Mathematics

Grade - 9

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Preface

The curriculum and curricular materials have been developed and revised on a regular basis with the aim of making the education objective-oriented, practical, relevant and job oriented. It is necessary to instill the feelings of nationalism, national integrity and democratic spirit in students and equip them with morality, discipline and self reliance so as to develop in them social, personal skills and the basic competencies of language, mathematics, science, information and communication technology, environment and health, and soft skills. Education should help them appreciate and make them aware of arts and aesthetics; preserve and promote social norms, values and ideals; equip them with creative skills and have due respect for ethnicity, languages, religions, cultures, disabilities, regional diversity and human rights so as to make them capable of playing the role of responsible citizens. This textbook is the translated version of the Mathematics textbook of grade 9 which had been developed in line with the Secondary Level Mathematics Curriculum, 2014.

In bringing out the textbook in this form, the contribution of the Executive Director of CDC Mr. Baburam Paudel, Prof. Dr. Min Bahadur Shrestha, Prof. Dr. Lekh Nath Sharma, Baikunth Khanal, Bijaya Baniya, Barun Baidya, Goma Shresth, Damber Dhog Angdembe and Pusp Raj Dhakal is highly acknowledged. The book is translated in to English by a team of Krishna Bahadur Bista, Nisha Parajuli, Rajkumar Mathema and Rajyalaxmi Shrestha. The subject matter and language of this book were edited by Chandra Kanta Bhusal and Harish Pant. The layout and illustrations of the book were done by Jayram Kuikel. CDC extends sincere thanks to all those who have contributed to develop this textbook.

This book contains a variety of learning materials and exercises which will help learners to achieve the competency and learning outcomes set in the curriculum. Each unit deals with all mathematical skills and the subject matters required to practise various learning activities. There is uniformity in the presentation of the activities which will certainly make it convenient for the students. The teachers, students and other stakeholders are expected to make constructive comments and suggestions to make it a more useful learning material.

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Contents

Lesson 1

Sets

1.0 Review

The given figure represents a set of students who take part in the game on the ocassion of annual day of a school. Answer the following questions by studying the figure.

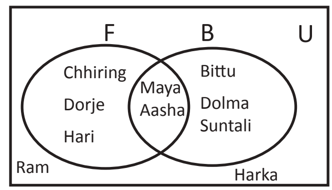
(a) How many members are there in the sets B and F in total? List them out.

(b) How many subsets can be formed from the set F?

(c) How many members are there in the set B?

(d) Who are the common members of both sets B and F?

(e) Who are not the members of any sets B and F?



(f) How many members are there in the given Venn diagram all together?

Chhiring, Dorje, Hari, Maya and Aasha are the members of football team. It can be written as F = {Chhiring, Dorje, Hari, Maya, Aasha} and the members of basketball team as B = {Bittu, Dolma, Suntali, Maya, Aasha} by using listing method.

The teams of the members who take part in football and basketball are donated by English alphabet capital letters 'F' and 'B' respectively.

> A well defined collection of the objects is called set.

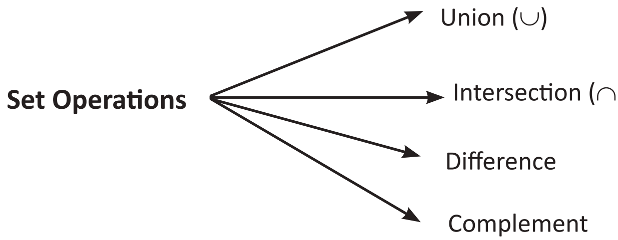
In the above Venn diagram, Ram and Harka are not the members of sets F and B. In this discussion the set that contains all the members of the given figure is called universal set. It is denoted by U. All the members of F and B are also the members of U. So, F and B are proper subsets of universal set U.

> A set from which all subsets are formed in a particular context is called universal set of the subsets. The sets which are formed from an universal set are called subsets.

German mathematician George Cantor (1844-1918) developed the ideas of set as mathematical theory in ninteenth century. John Euler Venn developed the method of representing the sets by closed figures called Venn diagrams. Now, we use Venn diagrams to represent set, set relations and set operations in a simplified form.

1.1 Set Operations

Now, we study the following operations on sets.

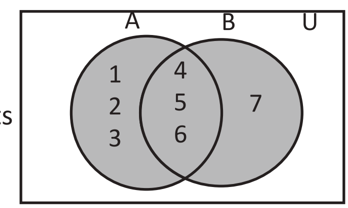


1.1.1 Union of Sets

Union of two sets A and B is a set which contains all the elements of A and B. The union of sets is denoted by mathematical symbol $\cup$.

If A = {1, 2, 3, 4, 5, 6} and B = {4, 5, 6, 7}, then 1, 2, 3, 4, 5, 6, 7 lie in the union of A and B.

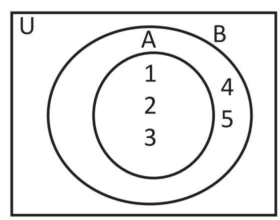
$A \cup B = \{1, 2, 3, 4, 5, 6, 7\}$.



In the figure, shade $A \cup B$.

Here, the elements 4, 5, 6 are common in the both sets A and B. Therefore, these common elements are written only once.

In set builder notation, $A \cup B = \{x : x \in A \text{ or } x \in B\}$.



If $A \subset B$, then $A \cup B = B$.

In the figure, $A \subset B$. Therefore

$B = A \cup B$.

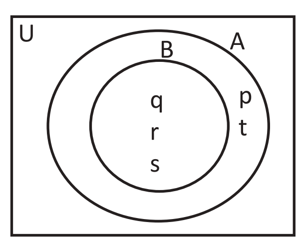
Can it be shown by shading?

Similarly, if $B \subset A$, then $A \cup B = A$.

Also, $A \cup U = U$,x since, every set is the subset of its universal set.

In the figure alongside, $B \subset A$.

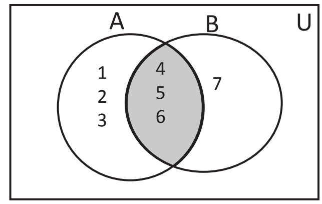
Therefore $A \cup B = A$.



1.1.2 Intersection of sets

Intersection of two sets A and B is a set which contains all the common elements of both sets.

The intersection of two sets A and B is denoted by $A \cap B$.



For Example, A = {1, 2, 3, 4, 5, 6} and B = {4, 5, 6, 7}, then $A \cap B$ ={4, 5, 6}. (In the figure, the shaded part indicates the set of common elements of both sets.)

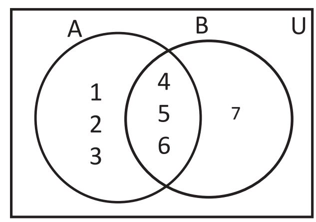
Intersection is represented in set builder notation as,

$A \cap B = \{x : x \in A \text{ and } x \in B\}$

If $A \subset B$, then $A \cap B = A$. Also, $A \cap U = A$

because every set is the subset of its universal set.

1.1.3 Difference of Sets



The difference of two sets A and B is said to be a set of the elements which are contained in A but not in B. It is denoted by A - B. For example: A ={1, 2, 3, 4, 5, 6} and B ={4, 5, 6, 7} are two sets, then A - B ={1, 2, 3}.

In the figure, shade A-B.

(A - B) can be written in another form as

$A - B = A - (A \cap B) = \{1, 2, 3)$. Similarly, $B - A = \{7\} = B - (A \cap B)$.

If the union of A - B and B - A is taken, then another set is formed. This set is called symmetric difference set. It is denoted by $A \Delta B$.

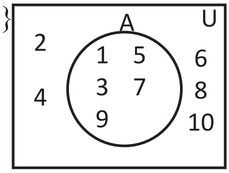
$A \Delta B = (A - B) \cup (B - A)$

$= \{1, 2, 3\} \cup \{7\} = \{1, 2, 3, 7\} = (A \cup B) - (A \cap B)$

1.1.4 **Complement of a Set**

A set of elements which are not contained in set A but contained in universal set U is called complement of set A. The complement of set A is denoted by $\overline{A}$. $\overline{A}$ is contained in U but not in A. So, it is also written as $\overline{A} = U - A$.

For example : U ={1, 2, 3, 4, 5, 6, 7, 8, 9, 10} and A ={1, 3, 5, 7, 9}



Then $\overline{A}$ ={2, 4, 6, 8, 10}

According to the set builder method,

$\overline{A} = \{x : x \in U, x \notin A\}$.

In the figure, shade $\overline{A}$ where the elements of universal set are not contained in set A.

By observing the Venn diagram, we can say that

$U = \overline{A} \cup A$ and $\overline{A} \cap A = \phi$

Example 1

If U ={x : x is a positive integer from 1 to 20},

$A = \{x: 6 \le x \le 20\}$, $B = \{x: x \le 8\}$

$C = \{x: 10 < x < 15\}$ ={the integers between 10 to 15}.

Carry out the following operations.

i. $A \cup B$ ii. $A \cap B$ iii. $B \cup C$ iv. $A - B$ v. $\overline{A \cup B}$

Solution :

Here, U ={x : x is a positive integer from 1 to 20}

$= \{1, 2, 3, ..., 20\}$

$A = \{x: 6 \le x \le 20\} = \{6, 7, 8, \text{-------} 19, 20\}$

$B = \{x: x \le 8, x \not\in N\} = \{1, 2, 3, \text{--} 7, 8\}$

$C = \{x: 10 < x < 15\} = \{11, 12, 13, 14\}$

Now, i. $A \cup B = \{6, 7, 8, \text{---} 19, 20\} \cup \{1, 2, 3, \text{---} 7, 8\}$,

$= \{1, 2, 3\ 4, 5, 6, 7, 8, 9, 10, \ldots, 20\} = \{x: 1 \le x \le 20\}$,

ii. $A \cap B = \{6, 7, 8, \text{---} 19, 20\} \cap \{1, 2, 3, \text{---} 7, 8\}$,

$= \{6, 7, 8\} = \{x: 6 \le x \le 8\}$

iii. $B \cup C = \{1, 2, 3, \text{- - -} 7, 8\} \cup \{11, 12, 13, 14\}$

$= \{1, 2, 3, 4, 5, 6, 7, 8, 11, 12, 13, 14\}$

iv. $A - B = \{6, 7, 8, \text{- - -} 19, 20\} - \{1, 2, 3, \text{- - -} 7, 8\}$

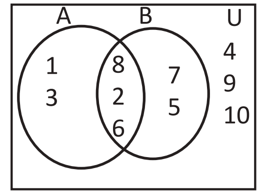
$= \{9, 10, 11, \text{- - -} 19, 20\}$

v. $\overline{A \cup B} = U - (A \cup B) = \{1, 2, 3, \text{- - -} 20\} - \{1, 2, 3, 4, 5, 6, 7, 8, \text{- - -} 19, 20\}$

$= \{\ \}$ or $\varnothing$

Exercise 1.1

1. Observe the given Venn diagram and write the following sets by listing method. Also find every relation and operations and express them in separate Venn diagrams by shading.



i. $A \cup B$ ii. $A \cap B$ iii. $\overline{A \cup B}$ iv. $\overline{A \cap B}$

v. $\overline{A} \cup \overline{B}$ vi. $A - B$ vii. $U - \overline{A}$ viii. $U - \overline{B}$

2. If U = {a, b, c, d, e, f, g, h, i, j, k},

   A = {a, c, d, f }, B ={g, h, i} and C = {e, i},

   (a) construct the following sets :

   i. $\overline{A \cup B}$ ii. $\overline{A \cap B}$ iii. $\overline{A} \cup \overline{B}$ iv. $\overline{A} \cap \overline{B}$

   (b) which of the sets in Q. No. 2 (a) are equal and why?

   (c) construct the following sets and then express in a Venn diagram.

   i. $\overline{A}$ ii. $\overline{B}$ iii. $\overline{\overline{A}}$

3. If U = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}, A = {1, 2, 6, 7} , B = {2, 3, 5, 6} and C = {4, 5, 6, 7}, then verify that :

   i. $\overline{A \cup B} = \overline{A} \cap \overline{B}$ ii. $\overline{A \cap B} = \overline{A} \cup \overline{B}$ iii. $(A \cup B) \cap C = (A \cap C) \cup (B \cap C)$

4. If U = {1, 2, 3, ..., 19, 20},

   P = {x: x is a prime number.}

   Q = {y: y, is a factor of 18} and

   R = {z: z, is a multiple of 3 less than 20}

   (a) prepare the list of the elements of the following sets by drawing separate Venn- diagrams.

   i. $P \cup Q$ ii. $(P \cup Q) \cap R$ iii. $\overline{P \cup Q}$ iv. $\overline{P} \cap \overline{Q}$

   v. $P - Q$ vi. Q-P   vii. $P \cup (Q \cap R)$

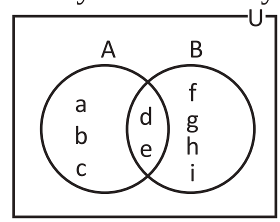
   (b) which sets are equal in Q. No. 4 (a)?

1.2 Cardinality of Sets

1.2.1 What is the cardinality of a set?

How many elements are contained in a set? That number of elements of the set is called cardinality of the set. For example : a set of all vowels of English alphabet (V) = {a, e, i, o, u}. There are 5 elements in this set. Therefore the cardinality of set V is 5. Symbolically it is written as n(V) = 5.

If any two sets A and B are intersecting sets then the number of elements of set A only is denoted by $n_o(A)$ and the number of elements of set B only is denoted by $n_o(B)$. In the figure,



$n_o(A) = 3$. Therefore $n_o(A) = n(A - B)$

It is also written as $n_o(A) = n(A) - n(A \cap B)$.

Again,

$n_o(B) = 4$. So, $n_o(B) = n(B - A) = n(B) - n(A \cap B)$.

Here, $n(A \cup B) = 9$ and $n(A \cap B) = 2$.

If a set is proper subset of another set among any two sets or if $A \subset B$, then $n(A \cap B) = n(A)$ and $n(A \cup B) = n(B)$. Also, when $B \subset A$ then $n(A \cap B) = n(B)$ and $n(A \cup B) = n(A)$.

1.2.2 Method to find the Cardinality of sets

Study the following Venn diagrams and answer the following questions.

i. $n(A) = ?$ ii. $n(B) = ?$ iii. $n(A \cup B) = ?$

iv. $n(A \cap B) = ?$ v. $\overline{n(A \cup B)} = ?$

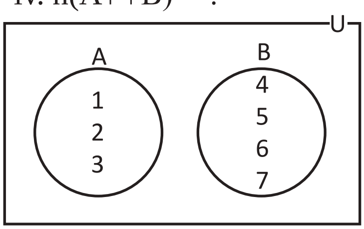


Fig 1

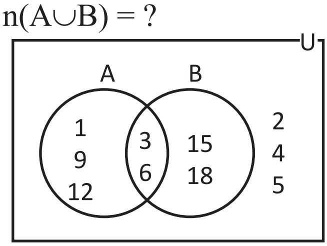


Fig 2

In fig.1 there is no any common element in sets A and B. So, the sets are disjoint sets. Here, in set A, there are three elements. So, n(A) =3. In the same way, n(B) = 4 and $n(A \cup B) = 7$. There is no any element in $(A \cap B)$. So, $n(A \cap B) = 0$

Observe the fig. 2, there are also other elements except the elements of sets A and B in U. Therefore, U ={1, 2, 3, 4, 5, 6, 9, 12, 15, 18}. So, n(U) = 10. By taking the elements except the elements of A and B, a set {2, 4, 5} is formed, which do not lie in $(A \cup B)$ but lie in U.

Therefore, $\overline{(A \cup B)} = \{2, 4, 5\}$. So, $n\overline{(A \cup B)} = 3$

Again in the above Venn diagram 1, there are elements of A and B which are inside U. So, there is not any elements in $\overline{(A \cup B)}$. Therefore, $\overline{(A \cup B)} = \phi$. So, $n\,\overline{(A \cup B)} = 0$.

The cardinality of above mentoined sets can be written as the following formula.

i. If A and B are two disjoint sets, then $n(A \cup B) = n(A) + n(B)$.

ii. If A and B are two intersecting sets, then $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

or, $n(A \cup B) = n_o(A) + n(A \cap B) + n_o(B)$.

iii. If the elements of A and B are only in U, then $n(U) = n(A \cup B)$ only.

iv. If there are other elements also except the elements of A and B inside U, then

$n(A \cup B) = n(U) - n\overline{(A \cup B)}$ and $n(U) = n(A \cup B) + n\overline{(A \cup B)}$.

v. In the intersecting sets, $n_o(A) = n(A) - n(A \cap B)$ and $n_o(B) = n(B) - n(A \cap B)$

Example 1

In a survey of 500 population of a village, 325 people drink filtered water and 230 people drink boiled water. How many people drink both filtered and boiled water? Find it by using a Venn diagram.

Solution :

Let the set of total population, set of population who drink filtered water and set of population who drink boiled water be denoted by U, F and B respectively.

According to the question,

$n(U) = 500 = n(F \cup B)$, $n(F) = 325$ and $n(B) = 230$

The number of population who drink both kind of water, $n(F \cap B) = ?$

Let $n(F \cap B) = x$

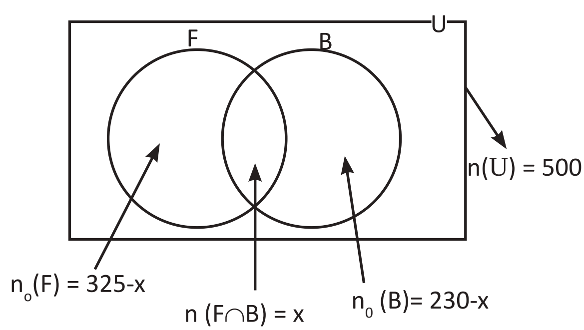
Now, according to the figure,

$n(F \cup B) = n_o(F) + n_o(B) + n(F \cap B)$

or, $500 = 325 - x + 230 - x + x$

or, $x = 55$

$\therefore\ n(F \cap B) = 55$



Hence, the number of people who drink both filtered and boiled water is 55.

Example 2

In a group of 120 students, 90 students study Mathematics and 72 students study Science. If 10 students do not study any one of the subject,

i. calculate the number of students who study at least one subject.

ii. calculate the number of students who study both the subjects.

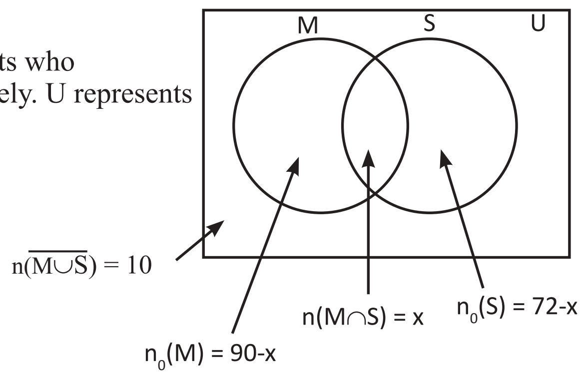
iii. how many students will be there who study Mathematics only?

iv. how many students will be there who study Science only?

Find it by drawing a Venn diagrams.

Solution :

Here, M and S represent the set of students who study Mathematics and Science respectively. U represents the set of all students.



So, $n(U) = 120$, $n(M) = 90$, $n(S) = 72$

and $n(\overline{M \cup S}) = 10$

Let $n(M \cap S) = x$

Now, according to the formula,

i. $n(M \cup S) = n(U) - n(\overline{M \cup S}) = 120 - 10 = 110$.

According to the Venn diagram,

ii. $n(M \cup S) = n_o(M) + n(M \cap S) + n_o(S)$

or, $110 = 90 - x + x + 72 - x$

or, $x = 162 - 110 = 52$

$\therefore$ $n(M \cap S) = 52$

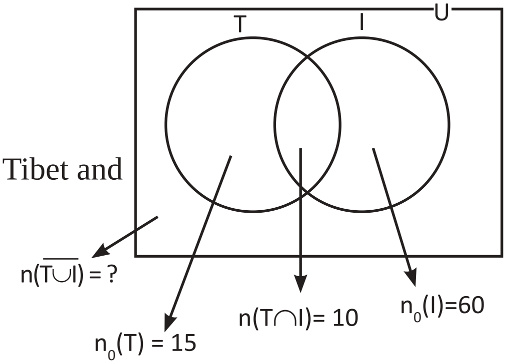
iii. Again, $n_o(M) = 90 - x = 90 - 52 = 38$

iv. Similarly, $n_o(S) = 72 - x = 72 - 52 = 20$

Example 3

In a survey of 100 Nepalese tourists, 15% of them visit Tibet only and 60 % of them visit India only. If 10% of them visit both the places, how many percent of tourists do not visit both the places? Find it by using a Venn diagram.

Solution,



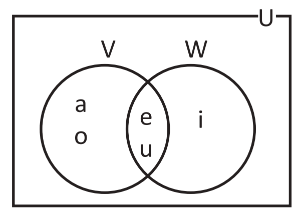
Here, let a set of all Nepalese tourist be U.
$\therefore$ n(U) = 100
Let us suppose the sets of tourists who visit Tibet and India be T and I respectively.

Then,
$n_o(T) = 15\%$ of $100 = 15$
$n_o(I) = 60\%$ of $100 = 60$
$n(T\cap I) = 10\%$ of $100 = 10$
and $n(\overline{T\cup I}) = ?$

According to the formula,
$n(T\cup I) = n_o(T) + n_o(I) + n(T\cap I) = 15 + 60 + 10 = 85$
$n(\overline{T\cup I}) = n(U) - n(T\cup I) = 100 - 85 = 15$

Hence, 15% of the tourists do not visit both the places. In the Venn diagram, 15% tourists lie out side of the union of sets I and T.

Exercise 1.2



1. In the given figure, find the values of n(V), n(W), $n_o(V)$, $n_o(W)$, $n(V\cup W)$ and $n(V\cap W)$.

2. If n(A) = 37, n(B) = 50 and $A\subset B$, find the value of $n(A\cap B)$.

3. In a survey of a market, 143 people use white tooth paste and135 use red tooth paste. If 70 people use both types of tooth pastes, find the number of people who use at least one type of tooth paste.

4. If U ={x : x is a positive integers less than or equal to 20},

   A ={y : y is a prime number},

   B ={z : z is a factor of 18} and

   C ={p : p is a multiple of 3 less then 20},

   find the cardinality of the following sets by drawing the Venn diagram :

   i. $n(A\cup B)$ ii. $n(B\cup C)$ iii. $n(A\cup B\cup C)$ iv. $n(A\cap B\cap C)$

   v. $n_o(A)$ vi. $n_o(B)$ vii. $n_o(C)$

5. According to a survey of 1000 families of any town in 2010 A.D., 794 families have radio and 187 families have television. If 63 families do not have any one of them, then how many families will have both radio and television? Find it.

6. Out of 300 students in a class, 60% students study Physics, 35% students study Chemistry and 20% students do not study both of the subjects.

   i. How many students study both subjects?

   ii. How many students study Physics only?

   iii. How many students study Chemistry only?

   iv. What is the number of students who study only one subject? Find it.

7. The students must take part in at least one of the activities among the extra curricular activities in a school. Out of 40 students of a class, 25 students take part in debate and 30 students take part in quiz contest, then

   i. show the above information in a Venn diagram.

   ii. how many students have taken part in both activities?

   iii. how many students have taken part in only one activity?

8. According to survey in 600 consumers of a village, 300 consumers are buying tea of own nation, 250 consumers are buying international tea and 150 consumers are buying both brands of tea. Find the number of consumers who do not buy any brand of tea.

9. In a ceremony, out of 800 participants, 620 drank milk, 350 drank tea and 50 did not drink any of them. Find out how many people drank the both drinks using a Venn diagram?

10. In a survey of 2000 people, 50% of them visited the country A and 40% of them visited the country B. If 15% of them do not visit both A and B, then find

   i. how many people visited both the countries?

   ii. how many people visit only one country?

   iii. Express the above statement in a Venn diagram.

Lesson

2 Profit and Loss

2.0 Review

Sabita bought a bag for Rs. 500 and sold it at the profit of 25%. At what price did she sell it? Kabir bought a watch for Rs. 4000 and sold it for Rs. 3200. What was the loss in percentage?

The profit and loss in any article is the certain percent amount increase or decrease of the cost price of the article. We have already calculated profit and loss in the previous classes by using the following formulae.

(a) Actual Profit = Selling price – Cost price

(b) Actual Loss = Cost price – Selling price

(c) Profit Percent = $\dfrac{\text{Actual profit}}{\text{Cost price}}$ X 100%

(d) Loss Percent = $\dfrac{\text{Actual loss}}{\text{Cost price}}$ X 100%

By using these formulae other many formulae can be derived. Explore as much as possible. How can we write the above formulae (a), (b), (c) and (d) if the actual profit, actual loss, cost price and selling price are denoted by P, L, CP and SP respectively.

2.1 Daily life problems related to profit and loss

The problems on profit and loss occur directly or indirectly in our daily life. These problems can be solved mathematically.

Activity 1 :

Divide yourself into groups of 4 to 6 students in the class. Every group make word problems related to profit and loss on your knowledge while buying things in the market. Exchange the problems among the groups.

Now, solve the problems on the basis of the following questions :

(a) Is it understandable the meaning of the question?

(b) What are the given informations in the problem?

(c) What is to be found out in the problem?

(d) Can you guess different possible methods of solving those problems using figure, formula, table to make easy to understand the problem. If yes, what can you do? Think, try to solve the problem by all possible methods. Ask the following questions after solving the problem by all possible methods :

i. Did you check every step of the solution?

ii. Is every step of solution correct?

iii. Can you prove or justify every step with reason?

At last ask the teacher whether the method is correct.

Study the following example to understand clearly the problem solving steps.

Bimal bought a doko (basket) at Rs. 300 and sold it at Rs. 390. How much percent profit did he make in the business of doko? Find it.

Solution :

Step 1 : Try to understand the problem and make a plan.

(a) What is the problem about? What is asked to do in the problem?

(b) Here, the given informations are cost price (CP) = Rs. 300

Selling price (SP) = Rs. 390

(c) In the problem, it is asked to find the profit percent while doing the business of doko.

(d) For understanding the problem easily it can be expressed in the table as follows :

Name of object	Cost price (CP)	Selling price (SP)	Profit percent
Doko (Basket)	Rs. 300	Rs. 390	?

(e) Use the formula, profit percent = $\frac{\text{Actual Profit}}{\text{Cost price}}$ X 100% to get the solution.

Step 2 : Solution of problem (apply the plan)

According to the formula,

$$\text{Profit percent} = \frac{\text{Profit amount}}{\text{Cost price}} \times 100\%$$

$$= \frac{\text{Selling price (SP) - Cost price (CP)}}{\text{Cost price (CP)}} \times 100\%$$

$$= \frac{390 - 300}{300} \times 100\%$$

$$= 30\%$$

Hence, Bimal made a profit of 30% in the business of doko.

Step 3 : To see back or check the solution.

(a) Is the information drawn from the problem correct?

(b) Is the chosen formula relevant to the problem?

(c) Is the calculation made is also correct?

Step 4 : Write the conclusion according to the question.

The above activity and example are related to the steps of the problem solving method. When we solve the questions given in the exercise by these steps, it becomes easy to solve the word problems.

Example : 1

Deepak has a stationary shop. He has bought one dozen copy at Rs. 360. He sold 2 copies at Rs. 62 after somedays. How many rupee does he get profit or loss in a copy? Find it.

Solution :

Here, the cost price of 12 copies = Rs. 360

Cost price of 1 copy = Rs. $\frac{360}{12}$

= Rs. 30

Again, the Selling price of 2 copies = Rs. 62

Selling price of 1 copy = Rs. $\frac{62}{2}$ = Rs.31

Here, the profit in one copy = Selling price - Cost price = Rs. 31 - Rs. 30 = Re. 1

Hence, Deepak has Re.1 profit in one copy.

Example : 2

China bought 250 kg mangoes at the rate of Rs. 40 per Kg. She paid Re. 1 per kg as the fare to bring mangoes. If she sold all mangoes for Rs. 12000, (a) at what price did she sell 1 kg. mangoes? (b) what percent profit or loss she got after selling all mangoes?

Solution :

Here, cost price of 250 kg mangoes (CP) = Rs. 40 x 250 = Rs. 10,000

The transportation fare of 1 kg mangoes = Re. 1

The transportation fare of 250 kg mangoes = Rs.1 x 250 = Rs. 250

Therefore, cost price of 250 kg mangoes = Rs. 10,000 + Rs. 250

= Rs. 10,250

Again, selling price of 250 kg mangoes = Rs. 12,000

(a) Selling price of 1 kg mangoes $= \text{Rs.} \frac{12000}{250} = \text{Rs. } 48$

Hence, the selling price of 1 kg mangoes is Rs. 48.

(b) Profit percent (P%) $= \frac{\text{Selling Price} - \text{Cost price}}{\text{Cost price}} \times 100\%$

$$= \frac{12{,}000 - 10{,}250}{10{,}250} \times 100\%$$

$$= \frac{1750}{10250} \times 100\%$$

$$= 17.07\ \%$$

Alternative method,

Here, cost price of mangoes = Rs. 40 /kg

And transportation cost = Re. 1/kg

Now, actual cost price of 1 kg mangoes (CP) = Rs. (40 + 1)

= Rs. 41

Again, Selling price of 250 kg mangoes = Rs. 12,000

Selling price of 1 kg mangoes (SP) $= \text{Rs. } \frac{12{,}000}{250}$

= Rs. 48

Hence, Profit Percent $= \frac{SP - CP}{CP} \times 100\%$

$$= \frac{48 - 41}{41} \times 100\%$$

$$= 17.07\ \%$$

Example : 3

Heera bought 3000 books. Out of them she gave 300 books to a school at free of cost. She sold the remaining books with 10% profit at the rate of Rs. 66 per book. What will be the cost price of a book? Find it.

Solution :

Here, suppose the cost price of a book is Rs. x.

Then, the cost price of 3000 books = Rs. $3000x$

Profit = 10% of cost price

$$= \frac{10}{100} \times 3000\,x$$

$$= 300\,x$$

Selling price = Cost price + profit

$$= 3000x + 300x = 3300x$$

But, she gave 300 books to the school at free of cost. So she sold only 2700 books.

The selling price of 2700 books at the rate of Rs. 66 per book = Rs. 66 x 2700.

Therefore, $3300\,x = 66 \times 2700$

or, $x = 54$

Hence, the cost price of one book is Rs. 54.

Note : The selling price of an article is more than one but cost price of it is always one and only one. Why? Discuss it.

Example : 4

There is a loss of 20% while selling a photocopy machine for Rs. 60,000. At what price will the machine be sold to gain 12%? Find it.

Solution:

Here, selling price of the photocopy machine = Rs. 60,000

Let cost price = Rs. x

Loss = 20% of cost price = $\frac{20}{100} \times x = x/5$

Selling price = Cost price - loss

or, $60{,}000 = x - \frac{x}{5}$

or, $60{,}000 = \frac{5x - x}{5}$

or, $60{,}000 = \frac{4x}{5}$

or, $\frac{60,000 \times 5}{4} = x$

or, $x = 75,000$

Here, Cost price of the photocopy machine = Rs. 75,000

Again, Profit = 12% of cost price

or, profit = $\frac{12}{100}$ x Rs. 75,000 = Rs. 9,000

Now,

Selling price = Cost price + Profit

= Rs. 75,000 + Rs. 9,000

= Rs. 84,000

Hence, the photocopy machine must be sold for Rs. 84,000 to earn the profit of 12%.

Exercise 2

1. Find the value of variables in the given table.

S.No.	Particulars/ Items	Cost price	Selling price	Profit/Loss	Profit/Loss percent
a.	Football	Rs. 500	Rs. 625	?	?
b.	Bag	Rs. 300	Rs. 270	?	?
c.	Sandle	Rs. 120	Rs. 144	?	?
d.	Pen	Rs. 50	?	?	15% (profit)
e.	Copy	?	Rs. 40	Rs. 10 (loss)	?

2.(a) Kapil bought 40 dolls for Rs. 4800. He sold all the dolls at the rate of Rs. 700 per 5 dolls after somedays. How much profit or loss did he get in one doll? Find it.

(b) There is total expenditure of Rs. 25000 to show a dance program. The dance team made a profit of Rs. 12,000 except the expenditure. If 370 people observed the dance by paying the money, how much money did a person pay for the show? Find it.

3.(a) A fruit seller bought 30 kg apples at the rate of Rs. 88 per kg from the market. He paid is Rs. 60 to bring the apples to his shop from the fruit market. He sold all apples for Rs. 3240. How much profit did he get in 1 kg apples? How much percent profit did he get by selling all apples? Find it.

(b) Newton has bought 12 dozen copies in Rs 4320. His expenditure to bring the copies to his shop is Rs. 288. How much profit does he make in one copy after selling all copies in Rs. 5760? Also how much profit in percent did he get in the business of copy? Find it.

4. (a) Rehaman bought 1200 copies at Rs. 60 per copy. He gave 300 copies out of these copies in the form of scholarship. If he sold the remaining copies at 5% loss, what was the selling price of each copy? find it.

(b) Parwati brought 60 glasses. 20 of them were broken. She sold the remaining glasses at the rate of Rs. 25 per glass making 20% profit. What was the cost price of one glass? Find it.

5. (a) A man has profit of 15% by selling a table at Rs 1035. If he sold the table at the loss of 8%, at what price would he sell it? Find it.

(b) Heera sold a pen at the loss of 15%. Had she sold it in Rs. 3 more, she should have 5% profit. What would be the cost price of the pen? Find it.

6. Find the cost price and selling price of copy, pen and books that you have asking with the nearby shopkeepers. Fill in the given table after finding profit or loss percent to obtain result from the discussion.

S.No.	Things	Cost price	Selling price	Profit/Loss	Profit/Loss percent

Lesson

3 Commission and Taxation

3.0 Review

What will be 13% of Rs. 12000?

What percent is Rs. 400 of Rs. 20,000?

10% of what amount will be Rs. 1000?

Generally a certain percent discount is offered in the price while buying things in the market. People spent a certain percent of their income and save the remaining amount. The fixed percentages of different gases are mixed in air. These and like other examples are related with the use of percentage in our daily life directly and indirectly. A citizen has to pay the certain percent of his/her income to the government. While purchasing things, certain amount is added or lessened on the selling price in the bill. Percent count is used in all these examples. We have already studied calculations using percentage in the previous classes.

3.1 Commission

Phulmaya is a farmer. She has 12 ropani land, one day her son was sick but she did not have money for the treatment of her son. She decided to sell 5 ropani land out of 12 ropanies. Jagat Bahadur is an excellent business man. He had a plan of vegetable farming and was in search of buying 5 ropani land. But he does not have time to find appropriate land for buying. Jagat Bahadur requested Sonam to find 5 ropani land for him. Sonam went to Phulmaya and made agreement to sell 5 ropani land of Phulmaya to Jagat Bahadur and take 5 % of the selling amount for himself. Here, Phulmaya is a seller, Jagat Bahadur is a buyer and Sonam is an agent.

> Agent takes a certain percent of selling amount that is called commission. The buyer, seller or both usually give the commission to the agent.

Generally, in business transactions of real-state, banking and commercial organizations, insurance companies and other daily economic trade and business, the provision of commission is in practice.

Activity :1

Among the present students in the classroom, divide yourself into group of 3/3 students. Assign the role of seller, buyer and agent to the students of each group. Fix the selling price and the rate of commission of the things which are sold in your local market in group discussion. Calculate the amount of commission and the selling price after commission by using the identified rate. Present the work of each group in the classroom.

The following formulas are used to solve the problem related to commission :

1. Amount of commission = Fixed commission percent of selling price.
2. The price after commission = Selling price - amount of commission.
3. Commission percent = $\dfrac{\text{Amount of commission}}{\text{Selling Price}} \times 100\%$

Example : 1

An agent gets 1 percent commission on the selling price Rs. 18000.

(a) How much amount does the agent get as the commission? Find it.

(b) How much amount does the sand business man get by selling the sand? Find it.

Solution :

Here, Selling price of the sand = Rs. 18000

Rate of commission = 1 %

Amount of commission = ?

Price after commission = ?

We know that,

Amount of commission = Selling price x Rate of commission

= Rs. 18000 x 1%

= Rs. 18000 x $\dfrac{1}{100}$

= Rs. 180

Price after Commission = Selling price - amount of commission

= Rs. 18000 - Rs. 180

= Rs. 17820

Hence, (a) the agent gets Rs. 180 as the commission.

(b) the sand businessman gets = Rs. 17820 by selling the sand.

Example : 2

The monthly salary of a staff of a clothing shop is Rs. 12,000. The shopkeeper gives him commission of 1% in a month on the selling amount that is of more than Rs. 6 lakh. If the total selling amount of the shop in a month is Rs. 10 lakh, what is the total income of the staff in the month? Find it.

Solution :

Here, Monthly salary of the staff = Rs. 12,000

Monthly Selling amount = Rs. 10,00,000

Selling amount in which commission is not given = Rs. 6,00,000

Rate of commission = 1%

Total monthly income of the staff =?

Selling amount in which commission is given = Rs. 10,00,000 - Rs. 6,00,000

= Rs. 4,00,000

Amount as commission = 1% of Rs. 4,00,000

$= \frac{1}{100} \times$ Rs. 4,00,000

= Rs. 4,000

Total income of the staff in that month = Salary + commission

= Rs. 12,000 + Rs. 40,00 = Rs. 16,000

Example : 3

The monthly salary of a staff who is working in a copy production company is Rs. 11,000. He earns the total amount of Rs. 21,000 in a month with commission. If the amount of total selling amount of copy is Rs. 5,00,000 in that month, what is the rate of commission? Find it.

Solution :

Here, the monthly salary of the staff = Rs. 11,000

Total monthly income = Rs. 21,000

Total selling amount of the month = Rs. 5,00,000

Rate of commission =?

Amount as the commission = Rs. 21,000 - Rs. 11,000

$= \text{Rs. } 10{,}000$

$$\text{Rate of commission} = \frac{\text{Amount as commission}}{\text{Total selling amount}} \times 100\%$$

$$= \frac{10000}{500000} \times 100\% \quad = 2\%$$

Hence, the rate of commission = 2% on the total amount of selling price.

Example : 4

A factory allows commission to a whole seller as 0.5 percent in the selling amount upto Rs. 10 lakh, 1 percent in the selling amount of Rs. 10 lakh to Rs. 15 lakh and 2 percent in the selling amount above Rs. 15 lakh. If the whole seller bought the things of Rs. 25,00,000 from the factory, how much amount as commission does he get? Find it.

Solution :

Here, total selling amount = Rs. 25,00,000

(a) Selling amount in which commission is 0.5 % = Rs. 10,00,000

Commission amount = 0.5 % of Rs. 10,00,000

$$= \frac{0.5}{100} \times 10{,}00{,}000$$

= Rs. 5,000

(b) Selling amount in which commission is 1 % = Rs. 10 lakh to Rs. 15 lakh

= Rs. 15,00,000 - Rs. 10,00,000

= Rs. 5,00,000

Commission amount = 1 % of Rs. 5,00,000

$$= \frac{1}{100} \times 5{,}00{,}000$$

= Rs. 5,000

(c) Selling amount in which commission is 2 % = above 15 lakhs

= Rs. 2500000 - Rs. 1500000

= Rs. 10,00,000

Commission amount = 2 % of Rs. 10,00,000

$$= \frac{2}{100} \times 10{,}00{,}000$$

= Rs. 20,000

Total amount as commission = Rs. 5,000 + Rs. 5,000 + Rs. 20,000 = Rs. 30,000.

Exercise 3.1

1. Find the amount of commission and the price after commission in each of the following items.

S.No.	Particulars (Items)	Total selling price	Rate of commission
a.	House	Rs. 80 lakh	3 %
b.	Printing machine	Rs. 1 crore	10 %
c.	ShareTransaction	Rs. 1 crore 20 lakh	2 %
d.	Land	Rs. 5 crore	5 %

2. (a) The monthly salary of a staff who is working in a cement shop is Rs. 8,000. He gets the commission of 1.5 percent in the selling of above Rs. 10 lakh in a month. If the selling of the cement is Rs. 24,00,000 in the month, what is his monthly income? Find it.

   (b) The monthly salary of a marketing representative who is working in a paper factor is Rs. 15,000. She gets 2% commission in the sale above Rs.30 lakh in a month. If the sale is Rs. 48 lakhs in a month, what is her monthly income of that month? And how much sale will be required so that her income is Rs. 59,000 in a month? Find it.

3. (a) The monthly salary of a staff working in a book shop is Rs. 13,000. He gets certain percent as commission in the sale amount. If the sale in a month is Rs. 10,00,000 and his total income is Rs. 28,000, find the rate of commission?

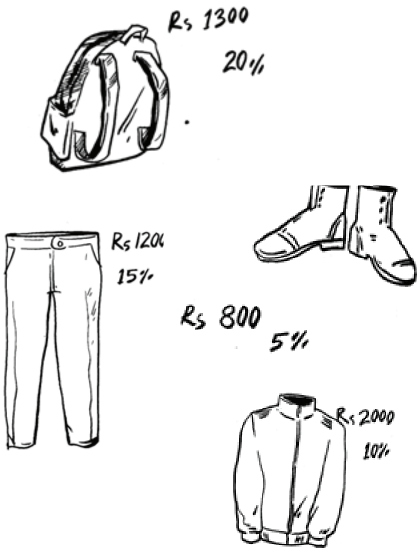
   (b) The monthly salary of a staff working in a doll shop is Rs. 7,500. She sold the dolls of Rs. 1,20,000 in a month. If she earned Rs. 13,500 in a month, find the rate of commission?

4. A company provides commission to its agent at the following rate.
   - 0.5 % in the sale upto Rs. 15 lakh.
   - 1 % in the sale from 15 lakh to Rs. 25 lakh.
   - 1.5 % in the sale from Rs.25 lakh to Rs. 40 lakh.
   - 2 % in the sale above Rs. 40 lakh

Calculate the amount of commission that can be obtained from the following sale amount by using the above rate of commission.

(a) Rs. 12 lakh (b) Rs. 24 lakh (c) Rs.38 lakh (d) Rs. 60 lakh

3.2 Discount



In the adjoining figure, different discount percent are mentioned in the things which are kept for selling in Ranjan's shop. At what amount the things can be purchased by reducing the price according to the mentioned discount percent? Discuss it. In business, the shopkeepers provide offer to sell the things in the less price than the marked price of the things to the customer in cash purchase. This offer amount is called the discount. The marked price is tagged on the things which is certain percent amount more in the cost price of the things. So that there is profit after selling that things. There are two types of discount which is cash discount and trade discount. We have study about cash discount only in class 9. The business people gives the offer of certain percent amount of the marked price in the things which are sold to the customer. This offer is called cash discount. The discount is offered on the basis of time, situation and type of things. The more amount as discount is given on the occasion of different festivals and the less amount as discount is given in general time.

Activity : 2

Lets divide few students of a class in a group of four students. Each group has to identify any five things with their market selling price which are available in the local area. The group are mentioned the discount percent in the things which are possible to offer. Try to create the situation so that the discount percent mentioned by one group is not known by another group. That four groups are named by four different shop. The students who are not in the group are made the list of the price of the things which can buy at what price in there shops. In which shop does every things get in cheaper price and high price? Find it and present in class.

The following formulae can be used to solve the problems related to discount :

Discount amount = certain percent of marked price

Selling price (price after discount) = marked price - discount amount

Discount amount = marked price - selling price (price after discount)

$$\text{Discount percent} = \frac{\text{Discount amount}}{\text{Marked price}} \times 100\ \%$$

Example 1

At what price can a book of marked price Rs. 450 be sold after allowing the discount of 10%? Calculate.

Solution :

Here, the marked price of the book = Rs. 450

Discount percent = 10%

Discount amount = 10% of Rs. 450

$= \frac{10}{100} \times$ Rs. 450

= Rs. 45

The price of book after discount (selling price) = Rs. 450 - Rs 45 = Rs. 405

Example 2

A shirt of marked price Rs. 500 can be bought at Rs. 400 after allowing discount. How much is the discount percent? Find it.

Solution :

Here, the marked price of the shirt = Rs. 500

The price after discount = Rs. 400

Discount amount = Rs. 500 - Rs. 400

= Rs. 100

Suppose, discount = x

Then,

Rs. 100 = x % of Rs. 500

or, $100 = \frac{x}{100} \times 500$

or, 100 = 5 x %

or, x = 20

So, discount percent = 20%

Alternative method

Here, Marked price = Rs. 500

The price after discount = Rs 400

Discount amount = Rs 500 - Rs 400

= Rs 100

Discount percent = $\frac{\text{discount amount}}{\text{marked price}} \times 100\%$

$= \frac{100}{500} \times 100\ \%$

= 20 %

Example 3

The marked price of a watch is 25% more than its cost price. If the watch is sold after allowing the discount of 15% , then the profit is Rs. 200. What will be the marked price of that watch? find it.

Solution :

Suppose, the cost price = Rs. x

Then, marked price = x + 25% of x

$$= x + \frac{25}{100}x = \frac{5x}{4}$$

Discount amount = 15% of marked price = 15% of $\frac{5x}{4}$

$$= \frac{15}{100} \times \frac{5x}{4}$$

$$= \frac{3x}{16}$$

Selling price = Marked price - discount amount

$$= \frac{5x}{4} - \frac{15x}{80}$$

$$= \frac{100x - 15x}{80}$$

$$= \frac{85x}{80} = \frac{17x}{16}$$

Profit = Rs. 200

or, Selling price - cost price = Rs 200

or, $\frac{17x}{16} - x$ = Rs. 200

or, $\frac{x}{16}$ = Rs. 200

or, x = Rs. 200 X 16

or x = Rs. 3200

$\therefore$ Marked price = $\frac{5x}{16} = \frac{5 \times 3200}{4}$

= Rs. 4000

Example 4

If a suitcase is sold by allowing the discount of 15%, there is profit of Rs. 250. If the suitcase is sold by allowing the discount of 30%, then there is loss of Rs. 500. Find the marked price of the suitcase.

Solution :

Suppose, the marked price of the suitcase = Rs x

Case 1, Discount percent = 15%

Then, discount amount = 15% of x = $\frac{15}{100} \times x = \frac{3x}{20}$

Selling price 1 = $x - \frac{3x}{20}$ [marked price - discount amount]

$= \frac{17x}{20}$

Profit = Selling price - cost price

or, 250 = $\frac{17x}{20}$ - Cost price

or, Cost price = $\frac{17x}{20} - 250$.............. (1)

Case 2, Again, discount amount = 30 % of the marked price.

Then, discount amount = 30 % of x

$= \frac{30}{100} x = \frac{3x}{10}$

Selling Price 2 = Marked price - discount amount

$= x - \frac{3x}{10} = \frac{7x}{10}$

Since loss = Rs. 500

or, Cost price - Selling price 2 = Rs. 500

or, $\frac{17x}{20} - 250 - \frac{7x}{10}$ = Rs. 500 [From equation (1)]

or, $17x - 5000 - 14x = 10000$

or, $3x = 15000$

or, $x = \frac{15000}{3}$

or, $x = 5000$

Hence, the marked price of the suitcase = Rs. 5000.

Exercise 3.2

1. Find the discount amount and the price after discount offered in each of the following condition.

S.No.	Name of items	Marked price	Rate of discount
a.	Sack of rice	Rs. 1300	5%
b.	Mustard oil	Rs. 80	2%
c.	Slipper (one pair)	Rs. 150	15%
d.	Book	Rs. 180	8%
e.	Pants	Rs. 700	20%

2. Find the discount amount and the rate of discount on the basis of the following information.

S.No.	Name of items	Marked price	Price after discount
a.	Shoes (one pair)	Rs. 400	Rs. 360
b.	Beaten rice (3 Kg.)	Rs. 300	Rs. 276
c.	Sugar (15 Kg.)	Rs. 1050	Rs. 1029
d.	Potatoes (30 Kg.)	Rs. 1200	Rs. 1188

3. (a) The marked price of a radio is 20% more than its cost price. The radio is sold to make the profit of 10%. If the profit is Rs. 80, what is the marked price? Find it.

   (b) The marked price of a cap is 25% more than its cost price. If the cap is sold at the discount of 10% , there is profit of Rs. 37.50. Find the marked price of the cap.

4. (a) There is a loss of Rs. 5 when a watch is sold at the discount of 10%. If the watch is sold at the discount of 5% , there is a profit of Rs. 22.50. Find the marked price of the watch.

   (b) if a mobile set is sold at the discount of 15%, there is profit of Rs. 100. If the mobile set is sold at the discount of 20%, there is loss of Rs. 800. Find the marked price of the mobile set.

5. What percent discount is given in the daily consumption goods in a shop near of your house? Make a table of goods price and discount. Present your result in the classroom.

3.3 Tax

The following questions are the curiosities of Dolma. Let's think about these curiosities.

(a) How does a government manage budget for administrative works, health, education sectors etc?

(b) The government collects some fund from its people, why? Every household has to pay land revenues or tax, why? The finance minister presents budget speech through different media that talks about taxes, why?

(c) What happens when a person's or the group of persons annual income is higher than that fixed by the government?

(d) We often hear 'Tax payer friendly tax administration, prosperity and good governance, our campaign' on radio and television. What does it mean?

These questions that have arisen in Dolma's mind are all related to tax. Tax is the money that a government or the government agencies collect from their people based on the laws, statute or constitution. Japan is the first country which initiated tax collection. Our government collects different types of taxes such as income tax, land revenue, property tax, labour tax, vehicle tax, value added tax and custom duty.

There are two types of taxes : Direct tax and Indirect tax. Direct tax is the kind of tax that is not transferable to others. The government of Nepal imposes direct tax on employment investment, property, real estate, land revenue, sales, house rent, sudden incomes etc. on contrary. The indirect tax can be transferred to others. The government of Nepal collects this kind of tax in the form of entertainment tax, value added tax etc. The person or institution does not pay such taxes but charges to the users.

Since Asoj, 2065, the government of Nepal started imposing tax on the fees collected by institutional schools. This tax is collected in the following ways :

(a) The institutional schools have to pay 1% of the total of admission fees and the monthly tuition fees as the tax.

(b) In case of the students who are going for abroad studies, 1% tax is imposed when they exchange foreign currency.

Here, we are going to study about income tax. The tax imposed by the government on the income of an individual or the group of people is called income tax. The government imposes certain percent of tax on the income amount that is more than the ceiling fixed by the government. In Nepal, the responsibility of managing tax is given to Inland Revenue Department.

According to the Inland Revenue Department, income tax is calculated by the following rule. Let's study this table.

For single person		For couple	
Title	Tax %	Title	Tax %
1. Upto Rs. 3,50,000 of the first taxable amount of the annual income.	1%	1. Upto Rs. 4,00,000 of the first taxable amount of the annual income	1%
2. The annual income of above Rs. 3,50,000 upto Rs. 4,50,000 only	15%	2. Above Rs. 4,00,000 as the annual income of upto Rs. 5,00,000 only	15%
3. Above Rs. 4,50,000 as the annual income upto Rs. 25,00,000 only.	25%	3. Above Rs. 5,00000 as the annual income upto Rs. 25,00,000 only.	25%
4. On above Rs. 25,00,000	35%	4. On above Rs. 25,00,000	35%

Source : http :\\ www.ird.gov.np

The concession on the property tax is given on the following headings :

(a) On the amount deposited at Employees provident fund.

(b) On the amount deposited at citizen investment trust.

(c) On the premium expenses paid as life insurance.

(d) On the donated amount.

(e) On the obtained amount as remote allowance.

(f) On the amount of 75 % of foreign allowance.

(h) On the expenses made for medical treatment etc.

The tax is charged on the remaining annual income amount after deducting the amount paid or obtained annually in the above titles.

Activity : 3

Make groups of four out of the present students in your classroom. Then discuss about any tax items which the group knows about like income tax, property tax, sales tax and internal tax. Present the result of discussion in the classroom. Ask to the teacher for more information.

Example : 1

Ram Awatar got a discount of 15% in a suitcase of price Rs. 12,000. How much did he pay the amount of tax at the rate of 13 % value added tax in the price after discount? Find it.

Solution :

Here, the marked price of suitcase = Rs. 12,000

Discount amount = 15 % of Rs. 12,000

$= \frac{15}{100} \times \text{Rs. } 12{,}000$

$= \text{Rs. } 1800$

The price after discount = Rs. 12,000 - Rs. 1,800

= Rs. 10,200

The amount of tax = 13 % of Rs. 10,200

$= \frac{13}{100} \times \text{Rs. } 10{,}200$

$= \text{Rs. } 1326$

Example : 2

600 students are studying in a private school. The average monthly fee of a student is Rs. 1200. How much amount for tax does the school pay to the government as education service tax at the rate of 1% in a year? Find it.

Solution :

Here, the average monthly fee of a student $= \text{Rs. } 1200$

The fee of 600 students $= \text{Rs. } 600 \times 1200$

$= \text{Rs. } 7{,}20{,}000$

The yearly fee of 600 students $= 12 \times \text{Rs. } 720{,}000$

$= \text{Rs. } 86{,}40{,}000$

The amount of tax $= 1\% \text{ of Rs. } 8640000$

$= \text{Rs. } 8640000 \times \frac{1}{100}$

$= \text{Rs. } 86{,}400$

Example 3

The monthly salary of an unmarried staff who is working in a bank is Rs. 75000. But 67% of his/her income is tax free. How much income tax does he/she pay yearly as per the rate which is given in the previous page.

Solution :

Here, monthly income = Rs. 75000

Yearly income = Rs. 75000 X 12

= Rs. 9,00,000

Tax free amount = 67% of Rs. 9,00,000

$= \frac{67}{100} \times$ Rs. 9,00,000

= Rs. 6,03,000

Tax payable income = Rs. 9,00,000 - Rs. 6,03,000

= Rs. 2,97,000

According to tax rate, tax will be charged on Rs. 250000

Tax amount = 1% of Rs. 2,50,000

= Rs. 2500

Remaining income = Rs. 2,97,000 - Rs. 2,50,000

= Rs. 47,000

Again, the amount of tax = 15% of Rs. 47000 = Rs. 7050

Hence the total tax = Rs.2500 + Rs.7050 = Rs.9550.

Exercise 3.3

1. Find the amount of tax on the price after allowing discount in the following items :

S.No.	Items	Marked price	Discount percent	Rate of tax
a.	Sticking machine	Rs. 12000	15%	13%
b.	Photocopy machine	Rs. 40000	10%	13%
c.	Computer	Rs. 60000	5%	13%
d.	Printer	Rs. 12000	8%	13%

2. The monthly fee and admission fee which is paid by a student studying in class 1 to 10 of a private school are given in the following table. How much total amount does the school pay as tax at the rate of 1% education service tax on the total annual amount of admission fee and monthly fees? Find it.

Grade	Number of students	Monthly fees	Admission fee (once in a year)
1	40	Rs. 1200	Rs. 3000
2	38	Rs. 1300	Rs. 3000
3	39	Rs. 1400	Rs. 3000
4	42	Rs. 1500	Rs. 3000
5	45	Rs. 1600	Rs. 3000
6	36	Rs. 1700	Rs. 3000
7	37	Rs. 1800	Rs. 3000
8	34	Rs. 1900	Rs. 4500
9	32	Rs. 2000	Rs. 5000
10	30	Rs. 2100	Rs. 5000

3. The monthly income amount and annual tax free amount of the persons who are working in different occupations are given in the following table. Find the amount of annual income and the amount of salary of a month after deducting the tax according to the table of income tax which is given in the previous page.

S.No.	Name	Monthly income	Tax free amount	Martial status
1.	Shivanarayan	Rs. 25,000	15 % of income	Unmarried
2.	Lal babu	Rs. 27,000	Rs. 60,000	Unmarried
3.	Pasang	Rs. 32,000	Rs. 1,08,000	married
4.	Himmat	Rs. 40,000	Rs. 2,00,000	married
5.	Laxmi	Rs. 45,000	33 % of income	married
6.	Salama	Rs. 18,000	10 % of income	Unmarried

3.4 Dividend

Smirti has invested in a bank. The bank provides her 5,000 shares. There are 1 lakh total number of shares in the bank. The price of each share is Rs. 100. The bank earned Rs. 1 Crore as a profit in a year and decided to distribute 20% dividend. Then how much amount did Smirti get for the dividend? Calculate it.

Here, total annual profit of the bank = Rs. 1 crore

$$20\ \%\text{ of Rs. 1 crore} = \frac{20}{100} \times \text{Rs. } 1,00,00,000$$

$$= \text{Rs. } 20,00,000$$

Again, profit of 100000 shares = Rs. 20,00,000

profit of 1 share = Rs 20,00,000/100000

= Rs. 20

Profit of 5,000 = Rs. 20 x 5,000

= Rs. 1,00,000

Therefore, Smirti gets Rs. 1,00,000 as dividend

The share holders of any company or profitable co-operative get certain amount of the profit amount which is earned by the company or co-operative. The amount which is obtained by the shareholders is called dividend.

The different banks and co-operatives of Nepal distribute the dividend every year by taking permission of central bank i.e. Nepal Rastra Bank. This dividend is provided as cash dividend and share dividend. Which companies provide the dividend? Discuss it.

Example 1

There are 20,000 shares at the rate of Rs. 100 each in a Maharudra Co-operative. That co-operative earned the profit of Rs. 2,00,000 in a year. If the co-operative decided to distribute 1% cash dividend to the shareholders from the total profit, how much dividend did each shareholder get? Find it.

Solution :

Here, total profit = Rs. 2,00,000

Total cash dividend = 1 % of Rs. 2,00,000

= 1/100 X Rs. 2,00,000

= Rs. 2,000

Again, total number of shares = 20,000

Therefore, the dividend of 20,000 shares = Rs. 2,000

Hence, the cash dividend of 1 share = Rs. 2,000/20,000

= Rs. 0.10

= 10 paisa

Exercise 3.4

1. Find the total dividend and dividend of each share from the following table ::

Company	Total Shares	Rate of Dividend	Total Profit
A	2,50,000	23%	Rs. 8,00,00,000
B	3,00,000	21.5%	Rs. 7,50,00,000
C	2,50,000	12%	Rs. 12,00,00,000
D	2,20,000	34.75%	Rs. 2,80,00,000
E	2,40,000	51%	Rs. 50,00,000

2. (a) There are 20,000 shares at the rate of Rs. 100 in Mayur Co-operative. Aakash has 500 shares in that co-operative. The co-operative earned Rs. 6,00,000 in a year and decided to distribute 10% cash dividend of total profit to the share holders. How much cash dividend did Aakash get? Find it.

(b) A hydropower company has 25,000 shares of Rs. 100 per share. That company earned Rs. 5,00,000 in each year. If the company decided to distribute 25% cash dividend, how much cash dividend did the shareholder who has 300 number shares get? Find it.

3. Find the provision of distribution of dividend by the bank, co-operative or the company who opened the shares in your surroundings. According to the company which provides the dividend, how many primary shares can be bought in Rs. 50 thousand and how much dividend do those share get in a year? Find it and present in the class.

Lesson 4

Household Arithmetic

4.0 Review

If 12 pens cost Rs. 360, what is the price of 1 pen? What will be the total amount if 10% fine is added to a charge of Rs. 500? What will be the amount after allowing 20 % discount in Rs. 300?

These questions are related with unitary method and percentage which we have already studied in previous classes. After getting the price of unit we can find the price of many things of same kind on the basis of it. Similarly in the condition of knowing the price of same kind, unit price can be deduced.

We can solve the above problems by the relation of the price of many articles of same kind = price of unit article X the number of articles. Similarly, we can find the required amount by using the addition, subtraction, multiplication and division operations after finding the amount of certain percent increasing and certain percent decreasing of any number. The unitary method and percentage are used while paying the monthly bill of electricity, water and telephone which are used in our houses.

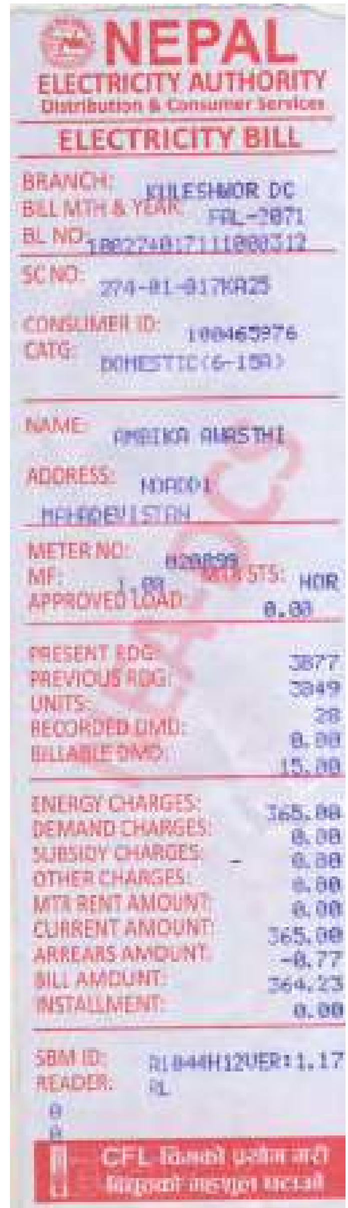
The customer can get information and pay the bill of electricity, water and telephone etc. from their houses by using information and communication technology (ICT). For example : to know about the amount of bill of PSTN, we can dial 1606 using the same telephone. The main objectives of introducing ICT is that to make customer used paying their bills through online but not in the queue (line).

4.1 House hold expenses for use of electricity :

Nepal Electricity Authority provides the bill according to consumption of electricity. The sample of bill that the Authority provides is shown in the following figure. The customers have to pay the bill in the near by branch of Nepal Electricity Authority office. The meter of the lower voltage level 220-400 volts connected to our homes. The capacity, minimum units and minimum charge to be paid is given in the following table.

Energy meter capacity	Minimum charge	Minimum unit
Up to 5 Ampere	Rs. 80	20
Up to 6 - 15 Ampere	Rs. 365	50
Up to 16 - 30 Ampere	Rs. 795	100
Up to 31 - 60 Ampere	Rs. 1765	200

Electricity bill is charged after consuming electricity according to the given bill in 2071 B.S.



NEPAL
ELECTRICITY AUTHORITY
Distribution & Consumer Services
ELECTRICITY BILL

BRANCH: KULESHWOR DC
BILL MTH & YEAR: FAL-2071
SC NO: 274-01-017KA25
CONSUMER ID: 100465976
CATG: DOMESTIC(6-15A)

NAME: AMBIKA AWASTHI
ADDRESS:

METER NO:
MF:
APPROVED LOAD: 0.00

PRESENT RDG:	3877
PREVIOUS RDG:	3849
UNITS:	28
RECORDED DMD:	0.00
BILLABLE DMD:	15.00
ENERGY CHARGES:	365.00
DEMAND CHARGES:	0.00
SUBSIDY CHARGES:	0.00
OTHER CHARGES:	0.00
MTR RENT AMOUNT:	0.00
CURRENT AMOUNT:	365.00
ARREARS AMOUNT:	-0.77
BILL AMOUNT:	364.23
INSTALLMENT:	0.00

SBM ID:
READER:

1. At the rate of Rs. 4 per unit upto 20 units.
2. Rs. 7.30 per unit from 21 to 50 units, after consuming 30 units, upto 20 units at the rate of Rs. 4 per unit and 21 to 30 units at the rate of Rs. 7.30 per unit has to be paid.
3. Rs 8.60 per unit rate from 5.1 to 150 units, at the rate of Rs. 7.30 per unit from the very first unit incase of 0 - 50 unit and Rs. 8.60 per unit rate from 51 - 150 unit bill is taken.
4. 0 - 150 units at the rate of Rs 8.60 per unit from the very first unit and Rs. 9.50 per unit from 151 - 250 units bill is taken.
5. At the rate of Rs. 11.50 per unit from 251 unit at above Rs. 9.50 per unit rate from 0 - 250 units and at the rate of Rs. 11.50 per unit from more than 250 units consumed bill is taken.

While paying the bill customer has the following kinds of discount and extra charge conditions :

1. 3 % discount if the bill is paid within 7 days of the me ter reading date but the total of Rs. 4 discount if the minimum bill is of Rs. 80.
2. Amount is taken according to the bill if it is paid from 8th day to 22nd day from the meter reading date.
3. 5 % extra charge is added in the amount of bill if the bill is paid from 23rd to 30th day from the meter reading date.
4. 10 % extra charge is added in the amount of bill if the bill is paid from 31st day to 40th day from the meter reading date.
5. 25 % extra charge is added in the bill amount if the bill is paid from 41st day to 60th days from the meter reading date.
6. Electricity line is cut off if the bill is not paid upto 60th day from the meter reading date.

The medium voltage level meter is fixed to conduct big factories and electricity bill also differs. We can get more information about it from the near by Nepal Electricity Authority office.

Example : 1

The details of the electricity bill of Ambika's house of 2071 Mangshir are as follows :

Capacity of meter	Domestic (6 – 15) A
Previous reading	3568
Current reading	3697

Find out the answer of the following questions on the basis of the above bill :

(a) How much unit of electricity has been consumed in the month of mangshir in Ambika's house?

(b) What was the tariff of the consumed units of electricity?

(c) How much amount the bill was paid on 5^{th} day from the meter reading date?

(d) How much amount she had to pay if the bill was paid on 18^{th} day from the meter reading date?

(e) How much extra charge she had to pay if the bill was paid on 24^{th} day from the meter reading date?

(f) How much extra charge she had to pay if the bill was paid on 35^{th} day from the meter reading date?

(g) How much total amount she had to pay if the bill was paid on 45^{th} day from the date of meter reading?

Solution :

Here, previous reading = 3568

Current reading = 3697

(a) Consumed unit = Current reading - previous reading

= 3697 - 3568

= 129

Hence, 129 units electricity is consumed in the month of Mangshir in Ambika's house.

(b) Here, the total 129 units electricity was consumed in Ambika's house.

According to energy fee (Single phase), the total amount of bill upto 50 units is paid at the rate of Rs. 7.30 per unit = 50 X 7.30 = Rs. 365.

The capacity of meter in her house is 6 - 15 Ampere, so the minimum charge can also be written as Rs. 365.

The amount of bill is calculated at the rate of Rs. 8.60 per unit from 51 unit to 150 units.

The total units from 51 to 129 units = 79 units.

The extra total bill amount of 79 units = Rs. 8.60 X 79 = Rs. 679.40.

Now, the total bill amount of 129 units (In the bill which she got at the meter reading day) = Rs. 365 + Rs. 679.40

= Rs. 1044.40

(c) 3 % discount is given if the bill is paid within 7 days from the date of meter reading. But she has paid the bill on 5^{th} day. Therefore, the total amount paid by her = Rs. 1044.40 - 3 % of Rs. 1044.40

= Rs. 1044.40 - Rs. 31.33

= Rs. 1013.07

(d) The amount is paid according to the bill if it is paid on 8^{th} day to 22^{nd} day from the date of meter reading. If she paid the bill on 18^{th} day, She had paid the total amount Rs. 1044.44. No discount or additional charge is added.

(e) 5 % extre charge has to be paid if the bill is paid on 23^{rd} to 30^{th} day from the date of meter reading. If she has paid the bill on 24^{th} day, she paid 5 % extra charge.

Therefore, the extra charge amount paid = 5 % of Rs. 1044.4

$= \frac{5}{100}$ x Rs. 1044.4

= Rs. 52.22

(f) Since 10 % extra charge has to be paid if the bill amount is paid on 31^{st} day to 40^{th} day from the date of meter reading. If she had paid the bill on 35^{th} day, she had paid 10 % extra charge.

Therefore, the extra charge paid = 10 % of Rs. 1044.40

$= \frac{10}{100}$ X Rs. 1044.40

= Rs. 104.44

(g) 25 % extra charge has to be paid if the bill amount is paid on 41^{st} day to 60^{th} day from the date of meter reading. If she paid the bill on 45^{th} day, she had paid 25 % extra charge in Rs. 1044.40.

Therefore, the total amount of bill paid = Rs. 1044.40 + 25 % of Rs. 1044.40

= Rs. 1044.40 + $\frac{25}{100}$ X Rs. 1044.40

= Rs. 1044.40 + Rs. 261.10

= Rs. 1305.50

Note : While calculating the billing amount of any month if decimal appears in the meter reading then the amount after the decimal is counted on the next month. For example; 10345 is only taken from 10345.67. The rest 0.67 unit is added in the next month.

Exercise 4.1

1. The meter reading of Heera's house from the month of Baishak to Ashoj is given in the following table. Write the answer of the given questions on the basis of following table.

Month	9^{th} Baishak	9^{th} Jetha	9^{th} Asar	9^{th} Sawan	9^{th} Bhadra	9^{th} Ashoj
Meter reading	1024	1099	1182	1284	1419	1484

   (a) What is the total unit consumed on the month of Jetha?

   (b) In which month electricity consumption is maximum?

   (c) In which month electricity consumption is minimum?

   (d) What is the total units of electricity consumed from Jetha to Ashoj?

2. A meter of capacity (6 - 15) A is fixed in Kamal's house. Find the total amount of tarrif deposited to Electricity Authority from Kamal's house for any one month in the given condition of current reading and previous reading of the meter.

   (a) Current reading : 23452, previous reading : 23272, the bill is paid on third day from the date of meter reading.

   (b) Current reading : 24903, previous reading : 24782, the bill is paid on fifteenth day from the date of meter reading.

   (c) Current reading : 27999, previous reading : 27819, the bill is paid on twenty eighth day from the date of meter reading.

   (d) Current reading : 30402, previous reading : 30154, the bill is paid on thirty fifth day from the date of meter reading.

   (e) Current reading : 32532, previous reading : 32337, the bill is paid on fourteenth day from the date of meter reading.

   (f) Current reading : 42873, previous reading : 42516, the bill is paid on fourty eighth day from the date of meter reading.

3. Collect the electricity bills of 6 months of your house and of your friend's house. Answer the following questions after studying the bill:

   (a) In which month electricity consumed is minimum? Why?

   (b) In which month electricity consumed is maximum? Why?

   (c) What short of techniques can be applied to consume less electricity?

4. How do people use ICT to pay electricity bill in your neighbors? What is the benefit of using ICT according to them? Discuss about your information in the class room.

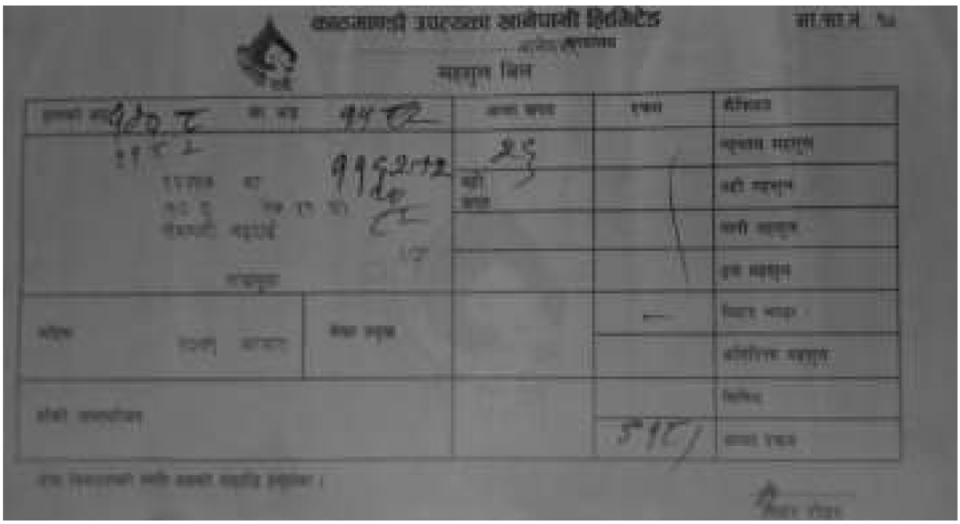
4.2 Household Expenses for Use of Water

Discuss about the following questions after studying the bill given below :

(a) How much is total unit of water consumed?

(b) How much is total amount in the bill?

The amount of the bill is calculated on the basis of the following rate :



Size of pipe (In inch)	Minimum consumption (In litre)	Meter installed pipe lines	
		Minimum price (Rs.)	On the basis of volume of water consumed (per litre) (Rs.)
½"	10,000	100	32
¾"	27,000	1910	71
1"	56,000	3960	71
1.5"	1,55,000	10950	71
2"	3,20,000	22600	71
3"	8,81,000	62240	71
4"	1,810,000	127865	71

In the given bill, the minimum 10,000 litre water or 10 units (1 unit = 1000 litre) water is consumed in 1/2 inch pipe. After that 16,000 litres (16 units) more water is used. The charge for minimum 10 units is Rs. 100 and the charge for extra 16 units = Rs. 32 x 16 = Rs. 512. The total amount for the total consumed 26 units = Rs. 100 + Rs. 512 = Rs. 612.

The total amount of bill with 50 % charge for drain service

= Rs. 612 + 50 % of Rs. 612 = Rs. 612 + Rs. 306 = Rs. 918.

The pipes of size more than 1/2 inch are used in big houses, hotels, hospitals, housings etc. The discount and extra charge in the bill of water supply are mentioned in the following table :

Paying (From date of bill provided)	**Discount/Extra charge**
Within first and second month	3 % discount
Within third month	Neither discount nor extra charge
Within fourth month	10 % extra charge
Within fifth month	20 % extra charge
After fifth month	50 % extra charge

Example 1

The meter reading of water in the month of Bhadra in Pradip's house is : previous reading 1340 and current reading 1372. How much amount of the bill has to be paid including 50 % of the additional amount for the drainage services? Find it.

Solution :

Here, current reading = 1372

Previous reading = 1340

Consumed units = 1372 - 1340 = 32

The price of minimum 10 units (10,000 litres) = Rs. 100

The price of remaining 22 units (22,000 litres) = Rs. 32 x 22

= Rs. 704

Total water bill = Rs. 100 + Rs. 704 = Rs. 804

The bill for drainage service = 50 % of Rs. 804

$= \frac{50}{100} \times \text{Rs. } 804 = \text{Rs. } 402$

Total amount of bill = Rs. 804 + Rs. 402 = Rs. 1206

Therefore, the amount of bill on the month of Bhadra was deposited by Pradip is Rs. 1206.

Example 2

The total 112 units of water is consumed through a pipe of size 3/4 inch in Karnali hotel. How much amount of the bill has to be paid if the bill was deposited on fifth month from the meter reading date? Find it.

Solution :

Total consumed unit = 112

The amount for minimum 27 units (27,000 litres) of water = Rs. 1910

Remaining units = 112 - 27 = 85 units.

The amount of 85 units (85,000 litres) of water = Rs. 71 x 85 = Rs. 6035

The amount of 112 units of water = Rs. 1910 + Rs. 6035 = Rs. 7945

The drainage service charge = 50 % of Rs. 7945

$= \frac{50}{100} \times$ Rs. 7945

= Rs. 3972.50

The amount of bill of water with drainage service = Rs. 7945 + Rs. 3972.50

= Rs. 11917.50

According to rule, 20 % extra charge is added if the bill is paid on the fifth month from the meter reading date.

So, the extra charged amount = 20 % of Rs. 11917.50

$= \frac{20}{100} \times$ Rs. 11917.50

= Rs. 2383.50

The total amount of the bill = Rs 11917.50 + Rs. 2383.50

= Rs. 14,301

Exercise 4.2

1. There is 1/2 inch water supply pipe line in Sunamaya's house. The current and previous reading in Ashoj are 2382 and 2345 respectively. Find the answer of the following questions on the basis of above given bill chart, discount and extra charge table.

(a) How much amount does Sunamaya pay with 50 % drain service charge?

(b) How much discount does Sunamaya get if she has to pay the bill within first and second month from the billing date?

(c) How much does she pay as the extra charge if she had paid the bill on the fourth month from the date of meter reading?

2. The water is supplied through 1 inch pipe line in the Park Village Hotel. A total of 560 units water is consumed in a month in that hotel.

   (a) Calculate the total billing amount if the bill is paid with 50 % drainage service charge.

   (b) How much should be paid if the bill is paid within the first month of meter reading date.

   (c) How much amount is paid if the bill is paid on 6^{th} month after meter reading date?

3. Collect the bill of three months of water supply either of your own house or from your neighbours. Find out the months when most and least volume of water was consumed.

4.3 Household Expenses for Use of Telephone

We use pre - paid and post paid telephone services these days. Pre - paid service using recharge card and post paid service using bill. Study the following PSTN/landline bill.


टेलिफोन विवरण
NEPAL TELECOM
NEPAL DOORSANCHAR COMPANY LTD.
NT PAN 300044614
PAGE NO
C. VAT
NO
DATE 67415107 STATEMENT FOR THE MONTH
2071/10/01 Poush , 2071
TEL NO. ACCOUNT CODE
NAME 15525925 A200309150484
ADDRESS NTC TEST NUMBER
CLASS PATAN
TYPE PERMANENT STATUS
ORDINERY LINE SERVICE

LOCAL CALL DETAILS

PREVIOUS	CURRENT	TOT CALL	RENTAL RS	EX CALL	TOTAL RS
1654	1654	0	200.00	0	200.00

CHARGE SUMMARY Rs.

LOCAL	
STD	200.00
ISD	0.00
NAT. OA	0.00
INT. OA	0.00
	0.00
ADJUSTMENT	
SUB TOTAL	0.00
TSC	200.00
TOTAL	20.00
VAT 13%	220.00
GRAND TOTAL	28.60
	248.60

This bill is of local call tariff. The tariff is according to the '' local call'' rules as follows:

Rs. 200 is charged for the minimum 175 calls and after that Re 1 per call extra amount is charged. It includes telephone service cost (TSC) and value added tax (VAT). 13 % VAT amount is deposited in Nepal Government Revenue Account. 10 % service charge is deposited in the company account which provides telephone service. In the above bill there is no call made through this telephone but minimum tariff equal to 175 calls is charged and has been paid.

Example : 1

The previous reading and current reading of a local call for a month of a simple telephone line are 2052 and 2276 units respectively. Answer the following questions on the basis of it.

(a) How many total calls were there in the month?

(b) How much amount of bill was for the telephone calls only?

(c) How much amount of bill is with 10 % telecome service charge?

(d) How much total amount of bill is with 13 % value added tax?

Solution :

Here, previous reading = 2052

Current reading = 2276

(a) Total consumed calls = Current reading - previous reading

= 2276 - 2052

= 224

(b) The minimum charge of first 175 calls for telephone = Rs. 200

Remaining calls = 224 - 175 = 49

Charge of 49 calls = Rs. 1 x 49 = Rs. 49

$\therefore$ The total amount for doing telephone = Rs. 200 + Rs. 49 = Rs. 249

(c) Telecome service charge = 10 % of Rs. 249

$= \frac{10}{100}$ x Rs. 249

= Rs. 24.90

The amount of bill with telecome service charge = Rs. 249 + Rs. 24.90

= Rs. 273.90

(d) Value Added Tax (VAT) = 13 % of Rs. 273.90

$= \frac{13}{100}$ x Rs. 273.90

= Rs. 35.60

Therefore, telephone bill with Value Added Tax = Rs. 273.90 + Rs. 35.60

= Rs. 309.50

Example : 2

For the first 175 telephone calls tariff is Rs. 200 and for additional calls in every Re. 1 is charged per call. Kapil has telephoned of Rs. 720 except telephone service charge and Value Added Tax. How many phone calls has he made? Find it.

Solution :

Here, 175 calls telephone can be made in minimum Rs. 200. So 520 additional calls telephone can be made at the rate of Re. 1 per call for the rest amount Rs. (720 - 200) = Rs. 520. Therefore, total number of calls = 175 + 520 = 695

Exercise 4.3

1. The previous reading telephone calls from Nima's house of Chaitra month is 3452 and current reading is 3789. Now calculate :
   (a) how many total phone calls have been made?
   (b) what is the total tariff of the calls if Rs. 200 for 175 calls and after that Re. 1 per call?
   (c) what will be the amount of bill on adding 10% telecome service charge?
   (d) the amount of bill on adding 13% value added tax in the bill.
2. (a) Minimum bill amount is Rs. 200 for 175 telephone calls and Re 1 per call is charged for additional calls. How many telephone calls can be done in Rs. 575? Find it.(excluding TSC and VAT)
   (b) Find out how many telephone calls can be done of Rs. 780 if minimum 175 telephone calls can be done in Rs. 200 and at the rate of Re. 1 per additional calls.
3. Find out the tariff of 650 telephone calls if 175 calls cost Rs. 200 and additional call is charged as Re.1 per call. What is the amount of the bill after adding 10 % telecome service charge with 13% value added tax? Find it.
4. How many telephone calls can be donefrom the following amount of money. If the first 175 calls made in Rs. 200 and after that each call at the rate of Re. 1? Find it. (Telecome service charge (TSC) and value added tax (VAT) are excluded.)
   (a) Rs. 250 (b) Rs. 425 (c) Rs. 775
5. Study the PSTN telephone bill of your own house or your neighbours from the last 6 month to 1 year. What difference do you find in the telephone bill? Wrie about it.
6. Discuss about the recharging process of telephone sets, TV and internet cable in your houses with your friends and prepare a report in groups.

4.4 Calculation of Taxi fares

There are two types of taxies in the city. One is with the meter installed and the other without any meter. We will study here about taxi with meter.

Nepal Bureau of Standards and Metrology (NBSM), Kathmandu fixes the meter for the convenience of public to use taxi. One should pay the taxi fare as the following :

Time	Minimum fare	Per 200 meter's fare
6 am to 9 pm	Rs. 14	Rs. 7.20
9 pm to 6 am	Rs. 21	Rs. 10.80

Extra charge of Rs. 7.20 is added after each 2 minutes in case of keeping the taxi in waiting during day time. The information about the taxi fare is given on the hoarding board of passenger's junctions like airpot, buspark, main chowks. One can complain about the inconvenience of taxi fare to the nearby traffic police station.

Example 1

Pemba took a taxi from Singhadurbar, Kathmandu to Kalanki for 5 km. At the beginning, taxi meter charged Rs. 14 as minimum and after that Rs. 7.20 per 200 meter. In the course of travelling 10 minutes waiting charge at the rate of Rs. 7.20 per 2 minutes was charged. How much fare did Pemba pay to the taxi? Find it.

Solution :

Here, the minimum charge = Rs. 14

The distance travelled by taxi = 5 km

= 5000 meter

Here, the fare of 200 meter = Rs. 7.20

The fare of 1 meter = $\frac{\text{Rs. } 7.20}{200}$

The fare of 5000 meter = Rs. $\frac{\text{Rs. } 7.20}{200}$ x 5000 = Rs. 180

The amount for waiting charge = $\frac{\text{Rs. } 7.20}{2}$ x 10 = Rs. 36

Therefore, Pemba paid total amount for fare = Rs. 14 + Rs. 180 + Rs. 36

= Rs.230

Exercise 4.4

1. (a) Hemraj used taxi for the distance of 7 kilometer from Singhadurbar, Kathmandu to Sanothimi, Bhaktapur. Taxi meter charged the minimum Rs. 14 at first and the fare at the rate of Rs. 7.20 per 200 meter. In the period of travelling, there was waiting time charged Rs. 7.20 per 2 minutes for 4 minutes waiting. How much amount did Hemraj pay for the fare? Calculate.

   (b) Aanand takes a taxi for 8 kilometer distance from Satdobato of Lalitpur to Kritipur of Kathmandu. The taxi driver charges the fair at first Rs. 14 as minimum charge and Rs. 7.20 for every 200 meter. The waiting charge of 12 minutes is charged at the rate of Rs. 7.20 per 2 minutes. How much total fair does Aanand pay? Find it.

2. Find the taxi fare according to the given rate in the previous page.

S.No.	Distance (km)	Time	Waiting time (minute)
a.	7	Morning 6 o'clock	4
b.	10	Afternoon 12 o'clock	12
c.	12.5	Night 10 o'clock	2
d.	4.6	Night 2 o'clock	–

3. (a) Ram Bahadur uses a taxi by paying Rs. 86. He pays the taxi fair by using the taxi fare of NBSM. What distance does Ram bahadur travel during day time? Find it.

(b) Manamati takes a taxi by paying Rs. 374. She paid the taxi Rs. 14 for minimum charge and Rs. 7.20 for each 200 meters. What distance did she travel? Find it.

Lesson 5

Area

5.0 Review

Divide the students of a class in group of 3 or 4 students. Distribute the paper in which one of the following topics is written to each group.

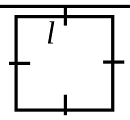
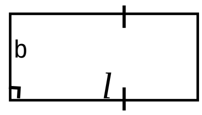
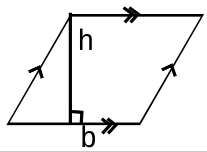
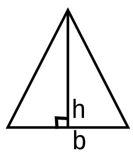
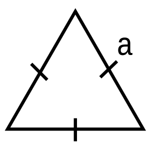
(a) Square (b) Rectangle (c) Parallelogram (d) Triangle

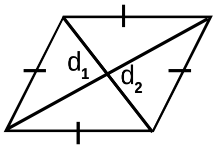
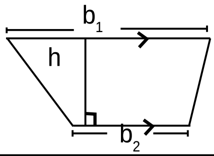
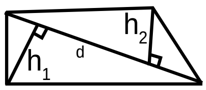
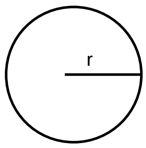
(e) Equilateral triangle (f) Scalane quadrilateral

(g) Trapezium (h) Rhombus (i) Circle

Let each group calculate the area and perimeter or circumferene of the plane shape of paper provided discussing in the group. Present the group work in classroom .

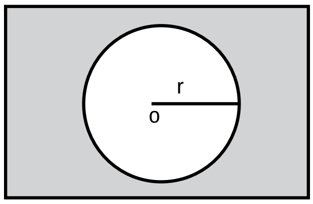
Now fill up the group work presented by the students in the following table :

Name of shape	Shape	Area	(Perimeter/ circumference)
Square		A = l^2 Square unit	P = $4l$ Unit
Rectangle		A =..........Square unit	P =2(l + b) Unit
Parallelogram		A =..........Square unit	P = ... Unit
Triangle		A =..........Square unit	P = ... Unit
Equilateral Triangle		A =.......Square unit	P = ... Unit

Rhombus		A =....... Square unit	P = ... Unit
Trapezium		A =.......... Square unit	p = Sum of the length of all four sides
Quadrilateral		A =.......... Square unit	p = Sum of the length of all four sides
Circle		A= πr^2 Square unit A= $\pi d^2 \div 4$ Square unit	Circumference (C) = $2\pi r$ = πd

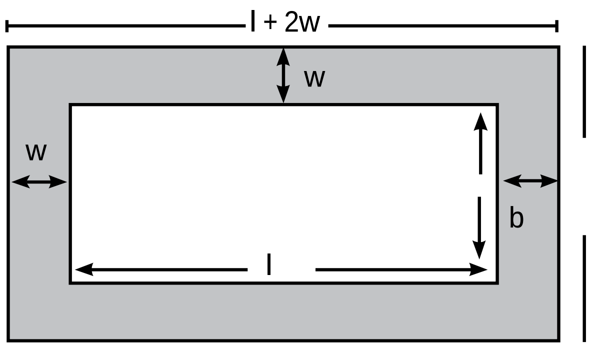
5.1 Area of Pathways

There is a rectangular field. A circular cricket ground will be constructed inside the field and the herb (dubo) will be planted in the remaining part of the field. Now, how can you find the area of the field where herb is planted? Present the conclusion after discussion in the group.



A. Outer path around

There is a rectangular garden length and breadth of which are *l* and *b* respectively. If a path of width w runs around outside of the garden, how can you find the area of that path? For this case, first of all we have to be clear about the shape of the garden that is rectangular and length and breadth of which are *l* and *b* respectively. Similarly, the length and breadth of the rectangular garden with path are (l + 2w) and (b + 2w) respectively.



Therefore, the area of the path (A) is said to be the difference of area of the garden with path (A_1) and the area of the garden without path (A_2).

Now, $A_1 = (l + 2w)(b + 2w)$ and $A_2 = l \times b$.

Therefore, area of the path (A) $= A_1 - A_2 = (l + 2w)(b + 2w) - l \times b$

$= l \times b + 2lw + 2bw + 4w^2 - l \times b$

$= 2w(l + b + 2w)$ square unit.

Hence, the area of the path (A) $= 2w(l + b + 2w)$ square unit.

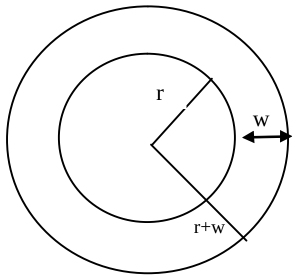
Note : If the shape of the field is square, what will be the area of the path?

In this case, $l = b$. So, the area of the path $= 2w(l + l + 2w) = 4w(l + w)$

$= 4w(l + w)$ square unit.

Again, if a circular path of wide (w) unit runs outside of a circular field, what will be the area of the path? Find it after observation of the adjoining figure.

Here, the radius of big circle is (r+w) and the radius of small circle is r.



We know that, area of a circle $= \pi r^2$.

Therefore, the area of the circular path is the difference of the area of big circle and area of small circle.

Area of the path (A) $= \pi(r + w)^2 - \pi r^2$ square unit

$= \pi(r^2 + 2rw + w^2 - r^2)$ square unit

$= \pi(2rw + w^2)$ square unit

$= \pi w(2r + w)$ square unit

In case, the diameter of the field is d, then the area of the path $= \pi w(d + w)$ square unit.

Example 1

A 2m wide path is constructed from outside around a rectangular field of length 20m and breadth 14m. Find the area of the path.

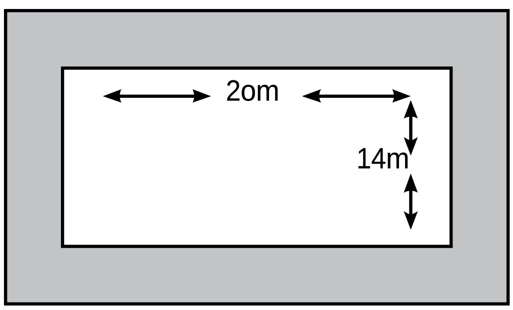
Solution :

Here, the length of the field (l) = 20m

Breadth of the field (b) = 14m

Width of the path (w) = 2m

Area of the path (A) =?



We know that, area of the outer path (A) = $2w(l + b + 2w)$ square unit

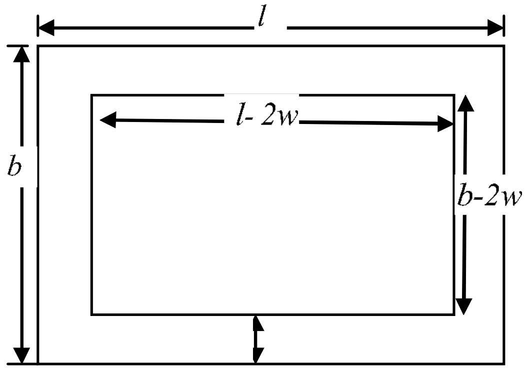
$= 2 \times 2(20 + 14 + 2 \times 2)\ m^2$

$= 4(20 + 18)\ m^2$

$= 4 \times 38\ m^2$

$= 152\ m^2$

B. Area of Inner Paths



A path of width (w) is constructed inside a park throughout the edges of the park having length (l) and breadth (b), what will be the area of the path?

Here, the length and breadth of a rectangular part inside the park except the constructed path are (l-2w) units and (b-2w) units respectively.

Therefore, area of inner path (A) = area of outer rectangle - area of inner rectangle

$= l \times b - (l - 2w)(b - 2w)$

$= lb - lb + 2wl + 2wb - 4w^2$

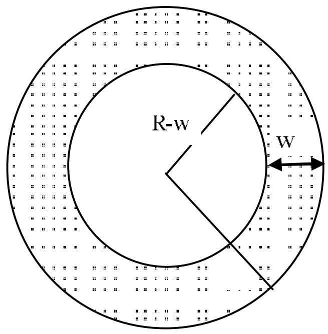
$= 2w(l + b - 2w)$ square unit

Note : If the shape of the land is square then $l = b$.

Therefore, area of the path (A) = $2w(l + l - 2w)$

$= 2w(2l - 2w)$

$= 4w(l - w)$ square unit.



Again, if a path of width (w) unit is constructed inside a circular land, then the area of the path can be found from the given figure.

Area of outer circle $= \pi R^2$ square unit

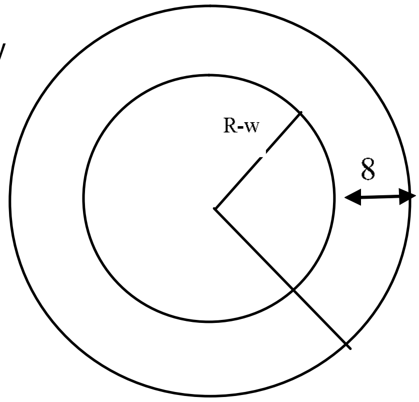
Area of inner circle $= \pi (R-w)^2$ square unit

Therefore area of the path = $\pi R^2 - \pi (R-w)^2 = \pi w(2R-w)$ square unit.

If the diameter (d) of the land is given, then area of the path (A) = $\pi w(d-w)$ square unit.

Example 2

A path of width 8 meter is made around a circular play ground of radius 70 meter from inside. What will be the area of that path? FInd it. (π = 3.14)



Solution :

Here, radius of the circular play ground (R) = 70 m.

Wide of the path (w) = 8m

Area of the path (A) =?

We know that ,

Area of inner circular path (A) = $\pi w(2R-w)$ square unit

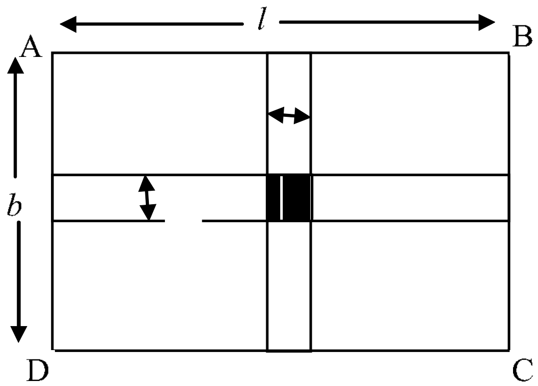
$= 3.14 \times 8 (2 \times 70 - 8)$ m^2

$= 25.12 \times 132$ m^2

$= 3315.84$ m^2

C. Area of the paths which cross each other perpendicularly

In the given figure, ABCD is a rectangular field of length AB = l unit and breadth BC = b unit. The cross paths of width w unit are made at the middle of it from both sides. Now, what will be the area of the path?



Here,

Area of the path which is made in its length = l x w square unit.

Area of the path which made in its breadth = b x w square unit.

Area of the square part where the paths cross each other = w^2 square unit.

Now,

Area of the path (A) = $(l \times w + b \times w - w^2)$

$= w(l + b - w)$ square unit.

Example 3

A path of width 5 m is made at the middle of both sides of a field having length of 70m and breadth 65 m. What will be the area of the path? Find it.

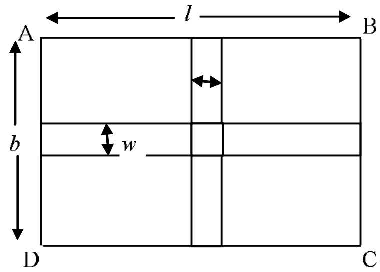
Solution :

Here, length of the field (l) = 70 m

Breadth of the field (b) = 65 m

Width of the path (w) = 5 m

Area of the path (A) =?



We know that,

Area of the cross path (A) = w(l + b - w) square unit

$= 5(70 + 65 - 5)\ m^2$

$= 5 \times 130\ m^2$

$= 650\ m^2$

D. Cost of Paving Paths

The tiles are needed to pave on the path of a house. How many tiles are required for that purpose? What will be the total cost for that work? How can we estimate the number of tiles and their cost? Discuss on the given questions in group. Present the conclusion of the group in the classroom and exchange feedback one another in the group.

The above discussion can be generalized in the following way :

First of all it is necessary to find the area of the path (A).

After that, find the area of each tile (a).

Then, the area of the paths (A) is divided by the area of each tile (a).

Total number of tiles (N) = Area of the path (A) / area of each tile (a).

$= A/a$

Total expenditure of the tiles = N x Cost of each tile.

Example 4

A path of width 10 ft. is made around a square field of edge 70 ft. from inside for the purpose of housing.

(a) Find the area of the path.

(b) How many stones will be required to pave on that path where the length and breadth of each stone are 2 ft. and 1.5 ft. respectively? Find it.

(c) If the cost of a stone is Rs. 105, find the total cost for paving stones on the path.

Solution :

Here, length of the square field (l) = (b) = 70 feet

Width of the path (w) = 10 feet

Area of the path (A) =?

(a) Area of the path (A) = 4w (l - w) square unit

$= 4 \times 10\ (70 - 10)\ ft^2$

$= 40\ (60)\ ft^2$

$= 2400\ ft^2$

(b) Length of a stone = 2 feet

Breadth of the stone = 1.5 feet

Area of a stone (a) = $2 \times 1.5\ ft^2$

$= 3\ ft^2$

Total number of stones (N) = $\frac{A}{a} = \frac{2400}{3} = 800$ pieces

Hence, the total 800 stones are required to pave the stones on the path.

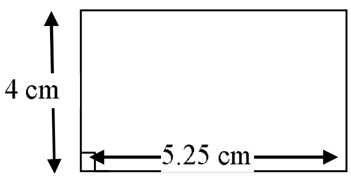
(c) The cost of paving a stone = Rs. 105

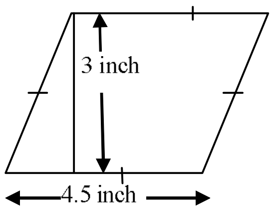
The cost for paving 800 stones = Rs. 105 x 800

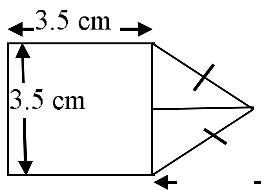
= Rs, 84,000

Exercise 5.1

1. Find the area and perimeter of the given plane figures :

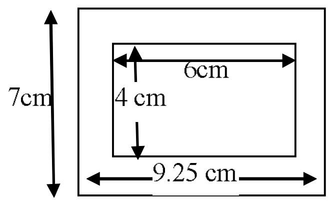




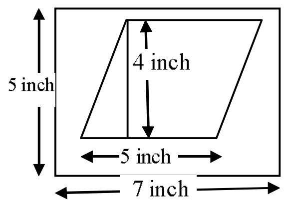


2. Find the area of the shaded part of the following figures.

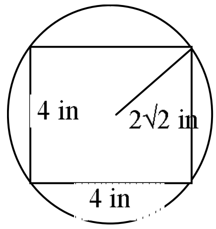
a)



b)



c)



3. The length and breadth of a rectangular field are 38 m and 32 m respectively. A path of 3 m wide is made around the field from outside. Find the area of the path.

