1.2.2 Quiz: Computing the Taylor Approximation

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In this last section, Mboyo introduced the new variable $x = \frac{v}{c}$. (He called this a "parameter", but it is a variable for our purposes). He suggested finding the Taylor approximation for the energy-mass equation by

- Finding the Taylor approximation for:

  $$f(x) = \frac{1}{\sqrt{1-x^2}}$$

  around the center zero $x = 0$

- Substituting in $x = \frac{v}{c}$ to

  $$E(x) = \frac{m_0 c^2}{\sqrt{1-x^2}}$$

  to get the Taylor approximation for the original form of the energy equation:

  $$E = \frac{m_0 c^2}{\sqrt{1-\frac{v^2}{c^2}}}$$

  .

We'll do this step by step here.

Question 1

1/1 point (graded)

Why does Mboyo claim that $\frac{v}{c}$ is a "very small number?" Choose the most complete answer.

- ○ For most moving objects, the speed v is close to 0, so $\frac{v}{c}$ is very close to zero (very small).
- ● For most moving objects, the speed v is small compared to the speed of light, so $\frac{v}{c}$ is very close to zero (very small). ✔

- For most moving objects, the speed v is close to the speed of light, so $\frac{v}{c} \approx 1$ which is a very small number.
- None of the above.

Explanation

$\frac{v}{c}$ can be considered to be very small, as long as v is small compared to c, the speed of light. The speed of light, is very large ($c \approx 300,000$ km/sec), and most moving objects have speed much less than this.

This means the approximation with $\frac{v}{c}$ as the variable will be better than if we use v as the center because the value of our variable will be closer to the center value of 0.

Submit You have used 1 of 2 attempts

Answers are displayed within the problem

Question 2

1/1 point (graded)

Compute the Taylor approximation of $f(x) = \frac{1}{\sqrt{1-x^2}}$ around the center $x = 0$, up to the degree two term.

- x
- $1 + x$
- $1 + \frac{1}{2}x^2$ ✔
- $1 + x + \frac{1}{2}x^2$
- None of the above.

Explanation

The Taylor series of $f(x)$ up to the degree two term is $1 + \frac{x^2}{2}$. In other words, the degree 2 Taylor polynomial is $1 + \frac{x^2}{2}$.

Here are the computations:

The Taylor series of $f(x)$ around 0 is the 'infinite polynomial'

$$T(x) = a_0 + a_1x^1 + a_2x^2 + a_3x^3 \dots$$

where

$$a_n = \frac{1}{n!} f^{(n)}(0).$$

Here $f^{(n)}$ is the nth derivative of $f(x)$. Let's start by finding the first and second derivatives of $f(x)$. Rewriting $f(x) = (1-x^2)^{-1/2}$ means we can just use power and chain rule, so we get

$$f'(x) = -\frac{1}{2}(1-x^2)^{-3/2} \cdot (-2x) = (1-x^2)^{-3/2} x.$$

To find the second derivative, we need product rule as well as the power and chain rule:

$$f''(x) = -\frac{3}{2}(1-x^2)^{-5/2} \cdot (-2x) \cdot x + (1-x^2)^{-3/2}.$$

So we have

n	nth derivative, $f^{(n)}(x)$	$f^{(n)}(0)$	nth coefficient of Taylor series, $\frac{f^{(n)}(0)}{n!}$
0	$(1-x^2)^{-1/2}$	1	1
1	$(1-x^2)^{-3/2} x$	0	0
2	$3(1-x^2)^{-5/2} x^2 + (1-x^2)^{-3/2}$	1	$\frac{1}{2}$

Thus the Taylor series up to the second degree term of $f(x) = \frac{1}{\sqrt{1-x^2}}$ around $x = 0$ is

$$1 + 0 \cdot x + \frac{1}{2} \cdot x^2 + \ldots$$

which simplifies to

$$1 + \frac{1}{2} x^2 + \ldots$$

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Answers are displayed within the problem

Question 3: Think About It...

1/1 point (graded)

Using your work for $f(x) = \frac{1}{\sqrt{1-x^2}}$, write the Taylor series of $E(x) = \frac{m_0 c^2}{\sqrt{1-x^2}}$ around $x = 0$, up to the degree two term. Then substitute in $x = \frac{v}{c}$ and simplify. Keep this expression handy as you watch the next video. Do you recognize anything familiar in any of the terms?

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E = m_0.c^2(1 + v^2 / 2c^2) = m_0.c^2 + m_0.v^2 / 2
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Thank you for your response.

Submit You have used 1 of 2 attempts

✔ Correct (1/1 point)

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