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MATH 2220
EXAM 1

1. Define the following sets as:

 - $A = \{x \in Z : x \text{ is an integer multiple of } 3\}$
 - $B = \{x \in Z : x \text{ is a perfect square}\}$
 - $C = \{4, 5, 9, 10\}$
 - $D = \{2, 4, 11, 14\}$
 - $E = \{3, 6, 9\}$
 - $F = \{4, 9, 16\}$

 Indicate which statements are true:

 (a) $D \cap A = \emptyset$ a)true

 (b) $E \subset A$ b)true

 (c) $F \subset B$ c)true

 (d) $(E \cap F) \subseteq (A \cap B)$ d)true

 (e) $C \cap D \cap E = \emptyset$ e)true

2. Given $A = \{1, 2, 3, 4, 5, 8, 10\}$, $B = \{2, 4, 6, 8\}$, and $C = \{a, b, c, 3, 5, 9\}$.

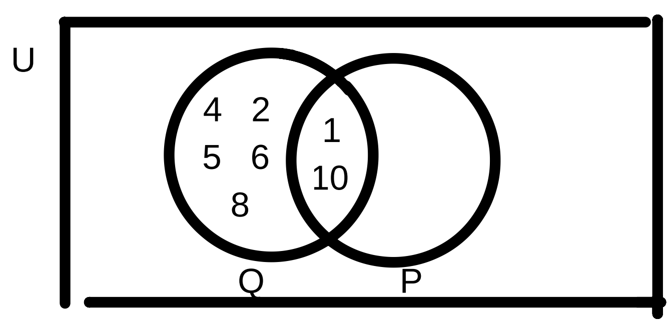
 (a) Find the set $P = A - (B \cup C)$

 ={1,10}

 (b) Find the set $Q = (A \cup B) - (A \cap C)$

 = {1, 2, 4, 5, 6, 8, 10)

 (c) Use the Venn diagrams to represent the set P and the set Q.



3. Determine whether each of these following statments is true or false:

(a) $\{0\} \subset \{\varnothing\}$

(b) $\{\varnothing\} \subseteq \{\varnothing\}$

(c) $\{x\} \in \{x, \{x\}\}$

(d) $\{\{\varnothing\}\} \subset \{\varnothing, \{\varnothing\}\}$

(e) If $A = \{\phi, 3, 5, 9\}, B = \{\phi, 9, 3, 3, 5, \{\phi\}\}$, then $A = B$.

a)false
b)true
c)true
d)true
e)false

4. Which of the following statements are propositions?

(a) $x^n + y^n = z^n$.

(b) How are you doing today?

(c) Do not text and drive!

(d) $2^{5x} = 2^{1000}$.

(e) The number 101 is a prime number.

(f) The number π is even if and only if the number e is odd.

a)yes
b)no
c)no
d)yes
e)yes
f)yes

5. Determine the truth value of each of the following statements if **the domain of each variable consists of all integers**:

(a) For some x, if $x > 0$, then $\frac{x}{x^2+1} < \frac{1}{3}$

true

(b) $\exists x \forall y (x < y^2)$

true

(c) $\forall x \forall y (x - y = 5)$

false

(d) $\forall x (x \neq 0 \rightarrow \exists y (xy = 1))$

true

6. Determine the truth value of each of the following statements if **the domain of each variable consists of all *real* numbers**:

   (a) $\forall x \exists y (x = y^2)$

   false

   (b) $\forall x \exists y (x + y = 1)$

   true

   (c) $\forall x (x \neq 0 \rightarrow \exists y (xy = 1))$

   true

   (d) $\exists x \forall y (x < y^2)$

   true

7. The propositional variables p, q, and s have the following truth assignments:

   - $p = T$,
   - $q = T$,
   - $s = F$.

   Give the **truth value** for the following compound propositions. Be sure to show all steps of your work.

   (a) $p \vee \neg q$

   T v F --> T

   (b) $p \vee \neg(q \wedge s)$

   T v ÿ(T ^ F) --> T v ÿ(F) --> T v T --> .

   (c) $\neg(q \wedge p \wedge \neg s)$

   ÿ(T ^ T ^ ÿF) --> ÿ(T ^ T ^ T) --> ÿT -->

   (d) $(p \rightarrow s) \oplus (q \rightarrow s)$

   (T->F) ⊕ (T->F) --> F ⊕ F --> T

8. Predicates P and Q are defined below. The domain of discourse is the set of all positive integers.

- $P(x)$: x is prime
- $Q(x)$: x is a perfect square.

  Are the following logical expressions propositions? If the answer is yes, indicate whether the statement is true or false.

(a) $P(x)$ is not a proposition

(b) $P(2)$ is a proposition, and true

(c) $\forall x(Q(x) \rightarrow \neg P(x))$ is not a proposition

(d) $(\forall x Q(x)) \wedge P(x)$ is not a proposition

(e) $P(3) \wedge Q(49)$ is a proposition, and is false

9. Given the the following 2 propositions

- $(p \vee q) \rightarrow s$
- $(p \rightarrow s) \wedge (q \rightarrow s)$

  (a) Construct the truth table for the above 2 propositions.

p	q	s	p V q	(p V q) -> s
T	T	T	T	T
T	T	F	T	F
T	F	T	T	T
T	F	F	T	F
F	T	T	T	T
F	T	F	T	F
F	F	T	F	T
F	F	F	F	T

p	q	s	(p -> s)	(q -> s)	(p -> s) v (q -> s)
T	T	T	T	T	T
T	T	F	F	F	F
T	F	T	T	T	T
T	F	F	F	T	F
F	T	T	T	T	T
F	T	F	T	F	F
F	F	T	T	T	T
F	F	F	T	T	T

(b) Determine whether the 2 given propositions are logically equivalent to each other.

the truth value of the 2 propositions are the same, they are equivalent

10. Using the laws of logic to determine whether the following logical expressions are tautologies, contradictions or neither. Be sure to specify the law you use for each step. You can use the Laws of Propositional Logic sheet as reference.

(a) $(p \to q) \lor p$

(ÿp v q) v p   --> Conditional Identities
ÿp v (q v p)   --> associative law
ÿp v (p v q)   --> Communitive Law
(ÿp v q) v q   --> Associative law
T v q   --> Complement Law
T   --> Identity Law

The logical expression is tautologies

(b) $[\neg q \land (p \to q)] \to \neg p$

ÿ(ÿq ^ (p -> q)) v ÿp --> Conditional identities
(ÿ(ÿq) v ÿ(p->q)) vÿp --> De Morgan's Law
(q v ÿ(p -> q)) v ÿp   --> Double Negation Law
(q v ÿ(ÿp ^ q)) v p   --> Conditional Identities
(q v (ÿ(ÿp) v ÿq)) v ÿp   --> De Morgan's Law
(q v (p v ÿq)) v ÿp   --> Double Negation Law
(q v (ÿq v p)) v ÿp   --> Commutative Law
((q v ÿq) v p) v ÿp --> Associative Law
(T v p) v ÿp --> Complement Law
(p v T) v ÿp   --> Commutative Law
T v ÿp --> Identity Law
T   --> Identity Law

The logical expression is tautologies.

Table 2.4.4: Laws of propositional logic.

Idempotent laws:	$p \vee p \equiv p$	$p \wedge p \equiv p$
Associative laws:	$(p \vee q) \vee r \equiv p \vee (q \vee r)$	$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$
Commutative laws:	$p \vee q \equiv q \vee p$	$p \wedge q \equiv q \wedge p$
Distributive laws:	$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$	$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$
Identity laws:	$p \vee F \equiv p$ $p \vee T \equiv T$	$p \wedge F \equiv F$ $p \wedge T \equiv p$
Double negation law:	$\neg\neg p \equiv p$	
Complement laws:	$p \vee \neg p \equiv T$ $\neg T \equiv F$	$p \wedge \neg p \equiv F$ $\neg F \equiv T$
De Morgan's laws:	$\neg(p \vee q) \equiv \neg p \wedge \neg q$	$\neg(p \wedge q) \equiv \neg p \vee \neg q$
Absorption laws:	$p \vee (p \wedge q) \equiv p$	$p \wedge (p \vee q) \equiv p$
Conditional identities:	$p \rightarrow q \equiv \neg p \vee q$	$p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$