WIRELESS COMMUNICATION SYSTEMS-LECTURE 3

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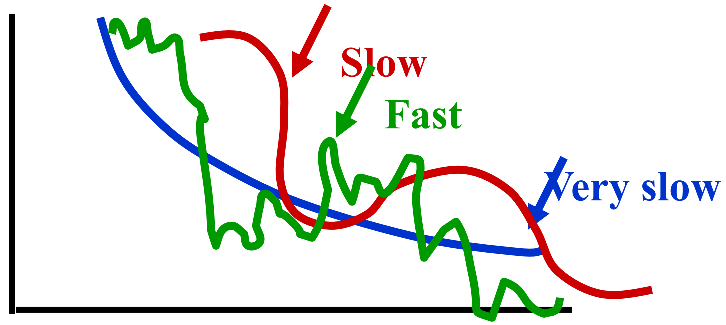
REVIEW OF LAST LECTURE

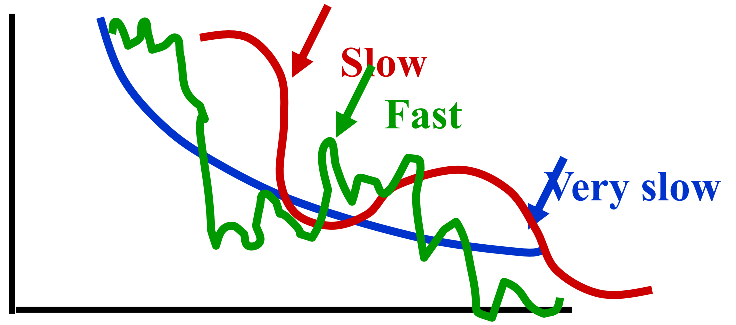
Introduction to mathematical modeling of channel

- Eq. low pass model
- Large-scale fading:
  - Free-space prop., empirical models, ray-tracing
  - Shadow fading, outage probability



GOAL OF THIS LECTURE

- Mathematical Modeling of a Stochastic Channel
- Time-varying Channel Impulse Response
- **Narrowband** Fading Models



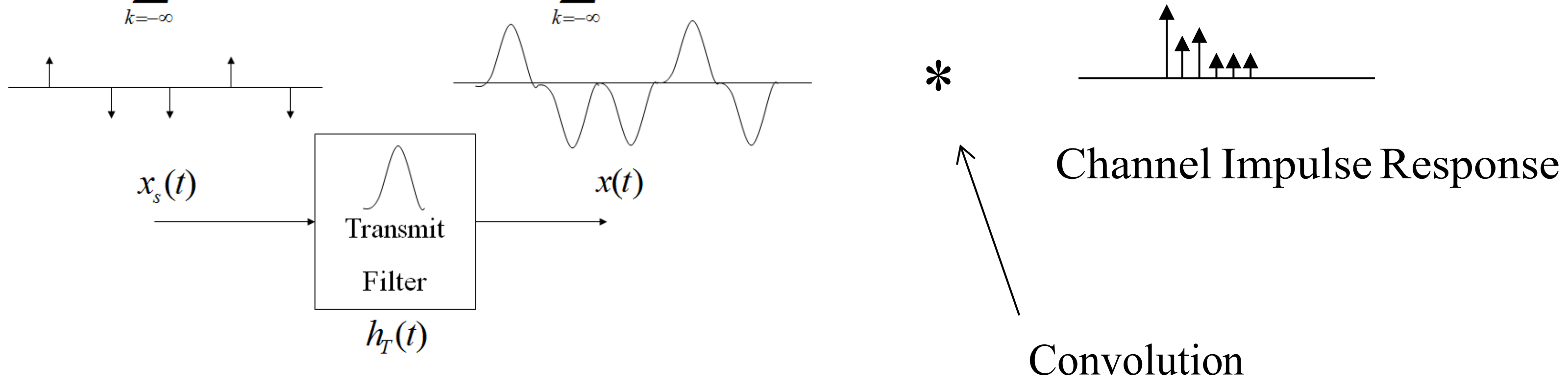
GOAL OF THIS LECTURE - DETAILED

- Mathematical Modeling of a Stochastic Channel
- Time-varying Channel Impulse Response
- **Narrowband** Fading Models
  - In-Phase and Quad Signal Components
  - Auto and Cross-correlation of received signal
  - Correlation and Power Spectral Density in uniform scattering
  - Signal Envelope Distribution
  - Finite-state Markov Models for Fading Channels

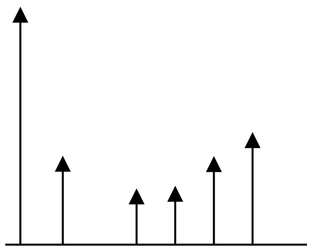
SETTING TODAY'S SCENE

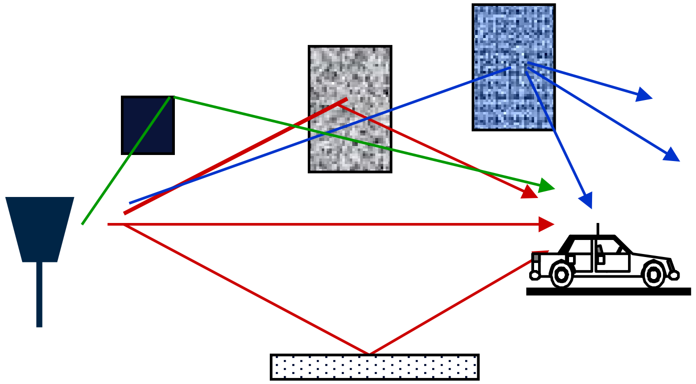
- Today we will develop a statistical model that can describe the fading of constructive and destructive additions of multi-path signal components.
- Last lecture, deterministic approach – with ray tracing, however rarely available in real life.

$$x_s(t) = \sum_{k=-\infty}^{\infty} a_k \delta(t - kT) \qquad x(t) = \sum_{k=-\infty}^{\infty} a_k h_T(t - kT)$$



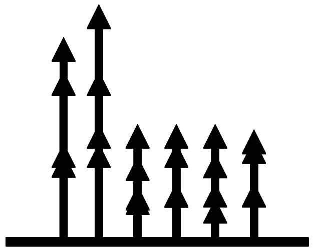
STATISTICAL MULTIPATH MODEL





- Random # of multipath components, each with
  - Random amplitude
  - Random phase
  - Random Doppler shift
  - Random delay
- Random components change with time
- Leads to time-varying channel impulse response

TIME-VARYING IMPULSE RESPONSE



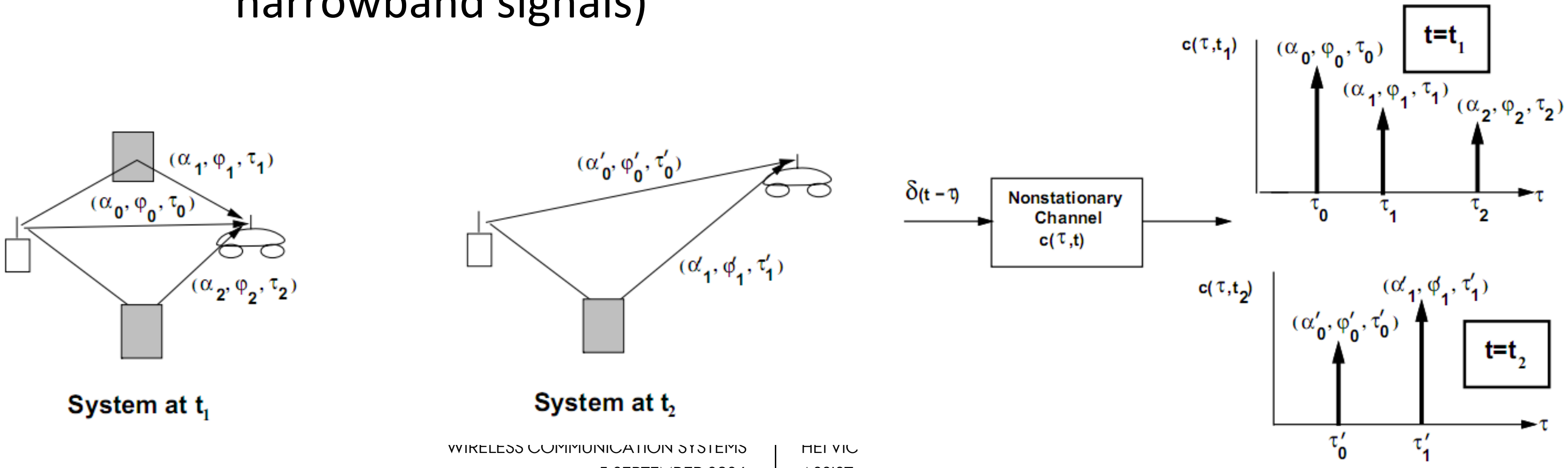
- The equivalent-low-pass-channel-response at time t to impulse at t-τ:

$$c(\tau,t)=\sum_{n=1}^{N}\alpha_n(t)e^{-j\varphi_n(t)}\delta(\tau-\tau_n(t)) \qquad (3.6)$$

