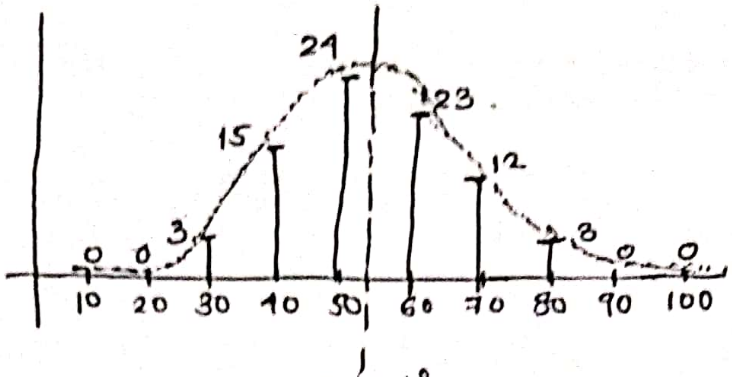
Standard Error of Mean (Standard Error)

Standard Error – It is a measure of uncertainity in sample mean. Higher the standard error. High is we are less confident.

Distribution of marks acheived by the students in a class exam.

10	0
20	0
30	3
40	15
50	24
60	23
70	12
80	3
90	0
100	0
Total	80



Population mean.

$$\boxed{SE(\bar{x}) = \frac{s}{\sqrt{n}}}$$

$s \rightarrow$ standard deviation
$n \rightarrow$ number of samples

Standard error $\Rightarrow$ Population mean $\neq$ Sample mean.

Population mean = 53/54

Sample mean = Most of the high quantity sample are 40, 50, 60, 70.

Q) You want to know the average IQ of statistics students.

Take 5 random students sit in IQ test = [127, 109, 121, 94, 109] . $\bar{x} = 112.0$

So, 112 is the average IQ of student. How confidence are you? Sample is too small?

Suppose for 50 students, $\bar{x} = 115.3$. How confidence are you? Little bit more confident.

Then 500 students, $\bar{x} = 114.7$. I am very much confident.

So, higher the observation, confidence go up and up. so $SE(\bar{x}) = \frac{S}{\sqrt{n}}$ High the value of n, it will decrease SE.

So, for 5 student, suppose SD or $S = 12.72$, then $SE(\alpha) = \frac{12.72}{\sqrt{5}} = 5.69$.

n	Sample mean	Std error of sample mean.	95% confidence Interval
5	112.0	5.69	[96.2, 127.8]
50	115.30	1.74	[108.4, 115.6]
500	114.7	0.55	[110.8, 113.1]

getting narrower.

$n = 5$
$\bar{x} = 112$
$SE(\alpha) = 12.72$.

To find confidence Interval, $\bar{x} \pm SE(\bar{x}) + t_{0.975, n-1}$

$= 112 \pm [5.69 \times t_{0.975, 4}] \Rightarrow$ one ans for + and one ans for −.

$= 96.2, 127.8$.

Here we use t test because normally IQ are normally distributed among human being & we use sample.

So, we can say that we are 95% confident that true mean lies between 96.2 and 127.8.

SAMPLE ERROR OF PROPORTION

One hundred voters are sampled, and 65 said they were voting for a major party. Find the standard error of sample proportion of major party voters and a 95% confidence interval.

$p = \frac{65}{100} = 0.65$

$SE(p) = \sqrt{\frac{(1-P)P}{n}}$

$SE(P) = \sqrt{\frac{(0.65)(0.35)}{100}}$

$SE(P) = 0.0477$

95% confidence interval $= p \pm SE(P) \times Z_{0.975}$.

Here z is used because n is large, and according to central limit theorem when n is large it follows normal distribution.

– Even if we don't know distribution of this set (voting majority party) It follow normal distribution.

$= 0.65 \pm 0.4777 \times 1.96 = [0.557, 0.743]$

So we are 95% confident that between 55.7% and 74.3% of voters vote for majority party.