Total number of printed pages–4

44 (4) BCA–HC–4026

2023

MATHEMATICS–II

Paper : BCA–HC–4026

Full Marks : 80

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. (a) For the non-empty sets A, B, C prove that

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ 3

(b) If $f : R \to R$ and $g : R \to R$ define by $f(x) = x^2$ and $g(x) = \sin x$

Examine whether $g \circ f = f \circ g$ 3

(c) If $R = \{(x, y) \mid x - y$ is divisible by 3 $x, y \in I\}$ prove that R is an equivalence relation. 3

Contd.

(d) Prove that

$$1^3+2^3+3^3+\ldots+n^3=\left\{\frac{n(n+1)}{2}\right\}^2. \qquad 3$$

(e) For non-empty sets A and B that

$$n(A \oplus B)=n(A)+n(B)-2n(A \cap B) \qquad 3$$

2. (a) Define complete graph and draw K_5. 1+2=3

(b) Define tree. Show that a tree with n number of vertices have $(n-1)$ number of edges. 1+4=5

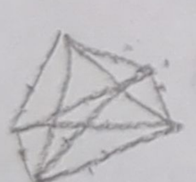
(c) Define cut vertex and cut edge. 2

(d) Draw a bipertite graph with 8 vertices. 2

(e) Prove that in a graph the number of odd vertices is always even. 3

3. (a) Prove that in a square matrix can be expressed uniquely as a sum of symmetric and skew symmetric matrix. 3

(b) For the matrix $A=\begin{bmatrix} 1 & -2 \\ -5 & 4 \end{bmatrix}$ 4



Find the eigenvalue and eigenvectors.

(c) For the matrix $A=\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$; Express $2A^5-3A^4+A^2-4I$ as linear expression. 4

(d) If A be an arbitrary matrix with disting eigenvalues then prove that there exist a smilar transformation such that

$$P^{-1}AP = D$$

where P is the diagonal matrix whose diagonal elements are the eigenvalues of A. 4

4. Answer ***any three*** parts from the following : $4 \times 3 = 12$

(a) Define Tautology, Contradiction, Contingency.

(b) (i) Prove that $(p \wedge q) \rightarrow p$ is a tautology.

(ii) Prove that the following statement are logically equivalent $\sim(p \rightarrow q)$ and $p \wedge \sim q$.

(c) Prove that in a Boolean algebra the complement of an element is unique.

(d) (i) Simplify : $z(y+z)(x+y+z)$

(ii) Write the duals of the following :

(a) $(p \wedge q) \vee r$ (b) $\sim(p \wedge q)$

(e) Translate the following statement (quantifiers) :

(i) All rational are real. $\forall x$: ratio$(x) \rightarrow$ real(x)

(ii) No rational are real.

(iii) Some rational are real. $\exists x$: ration$(x) \rightarrow$ re(x)

(iv) Some rational are not real.

5. *(a)* If $n \geq 0$ then prove that

$$\sum_{k=0}^{n} \binom{n}{k}^2 = \binom{2n}{n}$$ 4

(b) Find the number of sides of a polygone with 27 diagonals. 3

(c) State pigeonhole principle and explain with an example. 2+2=4

(d) Find the number of triangles that can be drawn from the set of 15 points. If p is the common point for all the triangles. 3

6. *(a)* Show that the set w of all elements of vector space $V_3(F)$ of the form $(x, 2y, 3z)$ where $x, y, z \in F$ is a subspace of $V_3(F)$. 3

(b) Examine that the following vector is linearly dependent or independent $\{(1, 0, 0), (0, 1, 0) (0, 0, 1)\}$. 2

(c) Let $V(F)$ be a vector space, prove that

(i) $a(-\alpha) = -(a.\alpha)$

(ii) $a.(\alpha - \beta) = a\alpha - a\beta$, $\alpha, \beta \in V, a \in F$

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