# 6. The Effects of Measurement Noise

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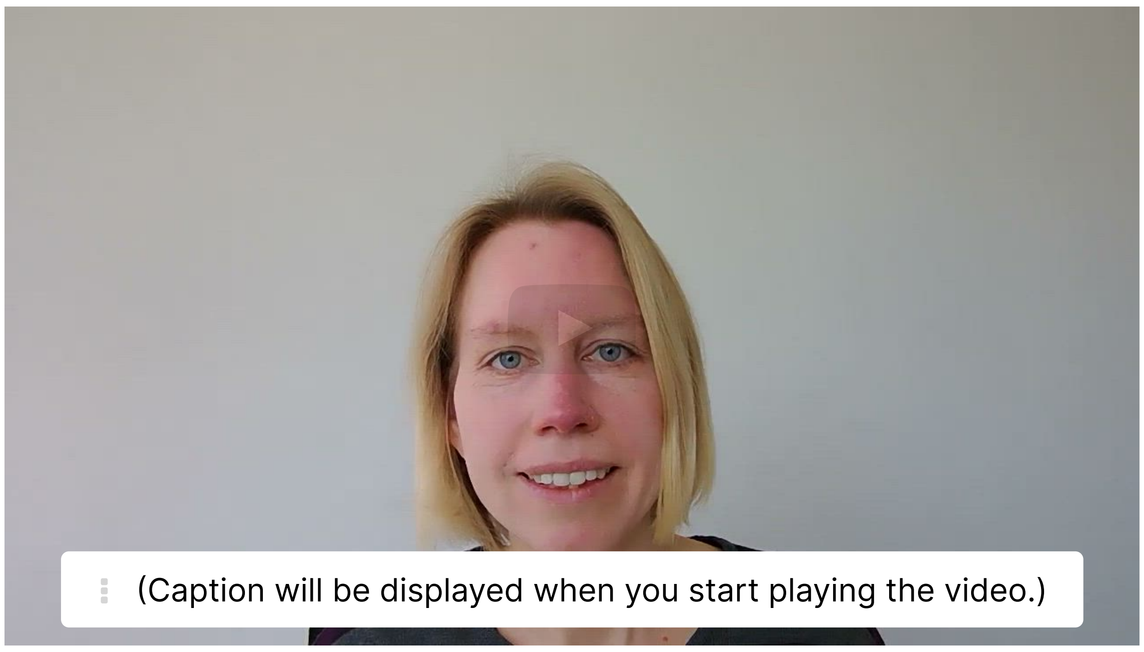
Exercises due Dec 1, 2021 17:29 IST Completed

**The Effects of Measurement Noise**

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Prof Jegelka: So finally, I would like

to go through two other choices of modeling

that you can consider when you look at your data.

And these are measurement noise and non-stationarity

in your data.

So let's first remind ourselves about stationarity.

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Thus far, we have assumed that the kernel is the only contribution to the covariance matrix of the data. Although is was not stated explicitly, this means we took the data, or the observations, as exact.

However, what happens if our sensors are faulty or imprecise? How can one introduce in the presented framework the uncertainty in the observed values itself?

Let us return to the original example of temperature measurements. Imagine the sensor providing the measurements is not perfect, and each time it takes a measurement, it induces some additional noise due to its inherit limitations, such as thermal noise. One can characterize such noise, for example, as an additional variable $\varepsilon$ that is also Normally distributed with mean zero and standard deviation $\tau$. We can denote this as $\varepsilon \sim \mathcal{N}\left(0, \tau^2\right)$.

Now, using the same notation as before, instead of directly observing the realizations of the random variable $\mathbf{X}_2$, we observe the realizations $\mathbf{y}_2$ of the random variable $\mathbf{Y}_2 = \mathbf{X}_2 + \varepsilon$. By definition, we will assume the random variable $\varepsilon$ is independent of $\mathbf{X}_2$.

