Name:

MATH 2220
Quiz 2

1. (6 points) Geometric sequences

   (a) Let $\{s_n\}$ be a geometric sequence that starts with an initial index of 0. The initial term is 2 and the common ratio is 5. What is s_2?

   s2 is 2 * 5^2 = 50

   (b) Let $\{s_n\}$ be a geometric sequence that starts with an initial index of 0. The initial term is 16 and the common ratio is $\frac{1}{2}$. What is s_3?

   s3 = 16 / (2^3) = 2

   (c) Consider the geometric sequence: $3, 6, 12, ...$ What is the common ratio?

   r = s(i+1) / si --> 6 /3 =2

2. (4 points) Arithmetic sequences

   (a) Let $\{s_n\}$ be an arithmetic sequence that starts with an initial index of 0. The initial term is 3 and the common difference is -2. What is s_2?

   s2 --> 3 + 2 * (-2) = -1

   (b) Consider the arithmetic sequence: 7, 4, 1, ... What is the next term in the sequence?

   d = 4 - 7 = -3 --> 1 + d = 1 + (-3) = -2

3. (8 points) Evaluate the following summations. For parts c and d, be sure to use the Closed Forms of Summation Formulae to find exact answers:

   (a) $\sum_{k=2}^{4}(2 + k^2)$ =35

   (b) $\sum_{k=0}^{50} 3k^2$ = 128775

(c) $\sum_{j=0}^{25}(5^j - 3^j)$ = $\frac{(5\text{^}26 \text{ -}2(3\text{^}26 + 1))}{4}$

(d) $\sum_{j=25}^{50} 3^j$ = $\frac{3\text{^}51 \text{ - } 3\text{^}25}{2}$

4. (3 points) Express the following sums using summation notation:

(a) $(-2) + (-1) + 0 + 1 + 2 + 3 + 4$ $\sum_{i = -2}^{4}$ i

(b) $2^2 + 2^3 + 2^4 + 2^5 + 2^6$ $\sum_{i = 2}^{6}$ 2^i

(c) $0^3 + 1^3 + 2^3 + 3^3 + 4^3 + \bullet\bullet\bullet + (21)^3$ $\sum_{i = 0}^{21}$ (i)^3

5. (2 points) Define the sequence $\{b_n\}$ as follows:

- $b_0 = 1$
- $b_k = 2 \cdot b_{k-1} + 1$

Prove that for $n \geq 0$; $b_n = 2^{n+1} - 1$

$b_{m+1} = 2b_{(m+1)-1}+1 = 2b_m + 1$
$= 2 * (2^{m+1} - 1) + 1$
$=2^{m+2} – 2 + 1$
$=2^{(m + 1) + 1} – 1$

Statement is proved true assuming $b_m = 2^{m+1} – 1$

6. (2 points) Prove that for every positive integer n:

$$\sum_{k=1}^{n} k \cdot 2^k = (n-1)2^{n+1} + 2$$

Let n = 1.

$$\sum_{k=1}^{1} k * 2^k = 1 * 2^1 = 2$$

And

$(1-1) * 2^{1+1} + 2 = 2$

Solve for n = m+1

$$\sum_{k=1}^{m+1} k * 2^k$$

$= (\sum_{k=1}^{m} k * 2^k) + (m+1) * 2^{m+1}$

$= [(m-1)2^{m+1} + 2] + (m+1) * 2^{m+1}$

$= (m - 1 + m + 1)\ 2^{m+1} + 2$

$= (2m)\ 2^{m+1} + 2$

$=(m)\ 2^{m+1+1} + 2$

$= ((m+1) - 1)\ 2^{(m+1)+1} + 2$

Statement is true and proved