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3. Asymptotics
LTI Consumer (External resource) (1.0 / 1.0 points)

Asymptotics for large time

Consider again three connected medicine storage tanks where tank 1 has a leak. For large time, the volume of medicine in each of the three tanks will decay to zero. This is because when we write our problem in matrix form $\dot{\mathbf{x}} = \mathbf{A}\mathbf{x}$ the matrix $\mathbf{A}$ has purely negative eigenvalues.

In the previous problem, we saw that initially solutions may still grow. This is known as the transient regime. Calculating what happens in the transient regime often requires knowing the exact solution of the problem, and the behavior may depend strongly on the initial conditions.

However, the long time solution of any problem of the form $\dot{\mathbf{x}} = \mathbf{A}\mathbf{x}$ will generically be dominated by the part of the solution corresponding to the eigenvalue with the largest real part. That is, if $\lambda_1 > \lambda_2 > \lambda_3$, with corresponding eigenvectors $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ then almost any long time solution will look like

$$\mathbf{x} \sim c e^{\lambda_1 t} \mathbf{v}_1$$

where c is a number that depends on the initial value and the eigenvector matrix.

Unfortunately, the output of MATLAB's "eig" function does not output the eigenvalues in any particular order. In this problem we will construct an algorithm to find the eigenvalue with largest real part. We will then compare the asymptotic solution with the exact solution to see how well it captures the decay of the solution.

Your Script

🖫 Save   ⟳ Reset   ▣ MATLAB Documentation (https://www.mathworks.com/help/)

```
1  A = [ -2 1 0; 1 -2 1; 0 1 -1];
2  x0 = [0.5;0;0.5];
3  t = linspace(0,4,100);
4  % Store the eigenvectors and eigenvalues in matrices V and D respectively.
5  [V,D] = eig(A);
6
7  %Define I to be the index of the largest eigenvalue of A. That is the largest eige
8  % Hint try using sort() or max() and look into the MATLAB documentation for help.
9
10 [~, ind] = sort(diag(D),'descend');
11
12 I = ind(1);
13
14 % Defined lambda to be the maximum eigenvalue, and v to be the maximum eigenvecto
15
16
17 lambda = D(I,I);
18 v = V(:,I);
19 %
```

```
19 %
20 C = inv(V)*x0;
21 c = C(I,1);
22 asymp = c*exp(lambda*t)*v(1,1);
23 % use the matrix exponential to find the solution for the given initial condition
24 for m=1:length(t)
25     x(:,m) = expm(A*t(m))*x0;
26 end
27 %
28
29 plot(t,x(1,:))
30 hold on
31 plot(t,asymp)
32 legend('x','asymp')
33 set(gca,'fontsize',18)
34 xlabel('Time')
35 ylabel('Volume')
```

▶ Run Script ❓ ()

Previous Assessment: Correct

Submit ❓ ()

✅ **V is correct**

✅ **D is correct**

✅ **I is correct**

✅ **lambda is correct**

✅ **v is correct**

✅ **Exponential matrix used**

✅ **x is correct**

3. Asymptotics

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💬 **matrix exponential question**
I didn't fully understand how we're supposed to perform in the last coding line. Up to now I've used e...

4

☑ **Dimension of Variable I**
If variable I is supposed to be the index of the entry of matrix **D** that contains the largest eigenv...

4

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