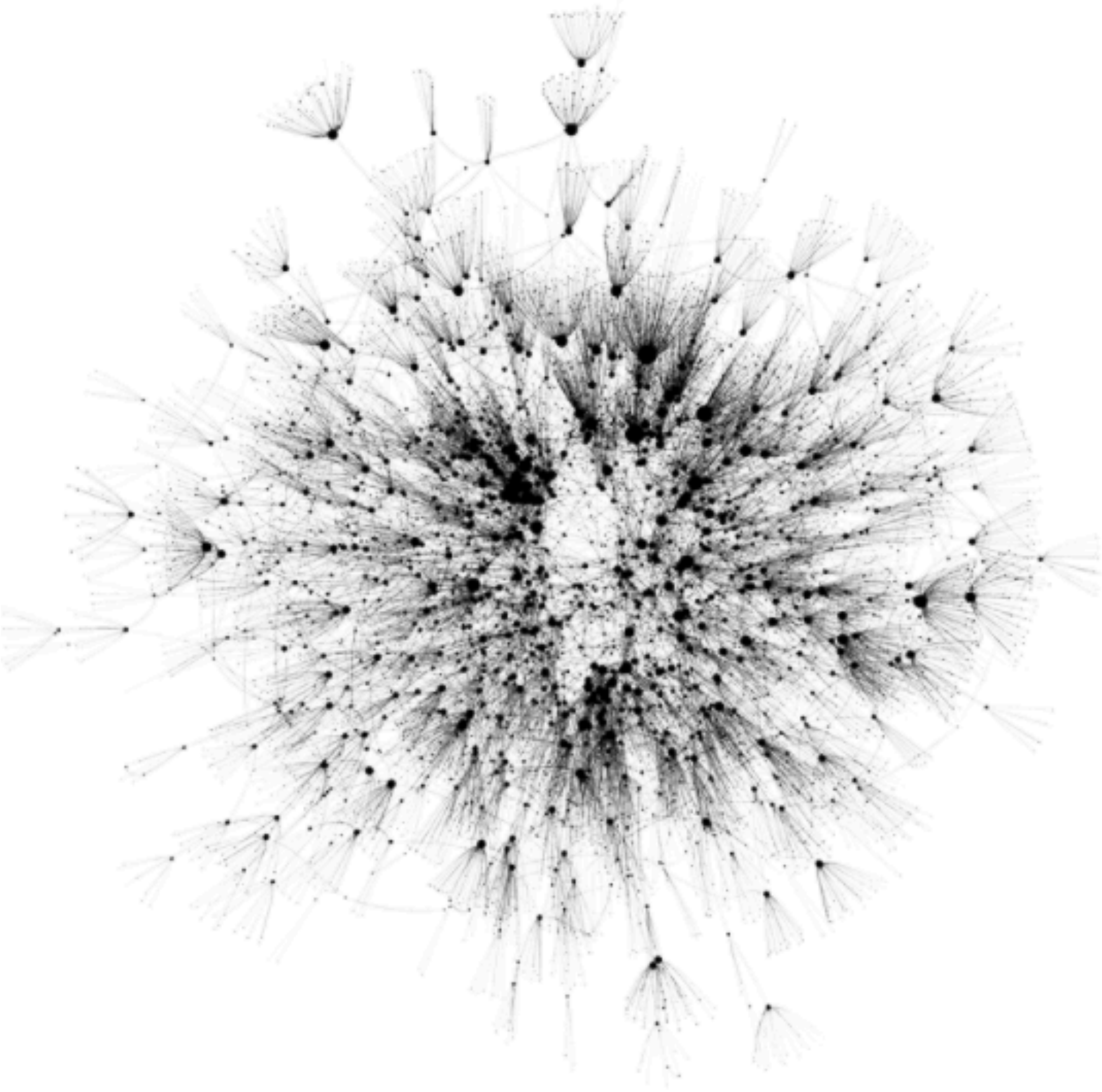
Social Network Analysis - Lecture 11

2162-F23

2023-03-13

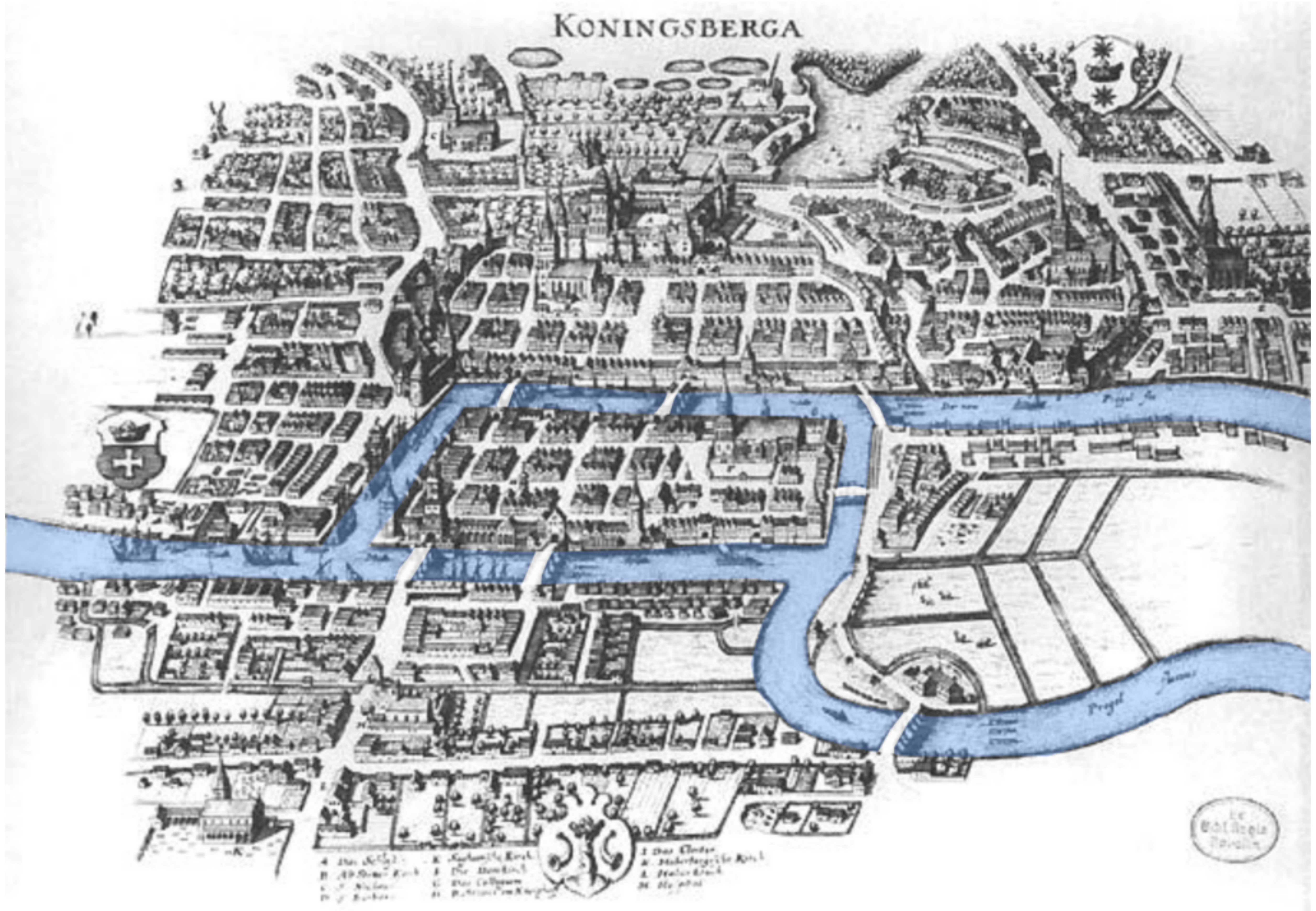


Art Network, http://netwonder.net/

Today’s plan!

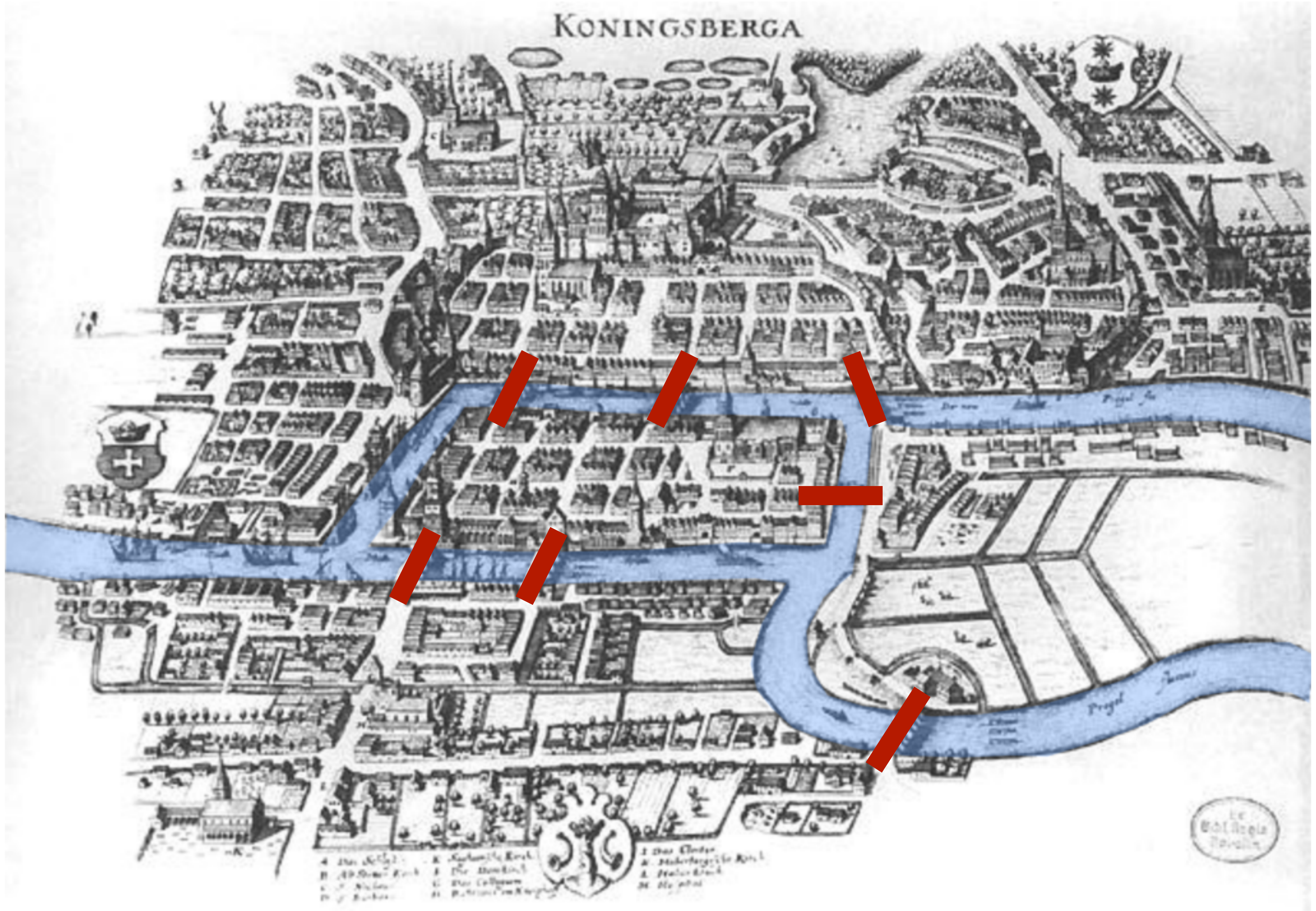
1. Introduction to the course: why networks?
2. Key concepts: Nodes, links, paths
3. Network representation: edge list, matrix
4. Linear Algebra quick recap

Königsberg bridges

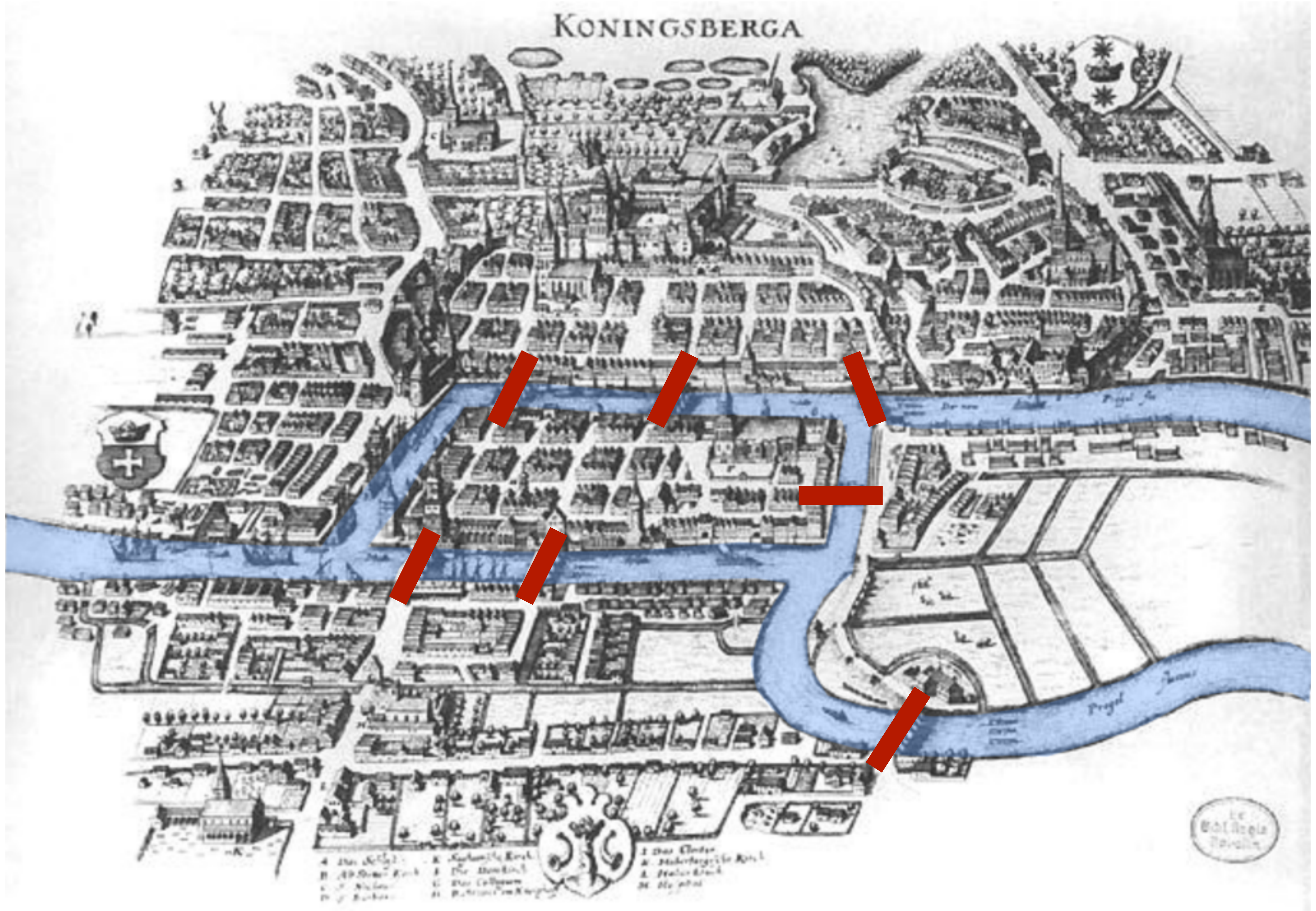


1735

Königsberg bridges

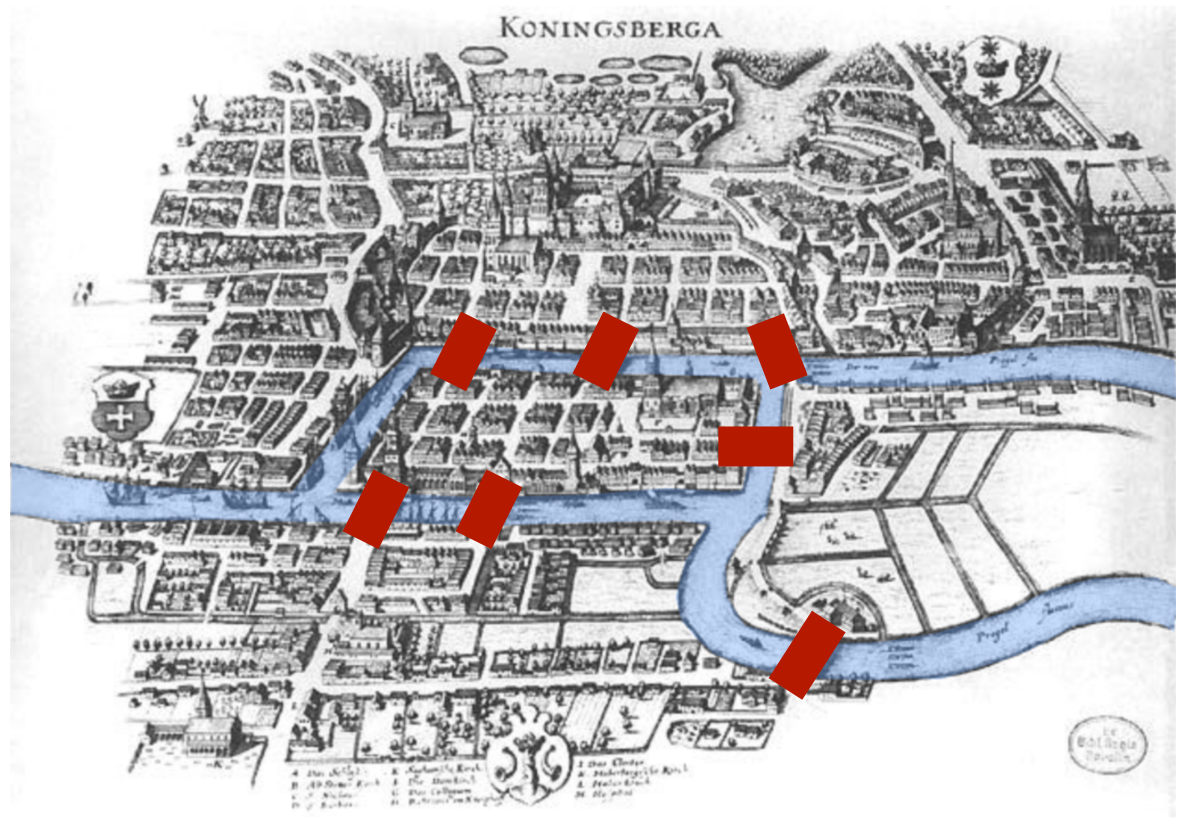


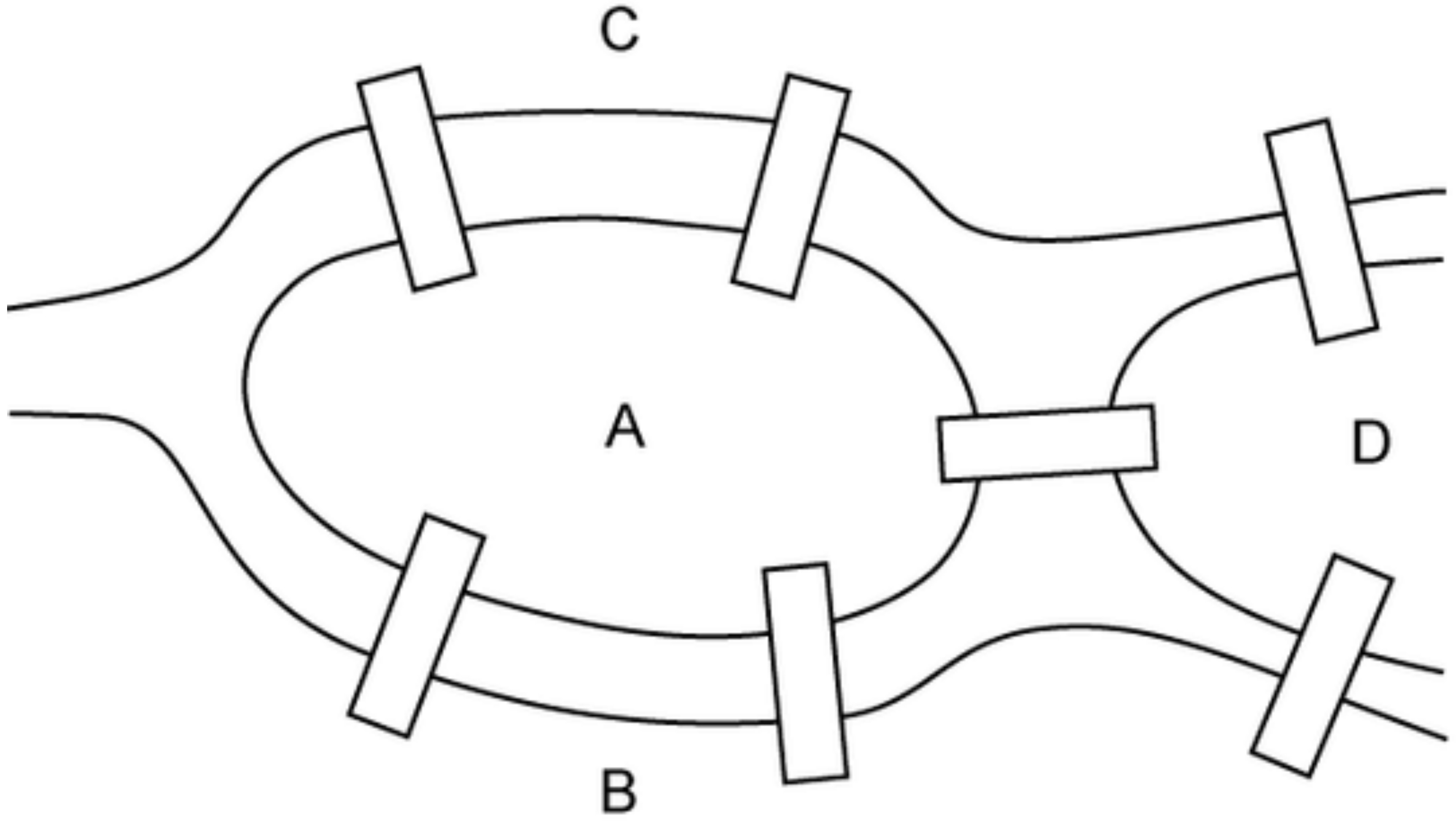
Königsberg bridges



Q: Is it possible to devise a walk through the city that would cross each of those bridges once and only once?

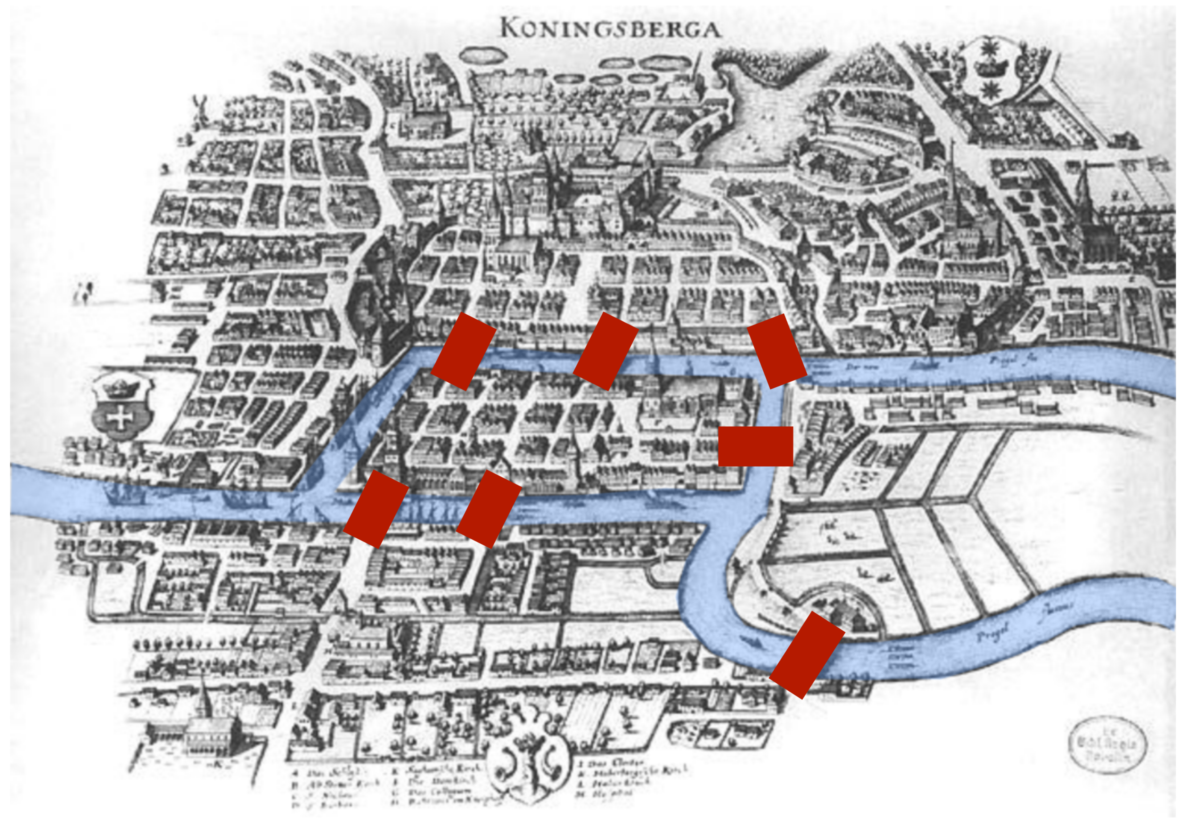
Königsberg bridges

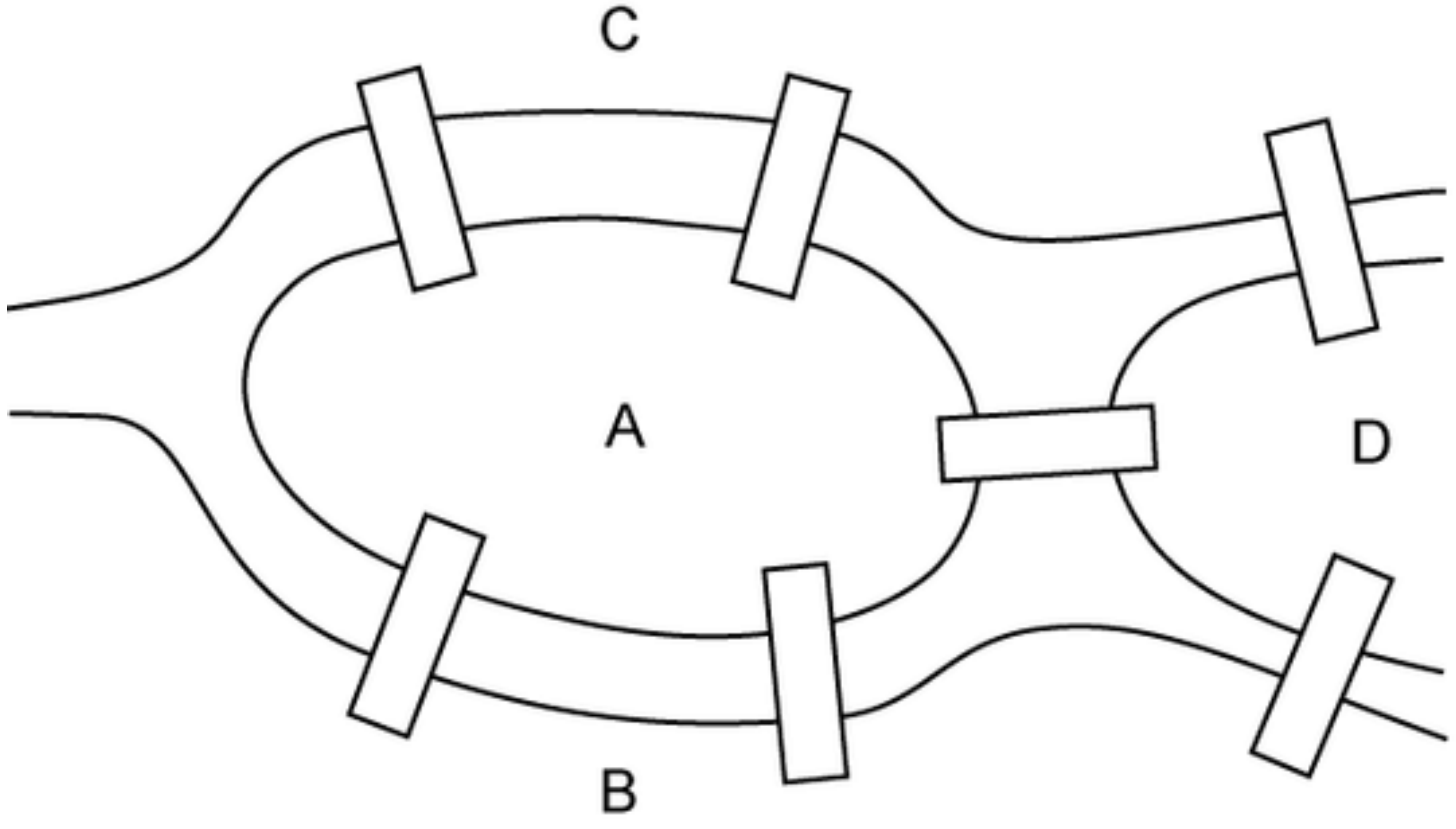


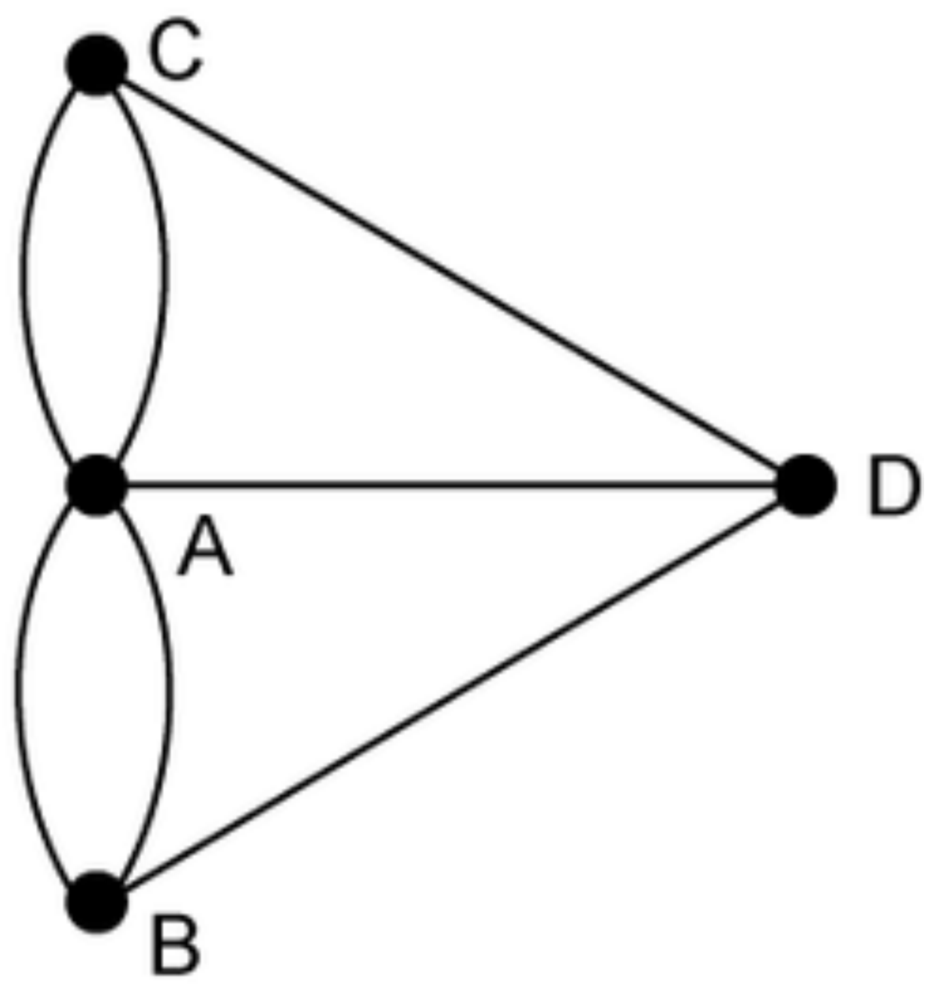


Q: Is it possible to devise a walk through the city that would cross each of those bridges once and only once?

