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Role of information and topology in agent-based competitive models with limited resources

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Alla mia famiglia

Abstract

The problem of competition, cooperation and the emergence of collective behaviour in the presence of limited resources is quite general, and one of cornerstones of evolutionary dynamics both in the natural and artificial worlds alike. For instance, trading is nowadays mainly performed by algorithms which act autonomously and form an ecosystem of their own. These models exhibit complex phenomena, governed by various parameters that describe the quantity and the quality of agents involved. We study how these parameters influence the efficiency of the models, measured as efficient distribution of limited resources, and how the additional information like vicinity, it's structure, the number and the cognitive abilities of participants modify these models efficiency.

We extend the classical prototype of such environment, i.e, minority games, by allowing agents to process additional information. We analyse how this exchange of information affects the dynamics of the system. The main focus is put on the role of the information from each agents vicinity and to study the influence of different community structures on the model. We investigate several topologies like simple patch vicinity, von Neumann vicinity, small world, scale-free networks and a hierarchical small world network.

We have found that there is a distinct relation between the structure and dimension of the vicinity, ie. the information given to each agent, and the efficiency of the model. These results could be used to optimise any kind of distributed algorithmic ecosystem that has a finite resources and needs an efficient use of it.

We further investigate how these finding can be used in algorithmic ecosystems. The context within which we elaborate some of our ideas is high-frequency financial markets, that are run by an enormous number of trading algorithms. Among other possible applications we consider optimizing the routing protocols for Delay Tolerant Networks, more efficient smart-grid energy systems and so on.

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Chapter 1

Introduction

During this thesis we have worked on a problem of information importance within competitive systems with limited resources. We have modelled a competitive environment with classic minority games, introduced in Section 1.2, and by modifying the basic implementation with a vicinity information, while studying it's structure as introduced in Section 1.5, we have studied how it affects the efficiency of the model. These models can be used to analyse any kind of competitive system that has a well defined resource, such as bandwidth in communication systems, mobility in transports, buy/sell decision-making in finance, and so on. The basic idea behind the study of financial markets is introduced in Section 1.3, while in Section 1.4 we introduce a principle that has inspired some assumptions made during the thesis.

1.1 Competitive systems and finite resources

The definition in ecology of a competitive system is the one where one species tries to dominate others while competing for the same resources. In the

Gause's law of competitive exclusion or just Gause's law, [1] it is stated that two species competing for the same resource cannot coexist at constant population values, if other ecological factors remain constant.

This definition can be applied to any human or human-made system where the agents involved act in the self-best interest and are competing for the same resource. There will be the losing side that will get excluded in the long run and a winning side whose behaviour will probably be replicated by others.

In many natural systems humans cannot exhibit enough control to prevent the destruction of less efficient species, and one can think that it is not even a wise thing to do, however there are other areas where one can intervene. Many human and algorithmic systems should be rendered more efficient, rather than wait for the exclusion of less able agents. If we take the example of human transport system, we can apply some sort of control over the system, whether by tackling modern navigation systems, maps or the physical structure of the transport network, rather than leave the poor performance agents, in this example human drivers, to their own devices.

1.2 Minority Games

Minority Games are a model of a competitive system, formulated by Damien Challet and Yi-Cheng Zhang in 1997 [2], based on the El Farol Bar problem. The basic model was proposed by Brian Arthur in 1994 [3] and it was inspired by the decision making of people in a small community of El Farol. Suppose that there is a cultural event being held every week in the El Farol Bar. The bar has finite space, so whether a single person enjoys the evening is determined by the quantity of other people at the bar. A certain limit is defined, 60% in the original paper, and when it is saturated it can be said that people present would rather be satisfied staying at home. Same can be said if the attendance is bellow the determined limit and the person has decided to stay at home, ie. decision to stay at home is considered a losing one.

Minority games set the limit to 50%, so that the losing side is always the majority, while the winning side is the minority. In this way the model becomes frustrated, meaning that most of the agents can not be satisfied. It is also called a negative-sum-game, as with time only the minority can win and be rewarded points, while the majority will have a negative score.

The simplest model consists of N agents, where N is an odd integer, that have to make a decision between two possibilities at each round. Each agent has S deterministic strategies from which he can choose. At each round every agent follows the strategy with the highest score among the ones in his/hers expertise After the evaluation of the majority and minority sides, every agent updates his/hers strategies by increasing the score of those that predicted the winning side, while decreasing the score of those that have failed to guess the correct decision.

The information given to each agent can be external or generated by the model, depending on the goal of the study. In classic minority games the information is internal, and it is a string of M past minority outcomes, where M is the brain size, or memory, of each agent, for example $'101100'$ if the agents have memory 6. There have been other studies where the information given to agents was generated by some external mechanism, or purely random sequences, and in these papers it has been proven that the source of information does not influence the important characteristics of the model.

0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	0

Table 1.1: Example strategy with brain size 3

The strategies of the agents are based on the assumption that by remembering past outcomes of the game, the strategy can predict the outcome at the next step. A strategy is defined as a function $f : 2^M \rightarrow \{0, 1\}$, where M is the memory of the agent, and $\{0, 1\}$ is the set of possible decisions. A sample strategy can be seen in Table 1.1.

Minority games have been mainly used to model financial markets, as they offer more insight into how the decisions are formulated compared to other models that offer only the possibility to study the time series of outcomes.

1.3 Algorithmic Trading and High Frequency Trading

Algorithmic trading and High Frequency trading are two types of capital and stock exchange that usually go together. High frequency trading (HFT) is defined as a large quantity of exchanges of capital in small intervals of time. The advent of HFT has brought us a market with ever increasing number of trading operations while the values of exchanged goods has decreased in proportion. Because of it's nature as a high speed approach in a response to constantly evolving market conditions, HFT has become dependent of algorithmic trading.

Algorithmic trading (or black-box trading), as defined in [4], is the process of using computers programmed to follow a defined set of instructions for placing a trade in order to generate profits at a speed and frequency that is impossible for a human trader. The defined sets of rules are based on timing, price, quantity or any mathematical model. Apart from profit opportunities for the trader, algo-trading makes markets more liquid and makes trading more systematic by ruling out emotional human impacts on trading activities. Human decision making has proven to be too slow for modern computers, thus high frequency trading cannot be implemented without algorithmic trading.

There are two main strategies with which traders, be they human or algorithmic, make profit in the stock exchange. One is called *market making* and it is applied by being an active influence on the trading system, this means trying to create trends by buying or selling certain quantity of stocks and exploiting that trend in the future. Second type of strategy is called *statistical arbitrage*. Arbitrage is defined as simultaneous purchase and sale of an asset in order to profit from a difference in the price, whether in space (different markets) or in time. Statistical arbitrage makes use of mathematical models to find arbitrage within existing market and use it to make profit.

The main problem with algorithmic trading is the instability and high volatility it presents, [5]. Due to the high frequency with which the decisions have to be made only a small part of information can be processed which causes most of the algorithms to look alike. This in return causes the algorithms to respond to same inputs with same outputs resulting in ultra-fast crashes and spikes.

Our concern in this work is the study of the impact of algorithmic trading, and some of it's assumptions, through models created by minority games. Certain behaviour observed within these models can help us better understand the context of high frequency trading, and the vicinity analysis has shown that it can influence greatly the outcome of a competitive system, such as a financial market.

1.4 Bounded rationality and Overfitting

Efficient market hypothesis states that it is impossible to "beat the market", [6]. Among many assumptions that this theory makes is the one that the actual state of the market reflects all the past information, and that the agents involved are rational. Other assumptions claim that all the changes of information are instantly reflected in the market, and even that the hidden information present within the market is reflected in the prices. Although it has been a guideline theory for investors during the last few decades, this theory that explains the market as being highly rational is being heavily criticized after the 2007 financial crash.