  - N is the number of multi-path components
  - t is time when impulse response is observed
  - t- τ is time when impulse put into the channel
  - τ is how long ago impulse was put into the channel for the current observation (path-length divided by c)
    - Path-delay for multipath component currently observed

RECEIVED SIGNAL CHARACTERISTICS

- Received signal consists of many multipath components
- Amplitudes change slowly
- Phases change rapidly
  - Constructive and destructive addition of signal components
  - Amplitude fading of received signal (both wideband and narrowband signals)

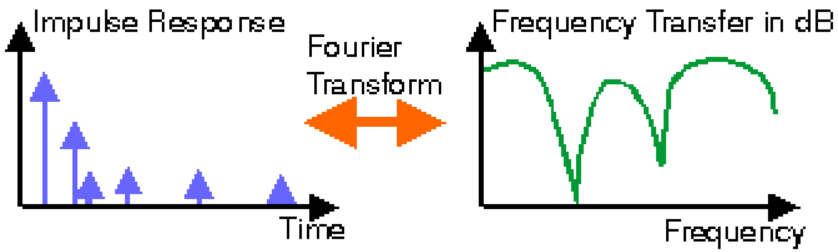


TIME-VARYING IMPULSE RESPONSE - EXAMPLE 3.1

Example 3.1: Consider a wireless LAN operating in a factory near a conveyor belt. The transmitter and receiver have a LOS path between them with gain α_0, phase ϕ_0 and delay τ_0. Every T_0 seconds a metal item comes down the conveyor belt, creating an additional reflected signal path in addition to the LOS path with gain α_1, phase ϕ_1 and delay τ_1. Find the time-varying impulse response $c(\tau, t)$ of this channel.

DELAY SPREAD – SOME DEFINITIONS

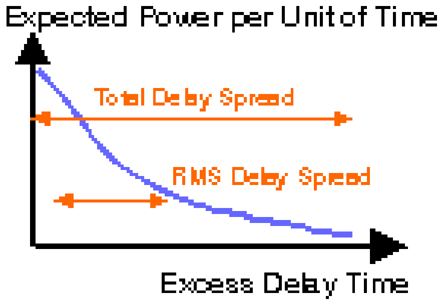
- In system evaluations, we typically prefer to address a class of channels with properties that are likely to be encountered, rather than one specific impulse response. Therefore we define the (local-mean) average power which is received with an excess delay that falls within the interval (T, T + dt). Such characterization for all T gives the "delay profile" of the channel.



- The delay profile determines the frequency dispersion, that is, the extent to which the channel fading at two different frequencies f1 and f2 is correlated.

DEFINITIONS:

- The maximum delay time spread is the total time interval during which reflections with significant energy arrive.
- The r.m.s. delay spread (T_{RMS}) is the **standard deviation value of the delay of reflections, weighted proportional to the energy** in the reflected waves.



- For a digital signal with high bit rate, this dispersion is experienced as frequency selective fading and inter-symbol interference (ISI). No serious ISI is likely to occur if the symbol duration is longer than, say, **ten times the r.m.s. delay spread**.

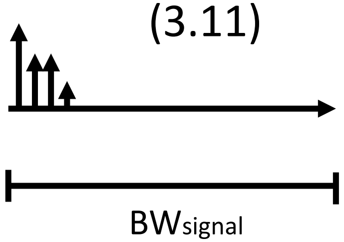
NARROWBAND MODEL

- Assume max$_{m,n}$ delay spread $|\tau_n(t) - \tau_m(t)| \ll 1/BW_{signal}$

….then the equivalent lowpass signal $u(t) \approx u(t-\tau)$

- The received signal is given by

$$r(t) = \Re\left\{ u(t) e^{j2\pi f_c t} \left[\sum_{n=0}^{N(t)} \alpha_n(t) e^{-j\phi_n(t)} \right] \right\} \quad (3.11)$$