Königsberg bridges

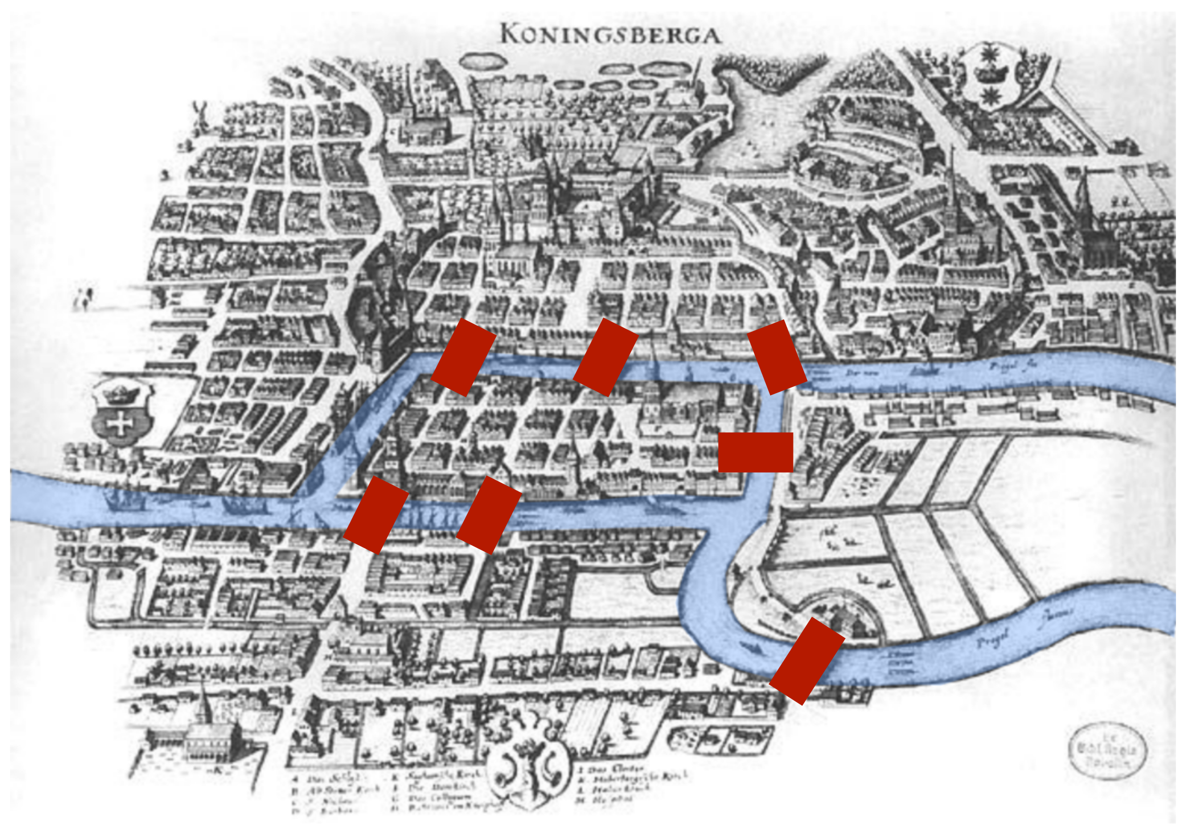


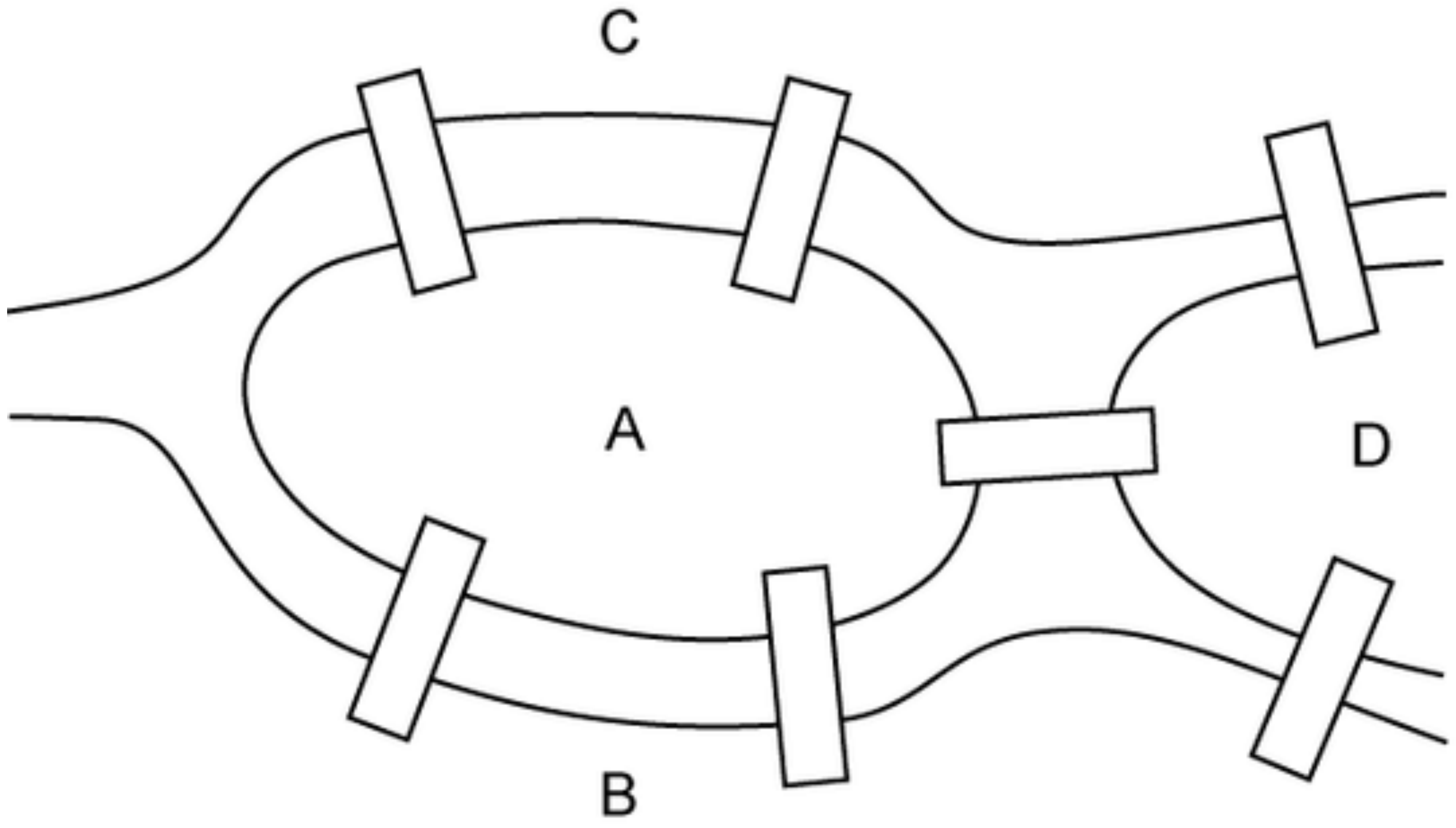


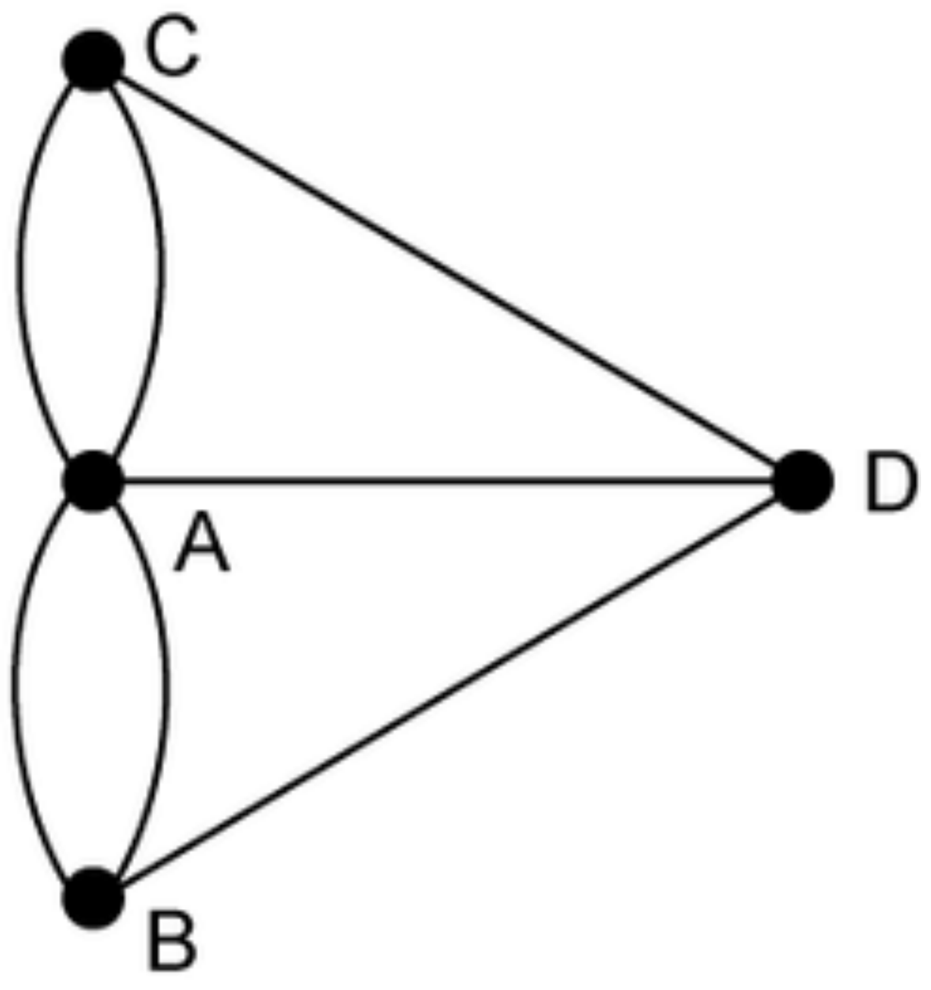


Q: Is it possible to devise a walk through the city that would cross each of those bridges once and only once?

Königsberg bridges







Q: Is it possible to devise a walk through the city that would cross each of those bridges once and only once?

A: No. At most two nodes (start, end) may have odd # of links

The history of Network Analysis

Graph theory: 1735, Euler

The history of Network Analysis

Graph theory: 1735, Euler

Social Network Research: 1930s, Moreno

Communication networks/internet: 1960s

Ecological Networks: 1979, May

The history of Network Analysis

Graph theory: 1735, Euler

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Communication networks/internet: 1960s

Ecological Networks: 1979, May

Today?

The Emergence of Network Data

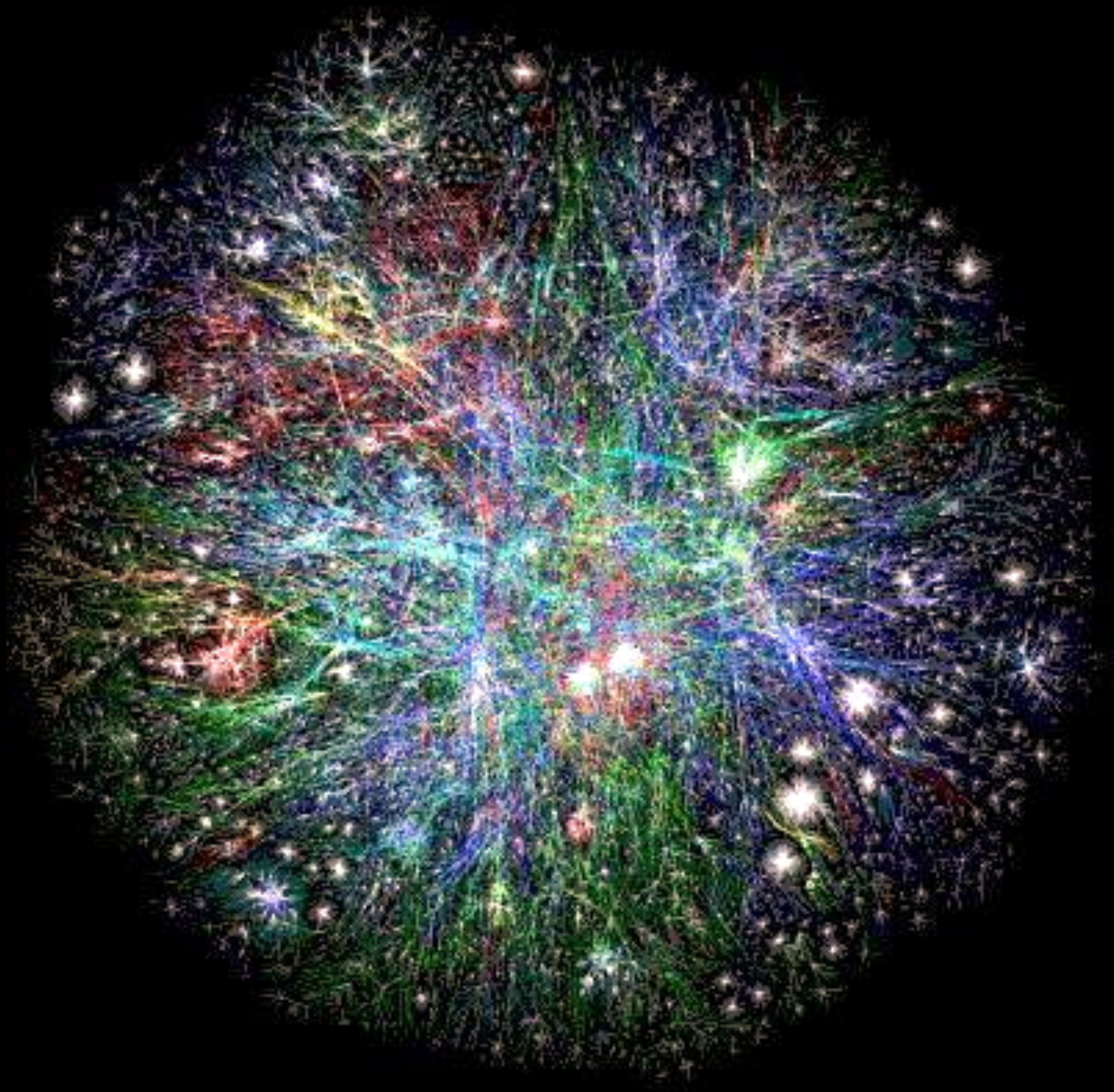
The Emergence of Network Data

Facebook

The Emergence of Network Data

Brain

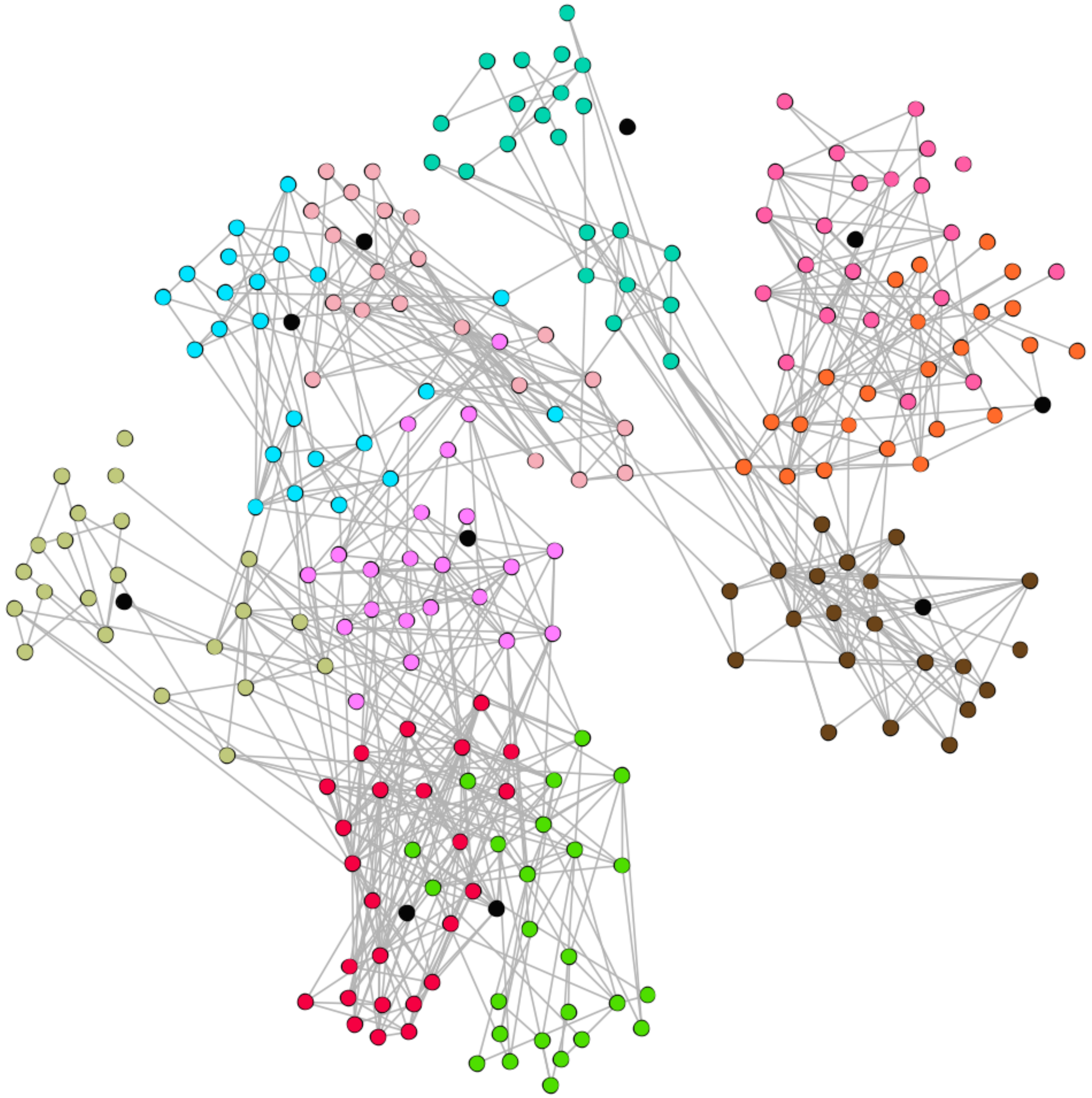
The Emergence of Network Data



WWW

The emergence of network maps

A network made of kids…



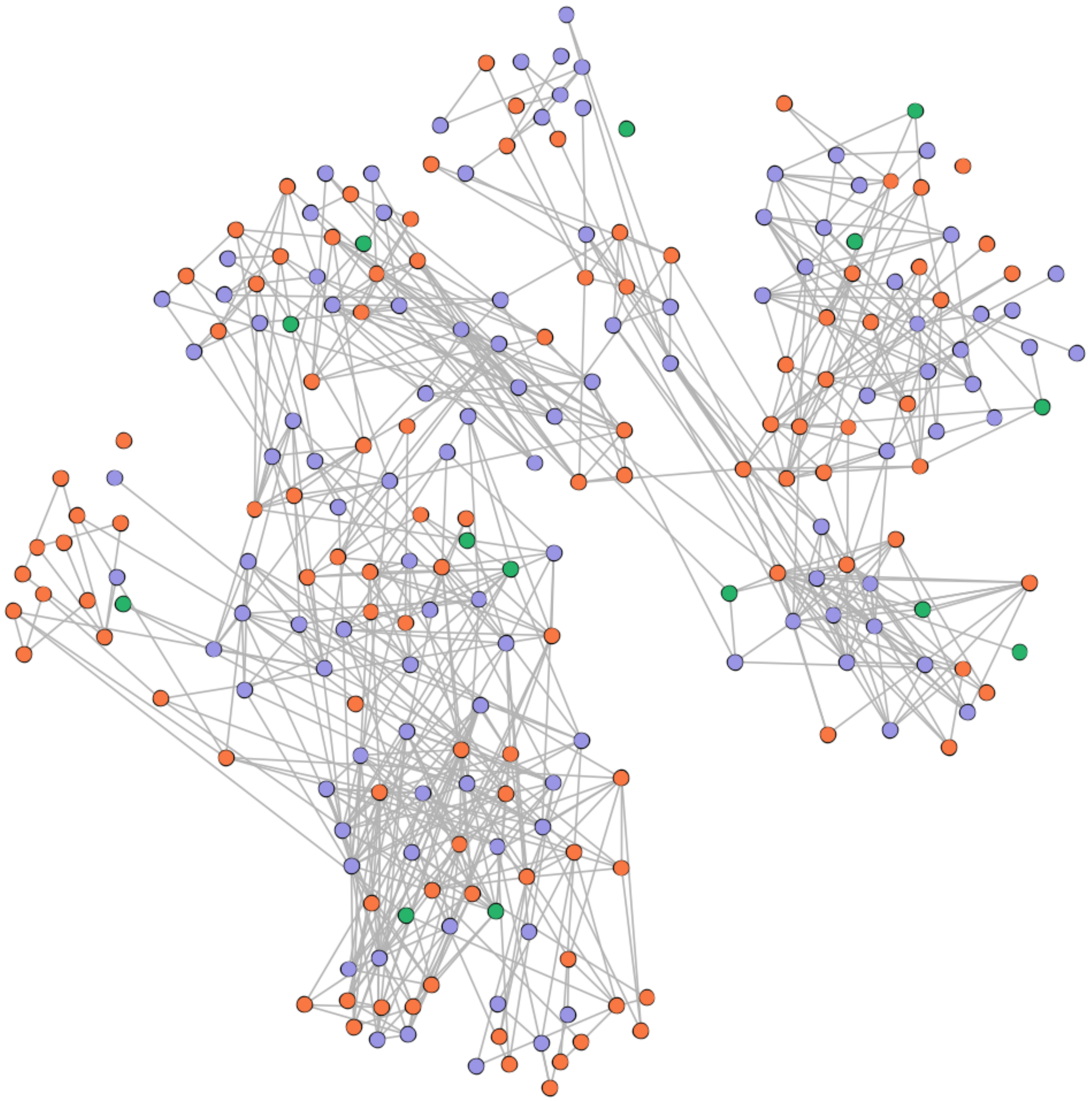
<u>Nodes</u>: Students at a primary school (1 day) + Teachers
<u>Edges</u>: Physical proximity (more the 60 secs)

<u>Legends</u>: color represent the class, teachers are represented as black nodes.

Soource: High-Resolution Measurements of Face-to-Face Contact Patterns in a Primary School.
Juliette Stehlé, Nicolas Voirin, Alain Barrat, Ciro Cattuto, Lorenzo Isella, Jean-François Pinton, Marco Quaggiotto, Wouter Van den Broeck, Corinne Régis, Bruno Lina and Philippe Vanhems. PLOS ONE 6(8): e23176 (2011).
doi:10.1371/journal.pone.0023176

The emergence of network maps

What could we ask?



<u>Nodes</u>: Students at a primary school (1 day) + Teachers
<u>Edges</u>: Physical proximity (more the 60 secs)

<u>Legends</u>: color represent the gender (green = unkwown)

Soource: High-Resolution Measurements of Face-to-Face Contact Patterns in a Primary School.
Juliette Stehlé, Nicolas Voirin, Alain Barrat, Ciro Cattuto, Lorenzo Isella, Jean-François Pinton, Marco Quaggiotto, Wouter Van den Broeck, Corinne Régis, Bruno Lina and Philippe Vanhems. PLOS ONE 6(8): e23176 (2011).
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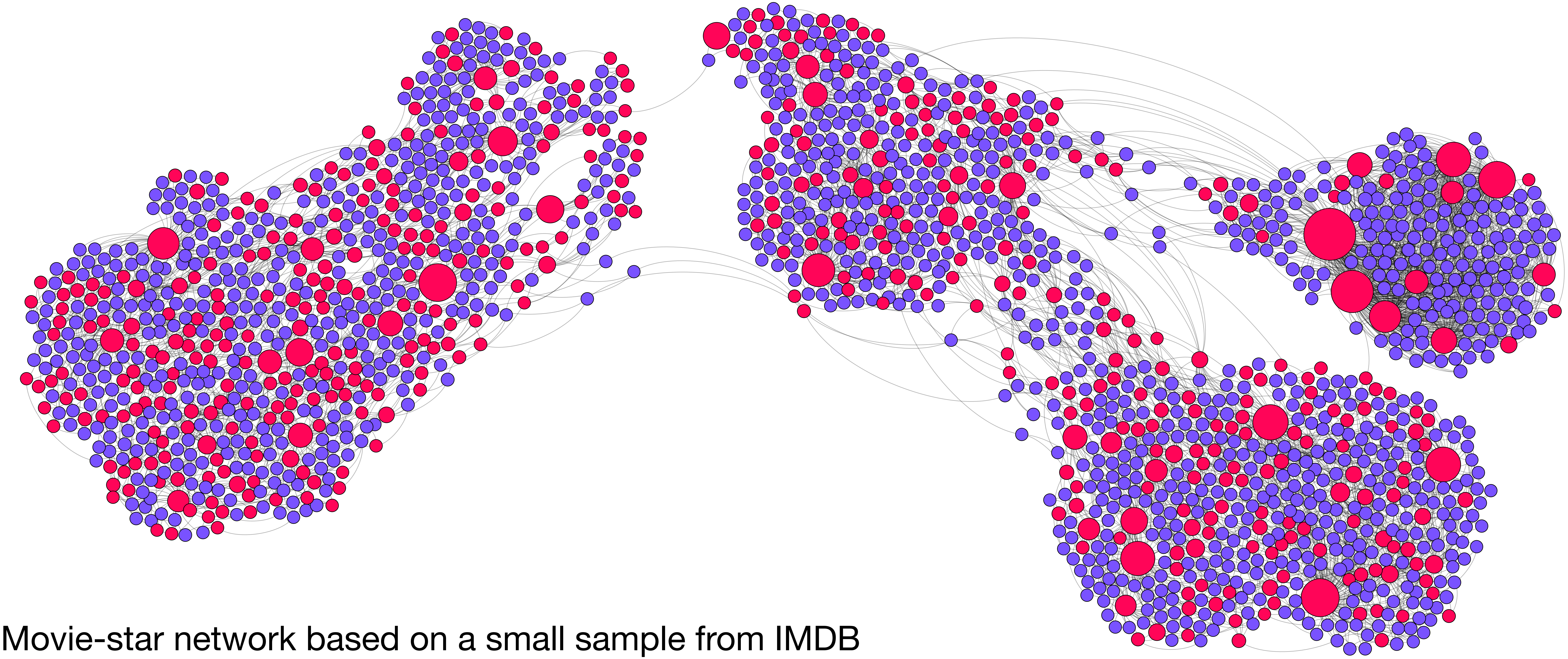
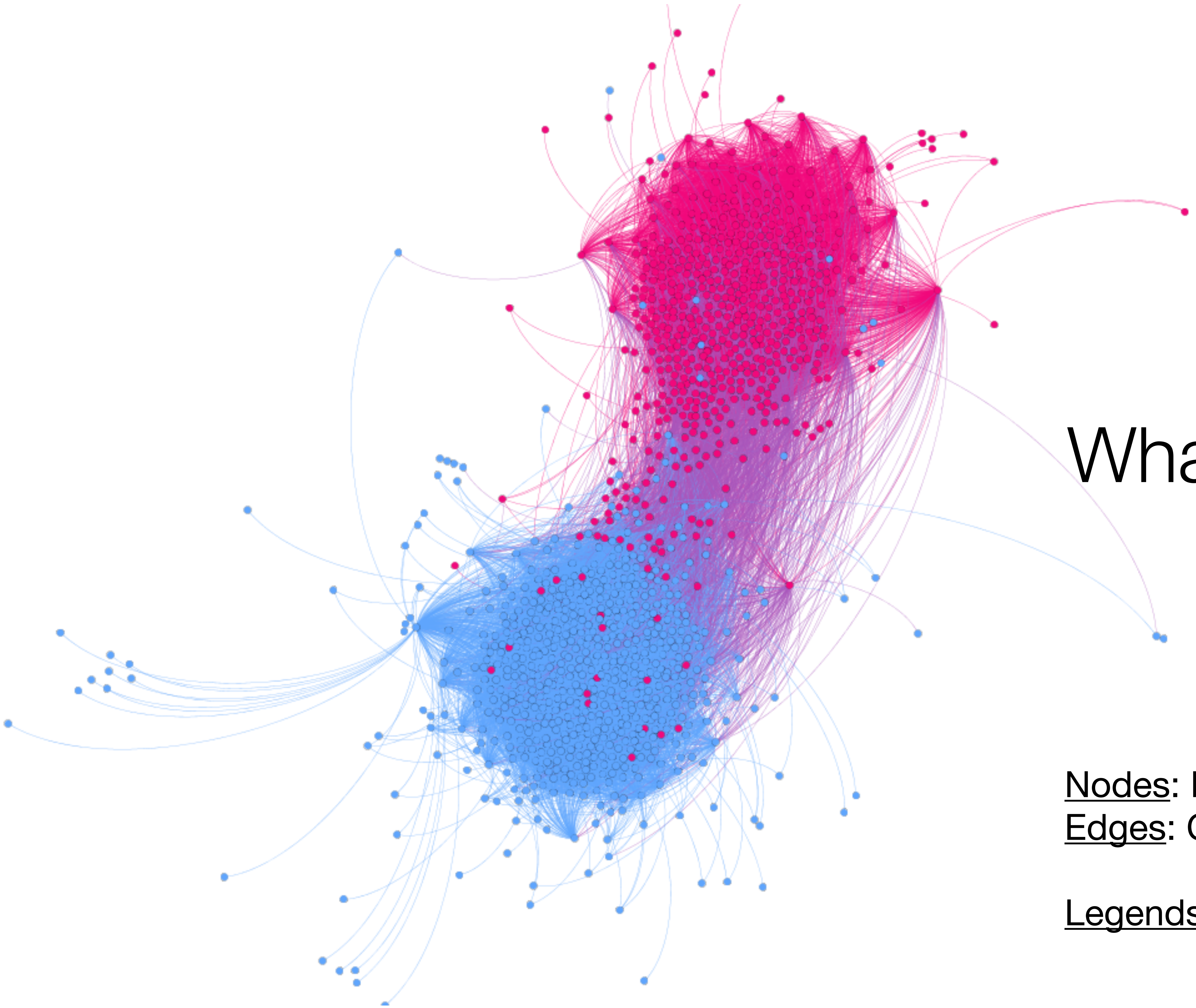
Give me a network!



