Statistical Inference for Data Science

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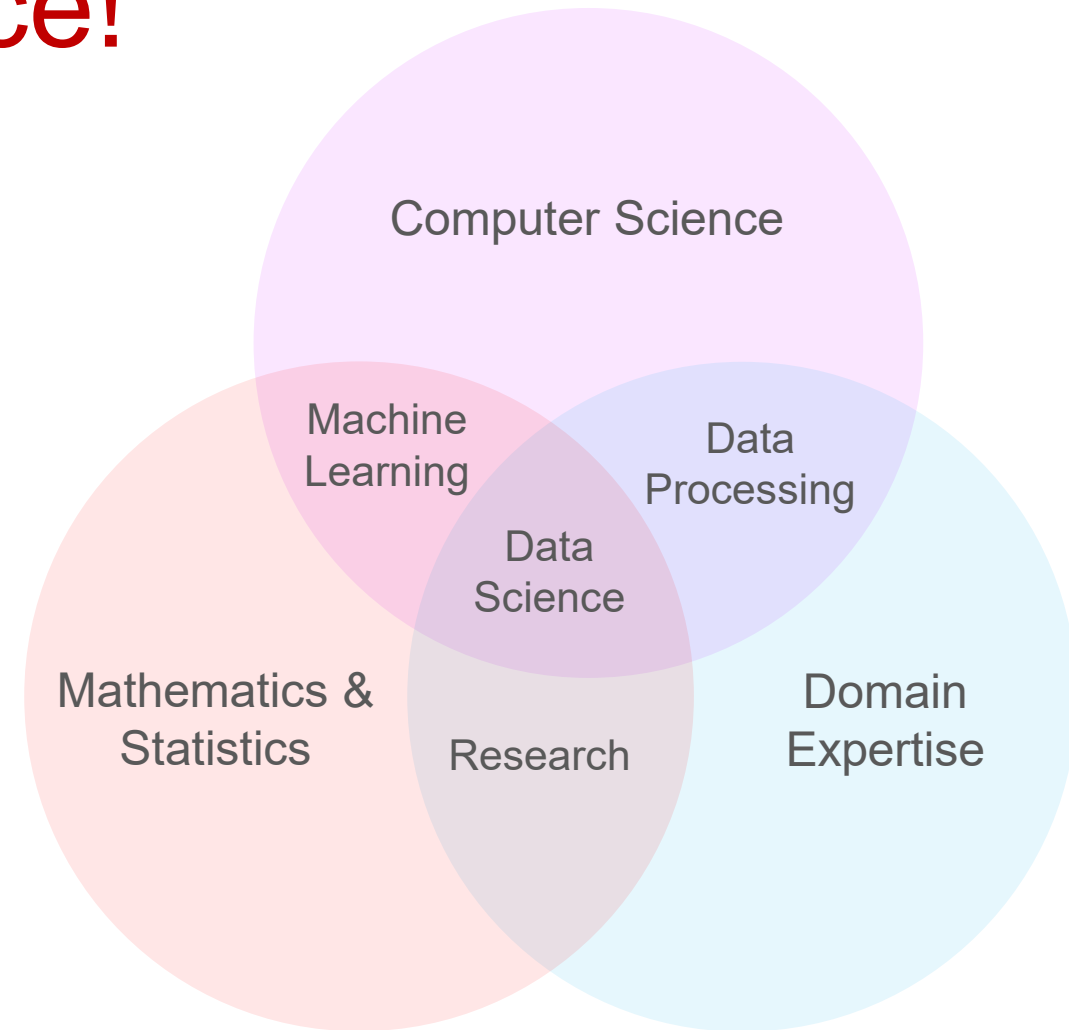
Welcome to Data Science!

Data Science uses

- Mathematics and Statistics
- Computer Science
- Domain expertise

on data to build information and extract knowledge.

Module 2

Tuesday
09:00 - 12:30 Descriptive Statistics
13:30 - 17:00 Notebook 2 (self study)

Wednesday
09:00 - 12:30 Parameter estimation
13:30 - 17:00 Self study

Thursday
09:00 - 12:30 Hypothesis testing
13:30 - 17:00 Prepare test for presentation (self study)

Friday
09:00 - 10:30 Presentations
10:30 - 11:00 Coffee Break
11:00 - 12:30 Notebook 5

! Caution

- This module aims to give a brief overview on basic statistics.

- That means in a short amout of time we'll see lot.

- While this may be repetition for some,

- For others there may be a lot of new things.

- I'll try my best to accomodate everyones needs.

Teaching

- Introductionary lectures

- In-depth self-study of the content with notebooks

- Discussion sessions based on your questions
  Please ask questions ☺

- I am open to modifications if wished for!

