Question

Assemble the stiffness $[K]$ and mass $[M]$ matrices for the 5-storey shear building shown below. Use the following properties:
Masses: $m_1 = 35000$ kg, $m_2 = 30000$ kg, $m_3 = 30000$ kg, $m_4 = 25000$ kg, $m_5 = 20000$ kg
Heights: $h_1 = 5.0$ m, $h_2 = 4.0$ m, $h_3 = 4.0$ m, $h_4 = 4.0$ m, $h_5 = 3.8$ m
Moments of Inertia: $I_{1A} = 2.2 \times 10^9$ mm^4, $I_{1B} = 3.4 \times 10^9$ mm^4, $I_2 = 2.2 \times 10^9$ mm^4, $I_3 = 2.2 \times 10^9$ mm^4, $I_4 = 1.8 \times 10^9$ mm^4, $I_5 = 1.4 \times 10^9$ mm^4
Elastic Modulus: $E = 210$ GPa
Gravity: $g = 9.81$ m/s^2

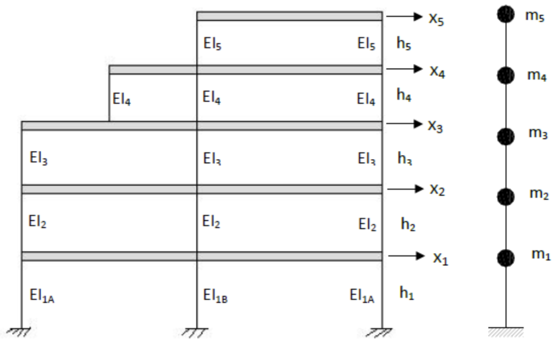

Figure 1: Schematic of the 5-storey shear building.

Solve for the natural frequencies and mode shapes for all five modes of vibration of the structure. Normalize the individual mode shapes by the largest displacement for a given mode shape. Draw the mode shapes and determine the free vibration response.

Solution

Theoretical Approach

Mass Matrix Assembly

The mass matrix $[M]$ is a diagonal matrix where each diagonal element represents the mass of a corresponding storey:

$$[M] = \begin{bmatrix} m_1 & 0 & 0 & 0 & 0 \\ 0 & m_2 & 0 & 0 & 0 \\ 0 & 0 & m_3 & 0 & 0 \\ 0 & 0 & 0 & m_4 & 0 \\ 0 & 0 & 0 & 0 & m_5 \end{bmatrix}$$

Stiffness Matrix Assembly

The stiffness matrix $[K]$ for a shear building is formed by the stiffness coefficients of each storey and the interaction between adjacent storeys. For each storey i, the stiffness k_i is calculated using the formula $k_i = \frac{12EI_i}{h_i^3}$, where E is the modulus of elasticity, I_i is the moment of inertia, and h_i is the height of the storey. The stiffness matrix is then constructed as a symmetric tridiagonal matrix:

$$[K] = \begin{bmatrix} k_1 + k_2 & -k_2 & 0 & 0 & 0 \\ -k_2 & k_2 + k_3 & -k_3 & 0 & 0 \\ 0 & -k_3 & k_3 + k_4 & -k_4 & 0 \\ 0 & 0 & -k_4 & k_4 + k_5 & -k_5 \\ 0 & 0 & 0 & -k_5 & k_5 \end{bmatrix}$$

Here, k_1, k_2, k_3, k_4, and k_5 are the stiffness coefficients for the floors from bottom to top. The off-diagonal elements represent the coupling effect of the shear forces between the floors, and are negative because they are opposing the diagonal stiffness terms.

Solving for Natural Frequencies and Mode Shapes

The system's natural frequencies and mode shapes are found by solving the eigenvalue problem:

$$[K]\{X\} = \omega^2[M]\{X\}$$

where $\{X\}$ is a mode shape vector and ω^2 are the eigenvalues corresponding to the square of the natural frequencies.

Normalizing the Mode Shapes

Mode shapes are normalized such that the maximum displacement in each mode is 1. This is done by dividing each element of the mode shape vector by the maximum absolute value of that vector.

Determining the Free Vibration Response

The free vibration response of the system is given by:

$$\{x(t)\} = \sum_{i=1}^{n}\{X_i\}q_i(t)$$

where $\{X_i\}$ are the normalized mode shapes, and $q_i(t)$ are the modal coordinates which can be expressed as:

$$q_i(t) = A_i \cos(\omega_i t + \phi_i)$$

A_i and ϕ_i are determined from initial conditions.

Numerical Solution

Given the specified properties, we can assemble the matrices and solve for the natural frequencies and mode shapes using MATLAB.

Mass Matrix $[M]$: The mass matrix is assembled as a diagonal matrix with the provided mass values.

Stiffness Matrix $[K]$: The stiffness matrix is assembled using the provided moments of inertia, converting them to m^4 from mm^4 and applying the stiffness formula.

Eigenvalue Problem Solution: By solving the eigenvalue problem in MATLAB, we obtain the natural frequencies in Hz and the corresponding mode shapes.

Mode Shape Normalization: The mode shapes are then normalized, and the results are printed in the MATLAB command window.

Results

The results from the MATLAB command window are as follows:

Mass Matrix $[M]$ **in kg**:

$$\begin{bmatrix} 35000 & 0 & 0 & 0 & 0 \\ 0 & 30000 & 0 & 0 & 0 \\ 0 & 0 & 30000 & 0 & 0 \\ 0 & 0 & 0 & 25000 & 0 \\ 0 & 0 & 0 & 0 & 20000 \end{bmatrix}$$

Stiffness Matrix $[K]$ **in N/m**:

$$\begin{bmatrix} 2.6485 \times 10^8 & -2.2050 \times 10^8 & 0 & 0 & 0 \\ -2.2050 \times 10^8 & 3.0713 \times 10^8 & -0.8662 \times 10^8 & 0 & 0 \\ 0 & -0.8662 \times 10^8 & 1.5750 \times 10^8 & -0.7087 \times 10^8 & 0 \\ 0 & 0 & -0.7087 \times 10^8 & 1.3517 \times 10^8 & -0.6430 \times 10^8 \\ 0 & 0 & 0 & -0.6430 \times 10^8 & 0.6430 \times 10^8 \end{bmatrix}$$

Mode	Frequency (Hz)	Time Period (s)
Mode 1	2.3915	0.4181
Mode 2	6.3303	0.1580
Mode 3	11.0453	0.0905
Mode 4	14.8683	0.0673
Mode 5	20.3350	0.0492

Mode Shape Plots

The mode shapes and free vibration response plots are shown below.

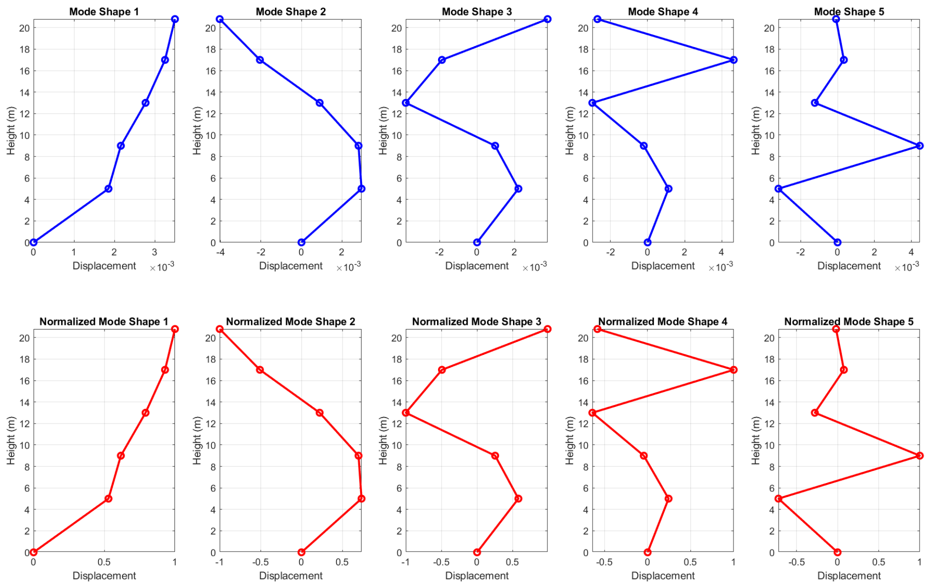

Figure 2: Mode shapes of the 5-storey shear building.

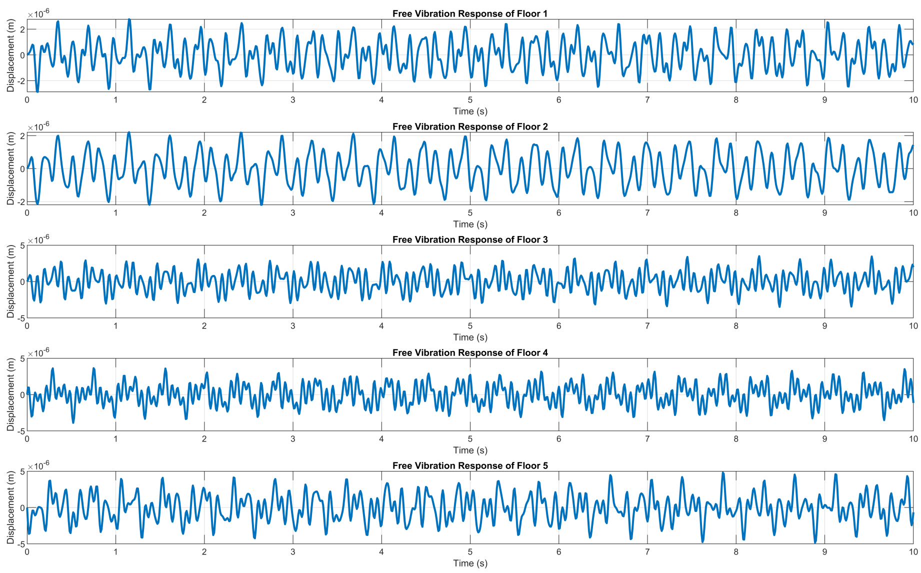

Figure 3: Free vibration response of the 5-storey shear building.