- No signal distortion (spreading in time)
- Multipath affects complex scale factor in brackets
- Characterize scale factor by setting $u(t) = e^{j\phi_0}$

IN-PHASE AND QUADRATURE UNDER CLT* APPROXIMATION

- We can re-write/approximate (3.11) through in-phase and quadrature signal components to (for Non-LOS):

$$r_I(t) = \sum_{n=0}^{N(t)} \alpha_n(t) e^{-j\phi_n(t)} \cos(2\pi f_c t),$$

$$r_Q(t) = \sum_{n=0}^{N(t)} \alpha_n(t) e^{-j\phi_n(t)} \sin(2\pi f_c t)$$

- We know from CLT (sum of large # of random variables) that for a large *N(t)*, $r_I(t)$ and $r_Q(t)$ jointly Gaussian distributed. That means:
  - Received signal characterized by its mean, autocorrelation, and cross correlation.
  - If $\phi_n(t)$ uniform, the in-phase/quad components are mean zero, independent and stationary.

* Central limit theorem (CLT), see Appendix B, page 582

AUTO- AND CROSS-CORRELATION OF RECEIVED SIGNAL

- Assume $\phi_n \sim U[0,2\pi]$ and re-call that Θ_n is the multipath angle of arrival
- Auto-correlation (A) of the received in-phase and quadrature components are

$$A_{r_I}(t,t+\tau) = A_{r_Q}(\tau) = PE_{\theta_n}[\cos 2\pi f_{D_n}\tau], \quad f_{D_n} = v\cos\theta_n/\lambda \tag{3.21}$$

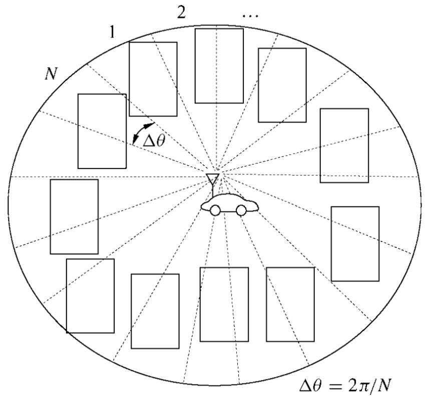
P = normalized received power ($E[\alpha_n^2]$), Doppler frequency assumed const.

- In same way cross-correlation of the received in-phase and quadrature signal can be described as

$$A_{r_I,r_Q}(\tau) = PE_{\theta_n}[\sin 2\pi f_{D_n}\tau] = -A_{r_I,r_Q}(\tau) \tag{3.22}$$

- Autocorrelation of received signal is

$$A_r(\tau) = A_{r_I}(\tau)\cos(2\pi f_c\tau) - A_{r_I,r_Q}(\tau)\sin(2\pi f_c\tau) \tag{3.23}$$



FURTHER SIMPLIFICATIONS OF (3.21) AND (3.22)

We assume uniform angle-of-arrivals (Figure 3.4):

- Under uniform scattering, in-phase and quadrature components have no cross-correlation and the autocorrelation is :

