Explanations to Math Problems – LD SAT Study Guide

Page 95

1. When multiplying two numbers with the same base, add the exponents.
$(u^2)(u^3) = u^{2+3} = u^5$

2. In this case we have a power, k^3, squared. This is a power to a power; we multiply the powers to find the solution.
$(k^3)^2 = k^{(3\times2)} = k^6$

3. In this case we have a power, a^2, cubed. This is a power to a power; we multiply the powers to find the solution.
$(a^2)^3 = a^{(2\times3)} = a^6$

4. The two terms, s^2 and s^3, are of two different powers. These are not like terms and cannot be added.

5. The two terms $2u^2$ and $3u^2$ each share a u^2, these are like terms. We can combine like terms by adding the coefficients.
$2u^2 + 3u^2 = 5u^2$

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8. Any number squared (or raised to an even power) will be positive; for example $(-2)^2 = (-2)(-2) = 4$ and $(-2)^4 = 16$. The square root of a negative number asks us to find a number which is negative when squared. These numbers do not exist and therefore the square root (or even root) of a negative number has no real solution.

9. A negative number cubed or raised to an odd power will stay negative. For example: $(-3)^3 = (-3)(-3)(-3) = -27$. This allows us to take the cube root (or an odd root) of a negative number and find a solution.
$\sqrt[3]{-27} = -3$

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1. Check by dividing.
$9 \div 3 = 3$.
The result is a whole number. Yes, 3 goes into 9 three times.

2. Check by dividing.

$10 \div 6 = 1\frac{2}{3}$.
The result is not a whole number. 6 does not go into 10.

3. Check by dividing:
$2 \div 6 = \frac{1}{3}$.
The result is not a whole number. 6 does not go into 2.

4. A factor of 18 is any number which goes into 18. Use your calculator's FACTORS program to find the factors of 18:
1, 2, 3, 6, 9, 18.

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1. Use the FACTORS program on your calculator to find all the numbers that go into 24.
1, 2, 3, 4, 6, 8, 12, 24.

2. Use the FACTORS program on your calculator to find the factors 20:
1, 2, 4, 5, 10, 20
Once again use FACTORS to find the factors of 8:
1, 2, 4, 8
1, 2, and 4 are factors of both 20 and 8.

3. Use the FACTORS program on your calculator to find all the numbers that go into 96:
1, 2, 3, 4, 6, 8, 12, 16, 24, 32, 48, 96
Between 10 and 20 are only the factors 12, and 16.

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2. The figure is not drawn to scale so we cannot find a value for all the tick marks. Since *k* is the distance between tick marks 4 and 1 we can solve: *k* = 4 – 1 = 3. Since the distance between *j* and 4 is 2, we can solve: *j* = 4+2 = 6.

Page 113

1. First, solve for *x* by subtracting 7 from each side of the equation.
$x + 7 = 13$
-7 -7
$x = 6$
Check by plugging 6 back into the original equation:

6+7=13
13=13
This works, x=6.

2. First, solve for z: add 9 to each side.
$6z - 9 = 3$
$\underline{\quad +9 \quad +9}$
6z=12
Now, divide each side by 6:
$\frac{6z}{6} = \frac{12}{6}$
z=2
Check by plugging 2 back into the original equation:
6(2) – 9 = 3
12 – 9 = 3
3 = 3
This works, z=2.

3. First, solve for r: divide each side of the equation by 3.
$\frac{3r^2}{3} = \frac{48}{3}$
$r^2 = 16$
Now, take the square root of each side.
$\sqrt{r^2} = \sqrt{16}$
$r = \pm 4$
Remember, that the square root means that *r* can be either positive or negative.
Check by plugging 4 and -4 back into the original equation.
$3(4)^2 = 48$
3(16) = 48
48=48
This works, r=4 is a solution.
$3(-4)^2 = 48$
3(16) = 48
48=48
This works, r=-4 is the second solution.

4. Solve for *q*: divide each side by 7
$\frac{35}{7} = \frac{7(\sqrt{q})}{7}$
$5 = \sqrt{q}$
Now, square each side of the equation.
$5^2 = (\sqrt{q})^2$
$25 = q$

Check by plugging 25 back into the original equation.

$35 = 7(\sqrt{25})$

$35 = 7(5)$

$35 = 35$

This works, *q* = 25 is a solution.

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2. 2x and -4 in the numerator and 8, the denominator, can be reduced by 2.

$\frac{2x-4}{8} = \frac{x-2}{4}$.

4. The numerator, 6, and both denominator terms, 6 and -12x, can be reduced by 6.

$\frac{6}{6-12x} = \frac{1}{1-2x}$

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2. Solving for e means moving the rest to the other side.
First, subtract 3f from each side:

$2e + 3f = 6 + 4f$

-3f -3f

$2e = 6 + f$

Next, divide each side by 2:

$\frac{2e}{2} = \frac{6+f}{2}$

$e = \frac{6+f}{2}$

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2. "Cross multiply" to find:
64d = 4(48)
64d = 192
d=2

3. "Cross multiply" to find:
4i = 5(25)
4i = 125
i = 31.25

Page 123

1. There are three slots, pants, shirts, and shoes. Multiply the three to find the solution:
3*5*3 = 45 different outfits.

Page 132

2. Use $C = 2\pi r$ from page 128. $C = 2\pi(6) = 12\pi$.

3. Use $V = \pi r^2 h$ from page 129. $V = \pi(25)^2(40) = 25000\pi$.

Page 136

1. The figure states that CDEF is a square. By definition, all four sides of a square are exactly equal. So, x =5, exactly.

