

MITx: 6.041x Introduction to Probability - The Science of Uncertainty



Unit 0: Overview

- EntranceSurvey
- Unit 1: Probability models and axioms

Lec. 1: Probability models and axioms

Exercises 1 due Feb 10, 2016 at 23:59 UTC

Mathematical background: Sets; sequences, limits, and series; (un)countable sets.

Solved problems

Problem Set 1

Problem Set 1 due Feb 10, 2016 at 23:59 UT

- Unit 2: Conditioning and independence
- Unit 3: Counting
- Unit 4: Discrete random variables
- Exam 1

Unit 7: Bayesian inference > Problem Set 7a > Problem 3 Vertical: Hypothesis test with a continuous observation

■ Bookmark

Problem 3: Hypothesis test with a continuous observation

(5/5 points)

Let Θ be a Bernoulli random variable that indicates which one of two hypotheses is true, and let $\mathbf{P}(\Theta=1)=p$. Under the hypothesis $\Theta=0$, the random variable X is uniformly distributed over the interval [0,1]. Under the alternative hypothesis $\Theta=1$, the PDF of X is given by

$$f_{X\mid\Theta}(x\mid 1) = \left\{egin{array}{ll} 2x, & ext{if } 0 \leq x \leq 1, \ 0, & ext{otherwise.} \end{array}
ight.$$

Consider the MAP rule for deciding between the two hypotheses, given that $oldsymbol{X} = oldsymbol{x}$.

1. Suppose for this part of the problem that p=3/5. The MAP rule can choose in favor of the hypothesis $\Theta=1$ if and only if $x\geq c_1$. Find the value of c_1 .

$$c_1 = \boxed{$$
 1/3 \checkmark

2. Assume now that p is general such that $0 \le p \le 1$. It turns out that there exists a constant c such that the MAP rule always decides in favor of the hypothesis $\Theta = 0$ if and only if p < c. Find c.

3. For this part of the problem, assume again that p=3/5. Find the conditional probability of error for the MAP decision rule given that the hypothesis $\Theta=0$ is true.

$$\mathbf{P}(\text{error} \mid \Theta = 0) = \boxed{2/3}$$

4. Find the probability of error associated with the MAP rule as a function of p. Express your answer in terms of p using standard notation .

► Unit 5: Continuous	When $p \leq 1/3$, $\mathbf{P}(\mathbf{error}) = igcap_{eta}$	
random variables	When $p \ge 1/3$, $\mathbf{P}(\mathbf{error}) = \boxed{3/2-1/(4*p)-(5*p)/4}$	
Unit 6: Further topics on random variables	You have used 1 of 2 submissions	
	DISCUSSION	
Unit 7: Bayesian inference	Click "Show Discussion" below to see discussions on this problem.	

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