Project

Formal

- Group of 2-3 people

- 15min presentation, 15min discussion

- Half-day presence on presentation session

Content

- Choose your own data set

- answer research questions using statistics
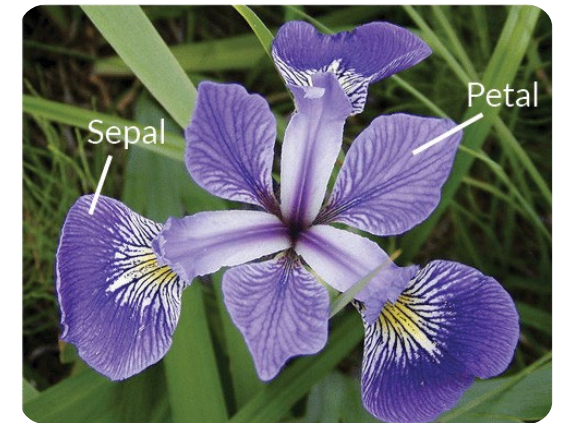
Iris data set

- *Due to time restrictions we use a single data set in this module*

- 3 classes: versicolor, setosa, virginica

- 4 characteristics
  petal: *length, width*
  sepal: *length, width*

Iris setosa

Iris virginica
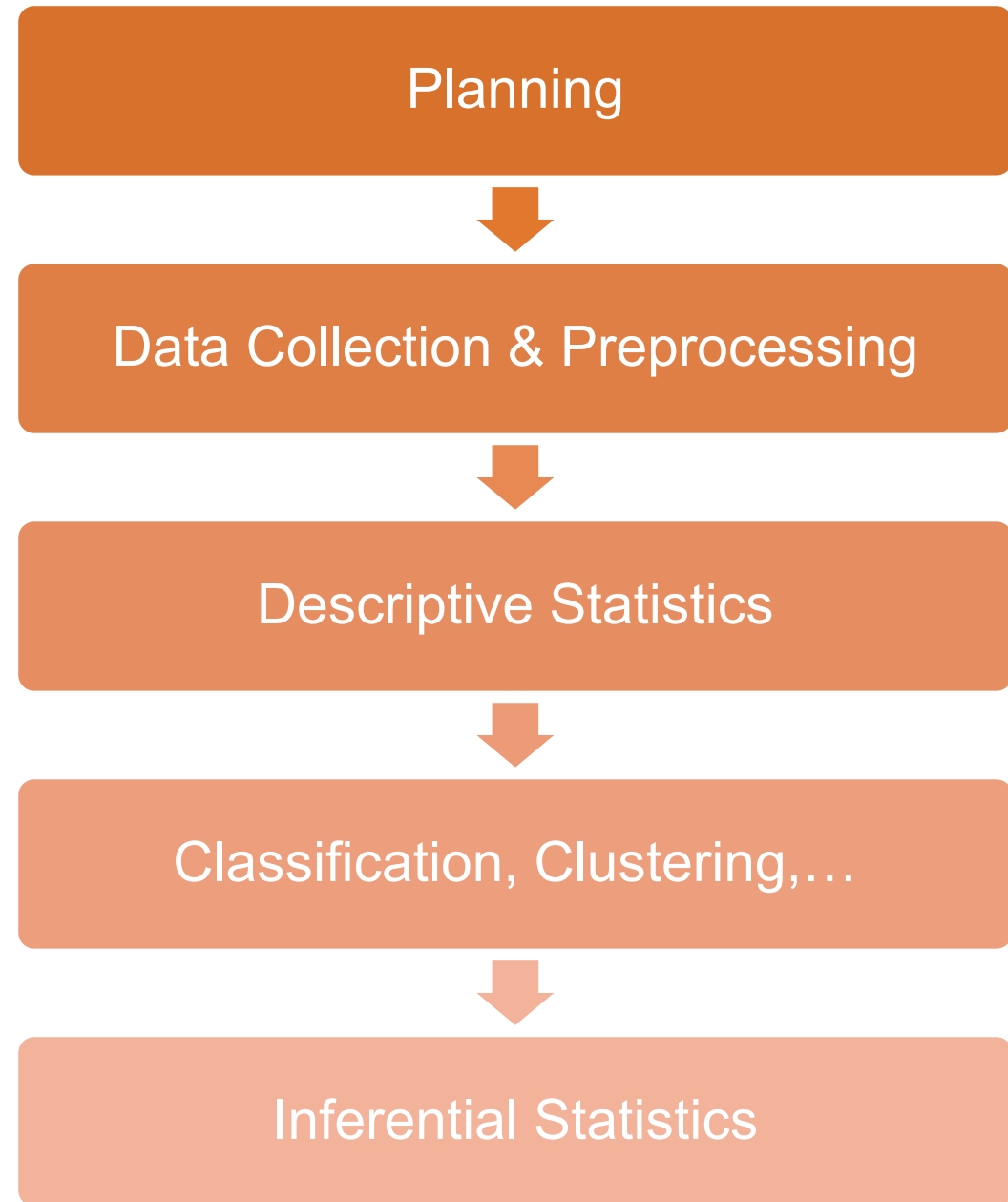

Iris versicolor

Any questions so far?

General Procedure

Planning

↓

Data Collection & Preprocessing

↓

Descriptive Statistics

↓

Classification, Clustering,…

↓

Inferential Statistics

Descriptive Statisics

Why?

