$$f(x) = sin(x^2), \quad -1 \le x \le 1$$

Midpoint Rule:

n = 10:

$$\Delta x = \frac{b - a}{n}$$
$$= \frac{1 - (-1)}{10}$$
$$= 0.2$$

$$\int_{-1}^{1} f(x)\, dx \approx M_n = \Delta x \left[f(\overline{x}_1) + f(\overline{x}_2) + \dots + f(\overline{x}_{10}) \right]$$
$$= 0.2 * 3.084391$$
$$= 0.6168782$$

n = 20:

$$\Delta x = \frac{b - a}{n}$$
$$= \frac{1 - (-1)}{20}$$
$$= 0.1$$

$$\int_{-1}^{1} f(x)\, dx \approx M_n = \Delta x \left[f(\overline{x}_1) + f(\overline{x}_2) + \dots + f(\overline{x}_{20}) \right]$$
$$= 0.1 * 6.196326$$
$$= 0.6196326$$

n = 40:

$$\Delta x = \frac{b - a}{n}$$
$$= \frac{1 - (-1)}{40}$$
$$= 0.05$$

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$$\int_{-1}^{1} f(x)\,dx \approx M_n = \Delta x\left[f(\overline{x}_1) + f(\overline{x}_2) + \ldots + f(\overline{x}_{40})\right]$$
$$= 0.05 * 12.406225$$
$$= 0.62031125$$

Trapezoidal Rule:

n = 10:

$$\Delta x = \frac{b-a}{n}$$
$$= \frac{1-(-1)}{10}$$
$$= 0.2$$

$$\int_{-1}^{1} f(x)\,dx \approx T_{10} = \frac{\Delta x}{2}\left[f(x_0) + 2f(x_1) + 2f(x_2) + \ldots + 2f(x_{n-1}) + f(x_{10})\right]$$
$$= \frac{0.2}{2} * 17.670891$$
$$= 1.7670891$$

n = 20:

$$\Delta x = \frac{b-a}{n}$$
$$= \frac{1-(-1)}{20}$$
$$= 0.1$$

$$\int_{-1}^{1} f(x)\,dx \approx T_{20} = \frac{\Delta x}{2}\left[f(x_0) + 2f(x_1) + 2f(x_2) + \ldots + 2f(x_{n-1}) + f(x_{20})\right]$$
$$= \frac{0.1}{2} * 33.658839$$
$$= 1.68294195$$

n = 40:

$$\Delta x = \frac{b-a}{n}$$
$$= \frac{1-(-1)}{40}$$
$$= 0.05$$

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$$\int_{-1}^{1} f(x)\,dx \approx T_{40} = \frac{\Delta x}{2} \left[f(x_0) + 2f(x_1) + 2f(x_2) + ... + 2f(x_{n-1}) + f(x_{40}) \right]$$

$$= 0.025 * 67.317679$$

$$= 1.682941975$$

Simpson's Rule:

n = 10:

$$\Delta x = \frac{b-a}{n}$$

$$= \frac{1-(-1)}{10}$$

$$= 0.2$$

$$\int_{-1}^{1} f(x)\,dx \approx S_{10} = \frac{\Delta x}{3} \left[f(x_0) + 4f(x_1) + 2f(x_2) + ... + 2f(x_8) + 4f(x_9) + f(x_{10}) \right]$$

$$= \frac{0.2}{3} * 15.696044$$

$$= 1.04640293$$

n = 20:

$$\Delta x = \frac{b-a}{n}$$

$$= \frac{1-(-1)}{20}$$

$$= 0.1$$

$$\int_{-1}^{1} f(x)\,dx \approx S_{10} = \frac{\Delta x}{3} \left[f(x_0) + 4f(x_1) + 2f(x_2) + ... + 2f(x_8) + 4f(x_9) + f(x_{20}) \right]$$

$$= \frac{0.1}{3} * 28.150280$$

$$= 0.93834267$$

n = 40:

$$\Delta x = \frac{b-a}{n}$$

$$= \frac{1-(-1)}{40}$$

$$= 0.05$$

$$\int_{-1}^{1} f(x)\,dx \approx S_{10} = \frac{\Delta x}{3}\left[f(x_0) + 4f(x_1) + 2f(x_2) + ... + 2f(x_8) + 4f(x_9) + f(x_{40})\right]$$

$$= \frac{0.05}{3} * 52.990672$$

$$= 0.883177867$$

Error Bounds:

$$|f''(x)| = |(f'(x))'|$$
$$= |(2x\,cos(x^2))'|$$
$$= |-4x^2\,sin(x^2) + 2\,cos(x^2)|$$
$$= 4x^2|sin(x^2)| + 2|cos(x^2)|$$

Because $-1 \leq x \leq 1$, then:

$$|f''(x)| \leq 4\,(1)^2\,(1) + 2\,(1)$$
$$\leq 6$$

We have $K = 6$, then for Midpoint Rule:

$$|E_M| \leq \frac{K(b-a)^3}{24n^2}$$
$$\leq \frac{6\,(2)^3}{24n^2}$$
$$\leq \frac{2}{n^2}$$

n = 10:

$$|E_M| \leq 0.02$$

n = 20:

$$|E_M| \leq 0.005$$

n = 40:

$$|E_M| \leq 0.000625$$

For Trapezoidal Rule:

$$|E_T| \leq \frac{K(b-a)^3}{12n^2}$$
$$\leq \frac{6\,(2)^3}{12n^2}$$
$$\leq \frac{4}{n^2}$$

n = 10:

$$|E_T| \leq 0.04$$

n = 20:

$$|E_T| \leq 0.01$$

n = 40:

$$|E_T| \leq 0.0025$$

For Simpson's Rule:

$$|f^{(4)}(x)| = |-12sin(x^2) - 48x^2cos(x^2) + 16x^4sin(x^2)|$$
$$= 12|sin(x^2)| + 48x^2|cos(x^2)| + 16x^4|sin(x^2)|$$

Because $-1 \leq x \leq 1$, then:

$$|f^{(4)}(x)| \leq 12\,(1) + 48\,(1) + 16\,(1)$$
$$\leq 76$$

We have $K = 76$, then

$$|E_S| \leq \frac{K(b-a)^5}{180\,n^4}$$
$$|E_S| \leq \frac{76(1-(-1))^5}{180\,n^4}$$
$$\leq \frac{2432}{180n^4}$$

n = 10:

$$|E_T| \leq 0.001351111$$

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n = 20:

$$|E_T| \leq 0.000084444$$

n = 40:

$$|E_T| \leq 0.000005277$$