$$A_{r_I}(\tau) = A_{r_Q}(\tau) = PJ_0(2\pi f_D \tau) \qquad (3.26)$$

$J_0(x)$ = Bessel function of

zero'th order

With these assumptions $J_0(x)$ de-correlates over

roughly half (~0.4) a wavelength

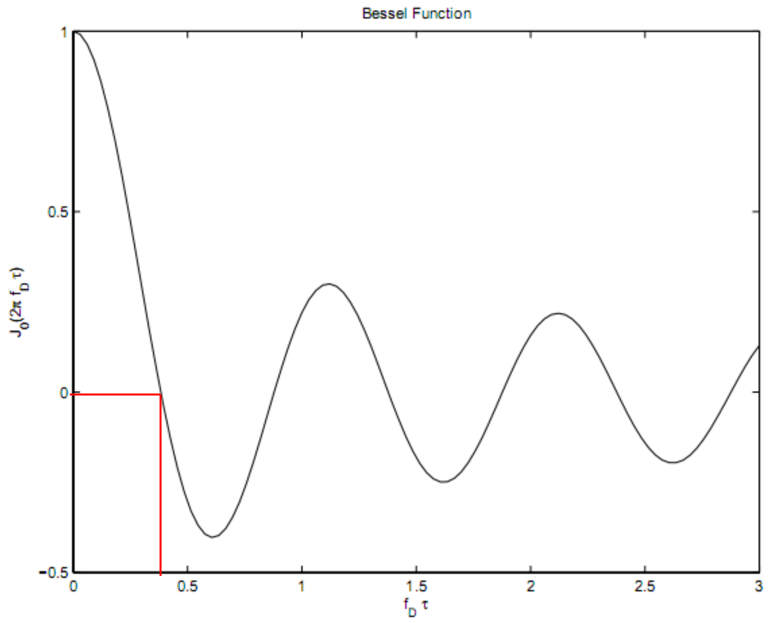
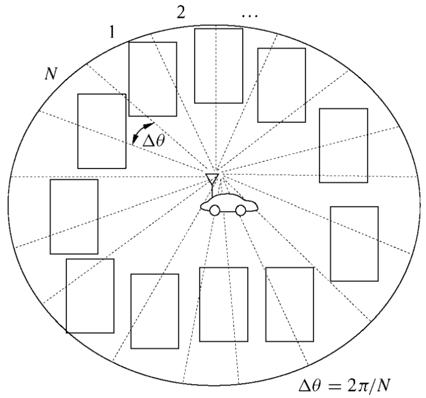


Figure 3.5: Bessel Function versus $f_d\tau$



FURTHER SIMPLIFICATIONS OF (3.21) AND (3.22)

We assume uniform Angle-of-arrivals (Figure 3.4):

- The Power Spectral Density of the received signal S(f) are from the autocorrelation of their respective Fourier transform :

$$S_{r_I}(f) = S_{r_Q}(f) = \mathcal{F}[PJ_0(2\pi f_D \tau)] \quad (3.28)$$

$$S_r(f) = .25[S_{r_I}(f - f_c) + S_{r_I}(f + f_c)] \quad (3.29)$$

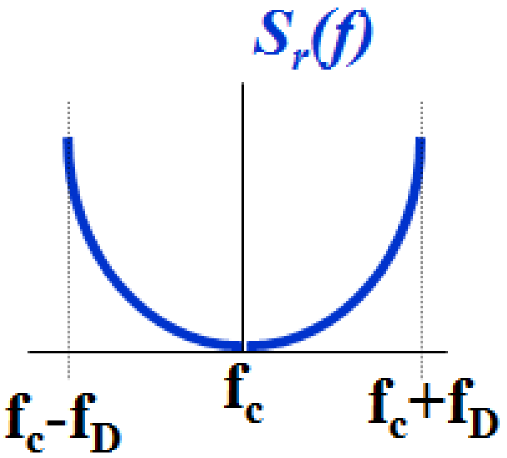
The PSD is useful to generate simulation

