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## subgroups of the group of pentagon symmetries

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### ? Question

The pentagon has 5 line symmetries and therefore we will have 10 symmetries. So, we let the group  $G$  with order 10 denote the symmetry group of a pentagon.

A subset  $H$  of  $G$  is a subgroup  $(H, *)$  to the group  $(G, *)$  if and only if  $(H, *)$  is a group. To determine how many subgroups there are, I can use the Lagrange's [theorem](#) which tells us that the subgroups of a group with order  $n$  have a order  $m$  such that  $m|n$ .

By this theorem we get know that the group  $G$  has 5 subgroups since the divisors  $m$  of 10 is  $m = 1, 2, 5, 10$ .

The question is, how shall I sketch the lattice of subgroups? Shall I [check](#) every possible subset and then check if they are subgroups or is there a faster way?

I know that the 10 symmetries are the identity element, 4 rotation and 5 reflections. We can see these operation of the transformation as permutation.

The first transformation that rotates 72 degrees are the permutation (ABCDE), 144 degrees are the permutation (ABCDE)<sup>2</sup> etc. This permutation have the order 5 since if we rotate 5 times we [will get](#) the identity ("the original pentagon").

The reflection transformations are permutations with 2 cycles with length 2.

But [how do I](#) know that the subgroups with order 5 are the ones with identity and 4 rotations?

#### EDIT

The order 2 is easy to determine. The subsets  $\{i, g\}$  there  $g \in G - \{\text{reflections}\}$  are not groups because the [inverse](#) of  $g$  is not in the subset. BUT if  $g$  is one of the reflections then we have a group, since the reflections have the order 2 which means that [the element](#)  $g$  is the inverse of it self. This means that the subset is a subgroup because (i) it is closed and (ii) the inverse is in the subset.

#### EDIT 2

Can I think like this? We let a subset be

$$H = \{\text{id}, r, r^2, r^3, z\}$$

there  $r$  are the rotation and  $z$  is the reflection. By checking [the properties](#) for a group, we can see that the properties about that every element in  $H$  has an inverse does not hold. the element  $r$  has the inverse  $r^4$  which is not in  $H$  and therefore the subset  $H$  cannot be a subgroup. So, by removing one of the rotations and putting one of the reflections, we see that we are removing some inverses. So, the only subgroup of order 5 are

$$K = \{\text{id}, r, r^2, r^3, r^4\}$$

Am I thinking [correctly](#)? Or is it better to write down the group table to find the subsets which are subgroups?

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## Answer

Your thinking is more or less correct. Generally, when we wish to think about the subgroups of a group  $G$ , we take different subsets of  $G$  and look at what groups those subsets generate.

The obviously thing to do is start with one element. Let's say I start with a rotation  $r^k$  for some  $k = 1, 2, 3, 4$ . Since 5 is a prime, the numbers 1, 2, 3, 4 have multiplicative inverses mod 5. This implies that  $\langle r^k \rangle$  actually contains  $r$ . Hence,  $\langle r^k \rangle = \langle r \rangle = \{1, r, r^2, r^3, r^4\}$  for each  $k$ . Now, let's say I start with a reflection  $z$  instead. It's easy to see that  $\langle z \rangle = \{1, z\}$ . Thus, we have 5 different subgroups of order 2, one for each reflection. The only remaining case is  $\langle 1 \rangle = \{1\}$ .

Now let's consider a generating set with two elements. If we take  $\langle r^k, r^l \rangle$  for some  $k, l$ , this is clearly just  $\langle r \rangle$  again. Suppose instead we take  $\langle r^k, z \rangle$ . Then this subgroup contains  $r$  and  $z$ , hence it contains  $r^l z$  for each  $l = 0, 1, 2, 3, 4$ , i.e. it contains every reflection. It also contains the powers of  $r$ , which are the rotations, hence  $\langle r^k, z \rangle = G$ . Finally, we could take  $\langle z_1, z_2 \rangle$  for two reflections  $z_1, z_2$ . But then  $z_1 z_2$  is a rotation which is in  $\langle z_1, z_2 \rangle$ , hence by the previous case we have  $\langle z_1, z_2 \rangle = G$ .

From here it's easy to see that we've found every subgroup of  $G$ . Obviously this took a lot less time than checking all  $2^{10}$  subsets of  $G$ .

If you find this answer useful please share it with other students.

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