7. Jeffreys Prior II: Examples

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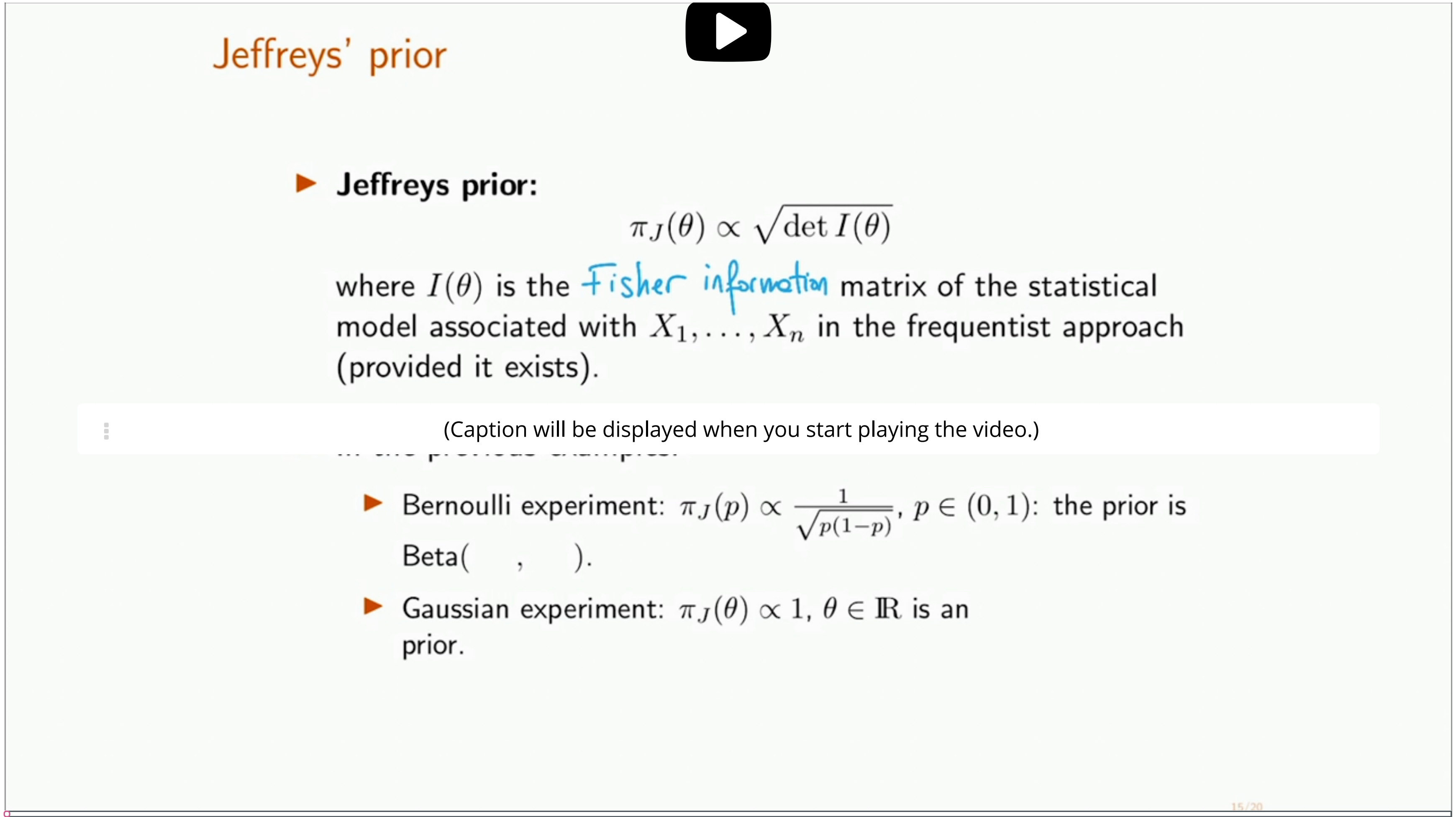

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Computing Jeffreys Prior

2/2 points (graded)

Let N be a Poisson random variable. That is,

$$p_N(n|\lambda) = e^{-\lambda}\frac{\lambda^n}{n!},$$

where $p_N(n|\lambda)$ denotes the conditional pmf of N given the parameter λ.

- Evaluate the Jeffreys prior, $p(\lambda)$ up to a proportionality constant, which only is a function of λ. Remove outside constants in your answer such that $I(1) = 1$.

1/sqrt(lambda)

$\frac{1}{\sqrt{\lambda}}$

✔ **Answer:** 1/sqrt(lambda)

- Is the Jeffrey's prior proper?

○ Yes

◉ No

✔

STANDARD NOTATION

Solution:

- We begin by computing Fisher information, $I(\lambda)$, as follows.

$$\frac{d}{d\lambda}\log p_N(n|\lambda) = -1 + \frac{n}{\lambda}.$$

Therefore,

$$I(\lambda) = \mathbb{E}\left[\left(\frac{d}{d\lambda}\log p_N(n|\lambda)\right)^2\right] = \mathbb{E}\left[\frac{(N-\lambda)^2}{\lambda^2}\right] = \frac{1}{\lambda^2}\mathrm{Var}(N).$$

Since $\mathrm{Var}(N) = \lambda$, for a Poisson random variable, we arrive at,

$$I(\lambda) = \frac{1}{\lambda} \implies p(\lambda) \propto \sqrt{I(\lambda)} = \frac{1}{\sqrt{\lambda}}.$$

- Since $\lambda \in (0, \infty)$, we can check that,

$$\int_0^\infty \frac{1}{\sqrt{\lambda}}\, d\lambda = \infty,$$

and therefore, Jeffreys prior for this problem is improper.

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You have used 2 of 3 attempts

Answers are displayed within the problem

Jeffreys Prior for Matrix Case

3/3 points (graded)

In this problem we will consider a model which has a two-dimensional parameter. Then you will calculate Jeffrey's prior using the Fisher information matrix.

Suppose that $X_1, \ldots, X_n \overset{iid}{\sim} N(\mu, \sigma^2)$, where μ and σ^2 are unknown. In this case, the Fisher information matrix $I(\theta)$ for $\theta = (\mu, \sigma^2)^T$ will be a 2×2 matrix, where the off-diagonal entries are 0.

- Find $(I(\theta))_{11}$.

1/sigma^2

✔ **Answer:** 1/(sigma^2)

$\frac{1}{\sigma^2}$

- Find $(I(\theta))_{22}$.

1/(2*sigma^4)

✔ **Answer:** 1/(2*sigma^4)

$\frac{1}{2\cdot\sigma^4}$

- Using your answers to the previous part, determine Jeffreys prior, $\pi(\theta)$, in terms of μ and σ. Express your answer in such a form that $\pi\left((1,1)^T\right) = 1$.

1/sigma^3

✔ **Answer:** 1/(sigma^3)

$\frac{1}{\sigma^3}$

STANDARD NOTATION

Solution:

- Clearly, the likelihood model is of form,

$$p_{Y|\mu,\sigma^2} = \frac{1}{\sqrt{2\pi\sigma^2}}\exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right) \implies \frac{1}{2}\log p_{Y|\mu,\sigma^2} = \log\left(\frac{1}{2\pi\sigma^2}\right) - \frac{(x-\mu)^2}{2\sigma^2}.$$

In particular,

$$(I(\theta))_{11} = -\mathbb{E}\left[\frac{\partial^2}{\partial\mu^2}\log p_{Y|\mu,\sigma^2}\right] = \frac{1}{\sigma^2},$$

using the fact that,

$$\frac{\partial}{\partial \mu} \log p_{Y|\mu,\sigma^2} = \frac{x-\mu}{\sigma^2} \implies \frac{\partial^2}{\partial \mu^2} \log p_{Y|\mu,\sigma^2} = -\frac{1}{\sigma^2}.$$

- Using the exact same strategy as above, and the fact that, $\mathbb{E}\left[(X-\mu)^2\right] = \mathrm{Var}\,(X) = \sigma^2$, we obtain that,

$$(I(\theta))_{22} = \frac{1}{2\sigma^4}.$$

- Since $\pi(\theta) \propto \sqrt{\det I(\theta)}$, we obtain that,

$$\det I(\theta) = \frac{1}{2\sigma^6} \implies \pi(\theta) \propto \frac{1}{\sigma^3}.$$

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You have used 2 of 3 attempts

Answers are displayed within the problem

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question posted 3 days ago by nch1993

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Sorry we are going back to fisher information. I have no problems computing fisher information. But what is Var(n)? and why does it appear? Am I missing anything? I thought we are taking the expectation of the 2nd derivative on lambda. Help please.

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1 response

stelstoy
3 days ago

Var(N) is just the variance of the variable in this question - notice they have called it N (not X, as usual). In this particular case think of N not as number of trials or variables, but as the variable itself (so where you see N - think X, small n is small x). This is all computed for one instance, so n in the usual case (as number of variables) doesn't appear.

You can take the second derivative with respect to λ and take expectation and arrive at the same result. Here N is a r.v just as X.

posted 3 days ago by **Analesdey**

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