STAT 123: Quantitative Finance, Spring 2022 Prof. Stephen Blyth

Section 10

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Section: Monday 10 - 11AM Office Hour: Monday 11AM - noon

1 Bermudan Swaptions

- Definition: A T_0 into $T_n - T_0$ Bermudan payer swaption with strike K gives the buyer the option at times $T_0, T_1, ..., T_{n-1}$ to enter into a swap from that time until T_n, paying fixed rate K. If the buyer exercises at T_i, she is locked into a swap from T_i to T_n, with no further optionality. If the buyer does not exercise at T_i, she can decide again at T_{i+1} whether to exercise.
- Pricing a Bermudan swaption is complex, but we can still put bounds on the value of a swaption. Let $C_K(t, T_0, T_n)$ be the value at time t of a K-strike T_0 by T_n cap, $\Psi_K(t, T_i, T_n)$ be the value at time t of a K-strike T_j into $T_n - T_j$ European payer swaption. Then, the bounds on price:
  - $B_K(t, T_0, T_n) \leq C_K(t, T_0, T_n)$
  - $B_K(t, T_0, T_n) \geq \underset{0 \leq i \leq n-1}{max} \Psi_K(t, T_i, T_n)$
  - $B_K(t, T_0, T_n) \leq \sum_{i=0}^{n-1} \Psi_K(t, T_i, T_n)$
- If exercise, we get $(y_{T_i}[T_i, T_n] - K) P_{T_i}[T_i, T_n]$, if not exercise, we get $B_K(T_j, T_{j+1}, T_n)$. However, $B_K(T_j, T_{j+1}, T_n)$ is unknown. The Do-Not-Exercise criteria:
  - Do not exercise at T_i if $y_{T_i}[T_i, T_n] < K$
  - Do not exercise at T_i if $(y_{T_i}[T_i, T_n] - K) P_{T_i}[T_i, T_n] < \underset{i+1 \leq j \leq n+1}{max} \Psi_K(T_i, T_j, T_n)$
  - Do not exercise at T_i if $y_{T_i}[T_i, T_j] < K$ for any j: $i + 1 \leq j \leq n$

2 Cancellable Swaps / Bermudan Cancellable Swaps

- European Cancellable Swap: the party is in a swap and pays fixed K and receives libor from $T = T_0$ to T_n, but at a single time T_j , the party has the option to cancel the swap. If the option is exercised, no more swap payments are made after T_j.

  We can construct a European cancellable swap from a swap and a European swaption in several ways:

  1) enter into swap paying K from T to T_n, plus long a T_j into $T_n - T_j$ European receiver swaption

  2) enter into swap paying K from T to T_j , plus long a T_j into $T_n - T_j$ European payer swaption

- Bermudan Cancellable Swaps: A T_n noncall T_j Bernumdan cancellable swap is a swap from T_0 to T_n where the party who pays fixed has the right to cancel at $T_j, T_{j+1}, \cdots, T_{n-1}$.
- Results
  - K{6nc2 Berm q} $\geq$ K{5nc2 Berm q} $\geq$ K{5nc2 Berm s/a } $\geq$ K{ 5nc3 Berm s/a } $\geq$ K{5nc3 Euro} $\geq$ five-year swap rate
  - K{5nc3 Euro} $\geq$ three-year swap rate
  - K{5nc2 Berm s/a } $\geq$ K{5nc2 Euro} $\geq$ two-year swap rate

3 Exercises

3.1 Bounds on Bermudan Swaption

Prove the given bounds on the value of a Bermudan swaption

- $B_K(t, T_0, T_n) \leq C_K(t, T_0, T_n)$
- $B_K(t, T_0, T_n) \geq \underset{0 \leq i \leq n-1}{max} \Psi_K(t, T_i, T_n)$
- $B_K(t, T_0, T_n) \leq \sum_{i=0}^{n-1} \Psi_K(t, T_i, T_n)$

Solution:

3.2 Bermudan Cancellable Swaps

Rank the following portfolios. Assume annual exercises, annual cancellations, and annual payment dates throughout.

- I 5-nc-2 European cancellable swap
- II 5-nc-2 Bermudan cancellable swap
- III swap from 0 to 3
- IV swap from 0 to 5, plus 2-into-3 Bermudan receiver swaption
- V swap from 0 to 2, plus 2-into-3 Bermudan payer swaption
- VI 5-nc-4 Bermudan cancellable swap
- VII 6-nc-4 Bermudan cancellable swap

I II

III II

VI II

VI VII

II IV

I V

Solution: