

# ROBOT LOCALISATION



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UNIVERSITY

DEPARTMENT OF ELECTRICAL AND COMPUTER  
ENGINEERING

AUTONOMOUS MOBILE ROBOTICS  
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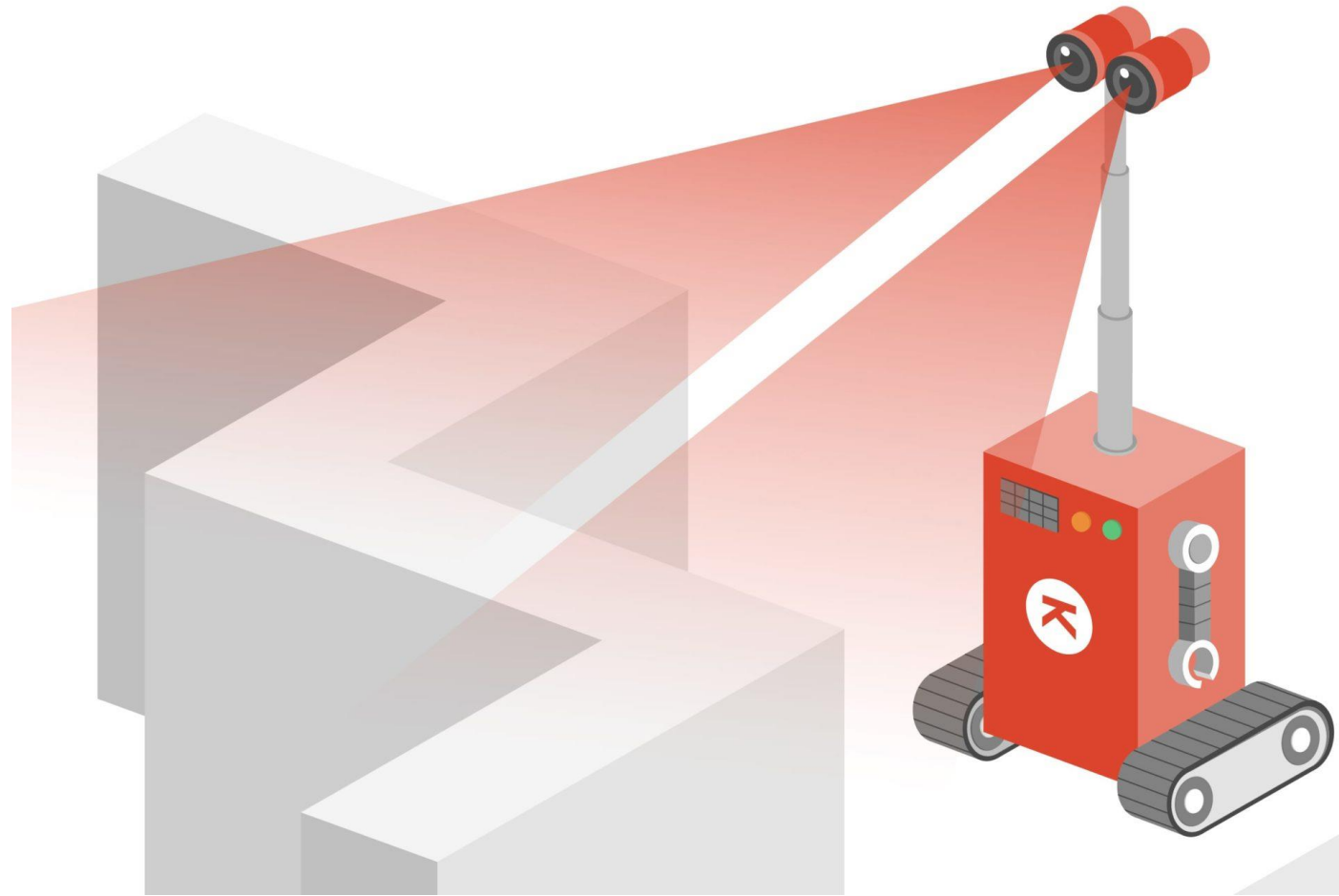
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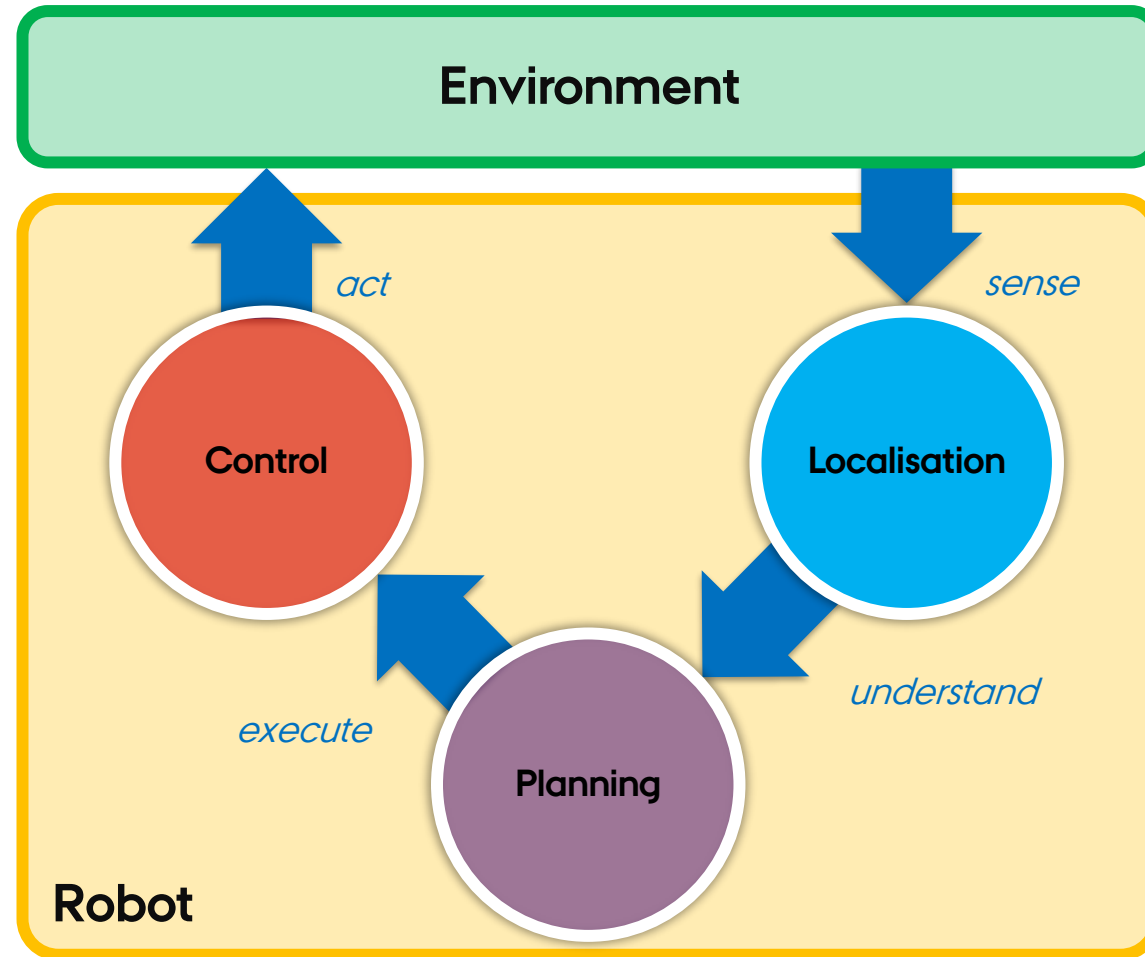
# ROBOT LOCALISATION

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- **Robot localization** is the process of determining where a mobile robot is located with respect to its environment.