- Get an overview of the data
- Identify Patterns
- Identify possible problems eg. outliers
- Get a feeling for the quality of the data

➡ good description is the basis for good inference

Descriptive Statisics

The two main tasks of descriptive statistics are

- the quantitative description and summary, and

- the graphical representation of data

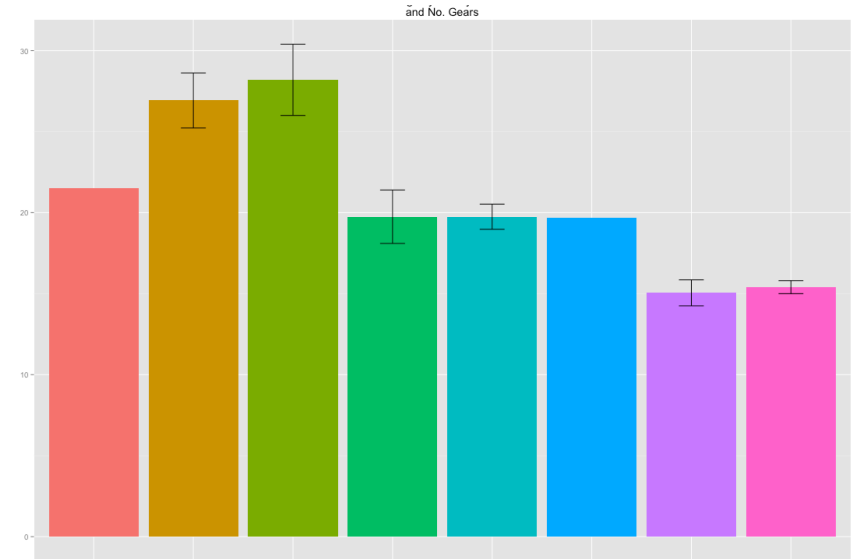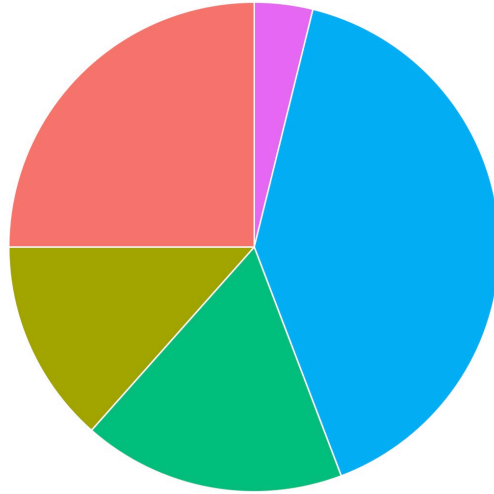What tools are suitable depends on the type of the variable we want to describe.

Categorical Variables

(quantitative)

- Absolute frequency (eg. number of female participants)

- Relative frequency (eq. number of female participants divided by the sample size)

Categorical Variables

(graphical)

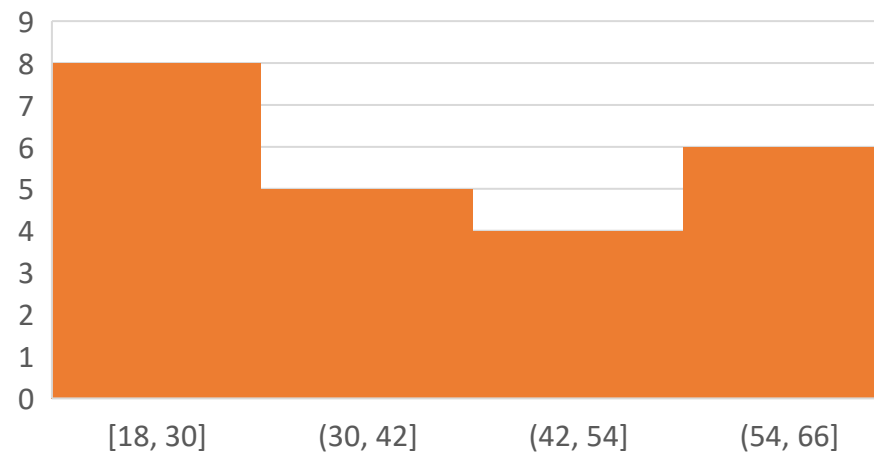(Either absolute or relative frequencies can be displayed)

Numerical Variables

(categorization)

Summary tables

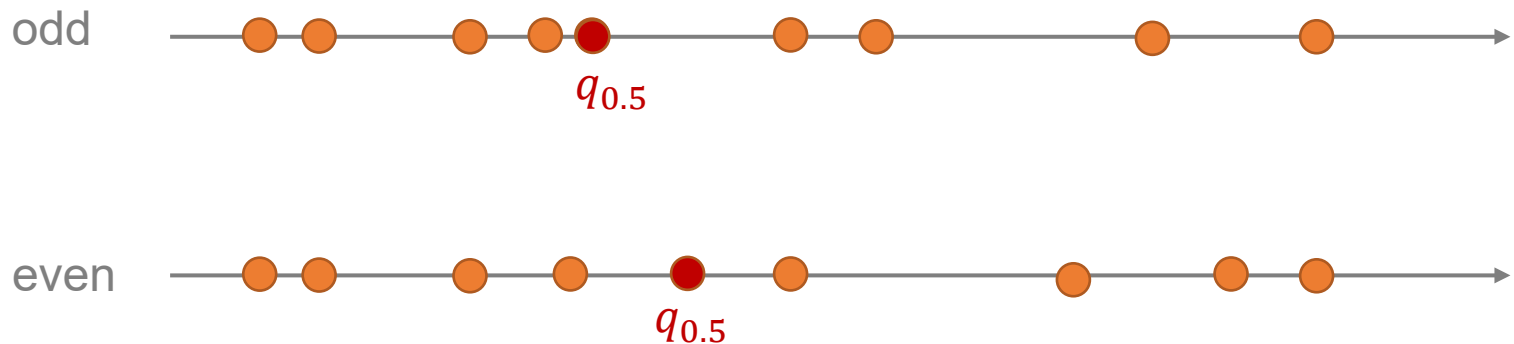| Age	Nr. of People
18-30	8
30-42	5
42-54	4
54-66	6

Histograms

Location

(Numerical Variables)

What are typical values for the variable X?

