

<u>Lecture 14: Wald's Test, Likelihood</u> <u>Ratio Test, and Implicit Hypothesis</u>

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> 13. Testing Implicit Hypotheses II

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13. Testing Implicit Hypotheses II

Testing Implicit Hypotheses III: Slutsky's Theorem

2/2 points (graded)

As above, we have that

$$\sqrt{n}\left(\hat{ heta}_{n}- heta^{*}
ight) \xrightarrow[n o \infty]{(d)} \mathcal{N}\left(\mathbf{0},\Sigma\left(heta^{*}
ight)
ight), \quad \Sigma\left(heta^{*}
ight) \in \mathbb{R}^{d imes d}.$$

and

$$\sqrt{n}\left(g\left(\hat{ heta}_{n}
ight)-g\left(heta^{*}
ight)
ight) \xrightarrow[n o \infty]{(d)} \mathcal{N}\left(\mathbf{0},\Gamma\left(heta^{*}
ight)
ight), \quad \Gamma\left(heta^{*}
ight) \in \mathbb{R}^{k imes k}.$$

In particular, $\hat{\theta}_n$ is a consistent estimator for θ^* .

Assume that $\Gamma(\theta)^x$ is a continuous function of $\theta \in \mathbb{R}^d$ for all $x \in \mathbb{R}$.

Which of the following is a consistent estimator for $\Gamma(heta^*)^{-1/2}$?







$$ullet$$
 $\Gamma(\hat{ heta}_{\,n})^{-1/2}$



Applying Slutsky's theorem and the result of the previous problem, to what distribution does the random vector

$$\sqrt{n}\Gamma(\hat{ heta}_{n})^{-1/2}\left(g\left(\hat{ heta}_{n}
ight)-g\left(heta^{st}
ight)
ight)$$

converge to as $n o \infty$?

leftleft $\mathcal{N}\left(\mathbf{0},I_{k}
ight)$

 $\bigcirc \, \mathcal{N} \left(\mathbf{0}, I_d
ight)$

 $\bigcirc \, \chi^2_d$

 $igcup \chi^2_k$

Solution:

Since $\hat{\theta}_n$ is a consistent estimator for θ^* , by continuity of $\theta \mapsto \Gamma(\theta)^{-1/2}$, this implies that $\Gamma(\hat{\theta}_n)^{-1/2}$ is a consistent estimator for $\Gamma(\theta)^{-1/2}$.

By the result of the previous problem,

$$\sqrt{n}\Gamma(heta^*)^{-1/2}\left(g\left(\hat{ heta}_{\,n}
ight)-g\left(heta^*
ight)
ight) \xrightarrow[n o\infty]{(d)} \mathcal{N}\left(\mathbf{0},I_k
ight).$$

So by Slutsky's theorem,

$$\sqrt{n}\Gamma(\hat{ heta}_{n})^{-1/2}\left(g\left(\hat{ heta}_{n}
ight)-g\left(heta^{st}
ight)
ight) \xrightarrow[n o \infty]{(d)} \mathcal{N}\left(\mathbf{0},I_{k}
ight).$$

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• Answers are displayed within the problem

Testing Implicit Hypotheses IV: Performing the Test

2/2 points (graded)

We would like to hypothesis test between the following null and alternative:

$$H_0:g(heta^*)_-=0$$

$$H_{1}:g\left(heta ^{st }
ight)
eq 0.$$

where $heta^* \in \mathbb{R}^d$.

To do so, we consider the test statistic

$$T_n := \left| \sqrt{n} \Gamma(\hat{ heta}_n)^{-1/2} \left(g\left(\hat{ heta}_n
ight)
ight)
ight|_2^2 = n g(\hat{ heta}_n)^T \Gamma(\hat{ heta}_n)^{-1} g\left(\hat{ heta}_n
ight)$$

and design the test

$$\psi = \mathbf{1}\left(T_n > C
ight)$$

where C is a threshold to be determined.

Under the null hypothesis, to what distribution does the test-statistic T_n converge?

 $\bigcirc \, \mathcal{N} \left(\mathbf{0}, I_k \right)$

 $\bigcirc \, \mathcal{N} \left(\mathbf{0}, I_d \right)$

 $\bigcirc \, \chi^2_d$

 $igotimes \chi^2_k$



Supposing that d=6 and k=3, what value of C should be chosen so that ψ is a test of asymptotic level 5%?

(You should consult a table, e.g. https://people.richland.edu/james/lecture/m170/tbl-chi.html) or use software, e.g. R.)

7.815 **Answer:** 7.815

Solution:

Under the null-hypothesis, we have that $g\left(\theta^{*}\right)=0$, so by the previous problem,

$$\sqrt{n}\Gamma(\hat{ heta}_{n})^{-1/2}g\left(\hat{ heta}_{n}
ight) \stackrel{(d)}{\longrightarrow} \mathcal{N}\left(\mathbf{0},I_{k}
ight).$$

By definition, $\left|\mathcal{N}\left(\mathbf{0},I_{k}
ight)\right|_{2}^{2}\sim\chi_{k}^{2}$, so we have by continuity that

$$\left|\sqrt{n}\Gamma(\hat{ heta}_n)^{-1/2}g\left(\hat{ heta}_n
ight)
ight|_2^2 = ng(\hat{ heta}_n)^T\Gamma(\hat{ heta}_n)^{-1}g\left(\hat{ heta}_n
ight) \stackrel{(d)}{\longrightarrow} \chi_k^2.$$

Indeed, the test statistic T_n converges to χ^2_k in distribution.

When k=3, then $T_n \xrightarrow[n \to \infty]{(d)} \chi_3^2$. The test $\psi=\mathbf{1}\,(T_n>C)$ will have asymptotic level 5% precisely when C is the 5%-quantile $q_{0.05}$ of χ_3^2 . Consulting a table, we have that $q_{0.05}=7.815$.

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1 Answers are displayed within the problem

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