

05.1 CHAPTER 5: CALCULATING NEURAL NETWORK WITH LOSS

- With a randomly initialized model, we train the model over time. To train model we tweak weight and bias to improve model accuracy & confidence.
- Loss function -
 - We calculate how much the error the model has.
 - Loss function also referred as the cost function, is the algorithm that quantifies how wrong the model is.
 - Since loss is a metric, model's error we ideally want it to be 0.

Categorical cross entropy loss -

- Categorical cross entropy is explicitly used to compare a "ground-truth" / reality probability (y or "targets") and model predicted distribution (\hat{y} or predictions).

Categorical cross entropy of y (actual) and \hat{y} (predicted) is $L_1 = -\sum y \times \log(\hat{y})$

- where L denotes Loss Value, y represent actual and \hat{y} represent predicted.
- In our case, we have a classification model that returns a probability distribution over all the outputs. Cross entropy compares two probability distributions. So that is why it is called Categorical cross entropy loss.
- ↑ Target Variable compare Calculate loss
 actual predicted

Example - Softmax Output of $[0.7, 0.1, 0.2]$ and target $[1, 0, 0]$ suggests 3 target class and softmax has output probability.

$$L_1 = -\sum y \times \log(\hat{y}) = -(1 \times \log(0.7) + (0 \times \log(0.1)) + (0 \times \log(0.2))) \\ = -(-0.35 + 0 + 0) = 0.35$$

In python, import math softmax-output = $[0.7, 0.1, 0.2]$ target = $[1, 0, 0]$

$$\text{loss} = -(\text{math.log}(\text{softmax-output}[0]) \times \text{target}[0] + \text{math.log}(\text{softmax-output}[1]) \times \text{target}[1] + \text{math.log}(\text{softmax-output}[2]) \times \text{target}[2])$$

print(loss) # 0.35

So we can say when loss = 0, model is perfect. Indirectly probability should be 1.

print(math.log(softmax-output) * target) = math.log(1) = 0. So here loss = 0.

It should be 1, so that output is 0

So what is log?

- Log is short for logarithm and is defined as the solution for x term in an equation of the form $a^x = b$.

For example, $10^x = 100$ can be solved by \log : $\log_{10}(100) = 2$.

This log property is extremely useful when e (Euler number or -2.71828) is used in base.

- Logarithm with e as its base is referred to natural logarithm, natural log or simply \log . Can be written as $\ln(x) = \log(x) = \log_e(x)$

So something like $e^x = b$ for example $e^x = 5.2$ is solved by $\log(5.2)$.

import numpy as np.

b = 5.2

print(np.log(b)) # 1.648.

import math.

print(math.e**1.648)

5.19999 \approx 5.2 so both are same.

- Consider a scenario with a neural network that perform classification between three classes. and neural network classifies in batch of three. After running softmax activation function with a batch of 3 samples and 3 classes (dog, cat, humans)

softmax_output = np.array([0.7, 0.1, 0.2],
[0.1, 0.5, 0.4],
[0.02, 0.9, 0.08])

class_targets = [0, 1, 1] # dog, cat, cat

In class_targets, first value is 0 means softmax output distribution intended was 0th index of [0.7, 0.1, 0.2], the model has 0.7 confidence that this observation is dog.

For second, second value is 1 so 1st index of [0.1, 0.5, 0.4], so model has 0.5 confidence it is cat.

Third, same as second.

	dog	cat	humans
0.7	0.7	0.1	0.2
0.1	0.1	0.5	0.4
0.02	0.02	0.9	0.08

for targ_ind, distribution in zip(class_targets, softmax_outputs):

print(distribution[targ_ind]) # 0.7, 0.5, 0.9

Other way to write is

pr print(softmax_outputs[range(len(softmax_outputs), class_targets)])

[0.7 0.5 0.9], same output return.

05-3 Finally, we want to average loss per batch to have an idea how our model is doing after training.

arithmetic mean: $\text{sum}(\text{iterable}) / \text{len}(\text{iterable})$

So in python, $\text{neg-log} = -\text{np.log}(\text{softmax-output}[\text{range}(\text{len}(\text{softmax outputs}), \text{class-targets})])$

$\text{average_loss} = \text{np.mean}(\text{neg-log})$

$\text{print}(\text{average_loss}) \# 0.385$

One hot encoded

- One hot encoded is where all the values except for one, are zeroes and the correct label's position is filled with 1.

last example
softmax-output
[0.7, 0.1, 0.2]
[0.1, 0.5, 0.4]
[0.02, 0.9, 0.08]

→ One hot encoded
[1, 0, 0]
[0, 1, 0]
[0, 1, 0]

For $\log(0)$ it will give error. i.e., $-\text{np.log}(0)$ will give error. Normally, $\log(0)$ is undefined. $y = \log(x)$ then $e^y = x$ in this case $y = \log(0)$ is same as $e^y = 0$. e raised to any power is positive number and there is no y resulting $e^y = 0$. This means $\log(0)$ is undefined and equal to very big number. so $-\text{np.log}(0) = \text{inf}$ #infinite.

then $\text{np.e}^{xx}(-\text{np.inf}) = 0.0$

So we could add a very small value to the confidence to prevent it from being zero.

For example, $1e-7$ $-\text{np.log}(1e-7) \# 16.118$

Adding a very small value, to the confidence at its far edge will insignificant impact the result, but leads to 2 additional issues.

First is when Confidence Value is 1 $\rightarrow -\text{np.log}(1 + 1e-7) \# -9.999e-8 \approx 0$

When model is fully correct in prediction and predicted all correct label, 1. but negative value $(-1e-7)$ $(1e-7)$ is shifting its confidence even if it is very small. Ideal in this case should be 0.

$-\text{np.log}(1 - 1e-7) \# 1.0000199e-7 \approx 0$

So this will prevent being exactly 0 making it very small value

Therefore we will clip values from both sides by same number, $1e-7$ in our case.

$y\text{-pred-clipped} = \text{np.clip}(y\text{-pred}, 1e-7, 1-1e-7)$

Categorical Cross Entropy Loss Class

Common Loss class

class Loss:

Calculate the data and regularization losses, model output and reality.
def calculate(self, output, y):

Calculate sample loss

sample-losses = self.forward(output, y)

Calculate mean loss

data-loss = np.mean(sample-losses)

return data-loss

since we implemented both probability and one-hot we need to first check whether its probability or one-hot.

so if targets are single dimensional (like a list) it is probability and if it is of 2 dimensions then it is one-hot. 2 means list of list.

sample - softmax-output = np.array([[0.7, 0.1, 0.2],
[0.1, 0.5, 0.4],
[0.02, 0.9, 0.08]])

class-targets = np.array([[1, 0, 0],
[0, 1, 0],
[0, 1, 0]])

for probability

if len(class-targets.shape) == 1:

correct-confidences = softmax-outputs[range(len(softmax-output)), class-target]

if One-hot

elif len(class-targets.shape) == 2:

correct-confidences = np.sum(softmax-outputs * class-targets, axis=1)

losses

neg-log = -np.log(correct-confidences)

average-loss = np.mean(neg-log)

when == 2, we multiply confidence by target, zeroing out all values except the one at correct labels, performing a sum along row axis (axis 1).

Accuracy Calculation →

- While loss is a useful metric for optimizing model, Accuracy is also a good measure.
- Accuracy often describe how often the largest confidence is correct in terms of fraction.

Model output

[0.7, 0.2, 0.1],
[0.5, 0.1, 0.4]
[0.02, 0.9, 0.08]

Actual

[0,
1
1]

Accuracy

✓ [0.7 (pred) = 0.7 Actual]

✗ [0.5 (pred) = 0.1 Actual]

✓ [0.9 (pred) = 0.9 Actual]

= $\frac{2}{3}$ = 0.66, Accuracy = 0.66

~~loss = 1 + 0.66 = 0.34~~