4. A path of width 5 ft. is made around a square ground having edges of 60 ft from inside. Find the area of the path.

5. There is 3m wide path around a circular cricket ground having the diameter of 137 m. Find the area of the path.

6. White colour is painted in 2 ft. wide which lies inside from both sides in a basketball court of length 78 ft. and breadth 46 ft. Find the area of the painted part.

7. There is a path of width 2.5 m inside around a square garden of length 45m.

   (a) Find the area of the path.

   (b) How many tiles will be required to pave in the path by the square tiles of length 0.5m? Find it.

8. The paths of width 3 m are made perpendicular to each other at the mid-point from the both sides of a rectangular ground of length 50 m and breadth 40 m. Find the area of the paths.

9. A 2m wide path is made around a square pound of area 6400 m^2.

   (a) Find the area of the path.

   (b) How many tiles are required to pave on the path by the tile of area 0.40 m^2? Find it.

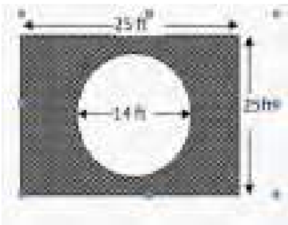
   (c) If the cost of a tile is Rs. 7.50, how much total cost will be required to pave the tiles on the path? Find it.

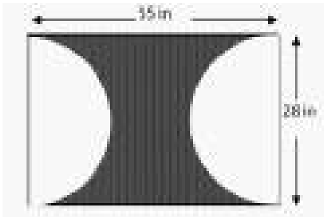
10. The silver carving of width 4 cm is made around outside of the frame of a photo of 24 cm x 28 cm.

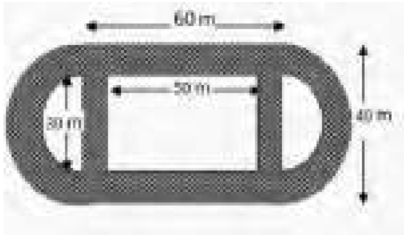
    (a) Find the area of the silver part.

    (b) If the cost of 1 cm^2 silver carving is Rs. 550, what will be the total cost required for the whole carving? Find it.

11. Find the area of the shaded part of the following figures.







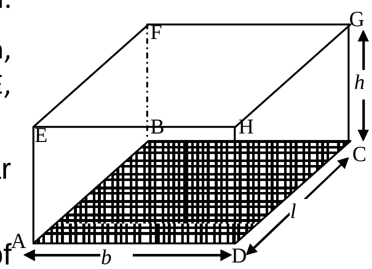
12. Measure the paths which cross each other or are parallel to each other in a garden or play ground of your school and find their areas.

13. How many walls are there in your classroom? What are the shapes of the floor and ceiling in your classroom? How can you find the area of each shapes? Prepare the answers individually and discuss in your group. Present the conclusion of group in the class.

5.2.1 Area of the Four Walls, Floor and Ceiling

There are four walls, one floor and one ceiling in a rectangular room. The opposite sides of a rectangle are equal. Therefore, the opposite walls of a rectangular room are equal and congruent. The area of the floor and ceiling are also equal.

Let us suppose, the adjoining figure is a model of a room, ABCD is the floor and EFGH is the ceiling. Similarly, ABFE, BCGF, CDHG and ADHE are four walls.



Area of four walls = Sum of the area of four rectangular surface walls

= area of ABFE + area of BCGF + area of CDHG + area of ADHE

= l x h + b x h + l x h + b x h

= 2 l x h + 2 b x h = 2 lh + 2 bh

= 2h (l + b) square unit

Area of floor and ceiling = area of ABCD + area of EFGH

= l x b + l x b = 2(l x b) square unit

Area of four walls, floor and ceiling = 2h (l + b) + 2(l x b) square unit

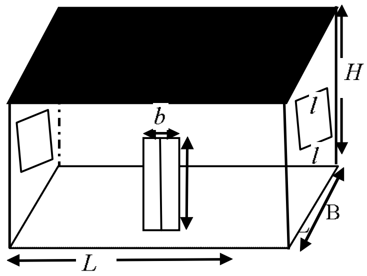
= 2(lh + bh + lb) square unit

Similarly to find the area of four walls excluding door and window.

(a) Find the area (A) of four walls.

(b) Find the total area (a) of the doors and windows and

(c) Subtract the area of the door and window from the area of four walls i.e. A - a



Example 1

A room is 5.5 meter long, 3.5 meter broad and 4 meter high. Find

(a) area of four walls (b) area of the floor and ceiling.

Solution :

Here, length of the room (l) = 5.5 m

Breadth of the room (b) = 3.5 m

Height of the room (h) = 4 m

We know that

(a) Area of four walls of the room = 2h (l + b) square unit

= 2 x 4 (5.5 + 3.5) square meter

= 8 (9) square meter

= 72 square meter

(b) Area of the floor and ceiling = 2 (l x b) square unit

= 2 (5.5 x 3.5) square meter

= 2 (19.25) square meter

= 38.50 square meter

Example 2

The length, breadth and height of a room are 14 ft, 10 ft and 9 ft respectively. There are two square shaped windows having an edge of 3 ft. and a door of 6 ft x 2 ft in the room. Four walls of the room are painted, find the area of the painted walls.

Solution :

Here, length of the room (L) = 14 ft, breadth (B) = 10 ft and height (H) = 9 ft

Length of the window (l) = 3 ft , height of the door (h) = 6 ft and

Breadth of the door (b) = 2 ft

We know that,

Area of four walls of the room (A) = 2H (L + B) square unit

= 2 x 9 (14 + 10) square feet

= 18 (24) square feet

= 432 square feet

Now,

Area of two square windows and a door (a) = (2 x l^2 + h x b) square unit

= (2 x 3^2 + 6 x 2) square feet

= (18 + 12) square feet

= 30 square feet

Again,

Area of the painted part = area of four walls (A) - area of the windows and door (a)

= (432 - 30) square feet

= 402 square feet.

5.2.2 Cost of Carpeting, Plastering, Painting, Papering etc.

How can we keep the carpet of length (l) and breadth (b) on the floor of a room having length (L), breadth (B) and height (H)? Find it after discussion in the group.

Here, the length , breadth and height of a room are L, B and H respectively. Similarly, the length and breadth of the carpet are l and b respectively.

To keep the carpet on the floor, the area of the floor and the area of the carpet must be equal. Therefore, L x B = l x b.

Now, how can we find the total cost (C)? Think it. The rate of cost for the area of one square unit is R, then the total cost of the area of A square unit is A x R.

If the value of any two quantities among three variables C, A and R are known then we can find third one by using following relation.

C = A x R; $A = \frac{C}{R}$ and $R = \frac{C}{A}$.

Example 3

There is a room of length 7m , breadth 5m and height 4m. There are two circular windows of radius 1.4m. and a door of height 2.5m. and breadth 1.4m. in the room.

(a) If the cost of per square meter of carpet is Rs. 250, what will be the total cost for carpeting on the floor of the room? Find it.

(b) Find the total cost estimation for the painting on the four walls at the rate of Rs. 175 per square meter. (π = 22/7)

Solution :

Here, Length of the room (l) = 7m.

Breadth of the room (b) = 5m.

Height of the room (h) = 4m.

Radius of circular window (r) = 1.4m.

Height of the door (h') = 2.5m.

Breadth of the door (b') = 1.4m.

(a) We know that ,

Area of the floor of the room = l x b square unit

$= 7 \times 5m^2$

$= 35m^2$

Now, the cost for carpetting is Rs. 250 per square meter.

So, the total cost for $35m^2$ carpet = Rs. 250 x 35

= Rs. 8750

Hence, the total cost for carpeting (c) = Rs. 8750

(b) Area of four walls of the room (A) = 2h (l + b) square unit

$= 2 \times 4\ (7 + 5)m^2$

$= 8\ (12)m^2$

$= 96m^2$

Area of circular window = πr^2 square unit

$= 22/7 \times (1.4)^2\ m^2$

$= 22/7 \times 1.96m^2$

$= 22 \times 0.28m^2$

$= 6.16m^2$

Total area of two windows = $2 \times 6.16m^2$

$= 12.32m^2$

Area of a door = b' x h' square unit

$= 1.4 \times 2.5m^2 = 3.5m^2$

Total area of two windows and a door (a) = $(12.32 + 3.5)m^2$

$= 15.82m^2$

Again, area of four walls without windows and door = Area of walls for painting

= (A - a) square unit

$= (96 - 15.82)m^2$

$= 80.18m^2$

The cost for painting in one square meter is Rs. 175. So, the total cost for painting in $80.18m^2$ = Rs. 175 x 80.18

= Rs. 14031.50

Example 4

Kopila wants to paint her room having length, breadth and height 26 ft, 22 ft and 8 ft respectively.

(a) If 96 square ft of area is painted with a packet of paint, how many packets of paint does she require to paint in the four walls of the room?

(b) The cost of each packet paint is Rs. 1230, how much total amount does she require? Find it.

Solution :

Here, length of room (l) = 26 ft.

Breadth of room (b) = 22 ft.

Height of room (h) = 8 ft.

We know that ,

(a) Area of four walls of the room = part of painting area

$= 2h\ (l+b)\ ft^2$

$= 2 \times 8\ (26 + 22)\ ft^2$

$= 16\ (48)\ ft^2$

$= 768\ ft^2$

Again, for 96 square feet area 1 packet paint is sufficient.

For 1 square feet area 1/96 packet is sufficient.

For 768 square feet area 1/96 x 768 packets paint is sufficient.

= 8 packets paint is sufficient.

(b) It is given that the cost of one packet paint is Rs. 1230.

Therefore, Kopila needs for the painting,

= Rs. 1230 x 8 = Rs. 9,840

Exercise 5.2

1. Solve the following problems.

   (a) What will be the total cost for carpeting a room of length 9 ft. and breadth 6 ft. at the rate of Rs. 125 per square ft? Find it.

   (b) What will be the total cost for papering on the ceiling of a classroom 15 m. length and 10 m breadth at the rate of Rs. 34.50 per square meter? Find it.

   (c) What will be the total cost for tiling on the walls upto 4 ft. high of a kitchen having dimensions 10 ft. x 8 ft. at the rate of Rs. 350 per square feet? Find it.

   (d) The height of a house having three rooms of the dimension 9 ft. x 12 ft. , 14 ft. x 16 ft. and 12 ft. x 14 ft. respectively is 9 ft. What will be the amount required to paint on all walls from inside at the rate of Rs. 160 per square feet? Find it.

2. The area of four walls of a room is 450 m^2. If the length and breadth of the room are 15 m and 12 m respectively, find its height.

3. The inner length, breadth and height of a hall are 50m, 45m and 4m, respectively. There are five windows of 2.5m x 2m and two doors of 2m x 3m.

   (a) Find the area of floor and ceiling of the hall?

   (b) Find the area of four walls without door and windows?

   (c) Four carvings are made in each square meter in the ceiling. How many carvings altogether will be there? Find it.

   (d) How many tiles or stones will be required to pave the floor if 8 tiles or stones are paved in each square meter of the floor? FInd it.

   (e) How much amount will be required to plaster on the four walls at the rate of Rs. 315 per square meter? FInd it.

4. There is a door of dimension 2m x 1.5m and two windows of dimension 1m x 1.5m in a room having length 7m and breadth 5m. If the total cost for painting on the four walls at the rate of Rs. 7.50 per square meter is Rs. 495, find the height of the room.

5. If square shaped turfs (*Dubo ko chapari*) having 30cm in length are placed on a 18m X 9m sized volleyball court, then find out:

   (a) how many turfs will be required in total?

   (b) if the cost of one turf is Rs. 225 then what will be the total cost for turfing the court?

6. Take the measurement of your bed room.

   (a) Find the area of four walls, floor and ceiling of the room

   (b) How much total expenditure is required to paste papers on the four walls at the rate of Rs. 25 per square unit? Find it.

   (c) How much amount is required for making inner ceiling on the ceiling of your room at the rate of Rs. 275 per square unit? Find it.

   (d) How much money is required for the carpeting on the floor at the rate of Rs. 166 per square unit? Find it.

7. Divide the students of class in the group of 5/5 students. Measure the length, breadth and height of the classroom where you are studying and also measure the length and breadth of the door and window. Find the solution of the following problems in the group.

   (a) Find the area of the floor and ceiling.

   (b) Find the area of four walls.

   (c) Find the area of four walls excluding door and windows.

   (d) Find the total cost required for painting on the four walls and ceiling at the current market rate.

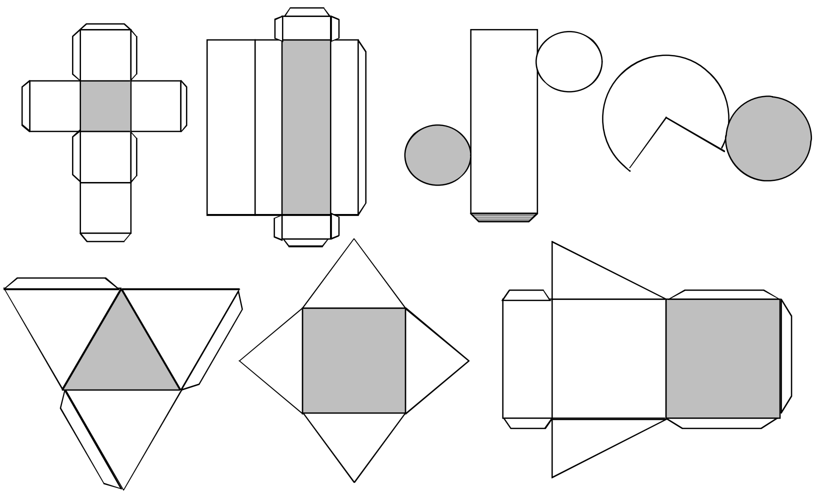
   (e) Find the total cost required for plastering on the floor and four walls after finding the rate of plaster per square unit.

Lesson 6

Solids

6.0 Review

Lets us cut off the given figures and fold them as marked. Let us discuss on the following questions.



(a) What solid models can be made from these nets?

(b) How many faces are there in each solid?

(c) How many vertices are there in each solid?

(d) What difference do you find after and before folding?

(e) What difference do you find in the shape of these solids?

Prepare a table to present the conclusion of above discussion.

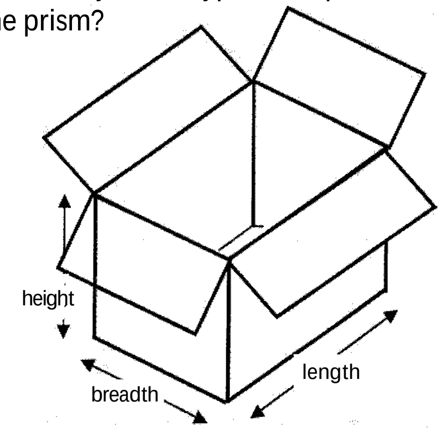
6.1 Surface Area and Volume of a Prism

What is a prism? How does its shape look like? Let's draw and present the figure of prisms in the class. We have studied about the area and volume of cuboid and cubes in the previous classes.

In this chapter, we will learn about the total surface area, cross - sectional area, lateral surface area and volume of different type of prisms.

6.1.1 Total Surface Area of Prism

Take a chalk or inkpot box or a cartoon. Open its faces slowly. What type of shape does it form? How many congruent surfaces are there in the prism? Point them out.



Measure its side and calculate area of each face.

If length (l), breadth (b) and height (h) are the sides of the given solid box, find its total surface area.

Discuss in group and find the solution.

Here, number of surface = 6 in which 3 pairs of them are congruent.

Therefore, total surface area,

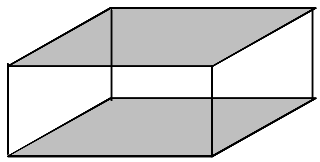
$$A = 2(l \times b) + 2(b \times h) + 2(l \times h) \text{ sq. unit}$$

$$= 2(lb + bh + lh) \text{ sq. units}$$

Note : If any solid is irregular we need to add each surface area to find the total surface area of the solid.

Example 1

In Harishiddhi School, there is a water tank with length and breadth 10 ft each and height 2 ft. Find the total surface area of it.



Solution :

Here, length of the water tank (l) = 10 ft

Breadth of the water tank (b) = 10 ft

Height of the water tank (h) = 2 ft

Total surface area of the tank (A) =?

We have, total surface area (A) = $2(l \times b + b \times h + l \times h)$ sq. units

$= 2(10 \times 10 + 10 \times 2 + 10 \times 2)\ ft^2$

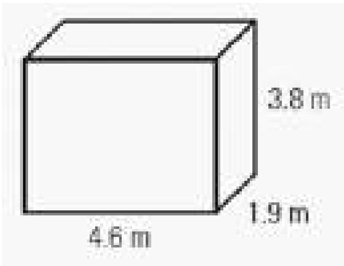
$= 2(100 + 20 + 20)\ ft^2$

$= 280\ ft^2$

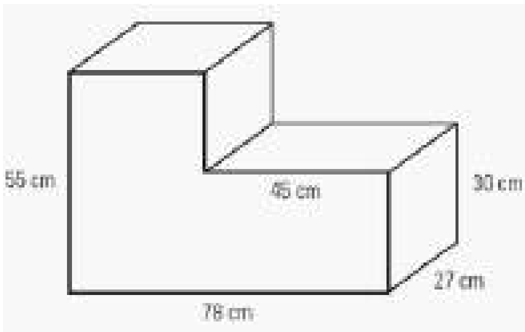
Example: 2

Find the total surface area of the following solids :

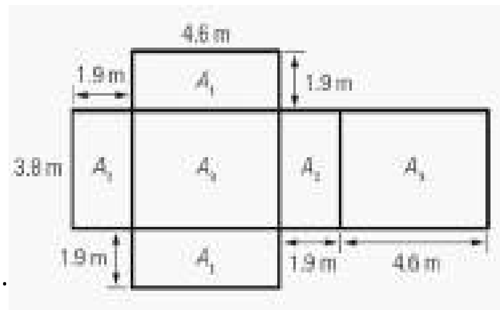
(a)



(b)



Solution :



(a) The given prism can be resolved as plane surfaces shown like here.

Therefore, the total surface area of the prism is the sum of all rectangles.

Now, to find the surface area of each rectangle.

Total surface area (A)

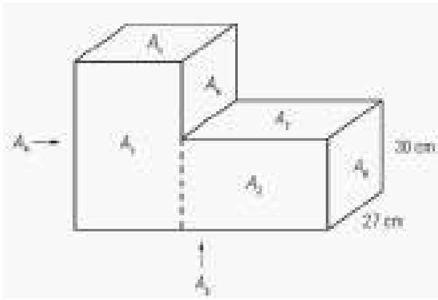
$= 2(l \times b + b \times h + l \times h)$

$= [2 \times (3.8 \times 1.9) + 2 \times (1.9 \times 4.6) + 2 \times (4.6\ x^3.8)]m^2$

$= [14.44 + 17.48 + 34.96]\ m^2$

$= 66.88\ m^2$

(b) Find the sum of rectangular surface area of given prism.



Total surface area

$= 2A_1 + 2A_2 + A_3 + A_4 + A_5 + A_6 + A_7 + A_8$

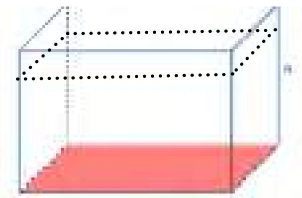
$= 2\ (55 \times 33) + 2\ (45 \times 30) + (27 \times 78) + (55 \times 27) + (27 \times 33) + (25 \times 27) + (45 \times 27) + (27 \times 30)\ cm^2$

$= (3630 + 2700 + 2106 + 1485 + 891 + 675 + 1215 + 810)\ cm^2$

$= 13512\ cm^2$

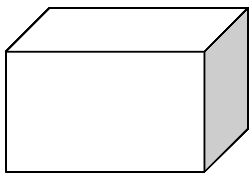
6.1.2 Cross Section Area of Prism

What do you mean by cross-section area? How can you find it? Let's observe a prism which can be cut into congruent slices.



Here, a prism or any solid which can be cut into congruent slices by a plane surface is called its cross-section. Thus, the prisms have uniform cross-sections. The congruent surfaces thus formed are parallel to each other.

6.1.3 Lateral Surface Area of Prism



Take a cuboid or prism in which length (l), breadth (b) and height (h). Find the area of all surfaces except base surface and parallel to it.

Now, let's find the conclusion of the sum of surface areas in the classroom.

Hence, the sum of 4 lateral surface areas is lateral surface area of prism.

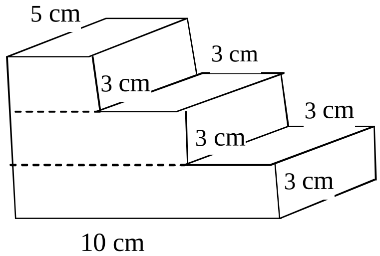
Lateral surface area (S) $= 2\ (l \times h) + 2(b \times h)$

$= 2h(l + b)$ square units

= perimeter of cross-section x height

Example 3

Calculate the cross-sectional area and lateral surface area of the following prism.



Solution :

According to figure,

Area of cross-section = (10 x 3 + 7 x 3 + 4 x 3) cm^2

= (30 +21 + 12) cm^2

= 63 cm^2

Now, perimeter of cross-section = (10 + 3 + 3 + 3 + 3 + 3 + 4 + 9) cm

= 38 cm

Height (h) = 5 cm

Lateral surface area of prism = perimeter of cross-section x height

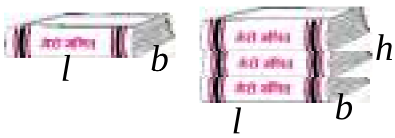
= 38 x 5 cm^2

= 190 cm^2

6.1.4 Volume of Prism

Divide the students in class into a group of 5 students in each. Measure your books' length and breadth and find their area. Pile up 5 mathematics books as shown. This pile looks like a prism.

Now, with help of ruler, measure the prism's length, breadth and height.



After measuring the prism, let each group find the product of length, breadth and height. Let's exchange the conclusion among the groups and share the feedback.

Volume of prism (V) = base area of prism x height

V = A x h cubic units.

Let us study the following table.

S. No.	Solids	Area of base or cross section	Lateral Surface Area	Total surface area	Volume
1.	Cuboid	$A = l \times b$	$2h(l + b)$	$2(l \times b + b \times h + l \times h)$	$V = l \times b \times h = A \times h$
2.	Cube	$A = l^2$	$4l^2 = 4A$	$6l^2 = 6A$	$V = l^3$
3.	Prism	A = Base area = Cross section	Perimeter of base x Height = P x h	P x h + 2A	Area of base x Height ($V = A \times h$)

Example 4

Find the total surface area and volume of the given figure.

Solution :

Here, base area of prism = 5(3 x 3) square inch

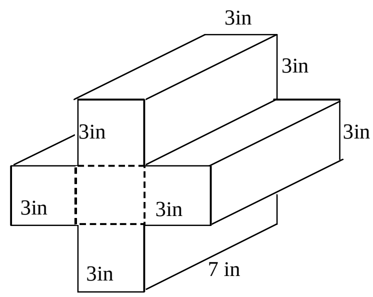
= 45 inch^2

Total surface area of prism = 2 x base area of prism + remaining surface area

= 2 x 45 + 12(3 x 7)

= 90 + 252 inch^2

= 342 inch^2



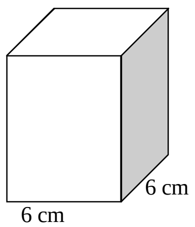
Again, volume of prism = base area x height

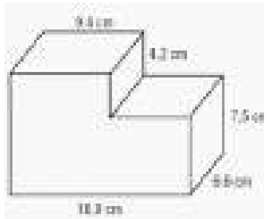
= 45 x 7 inch^3

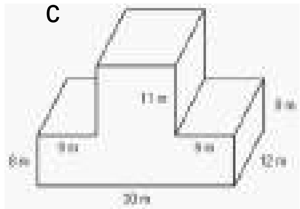
= 315 inch^3

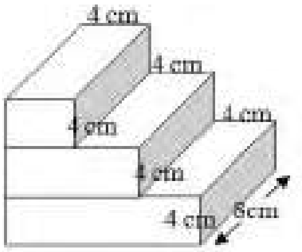
Exercise 6.1

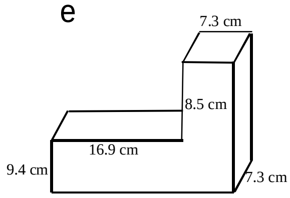
1. Find the total surface area of the following prisms.

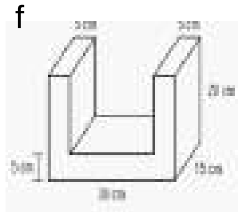
a


b


c


d


e


f


2. Find (a) lateral surface area (b) cross-section area of a prism with length 5 cm, breadth 6 cm and height 8 cm.
3. Calculate the lateral surface area of a prism with perimeter of cross-section 56 cm and height 12 cm.
4. For the solids in question no. 1, find
   i. Cross-sectional area
   ii. Lateral surface area
   iii. Volume
5. List out the name of any three prism shape objects which are found in your house or surroundings. Measure the length of their dimensions and calculate the following.
   i. Cross-section area
   ii. Lateral surface area
   iii. Volume of prism
6. A match box is 4cm X 2.5 cm X 1.5 cm. Find its volume.
7. In a village, there is a cuboid tank of length 6m, breadth 5m and height 3m. How much water can be filled in this tank?
8. Divide a timber cube of 20cm long into 8 equal parts. Find the length of each piece.

9. There is a cubical tank of volume 125 m^3 in a town. Find its total surface area.

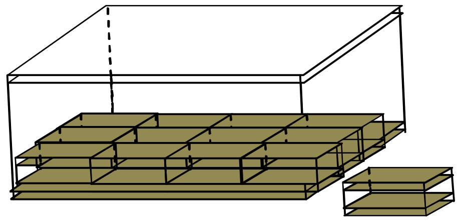
10. The base area of a rectangular wood is $120m^2$. If its volume is $720m^3$, find its thickness.

11. Find the surface area and volume of chautara or stage in your school's ground.

6.2 Estimation of Numbers, Cost and Volume

Take a cuboid of inner sides 18 cm long, 20 cm wide and 6 cm high. How many matches box of 4.5 cm long, 4 cm wide and 1.5 cm high are required to fill up the cuboid?

What is the total cost of match boxes to fill up the cuboid at Rs. 2/box?



To estimate the number of required bricks while building a wall, it is required to find the volume of the wall and the volume of a brick. If length L, breadth B and height H of the wall and length l, breadth b and height h of a brick, then

Volume of the wall (V) = L x B x H cubic units

Volume of a brick (v) = l x b x h cubic units

Therefore, no. of required bricks (N) = $\frac{V}{v}$

Volume of a brick (v) = $\frac{V}{N}$

Volume of a wall (V) = N x v

If there is a door and a window in a wall, what should be done? Let's have a group discussion. If a cost of a brick is Rs. C, total cost = N x C.

Example 1

Find the volume of a wall built from 4000 bricks of dimension 20 cm x 10 cm x 4 cm.

Solution :

Here, volume of one brick (v) = 20 cm x 10 cm x 4 cm

Number of bricks (N) = 4000

Volume of wall (V) = N x v cm^3

= 4000 x 800 cm^3

= 32,00,000 cm^3

Example 2

To build a wall of 11m long, 1m wide and 5m height.

(a) How many bricks of size 22 cm x 10 cm x 5 cm are required?

(b) What is the total cost if the rate is Rs. 12.50 per brick is determined?

Solution :

Here, length of a wall (l) = 11 m

Width (w) = 1 m

Height (h) = 5 m

Hence, volume of the wall (V) = 11 x 1 x 5 m^3

= 55 m^3

Again, volume of a brick (v) = 22 cm x 10 cm x 5 cm

= 0.22 m x 0.10 m x 0.05 m

= 0.0011 m^3

(a) number of bricks required (N) = $\frac{V}{v} = \frac{55}{0.0011}$

= 50,000

(b) Cost of a brick (C) = Rs. 12.50

Hence the total cost of bricks (T) = N x C

= Rs. 50,000 x 12.50

= Rs. 625,000

Example 3

A rectangular tank is 3m long and 1.5 m wide contains 9000 liters of water. Find its height. (Hints : $1m^3 = 1000$ litre)

Solution :

Here, length of the tank (l) = 3m

breadth of the tank (b) = 1.5m

Amount of water in the tank (V) = 9000 litre

height of the tank (h) =?

We know, 1000 litre = $1m^3$

1 litre = $\frac{1}{1000}$ m^3

9000 litre = $\frac{1}{1000}$ X 9000 = 9 m^3

Hence, the volume of the tank = length x breadth x height

or, 9 m^3 = (3 x 1.5) m^2 x h

or, h = $\frac{9}{4.5}$ m = 2m

Hence, the height of the tank = 2m

Example 4

The volume and height of square shaped room is 87.5 m^3 and 3.5 m respectively. What is the total cost of plastering its four walls at the rate of Rs. 150 per square meter?

Solution :

Here, volume of the room (V) = 87.5 m^3

and height (h) = 3.5m

We know, volume of the room (V) = Area of floor x height

or, Area of floor x height = $87.5m^3$

or, Area of floor x 3.5 m = $87.5m^3$

or, Area of the floor (A) = $\frac{87.5m^3}{3.5m}$ = $25m^2$

Since the room is square shaped,

So, its length = breadth = $\sqrt{\text{Area}}$

= $\sqrt{25m^2}$

= 5m

Now the area of four walls (A) = 2 h (l + b)

$= 2 \times 3.5\ (10)\ m^2$

$= 70\ m^2$

Again, rate of plastering (C) = Rs. 150/square meter

Hence the total cost of plastering the four walls (T) = C x A

= Rs. 70 x 150

= Rs. 10,500

Example 5

A room's length is double of its width contains $396m^3$ air. The ceiling of the room is painted with Rs. 30 per square meter costs Rs. 2160. How much will be the total cost if the four walls are painted with Rs. 60 per square meter?

Solution :

Let, the breadth of the room (b) = x

So, the length of room (l) = 2x

Si nce the rate of coloring the ceiling is Rs. 30 per square meter.

Hence with Rs. 30 we can color = $1\ m^2$

Again with Re. 1 we can color = $\frac{1}{30}\ m^2$

And with Rs. 2160 we can color = $\frac{1}{30} \times 2160 = 72\ m^2$

Hence the area of the ceiling (A) = $72m^2$

or, $2x \times x = 72m^2$

or, $x^2 = 36m^2$

or, $x = 6m$

Hence the breadth of the room (b) = 6 m

So the length (l) = 2 x 6 = 12m

Again, the volume of the room (V) = $396m^3$

or, $l \times b \times h = 396m^3$

or, $h = \frac{396}{72}\ m = 5.5m$

Now, the area of four walls (A) = $2h(l + b)$

$= 2 \times 5.5\ (12 + 6)\ m^2$

$= 11 \times 18\ m^2$

$= 198\ m^2$

Here, to color 1 m^2 wall the cost is (C) = Rs. 60

So, to color 198 m^2 walls the cost is (T) = C x A

= Rs. 60 x 198

= Rs. 11,880

Exercise 6.2

1. Calculate the volume of a wall constructed of 4000 bricks each having 20 cm length, 10 cm width and 4 cm height.
2. A wall is 100m long, 0.3m broad and 4.5m high. How many bricks of size 15cm long, 5cm width and 5cm thick are required to construct the wall?
3. A store room is 40m long, 25m broad and 10m high. How many packets of biscuits having 1.5m x 1.24m x 0.5m can be stored there?
4. The population of Barakot village is 40,000. If 15 litre of water per person per day is required, how longer does 20m x 15m x 6m of a water tank can supply the water?
5. A wall is 50 m long, 0.2m wide and 2m high having two windows each of size 1m x 0.20m x 0.5m. How many bricks of size 22cm x 10cm x 5cm are required to build the wall?
6. 2450 pieces of blocks each of side 20cm are required to construct a wall 15m long and 40 cm width. If the wall has a door of size 2.5m x 1m and a window of size 2m x 3m, what is the height of the wall?
7. The volume of a room having 15m length and 8m width is 600m^3. Find the cost of designing its four walls at Rs. 38/m^2.
8. The volume of air occupied inside a room is 405 m^3. The cost of carpeting the room at Rs. 55/m^2 is Rs. 4455. Find the cost of painting 4 walls of the room at Rs. 25.50/m^2.
9. The cost of constructing a 63m^3 wall is Rs. 84000. Find the volume of each brick if the cost of a brick is Rs. 13.
10. The volume of a room is 550 m^3. If the cost of plastering the room at Rs. 240/m^2 is Rs. 26400, what is the height of the room?
11. In the west block of Bhartibhawan Higher Secondary School, a wall of bricks having 0.5m wide and 1.5m is to be constructed surrounding a ground 200m long. To build that wall, bricks of size 20cm x 10cm x 5cm are required. If the cost of a brick, transportation charge and wages are Rs. 12, Rs. 1.75 and Rs. 0.25 respectively. Find

the total cost of constructing the wall.

12. The volume of a square room has a door of 2m x 1.5m and two windows, each of 1m x 1.5m is $900m^3$. If the cost of paving tiles including wage at Rs. $7.50/m^2$ is Rs. 1687.50, find the cost of colouring 4 walls at Rs. $12.50/m^2$.

13. Divide students into groups of 5 members in each. Make each group to measure the assigned walls of the ground or a building. Then provide each group a piece of brick or cement block to measure its length, width and height. Ask them to find out how many bricks or blocks are required to build the walls. Also ask them to find out the total cost required to build assigned wall at the current price per piece of brick or block in the market.

Lesson 7 Algebraic Expressions

7.0 Review

Let us discuss about the following figures and find the area of the each figures.

(a)

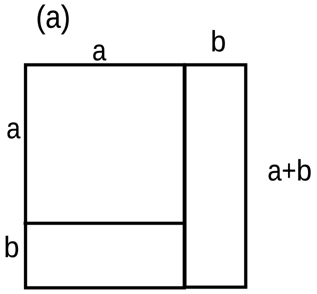


Fig. 1

(b)

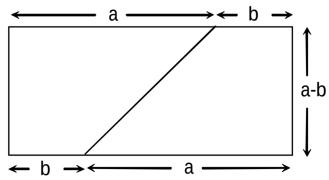


Fig. 2

(c)

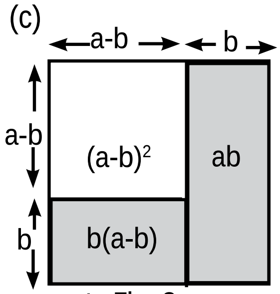


Fig. 3

(d)

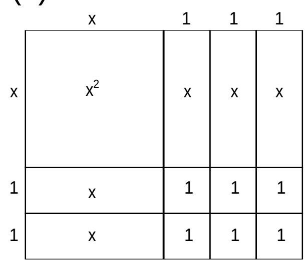


Fig. 4

Based on the above discussion, now factorize the following expressions.

1. $a^2 - b^2 =$ .... ?
2. $x^2 - 4y^2 =$ .... ?
3. $z^2 - 10z + 25 =$ .....?
4. $y^2 + 6y + 9 =$ .....?
5. $a^2 - 2ab + b^2 =$ ......?
6. $x^2 - 5x + 6 =$ .....?
7. $a^3 - b^3 =$ .....?
8. $x^3 - 8y^3 =$ ......?
9. $x^3 + y^3 =$ ......?
10. $8z^3 - 1 =$.....?
11. $8a^3 + 36a^2b + 54ab^2 + 27 =$.....?

7.1 Factorization of the Expression in the Form of $a^4 + a^2b^2 + b^4$

Here, $a^4+a^2 b^2+b^4 = (a^2)^2 + 2a^2 b^2 - a^2 b^2 + (b^2)^2$

$= (a^2 + b^2)^2 - (ab)^2 \quad [\because a^2+2ab+b^2 = (a+b)^2]$

$= (a^2 + b^2 + ab)(a^2 + b^2 - ab) \quad [\because a^2 - b^2 = (a+b)(a-b)]$

$= (a^2+ab+b^2)(a^2-ab+b^2)$

Hence, $a^4+a^2 b^2+b^4 = (a^2+ab+b^2)(a^2-ab+b^2)$

Example 1

Factorize : $x^4 + x^2 + 1$

Solution : $x^4 + x^2+1 = (x^2)^2 + 2x^2 - x^2 + 1$

$= (x^2)^2 + 2.x^2.1 + 1^2 - x^2$

$= (x^2+1)^2 - x^2$

$= (x^2 + 1 + x)(x^2 + 1 - x)$

$= (x^2 + x + 1)(x^2 - x + 1)$

Example 2

Factorize : $x^4 + 4$

Solution : Here,

$x^4 + 4 = (x^2)^2 + 2.x^2.2 + (2)^2 - 2.x^2.2$

$= (x^2 + 2)^2 - (2x)^2$

$= (x^2 + 2 + 2x)(x^2 + 2 - 2x)$

$= (x^2 + 2x + 2)(x^2 - 2x + 2)$

Example 3

Factorize : $x^4 - 3x^2 + 1$

Solution :

Here, $x^4 - 3x^2 + 1$

$= x^4 - 2x^2 - x^2 + 1$

$= (x^2)^2 - 2.x^2.1 + 1 - x^2$

$= (x^2 - 1)^2 - (x)^2$

$= (x^2 - 1 + x)(x^2 - 1 - x)$

$= (x^2 + x - 1)(x^2 - x - 1)$

Example 4

Factorize : $\frac{x^2}{y^2} + 1 + \frac{y^2}{x^2}$

Solution : Here,

$$\frac{x^2}{y^2} + 1 + \frac{y^2}{x^2} = \left(\frac{x}{y}\right)^2 + 2\frac{x}{y} \times \frac{y}{x} - \frac{x}{y} \times \frac{y}{x} + \left(\frac{y}{x}\right)^2$$

$$= \left(\frac{x}{y}\right)^2 + 2\frac{x}{y} \times \frac{y}{x} + \left(\frac{y}{x}\right)^2 - \frac{x}{y} \times \frac{y}{x}$$

$$= \left(\frac{x}{y} + \frac{y}{x}\right)^2 - 1$$

$$= \left(\frac{x}{y} + \frac{y}{x}\right)^2 - (1)^2$$

$$= \left(\frac{x}{y} + \frac{y}{x} + 1\right)\left(\frac{x}{y} + \frac{y}{x} - 1\right)$$

$$= \left(\frac{x}{y} + 1 + \frac{y}{x}\right)\left(\frac{x}{y} - 1 + \frac{y}{x}\right)$$

Example 5

Factorize : $\frac{x^4}{y^4}+1-\frac{7x^2}{y^2}$

Solution : Here,

$$\frac{x^4}{y^4}+1-\frac{7x^2}{y^2}$$

$$=\left(\frac{x^2}{y^2}+1\right)^2-2.\frac{x^2}{y^2}.1-\frac{7x^2}{y^2} \qquad [\because a^2+b^2=(a+b)^2-2ab]$$

$$=\left(\frac{x^2}{y^2}+1\right)^2-\frac{2x^2}{y^2}-\frac{7x^2}{y^2}$$

$$=\left(\frac{x^2}{y^2}+1\right)^2-\frac{9x^2}{y^2}$$

$$=\left(\frac{x^2}{y^2}+1\right)^2-\left(3\frac{x}{y}\right)^2$$

$$=\left(\frac{x^2}{y^2}+1+3\frac{x}{y}\right)\left(\frac{x^2}{y^2}+1-3\frac{x}{y}\right) \qquad [\because a^2-b^2=(a+b)(a-b)]$$

$$=\left(\frac{x^2}{y^2}+3\frac{x}{y}+1\right)\left(\frac{x^2}{y^2}-3\frac{x}{y}+1\right)$$

Example 6

Factorize : $x^2 - 10x + 24 + 6y - 9y^2$

Solution : Here,

$x^2 - 10x + 24 + 6y - 9y^2$

$= x^2 - 2.x.5 + 25 - 1 + 2.1.3y - (3y)^2$

$= (x-5)^2 - \{1^2 - 2.1.3y + (3y)^2\}$

$= (x-5)^2 - (1-3y)^2$

$= (x - 5 + 1 - 3y)\{(x-5) - (1-3y)\}$ $\qquad [\because a^2 - b^2=(a+b)(a-b)]$

$= (x - 4 - 3y)(x - 5 - 1 + 3y)$

$= (x - 4 - 3y)(x - 6 + 3y)$

$= (x - 3y - 4)(x + 3y - 6)$

Exercise 7

Factorize.

1. $y^4 + y^2 + 1$
2. $x^4 + x^2y^2 + y^4$
3. $x^8 + x^4 + 1$
4. $x^4 + 4y^4$
5. $81x^4 + 64y^4$
6. $64x^4 + y^4$
7. $x^4 - 7x^2 + 1$
8. $x^4 - 5x^2y^2 + 4y^4$
9. $49x^4 - 154x^2y^2 + 9y^4$
10. $25a^4 - 34a^2x^2 + 9x^4$
11. $256x^4 - x^2y^2 + 49y^4$
12. $1225x^4 + 31x^2y^2 + 64y^4$
13. $2025x^4 + 185x^2y^2 + 2401y^4$
14. $4a^4 + 35a^2b^2 + 121b^4$
15. $4x^4 + 8x^2y^2 + 9y^4$
16. $x^4 + 9x^2 + 81$
17. $x^4 + 1 + \frac{1}{x^4}$
18. $\frac{x^4}{y^4} + \frac{x^2}{y^2} + 1$
19. $\frac{x^4}{y^4} + \frac{y^4}{x^4} + 1$
20. $x^4 - 8x^2 - 33 - 14y - y^2$
21. $x^4 - 6x^2 - 7 - 8x - x^2$
22. $x^4 - 12x^2 - 28 + 16y - y^2$
23. $x^4 - 10x^2 + 24 + 6y^2 - 9y^4$
24. $x^2 - 10xy + 16y^2 - z^2 + 6yz$
25. $4225x^4 - 130x^2 - 3 + 36y^2 - 81y^4$
26. $x^2 - 90xy + 2000y^2 - 550yz - 3025z^2$
27. $169x^2 - 52x - 56y - 196y^2$
28. $4225x^2 - 130xy - 3y^2 - z^2 - 4yz$
29. $289x^2 + 170x + 24 - 38y - 361y^2$
30. $x^2 + 50xy + 609y^2 - 8yz - z^2$

Lesson 8

Indices

8.0 Review

Let us find out the value of the following expressions.

1. $x^2 \times x^3 = ?$ $\quad y^5 \times y^2 = ?$ $\quad a^4 \times a^7 = ?$
2. $x^5 \div x^3 = ?$, $\quad x^7 \div x^4 = ?$, $\quad 2^5 \div 2^2 = ?$
3. $3^{2-2} = ?$ $\quad 5^{3-3} = ?$ $\quad n^{5-5} = ?$
4. $(x^3)^2 = ?$ $\quad (2^2)^3 = ?$ $\quad (5^2)^2 = ?$
5. $(x^3)^2 = ?$ $\quad (2^2)^3 = ?$ $\quad (5^2)^2 = ?$
6. $\left(\frac{2}{x}\right)^3 = ?$ $\quad \left(\frac{7}{x}\right)^3 = ?$ $\quad \left(\frac{x}{5}\right)^4 = ?$

8.1 Problems Related to Indices

Let us discuss on the following questions.

1. What is the difference between 10^3 and $\frac{1}{10 \times 10 \times 10}$?
2 . What is the difference between $(5)^3$ and $(5)^{-3}$?
3. What is the difference between $\sqrt[3]{64}$ and $\sqrt{64}$?

From the above discussion we can conclude ;

(a) $x^{-m} = \frac{1}{x^m}$ $\quad$ (b) $\sqrt[n]{x} = x^{\frac{1}{n}}$ $\quad$ (c) $\sqrt[n]{x^m} = x^{\frac{m}{n}}$

Example 1

Find the value of:

(a) 4^{-2} $\quad$ (b) $\left(\frac{3}{2}\right)^{-4}$ $\quad$ (c) $\sqrt[3]{64}$

Solution : Here,

(a) $4^{-2} = \frac{1}{4^2} = \frac{1}{16}$

(b) $\left(\frac{3}{2}\right)^{-4} = \frac{3^{-4}}{2^{-4}}$

$= \frac{2^4}{3^4} = \frac{16}{81}$

(c) $\sqrt[3]{64}$

$= (64)^{\frac{1}{3}}$

$= (4^3)^{\frac{1}{3}}$

$= 4^{3\times\frac{1}{3}}$

$= 4^1$

$= 4$

Example 2

Simplify : $\left(\frac{x^a}{x^b}\right)^{a+b} x \left(\frac{x^b}{x^c}\right)^{b+c} x \left(\frac{x^c}{x^a}\right)^{c+a}$

Solution : Here,

$$\left(\frac{x^a}{x^b}\right)^{a+b} x \left(\frac{x^b}{x^c}\right)^{b+c} x \left(\frac{x^c}{x^a}\right)^{c+a}$$

$= (x^{a-b})^{a+b} \text{ x } (x^{b-c})^{b+c} \text{ x } (x^{c-a})^{c+a}$ [$x^m \div x^n = x^{m-n}$]

$= x^{(a-b)(a+b)} \times x^{(b-c)(b+c)} \times x^{(c-a)(c+a)}$ [$(x^m)^n = x^{mn}$]

$= x^{(a^2-b^2)} \times x^{(b^2-c^2)} \times x^{(c^2-a^2)}$ [(a+b) (a-b)=a^2 - b^2]

$= = x^{(a^2-b^2+b^2-c^2+c^2-a^2)}$ [$(x^m \times x^n = x^{(m+n)}$]

$= x^0$

$= 1$

Example 3

Simplify : $\left(\frac{x^a}{x^b}\right)^{a^2+ab+b^2} \times \left(\frac{x^b}{x^c}\right)^{b^2+bc+c^2} \times \left(\frac{x^c}{x^a}\right)^{c^2+ca+a^2}$

Solution : Here,

$$\left(\frac{x^a}{x^b}\right)^{a^2+ab+b^2} \times \left(\frac{x^b}{x^c}\right)^{b^2+bc+c^2} \times \left(\frac{x^c}{x^a}\right)^{c^2+ca+a^2}$$

$= \left(x^{a-b}\right)^{\left(a^2+ab+b^2\right)} \times \left(x^{b-c}\right)^{\left(b^2+bc+c^2\right)} \times \left(x^{c-a}\right)^{\left(c^2+ca+a^2\right)}$ [$\because x^m \div x^n = x^{m-n}$]

$$= x^{(a-b)(a^2+ab+b^2)} \times x^{(b-c)(b^2+bc+c^2)} \times x^{(c-a)(c^2+ca+a^2)} \qquad [\because (x^m)^n = x^{mn}]$$

$$= x^{(a^3-b^3)} \times x^{(b^3-c^3)} \times x^{(c^3-a^3)}$$

$$= x^{a^3-b^3+b^3-c^3+c^3-a^3}$$

$$= x^0 = 1$$

Example 1

Prove that : $\sqrt[a+b]{x^{a^2-b^2}} \times \sqrt[b+c]{x^{b^2-c^2}} \times \sqrt[c+a]{x^{c^2-a^2}} = 1$

Solution :

Here, L.H.S. = $\sqrt[a+b]{x^{a^2-b^2}} \times \sqrt[b+c]{x^{b^2-c^2}} \times \sqrt[c+a]{x^{c^2-a^2}}$

$$= \left(x^{a^2-b^2}\right)^{\frac{1}{a+b}} \times \left(x^{b^2-c^2}\right)^{\frac{1}{b+c}} \times \left(x^{c^2-a^2}\right)^{\frac{1}{c+a}}$$

$$= x^{(a-b)(a+b)\times\frac{1}{a+b}} \times x^{(b-c)(b+c)\times\frac{1}{b+c}} \times x^{(c-a)(c+a)\times\frac{1}{c+a}}$$

$$= x^{a-b} \times x^{b-c} \times x^{c-a}$$

$$= x^{a-b+b-c+c-a}$$

$$= x^0$$

= 1 = R.H.S. proved

Exercise 8.1

1. Find the value of :

(i) $3^4 \times 3^{-4}$ (ii) $5^5 \times 5^{-5}$ (iii) $7^3 \times \frac{1}{7^3}$ (iv) $7^2 \times 7$

v) $5^3 \times 5^2$ (vi) 6×6^3 (vii) $(16)^{\frac{1}{4}} \times \left(\frac{1}{8}\right)^{\frac{4}{3}}$

viii) $(64)^{3/6}$ (ix) $\left(\frac{1}{128}\right)^{\frac{1}{7}} + \left(\frac{1}{64}\right)^{\frac{1}{6}}$ (x) $\left(\frac{81}{16}\right)^{\frac{-3}{4}}$

(xi) $\left(\frac{625}{1296}\right)^{\frac{3}{4}}$ (xii) $\sqrt[5]{\frac{243}{1024}}$

:

2. **Simplify :**

i. $\sqrt{81x^2y^2}$

ii. $\sqrt{7225x^{-4}y^4}$

iii. $\sqrt[4]{1296x^8y^{12}}$

iv. $x^{a-b} \times x^{(b-c)} \times x^{(c-a)}$

v. $x^{(a-b-c)} \times x^{(b-c-a)} \times x^{(c-a-b)}$

vi. $x^{b^2+2ca} \times x^{c^2+2ab} \times x^{a^2+2bc}$

vii. $x^{a(a-c)} \times x^{b(c-a)} \times x^{c(a-b)}$

viii. $\dfrac{1}{1-x^{a-b}} + \dfrac{1}{1-x^{b-a}}$

ix. $\dfrac{1}{1-x^{a-b}} + \dfrac{1}{1-x^{b-a}}$

x. $\left(\dfrac{x^b}{x^c}\right)^a \times \left(\dfrac{x^c}{x^a}\right)^b \times \left(\dfrac{x^a}{x^b}\right)^c$

3. Simplify :

i. $\left(\dfrac{x^{-b}}{x^{-a}}\right)^c \times \left(\dfrac{x^{-c}}{x^{-b}}\right)^a \times \left(\dfrac{x^{-a}}{x^{-c}}\right)^b$

ii. $\left(\dfrac{x^{-b}}{x^{-a}}\right)^{a+b} \times \left(\dfrac{x^{-c}}{x^{-b}}\right)^{c+b} \times \left(\dfrac{x^{-a}}{x^{-c}}\right)^{c+a}$

iii. $\left(\dfrac{x^a}{x^{-b}}\right)^{a-b} x \left(\dfrac{x^b}{x^{-c}}\right)^{b-c} x \left(\dfrac{x^c}{x^{-a}}\right)^{c-a}$

iv. $\left(\dfrac{x^a}{x^{-b}}\right)^{a^2-ab+b^2} \times \left(\dfrac{x^b}{x^{-c}}\right)^{b^2-bc+c^2} \times \left(\dfrac{x^c}{x^{-a}}\right)^{c^2-ca+a^2}$

v. $\left(\dfrac{x^p}{x^q}\right)^r x \left(\dfrac{x^q}{x^r}\right)^p x \left(\dfrac{x^r}{x^p}\right)^q$

vi. $\left(\dfrac{a^x}{a^y}\right)^{x+y} x \left(\dfrac{a^y}{a^z}\right)^{y+z} x \left(\dfrac{a^z}{a^x}\right)^{z+x}$

vii. $\left(\dfrac{a^x}{a^y}\right)^{x^2+xy+y^2} \times \left(\dfrac{a^y}{a^z}\right)^{y^2+yz+z^2} \times \left(\dfrac{a^z}{a^x}\right)^{z^2+zx+x^2}$

viii. $\left(\dfrac{a^x}{a^{-y}}\right)^{x^2-xy+y^2} \times \left(\dfrac{a^y}{a^{-z}}\right)^{y^2-yz+z^2} \times \left(\dfrac{a^z}{a^{-x}}\right)^{z^2-zx+x^2}$

ix. $\sqrt[x+y]{a^{x^2-y^2}} \times \sqrt[x+z]{a^{y^2-z^2}} \times \sqrt[z+x]{a^{z^2-x^2}}$

x. $\dfrac{1}{1+a^{x-y}+a^{z-y}} + \dfrac{1}{1+a^{y-z}+a^{x-z}} + \dfrac{1}{1+a^{z-x}+a^{y-x}}$

8.2 Exponential Equation

Let us discuss on these questions;

What is the value of x in $a^x = a^2$?
Similarly what is the value of a in $a^x = 2^x$?
Is a = 2 the solution of this equation? Discuss.

We know, if $x^y = x^z$ then, y = z , Hence when the bases of exponential equation are equal, their exponents are also equal.

Example 1

Solve : $2^x = 32$

Solution :

Here, $2^x = 32$

or, $2^x = 2^5$

$\therefore x = 5$ [If $x^a = x^b$ then a = b]

Example 2

Solve : $3^{-x} = \frac{1}{243}$

Solution :

Here, $3^{-x} = \frac{1}{243}$

or, $3^{-x} = \frac{1}{3^5}$

or, $3^{-x} = 3^{-5}$

or, $-x = -5$

$\therefore x = 5$

Example 3

Solve : $27^x = 3^{x+4}$

Solution :

Here, $27^x = 3^{x+4}$

or, $(3^3)^x = 3^{x+4}$

or, $3^{3x} = 3^{x+4}$

or, $3x = x + 4$

or, $3x - x = 4$

or, $2x = 4$

or, $x = \frac{4}{2}$

$\therefore x = 2$

Example 4

Solve : $2^x + 2^{x+2} = 5$

Solution :

Here, $2^x + 2^{x+2} = 5$

or, $2^x + 2^x \times 2^2 = 5$

or, $2^x(1 + 2^2) = 5$

or, $2^x(1 + 4) = 5$

or, $2^x . 5 = 5$

or, $2^x = 1$

or, $2^x = 2^0$

$\therefore x = 0$

Exercise 8.2

Solve and check

1. $3^x = 81$
2. $8^x = 2^{2x+1}$
3. $3 \times 81^x = 9^{x+2}$
4. $5 \times 125^x = 5^{2x-2}$
5. $3^{x+1} + 3^x = 108$
6. $5^{2x+1} + 5^{2x} = 150$
7. $3^{x+1} \times 2^{2x+1} = 6$
8. $2^{x+3} + 2^x = 18$
9. $2^x + \frac{2}{2^x} = 3$
10. $5^x + \frac{1}{5^x} = 5\frac{1}{5}$
11. If $a^x = b^y = c^z$ and $b^2 = ac$, then prove that $\frac{2}{y} = \frac{1}{x} + \frac{1}{z}$

Lesson 9 Ratio and Proportion

9.0 Review

Suppose, white button mushroom costs Rs. 300 per kg and oyster mushroom costs Rs. 150 per kg. How many times the cost of white button mushroom is more than the cost of oyster mushroom? How can we express this ratio?

Is the ratio of 2 and 4 equal to the ratio of 4 and 8? If two or more ratios are equal, the ratios are called in proportion.

9.1 Properties of Proportion

i. The property of the proportion in which if a:b = c:d, then b:a = d:c is known as invertendo. For example 2 : 4 = 8 : 16, then 4 : 2 = 16 : 8

ii. The property of the proportion in which if a:b = c:d , then a:c =b:d is known as alternendo.

iii. The property of the proportion in which if a:b =c:d, then (a+b):b = (c+d):d is known as componendo.

iv . The property of the proportion in which if a:b =c:d, then (a - b):b =(c - d):d is known as dividendo.

v. The property of the proportion in which if a:b = c:d, then (a + b):(a - b) = (c + d):(c - d) is known as componendo and dividendo.

vi. The property of the proportion in which if a:b = c:d, then $\frac{a+c}{b+d}=\frac{a}{b}=\frac{c}{d}$ is known as addendo.

Example 1

If $\frac{a}{b}=\frac{3}{4}$, then find the value of $\frac{4a+2b}{4a+3b}$

Solution :

Here, $\frac{a}{b}=\frac{3}{4}$ so, we get 4a = 3b

Now, $\frac{4a+2b}{4a+3b}=\frac{3b+2b}{3b+3b}$ [$\therefore 4a = 3b$]

$= \frac{5\ b}{6\ b}$

$= \frac{5}{6}$

Hence, (4a + 2b) : (4a + 3b) = 5:6

Example 2

If $\frac{a}{b} = \frac{5}{6}$ then, find the value of $\frac{5a-2b}{6a+2b}$

Solution :

Here, $\frac{5a-2b}{6a+2b} = \frac{5\frac{a}{b}-2\frac{b}{b}}{6\frac{a}{b}+2\frac{b}{b}}$ [Dividing the numerator and denominator by b]

$= \frac{5\times\frac{5}{6}-2}{6\times\frac{5}{6}+2}$ [$\frac{a}{b}=\frac{5}{6}$]

$= \frac{\frac{25}{6}-2}{\frac{30}{6}+2} = \frac{25-12}{6}\times\frac{6}{30+12}$ [$\frac{x}{\frac{1}{x}}=\times\frac{x}{1}$]

$= \frac{13}{42}$

$\therefore$ (5a – 2b) : (6a + 2b) = 13:42

Example 3

If (4a – 5b) : (5a + 4b) = 1:3, then find the value of a : b.

Solution :

Here, (4a – 5b) : (5a + 4b) =1:3

or, $\frac{4a-5b}{5a+4b}=\frac{1}{3}$

or, 12a – 15b = 5a + 4b

or, $12a - 5a = 4b + 15b$

or, $7a = 19b$

or, $a : b = 19 : 7$

Example 4

If the ratio of two digits is 5:6 and their sum is 33, find the numbers.

Solution :

Let's suppose the numbers be x and y. Then

$\therefore \quad \frac{x}{y} = \frac{5}{6}$ and $x + y = 33$

or, $6x = 5y$ and $y = 33 - x$

Now, $6x = 5(33 - x)$ [substituting the value of $y = 33 - x$]

or, $6x = 165 - 5x$

or, $6x + 5x = 165$

or, $11x = 11 \times 15$

$\therefore x = 15$

Again $y = 33 - x$

or, $y = 33 - 15$

$= 18$

Hence the required numbers are 15 and 18

Example 5

Divide the number 455 in the ratio of 6 : 7.

Solution :

Let 6k and 7k be the two numbers. Then,

Or, $6k + 7k = 455$

Or, $13k = 13 \times 35$

$\therefore \quad k = 35$

By substituting the value of k, $6k = 6 \times 35 = 210$

and $7k = 7 \times 35 = 245$

Hence the required numbers are 210 and 245.

Eercise 9.1

1. If a:b = 1:2 then find the value of (3a + b) : (4a + 2b).
2. If a:b = 3:4 then find the value of (5a + 3b) : (5a + 4b).
3. If a:b = 4:5 then find the value of (6a – 3b) : (5a + 2b).
4. If (3a – 5b) : (3a + 5b) = 1:4 then find the value of a : b.
5. If (5a – 3b) : (7a – 4b) = 9:13 then find the value of a : b.
6. Two numbers are in the ratio 3:2 and their difference is 5. Find the numbers.
7. Two numbers are in the ratio 7:9 and their sum is 80. Find the numbers.
8. Two numbers are in the ratio 9:13 and their difference is 20. Find the numbers.
9. The present age of a father and son are in the ratio 5:3 and the difference is 30. Find their present ages.
10. The present ages of mother and daughter is in the ratio 3:1 After 5 years, if the ratio of their ages will be 5:2, find their present ages.
11. The ratio of the present ages of two persons is 4:5. Before 6 years, if the ratio of their ages was 7:9, find their present ages.
12. The ratio of the present market price rate per kg of oyster mushroom and white button mushroom is 1:3 If the prices of mushroom rises Rs. 50 per kg, the ratio of their market price becomes 3:7. What is the present market price rate of each type of mushroom? Find it.
13. Divide Rs. 810 in the ratio of 7:8.
14. Divide 192 oranges in 7:9 ratio.
15. Two numbers are in the ratio 8:5. If 6 is added to each number, the ratio becomes 3:2. Find the numbers

9.2 Continued Proportion

Let's consider the roll numbers of class - 9 students as follows :

2, 3, 4, 6, 8, 12, 16, 24, 1, 9, 27

How many different ratios can be formed from the above numbers? How many proportions can be formed from those ratios? In class, discuss with your friend and list out ratio, proportion and continued proportions. Here are some examples of proportions $\frac{2}{4}=\frac{4}{8}$ and $\frac{3}{6}=\frac{6}{12}=\frac{12}{24}$

What are the characteristics in these proportion as compared to previous lesson? DIscuss and find out the characteristics. Do these characteristics help us to understand continued proportions? Discuss.

Here, the proportion, which can be expressed as $\frac{2}{4}=\frac{4}{8}=\frac{8}{16}$, is called continued proportion. If we can show $\frac{a}{b}=\frac{b}{c}=\frac{c}{d}$ from a, b, c and d, then a, b, c, and d are said to be in continued proportion.

Example 1

If $\frac{a}{b}=\frac{c}{d}$ then prove that $\frac{a+b}{a-b}=\frac{c+d}{c-d}$

Solution :

Let $\frac{a}{b}=\frac{c}{d}=k$

Then, $\frac{a}{b}=k$ and $\frac{c}{d}=k$

or, a = bk and c = dk

Now, substituting the values of a and c we get.

$$\text{LHS} = \frac{a+b}{a-b} = \frac{bk+b}{bk-b} = \frac{b(k+1)}{b(k-1)} = \frac{k+1}{k-1}$$

$$\text{and RHS} = \frac{c+d}{c-d} = \frac{dk+d}{dk-d} = \frac{d(k+1)}{d(k-1)} = \frac{k+1}{k-1}$$

$\therefore$ Hence LHS = RHS proved.

Example 2

If $\frac{a}{b}=\frac{c}{d}$, then prove that $\frac{7a-5b}{7a+5b}=\frac{7c-5d}{7c+5d}$

Solution: Let, $\frac{a}{b}=\frac{c}{d}$ = k and,

Then, a = bk and c = dk

$$\text{LHS} = \frac{7a-5b}{7a+5b}$$

$$= \frac{7.bk-5b}{7.bk+5b}$$

$$= \frac{b(7k-5)}{b(7k+5)}$$

$$= \frac{7k-5}{7k+5}$$

and $\text{RHS} = \frac{7c-5d}{7c+5d}$

$$= \frac{7.dk-5d}{7.dk+5d}$$

$$= \frac{d(7k-5)}{d(7k+5)}$$

$$= \frac{7k-5}{7k+5}$$

Hence LHS = RHS proved.

Example 3

If $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}$ then, prove that $\frac{a+2c+3e}{b+2d+3f}=\sqrt[3]{\frac{ace}{bdf}}$

Solution :

Let, $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}$ = k

Then, a = bk, c = dk and e = fk

Now, $\text{LHS} = \frac{a+2c+3e}{b+2d+3f}$

$$= \frac{bk+2dk+3fk}{b+2d+3f}$$

$$= \frac{k(b+2d+3f)}{b+2d+3f}$$

$$= k$$

and $\text{RHS} = \sqrt[3]{\frac{ace}{bdf}}$

$$= \sqrt[3]{\frac{bk.dk.fk}{bdf}}$$

$$= \sqrt[3]{k^3}$$

$$= k$$

$\therefore$ Hence LHS = RHS proved.

Example 4

If $\frac{a}{b}=\frac{b}{c}$, then prove that $\frac{a^2+b^2+2ab}{b^2+c^2+2bc}=\frac{a}{c}$

Solution: Let $\frac{a}{b}=\frac{b}{c}$ = k

Then, b = ck and a = bk = ck.k = ck^2

Now **L.H.S** = $\frac{a^2+b^2+2ab}{b^2+c^2+2bc}$

$= \frac{(ck^2)^2+(ck)^2+2ck.ck^2}{(ck)^2+c^2+2.ck.c}$

$= \frac{c^2k^2(k^2+1+2k)}{c^2(k^2+1+2k)}$

$= k^2$

and **R.H.S** = $\frac{a}{c}$

$= \frac{ck^2}{c}$

$= k^2$

∴ Hence LHS = RHS proved.

Example 5

If $\frac{a}{b}=\frac{b}{c}=\frac{c}{d}$, then prove that $\frac{a^3+b^3+c^3}{b^3+c^3+d^3}=\frac{a}{d}$

Solution:

Let, $\frac{a}{b}=\frac{b}{c}=\frac{c}{d}$ = k

Then, c = dk

b = ck = dk.k = dk^2

a = bk = dk^2.k = dk^3

Now, LHS = $\frac{a^3+b^3+c^3}{b^3+c^3+d^3}$

$= \frac{d^3k^9+d^3k^6+d^3k^3}{d^3k^6+d^3k^3+d^3}$

$= \frac{d^3k^3(k^6+k^3+1)}{d^3(k^6+k^3+1)}$

$= k^3$

and RHS = $\frac{a}{d}$

$= \frac{dk^3}{d}$

$= k^3$

∴ Hence LHS = RHS proved.

Exercise 9.2

(a) If $\frac{a}{b}=\frac{c}{d}$, then prove that.

1. $\frac{a-b}{a+b}=\frac{c-d}{c+d}$

2. $\frac{5a+3b}{5a-3b}=\frac{5c+3d}{5c-3d}$

3. $\frac{7a+5c}{7a-5c}=\frac{7b+5d}{7b-5d}$

4. $\frac{a^2+ab+b^2}{a^2-ab+b^2}=\frac{c^2+cd+d^2}{c^2-cd+d^2}$

5. $\frac{a^2+b^2}{a^2-b^2}=\frac{c^2+d^2}{c^2-d^2}$

6. $\frac{a^2+c^2}{b^2 \top d^2}=\frac{a^2-c^2}{b^2-d^2}$

7. $\frac{a^2+c^2}{b^2+d^2}=\frac{ca}{db}=\frac{c^2}{d^2}$

8. $\frac{(a+c)^2}{(b+d)^2}=\frac{a^2+c^2}{b^2+d^2}=\frac{c^2}{d^2}=\frac{a^2}{b^2}$

9. $\frac{a^2-ca+c^2}{b^2-bd+d^2}=\frac{(c+a)^2}{(b+d)^2}$

10. $\frac{(c+a)^3}{(b+d)^3}=\frac{b(a-c)^4}{a(b-d)^4}$

(b) If $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}$, then prove that

1. $\frac{a+c+e}{b+d+f}=\frac{e}{f}$

2. $\frac{a^2+c^2+e^2}{b^2+d^2+f^2}=\frac{ca}{bd}$

3. $\frac{a^3+c^3+e^3}{b^3+d^3+f^3}=\frac{ace}{bdf}$

4. $\left(\frac{a+c+e}{b+d+f}\right)^3=\frac{ace}{bdf}$

5. $\frac{a^3+c^3+e^3}{b^3+d^3+f^3}=\left(\frac{a-2c+3e}{b-2d+3f}\right)^3$

6. $\frac{a^2+c^2+e^2}{b^2+d^2+f^2}=\frac{ac+ce+ea}{bd+df+fb}$

7. $\sqrt{\frac{a^3c^3+c^3e^3+e^3a^3}{b^3d^3+d^3f^3+f^3b^3}}=\frac{ace}{bdf}$

8. $\frac{a^2+c^2+e^2+2ac+2ce+2ea}{b^2+d^2+f^2+2bd+2df+2fb}=\frac{ac}{bd}$

9. $\frac{a^3-c^3-e^3}{b^3-d^3-f^3}=\left(\frac{a-c}{b-d}\right)^3$

10. $\frac{a^2c-c^2e-e^2a}{b^2d-d^2f-f^2b}=\frac{ace}{bdf}$

(c) If $\frac{a}{b}=\frac{b}{c}$ then prove that.

1. $\frac{a-b}{b-c}=\sqrt{\frac{a^2+b^2}{b^2+c^2}}$

2. $\frac{a(a-b)}{b(b-c)}=\frac{a^2+b^2}{b^2+c^2}$

3. $\frac{a(a-b)}{c(b-c)}=\frac{a^3-b^3}{b^3-c^3}$

4. $\frac{a^2+ab+b^2}{b^2+bc+c^2}=\frac{a}{c}$

5. $\frac{a(a+b)}{c(b+c)}=\sqrt{\frac{a^3}{c^3}}$

6. $abc(a+b+c)^3=(ab+bc+ca)^3$

7. $a^2b^2c^2\left(\frac{1}{a^3}+\frac{1}{b^3}+\frac{1}{c^3}\right)=a^3+b^3+c^3$

8. $\frac{a^3+b^3}{b^3+c^3}=\frac{a(a+b)}{c(b+c)}$

9. $\frac{a^3+b^3+c^3}{a^2b^2c^2}=\frac{1}{a^3}+\frac{1}{b^3}+\frac{1}{c^3}$

10. $abc(a^3-b^3-c^3)=(a^3b^3-b^3c^3-c^3a^3)$

(d) If $\frac{a}{b}=\frac{b}{c}=\frac{c}{d}$ then prove that.

1. $\frac{a^3+b^3+c^3}{b^3+c^3+d^3}=\frac{bc}{d^2}$

2. $\frac{a^2b+b^2c+c^2a}{b^2c+c^2d+d^2b}=\frac{a}{d}$

3. $\frac{a^3+b^3+abc}{b^3+c^3+bcd}=\frac{a^3+b^3+c^3}{b^3+c^3+d^3}$

4. $\frac{a^2+b^2+c^2}{b^2+c^2+d^2}=\frac{a-b}{c-d}$

5. $\frac{a^2b+b^2c+c^2a}{b^2c+c^2d+d^2b}=\frac{a^3+b^3+c^3}{b^3+c^3+d^3}$

6. $\frac{a-b}{b-c}=\frac{b-c}{c-d}$

7. $\frac{a^3}{b^3}=\frac{a}{d}$

8. $\frac{(a-b)^2}{ab}=\frac{(c-d)^2}{cd}$

9. $\frac{a^2-ab}{b^2}=\frac{b^2-bc}{c^2}=\frac{c^2-cd}{d^2}$

10. $\sqrt{(a-b-c)(b-c-d)}=\sqrt{ab}-\sqrt{bc}-\sqrt{cd}$

(e) If a, b and c are in continued proportion, then prove that, $\frac{a}{c}=\frac{a^2}{b^2}$.

(f) If a, b, c, d, e and f are in continued proportion, then prove that $\frac{a}{f}=\frac{a^5}{b^5}$.

Lesson 10 Linear Equation

10.0 Review

A customer pays Rs. 1050 for 3 kg of oyster mushroom and 2 kg of white button mushroom. But another customer pays Rs. 1200 for 2 kg of oyster mushroom and 3 kg of white button mushroom. Then find the cost of each types of mushroom. Find the correct view. Can you solve this problem in the algebric expressions or not ? If yes, how can this problem be represented by the algebric expressions ? Show your teacher and take advise. On the basis of above price of mushrooms, let's express and solve this problem using linear equation.

i. Cost of 3kg oyster mushroom and 2kg white button mushroom = Rs. 1050

ii. Cost of 2kg oyster mushroom and 3kg white button mushroom = Rs. 1200

Suppose, cost of 1 kg. of oyster mushroom = Rs. x

and cost of 1 kg. of white button mushroom = Rs. y

So we have, $3x + 2y = 1050$ … … … (i)

and $2x + 3y = 1200$ … … … (ii)

In the above equations, x and y represents as variables. Since, these equations represented the first degree equations, they lie on the straight line in the graph. The value of the variables x and y satisfied in both equations. Finding the values of x and y is known as solution of equations. If two equations usually have only one pair of solution which satisfies both the equations are called "simultaneous equations". Here, simultaneous equations are also known as "simultaneous equations in two variables" as they contain two variables.

10.1 Methods of Solving Simultaneous Linear Equations

Let $x + y = 12$ and $x - y = 2$ are two equations. Here $x - y = 2$ from the 2nd equation, we get $x = y + 2$ and putting the value of x in the 1st equation, how can you find the value of y. Find it.

The methods to solve simultaneous equations are as follows :

(a) Substitution method

(b) Elimination method

(c) Graphical Method

10.1.1 Substitution Method

Example 1

Solve :

$3x + 2y = 1050$ … … … (i)

$2x + 3y = 1200$ … … … (ii)

Solution, Here from equation (i)

$3x = 1050 - 2y$

Or, $x = \frac{1050 - 2y}{3}$

Now, substituting the value of x in equation (ii) we get,

$$\frac{2(1050 - 2y)}{3} + 3y = 1200$$

Or, $\frac{2100 - 4y + 9y}{3} = 1200$

Or, $2100 + 5y = 3600$ [Cross multipication]

Or, $5y = 3600 - 2100$

Or, $y = 1500/5$

Or, $y = 300$

$\therefore$ $y = 300$

Now, substituting the value of y in equation (ii)

[But we can also substitute the value of y in any of the equation (i) or (ii)]

i.e. $2x + 3 \times 300 = 1200$

Or, $2x + 900 = 1200$

Or, $2x = 1200 - 900$

Or, $2x = 300$

Or, $x = \frac{300}{2}$

$\therefore$ $x = 150$

Let us check the solutions;

In the First equation :

$$3x + 2y = 1050$$

Or, $3 \times 150 + 2 \times 300 = 1050$

Or, $450 + 600 = 1050$

Or, $1050 = 1050$

$\therefore$ L.H.S = R.H.S

Again, in the second equation :

$$2x + 3y = 1200$$

Or, $2 \times 150 + 3 \times 300 = 1200$

Or, $300 + 900 = 1200$

Or, $1200 = 1200$

$\therefore$ L.H.S. = R.H.S.

Here, the values of x and y satisfied in both the equations. So, the process and solution is correct.

10.1.2 Elimination Method

The total cost of 4 small and 3 large copies is Rs. 100. The total cost of 5 small and 2 large copies is Rs. 90. Find the cost of each small and large copy using elimination method.

Let the cost of small copy be Rs. x and the cost of large copy be Rs. y.

Then, $4x + 3y = 100$ … … … (i)

and $5x + 2y = 90$ … … … (ii)

Now to eliminate y, multiplying equation(i) by 2 and equation (ii) by 3, we get

$[4x + 3y = 100] \times 2$

and $[5x + 2y = 90] \times 3$

Or, $8x + 6y = 200$ … … … (iii)

$15x + 6y = 270$ … … … (iv)

Now,, subtracting equation (iii) from equation (iv), we have

$$\therefore \quad 15x + 6y = 270 \; \dots \dots \dots \text{(iv)}$$

$$\underline{\underset{-}{(}8x \underset{-}{+} 6y \underset{-}{=} 200) \; \dots \dots \dots \text{(iii)}}$$

$$7x \qquad = 70$$

Or, $x = \dfrac{7 \times 10}{7}$

$\therefore x = 10$

Now, substituting the value of x in equation (ii), we get

$$5 \times 10 + 2y = 90$$

or, $2y = 90 - 50$

or, $y = \dfrac{40}{2}$

$\therefore$ $y = 20$

Hence the price of small copy is Rs. 10 and the price of large copy is Rs. 20.

Example 2

Solve : $3x + 2y = 11 \; \dots \dots \dots$ (i)

$4x - 3y = 9 \; \dots \dots \dots$ (ii)

Solution : Here multiplying equation (i) by 3 and equation (ii) by 2, we get the coefficients of y same in both the equations.

Here, $[3x + 2y = 11] \times 3$

$[4x - 3y = 9] \times 2$

Or, $9x + 6y = 33 \; \dots \dots \dots$ (iii)

and $8x - 6y = 18 \; \dots \dots \dots$ (iv)

Let us add equation (iii) and (iv),

$$\therefore \quad 9x + 6y = 33$$

$$+ \quad \underline{8x - 6y = 18}$$

$$17x \qquad = 51$$

or, $x = \dfrac{51}{17} = \dfrac{3 \times 17}{17}$

$\therefore$ $x = 3$

Now, substituting the value of x in equation (ii), we get

$$4 \times 3 - 3y = 9$$

Or, $12 - 3y = 9$

Or, $3 = 3y$

$\therefore$ $y = 1$

To check the solution, put $x = 3$ and $y = 1$ in the given equations.

In the first equation $3x + 2y = 11$

Or, $3 \times 3 + 2 \times 1 = 11$

Or, $9 + 2 = 11$

Or, $11 = 11$

Hence, L.H.S. = R.H.S.

Again in the second equation $4x - 3y = 9$

Or, $4 \times 3 - 3 \times 1 = 9$

Or, $12 - 3 = 9$

Or, $9 = 9$

Hence, L.H.S. = R.H.S.

Hence the values of x and y are correct.

10.1.3 Graphical Method

The pair of solutions of each equation are plotted in a graph and by joining them two separate straight lines are obtained. The coordinates of the point of intersection of two straight lines are the solutions of the given equations. Therefore, the process of finding the coordinates of the point of intersection of two straight lines in graph is known as graphical method.

Example 1

Solve graphically $2x + 3y = 12$ … … … (i) and $x + 2y = 7$ … … … (ii)

Solution : Here, First of all the equations (i) and (ii) are expressed in the form of $y = mx + c$ to plot on the graph.

$2x + 3y = 12$ and $x + 2y = 7$

$3y = -2x + 12$ $2y = 7 - x$

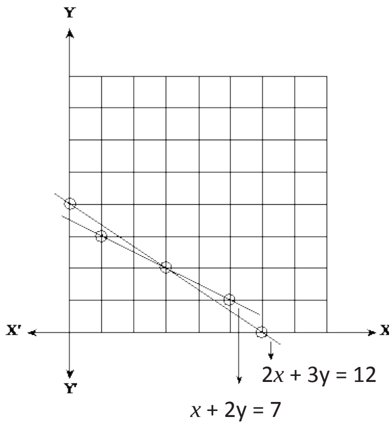
$\therefore\ Y = \dfrac{-2x+12}{3}$ $\therefore\ Y = \dfrac{-x+7}{2}$

The different values of y can be obtained by choosing the different values of x, which can be shown in the following tables.

x	0	3	6
y	4	2	0

x	1	3	5
y	3	2	1

Here, the common point of two straight lines is (3, 2).



The coordinates of the point of intersection of the two straight lines represented by the equations is (3, 2). Therefore, the solution of the equatios is (3, 2).

Hence , x = 3 and y = 2.

Exercise 10

(a) Solve the pair equations by substitution method and check the solutions.

1. $x + y = 7$, $3x + y = 15$
2. $3x - 2y = 11$, $x + 3y = 11$
3. $3x - 2y = 3$, $3x + 5y = 24$
4. $3x - 2y = 8$, $x + 2y = 8$
5. $4x - 3y = 2$, $3x + 4y = 39$
6. $5x - 2y = 20$, $3x + 5y = 43$
7. $3x + 7y = 13$, $9x + y = 19$
8. $4x - 3y = -1$, $3x + 2y = 12$
9. $y = 5x + 1$, $2x - 5y = -51$
10. $3x + 4y = 27$, $7x - 2y = 29$

(b) Solve the pair equations by elimination method and check the solutions.

1. $7x + 9y = 41$, $2x + 3y = 13$
2. $5x + 6y = 27$, $3x + 4y = 17$
3. $2x - y = 1$, $y = 3x$
4. $3x - 2y = 11$, $x - 4y = -3$
5. $3x + 2y = 100$, $x + 4y = 120$
6. $9x - 8y = 12$, $2x + 3y = 17$
7. $x - 6y + 36 = 0$, $3x - 4y = 4$
8. $3x + 2y - 340 = 0$, $2x + 3y = 360$
9. $2x + y - 12 = 0$, $y = -0.5x + 6$
10. $x + 2y = 7$, $2x - y = 4$

(c) Solve the pair of linear equations by graphical method and check the solutions.

1. $x + y = 16$, $x - y = -4$
2. $3x + y = 15$, $2x + 3y = 17$
3. $3x - 2y = 4$, $5x - y = 23$
4. $4x - 3y = 6$, $3x - 4y = 1$
5. $x + y = 25$, $x - y = 5$
6. $x + y = 9$, $x + 2y = 12$
7. $2x + y = 6$, $x + 2y = 6$
8. $x + 2y = 10$, $2x - y = 0$
9. $5x + 2y = 165$, $5x - 3y = 65$
10. $3x + 2y = 8$, $2x + y = 5$

(d) If the total cost of 2 kg of chicken and 1 kg of mutton is Rs. 1500. And the total cost of 1 kg of chicken and 2 kg of mutton is Rs. 1950. Find the cost of per kg chicken and mutton.

(e) The breadth of a rectangular play ground of a school is one-third of its length. If the perimeter of the play ground is 32 m, find its length and breadth.

Lesson

11 Quadratic Equation

11.0 Review

Nima said to her sister,"I have some money but if you give me Rs. 5, I will have Rs. 10." can it be solved in equation form? Discuss.

Which of $0x^2 + 3x + 5 = 0$ and $x^2 - 12x + 20 = 0$ is a quadratic equation? Is $ax^2 + bx + c = 0$ a quadratic equation? Discuss. In $x^2 - 25 = 0$, out of - 5, 0, 5, 25 which is the value of x? The equation $x^2 - 25 = 0$ is said to be Pure Quadratic Equation and the equation $x^2 - 5x + 6 = 0$ is said to be adfected Quadratic Equation. Thus, $ax^2 - c = 0$ as a pure quadratic equation in which the second variable/term bx is not included. The general or standard form of quadratic equation is $ax^2 + bx + c = 0$. Since the maximum power of variable x is 2, this equation is known as Second Degree. In this type of equation, x has always two values. The values of x are identified as roots of the quadratic equation. Quadratic equation can be solved by different methods. The three methods to solve the quadratic equation are as given below ;

i. Factorization Method.

ii. Completing square Method.

iii. Formulae using Method.

11.1 Solution of the Quadratic Equation by Factorization Method

Example 1

Solve : $x^2 - 3x + 2 = 0$

Solution : Here, $x^2 - 3x + 2 = 0$

Or, $x^2 - (2 + 1)x + 2 = 0$

Or, $x^2 - 2x - x + 2 = 0$

Or, $x(x - 2) - 1(x - 2) = 0$

Or, $(x - 2)(x - 1) = 0$

Either, $x - 2 = 0$ ... (i), Or, $x - 1 = 0$ ... (ii)

or $x = 2$ and $x = 1$

Here from the first equation, the value of x is 2 and from the second equation, 1 is the value of x.

Example 2

Solve : $x^2 - 5x + 6 = 0$

Solution : Here, $x^2 - 5x + 6 = 0$

Or, $x^2 - 3x - 2x + 6 = 0$

Or, $x(x - 3) - 2(x - 3) = 0$

Or, $(x - 3)(x - 2) = 0$

Either, $x - 3 = 0$ Or, $x - 2 = 0$

Or, $x = 3$ Or, $x = 2$

Hence the roots of the quadratic equation are 2 and 3.

Example 3

Solve : $x^2 + 3x + 2 = 0$

Solution : Here, $x^2 + 3x + 2 = 0$

Or, $x^2 + 2x + x + 2 = 0$

Or, $x(x + 2) + 1(x + 2) = 0$

Or, $(x + 2)(x + 1) = 0$

Either $x + 2 = 0$ ... ... (i),

Or, $x + 1 = 0$ ... ... (ii)

From equation (i), $x = -2$ and from equation (ii) $x = -1$

$\therefore$ Hence -1 and -2 are the roots of the quadratic equation.

Example 4

Solve : $x^2 + 4x + 4 = 0$

Solution : Here, $x^2 + 4x + 4 = 0$

Or, $x^2 + 2x + 2x + 4 = 0$

Or, $x(x + 2) + 2(x + 2) = 0$

Or, $(x + 2)(x + 2) = 0$

Either $x + 2 = 0$ … … … (i) ,

Or, $x + 2 = 0$ … … … (ii)

Here, from both the equations x = - 2. Hence, x = - 2 is the root of the given quadratic equation.

11.2 Solving Quadratic Equation by Completing Square

Example 5

Solve the quadratic equation by completeing square : $x^2 - 4x = -4$

Solution :

Here, $x^2 - 4x = -4$

Or, $x^2 - 4x + 4 = 0$

Or, $x^2 - 2.x.2 + 2^2 = 0$

Or, $(x - 2)^2 = 0$

Either, $x - 2 = 0$ Or, $x - 2 = 0$

$\therefore$ $x = 2$ and $x = 2$

Hence, $x = 2$ is the root of $x^2 - 4x = -4$

Example 6

Solve the quadratic equation by completing square : $x^2 - 7x + 12 = 0$

Solution :

Here, $x^2 - 7x + 12 = 0$

Or, $x^2 - 7x = -12$

Or, $x^2 - 2.x.\frac{7}{2} + \left(\frac{7}{2}\right)^2 = -12 + \left(\frac{7}{2}\right)^2$ [Adding $\left(\frac{7}{2}\right)^2$ in both sides]

Or, $$\left(x-\frac{7}{2}\right)^2=\frac{-12\times4+49}{4}$$ [$\because$ $a^2 - 2ab + b^2 = (a - b^2)$]

Or, $$\left(x-\frac{7}{2}\right)^2=\frac{1}{4}$$

Or, $$\left(x-\frac{7}{2}\right)^2=\left(\pm\frac{1}{2}\right)^2$$

$\therefore$ $x-\frac{7}{2}=\pm\frac{1}{2}$

Now, taking +ve sign,

$x-\frac{7}{2}=\frac{1}{2}$

Or, $x=\frac{1}{2}+\frac{7}{2}$

Or, $x=\frac{1+7}{2}$

Or, $x=\frac{8}{2}$

$\therefore$ $x = 4$

And, taking -ve sign,

$x-\frac{7}{2}=-\frac{1}{2}$

Or, $x=-\frac{1}{2}+\frac{7}{2}$

Or, $x=\frac{-1+7}{2}$

Or, $x=\frac{6}{2}$

$\therefore x = 3$

$\therefore$ Hence 3 and 4 are the roots of $x^2 - 7x + 12 = 0$

Example 7

Solve the quadratic equation $ax^2 + bx + c = 0$ by completing square.

Solution : Here, $ax^2 + bx + c = 0$

Or, $ax^2 + bx = -c$

Or, $\frac{ax^2}{a}+\frac{bx}{a}=\frac{-c}{a}$ [Dividing by a]

Or, $x^2 + 2.x.\frac{b}{2a}+\left(\frac{b}{2a}\right)^2=\frac{-c}{a}+\left(\frac{b}{2a}\right)^2$ [Adding $\left(\frac{b}{2a}\right)^2$ in both sides]

Or, $\left(x+\frac{b}{2a}\right)^2=-\frac{c}{a}+\frac{b^2}{4a^2}$ [$(a + b)^2 = a^2 + 2ab + b^2$]

Or, $\left(x+\frac{b}{2a}\right)^2=\frac{-4ac+b^2}{4a^2}$

Or, $\left(x+\frac{b}{2a}\right)^2=\left(\pm\frac{\sqrt{b^2-4ac}}{2a}\right)^2$

Or, $x+\frac{b}{2a}=\pm\frac{\sqrt{b^2-4ac}}{21}$

Or, $x=\frac{-b}{2a}\pm\frac{\sqrt{b^2-4ac}}{2a}$

$\therefore \quad x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

Hence the roots of $ax^2 + bx + c = 0$ are $\frac{-b+\sqrt{b^2-4ac}}{2a}$ and $\frac{-b-\sqrt{b^2-4ac}}{2a}$

Hence to solve the quadratic equation $ax^2 + bx + c = 0$, we can use the formula

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

Let, us compare the equation $x^2 - 5x - 6 = 0$ with $ax^2 + bx + c = 0$, we get $a = 1$, $b = -5$ and $c = -6$

From the formula, $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

Now, substituting the values of a, b, and c, we get

$$x=\frac{-(-5)\pm\sqrt{(-5)^2-4(-6)(1)}}{2.1}$$

$$=\frac{5\pm\sqrt{25+24}}{2}$$

$$=\frac{5\pm\sqrt{49}}{2}$$

$$=\frac{5\pm 7}{2}$$

Now, taking +ve sign,

$$x = \frac{5+7}{2}$$

$$= \frac{12}{2}$$

$$= 6$$

And, taking -ve sign,

$$x = \frac{5-7}{2}$$

$$= \frac{-2}{2}$$

$$=-1$$

Hence 6 and - 1 are the roots of $x^2 - 5x - 6 = 0$

11.3 Solving Quadratic Equation Using Formula.

Example 8

Solve using formula : $x^2 - 5x - 24 = 0$

Solution :

Comparing the equation $x^2 - 5x - 24 = 0$ with $ax^2 + bx + c = 0$ we get a = 1, b = –5 and c = –24

Then, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4.(-24).1}}{2 \times 1}$$ [Substituting the value of a, b, and c]

$$= \frac{5 \pm \sqrt{25+96}}{2}$$

$$= \frac{5 \pm \sqrt{121}}{2}$$

$$= \frac{5 \pm 11}{2}$$

Now, taking +ve sign

$$x = \frac{5+11}{2}$$

$$= \frac{16}{2}$$

$$= 8$$

And, taking -ve sign

$$x = \frac{5-11}{2}$$

$$= \frac{-6}{2}$$

$$= -3$$

$\therefore$ Hence –3 and 8 are the roots of $x^2 - 5x - 24 = 0$

Exercise 11

1. Solve the quadratic equations and check the solutions.

   i. $(x-1)(x+2)=0$ ii. $(x-2)(x-3)=0$

   iii. $(x-5)(x-3)=0$ iv. $(3x-9)(x-5)=0$

2. Solve by factorization method.

   i. $x(2x+1)=3x$ ii. $x(x-3)+4x=0$

   iii. $x^2-x-12=0$ iv. $x^2-x-20=0$

   v. $x^2+11x+30=0$ vi. $x^2-7x+12=0$

   vii. $x^2-2x-35=0$ viii. $x^2-13x+42=0$

   ix. $-x^2+16x-63=0$ x. $x^2=1225$

3. Solve by completing the square.

   i. $x^2-10x+25=0$ ii. $x^2-18x+81=0$

   iii. $x^2-4x-21=0$ iv. $x^2-4x-45=0$

   v. $-x^2+4x+77=0$ vi. $x^2+117=22x$

   vii. $x^2+\frac{x}{12}=\frac{1}{2}$ viii. $x^2-2x+\frac{3}{4}=0$

   ix. $x^2+\frac{15}{16}=2x$ x. $x^2+\frac{2x}{3}=\frac{35}{9}$

4. Solve by using the formula.

   i. $x^2-10x+21=0$ ii. $x^2-17x+72=0$

   iii. $x^2=2x+143$ iv. $x^2-30x+221=0$

   v. $x^2+3x=28$ vi. $x^2-5x=66$

   vii. $x\left(x+\frac{2}{7}\right)=\frac{3}{49}$ viii. $x^2+2x=323$

   ix. $\frac{1}{x-2}+\frac{1}{x+3}=\frac{1}{5}$ x. $\frac{1}{x+2}+\frac{1}{x-3}=\frac{1}{10}$

Lesson 12 Triangle

12.0 Review

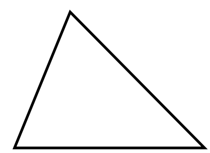
Let us take 5 thin bamboo (wooden) sticks of lengths 1 inch, 2 inches, 3 inches, 4 inches and 5 inches. In how many ways can we form triangular shapes by joining any three of them? Can 1 inch, 2 inches and 3 inches sticks be joined to form a triangle? If not why? Discuss with your friends to find out correct answer. Draw different types of triangles on your copy with sides having lengths given above with the help of ruler and compass. Now discuss about the lengths of line segments to form triangles.

Definition of a triangle : A closed plane figure bounded by the three line segments is called a triangle. There are various types of triangles. We classify the triangles on the basis of their sides and angles.

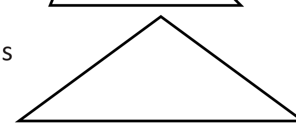
Classification of triangles on the basis of sides

On the basis of sides there are three types of triangles

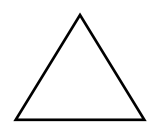
i. Scalene Triangle : A triangle having unequal sides is called scalene triangle.



ii. Isosceles triangle : A triangle having two sides equal is called an isosceles triangle.

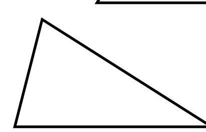


iii. Equilateral Triangle : A triangle having all sides equal is called an equilateral triangle

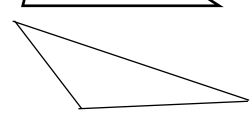


Classification of triangles on the basis of angles

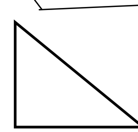
i. Acute - angled triangle : A triangle having all angles less than 90° is called an acute - angled triangle.



ii. Obtuse - angled triangle : A triangle having one angle greater than 90° is called an obtuse - angled triangle.



iii. Right - angled triangle : A triangle having one angle exactly 90° is called a right - angled triangle.



12.1 Verification of Geometrical Properties

Pradeep and Dina study in grade 9. A part of the conversation that took place between them on the second day of geometry class is given below.

Pradeep : What properties does a triangle have?

Dina : I think, properties of a triangle deal about its angle relationship, side relationship and angle - side relationship. For example : the sum of internal angle of a triangle is 180°. The sum of two sides of a triangle is always greater than the third side.

Pradeep : What do you understand by the experimental verification and theoretical proof of geometrical properties?

Dina : So far as I think, the experimental verification is a method of demonstrating geometrical properties on the basis of drawing and measurement of geometrical figures . For example the triangle - angle- sum property that we demonstrated by drawing different triangles and measuring interior angles, add them and conclude results is an example of experimental verification and it is called inductive approach. But in theoretical proof, we need to do differently rather than drawing and measuring. I think we need to demonstrate geometrical properties on the basis of definitions (of geometrical terms), basic properties (such as angle relationships in parallel lines intersected by transversal) together with some self evident truths, axoims or postulates. It is also known as deductive approach.

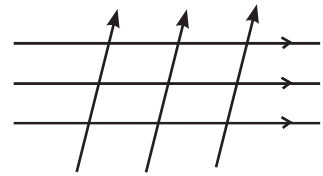
So far as I have heard from our senior friends, such kind of demonstration is called deductive proof which is completely different from experimental verification which is said to be based on inductive method.

Pradeep : You seem to have more knowledge about theoretical proof. Can we make more discussion about it?

Dina : I also think so as you are thinking about.

Teacher : After reading the above conversation that took place between Pradeep and Dina. let us discuss about how we established or proved different geometrical properties in the previous classes and how we will use these approaches.

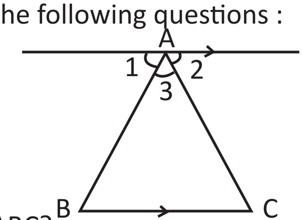
12.2 Angle Relation on Parallel lines



Draw atleast three parallel lines with the help of a ruler and set square in the horizontal direction. Also intersect these lines by atleast three other parallel lines in the oblique (cross) direction. Discuss about the geometrical figures and angles so formed

After this, find out which angles are equal in the figure, the relations between alternate angles, internal angles and external angles. Also find out the sum of internal angles of the triangle ABC as shown in figure.

In the triangle ABC shown in the figure, answer or answer the following questions :



(a) Which are the alternate angles of $\angle 1$ and $\angle 2$?

(b) $\angle ACB + \angle ABC + \angle BAC = \angle 1 + \angle 2 + \angle 3$ (why?)

(c) $\angle 1 + \angle 2 + \angle 3 =$ ....?

(d) How much is the sum of internal angles of the triangle ABC?

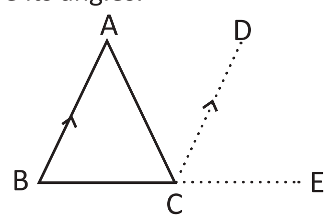
(e) Likewise, draw other different triangles and find out sum of their internal angles. How are the angles made by the parallel lines used to prove that the sum of internal angles of a triangle is 180^0

Theorem 1 : The sum of internal angles of a triangle is equal to 180^0.

Given : ABC is a triangle in which $\angle BAC$, $\angle ABC$ and $\angle ACB$ are its angles.

To prove : $\angle BAC + \angle ABC + \angle ACB = 180°$.

Construction : Produce BC to E and draw CD || BA.

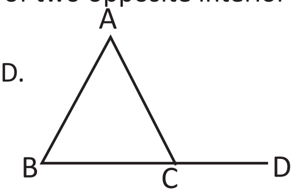


Proof :

Statements	Reasons
1. $\angle BAC = \angle ACD$	1. Alternate angles (BA || CD)
2. $\angle ABC = \angle DCE$	2. Corresponding angles (BA || CD)
3. $\angle BCA + \angle ACD + \angle DCE = 180^0$	3. All angles make a straight angle on one side of the line BE.
4. $\angle BCA + \angle BAC + \angle ABC = 180^0$	4. From statements 1, 2 and 3.

proved.

Theorem 2 : The exterior angle of a triangle is equal to the sum of two opposite interior angles.



Given : ABC is a triangle whose side BC is produced to the point D. $\angle ACD$ is an exterior angle and $\angle ABC$ and $\angle BAC$ are opposite interior (non-adjacent) angles.

To prove : $\angle ACD = \angle ABC + \angle BAC$

Proof :

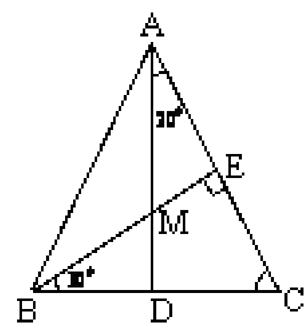
Statements	Reasons
1. $\angle ABC + \angle ACB + \angle BAC = 180°$	1. Sum of interior angles of triangle (theorem -1
2. $\angle ACB + \angle ACD = 180°$	2. The sum of adjacent angles on a straight line (Linear pair)
3. $\angle ABC + \angle ACB + \angle BAC = \angle ACB + \angle ACD$	3. From statements 1 and 2
4. $\angle ABC + \angle BAC = \angle ACD$	4. Subtracting $\angle ACB$ from both sides of statement 3

Proved

Example 1

In the given figure, $BE \perp AC$. If $\angle EBC = 30°$ and $\angle DAC = 20^0$.

find the value of (a) $\angle ACD$ (b) $\angle ADB$ (c) $\angle AMB$



Solution :

Here, $BE \perp AC$. So, $\angle BEC = 90°$

Again, in the triangle BEC, $\angle BEC + \angle EBC + \angle ECB = 180°$

Or, $90° + 30° + \angle ACD = 180°$ [Why, give reasons.]

Or, $\angle ACD + 120° = 180°$ [Why, give reasons.]

Or, $\angle ACD = 180° - 120°$ [Why, give reasons.]

Or, $\angle ACD = 60°$

b) In ΔADC, $\angle ADB = \angle DAC + \angle ACD$ [By theorem 2 and above]

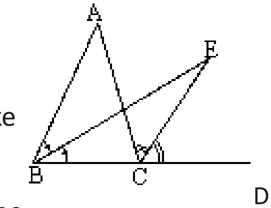
Or, $\angle ADB = 20° + 60^0 = 80^0$

c) In ΔBMD, $\angle AMB = \angle MBD + \angle BDM$

Or, $\angle AMB = 30^0 + 80^0 = 110^0$

Example 2

In the given figure, the side BC is produced to D. If $\angle ABC$ and $\angle ACD$ are bisected by the lines BE and CE respectively, prove that $\angle BAC = 2\angle BEC$.



Solution : Here, $\angle ABC = 2\angle EBC$ and $\angle ACD = 2\angle ECD$ [BE and CE make equal parts of angle]

$\angle BAC + \angle ABC = \angle ACD$ [Exterior angle is equal to the sum of the non-adjacent interior angles]

Or, $\angle BAC = \angle ACD - \angle ABC$

Or, $\angle BAC = 2\angle ECD - 2\angle EBC = 2(\angle ECD - \angle EBC)$...................(i)

$\angle EBC + \angle BEC = \angle ECD$ [Exterior angle is sum of two non adjacent interior angles of the triangle EBC]

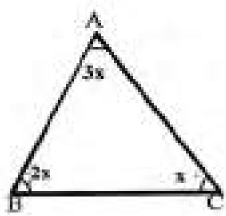
Or, $\angle BEC = \angle ECD - \angle EBC$..(ii)

Or, $\angle BAC = 2\angle BEC$ [From equations (i) and (ii)]

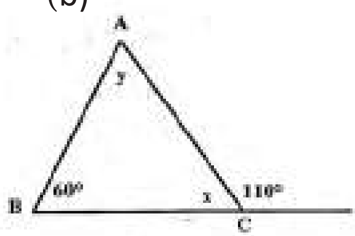
Exercise 12.1

1. Find the values of x,y and z in the following figures.

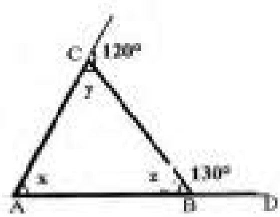
(a)



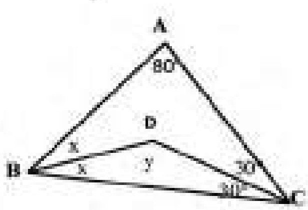
(b)



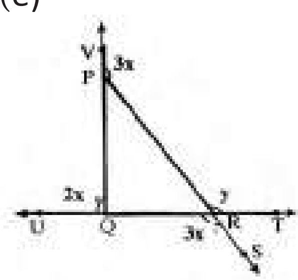
(c)



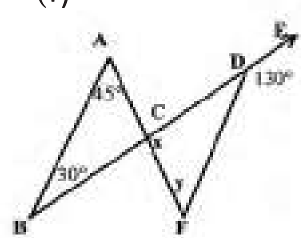
(d)



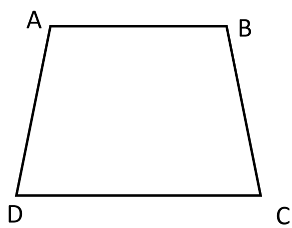
(e)

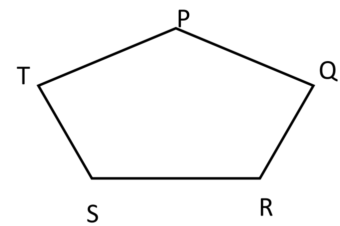


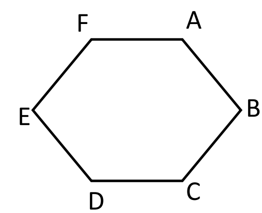
(f)



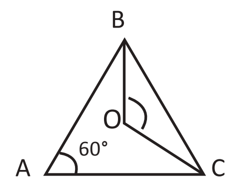
2. By using the property of sum of interior angles of a triangle, find the sum of interior angles of the following polygons.

a) 

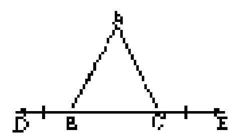
b) 

c) 

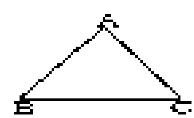
3. In the given figure, BO and CO are the bisectors of $\angle ABC$ and $\angle ACB$ respectively. If $\angle BAC = 60^0$, prove that $\angle BOC = 120^0$



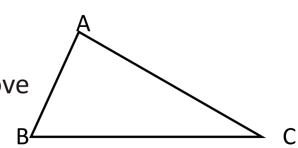
4. In the given figure, the side BC is produced to the points D and E both sides. Prove that $\angle ABE + \angle ACD + \angle BAC = 180°$



5. By producing the sides of a triangle ABC i.e. AB, BC and AC both ways how many different exterior angles are formed? How much is the sum of all three exterior angles? Find their sum. Also find the relation of interior and exterior angle-sums.



6. In the given triangle ABC, draw the straight lines through the vertices A, B and C parallel to BC, CA and AB respectively. Prove that the sum of internal angles of the triangle ABC is 180^0 for each of the parallel lines drawn as above.



12.3 Properties of Isosceles Triangle

An isosceles triangle is a type of triangle in which any two of the sides are equal. Because of two equal sides it becomes a symmetric triangle. As a result, it has many properties. In an isosceles triangle, the angles opposite to equal sides are equal. These properties are proved as theorem. In these theorem, we use the properties of congruent triangles. So, we review some of the conditions for the triangles to be congruent here.

Conditions for congruency of triangles

Discuss about the following questions among the friends and draw conclusions.

(a) What will happen if one triangle perfectly resembles the other triangle?

(b) What are S.A.S, A.S.A, S.S.S conditions? Why are these conditions required?

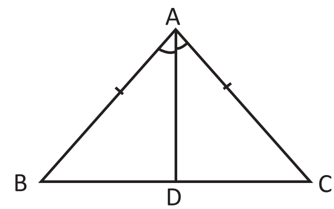
(c) Can we write 'right angle, hypotenuse and one side' as R.H.S?

(d) Can a triangle be perfectly overlapped by another triangle when their two corresponding angles and the sides between them are equal?

Similarly, if two triangles are congruent under the condition S.A.S, can they be congruent under the condition S.S.A.? Verify them.

Can we verify these conditions with the help of compass and ruler? Lets try once.

Theorem - 3



The base angles of an isosceles triangle are equal.
Given : ABC is an isosceles triangle with AB = AC.
To prove : $\angle ABC = \angle ACB$
Construction : Bisect $\angle BAC$ by AD

Proof :

Statements	Reasons
1. In the triangles ABD and ACD	1.
i. AB = AC (S)*	i. Given equal sides
ii. $\angle BAD = \angle CAD$ (A)*	ii. AD bisects $\angle BAC$, from construction
iii. AD = AD (S)*	iii. Common side
2. $\Delta BAD \cong \Delta CAD$	2. S.A.S. condition
3. $\angle ABD = \angle ACD$	3. Corresponding angles of congruent triangles are equal.

***Note :** S - side, A- angle

Proved.

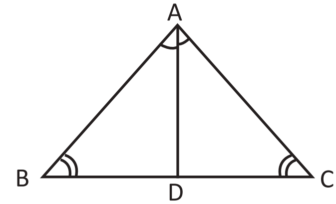
Theorem - 4 (converse of theorem 3)

If two angles of a triangle are equal then their opposite sides are also equal.

Given : In ΔABC, $\angle ABC = \angle ACB$

To prove : AB = AC

Construction : Bisect $\angle BAC$ by AD.



Proof :

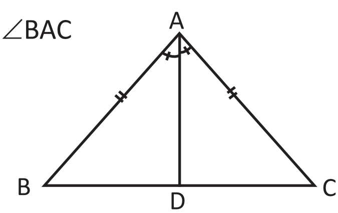
Statements	Reasons
1. In ΔABD and ΔACD	1.
i. $\angle ABD = \angle ACD$	i. Given equal angles.
ii. $\angle BAD = \angle CAD$	ii. AD bisects $\angle BAC$, from construction
iii. AD = AD	iii. Common side
2. $\Delta ABD \cong \Delta ACD$	2. A.A.S. condition
3. AB = AC	3. The corresponding sides of congruent triangles are equal.

Proved.

Theorem - 5 : The bisector of the vertical angle of an isosceles triangle bisects the opposite side perpendicularly.

Given : In the isosceles triangle ABC, AB = AC and AD bisects $\angle BAC$

To prove : BD = DC and $AD \perp BC$. ($\angle BDA = \angle CDA = 90^0$)

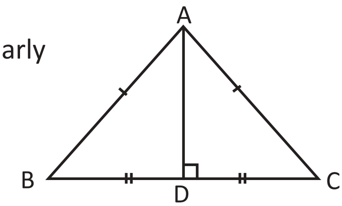


Proof :

Statements	Reasons
1. In ΔABD and ΔACD	1.
i. AB = AC	i. Given equal sides
ii. $\angle BAD = \angle CAD$	ii. Given AD bisects $\angle BAC$
iii. AD = AD	iii. Common side
2. $\Delta ABD \cong \Delta ACD$	2. S.A.S. condition
3. BD = DC	3. Corresponding sides of the congruent triangles ABD and ACD
4. $\angle BDA = \angle CDA$	4. Corresponding angles of the congruent triangles ABD and ACD.
5. $\angle BDA + \angle CDA = 180^0$	5. Sum of adjacent angles on one side of a straight line is equal to 180^0
6. $\angle BDA = \angle CDA = 90^0$	6. From statements 4 and 5
7. $AD \perp BC$	7. From statement 6.

Proved.

Theorem - 6 : In an isosceles triangle, the perpendicular bisector of the base bisect the vertical angle (converse of theorem 5).



Given : In ΔABC, AB = AC and AD bisects BC perpendicularly

To prove : $\angle BAD = \angle CAD$.

Proof :

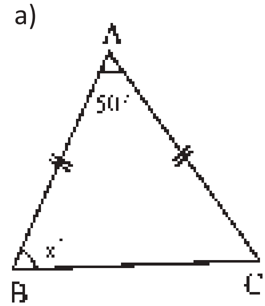
Statements	Reasons
1. In ΔABD and ΔACD	1.
i. $\angle ADB = \angle ADC$ (R)	i. Both right angles, $AD \perp BC$
ii. AB = AC (H)	ii. Given equal sides
iii. BD = DC (S)	iii. AD bisects BC perpendicularly
2. $\Delta ABD \cong \Delta ACD$	2. By R.H.S. axiom
3. $\angle BAD = \angle CAD$	3. Corresponding angles of congruent triangles are equal.

Proved.

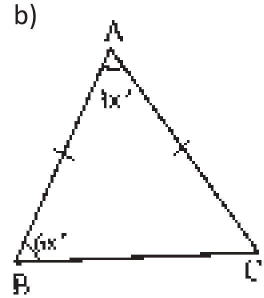
Example 3

In the given figure, if AB = AC, find the of value x°

a)



b)



Solution :

(a) i. $\angle BAC = 50°$, AB = AC [Given]

ii. $\angle ABC = \angle ACB$ [Base angles of isosceles triangles]

iii. $\angle ABC + \angle ACB + \angle BAC = 180°$ [Sum of interior angles of a triangle]

Or, $x° + x° + 50° = 180°$

Or, $2x° = 130°$

Or, $x° = \frac{130°}{2}$ $\therefore$ $x° = 65°$

(b) i. $\angle ABC = 6x°$, $\angle BAC = 3x°$, AB =AC [Given]

ii. $\angle ABC = \angle ACB$ [Base angles of isosceles triangles]

iii. $\angle ABC + \angle ACB + \angle BAC = 180°$ [Sum of interior angles of triangle]

Or, $6x° + 6x° + 3x° = 180°$

Or, $15x° = 180°$

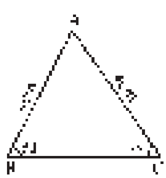
Or, $x^0 = \frac{180^0}{15}$

$\therefore$ $x° = 12°$

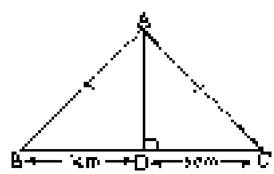
Example 4

According to the information given in the following figures, find the value of x.

(a)



(b)



Solution :

(a) i. $\angle ABC = \angle ACB = 50°$; AC = 4cm, AB = x cm [Given]

ii. AB = AC [Sides opposite to the equal angles of a triangle]

Or, x cm = 4 cm

Or, $x = 4$

(b) i. AB = AC, AD $\perp$ BC, DC = x cm, BD = 3cm [Given]

ii. BD = DC [The perpendicular from the vertex to the base of an isosceles triangle bisect the base]

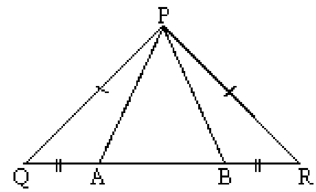
Or, 3cm = x cm

$\therefore$ $x = 3$

Example 5

In the given figure, If **PQ = PR** and **AQ = BR,** prove that

(a) AP = BP (b) $\angle$PAB = $\angle$PBA



Solution :

Given : PQ = PR and AQ = BR

To prove : (a) AP = BP and

(b) $\angle$PAB = $\angle$PBA

Proof:

1. In ΔAPQ and ΔBRP

i. PQ = PR (S)	[Given]
ii. $\angle$PQA =$\angle$PRB (A)	[Base angles of isosceles triangle PQR]
iii. QA = BR (S)	[Given]
2. ΔAPQ $\cong$ ΔBRP	[S.A.S. axiom]
3. AP = BP;	[Corresponding sides of congruent triangles APQ and RPB]
4. $\angle$PAB =$\angle$PBA	[Base angles of isosceles triangle APB] (from st. 3)

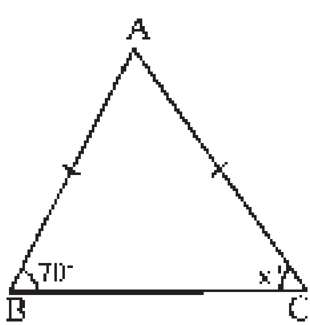
$\therefore$ (a) AP = BP

(b) $\angle$PAB = $\angle$PBA [From 3 and 4]

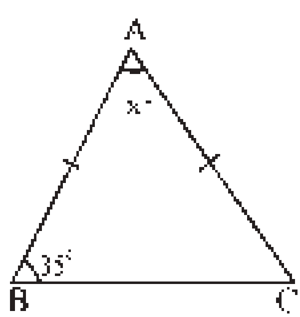
Exercise 12.2

1. On the basis of the information given in the following figures, find the value of x

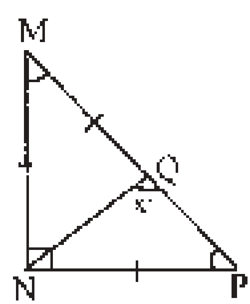
(a)



(b)

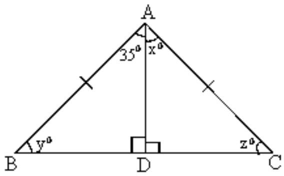


(c)

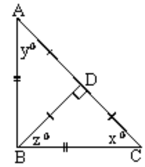


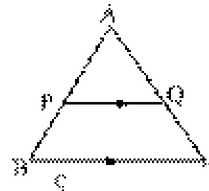
2. Using the information given in the following figures, find the values of x°, y°and z°.

a)

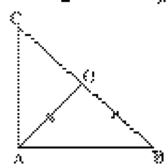


b)

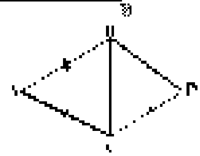




3. (a) In the given figure, **AB = AC** and **PQ //BC,** prove that AP = AQ**.**



(b) In the given figure, AO = OB = OC, prove that $\angle$**BAC = 90⁰**.



(c) In the given figure, if Δ**ABC** and Δ**BCD** are isosceles. Prove that $\angle$**ABD** = $\angle$**ACD** .

4. Prove that all the angles of an equilateral triangle are equal. Also write of statement its converse and prove it.

5. Is equilateral triangle an isosceles triangle? Explain.

6. Make a list of geometrical objects in which isosceles triangles can be seen.

12.3 Relation Between Sides and Angles of a Triangle

Activity : Draw a line segment AB as shown in the figure 1. From the point A draw an arc of certain length with the help of your compass. Show the points C, D, E, F on that arc.

Figure (1)

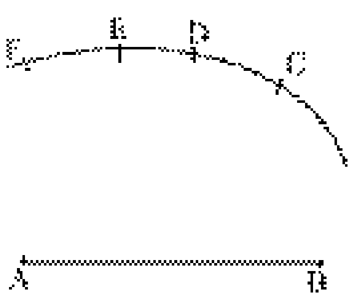
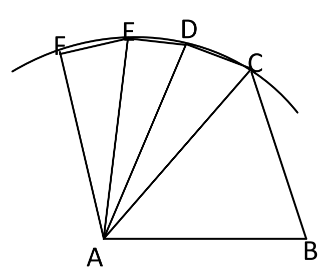
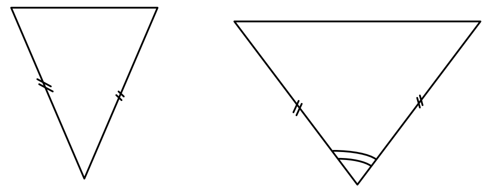


Figure (2)



Join the points C, D, E, and F to the points A and B as shown in the figure 2. Try to answer the following questions by the observation and measurement.

i. How many triangles are formed on the base AB?
ii. Which is the longest side in each triangle?
iii. Which is the shortest side in each triangle?
iv. Which is the greatest angle in each triangle?
v. Which is the smallest angle in each triangle?



vi. How are the measurements of the angles formed on the points A and B changed?

Relation among sides in a triangle

Experimental work 1(a) : Let us distribute some bundles of thin bamboo sticks of different length (from 1 inch to 10 inch) among the group of students. Involve each group of students to make triangular shapes by using the bamboo sticks of different lengths. Tell the students to keep the bamboo sticks which makes or do not make triangular shapes separately. By performing the above activities, examine what relations are found to exist among the sides of the triangles? Try experimentally to draw conclusions in groups and present them in the classroom. After this, practise the following experimental activities.

Experimental work 1(b)

Draw three triangles of different sizes as shown in the figure.

Figure (a)

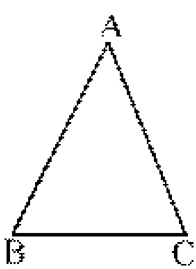


Figure (b)

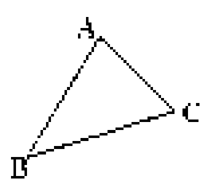
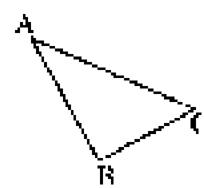


Figure (c)



Measure the lengths of the sides of each triangle and get the sum of length of any two sides. Fill up the following table from the results obtained above.

Triangle	AB	BC	AC	AB + AC	AB + BC	BC + AC
a.						
b.						
c.						

Conclusion ....

Experimental Activity 2 (a)

Draw three scalene triangles of different measurement (size) as shown in the figure.

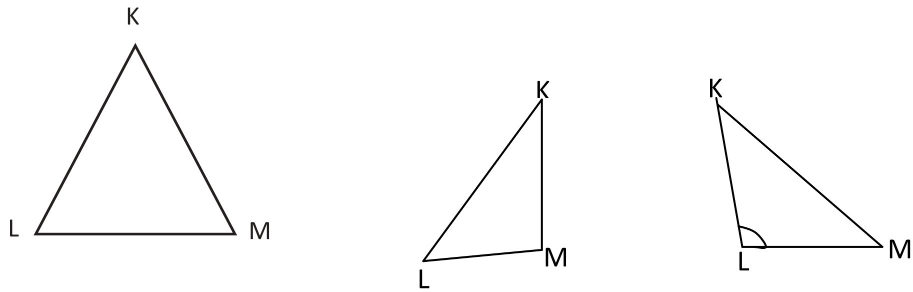


Figure 2(a) Figure 2(b) Figure 2 (c)

Measure the longest side and its opposite angle and shortest side and its opposite angle in each triangle and fill up the following table with these measurements.

Triangle	Shortest side	Angle opposite to shortest side	Longest side	Angle opposite to longest side	Measurement of angles & sides
a.					
b.					
c.					

Conclusion ....

Experimental Activity 2 (b)

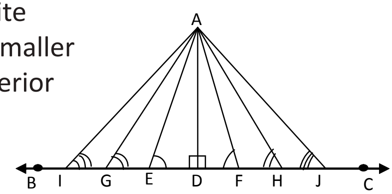
In the above experimental work we measured and observed the relations of longest and shortest sides with their opposite angles. As a conclusion, we found out that the angle opposite to the longest side is greater than the angle opposite to the shortest side. Express the converse of above relation of the sides and the opposite angles and show experimentally that the side opposite to the greater angle is longer than the side opposite to the smaller angle.

Experimental Activity 3

The perpendicular is the shortest line segment among all the line segments drawn from some external point to a given line. With the help of set square, draw a perpendicular AD from A to the line segment BC. On both side of the perpendicular AD, draw other line segments AE and AF, AG and AH, AI and AJ. Out of these line segments which one is the shortest and which one is the longest distance from A to the line segment BC. Discuss among the friends and find the conclusion.

In the given figure, how can you prove that AD < AF< AH < AJ on the basis of their lengths? Explain it.

In any triangle, prove the statement that the side opposite to greater angle is longer than the side opposite to the smaller angle (by using the already proved theorem that the exterior angle of a triangle is equal to interior angle). Also write the converse of the above statements and discuss among the friends.



Relation among the sides of a right-angled triangle

Form groups of students and draw a right-angled triangle in each group by using a set square. Draw the squares on all sides of these right-angled triangles. Find the areas of all these squares and compare the area of the square made on hypotenuse with the areas of the squares made on the other sides. Draw the right angled triangles of sides 3, 4, 5 and 6, 8, 10 cm, on the graph paper and verify the above relations of the squares made on the sides of the right-angled triangles.

Experimental Activity 4

Draw three right-angled triangles of different measurements with the help of set square.

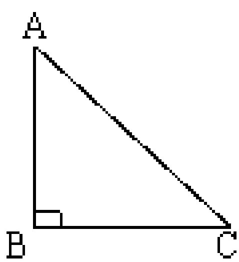


Figure (a)

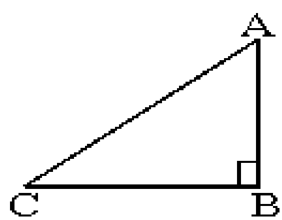


Figure (b)

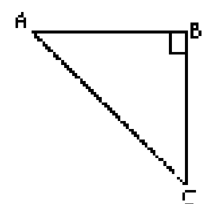


Figure (c)

Measure the lengths of each sides and find their squares. fill up the following table

Figure	AB	BC	AC	$AB^2 + BC^2$	AC^2
a.					
b.					
c.					

Conclusion ....

Compare the above conclusions with the conclusions given in the following table.

Experimental Activity	Conclusion
1	The sum of any two sides of a triangle is greater than the third side.
2	The sides oppsoite to the greater angle is longer than the side opposite to the smaller angle of a triangle.
3	The angle opposite to the longer side is greater than the angle opposite to the smaller side of a triangle.
4	In a right angled triangle the sum of squares of the base and perpendicular is equal to the squares of its hypotenuse.

The conclusion of experimental work 4 is called 'Pythagoras theorem' because it was proved by a mathematician Pythagoras for the first time. It was used not only in geometry but also in every field of mathematics. To make the concept of relation of sides and angles of a triangle more clear discuss on the basis of the following questions among your friends.

(a) While increasing or decreasing the size of angle, what happens to its opposite side?

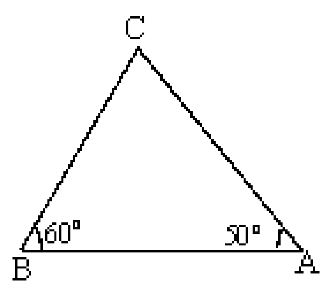
(b) How we apply Pythagoras theorem in taking short routes in the paths?

(c) In any right-angled triangle, the square of hypotenuse = square of base + square of perpendicular. How can you show this relation on graph paper?

(d) While drawing a triangle, what will happen if the sum of two sides is not greater than the third side?

Example 1

In **ΔABC,** if **$\angle$BAC = 50^0** and **$\angle$ABC = 60^0,** which will be the greatest and shortest sides? Find them.



Solution :

Here, $\angle BAC = 50^0$, $\angle ABC = 60^0$.

We know that, $\angle BAC + \angle ABC + \angle ACB = 180^0$

Or, $50^0 + 60^0 + \angle ACB = 180^0$

Or, $110° + \angle ACB = 180^0$

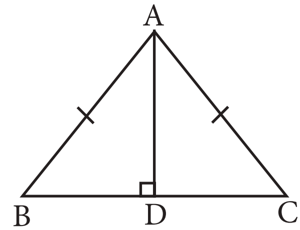
Or, $\angle ACB = 180^0 - 110^0$

Or, $\angle ACB = 70^0$

Here, $\angle$ACB is the greatest and $\angle$BAC is the smallest angle. So, AB is the longest side and BC is the shortest side of triangle ABC.

Example 2

In the given figure, AB = AC = 25cm and BC = 14cm. If AD⊥BC, find the length of AD.



Solution :

Here,

AB = AC = 25cm, BC = 14cm and AD⊥ BC [Given]

BD = DC $= \frac{1}{2} \times 14$cm

= 7cm [The perpendicular drawn from the vertex of an isosceles triangle to the base bisects it]

Again, $AB^2 = AD^2 + BD^2$ [In ΔADB, using Pythagoras theorem]

Or, $(25)^2 = (AD)^2 + (7)^2$

Or, $625 = (AD)^2 + 49$

Or, $576 = (AD)^2$

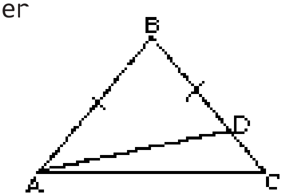
Or, $\sqrt{576} = AD$

Or, 24 = AD

∴ AD = 24cm

Example 3

In the given figure, if AB = BC, what will be the relation between AD and CD? Find it.



Solution :

Here, in the triangle ABC, AB = BC. So, ∠BAC = ∠BCA

Again, ∠DAC < ∠BAC [∠DAC being the part of ∠BAC whole part axiom]

∠DAC < ∠BCA [∠BAC = ∠BCA]

i.e. ∠DAC < ∠DCA

So, DC < AD [The side opposite to the greater angle is longer than the side opposites to the smaller angle.]

In short

i. ∠BAC = ∠BCA ii. ∠DAC < ∠BCA iii. CD < AD

Exercise 12.3

1. In the Δ**ABC,** the measurements are as given in (a), (b) and (c) below. Write.

   i. The longest side ii. The shortest side

   iii. The greatest angle iv. The smallest angle in each case

   (a) $\angle BAC = 90°$, $\angle ABC = 50°$

   (b) $\angle ABC = 2\angle ACB$, $\angle BAC = 80°$

   (c) $\angle ABC + \angle ACB = 120°$, $\angle ABC + \angle BAC = 90°$, $\angle CAB + \angle ACB = 150°$

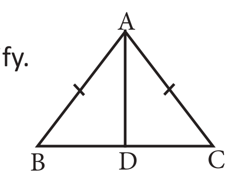
2. (a) In the ΔABC, if $\angle BAC = 90°$, AB = 9 cm and AC = 40 cm find the length of BC.

   (b) Whether a triangle having sides 10 cm, 12 cm and 16 cm is a right-angled triangle or not? Find out.

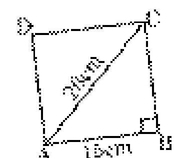
   (c) Prove that the triangle having lengths of sides as $m^2 + n^2$, $m^2 - n^2$ and 2mn is a right-angled triangle, where m and n are the positive numbers.

   (d) Find the lengths of the perpendicular drawn from the vertex to the base of an equilateral triangle of side 10 cm.

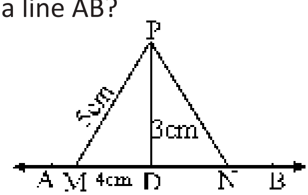
3. (a) In the given figure, AB = AC. Can we write AB>AD? Why? Justify.



   (b) In the given figure, ABCD is a rectangle. If AC=20 cm and AB=16 cm, find the lengths of BD and AD.



   (c) How many line segments can be drawn from a point P to a line AB? Which is the shortest line segment among them?



Experimental Work

1. Which of the following measurements in cm/inch can or can not form a triangle? Find out with reasons.

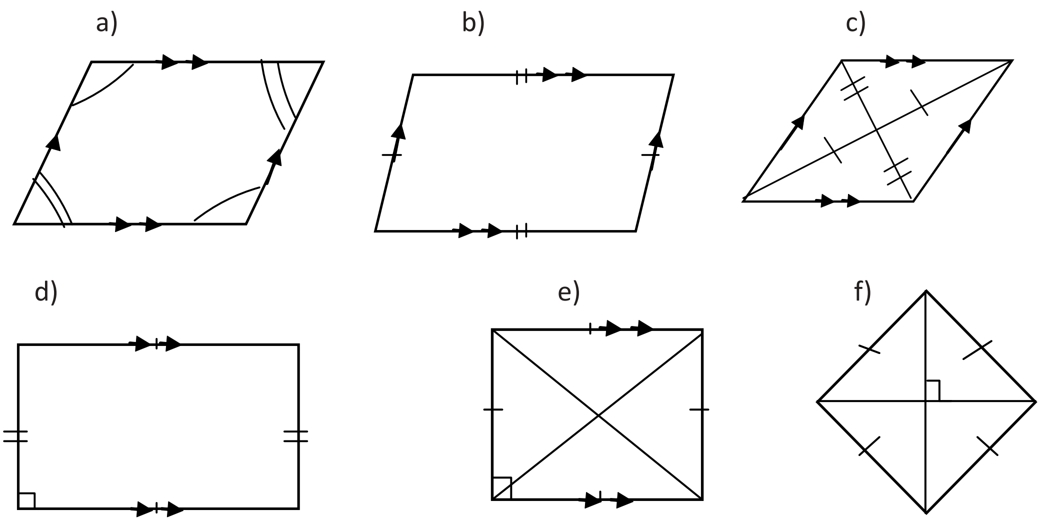
   i. 1, 2, 3 ii. 2, 3, 4 iii. 3, 4, 5

   iv. 3, 4, 8 v. $3\frac{1}{2}, 4\frac{1}{2}, 7$

Lesson 13 Parallelograms

13.0 Review

Paste (stick) the following figures on the walls of your classroom at different places separately. Make the students to observe these figures one by one and write the conclusion on their own copies. Tell the students to discuss in group about the information given by these figures and write everything pointwise together with the figures in their copies.



After preparing the above points, make the students to present them in the classroom group wise and give feedback on the presentation of each other.

The conclusions of the above discussion can be presented as follows :

A quadrilateral opposite sides of which are parallel is called parallelogram. On the basis of this definition and other geometrical facts and theorem, we can prove different characteristics of a parallelogram. The characteristics of a parallelogram are as follows

- The opposite sides of a parallelogram are equal.
- The opposite angles of a parallelogram are equal.
- The diagonals of a parallelogram bisect each other.

We have already performed the experimental verification of the above properties of a parallelogram in the previous classes. Here, we will prove these statements theoretically.

13.1 Properties of parallelogram

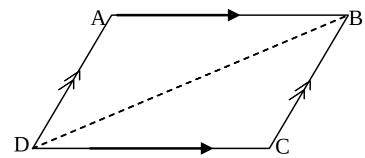
Theorem 1: The opposite sides and angles of a parallelogram are equal

Given : ABCD is a parallelogram in which AB // CD and AD// BC

To prove :

1. $\angle ABC = \angle ADC$, $\angle DAB = \angle BCD$
2. AB = CD, AD = BC

Construction : Join B and D



Proof :

Statements		Reasons	
1.	In ΔABD and ΔBCD	1.	
i. ii iii	$\angle ABD = \angle BDC$ (A) AC = AC (S) $\angle ADB = \angle CBD$ (A)	i. ii. iii.	Alternate angles of AB//CD Common side Alternate angles of AD//BC
2	$\Delta ABD \cong \Delta BCD$	2.	A.S.A axiom
3.	AB = CD and AD = BC	3.	Corresponding sides of congruent triangles are equal.
4.	$\angle DAB = \angle BCD$	4	Corresponding angles of congruent triangles are equal.
5.	$\angle ABD + \angle DBC = \angle BDC + \angle ADB$	5.	Adding equal angles to equal angles we again get equal angles. [From 1(i) and (iii)]
6.	$\angle ABC = \angle ADC$	6.	From statement 5
Hence AB = CD, AD = BC and $\angle ABC = \angle ADC$, $\angle DAB = \angle BCD$			

Proved

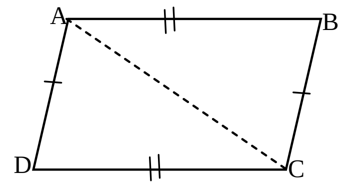
Theorem 2 : (converse of theorem 1(a))

A quadrilateral whose opposite sides are equal is a parallelogram.

Given : ABCD is a quadrilateral in which AB=CD and AD = BC

To prove : ABCD is a parallelogram i.e AB // CD and AD // BC

Construction : join A and C



Proof :

Statements		Reasons	
1.	In ΔABC and ΔACD	1.	
i. ii. iii.	AB = CD AD = BC AC =AC	i. ii. iii.	Given Given Common side
2.	ΔABC ≅ΔACD	2.	By S.S.S. statement (axiom)
3.	∠ACB = ∠DAC and ∠BAC = ∠ACD	3.	Corresponding angles of congruent triangles.
4.	AB // CD , AD //BC	4.	From (3) alternate angles are equal
5.	ABCD is a parallelogram	5.	From 5, opposite sides are parallel.

Proved

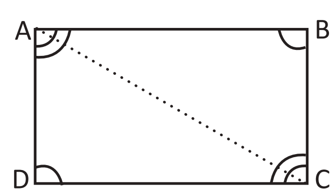
Theorem 2 : (converse of theorem 1(b))

A quadrilateral whose oppsoite angles are equal is a parallelogram

Given : ABCD is a quadrilateral in which ∠ABC = ∠ADC and ∠DAB = ∠BCD

To prove : ABCD is a parallelogram i.e. AB // CD and AD // BC

Construction : Join AC



Proof :

Statements		Reasons	
1.	$\angle ABC + \angle BCD + \angle CDA + \angle DAB = 360^0$	1.	Sum of internal angles of a quadrilateral is equal to 360^0
2.	$\angle ABC + \angle BCD + \angle ABC + \angle BCD = 360^0$ Or, $2\angle ABC + 2\angle BCD = 360^0$ Or, $\angle ABC + \angle BCD = 180^0$	2.	$\angle ABC = \angle CDA$ and $\angle DAB = \angle BCD$
3.	AB//CD	3.	Sum of consecutive interior angles is 180^0 (statement 2)
4.	Similarly, $\angle BCD + \angle CDA = 180°$	4.	Same as statement 1 and 2
5.	BC // AD	5.	Sum of consecutive interior angles is 180^0 (statement 4)
6.	AB // CD , BC//AD	6.	From statement 3 and 5
7.	ABCD is a parallelogram	7.	From statement 6, opposite sides are parallel

Proved

Theorem 4

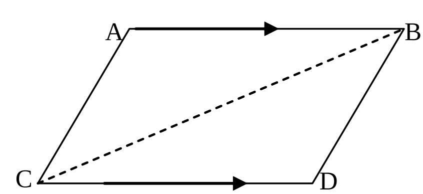
Two line segments joining the end points of two equal and parallel line segments towards the same side are also equal and parallel.

Given :

AB and CD are equal and parallel i.e. AB = CD and AB//CD where AC and BD are joined.

To prove : AC = BD and AC// BD

Construction : Join B and C



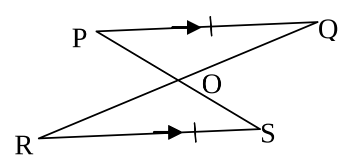
Proof :

Statements		Reasons	
1.	In ΔABC and ΔBCD	1	
i. ii. iii.	AB = CD (S.) $\angle ABC = \angle BCD$ (A.) BC = BC (S.)	i. ii. iii.	Given AB//CD (alternate angles) Common side
2.	$\Delta ABC \cong \Delta BCD$	2.	By S.A.S. statement (axiom)
3.	AC = BD	3.	Corresponding sides of congruent triangles are equal.
4.	$\angle ACB = \angle CBD$	4.	Corresponding angles of congruent triangles are equal.
5.	AC // BD	5.	Alternate angles are equal from 4
6.	AC= BD and AC // BD	6.	From 3 and 5

Proved

Theorem 5 :

Two line segments joining the opposite end points of two equal and parallel line segments are bisected by each other i.e. bisect each other.



Given : PQ and RS are equal and parallel line segments i.e. PQ = RS and PQ // RS where PS and QR are joined to meet each other at the point O.

To prove : PO = OS and QO = OR i.e. PS and QR bisect each other

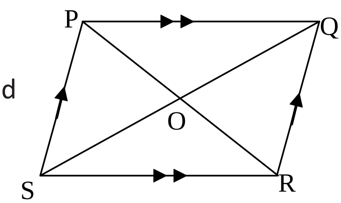
Proof :

Statements		**Reasons**	
1.	In ΔPOQ and ΔROS	1.	
i. ii. iii.	$\angle OPQ = \angle OSR$ (A.) PQ =RS (S.) $\angle PQO = \angle SRO$ (A.)	i. ii. iii.	Alternate angles of PQ//RS Given Alternate angles of PQ//RS
2.	$\Delta POQ \cong \Delta ROS$	2.	A.S.A. axiom
3.	PO = OS , QO = OR	3.	Corresponding sides of congruent triangles are equal.
4.	PS and QR bisects each other	4.	From statement 3.

Proved

Theorem 6 :

The diagonals of a parallelogram bisect each other.



Given : PQRS is a parallelogram in which the diagonals PR and QS intersect each other at O

To prove : PO = OR and SO = OQ

Proof :

Statements		Reasons	
1.	In ΔPOQ and ΔROS	1.	See figure
i. ii. iii.	$\angle OPQ = \angle ORS$ (A.) PQ = RS (S.) $\angle OQP = \angle OSR$ (A.)	i. ii. iii.	Alternate angles, PQ // SR Opposite sides of a parallelogram PQ// SR, alternate angles
2.	$\Delta POQ \cong \Delta ROS$	2.	A.S.A. statement
3.	PO = OR, QO= OS	3.	Corresponding sides of congruent triangles are equal
4.	PR and QS bisect each other.	4.	From the statement 3.

Proved

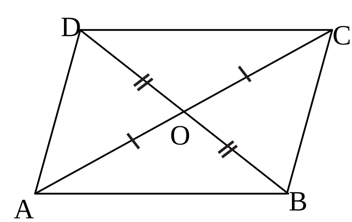
Theorem 7 : (converse of theorem 6)

A quadrilateral whose diagonals bisect each other is a parallelogram

Given : ABCD is a quadrilateral in which AC and BD bisect each other at O i.e. AO = OC and DO = OB

To prove : ABCD is a parallelogram

Proof :



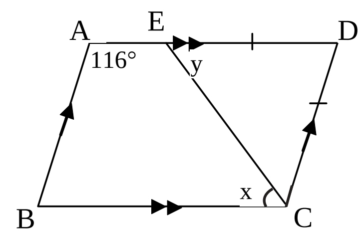
Statements		Reasons	
1.	In ΔAOB and ΔDOC	1	
i. ii. iii.	AO = OC (S.) $\angle AOB = \angle COD$ (A.) OB = OD (S.)	i. ii. iii.	Given Vertically opposite angles Given
2.	$\Delta AOB \cong \Delta DOC$	2.	By S.A.S axiom

3.	AB = DC	3.	Corresponding sides of congruent triangles are equal
4.	$\angle OBA = \angle ODC$	4	Corresponding angles of congruent triangles are equal
5.	AB // DC	5.	From statement 4, alternate angles are equal
6.	AD // BC , AD = BC	6.	AB // DC and AB = DC from 3 and 5
7.	ABCD is a parallelogram	7.	Opposite sides are equal and parallel from statement 6.

Proved

Example 1

Find the value of x and y in the given figure.



Solution,

We know that,

i. $\angle BAE + \angle EDC = 180^0$

[$\because$ AB //CD, consecutive interior angles)

Or, $\angle EDC = 180^0 - \angle BAE = 180^0 - 116^0 = 64^0$

ii. $\angle DEC + \angle ECD + \angle EDC = 180^0$ [$\because$ Sum of angles of triangle CDE]

Or, $y + \angle ECD + 64^0 = 180^0$

Or, $y + y = 180^0 - 64^0$

Or, $2y = 116^0$

Or, $y = 58^0$

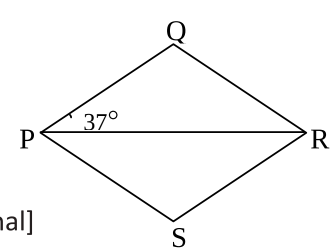
Again, $\angle BCD = \angle BAD = 116^0$

Or, $x + y = 116^0$ [$\because$ Opposite angles of parallelogram ABC are equal]

Or, $x = 116^0 - y = 116^0 - 58^0 = 58^0$

Example 2

In the adjoining rhombus, find the value of $\angle$**PSR**



Solution :

Here, $\angle QPR = \angle QRP = 37^0$[Opposite angles of a rhombus are equal and bisected by the diagonal]

$\angle QPR + \angle QRP + \angle PQR = 180^0$ [$\because$ In triangle PQR, sum of interior angles]

Or, $37^0 + 37^0 + \angle PQR = 180^0$

Or, $\angle PQR = 180^0 - 74^0 = 106^0$

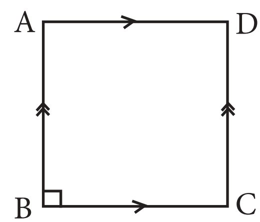
Again, $\angle PSR = \angle PQR = 106^0$ [Opposite angles of rohmbus PQRS]

$\therefore \angle PSR = 106^0$

Example 3

If one angle of a parallelogram is 90^0 then its all angles are 90^0. Prove it.

Given :



ABCD is a parallelogram in which $\angle ABC = 90^0$

To prove : $\angle ABC = \angle BCD = \angle ADC = \angle BAD = 90^0$

Proof :

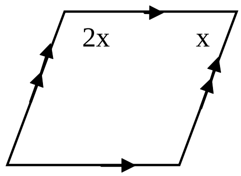
	Statements		Reasons
1.	$\angle BAD + \angle ABC = 180°$ Or, $\angle BAD + 90° = 180°$ Or, $\angle BAD = 90°$	1.	AD // BC, sum of consecutive interior angles = 180°
2.	$\angle BAD = \angle BCD = 90^0$	2.	Opposite angles of a parallelogram are equal
3.	$\angle ADC = \angle ABC = 90^0$	3.	Opposite angles of a parallelogram are equal
4.	$\angle ABC = \angle BCD = \angle CDA = \angle DAC = 90^0$	4.	From the statements 2 and 3.

Proved

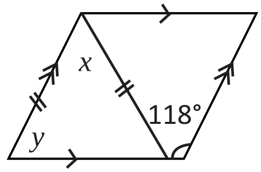
Exercise 13.1

1. For which the value of x and y, the following figure will be parallelogram

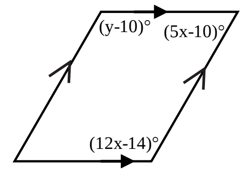
(a)



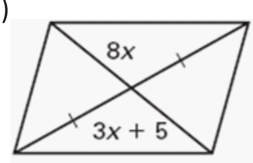
(b)



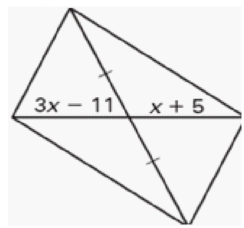
(c)



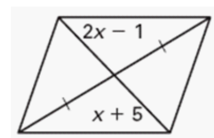
(d)



(e)

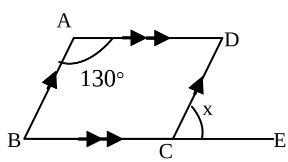


(f)

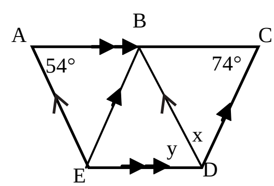


2. Find the value of x, y and z.

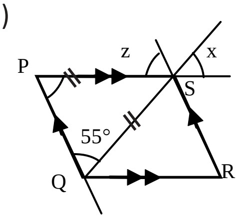
(a)



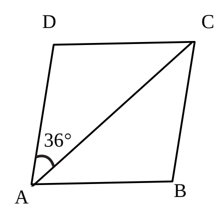
(b)



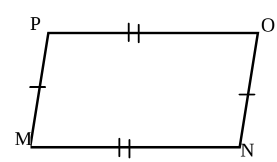
(c)



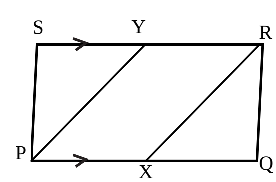
3. In the adjoining figure, ABCD is a rhombus. If $\angle DAC = 36^0$, find its remaining angles.



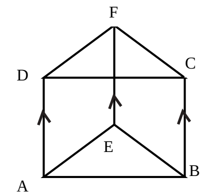
4. In the adjoining figure, MN = OP and PM = NO. Prove that MNOP is a parallelogram.



5. PQRS is a parallelogram. If X and Y are the mid-points of PQ and RS, prove that PXRY is a parallelogram.

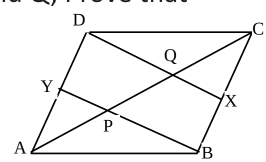


6. In the given figure, if AD = EF, AD//EF , EF = BC and EF//BC, prove that ABCD is a parallelogram.



7. A quadrilateral formed by joining the mid-points of the sides of a quadrilateral is a parallelogram. Prove it

8. In the given figure, ABCD is a parallelogram and X and Y are the mid-points of BC and AD. If BY and DX intersect the diagonal AC at the points P and Q, Prove that AP = PQ = QC.



9. If in the quadrilateral SLOW, SL = LO = OW = SW, prove that SLOW is a parallelogram

10. In the quadrilateral MOAT, the diagonal MA intersects OT at R and MR = RA and TR = OR. Prove that this quadrilateral is a parallelogram.

11. Prove that all the rectangles are parallelogram. Also write its converse. Can you prove this converse ? Explain.

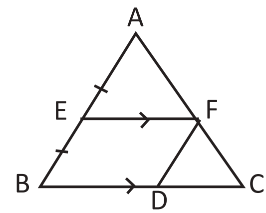
12. Make a list of parallelograms, rectangles and squares shaped objects. Among them find the properties of parallelogram.

13.2. Mid–Points of a Triangle

The properties of parallelogram which are proved above can be used to prove some more properties related to the triangles. How many parallelograms can be formed by joining the mid-points of the sides of a triangle ? Discuss among friends.

Theorem 8

A line segment drawn through the mid-point of one side and parallel to another side of a triangle bisects the third side.



Given : In ΔABC, AE = BE and EF//BC

To prove : AF = FC

Construction : Draw FD//EB and meeting BC at D.

Proof :

Statements	Reasons
1. EFDB is a parallelogram.	1. EF//BD and EB//FD
2. EB = FD	2. Being opposite sides of parallelogram
3. AE = EB	3. Given
4. AE = FD	4. From statements 2 and 3
5. In ΔAEF and ΔCFD	5.
i. AE = FD	i. From statements 4
ii. $\angle EAF = \angle DFC$	ii. Corresponding angles (AB//FD)
iii. $\angle AFE = \angle DCF$	iii. Corresponding angles (EF//BC)
6. $\Delta AEF \cong \Delta CFD$	6. By S.A.A. statement
7. AF = FC	7. Corresponding sides of congruent tringles.

Proved.

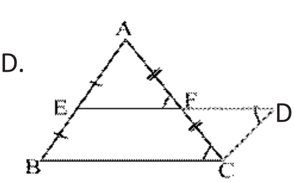
Theorem 9 : (converse of theorem 8)

A line segment joining the mid-point of any two sides of a triangle is parallel to the third side.

Given : In ΔABC, AE = EB and AF = FC

To prove : EF // BC

Construction : Produce EF to D such that FD = EF and join C and D.



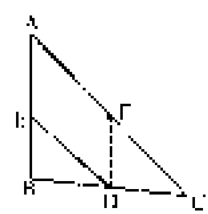
Proof :

Statements	Reasons
1. In ΔAEF and ΔCFD	1.
i. AF = CF (S)	i. Given
ii. $\angle AFE = \angle CFD$ (A)	ii. Vertically opposite angles are equal
iii. EF = FD (S)	iii. From construction
2. $\Delta AEF \cong \Delta DFC$	2. By S.A.S. statement (axiom)
3. $\angle AEF = \angle CDF$	3. Corresponding angles of congruent tringles are equal
4. BE // CD	4. From (3), alternate angles are equal.
5. AE = CD	5. Corresponding sides of congruent tringles are equal
6. AE = BE	6. Given
7. BE = CD	7. From statements 5 and 6
8. BCDE is a parallelogram.	8. BE = CD and BE//CD, from 4 and 7
9. BC // ED or BC // EF	9. Opposite sides of a parallelogram are parallel.

Proved.

Note : Is EF = $\frac{1}{2}$BC ? Prove it by discussion.

Example 1



In the given figure, D, E and F are the mid-points of BC, AB and AC. If EF = 2 cm, ED = 2.6 cm and FD = 1.5 cm, find the lengths of AB, BC and AC.

Solution :

We know that, EF = $\frac{1}{2}$ BC [The line segment joining the mid-points of two sides is half of the third side of a triangle]

Or, 2cm = $\frac{1}{2}$ BC

Or, BC = 4cm

Similarly, $ED = \frac{1}{2} AC$

Or, $AC = 2ED$

$= 2 \times 2.6$

$= 5.2$ cm

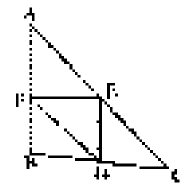
$FD = \frac{1}{2} AB$

Or, $AB = 2 FD$

$= 2 \times 1.5 = 3$ cm

Example 2

In the given figure D, E and F are the midpoints of BC, AC and AB. Are four congruent triangles formed by joining D, E and F ? If yes, prove it.



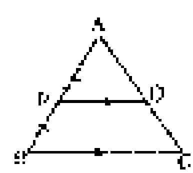
Solution :

1. DE = BF , DE // BF [$\because$ D and E are the mid-ponts of BC and AC respectively]
2. $\Delta EFD \cong \Delta EDC$ [$\because$ The diagonal ED bisects the parallelogram DCEF into two congruent triangles]
3. $\Delta DEF \cong \Delta BFD$[$\because$ The diagonal FD divides the parallelogram BDEF into congruent triangles]
4. $\Delta AEF \cong \Delta DEF$ [$\because$ By S.S.S. statement]

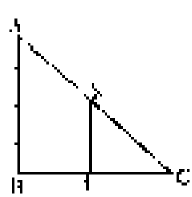
So, the ΔAEF, ΔDEF, ΔBFD and ΔCED are congruent triangles.[From statement 2, 3 and 4

Exercise 13.2

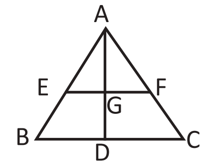
1. (a) In the given figure, AP = PB and PQ // BC. If QC = 2cm, what will be the value of CA ? If BC = 4.8cm, what will be the value of PQ ?



(b) In the given figure, CE = 2.4cm, AC = 4.8cm, BF = 1.8cm, BC = 3.6cm and $\angle FEC = 52°$. Find the value of $\angle BAC$.



2. In the given figure, E and F are the mid-points of AB and AC. AD meets BC at D and EF at G. prove that AG = GD.

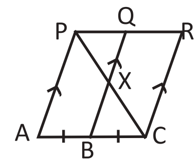


3. In the given figure, AB = BC and AP//BQ//CR

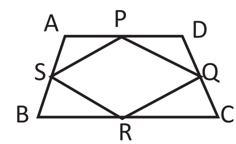
Prove that :

(a) $BX = \frac{1}{2} AP$

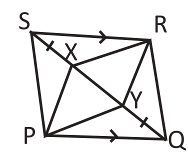
(b) $AP + CR = 2BQ$



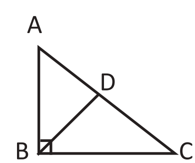
4. In the quadrilateral ABCD, P, Q, R and S are the mid-points of the sides AB, BC, CD and DA respectively. Prove that PQRS is a parallelogram.



5. In the adjoining figure, PQRS is a parallelogram. X and Y are any two points in the diagonal QS such that QX = SY. Prove that PXRY is a parallelogram.



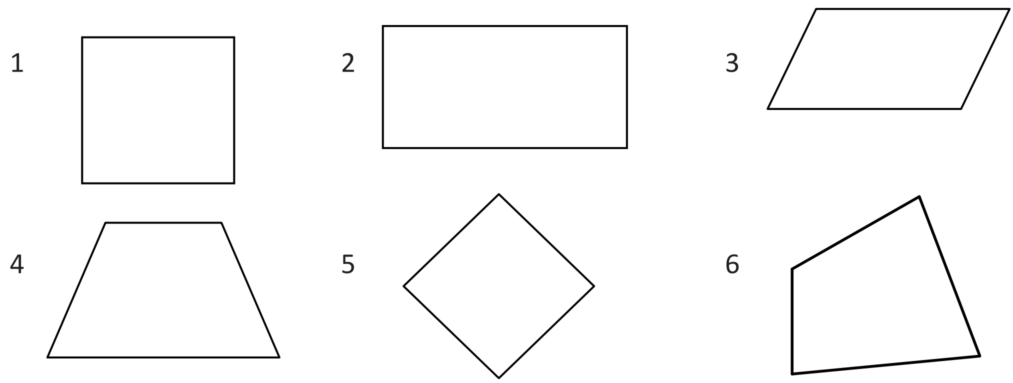
6. In the triangle ABC right-angled at B, if AD = CD, prove that BD = 1/2 AC



Lesson 14 Construction

14.0 Review

Take the students to the play ground with their notebooks. Make the groups of six students each. Name the members of each group as 1, 2, 3, 4, 5 and 6. Make the following figures available to each group. Distribute the numbered figures to the students matching their own numbers. For example : student-1 should get 1^{st} figure, student-2 should get 2^{nd} figure and so on.



Now, make the groups of the students having figure1, a group of students having figure 2 and so on. Make these members of new group to discuss about the quadrilateral they have and answer the following questions. Draw a common conclusion.

(a) What is the name of the quadrilateral?

(b) What are the characteristics of the quadrilateral?

After concluding in the new group, go to the respective previous groups and present the characteristics of the quadrilateral that you have learnt in the new groups. After this, prepare the group report of all the quadrilaterals and their properties and present them in the class room group wise. Like this, we have already studied many properties and characteristics of different quadrilaterals in the previous classes. In this chapter, we will study about the constructions of different types of quadrilaterals.

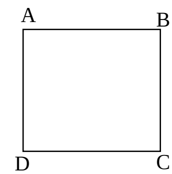
14.1 Construction of Quadrilaterals

We know that quadrilateral is a plane figure bounded by four line segments. In the above activity quadrilaterals are classified according to their special characteristics such as square, rectangle, parallelogram, rhombus and trapazoid. Let us think how to construct them using their special features. We should begin from square, the most special case of quadrilateral. Before the construction we must understand how to perform Construction in geometry and what tools and basic skills we use and why?

Note : Explain each step of construction with reasons why it is done in all the examples given below.

14.1.1 Construction of a Square

What is a square? What about its angle and sides? We all know the answers of such questions. Now, we are going to construct the squares on the basis of different given conditions.

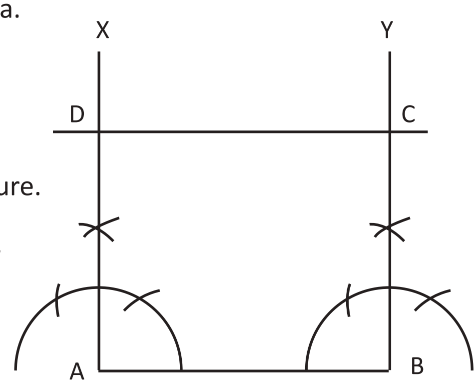


(a) To construct a square when its one side is given

Construct a square ABCD of which the length of a side AB is given.

Construction Steps :

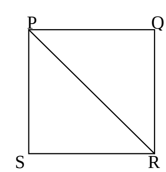
1. Make a rough sketch of a square of the given data.
2. Draw a line segment AB of the given length.
3. Draw 90^0 angles at the points A and B with the help of compass. Name the lines as AX and BY making 90^0 angles at A and B as shown in the figure.
4. Take an arc of radius equal to AB from the points A and B with the help of a compass and mark on the lines AX and BY respectively. Name these marks as the points D and C as shown in the figure. Join the points D and C.



In this way we obtain the required square ABCD.

(b) To construct a square when its length of a diagonal is given.

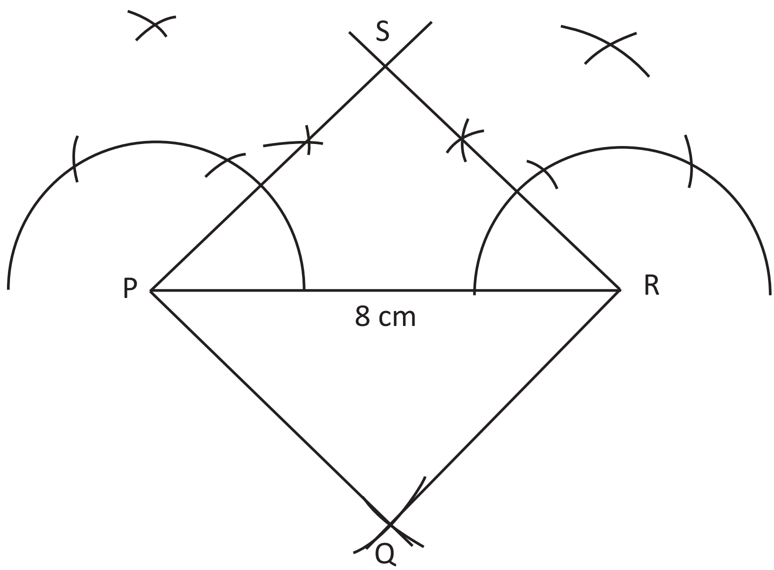
Construct a square PQRS when length of its diagonal PR is given.



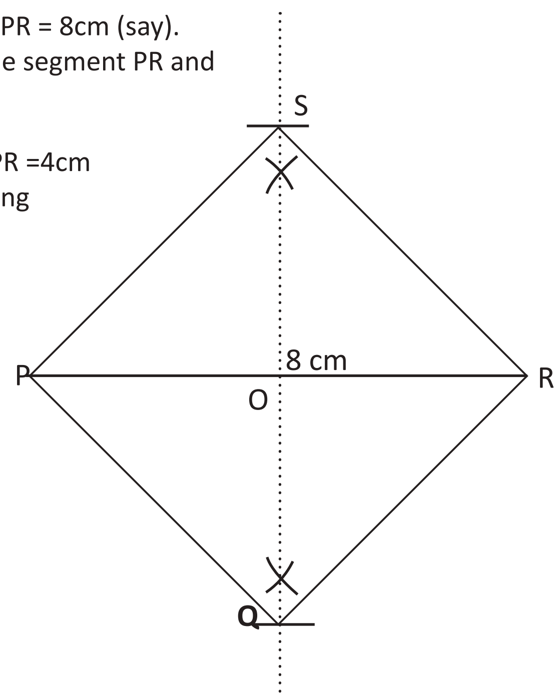
Construction Steps :

1. Make a rough sketch of the square from the given data.
2. Draw a line segment PR of the given length PR = 8cm (say)
3. As we know that the diagonal of a square bisects its opposite angles which are of 90^0 each. So draw angles of 45^0 at the points P and R and mark the point of intersection by S.
4. Draw an arc of radius equal to PS from the points P and R on opposite side of S with the help of a compass and denote the intersecting point of the arcs by Q.
5. Join PQ and QR.

In this way, the required square PQRS is obtained.



Another method



1. Draw a line segment PR of given length PR = 8cm (say).
2. Draw a perpendicular bisector of the line segment PR and name the point of intersection by O.
3. Take the arcs of radius equal to half of PR =4cm from the point 'O' on its both sides cutting the perpendicular bisector of RP at two points as Q and S as shown in figure. We do so because the diagonals of a square bisect each other perpendicularly.
4. Join PS, RS, PQ and QR. In this way, we get the required square of the given measurement.
5. Why PQRS is a square? Justify.

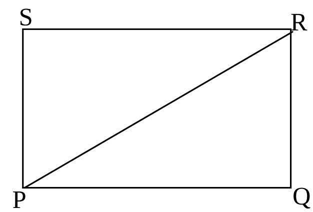
14.1.2 Construction of a Rectangle

We have studied the characteristics of rectangles. Here, we will study about how different rectangles can be constructed on the basis of given data.

Definition : A parallelogram one angle of which is 90^0 called a rectangle.

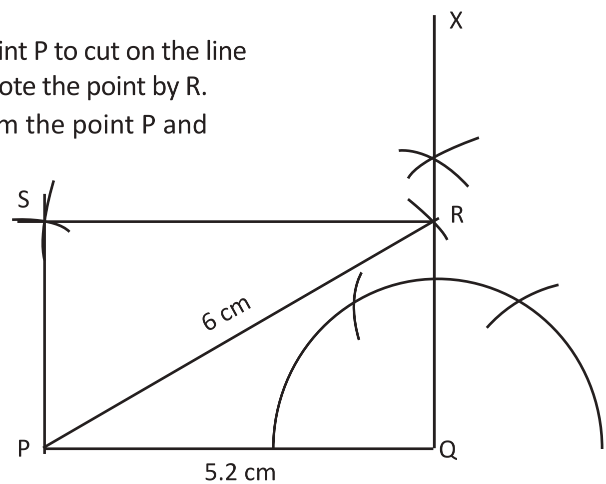
(a) To construct a rectangle when its lengths of a side and a diagonal are given

Construct a rectangle PQRS where PQ = 5.2 cm and diagonal PR = 6 cm



Construction Steps :

1. Make a tentative rough sketch of the rectangle of the given measurement.
2. Draw a line segment PQ = 5.2cm.
3. With the help of a compass, draw angle PQX = 90^0 at the point Q.
4. Take an arc of radius 6 cm from the point P to cut on the line QX with the help of a compass and denote the point by R.
5. Take an arc of radius equals to QR from the point P and another arc of radius equals to 5.2cm from the point R and make the intersecting point of the arcs by S as shown in the figure.
6. Join PS and SR. Thus, we obtain the required rectangle PQRS.



14.1.3 Construction of Parallelograms

We have discussed about a parallelogram and its properties in the previous chapters. In this chapter we will study about how to construct the parallelogram on different given conditions.

(a) To construct a parallelogram when its two adjacent sides and the angle between them are given

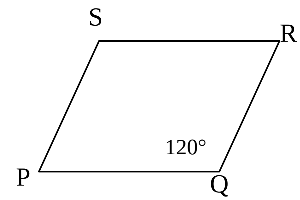
Construct a parallelogram PQRS where PQ = 6.2 cm, QR = 4.5 cm and $\angle PQR = 120^0$.

Construction Steps :

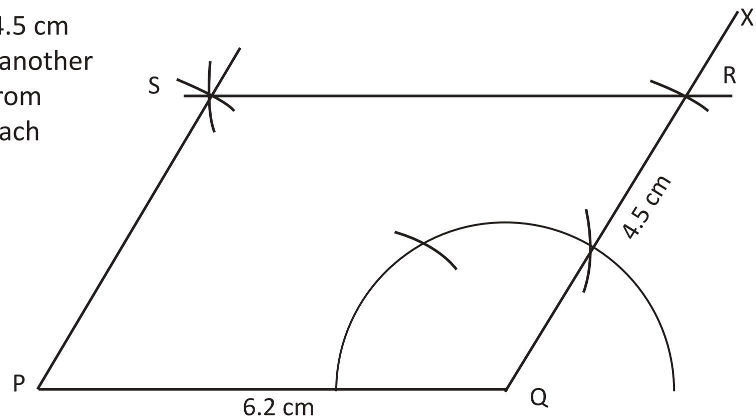
1. Make a tentative sketch of the parallelogram according to the given measurement.

2. Draw a line segment PQ = 6.2cm.

3. Draw an angle PQX = 120^0 at the point Q with the help of a compass.



4. Take an arc of radius 4.5 cm from the point Q to cut on the line QX and name the point as R.
5. Take an arc of radius 4.5 cm from the point P and another arc of radius 6.2 cm from the point R to meet each other at the point S.



6. Join PS and RS. Thus, we get the required parallelogram PQRS.

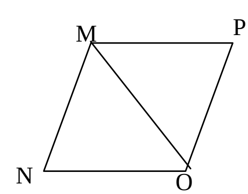
7. Why the quadrilateral PQRS is a parallelogram according to the given condition? Justify.

(b) To construct a parallelogram when its two adjacent sides and a diagonal are given

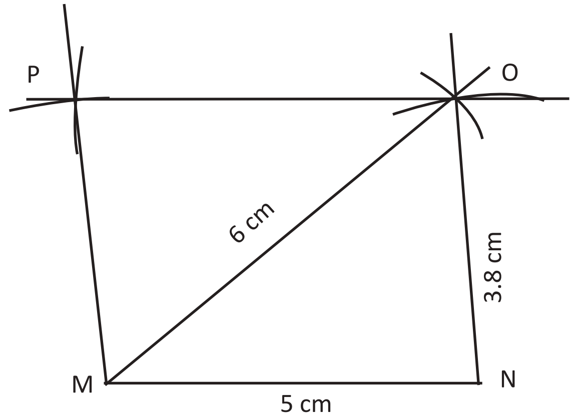
Construct a parallelogram MNOP in which MN = 5 cm, NO = 3.8 cm and MO = 6 cm.

Construction Steps :

1. Draw a tentative figure of a parallelogram having the given measurement.
2. Draw a line segment MN = 5 cm.



3. Take an arc of radius 6 cm from the point M and another arc of radius 3.8 cm from the point N to cut each other at the point O.
4. Join NO.
5. Take arcs of radius 5 cm and 3.8 cm from the points O and M respectively to meet each other at the point P.
6. Join PM and PO. Now MNOP is the required parallelogram.
   Justify why MNOP is the required parallelogram according to the given data.

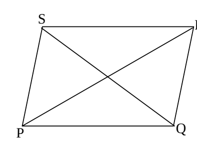


(c) To construct a parallelogram when the lengths of its one side and two diagonals are given.

Construct a parallelogram PQRS in which PQ = 6.3 cm and diagonals PR = 8.8 cm and QS = 8cm.

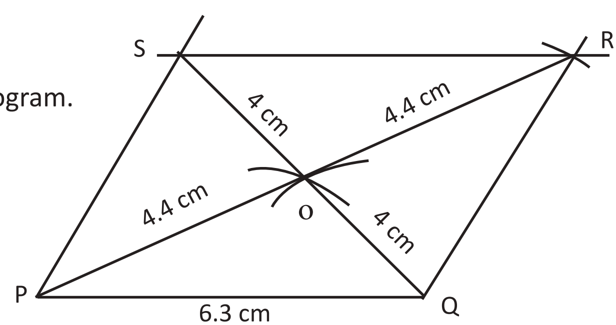
Construction Steps :

1. Make a tentative rough sketch of a parallelogram PQRS according to the given data.



2. Draw a base line segment PQ = 6.3 cm.
3. Draw an arc of radius 4.4 cm from the point P and an arc of radius 4 cm from the point Q to cut each other at the point O as shown in the figure.
4. Join PO and QO.
5. Produce PO to R making PO = OR.
6. Produce QO to S making QO = SO.
7. Join PS, QR and RS.

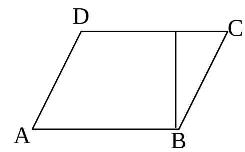
Hence PQRS is the required parallelogram.



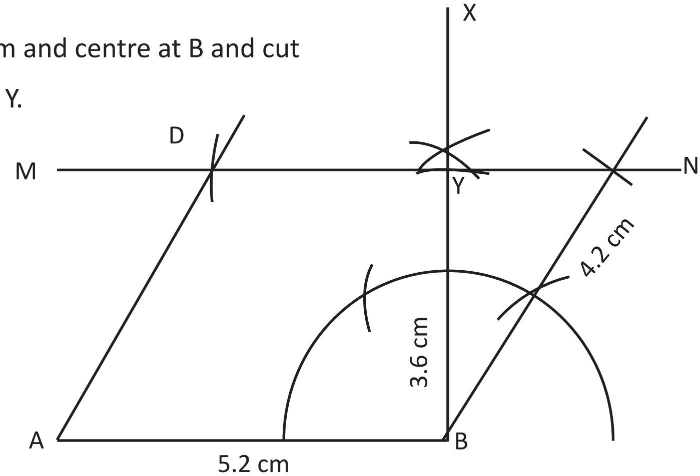
(d) To construct a parallelogram when its height and lengths of two adjacent sides are given.

Construct a parallelogram ABCD in which AB = 5.2 cm, BC = 4.2 cm and the height = 3.6 cm.

Construction Steps :



1. Draw a tentative sketch of the parallelogram.
2. Draw the base line segment AB = 5.2 cm.
3. Draw an angle ABX = 90^0 at the point B with the help of a compass.
4. Take an arc of radius 3.6 cm and centre at B and cut on the line BX at the point Y.
5. Draw a line segment MN parallel to AB and passing through Y.
6. Take an arc of radius 4.2 cm from the point B cutting MN at the point C.
7. Take an arc of radius 5.2 cm from the point C to cut MN at the point D.



8. Join BC and AD to get the required parallelogram ABCD.

14.1.4 Construction of a Rhombus

What is a rhombus? What are its properties? Before constructing the rhombus, we need to know about its different properties. A rhombus is a quadrilateral all sides of which are equal. Its characteristics are as follows :

1. All sides are equal in length.
2. Opposite angles are equal.
3. Diagonals bisect at right angle to each other i.e. they are the perpendicular bisector of each other.

A quadrilateral having all sides equal is called a rhombus.

Now, we construct a rhombus on different conditions.

(a) To construct a rhombus when the length of its one side is given

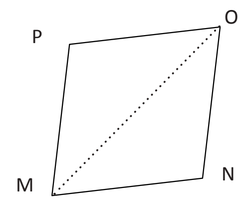
Can you construct a rhombus of a side having length x cm with the help of compass and ruler? Discuss. If not, what other measurements and information should be given? Why are they required? Mention the possible reasons.

(b) To construct a rhombus one side and one diagonal of which are given

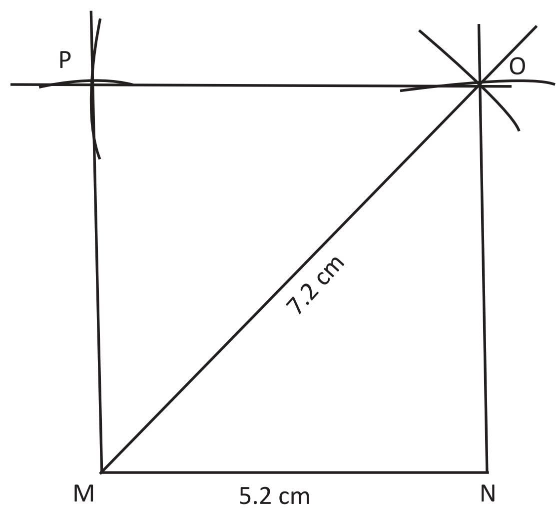
Construct a rhombus MNOP where MN = 5.2 cm and MO = 7.2 cm.

Construction Steps :

1. Make a rough sketch of the rhombus of given measurement
2. Draw a line segment MN = 5.2 cm
3. Take an arc of radius 7.2 cm from the point M and another arc of radius 5.2 cm from the point N to cut each other at the point O as shown in the figure.



4. Join MO and NO
5. Take two equal arcs of radius 5.2 cm from the points M and O to meet each other at the point P
6. Join PM and PO to get the required rhombus MNOP.



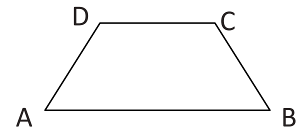
14.1.5 Construction of ax Trapezoid

Before starting the construction of a trapezium, we need to know what a trapezium is and what are its properties. A quadrilateral one pair of oppsoite sides of which are parallel is called a trapezium.

Now, we start constructing different trapeziums on the basis of different given conditions and measurements.

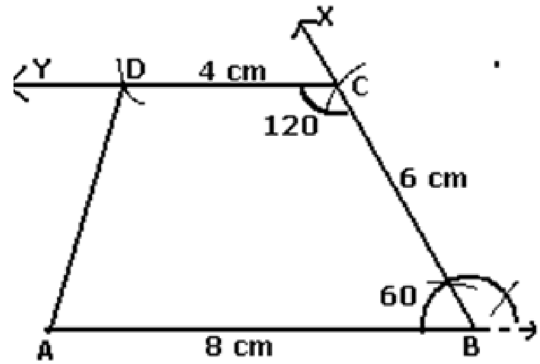
(a) To construct a trapezium when its three sides and an angle are given

Construct a trapezium ABCD in which AB // CD, AB = 8 cm, BC = 6.0 cm, and CD = 4 cm



Construction Steps :

1. Make tentative rough figure of the trapezium according to given measurements.
2. Draw a line segment AB = 8 cm
3. Draw an angle $\angle ABX = 60°$ at the point B.
4. Take an arc of radius 6 cm and centre at B and mark a point C on BX.
5. At point C, draw $\angle BCY = 120^0$.
   ($\angle B + \angle C = 180°$ or $\angle C = 180° - 60° = 120°$)
6. Mark the point D on CY by taking an arc of radius 4 cm from the point C.
7. Join AD



Thus, we obtain the required trapezium ABCD.

Exercise 14

1. Construct the squares on the following conditions with justifications.

(a) Length of side is 6.4 cm

(b) A square ABCD in which AB = 6 cm

(c) Length of a diagonal PR = 5.8 cm

(d) Length of a diagonal is 6 cm

2. Construct the rectangles using the following data

(a) A rectangle ABCD having the lengths of adjacent sides 11 cm and 8.5 cm

(b) A rectangle PQRS in which QR = 3.6 cm and diagonal PR = 6 cm

(c) A rectangle WXYZ in which WX = 7cm and XY = 4cm.

3. Construct the parallelograms of the following measurements.

(a) A parallelogram ABCD in which BC = 5 cm, $\angle BCD = 120°$ and CD = 4.8 cm

(b) A parallelogram PQRS in which PQ = 4.3 cm, PS = 4 cm and PR = 6.8 cm

(c) A parallelogram in which one side = 4.4 cm and diagonals are 5.6 cm and 7 cm

(d) A parallelogram WXYZ in which diagonal WY = 3.8 cm, diagonal XZ = 4.6 cm and angle between them is 60^0

(e) A parallelogram ABCD in which AB = 6.5 cm, BC = 3.4 cm and height AL= 2.5cm

(f) A parallelogram MNOP in which MN = 5.5 cm, MO = 5.2 cm and height 3.5cm.

4. Construct the rhombus from the following measurements.

(a) A rhombus PQRS in which one angle is 60^0 and length of a side is 7.2 cm

(b) A rhombus ABCD in which AB = 4cm and AC = 6.5cm

(c) A rhombus ABCD in which diagonals AC=8 cm and BD = 6 cm

(d) A rhombus MNOP in which MN = 6cm and $\angle N = 75^0$

5. Construct a trapezium from the following data.

(a) A trapezium ABCD in which AB = 6 cm, BC = 4 cm, CD = 3.2 cm, $\angle B = 75°$ and DC//AB.

(b) A trapezium ABCD in which AB // DC, AB = 7 cm, BC = 5 cm, AD = 6.5 cm and $\angle B = 60°$.

(c) A trapezium ABCD in which AB // CD, AB = 8 cm, BC = 6 cm, CD = 4 cm and $\angle C = 120^0$

6. Construct quadrilaterals from the following data.

(a) A quadrilateral ABCD in which AB = AD = 3 cm, BC = 2.5 cm, AC = 4 cm and BD = 5 cm.

(b) A quadrilateral MNOP in which MO = MP = 6 cm, NO = 7.5 cm, OP = 5 cm and NP = 10 cm.

(c) A quadrilateral PQRS in which PQ = 3.5 cm, QR = 2.5 cm , RS = 4 cm, $\angle Q = 75^0$ and $\angle R = 120^0$

7. Can a quadrilateral be constructed by using the line segments of lengths 3 cm, 4 cm, 5 cm, and 6 cm. Try to construct by using a compass and ruler. Discuss with the friends in the classroom about whether it is possible to construct or not and find out different alternatives (ways) to construct this quadrilateral.

8. If the statement,'Diagonals of a rhombus bisect each other' is correct then whether or not its converse is true? Explain by proper construction.

Lesson

15 Similarity

15.0 Review

Niraj has kept maps of Nepal of different sizes in his shop. Neema has kept photos of great poet of Nepal of different sizes in her studio. These maps and photos of different sizes but of same object, what properties can be similar and common? Discuss among the friends.

Likewise, here are some figures given below :

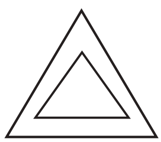


Figure 15.0 (i)

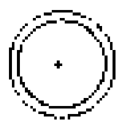


Figure 15.0 (ii)

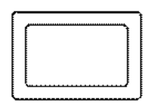


Figure15.0 (iii)

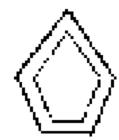


Figure15.0 (iv)

Geometrical figures of different size but having the same shape are called similar figures.

Let us have discussion with each other on the following relations to develop the concept of similarity.

(a) Do all circles have same shape?

(b) Are all squares of same shape?

(c) Are all rhombus of same shape?

(d) Do all rectangles have same shape?

(e) What are the conditions for any two triangles to be similar?

After the discussion, draw conclusions on each questions. Let ABC be a triangle of sides AB = 4cm, BC = 6cm and AC = 5cm. Similarly, the other triangle PQR is of sides PQ = 6cm, QR= 9cm and PR = 7.5cm. Are these two triangles similar in shape? Construct them and write their corresponding sides and angles if they are similar. Likewise, there is Nistha's house. In her yard, there are two triangular figures of different measurements. One of them has two angles 30° and 60° and the other has two angles 60° and 90°. Are these two triangles similar?

In the both conditions given above the triangles are similar.
In the following two conditions the triangles become similar.

(a) If the corresponding sides of two triangles are proportional, or

(b) If the two corresponding angles of two triangles are equal.

15.1 Problems Related to Similar Polygons

Here, we will study about the problems related to the conditions in which not only the triangles but also the polygons become similar.

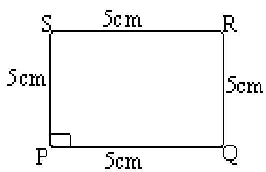


Figure 15.1(i)

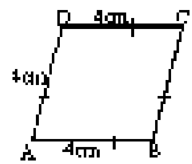


Figure 15.1(ii)

Here are two figures. In the given figure 15.1(i), there is a square having length of a side 5 cm and in the figure 15.1(ii), there is a rhombus having length of a side 4 cm. On the basis of this information, discuss the following questions :

(a) Are the ratios of any two sides equal in these figures?
(b) Are their ratios of sides and perimeters equal?
(c) Are both figures similar? If not why?

Likewise in the figure No 15.1(iii) and figure No 15.1(iv), there are two regular hexagons having lengths of sides 4.5 cm and 3 cm respectively. On the basis of this information, answer the following questions :

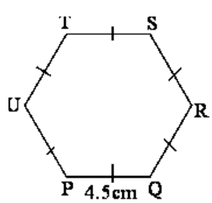


Figure 15.1(iii)

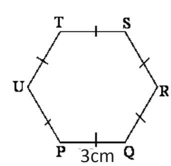


Figure 15.1(iv)

(a) Are the interior angles of both polygons equal?
(b) Are the ratios of corresponding sides of both polygons equal?
(c) Are the ratios of sides and perimeters of both polygons equal?
(d) Are both figures similar? If yes, why?

The above figures 15.1(i) and 15.1(ii) are not similar and 15.1(iii) and 15.1(iv) are similar. So, the following conditions are required for the polygons to be similar :

i. The number of sides of the polygons must be equal.

ii. Corresponding angles of the polygons must be equal.

iii. Ratios of the corresponding sides and the perimeters must be equal.

Example 1

Dipika, a girl of height 1.2 m, is standing in front of the lamppost of height 3.9 m. What will be the length of her shadow when the length of the shadow of the lamp post is 6.5 m?

Solution:

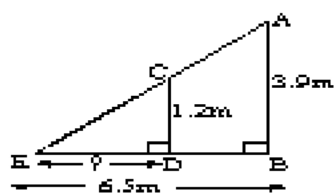
In the figure, let AB be the height of lamp post and CD be the height of Dipika, BE be the length of shadow of lamp post and DE be the length of shadow of Dipika.

By the question, AB = 3.9 m, BE = 6.5m, CD = 1.2m, ED =?

Here, ΔCDE and ΔABE are similar. [$\angle$ABE = $\angle$CDE = 90^o and $\angle$E is common angle]

So, $\frac{AB}{CD} = \frac{BE}{DE}$ [Corresponding sides of similar triangles are proportional]

Or, $\frac{3.9m}{1.2m} = \frac{6.5m}{DE}$



Or, 3.9 DE = 6.5 x 1.2 m

Or, DE = $\frac{6.5x1.2}{3.9}$ m = 2 m

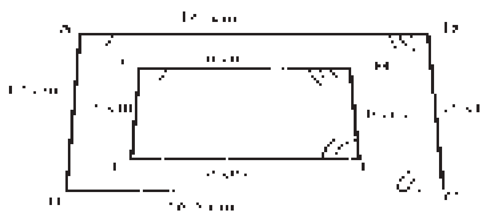
So, the length of shadow of Dipika will be 2m at that time.

Example 2

In the given figure, the quadrilaterals ABCD and EFGH are similar. If AD = 16cm, CD = 15cm, AB = 12cm, BC = 16.5cm, EF = 5cm, EH = a cm ,GH = b cm and FG = c cm then find the value of a, b and c.

Solution :

Here, the quadrilateral ABCD and EFGH are similar. Therefore, the corresponding sides must be proportional.



So, $\frac{AB}{EF}=\frac{AD}{EH}=\frac{DC}{GH}=\frac{BC}{FG}$

Or, $\frac{12}{5}=\frac{16}{a}=\frac{15}{b}=\frac{16.5}{c}$

i. Taking 1st and 2nd ratios, we get

$$\frac{12}{5}=\frac{16}{a}$$

Or, $12a = 16 \times 5$

Or, $a = \frac{16\times5}{12}=\frac{20}{3}=6\frac{2}{3}$

$\therefore$ EH = $6\frac{2}{3}$ cm

ii. Taking 1st and 3rd ratios, we get

$$\frac{12}{5}=\frac{15}{b}$$

Or, $12b = 75$

Or, $b = \frac{75}{12}$

Or, $b = \frac{25}{4}=6\frac{1}{4}$

$\therefore$ GH = $6\frac{1}{4}$ cm

iii. Taking 1st and 4th ratios, we get

$$\frac{12}{5}=\frac{16.5}{c}$$

Or, $12c = 16.5 \times 5$

Or, $c = \frac{16.5\times5}{12}=6.88$

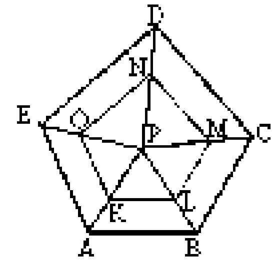
$\therefore$ FG = 6.88cm

Example 3

In the given figure, the pentagons ABCDE and KLMNO are similar

(a) Write the names of five pairs of similar triangles.

(b) If CD = 10cm, ΔPMN has perimeter of 12.5cm and ΔPCD has perimeter 25cm, find the value of MN.



Solution :

Here, the pentagons ABCDE and KLMNO are similar and the corresponding vertices of both pentagons are joined to a common point P as shown in the figure.

(a) The five pairs of similar triangles are i. ΔPMN and ΔPCD, ii. ΔPON and ΔPED, iii. ΔPOK and ΔPEA, iv. ΔPKL and ΔPAB and v. ΔPLM and ΔPBC

(b) $\dfrac{MN}{CD} = \dfrac{\text{perimeter of } \Delta PMN}{\text{perimeter of } \Delta\ PCD}$

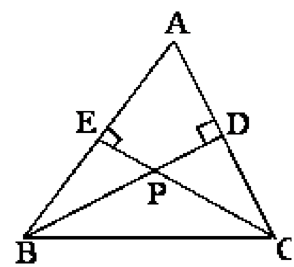
Or, $\dfrac{NM}{10cm} = \dfrac{12.5cm}{25cm}$

Or, $MN = \dfrac{1}{2} \times 10 = 5$

$\therefore$ MN = 5cm

Example 4

In the given figure, ΔBDC and ΔBEC are right-angled triangles, Prove that BP x PD = EP x PC



Solution :

1. In ΔBPE and ΔPDC

   i. $\angle BEP = \angle CDP$ (A) [Both being 90^0]

   ii. $\angle EPB = \angle DPC$ (A) [Vertically opposite angles are equal]

   iii. $\Delta BPE \sim \Delta CPD$ [By A.A.A. statement]

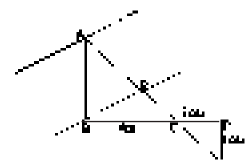
2. $\dfrac{EP}{PD} = \dfrac{BP}{CP}$ [Corresponding sides of similar triangles]

   Or, BP x PD = EP x CP Proved.

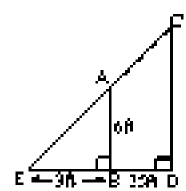
Exercise 15

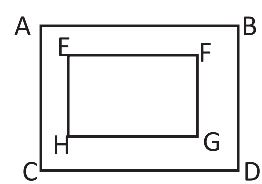
1. (a) The length and breadth of a rectangular room are 20 ft and 18 ft respectively. If the length of the map of the room is 5 cm, what will be the breadth of that map?

(b) In the given figure, A and B are any two points on the opposite edges of a stream. If BC = 6m, CD = 3m, DE = 4cm and AB // DE then what is the actual breadth (AB) of the stream.



(c) In the given figure, a man of height 6 ft (AB) is standing 25 ft away (BD) from a pole (CD) during a day time. If his shadow (BE) is 3 ft and concide with the shadow of the pole, find out the height of the pole (CD).



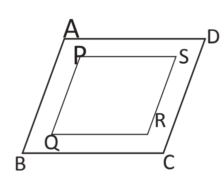


2. (a) In the given figure, ABCD and EFGH are any two rectangular frames where the breadth of outer frame is double than that of the inner frame. If the length of the inner frame is 3 cm, find out the length of the outer frame.

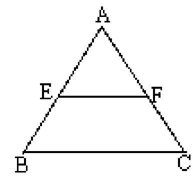
(b) In the given figure, ABCD and PQRS are any two similar rhombus in which PS//AD and PQ// AB

i. Write the name of two trapeziums.

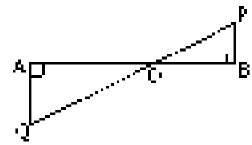
ii. If PQ : AB = 2:3 and AB = 5cm, find the length of AD.



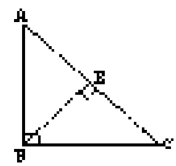
3. (a) In the given figure, EF//BC. If EF = 3cm, AC = 12cm, AE = 4.5cm and BC =6cm, find the lengths of AF and AB.



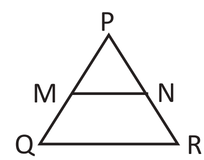
(b) In the given figure, PB and QA are perpendicular to AB. If OA = 10cm, BO = 6cm and PB = 9cm, find the value of AQ.



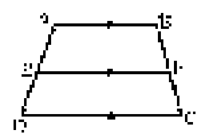
(c) In the given figure, $\angle ABC = 90^0$ and $BE \perp AC$. If AE = 4cm and CE = 9cm, find the length of BE.



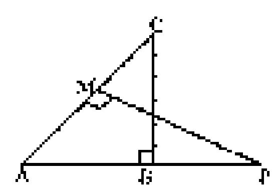
4. (a) In the given figure, MN//QR. Prove that $\Delta PMN \sim \Delta PQR$



(b) In the given figure, ABCD is a trapezium. If AB// EF//DC, Prove that $\frac{AE}{ED} = \frac{BF}{FC}$



(c) In the given figure, ΔABC and ΔAMP are right an gled triangles where right-angled at B and M respectively. Prove that



i. $\Delta ABC \sim \Delta AMP$

ii. $\frac{CA}{PA} = \frac{BC}{MP}$

Lesson 16

Circle

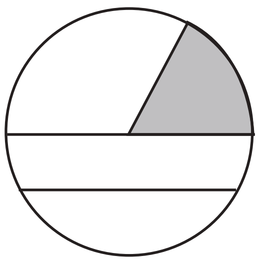
16.0 Review

Discuss, what is formed when we trace or stamp a two-rupee coin on the copy. How can you draw similar figures with the help of compass? In the figures like this drawn with the help of a compass, can we show the following parts?

i. Radius

ii. Circumference

iii. Chord



iv. Diameter

v. Arc

vi. Semi-circle

vii. Sector

viii. Segment. Discuss in groups by drawing the figures.

A locus traced out by a moving point in a plane under the geometrical condition that it remains equidistant from some a point is called a circle.

The different parts of a circle are as given above about which we have studied in detail in the previous classes. Think of the following given conditions and discuss.

(a) Is there a single circle with the same centre and different radii?

(b) How many radii and diameters can be drawn in a circle? What can be the relations among them?

(c) Is there a common centre of two circles having equal radii?

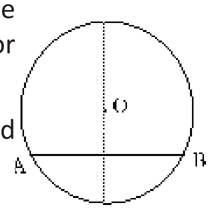
(d) Does a diameter bisects a circle?

(e) Which can be the longest chord of a circle?

(f) Does a straight line intersect a circle at more than two points?

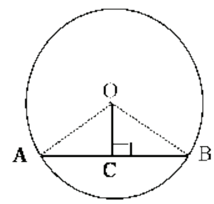
16.1 Theorems Related to Chord of a Circle

A line segment joining any two points of the circumference of a circle is called its chord. Diameter is the longest chord of a circle. We will study about the theoretical proofs of the theorems related to the chord of a circle here. As shown in the figure, draw a circle with centre O on the tracing paper or a white paper. Draw a chord AB to the circle. Fold the circle along the line passing through the centre and perpendicular to the chord AB. Whether the line made by the paper bisects the chord AB or not? Observe it with measurements.



Theorem 1 : A perpendicular drawn from the centre of a circle to a chord bisects the chord.

Given : In the given figure, O is the centre and AB is the chord where $OC \perp AB$



To prove : AC = BC

Construction : Join AO and BO.

Proof :

Statements	Reasons
1. In ΔOAC and ΔOBC	1.
i. $\angle OCA = \angle OCB$	i. $OC \perp AB$, each angle = 90^0
ii. OA = OB	ii. Radii of the same circle
iii. OC = OC	iii. Common side
2. $\Delta OAC \cong \Delta OBC$	2. From R.H.S. statement
3. AC = BC	3. Corresponding sides of congruent triangles.

Q.E.D

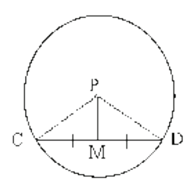
Can you prove the above theorem by the different names of the centre and its chord? If yes, prove it.

Theorem 2 : A line segment joining the centre of a circle to the mid-point of its chord is perpendicular to the chord.

Given : In the drawn figure P is the centre and PM bisects the chord CD.

To prove : $PM \perp CD$

Construction : Join PC and PD.

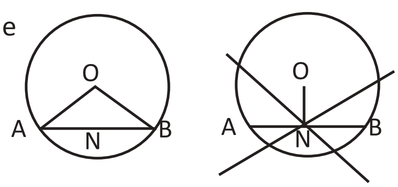


Proof :

Statements	Reasons
1. In ΔPMC and ΔPMD	1.
i. PC = PD	i. Being the radii of the same circle
ii. PM = PM	ii. Common side
iii. CM = DM	iii. By given statement
2. ΔPMC ≅ ΔPMD	2. By S.S.S. statement
3. ∠PMC = ∠PMD	3. Corresponding angles of congruent triangles
4. ∠PMC + ∠PMD = 180°	4. Sum of adjacent angles made on one side of a straight line.
5. ∠PMC + ∠PMC = 180^0 or, 2∠PMC = 180^0 or, ∠ PMC = 90^0	5. From statement (3) and (4)
6. PM ⊥CD	6. From statement (5), ∠ PMC = 90^0

Q.E.D

Draw a circle with the help of a compass with centre O and chord AB. Draw the bisectors of AB making one of them pass through the origin. Measure the angle made by the bisector passing through the centre O with the chord AB. Write the result so obtained. Can you do the same work by folding a paper? Try it.

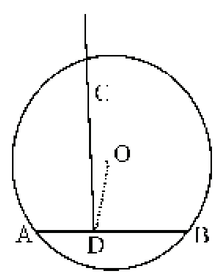


Theorem 3 : The perpendicular bisector of a chord of a circle passes through the centre of that circle.

Given : Here O is the centre of a circle where the chord AB of which is perpendicularly bisected

by CD at D.

To prove : CD passes through the centre O i.e. O lies on CD.



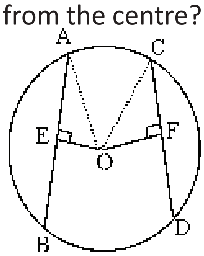
Construction : Suppose O does not lie on CD. Then draw OD.

Proof :

Statements	Reasons
1. CD $\perp$ AB	1. By given statement
2. OD$\perp$AB	2. O is the centre and D is the mid-point of AB, (theorem 2)
3.i. $\angle$CDB =$\angle$ODB	3.i. From 1 and 2, both being 90^0
ii. It is not true unless CD and OD coincide with each other.	ii. $\angle$CDO = 0^0, according to construction
4. CD and OD must lie on the same straight line i.e. CD passes through the centre of the circle.	4. From statement (3), our supposition must be wrong.

Q.E.D.

Draw a circle of centre O with equal chords AB and CD. Now cut the circle into two semi-circles and fold the semi-circles such that dividing the chords AB and CD into two equal halves. By folding this way, are the points lying on AB and CD equidistant from the centre? Observe with necessary measurements.



Theorem 4 : The equal chords of a circle are equidistant from the centre of the circle.

Given : O is the centre of a circle where AB and CD are its two equal chords whose distances from the centre are OE and OF i.e.

OE $\perp$ AB and OF $\perp$ CD.

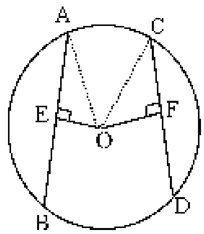
To prove : OE = OF

Construction : Join AO and CO.

Proof :

Statements	Reasons
1. In ΔAEO and ΔCFO	1.
i. $\angle$AEO = $\angle$CFO	i. Both being 90^0
ii. AO = CO	ii. Being the radii of the same circle
iii. AE = CF	iii. Both being halves of equal chords AB and CD.
2. ΔAEO $\cong$ ΔCFO	2. R.H.S. axiom
3. OE = OF	3. Corresponding sides of congruent triangles are equal.

Theorem 5 : (Converse of theorem 4) In any circle, the chords that are equidistant from the centre are equal to each other.



Given : AB and CD are any two chords of a circle with centre O in which OE $\perp$AB, OF $\perp$ CD, OE = OF.

To prove : AB = CD

Construction : Join OA and OC.

Proof :

Statements	Reasons
1. In ΔOAE and ΔOCF	1.
i.$\angle$AEO = $\angle$CFO	i. OE$\perp$AB and OF$\perp$CD, both being 90^0
ii.OA = OC	ii. Radii of the same circle are equal
iii. OE = OF	iii. By the given statement
2.ΔOAE $\cong$ ΔOCF	2. R.H.S. axiom
3. AE = CF	3. Corresponding sides of congruent triangles are equal.
4. 2AE = 2CF i.e. AB = CD	4. The perpendicular from the centre to the chord bisects it.

Q.E.D.

Example 1 :

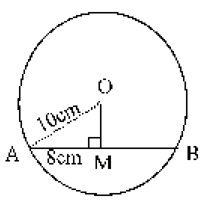
If the radius of a circle is 10 cm and the length of its chord is 16 cm, find the distance of the chord from the centre of the circle.

Solution :

Let O is the centre of the circle whose chord is AB.

By the question, AB = 16 cm and OA = 10 cm. Draw OM$\perp$AB.