2. From the scale figure, we can approximate the length of GH to be about three times as long as GF. So, x=12, approximately.

3. The figure is not drawn to scale. The value of x cannot be determined from the given information.

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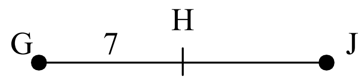
1. A) The parts of UW added together equal the length of UW, so:
UW = UV+VW = 5+8 = 13.
B) The parts of UX added together equal the length of UX, so:
UX = UV+VW+WX = 3+6+2=11.
C) The line segment UW minus the part UV leaves the part WV, so:
WV = UW-UV = 10-4 = 6.
From the figure we see that the parts of UW added together equal the length of UW, so:
UW = UV+WV = 5+8 = 13. `

2. The perimeter is the sum of the sides, P = ST+TR+RS = 6+14+10 = 30.

4. Draw a picture:.



Since H is the midpoint, HJ = GH = 7. The line segment GJ = GH + HJ = 7+7 = 14.

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1. Angle A has a square mark; this means that it must be a right angle.
Angle B has no square mark; just because the angle *looks* right doesn't mean that it is. Because no additional information is given, we cannot conclude that Angle B is right.
Just because Angle C does not look right does not mean it is not – remember that the figure is not drawn to scale. Line segment MN is perpendicular to line segment NO, so Angle C must be right.

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1. All of the angles along a straight line must add up to 180°. This means:
$75 + x + 50 = 180$
Now, solve for *x*:
$x + 125 = 180$
$x = 55$

2. We will treat this as a full circle. Just because the figure looks like is a straight line through the middle doesn't mean that it really is. In fact, if we add the angles to the right of our "straight" line ($59° + 125° = 184°$) we find that they do not add to 180° and therefore the line is not straight.
All of the angles around a circle must add up to 360°. This means:
$57 + 125 + 59 + z + 61 = 360$
Solving for *z*:
$302 + z = 360$
$z = 58$.

3. All of the angles along a straight line must add up to 180°. This means:
$39 + e + 2e = 180$
As normal, solve for *e*.
Simplify.
$39 + 3e = 180$
Subtract 39 from each side.
$3e = 141$
Divide each side by 3.
$e = 47$.

Page 150

1. The crossing lines make vertical angles that are equal.
e = 50.

2. Angle d is adjacent to 50°; these angles must add up to 180°.
d + 50 = 180
Subtract 50 from each side.
d = 130.

3. Angles e and f are adjacent; these angles must add up to 180°.
e + f = 180.

4. A circle of angles must add up to 360°.
e + f + 50 + d = 360.

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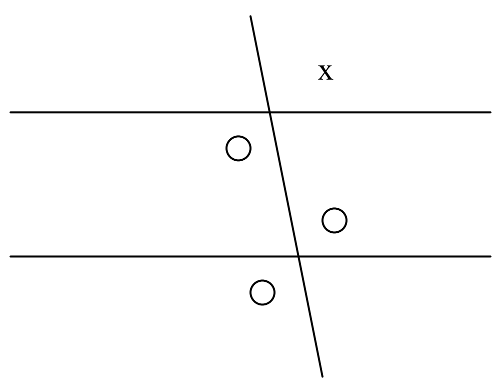
1. Because the two vertical lines are assumed to be parallel, all angles must be either equal to A or add up to 180° with A. "A" is a big angle, all other big angles must be equal to A.

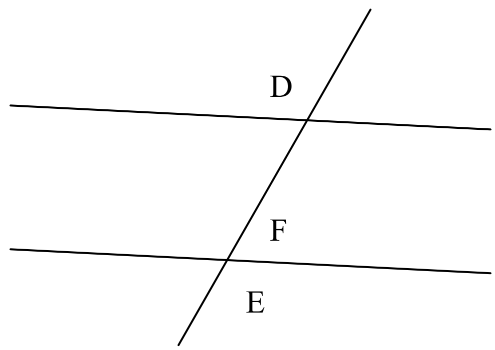
2. We know the two horizontal lines to be parallel. Although the third line looks perpendicular to the two, we cannot be sure. To find the equal angles, we will redraw the picture with x as the big angle:



All other big angles are equal to x. These are shown as circles.

3. We know the slanted horizontal lines to be parallel. All angles must be equal or add up to 180°. From the drawing we can see that *P* is a big angle and 70 is a small angles; these cannot be equal so they must sum to 180°.
70° + *P*° = 180°
P = 110°.

4. We know the slanted horizontal lines to be parallel. Although the third line looks perpendicular to the two, we cannot be sure. To find the equal angles, we will redraw the picture with D as the big angle. From our new drawing it is clear that D equals the other big angle, E, and must add up to 180° with the small angle, F.



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2. For each of these shapes, first use the "sum of internal angles" formula:
S = 180° (n-2)
But be careful! This tells you the sum of all the internal angles–not just h or j. To find the value of h or j we must divide the total sum by the number of angles.

A triangle has 3 sides, n=3.
S= 180° (3-2) = 180°
A triangle also has three angles, so each angle must measure:
180° / 3 = 60°
h = 60°

A hexagon has 6 sides, n=6.
S=180°(6-2) = 180°(4) = 720°
A hexagon also has 6 angles, so each angle must measure:
720°/6 = 120
j = 120°.

Page 158

1. All internal angles of a triangle must add up to 180°.
66 + 86 + m = 180

m = 28

2. All *internal* angles of a triangle must add up to 180°. *c* is not an internal angle, *a, b,* and *c* will not add up to 180°.

