STAT 123: Quantitative Finance, Spring 2022 Prof. Stephen Blyth

Final Review – Practice Questions

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Final Review Section: April 29 1 - 3 PM

1 Relating the Interest Rate Derivatives

Using words and equations, write down any relationships you can think of between the following derivatives:

FRAs, Caplets/Caps and Floorlets/Floors, Libor-in-Arrears, Swaps, Swaptions, Cancellable Swaps

Hint: Consider put-call parity involving caps and floors, European payer and receiver swaptions.

Solution:

2 Volatility (Sample Final Question 3)

For each of the following positions A-N, determine whether:
I. HMC is long volatility (ie the position has positive vega and thus increases in value if volatility increases);
II HMC is short volatility (ie the position has negative vega);
III There is no volatility exposure in the trade;
IV The volatility exposure is indeterminate with the information given.
Glossary: 3mL means 3-month libor; 1mL 1-month libor; Options in A-G all have maturity T on a stock that pays no dividends, and $K_1 < K_2$.

- A. HMC is long a K-strike call
- B. HMC is long a K-strike call and long a K-strike put

- C. HMC is short a K-strike call and long a K-strike put
- D. HMC is long a K_1, K_2 call spread
- E. HMC is long a $K_1, (K_1 + K_2)/2, K_2$ call butterfly
- F. HMC is short a K_1-strike call which knocks out if $S_t > K_2$ at any time
- G. HMC is short a K_2-strike call which knocks out if $S_t > K_1$ at any time
- H. HMC is long a forward contract on a stock
- J. HMC is long a futures contract on a stock
- K. HMC is long a futures contract on a fixed rate bond
- L. HMC pays 3mL q, receives 3mL in arrears q, for 10yrs
- N. HMC pays 1mL q, receives 3mL q, for 10yrs

Solution:

3 Terms Comparison

Comparing Terms For Each Pair – choose the relationship $(\leq, \geq, =, ?)$ that best describes the connection between the two terms at time t. Assume $t < T_1 < T_2 < T_3$ and non-negative interest rates.

(a) $Z(t, T_3) \quad Z(T_2, T_3)$

(b) $V_{F(T_1,T_2)}(T_1, T_2) \;\; Z(T_1, T_2)$

(c) $F(t, T_1, T_1) \;\; F(t, T_2, T_2)$

(d) $L_t[T_1, T_2] \;\; L_T[T_1, T_2]$

(e) $P_t[T_1, T_2] \;\; P_t[T_1, T_3]$

Solution:

4 Caplet in Arrears

A caplet in arrears has payout $\alpha(L_T - K)^+$ at time T. Show that the price of the caplet in arrears at t can be expressed as:

- the price of X number of regular caplets with strike K. (Please determine X)
- the price of a "square-everything caplet" whose payout at $T + \alpha$ is $\alpha^2(L_T^2 - K^2)$ if $L_T > K$ and 0 otherwise.

Solution:

5 Bermudan Cancellable Swaps

Rank the following portfolios. Assume annual exercises, annual cancellations, and annual payment dates throughout.

I 5-nc-2 European cancellable swap

II 5-nc-2 Bermudan cancellable swap

III swap from 0 to 3

IV swap from 0 to 5, plus 2-into-3 Bermudan receiver swaption

V swap from 0 to 2, plus 2-into-3 Bermudan payer swaption

VI 5-nc-4 Bermudan cancellable swap

VII 6-nc-4 Bermudan cancellable swap

I II

III II

VI II

VI VII

II IV

I V

Solution:

6 Binomial Tree and Risk-Neutral Probability

(a) Let us say we have a stock such that $S_0 = 110$. At time $T = 1$, in the up state, $S_1 = 135$ and in the down state, $S_1 = 90$. Also $r = 5\%$, compounded annually. We want to find $P_{105}(0, 1)$ the fair price of the 105 put.

(b) What is the risk-neutral probability p^*.

Solution:

7 Swap Value (IQF Chapter 4, Exercise 1 (b))

By expressing a swap as a difference between a floating rate bond and fixed rate bond, prove that, for a given K, the value of swap $V_K^{SW}(t)$ is bounded, that is, there exists finite l and u independent of interest rates such that $l \leq V_K^{SW}(t) \leq u$. For $t = T_0 = 0$ (a spot starting swap), $T_n = n$ and frequency $\alpha = 1$, find bounds in terms of n and K.

Solution: