UNIVERSITY OF MARYLAND,COLLEGE PARK

PROJECT REPORT

ENPM 667 CONTROL OF ROBOTIC SYSTEM

Sandip Sharan Senthil Kumar, Kiran Ajith

November, 2022

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Abstract

The concept of coordinated motion control for space manipulators and their spacecraft is explained in the paper. The system barycentric vectors are used in this paper to write the dynamics of a free-flying robotic system. This paper develops a new control method that considers the orientation and position of the spacecraft and its manipulators. To control the motion of the robotic system, a new Transposed-Jacobian controller with inertial feedback is created. An example of this method is also provided in this study.

1

Introduction

The free-flying space robotic system is one in which a spacecraft is mounted with robotic manipulators. This system is intended for building, maintaining, and performing repairs in space. In this study, a manipulator is fitted on a free-flying spacecraft with a reaction jet attitude control system. Robotic manipulators on a spacecraft are anticipated to be used on the next space missions to perform satellite servicing tasks. When analyzing the motion of various robotic systems, control is a key aspect.

The position and attitude of the spacecraft may be disturbed by the movements of the robotic manipulator placed on the spacecraft. The system's attitude control reaction jets may not be able to control these disturbances when the manipulator's movement speed increases. The control method described here considers the reaction jet saturation limitations as well as the joint actuator saturation limits of a manipulator. It offers the system's ideal position and velocity command profiles as well as the minimum possible time-and-space motions for the manipulator.

In order to eliminate the need for reaction jet fuel, there are numerous control strategies that allow the spacecraft to be completely uncontrolled. However, these techniques are constrained by the impacts of uncontrolled spacecraft on their surroundings, and the workspace of such a system is constrained by dynamic singularities. A dynamic singularity occurs when the spacecraft responds to manipulator motions without obtaining compensation from its reaction jet control system. You can achieve unlimited workspace using a different control technique that switches the system between a free-flying mode and considers it as a redundant manipulator with a pseudo-inverse Jacobian-based controller. In this instance, a restriction is placed on the consumption of response jet fuel.

The focus will be on a system consisting of a spacecraft and one N-DoF manipulator. Some of the formulation's basic characteristics are:

- Lagrangian formulation is used to keep the dynamic equations describing the motion of robotic systems in a compact form.
- The system CM is chosen to represent the translation of the system.
- To characterize the system's orientation and configuration, the spacecraft's attitude and related angles are chosen.
- The resulting kinematic and dynamic equations are simplified using the concept of barycenters.
- To eliminate needless derivations using transformations, the findings are presented in a compact manner using a dyadic vectorial form.

With this control method, the manipulator's workspace can be extended to infinity while the system's motion is planned to prevent collisions and retain a beneficial posture.

2

SYSTEM KINEMATICS

2.1 Position

The primary goal of this project is to obtain analytical equations for the position and velocity of a space manipulator system's arbitrarily positioned point, as shown in Figure 1 (The point is marked as"m"). The barycentric vectors, a minimal amount of body-fixed vectors, are used to represent the position vectors as vectorial sums. These once-calculated barycentric vectors are utilized to determine the kinematic and dynamic quantities of space systems.

By differentiating position vectors, it is then possible to determine the linear velocities. Transformation matrices are used to define the orientation of the various linkages, and the angular velocities are expressed as a function of the controlled angular velocities.

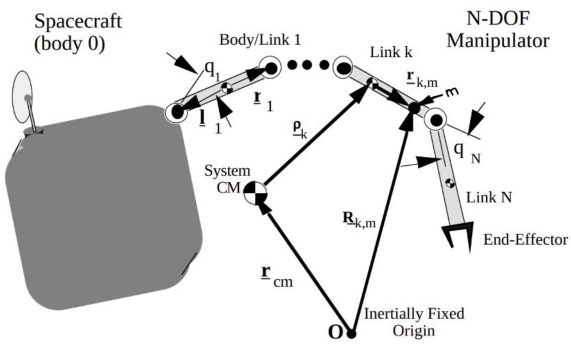

Figure 2.1: A spatial free-flying robotic manipulator system

The body 0 in Figure 1 represents the spacecraft, and the bodies k (k=1,...,N) represent the manipulator links. The manipulator is an open-chain configuration so that a system with an N degree-of-freedom (DoF) manipulator will have in total of (N+6) DoF, the additional 6 DoF is from the spacecraft position and attitude. From Figure 1, the spacecraft is mounted with an N-DoF manipulator which means that the last link is marked with the letter "N". An inertially fixed point "O" is considered as the Fixed Origin point. A

matrix representation of the kinematics and dynamics equation is obtained by attaching a reference frame to each center of mass with axes parallel to each body's principal axes. Left superscripts must be interpreted as "expressed in frame," and if a left superscript is missing then it implies a column vector expressed in the inertial frame.