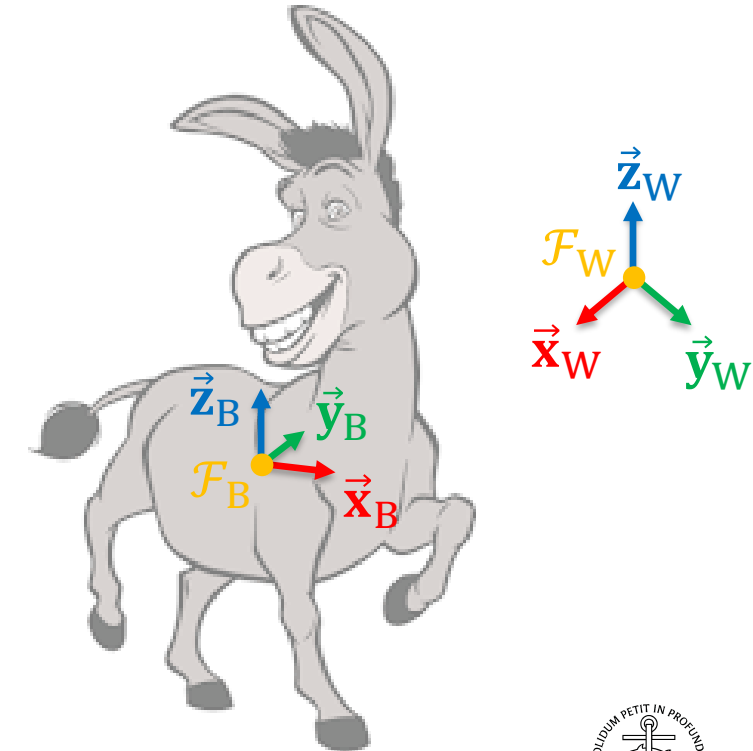
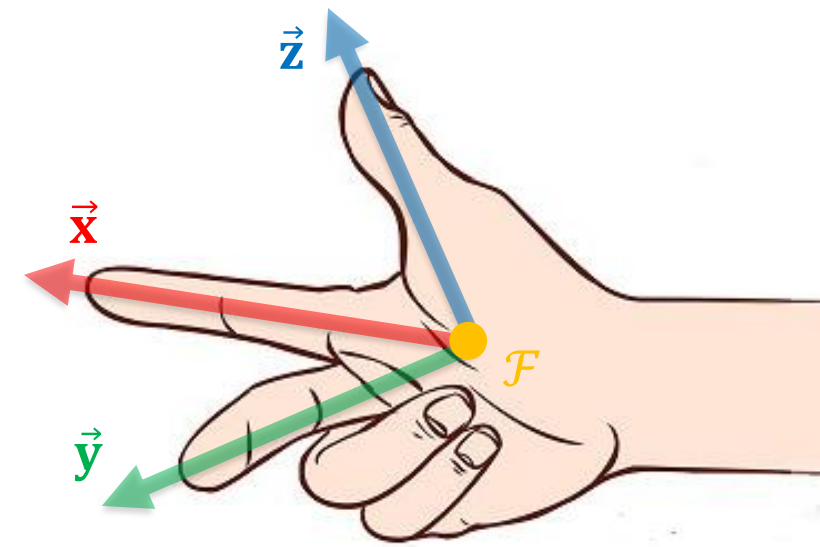


# NAVIGATION PARADIGM



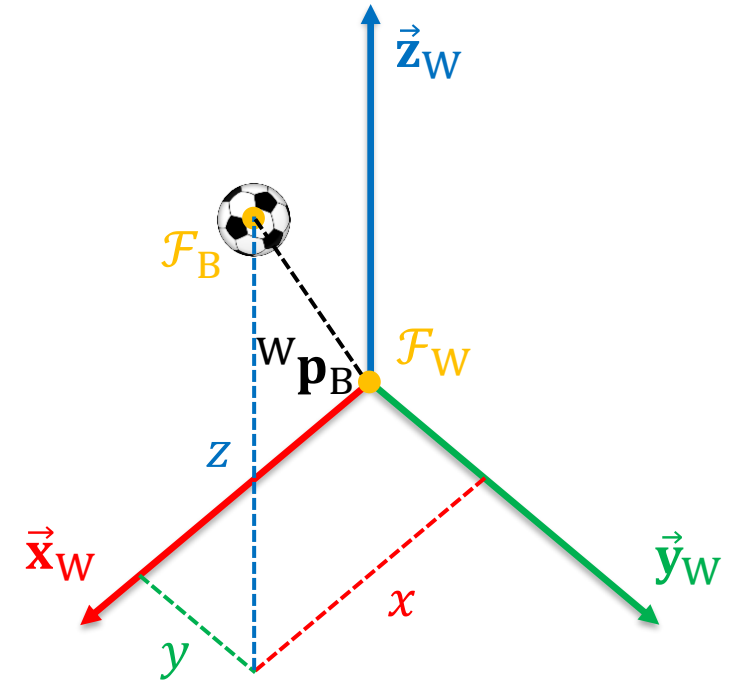
# REFERENCE FRAME

- **Reference frame** is a coordinate system.
- Reference frames are used to describe state of the objects, like *position*, *orientation* and *velocity*, in that frame.
- Each frame  $\mathcal{F}$  is defined by its origin and three orthogonal vectors  $\vec{x}$ ,  $\vec{y}$  and  $\vec{z}$ .
- A fixed global frame is called world frame  $\mathcal{F}_W$ .
- A moving reference frame is called body frame  $\mathcal{F}_B$ .
- We assume that frames follow the right-hand rule.



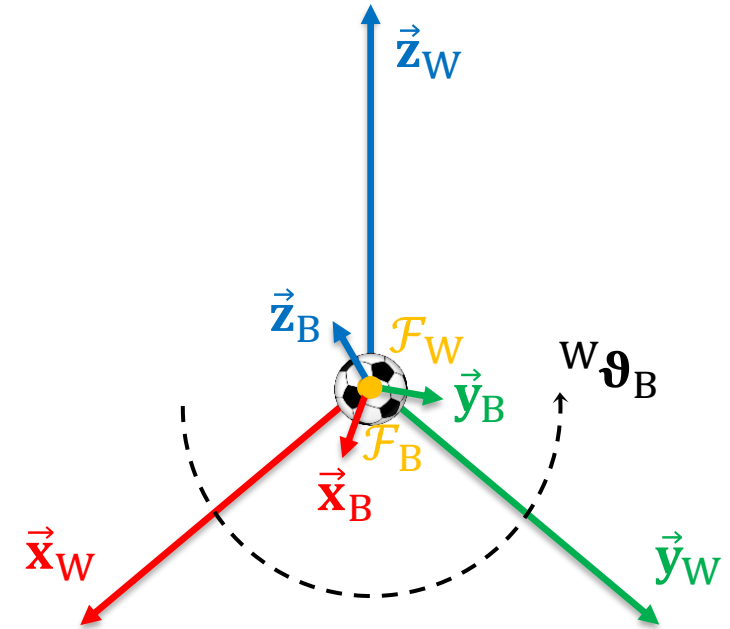
# ROBOT POSITION

- **Position** is the translational offset of an object w.r.t a *reference frame*.
- Position is defined by three Cartesian coordinates  $x$ ,  $y$  and  $z$ .
- Positions are represented by vectors  $\mathbf{p} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \in \mathbb{R}^3$ .
- Position of a frame  $\mathcal{F}_B$  with respect to a frame  $\mathcal{F}_A$  is indicated by  ${}^A\mathbf{p}_B$ .
- We assume that positions are expressed in the world reference frame.
- Position is measured in *meters* [m].



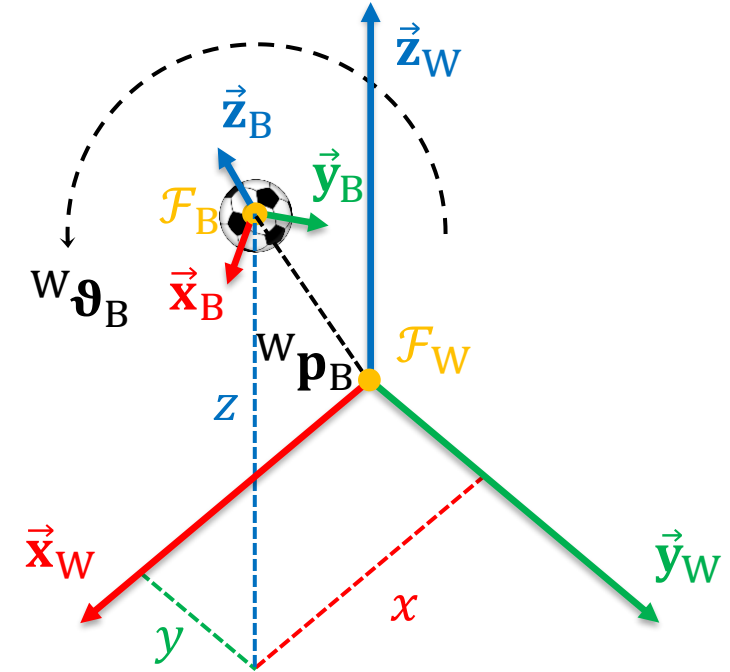
# ROBOT ORIENTATION

- **Orientation** is the directional displacement of an object w.r.t. a *reference frame*.
- Orientation can be parametrized by three Euler's angles: roll  $\phi$ , pitch  $\theta$  and yaw  $\psi$ .
- Orientation of a frame  $\mathcal{F}_B$  with respect to a frame  $\mathcal{F}_A$  is indicated by  ${}^A\boldsymbol{\vartheta}_B$ .
- Orientation can be described by a rotation matrix  ${}^A\mathbf{R}_B$ .
- We assume that orientations are expressed in the world reference frame.
- Orientation is measured in *radians* [rad].



# ROBOT POSE

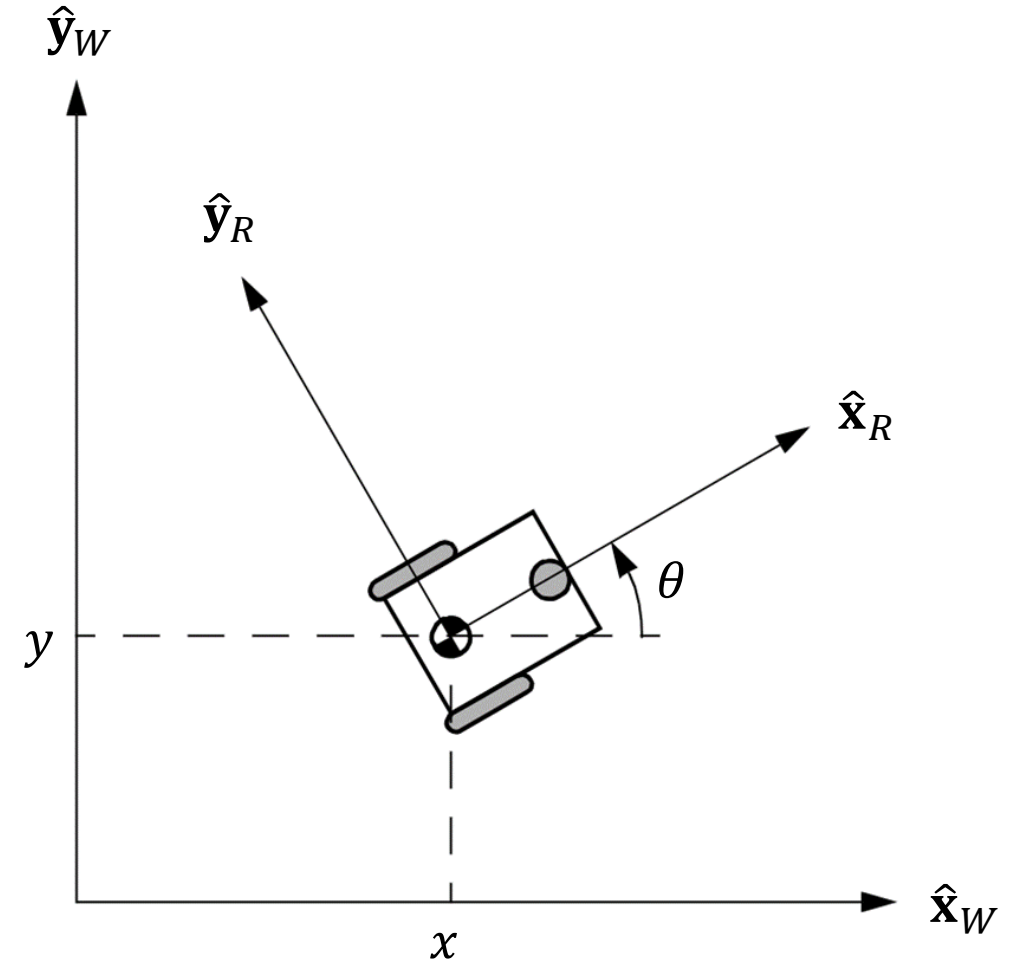
- **Pose** is the combination of *position* and *orientation*.
- Pose fully describes the configuration of a body in the space.
- Pose can be interpreted as a homogeneous transformation matrix  ${}^A\mathbf{T}_B = \begin{bmatrix} {}^A\mathbf{R}_B & {}^A\mathbf{p}_B \\ \mathbf{0} & 1 \end{bmatrix} \in \mathbb{R}^{4 \times 4}$ .
- To apply a transformation  ${}^A\mathbf{T}_B$  to a point  ${}^A\mathbf{p}$ :  ${}^B\mathbf{p} = {}^A\mathbf{T}_B {}^A\mathbf{p}$ .



# ROBOT STATE

- State of a wheeled robot is

$$\begin{bmatrix} x \\ y \\ \theta \end{bmatrix}$$





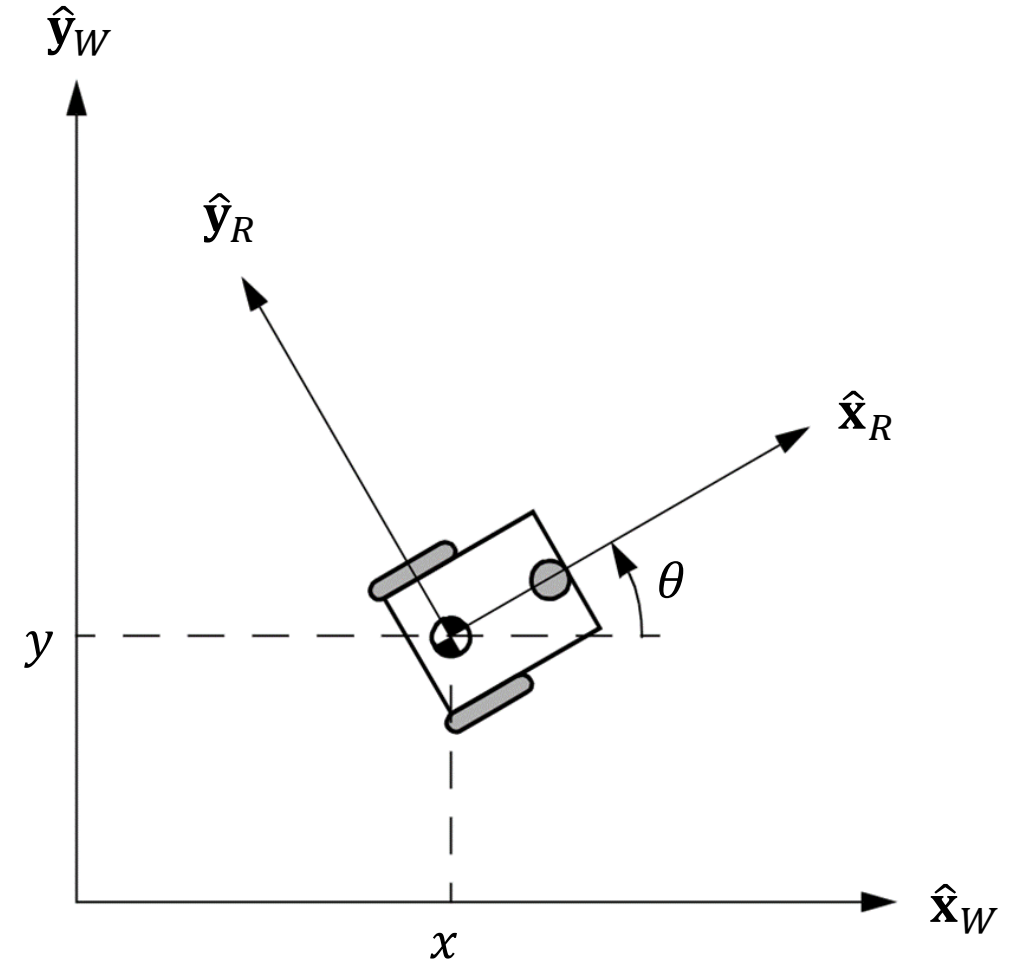
# ROBOT KINEMATICS

- **Kinematics** is the description of the rules of bodies motion.
- Kinematics of a wheeled robot is

$$\begin{cases} \dot{x} = v \cos \theta \\ \dot{y} = v \sin \theta \\ \dot{\theta} = \omega \end{cases}$$

- With discrete time  $\Delta t[k] = t[k] - t[k - 1]$ :

$$\begin{cases} x[k + 1] = x[k] + v[k] \cdot \cos \theta[k] \cdot \Delta t[k] \\ y[k + 1] = y[k] + v[k] \cdot \sin \theta[k] \cdot \Delta t[k] \\ \theta[k + 1] = \theta[k] + \omega[k] \cdot \Delta t[k] \end{cases}$$

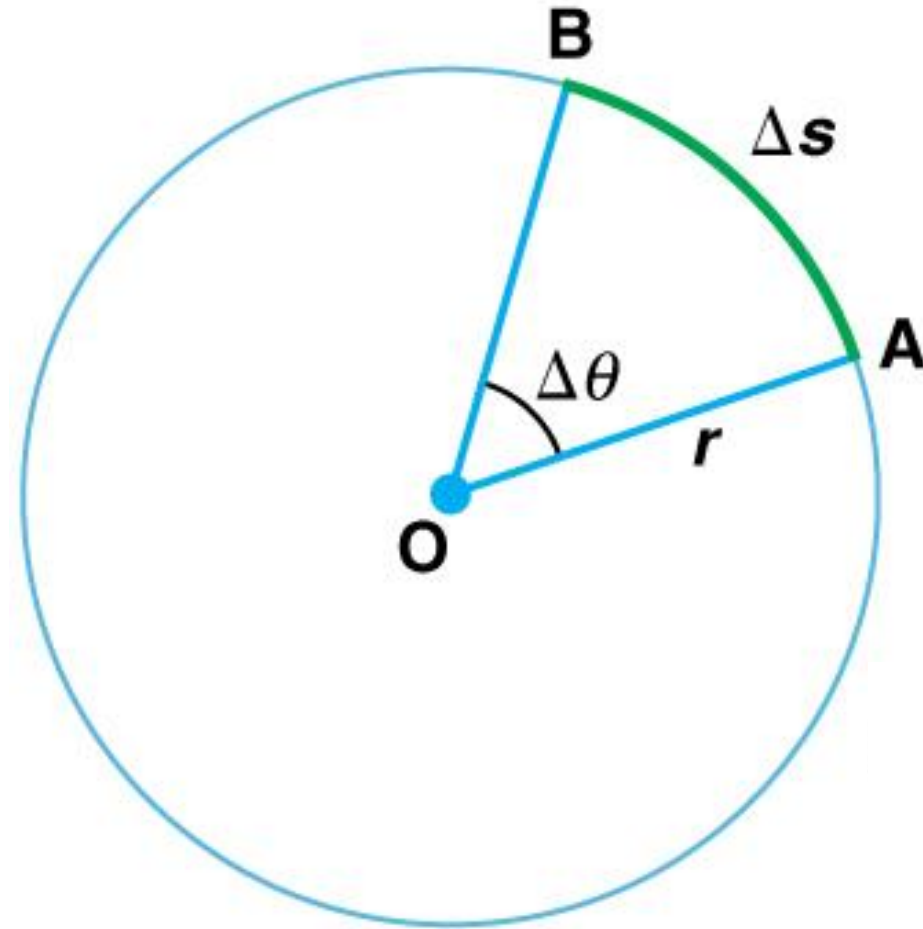


# WHEEL ENCODER



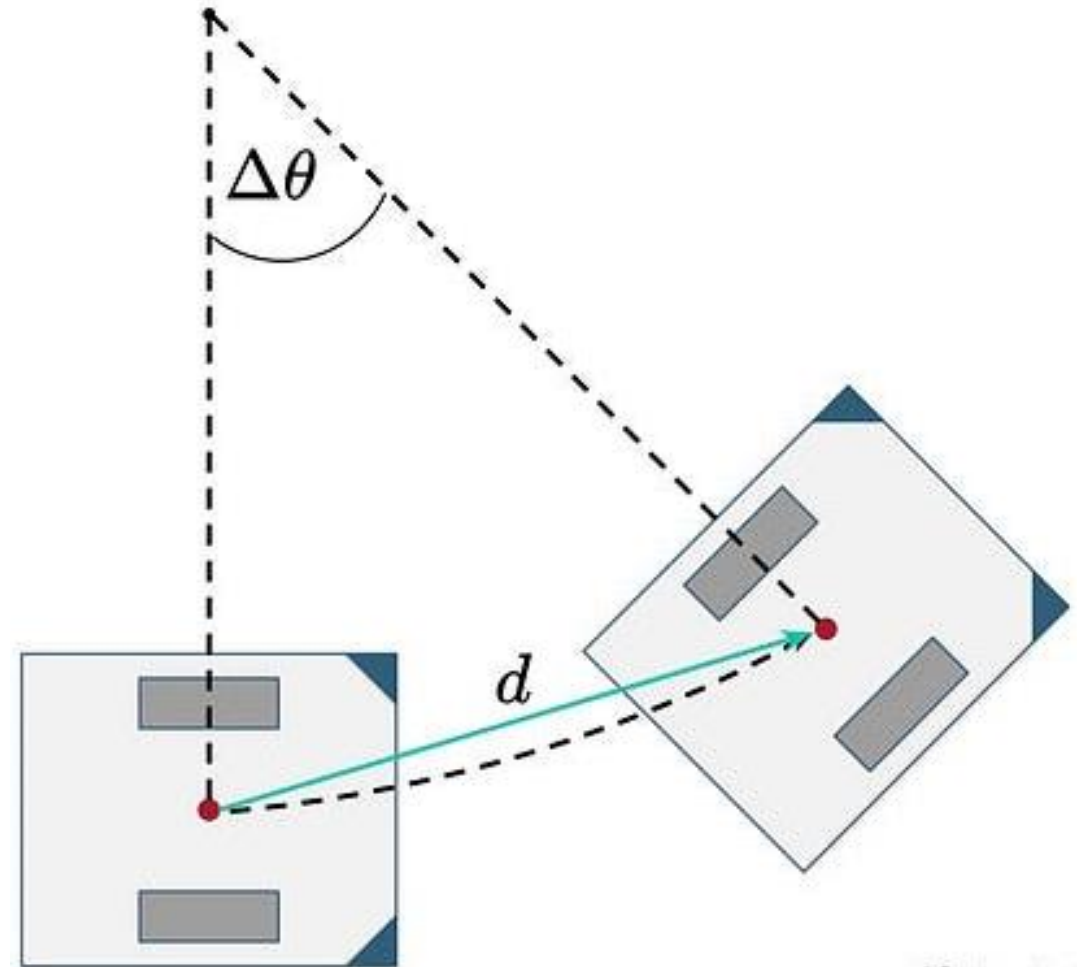
- **Wheel encoder** is an electro-mechanical device that measures the angular position of the wheel.
- Wheel encoders allow to estimate the travelled distance:

$$\Delta s = r \cdot \Delta \theta$$



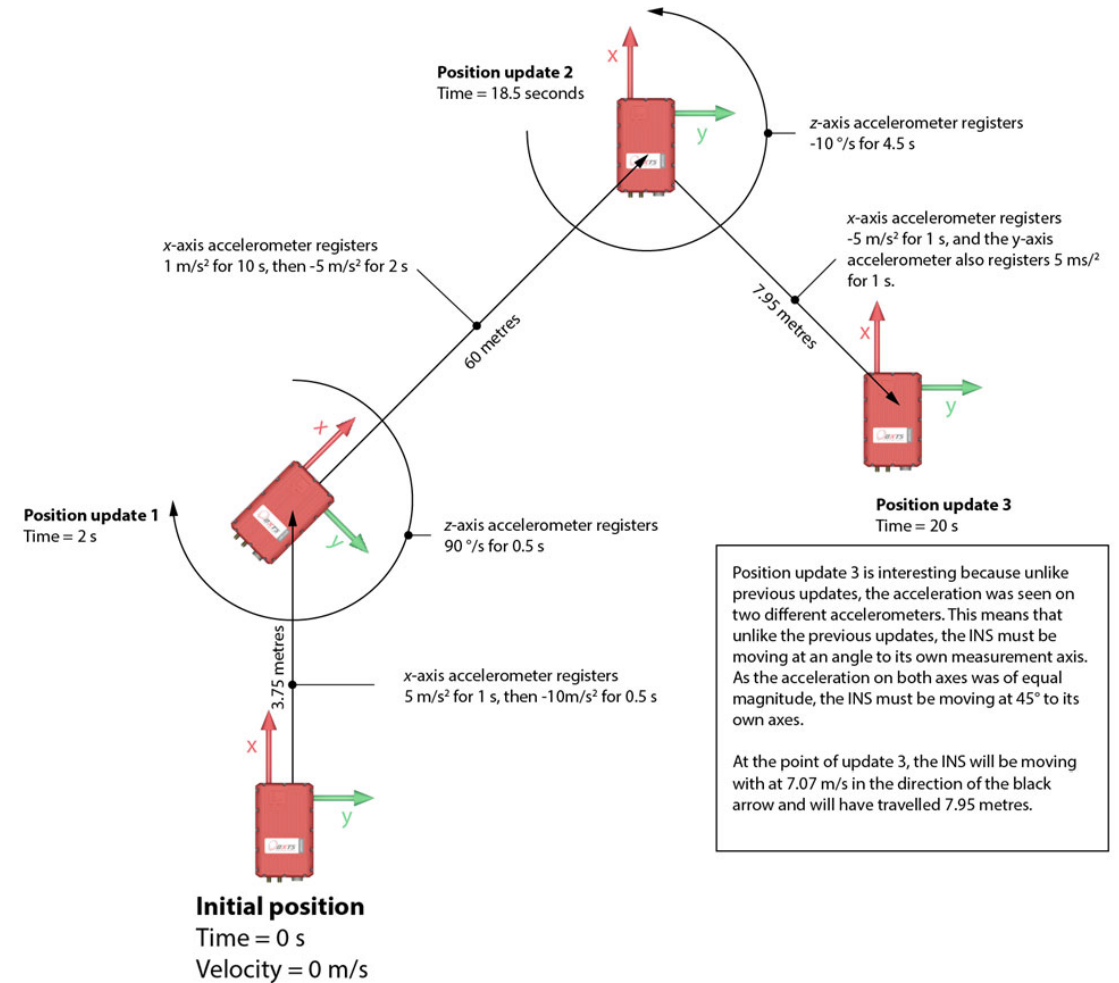
# WHEEL ODOMETRY

- **Wheel odometry** is the use of data from wheel encoders to estimate change in *position*.
- Wheel odometry is used for wheeled robots to estimate their *position* relative to a starting location.
- Wheel odometry is sensitive to errors due to the integration of velocity measurements over time.
- Wheel odometry is sensitive to wheel slip.
- Wheel odometry cannot solve the kidnapped robot problem.