3. All internal angles of a triangle must add up to 180°.
$p + p + 70 = 180$
$p = 55$

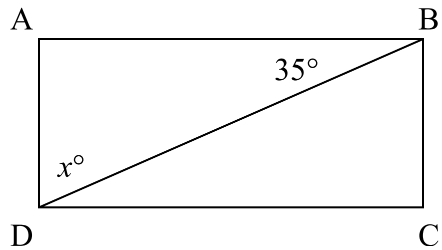
Pages 159 - 160

1. There is a vertical straight line. Along a straight line, all angles must add up to 180°.
$(t - 50) + (t) = 180$
Combine like terms and add 50 to each side.
$2t = 230$
$t = 115$; (A)

2. When two lines cross, the vertical angles are equal.
$2m + 21 = 65$
Subtract 21 from each side.
$2m = 44$
$m = 22$; (A)

3. In a triangle, all internal angles must add up to 180°. This can be used to find the measurement of a third angle when two are known. Take 80° and one of the multiple choices as our two angles and calculate what the third angle would have to be:
(A) 180 – 80 – 190 = -90 This cannot be.
(B) 180 – 80 – 70 = 30 This is below 39° and out of range
(C) 180 – 80 – 60 = 40 This is within our 39° to 59° range and is a valid choice for the third angle.
(D) 180 – 80 – 40 = 60 This is above 59° and out of range.
(E) 180 – 80 – 10 = 90 This is above 59° and out of range.
The correct answer must be angle (C).

4. Draw a picture:



From our picture we see that triangle ABD is a right triangle with $\angle DAB = 90$.
The internal angles of a triangle must add up to 180°.
$80 + 35 + x = 180$
$115 + x = 180$
Subtract 115 from each side.
$x = 65$; (E)

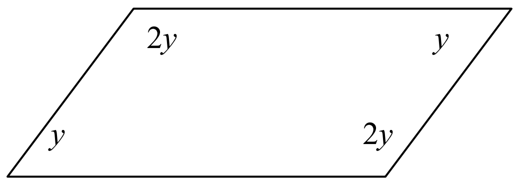
5. A quadrilateral has 4 sides, the sum of the internal angles must be:
$S = 180(4 - 2) = 360$
Let the final angle be x and set the sum of all 4 angles equal to 360.
$x + 85 + 85 + 85 = 360$
$x + 255 = 360$
Subtract 255 from each side.
$x = 105$; (D)

6. First, calculate the sum of the internal angles of a regular hexagon (6 sides):
$S = 180(6 – 2) = 720$
Let each angle have a value of x; all 6 angles must add up to 720:
$6x = 720$
Divide by 6
$x = 120$; (E).

7. The angle vertical to 60° is equal and must also measure 60°. All angles around a circle add up to 360°:
$m + 60 + n + 60 = 360$
$m + n + 120 = 360$
Subtract 120 from each side
$m + n = 240$; (E)

8. x is in a triangle with two other angles. One of these angles (top) is vertical to 40° and is therefore equal to 40°. The other angle (right) is at the corner of two perpendicular lines and therefore measures 90°. The internal angles of a triangle must add up to 180°, so:
$x + 40 + 90 = 180$
$x + 130 = 180$
Subtract 130 from each side.
$x = 50$; (C)

9. Redraw this figure with the new information such that each angle is in terms of *y*. Note that if we multiply each side by 2 we find that $y = \frac{1}{2}z$ is the same as $z = 2y$.



The sum of the internal angles of a quadrilateral (4 sides) must add up to:
S = 180(4 – 2) = 360
If we equate these four angles to 360 we find:
$2y + y + 2y + y = 360$
$6y = 360$
Divide each side by 6.
$y = 60$; (D).

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2. The formula for circumference is:
$C = 2\pi r = 2\pi(4) = 8\pi \approx 25.1$
The diameter is twice the circumference:
$D = 2r = 2(4) = 8$
The formula for the area is:
$A = \pi r^2 = \pi(4)^2 = 16\pi \approx 50.3$

3. Since line segment *pr* crosses the center of the circle, we know it's a diameter.
$D = pr = 12$
A radius is half a diameter.
$r = D/2 = 12/2 = 6$
The formula for the area is:
$A = \pi r^2 = \pi(6)^2 = 36\pi \approx 113$
The formula for circumference is:
$C = 2\pi r = 2\pi(6) = 12\pi \approx 37.7$
Since *q* is a center point and *s* is on the circle, we know that the line segment *qs* is a radius.
$qs = r = 6$

4. Use the formula for the area to solve for the radius, and then use the radius to find the circumference.
$A = \pi r^2$
$100\pi = \pi r^2$
Divide each side by π.

$100 = r^2$
Take the square root of each side
$r = 10$
Now, use the formula for circumference:
$C = 2\pi r = 2\pi(10) = 20\pi \approx 62.8$

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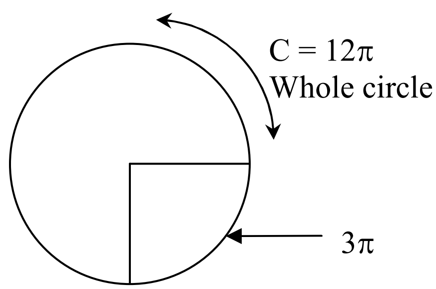
3. Set up a slice fraction; remember that a whole circle is 360°.

$$slice\ \ fraction = \frac{slice}{circle} = \frac{20°}{360°} = \frac{1}{18}$$

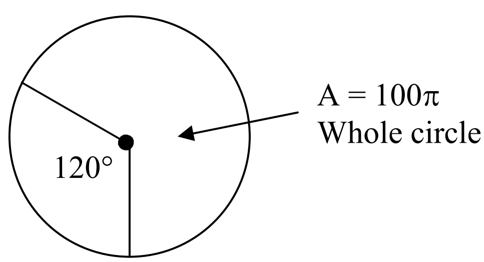
4. Draw a picture



Set up a slice fraction; compare the arc length to the total circumference.

$$slice\ \ fraction = \frac{arc\ length}{circumference} = \frac{3\pi}{12\pi} = \frac{3}{12} = \frac{1}{4}$$

5. Draw a picture



Set up a slice fraction comparing the central angle and 360°.

$$slice\ \ fraction = \frac{slice}{circle} = \frac{120°}{360°} = \frac{1}{3}$$

Now use this fraction, 1/3, to find the unknown slice area and arc length.

$$\frac{1}{3} = \frac{area}{100\pi}$$

$$area = \frac{100\pi}{3} \approx 33.3\pi \approx 105$$

To find the arc length we must first calculate the radius of the circle using the total area (100π) and the formula for the area of a circle.

$A = \pi r^2$

$100\pi = \pi r^2$

Divide each side by π.

$100 = r^2$

Take the square root of each side

$r = 10$

Now, use the formula for circumference:

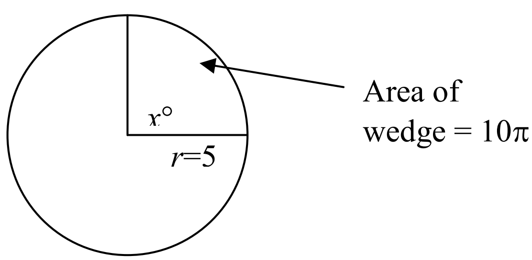
$C = 2\pi r = 2\pi(10) = 20\pi \approx 62.8$

Next, use the slice fraction (1/3) and the circumference (20π) to calculate the arc length.

$$\frac{1}{3} = \frac{arc\ length}{20\pi}$$

$$arc\ length = \frac{20\pi}{3} \approx 6.67\pi \approx 20.9$$

6. Draw a picture. Let the unknown central angle be $x°$.



First, use the radius (5) to calculate the area of the whole circle.

$A = \pi r^2 = \pi(5)^2 = 25\pi$

Set up a slice fraction which compares the area of the wedge (10π) to the area of the whole circle (25π).

$$slice\ fraction = \frac{slice}{circle} = \frac{10\pi}{25\pi} = \frac{2}{5}$$

Now, use the slice fraction to find x by comparing x with the degrees of the full circle (360°).

$$\frac{2}{5} = \frac{x°}{360°}$$

$$x° = \frac{2}{5} \cdot 360° = 144°$$

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1. The area of a semicircle of radius 6 is one half the area of a full circle of radius 6. First calculate the area of a full circle with radius 6.

$A = \pi r^2 = \pi(6)^2 = 36\pi$
Now, find half of this area.
$A_{semi}=0.5(36\pi)=18\pi$; (D).

2. Since line AB goes through the center and touches each side of the circle, it is a diameter. Use the area of the circle (10π) to calculate the radius and this to find the diameter.
$A = \pi r^2$
$10\pi = \pi r^2$
Divide each side by π.
$10 = r^2$
Take the square root of each side
$r \approx 3.16$
The diameter is twice the radius
$D = 2r \approx 2(3.16) \approx 6.3$; (A).

3. In one hour, the minute hand will trace one circle. The radius of this circle is the length of the minute hand, 16 inches. The circumference of the circle is equal to how far the tip has traveled in one circle.
$C = 2\pi r = 2\pi(16) = 32\pi$; (E).

4. If the area of circle X is three times that of circle Y, that means that the area of circle Y is one third the area of circle X.
$A_y = (1/3)A_x = (1/3)3\pi = \pi$
Use the area of circle Y (π) to find the radius of circle Y.
$A = \pi r^2$
$\pi = \pi r^2$
Divide each side by π.
$1 = r^2$
$1 = r$
Use this to find the circumference of circle Y.
$C = 2\pi r = 2\pi(1) = 2\pi$; (C).

5. Set up a slice fraction that compares the central angle (120°) to the full circle (360°).

$$slice\ fraction = \frac{slice}{circle} = \frac{120°}{360°} = \frac{1}{3}$$

Use this same fraction to compare the arc length to the circumference (81π).

$$\frac{1}{3} = \frac{arc\ length}{81\pi}$$

Multiply each side by 81π.

$$arc\ length = \frac{81\pi}{3} = 27\pi\ ;$$

6. Segment MN starts in the center and ends at the circle, it is a radius with length 7. First, use the radius (7) to find the total area.
A = πr^2 = $\pi(7)^2$ = 49π
Now, set up a slice fraction with the wedge area (20π) and the total area (49π).

$$slice\ fraction = \frac{slice}{circle} = \frac{20\pi}{49\pi} \approx 0.41$$

Use this fraction (0.41) to calculate the central angle by comparing it with the degrees in a full circle (360°).

$$0.41 = \frac{angle\ LMN}{360°}$$

Multiply each side by 360°.
$angle\ LMN = 0.41 \cdot 360° = 147°$; (A).

