



## MITx: 6.041x Introduction to Probability - The Science of Uncertainty



Bookmarks

- ▶ Unit 0: Overview
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Unit overview

Lec. 11: Derived distributions

Exercises 11 due Mar 30, 2016 at 23:59 UTC

Unit 6: Further topics on random variables > Lec. 12: Sums of independent r.v.'s; Covariance and correlation > Lec 12 Sums of independent r v s Covariance and correlation vertical7



Bookmark

## Exercise: Correlation properties

(6/6 points)

As in the preceding example, let  $Z$ ,  $V$ , and  $W$  be independent random variables with mean  $0$  and variance  $1$ , and let  $X = Z + V$  and  $Y = Z + W$ . We have found that  $\rho(X, Y) = 1/2$ .

a) It follows that:

$$\rho(X, -Y) =$$

-1/2



Answer: -0.5

$$\rho(-X, -Y) =$$

1/2



Answer: 0.5

b) Suppose that  $X$  and  $Y$  are measured in dollars. Let  $X'$  and  $Y'$  be the same random variables, but measured in cents, so that  $X' = 100X$  and  $Y' = 100Y$ . Then,

$$\rho(X', Y') =$$

1/2



Answer: 0.5

c) Suppose now that  $\tilde{X} = 3Z + 3V + 3$  and  $\tilde{Y} = -2Z - 2W$ . Then

$$\rho(\tilde{X}, \tilde{Y}) =$$

-1/2



Answer: -0.5

d) Suppose now that the variance of  $Z$  is replaced by a very large number. Then

$$\rho(X, Y) \text{ is close to}$$

1



Answer: 1

e) Alternatively, suppose that the variance of  $Z$  is close to zero. Then

$$\rho(X, Y) \text{ is close to}$$

0



Answer: 0

Answer:

**Lec. 12: Sums of independent r.v.'s; Covariance and correlation**

Exercises 12 due Mar 30, 2016 at 23:59 UTC

**Lec. 13: Conditional expectation and variance revisited; Sum of a random number of independent r.v.'s**

Exercises 13 due Mar 30, 2016 at 23:59 UTC

**Solved problems**

**Additional theoretical material**

**Problem Set 6**

Problem Set 6 due Mar 30, 2016 at 23:59 UTC

**Unit summary**

We saw that a linear transformation  $x \mapsto ax + b$  of a random variable does not change the value of the correlation coefficient, except for a possible sign change if the coefficient  $a$  is negative. Note that in the case of  $\rho(-X, -Y)$ , we have two sign changes, hence no sign change.

For the last two parts, if  $Z$  has a very large variance, then the terms  $V$  and  $W$  become insignificant, and  $\rho(X, Y) \approx \rho(Z, Z) = 1$ . And if  $Z$  has very small variance, then  $X$  and  $Y$  are approximately independent, so that  $\rho(-X, -Y) \approx 0$ . (These conclusions can also be justified by an exact calculation.)

*You have used 1 of 2 submissions*

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