



Bookmarks

- ▶ Unit 0: Overview
- ▶ Entrance Survey
- ▶ Unit 1: Probability models and axioms
- ▶ Unit 2: Conditioning and independence
- ▶ Unit 3: Counting
- ▶ Unit 4: Discrete random variables
- ▶ Exam 1
- ▶ Unit 5: Continuous random variables
- ▶ Unit 6: Further topics on random variables
- ▼ Unit 7: Bayesian inference

Unit 7: Bayesian inference > Lec. 15: Linear models with normal noise > Lec 15 Linear models with normal noise vertical5



Bookmark

Exercise: The effect of a stronger signal

(1/1 point)

For the model $\mathbf{X} = \boldsymbol{\Theta} + \mathbf{W}$, and under the usual independence and normality assumptions for $\boldsymbol{\Theta}$ and \mathbf{W} , the mean squared error of the LMS estimator is

$$\frac{1}{(1/\sigma_0^2) + (1/\sigma_1^2)},$$

where σ_0^2 and σ_1^2 are the variances of $\boldsymbol{\Theta}$ and \mathbf{W} , respectively.

Suppose now that we change the observation model to $\mathbf{Y} = 3\boldsymbol{\Theta} + \mathbf{W}$. In some sense the "signal" $\boldsymbol{\Theta}$ has a stronger presence, relative to the noise term \mathbf{W} , and we should expect to obtain a smaller mean squared error. Suppose $\sigma_0^2 = \sigma_1^2 = 1$. The mean squared error of the original model $\mathbf{X} = \boldsymbol{\Theta} + \mathbf{W}$ is then $1/2$. In contrast, the mean squared error of the new model $\mathbf{Y} = 3\boldsymbol{\Theta} + \mathbf{W}$ is



Answer: 0.1


Hint: Do not solve the problem from scratch. Think of an alternative observation model in which you observe $\mathbf{Y}' = \boldsymbol{\Theta} + (\mathbf{W}/3)$.

Answer:


Since \mathbf{Y}' is just \mathbf{Y} scaled by a factor of $1/3$, \mathbf{Y}' carries the same information as \mathbf{Y} , so that $\mathbf{E}[\boldsymbol{\Theta} | \mathbf{Y}] = \mathbf{E}[\boldsymbol{\Theta} | \mathbf{Y}']$. Thus, the alternative observation model $\mathbf{Y}' = \boldsymbol{\Theta} + (\mathbf{W}/3)$ will lead to the same estimates and will have the same mean squared error as the unscaled model $\mathbf{Y} = 3\boldsymbol{\Theta} + \mathbf{W}$. In the equivalent \mathbf{Y}' model, we have a noise variance of $1/9$ and therefore the mean squared error is

$$\frac{1}{\frac{1}{1} + \frac{1}{1/9}} = \frac{1}{10}.$$


Unit overview**Lec. 14:
Introduction to
Bayesian inference**

Exercises 14 due Apr
06, 2016 at 23:59 UTC 


**Lec. 15: Linear
models with
normal noise**

Exercises 15 due Apr
06, 2016 at 23:59 UTC 


Problem Set 7a

Problem Set 7a due
Apr 06, 2016 at 23:59
UTC 


**Lec. 16: Least
mean squares
(LMS) estimation**

Exercises 16 due Apr
13, 2016 at 23:59 UTC 

**Lec. 17: Linear
least mean
squares (LLMS)
estimation**

Exercises 17 due Apr
13, 2016 at 23:59 UTC 

Problem Set 7b

Problem Set 7b due
Apr 13, 2016 at 23:59
UTC 

Solved problems**Additional
theoretical
material****Unit summary**

You have used 1 of 3 submissions

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