7. First, set up a slice fraction with the central angle (14°) and the degrees around the full circle (360°).

$$slice\ fraction = \frac{slice}{circle} = \frac{14°}{360°} \approx 0.0389$$

Treat weight as you would any circle property. Use the slice fraction (0.0389) to find the total weight by comparing it with the slice weight (20π). Note that slice weight goes on top and the total weight is on the bottom of our fraction.

$$0.0389 = \frac{20\pi}{total\ weight}$$

Multiply both sides by total weight, then divide each side by 0.0389.
$total\ weight = \frac{20\pi}{0.0389} = 514\pi$; (E).

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1. This is an isosceles triangle with two equal angles (50°), the sides opposite these angles must also be equal.
k = 8

2. The sum of the two short sides must be greater than the longest side. Find the maximum value of m by adding the other two sides
m < 8+8
m < 16
m cannot be 18 because 18 is greater than 16.

Page 181

1. We are given the two legs (6 and 9) and must solve for the hypotenuse (x).
$a^2 + b^2 = x^2$

$6^2 + 9^2 = x^2$
$36 + 81 = x^2$
$117 = x^2$
$x = \sqrt{117} \approx 10.8$

2. We are given the hypotenuse (12.1) and one leg (11) and asked to find the second leg (*n*)
$n^2 + b^2 = c^2$
$n^2 + 11^2 = 12.1^2$
$n^2 + 121 = 146.41$
$n^2 = 25.41$
$n = 5.0$

3. The internal angles of a triangle add up to 180°. A right angle is 90°.
$j + k + 90° = 180°$
$32° + k + 90° = 180°$
$k + 122° = 180°$
$k = 58°$

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1. This is a 30-60-90 triangle. We are given the short leg, 9. The hypotenuse, *k*, is the short leg times 2.
$k = 9 \cdot 2 = 18$
The long leg, *m*, is the short leg times $\sqrt{3}$
$m = 9\sqrt{3} \approx 15.6$

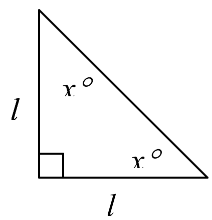
2. This is a 45-45-90 triangle with hypotenuse 5. The leg, *x*, is the hypotenuse divided by $\sqrt{2}$.
$$x = \frac{5}{\sqrt{2}} = \frac{5\sqrt{2}}{2} \approx 3.54$$
The perimeter is the sum of the three sides. The bottom leg is equal to *x*.
P = 3.54 + 3.54 + 5 = 12.1
The area is one half the base times the height. In a right triangle, the base and the height are the two legs.
$$A = \frac{1}{2}bh = \frac{1}{2}(3.54)(3.54) = 6.25$$

3. This is a 30-60-90 triangle. First, use the long leg, 12, to find the short leg *y* by dividing by $\sqrt{3}$.
$$y = \frac{12}{\sqrt{3}} = \frac{12\sqrt{3}}{3} = 4\sqrt{3} \approx 6.93$$

Now, multiply the short side, *y*, by 2 to find the hypotenuse, *x*.

$x = 2y = 2 \cdot 4\sqrt{3} = 8\sqrt{3} \approx 13.9$

4. Draw a picture. An isosceles right triangle has two equal angles, *x*, and one right angle.



The interior angles of a triangle add up to 180° .
x + *x* + 90° = 180°
2*x* = 90°
x = 45°
This is a 45-45-90.
The area is one half the base times the height. In a right triangle, the base and the height are the two legs, *l*, which are equal.

$A = \frac{1}{2}bh = \frac{1}{2}l^2$

We are given the area is 84.5

$84.5 = \frac{1}{2}l^2$

$169 = l^2$

$13 = l$

Pages 189 - 191

1. The interior angles of a triangle add up to 180°. AB and BC make a right angle which measures 90°.
h + 85° + 90° = 180°
h + 175° = 180°
h = 5°; (A).

2. First, find the length of the leg *LM* from the leg *LN* ($\sqrt{28}$), and the hypotenuse *MN* (8).

$$LM^2 + LN^2 = MN^2$$
$$LM^2 + (\sqrt{28})^2 = (8)^2$$
$$LM^2 + 28 = 64$$
$$LM^2 = 36$$
$$LM = 6$$

The area is one half the base times the height. In a right triangle, the base and the height are the two legs.

$A = \frac{1}{2}bh = \frac{1}{2}(6)(\sqrt{28}) = 3\sqrt{28}$; (B).

3. An isosceles triangle has two sides that are equal. Side RS must be equal to either side ST (3) or TR (7). Calculate the perimeter (the sum of the three sides) for both cases and compare with the three choices. The p
RS = ST = 3
P = RS + ST + TR = 3 + 3 + 7 = 13
or
RS = TR = 7
P = RS + ST + TR = 7 + 3 + 7 = 17
I and III are possible perimeters, II is not; (D).

4. The sum of the two short sides must be greater than the longest side. To find the maximum value of RS, assume that RS is the longest side. If we assume this,
RS < RT + ST
RS < 5 + 12
RS < 17
To find the minimum value of RS, assume that RS is not the longest side. The longest side must be ST and the following must be true:
ST < RS + RT
12 < RS + 5
8 < RS
So, 8 < RS < 17; (B).