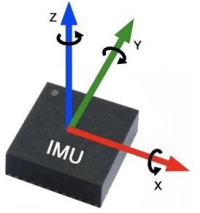


# DEAD RECKONING

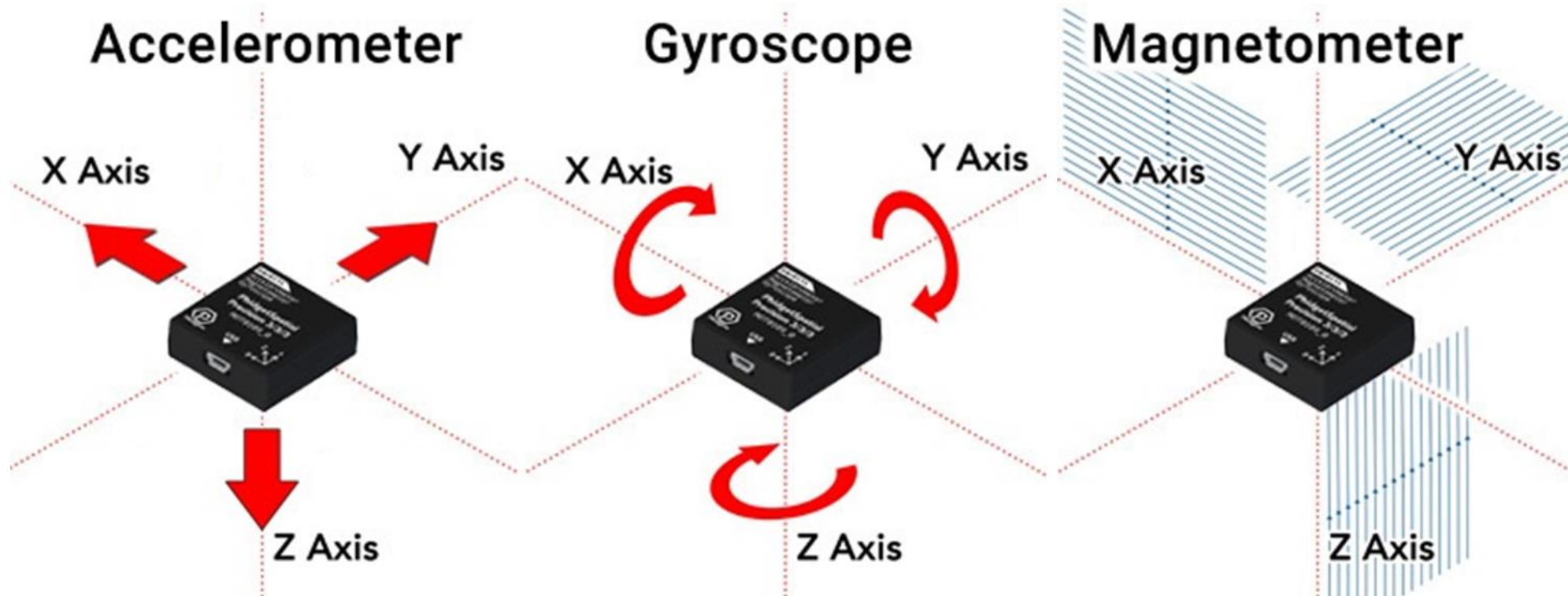
- **Dead reckoning** is the estimation of a robot's pose based on its estimated *acceleration*, *direction* and *time* of travel with respect to a previous estimate.
- Dead reckoning can be implemented by using only inertial (IMU) data: *acceleration* and *orientation*.



# INERTIAL MEASUREMENT UNIT



- **Inertial measurement unit (IMU)** combines 3 accelerometers for *linear acceleration*, 3 gyroscopes for *angular velocity* and 3 magnetometers for *orientation*.



# INERTIAL ODOMETRY

- **Inertial odometry** is the process of determining the position and orientation of a robot from robot's inertial information.

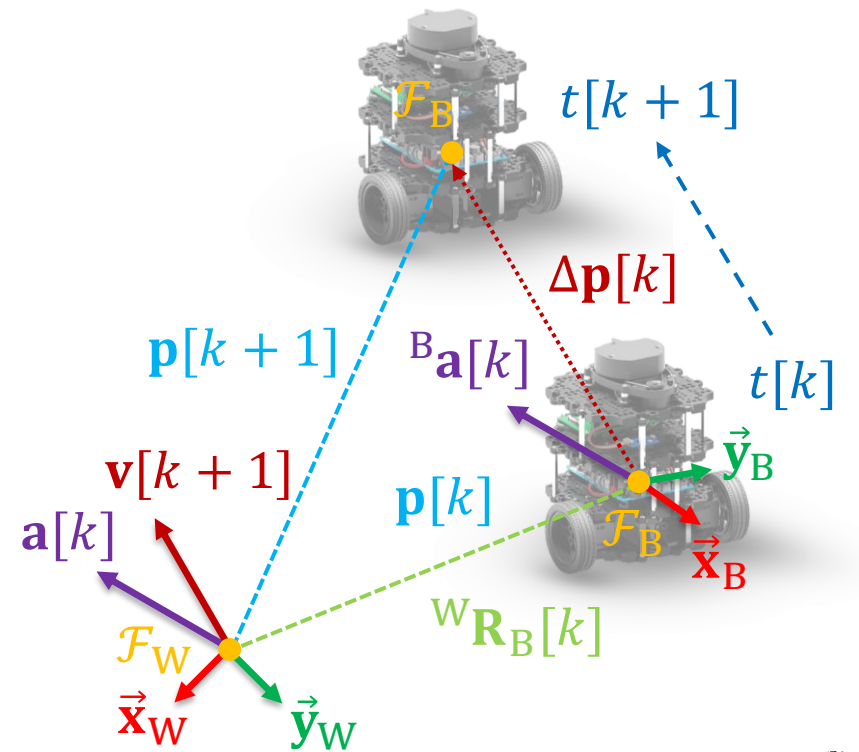
- *Position* calculation:

$$\mathbf{a}[k] = {}^B\mathbf{R}_W[k] \cdot {}^B\mathbf{a}[k]$$

$$\mathbf{v}[k+1] = \mathbf{v}[k] + \mathbf{a}[k] \cdot \Delta t[k]$$

$$\mathbf{p}[k+1] = \mathbf{p}[k] + \underbrace{\mathbf{v}[k] \cdot \Delta t[k] + \frac{1}{2} \mathbf{a}[k] \cdot \Delta t[k]^2}_{\Delta \mathbf{p}[k]}$$

