04.1 04 KHAN ACADEMY : PRE CALCULUS

PROBABILITY

$$\text{PROBABILITY} = \frac{\text{No of conditions meet}}{\text{Total no of conditions}}$$

Example – Take Card game

4 suits, Each suits have 13 types of card – Ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, Jack, Queen, King
(type)

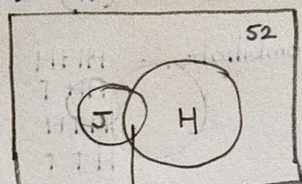
♤ ♢ ♧ ♡ Total = 4×13 = 52 cards

Spade Diamond Club Heart

$P(\text{Jack}) = \frac{4}{52} = \frac{1}{13}$ $P(♡) = \frac{13}{52} = \frac{1}{4}$

$P(\text{J and Heart}) = \frac{1}{52}$ $P(\text{J or H}) = \frac{4+13-1}{52}$ ← Jack is also in heart

$= \frac{17-1}{52} = \frac{16}{52} = \cancel{\frac{4}{4}}\ \frac{4}{13}$




$$= \frac{J + H - 1}{52}$$

Jack which is also heart = 1

Additional rule of propability

Take Example – Green Cube – 8, Green Sphere – 9, ~~G~~ Yellow Cube – 5, Yellow Sphere – 7.

Mix ~~each~~ everything in a bag.

$P(\text{Cube}) = \frac{8+5}{8+9+5+7} = \frac{13}{29}$. $P(\text{yellow}) = \frac{7+5}{29} = \frac{12}{29}$

$P(\text{Yellow and Cube}) = \frac{5}{29}$

no of yellow, no of cube, yellow cube is already covered in no of yellow

$P(\text{Yellow or Cube}) = \frac{12 + 13 - 5}{29} = \frac{20}{29}$

$P(\text{Yellow or Cube}) = \frac{12}{19} + \frac{13}{29} - \frac{5}{29}$

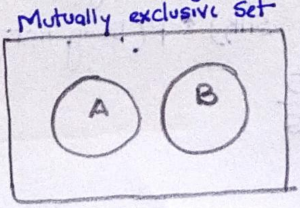
$P(\text{Yellow or Cube}) = P(Y) + P(C) - P(Y \text{ AND } C)$

Generally, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ ← Additional rule.

So, if there is something overlapping, we need to subtract the common thing.

If there is no overlap then it is mutually exclusive set. i.e, $P(A \text{ and } B) = 0$

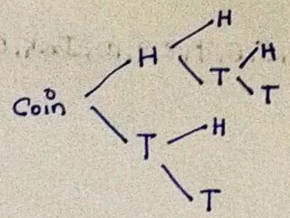
Mutually exclusive set



# MULTIPLICATION RULES FOR PROBABILITIES

Example – Take example of Coin.

Toss two times –




All possibility = HH, HT, TH, TT.

So $P(H,H) = \frac{1}{4}$.

Another way to consider is, they are independent events.

P(Head getting on first flip) * P(Head getting on second flip)

$= P(H_1) * P(H_2) = \frac{1}{2} * \frac{1}{2} = \frac{1}{4}$

P(THT) = This also ~~indepent~~ independent events.

$= P(T_1) * P(H_2) * P(T_1) = \frac{1}{2} * \frac{1}{2} * \frac{1}{2} = \frac{1}{8}$.

Verify – All possible combination – HHH, HHT, HTH, HTT, THH, THT✓, TTH, TTT

$= \frac{\text{no of condition met}}{\text{Total conditions}} = \frac{1}{8}$

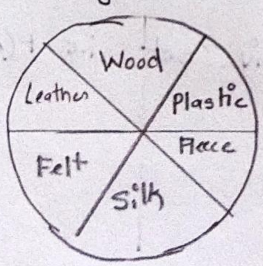
Eg – On a multiple choice test; problem 1 has 4 choices. and problem 2 has 3 choices. Each problem has only one correct answer. What is the probability of randomly guessing the correct answer to both problems?

– They are independent events, as problem 1 is not depend on problem 2

P(correct on #1 and #2) = P(correct on #1) * P(correct on #2)

$= \frac{1}{4} * \frac{1}{3} = \frac{1}{12}$

Eg – Maya and Doug are finalist in crafting competition. For the final round, each of them spin a wheel to determine what star material must be in craft. Maya and Doug both want to get silk as their star material. Maya will spin first, followed by Doug. What is the probability that Neither contestant get silk?