5. The interior angles of a triangle add up to 180°.
$r^\circ + s^\circ + 60^\circ = 180$
$r^\circ + s^\circ = 120$; (A) must be true.
The side opposite s (NO=6) is larger than the side opposite r (MO=5), therefore angle r is larger than angle s,
$s > r$; (B) must be true.
Start with equation (A) to find an equation for s:
$r^\circ + s^\circ = 120^\circ$
$s^\circ = 120^\circ - r^\circ$
Substitute the r in equation (B) for the above equation.
$s > r$
$120 - r > r$

120 > 2*r*
60 > *r*
r < 60; (C) must be true.
Similarly, solve for *r* in equation (A) and substitute once equation (B) to find the restrictions on *s*.
s > 120 – *s*
2*s* > 120
s > 60; (D) must be false.
Since 60 is greater than *r*° (equation C), the side opposite 60° (MN) must be greater than the side opposite r° (MO).
MN > MO
MN > 5; (E) must be true.

6. Suppose that m∠UVW = 90°. The area is one half the base times the height. In a right triangle, the base (VW) and the height (UV) are the two legs.

$$A = \frac{1}{2}bh = \frac{1}{2}(10)(7) = 35$$

But, because m∠UVW is not 90°, we know that the height of the triangle must be less than 7 and therefore that the area must be less than 35. (A) is the only answer which fits this criteria.

7. This is a 30-60-90 triangle. First, calculate the value of the short leg (*n*) by diving the long side by $\sqrt{3}$.

$$n = \frac{m}{\sqrt{3}} = \frac{3}{\sqrt{3}} = \frac{3\sqrt{3}}{3} = \sqrt{3}$$

Now calculate the hypotenuse (*p*) by multiplying the short side (*n*) by 2.

$$p = 2n = 2\sqrt{3}$$

$n + p = \sqrt{3} + 2\sqrt{3} = 3\sqrt{3}$; (E).

8. The bisected equilateral triangle forms two 30-60-90 triangles.
(A) The perimeter of AXC is sufficient information to solve for the length AB and therefore the perimeter of ABC.
(B) AB = BC = CA. The length of AB is sufficient information to solve for the perimeter of ABC.
(C) The ratio $\frac{AX}{XB}$ is the same for any 30-60-90 triangle. This will not help us find any length or the perimeter of ABC.
(D) The area of ABC can be used to find the lengths of the 30-60-90 triangle and therefore the perimeter of ABC.
(E) The length XC is half of CB which can be used to find the perimeter of ABC.

9. This is a 30-60-90 triangle.
(A) The longest side, *e*, must always be shorter than the sum of the other sides, *f* + *g*. This statement must be true.

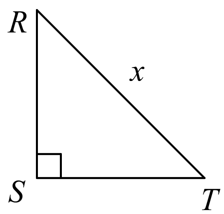
(B) The long leg is the short leg multiplied by $\sqrt{3}$. There is no number *f* such that $f + f\sqrt{3}$ is an integer. This statement must be false.
(C) The short side, *f*, is the long side divided by $\sqrt{3}$ and the hypotenuse is twice the short side.

$$f = \frac{g}{\sqrt{3}}$$

$$e = 2\frac{g}{\sqrt{3}} = \frac{2g\sqrt{3}}{3}$$

If *g* is an integer, *f* and *e* must not be integers. This statement must be true as well as statement (D).

10. Draw a picture.



The internal angles of a triangle add up to 180°. A right angle is 90°.
m∠TRS + m∠RTS + 90° = 180°
2·m∠RTS = 90°
m∠RTS = 45°
m∠TRS = 45°
This is a 45-45-90 triangle. The leg *ST* is the hypotenuse *RT* divided by $\sqrt{2}$.

$$ST = \frac{RT}{\sqrt{2}} = \frac{\sqrt{2}}{2}RT$$

If *RT* = 2, $ST = \sqrt{2}$
If *RT* = 6, $ST = 3\sqrt{2}$
But, no integer value of *RT* could make $ST = 3\sqrt{3}$; (C) I and II.

11. Let RP=*x*. In a 45-45-90 triangle a leg is the hypotenuse, *RP*, divided by $\sqrt{2}$.

$$RQ = QP = \frac{RP}{\sqrt{2}} = \frac{x}{\sqrt{2}} = \frac{\sqrt{2}}{2}x$$

The area of a triangle is one half the base times the height. In a right triangle, the base and the height are the two legs.

$$A = \frac{1}{2}bh = \frac{1}{2}(\frac{\sqrt{2}}{2}x)(\frac{\sqrt{2}}{2}x) = \frac{2}{8}x^2 = \frac{1}{4}x^2$$

We are told that the area of the triangle is numerically equal to *RP*.

$RP = A$

$x = \frac{1}{4}x^2$

$0 = \frac{1}{4}x^2 - x$

$0 = x(\frac{1}{4}x - 1)$

$x = 0 \; and \; x = 4$

Discard x=0; (E).

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1. First, find the length AC by realizing it is a diameter of the circle. Use the formula for circumference and solve for r:

$C = 2\pi r$

$6\pi = 2\pi r$

$3 = r$

$AC = Diameter = 2r = 2(3) = 6$

$P = AC + BC + AB = 6 + 5 + 3 = 14$

2. There are a few different ways to approach this problem. One way is to notice that the four triangles formed by the small square have angles 45-45-90 with a hypotenuse of 10. Divide the hypotenuse by $\sqrt{2}$ to find the length of a leg.

$leg = \frac{10}{\sqrt{2}} = \frac{10\sqrt{2}}{2} = 5\sqrt{2}$

The leg of the triangle is half of a side of the big square. So, each side must measure $10\sqrt{2}$. The area of the big square is a side squared:

$A = (10\sqrt{2})^2 = 100 \cdot 2 = 200$

Another approach is to notice that the area of the big square is the area of the small square plus the area of the four triangles. If rearranged, the four triangles form a square with each side equal to 10 and an area of 100.