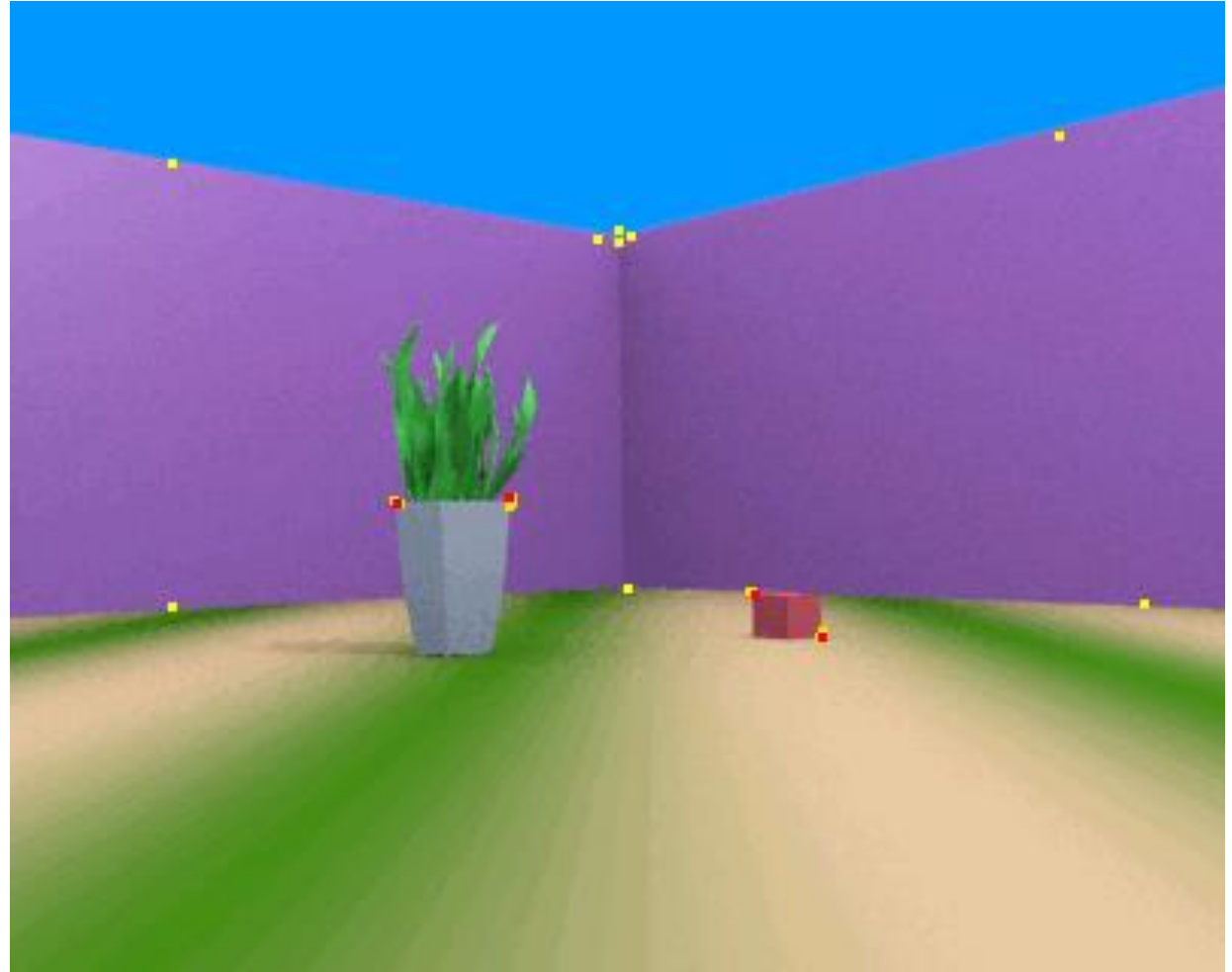
- *Orientation* can be obtained directly from the magnetometer.



# VISUAL ODOMETRY

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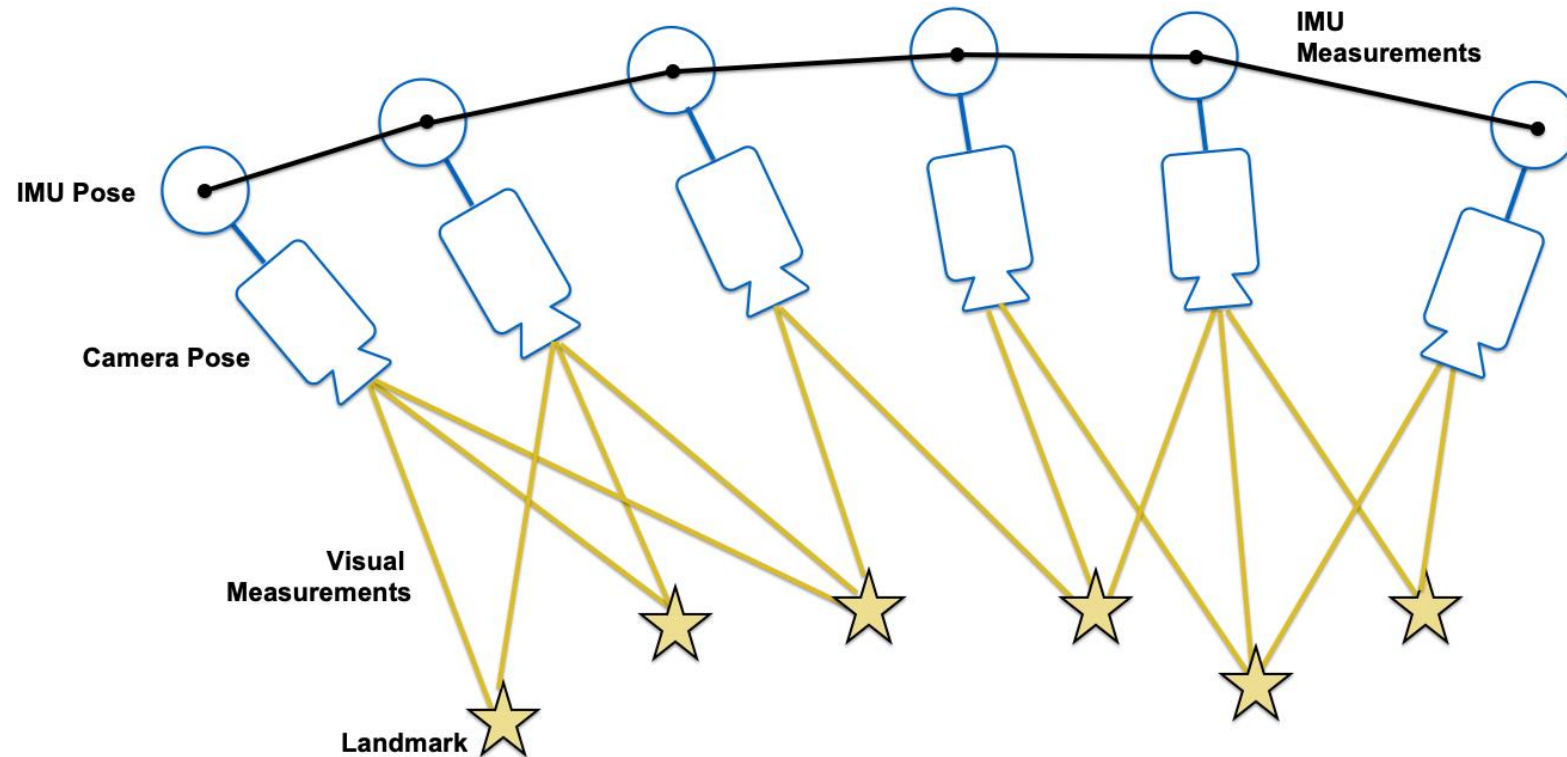
- **Visual odometry** is the process of determining the position and orientation of a robot by analyzing the associated camera images.
- Visual odometry extracts the image feature points and tracks them in the image sequence.
- Depending on the camera setup, visual odometry can be *monocular* (single camera) or *stereo* (two cameras in stereo setup).