Bounded rationality on the other hand is the theory proposing that when humans make decisions, their rationality is limited by their cognitive abilities, information available and the time at their disposition. This view has been first theorized by Herbert A. Simon which he proposed as an alternative approach of modelling decision-making in economics, politics and other social areas. Simon claims that the human mind uses it's extensive knowledge of the structure of the problem at hand to make the decision, thus usually resulting in satisfactory although generally not optimal behaviour.

In machine learning, when faced with the problem to find the underlying relationship between data, a statistical model is obtained by making it fit to the data at hand. During this process one can decide how much data should be available for learning, with which parameters it is to be done, and what family of functions will be used to produce the statical model to fit the data. If too much data is given, or an inappropriate family of functions is used, a phenomenon of overfitting can occur. This means that the statistical model describes not only the data available but also the noise and eventual errors present in them. This phenomenon presents a problem when we try to use the apprehended model to predict the outcome of unseen data, where it will perform poorly as it has not generalized the underlying relationship.

The bounded rationality and overfitting concepts are rather similar for the purpose of this thesis. The bounded rationality tells us that humans make decision using clever heuristics because they cannot process all the information, whether because they don't have the cognitive capabilities, the time or simply the information is not available. On the other hand overfitting phenomenon tells us that machines should use limited number of parameters for their models, lest they try predicting all the errors and noise, thus making terrible decision making algorithms. The duality between these two approaches is evident, both of them tell us that there is a certain limit to the information that should be used in a decision-making process and the model should reflect that limit in it's complexity.

1.5 Vicinity Structure

When modelling a competitive agent based systems one has to decide how to implement the structure and the relationships between agents. One way is to consider the agents as an independent set, whose only way of communicating with each other is through the global information passed to every agent in the same form. Another approach is to model the set of agents as a graph, where each vertex represents an agent and each edge the passage of information between two agents. Of course, one can also model the system by merging the two approaches so that the agents have access to global information but also to the local one through their neighbours.

In the classical minority games vicinity is not considered, meaning that agents have access only to the global information. We find this kind of approach lacklustre when it comes to modelling financial markets and most of the other problems to which minority games have been applied. One important factor is the structure of the vicinity that is modelled as it influences the way information is passed around. We have tried various types of network ranging from the simplest to more sophisticated ones.

The simplest approach is to use a one dimensional array that represents a list of agents and divide it into a number of communities that we want to model. Another similar method, but with different characteristics, is to use a sliding window on the one dimensional array of agents, so that each agents has a personal neighbourhood. This way the number of communities is the same as the number of agents and makes the passage of information between them a bit slower.

Observing the phenomenons that are being modelled it is easy to notice that they do not have a one dimensional structure. This has pushed us in the direction to try different kind of vicinity structures, mainly using well known von Neumann and Moore neighbourhood. Defined as a set of point with Manhattan distance equal to 1, the von Neumann neighbourhood can be extended to a vicinity of a point of radius R defined as a set of points with Manhattan distance less than or equal to R. Moore neighbourhood on the other hand uses the Chebyshev distance, defined as a minimum distance along any axis between two points. As with the von Neumann neighbourhood, we can extend the vicinity defined with Chebyshev distance to the set of point where it is less than or equal to R.

More complex ways are based on graph theory that define the way vertexes are chosen when establishing connections and the clustering factor of the network. Most eminent examples are the small world networks where number of edges is small relative to the number of vertexes, however the distance between two random nodes grows like a logarithmic, ie.

$$L \propto \log N$$

The Watts–Strogatz model is an example of a small world network and it has been use in this thesis for vicinity generation.

Another complex model taken from the networks theory is the scale free model. The basic idea behind these models is that the distribution of number of connections per node follow a power law. One example is the Barabási–Albert model that generates random graphs by using a preferential attachment mechanism. Scale-free networks are widely observed in natural and human-made systems, including the Internet, the world wide web, citation networks, and some social networks which makes them an excellent candidate for the purposes of our study.

1.6 Structure of the thesis

This thesis is structured as follows. In Chapter 2 we expand on the introduction of the minority games made in this chapter, by describing some properties of the model, control parameters and possible modifications to better suite our intents. Next we describe more in detail the vicinity structure in Chapter 3, how it is generated and what are their properties.

Chapter 2

Minority games

Minority games (MG) is a simple multi-agent based approach to simulating financial markets. It was first introduced by Challet and Zhang in [2], and has since evolved in it's many forms. Although it has been mainly used to simulate financial markets, with certain modifications this model can be used to simulate any kind of system where agents act independently and in their best interest, while the resource for which they are competing is limited. Humans and machine solve these kinds of problems everyday and some of the example are the choice of a road to take to evade traffic, or the routing a packet takes in the network to evade delays.

Main idea behind minority games is that each agent acts in his own best interest by following a certain set of strategies defined for each agent. These strategies are deterministic, and each agent has a number of them. It has been noted that the number of strategies per agent, as long as it is greater than one, has no effect on the qualitative properties of the model, so in most works it is enough to give two strategies per agent to test various hypothesis.

The agents use the history of the game to decide at each round which action to compute, A or B, and the history itself is generated by the agents.

At each round a minority is calculated and is defined as a winning side, so if fewer agents have chosen B as their action it becomes the winning side, and all the agents and strategies that have made that decision are awarded points.

Most of the economics models are deductive in nature and have proven difficult to analyse with conventional physicist models. Since the agents in minority games are inductive, the model has proven popular among physicists to study and analyse financial markets by using some conventional physicists models [7].

Another major feature of the MG model are two distinctive phases which characterize the game. In the two phases there are clearly different collective behaviours of the agents that can be explained by the quantity and cognitive abilities of the agents.

Further noted is the fact that with a simple model as this, and small modifications, various financial market characteristics are observable. Macroscopic behaviour characteristic of financial markets, such as fat tail price return as volatility clustering, can be observed in MG models. Aside from allowing a macroscopic analysis of the simulated financial markets, minority games offer an opportunity to study the microscopic properties, mainly how the decision-making process used by every agent. All of these aspects have made minority games popular among physicists interested in the study and analysis of financial markets, and have brought around a new field of research known as econophysics.

There are certain variations that have been proposed in literature to bring the model even closer to the financial markets. One of the main versions of the game is Grand Canonical MG, that adds the possibility for agents to abstain from the market if they find the game unfavourable to them. Due to the simple nature of the basic model there is great freedom in modifying the behaviour of the agents and thus the model, and many variations are available in the academic literature.

In this chapter we describe the basic model of the game 2.1, give a more detailed definition in subsection 2.2. After that we explain the major features of the model in 2.3, and conclude the chapter with two subsections that introduce the variations of the model used to simulate financial markets in 2.4 and our own model that add the vicinity structure to the game in **??**.

2.1 The Basic Model

The basic model of the minority games are based on the El Farol Bar problem, defined by Brian Arthur in [3]. In the El Farol community every Thursday there is a cultural event that people like to attend. However if more that 60% of the population goes to the Bar they will not have fun for it is overcrowded, and a better decision would be to have stayed home. If less than 60% of the population is present at the Bar than the will have good time, and staying at home is considered a less favourable decision. Every person has to decide independently based only on their knowledge of past week. This makes their behaviour inductive, as they can only remember a finite amount of weeks and the attendance at the bar. The agents act in their own best interest and try to predict every week what the attendance will be at the bar, and then decide whether to attend or stay at home. This model is also self contained as the new information, ie. the attendance at the bar in current week, is generated by the population.

This idea has been modified by Challet and Zhang into the first model of Minority Games. Mainly the limit for deciding the winning side has been lowered to 50%, which makes the winning side the minority one, hence the name Minority Games. The use of MGs as a model to simulate financial markets is justified by a simple consideration of the nature of economic system. The basic assumption in the system is the supply and demand phenomenon. This economic concept explain that when the supply is high, the price will be driven low so it is considered a good choice to buy. Viceversa, if the demand is high it will drive the price high and a strategy to sell is considered good. This simple mechanism, to buy or sell based on the fact whether other participants of the market are buying or selling is perfectly simulated by the minority rules.