values for the fading process. Often using

2 independent White Gaussian Noise

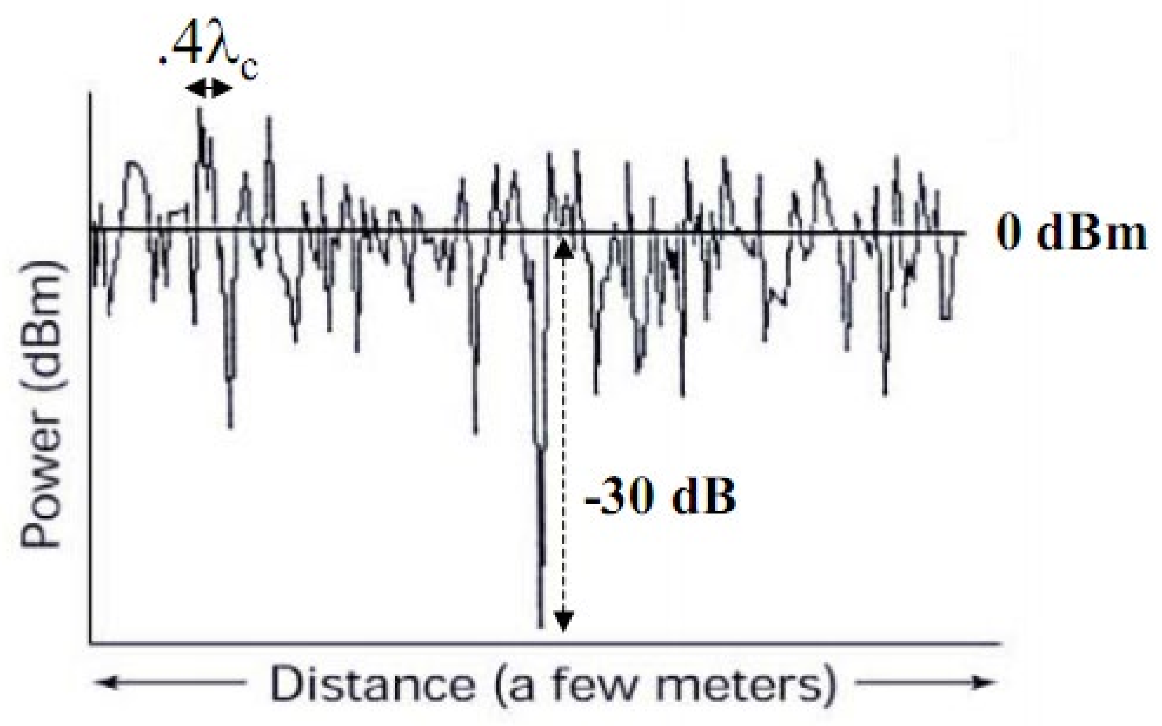
signals with PSD $N_0/2$ Low-pas filter

- e.g. MATLAB.



SIMULATION MODEL : POWER VS. DISTANCE FOR SIGNALS EXHIBITED IN NARROWBAND WIRELESS CHANNELS

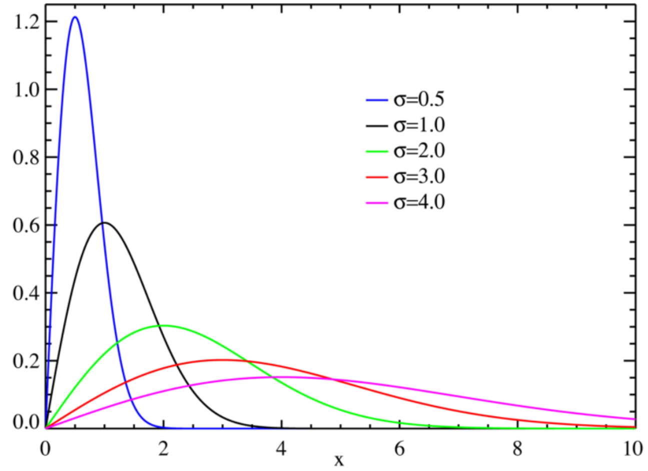
Combined path-loss, shadowing and narrowband fading:



SIGNAL ENVELOPE (AMPLITUDE) DISTRIBUTION – NON-LOS

- For any 2 Gaussian random variables (X) and (Y) both with zero mean and equal variance (σ^2) it can be shown that $Z = \sqrt{X^2 + Y^2}$ is Rayleigh distributed and that Z^2 is exponentially distributed

$$f(z;\sigma) = \frac{z}{\sigma^2} e^{-x^2/2\sigma^2} \quad (3.32)$$



- In other words, the received signal has a Rayleigh-distributed amplitude and a uniform phase.

SIGNAL ENVELOPE (AMPLITUDE) DISTRIBUTION – NON-LOS

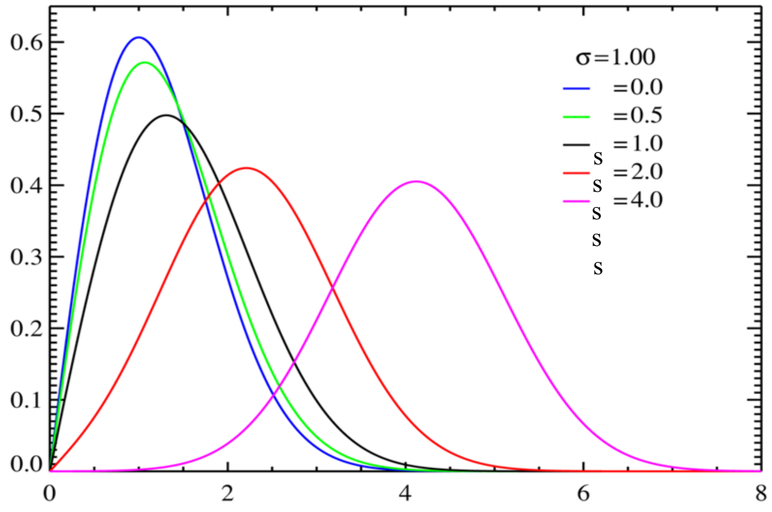
Example 3.2: Consider a channel with Rayleigh fading and average received power $P_r = 20$ dBm. Find the probability that the received power is below 10 dBm.