Movie-star network based on a small sample from IMDB
Blue nodes represent movies
Red nodes represent actors

What's the difference?
What could we ask?

More networks



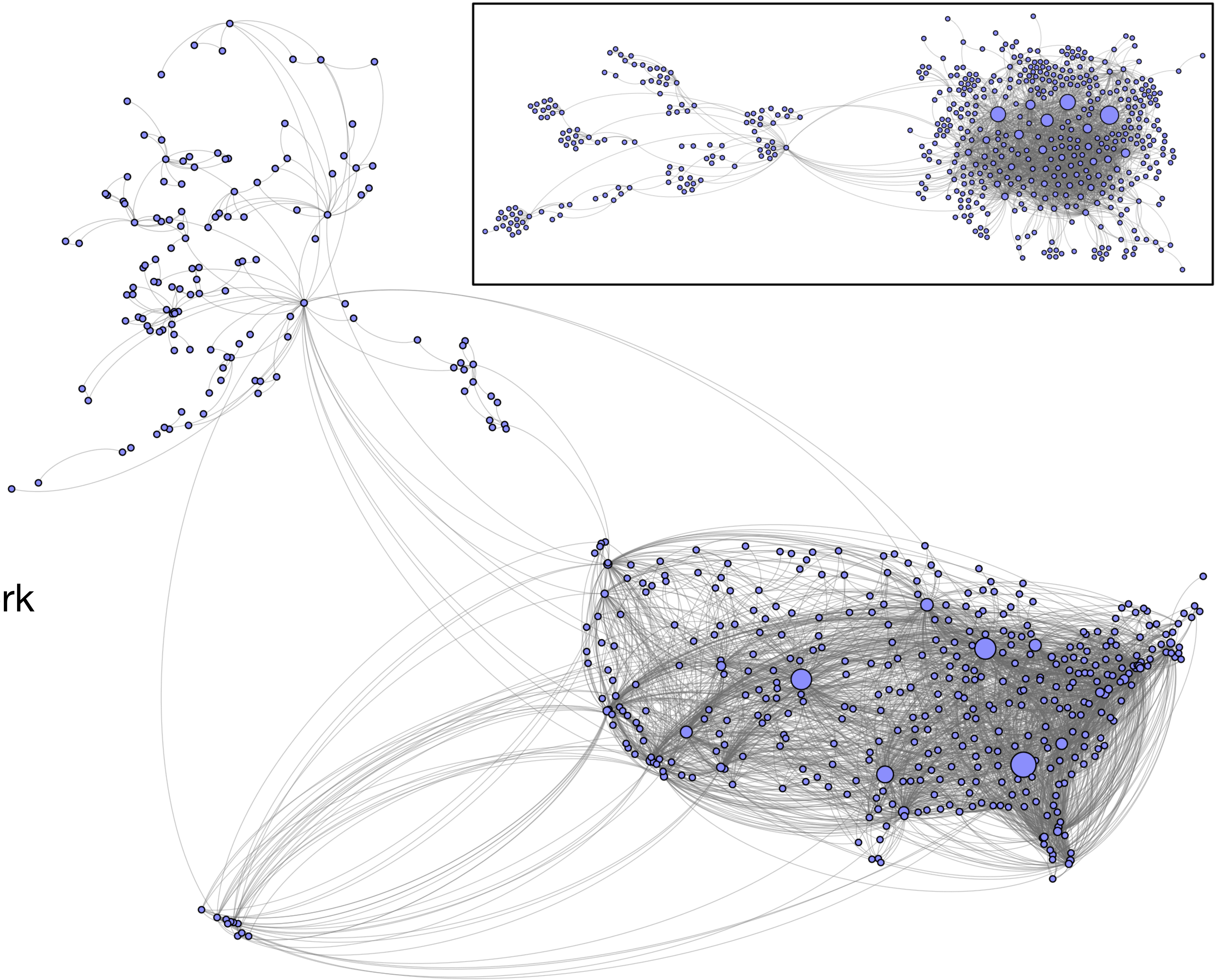
What could we ask?

Nodes: Political Blogs (US 2004)
Edges: Co-link

Legends: color represent the political leaning (Rep / Dem)

Source: Adamic, L. A., & Glance, N. (2005, August). The political blogosphere and the 2004 US election: divided they blog. In *Proceedings of the 3rd international workshop on Link discovery* (pp. 36-

More networks

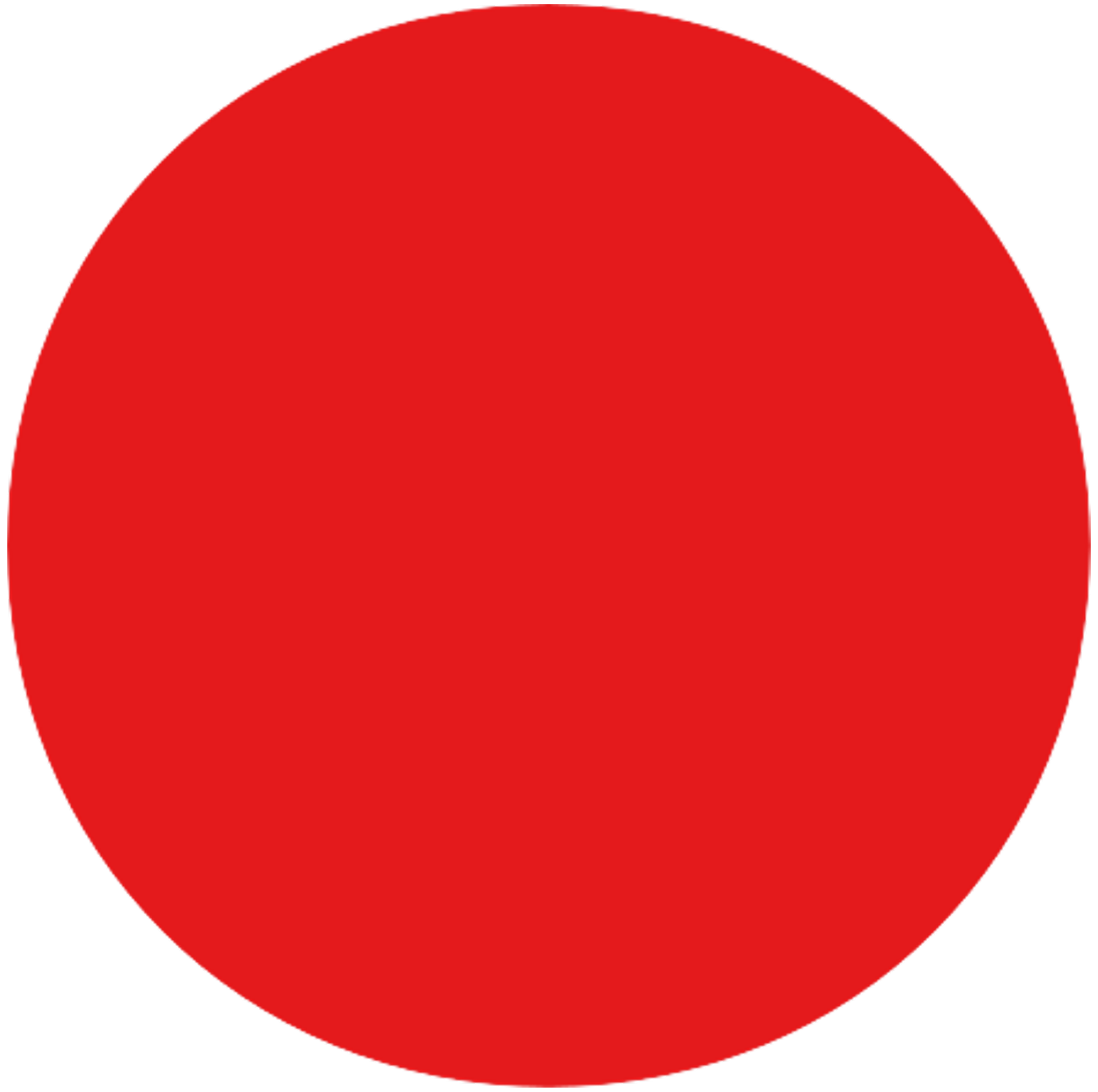


US air transportation network
What do nodes represent?
What do links represent?
What can we ask?

Definitions

Node

Vertex



Actor

Entity

From Michele Coscia, ITU Network Analysis course. https://www.networkatlas.eu

Notation

G = graph

V = set of vertices

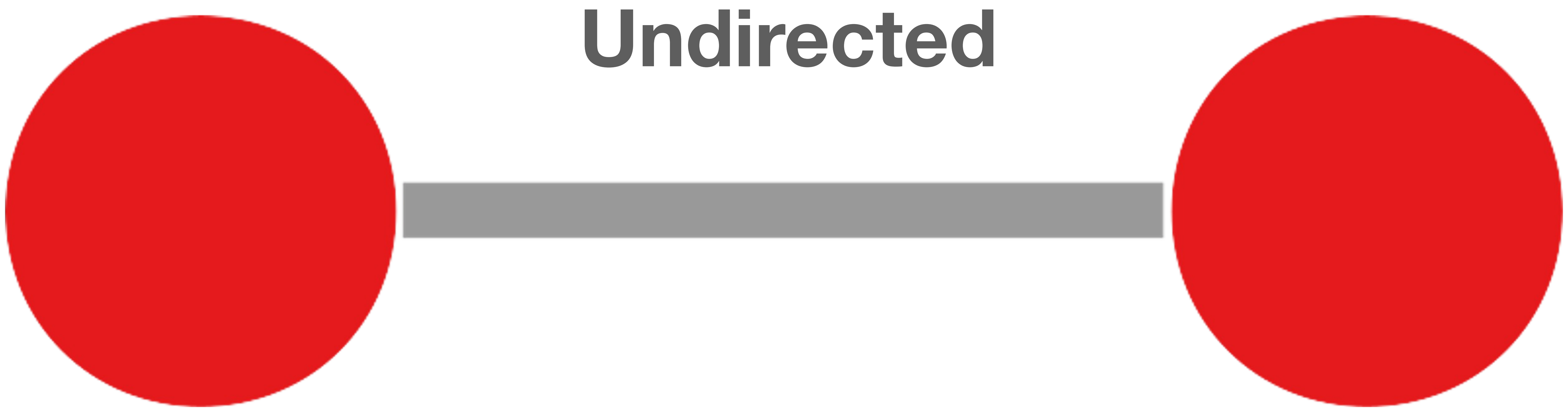
v, u = single vertices

N or |V| = number of vertices

Definitions

Edge

Link



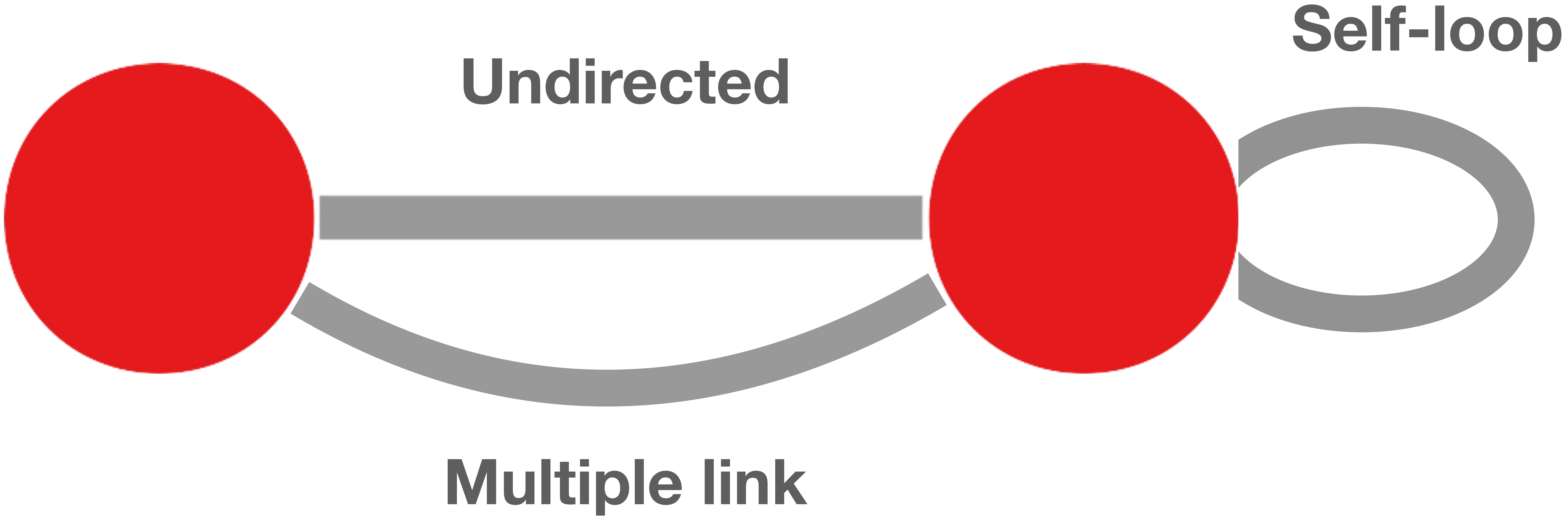
Connection

Arc

Definitions

Edge

Link



Connection

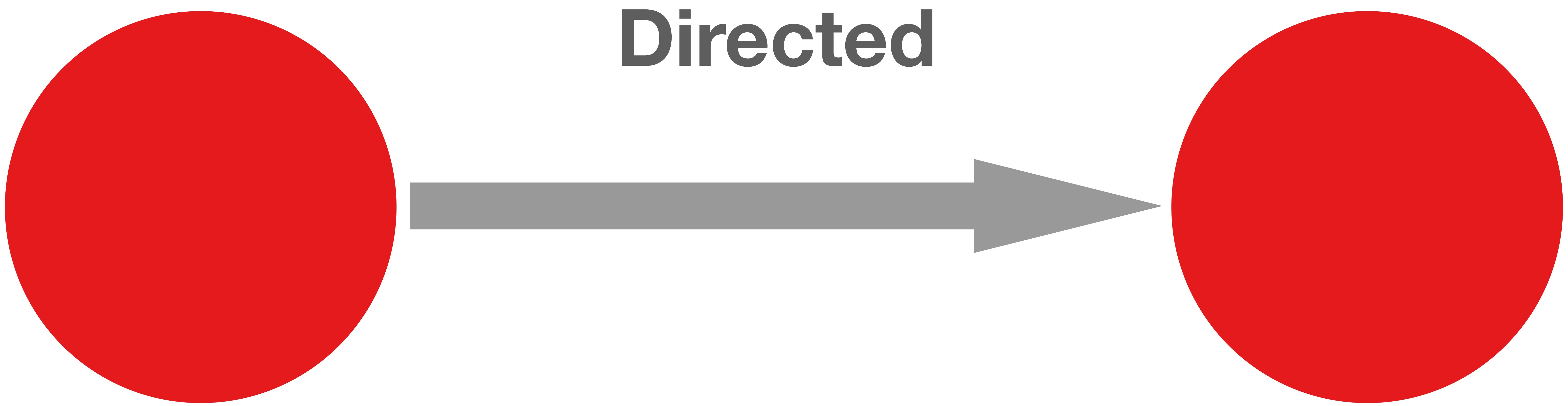
Arc

From Michele Coscia, ITU Network Analysis course. https://www.networkatlas.eu

Definitions

Edge

Link



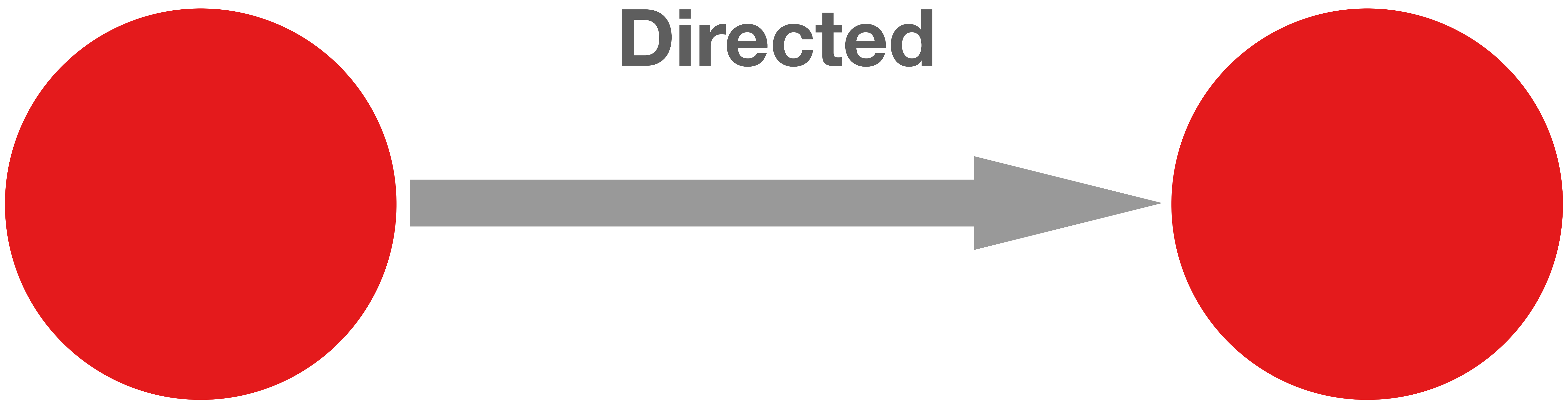
Connection

Arc

Definitions

Edge

Link



Complicating everything since 1735!
Only use if needed!

Connection

Arc

Notation

E = set of edges

(u, v) = single edge: a pair of vertices

L or (|E|) = number of edges

So: G = (V, E)

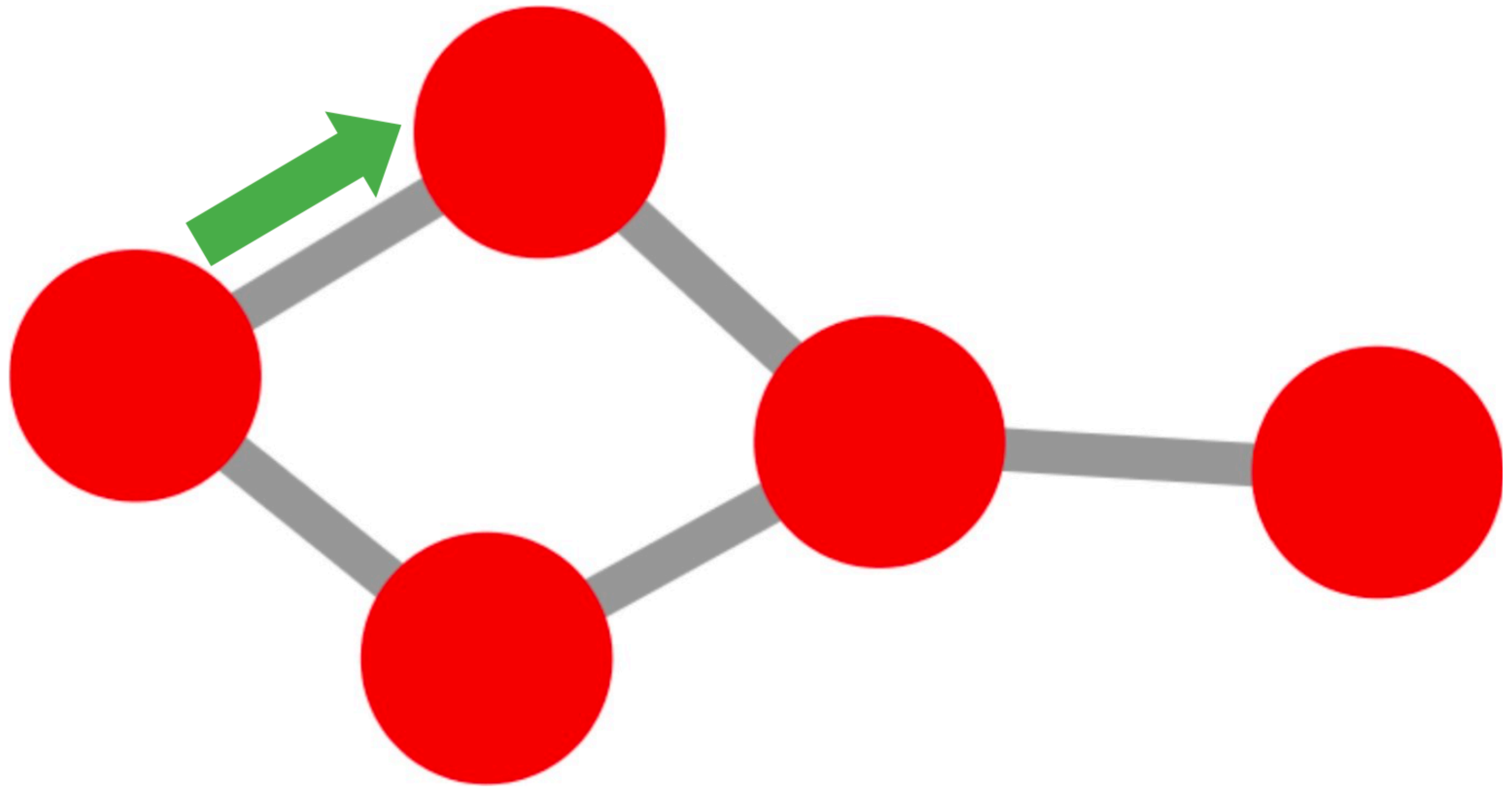
A **Graph** is a set of nodes and a set of edges
Digraph if edges are directed

Network	**Graph**
Node	**V**ertex
Link	**E**dge

I will mostly use i, j to refer to nodes

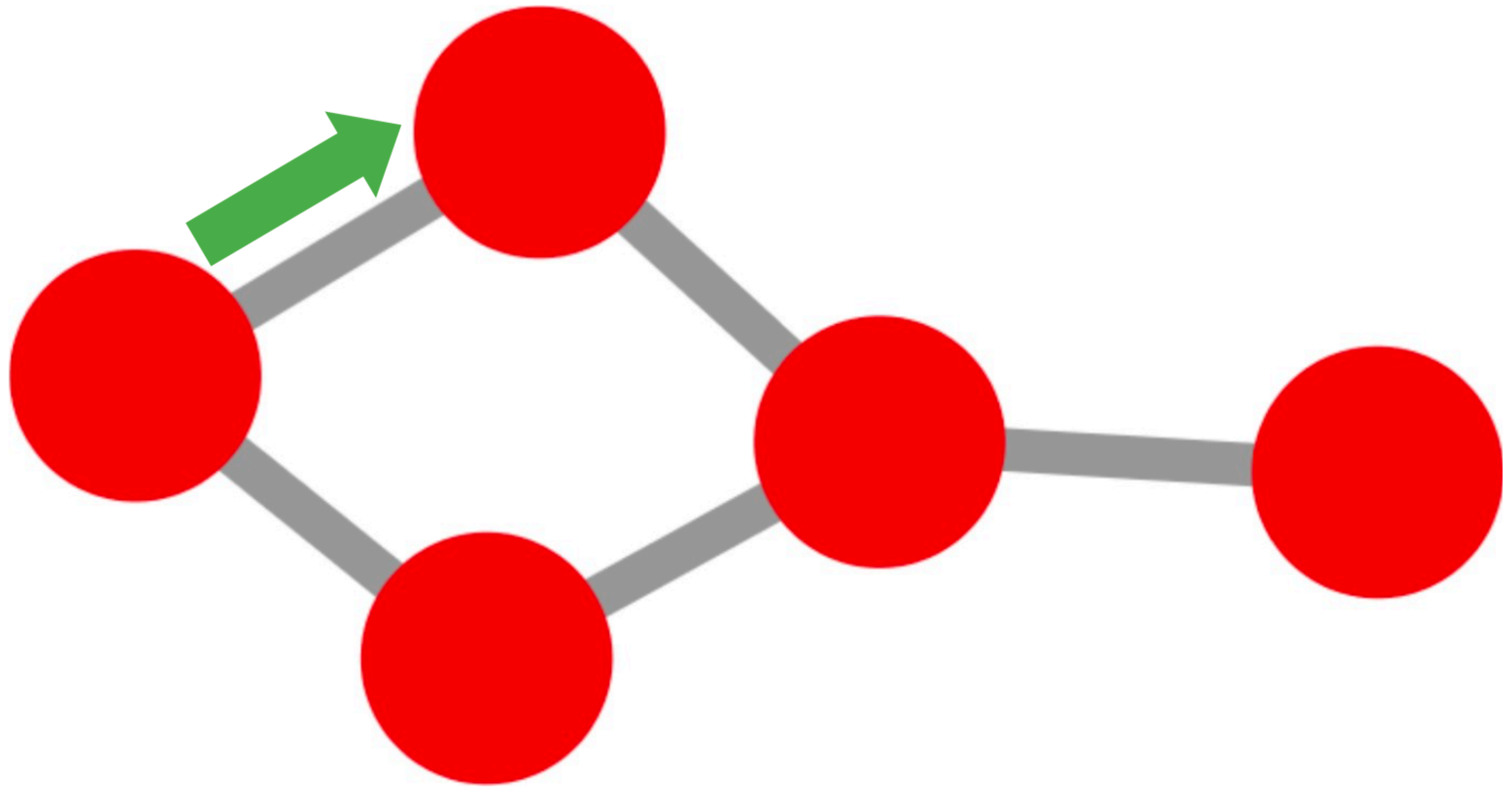
Definitions

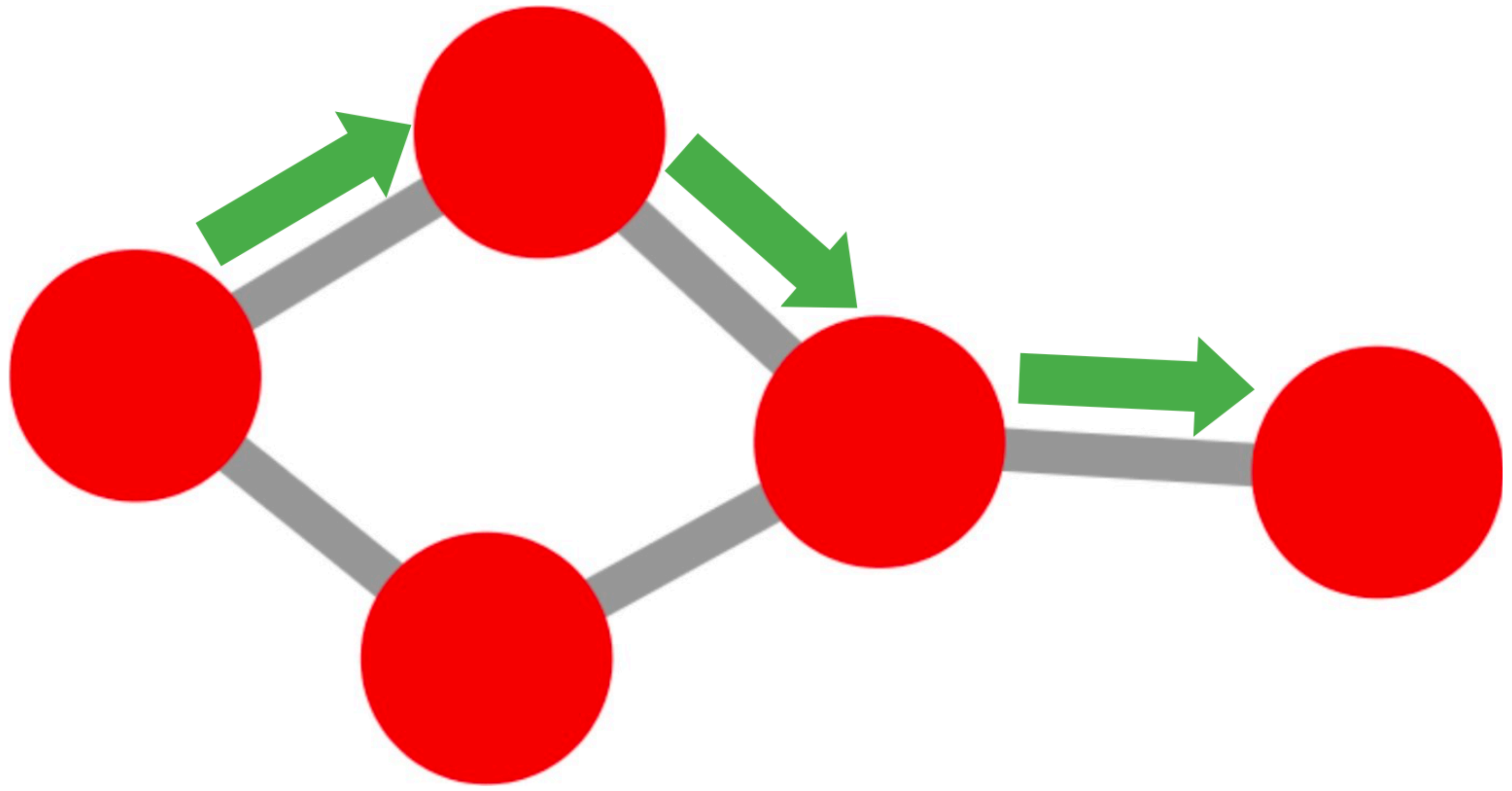
Path: (walk, trail, …)



Definitions

Path: (walk, trail, …)





Definitions

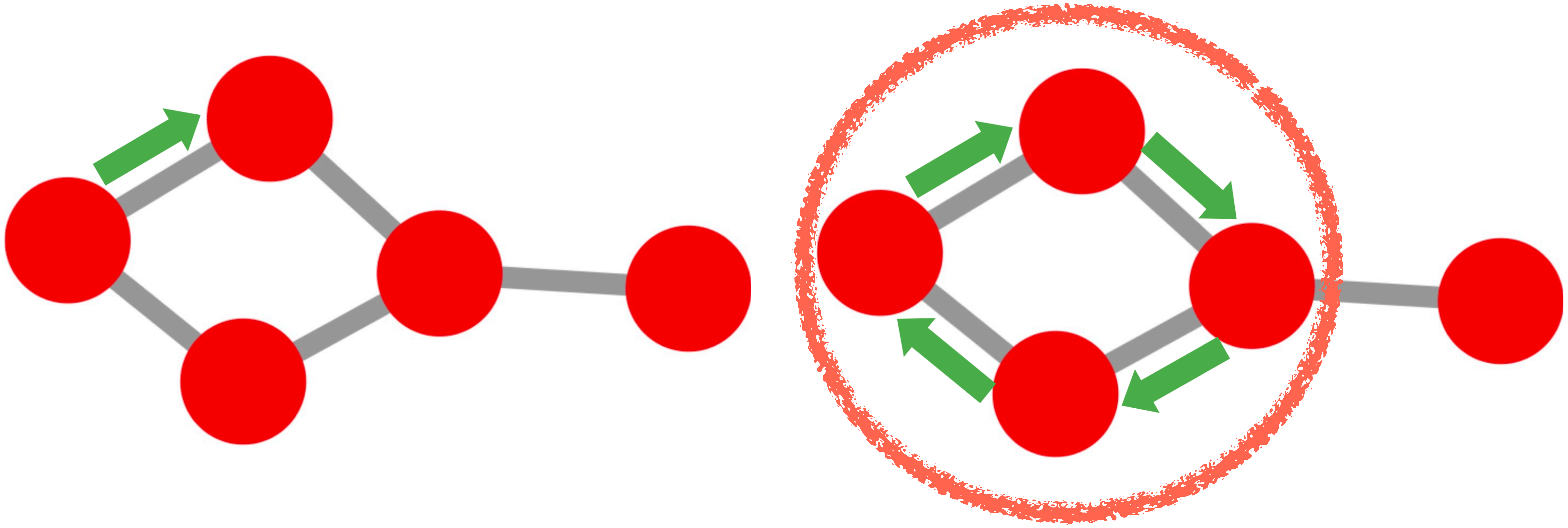
How many edges visited? Path length 6

Shortest path from 1 to 5? 3 edges
{(1,2),(2,3),(3,4)}

Other definitions with different constraints: Eulerian Path, Hamiltonian Path

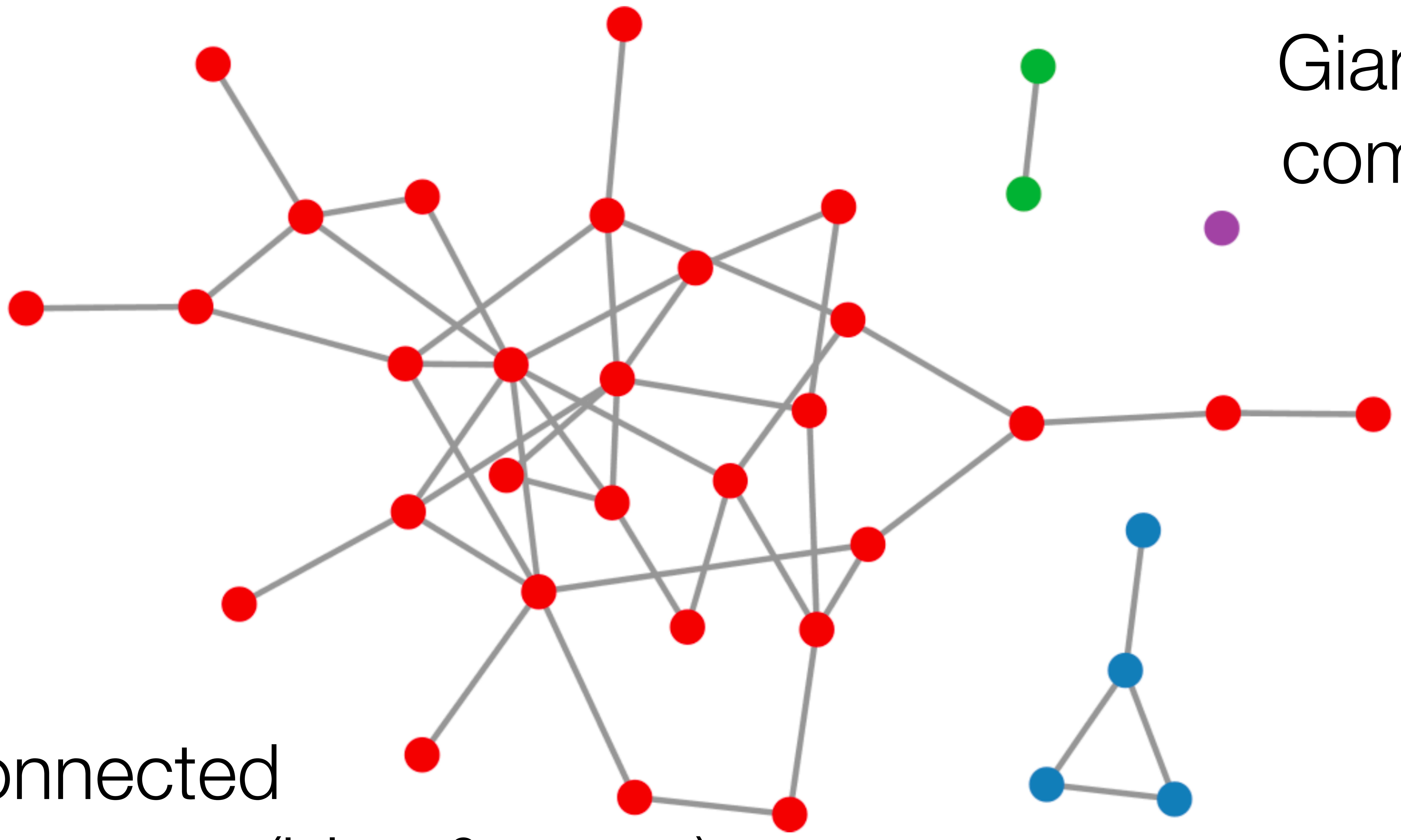
Definitions

Cycle: (Special path!)



From Michele Coscia, ITU Network Analysis course. https://www.networkatlas.eu

Definitions

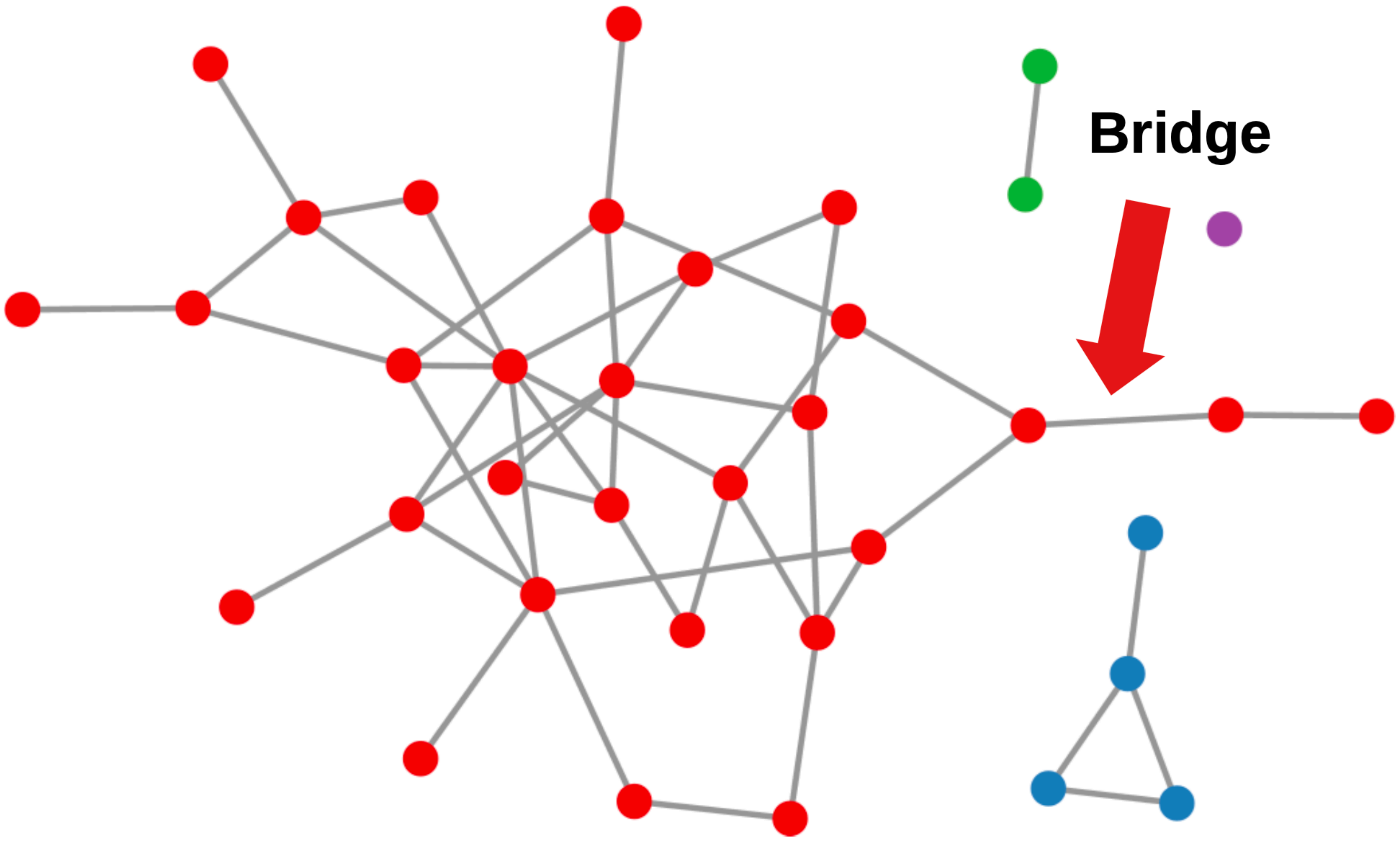


Giant connected component (red)

Disconnected components (blue & green)

From Michele Coscia, ITU Network Analysis course. https://www.networkatlas.eu

Definitions

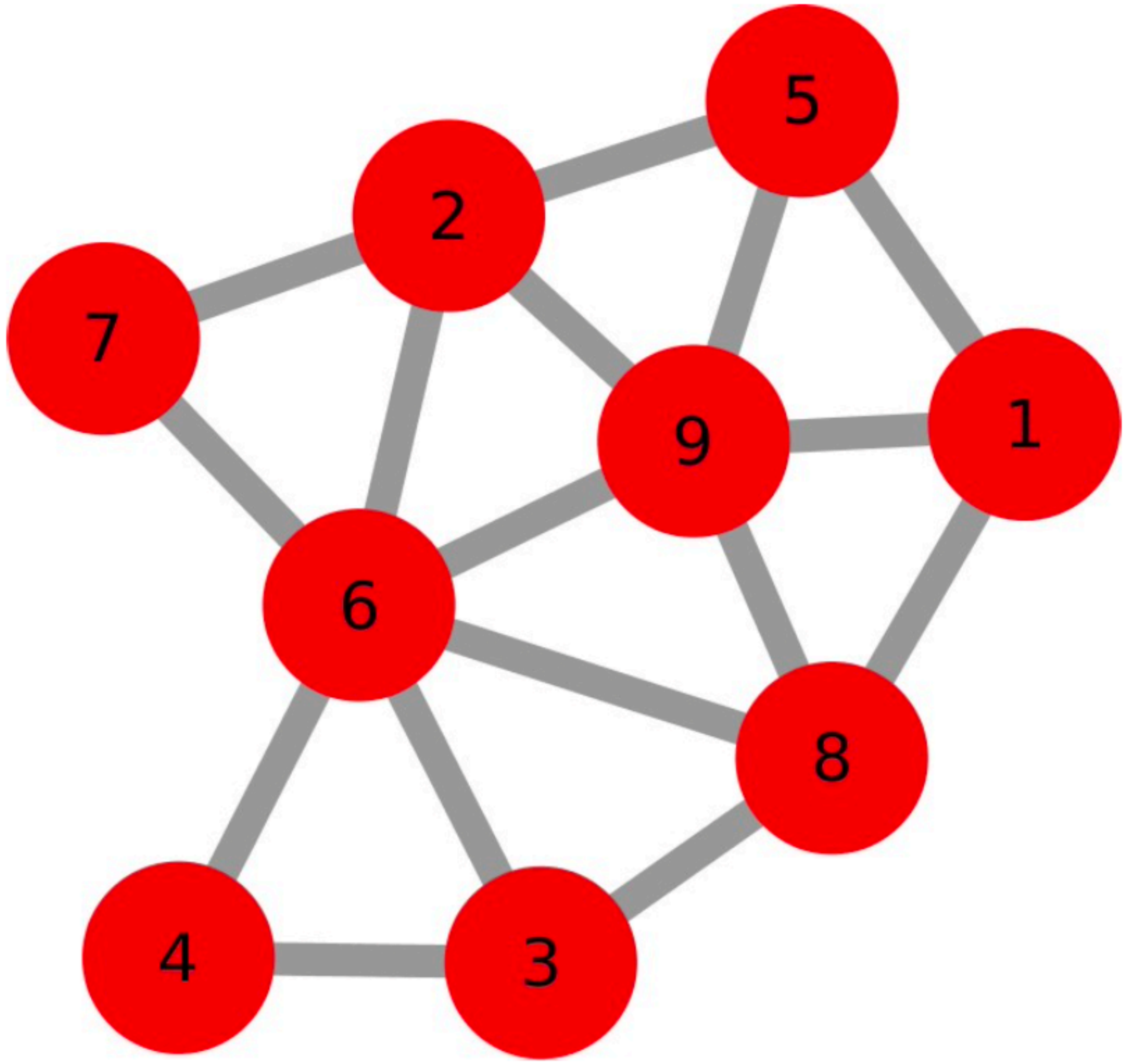


From Michele Coscia, ITU Network Analysis course. https://www.networkatlas.eu

[redacted]

Break?

Network representation



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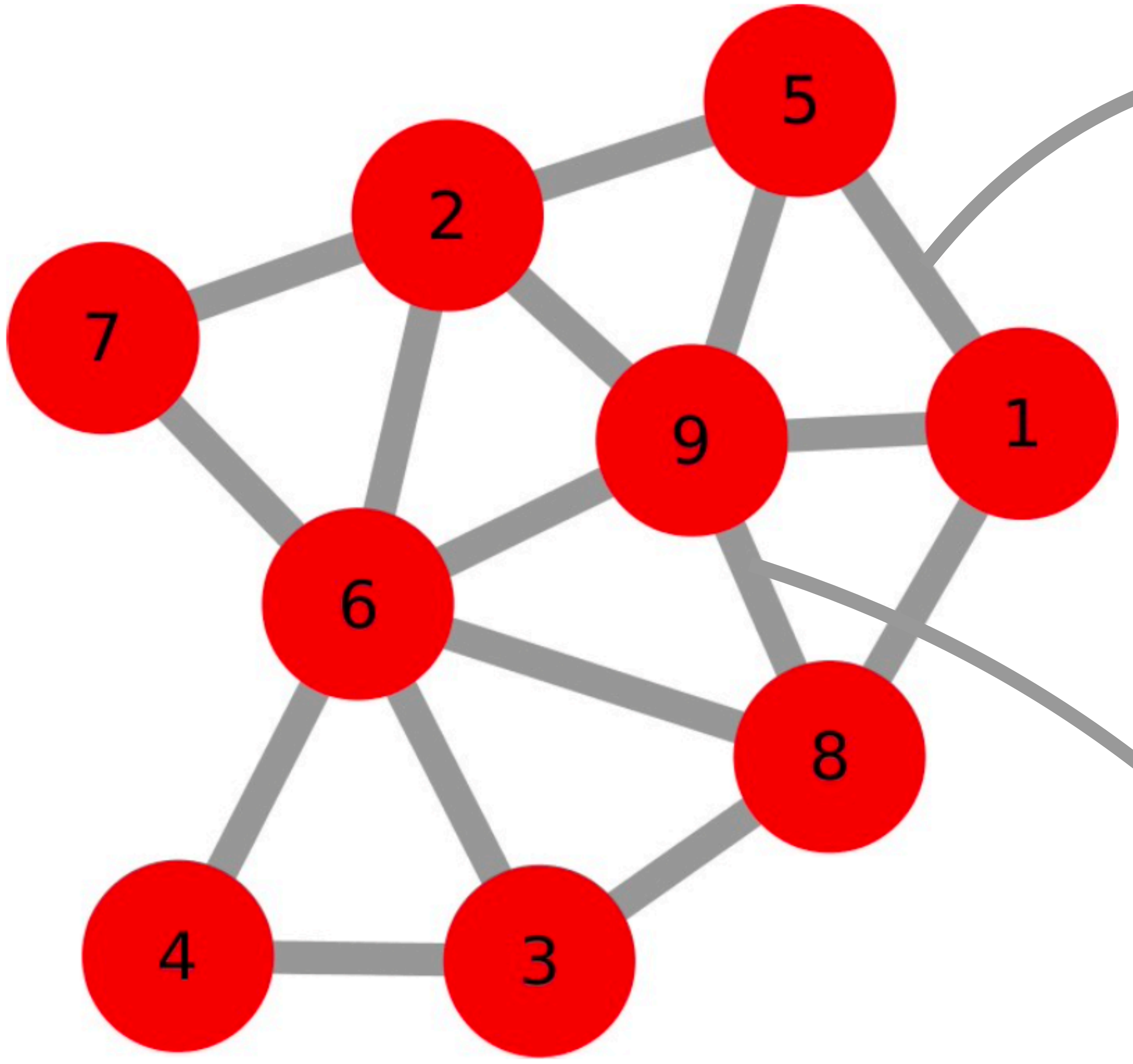
Network representation



{ (1,5),
(1,8),
(1,9),
(2,5),
(2,6),
(2,7),
(2,9),
…
(9,5),
(9,6),
(9,8) }

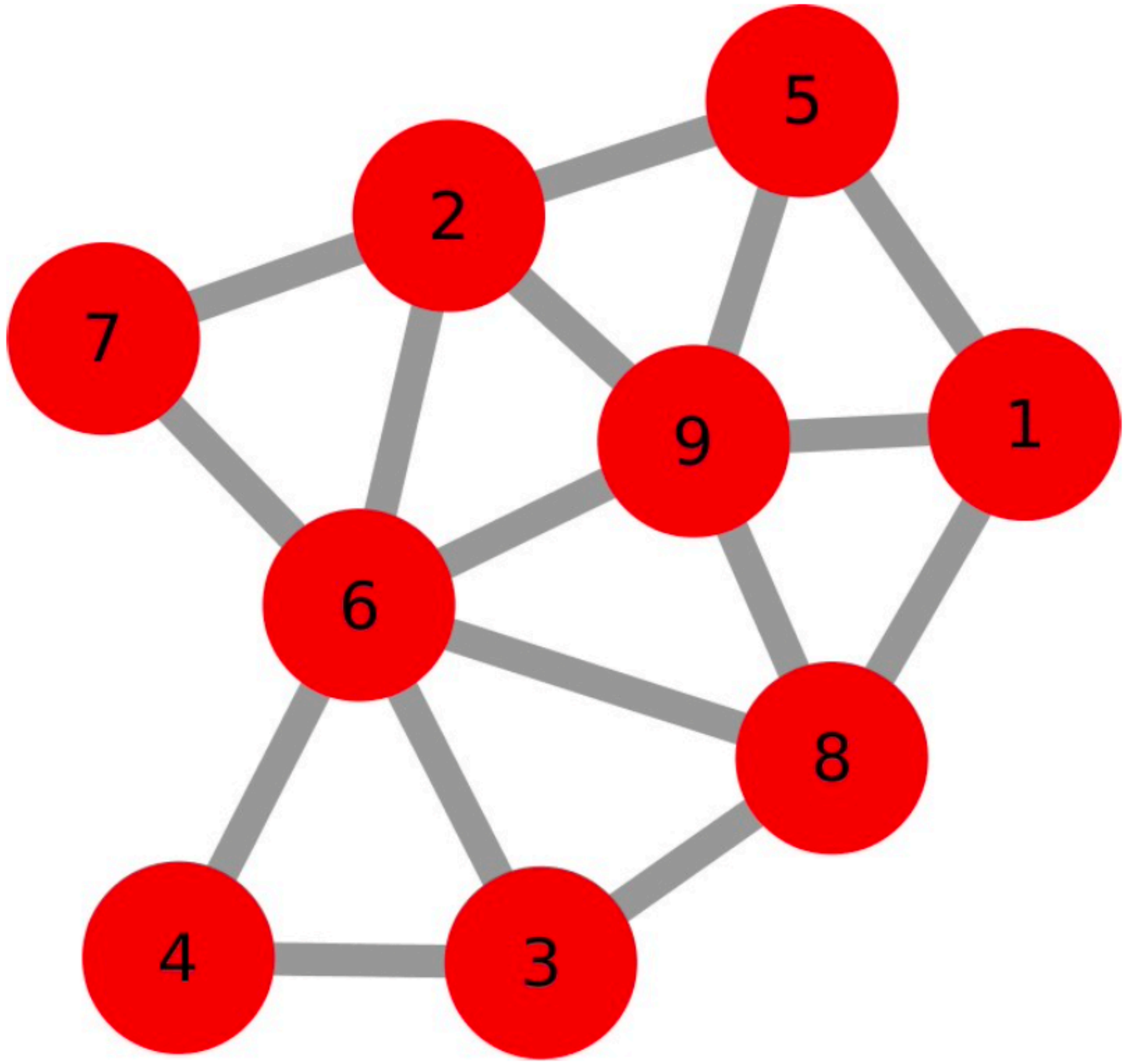
Edge list

(undirected)

pairs (i,j)

From Michele Coscia, ITU Network Analysis course. https://www.networkatlas.eu

Network representation



0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0

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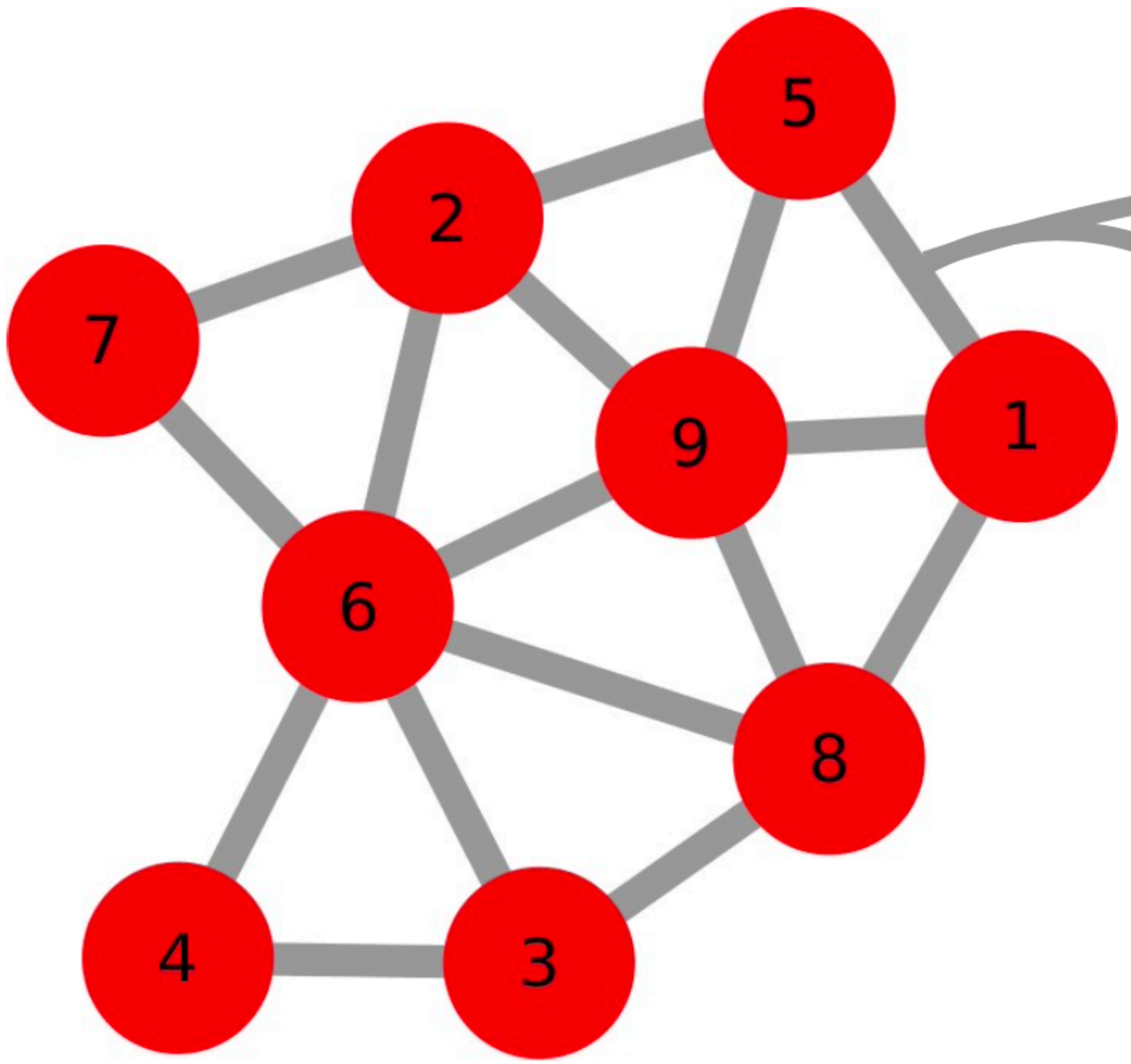
Network representation



	1	2	3	4	5	6	7	8	9
1	0	0	0	0	1	0	0	0	0
2	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0
5	1	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0

