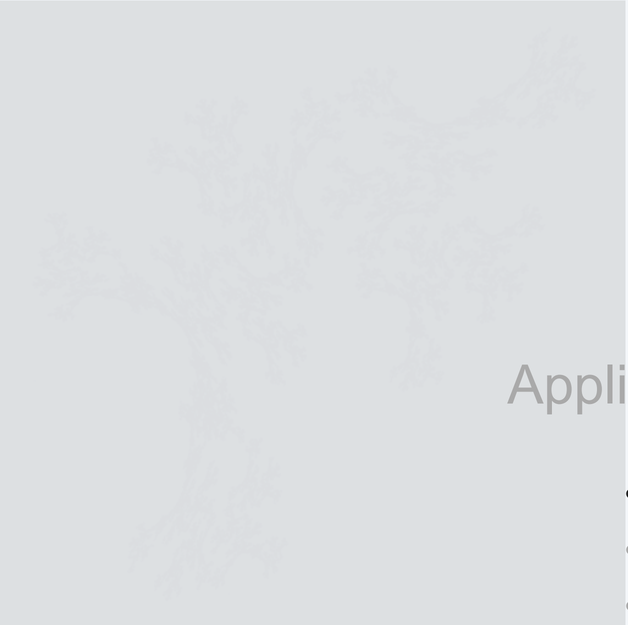
Applications of Propositions

1.1-1.2

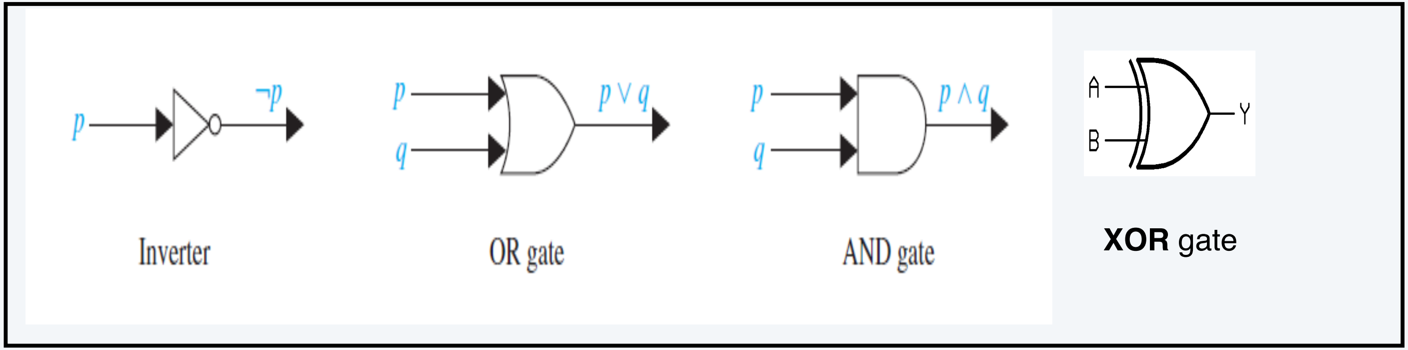


Applications of Propositions

- Logic Gates
- Natural language to propositions
- Consistent systems
- puzzles

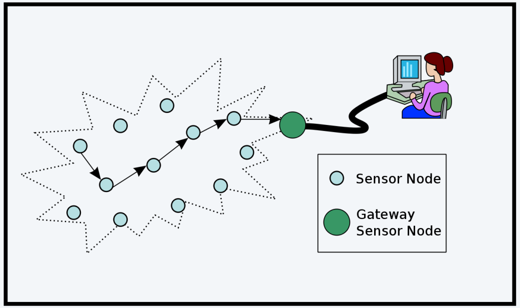
Logic gates

Logic gates are used in computer hardware design



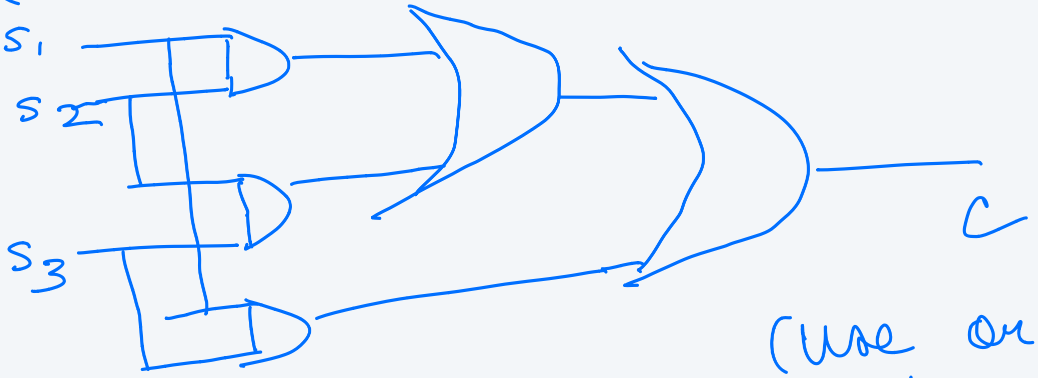
Sensor network

Requirements. Assume there is a 3-sensor network. If 2 or more sensors are true, then we must send TRUE to control station.



Design a logic circuit that meets these requirements

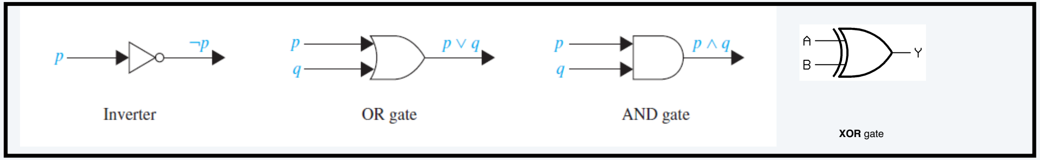
$$((S_1 \wedge S_2) \vee (S_2 \wedge S_3)) \vee (S_1 \wedge S_3)$$

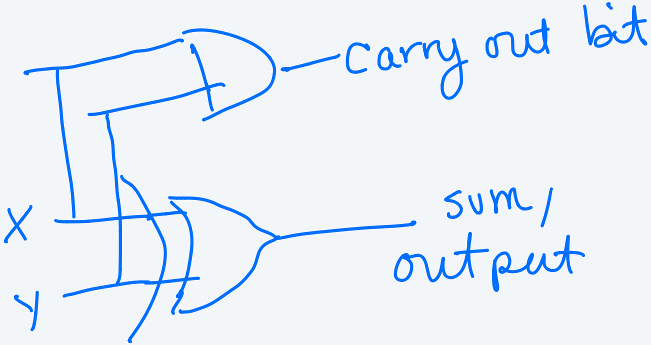


(use or gates not exclusive or)

Half Adder

Design a logic circuit that adds together two bits. It should have an output bit and carry-out bit.





INTRODUCTION TO DISCRETE STRUCTURES

- introduction
- Natural language to propositions
- Consistent systems
- puzzles

Many ways of saying p → q

If p, then q

p is sufficient for q

q if p

q when p

a necessary condition for p is q

p implies q

p only if q

a sufficient condition for q is p

q whenever p

q is necessary for p

q follows from p

Examples

Convert the following statements to logical propositions

roads will be wet, if it rains

q p

q if p

$p \to q$

you can only pass the exam if you study tonight

$p \to q$

p only if q

It is below freezing and snowing

$p \land q$

It is either below freezing or snowing, but not both

$p \otimes q$

2.1 Applications of Propositions

- introduction
- Natural language to propositions
- **Consistent systems**
- puzzles

Consistency

- A set of compound propositions are consistent if there exists truth assignment such that all propositions are satisfied at the same time.
- First, we will show how two compound propositions using the atomic propositions, p and q, are consistent.
- Then we will introduce a third proposition to show when they are not longer consistent.

Example: p = “There was a heatwave in Los Angeles in July 2019. ”
q = “There was a heatwave in London in July 2019.”

$(p \vee q) \wedge (p \vee \neg q)$

p	q	$p \vee q$	$p \vee \neg q$	$\neg p$
T	T	T	T	F
T	F	T	T	F
F	T	T	F	T
F	F	F	T	T

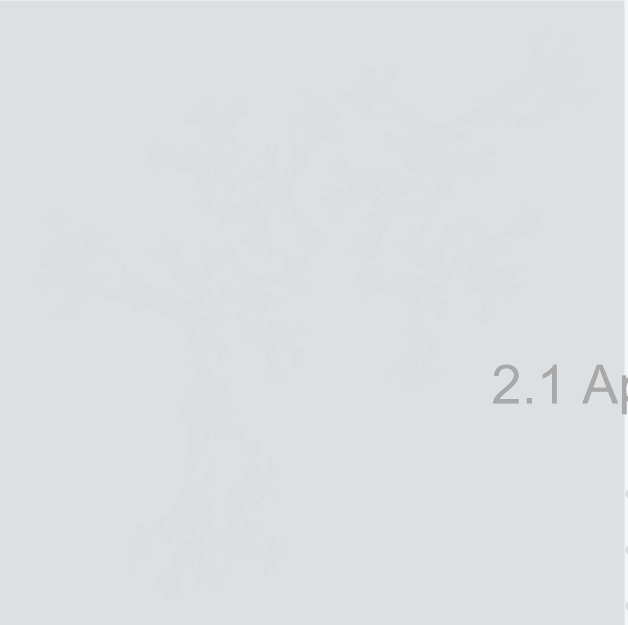
$(p \vee q) \wedge (p \vee \neg q) \wedge \neg p$

Are these system specifications consistent? $(p \to \neg q) \land (q \to r) \land (\sim r \to \sim p)$

- Whenever the **system software is being upgraded(p)**, users cannot **access the file system(~q)**. ($p \to \sim q$)
- If users **can access the file system**(q), then they **can save new files (r)**. ($q \to r$)
- If users cannot save new files(~r), then the system software is not being upgraded(~p).” ($\sim r \to \sim p$)

$\neg$ or $\sim$

p	q	r	$p \to \neg q$	$q \to r$	$\neg r \to \neg p$
T	T	T	F		
T	T	F	F		
T	F	T	T	T	T
T	F	F	T	T	F
F	T	T	T	T	T
F	T	F	T	F	
F	F	T	T	T	T
F	F	F	T	T	T



2.1 Applications of Propositions

- applications
- Natural language to propositions
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- puzzles

Puzzle

There is an island with two kinds of inhabitants. Those who always tell the truth (knights) and those who always lie (knaves). You encounter two people A and B.

What are A and B, if A says, “B is a knight” and B says “two of us are opposite types”

A B

A Knight $\wedge$ B is knight

$\wedge$ B says opposites

A	B
✗ Knight	Knight ✗
✗ Knight	Knave ✗
✗ Knave	Knight ✗
Knave	Knave $\wedge$ B knave

A knave $\wedge$ A lies saying B knight $\Rightarrow$ B knave $\wedge$ B lies and say opposite

Practice problem

A tourist come to a Y junction and the city may be to the left or to the right. There is a native person standing at the junction who knows the answer. But the person may be lying or telling the truth and they only answer with YES or NO.

What question can the tourist ask, so that if the answer is "yes' he will go left and if the answer is no, then he will go right.

Q: If I were to ask you whether the city is to the left would you answer yes?

honest local: Yes

dishonest local: Yes

	city left	city right
honest	Yes	no
dishonest	Yes	no