SIGNAL ENVELOPE DISTRIBUTION - LOS

- When LOS component present received signal cannot be assumed to be zero-mean variables !
- Therefore a Ricean distribution is used to model the received signal envelope

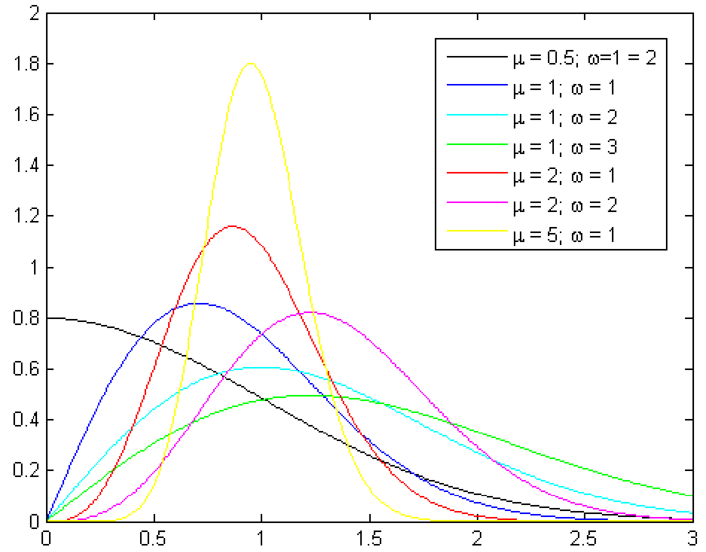
$$p_Z(\mathrm{z}) = \frac{z}{\sigma^2} \exp\left[\frac{-(z^2+s^2)}{2\sigma^2}\right] I_0\left(\frac{zs}{\sigma^2}\right) \qquad (3.34)$$

- I_0 is modified Bessel function.
- s^2 is the power of LOS component (if s = 0 , then equal to Rayleigh dist.).

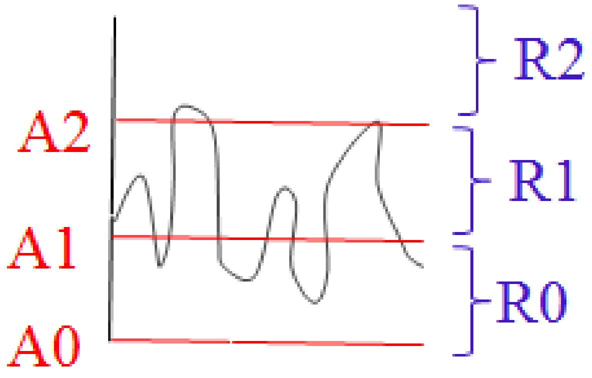


SIGNAL ENVELOPE DISTRIBUTION – REAL-LIFE, WHEN RAYLEIGH /RICEAN DISTRIBUTION IS NOT USABLE

- Some experimental data does not fit to either of the two distributions.
- A more general distribution has been developed : Nakagami (3.38)
  - Similar to Ricean, but can model “worse than Rayleigh



FINITE-STATE MARKOV CHANNEL MODELS FOR FADING



- If simpler models necessary,
  state-machines can be nice to model fading dynamics
  - Simplifies performance analysis

- We divides range of fading power into discrete regions

$$R_j = \{\Upsilon\colon\ A_j \leq \Upsilon\ <\ A_{j\ +\ 1}\}$$

  - $A_j\,, s\ \ and\ \ '\#\ of\ regions'$ are functions of the model that allows us to describe the transition probability from one state to the other, see (3.48).

MAIN POINTS

- Statistical multipath model leads to a time-varying channel impulse response
- Narrowband model has in-phase and quadrature components that are zero-mean stationary Gaussian processes
  - Auto and cross correlation depends on angle-of-arrival of multipath
- Uniform scattering makes autocorrelation of in-phase and quadrature follow Bessel function
  - Signal components de-correlate over half wavelength
  - Cross correlation is zero (in-phase/quadrature independent)
- PSD of received signal has bowl shape centered at carrier frequency: useful for simulations
- Fading distribution depends on environment; Rayleigh, Ricean & Nakagami all common
- Markov model approximates fading dynamics.

NEXT LECUTE

- Wideband Fading Models
- Discrete-Time Model
- Space-Time Channel Models

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