- Sample Mean:

$$\bar{x} = \frac{1}{n} \sum_{i=1}^{n} x_i = \frac{x_1 + x_2 + \cdots + x_n}{n}$$

- Sample Median: «center of the observations»

odd

$q_{0.5}$

even

$q_{0.5}$

➡ median ist more robust than the mean
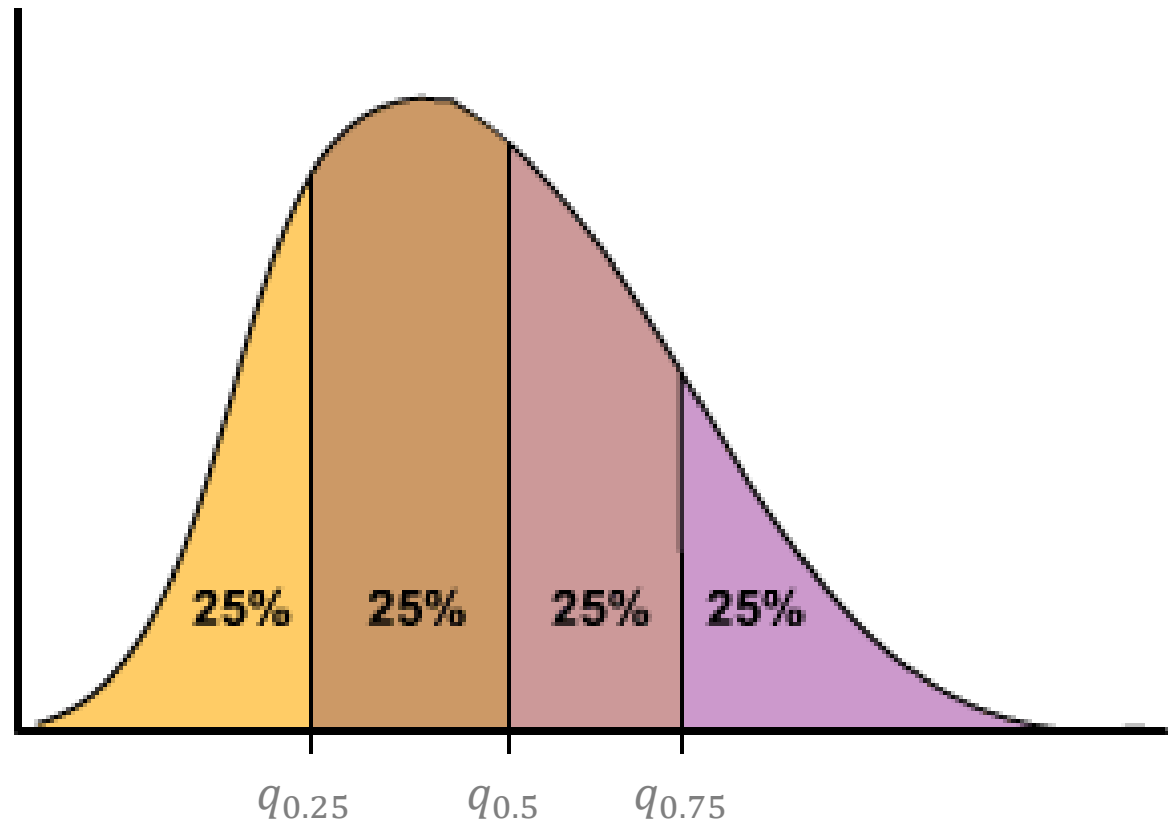
Generalizing the idea of the median to other fractions.