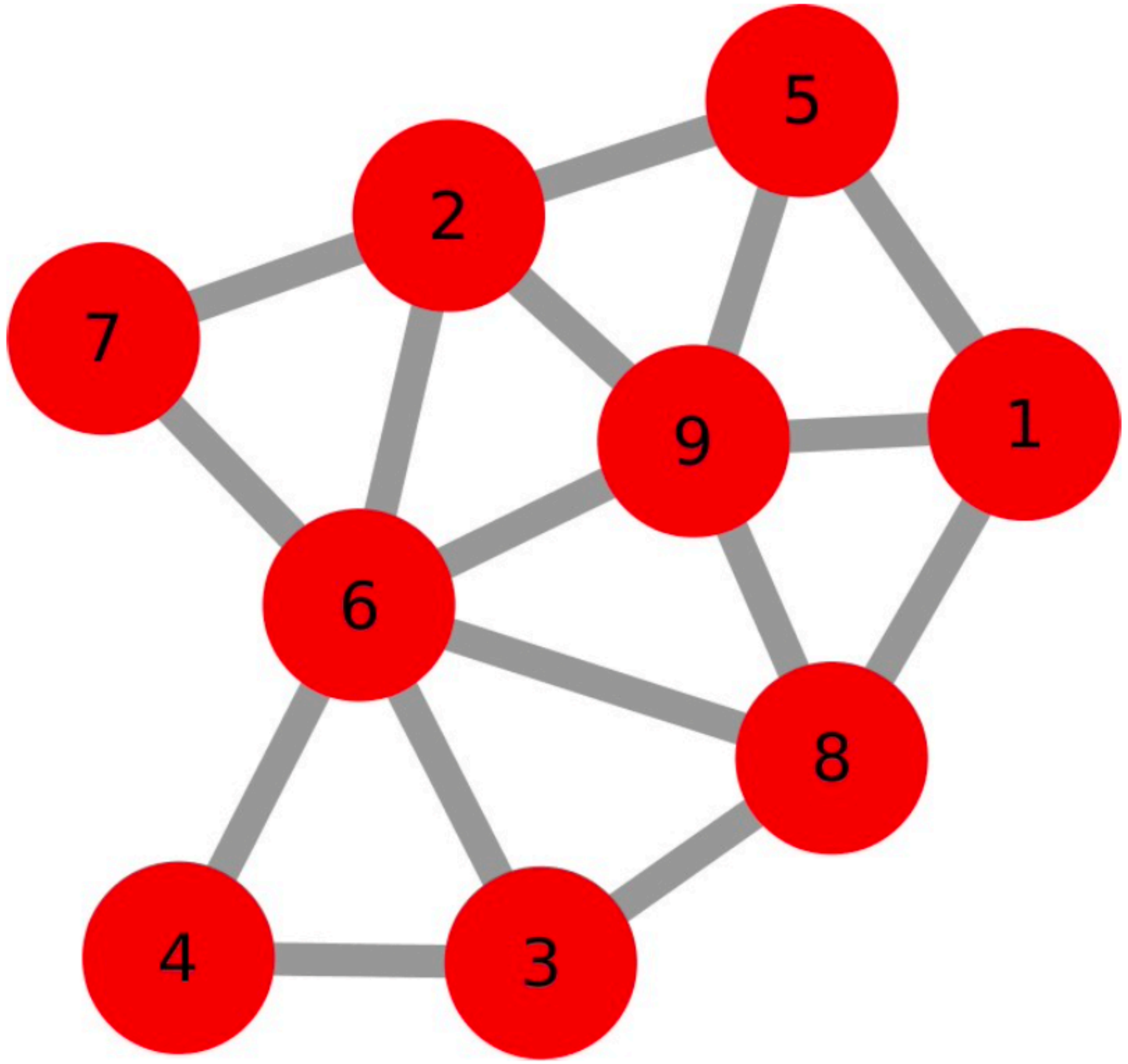
A_{ij}, A_{ji} =1
if (5,1) are connected

0
otherwise

Undirected adjacency matrix $\mathbf{A}$

From Michele Coscia, ITU Network Analysis course. https://www.networkatlas.eu

Network representation



$\mathbf{A}$ is symmetric & has diagonal = 0

	1	2	3	4	5	6	7	8	9
1	0	0	0	0	1	0	0	1	1
2	0	0	0	0	1	1	1	0	1
3	0	0	0	1	0	1	0	1	0
4	0	0	1	0	0	1	0	0	0
5	1	1	0	0	0	0	0	0	1
6	0	1	1	1	0	0	1	1	1
7	0	1	0	0	0	1	0	0	0
8	1	0	1	0	0	1	0	0	1
9	1	1	0	0	1	1	0	1	0

Undirected adjacency matrix $\mathbf{A}$

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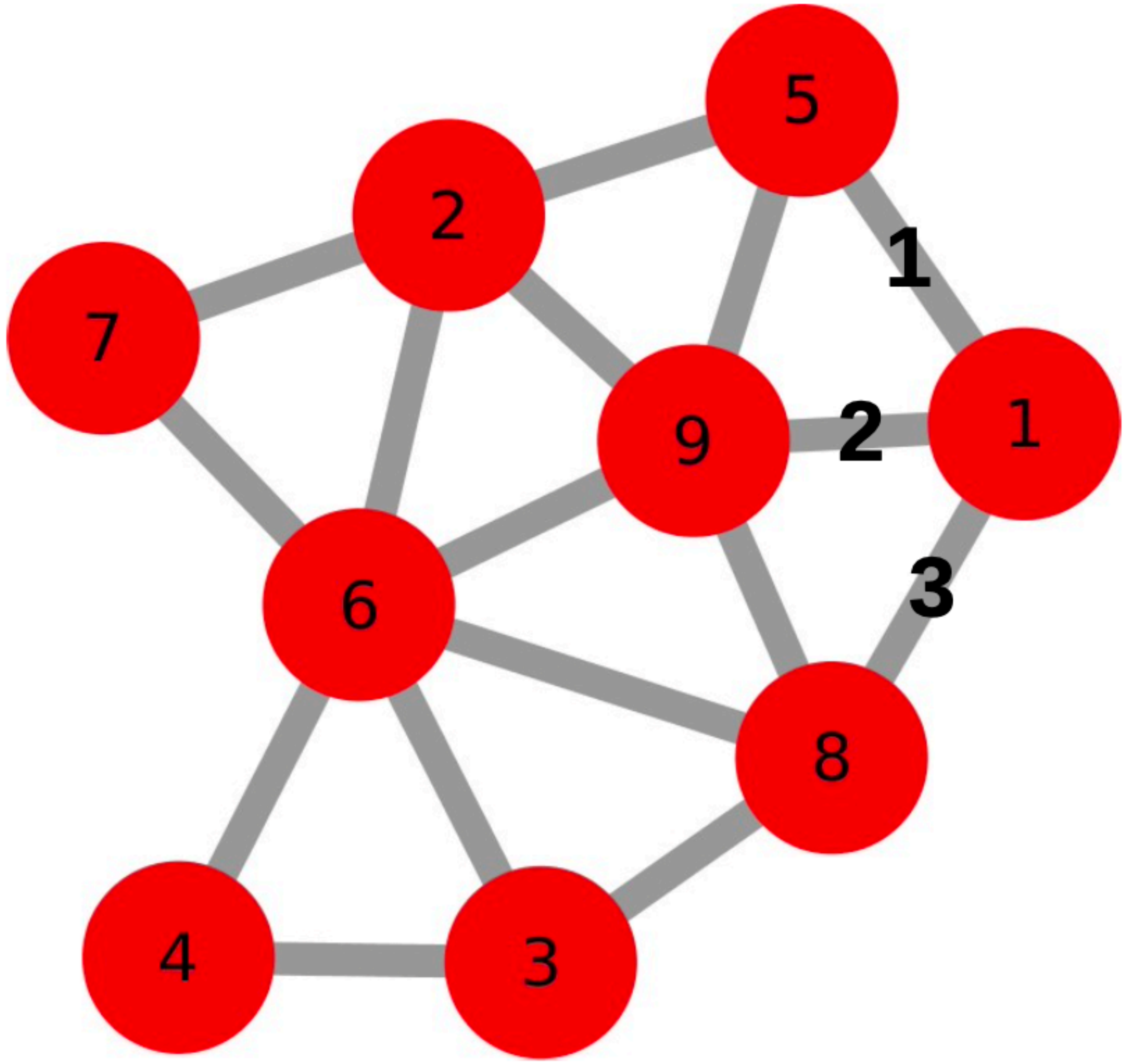
Network representation



A is squared: **NxN**

0	0	0	0	1	0	0	1	1
0	0	0	0	1	1	1	0	1
0	0	0	1	0	1	0	1	0
0	0	1	0	0	1	0	0	0
1	1	0	0	0	0	0	0	1
0	1	1	1	0	0	1	1	1
0	1	0	0	0	1	0	0	0
1	0	1	0	0	1	0	0	1
1	1	0	0	1	1	0	1	0

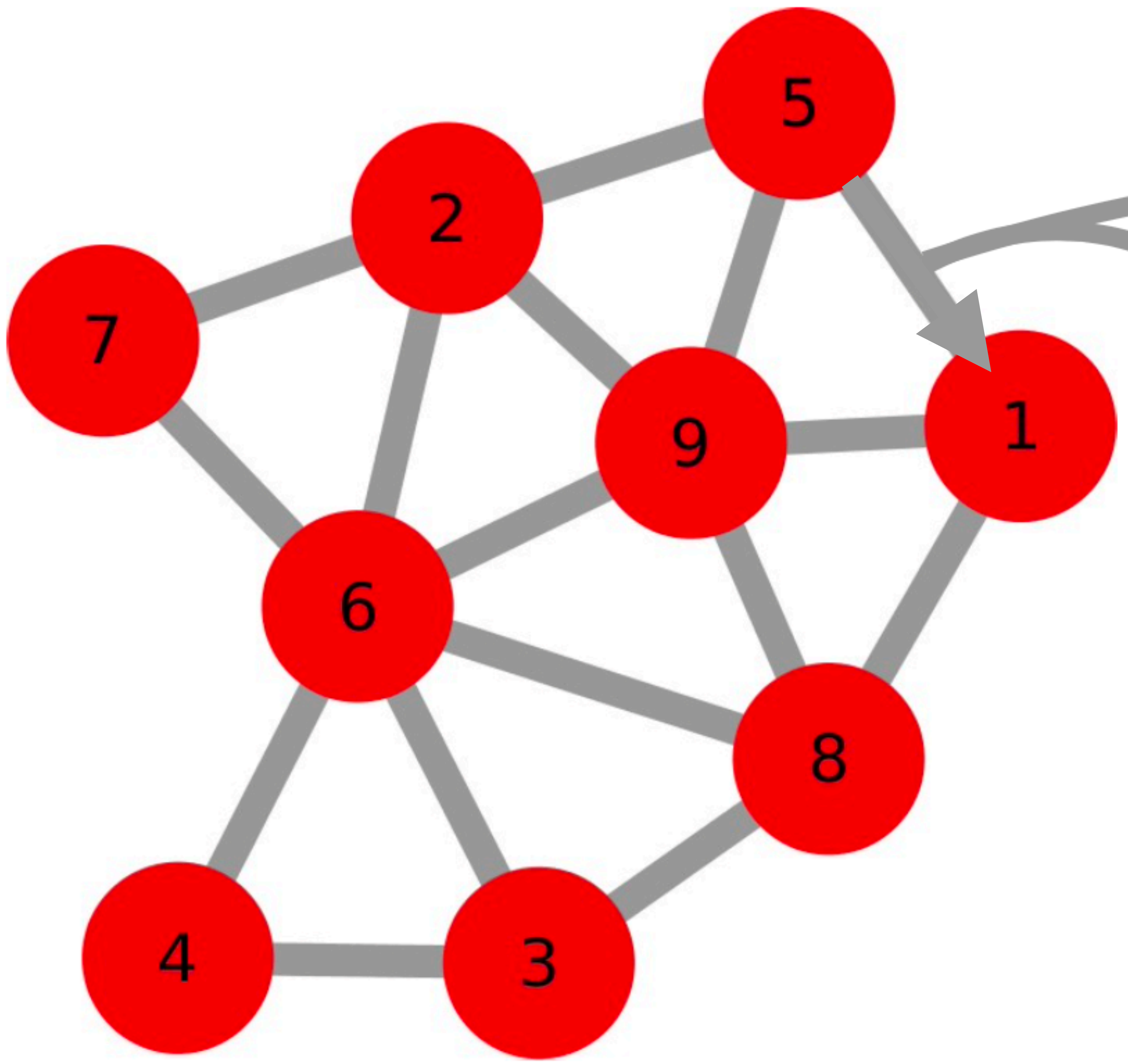
3

Links of node i

Undirected adjacency matrix **A**

From Michele Coscia, ITU Network Analysis course. https://www.networkatlas.eu

Network representation



	1	2	3	4	5	6	7	8	9
1	0	0	0	0	**0**	0	0	0	0
2	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0
5	1	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0

A_{51} = 1
there's a link from 5 to 1

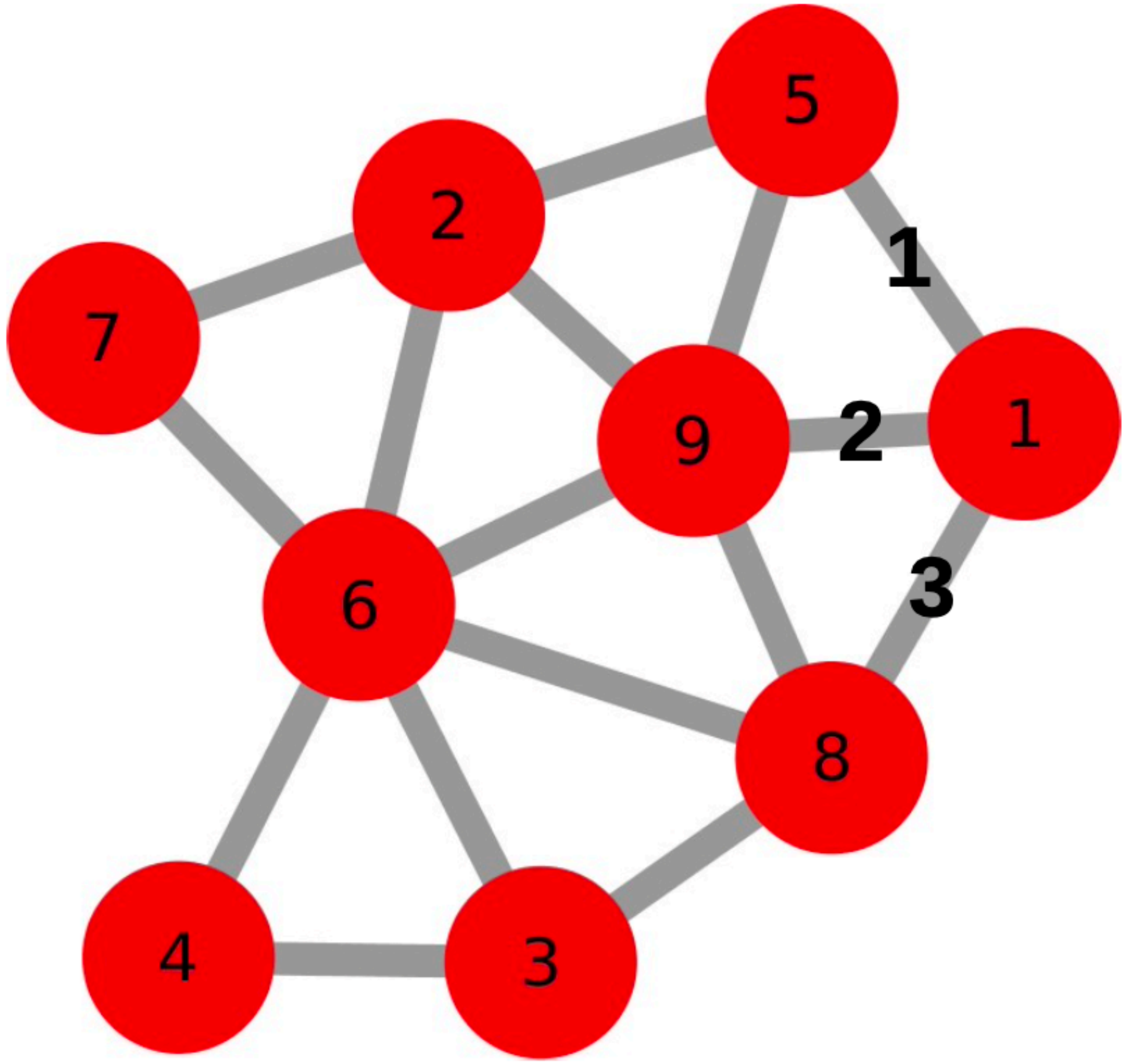
A_{15} = 0
no link from 1 to 5

Directed adjacency matrix $\mathbf{A}$

From Michele Coscia, ITU Network Analysis course. https://www.networkatlas.eu

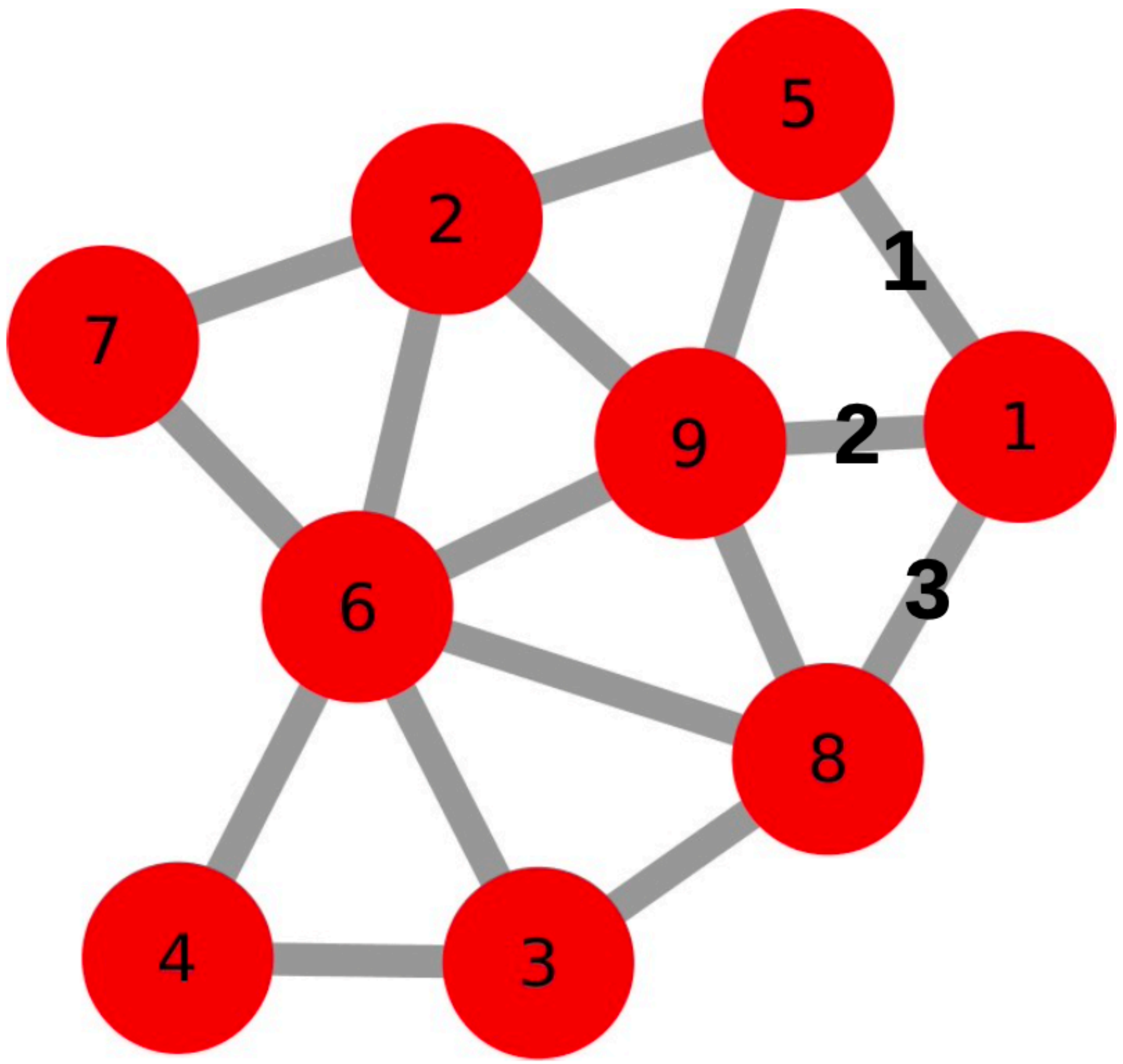
Network representation

Adding weights



0	0	0	0	0.33	0	0	0.33	0.33
0	0	0	0	0.25	0.25	0.25	0	0.25
0	0	0	0.33	0	0.33	0	0.33	0
0	0	0.5	0	0	0.5	0	0	0
0.33	0.33	0	0	0	0	0	0	0.33
0	0.17	0.17	0.17	0	0	0.17	0.17	0.17
0	0.5	0	0	0	0.5	0	0	0
0.25	0	0.25	0	0	0.25	0	0	0.25
0.2	0.2	0	0	0.2	0.2	0	0.2	0

Network representation



If you normalise the $\boldsymbol{A_i}$ entries to 1, they become probabilities.

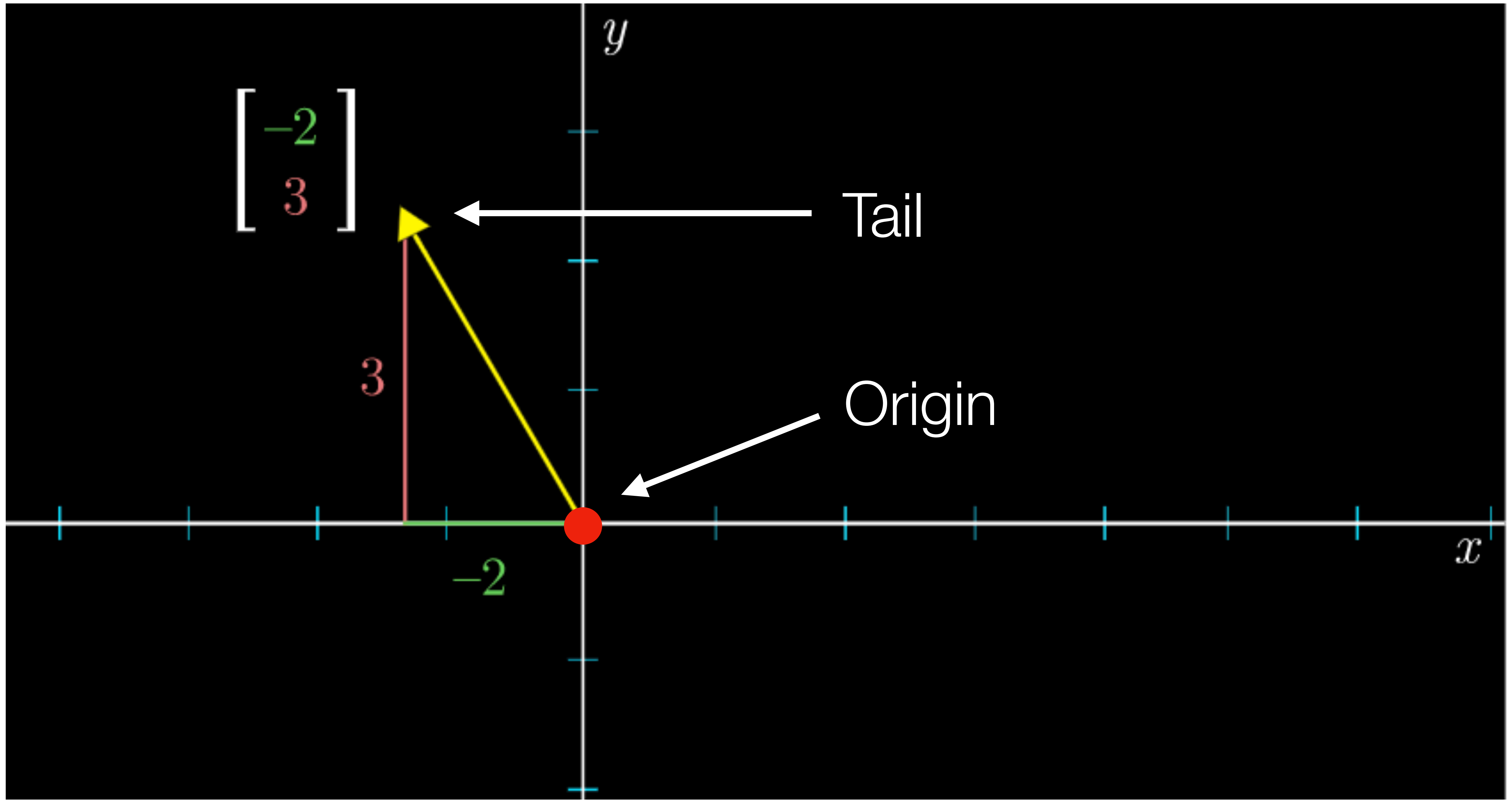
0	0	0	0	0.33	0	0	0.33	0.33	**1**
0	0	0	0	0.25	0.25	0.25	0	0.25	
0	0	0	0.33	0	0.33	0	0.33	0	
0	0	0.5	0	0	0.5	0	0	0	
0.33	0.33	0	0	0	0	0	0	0.33	
0	0.17	0.17	0.17	0	0	0.17	0.17	0.17	
0	0.5	0	0	0	0.5	0	0	0	
0.25	0	0.25	0	0	0.25	0	0	0.25	
0.2	0.2	0	0	0.2	0.2	0	0.2	0	

Stochastic matrix (not covered)

Linear Algebra recap

One step back

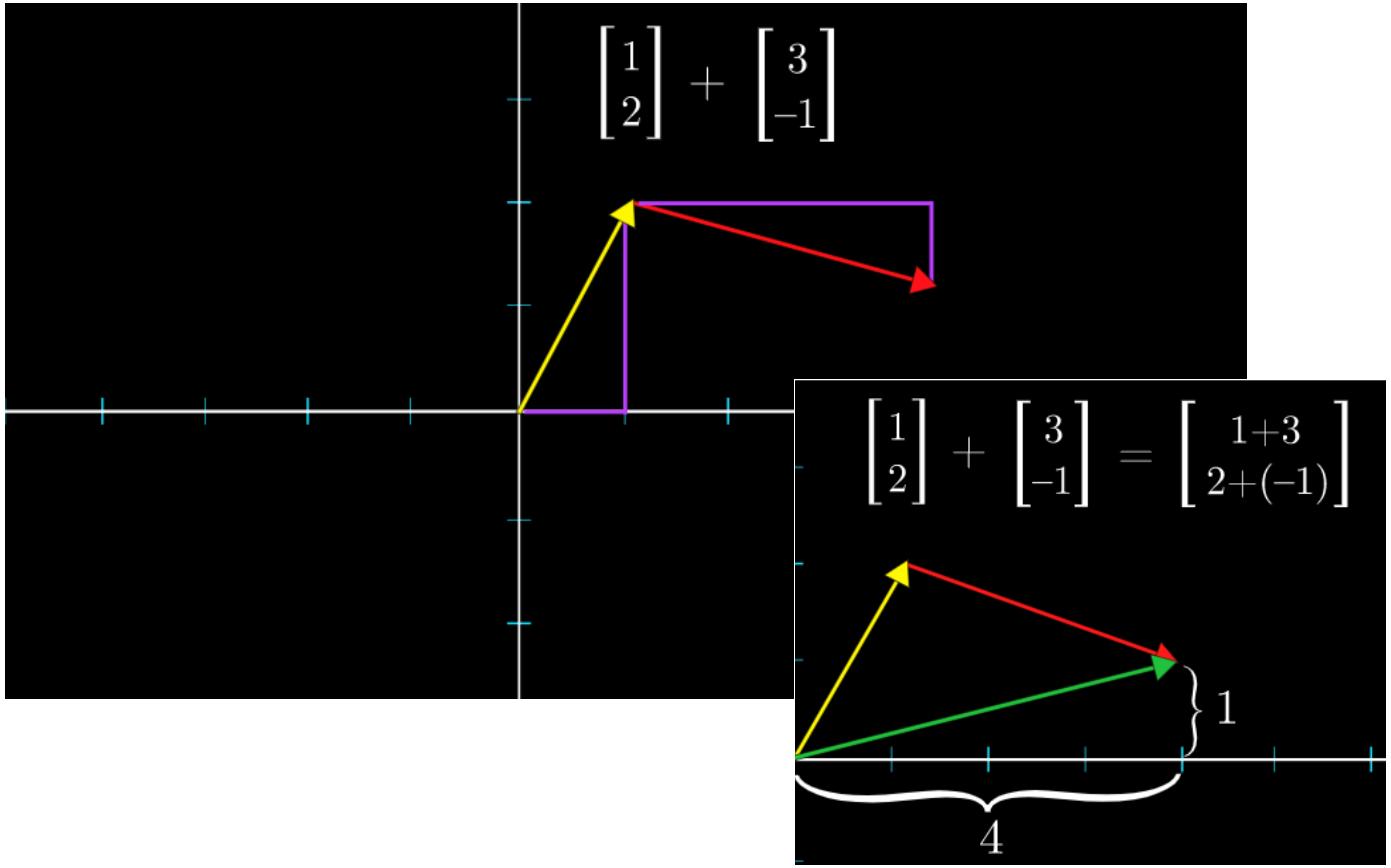
Vectors



Vector: coordinates of the tail and origin in a space

$$\vec{\mathbf{v}} \qquad \nwarrow = \begin{bmatrix} -2 \\ 3 \end{bmatrix} \neq (2, 3)$$

Vectors

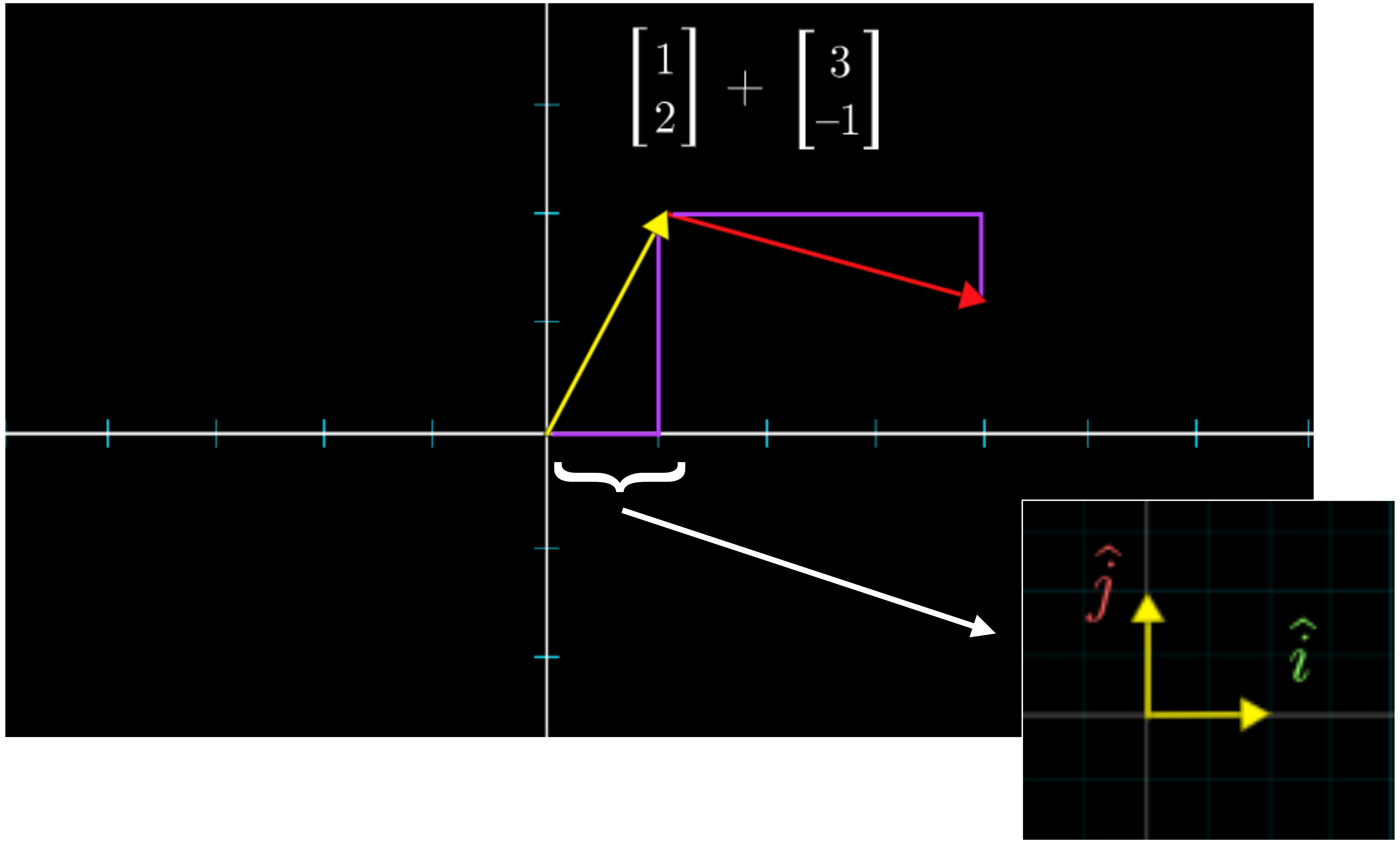


$$\begin{bmatrix}x_1\\y_1\end{bmatrix}+\begin{bmatrix}x_2\\y_2\end{bmatrix}=\begin{bmatrix}x_1+x_2\\y_1+y_2\end{bmatrix}$$ add

$$2\vec{\mathbf{v}} = 2\cdot\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}2x\\2y\end{bmatrix}$$ Multiply

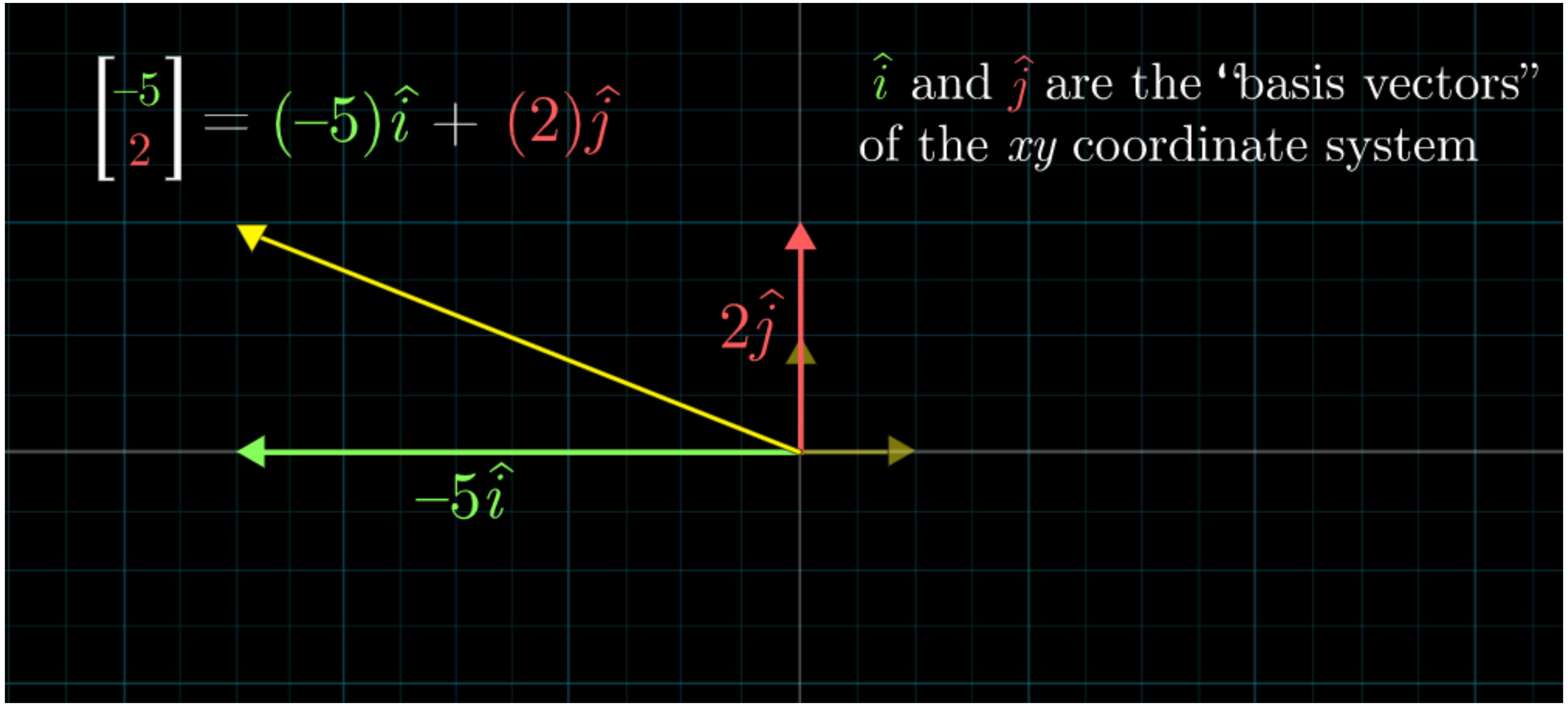
Stretch and squish vectors: **scaling**

Vectors



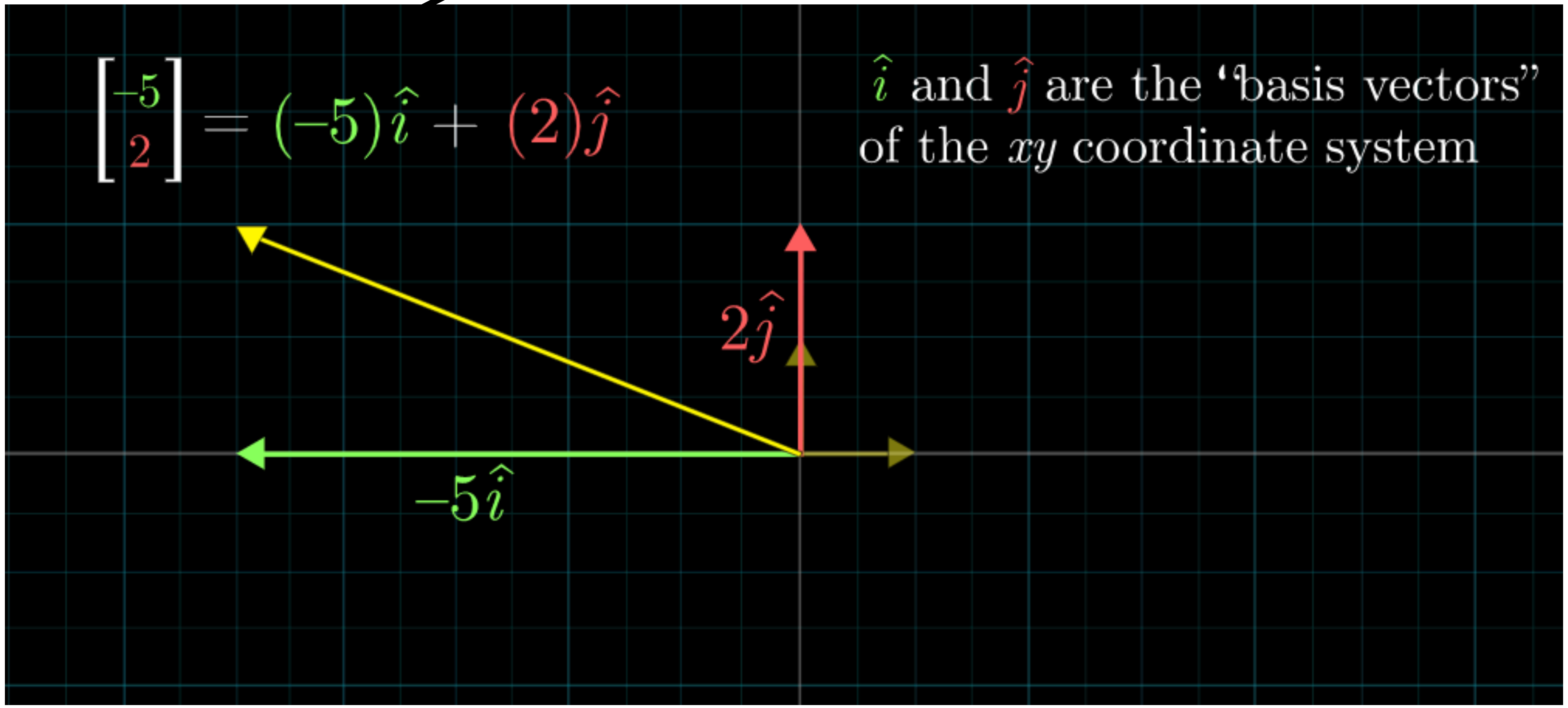
Basis vector = unit length

Linear Algebra recap



Flipping x + scaling by 5, scaling y by 2

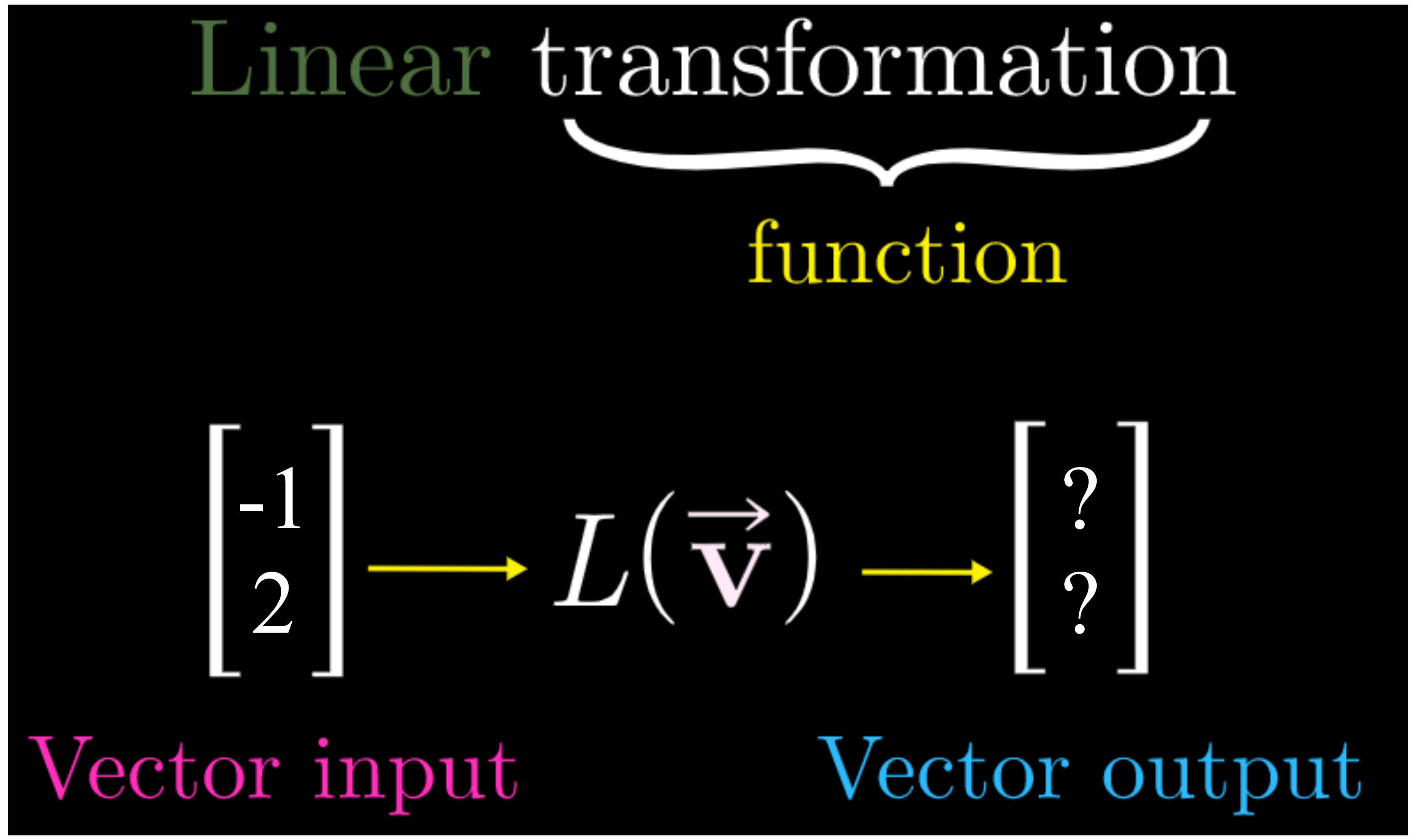
Linear transformation



Linear transformation

Flipping x + scaling by 5, scaling y by 2

Linear transformation

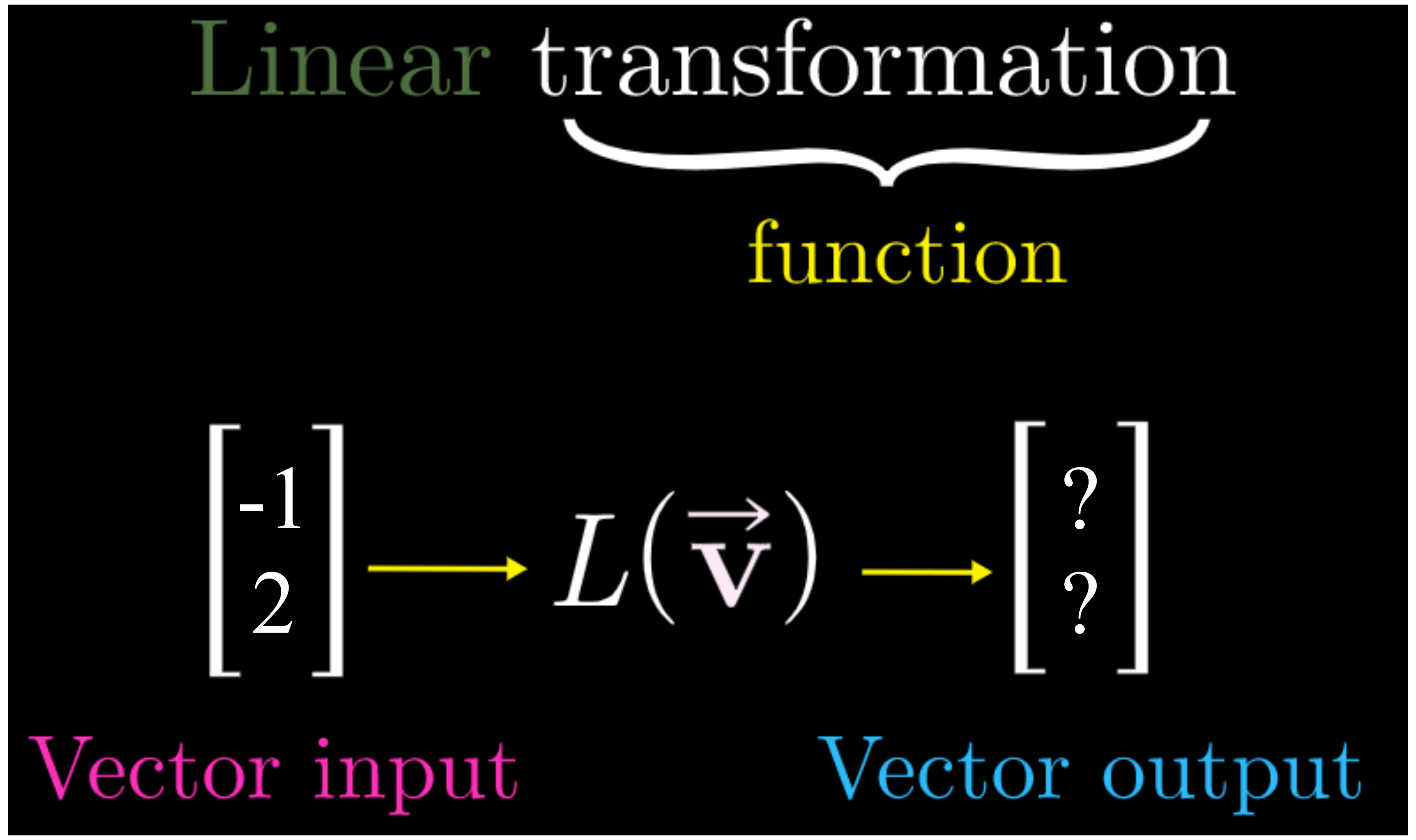


L preserves sums: $L(\vec{\mathbf{v}} + \vec{\mathbf{w}}) = L(\vec{\mathbf{v}}) + L(\vec{\mathbf{w}})$

L preserves scaling: $L(s\vec{\mathbf{v}}) = sL(\vec{\mathbf{v}})$

$$\vec{\mathbf{v}} = \begin{bmatrix} -1 \\ 2 \end{bmatrix} \qquad \vec{\mathbf{v}} = -1\hat{\imath} + 2\hat{\jmath}.$$

Linear transformation



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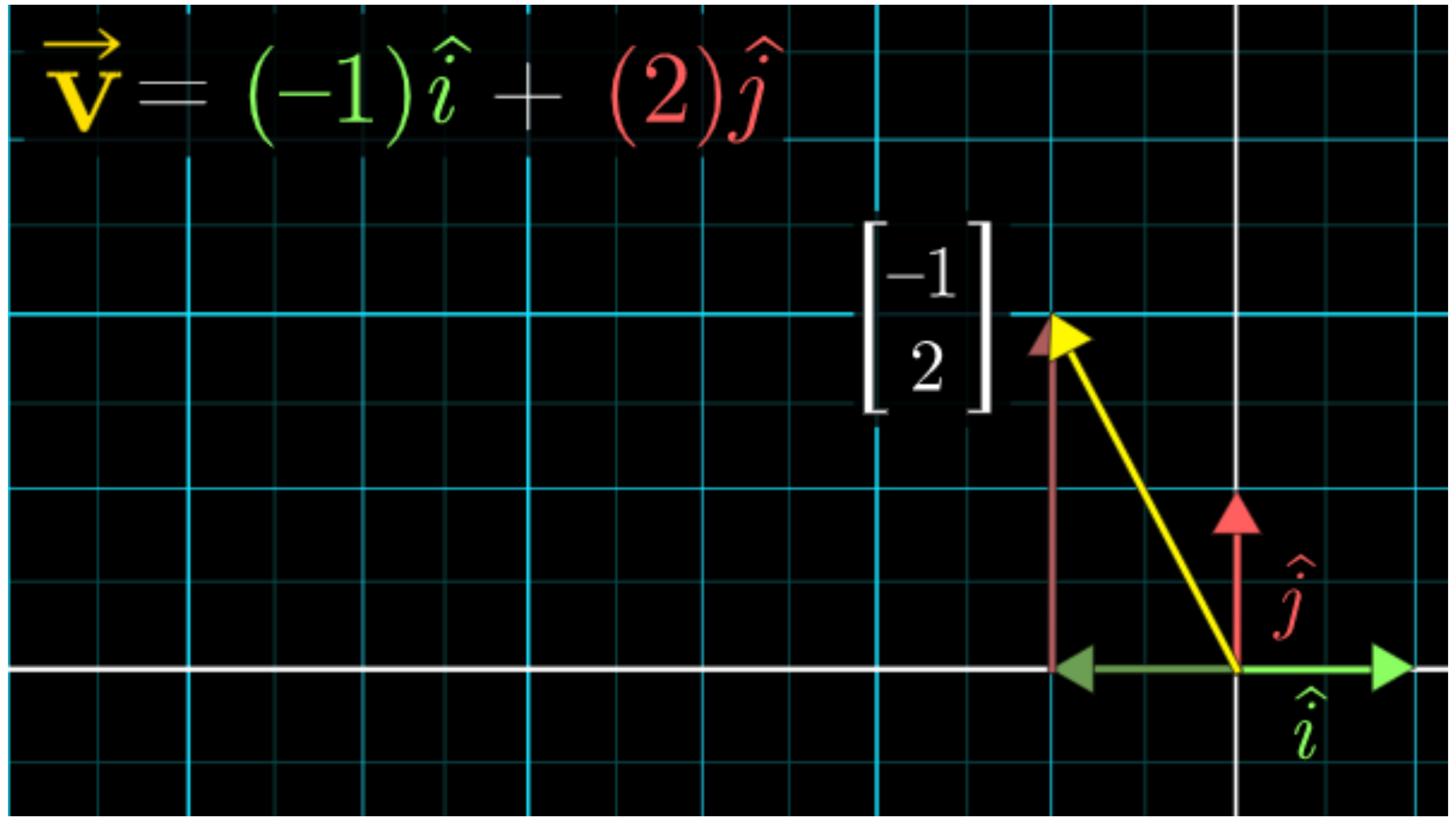
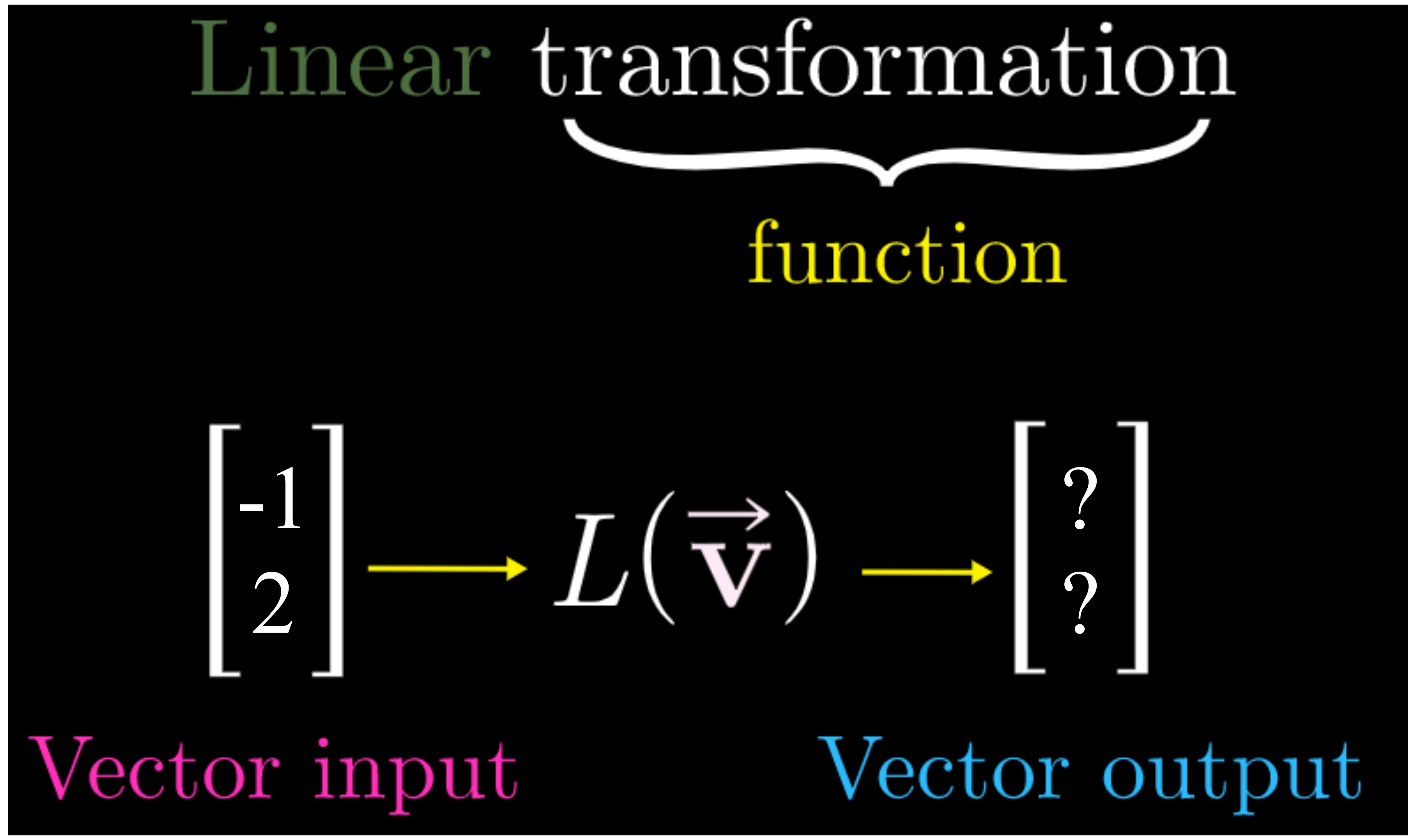
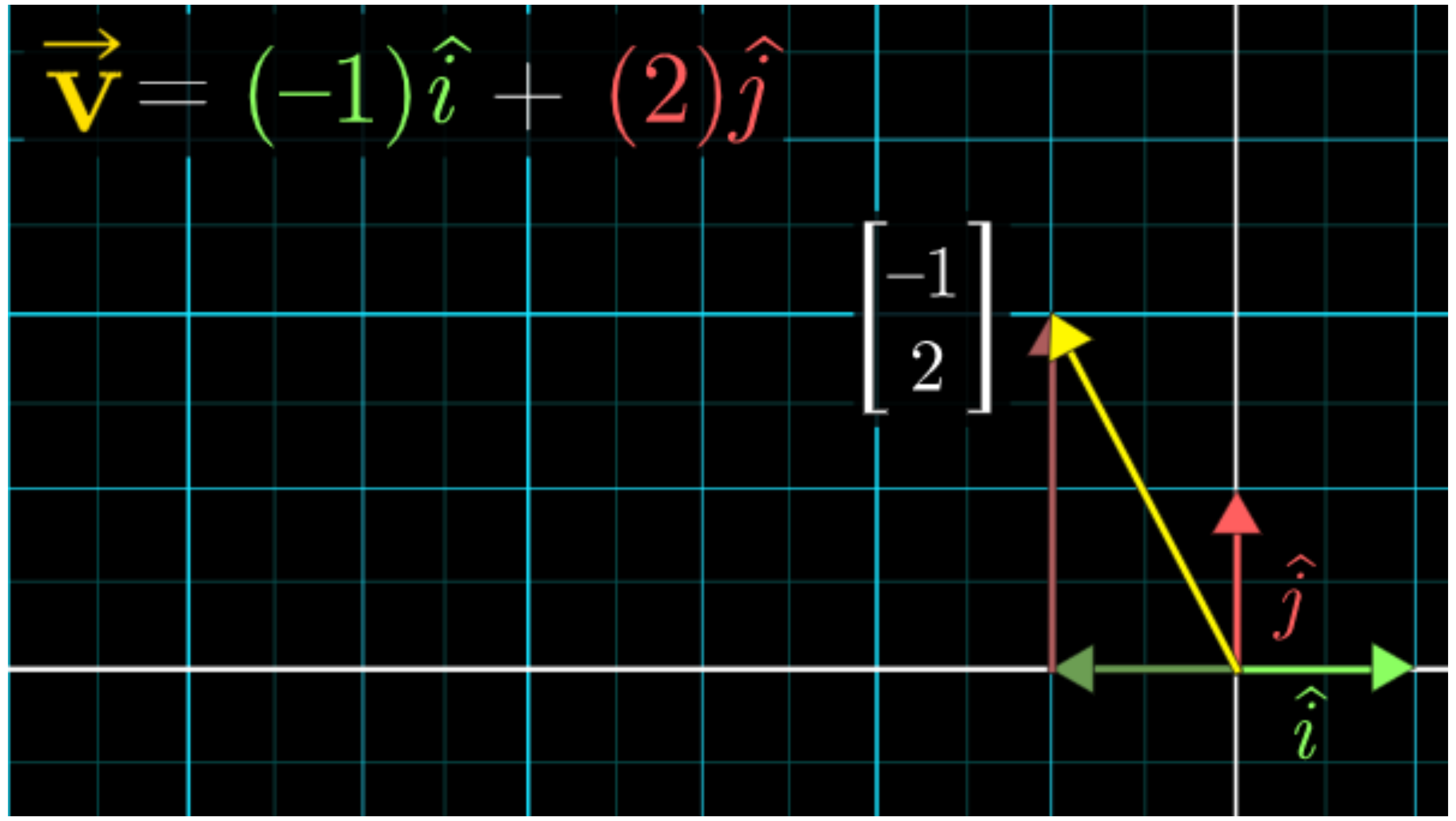


$$L(\hat{\imath}) = \begin{bmatrix} 1 \\ -2 \end{bmatrix} \qquad L(\hat{\jmath}) = \begin{bmatrix} 3 \\ 0 \end{bmatrix}$$

Linear transformation





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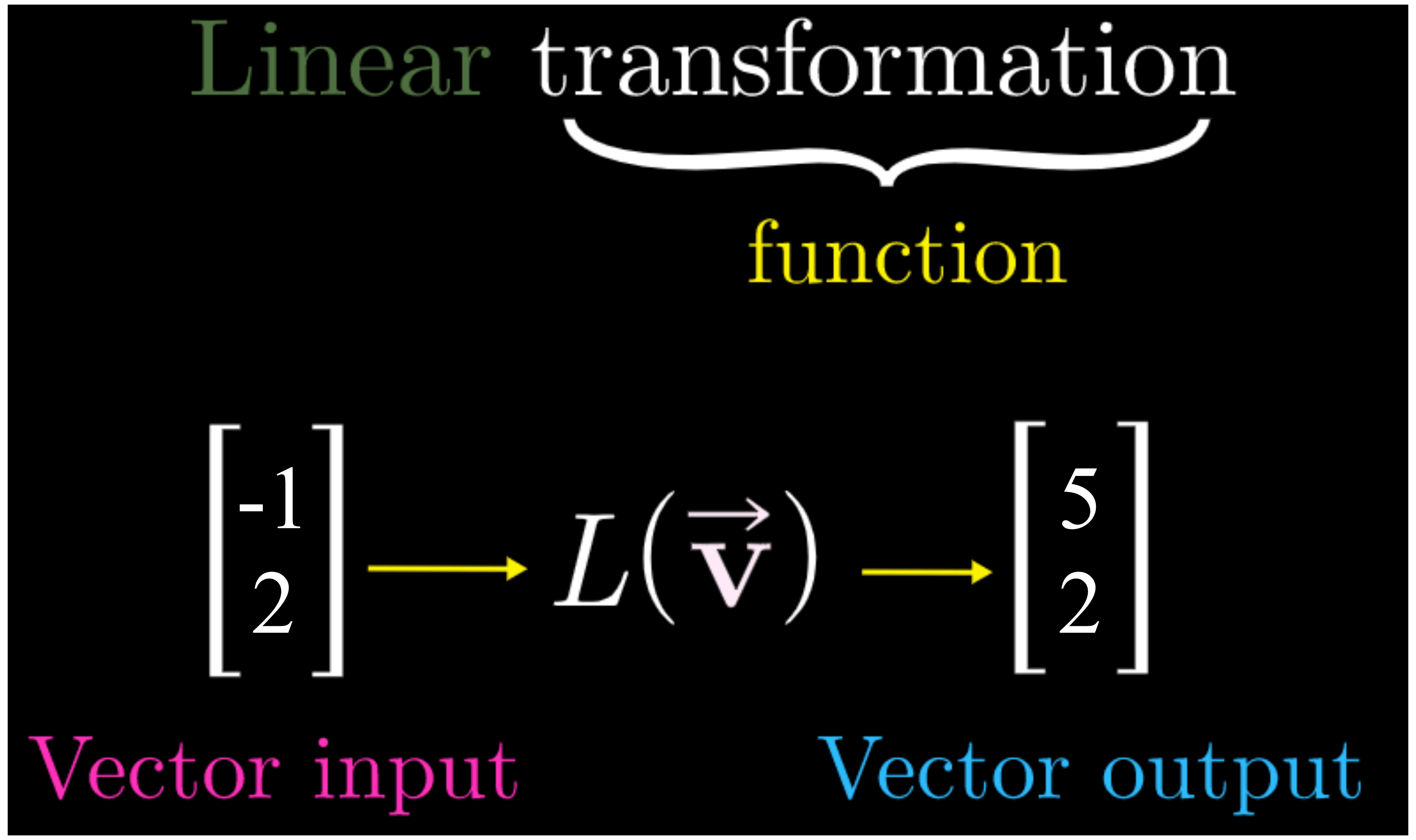
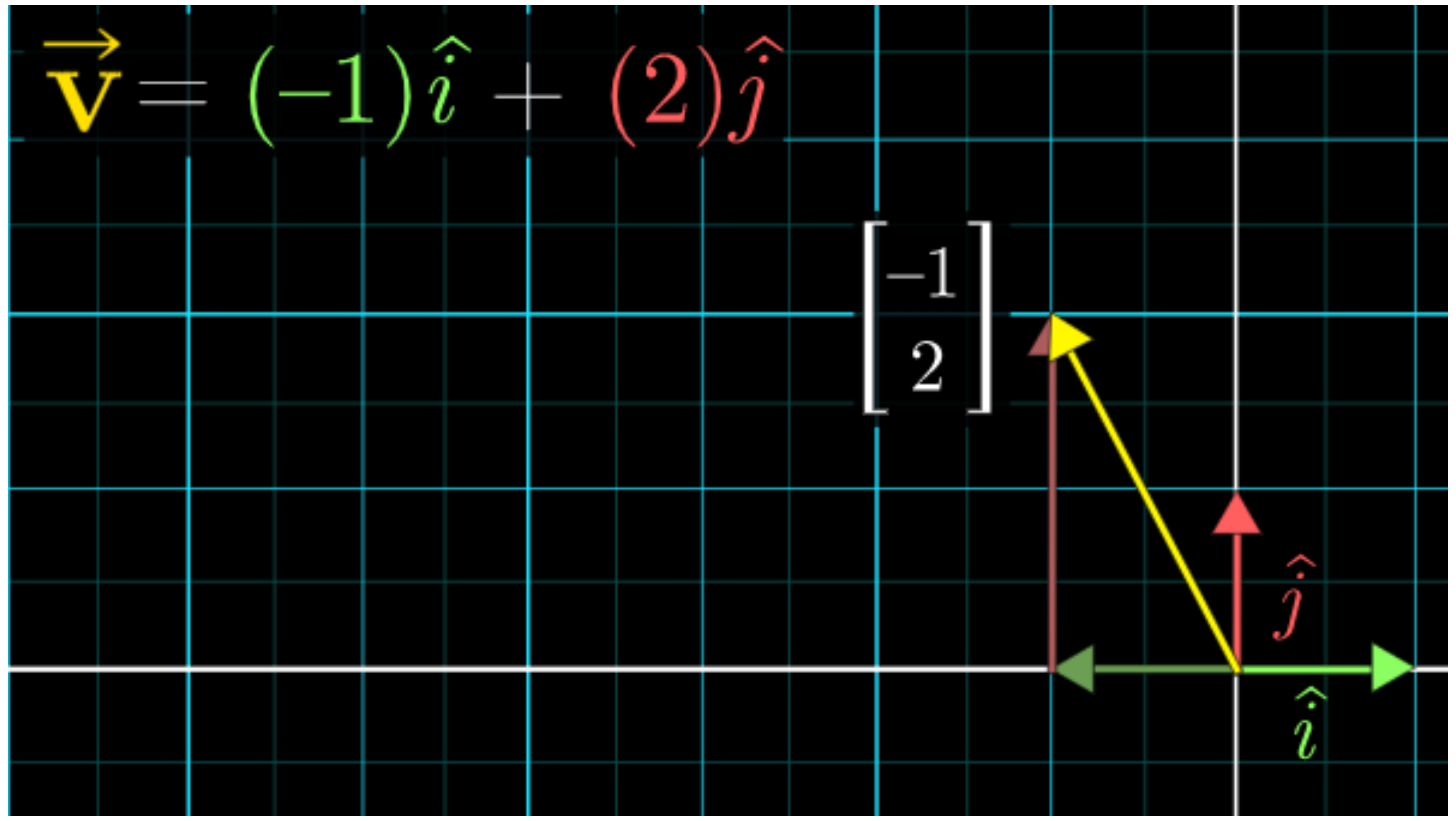
$$L(\hat{\imath}) = \begin{bmatrix} 1 \\ -2 \end{bmatrix} \qquad L(\hat{\jmath}) = \begin{bmatrix} 3 \\ 0 \end{bmatrix}$$

$$\begin{aligned} L(\vec{\mathbf{v}}) &= L(-1\hat{\imath} + 2\hat{\jmath}) \\ &= -1 \cdot L(\hat{\imath}) + 2 \cdot L(\hat{\jmath}) \\ &= -1 \cdot \begin{bmatrix} 1 \\ -2 \end{bmatrix} + 2 \cdot \begin{bmatrix} 3 \\ 0 \end{bmatrix} \quad = \begin{bmatrix} 5 \\ 2 \end{bmatrix} \end{aligned}$$

Linear transformation





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$$\vec{\mathbf{v}} = \begin{bmatrix}-1\\2\end{bmatrix} \qquad \vec{\mathbf{v}} = -1\hat{\imath} + 2\hat{\jmath}.$$

$$L(\vec{\mathbf{v}}) = L(-1\hat{\imath} + 2\hat{\jmath})$$

$$= -1 \cdot L(\hat{\imath}) + 2 \cdot L(\hat{\jmath})$$

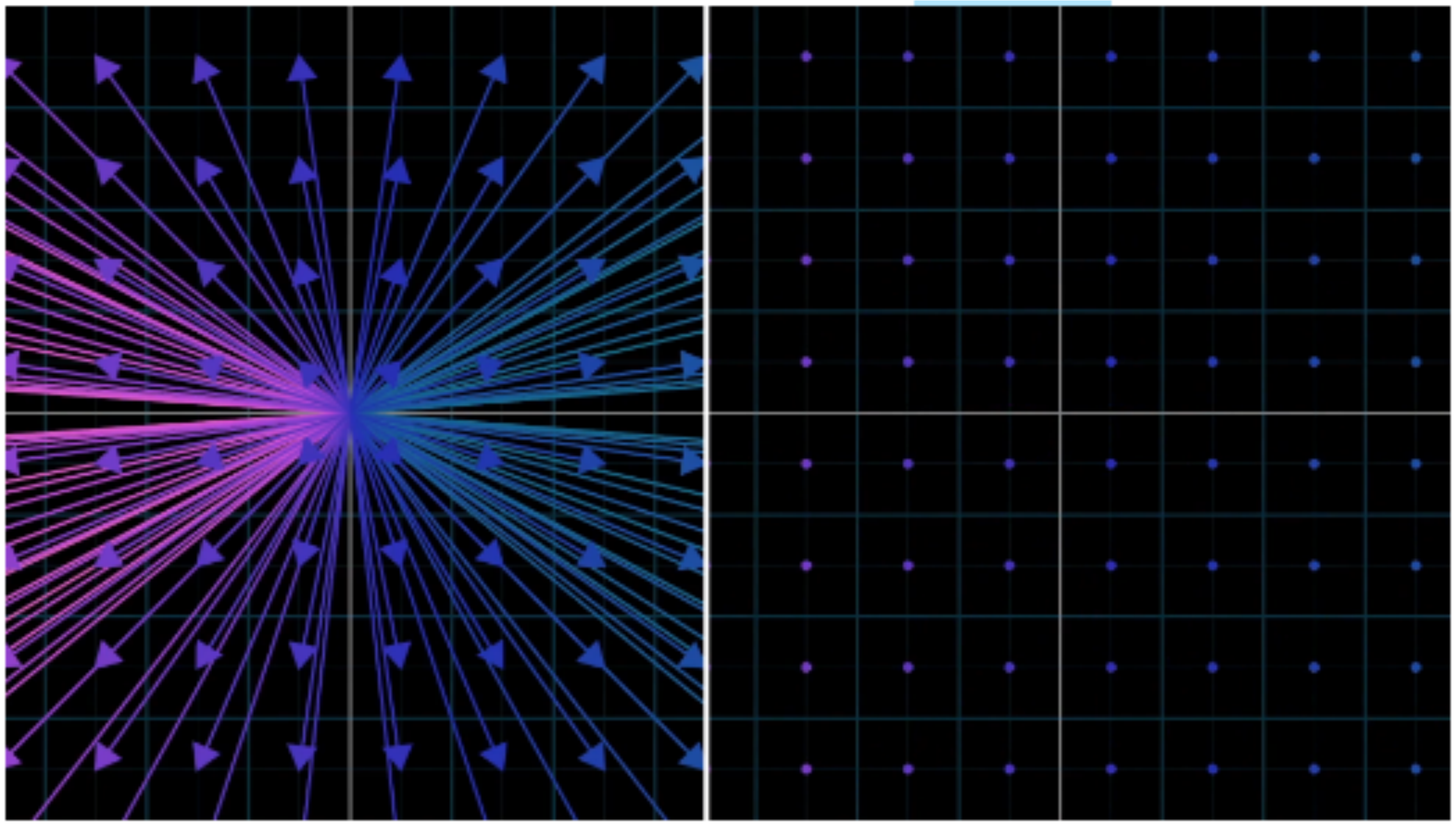
$$= -1 \cdot \begin{bmatrix}1\\-2\end{bmatrix} + 2 \cdot \begin{bmatrix}3\\0\end{bmatrix} = \begin{bmatrix}5\\2\end{bmatrix}$$

$$\underbrace{L(\hat{\imath}) = \begin{bmatrix}1\\-2\end{bmatrix} \qquad L(\hat{\jmath}) = \begin{bmatrix}3\\0\end{bmatrix}}_{\text{“}2 \times 2 \text{ Matrix”}}$$

$$\begin{bmatrix}1 & 3\\-2 & 0\end{bmatrix}$$

Many known matrices encoding common transformations

Linear transformation



Intuition: Vectors as points in a coordinate space.
We can store only the tip of the vector as **points** —> 2D space (matrix).
Linear transformations manipulate this space.

Matrix operations

$$\begin{bmatrix} 3 & 2 \\ -2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 5 \\ 7 \end{bmatrix} = 5 \begin{bmatrix} 3 \\ -2 \end{bmatrix} + 7 \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 29 \\ -3 \end{bmatrix}$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = x \begin{bmatrix} a \\ c \end{bmatrix} + y \begin{bmatrix} b \\ d \end{bmatrix} = \begin{bmatrix} ax + by \\ cx + dy \end{bmatrix}$$

Matrix multiplication by a vector

Matrix operations

$$\underbrace{\begin{bmatrix} a & b \\ c & d \end{bmatrix}}_{M_2} \underbrace{\begin{bmatrix} e & f \\ g & h \end{bmatrix}}_{M_1} = \begin{bmatrix} 0 & 2 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & -2 \\ 1 & 0 \end{bmatrix}$$

$$= 1 \begin{bmatrix} 0 \\ 1 \end{bmatrix} + 1 \begin{bmatrix} 2 \\ 0 \end{bmatrix} \qquad = -2 \begin{bmatrix} 0 \\ 1 \end{bmatrix} + 0 \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 2 \\ 1 \end{bmatrix} \qquad = \begin{bmatrix} 0 \\ -2 \end{bmatrix}$$

Matrix multiplication by a matrix

Matrix operations

$$\underbrace{\begin{bmatrix} a & b \\ c & d \end{bmatrix}}_{M_2} \underbrace{\begin{bmatrix} e & f \\ g & h \end{bmatrix}}_{M_1} =$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} e \\ g \end{bmatrix} = e \begin{bmatrix} a \\ c \end{bmatrix} + g \begin{bmatrix} b \\ d \end{bmatrix} = \begin{bmatrix} ae + bg \\ ce + dg \end{bmatrix}$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} f \\ h \end{bmatrix} = f \begin{bmatrix} a \\ c \end{bmatrix} + h \begin{bmatrix} b \\ d \end{bmatrix} = \begin{bmatrix} af + bh \\ cf + dh \end{bmatrix}$$

$$= \underbrace{\begin{bmatrix} ae + bg & af + bh \\ ce + dg & cf + dh \end{bmatrix}}_{M_2 M_1}$$

Matrix multiplication by a matrix

Matrix operations

<u>Rules</u>

$$(A+B)+C = A+(B+C) \quad \text{(associativity)}$$

$$A+B = B+A \quad \text{(commutativity)}$$

$$d\cdot(c\cdot A) = (d\cdot c)\cdot A$$

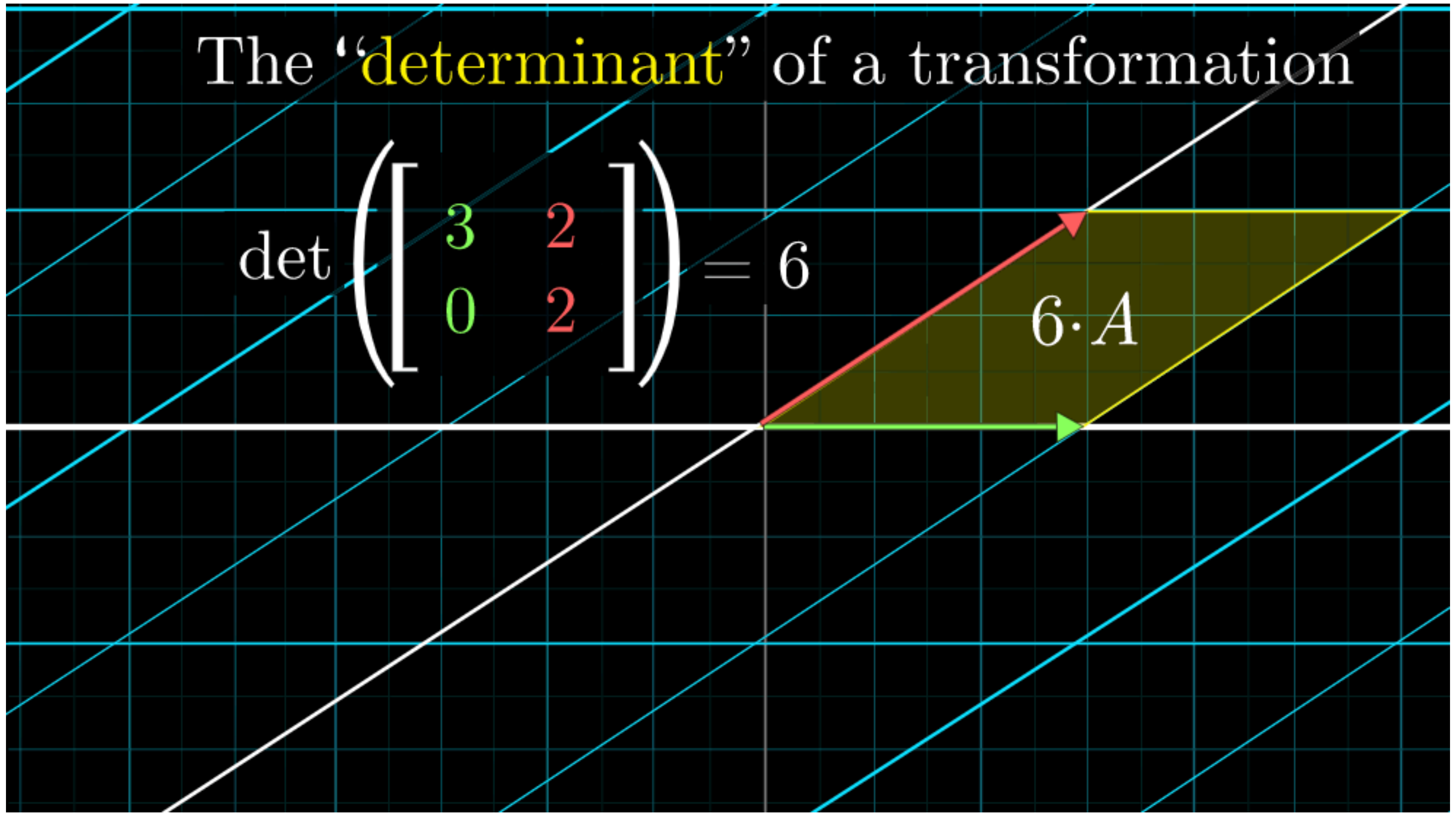
$$1\cdot A = A$$

$$c\cdot(A+B) = c\cdot A + c\cdot B \quad \text{(distributive law)}$$

$$(c+d)\cdot A = c\cdot A + d\cdot A \quad \text{—— " ——}$$

Matrix properties

Matrix operations

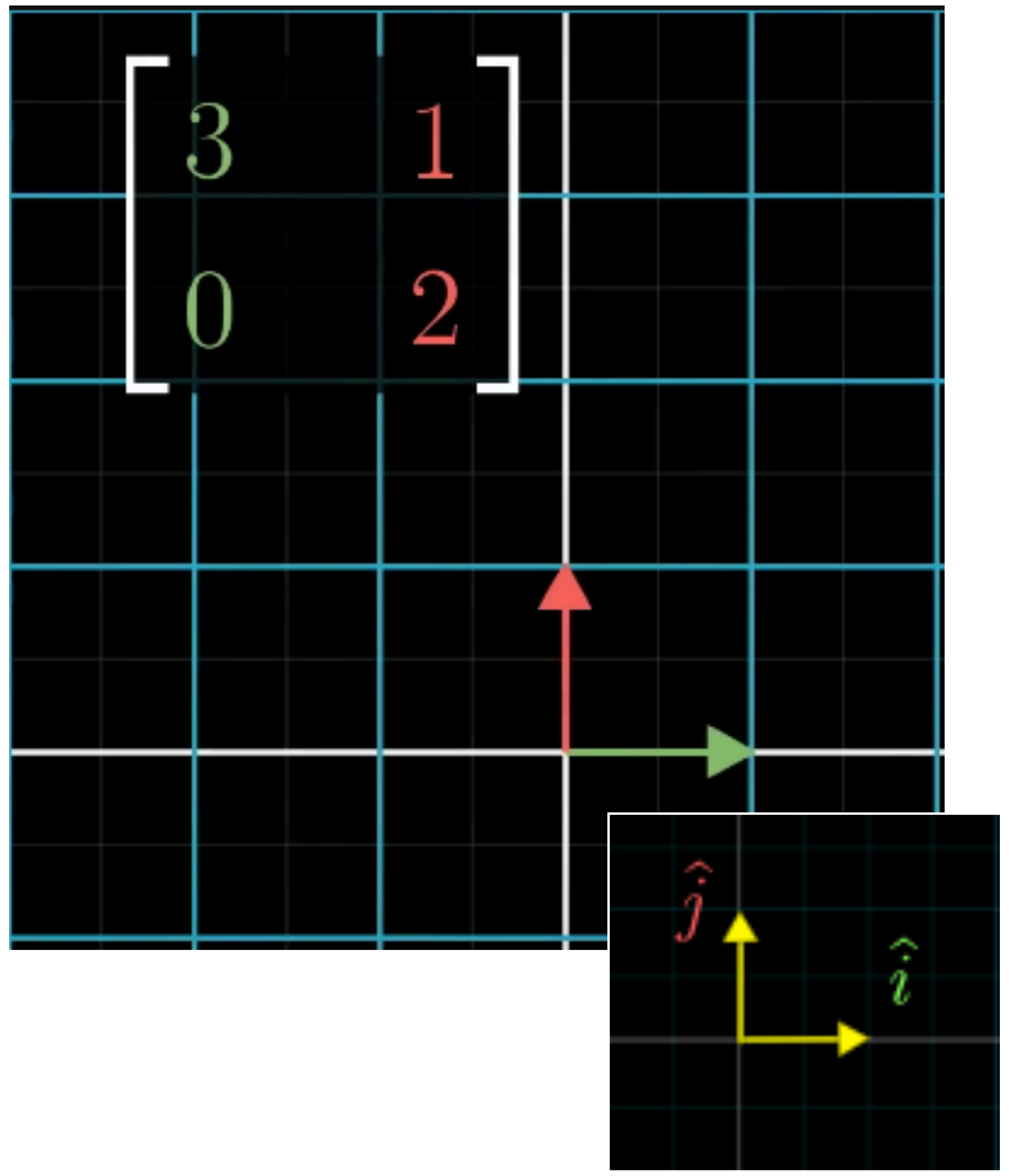


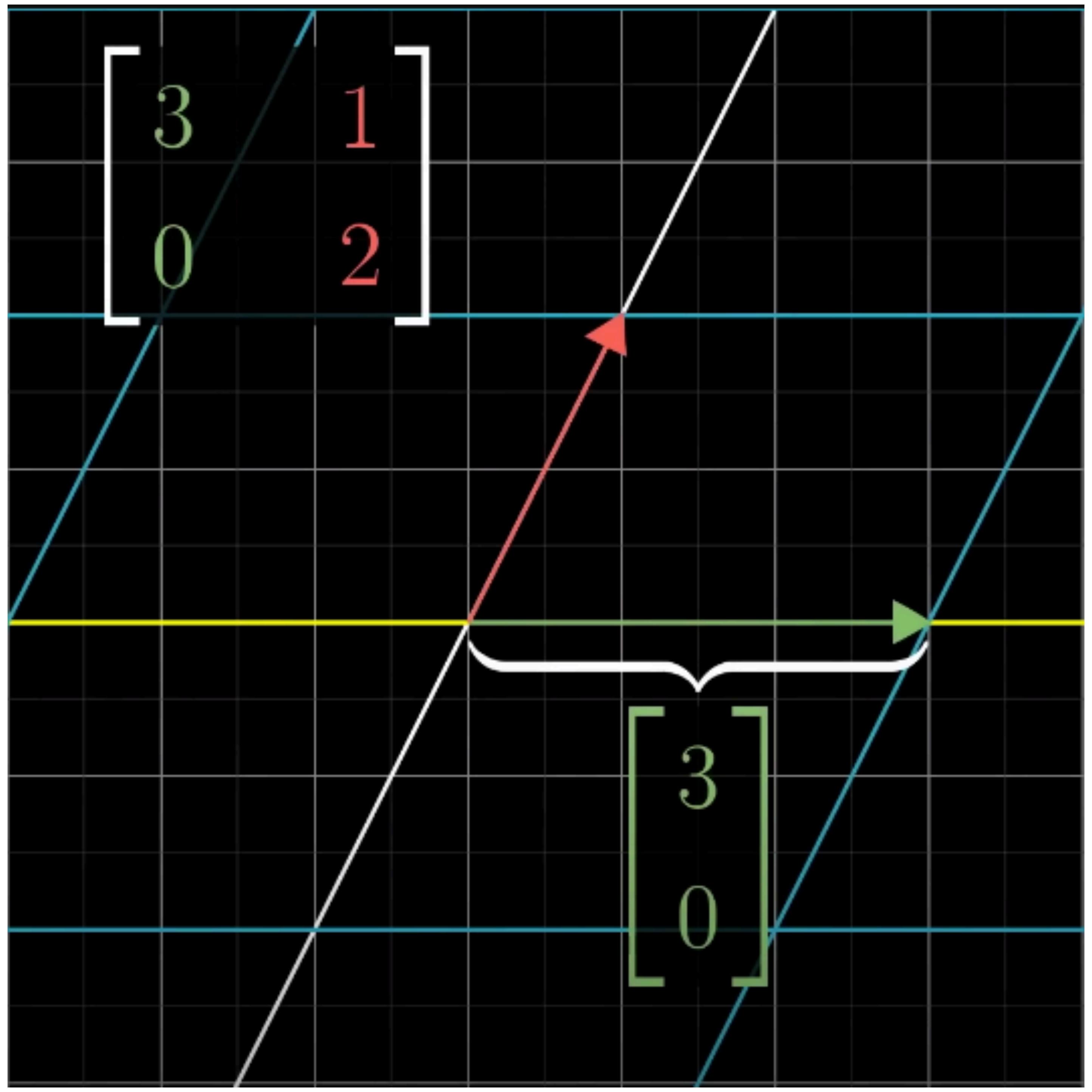
If $\boldsymbol{det(L(A)) = 0}$,

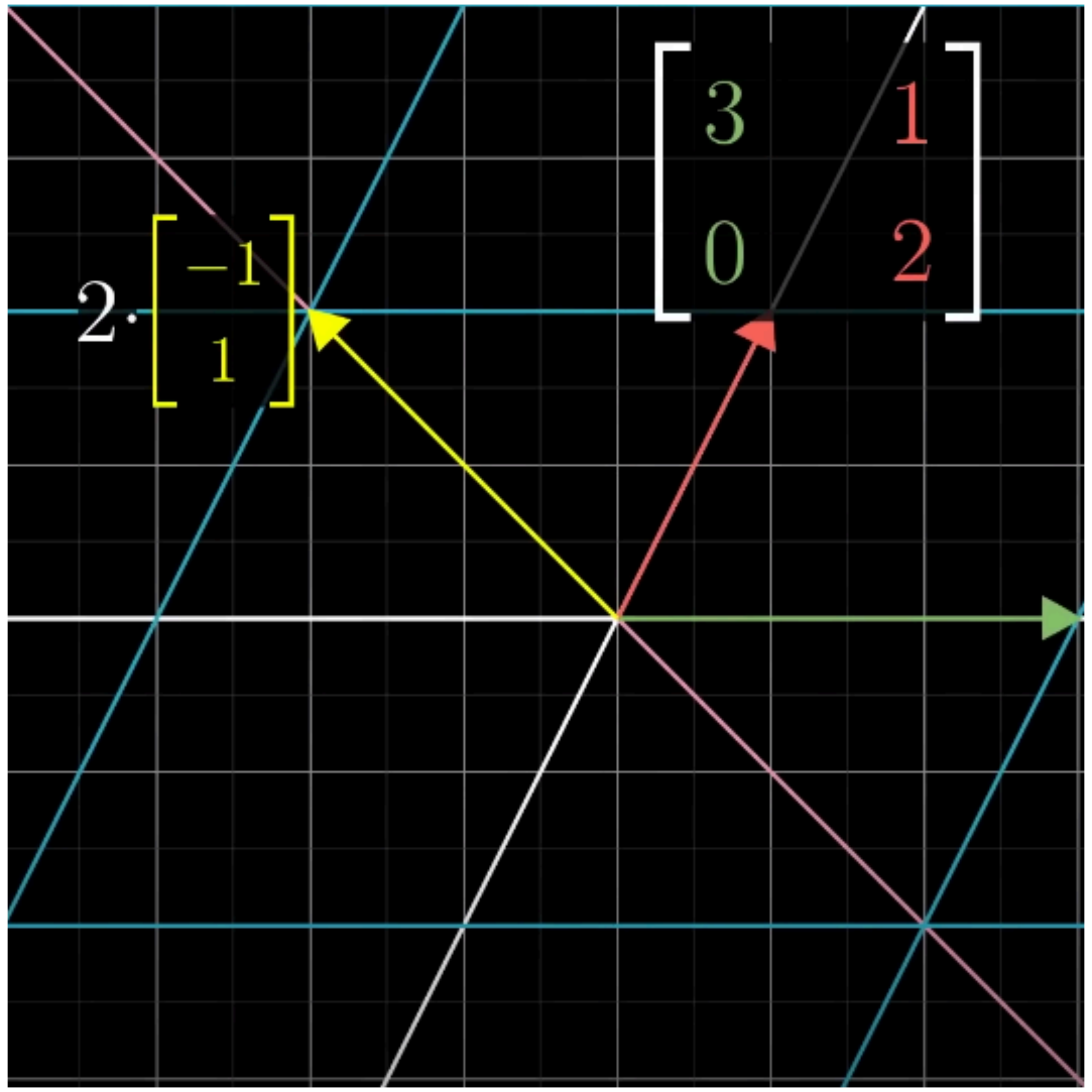
it squishes all of space onto a lower dimension, here a (line).

How much an area scaled?

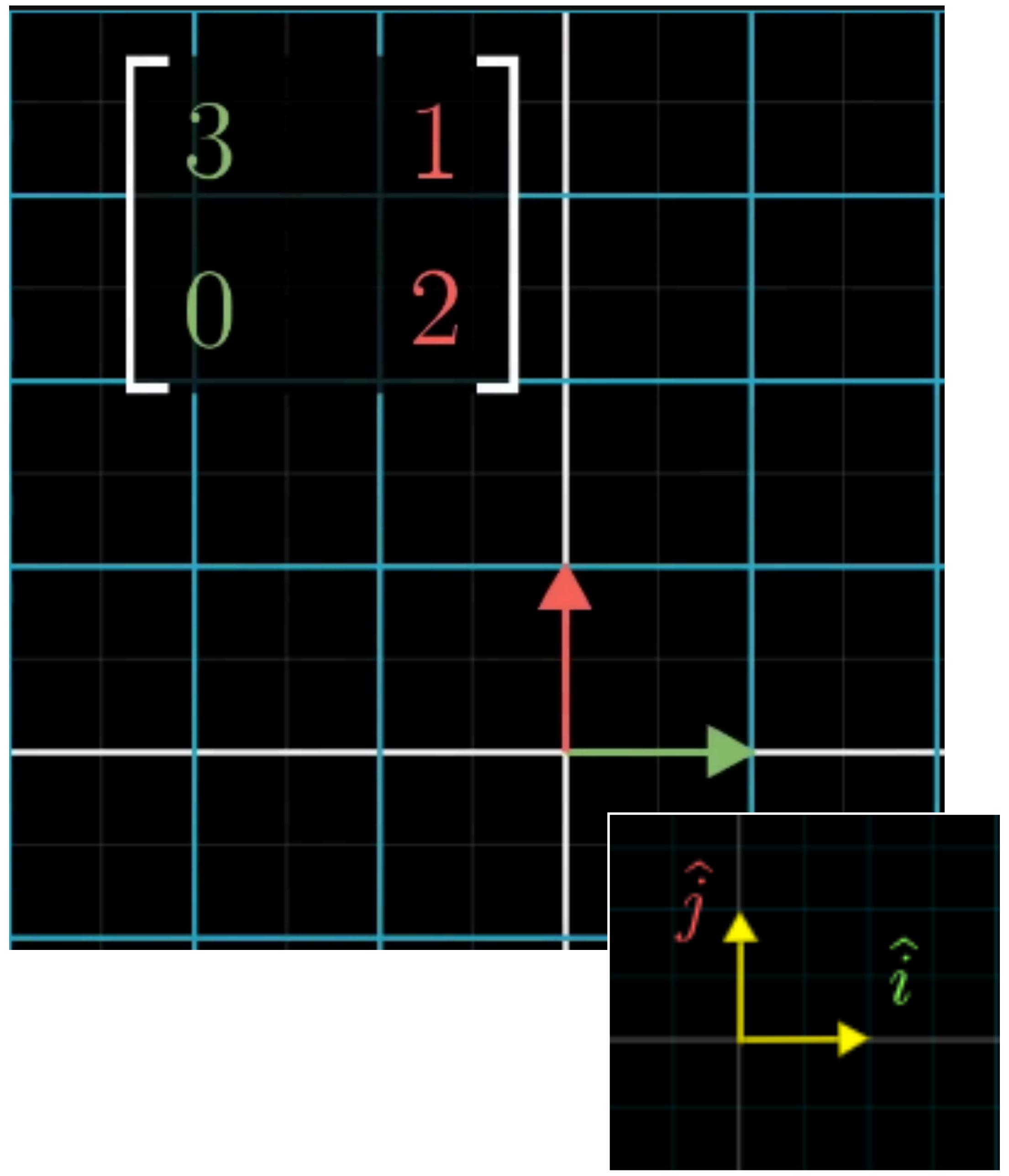
Eigenvalues & Eigenvectors

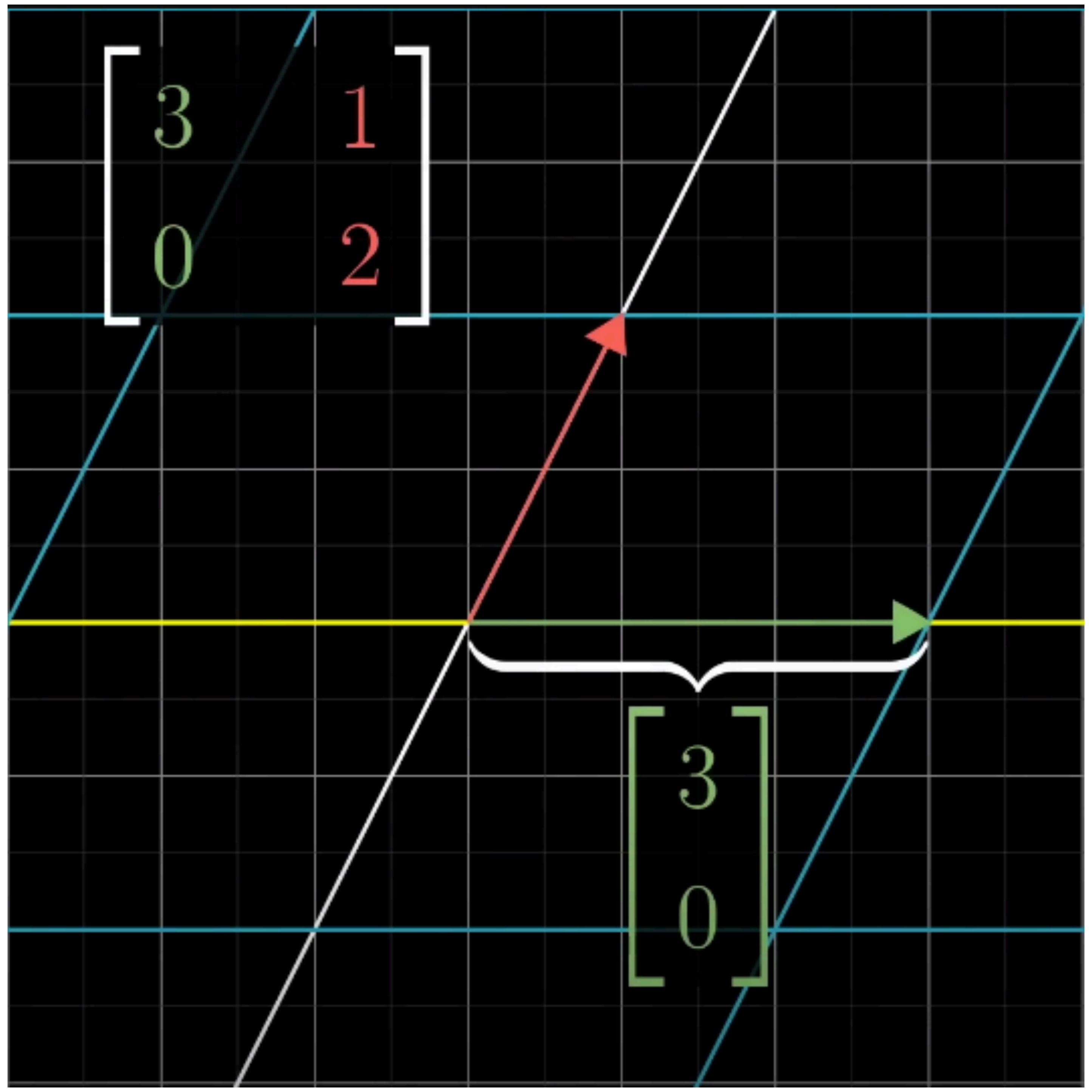


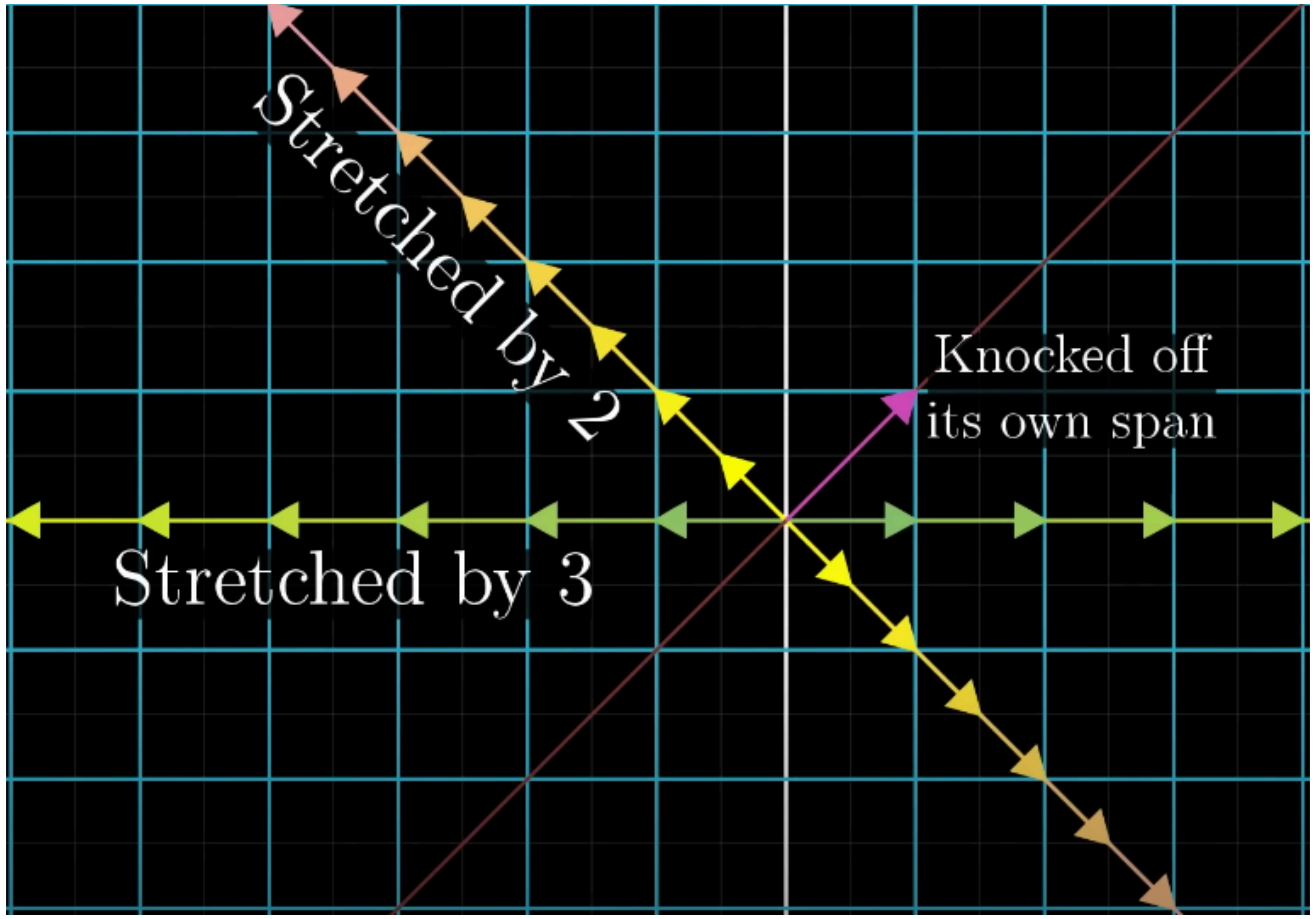




Eigenvalues & Eigenvectors



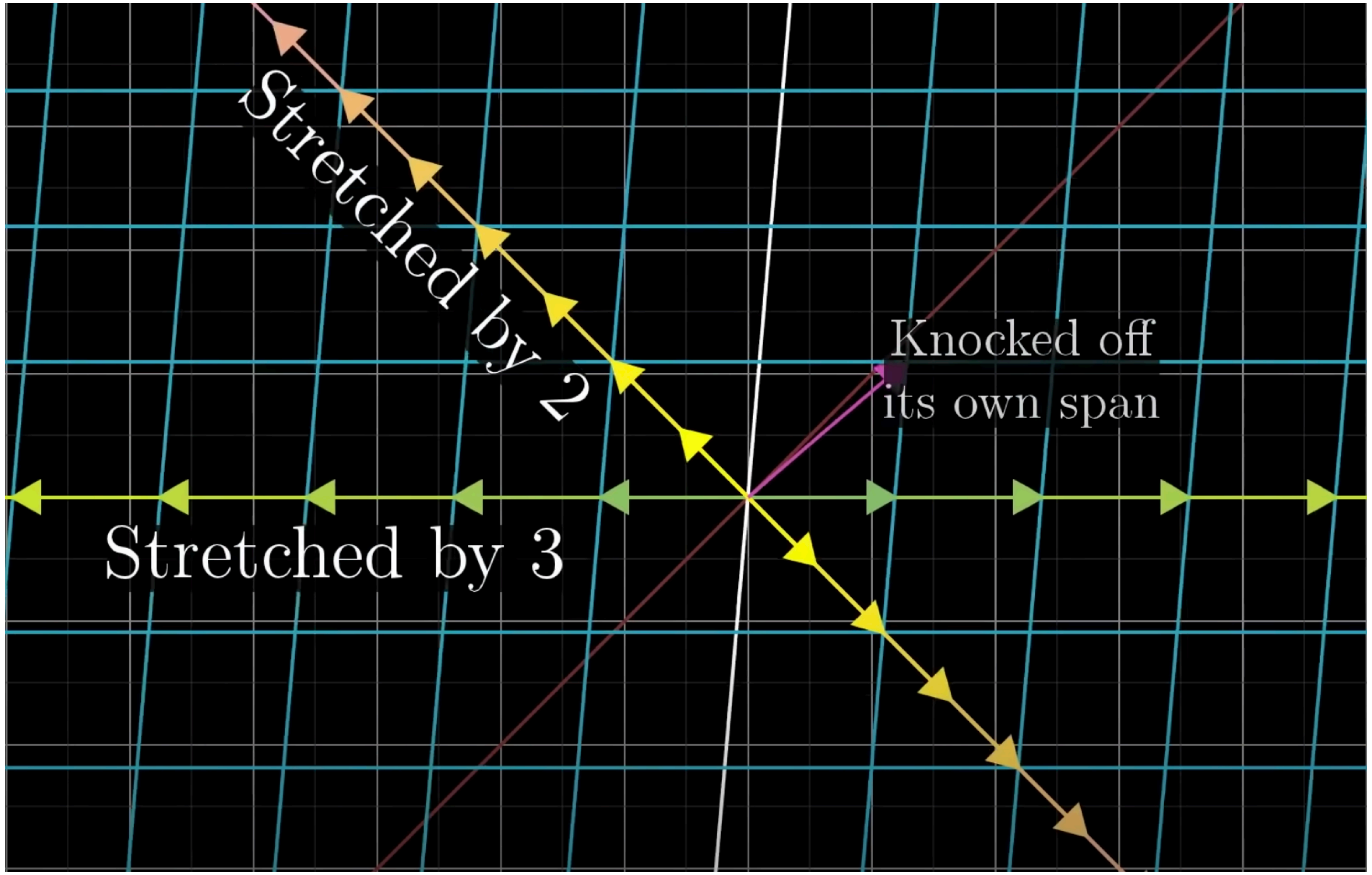




Eigenvalues & Eigenvectors

Eigenvectors

with eigenvalues λ2 and λ3



Gives us an orientation point

No change, Axis of rotation

Eigenvalues & Eigenvectors

Solve: Find $\vec{\mathbf{v}}$ and λ where this is true

Matrix multiplication

$$\overbrace{A}^{\text{Transformation matrix}}\vec{\mathbf{v}} = \overbrace{\lambda}^{\text{Eigenvalue}}\vec{\mathbf{v}}$$

Eigenvector

equivalent to scaling $\vec{\mathbf{v}}$ by λ

Rewrite the right-hand side as matrix vector multiplication

Scaling by λ <> Matrix multiplication by

$$\begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix}$$

Eigenvalues & Eigenvectors

Solve: Find $\vec{\mathbf{v}}$ and λ where this is true

Rewrite the right-hand side as
matrix vector multiplication

Matrix
multiplication

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equivalent to
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Scaling by λ <> Matrix
multiplication by

$$\begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} \qquad \lambda \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{I}$$

Factor out λ
Identity matrix I

Eigenvalues & Eigenvectors

Solve: Find $\vec{v}$ and λ where this is true

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Factor out λ

Identity matrix I

Both sides as matrix vector multiplication

$$A\vec{\mathbf{v}} = (\lambda I)\vec{\mathbf{v}}$$

$$A\vec{\mathbf{v}} - (\lambda I)\vec{\mathbf{v}} = \vec{0}$$

$$(A - \lambda I)\vec{\mathbf{v}} = \vec{0}$$

Eigenvalues & Eigenvectors

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Scaling by λ <> Matrix multiplication by

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Factor out λ

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$$\det(A - \lambda I) = 0$$

Eigenvalues & Eigenvectors

Tweaking values until the det(area) = 0 > compressing to a vector.

$$A\vec{\mathbf{v}} = \lambda\vec{\mathbf{v}}$$

$$A\vec{\mathbf{v}} - \lambda I\vec{\mathbf{v}} = 0$$

$$(A - \lambda I)\vec{\mathbf{v}} = 0$$

$$\det(A - \lambda I) = 0$$

$$\det\left(\begin{bmatrix} 3-\lambda & 1 \\ 0 & 2-\lambda \end{bmatrix}\right) = \underbrace{(3-\lambda)(2-\lambda) = 0}_{\lambda = 2 \text{ or } \lambda = 3}$$

Seeking eigenvalue λ

$$\begin{bmatrix} 3-2 & 1 \\ 0 & 2-2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

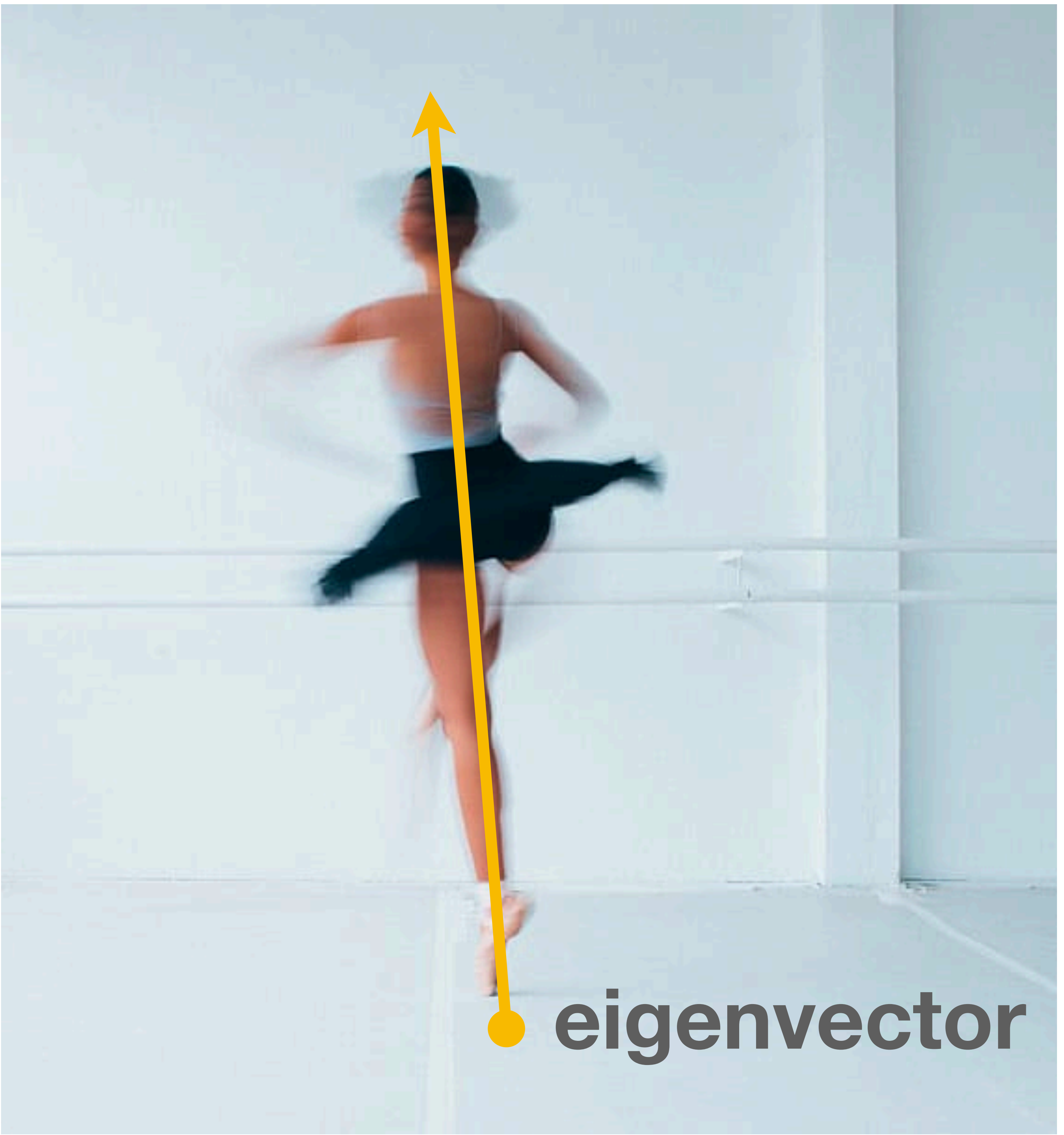
$$\lambda = 2$$

Plugin the eigenvalue and solve to get the eigenvectors

Finding these eigen-things

Linear Algebra recap

If none of that makes sense:
remember this



Useful for:

Change coordinate systems, Computer vision, finding centrality and components in a network…

and for Google making loads of money (PageRank)

Course logistics

Syllabus

1. **2023/03/13 Mon - Basic Graph Theory [Sandro]**
    1. Motivation: What is Network Science, why is it relevant, and what examples?
    2. Key concepts: Nodes & Links
    3. Origins: Leonhard Euler - Seven Bridges of Königsberg (Path, Link directions, Connected components)
    4. Network representation: Edge list, Adjacency matrix, Adding weights
    5. Linear Algebra short recap: Vectors, Matrix, Matrix operations, Eigenvectors & eigenvalues

2. **2023/03/15 Wed - Network identity card & Centrality [Sandro]**
    1. Identity card Part 1: Nodes and edges, Unipartite/bipartite, Clustering coefficient, Shortest paths, Average path length, Diameter, Density
    2. Identity card Part 2: Node degree, In-degree/ Out-degree, Weights
    3. Network centrality: Degree centrality, Betweenness centrality, Eigenvector centrality

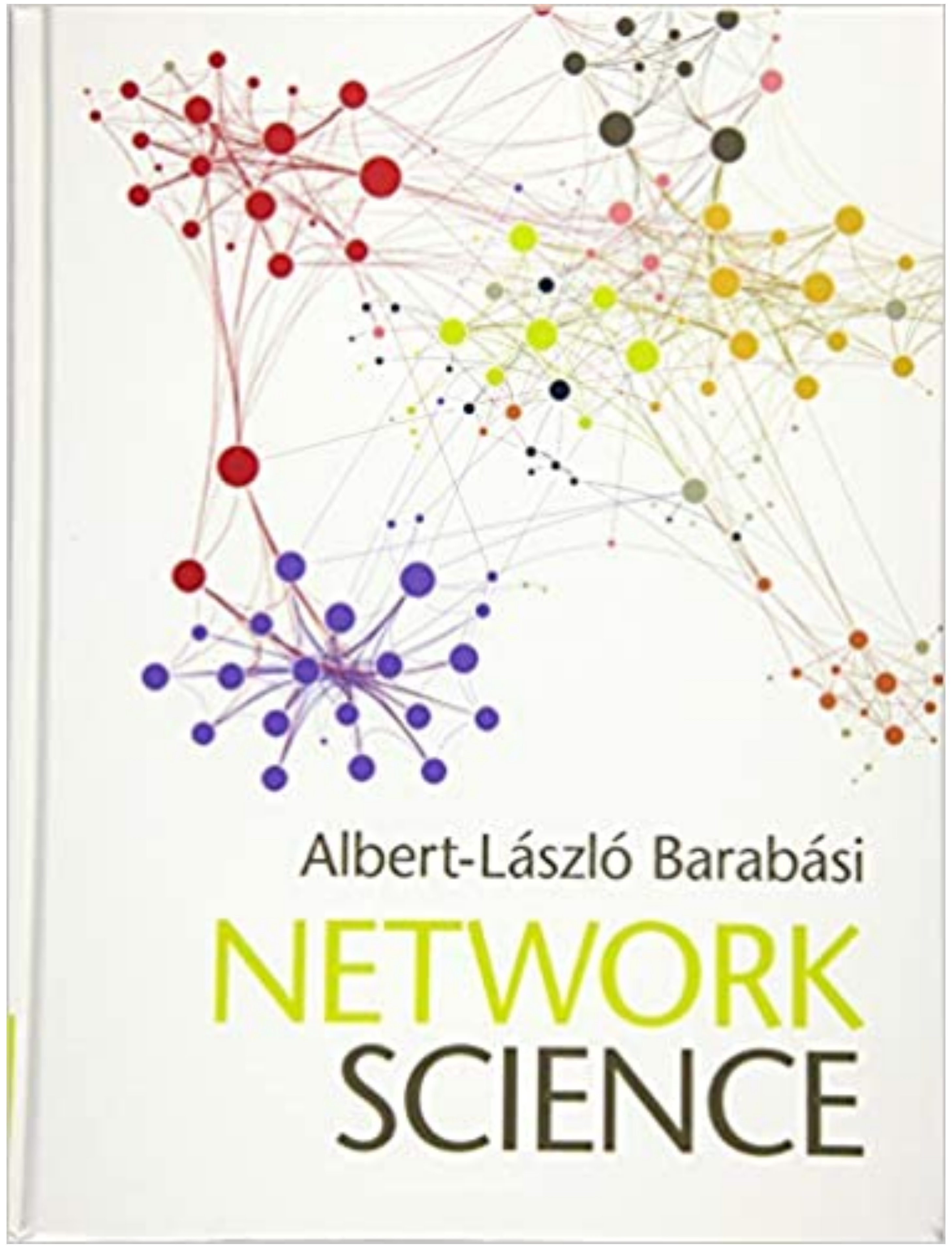
3. **2023/03/20 Mon - Network statistical distributions [Sandro]**
    1. Statistical distribution: Definition, Calculation, Interpretation, Degree distribution
    2. Random network model: Parameters, Degree distribution
    3. Stanley Milgram (Small-world experiment): Shortest path in real-world

4. **2023/03/22 Wed - Network mesoscale structure [Roberta]**
    Ego network, Homophily, Assortativity, Communities

Exercises

- Work on the ipynb files provided.
- Writing tasks: Be reflexive; it’s about understanding the concepts instead of getting the right answer.
- Coding tasks: there are many ways to operationalise an idea; use what you are comfortable with.

Book



Available for free at:

http://networksciencebook.com/

Or you can purchase a printed copy.