Recall that the parameters of the conditional distribution of $\mathbf{X}_1$ given $\mathbf{X}_2$ are

$$\begin{aligned} \mu_{\mathbf{X}_1|\mathbf{X}_2} &= \mu_1 + \Sigma_{12}\Sigma_{22}^{-1}\left(\mathbf{x}_2 - \mu_2\right) \\ \Sigma_{\mathbf{X}_1|\mathbf{X}_2} &= \Sigma_{11} - \Sigma_{12}\Sigma_{22}^{-1}\Sigma_{21}. \end{aligned}$$

Adding $\varepsilon$ to $\mathbf{X}_2$ alters the covariance matrix:

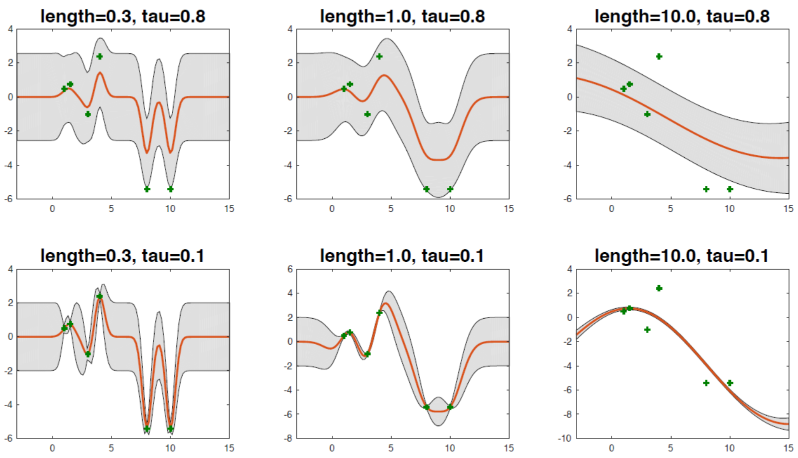
$$\mathbf{\Sigma} = \begin{bmatrix} \mathbf{\Sigma}_{11} & \mathbf{\Sigma}_{12} \\ \mathbf{\Sigma}_{21} & \mathbf{\Sigma}_{22} + \tau^2 I \end{bmatrix}$$

such that diagonal elements of magnitude $\tau^2$ are added to the $\mathbf{\Sigma}_{22}$ component.

This changes the conditional equations to be

$$\begin{aligned}\mu_{\mathbf{X}_1|\mathbf{Y}_2} &= \mu_1 + \Sigma_{12}(\Sigma_{22}+\tau^2 I)^{-1}(\mathbf{y}_2 - \mu_2)\\ \Sigma_{\mathbf{X}_1|\mathbf{Y}_2} &= \Sigma_{11} - \Sigma_{12}(\Sigma_{22}+\tau^2 I)^{-1}\Sigma_{21}.\end{aligned}$$

The below figure shows a Guassian process for two different values of $\tau$, $\tau = 0.8$ and $\tau = 0.1$.




**35**: The effects of noisy measurements

We can note that as $\tau$ is increases, so too does the variance on the estimate increase.

## Observational Noise 1

1/1 point (graded)

What happens in the extreme case where $\tau \to \infty$? What happens to the mean of the estimates compared to the prior assumed mean?

- (selected) Mean remains the same as the prior
- Mean decreases compared to the prior
- Mean increases compared to the prior

**Solution:**

As $\tau \to \infty$ we get

$$\mu_{\mathbf{X}_1|X_2} = \mu_1 + \Sigma_{12}(\Sigma_{22}+\tau^2 I)^{-1}(x_2 - \mu_2) \to \mu_1,$$

and so the mean remains the same as the prior assumed mean of $\mu_1$.

Submit You have used 1 of 2 attempts

Answers are displayed within the problem

## Observational Noise 2

1/1 point (graded)

Consider, again, the extreme case where $\tau \to \infty$. If we increase the number of observations in this limit, do the additional observations reduce the variance on the estimate?

- Variance decreases
- (selected) Variance remains the same

**Solution:**

As $\tau \to \infty$ we get

$$\Sigma_{\mathbf{X}_1|X_2} = \Sigma_{11} - \Sigma_{12}(\Sigma_{22} + \tau^2 I)^{-1}\Sigma_{21} \to \Sigma_{11},$$

and so the variance does not decrease with additional observations.

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