You are taking "Exam (Timed, No Correctness Feedback)" as a timed exam. Show more

End My Exam 43:46:15

1. Directional Derivatives

Bookmark this page

Recitation due Aug 18, 2021 20:30 IST Completed

Practice

1.

1/1 point (graded)

Imagine a mountain landscape where the height at a point (x, y) is $h(x, y) = x^2 + x + y^4$. There are two trails in this landscape starting at $(1, 1)$. One trail goes in a straight line to $(1, 2)$ and one trail goes in a straight line to $(2, 2)$. Near $(1, 1)$, which trail is steeper?

- ( ) The trail from $(1, 1)$ to $(1, 2)$ is steeper.
- (•) The trail from $(1, 1)$ to $(2, 2)$ is steeper.

✔

Solution:

The first trail points in the direction $\langle 0, 1\rangle$ and the second points in the direction $\langle 1, 1\rangle$. Computing the directional derivative at $(1, 1)$ in each of these directions, we find:

$$\begin{aligned} \nabla h(x, y) &= \langle 2x + 1, 4y^3 \rangle \\ \nabla h(1, 1) &= \langle 3, 4\rangle \\ D_{\langle 0,1\rangle} h(1, 1) &= \nabla h(1, 1) \cdot \langle 0, 1\rangle = 4 \\ D_{\langle 1,1\rangle} h(1, 1) &= \nabla h(1, 1) \cdot \langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \rangle = \frac{7}{\sqrt{2}} \end{aligned}$$

Since $7/\sqrt{2} > 4$, we conclude that the slope is steeper in the direction $\langle 1, 1\rangle$.

Submit You have used 1 of 1 attempt

ℹ Answers are displayed within the problem

Could not format HTML for problem. Contact course staff in the discussion forum for assistance.

3.

3.0/3 points (graded)

As above let $h(x, y) = x^2 + x + y^4$.

Let c be the curve defined by $h(x, y) = 3$. (Notice that $(1, 1)$ is on the curve c.)

a.) Find a unit vector which is normal to c at $(1, 1)$.

[3/5,4/5] ✔ **Answer:** [0.6,0.8]

b.) Find a unit vector $\hat{u}$ which is tangent to c at $(1, 1)$.

[-4/5,3/5] ✔ **Answer:** [0.8,-0.6]

Calculator Hide Notes

c.) Compute $D_{\hat{u}} h(1,1)$ (where $\hat{u}$ is the vector you found in part (b).)

0 **Answer:** 0

d.) (Self reflection.) Can you explain why you got the answer that you did?

Solution:

a) We learned that the gradient $\nabla h(1,1)$ points perpendicular to the level curve of $h(x,y)$ that passes through $(1,1)$. Since the question asked for a unit vector, we rescale $\nabla h(1,1) = \langle 3,4 \rangle$ to have unit length, giving the answer $\langle 3/5, 4/5 \rangle$.

b) Since we already have a unit vector that is normal to the curve, rotating this vector by 90 degrees (clockwise or counter-clockwise) gives a vector that is tangent to the curve. Rotating a vector $\langle a,b \rangle$ 90 degrees clockwise gives the vector $\langle b, -a \rangle$. Therefore, the answer is $\langle 4/5, -3/5 \rangle$, or its negative.

c) $D_{\langle 4/5,-3/5 \rangle} h(1,1) = \langle 3,4 \rangle \cdot \langle 4/5, -3/5 \rangle = 0$.

d) Since the vector we found in part (b) is perpendicular to the gradient, the dot product between the two will be zero.

Submit You have used 1 of 5 attempts

Answers are displayed within the problem

1. Directional Derivatives

Hide Discussion

Topic: Unit 2: Geometry of Derivatives / 1. Directional Derivatives

Add a Post

Show all posts by recent activity

?	3d I was wondering if my intuition for 3d is correct: since we're finding the direction derivative along the unit vector tangent to the level ...	3
?	Question 1 flawed (?) Yes I understand the solution and the thought process behind it, but imagining the problem as a mountain landscape, shouldn't the fi...	2
	Error < signed, not allowed For Q2b) What unit vector u makes Duh<1,1> the biggest? I am not able to type the vector because of the error, "Error < signed, not ...	3
	a pair of small typos Hello. In the solution to 3, perhaps change "perpindicular" to perpendicular in two places. Very best wishes.	2
?	Question 1 -- easier solution Isn't the question of which is steeper more directly answered by simply comparing h(1,2) vs h(2,2) ?	4
	[Staff] Imprecise Terminology	6
	[Staff] Bug	8
	Stumped on Q3 For some reason, these tangent and normal vectors have my head spinning in circles. We could use a hint to which lecture to refer to...	6
?	Problem 1:	3

Calculator Hide Notes

< Previous Next >