Then, AM = $\frac{1}{2}$ AB = 8cm [$\because$ AM = $\frac{1}{2}$ x 16 cm = 8 cm]



In the right angled ΔOMA, we know, $OA^2 = OM^2 + AM^2$

or, $10^2 = OM^2 + 8^2$ or, $100 = OM^2 + 64$

or, $100 - 64 = OM^2$

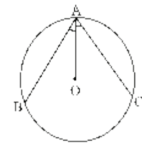
or, $36 = OM^2$

or, $6 = OM$

$\therefore$ OM = 6 cm which is the required distance.

Example 2

In the given figure, AB and AC are the chords of a circle with centre O. If $\angle$**BAO =** $\angle$**CAO** Prove that **AB = AC**

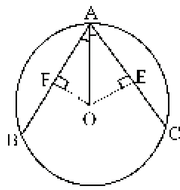


Solution :

Given : Here, $\angle BAO = \angle CAO$ and O is the centre of the circle.

To prove : AB = AC

Construction : Draw OF $\perp$ AB and OE$\perp$ AC.



Proof :

Statements	Reasons
1. In ΔOAF and ΔOAE	1.
i. OA = OA	i. Common side
ii. $\angle OAF = \angle OAE$	ii. By given condition
iii. $\angle AFO = \angle AEO$	iii. Being both angle 90^0
2. $\Delta OAF \cong \Delta OAE$	2. From S.A.A. axiom
3. AF = AE	3. Corresponding sides of congruent triangles
4. AF = BF and AE = EC	4. OF$\perp$AB and OE$\perp$AC
5. 2AF = 2AE or, AB = AC	5. From statement (3) and (4)

Q.E.D

Example 3

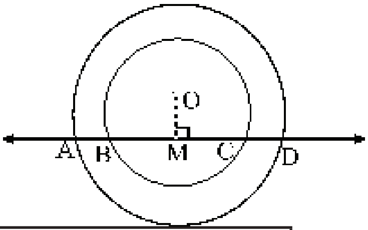
In the given figure, a line intersects two concentric circles of centre O at the points A,B,C and D as shown in figure. Prove that AB = CD.

Solution :

Given : A straight line has intersected two concentric circles with centre O at the points A, B, C and D.

To prove : AB = CD

Construction : Draw OM$\perp$ AD.



Proof :

Statements	Reasons
1. AM = DM	1. A perpendicular from a centre to a chord bisects the chord
2. BM = CM	2. Same as statement (1)
3. AB = CD	3. Subtracting statement (2) from (1).

Proved

Exercise 16

1. (a) What will be the length of a chord which is 4 cm away from the centre of a circle with radius 5cm?

   (b) A circle has radius 26 cm and length of its chord is 48 cm. Find how far is the chord from the centre of the circle?

   (c) The length of two parallel chords AB and CD on the same side of the centre O of a circle are 6cm and 12 cm respectively. If the distance between AB and CD is 3 cm, what will be the radius of the circle?

2. (a) In the adjoining figure, O is the centre of a circle. The chords AB and CD are equal and intersected at the point X. Prove that AX=CX and BX=DX.

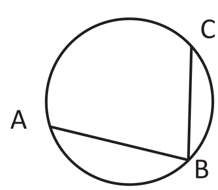
   (b) In the given figure, O is the centre of a circle. If OD $\perp$ AB, OE$\perp$ AC and AB = AC, Prove that ΔADE is an isosceles triangle.

3. In the given figure, X and Y are the centres of two circles. XY $\perp$ CD and CD intersects the circle with center X at the points M and N and XY at the point P. Prove that

   i. CM = DN ii. CN = DM

4. Three students Ahmed, Nimba and Krishna are playing a game standing in a circular path of radius 10 m. Ahmed threw a ball towards Nimba and Nimba threw it towards Krishna one by one. The distance between Ahmed and Nimba is 12 m and Nimba and Krishna is also 12 m. What will be the distance between Ahmed and Krishna? Find the distance making a proper diagram.

5. In the given figure, AB and BC are any two chords of a circle. Trace that circle on your copy and find out the centre of the circle.

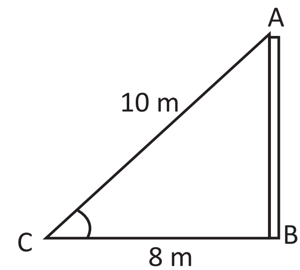


Lesson

17 Trigonometry

17.0 Review

A part of conversation between a teacher and his/her students at a ground of school is given below.



Teacher : Dilmaya, can you say what is the height of that volley ball pole?

Dilmaya : Sir, the pole is tall and smooth. We cannot climb, how to measure?

Teacher : Why to climb? It's shadow has reached up to here. We can say by measuring it's distance.

Jeewan : Sir, the distance between foot of pole and end of shadow is 8 m. Is this the height of the pole?

Teacher : No, thats not. If we put a bamboo stick up to top of the pole from here, we find the length of bamboo stick which is 10 m. Now, Jeewan can you say what kind of picture will be formed between them?

Jeewan : A right angled triangle will beformed sir. Now I undersand it. We can find the height of pole by using pythagorous theorem which we had studied in geometry.

Dilmaya : Yes, bamboo is opposite to right angle. So hypotenuse(h) = 10 m. The angle between shadow and bamboo is C. It is called reference angle. Pole is opposite side of $\angle$ C. So, perpendicular (AB) = p. Length of shadow = Adjacent side or base (BC) = b = 8 cm.

Here, from Pythagorous theorem,

$(\text{hypotenuse})^2 = (\text{perpendicular})^2 + (\text{base})^2$

or, $(10)^2 = p^2 + (8)^2$

or, $100 = p^2 + 64$

or, $p = \sqrt{100-64} = \sqrt{36} = 6\text{m}$

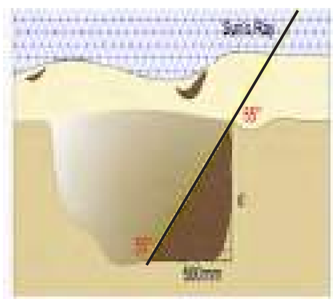
In this way, height of the pole is 6 m , isn't it sir?

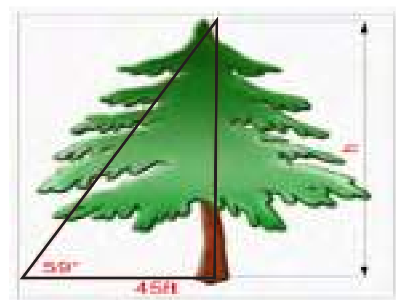
Teacher : Yes, Dilmaya, you found the right solution.

17.1 Trigonometry

From above facts, trigonometry is related to measurement of angles and sides of triangles. Trigonometry studies the relation between sides and angles of triangle. Trigonometry is used to measure the height and length of those objects which cannot be measured easily.

For example, in the given figure, we have to measure depth of river and height of tree or pole, trigonometry is used to find height and distance up to foot of the pole with the help of horizontal line from the tree or pole to any point and clinometer. (An instrument which is used to measure angle from the surface of the ground to the top of tree etc). The development and extension of mathematics, physics and engineering field is impossible without the help of trigonometry. Therefore, trigonometry is taken as an important part of mathematics and science.





Activity 1

Fill the table given below, by measuring the sides and angles of triangle given in the figure

Fig 1

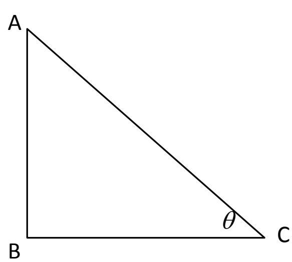


Fig 2

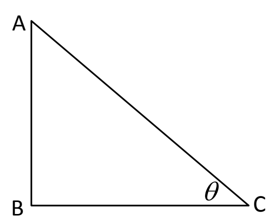
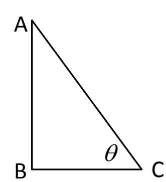


Fig 3



	AB	BC	CA	$\angle A$	$\angle C$	$\frac{AB}{AC}$	$\frac{BC}{AC}$	$\frac{AB}{BC}$
Fig 1								
Fig 2								
Fig 3								

After filling the above table, answer the following questions.

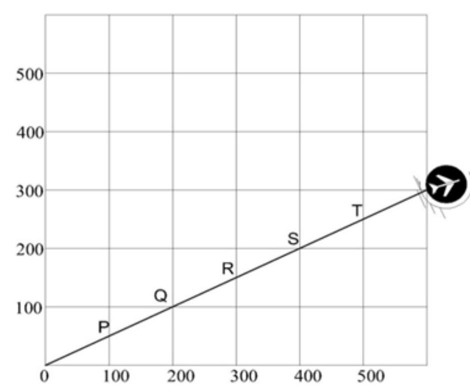
1. What are the right angle and reference angle in ΔABC?

2. What are the hypotenuse, opposite side and adjacent sides in ΔABC? Are they in proportion?

Hence, we can find the fact that there is direct relation between angle made by hypotenuse with base i.e. 'b' and adjacent side 'p' in right angled triangle ABC.

Example 1

This is the graph of a plane flying from Kathmandu to Surkhet to rescue the victims of flood in 2071 B.S. Find the height of the plane when it covers 500 m distance after the flight.



When the plane reaches to point P. it covers 100 m distance in ground and 50 m high above the ground. At that time, the ratio of height and distance (distance covered in ground)

or, height : distance = $\frac{50}{100}$

When the plane covers 200, 300, 400, 500 metre distance respectively in the ground, the plane will be at the height of 100, 150, 200 and 250 metres respectively. Fill the ratios in the following table by studying the figure.

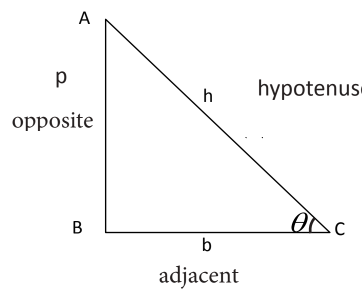
Position of plane	Height	Distance	Ratio (height : distance)	Angle made with ground
P	50	100	1:2 = 0.5	
Q	100			
R	150			
S	200			
T	250			

From the above table we can conclude that in any right angled triangle, there is direct relation between the opposite side (p) of angle made by hypotenuse with base (b). For a constant angle, height : distance = $\frac{p}{b}$ is also constant. The constant angle is called angle of reference. It is denoted by a Greek letter θ. When θ increases, ratio increases and when θ decreases, ratio decreases where $(0 \le \theta \le 90^0)$. Thus, if we know (height : distance) or $\frac{p}{b}$, we can find θ. In trigonometry, generally Greek letters are used to denote angles which are given below.

Small Greek letters	Name
α	Alpha
β	Beta
γ	Gamma
θ	Thita
ϕ	Phi
ψ	Psi

17.1.2: Fundamental Trigonometric Ratios

In the given figure of right angle triangle ABC, opposite side of reference angle θ = p = AB, adjacent side = b = BC and hypotenuse = h = AC.



Now, Ratio $\frac{AB}{AC}$ is called sine of θ

Symbolically, it is written as sin θ.

In right angled ΔABC,

Sine of θ or, $\sin\theta = \frac{\text{Perpendicular}}{\text{hypotenuse}} = \frac{AB}{AC} = \frac{p}{h}$

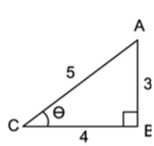
Similarly, other two basic ratios can be defined as :

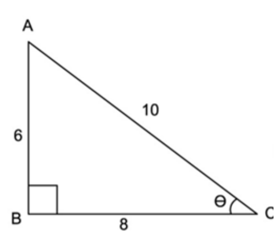
Cosine of θ or, $\cos\theta = \frac{\text{base}}{\text{hypotenuse}} = \frac{BC}{AC} = \frac{b}{h}$

Tangent of θ or, $\tan\theta = \frac{\text{Opposite or Perpendicular}}{\text{adjacent}} = \frac{AB}{BC} = \frac{p}{b}$

Thus, these three ratios $\sin\theta$, $\cos\theta$, and $\tan\theta$ are called basic trigonometric ratios. Beside this, other ratios in triangle are $\frac{AC}{AB}, \frac{AC}{BC}$ and $\frac{BC}{AB}$. They are denoted by cosec θ, $\sec\theta$ and $\cot\theta$ respectively.

Some important properties of trigonometric ratios can be shown by following examples.





In smaller ΔABC

$\sin\theta = \frac{p}{h} = \frac{3}{5}$,

$\cos\theta = \frac{b}{h} = \frac{4}{5}$

$\tan\theta = \frac{p}{b} = \frac{3}{4}$,

In greater ΔABC

$\sin\theta = \frac{p}{h} = \frac{6}{10} = \frac{3}{5}$

$\cos\theta = \frac{b}{h} = \frac{8}{10} = \frac{4}{5}$

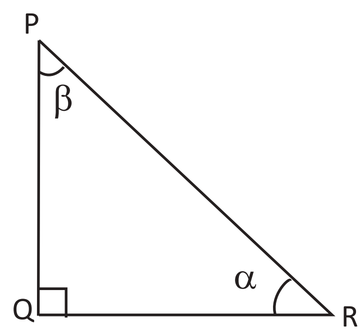
$\tan\theta = \frac{p}{b} = \frac{6}{8} = \frac{3}{4}$,

Size of triangle doesnot affect the value of trigonometric ratios in right angled triangles.

Activity 2

By observing the adjoining figure, match the following data :

i.	Perpendicular for α	PQ
ii.	Perpendicular for β	QR
iii.	Hypotenuse	PR
iv.	$\sin\alpha$	$\frac{PQ}{PR}$
v.	$\cos\beta$	$\frac{PQ}{PR}$
vi.	$\tan\alpha$	$\frac{PQ}{QR}$
vii.	$\tan\beta$	$\frac{QR}{PQ}$



Example 2

Find the value of :

(a) If $\sin\theta = \frac{3}{5}$ and h = 20 , p =?

(b) If $\cos\theta = \frac{4}{5}$ and b = 8 , h =?

(c) If $\tan\theta = \frac{3}{4}$ and b = 8 , p =?

(d) If $\sin\theta = \frac{3}{5}$, find the value of $\cos\theta$ and $\tan\theta$?

Solution :

(a) Here,

$\sin\theta = \frac{p}{h}$ or, $\frac{p}{h} = \frac{3}{5}$

h = 20 Then, $\frac{P}{20} = \frac{3}{5}$ Or, 5p = 3 x 20 or, $P = \frac{3 \times 20}{5} = 3 \times 4 = 12$

Therefore, p = 12

(b) Here, $\cos\theta = \frac{b}{h} = \frac{4}{5}$

or, $\frac{b}{h} = \frac{4}{5}$, if b = 8, Then $\frac{8}{h} = \frac{4}{5}$

or, 4h = 5 x 8

So, h = 10

(c) Here, $\tan\theta = \frac{p}{b} = \frac{3}{4}$

or, $\frac{p}{b} = \frac{3}{4}$, b = 8 or, $\frac{p}{8} = \frac{3}{4}$

or, 4p = 8 x 3

So, p = 6

(d) If $\sin\theta = 3/5$, then p = 3, h = 5 (Let)

Using Pythagorous theorem,

$h^2 = p^2 + b^2$

or, $5^2 = 3^2 + b^2$

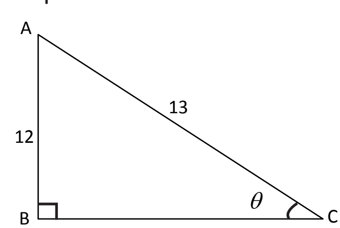
or, $b^2 = 25 - 9 = 16$, So, b = 4

Now, $\cos\theta = \frac{b}{h} = \frac{4}{5}$ and $\tan\theta = \frac{p}{b} = \frac{3}{4}$

Example 3

In ΔABC, $\angle B = 90^0$, $\angle C = \theta$, AB = 12 cm and AC = 13 cm, then prove that:

$\sin^2\theta + \cos^2\theta = 1$



Solution :

Here, AB = p = 12 cm, AC = h = 13 cm, BC = b =?

According to Pythagorous theorem,

$h^2 = p^2 + b^2$

or, $13^2 = 12^2 + b^2$

or, $b^2 = 169 - 144 = 25$

$\therefore$ b = 5 cm

Now, $\sin^2\theta + \cos^2\theta$

$$= \frac{p^2}{h^2} + \frac{b^2}{h^2} = \frac{(12)^2}{(13)^2} + \frac{(5)^2}{(13)^2} = \frac{169}{169} = 1$$

Hence, proved

Example 4

Express tanθ in terms of cosθ.

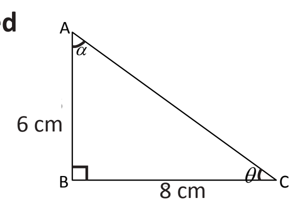
Solution :

We know that $\sin\theta = \frac{p}{h}$ and $\cos\theta = \frac{b}{h}$,

$$\tan\theta = \frac{p}{b} = \frac{\frac{p}{h}}{\frac{b}{h}} = \frac{\frac{\sqrt{h^2 - b^2}}{h}}{\frac{b}{h}} = \frac{\sqrt{\frac{h^2}{h^2} - \frac{b^2}{h^2}}}{\frac{b}{h}} = \frac{\sqrt{1 - (\frac{b}{h})^2}}{\frac{b}{h}} = \frac{\sqrt{1 - \cos^2\theta}}{\cos\theta}$$

Exercise 17.1

1. **Answer the following questions from the given right angled triangle.**



(A) i. Which one is the longest side? What is it's length?

ii. What are the perpendicular and base for angle α?

iii. What are the perpendicular and base for angle θ?

(B) Find the value of :

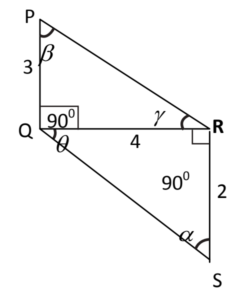
i. $\sin\alpha$, $\cos\alpha$, $\tan\alpha$

ii. $\sin\theta$, $\cos\theta$, $\tan\theta$

(C) Prove that :

i. $\sin^2\alpha + \cos^2\alpha = 1$ ii. $\sin^2\theta + \cos^2\theta = 1$

2. In the given figure, find the ratios of sine, cosine and tangent from the relation between angles α, β, γ, θ, and their sides



3. Caculate :

i. Express tanA in terms of sinA.

ii. Express sinA in terms cosA.

iii. If $\tan\theta = \frac{3}{4}$, find the value of $\sin\theta$ and $\cos\theta$.

iv. If $\sin\theta = \frac{\sqrt{3}}{2}$, find the value of $\cos\theta$ and $\tan\theta$.

v. If $\sin\theta = \frac{3}{5}$, find the value of $\cos\theta$.

vi. If $\tan\theta = \frac{1}{\sqrt{3}}$, find the value of $\sin\theta$ and $\cos\theta$.

vii. If $\sin\theta = \frac{p}{h} = \frac{6}{10}$ and h = 20, then b = ?

viii. If $\cos\theta = \frac{2\sqrt{3}}{4}$ and p = 6, then h =? and b =?

17.2 Trigonometric Ratio of Some Special Angles

We can solve the problems of triangles with the help of special angles from 0^0 to 90^0. Among the special angles, we will learn to find the values of 0^0, 30^0, 45^0, 60^0 and 90^0.

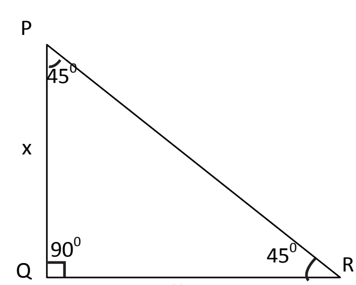
17. 2.1: Values of trigonometric ratios of angle 45^0

Here, ΔPQR is a right angled isosceles triangle

where $\angle P = 45^0$ and $\angle R = 45^0$

So, let us supposeside PQ = QR = x

Now, using Pythagorous theorem,



We get, $PR^2 = PQ^2 + QR^2 = x^2 + x^2 = 2x^2$

or, PR = $\sqrt{2}x$

In right angled ΔPQR

$$\sin R = \sin 45^0 = \frac{p}{h} = \frac{PQ}{PR} = \frac{x}{\sqrt{2}x} = \frac{1}{\sqrt{2}} = 0.707$$

$$\cos R = \cos 45^0 = \frac{b}{h} = \frac{QR}{PR} = \frac{x}{\sqrt{2}x} = \frac{1}{\sqrt{2}} = 0.707$$

$$\tan R = \tan 45^0 = \frac{p}{b} = \frac{PQ}{QR} = \frac{x}{x} = 1 = 1.000$$

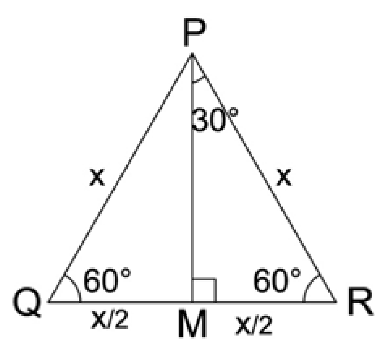
17.2.2 Values of trigonometric ratios of the angles 30^0 and 60^0

To find the values of trigonometric ratios of the angles 30^0 and 60^0, let ΔPQR be an equilateral triangle where the sides PQ = QR = PR = x. Right angled triangle is required for trigonometric ratios. So, let us draw a perpendicular PM from P to base QR which divides the base QR in two equal parts and here $QM = \frac{x}{2} = MR$.

Now, In right angled ΔPMR, If $\angle R = 60^0$ and $\angle M = 90^0$, then $\angle MPR = 30^0$

From Pythagorous theorem,

In ΔPMR,



$(PM)^2 = (PR)^2 - (MR)^2$

$(PM)^2 = x^2 - \left(\frac{x}{2}\right)^2 = x^2 - \frac{x^2}{4}$

$(PM)^2 = \frac{3x^2}{4}$

$PM = \sqrt{\frac{3x^2}{4}} = \frac{\sqrt{3}}{2}x$

Again, for angle of reference, $\angle MPR = 30^0$, p = $MR = \frac{x}{2}, h = PR = x, b = PM = \frac{\sqrt{3}x}{2}$

Therefore, $\sin 30° = \frac{p}{h} = \frac{MR}{PR} = \frac{\frac{x}{2}}{x} = \frac{x}{2} \times \frac{1}{x} = \frac{1}{2} = 0.5$

$$\cos 30° = \frac{b}{h} = \frac{PM}{PR} = \frac{\frac{\sqrt{3}}{2}x}{x} = \frac{\sqrt{3}}{2} = 0.866$$

$$\text{Tan}30^\circ = \frac{\text{p}}{\text{b}} = \frac{\frac{x}{2}}{\frac{\sqrt{3}}{2}x} = \frac{x}{2} \times \frac{2}{\sqrt{3}x} = \frac{1}{\sqrt{3}} = 0.577$$

Again, in right angled ΔPMR, for reference angle R = 60°,

We have, $\text{P} = \text{PM} = \frac{\sqrt{3}}{2}x, \text{b} = \text{MR} = \frac{x}{2}$ and h = PR $= x$

$$\text{Sin}60^\circ = \frac{\text{p}}{\text{h}} = \frac{\text{PM}}{\text{PR}} = \frac{\frac{\sqrt{3}x}{2}}{x} = \frac{\sqrt{3}x}{2x} = \frac{\sqrt{3}}{2} = 0.886$$

$$\text{Cos } 60^\circ = \frac{\text{b}}{\text{h}} = \frac{\text{MR}}{\text{PR}} = \frac{\frac{x}{2}}{x} = \frac{x}{2} \times \frac{1}{x} = \frac{1}{2} = 0.5$$

$$\text{Tan}60^\circ = \frac{\text{p}}{\text{b}} = \frac{\text{PM}}{\text{MR}} = \frac{\frac{\sqrt{3x}}{2}}{\frac{x}{2}} = \frac{\sqrt{3x}}{2} \times \frac{2}{x} = \sqrt{3} = 1.732$$

Hence, sin30°= $\frac{1}{2}$ = cos60° = Cos (90° – 30°)

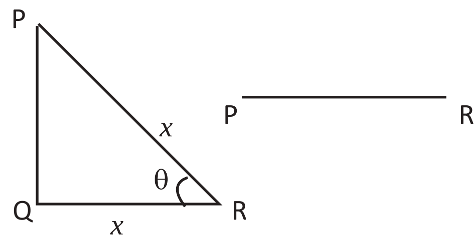
$\therefore$ Sinθ = Cos (90°–θ)

Similarly, Cos30° = $\frac{\sqrt{3}}{2}$ = Sin60° = Sin (90° – 30°)

$\therefore$ Cosθ = Sin (90°–θ)

17.2.3 Values of trigonometric ratios of the angles 0^0 and 90^0

In right angled ΔPQR, when PR is rotating towards QR, then $\angle PRQ$ becomes smaller and smaller and finally P coincides with point Q. At this time $\angle PRQ$ approaches to 0^0 and PQ = 0 or, $PR = x$, $QR = x$, $PQ = 0$



Therefore, $\sin 0° = \frac{PQ}{PR} = \frac{0}{x} = 0$

$$\cos 0° = \frac{QR}{PR} = \frac{x}{x} = 1$$

$$\tan 0° = \frac{PQ}{QR} = \frac{0}{x} = 0$$

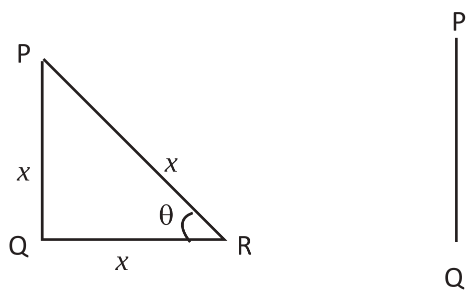
Similarly,

In ΔPQR, when $\angle PRQ$ becomes greater and greater from 0^0 and finally approaches to $\angle\theta = 90^0$, PQ and PR coincides with each other and if $PR = x$, $PQ = x$ and $QR = 0$.

$$\sin 90° = \frac{PQ}{PR} = \frac{X}{X} = 1$$

$$\cos 90° = \frac{QR}{PR} = \frac{O}{X} = 0$$

$$\tan 90° = \frac{PQ}{QR} = \frac{X}{O} = \infty \quad \text{(Undefined)}$$



The values of trigonometric ratios of special angles can be shown in the following table :

Angle / Ratio	0^0	30^0	45^0	60^0	90^0
Sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
Cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
Tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞ (undefined)

Example 4

If Sin90° = 1, Cos90° = 0, sin45^0 = $\frac{1}{\sqrt{2}}$, cos60^0 = $\frac{1}{2}$, then prove that :

i. Sin(90°–60°) = Sin90°.Cos60° – Cos90°.Sin60°

ii. Cos (45°+45°) = Cos45°.Cos45° – Sin45°.Sin45°

Solution :

Here, if sin45^0 = $\frac{1}{\sqrt{2}}$, , then p = 1 and h = $\sqrt{2}$

then, cos45^0 = $\frac{\sqrt{h^2 - p^2}}{h} = \frac{\sqrt{2-1}}{\sqrt{2}} = \frac{1}{\sqrt{2}}$

If cos60^0 = $\frac{1}{2}$, then p = 1 and h = 2

then, sin60^0 = $\frac{\sqrt{h^2 - b^2}}{h} = \frac{\sqrt{4-1}}{2} = \frac{\sqrt{3}}{2}$

If sin90^0 = 1, then p = 1 and h = 1

then, cos90^0 = $\frac{\sqrt{h^2 - p^2}}{h} = \frac{\sqrt{1-1}}{1} = \frac{0}{1} = 0$

i. LHS = sin(90°–60°) = sin30° = $\frac{1}{2}$

RHS, = sin 90°.cos 60° – cos 90°.sin 60°

$= 1x\ \frac{1}{2} - 0\ x\ \frac{\sqrt{3}}{2} = \frac{1}{2}$

Hence, L.H.S. = R.H.S. proved.

ii. L.H.S = cos (45°+45°) = cos90^0 = 0

R.H. S = cos 45°.cos 45° – sin 45°.sin 45°

$= \frac{1}{\sqrt{2}}\ X\ \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}\ X\ \frac{1}{\sqrt{2}} = 0$

Hence, L.H.S. = R.H.S. proved.

Exercise 17.2

1. Evaluate the following trigonometric ratios using the table :

i. $\tan 60^0$ ii. $\tan 30^0$ iii. $\sin 90^0$

iv. $\cos 45^0$ v. $\cos 60^0$ vi. $\tan 45^0$

vii. $\tan 60^0 + \cos 60^0$ viii. $2 \sin 30^0 \cos 30^0$ ix. $\sin 60^0$

2. Find the value of acute angles of right angled triangle for the following values by drawing and measuring figure.

i. $\sin\theta = 0.5$

ii. $\cos\theta = 0.5$

iii. $\tan\theta = 1$

3. Find the value of :

i. $\sin 0^0 + \sin 30^0 + \sin 45^0 + \sin 60^0$ ii. $\cos^2 45^0 + \sin^2 45^0$

iii. $\dfrac{2\tan 30^\circ}{1-\tan^2 30^\circ}$ iv. $\dfrac{\tan 30^\circ + \tan 45^\circ}{1-\tan 30^\circ \tan 45^\circ}$

4. Prove that :

i. $\cos^2 30^0 + \sin^2 30^0 = 1$ ii. $\sin 90^0 = 2\sin 45^0 \cos 45^0$

iii. $\tan 60^0 = \dfrac{2\tan 30^0}{1-\tan^2 30^0}$ iv. $\cos 60^0 = 1 - 2\sin^2 30^0 = 2\cos^2 30^0 - 1$

5. Solve :

i. $\cos\theta = \dfrac{\sqrt{3}}{2}\tan 30^\circ$ ii. $2\sqrt{3}\cos\theta = 3$ iii. $4\cos^2\theta = 1$

6. In a right angled triangle, length of hypotenuse is 5 cm and angle of reference is 30^0, find the value of remaining angles and sides.

7. From an equilateral triangle, find the value of $\sin 30^0$, $\cos 30^0$ and $\sin 60^0$.

8. The top of the clock tower of height 36 cm is observed from a point $12\sqrt{3}$ m away from its foot. Determine the angle of reference.

Lesson 18

Statistics

18.0 Review

The word 'statistics' came from the Latin language and its meaning is "Statute of state". During ancient period, statistics was used to study the population and poverty. Nowadays it is not limited to study the population and poverty but also in economic, administrative, finance, business, research, banking and all other fields of the state. So, importance of the statistics is increasing day by day.

The following are the marks obtained in mathematics by 15 students of class 8 in district level examination :

45, 46, 67, 78, 85, 92, 49, 65, 79, 58, 59, 45, 67, 85, 78,

In how many ways the above data can be presented. Discuss in groups.

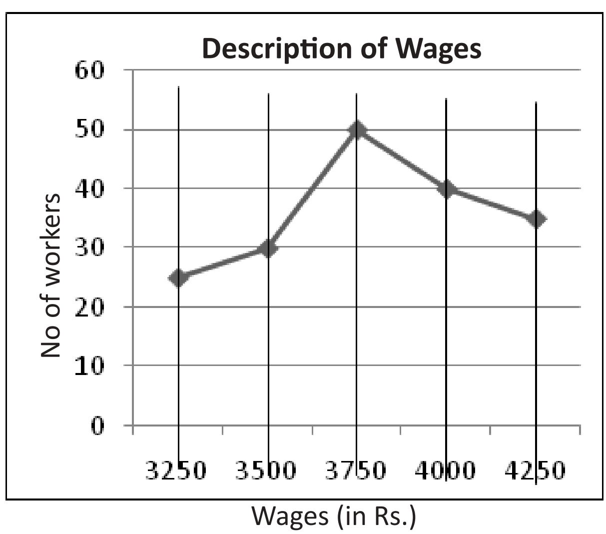
We have presented the obtained marks of the students of a class in the table. From this we can understand that the collection of numbers having fact is understood that the collection of numbers having fact is called data. In fact, the set of collected quantitative information is data. In this way, we can present the conclusion of above discussion in individual, discrete and continuous series.

18.1 Line Graph and Pie Chart

18.1.1 Line Graph

Discuss the questions given below by studying the given line graph and find the answer in group.

1. What is the amount of minimum wage? How many workers will get minimum wage?
2. How many workers will get maximum wage?
3. What is the difference between maximum and minimum wages?



Thus, after the above discussion, line graph can be defined as :

> The graph which represents any data and informations like change in temperature in a day, change of population in different year, wages of workers of a company in the form of chain of points is known as line graph. In line graph, every points are joined by straight line as shown in the above figure.

Method of constructing a line graph

Example 1

The number of families with respect to the number of children are shown in the table given below. Represent this data in a line graph.

No. of children	1	2	3	4	5
No. of family	5	6	4	3	2

Solution :

The above data can be represented in line graph by following steps.

Step 1 : Write the number of children in X-axis and number of family in Y-axis.

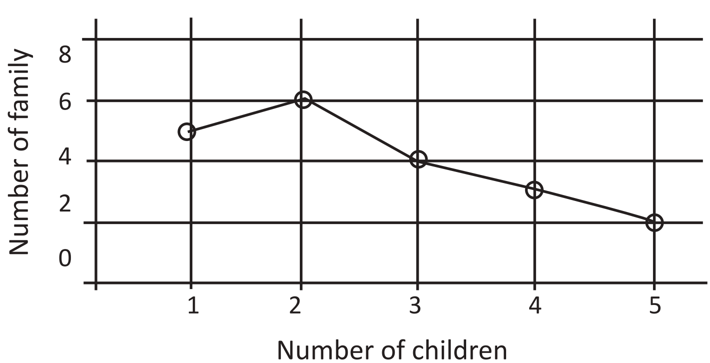
Step 2 : For the first point, plot the point such 1- unit along X-aixs and 5 unit along Y-axis to represent number of children and number of family respectively. Similarly, points for number of family having 2 children is 6, number of family having 3 children is 4, number of family having 4 children is 3 and number of family having 5 children is 2 are plotted in the graph.

Step 3 : Join every points in order by the straight line.

Now, the following line graph is formed.

I

Number of children and family



18.1.2 Pie Chart (Angular Diagram)

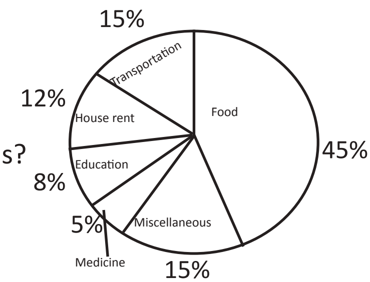
Example: 2

The monthly income of a family is Rs. 4000. The details of monthly expenditure of this family are shown in the pie chart given alongside. Answer the following question by studying the pie chart.

(a) What is the expenditure on food?

(b) What amount of expenditure is spent on house rent?

(c) What amount of expenditure is spent on miscellaneous?



Solution :

We know that, in pie chart 360^o or 100% denotes total frequency (N)

In figure, 100% = Rs. 4000

or, 1% = Rs. 40

(a) Expenditure on food = 45% = 45 x Rs. 40 = Rs. 1800

(b) Expenditure on house rent = 12% = 12 x Rs. 40 = Rs. 480

(c) Expenditure on Miscellaneous = 15% = 15 x Rs. 40 = Rs. 600

From the above discussion, pie chart can be defined in the following way.

The two dimensional circular diagram is called pie chart in which the angle of sectors represents the frequencies of corresponding item. We can easily study how much the government spend on different headings like agriculture, industry, transportation, irrigation etc in the budget with the help of pie-chart.

Similarly, we can study how much percentages of income was spent in different headings of expenditure by a person.

Method of constructing a Pie chart

Example 3

The number of people participating in the blood donation programme of a village is 2200. During the blood test, it was found that 323 have group 'A', 220 have group 'B', 850 have group 'O' and 807 have group 'AB'. Represent this data in the pie-chart

Solution :

It can be represented in following three steps.

Step 1

We have to show total number of blood donors i.e. 2200 in the whole pie chart.

Therefore, 2200 = 360^0

Consequently, 1 person = $\frac{360^o}{2200}$

Step 2

Using formula,

Central angle (Angle of sector) = $\frac{\text{Frequency of corresponding group}}{\text{Total frequency}} \times 360°$

Which is shown in the following table :

Blood group	No. of people	Central angle	Rounded value
A	323	$\frac{323}{2200}\times 360° = 52.85°$	53^o
B	220	$\frac{220}{2200}\times 360° = 36°$	36^o
O	850	$\frac{850}{2200}\times 360° = 139°$	139^o
AB	807	$\frac{807}{2200}\times 360° = 132°$	132^o
Total	2200	$\frac{2200}{2200}\times 360° = 360°$	360^0

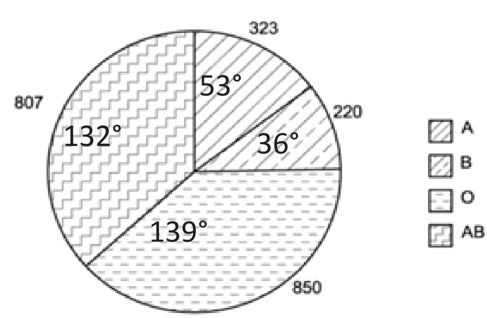
Step 3

i. Construct a circle (radius = 3 cm)

ii. Draw a radius and construct an angle of 53^o with the help of proctactor by taking the same radius as base line. Write blood group A on that sector.

iii. Then after, divide the circle by constructing the angles 36°, 139° and 132° respectively and indicate the corresponding groups with different index.

No of blood donors



Exercise 18.1

1. The data of the number of goals secured in 30 football matches is given below. Represent it in table.

   1, 0, 2, 3, 4, 2, 2, 1, 2, 0, 2, 0, 4, 0, 4, 2, 2, 3, 6, 2,2, 3, 1, 5, 5, 1, 1, 2, 6, 3

2. Following is the data of pulse rate of 37 persons present in a health camp.

   59, 52, 60, 66, 82, 68, 70, 59, 65, 72, 61, 56, 73, 78, 61, 66, 59, 51, 68, 84, 75, 79, 58, 68, 62, 71, 53, 57, 74, 70, 50, 66, 71, 64, 72, 58, 58

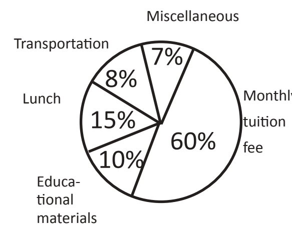
   Represent the above data in the frequency distribution table by taking class intervals 50 - 55, 55 - 60, 60- 65,...,

3. The monthly expenditure of a student studying in a private school is Rs. 7500 which is shown in the pie chart given below.

   (a) How much is the monthly tuition fee?

   (b) What is the amount of expenditure on educational materials and transportation?

   (c) In which heading there is maximum expenditure?



4. The table given below shows the production of rice in different districts. Represent this data in Pie-chart.

District	Jhapa	Morang	Dhanusa	Kathmandu	Kabhre
Production (metric ton)	1500	1250	1550	500	1200

5. The table given below shows the data of export of Nepali home made clothes to different countries. Represent this in Pie-chart.

Country	America	Australia	U.K.	France	others	Total
Export (Rs. in thousand)	530	55	83	48	24	740

6. The height of different mountains in Nepal are given in the table. Construct a line graph for this data.

Mountain	Dhaulagiri	Annapurna	Manaslu	Sagarmatha	Lhotse	Makalu	Kanchanjanga
Height (m)	8172	8078	8156	8848	8501	8470	8598

7. Collect the data of number of school students from six families near by your house in your community and represent the data in a line graph and a table.
8. Collect the result of SLC examination of last five years of your school and represent it in pie - chart.
9. What are the monthly expenditure on food, clothes, education, health and miscellaneous of your family? Represent the data in pie chart by consulting your guardian.

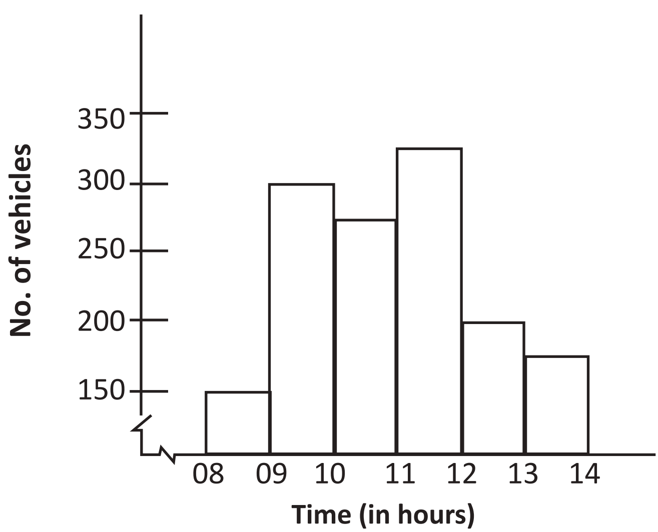
18.2 Histogram and Cumulative Frequency Curve

18.2.1 Histogram

Answer the following questions by studying the following table and histogram.

The number of vehicles going outside the valley from Kalanki, Kathmandu in different time intervals are as follows :

Time (hours)	08 - 09	09 – 10	10 – 11	11 -12	12 - 13	13 - 14
No. of vehicles	150	300	275	325	200	175



(a) What is represented by the numbers in X - axis?

(b) What is represented by the numbers in Y - axis?

(c) What is the lowest number of vehicles going outside? At what time they went out?

(d) How many vehicles will go out in the time interval of 09 - 10 O'clock?

(e) What is the highest number of vehicles going outside? At what time they went out?

After observing the above histogram, we can get the following conclusions :

The first bar shows that the number of vehicles going outside from 08 to 09 O'clock is 150. Similarly, second bar shows that the number of vehicles going outside from 09 to 10 O'clock is 300. The tallest bar denotes the highest number of vehicles and the shortest bar denotes the lowest number of vehicles.

Thus, the given or collected data are represented in continuous frequency distribution table having different class intervals. Bars having breadth equal to each class interval in X - axis and height equal to frequency in Y - axis are drawn. This diagram is called histogram. It is used for continuous data.

Method to construct a Histogram

The monthly wages of workers of a company are as follows. Represent this data in a histogram by tabulating the data in continuous frequency distribution table.

Wages (in thousand Rs.) 26, 24, 25, 35, 26, 24, 23, 22, 20, 23, 24 36, 42, 45, 54, 55, 33, 37, 32, 23, 24, 40, 42, 52, 50, 53, 57, 58, 50, 39, 23, 24

Step 1

Here, the lowest number is 20 and highest number is 58. There are 4 classes to include the whole data with class interval of 10/10.

Now, represent the data in grouped frequency distribution table

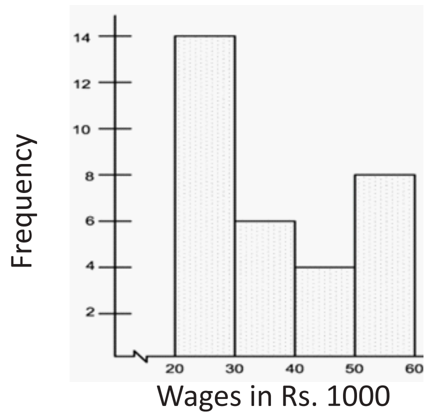
Wages (in thousand Rs.)	Tally Marks	Frequency
20 - 30	~~||||~~ ~~||||~~ ||||	14
30 - 40	~~||||~~		6
40 - 50	||||	4
50 - 60	~~||||~~ |||	8

Step 2

Here, upper limit of every class interval is excluded. Now, fix the origin X - axis. Then, take a scale as 1 cm = 5 units and mark the points for the class intervals 20 - 30, 30 - 40, 40 - 50, 50 - 60

Step 3

The highest frequency is 14. So, taking scales as 2cm = 1 unit mark the points 2, 4, 6, 8, 10, 12 and 14 respectively in Y - axis above the origin.



Step 4

Draw a bar for class interval (20 - 30) in X - axis having breadth 2 cm and height 14 units. Similarly, for class interval (30 - 40), (40 - 50) and (50 - 60) draw the bars having heights 6 units, 4 units, and 8 units respectively

Hence, required histogram is constructed.

18.2.2 Cumulative Frequency Distribution Table

The following are the marks obtained by 28 students of class 9 of Shree Krishna Higher Secondary School in Mathematics in second terminal examination.

35, 40, 45, 40, 35, 55, 60, 55, 40, 35, 60, 65, 45, 55, 45, 65, 45, 35, 55, 45, 55, 65, 60, 45, 60, 65, 35, 65

This data can be presented in frequency distribution table by using tally marks in the following way :

Marks obtained	Tally Marks	Frequency (No. of students)
35	$\cancel{\vert\vert\vert\vert}$	5
40	$\vert\vert\vert$	3
45	$\cancel{\vert\vert\vert\vert}\ \vert$	6
55	$\cancel{\vert\vert\vert\vert}$	5
60	$\vert\vert\vert\vert$	4
65	$\cancel{\vert\vert\vert\vert}$	5

We can classify the table in 4 classes of class interval in the following way :

Marks obtained	Tally Marks	Frequency
30 – 40	卌	5
40 – 50	卌 ||||	9
50 – 60	卌	5
60 – 70	卌 ||||	9

In above table, the counting has done by excluding the upper limit of the class interval. 40 is excluded in 30-40. Similarly last number or upper limit of each class has been excluded. In table, the tally sign denotes the number which is repeated frequently, so it is called frequency. This denotes the number of students.

From the above table, we can calculate the cumulative frequency in the following ways.

- First of all, let us write the data in ascending or descending order.
- Then, add the frequency of each one after another.

Marks obtained	Frequency	Cumulative Frequency (c.f.)
Less than 40 (< 40)	5	5
Less than 50 (< 50)	9	5 + 9 = 14
Less than 60 (< 60)	5	14 + 5 = 19
Less than 70 (< 70)	9	19 +9 = 28
Total	28	

Thus, the sum obtained by adding the frequency of each class one after another is called cumulative frequency. It is denoted by c.f. The above table is called cumulative frequency table.

18. 2. 3 Cumulative Frequency Curve or Ogive

The value of blood pressure of some females between 20 - 45 years is given below :

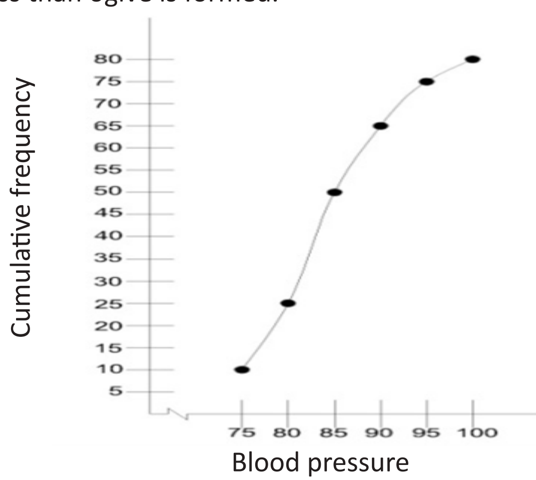
Diastolic B.P. (mmHg)	70 - 75	75 - 80	80 - 85	85 – 90	90 - 95	95 -100
No. of females	10	18	23	15	8	5

We can present the given table in less than cumulative frequency table in the following way :

Lower limit of blood pressure	Frequency
70 - 75	10
75 - 80	18
85 - 90	15
90 -95	8
95 -100	5

Lower limit of blood pressure	c.f.
less than 75	10
less than 80	10 +18 = 28
less than 85	28 + 23 = 51
less than 90	51 + 15 = 66
less than 95	66 + 8 = 74
less than 100	74 + 5 = 79

Now, as in the above cumultative frequency table, the data of each class is written in ascending order and frequencies are added one after another. When it is represented in line graph the following less than ogive is formed.



Here, upper values from 75 to 100 are written in X - axis and the relative requency is in Y - axis. Points are plotted for less than ogive in graph. Each points are joined respectively by free hand. Thus constructed line graph is called less than ogive. Hence, less than ogive is always increasing curve sloping upwards from left to right.
As like less than ogive, we can construct more than ogive. For this, let us observe the table given below.

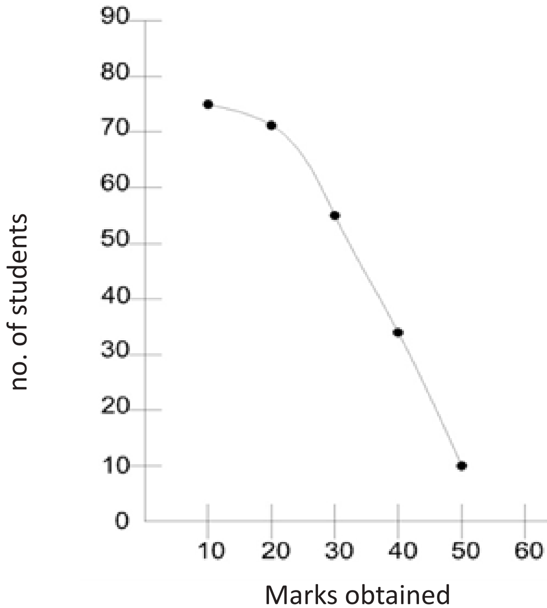
Marks obtained	10 -20	20 -30	30 - 40	40 -50	50 - 60
No. of students	5	14	25	23	10

Now, presenting this in more than cumulatative frequency we get,

Marks obtained	**Frequency**
10 - 20	5
20 - 30	14
30 – 40	25
40 - 50	23
50 - 60	10
Total	77

Marks obtained	**Cumulative Frequency (c.f)**
More than 10	77
More than 20	77 - 5 = 72
More than 30	72 - 14 = 58
More than 40	58 - 25 = 33
More than 50	33 - 23 = 10

Thus, in constructing more than ogive, there are all the numbers more than 10 i.e. the lowest number. Therefore, all the frequencies are added. Then, as like in the table, frequency of each class is subtracted from the total frequency one after another and at last, cumulative frequency of more than 50(50 - 60) will be 10. When it is represented in cumulative frequency curve, the following more than ogive is formed. Hence, more than ogive is always decreasing curve sloping downwards from left to right.



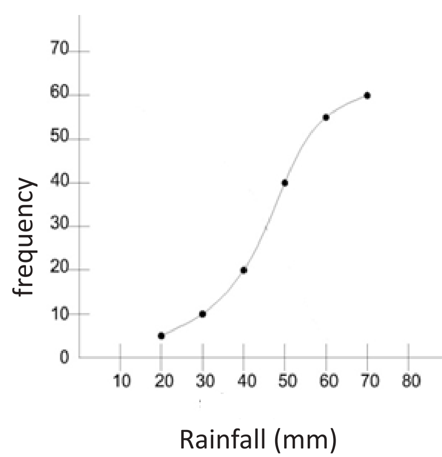
Note : We can represent the more than ogive and less than ogive of given grouped data on the same graph.

Exercise 18.2

1. Complete the following cumulative table by studying the cumulative frequency curve given below :

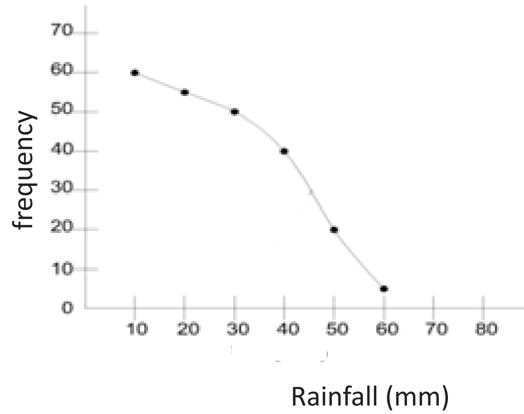
(a)

Less than Ogive	
Rainfall (mm)	**Less than c.f.**
Less than 20	
Less than 30	
Less than 40	
Less than 50	
Less than 60	
Less than 70	



(b)

More than Ogive	
Rainfall (mm)	**More than c.f.**
More than 20	
More than 30	
More than 40	
More than 50	
More than 60	
More than 70	



2. The weight (in Kg) of 30 boys are as follows :

   20, 14, 18, 35, 47, 33, 18, 16, 25, 14, 30, 14, 30, 27, 11, 29, 20, 22, 15, 29, 25, 20, 29, 14, 39,19, 18, 10, 25, 26

   (a) Prepare the frequency table by taking class interval of 10 from above weights (kg) of 30 boys.

   (b) Construct the histogram from the above table (a). Then identify the classes of boys having lowest weight and highest weight.

   (c) Construct less than c.f table on the basis of table (a) and represent it in cumulative frequency curve (ogive).

3. Prepare the grouped frequency distribution table from less than c.f table given below and represent it in histogram.

Daily wage (In Rs.)	Less than 50	Less than 100	Less than 150	Less than 200	Less than 250	Less than 300	Less than 350
No.of workers (c.f.)	10	24	30	50	70	90	100

4. The given data is of 60 diabetic patients found during the health check up of 300 people in a health camp of a community. Now, construct an ogive by making cummulative frequency table based on this data.

Age (years)	20 - 30	30 - 40	40 - 50	50 - 60	60 – 70
No. of patients (f)	2	5	15	25	13

18.3 Measures of Central Tendency

18.3.1 Arithmetic Mean

Write the marks you obtained in examination of all subjects of class 8. Add all the marks and divide it by the total number of subjects. The number obtained is arithmetic mean or simply mean. It is dented by $\overline{X}$.

For example, the weight (in kg) of 7 girls are given below. Calculate the mean from the following data.

50, 45, 31, 38, 40, 58, 53

Here,

Total number of data (N) = 7

Now,

Adding all the weights we get,

50 + 45 + 31 + 38 + 40 + 58 + 53 = 315 Kg

Dividing it by 7 we get, $\frac{315}{7} = 45$ Kg

$\therefore$ Mean = 45 Kg

i. Mean of Individual series

If $x_1, x_2, x_3 .. x_n$ denotes the given items, then

$$\text{Mean} = \left(\overline{x}\right) = \frac{x_1 + x_2 + ... + x_n}{N}$$

$$\therefore \overline{x} = \frac{\sum x}{N}$$

Where, $\sum x$ = *Sum of all items*

N = total number of items.

Example 1

The average age of 10 students of class 9 is 12 years. Find the value of y if their ages are

8, 9, 10, 11, 12, 13, y, 14, 15, 16.

Solution :

Here, total number of students (N) = 10

Mean ($\overline{X}$) = 12

Given data = 8, 9, 10, 11, 12, 13, y, 14, 15, 16

or, $\sum x$ = 8 + 9 + 10 + 11 + 12+ 13 + y + 14 + 15 + 16 = 108 + y

and N = 10

Now, using the formula,

$$\therefore \bar{x} = \frac{\sum x}{N}$$

or, $12 = \dfrac{108 + y}{10}$

or, $120 = 108 + y$

or, $y = 120 - 108 = 12$

$\therefore$ $y = 12$

So, the data given without frequency is known as individual series. If there is any data given with frequency, it is called discrete series which is already mentioned in definition of statistics.

ii. Mean of discrete series

If $f_1, f_2, ... f_n$ are the corresponding frequencies of the items $x_1, x_2,.... x_n$ respectively in the given table.

X	x_1	x_2	x_3	x_4	x_5	....	x_n
f	f_1	f_2	f_3	f_4	f_5	....	f_n

Arithmetic Mean $\left(\bar{x}\right) = \dfrac{x_1 f_1 + x_2 f_2 + ... + x_n f_n}{f_1 + f_2 + ... + f_n}$

or, $\bar{x} = \dfrac{\sum fx}{\sum f} = \dfrac{\sum fx}{N}$

Where, Σfx = Sum of product of each item and corresponding frequencies.

$N = \sum f$ = Sum of frequencies

Example 2

Find the average bonus received by members of a Co-operative Company from the given data.

Bonous (X)	45	50	60	65	70	80	90	100
No. of members (f)	2	3	6	8	7	5	3	1

Solution :

The above data can be presented in the following frequency table as,

Bonous (X)	No. of members (f)	fx
45	2	90
50	3	150
60	6	360
65	8	520
70	7	490
80	5	400
90	3	270
100	1	100
	$\Sigma f = 35$	$\Sigma fx = 2380$

Here, $N = \Sigma f = 35$ and $\Sigma fx = 2380$

Now, using the formula, Mean $\bar{x} = \frac{\sum fx}{N} = \frac{2380}{35} = 68$ $\therefore \bar{X} = 68$

Example 3

If $\bar{X} = 7.3$, find the value of x in the data given below

x	5	6	7	8	9
f	4	6	12	x	8

Solution :

Here, $\bar{X} = 7.3$

Now, making the frequency table

x	f	fx
5	4	20
6	6	36
7	12	84
8	x	8x
9	8	72
	$\Sigma f = 30 + x$	$\Sigma fx = 212 + 8x$

So, from the formula, Mean, $= \frac{\sum fx}{N}$

Or, $7.3 = \frac{212 + 8x}{30 + x}$

Or, $219.0 + 7.3x = 212 + 8x$
Or, $0.7x = 7$

$$x = \frac{7}{0.7} = 10$$

$\therefore$ The value of x is 10.

18.3.2 Median

The value of middle most part of a regular data is called the median. The data must be arranged in either ascending or descending order before calculating the median.

i. Median of individual data

Example 1

If 2, 1, 3, 7, 4 is an individual data, arrange the data in ascending order and calculate the median.

Solution :

The given data can be arranged in ascending order as, 1, 2, 3, 4, 7

Here, total number of data is 5, i.e., N = 5

Now, from the formula, median = $\frac{(N+1)}{2}^{th}$ place value.

So, Median = $\frac{(5+1)}{2}^{th}$ place value.

Median = 3^{rd} place

$\therefore$ Median = 3

First of all we have to calculate the place of median. Then after, the number in that place will be the value of median. If the number of the data is even in the data, median will be at the middle of any two numbers or the average value of the two numbers.

Example 2

Find the median value of the given data.

11, 12, 13, 14, 16, 17.

Solution :

Here, total number (N) = 6

The data is in ascending order,

Place of median = $\frac{(N+1)^{th}}{2}$ item

$= \frac{(6+1)^{th}}{2}$ item = 3.5^{th} item

Median = average of 3^{rd} and 4^{th} item

$$= \frac{(13+14)}{2} = 13.5$$

$\therefore$ Median $(M_d) = 13.5$

Example 3

If median = 32, find the value of x in the given data.

22, 26, 30, x + 3, x + 5, 34, 37, 40

Solution :

We know from the formula,

$$\text{Value of median } = \frac{(N+1)}{2}^{th} \text{ item } = \left(\frac{8+1}{2}\right)^{th} \text{ item}$$

$= 4.5^{th}$ place value.

So the median lies in between fourth and fifth place

or, median = $\frac{(x+3)+(x+5)}{2} = \frac{(2x+8)}{2} = (x+4)$

or, Median = $x + 4$

or, 32 = $x + 4$

or, $x = 32 - 4 = 28$

ii. Median of discrete series

Example 4

Write the data in ascending order. Then prepare the cumulative frequency table.

x	12	15	18	21	24	27	30
f	4	8	13	10	8	7	5

Solution :

x	*f*	*c.f*
12	4	4
15	8	4 + 8 = 12
18	13	12 + 13 = 25
21	10	25 + 10 = 35
24	8	35 + 8 = 43

27	7	43 + 7 = 50
30	5	50 + 5 = 55
	N = 55	

Now, place of median = $\frac{(N+1)}{2}^{th}$ item

$= \frac{55+1}{2}^{th} = \frac{56}{2}^{th} = 28^{th}$ item

If c.f is 28, then the corresponding value to it is median. If c.f is not 28, then the corresponding value just greater than 28 is median. Here, the value is just greater than 28 is 35. So, corresponding value of 35 is 21. Therefore, median is 21.

Example 5

The age and number of patient visiting a health post is given below. Calculate the median age from it.

Age (year)	30	32	35	40	48	50	52
No. of patients	1	4	2	5	3	4	6

Solution :

Prepare cumulative frequency table from given frequency table

Age (x)	**No of patients (f)**	***c.f***
30	1	1
32	4	5
35	2	7
40	5	12
48	3	15
50	4	19
52	6	25
	N = 25	

Place of median = $\frac{N+1}{2}$ th item = $\frac{25+1}{2}$ th item

= $\frac{26}{2}$ th item =13^{th} item

Here, c.f just greater than 13^{th} item is 15. So, the corresponding value of 15^{th} item is 48. Therefore, median age is 48 years.

18.3.3 Quartiles

The values which divide the data in four equal parts is called quartiles. To calculate the values of quartile, the data is placed in ascending order.

First quartile (Q_1) indicates the maximum value of 25 % data from bottom in the data arranged in ascending order and the third quartile (Q_3) indicates the maximum value of 75 % data from the minimum value.

Representing it in a straight line,

Q_1	Q_2		Q_3
25%	25%	25%	25%

i. Quartile of individual series

Place of Q_1 = $\frac{N+1}{4}$ th item

Place of Q_2 = $\frac{2(N+1)}{4}$ th item $=\frac{N+1}{2}$ item = Median

Place of Q_3 = $\frac{3(N+1\ )}{4}$ th item

Now study the following examples.

Example 5

Find the value of first and third quartiles of the given data.

7, 18, 55, 33, 67, 41, 29

Solution :

Writing the data in ascending order

7, 18, 29, 33, 41, 55, 67

Here, total number of items, (N) = 7

First quartile = $\frac{N+1}{4}$ th item = $\frac{7+1}{4}$ th item = $\frac{8}{4}$ th item = 2^{nd} item = second item

$\therefore\ Q_1 = 18$

Similarly, Q_3 = $\frac{3(N+1)}{4}$ th item = $\frac{3(7+1)}{4}$ th item = 6^{th} item

$\therefore\ Q_3 = 55$

ii. Quartile of discrete series :

The data in ascending order is presented in cumulative frequency table. Then, quartiles are obtained by identifying the place of first, second and third quartile. Now study the following example.

Example 6

Find the value of first and third quartiles of the given data.

Wages Rs. (x)	20	22	23	25	27	28	30
No. of workers (f)	8	10	11	16	20	25	15

Solution :

Writing the data in cummulative frequency table,

Wages (x)	**No. of workers (f)**	***c.f.***
20	8	8
22	10	18
23	11	29
25	16	45
27	20	65
28	25	90
30	15	105
	N =105	

From fromula,

First quartile = $\frac{N+1}{4}$ th item = $\frac{105+1}{4}$ th item = 26.5^{th} item

Here, c.f. just greater than 26.5 is 29. So, corresponding value of 29 is 23

$\therefore$ Q_1 = Rs. 23

Similarly, $Q_3 = \frac{3(N+1)}{4}$ item = 3 x 26.5 = 79.5^{th} item

Here, c.f. just greater than 79.5 is 90. $\therefore$ Q_3 = Rs. 28

18.3.4 Mode

The most repeated number of given data is called mode. In discrete series, the number having the highest frequency is mode. Similarly, the class having the highest frequency in continuous class interval is called modal class.

i. Mode of individual series

The given data is 1, 1, 1, 2, 2, 3, 3, 3, 4, 5, 5, 6, 6, 6, 6

In this data

1 is repeated 3 times

2 is repeated 2 times

3 is repeated 3 times

4 is repeated 1 time

5 is repeated 2 times

6 is repeated 4 times

The most repeated number is 6. Therefore, mode is 6.

ii. Mode of discrete series

Let us study the given discrete data.

Size of object	10	11	12	13	14	15	16	17
Frequency	2	5	7	9	10	6	4	3

Here, 14 has the highest frequency i.e. 10. That means, object of size 14 has been repeated time to time or it is repeated the highest 10 times. So, the mode is 14.

Exercise 18.3

1. Find the mean, median and mode from the following data.

   (a) 23, 22, 20, 25, 16, 17, 18, 21, 22, 25, 22, 18, 22, 25

   (b) 8, 16, 28, 60, 30, 60, 8, 12, 8

   (c) 110, 105, 100, 150, 250, 175, 225, 275, 110, 150, 100, 110

2. The weight (in Kg) of 10 children coming for regular check up in a hospital is as follow

   10, 15, 20, 22, 12, 25, 30, 32, 35, 19

Now, calculate the average weight (mean and median) of the above 10 children.

3. The mean of 6, 7, 9, x, 12, 14 is 11. Find the value of x.

4. If $\bar{x} = 20$ and $\sum x = 120$, calculate the total number of data.

5. Compute mean, median, mode, Q_1 and Q_3 from the discrete data below

(a)

Marks obtained	5	6	7	10	9	8
Frequency	2	3	1	2	3	5

(b)

Temperature (°C)	18	17	25	35	37
No. of days	12	15	28	25	20

(c)

Monthly income (in Rs. 100)	100	150	170	180	200
No. of employees	6	4	10	9	6

6. From the data given below, find the value of y whose mean $\overline{X}$ = 19.6

Marks obtained	5	10	15	20	25	30
No. of students	2	3	y	8	5	4

7. Collect the internal examination fee and number of students of every class in your shool. Calculate the total examination fee, total number of students, average examination fee, average number of students per class, average examination fee perstudent. Find the mean, median and mode of the data.
8. Divide the students of class in two groups and note the marks obtained in mathematics in final examination of class 8. Then, draw a histogram by representing the data in grouped frequency table with suitable class interval.
9. Collect the ages of people below 100 years from your surrounding and fill in the table given below.

Age group	No. of people
0-10	
10 - 20	
20 - 30	
30 -40	
40 - 50	
50 - 60	
60 - 70	
70 - 80	
80 - 90	
90 - 100	

Then, from the table, construct the cumulative frequency curves in the same graph by making the cumulative frequency tables for less than (ascending order) and more than (descending order).

Lesson

19 Probability

19.0 Review

How many surfaces are there in a coin?

What surfaces can be observed when we throw a dice?

What are the colours of playing cards?

What is the probability of occurring an ace of heart or a king when a card is randomly taken from the deck of playing cards?

It is found that from ancient period people used to estimate different events which can happen in their daily life. The tradition of being happy if outcomes of estimated events are true and being sad if outcomes are wrong. This is human nature till now. We know about the various sad events happened because of the wrong or bad estimation. Thus every estimation of every person may not be true or good. Estimation themselves are uncertain forecast and possibilities. So, this uncertain possibility is probability. For example, the sky is cloudy today and there is possibility of rainfall. So, umbrella should be carried. Here, possibility of rainfall is only estimation (guess). It is not sure. Therefore, clouds in sky is an indication of probability of rainfall. The word probability is used broadly in our daily life. In general, the statistics which is used to describe and analyse the uncertain events that can occur in future.

19.1 Some defined terms in probability

1. Experiment

In general, an action which is used to observe the possible outcomes for the proability of any event is called experiment. Discuss the different examples of experiment in your classroom.

2. Random Experiment

Any experiment the outcome of which can not be predicated or determined definitely (distinctly) in advance is called a random experiment. For example : Perform some random experiments in classroom by using coin, dice etc.

3. Outcomes

The results of an experiment are called outcomes. For example while tossing a coin, the occurrence of head or tail is the outcome. Discuss about other instructional materials besides coin and dice that can be used in class work.

4. Sample Space

The set of all possible outcomes of an experiment is called sample space. For example, the possible outcomes of a student's result of an exam are A^+, A, B^+, B, C^+, C, D^+, D or E grades and it is denoted by S = {A^+, A, B^+, B, C^+, C, D^+, D, E}

5. Event

Any subset of sample space of an experiment is called an event. For example, while tossing a coin the sample space S = {H,T} then {H}, {T}, {H,T}, {} are events.

6. Sample Point

Each result or event of a sample space of an experiment is called sample point. For example, when a dice is tossed, 1, 2, 3, 4, 5, 6 all are sample points in sample space. So

S = {1, 2, 3, 4, 5, 6}

7. Elementary Event

If the number of element of an event is only one then the event is called an elementary event. For example while rolling a dice, {1}, {2},... etc are elementary events in the sample space S = {1, 2, 3, 4, 5, 6}.

8. Equally Likely Outcomes

The outcomes are said to be equally likely outcomes if the probability of possible events of an experiment are equal (or same). For example, while tossing a coin, the probability of occurring head (H) or tail (T) is equal.

9. Number of Favourable Outcomes

The number of elements of possible events which are desirable to us is called number of favourable outcomes.

$\therefore$ Probability of an event $P(E) = \frac{n(E)}{n(S)} = \frac{m}{n}$

$$\therefore P(E) = \frac{\text{No. of favourable outcomes}}{\text{Total no. of possible outcomes}}$$

For example, 40 students of a community school had participated in SLC examination. Among them, 25 passed in grade A and 15 passed in grade B. If only one student is selected from those students then probability of selecting the students with grade A,

i.e. $P(A) = \frac{\text{No. of students passed in grade A}}{\text{Total no. of students}} = \frac{n(A)}{n(S)} = \frac{25}{40} = \frac{5}{8}$

Similarly, Probability of choosing the student with grade 'B'

i.e. $P(B) = \frac{n(B)}{n(S)} = \frac{15}{40} = \frac{3}{8}$

Example 1

What is the probability of getting a head in tossing of a fair coin?

Solution :

Here, sample space (S) = {H, T}

Total no. of possible outcomes, n(S) = 2

Favourable outcome (E) = {H}

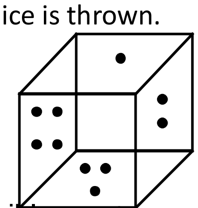
No. of favourable outcomes, n(E) = 1

Probability of getting head, $p(E) = \frac{n(E)}{n(S)}$

or, $p(H) = \frac{1}{2}$

Example 2

Answer the following questions based upon the events when a cubical dice is thrown.



i. Write the sample space of events which can turn up.

ii. Find the probability of turning up 3?

Solution :

i. Here, when a fair cubical dice is thrown, then the sample space of possible events

S = {1, 2, 3, 4, 5, 6}

$\therefore$ n(S) = 6

ii. There is only one 3 in a cubical dice or event (E) = {3} and n(E) = 1

$\therefore$ n(E) = 1, which is no. of favourable outcomes.

$\therefore$ Probability of turning up 3 is,

$$P(E) = \frac{n(E)}{n(S)} = \frac{1}{6}$$

Example 3

When a card is drawn at random from a deck of well shuffled 52 cards, find the probability that the card is king?

Solution :

Here, Total number of possible outcomes, n(S) = 52

The number of favourable outcomes or King ,n(E) = 4

Probability of getting a king, p(King) = $\frac{n(E)}{n(S)}$

$\therefore$ p(K) $= \frac{4}{52} = \frac{1}{13}$

Example 4

What is the probability of occuring a day is Sunday?

Solution :

Here, days in a week, S = {Sunday, Monday, Tuesday, Wednesday, Thursday, Friday, Saturday}

$\therefore$ n(S) = 7

There is only one Sunday in a week.

Event (E) = {Sunday}

$\therefore$ n(E) = 1

$\therefore$ Probability of occuring Sunday = $\frac{n(E)}{n(S)}$

$\therefore$ P(E) = $\frac{1}{7}$

Example 5

What is the probability of birthday of a child on Saturday?

Solution :

There is only one saturday in a week, n(E) = 1 which is the number of favourable outcomes. There are 7 days in a week. So, n(s) = 7 is the total number of possible outcomes.

$\therefore$ Probability of birthday on saturday P(E) = $\frac{n(E)}{n(S)} = \frac{1}{7}$

Example 6

There are 2 red, 3 black and 4 green coloured marbles of same size in a bag. Now, when a marble is drawn from the bag randomly:

i. what is the probability of getting a red marble?

ii. what is the probability of getting a green marble?

Solution :

i. Here, total number of marbles = n(S) = 2 + 3 + 4 = 9

or, total number of possible outcomes = n(S) = 9

Number of red marbles, n(R) = 2

Number of favourable outcomes, n(R) = 2

$\therefore$ Probability of getting red marble, $p(R) = \frac{n(R)}{n(S)} = \frac{2}{9}$

ii. Number of green marbles, n(G) = 4

or, Number of favourable outcomes, n(G) = 4

$\therefore$ Probability of getting green marble = $p(G) = \frac{n(G)}{n(S)} = \frac{4}{9}$

Example 7

There are 10 white marbles of same size marked from 1 to 10. If one marble is drawn from those group of marbles, what is the probability of getting marble marked with prime number?

Solution :

Here, the number of marbles having same size, n(S) = 10

Marbles marked with prime numbers, E = {2, 3, 5, 7}

$\therefore$ n(E) = 4

$\therefore$ Probability of getting marble marked with prime number,

$$p(E) = \frac{n(E)}{n(S)} = \frac{4}{10} = \frac{2}{5}$$

19.2 Introduction of Probability Scale

Activity 2

Let us put a red, a green and a white table tennis balls of same size in a bag or box. Discuss about the possible events when a tennis ball is drawn randomly by a student in the class room. Give duty to a student to write on the board. After finding the probability of 1, 1 ball of three colours separately, let us make to add probabilities of all. What is the total probability? Find the answer from students. Now ask the following questions to students after the discussion and class activity

i. What is the probability of getting each table tennis ball?

Expected answers are p(W) = $\frac{1}{3}$, P(R) = $\frac{1}{3}$ and P(G) = $\frac{1}{3}$

ii. What is the probability of getting red or white ball?

Expected answer = P(R) + P(W) = $\frac{1}{3} + \frac{1}{3} = \frac{2}{3}$

iii. What is the probability of red or white or green table tennis ball?

Expected answer = P(W or R or G)

$$= P(W) + P(R) + P(G)$$

$$= \frac{1}{3} + \frac{1}{3} + \frac{1}{3}$$

$$= \frac{1+1+1}{3} = \frac{3}{3} = 1$$

iv. If ball is not drawn from anyone, then what is the probability of getting in that case? Or what is the probability of getting black ball? (Since there is no black ball, its probability is zero (0)).

v. Let us make to show answers of above questions in a number line

0	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{3}{3} = 1$
P (black)	P(W) or P(R) or P(G)	P(W or R) P(R or G) P(G or W)	P(W or R or G)

From the analysis of the above activity, it is clear that the minimum probability is zero and maximum is one, or $0 \le P \le 1$.

or, $0 < \frac{1}{3} < \frac{2}{3} < 1$ (from the experiment of 3 events)

or, $0 < \frac{1}{4} < \frac{2}{4} < \frac{3}{4} < \frac{4}{4} < 1$ (from the experiment of 4 events)

Note :

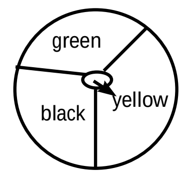
The probability can never be less than zero (0) and more than one (1). Thus, the scale, from 0 to 1 which is used to measure probability is called probability scale. If P(E) is the probability of any event, then probability of non - happening of that event is $P(\overline{E}) = 1 - P(E)$

Example 8

In spinning of a spinner coloured green, yellow and black in equal sectors, What is the probability of pointer to stop on green? What is the probability of pointer do not stop on green?

Solution :

Here, three equal parts of spinner are : green, yellow and black.



$\therefore$ Total number of possible outcomes, n(S) = 3

Here, the green sector is only one.

So, the number of favourable outcomes n(E) = 1

$\therefore$ Probability of pointing green $P(E) = \frac{n(E)}{n(S)} = \frac{1}{3}$

Probability of not pointing green $P(\overline{E}) = 1 - P(E)$

$= 1 - 1/3$

$= 2/3$

19.3 Emperical Probability

We know that when a coin is tossed, the probability of occuring head is 1/2 or P(H) = 1/2 and its complement P(T) = 1 - 1/2 = 1/2.

Based upon this theory if a coin is tossed 10 times, the probability of occurence of head should be 10 X 1/2 = 5. However, when the experiment is done practically, the result obtained may be different from this. Thus the probability estimated (calculated) from the actual experiments is known as emperical probability or the process of estimating (calculating) the probability based on experiment is called emperical probability. To determine emperical probability, the following formula is used.

$$\text{Emperical probability} = \frac{\text{No. of favourable outcomes obtained from experiment}}{\text{Total no. of experiments}}$$

Example 9

The results obtained in tossing a coin 50 times are given below in the table

Results obtained from experiment (E)	Head (H)	Tail (T)
Number of outcomes (f)	23	27

What are the empirical probability of head (H) and tail (T)?

Solution :

Here, Total no. of experiments n(S) = 50

No. of occurrence of head n(H) = 23

No. of occurrence of Tail n(T) = 27

So, empirical probability $P(H) = \frac{n(H)}{n(S)} = \frac{23}{50}$

whereas, $P(T) = \frac{n(T)}{n(S)} = \frac{27}{50}$

Example 10

While unloading 2000 bricks from a mini truck, the probability of breaking of brick is 0.1. Find the number of non-breaking bricks.

Solution :

Here, the total number of bricks, n(S) = 2000

Probability of breaking of brick, P(E) = 0.1

Probability of non - breaking of brick, $P(\overline{E}) = 1 - 0.1 = 0.9$

$\therefore$ No. of non - broken bricks $= n(S) \times P(\overline{E})$

$= 2000 \times 0.9$

$= 1800$

Exercise 19

1. Find the probability of getting an ace when a card is drawn randomly from a well shuffled pack of 52 cards.
2. When a dice is thrown freely, what is the probability of getting a face numbered 2? Also, find the probability of the face numbered 3.
3. What is the probability of giving birth to a child by a pregnant woman on sunday?
4. A bag contains 2 blue, 3 black and 5 red marbles of same size. If one marble is drawn randomly from the bag what is the probability of getting a black marble? Also, find the probability of not getting a black marble.
5. From the cards numbered 2 to 25, a card is drawn at random. Find the probability of getting the card numbered multiple of 3. Also, find the probability of getting the card numbered with an even number.

6. There are 50 students in a class, out of them 28 are girls. If a student is chosen at random to participate in essay competition, what is the probability of choosing girl?
7. Among the 35 students of class 9, 25 students can speak and write English properly. If a student is chosen for quiz contest, what is the probability of choosing the student who can speak and write english properly?
8. Answer the following questions based on the sample space of two coins when tossed simultaneously.
   (a) What is the probability of getting two heads?
   (b) What is the probability of getting two tails?
   (c) What is the probability of getting tail in one coin and head in another?
9. Make a sample space of possible outcomes when a cubical dice is rolled and find the probability of:
   (a) turning up the face numbered 5.
   (b) getting the face marked by even number.
   (c) getting the face marked by odd number.
10. A cubical dice is thrown 60 times and the results obtained are given below in the table.

Outcomes	1	2	3	4	5	6
Frequency	8	9	13	15	11	4

   Find the empirical probability of :

   (a) Getting 4 (b) Getting greater than 5

   (c) Getting less than 3 (d) Getting less than 7.
11. Among the 100 newly born babies in a hospital, 35 are daughters. What is the empirical probability that the newly born baby is boy?
12. Ask your parents and list the events which have the highest and the lowest probabilities of occurrence, e.g. probability of rain fall in Sravan and Paush.

Answers

Exercise 1.1

1.(i) {1, 2, 3, 5, 6, 7, 8} (ii) { 2, 6, 8} (iii) { 4, 9, 10} (iv) { 1, 3, 4, 5, 7, 9, 10}

(v) { { 1, 3, 4, 5, 7, 9, 10} (vi) {1, 3} (vii) { 1, 2, 3, 6, 8} (viii) { 2, 5, 6, 7, 8}

Show the Venn diagrams to your teacher.

2. (a) (i) { b, e, i, k } (ii) {a, b, c, d, e, f, g, h, i, j, k }

(iii) {a, b, c, d, e, f, g, h, i, j, k } (iv) {b, e, i, k}

(b) (i) , (iv) and (ii) , (iii) are equal sets.

(c) (i) {b, e, g, h, i, j, k} (ii) {a, b, c, d, e, f, j, k} (iii) {a, c, d, f} = A

Show the Venn diagrams to your teacher.

4. (a) (i) {1, 2, 3, 5, 6, 7, 9, 11, 13, 17,18, 19} (ii) {3, 6, 9, 18}

(iii) {4, 8, 10, 12, 14, 15, 16,20}

(iv) {4, 8, 10, 12, 14, 15, 16, 20} (v) {2, 5, 7, 11, 13, 17, 19}

(vi) Ø (vii) {1, 2, 3, 5, 6, 7, 9, 11, 13, 17, 18, 19}

(b) (i) , (vii) and (iii) , (iii) are equal sets.

Exercise 1.2

1. $n(V) = 4$, $n(W) = 3$, $n_o(V) = 2$, $n_o(W) = 1$, $n(V \cup W) = 5$, $n(V \cap W) = 2$

2. 37

3. 108

4. (i) 12 (ii) 8 (iii) 14 (iv) 1 (v) 6 (vi) 1 (vii) 2

Show the venn diagrams to your teacher.

5. 44

6. (i) 45 (ii) 135 (iii) 60 (iv) 195

Show the venn diagrams to your teacher.

7. (i) Show the venn diagrams to your teacher. (ii) 15 (iii) 25 8. 200 9. 220 10. (i) 100 (ii) 1600

Exercise 2

1. (a) Rs. 125, 25% (b) Rs. 30, 10% (c) Rs. 24,20% (d) Rs. 57.5, Rs. 7.5 (e) Rs. 50, 20%

2. (a) Profit Rs. 20 (b) Rs. 100

3. (a) Rs. 18, 20% (b) Rs. 8, 25%

4. (a) Rs. 57 (b) Rs. 13.88

5. (a) Rs. 828 (b) Rs. 15

6. Show the Venn diagrams to your teacher.

Exercise 3.1

1. (a) Rs. 2,40,000, Rs. 77,60,000 (b) Rs. 10,00,000, Rs. 90,00,000 (c) Rs. 2,40,000, Rs. 1,17,60,000

   (d) Rs. 25,00,000, Rs. 4,75,00,000

2. (a) Rs. 29,000 (b) (i) Rs. 51,000 (ii) Rs. 52 Lakh

3. (a) 1.5% (b) 5%

4. (a) Rs. 6,000 (b) Rs. 16,500 (c) Rs. 37,000 (d) Rs. 80,000

Exercise 3.2

1. (a) Rs. 65, Rs.1235 (b) Rs. 1.60, Rs.78.40 (c) Rs. 22.5, Rs. 127.50 (d) Rs. 14.40, Rs. 165.60
   (e) Rs. 140, Rs. 560

2. (a) Rs. 40, 10% (b) Rs. 24, 8% (c) Rs. 21, 2% (d) Rs. 12, 1%

3. (a) Rs. 960 (b) Rs. 375

4. (a) Rs. 550 (b) Rs. 18,000

Exercise 3.3

1. (a) Rs. 1326 (b) Rs. 4680 (c) Rs. 7410 (d) Rs. 1435.20
2. Show to your teacher.

3. Show to your teacher.

Exercise 3.4

1. (a) Rs. 1,84,00,000, Rs. 73.6 (b) Rs. 61,25,000, Rs. 53.75 (c) Rs. 1,44,00,000, Rs. 57.60

   (d) Rs. 97,30,000, Rs. 44.22 (e) Rs. 25,50,000 ,Rs.10.62

2. (a) Rs. 1500 (b) Rs. 1500

Exercise 4.1

1. (a) 75 (b) Bhadra (c) Ashoj (d) 460
2. Show to your teacher.

Exercise 4.2

1. (a) 1446 (b) Rs. 43.38 (c) Rs. 144.60

2. (a) Rs. 59616 (b) Rs. 57827.52 (c) Rs. 89424

Exercise 4.3

1. (a) 337 (b) Rs. 362 (c) Rs. 398.20 (d) Rs. 449.96

2. (a) 550 (b) 755 3. Rs. 839.02

4. (a) 215 (b) 400 (c) 750

Exercise 4.4

1. (a) Rs. 280.40 (b) Rs. 345.20

2. (a) 280.40 (b) Rs. 417.20 (c) Rs. 703.20 (d) Rs. 269.40

3. (a) 2 km (b) 10 km

Exercise 5.1

1. a) 21cm2, 18.5 cm b) 13.5 sq inch, 18 inch c) 16.45 cm2, 16.44 cm

2. a) 40.75 cm^2 b) 15 $inch^2$ c) 9.14 $inch^2$

3. 456 m^2 4. 1100 sq ft 5. 1320 m^2 6. 480 ft^2

7. (a) 425m^2 b)1700 8. 261 m^2

9. a) 656 m^2 b) 1640 c) Rs. 12,300 10. a) 480 cm^2 b) Rs. 2,64,000

11. a) 401 ft^2 b) 924 $inch^2$ c) Rs. 2800 m^2 12, 13. Show to your teacher.

Exercise 5.2

1. (a) 6,750 (b) 5,175 (c) Rs. 50,400 (d) 1,73,760

2. 8.33 m 3. (a) 4500m^2 (b) 723 m^2 (c) 9,000 (d) 18,000 (e) Rs. 2,27,745

4. 3 m 5. (a) 1800 (b) Rs. 405,000

6, 7 . Show to your teacher.

Exercise 6.1

1. (a) 288 cm^2 (b) 707.16 cm^2 (c) 1920m^2 (d) 576 cm^2

   (e) 1193.72 cm^2 (f) 2400 cm^2

2. (i) 176 cm^2 (ii) 30 cm^2

3. 672 cm^2

4. (a) 36 cm^2 , 216 cm^2 , 324 cm^3 (b) 165.48 cm^2, 376.2 cm^2, 1092.168cm^3

   (c) 276m^2, 1176 m^2, 4464 m^3 (d) 96 cm^2, 384 cm^2, 768 cm^3

   (e) 289.53 cm^2, 614.66 cm^2, 2113.569 cm^3 (f) 300 cm2, 1950 cm^2, 4500cm^3

5. Show to your teacher. 6. 180 cm^3

7. 90,000 Liters 8. 10cm 9. 150m^2 10. 6 cm

11. Show to your teacher.

Exercise 6.2

1. 3.2 m^3	2. 3,60,000	3. 10,753	4. 3 days
5. 17,955	6. 3.83 m	7. Rs. 8740	8. Rs. 4590
9. 0.009 m^3	10. 5 m	11. 21 Lakhs	12. Rs. 2925

13. Show to your teacher.

Exercise 7

1. $(y^2 + y + 1)(y^2 - y + 1)$	2. $(x^2 + xy + y^2)(x^2 - xy + y^2)$
3. $(x^2 + x + 1)(x^2 - x + 1)(x^4 - x^2 + 1)$	4. $(x^2 + 2xy + 2y^2)(x^2 - 2xy + 2y^2)$
5. $(9x^2 + 12xy + 8y^2)(9x^2 - 12xy + 8y^2)$	6. $(8x^2 + 4xy + y^2)(8x^2 - 4xy + y^2)$
7. $(x^2 + 3x + 1)(x^2 - 3x + 1)$	8. $(x^2 + 3x + 2y^2)(x^2 - 3x + 2y^2)$
9. $(7x^2 + 14xy + 3y^2)(7x^2 - 14xy + 3y^2)$	10. $(5a^2 + 2ax - 3x^2)(5a^2 - 2ax - 3x^2)$
11. $(16x^2 + 15xy + 7y^2)(16x^2 - 15xy + 7y^2)$	12. $(35x^2 + 23xy + 8y^2)(35x^2 - 23xy + 8y^2)$
13. $(45x^2 + 65xy + 49y^2)(45x^2 - 65xy + 49y^2$	14. $(2a^2 + 3ab + 11b^2)(2a^2 - 3ab + 11b^2)$
15. $(2x^2 + 2xy + 3y^2)(2x^2 - 2xy + 3y^2)$	16. $(x^2 + 3x + 9)(x^2 - 3x + 9)$
17. $(x^2 + 1 + 1/x^2)(x^2 - 1 + 1/x^2)$	18. $(x^2/y^2 + x/y + 1)(x^2/y^2 - x/y + 1)$
19. $(x^2/y^2 + 1 + y^2/x^2)(y^2/x^2 - 1 + y^2/x^2)$	20. $(x^2 + y + 3)(x^2 - y - 11)$
21. $(x^2 + x + 1)(x^2 - x - 7)$	22. $(x^2 + y - 14)(x^2 - y + 2)$
23. $(x^2 + 3y^2 - 6)(x^2 - 3y^2 - 4)$	24. $(x - 2y - z)(x - 8y + z)$
25. $(65x^2 - 9y^2 + 1)(65x^2 + 9y^2 - 3)$	26. $(x - 40y + 55z)(x - 50y - 55z)$
27. $(13x + 14y)(13x - 14y - 4)$	28. $(65x + y + z)(65x - 3y - z)$
29. $(17x + 19y + 6)(17x - 19y + 4)$	30. $(x + 29y + z)(x + 21y - z)$

Exercise 8.1

1. (i) 1 (ii) 1 (iii) 1 (iv) 34 (v) 3125 (vi) 1296
   (vii) 1/8 (viii) 8 (ix) 1 (x) 8/27 (xi) 125/216 (xii) 3/4
2. (i) $9xy$ (ii) $85y^2/x^2$ (iii) $6x^2y^3$ (iv) 1 (v) $1/x$a+b+c
   (vi) $x^{(a+b+c)^2}$ (vii) $x^{a(a-b)}$ (viii) 1 (ix) 1 (x) 1
3. (i) 1 (ii) 1 (iii) 1 (iv) $x^{2(a^3+b^3+c^3)}$ (v) 1
   (vi) 1 (vii) 1 (viii) $a^{2(x^3+y^3+z^3)}$ (ix) 1 (x) 1

Exercise 8.2

1. x = 4	2. x = 1	3. x = 1.5	4. x = -3	5. x = 3
6. x = 1	7. x = 0	8. x = 1	9. x = 1, 0	10. x = 1, -1

Exercise 9.1

1. 5:8   2. 27:31   3. 3:10   4. a:b = 25 :9   5. a:b = 3:2

6. 15 and 10   7. a = 35, b = 45   8. a = 45, b = 65   9. Father is 75 years and Son is 45 years old

10. Mother is 45 years and daughter is 15 years old.   11. 48 years and 60 years.

12. Oyster mushroom Rs. 100 and White button mushroom Rs. 300   13. 378 and 432   14. 108 and 84

15. 48 and 30

Exercise 10

a) 1. x = 4, y = 3   2. x = 5, y = 6   3. x = 3, y = 3   4. x = 4, y = 2   5. x = 5, y = 6

6. x = 6, y = 5   7. x = 2, y = 1   8. x = 2, y = 3   9. x = 2, y = 11   10. x = 5, y = 3

b) 1. x = 2, y = 3   2. x = 3, y = 2   3. x = -1, y = -3   4. x = 5, y = 2   5. x = 16, y = 26

6. x = 4, y = 3   7. x = 12, y = 8   8. x = 60, y = 80   9. x = 4, y = 4   10. x = 3, y = 2

c) 1. x = 6, y = 10   2. x = 4, y = 3   3. x = 6, y = 7   4. x = 3, y = 2   5. x = 15, y = 10

6. x = 6, y = 3   7. x = 2, y = 2   8. x = 2, y = 4   9. x = 25, y = 20   10. x = 2, y = 1

d) x = 350, y = 800   e) l = 12m, b = 4m

Exercise 11

1. (i) x = 1, -2   (ii) x = 2,3   (iii) x = 3,5   (iv) x = 3,5

2. (i) x = 0,1   (ii) x = 0, -1   (iii) x = -3, 4   (iv) x = -4,5
(v) x = -5, -6   (vi) x = 3,4   (vii) x = -5,7   (viii) x = 6,7   (ix) x = 7,9
(x) x = -35,35

3. (i) x = 5,5   (ii) x = 9,9   (iii) x = -3,7   (iv) x = -5,9   (v) x = -7,11

(vi) x = 13,9   (vii) x = 2/3, -3/4   (viii) x = 1/2,3/2   (ix) 3/4,5/4   (x) 5/3,-7/3

4. (i) x = 3,7   (ii) x = 8,9   (iii) x = -11,13   (iv) 13,17   (v) x = 4,-7

(vi) x = -6,11   (vii) x = 1/7,-3/7   (viii) x = 17,19   (ix) $\frac{9 \pm 5\sqrt{5}}{2}$ (x) $\frac{21 \pm 9\sqrt{5}}{2}$

Exercise 12.1

1. (a) 30^0 (b) $x^0 = 70^0$, $y^0 = 40^0$ (c) $x^0 = 70^0$, $y^0 = 60^0$, $z = 50^0$ d) $x^0 = 20^0$, $y = 130^0$

(e) $x^0 = 45^0$, $y^0 = 135^0$ (f) $x^0 = 105^0$, $y^0 = 25^0$

2. (a) 360^0 (b) 540^0 (c) 720^0

Exercise 12.2

1. (a) x = 70   (b) x = 110   (c) x = 112.5

2. (a) $x^0 = 35^0$, $y^0 = 550$, $z^0 = 55^0$ (b) $x^0 = y^0 = z^0 = 45^0$, $z = 75^0$

Exercise 12.3

1. (a) i. BC   ii. AB   iii. $\angle BAC$ iv. $\angle ACB$

(b) i. BC   ii. AB   iii. $\angle BAC$ iv. $\angle ACB$

(a) i. AB ii. AC iii. ∠ACB iv. ∠ABC

2. (a) 41cm (b) So (d) 53 cm

3. (a) Yes (b) 20cm, 12cm (c) Innumerable

Exercise 13.1

1 a) 60^0 b) $56^0, 62^0$ c) 12, 140 d) 1 e) 8 f) 6

2. a) 50^0 b) $52^0, 54^0$ c) 60^0 65^0 3. $72^0, 108^0$

Exercise 13.2

1 a) 4cm, 2.4cm b) 52^0

Exercise 15

1. (a) 4.5cm (b) 8m (c) 56 Ft

2. (a)16cm (b) i. APSD, BQRC and APQB, CDSR ii. 7.5cm

3. (a)6cm, 9cm (b)15cm (c) 6cm

Exercise 16

1. (a) 6cm (b) 10cm (c) 31.24cm (d)6.7cm

4. Show to your teacher.

Exercise 17.1

1. (a) (i) AC,10 cm (ii) For angle $\propto$, BC is perpendicular and AB is base. (iii) For angle θ, AB is perpendicular and BC is base.

(b) (i) $\text{Sin}\propto = \frac{4}{5}$, $\cos\propto = \frac{3}{5}$ $\tan\propto = \frac{4}{3}$,

(ii) $\text{Sin}\theta = \frac{3}{5}$ $\cos\theta = \frac{4}{5}$, $\tan\theta = \frac{3}{4}$

2. (i) $\sin\propto = \frac{2}{\sqrt{5}}$ $\cos\propto \frac{1}{\sqrt{5}}$, $\tan\propto = 2$ (ii) $\text{Sin}\beta = \frac{4}{5}$, $\cos\beta = \frac{3}{5}$, $\tan\beta = \frac{4}{3}$

(iii) $\sin\gamma = \frac{3}{5}$, $\cos\gamma = \frac{4}{5}$, $\tan\gamma = \frac{3}{4}$ (iv) $\sin\theta = \frac{1}{\sqrt{5}}$, $\cos\theta = \frac{2}{\sqrt{5}}$, $\tan\theta = \frac{1}{2}$

3. (i) $\frac{\sin A}{\sqrt{1-\sin^2 A}}$ (ii) $\sqrt{1-\cos^2 A}$ (iii) $\sin\theta = \frac{3}{5}$, $\cos\theta = \frac{4}{5}$

(iv) $\cos\theta = \frac{1}{2}$, $\tan\theta = \sqrt{3}$ (v) $\cos\theta = \frac{4}{5}$, (vi) $\sin\theta = \frac{1}{2}$, $\cos\theta = \frac{\sqrt{3}}{2}$ (vii) 16

Exercise 17.2

1. (i) $\sqrt{3}$ (ii) $\frac{1}{\sqrt{3}}$ (iii) 1 (iv) $\frac{1}{\sqrt{2}}$ (v) $\frac{1}{2}$ (vi) 1 (vii) $\frac{3\sqrt{3}+1}{2}$ (viii) $\frac{\sqrt{3}}{2}$ (ix) $\frac{\sqrt{3}}{2}$ 2. 30^0, 60^0 45^0,

3. (i) $\frac{3+\sqrt{2}+\sqrt{3}}{2}$ (ii) 1 (iii) $\sqrt{3}$ (iv) 2+ $\sqrt{3}$

4. (i) θ= 60^0 (ii) θ= 30^0 (iii) θ= 45^0 6.(i) , 4cm, cm 7. 8. 60^0

Exercise 18.1

1. Show to your teacher.

2. Show to your teacher.

3. (a) Rs. 4500 (b) Rs. 750 in instructional materials and Rs. 600 on transportation (C) Monthly fee

4-8 show to your teacher.

Exercise 18.2

1. (a)

Less than ogive	
Rainfall (mm)	less than c.f.
Less than 20	5
Less than 30	10
Less than 40	20
Less than 50	40
Less than 60	60
Less than 70	65

(b)

More than ogive)	
Rainfall (mm)	less than c.f.
More than 10	60
More than 20	55
More than 30	50
More than 40	40
More than 50	20
More than 60	5

2. (a)

Mass (kg)	Tally marks	Frequency
10 – 20	𝍸 𝍸 ||	12
20 – 30	𝍸 𝍸 ||	12
30 – 40	𝍸	5
40- 50	|	1

(a) Show to your teacher. (c) Show to your teacher.

3. Show to your teacher.

4. Show to your teacher.

Exercise 18.3

1. (a) Mean $(\overline{X})$ = 21.14, Median (M_d) = 22.5, Mode (M_o) = 22

(b) Mean $(\overline{X})$ = 25.56, Median (M_d) = 16, Mode (M_o) = 8

(c) Mean $(\overline{X})$ = 155, Median (M_d) = 130, Mode (M_o) = 110

2. (a) Mean $(\overline{X})$ = 22, Median (M_d) = 21,

3. x = 7,

4. N = 6

5. (a) Mean $(\overline{X})$ = 7.63, Median (M_d) = 8, Mode (M_o) = 8, Q_1 = 6, Q_3 = 9

(b) Mean $(\overline{X})$ = 27.86, Median (M_d) = 25, Mode (M_o) = 25, Q_1 = 18, Q_3 = 35

(C) Mean $(\overline{X})$ = 163.43, Median (M_d) = 170, Mode (M_o) = 170, Q_1 = 150, Q_3 = 180

6. y = 3 7-9. Show to your teacher.

Exercise 19.1

1. 1/13 2. 1/6,1/6 3. 1/7 4. 3/10, 7/10 5. 1/3, 1/2

6. 14/25 7. 5/7 8. (a) 1/4 (b) 1/4 (c) 1/2

9. (a) 1/6 (b) 1/2 (c) 1/2

10. (a) p(4) = 1/4 (b) p(greater than5) =1/15 (c) p(less than 3) = 17/60 (d) p (less than 7) = 1

11. p(s) = 13/20 12. Show to your teacher.