Typical for descripitve analyses: $q_{0.25}, q_{0.5}, q_{0.75}$

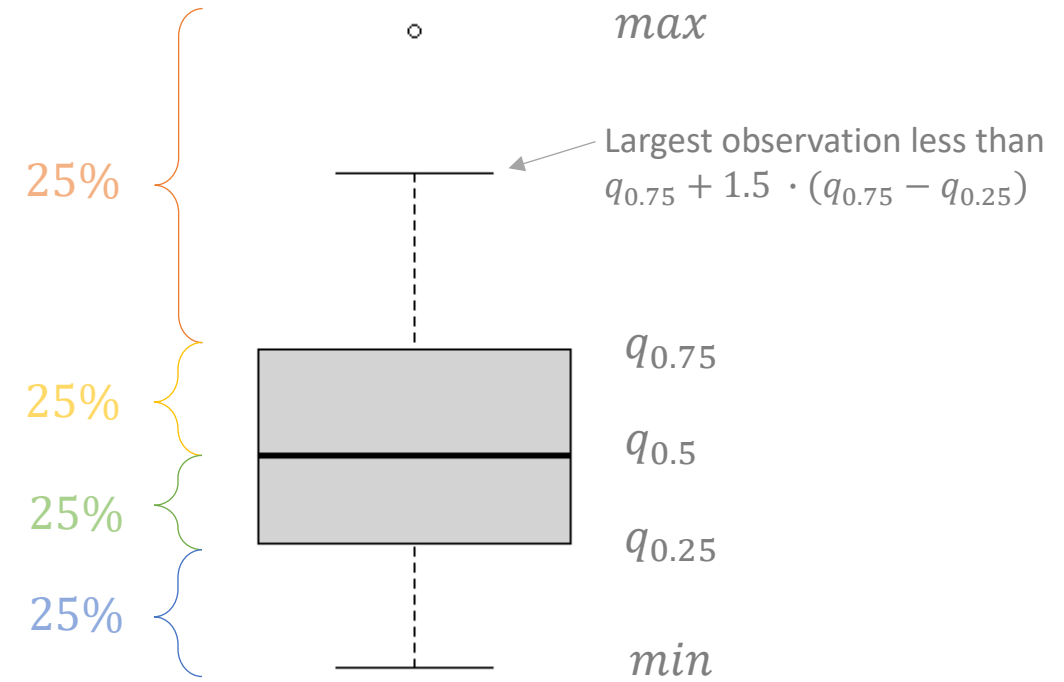Typical for hypothesis testing: $q_{0.01}, q_{0.05}, q_{0.95}, q_{0.99}$

Quantiles

(Numerical Variables)

Boxplots

(Numerical Variables)

max

Largest observation less than
$q_{0.75} + 1.5 \cdot (q_{0.75} - q_{0.25})$

25%

25% $q_{0.75}$

25% $q_{0.5}$

25% $q_{0.25}$

25%

min

Spread

(Numerical Variables)

How strong is the deviation from the center?

- Sample standard deviation:

$$s = \sqrt{\frac{1}{n-1} \sum_{i=1}^{n}(x_i - \bar{x})^2}$$

- IQR (inter quartile range):

$$IQR = q_{0.75} - q_{0.25}$$
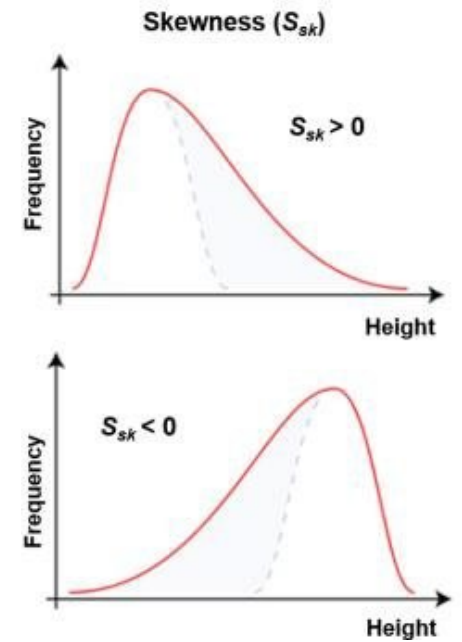
$S = 1.16, IQR = 1.34$

$S = 4.05, IQR = 5.93$

Shape

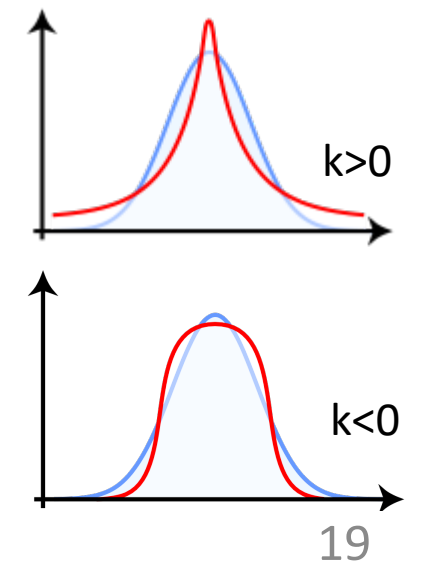(Numerical Variables)

Is the distribution symmetric?

- Skewness:

$$S_{sk} = \frac{1}{n} \sum_{i=1}^{n} \frac{(x_i - \bar{x})^3}{s^3}$$

Skewness (S_{sk})

$S_{sk} > 0$

$S_{sk} < 0$

Does the distribution look like a bell curve?

- Kurtosis:

$$k = \frac{1}{n} \sum_{i=1}^{n} \frac{(x_i - \bar{x})^4}{s^4} - 3$$

k>0

k<0

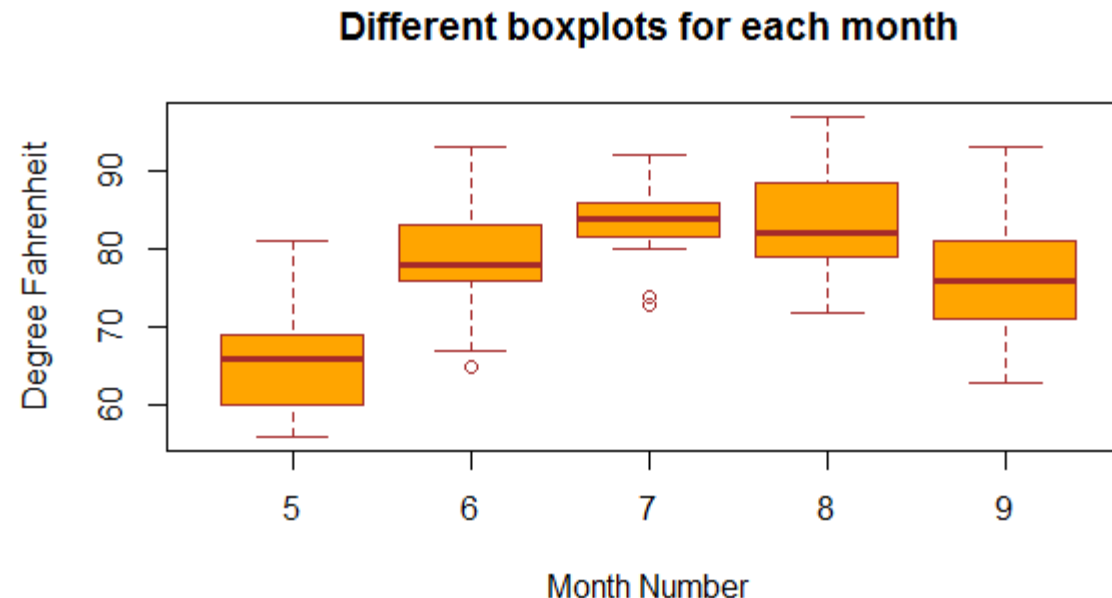# Simultaneous description

(of two features)

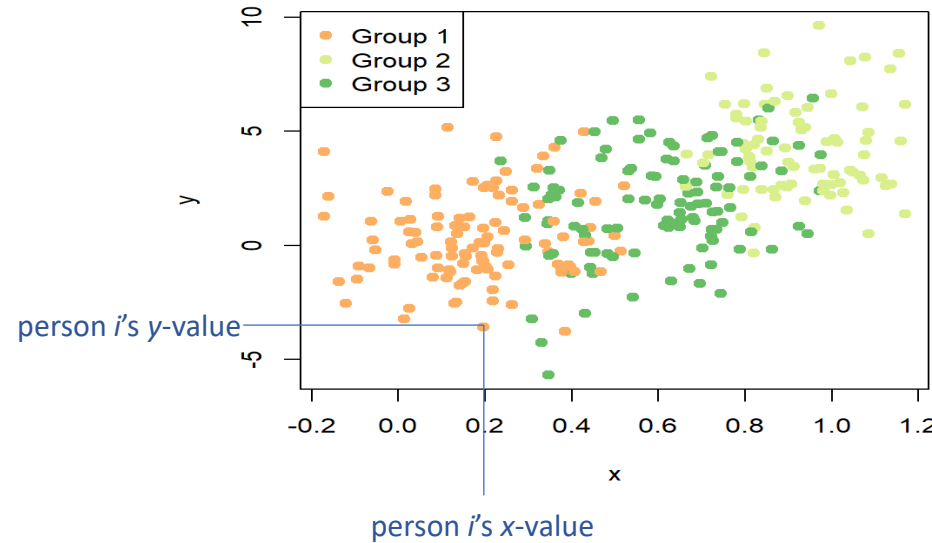- Contigency table (2 categorical features)

	Male	Female	Total
Blonde	4	8	12
Brunette	7	9	16
Total	11	17	28

- Boxplots (1 categorical and 1 numerical feature)

Different boxplots for each month

Simultaneous description

(of two features)

