STAT 123: Quantitative Finance, Spring 2022 Prof. Stephen Blyth

Midterm Review – Practice Questions

Xin Zeng (xinzeng@fas.harvard.edu) 26th February 2022

Section: Monday 10 - 11AM Office Hour: Monday 11AM - noon

1 Properties of Call / Put Options (Sample Midterm 2 Question 1)

(a) Assume no arbitrage. By considering a portfolio of an amount of ZCB and a call option prove that the value at time $t < T$ of a call option with strike K on a stock that pays no dividends satisfies

$$(S_t - KZ(t,T))^+ \leq C_K(t,T)$$

(b) Hence, prove that if $t \leq T_1 \leq T_2$, $C_K(t,T_2) \geq C_K(t,T_1)$. Hint, consider the case $t = T_1$.

(c) Does the same result hold for puts? That is: prove or find a counterexample to the statement $P_K(t,T_2) \geq P_K(t,T_1)$ for $t \leq T_1 \leq T_2$.

Solution:

2 Ranking Problem in Options (Sample Midterm 2 Question 4)

Consider the following 10 option portfolios. Let $K_{i+1} = K_i + \beta$ for $i = 1, 2$ for a constant $\beta > 0$:

a) K_1 call

b) K_1 put

c) K_3, K_2 put spread

d) K_1, K_2, K_3 call butterfly (Number of K_1 call is 1.)

e) K_1 call that knocks out if $S_T \geq K_2$

f) Digital call with strike K_1 and payout β

g) K_1, K_3 digital call spread both with payout β

h) Digital put with strike K_3 and payout β that knocks out if $S_T \leq K1$

i) A portfolio of $+1$ K_1 call and -2 K_2 calls, all of which knock out if $S_t < K_3$ for any $0 \leq t \leq T$

j) A portfolio of $+1$ K_1 call and -2 K_2 calls, all of which knock out if $S_T > K_3$

For each pair, choose the most appropriate relationship between prices at time $t \leq T$ out of $=, \geq, \leq$, and ?, where ? means the relationship is indeterminate.

- a b
- b c
- c d
- d e
- d j
- g h
- c h
- d g
- b i
- b j

Solution:

3 Annuity Formula

Recall that at time t the value V of an annuity which pays coupons C at times $T_1, ..., T_n$ is $V = C\sum_{i=1}^{n} Z(t, T_i)$. If the coupons are paid annually for M years and the annually compounded interest rate is fixed at r, then derive V in terms of C and r

Solution:

4 Forward Contract Value

If I went long a forward contract at the fair price and the stock doesn't move, then at time T I will MAKE/LOSE money (circle one). Explain. Assume no income/dividends, and positive interest rate.

Solution:

5 Terms Comparison

Comparing Terms For Each Pair – choose the relationship ($\leq, \geq, =, ?$) that best describes the connection between the two terms at time t. Assume $t < T_1 < T_2 < T_3$ and non-negative interest rates.

(a) $Z(t, T_3) \quad Z(T_2, T_3)$

(b) $V_{F(T_1,T_2)}(T_1, T_2) \quad Z(T_1, T_2)$

(c) $F(t, T_1, T_1) \quad F(t, T_2, T_2)$

(d) $L_t[T_1, T_2] \quad L_T[T_1, T_2]$

Solution:

6 Forward Rates

(a) The one-year and two-year zero rates are 1% and 2% respectively. What is the one-year forward one-year rate (that is, f_{11})? Assume all rates are annually compounded.

(b) If the two-year forward one-year rate (f_{21}) is 3%, what is the three-year zero rate?

Solution:

7 Forwards and Carry

(a) Use arbitrage arguments involving two forward contracts with maturity T to prove that

$$V_K(t,T) = (F(t,T) - K)e^{-r(T-t)}$$

(b) Verify that $V_K(T,T)$ equals the payout of a forward contract with delivery price K. For an asset that pays no income, substitute the expression for its forward price into the above equation and give an intuitive explanation for the resulting expression.

(c) Suppose at time t_0 you go short a forward contract with maturity T (and with delivery price equal to the forward price). At time t, $t_0 < t < T$, suppose both the price of the asset and interest rates are unchanged. How much money have you made or lost? (This is sometimes called the carry of the trade.) How does your answer change if the asset pays dividends at constant rate q?

Solution:

8 Binomial Tree and Risk-Neutral Probability

(a) Let us say we have a stock such that $S_0 = 110$. At time $T = 1$, in the up state, $S_1 = 135$ and in the down state, $S_1 = 90$. Also $r = 5\%$, compounded annually. We want to find $P_{105}(0, 1)$ the fair price of the 105 put.

(b) What is the risk-neutral probability p^*.

Solution:

9 Butterflies, Condors and Call Ladders (IQF Chapter 7, Exercise 3)

(a) Recall that a call butterfly with strikes $(K_1, K_1+\beta, K_1+2\beta)$, for some fixed $\beta > 0$, is a portfolio consisting of +1 K_1 call, +1 $(K_1+2\beta)$ call and –2 $(K_1+\beta)$ calls. Using put-call parity or otherwise, restate the call butterfly as a portfolio consisting solely of puts.

(b) A call condor is a portfolio consisting of +1 K call, –1 $(K + \beta)$ call, –1 $(K + 2\beta)$ call and +1 $(K + 3\beta)$ call. Draw the payout of the condor, and express the condor as a portfolio consisting solely of call butterflies.

(c) A call ladder consists of $+1$ K call, -1 $(K+\beta)$ call and -1 $(K+2\beta)$ call. What relationships hold between the prices at time $t \leq T$ of the call ladder, butterfly and condor with common maturity T?

Solution: