Alexandra Anderson(aea84)
CS 4620: a6-Animation
Written Questions

1.

a.

$$M_0 = \begin{pmatrix} 0.5 & 0 \\ 0 & 1.5 \end{pmatrix} \qquad M_1 = \begin{pmatrix} 0 & 1.5 \\ -0.5 & 0 \end{pmatrix}$$

$$M_{0.5}^{lin} = \begin{pmatrix} 0.25 & 0.75 \\ -0.25 & 0.75 \end{pmatrix}$$

$$R_0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \qquad S_0 = \begin{pmatrix} 0.5 & 0 \\ 0 & 1.5 \end{pmatrix}$$

$$R_1 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \qquad S_1 = \begin{pmatrix} 0.5 & 0 \\ 0 & 1.5 \end{pmatrix}$$

$$R_{0.5} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} \qquad S_{0.5} = \frac{1}{2} \begin{pmatrix} 1 & 0 \\ 0 & 3 \end{pmatrix}$$

$$M_{0.5}^{pol} = \frac{1}{2\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -1 & 3 \end{pmatrix}$$

b.

We observe that M_0 and M_1 can easily be written as a composition SR, where S is a scale of x=0.5 and y=1.5, and where R is a rotation 45 degrees clockwise or counter-clockwise. Calculations follow from this premise.

$$M_0 = \frac{1}{2\sqrt{2}} \begin{pmatrix} 1 & -1 \\ 3 & 3 \end{pmatrix} \qquad M_1 = \frac{1}{2\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -3 & 3 \end{pmatrix}$$

$$M_{0.5}^{lin} = \frac{1}{2\sqrt{2}} \begin{pmatrix} 1 & 0 \\ 0 & 3 \end{pmatrix}$$

We calculate the polar decomposition by noting that $R = M^T S$, where S is symmetric. Thus, we derive (in lecture) that $(s, c) = norm(m_{21} - m_{12}, m_{11} + m_{22})$. For M_0, the un-normalized vector is $(m_{21} - m_{12}, m_{11} + m_{22}) = (\sqrt{2}, \sqrt{2})$, and the normalization is $(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$. This gives us the value of R, and S is easily computable from there.

For M_1, our key vector is $(m_{21} - m_{12}, m_{11} + m_{22}) = (-\sqrt{2}, \sqrt{2})$, and it normalizes to $(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$.

$$
\begin{aligned}
R_0 &= \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \\
S_0 &= \frac{1}{2}\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} \\
R_1 &= \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} & S_1 &= \frac{1}{2}\begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix} \\
R_{0.5} &= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} & S_{0.5} &= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \\
M_{0.5}^{pol} &= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}
\end{aligned}
$$

c.

$$
\begin{aligned}
M_0 &= \frac{1}{2\sqrt{2}}\begin{pmatrix} 1 & -1 \\ 3 & 3 \end{pmatrix} & M_1 &= \frac{1}{2\sqrt{2}}\begin{pmatrix} 3 & 3 \\ -1 & 1 \end{pmatrix} \\
M_{0.5}^{lin} &= \frac{1}{2\sqrt{2}}\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}
\end{aligned}
$$

We calculate our polar decomposition again. For M_0, the vector is $(m_{21} - m_{12}, m_{11} + m_{22}) = (\frac{2}{\sqrt{2}}, \frac{2}{\sqrt{2}})$, which normalizes to $(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$.

For M_1, the vector is $(m_{21} - m_{12}, m_{11} + m_{22}) = (-\frac{2}{\sqrt{2}}, \frac{2}{\sqrt{2}})$, which again normalizes to $(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$.

$$
\begin{aligned}
R_0 &= \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} & S_0 &= \frac{1}{2}\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} \\
R_1 &= \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} & S_1 &= \frac{1}{2}\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} \\
R_{0.5} &= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} & S_{0.5} &= \frac{1}{2}\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} \\
M_{0.5}^{pol} &= \begin{pmatrix} 1 & 0.5 \\ 0.5 & 1 \end{pmatrix}
\end{aligned}
$$

(Graphs)

On the following pages are graphs. The first page has linear interpolations for a, b, and c. The second page has polar interpretations for a, b, and c.

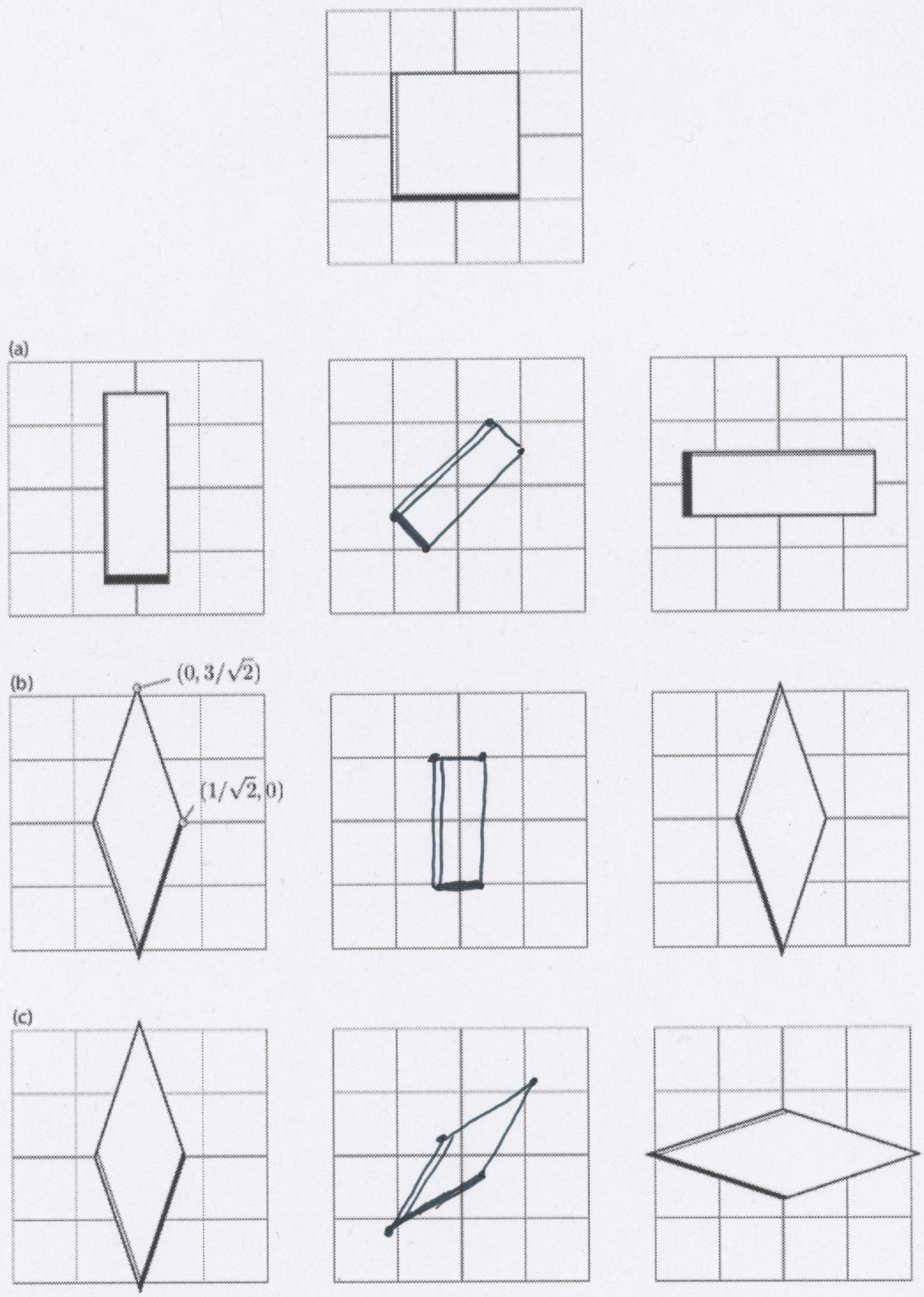

Figure 1: Written Problem 1

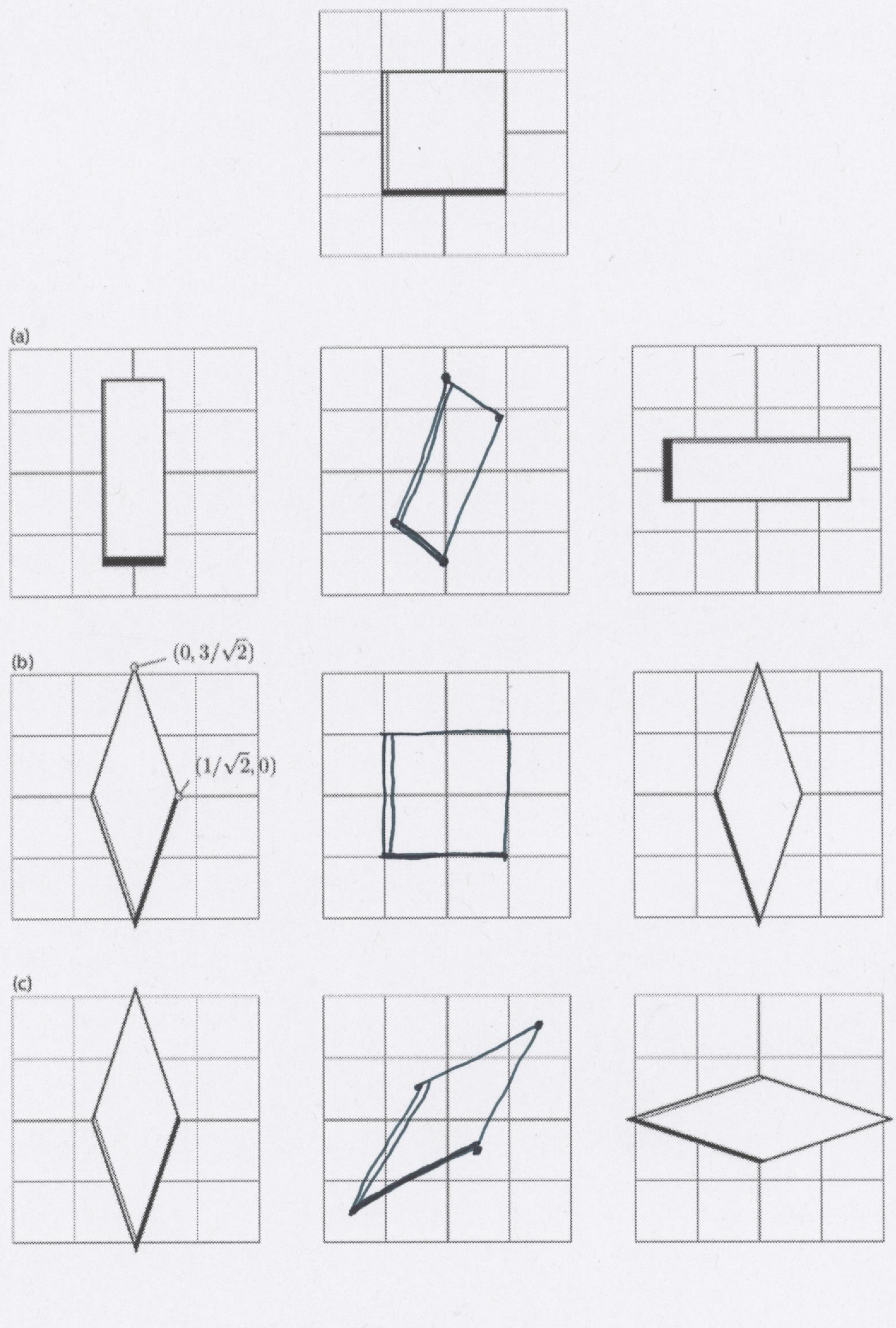

Figure 1: Written Problem 1

2.

1. Rotation of 0 degrees around the X-axis : (1, 0, 0, 0) Rotation of 180 degrees around the X-axis : (0, 1, 0, 0)

2. Their spherical interpolation one-quarter of the way is : $(-0.5411961, 1.3065629, -0.5411961, 1.3065629)$, which corresponds to 45 degrees around the X-axis.

3. We convert $(0, \sqrt{2}, 0, \sqrt{2})$ to a 3x3 matrix:

$$\begin{pmatrix} 0 & 0 & 1 \\ -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \end{pmatrix}$$

Yes, this is the matrix that would be generated for 45-degrees around the x-axis.

4. The matrix for a x-rotation of 90 degrees followed by a y-rotation of 90 degrees is :

$$\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & -1 \\ -1 & 0 & 0 \end{pmatrix}$$

Its quaternion is $q_4 = (0.5, 0.5, 0.5, -0.5)$.

The matrix for a y-rotation of 90 degrees followed by a z-rotation of 90 degrees is :

$$\begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & 1 \\ -1 & 0 & 0 \end{pmatrix}$$

Its quaternion is $q_5 = (-0.5, 0.5, -0.5, -0.5)$.

5. $q_6 = (0, 0.7653668, -1.847759, 0)$. This corresponds to the axis (0.67859834, -0.28108466, -0.67859834) and angle 1.2967819 in radians, or 74 degrees.

6. q_7, the rotation from q_4 to q_6, is (-0.38268343, 0.0, 0.0, 0.92387956).

7. q_7 corresponds to the axis (0.0, 0.0, 1.0) and angle 1.1780974 radians.