Both are independent events as even if maya get silk doesn't mean Doug will not get silk.

= P(Maya not getting silk) * P(Doug not getting silk)

$= \frac{5}{6} * \frac{5}{6} = \frac{25}{36}$

04.3 Dependent Probability

Suppose we have a bag of 5 marbles. In that we have 3 Green marbles and 2 orange Marbles. Suppose if we have to take 2 Green marbles in first two counter, we will win ₹100, entry fee of 2 picks is ₹35. so we should play or not?

→ $P(1^{st}\text{green} \cap 2^{nd}\text{green}) = P(1^{st}\text{green}) \cdot P(2^{nd}\text{green} \mid 1^{st}\text{green})$

It is dependent probability because second pick is dependent on first. Suppose we took green as first pick, we remain with 4 marbles

So, $P(2^{nd}\text{green} \mid 1^{st}\text{green}) = \frac{2}{4}$.

$P(1^{st}\text{green} \cap 2^{nd}\text{green}) = P(1^{st}\text{green}) \cdot P(2^{nd}\text{green} \mid 1^{st}\text{green})$

$= \frac{3}{5} \times \frac{2}{4} = \frac{3}{5} \times \frac{1}{2} = \frac{3}{10} = 0.3$

So, 30% chance we will get first 2 pick as green.

So, if we play 100 times, 30 times may be we will win.

$= \underset{\text{played}}{100} * \underset{\text{entry fee}}{(35)} = \underset{\text{given amount}}{3500}$ ; $\underset{\text{win}}{30} * \underset{\text{win amount}}{(100)} = \underset{\text{Total win amount}}{3000}$

So, we gave 3500 but won 3000 so we should not play the game.

Eg – Maya and Doug are finalist in crafting competition. For final round, each of them will randomly select a card without replacement, that will reveal what the star material must be in their craft.

Here the card Available : Leather, Wood, Plastic, Felt, Silk, Fleece

Maya and doug both want silk as their star material. Maya will draw first, followed by Doug. What is the probability that Neither contestant draws silk? It is dependent event, because when maya pick it is without replacement

→ P(Maya No Silk and Doug no Silk) = P(MNS) * P(DNS | MNS)

$P(MNS) = \frac{5}{6}$ ← Maya pick 1 but not silk = 6−1 = 5.

$P(DNS \mid MS) = \frac{4}{5}$ ← Total 6, Maya pick 1 but not silk = 5. Doug ~~cannot take~~ not take silk = 5−1 = 4

$P(MNS) * P(DNS \mid MNS) = \cancel{\frac{5}{6} \times \frac{4}{5}} = \frac{5}{6} * \frac{4}{5} = \frac{\cancel{5}}{\cancel{6}_3} * \frac{\cancel{4}^2}{\cancel{5}} = \frac{2}{3}$

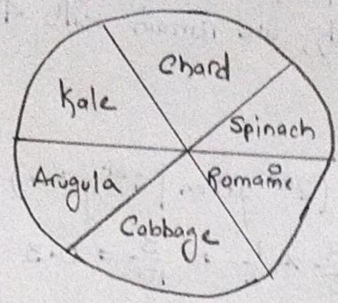
Independent event, $P(A \text{ and } B) = P(A) * P(B)$.

Dependent event, $P(A \text{ and } B) = P(A) * P(B|A)$

or

$= \cancel{P(B)}\ P(A|B) * P(B)$

Eg - Two contestants are finalist in a cooking competition. For the final round, each of them spin a wheel to determine what star ingredient must be in their dish:




| Event | Meaning |
| --- | --- |
| $K_1$ | First contestant land on Kale |
| $K_2$ | Second contestant land on Kale. |
| $K_1^c$ | First contestant does not land on Kale |
| $K_2^c$ | Second contenstant does not land on Kale |

C → Complement / opposite

- Using general multiplication rule, express symbollically probability neither Contestant land on Kale. → This is an ~~od~~ independent event

$$P(K_1^c \text{ and } \cancel{P}K_2^c) = P(K_1^c) * P(K_2^c)$$

- Explain $P(K_1^c \text{ and } K_2) = P(K_1^c) \cdot P(K_2 | K_1^c)$

$P(K_1^c \text{ and } K_2)$ → Probability 1st does not get Kale but second contestant get Kale.

$P(K_1^c)$ → Probability 1st does not get Kale.

$P(K_2 | K_1^c)$ → Probability 2nd get Kale given 1st does not get Kale.

04.5 PERMUTATIONS

Three seats $\underline{\quad}$ $\underline{\quad}$ $\underline{\quad}$ Suppose we have 3 person. A, B, C
1 2 3

In what combination we can make them seat.

Possible Combination — ABC
ACB
BAC
BCA
CAB
CBA

This is all permutations.

But it will get complicate if we have 100 seats.

So choose any way —
So in seat 1, how many people can sit.
In seat 2, one already occupied Seat 1.
In Seat 2, two already sat down only one.

$\underset{\text{Seat 1}}{\underline{3}} \times \underset{\text{Seat 2}}{\underline{2}} \times \underset{\text{Seat 3}}{\underline{1}} = 3\times2\times1 = 6$
$= 3!$

Suppose we have 5 ~~feat~~ seats then $5! = 5\times4\times3\times2\times1 = 120$.

- Suppose we have 5 people and 3 chairs. So in how many ways they can sit.

$\underset{\text{Seat 1}}{\underline{5}} * \underset{\text{Seat 2}}{\underline{4}} * \underset{\text{Seat 3}}{\underline{3}} = 60$ ways / permutations

Formula wise, $\boxed{\text{Permutation } (n, r),\ {}^nP_r = \frac{n!}{n-r!}}$. In above case, n = no of person
r = no of chair

Eg - Take English alphabet / characters. How many are 3 letter words.
Total letter = 26.

$\underset{\text{Letter 1}}{\underline{26}} * \underset{\text{Letter 2}}{\underline{26}} * \underset{\text{Letter 3}}{\underline{26}} = (26)^3$.

If all different letter $= \underset{\text{letter 1}}{\underline{26}} * \underset{\text{letter 2}}{\underline{25}} * \underset{\text{letter 3}}{\underline{24}} = 26 * 25 * 24$

By formula wise, $n = 26$, $r = 3$. $\Rightarrow {}^{26}P_3 = \frac{26!}{(26-3)!} = \frac{26!}{23!} = 26\times25\times24$

Zero Factorial or 0!

Normally, $n! = n\cdot(n-1)(n-2)\ldots 1!$ eg $3! = 3\times2\times1$, $2! = 2\times1$, $1! = 1$.

$0! = 0$ No but $0! = 1$

${}^nP_k = \frac{n!}{(n-k)!}$ here $n = k$; $= \frac{n!}{(n-n)!} = \frac{n!}{0!}$ so it cannot go to infity due to 0. $\therefore$ we consider $0! = 1$.

Eg - Possible colors are Blue, Yellow, White, Red, Orange and Green. How many 4-colour codes can be made if the colors cannot be repeated.

$\underset{\text{Code 1}}{\underline{6}} \times \underset{\text{Code 2}}{\underline{5}} \times \underset{\text{Code 3}}{\underline{4}} \times \underset{\text{Code 4}}{\underline{3}} = 6\times5\times4\times3 = 30\times12 = 360$ ways or permutation

Eg - A club of 9 people wants to choose a board of three officers: President, Vice president, secretary. How many ways are there to choose from 9 peoples?

$\underset{\text{President}}{\underline{9}}, \underset{\text{VP}}{\underline{8}}, \underset{\text{Secretary}}{\underline{7}} = 9\times8\times7 = \underset{72}{\cancel{82}}\times7 = 504$ permutation / ways

# COMBINATIONS

3 chairs and ~~6~~ 5 peoples.

Total way → $\underset{\text{Chair 1}}{6}$ $\underset{\text{Chair 2}}{5}$ $\underset{\text{Chair 3}}{4}$ = 120 permutations.

Suppose first we choose 3 people : ABC BAC CAB
ACB BCA CBA

For each 3 seat, we have one set which contain 6 combinations
BCF CBF FBC
BFC CFB FCB and there are total 120.

If only 3 people choose to seat and in any combination between them

$= \frac{120}{6}$ ← Total ways / permutation ← Number of ways to arrange 3 people

$= 20$

Formula wise, 6 six people to choose 3 chair $= {}^{6}C_{3} = 20$ ways

4 chairs and 6 people

$\underset{\text{Chair 1}}{6}$ $\underset{\text{Chair 2}}{5}$ $\underset{\text{Chair 3}}{4}$ $\underset{\text{Chair 4}}{3}$ $= 6\times5\times4\times3 = \frac{6!}{(6-4)!} = \frac{6!}{2!} = 6\times5\times4\times3$

$= 360$

↑ permutation formula ⇒ ${}^{n}P_{K} = \frac{n!}{(n-K)!}$

$n = 6$, $K = 4$
people chair

Combinations,

$$\boxed{{}^{n}C_{K} = \frac{\frac{n!}{(n-K)!}}{K!} = \frac{n!}{K!\,(n-K)!}}$$

# Number of ways to arrange K things in K spots

$\frac{K}{1} * \frac{K-1}{2} * \frac{K-2}{3} * \frac{K-3}{4} * \dots * \frac{1}{K} = K!$

How many ways to choose people from 6 people → ${}^{6}C_{4} = \frac{6!}{4!\,(6-4)!}$

Here people = 6, chair = 4.
$n = 6$, $K = 4$.

$= \frac{6!}{4!\,2!} = \frac{5\times\cancel{6}^{3}}{\cancel{2}} = 15$

~~Theare~~ There are 4 people A, B, C and D. Each one handshake with other once. How many hand shakes will be there?

1. A B
2. A C
3. A D
4. B C
5. B D
6. C D

How many people to choose 2 from 4.
$n = 4$, $K = 2$ ${}^{4}C_{2} = \frac{4!}{2!\,(4-2)!}$

$= \frac{4\times3\times\cancel{2}\times1}{\cancel{2}\times\cancel{1}}$

$= 6$

Suppose we find ~~can~~ permutation

$\underset{\text{person 1}}{4}$ $\underset{\text{person 2}}{3}$ $= 4 * 3 = 12$ permutation

Eg, A B which is same
B A

So it is rising double counting.
Therefore it is a combination problem.

Logically, $= \frac{\text{total possible way}}{\text{No of ways to arrange 2 people}} = \frac{4\times3}{2!} = \frac{12}{2} = 6$

047 A card games using 36 unique cards : four suits (diamonds, hearts, clubs, and spades) with card numbered from 1 to 9 in each suit. A hand is a collection of 9 cards, which can be sorted however the player chooses. How many 9 cards hands are possible?

36 35 34 33 32 31 30 29 28

suppose 1 combination - 9spade 1 2 3 4 5 6 7 8

Another combination - 8 7 6 5 4 3 2 1 9spades.

Here, we got the same combination only it is not sorted, if it is sorted it is a same combination.

So problem in $36 \times 35 \times 34 \times 33 \times 32 \times 31 \times 30 \times 29 \times 28$ is duplication. (contains)

So, divide by 9 cards can be arrange $= 9!$

Answer is $= \dfrac{36 \times 35 \times 34 \times 33 \times 32 \times 31 \times 30 \times 29 \times 28}{9!} = 94{,}143{,}280$ possible 9 card hand.

Formula wise, $n = 36$ = total cards. $k = 9$ = we have to choose

$^{36}C_9 = \dfrac{36!}{(36-9)!\,9!} = \dfrac{36!}{27!\,9!}$

$= \dfrac{36 \times 35 \times \cdots \times 28}{9!}$

Same answer

PROBABILITY USING COMBINATIONS

What is the probability to get 3 Heads when flipping 8 times?

~~How many~~ What is the probability to get 3 Heads when flipping 8 times?

$P\left(\dfrac{3\text{heads}}{8\text{ times flip}}\right) = P\left(\dfrac{3H}{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2}\right) = P\left(\dfrac{3H}{2^8}\right) = \dfrac{8 \times 7}{2^8} = \dfrac{2^3 \times 7}{2^8} = \dfrac{7}{2^5} = \dfrac{7}{32} = 0.218 = 21\%$

Here $N = 8$, $K = 3$. Because 8 flips and we need 3 Heads.

$^{N}C_{K} = \dfrac{N!}{(N-K)!\,K!} = \dfrac{8!}{3!\,5!} = \dfrac{8 \times 7 \times 6}{3!} = \dfrac{8 \times 7 \times \cancel{6}}{\cancel{3} \times \cancel{2} \times 1} = 56$ different ways to pick 3 heads out of 8 flip.

To win a particular lottery game, a player chooses 4 numbers from 1 to 60. Each number can only be choosen once. If all 4 numbers match the winning numbers, regardless of order, the player wins. What is the probability that the winning numbers are 3, 15, 46 and 49?

$- N = 60,\ K = 4;\ \rightarrow \dfrac{60!}{(60-4)!\,4!} = \dfrac{60!}{56!\,4!} = \dfrac{60 \times 59 \times 58 \times 57 \times \cancel{56!}}{\cancel{56!}\ 4!} = \dfrac{60 \times 59 \times 58 \times 57}{4!}$

So why not $\rightarrow \dfrac{60}{\text{Number 1}} \times \dfrac{59}{\text{Number 2}} \times \dfrac{58}{\text{Number 3}} \times \dfrac{57}{\text{number 4}}$. but addition to that we have to also divide by 4 slots (regardless of order)

Here we are considering the order, first one out -1, second one out -1

But the sequence is not important ie, output (3, 15, 46, 49 / 49, 15, 3, 46 / ....)

So divide by $4! = \dfrac{4}{\text{slot 1}} \times \dfrac{3}{\text{slot 2}} \times \dfrac{2}{\text{slot 3}} \times \dfrac{1}{\text{slot 4}}$. So ans is $\dfrac{60 \times 59 \times 58 \times 57}{4!}$

$= \frac{60 \times 59 \times 58 \times 57}{4 \times 3 \times 2 \times 1} = 487,635$ combinations total

To get output $(3, 15, 46, 49) = \frac{1}{487,635}$ win probability

Eg → A club of 9 peoples wants to choose a ~~ba~~ board of three officers : President, Vice President, and Secretary. Assuming the officers are choosen at random, what is the probability that officers are Marsha for President, Sabita for Vice President and Robert for secretary?

→ $P\begin{pmatrix} \text{President} = \text{Marsha} \\ \text{VP} = \text{Sabita} \\ \text{Secretary} = \text{Robert} \end{pmatrix} = \frac{1}{\text{Total no of possible}} = \frac{1}{504}$

$\frac{9}{P} \times \frac{8}{VP} \times \frac{7}{S} = 504$ possibilities.

Eg - Samara is setting up an olive oil tasting competition for a festival. From 15 distinct varities, Samara will choose 3 different olives oil and blend them together. A contestant will taste the blend and try to identify which 3 of the 15 varities were used to make it.

Assume that a contestant can't taste any differences & is randomly guessing. What is the probability that a contestant correctly guesses which 3 variety was used?

$\frac{15}{\text{task 1}} \times \frac{14}{\text{task 2}} \times \frac{13}{\text{taste 3}}$ → This is wrong, because here we have to find exact 3 variety not how many way possible to do it.

So, $P(\text{exact 3 variety}) = \frac{\text{Number of correct combination}}{\text{Total combination}} = \frac{1}{\frac{15!}{12!\,3!}}$

Total combination, $n = 15,\ k = 3,\ {}^{n}C_{k} = {}^{15}C_{3} = \frac{15!}{12!\,3!}$

$= \frac{12!\,3!}{15!}$

$= \frac{3 \times 2}{15 \times 14 \times 13}$ (5, 7)

$= \frac{1}{35 \times 13} = \frac{1}{455}$

$1/455$ is the probability that contestant will correctly guess which 3 variety was used.

04.9 Kyra works on a team of 13 total people. Her manager is randomly selecting 3 members from her team to represent the company at a conference. What is the probability that Kyra is choosen for the conference?

→ $\dfrac{\text{Number of team which Kyra can be part of}}{\text{Total no of teams}}$.

Total number of teams, $n=13$, $K=3$, 13 people pick combination of 3.
$\Rightarrow {}^{13}C_3$, logically $13\times12\times11$, no specific combination $= \dfrac{13\times12\times11}{3!}$

Number of team which Kyra can be part of, $n=12$, $K=2$, 1 is Kyra. So remaining is 12 and out of 12 we need to choose 2 people to make a team of 3. $\Rightarrow {}^{12}C_2$, logically = one position is Kyra so remaining $12\times11$, we don't have any combination $\dfrac{12\times11}{2!}$

$$\frac{\text{No of team in Kyra}}{\text{Total no of team}} = \frac{{}^{12}C_2}{{}^{13}C_3} = \frac{\frac{12!}{10!\,2!}}{\frac{13!}{10!\,3!}} = \frac{\frac{12\times11}{2\times1}}{\frac{13\times12\times11}{3\times2\times1}} = \frac{3}{13}$$ Kyra is choosen for conference

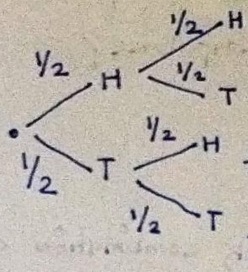
A standard deck of 52 playing cards includes 4 aces, 4 kings and 44 other card. Suppose that Luis randomly draw 4 cards without replacement. What is the probability that Luis get 2 aces and 2 kings (in any order)?

$$\frac{\text{Number of draws with 2 Aces and 2 Kings}}{\text{Total possible draws of 4 cards}} = \frac{({}^{4}C_2)\times({}^{4}C_2)}{{}^{52}C_4}$$

~~Proba~~ General Multiplication rule in Probability –
When we calculate probabilities involving one event AND another event occuring, we multiply their probabilities.
In some cases, the first event happening impacts the probability of the second event. We called these dependent events.
In other cases, the first event happening does not impact the probability of the seconds. We call these independent events.

Independent events: Flipping a coin twice
What is the probability of flipping a fair coin and getting "heads" twice in a row? That is, what is the probability of getting heads on the first flip AND heads on the second flip?
- Imagine we had 100 people simulate this and flip a coin twice. On average 50 people would get heads on first flip and then 25 of them would get heads again. So 25 out of the original 100 people – or 1/4 of them – would get heads twice in a row.

The number of people, we can represent with a tree diagram.




$\frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$

$\frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$

$\frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$

$\frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$

We multiply the probabilities along the branches to find the overall probability of one event AND the next even occuring.

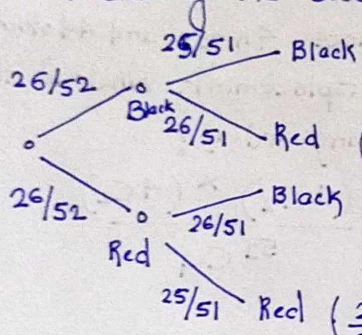
$P(H \text{ AND } H) = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$

Be careful only when two events are independent, $P(A \text{ AND } B) = P(A) \cdot P(B)$

Find the probability that both dice shows a 3 — $\left(\frac{1}{6}\right) \times \left(\frac{1}{6}\right) = \frac{1}{36}$

Dependent events: Drawing a card

- Consider drawing two cards, without replacement ~~for~~ from a standard deck of 52 cards. That means we are drawing the first card, leaving it out and then drawing the second card. What is the probability both cards are black?




$\left(\frac{26}{52}\right)\left(\frac{25}{51}\right) = 0.245$

$\left(\frac{26}{52}\right)\left(\frac{26}{51}\right) = 0.255$

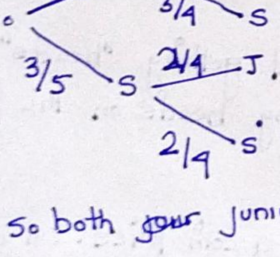
$\left(\frac{26}{52}\right)\left(\frac{26}{51}\right) = 0.255$

$\left(\frac{26}{52}\right)\left(\frac{25}{51}\right) = 0.245$

$P(\text{both black}) = \frac{26}{52} \cdot \frac{25}{51}$

$\rightarrow 0.245$

Eg – A table of 5 students has 3 seniors and 2 juniors. The teacher is going to pick 2 students at random from this group to present homework solution. Find the probability both are juniors.




→ 1 Junior is already selected. So remaining 1 Junior $= 1/4$.

→ 0 senior is selected (in last phase). 3 senior $= 3/4$.

→ 0 Junior is selected (last phase). 2 Junior $= 2/4$.

→ 1 senior is selected. Remaining ~~3~~ 2 $=$ ~~3~~ 2/4.

So both ~~your~~ junior $= \left(\frac{2}{5}\right) \times \left(\frac{1}{4}\right) = \frac{2}{20} = \frac{1}{10}$

04.11 Conditional Probability

$$P(A \text{ AND } B) = P(A|B) \cdot P(B) = P(B|A) \cdot P(A)$$

Eg - Rahul two favourite foods are bagels and pizza. Let A represent the events that he eats a bagel for breakfast and B represent the event that he eats pizza for lunch.