2.2 Definition of the Model

The model is defined as a set of N agents, where N is an odd integer. This constrain is used to be able to determine the minority side. Population of agents is involved in a series of repeated games where at each round every agent has to make a choice between two actions. These to actions can model various resources, as mentioned in 1.1, and in the case of computational representation we have chosen to use "0" and "1". Note that in some literature the convention for simulating minority games is to use "-1" and "1" as the opposed actions possible, hence some definitions have to be change to reflect a different choice representation. The action that the agent takes at step t is also referenced as *bid* in literature and is denoted by $a_i(t)$, corresponding to the bid of the agent i at time t.

Each agent makes his decisions based on a set of strategies that are available. When the game starts agents draw a number of strategies, equal to S, from the set of available strategies. A strategy is defined as a discrete function, $f : 2^M \to \{0, 1\}$, where M is the *memory* or the *brain size* of each agent. The memory represents how much past information can each agent store and use in order to predict future outcomes. An example strategy of brain size 3 can be seen in 1.1. For a game with memory M the total amount of possible signals is 2^M, hence the total number of strategies in the strategy pool is 2^{2^M}.

The history, denoted as $\mu(t)$, is a string of M bits that records the winning actions of the past M steps. It is also called the *information* as it can be of external or internal origin, or be a mix of the two. So if an agent is using this strategy defined in 1.1 to predict future outcome, and the *information* is $'101'$, it will predict that the next correct action should be 1. Of course, to see whether this action is really the winning action we have to look at the decisions made by all the agents.

The sum of all the agent's decisions is called *attendance*, denoted as $A(t)$ and defined as:

$$A(t) = \sum_{i=1}^{N} a_{i,s_i}^{\mu(t)}(t) = \sum_{i=1}^{N} a_i(t)$$

Where $a_{i,s_i}^{\mu(t)}(t)$ is the decision made by agent i at time t using the best strategy $s_i(t)$ with the information $\mu(t)$.

To define how the best strategy is calculated between S strategies of the agent, we first need to introduce the concept of *cumulated payoff*, referred also as *virtual score* of the strategy. The idea behind the virtual score is to follow the decision making of the strategy through time, whether it is used or not, and confront it to the winning choices. If the strategy predict the winning side correctly, even if it is not used, it is rewarded a certain amount known as *payoff* to it's virtual score, hence the name cumulated payoff. Viceversa, when the strategy makes an erroneous prediction same amount is detracted from it's cumulated payoff. This way agent can see which strategy would have brought him best win ratio over time, and chooses to use it in the next step. If more than one strategy has the maximum virtual score at time t, then one of the best strategies is chosen randomly. The best strategy is defined as:

$$s_i(t) = \arg\max_s U_{i,s}(t)$$

Where $U_{i,s}(t)$ is the cumulated payoff of strategy s of agent i. This parameter starts from an arbitrary value, usually zero, and is defined as:

$$U_{i,s}(t+1) = U_{i,s}(t) - \text{sign}[(2a_{i,s}^{\mu(t)}(t+1) - 1)(A(t) - \frac{N}{2})]$$

Note that the convention we are using for representing actions is "0" and "1" so our attendance is always positive and has to be confronted with $\frac{N}{2}$. Same goes for the agents action that should be brought to the "-1" and "1" representation to be able to calculate whether the agent made the winning decision. If we were using the $(-1, 1)$ convention we could calculate the cumulated payoff at time $t+1$ with

$$U_{i,s}(t+1) = U_{i,s}(t) - \text{sign}[a_{i,s}^{\mu(t)}(t+1)A(t)]$$

With the mechanism of choice between different strategies each agent becomes adaptive. Of course there is a problem with randomly drawing strategies for it is possible for an agent to draw two very similar strategies and hence cannot use the information to it's fullest as his strategies act in similar fashion.

Being the total number of agents equal to an odd integer, a minority side can be calculated at each step. As the number of winner is always smaller than the number of losers the minority game is a *negative-sum-game*. Since the two actions are symmetric it can be noted that the average of attendance over time is equal to $\frac{N}{2}$ (or 0 when $(-1, 1)$ convention is used). It is therefore more interesting to study other moments of the model, mainly the fluctuation of attendance. The variation of attendance is defined as

$$\sigma^2 = \langle A^2 \rangle - \langle A \rangle^2$$

It is one of the main parameters when studying minority games, and has been proven that the variance of attendance in a model is not influenced by the source of information. So whether an exogenous or an endogenous model is simulated the observed properties of the variance remain the same.

2.3 Major features

Most important feature of the minority game model is the characteristic two phases that occur described in 2.3.2, but first we describe the logical approach took to define the control parameters of this phenomenon.

2.3.1 Standard deviation of attendance

As the average of the attendance of a basic model tends to $\frac{N}{2}$ as noted in 2.2 it is not very useful when studying the model. There are two major aspect that can be studied in the model, M and N that respectively represent the memory of the agent and the total number of agents. One can choose to study what happens when cognitive abilities of agents change, ie. the quantity of information that can be remembered and processed. Let's assume that N is fixed, and that we vary M within certain interval. One simple value that we can track is the standard deviation of attendance. The graph of standard deviation against brain size is shown in figure 2.1. The data have been generated using basic minority model, with $N = 101$ and $M \in [2, 12]$. We have done in total 32 runs to be able to see the trend in the data.

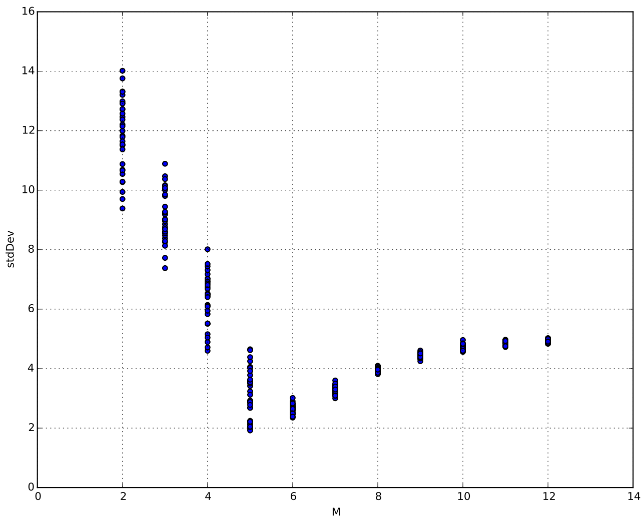


Figure 2.1: Plot of standard deviation of attendance for a model with 101 agents and 32 runs. In the axis memory (brain size) of agents

We can see that there is a minimum of the function somewhere between value 5 and 6 for memory. Before this point standard deviation is definetly high with respect to the rest of the graph, and after the critical point it seems to converge to some value.

It is already evident that there is certain connection between the memory of agents and their ability to perform efficiently, intended as a efficient distribution of the resource they are competing for. In this graph however N is a fixed value, so let us see what happens when we start varying both parameters.

2.3.2 Phase Transition

After further studies Savit, Manuca and Riolo [8] have noted that the macroscopic behaviour of the model does not depend independently on the single parameters M and N, but rather on the rapport between the two. They have discovered that a new control parameter α defined as $\alpha = \frac{2^M}{N}$ defines the volatility of the model. Volatility is defined as a normalized variance:

$$\frac{\sigma^2}{N}$$

The volatility depends only on the ratio between 2^M and N and it is no influenced by the source of information, meaning that it maintains it's characteristic behaviour in endogenous and exogenous games.