$A = A_{small} + A_{triangles} = 100 + 100 = 200$

3. This is a triangle bowtie with an equal vertical angle. The interior angles of a triangle add up to 180°. Apply this to the triangle on the left to find the missing angle that we will call v.

$50° + 80° + v° = 180$

$v° = 50°$

This is the same as the missing angle in the triangle to the right.

$50° + g° + g° = 180$

$2g° = 130$

$g° = 65°$

Page 206

1. The area of the polygon is the sum of the shapes which make it up: Triangle ABC, Rectangle BCDE and Square DEFG.
A = 30 + 60 + 40 = 130

2. Consider the larger triangle which is formed by both the smaller triangles. The interior angles of a triangle add up to 180°. The top angle is in both triangles and has a value which is the sum of each (23° + *x*°).
40° + 60° + (23° + *x*°) = 180
123° + x° = 180
x° = 57°

Page 207

1. The shaded region is the area of the circle minus the area of the triangle. The triangle is a right triangle in which both legs are a radius of the circle. The area of a triangle is one half the base times the height. In a right triangle, the base and the height are the two legs.

$$A = \frac{1}{2}bh = \frac{1}{2}(7)(7) = \frac{1}{2} \cdot 49 = 24.5$$

The area of a circle is πr^2 where the radius is AB = 7.

$$A = \pi r^2 = \pi(7)^2 = 49\pi$$

The area of the shaded region is the area of the circle minus the area of the triangle.
$A = 49\pi - 24.5 \approx 129.4$

Page 209
Check your understanding: Similar Shapes

1. First find the ratio of the lengths by dividing the top of the big rectangle by the top of the small rectangle.

$$ratio = \frac{8}{3}$$

Set up an equal fraction to find the height of the small rectangle.

$$\frac{8}{3} = \frac{4}{u}$$

Cross multiply.

$$8u = 3 \cdot 4$$
$$8u = 12$$
$$u = \frac{12}{8} = 1.5$$

Pages 209 – 210
Sample SAT problems: Multiple Shapes

1 Use the equation for the area of a circle to solve for the radius.

$$A = \pi r^2$$
$$12.25\pi = \pi r^2$$
$$12.25 = r^2$$
$$3.5 = r$$

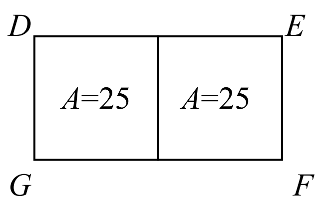
The diameter of the circle is the same as the height of the rectangle, *CB*.

$$BC = D = 2r = 2(3.5) = 7$$

Now calculate the area of the rectangle

$A = l \cdot w = CD \cdot BC = 8 \cdot 7 = 56$; (D).

2. Draw a picture.



Use the equation of the area of a square to find the length of a side, *s*.

$$A = s^2$$
$$25 = s^2$$
$$5 = s$$

Calculate the perimeter.

P = *DE* + *EF* + *FG* + *DG* = 2*s* + *s* + 2*s* + *s* = 6*s* = 6(5) = 30; (E).

3. The area of a triangle is one half the base times the height. In a right triangle, the base and the height are the two legs. Use this to find the length of side *AB*.

$$A = \frac{1}{2}bh$$
$$18 = \frac{1}{2}(6)(AB)$$
$$18 = 3AB$$
$$6 = AB$$

Side *AB* is also a diameter of Circle O. Calculate the circumference.

$C = \pi D = \pi(AB) = 6\pi$; (B).

4. The height of triangle KLN which is perpendicular to KL (not shown) and connects to point N is the same length as the height of the rectangle (LM or KO). Use the equation for the area of a triangle with KL as a base and LM as a height to find a value for the product of *KL* and *LM*. This product is the area of rectangle KLMO.

$A = \frac{1}{2}bh$

$5 = \frac{1}{2}(KL)h$

$5 = \frac{1}{2}(KL)(LM)$

$10 = (KL)(LM)$

The rectangle has an area of 10; (C).

5. Use the areas of the semiclrcles to find their diameters.
For the smaller semicircle:

$A = \frac{1}{2}\pi r^2$

$8\pi = \frac{1}{2}\pi r^2$

$16 = r^2$

$4 = r$

$D = 2r = 2(4) = 8$

For the larger semicircle:

$A = \frac{1}{2}\pi r^2$

$18\pi = \frac{1}{2}\pi r^2$

$36 = r^2$

$6 = r$

$D = 2r = 2(6) = 12$

The area of a triangle is one half the base times the height. In a right triangle, the base and the height are the two legs.

$A = \frac{1}{2}bh = \frac{1}{2}(8)(12) = 48$; (5).

6. Use the equation of the circumference of a circle to calculate the diameter of circle O.

$$C = \pi D$$
$$4\pi = \pi D$$
$$4 = D$$

This diameter is the same length as the sides of square ABFE and of length CD. Next, calculate the area of ABFE by squaring a side.

$$A = s^2 = 4^2 = 16$$

The area of rectangle BCDF is the area of rectangle ACDE minus the area of square ABFE.

$$BCDF = ACDE - ABFE = 60 - 16 = 44$$

The area of rectangle BCDF is also the product of its sides, BC and CD. Recall that CD is the same length as the diameter, 4.

$$BCDF = (BC)(CD)$$
$$44 = (BC)(4)$$
$$11 = BC$$

(A).