- Scatterplot (2 numerical features)
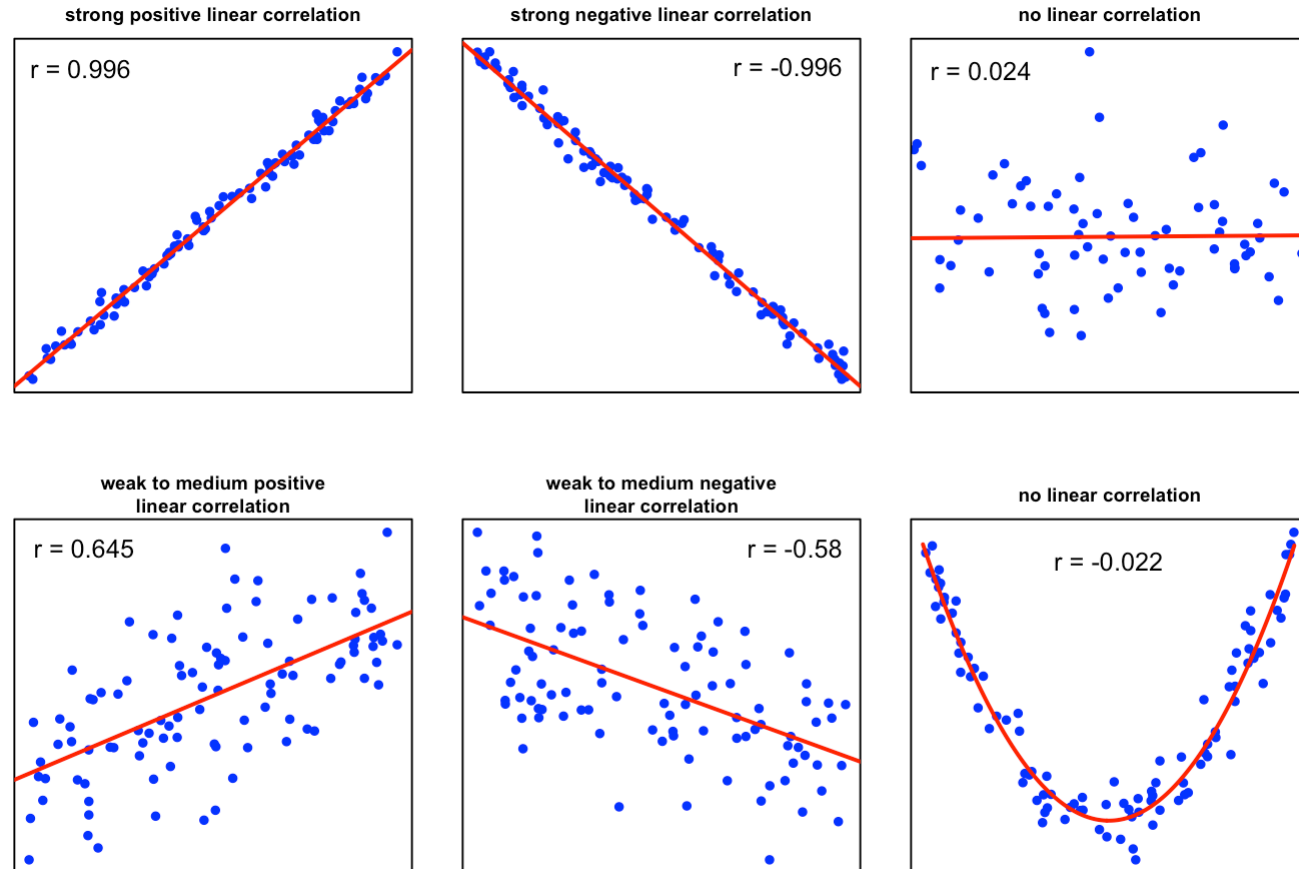

person i's y-value

person i's x-value

- Pearson Correlation (2 numerical features)

$$r = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^{n}(x_i - \bar{x})^2}\sqrt{\sum_{i=1}^{n}(y_i - \bar{y})^2}}$$

Simultaneous description

(of two features)

- Pearson Correlation (2 numerical features)

Probability

- Descriptive statistic is an important first step but does not provide us with the means we aim for eventually.

- In general, we want confirm a hypothesis on a population based on sample of said population.

- To this end, we need a mathematical framework for dealing this uncertainty.

- To quantify the uncertainty one often works with probability distributions.
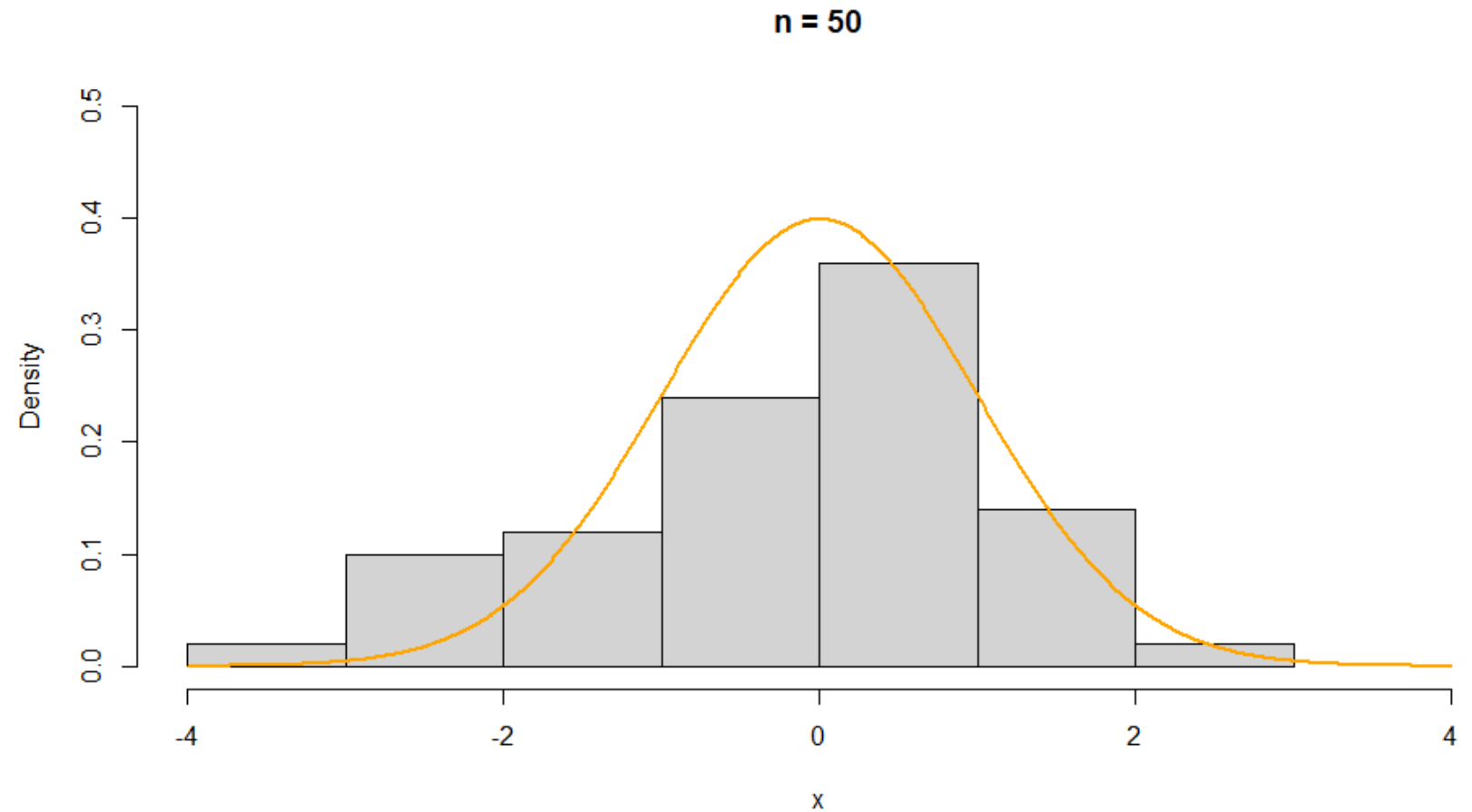
Probability