Figure 1 shows the vector from the arbitrarily chosen point m on the link/body k to the inertially fixed origin O, $\underline{\mathbf{R}}_{km}$ is given by:

$$\underline{\mathbf{R}}_{km} = \underline{\mathbf{R}}_k + \underline{\mathbf{r}}_{km} \qquad k = 0, ..., N \tag{2.1}$$

where $\underline{\mathbf{R}}_k$ is a vector from the inertially fixed origin O to a k body's CM, and $\underline{\mathbf{r}}_{km}$ is a barycentric vector from k body's CM to the point m. Equation (2.1) can be further decomposed as:

$$\underline{\mathbf{R}}_k = \underline{\mathbf{r}}_{cm} + \underline{\rho}_k \qquad k = 0, ..., N \tag{2.2}$$

where $\underline{\mathbf{r}}_{cm}$ and $\underline{\rho}_k$ are defined from the Figure 1. The CM locations of the individual links and the spacecraft with respect to system CM are defined uniquely by some system configuration and it is possible to express $\underline{\rho}_k$ in terms of $\underline{\mathbf{r}}_i$ and $\mathbf{l}_i$ where i = 0,...N. Therefore,

$$\underline{\rho}_k - \underline{\rho}_{k-1} = \underline{\mathbf{r}}_{k-1} - \underline{\mathbf{l}}_k \qquad k = 1, ..., N \tag{2.3}$$

Since the $\underline{\rho}_k$ vectors are defined with respect to the system CM, it holds that:

$$\sum_{k=0}^{N} m_k \rho_k = 0 \tag{2.4}$$

where m_k is the mass of the body k. Equations (2.3 & 2.4) can be solved to yield:

$$\underline{\rho}_k = \sum_{i=1}^{k} (\underline{\mathbf{r}}_{i-1} - \mathbf{l}_i)\mu_i - \sum_{i=k+1}^{N} (\underline{\mathbf{r}}_{i-1} - \mathbf{l}_i)(1 - \mu_i) = 0 \qquad k = 0, ..., N \tag{2.5}$$

where μ_i represents the mass distribution defined by:

$$\mu_i = \left\{ \begin{matrix} 0 & i = 0 \\ \sum_{j=0}^{i-1} \frac{m_i}{M} = 0 & i = 1, ..., N \\ 1 & i = N + 1 \end{matrix} \right\} \tag{2.6}$$

M is the total mass of the system. From [1] we get that the (2.5) can be written as the sum of the mass-weighted vectors and the resulting complex equation can be simplified using barycenters. The barycenter location for the i^{th} body can be given by:

$$c_i{}^* = l_i\mu_i + r_i(1 + \mu_{i+1}) \tag{2.7}$$

Figure 2 also shows a set of body fixed vectors, called the barycentric vectors in this paper is defined by:

$$\begin{aligned} c_i{}^* &= -c_i \\ r_i{}^* &= r_i - c_i \\ l_i{}^* &= l_i - c_i \end{aligned} \tag{2.8}$$

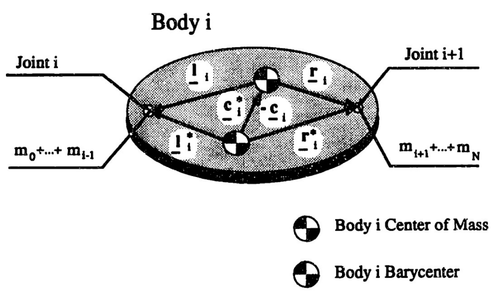

Figure 2.2: Barycenter and barycentric vectors

Using all the definitions, Equation 2.5 can be written generally as:

$$\rho_k = \sum_{i=0}^{N} \mathbf{v}_{ik} \qquad k = 0, ..., N \tag{2.9}$$

where the barycentric vectors v_{ik} are given by the following expression,

$$v_{ik} = \left\{ \begin{matrix} r_i{}^* & i < k \\ c_i{}^* & i = k \\ l_i{}^* & i > k \end{matrix} \right\} \tag{2.10}$$

The expression ρ_k can be substituted in the Equations (2.1) & (2.2) to yield $\underline{\mathbf{R}}_k$ & $\underline{\mathbf{R}}_{k,m}$:

$$\underline{\mathbf{R}}_{k,m} = \underline{\mathbf{r}}_{cm} + \rho_k + \underline{\mathbf{r}}_{k,m}$$

$$= \underline{\mathbf{r}}_{cm} + \sum_{i=0}^{N} \mathbf{v}_{ik,m} \tag{2.11}$$

where the vector $\mathbf{v}_{ik,m}$ just absorbs $\mathbf{r}_{k,m}$ according to the formula given below:

$$\mathbf{v}_{ik,m} = \mathbf{v}_{ik} + \delta_{im} + \mathbf{r}_{k,m} \qquad i,k = 0,...,N \tag{2.12}$$

δ_{im} is a Kronecker delta .The position of the end effector can be found using the above equation[1]. The orientations can be described by the transformation matrices.

2.2 Velocity

In order to write the expressions for linear velocities of arbitrary system points, derivatives of ρ_k must be calculated first. The time derivative of ρ_k can be written as:

$$\dot{\rho}_k = \sum_{i=0}^{N} \omega_i \times \mathbf{v}_{ik} \qquad k = 0,...,N \tag{2.13}$$

Similarly, differentiating Equation (2.11) yields the following expression for the velocity of a point m in body k, $\dot{\underline{\mathbf{R}}}_{k,m}$.

$$\dot{\underline{\mathbf{R}}}_{k,m} = \dot{\underline{\mathbf{r}}}_{cm} + \sum_{i=0}^{N} \underline{\omega}_i \times \underline{\mathbf{v}}_{iN,E} \tag{2.14}$$

The velocity of the end effector, $\dot{\underline{\mathbf{r}}}_E$ can be interpreted similarly as

$$\dot{\underline{\mathbf{r}}}_E = \dot{\underline{\mathbf{r}}}_{cm} + \sum_{i=0}^{N} \underline{\omega}_i \times \underline{\mathbf{v}}_{iN,E} \tag{2.15}$$

Equations (2.14) and (2.15) can be written as a function of relative angular velocities which correspond to the controlled manipulator joint rates: $\dot{\mathbf{q}} = \frac{d}{dt}(q_1,...,q_n)^T$. The angular velocity of body "k" can be written as:

$$\underline{\omega}_k = \underline{\omega}_0 + \sum_{i=1}^{k} \underline{\omega}_i{}^{i-1} \qquad j = 1,...,N \tag{2.16}$$

where the $\underline{\omega}_0$ is the spacecraft angular velocity. The end-effector angular velocity can be interpreted similarly as:

$$\underline{\omega}_E = \underline{\omega}_0 + \sum_{i=1}^{N} \underline{\omega}_i{}^{i-1} \tag{2.17}$$

The 3×1 column vectors ${}^{i}v_{ik}$ expressed in frame i are represented as:

$${}^{0}\mathbf{v}_{ik} = {}^{0}\mathbf{T}_i + {}^{i}\mathbf{v}_{ik} \tag{2.18}$$

where T_i is a 3×3 transformation matrix that describes the orientation of the i_{th} frame with respect to the inertial frame. Using Equation (2.18), equations(2.14) &(2.16) can be written as[1]:

$$\dot{\mathbf{R}}_{k,m} = \dot{\mathbf{r}}_{cm} + \mathbf{T}_0({}^0\mathbf{J}_{11k,m}{}^0\omega_0 + {}^0\mathbf{J}_{12k,m}\mathbf{q}) \tag{2.19}$$

$$\omega_k = \mathbf{T}_0 + ({}^0\omega_0 + {}^0\mathbf{J}_{22k,m}\dot{\mathbf{q}}) \tag{2.20}$$

where:

$${}^0\mathbf{J}_{11k,m} = -\sum_{i=0}^{N}[{}^0\mathbf{T}_i{}^i\mathbf{v}_{ik,m}]^\times \tag{2.21}$$

$${}^0\mathbf{J}_{12k,m} = -\sum_{i=0}^{N}[{}^0\mathbf{T}_i{}^i\mathbf{v}_{ik,m}]^{\times\,0}F_i \tag{2.22}$$

$${}^0\mathbf{J}_{22k,m} = {}^0F_k \tag{2.23}$$

where ${}^0\mathbf{J}_{11k,m}$ is a skew-symmetric 3×3 matrix whose elements corresponds to the vector from the system CM to point m, expressed in the spacecraft frame. ${}^0\mathbf{J}_{12k,m}$ & ${}^0\mathbf{J}_{22k,m}$ are 3×N matrices describing the effect of joint motions on the motion of point m. Equations (2.19) &(2.20) can be combined in one single matrix equation as follows:

$$\dot{\mathbf{x}}_{k,m} = \begin{bmatrix} \dot{\mathbf{R}}_{k,m} \\ \omega_k \end{bmatrix} = \mathbf{J}^+{}_{k,m} \begin{bmatrix} {}^0\dot{\mathbf{r}}_{cm} \\ {}^0\omega_0 \\ \dot{\mathbf{q}} \end{bmatrix} \tag{2.24}$$

where:

$${}^0\mathbf{J}^+{}_{k,m}(\mathbf{q}) = \begin{bmatrix} 1 & {}^0\mathbf{J}_{11k,m} & {}^0\mathbf{J}_{12k,m} \\ 0 & 1 & {}^0\mathbf{J}_{22k,m} \end{bmatrix} \tag{2.25}$$

Here 1 is the unity 3×3 matrix and 0, the zero is a 3×3 matrix. ${}^0\mathbf{J}^+{}_{k,m}$ is a function of the barycentric vectors of the system, which are functions of the masses and geometry of all the system bodies, including spacecraft. Its size is $6 \times (N + 6)$, so even when N=6, it is a non-square matrix. Similar equation can be written for the end-effector, by noting that the body k in this case is body N and the point m is the end-effector point E. Therefore,

$$\dot{\mathbf{r}}_E = \dot{\mathbf{r}}_{cm} + \mathbf{T}_0({}^0\mathbf{J}_{11}{}^0\omega_0 + {}^0\mathbf{J}_{12}\mathbf{q}) \tag{2.26}$$

$$\omega_E = \mathbf{T}_0 + ({}^0\omega_0 + {}^0\mathbf{J}_{22}\dot{\mathbf{q}}) \tag{2.27}$$

where:

