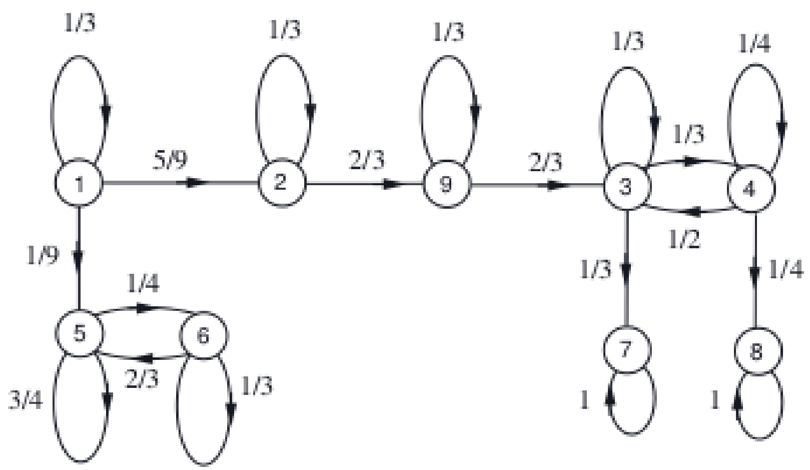
Exercise: Steady-state approximation

(3 points possible)

Consider a Markov chain with the following transition probability graph:



Find an approximation to $\mathbf{P}(X_{10000} = 5 \mid X_0 = 1) = r_{15}(10000)$.
Hint: First find an (exact) equation relating $r_{15}(10000)$, $r_{15}(9999)$ and $r_{55}(9999)$.

$r_{15}(10000) \approx$ 1/12 ✖ **Answer:** 0.12121

Answer:
Following the hint, we look one transition ahead: going from state 1 to state 5 after 10000 transitions can be achieved by either (i) staying at state 1 after the first transition and then going from state 1 to state 5 in 9999 transitions, or (ii) going from state 1 to state 5 after the first transition and then ending back in state 5 after 9999 transitions. The first transition cannot go to state 2 because then there would be no way to end up in state 5. Hence,

$$r_{15}(10000) = p_{11}r_{15}(9999) + p_{15}r_{55}(9999) = \frac{1}{3}r_{15}(9999) + \frac{1}{9}r_{55}(9999).$$

Since 9999 and 10000 are both large numbers of transitions, we use two approximations: (i) $r_{15}(9999) \approx r_{15}(10000)$ and (ii) $r_{55}(9999) \approx \pi_5$, the steady-state probability of being in state 5 when we consider the aperiodic recurrent class $\{5, 6\}$. With these approximations, we have

$$r_{15}(10000) \approx \frac{1}{3}r_{15}(10000) + \frac{1}{9}\pi_5 \Rightarrow r_{15}(10000) \approx \frac{1}{6}\pi_5.$$

The steady-state probabilities π_5 and π_6 are obtained by solving the system of equations

$$\begin{aligned}\frac{1}{4}\pi_5 &= \frac{2}{3}\pi_6\\ \pi_5 + \pi_6 &= 1,\end{aligned}$$

Lec. 26: Absorption probabilities and expected time to absorption
Exercises 26 due May 18, 2016 at 23:59 UTC

Solved problems

Problem Set 10
Problem Set 10 due May 18, 2016 at 23:59 UTC

Exit Survey

which leads to $\pi_5 = 8/11$ and $\pi_6 = 3/11$.

Therefore, $r_{15}(10000) \approx \frac{4}{33}$.

You have used 2 of 2 submissions