# VISUAL-INERTIAL ODOMETRY

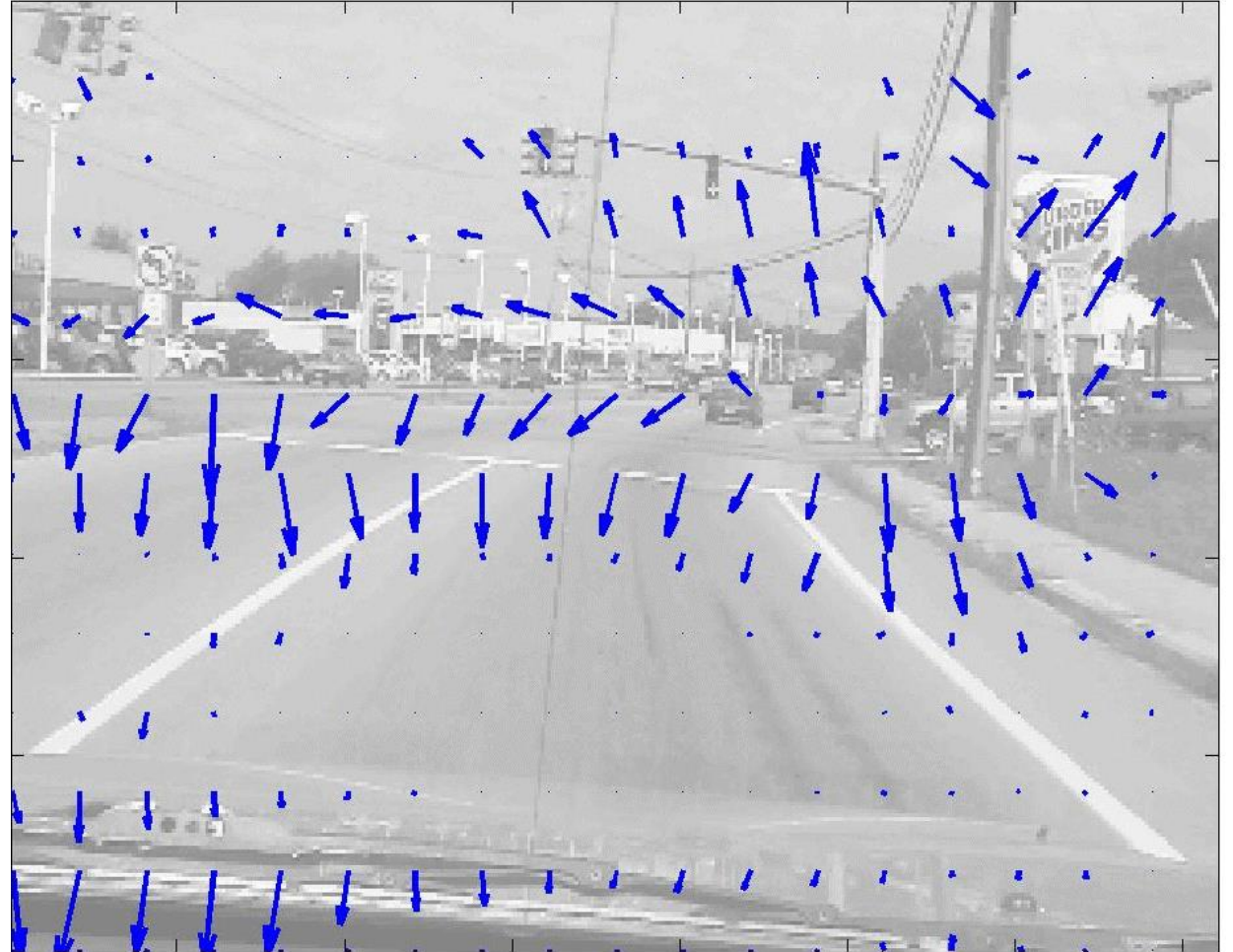
- **Visual-inertial odometry** (VIO) is the process of estimating the pose of a robot by using a camera and an IMU.
- VIO uses images from a camera to estimate translation and data from IMU to estimate rotation.





# OPTICAL FLOW

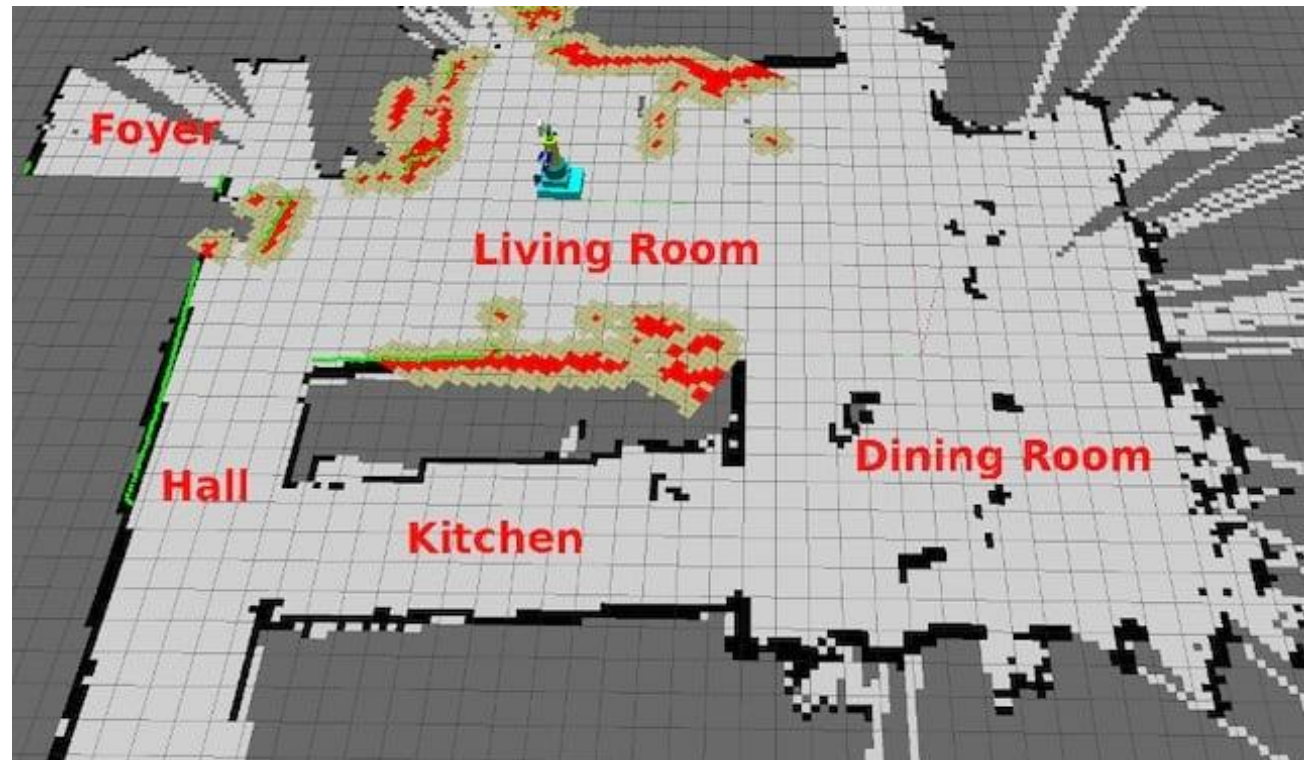
- **Optical flow** is the pattern of apparent motion of objects in a visual scene caused by the relative motion between the camera and the scene.
- Optical flow can also be defined as the distribution of apparent velocities of movement of pixels in an image.
- Optical flow allows to estimate the linear (translational) *velocity* of the robot.



# ROBOTIC MAPPING

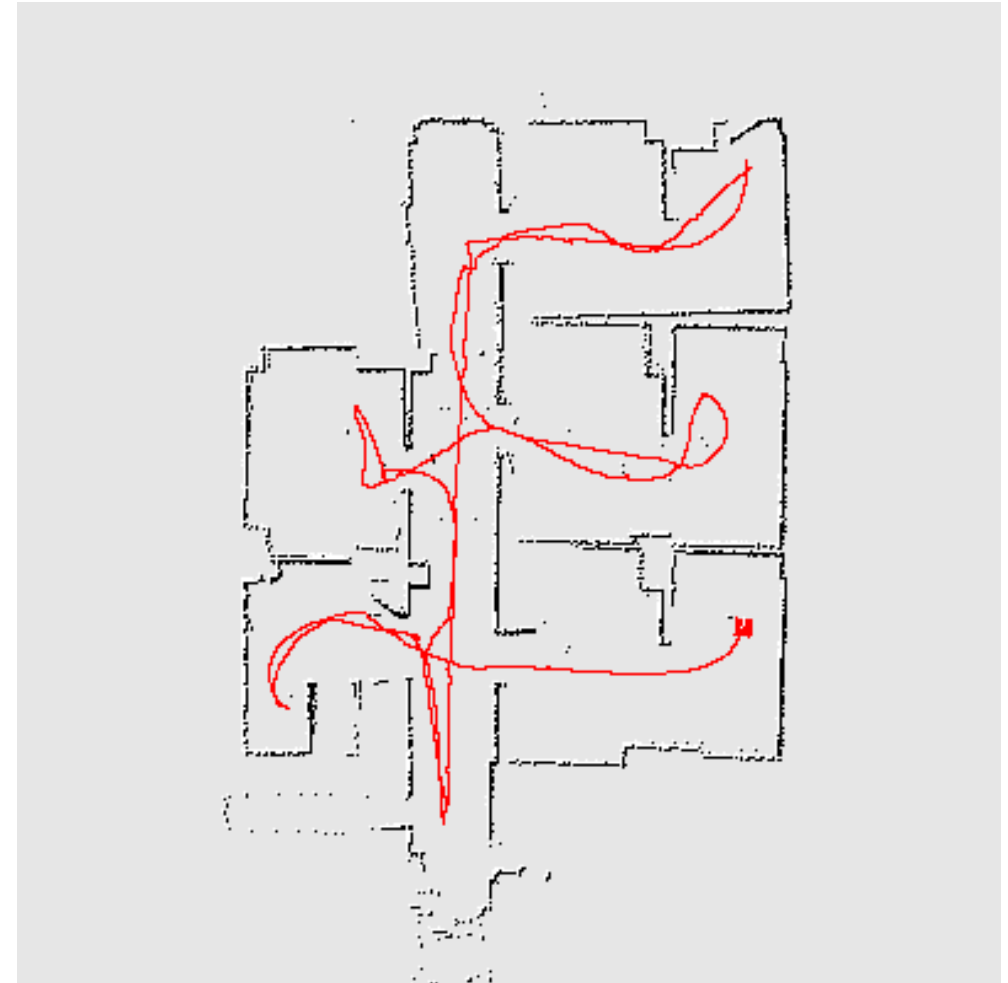
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- **Robotic mapping** is a problem of constructing a map by an autonomous robot.
- Mapping can be performed by moving lidar or cameras in the environment.



