DESIGN AND ANALYSIS OF ALGORITHMS

CS 4120/5120

APPLICATION OF ASYMPTOTIC NOTATIONS

AGENDA

- Pickup what we left for standard functions.
- Use the definition of the asymptotic notation to bound a given function.
- Take notes of today's practice work.

STANDARD FUNCTIONS
PRACTICE

- Rewrite the following expression in the form of one number, or a single exponentiation/logarithm with a possible coefficient.
  - $8^3 \cdot (2^8)^{1/2}$
    - Hint: $(a^m)^n = a^{mn}, a^m a^n = a^{m+n}$

STANDARD FUNCTIONS PRACTICE

- Rewrite the following expression in the form of one number, or a single exponentiation/logarithm with a possible coefficient.
  - $\log_4 27 \cdot \log_3 4$
    - Hint: $\log_a b = \frac{1}{\log_b a}, log_b a = \frac{log_c a}{log_c b}$

STANDARD FUNCTIONS PRACTICE

- Rewrite the following expression in the form of one number, or a single exponentiation/logarithm with a possible coefficient.
  - $n^{\frac{1}{\log_m n}}$
    - Hint: $log_b\, a = \frac{1}{log_a\, b}$, $n^{\log_n m} = m$

REVIEW OF ASYMPTOTIC

- For a given function $g(n)$, we denote by $\boldsymbol{\Theta}(\boldsymbol{g}(\boldsymbol{n}))$ the set of functions $\Theta(g(n)) = \{f(n)$: there exist **positive constants $c_1, c_2,$ and n_0** such that $\mathbf{0 \leq c_1 g(n) \leq f(n) \leq c_2 g(n)}$ **for all $n \geq n_0$**$\}$

- For a given function $g(n)$, we denote by $\boldsymbol{O}(\boldsymbol{g}(\boldsymbol{n}))$ the set of functions $O(g(n)) = \{f(n)$: there exist **positive constants c and n_0** such that $\mathbf{0 \leq f(n) \leq cg(n)}$ for all $\boldsymbol{n \geq n_0}\}$.

- For a given function $g(n)$, we denote by $\Omega(g(n))$ the set of functions $\Omega(g(n)) = \{f(n)$: there exist **positive constants c and n_0** such that $\mathbf{0 \leq cg(n) \leq f(n)}$ for all $\boldsymbol{n \geq n_0}\}$.

SOLVING ASYMPTOTIC QUESTIONS

Warm up

Definitions of the asymptotic notations

A. Function $g(n) = \Theta(h(n))$ indicates that ____________ is an asymptotic tight bound of ____________.

B. Function $f(n) = \Omega(h(n))$ indicates that ____________ is lower-bounded by ____________.

C. Function __________________ indicates that $f(n)$ is an asymptotic upper-bound of $g(n)$.

SOLVING ASYMPTOTIC QUESTIONS

Key steps:

Plug the given function(s) in the definition.

Choose appropriate values of the constants (c_1, c_2, n_0) or (c, n_0).

Note: No need to find the minimum/maximum possible value of c_1/ c and n_0

- Is $\frac{1}{2}n^2 - 3n = \Theta(n^2)$? Justify.
  - Step 1: Identify $f(n) =$ ______, $g(n) =$ ______.

SOLVING ASYMPTOTIC QUESTIONS

Key steps:

Plug the given function(s) in the definition.

Choose appropriate values of the constants (c_1, c_2, n_0) or (c, n_0).

Note: No need to find the minimum/maximum possible value of c_1/ c and n_0

- Is $\frac{1}{2}n^2 - 3n = \Theta(n^2)$? Justify.
  - Step 2: By the definition of __-notation, there exist positive constants c_1, c_2 and n_0 such that ______________________________ for all _______.

- Is $\frac{1}{2}n^2 - 3n = \Theta(n^2)$? Justify.
  - Step 3: Solve the inequality for c_1, c_2 and n_0.

SOLVING ASYMPTOTIC QUESTIONS

Key steps:

Plug the given function(s) in the definition.

Choose appropriate values of the constants (c_1, c_2, n_0) or (c, n_0).

Note: No need to find the minimum/maximum possible value of c_1/ c and n_0

SOLVING ASYMPTOTIC QUESTIONS

Key steps:

Plug the given function(s) in the definition.

Choose appropriate values of the constants (c_1, c_2, n_0) or (c, n_0).

Note: No need to find the minimum/maximum possible value of c_1/ c and n_0

- Is $\frac{1}{2}n^2 - 3n = \Theta(n^2)$? Justify.
  - Step 4: Conclude with either of the following.
    - Option 1: Let c_1 be ________, c_2 be ________, and n_0 be ________.
      Inequality ________________________ holds true for all $n \geq n_0$. Therefore, ________________________.
    - Option 2: There does not exist positive constants c_1, c_2 and n_0 such that ______________________ for all $n \geq n_0$. Therefore, ______________________.

- Show that $k \ln n = \Theta(n)$ implies $k = \Theta(\frac{n}{\ln n})$.

SOLVING ASYMPTOTIC QUESTIONS

Key steps:

Plug the given function(s) in the definition.

Choose appropriate values of the constants (c_1, c_2, n_0) or (c, n_0).

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SOLVING ASYMPTOTIC QUESTIONS

Key steps:

Plug the given function(s) in the definition.

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- Breakout session

  A. Is $n^2 = O\left(2^{\frac{n}{2}}\right)$? Justify.

  B. For the following functions: $f(n) = 7\log n$ and $g(n) = \log n^3 + 56$, indicate whether it is one of the three cases: $f(n) = O(g(n))$, $f(n) = \Omega(g(n))$, or $f(n) = \Theta(g(n))$.

- Come back in 10 minutes.

NEXT UP
DESIGN TECHNIQUE

- Divide and Conquer

REFERENCE

- https://www.youtube.com/watch?v=SEbzTe0CzT8
- https://www.yourdictionary.com/asymptotic#:~:text=adjective,are%20asymptotic%20to%20each%20other.