In figure 2.2 volatility is plotted versus the control parameter α. The red line in the graph represents the volatility of a *random-choice* model. If we create a model where all the agents make random decision at every step the volatility that we obtain is:

$$\frac{\sigma^2}{N} = \frac{Np(1-p)}{N} = 0.5(1-0.5) = 0.25$$

This is calculated assuming a binomial distribution of agent's actions with probability $p = 0.5$.

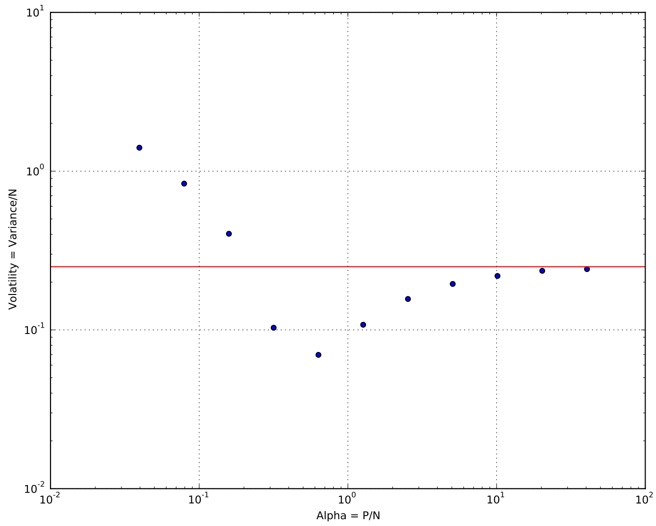


Figure 2.2: Plot of normalized variance versus control parameter α

Looking at the graph 2.2 we can see that for low values of our control parameter α agents perform worse than they would if only random decisions where made. By incrementing the control parameter, either by raising the memory available or remove certain quantities of agents from the model, the volatility pummels to it's minimum at the critical value of α (α_c). This critical value has been calculated in [9] by Marsili et al. and is equal to 0.3347... for $S = 2$. By incrementing further the control parameter the volatility starts incrementing again and converges to the random choice limit.

This behaviour can be observed if we look at the plots of attendance for models with different values of α. In figures 2.3, 2.4 and 2.5 graphs of attendance can be seen for models consisting of 101 agents but with varying memory size. Different brain sizes plotted here are 2, 5 and 9, which gives us α values of 0.03960, 0.3168316 and 5.0693069 respectively. For α bellow it's critical value we can see that the variance of the attendance is rather high, it becomes minimum for $M = 5$ and then again as we increase the brain size it start to have a volatile nature.

The α_c identifies a separation between two phases of minority games. To characterize better the two phases let's look at the information available to agents in different phases. We plot the probability of "1" being the winning choice given a certain history, $P(1|\mu)$ in figures 2.6 and 2.7. When the control parameter is below it's critical value α_c the probability of "1" being the winning side is equal to 0.5 for all values of history μ. This shows the fact that there is no information to be extracted from the model for agents of that particular brain size for all outcomes seem to be random. For reasons expressed, this phase is called *unpredictable* phase or also *symmetric* phase for the symmetry present in the histogram of probability given a certain history. If we look at the graph of probability given a certain history when

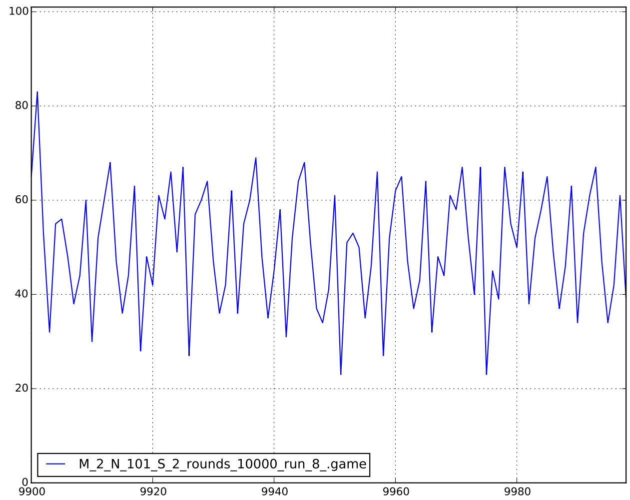


Figure 2.3: Plot of attendance over time for a model with agents with $M = 2$ and $N = 101$

the control parameter α is above it's critical value, given in figure 2.7, we can see that there is information available to be exploited in this phase. This phase hence is called *asymmetric* or *predictable*. In this phase agents act better than when making random choices, and even though each agent acts in his self best interest we can say that a phenomenon of cooperation emerges as the agents are able to distribute themselves on both sides with rather small variance. Note that even in the asymmetric phase model retains it's *negative-sum-game* nature and majority of the agents continue to lose, however the number of losing agents is brought to it's minimum.

It is important to note that the information is present within the model even in the symmetric phase, but the agents don't have the capabilities to exploit that information. In fact, if we introduce an agent with M greater that those of agents already in the *symmetric* phase we can expect that he will be able to exploit his advantage of greater memory. The information

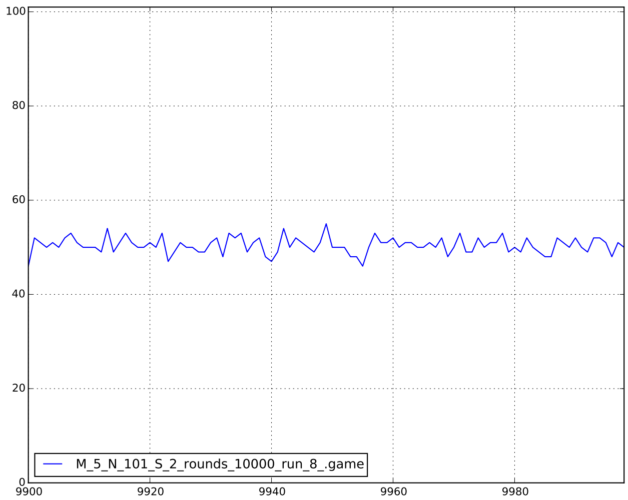


Figure 2.4: Plot of attendance over time for a model with agents with $M = 5$ and $N = 101$

present in the model for an agent with larger brain size can be seen in figure 2.8 that plots the probability that "1" will be the minority size versus the possible history.

2.3.3 Crowds and anti-crowds

The nature of *symmetric* phase brings about the phenomenon of the formation of crowds and anti-crowds. When we find ourselves in the symmetric phase the α is below it's critical level, meaning that 2^M is roughly one order of magnitude smaller than N. The consequence is that the number of available strategies 2^{2^M} is low when compared to the number of agents, hence probability of different agents having same strategies is higher than in the *asymmetric* phase. Another way to explain the phenomenon is to say that when M is low agents are able to efficiently elaborate the information, however since the information is short many agents will come to same predictions,

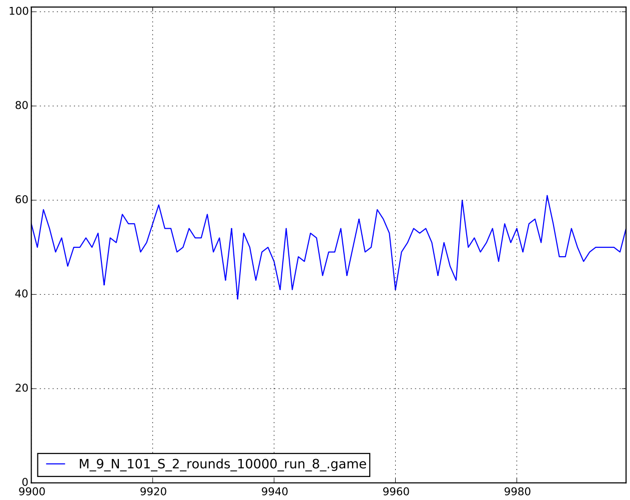


Figure 2.5: Plot of attendance over time for a model with agents with $M = 9$ and $N = 101$

and behave in the same fashion.

The phenomenon of crowds and anti-crowds thus happens in symmetric phase and when it occurs most of the agents behave in the same way, giving rise to high volatility. If number of agents is large and the number of available strategies is small, it is more opportune to make decisions randomly rather than use deterministic strategies.

2.4 Variations for financial markets

The basic minority games introduce a very simple model for the financial markets, however there are certain modification to be done before we can start comparing it to a real market. Let us assume that the action "1" stands for "buy" and action "0" stands for sell. Attendance can now be seen as a number of agents that participate in the market as buyers, while $N - A(t)$ is the number of sellers. The quantity $A(t) - \frac{N}{2}$ is the excess demand

Figure 2.6: Plot of $P(1|\mu)$ versus μ using a sliding window of length 4 on a model where agents have $M = 4$

in the market. So if that quantity is positive most of the agents involved are buyers and it is profitable at that moment to sell, and viceversa. This is an economical point of view to the basic mechanism of minority games. The payoff that is given to agents at each round, introduced in [10], is defined as

$$g_i(t) == a_i(A(t) - \frac{N}{2})$$

This captures the fact that the agent on the minority side are awarded proportionally to their investment.

To model the prices Challet et al. have used this price dynamics in their original paper:

$$\log p(t+1) = \log p(t) + \frac{A(t)}{\lambda}$$

where λ is related to the market depth.

The most important modification of the basic model when trying to simulate financial markets is the introduction of two types of agents, *producers*

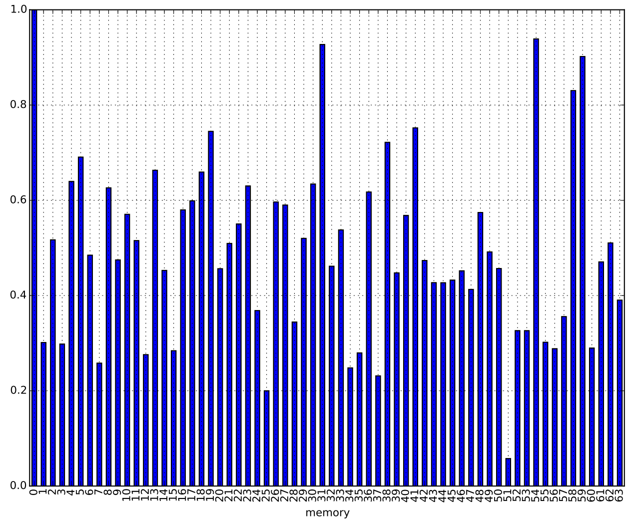


Figure 2.7: Plot of $P(1|\mu)$ versus μ using a sliding window of length 6 on a model where agents have $M = 6$

and *speculators* that interpret different approaches to the real market.

2.4.1 Producers

Producers, per definition, contribute to the market always following a predetermined behaviour. They model the agents that produce goods and services, and as such participate in the market at all times. Since their scope is not to speculate on other agents behaviour but to use market to sell their goods and buy other goods needed, they have a deterministic behaviour, acting always in the same fashion for same values of $\mu(t)$. Inside the model of the market they create information that is then exploited by speculators.

To model producers starting from our basic model is rather simple, only one strategy is given to the agent and no other changes are made. Participation is already obligatory in the basic model and having only one strategy makes his deterministic.

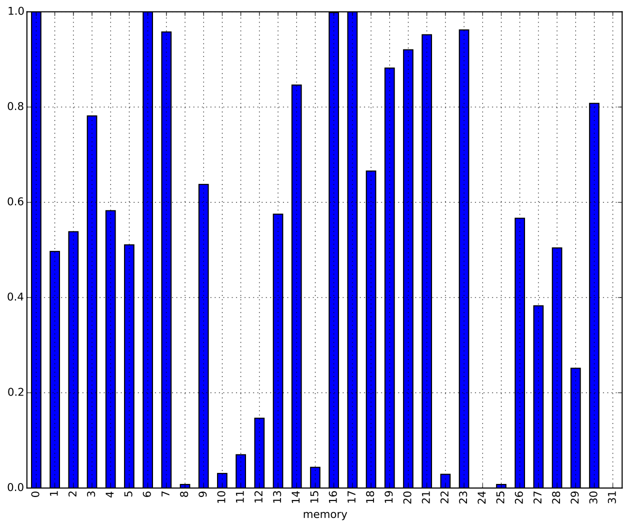


Figure 2.8: Plot of $P(1|\mu)$ versus μ using a sliding window of length 5 on a model where agents have $M = 3$

2.4.2 Speculators

Speculators on the other hand do not produce goods or services, but use the market to exploit the information injected in the model by the producers to gain profit. Two main differences between speculators and producers are the facts that speculators have adaptive behaviour and that they can choose not to participate in the market if they find it unprofitable.

In order to model adaptive behaviour S strategies are given to each speculator, drawn randomly from 2^{2^M}, from which he can choose. As for the ability to abstain from the market in unfavourable conditions we add yet another strategy, called *0-strategy* that tells the speculator not to interact with the market at chosen time.

At each round the speculator chooses the best strategy by picking the one with the highest virtual score, calculated with:

$$U_{i,s}(t+1) = U_{i,s}(t) - [(2a_{i,s}^{\mu(t)}(t+1) - 1)(A(t) - \frac{N}{2})] + \epsilon\delta_{s_i(t),0}$$

The first part of the formula is the same as the virtual score calculation for the basic model, ie. a strategy is awarded points if it correctly predicts the minority side. The second part models the virtual score of the zero strategy that get incremented at every step by ϵ. This new parameter models the interest rate for the speculators, making them active participants in the market with only the strategies that can guarantee a profit over time that is larger than ϵt, with t the number of rounds thus far. Note that all the considerations above with the assumption that ϵ is positive, in fact if we set ϵ to be infinitely negative speculators act as agents in the basic model.

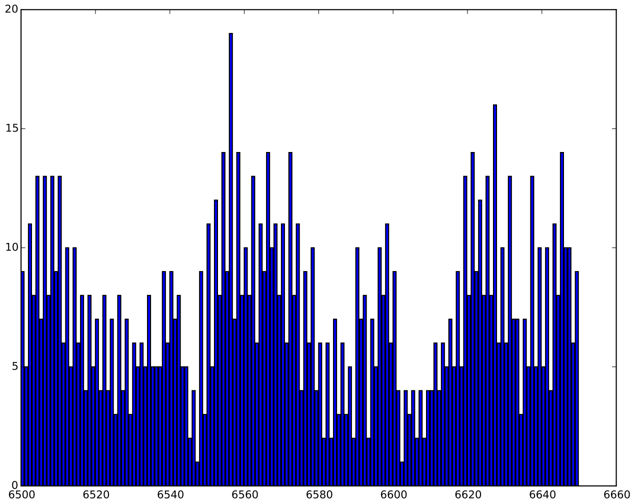
2.4.3 Market ecology and crash simulation

To simulate markets a minority game is defined with a certain number of producer and speculator agents. The market is characterized by the rapport between these two quantities. As producers behave in a deterministic fashion we can say that they introduce certain information inside the model that can be used by speculators. Speculators on the other hand participate when they have a strategy that can consistently outperform the producers, ie. if the virtual score of the strategy is larger than ϵt, where t is the time and ϵ models the interest rate.

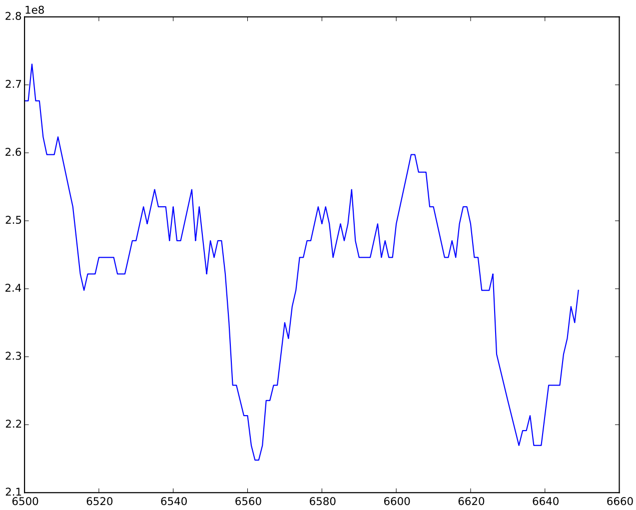
If the number of producers is sufficiently high and greater than the number of speculators, than a majority of speculators will be always active as the information introduced by producers is high as well as the possibility to have a strategy that can use that information. On the other hand when the number of speculators is higher than the number of producers, a majority of speculators will not participate in the market.

The dynamic when a great quantity of speculators is present when compared to producers causes fluctuations in the market. In the beginning the speculators participate in the market, but after certain period t_1 the zero-strategy starts to have higher virtual score than the rest of strategies, causing speculators to abstain. After t_1 then the majority of agents involved are producers that introduce information in the model, so after a second interval at time t_2 certain number of speculators start acting inside the market again and reduce the information available.

This behaviour of speculators causes the price to fluctuate and can cause crashes and spikes inside the market. An example is shown in figure 2.9, where in 2.9.(a) we can see the number of active speculators and in 2.9.(b) the market price generated with the mechanism described in 2.4, We can see that the sudden participation of a great number of speculators in the model can cause crashes.



(a) Number of active speculators in a game with 10 producers and 300 speculators



(b) Market price for the same game in the same interval

Figure 2.9: Active speculators and market price

Chapter 3

Topology of the model and vicinity information

In the basic model of minority games all agents act independently without the ability to communicate with each other. Only means of passing information between them is by using the history of the model that encapsulates all the decisions made by agents in a string of bits. This simple model allows us to study certain characteristics of competitive systems with limited resources, however it is not a realistic assumption to think of agents as isolated elements that don't communicate with their neighbours.

Once the information of the neighbouring agents is added to the model, two important factors start to influence the dynamics of the system, (i) the dimension of the vicinity, ie. the quantity of neighbouring agents and (ii) the topology of the network generated.

In this chapter we introduce various structures used to model vicinity and how they influence the information flow in the system. Before diving into vicinity structures, the modifications of the basic model of minority games to include local information is defined in 3.1. First sections, 3.2 and 3.3, refer to rather simple structures that are computationally easy to implement and maintain, however they do not express much similarity with real world applications. Another similar structure is explained in 3.4 based on von Neumann distance. In the later sections, 3.5, 3.6 and 3.7, more complex structures are introduced based on graph theory that simulate real world examples more closely.

3.1 Model modifications to include local information

Once decided to make available new information to agents, we need to decide what that information should be and how will it be used by agents.

Information that we decided to include is the results of reduced minority games consisting of agents in the community. In addition to global minority game, additional C minority games are added, where C is the number of communities that we want to include. Each minority game follows the same definitions from Chapter 2 and the information generated is available to the agents that define the community. Another convention used here is that each minority game contains the same quantity of information available as the global minority game, meaning that if the length of the string of bits representing history is equal to M, then also the local history remembered should be M bits long.

To incorporate this change in our basic model the simplest way is to create new kind of agents, called *community agents*, that have the same behaviour, but their brain size is doubled. By doubling the brain size, we do not have to change the implementation of strategies, only the history that is passed as argument to each strategy. For example, a strategy of a new community

local		global		prediction
0	0	0	1	1
0	0	1	1	0
0	1	0	0	0
0	1	1	1	1
1	0	0	0	0
1	0	1	1	0
1	1	0	0	1
1	1	1	0	0

Table 3.1: Example strategy of a community agent with brain size 2

agent with brain size 2 is generated is the same fashion as a strategy of a basic agent with brain size 4 is generated. A strategy for community agent with brain size 2 is visible in figure 3.1 where first 2 bits, denominated local, are generated by the community minority game of the agent, and second 2 bits, denominated global, are generated by minority game consisting of all the agents.

One particular case that should be noted is that by dividing the agents in communities, depending on the procedure used, a minority game where the number of agents is pair can occur. In this case additional guard should be put in the simulation to generate a random winning side when the attendance is equal to $\frac{N}{2}$. Of course certain procedures are implemented to reduce the number of community minority games with pairwise number of agents to minimum in order to reduce the influence on the whole model. Using simple structures this condition does not usually present itself, however when more complex structures are implemented it occurs with higher frequency. Ad hoc procedures have been implemented in this thesis to reduce the number of

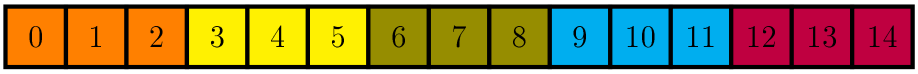


Figure 3.1: Example of 15 agents divided in 5 communities, consisting of 3 agents each

pairwise minority games when using scale free and small world structures.

3.2 Fixed one-dimensional communities

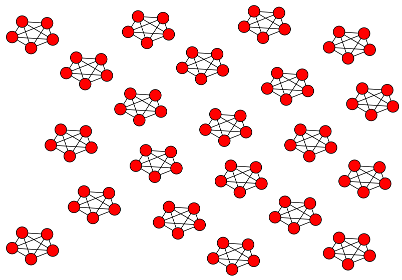
The simplest approach to include local information inside the model is to divide agents in certain number of communities. By using a one-dimensional list to represent agents we can divide the array in equal parts to model communities. Let's assume N the number of agents, and C number of communities, the communities are defined as

$$C_i = \bigcup agent_j \qquad | \qquad j \in [i, i + \frac{N}{C})$$

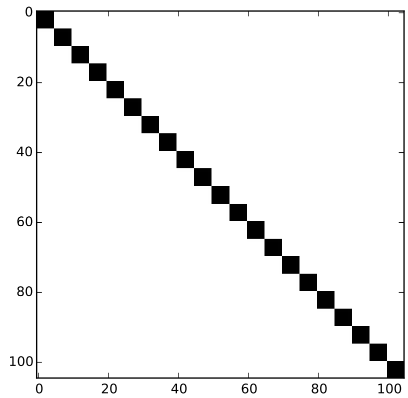
An example division can be seen in figure 3.1 where a set of 15 agents is divided in 5 communities. When the rest of $\frac{N}{C}$ is different from zero, last N mod C agents are assigned to their own community.

This structure creates isolated communities and only way for information to flow between communities is through the global state. Same local information is available for each agent of the community and it is expected that this information will be used efficiently as they are all involved in the same minority game at a local level. However this sort of connectivity between agents does not represent real world examples as in most scenarios communities are not isolated from each other.

An example network generated by this division is shown in Figure 3.2, were the isolation of communities from each other is clearly visible. The same



(a) The graphu



(b) The adjacency matrix

Figure 3.2: Graph and the adjacency matrix of a set of 101 agents divided into 21 fixed one-dimensional communities

result can be seen by looking at the adjacency matrix in subfigure 3.2b.

3.3 Sliding window communities

To overcome the difficulty of having isolated communities we take different approach and create a community for each agent. This is more representative of real world cases and also of the way minority games are defined.

Sliding window technique is used to create communities by creating a local neighbourhood for each agent. Another possibility opens up here in deciding how many dimension we want to use to represent the set of agents. We have used one-dimensional arrays and two-dimensional matrix to group agents into communities, and have not delved in higher dimensions as it does not bring any qualitative change, only modifies the graphical representation of agents.

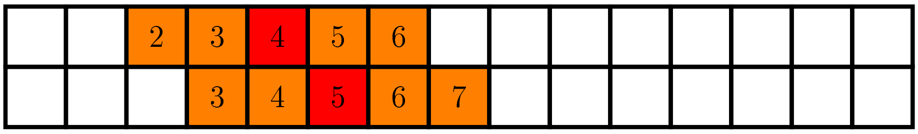
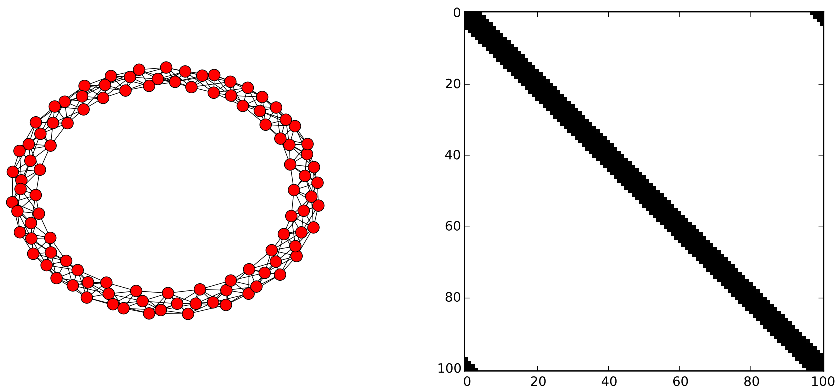


Figure 3.3: Example of communities created using sliding window on one-dimensional array for agents 4 and 5 where each community consists of 5 agents

3.3.1 Sliding window on one-dimensional array

Let's assume N is the number of agents and V is an odd integer representing the number of neighbours in each community. For each agent a_i a community C_i is created from a sliding window of length V centred on agent a_i. an example of this simple procedure can be seen in figure 3.3



(a) The graph

(b) The adjacency matrix

Figure 3.4: Graph and the adjacency matrix of a set of 101 agents divided into 101 sliding window one-dimensional communities

The network created in this way allows the information to flow between communities. This kind of behaviour renders the efficient use of information more difficult when minority games are concerned, but it is more representative of human and human-made systems. An example of a network can be seen in Figure 3.4 where the graph of a model with 101 agents, with each agent having a community of 5 neighbours, and the adjacency matrix are drawn.

3.3.2 Sliding window on a matrix

Different approach is to represent agents in two-dimensional array and construct neighbourhoods on the matrix generated. This opens possibilities to use different rules for generating the vicinity, such as using the Manhattan distance to generate von Neumann vicinity as described in 3.4. Another simple metric that can be used to generate neighbourhood is Chebyshev distance. The Chebyshev distance, named after Pafnuty Chebyshev, also called chessboard distance, is defined as the greatest difference of distance along any coordinate dimension between two vectors.

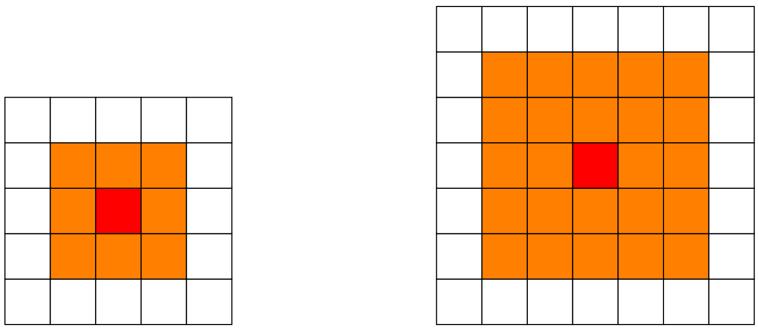
$$D_{Chebyshev}(p,q) := max_i(|p_i - q_i|)$$

For each agent a community is generated that includes all agents with $D_{Chebyshev}$ less or equal than R, where R is the radius of the neighbourhood.

3.4 Von Neumann Vicinity

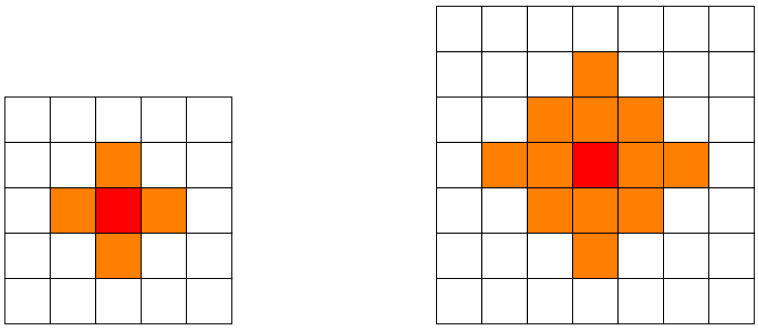
Along with the chessboard metric, Manhattan distance is another option when constructing vicinities. Called also taxicab distance, this metric is defined as the sum of the lengths of the projections of the line segment between the points onto the coordinate axes.

$$d_1(p,q) = ||p - q||_1 = \sum_{i=1}^{n} |p_i - q_i|$$



(a) Moore vicinity of radius 1 (b) Moore vicinity of radius 2

Figure 3.5: Vicinity calculated with Chebyshev distance, called Moore neighbourhood

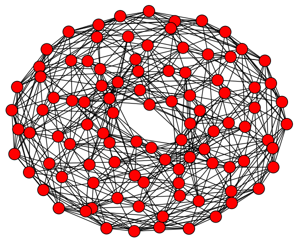


(a) Radius 1 (b) Radius 2

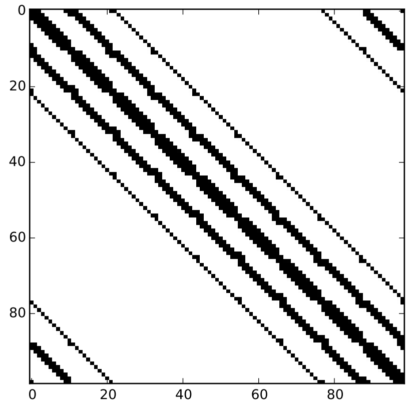
Figure 3.6: von Neumann vicinity with radius 1 and 2

The vicinity is constructed by using a sliding window mechanism on a two-dimensional matrix of agents. For each agent p is composed by a set of agents q such that $d_1(p, q)$ is less than or equal to R, where R is the radius of the von Neumann neighbourhood. Such vicinities, with R equal to 1 and 2, are shown in 3.6.

The main distinction between using a one-dimensional array or two-dimensional matrix to represent the set of agents is that more possible metrics for constructing the vicinity are available as the dimensions get higher. The



(a) The graph



(b) The adjacency matrix

Figure 3.7: Graph and the adjacency matrix of a set of 101 agents divided into 101 von Neumann two-dimensional communities with $R = 2$

downside of higher dimension vicinities is that usually the number of agents included in the neighbourhood is fixed. For example, when constructing von Neumann vicinities the number of agents inside the community is 5 if $R = 1$ is used, 13 if $R = 2$ is used, 41 for $R = 3$ and so on. This sort of progression can prove to be inadequate when we want to study the dynamics of a system based on the number of agents in the communities. A more contained approach that enables us to study the model while increasing the number of neighbours in a linear fashion is more favourable.

The graph defined by agents as nodes and where edges represent the membership of two agents to the same community can be seen in Figure 3.7, while the adjacency matrix is shown in subfigure 3.7b. The graph exhibits similar properties to the graph defined by one-dimensional sliding window vicinities, with the difference that the communities consist of more agents, as $R = 2$ is used, ie. neighbourhood consists of 13 agents. The adjacency matrix seems to exhibit different characteristics, but the difference is mainly due to the fact that the vicinity is constructed on a two-dimensional matrix instead of a one-dimensional array. The qualitative properties do not show significant change between different dimensional approaches.

