Exercise: Memorylessness of the exponential

(3/3 points)

Let X be an exponential random variable with parameter λ.

a) The probability that $X > 5$ is

- $\lambda e^{-5\lambda}$
- $e^{-5\lambda}$ ✔
- none of the above

b) The probabilty that $X > 5$ given that $X > 2$ is

- $\lambda e^{-5\lambda}$
- $e^{-5\lambda}$
- $\lambda e^{-3\lambda}$
- $e^{-3\lambda}$ ✔
- none of the above

c) Given that $X > 2$, and for a small $\delta > 0$, the probability that $4 \leq X \leq 4 + 2\delta$ is approximately

- $\lambda\delta$
- $2\lambda\delta$
- $\delta e^{-4\lambda}$

- $\lambda\delta e^{-4\lambda}$
- $\lambda\delta e^{-2\lambda}$
- $2\lambda\delta e^{-2\lambda}$ ✔
- none of the above

Answer:
a) We have seen in the past that for an exponential random variable with parameter λ, $\mathbf{P}(X > a) = e^{-\lambda a}$, and so $\mathbf{P}(X > 5) = e^{-5\lambda}$.

b) Because of the memorylessness property, given that $X > 2$, the remaining time $X - 2$ is again exponential with the same parameter. Thus,
$\mathbf{P}(X > 5 \,|\, X > 2) = \mathbf{P}(X - 2 > 3 \,|\, X > 2) = \mathbf{P}(X > 3) = e^{-3\lambda}$
.

c) By memorylessness, this is the same as the unconditional probability that an exponential takes values in the interval $[2, 2 + 2\delta]$, which is approximately the length, 2δ, of the small interval times the density evaluated at 2, yielding $2\lambda\delta e^{-2\lambda}$.

You have used 1 of 2 submissions