On a randomly selected day, the probability that Rahul will eat a bagel for breakfast, $P(A)$ is 0.6, the probability that he will eat pizza for lunch, $P(B)$ is equal to 0.5, and conditional probability that he eats a bagel for breakfast, given that he eats pizza for lunch, $P(A|B)$ is equal to 0.7.

Based on this information, what is $P(B|A)$, the conditional probability that Rahul eats pizza for lunch, given that he eats a bagel for breakfast?

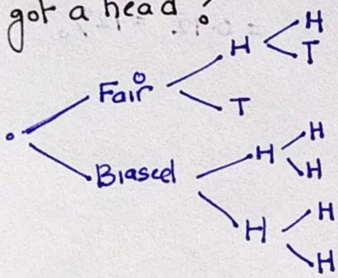
$P(A) = 0.6$, $P(B) = 0.5$, $P(A|B) = 0.7$.

$$P(A|B) \times P(B) = P(B|A) * P(A)$$

$$(0.7) \times (0.5) = P(B|A) * (0.6).$$

$$P(B|A) = \frac{0.35}{0.6} \approx 0.58$$

Suppose there are two coins, one is fair (1H, 1T) and other one is biased (1H, 1H → 2H). What is the probability that he choose a fair coin, given he got a head?




$$P(\text{fair} | \text{Head}) = \frac{\text{Possible outcomes of Fair AND H}}{\text{Possible outcomes to get H}}$$

$$= \frac{1}{3}$$

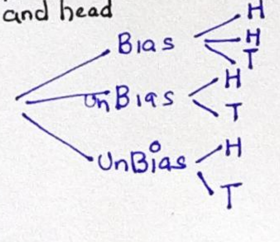
Suppose again he flipped (2 time toss)

$$P(\text{fair} | HH) = \frac{1}{5}$$

So as we are increasing the toss, probability of getting Head is decreasing.

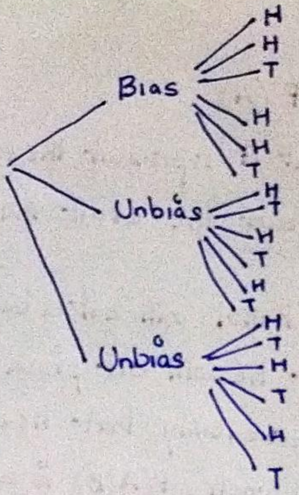
Therefore, as we increase the toss, probability of head will go down but never 0

Bob has 3 coin. 1 biased and 2 unbias. What is the probability he choose bias coin and head. Unbiased coin has 3 sides, H, H, T.




Trick is we cannot have different leaves for each node. So we have to multiply to get equal number of leaves.

Hint multiply by 2 and 3.




So, now ~~we~~ for all nodes we have 6 leaves

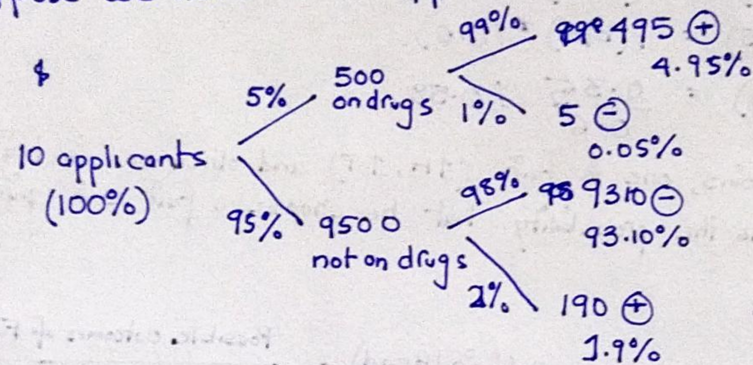
$$P(\text{Bias} \mid \text{head}) = \frac{4}{10} = \frac{P(\text{bias and head})}{P(\text{head})}$$

So choose Conditional Probobility by Bayes theorem.

$$P(A \mid B) = \frac{P(B \mid A) * P(A)}{P(B)}$$

- A company screen jobs applicants for Illegal drugs at a certain stage in their hiring process. They said False positive rate = 2% and false negative rate = 1%. Suppose they have 5% of actuall illegal drugs. Given the applicants test positive, what is probability they are actually on drugs?

Suppose we have 10,000 applicants.




P(Actuall on drugs | test positive)

$$= \frac{495}{495 + 190}$$

$$= 0.72 = 72\%$$