Total number of printed pages–7

44 (Sem–2) M–I (HC–2016) N

2022

MATHEMATICS (I)

Paper : BCA–HC–2016

Full Marks : 80

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following : $1 \times 7 = 7$

   (a) Define non-zero matrix and transpose of a matrix.

   (b) What is symmetric matrix? Give *one* example.

   (c) Define orthogonal matrix.

   (d) When are two matrices conformable for multiplication?

   Or

   Prove that $(A')' = A$, A be any matrix.

Contd.

(e) Find the transpose of $\begin{bmatrix} 5 \\ \frac{1}{2} \\ -1 \end{bmatrix}$.

(f) Find the co-factors of a^3 in $\begin{vmatrix} 1 & a & x \\ 1 & a^2 & x^2 \\ 1 & a^3 & x^3 \end{vmatrix}$.

(g) Find x if $\begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix} = \begin{vmatrix} x & 3 \\ 2x & 5 \end{vmatrix}$.

2. Define idempotent matrix, involutory matrix and nilpotent matrix. Also give example of each. 6

3. Find x, y and z : 5

$$\begin{aligned} x - y + z &= 4 \\ 2x + y - 3z &= 0 \\ x + y + z &= 2 \end{aligned}$$

4. Answer ***any two*** : 5×2=10

(a) If A and B are two invertible matrices, then prove that AB is also invertible and $(AB)^{-1} = B^{-1}A^{-1}$.

(b) If B is an idempotent matrix, then show that $A = I - B$ is also idempotent. Also show that $AB = BA = 0$.

(c) Verify the result

$A(adj\ A) = |A|\ I = (adj\ A)\ A$ for the following matrix :

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$$

(d) If $A = \begin{bmatrix} 3 & -4 \\ 1 & 1 \end{bmatrix}$, then prove that

$A^n = \begin{bmatrix} 1+2n & -4n \\ n & 1-2n \end{bmatrix}$, where n is any +ve integer.

5. Solve by Cramer's rule 6

$$x + y - z = 4$$
$$2x - y + 5z = 12$$
$$3x + 7y - 2z = 17$$

Or

State Cayley-Hamilton theorem. Verify Cayley-Hamilton theorem for the matrix

$\begin{pmatrix} 11 & -4 & -7 \\ 7 & -2 & -5 \\ 10 & -4 & -6 \end{pmatrix}$. Also find its inverse if possible.

6. *(a)* For any three complex numbers z_1, z_2, z_3 prove that 3

$$z_1(z_2+z_3) = z_1z_2 + z_1z_3$$

(b) If $(x+iy)^3 = u+iv$, then show that

$$\frac{u}{x} + \frac{v}{y} = 4(x^2-y^2)$$ 3

(c) Evaluate : $\left[i^{18} + \left(\frac{1}{i}\right)^{25}\right]^3$ 3

Or

Express in polar form $\sqrt{3}+i$.

(d) If z_1 and z_2 are two complex numbers such that $\frac{z_1}{z_2}$ are purely imaginary, prove that

$$|z_1 + z_2|^2 = |z_1|^2 + |z_2|^2$$ 3

Or

For any two complex numbers z_1 and z_2, prove that

$$arg(z_1 z_2) = arg(z_1) + arg(z_2).$$

7. (a) Find $\frac{dy}{dx}$ of the following : ***(any two)*** 3×2=6

(i) $y = tan^{-1}\left(\frac{sin\,x}{1 + cos\,x}\right)$

(ii) $x^{2/3} + y^{2/3} = a^{2/3}$

(iii) $y = cos\,x^3 \cdot sin^2(x^5)$

(b) Evaluate : 2

$$\lim_{x \to 0} \frac{(x+1)^5 - 1}{x}$$

(c) A function $f(x)$ is defined as follows

$$f(x)=\begin{cases}\frac{1}{2}-x, & 0<x<\frac{1}{2}\\ \frac{1}{2}, & x=\frac{1}{2}\\ \frac{3}{2}-x, & \frac{1}{2}<x<1\end{cases}$$

show that $f(x)$ is discontinuous at $x=\frac{1}{2}$. 3

(d) Find the derivative of : 2×2=4

(i) $y=\dfrac{\sin(ax+b)}{\cos(cx+d)}$

(ii) $y=\cos\left(5x^4+3x^3+2\right)$

8. Answer ***any one*** : 4

(a) Find the maximum and minimum values of $f(x)=x^5-5x^4+5x^3+12$.

(b) Verify Rolle's theorem for : $f(x)=x^3-6x^2+11x-6$

9. State Rolle's theorem and give its geometrical representation. 5

10. Evaluate :

(a) $\lim_{x \to \pi/2} (1 - \sin x) \tan x$ 2

(b) $\lim_{x \to 0} \frac{\tan x - x}{x - \sin x}$ 3

Or

If $\lim_{x \to 0} \frac{\sin 2x + a \sin x}{x^3}$ be finite, find the values of a and the limit. 5

11. Using definition of derivative, find the derivative of $f(x) = x^3$. 3

12. Find the intervals in which the function f is given by $f(x) = x^2 - 4x + 6$ is 2

(a) increasing

(b) decreasing
