MITx: 6.041x Introduction to Probability - The Science of Uncertainty

- Unit 0: Overview
- Entrance Survey
- Unit 1: Probability models and axioms
- Unit 2: Conditioning and independence
- Unit 3: Counting
- Unit 4: Discrete random variables
- Exam 1
- Unit 5: Continuous random variables

Problem 3: Shuttles

(5/5 points)

In parts 1, 3, 4, and 5 below, your answers will be algebraic expressions. Enter 'lambda' for λ and 'mu' for μ. Follow standard notation .

1. Shuttles bound for Boston depart from New York every hour on the hour (e.g., at exactly one o'clock, two o'clock, etc.). Passengers arrive at the departure gate in New York according to a Poisson process with rate λ per hour. What is the expected number of passengers on any given shuttle? (Assume that everyone who arrives between two successive shuttle departures boards the shuttle immediately following his/her arrival.)

   lambda ✔ **Answer:** lambda

2. Now, and for the remaining parts of this problem, suppose that the shuttles are not operating on a deterministic schedule. Rather, their interdeparture times are independent and exponentially distributed with common parameter μ per hour. Shuttle departures are independent of the process of passenger arrivals. Is the sequence of shuttle departures a Poisson process?

   Yes, it is a Poisson process. ▾ ✔ **Answer:** Yes, it is a Poisson process.

- Unit 6: Further topics on random variables
- Unit 7: Bayesian inference
- Exam 2
- Unit 8: Limit theorems and classical statistics
- **Unit 9: Bernoulli and Poisson processes**

Unit overview

Lec. 21: The Bernoulli process
Exercises 21 due May 11, 2016 at 23:59 UTC

Lec. 22: The Poisson process
Exercises 22 due May 11, 2016 at 23:59 UTC

Lec. 23: More on the Poisson process
Exercises 23 due May 11, 2016 at 23:59 UTC

3. Let us say that an “event" occurs whenever a passenger arrives or a shuttle departs. What is the expected number of “events" that occur in any one-hour interval?

mu+lambda ✔ **Answer:** lambda+mu

4. If a passenger arrives at the gate and sees 2λ people waiting (assume that 2λ is an integer), what is his/her expected waiting time until the next shuttle departs?

1/mu ✔ **Answer:** 1/mu

5. Find the PMF, $p_N(n)$, of the number, N, of people on any given shuttle. Assume that $\lambda = 20$ and $\mu = 2$.

For $n \geq 0$, $p_N(n) =$ (1/11)*(10/11)^n ✔ **Answer:** 2*20^n/(22^(n+1))

Answer:

1. The number of people who arrive within an hour is Poisson-distributed with parameter λ, and its expected value is λ.

2. If the interdeparture times for the shuttles are independent and exponentially distributed with common parameter μ, then shuttle departures form a Poisson process with rate μ per hour.

3.

Solved problems

Additional theoretical material

Problem Set 9
Problem Set 9 due May 11, 2016 at 23:59 UTC

Unit summary

- Unit 10: Markov chains

- Exit Survey

Here, we are merging two independent Poisson processes, which results in a Poisson process of rate $\mu + \lambda$ per hour. Therefore, the expected number of “events" occurring in one hour will be $\mu + \lambda$.

4. The number of people waiting conveys some information on the time since the last departure. On the other hand, since the 2 processes are independent, the memorylessness property of the exponential distribution tells us that this number is independent of the time until the next departure. Thus, the expected waiting time is just $1/\mu$, irrespective of how many people are waiting.

5. Each “event" has probability $\lambda/(\lambda+\mu)$ of being a passenger arrival (“failure") and probability $\mu/(\lambda+\mu)$ of being a shuttle departure (“success"). Furthermore, different events are independent. The number of passengers on a shuttle is the number of failures until the first success and is distributed as $K-1$, where K is a geometric random variable with parameter $\mu/(\lambda+\mu)$. Thus, the PMF of the number of people on the shuttle is

$$p_N(n) = \left(\frac{\lambda}{\lambda+\mu}\right)^n \left(\frac{\mu}{\lambda+\mu}\right), \qquad n = 0, 1, \ldots.$$

Assuming that $\lambda = 20$ and $\mu = 2$, this simplifies to

$$p_N(n) = \left(\frac{20}{22}\right)^n \left(\frac{2}{22}\right), \qquad n = 0, 1, \ldots.$$

You have used 1 of 3 submissions

DISCUSSION

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