3.5 Scale free network vicinities

Scale free networks are characterized by a degree distribution, ie. the connectivity of nodes, as a power law. This means that the probability of an agent having k neighbours follows

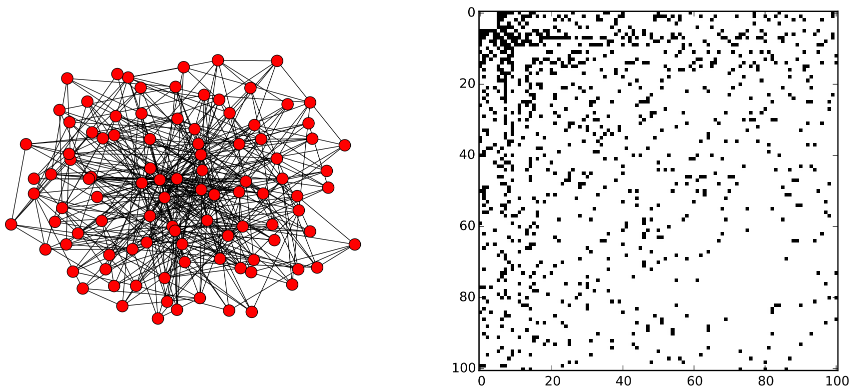
$$P(k) \sim k^{-\gamma}$$

The scale free properties are observed in many ecosystems that are the subject of this thesis, as social networks, financial markets and computer networks. The assumption used to generate and explain these kinds of network is called preferential attachment. This mechanism explains how when new nodes are added to the network they are more likely to connect to a node with high degree, rather than to a unknown node. Example of a computer network is the network of links between websites, where when new website is created it is more probable it will contain links to well known websites such as Wikipedia, rather than to some obscure nodes of the network. In human networks we can think of financial agents and agencies as nodes and the flow of information between them as nodes. As new agents enter the market they will probably be more interested in connecting to, ie. obtaining information from, a well known agency than from some obscure sources.

To construct an artificial scale-free network a Barabasi-Albert algorithm [11] is used. The network is initiated with n_0 connected nodes and new nodes are added one at a time. When generating the network, aside from the number of total nodes, a number e of edges per node is given. Each time a node is added it is connected to e nodes with probability of being connected to node i equal to

$$p_i = \frac{k_i}{\sum_j k_j}$$

where k_i is the degree of node i.



(a) The graph (b) The adjacency matrix

Figure 3.8: Scale free graph and the adjacency matrix of a set of 101 agents

We have opted to use this network structure because it closely models some of the systems that we are trying to optimize. An example graph constructed with Barabasi-Albert algorithm can be seen in Figure 3.8. Event though the number of nodes and edges makes it more difficult to see the nature of the graph, it can be noted that central nodes, called *hubs*, are the ones more connected than the peripheral ones. This property can be more easily seen examining the adjacency matrix of the graph, in subfigure 3.8b, where it is obvious that the nodes that are added initially are the ones well connected and become *hubs* as the graph is populated.

3.6 Small world network vicinities

Small world networks are a type of graphs where most of the nodes are not the neighbours of each other but are easily reachable with a limited number of hops. They have been introduced by Duncan Watts and Steven Strogatz in their joint work [12] and the two main characteristics of such a network are short average path length between nodes, and high clustering effect. The average number of hops required to reach node j starting from node i grows as a logarithm of the total number of agents, ie. $L \propto log(N)$ and hence the first property. The second property is obtained through the network generation proposed by Watts and Strogatz that differs from previously presented random generation algorithms.

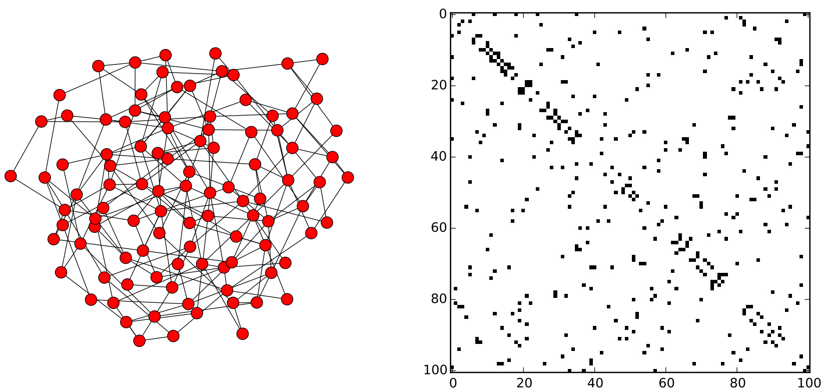
Small world networks have been observed in social networks as well as other ecosystems, human-made and natural. In the social network context the small world properties of short average node-to-node distance and high clustering are a result of strangers being linked through mutual acquaintances, and communities clustering in same space and time. The probability of two persons being linked directly is significantly higher if they inhabit the same community (or city, country, etc.) as well as if they are close in time, ie. similar age, hence explained the clustering effect.

Watts and Strogatz algorithm to generate small world network takes in input the number of nodes N, average degree of a node K and the special parameter β such that $0 \leq \beta \leq 1$ and $N \gg K \gg ln(N) \gg 1$. The graph is constructed by first creating a regular lattice ring, a set of nodes N each connected to the $K/2$ neighbours before and after if a set of nodes is represented as an array. This mechanism is the same as the one describes as a one-dimensional sliding window in Section 3.3. After the first step for every node n_i every edge (n_i, n_j) is taken and rewired with probability β.

The rewiring is done by replacing (n_i, n_j) with (n_i, n_k) where k is chosen randomly until a condition that $k \neq k'$ if $(n_i, n_{k'})$ exists.

Since there is still debate as to whether different social and computer networks are better modelled with scale-free or small world graphs, both have been used to study how their structure influences the community minority games. With small world we can expect that the information is used rather efficiently as the agents are clustered into local communities, and that some flow of information is permitted through the rewired nodes between different communities.

An example of a small world graph can be seen in Figure 3.9. Better understanding of this network is obtained if we look at the adjacency matrix in subfigure 3.9b, where we can see the resemblance with the sliding window techniques, and the randomness introduced by the parameter β. For different values of β, the similarity with sliding window vicinities changes, as can be seen in Figure 3.10 where for a low probability of rewiring the resemblance is greater that in the case shown in subfigure 3.9b where $\beta = 0.75$.



(a) The graph (b) The adjacency matrix

Figure 3.9: Small world graph and the adjacency matrix with $N = 101$, $K = 5$ and $\beta = 0.75$

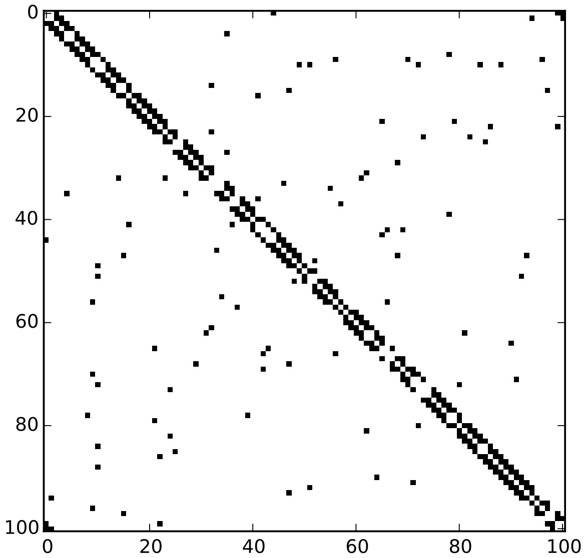


Figure 3.10: Small world graph adjacency matrix with $N = 101$, $K = 5$ and $\beta = 0.25$

3.7 Hierarchical vicinity structure

Chapter 4

Results of competitive systems with different topologies

Chapter 5

Real world applications of the research results

Chapter 6

Conclusions

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