Page 213

1. Find the result of the equation when *x* = 4.

$$f(x) = 2x - 3$$
$$f(4) = 2(4) - 3 = 8 - 3 = 5$$
$$f(4) = 5$$

2. Find the result of the equation when *x* = 6.

$$f(x) = \frac{x+2}{4}$$
$$f(6) = \frac{6+2}{4} = \frac{8}{4} = 2$$
$$f(6) = 2$$

Page 214

1. Plug in 2 for *x* and 4 for *y*.

$$f(x, y) = 4x - y$$
$$f(2,4) = 4(2) - 4 = 8 - 4 = 4$$
$$f(2,4) = 4$$

Plug in -2 for *x* and 4 for *y*.

$$f(x, y) = 4x - y$$
$$f(-2,4) = 4(-2) - 4 = -8 - 4 = -12$$
$$f(-2,4) = 12$$

Page 215

2. The problem wants the height 4 seconds after the ball is thrown. This is asking for *h*(4). Find the result of the equation when *t*=4.

$h(t) = 6t - t^2$

$h(4) = 6(4) - (4)^2 = 24 - 16 = 8$

$h(4) = 8$

The height of the ball is 8 feet.

Page 219

1. Start with *f*(*j* + 1). This means plug (*j* + 1) in for *x*.

$f(x) = 3x + 4$

$f(j+1) = 3(j+1) + 4 = 3j + 3 + 4 = 3j + 7$

$f(j+1) = 3j + 7$

Now try *f*(2 - *k*). This means plug (2 - *k*) in for *x*.

$f(x) = 3x + 4$

$f(2-k) = 3(2-k) + 4 = 6 - 3k + 4 = 10 - 3k$

$f(j+1) = 10 - 3k$

2. Start with $f(r^2)$. This means plug r^2 in for *x*.

$f(x) = 5x$

$f(r^2) = 5r^2$

Now try $f(\sqrt{2t+6})$. This means plug $\sqrt{2t+6}$ in for *x*.

$f(x) = 5x$

$f(\sqrt{2t+6}) = 5\sqrt{2t+6}$

Page 220

1. To find 4*f*(m), use *f*(m) as a single variable with a value of 5. Substitute 5 where you see *f*(m).

4*f*(m) = 4(5) = 20

Page 225

1. First, rewrite the weird function as a normal function of *x* and *y*:

f(*x*, *y*) = 2*x* + 3*y*

The function is two times the first number added to three times the second number. Notice that input 1 ♣ 4 is the same as $f(1,4)$ in our new function.
$f(1, 4) = 2(1) + 3(4) = 2 + 12 = 14$
$f(1, 4) = 14$; (D).

2. First, rewrite the weird function as a normal function of x and y.
$$f(x,y) = \frac{|x|}{x-y}$$
The input $3 \, \varepsilon \, t$ is the same as $f(3, t)$ in our new function.
$$f(3,t) = \frac{|3|}{3-t}$$
We know that this must be less than zero.
$$\frac{|3|}{3-t} < 0$$
Since the numerator is positive 3, for this to be true the denominator, $3 - t$, must be less than zero. t must be greater than 3; (A).

3. First, rewrite the weird function as a normal function of x.
$f(x) = x - x^2$
The input $\lceil p$ is the same as $f(p)$ in our new function.
$f(p) = p - p^2$
We know that this input, $f(p)$, is greater than 0.
$f(p) > 0$
$p - p^2 > 0$
Plug in the given answers to see which will make the inequality true.
$f\left(\frac{1}{2}\right) = \frac{1}{2} - \left(\frac{1}{2}\right)^2 = \frac{1}{2} - \frac{1}{4} = \frac{1}{4} > 0$; (B).

4. The question tells you that $(r,s)\boxed{\exists}\,u$ is true if $rs > u^2 > ru$. Find out what $(3,6)\boxed{\exists}\,y$ is:
$$3 \cdot 6 > y^2 > 3y$$
$$18 > y^2 > 3y$$
Plug in the given answers to see which will make the inequality true. Try $y = 4$.
$$18 > (4)^2 > 3(4)$$
$$18 > 16 > 12$$
This works; (D).

5. First, rewrite the weird function as a normal function of x and y.
$$f(x,y) = xy - y$$
The input $r \, \chi \, 4$ is the same as $f(r,4)$ in our new function. We are told that this is equal to zero.

$$f(r,4)=0$$
$$4r-4=0$$
$$4r=4$$
$$r=1$$
This works only if r = 1; (B).

Page 233

1. Line *b* is parallel to line *a*. Therefore its slope is equal to the slope of line *a*.

Slope of line *b* = $\frac{2}{5}$

Line *c* is perpendicular *a*. Therefore its slope is the opposite reciprocal of the slope of line *a*. To find this, multiply the slope of *a* by -1 and flip over the fraction.

Slope of line *c* = -$\frac{5}{2}$

Page 241

1. Compare the white bars (rainfall in 1990). The tallest white bar is City B. City B had the greatest average monthly rainfall in 1990.

2. Compare the white bars (rainfall in 1990) with the shaded bars (rainfall in 2000). We are looking for the city with an increase in rainfall between 1990 and 2000; this city will have a shorter white bar than a shaded bar. City A had an increase in average monthly rainfall from 1990 to 2000.

3. Compare the white bars (rainfall in 1990) with the shaded bars (rainfall in 2000). We are looking for a city with very little change in rainfall between these two times; this city will have a white bar and a shaded bar that is nearly the same length. City D has the least amount of change in average monthly rainfall from 1990 to 2000.