# SIMULTANEOUS LOCALIZATION AND MAPPING

- **Simultaneous localization and mapping** (SLAM) is an approach for constructing a map of an unknown environment while simultaneously keeping track of robot's location.
- Popular approximate solution methods include the *particle filter*, *extended Kalman filter* and *covariance intersection*.
- SLAM algorithms are based on concepts in computational geometry (and computer vision).

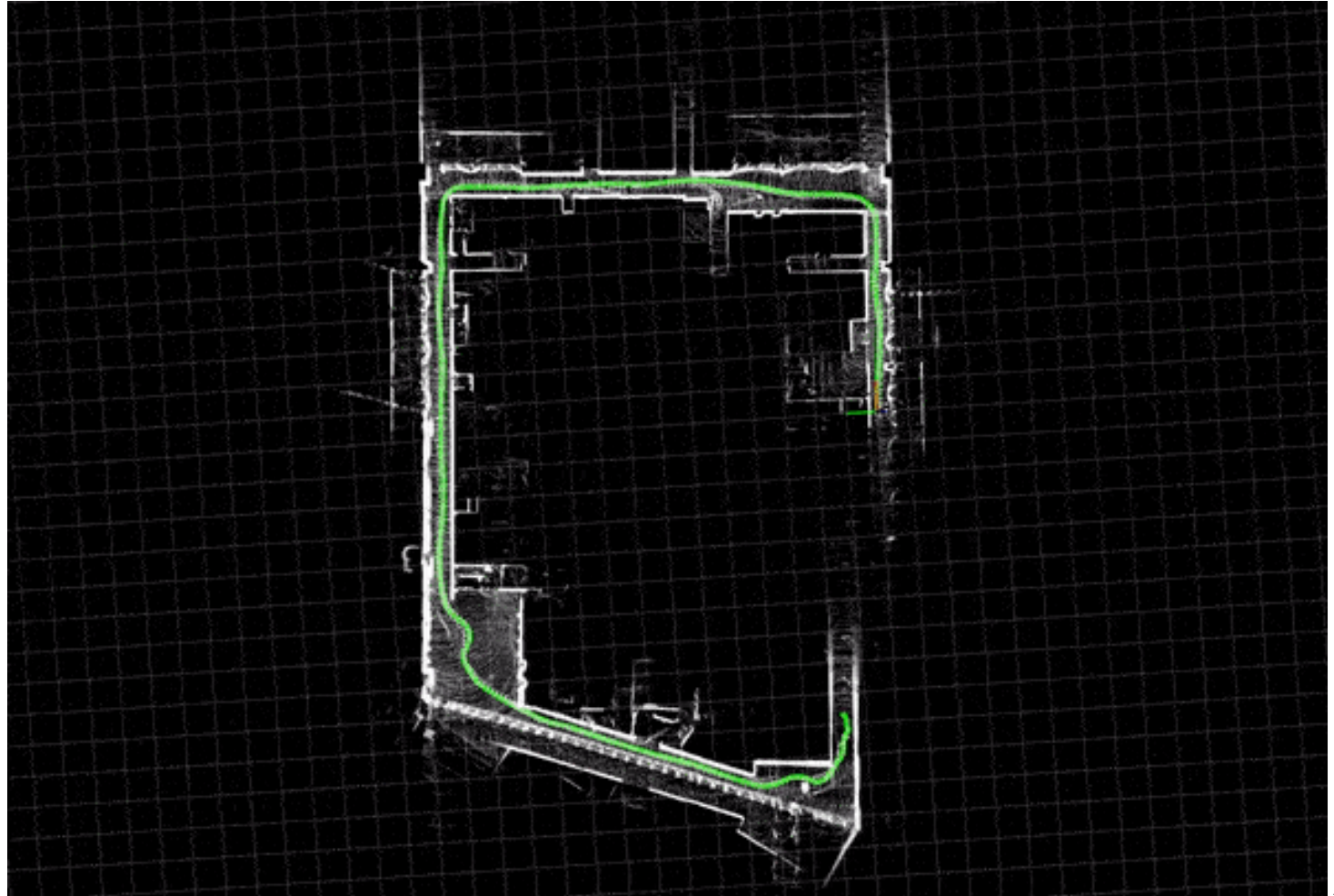




# LOOP CLOSURE

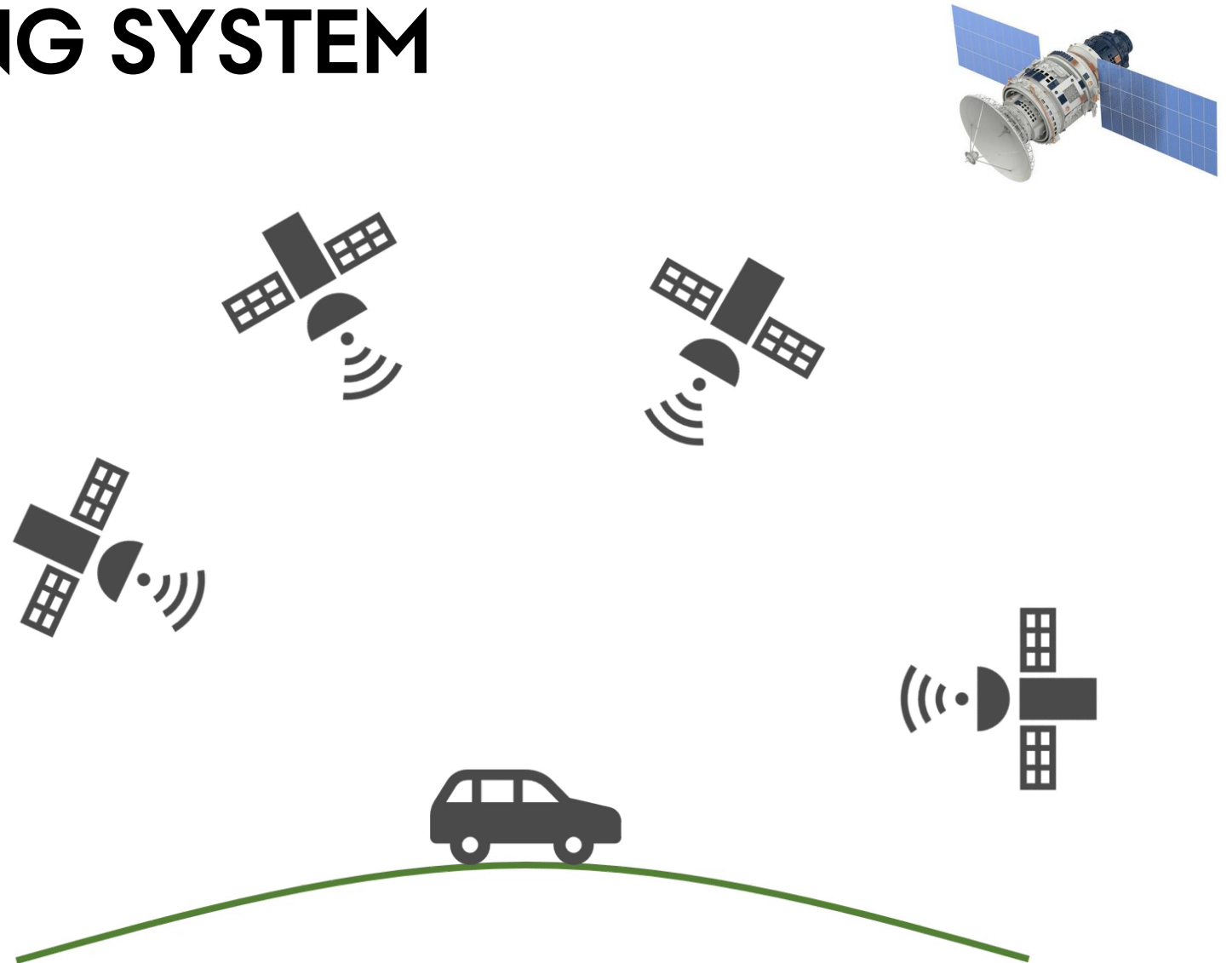
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- **Loop closure** is a sub algorithm of SLAM for identifying previously visited locations to correct the accumulated errors in the robot's pose estimation.
- Loop closure is an iterative optimization algorithm.



# GLOBAL POSITIONING SYSTEM

- **Global positioning system (GPS)** provides geolocation and time information to a GPS receiver anywhere on the Earth where there is an unobstructed line of sight
- GPS provides localization in the global reference frame.
- GPS has no coverage indoors.





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