Probability density function (pdf)

Histogram With Fitted Density Curve, bw=.5
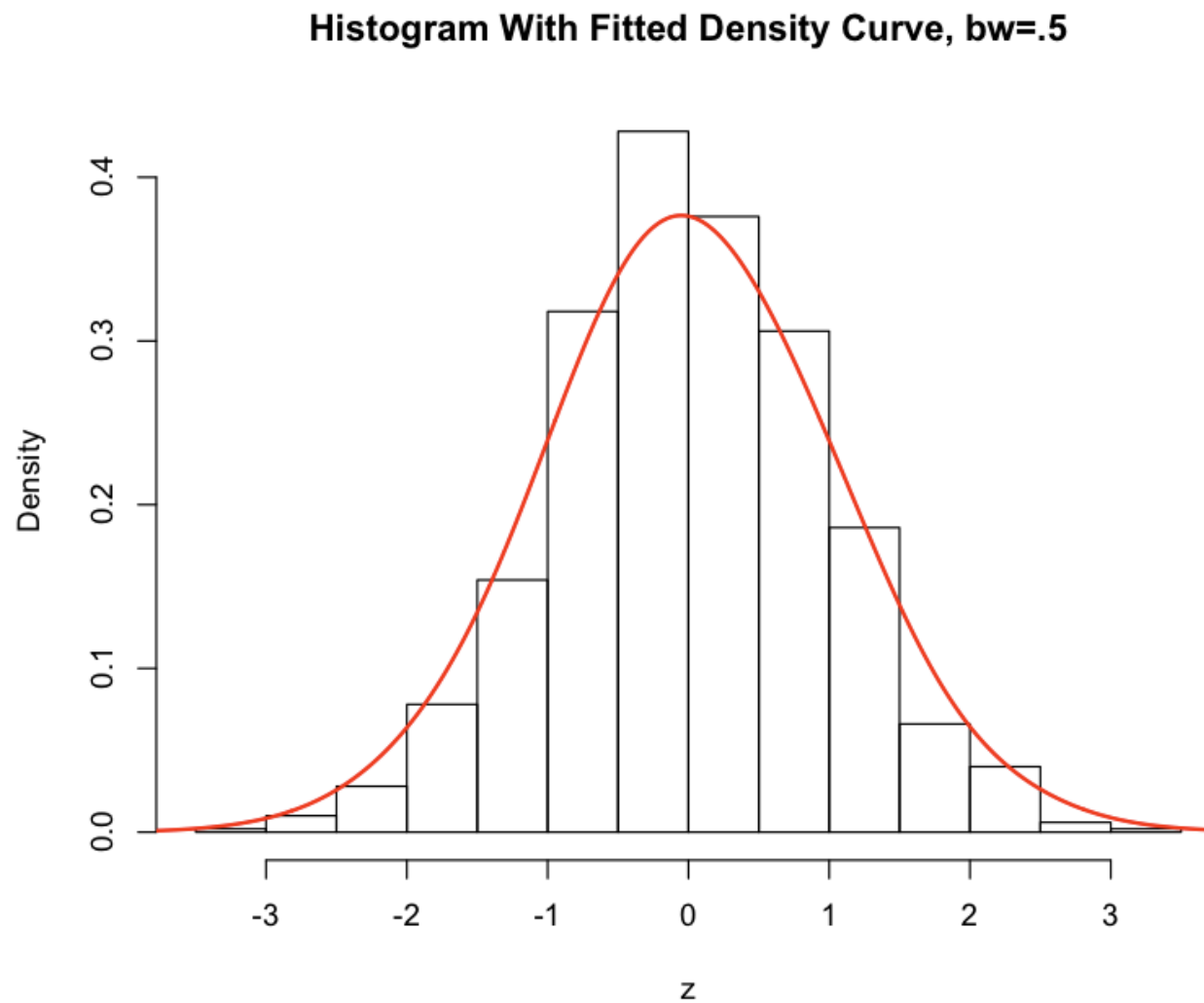
Probability

Probability density function (pdf)

Sketch of idea