$${}^0\mathbf{J}_{11} = -\sum_{i=0}^{N}[{}^0\mathbf{T}_i{}^i\mathbf{v}_{iN,E}]^\times \tag{2.28}$$

$$^0\mathbf{J}_{12} = -\sum_{i=0}^{N}[^0\mathbf{T}_i{}^i\mathbf{v}_{iN,E}]^{\times 0}F_i \tag{2.29}$$

$$^0\mathbf{J}_{22} = {}^0F_N \tag{2.30}$$

and the matrices $^0\mathbf{J}_{11}\&^0\mathbf{J}_{12}\&^0\mathbf{J}_{22}$ are similar to the matrices for point m. Therefore,

$$\dot{\mathbf{x}} = \begin{bmatrix} \dot{\mathbf{r}}_E \\ \omega_E \end{bmatrix} = \mathbf{J}^+ \begin{bmatrix} ^0\dot{\mathbf{r}}_{cm} \\ ^0\omega_0 \\ \dot{\mathbf{q}} \end{bmatrix} \tag{2.31}$$

$$^0\mathbf{J}^+(\mathbf{q}) = \begin{bmatrix} 1 & ^0\mathbf{J}_11 & ^0\mathbf{J}_12 \\ 0 & 1 & ^0\mathbf{J}_22 \end{bmatrix} \tag{2.32}$$

Note that the rank of J^+ is always six because its first six columns always contain six independent column vectors. This reflects the fact that even when the manipulator does not move, its end-effector can reach any position or orientation by moving the spacecraft alone. Equations (2.25) & (2.32) describe the differential kinematics of any point in the system, including the end-effector. Kinematic equations related to the particular orientation representation also must be used. If the 321 sequences of Euler angles are used to represent the orientation Θ_k of the k^th body, then the following holds, [2]:

$$\dot{\Theta}_k = \frac{d}{dt}[\Theta_{1,k}, \Theta_{2,k}, \Theta_{3,k}]^T = \mathbf{S}^{-1}{}_k(\Theta_k)\omega_k \qquad k = 0, ..., N \tag{2.33}$$

where $\mathbf{S}_k$ is an invertible matrix[1]. A four parameter representation of the orientation can be used if representation singularities can be a problem[2].

3

SYSTEM DYNAMICS

The kinematic equations of motion are written in the Lagrangian approach. As the system is rigid, potential energy due to flexibility is zero. Potential energy due to gravity is also zero. Hence the expression for kinetic energy is only required to form the Lagrangian system.

The kinetic energy of the system, denoted by T, can be written as:

$$T = \frac{1}{2}\,{}^0\omega_0^T\,{}^0D\,{}^0\omega_0\ +^0\omega_0 \tag{3.1}$$

The generalized momentum that corresponds to spacecraft angular velocity, ${}^0\omega_0$, is obtained by differentiating the Lagrangian equation given above.

As the angular momentum is conserved in the system, the generalized momentum is given by,

$$\frac{\partial T}{\partial^0\omega_0} =^0 D^0\omega_0 +^0 D_q\dot{q} = T_0^T h_{cm,0} =^0 h_{cm,0} \tag{3.2}$$

If the system is at rest initially, $h_{cm,0}$ is zero, and angular momentum is conserved in both the inertial and spacecraft frames. As $h_{cm,0}$ and ${}^0h_{cm,0}$ are perpendicular to the plane of motion, the condition holds true even if $h_{cm,0}$ is non-zero.

The compact form of (3.1) can be written as

$$T = \tfrac{1}{2}\dot{z}_0{}^T\ H^+(q)\dot{z}_0$$

where $H^+(q)$ is the system inertia matrix given by,

$$H^{+}(q) \;=\; {}^{0}D_{qq} \,-^{0} d_{q}^{T}\, {}^{0}D_{q}^{-}1\, {}^{0}D_{q}$$

$$\text{and } \dot{z_0} = \begin{bmatrix} {}^{0}\dot{r_{cm}} \\ {}^{0}\omega_0 \\ \dot{q} \end{bmatrix}$$

${}^{0}D_q$ is a $3 \times N$ matrix and ${}^{0}D_{qq}$ is a $N \times N$ matrix which are expressed as:

$${}^{0}D_j = \sum_{i=0}^{N} {}^{0}D_{ij} \tag{3.3}$$

$${}^{0}D_{qq} = \sum_{j=0}^{N} {}^{0}F_j^T\, {}^{0}D_j\, {}^{0}F_j \tag{3.4}$$

The expression for Kinetic energy given by equation(3.1) is the system Routhian, which is the Lagrangian function of the system. The kinetic energy of the system is a function of manipulator joint angles and velocities. Hence, Lagrange's equation can be written as:

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}}\right) - \frac{\partial T}{\partial q} = \tau \tag{3.5}$$

τ is the generalized force vector, which is equal to the applied force vector $[\tau_1, \tau_2....\tau_n]^T$. Applying the generalized force vector equation to the Lagrangian function results in a set of N dynamic equations of the form:

$$H^{+}(q)\, \ddot{z}_0 + C^{+}(q, {}^{0}\omega_0, \dot{q}) = Q \tag{3.6}$$

C^{+} contains the nonlinear terms of the equations of motion. Equation 3.5 represents N+6 equations that explain how a free-flying manipulator system moves while being affected by both internal actuator torques and external forces and torques. The generalized Q forces are a combination of disturbance forces Q_d and control forces Q_c. Therefore, we can write

$$Q = Q_c + Q_d \tag{3.7}$$

The control forces of a spacecraft comprises of thruster forces ${}^{0}f_s$ and torques,${}^{0}n_s$ and the manipulator's joint torques τ. Using the equations in the previous section, the control forces can be written as:

$$Q_c = J_q^T \begin{bmatrix} {}^{0}f_s \\ {}^{0}n_s \\ \tau \end{bmatrix} = \begin{bmatrix} 1 & {}^{0}J_{11.S} & {}^{0}J_{12.s} \\ 0 & 1 & {}^{0}J_{22.s} \\ 0 & 0 & 1 \end{bmatrix}^T \begin{bmatrix} {}^{0}f_s \\ {}^{0}n_s \\ \tau \end{bmatrix} \tag{3.8}$$

S represents the CM of the matrix and J_q is square and always invertible.

4

CONTROL OF SPACE MANIPULATOR

4.1 CONTROL MODES FOR SPACE MANIPULATOR SYSTEMS

It is well known that one generally needs three basic elements in order to control a fixed-base manipulator. As shown in Figure 3, first, an invertible representation of manipulator kinematics, which can in form of a Jacobian. Second, a set of dynamic equations describing the response of manipulator joint angles to actuator torques or forces. Third, a control algorithm using internally or externally provided sensory information in order to calculate torques/forces required to achieve the desired task.

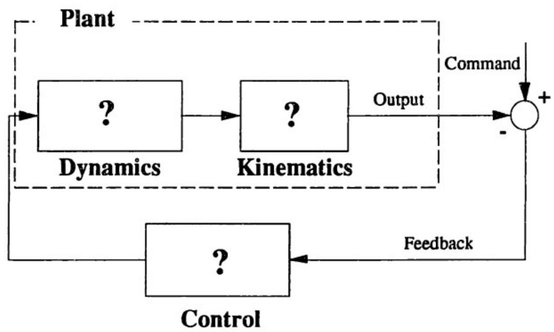

Figure 4.1: General block diagram for a closed loop control

Usually, two modes of motion control are implemented in fixed-based systems. In the first mode, joint angles of a manipulator are commanded to some desired angles, in other words, the manipulator is controlled in its joint space. In the second mode, the end-effector is commanded to move with respect to some fixed coordinated frame. The end-effector is commanded in some cartesian or operational space and the command is some cartesian space position and/or velocity.

Similarly to fixed-based systems, a space manipulator may operate under different modes, differing by the control objective. As in the fixed-based case, kinematics are not needed to achieve this task and a simple PD joint controllers are enough. Unlike fixed-based systems, there are two types of cartesian space motion control: Spacecraft-Referenced End-Point Motion Control and Inertially-Referenced End-Point Motion Control.

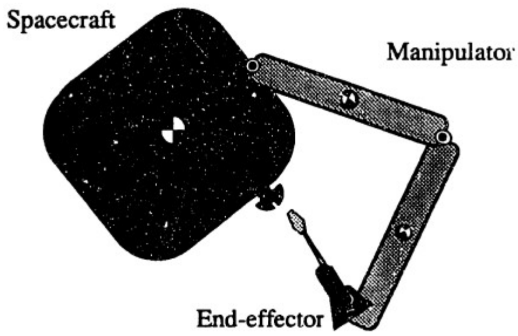

Figure 4.2: Spacecraft-Referenced End-Point Motion Control

Spacecraft-Referenced End-Point Motion Control is the mode of control where the manipulator end-point is commanded to move to a location fixed to its own spacecraft. That is, the position of the target or the reference trajectory is defined with respect to the moving spacecraft. For example in Figure 4, the task is to move the end-effector to close to the screw which is fixed with respect to the spacecraft.

Inertially-Referenced End-Point Motion Control is the mode where the manipulator end-point is commanded to move with respect to inertial space. For example, In Figure 5, the desired trajectory is fixed in inertial space, and the controllers receive feedback from instrumentation fixed on the spacecraft(an inertially-fixed camera)

The appropriate block diagram is constructed using the results from kinematics & dynamics and is depicted in Figure 6. Kinematic equations relating to attitude variables to angular velocities also must be used and these equations are represented here by an integrator block in Figure 6. To, draw a conclusion about the behavior of the whole system, the remaining system states must be examined. Since, it is not possible to control the position and velocity of the system CM with internal forces, such as manipulator

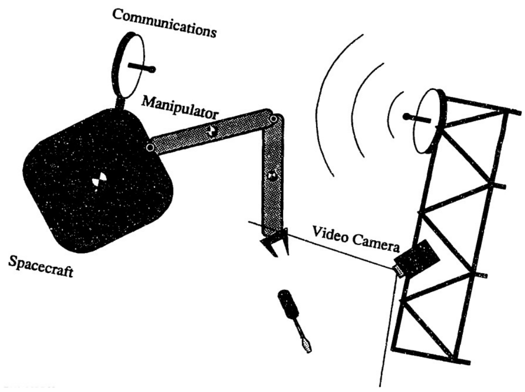

Figure 4.3: Inertially-Referenced End-Point Motion Control

joint torques, $\mathbf{r}_{cm}$ & $\dot{\mathbf{r}}_{cm}$ are uncontrollable. These states correspond to a double integrator system & the states left are the spacecraft's angular velocity and attitude.

4.2 COORDINATED CONTROL FOR FREE-FLYING MANIPULATORS

In this section it is assumed that the spacecraft is using its actuators which are capable of applying a force at its CM as well as a torque around it. Forces can be generated by thruster (jet) actuators, while torques may be generated by thruster actuators, momentum gyros or reaction wheels. Spacecraft actuators are required for the following reasons: (a) To keep a spacecraft's position and attitude fixed. (b) To move the entire system freely in space. (c) To compensate for external disturbances.

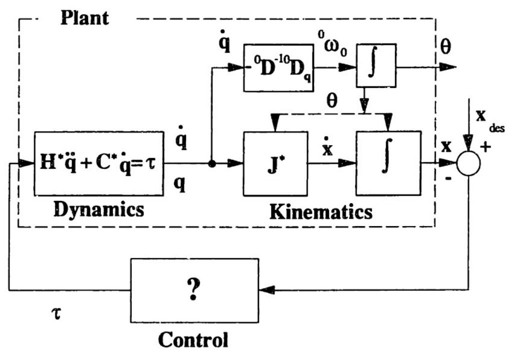

Figure 4.4: Block diagram of a free-floating system operating in Inertially-Referenced End-Point Motion Control

The similarity between Equation (3.6) and the equations of motion that correspond to a fixed-based manipulator leads to an investigation of whether control laws that are applicable in the latter case also can be used in the control of space robotic systems. However, two differences between the two situations must be pointed out. The first is that an appropriate representation of a spacecraft's attitude is needed, e.g. Euler angles, see Equation (2.33), or Euler parameters. The second difference is that, due to a spacecraft's mobility, a space robotic system is inherently redundant. This redundancy can be used to achieve additional tasks; here it is used to control a spacecraft's location and attitude by augmenting the system output.

As noted in Chapter 2, even when the manipulator has six DoF, (N=6), J^+ is not a square matrix and therefore is not invertible, although its rank is always six. If N=6, then J^+ is a 6 × 12 matrix. The Jacobian is obtained by setting the subscript S to stand spacecraft's CM, (k=0 and m=CM).

$$\dot{z_1} = \begin{bmatrix} \dot{r_E} \\ \omega_E \\ \dot{R_S} \\ \omega_S \end{bmatrix} = diag(T_0, T_0, T_0, T_0) \begin{bmatrix} 1 & {}^0J_{11} & {}^0J_{12} \\ 0 & 1 & {}^0J_{22} \\ 1 & {}^0J_{11,S} & {}^0J_{12,S} \\ 0 & 1 & 0 \end{bmatrix} \dot{z_0} \tag{4.1}$$

$$\dot{z_1} = J_z \dot{z_0} \tag{4.2}$$

where $\dot{z_1}$ is the vector of output velocities. The Jacobian J_z is an invertible 12 × 12 matrix, unless the manipulator is kinematically singular, and relates an input to output velocities providing a basis for controlling a free-flying system.

Equations of motion (3.6) and (4.1) can be used to apply various standard motion control techniques, in a way similar to Khatib's "operational" approach [2]. In this case equation (4.1) is used as a map between the selected state variables $\dot{z}_0$ and the output variables $\dot{z}_1$.

Here, a Transposed-Jacobian type controller with inertial feedback is designed to demonstrate this technique. The equations of motion in the z_1 domain can be found by substituting Equation (4.1) into equation (3.6) to obtain the form:

$$\tilde{\mathbf{H}}\ddot{z}_1 + \tilde{\mathbf{C}} = (\mathbf{J}_z^{-1})^T \mathbf{Q}_c \tag{4.3}$$

where $\tilde{\mathbf{C}}$ contains the nonlinear terms and $\tilde{\mathbf{H}}$ is given by:

$$\tilde{\mathbf{H}} = (\mathbf{J}_z^{-1})^T \mathbf{H}^+ \mathbf{J}_z{}^-1 \tag{4.4}$$

The inertia matrix is positive definite if $\mathbf{J}_z$ is non-singular. An error $\mathbf{e}$ is defined in the output domain $\mathbf{z}_1$ as:

$$\mathbf{e} = \mathbf{z}_{1,des} - \mathbf{z}_1 \tag{4.5}$$

where $\mathbf{z}_1$ is provided by inertial feedback, and $\mathbf{z}_{1,des}$ is the desired inertial point. It is assumed here that inertial measurements of the position and orientation of the spacecraft and the end-effector are available. Then, if the input $\mathbf{Q}_c$ is taken to be:

$$\mathbf{Q}_c = \mathbf{J}_z^T[\tilde{\mathbf{H}}(\mathbf{K}_p\mathbf{e} + \mathbf{K}_d\dot{\mathbf{e}} + \ddot{z}_1, des) + \tilde{\mathbf{C}}] \tag{4.6}$$

where $\mathbf{K}_p$ & $\mathbf{K}_d$ are positive definite diagonal matrices, then the error dynamics are given by:

$$\ddot{\mathbf{e}} + \mathbf{K}_d\dot{\mathbf{e}} + \mathbf{K}_p\mathbf{e} = 0 \tag{4.7}$$

and therefore the error converges asymptotically to zero. Equation (4.6) is a modification of the operational space controller [3]. If high enough gains are used, the simpler Transposed Jacobian controller can be used[4]:

$$\mathbf{Q}_c = \mathbf{J}_z^T(\mathbf{K}_p\mathbf{e} + \mathbf{K}_d\dot{\mathbf{e}}) \tag{4.8}$$

Note that if a singularity is encountered, the controllers given by Equation (4.6) and (4.8) will result in errors but will not fail fatally. If some disturbance acts on the system, a small steady-state error is expected, because these controllers are basically PD controllers. Finally, the reaction jet forces and torques and the joint torques can be found out by:
Assuming that Equation (4.8) is used, these are given by:

$$\begin{bmatrix} {}^0f_s \\ {}^0n_s \\ \tau \end{bmatrix} = (\mathbf{J}_q^T)^{-1}\mathbf{J}_z^T(\mathbf{K}_p\mathbf{e} + \mathbf{K}_d\dot{\mathbf{e}}) \tag{4.9}$$

The inversion of $\mathbf{J}_q$ is possible since this Jacobian is always non-singular. Equation (4.9) is the final result that permits coordinated control of both the spacecraft and its manipulator, based on inertial measurements of the spacecraft and end-effector locations and orientations. If no such measurements are available, the error $\mathbf{e}$ can be estimated by integrating the equations of motion in real-time. Still, then errors due to model uncertainties will be introduced. This method of motion control is demonstrated with an example.

5

Example

As an example of the algorithm described in the previous sections, a controller capable of simultaneously controlling a spacecraft's motion and its manipulator's end-effector motion was designed. The system in the example is planar and consists of a two DOF manipulator mounted on a three DOF spacecraft. Its parameters including mass, inertia, and kinematic properties are given in the table.

Table I. System parameters for the example.

Body	l_i (m)	r_i (m)	m_i (Kg)	I_i (Kg m^2)
0	.5	.5	40	6.667
1	.5	.5	4	0.333
2	.5	.5	3	0.250

The spacecraft is assumed to have reaction jets that can produce forces and torques proportional to the specified control input. In this example, the independent coordinates vector, z_0, and the vector of controlled coordinates, z_1, are as follows:

$$z_0 = [{}^0x_{cm}, {}^0y_{cm}, \theta, q1, q2]^T,\ z_1 = [x_s, y_s, \theta, x_E, y_E]^T \tag{5.1}$$

In the above equation, ${}^0x_{cm}$ and ${}^0y_{cm}$ are the coordinates of the system CM with reference to the inertial frame, and θ is the spacecraft's attitude. $q1$ and $q2$ are the joint angles of the manipulator, and J_q is always non singular.
The controller equations derived in the previous sections are used to compute the reaction jet forces and toques, ${}^0f_s$ and ${}^0n_s$, and the manipulator joint torques,τ. Using these values, the required trajectory of the spacecraft as well as the manipulator can be con-

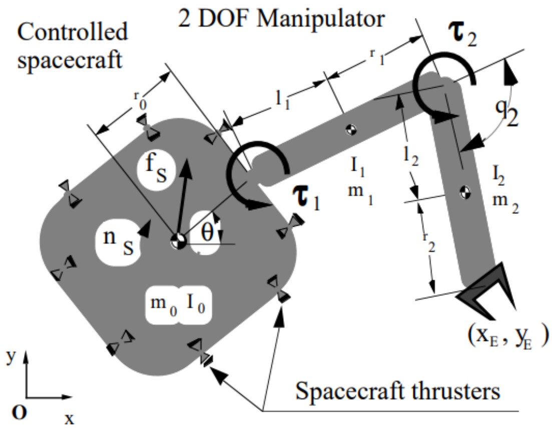

Figure 5.1: A two-DOF manipulator on a three-DOF spacecraft

structed. Even if the given trajectory cannot be achieved, the controller tries to mimic the given trajectory by getting as close as possible to the specified path.
In the following simulations, initially, the inertial fixed frame is set at the position of system CM, ie when $(\theta, q1, q2) = (21, -58, 60.3)$ and the end-effector position is at $(2.0, 0.0)$. The end-effector is then made to move to the coordinate $(1.5, 1.5)$ and the spacecraft is made to move to $(\theta, x_s, y_s) = (15, 0.5, 0.5)$.
Figure 5.2 given below shows the motion of the system in inertial space. The error input values for this simulation have been manually calculated using transformation matrices to find out the spacecraft's position and attitude by the end-effector position values [1]. We observe that the spacecraft maintains the required location and attitude, while the manipulator moves along a straight line to the desired location. It should be noted that if the spacecraft had remained in its initial position, the end-effector would move to point B in any configuration.
Due to the presence of dynamic singularities, if the spaceship was free-floating and its reaction jets were turned off, point B could not have been reached from point A by the straight line path.

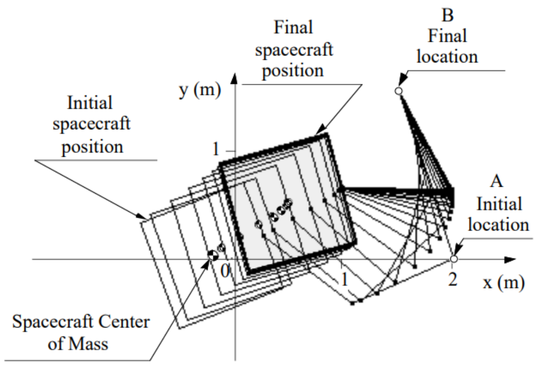

Figure 5.2: Coordinated spacecraft motion example system

Figures 5.3 shows the spacecraft's position and attitude error during the end-effector motion. These errors nullify in 12 seconds.

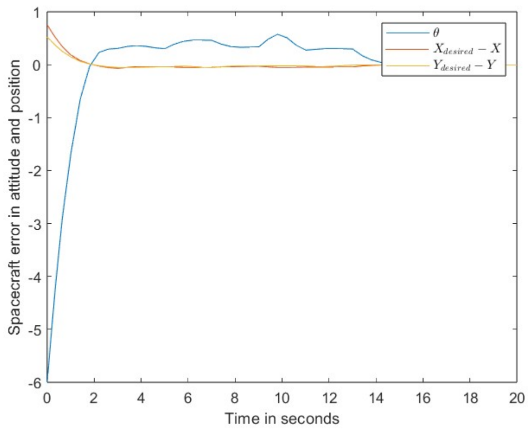

Figure 5.3: Motion of spacecraft during the maneuver

Figure 5.4 shows the thruster forces required to move the spacecraft during the first five seconds of the maneuver. Since the error converges to zero asymptotically and there are no external disturbances acting on the system, the reaction jet forces approach asymptotically to zero as soon as the motion ceases.

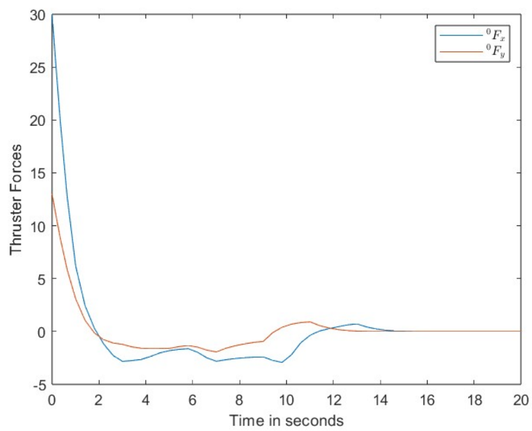

Figure 5.4: Thruster forces required during the maneuver

6

CONCLUSION AND FUTURE SCOPE

6.1 FUTURE RESEARCH

It is mentioned that dynamic singularities can be avoided by using manipulator redundancy. Such redundancy also can be used to minimize fuel expenditure in free-flying or to improve a system's reliability, an important and under-explored issue.
Numerous intriguing path-planning-related issues are still open for discussion. For instance, if a system is free-floating, an end-effector path can be used to additionally regulate the attitude of the system's spacecraft. Such pathways are still difficult to find despite some prior efforts. It would also be desirable to have useful end-effector pathways that would enable a free-floating system to use its Path Dependent Workspace without encountering any issues.
Because it is exceedingly difficult to apply closed-loop feedback and hence decrease system uncertainty, system calibration is necessary. This problem is brought on by the dearth of efficient end-point sensing methods. In space, methods to lessen this issue would be tremendously beneficial.
While focusing on the principles of space manipulators, this thesis made idealistic assumptions about these systems. Practically speaking, further significant difficulties must be addressed. The control of numerous manipulator systems, joint or link flexibilities, interactions between manipulators and payloads during grasping, sensing and sensor data fusion, communication delays, telerobotic and human factor concerns, as well as safety and reliability issues, may be a few of these.

6.2 CONCLUSIONS

The investigation of the structure, specifications, and constraints of controllers to be employed in the management of spacecraft-borne manipulators was the primary driving force behind this thesis. A modeling methodology that produced an explicit and compact kinematic and dynamic description for free-flying systems, taking into account forces due to spacecraft thrusters, to end-effector forces, and to disturbances, was based on the fundamental premise that effective control is only possible if based on knowledge of the structure of a system's "plant". This study gives compactly constructed functions of the system's barycentric vectors that describe the dynamics of free-flying space robotic systems. The basis for coordinated control of a spacecraft and its manipulator is the addition of the spacecraft's position and attitude to the control needs. This plan can be used to prevent a robotic system from negatively impacting its surroundings or to keep a beneficial manipulator configuration. These methods are shown via a Transposed-Jacobian type controller with inertial feedback.

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