to some positive function  $h_{\mathbf{X}}(x)$  for  $x \geq 0$  and is zero for  $x < 0$ . Similarly,  $f_{\mathbf{Y}}(y)$  is equal to some positive function  $h_{\mathbf{Y}}(y)$  for  $y \geq 0$ , and is zero for  $y < 0$ . Then, the convolution integral  $\int_{-\infty}^{\infty} f_{\mathbf{X}}(x)f_{\mathbf{Y}}(z-x)dx$  is of the form

$$\int_a^b h_{\mathbf{X}}(x)h_{\mathbf{Y}}(z-x)dx,$$

for suitable choices of  $a$  and  $b$  determined by  $z$ . Fix some  $z \geq 0$ . Find  $a$  and  $b$ . (Your answer can be an algebraic function of  $z$ .)

 $a =$ 

0



Answer: 0

 $b =$ 

z



Answer: z

Answer:

The integrand is equal to  $h_{\mathbf{X}}(x)h_{\mathbf{Y}}(z-x)$  only for those choices of  $x$  for which the arguments of the functions  $h_{\mathbf{X}}$  and  $h_{\mathbf{Y}}$  are nonnegative; that is, when  $x \geq 0$  and  $z-x \geq 0$ , which yields  $0 \leq x \leq z$ . Thus, we should only integrate from 0 to  $z$ .

Graphically, the PDF of  $\mathbf{X}$  extends from 0 to  $\infty$ . Also, when we flip the PDF of  $\mathbf{Y}$ , the resulting PDF extends from  $-\infty$  to 0, and when we shift to the right it by  $z$ , it will extend from  $-\infty$  to  $z$ . Thus the two PDFs that we need to multiply in the convolution integral overlap only for values from 0 to  $z$ .

*You have used 1 of 2 submissions*

**Lec. 12: Sums of independent r.v.'s; Covariance and correlation**

Exercises 12 due Mar 30, 2016 at 23:59 UTC

**Lec. 13: Conditional expectation and variance revisited; Sum of a random number of independent r.v.'s**

Exercises 13 due Mar 30, 2016 at 23:59 UTC

**Solved problems****Additional theoretical material****Problem Set 6**

Problem Set 6 due Mar 30, 2016 at 23:59 UTC

**Unit summary**

Your answers have been saved but not graded. Click 'Final Check' to grade them.

© All Rights Reserved



© edX Inc. All rights reserved except where noted. EdX, Open edX and the edX and Open EdX logos are registered trademarks or trademarks of edX Inc.

POWERED BY  
**OPENedX**